PEARSON **mathematics**

S.B.

2ND EDITION

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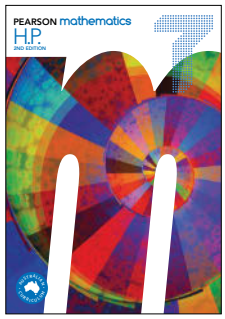

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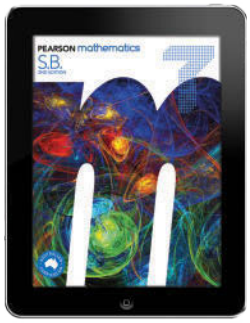
Lightbook Starter



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Student Book

The Second Edition Student Book includes updated questions, activities and design, with full coverage of the Australian Curriculum: Mathematics as well as the Victorian Curriculum: Mathematics.

It incorporates the latest research as well as feedback from teachers and learners across Australia.

Content caters for students of all abilities, with improved differentiation of all exercise questions and more questions for students consolidating their skills.

Homework Program

The Homework Program provides a collection of tear-out worksheets for students to practise and revise mathematical concepts.

Teacher Companion

The Teacher Companion makes lesson preparation easy by combining full-colour Student Book pages with teacher support including improved contextual teaching suggestions and strategies, class activities, extra questions, worked solutions and answers for every question in the Student Book.

AUSTRALIAN CURRICULUM

Pearson

Pearson Lightbook Starter

Lightbook Starter is an innovative digital resource powered by Pearson's award-winning Lightbook technology. It has been developed to help students learn key mathematical concepts, evaluate their understanding and track their progress. 'Before you begin' sections assess learner readiness before each chapter topic, while 'Check-in' questions can be used to evaluate learner understanding and practice after every chapter section.

Auto-correcting questions are linked to the Progress Tracker dashboard for easy analysis and viewing of results, which are mapped to progression through the Student Book as well as to Australian Curriculum: Mathematics and Victorian Curriculum: Mathematics content descriptions.

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We believe in learning.
All kinds of learning for all kinds of people,
delivered in a personal style.
Because wherever learning flourishes, so do people.

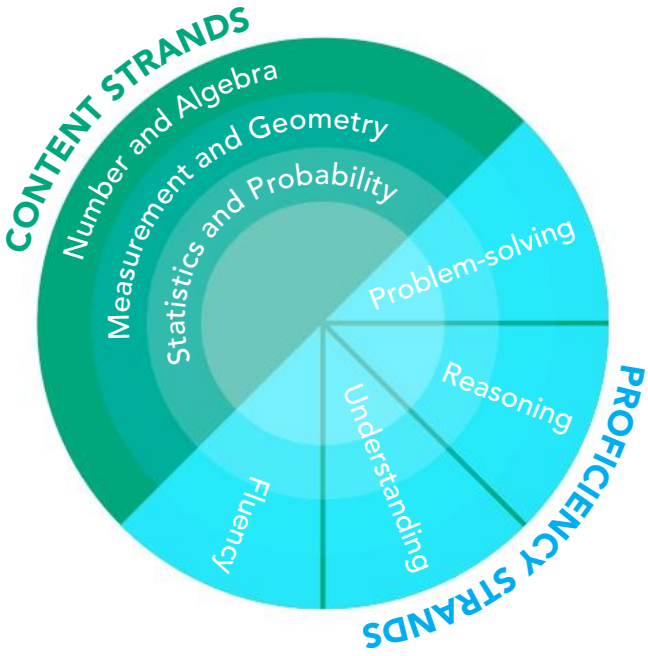
Using PEARSON mathematics

2ND EDITION

The Australian Curriculum: Mathematics aims to ensure that students:

- *are confident, creative users and communicators of mathematics, able to investigate, represent and interpret situations in their personal and work lives and as active citizens*
- *develop an increasingly sophisticated understanding of mathematical concepts and fluency with processes, and are able to pose and solve problems and reason in number and algebra, measurement and geometry, and statistics and probability*
- *recognise connections between the areas of mathematics and other disciplines and appreciate mathematics as an accessible and enjoyable discipline to study.*

Pearson Mathematics Second Edition is a compelling research-based series, written by experienced and practising Australian teachers with the support of Australia's leading mathematics education experts. It has been specifically designed to scaffold students' fluency development, conceptual understanding, reasoning and problem-solving skills.



Pearson Mathematics shares these aims.

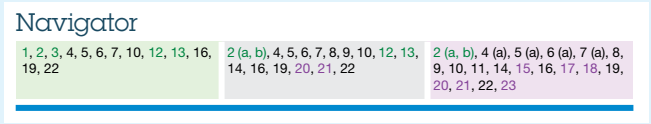
Supporting all learners

Differentiation

Pearson Mathematics Second Edition has been written and designed for the needs of the full ability spectrum of students in Australian classrooms.

Recall—Each chapter begins with a review of assumed and necessary knowledge for the chapter. For students needing extra revision, Recall Worksheets are available for each Recall question, with explanations to refresh understanding and exercises to practise skills.

Exercises—All Exercises include questions according to the Australian Curriculum Proficiency Strands: Fluency, Understanding and Reasoning. They have been carefully paced to help students build skills, develop deep conceptual understanding and apply learning. Every Exercise also has 'Open-ended' questions to encourage students' creative thinking and ability to communicate mathematics effectively.



Navigator—Three optional graded pathways through *every* Exercise, with every question rated as Foundation, Standard or Advanced level.

Challenge—Every chapter includes a full page Challenge section for early finishers and advanced students.

Regular revision and reinforcement

Pearson Mathematics Second Edition has a broad range of cumulative and chapter-based revision.

Half-time—A mid-chapter review of chapter content so far.

Chapter review—An end-of-chapter thorough review of chapter content.

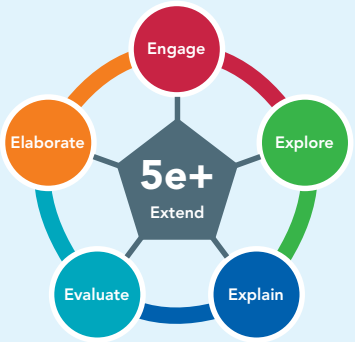
Mixed review—Cumulative revision that mixes content from previous chapters.

Encouraging inquiry and problem solving

Investigation

These are scaffolded to be accessible to all learners and structured following a 5e+ format.

Investigation



Exploration Spreadsheet and CAS

Pearson Mathematics Second Edition understands the importance places of students developing technological literacy. Explorations with CAS technology and spreadsheet software allow students to learn and practise their technical proficiency, while also helping to develop students' deeper understanding of the mathematical concepts covered.

Exploration Spreadsheet

Exploration CAS

Problem solving, Puzzles and Games

These are located throughout every chapter and give students fun opportunities to develop their problem-solving skills and logical reasoning.

Problem solving

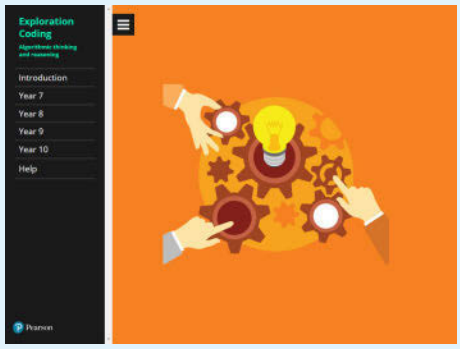
Puzzle

Game

Exploration Coding

Algorithmic thinking is an increasingly relevant component of mathematics education for the 21st century. Algorithmic thinking and coding tasks encourage learners to understand and develop reasoning skills with computational procedures, algorithms and logical problem-solving, exploring a coding environment within a mathematical context.

Exploration Coding



Real-life contexts and engagement

Exploration Task and STEM

Practical contexts provide useful learning opportunities and natural pathways into interesting, relevant mathematics.

Exploration tasks are rich tasks that encourage student engagement, questioning and creative thinking. STEM activities allow students to explore topics related to Science, Technology, Engineering and Design with a Mathematics perspective.

Exploration Task

Exploration STEM

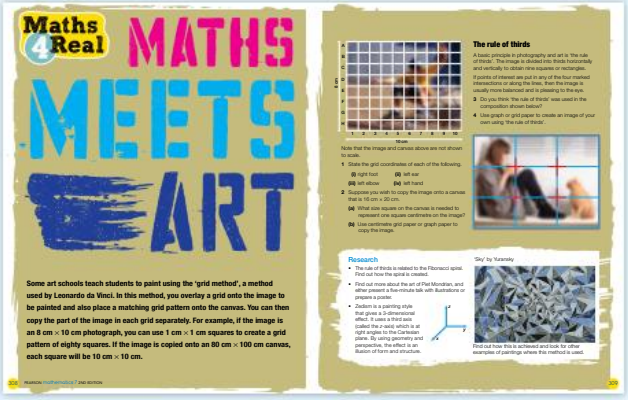
Home Page

Anticipate the 'Why learn this?' question and provide a motivating entry into every new chapter topic, including discussion-provoking Forum questions.

Pearson Mathematics Second Edition has been designed to capture students' interest, with the incorporation of Maths 4 Real, Gamespace and more.

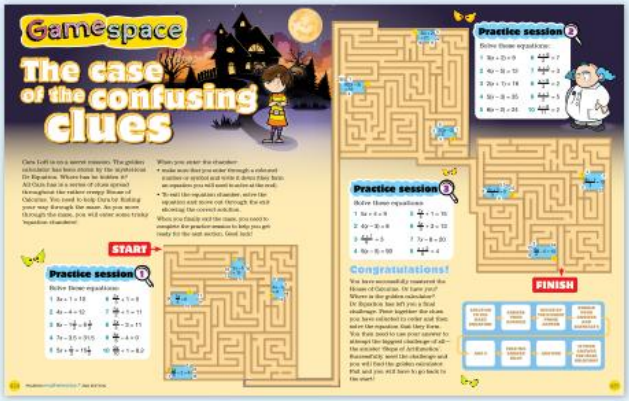
Maths 4 Real

Scenarios that help students make connections with mathematics in the real world.



Gamespace

Skill consolidation and reinforcement wrapped up in fun and quirky scenarios, including multi-player maths board games and solve-the-riddle, find-the-clue tasks and games.



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Whole numbers

It's a Santa stampede! Another Guinness World Record, this time for 'the largest gathering of Santa Claus'. (18 112 of them!)

For decades, *Guinness World Records* has been recording extreme and bizarre feats, including 'the most cockroaches eaten in one minute' (36), and 'the most consecutive skateboard frontside ollies off a half-pipe ramp' (348). The book itself holds the record for being the best-selling copyrighted series of all time. People are fascinated to know the most, the biggest, the fastest and the tallest.

To keep records, you need a system of numbers to count, order, measure and calculate with. People also need numbers for our personal lives: how much money we spend, save and earn; how far and how often we travel; and how much of the Earth's resources we use. Every day, people need to find totals, work out differences, divide quantities, and make estimates. To live in this world, you need to be able to work with numbers!

Forum

Is it important to keep 'world records'? If so, then what kinds of of achievements should records be kept for?

What other kinds of records are good to keep?

Why learn this?

You are buying two $18 T-shirts and a $65 pair of jeans. Will $100 be enough? If you swim 12 laps of a 50 metre pool 4 times a week, how many metres per week is that? Working with numbers means more than just being able to add, subtract, multiply and divide. It means being able to choose which skill to use, to estimate and round numbers, and to have a range of mental skills so that you can work things out efficiently and accurately.

After completing this chapter you will be able to:

- choose and use a range of mental strategies for calculations
- understand how the properties of numbers can be used to calculate efficiently
- interpret and work with numbers in index form
- estimate answers to problems using estimating and rounding strategies
- apply the order of operations
- solve problems involving whole numbers.

Recall 1

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

1 What is the place value of the red digit shown?

(a) 45 783

A eight **B** tens **C** eighty-three **D** hundreds

(b) 1 264 184

A two **B** hundreds **C** ten-thousands **D** hundred-thousands

2 **(a)** Rearrange the following numbers in ascending order (from smallest to largest).

567, 4500, 0, 74, 11 100, 6008, 12, 602

(b) Rearrange the following numbers in descending order (from largest to smallest).

1200, 204, 987, 2196, 240, 95, 2400, 1010

3 Find:

(a) 50 000 + 6000 + 800 + 90 + 5 **(b)** 7 000 000 + 20 000 + 5000 + 70 + 3

4 **(a)** Round 1245 to the nearest:

(i) 10 **(ii)** 100 **(iii)** 1000

(b) Round 8983 to the nearest:

(i) 10 **(ii)** 100 **(iii)** 1000

5 Find:

(a) $3 \times 2 \times 3$ **(b)** $5 \times 3 \times 3 \times 2$ **(c)** $2 \times 2 \times 2$ **(d)** $10 \times 10 \times 10 \times 10$

6 Set out these calculations in your preferred way and work out the answers.

(a) 456 + 56 **(b)** 16 + 2047 **(c)** 90 + 1267 + 341

7 Set out these calculations in your preferred way and work out the answers.

(a) 298 – 123 **(b)** 854 – 227 **(c)** 1406 – 249

8 Set out these calculations in your preferred way and work out the answers.

(a) 45×7 **(b)** 134×5 **(c)** 34×95

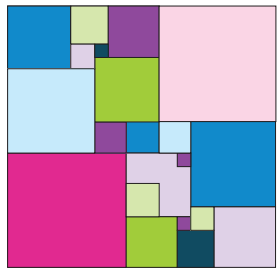
9 Set out these calculations in your preferred way and work out the answers.

(a) $844 \div 4$ **(b)** $3708 \div 9$ **(c)** $897 \div 7$

Exploration Task

You can download this activity from the eBook or the Pearson Places website.

The root of the problem



In this activity, you will explore the relationship between square roots and the geometric properties of squares.

Mental strategies

1.1

This section looks at some strategies that are useful for doing calculations in your head. Try writing them out first, before you do them only in your head. You might find some methods easier than others, or you might have different strategies of your own. That's okay—people are different in the way they like to approach things.

Strategy 1—Make easy numbers

This strategy uses these two properties of numbers:

1 The order in which you add or multiply any two numbers does not change the result.

For example: $2 \times 3 = 6$ and $3 \times 2 = 6$

$4 + 5 = 9$ and $5 + 4 = 9$

This is known as the **commutative law**.

2 The order in which three or more numbers are added, or the order in which they are multiplied, also does not change the result.

In the following examples, brackets are used to show which pair of numbers is added or multiplied first.

Multiplication:

$(2 \times 3) \times 5$ and $2 \times (3 \times 5)$
$= 6 \times 5$ $= 2 \times 15$
$= 30$ $= 30$

Addition:

$(6 + 7) + 8$ and $6 + (7 + 8)$
$= 13 + 8$ $= 6 + 15$
$= 21$ $= 21$

This is known as the **associative law**.

It is important to understand that the commutative and associative laws do *not* apply to subtraction and division, which both need to be worked from left to right.

For example: $(9 - 5) - 4$ but $9 - (5 - 4)$
$= 4 - 4$ $= 9 - 1$
$= 0$ $= 8$

$(40 \div 10) \div 2$ but $40 \div (10 \div 2)$
$= 4 \div 2$ $= 40 \div 5$
$= 2$ $= 8$

The order in which you add numbers and the order in which you multiply numbers does not change the result. The order only changes the result for subtraction or division.

You can use the commutative and associative laws to 'shuffle' or rearrange the calculation to create multiples of 10, which are easier to add and multiply.

For example:

$7 + 9 + 3$ is the same as: $3 + 7 + 9$
$= 10 + 9$

$5 \times 37 \times 2$ is the same as: $5 \times 2 \times 37$
$= 10 \times 37$

You can also split one number into two parts, then use the associative law to add one part to the other number to create a multiple of 10.

For example: 135 + 46
= 135 + 5 + 41 (splitting 46 into 5 and 41)
= 140 + 41 (adding 5 to 135)

Worked example 1 W.E. 1

Calculate the following using the 'make easy numbers' strategy.

(a) 7 + 32 + 13 **(b)** 2 × 13 × 5 **(c)** 293 + 568

Thinking			Working
(a)	**1**	Rearrange the addition to form 'easy' numbers, such as multiples of 10.	*(a)* 7 + 32 + 13 = 7 + 13 + 32
	2	Perform these calculations first.	= 20 + 32
	3	Complete the question.	= 52
(b)	**1**	Rearrange the multiplication to form 'easy' numbers, such as multiples of 10.	*(b)* 2 × 13 × 5 = 2 × 5 × 13
	2	Perform these calculations first.	= 10 × 13
	3	Complete the question.	= 130
(c)	**1**	Split one number into two parts, then add one part to the other number to create an 'easy' number. (Here, we have split 7 away from 568 and added it to 293.)	*(c)* 293 + 568 = 293 + 7 + 561 = 300 + 561
	2	Perform the calculation to complete the question.	= 861

Strategy 2—Use the distributive law

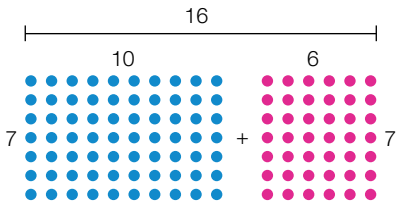
This strategy uses a property of numbers called the **distributive law**. The distributive law lets you multiply a large number by splitting it up into 10s and 1s (or 100s, 10s and 1s), multiplying each part separately, then adding or subtracting each of the products.

(When two numbers are multiplied together, the result is called the **product**.)

For example, you can split 7 × 16 into 7 lots of 10 plus 7 lots of 6.

You can represent the multiplication in an array diagram:

7 × 16
= 7 × (10 + 6) (because 16 = 10 + 6)
= 7 × 10 + 7 × 6
= 70 + 42
= 112

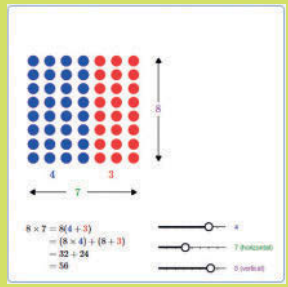


Interactive

The distributive law

Explore visually how the distributive law works with changing numbers.

Go to the eBook or the Pearson Places website to access this interactive.

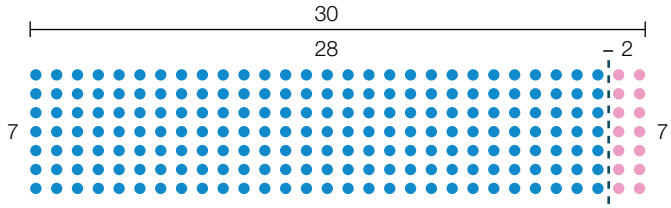


The distributive law means that $7 \times 16 = 7 \times 10 + 7 \times 6$.

In a similar way, you can write 19 as 20 – 1, or 28 as 30 – 2. For example, you can split 7×28 into 7 lots of 30 minus 7 lots of 2.

You can represent this multiplication as an array diagram (the number subtracted is on the right of the dotted line).

7×28
$= 7 \times (30 - 2)$ (because $28 = 30 - 2$)
$= 7 \times 30 - 7 \times 2$
$= 210 - 14$
$= 196$



To multiply a large number, split it up into 10s and 1s (or 100s, 10s and 1s). Multiply by these separately, then add or subtract each of the products.

Worked example 2

W.E. 2

Evaluate the following using the distributive law.

(a) 7×22 **(b)** 15×9

Thinking		Working
(a) **1**	Split the number you are multiplying by into 10s and 1s, writing it in brackets.	*(a)* 7×22 $= 7 \times (20 + 2)$
2	Multiply the 10s and the 1s by the number in front of the brackets.	$= 7 \times 20 + 7 \times 2$ $= 140 + 14$
3	Add the two products together.	$= 154$
(b) **1**	Round one of the numbers to the nearest multiple of 10, then write it as a subtraction in brackets. Here, you write 9 as (10 – 1).	*(b)* 15×9 $= 15 \times (10 - 1)$
2	Multiply the numbers inside the brackets by the number in front.	$= 15 \times 10 - 15 \times 1$ $= 150 - 15$
3	Complete the subtraction.	$= 135$

1.1 Mental strategies

Navigator

Answers p. 646

1 (columns 1–2), 2 (columns 1–2), 3 (columns 1–2), 4 (a–g), 5, 6, 11, 12, 13, 15	1 (columns 2–3), 2 (columns 2–3), 3 (columns 2–3), 4 (column 1), 5, 6, 8, 10, 11, 12, 13, 14, 15, 16, 17	1 (column 3), 2 (column 3), 3 (column 1), 4 (column 2), 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17

Fluency

W.E. 1

1 Calculate the following using the 'make easy numbers' strategy.

(a) $5 + 32 + 5$ **(b)** $1 + 28 + 9$ **(c)** $7 + 24 + 33$

(d) $2 \times 9 \times 5$ **(e)** $5 \times 7 \times 2$ **(f)** $5 \times 18 \times 2$

(g) $8 + 23 + 42$ **(h)** $15 + 57 + 35$ **(i)** $64 + 79 + 56$

(j) $5 \times 6 \times 2$ **(k)** $4 \times 6 \times 5$ **(l)** $2 \times 42 \times 5$

(m) $5 \times 7 \times 6$ **(n)** $5 \times 3 \times 8$ **(o)** $5 \times 14 \times 4$

(p) $47 + 73$ **(q)** $124 + 56$ **(r)** $211 + 169$

(s) $37 + 128 + 63$ **(t)** $77 + 78 + 23$ **(u)** $89 + 116 + 11$

W.E. 2

2 Evaluate the following using the distributive law.

(a) 2×13 **(b)** 4×12 **(c)** 2×43

(d) 3×13 **(e)** 3×42 **(f)** 3×24

(g) 17×9 **(h)** 19×8 **(i)** 49×6

(j) 6×31 **(k)** 7×52 **(l)** 5×43

(m) 99×9 **(n)** 77×3 **(o)** 57×8

(p) 14×11 **(q)** 15×13 **(r)** 16×12

(s) 101×8 **(t)** 113×5 **(u)** 124×11

3 Use any appropriate mental strategy to work out the following.

(a) $33 + 4 + 7$ **(b)** $135 + 45$ **(c)** $2 \times 24 \times 5$

(d) $4 \times 7 \times 5$ **(e)** 3×85 **(f)** 5×199

(g) $23 + 41 + 57$ **(h)** $347 + 156$ **(i)** $335 - 170$

(j) $8 \times 9 \times 5$ **(k)** 14×7 **(l)** 21×9

(m) 103×6 **(n)** 22×11 **(o)** 3×194

(p) $147 + 213$ **(q)** 19×14 **(r)** $4 \times 7 \times 15$

Understanding

4 Use your calculator to find whether each equation is true (T) or false (F). If it is true, then state which law makes it true: commutative law, associative law or distributive law.

(a) $435 \times 543 = 543 \times 435$ **(b)** $24 \div 6 = 6 \div 24$

(c) $24 + (45 + 76) = (24 + 45) + 76$ **(d)** $325 - (85 - 75) = (325 - 85) - 75$

(e) $45 \times (53 \times 26) = (45 \times 53) \times 26$ **(f)** $(48 \div 6) \div 2 = 48 \div (6 \div 2)$

(g) $45 \times (53 + 26) = 45 \times 53 + 45 \times 26$ **(h)** $45 \times (53 - 26) = 45 \times 53 - 45 \times 26$

(i) $45 + (53 \times 26) = (45 + 53) \times (45 + 26)$

5 Choose the correct answer to the following question.

23×7 could be calculated by:

A multiplying 3 and 7, then adding 20

B multiplying 20 and 7, then adding 3

C multiplying 20 and 7, multiplying 3 and 7, then adding the products together

D multiplying 20, 3 and 7 all together.

6 Bilal has completed the first 3 stages of a bike rally. He rode 87 km in Stage 1, 95 km in Stage 2, and 63 km in Stage 3. Use mental strategies to calculate:

(a) the total distance that Bilal has ridden so far

(b) how far Bilal still has to ride, if the total rally distance is 480 km.

7 Year 7 students at Mountain View Secondary College are doing a project to improve their environment. Each student will plant 5 seedlings of a native plant. Use a mental strategy to calculate how many seedlings will be needed for 8 classes of 25 students.

8 Jason is saving $8 every week for some new cricket gear. Use mental strategies to calculate:

(a) how much Jason has saved after 17 weeks

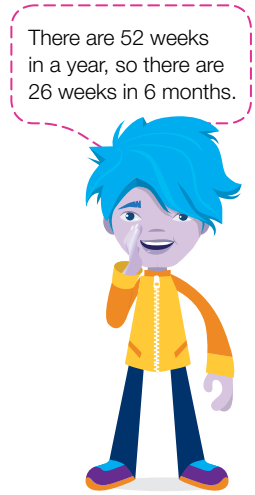
(b) how much he still has to save if the cricket gear he wants costs $189.

9 Carlos is monitoring traffic on a busy road. Twelve cars go past him in 1 minute.

(a) Use a mental strategy to calculate how many cars Carlos can expect to go past in 1 hour, based on his 1 minute count.

(b) List two reasons why the actual number of cars might be more or less than your answer to **(a)**.

10 Jessica earns $5 every time she walks her neighbour's dog. If she walks the dog 3 times a week, how much will she earn in 6 months?



11 Alicia is ordering stationery for her office cupboard. Use mental strategies to calculate the cost of each of the following, in dollars.

(a) 8 notepads at 98 cents each

(b) 3 gluesticks at 77 cents each

(c) 12 pens at 59 cents each

(d) 5 boxes of paperclips at 82 cents each

Reasoning

12 Below are some mistakes made by students on a test, and their explanation of the method they used. Write what each student has done incorrectly and what the answer should be.

(a) $21 \times 7 = 161$. Kate: 'I multiplied 7 by 20, and this gave me one less lot of 21 than I needed. So, then I added 21.'

(b) $35 \times 3 = 140$. Sam: 'I doubled 35, then doubled my answer to get 140.'

(c) $256 - 65 = 209$. Leah: 'I first subtracted 56 to get back to 200, and then added the remaining 9.'

13 Four students were asked to write a mental maths strategy for calculating 9×29. Here are their suggestions:

Alex: $10 \times 29 - 29$ Indrah: $10 \times 29 - 9$
Khalid: $9 \times 30 - 9$ Lucy: $9 \times 30 - 30$

(a) Who has written a correct strategy?

(b) Why do two different strategies give the same answer?

(c) Explain what is wrong with the other strategies.

14 Rosa writes her calculation of 23×75 as a 'long multiplication', as shown.

Evaluate $23 \times (70 + 5)$ using the distributive law. Use your solution to explain why Rosa's 'long multiplication' gives the correct answer.

```
  23
× 75
 115
1610
1725
```

Open-ended

15 Tranh is in the hardware shop buying some supplies. He has 5 picture hooks, which are 28 cents each, 12 curtain rings, which are 15 cents each, and a small hammer costing $6. As he walks to the checkout, Tranh wonders if he has enough money, as he only has $10 in his pocket. Describe the mental calculations Tranh could do to be sure he has enough money to pay for his items. Is $10 enough?

16 Describe how you could use a calculator to work out 11×23 if the '1' key was broken.

17 Brendan has conducted a survey of the number of pets owned by each member of his class. His results are: 2, 2, 2, 1, 1, 2, 3, 3, 2, 2, 1, 4, 1, 3, 5, 2, 4, 1, 2, 1, 3, 2, 1.

Brendan has been trying to use his calculator to add up his list of numbers, but he keeps losing his place in the list. Suggest a method that Brendan could use to organise his numbers and use his calculator more efficiently. Use your method to add up Brendan's list.

Game

Greed

Equipment required: 2 dice

Aim of the game:
The winner is the first player to reach or pass 500.

How to play:
On your turn, roll the dice and make a two-digit number from the numbers that you roll. (For example, if you roll a 4 and a 3, you can make the number 34 or 43). Write this number.

Roll again, write another two-digit number and add it to your previous number.

Keep rolling, recording the numbers, and adding up the total as you go. BUT, if you roll a 'double' (two of the same number), then all your score for the turn is lost and your turn ends.

To avoid losing your score by rolling a double, you can choose to end your turn at any time and save your score. On your next turn, you will start with your saved score.

How many times should you roll the dice each turn to get the highest score without rolling a double?

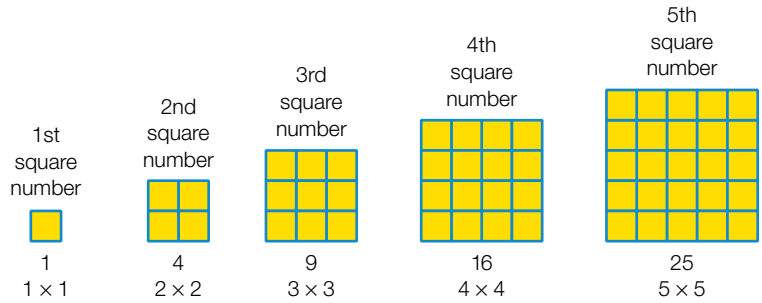
To help keep track of the scores, be sure to write down every number and circle the score when you 'save' it.

Indices

1.2

Square numbers

Square numbers (or **perfect squares**) are numbers that can be represented by squares of different whole number side lengths, as shown below. 1, 4, 9, 16 and 25 are square numbers.



The number of small squares in each larger square is equal to the number of smaller squares along one side, multiplied by itself. You can think of it as multiplying the length and width of the larger square.

There is a shorthand way of writing square numbers. For example, 16 is equal to 4×4 and can be written as 4^2. The small '2' means that two 4s are multiplied together. We say 4^2 as 'four squared'.

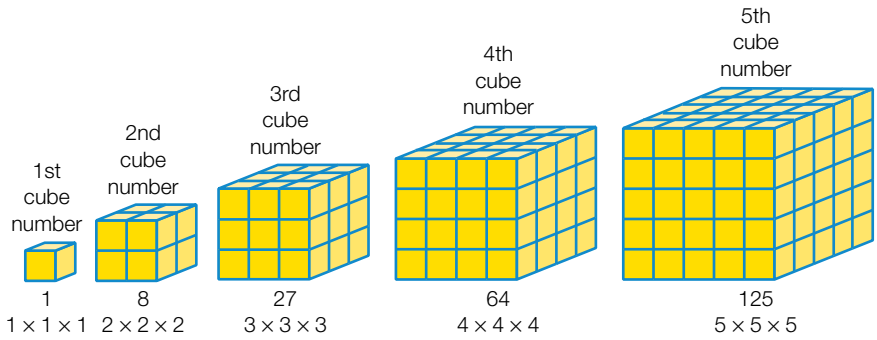
You can write the sequence of perfect squares as:

$1^2, \quad 2^2, \quad 3^2, \quad 4^2, \quad 5^2, \quad 6^2, \quad 7^2, \ldots$

$= 1, \quad 4, \quad 9, \quad 16, \quad 25, \quad 36, \quad 49, \ldots$

Cube numbers

Cube numbers (or **perfect cubes**) are numbers that can be represented by blocks arranged in a cube pattern, as shown below. 1, 8, 27 and 64 are cube numbers.



The number of smaller cubes in each larger cube is equal to the number of smaller cubes along one side, multiplied by itself, and itself again. You can think of it as multiplying the length, width and height of the cube.

(Notice that when a number is cubed, the number is written out three times.)

Using shorthand, you can write the number 8 as $2 \times 2 \times 2$ or 2^3. The small '3' indicates that three 2s are multiplied together. We say 2^3 as 'two cubed'.

You can write the sequence of perfect cubes as:

$1^3, \quad 2^3, \quad 3^3, \quad 4^3, \quad 5^3, \ldots$

$= 1, \quad 8, \quad 27, \quad 64, \quad 125, \ldots$

To 'square' a number, multiply it by itself:

$$6^2 = 6 \times 6$$
$$= 36$$

To 'cube' a number, multiply it by itself and itself again, so that the number is written three times in the multiplication:

$$4^3 = 4 \times 4 \times 4$$
$$= 64$$

Do not confuse 4^3 with 4×3.

Square roots and cube roots

Finding the **square root** or **cube root** of a number is the reverse of squaring or cubing a number. For example, the square root of 16 is the number that gives 16 when multiplied by itself. In this case, that number is 4, because $4^2 = 16$. We write $\sqrt{16} = 4$, where the symbol $\sqrt{\ }$ means 'the square root of'. The cube root of 8 is 2, because $2^3 = 8$. We write $\sqrt[3]{8} = 2$, where $\sqrt[3]{\ }$ means 'the cube root of'.

Only the roots of perfect squares and perfect cubes are whole numbers. Many roots are not whole numbers and a calculator can be used to find them.

Index notation

You can extend your knowledge of square numbers and cube numbers to write other large numbers produced by repeated multiplication, by continuing the pattern.

If:	$3^2 = 3 \times 3$	$(= 9)$
and	$3^3 = 3 \times 3 \times 3$	$(= 27)$
then,	$3^4 = 3 \times 3 \times 3 \times 3$	$(= 81)$
	$3^5 = 3 \times 3 \times 3 \times 3 \times 3$	$(= 243)$

This notation is called **index form**. The number being multiplied repeatedly is called the **base**. The number written up high to the right of the base is called the **power**, or **index**. (The plural of index is **indices**.) The index tells you how many times the base will appear when written in **expanded form** (that is, written as a series of multiplications).

Index form:

5^4 ← index (power)
5 ← base

Expanded form: $5 \times 5 \times 5 \times 5$

Say: '5 to the power of four', or '5 to the fourth' or 'base 5, index 4'.

The value of 5^4 is 625.

Any number to the power of 1 is itself. $5^1 = 5$.

Worked example 3 W.E. 3

Write each of the following numbers in expanded form, then find its value.

(a) 4^3 **(b)** 8^5

Thinking			Working
(a)	**1**	Identify the base and the index. Multiply the base by itself according to what the index is.	(a) $4^3 = 4 \times 4 \times 4$
	2	Perform the multiplication.	$= 64$
(b)	**1**	Identify the base and the index. Multiply the base by itself according to what the index is.	(b) $8^5 = 8 \times 8 \times 8 \times 8 \times 8$
	2	Perform the multiplication.	$= 32\,768$

Indices on a calculator

Most calculators have a key to help you work with indices. It usually looks like [^] or [x^y]. For example, to calculate 12^4, you would enter: [1] [2] [^] [4] [=] or [1] [2] [x^y] [4] [=].

Some calculators also have an [x^2] key and an [x^3] key for calculating square and cube numbers. For example, to find 17^2, you could enter: [1] [7] [x^2] [=].
To find 2^3, you could enter: [2] [x^3] [=].

The [^] or [x^y] key can also be used to calculate squares and cubes.

Square roots and cube roots on a calculator

Scientific calculators have a square root key, which usually looks like this: [$\sqrt{\ }$].

To find the square root of 256, you would enter: [$\sqrt{\ }$] [2] [5] [6] [=].
(If you try this, you should find that the answer is 16.) For some calculators, you may need to enter the number first, then the [$\sqrt{\ }$] key.

To find a cube root on the calculator, use the [$\sqrt[x]{\ }$] key. To find the cube root of 512, you could enter: [3] [$\sqrt[x]{\ }$] [5] [1] [2] [=]. (If you try this, you should find that the answer is 8.)

Powers of 10

The number system that we use for counting is based on powers of ten:

$10^1 = 10$
$10^2 = 10 \times 10 = 100$
$10^3 = 10 \times 10 \times 10 = 1000$
$10^4 = 10 \times 10 \times 10 \times 10 = 10\,000$ and so on.

Here, the word 'power' means the actual value of the number in index form.
So, the first four powers of 10 are: 10, 100, 1000, 10 000.

Notice that when you write the value of the number in index form, the number of zeros is the same as the index. For example, 10^3 has an index of 3, and its value 1000 has 3 zeros.

1.2 Indices

Navigator

Answers p. 647

1, 2 (columns 1–2), 3, 4 (columns 1–2), 5, 6, 7, 8 (columns 1–2), 9, 10, 11, 12 (a), 13, 14 (column 1), 15, 17, 18 (columns 1–2), 19, 20, 23, 24, 27	1, 2 (columns 2–3), 3, 4 (columns 2–3), 5, 6, 7, 8 (columns 2–3), 9, 10 (columns 1–2), 11, 12, 13, 14 (column 2), 15, 16 (a–b), 17, 18 (columns 2–3), 19, 20, 21, 23, 24, 26, 27	1, 2 (j–r), 3 (column 2), 4 (column 3), 6, 7, 8 (columns 3–4), 9, 10 (columns 2–3), 11, 12, 13, 14 (column 3), 15, 16, 17, 18 (column 3), 19, 20, 21, 22, 23, 24, 25, 26

Equipment required: calculator

Fluency

1 Write whether each statement is true (T) or false (F).

(a) 3^2 can be said as 'two cubed'.

(b) 3^2 can be said as 'three squared'.

(c) 3^5 can be said as 'three to the power 5'.

(d) 3^5 is in index form.

(e) $10 \times 10 \times 10$ is in expanded form.

(f) 25 is a perfect square.

(g) 10 is a perfect square.

(h) 64 is not a perfect square.

(i) 64 is a cube number.

W.E. 3

2 Write each of the following numbers in expanded form, then find its value.

(a) 4^2 **(b)** 10^2 **(c)** 10^3

(d) 3^2 **(e)** 1^2 **(f)** 5^3

(g) 2^3 **(h)** 2^4 **(i)** 2^6

(j) 1^8 **(k)** 0^7 **(l)** 10^3

(m) 6^4 **(n)** 5^5 **(o)** 8^6

(p) 11^3 **(q)** 12^4 **(r)** 14^5

3 Write each of the following in index form.

(a) $8 \times 8 \times 8$

(b) $4 \times 4 \times 4 \times 4 \times 4 \times 4$

(c) $12 \times 12 \times 12 \times 12 \times 12$

(d) $16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16$

(e) seventeen cubed

(f) nineteen squared

(g) eight to the power of 4

(h) thirteen to the power of seven

(i) base 11, index 7

(j) base 9, index 6

4 Find the value of each of the following.

(a) 4^7 **(b)** 6^6 **(c)** 9^4

(d) 8^3 **(e)** 7^5 **(f)** 15^3

(g) seventeen squared **(h)** fifty cubed **(i)** the cube of 4

(j) 5×10^2 **(k)** 4×10^3 **(l)** $10^3 \times 2$

(m) 2 to the power of 10 **(n)** 3 to the power of 9 **(o)** base 1, index 8

5 Write in words how you could say each of the following.

(a) 5^2 **(b)** 31^2 **(c)** 3^3 **(d)** 27^3

(e) 4^5 **(f)** 9^6 **(g)** 1^4 **(h)** 7^7

6 If 1 is the first square number and 4 is the second, write the 5th, 6th and 7th square numbers.

7 If 1 is the first cube number and 8 is the second, write the 4th, 5th and 6th cube numbers.

8 Evaluate:

(a) $\sqrt{25}$ **(b)** $\sqrt{49}$ **(c)** $\sqrt{100}$ **(d)** $\sqrt{121}$

(e) $\sqrt{1}$ **(f)** $\sqrt{0}$ **(g)** $\sqrt{4900}$ **(h)** $\sqrt{400}$

(i) $\sqrt[3]{8}$ **(j)** $\sqrt[3]{64}$ **(k)** $\sqrt[3]{1000}$ **(l)** $\sqrt[3]{1}$

(m) $\sqrt[3]{0}$ **(n)** $\sqrt[3]{125}$ **(o)** $\sqrt[3]{8000}$ **(p)** $\sqrt[3]{27\,000}$

9 **(a)** In $2^3 = 8$, the base number is:

A 2 **B** 3 **C** 6 **D** 8

(b) The square of 5 is:

A 2.2 **B** 10 **C** 25 **D** 125

(c) The cube root of 729 is:

A 9 **B** 27 **C** 243 **D** 2187

Understanding

10 Evaluate the following without using a calculator.

(a) $1^4 + 2^2$ **(b)** $2^3 - 1^6$ **(c)** $3^4 - 2^4$

(d) $2^3 \times 2^2$ **(e)** $2^2 \times 2^4$ **(f)** $3^3 \times 3^2$

(g) $2^5 + 5^2 - 6^2$ **(h)** $10^2 + 3^2 - 4^3$ **(i)** $5^2 - 1^6 + 3^3$

11 Write whether each of the following is true (T) or false (F).

(a) 2^{10} is greater than 10^3. **(b)** $2^3 \times 2^4 = 2^7$

(c) $\sqrt{5^4} = 52$ **(d)** $\sqrt{2^6} = 2^3$

(e) $\sqrt{2}$ is a whole number. **(f)** $\sqrt{121}$ is a whole number.

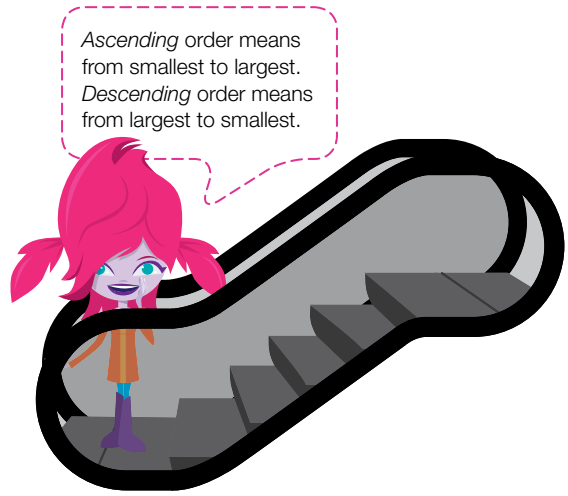
(g) $\sqrt[3]{1000000}$ is a whole number.

12 **(a)** Arrange these numbers in ascending order:

$4^5, 5^4, 1^{200}, 10^3, 4^6, 5^5$

(b) Arrange these numbers in descending order:

$100^2, 10^5, 1^{1000}, 0^{100}, 3^2, 2^3$



13 **(a)** Write two numbers between 5 and 40 that are both even and square.

(b) Write two numbers between 30 and 90 that are both odd and square.

14 Evaluate these with your calculator, using the most efficient method possible:

(a) $16^3 - 4096$ **(b)** $21^4 - 4481$ **(c)** $15^4 - 5625$

(d) $36^3 \times 53$ **(e)** $14^4 \times 14$ **(f)** $19^5 \times 21$

(g) $2^{12} + 2^{18}$ **(h)** $3^{10} + 12^4$ **(i)** $4^9 + 3^{11}$

15 Only the square roots of perfect squares are whole numbers. Square roots of other numbers are in between the perfect square roots (they are decimal numbers). For example, $\sqrt{16} = 4$ and $\sqrt{25} = 5$, so $\sqrt{20}$ would be between 4 and 5. Find which two consecutive whole numbers the following are between. (Consecutive numbers come one after the other, e.g. 8 and 9.)

(a) $\sqrt{10}$ **(b)** $\sqrt{5}$ **(c)** $\sqrt{20}$ **(d)** $\sqrt{62}$

(e) $\sqrt{99}$ **(f)** $\sqrt{2}$ **(g)** $\sqrt{70}$ **(h)** $\sqrt{108}$

16 Arrange the following in ascending order.

(a) 2^4, $\sqrt[3]{64}$, 5, $\sqrt{49}$ **(b)** 7^2, $\sqrt[3]{125}$, 9, $\sqrt{60}$

(c) $\sqrt[3]{8}$, 10, $\sqrt{90}$, 4^2 **(d)** $\sqrt{80}$, 8, $\sqrt{105}$, 3^3

17 (a) Complete the following table of the powers of 10.

Index form	Expanded form	Value
10^1	10	10
10^2	10×10	
10^3		1000
10^4		

(b) For each row of the table, compare the index numbers in the first column with the number of zeros in the third column. Describe the pattern.

18 For each of the following, find the missing power of 10.

(a) $20\,000 = 2 \times 10^{\square}$ **(b)** $400 = 4 \times 10^{\square}$ **(c)** $5000 = 5 \times 10^{\square}$

(d) $300\,000 = 3 \times 10^{\square}$ **(e)** $80 = 8 \times 10^{\square}$ **(f)** $7\,000\,000 = 7 \times 10^{\square}$

(g) $150\,000 = 15 \times 10^{\square}$ **(h)** $91\,500 = 915 \times 10^{\square}$ **(i)** $2\,340\,000 = 234 \times 10^{\square}$

Reasoning

19 (a) Use your calculator to work out $5^2 \times 5^3$ and $5 \times 5 \times 5 \times 5 \times 5$.

(b) Now, try it with $2^3 \times 2^6$ and $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$.

(c) What can you conclude?

20 How many whole numbers have the value of their square root in between:

(a) 2 and 3 **(b)** 5 and 6 **(c)** 8 and 9?

21 (a) Use your calculator to answer true (T) or false (F) for each of the following statements.

(i) 4^6 is bigger than 6^4 **(ii)** 2^{10} is bigger than 10^2

(iii) 3^9 is bigger than 9^3 **(iv)** 19^2 is bigger than 2^{19}

(b) Look at your answers for part **(a)**. Then, without using your calculator, answer true (T) or false (F) for each of the following statements.

(i) 9^8 is bigger than 8^9 **(ii)** 2^{100} is bigger than 100^2

22 **(a)** The number 10^{100} was named a 'googol' by Milton Sirotta, the 9-year-old nephew of the mathematician Edward Kasner.

(i) How many 10s are multiplied together to give a googol?

(ii) How many zeros would follow the 1 in a googol?

(b) If you raise the number ten to the power of a googol, you get a number called a googolplex.

(i) How many 10s are multiplied together to give a googolplex?

(ii) How many zeros would follow the 1 in a googolplex? How much time do you think you save by writing a googolplex in index form?

23 **(a)** Complete the following.

$11^2 =$ ____ $111^2 =$ ____ $1111^2 =$ ____

(b) Look at the pattern in part **(a)** and, *without* using a calculator, copy and complete the following.

$11\,111^2 =$ ____ $111\,111^2 =$ ____ $1\,111\,111^2 =$ ____

Open-ended

24 The number 1 is a perfect square and also a perfect cube, because $1 = 1 \times 1 = 1^2$ and also $1 = 1 \times 1 \times 1 = 1^3$.

The number 64 is a perfect square and also a perfect cube, because $64 = 8 \times 8 = 8^2$ and also $64 = 4 \times 4 \times 4 = 4^3$.

(a) Write 64 as a power of base 2, in expanded form and also in index form.

(b) Write 729 as a power of base 3, in expanded form and also in index form.

(c) Show that 729 is a perfect square and also a perfect cube.

(d) Write 4096 as a power of base 4. Show that 4096 is a perfect square and also a perfect cube.

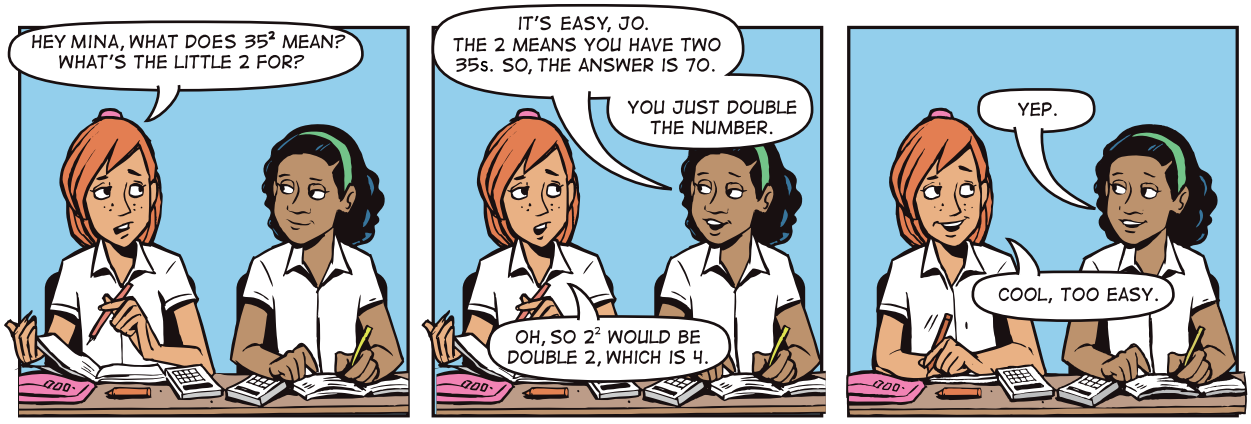
(e) Can you find any larger numbers that are perfect squares as well as perfect cubes?

25 Use the digits 1, 2 and 3 to create at least three numbers that are greater than 500.

26 **(a)** A number has two digits when it is squared and three digits when it is cubed. What might be the number?

(b) A number has three digits when it is squared and four digits when it is cubed. What might be the number?

27



(a) What is wrong with Mina's explanation? Write a better explanation that Mina could give to Jo.

(b) What did Jo get right? Why is this right, even though the explanation is wrong?

Problem solving

Teacher's age

Cheryl, who is a teenager, is curious about her maths teacher's age.

'Well', said the teacher, 'Right now the second digit of my age is a square number and the product of my age's digits is a cube number. In ten years' time, my age will be a square number. Twenty-five years from now my age will be a number that is cubed.'

'I know your age,' Cheryl said with confidence. 'It's also interesting that the difference between our ages is one more than an odd number that is squared.'

How old is the teacher?

How old is Cheryl?

Strategy options

- Guess and check.
- Make a table.
- Work backwards
- Test all possible combinations.

Investigation

How many squares on a chessboard?

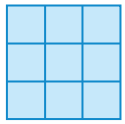
Equipment required: graph or grid paper

The Big Question

A chessboard is an 8×8 grid of squares. What is the total number of squares on a chessboard?

Engage

1 How many squares are in this 3×3 grid?



People who are tricked by this puzzle forget there are squares of more than one size. Also, several squares overlap each other.

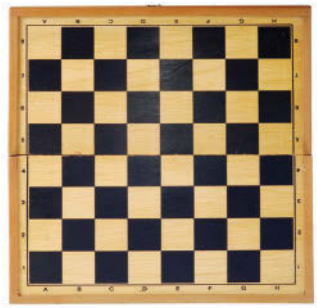
By copying the grid and shading, show that there are 14 squares of three different sizes in the 3×3 grid.

Explore

2 To solve a puzzle like this, it helps to have a system. Use your results from **1** to fill in the following table. The 1×1 and 2×2 grids have been done for you.

		Number of squares of each type				
		Side of length 1 unit	Side of length 2 units	Side of length 3 units	Side of length 4 units	Total squares in whole grid
Grid size	1×1 	1				1
	2×2 	4	1			5
	3×3 					

3 Now, extend the system of counting the number of squares of each size to work out how many squares there are in a 4×4 grid and a 5×5 grid … up to an 8×8 grid (a chessboard). Extend your table to record the number of each size of square, and the total number of squares in each grid.



Strategy options

- Draw a diagram.
- Look for a pattern.
- Break problem into manageable parts.

Explain

4 Describe any patterns you can see in your table of results. Consider the numbers of each size square in a grid and how this changes as the grid gets larger.

Elaborate

5 State your answer to the Big Question. Present your results so that they clearly show how you arrived at your answer.

6 Use your pattern to predict the total number of squares on: **(a)** a 10×10 grid **(b)** any size grid.

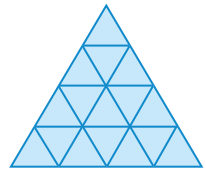
Evaluate

7 **(a)** Consider how you worked on this investigation and the methods you used. Describe the strategy or method you used to make sure that you counted all the squares. How confident are you that your answer is correct?

(b) Did any of your methods or strategies change as you went? Did you develop any shortcuts that made your working easier? What were they?

Extend

8 Using a similar method, work out how many triangles there are in this pattern.



Maths 4 Real

Numbers around the world

Throughout history, different groups of people have developed different systems and symbols for numbers. Some of these number systems travelled around the world as people migrated and traded, while other number systems were only used by small groups.

The Hindu–Arabic system

0 1 2 3 4 5 6 7 8 9

You should be familiar with these ten number symbols that are used to represent numbers, the digits from 0 (zero) to 9 (nine). These symbols come from the 'Brahmi numerals', which were used in India from around the 3rd century BCE. They were adopted and modified by Arabic traders, who brought them to Europe during the Middle Ages. Because of this history, this number system is called the 'Hindu–Arabic system'.

An early version of the Brahmi numerals are shown in the second row of the table below. They evolved over time to become the more familiar Hindu–Arabic number symbols in the top row.

1	2	3	4	5	6	7	8	9
—	=	≡	𑁤	𑁥	𑁦	𑁧	𑁨	𑁩

The Roman system

The ancient Romans used letters as their symbols for numbers. Roman numerals are still sometimes used today for clocks, monuments and other situations where people want numbers to seem more 'historical' or significant.

1	2	3	4	5	6	7	8	9
I	II	III	IV	V	VI	VII	VIII	IX

10	50	100	500	1000
X	L	C	D	M

In Roman numerals, letters are written next to each other to add them together. For example, **II** means **I** + **I** = 1 + 1 = 2. In the same way, **XV** means **X** + **V** = 10 + 5 = 15. You always write the higher-value numerals before the lower-value numerals.

However, in Roman numerals you can also write a letter **I**, **X** or **C** before a higher value letter to take it away from the higher value. The rule is that you can only subtract a Roman numeral like this from the next two higher-level Roman numerals. For example, **IV** means **V** – **I** = 5 – 1 = 4. In the same way, **IX** means **X** – **I** = 10 – 1 = 9. However, 49 is not **IL**. Instead, 49 is **XLIX**, made up of **XL** for 40 and **IX** for 9.

So, **I** can be subtracted from **V** and **X**, **X** can be subtracted from **L** and **C**, and **C** can be subtracted from **D** and **M**.

39 is **XXXIX**
(10 + 10 + 10 + 9)

2644 is **MMDCXLIV**
(1000 + 1000 + 500 + 100 + (50 – 10) + (5 – 1))

1 Write these numbers using Roman numerals:
(a) 8 **(c)** 439 **(b)** 84 **(d)** 1975

2 Calculate the following. Write your answers using Roman numerals.
(a) XXIX + CLII
(b) MDCCLX – MCDXCIX

3 What is the largest Roman numeral you can make that uses one of each of the different symbols: **C**, **L**, **M**, **X**, **D**, **V**, **I**? Write your answer in Roman numerals and in Hindu–Arabic numerals.

The Chinese character system

0	1	2	3	4	5	6
〇	一	二	三	四	五	六

7	8	9	10	100	1000	10000
七	八	九	十	百	千	万

The Chinese character system for numbers uses two symbols for each 'digit', when multiple digits are needed: the first symbol shows how many, and the second symbol shows the place value. However, when there is only one digit then only one symbol is used. Also, these numbers are traditionally written vertically. For example:

270		19		8005	
二 百 七 十	two (lots of) one hundred and seven (lots of) ten	十 九	ten and nine	八 千 五	eight (lots of) one thousand and five

4 Write these numbers using the Chinese character system:

(a) 37 **(b)** 823 **(c)** 1053 **(d)** 6400

5 Calculate the following. Write your answers using Chinese characters.

(a) 五百六十 + 八百九 **(b)** 千 − 三百二十七

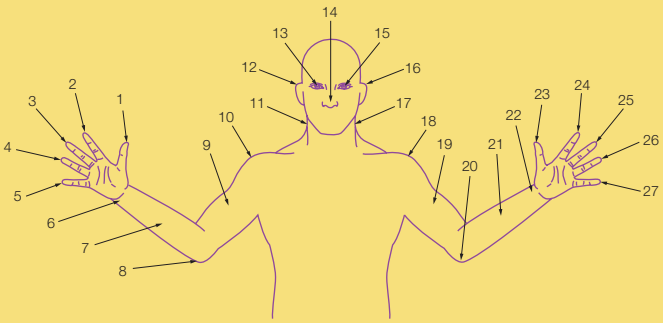
6 Did you find it difficult to perform calculations with the Roman and Chinese systems? What are the advantages of using the Hindu–Arabic system for calculations?

Other systems

The Hindu–Arabic system is a 'base 10' or 'decimal' system. The place value of each digit in a number is 10 times bigger than the next digit. For example: 367 = 3 'hundreds' + 6 'tens' + 7 'ones', or 300 + 60 + 7.

Some indigenous cultures of Australia and Papua New Guinea have counting systems that use body parts to indicate numbers. These systems are not base 10 systems. For example, the people of Sandaun province, Papua New Guinea (speaking the Oksapmin language) use the parts of the upper body to count in lots of 27, as shown:

Counting in Oksapmin



1	*tipun* 'thumb'
2	*ləwatipun* 'index finger'
3	*bumlip* 'middle finger'
4	*xətlip* 'ring finger'
5	*xətxət* 'little finger'
6	*xadəp* 'wrist'
7	*bes* 'forearm'
8	*amun* 'elbow'
9	*tuwət* 'upper arm'
10	*kat* 'shoulder'
11	*gwel* 'side of neck'
12	*nat* 'ear'
13	*kin* 'eye'
14	*lum* 'nose'

15	*kin tən* 'other-side eye'
16	*nat tən* 'other-side ear'
17	*gwel tən* 'other-side of neck'
18	*kat tən* 'other-side shoulder'
19	*tuwət tən* 'other-side upper arm'
20	*amun tən* 'other-side elbow'
21	*bes tən* 'other-side forearm'
22	*xadəp tən* 'other-side wrist'
23	*tipun tən* 'other-side thumb'
24	*ləwatipun tən* 'other-side index finger'
25	*bumlip tən* 'other-side middle finger'
26	*xətlip tən* 'other-side ring finger'
27	*xətxət tən* 'other-side little finger'

7 **(a)** In Oksapmin, what word would you use to say '15'?

(b) If you wanted to say 'I have eight chickens' in Oksapmin, what body part would you point to?

8 The Hindu–Arabic number system is used almost everywhere across the world today. List some advantages and disadvantages of having everyone use the same number system.

Research

- Investigate the number systems of other ancient civilisations (such as the Mayans or Babylonians) or other cultures.
- Find out how to write and say numbers in another language, such as Japanese or Greek.

1.3 More strategies for multiplication and division

The result of a multiplication calculation is called the product. The numbers being multiplied together are sometimes called multipliers.

The result of a division calculation is called the **quotient**. The number being divided is the **dividend** and the number it is divided by is the **divisor**. Sometimes after a division you are left with a **remainder**.

18 multiplier	×	27 multiplier	=	486 product	
32 dividend	÷	10 divisor	=	3 quotient	rem 2 remainder

You should be familiar with multiplying any two numbers from 0 to 10 together, such as 4×3, 8×6 or 9×5. If you know a lot of multiplication facts, you also know a lot of division facts: for example, $63 \div 7 = 9$ because $9 \times 7 = 63$.

Following on from strategies 1 and 2 covered in section 1.1, here are two more strategies useful for multiplying and dividing numbers. You may have other methods that you like to use.

Strategy 3—Work in stages

Multiplying a number by 4 is the same as multiplying by 2 twice ($4 = 2 \times 2$); that is, doubling and then doubling again. To multiply by 6, you could multiply by 3, and double the answer ($6 = 3 \times 2$). Similarly, you could divide by 4 by dividing by 2 twice. Dividing by 6 is the same as dividing by 2, and then dividing by 3 (you won't want to use this strategy if the number you are dividing is odd).

Multiply or divide in stages by breaking the multiplier or divisor into its parts and doing a series of simpler multiplications or divisions.

Worked example 4 W.E. 4

Calculate the following using the 'work in stages' strategy.

(a) 34×8 **(b)** $900 \div 15$

Thinking	Working
(a) 1 Break the multiplier down into a series of simpler multiplications.	*(a)* 34×8 $= 34 \times 4 \times 2$ $= 34 \times 2 \times 2 \times 2$
2 Perform one of these multiplications.	$= 68 \times 2 \times 2$

	3	Perform the next multiplication (× 2 again).	= 136 × 2
	4	Keep multiplying until the multiplication is complete (× 2 again).	= 272
(b)	1	Break the divisor down into a series of simpler divisions.	(b) 900 ÷ 15 = 900 ÷ 3 ÷ 5
	2	Perform one of these divisions (÷ 3).	= 300 ÷ 5
	3	Perform the next division (÷ 5). Keep dividing until the division is complete.	= 60

Strategy 4—Use an array

Multiplying two two-digit or three-digit numbers is not as easy as multiplying two one-digit numbers. You can extend the 'use the distributive law' strategy from section 1.1 to both numbers and multiply them in an array or grid, as shown in the examples below.

Worked example 5

W.E. 5

Calculate 23 × 47 using the 'use an array' strategy.

Thinking	Working
1 Split the numbers to be multiplied into 10s and 1s. Draw up a grid. Place one broken-up number on top of the columns and one to the left of the rows.	23 = 20 + 3 47 = 40 + 7 20	3 40 | | | 7 | | |
2 Multiply each of the 10s and 1s with each of the other 10s and 1s, to get a product in each of the four grid spaces.	20	3 40 | 800 | 120 | 7 | 140 | 21 |
3 Add the two products in each of the rows. (Alternatively, you could add the columns.)	20	3 40 | 800 | 120 | 920 7 | 140 | 21 | 161
4 Add these two sums to get your final answer.	920 + 161 1081

Multiplying by multiples of 10

Multiplying a whole number by a power of 10 such as 10, 100 or 1000 will make the value of each digit in the number 10, 100 or 1000 times bigger. Because this is the same as moving each digit across to increase the place value, you can show this by adding zeros onto the end of the number. The number of zeros added is the same as the number of zeros in the power of 10 (and the same as the power number).

For example: $3 \times 10 = 30$ (3×10^1)
$7 \times 1000 = 7000$ (7×10^3)
$15 \times 100 = 1500$ (15×10^2)

To multiply a whole number by a multiple of 10 such as 40, 800 or 12 000, or to multiply two multiples of 10 together, you can use the associative law to rearrange the calculation and multiply in stages.

For example: $7 \times 40 = 7 \times 4 \times 10$
$= 28 \times 10$
$= 280$

30×600
$= 3 \times 10 \times 6 \times 100$
$= 3 \times 6 \times 10 \times 100$
$= 18 \times 1000$
$= 18\,000$

To multiply whole numbers by numbers that are multiples of 10:

1. write each multiple of 10 as the product of a number and a power of 10
2. rewrite the multiplication, grouping the powers of 10 together
3. multiply the other numbers, using the total number of zeros in the powers of 10 to write the correct number of zeros in the answer.

Dividing by multiples of 10

Consider the following divisions.

$50 \div 10 = 5$
$500 \div 100 = 5$
$500 \div 10 = 50$
$50\,000 \div 100 = 500$
$5\,000\,000 \div 10\,000 = 500$

The number of zeros in the answer is the difference between the number of zeros in the numbers being divided.

A quick way to show this type of division is to draw a line through them. This is often called 'cancelling zeros' or 'crossing zeros out'. For example: $50\,0\cancel{00} \div 1\cancel{00}$
$= 500 \div 1$
$= 500$

It is important to remember what 'cancelling zeros' means—you are dividing both numbers by a power of 10. For example, crossing out three zeros means that you are dividing by 1000.

To divide by whole numbers that are multiples of 10:

1. divide both numbers by 10 until one number is no longer a multiple of 10 (show this by cancelling zeros)
2. do the simplified division.

Worked example 6 W.E. 6

Calculate the following.

(a) 7×900 **(b)** 6000×30 **(c)** $4500 \div 90$

Thinking			Working
(a)	**1**	Rewrite the multiple of 10 as the product of a power of 10 and another number.	*(a)* 7×900 $= 7 \times 9 \times 100$
	2	Multiply the first two numbers.	$= 63 \times 100$
	3	Multiply by the power of ten.	$= 6300$
(b)	**1**	Rewrite each multiple of 10 as the product of a number and a power of 10.	*(b)* 6000×30 $= 6 \times 1000 \times 3 \times 10$
	2	Rearrange the calculation, placing the powers of 10 together and the other numbers together.	$= 6 \times 3 \times 1000 \times 10$
	3	Do the multiplications.	$= 18 \times 10\,000$ $= 180\,000$
(c)	**1**	Divide both numbers by 10 until one number is no longer a multiple of 10.	*(c)* $450\cancel{0} \div 9\cancel{0}$
	2	Write the simpler division that results.	$= 450 \div 9$
	3	Do the division.	$= 50$

1.3 More strategies for multiplication and division

Navigator

1 (columns 1–2), 2 (columns 1–2), 3 (columns 1–2), 4 (columns 1–2), 5, 6, 7, 8, 9, 10, 14, 15, 16, 18	1 (columns 2–3), 2 (columns 2–3), 3 (columns 2–3), 4 (columns 2–3), 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18	1 (column 3), 2 (column 3), 3 (column 3), 4 (column 3), 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18

Answers p. 648

Fluency

1 Calculate the following using the 'work in stages' strategy. W.E. 4

(a) 25×4 **(b)** 35×6 **(c)** 45×8

(d) $1000 \div 4$ **(e)** $200 \div 8$ **(f)** $30\,000 \div 12$

(g) 27×4 **(h)** 41×6 **(i)** 62×8

(j) $600 \div 4$ **(k)** $153 \div 9$ **(l)** $864 \div 16$

(m) 35×9 **(n)** $390 \div 6$ **(o)** 70×15

(p) 15×12 **(q)** $450 \div 25$ **(r)** $360 \div 24$

W.E. 5

2 Calculate the following using the 'use an array' strategy.

(a) 11×11	**(b)** 21×11	**(c)** 12×23
(d) 42×25	**(e)** 42×35	**(f)** 72×63
(g) 12×41	**(h)** 17×26	**(i)** 19×68
(j) 23×61	**(k)** 31×49	**(l)** 39×56
(m) 42×47	**(n)** 49×53	**(o)** 55×55
(p) 61×82	**(q)** 72×77	**(r)** 96×51

W.E. 6

3 Calculate the following.

(a) 2×40	**(b)** 3×4000	**(c)** 300×5
(d) 200×60	**(e)** 700×4000	**(f)** 6000×5000
(g) $200 \div 50$	**(h)** $60\,000 \div 200$	**(i)** $10\,000 \div 50$
(j) 700×3	**(k)** 400×5	**(l)** 2000×13
(m) $360 \div 20$	**(n)** $5400 \div 200$	**(o)** $8000 \div 500$
(p) 1200×400	**(q)** $400 \times 32\,000$	**(r)** $7000 \times 20\,000$
(s) $12\,000 \div 20$	**(t)** $350\,000 \div 700$	**(u)** $8\,400\,000 \div 4000$

4 Calculate the following using any suitable strategy.

(a) $180 \div 20$	**(b)** $450 \div 90$	**(c)** $800 \div 25$
(d) 40×19	**(e)** 53×20	**(f)** 102×18
(g) $2700 \div 18$	**(h)** $1075 \div 25$	**(i)** $2250 \div 15$
(j) 120×12	**(k)** 59×72	**(l)** 98×25

Understanding

5 Answer the following using any suitable strategy.

(a) Fish tanks cost $31 each at a pet shop. Suppose you can't remember your 8 times table, but you can remember your 2 times table (you have left your calculator and phone at home). Find the cost of 8 fish tanks.

(b) An aircraft travels at 700 kilometres per hour, non-stop, for 20 hours. How many kilometres does it travel?

(c) A farmer sells a herd of 200 cows for $50 000. How much money is this for each cow?

(d) A student earns $12 per hour working at a supermarket. She works 20 hours a week and 50 weeks each year. If she saves all her earnings, how much will she have after 5 years? (Assume that she pays no tax and does not earn interest on her bank account.)

6 Charlotte is a distance runner in training. She runs 20 laps of a 400 metre race track 6 times a week. How many metres does Charlotte run in a week?

7 Year 7 students are practising for sports day. They need to be divided into five equal groups. There are 130 students in the year. Explain a quick way the teacher could use to work out how many students will be in each group, and find the answer.

8 Marie's monthly phone bill is $52. If she continued to pay this amount every month, use a mental or written strategy to calculate how much Marie will have paid in phone bills by the end of her 2 year contract.

9 A group of eight friends have won $2000 and want to divide it equally between them. How much will each receive?

10 Rehan is paying back a bank loan that he used to buy a house. His repayments are set at $1800 per month for 30 years. How much money will Rehan have paid back at the end of this time?

11 Dharma has printed 3700 brochures advertising his business. He wants to place them in letterboxes of houses in the local area. Dharma estimates that there are 40 houses in each street. How many streets will he be able to cover with 3700 brochures?

Reasoning

12 A large new apartment building contains 1300 apartments. Each apartment is to be sold for $580 000. If all the apartments are sold, then how much money will this be? If the total cost of building the apartments was $500 000 000, then how much profit will this make?

13 Given that $4 \times 25 = 100$, $8 \times 125 = 1000$ and $16 \times 625 = 10\,000$, use this to calculate the following.

$16 \times 125 \times 25 \times 625 \times 4 \times 8 \times 7$

14 A travelling salesperson needs to work out the individual cost of an item that is packaged as groups of 12 items, and costs $660 for each pack.

(a) The salesperson decides to approximate by dividing by 10. Find this approximation.

(b) Calculate the actual cost of each individual item.

(c) The salesperson sells five items at the approximated cost. How much money is lost or gained, compared to the cost in packs?

15 The 'use an array' method of multiplication can be extended from two-digit to three-digit numbers by adding an extra row and an extra column to the grid. For example, 213×145 can be calculated using the following array.

	200	10	3
100			
40			
5			

Use an extended grid to calculate the following.

(a) 162×246 **(b)** 356×412 **(c)** 107×560

16 Which of the following is an *incorrect* method for calculating $240 \div 4$?

A Halve 240, then halve the answer.

B Calculate $2 \div 4$, $4 \div 4$ and $0 \div 4$, then add the results together.

C Calculate $200 \div 4$ and $40 \div 4$, then add the quotients.

D Calculate $24 \div 4$, then multiply the answer by 10.

Open-ended

17 ◆ × ● = 1620

(a) What could ◆ and ● be? Find at least seven different combinations of whole numbers.

(b) Find at least three combinations where both ◆ and ● are two-digit numbers.

18 Here is part of Kim's maths homework. She had been learning about splitting up numbers to be multiplied, so she decided to use that strategy to complete the following.

Multiplication

(1) 9×24
$= 9 \times 2 + 9 \times 4$
$= 18 + 36$
$= 54$

(2) 12×43
$= 12 \times 4 + 12 \times 3$
$= 48 + 36$
$= 84$

(a) Kim is a bit worried that her answers don't look big enough. What are the correct answers?

(b) Explain to Kim the mistake she has made in both questions.

(c) Give Kim some advice so that she can avoid similar mistakes in the future.

Problem solving

Wizard maths

Henry and his friends Hilary and Rob are shopping on Oblique Lane. Oblique Lane is a street of wizard shops where goods are purchased using wizard money. But wizard money is different to normal money.

Wizard money comes in 3 types: aureas, denaris and cuprums.

1 aurea is worth 17 denaris.
1 denari is worth 29 cuprums.

1 Henry, Hilary and Rob each have 10 aureas to spend. Here are their shopping lists. Add up the total cost of each list, then work out how much wizard money will be left over at the end, in aureas, denaris and cuprums.

Remember to use the information above to help you with your calculations.

2 Invent a wizard shopping list of your own and give it to a friend to add up. (However, make sure that you can work out the answer yourself!)

Henry

Item	*Price*
1 cloak:	2 aureas, 4 denaris
1 large bag owl pellets:....	3 aureas, 15 cuprums
2 bottles elderflower juice:	14 denaris, 27 cuprums

Hilary

Item	*Price*
10 scrolls of parchment:.....	1 aurea, 14 denaris,
1 box of quills:	11 denaris, 23 cuprums
Reading Runes book:	5 aureas, 13 denaris, 10 cuprums

Rob

Item	*Price*
Wizard Monthly magazine:	9 denaris, 21 cuprums
New wand:.....	6 aureas, 16 denaris
1 box of tricky toffee:.....	1 aurea, 9 denaris, 17 cuprums

Half-time 1

1 Calculate the following by using an appropriate mental strategy. **1.1, 1.3**

(a) $4 \times 17 \times 5$ **(b)** $15 \times 23 \times 2$ **(c)** $33 + 45 + 107$

(d) 40×19 **(e)** $37 + 49 + 153$ **(f)** $127 + 65 + 69$

2 **(a)** Write the value of the following. **1.2**

(i) 6^3 **(ii)** 5^5 **(iii)** 10^1 **(iv)** 7^2

(v) $\sqrt{49}$ **(vi)** $\sqrt[3]{8}$ **(vii)** $\sqrt{100}$ **(viii)** $\sqrt[3]{64}$

(b) Which of your answers to **(a)** is:

(i) a perfect square **(ii)** a perfect cube?

3 Calculate the following. Use mental strategies if possible. **1.1, 1.3**

(a) 19×9 **(b)** 41×8 **(c)** 5×38 **(d)** 22×17

(e) 147×3 **(f)** 12×53 **(g)** 67×71 **(h)** 92×84

4 Perform the following divisions. Use mental strategies if possible. **1.3**

(a) $136 \div 4$ **(b)** $174 \div 6$ **(c)** $580 \div 5$ **(d)** $940 \div 20$

(e) $128 \div 8$ **(f)** $351 \div 9$ **(g)** $270 \div 15$ **(h)** $1344 \div 24$

5 Write the following in index form. **1.2**

(a) $3 \times 3 \times 3 \times 3 \times 3 \times 3$ **(b)** 6 to the power of 5 **(c)** base 4, index 7

(d) fifteen squared **(e)** twenty-one cubed **(f)** $41 \times 41 \times 41$

6 Calculate the following. **1.3**

(a) 70×60 **(b)** 150×20 **(c)** 400×300

(d) 250×800 **(e)** $3600 \div 40$ **(f)** $9000 \div 300$

(g) $7500 \div 1500$ **(h)** $124\,000 \div 200$ **(i)** $60\,000 \div 400$

7 13 000 tickets were sold to a music concert at a cost of $52 each. Use a combination of strategies to calculate the amount made by the promoters of the concert from ticket sales. **1.1, 1.3**

1.4 Estimating and rounding

Often, you do not need to know the exact answer to a problem. An **estimate**, or approximate answer, is accurate enough.

Estimating by rounding to the first digit

Rounding is an important skill for making good estimations.

Consider the following.

46 is *rounded up* to 50 (because it is in between 40 and 50, and is closer to 50).

44 is *rounded down* to 40 (because it is in between 40 and 50, and is closer to 40).

When rounding, look at the digit to the right of the digit being rounded. If it is 0, 1, 2, 3 or 4 then round down. If it is 5, 6, 7, 8 or 9 then round up.

This means that (when rounding to the first digit):

173 is rounded *up* to 200

137 is rounded *down* to 100

4500 is rounded *up* to 5000

4499 is rounded *down* to 4000

25 374 is rounded *up* to 30 000

24 985 is rounded *down* to 20 000

This type of rounding is called 'rounding to the first digit' because the rounded number has only zeros after the first digit.

Rounding to the first digit can help you estimate the answers to multiplication and division questions.

To round a number to the first digit, look at the second digit of the number.
If it is 0,1, 2, 3 or 4, keep the first digit the same and replace the other digits with zeros.
If it is 5, 6, 7, 8 or 9, increase the first digit by 1, and replace the other digits with zeros.

When rounding, use the symbol ≈ which means 'approximately equal to'.

Worked example 7 W.E. 7

Round the following numbers to the first digit.

(a) 361 **(b)** 2050 **(c)** 8

Thinking	Working
(a) Look at the second digit. It is 5 or greater so increase the first digit by one and replace the following digits with zeros.	(a) $361 \approx 400$
(b) Look at the second digit. It is less than 5 so keep the first digit the same and replace the following digits with zeros.	(b) $2050 \approx 2000$
(c) There is only one digit so this number is already rounded to the first digit.	(c) 8

Estimating with multiplication and division

Multiplication and division can be estimated by rounding to the first digit. This will not give an exact answer to a problem, but an approximate answer.

Worked example 8 W.E. 8

Calculate an approximate answer to the following by first rounding each number to the first digit.

(a) 368×52 **(b)** $77\,483 \div 421$

Thinking	Working
(a) 1 Round both numbers to the first digit.	(a) 368×52 $\approx 400 \times 50$
2 Multiply the two rounded numbers together and write the approximate answer.	$\approx 4 \times 100 \times 5 \times 10$ $\approx 20 \times 1000$ $\approx 20\,000$
(b) 1 Round both numbers to the first digit.	(b) $77\,483 \div 421$ $\approx 80\,000 \div 400$
2 Divide the two rounded numbers by an appropriate multiple of 10 by 'cancelling zeros'. (Here, we have divided both numbers by 100.)	$\approx 80\,000 \div 400$ $\approx 800 \div 4$
3 Do the simple division and write the approximate answer.	≈ 200

Estimating using other ways of rounding

Instead of rounding to the first digit, you could round to a closer multiple of 5 or 10 that is convenient to work with. This would give a more accurate estimate of the actual answer.

Worked example 9

W.E. 9

Calculate an approximate answer for each of the following by first rounding each number to a convenient multiple of 5, 10, 100 or 1000.

(a) $4587 \div 49$ **(b)** 197×23

Thinking		Working
(a) **1**	Round the numbers that you are dividing to convenient numbers. (Here, one number is rounded to the nearest 100, the other to the nearest 10.)	*(a)* $4587 \div 49$ $\approx 4600 \div 50$
2	Perform the division with the rounded numbers, cancelling zeros first.	$\approx 460\not{0} \div 5\not{0}$ $\approx 460 \div 5$
3	Change the non-zero digits of the larger number to the nearest multiple of the smaller number.	$\approx 450 \div 5$
4	Do the simple division and write the approximate answer.	≈ 90
(b) **1**	Round the numbers that you are multiplying to convenient numbers. (Here, one number is rounded to the nearest 10, the other to the nearest 5.)	*(b)* 197×23 $\approx 200 \times 25$
2	Do the simplified multiplication.	$\approx 2 \times 100 \times 25$ $\approx 2 \times 25 \times 100$ $\approx 50 \times 100$
3	Write the approximate answer.	≈ 5000

Overestimating and underestimating

You can use an estimate of the answer to a problem to check whether your calculated answer is 'reasonable'. It is useful to be able to judge whether your estimate is greater than the actual answer (an 'overestimate') or less than the actual answer (an 'underestimate'). To do this, you need to consider how the numbers were rounded.

Consider these examples of rounding to the first digit.

(a) 157×66
$\approx 200 \times 70$
$\approx 14\,000$

Actual answer: 10 362

(b) 218×46
$\approx 200 \times 50$
$\approx 10\,000$

Actual answer: 10 028

In **(a)**, the actual answer is much less than the estimate! This is because both numbers were rounded up by a large amount.

In **(b)**, the actual answer is very close to the estimate. This is because one number was rounded up and the other rounded down, by small amounts.

When dividing, you need to be aware of the effect of rounding up or down.

Consider these examples of rounding to the first digit.

(a) $2320 \div 9$
$\approx 2000 \div 10$
$= 200$

Actual answer: 257.8

(b) $6432 \div 33$
$\approx 6000 \div 30$
$= 200$

Actual answer: 194.9

In **(a)**, the big difference between the approximate answer and the actual answer is because the rounded calculation gave a smaller number being divided by a larger number.

In **(b)**, the approximate answer is very close to the actual answer. This is because both numbers were rounded down, giving a smaller number divided by a smaller number.

To find a reasonable approximate answer:

- when multiplying, round one number up and the other number down
- when dividing, round both numbers up or both numbers down.

1.4 Estimating and rounding

Navigator

1 (columns 1–2), 2 (columns 1–2), 3 (columns 1–2), 4, 5, 6 (a–d), 7, 8, 9, 12, 14, 15, 16	1 (columns 2–3), 2 (columns 2–3), 3 (columns 2–3), 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16	1 (column 4), 2 (column 3), 3 (column 3), 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17

Answers p. 648

Fluency

1 Round the following numbers to the first digit.

W.E. 7

(a) 24	**(b)** 56	**(c)** 75	**(d)** 99
(e) 68	**(f)** 74	**(g)** 10	**(h)** 6
(i) 619	**(j)** 650	**(k)** 483	**(l)** 970
(m) 4846	**(n)** 3723	**(o)** 9510	**(p)** 9643
(q) 75 000	**(r)** 716 599	**(s)** 800 050	**(t)** 1 801 021

2 Calculate an approximate answer to the following by first rounding each number to the first digit.

W.E. 8

(a) 27×42	**(b)** 31×50	**(c)** 95×97
(d) $89 \div 32$	**(e)** 607×85	**(f)** 48×580
(g) 681×41	**(h)** 547×84	**(i)** 62×819
(j) $2940 \div 51$	**(k)** $3199 \div 62$	**(l)** $1955 \div 78$
(m) 141×837	**(n)** 104×8946	**(o)** 7340×250
(p) 9×6511	**(q)** 250×950	**(r)** $75\,342 \div 80$
(s) $11\,397 \div 213$	**(t)** $27\,496 \div 487$	**(u)** $2547 \times 30\,995$

W.E. 9

3 Calculate an approximate answer for each of the following, by first rounding each number to a convenient multiple of 5, 10, 100 or 1000.

(a) 41×26 **(b)** 122×54 **(c)** 191×14

(d) $387 \div 43$ **(e)** $379 \div 22$ **(f)** $248 \div 52$

(g) 1587×23 **(h)** $33\,997 \times 21$ **(i)** 4036×253

(j) $3527 \div 52$ **(k)** $10\,803 \div 95$ **(l)** $44\,895 \div 14$

(m) $23\,938 \div 356$ **(n)** $15\,337 \div 2498$ **(o)** 5089×2492

4 In each case, which of these estimates do you think is the closest? Don't try to count or measure.

(a) About how many people are there in the photo?

A 700

B 4000

C 30 000

D 100 000

(b) Approximately how far is it:

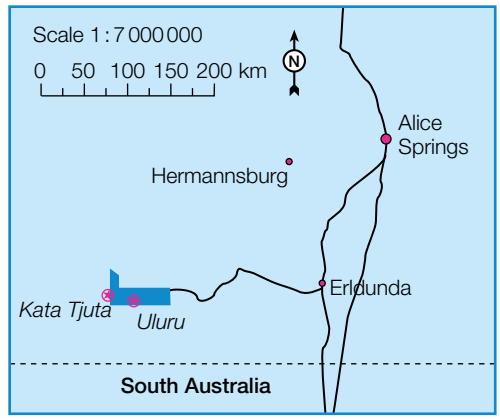
(i) from Alice Springs to the South Australia border, by the roads shown?

A 75 km

B 150 km

C 300 km

D 600 km



(ii) directly from Alice Springs to Uluru (in a straight line)?

A 40 km

B 350 km

C 789 km

D 1000 km

Understanding

5 Complete the following.

(a) Estimate the answer to 78×63 by rounding both numbers to the nearest ten.

(b) Estimate the answer to $88 \div 33$ by rounding both numbers to the nearest ten.

(c) Estimate the answer to 6378×542 by rounding both numbers to the nearest hundred.

(d) Estimate the answer to 854×76 by rounding both numbers to the nearest ten.

(e) Estimate the answer to $754 \div 52$ by rounding both numbers to the nearest ten.

(f) Estimate the answer to 63×52 by rounding both numbers to the nearest multiple of 5.

6 Estimate the following by rounding the smaller number to the first digit and the larger number to that same place value. (For example, if the smaller number is rounded to the nearest 10, then round the larger number to the nearest 10 also.)

(a) $1247 + 323$ **(b)** $10\,290 + 178$ **(c)** $14\,532 + 2788$

(d) $823 - 79$ **(e)** $1176 - 241$ **(f)** $453 - 276$

7 For each of the following, find an estimate for the answer by first rounding each number to the first digit. State whether the actual answer will be higher or lower than the estimate.

(a) 364×57 **(b)** 2398×426 **(c)** 153×16

(d) $1996 \div 24$ **(e)** $943 \div 86$ **(f)** $597 \div 136$

8 In each case, choose the best estimate from the alternatives given. Don't do the actual calculation.

(a) 321×73

A 210 **B** 2100 **C** 2163 **D** 21 000

(b) 405×950

A 360 000 **B** 400 000 **C** 450 000 **D** 3 600 000

(c) $7865 \div 24$

A 400 **B** 900 **C** 1600 **D** 210 000

(d) $999\,160 \div 527$

A 220 **B** 330 **C** 660 **D** 2000

9 In one weekend, 255 345 people attended AFL matches and paid on average $22 each to attend each game.

(a) Round the attendance to the first digit.

(b) Use your answer to **(a)**, and a rounded estimate for the ticket price, to find an estimate for the total money spent on tickets.

10 **(a)** Tahnee earns $38 295 per year. Approximately how much is this per week? Calculate by rounding to convenient numbers first. (There are 52 weeks in a year.)

(b) Catriona earns $175 per week from her part-time job. Approximately how much is this per year? Calculate by rounding to convenient numbers first.

Reasoning

11 A large mango farm in Western Australia has 120 000 trees. Each tree will produce at least 74 mangos per year.

(a) If each mango sells for $1, estimate (by rounding to the first digit) the farm's income from mango sales each year. Is your estimate an overestimate or an underestimate?

(b) The plantation covers an area of 7600 hectares. Estimate (by rounding to the nearest thousand) the number of trees on each hectare of the plantation. Is your estimate an overestimate or an underestimate?

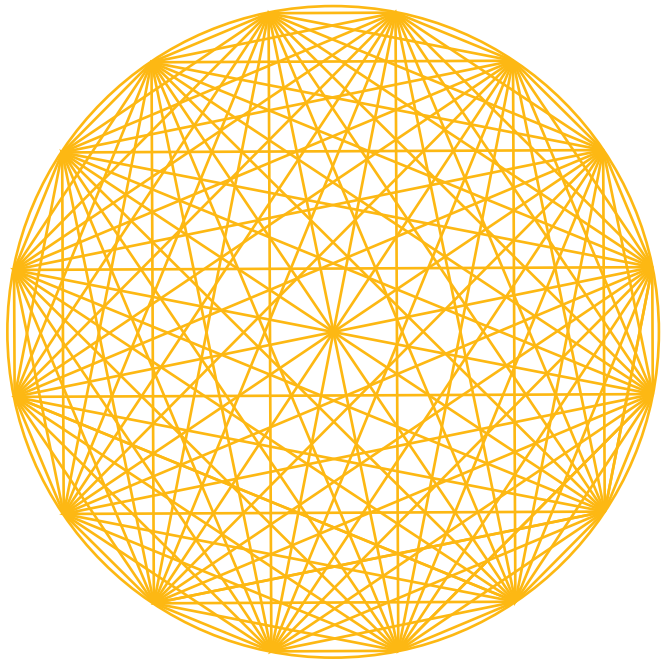
12 **(a)** Approximately how many straight lines are there in this picture?

A 100

B 500

C 5000

D 10 000



(b) Explain a way of finding the exact number of lines without counting them all.

13 79×5003 and 76×5488 both give the same approximate answer of 400 000 when each number is rounded to the first digit. Which of the actual answers to the two calculations will be closer to 400 000? Explain how you arrived at your answer.

14 **(a)** Use rounding to the first digit to find an approximate answer for $1344 \div 21$.

(b) Find an approximate answer for $1344 \div 21$ by first rounding to convenient numbers, such as multiples of 10.

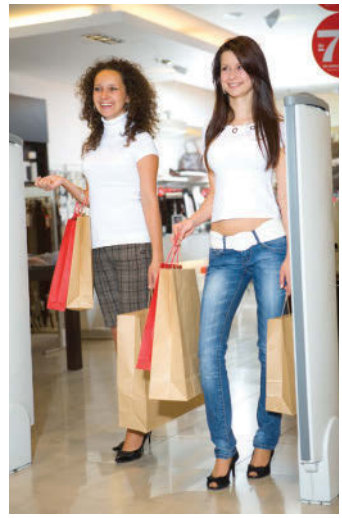
(c) Which method do you think has given a closer estimate to the actual answer? Give a reason for your prediction.

(d) Work out the actual answer and see if you were correct.

15 Ms Kelly, the assistant principal at Summerhill Secondary, is organising buses to transport students to the pool for the swimming carnival. Each bus can seat 52 people. There are 428 staff and students going (although Ms Kelly doesn't know how many might be sick or away on the day). How many buses should Ms Kelly order? Why is estimating by rounding not helpful in this situation?

Open-ended

16 Natalie is out shopping. She is heading towards the checkout when she suddenly wonders if she has enough money. She mentally rounds her purchases in her head and adds them up, like this:



T-shirt ≈ $20
Earrings ≈ $10
Necklace ≈ $10
Lip gloss ≈ $10
Total ≈ $50

'Terrific!' thinks Natalie. 'I've got exactly enough!'

The actual prices of the items are: T-shirt $17, earrings $14, necklace $13, lip gloss $9.

(a) There is nothing wrong with Natalie's rounding. Why is she in trouble when she gets to the checkout?

(b) Describe a 'safer' method for rounding money.

17 Write three pairs of numbers that can be rounded to the first digit and then multiplied together to give:

(a) 3200 **(b)** 200 **(c)** 100 000 **(d)** 42 000

Game

Estimation frustration

Equipment required: 2 dice, 1 calculator

Aim of the game:
The winner is the first person to 40 points.

How to play:

1. Roll the dice. Make a two-digit number from the numbers on the dice. (For example, rolling a 4 and a 3 means you can make 34 or 43—your choice.) Record this number.
2. Roll again to record another two-digit number.
3. Show these two numbers to your partner. They now have just 10 seconds to write an estimate of the product of these two numbers.
4. Use a calculator to work out the actual product of the numbers, and compare this to your partner's estimate.
5. Score according to the table below.

How close is the estimate?	Points
Exactly correct	20 points
Less than 100 off	10 points
Between 100 and 200 off	5 points
More than 200 off	0 points

6. Continue to play, taking turns with your partner to roll the dice and estimate the product. Finish when your score reaches 40 points.

1.5 Order of operations

Jamie, Josh and Ali all worked out the following problem: $1 + 2 \times 3 + 4$.

- Jamie's answer was 13.
- Josh's answer was 11.
- Ali's answer was 15.

Why were there different answers?

It is not useful for different people to get different answers for the same calculation. To make calculations the same for everyone, mathematicians agree on rules about the order in which to do the four operations $\times$, $+$, $-$ and $\div$. Rules that mathematicians agree to use are called **mathematical conventions**.

The **order of operations** rules:

1. Always do calculations in brackets first.
2. Next, do any calculations with indices (powers).
3. Then do multiplication and division, in order from left to right.
4. Finally do addition and subtraction, in order from left to right.

According to the order of operations rules, who had the correct answer to the problem above: Jamie, Josh or Ali?

Worked example 10

W.E. 10

Calculate the following, showing all steps of working.

(a) $24 + 6 \div 2 - 1 \times 4$ **(b)** $14 - 9 + 6^2 \div (7 + 2) \times 3$

Thinking	Working
(a) 1 There are no brackets and no indices, so do multiplications and divisions in the order in which they appear.	*(a)* $24 + 6 \div 2 - 1 \times 4$ $= 24 + 3 - 1 \times 4$ $= 24 + 3 - 4$
2 Now, do additions and subtractions in the order in which they appear.	$= 27 - 4$ $= 23$

(b) 1 Do the calculation in brackets first.

2 Evaluate any numbers in index form.

3 Do multiplications and divisions in the order in which they appear.

4 Do additions and subtractions in the order in which they appear.

(b) $14 - 9 + 6^2 \div (7 + 2) \times 3$
$= 14 - 9 + 6^2 \div 9 \times 3$
$= 14 - 9 + 36 \div 9 \times 3$
$= 14 - 9 + 4 \times 3$
$= 14 - 9 + 12$
$= 5 + 12$
$= 17$

1.5 Order of operations

Navigator

1, 2 (column 1), 3 (column 1), 4, 5, 6, 7, 8 (a–h), 9, 11, 12 (a–d), 13, 14, 15, 17, 18	1 (column 1), 2 (column 2), 3 (column 2), 4, 6, 7, 8 (column 1), 9, 11, 12 (a–f), 13, 14, 15, 17, 18	2 (e–h), 3, 6, 7, 8 (column 2), 9, 10, 11, 12, 13, 14, 15, 16, 17, 18

Answers p. 649

Fluency

W.E. 10

1 Calculate the following, showing all steps of working.

(a) $2 + 5 - 4$ **(b)** $20 \times 10 \div 4$ **(c)** $20 - 13 - 4$

(d) $20 - (13 - 4)$ **(e)** $20 \times (13 - 4)$ **(f)** $4 + 5^2$

(g) $(6 + 5 \times 3) \times 2^3$ **(h)** $(6 + 5 \times 4) \div (12 \div 6)$

2 Calculate the following, showing all steps of working.

(a) $1 + 8 \times 3$ **(b)** $6 \times 2 - 1$

(c) $25 - 2 \times 11$ **(d)** $8 - 24 \div 12 + 3$

(e) $9 \times (10 - 7) \div 3$ **(f)** $88 \div 8 - 6 \times (5 - 4)$

(g) $38 - 7 \times 2^2 + 13 - 4 \div 2$ **(h)** $23 - 5 + (17 - 2) \times 3 + 5^2$

3 Calculate the following, showing all steps of working.

(a) $8 - 5 \div 5$ **(b)** $8 - 4 \times 2$

(c) $6 \div 3 + 3 \times 5$ **(d)** $8 \times 3 \div 4 \times 2$

(e) $24 \div (7 + 5) \times 5^2$ **(f)** $12 \times 5 + 4 \times (10 - 4)$

(g) $9 - 2 + 5 + 3 \times 4 \div 6$ **(h)** $28 \div 7 \times 3^2 + (5 - 1) \div 2 + 3$

4 State true (T) or false (F) for the following.

(a) For $2 + 6 \times 4$ you would calculate $2 + 6$ first.

(b) For $6 + 12 \div 3$ you would calculate $12 \div 3$ first.

(c) For $8 + 40 \div (3 + 5) \times 10^2$ you would calculate $8 + 40$ first.

(d) For $24 + 6 \div 2 - 1 \times 4$ you would calculate $6 \div 2$ first.

(e) $4 + 12 \div 2$ simplifies to 8.

(f) $20 \div 5 - 1$ simplifies to 3.

5 (a) $8 + 6 - 4 \div 2$ is equal to:

A 5 **B** 9 **C** 12 **D** 16

(b) $12 - 8 \div 4 + 2^2$ is equal to:

A 5 **B** 6 **C** 9 **D** 14

Understanding

6 **(a)** Which calculation would you do first and which would you do second in each of these questions?

(i) $14 - 2 \times 6 + 2 \times 2$ **(ii)** $(15 \div 5) + 6 \times 3$

(b) Calculate the value of the expressions in **(a)**.

7 Complete the following.

(a) Calculate $2 + 3 \times 4 + 5$, showing all the steps of the working.

(b) By using brackets to change the order of operations, you can get different results from $2 + 3 \times 4 + 5$. Can you use brackets to find three other possible results? Show your working to find each different result.

8 Put brackets into these statements, where necessary, to make them true.

(a) $6 + 6 \times 3 = 36$ **(b)** $10 - 4 \times 5 = 30$

(c) $9 - 8 \times 6 + 4 = 10$ **(d)** $12 + 6 \div 7 - 4 = 14$

(e) $6 \div 3 + 3 \times 5 = 5$ **(f)** $3 \times 6 \div 8 - 4 + 5 = 2$

(g) $3 \times 10 - 7 \div 9 + 12 = 13$ **(h)** $18 \div 3 \times 5 - 3 + 2 = 14$

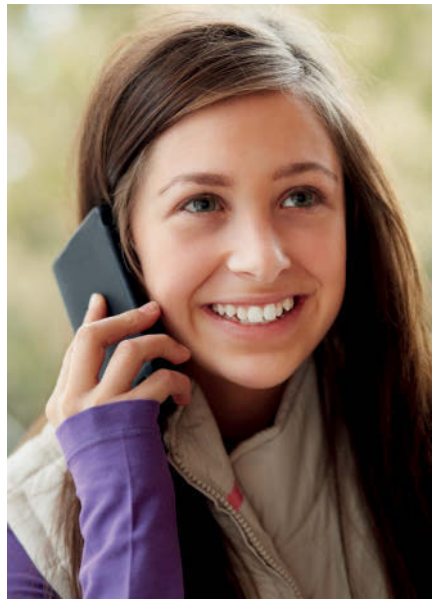
(i) $7 + 3 \div 4 + 1 = 2$ **(j)** $5 - 3 \times 8 - 6 \div 2 = 2$

9 For each taxi ride, a taxi company charges \$4 'flagfall' (an initial fee) plus \$2 for each kilometre travelled. If you take a taxi to travel 7 km to the library, another taxi for 5 km to a restaurant for lunch and then a final taxi home, 8 km away, which of the following calculations gives the total cost for the 3 taxi rides?

A $4 + 2 \times (7 + 5 + 8)$ **B** $4 + 2 \times 7 + 4 + 2 \times 5 + 4 + 2 \times 8$

C $3 \times 4 + (7 + 5 + 8) \times 2$ **D** $3 \times 2 + (7 + 5 + 8) \times 4$

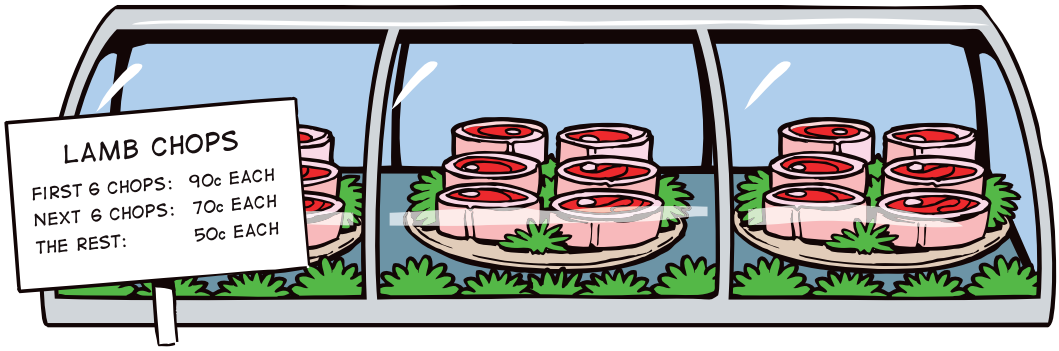
10 During 'peak' time, Shania's phone company charges 73c per minute for each voice call, plus a 29c 'flagfall' (a single amount charged as soon as the call is connected). During 'off-peak' time, there is no flagfall, and the charge is 50c per minute. Shania has \$20 credit on her phone account. She makes one 5-minute call during 'peak' time, and one 8-minute call during 'off-peak' time.



(a) Calculate the total cost of the two calls. Show your working clearly.

(b) Using the symbols +, –, × and at least one pair of brackets, write a calculation for Shania to work out how much credit remains on her phone account after the two calls are made.

11 A butcher has too many lamb chops, so the butcher decides to tempt people to buy more by putting up the following sign:



Stuart bought 8 chops, Mahalia bought 11 chops, and Sofia bought 17 chops.

The amount Stuart paid can be worked out using the following calculation:

$6 \times 90 + 2 \times 70$. This gives an answer of 680 cents, which is $6.80.

(a) Write a calculation similar to the one above to work out how much Mahalia paid, then do the calculation.

(b) Write a calculation similar to the one above to work out how much Sofia paid, then do the calculation.

(c) How much more than Stuart did Sofia pay?

12 Replace each $*$ with one of the four symbols $(+, -, \times, \div)$ to make the statement true.

(a) $2 + 21 * 3 = 9$

(b) $15 - 6 * 2 = 12$

(c) $14 - 8 * 6 = 0$

(d) $7 * 5 * 6 = 29$

(e) $14 * 3 * 2 = 15$

(f) $(24 * 6) * 10 = 3$

(g) $8 * 5 * 2 - 6 = 12$

(h) $12 * 2 + 1 * 9 = 15$

13 Replace each $*$ with either $<$, $>$ or $=$ to make the statement true.

(a) $6 \times (4 \div 2) \times 3 * (6 \times 4) \div 2 \times 3$

(b) $(1 + 4) \times 20 \div 5 * 1 + (4 \times 20) \div 5$

(c) $100 + 10 \div 10 * (100 + 10) \div 10$

(d) $36 \div 6 \times (3 - 3) * 36 \div 6 \times 3 - 3$

Reasoning

14 Remove the unnecessary brackets from these statements.

(a) $(4 \times 3) + (6 - 2) = 16$

(b) $8 + (4 \div 2) + (6 \times 2) = 22$

(c) $4 + (20 \div 5) \times (3 + 7) = 44$

(d) $6 \times (3 + 2) - (12 \div 2) = 24$

15 Sean typed the following into his calculator: $8 + 6 \div 2 + 3 \times 5$. Shazhad typed the same into the calculator app on a phone. Sean's answer was 26. Shazhad's answer was 50.

(a) Do the calculation to decide which answer was correct.

(b) How did the other calculator find a different answer?

Open-ended

16 By placing one pair of brackets in different positions, show all the possible answers for this question: $8 + 4 \times 6 \div 2 - 1$.

17 Using the digits 1, 2 and 3 exactly once (in any order), and using the operations +, −, ×, ÷ as well as indices and brackets, it is possible to write calculations with many different results. For example, $21 + 3 = 24$ and $3^{(2+1)} = 27$.

(a) Can you find a way to calculate each of the numbers 1 to 10 using only the digits 1, 2, and 3?

(b) What is the largest number that you can calculate in this way?

(c) Compare your answers to part **(a)** to other students' answers. See how many different ways you can find to calculate each of the numbers from 1 to 10.

18 Jing has worked out that the answer to $30 \div 2 \times (2 + 3)$ is 3. Here is her working:

$30 \div 2 \times (2 + 3)$
$= 30 \div 2 \times 5$ Step 1
$= 30 \div 10$ Step 2
$= 3$ Step 3

Jing's teacher has marked this as incorrect. What is the correct answer? Explain to Jing which step of the working is incorrect, what should have been done instead, and why.

Problem solving

Four 4s = 100?

1 Using the number 4 exactly four times, together with any of the four operations (+, −, ×, ÷) and brackets if you need them, see if you can make each of the numbers 0 to 9. A way to make 5 has been done for you. There is more than one way in many cases.

0 =
1 =
2 =
3 =
4 =
5 = $(4 \times 4 + 4) \div 4$
6 =
7 =
8 =
9 =

2 See how many numbers up to 100 you can find using just four 4s like this. You might like to work with a partner or in a small group.

3 Using each digit of the number 3210 exactly once, together with brackets and any of the four operations, see how many of the numbers from 1 to 10 you can make.

e.g. $(3 + 1) \times 2 + 0 = 8$

Strategy options

- Guess and check.
- Test all possible combinations.

Mixed whole number problems

1.6

Maths problems in daily life do not appear as final calculations all set out for you to do. Real maths problems come from situations that you need to understand and think about first, to decide which skills and methods to use.

Some words that are associated with the four operations are listed below. You might be able to think of some others. Identifying these words in a problem will help you to apply the right operation.

+ Sum	− Difference	× Product	÷ Quotient
add	subtract	multiply	divide
total	less than	how many times more	goes into
altogether	minus	lots of	shared between

This section contains problems that will require you to read the information carefully and decide which skills to use. Many problems require more than one step or calculation. You should use the skills and strategies covered in this chapter to help you.

1.6 Mixed whole number problems

Navigator

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 15, 16, 17, 22	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 22	2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22

Answers p. 650

Fluency

1 **(a)** The Australian cricket team made 425 runs in their first innings and 299 in their second. How many runs did they make in total for the match?

(b) A new car cost $28 500. Five years later it was worth $19 275. By how much had the value of the car decreased in that time?

(c) A piano teacher charges $27 for each lesson. How much would 13 lessons cost?

(d) Sula is paid $12 for every piece of work she does. How many pieces of work did Sula do in a week when her pay was $372?

2 Harvey 'Scoop' Roberts, a journalist with the *Monthly Farm News*, can type 60 words a minute. How long does it take him to type an article of 1800 words?

3 Wendy is training to be an Olympic swimmer. Every morning she swims 3600 m in a 50 m pool. How many laps is that? (1 lap is 1 length of the pool.)

4 A leap year contains 52 weeks and 2 days. How many days are in a leap year?

5 A student earns $8 for every homemade shirt that she can sell. She sells 3 shirts on Monday, 4 shirts on Thursday, 5 shirts on Friday and 3 shirts on Saturday morning.

(a) How many shirts does she sell this week? Can you see a way to check the total using the 'make easy numbers' strategy?

(b) How much money does she earn this week?

Understanding

6 Little Lucy is exactly 5 weeks' old. How many minutes old is she?

7 The highest mountain in the world, measured from sea level, is the Himalayan peak of Mount Everest. It is 8848 m above sea level. If we measure mountains that start under the ocean, the tallest mountain in the world from base to tip is Mauna Kea on the island of Hawaii. It measures 10 203 m, of which 4205 m is above sea level.

(a) How much of Mauna Kea is below sea level?

(b) If we don't count the part of Mauna Kea that is under water, how much higher is Mount Everest?

8 Cathy is out shopping for bargains. She bought two pairs of $68 shoes in a store that said 'buy one pair, get the second pair half price'. She bought two $19 T-shirts where the special deal was 'buy one, get $10 off the second'. Cathy also bought a pair of earrings for $25. What was the total cost of her shopping spree?

9 Mick is a baker in a supermarket. He makes hot cross buns and packs them in bags of 6. He stacks 4 shelves with 8 bags on each shelf. At the end of the day, Mike has 3 bags of buns left. How many hot cross buns did he sell?

10 The two longest rivers in the world are the Amazon (6448 km) and the Nile (6670 km). The longest river in Australia is the Darling (2739 km).

(a) How much longer is the Nile than the Amazon?

(b) How much longer is the Nile than the Darling?

(c) How much longer is the Amazon than the Darling?

11 The width of a painting including the frame is 85 cm. If the frame is 6 cm wide all the way around, what is the width of the unframed painting?

12 The Pizza Pit-Stop employs five people. The two cooks work 36 hours each per week for $18 an hour. The three waiters work 30 hours each per week for $15 an hour. What does the Pizza Pit-Stop pay its five employees in total per week?

13 Complete the following.

(a) The speed of radio signals through empty space is approximately 299 792 kilometres per second. The planet Mars is sometimes as close as 55 million km to Earth. Estimate how much time it takes, in seconds, for a radio signal to reach Mars from Earth.

(b) Radio waves are a form of light, which always travels at the same speed. Astronomers often use the distance that light travels in one year as a unit of distance, called a 'light year'. A year is 365 days, each day has 24 hours and each hour has 60×60 seconds. Write the calculation for the number of kilometres in a light year, then round this to the first digit to calculate your answer.

14 Complete the following.

(a) One billion dollars is 1000 million dollars. What is one billion dollars when written as a power of ten?

(b) One trillion dollars is 1000 billion dollars. What is one trillion dollars when written as a power of ten?

Reasoning

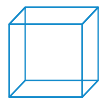
15 A dozen eggs is 12 eggs. However, bakeries sometimes sell a 'baker's dozen' of bread, which is 13 bread rolls at a special cheaper price.

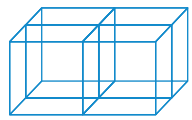
(a) You plan to make egg rolls for the school breakfast program. Each day you buy 28 dozen eggs and 27 baker's dozen bread rolls. Bread rolls cost $4 for each baker's dozen, while eggs cost $3 per dozen. How much money will you spend each day on these ingredients?

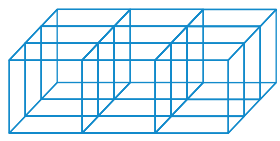
(b) Each egg roll serving uses 1 egg and 1 bread roll. With 28 dozen eggs and 27 baker's dozens of bread rolls, how many servings can you make each day? What ingredients, if any, will be left over? Can you make the same number of egg rolls, but reduce expenses by buying less ingredients? If so, how?

16 Sam has 28 model planes on a display shelf. Mark has twice Sam's number of model planes. Harvey has half as many planes as Sam. What is the difference between the number of model planes that Harvey and Mark have?

17 A student is playing with some blocks called 'flat squares', which are each 1 cm thick but their length × width dimensions vary. A 1×1 flat square has dimensions 1 cm × 1 cm × 1 cm, so is made of 1 centimetre cube. A 2×2 flat square has dimensions 1 cm × 2 cm × 2 cm, so it is made of 4 centimetre cubes. A 3×3 flat square has dimensions 1 cm × 3 cm × 3 cm, so it is made of 9 centimetre cubes, and so on.







The student realises that the flat squares can be piled up to make interesting pyramid-like shapes. She decides to investigate the number of different flat squares needed to make different pyramids, so she creates the table below for her results.

She calls the number of centimetre cubes the 'pyramid numbers', so 5 is the second pyramid number, 14 is the third pyramid number and 30 is the fourth pyramid number.

Pyramid height	Number of cm cubes
1	$1^2 = 1$
2	$1^2 + 2^2 = 1 + 4 = 5$
3	$1^2 + 2^2 + 3^2 = 1 + 4 + 9 = 14$
4	$1^2 + 2^2 + 3^2 + 4^2 = 1 + 4 + 9 + 16 = 30$

(a) What is the fifth pyramid number?

(b) Write the first ten square numbers. Can you show how to use a 'make easy numbers' strategy to find the total quickly and accurately? From this, find the tenth pyramid number.

18 Hiram is moving house. He has 13 boxes of possessions, but can only fit 4 boxes at a time in his car.

(a) Beginning the day at his old house and ending it at his new house, how many times will Hiram need to travel between the two houses to move all his boxes?

(b) If the distance between the two houses is 7 km, how far will he travel?

19 Jules Verne wrote about travelling around the world in 80 days.

(a) Write 80 days as a number of weeks plus a number of days.

(b) How many months, weeks and days is this, if you assume there are 30 days in a month?

(c) How many months, weeks and days is this, if you assume there are 4 weeks in a month?

(d) Explain why your answers to **(b)** and **(c)** are different.

20 Weighing Jason's pet dog Scruffy was a problem, as Scruffy kept jumping off the scales. The vet held Scruffy and stood on the scales. Then, Jason held Scruffy and stood on the scales. Finally, the vet, Jason and Scruffy all stood on the scales.

The scales read as follows:
The vet and Scruffy = 91 kg
Jason and Scruffy = 56 kg
The vet, Jason and Scruffy = 138 kg

Work out how much Scruffy weighs.

Open-ended

21 **(a)** Write two numbers that have a sum greater than 90 but a product less than 1800.

(b) Write two numbers that have a difference of less than 10 but have a sum greater than one-quarter of their product.

22 Simon owns a company that produces fruit juice. The juice comes in three different-sized bottles: 300 mL, 600 mL and 1000 mL (1 L), weighing 300 g, 600 g and 1000 g (1 kg) respectively. Simon packs same-size bottles into boxes of 20, and delivers them by truck to shops. Simon's truck can hold a maximum load of 1000 kg.

(a) For each of the different-sized bottles, write the mass in kg of a box of 20 bottles. (Remember, 1000 g = 1 kg.)

(b) How many boxes of each type of juice can Simon get on the truck to make a load of 1000 kg?

(c) Find at least two different combinations of boxes that would make a load of 1000 kg.

Problem solving

Nine hundred and ninety-nine

Copy the following equation with boxes into your workbook.

$$\square\square\square + \square\square\square = 999$$

Using each of the digits from 1 to 9 once in each of the boxes, create three three-digit numbers whose sum is 999.

(It may help to write the digits 1 to 9 on small pieces of paper, so you can move them around the boxes easily.)

How many different combinations can you find?

Aim to find at least four combinations of different three-digit numbers.

Strategy options

- Test all possible combinations.
- Break problem into manageable parts.

Challenge 1

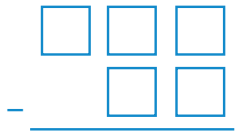
1 The digits 5, 6, 7, 8 and 9 can be arranged to form five-digit even numbers. The tens digit in the smallest of these numbers is:

A 6 **B** 7 **C** 8 **D** 9

2 How many different numbers can you make by multiplying two one-digit even numbers?

A 6 **B** 8 **C** 9 **D** 10

3 Each of the digits 3, 5, 6, 7 and 8 is placed in a box in the diagram opposite. If the two-digit number is subtracted from the three-digit number, the smallest possible difference is:



A 261 **B** 269 **C** 271 **D** 278

4 Aria and Jackson are given savings accounts by their grandmother, with the same amount of money in each. Aria is a good saver and puts an extra $5 into her account each week. Jackson takes $1 out of his account every week. After 10 weeks, Aria has 3 times as much in her account as Jackson has in his account. What was the starting amount in each account?

5 The product of the ages of two teenagers and their father is 15 181. How old is the father?

6 Look at the following number arrangement.

1

3 5

7 9 11

13 15 … …

What will be the sum of the seventh row? Look for a pattern in the row totals. What will be the sum of the one-hundredth row?

7 How many square numbers are there between 8 and 8008?

8 Find the next three terms in this number pattern 4, 7, 11, 18, 29, …

9 The counting numbers are arranged in six columns as shown. In which column will 1001 appear?

A	B	C	D	E	F
1	2	3	4	5	6
12	11	10	9	8	7
13	14	15	16	17	18
…	…	…	…	20	19

10 **(a)** Write the values of 0^2, 1^2, 2^2, 3^2, … , 9^2.

(b) Consider the last digit of each of the values found in **(a)**. What are the only digits that a square number can end in?

(c) What digits can't a perfect square end in?

(d) Which of the following numbers can't be perfect squares?

A 327 **B** 324 **C** 343 **D** 289 **E** 732

(e) Without using a calculator, write 961 as a perfect square by first deciding what the square root could end in, then thinking about the factors of 900.

Gamespace

The operation theatre

There are many weird and interesting facts about the human body. Calculate the following expressions using the correct order of operations to discover some of these facts.

1 The number of muscles it takes to produce human speech.

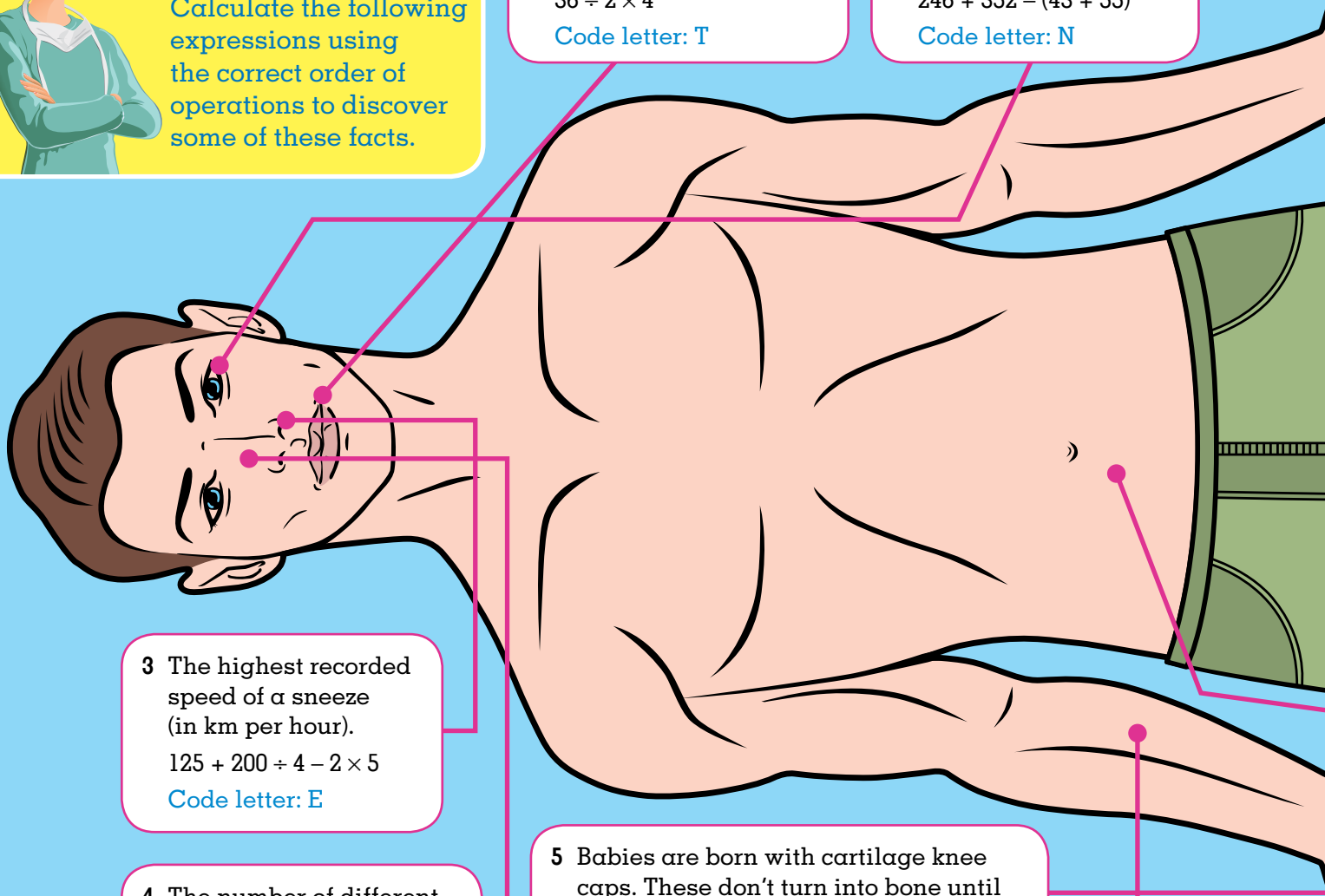
$36 \div 2 \times 4$

Code letter: T

2 The number of shades of grey that a human eye can distinguish.

$246 + 352 - (43 + 55)$

Code letter: N



3 The highest recorded speed of a sneeze (in km per hour).

$125 + 200 \div 4 - 2 \times 5$

Code letter: E

4 The number of different scents your nose can detect.

$(6000 + 1500 \div 5) - 43 \times 100$

Code letter: W

5 Babies are born with cartilage knee caps. These don't turn into bone until the child is between which two ages?

(a) $3 \times 4 \div 12 \times 2$ Code letter: Y

(b) $30 - 6 \times 8 \div 12 \times 6$ Code letter: B

6 **(a)** The number of bones you are born with.

$200 - 50 + (30 - 1) \times 5 + 5$ Code letter: H

(b) The number of bones an adult has.

$618 \div (56 - 47) \times 3$ Code letter: G

7 The average human will shed approximately how many billion flakes of skin a year?

$30 - 7 \times 3 + 10 - 36 \div 4$

Code letter: I

8 The number of million blood cells destroyed in the human body every few seconds.

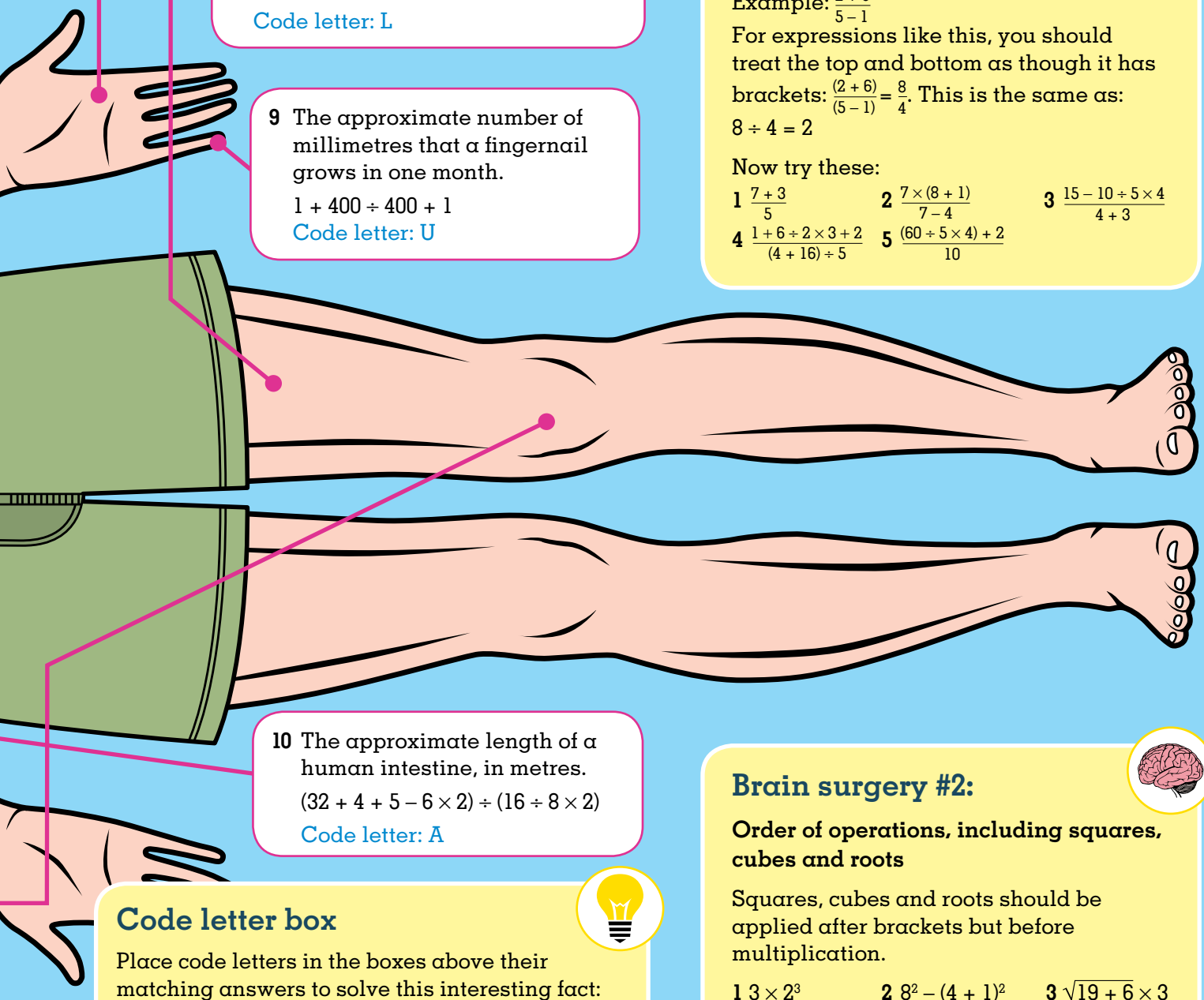
$90 \div (7 + 11) \times 3$

Code letter: L

9 The approximate number of millimetres that a fingernail grows in one month.

$1 + 400 \div 400 + 1$

Code letter: U



10 The approximate length of a human intestine, in metres.

$(32 + 4 + 5 - 6 \times 2) \div (16 \div 8 \times 2)$

Code letter: A

Code letter box

Place code letters in the boxes above their matching answers to solve this interesting fact: Your brain generates approximately

☐	☐	☐	☐	☐	☐	watts
72	2000	165	500	72	2	

of power while you're awake—enough to illuminate

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
7	15	10	206	300	72	6	3	15	6

WARNING: Advanced order of operations

Brain surgeons need to be smart, so see if you can calculate these trickier expressions.

Brain surgery #1:

Order of operations using a fraction bar

Example: $\frac{2+6}{5-1}$

For expressions like this, you should treat the top and bottom as though it has brackets: $\frac{(2+6)}{(5-1)} = \frac{8}{4}$. This is the same as:

$8 \div 4 = 2$

Now try these:

1 $\frac{7+3}{5}$ **2** $\frac{7 \times (8+1)}{7-4}$ **3** $\frac{15 - 10 \div 5 \times 4}{4+3}$

4 $\frac{1+6 \div 2 \times 3+2}{(4+16) \div 5}$ **5** $\frac{(60 \div 5 \times 4)+2}{10}$

Brain surgery #2:

Order of operations, including squares, cubes and roots

Squares, cubes and roots should be applied after brackets but before multiplication.

1 3×2^3 **2** $8^2 - (4+1)^2$ **3** $\sqrt{19+6} \times 3$

4 $\frac{4+3^2 \div 3}{\sqrt{16}+3}$ **5** $\left(\frac{3 \times (8 \div 2 \times 4)}{\sqrt{64}}\right)^2$

Chapter review 1

Maths literacy

associative law	divisor	mathematical conventions	quotient
base	estimate	order of operations	remainder
commutative law	expanded form	perfect cubes	rounding
cube root	index	perfect squares	square root
distributive law	index form	power	
dividend	indices	product	

Copy and complete the following using the words and phrases from this list, where appropriate. A word or phrase may be used more than once.

1 For the number 7^3, 7 is the _____ and 3 is the _____, or _____.

2 1, 9 and 25 are all examples of _____. 1, 8 and 27 are all examples of _____.

3 The _____ of 8 is 2.

4 In the statement $97 \div 3 = 32$ rem 1, 97 is the _____, 3 is the _____, 32 is the _____, and 1 is the _____.

5 One strategy for making multiplication easier is to use the _____; for example, $6 \times 17 = 6 \times (10 + 7)$.

To find the value of a statement such as $(9 + 5) \times 3 - 2$, you would use the _____.

6 The 'long way' of writing a number in index form is in _____.

7 The _____ of 16 is 4, because $4 \times 4 = 16$.

8 $7 \times 5 = 5 \times 7$ is an example of the _____.

9 When two or more numbers are multiplied together, the answer is called the _____.

10 You can obtain an _____ of the answer to a calculation by _____ to the first digit.

Fluency

1 Use an appropriate mental strategy to help calculate each of the following. **1.1**

(a) $89 + 53$ (b) $71 + 67$ (c) $93 + 125 + 7$ (d) $5 \times 12 \times 2$

(e) 21×15 (f) 19×23 (g) $4 \times 9 \times 5$ (h) 32×17

2 Find the value of the following. **1.2**

(a) 12^2 (b) 5^3 (c) 10^5

(d) $\sqrt{64}$ (e) $\sqrt{900}$ (f) $\sqrt[3]{27}$

(g) $5^3 + 2^2$ (h) $8^4 - 3^5$ (i) $2^3 \times 3^2$

3 Write each of the following numbers in index form. **1.2**

(a) $7 \times 7 \times 7 \times 7 \times 7$ (b) ten cubed

(c) five squared (d) twelve to the power of eight

4 Calculate the following using any suitable strategy. **1.3**

(a) 35×40 **(b)** 850×200 **(c)** 757×7 **(d)** 98×31

(e) $4400 \div 200$ **(f)** 27×59 **(g)** $612 \div 4$ **(h)** $756 \div 7$

5 Use rounding to the first digit to find approximate answers to the following. **1.4**

(a) 35×241 **(b)** 1763×27 **(c)** $3299 \div 64$ **(d)** $57\,008 \div 192$

(e) 159×286 **(f)** 3427×963 **(g)** $4398 \div 487$ **(h)** $86\,005 \div 334$

6 Find approximate answers to the following by first rounding each number to a convenient multiple of 10 or 100. **1.4**

(a) $554 \div 7$ **(b)** $2136 \div 19$ **(c)** $15\,624 \div 216$ **(d)** 28×604

(e) 157×419 **(f)** 523×1998 **(g)** $54\,327 \div 915$ **(h)** $12\,794 \div 183$

7 Calculate: **1.5**

(a) $9 \div (2 + 1) - 2$ **(b)** $(3 \times 8) \div 4 + 7$ **(c)** $12 - 6 \times 2 + 11$

(d) $7 + 12 \div 4 - 1 \times 2^2$ **(e)** $(13 - 5 \times 2) + (20 \div 10)$ **(f)** $[5 \times (9 + 1)] - 3^2$

8 Misha pays $112 for two T-shirts and a pair of jeans. If the T-shirts were $23 each, how much did the jeans cost? **1.6**

Understanding

9 Cartons of milk are packed into boxes that are then stacked into larger crates. If there are 20 cartons in a box, and 9 boxes in a crate, use mental strategies to calculate how many cartons of milk are delivered to a supermarket that ordered 4 crates. **1.1**

10 Use rounding to the first digit to estimate the answers to the following, then state whether the actual answer will be higher, lower, or close to your estimate. **1.4**

(a) 3741×22 **(b)** 265×341 **(c)** $25\,736 \div 49$ **(d)** $96\,001 \div 17$

11 Sasha runs a small free-range egg farm. On Tuesday, she collected 283 eggs from her hens. **1.3**

(a) How many cartons will she be able to fill if each carton can hold 12 eggs?

(b) How many eggs will she have left over?

12 In the calculation of $2 \times (30 \div (4 - 1)) + 6$ the first operation to do is: **1.5**

A $+$ **B** $-$ **C** $\times$ **D** $\div$

13 Put brackets into these statements, where necessary, to make them true. **1.5**

(a) $4 \times 2 + 3 \div 5 - 1 = 3$ **(b)** $5 + 1 \div 6 + 4 + 2 = 7$

14 Melissa's salary is $57 935 per year. **1.4**

(a) Approximately how much is this per week? Calculate by rounding to the first digit first.

(b) Consider how you rounded the numbers in **(a)**. Will Melissa's actual weekly salary be more or less than the approximate answer you calculated?

15 Which two consecutive numbers are the following between? **1.2**

(Hint: Consider the perfect squares on either side of the number.)

(a) $\sqrt{12}$ **(b)** $\sqrt{30}$ **(c)** $\sqrt{88}$ **(d)** $\sqrt{150}$

16 The local restaurant pays its casual employees $25 per hour. The following table shows the number of hours worked by some of the casual staff. Work out the pay for each of these workers and the total of the wages bill. **1.6**

Employee	Number of hours worked
Jessica	12
Amy	11
Aaron	9
Nikkita	6
Josef	14

17 Replace each * with one of the four operations (+, –, ×, ÷) to make the statement true. **1.5**

(a) 9 * 7 * 3 = 30 **(b)** 16 * 4 × 2 * 2 = 12

18 As Eli drove into a carpark, he noticed that the cost of parking was $6 for the first hour, then $3 for every hour after that. Eli's car was parked for 6 hours. **1.6**

(a) Write a calculation that Eli could use to work out how much his parking has cost him.

(b) As Eli drives out of the carpark, he gives the attendant $40. Use your answer to part **(a)** to calculate how much change he will receive.

19 Write these numbers in expanded form and then work out the answer. **1.2**

(a) $5^3 + 2^2$ **(b)** $8^4 - 3^5$ **(c)** $(3^2 - 2^3) \times 6^2$

20 **(a)** Complete this sequence of the powers of 2: **1.2**

$2^1 = 2$ $2^4 =$ _____

$2^2 = 4$ $2^5 =$ _____

$2^3 = 8$ $2^6 =$ _____

(b) Will 2^{10} be larger or smaller than 1000? Write the difference.

21 Write these numbers in ascending order: 2^4, $\sqrt{121}$, 10^2, 3^3, 4, $\sqrt{81}$ **1.2**

Reasoning

22 Michelle has $17 in her purse. Michelle's dad empties out the 'loose change' jar and divides the money equally between his 3 children. Michelle now has $24 in her purse. How much loose change was in the jar? **1.6**

23 The train to Suntown has broken down and 224 passengers are stranded. Carly, the train controller, needs to order replacement buses. Each bus can seat 42 people. How many buses should Carly order? Explain why rounding down to the first digit is not useful in this situation. **1.4**

24 How many whole numbers have the value of their square root between 4 and 5? **1.2**

25 Hamish has a collection of 12 alien figurines. His friend Ben has half this number. Hamish's brother Sean has three times the number that Ben has. The difference between the number of aliens in Sean's collection and the number in Hamish's is: **1.6**

A 6 **B** 12 **C** 18 **D** 24

26 Explain why 3^2 is not equal to 3×2. **1.2**

27 Will the actual answer to 57×196 be greater or less than the approximate answer from rounding to the first digit? Explain. **1.3**

Numeracy practice 1

Non-calculator

1 Another way of writing 7^3 is:

A $7 + 3$ **B** 7×3

C $7 \times 7 \times 7$ **D** $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

2 What is the best way to estimate the total cost of the items below?
skateboard = \$87, knee pads = \$31, helmet = \$54

A \$80 + \$30 + \$50

B \$80 + \$30 + \$60

C \$90 + \$30 + \$50

D \$90 + \$40 + \$60

3 The correct answer to $18 + 12 \div (5 + 1)$ is:

A 5 **B** 6 **C** 7 **D** 20

4 The answer to 24×379 would be closest to:

A 1000 **B** 2000 **C** 5000 **D** 10 000

5 Which expression has the same value as 13×5?

A $10 + 3 + 5$ **B** $10 \times 3 + 5$ **C** $10 \times 5 + 3$ **D** $10 \times 5 + 15$

Calculator allowed

6 'Mini-Munch' chocolate bars can be bought individually for 85c each or in packs of 6 for \$5. You need to buy 34 Mini-Munches for a party. What is the least amount you can pay?

7 Tom is stacking a drinks fridge in the supermarket with cans of cola. The fridge has 3 shelves. On each shelf Tom can place 5 rows of 12 cans. How many cans will fit in the fridge?

8 The area of New Zealand is 268 676 square kilometres. What is this area rounded to the nearest thousand square kilometres?

A 268 700 **B** 268 000 **C** 269 000 **D** 270 000

9 The government of Victoria asked everyone in the state to reduce their water usage to less than 155 L per person per day. Which of the following families did not achieve this target?

Family name	Number of people in the family	Total water used by the family in 1 day
Bennett	5	760 L
Farah	4	632 L
Nicolai	6	918 L
Zhang	3	459 L


2

Integers

2

A universal language?

Mathematics might be used to communicate with aliens. How would this work?

How could we use maths to discover other forms of intelligent life in the universe? Our number system is based on tens (mainly because we have ten fingers), but we cannot assume that an alien number system would be the same.

It is believed that the best way to send a universal message might be to use prime numbers. Prime numbers, such as 2, 3, 5 and 7, have only two factors: 1 and the number itself. This property means that prime numbers will be the same in any number system.

In 1974, the Arecibo telescope in Puerto Rico broadcast a message into a star cluster 21 000 light years away. The message consisted of 1679 'bits' of data, which can be arranged into 73 lines of 23 characters (73 and 23 are prime numbers). No answer has been detected yet; this is not surprising given the distance it will have to travel. Later in this chapter you can learn about another way prime numbers are used to send information.

Forum

If you had the opportunity to send the first message to an alien species, what would you say?

Our number system is based on multiplying and dividing by 10. However, sometimes we count by 2, 12, 60, 360 and 365. What do we count using these numbers?

Why learn this?

Understanding relationships between numbers allows you to work with them confidently and efficiently, often without the need for a calculator. A knowledge of factors, multiples and prime numbers is a good foundation for study of many other areas of mathematics. Negative numbers are another important set of numbers,used for example when working with temperatures, elevations, score differences and money.

After completing this chapter you will be able to:

- find the lowest common multiple of a group of numbers
- find the highest common factor of a group of numbers
- use divisibility tests to assist in finding factors
- identify prime and composite numbers
- find the prime factors of a number
- use positive and negative numbers (integers) to represent quantities
- compare and order integers
- add and subtract integers.

Recall 2

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

1 Copy and complete these within 3 minutes.

(a)	$6 \times 7 =$	$6 \times 6 =$	$6 \times 4 =$	$6 \times 11 =$	$6 \times 8 =$
(b)	$7 \times 11 =$	$7 \times 7 =$	$7 \times 5 =$	$7 \times 2 =$	$7 \times 3 =$
(c)	$8 \times 7 =$	$8 \times 6 =$	$8 \times 4 =$	$8 \times 10 =$	$8 \times 8 =$
(d)	$9 \times 12 =$	$9 \times 3 =$	$9 \times 5 =$	$9 \times 11 =$	$9 \times 8 =$
(e)	$12 \times 7 =$	$12 \times 6 =$	$12 \times 12 =$	$12 \times 9 =$	$12 \times 11 =$

2 **(a)** List all the digits that an even number can end with.

(b) List all the digits that an odd number can end with.

3 Copy and complete each of the following by writing a < (less than) or > (greater than) symbol between the given values.

(a) 10 _____ 7 **(b)** 3 _____ 6 **(c)** 2 _____ 0 **(d)** 0 _____ 5

4 Calculate:

(a) $3 + 8 + 12$ **(b)** $22 + 19 - 7$ **(c)** $22 - 9 + 87 - 35$

(d) $18 - 9 - 4$ **(e)** $72 - 39 + 14$ **(f)** $51 + 43 - 11 - 7$

5 Write the following temperatures in order from coldest to warmest.

(a) 15 °C, 7 °C, 0 °C, -4 °C, 21 °C, -11 °C

(b) 5 °C, -3 °C, 10 °C, -25 °C, 32 °C, -14 °C

6 Write the following in expanded form, then evaluate.

(a) 7^2 **(b)** 3^4 **(c)** 2^6 **(d)** 1^9

7 Calculate the following.

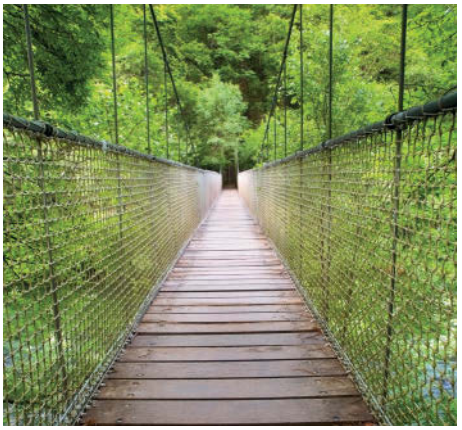
(a) $3^2 \times 5^2$ **(b)** $4^3 \div 2^3$ **(c)** $8^2 + 6^2$ **(d)** $9^2 - 7^2$

Exploration Task

You can download this activity from the eBook or the Pearson Places website.

Is adding always more? Is subtracting always less?

In this activity, you will explore the relationship between the addition and subtraction of negative numbers and the direction of the number line.



Multiples, factors and divisibility

2.1

Multiples and factors

The numbers 1, 2, 3, 4, 5, … are often called whole numbers or counting numbers. (When … is written in mathematics, this means that the pattern goes on forever.)

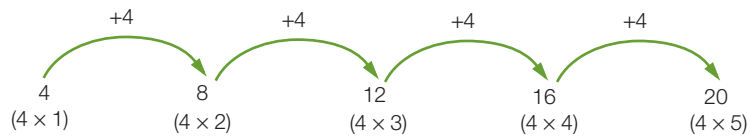
You can find the **multiples** of a whole number by multiplying it by another whole number.

For example, the multiples of 7 are:

	1×7	2×7	3×7	4×7	5×7	…
Multiples of 7	7	14	21	28	35	…

Another way to create a list of multiples of a number is to start with the number and add it repeatedly.

For example, the multiples of 4 are:



The first in the sequence of multiples of a number is always the number itself. You can see from the above table and sequence that the first multiple of 7 is 7 (1×7), and the first multiple of 4 is 4 (1×4).

A **factor** is a number that divides exactly into another number.

'Exactly' means that there is no remainder left after the division.

You can think of the process of finding factors as the reverse of finding multiples.

By reversing (flipping) the above table, you can see some factors:

	7	14	21	28	35	…
Some factors	1, 7	2, 7	3, 7	4, 7	5,7	…

This means that the factors of 7 are 1 and 7, some factors of 14 are 2 and 7 etc.

It is often important to find *all* the factors that a number has. You can see from the table that 28 has factors of 4 and 7, because 4 and 7 multiply to give 28.

However, 28 has other factors as well:

$28 = 4 \times 7$
and $28 = 2 \times 14$
and $28 = 1 \times 28$

So, 28 has six factors: 1, 2, 4, 7, 14 and 28.

Worked example 1 W.E. 1

Find all the factors of each of the following numbers.

(a) 12 **(b)** 110

Thinking		Working
(a) **1**	Write the pairs of numbers that multiply to give the original number. The number will always be divisible by 1, so write 1 × original number as the first pair, then consider whether there are pairs beginning with 2, 3 etc.	(a) $1 \times 12 = 12$ $2 \times 6 = 12$ $3 \times 4 = 12$
2	List the factors from smallest to largest.	Factors of 12: 1, 2, 3, 4, 6, 12.
(b) **1**	Write the pairs of numbers that multiply to give the original number. The number will always be divisible by 1, so write 1 × original number as the first pair, then consider whether there are pairs beginning with 2, 3 etc.	(b) $1 \times 110 = 110$ $2 \times 55 = 110$ $5 \times 22 = 110$ $10 \times 11 = 110$
2	List the factors from smallest to largest.	Factors of 110: 1, 2, 5, 10, 11, 22, 55, 110.

Sometimes, two of the same factor are multiplied to give the original number. For example, $7 \times 7 = 49$. You include 7 only once in the list of factors for 49.

Divisibility

Another way of considering factors and multiples is to talk about divisibility. A larger number is **divisible** by a smaller number if dividing by the smaller number gives an exact whole number answer with no remainder. The following sentences all describe the same idea.

- Two factors of 35 are 5 and 7.
- 35 is divisible by 5 and 7.
- Both 5 and 7 go into 35 exactly (without any remainder).
- 5 multiplied by 7 gives 35.
- 35 is a multiple of 5 and also a multiple of 7.

A good knowledge of factors and multiples will help you determine which numbers are divisible by others. For larger numbers, you can use tests to determine whether one number is divisible by another. These tests are summarised in the following table.

A number is divisible by …	… if it passes this divisibility test
2	The number is an even number (ends in 0, 2, 4, 6 or 8).
3	The sum of the digits is divisible by 3.
4	The number formed by the last two digits is divisible by 4.
5	The last digit is 0 or 5.
6	The number is even (divisible by 2) and also divisible by 3.
8	The number formed by the last 3 digits is divisible by 8.
9	The sum of the digits is divisible by 9.
10	The last digit is 0.

Worked example 2

W.E. 2

Using the divisibility tests, determine which of the numbers 75, 98, 110 and 132 are divisible by each of the following.

(a) 3 **(b)** 4 **(c)** 5 **(d)** 6

Thinking	Working
(a) 1 To test if a number is divisible by 3, add the digits in each of the numbers. If the sum of the digits is divisible by 3, the number is divisible by 3.	(a) 75: 7 + 5 = 12 ✓ 98: 9 + 8 = 17 ✗ 110: 1 + 1 + 0 = 2 ✗ 132: 1 + 3 + 2 = 6 ✓
2 State the answer for each number.	75 and 132 are divisible by 3. 98 and 110 are not divisible by 3.
(b) 1 To test if a number is divisible by 4, look at the number formed by the last two digits. If that number is divisible by 4, then the whole number is divisible by 4.	(b) 75 ✗ 98 ✗ 110 ✗ 132 ✓
2 State the answer for each number.	132 is divisible by 4. 75, 98 and 110 are not divisible by 4.
(c) 1 To test if a number is divisible by 5, is the last digit 5 or 0?	(c) 75 ✓ 98 ✗ 110 ✓ 132 ✗
2 State the answer for each number.	75 and 110 are divisible by 5. 98 and 132 are not divisible by 5.
(d) 1 To test if a number is divisible by 6, perform the tests for divisibility of 2 and 3. Write the even numbers (numbers divisible by 2). Add the digits in each of these numbers and see whether the number is divisible by 3.	(d) Using the working from (a): 98: 17 ✗ 110: 2 ✗ 132: 6 ✓
2 State the answer for each number.	132 is divisible by 6. 75, 98 and 110 are not divisible by 6.

Multiples of a whole number are found by multiplying it by another whole number.

A factor is a number that divides exactly into another number.

Divisibility tests can help find the factors of a whole number.

Common multiples

A **common multiple** of two numbers is a number that both of them divide into exactly. (In other words, the multiple is 'common' to both of them.) Changing the multiple table from the start of the section slightly, you get:

	1 and 7	2 and 7	3 and 7	4 and 7	5 and 7	...
A common multiple	7	14	21	28	35	...

This table only gives one common multiple for each pair of numbers. There are an infinite number of others. The **lowest common multiple (LCM)** of two numbers is the *smallest* number that both of the numbers divide into exactly. The common multiples of 2 and 7 are 14, 28, 42, 56,… so the LCM of 2 and 7 is 14. Because numbers can always be higher, there is no such thing as a highest common multiple.

Worked example 3 W.E. 3

Find the lowest common multiple (LCM) of the following numbers, by first listing the multiples of each: 4 and 6.

Thinking	Working
1 List the first few multiples of the first number.	4: 4, 8, (12), 16, 20, 24, …
2 List the first few multiples of the second number.	6: 6, (12), 18, 24, 30, 36, …
3 Circle the first number that appears in both lists. This is the LCM.	LCM of 4 and 6 is 12.

Common factors

A **common factor** of two numbers is a number that divides exactly into both of them. Common factors should not be confused with common multiples. Consider the following.

	7 and 14	4 and 20	9 and 15	8 and 40	12 and 18
Common factors	1, 7	1, 2, 4	1, 3	1, 2, 4, 8	1, 2, 3, 6

The number 1 will always be a common factor of any set of numbers.

It is often useful to find the **highest common factor (HCF)** of a pair of numbers. From the above table, you can see that the HCF of 7 and 14 is 7, the HCF of 9 and 15 is 3, the HCF of 12 and 18 is 6 etc.

If the smaller number in the pair is a factor of the larger number, then the smaller number is the HCF. For example, the HCF of 4 and 20 is 4. The HCF of 8 and 40 is 8. The HCF of a pair of numbers cannot be bigger than the smaller number of the pair.

Worked example 4 W.E. 4

Find the highest common factor (HCF) of the following pairs of numbers, by first listing the factors of each number: 12 and 18.

Thinking	Working
1 List all factors of the first number.	12: (1), (2), (3), 4, (6), 12
List all factors of the second number.	18: (1), (2), (3), (6), 9, 18
2 Circle the factors appearing in both lists. These are the common factors.	
3 Select the largest number that appears in both lists. This is the HCF.	HCF of 12 and 18 is 6.

The lowest common multiple (LCM) of two numbers is the smallest number that both of the numbers divide into exactly.

The highest common factor (HCF) of two numbers is the largest number that divides exactly into both of the numbers. The highest common factor is sometimes also called the greatest common divisor (GCD).

2.1 Multiples, factors and divisibility

Navigator

1 (columns 1–2), 2, 3, 4 (columns 1–2), 5 (a–h), 6, 7, 8, 9, 10, 11, 12, 13, 14 (a), 15, 16, 17, 19 (a–b), 21, 22, 25, 26, 28	1 (columns 2–3), 2, 3, 4 (columns 2–3), 5 (columns 2–3), 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19 (a–c), 21, 22, 23, 25, 26, 28	1 (columns 3–4), 3, 4 (j–o), 5 (i–l), 7, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28

Answers p. 652

Fluency

1 Find all the factors of each of the following numbers. W.E. 1

(a) 18 **(b)** 16 **(c)** 23 **(d)** 24

(e) 20 **(f)** 35 **(g)** 36 **(h)** 42

(i) 53 **(j)** 60 **(k)** 77 **(l)** 84

2 How can you check whether a number is divisible by the following?

(a) divisible by 2 **(b)** divisible by 10 **(c)** divisible by 3

(d) divisible by 6 **(e)** divisible by 4 **(f)** divisible by 5

3 Determine which of the numbers 92, 108, 245 and 3100 are divisible by each of the following. W.E. 2

(a) 3 **(b)** 4 **(c)** 5 **(d)** 8 **(e)** 9

W.E. 3

4 Find the lowest common multiple (LCM) of the following sets of numbers, by first listing the multiples of each.

(a) 2 and 5 **(b)** 3 and 9 **(c)** 5 and 25
(d) 5 and 6 **(e)** 4 and 7 **(f)** 8 and 12
(g) 7 and 9 **(h)** 10 and 12 **(i)** 6 and 11
(j) 9 and 12 **(k)** 20 and 50 **(l)** 8 and 14
(m) 3, 4 and 5 **(n)** 2, 25 and 50 **(o)** 20, 50 and 60

W.E. 4

5 Find the highest common factor (HCF) of the following pairs of numbers, by first listing the factors of each number.

(a) 10 and 15 **(b)** 8 and 24 **(c)** 5 and 12 **(d)** 26 and 36
(e) 11 and 33 **(f)** 28 and 70 **(g)** 44 and 22 **(h)** 10 and 30
(i) 40 and 70 **(j)** 32 and 60 **(k)** 35 and 70 **(l)** 42 and 48

6 **(a)** The lowest common multiple of 8 and 1 is:

A 8 **B** 16 **C** 24 **D** 80

(b) Which of the following is a factor of 34?

A 4 **B** 12 **C** 17 **D** 68

7 **(a)** A number divisible by 2, 3 and 5 is:

A 6 **B** 15 **C** 60 **D** 65

(b) Which pair of numbers are both divisible by 4?

A 38 and 42 **B** 38 and 52 **C** 38 and 60 **D** 52 and 60

8 Which list of numbers only has multiples of 30?

A 31, 32, 33, 34, 35 **B** 30, 40, 50, 60
C 30, 60, 90, 120 **D** 1, 2, 3, 5, 6, 10, 15, 30

Understanding

9 **(a)** Which one of the following numbers is not a multiple of 8?

A 4 **B** 24 **C** 72 **D** 88

(b) Which of the following is not a factor of 42?

A 1 **B** 6 **C** 21 **D** 84

10 How many factors does the number 18 have?

A 2 **B** 3 **C** 5 **D** 6

11 State true (T) or false (F) for the following.

(a) 346 is a multiple of 3. **(b)** 872 is divisible by 6.
(c) 2 is a factor of 348. **(d)** 52 is a multiple of 4.
(e) 854 is divisible by 9. **(f)** 3 is a factor of 56 902.

12 For each group of numbers, find **(i)** the LCM and **(ii)** the HCF.

(a) 4, 6 and 10 **(b)** 6, 8 and 12
(c) 8, 12 and 16 **(d)** 10, 25 and 40

13 Complete the following sentences by using the words 'multiple', 'factor' or 'divisible'.

(a) 32 is a multiple of 8 because it is _____ by 8.

(b) 6 is a _____ of 54, so 54 is a multiple of 6.

(c) 72 is divisible by 9, so that makes it a _____ of 9.

(d) 4 is a factor of 60, so 60 is _____ by 4.

14 **(a)** If 24 lollies are placed into bags so that each bag contains the same number, how many lollies can be in each bag? List all possible answers.

(b) If 36 lollies are placed into bags so that each bag contains the same number, how many lollies can be in each bag? List all possible answers.

15 Ms Williams wants to arrange the seating in the hall for the Year 7s. There must be the same number of chairs in each row. She wants the students to take up all the seats in a row. There are 96 students.

(a) How many rows could there be, and how many seats are in each row? Give all possible combinations, including impractical ones.

(b) Ms Williams would like the arrangement to be as 'square' as possible. Which arrangement is best for this?

16 Mr Rasheed is putting his students into groups to work on a project. Students must be in groups of 3 or 4. He has 26 students in his class. Find the two different ways Mr Rasheed can divide up his class.

17 The smallest number divisible by 3, 4 and 5 is:

A 12 **B** 24 **C** 30 **D** 60

18 In a lighting display, one light flashes every 25 seconds while another light flashes every 60 seconds. If both lights are turned on at the same time, then write the next three times when both lights will flash together. (Hint: The lights will flash together at a common multiple of their flashing times.)

19 **(a)** Find the lowest number greater than 50 that is divisible by 7.

(b) Find the lowest number greater than 100 that is divisible by 11.

(c) Find the first common multiple of 2 and 7 that is greater than 100.

(d) Find the first common multiple of 2, 5 and 7 that is greater than 200.

Reasoning

20 Peter power-walked around an oval while Mei Ling jogged. They started and finished at the same time. They started on the same spot and went in the same direction, keeping up a constant speed for 1 hour. Peter walked 8 laps and Mei Ling jogged 24 laps in the hour.

(a) How many times did Mei Ling pass Peter?

(b) How many times did Mei Ling pass Peter exactly on the spot where they started?

(c) At the beginning of which laps did Mei Ling pass Peter exactly on the spot where they started?

21 **(a)** Copy the following table and do the divisibility tests on the numbers in the left column. Circle the number if the original number is divisible by it. The first one has been done for you.

	Divisible by…							
100 000	(2)	3	(4)	(5)	6	(8)	9	(10)
202 008	2	3	4	5	6	8	9	10
12 121 212	2	3	4	5	6	8	9	10
300 300 300	2	3	4	5	6	8	9	10
7 500	2	3	4	5	6	8	9	10
900 090	2	3	4	5	6	8	9	10
123 456 789	2	3	4	5	6	8	9	10

(b) Complete the following.

(i) If a number is divisible by 4, then it is also divisible by _____.

(ii) If a number is divisible by 9, then it is also divisible by _____.

(c) Explain your answers to **(b)**.

22 To test whether a number is divisible by 6, you must test whether it is divisible by 2 *and* 3. Explain why this test works.

23 A 'perfect' number is a number that has the sum of its factors (excluding itself) equal to itself. The first perfect number is 6, because its factors are $1 + 2 + 3 = 6$.

(a) What is the next perfect number? It is less than 40.

(b) The next perfect number is between 490 and 510. See if you can find it.

24 An 'abundant' number is a number for which the sum of its factors is greater than two times the number itself. The first abundant number is 12, as $1 + 2 + 3 + 4 + 6 + 12 = 28$, which is greater than 2×12. Find the next two abundant numbers. (Both are less than 40.)

25 **(a)** How can you always find a common multiple of a pair of numbers?

(b) How can you check if this number is the lowest common multiple?

Open-ended

26 Darren is designing a box for 60 identical chocolates to be placed in rows.

(a) Find three ways Darren could arrange the chocolates in the box.

(b) Which of your arrangements do you think is the most practical for a chocolate box? Explain your answer.

27 Zena is five years of age and Sam is less than 90 years old. Sam's age is a multiple of three and is also a multiple of Zena's age. Find three possible ages Sam could be.

28 Is it possible to find the highest common multiple of two or more numbers? Explain your answer.

Primes and composites

2.2

A number that has more than two factors is called a **composite number**.

A whole number greater than 1 that has exactly two factors, itself and 1, is a **prime number**.

The number 7 is a prime number as its factors are 1 and 7. The number 8 is a composite number as its factors are 1, 2, 4 and 8.

Two numbers are said to be **co-prime** if their highest common factor is 1.

The sieve of Eratosthenes

Eratosthenes was a Greek mathematician who lived more than 2200 years ago. He is believed to be the first person to calculate a value for the circumference of the Earth. He is also famous today for his 'sieve', a method for finding prime numbers.

To use the sieve of Eratosthenes, copy the table and follow the instructions.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Step 1 Cross out the number 1.

Step 2 Go to the next number, which is 2, and circle it. Then, cross out all of the other multiples of 2.

Step 3 Go to the next number that isn't crossed out. This should be 3. Circle it. Then, cross out all of the other multiples of 3.

Step 4 Go to the next number that isn't crossed out, circle it, then cross out all of its multiples.

Step 5 Repeat for the next number that isn't crossed out. Keep repeating this process until there is no 'next number'.

Step 6 Write the factors of each of the circled numbers. What types of numbers are these?

Step 7 Write the factors of any five of the crossed out numbers, except for 1.

Step 8 Which type of number—circled or crossed out—has more factors? Explain why.

2.2 Primes and composites

Navigator

Answers p. 653

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14 (a), 16, 17	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18	1, 3, 4 (b), 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18

Use 'The sieve of Eratosthenes' on the previous page to help you answer Questions **1–6**.

Fluency

1 Write the prime numbers between 1 and 20.

2 How many single-digit prime numbers are there? List them.

3 List all the primes between 20 and 60.

4 **(a)** The first prime number after 50 is:

A 51 **B** 53 **C** 55 **D** 57

(b) A number co-prime with 18 is:

A 9 **B** 21 **C** 24 **D** 25

Understanding

To show that a number is composite, you only need to show that one of the divisibility tests works.

5 **(a)** What is the next prime number after 60?

(b) What is the next composite number after 60?

(c) What are the two odd composite numbers less than 20?

(d) What is the largest prime number less than 50?

6 Write true (T) or false (F) for each of the following statements.

(a) 21 is prime.

(b) 38 is composite.

(c) 59 is prime.

(d) 49 is prime.

(e) 5 and 7 are co-prime.

(f) 5 and 6 are co-prime.

(g) All even numbers greater than 2 are composite.

(h) All prime numbers are odd.

7 Name a divisibility test that shows that the following numbers are composites.

(a) 410 **(b)** 621 **(c)** 9909

(d) 4 516 803 **(e)** 87 912 404 **(f)** 2 871 025

8 Are the following pairs of numbers co-prime? Give reasons for your answer.

(a) 9 and 17 **(b)** 8 and 11 **(c)** 13 and 52 **(d)** 27 and 63

Reasoning

9 Explain why any pair of prime numbers is co-prime.

10 2 is the only even prime number. Explain why.

11 Explain why it is easy to tell that 4 567 278 is a composite number.

12 Explain why 2 and 3 are the only two consecutive prime numbers.

13 What is the smallest difference between any two consecutive composite numbers?

14 **(a)** Find the numbers closest to 100 that are co-prime with 100.

(b) Find the numbers closest to 36 that are co-prime with 36.

15 Will a prime number always be co-prime with any other whole number? Explain your answer.

16 If one number is a multiple of another number and both numbers are greater than 1, explain why they cannot be co-prime.

Open-ended

17 A conjecture is a mathematical statement that is believed to be true, but has not yet been proven. Goldbach's conjecture (named after the mathematician Christian Goldbach) states that 'every even number greater than 2 can be written as the sum of two primes'. Choose 10 even numbers, and use them to demonstrate Goldbach's conjecture.

18 A pair of 'Sophie Germain primes' (named after the mathematician) is a pair of prime numbers where one number is exactly one more than double the other number. For example, 11 and 23 are Sophie Germain primes, because $11 \times 2 + 1 = 23$. Find two more pairs of Sophie Germain primes.

Puzzle

Gold digger

How to play

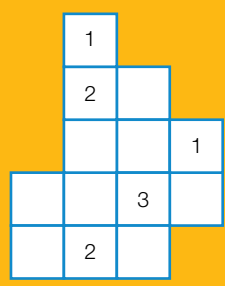
It's the final day of the 16th annual gold-digging competition. Carmen, your partner for the competition, has almost worked out where the gold is located. On two separate maps drawn as square grids, she has marked the squares that have gold nearby.

If a number is written in a grid square on the map, then it has no gold, but there must be gold in that number of the squares touching it (either horizontally, vertically or diagonally, sharing an edge or a corner). So if a square is numbered 3, then there must be gold in 3 of the squares that touch it. No square contains more than one piece of gold.

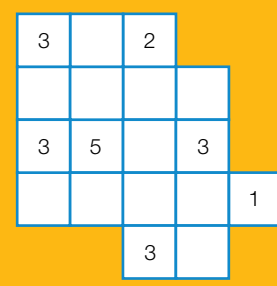
Your task is to find exactly which squares have gold, so your team can get all the gold and win the competition.

Now, copy the following maps and find the gold.

Map (a)



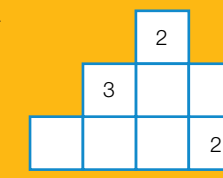
Map (b)



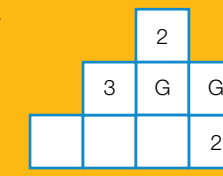
Hints for finding gold

Use the numbered squares to eliminate the possibilities.

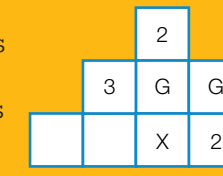
Here, the top square (numbered 2) is only touching two other empty squares, so both of these must contain pieces of gold. Mark these squares with a 'G' to signify this.



Now, look at the 2 in the bottom right-hand corner. It is already next to 2 pieces of gold, so the other square it touches must be empty. Mark this square with an X.



Now, look at the 3. It is already next to one piece of gold, and it is only touching two other empty squares, so both of these squares must contain gold.



Gamespace

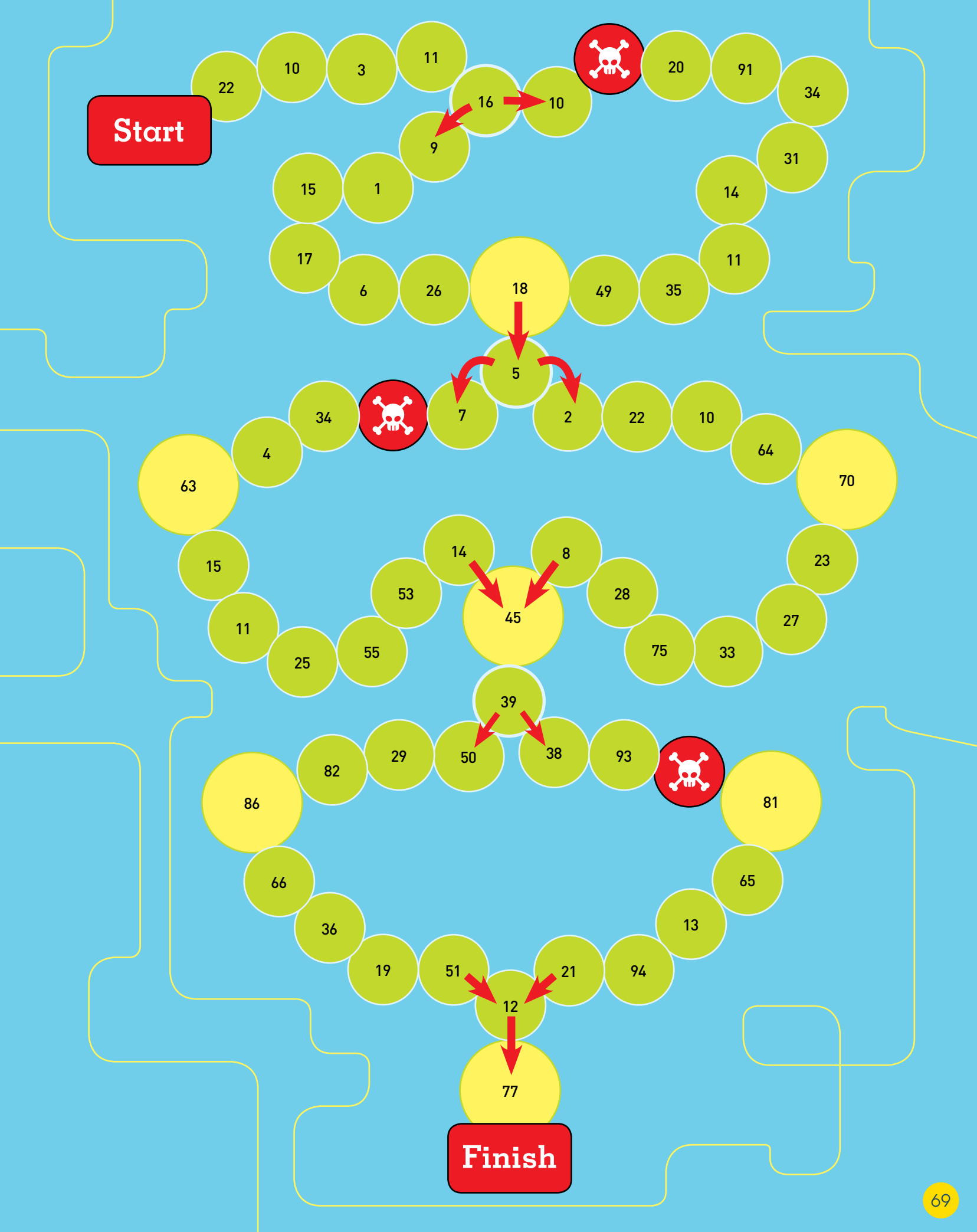
FACTOR-PILLAR

Equipment required: 1 counter per player, 1 die, calculator (optional)

How to play:

- Roll the die to decide who goes first.
- Players take turns to roll the die and move forward the number of spaces shown. You cannot move backwards. When you reach a circle with two arrows leading from it, you can decide which path to take.
- The aim is to move from start to finish and get as many points as possible by finding factors of numbers.
- When you land on a number, you must list all the factors of that number. The number of factors is your score. For example, a player lands on 6: factors are 1, 2, 3 and 6; score = 4 points.

  But beware! If another player notices you have missed one or more factors, then they gain five points. If they suggest a factor that is wrong, then they lose five points.
- **LARGE CIRCLE:** If a player correctly finds all the factors of the number in a large circle, they earn double the points for that number.
- **CROSSBONES:** If a player lands on a circle containing a skull and crossbones, they miss a turn.
- **SQUARE NUMBER:** If a player lands on a square number and calls 'Square Power' within two seconds, they have an extra turn.
- **PRIME NUMBER:** If a player lands on a prime number and calls 'Prime Power' within two seconds, they can double their points for that number. If they don't call 'Prime Power' then they only get two points for the two factors.
- The game ends when one player reaches or crosses the finish line. The player with the highest number of points at that moment is declared the winner (even if they haven't reached the finish line).


Start
22
10
3
11
16
10
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20
91
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31
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26
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49
35
11
5
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4
63
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28
45
11
27
25
55
75
33
39
29
50
38
93
82
86
81
66
65
36
13
19
51
21
94
12
77
Finish

2.3 Prime factors

A **prime factor** of a number is a factor that is also a prime number.

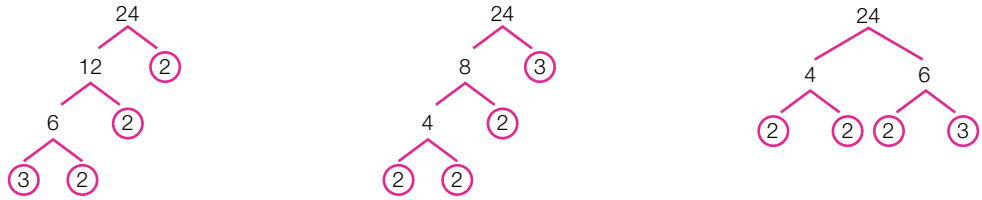
Every whole number can be written as the unique product of its prime factors.

Using a factor tree to find prime factors

A factor tree is a useful way of finding the prime factors of a number. To construct a factor tree:

1. Draw two lines ('branches') extending out from the number, in an upside-down 'V' shape.
2. Write any two factors of the number at the end of the branches.
3. If one of the factors is prime, circle it. If it is composite, draw another two branches and split the factor into two smaller factors.
4. Repeat step **3** until all branches end in a circled prime number.

There is often more than one way to make a factor tree. Three versions of the factor tree for 24 are shown here. These factor trees look different, but the prime factors on the ends of the branches are always the same: 2, 2, 2, 3.



When all of the branches end in prime factors, then you have found all of the prime factors, so you can write the number as the product of those prime factors. You should write the factors in ascending order (smallest to largest) and write any repeated factors in index form.

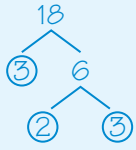
For example: $24 = 2 \times 2 \times 2 \times 3$

$= 2^3 \times 3$

Worked example 5

W.E. 5

Draw a factor tree for the number 18, then express the number as a product of its prime factors in index form.

Thinking	Working
1 Write the number as the product of two factors.	$18 = 3 \times 6$
2 Show each factor at the end of a branch and circle it if it is prime.	
3 Split any composite factor into two smaller factors, continuing until the factors at the end of each branch are prime.	
4 Write the number as a product of the primes in index form and in ascending order (smallest to largest).	$18 = 2 \times 3 \times 3$ $= 2 \times 3^2$

Using repeated division to find prime factors

You can also find the prime factors of a number by doing a series of divisions by small prime numbers, such as 2, 3 or 5. You can use the divisibility rules to identify these smaller prime factors. For example, if the number is even, you can divide by 2. The divisibility rules are listed in section 2.1 of this chapter.

Keep dividing by the same or different prime factors until the final answer is 1. The list of all the divisors will be the list of the prime factors.

Worked example 6

W.E. 6

Use repeated division to find the prime factors of 84. Express the number as a product of its prime factors in index form.

Thinking	Working
1 Identify a prime factor of the number and divide it into the number. Continue dividing by the same or a different prime factor until the last answer is 1.	$2\overline{)84}$ $2\overline{)42}$ $3\overline{)21}$ $7\overline{)7}$ 1
2 The prime factors are listed down the left side of the division calculation. Write the number as a product of these.	$84 = 2 \times 2 \times 3 \times 7$
3 Write the number as a product of the prime factors in index form and in ascending order.	$84 = 2^2 \times 3 \times 7$

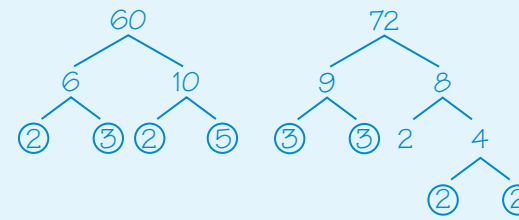
Using prime factors to find the HCF (highest common factor)

If you have two numbers both written as a product of their prime factors, then their HCF must be the product of all their common prime factors (that is, the prime factors that appear in both).

Worked example 7

W.E. 7

Use prime factors to find the HCF of the following pair of numbers: 60 and 72.

Thinking	Working
1 Use factor trees to find the prime factors.	
2 Write each number as a product of its prime factors. Do not use index form.	$60 = (2 \times 2 \times 3) \times 5$ $72 = 2 \times (2 \times 2 \times 3) \times 3$
3 Circle the factors that are common to both numbers (there must be the same number of factors in each group).	
4 Find the product of the common prime factors. This is the HCF.	$HCF = 2 \times 2 \times 3$ $= 12$

2.3 Prime factors

Navigator

Answers p. 653

1 (columns 1–2), 2 (columns 2–3), 3 (a–h), 4, 5, 6 (a–b), 7, 8, 9, 10, 11, 12, 13, 15	1 (columns 2–3), 2 (columns 1, 3), 3 (columns 1–2), 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15	1 (columns 3–4), 2 (columns 3–4), 3 (columns 2–3), 5 (d–f), 6, 7, 8, 9, 10, 11, 13, 14, 15, 16

Fluency

W.E. 5

1 Draw a factor tree for each of the following numbers, then express the number as a product of its prime factors in index form.

(a) 8	**(b)** 12	**(c)** 20	**(d)** 48
(e) 14	**(f)** 26	**(g)** 68	**(h)** 44
(i) 64	**(j)** 72	**(k)** 108	**(l)** 144
(m) 200	**(n)** 750	**(o)** 1000	**(p)** 1236

W.E. 6

2 Use repeated division to find the prime factors of the following numbers. Express each number as a product of its prime factors in index form.

(a) 8	**(b)** 12	**(c)** 28	**(d)** 36
(e) 39	**(f)** 77	**(g)** 51	**(h)** 38
(i) 30	**(j)** 63	**(k)** 96	**(l)** 132
(m) 168	**(n)** 198	**(o)** 288	**(p)** 212

W.E. 7

3 Use prime factors to find the HCF of the following pairs of numbers.

(a) 8 and 20	**(b)** 12 and 18	**(c)** 12 and 36
(d) 36 and 48	**(e)** 24 and 56	**(f)** 28 and 84
(g) 64 and 96	**(h)** 70 and 98	**(i)** 60 and 105
(j) 66 and 110	**(k)** 80 and 128	**(l)** 130 and 156

4 **(a)** 120 written as the product of its prime factors is:

A $2 \times 4 \times 15$ **B** $3 \times 8 \times 5$ **C** $2 \times 2 \times 5 \times 6$ **D** $2 \times 2 \times 2 \times 3 \times 5$

(b) The prime factors of 550 are:

A 2, 5, 11 **B** 2, 10, 55 **C** 5, 10, 11 **D** 2, 11, 25

5 Which numbers are the product of these prime factorisations?

(a) $2^3 \times 3^2$	**(b)** $2^2 \times 3 \times 5^2$	**(c)** $2^4 \times 3^3 \times 5^3$
(d) $3^2 \times 5 \times 7^2$	**(e)** $2^3 \times 5^2 \times 7 \times 11$	**(f)** $2^6 \times 11^2$

6 $6 = 2 \times 3$ and $8 = 2^3$. Use these facts to write the following numbers as products of prime numbers.

(a) 48 **(b)** 96 (48×2) **(c)** 144 (48×3)

7 $33 = 3 \times 11$ and $10 = 2 \times 5$. Use these facts to write the following numbers as products of prime numbers.

(a) 330 **(b)** 660 **(c)** 990

Understanding

8 Explain the difference between factors and prime factors, using the number 24 as an example.

9 The prime factors of 192 are 2 and 3, while the prime factors of 42 are 2, 3 and 7. List the prime factors of 192×42.

10 **(a)** The HCF of a pair of numbers can also be used to find the lowest common multiple (LCM). Copy and complete the following method.

Prime factors of 36: $2 \times \_\_ \times \_\_ \times \_\_$

Prime factors of 100: $2 \times \_\_ \times \_\_ \times \_\_$

Highest common factor (HCF) of 36 and 100: $2 \times \_\_ = \_\_$

Factors of 36 not used to find the HCF: $\_\_ \times 3 = \_\_$

Factors of 100 not used to find the HCF: $5 \times \_\_ = \_\_$

HCF × all unused factors of 36 × all unused factors of 100: $\_\_ \times 3 \times \_\_ \times 5 \times \_\_ = 900$

The result is the lowest common multiple of 36 and 100.

(b) Use this method to find the LCM for the following pairs of numbers.

(i) 8 and 18	**(ii)** 12 and 18	**(iii)** 8 and 20
(iv) 8 and 72	**(v)** 20 and 26	**(vi)** 20 and 800
(vii) 20 and 1000	**(viii)** 400 and 800	**(ix)** 800 and 1000

11 Marcus is making identical balloon arrangements for a party. He has 84 red balloons and 54 blue balloons. He would like to sort his balloons into groups, so that each group has the same number of red and blue balloons, with no balloons left over.

(a) Use prime factors to find the highest common factor of 84 and 54.

(b) If Marcus split his balloons into this number of groups, then how many red and blue balloons would be in each group?

Reasoning

12 A number less than 550 is the product of four prime factors: 2, 3, 5 and a fourth prime factor between 15 and 20. What is the fourth factor, and what is the number?

13 **(a)** Find the smallest number that can be written in prime factor form with:

(i) exactly two different prime factors

(ii) exactly three different prime factors

(iii) exactly four different prime factors.

(b) Repeat **(a)**, but remove the condition that the factors are different.

14 140 tickets were sold for the school concert. Each ticket sold was numbered from 1 to 140. A lucky door prize was awarded to each person who had a ticket with a number that was a multiple of 15, but not a multiple of 9, 10 or 25. Five friends had the ticket numbers 45, 80, 90, 105 and 135.

(a) Do any of them have a winning ticket? If so, what is their winning number?

(b) If 250 tickets were sold, how many winning tickets would there be? Find the winning numbers between 1 and 250.

Open-ended

15 A number between 100 and 200 is the product of four primes and has three prime factors (one factor is repeated). Explain how you might go about finding this number and write down two such numbers.

16 To find the prime factors of a number, Ray uses the divisibility tests. He then writes down the number from each divisibility test that worked.

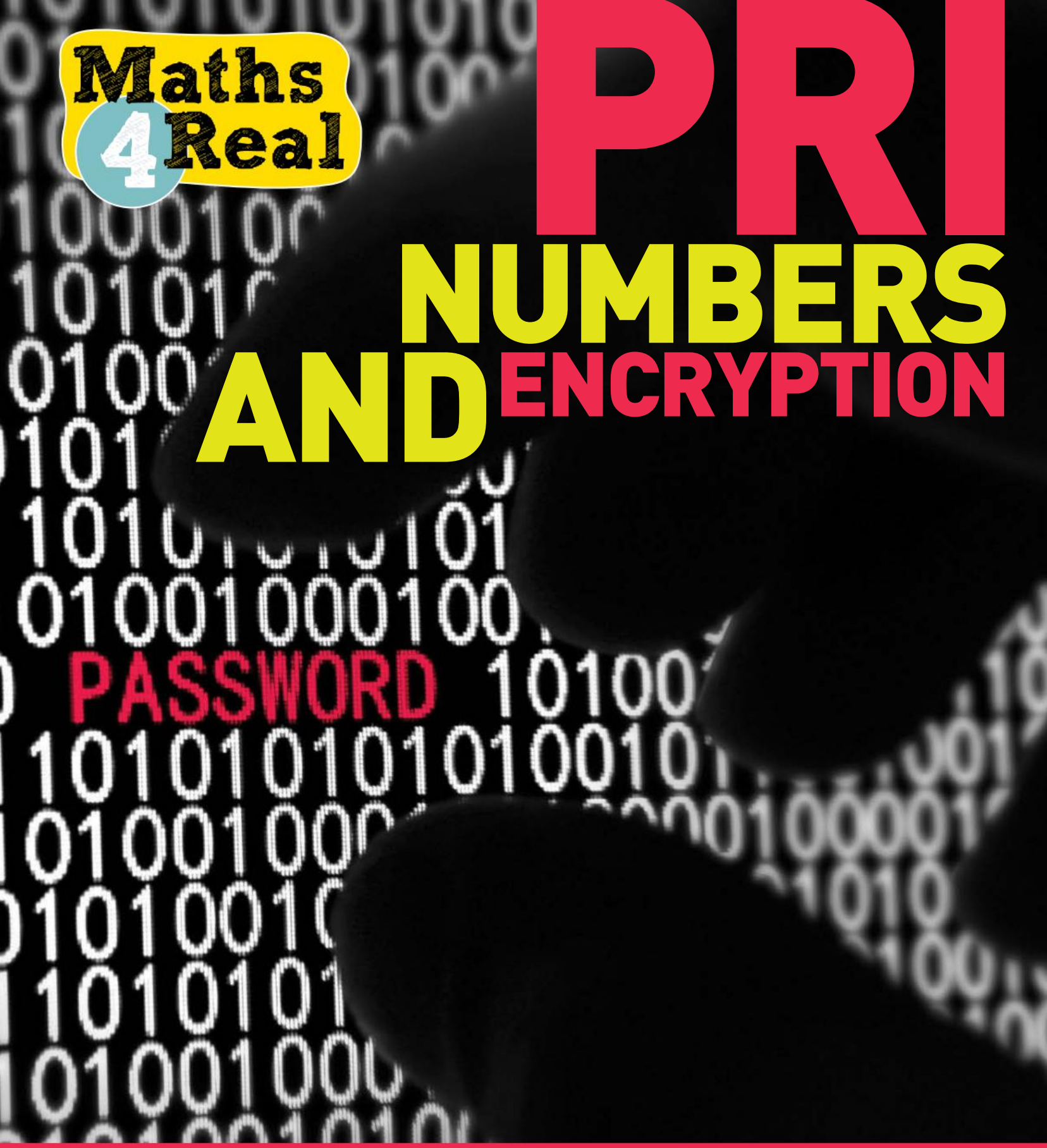
(a) Using this method, what would Ray say are the prime factors of 20?

(b) What is wrong with Ray's method?

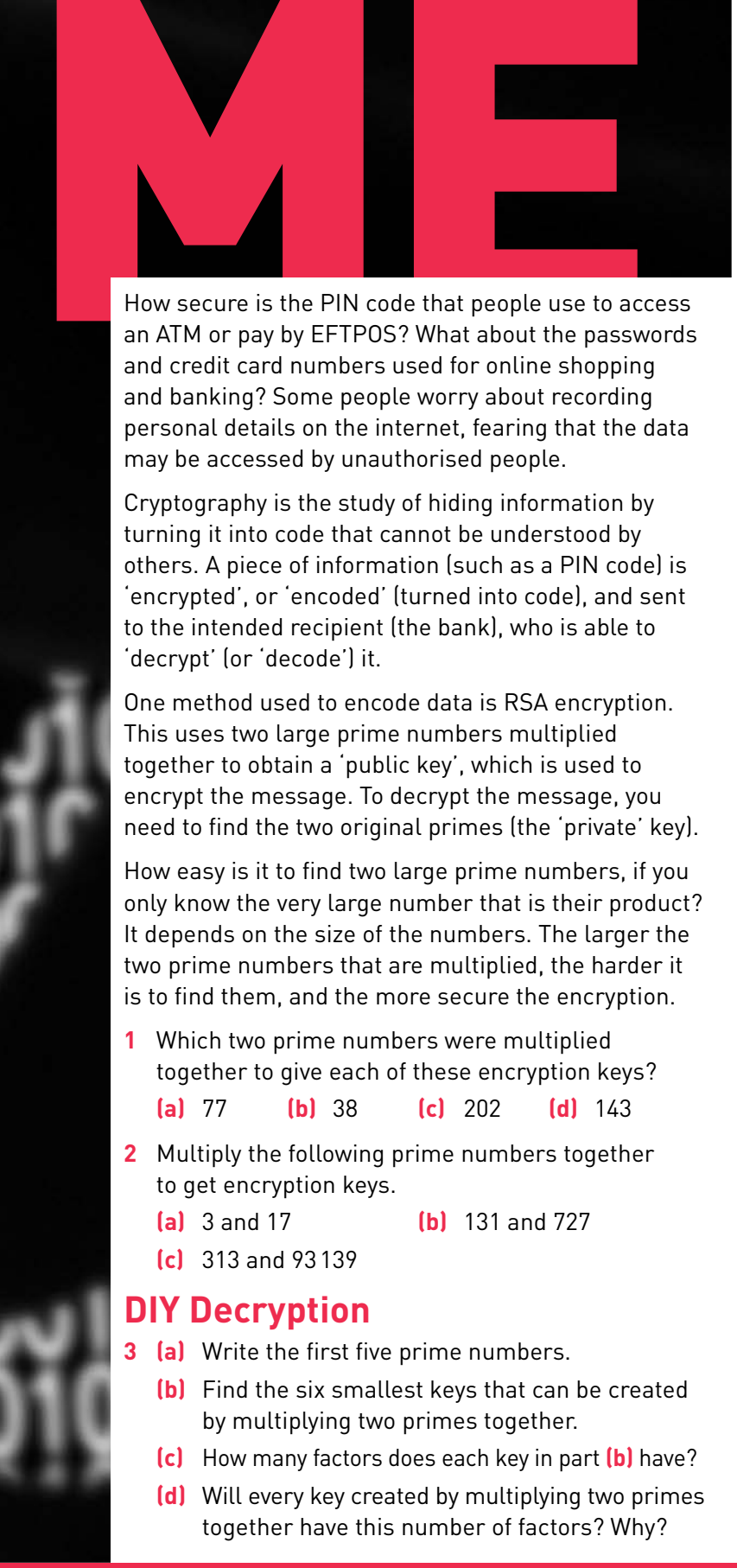
(c) Suggest how Ray's method can be improved so that it is accurate.

Maths 4 Real

PRIME NUMBERS AND ENCRYPTION



{A B C D E F G H I J K L M



How secure is the PIN code that people use to access an ATM or pay by EFTPOS? What about the passwords and credit card numbers used for online shopping and banking? Some people worry about recording personal details on the internet, fearing that the data may be accessed by unauthorised people.

Cryptography is the study of hiding information by turning it into code that cannot be understood by others. A piece of information (such as a PIN code) is 'encrypted', or 'encoded' (turned into code), and sent to the intended recipient (the bank), who is able to 'decrypt' (or 'decode') it.

One method used to encode data is RSA encryption. This uses two large prime numbers multiplied together to obtain a 'public key', which is used to encrypt the message. To decrypt the message, you need to find the two original primes (the 'private' key).

How easy is it to find two large prime numbers, if you only know the very large number that is their product? It depends on the size of the numbers. The larger the two prime numbers that are multiplied, the harder it is to find them, and the more secure the encryption.

1 Which two prime numbers were multiplied together to give each of these encryption keys?

(a) 77 **(b)** 38 **(c)** 202 **(d)** 143

2 Multiply the following prime numbers together to get encryption keys.

(a) 3 and 17 **(b)** 131 and 727

(c) 313 and 93 139

DIY Decryption

3 **(a)** Write the first five prime numbers.

(b) Find the six smallest keys that can be created by multiplying two primes together.

(c) How many factors does each key in part **(b)** have?

(d) Will every key created by multiplying two primes together have this number of factors? Why?

4 Here is a simplified version of how decryption using prime numbers can work. Suppose the encrypted message is W L R G Y S B E M R K C R E B J C V and your key is 713.

Find the two prime numbers whose product is 713 (23 and 31). Now write the digits of these primes under the code as follows:

W	L	R	G	Y	S	B	E	M	R	K	C	R	E	B	J	C	V
2	3	3	1	2	3	3	1	2	3	3	1	2	3	3	1	2	3

The digits tell you how many places to shift each letter along the alphabet. The 2 under the W means that you shift two letters forward through the alphabet from W to Y so that the decoded message has Y as its first letter. To decode the second letter, you move 3 places forward from L in the alphabet, which is the letter O.

(a) Continue decrypting the code to get the full message.

(b) Describe what you did with the coded letter Y.

5 Decode this message using 943 as the key.

K X P G Q F O N D M N H K B E L N L N S Y K Y D

Note: You will need to decide which order to write the two primes. Only one way will work.

6 Encrypt your own message using this method. Use prime numbers less than 100 and a message with less than 25 letters. Give your coded message and key to someone to decrypt.

7 Why are prime numbers used as encryption keys? Why can't you use any number, such as 120, as a key?

Research

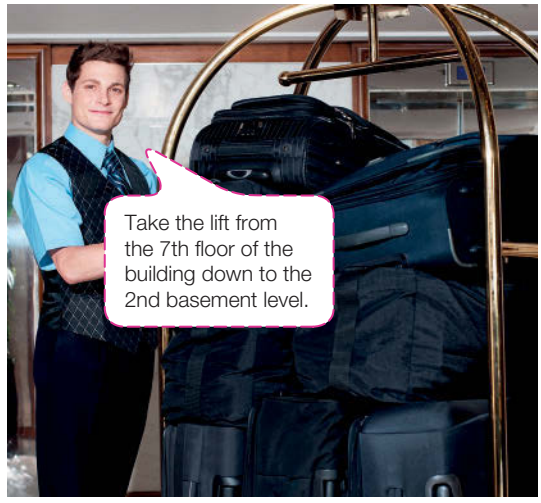
Find out about GIMPS (Great Internet Mersenne Prime Search) and how prime numbers are used in email encryption.

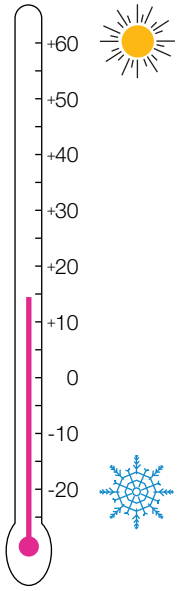
Do you think governments should ban encryption so that intelligence agencies can read everyone's emails?

Make a list of the potential problems and dangers that can arise from placing personal information on the internet.

N O P Q R S T U V W X Y Z }

2.4 Introduction to integers





All of the above statements describe more than just the size of a number—they also give a direction. You can use a + (**positive**) sign or a – (**negative**) sign to show this direction. Below is a list of words usually associated with + or –.

+ (positive)	– (negative)
up	down
increase	decrease
gain	lose
deposit	withdrawal
profit	loss
above	below

The words **deposit** and **withdrawal** are banking words used to describe the movement of money into and out of a bank account. **Profit** and **loss** are words used to describe whether money has been made or lost by a business.

Integers

The **integers** include all of the whole numbers that you are familiar with (1, 2, 3, …), and negative whole numbers (-1, -2, -3, …), and zero (0), which is neither positive nor negative.

You indicate whether a number is positive or negative by placing a '+' or '-' in front of the number. For example, you would write '+4' and say 'positive four'. You would write '-9', and say 'negative nine'.

Worked example 8 W.E. 8

Write an integer suggested by each of the following.

(a) depositing $660 into your bank account

(b) diving 5 m below the surface of the ocean

Thinking		Working
(a) 1	Does the statement suggest an increase or a decrease?	(a) In this case, 'deposit' suggests an increase, so use a positive sign.
2	Write the number with the appropriate sign.	+660
(b) 1	Does the statement suggest an increase or a decrease?	(b) In this case, 'below' suggests a decrease, so use a negative sign.
2	Write the number with the appropriate sign.	-5

Comparing integers

The number line is very useful when working with integers.

Before now, you may have only used the positive section of the number line, to the right of zero. Now you can extend the number line to the left of zero, to include negative numbers.

Notice that the negative numbers form a mirror image of the positive numbers on the other side of zero.

Along the number line, the numbers get larger as you go to the right (more positive), and smaller as you go to the left (more negative).

Look at the positions of +5 and -10 on the number line shown. +5 is further to the right than -10. You say that +5 *is greater than* -10, and write +5 > -10. You could also say that -10 *is less than* +5, and write -10 < +5.

Note: Usually, the + sign is not written on positive numbers. For example, 6 is the same as +6.

If a number has no sign in front of it, assume it is positive. 7 = +7

'>' is the symbol that means 'is greater than'.

'<' is the symbol that means 'is less than'.

Worked example 9

W.E. 9

Write a > or < symbol between each pair of integers to make a correct statement.

(a) -9 _____ -4 **(b)** 0 _____ -10 **(c)** 7 _____ -48

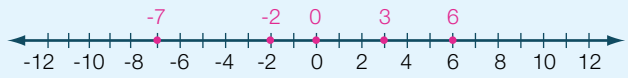
Thinking	Working
(a) Consider the position of the pair of integers on a number line. Is the first number to the right or to the left of the second number? (Here, -9 is to the left of -4.) If it is to the right, insert >. If it is to the left, insert <.	(a) $-9 < -4$
(b) Consider the position of the pair of integers on a number line. Is the first number to the right or to the left of the second number? (Here, 0 is to the right of -10.) If it is to the right, insert >. If it is to the left, insert <.	(b) $0 > -10$
(c) Consider the position of the pair of integers on a number line. Is the first number to the right or to the left of the second number? (Here, 7 is to the right of -48.) If it is to the right, insert >. If it is to the left, insert <.	(c) $7 > -48$

Worked example 10

W.E. 10

Arrange the following integers in ascending order (from smallest to largest).

-2, 3, -7, 0, 6

Thinking	Working
1 Mark the integers on a number line as shown.	
2 Now, list the marked integers in order, as they appear on the number line, moving from left to right.	-7, -2, 0, 3, 6

The further to the right that a number is on a horizontal number line, the greater its value.

The further to the left that a number is on a horizontal number line, the smaller its value.

Vertical number lines can also represent directed numbers, so that the higher a number is on the line, the greater its value. A traditional thermometer is a common example of a vertical number line.

2.4 Introduction to integers

Navigator

1 (column 1), 2 (columns 1–2), 3 (a–d), 4 (a–d), 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18	1 (column 1), 2 (columns 1, 3), 3 (a–c), 4 (a–c), 5 (a–f), 6, 7, 8, 9 (column 1), 10, 11, 12, 13, 14, 15, 16, 17, 18	1 (column 2), 2 (columns 2, 4), 3 (d–f), 4 (d–f), 5 (column 3), 7, 8, 9 (column 2), 11, 12, 13, 14, 15, 16, 17, 18

Answers p. 654

Fluency

1 Write an integer suggested by each of the following.

W.E. 8

(a) a profit of $300
(b) a loss of $50 000
(c) 8 seconds before blast-off
(d) 4 hours after take-off
(e) 36 m above sea level
(f) a deposit of $45
(g) a withdrawal of $20
(h) a win by 12 points
(i) a 2 point loss
(j) down 7 floors
(k) up 28 floors
(l) I owe the bank $40 000.
(m) I have $225 in the bank.
(n) The water level dropped by 9 metres.

2 Write a > or < symbol between each pair of integers to make a correct statement.

W.E. 9

(a) +6 _____ -3
(b) +1 _____ -3
(c) -7 _____ +5
(d) -2 _____ +6
(e) -12 _____ -4
(f) -7 _____ -1
(g) -2 _____ -11
(h) -9 _____ 0
(i) 0 _____ -2
(j) -35 _____ +7
(k) -4 _____ +67
(l) -11 _____ +75

3 Arrange the following integers in ascending order (from smallest to largest).

W.E. 10

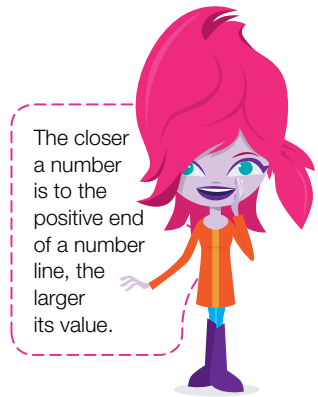
(a) -3, -12, 6, -4, -1
(b) -11, 10, 9, 2, -10
(c) 1, -6, -4, 9, 2
(d) -8, -5, 7, 1, -6
(e) -1, -4, -7, -8, -12
(f) -56, 23, -79, 0, 6

4 Arrange the following integers in descending order (from largest to smallest).

(a) -5, -6, 3, 7, 8
(b) -8, 7, 10, 1, -12
(c) 1, -8, -4, -9, -6
(d) -12, 0, -5, 7, -11
(e) -11, -5, -2, -12, -9
(f) 0, 79, -54, 4, -9

5 State the opposite of:

(a) west 300 km
(b) 3 days late
(c) 7 °C below zero
(d) add 5
(e) subtract 22
(f) go left 3 m
(g) -2
(h) +16
(i) -350



6 **(a)** The directed number +2 is suggested by:

A going up two flights of stairs

B going down two flights of stairs

C losing 2 kg in weight

D the temperature going from 19 °C to 17 °C.

(b) The directed number -4 is suggested by:

A going up four flights of stairs

B the rain gauge filling with 4 mm of rain

C gaining 4 kg in weight

D the temperature going from 22 °C to 18 °C.

7 Write true (T) or false (F) for each of the following.

(a) $58 > -60$ **(b)** $-44 < -50$ **(c)** $-29 > 30$ **(d)** $-31 < -29$

(e) $-75 > 70$ **(f)** $-92 > -11$ **(g)** $19 > -100$ **(h)** $82 > -99$

Understanding

8 Write all the integers that are between the following pairs.

(a) -4 and 3 **(b)** -2 and 2 **(c)** -9 and -4

(d) -5 and 0 **(e)** -37 and -42 **(f)** -120 and -115

9 Find the next three numbers in each pattern.

(a) -18, -16, -14, ____, ____, ____ **(b)** -20, -15, -10, ____, ____, ____

(c) 9, 6, 3, ____, ____, ____ **(d)** -6, -12, -18, ____, ____, ____

(e) 30, 20, 10, ____, ____, ____ **(f)** 12, 7, 2, ____, ____, ____

10 The balances of four bank accounts show the following figures.

Aran	$428	Zoe	-$23
Orlando	-$260	Serena	$325

(a) Who has the largest amount of money in their account?

(b) Who owes their bank the most?

11 The minimum and maximum temperatures recorded at Uluru during one week were:

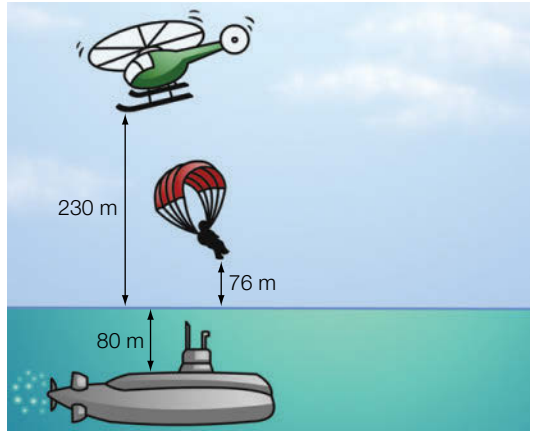
Day	Maximum (°C)	Minimum (°C)
Monday	19	-3
Tuesday	16	-7
Wednesday	22	0
Thursday	20	-1
Friday	23	2
Saturday	20	-2
Sunday	18	-4

(a) On which day was the lowest minimum recorded?

(b) When was the highest minimum recorded?

(c) Which day had the greatest difference between the minimum and the maximum?

12 A helicopter and a submarine are in place at the levels shown. A parachutist has 76 m to fall before hitting the ocean.



(a) How far is the parachutist from the submarine?

(b) What distance from the helicopter is the parachutist?

(c) How far apart are the helicopter and the submarine?

Reasoning

13 **(a)** You ride 5 km west, then 3 km east, then 2 km west. What single journey could you have taken instead to get from the start to the finish?

(b) What is the opposite of this single journey?

14 **(a)** You get in an elevator on the ground floor, then travel 7 floors up, then 2 floors up, then 6 floors down, then finally 1 floor up. What single journey could you have taken instead to get to your final floor?

(b) What is the opposite of this single journey?

15 On Monday, Sam withdrew $80 from his bank account at an ATM. On Tuesday, his pay of $350 was deposited into the account, and he also deposited $40 that he had received for his birthday. On Thursday, Sam used the account to pay a $120 bill online, and on Friday he withdrew $50 from an ATM.

(a) Did Sam have more or less money at the end of the week than at the start?

(b) How much more or less did he have?

Open-ended

16 Write any three negative integers greater than -8.

17 The game of indoor cricket is similar to regular cricket, in that you can score 1, 2, 3, 4 or 6 runs from each ball that is bowled. It is different in that if you lose your wicket, or 'get out', then 5 runs are subtracted from your score. When batting, cricketers bat together in pairs for 4 overs (an 'over' is a set of 6 balls bowled).

(a) In their first over, Angelo and Murray scored as follows: 2, wicket, 1, 2, 4, wicket. What was their overall score at the end of the over?

(b) At the end of their first over, Saul and Jeremy's score was 4. On each ball of the over, they had either scored runs or lost their wicket. List at least three ways they could have reached 4 in six balls.

18 Sharif is a computer technician. He works in an office building with 25 floors, helping to fix computer problems for other employees. Write a short story using this sequence of directed numbers, describing a day at work for Sharif.

0, +21, -13, +7, 0, +4, 0, -2, +5, 0, -1, -2, 0

Puzzle

Temperature tangle

Equipment required: access to the internet or a library may be necessary

Australia is known for its warm summers, but it can get cold here too!

Solve this puzzle to discover the place that holds the record for the coldest recorded temperature in Australia.

1. For the 13 situations listed below, find the matching temperature and code letter.
2. If the temperatures and their corresponding letters are written in the correct order, you will spell out the name of Australia's coldest place.

1	Boiling point of water	70 °C	P
2	Oven temperature to bake a cake	37 °C	E
3	Temperature inside a household fridge	0 °C	O
4	Coldest air temperature recorded on Earth (Vostok Station, Antarctica)	180 °C	H
5	Temperature of the surface of the Sun	51 °C	A
6	Freezing point of water	-18 °C	T
7	Hottest air temperature recorded on Earth (Al 'Aziziyah, Libya)	4 °C	A
8	Temperature inside a household freezer	100 °C	C
9	Body temperature of a healthy human	-196 °C	S
10	Drinking temperature of a cup of tea	-30 °C	S
11	Hottest temperature recorded in Australia (Oodnadatta, SA)	58 °C	T
12	Temperature of a freezer in an ice-cream factory	5800 °C	L
13	Temperature of liquid nitrogen	-95 °C	R

Half-time 2

1 **(a)** Find the lowest common multiple (LCM) of 9 and 15. **2.1**

(b) Find the highest common factor (HCF) of 48 and 72.

2 The common factors of 18 and 30 are: **2.1**

A 1, 18, 23 **B** 1, 2, 3, 6 **C** 1, 9, 18, 30 **D** 2, 3, 9, 18

3 Write each of the following as either a positive or a negative integer. **2.4**

(a) a deposit of $85 into a bank account **(b)** a weight loss of 7 kg

(c) a win by 3 goals **(d)** a profit of $28 000

(e) 40 m below sea level **(f)** going up 11 floors in a building

4 List all the prime numbers between 30 and 50. **2.2**

5 Write a < (less than) or > (greater than) symbol between each of the following to make a true statement. **2.4**

(a) 6 _____ 10 **(b)** -4 _____ 7 **(c)** 3 _____ -12 **(d)** -8 _____ -15

6 Draw a factor tree for each of the following, then express each number as a product of its prime factors in index form. **2.3**

(a) 54 **(b)** 60 **(c)** 98 **(d)** 104

7 **(a)** A number is divisible by 15 if it is divisible by both 3 and 5. Use divisibility tests to decide if 76 905 is divisible by 15. **2.1**

(b) Show, using two divisibility tests, that 4734 is divisible by 6.

8 Write the following sets of integers in ascending order (smallest to largest). **2.4**

(a) -37, 7, -30, 0, -3, 3 **(b)** 54, -20, -1, -5, -40

9 Use prime factors to find the HCF of the following pair of numbers: 48 and 120. **2.3**

10 The sum of two prime numbers is 30. The numbers cannot be: **2.2**

A 7 and 23 **B** 11 and 19 **C** 13 and 17 **D** 14 and 16

11 Three racing cars, Honda, Holden and Ford, take 25 seconds, 30 seconds and 50 seconds respectively to complete each lap of a circuit. They start the race at the same time and from the same position. **2.1**

(a) How many seconds will pass before all three cars are at the same position again on the track?

(b) How many laps has each car completed by this time?

(c) How many laps does the Honda complete in the time taken by the Ford to complete 6 laps?

2.5 Adding and subtracting positive integers

In the following sections you will learn how to perform calculations with integers. A number line can be very useful for this.

Adding positive integers

Adding two positive integers is the normal addition you have been doing for years.

For example, +1 + +2 = +3, which can simply be written as 1 + 2 = 3.

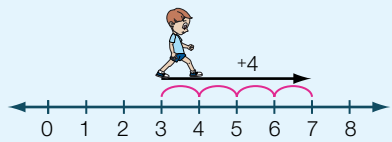
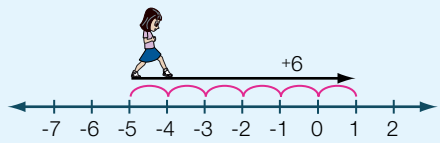
To add a positive number, move that many spaces along the number line to the right (the positive direction) from the starting position.

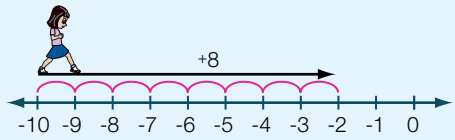
The steps to do an addition are the same, whether you start with a positive or a negative integer.

Worked example 11 W.E. 11

Use a number line to perform each of the following additions.

(a) 3 + 4 **(b)** -5 + 6 **(c)** -10 + 8

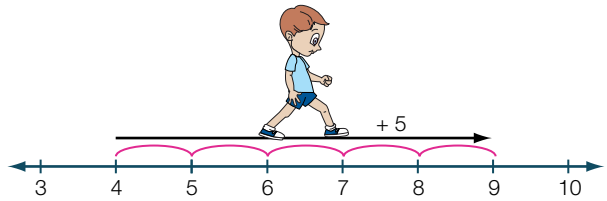
Thinking		Working
(a) **1**	Draw or imagine a number line. Start at the first number (3) and walk the number of steps indicated by the second number (4) in the positive direction (to the right).	*(a)* 
2	Write the addition as a number sentence.	3 + 4 = 7
(b) **1**	Draw or imagine a number line. Start at the first number (-5) and walk the number of steps indicated by the second number (6) in the positive direction (to the right).	*(b)* 
2	Write the addition as a number sentence.	-5 + 6 = 1

(c)	**1**	Draw or imagine a number line. Start at the first number (-10) and walk the number of steps indicated by the second number (8) in the positive direction (to the right).	*(c)* 
	2	Write the addition as a number sentence.	*-10 + 8 = -2*

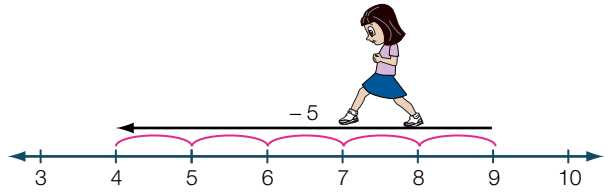
Subtracting positive integers

Subtraction and addition are opposite operations. We have shown addition as starting on a point on the number line and walking along a number of spaces to the right. Because subtraction is the opposite of addition, we subtract by moving to the left on the number line.

Addition:
Move to the right
4 + 5 = 9



Subtraction:
Move to the left
9 – 5 = 4

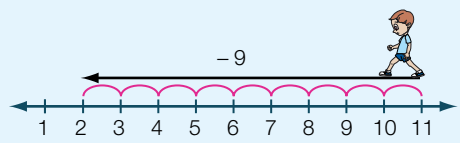
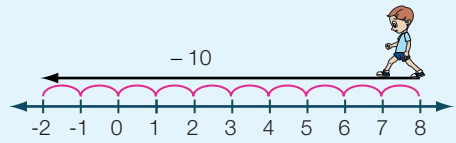


To subtract a positive number, move that many spaces along the number line to the left (the negative direction) from the starting position.

Worked example 12 W.E. 12

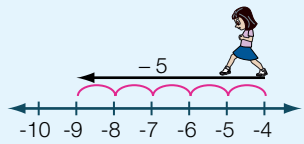
Use a number line to perform each of the following subtractions.

(a) 11 – 9 **(b)** 8 – 10 **(c)** -4 – 5

Thinking			Working
(a)	**1**	Draw or imagine a number line. Start at the first number (11) and walk the number of steps indicated by the second number (9) in the negative direction (to the left).	*(a)* 
	2	Write the subtraction as a number sentence.	*11 – 9 = 2*
(b)	**1**	Draw or imagine a number line. Start at the first number (8) and walk the number of steps indicated by the second number (10) in the negative direction (to the left).	*(b)* 
	2	Write the subtraction as a number sentence.	*8 – 10 = -2*

(c) 1 Draw or imagine a number line. Start at the first number (-4) and walk the number of steps indicated by the second number (5) in the negative direction (to the left).

(c)



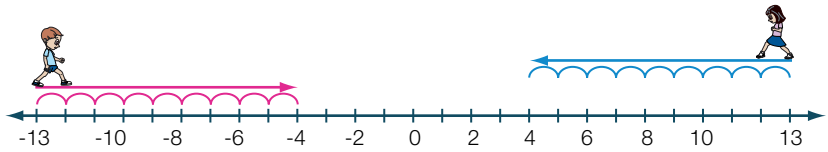
2 Write the subtraction as a number sentence.

$-4 - 5 = -9$

Part **(b)** of the example above shows that if you subtract a larger number (such as 10) from a smaller one (such as 8), you will walk past the zero and into the negative side of the number line.

Using number line symmetry

The number line can be thought of as being a reflection, or mirror image, around zero (0). This symmetry can be useful when working with integers. For example, the journey to calculate -13 + 9 (moving to the right) is the mirror image of the journey to calculate +13 – 9 (moving to the left), with answers of -4 and +4, respectively.



So, to calculate -85 + 23, you could simply calculate 85 – 23, then place a negative sign in front of the answer.

$85 - 23 = 62$ $\qquad$ $-85 + 23 = -62$

2.5 Adding and subtracting positive integers

Navigator

Answers p. 655

1 (a–l), 2 (a–l), 3, 4, 5 (columns 1–2), 6, 7, 8, 9, 11, 12, 13 (a–b), 14, 15	1 (columns 1–3), 2 (columns 1–3), 3, 4, 5 (columns 2–3), 6, 7, 8, 9, 10, 11, 12, 13, 14, 15	1 (columns 3–4), 2 (columns 3–4), 3, 4, 5 (column 3), 6, 7, 8, 9, 10, 12, 13, 14, 15

Fluency

W.E. 11

1 Use a number line to perform each of the following additions.

(a) 8 + 3	**(b)** 7 + 1	**(c)** 3 + 8	**(d)** 1 + 7
(e) -9 + 7	**(f)** -11 + 6	**(g)** -10 + 5	**(h)** -12 + 6
(i) -8 + 14	**(j)** -7 + 19	**(k)** -5 + 30	**(l)** -20 + 40
(m) -30 + 20	**(n)** -17 + 13	**(o)** -75 + 55	**(p)** -87 + 56

W.E. 12

2 Use a number line to perform each of the following subtractions.

(a) 5 – 3	**(b)** 8 – 2	**(c)** 12 – 7	**(d)** 11 – 11
(e) 8 – 14	**(f)** 6 – 17	**(g)** 10 – 18	**(h)** 11 – 15
(i) 30 – 40	**(j)** -3 – 5	**(k)** 2 – 22	**(l)** -5 – 7
(m) -9 – 3	**(n)** -24 – 6	**(o)** -35 – 34	**(p)** -66 – 58

3 **(a)** -5 + 3 may be calculated by following which of these instructions?

A start at +5 and walk 3 steps to the right, arriving at 8

B start at -5 and walk 3 steps to the right, arriving at -2

C start at +5 and walk 3 steps to the left, arriving at 2

D start at -5 and walk 3 steps to the left, arriving at -8

(b) -15 – 10 may be described by which number line journey?

A start at -15 and walk 10 steps to the right, arriving at -5

B start at +10 and walk 15 steps to the left, arriving at -5

C start at +15 and walk 10 steps to the left, arriving at +5

D start at -15 and walk 10 steps to the left, arriving at -25

4 **(a)** -5 + 9 is the 'mirror image' of:

A 5 + 9 **B** 5 – 9 **C** -5 – 9

(b) 23 – 7 is the 'mirror image' of:

A -23 + 7 **B** -23 – 7 **C** 23 + 7

5 Calculate the following.

(a) 3 + 4 + 5 **(b)** 5 + 2 + 4 **(c)** 7 + 1 + 4

(d) -8 + 6 + 9 **(e)** -29 + 5 + 6 **(f)** -37 + 7 + 8

(g) 6 – 7 – 4 **(h)** 3 – 5 – 8 **(i)** 12 – 16 – 4

(j) 14 + 9 – 10 **(k)** 23 – 30 + 6 **(l)** 44 – 51 + 8

Understanding

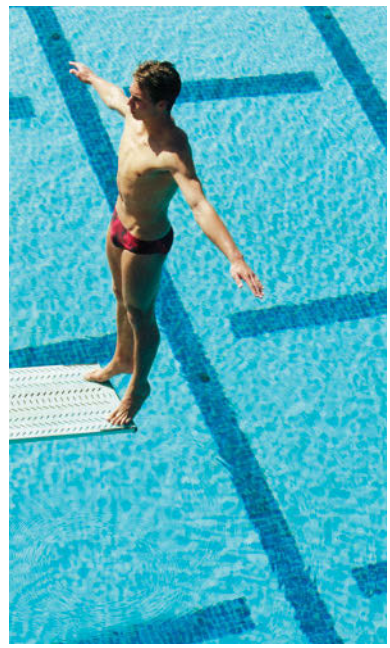
6 The temperature in the desert is -12 °C, but rises by 20 degrees. What is the new temperature?

7 A lift is at the third basement floor (3 floors below ground level). It moves up 14 levels, then stops. On which floor does it stop?

8 A submarine is 30 m below the surface of the ocean. It rises 16 m, then dives 20 m. What depth is it at now?

9 A diver jumps from a platform 12 m high and dives 17 m down to the bottom of a pool. How deep is the pool?

10 You have $5 in your pocket, but you owe a friend $12 and someone else owes you $20. If everyone pays up, how much will you have at the end?



11 Write the following number sentences in words.

(a) +5 – 7 = -2 **(b)** -8 + 9 = +1 **(c)** -4 – 3 = -7

Reasoning

12 **(a)** Show the following pairs of calculations on number lines.

(i) -1 + 2 and 2 – 1 **(ii)** -7 + 4 and 4 – 7 **(iii)** -13 + 6 and 6 – 13

(b) What do you notice? Comment on your observation.

(c) Use your observation from **(b)** to write the answers to the following without using a number line.

(i) -37 + 86 **(ii)** -72 + 95 **(iii)** -104 + 119

13 **(a)** Draw a number line from -12 to 12. Draw a number line journey showing 5 – 3 = 2 and the 'mirror image' journey showing -5 + 3 = -2.

(b) Use the number line to help you write the 'mirror image' journeys for the journeys represented by these number sentences:

(i) 8 – 6 = 2 **(ii)** 5 – 4 = 1 **(iii)** -10 + 6 = -4 **(iv)** -12 + 9 = -3

(c) Calculate these additions on the negative side of the number line by considering their positive 'mirror image' journeys.

(i) -23 + 17 **(ii)** -48 + 27 **(iii)** -59 + 34 **(iv)** -62 + 43

Open-ended

14 The thermometer at a weather station in the desert near Alice Springs reads -4 °C at 6 am. The temperature then rises until it reaches a maximum of 25 °C at 12 pm. What might the thermometer readings have been at 7, 8, 9, 10 and 11 am?

15 Write three different positive integers that give a negative answer when added to -13.

Problem solving

The slippery snail

A snail is at the bottom of a low garden wall 40 cm high. If the snail manages to move 4 cm up the wall each day, but slips down 3 cm each night, then how much time does it take the snail to reach the top of the wall?

Strategy options

- Draw a diagram.
- Act it out.
- Look for a pattern.

Adding and subtracting negative integers

2.6

The '+' and '–' signs can be used to show an operation (addition or subtraction), or a direction (positive or negative). To avoid confusion, brackets can be written around the number and the direction sign in front of it, to separate this from an operation symbol.

For example, +2 + (-3) is said as 'positive 2 add negative 3'.

-4 – (+7) is said as 'negative 4 subtract positive 7'.

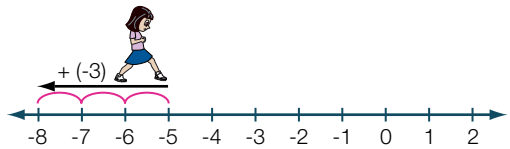
Adding negative integers

Imagine that you owe a friend $5. You could write this as -5. Now, imagine that you owe a second friend $3, which you can write as -3. How much do you owe altogether? The answer is $8, which you can write as -8.

'Add up' your total debt by calculating the sum: -5 + (-3) = -8. You can see that adding a negative number gives a 'more negative' result, which is a number that is further towards the negative end of the number line.

You can show this on the number line in the following way.

1. Start at -5.
2. You are adding, so face to the right.
3. Then, because it is a negative number, walk *backwards* along the number line 3 units (to the left).
4. Arrive at the answer of -8.



-5 + (-3) on the number line shows the same journey as the subtraction -5 – 3, or -5 – (+3).

The movement along the number line is the same, and the end result of -8 is the same.

This means that adding -3 and subtracting +3 are exactly the same process.

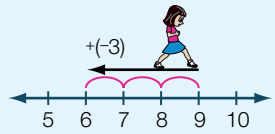
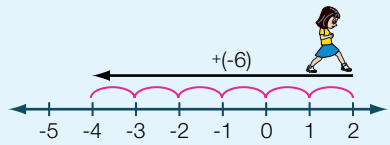
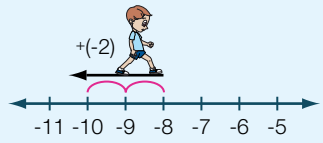
To add a negative number, subtract it as a positive number.

-4 + (-7)	+2 + (-3)
= -4 – 7	= +2 – 3
= -11	= -1

Worked example 13 W.E. 13

Use a number line to perform each of the following additions.

(a) 9 + (-3) **(b)** 2 + (-6) **(c)** -8 + (-2)

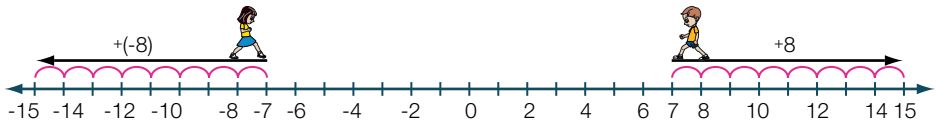
Thinking		Working
(a) **1**	Draw or imagine a number line. Start facing right at the first number (9), then walk backwards the number of steps indicated by the second number (-3).	*(a)* 
2	Write the addition as a number sentence.	*9 + (-3) = 6*
(b) **1**	Draw or imagine a number line. Start facing right at the first number (2), then walk backwards the number of steps indicated by the second number (-6).	*(b)* 
2	Write the addition as a number sentence.	*2 + (-6) = -4*
(c) **1**	Draw or imagine a number line. Start facing right at the first number (-8), then walk backwards the number of steps indicated by the second number (-2).	*(c)* 
2	Write the addition as a number sentence.	*-8 + (-2) = -10*

Using number line symmetry

The symmetry of the number line can be useful when adding negative numbers.

To calculate 7 + 8 on the number line, start at 7 and move to the right 8 steps to 15.

-7 + -8 is the mirror image of this. On the number line start at -7, facing to the right, then move to the left 8 steps to -15.



So, to calculate -47 + (-59), you can perform the 'mirror image' calculation of 47 + 59, then place a negative sign in front of the answer.

Subtracting negative integers

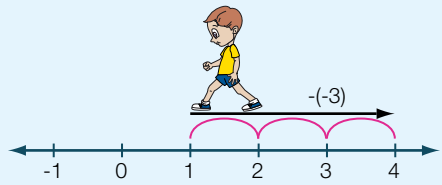
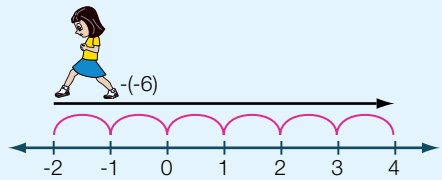
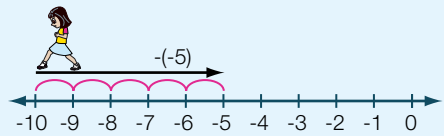
It can be difficult to imagine what it means to subtract a negative number. But you have already used a number line for adding and subtracting, so you can use the same ideas for subtracting a negative number.

1. On the number line, start at the first number. You are subtracting, so face to the left.
2. Then, because it is a negative number, walk *backwards* the number of of steps (to the right).

Worked example 14 W.E. 14

Use a number line to perform each of the following subtractions.

(a) 1 – (-3) **(b)** -2 – (-6) **(c)** -10 – (-5)

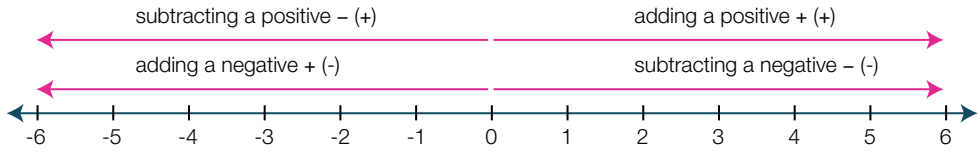
Thinking	Working
(a) 1 Draw or imagine a number line. Start facing left at the first number (1), then walk backwards the number of steps indicated by the second number (-3).	*(a)* 
2 Write the subtraction as a number sentence.	$1 - (-3) = 4$
(b) 1 Draw or imagine a number line. Start facing left at the first number (-2), then walk backwards the number of steps indicated by the second number (-6).	*(b)* 
2 Write the subtraction as a number sentence.	$-2 - (-6) = 4$
(c) 1 Draw or imagine a number line. Start facing left at the first number (-10), then walk backwards the number of steps indicated by the second number (-5).	*(c)* 
2 Write the subtraction as a number sentence.	$-10 - (-5) = -5$

Note that the above examples are said in the following way.

- +1 – (-3) is 'positive one subtract negative three'
- -2 – (-6) is 'negative two subtract negative six'
- -10 – (-5) is 'negative ten subtract negative five'

The number line journeys above show that subtracting a negative number results in a journey in the positive direction of the number line. This is the same journey as adding the positive equivalent of the number.

You can think of subtracting a negative number as making the other number 'less negative', which is the same as making it 'more positive'.



To subtract a negative number, add it as a positive number.

$3 - (-1)$
$= 3 + 1$
$= 4$

$-7 - (-5)$
$= -7 + 5$
$= -2$

2.6 Adding and subtracting negative integers

Navigator

Answers p. 656

1 (a–l), 2 (a–l), 3, 4 (columns 1–2), 5, 6, 7, 8, 9, 10, 11, 12, 13 (a–b), 14, 15	1 (columns 1–3), 2 (columns 1–3), 3, 4 (columns 2–3), 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16	1 (columns 3–4), 2 (columns 3–4), 3, 4, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16

Fluency

W.E. 13

1 Use a number line to perform each of the following additions.

(a) 5 + (-4)	**(b)** 3 + (-2)	**(c)** 12 + (-3)	**(d)** 8 + (-2)
(e) 2 + (-6)	**(f)** 4 + (-9)	**(g)** 10 + (-17)	**(h)** 9 + (-20)
(i) -2 + (-7)	**(j)** -3 + (-8)	**(k)** -8 + (-12)	**(l)** -10 + (-24)
(m) -5 + (-5)	**(n)** -30 + (-60)	**(o)** -17 + (-12)	**(p)** -62 + (-71)

W.E. 14

2 Use a number line to perform each of the following subtractions.

(a) 10 – (-1)	**(b)** 4 – (-2)	**(c)** 5 – (-5)	**(d)** 6 – (-6)
(e) -3 – (-11)	**(f)** -9 – (-12)	**(g)** -4 – (-20)	**(h)** -17 – (-11)
(i) -3 – (-15)	**(j)** -9 – (-8)	**(k)** -5 – (-13)	**(l)** -19 – (-13)
(m) -30 – (-20)	**(n)** -55 – (-45)	**(o)** -67 – (-41)	**(p)** -81 – (-53)

3 **(a)** The finishing point for the number line journey +6 + (-12) may be found by following which of these instructions?

A start at +6 and walk 12 steps to the left, finishing at -6

B start at +6 and walk 12 steps to the right, finishing at +18

C start at -12 and walk 6 steps to the left, finishing at -18

D start at -6 and walk 12 steps to the right, finishing at +6

(b) The answer to +3 – (-20) may be found by following which of these instructions?

A start at +3 and walk 20 steps to the left, arriving at -17

B start at +3 and walk 20 steps to the right, arriving at 23

C start at -20 and walk 3 steps to the right, arriving at -17

D start at -3 and walk 20 steps to the left, arriving at -23

4 Calculate the following.

(a) 9 + (-7) + (-2)	**(b)** 6 + (-5) + (-7)	**(c)** 9 + (-12) + (-4)
(d) -1 + (-4) + (-5)	**(e)** 10 + (-8) + (-7)	**(f)** 24 + (-12) + (-6)
(g) 2 – (-1) – (-2)	**(h)** 1 – (-6) – (-2)	**(i)** -4 – (-7) + (-6)
(j) -7 – (-10) + (-5)	**(k)** 12 + (-16) + (-4)	**(l)** 48 + (-32) – (-6)

5 **(a)** 12 – (-7) is equal to:

A 12 – 7 **B** -12 – (-7) **C** 12 + 7

(b) -81 + (-63) is equal to:

A -81 – 63 **B** 81 + 63 **C** -81 + 63

Understanding

6 The temperature at sunset was 12 °C. During the night it fell by 16 °C to the minimum temperature. What was the minimum temperature?

7 A worker steps into an elevator on the 7th floor of an office building and travels down 10 levels. Which level does the worker finish at? Write your answer as an integer.

8 Your company owes $2000. You then receive a notice that you owe the bank another $1500. What is your financial state now?

9 A farmer lowers a bucket from the top of a 1 m high well using a 10 m rope. If she lowers the whole length of rope, how far below ground does the bucket reach?

10 A bird flying 6 m above sea level suddenly sees a fish and dives 9 m straight down to catch the fish. How far below the surface was the fish?

11 A miner is sitting on a ledge 8 m below the top of a vertical mine shaft when he drops his torch. If the torch falls 23 m before hitting the bottom of the shaft, how deep is the shaft?

12 Write the following number sentences in words.

(a) $-5 + (-6) = -11$ **(b)** $+7 - (-8) = +15$ **(c)** $-9 - (-14) = +5$

Reasoning

13 (a) Draw a number line from -12 to +12. Draw number line journeys showing $2 + 3 = 5$ and $-2 + (-3) = -5$.

(b) Use the number line to help you write the 'mirror image' journeys for the journeys represented by these number sentences.

(i) $1 + 6 = 7$ **(ii)** $4 + 7 = 11$ **(iii)** $-3 + (-8) = -11$ **(iv)** $-5 + (-4) = -9$

(c) Work out these calculations on the negative side of the number line by considering their positive 'mirror image' journeys.

(i) $-21 + (-15)$ **(ii)** $-28 + (-27)$ **(iii)** $-36 + (-41)$ **(iv)** $-65 + (-29)$

14 (a) Evaluate the following.

(i) $12 - (+6)$ **(ii)** $12 - (-6)$ **(iii)** $-3 - (+6)$ **(iv)** $-3 - (-6)$

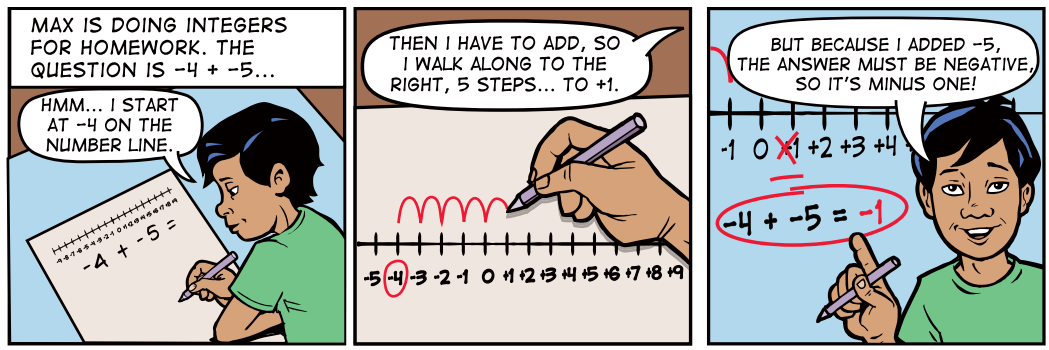
(b) Copy and complete the following sentence by inserting the words 'smaller', 'larger' or 'unchanged'.

Whether you start with a positive or a negative integer, if you subtract a positive number, then the result is _____ and if you subtract a negative number, then the result is _____.

Open-ended

15 Write integers that give a positive answer when subtracted from -21.

16



(a) What is the correct answer?

(b) Explain to Max where he went wrong, and how to do these types of questions correctly in the future.

Investigation

Heat beads and ice blocks

Equipment required: 10 red and 10 blue counters (or any other pair of colours)

The Big Question

Positive and negative integers can be represented by different coloured counters. How can you use them to show integer addition and subtraction?

Engage

Use the red counters to represent positive integers and the blue counters to represent negative integers.

For example:  = +3

 = -5

Because they are opposites, 1 blue counter will cancel out 1 red counter. In other words, 1 blue + 1 red = 0. You might think of this as 1 'heat bead' (red) cancelling out 1 'ice block' (blue).

You can write this as a number sentence: 1 + (-1) = 0.

You can use combinations of red and blue counters to represent integers, by cancelling red and blue pairs.

For example:

 -7

+

 +4

= -3

(It is shown here that 4 red and 4 blue pairs cancelled each other.)

1 Place 3 red counters on the table to represent +3. Now, place 2 blue counters underneath them.

What number is represented now (remembering that 2 blues will cancel 2 reds)?

Complete the number sentence: +3 + (-2) = _____ .

2 Use your counters to do the following. Draw diagrams to show each addition, cancelling red and blue pairs.

(a) +3 + (-4) **(b)** +2 + (-5) **(c)** +4 + (-6)

3 Use your counters to do the following. Draw diagrams to show each addition, cancelling red and blue pairs.

(a) -4 + (+1) **(b)** -7 + (+3) **(c)** -9 + (+2)

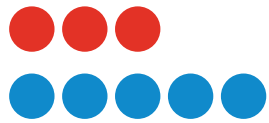
Explore

4 So far, you have used the counters to model addition. Modelling subtraction can be a little more challenging.

Strategy options

- Draw a diagram.
- Act it out.

(a) What number is shown by this collection of counters?



(b) If you removed the red counters, what number would be shown now?

Complete the number sentence:
-2 – (+3) = _____

5 Notice that in order to perform this subtraction, you had to have 3 red counters present, so that you could show the subtraction by removing them. They were balanced by 3 of the blue counters, so they had no effect on the starting number.

Use your counters to do the following subtractions. You may need to add extra pairs of red and blue counters to have enough to remove.

(a) -4 – (+2) **(b)** -1 – (+3) **(c)** -2 – (+5)

6 Use your counters to do the following subtractions. Again, you may need to add pairs of counters at the beginning.

(a) +3 – (+4) **(b)** +2 – (+5) **(c)** +4 – (+6)

7 Looking at the counters diagram in Question **4**, what number would be shown if you removed all of the blue counters? Write this as a subtraction.

8 Use your counters to do the following subtractions.

(a) -4 – (-1) **(b)** -7 – (-3) **(c)** -9 – (-2)

9 Use your counters to do the following subtractions.

(a) -2 – (-4) **(b)** -5 – (-1) **(c)** -3 – (-7)

10 Use your counters to do the following subtractions.

(a) +2 – (-4) **(b)** +5 – (-1) **(c)** +3 – (-7)

Explain

11 **(a)** What do you notice about the answers to Question **2** and Question **6**? What conclusion can you draw from this?

(b) What do you notice about the answers to Question **3** and Question **8**? What conclusion can you draw from this?

(c) What do you notice about the answers to Question **9** and Question **10**? Why is this the case?

12 Why was it necessary to use counters of both colours to represent your starting number in Questions **5** and **6**, and **9** and **10**?

13 Helen wants to use her red and blue counters to show +3 – (-5). She has lined up 3 red counters to show +3.

She has no blue counters to remove. Explain how Helen can include 5 blue counters in her representation of +3.

Elaborate

14 For each of the following, choose an integer between -10 and +10, and use counters of one or both colours to represent it. (For example, you might choose -3, which you could show with 3 blue counters, or 2 red and 5 blue counters.) Then, add or subtract a second number of counters to show the operation described. Write a number sentence for the operation you have modelled, and draw a diagram of the counters.

(a) adding a positive integer to a negative integer

(b) adding a negative integer to a positive integer

(c) subtracting a positive integer from a negative integer

(d) subtracting a negative integer from a positive integer

15 Sam has made a number using some of his 10 red and 10 blue counters. He has used twice as many of one colour than the other. Draw two different arrangements of counters to show what Sam's number could be.

Evaluate

16 Using counters in this way is one method of learning how to add and subtract negative numbers. Another method is to use a number line, which is presented elsewhere in this chapter.

Do you find one method easier to work with than the other? Which one? Why do you think this is?

Extend

17 Can you use your counters to show multiplication?

(a) Start by showing 2 × -4 and 3 × -3. It may be useful to remember that multiplication means 'groups of'.

(b) Can you show -4 × -2 or -3 × -3? How is this method of using counters limited here?

2.7 Simplifying addition and subtraction

As seen, you can simplify operations with directed numbers by omitting the '+' sign in front of positive numbers. For example, +7 can be written simply as 7. This can be used to write additions and subtractions more simply.

+4 + (+5) = +9 can be written as 4 + 5 = 9

+4 – (+5) = -1 can be written as 4 – 5 = -1

You can simplify other expressions by replacing the two signs between the numbers with a single symbol.

As seen, adding a negative number and subtracting a positive number are the same. Both result in a movement to the left on the number line; that is, subtraction.

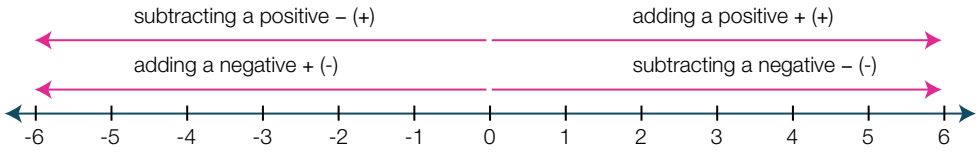
7 + (-3) is the same as 7 – (+3). Both can be written as 7 – 3.

+ (-) and – (+) can be replaced by –

As also seen, adding a positive number and subtracting a negative number are the same. Both result in a movement to the right on the number line; that is, addition.

6 + (+2) is the same as 6 – (-2). Both are equal to 6 + 2.

+ (+) and – (-) can be replaced by +



These observations can be summarised with the following rules for adding and subtracting integers:

If the two signs are the same, the result is addition.
+ (+) = +
– (-) = +

If the two signs are different, the result is subtraction.
– (+) = –
+ (-) = –

To apply these rules, the two signs must be *next* to each other when the calculation is written. The rules do not apply otherwise.

A visual method

Remember you can always use a number line if you do not remember the rules.

1. Draw or imagine a number line. Place your pen, or imagine you are standing, on the first number.
2. Look at the operations symbol:

   If it is +, turn and face right (towards the positive end of the number line).

   If it is –, turn and face left (towards the negative end of the number line).
3. Look at the sign of the second number:

   If it is positive, walk forwards that many units.

   If it is negative, walk backwards that many units.

Worked example 15 W.E. 15

Simplify each of the following by writing a single symbol between the values, then calculate the answer. Use a number line if necessary.

(a) 4 + (+9) **(b)** -1 + (-3) **(c)** 10 – (+1) **(d)** -6 – (-5)

Thinking			Working
(a)	**1**	Adding a positive number is straightforward addition. Replace + (+) with +.	(a) 4 + (+9) = 4 + 9
	2	Calculate.	= 13
(b)	**1**	Adding a negative number is equivalent to subtraction. Replace + (-) with –.	(b) -1 + (-3) = -1 – 3
	2	Calculate.	= -4
(c)	**1**	Subtracting a positive number is straightforward subtraction. Replace – (+) with –.	(c) 10 – (+1) = 10 – 1
	2	Calculate.	= 9
(d)	**1**	Subtracting a negative number is equivalent to addition. Replace – (-) with +.	(d) -6 – (-5) = -6 + 5
	2	Calculate.	= -1

2.7 Simplifying addition and subtraction

Navigator

Answers p. 656

1 (columns 1–2), 2, 3 (a–h), 4, 5 (columns 1–3), 6, 7, 8, 9, 12 (a), 13	1 (columns 2–3), 2, 3 (columns 1–3), 4, 5 (columns 2–3), 6, 7, 8, 9, 10, 12, 13	1 (column 4), 3 (e–l), 5 (column 4), 6, 7, 8, 9, 10, 11, 12, 13

Fluency

W.E. 15

1 Simplify each of the following by writing a single symbol between the values, then calculate the answer. Use a number line if necessary.

(a) +5 + (+2) **(b)** -1 + (+4) **(c)** +5 + (+6) **(d)** -9 + (+7)

(e) +5 – (+3) **(f)** -8 – (+4) **(g)** +1 – (+12) **(h)** -3 – (+7)

(i) +12 + (-3) **(j)** +10 + (-4) **(k)** +7 + (-9) **(l)** +4 + (-8)

(m) -7 – (-6) **(n)** -3 – (-9) **(o)** +2 – (-10) **(p)** +1 – (-1)

(q) -7 + (-3) **(r)** -9 + (-3) **(s)** -4 – (-9) **(t)** -2 – (-6)

2 State true (T) or false (F) for the following.

(a) 25 + (+5) = 25 – 5 **(b)** 30 + (-10) = 30 – 10

(c) 15 – (+20) = 15 – 20 **(d)** 40 – (-50) = 40 – 50

3 Calculate:

(a) 6 – 18 **(b)** 9 – 20 **(c)** -5 + 9 **(d)** -3 + 6

(e) -12 + 7 **(f)** -10 + 2 **(g)** -9 – 4 **(h)** -7 – 5

(i) -60 + 30 **(j)** -52 + 21 **(k)** -27 + 84 **(l)** -28 + 93

4 **(a)** -9 – (-6) – (+4) simplifies to:

A -9 – 6 – 4 **B** -9 + 6 + 4 **C** -9 + 6 – 4 **D** 9 + 6 + 4

(b) 14 – (+4) + (-3) is the same as:

A -14 – 4 – 3 **B** 14 – 4 – 3 **C** 14 – 4 + 3 **D** 14 + 4 + 3

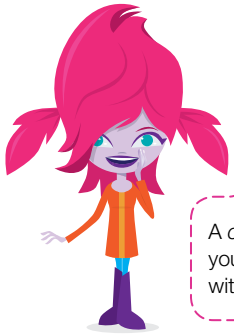
5 Calculate:

(a) -6 + 7 + 5 **(b)** -4 + 6 + 5 **(c)** -12 + 3 + 2 **(d)** -11 + 3 + 4

(e) -8 + 5 – 3 **(f)** -6 + 2 – 4 **(g)** -4 + 12 – 4 **(h)** -3 + 10 – 6

(i) -5 – 1 – 6 **(j)** -2 – 5 – 4 **(k)** 36 – 40 – 5 **(l)** 43 – 50 – 2

Understanding



6 Your monthly bank statement shows a balance of $260 at the start of a particular month, followed by the transactions for the month listed below.

Credit	$30
Debit	-$80
Credit	$200
Debit	-$60
Debit	-$500

What is your account balance at the end of the month?

7 A maintenance worker in an office block starts work on the ground floor, and then travels as follows.

Up 10 floors
Down 7 floors
Down 5 floors
Up 4 floors
Down 6 floors
Up 12 floors

(a) What was the highest floor the worker reached?

(b) What was the lowest floor the worker travelled to?

(c) Which floor did the worker finish on?

8 Diana makes and sells soft toys. She keeps records of her profits and losses over a 10-week period. These records are shown in the table. Find Diana's overall profit or loss during this time.

Week	Result	
1	Profit	$120
2	Profit	$25
3	Loss	-$70
4	Profit	$210
5	Loss	-$150
6	Loss	-$180
7	Loss	-$90
8	Profit	$40
9	Loss	-$160
10	Loss	-$95

Reasoning

9 Complete the following addition and subtraction grids. Begin with the number at the top of each column, then either add or subtract the numbers at the left of each row.

(a)

+	-6	+20		31
+4				
		+13		
	+2			
			-23	22

(b)

–	8	-11		
	5			
-7			26	
+5				-30
		-1		

10 A 'magic square' is a square grid of numbers where the sum of each of the rows, columns and diagonals is the same number.

(a) What is the magic sum for this 3×3 magic square?

(b) Complete this magic square.

	6	-1
	2	
5		

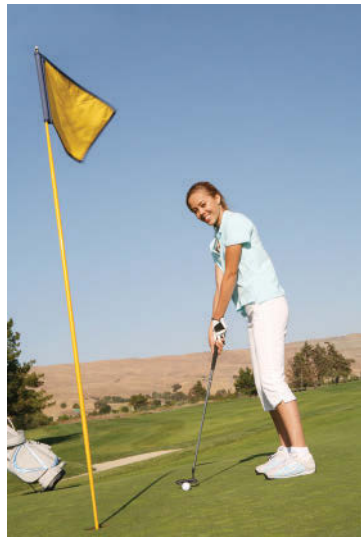
11 You can make new magic squares by adding or subtracting the same number from every number in an existing magic square.

(a) Make a new 3×3 magic square by subtracting 4 from each number in the magic square in the previous question.

(b) What is the new magic sum? Explain why the new magic sum is not 4 less than the old magic sum.

Open-ended

12 In the sport of golf, each hole has a particular number of shots that are expected to get the ball from the tee to the hole. This expected number of shots is called the 'par' for the hole. If you take this number of shots to get the ball in the hole, your score for the hole is 0. If your number of shots for the hole is *less than par*, this is shown on the bottom row of the scorecard by a *negative number*. If your number of shots is *greater than par*, this is shown with a *positive number*.



(a) Carla played nine holes of golf. She scored under par on four holes, par on three holes and over par for the rest. If Carla took a total of 35 shots to complete her round, fill in what her scorecard may have looked like.

Hole number	1	2	3	4	5	6	7	8	9	Total
Par for the hole	4	4	5	3	4	5	4	3	4	36
Shots taken										35
Score compared to par										

(b) Carla played another round of nine holes. Her overall score was -1 (1 under par). What could her scorecard have looked like, if she did not score par (0) on any hole?

13 Find two numbers, one positive, one negative, that have a difference of 31. Find two more such pairs.

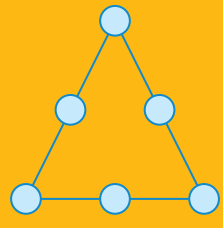
Puzzle

Pattern sums

Copy and complete each of the following patterns. Arrange the numbers given, one in each circle in the pattern, so that the sum of each connected straight line is equal to the given sum value.

1 -4, -3, -2, -1, 0, 1, 2; sum = -3

2 -2, -1, 0, 1, 2, 3; sum = 0



Challenge 2

1 What are the next three numbers in each pattern?

(a) 17, 13, 9, 5, …

(b) -31, -25, -19, -13, …

2 The product of three brothers' ages is 72 and their sum is 14. The youngest of the brothers are twins. What are the ages of the brothers?

3 Find three consecutive numbers (numbers that come one after another, such as 5, 6, 7) so that the sum of the first and third numbers is 172.

4 In a school there are 150 students in Year 7. For a performance in the school hall, chairs are arranged in rows, with the same number of chairs in each row, so that all 150 students are seated with no spare seats. If 10 more chairs were added to each row, everyone could be seated in 4 fewer rows, allowing the people in the back row to be closer to the stage. How many chairs were in each row in the original seating arrangement?

5 What is the sum of all the digits in the numbers from 1 to 100?

6 Jamie needs to know the total amount of money her friends have raised for the 'Save the Koala' fund. All they will tell her is this:

Siena and Levi have collected $130 between them.
If Levi and Marwa pooled their money, they would have $150.
Marwa and Allison have $100 altogether.
Allison and Kaya have just $70 in total.
If Kaya and Siena combined their money, they would have $90.
Tell Jamie what she needs to know.

7 A number less than 100 gives a remainder of 2 when divided by 4, a remainder of 3 when divided by 5, and a remainder of 4 when divided by 6. Find the number.

8 41 is a prime number. If the order of its digits is reversed, it becomes 14, which is *not* a prime number. How many two-digit prime numbers *do* give a prime number when their digits are reversed?

A 8 **B** 9 **C** 10 **D** 11

9 (a) How many zeros are at the end of the number given by the following multiplication?

$10 \times 11 \times 12 \times 13 \times 14 \times 15 \times 16 \times \ldots \times 19$

A 1 **B** 2 **C** 3 **D** 4

(b) How many zeros are at the end of the number given by the following multiplication?

$10 \times 11 \times 12 \times 13 \times 14 \times 15 \times 16 \times \ldots \times 29$

A 3 **B** 4 **C** 5 **D** 6

(c) How many zeros are at the end of the number given by the following multiplication?

$10 \times 11 \times 12 \times 13 \times 14 \times 15 \times 16 \times \ldots \times 59$

A 9 **B** 10 **C** 11 **D** 12

(d) How many zeros are at the end of the number given by the following multiplication?

$10 \times 11 \times 12 \times 13 \times 14 \times 15 \times 16 \times \ldots \times 99$

A 18 **B** 19 **C** 20 **D** 21

Chapter review 2

Maths literacy

common factor	factor	negative
common multiple	highest common factor (HCF)	positive
composite number	integers	prime factor
co-prime	loss	prime number
deposit	lowest common multiple (LCM)	profit
divisible	multiple	withdrawal

Copy and complete the following using the words and phrases from this list, where appropriate. A word or phrase may be used more than once.

1 A _____ of 6 is 18. The _____ of 6 and 4 is 12.

2 The _____ are all of the positive and negative whole numbers, and zero, which is neither positive nor negative.

3 1, 2, 3, 6, 9 and 18 are the _____s of 18. The _____ of 18 and 27 is 9.

4 A number that is not _____ by any numbers other than 1 and itself is called a _____.

5 A number with more than two factors is called a _____.

6 When you put money into a bank account, you are making a _____.

7 The addition of two negative numbers will always give a _____ answer.

8 If you sell something for less money than you bought it for, you have made a _____.

9 Every whole number greater than 1 can be written as the unique product of its _____s.

10 You make a _____ when you take money out of your bank account.

11 If you sell something for more than you bought it for, you have made a _____.

12 Two numbers are _____ if their highest common factor is 1.

Fluency

1 Find the LCM of: **2.1**

(a) 9 and 6 **(b)** 9 and 12 **(c)** 10 and 15

2 List all the factors of: **2.1**

(a) 36 **(b)** 48 **(c)** 51 **(d)** 100

3 Find the HCF of: **2.1**

(a) 24 and 56 **(b)** 18 and 72 **(c)** 45 and 80

4 Copy the following table and do the divisibility tests. Circle the numbers 2 3 4 5 6 9 10 if the number at the left is divisible by it. **2.1**

5301	2	3	4	5	6	9	10
10 000	2	3	4	5	6	9	10
333 333	2	3	4	5	6	9	10
31 700	2	3	4	5	6	9	10
43 521 820	2	3	4	5	6	9	10

5 State whether each of the following is a prime number or a composite number, and explain why. **2.2**

(a) 5 **(b)** 16 **(c)** 77 **(d)** 276 350

6 By drawing a factor tree or using the 'repeated division' method, express each number as a product of its prime factors. **2.3**

(a) 24 **(b)** 30 **(c)** 88 **(d)** 200

7 Use prime factors to find the HCF of the following. **2.3**

(a) 27 and 36 **(b)** 72 and 96 **(c)** 108 and 240

8 Write an integer to represent the following. **2.4**

(a) 14 degrees below zero **(b)** an altitude of 200 metres

9 State the opposite of: **2.4**

(a) north 5 km **(b)** adding 27

10 Write < or > between the following pairs of numbers to make a true statement. **2.4**

(a) -52 _____ 25 **(b)** 19 _____ -20

11 Write the following numbers in ascending order. **2.4**

(a) -7, 12, 0, -9, 7 **(b)** 4, -4000, 40, 400

12 Calculate: **2.5**

(a) +16 + 2 **(b)** -3 + 18 **(c)** -15 + 5 **(d)** +9 – 3

(e) +1 – 5 **(f)** +16 – 8 **(g)** +7 – 12 **(h)** -14 – 18

13 Calculate: **2.6**

(a) +7 + (-10) **(b)** +9 + (-6) **(c)** -11 – (-4) **(d)** -4 – (-4)

(e) -12 – (-5) **(f)** +5 + (-3) **(g)** -8 – (-5) **(h)** -5 + (-7)

14 Rewrite the following with a single sign between the integers, then evaluate. **2.7**

(a) 9 – (+11) **(b)** -3 + (+10) **(c)** -10 – (-21) **(d)** 8 + (-12)

(e) -4 – (-41) **(f)** -14 + (+28) **(g)** -5 – (+8) – (-2) **(h)** 4 + (-9) + (+2)

Understanding

15 Use the words 'multiple', 'factor' and 'divisible' to complete the following sentences. **2.1**

(a) 45 is _____ by 9, so that makes it a _____ of 9.

(b) 8 is a _____ of 56, so 56 is a _____ of 8.

(c) 27 is _____ by 3, so that makes it a _____ of 3.

16 If 96 lollies are to be divided into packets so that each packet contains the same number, how many lollies can be in each packet? Give all possible combinations. **2.1**

17 Use the symmetry of the number line to help you calculate the following. **2.6**

(a) -31 + 19 **(b)** -54 – 27 **(c)** -22 + (-38) **(d)** -9 – (61)

18 Describe the number line journey you could follow to find the value of: **2.6**

(a) +3 + (-8) **(b)** -6 – (+5)

19 For each pair of numbers, state whether or not they are co-prime. If not, explain why. **2.2**

(a) 11 and 27 **(b)** 51 and 63 **(c)** 14 and 35 **(d)** 24 and 55

20 In the game of indoor cricket, 5 runs are subtracted from a team's score every time a wicket is taken. Here is what happened in the first over of a game: **2.6**

Ball 1: 2 runs
Ball 2: wicket
Ball 3: 1 run
Ball 4: wicket
Ball 5: 1 run
Ball 6: 4 runs

What was the score at the end of the over?

21 There are 84 junior and 108 senior club members at an official club dinner. The dinner organiser wants to have an equal number of junior and senior members at each table. **2.3**

(a) Use prime factors to find the HCF of 84 and 108, and so find the number of tables required.

(b) Use your answer from **(a)** to find the number of junior and senior members at each table.

22 Find the first common multiple of 2, 7 and 9 that is greater than 500. **2.1**

23 **(a)** What is the first prime number after 70? **2.2**

(b) What is the first composite number after 70?

24 Michelle made deposits of $210, $25, $45 and $66 into her bank account during one month, and withdrawals of $35, $56, $214 and $102 during the same period. **2.6**

(a) At the end of the period, had her balance increased or decreased?

(b) By how much had it increased or decreased?

25 Joanna and Petra are on two different ferris wheels, both rotating clockwise. The first wheel takes 25 seconds to make a rotation and the other takes 30 seconds. If Joanna and Petra were both at eye level at the bottom of each of their ferris wheels when they start turning, how many seconds will pass until they are again both at the bottom at eye level? **2.6**

26 Miners in a copper mine are working 900 m underground. They get in a lift and travel a further 250 m down. What depth are they working at now? Write your working and answer using negative integers. **2.6**

Reasoning

27 Copy the following and write <, = or > to make true statements. **2.7**

(a) $4 - (-7) \_\_\_\_ 4 + 7$ **(b)** $-3 + 2 \_\_\_\_ 3 - 2$

(c) $5 + (-3) \_\_\_\_ 5 - (+3)$ **(d)** $-8 - 9 \_\_\_\_ -8 + 9$

28 If you know that a number is divisible by 8, what other numbers do you also know it is divisible by? **2.1**

29 A number between 900 and 1000 has four prime factors: 2, 5, 7 and one other factor. What is the number, and what is the missing factor? **2.1**

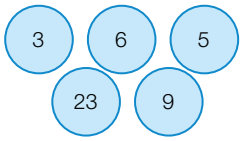
Numeracy practice 2

Non-calculator

1 Alicia is standing at -4 on a large number line placed on the floor. She walks 11 steps in the positive direction. At which number is she standing now?

A -15 **B** -7 **C** 7 **D** 15

2



The sum of the composite numbers in the group shown above is:

A 8 **B** 15 **C** 18 **D** 31

3 A maintenance worker in a city office building gets in a lift in the 3rd basement level (3 floors below ground level) and goes up 11 levels. What floor does the worker get out on?

4 The number 42 written as a product of prime factors is:

A 21×2 **B** $2 \times 3 \times 7$ **C** 1×42 **D** 6×7

5 On a sunny winter's day in Moscow, the temperature at midday was 3°C. By midnight it had dropped to -9°C. The integer that represents this change is:

A -12 **B** -3 **C** 3 **D** 12

Calculator allowed

6 In a remote town, once every week a train passes and every 10 days an aircraft flies overhead. If the train and the aircraft are observed at the town on a certain day, then how many more days will it be until both appear again at the same time?

A 10 **B** 11 **C** 50 **D** 70

7 Vin has $260 in his bank account. During one month, he makes the following transactions.

Deposit:	$55
Withdrawal:	-$75
Withdrawal:	-$33
Deposit:	$85
Withdrawal:	-$27

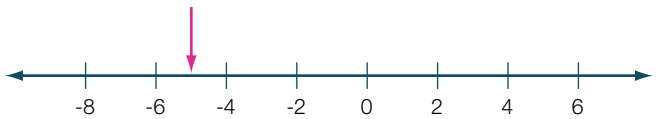
How much does Vin have in his account at the end of the month?

8 The ages (in years) of three people are 65, 39 and 52. The highest common factor of the three ages is:

A 1 **B** 13 **C** 39 **D** 165

9 The arrow is pointing to an integer on the number line.

What number is at this position?



Mixed review A

Fluency

1 Write these integers in ascending order. **2.4**

8, 17, -10, 0, -25, 32, -48

2 Write the following in index form. **1.2**

(a) 9 squared **(b)** $7 \times 7 \times 7 \times 7$ **(c)** 4 cubed

3 List all numbers divisible by both 8 and 6 that are less than 100. **2.1**

4 Write an integer to represent the following. **2.4**

(a) a bank withdrawal of $570 **(b)** a win by 5 points

5 Write < or > between the following pairs of numbers to make a true statement. **2.4**

(a) -27 ____ 14 **(b)** 0 ____ -35

6 Calculate: **1.3**

(a) $8000 \div 200$ **(b)** $1200 \div 4$ **(c)** $45\,000 \div 90$

(d) 30×120 **(e)** 400×1500 **(f)** 2000×5000

7 Use a mental strategy to calculate the following. **1.1**

(a) $4 \times 17 \times 5$ **(b)** $183 + 220$ **(c)** 42×19

(d) 36×11 **(e)** $169 + 71$ **(f)** $5 \times 24 \times 8$

8 Evaluate: **1.5**

(a) $6 \times 4 \div 2 \times 6$ **(b)** $5 + 6 \times 7$ **(c)** $18 + 12 - 7 + 6$

(d) $2 + 5 \times 9$ **(e)** $18 \div 6 - 3$ **(f)** $8 \times (15 - 5)$

9 Calculate the following. **2.5, 2.6**

(a) $-9 + 7$ **(b)** $5 + (-8)$ **(c)** $-3 - (-7)$ **(d)** $-6 - 11$

10 Estimate the answers to the following by rounding to the first digit. **1.4**

(a) 17×93 **(b)** 46×281 **(c)** 337×240

(d) $953 \div 11$ **(e)** $8195 \div 237$ **(f)** $12\,495 \div 5400$

11 Arrange the following numbers in ascending order (from smallest to largest). **2.4**

(a) 5, 0, -15, 10, -5 **(b)** -300, 3, 0, -30, 3000

12 Find the lowest common multiple of: **2.1**

(a) 8 and 12 **(b)** 12 and 16

13 Find the highest common factor of: **2.1**

(a) 36 and 27 **(b)** 64 and 72

14 List the factors of each of these numbers and state whether each number is prime or composite. **2.2**

(a) 18 **(b)** 23 **(c)** 44 **(d)** 79

15 Write each number as the product of its prime factors in index form. **2.3**

(a) 63 **(b)** 48 **(c)** 72 **(d)** 120

16 Simplify the following by writing a single sign between the values, then calculate the answer. **2.7**

(a) $-6 + (-7)$ **(b)** $4 - (-11)$ **(c)** $-2 + (+7)$

(d) $22 + (-9)$ **(e)** $-5 - (-10)$ **(f)** $18 + (+3)$

Understanding

17 A submarine 110 m below the surface of the water rises 80 m, then dives 150 m. What depth is it at now? **2.5**

18 Which of the following numbers are: **2.1, 2.2**

(a) prime

(b) perfect squares

(c) powers of 2?

3, 7, 9, 24, 11, 16, 19, 43, 32, 28, 13, 8, 2, 25

19 What is the first perfect cube that is divisible by both 3 and 4? **1.2, 2.1**

20 Calculate an approximate answer for the following by rounding to the first digit, then state whether the actual answer will be higher or lower. **1.4**

(a) 256×37 **(b)** 1379×24 **(c)** $5498 \div 46$

21 The Royal Easter Show runs for 7 days. The total attendance at one year's show was 62 982. **1.4**

(a) Approximately how many people per day was this? Use rounding to a convenient multiple of 1000 to calculate your answer.

(b) If each person paid an average ticket price of $12, use your answer from **(a)** to calculate how much money the show organisers made from ticket sales. Use some mental or written strategies to calculate your answer.

Reasoning

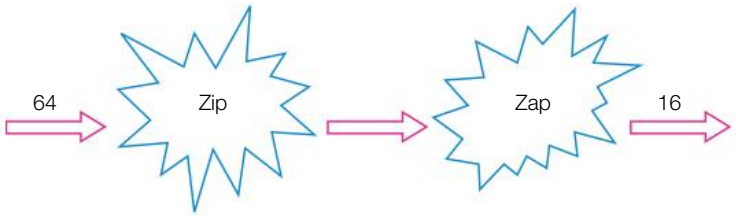
22 On Monday, Kiran withdrew $100 from his bank account at an ATM. On Tuesday, he used his account to pay his $85 phone bill online. On Wednesday, he deposited $250. On Friday, he withdrew another $60 from the account. **2.5**

(a) By the end of the week, did Kiran have more or less money in his bank account than at the start?

(b) How much more or less?

23 The number 64 passes through two 'magic clouds' Zip and Zap. It then emerges as the number 16. **1.2**

Which of the following can describe what Zip and Zap did to the number passing through?



A Zip: square root, Zap: square

B Zip: cube root, Zap: square

C Zip: square root, Zap: nothing

D Zip: nothing, Zap: cube root


3

Fractions

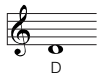
3

Frets make fractions.

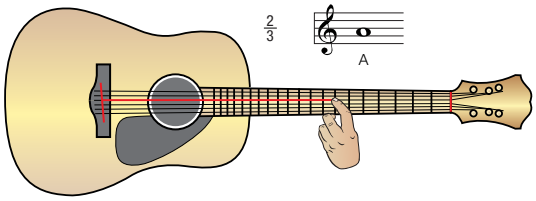
Learning the guitar or violin could help your understanding of fractions.

Musicians who play guitars, violins or other stringed instruments know that by pressing a finger to the string along the neck of the instrument, the string is shortened and so it makes a higher-sounding note. What they may not realise is that when the notes are in tune, the shortened string length is an exact fraction of the full ('open') string length

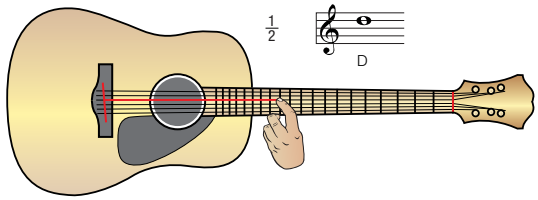
For example, on a guitar with standard tuning, plucking the third string gives the note 'D':



Shortening the string to $\frac{2}{3}$ of its length gives the note 'A'.



Shortening the string to $\frac{1}{2}$ of its length gives the note 'D' but an octave higher.



Guitar makers use these fractions to work out where to place the frets on the guitar neck. Violins do not have frets, so players must learn exactly where to place their fingers to make different notes. You will find out other ways in which fractions are involved in music later in this chapter.

Forum

Some studies claim that learning mathematics and learning music uses the same parts of the brain. What do you think? What could you do to investigate this theory?

Why learn this?

Whether with money, measurement or mixing a cake, we do not always work with whole numbers. Fractions are numbers that let you work with parts of wholes, so you need to be able to estimate and calculate with them.

After completing this chapter you will be able to:

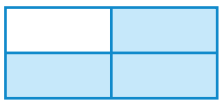
- identify and calculate fractions of a whole
- write one amount as a fraction of another
- convert between fractions, whole numbers and mixed numbers
- simplify fractions and find equivalent fractions
- use strategies to visualise, estimate, order and compare fractions
- add, subtract, multiply and divide fractions
- solve problems involving fractions.

Recall 3

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

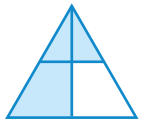
1 Which of the following have three-quarters of the whole shape shaded in?

A



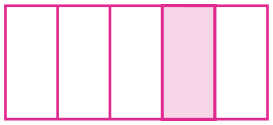
B

C

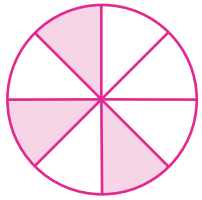


2 Write a fraction to represent each of the following diagrams or descriptions.

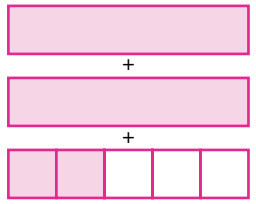
(a)



(b)



(c)



(d) One person's share of a birthday cake that was divided equally between 10 people.

(e) Joseph eats five out of eight equal-sized slices of a pizza.

(f) two-thirds **(g)** twelve-fifths **(h)** seven-sixteenths

3 (a) Write these in descending order (from largest to smallest): $\frac{3}{4}$, 1, 0, 2, $1\frac{1}{4}$

(b) Write these in ascending order (from smallest to largest): $\frac{3}{8}$, $\frac{1}{8}$, 0, $\frac{7}{8}$, $\frac{11}{8}$, 1, $\frac{9}{8}$

4 Calculate: **(a)** $\frac{4}{7}+\frac{1}{7}$ **(b)** $\frac{8}{11}-\frac{2}{11}$ **(c)** $\frac{2}{3}+\frac{2}{3}$

5 (a) Write the first five multiples of 8. **(b)** Write the first five multiples of 12.

(c) Write the lowest common multiple (LCM) of 8 and 12.

(d) Write all the factors of 24. **(e)** Write all the factors of 36.

(f) Write the highest common factor (HCF) of 24 and 36.

Exploration Task

You can download this activity from the eBook or the Pearson Places website.

Sharing again and again … makes more?

In this activity, you will explore what it means to divide a fraction by another fraction, to see whether this gives a smaller or larger result.

Understanding fractions

3.1

What is a fraction?

A **fraction** is a number that is used to show parts of a whole.

A whole might be an object, a collection of objects or a section of a number line.

If you are dividing a whole into parts to show a fraction, each of the parts must be equal in size.

If you are dividing up a collection of objects into groups to show a fraction, there must be the same number of objects in each group.

To write a fraction, two numbers are required.

$\frac{3}{5}$ ← The **numerator** tells you how many equal parts there are.

$\frac{3}{5}$ ← The **denominator** tells you how many equal parts one whole has been divided into.

Worked example 1 W.E. 1

How many smileys are there in $\frac{4}{5}$ of this collection?

Thinking	Working
1 The denominator shows us how many equal parts to divide the collection into. (Dividing 15 into 5 groups gives 3 smileys in each group.)	$15 \div 5 = 3$ $\frac{1}{5}$ is 3 smileys 
2 The numerator tells us how many of these 'equal parts' to count for the total. ($\frac{4}{5}$ means take 4 of the 5 groups.)	$4 \times 3 = 12$ 
3 Write the answer.	$\frac{4}{5}$ is 12 smileys

Types of fractions

Proper fractions have a numerator that is less than the denominator. They have values less than 1. Examples: $\frac{1}{2}$ and $\frac{3}{5}$.

Improper fractions have a numerator that is greater than or equal to the denominator. They have values greater than or equal to 1. Examples: $\frac{3}{2}$, $\frac{9}{9}$ and $\frac{17}{6}$.

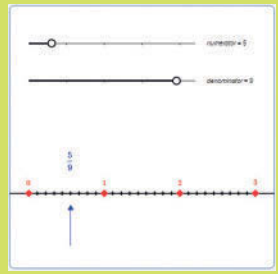
Mixed numbers have whole number parts and fraction parts written separately. Examples: $1\frac{1}{2}$, $2\frac{5}{6}$ and $34\frac{2}{7}$.

Interactive

Understanding fractions

Explore fractions on a number line by changing the numerator and the denominator.

Go to the eBook or the Pearson Places website to access this interactive.



Every whole number can be written as an improper fraction.

If an object is divided into 5 parts, then those 5 parts will make up the whole.

Using fractions to show this: $\frac{5}{5} = 1$, $\frac{10}{5} = 2$, $\frac{15}{5} = 3$ and so on.

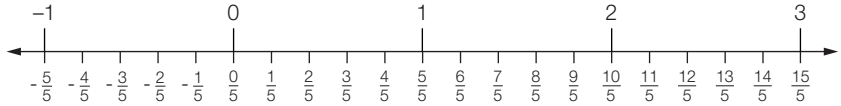
This also shows that a fraction is similar to a division:

$5 \div 5 = 1$, $10 \div 5 = 2$, $15 \div 5 = 3$.

Writing $\frac{5}{5}$ is the same as writing $5 \div 5$.

The simplest way to write a whole number as an improper fraction is to write it with a denominator of 1. For example, $3 = \frac{3}{1}$ (as $3 \div 1 = 3$).

Writing whole numbers as improper fractions



Looking at the positions of the whole numbers -1, 0 1, 2 and 3 on the number line above, notice that there is an improper fraction for every value greater than or equal to 1. Similarly there will be an improper fraction for every value less than or equal to -1.

$\frac{5}{5} = 1$, $\frac{10}{5} = 2$, $\frac{15}{5} = 3$, $-\frac{5}{5} = -1$ and $\frac{0}{5} = 0$

Remember that $5 \div 5 = 1$, $10 \div 5 = 2$, $15 \div 5 = 3$, $-5 \div 5 = -1$ and $0 \div 5 = 0$.

Worked example 2 W.E. 2

Write the whole number 9 as an improper fraction with a denominator of 4.

Thinking	Working
1 Which number divided by the denominator gives the required whole number? (Which number divided by 4 gives 9?)	$9 = \frac{\square}{4}$
2 Create the numerator by multiplying the whole number by the denominator required.	$\frac{9 \times 4}{4} = \frac{36}{4}$
3 Write the answer.	$9 = \frac{36}{4}$

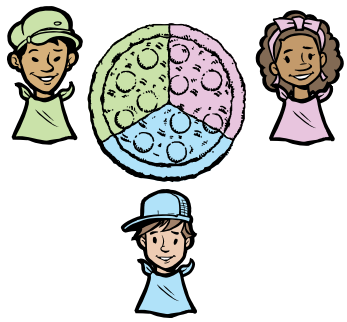
Drawing the line between the numerator and the denominator is equivalent to writing the division sign, ÷.

Fractions and division

You can understand all fractions in terms of division. Consider the following situation: 1 pizza is shared equally between 3 friends. How much of the pizza does each person get?

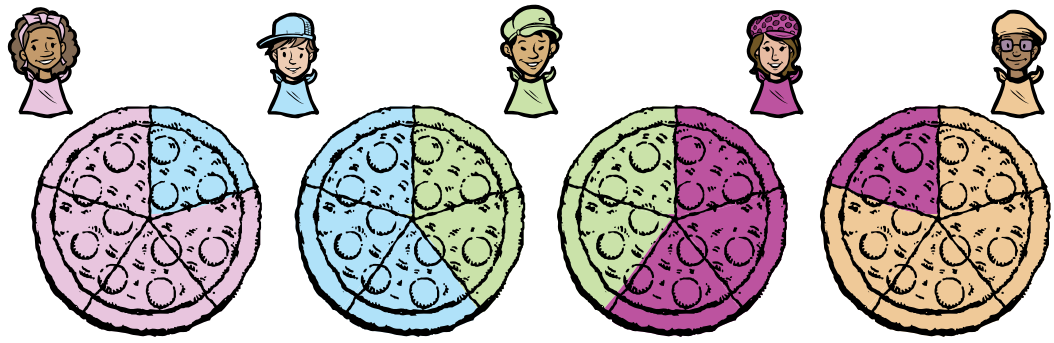
By drawing lines to divide the pizza into 3 equal pieces, you can see that each person will get $\frac{1}{3}$ of the pizza.

The different colours show which piece goes to which person.



You can write the division of 1 whole pizza among 3 friends as $1 \div 3 = \frac{1}{3}$.

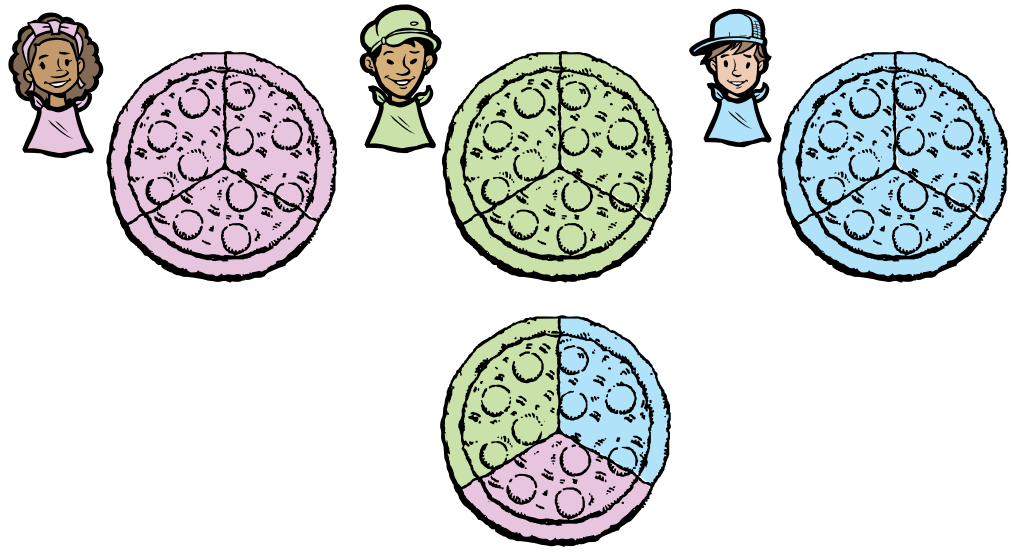
Similarly, if 4 pizzas are shared between 5 friends, each person gets $\frac{4}{5}$ of a pizza: $4 \div 5 = \frac{4}{5}$



If you have a greater number of pizzas than people, each person gets more than a whole pizza.

For example, 4 pizzas shared between 3 friends gives $4 \div 3 = \frac{4}{3}$ of a pizza each.

From the diagram, you can see that this is equal to 1 whole and $\frac{1}{3}$ of a pizza each, or $1\frac{1}{3}$.



Worked example 3

W.E. 3

For the following, write the amount each person receives as a fraction (or as a mixed number if appropriate).

5 blocks of chocolate are shared equally between 8 people.

Thinking	Working
1 Identify the whole and how many it is being shared between. (The whole is 5 and it is being shared between 8.) Write a division that shows this.	$5 \div 8$
2 Write this division as a fraction.	$= \frac{5}{8}$

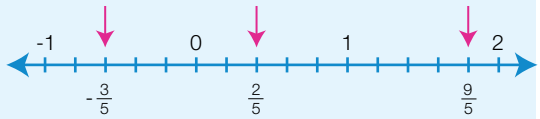
Using a number line to represent fractions

Placing fractions on a number line can help to compare their sizes to other numbers. Just like whole numbers, fractions can be positive or negative.

Drawing a number line is easier if you can make the distance between whole numbers the same as the denominator. For example, if dividing into thirds, make the distance 3 cm. Then, you can mark a third for every centimetre.

Worked example 4 W.E. 4

Copy this number line and show the positions of these fractions: $-\frac{3}{5}$, $\frac{2}{5}$ and $\frac{9}{5}$.

Thinking	Working
1 Copy the number line.	
2 The denominator tells us how to divide up the spaces between the whole numbers (in this case, in fifths). Make sure the distance between each marked division is the same.	
3 For each fraction, look at the numerator and count that many parts along from zero. Indicate the location of the fraction with an arrow.	

3.1 Understanding fractions

Navigator

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14 (a–d), 15, 17, 18 (a), 19, 20	1, 2, 3 (a), 4, 5, 6, 7 (columns 1–2), 8, 9, 11, 12, 13, 14, 15, 17, 18, 19, 20	2, 3 (b), 4, 5, 7 (column 3), 8 (c–e), 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20

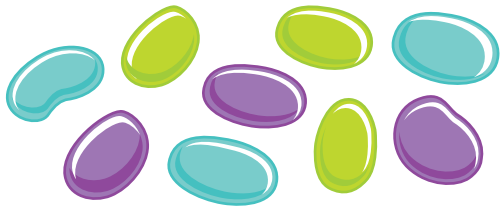
Answers p. 659

Fluency

1 (a) How many smileys are there in $\frac{1}{4}$ of this collection? W.E. 1

Hint: The denominator is 4, so divide the collection into 4 equal groups.

(b) How many jelly beans are there in $\frac{2}{3}$ of this collection?



W.E. 2

2 (a) Copy this number line and show the positions of these fractions: $\frac{3}{4}, \frac{8}{4}, \frac{5}{4}$ and $-\frac{1}{4}$.

(b) Copy this number line and show the positions of these fractions: $\frac{5}{6}, \frac{6}{6}, \frac{1}{6}$ and $-\frac{3}{6}$.

W.E. 3

3 (a) Write the whole number 5 as an improper fraction with a denominator of:

(i) 2 **(ii)** 7 **(iii)** 11 **(iv)** 5 **(v)** 1

(b) Write the whole number 13 as an improper fraction with a denominator of:

(i) 2 **(ii)** 5 **(iii)** 8 **(iv)** 13 **(v)** 1

W.E. 4

4 (a) For each of the following, write the amount each person receives as a fraction (or as a mixed number if appropriate).

(i) 1 pizza is shared equally between 2 people.

(ii) 2 apples are shared equally between 3 people.

(iii) 6 packets of lollies are shared equally between 5 people.

(iv) 10 packets of biscuits are shared equally between 7 people.

(b) In which of the above situations does a person receive more than one whole?

Understanding

5 Here are 16 lollies. How many will you eat if you eat these fractions of the total?

(a) $\frac{1}{4}$

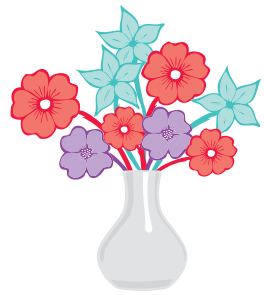
(b) $\frac{3}{4}$

(c) $\frac{1}{8}$

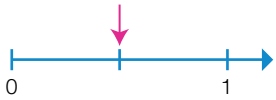
(d) $\frac{5}{8}$

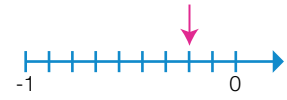
6 (a) What fraction of the bananas in this bunch are rotten?

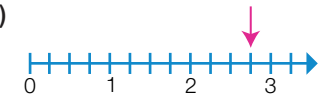
(b) What fraction of the flowers in this vase are red?

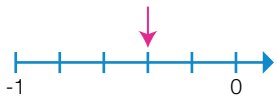


7 Write the value of the fraction indicated by the arrow on each of the number lines below.

(a)


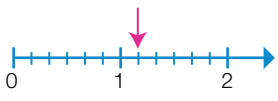
(b)


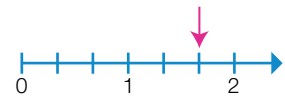
(c)


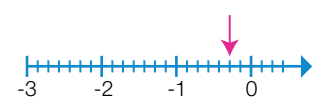
(d)


(e)


(f)


(g)


(h)


(i)


8 Write a fraction or a mixed number to show each of these:

(a) a numerator of 8 and a denominator of 17

(b) denominator of 4 and a numerator of 15

(c) nine stars in a packet of 20 stickers

(d) two wholes and two thirds

(e) 3 whole 24-piece blocks of chocolate, with 7 extra pieces.

9 Write a fraction to show each of these:

(a) 421 L of water in a 500 L rainwater tank.

(b) 157 mL of juice drunk from a 375 mL bottle

(c) 1 second out of a whole minute

(d) 17 minutes out of a whole hour

(e) the weekend days as a fraction of a whole week

(f) 1 hour out of a whole day

10 What fraction of this collection of shapes are:

(a) stars

(b) stars or hearts

(c) not hearts?

11 **(a)** Draw a diagram to show that if 5 blocks of chocolate are shared equally between 4 friends, then each person receives 1 full block and $\frac{1}{4}$ of a second block.

(b) Draw a diagram to show that if 5 pizzas are shared equally between 6 friends, then each person gets $\frac{5}{6}$ of a pizza.

12 Tim is 149 cm tall. His dad is 185 cm tall.

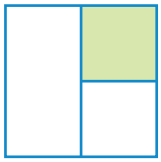
(a) Write Tim's height as a fraction of his dad's height.

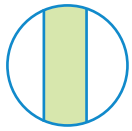
(b) Write his dad's height as a fraction of Tim's height.

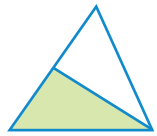
Reasoning

13 Four teenagers share 5 pizzas evenly between them. Five adults share 4 pizzas (of the same size) evenly between them. Write the fraction of pizza that each teenager and each adult gets. Who gets more pizza, a teenager or an adult?

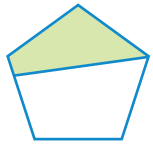
14 To show fractions, a whole must be divided into equal parts. For each of the shapes below, consider whether the shaded section represents one or more equal parts. State the fraction shown. If you don't think it is possible to state a definite fraction, explain why.

(a) 

(b) 

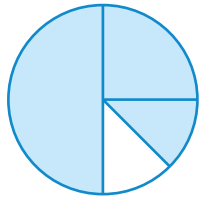
(c) 

(d) 

(e) 

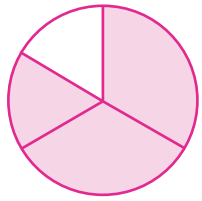
(f) 

15 The fraction of this circle that is shaded is:



A $\frac{3}{4}$ **B** $\frac{4}{5}$ **C** $\frac{5}{6}$ **D** $\frac{7}{8}$

16 The fraction of this circle that is shaded is:



A $\frac{3}{4}$ **B** $\frac{4}{5}$ **C** $\frac{5}{6}$ **D** $\frac{7}{8}$

Open-ended

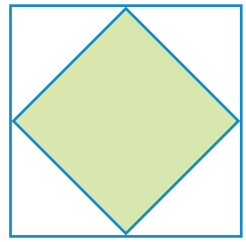
17 Draw a diagram that shows:

(a) three-eighths of an object **(b)** $\frac{5}{6}$ of a collection of objects **(c)** $\frac{9}{3}$

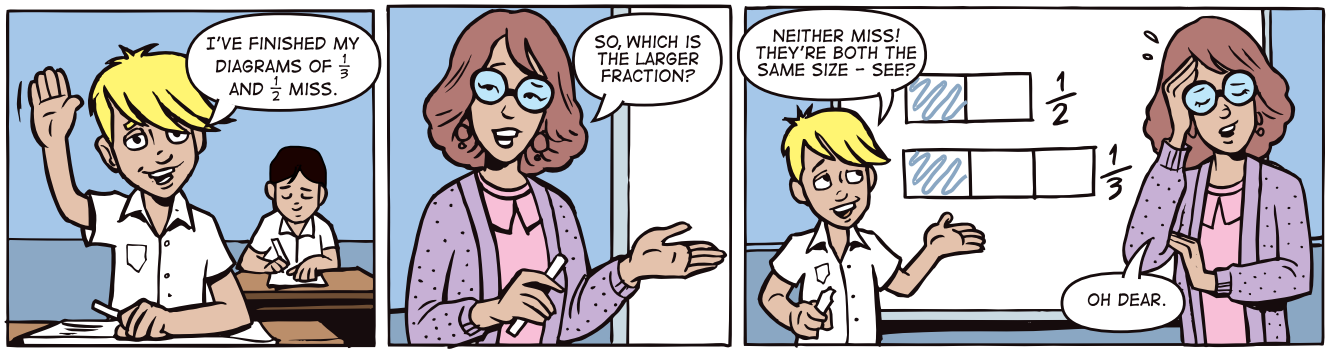
18 (a) This shape  represents $\frac{1}{5}$ of a whole. Draw two examples of what one whole could look like.

(b) This shape  represents $\frac{1}{4}$ of a whole. Draw two examples of what two wholes could look like.

19 Draw a diagram to help explain why the fraction of this tile that is shaded is $\frac{1}{2}$.



20



Oliver's diagram is supposed to show which of the fractions $\frac{1}{2}$ or $\frac{1}{3}$ is the larger fraction.

(a) Explain to Oliver what is wrong with his diagram.

(b) Draw a more accurate diagram to help you explain where he has gone wrong.

Puzzle

Fractionally funny fiction

Q: Which English ruler invented fractions?

To find out, write the letters that correspond to the fractions of each of the following words.

For example, 'the first $\frac{1}{4}$ of 'TERRIFIC' would be the letters 'TE' ('TE-RR-IF-IC')

The letters in each of the 4 boxes form a word.

When you have put together four words, rearrange them to find the answer.

The second $\frac{1}{5}$ of ENTHUSIASM
The middle $\frac{1}{5}$ of WHEEL

The first $\frac{1}{5}$ of KNIFE
The middle $\frac{1}{3}$ of FRINGE
The last $\frac{1}{4}$ of SONG

The last $\frac{1}{6}$ of ENDURE
The third $\frac{1}{6}$ of REFRIGERATOR
The last $\frac{1}{3}$ of HEIGHT
The middle $\frac{1}{7}$ of WASHING

The first $\frac{1}{6}$ of HEAVEN
The middle $\frac{1}{5}$ of CONVENTION
The last $\frac{1}{3}$ of PANTRY

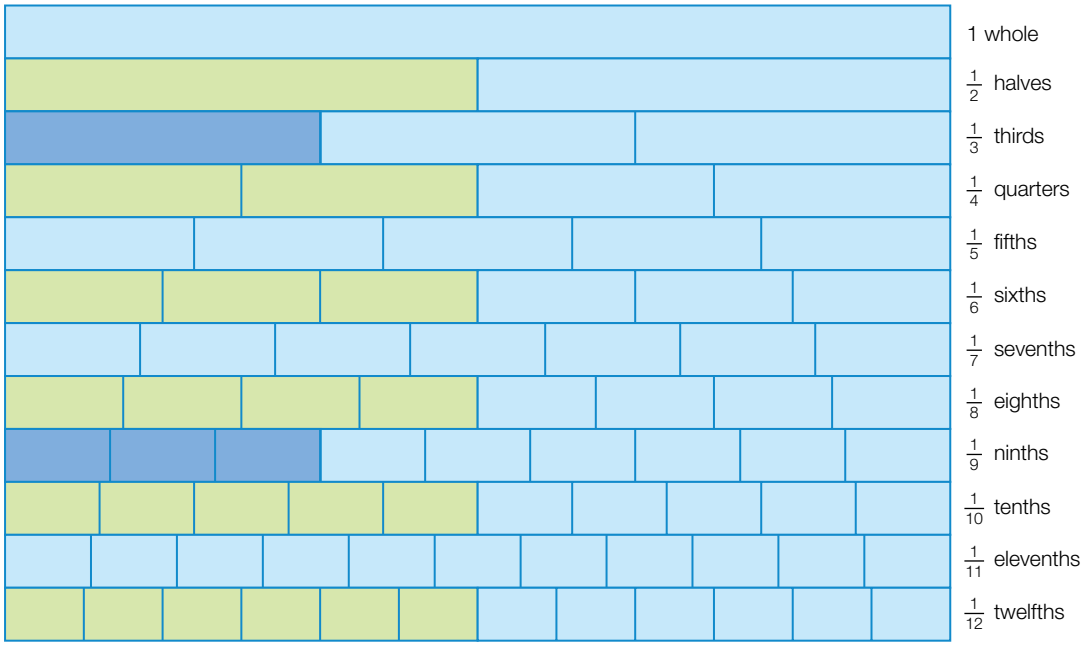
Figured it out?

Make up your own puzzle that uses fractions of words to make the answer to a joke or fact.

3.2 Working with fractions

The fraction wall

This fraction wall is made up of 12 layers of identical rectangles. One layer is left whole at the top, then each layer is divided into halves ($\frac{1}{2}$), thirds ($\frac{1}{3}$), quarters ($\frac{1}{4}$), fifths ($\frac{1}{5}$) … all the way down to twelfths ($\frac{1}{12}$). You can see that the size of the individual fractions (the 'bricks' in the wall) get smaller as you divide the whole into a greater number of pieces.



Equivalent fractions

If you examine the small vertical lines on the fraction wall, you can see that some lie exactly underneath each other (holding a ruler against the lines can help you see this more clearly). These lines in identical positions across the wall indicate **equivalent fractions**—fractions that represent the same amount, but with different numerators and denominators.

For example, if you consider $\frac{1}{2}$, you can see that $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$ (these fractions are all shaded in green on the wall).

If you consider $\frac{1}{3}$, you can see that $\frac{1}{3} = \frac{3}{9}$ (as $\frac{1}{3}$ is also equivalent to $\frac{2}{6}$ and $\frac{4}{12}$).

Can you see the pattern that links the numerators and the denominators of the fractions that are equivalent to $\frac{1}{2}$ and $\frac{1}{3}$?

Multiplying the numerator and the denominator of $\frac{1}{2}$ by 2 gives $\frac{2}{4}$: $\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$.

Similarly, multiplying the numerator and the denominator of $\frac{1}{2}$ by 3, 4, 5 and 6 gives $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$ and $\frac{6}{12}$.

Dividing the numerator and the denominator by a common factor also produces equivalent fractions: $\frac{3}{9} \div \frac{3}{3} = \frac{1}{3}$.

(As $\frac{2}{2}$ and $\frac{3}{3}$ are equivalent to 1 whole, multiplying or dividing a fraction by them is the same as multiplying or dividing by 1, which does not change the value of the fraction.)

Equivalent fractions are found either by multiplying or dividing both the numerator and the denominator by the same number.

Worked example 5 W.E. 5

Write pairs of equivalent fractions by copying and completing the following.

(a) $\frac{7}{10} = \frac{\square}{40}$ **(b)** $\frac{27}{36} = \frac{3}{\square}$

Thinking	Working
(a) 1 Compare the two denominators to determine what you need to multiply the first denominator by to get the second. (To get 40, multiply 10 by 4.)	(a) $\frac{7}{10} \overset{\times 4}{=} \frac{\square}{40}$ ($\times 4$)
2 Multiply the numerator by the same number to complete the equivalent fraction.	$\frac{7}{10} \overset{\times 4}{=} \frac{28}{40}$ ($\times 4$)
(b) 1 Compare the two numerators to determine what you need to divide the first numerator by to get the second. (To get 3, divide 27 by 9.)	(b) $\frac{27}{36} \overset{\div 9}{=} \frac{3}{\square}$ ($\div 9$)
2 Divide the denominator by the same number to complete the equivalent fraction.	$\frac{27}{36} \overset{\div 9}{=} \frac{3}{4}$ ($\div 9$)

Simplifying fractions

$\frac{1}{4}$ and $\frac{5}{20}$ are equivalent fractions: $\frac{1}{4} \times \frac{5}{5} = \frac{5}{20}$.

Although they both represent the same amount, $\frac{1}{4}$ is the **simplest form** of this pair of fractions.

A fraction in simplest form is often the easiest to understand and visualise.

To write a fraction in its simplest form, or to **simplify** it, divide the numerator and the denominator by their highest common factor (HCF).

Answers to fraction questions should always be written in simplest form.

For example, the HCF of 5 and 20 is 5, so dividing the numerator and denominator by 5 gives $\frac{1}{4}$, which is the simplest form of $\frac{5}{20}$.

The process of simplifying fractions is often called '**cancelling**' and can be shown in the following way. Divide the numerator by the HCF, cross it out and write the result of the division next to it.

Repeat this for the denominator.

For example: $\frac{\overset{1}{\cancel{5}}}{\underset{4}{\cancel{20}}} = \frac{1}{4}$

Sometimes, if the HCF is not obvious, you can do several divisions by smaller factors, such as 2 or 3. This is shown in Method 2 of the example below.

Worked example 6 W.E. 6

Write the following fraction and mixed number in their simplest form.

(a) $\frac{24}{60}$ **(b)** $3\frac{12}{28}$

Method 1: Find the HCF

Thinking		Working
(a) **1**	Find the HCF of the numerator and the denominator.	(a) HCF of 24 and 60 is 12
2	Divide both the numerator and the denominator by the HCF.	$\frac{24}{60} = \frac{2}{5}$ (÷ 12 above and below)
3	Write the answer.	$\frac{2}{5}$
(b) **1**	Find the HCF of the numerator and the denominator of the fraction part.	(b) HCF of 12 and 28 is 4
2	Divide both the numerator and the denominator by the HCF leaving the whole number part untouched (this is shown here as cancelling).	$3\frac{\overset{3}{\cancel{12}}}{\underset{7}{\cancel{28}}}$
3	Write the answer including the whole number part.	$= 3\frac{3}{7}$

Method 2: Do several small divisions

Thinking		Working
(a) **1**	Divide by any small common factor such as 2, 3 or 5. (Here, both the numerator and the denominator are even, so divide by 2.)	(a) $\frac{24}{60} = \frac{12}{30}$ (÷ 2 above and below)

2 Continue to divide by this or another small factor until you cannot divide any further. (Here, divide by 2 again, then by 3.)

$$\frac{12}{30} \overset{\div 2}{=} \frac{6}{15}$$

$$\frac{6}{15} \overset{\div 3}{=} \frac{2}{5}$$

3 Write the answer. $= \frac{2}{5}$

(b) 1 Divide by any small common factor such as 2, 3 or 5. (Here, both the numerator and the denominator are even, so divide by 2.)

(b) $3\frac{\cancel{12}^{6}}{\cancel{28}_{14}}$

2 Continue to divide by this or another small factor until you cannot divide any further. (Here, divide by 2 again.)

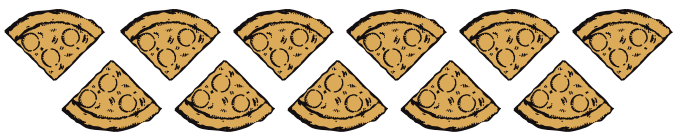
$= 3\frac{\cancel{6}^{3}}{\cancel{14}_{7}}$

3 Write the answer. $= 3\frac{3}{7}$

Mixed numbers and improper fractions

Improper fractions that have a value greater than 1 can be written as mixed numbers. To write an improper fraction as a mixed number, you need to see how many 'wholes' you can make, and what fraction you have left over.

For example, here are 11 slices of pie. Each slice represents $\frac{1}{4}$, so the total here is $\frac{11}{4}$.



With 11 quarter slices, you can make 2 whole pies with 3 slices left over, or $2\frac{3}{4}$.

This is the same as doing $11 \div 4 = 2$ remainder 3. The 3 becomes the numerator of the fraction part of the mixed number with the same denominator, 4.

To write an improper fraction as a mixed number, divide the numerator by the denominator to get the whole number part. Then use the remainder as the numerator of the fraction part.

Worked example 7 W.E. 7

Write the improper fraction $\frac{34}{7}$ as a mixed number.

Thinking	Working
1 Divide the numerator by the denominator. Include the remainder with your answer.	$34 \div 7 = 4$ rem 6
2 Write the whole number part and the remainder becomes the numerator of the fraction part.	$\frac{34}{7} = 4\frac{6}{7}$

Writing a mixed number as an improper fraction is the opposite of the above process. Instead of forming wholes from fraction parts, 'cut up' whole numbers into fraction parts, and count the total number of parts there are.

To write a mixed number as an improper fraction, write the whole number part as an improper fraction with the same denominator. Then, add the proper fraction part.
Note: When adding fractions, if the denominator is the same, add the numerators.

Worked example 8 W.E. 8

Write the mixed number $7\frac{3}{5}$ as an improper fraction.

Thinking	Working
1 Write the whole number part as an improper fraction by multiplying the whole number by the denominator. (Here, 7×5 tells you how many fifths there are in 7.)	$7 = \frac{7 \times 5}{5}$
2 Add the two fractions by adding the numerators.	$\frac{35}{5} + \frac{3}{5} = \frac{38}{5}$
3 Write the mixed number with its equivalent proper fraction.	$7\frac{3}{5} = \frac{38}{5}$

Can you see a shortcut for these types of questions? Multiply the whole number by the denominator, add the numerator, and place the answer over the denominator:

$7\frac{3}{5} = \frac{38}{5}$.

3.2 Working with fractions

Navigator

1 (columns 1–3), 2 (columns 1–3), 3 (a–h), 4 (a–h), 5, 6, 7, 8, 9, 10, 11, 12 (a–b), 14, 15, 17, 18 (a–c), 21, 22	1 (columns 2–4), 2 (columns 1–3), 3 (columns 3–4), 4 (columns 1–3), 5, 6, 7, 8, 9, 10, 11, 12 (a–c), 13, 14, 15, 16, 17, 18, 19, 21, 22	1 (columns 3–4), 2 (columns 3–4), 3 (columns 1–2), 4 (columns 3–4), 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22

Answers p. 660

Fluency

1 Write pairs of equivalent fractions by copying and completing the following.

W.E. 5

(a) $\frac{3}{4} = \frac{\square}{20}$ (b) $\frac{1}{3} = \frac{7}{\square}$ (c) $\frac{1}{2} = \frac{\square}{52}$ (d) $\frac{2}{5} = \frac{16}{\square}$

(e) $\frac{8}{100} = \frac{\square}{25}$ (f) $\frac{15}{100} = \frac{\square}{20}$ (g) $\frac{42}{70} = \frac{6}{\square}$ (h) $\frac{7}{11} = \frac{\square}{99}$

(i) $\frac{4}{3} = \frac{\square}{27}$ (j) $\frac{28}{21} = \frac{4}{\square}$ (k) $\frac{24}{16} = \frac{\square}{4}$ (l) $\frac{72}{36} = \frac{\square}{9}$

(m) $\frac{8}{\square} = \frac{88}{55}$ (n) $\frac{\square}{5} = \frac{42}{35}$ (o) $\frac{54}{\square} = \frac{9}{6}$ (p) $\frac{\square}{36} = \frac{4}{9}$

2 Write the following fractions and mixed numbers in their simplest form.

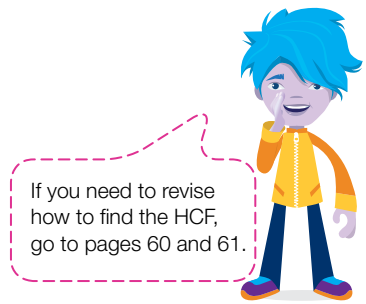
W.E. 6

(a) $\frac{5}{10}$ (b) $\frac{3}{9}$ (c) $\frac{3}{12}$ (d) $\frac{7}{21}$

(e) $\frac{4}{10}$ (f) $\frac{8}{22}$ (g) $\frac{10}{16}$ (h) $\frac{36}{24}$

(i) $2\frac{14}{21}$ (j) $3\frac{5}{30}$ (k) $1\frac{6}{20}$ (l) $12\frac{24}{30}$

(m) $6\frac{20}{45}$ (n) $3\frac{25}{100}$ (o) $2\frac{35}{42}$ (p) $100\frac{12}{84}$



3 Write these improper fractions as mixed numbers.

W.E. 7

(a) $\frac{7}{5}$ (b) $\frac{13}{6}$ (c) $\frac{23}{4}$ (d) $\frac{15}{7}$

(e) $\frac{37}{10}$ (f) $\frac{48}{5}$ (g) $\frac{44}{7}$ (h) $\frac{59}{10}$

(i) $\frac{107}{100}$ (j) $\frac{97}{40}$ (k) $\frac{35}{3}$ (l) $\frac{79}{8}$

4 Write these mixed numbers as improper fractions.

W.E. 8

(a) $1\frac{3}{4}$ (b) $3\frac{1}{5}$ (c) $6\frac{2}{3}$ (d) $5\frac{9}{10}$

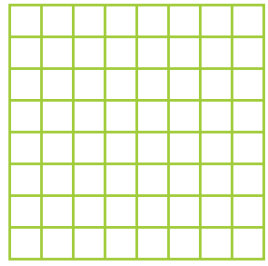
(e) $4\frac{3}{8}$ (f) $10\frac{2}{7}$ (g) $6\frac{8}{11}$ (h) $3\frac{9}{100}$

(i) $6\frac{8}{9}$ (j) $2\frac{4}{7}$ (k) $7\frac{1}{6}$ (l) $9\frac{4}{15}$

5 Talia shaded a fraction of this long rectangle.

(a) What fraction of this long rectangle is shaded?

(b) How many squares on this grid should Talia shade to show a fraction equivalent to the fraction she shaded in the rectangle above?



6 (a) Which one of the following is the simplest form of $3\frac{35}{45}$?

A $3\frac{5}{9}$ **B** $3\frac{3}{4}$ **C** $3\frac{7}{9}$ **D** $3\frac{70}{90}$

(b) Which of the following fractions is not equivalent to $\frac{10}{15}$?

A $\frac{2}{3}$ **B** $\frac{30}{45}$ **C** $\frac{40}{60}$ **D** $\frac{20}{25}$

(c) As a mixed number, $\frac{33}{4}$ equals:

A $3\frac{3}{4}$ **B** $7\frac{4}{5}$ **C** $8\frac{1}{4}$ **D** $8\frac{1}{3}$

(d) As an improper fraction, $2\frac{5}{8}$ equals:

A $\frac{7}{8}$ **B** $\frac{21}{8}$ **C** $\frac{25}{8}$ **D** 21

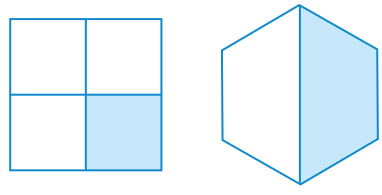
Understanding

7 Use the fraction wall on page 120 to identify the fraction or fractions that are equivalent to:

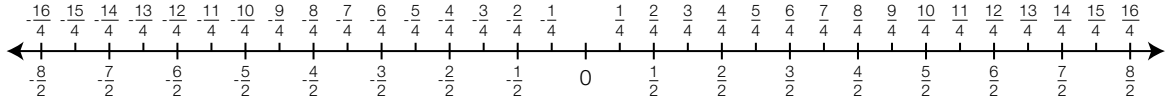
(a) $\frac{3}{5}$ **(b)** $\frac{2}{8}$ **(c)** $\frac{5}{6}$ **(d)** $\frac{9}{12}$

8 (a) Copy each of these figures. Write the fraction that is shaded.

(b) In each figure, rule two diagonal lines joining the opposite vertices (corners). Name the equivalent fraction of the shaded part that you have created.

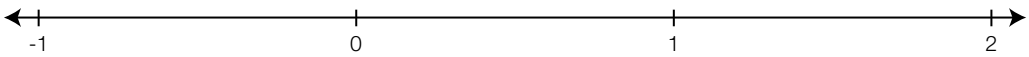


9 **(a)** The top side of this number line is divided into quarters, and the bottom side into halves.



What do you observe about the positions of the equivalent pairs of fractions such as $\frac{1}{2}$ and $\frac{2}{4}$, $\frac{2}{2}$ and $\frac{4}{4}$, $\frac{3}{2}$ and $\frac{6}{4}$, $-\frac{1}{2}$ and $-\frac{2}{4}$, and $-\frac{4}{2}$ and $-\frac{8}{4}$?

(b) Copy the number line below. Divide the top side into thirds and the bottom side into ninths. Use your observation from **(a)** to identify pairs of equivalent fractions.



10 Monica, Michelle and Travis are sharing an apple pie that has been cut into 12 equal slices. Monica and Michelle decide to have $\frac{1}{4}$ of the apple pie each and give Travis the remainder. How many slices of pie did each person receive? Draw a diagram that shows this information.

11 A block of chocolate has 24 individual squares. How many squares of the chocolate will be gone if $\frac{7}{12}$ of the chocolate is eaten?

12 Write your answers in simplest form.

(a) What fraction of an hour has passed from 3:49 pm to 3:51 pm?

(b) What fraction of an hour has passed from 11:03 am to 11:23 am?

(c) What fraction of an hour has passed from 8:13 pm to 8:25 pm?

(d) What fraction of an hour has passed from 2:36 am to 2:45 am?

13 Imagine that the whole rectangle on the top of the fraction wall at the start of this section (page 120) represents 1 hour. How many minutes would each of the individual fractions on the wall represent? Write any answers that are not whole numbers as mixed numbers.

Reasoning

14 Six pizzas were each sliced into 8 equal pieces.

(a) If $4\frac{3}{8}$ of the pizzas were eaten, how many pieces is that?

(b) Sara, Will, Josh and Ali ate 29 pieces between them. How many pizzas is this? Write your answer as a mixed number.

15 How many whole numbers would appear on the section of the number line between $\frac{4}{3}$ and $\frac{51}{9}$? (Hint: Convert these improper fractions to mixed numbers.)

16 Which of the following fractions could not be the simplified form of an original fraction that had a denominator of 40?

A $\frac{3}{8}$ **B** $\frac{2}{5}$ **C** $\frac{3}{7}$ **D** $\frac{9}{10}$

17 Look at the 'sevenths' row of the fraction wall at the start of this section (page 120). None of the little lines along the row have any matching lines in the same position in other rows. Are there any other rows like this? Explain what this shows.

18 For each of the following problems, it may be useful to draw and label a number line with the known fractions. Then, locate the position of the unknown fraction and identify it. Equivalent fractions can help you do this.

Which fraction is exactly in-between the following fractions on the number line?

(a) $\frac{2}{5}$ and $\frac{3}{5}$ **(b)** $\frac{3}{7}$ and $\frac{4}{7}$ **(c)** $\frac{1}{4}$ and $\frac{2}{4}$

(d) $\frac{6}{10}$ and $\frac{7}{10}$ **(e)** $-\frac{6}{9}$ and $\frac{6}{9}$ **(f)** $\frac{2}{11}$ and $\frac{7}{11}$

Open-ended

19 Write three fractions that are equivalent to $\frac{4}{5}$.

20 Write three improper fractions that simplify to $1\frac{2}{3}$.

21 For each of these pairs of fractions, draw a diagram that shows they are equivalent.

(a) $\frac{1}{3}, \frac{5}{15}$ **(b)** $\frac{3}{4}, \frac{12}{16}$

22 From the fraction wall at the start of this section (page 120), choose three fractions with three different denominators that have no equivalent fractions in the wall.

Puzzle

What fractions are we?

1 I am greater than $\frac{1}{2}$.

My numerator and denominator are both square numbers.

My numerator is an odd number.

The difference between my numerator and denominator is seven.

What fraction am I?

2 My numerator is a single-digit odd number.

My denominator is a two-digit number with both digits the same.

If you add 1 to both my numerator and denominator, you get a fraction equivalent to $\frac{1}{2}$.

What fraction am I?

3 My numerator is a multiple of 5.

My denominator is the LCM of 6 and 9.

I am less than 1.

I am in simplest form.

What fraction am I?

3.3 Estimating and comparing fractions

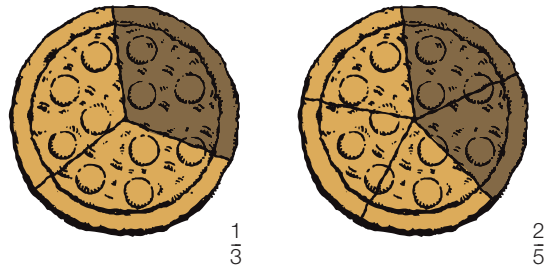
Comparing fractions using the lowest common denominator (LCD)

If two fractions have the same denominator, it is clear which fraction is bigger.

$\frac{2}{8}$ of a pie is obviously more pie than $\frac{1}{8}$ of a pie. (It is twice as big.)

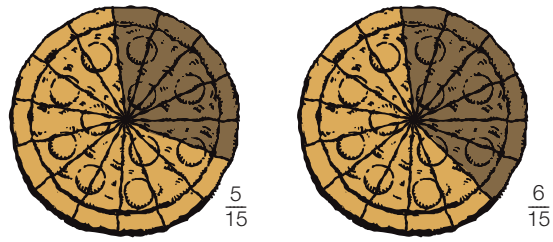
Comparing fractions with different denominators is harder. Having different denominators is like having different-sized pieces of pie.

For example, which is larger, $\frac{1}{3}$ or $\frac{2}{5}$?



We need to rewrite the fractions so they have the same denominator. This is like cutting the pie pieces into smaller, but equal-sized pieces. One way to do this is to find the **lowest common denominator (LCD)**. The LCD is the lowest common multiple of the denominators.

In this example the LCD is 15, so you should cut both pies into 15 pieces, then count how many pieces make up each fraction:



$\frac{1}{3} = \frac{5}{15}$ $\frac{2}{5} = \frac{6}{15}$

$\frac{2}{5}$ is the larger fraction, so write $\frac{2}{5} > \frac{1}{3}$.

To compare fractions using the lowest common denominator (LCD):

- find the lowest common multiple of the denominators
- rewrite each fraction as an equivalent fraction with this multiple as the new denominator
- write > (greater than), < (less than) or = (equal to) between the fractions.

If working with mixed numbers, write them as improper fractions first.

Worked example 9 W.E. 9

By finding the LCD, determine which fraction in each of the following pairs is larger.

(a) $\frac{3}{5}$ or $\frac{4}{7}$ (b) $\frac{15}{6}$ or $2\frac{7}{12}$

Thinking		Working
(a) 1	Write the list of multiples for each denominator.	(a) $\frac{3}{5}$ or $\frac{4}{7}$ Multiples of 5: 5, 10, 15, 20, 25, 30, 35, 40, … Multiples of 7: 7, 14, 21, 28, 35, 42, …
2	Find the lowest common multiple of these denominators (LCD).	LCD = 35
3	Multiply each fraction to get an equivalent fraction with the LCD as the denominator.	$\frac{3}{5} \times \frac{7}{7} = \frac{21}{35}$ $\frac{4}{7} \times \frac{5}{5} = \frac{20}{35}$
4	Use < or > to show which of the two fractions is larger.	$\frac{21}{35} > \frac{20}{35}$
5	Use < or > to show which of the original fractions is larger.	So, $\frac{3}{5} > \frac{4}{7}$
(b) 1	Write any mixed numbers as improper fractions.	(b) $2\frac{7}{12} = \frac{31}{12}$ Which is larger, $\frac{15}{6}$ or $\frac{31}{12}$?
2	Find the LCD.	LCD = 12
3	Multiply to get equivalent fractions with the LCD as the denominator. (Only one fraction needs to be multiplied here.)	$\frac{15}{6} \times \frac{2}{2} = \frac{30}{12}$
4	Use < or > to show which of the two fractions is larger.	$\frac{30}{12} < \frac{31}{12}$
5	Use < or > to show which of the original fractions is larger.	So, $\frac{15}{6} < 2\frac{7}{12}$

Comparing fractions by estimating and visualising

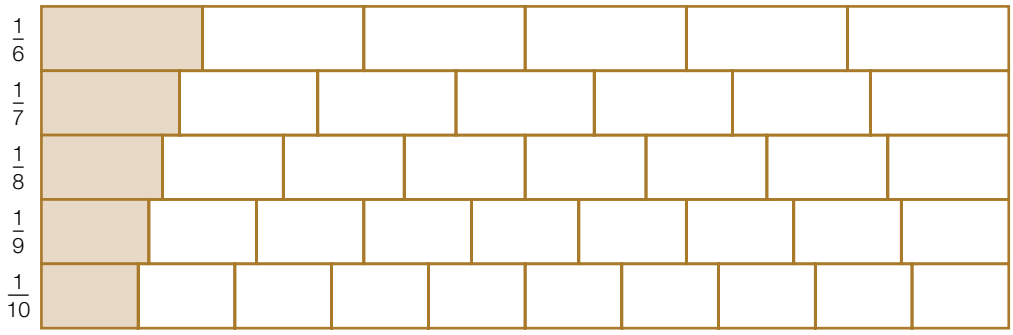
Often, you do not need to know an exact value of a fraction—it is enough to have an estimate. Similarly, when you compare two fractions, you only need to know which fraction is larger, not how much larger it is.

The following strategies can help you visualise what fractions 'look like', so you can estimate and compare them.

Strategy 1—Use the fraction wall

A fraction wall is very useful for comparing fractions. A section of the fraction wall from the start of this section (page 120) is shown here. It shows that as you divide a whole into more parts, the sizes of the parts (the 'bricks' in the wall) get smaller. (You are possibly already aware of this—the more people you share a pizza with, the smaller the amount that each person gets!)

The shaded parts of each layer of the fraction wall demonstrate that: $\frac{1}{6} > \frac{1}{7} > \frac{1}{8} > \frac{1}{9} > \frac{1}{10}$.



Looking at the fraction wall, you can see that:

$\frac{2}{6}$ is slightly larger than $\frac{2}{7}$, and much larger than $\frac{2}{9}$

$\frac{5}{6}$ is larger than $\frac{5}{8}$ and also larger than $\frac{6}{8}$.

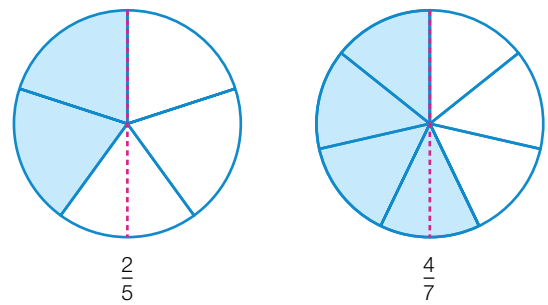
As the denominator numbers get larger, the fraction sizes get smaller.

If two fractions have the same numerator but different denominators, the *larger fraction* is the one with the *smaller number as the denominator*. For example, $\frac{5}{9} > \frac{5}{12}$, $\frac{3}{7} > \frac{3}{8}$.

Strategy 2—Compare to a common number

Visualise what a common number such as $\frac{1}{2}$, $\frac{1}{4}$, 1 or 2 'looks like' by imagining a shape, such as a circle or a rectangle, with that fraction shaded in. How would your fraction look in comparison? Would it be bigger or smaller?

For example, you can see that $\frac{2}{5} < \frac{4}{7}$ if you compare them to $\frac{1}{2}$.



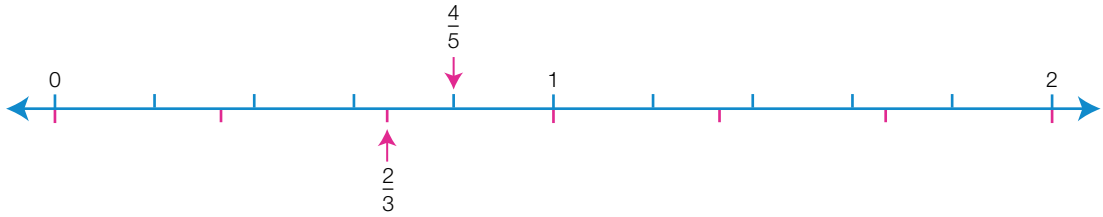
$\frac{2}{5}$ is less than $\frac{1}{2}$ and $\frac{4}{7}$ is greater than $\frac{1}{2}$, so $\frac{2}{5} < \frac{4}{7}$.

Strategy 3—Compare by using a number line

One number is larger than another if it lies to the right of it on the number line.

It is not always necessary to measure up the number line exactly—sometimes an accurate sketch will be sufficient.

For example, you can show that $\frac{4}{5} > \frac{2}{3}$ by sketching a number line and dividing one side into fifths, and the other side into thirds. You can then show that $\frac{4}{5}$ lies to the right of $\frac{2}{3}$, which means $\frac{4}{5}$ is larger than $\frac{2}{3}$.

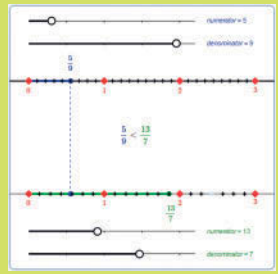


Interactive

Comparing fractions

Explore how to compare fractions by selecting different fractions on two number lines.

Go to the eBook or the Pearson Places website to access this interactive.



3.3 Estimating and comparing fractions

Navigator

Answers p. 661

1, 2, 3 (columns 1–3), 4, 5 (columns 1–3), 6, 8, 9, 11, 12 (a), 13, 14, 15, 18	1, 2, 3 (columns 2–3), 4 (columns 1–3), 5 (columns 1–2), 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 18	1, 3 (column 4), 4 (columns 3–4), 5 (columns 3–4), 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18

Equipment required: ruler for Question **1**

Fluency

1 (a) Draw a number line from -1 to 2. Make it 9 cm long, showing quarters along the top and thirds along the bottom. Use arrows to show the positions of the following fractions on the number line, then use the number line to write the fractions in ascending order (smallest to largest).

$\frac{2}{3}, \frac{6}{4}, -\frac{3}{4}, 1\frac{1}{3}$

(b) Draw a number line from -2 to 1. Make it 15 cm long, showing halves along the top and tenths along the bottom. Use arrows to show the positions of the following fractions on the number line, then use the number line to write the fractions in ascending order (smallest to largest).

$-1\frac{1}{2}, \frac{7}{10}, \frac{1}{2}, -\frac{3}{10}$

2 For each fraction state the nearest whole number.

(a) $\frac{2}{3}$ **(b)** $\frac{-9}{10}$ **(c)** $\frac{13}{12}$ **(d)** $7\frac{1}{3}$

(e) $-\frac{6}{3}$ **(f)** $\frac{1}{3}$ **(g)** $\frac{99}{100}$

3 By finding the LCD, determine which fraction in each of the following pairs is larger.

W.E. 9

(a) $\frac{3}{5}, \frac{7}{15}$ **(b)** $\frac{13}{12}, \frac{5}{4}$ **(c)** $\frac{7}{8}, \frac{5}{6}$ **(d)** $\frac{5}{9}, \frac{7}{12}$

(e) $\frac{2}{3}, \frac{5}{7}$ **(f)** $\frac{1}{6}, \frac{2}{9}$ **(g)** $\frac{3}{8}, \frac{5}{12}$ **(h)** $\frac{7}{4}, \frac{11}{6}$

(i) $1\frac{3}{4}, \frac{11}{8}$ **(j)** $\frac{5}{2}, 2\frac{1}{4}$ **(k)** $\frac{16}{7}, 2\frac{1}{5}$ **(l)** $2\frac{2}{7}, 2\frac{4}{9}$

4 Use the fraction wall on page 120 to determine which fraction in each of the following pairs is larger.

(a) $\frac{3}{7}$ or $\frac{3}{10}$ **(b)** $\frac{7}{9}$ or $\frac{7}{8}$ **(c)** $\frac{4}{5}$ or $\frac{5}{6}$ **(d)** $\frac{1}{3}$ or $\frac{2}{5}$

(e) $\frac{9}{12}$ or $\frac{3}{4}$ **(f)** $\frac{5}{11}$ or $\frac{4}{9}$ **(g)** $\frac{6}{7}$ or $\frac{7}{8}$ **(h)** $\frac{3}{5}$ or $\frac{7}{11}$

5 Copy the following fraction pairs. Use any appropriate strategy to decide which fraction is larger, and show this by writing < or > between them. If the fractions are equivalent, write = between them.

(a) $\frac{2}{7}$ _____ $\frac{5}{7}$ **(b)** $\frac{4}{5}$ _____ $\frac{4}{7}$ **(c)** $\frac{1}{2}$ _____ $\frac{5}{8}$ **(d)** $\frac{8}{11}$ _____ $\frac{9}{10}$

(e) $\frac{2}{4}$ _____ $\frac{3}{6}$ **(f)** $\frac{6}{12}$ _____ $\frac{7}{10}$ **(g)** $\frac{3}{7}$ _____ $\frac{5}{8}$ **(h)** $\frac{5}{9}$ _____ $\frac{2}{5}$

(i) $\frac{5}{6}$ _____ $\frac{7}{8}$ **(j)** $\frac{7}{11}$ _____ $\frac{8}{10}$ **(k)** $\frac{6}{10}$ _____ $\frac{3}{5}$ **(l)** $\frac{4}{5}$ _____ $\frac{6}{9}$

(m) $\frac{2}{4}$ _____ $\frac{4}{2}$ **(n)** $\frac{24}{6}$ _____ $\frac{8}{2}$ **(o)** $\frac{18}{11}$ _____ $\frac{9}{5}$ **(p)** $\frac{9}{7}$ _____ $\frac{11}{9}$

Understanding

6 Ali gets 7 out of 8 shots at goal in the basket. Joseph gets 10 out of 12 shots in. By comparing fractions, determine who is the more accurate shooter.

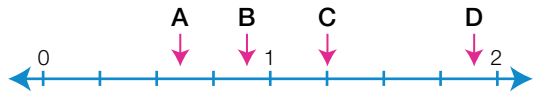
7 In a box containing 16 chocolates, 5 of them contain nuts. A larger box with 64 chocolates contains 23 nut chocolates. Which box contains the greater fraction of chocolates with nuts?

8 Use the fraction wall on page 120 to determine which fraction or fractions on the wall lie in between the following (include all equivalent fractions).

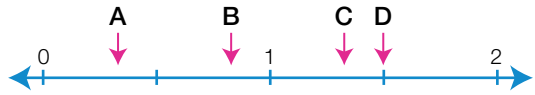
(a) $\frac{9}{10}$ and $\frac{11}{12}$ **(b)** $\frac{5}{8}$ and $\frac{7}{10}$

(c) $\frac{3}{8}$ and $\frac{4}{9}$ **(d)** $\frac{2}{11}$ and $\frac{3}{11}$

9 (a) Which arrow is pointing closest to the location of $\frac{9}{10}$ on this number line?



(b) Which arrow is pointing closest to the location of $1\frac{1}{3}$?



10 (a) Which one of the following is closest to $\frac{5}{8}$?

A $\frac{2}{9}$ **B** $\frac{1}{3}$ **C** $\frac{1}{2}$ **D** $\frac{7}{8}$

(b) Which one of the following is closest to 2?

A $\frac{11}{12}$ **B** $\frac{3}{2}$ **C** $\frac{7}{4}$ **D** $\frac{7}{3}$

Reasoning

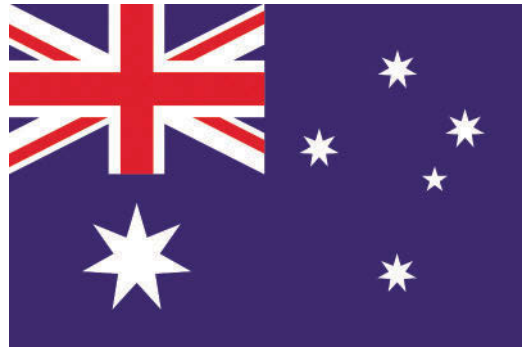
11 Write each of these lists in ascending order (from smallest to largest). Explain using a series of brief points, or steps, how you decided the order.

(a) $\frac{2}{3}, \frac{5}{6}, \frac{7}{8}, \frac{1}{2}, \frac{3}{4}, \frac{10}{11}$ **(b)** $\frac{4}{7}, 1, \frac{7}{4}, \frac{6}{7}, \frac{7}{8}, 0$

12 Write each of these lists in descending order (from largest to smallest). Explain briefly how you decided the order.

(a) $\frac{4}{7}, \frac{2}{9}, \frac{2}{3}, 1, \frac{11}{21}, \frac{45}{63}$ **(b)** $\frac{21}{16}, 1\frac{3}{80}, \frac{5}{4}, 1\frac{7}{20}, \frac{39}{40}, 1\frac{3}{10}$

13 **(a)** Estimate the fraction of this Australian flag that is blue.



(b) Estimate the fraction of Australia that is taken up by the state of Queensland.

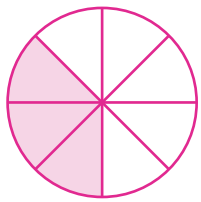
(c) Estimate the fraction of this jug that has juice in it.

14 For each of the following pairs of identical shapes, state whether the fraction of the shape that is shaded is larger in **A** or in **B**, or the same.

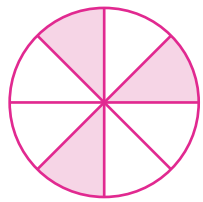
(a) **A**

B

(b) **A**

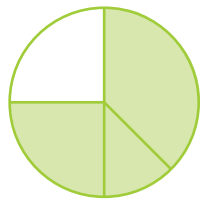


B

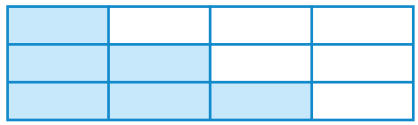


(c) **A**

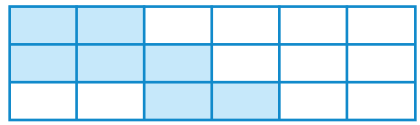
B



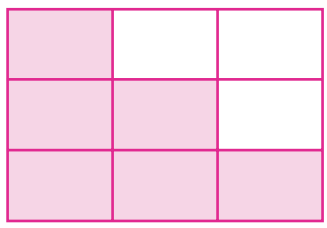
(d) **A**



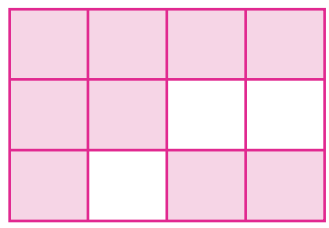
B

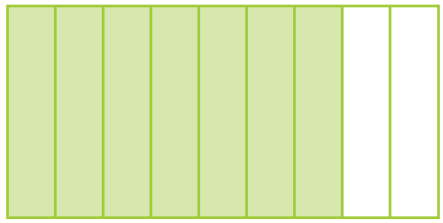
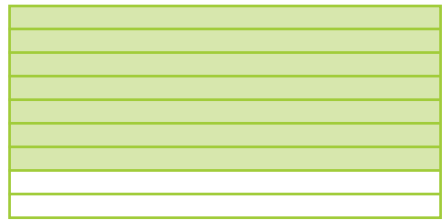


(e) **A**



B



(f) A  B 

15 Angus has 100 mL left of his 500 mL bottle of water. Gianni has 50 mL left in his 300 mL bottle.

(a) Who has drunk more water?

(b) Who has drunk a greater fraction of the water in their bottle?

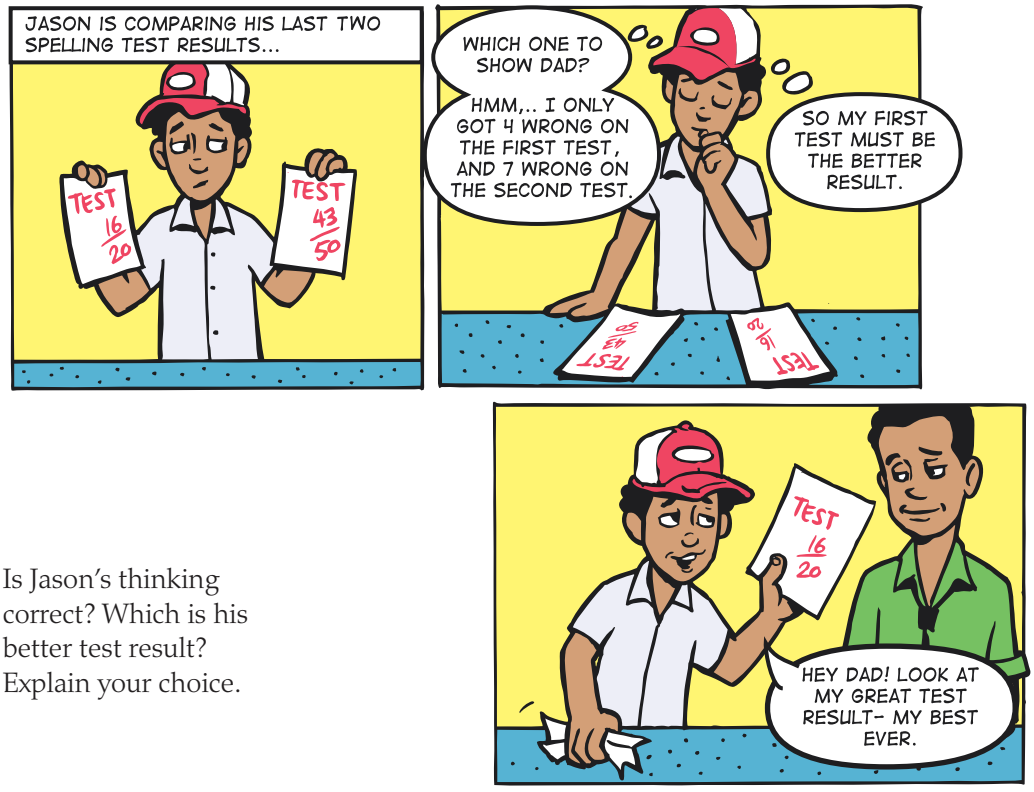
Open-ended

16 **(a)** A proper fraction is greater than $\frac{3}{4}$ but less than 1. What might the fraction be? Give two possibilities.

(b) An improper fraction is greater than 2 but less than 3. What might the fraction be? Give two possibilities.

17 Write three fractions that are greater than $\frac{2}{5}$ but less than $\frac{4}{5}$.

18



Is Jason's thinking correct? Which is his better test result? Explain your choice.

Game

Flatlining

Equipment required: 1 die, small counters or pieces of paper of two different colours or initials, a copy of the number line shown

Aim of the game:
The first player to get three circles in a row on the number line is the winner.

How to play:
Taking turns, roll the die and look up the number you rolled in the table below. Choose one of the fractions from the list. Find that fraction's approximate position on the number line and cover the circle above it with one of your counters or pieces of paper.

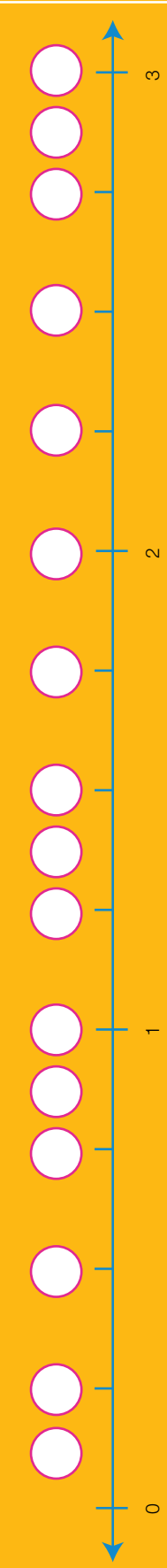
If you can't cover a circle you miss your turn.

Make sure your opponent locates their fraction correctly.

Once a circle is covered it cannot be taken by another player.

If all of the circles have been covered and neither player has three circles in a row, then the game is a draw.

Dice roll	Fractions to choose from		
1	$\frac{1}{4}$	$1\frac{1}{2}$	$\frac{7}{4}$
2	$\frac{1}{2}$	$\frac{5}{4}$	$2\frac{4}{5}$
3	$\frac{4}{3}$	$\frac{3}{4}$	$\frac{9}{4}$
4	3	$\frac{4}{5}$	$\frac{5}{2}$
5	2	$\frac{11}{4}$	$\frac{3}{3}$
6	1	$\frac{1}{5}$	$1\frac{3}{4}$



3.4 Adding and subtracting fractions

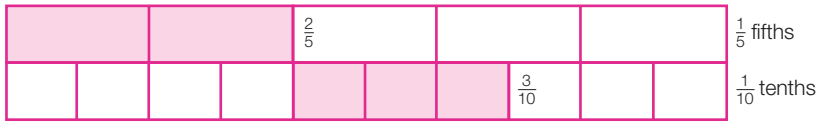
To add or subtract different fractions, they must have the same denominator. If the denominators are the same, the size of the fraction parts is the same, and you can simply count how many of each part you have.

For example: $\frac{3}{7} + \frac{2}{7} = \frac{5}{7}$ $\frac{3}{7} - \frac{2}{7} = \frac{1}{7}$.

The fraction wall can be useful for learning to add fractions.

For example, you can show $\frac{2}{3} + \frac{1}{6}$ by moving along the wall to $\frac{2}{3}$, then along $\frac{1}{6}$ to arrive at $\frac{5}{6}$. This sum is shaded in blue on the fraction wall below.

You can also show $\frac{2}{5} + \frac{3}{10}$ by moving along the wall to $\frac{2}{5}$, then along $\frac{3}{10}$ to arrive at $\frac{7}{10}$. This sum is shaded in pink on the fraction wall below.



These additions can be shown in this way because the denominators of the two fractions are related—you can write one of the fractions as an equivalent fraction with the same denominator as the second fraction.

$\frac{2}{3} + \frac{1}{6}$

$= \frac{4}{6} + \frac{1}{6}$

$= \frac{5}{6}$

$\frac{2}{5} + \frac{3}{10}$

$= \frac{4}{10} + \frac{3}{10}$

$= \frac{7}{10}$

Adding or subtracting fractions with different, unrelated denominators requires some additional steps. It is necessary to change both fractions into equivalent fractions with the same denominator. This new denominator will be a multiple of the denominators of the original fractions. The lowest common denominator (LCD) is often the most efficient one to use.

For example: $\frac{1}{3}+\frac{2}{5}$ LCD $= 3 \times 5 = 15$

$\frac{1}{3} = \frac{5}{15}$ $\frac{2}{5} = \frac{6}{15}$

Each fraction is now written in fifteenths, so you can find how many fifteenths there are:

$\frac{5}{15}+\frac{6}{15} = \frac{11}{15}$

Estimating answers to fraction problems

To check whether your answer to a fraction addition or subtraction is 'reasonable', estimate the answer before doing the calculation.

For example, the answer to $\frac{3}{5}+\frac{3}{7}$ will be approximately 1, as $\frac{3}{5}$ is slightly bigger than $\frac{1}{2}$ and $\frac{3}{7}$ is slightly smaller. (The actual answer is $\frac{36}{35}$ or $1\frac{1}{35}$, which is very close to 1.)

Worked example 10 W.E. 10

Calculate the following, giving your answers in simplest form. Use estimation to judge whether your answers are reasonable.

(a) $\frac{2}{3}+\frac{3}{4}$ **(b)** $\frac{7}{10}-\frac{1}{5}$

Thinking	Working
(a) 1 Estimate the answer first. (Here, both fractions are greater than $\frac{1}{2}$ but less than 1. The answer should lie between 1 and 2.)	(a) Estimate: between 1 and 2
2 Rewrite the fractions with the LCD as the denominator by multiplying the numerators and denominators by the necessary factors. (In this case, 4 and 3.)	$\frac{2}{3}+\frac{3}{4}$ LCD $= 3 \times 4 = 12$ $= \frac{2 \times 4}{3 \times 4}+\frac{3 \times 3}{4 \times 3}$ $= \frac{8}{12}+\frac{9}{12}$
3 Add the numerators.	$= \frac{17}{12}$
4 Write the answer as a mixed number if appropriate.	$= 1\frac{5}{12}$
5 Check your answer against your estimate. Is it reasonable?	Reasonable

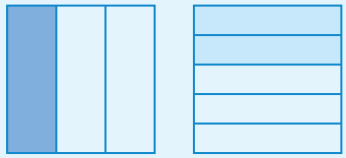
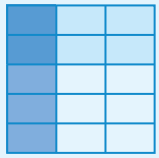
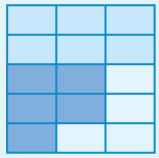
(b) 1	Estimate the answer first. (Here, we are subtracting $\frac{1}{5}$ from a fraction that is less than 1, so the answer will be about $\frac{1}{2}$.)	(b) Estimate: $\frac{1}{2}$
2	Rewrite the fractions with the LCD as the denominator by multiplying the numerators and denominators by the necessary factors. (Here, only one fraction needs to be rewritten.)	$\frac{7}{10} - \frac{1}{5}$ LCD = 10 $= \frac{7}{10} - \frac{2}{10}$
3	Subtract the numerators.	$= \frac{5}{10}$
4	Simplify the answer.	$= \frac{1}{2}$
5	Check your answer against your estimate. Is it reasonable?	Exact

The grid method is a visual way of adding fractions, as demonstrated in the following example. (However, the grid method does not work so well for subtracting fractions.)

Worked example 11

W.E. 11

Add $\frac{1}{3}$ and $\frac{2}{5}$ by using the grid method.

Thinking	Working
1 Draw two identical squares or rectangles. Divide them up and shade in each fraction.	$\frac{1}{3} + \frac{2}{5}$ 
2 Lay one square or rectangle on top of the other to form a grid.	
3 Where the grid squares are shaded twice due to overlap, shade an equal number of blank grid squares to 'remove' the overlap. The resultant shaded grid represents the fraction sum.	
4 Write the answer to the fraction sum.	$\frac{11}{15}$

Adding and subtracting mixed numbers

To add or subtract mixed numbers, you can either:

- add or subtract the whole number parts and fraction parts separately (Method 1), or
- convert the mixed numbers to improper fractions, then add or subtract them in the usual way (Method 2).

Worked example 12 W.E. 12

Calculate $4\frac{5}{6} + 2\frac{3}{4}$. Write your answer as a mixed number in simplest form.

Method 1: Add wholes and fractions separately

Thinking	Working
1 Estimate the answer first. Here, we have 6 wholes, and 2 fractions that are each bigger than $\frac{1}{2}$. The answer lies between 7 and 8.	Estimate: between 7 and 8 $4\frac{5}{6} + 2\frac{3}{4}$
2 Rearrange the addition by separating the whole numbers from the fractions.	$= 4 + 2 + \frac{5}{6} + \frac{3}{4}$
3 Add the whole numbers.	$= 6 + \frac{5}{6} + \frac{3}{4}$
4 Rewrite the fractions with the LCD as the denominator.	$= 6 + \frac{10}{12} + \frac{9}{12}$
5 Add the fractions and simplify if possible.	$= 6 + \frac{19}{12}$
6 If the fraction sum is an improper fraction, convert it to a mixed number.	$= 6 + 1\frac{7}{12}$
7 Add the whole numbers.	$= 7\frac{7}{12}$
8 Check your answer against your estimate. Is it reasonable?	Reasonable

Method 2: Use improper fractions

Thinking	Working
1 Estimate the answer first. Here, we have 6 wholes, and 2 fractions that are bigger than $\frac{1}{2}$. The answer lies between 7 and 8.	Estimate: between 7 and 8 $4\frac{5}{6} + 2\frac{3}{4}$
2 Write the mixed numbers as improper fractions.	$= \frac{29}{6} + \frac{11}{4}$
3 Rewrite the fractions with the LCD as the denominator.	$= \frac{58}{12} + \frac{33}{12}$
4 Add the numerators, simplify if possible.	$= \frac{91}{12}$
5 Convert to a mixed number.	$= 7\frac{7}{12}$

Worked example 13

W.E. 13

Calculate $6\frac{5}{8} - 2\frac{3}{4}$. Write your answer as a mixed number in simplest form.

Method 1: Subtract wholes and fractions separately

Thinking	Working
1 Estimate the answer first. Here, you are subtracting $2\frac{3}{4}$ from a number less than $6\frac{3}{4}$, so the answer will be less than 4.	Estimate: less than 4 $6\frac{5}{8} - 2\frac{3}{4}$
2 Rearrange the subtraction by separating the whole numbers from the fractions.	$= 6 - 2 + \frac{5}{8} - \frac{3}{4}$
3 Subtract the whole numbers.	$= 4 + \frac{5}{8} - \frac{3}{4}$
4 Rewrite the fractions with the LCD as the denominator.	$= 4 + \frac{5}{8} - \frac{6}{8}$
5 Subtract the fractions. If you cannot do the subtraction without getting a negative answer, 'borrow' 1 whole. (Here, 1 whole is $\frac{8}{8}$.)	$= 3 + \frac{8}{8} + \frac{5}{8} - \frac{6}{8}$ $= 3 + \frac{13}{8} - \frac{6}{8}$
6 Write the answer and simplify it if possible.	$= 3\frac{7}{8}$
7 Check your answer against your estimate. Is it reasonable?	Reasonable

Method 2: Use improper fractions

Thinking	Working
1 Make an estimate of the answer first, as for Method 1.	Estimate: less than 4 $6\frac{5}{8} - 2\frac{3}{4}$
2 Write the mixed numbers as improper fractions.	$= \frac{53}{8} - \frac{11}{4}$
3 Rewrite the fractions with the LCD as the denominator (only one fraction needs to be rewritten here).	$= \frac{53}{8} - \frac{22}{8}$ LCD = 8
4 Subtract the numerators.	$= \frac{31}{8}$
5 Simplify, if possible, and write your answer as a mixed number.	$= 3\frac{7}{8}$
6 Check your answer against your estimate. Is it reasonable?	Reasonable

Fractions on a calculator

The key for entering fractions into your calculator will usually look like $\square\!\!\square$. To enter a fraction, you would enter $\square\!\!\square$, then enter the numerator and the denominator, using the arrow keys to move between them. To enter a mixed number, you would usually enter SHIFT $\square\!\!\square$.

On some calculators, the key may look like $a\,{}^{b}\!/_{c}$ or ${}^{a}\!/_{b}$. To enter a fraction, such as $\frac{5}{6}$, you would enter 5 $a\,{}^{b}\!/_{c}$ 6. You may need to press = for the fraction to be displayed on the screen. To enter a mixed number, such as $4\frac{3}{8}$, you would enter 4 $a\,{}^{b}\!/_{c}$ 3 $a\,{}^{b}\!/_{c}$ 8. The calculator may use symbols such as ⌐, ⌙ or _ to separate the whole numbers, numerator or denominator.

Check that you can work with fractions correctly on your calculator by entering some additions and subtractions for which you already know the answers. For example, if you enter $\frac{3}{8} + 1\frac{2}{3}$ correctly, the answer of $2\frac{1}{24}$ should appear on the screen.

To convert a mixed number to an improper fraction, you can usually enter the mixed number followed by SHIFT and $a\,{}^{b}\!/_{c}$ or SHIFT and S⇔D. (You might need to press = to see the result.)

You may also notice that pressing the S⇔D key or pressing the $a\,{}^{b}\!/_{c}$ key twice will convert a fraction to its decimal form.

3.4 Adding and subtracting fractions

Navigator

1, 2 (a–l), 3, 4 (a–h), 5 (a–h), 6, 7, 8, 9, 10, 11, 13, 17	1, 2 (columns 1–2), 3, 4 (columns 1–2), 5 (columns 1–2), 6, 8, 9, 10, 11, 13, 14, 15, 16, 18	2 (columns 3–4), 3, 4 (columns 3–4), 5 (columns 3–4), 6, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18

Answers p. 662

Fluency

W.E. 10

1 Calculate the following, giving your answers in simplest form. Use estimation to judge whether your answers are reasonable.

(a) $\frac{2}{5} + \frac{1}{5}$ (b) $\frac{8}{7} + \frac{1}{7}$ (c) $\frac{3}{10} + \frac{5}{10}$ (d) $2\frac{1}{4} + 5\frac{3}{4}$

2 Calculate the following, giving your answers in simplest form. Use estimation to judge whether your answers are reasonable.

(a) $\frac{5}{6} + \frac{1}{3}$ (b) $\frac{2}{5} - \frac{7}{20}$ (c) $\frac{5}{8} - \frac{7}{40}$ (d) $\frac{4}{55} + \frac{2}{11}$

(e) $\frac{5}{14} + \frac{1}{7}$ (f) $\frac{15}{49} + \frac{5}{7}$ (g) $\frac{3}{5} + \frac{13}{20}$ (h) $\frac{3}{4} + \frac{1}{5}$

(i) $\frac{1}{4} + \frac{1}{6}$ (j) $\frac{5}{9} + \frac{3}{4}$ (k) $\frac{1}{6} + \frac{3}{7}$ (l) $\frac{11}{15} + \frac{7}{20}$

(m) $\frac{23}{30} - \frac{7}{20}$ (n) $\frac{11}{12} + \frac{3}{10}$ (o) $\frac{3}{5} + \frac{9}{14}$ (p) $\frac{41}{45} - \frac{11}{18}$

W.E. 11

3 Add the following fractions using the grid method.

(a) $\frac{2}{3}+\frac{1}{2}$ (b) $\frac{3}{4}+\frac{1}{7}$

W.E. 12

4 Calculate the following. Write your answers as mixed numbers in simplest form.

(a) $3\frac{1}{7}+4\frac{2}{7}$ (b) $8\frac{1}{9}+5\frac{5}{9}$ (c) $2\frac{3}{8}+7\frac{1}{4}$ (d) $6\frac{7}{10}+1\frac{1}{5}$

(e) $1\frac{3}{4}+1\frac{1}{2}$ (f) $3\frac{1}{5}+2\frac{9}{10}$ (g) $4\frac{1}{2}+2\frac{3}{4}$ (h) $1\frac{7}{12}+7\frac{2}{3}$

(i) $6\frac{4}{5}+5\frac{1}{2}$ (j) $3\frac{8}{9}+7\frac{1}{6}$ (k) $1\frac{4}{11}+3\frac{2}{5}$ (l) $1\frac{4}{25}+2\frac{3}{20}$

W.E. 13

5 Calculate the following. Write your answers as mixed numbers in simplest form.

(a) $3\frac{5}{7}-2\frac{1}{7}$ (b) $3\frac{2}{9}-2\frac{5}{9}$ (c) $5\frac{7}{9}-1\frac{1}{3}$ (d) $7\frac{11}{12}-6\frac{1}{4}$

(e) $3\frac{1}{12}-1\frac{5}{6}$ (f) $4\frac{7}{10}-2\frac{1}{5}$ (g) $4\frac{1}{10}-2\frac{4}{5}$ (h) $6\frac{1}{18}-4\frac{1}{6}$

(i) $2\frac{1}{9}-1\frac{1}{6}$ (j) $2\frac{1}{4}-1\frac{2}{5}$ (k) $2\frac{1}{8}-1\frac{1}{6}$ (l) $5\frac{1}{8}-2\frac{2}{3}$

6 Find the following. You might like to use your calculator to check your answers.

(a) $\frac{3}{8}+\frac{1}{6}+\frac{1}{3}$ (b) $\frac{1}{2}+\frac{3}{4}-\frac{1}{3}$ (c) $\frac{7}{10}+\frac{4}{5}-\frac{1}{2}$

(d) $1\frac{1}{2}+\frac{2}{5}+2\frac{3}{10}$ (e) $2\frac{1}{3}+5\frac{3}{4}-1\frac{2}{5}$ (f) $3\frac{1}{4}+2\frac{1}{5}-3\frac{1}{3}$

7 Choose the correct answer to each of the following.

(a) $\frac{5}{8}-\frac{1}{3}=$

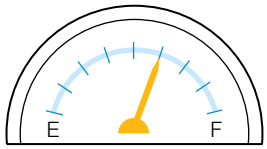
A $\frac{4}{24}$ B $\frac{7}{24}$ C $\frac{4}{5}$ D $\frac{23}{24}$

(b) $1\frac{2}{3}+\frac{3}{20}=$

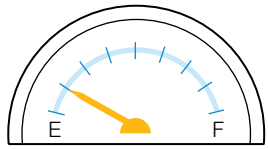
A $1\frac{1}{17}$ B $1\frac{5}{23}$ C $1\frac{5}{20}$ D $1\frac{49}{60}$

Understanding

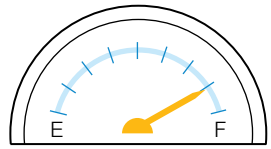
8 When Eric got in his car and started driving, his fuel gauge looked like this:



Eric stopped to get petrol when his fuel gauge looked like this:



After putting some petrol in his car, Eric's fuel gauge looked like this:



Write a fraction addition or a subtraction to answer the following.

(a) What fraction of the tank of fuel did Eric use between the first two gauge readings? Write your answer in simplest form.

(b) What fraction of the tank of petrol did Eric add when he was at the petrol station? Write your answer in simplest terms.

9 What fraction of a large pizza was eaten if Steven had $\frac{5}{12}$ of the pizza and Lara had $\frac{1}{3}$ of it? Write your answer in simplest form.

10 Jaydeep, Corey and Isaac were sharing the driving on a trip. If Jaydeep drove $\frac{3}{5}$ of the distance and Isaac drove $\frac{3}{20}$, what fraction of the total distance did Corey drive?

11 Belinda worked part-time at a cafe. Her hours for three days she worked one week were:

$2\frac{1}{4}$, $3\frac{1}{2}$ and $4\frac{1}{3}$.

Using fractions, calculate the total number of hours she worked for the week.

12 Five large packets of cereal were bought for a Year 7 camp. On the first day $1\frac{1}{3}$ packets were eaten, on the second day $\frac{3}{4}$ of a packet was eaten and on the third day $2\frac{1}{6}$ packets were eaten. How much cereal was available for the fourth day?

Reasoning

13 A fraction is added to $\frac{3}{5}$. The lowest common denominator for the two fractions is 40.

Which of the following fractions was added?

A $\frac{2}{6}$ **B** $\frac{3}{8}$ **C** $\frac{13}{20}$ **D** $\frac{11}{15}$

14 A fraction is added to $\frac{3}{10}$. Which of the following would give an improper fraction as the result?

A $\frac{7}{15}$ **B** $\frac{7}{12}$ **C** $\frac{3}{5}$ **D** $\frac{18}{25}$

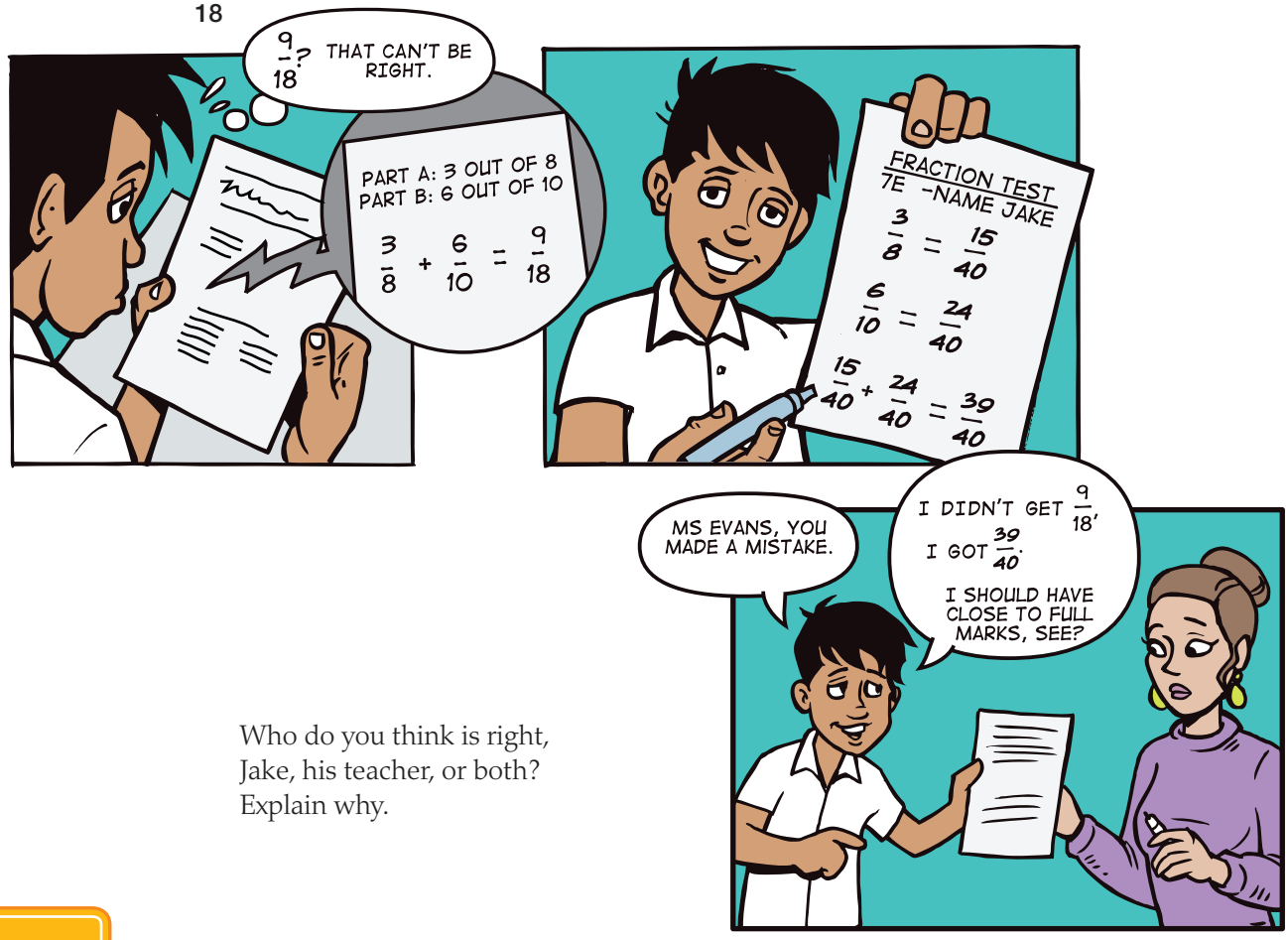
Open-ended

15 This question appeared on a Year 7 maths test: 'What whole number is $\frac{7}{8} + \frac{12}{13}$ approximately equal to?' Several students answered 19 or 21, which are both incorrect. How might the students have arrived at their incorrect answers? Describe how you could use estimation to arrive at the correct answer.

16 Find two fractions that add to $\frac{3}{4}$ and do not have a denominator of 4.

17 Use a diagram to help you explain why $\frac{3}{10} + \frac{4}{10} = \frac{7}{10}$, not $\frac{7}{20}$. Why should you add the numerators, but not the denominators?

18



Who do you think is right, Jake, his teacher, or both? Explain why.

Game

Race to 10

Equipment required: 2 dice

How to win:
The aim is to be the person to get to a total as close to 10 as possible. Beware, because if you go over 10 you are out!

How to play:
Take turns to roll both dice. Choose one number to be the numerator of a fraction and the other to be the denominator. It is your choice whether you make a proper or an improper fraction.

For example, if the first die rolls 5 and the second die rolls 4, then the fraction is either $\frac{5}{4}$ or $\frac{4}{5}$.

Keep a running total of your fractions. When your total is as close to 10 as you dare, say 'I'll sit'. If you choose to roll and your total goes over 10, you are out.

Play three games to find the winner.

Half-time 3

1 There are 15 cookies in a bag. **3.1**

(a) If I eat 7 cookies, what fraction of the total cookies is this?

(b) If I eat $\frac{2}{5}$ of the cookies in the bag, how many cookies do I eat?

2 Simplify the following. **3.2**

(a) $\frac{18}{48}$ **(b)** $\frac{9}{36}$ **(c)** $3\frac{9}{12}$ **(d)** $7\frac{35}{50}$

3 Convert the following improper fractions to whole numbers or mixed numbers. **3.2**

(a) $\frac{27}{4}$ **(b)** $\frac{40}{8}$ **(c)** $\frac{50}{10}$ **(d)** $\frac{37}{7}$

4 Determine which fraction in each of these pairs is the larger number, and write a < or > sign between them. If they are equivalent, write an '=' sign between them. **3.3**

(a) $\frac{5}{8}$ ____ $\frac{2}{3}$ **(b)** $\frac{4}{9}$ ____ $\frac{15}{27}$ **(c)** $\frac{48}{56}$ ____ $\frac{6}{7}$ **(d)** $\frac{11}{5}$ ____ $\frac{9}{4}$

5 Draw a number line from 0 to 2. Divide the bottom side of the number line into quarters and the top side into eighths. **3.1, 3.2, 3.3**

(a) Show the positions of these fractions with a labelled arrow: $\frac{3}{4}, \frac{5}{8}, \frac{5}{4}, \frac{9}{8}, \frac{7}{4}, \frac{14}{8}$.

(b) Identify five pairs of equivalent fractions that are shown on the number line.

(c) Use the number line to determine which fraction out of the following pairs is larger.

(i) $\frac{5}{4}$ or $\frac{9}{8}$ **(ii)** $\frac{11}{8}$ or $\frac{7}{4}$.

6 Calculate the following, writing your answers in simplest form. **3.4**

(a) $\frac{3}{8} + \frac{3}{4}$ **(b)** $\frac{7}{15} - \frac{3}{10}$ **(c)** $\frac{7}{9} + \frac{5}{6}$ **(d)** $\frac{11}{18} - \frac{5}{12}$

7 Write the following in ascending order by first locating them on a number line. **3.3**

$\frac{1}{3}, \frac{6}{3}, \frac{7}{4}, -\frac{3}{4}, 1\frac{2}{3}$

8 7 pizzas were each sliced into 6 equal pieces. **3.2**

(a) If $4\frac{5}{6}$ of the pizzas were eaten, how many slices was that?

(b) If $3\frac{1}{2}$ of the pizzas were eaten, how many slices were left over?

9 Calculate the following. **3.4**

(a) $2\frac{7}{12} + 2\frac{1}{10}$ **(b)** $1\frac{16}{25} + 2\frac{13}{20}$ **(c)** $2\frac{13}{20} - 1\frac{3}{100}$ **(d)** $3\frac{9}{10} - 2\frac{8}{25}$

10 Dave is leading a group of 12 people on a bushwalk. He has brought 30 L of water to supply the group. **3.1, 3.2**

(a) How many litres does each group member receive? Write your answer as both an improper fraction and as a mixed number in simplest form. **3.4**

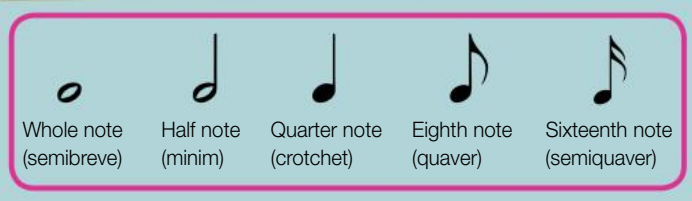
(b) One of the group members has drunk $\frac{3}{4}$ of a litre by lunchtime. How many litres does this person have left?

Maths 4 Real

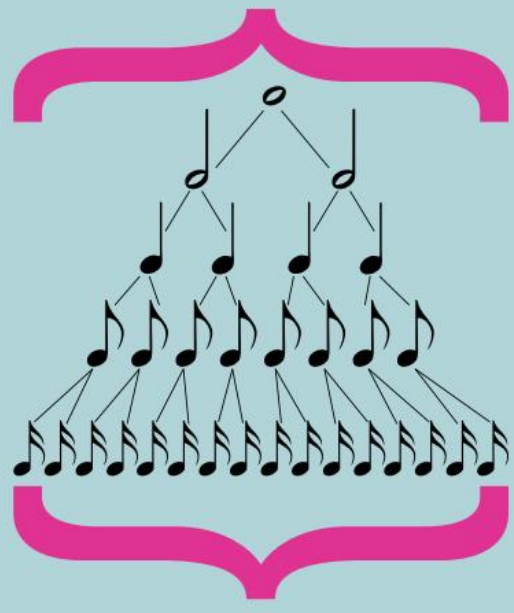
MUSICAL FRACTIONS

Traditionally, European-style music is written using symbols called 'notes'. Music notes have different shapes that indicate sounds of different durations, to tell the musician the length of time that the sound should be played.

Musical note names and symbols



The note names are based on fractions, because music notes 'add' together in the same way as fractions. This tree diagram shows how shorter notes add together to make the same time duration as longer notes.



From the diagram, you can see that:

2 half notes are equal in length to 1 whole note:	4 quarter notes are equal in length to 1 whole note:	2 quarter notes are equal in length to 1 half note:
$\frac{1}{2}+\frac{1}{2}=1$	$\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=1$	$\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

Note that:

Eighth notes and sixteenth notes are often joined in pairs, like this:

Or joined in groups of four, like this:

1 Using the tree diagram and examples above, copy and complete the questions below by writing one equivalent note after the equals sign. Write the fraction sum underneath.

(a) **(b)** **(c)** **(d)**

You can also add different types of notes together:

$\frac{1}{4}+\frac{1}{4}+\frac{1}{2}=1$ $\frac{1}{4}+\frac{1}{8}+\frac{1}{8}=\frac{1}{2}$

2 Copy and complete the questions below by writing one note in the space. Write the fraction sums underneath.

(a) ♩♫♫ = ☐ **(b)** ♫♪♩ = ☐

(c) ☐ ♫♫♫♫ = 𝅝 **(d)** ♪ ☐ ♩ = 𝅗𝅥

3 Copy and complete the questions below by writing two or more different notes in the spaces (there are several ways this can be done).

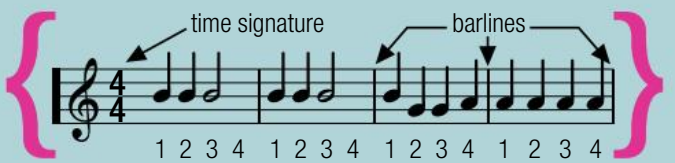
(a) ♫♫♫ = ☐ **(b)** ♩𝅗𝅥 = ♬ ☐

(c) ♩♫♫ ☐ = 𝅗𝅥𝅗𝅥 **(d)** ♩♩♩ ☐ = 𝅝𝅝

Time signatures

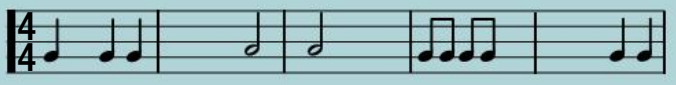
Music is written on 5 parallel lines called a 'stave' or 'staff' and in sections called 'bars'. Each bar contains the same number of 'beats', or counts. 'Barlines' are drawn across the lines of the stave to mark each bar. The piece of music below shows 4 bars and barlines.

The 'time signature' is written at the beginning of the first bar of a piece of music, and looks like a fraction. The top number tells the number of beats in each bar, while the bottom number tells what type of note counts as one beat.



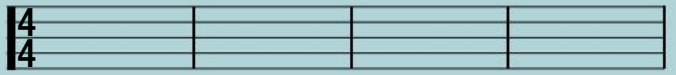
A common time signature is $\frac{4}{4}$. This means there are 4 beats in each bar, and each beat is worth 1 quarter note. In $\frac{4}{4}$ time, the note values in each bar must add to 4 quarter notes (which is equal to 1 whole note).

4 (a) Copy these bars, then use quarter notes or half notes to fill in the missing beats. Check that the fraction sum for each bar adds up to $\frac{4}{4}$.



(b) Copy these bars, then use any combination of half, quarter, eighth or sixteenth notes to fill in the missing beats. Make sure you check the fraction sum of each bar.

5 Write your own piece of music in the four bars below. Use any number or combination of notes, just make sure that every bar adds to 4 quarter notes.



Research

Find out about other systems of musical notation, such as for traditional Chinese or Indian music. Write some simple questions or exercises like the ones in this task to teach others what you have learnt.

Investigation

Tangram teaser

Equipment required: paper, scissors, tangram (draw your own copy of the tangram shapes shown, or download a template from the eBook or the Pearson Places website)

The tangram is an ancient Chinese puzzle. It has seven pieces that can be arranged to make hundreds of different shapes and patterns, including the large square you can see here.

The Big Question

How many different squares can be made by using various combinations of the seven tangram pieces? If the largest square is one whole, what are the fraction values of the smaller squares?

Engage

The tangram has seven individual pieces of five different shapes: two large triangles, two small triangles, one medium triangle, one small square and one parallelogram.

1 If the large square that is made of all the pieces represents one whole, estimate the fraction that is represented by:

   (a) a large triangle

   (b) the medium triangle

   (c) a small triangle

   (d) the small square

   (e) the parallelogram.

2 Check your estimates by cutting out the parts on your tangram template and using them to visually show what fraction of the large square they cover.

3 Not all seven pieces need to be used to make a square. A smaller square can be made using four pieces, for example:

By adding the fractions represented by each piece, work out what fraction of the large seven-piece square is taken up by this four-piece square. Write your answer in simplest form.

Explore

4 Experiment with different numbers and combinations of pieces to make as many different squares as possible. Make sure you keep a visual record of each different square that you make. (Hint: There is more than one way to make the big, seven-piece square.)

Strategy options

- Draw a diagram
- Make a table.
- Test all possible combinations.

Explain

5 Accurately draw all the squares you make, clearly showing the parts in each. Underneath your drawings, write a fraction sum that shows how each of the pieces add to give a fraction of the big, seven-piece square. How many different squares have you found?

Elaborate

6 Sometimes, the same four or five pieces can be used to make more than one square. How did you decide whether two squares were 'different' to each other?

7 Using your definition of 'different,' summarise your findings by answering the Big Question.

Evaluate

8 Consider how you worked on the investigation and the methods you used. Do you think you have the complete set of solutions to this problem? Explain your answer.

9 **(a)** Did you find this task challenging, or straightforward?

(b) What was the most difficult part of the task?

(c) Would you tackle puzzles like the tangram in your own time?

Extend

10 Try any or all of the following.

(a) Can you form a triangle using:

(i) two tangram pieces

(ii) three tangram pieces

(iii) four tangram pieces

(iv) five tangram pieces

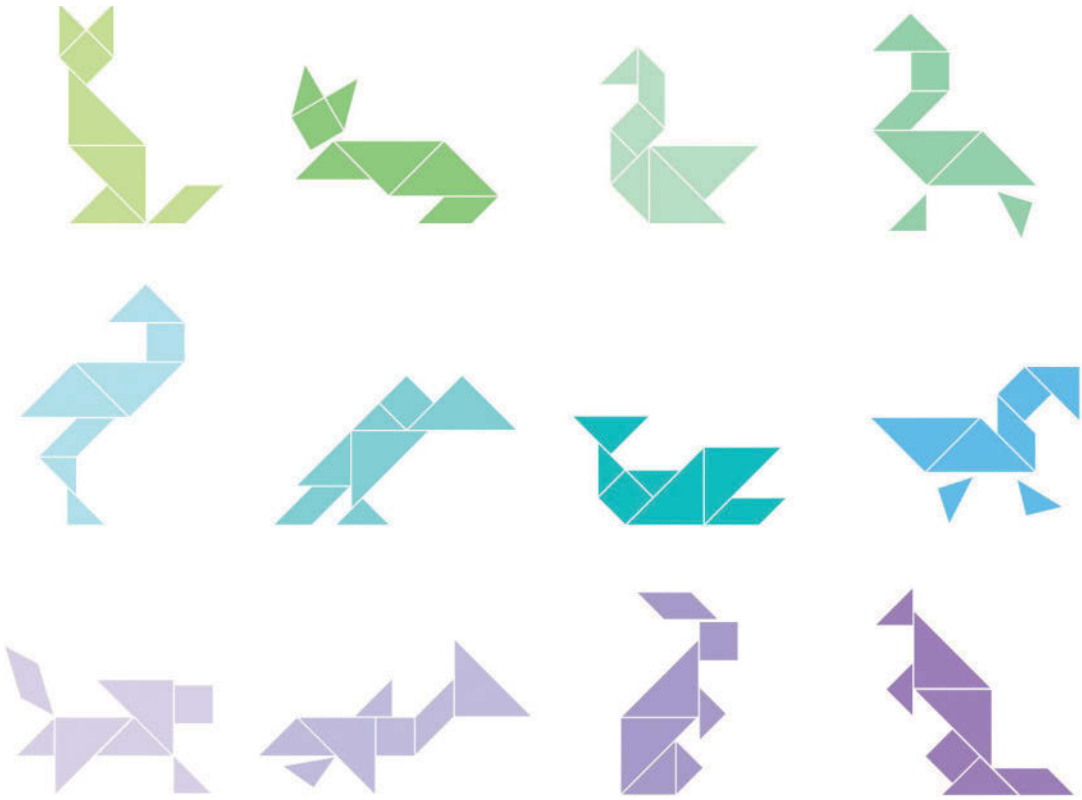
(v) six tangram pieces

(vi) all seven tangram pieces?

Draw a diagram of each triangle formed and write a fraction sum to show what fraction of the large square each triangle represents.

(b) Use the tangram pieces to make one or both of your initials.

(c) Use the tangram designs below as inspiration to make your own tangram designs.



3.5 Multiplying fractions

The multiplication sign × means '*lots of*', or simply '*of*'.
For example, 8 lots of \$5 is: $8 \times \$5 = \$40$

Finding a fraction of a whole amount

Three-quarters of an hour is 45 minutes: $\frac{3}{4}$ of $60 = 45$.

This can be calculated in two ways:

1 Find one-quarter of an hour, then multiply by three.

$\frac{1}{4}$ of $60 = 60 \div 4 = 15$

$\frac{3}{4}$ of $60 = 3 \times 15 = 45$

This method is called the 'unit fraction method'.
A **unit fraction** is a fraction with a numerator of 1.
In the above example, the unit fraction is $\frac{1}{4}$.

(This method works well if the denominator of the fraction is a factor of the whole number. This makes the division step straightforward. In the example above, 4 is a factor of 60. $60 \div 4 = 15$.)

2 Replace the word 'of' with the multiplication sign × and multiply:

$$\frac{3}{4} \text{ of } 60 = \frac{3}{4} \times \frac{60}{1} \quad \text{(writing 60 as an improper fraction with a denominator of 1)}$$

$$= \frac{3}{\cancel{4}_{1}} \times \frac{\cancel{60}^{15}}{1} \quad \text{(cancelling any common factors first, then multiplying the numerators together and the denominators together)}$$

$$= \frac{45}{1}$$

$$= 45$$

To find a fraction of a whole number, you can use two methods:

- Find the unit fraction (by dividing both parts of the fraction by the denominator), then multiply the whole number by the numerator of this unit fraction.
- Write the whole number as an improper fraction with a denominator of 1. Taking the first fraction and this improper fraction, multiply the numerators together and the denominators together. To avoid working with large numbers, cancel any common factors between numerators and denominators before multiplying.

Worked example 14 W.E. 14

Find $\frac{2}{5}$ of 40.

Method 1: Find the unit fraction, then multiply

Thinking	Working
1 Find the unit fraction (in this case, $\frac{1}{5}$) by dividing the whole number by the denominator.	$\frac{1}{5}$ *of 40:* $40 \div 5 = 8$
2 Multiply the unit fraction amount by the numerator.	$2 \times 8 = 16$
3 Write the answer.	$\frac{2}{5}$ *of* $40 = 16$

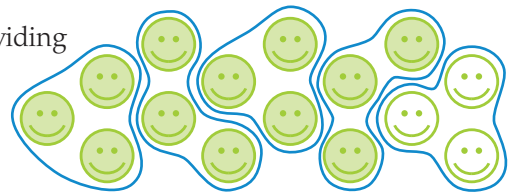
Method 2: Multiply two fractions

Thinking	Working
1 Replace 'of' with '×' and write the whole number as an improper fraction with a denominator of 1.	$\frac{2}{5}$ *of 40*
2 Cancel any common factors (here, cancel a common factor of 5). Multiply the numerators and denominators together.	$= \frac{2}{\cancel{5}_{1}} \times \frac{\cancel{40}^{8}}{1}$ $= \frac{16}{1}$
3 Simplify if possible.	$= 16$

Finding a fraction of a fraction

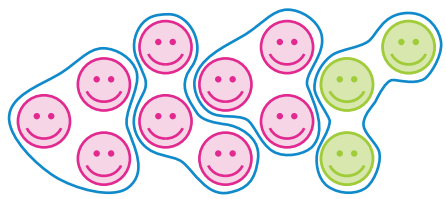
The 'smiley collection' from the start of this chapter (page 111) can help you understand what it means to find a fraction of a fraction.

In worked example 1 (on page 111), $\frac{4}{5}$ of 15 was found by dividing the collection into five equal groups and counting how many smileys were in four of those groups.



$\frac{4}{5}$ of 15 = 12

If you wanted to find $\frac{3}{4}$ of $\frac{4}{5}$, you would take the 12 smileys *as a new whole*, ignoring the fifth group. Finding $\frac{3}{4}$ of this new whole means taking three of the four groups, or nine smileys.



$\frac{3}{4}$ of 12 = 9

These 9 of the 15 original smileys represent $\frac{3}{4}$ of $\frac{4}{5}$.

$\frac{3}{4}$ of $\frac{4}{5}$ of 15 = 9

They also represent $\frac{3}{5}$ of the original collection: $\frac{3}{5}$ of 15 = 9.

The following example shows how multiplying $\frac{3}{4}$ and $\frac{4}{5}$ gives $\frac{3}{5}$.

Worked example 15

W.E. 15

Find $\frac{3}{4}$ of $\frac{4}{5}$.

Thinking	Working
1 Change the 'of' to ×.	$\frac{3}{4}$ of $\frac{4}{5}$
2 Cancel any common factors (here, the common factor is 4). Multiply the numerators and denominators together.	$= \frac{3}{\cancel{4}_1} \times \frac{\cancel{4}^1}{5}$ $= \frac{3}{5}$

To find a fraction of a fraction:

- write mixed numbers as improper fractions before multiplying
- replace 'of' with a '×' symbol
- cancel any common factors
- multiply the numerators together and the denominators together
- simplify the answer further if possible.

Worked example 16

W.E. 16

Find $4 \times 2\frac{3}{20}$. Write your answer as a mixed number, if appropriate.

Thinking	Working
1 Write both numbers as improper fractions. (Whole numbers can be written as fractions with a denominator of 1.)	$4 \times 2\frac{3}{20}$ $= \frac{4}{1} \times \frac{43}{20}$
2 Cancel any common factors between numerators and denominators. (Here, the common factor is 4.)	$= \frac{\cancel{4}^1}{1} \times \frac{43}{\cancel{20}_5}$
3 Multiply the simplified numerator and the denominator.	$= \frac{1 \times 43}{1 \times 5} = \frac{43}{5}$
4 Write the answer as a mixed number.	$= 8\frac{3}{5}$

3.5 Multiplying fractions

Navigator

1 (columns 1–3), 2 (columns 1–3), 3 (columns 1–3), 4 (a–d), 5, 6, 8, 9, 10, 11, 14, 15, 16, 18, 20	1 (columns 2–3), 2 (columns 2–3), 3 (columns 1–2), 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 16, 18, 20	1 (column 4), 2 (columns 3–4), 3 (columns 3–4), 4, 5, 7, 8, 10, 11, 12, 13, 16, 17, 18, 19, 20

Answers p. 663

Fluency

1 Find the following.

W.E. 14

(a) $\frac{1}{3}$ of 18 **(b)** $\frac{1}{4}$ of 12 **(c)** $\frac{1}{6}$ of 18 **(d)** $\frac{1}{9}$ of 36

(e) $\frac{3}{4}$ of 28 **(f)** $\frac{4}{5}$ of 30 **(g)** $\frac{6}{7}$ of 56 **(h)** $\frac{3}{8}$ of 72

(i) $\frac{3}{2}$ of 20 **(j)** $\frac{6}{5}$ of 100 **(k)** $\frac{11}{10}$ of 50 **(l)** $\frac{5}{4}$ of 60

2 Find the following.

W.E. 15

(a) $\frac{1}{2}$ of $\frac{2}{13}$ **(b)** $\frac{2}{9} \times \frac{1}{2}$ **(c)** $\frac{4}{5} \times \frac{5}{9}$ **(d)** $\frac{3}{7} \times \frac{7}{13}$

(e) $\frac{5}{7} \times \frac{1}{10}$ **(f)** $\frac{3}{5}$ of $\frac{10}{11}$ **(g)** $\frac{5}{12}$ of $\frac{6}{7}$ **(h)** $\frac{5}{6}$ of $\frac{1}{11}$

(i) $\frac{6}{7} \times \frac{3}{8}$ **(j)** $\frac{9}{10} \times \frac{5}{6}$ **(k)** $\frac{4}{7}$ of $\frac{2}{3}$ **(l)** $\frac{3}{10} \times \frac{9}{25}$

3 Find the following. Write your answers as mixed numbers, if appropriate.

W.E. 16

(a) $3 \times \frac{2}{9}$ **(b)** $\frac{3}{4} \times 8$ **(c)** $\frac{5}{8} \times 2$ **(d)** $3 \times \frac{5}{12}$

(e) $5 \times 2\frac{3}{10}$ **(f)** $8 \times 1\frac{3}{4}$ **(g)** $1\frac{4}{5} \times 15$ **(h)** $5\frac{1}{12} \times 8$

(i) $5\frac{1}{3} \times \frac{3}{8}$ **(j)** $3\frac{1}{2} \times 3\frac{1}{5}$ **(k)** $4\frac{1}{5} \times 4\frac{2}{7}$ **(l)** $1\frac{1}{8} \times 3\frac{5}{9}$

When cancelling common factors, don't cancel out two things on the same line—cancel something on the bottom with something on the top.

4 Calculate the following, giving your answers in simplest form.

(a) $\frac{3}{11} \times \frac{11}{5} \times \frac{1}{4}$ **(b)** $\frac{2}{7} \times \frac{7}{9} \times \frac{1}{5}$ **(c)** $\frac{6}{7} \times \frac{5}{8} \times \frac{2}{3}$

(d) $1\frac{2}{3} \times \frac{12}{13} \times \frac{1}{2}$ **(e)** $3\frac{2}{5} \times \frac{5}{6} \times \frac{7}{17}$ **(f)** $\frac{4}{9} \times 3\frac{1}{2} \times \frac{6}{7}$

5 **(a)** $\frac{2}{5}$ of $2\frac{1}{2}$ cups of sugar is:

A $\frac{1}{5}$ cup **B** $\frac{4}{10}$ cup **C** 1 cup **D** $2\frac{1}{5}$ cups

(b) $\frac{1}{5}$ of $\frac{1}{3}$ of 15 is:

A 1 **B** 2 **C** 3 **D** 5

Understanding

6 A group of 7 workers shared a prize in a local raffle. The prize was worth $2170.

(a) What fraction of the prize money does each worker receive?

(b) How much money does each worker get?

7 A $48 000 inheritance was to be shared between four cousins.

Each cousin was to receive a specific fraction of the inheritance. Michael was to receive $\frac{5}{24}$, Marcus $\frac{1}{12}$, Lily $\frac{3}{8}$ and Julie $\frac{1}{3}$. How much money did each cousin receive?

8 A chocolate bar was made up of 45 equal pieces. Ling had to share it equally with her brother and sister, Gao and Chen.

(a) **(i)** Copy this chocolate block and divide it up to show each person's share.

(ii) What fraction of the block did each person receive?

(iii) How many pieces did each person receive?

(iv) Use your answers to **(ii)** and **(iii)** to complete the following multiplication:

$$\frac{\square}{\square} \times \frac{45}{1} = \square$$

(b) Ling then decided to divide her share of chocolate equally between herself and four of her friends.

(i) What fraction of Ling's share does each person receive?

(ii) How many pieces did each person receive?

(iii) What fraction of the original block is this?

(iv) On your drawing of the chocolate bar, show the fraction that Ling and each of her friends receive.

(v) Use your answers to **(i)** and **(iii)** to complete the following multiplication:

$$\frac{\square}{\square} \times \frac{1}{3} = \frac{\square}{\square}$$

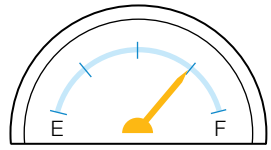
9 Annika cut one-quarter of an apple pie in halves to share with her brother. What fraction of the original whole pie do they each now have?

10 Draw a rectangle that is 5 cm long and 4 cm wide. Divide it lengthways into fifths. Shade in $\frac{1}{5}$. Now, take the shaded section and divide it into quarters. Shade in $\frac{3}{4}$ of this section.

(a) What fraction of the original rectangle have you shaded twice?

(b) Write the fraction multiplication that is shown by your diagram.

11 A petrol tank was filled to its capacity of 52 litres but now has the petrol gauge reading shown.



(a) How much petrol has been used?

(b) How much petrol is still in the tank?

12 Melissa surveyed 75 Year 7 students. Of these students, $\frac{2}{3}$ said they enjoyed studying mathematics. $\frac{4}{5}$ of those who enjoyed mathematics also said they enjoyed studying science.

(a) How many students said they enjoyed mathematics?

(b) How many students said they enjoyed mathematics and science?

13 Angelina is making cookies. Below are the ingredients that make 24 cookies:

180 g butter $\frac{1}{2}$ cup icing sugar $1\frac{1}{4}$ cups self-raising flour $\frac{1}{3}$ cup custard powder

(a) What mixed number should 24 be multiplied by, to get 60?

(b) Rewrite the list of ingredients above to make 60 cookies.

Reasoning

14 Without doing the calculation, which is larger: $1\frac{1}{2} \times 12$ or $\frac{1}{3} \times 9$?

15 Without doing any calculations, say which of the following would give the biggest answer.

A $\frac{1}{2} \times \frac{3}{4}$ **B** $\frac{2}{5} \times \frac{2}{3}$ **C** $\frac{3}{4} \times \frac{6}{5}$ **D** $1\frac{1}{2} \times 1\frac{2}{5}$

16 For each of the following multiplications:

(i) Without doing any calculations, state which multiplication will give a larger number.

(ii) Explain how you could tell without calculating.

(a) 4×3 or $\frac{1}{4} \times 3$ **(b)** $2\frac{1}{2} \times 7$ or $\frac{5}{6} \times 7$ **(c)** $1\frac{1}{2} \times 12$ or $1\frac{1}{2} \times 9$

17 The missing number in the statement $\frac{5}{12} \times \frac{\square}{10} = \frac{5}{8}$ is:

A 4 **B** 5 **C** 10 **D** 15

Open-ended

18 Write two fractions that multiply to give $\frac{1}{6}$ as the simplified answer.

19 Fill each of the boxes with a single digit (0–9) to make the statement correct. There are six possible combinations. Try to find at least three of them.

$\frac{1}{\square} \times 3\square = 1\square$

20 Asher was asked to find $\frac{3}{5}$ of 60. His answer was 4. Here is his working:

$\frac{\cancel{3}^{1}}{5} \times \frac{\cancel{60}^{20}}{1} = \frac{20}{5} = 4$

Asher knows his answer is not correct, because he knows that $\frac{3}{5}$ is bigger than one half, and 4 is nowhere near half of 60! Explain where Asher has gone wrong. Include the correct working and answer to the problem.

Game

Fringo!

Equipment required: 2 dice

How to win:
Fringo is fraction bingo. The winner is the first person to call out 'Fringo!' after they have four fractions in a row (horizontal, vertical or diagonal) crossed off their grid.

How to play:
Draw a 4×4 grid into your book. Fill the squares in your grid with any 16 of the numbers, fractions and mixed numbers from this list:

1	2	3	4	5	6
$\frac{1}{2}$	$1\frac{1}{2}$	$2\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$1\frac{1}{3}$
$1\frac{2}{3}$	$\frac{1}{4}$	$\frac{3}{4}$	$1\frac{1}{4}$	$\frac{1}{5}$	$\frac{2}{5}$
$\frac{3}{5}$	$\frac{4}{5}$	$1\frac{1}{5}$	$\frac{1}{6}$	$\frac{5}{6}$	

Take turns to roll the dice. Write the first number you roll as the denominator of a fraction. For example, if you roll a 4, write $\frac{1}{4}$. Multiply this fraction by the second number you roll. Write the answer in simplest form.

For example, Roll 1 = 4, Roll 2 = 2. Write $\frac{1}{4} \times 2$ and calculate the answer $\frac{2}{4}$, which simplifies to $\frac{1}{2}$.

If the answer to your fraction multiplication is on your grid, cross it off.

(Hint: You may realise after playing a few rounds that some answers come up more than others. Thinking about the best places to put these on your grid will increase your chance of winning. You could also investigate how many ways the fractions from the list can be made.)

Problem solving

Fill in the blanks

1 Fill in the blanks with positive whole numbers to make mathematically true statements. Do not use the same number twice within a statement. See if you can come up with at least two different combinations that are both true statements.

(a) $\frac{\square}{4} + \frac{1}{\square} = \frac{\square}{20}$ **(b)** $\frac{\square}{\square} - \frac{\square}{6} = \frac{\square}{12}$

2 Use four of the digits 1, 3, 4, 5, 6, 7 to make two fractions whose sum is close to but less than 1.

$$\frac{\square}{\square} + \frac{\square}{\square} < 1$$

Try to come up with at least three combinations. Which combination is the closest to 1?

Strategy options

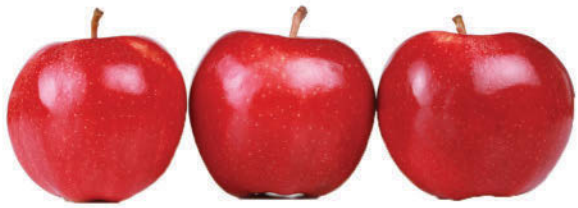
- Guess and check.
- Test all possible combinations.

Dividing fractions

3.6

Whole numbers divided by fractions

Here are three whole apples.



If you cut each apple into quarters, how many apple quarters will there be?

You can think of this as 'how many quarters in 3'?

Write it as $3 \div \frac{1}{4} = ?$

There are 4 quarters in each apple, so there are 12 quarters in 3 apples: $3 \div \frac{1}{4} = 12$.

If you cut the apples into fifths, you would have fifteen fifths: $3 \div \frac{1}{5} = 15$.

If you cut the apples into sixths, you would have eighteen sixths: $3 \div \frac{1}{6} = 18$.

Dividing a whole number by a unit fraction is exactly the same as multiplying the whole number by the denominator of the fraction.

i.e.	$3 \div \frac{1}{4}$	$3 \div \frac{1}{5}$	$3 \div \frac{1}{6}$
	$= 3 \times \frac{4}{1}$	$= 3 \times \frac{5}{1}$	$= 3 \times \frac{6}{1}$
	$= 12$	$= 15$	$= 18$

The fractions have been turned upside down, or 'inverted'.

Inverting a fraction (turning it upside down) is also known as 'finding the **inverse**'.

The inverse of $\frac{1}{4}$ is $\frac{4}{1}$ or simply 4.

What about dividing by fractions where the numerator is not 1, such as $3 \div \frac{3}{4}$?

If you put 12 apple quarters into groups of 3 quarters, there will be 4 groups.

You can achieve the same answer by multiplying 3 by $\frac{4}{3}$, the inverse of $\frac{3}{4}$.

$$3 \div \frac{3}{4}$$

$$= \frac{\overset{1}{\cancel{3}}}{1} \times \frac{4}{\underset{1}{\cancel{3}}}$$

$$= \frac{4}{1}$$

$$= 4$$

To divide a whole number by a fraction, multiply by the inverse of the fraction.

The result of a division calculation is called the 'quotient'.

$\frac{3}{4}$ is three times as big as $\frac{1}{4}$, the answer to $3 \div \frac{3}{4}$ is one-third as big as $3 \div \frac{1}{4}$.

Another way of dividing a fraction (or any number) by a fraction is to convert both numbers to the same type of fraction—that is, fractions with the same denominator (the LCD). When you divide fractions with the same denominator, the denominators cancel each other, so you only need to divide the numerators. For example:

$$4 \div \frac{2}{3} = \frac{12}{3} \div \frac{2}{3}$$

$$= \frac{12}{\underset{1}{\cancel{3}}} \times \frac{\overset{1}{\cancel{3}}}{2}$$

$$= \frac{12}{2}$$

$$= 6$$

This is also demonstrated in Method 2 of the example below.

Worked example 17

W.E. 17

Calculate $9 \div \frac{3}{8}$.

Method 1: Multiply by the inverse

Thinking	Working
1 Write the whole number as an improper fraction with a denominator of 1.	$\frac{9}{1} \div \frac{3}{8}$
2 Find the inverse of the second fraction (turn it upside down) and change the ÷ to ×.	$= \frac{9}{1} \times \frac{8}{3}$
3 Cancel any common factors between numerators and denominators. (Here, the common factor is 3.)	$= \frac{\overset{3}{\cancel{9}}}{1} \times \frac{8}{\underset{1}{\cancel{3}}}$
4 Multiply the simplified numerators and denominators.	$= \frac{3 \times 8}{1 \times 1} = \frac{24}{1}$
5 Write the answer.	$= 24$

Method 2: Use equivalent fractions with the same denominator

Thinking	Working
1 Write the whole number as an improper fraction with the same denominator as the fraction (in this case, 8).	LCD = 8 $9 = \frac{72}{8}$
2 Rewrite the division using the two fractions.	$9 \div \frac{3}{8}$ $= \frac{72}{8} \div \frac{3}{8}$
3 Now that the denominators are the same, they will cancel out, so you only need to divide the numerators.	$= 72 \div 3$ $= 24$

Fractions divided by fractions

- Convert mixed numbers to improper fractions before dividing.

To divide a fraction by another fraction:

- Method 1: Invert the fraction you are dividing by (that is, the second fraction) and multiply the two fractions together.
- Method 2: Convert the fractions to equivalent fractions with the same denominator (the LCD), then the denominators cancel each other and you can divide the numerators.

Worked example 18 W.E. 18

Calculate $1\frac{1}{6} \div \frac{2}{3}$.

Method 1: Multiply by the inverse

Thinking	Working
1 If there are any mixed numbers, convert them to improper fractions.	$1\frac{1}{6} = \frac{7}{6}$ $\frac{7}{6} \div \frac{2}{3}$
2 Find the inverse of the second fraction (turn it upside down) and change the ÷ to ×.	$= \frac{7}{6} \times \frac{3}{2}$
3 Cancel common factors and perform the simplified multiplication.	$= \frac{7}{\cancel{6}_{2}} \times \frac{\cancel{3}^{1}}{2}$ $= \frac{7 \times 1}{2 \times 2}$ $= \frac{7}{4}$
4 Write the answer as a mixed number.	$= 1\frac{3}{4}$

Method 2: Use equivalent fractions with the same denominator

Thinking	Working
1 Find the LCD. Write the mixed number as an improper fraction with the LCD. Write the second fraction as an equivalent fraction with the LCD.	LCD = 6 $1\frac{1}{6} = \frac{7}{6}$ $\frac{2}{3} = \frac{4}{6}$
2 Rewrite the division using the fractions with the LCD.	$1\frac{1}{6} \div \frac{2}{3}$ $= \frac{7}{6} \div \frac{4}{6}$
3 Perform the division with just the numerators, and write the answer.	$= 7 \div 4$ $= 1\frac{3}{4}$

3.6 Dividing fractions

Navigator

Answers p. 663

1 (columns 1–3), 2 (columns 1–3), 3, 4, 5, 6, 8, 9, 10, 12, 14, 16, 17	1 (columns 2–3), 2 (columns 1–2), 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 16, 17	1 (column 4), 2 (columns 3–4), 3, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17

Fluency

W.E. 17

1 Calculate the following.

(a) $6 \div \frac{1}{4}$ **(b)** $2 \div \frac{1}{6}$ **(c)** $3 \div \frac{1}{7}$ **(d)** $4 \div \frac{1}{9}$

(e) $4 \div \frac{2}{9}$ **(f)** $4 \div \frac{4}{5}$ **(g)** $5 \div \frac{3}{7}$ **(h)** $7 \div \frac{4}{9}$

(i) $6 \div 1\frac{1}{8}$ **(j)** $8 \div 2\frac{2}{3}$ **(k)** $6 \div 3\frac{1}{5}$ **(l)** $3 \div 2\frac{1}{4}$

W.E. 18

2 Calculate the following.

(a) $\frac{1}{2} \div \frac{1}{4}$ **(b)** $\frac{1}{4} \div \frac{1}{6}$ **(c)** $\frac{3}{4} \div \frac{2}{3}$ **(d)** $\frac{7}{10} \div \frac{1}{2}$

(e) $\frac{1}{3} \div 2$ **(f)** $\frac{1}{5} \div 4$ **(g)** $\frac{3}{4} \div 3$ **(h)** $\frac{9}{10} \div 6$

(i) $\frac{9}{10} \div 1\frac{3}{5}$ **(j)** $\frac{7}{6} \div 2\frac{1}{12}$ **(k)** $\frac{4}{5} \div 1\frac{8}{9}$ **(l)** $\frac{9}{10} \div 2\frac{2}{3}$

(m) $2\frac{2}{3} \div 1\frac{1}{3}$ **(n)** $5\frac{1}{2} \div 5\frac{3}{8}$ **(o)** $4\frac{1}{2} \div 1\frac{1}{5}$ **(p)** $4\frac{5}{7} \div 2\frac{3}{4}$

3 Choose the correct answer to each of the following.

(a) $5 \div \frac{15}{8} =$

A $\frac{8}{75}$ **B** $\frac{3}{8}$ **C** $2\frac{2}{3}$ **D** $\frac{75}{8}$

(b) $\frac{16}{7} \div \frac{20}{49} =$

A $\frac{5}{28}$ **B** $\frac{320}{343}$ **C** $\frac{343}{320}$ **D** $5\frac{3}{5}$

4 Copy and complete the following using your preferred method.

(a) There are _____ halves in one, so $1 \div \frac{1}{2} =$ _____.

(b) There are _____ halves in three, so $3 \div \frac{1}{2} =$ _____.

(c) There are _____ quarters in one, so $1 \div \frac{1}{4} =$ _____.

(d) There are _____ quarters in two, so $2 \div \frac{1}{4} =$ _____.

5 The inverse of $\frac{1}{3}$ is:

A $\frac{1}{6}$ **B** $\frac{2}{3}$ **C** 1 **D** 3

Understanding

6 Draw three rectangles, each with a length of 6 cm and a width of 2 cm. Divide each rectangle into thirds.

(a) What is $3 \div \frac{1}{3}$? (To find the answer, count how many thirds ($\frac{1}{3}$) there are in the 3 whole rectangles.)

(b) What is $3 \div \frac{2}{3}$? (To find the answer, shade in or circle as many lots of $\frac{2}{3}$ as possible. How many lots are there, and what is left over?)

7 William has $\frac{4}{5}$ of a bag of dog food. His dog eats $\frac{3}{10}$ of the bag every day. How many days will William's bag last? Write your answer as a mixed number.

8 There is $\frac{5}{8}$ of a metre of ribbon in the class cupboard. Students need $\frac{1}{10}$ of a metre each for their projects. How many students will be able to use the ribbon?

9 Scientists in a laboratory work in shifts of $3\frac{1}{2}$ hours each. How many whole shifts are to be worked during a period of 42 hours?

10 Hannah is a baker whose speciality is scones. She has a sack containing 80 cups of flour. Her scone recipe uses $2\frac{2}{3}$ of a cup of flour for each batch of scones. How many batches of scones will Hannah get from her sack of flour?

11 On a 15 km fun run course, drinks stations are placed every $2\frac{1}{2}$ km from the start. How many drinks stations will there be on the course? (There are no drinks stations at the start or finish lines.)

12 The hit movie 'Exterminator 5' runs for $1\frac{3}{4}$ hours. A local cinema plans to screen it back to back. The first screening will start at 1:30 pm. The last screening must finish by 11 pm. How many times can the movie be screened within this time period?

Reasoning

13 The missing number from the statement $\frac{3}{4} \div \frac{\square}{8} = 2$ is:

A 1 **B** 2 **C** 3 **D** 4

14 Without doing a calculation, state which will give the larger number as a result:
$\frac{4}{5} \div \frac{1}{3}$, or $\frac{4}{5} \div \frac{2}{3}$?

15 Consider this sequence: $5 \div \frac{3}{4} = 6\frac{2}{3}$, $5 \div \frac{3}{5} = 8\frac{1}{3}$, $5 \div \frac{3}{6} = 10$, $5 \div \frac{3}{7} = 11\frac{2}{3}$.

(a) Why does the quotient (the answer to the division) increase as the denominator of the fractions in red is increased?

(b) Predict the answer to the next term in the sequence: $5 \div \frac{3}{8}$ will be …

Open-ended

16 Use digits from 0 to 9 to fill in the boxes to make a correct statement. Find at least two different combinations.

$4 \div \frac{2}{\square} = 1\square$

17 Here is Gabi's work on a fraction division problem:

$3 \div \frac{1}{4} = 3 \div 4 = \frac{3}{4}$

Her friend Natalie has worked it out differently. This is what Natalie wrote:

$3 \div \frac{1}{4} = \frac{3}{1} \times \frac{4}{1} = 12$

'That can't be right,' said Gabi, '12 is way too big to be the answer!'

Which working is correct? Explain what the other has done wrong.

Mixed fraction problems

3.7

Fractions and the order of operations

The order of operations rules that apply to whole numbers also apply to fractions.

The first step in any calculation is to complete any operation within brackets. Working from left to right, do any multiplication or division as you come to it. Then working from left to right, do addition or subtraction as you come to it.

Worked example 19

W.E. 19

Simplify the following using the correct order of operations.

(a) $3+\frac{2}{5}\times\frac{1}{2}$ **(b)** $\frac{6}{7}\times\left(1\frac{2}{3}+\frac{1}{4}\right)-\frac{3}{7}$

Thinking	Working
(a) 1 As there are no brackets, do the multiplication first.	(a) $3+\frac{2}{5}\times\frac{1}{2}$ $=3+\frac{{}^{1}\cancel{2}\times 1}{5\times{}^{1}\cancel{2}}$ $=3+\frac{1}{5}$
2 Do the addition and state your answer in simplest form.	$=3\frac{1}{5}$
(b) 1 Perform the operation in the brackets first.	(b) $\frac{6}{7}\times\left(1\frac{2}{3}+\frac{1}{4}\right)-\frac{3}{7}$ $=\frac{6}{7}\times\left(\frac{5}{3}+\frac{1}{4}\right)-\frac{3}{7}$ $=\frac{6}{7}\times\left(\frac{20}{12}+\frac{3}{12}\right)-\frac{3}{7}$ $=\frac{6}{7}\times\frac{23}{12}-\frac{3}{7}$
2 Next, perform any multiplication or division, cancelling any common factors.	$=\frac{\cancel{6}^{1}\times 23}{7\times{}^{2}\cancel{12}}-\frac{3}{7}$ $=\frac{23}{14}-\frac{3}{7}$
3 Finally, perform any addition or subtraction. Write the answer in simplest form.	$=\frac{23}{14}-\frac{6}{14}$ $=\frac{17}{14}=1\frac{3}{14}$

3.7 Mixed fraction problems

Navigator

Answers p. 664

1 (columns 1–2), 2, 3, 4, 5, 6, 11, 12, 13, 14	1 (columns 2–3), 2, 3, 4, 5, 6, 7, 9, 11, 12, 13, 14	1 (e–l), 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14

Fluency

W.E. 19

1 Simplify the following using the correct order of operations. Write your answers as mixed numbers where appropriate.

(a) $2+\frac{3}{4}\times\frac{1}{3}$ **(b)** $5-\frac{7}{8}\times\frac{1}{7}$ **(c)** $\frac{5}{11}\div(5+4)$

(d) $\frac{3}{4}\times\left(\frac{1}{3}+\frac{1}{6}\right)\times\frac{1}{2}$ **(e)** $\frac{7}{8}\times\left(\frac{9}{14}-\frac{2}{7}\right)$ **(f)** $\frac{5}{6}\times\left(\frac{2}{5}+\frac{1}{3}\right)$

(g) $2\frac{3}{4}+\left(4\frac{1}{2}-3\frac{1}{4}\right)\times\frac{1}{2}$ **(h)** $1\frac{5}{8}+\left(1\frac{3}{4}-1\frac{1}{8}\right)\times5$ **(i)** $1\frac{3}{5}+\left(3\frac{3}{10}-1\frac{1}{5}\right)\div2$

(j) $1\frac{1}{4}-\left(3\frac{1}{3}-2\frac{5}{6}\right)\div2$ **(k)** $3\frac{3}{8}+\left(8\frac{1}{4}-6\frac{3}{8}\right)\div3$ **(l)** $\frac{7}{9}\div\left(1\frac{5}{6}-1\frac{3}{5}\right)\times\frac{1}{5}$

2 Choose the correct answer to each of the following.

(a) $\frac{3}{7}\times\frac{1}{3}+5=$

A $5\frac{1}{7}$ **B** $5\frac{1}{3}$ **C** $5\frac{5}{21}$ **D** $5\frac{7}{9}$

(b) $6-\frac{9}{13}\div\frac{3}{4}=$

A $5\frac{1}{13}$ **B** $5\frac{25}{52}$ **C** $5\frac{12}{13}$ **D** $7\frac{1}{13}$

Understanding

3 Write the following questions using fractions, the operations symbols (×, ÷, +, –) and brackets where necessary. Then, evaluate using the correct order of operations.

(a) Add $\frac{7}{8}$ and $\frac{5}{6}$, and then multiply the total by 2.

(b) Find the product of $\frac{4}{5}$ and $\frac{1}{3}$, then add $1\frac{1}{2}$.

(c) Find the difference between $2\frac{4}{5}$ and $1\frac{2}{3}$, then multiply by 3.

(d) Find out how many $\frac{2}{3}$s in 5, then multiply by 4.

4 A survey was taken of 120 people as they left Wally's Sandwich Bar. Write your answers to the following in simplest form.

(a) If 80 of the people surveyed were male, what fraction were male?

(b) What fraction were not male?

(c) If 45 were under twenty years of age, what fraction were under twenty?

(d) What fraction were twenty or over?

(e) If $\frac{3}{4}$ had bought a drink, how many had bought a drink?

(f) Of those who bought a drink, $\frac{2}{5}$ had bought a salad roll. How many people had a drink and a salad roll?

5 Grant is training in his backyard pool for the 1500 m freestyle. Unfortunately, the pool is only 20 m long.

(a) What fraction of the total 1500 m race would he have swum after completing just 1 lap?

(b) How many laps would he have to complete to swim 1500 m?

(c) What fraction of the total 1500 m race would he have swum after completing 6 laps?

(d) How far would he have swum if he had completed $\frac{1}{10}$ of the race distance?

(e) How far would he have swum if he had completed $\frac{3}{4}$ of the race distance?

6 Valerie is running laps of her school oval to train for a charity run. She ran $4\frac{1}{2}$ laps before having to rest. After her rest, she ran another $3\frac{1}{4}$ laps before stopping again. Then, she struggled through another $\frac{1}{3}$ of a lap. How many laps did she complete altogether?

7 One weekend, Louisa walks from Ferndale to Greenhill and back, a total of $2\frac{3}{4}$ km. The next weekend she walks from Ferndale through Greenhill to Highvale, which is $5\frac{1}{3}$ km.

How far (in km) is it from:

(a) Ferndale to Greenhill

(b) Greenhill to Highvale

(c) Ferndale to Highvale and back again?

Reasoning

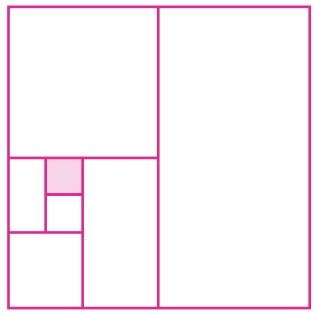
8 Theresa, Isabelle and Bryce worked on a mathematical problem and came up with the answers $8\frac{1}{2}$, $8\frac{2}{3}$ and $8\frac{3}{5}$, respectively. The correct answer was $8\frac{7}{12}$. Which of the three students was closest to the correct answer?

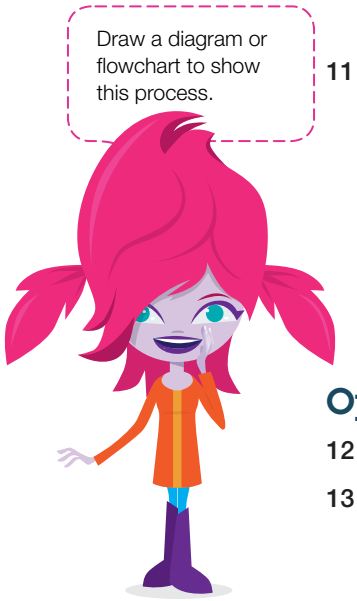
9 Despina's department store advertises a winter sale in which they claim everything is between $\frac{1}{3}$ off and $\frac{1}{2}$ off. For which of the following items is this incorrect?

Item	Normal price	Sale price
Bath towels	$22	$12
Camera	$900	$600
Cutlery set	$250	$120
Microwave oven	$540	$350
Jeans	$60	$35

10 A farming cooperative has divided its square block of land in the way shown in the diagram. Each section of land is exactly half of the section next to it.

The May family has been given the shaded region to farm. If the total block of land has an area of 1 square unit, then what fraction of the total do the May family *not* farm?





11 Prashanth agrees to sell badges for a charity. He receives a large box and places them at the school reception. After several days, Prashanth takes an estimate of the number of badges he has left. He empties out the box, divides the pile of badges roughly into halves, and puts half back in the box. He then halves the pile he has left and returns half to the box. He does this one more time, after which he counts the number of badges in the pile and finds he has 13.

(a) What fraction of the total number of badges does 13 represent?

(b) How many badges did Prashanth have in the box, approximately?

Open-ended

12 Write three numbers in fraction form that are less than $\frac{7}{8}$ but greater than $\frac{1}{2}$.

13 A and B are two different numbers selected from the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, where A is always greater than B.

(a) What values can $\frac{A+B}{A-B}$ have? Give three possibilities.

(b) What is the largest value this fraction can have?

14 'Ideal fractions' are pairs of fractions whose sum and product are the same. (They give the same number when added as when multiplied.)

(a) Show that $\frac{7}{3}$ and $\frac{7}{4}$ are ideal fractions.

(b) Show that $\frac{5}{3}$ and $\frac{5}{2}$ are ideal fractions.

(c) Look carefully at the numerators and denominators of the fraction pairs in **(a)** and **(b)**. Try to find a pattern.

(d) Write another pair of ideal fractions.

Problem solving

Mega-equivalents

1 Use four of the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 to make a fraction equivalent to $\frac{1}{2}$ that has two digits in the numerator and in the denominator (e.g. $\frac{34}{68}$).

2 Now, use six digits to make another fraction equivalent to $\frac{1}{2}$ that has three digits in the numerator and denominator.

3 See if you can form another fraction equivalent to $\frac{1}{2}$ using eight digits (four in numerator and denominator).

4 Can you make a fraction equivalent to $\frac{1}{2}$ using all 10 digits?

Strategy options

- Guess and check.
- Test all possible combinations.

Challenge 3

1 If $\frac{1}{3}$ of a number is 16, then $\frac{3}{8}$ of this number is:

A 8 **B** 16 **C** 18 **D** 24

2 Kerry bought a new water tank for her garden. The tank is initially empty, but fills to $\frac{2}{3}$ of capacity after rain. The tank now holds 480 litres of water. How many litres does the tank hold when full?

3 If $\frac{\blacksquare}{16}$ lies between $\frac{1}{4}$ and $\frac{3}{8}$, and $\blacksquare$ is a whole number, then $\blacksquare$ equals:

A 5 **B** 6 **C** 7 **D** 8

4 Two proper fractions (in simplest form) are subtracted, the first with a denominator of 10, and the second with a denominator of 6. If the answer is given in simplest form, the largest possible numerator that it could have is:

A 8 **B** 9 **C** 11 **D** 22

5 The number half-way between $\frac{1}{5}$ and $\frac{1}{20}$ is:

A $\frac{1}{10}$ **B** $\frac{1}{8}$ **C** $\frac{7}{40}$ **D** $\frac{1}{4}$

6 Two frogs live together in a backyard pond. One frog croaks every 4 minutes, the other frog croaks every $2\frac{1}{2}$ minutes. If they both croak together at 11 am, what is the first time after 1 pm that they will croak together?

7 $2 + \dfrac{6}{1 + \frac{1}{2}}$ equals:

A 2 **B** 4 **C** 6 **D** 8

8 Express this fraction in its simplest form:

$$\cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{3 + \frac{1}{3}}}}$$

9 If 60 is added to one-third of a number, the resulting value is double the number. What is the number?

10 Five numbers are put in a row from smallest to largest. The difference between each adjacent number (a number and the number next to it) is the same. If the first number is $\frac{1}{8}$ and the last is $\frac{1}{6}$, what are the three numbers in between, in simplest form?

11 Tom was born into a strange mathematical family. His grandmother sent him a money voucher for his birthday. The card said:

Dear Tom,
I am giving you the following amount to spend as your wish
$\frac{1}{2}$ *of* $\frac{2}{3}$ *of* $\frac{3}{4}$ *of* $\frac{4}{5}$ *of* $\frac{5}{6}$ *of* $\frac{6}{7}$ *of* $\frac{7}{8}$ *of* $\frac{8}{9}$ *of* $\frac{9}{10}$ *of* \$1000.
Love Grandma

How much birthday money did Tom receive?

Gamespace

FRACTURED RAVINE

While exploring the mountains with your friends you fall down into a deep cave! Armed with your fraction skills, can you survive the perils that await you?

Equipment: 1 die

Rules:

- Each player begins with 100 life points.
- If you lose all your life points you are out (dead).
- Take turns to roll the die and move forward that number of spaces. Follow the directions given in the space that you land on.
- All players must stop at the 'Great Leap' regardless of what number they have rolled.
- The goal: to be the first to reach sunlight.

START

You sprain an ankle.

$1\frac{1}{2} + 3\frac{1}{4} + 4\frac{2}{8} =$

Deduct the solution from your life points total.

You call out 'Cooee' and cause a rock fall.

$\frac{3}{4}$ of 16 =

Deduct the answer from your life points total.

Your torch runs out of batteries.

$\frac{1}{3}$ of 60 =

Deduct the answer from your life points total.

You run out of drinking water.

$12\frac{1}{3} + 3\frac{1}{6} - 4\frac{1}{2} =$

Deduct the solution from your life points total.

The oxygen level decreases rapidly.

Roll the die twice. Turn these two numbers into a proper fraction.

e.g. Roll 1 = 3, Roll 2 = 4, Fraction = $\frac{3}{4}$.

Find the fraction of 60 that this would be,

e.g. $\frac{3}{4}$ of 60 = 45.

Deduct this from your life points total.

Your abseil rope gets caught on a sharp rock.

$\frac{2}{5}$ of 30 =

Deduct the answer from your life points total.

You begin to feel claustrophobic.

Roll the die and multiply the number you rolled by $\frac{2}{5}$. Round the answer to the nearest whole number.

Deduct this from your life points total.

You eat the last of the food supply.

Roll the die and multiply the number you rolled by $\frac{7}{2}$. Round the answer to the nearest whole number.

Deduct this from your life points total.

A rock falls and breaks all the bones in one of your feet.

The human body contains 206 bones. If approximately $\frac{1}{8}$ of these are in each foot, how many bones did you break (rounded to the nearest whole number)?

Deduct this from your life points total.

GREAT LEAP

ALL EXPLORERS STOP HERE

There is a large crevice ahead, but it's the only way forward—you need to find the courage to jump!

Roll the die and substitute the number you roll into this fraction $\frac{3}{\square}$.

If the fraction created is greater than (or equal to) 1 you've made it across.

If the fraction created is less than 1 you must wait until you can roll again.

You reach a ladder and see sunlight above.

All explorers must stop at each rung of the ladder.

(Players don't need to roll to move.)

Rung 1:

Find what fraction of your first name is made up of vowels. e.g. 'Natalie' $= \frac{4}{7}$. Do the same for your last name . Multiply one of these fractions by 50 and round to the nearest whole number.

Deduct this from your life points total.

Rung 2:

If the fraction of your first name that is made up of vowels is less than or equal to $\frac{1}{2}$, *deduct 10 from your life points total*. If it's greater than $\frac{1}{2}$, *deduct 15.*

Rung 3:

You are on the final rung.

Roll the die twice. Turn these two numbers into a proper fraction.

e.g. Roll 1 = 3, Roll 2 = 4, Fraction = $\frac{3}{4}$. You need to roll a fraction larger than $\frac{2}{3}$ to make the final step out of the ravine.

Every time you fail you must deduct 10 points from your life points total.

Chapter review 3

Maths literacy

cancelling	improper fraction	mixed number	simplest form
denominator	inverse	numerator	simplify
equivalent fractions	lowest common denominator (LCD)	proper fraction	unit fraction
fraction			

Copy and complete the following using the words and phrases from this list, where appropriate. A word or phrase may be used more than once.

1 In a fraction, the _____ tells us how many parts there are, and the _____ tells us the size of each part.

2 A _____ has a value between 0 and 1.

3 To _____ a fraction, divide the numerator and denominator by a common factor. This process is known as _____.

4 Dividing by a fraction is the same as multiplying by the _____ of the fraction.

5 An _____ has a numerator that is greater than or equal to the denominator.

6 _____ are located at exactly the same position on a number line.

7 A _____ is a fraction with a numerator of 1.

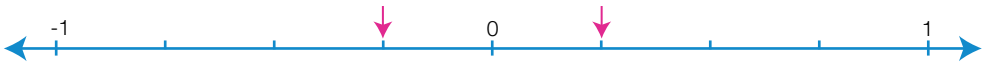
Fluency

1 Here are 3 dozen (36) eggs. **3.1, 3.2**

(a) If $\frac{4}{9}$ of the eggs are cracked, then how many eggs are cracked?

(b) The farmer's 3 best hens laid 15 of the eggs pictured. What fraction of the total is this? Write your answer in simplest form.

2 Write the value of the fractions shown by the arrows on this number line: **3.1**



3 Write the numbers **(a)** 6 and **(b)** 11 as improper fractions with denominators of: **3.1**

(i) 3 **(ii)** 10 **(iii)** 1

4 **(a)** 3 pizzas are shared between 5 friends. Write the amount of pizza each person gets as a fraction. **3.1**

(b) 7 packets of stickers are shared between 4 students. Write the number of packets that each student receives as a mixed number.

5 Find equivalent fractions by copying and completing the following. **3.2**

(a) $\frac{2}{7} = \frac{12}{\square}$ **(b)** $\frac{12}{18} = \frac{\square}{6}$ **(c)** $\frac{15}{30} = \frac{45}{\square}$ **(d)** $\frac{\square}{24} = \frac{5}{4}$

6 Write the following fractions in their simplest form. **3.2**

(a) $\frac{12}{15}$ **(b)** $\frac{48}{20}$ **(c)** $3\frac{60}{72}$ **(d)** $1\frac{8}{40}$

7 **(a)** Write $3\frac{2}{7}$ as an improper fraction. **3.2**

(b) Write $\frac{50}{9}$ as a mixed number.

8 Copy and complete the following by inserting <, > or =. **3.3**

(a) $\frac{5}{13}$ _____ $\frac{1}{2}$ **(b)** $\frac{6}{16}$ _____ $\frac{30}{80}$ **(c)** $\frac{5}{12}$ _____ $\frac{3}{8}$ **(d)** $\frac{8}{9}$ _____ $\frac{9}{10}$

9 Evaluate the following. Estimate first to check that your answers are reasonable. **3.4**

(a) $\frac{5}{12} + \frac{7}{8}$ **(b)** $\frac{7}{18} - \frac{2}{9}$ **(c)** $\frac{2}{3} - \frac{2}{7}$ **(d)** $\frac{1}{6} + \frac{7}{10}$

10 Evaluate the following. Write your answers as mixed numbers in simplest form. **3.4**

(a) $3\frac{2}{5} - 1\frac{3}{4}$ **(b)** $2\frac{1}{4} + 5\frac{11}{12}$ **(c)** $8 - 2\frac{5}{7}$ **(d)** $4\frac{5}{6} - 4\frac{2}{3}$

11 Calculate the following. **3.5**

(a) $\frac{3}{7}$ of \$28 **(b)** $\frac{2}{9}$ of $\frac{3}{4}$

12 Evaluate the following. **3.5**

(a) $\frac{6}{11} \times \frac{55}{18}$ **(b)** $2\frac{4}{9} \times 5$ **(c)** $\frac{2}{3} \times \frac{9}{16} \times 1\frac{1}{7}$

13 Evaluate the following. **3.6**

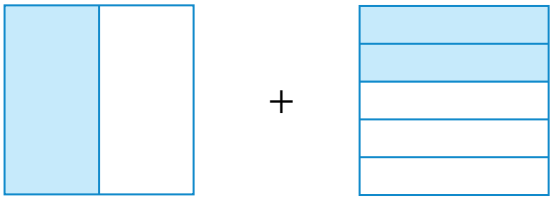
(a) $9 \div \frac{3}{4}$ **(b)** $21 \div 3\frac{2}{7}$ **(c)** $\frac{6}{5} \div \frac{32}{15}$ **(d)** $3\frac{1}{4} \div 3\frac{1}{3}$

14 Simplify the following. Remember to use the correct order of operations. **3.7**

(a) $4 + \frac{5}{6} \times 2\frac{1}{3}$ **(b)** $7\frac{2}{5} - \left(3\frac{1}{10} - \frac{3}{5}\right)$ **(c)** $\frac{7}{8} \times \left(\frac{3}{4} + 4 \times \frac{3}{4}\right)$

Understanding

15 **(a)** What fraction sum is represented by the following two grids? **3.3**



(b) Calculate the fraction sum using the grid method or any other method.

16 Write a fraction or a mixed number in simplest form to show each of these: **3.1**

(a) eight chocolate biscuits in a packet of 24

(b) two complete 'six-packs' of soft drink cans, with one can left over

(c) one packet of twelve colour pencils and four pencils in a second packet

(d) nine hours sleep in one day.

17 There are 12 dogs and 15 cats in an animal shelter. 6 of the animals have brown fur, and 2 have red fur. Write the following as fractions of the whole animal shelter, in simplest form: **3.2**

(a) the number of cats

(b) the number of brown-furred animals

(c) the number of animals with neither brown nor red fur.

18 Write each set of numbers in order from smallest to largest. **3.3**

(a) $\frac{3}{4}, \frac{1}{2}, 1, \frac{1}{3}, \frac{3}{5}$ **(b)** $2, \frac{13}{5}, \frac{5}{4}, \frac{30}{50}, 1\frac{2}{5}$

19 Jamal gets 7 out of 9 shots at goal in the netball ring, whereas Kayla gets 9 out of 12 shots in. Determine who is the more accurate shooter by comparing fractions. **3.3**

20 Elise and Reece each have an identical block of chocolate. Elise eats $\frac{2}{5}$ and Reece eats $\frac{2}{3}$. **3.3**

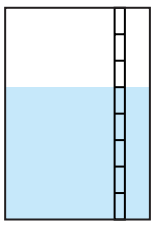
(a) How much more has Reece eaten, as a fraction of a block?

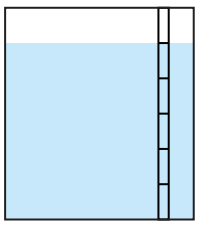
(b) Together, have Elise and Reece eaten more or less than a whole block of chocolate? Estimate, then calculate your answer.

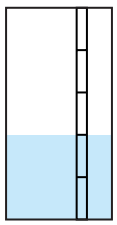
21 Gary had $1\frac{1}{2}$ bags of cement in his shed. He used $\frac{2}{3}$ of a bag to mix up some concrete. What fraction of a full bag does he have left? **3.4**

22 Jarrod owns a petrol station that has three main fuel tanks. There is a dip stick in each tank that shows the level of fuel remaining in each tank. How many litres are left in each tank? **3.5**

(a) Full tank = 48 000 L **(b)** Full tank = 60 000 L **(c)** Full tank = 30 000 L







23 Here are the ingredients for a cake: **3.6**

180 g butter	$\frac{2}{3}$ cup caster sugar	4 eggs
$1\frac{3}{4}$ cups self-raising flour	2 tablespoons cocoa	$\frac{1}{2}$ cup orange juice

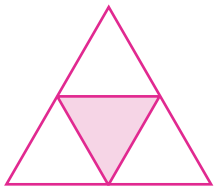
Rewrite the ingredients to make a cake that is one-quarter of the size of this cake.

Reasoning

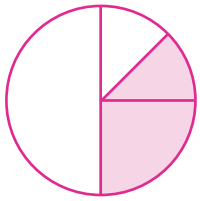
24 State the fraction of the area of these shapes that is shaded. **3.1**

(a)

(b)



(c)



25 45 minutes into a fun run, Katya had completed 4 km of the 10 km course, while Mabok had completed 7 km of the 15 km course. **3.2**

(a) Who had the least distance still to run?

(b) Who had completed a bigger fraction of their particular course?

26 Which of the following will give an answer of 1? **3.6**

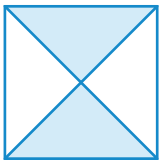
A $\frac{1}{4} \div \frac{1}{4}$ **B** $\frac{1}{2} \div \frac{1}{4}$ **C** $1 \div \frac{1}{4}$ **D** $4 \div \frac{1}{4}$

Numeracy practice 3

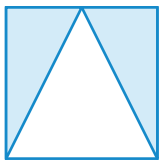
Non-calculator

1 Which diagram does not have exactly $\frac{1}{2}$ of the area shaded?

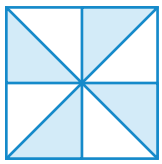
A



B

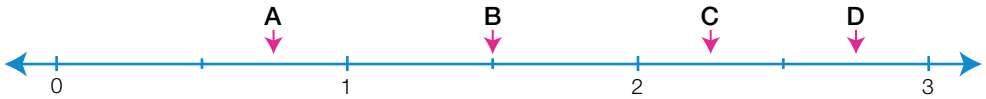


C



D

2 Which arrow is pointing closest to the location of $\frac{11}{8}$ on this number line?



3 Kevin is mixing sand, gravel and cement to make concrete. Sand makes up $\frac{1}{2}$ of the mixture, and gravel is $\frac{1}{3}$. What fraction of the mixture is cement?

A $\frac{1}{6}$ **B** $\frac{1}{5}$ **C** $\frac{1}{4}$ **D** $\frac{1}{3}$

4 Esther is making vanilla ice-cream using these ingredients that make 6 servings:

5 eggs
1 cup sugar
2 cans condensed milk
6 cups milk
$1\frac{1}{2}$ teaspoons vanilla

How many teaspoons of vanilla will Esther need to make 30 servings?

A $5\frac{1}{2}$ **B** $7\frac{1}{2}$ **C** $30\frac{1}{2}$ **D** 45

Calculator allowed

5 A school has 200 students. 48 of the students are involved in the school production. The fraction of students who are involved in the school production is closest to:

A one-fifth **B** one-quarter **C** one-third **D** one-half

6 Which of the following fractions is greater than 2 but less than 3?

A $\frac{3}{4}$ **B** $\frac{12}{9}$ **C** $\frac{17}{7}$ **D** $\frac{15}{5}$

7 Imran ran $\frac{1}{6}$ of a cross-country course. He ran 3 km. How long is the course?

A 6 km **B** 9 km **C** 12 km **D** 18 km

8 Which fraction is exactly half-way between $\frac{3}{8}$ and $\frac{4}{8}$ on the number line?

A $\frac{3}{16}$ **B** $\frac{1}{4}$ **C** $\frac{1}{3}$ **D** $\frac{7}{16}$

Exploration STEM

You can download this activity from the eBook or the Pearson Places website.

The secret message

In real life, secret data is turned into secure codes using prime numbers. Your task is to encode and decode your own top-secret communications using a system of prime number factorisation.

Exploration STEM

You can download this activity from the eBook or the Pearson Places website.

Electric fonts

Digital light displays (like LEDs on a clock) use regular patterns of shapes to create a 'digital font' of numbers and letters. How many individual shapes (pixels) are needed to create each of the numbers and letters? Which digital fonts use the least pixels? Which fonts look the best and which are easiest to read? Your task is to investigate these fonts and create your own.

Exploration STEM

You can download this activity from the eBook or the Pearson Places website.

The best buys

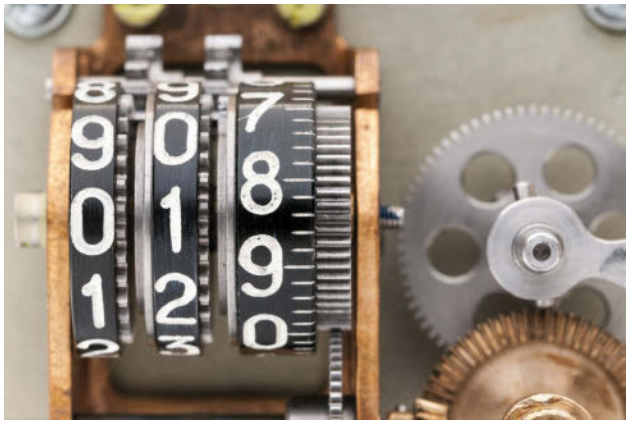
Whether you are buying eggs for your kitchen or RAM for your computer, the variety of options available can be confusing. Your task is to explore the options and their differences, to see how to find the best and cheapest items on offer.

Exploration Coding

You can access this activity from the eBook or the Pearson Places website.

Adding consecutive numbers

Is there an easy way to add consecutive numbers? Explore an algorithm that will find the sum of a set of consecutive numbers.



4

Decimals, percentage and ratio

4

When is a decimal point not a decimal point?

A number may contain a point, but this does not always make it a decimal point.

Usually in mathematics numbers represent quantities or amounts, so you can calculate with them. But numbers can also be used in a less mathematical way, as labels or 'tags' that help to identify things. Room numbers, phone numbers and football scores are all examples of this.

Some numbers like this include a point that looks like a decimal point, but is actually just a 'separator', placed between numbers that are labels for different things. For example, a score of 3.6 in an AFL game means 3 goals and 6 behinds, not 3 goals plus 6 tenths of a goal. A large building such as a hotel might have a room number written as 2.13, meaning the 13th room on the 2nd floor, not 2 rooms plus 13 hundredths of a room.

Forum

The time written as 7:30 or 7.30 is read as 'seven thirty' or 'half past seven'. If you wanted to write this time as a proper decimal number, what would you write? How many minutes would 0.30 hours be? Can you think of other examples where numbers may have a point and look like decimal numbers, but mean something different?

Amounts of money, such as $4.35, are written with a point. Is this a decimal point?

Why learn this?

An understanding of decimal numbers can help you to measure the wood to build a new shelf, work out who won the 100 m freestyle or calculate a household budget. Percentages have many everyday uses, such as measuring performance, advertising discounts or presenting survey results. Ratios and rates help us compare and calculate quantities of the same and different types, such as when determining which products are the best value for money.

After completing this chapter you will be able to:

- compare, order and round decimal numbers
- add, subtract, multiply and divide decimal numbers
- use estimations to check that answers are reasonable
- convert between decimals, fractions and percentages
- use percentages to solve problems
- understand the relationship between ratios, fractions and percentages
- use ratios and rates to compare and calculate amounts
- calculate unit prices and determine 'best buys'.

Recall 4

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

1 Copy and complete each of the following by writing < (less than), > (greater than), or = (equal to) between the given numbers.

(a) 0.1 _____ 0.01 **(b)** 2 _____ 2.2 **(c)** 0.3 _____ 0.1

(d) 3.2 _____ 4.1 **(e)** 0.008 _____ 0.09 **(f)** 0.7 _____ 0.07

2 The number 43 can be written in expanded form as 'four tens and three ones'. Write the following in expanded form.

(a) 72 **(b)** 603 **(c)** 9251 **(d)** 11 080

3 Write each of the following **(i)** in words and **(ii)** as decimals.

(a) $\frac{7}{10}$ **(b)** $\frac{8}{1000}$ **(c)** $\frac{3}{100}$ **(d)** $1\frac{9}{10}$

4 Calculate:

(a) $\begin{array}{r} 34 \\ +\ 76 \\ \hline \\ \hline \end{array}$ **(b)** $\begin{array}{r} 925 \\ +\ 610 \\ \hline \\ \hline \end{array}$ **(c)** $\begin{array}{r} 67 \\ 409 \\ +\ \ 3 \\ \hline \\ \hline \end{array}$ **(d)** $\begin{array}{r} 459 \\ 6013 \\ +\ \ 27 \\ \hline \\ \hline \end{array}$

5 Calculate:

(a) $\begin{array}{r} 74 \\ -\ 25 \\ \hline \\ \hline \end{array}$ **(b)** $\begin{array}{r} 823 \\ -\ 376 \\ \hline \\ \hline \end{array}$ **(c)** $\begin{array}{r} 8289 \\ -\ \ 384 \\ \hline \\ \hline \end{array}$ **(d)** $\begin{array}{r} 2000 \\ -\ \ 352 \\ \hline \\ \hline \end{array}$

6 Calculate: **(a)** 2×17 **(b)** 25×96 **(c)** 51×800

7 Calculate: **(a)** $362 \div 2$ **(b)** $9459 \div 9$ **(c)** $5600 \div 2000$

8 Calculate:

(a) 70×100 **(b)** $12 \times 10\,000$ **(c)** 0.0427×1000

(d) $58 \div 10$ **(e)** $901 \div 100$ **(f)** $76.2 \div 1000$

9 Write these percentages as fractions in simplest form.

Exploration Task

You can download this activity from the eBook or the Pearson Places website.

Which deal is best?

In this activity, you will investigate different kinds of discount deals to decide which option is best.

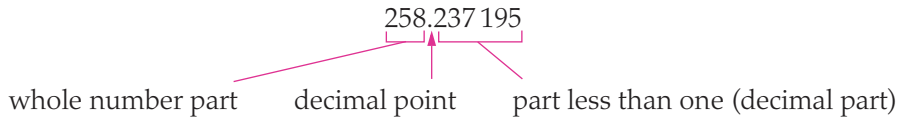
Place value and comparing decimals

4.1

Decimal place value

Like fractions, decimals are used to write numbers that have a part that is less than one. The **decimal point** is used to separate the whole number part from the part that is less than one.

The word decimal comes from the Latin word *decima* meaning 'tenth part', which comes from *decem* meaning ten. December was the tenth month in the original Roman calendar.



The number 258.237 195 is made up of nine individual **digits**. Because six of the digits are after the decimal point, this means the number has 6 **decimal places**.

More examples: 3.25 has three digits and 2 decimal places
45.079 has five digits and 3 decimal places.

You can write each digit of a number into a 'place value table' to show this:

Hundreds 100	Tens 10	Ones 1	•	Tenths $\frac{1}{10}$ (0.1)	Hundredths $\frac{1}{100}$ (0.01)	Thousandths $\frac{1}{1000}$ (0.001)	Ten-thousandths $\frac{1}{10\,000}$ (0.0001)	Hundred-thousandths $\frac{1}{100\,000}$ (0.000 01)	Millionths $\frac{1}{1\,000\,000}$ (0.000 001)
2	5	8	•	2	3	7	1	9	5

The **place value** of each column (hundreds, tens, ones, tenths, hundredths etc.) is one-tenth of each column to the left. As you move across the columns from left to right, divide by 10 each time. This pattern is continued past the decimal point to get the values of the decimal places.

You are probably familiar with the first 3 decimal places of tenths, hundredths and thousandths. You can keep dividing by 10 to get ten-thousandths, hundred-thousandths, millionths and so on.

If there is no digit for a particular place value in a decimal number, use a zero to show this. For example, five and two tenths and three thousandths is written as 5.203. It is important to include the zero between the 2 and the 3. Leaving it out would make 5.23, which is a different number.

Writing decimals

Decimal numbers can be written in several ways:

- Decimal form: 258.237 195
- Expanded fractional form: $258 + \frac{2}{10} + \frac{3}{100} + \frac{7}{1000} + \frac{1}{10\,000} + \frac{9}{100\,000} + \frac{5}{1\,000\,000}$
- Expanded word form: *two* hundreds, *five* tens, *eight* ones, *two* tenths, *three* hundredths, *seven* thousandths, *one* ten-thousandth, *nine* hundred-thousandths and *five* millionths.

Saying decimals

- Say each digit after the decimal point as if it is a separate number. For example, 14.625 is 'fourteen point six two five', not 'fourteen point six hundred and twenty five'.
- Fraction equivalents can also be used. For example, 0.63 can be said as 'sixty-three hundredths'. This is the same as saying 'six tenths and three hundredths'.

$\frac{63}{100} = \frac{6}{10} + \frac{3}{100}$

Worked example 1 W.E.1

(a) Write $4 + \frac{3}{10} + \frac{6}{1000} + \frac{5}{10\,000}$ as a decimal.

(b) Write 6.2807 in expanded fractional form.

Thinking

(a) Imagine the numerators of each fraction in their place-value columns. (As there are no $\frac{1}{100}$s, write a zero in the 'hundredths' column.)

(b) 1 Imagine the decimal digits in their place-value columns.

2 Write a series of fractions using the headings of the place-value columns. There is no need to include columns with 0 (e.g. $\frac{0}{1000}$).

Working

(a)

Ones 1	•	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$	Ten-thousandths $\frac{1}{10\,000}$
4	•	3	0	6	5

$4 + \frac{3}{10} + \frac{6}{1000} + \frac{5}{10\,000}$

$= 4.3065$

(b)

Ones 1	•	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$	Ten-thousandths $\frac{1}{10\,000}$
6	•	2	8	0	7

$= 6 + \frac{2}{10} + \frac{8}{100} + \frac{7}{10\,000}$

Worked example 2

W.E. 2

(a) Write 'seven units, three tenths, six hundredths, seven thousandths and four hundred-thousandths' as a decimal.

(b) Write 28.0045 in expanded word form.

Thinking

(a) Look at a place-value table. Put the digits in the corresponding place-value columns.

(b) 1 Look at a place-value table. Put the digits in their place-value columns, including zero.

2 Write the non-zero digits in words, with the place-value column heading after each one.

Working

(a)

Ones 1	•	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$	Ten-thousandths $\frac{1}{10\,000}$	Hundred-thousandths $\frac{1}{100\,000}$
7	•	3	6	7	0	4

7.367 04

(b)

Tens 10	Ones 1	•	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$	Ten-thousandths $\frac{1}{10\,000}$
2	8	•	0	0	4	5

28.0045
= Two tens, eight ones, four thousandths and five ten-thousandths

Worked example 3

W.E. 3

(a) Write the value of the 6 in 4.368 as a fraction.

(b) Write the value of the 9 in 0.0109 in words.

Thinking

(a) Identify the place value of the digit, then write a fraction with the digit as the numerator and the place value as the denominator.

(b) Identify the place value of the digit, then write in words the digit, followed by the place value.

Working

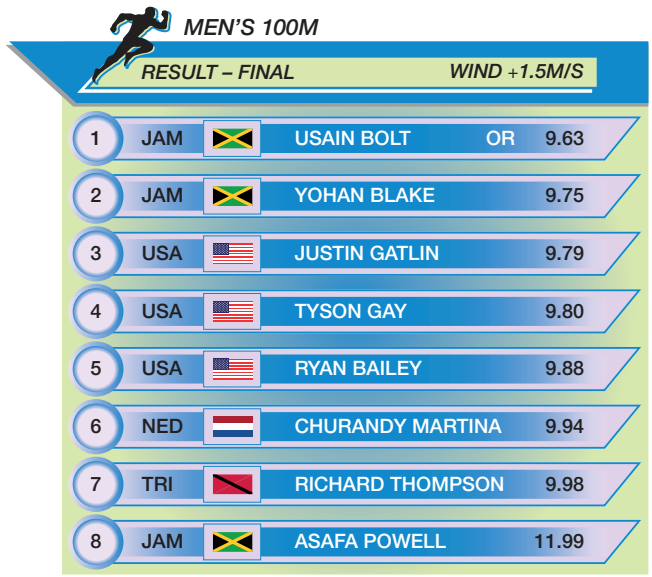
(a) $\frac{6}{100}$

(b) nine ten-thousandths

Comparing decimals

You often need to compare two decimal numbers to decide which number is greater. You might be comparing prices of goods or seeing who won a race. Look at the decimal numbers in these results for an Olympic 100 m race. Usain Bolt's time is first because it is the smallest decimal number. How much did Bolt win by?

Use the relation symbols of less than (<), greater than (>) and equals (=) when comparing decimals.

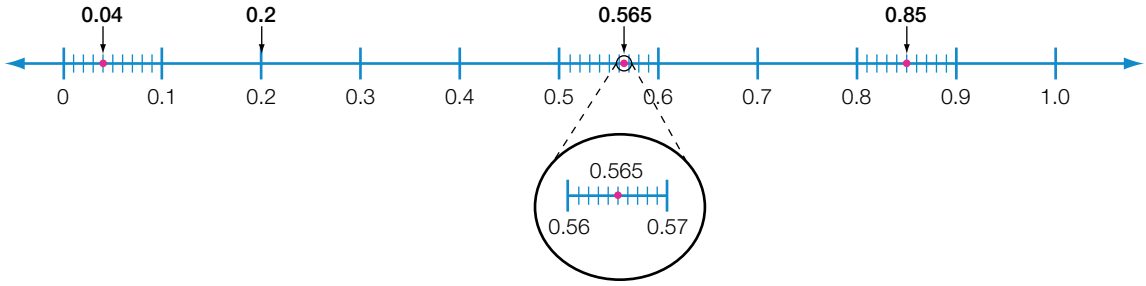


Comparing decimals using a number line

You can use a number line to compare decimals. Numbers get larger as you move along the number line from left to right.

The number line below is marked in tenths. The space between 0 and 0.1 and between 0.5 and 0.6 shows 1 tenth divided into 10 parts, each equal to 1 hundredth: $\frac{1}{10} = \frac{10}{100}$

The positions of the decimal numbers 0.04, 0.2, 0.565 and 0.85 are shown with arrows.



The interval between 0.56 and 0.57 is magnified to show the position of 0.565 more clearly.

It shows 1 hundredth divided into 10 parts, each equal to 1 thousandth: $\frac{1}{100} = \frac{10}{1000}$

From the decimal numbers shown on the number line, you can see that:

The decimal number with the most digits is not necessarily the largest.

For example, $0.565 < 0.85$.

A number's size depends on the value and position of the digits.

For example, $0.2 > 0.04$.

Comparing decimals by comparing digits

For each digit, the higher its place value, the greater the value. For example, $0.2 > 0.04$ because $\frac{2}{10} > \frac{4}{100}$, so on the number line 0.2 is further to the right than 0.04. To find which number is bigger, compare the digits in each place value column, beginning with the highest place values.

Worked example 4 W.E. 4

Write < or > between each of the following pairs of decimals to make a true statement.

(a) 5.173 _____ 5.0731 **(b)** 0.472 39 _____ 0.4731 **(c)** 3.5 _____ 3.51

Thinking		Working
(a) **1**	Compare the whole number parts of each decimal.	(a) The whole number parts are the same number, 5.
2	If the whole number parts are the same, compare the tenths digits to see which is greater.	1 is greater than 0, so 5.173 is the larger number.
3	Write a relation symbol (< or >) between the two numbers.	5.173 > 5.0731
(b) **1**	Compare the whole number parts of each decimal.	(b) The whole number parts are the same, 0.
2	If the whole number parts are the same, compare the tenths digits.	The tenths digits are the same, 4.
3	If the tenths digits are the same, compare the hundredths digits.	The hundredths digits are the same, 7.
4	Repeat until you get different digits.	The thousandths digits are 2 and 3. So, 0.472 39 is less than 0.4731.
5	Write a relation symbol beween the two numbers.	0.472 39 < 0.4731
(c) **1**	Add zeros to the end of one of the decimals to help you compare. This doesn't change its value.	(c) 3.5 is the same as 3.50.
2	Compare the two decimals as in previous examples.	3.50 has the smaller hundredths digit, so it is the smaller decimal.
3	Write < or > between the two numbers.	3.5 < 3.51

4.1 Place value and comparing decimals

Navigator

Answers p. 665

1, 2, 3, 4, 5, 6 (column 1), 7, 8, 9, 10, 11 (a–f), 13, 14, 15, 16, 17, 18, 19	1, 2, 3, 4, 5, 6 (column 1), 7, 8, 9, 10, 11 (column 1), 12, 13, 14, 16, 17, 18, 19, 20	1, 2, 3, 4, 6 (column 2), 7, 10, 11 (column 2), 12, 13, 14, 16, 17 (a–b), 18, 19, 20, 21

Fluency

1 Copy the table below, and for each of the following:

(i) place the whole numbers and numerators into their place-value columns

(ii) write the number in decimal form.

Tens 10	Ones 1	•	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$
		•			

(a) $3 + \frac{1}{10} + \frac{2}{100} + \frac{6}{1000}$ **(b)** $17 + \frac{8}{10} + \frac{3}{1000}$ **(c)** $\frac{2}{100} + \frac{8}{1000}$

2 Copy the table below, and for each of the following:

(i) place the digits into their place-value columns

(ii) write the number in expanded fractional form.

Hundreds 100	Tens 10	Ones 1	•	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$
			•			

(a) 312.76 **(b)** 2.093 **(c)** 0.671

W.E. 1

3 (a) Write each of the following as a decimal.

(i) $45 + \frac{4}{10} + \frac{6}{100} + \frac{2}{1000}$

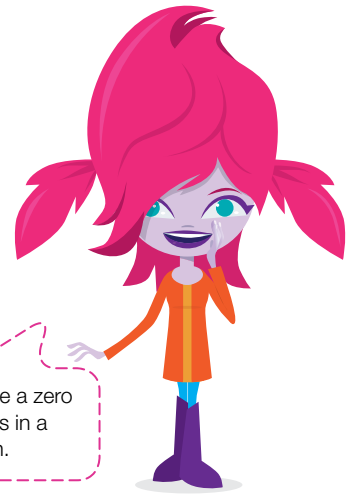
(ii) $12 + \frac{5}{10} + \frac{1}{100} + \frac{9}{1000} + \frac{3}{10\,000} + \frac{7}{100\,000} + \frac{2}{1\,000\,000}$

(iii) $3 + \frac{7}{10} + \frac{9}{100} + \frac{8}{1000} + \frac{5}{10\,000}$

(iv) $1 + \frac{8}{1000} + \frac{3}{10\,000} + \frac{3}{100\,000} + \frac{2}{1\,000\,000}$

(v) $\frac{7}{10} + \frac{8}{100} + \frac{6}{1000} + \frac{6}{100\,000}$

(vi) $7 + \frac{3}{100} + \frac{3}{1000} + \frac{4}{100\,000} + \frac{7}{1\,000\,000}$



(b) Write each of the following in expanded fractional form.

(i) 6.63 **(ii)** 0.921 **(iii)** 0.7345

(iv) 7.826 **(v)** 23.913 04 **(vi)** 45.004 589

4 (a) Write each of the following as a decimal. **W.E. 2**

(i) six ones and five tenths

(ii) nine tenths and seven hundredths

(iii) two tenths, seven hundredths and three thousandths

(iv) three tens, seven ones, four tenths, two hundredths and one ten-thousandth

(v) one ten, four ones, nine tenths, five hundredths, seven thousandths, six ten-thousandths, two hundred-thousandths and three millionths

(vi) seven hundreds, four thousandths, five ten-thousandths and nine hundred-thousandths

(b) Write each of the following in expanded word form.

(i) 5.2 **(ii)** 4.9 **(iii)** 34.17 **(iv)** 0.61

(v) 2.794 **(vi)** 7.5092 **(vii)** 35.865 43 **(viii)** 0.820 027

5 (a) Write the value of the 2 in each of the following as a fraction. **W.E. 3**

(i) 6.012 **(ii)** 0.004 52 **(iii)** 3.287 **(iv)** 2.034

(b) Write the value of the 7 in each of the following in words.

(i) 5.734 **(ii)** 0.0076 **(iii)** 1.2037 **(iv)** 8.130 037

6 Write < or > between each of the following pairs of decimals to make a true statement. **W.E. 4**

(a) 2.4 _____ 0.42 **(b)** 2.32 _____ 1.955

(c) 0.65 _____ 0.57 **(d)** 0.3003 _____ 0.333

(e) 4.7038 _____ 4.7312 **(f)** 8.251 _____ 8.2501

(g) 7.02 _____ 7.002 **(h)** 4.7367 _____ 4.7376

(i) 0.927 _____ 0.927 34 **(j)** 6.013 _____ 6.01

(k) 3.406 _____ 3.4063 **(l)** 0.9995 _____ 0.9986

7 Write true (T) or false (F) for each of the following.

(a) $7.5 < 5.77$ **(b)** $4.1 > 4.12$

(c) $6.08 > 6.8$ **(d)** $67.54 < 67.504$

(e) $3.023 < 3.203$ **(f)** $0.547 > 0.547\,08$

(g) $2.000\,012 < 2.0001$ **(h)** $4.145\,29 > 4.200\,01$

8 53.017 expressed in expanded word form is:

A five tens, three ones, one tenth and seven hundredths

B five tens, three ones, one hundredth and seven thousandths

C five tenths, three hundredths, one ten-thousandth and seven hundred-thousandths

D five tens, three ones, one tenth and seven thousandths

9 Nine hundredths, four thousandths and three ten-thousandths is equal to:

A 0.090 43 **B** 0.0943 **C** 0.943 **D** 9.43

10 For each of the following sets of numbers, copy the number line shown, then mark the positions of the numbers with a labelled arrow. (You may have to estimate the position of some.)

(a) 2.05, 2.09, 2.6, 2.12, 2.59

(b) 2.0, 2.8, 2.88, 2.9, 2.805

(c) 2.7, 2.4, 2.07, 2.04, 2.407

(d) 2.2, 2.4, 2.85, 2.35, 2.05

11 Write each set of decimals in order from smallest to largest.

(a) 2.3, 2.03, 2.13

(b) 8.7, 8.007, 8.67

(c) 6.646, 6.6403, 6.64

(d) 0.0095, 0.0905, 0.0509

(e) 5.3281, 5.38, 5.003 821

(f) 3.616, 3.116, 3.661

(g) 0.92, 0.29, 0.092

(h) 0.85, 0.815, 0.086

Understanding

12 Joel wrote a five-digit number as he saw it on his stopwatch: 37901. However, he forgot to write the decimal point. Anna knew that the race was timed to a thousandth of a second, so she was able to write the decimal point in the correct place. Write the number with the decimal point in the correct place.

13 The difference in times between two skiers in a downhill race was 0.437 of a second. Write this:

(a) in expanded fraction form

(b) as a single fraction.

14 For each of the following sets of decimals, draw a section of the number line, marked in either tenths or hundredths, and indicate the position of each number in the set.

(a) 0.4, 0.72, 1.01

(b) 1.7, 1.25, 0.95

(c) 0.06, 0.045, 0.038

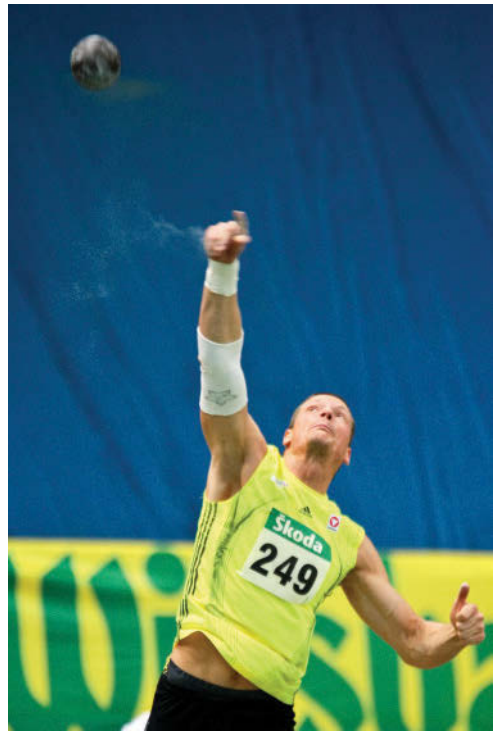
(d) 2.9, 3.2, 2.75, 3.06

15 Jane records her three best practice times for the 100 m sprint. They are as follows. 13.95 seconds, 13.08 seconds and 13.69 seconds.

(a) Which was Jane's fastest time?

(b) Which was Jane's slowest time?

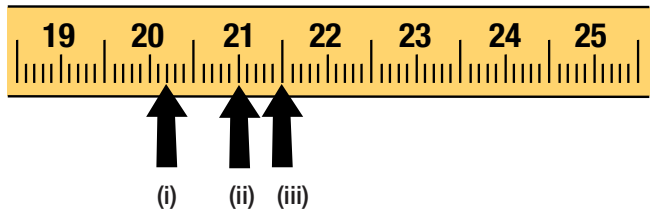
16 Egor throws a shot-put the following distances during a competition: 25.6 m, 25.56 m and 25.081 m. Which distance was Egor's longest throw?



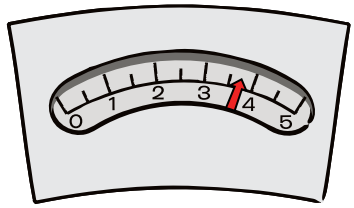
Reasoning

17 Write the value of the decimals indicated by the arrows on the following scales.

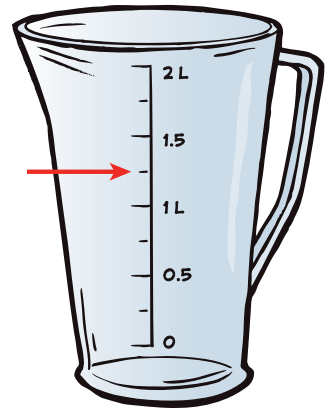
(a)



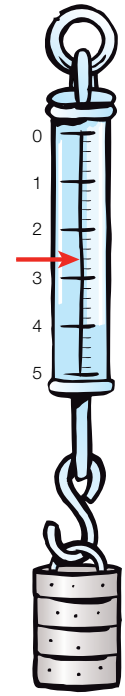
(b)



(c)



(d)



18 State the mistakes that the following students have made.

(a) Minh says that the decimal 34.162 has a 2 in the hundredths column.

(b) Al writes four hundreds, nine ones, six tenths and seven hundredths as 409.067.

(c) Max writes seven ones, eight hundredths and nine ten-thousandths as 0.7809.

(d) Polly writes 0.9056 as $\frac{9}{10} + \frac{5}{100} + \frac{6}{1000}$.

Open-ended

19 **(a)** Write at least six numbers between 4.5 and 4.7.

(b) Write your numbers from **(a)** in ascending order.

20 **(a)** Use any digit from 0 to 9 in each box to make a correct statement. Digits can be repeated. Find at least three different answers.

$\square.\square\square < 5.\square$

(b) Explain any limitations on the other numbers that can be used when certain numbers are used in some boxes.

21 Donna has cut out digits from coloured paper to stick on a poster to show a decimal number. If she uses the digits 2, 5 and 7, as well as a decimal point:

(a) How many different decimal numbers with 2 decimal places could she make?

(b) How many decimal numbers greater than 5.4 could she make?

Game

Duelling decimals

Equipment required: 1 die

Aim of the game:
The winner is the first player to 5 points.

How to play:

1 One player rolls a die to determine the number of digits in the round. Each player then draws the appropriate number of boxes on their paper after the decimal point.

For example, if a 3 is rolled, each player draws:

0 . ☐☐☐ 0 . ☐☐☐

2 Each player then takes turns to roll the die and write the number they roll into a box of their choice.

When all of the boxes have been filled, the player who has created the biggest number wins a point for that round.

For example, the players might have filled in their boxes like this:

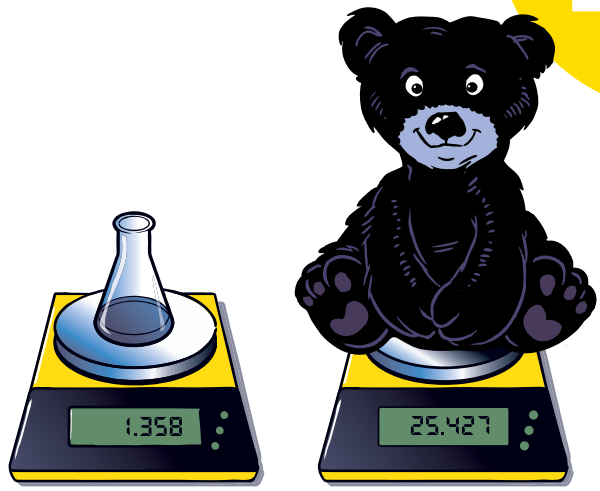
0 . 2 5 3 0 . 6 3 1

Here, the second player would win the point.

3 The game is repeated, with players taking turns to roll the number of digits for each round.

Rounding decimals

4.2



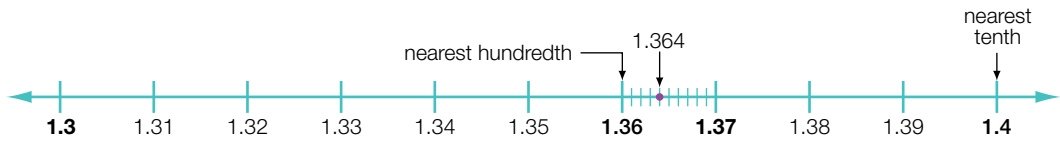
Decimal numbers sometimes contain more decimal places than are required. It depends on the situation.

A scientist might need to know the weight of a chemical accurate to a thousandth of a gram, or 3 decimal places. They could weigh the chemical on a scale and get a reading such as 1.358 g.

A zookeeper wanting to find the mass of a bear cub could put the cub on a similar scale and get a reading such as 25.427 kg. However, 2 hundredths or 7 thousandths of a kilogram is an insignificant amount compared to the whole mass of the cub. The zookeeper would write the bear cub's mass as 25.4 kg, or just 25 kg. This is called **rounding**.

The number line below shows how the number 1.364 could be rounded:

- To the nearest tenth (1 decimal place): 1.364 is between the tenths values of 1.3 and 1.4. The 6 in the hundredths column puts it closer to 1.4.
  1.364 rounded to the nearest tenth is *rounded up* to 1.4.
- To the nearest hundredth (2 decimal places): 1.364 is between the hundredths values of 1.36 and 1.37. The 4 in the thousandths column puts it closer to 1.36.
  1.364 rounded to the nearest hundredth is *rounded down* to 1.36.



To round decimal numbers:

Step 1 Determine the number of decimal places that you need to round to. The digit in this place will either stay the same, or increase by one. This is the digit being rounded.

Step 2 Look at the next digit, to the right of the digit being rounded. If this digit is:

- 0, 1, 2, 3 or 4, then leave the digit being rounded as it is and delete all digits after it
- 5, 6, 7, 8 or 9, then increase the digit being rounded by one and delete all digits after it.

If the number in the place value you are rounding to is 9 and you need to increase it, make the 9 into 0, and add one to the digit in front of it. For example, 1.497 rounded to the nearest hundredth is 1.50. Write a 0 in the hundredths column to show that this number is to the nearest hundredth (even though 1.50 is the same value as 1.5).

Worked example 5 W.E. 5

Round the following decimals to the number of decimal places shown in brackets.

(a) 3.785 (2) **(b)** 0.958 34 (3) **(c)** 6.214 96 (4)

Thinking			Working
(a)	**1**	Decide which digit you will be rounding. (Here, it is the 8, so the answer will either be 3.78 or 3.79.)	(a) 3.785
	2	Look at the digit to the right of this digit and consider whether you need to round up (5–9) or down (0–4). (Here, the 5 tells us to round up, so the 8 becomes a 9.)	3.785
	3	Either increase the digit by one, or leave it as it is. Delete all digits following the one that you are rounding to.	3.79
(b)	**1**	Decide which digit you will be rounding. (Here, it is the 8, so the answer will either be 0.958 or 0.959.)	(b) 0.958 34
	2	Look at the next digit to the right of this digit and consider whether you need to round up (5–9) or down (0–4). (Here, the 3 tells us that we round down, so we leave the 8 as it is.)	0.958 34
	3	Either increase the digit by one, or leave it as it is. Delete all digits following the one that you are rounding to.	0.958
(c)	**1**	Decide which digit you will be rounding. (Here, it is the 9, so the answer will either be 6.2149 or 6.2150.)	(c) 6.214 96
	2	Look at the next digit to the right of this digit and consider whether you need to round up (5–9) or down (0–4). (Here, the 6 tells us to round up, but rounding the 9 up will give 10 ten-thousandths, which is equal to 1 thousandth. This means that the thousandths digit is increased by 1.)	6.214 96
	3	Either increase the digit by one, or leave it as it is. Delete all digits following the one that you are rounding to. (Here, we leave the zero in the rounded decimal place.)	6.2150

Rounding money

Money is written using decimal notation, usually with 2 decimal places. Because \$1 = 100c, or ten lots of 10c, the tenths place value column shows how many lots of 10c make up the amount, and the hundredths column shows how many lots of 1c make up the amount.

In the 1990s, Australia stopped using 1-cent and 2-cent pieces, making the 5-cent coin the smallest coin in use. This means that all money amounts paid in cash must be rounded to the nearest 5 cents (5c).

If money is being paid in cash, the following rules apply.

If the amount ends in…	*then it is rounded…*	*the last digit becomes…*
1 or 2 cents	down	0
3 or 4 cents	up	5
6 or 7 cents	down	5
8 or 9 cents	up	0

If the amount ends in 0 or 5 cents, then the exact amount is paid.

Worked example 6

W.E. 6

Round the following amounts of money to the nearest 5 cents.

(a) \$8.53 **(b)** \$7.02 **(c)** \$4.99

Thinking	Working
(a) Note the value of the digit in the 1c (hundredths) column. Round it up or down according to the rules. (Here, we round the 3 up to the nearest 5 cents.)	(a) \$8.55
(b) Note the value of the digit in the 1c (hundredths) column. Round it up or down according to the rules. (Here, we round the 2 down to the nearest multiple of 10 cents, which is 0.)	(b) \$7.00
(c) Note the value of the digit in the 1c (hundredths) column. Round it up or down according to the rules. (Here, we round the 9 up to the nearest multiple of 10 cents, which is 100.)	(c) \$5.00

4.2 Rounding decimals

Navigator

Answers p. 667

1, 2, 3 (columns 1–2), 4 (columns 1–3), 6, 7, 8, 9, 10, 11, 13, 14, 15	1, 2, 3 (columns 2–3), 4 (columns 1–2), 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16	2 (column 2), 3 (columns 2–3), 4 (columns 3–4), 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18

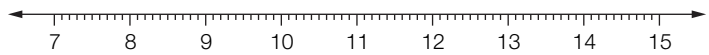
Fluency

1 For each of the following decimals:

(i) draw an arrow to show where the decimal is on the number line

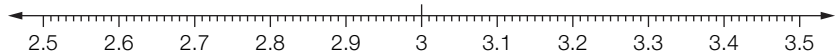
(ii) round the decimal to the number of decimal places shown in brackets.

(a) 10.4 (0)

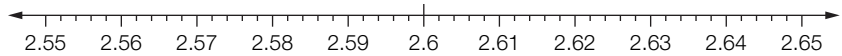


(b) 5.57 (1)

(c) 3.05 (1)



(d) 2.608 (2)



2 For each of the following decimals, state which of the two values in brackets it is closest to.

(a) 7.37 (7.3 or 7.4)

(b) 13.8451 (13.84 or 13.85)

(c) 0.296 (0.29 or 0.30)

(d) 4.4584 (4.45 or 4.46)

(e) 2.764 19 (2.764 or 2.765)

(f) 0.087 92 (0.087 or 0.088)

W.E. 5

3 Round the following decimals to the number of decimal places shown in brackets.

(a) 4.88 (1)

(b) 6.72 (1)

(c) 7.635 (2)

(d) 4.552 (2)

(e) 0.6416 (3)

(f) 3.2772 (2)

(g) 0.314 42 (4)

(h) 11.828 55 (2)

(i) 2.917 96 (2)

(j) 18.499 95 (3)

(k) 90.899 99 (4)

(l) 18.999 999 (3)

(m) 23.4097 (2)

(n) 57.0804 (3)

(o) 259.899 99 (4)

W.E. 6

4 Round the following amounts of money to the nearest 5 cents.

(a) $4.52

(b) $2.76

(c) $11.03

(d) $23.88

(e) $176.35

(f) $542.06

(g) $789.54

(h) $56.75

(i) $6774.99

(j) $1149.97

(k) $8989.99

(l) $9999.99

5 0.837 256 rounded to the nearest ten-thousandth is:

A 0.837

B 0.8372

C 0.837 26

D 0.8373

Understanding

6 At the 2009 World Athletics Championships, Usain Bolt set a 100 m record time of 9.58 seconds. Round this time to the nearest tenth of a second.

7 The total for Russell's grocery shopping is $53.67. If Russell is paying in cash, how much money will he have to hand over?

8 Sir Donald Bradman's cricket batting average was 99.943 (rounded to the nearest thousandth). What would be his average rounded to the nearest:

(a) tenth **(b)** hundredth?

9 The top temperature recorded one day in Mildura was 41.6 °C. What figure should a weather reporter say as the day's maximum, if the temperature is rounded to the nearest whole number (no decimal places)?

10 The price of a box of chocolates is $11.97. If Beau buys the chocolates and pays with cash, how much will he pay?

11 A physics student measures the speed of sound to be 352.153 metres per second. Because her measuring equipment is not accurate enough for this to be reliable, she decides to round this measurement to the nearest tenth. What value does she write in her report?

Reasoning

12 Ruth calculates the length of shelving required for a wall unit to be 1.266 66 m. As her tape measure is only accurate enough to measure in centimetres (cm), she needs to round this figure to the nearest centimetre. If 1 cm = 0.01 m, what length should she measure?

13 In a cycling race, the following results were recorded: 12.149 s, 12.143 s, 12.138 s, 12.152 s and 12.157 s.

(a) Write these in order from fastest to slowest.

(b) It was found that there was a small error in the timing equipment and the results were to be rounded to the nearest hundredth. What were the new times?

(c) Does this affect the race result?

14 **(a)** The total cost of Rani's shopping was $20.34. Round this amount to the nearest 5 cents.

(b) How much more would Rani have paid if the individual prices of the items were each rounded to the nearest 5 cents?

The individual costs of the items Rani bought were:

Cereal	$6.93
Yoghurt	$4.78
Bananas	$2.51
Juice	$3.63
Pasta	$2.49

Open-ended

15 Write at least three different decimal numbers that, when rounded, would give the number 3.79.

16 Write two prices that would give $100 when rounded to:

(a) the nearest 5 cents **(b)** the nearest dollar.

17 Taking 2.548 46 and rounding it to 4 decimal places gives 2.5485, then rounding this to 3 decimal places gives 2.549.

What answer do you get when rounding 2.548 46 to 3 decimal places?

Explain why this is different, and what is wrong with doing it the first way.

18 Why do you think 1-cent and 2-cent coins were taken out of circulation? List two or three possible reasons why a government might make this decision.

4.3 Decimals and fractions

Writing a decimal as a fraction

Any decimal can be written in expanded fractional form. This expanded form can then be simplified to show the decimal as a single fraction or as a mixed number.

For example: $3.72 = 3 + \frac{7}{10} + \frac{2}{100}$

$= 3 + \frac{70}{100} + \frac{2}{100}$

$= 3 + \frac{72}{100}$

$= 3\frac{72}{100}$ or $\frac{372}{100}$

$= 3\frac{18}{25}$ or $\frac{93}{25}$

The place value of the last digit (hundredths) gives the denominator of the unsimplified fraction (100), while the digits themselves give the numerator (72).

Remember that fractions should always be given in simplest form.

Worked example 7 W.E. 7

Convert the following decimals to fractions, giving your answers in simplest form.

(a) 6.28 **(b)** 0.3125

Thinking	Working
(a) 1 What is the place value of the last digit? This gives the denominator of the fraction. Write the digits as the numerator.	(a) 6.28 $= 6\frac{28}{100}$ or $\frac{628}{100}$
2 Simplify the fraction if possible. (Here, a common factor of 4 has been cancelled.)	$= 6\frac{7}{25}$ or $\frac{157}{25}$
(b) 1 What is the place value of the last digit? This gives the denominator of the fraction. Write the digits as the numerator.	(b) 0.3125 $= \frac{3125}{10\,000}$
2 Simplify the fraction if possible. (Here, we cancel common factors of 5, then 25, then 5 again.)	$= \frac{625}{2000}$ $= \frac{25}{80}$ $= \frac{5}{16}$

Writing a fraction as a decimal

There are two methods that can be used to convert a number in fraction form to decimal form:

Method 1: Write the fraction as an equivalent fraction (or mixed number) with a denominator of 10, 100, 1000 etc. Then, write the digits of the numerator of this new fraction as the decimal place values.

For example: $\frac{4}{5} = \frac{8}{10} = 0.8$

$\frac{13}{20} = \frac{65}{100} = 0.65$

This method is only convenient for fractions with denominators that are factors of 100: 2, 4, 5, 10, 20, 25, 50.

Method 2: The line between the numerator and the denominator is equivalent to the division symbol, ÷. To convert a fraction to a decimal, simply perform the division. This method can be used for all fractions.

For example: $\frac{7}{8} = 7 \div 8 = 0.875$

$\frac{37}{40} = 37 \div 40 = 0.925$

It seems the decimal point was first used in the 1600s by mathematician Bartholomaeus Pitiscus. Thanks, Bart!

Worked example 8 W.E. 8

Convert each of the following fractions to decimals.

(a) $\frac{7}{5}$ (b) $\frac{3}{8}$ (c) $\frac{4}{3}$

Thinking		Working
(a) 1	Is the denominator a factor of 10, 100 or 1000? (Yes)	(a) $\frac{7}{5}$
2	If the fraction is improper, write it as a mixed number.	$= 1\frac{2}{5}$
3	Write the fraction as an equivalent fraction with a denominator of 10, 100, or 1000 (10 in this case).	$= 1\frac{4}{10}$
4	Write any whole numbers on the left of a decimal point. (If there is no whole number part, write a zero.) Write the numerator of the fraction on the right of the decimal point.	$= 1.4$ $\frac{7}{5} = 1.4$

(b)	**1**	Is the denominator a factor of 10, 100 or 1000? (No)	(b) $\frac{3}{8}$	
	2	Write the fraction as a division calculation.	$3 \div 8$	$8\overline{)3}$
	3	Perform the division. If you are unable to begin because the divisor is a greater number (here, $8 > 3$), add a decimal point and a zero to the dividend. Put a decimal point in the quotient and a zero in front of it.	$3.0 \div 8$	$8\overline{)3.0}$ (0.)
	4	Continue to divide as though the decimal point was not there, adding extra zeros where required.	$3.000 \div 8$	$8\overline{)3.0^{6}0^{4}0}$ (0.375)
	5	State the answer.	$\frac{3}{8} = 0.375$	
(c)	**1**	Is the denominator a factor of 10, 100 or 1000? (No)	(c) $\frac{4}{3}$	
	2	Write the fraction as a division calculation.	$4 \div 3$	$3\overline{)4}$
	3	Perform the division. Add a decimal point and zeros to the dividend to continue the division, placing a corresponding decimal point in the quotient.	$4.000 \div 3$	$3\overline{)4.^{1}0^{1}0^{1}0}$ (1.333)
	4	If the division gives a repeating pattern, stop. Write the answer with a dot above the repeating digit.	$\frac{4}{3} = 1.\dot{3}$	

Recurring decimals

Dividing the numerator by the denominator sometimes gives a **recurring decimal**. ('Recurring' means 'repeating'.)

A recurring decimal is a decimal that has a repeating pattern of digits, such as in part **(c)** in the previous example. To write a recurring decimal, you put dots or a line above the digits that repeat.

For example: $\frac{2}{3} = 0.666\,666\ldots = 0.\dot{6}$, $\frac{1}{12} = 0.083\,333\ldots = 0.08\dot{3}$,

$$\frac{4}{7} = 0.571\,428\,571\,428\,571\,428\ldots = 0.\overline{571\,428} \text{ or } 0.\dot{5}71\,42\dot{8}$$

A recurring decimal can also be rounded to give an approximate value. For example,

$0.\dot{6}$ rounded to 3 decimal places is 0.667.

A decimal that is not recurring, but has a finite number of digits (such as 1.4 or 0.375 is called a **terminating decimal**. ('Terminating' means 'stopping'.)

Fractions and decimals on a calculator

On most scientific calculators, there is a $a^b/_c$ or S⇔D key that can be used to convert back and forth between fraction and decimal forms of a number.

4.3 Decimals and fractions

Navigator

1, 2 (columns 1–3), 3 (a–l), 4, 6, 7, 8, 9, 10, 11, 12, 13 (a), 14	1, 2 (columns 1, 2), 3 (columns 1–3), 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14	1, 2 (columns 3–4), 3 (e–p), 5, 6, 7, 8, 9, 10, 12, 13, 14

Answers p. 667

Equipment required: calculator may be used for Question **3 (i)–(p)**

Fluency

1 Using the following table headings, write each decimal as a single fraction or mixed number. Give your answers in simplest form. (Hint: Use the position of the last digit as the denominator.)

Ones 1	.	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$	Ten thousandths $\frac{1}{10000}$
	•				

(a) 0.02 **(b)** 0.6 **(c)** 2.04 **(d)** 6.0002 **(e)** 0.26 **(f)** 0.035

2 Convert each of the following decimals to fractions or mixed numbers, giving your answers in simplest form.

W.E. 7

(a) 4.1 **(b)** 6.5 **(c)** 8.6 **(d)** 9.4

(e) 4.71 **(f)** 2.37 **(g)** 0.35 **(h)** 2.48

(i) 5.009 **(j)** 0.884 **(k)** 6.128 **(l)** 3.172

(m) 7.045 **(n)** 0.088 **(o)** 6.0015 **(p)** 1.0075

3 Convert each of the following fractions to decimals.

W.E. 8

(a) $\frac{7}{10}$ **(b)** $\frac{9}{100}$ **(c)** $\frac{17}{100}$ **(d)** $\frac{123}{1000}$

(e) $\frac{1}{5}$ **(f)** $\frac{3}{25}$ **(g)** $\frac{13}{50}$ **(h)** $\frac{7}{4}$

(i) $\frac{7}{8}$ **(j)** $\frac{11}{16}$ **(k)** $\frac{23}{40}$ **(l)** $\frac{47}{60}$

(m) $\frac{1}{9}$ **(n)** $\frac{7}{12}$ **(o)** $\frac{17}{6}$ **(p)** $\frac{16}{15}$

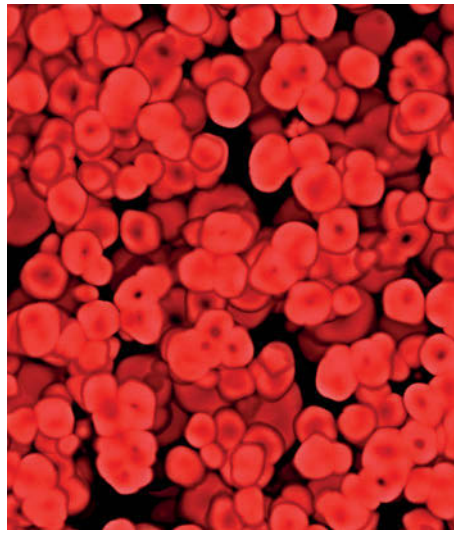
4 0.55 expressed as a fraction in simplest form is:

A $\frac{55}{1000}$ **B** $\frac{11}{200}$ **C** $\frac{11}{25}$ **D** $\frac{11}{20}$

5 2.000 47 is equal to:

A $2\frac{47}{100}$ **B** $2\frac{47}{1000}$ **C** $2\frac{47}{10\,000}$ **D** $2\frac{47}{100\,000}$

Understanding



6 As part of a science experiment, Galina's reaction time was measured to be 0.48 seconds. What fraction of a second is this?

7 One inch is approximately equal to 2.54 centimetres. Express 2.54 as a fraction in simplest form.

8 Hair grows at an average rate of 1.35 cm per month. Write this as a mixed number in simplest form.

9 A virus has a diameter of 0.000 024 mm. Write this as a fraction in simplest form.

10 A red blood cell's diameter is $\frac{1}{200\,000}$ of a metre. Write this measurement in decimal form.

Reasoning

11 A bank offers its customers an interest rate of 5.85%. Write this rate in single fraction form. Is this rate higher than the $5\frac{3}{4}$% offered by another bank?

12 Jeremy has five different containers and is trying to put them in ascending order of size. The volumes of all five containers are measured in litres but some are given using decimals and others using fractions. The tubs have the following volumes: $4\frac{1}{5}$, 4.375, 4.865, $4\frac{9}{10}$, $4\frac{1}{2}$. Help Jeremy put them in order from the smallest to the largest.

Open-ended

13 **(a)** Write three different fractions that have decimal values between 3.4 and 3.5 and have a denominator of 100.

(b) Write three different fractions that have decimal values between 3.4 and 3.5 where each fraction has a different denominator that is not a power of 10 (such as 100 or 1000).

14 Two different numbers are rounded to 1 decimal place to give 5.6. What could those two numbers be? Give your answers in fraction form.

Puzzle

What are the neighbours having for dinner?

Four children from the same class in school live on the same street. Use these facts to find out which house each child lives in, and what they had for dinner yesterday evening.

- Steve didn't have pizza for dinner, which was the meal eaten at house number 15.
- Fish and chips were eaten at house number 1, while Joseph lives at house number 11.
- The salad was eaten in a house with a lower number than the house where lasagne was eaten.
- One of Rowena's classmates lives in house number 5.
- Harriet lives in the highest number house of the four classmates.

Decimal addition and subtraction

4.4

Decimal numbers are added and subtracted in the same way as whole numbers. As with whole numbers, be sure to add or subtract digits of the same place value. The best way to do this is to line up the decimal points underneath each other.

To add or subtract decimals, line up the decimal points so that digits with the same place value are underneath each other.

If the numbers have different numbers of decimal places, zeroes can be written in the 'empty' place value columns to help them line up correctly.

Worked example 9

W.E. 9

Calculate: 12.45 + 6 + 0.3678

Thinking	Working
1 Put the numbers underneath each other with the decimal points lined up. Fill empty place value columns with zeros if necessary.	$\begin{array}{r} 12.^{1}4500 \\ 6.0000 \\ +\ 0.3678 \\ \hline 18.8178 \end{array}$
2 Add as though the values are whole numbers.	
3 Put a decimal point in the answer so it lines up with the other decimal points.	

Worked example 10

W.E. 10

Calculate: 15 – 2.147

Thinking	Working
1 Write the second number underneath the first with the decimal points lined up. (Write 15 as 15.000.)	$\begin{array}{r} {\scriptstyle 4\ 9\ 9\ 1} \\ 15.000 \\ -\ 02.147 \\ \hline 12.853 \end{array}$
2 Fill empty place value columns with zeros.	
3 Subtract, lining up the decimal point in the answer with those in the question.	

4.4 Decimal addition and subtraction

Navigator

Answers p. 668

1, 2 (columns 1–2), 3 (columns 1–2), 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 (a)	1, 2 (columns 2–3), 3 (columns 2–3), 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17 (a)	1, 2 (g–l), 3 (g–l), 5, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18

Fluency

1 For each of the following:

(i) find the approximate answer by rounding numbers before calculating

(ii) find the exact answer.

(a) 2.4 + 1.5 **(b)** 8.3 + 7.4 **(c)** 2.15 – 0.26

(d) 3.91 + 4.8 **(e)** 4.7 – 1.06 **(f)** 4 + 3.65

W.E. 9

2 Calculate:

(a) 8.3 + 7.9 **(b)** 4.85 + 8.09 **(c)** 0.237 + 0.677

(d) 25.61 + 0.038 **(e)** 5.098 + 21.32 **(f)** 0.025 + 39.786

(g) 9.703 + 5.624 + 7.5 **(h)** 7.35 + 0.609 + 2.3 **(i)** 0.648 + 7.31 + 0.9

(j) 6 + 5.017 + 12.9 **(k)** 2.59 + 15 + 0.005 **(l)** 0.0004 + 3.583 + 8

W.E. 10

3 Calculate:

(a) 5.7 – 3.8 **(b)** 7.05 – 2.93 **(c)** 23.982 – 11.735

(d) 9.663 – 2.7 **(e)** 4.984 – 1.8 **(f)** 7.238 – 3.4

(g) 9.5 – 2.24 **(h)** 16.2 – 8.75 **(i)** 3.7 – 0.931

(j) 8 – 7.44 **(k)** 3 – 2.92 **(l)** 93 – 0.698

4 Which of the following shows the correct way to set out the addition of 0.56, 15.092 and 2.7?

A
$$\begin{array}{r} 0.5600 \\ 15.092 \\ +\ 2.7000 \\ \hline \end{array}$$

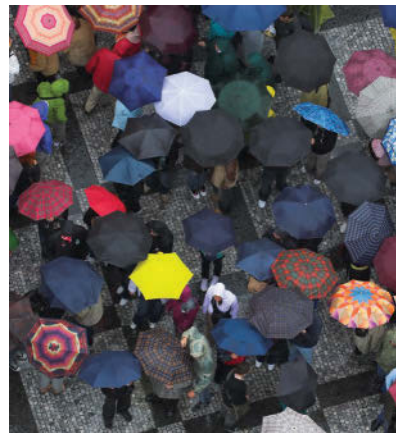
B
$$\begin{array}{r} 000.56 \\ 15.092 \\ +\ 0002.7 \\ \hline \end{array}$$

C
$$\begin{array}{r} 00.560 \\ 15.092 \\ +\ 2.7000 \\ \hline \end{array}$$

D
$$\begin{array}{r} 0.560 \\ 15.092 \\ +\ 2.700 \\ \hline \end{array}$$

Understanding

5 Daily rainfall totals for three days over a long weekend were 3.78, 2.5 and 6.42 millimetres. What was the total rainfall over this three-day period?



6 John loads his shopping trolley with several items priced as shown.

1 frozen lasagne	$5.71
2 L orange juice	$4.25
1 tub of yoghurt	$3.77
1 packet of cereal	$6.14

(a) Find the total cost of John's purchases.

(b) Round the total to the nearest 5 cents.

7 Cristina's bank account balance was $335.96 just before she withdrew $40.45 to pay a bill. How much did she have left in her account after the withdrawal?

8 In a gymnastics competition, Adelia scores the following from the five judges: 7.5, 8.5, 7.9, 8 and 8.6. Find her total score.

9 A family drives to a holiday resort and records the car's odometer readings as shown.

Departure: 234.8 km Arrival: 502.7 km

How far did they travel to get to the resort?

10 During a car tour of Tasmania, Helena travelled distances of 25.64 km, 165.35 km and 5.97 km all in one day. How many kilometres did she travel altogether that day?

11 Georgina pays for $36.35 worth of groceries with a $100 note. How much change should she receive?

12 Nico pours 1.625 litres of milk from a full 2-litre container. How much milk is left in the container?

Reasoning

13 Amanthi's credit card bill shows the following purchases: $23.56, $40.15, $7.89, $18.48, $45.50, $21.73 and $8.59. The total at the bottom of her statement is $184.38, which she is sure is wrong. What should the total be?

14 Tina sells flowers at a market stall. One day, she takes in $473.50 from the sale of flowers. If the flowers cost Tina $128.35, and she had to pay a fee of $14.50 to rent the stall for 1 day, what profit did she make for the day?

Open-ended

15 **(a)** Write two numbers that add up to 2.871.

(b) Write three numbers, each with 3 decimal places, that add up to 3.86.

(c) Explain how you chose the last digit of the numbers you used in **(b)**.

16
$$\begin{array}{r} 4.\square \\ +\ \square.8\square \\ \hline 7.\square 5 \end{array}$$

(a) Choose digits from 0 to 9 to fill each box, to make a correct sum. Find at least three different combinations.

(b) Was your choice of digits restricted? Explain how.

17 **(a)** Write 2 decimal numbers, each with 3 decimal places, that have a difference of 8.712.

(b) Write 2 decimal numbers, each with 3 decimal places, that have a difference of 8.71.

(c) Explain how you chose the last digit of the numbers you used in **(b)**.

18 Two Australian swimmers finish first and second in the women's 100 m freestyle at the Olympics. The difference in their times is 0.04 seconds. The Olympic record was 53.52 seconds. Assuming that they both break this record by less than $\frac{2}{10}$ of a second, what could their times be?

Game

Closer than you

Equipment required: 2 dice

How to win: The aim is to get a score as close to 10 as possible.

How to play:

1 Each player makes an addition table like the one on the right.

2 Take turns to roll two dice and enter the numbers rolled into the first row of the table, on each side of the decimal point, in whatever order you like. As you continue to roll the dice, fill in the next rows of the table, and calculate the additions as you go. You can choose to stop at any point.

3 When both players have stopped (or reached the end of the table), then the winner is the player who has a final addition value closest to 10. Play the best of three.

	●
+	●
=	●
+	●
=	●
+	●
=	●
+	●
=	●

Game

Decimal zilch

Equipment required: 3 dice

How to win: The aim is to get the closest score to zero (but not less than zero).

How to play:

1 Each player makes a subtraction table like the one on the right.

2 Take turns to roll the three dice. Enter the numbers rolled into the first row of the table in any order you choose. As you continue to roll the dice, fill in the next rows of the table, and calculate the subtractions as you go. You can choose to stop at any point.

3 Play the best of three.

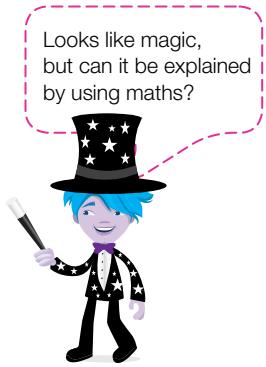
10	0	0
−		
=		
−		
=		
−		
=		

Investigation

Ten dollars and eighty-nine cents

The Big Question

Why is the answer to a seemingly random collection of calculations always the same?



Engage

1 **(a)** Choose an amount of money less than $10.00 (e.g. $7.25) and write it down.

(b) Reverse the digits and write the new amount formed, keeping the decimal point after the first digit (e.g. $5.27).

(c) Subtract the smaller amount from the larger amount (e.g. $7.25 – $5.27 = $1.98).

(d) Reverse the digits in your answer to part **(c)** and add the result to your answer to part **(c)** (e.g. $1.98 + $8.91 = $10.89).

(e) Repeat parts **(a)** to **(d)** with a different starting amount. Do you get the same final answer?

Explore

2 To investigate further, write at least 10 different amounts (all less than $10.00) using different combinations of digits. Repeat the same steps from **1** for each of them. Keep a record of which amounts produced the same final answer and which didn't.

3 Can you see a pattern in the calculations that can explain why the same answer keeps appearing? If you can, try to describe it. If you can't, write some new amounts and test them. Remember that the three digits you use do not have to all be different —try making some the same and see what happens to the result.

Strategy options

- Look for a pattern.
- Seek an exception.
- Break problem into manageable parts.

Explain

4 For your calculations that gave $10.89 as the final answer, explain the following.

(a) Look at the subtractions of the digits in the hundredths column. What do they all have in common? Can you explain why?

(b) Look at the middle digit that results after the subtraction step. What do you notice? Can you explain why?

(c) Did you find amounts for which the answer was not $10.89? What did the digits of these numbers have in common?

Elaborate

5 **(a)** For amounts that add to $10.89, what is the sum of the first and last digits of the answer to the subtraction? Explain why.

(b) Explain why the final addition step produces $10.89. Why does the addition of the tenths digits always result in an 8?

6 Summarise what you have found in this investigation by answering the Big Question.

Evaluate

7 **(a)** Did you find the results to your first few calculations surprising? Do you have more of an understanding now of how the 'magic' works?

(b) How many calculations did it take for you to see patterns emerging? What did you first start to notice?

(c) Did you feel frustrated at any time in this investigation? If so, how did you deal with the frustration?

Extend

8 Try this.

(a) Select any three single-digit decimal numbers (e.g. 0.4, 0.6, 0.1).

(b) Use these numbers to make six two-digit numbers with one number the units and the other number the tenths (e.g. 4.6, 4.1, 6.1, 6.4, 1.4, 1.6).

(c) Add these numbers (e.g. 24.2).

(d) Add your original numbers (e.g. 1.1).

(e) Divide this into the sum you obtained in part **(c)** (e.g. 24.2 ÷ 1.1).

Did you get 22?

4.5 Decimal multiplication

Multiplying a decimal by a whole number

To multiply a decimal by a whole number:

Multiply as though both values are whole numbers. Then, put the decimal point in the answer so that it has the same number of decimal places as the decimal number in the question.

To check that your decimal point is in the correct place, it is a good idea to estimate your answer before calculating it.

Worked example 11 W.E. 11

Calculate:

(a) 4.172×3 **(b)** 52.63×14 **(c)** 0.0003×2

Thinking		Working
(a) **1**	Use rounding to estimate the answer.	(a) $4.172 \times 3 \approx 4 \times 3$ ≈ 12
2	Multiply as though both values are whole numbers.	$\begin{array}{r} 4.172 \\ \times \quad 3 \\ \hline 12.516 \end{array}$ (carry: 2)
3	Make sure that there are the same number of decimal places in the answer as in the question (in this case, 3).	$4.172 \times 3 = 12.516$
4	Check your answer against your estimate to see whether it is reasonable.	Reasonable
(b) **1**	Use rounding to estimate the answer.	(b) $52.63 \times 14 \approx 50 \times 14$ $\approx 14 \times 5 \times 10$ ≈ 700
2	Multiply as though both values are whole numbers.	$\begin{array}{r} 52.63 \\ \times \quad 14 \\ \hline 21\,052 \\ 52\,630 \\ \hline 736.82 \end{array}$ (carry: 1 2 1)

	3	Make sure that there are the same number of decimal places in the answer as in the question (in this case, 2).	$52.63 \times 14 = 736.82$
	4	Check your answer against your estimate to see whether it is reasonable.	Reasonable
(c)	**1**	An estimate is not needed here, as we can see that the only non-zero digit will be a 6 (3×2).	(c)
	2	Multiply as though both values are whole numbers. Put a decimal point in the answer so that there are the same number of decimal places as in the decimal number in the question (in this case, 4).	$\begin{array}{r} 0.0003 \\ \times \quad 2 \\ \hline 0.0006 \end{array}$ $0.0003 \times 2 = 0.0006$

Multiplying and dividing decimals by powers of 10

When multiplying a number by 10, make each digit in the number 10 times its previous value.

In the place value table, each digit is moved up one place value column to the left to show this. Hundredths become tenths, tenths become ones, ones become tens etc.

For example, $43.25 \times 10 = 432.5$ as shown:

Hundreds 100	Tens 10	Ones 1	•	Tenths $\frac{1}{10}$ (0.1)	Hundredths $\frac{1}{100}$ (0.01)		Hundreds 100	Tens 10	Ones 1	•	Tenths $\frac{1}{10}$ (0.1)
	4	3	•	2	5	$\times 10 =$	4	3	2	•	5

$\times 10$ $\times 10$ $\times 10$ $\times 10$

When dividing by 10, make each digit one-tenth of its previous value. In the place value table, each digit is moved down one place value column to the right to show this. Tenths become hundredths, ones become tenths, tens become ones and so on.

For example, $65.1 \div 10 = 6.51$ as shown:

Tens 10	Ones 1	•	Tenths $\frac{1}{10}$ (0.1)	Hundredths $\frac{1}{100}$ (0.01)		Tens 10	Ones 1	•	Tenths $\frac{1}{10}$ (0.1)	Hundredths $\frac{1}{100}$ (0.01)
6	5	•	1		$\div 10 =$		6	•	5	1

$\div 10$ $\div 10$ $\div 10$

You can think of this as moving the decimal point one place to the right (for multiplication) or moving it one place to the left (for division). However, you must remember it is the digits that move as their values increase or decrease. The decimal point is always really in the same place.

Multiplying or dividing by 100 is like multiplying or dividing by 10 twice (because $100 = 10 \times 10$). Each digit moves two place value columns. Again, you can think of this as moving the decimal point 2 places to the right or left.

To *multiply* a decimal by a power of 10:

Move the decimal point to the *right* the same number of place values as the zeroes in the power of 10. Write a zero in any empty place values.

$0.45 \times 10\ (10^1) = 4.5$

$0.45 \times 100\ (10^2) = 45$

$0.45 \times 1000\ (10^3) = 450$

To *divide* a decimal by a power of 10:

Move the decimal point to the *left* the same number of place values as zeroes in the power of 10. Fill in any empty place values with zeroes.

$78.9 \div 10\ (10^1) = 7.89$

$78.9 \div 100\ (10^2) = 0.789$

$78.9 \div 1000\ (10^3) = 0.0789$

Decimal 'names'

Each of the powers of 10 has its own name, as shown below for the first nine powers:

10^1 = ten	10^4 = ten thousand	10^7 = ten million
10^2 = hundred	10^5 = hundred thousand	10^8 = hundred million
10^3 = thousand	10^6 = million	10^9 = billion (thousand million)

To write very large numbers, we sometimes write a smaller decimal number and the name of the power. For example, instead of writing 2 300 000, we could write 2.3 million, or 2.3×10^6. This is the same as writing $2.3 \times 1\,000\,000$, or 2 300 000.

Multiplying decimals by other multiples of 10

Multiples of 10, such as 20, 190, 1700, 8000 etc., can be written as a number multiplied by a power of 10: $20 = 2 \times 10$, $190 = 19 \times 10$, $1700 = 17 \times 100$, $8000 = 8 \times 1000$.

Writing them in this way means we can multiply by the number first, then the power of 10.

Worked example 12

W.E. 12

Calculate:

(a) 0.6295×7000 **(b)** $9.81 \times 340\,000$

Thinking	Working
(a) 1 Rewrite the whole number as a product of a number and a power of 10.	*(a)* $7000 = 7 \times 1000$
2 Multiply this number in the product by the decimal, remembering that the number of decimal places in the answer is the same as in the decimal being multiplied.	$0.6295 \times 7 = 4.4065$ (4 2 6 3)
3 Multiply the answer by the power of 10 by moving the decimal point to the right. (In this case, 3 places.)	4.4065×1000 (3 places) $= 4406.5$

(b)	**1**	Rewrite the whole number as a product of a number and a power of 10.	(b) $340\,000 = 34 \times 10\,000$
	2	Multiply this number in the product by the decimal, remembering that the number of decimal places in the answer is the same as in the decimal being multiplied.	$\begin{array}{r} 9.81 \\ \times\ {}^{23}\ 34 \\ \hline 3924 \\ {}_{1}2_{1}9430 \\ \hline 333.54 \end{array}$
	3	Multiply by the power of 10 by moving the decimal point to the right. (In this case, 4 places.) Write zeros into the empty place value columns.	$333.54 \times 10\,000$ $= 3\,335\,400$

Multiplying a decimal by another decimal

To see how to multiply one decimal by another, it is useful to look at some number patterns. When you multiply by powers of 10, each digit in the number moves by the same number of place values as the power number. You show this by moving the decimal point. For example:

$0.23 \times 10 = 2.3$ $0.23 \times 100 = 23$ $0.23 \times 1000 = 230$

You can reverse the above calculations by dividing by the powers of 10:

$2.3 \div 10 = 0.23$ $23 \div 100 = 0.23$ $230 \div 1000 = 0.23$

Dividing by powers of 10 corresponds to moving the decimal point to the left by the same number of places as there are zeroes in the power of 10.

Another way to multiply two decimals is to first count the total number of decimal places used in both numbers. Then, multiply the numbers as if they are whole numbers. Put the decimal point back into the product by counting the same total number of decimal places from the left.

Both ways are shown in the example below.

Worked example 13

W.E. 13

Calculate:

(a) 0.63×0.4 **(b)** 0.014×0.002

Thinking			Working
(a)	**1**	Write both numbers as the division of a whole number by a power of 10.	(a) $0.63 = 63 \div 100$ $0.4 = 4 \div 10$
	2	Multiply as though both numbers are whole numbers. (We do not need to multiply by the zero in 0.4, as it does not affect the outcome.)	$\begin{array}{r} 63 \\ \times\ 4 \\ \hline 252 \end{array}$

3	Divide the product by the powers of 10 identified in step **1**. Show this division by moving the decimal point. (Here, divide by 100 and 10, which is the same as dividing by 1000.) If necessary, put a zero in the units place to clearly show the position of the decimal point.	0.63×0.4 $= 252 \div 10 \div 100$ $= 252 \div 1000$ $= 0.252$
(b) 1	Count the total number of decimal places. (0.014 has 3 decimal places and 0.002 has 3 decimal places so there is a total of 6 decimal places.)	(b) 0.014: 3 decimal places 0.002: 3 decimal places total: 6 decimal places
2	Multiply as though both numbers are whole numbers (ignoring the zeroes).	$\begin{array}{r} 14 \\ \times\ 2 \\ \hline 28 \end{array}$
3	Using the product, count the total number of decimal places from the left and place a decimal point. (Here we count 6 decimal places.) Put zeroes in the empty place values and a zero in the units place.	0.014×0.002 $= 0.000\,028$

To multiply one decimal by another:

Step 1 Write both numbers as the division of a whole number and a power of 10.

Step 2 Multiply as though both numbers are whole numbers.

Step 3 Divide the product by the powers of 10 identified in **Step 1**. Show this by moving the decimal place to the left.

Or:

Step 1 Count the total number of decimal places used in the decimals.

Step 2 Multiply as though both numbers are whole numbers.

Step 3 Put the decimal point into the product by counting the total number of decimal places from the left.

4.5 Decimal multiplication

Navigator

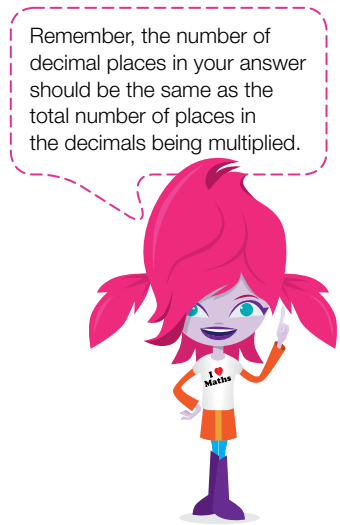
1 (columns 1–2), 2, 3 (columns 1–2), 4, 5 (columns 1–2), 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 23, 25, 27 (a)	1 (columns 2–3), 2, 3 (columns 2–3), 4, 5 (columns 2–3), 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 23, 25, 26, 27	1 (column 3), 2 (d–f), 3 (column 3), 4, 5 (column 3), 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27

Answers p. 668

Fluency

1 Calculate:

W.E. 11



(a) 7.6×4 **(b)** 5.3×7 **(c)** 6.5×9

(d) 5×4.62 **(e)** 0.54×4 **(f)** 6×12.33

(g) 8.047×8 **(h)** 0.619×7 **(i)** 3×18.309

(j) 22.08×13 **(k)** 48.76×25 **(l)** 37.09×32

(m) 0.0004×2 **(n)** 0.0005×7 **(o)** 1.0006×3

2 Calculate the following.

(a) 2.75×10 **(b)** 0.073×10 **(c)** 1.765×100

(d) 0.047×1000 **(e)** 0.1345×1000 **(f)** 7.099×100

3 Calculate:

W.E. 12

(a) 2.4×60 **(b)** 3.27×70 **(c)** 2.991×40

(d) 0.57×200 **(e)** 4.34×500 **(f)** 6.625×400

(g) 4.591×6000 **(h)** 6.38×4300 **(i)** 3.14×9300

(j) $0.571 \times 68\,000$ **(k)** $0.9465 \times 71\,000$ **(l)** $7.4832 \times 55\,000$

4 For each of the following:

(i) estimate an answer by rounding each number to the nearest whole number before multiplying

(ii) use a calculator to find the exact answer.

(a) 3.3×4.2 **(b)** 2.4×0.76 **(c)** 0.57×6.8

5 Calculate:

W.E. 13

(a) 0.6×0.8 **(b)** 0.4×0.6 **(c)** 0.9×0.4

(d) 9.32×0.7 **(e)** 4.12×0.3 **(f)** 5.26×0.8

(g) 7.74×0.31 **(h)** 9.46×0.22 **(i)** 3.76×0.18

(j) 0.004×0.6 **(k)** 0.0308×0.3 **(l)** 0.072×0.05

6 A reasonable estimate for the answer to 1.8×15 is:

A 3 **B** 16.8 **C** 30 **D** 300

7 How many decimal places will there be in the answer to 12.789×41?

A 1 **B** 2 **C** 3 **D** 4

8 How many decimal places will there be in the answer to 0.36×1.04?

A 2 **B** 3 **C** 4 **D** 5

Understanding

9 Pearl buys four notebooks for $3.15 each. How much does she pay in total?

10 Renée buys 52 components from an electronics shop for $0.35 each. What is the total cost of the components?

11 A pill bottle contains 37 pills, each of mass 0.287 grams. What is the total mass of the bottle's contents?

12 Write the following as whole numbers.

(a) 3.5 million **(b)** 4.26 billion **(c)** 1.3 trillion (1 trillion = 1000 billion)

(d) 57.08 million **(e)** 10.2 trillion **(f)** 20.045 billion

13 A snail crawls 0.041 metres in an hour. If it never slows down, how far can it move in a day?

14 Amy orders 27.5 metres of timber priced at $4.29 per metre from her local hardware store. How much will the timber cost her in total? (Round your answer to the nearest cent.)

15 Gavin sells 300 punnets of berries to a supermarket. If he is paid $1.72 for each punnet, how much money does he receive?

16 **(a)** Marita fills her car's fuel tank with petrol at a cost of $1.35 per litre. If she buys 70 litres, how much must she pay?

(b) Ali pumps 9.5 litres of petrol into his car's tank. How much does the fuel cost him if he pays the same price per litre as Marita? Round your answer to the nearest cent.

17 The instructions on a bag of fertiliser say to spread 0.75 kilograms per square metre of area to be fertilised. Raisa wants to fertilise 2600 square metres of property using this product. How many kilograms of fertiliser should she use?

18 Bryan has calculated that he needs 3.6 square metres of tiles for his bathroom. What will he have to pay if the tiles he wants cost $22.39 per square metre? (Round your answer to the nearest cent.)

Reasoning

19 Kim works in an ice-cream parlour, and one hot day sells 500 scoops of vanilla. If there is 0.185 litres of ice-cream in each scoop and the day started with 130 litres of vanilla in the freezer, how many litres of vanilla ice-cream are left at the end of the day?

20 Yoghurt contains 167 kilojoules per 100 grams. How many kilojoules would a 175 gram tub contain?

21 In May 2016, the Australian Bureau of Statistics calculated that the population of Australia was 24 061 920.

(a) Write this number as a decimal number, with the number of millions as the whole number part, and the rest of the number rounded to 1 decimal place.

(b) Why would your answer to **(a)** be the number you would expect to see reported in the news, instead of the 'exact' value?

22 **(a)** A woodwork teacher needs 50 pieces of pine, each 0.135 metres long, to give to students to make a particular model. What total length (in metres) of timber does the teacher need? (Ignore any lost length due to saw cuts.)

(b) If the teacher can buy the pine in 3.5 m lengths, how many pieces are needed, and how much is left over?

23 A multi-bag of mini chocolate bars costs $3.95 and contains 12 bars. Each mini chocolate bar can also be purchased individually for 40 cents.

(a) How much would 12 mini chocolate bars cost if they were purchased individually?

(b) How much do you save by purchasing the multi-bag?

(c) Charlotte wants to buy 30 mini bars for a party. Should she buy them individually, or in multi-bags? Which way is cheaper?

24 Wayne is following his grandfather's recipe for tomato sauce, which requires $3\frac{1}{2}$ pounds of tomatoes. (A pound is an old imperial unit of mass.) Because he only has scales that can measure mass in kilograms, Wayne decides to convert the mass to kilograms. A website tells him that 1 pound equals 0.454 kilograms. What answer should he get when he converts $3\frac{1}{2}$ pounds of tomatoes to kilograms? Round your answer to 1 decimal place.

25 Francesca read her electricity meter one Saturday, and again at the same time the next Saturday. She recorded the following readings.

First Saturday: 7562.3 kilowatt-hours
Second Saturday: 7638.6 kilowatt-hours

(a) How many kilowatt-hours of electricity did Francesca's household use in the week?

(b) If electricity costs $0.13 per kilowatt-hour, find the total cost of the electricity supplied in the week.

Open-ended

26 Jay has been asked to work out the length if students in Year 7 were laid end to end. He knows that there are 89 students in Year 7 and that their average height is 1.63 m.

This is his working out:

$89 \times 1 = 89$
$89 \times 6 = 534$
$89 \times 3 = 267$

$$\begin{array}{r} 89 \\ 534 \\ +\ 267 \\ \hline = 880 \end{array}$$

Answer: 8.80 m

(a) Where has Jay made his error? Explain how he could use estimation to see that he was incorrect.

(b) Explain how he could modify his method to get the correct answer.

(c) Show how you would complete this problem.

27 Two numbers are multiplied together to give 68.4.

(a) Find at least two possible pairs of numbers with one number being a multiple of 10.

(b) Find at least two possible pairs of numbers with neither being a multiple of 10.

Game

Multiplicity

Equipment required: 2 dice, 1 calculator

How to win:
The aim is to get to the tens and ones place value of the target number, no matter what comes after the decimal point.

How to play:

1. The first player rolls the dice to determine the two-digit target number (the first number followed by the second number).
2. The second player then rolls the dice to determine the starting number.
3. The first player puts the starting number into a calculator and can multiply this by any number of their choice, with the aim of reaching the target number.
4. If they reach the tens and ones place value of the target number, then they're the winner. If not, the other person takes their answer (leave it in the calculator display) and then chooses a number to multiply this by, to try to reach the target number.

For example, the first player rolls a 4 and a 3, so the target number is 43.

The second player then rolls two 1s, so the starting number is 11.

The first player chooses to multiply the 11 by 4, which gives an answer of 44—too big.

The second player chooses to multiply the 44 by 0.9, which gives an answer of 39.6—too small.

This continues until a player reaches an answer of 43 point something.

The game can be made more difficult by making the target number a three-digit number, or a two-digit number with one decimal point.

Half-time 4

1 Convert the following decimals into fractions (or mixed numbers where appropriate) in simplest form. **4.3**

(a) 0.25 **(b)** 0.3 **(c)** 3.54 **(d)** 9.345

2 Write the following decimals in order from smallest to largest. **4.1**

9.01, 19.12, 0.03, 0.321, 9.003 24

3 Calculate: **4.4**

(a) 12.5 + 7.9 **(b)** 124.54 + 8.379 **(c)** 0.32 + 2.5

(d) 15 – 8.98 **(e)** 127.061 – 43.2 **(f)** 6 – 4.9

4 Round the following numbers to 3 decimal places. **4.2**

(a) 4.8765 **(b)** 13.0037 **(c)** 2.3498 **(d)** 23.032 49

5 Calculate the following. **4.5**

(a) 7.6×4 **(b)** 28.3×5 **(c)** 1.37×9

(d) 45.2×30 **(e)** 0.26×2000 **(f)** 6.78×500

(g) 4.3×1.7 **(h)** 39.82×15.4 **(i)** 1.2×0.0005

6 Franji is saving up to purchase a games console. She has saved the following amounts over the last four weeks: \$25, \$47.50, \$18.70, \$9.35. **4.4**

(a) How much has she saved in total?

(b) If the console is \$149, how much more does Franji need?

7 Write the following in decimal form. **4.1**

(a) $23 + \frac{4}{10} + \frac{9}{100} + \frac{7}{10\,000}$ **(b)** $14 + \frac{235}{1000}$

(c) 2 hundreds, 9 tens, 3 ones, 4 tenths, 1 hundredth and 5 thousandths

(d) 9 tens, 0 ones, 6 tenths, 2 hundredths and 7 ten-thousandths

8 Round the following amounts of money to the nearest 5 cents. **4.2**

(a) \$1.93 **(b)** \$35.08 **(c)** \$57.44 **(d)** \$189.98

9 Write the following fractions as decimals, using the correct notation for recurring decimals if necessary. **4.3**

(a) $\frac{4}{5}$ **(b)** $\frac{17}{20}$ **(c)** $\frac{2}{3}$ **(d)** $\frac{7}{12}$

10 The times recorded by the finalists in a track cycling sprint race were (in seconds): 59.023 s, 59.105 s, 59.23 s, 59.125 s. Write the times in order from fastest to slowest. **4.1**

11 Melinda is buying ingredients for a cooking class. She buys: **4.4, 4.5**

12 packets of pasta at \$2.49 each
18 tins of tomatoes at \$1.35 each
6 kg of mince at \$8.99 per kg
3 packets of cheese at \$4.78 per packet

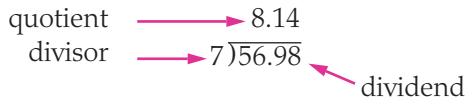
Find the total cost of Melinda's shopping.

4.6 Decimal division

When studying division, the following mathematical terms are useful.

- The dividend is the number that is being divided.
- The divisor is the number you are dividing by.
- The quotient is the result of the division (the answer).

These are shown in the example below.



When using long or short division to divide a decimal by a whole number:

Line up the decimal point in the quotient with the decimal point in the dividend.

Worked example 14 W.E. 14

Use short division to calculate the following. Round your answers to 3 decimal places if necessary.

(a) $5.28 \div 3$ **(b)** $0.41 \div 7$

Thinking		Working
(a) **1**	Write the question with the setting out shown.	(a) $3\overline{)5.28}$
2	Perform the division, and put a decimal point in the quotient so it lines up with the one in the dividend.	$3\overline{)5.^{2}2^{1}8}$ with quotient 1.76
(b) **1**	Write the question with the setting out shown.	(b) $7\overline{)0.41}$
2	Perform the division, remembering to put a zero in the quotient if the divisor goes zero times into the dividend. Add zeros to the divisor to continue the division, until there are four decimal places in the answer.	$7\overline{)0.4^{4}1^{6}0^{4}0}$ with quotient 0.0585...
3	Round the answer to 3 decimal places.	0.059 (to 3 d.p.)

Dividing decimals by multiples of 10

As done for multiplying decimals, you can write other multiples of 10 as the product of a number and a power of 10, and do two separate division steps.

Remember that you can show division by a power of 10 by moving the decimal point to the left the same number of places as the zeroes in the power of 10.

Worked example 15 W.E. 15

Calculate the following. Round your answers to 3 decimal places where necessary.

(a) 9.76 ÷ 400 **(b)** 14.7 ÷ 80

Thinking		Working
(a) **1**	Break the divisor down into a whole number and a power of 10.	(a) 9.76 ÷ 400 = 9.76 ÷ 4 ÷ 100
2	Divide by the whole number first (in this case, by 4).	$4\overline{)9.^{1}7^{1}6}$ → 2.44
3	Now, divide the quotient of the first division by the power of 10. Show this by moving the decimal point. (Here, it is moved 2 places left.)	2.44 ÷ 100 = 0.0244
4	Round your answer to the specified number of decimal places.	= 0.024 (to 3 d.p.)
(b) **1**	Break the divisor down into a whole number and a power of 10.	(b) 14.7 ÷ 80 = 14.7 ÷ 8 ÷ 10
2	Divide by the whole number first (in this case, by the 8). Add zeros to the divisor to continue the division until there are more decimal places in the answer than the rounded answer requires.	$8\overline{)14.^{6}7^{3}0^{6}0^{4}0}$ → 1.8375
3	Now, divide the answer by the power of 10. Show this by moving the decimal point. (Here, it is moved 1 place left.)	1.8375 ÷ 10 = 0.183 75
4	Round your answer to the specified number of decimal places.	= 0.184 (to 3 d.p.)

Dividing a decimal by another decimal

In order to divide a decimal by another decimal you need to convert it to a problem where you can divide a decimal by a whole number.

Recall that when working with fractions you can multiply or divide the numerator and the denominator of a fraction by the same number without changing the value of the fraction.

For example, $\frac{1}{2}$ is the same as $\frac{2}{4}$ or $\frac{10}{20}$. You can apply the same principle to dividends and divisors.

For example, 1 ÷ 2 gives the same answer as 2 ÷ 4 and 10 ÷ 20.

13.52 ÷ 1.2
= (13.52 × 10) ÷ (1.2 × 10)
(multiplying both numbers by 10)
= 135.2 ÷ 12
= 11.267 (to 3 decimal places)

7.5 ÷ 2.5
= (7.5 × 2) ÷ (2.5 × 2)
(multiplying by 2)
= 15 ÷ 5
= 3

To divide one decimal by another, multiply both numbers so that the divisor is no longer a decimal. Then, divide the new dividend by the new (whole number) divisor.

Multiplying both numbers by a power of 10 is often the easiest way to multiply. Multiply by the smallest power of 10 that gives a whole number divisor.

Worked example 16 W.E. 16

Calculate the following. Round your answer to 3 decimal places if necessary.

(a) $3.35 \div 0.005$ **(b)** $10.52 \div 0.9$

Thinking			Working
(a)	**1**	What does the divisor need to be multiplied by to make it a whole number? Multiply both the dividend and the divisor by this number. (Here, we multiply both by 1000.)	(a) $3.35 \div 0.005$ $= (3.35 \times 1000) \div (0.005 \times 1000)$
	2	Rewrite the question using the new divisor and new dividend.	$= 3350 \div 5$
	3	Calculate the quotient.	$5\overline{)33^{3}50}$ 6 70
	4	State the answer.	$3.35 \div 0.005 = 670$
(b)	**1**	What does the divisor need to be multiplied by to make it a whole number? Multiply both the dividend and the divisor by this number. (Here, we multiply both by 10.)	(b) $10.52 \div 0.9$ $= (10.52 \times 10) \div (0.9 \times 10)$
	2	Rewrite the question using the new divisor and new dividend.	$= 105.2 \div 9$
	3	Calculate the quotient to one more decimal place than the rounded number required.	$9\overline{)10^{1}5.^{6}2^{8}0^{8}0^{8}0}$ 1 1. 6 8 8 8
	4	Round the quotient to the required number of decimal places.	$= 11.689$ (to 3 d.p.)

4.6 Decimal division

Navigator

1 (columns 1–2), 2 (columns 1–2), 3 (columns 1–2), 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 20, 22	1 (columns 2–3), 2 (columns 2–3), 3 (columns 2–3), 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 22	1 (j–o), 2 (d–f), 3 (columns 2–3), 4, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25

Answers p. 669

Fluency

1 Calculate the following. Round your answers to 3 decimal places if necessary.

W.E. 14

(a) 8.48 ÷ 4 **(b)** 12.96 ÷ 3 **(c)** 14.22 ÷ 9

(d) 3.3 ÷ 6 **(e)** 2.4 ÷ 7 **(f)** 5.1 ÷ 9

(g) 39.42 ÷ 6 **(h)** 10.6 ÷ 7 **(i)** 0.47 ÷ 9

(j) 0.42 ÷ 6 **(k)** 1.22 ÷ 3 **(l)** 18.46 ÷ 7

(m) 154.33 ÷ 5 **(n)** 84.365 ÷ 8 **(o)** 347.507 ÷ 7

2 Calculate the following.

(a) 1.23 ÷ 10 **(b)** 15.4 ÷ 100 **(c)** 506.8 ÷ 100

(d) 813 ÷ 1000 **(e)** 0.009 ÷ 100 **(f)** 0.432 ÷ 10

3 Calculate the following. Round your answers to 3 decimal places if necessary.

W.E. 15

(a) 3.6 ÷ 40 **(b)** 8.1 ÷ 90 **(c)** 36.216 ÷ 60

(d) 96.58 ÷ 200 **(e)** 2.296 ÷ 400 **(f)** 23.4 ÷ 600

(g) 233.4 ÷ 3000 **(h)** 40.6 ÷ 8000 **(i)** 18.9 ÷ 7000

(j) 68.4 ÷ 120 **(k)** 3.19 ÷ 110 **(l)** 8.97 ÷ 1300

4 Calculate the following. Round your answers to 3 decimal places if necessary.

W.E. 16

(a) 5.14 ÷ 0.2 **(b)** 4.12 ÷ 0.4 **(c)** 5.1 ÷ 0.6

(d) 15.48 ÷ 0.04 **(e)** 9.018 ÷ 0.09 **(f)** 1.736 ÷ 0.07

(g) 2.382 ÷ 0.006 **(h)** 0.3996 ÷ 0.009 **(i)** 0.196 ÷ 0.008

(j) 7.7 ÷ 0.011 **(k)** 3.78 ÷ 0.012 **(l)** 1.16 ÷ 0.008

(m) 1.74 ÷ 0.0002 **(n)** 8.05 ÷ 0.0007 **(o)** 13.36 ÷ 0.0008

5 How many decimal places will there be in the answer to 298.12 ÷ 100?

A 1 **B** 2 **C** 3 **D** 4

6 Which of the following will give the answer to 3.2 ÷ 0.005?

A 0.0032 ÷ 5 **B** 320.0 ÷ 5 **C** 3200.0 ÷ 5 **D** 32 000.0 ÷ 5

Understanding

7 A block of metal of mass 2.312 kg is divided equally into 8 pieces. What is the mass of each piece?

8 A company makes a profit of 7.86 million dollars. If the profit is divided equally among the four company owners, how much does each owner get? (Answer in millions of dollars.)

9 The total cost for a wedding reception for 100 guests is $9398.55. How much is this cost per guest? (Round your answer appropriately.)

10 Anthony needs to mix 0.07 litres of anti-bug spray with water each time he fills the tank of his garden sprayer. If he has 0.84 litres of anti-bug spray left, how many times can he mix this with water to fill the tank?

11 A jeweller calculates that a popular style of necklace contains 0.009 g of gold. How many necklaces can she make using the 2.332 g of gold in her workshop?

12 Eleanor's car travels 37.29 km on three litres of fuel. How far can it travel on one litre of fuel?

13 A 4000 mL container of orange juice costs $8.90. Determine the cost per 100 mL. (Round your answer appropriately.)

14 Simon buys 7 kg of spicy sausages for $32.95. How much is this cost per kilogram?

15 Reggie has a tub containing 5.8 kilograms of worms which she uses for fishing. If the average worm has a mass of 0.003 kilograms, how many worms are in the tub?

16 Hai is making a long-distance call on his mobile. The call costs $0.90 per minute. For how many minutes can he continue the call if he has $5.85 credit on his phone?

17 A 1.2 kilometre taxi trip costs Joan $5.70. How much is this cost per kilometre?

Reasoning

18 The captains of two netball teams were discussing which of their teams was taller. The heights of Angie's team were 1.49 m, 1.54 m, 1.65 m, 1.67 m, 1.58 m, 1.73 m and 1.56 m. The heights of Jessica's team were 1.73 m, 1.58 m, 1.59 m, 1.62 m, 1.45 m, 1.51 m and 1.62 m. Add the seven heights and divide the result by seven to find the average (mean) height of each team. Which of the two teams was taller, on average? Answer to 2 decimal places.

19 Monica and Celeste were trying to determine the best bargain of three different jars of coffee. The 200 g jar of BestCafe cost $5.95, the 500 g jar of Maxine House was $14.85 and the 300 g jar of Brazilian Roast was $6.75. Which is the best buy in terms of price per 100 grams?

20 Karen was counting the coins in her cash register drawer and found that she had $1.65 in 5-cent coins, $2.30 in 10-cent coins, $5.20 in 20-cent coins and $7.50 in 50-cent coins. How many of each type of coin did she have?

21 Jacob received a bill from a rent-a-car firm that gave the total cost as $310. This included a daily cost of $24.80 plus $1.20 per kilometre travelled. If Jacob had rented the car for 2 days, how many kilometres did he travel?

Open-ended

22 Write three numbers, each with 3 decimal places, that when divided by 4 give an answer between 0.5 and 1.

23 For each of the following, write two equivalent divisions by multiplying both the divisor and the dividend by a number that is not a power of 10.

(a) $10.05 \div 0.5$ **(b)** $36.04 \div 0.02$

24 The area of a rectangle is calculated by multiplying the length and the width. If the area of a rectangle is 8.45 m^2, what might be the length and width of the sides? Give two possible values for the side dimensions. For each of the side dimensions, work out the related perimeter (distance along the boundary of the rectangle).

25 Tran divided 5.25 by 5 and wrote an answer of 1.5.

(a) What error did Tran make?

(b) Explain to Tran what he needs to do to calculate decimal divisions accurately.

Puzzle

Magic square

Copy this 6 × 6 table into your book.

Fill in the blank squares so that each row and each column adds up to 11.1.

2.1		2.5	2.7		
	2.5	2.8		0.5	
0.1	0.3		1.9	3.3	
0.4	0.2	1.7	1.8	3.6	
2.9	3.1	0.9	1.1	1.6	
3.2	3.0	1.2		1.3	0.8


Maths 4 Real
PERPLEXING PRICES

Imagine you are in your local supermarket, shopping for supplies for a party. You need 7 to 8 litres of soft drink and some lollies, so you move to the soft drink aisle first. A vast array of products and prices confronts you. You see that your favourite brand has three different-sized bottles:

1.25 L for $2.25

1.5 L for $2.78

2 L for $3.56

'Hmmm,' you think. 'The 2 L bottle gives me half a litre more than the 1.5 L, for only 78 cents extra. I could get two 1.5 L bottles, that's 3 L, for $5.56. Maybe I should get a couple of 1.25 L bottles instead. Which do I choose?'

1 At first glance, which bottle size looks to be the best value for money to you?

You remember that soft drink also comes in 375 mL cans. You move further down the aisle and see the following:

6-pack: $7.00 12-pack: $13.59 18-pack: $19.34 30-pack: $26.34

You begin to wonder: 'Should I buy cans instead? How many cans are equivalent to one bottle? Are they better value for money? Help!'

2 What is it that makes your 'value for money' decision difficult? What information would be useful to help you decide?

In December 2009, the Australian government introduced a new compulsory pricing system in supermarkets, designed to make decisions easier for shoppers. The system is called 'unit pricing'. As well as the regular price, supermarkets must display the price for a particular 'unit' of the product, such as 1 L, 1 kg, 100 g or 100 mL. Comparing the prices of different products is much easier when you are comparing the same price per unit.

You can calculate a unit price for the bottles of soft drink by calculating the price *per litre*. To do this, divide the price by the capacity of the bottle, in litres.

For example, the unit price for the 1.25 L bottle is: $2.25 ÷ 1.25 L = $1.80 per L.

3 (a) Calculate the price per litre of the other two bottle sizes.

(b) Which size bottle is the best value for money?

(c) Would it always be better to purchase this size? Explain your answer.

(d) Further down the aisle, you notice that a 1.5 L bottle of a rival brand of cola is on special for $2.39. Calculate the price per litre of this discounted bottle.

(e) You also notice that you can buy a 'multipack' of twelve 300 mL bottles for $11.86. Calculate the price per litre of the multipack. (300 mL is 0.3 L.)

(f) Why is the unit price of the multipack so different to the unit prices for individual bottles?

4 (a) Calculate the price per can of each of the packs.

(b) Which pack represents the best value for money? Did you find anything surprising in your results?

(c) The price of a single can of soft drink is $1.35. Why is there such a difference between the price of a single can and the price per can in the can packs?

5 (a) Calculate the price per litre of each of the can packs, remembering that each can contains 375 mL (0.375 L).

(b) Suggest a possible explanation for the difference in the price per litre between the cans and the bottles.

(c) When might people prefer to buy multipacks of cans instead of bottles?

You move down to the confectionery aisle and see a 190 g 'Party Mix' bag of lollies for $2.90, and a 'Jumbo' size 570 g bag for $5.82.

6 The unit price for products like bags of lollies is the price per 100 g.

(a) Calculate the unit price for the 190 g bag and the 570 g bag. (Hint: Work out what you need to divide 190 or 570 by to get 100, then divide the price by this amount.)

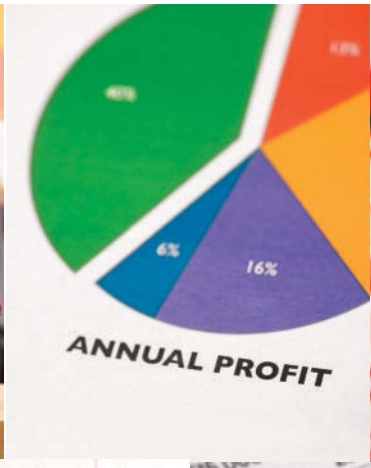
(b) Which is the better buy?

Research

- Visit a supermarket, use catalogues or go shopping online to investigate the cost of goods that come in different sizes, such as coffee, Vegemite™, washing powder and pet food. Use a suitable unit such as 100 g or 1 kg to calculate the unit prices.
- Comment on your findings. Is there a general rule or conclusion you can make? Discuss, with examples, why different people might buy different-sized products.

4.7 Percentages, fractions and decimals

Percentages are used in many everyday situations. Shopping discounts, survey results, business statistics and sporting league ladders all use percentages to show information. Percentages make it easier to visualise and compare quantities, especially when comparing parts from different-sized wholes.



The words **'per cent'** literally mean 'for every hundred' or 'out of one hundred'. The **percentage** symbol is %. Percentages can be written as fractions with a denominator of 100. Percentages can also be thought of as division by 100.

For example, 25% means 25 out of 100, and 9% means 9 out of 100. You can write percentages as fractions, some of which can be simplified.

$$25\% = \frac{25}{100} = \frac{1}{4} \qquad 9\% = \frac{9}{100}$$

You can divide the fraction to get the percentage in decimal form.

$$\frac{25}{100} = 25 \div 100 = 0.25 \qquad \frac{9}{100} = 9 \div 100 = 0.09$$

Writing percentages as fractions and decimals

To write a percentage as a fraction:

- Write the percentage value as the numerator of a fraction with a denominator of 100.
- Simplify the fraction if possible.

To write a percentage as a decimal:

- Divide by 100.

Worked example 17 W.E. 17

Write 12% as a fraction in simplest form.

Thinking	Working
1 Write the value of the percentage as the numerator over a denominator of 100.	$12\% = \frac{12}{100}$
2 Simplify the fraction if possible. (Here, this has cancelled a common factor of 4.)	$= \frac{3}{25}$

Worked example 18 W.E. 18

Write 45% as a decimal.

Thinking	Working
Divide the value of the percentage by 100. Write your answer without the percentage symbol.	$45\% = 45 \div 100$ $= 0.45$

Writing fractions as percentages

To write a fraction as a percentage, you can use one of two methods:

Method 1: Write the fraction as an equivalent fraction with a denominator of 100. Use the numerator of this new fraction as the percentage.

Method 2: Multiply the fraction by 100%.

Method 1 is convenient, but only works well with fractions that have denominators that are factors of 100: 2, 4, 5, 10, 20, 25, 50.

Method 2 can be used with any fraction.

Note: As the example below shows, not all percentages have whole number values. These percentage numbers are usually written as decimals. These percentages can also be written in fractional form; for example, $16.7\% = 16\frac{7}{10}\%$, but decimal form is usually easier to work with.

Remember that you may be able to use a $a\frac{b}{c}$ or S⇔D key on your calculator to convert between decimal and fraction form.

100% is equivalent to 1 whole, $\frac{1}{1}$ and 1.0.

Worked example 19

W.E. 19

Write each of the following fractions as percentages.

(a) $\frac{4}{5}$ **(b)** $\frac{27}{40}$

Thinking	Working
(a) 1 Is the denominator a factor of 100? (Yes.) Determine what number the denominator must be multiplied by to get 100.	(a) $\frac{4}{5} \times \frac{\quad}{20} = \frac{\quad}{100}$
2 Multiply the numerator by the same number.	$\frac{4}{5} \times \frac{20}{20} = \frac{80}{100}$
3 Write the numerator of the new fraction with the percentage symbol.	$= 80\%$
(b) 1 Is the denominator a factor of 100? (No.)	(b) $\frac{27}{40} \times \frac{100}{1}\%$
2 Multiply the fraction by 100. (Remember that you can write a whole number as an improper fraction with a denominator of 1.) Cancel common factors first to simplify the multiplication. (Here, a common factor of 20 is cancelled.)	$= \frac{27}{{}_{2}\cancel{40}} \times \frac{{}^{5}\cancel{100}}{1}\%$ $= \frac{135}{2}\%$
3 Divide the numerator of the answer by the denominator. Write your answer as a decimal (rounded if necessary). Write the answer with a percentage symbol.	$= 67.5\%$

Writing decimals as percentages

To write a decimal as a percentage, multiply it by 100 and write the % symbol.

Worked example 20

W.E. 20

Write each of the following decimals as percentages.

(a) 0.04 **(b)** 0.125

Thinking	Working
(a) Multiply the decimal by 100%.	(a) $0.04 = 0.04 \times 100\%$ $= 4\%$
(b) Multiply the decimal by 100%.	(b) $0.125 = 0.125 \times 100\%$ $= 12.5\%$

4.7 Percentages, fractions and decimals

Navigator

1, 2, 3, 4 (columns 1–3), 5, 6, 7, 8, 10 (a), 12, 13, 15, 16, 18	1, 2, 3, 4 (columns 2–4), 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 18	1, 2, 3 (column 2), 4 (columns 3–4), 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 17, 18

Answers p. 669

Fluency

1 Write each of the following percentages as fractions in simplest form. W.E. 17

(a) 15% **(b)** 24% **(c)** 18% **(d)** 64%

(e) 110% **(f)** 122% **(g)** 140% **(h)** 198%

2 Write each of the following percentages as decimals. W.E. 18

(a) 34% **(b)** 13% **(c)** 78% **(d)** 92%

(e) 120% **(f)** 123% **(g)** 254% **(h)** 320%

3 Convert the following fractions to percentages by filling in the missing numbers.

(a) $\frac{3}{5} \times \frac{20}{20} = \frac{\quad}{100} = \_\_\_\_\_\%$ **(b)** $\frac{7}{10} \times \frac{\quad}{10} = \frac{70}{\quad} = 70\%$

(c) $\frac{1}{4} \times \frac{\quad}{\quad} = \frac{25}{100} = \_\_\_\_\_\%$ **(d)** $\frac{2}{3} \times \frac{100}{1}\% = \_\_\_\_\_\% = 66.\dot{6}\%$

(e) $\frac{5}{8} \times \frac{\quad}{1}\% = \frac{500}{\quad}\% = 62.5\%$ **(f)** $\frac{7}{15} \times \frac{\quad}{\quad}\% = \frac{\quad}{15}\% = 46.\dot{6}\%$

4 Write each of the following fractions as percentages. W.E. 19

(a) $\frac{2}{5}$ **(b)** $\frac{3}{4}$ **(c)** $\frac{3}{10}$ **(d)** $\frac{7}{20}$

(e) $\frac{11}{10}$ **(f)** $\frac{3}{2}$ **(g)** $\frac{28}{25}$ **(h)** $\frac{6}{5}$

(i) $\frac{3}{8}$ **(j)** $\frac{1}{3}$ **(k)** $\frac{5}{6}$ **(l)** $\frac{4}{9}$

(m) $\frac{13}{15}$ **(n)** $\frac{19}{30}$ **(o)** $\frac{33}{40}$ **(p)** $\frac{44}{60}$

5 Write each of the following decimals as percentages. W.E. 20

(a) 0.87 **(b)** 0.34 **(c)** 0.65 **(d)** 0.96

(e) 1.34 **(f)** 3.58 **(g)** 5.32 **(h)** 1.21

6 Complete the following table.

Percentage	Fraction (simplest form)	Decimal
40%		
	$\frac{7}{10}$	
		0.05
	$\frac{1}{50}$	
75%		
	$\frac{5}{4}$	

Understanding

7 $\frac{1}{4}, \frac{1}{2}, \frac{1}{10}, \frac{1}{5}$ are all common fractions.

(a) Change each of these fractions into decimals.

(b) Change each of the decimals from the previous question part into percentages.

8 Ninety-five per cent of all the animal species in the world are insects. Write this as a fraction in simplest form.

9 At Valley High School, $\frac{5}{8}$ of the students are studying French. Write this fraction as a percentage.

10 Put the following in ascending order (smallest to largest).

(a) 82%, 0.03, $\frac{7}{50}$, $\frac{3}{5}$, 0.8, 57%

(b) $\frac{1}{2}$, 73%, 1.05, $\frac{3}{10}$, 43%, 0.05

11 The Petronas Towers in Kuala Lumpur are approximately 0.6 times the size of the tallest building in the world, the Burj Khalifa in Dubai.

(a) Write the height of the Petronas Towers as a fraction of the height of the Burj Khalifa.

(b) Now, write the equivalent percentage.

Reasoning

12 A rugby stadium contains 43 500 people. 37% of the crowd supports England and the rest supports Australia. What percentage of the crowd supports Australia?

13 Allison is a journalist. She has surveyed 150 people and found that six believe that ghosts exist.

(a) Write the survey result:

(i) as a fraction in simplest form

(ii) as a percentage

(iii) as a decimal.

(b) Which of the three forms of the result would you recommend that Allison uses as the headline for the article she is writing? Give a reason for your answer.

(c) Use your answer from **(b)** to write a potential headline.

14 $\frac{3}{4}$ is 3 lots of $\frac{1}{4}$, or $3 \times \frac{1}{4}$. How is this fact reflected in the percentage values for $\frac{1}{4}$ and $\frac{3}{4}$?

15 13% of a class have red hair, 62% of the class have black hair.

(a) What is the total percentage of the class that have red or black hair?

(b) What is this total as a decimal?

(c) What percentage of the class have a hair colour that is not red or black?

(d) Write your answer to **(c)** as a decimal.

Open-ended

16 Write at least three fractions that have equivalent percentage values between 10 and 40%.

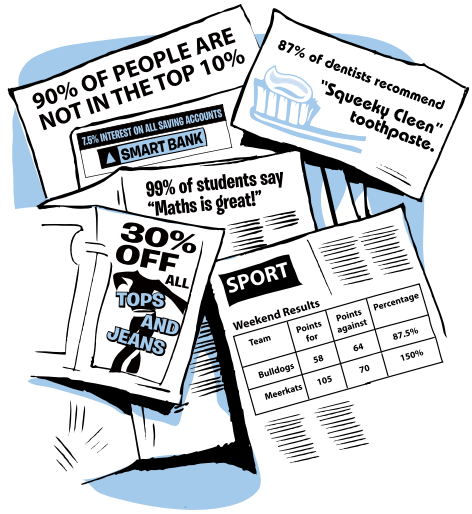
17 Find at least three percentages that have decimal values between 0.34 and 0.35.

18 Estimate the percentage of one weekday that you would spend:

(a) sleeping **(b)** eating **(c)** studying.

4.8 Using percentages

In this section you will learn two important skills that will help you to understand and work with two common uses of percentages.



Finding a percentage of a quantity

Statements involving percentages are common:

'32% of the students in the school have blue eyes.'
'48% of voters voted for this candidate.'
'The television is discounted by 20% of the original price.'

Knowing the fraction equivalents of some common percentages is very useful.

For example:

50% = $\frac{1}{2}$, so to find 50% of something, simply divide by 2.

25% = $\frac{1}{4}$, so find 25% by dividing by 4 (or divide by 2 twice).

10% = $\frac{1}{10}$, so divide by 10 to find 10%.

20% = $\frac{1}{5}$, so divide by 5 to find 20% (or divide by 10, then multiply by 2).

You can use these facts to find a percentage of a quantity. This is shown in Method 1 of the following example.

Another way to find a percentage of a quantity is to use the fact that in mathematics, the word 'of' means 'multiply'. This is shown in Method 2 of the following Worked example.

Worked example 21 W.E. 21

Calculate the following percentage: 15% of 70.

Method 1: Use basic percentages

Thinking	Working
1 Break the percentage up into basic percentages, such as 50%, 10%, 5% or 1%.	15% = 10% + 5%
2 Calculate each of the basic percentages.	10% of 70 = 70 ÷ 10 = 7 5% of 70 = (10% of 70) ÷ 2 = 7 ÷ 2 = 3.5
3 Use combinations of the basic percentages calculated to obtain the required percentage.	15% of 70 = 7 + 3.5 = 10.5

Method 2: Fraction or decimal multiplication

Thinking	Working
1 Replace the word 'of' with the multiplication symbol, ×.	$15\%\ of\ 70$ $= 15\% \times 70$
2 Convert the percentage to a fraction or decimal.	$= \frac{15}{100} \times 70$ *or* 0.15×70
3 Perform the multiplication.	$= \frac{\cancel{15}^{3}}{\cancel{100}_{2}} \times \frac{\cancel{70}}{1}$ $= \frac{3 \times 7}{2 \times 1}$ $= \frac{21}{2}$ $= 10.5$

Writing one amount as a percentage of another

Writing fractions of a whole as percentages makes them easier to visualise, compare and order. The results of surveys, tests and sports teams are often reported using percentages.

To write one amount as a percentage of another:

1. Write the first amount in as the numerator of a fraction, and the second amount as the denominator.
2. Convert this fraction to a percentage. If the percentage is not a whole number, then rounding to 1 decimal place gives an accurate enough answer in most cases.

Worked example 22 W.E. 22

Write the following as percentages. Round your answers to 1 decimal place, if necessary.

(a) Melissa got 21 out of 25 questions on her grammar test correct.

(b) 13 of the 24 students in class 7C have brown hair.

Thinking	Working
(a) 1 Write the first amount as the numerator and the second amount as the denominator. Is the denominator a factor of 100? (Yes)	*(a)* $\frac{21}{25}$
2 Determine what number the denominator must be multiplied by to get 100.	$\frac{21}{25} \times \frac{\ }{4} = \frac{\ }{100}$
3 Multiply the numerator by the same number.	$\frac{21}{25} \times \frac{4}{4} = \frac{84}{100}$
4 Write the numerator of the new fraction with a percentage symbol.	84%
5 Write your answer in a sentence.	*Melissa got 84% of the test questions correct.*

(b) 1 Write the first amount as the numerator and the second amount as the denominator. Is the denominator a factor of 100? (No)

(b) $\frac{13}{24}$

2 Convert the fraction to a percentage by multiplying by 100. Cancel common factors first to simplify the multiplication. (Here, a common factor of 4 has been cancelled.)

$\frac{13}{\cancel{24}_{6}} \times \frac{\cancel{100}^{25}}{1}\%$

$= \frac{325}{6}\%$

3 Divide the numerator of the answer by the denominator. Write your answer as a decimal, rounded to 1 decimal place.

54.2%

4 Write your answer in a sentence.

54.2% of the students in 7C have brown hair.

4.8 Using percentages

Navigator

Answers p. 670

1 (columns 1–2), 2, 3 (columns 1–2), 4, 5, 6, 7, 9, 10, 11, 13, 14, 16, 18, 20	1 (columns 2–3), 2 (a–f), 3 (columns 2–3), 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 20	1 (column 3), 2 (a–d), 3 (columns 3), 5, 6, 7, 8, 9, 11, 12, 13, 15, 16, 17, 18, 19, 20

Equipment required: calculator

Fluency

W.E. 21

1 Calculate each of the following percentages.

(a) 50% of $36
(b) 10% of 450 m
(c) 20% of 840 L
(d) 80% of 120 m
(e) 90% of 500 kg
(f) 2% of $8400
(g) 13% of 72 kg
(h) 27% of $354
(i) 21% of 876 m
(j) 28% of 231.25 m
(k) 34% of 210.4 L
(l) 4% of $32.65

W.E. 22

2 Write the following as percentages. Round your answers to 1 decimal place, if necessary.

(a) 17 out of 20 on a spelling quiz.
(b) 5 out of 22 students in the class have blond hair.
(c) There are 100 kittens at the animal shelter, 40 of which are female.
(d) 4 out of 5 students surveyed own a smartphone.
(e) 50 pairs of shoes were sold, of which 38 pairs were black.
(f) 54 out of 70 cars counted on a freeway had only 1 person in them.
(g) 15 of the 27 cars in the staff car park are white or silver.
(h) 3 out of every 4 people on the bus said they were travelling to work.
(i) 28 out of 34 people surveyed said they were non-smokers.

3 Calculate the following using the 'basic percentages method' or any other mental strategy.

(a) 50% of $25 **(b)** 10% of 75 kg **(c)** 25% of 60 km

(d) 20% of 40 L **(e)** 1% of 360 m **(f)** 5% of $120

(g) 60% of 350 kg **(h)** 75% of 800 mL **(i)** 35% of $400

4 Mehmet successfully shot 13 basketball goals out of 18 attempts. How would the calculation that converts this result to a percentage be set out?

A $\frac{13}{100} \times \frac{18}{1}$ **B** $\frac{13}{18} \times \frac{100}{1}$ **C** $\frac{18}{13} \times \frac{100}{1}$ **D** $\frac{100}{13} \times \frac{18}{1}$

5 20% of $420 is:

A $8.40 **B** $20 **C** $84 **D** $200

Understanding

6 40% of the 25 students in a class have black hair. How many students is this?

7 16 out of the 28 biscuits in a packet have cream centres. What percentage of the packet are cream-centred biscuits?

8 A 2-litre bottle of orange drink contains 35% orange juice. Work out how many litres of orange juice is contained in the bottle. (It may help to work in mL.)

9 A lion can sleep for up to 20 hours a day. What percentage of a whole day is this?

10 There are 11 boys and 15 girls in a class. Write the number of boys and the number of girls as percentages of the total number of students in the class.

11 Riley kicked 7 goals from 10 attempts. Franco kicked 11 goals from 16 attempts.

(a) Express the results as percentages.

(b) Who is the more accurate goalkicker?

12 Australia scored 381 runs in a cricket match against India. If Shane Watson scored 126 of those runs, what percentage of the team's total did he score?

13 Shops often uses percentages to advertise savings during sales. The percentage is used to calculate the amount that is removed from the price of the goods. This amount is known as the 'discount'.

(a) Calculate the dollar value of the discount on the following.

(i) $40 shirt, 20% discount **(ii)** $125 jacket, 25% discount

(iii) $65 jeans, 50% discount **(iv)** $239 suitcase, 10% discount

(v) $799 camera, 40% discount **(vi)** $1299 sofa, 5% discount

(b) Now, find the sale price of the above items by subtracting the discount amount from the original price.

Reasoning

14 Eight per cent of the apples in a crate were rotten. What percentage of the apples were not rotten?

15 Chelsea and Sonia are netballers. In one game, Chelsea scored 17 goals from 25 attempts, and Sonia scored 15 goals from 20 attempts. In the next game, Chelsea scored 19 goals from 27 attempts, and Sonia scored 17 goals from 23 attempts. Use percentages to show which one of the girls improved the accuracy of her goal shooting. Round your figures to 1 decimal place, if necessary.

16 The 180 students at St John's College all completed an online survey. The results showed that:

22% of the students had blue eyes

34% spoke two or more languages fluently

7% of them were left-handed.

(a) Calculate the number of students in each category using the above three percentages.

(b) Considering that we are dealing with numbers of students, what is strange about your answer to **(a)**?

(c) What might be the actual numbers of students represented by each of the percentages?

17 The energy in food can be measured in kilojoules (kJ). The National Heart Foundation of Australia recommends that no more than 30% of our daily energy intake should come from eating fats.

(a) A healthy, active person is told they should eat up to 9000 kJ of food per day. Following this advice, how many kJ in the form of fats could be eaten in 1 day?

(b) 1 gram of fat gives the body 37 kJ of energy. How many grams of fat would provide the maximum daily kilojoules allowed from fat? Round your answer to the nearest whole number.

Open-ended

18 Suggest some reasons as to why some stores use percentages to advertise savings, rather than dollar amounts.

19 Mark saved $5.00 when a store had a sale. Find at least three possible different percentage discounts and original prices for the item that Mark purchased.

20 Issa and Chiara were asked to show their working to the following question.
'Calculate 15% of 80'.

Here is Chiara's working:

$$\frac{15}{80} \times \frac{10\cancel{0}}{1}$$
$$= \frac{150}{8}$$
$$= 18.75$$

Here is Issa's working:

$$15\% \text{ of } 80$$
$$= 0.15 \times 80$$
$$= 12$$

Which of the students' working is correct? Write some advice to the other student to help them avoid similar mistakes in the future.

Problem solving

The great half-price sale

Sam and Carly were out shopping. They headed towards their favourite clothes shop. On the shop window, a sign said 'Take 50% off all marked prices!' Below that sign was another sign, saying 'Take another 50% off reduced prices!'

'Eh?' said Carly. '50% is one half. If I take half off the price, then take off another half, there won't be any price left! That means everything in the shop is free!'

'I don't think so', said Sam. 'How can you run a business like that?'

'Well', said Carly, 'If I pick up a pair of $100 jeans and take two lots of 50% off the price, what do they cost now?'

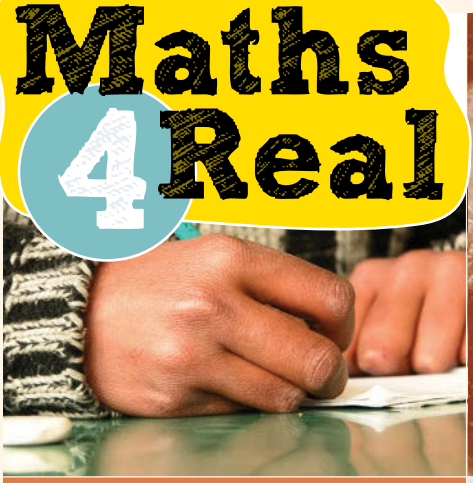
'Errr …', said Sam.

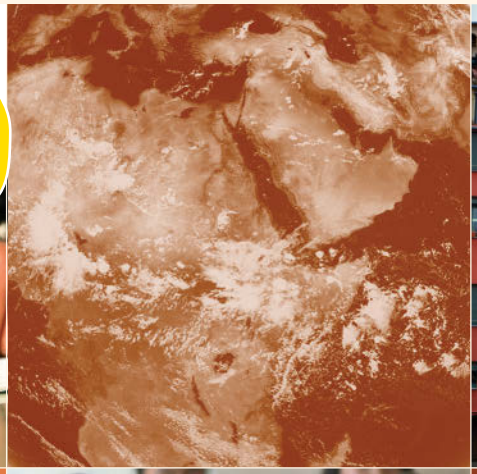
Can you work it out? How much would the jeans cost?

Strategy options

- Draw a diagram.
- Break problem into manageable parts.

Maths 4 Real





Our global village

Facts about our global village

There are more than 7 billion people on Earth today. Who is this large community composed of?

1 Convert the following facts to percentages to find out.

Region

- 0.61 Asia
- $\frac{12}{100}$ Europe
- 0.08 North America
- 0.05 South America and Caribbean
- 0.13 Africa
- $\frac{1}{100}$ Oceania (including Australia)

Conditions

- 0.37 live without basic sanitation
- $\frac{1}{10}$ don't have access to safe drinking water
- 0.11 are hungry and/or malnourished

Wealth

- 0.01 of the population own more than 50% of the entire wealth
- $\frac{1}{5}$ of the population have more than 75% of the income
- 0.53 of the population live on $2.50 or less per day

Education and technology

- 0.11 can't read
- 0.07 have a secondary education
- 0.48 have access to the internet

How rich are you?

If you have a bed to sleep on, food to eat and a roof over your head, you're richer than what percentage of the entire world's population?

2 To find out, complete this decimal calculation and *convert your answer to a percentage*.

1.2 – 2.7 + 3.5 – 1.25 =

How many people?

In 2016, there were approximately 7 125 000 000 people living on Earth, with the number increasing all the time.

3 Using the facts you now know about our world, calculate how many people:

(a) don't have access to clean water

(b) live on $2.50 or less per day

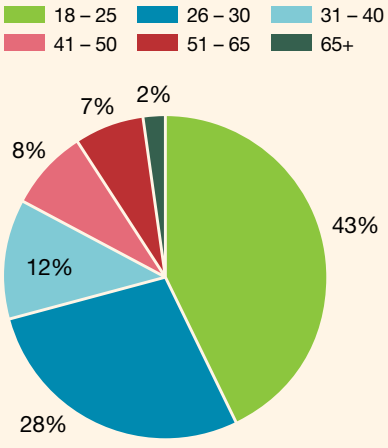
(c) have access to the internet

(d) can't read.

I want to help

Many charities and other organisations exist to try to create more equality in the world. Charities often rely on the generosity of ordinary people. Here is a pie chart of the age groups of a typical charity's volunteers.

Volunteers by age group



4 Using the pie chart of charity volunteers, find out the following.

(a) What percentage of volunteers are between 18 and 30?

(b) Why do you think most volunteers are in this age range?

(c) One of the challenges charities face is the need to pay for fundraising and administration costs. If a charity spends 24.4% of its income on fundraising and 8.9% on administration costs, what percentage of its income is available to go directly to community-aid projects?

In the future

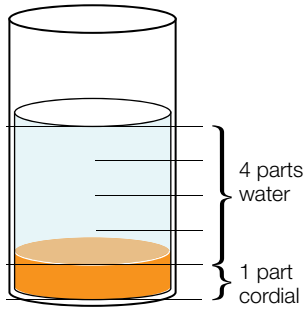
5 Imagine it is the year 2055. How do you hope the 'global village' percentages have changed? Write your predictions and ideas.

4.9 Ratio

A **ratio** is a comparison of amounts.

For example, when you mix a cordial drink, you add different amounts of cordial and water.

The amount of cordial compared to the amount of water may be written in many ways.



1 part cordial to 4 parts water

or

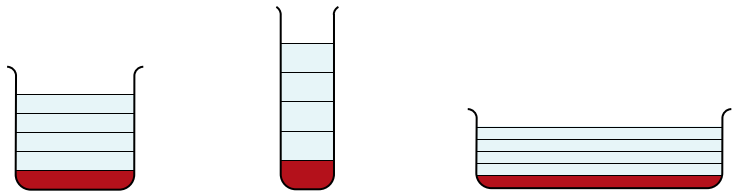
1 to 4

or using a ratio symbol,

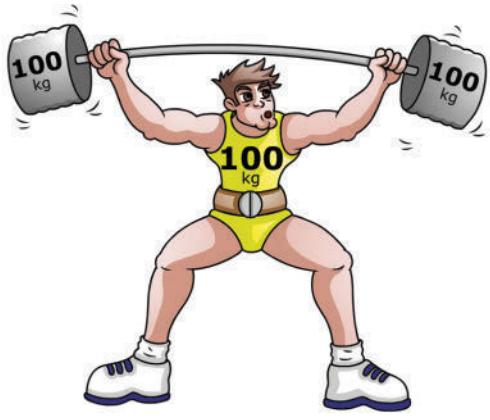
1 : 4

(The ':' can be read as 'to'.)

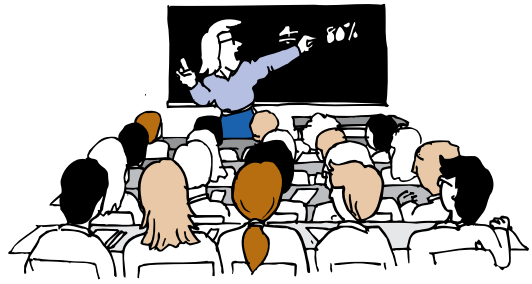
The actual volumes of cordial and water (the number of millilitres) are not given. It is the ratio of cordial compared to water that is important to get the right taste. Specifying actual quantities would not be useful, because cordial is made in glasses and jugs of many different sizes. But using proportions made from the ratio, a sip of cordial from each of the vessels below would taste the same after the mixtures are stirred.



Some more examples of ratios are given below.



The ratio of the mass being lifted to the mass of the lifter is 2 : 1.



The teacher-to-student ratio in the class is 1 : 20.

A ratio is a comparison of two or more amounts, written using a ':' symbol.

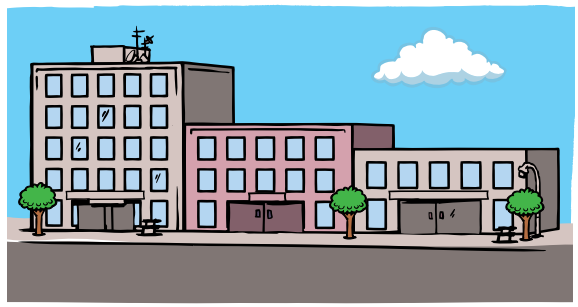
The order of the amounts in a ratio must be clear. If the ratio of teachers to students in a class is 1 : 20, then the ratio of students to teachers is 20 : 1.

Worked example 23 W.E. 23

Write a ratio for each of the following.

(a) the number of windows in the middle building compared to the number of windows in the smallest building

(b) the number of floors in the buildings, from left to right



Thinking			Working
(a)	**1**	Write the quantities in the correct order.	(a) Windows in middle building: 12 Windows in smallest building: 7
	2	Write as a ratio.	12 : 7
(b)	**1**	Write the quantities in the correct order.	(b) 5 floors to 3 floors to 2 floors
	2	Write as a ratio.	5 : 3 : 2

You can see from the example above that a ratio can compare more than two amounts.

Part : part and part : whole ratios

Suppose there is a class of 9 students with black hair and 11 students with brown. You could write a ratio comparing the number of brown-haired students to black-haired students as 11 : 9. This **part : part ratio** compares two parts of a whole.

You can also write another type of ratio that compares the number of brown-haired students to the total number of students in the class. There are 11 + 9 = 20 students in the class as a whole, so the ratio of brown-haired students to the class is 11 : 20. This is a **part : whole ratio** that compares one part to the whole.

A part : part ratio compares separate parts of a whole.

A part : whole ratio compares a part (or parts) to a whole.

Ratios, fractions and percentages

The numbers in part : whole ratios can be written as fractions. For example, the above ratio of 11 : 20 could be written as the fraction $\frac{11}{20}$ or as the percentage 55%. The remaining fraction would therefore be $\frac{9}{20}$ of the class, or 45%.

Worked example 24 W.E. 24

A large enclosure at the zoo contains 5 giraffes and 12 zebras.

(a) Write a ratio comparing the number of zebras to the number of giraffes.

(b) Write a ratio comparing the number of giraffes to the total number of animals.

(c) Write the number of giraffes as a fraction of the total number of animals.

(d) Write the number of zebras as a percentage of the total number of animals.

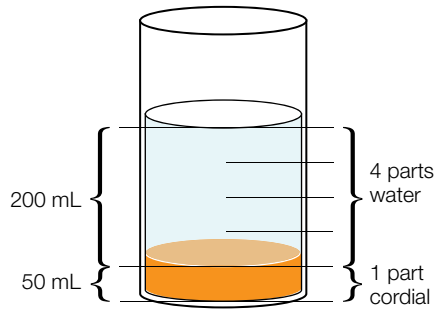
Thinking	Working
(a) Write the ratio in words, in the correct order, then write the numbers underneath.	(a) zebras : giraffes = 12 : 5
(b) 1 Find the number of parts in the whole (the total number of animals).	(b) 5 + 12 = 17
2 Write the ratio in words, in the correct order, then write the numbers underneath.	giraffes : total animals = 5 : 17
(c) Write one number in the part : whole ratio as a fraction of the other.	(c) $\frac{5}{17}$
(d) Write a new part : whole ratio as a fraction, then convert the fraction to a percentage, rounding to a sensible value.	(d) zebras : total animals = 12 : 17 $\frac{12}{17} \times \frac{100}{1}$ % = 71% (to nearest whole number)

Equivalent ratios

Imagine that you wanted to mix up a large quantity of a cordial drink using the cordial : water ratio of 1 : 4 that was considered at the beginning of this section. Mixing 1 mL of cordial and 4 mL would only give 5 mL of cordial; however, you can make 500 mL of cordial drink that would taste exactly the same if you mixed 100 mL of cordial with 400 mL of water.

1 : 4 and 100 : 400 are **equivalent ratios**. Equivalent ratios are created in the same way as equivalent fractions. You multiply each part in the ratio by the same number. Here 1 : 4 has been multiplied by 100 to obtain the ratio 100 : 400. Multiplying by 50 would give a ratio of 50 : 200.

$$\begin{array}{ccccc} & 1 & : & 4 & \\ \times 50 & \downarrow & & \downarrow & \times 50 \\ = & 50 & : & 200 & \end{array}$$



Simplifying ratios

Imagine that you have mixed a cordial drink using the above ratio of 50 : 200, which is equivalent to 1 : 4. Your friend Jack has made his drink by mixing 60 mL of cordial with 300 mL of water. He says his drink is stronger than yours, but it is not easy to tell by comparing the ratios of 50 : 200 and 60 : 300. It would be easier to compare the ratios in simplest form.

You can simplify a ratio in the same way you would simplify a fraction—by dividing each part in the ratio by a common factor. If you divide by the HCF, you will obtain the ratio in simplest form.

The HCF of the ratio 60 : 300 is 60.

$$\begin{array}{ccccc} & 60 & : & 300 & \\ \div 60 & \downarrow & & \downarrow & \div 60 \\ = & 1 & : & 5 & \end{array}$$

(you could also have obtained this ratio by doing a series of smaller divisions, such as ÷10, ÷6)

Now, instead of comparing 50 : 200 and 60 : 300, compare 1 : 4 with 1 : 5. The ratio 1 : 5 has one extra part of water to the same amount (1 part) of cordial, so Jack's drink is actually weaker than yours. (You may be able to tell this by looking at the colour, or by tasting it.)

Equivalent ratios are created by multiplying or dividing each part in the ratio by the same number.

Ratios can be simplified by dividing each part in the ratio by a common factor. Dividing by the highest common factor (HCF) will give the ratio in simplest form.

4.9 Ratio

Navigator

1, 2, 3 (columns 1–3), 4 (columns 1–3), 5, 6, 7, 8, 9, 11, 12, 13, 15, 17, 18	1, 2, 3 (columns 2–3), 4 (columns 2–3), 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 17, 18	1 (column 1), 2, 3 (columns 3–4), 4 (columns 3–4), 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19

Answers p. 670

Fluency

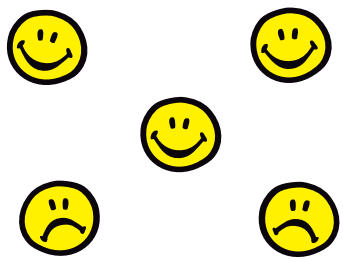
W.E. 23

1 Write a ratio for each of the following.

(a) circles to squares

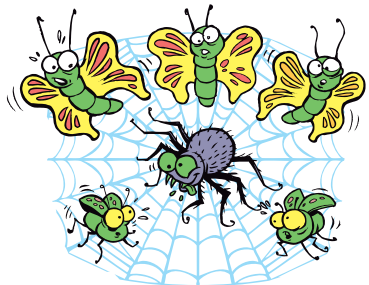
(b) blue circles to red circles

(c) happy faces to sad faces

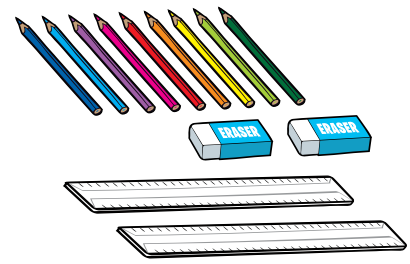


(d) ducks to swans

(e) spider to flies to butterflies

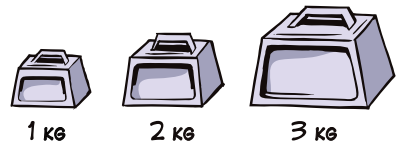


(f) rulers to pencils to erasers



(g) bees to flowers to leaves

(h) large mass to medium mass to small mass



W.E. 24

2 Sumi has 3 canaries and 2 mice as pets.

(a) Write a ratio comparing the number of mice to the number of canaries.

(b) Write a ratio comparing the number of canaries to the total number of pets.

(c) Write the number of canaries as a fraction of the total number of pets.

(d) Write the number of mice as a percentage of the total number of pets.

3 Create pairs of equivalent ratios by multiplying or dividing each part in the ratio by the number given in brackets beside it.

(a) 1 : 3 (×2)	**(b)** 1 : 8 (×2)	**(c)** 1 : 11 (×2)	**(d)** 1 : 33 (×2)
(e) 4 : 5 (×3)	**(f)** 3 : 7 (×4)	**(g)** 6 : 9 (÷3)	**(h)** 8 : 12 (÷4)
(i) 10 : 8 (÷2)	**(j)** 18 : 9 (÷9)	**(k)** 8 : 5 (×7)	**(l)** 12 : 11 (×5)
(m) 48 : 18 (÷6)	**(n)** 20 : 6 (×5)	**(o)** 60 : 54 (÷3)	**(p)** 15 : 13 (×5)

4 Write the following ratios in simplest form, by dividing by the highest common factor (HCF).

(a) 2 : 10	**(b)** 3 : 12	**(c)** 4 : 18	**(d)** 6 : 21
(e) 21 : 14	**(f)** 24 : 15	**(g)** 32 : 20	**(h)** 48 : 36
(i) 54 : 64	**(j)** 56 : 68	**(k)** 72 : 81	**(l)** 38 : 95
(m) 75 : 50	**(n)** 120 : 80	**(o)** 105 : 60	**(p)** 240 : 66

5 There are 14 cricketers and 9 basketballers in a Year 7 maths class.

(a) The ratio of basketballers to cricketers is:

A 14 : 9 **B** 9 : 14 **C** 14 : 23 **D** 9 : 23

(b) The fraction of the whole class that are cricketers is:

A $\frac{9}{23}$ **B** $\frac{9}{14}$ **C** $\frac{14}{23}$ **D** $\frac{14}{9}$

Understanding

6 A packet of jellybeans contains three colours: 12 red, 7 blue and 16 yellow.

(a) Write the ratio of red to blue to yellow jellybeans.

(b) How many jellybeans are in the packet?

(c) What fraction of the jellybeans in the packet are red?

(d) What percentage of the jellybeans are blue?

7 During a cricket season, Erika's team won 15 matches, drew 2 and lost 7.

(a) Write the won : lost : drew ratio for Erika's team for that season.

(b) What fraction of their games did Erika's team lose?

(c) Write your answer to **(b)** as a percentage, rounded to the nearest whole number.

8 The students at Hillview High School are voting for a school captain. The results show that for every 4 students who voted for Guy, 5 students voted for Dominic and 3 for Melissa.

(a) Write the ratio of the number who voted for Dominic to the number who voted for Guy, to the number who voted for Melissa.

(b) What fraction of the total vote did Dominic receive?

(c) Write your answer to **(b)** as a percentage, rounded to the nearest whole number.

9 There must be 1 instructor for every 6 students on a kayaking course.

(a) Write the comparison of students to instructors as a ratio.

(b) If 24 students want to do the course, use an equivalent ratio to determine how many instructors will be required.

(c) If 5 instructors are available, what is the maximum number of students that can do the course?

10 Tom's recipe for making concrete is to add gravel, sand and cement in the ratio 3 : 2 : 1 before adding water. Tom has 4 buckets of cement. Use an equivalent ratio to show how many buckets of sand and gravel he should add to get the correct concrete mixture.

11 'White gold' is commonly made by mixing gold and nickel in the ratio 9 : 1.

(a) Write the amount of nickel as a percentage of the total mixture.

(b) A jeweller wants to make 20 g of white gold. How much nickel will he require?

12 A survey found that 75% of Year 7 students were studying music.

(a) What percentage were not studying music?

(b) Use the two percentages to write a ratio that compares the group studying music to the group not studying music, in simplest form.

Reasoning

13

Which points divide the above line so that the ratio (shorter part) : (longer part) is the following:

(a) 1 : 7 **(b)** 5 : 3 **(c)** 1 : 3 **(d)** 1 : 1

14 There must be at least 1 teacher for every 20 students on a school excursion.

(a) If 68 students are going on an excursion, how many teachers are required? Give reasons for your answer.

(b) Use your answer to **(a)** to write the actual ratio of teachers : students, in simplest form.

15 Chloe made a cordial drink by mixing 30 mL of concentrate with 150 mL of water. Her friend Hannah made a cordial drink by mixing 80 mL of concentrate with 200 mL of water. Hannah said her cordial drink was 'stronger' than Chloe's. Chloe could not tell by looking.

(a) Write the ratio of concentrate : water for each drink, in simplest form.

(b) Use the ratios to state whether Hannah is correct, giving a reason for your answer.

16 Mia has three cats and five dogs as pets.

(a) What fraction of her pets are dogs?

(b) What fraction of her pets are cats?

(c) If Mia adopts another cat, what fraction of her pets are cats now?

(d) After adopting the new cat, Mia moves into a house with Shona who also has some cats and dogs. If the final ratio of cats to dogs is now 1 : 1, what is the smallest number of cats and dogs that Shona brings to the house?

Open-ended

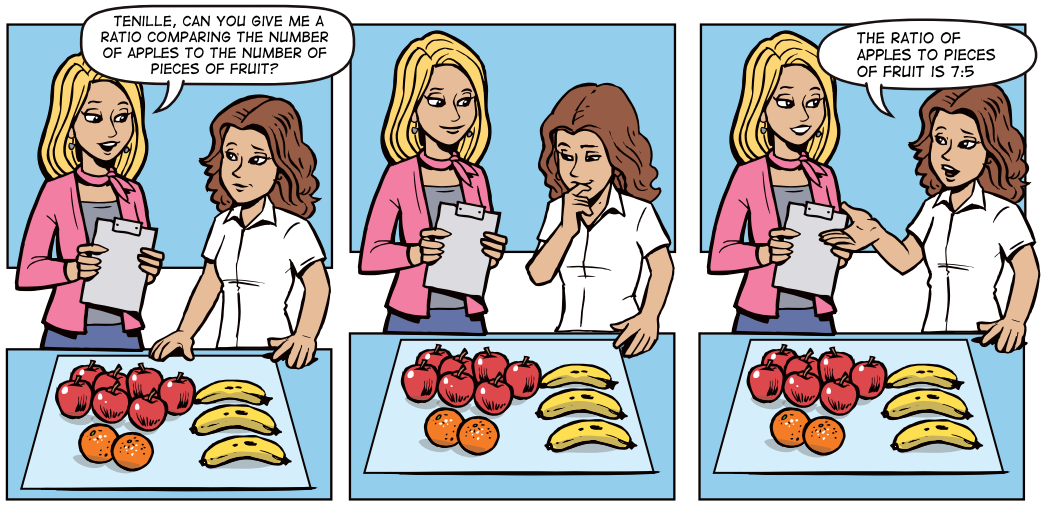
17 **(a)** What is the ratio of girls to boys in your class?

(b) Would the ratio be the same in all the Year 7 classes in your school? Explain your answer by giving examples.

(c) Would the ratio be the same for all school classes in Australia? Give some reasons for your answer.

18 Write three ratios that are equivalent to 4 : 7.

19



What mistake has Tenille made? What is the correct answer?

Rates

4.10

In the previous section, ratios were used to compare two or more amounts of the same type of quantity (e.g. millilitres of water and millilitres of cordial).

A **rate** is a way of comparing two amounts of different types of quantities. They usually involve the word *per* which means 'for each' and have the symbol / or the letter 'p'. A rate is written as the number of the first quantity for every 1 of the second quantity. Some examples of rates are:

If the price of petrol is $1.29/L, you will pay a rate of $1.29 for 1 litre of petrol.

Cricketers scoring at a rate of 4 runs per over will score an average of 4 runs in 1 over.

The unitary method

If you know how much 1 litre of petrol costs, you can calculate the cost of any other volume of petrol by multiplying by the cost of one litre.

If you know that 50 L of petrol costs $75, you can work out the cost of 4 L, 15 L, 37 L or any other volume of petrol by first calculating how much 1 L costs.

Solving problems by calculating a number (such as cost) 'per unit' is called the **unitary method**.

Worked example 25

W.E. 25

(a) If 8 kg of oranges costs $24, find the cost of 6 kg of oranges.

(b) A machine in a shampoo factory produces 210 bottles every hour. How many bottles would be produced in 40 minutes?

Thinking		Working
(a)	**1** Calculate the cost of one 'unit' of the amount (1 kg of oranges).	(a) $24 for 8 kg (÷ 8) = $3 for 1 kg (× 6)
	2 Multiply the cost per unit by the number of units.	= $18 for 6 kg

(b) 1	Calculate the number of items produced in 1 minute.	(b) 210 ÷ 60 = 3.5 bottles/min
2	Multiply the number produced in one minute by the number of minutes. Round your answer to a sensible value.	3.5 bottles/min × 40 min = 140 bottles in 40 min

Comparing prices

Many products come in different sizes, which makes it difficult to compare the prices. For example: Which is better value for money, a 500 g box of breakfast cereal for $5.99, or the larger 820 g box for $7.21?

You can use the unitary method to compare the prices. Divide the prices by the mass of the boxes to find the price per 100 g of the cereal, because this is how supermarkets usually display these unit prices.

A price per 100 g (or 100 mL, or 1 kg or 1 L) is called the **unit price**. Unit prices let you more easily compare sizes and brands to find products that are the best value for money.

To convert ordinary prices to prices per 100 g, divide the price by the mass to find the price for 1 g, then multiply by 100. For example:
500 g box: $5.99 ÷ 500 × 100 = $1.20 per 100 g
820 g box: $7.21 ÷ 820 × 100 = $0.88 per 100 g

However, a quicker method is to mentally divide the mass by 100 to find the number of 100 g 'lots'. You can then divide the price by this number:

500 ÷ 100 = 5
$5.99 ÷ 5 = $1.20 per 100 g

820 ÷ 100 = 8.2
$7.21 ÷ 8.2 = $0.88 per 100 g

Worked example 26

W.E. 26

Calculate the unit price (price per 100 g) of the following products, to determine which is better value for money:

- a 500 g packet of pasta for $2.42
- a 750 g packet for $3.44.

Thinking	Working
1 Divide the mass of the first product by 100.	500 g ÷ 100 g = 5
2 Divide the price of the first product by your answer to step **1**.	$2.42 ÷ 5 = $0.48/100 g
3 Repeat steps **1** and **2** for the second product.	750 g ÷ 100 g = 7.5 $3.44 ÷ 7.5 = $0.46/100 g
4 State your answer.	The 750 g packet is slightly better value for money (2c less per 100 g).

4.10 Rates

Navigator

1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 16 (a–c), 17, 20, 21	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 19, 20, 21	1, 2 (a–d), 3 (a–d), 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21

Answers p. 671

Fluency

1 Copy and complete the following.

(a) Find the cost of 5 kg of apples if 3 kg costs $15.

Cost	Amount	
$15	3 kg	(÷ 3)
$_____	1 kg	(× 5)
$_____	5 kg	

(b) Find the cost of 2 kg of lentils if 5 kg costs $12.50.

Cost	Amount	
$12.50	5 kg	(÷ _____)
$2.50	1 kg	(× _____)
$5	2 kg	

(c) Find the cost of 300 g of coffee if 500 g costs $30.

Cost	Amount	
$30	500 g	(÷ _____)
$_____	100 g	(× _____)
$_____	300 g	

2 **(a)** If 15 kg of potatoes costs $12, find the cost of 7 kg of potatoes. W.E. 25

(b) A machine in a canning factory produces 250 cans every hour. How many cans would be produced in 25 minutes?

(c) If 14 pens cost $5.60, find the cost of 3 pens.

(d) Farmer Harry buys 8 cows for $6800. How much would 10 cows have cost?

(e) If 21 bags of rice weigh 35 kg, how much do 15 bags of rice weigh?

(f) A printing machine prints 70 pages in 8 minutes. How many pages could it print in one hour?

3 Calculate the unit price (the price per 100 g) of the following pairs of products, and so determine which one is better value for money. W.E. 26

(a) a 600 g jar of jam for $4.09 or a 250 g jar for $2.68

(b) a 500 g packet of pasta for $2.81 or a 750 g packet for $3.45

(c) a 375 g jar of tomato paste for $1.28 or a 500 g jar for $3.25

(d) a 200 g jar of peanut butter for $3.21 or a 375 g jar for $4.93

(e) a 500 mL bottle of soy sauce for $2.71 or a 920 mL bottle for $4.29

(f) a 450 g bag of cat food for $3.78 or a 1 kg bag for $8.79

4 Nazeh has saved $2000 in 4 months. If he continues to save at this rate, how much will he have by the end of the year?

A $6000 **B** $8000 **C** $12 000 **D** $24 000

5 An entertainer receives $450 000 for a season of five performances, each of which lasts $1\frac{1}{2}$ hours. The rate of pay per minute is closest to:

A $60 000 **B** $5000

C $1500 **D** $1000

Understanding

6 A caterer knows that 4 loaves of bread can make enough sandwiches for 25 people. How many loaves will they need to make enough sandwiches for 120 people?

7 **(a)** Which is better value for money, a 280 g jar of jam for $5.15, or a 400 g jar for $6.42?

(b) The 280 g jar goes on special for $4.49. Does this change your answer to **(a)**?

8 A phone company charges 75c for each 30 seconds of a phone call. How much will a 4-minute phone call cost?

9 Rajid's cricket team needs 85 runs from 12 overs to win the game. What rate, in runs per over, must they score to win? (Answer to 1 decimal place.)

10 **(a)** Calculate the price per item in each pack of the following products, to determine which pack is better value for money.

(i) a packet of 6 muesli bars for $4.83, or a packet of 10 for $7.51

(ii) a 12-pack of cans of soft drink for $14.20, an 18-pack for $20.21, or a 30-pack for $27.51

(iii) a box of 50 tea bags for $3.20, or a box of 200 for $7.99

(b) Describe another way you could compare the prices of the tea bags in part **(a) (iii)**.

11 Lucy buys a 1-year subscription to *Australian Explorer* magazine. She pays $102 for 12 monthly magazines. The regular price for one issue of the magazine is $9.95. How much is Lucy saving:

(a) per magazine **(b)** over the 1-year period?

12 Andrea has a leaky tap in her bathroom. She puts a container underneath it and collects 1.2 litres of water in 8 hours. Use a mental or written method to calculate how much water will leak in:

(a) 1 day **(b)** 1 week.

13 A 200 g serve of yoghurt contains 6.5 g of fat. Use a mental or written method to calculate how much fat there is in:

(a) a 100 g serve **(b)** a 400 g serve **(c)** a 500 g serve.

14 A 5 GB data download costs $3.00. If the same rate is charged, use a mental or written method to calculate the cost of:

(a) a 10 GB data download **(b)** a 1 GB data download **(c)** a 13 GB data download.

Reasoning

15 Charlie's bakery sells hot cross buns in packets of 6. One packet costs $7.20. A customer wants to buy exactly 10 buns. Charlie is happy to take 4 buns from a second packet, but how much should he charge the customer?

16 Imagine that you are standing in a supermarket, comparing the following pairs of products. Do some mental calculations to determine which product is the better value for money. Rounding prices slightly may help to simplify some calculations. Show how you arrived at your answer.

(a) A 1 kg box of washing powder for $5.49, or a 4 kg box for $15.97.

(b) Four 250 mL cartons of orange juice for $1.25 each, or a 1 litre carton for $3.99.

(c) Buying 500 g of ham from the deli counter where the price is $13 per kg, or buying it in a packet for $5.83.

(d) A 400 g jar of honey for $7.99, or a 300 g jar for $6.54.

(e) Give some reasons why people might prefer to buy a product even though it is not the best value for money.

17 A phone company charges a rate of 85 cents per minute for pre-paid calls. Calls made on a post-paid plan are charged at 75 cents per minute, plus a 35-cent connection fee. Which of these options is cheaper for a 5-minute phone call?

18 During the month of January, the water level in a Northern Territory dam increased by 78 000 kilolitres (1 kilolitre = 1000 litres).

(a) How many kilolitres per day was this? (Round your answer to 2 decimal places.)

(b) If the dam continued to fill at this rate, use your answer to **(a)** to calculate how much water will have flowed into the dam by the end of the year. (Assume there are 365 days in a year.)

(c) Would it be sensible to use your answer to **(b)** as a prediction for the amount of water that will flow into the dam over the year? Give some reasons for your answer.

19 'Trim and Tasty' cheese comes in three different-sized blocks:

250 g for $4.50
500 g for $7.42
750 g for $9.76

(a) How many times bigger than the 250 g block is the 500 g block?

(b) If the prices of the 500 g block and the 750 g block were in the same proportion to the price of the 250 g block as their sizes, what would you expect their prices to be?

(c) Often (but not always), larger sizes of a product have cheaper unit prices. Suggest a reason why this is the case.

Open-ended

20 List four supermarket products whose unit price would be usefully calculated per 100 mL, or per L.

21 Kumar needs 4.5 kg of rice for the students who are attending his cooking classes. It is available in the following sizes and prices.

500 g for $1.24 1 kg for $2.35 1.5 kg for $3.59 2 kg for $4.52

(a) List at least three different combinations of sizes that would give Kumar 4.5 kg of rice.

(b) Which combination is the cheapest?

Problem solving

Numberrrrrrrrrs

1 Write your favourite single-digit number.

2 Multiply that number by 9.

3 Multiply this new number by 12 345 679. (If your calculator does not display more than 8 digits, then you will need to do this by hand to see the full effect.)

4 If you did choose your favourite number, then you should like the answer you see. Does this work for other single-digit numbers? Can you explain how this works?

Strategy options

- Guess and check.
- Work backwards.
- Test all possible combinations.

Challenge 4

1 What percentage of whole numbers from 6 to 25 inclusive (i.e. including 6 and 25) are exact multiples of 6?

A 5 **B** 20 **C** 25 **D** 30

2 The numbers 5, 8, 7 and 3 are written on cards. Where should they be put so that the following product has the greatest possible value?

$\square \times 0.\square\square\square$

3 Evaluate $0.1^2 - 0.1^3$.

4 Zain has $\frac{3}{4}$ of a dollar and Katrine has $\frac{3}{10}$ of a dollar. Together they have:

A \$0.95 **B** \$1.00 **C** \$1.05 **D** \$1.10

5 $200 \div 2.5 = 80$. Therefore, $20 \div 0.25 =$

A 0.80 **B** 8.0 **C** 80 **D** 800

6 When doing a series of additions using his calculator, Trent noticed that he added 49 095 instead of 49.95. In order to correct his error with a single entry, he should now:

A add 49.95 **B** subtract 49 045.05

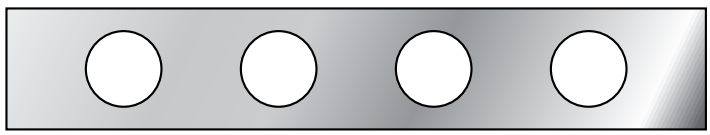
C add 49 045.05 **D** subtract 49 095

7 A decimal with 2 decimal places is multiplied by a decimal with 3 decimal places. Explain with number examples how the solution could have:

(a) 5 decimal places **(b)** 4 decimal places **(c)** 3 decimal places.

8 In Felicity's first six netball games she averaged 7.5 goals per game. If the least number of goals she scored was 4, what is the lowest possible value for the highest number of goals she scored?

9 Four holes are to be drilled along the centre of a strip of metal so that their centres are 2.25 cm apart. The centres of the two end holes are to be 3.35 cm from their corresponding ends. What is the length of the strip of metal?



10 What is the quotient when 0.01 is divided by 0.002?

11 Mikaela had her salary reduced by 10%. She was later promoted and her salary was increased by 10%. If her original salary was \$30 000, her present salary is:

A \$24 300 **B** \$27 000 **C** \$29 700 **D** \$30 000

12 $\frac{2009}{20.09}$ equals

A 100 **B** 10 **C** 0.1 **D** 0.01

13 A town has 5000 residents, of whom 60% voted in a local council election. The result was that, of those who voted, 37% voted for *P*, 32% for *Q* and 31% for *R*. Under the voting system, *P* was elected. The number of residents who voted for *P* was:

A 1850 **B** 1110 **C** 960 **D** 930

Chapter review 4

Maths literacy

decimal places	part : whole ratio	ratio
decimal point	per cent	recurring decimal
digit	percentage	terminating decimal
equivalent ratios	place value	unit price
part : part ratio	rate	unitary method

Copy and complete the following using the words and phrases from this list, where appropriate. A word or phrase may be used more than once.

1 4.87 has three _____s and two _____ .

2 When adding or subtracting decimals, it is important to line up the _____s.

3 _____ are created by multiplying each part in the ratio by the same number.

4 _____s compare two or more parts of a whole.

5 A _____ has a finite number of decimal places, a _____ has an infinite number of decimals, in a repeating pattern.

6 _____ literally means 'for every hundred'.

7 Calculating a _____ helps to compare the prices of different sizes and brands of products.

Fluency

1 Write in expanded fractional form. **4.1**

(a) 0.968 **(b)** 5.0702 **(c)** 6.005

2 Write as decimals: **4.1**

(a) six tenths, five hundredths, two thousandths and four ten-thousandths

(b) eight hundredths, nine hundred-thousandths and three millionths

3 Write as decimals: **4.1**

(a) $4 + \frac{3}{10} + \frac{8}{100} + \frac{6}{10\,000}$ **(b)** $50 + 7 + \frac{3}{100} + \frac{2}{1000} + \frac{9}{100\,000}$

4 Write each of the following in expanded word form. **4.1**

(a) 1.8531 **(b)** 0.070 06 **(c)** 61.0009

5 Write the value of the 9 in each decimal as **(i)** a fraction **(ii)** in words. **4.1**

(a) 5.091 **(b)** 0.0659 **(c)** 25.291

6 Write < or > in each of the following pairs of numbers to make a true statement. **4.1**

(a) 3.0427 _____ 3.0274 **(b)** 0.009 95 _____ 0.01

7 Write in order from smallest to largest. **4.1**

(a) 0.6055, 0.5506, 0.607 **(b)** 0.071, 0.701, 0.71

8 Copy this number line and show the positions of the following numbers: 0.25, 1.45, 0.09, 2.05. **4.1**

9 Round each of the following to the number of decimal places shown in the brackets. **4.2**

(a) 4.398 (1) **(b)** 34.6503 (2) **(c)** 23.098 (1) **(d)** 102.4728 (2)

10 Round the following to the nearest 5 cents. **4.2**

(a) $41.21 **(b)** $10.03 **(c)** $79.97 **(d)** $98.98

11 Write each of the following as a single fraction or mixed number in simplest form. **4.3**

(a) 3.9 **(b)** 0.62 **(c)** 2.45 **(d)** 0.0018

12 Write the following fractions as decimals, using the correct notation for recurring decimals if necessary. **4.3**

(a) $\frac{4}{5}$ **(b)** $\frac{19}{20}$ **(c)** $\frac{2}{9}$ **(d)** $\frac{10}{12}$

13 Calculate: **4.4**

(a) 23.6 + 4.79 **(b)** 3.768 + 10.9205 + 0.77 **(c)** 6 + 0.408 + 35.025

(d) 10.367 – 2.65 **(e)** 2.082 – 0.394 **(f)** 7 – 2.2198

14 Find the following products. **4.5**

(a) 3.376×4 **(b)** 2.99×35 **(c)** 6.09×33

(d) 23.6×2000 **(e)** 0.548×300 **(f)** 0.069×50

(g) 0.6×0.9 **(h)** 0.009×0.04 **(i)** 5.2×3.8

15 Calculate the following. Round answers to 3 decimal places where necessary. **4.6**

(a) 10.32 ÷ 4 **(b)** 7.028 ÷ 7 **(c)** 3.75 ÷ 8

(d) 480.6 ÷ 2000 **(e)** 8.23 ÷ 500 **(f)** 54.63 ÷ 300

(g) 0.48 ÷ 0.04 **(h)** 1.2 ÷ 0.5 **(i)** 11.9 ÷ 0.002

16 Write the following fractions as percentages. **4.7**

(a) $\frac{7}{10}$ **(b)** $\frac{5}{4}$ **(c)** $\frac{1}{8}$ **(d)** $\frac{2}{3}$

17 Convert the following decimals to percentages. **4.7**

(a) 0.38 **(b)** 0.96 **(c)** 3.55 **(d)** 0.09 **(e)** 3.234 **(f)** 0.5432

18 Calculate the following percentages. **4.8**

(a) 40% of $60 **(b)** 15% of 85 L **(c)** 83% of 200 m **(d)** 12.5% of $600

19 Write the following results as percentages, rounding answers to 1 decimal place if necessary. **4.8**

(a) 13 out of 20 **(b)** 35 out of 70

(c) 14 out of 40 **(d)** 28 out of 30

20 There are 56 grapes and 64 berries in a fruit bowl. **4.9**

(a) Write the ratio berries : grapes in simplest form.

(b) Write the number of grapes as a fraction of the total number of fruits, in simplest form.

(c) Write the number of berries as a percentage of the total number of fruits, to the nearest whole number.

21 **(a)** A 10 kg box of tomatoes costs $23.50. Use the unitary method to calculate the cost of a 3 kg box. **4.10**

(b) If a 250 g block of cheese costs $7.85, find the unit price (the price per 100 g).

Understanding

22 Write the populations of the following countries as whole numbers. **4.1**

(a) China: 1.381 billion **(b)** India: 1.32 billion

(c) Indonesia: 259.9 million **(d)** New Zealand: 4.56 million

23 An employer wants to buy each of her staff a uniform. If each uniform costs $75.59, how much will the uniforms cost: **4.5**

(a) for 40 staff **(b)** for 100 staff?

24 Kayla bought the following items: a $30 T-shirt discounted by 10% and a $60 pair of jeans discounted by 40%. **4.8**

(a) Calculate the dollar value of each of the discounts.

(b) Subtract the discount amount to find how much Kayla paid for each item.

25 **(a)** Which is better value for money: a 250 g jar of honey for $2.98, or a 400 g jar for $4.23? **4.10**

(b) The 250 g jar goes on special for $1.98. Does this change your answer to **(a)**?

(c) Give reasons why people might prefer to buy a certain size or product, even though it doesn't have the cheapest unit price.

26 A salad dressing is made by mixing oil and vinegar in the ratio 3 : 1. **4.9**

(a) How much oil should be mixed with 25 mL of vinegar?

(b) Sam mixes 60 mL of oil with 40 mL of vinegar. Is this an equivalent ratio to the one given above? Demonstrate your answer by writing it in simplest form.

27 In a diving competition, Claudia needed a score of 52 or more to take first position. For her final dive she chose a dive with a degree of difficulty of 2.7. The scores she received from the judges were 7.0, 8.0, 7.5, 8.5, 7.5, 8.0 and 8.0. **4.5, 4.6**

(a) Remove the highest and lowest scores and calculate the average of the remaining five scores (add them up and then divide by 5).

(b) Calculate the 'three-judge total' by multiplying this average by 3.

(c) Calculate her final score by multiplying the 'three-judge total' by the degree of difficulty. Did she do well enough to win?

Reasoning

28 Tim buys 6 individual cans of drink for $0.87 per can. **4.10**

(a) If he pays in cash, find the total cost of the cans.

(b) A pack of 6 cans costs $4.80. How much could Tim have saved by buying the 6 pack?

(c) What is the individual cost of the cans in the 6 pack?

(d) Tim needs to buy 40 cans for a party. What is the cheapest way to buy them?

29 There must be at least one supervisor for every 12 children at a childcare facility. If there are 54 children booked in for a day, how many supervisors will be needed? Give a reason for your answer. **4.9**

30 Sally is shopping for shoes. She has found the pair she wants in two different stores. Store A has them priced at $65, with a 'take 25% off' tag on them. Store B has them priced at $75, but the store is having a '30% off everything' sale. Where should Sally buy her shoes? Explain your choice by showing the relevant calculations.

Numeracy practice 4

Non-calculator

1 Which number is greater than 0.06?

A 0.0069 **B** 0.007 **C** 0.05 **D** 0.1

2 Geraldine buys 400 snack bars for the school camp. If each bar costs $0.64, how much would Geraldine pay for the 400 bars?

A $25.60 **B** $256 **C** $464 **D** $25 600

3 What is the answer to $5.4 \div 0.9$?

A 0.06 **B** 0.6 **C** 6.6 **D** 6

4 20% of a class have the flu. The class has 25 students. How many have the flu?

A 4 **B** 5 **C** 10 **D** 20

Calculator allowed

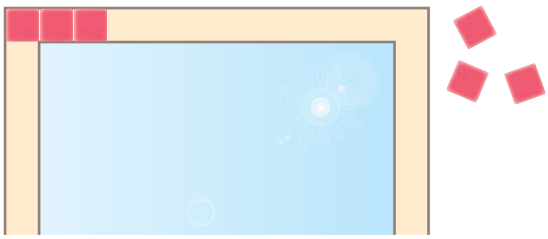
5 A pack of 10 candles costs $4.60.
A pack of 6 costs $3.20.
You need to buy 22 candles.

What is the least amount you can pay?

6 The price of grapes at the supermarket is $14 per kg. If Jack paid $3.50 for his grapes, how many grams did he buy?

A 0.25 g **B** 25 g **C** 100 g **D** 250 g

7 Lucy is adding a row of square tiles across the top of a mirror. Each tile is 4.5 cm wide. The mirror is 56 cm wide. Lucy wants to tile right across the top, with no gaps between the tiles. What calculation should she do to work out how many tiles she needs?



A $4.5 \div 56$ **B** $5.6 \div 4.5$ **C** $56 \div 4.5$ **D** 4.5×56

8 The material for the curtains in the classroom costs $21.30 per metre. 8.4 m is needed. How much will the material cost?

Mixed review B

Fluency

1 Write the following fractions in simplest form. **3.2**

(a) $\frac{12}{40}$ **(b)** $\frac{15}{50}$ **(c)** $\frac{48}{64}$ **(d)** $\frac{20}{65}$

2 Write a positive or a negative integer to describe the following. **2.4**

(a) a withdrawal from a bank account of $74 **(b)** a weight gain of 3 kg

(c) 215 m below sea level **(d)** an increase in length of 8 m

3 For each of the following numbers: **2.3**

(i) use a factor tree to find the prime factors

(ii) write the number as a product of its prime factors, using index notation.

(a) 76 **(b)** 108 **(c)** 250

4 Calculate approximate answers to the following by rounding to a convenient multiple of 5 or 10. **1.4**

(a) 238×21 **(b)** 107×31 **(c)** $478 \div 39$ **(d)** $604 \div 153$

5 For each pair of fractions, determine which one is larger, then write a > or < symbol between them. **3.3**

(a) $\frac{2}{3}$ _____ $\frac{2}{5}$ **(b)** $\frac{5}{12}$ _____ $\frac{5}{9}$ **(c)** $\frac{11}{15}$ _____ $\frac{3}{4}$

6 Find: **4.5, 4.6**

(a) 0.45×0.6 **(b)** $45.9 \div 3$ **(c)** $43.5 \div 0.1$

7 Calculate the following by first writing a single symbol between the integers. **2.7**

(a) $-1 + (-5)$ **(b)** $3 - (-11)$ **(c)** $-6 - (+4)$

(d) $9 + (-17)$ **(e)** $-8 - (-14)$ **(f)** $21 - (+30)$

8 Write each of the following as a fraction in simplest form. **4.3**

(a) 0.32 **(b)** 1.24 **(c)** 3.237

9 Write the following in index form. **1.2**

(a) $7 \times 7 \times 7 \times 7$ **(b)** 4 squared

(c) $2 \times 2 \times 3 \times 3 \times 3$ **(d)** 11 cubed

10 Find the lowest common multiples of the following numbers. **2.1**

(a) 5 and 6 **(b)** 3 and 6 **(c)** 2, 3 and 5

11 Write < or > between the following pairs of numbers. **2.4**

(a) -5 _____ 3 **(b)** -4 _____ -11 **(c)** -7 _____ 0

12 Find the HCF of the following numbers. **2.1**

(a) 12 and 30 **(b)** 16 and 40 **(c)** 24 and 60

13 Calculate the following. Write your answers in simplest form. **3.4**

(a) $\frac{1}{5} + \frac{3}{4}$ **(b)** $1\frac{2}{3} + 2\frac{3}{8}$ **(c)** $\frac{5}{6} - \frac{3}{8}$ **(d)** $3\frac{1}{4} - 2\frac{1}{2}$

Understanding

14 Use an appropriate mental strategy to calculate the following. **1.1**

(a) 29×7 **(b)** 44×11 **(c)** 13×15

(d) $260 \div 4$ **(e)** $2 \times 4 \times 7 \times 5$ **(f)** $440 \div 8$

15 Erin calculates that she needs 9 lots of 0.35 m of ribbon for her textiles project. She wants to round up to the nearest metre so that she makes sure she has enough. **3.5**

(a) Calculate how many metres she should buy.

(b) If the ribbon costs \$0.75 per m, how much will she pay?

16 Write **(i)** a fraction in simplest form and **(ii)** a percentage to represent the following situations. **4.7**

(a) 12 black jelly beans in a bag of 40

(b) 120 Year 7 students in a school of 520 students

17 A 250 g serve of soup contains 7 g of fat. Calculate the amount of fat contained in: **4.10**

(a) a 500 g serve **(b)** a 50 g serve **(c)** a 150 g serve.

18 7 pizzas were each sliced into 6 equal pieces. **3.7**

(a) If $4\frac{5}{6}$ pizzas were eaten, how many slices is that?

(b) How many slices are left over?

19 Camilla is on an athlete's diet where $\frac{2}{3}$ of her daily energy must come from carbohydrates, $\frac{1}{4}$ from protein, and the rest from fats. Her daily energy allowance is 9600 kJ. **3.7**

(a) Calculate how many kJ of carbohydrates she must have every day.

(b) Calculate how many kJ of protein she must have every day.

(c) What fraction of her daily kJ intake is fat?

20 Explain why $3^2 = 9$, but $3 \times 2 = 6$. **2.1**

Reasoning

21 A lighting display in the city has a set of blue lights that flash every 6 minutes, and a set of red lights that flash every 8 minutes. How many times an hour will the lights flash together? **3.2**

22 There is a large block of chocolate sitting in the Williams family pantry. Chloe breaks off and eats $\frac{1}{6}$, and returns the rest. Later, her brother Eamon breaks off and eats $\frac{2}{3}$ of the rest. **3.7**

(a) What fraction of the original block is left?

(b) If the original block contained 54 squares of chocolate, who ate more, Chloe or Eamon?

23 Shania surveyed 130 Year 7 students to find out which charity they wanted to fundraise for. Shania reported back to the Student Council that 65% of the Year 7 students wanted to support Oxfam. **4.7**

(a) What is 65% of 130?

(b) Considering that you are working with numbers of students, what is 'strange' about your answer to **(a)**?

(c) What might have been the actual number of students who said they wanted to support Oxfam? Explain how Shania obtained her figure of 65%.

24 How many whole numbers lie on the number line between $\frac{4}{3}$ and $\frac{38}{4}$? **3.3**

25 Jeff has 46 footy cards in his collection. His little brother Joseph has half as many in his collection, while his friend Shahin's collection is twice the size of Jeff's. How many more cards has Shahin than Joseph? **3.7**


5

Algebra

5

Algebra rules my kitchen! Can algebra help you cook the perfect pancake?

A mathematics lecturer at an English university has published a formula she believes will create the perfect pancake. Dr Ruth Fairclough included every detail that could affect the quality of the pancake in her formula. She says that the most important factor, apart from the batter's recipe, is the frying pan's temperature.

The formula is:

$$P = 100 - \frac{10L - 7F + C(k - C) + T(m - T)}{S - E}$$

where
P = the pancake score
L = number of lumps in the batter
C = batter consistency
F = flipping score
k = ideal batter consistency
T = temperature of the pan
m = ideal temperature of the pan
S = length of time the batter stands before cooking
E = length of time the cooked pancake sits before being eaten.

The closer you get to 100, the more delicious the pancake. Yum!

Forum

Would you use this formula to make pancakes at home?

Would a chef in a restaurant use this formula?

Where would you use a formula around the home or in the garden? Who would use formulas in their work?

Why learn this?

Algebra is a mathematical language. It uses letters and symbols to communicate general rules, describe patterns and solve problems. Algebra can help engineers to calculate stress forces on bridges, architects to design environmentally friendly buildings, nurses to calculate the correct doses of medicine and accountants to calculate how much tax needs to be paid.

After completing this chapter you will be able to:

- write and simplify algebraic expressions using pronumerals
- use a flowchart to describe algebraic rules
- generate tables of values using rules given in words or algebraic formulas
- change rules expressed in words to algebraic formulas using pronumerals
- substitute values into formulas
- use algebra to describe patterns and to solve problems
- use tables of values to plot points on a Cartesian plane
- use rules to plot points and to draw linear graphs
- interpret point and line graphs.

Recall 5

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

1 Find the value of the ☐ in each of the following number sentences. Where the ☐ appears more than once in a number sentence, the same number must be used.

(a) $3 + 7 = \square + 5$ **(b)** $5 - 1 = 2 \times \square$ **(c)** $3 + \square + 4 = \square \times 8$

(d) $5 - 4 + \square = 5$ **(e)** $10 \div \square = 5$ **(f)** $\square \times \square = 16$

2 **(a)** Multiply each of the following by 11 and subtract 3 from your answer.

(i) 4 **(ii)** 10 **(iii)** 200

(b) Multiply each of the following by itself and then take 6 from your answer.

(i) 4 **(ii)** 10 **(iii)** 8

3 Write the following sentences using only numbers, the operations (+, –, ×, ÷) and equals (=).

(a) The sum of eleven and seventeen is twenty-eight.

(b) The difference between nine and seven is two.

(c) The product of three and four is twelve.

(d) The quotient of sixteen and eight is two.

4 Write the following sentences using only numbers, the four operations and brackets. Then, use the order of operations to find the answer.

(a) Take the number six, multiply it by two, and then subtract four from it.

(b) Add five to the number ten, then divide what you get by five.

(c) Multiply the sum of six and five by three.

(d) The difference between twenty-three and twenty-one is multiplied by twelve.

5 Decode the following by writing the letters contained in each grid square.

(a) The fastest living creature
E2 E4 A1 E4 B3 A1 D3 C2 E4
A3 A4 A2 C4 D2 C2

(b) The largest land carnivore
E3 D2 D4 D3 A4 E3 B4 E4 A4 A1

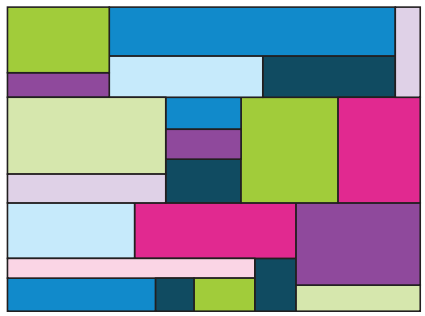
	A	B	C	D	E
4	a	b	c	d	e
3	f	g	h	i	k
2	l	m	n	o	p
1	r	s	t	u	y

Exploration Task

You can download this activity from the eBook or the Pearson Places website.

Areas of rectangles

In this activity, you will investigate how the area of a rectangle can change as its dimensions change.



Pronumerals and variables

5.1

The meaning of 'x' (or 'n' or ...)

Algebra is a language used by mathematicians to communicate mathematical ideas and information clearly. To become a mathematician, you need to learn the language in the same way that you learn any other language. You will begin this study of algebra by learning words and algebraic conventions (rules that everyone agrees to follow) that will help you to read, speak, write and understand mathematics.

Algebra is used to write general rules to describe patterns found with numbers and solve problems using these rules.

Writing algebra

- **Pronumerals** are letters or symbols used for numbers with values that you don't know or that may change. Pronumerals represent (stand for) a number, the same way that pronouns in English 'stand for' a noun.
- An **unknown** is the actual number that the pronumeral represents.
- A **variable** describes the unknown number if its value may change.

Using algebra, you can use x as a pronumeral to represent an unknown number of lollies in a packet. If the number of lollies in the packet can vary, x is called a variable.

If you are given 3 extra lollies, you now have $x + 3$ lollies.

If you buy another identical packet of lollies, you have $x + x$, or $2 \times x$ lollies.

If you have 2 identical packets and 3 extra lollies, you have $2 \times x + 3$ lollies in total.

When you have different variables, you need to use different pronumerals. For example, if you represent the number of students in a class with the pronumeral s, you need to use a different pronumeral, such as t, to represent the number of teachers in the class. The total number of students and teachers in the class is the sum of the two unknowns and can be written as $s + t$. Any two letters can be used to represent the number of students, so the number of people in the class could be written as $x + y$ or $p + r$.

Algebraic conventions

Mathematicians like to write algebra as simply as possible, so they have decided that it is okay to leave out the '×' sign between a number and a pronumeral and between different pronumerals. That means you can write $2 \times n$ as $2n$ and $x \times y$ as xy. It has also been agreed that a '÷' sign can be replaced by a fraction bar, so $n \div 2$ is written as $\frac{n}{2}$. Decisions like this are called algebraic conventions. They ensure that all mathematicians use the same mathematical language. Some conventions that are used in algebra are listed below.

- Leave out the '×' between a number and a pronumeral and between different pronumerals (e.g. $2 \times n$ is written as $2n$ and $x \times y$ as xy).
- Replace ÷ with a fraction bar (e.g. $n \div 2$ is written as $\frac{n}{2}$).
- Numbers are always written in front of the pronumeral (e.g. $x \times 2$ is written as $2x$).
- When multiplying more than one pronumeral, write the letters in alphabetical order (e.g. $f \times a \times c \times e$ is written as $acef$).
- Brackets show that addition or subtraction needs to be done before multiplication or division. They change the order of operations.
- Brackets can be left out when using a fraction bar; e.g. $(a + b) \div 2$ can be written as $\frac{a+b}{2}$.
- When a pronumeral is raised to a power, use the same notation as for numbers (e.g. $2 \times 2 \times 2 \times 2 = 2^4$ so you would write $c \times c \times c \times c = c^4$ and p squared as p^2).
- Numbers are usually written last in an expression (e.g. $6a + 7$).

When working with variables, the same words are used to describe the four operations as when working with numbers:

+	add, sum, total, plus, altogether, more, gain
−	subtract, less than, minus, difference, take away, loss
×	multiply, product, lots of, times, groups of
÷	divide, quotient, share equally, how many, split into

The order of operations, including the use of brackets, is also the same.

Worked example 1 W.E. 1

Write the following situations using algebra.

(a) Simon has x fish in his aquarium. He buys 10 more. How many fish does he have now?

(b) There are r cards in one pack. How many cards are there in three identical packs?

(c) Sonia has n dollars in her bank account. She withdraws $100. How much money is left in the account?

(d) There are d biscuits in a packet. Half of them are eaten. How many are left?

(e) A kitchen cupboard contains x plates and w bowls. What is the total number of plates and bowls in the cupboard?

Thinking	Working
(a) 'More' means add onto the unknown amount (add 10 to x).	(a) Number of fish now = $x + 10$.
(b) Multiplication is used to find 'lots of' the unknown amount (multiply r by 3).	(b) Number of cards in 3 packs = $3r$.

(c) 'Withdraw' means subtract from the unknown amount (subtract 100 from n).	(c) Amount of money left in Sonia's account $= \$(n - 100)$.
(d) 'Half' means divide the unknown amount by 2 (divide d by 2).	(d) Number of biscuits left $= \frac{d}{2}$.
(e) 'Total' means add all the unknown amounts together (add x and w).	(e) Number of plates and bowls $= x + w$.

Worked example 2

W.E. 2

Write each of the following using algebra.

(a) The product of e, f and 7.

(b) c is divided by 4, then 1 is subtracted.

(c) The sum of x and 5 is multiplied by 3.

Thinking	Working
(a) 'Product' means multiply. Write the number first, then the pronumerals in alphabetical order with no multiplication signs between them.	(a) $e \times f \times 7 = 7 \times e \times f$ $= 7ef$
(b) Division is done first, then subtraction. Use a fraction bar to show division.	(b) $c \div 4 - 1 = \frac{c}{4} - 1$
(c) Addition is done first, then multiplication. Brackets are needed to show this order of operations.	(c) $3(x + 5)$

5.1 Pronumerals and variables

Navigator

1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 15, 17, 18	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19	2 (e–l), 3, 5, 6, 7, 8, 10, 11, 13, 14, 15, 16, 17, 18, 19

Answers p. 673

Fluency

W.E. 1

1 Write the following situations using algebra.

(a) There are p books in Jacob's locker. Jacob takes out 3 books he needs to use in his class. How many books are left in his locker?

(b) There are k people in the line for concert tickets. Nine more people join the end of the line. How many people are lined up now?

(c) There are v chocolates in one box. How many chocolates are in 12 identical boxes?

(d) A group of four people won a prize of a dollars, which they shared equally. How much did each person receive?

(e) Kyle has d footy cards in his collection, while Khalid has e cards in his. If they combine their collections, how many cards do they have altogether?

W.E. 2

2 Write each of the following using algebra.

(a) x plus 3

(b) x minus 6

(c) sum of z and p

(d) x take away 4

(e) the difference between a and b

(f) 7 lots of t

(g) g groups of 5

(h) f split into 5

(i) share y by 6 then add 5

(j) The sum of r, s and t is divided by 3, then 11 is added.

(k) y is multiplied by itself, then 20 is added.

(l) x is squared, then multiplied by 8.

3 Write the following situations using algebra.

(a) A farmer had c chickens and bought 5 more chickens. How many chickens does the farmer have now?

(b) If an athlete trained for x hours in the morning and 2 hours in the afternoon every day, how many hours did she train each day?

(c) David put m marbles in his pocket. He had a hole in his pocket and lost 6 marbles. How many marbles does he have now?

(d) Eva's apple tree had a apples before she picked 3 apples and ate them. How many apples are left?

(e) If each sports team has p players, then what is the total number of players needed to make 7 teams?

(f) Each chicken produces e eggs every week. What is the total number of eggs produced by 26 chickens in a week?

(g) There are p coloured pens in a bucket. The pens are to be shared equally by 5 students in a class. How many pens does each student get?

4 Nadine has 7 bags with x lollies in each bag, plus 9 extra lollies. The total number of lollies Nadine has altogether can be written using algebra as:

A $x + 7 + 9$ **B** $7x + 9$ **C** $9x + 7$ **D** $x^7 + 9$

5 The sum of z and 6 is divided by 2. When written using algebra it is:

A $z + 3$ **B** $\frac{z}{2} + 6$ **C** $z + 6 \div 2$ **D** $\frac{z + 6}{2}$

6 How can this number sentence be written using algebra?
'The sum of r, x and y is divided by 4, then has 7 and q added to it.'

A $\frac{r + x + y}{4} + 7 + q$ **B** $r + x + y + 7 + q$ **C** $r + x + \frac{y}{4} + 7 + q$ **D** $r + x + y \div 4 + 7 + q$

Understanding

7 Write each of the following using algebra. Use brackets where necessary.

(a) 1 is added to a number n, and the result is multiplied by 5.

(b) 2 is added to a number d, and the result is divided by 9.

(c) The sum of two different numbers, a and b, is multiplied by 12.

(d) 3 is subtracted from the product of two different numbers, m and n.

8 There are x people standing in line at the post office. Half of them leave, thinking they will come back later.

(a) How many people are in the line now?

(b) Three other people walk in and join the line. How many people are in the line now?

9 **(a)** What is the cost of d metres of timber, if it costs \$6 per metre?

(b) What is the cost of t kilograms of oranges, if they cost \$2.50 per kilogram?

10 What is the cost to hire a car for 3 days, if the rental cost is d dollars per day plus an administration fee of \$50?

11 What is the cost of hiring the community hall for x hours, if the rate is \$55 per hour, plus a one-off charge of \$300?

12 What is the cost of buying b movie tickets, at v dollars per ticket?

13 The cost of 1 desk is t dollars, and the cost of 1 chair is s dollars. 12 new desks and 24 new chairs are bought for a classroom.

(a) What is the total cost of the desks?

(b) What is the total cost of the chairs?

(c) What is the total cost of the furniture?

Reasoning

14 Christina makes candles. For every candle she uses 1 jar (j), 2 wicks (w) and 50 g of candle wax (c). Use algebra to write the requirements to make 1 candle.

15 A purse contains n 5-cent coins and r 10-cent coins. There is no other money in it. Use algebra to write:

(a) the total value in cents of the 5-cent coins in the purse

(b) the total value in cents of the 10-cent coins in the purse

(c) the total value in cents of the money in the purse.

16 Emma makes and sells stationery gift packs. Her small pack has m pens, n pencils, q note pads and 1 eraser.

(a) Her large pack has twice as many of each item. Use algebra to write:

(i) the total number of items in the small pack

(ii) the total number of items in the large pack.

(b) Emma needs to make up 3 small and 2 large packs. Use algebra to write the number of pens that she will need altogether.

17 **(a)** The length of a rectangle is 5 cm longer than its width. If w represents the width of the rectangle:

(i) write the length of the rectangle using w as the variable

(ii) draw a rectangle and label each side using w as the variable.

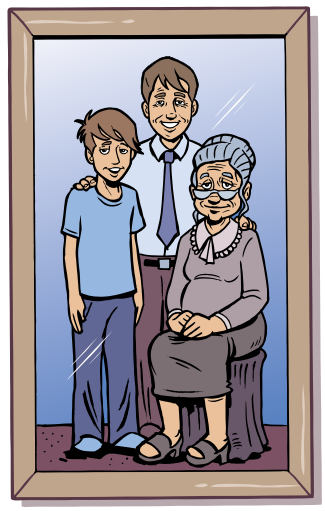
(b) Write, in terms of w, the total distance around the rectangle (the perimeter).

Open-ended

18 **(a)** Draw a diagram that could be represented by $3x + 5$. (Hint: Look at the 'lolly bag' diagrams on page 261.)

(b) What kind of diagram could you draw to show $\frac{x}{4}$?

Draw a diagram, adding any necessary descriptions or labels.



19 Rory's grandma says she is twice as old as the combined ages of Rory and his dad. Rory is r years old and his dad is d years old.

(a) Write Grandma's age using algebra.

(b) Write some possible ages for Rory, his dad and his grandma, if Rory is in primary school.

5.2 Terms, expressions and equations

The language of algebra

Here are some more mathematical words that you need to know when using algebra.

Constants

A number by itself is called a **constant**.

Here are some examples of constants: 2, 9, -37, $\frac{2}{7}$, 4.6

Terms

Terms have one or more pronumerals, or may be just a number. mn means $m \times n$ and is the same as $n \times m$ or nm. The pronumerals are often multiplied by a number that is written first.

Here are some examples of terms: $5a$, $7q$, $3pr$, z, abc, 4, $\frac{3}{11}$, $-2.32x$

Coefficients

The number written in front of a pronumeral is called the **coefficient**. The sign of the number is included. For example, in $9x$, the coefficient of x is 9. In $9x - 3y$, the coefficient of y is -3.

Expressions

Expressions are made by adding terms. The expression $5a + 7q - 12$ has three terms.

Expressions can also consist of just one term, so $5a$ or 4 could each be called an expression.

In section 5.1, we wrote $x + 3$, $2x$ and $2x + 3$. These are called algebraic expressions. Here are some more examples of expressions: $5a + 7q$, $5a + 7q - 12$, $9xy - z$, $2(p - 7)$.

Equations

Equations contain an equals sign. They are made by writing two expressions equal to each other.

Equations have a left-hand side (LHS) and a right-hand side (RHS) on either side of the equals sign.

Here are some examples of equations: $9xy + z = 17$, $y = 6h$, $2(p - 7) = p + 10$

$3xy - 7z^2 + 5 = 16 + x$

- This is an equation because it has an equals sign.
- x, y and z are pronumerals that represent variables.
- 3 is the coefficient of xy and -7 is the coefficient of z^2.
- $3xy - 7z^2 + 5$ and $16 + x$ are expressions.
- $3xy$, $-7z^2$, 5, 16 and x are all terms.
- 5 and 16 are constants.

Representing situations using algebra

When using algebra to describe practical situations, it is very important to **define** all the variables you are going to use so you know what they represent. For example, you may not know the cost per kilogram of apples and bananas, so they are the variables you need to define. You do know that 3 kg of apples and 2 kg of bananas were purchased and $15.20 was spent. If you define the cost of apples per kilogram as x ($) and the cost of bananas per kilogram as y ($), then you can write:

- $3x$ as the cost of apples ($)
- $2y$ as the cost of bananas ($)
- $3x + 2y = 15.2$ as the equation
- $(3x + 2y)$ and 15.2 as the total cost of the fruit ($).

In this situation, there are examples of:

a constant:	15.2	expressions:	$3x + 2y$, $3x$, $2y$
pronumerals:	x, y	an equation:	$3x + 2y = 15.2$
terms:	$3x$, $2y$, 15.2		

Worked example 3

W.E. 3

For $3p + 4q - 8pq = 7 + 2r$

(a) state whether this is an equation or an expression

(b) identify the coefficient of pq

(c) list all the variables

(d) write any constants

(e) list all its terms.

Thinking	Working
(a) Is there an equals sign? If yes, then it is an equation.	(a) It is an equation.
(b) Look for the number in front of pq. Include the sign.	(b) Coefficient of pq is -8.
(c) Look for all the different letters used and list them. (Note that pq is a product of p and q.)	(c) p, q and r are the variables.
(d) Look for a number by itself.	(d) The constant is 7.
(e) Look for all the parts of the equation separated by addition or subtraction.	(e) $3p$, $4q$, $-8pq$, 7 and $2r$ are the terms.

Worked example 4

W.E. 4

For the following situation:

(a) identify the variables

(b) define each variable using the given pronumerals

(c) write an equation.

A farmer has a number of sheep, x, and a number of ducks, y, in a paddock. There are five times as many sheep as ducks.

Thinking	Working
(a) Identify the variables.	(a) The variables are the number of sheep and the number of ducks.
(b) Define the variables.	(b) Let x = the number of sheep. Let y = the number of ducks.
(c) Write the equation. (Multiply y by 5 to get x.)	(c) $x = 5y$

- An algebraic expression may have one term or it may be made up of two or more terms that are added or subtracted.
- An equation uses an equals sign to show that one expression has the same value as another expression.

5.2 Terms, expressions and equations

Navigator

Answers p. 674

1, 2, 3, 4, 5, 6 (a, d), 7 (a–c), 8	1, 2, 3, 4, 5, 6, 7, 8, 9	2, 3, 4 (column 2), 5, 6, 7, 8, 9

Fluency

1 Match the following algebraic expressions with the most appropriate mathematical word.

(a) 6 **(b)** $7x + 2y$ **(c)** $y + 3 = 3x - 8$

A expression **B** constant **C** equation

W.E. 3

2 For each of the following:

(i) state whether it is an equation or an expression

(ii) identify the coefficient of x

(iii) list all the variables

(iv) write any constants

(v) list all the terms used.

(a) $2x + 3y - 7 + 4z$ **(b)** $32 - 4x = 3xy + 4z$

(c) $2xy - 3x^2 = 9 - 23x$ **(d)** $6 - x + 8y - 4xy$

W.E. 4

3 For each of the following situations:

(i) identify the variables

(ii) define each variable using the given pronumerals

(iii) write an equation.

(a) The number of boys, x, in the class is three times the number of girls, y.

(b) Fred is g years old. This is one less than three times James's age. James is h years old.

(c) At the market, an apple costs $\$m$ and a pear costs $\$n$. Two apples cost the same as three pears.

(d) A kilogram of potatoes costs $\$a$ and a watermelon costs $\$b$. Three kilograms of potatoes and a watermelon cost \$5.20.

(e) The distance, r, to Ranko's house is 5 km more than the distance, s, to Skye's house.

(f) You have v plants and w pavers. To make a path around a row of plants, you need to have twice as many pavers as you have plants plus 6 extra pavers.

4 Answer true (T) or false (F) for each of these statements.

(a) $6y$ is a term

(b) $7y - 9$ is an equation

(c) $y = 7x$ is a term

(d) ab is a term

(e) $r = 5t - 9$ is an expression

(f) $cd + 4$ is an expression

(g) $w = s - 5b$ is an equation

(h) $6g$ is an equation

(i) $rs = sr$

(j) $7 - 6xzy = 7 - 6yxz$

5 Which expression below matches the instruction given in each case?

(a) Choose a number and add any other number to it.

A $6 + a$ **B** $u + v$ **C** $c + 9$ **D** $a + b + 1$

(b) Choose a number and multiply it by any other number.

A $a - b$ **B** $7a$ **C** $4ab$ **D** ab

(c) Choose a number and multiply it by three, then subtract any other number.

A $3a - 1$ **B** $3n - y$ **C** $3m - 3$ **D** $3t$

(d) Choose a number and multiply it by two, add any other number, then subtract thirteen.

A $2f + 2d - 13$ **B** $2mn - 13$ **C** $2w + g - 13$ **D** $a + b - 13$

(e) Choose a number and add it to any other number, then multiply the answer by nine.

A $9xy$ **B** $9x + y$ **C** $9(x + y)$ **D** $9 + xy$

(f) Choose a number and multiply it by ten, choose any other number and multiply it by four, then add the two answers together.

A $w + v + 10 + 4$ **B** $14(v + w)$ **C** $10v + 4w$ **D** $40vw$

Understanding

6 (a) A bottle of mass 120 g contains 25 tablets. The total mass of the bottle and tablets is 195 g.

(i) If the variable is the mass of a tablet, use a pronumeral to represent the variable.

(ii) Write an equation you would use to find the mass of a tablet.

(iii) What is the mass of each tablet?
Hint: Determine the mass of the 25 tablets first.

(b) Half of Mei Lee's age is the age she was 8 years ago.

(i) If the variable is Mei Lee's age, use a pronumeral to represent the variable.

(ii) Write an equation you would use to find Mei Lee's age.

(iii) What is Mei Lee's age now?

(c) In 10 years' time, Beth will be half of her dad's age now.

(i) If one variable is Beth's age and the other variable is her dad's age, use two pronumerals to represent these variables.

(ii) Write an equation you would use to find Beth's age now.

(d) In a ball game, a goal is worth 8 points and a penalty goal is worth 1 point. At a match, one team scored 100 points.

(i) Define the variables and represent them with pronumerals.

(ii) Using these pronumerals, write an equation you would use to show the team's score.

Reasoning

7 A family on holiday travelled a certain distance on the first day. The next day they were able to travel twice as far as on the first day. On the third day, they travelled 480 km further than on the first day.

(a) Define the variable.

(b) Write an expression to show the distance travelled on the second day.

(c) Write an expression to show the distance travelled on the third day.

(d) If the distance travelled on the third day was three times the distance they travelled on the first day, write an equation to show this situation.

Open-ended

8 In Australian Rules football, a goal is worth 6 points and a behind is worth 1 point. A team has scored 114 points.

(a) Define any variables you may need to use.

(b) Using these variables, write an equation for the number of points scored.

(c) Find at least three different ways that the 114 points could have been scored.

9 A plain cake mixture (c) includes the following ingredients: flour, water, sugar and eggs. The amount of flour used is twice the amount of sugar. For every 150 g of sugar, you need one egg and 50 mL of water.

(a) What are the variables in the recipe?

(b) What is the formula for the recipe?

(c) If you are given 3 kg of flour, how much sugar, eggs and water do you need?

Problem solving

We all want ice-cream!

Summer had arrived and four industrious people decided to sell ice-creams at their local beach.

In total, there were four flavours and each person had 2 different flavours.

- Charli had vanilla.
- Vanessa did not have mint.
- Milad had strawberry but not chocolate, while Siham did not have vanilla.
- One person who had mint also had chocolate.
- One person who had vanilla also had strawberry.
- One of the people who had chocolate had no mint.
- Neither of the people who had vanilla had chocolate.
- No person had two of the same flavour, no two people had the same two flavours, and nobody had the flavour that starts with the first letter of their name.

Can you tell who had which flavoured ice-cream?

Using rules

5.3

Relationships

One of the main uses of algebra is to describe a **relationship** between two or more variables. A relationship means that the variables are connected in some way so that changing the value of one affects the value of the other.

Consider the following examples. Identify the two variables (values that can change) and decide whether a relationship exists between them. Does changing the value of one cause the other to change?

The number of blocks of chocolate bought, and the total cost of the chocolate.

The number of houses being built, and the number of bricks the builder will require.

The number of students sharing a packet of lollies, and the number of lollies each one receives.

The number of animals in a zoo enclosure, and the amount of space each one has to move around in.

Rules and flowcharts

Relationships are described by a rule or set of instructions that tells you how to calculate one variable if you know the other.

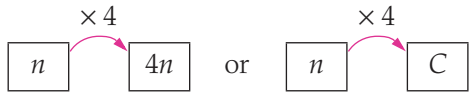
For example, if one block of chocolate costs $4, the rule for calculating the cost of several blocks of chocolate would be:

'Multiply the number of blocks of chocolate you are buying by 4'.

Flowcharts are step-by-step instructions for performing a task. They are used in industry to show the sequence of steps in building a product, such as a car on an assembly production line. Flowcharts are also used to write computer programs.

You can use a flowchart to represent the cost of the chocolate.

There are different ways of writing flowcharts, but in mathematics it is usually written as:



Reading from left to right, this flowchart means: 'Take the number of blocks (n) and multiply them by 4 to get the total cost (C)'.

Rules using algebra

Rules can be written in words, but it is much quicker to use algebra. Written in algebra, a rule for the cost of the chocolate would look like this:

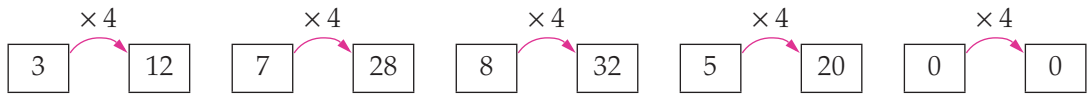
$C = 4n$

where C = total cost of the chocolate
and n = the number of blocks of chocolate bought at $4 a block.

You can use a **table of values** to show the cost of buying different numbers of blocks of chocolate.

Number of blocks, n	3	7	8	5	0
Cost, C	12	28	32	20	0

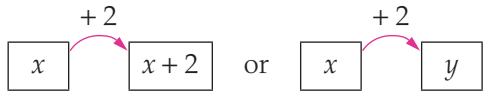
Each of the numbers in the top row of the table has gone through the flowchart (been multiplied by 4) to get the number in the bottom row.



Creating rules

If you choose any two pronumerals, such as x and y, you can create different rules that connect them by constructing different flowcharts.

For example, the rule 'y is equal to x plus 2' would have the flowchart:



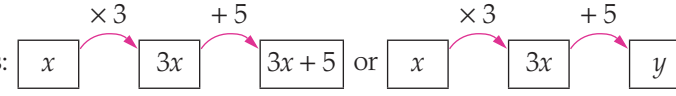
Written in algebra, the rule is: $y = x + 2$.

A 'table of values' for the rule could look like this:

x	4	10	6	9	2
y	6	12	8	11	4

The numbers in the bottom row of the table (y-values) are the result of adding 2 to each of the numbers in the top row of the table (x-values).

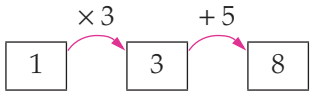
Some rules are two-step rules; for example, the rule 'to get y, multiply x by 3, then add 5' would have a flowchart that looks like this:



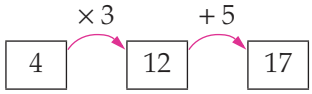
Written in algebra, the rule is: $y = 3x + 5$.

A table of values for this rule could look like this:

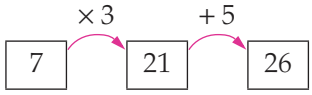
x	1	4	7	2
y	8	17	26	11



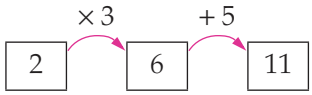
or $3 \times 1 + 5 = 8$



or $3 \times 4 + 5 = 17$



or $3 \times 7 + 5 = 26$



or $3 \times 2 + 5 = 11$

Worked example 5

W.E. 5

For each of the following rules, draw a flowchart, write the rule using algebra, and complete the given table of values.

(a) y is equal to x divided by 4.

x	20	16	4	12	10
y					

(b) 2 is added to the product of x and 5 to get y.

x	2	5	4	3	0
y					

Thinking

(a) 1 Decide how many operations are performed on x, what operation(s) they are and in which order (one operation, ÷ 4). Draw the flowchart.

2 Write y on one side of the rule. On the other side, show the operations being performed on x (that is, $\frac{x}{4}$).

3 For each value of x in the table, apply the rule by passing it through the flowchart (divide it by 4).

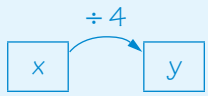
(b) 1 Decide how many operations are performed on x, what operation(s) they are and in which order (× 5, then + 2). Draw the flowchart.

2 Write y on one side of the rule. On the other side, show the operations being performed on x ($5x + 2$).

3 For each value of x in the table, apply the rule by passing it through the flowchart (multiply by 5, then add 2).

Working

(a)



$y = \frac{x}{4}$

x	20	16	4	12	10
y	5	4	1	3	2.5

(b)

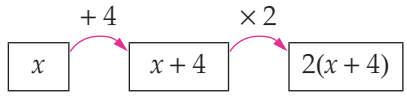
$y = 5x + 2$

x	2	5	4	3	0
y	12	27	22	17	2

Using brackets

Brackets may be needed to show the correct order of operations.

For example, if the rule is 'y is equal to twice the sum of x and 4', then the flowchart would look like this:



The rule written in algebra would be: $y = 2(x + 4)$

Brackets are needed to show that the addition is done before the multiplication.

5.3 Using rules

Navigator

Answers p. 674

1 (a–f), 2, 3, 4, 5 (a–f), 6 (a–d), 7, 8, 9, 11 (a), 12, 14, 15	1, 2 (column 1), 3, 4 (column 1), 5 (a–g), 6, 7, 8, 9, 10, 11, 12, 14, 15	1 (column 2), 2 (column 2), 3 (c), 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14

Fluency

1 Draw a flowchart to represent each of the following rules.

(a) $y = x + 2$

(b) $\frac{x + 3}{2} = y$

(c) $z + 3 = x$

(d) $y = x^2$

(e) $y = x - 5$

(f) $c = \frac{a + 3}{9}$

(g) $y = \frac{x}{2}$

(h) $w = u^2 + 2$

W.E. 5

2 For each of the following rules, draw a flowchart, write the rule using algebra, and complete the given table of values.

(a) y is equal to x plus 2.

x	13	11	7	28	1
y					

(b) To find y, subtract 5 from x.

x	6	18	9	85	5
y					

(c) To find y, double x.

x	2	3.4	10	11	101
y					

(d) y is equal to x divided by 3.

x	18	12	30	9	0
y					

(e) y is equal to 3 subtracted from the result of x multiplied by 5.

x	4	2	0	5	20
y					

(f) To find y, add 5 to x, then divide by 10.

x	5	25	45	10	33
y					

(g) To find y, divide x by 2, then subtract 1.

x	10	6	24	9	15
y					

(h) y is equal to x multiplied by itself.

x	3	11	7	6	10
y					

3 Choose the correct algebraic rule given in each case.

(a) y is equal to the sum of x and twelve.

A $y = x + 12$ **B** $y + 12 = x$ **C** $y = 12x$ **D** $y = \frac{x}{12}$

(b) To get y, subtract 50 from x.

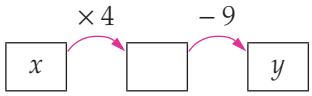
A $y = x - 50$ **B** $y = 50 - x$ **C** $y = x \times 50$ **D** $y = x + 50$

(c) To get y, add thirteen to x, then multiply by nine.

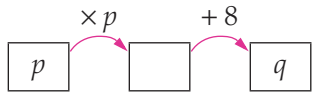
A $y = (x + 9) \times 13$ **B** $y = (x \times 13) + 9$ **C** $y = x + 13 \times 9$ **D** $y = 9(x + 13)$

4 Write the rule in algebra shown by each of these flowcharts. Make sure you use brackets where necessary.

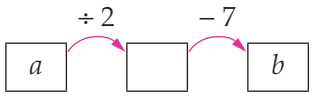
(a)



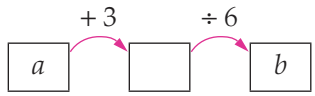
(b)



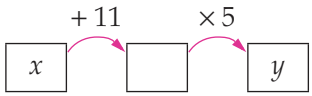
(c)



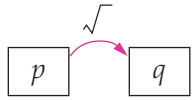
(d)



(e)



(f)



5 Rewrite each of these rules using algebra.

(a) To find y, subtract eighteen from x.

(b) y is equal to sixty multiplied by x.

(c) Divide x by seven to find y.

(d) To find y, add forty-three to x, then multiply by twenty.

(e) Multiply x by one hundred, then subtract fifty to find y.

(f) To find y, divide x by sixteen, then add thirteen.

(g) y is equal to x multiplied by itself.

(h) Subtract twelve from x, then divide by nine to find y.

(i) Multiply x by itself, then take away thirty-seven to find y.

Understanding

6 For the following rules, draw a flowchart and describe the rule using words.

(a) $y = 5x + 11$

(b) $y = \frac{x}{7} - 5$

(c) $b = 7(a + 6)$

(d) $b = \frac{a - 7}{12}$

(e) $d = \frac{e + 9}{4} - 13$

(The flowchart for this rule should have three steps.)

(f) $d = \frac{e^2 - 1}{5}$

(The flowchart for this rule should have three steps.)

7 Joe has some lollies. He gives half to his little brother and then eats four of the rest.

(a) Draw a flowchart to show how many lollies he has left. Use x for the number of lollies he starts with and y for the final number.

(b) Write the rule to describe the situation.

(c) Use the rule to find how many lollies Joe has now if he had 20 to start with.

8 Anita has some money in her wallet. Her mum gives her $20 for doing some chores and she decides to spend half of the money she has now on a birthday present for her sister. She spends another $4 on a birthday card.

(a) Draw a flowchart you would use to find how much money she has spent. Use x for the initial amount in Anita's wallet and y for the amount she has spent.

(b) Write a rule to describe the situation.

(c) Use the rule to find how much money Anita spent if she had $12 in her wallet initially.

9 Uncle Harry is twice your age, plus 5.

(a) Show this information in a flowchart. Use m to represent your age, and H to represent Uncle Harry's age.

(b) Write a rule to show this situation.

(c) If you are 12 years old, how old is Uncle Harry?

10 To produce jeans it costs $245 to set up the machine, and then $7 for each pair of jeans. So, for one pair of jeans to be produced, it costs $\$245 + \$7 \times 1 = \$252$.

(a) Using this information, fill in the table.

Number of pairs of jeans, n	10	50	150	200
Cost to produce the jeans, C				

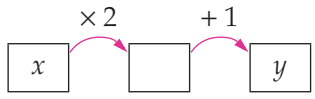
(b) Write a rule for the cost of producing the jeans. Use C for cost, and n for the number of pairs of jeans.

Reasoning

To work backwards along the flowchart, do the *opposite* operation to the one shown in the forwards direction.

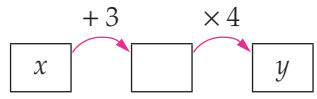
11 The following tables of values have only the y-values filled in. Work *backwards* along the flowchart to determine the values of x that were used, and complete the tables of values.

(a)



x					
y	5	11	15	9	21

(b)



x					
y	52	16	20	28	84

12 For each of the following two rules:

(i) y is equal to the sum of x and 3, which is then multiplied by 2

(ii) to find y, multiply x by 2, then add 6

(a) Draw a flowchart.

(b) Write the rule using algebra.

(c) Copy and complete the table of values below.

x	2	5	3	4	7
y					

(d) What do you notice about these two rules? Can you explain your observation?

Open-ended

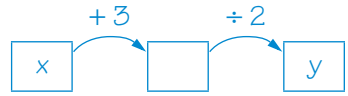
13 Using the rule $y = 3 + \frac{x}{5}$, construct a table of values with at least five different values for x.

14 **(a)** Using two pronumerals (such as x and y), at least two of the following operations ($+, -, \times, \div$) and some arrows, create at least three different flowcharts that show a rule connecting x to y. Use a different combination of operations for each flowchart.

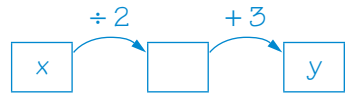
(b) Write the rules shown by your flowcharts using algebra.

15 Kim and Jai are working on the following question: 'Draw a flowchart that represents this rule: to obtain y, divide x by 2, then add 3'.

Kim's flowchart looks like this:



Jai's flowchart looks like this:



Who has the correct flowchart, and why? Are they both right?

Problem solving

Cutting string

Equipment required: 2 lengths of string, scissors

1 Imagine that you have a piece of string. How many pieces will you have if you cut the string once? If you cut one of the pieces again, how many pieces will you now have? Check your answer using some string and scissors, then complete the following table.

Number of cuts	0	1	2	3	4	5	6	c
Total number of pieces								

2 This time, fold a piece of string in half each time before you cut it. How many pieces at each cut do you have now? Remember: Cut only one piece at each time.

Number of cuts	0	1	2	3	4	5	6	c
Total number of pieces								

Strategy options

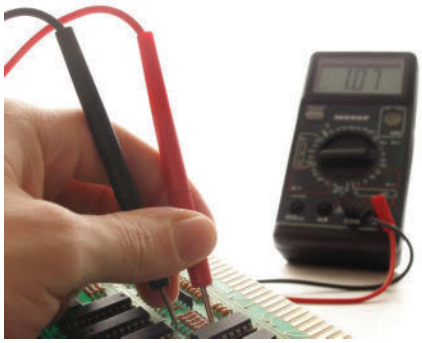
- Act it out.
- Look for a pattern.

5.4 Formulas and substitution

A **formula** is a mathematical rule that uses two or more variables. A formula is used to calculate the value of one variable when the value of the other variables are known. The plural of formula is formulas or formulae.

The rules we have been working with so far can all be called formulas. However, we usually think of a formula as a rule we use in a practical situation. Here are some examples of practical formulas, and what they are used for:

$V = IR$



The voltage (V) of an electric circuit can be found by multiplying the current (I) by the resistance (R).

$F = \frac{9C}{5} + 32$

To convert a temperature from degrees Celsius to degrees Fahrenheit, multiply the temperature in Celsius (C) by 9, divide by 5, and then add 32.

Substitution

Many team sports use substitution. Is it similar to the substitution we're doing here?

In mathematics, you can **substitute** a number for a variable. This allows us to find a value for the formula or expression, to **evaluate** it.

Worked example 6 W.E. 6

For each of the following formulas, evaluate y by substituting the given value of x.

(a) $y = 10x - 5, \quad x = 7$

(b) $y = \frac{x}{2} + 4, \quad x = 5$

Thinking		Working
(a)	**1** Write the formula. State the value you will substitute for x.	(a) $y = 10x - 5, x = 7$
	2 Replace the variable (x) in the formula with its given value (7). Insert the $\times$ sign between the coefficient and the number.	$y = 10 \times 7 - 5$
	3 Calculate the value of the other variable (y).	$y = 70 - 5$ $= 65$

(b)	**1**	Write the formula. State the value you will substitute for x.	(b) $y = \frac{x}{2} + 4, x = 5$
	2	Replace the variable (x) in the formula with its given value (5).	$y = \frac{5}{2} + 4$
	3	Calculate the value of the other variable (y).	$= 2\frac{1}{2} + 4$ $= 6\frac{1}{2}$ or 6.5

When substituting numbers into a formula, write any 'hidden' multiplication signs, then evaluate.

e.g. Substituting $a = 5$ into $b = 4a + 3$:

$b = 4a + 3$
$= 4 \times 5 + 3$
$= 23$

Worked example 7

W.E. 7

(a) The cost of hiring a community hall is calculated using the formula $C = 65t + 200$, where C = cost (in dollars) and t = the hire period (in hours). Find the cost of hiring the hall for 4 hours.

(b) The area of a rectangle (in square metres) is given by the formula $A = lw$ where l = length (in metres) and w = width (in metres). Calculate the area of a rectangular room that is 4.5 m long and 3 m wide.

Thinking			**Working**
(a)	**1**	Write the formula and the value of the variable to substitute.	(a) $C = 65t + 200, t = 4$
	2	Replace the variable in the formula with its given value (replace t with 4, the number of hours). Insert a multiplication sign between the coefficients and the numbers (65 and 4).	$C = 65 \times 4 + 200$
	3	Evaluate.	$= 260 + 200$ $= 460$
	4	State the answer.	The cost of hiring the hall for 4 hours is $460.
(b)	**1**	Write the formula and the values of the variables to substitute.	(b) $A = lw, l = 4.5, w = 3$
	2	Replace the variables in the formula with their given values. (Replace l with 4.5 and w with 3.) Insert a multiplication sign between the numbers (4.5 and 3).	$A = 4.5 \times 3$
	3	Evaluate.	$= 13.5$
	4	State the answer.	The area is 13.5 square metres.

5.4 Formulas and substitution

Navigator

Answers p. 676

1 (column 1), 2, 3, 4 (column 1), 5, 7, 8, 9 (a–b), 10, 11, 12	1 (column 1), 2, 3, 4 (column 1), 5, 6 (a–c), 7, 8, 9, 10, 11, 12	1 (column 2), 2, 3 (c–e), 4 (column 2), 6, 7, 9, 10, 11

Fluency

W.E. 6

1 For each of the following formulas, evaluate y by substituting the given value of x.

(a) $y = x + 3$, $x = 4$

(b) $y = x + 5$, $x = \frac{1}{2}$

(c) $y = 2x - 7$, $x = 3$

(d) $y = 3x - 1$, $x = 8$

(e) $y = x - 1$, $x = 3.6$

(f) $y = x - 5$, $x = 30$

(g) $y = 25 - 3x$, $x = 7$

(h) $y = 30 - 2x$, $x = 1$

(i) $y = 4x$, $x = \frac{1}{2}$

(j) $y = 8x$, $x = 4$

(k) $y = 3(x + 5)$, $x = 1$

(l) $y = 6(x + 2)$, $x = 18$

(m) $y = \frac{x}{5}$, $x = 20$

(n) $y = \frac{x}{8}$, $x = 2$

(o) $y = 11(x - 3)$, $x = 7$

(p) $y = 12(10 - x)$, $x = 3$

W.E. 7

2 (a) The cost of hiring power tools from the local hardware store is calculated using the formula $C = 35t + 60$, where C = cost (in dollars), and t = the hire period (in hours). Find the cost of hiring a sander for 6 hours.

(b) A bakery that makes birthday cakes has a daily set up cost of $45. The profit ($P$) for each cake ($c$) sold is $20. If the daily profit from birthday cakes is calculated using the formula $P = 20c - 45$, what is the profit or loss of selling 2 cakes in one day?

(c) Body mass index (B) is one way of measuring a person's health, and is calculated by the formula $B = \frac{m}{h^2}$, where m is the person's mass in kilograms, and h is their height in metres. Calculate the body mass index of a runner who is 1.7 metres tall and weighs 64 kg. (Round your answer to 2 decimal places.)

3 Answer true (T) or false (F) for each of the following statements.

(a) If you substitute $a = 4$ into $b = 5a$ you get $b = 20$.

(b) If you substitute $a = 9$ into $b = a + 11$ you get $b = 20$.

(c) If you substitute $u = 4$ into $v = 6u + 1$ you get $v = 65$.

(d) If you substitute $q = 10$ into $k = 13q - 8$ you get $k = 122$.

(e) If you substitute $x = 6$ into $y = 4(x - 5)$ you get $y = 19$.

(f) If you substitute $x = 12$ into $y = 5(14 - x)$ you get $y = 10$.

4 Use substitution to complete these tables of values for the given rules.

(a) $b = 4a$

a	11	20	5	9	50
b					

(b) $y = 7x$

x	6	4	10	20	101
y					

(c) $n = 3m + 2$

m	1	2	10	6	5
n					

(d) $k = 4j + 7$

j	2	5	11	10	100
k					

(e) $q = 2p - 10$

p	11	15	10	20	100
q					

(f) $s = 8r - 2$

r	1	2	3	0	200
s					

(g) $v = 4(u - 1)$

u	5	3	1	201	6
v					

(h) $n = 3(m - 2)$

m	5	10	11	102	52
n					

5 (a) For the rule $m = 3x - 5$, when $x = 3$, m would be equal to:

A 1 **B** 2 **C** 3 **D** 4

(b) For the rule $y = \frac{x}{3} + 4$, when $x = 6$, y would be equal to:

A $\frac{10}{3}$ **B** $4\frac{1}{2}$ **C** 6 **D** 22

(c) For the rule $l = 3(n + 2) - 1$, when $n = 1$, l would be equal to:

A 5 **B** 6 **C** 8 **D** 9

Understanding

6 Gemma helps her mother at their Italian restaurant. Every day she arrives early and makes 3 sample meatballs to make sure the size is right. Once they are approved, Gemma is able to make 7 meatballs each hour.

(a) Write a formula for the number of meatballs (m) that Gemma makes in a day of h hours working.

(b) Gemma's mother can make 11 meatballs an hour and does not need to make any sample meatballs first. Write a formula for the number of meatballs (b) Gemma's mother can make in a day of h hours working.

(c) Use the two formulas to work out how many meatballs Gemma and her mother can make in an 8-hour day (include set-up).

(d) Each meatball is sold for $2 and the daily set-up cost is $154. Write a formula for the profit (P) made from selling M meatballs?

(e) What is the daily profit based on the number of meatballs found in part **(c)**?

7 A watch salesperson earns $350 per week plus $10 for every watch sold.

(a) Write this as a formula with n representing the number of watches sold, and p representing the amount of money earned each week.

(b) If 15 watches are sold in the first week, use your formula to find how much the salesperson earns this week.

(c) If 20 watches are sold the next week, use your formula to find how much money the salesperson earns that week.

Reasoning

8 I buy three loaves of bread and a $2.60 carton of milk. It costs me $\$D$.

(a) If l represents the cost of a loaf of bread, write a formula to find the cost of my shopping (D) in terms of l.

(b) Use the formula to evaluate D if $l = \$3.25$.

9 Jasmine is a caterer. She uses formulas to work out how much food she needs to feed different numbers of people.

(a) To calculate the number of sausages she needs at a barbecue, Jasmine uses the following formula: 'Allow 2 sausages for every person and have an extra 10 sausages'. Write Jasmine's formula using algebra using n to represent the number of people and s to represent the number of sausages.

(b) Another one of Jasmine's formulas for barbecues is: 'One bowl of salad will feed 8 people'. Write this formula using algebra. Use n to represent the number of people, and b for the number of bowls of salad.

(c) Use the formulas you have written to determine the number of sausages and bowls of salad Jasmine will need to cater for 40 people.

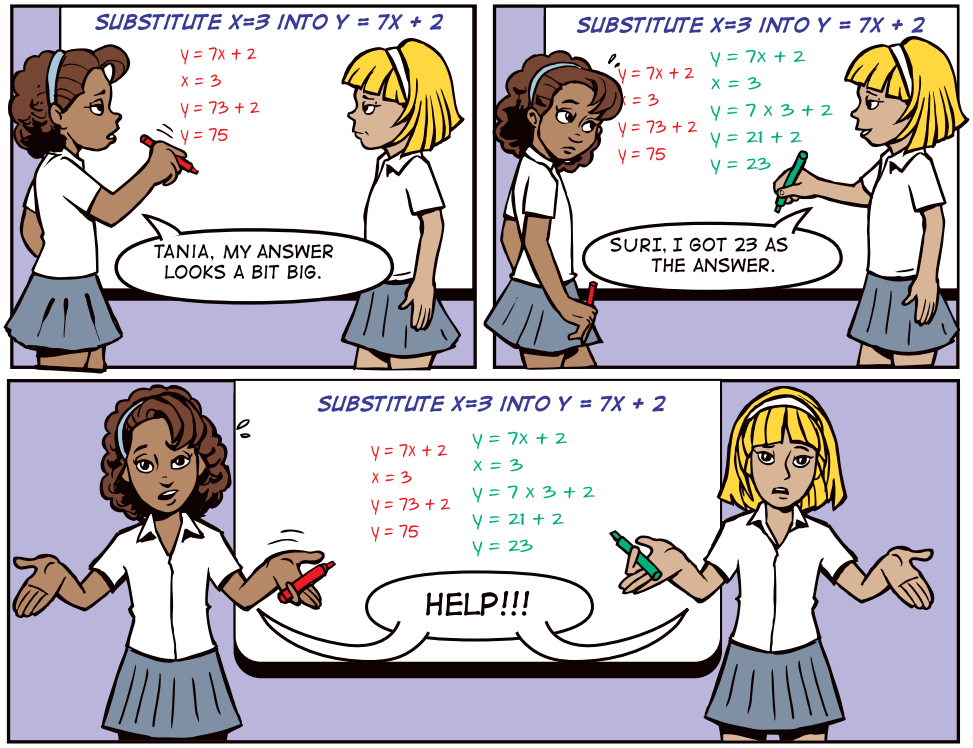
10 Show that the formulas $b = 3(a + 5)$ and $b = 3a + 5$ are different by substituting $a = 4$ into each of them. Explain how the two are different in terms of the order of operations.

Open-ended

11 Write two different formulas connecting x and y that use addition and multiplication. Use them to complete the following table for each one.

x	7	20	13	101
y				

12



Which of the two students has substituted correctly? What mistake has the other student made? Give them some advice so they can avoid a similar mistake in the future.

Game

Dicey formulas

Equipment required: 1 die

How to win:
Be the first to reach a total of 60 in each game. Play the best of 3 games to find the winner.

How to play:

1 Each player is given one of the formulas that follow.

Game 1:
Player 1, $A = 3n + 2$;
Player 2, $B = 3n - 2$

Game 2:
Player 1, $A = 3n + 4$;
Player 2, $B = 3(n + 2)$

Game 3:
Player 1, $A = 6 - n$;
Player 2, $B = 7 - n$

2 Take turns to roll the die. After each roll, substitute the value shown on the die into your formula and write the result.

3 Keep a running total by adding all your results together. Your opponent may check your answer and if it is wrong you lose a turn.

4 Stop rolling the die when you think that you can get no closer to 60.

5 The other player may keep going until they decide they can get no closer to 60.

6 The winner is the player who gets to exactly 60, or the nearest to 60 (but less than, not more than). If you score more than 60 then you lose, so you must decide when to stop.

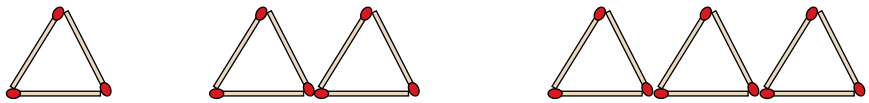
As a challenge, play this game where you have to get exactly 60. Or use 2 dice and be the first to reach a total of 150.

5.5 Patterns and rules

A strategy that is often useful in problem solving is to look for a pattern in the results. Algebra can help describe a pattern, which can often be written as a general rule, or formula. You can then use the formula to solve problems without having to draw out endless patterns.

Worked example 8 W.E. 8

(a) Here is a matchstick pattern of triangles. Copy and complete this table of values by continuing the pattern.



Number of triangles (t)	1	2	3	4	5
Number of matches (m)					

(b) Find a general rule that connects the number of triangles in the pattern (t) to the number of matches used (m). Write the rule in words and in algebra.

(c) Use your rule to find the number of matches required to make 100 triangles.

Thinking

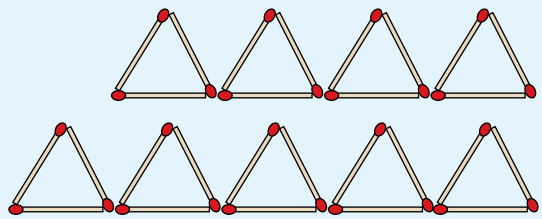
(a) 1 Construct the table of values. Fill in any information you know from the pattern provided.

Working

(a)

Number of triangles (t)	1	2	3	4	5
Number of matches (m)	3	6	9		

2 Continue the pattern by adding matches to create more triangles.



3 Count the number of matches used for the continued pattern (formed with four and five triangles) and fill in the table.

Number of triangles (t)	1	2	3	4	5
Number of matches (m)	3	6	9	12	15

(b) 1 Identify the number of matches that are being added on each time to make a new shape (3). (This number is also being added on to each number in the second row of the table.) This is the multiplication factor in the rule (× 3).

(b) $m = 3 \times t$

2	Write the rule that links the two variables (t and m) in the table, in words and in algebra.	The number of matches is 3 times the number of triangles. $m = 3t$
(c) 1	Substitute $t = 100$ into the rule, and evaluate.	(c) $t = 100$ $m = 3 \times 100$ $m = 300$
2	Write the answer in words.	300 matches are needed to make 100 triangles.

In the above example, you can see that one 'lot' of 3 matches is added every time to make a new triangle and the next number in the sequence. Therefore, you are multiplying the number of triangles by 3 to find the number of matches.

When trying to find a pattern, look for the number that is being added or subtracted every time. This will tell you what to multiply by in your rule.

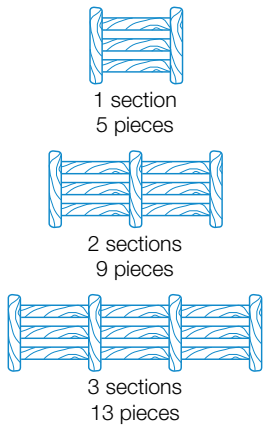
In some cases, the above procedure may not give you the complete rule. You must then look for the number that is required to complete the first pattern. You then need to add on this number to make the rule work; e.g. $m = 3t + 1$; $m = 3t + 2$; $m = 3t + 3$ etc. Worked example 9 shows this in detail.

Always try to understand how the rule works by looking at the pattern that has been formed.

Worked example 9

W.E. 9

Frederico is building a fence around Farside Farm. The fences are made up of pieces of timber as shown.



(a) Copy and complete this table of values by continuing the pattern.

Number of sections of fence (F)	1	2	3	4	5
Number of pieces of timber (P)	5	9			

(b) Find a general rule that connects the number of sections of fence (F) to the number of pieces of timber (P) needed. Write the rule in words and in algebra.

(c) Use your rule to find the number of pieces of timber required for a fence made up of 50 sections.

Thinking	Working
(a) 1 Construct the table of values. Fill in any values you know from the pattern provided. Continue the pattern to complete the table.	(a)

Number of sections of fence (F)	1	2	3	4	5
Number of pieces of timber (P)	5	9	13	17	21

(b)	**1**	Identify the number of pieces of the pattern that are needed to complete the new section (4). (This number is also being added on to each number in the second row of the table.) This number is the multiplication factor in the rule (× 4).	(b) 4 pieces are being added to make a new section. Therefore, 4*F* is in our rule.
	2	When you multiply *F* by 4, you do not have the numbers in the second row, so you now look for a number to add or subtract. How many pieces of the pattern were needed to start the first section? This number is added on to the rule. (1 piece of timber.)	$4F + 1$
	3	Write the rule that links the variables, both in words and in algebra.	The number of pieces is 4 times the number of sections plus 1 extra piece. $P = 4F + 1$
(c)	**1**	Substitute the value of *F* into the formula ($F = 50$).	(c) $P = 4F + 1$ $= 4 \times 50 + 1$
	2	Evaluate.	$P = 200 + 1$ $= 201$
	3	Write the answer in words.	To make a fence of 50 sections, 201 pieces of timber are needed.

5.5 Patterns and rules

Navigator

Answers p. 676

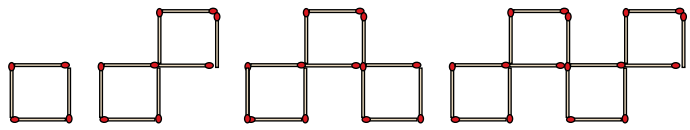
1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12	1, 2, 3, 4, 5, 6, 7, 8, 9 (a–d), 10, 11, 12	1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13

Equipment required: centimetre grid paper for Question **12**

Fluency

W.E. 8

1 **(a)** Here is a matchstick pattern of squares. Copy and complete the table of values by continuing the pattern.



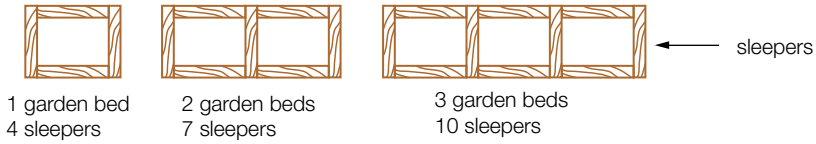
Number of squares (*s*)	1	2	3	4	5	6	7	8
Number of matches (*m*)								

(b) Find a general rule that connects the number of squares in the pattern (*s*) to the number of matches used (*m*). Write the rule in words and in algebra.

(c) Use your rule to find the number of matches required to make 100 squares.

W.E. 9

2 Larissa, a landscape gardener, uses sleepers to divide up gardens into separate beds as shown.



1 garden bed
4 sleepers

2 garden beds
7 sleepers

3 garden beds
10 sleepers

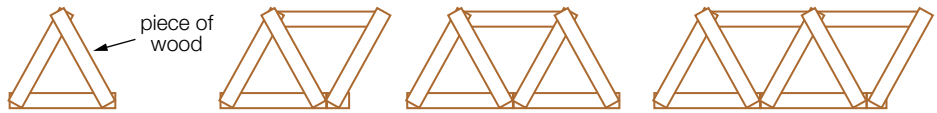
(a) Copy and complete this table of values by continuing the pattern.

Number of garden beds (B)	1	2	3	4	5
Number of sleepers (S)	4	7	10		

(b) Find a general rule that connects the number of garden beds (B) to the number of sleepers (S) needed. Write the rule in words and in algebra.

(c) Use your rule to find the number of sleepers required to divide the garden into a section made up of 21 beds.

3 Clarence the carpenter has been working on a new restaurant. The owners want a triangular woodwork design, like the one shown below, running across the walls.



1 triangle
3 pieces of wood

2 triangles
5 pieces of wood

3 triangles
7 pieces of wood

4 triangles
? pieces of wood

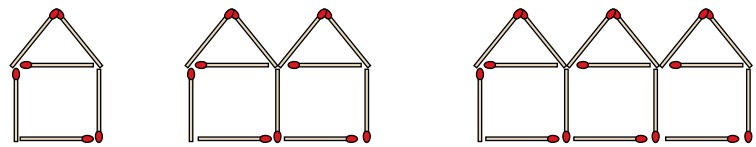
(a) Copy and complete this table of values by continuing the pattern.

Number of triangles (T)	1	2	3	4	5
Number of pieces of wood (P)	3	5	7		

(b) Find a general rule that connects the number of triangles to the number of pieces of wood needed.

(c) Use your rule to find how many pieces of wood are needed to make a total of 203 triangles.

4 Here is a matchstick pattern of houses.



1 house
6 matches

2 houses
11 matches

3 houses
? matches

(a) Copy and complete the table below.

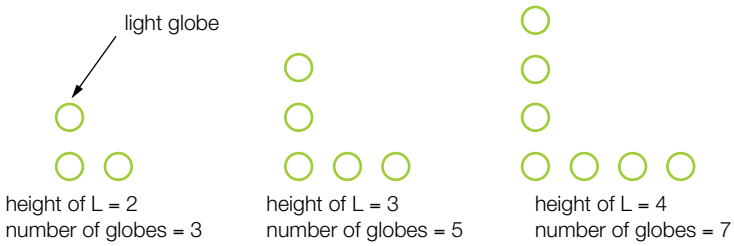
Number of houses (h)	1	2	3	4	5	6	7	8
Number of matches (m)	6	11						

(b) Find the general rule that connects the number of matches to the number of houses.

(c) Use your rule to find the number of matches needed to build 20 houses.

Understanding

5 Lightworks International Co., designers and manufacturers of large illuminated advertising signs, want to put a giant L made up of individual globes onto their largest building. They have already made some small Ls on some of their other buildings.



(a) Construct and complete a table of values for this pattern. Use the three patterns given here to begin your table, then continue the pattern for the next two Ls in the pattern.

(b) Find the rule that connects the height of the L to the number of globes required. Let H = height of the L and G = number of globes.

(c) Use your rule to find how many globes you would need to make an 'L' with a height of 120.

6 A rule is given as $L = 3n + 1$. Which statement is not true?

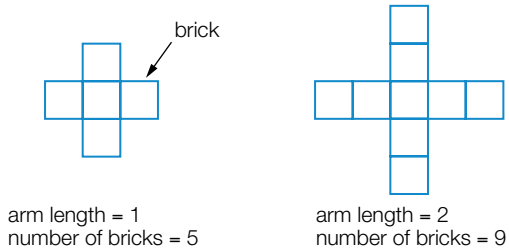
A When $n = 3$, $L = 10$.

B When $n = 4$, $L = 12$.

C When $n = 5$, $L = 16$.

D When $n = 6$, $L = 19$.

7 A charity organisation has a cross as its emblem. Their buildings all have a cross built into the brickwork using white bricks. Some smaller versions of the cross are shown.



(a) Draw the next two crosses in the pattern.

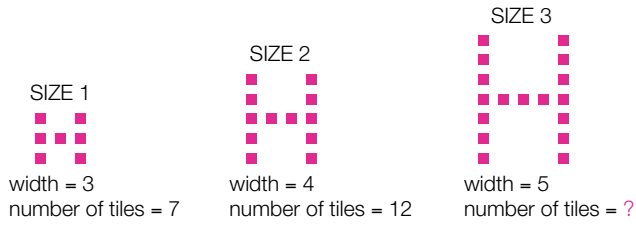
(b) Construct and complete a table of values connecting the arm length (a) and the number of white bricks (b) for these four different-sized crosses.

(c) Extend your table for arm lengths of 5 and 6.

(d) Find the rule that connects the two pronumerals.

(e) Find how many white bricks would be needed to make a cross with an arm length of 52.

8 Mr Harrison has decided that he wants to use tiles to form large letter Hs, which are built into the brickwork of the walls of his company offices. He must work out how many tiles are needed to make different-sized Hs.



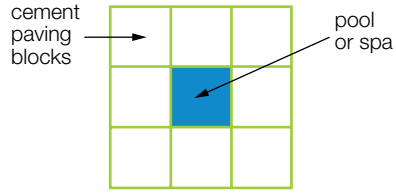
(a) Draw the next two Hs in the pattern.

(b) Construct and complete a table of values connecting the width of the H (w), and the total number of tiles (t).

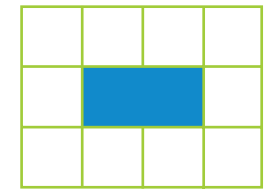
(c) Find the rule for these five different sizes that connects the width of the H (w) to the total number of tiles (t).

(d) Find the number of tiles needed to make an H that is 12 tiles wide.

9 A pool paving company specialises in large square cement paving blocks to surround swimming pools and outdoor spas, like the ones shown below.



length of spa = 1
number of paving blocks = 8



length of spa = 2
number of paving blocks = 10

(a) Draw the pattern's next two spas with paving blocks.

(b) Construct and complete a table of values up to 4 spa lengths, connecting the spa length (l) with the number of pavers used (p) for these four different spa sizes.

(c) Find a rule without brackets connecting the two pronumerals.

(d) Explain your rule by using diagrams of spas of different lengths. Which paving blocks does the constant represent? Where does the coefficient of the pronumeral representing spa length come from?

(e) Find a rule using brackets that also works.

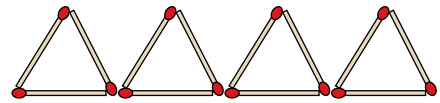
(f) The pool paving company has recently been asked by a swimming club to build a single-lane lap pool to help train its long-distance swimmers. It has been worked out that the lap pool is 345 paving blocks in length. Find out how many paving blocks will be needed to pave around the whole pool.

(g) For another job, the company uses 40 paving blocks to surround an outdoor spa. What is the length of this spa?

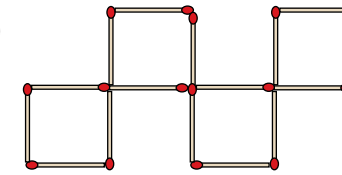
Reasoning

10 In each of the following matchstick patterns, the diagrams show the pattern after the fourth set of matchsticks has been added.

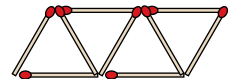
(i)



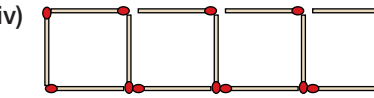
(ii)



(iii)



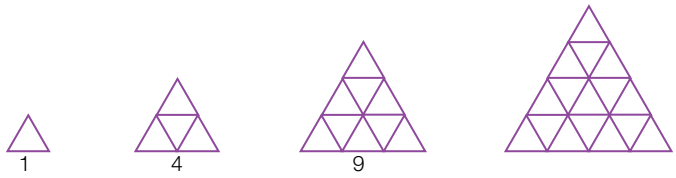
(iv)



(a) Write a general rule for each of the patterns if one more set of matchsticks is added to the pattern each time. Use s = number of shapes and m = number of matchsticks as pronumerals.

(b) Patterns **(i)** and **(iii)** both add a new triangle every time. However, the rules are different. Patterns **(ii)** and **(iv)** both add a new square every time. However, the rules are also different. How are patterns **(i)** and **(ii)** different from the patterns in **(iii)** and **(iv)**? How is this difference shown in the rule for each pattern?

11 **(a)** Count *all* the small triangles in the fourth large triangle.



(b) How many small triangles do you think there will be in the fifth large triangle?

(c) Copy and complete the following table. Try to find the pattern to help you.

Large triangle	1	2	3	4	5	6
Number of triangles	1	4	9			

Open-ended

12 Choose one of the following letters: P, T, A, X or Y. Create a design for the letter from small centimetre squares, and use it to create a pattern of letters that gradually increases in size. Try to find a rule that describes your pattern.

13 Sara has decided to make a closed matchstick shape with 6 matchsticks. She then adds matchsticks to make a pattern that has a rule $m = 5s + 1$ where m is the number of matchsticks and s is the number of repeated shapes.

(a) Construct a shape that Sara may have started with.

(b) Sara changes her shape and adds matchsticks to make a new pattern. She finds that the rule is now $m = 4s + 2$. Draw a second shape that she may have used.

Puzzle

Puzzling tables

The table contains multiple patterns, across each row and down each column.

Copy the table and write the correct number in each blank space.

Describe the pattern for each row and for each column.

Patterns like this often exist in tables of information.

This table shows a delivery company's costs to deliver a package to different places overseas, according to the weight of the package. Different parts of the table have different patterns. A pattern may only work in part of a row or column. Describe the patterns that you can find.

	Column 1	Column 2	Column 3	Column 4
Row 1	4	6	8	10
Row 2	7	10.5	14	
Row 3		15	20	25
Row 4	13		26	32.5
Row 5	16	24		40

Weight	To New Zealand	To Asia/Pacific	To USA/Canada/ Middle East	To Rest of World
Up to 250 g	$7.20	$8.40	$9.55	$11.35
Over 250 g up to 500 g	$11.00	$13.40	$15.70	$19.30
Over 500 g up to 750 g	$14.80	$18.40	$21.85	$27.25
Over 750 g up to 1000 g	$18.60	$23.40	$28.00	$35.20
Over 1000 g up to 1250 g	$22.40	$28.40	$34.15	$43.15
Over 1250 g up to 1500 g	$26.20	$33.40	$40.30	$51.10
Over 1500 g up to 1750 g	$30.00	$38.40	$46.45	$59.05
Over 1750 g up to 2000 g	$33.80	$43.40	$52.60	$67.00
Extra 500 g or part thereof	$4.00	$5.20	$7.45	$9.95

Half-time 5

1 Which rule describes the given information? **5.3**

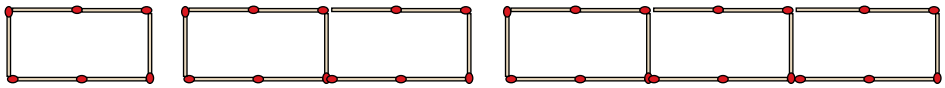
'Subtract 45 from x to get y.'

A $y = x + 45$ **B** $y = x - 45$ **C** $y = x \times 45$ **D** $y = x \div 45$

2 For each of the following rules, evaluate y by substituting the given value of x. **5.4**

(a) $y = x - 4$, $x = 6$ **(b)** $y = 2(x - 7)$, $x = -13$ **(c)** $y = 54 - 3x$, $x = -9$

3 If s represents the number of shapes formed and M is the number of matchsticks used, then which rule is correct for this pattern? **5.5**



A $M = 5s + 1$ **B** $M = 6s$ **C** $M = 5s - 1$ **D** $M = 4s + 2$

4 Write an equation to describe each of the following. **5.2**

(a) Tim has p footy cards. He collects 7 more. He now has 28 cards.

(b) 20 is obtained when 4 is subtracted from the product of x and 7.

(c) A bird aviary has w parrots and g canaries. When 4 of the canaries are sold, there are 14 birds left in the aviary. How many birds are in the aviary now?

5 If y is equal to 4 plus the quotient of x divided by 2: **5.3**

(a) draw a flowchart for this rule

(b) write the rule using algebra

(c) copy and complete this table of values for the rule.

x	2	8	10	5	13
y					

6 Nicola has 6 identical bags of lollies, plus an extra 11 loose lollies. If n represents the number of lollies in one bag, write an expression for the total number of lollies Nicola has. **5.1**

7 Given the rule $y = 4x - 3$, which statement is not true? **5.4**

A When $x = 6$, $y = 21$. **B** When $x = 5$, $y = 17$.

C When $x = 4$, $y = 13$. **D** When $x = 3$, $y = 0$.

8 The cost of an international phone call with a certain phone company is 35 cents for every minute, plus a 'flagfall' charge of 50 cents per call. **5.4**

(a) Write a rule to calculate the total cost of an international phone call. Use the pronumeral C for the cost and m for the length of the call, in minutes.

(b) Use your rule to calculate the cost of a 6-minute phone call. Write your answer in dollars.

(c) Ravi wants to call his parents in Singapore. He has $8. Will this be enough money for a 20-minute call?

Investigation

Richie's Restaurant

Richie has just bought 30 new rectangular tables for his restaurant. One table can seat six people. He needs to arrange them according to two conditions.

- The tables must be set up so that he can seat at least one group of 6, 8, 10, 12, 14, 16 and 18 people all at the same time.
- He wants to seat the maximum number of people possible.

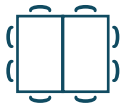
The Big Question

How should Richie arrange his tables to fulfil both of the conditions?

Engage

To seat more than six people, there are two ways in which Richie can join the tables together:

Lengthways or widthways



1 How many people can be seated around two tables if they are joined together:

(a) lengthways **(b)** widthways?

2 For each of the two ways of joining the tables, copy and complete the following table of values. Draw the tables and count the number of people around them if necessary.

Number of joined tables (t)	1	2	3	4	5	6	7
Number of people seated (p)							

Explore

3 If Richie had no restrictions on the way he could set up his tables, what is the maximum number of people he could seat?

4 If Richie now sets up his tables to fulfil the given conditions, draw up a seating plan of Richie's restaurant, showing your arrangement of the 30 tables. How many people can be seated using this arrangement?

Strategy options

- Draw a diagram.
- Guess and check.
- Make a table.
- Look for a pattern.
- Test all possible combinations.

Explain

5 Consider the two ways of joining the tables in order to seat 10 people. Which type of arrangement (lengthways or widthways) is more efficient, using fewer tables? Is this type of arrangement always more efficient?

6 Why does the way in which the tables are joined together (lengthways or widthways) affect the number of people that can be seated around them?

7 For each of the two arrangements, write the rule that connects the number of tables (t) to the number of people that can be seated (p).

Elaborate

8 Consider the coefficient and the constant that appears in each of your rules. Explain where they come from.

9 How do the two rules show that one type of table arrangement is more efficient than the other?

10 Given what you now know about the efficiency of different table arrangements, modify your plan from **4**, if necessary, to seat a greater number of people, while still fulfilling Richie's condition.

11 Draw your improved seating arrangement to answer the Big Question.

Evaluate

12 Consider the way in which you worked on this problem. How did you approach it? Could you have gone about it a different way?

13 Did you use your tables of values (from Question **2**) or the algebra rules to help you solve the problem?

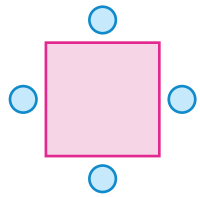
14 How confident are you that your final solution is the best solution?

15 Make a list of all the other factors you would need to consider if you were setting up tables in your own restaurant.

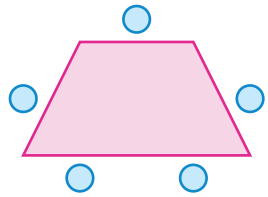
Extend

16 For his next restaurant, Richie would like to use some different-shaped tables. He could use:

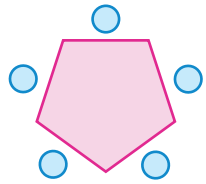
(a) square tables that sit 4 people



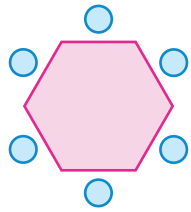
(b) trapezoidal tables that seat 5 people



(c) pentagonal tables that seat 5 people or



(d) hexagonal tables that seat 6 people.



Write a report to explain to Richie why none of these different table shapes would be suitable if he wants to seat the maximum number of people while fulfilling the given conditions and using no more than 30 tables.

5.6 Simplifying expressions with addition and subtraction

Like terms and non-like terms

Like terms:

- have the same pronumerals part
- can have different coefficients; $2x$ and $5x$ are like terms
- can have pronumerals in a different order; ab and ba are like terms
- can have pronumerals raised to exactly the same power; $7x^3$ and $4x^3$ are like terms.

Non-like terms:

- have different pronumeral parts; $2x$ and $2y$ are non-like terms
- can have the same pronumerals raised to different powers; x^2 and x are non-like terms.

Because pronumerals represent numbers, you can use number laws in algebra to change the order of pronumerals.

$2 \times 3 = 3 \times 2$, so $ab = ba$ (commutative law)
$(3 \times 2) \times 5 = 3 \times (2 \times 5)$, so $(ab)c = a(bc)$ (associative law)
$abc = acb = bac = bca = cab = cba$

When terms have different pronumeral parts, you usually write the pronumerals in alphabetical order. This helps to identify like terms.

For example, $6abc$, $3bca$, $4dab$, $8cad$, $11adb$, $7cda$ can be written as $6abc$, $3abc$, $4abd$, $8acd$, $11abd$ and $7acd$. You can now see that $6abc$ and $3bca$ are like terms, $4dab$ and $11adb$ are like terms and $8cad$ and $7cda$ are also like terms.

Worked example 10 W.E. 10

Which of the following are pairs of like terms?

(a) $4x$ and $7x$ **(b)** $5ab$ and $9b$ **(c)** x and $12x$

(d) 8 and $8z$ **(e)** h^3 and $6h^3$ **(f)** xy and x^2y

Thinking	Working
(a) Do the terms have exactly the same pronumeral part?	(a) $4x$ and $7x$ are like terms.
(b) Do the terms have exactly the same pronumeral part?	(b) $5ab$ and $9b$ are not like terms.

(c)	Do the terms have exactly the same pronumeral part?	(c) x and $12x$ are like terms.
(d)	Do the terms have exactly the same pronumeral part?	(d) 8 and $8z$ are not like terms.
(e)	Do the terms have exactly the same pronumeral part?	(e) h^3 and $6h^3$ are like terms.
(f)	Do the terms have exactly the same pronumeral part?	(f) xy and x^2y are not like terms.

Adding and subtracting like terms

Maddie bought 6 identical packets of party balloons for her birthday party. Each packet contained x balloons, so she bought $6x$ balloons. At home, Maddie found that she had 4 more identical packets, so she added $4x$ balloons to the $6x$ balloons she has just bought. Now, she had 10 packets, all containing x balloons, so she had a total of $10x$ balloons.

$6x$ and $4x$ are like terms. When you add $6x$ and $4x$ you can simplify them to $10x$ by adding their coefficients.

$6x + 4x = 10x$

Maddie didn't need all the balloons, so she took 2 packets back to the store. She returned $2x$ balloons. She now has 8 packets of balloons, so she has $8x$ balloons for her party.

$10x$ and $2x$ are like terms. When you subtract $2x$ from $10x$ you can simplify them to $8x$ by subtracting their coefficients.

$10x - 2x = 8x$

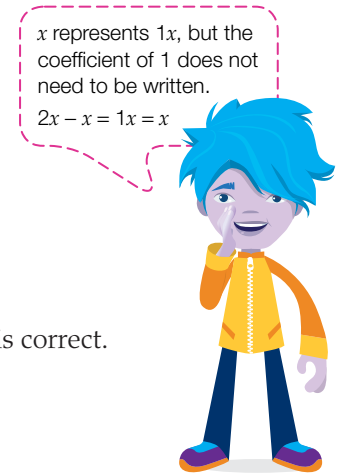
You can simplify an expression by adding or subtracting like terms. This is called 'collecting like terms'. You add or subtract like terms by adding or subtracting their coefficients.

If Maddie buys 3 identical packets of balloons that contain x balloons and 2 different packets that each contain y balloons, you can show that she now has $3x + 2y$ balloons. $3x + 2y$ cannot be simplified any further, as $3x$ and $2y$ are non-like terms.

Examples:

$6c + 9c = 15c$
$12y + 3y - 4y = 11y$
$3x + 4y + 5x = 8x + 4y$
$7ab + 2ba = 9ab$
$3ab + 4bc$ cannot be simplified, because ab and bc are not like terms.
$2x + 2x^2$ cannot be simplified, because x and x^2 are not like terms.

$8x - 3x = 5x$
$16abc - 13abc = 3abc$
$6x^2 + 4x^2 + 10x^2 = 20x^2$
$2x + 3x^2 - x = x + 3x^2$



Checking by substitution

You can replace a pronumeral with any number to check that your simplification is correct.

For example: $6x + 4x = 10x$
$10x - 2x = 8x$

These are true for all values of x.

If $x = 20$:
$6 \times 20 + 4 \times 20 = 10 \times 20$
$10 \times 20 - 2 \times 20 = 8 \times 20$

If $x = 7$:
$6 \times 7 + 4 \times 7 = 10 \times 7$
$10 \times 7 - 2 \times 7 = 8 \times 7$

This is an application of the distributive law.

Worked example 11

W.E. 11

Simplify each expression by collecting like terms.

(a) $4y + 7y$ **(b)** $2mn - 4mn$ **(c)** $8a + 4 - 3a$

Thinking		Working
(a) **1**	Identify like terms by looking for terms that have exactly the same pronumeral part.	(a) $4y + 7y$
2	Simplify like terms by adding the coefficients of the terms.	$= 11y$
(b) **1**	Identify like terms by looking for terms that have exactly the same pronumeral part.	(b) $2mn - 4mn$
2	Simplify like terms by subtracting the coefficients of the terms.	$= -2mn$
(c) **1**	Identify the like terms.	(c) $8a + 4 - 3a$
2	Rearrange the expression to collect the like terms together. When a term is moved, include the sign ($-3a$).	$= 8a - 3a + 4$
3	Simplify like terms by subtracting the coefficients of the terms.	$= 5a + 4$

Worked example 12

W.E. 12

Simplify each expression where possible by collecting like terms.

(a) $4x + 3y - 9x + 2y$ **(b)** $6ab - 4a + 3ab - 7$

Thinking		Working
(a) **1**	Rearrange the expression to collect the like terms together. When a term is moved, include the sign ($-9x$).	(a) $4x + 3y - 9x + 2y$ $= 4x - 9x + 3y + 2y$
2	Simplify the like terms by adding or subtracting the coefficients of the terms.	$= -5x + 5y$
(b) **1**	Rearrange the expression to collect the like terms together. When a term is moved, include the sign ($+3ab - 4a$).	(b) $6ab - 4a + 3ab - 7$ $= 6ab + 3ab - 4a - 7$
2	Simplify the like terms by adding the coefficients of the terms.	$= 9ab - 4a - 7$

- Only like terms can be added together or subtracted.
- The + or – belongs to the term that follows it.

5.6 Simplifying expressions with addition and subtraction

Navigator

1 (columns 1–2), 2 (columns 1–2), 3 (columns 1–2), 4, 5, 6, 7 (a–b), 9, 10 (a–b)	1 (column 2), 2 (column 2), 3 (columns 2–3), 4, 5, 6, 7 (a–b), 8 (a–c), 9, 10 (a–b), 11	1 (column 3), 2 (column 3), 3 (column 3), 5, 6, 7, 8, 9, 10, 11

Answers p. 678

Fluency

1 Are the following pairs of terms like terms? W.E. 10

(a) $3k$ and $5k$ **(b)** $4y$ and $5z$ **(c)** $11y$ and $12v$

(d) 17 and 8 **(e)** $6ab$ and $7bc$ **(f)** $2xy$ and $14yx$

(g) $4xyz$ and yzx **(h)** mnp and $3npm$ **(i)** 3, 5 and x

(j) $3c$ and $7c$ **(k)** $2x$ and $2x^2$ **(l)** $3h^3$ and $5h^2$

2 Simplify each expression by collecting like terms. W.E. 11

(a) $3x + 4x$ **(b)** $12y + 4y$ **(c)** $11a + 2a$

(d) $13y - 9y$ **(e)** $7m - 3m$ **(f)** $-10x - x$

(g) $3mn + 2nm$ **(h)** $8xy - 3xy$ **(i)** $-2z + 3z + 4$

(j) $x + 4y + 6x$ **(k)** $10w + 3z + 4w$ **(l)** $17a + 2b - 5a$

3 Simplify each expression where possible by collecting like terms. W.E. 12

(a) $3e + 4f + 7e - 6f$ **(b)** $12p + 8q - 4p - 12q$ **(c)** $8t + 15s - 16t - 3s$

(d) $6x - 4y + 3x + y^2$ **(e)** $12w - 2z + 3w - 3z^2$ **(f)** $10m + 9n - n - 5m^2$

(g) $3a + 17 + 4a - 8$ **(h)** $14x - 4 - 6x + 9$ **(i)** $21y - 36 - y - 17$

4 **(a)** A like term for $7x$ is:

A $12x$ **B** $3 + x$ **C** 7 **D** xy

(b) A like term for $6wxy$ is:

A $6w$ **B** $6x$ **C** $6y$ **D** $12xyw$

Understanding

5 Simplify each expression where possible.

(a) $5e + 14f + 8e + 6$ **(b)** $18x + 6y - 4y + 2$ **(c)** $4 + 6x + 5y - 3$

(d) $2x - 3y + 4$ **(e)** $9m - 2n + 3mn$ **(f)** $6a + 11b - 5c - 7$

6 For each of the following pairs of rules:

(i) write each rule in algebra

(ii) using a table of values, substitute the numbers 2, 3, 4 into each rule

(iii) determine whether the two rules give the same result or not.

(a) Rule 1: 'Take any number and multiply it by four. Take the same number and multiply it by six. Then add the two answers together.'

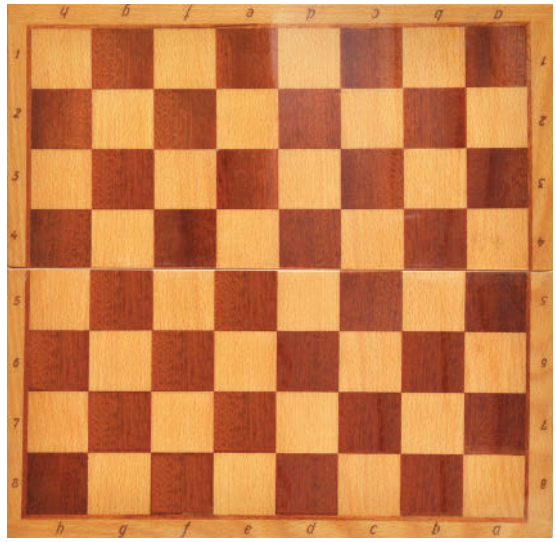
Rule 2: 'Take a number and multiply it by ten.'

(b) Rule 1: 'Take any number and multiply it by fifteen. Take the same number and multiply it by ten. Subtract the second answer from the first.'

Rule 2: 'Take a number and multiply it by five.'

Reasoning

7 A number of coins are put on a square of a chessboard. Twice as many coins are then put on the next square. On the third square, there are twice as many coins as there are on the second square. This is continued for six squares, doubling the number of coins each time.



(a) If x is the number of coins on the first square, how many are on the:

(i) second square

(ii) third square

(iii) sixth square?

(b) How many coins, in total, are there on the first six squares of the board?

(c) If each coin's value is $\$v$, what is the total value of the coins on the board?

(d) Find a number for x and a number for v that will make the total value of the coins \$378.

8 On each of her birthdays, Georgia is given as a present three times as much money as she was given for her previous birthday. On her first birthday she received d dollars.

(a) Write an expression for the amount of money Georgia received on her fourth birthday.

(b) How much money, in total, did Georgia receive for her first four birthdays? Write your answer in terms of d.

(c) How much money did Georgia receive altogether for her first four birthdays if $d = 8$?

(d) If Georgia receives \$567 on her fifth birthday, how much did she receive on her first birthday?

9 Kristian opens a bank account by depositing $\$y$. He then deposits $\$x$ every week and makes no withdrawals or other deposits.

The amount in his account after 5 weeks is:

A $x + 5y$ **B** $y + x + 5$ **C** $y + 5x$ **D** $5y + 5x$

Open-ended

10 This algebra pattern consists of the terms $x + y$, $2x + 3y$, $3x + 5y$, $4x + 7y$, … where the coefficient of x increases by 1 each time and the coefficient of y increases by 2 each time.

(a) Write an expression for the sixth term.

(b) Write an expression for the sum of the first five terms.

(c) Find two sets of values for x and y so that the sum of the first five terms is a multiple of 10.

(d) Find the sum for your chosen values.

11 A garden bed is made using rectangular timber planks. The planks are arranged to form square and rectangular garden beds as shown in the diagram. Let N be the total number of planks used to make the garden bed, L be the number of planks for the length of the bed and W be the number of planks for the width of the bed.

(a) Write a rule that will find the number of planks needed for square garden beds of any side length.

(b) Write a rule that will find the number of planks needed for a rectangular garden bed of length L and width W.

(c) Give examples of each type of garden bed using no more than 30 planks.

(d) If the length must always be double the width, rewrite the rule for the rectangular garden bed using only W as the pronumeral.

(e) Using this new rule, give examples of different types of rectangular garden beds using no more than 30 planks.

Puzzle

Latin squares

a	d	b	c
c	b	d	a
d	a	c	b
b	c	a	d

The sum of each row, column and diagonal is represented by $a + b + c + d$.

If you divide the table into quarters, what is the sum of the four squares in each quarter?

Can you find other 2×2 groups of squares in the table whose sum is $a + b + c + d$?

Draw the two diagonals on the table. What can you say about the remaining rows and columns; that is, the rows and columns that are not on the diagonals?

Draw some 4×4 tables.

Pick four different values for a, b, c and d and make up some Latin squares, using the Latin square above-left to help you. Make sure that you check that your answers are actually Latin squares (i.e. that the values that replace each letter are the same throughout).

Leave out some of the values and ask your partner to find the missing numbers to complete your Latin square.

Your complete Latin square might look like:

9	8	14	3
3	14	8	9
8	9	3	14
14	3	9	8

You might then change it to this one before asking your partner to complete the Latin square.

9	8		
3		8	
8		3	
14	3		8

Exploration Spreadsheet

Age matters

Equipment required: Microsoft® Excel or similar spreadsheet software. (For Casio ClassPad CAS or TI-Nspire CAS, you can download instructions from the eBook or the Pearson Places website.)

Poh, Quentin and Rasheed are three friends who share the same birthday. They are now twelve and a half years old (150 months) and they have decided, after learning some algebra, that they may be able to negotiate a better pocket money agreement with their parents. The agreement they reach with their parents will run for 24 months.

Poh thinks that she will do quite well if she can get her parents to agree to pay her using the following formula:

- $P = 2m - 285$ where m is her age in months and P is her monthly payment in dollars.

Quentin thinks he has a better plan. His formula is:

- $Q = 3m - 440$ where m is his age in months and Q is his monthly payment in dollars.

Rasheed disagrees with the other two. He believes he has the winning formula:

- $R = 4m - 595$ where m is his age in months and R is his monthly payment in dollars.

1 To see who has the best formula, open a new spreadsheet. Set up four columns as shown below, using P ($) to represent Poh's pocket money, Q ($) to represent Quentin's pocket money and R ($) to represent Rasheed's pocket money, with their age in months (m) in the first column.

2 In cell **A2**, enter 150 as the starting value for the age in months. To increase this value down the column, enter the formula **=A2+1** in cell **A3** and **Fill Down** to copy this formula down the column.

Then, enter the formulas for pocket money in the other columns: **=2*A2-285** in cell **B2**, **=3*A2-440** in cell **C2**, and **=4*A2-595** in cell **D2** as shown below. Use **Fill Down** to copy these formulas down their columns.

3 Find the total amount of pocket money Poh receives in the first 12 months by adding the values. To do this, you can enter the formula **=SUM(B2:B13)** as shown.

fx =A2+1

	A	B	C	D
1	m (age in months)	P ($)	Q ($)	R ($)
2	150	=2*A2-285	=3*A2-440	=4*A2-595
3	151			
4	152			
5	153			
6	154			
7	155			
8	156			
9	157			
10	158			
11	159			
12	160			
13	161			
14		=SUM(B2:B13)		

If you have entered the first row of formulas correctly, then your spreadsheet values should look like this.

	A	B	C	D
1	m (age in months)	P ($)	Q ($)	R ($)
2	150	15	10	5
3	151			
4	152			
5	153			
6	154			

Use your spreadsheet to answer the following questions.

4 How much would Poh, Quentin and Rasheed each get paid in the first month, if their parents agree to use these formulas?

5 Is there any time when they would all get the same amount for the same month? If so, when does this happen and how much do they receive that month?

6 How much would each of them get on their 13th birthday ($m = 156$)?

7 What is the total amount each of them would receive in the first 12 months of their agreement?

8 Who has received the most pocket money after 12 months?

9 Extend your formulas for another 12 months. Who has received the most pocket money over 24 months? Is your answer the same as in **8**?

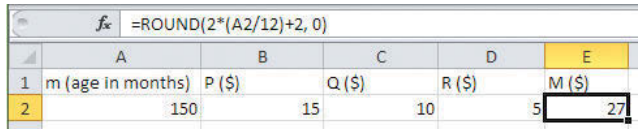
10 Which of the friends has the best formula? Give reasons for your choice. What do you think each of them would say to convince their parents to adopt their formula?

Taking it further

11 Poh's mum suggests a different formula: $M = 2y + 2$, where y is the age in years and M is the pocket money in dollars. She says she will round this to the nearest dollar.

(a) Add this formula to your spreadsheet, using the **ROUND(** function to round to the nearest dollar (rounding to 0 decimal places): **=ROUND(2*(A2/12)+2, 0)**. Can you see how this spreadsheet formula calculates the algebra formula above?

Determine the total amount she would pay in the first 12 months.



(b) Compare the amount of pocket money Poh would earn in the first 12 months using this formula with the other three formulas.

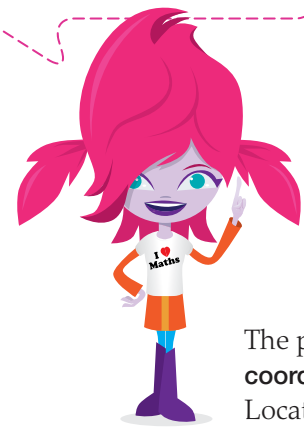
(c) Poh's older sister advises her to be careful before she decides which formula to use. Which formula would you advise her to use over a 2-year period? Justify your recommendation.

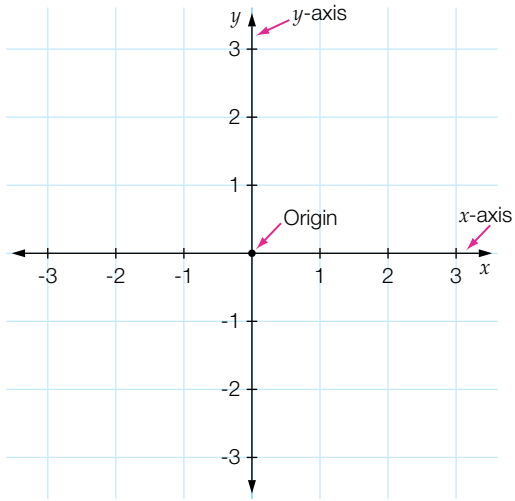
5.7 The Cartesian plane

In 1637, the French mathematician René Descartes developed a reference system for accurately locating any point on a plane. (A plane is any flat two-dimensional surface.) This is called the **Cartesian plane** after Descartes, although it is also known as the **number plane**. This was an exciting idea, as it allows algebra to be visualised and can also be used to solve geometrical problems.

A Cartesian plane is constructed by drawing two lines at right angles to each other, one horizontal and the other vertical. The point where they cross is called the **origin**. The horizontal line is called the x-axis and the vertical line is called the y-axis. Both **axes** (plural of axis) are number lines that extend infinitely in both directions. The integers (whole number values) on the axes form a grid that allows any point to be located in reference to the origin.

The Cartesian plane is more precise than alphanumeric grid systems used by street directories and spreadsheets, because the Cartesian plane locates points, while analphanumeric system finds an area within a square or cell.





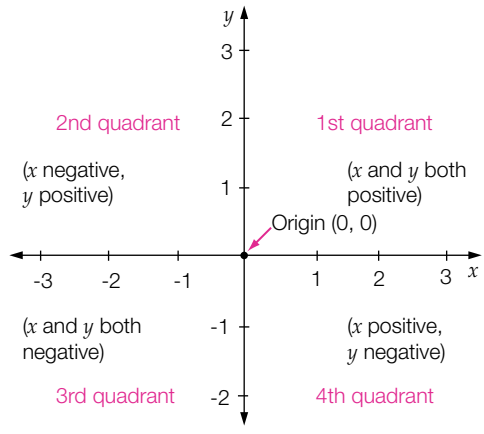
The position of any point on a number plane is described by a pair of numbers called the **coordinates** of the point. Coordinates are always written in brackets as an **ordered pair** (x, y). Locating any point on the plane involves two moves from the origin.

(x, y)

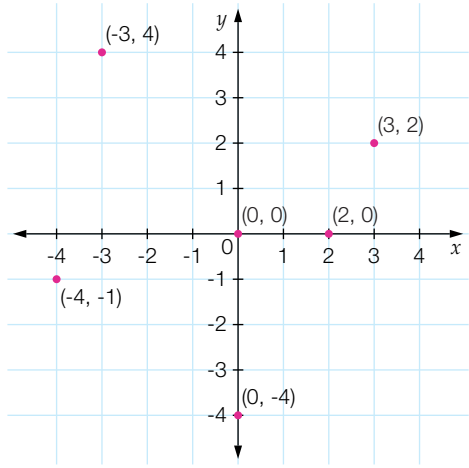
The x-coordinate shows the move to the left (–) or to the right (+) of the origin.

The y-coordinate shows the move up (+) or down (–) from the origin.

The x- and y-axes divide any plane into four **quadrants**. Number the quadrants starting with the quadrant in which both the x- and the y-coordinates have positive values. This is the 1st quadrant. Move in an anticlockwise direction to the 2nd, 3rd and 4th quadrants.



The following points and their Cartesian coordinates are shown on the Cartesian plane at right: (3, 2), (-3, 4), (-4, -1), (0, 0), (2, 0) and (0, -4).



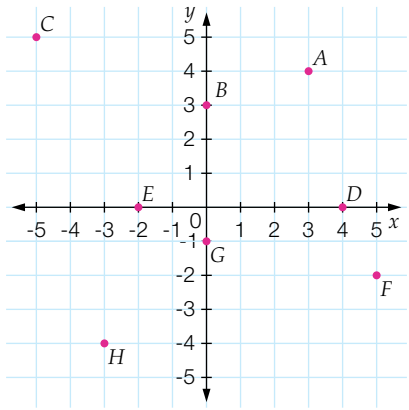
- Always move in the horizontal direction first and then in the vertical direction. This is a mathematical convention.
- The name 'ordered pair' tells you that order is important. The x-coordinate is always written first, and the y-coordinate is written second (x, y).
- The coordinates of the origin are (0, 0).

Worked example 13

W.E. 13

(a) Write the coordinates of each of the points A to H shown on the Cartesian plane.

(b) State the quadrant in which each point is located.



Thinking	Working
(a) For each point, find the number of units it is to the left or right of the origin. This is the x-coordinate of the point. Then, find the number of units it is up or down from the origin. This is the y-coordinate. The + and - signs indicate direction. (Point A is 3 units to the right of the origin and 4 units up from the origin.) Write the coordinates as an ordered pair.	(a) $A = (3, 4)$ $B = (0, 3)$ $C = (-5, 5)$ $D = (4, 0)$ $E = (-2, 0)$ $F = (5, -2)$ $G = (0, -1)$ $H = (-3, -4)$
(b) Identify the quadrant that each point is in. Points that lie on the x-axis or on the y-axis are not in a quadrant.	(b) A is in the 1st quadrant, C is in the 2nd quadrant, H is in the 3rd quadrant and F is in the 4th quadrant. B, D, E and G lie on an axis, so they are not in any quadrant.

5.7 The Cartesian plane

Navigator

Answers p. 678

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 15	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15	2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15

Equipment required: graph paper for Questions **9**, **10**, **15**

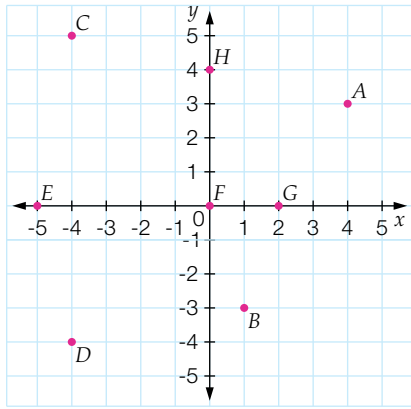
Fluency

1 Draw a Cartesian plane with a scale from -4 to 4 on both axes, then label the x-axis, y-axis and each quadrant.

W.E. 13

2 **(a)** Write the coordinates of each of the points A to H shown on the Cartesian plane.

(b) State the quadrant in which each point is located.



3 **(a)** State whether each of the following coordinates is in quadrant 1, quadrant 2, quadrant 3 or quadrant 4.

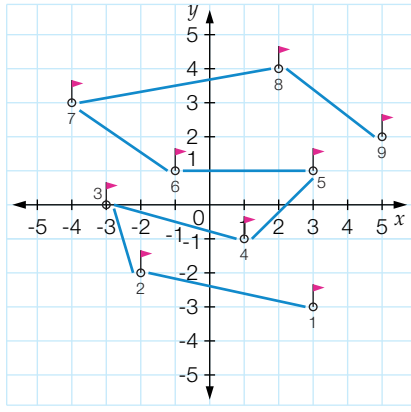
(i) (-2, 6) **(ii)** (5, -1) **(iii)** (-7, -4) **(iv)** (12, 10)

(v) (-32, 12) **(vi)** (51, -1) **(vii)** (-87, -90) **(viii)** (-2, 21)

(b) State whether each coordinate lies on the x-axis, the y-axis, or on both the x-axis and the y-axis.

(i) (0, 5) **(ii)** (0, -8) **(iii)** (0, 0) **(iv)** (-4, 0) **(v)** (15, 0)

4 The layout of a nine-hole golf course is shown on the number plane. Write in order the ordered pairs of the nine holes.



5 **(a)** A point is 1 unit right and 4 units up from the origin of a Cartesian plane. The coordinates of the point are:

A (4, 1) **B** (-4, 1)
C (1, 4) **D** (-1, 4)

(b) A point is 5 units left and 2 units up from the origin of a number plane. The coordinates of the point are:

A (-5, 2) **B** (2, -5) **C** (5, -2) **D** (-2, 5)

(c) A point is 4 units down and 3 units left of the origin of a Cartesian plane. The coordinates of the point are:

A (-4, -3) **B** (-3, -4) **C** (3, -4) **D** (-3, 4)

6 **(a)** The origin has the coordinates:

A (1, 0) **B** (0, 1) **C** (1, 1) **D** (0, 0)

(b) One coordinate that lies on the x-axis is:

A (0, 4) **B** (1, 1) **C** (2, 4) **D** (1, 0)

(c) One coordinate that lies on the y-axis is:

A (3, 0) **B** (3, 3) **C** (0, -3) **D** (-3, 0)

7 **(a)** Which of the following coordinates is in quadrant 3?

A (-3, -2) **B** (3, -2) **C** (0, 5) **D** (-3, 2)

(b) Which of the following sets of coordinates are all on the x-axis?

A (-3, -2), (-3, 5), (-3, 6) **B** (0, -2), (0, 3), (0, 0) **C** (5, 0), (-3, 0), (0, 0)

(c) Which of the following sets of coordinates are all on the y-axis?

A (7, 2), (6, 2), (3, 2) **B** (0, -2), (0, 3), (0, 0) **C** (5, 0), (-3, 0), (0, 0)

Understanding

8 **(a)** Which of the following points will give a vertical line passing through the x-axis when joined in a straight line to (10, -6)?

A (-6, -2) **B** (4, -6) **C** (10, 0) **D** (0, 10)

(b) Which of the following points will give a horizontal line passing through the y-axis when joined in a straight line to (8, 13)?

A (4, 2) **B** (8, -3) **C** (-5, 0) **D** (0, 13)

9 Use a ruler to draw a set of axes to form a Cartesian plane. Accurately draw a scale from -9 to 9 on the x-axis and -4 to 4 along the y-axis. Plot the following points and join them in the order given to form a picture.

(-4, -1) (-5, -1) (-5, 0) (-9, 1) (-6, 3) (-1, 2) (-4, -1) (-4, -2) (-1, -4) (0, -3) (-1, 2) (0, 3) (1, 2) (6, 3) (9, 1) (5, 0) (5, -1) (4, -1) (1, 2) (0, -3) (1, -4) (4, -2) (4, -1)

Now plot the points with coordinates (3, 4) joined to (0, 2.5) and (-3, 4) joined to (0, 2.5).

10 Use a ruler to draw a set of axes to form a Cartesian plane. Accurately draw a scale from -8 to 9 along the x-axis and -11 to 11 along the y-axis. Join each of the following sets of points in the order given. When you reach the word STOP, lift your pencil and start again from the next pair of coordinates.

Join (3, 0) (3, 5) (2, 5) (2, 4) (1, 4) (1, 5) (0, 5) (0, 4) (-1, 4) (-1, 5) (-2, 5) (-2, 4) (-3, 4) (-3, 5) (-4, 5) (-4, -9) (3, -9) (3, 0) (4, 0) (4, -1) (5, -1) (5, 0) (6, 0) (6, -1) (7, -1) (7, 0) (8, 0) (8, -9) (3, -9) STOP

Join (5, 0) (5, 7) (6.5, 10) (6.5, 11) (9, 10) (6.5, 10) (8, 7) (8, 0) STOP

Join (-7, -2) (-7, -9) (-4, -9) (-4, 3) (-5, 3) (-5, 2) (-6, 2) (-6, 3) (-7, 3) (-7, 6) (-7.5, 8) (-8, 6) (-8, -1) (-7, -2) STOP

Join (-2, -6) (-1, -5) (0, -5) (1, -6) (-2, -6) (-2, -9) (-3, -11) (2, -11) (1, -9) (1, -6) STOP

Join (3, 5) (3, 7) (3.5, 9) (4, 7) (4, 4) (3, 1) STOP Join (-4, 5) (-4, 9) (-2, 8) (-4, 8) STOP

Join (6, -5) (7, -5) (7, -2) (6, -2) (6, -5) STOP Join (6, 6) (7, 6) (7, 3) (6, 3) (6, 6) STOP

Join (2, -1) (2, 2) (1, 2) (1, -1) (2, -1) STOP Join (-3, -1) (-2, -1) (-2, -4) (-3, -4) (-3, -1) STOP

Join (-6, -3) (-5, -3) (-5, 1) (-6, 1) (-6, -3) STOP Join (3, 7) to (4, 7) STOP

Join (5, 7) to (8, 7) STOP Join (-8, 6) to (-7, 6) STOP

Join (-2, -6) to (-3, -1) STOP Join (1, -6) to (2, -11) STOP

What have you drawn?

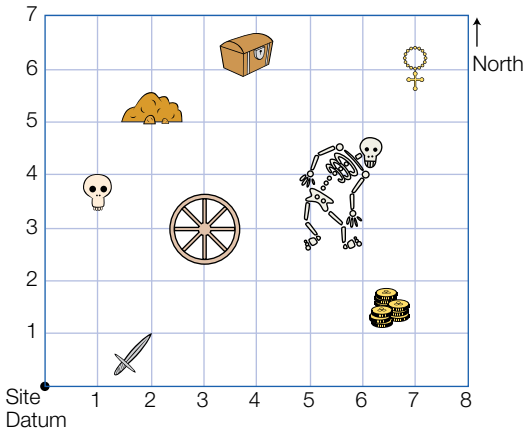
11 **(a)** Which of the following coordinate pairs lies furthest up on the Cartesian plane?

A (-3, 2) **B** (2, 3) **C** (3, -2) **D** (1, -3)

(b) Which of the following coordinate pairs is the furthest to the left on the Cartesian plane?

A (-3, 2) **B** (2, 3) **C** (3, -2) **D** (1, -3)

12 The grid shown allows an archaeologist to reconstruct the layout of an old tomb. (Archaeologists call the point (0, 0) the datum point.) State the point(s) at which each of the following is found.



(a) the centre of the base of the treasure chest

(b) the middle of the rock face base

(c) the tip of the dagger

(d) the centre of the chariot wheel

(e) the right wrist and the left shoulder of the skeleton

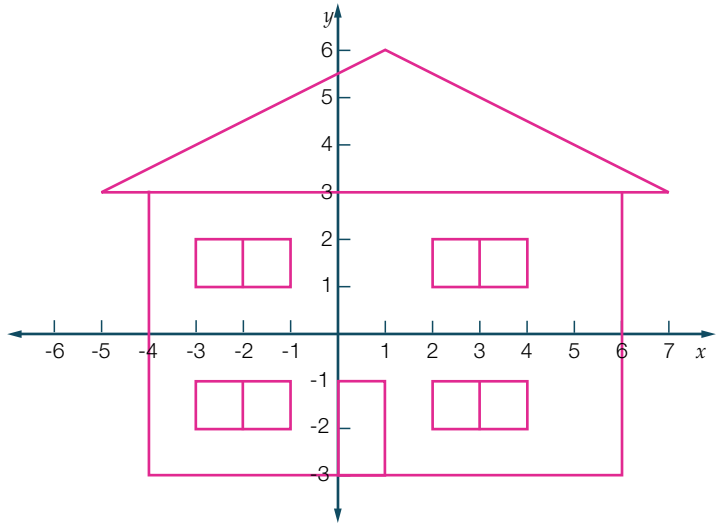
(f) the four grid points that enclose all the coins

(g) the top of the skull lying by itself

(h) the chain at the point where the cross is attached

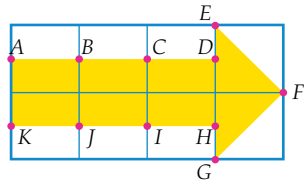
Reasoning

13 Write instructions using coordinates so that this drawing could be reproduced by another student.



Open-ended

14 Draw this arrow shape on a Cartesian plane so that it lies in at least three quadrants, and give the coordinates of each of the points A to K.



15 On a piece of grid or graph paper, rule a set of axes to form a Cartesian plane. Allow for a scale from -5 to +5 on both axes.

(a) Using more than one quadrant on your Cartesian plane, use straight lines to draw a simple design of your own.

(b) Write instructions using coordinates so that your design could be reproduced by another student.

(c) Give your instructions to another student to see if they can draw your design exactly.

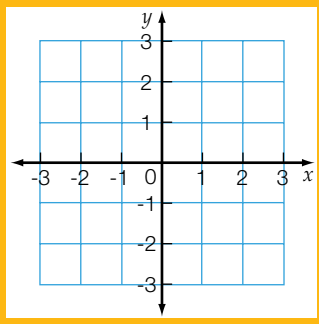
Game

Line up

Equipment required:
grid paper, die

How to win:
Five rounds are played and the player with the highest score is the winner.

How to play:



1 Copy the grid shown. Take turns to roll the die. The aim is to get three consecutive points in a row, a column or a diagonal (that is, in a straight line).

2 To find the coordinates for a point, use the number rolled on the die as the sum of the coordinate values (ignoring any negatives).

For example, if a 3 is rolled, the coordinates might be (0, 3), (3, 0), (0, -3), (-3, 0), (1, 2), (2, -1), (1, -2), (-1, 2), (-1, -2), (-2, -1).

The player can mark any one of these points.

3 Only coordinates that fit on the grid can be used.

For example, if a 6 is rolled, the only combinations that fit on the grid are (3, 3), (-3, -3), (-3, 3), (3, -3) but not, for example, (-5, 1) or (6, 0).

4 If a player cannot mark a point because it is already marked, they miss a turn.

5 When a player has three points next to each other in a line, the score earned is the total of all the coordinate values, ignoring any negative signs.

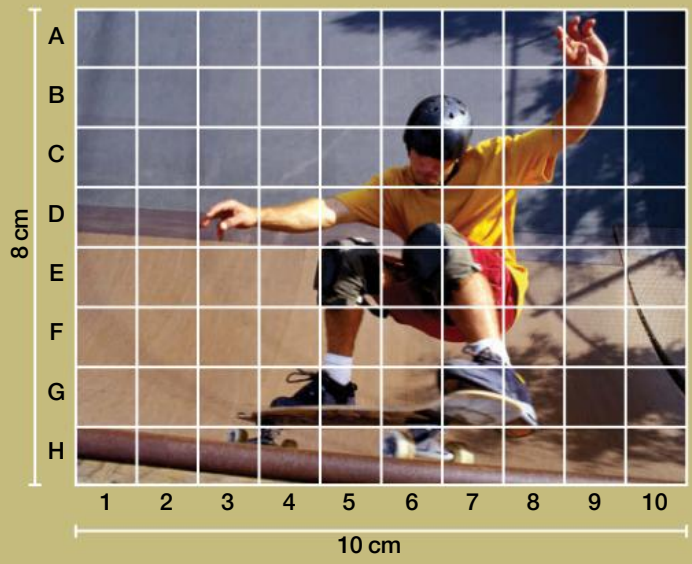
For example, for the coordinates (-3, 2), (-2, 2) and (3, 2), the score is 3 + 2 + 2 + 2 + 3 + 2 = 14.

6 After recording the score, draw a new grid for the next round.

Maths 4 Real

MATHS MEETS ART

Some art schools teach students to paint using the 'grid method', a method used by Leonardo da Vinci. In this method, you overlay a grid onto the image to be painted and also place a matching grid pattern onto the canvas. You can then copy the part of the image in each grid separately. For example, if the image is an 8 cm $\times$ 10 cm photograph, you can use 1 cm $\times$ 1 cm squares to create a grid pattern of eighty squares. If the image is copied onto an 80 cm $\times$ 100 cm canvas, each square will be 10 cm $\times$ 10 cm.



Note that the image and canvas above are not shown to scale.

1 State the grid coordinates of each of the following.

(i) right foot **(ii)** left ear

(iii) left elbow **(iv)** left hand

2 Suppose you wish to copy the image onto a canvas that is 16 cm × 20 cm.

(a) What size square on the canvas is needed to represent one square centimetre on the image?

(b) Use centimetre grid paper or graph paper to copy the image.

The rule of thirds

A basic principle in photography and art is 'the rule of thirds'. The image is divided into thirds horizontally and vertically to obtain nine squares or rectangles.

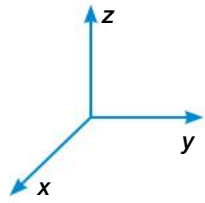
If points of interest are put in any of the four marked intersections or along the lines, then the image is usually more balanced and is pleasing to the eye.

3 Do you think 'the rule of thirds' was used in the composition shown below?

4 Use graph or grid paper to create an image of your own using 'the rule of thirds'.

Research

- The rule of thirds is related to the Fibonacci spiral. Find out how the spiral is created.
- Find out more about the art of Piet Mondrian, and either present a five-minute talk with illustrations or prepare a poster.
- Zedism is a painting style that gives a 3-dimensional effect. It uses a third axis (called the *z*-axis) which is at right angles to the Cartesian plane. By using geometry and perspective, the effect is an illusion of form and structure.



'Sky' by Yuransky

Find out how this is achieved and look for other examples of paintings where this method is used.

5.8 Patterns and plotting points

When points are plotted on a number plane they often appear to follow a pattern or shape. This shape may be a straight line or it may be a curve. If you can draw a straight line through all the points you form a **linear graph**. Any set of points with a definite shape, such as a straight line, can be described by an algebraic rule.

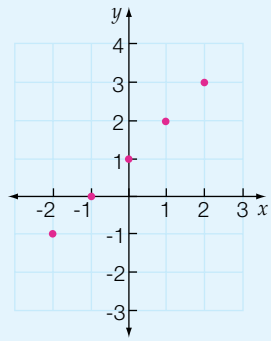
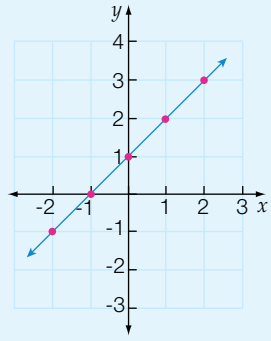
Worked example 14 W.E. 14

(a) Plot the points (-2, -1), (-1, 0), (0, 1), (1, 2) and (2, 3) on a number plane.

(b) Rule a straight line passing through all the points.

(c) Summarise the set of points in a table of values.

(d) Write the rule linking the x- and y-values.

Thinking	Working
(a) Use a ruler to draw and number the x-axis and y-axis on graph or grid paper. Label your axes and plot the points (-2, -1), (-1, 0), (0, 1), (1, 2) and (2, 3) moving left or right first, then up or down.	*(a)* 
(b) Join the points carefully with a ruler. You should have a straight line.	*(b)* 
(c) Construct a table with x-values on the top line and y-values on the second line.	*(c)*
(d) 1 Look for a link between the x- and y-values. (Each y-value is 1 more than the x-value.)	*(d)*
2 State the rule.	The rule is $y = x + 1$.

x	-2	-1	0	1	2
y	-1	0	1	2	3

Worked example 15

W.E. 15

(a) List the coordinates of the points given in the table of values below.

x	-2	-1	0	1	2
y	-3	-1	1	3	5

(b) Plot these points on a number plane.

(c) Rule a straight line passing through all the points.

(d) Write the rule linking the x- and y-values.

Thinking

(a) Write the coordinates as ordered pairs with the x-coordinate first and the y-coordinate next.

(b) Use a ruler to draw and number the x- and y-axes on graph or grid paper. Label your axes and plot the points (-2, -3), (-1, -1), (0, 1), (1, 3) and (2, 5) moving left or right first, then up or down.

(c) Join the points carefully with a ruler. You should have a straight line.

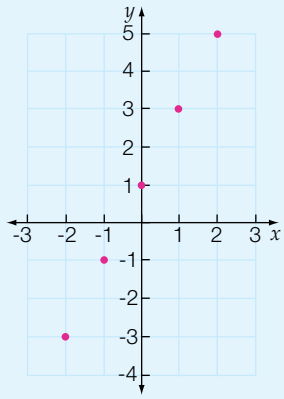
(d) **1** Look for a link between the x- and y-values. (The y-values go up by 2, so we are multiplying by 2. Each y-value is 1 more than twice the x-value.)

2 State the rule.

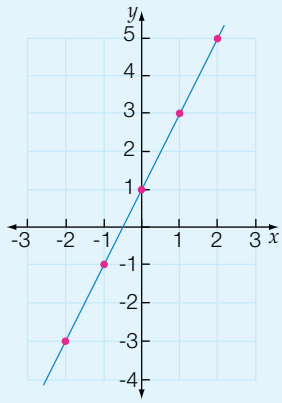
Working

(a) (-2, -3), (-1, -1), (0, 1), (1, 3), (2, 5)

(b)



(c)



(d)

The rule is $y = 2x + 1$.

5.8 Patterns and plotting points

Navigator

Answers p. 679

1, 2, 3, 4, 5, 6, 7, 9 | 1, 2, 3, 4, 5, 6, 7, 9, 10 | 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

Equipment required: graph paper

Fluency

W.E. 14

1 **(a)** Plot the points (-2, 0), (-1, 1), (0, 2), (1, 3) and (2, 4) on a number plane.

(b) Rule a straight line passing through all the points.

(c) Summarise the set of points in a table of values.

(d) Write the rule linking the x- and y-values.

W.E. 15

2 **(a)** List the coordinates of the points given in the table of values below.

x	-1	0	1	2	3
y	-3	-1	1	3	5

(b) Plot these points on a number plane.

(c) Rule a straight line passing through all the points.

(d) Write the rule linking the x- and y-values.

3 **(a)** Plot the points (1, 6), (2, 5), (3, 4), (4, 3), (5, 2) and (6, 1) on a number plane.

(b) Join the points in this order.

(c) Summarise the set of points in a table of values.

(d) Write the rule linking the x- and y-values.

Understanding

4 Which one of the points (-1, -1), (2, 2), (3, 4) and (5, 5) will not lie on the same straight line as the three other points?

A (-1, -1) **B** (2, 2) **C** (3, 4) **D** (5, 5)

5 **(a)** Plot the points (-2, 8), (0, 6) and (5, 1) on a number plane.

(b) Draw a straight line passing through all the points.

(c) What is the y-coordinate of a point on the line if its x-coordinate is -1?

(d) What is the x-coordinate of a point on the line if its y-coordinate is 5?

6 **(a)** Plot the points (1, 1), (1, 5), (5, 5) and (5, 1) on a number plane and join the points in order. (Starting at the first point, draw a straight line to the second point, then to the third point, then to the fourth point, and finally back to the first point.)

(b) Name the geometrical figure drawn in part **(a)**.

Reasoning

7 **(a)** Plot the points (0, 0), (2, 2) and (-5, -5) on a number plane.

(b) Draw a straight line passing through all the points.

(c) Write the coordinates of three other points that the line passes through.

(d) Summarise the set of six points in a table of values.

(e) Write the rule linking the x-values and y-values.

(f) By substituting $x = -3$ into the rule, show that the point (-3, -3) lies on the line.

8 (a) Plot the points (-1, -5) and (5, 13) on a number plane and join them with a straight line.

(b) Write the coordinates of three other points that the line passes through.

(c) Summarise the set of points in a table.

(d) Write the rule linking the x-values and y-values.

(e) Substitute $x = 0$ into the rule to show that (0, -3) does not lie on the line.

(f) Plot (0, -3) on the number plane to confirm it does not lie on the line.

Open-ended

9 (a) Plot the point (3, 3) on a number plane.

(b) Choose another point on the same number plane and mark it. Draw a straight line passing through your point and the given point.

(c) Write the coordinates of two other points that the line passes through.

(d) Write the rule linking the x-values and y-values for this straight line.

(e) Repeat **(b)**, **(c)** and **(d)** using (3, 3) and a different point to find a different rule.

10 Draw a Cartesian plane with a scale of -4 to 4 on the x- and y-axes.

(a) Choose a point (A) in the 2nd quadrant, mark it and write its coordinates.

(b) Choose two other points (B and C) in quadrants 3 and 1 that can form a straight line with point A. Write the coordinates for points B and C, then draw a straight line through the three points.

(c) Find the rule for that line.

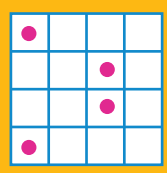
Puzzle

Gridlock

Equipment required: grid or graph paper

(a) In Grid 1 below, mark four squares so that no two marked squares lie in the same row, column or diagonal.

For example, this arrangement is NOT allowed, because it has two marked squares in the same column:



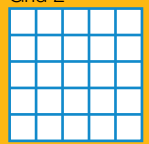
(b) In Grid 2, mark five squares so that no two marked squares lie in the same row, column or diagonal.

(c) Mark six squares in Grid 3 and seven squares in Grid 4 so that no two marked squares lie in the same row, column or diagonal.

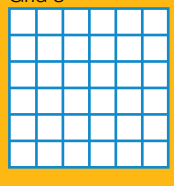
(d) The challenge: Mark eight squares in Grid 5 so that no two marked squares lie in the same row, column or diagonal.

Grid 1

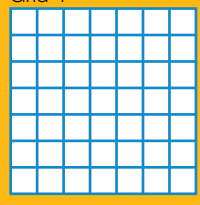
Grid 2



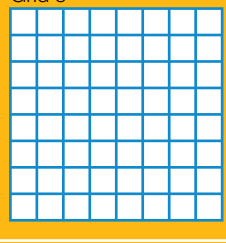
Grid 3



Grid 4



Grid 5



Gamespace

SPY vs SPY

Equipment required:

For task #1: graph paper, ruler

For task #3: 1 die, counters or small pieces of paper

While working on Operation Cartesian as a spy for the Confederation of Mathematicians, Freedo has been caught by the Anti-Maths forces deep in the jungle.

Task #1: Frame the Zero

Freedo's last communication from headquarters gave him a picture of 'Zero', the leader of the Anti-Maths conspiracy.

To decipher the picture, Freedo must apply his knowledge of Cartesian graphs. On your graph paper begin by creating a set of axes, from -8 to 8 along the *x*-axis and from -10 to 7 along the *y*-axis.

Use a ruler to join each of the following sets of coordinates, like a dot-to-dot drawing. Where there is a •, lift your pen and start a new line without joining to the previous lines drawn.

(0, 7), (2, 7), (4, 6), (6, 4), (6, 1), (7, 2), (8, 0), (7, -2), (6, -3), (5, -5) •

(4, -6), (3, -8), (1, -10), (0, -9) •

(0, -8), (1, -7), (3, -7), (1, -6), (0, -6) •

(3, -7), (1, -5), (0, -5) •

(0, 2), (2, 4), (5, 4), (3, 3), (1, 1), (1, -1), (2, -2), (1, -3), (0, -3) •

(0, -4), (1, -3), (3, -4), (6, -6), (4, -6), (1, -4), (0, -4) •

(6, -1), (7, -1), (7, 1), (6, 0) •

(1, 1), (2, 0), (5, 1), (4, 2), (2, 2), (2, 1), (3, 1), (3, 2) •

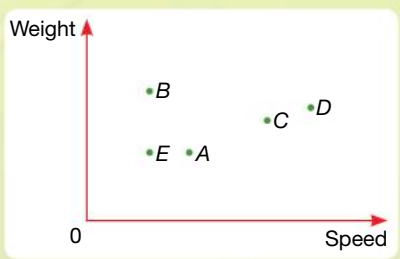
Complete the image by drawing a reflection of the current image in the *y*-axis.

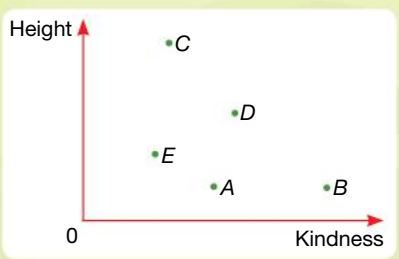
Task #2: Guard help

Freedo is locked in a cage. He was given the following information from headquarters:

'The guard who will help you is one of the two shortest guards and one of the two slowest guards.'

Looking at the following graphs, can you figure out which guard will help Freedo?





Task #3: Quadrant Os + Xs

Play this game to see if you are able to escape from the Anti-Maths conspiracy.

Choose who will play the role of Zero and who will play the role of Freedo. Will Freedo escape?

Your aim is to be the first to have 3 counters in a row, column or diagonal.

Take turns in rolling a die. Here are the rules for where you put your counters:

Rolling a 1 means you must put a counter in quadrant 1.

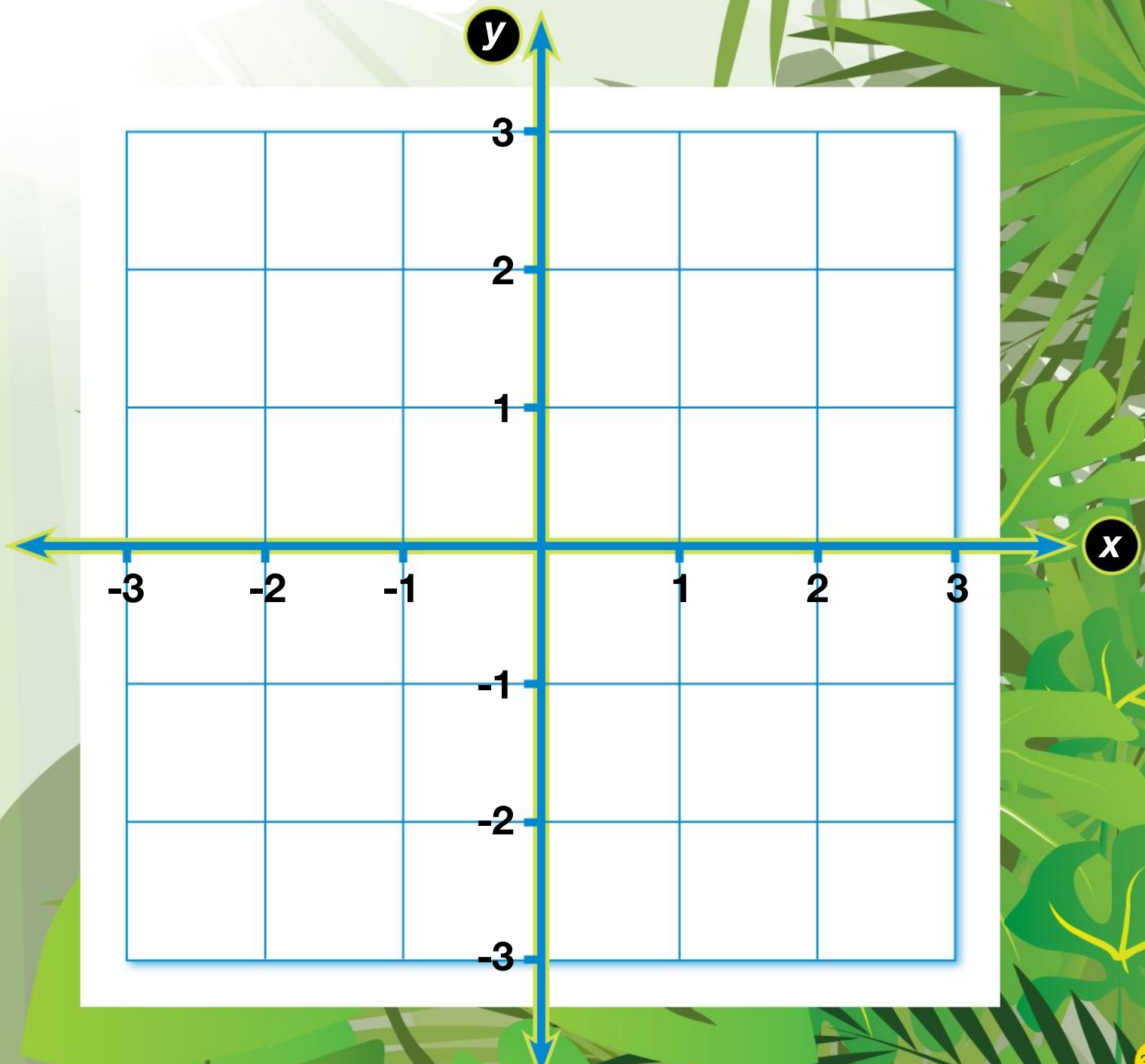
Rolling a 2 means you must put a counter in quadrant 2.

Rolling a 3 means you must put a counter in quadrant 3.

Rolling a 4 means you must put a counter in quadrant 4.

Rolling a 5 means you must put a counter on the x-axis.

Rolling a 6 means you must put a counter on the y-axis.



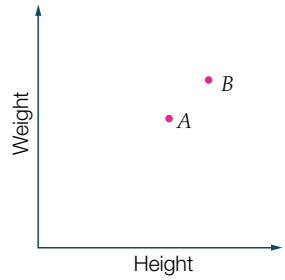
5.9 Interpreting graphs

Sometimes, when you plot points showing information relating to two variables, the points should not be joined.

This type of graph is called a **point graph**.

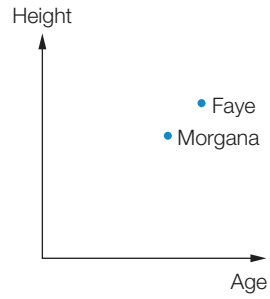
In the graph shown, the points labelled *A* and *B* tell us about the height and weight of Oscar and Theodore.

Point *A* matches Theodore (smaller height and lower weight) and point *B* matches Oscar (taller and heavier). The graph shows that Oscar is taller than Theodore and that Theodore weighs less than Oscar.



Worked example 16 W.E. 16

This graph shows the age and height of Faye and Morgana.



Answer true (T) or false (F) to each of these statements.

(a) Faye is taller than Morgana.

(b) Morgana is older than Faye.

(c) Faye is younger than Morgana.

(d) Morgana is shorter than Faye.

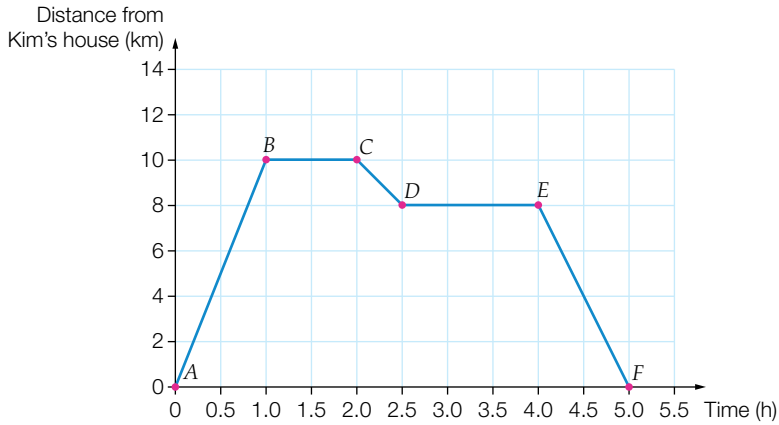
Thinking	Working
(a) Compare heights. Height increases the further you move up the vertical axis. The point for Faye is higher, so she is taller than Morgana.	(a) Faye is taller than Morgana. True
(b) Compare ages. Age increases the further you move to the right. The point for Faye is further to the right, so she is older than Morgana.	(b) Morgana is older than Faye. False
(c) Compare ages. Faye is older than Morgana as the point for Faye is further to the right.	(c) Faye is younger than Morgana. False
(d) Compare heights. The point for Morgana is lower, so she is shorter than Faye.	(d) Morgana is shorter than Faye. True

If you join points with straight lines, they may not form a linear graph, as the points may not all lie on the same straight line. These graphs are called **line graphs**. An example of line graphs are travel graphs where time is plotted on the horizontal axis and distance is plotted on the vertical axis. When lines are horizontal, the distance is not changing. When lines are steepest, the speed is greatest.

Worked example 17

W.E. 17

May and Kim go jogging. They leave from Kim's house, jog to their friend Jenny's house, stay there for a swim, then jog to May's house. They stay there for a while and watch a movie before jogging back to Kim's house.



Using the graph given above that shows their journey, answer the following questions.

(a) What are the graph coordinates of Kim's house?

(b) How far away from Kim's house does Jenny live?

(c) How long did it take to reach Jenny's house?

(d) How long did they stay at Jenny's house?

(e) How far from Kim's house does May live?

(f) How long did they stay at May's house?

(g) How long were they away from Kim's house?

(h) When were they jogging the fastest?

Thinking	Working
(a) Identify where the journey begins (A).	(a) (0, 0)
(b) Look for the distance to B on the y-axis. The y-coordinate of B gives that distance.	(b) 10 km
(c) Look for the time taken to get from A to B on the x-axis. The x-coordinate of B gives the time.	(c) 1 h
(d) The horizontal line BC shows that they stayed in the same place during this time, so the time will be the time difference between B and C.	(d) 1 h
(e) Look for the distance D on the y-axis. The y-coordinate of D gives the distance.	(e) 8 km

(f) The horizontal line DE shows that they stayed in the same place during this time, so the time will be the time difference between D and E.

(f) $1\frac{1}{2}$ h

(g) Look for the time to get from A to F on the x-axis. The x-coordinate of F gives the time.

(g) They were away for 5 hours.

(h) Look for the steepest line. This is the line from A to B at the start of the jog.

(h) They jogged fastest from Kim to Jenny's house.

5.9 Interpreting graphs

Navigator

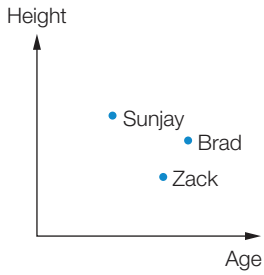
Answers p. 680

1, 2, 3, 4, 5, 6 (a), 8, 9, 10, 11, 14	1, 2, 3, 4, 5, 6, 7 (a), 8, 9, 10, 11, 12, 14	2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14

Fluency

W.E. 16

1 This graph shows the age and height of Sunjay, Zack and Brad.



Answer true (T) or false (F) to each of these statements.

(a) Brad is taller than Sunjay.

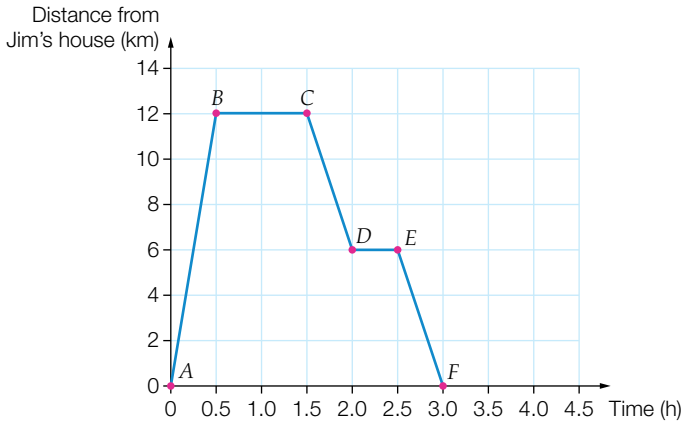
(b) Zack is the shortest.

(c) Brad is older than Zack.

(d) Sunjay is the youngest.

W.E. 17

2 Ray and Jim go for a bike ride. They leave from Jim's house, ride to their friend Terry's house, stay there for lunch, and then ride to Ray's house. After a short break, they ride back to Jim's house.



Using the graph given above that shows their journey, answer the following questions.

(a) What are the graph coordinates of Jim's house?

(b) How far away from Jim's house does Terry live?

(c) How long did it take to reach Terry's house?

(d) How long did they stay at Terry's house?

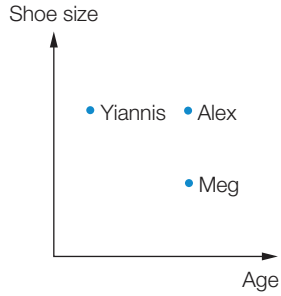
(e) How far from Jim's house does Ray live?

(f) How long did they stay at Ray's house?

(g) How long were they away from Jim's house?

(h) When were they riding the fastest?

3 Use the graph opposite to answer the following questions.



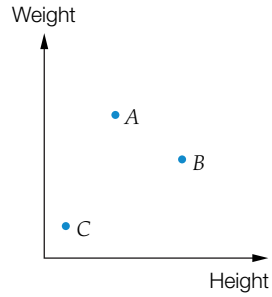
(a) Who is the youngest?

(b) Who takes the smallest shoe size?

(c) What can you say about the shoe sizes of Yiannis and Alex?

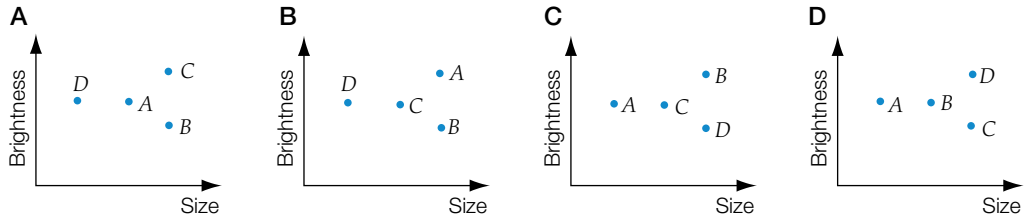
(d) What can you say about the ages of Meg and Alex?

4 Consider this graph showing the weight and height of an elephant, a giraffe and a kangaroo. Match each point with the animal it represents.

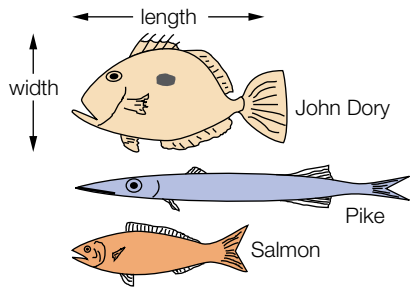


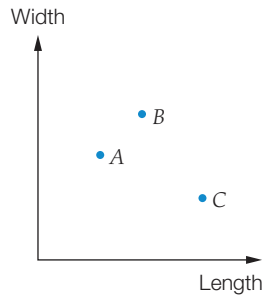
Understanding

5 Four light globes, *A*, *B*, *C* and *D* have the properties that globe *A* and globe *B* are equally bright and globe *C* and globe *D* are of equal size. Which graph can represent this situation?



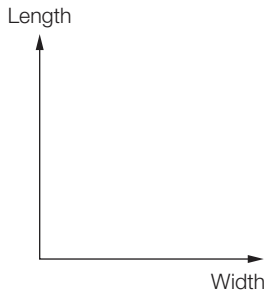
6 Consider this graph showing the widths and lengths of various fish.



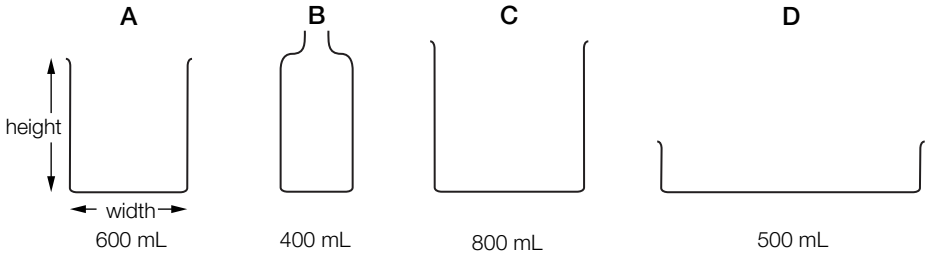


(a) Match each point with the fish it represents.

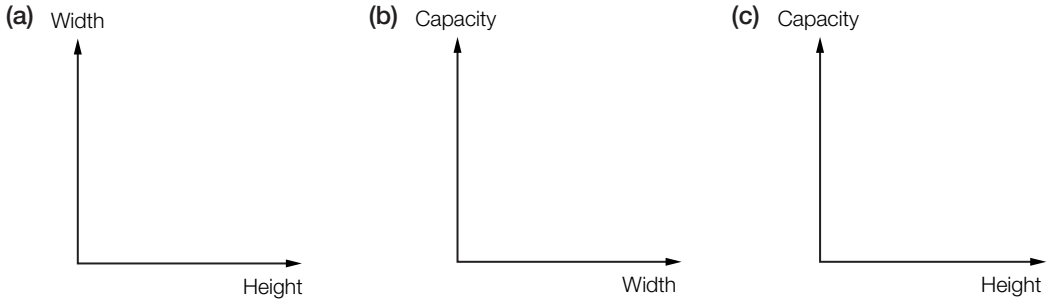
(b) Copy and complete this graph to show the positions of the points *A*, *B* and *C* for the three fish.



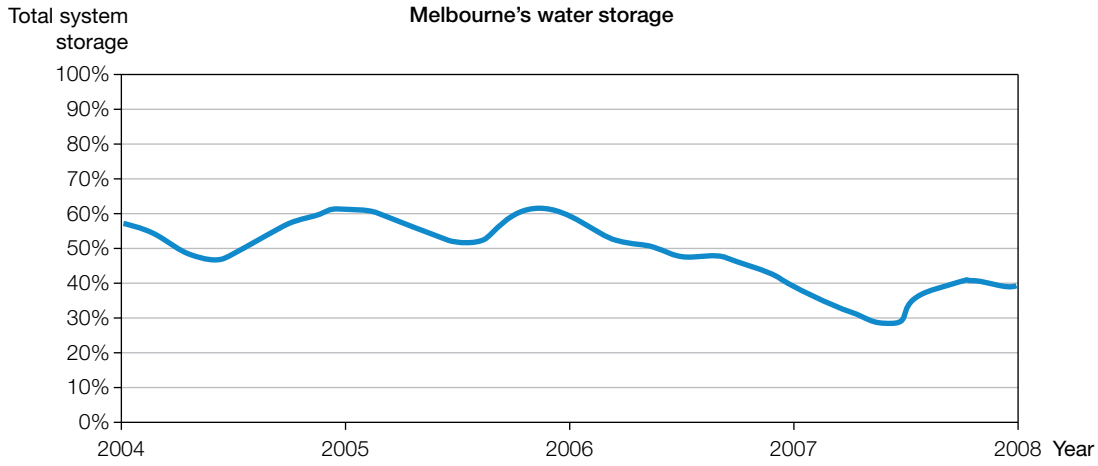
7 The capacity of each container is measured in mL and the size (width and height) is represented as shown.



Complete the following point graphs for the four containers.



8 Large cities are often surrounded by dams that collect rain to supply the city with water. The total amount of water stored in the dams is called the water storage. When this amount is calculated as a percentage of the total capacity of all the dams, it gives a measure of how full the dams are.



Above is a graph of Melbourne's water storage from 2004 to 2008. Use this graph to answer the following questions.

(a) What was the storage percentage at the start of 2004?

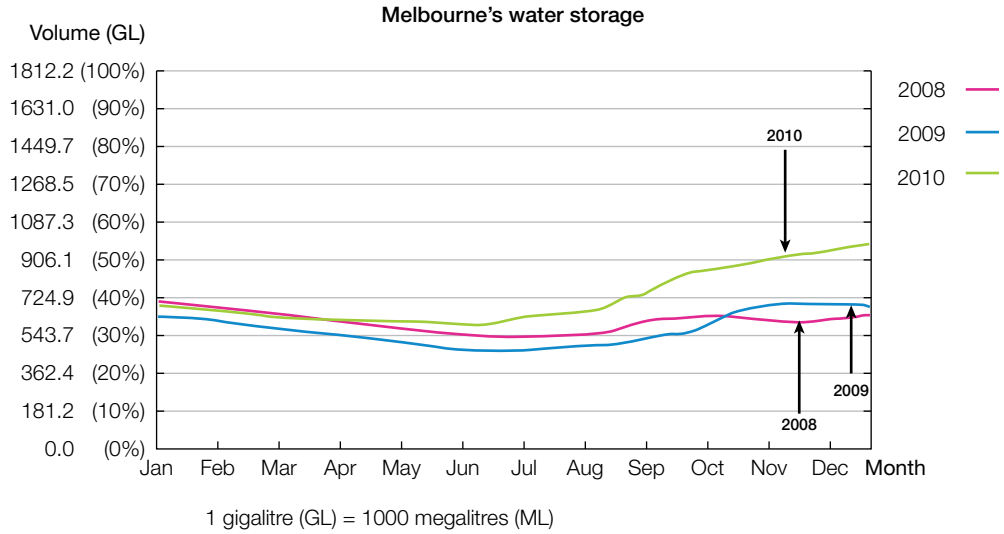
(b) The water storage percentage rose in the second half of 2004. What could explain this?

(c) How can you explain the increase in storage percentage in the second half each year (except for 2006)?

(d) What does the graph tell you about Melbourne's rainfall from the start of 2006 to mid-2007?

(e) When was the water storage the lowest in the time period 2004–2008? What was the lowest percentage level reached?

9 Below is a graph of Melbourne's water storage for the years 2008, 2009 and 2010.



Use this graph to answer the following questions.

(a) Compare the storage levels at the start of 2008, 2009 and 2010. Were the levels similar? Which year started with the lowest storage level? What was the difference between the highest and lowest storage percentage at the start of each year?

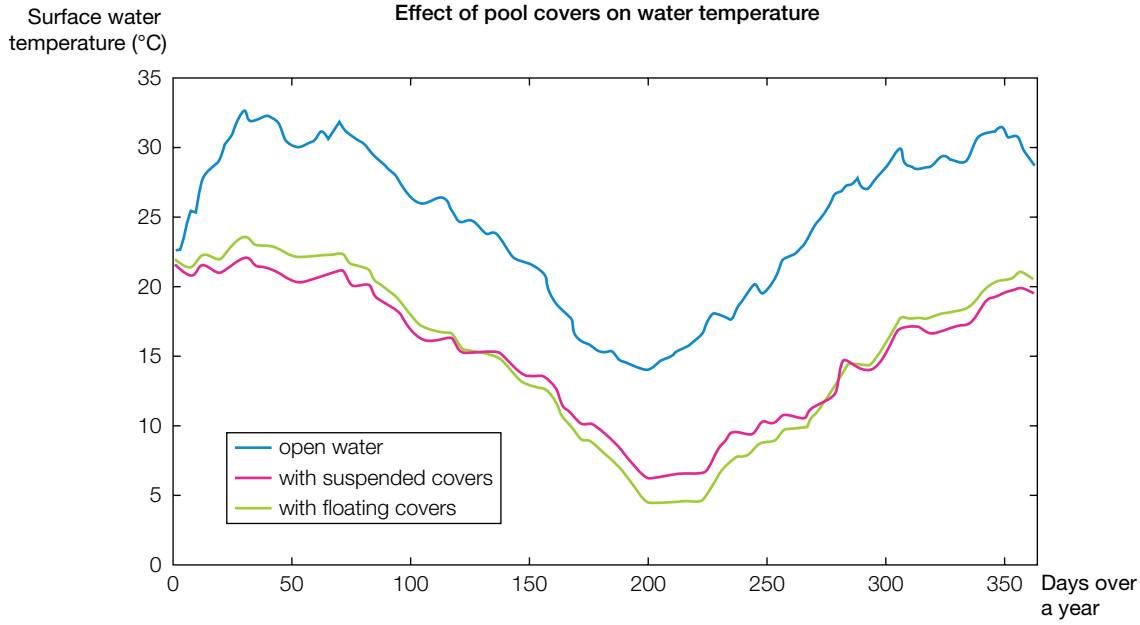
(b) Compare the storage levels at the end of 2009 and 2010. What is the difference in storage percentage?

(c) During which 3 months over the 3-year period was the storage level the highest?

(d) During which 3 months over the 3-year period was the storage level the lowest?

Reasoning

10 Evaporation is a major source of water loss. Water evaporates more as temperature increases, so the lower the temperature of the water, the less water will be lost to evaporation. The graph below shows how the surface temperature of water is affected by being covered or not throughout the year.



(a) Do you think covers reduce evaporation? Give reasons for your answer.

(b) Which cover is more effective over the colder days of the year?

(c) When is a suspended cover more effective than a floating cover?

(d) What overall effect does the use of covers have on the water temperature?

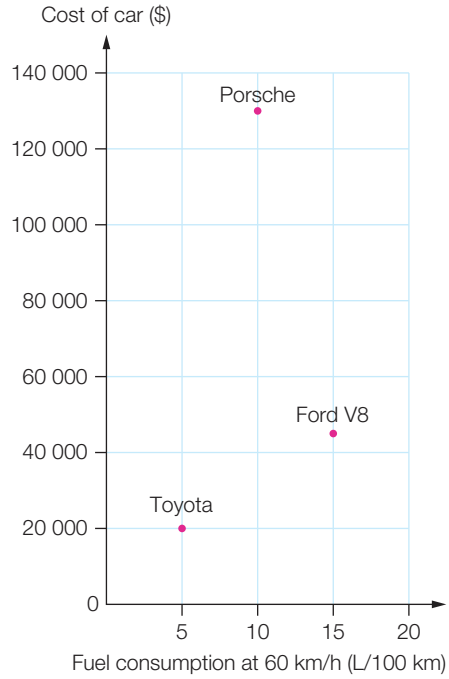
11 **(a)** What are the two variables shown in this graph?

(b) What happens to the shoe size as the height increases?

12 Use the graph shown to answer the following questions.

(a) Is there a relationship between the cost of a car and the fuel consumption? Explain your answer.

(b) Redraw the graph with the axes swapped (vertical for horizontal and horizontal for vertical).



Open-ended

13 Tom, Adam, Sheena and Toula play basketball. Sheena is shorter than Tom, and Toula scores more points than Adam. One point of view is that the taller the basketball player, the more points they score. Using the variables 'Height' and 'Points scored', draw two possible graphs that show the relationship.

14 Draw a travel graph of your own and write a few sentences to explain the journey.

Puzzle

Solve the grid

Each letter stands for a particular number. The sum of these values for the first row and for the first two columns is given.

D	*D*	*E*	*D*	39
E	*D*	*D*	*E*	
E	*E*	*E*	*X*	
X	*E*	*X*	*X*	
40	42			

1 What number does *X* represent?

2 Find the total of each of the other rows and columns.

Challenge 5

1 George has 15 coins, all 50c and 20c pieces. If the 20c pieces were 50c pieces and the 50c pieces were 20c pieces, then he would have $2.10 more. How many 50c pieces does he actually have?

2 If $p \bigstar q$ means $3(p + q)$, what is the value of $5 \bigstar (2 \bigstar 8)$?

3 Steven and Claire live next door to each other. The product of their house numbers is 483. What are their house numbers?

4 If $x > 4$, put these terms in order from smallest to largest.

$$\frac{x}{4} \qquad \frac{4}{x} \qquad \frac{4}{x+1} \qquad \frac{x+1}{4}$$

5 Each letter below represents one of the digits 1, 2, 3, 4 or 5.

$$\begin{array}{r} \text{K L} \\ \times \quad \text{M} \\ \hline \text{P N} \\ \hline \end{array}$$

The answer to this multiplication is correct.

Which statement is correct?

A K = 3 **B** L = 3 **C** M = 3 **D** N = 3

6 What are possible positive values for a and b if $(a \times b)^2 = a \times b^2$?

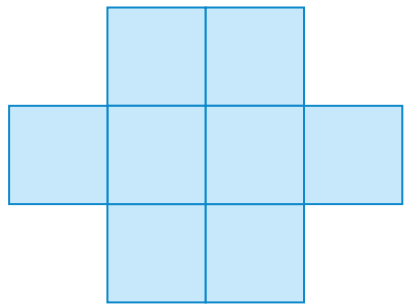
7 In a 30-question test you earn 9 marks for every correct answer and lose 5 marks for every wrong answer. You must attempt every question.

(a) How many marks do you get for 15 correct answers? (Assume you attempt all the questions.)

(b) What is the greatest number of marks you can get if you have more answers wrong than correct?

(c) Is it possible to score exactly zero on the test? Explain why or why not.

8 Show how one of the numbers 1, 2, 3, 4, 5, 6, 7 and 8 can be placed in each blue square so that no consecutive numbers are beside each other. (For example, 2 must not be next to a 3 either above, below, to the left or to the right.



9 If half the number represented by x is 24, what is the value of $2x$?

10 The sum of three consecutive integers (whole numbers) is 90. What is the smallest of the three integers?

A 28 **B** 29 **C** 30 **D** 31

11 Find two numbers that make 1 000 000 when multiplied together if neither of the numbers contain any zeros.

Chapter review 5

Maths literacy

axes	equation	linear graph	pronumeral	terms
Cartesian plane	evaluate	non-like terms	quadrant	unknown
coefficient	expression	number plane	relationship	variable
constant	formula	ordered pair	rule	
coordinates	like terms	origin	substitute	
define	line graph	point graph	table of values	

Copy and complete the following using the words and phrases from this list, where appropriate. A word or phrase may be used more than once.

1 A _____ is a letter or a symbol that represents a number.

2 Completing a _____ is helpful when finding the _____ that describes a pattern.

3 To find the value of the expression $2x + 3$, when $x = 1$, you need to _____ 1 for x, and _____ the expression.

4 The _____ is the point (0, 0). It is the point where the two _____ of the Cartesian plane intersect.

5 A pronumeral represents an _____ or a _____.

6 A number written next to a pronumeral is the _____ of the pronumeral. The pronumeral is multiplied by that number.

7 To simplify the expression $3x + 4x + 5$ you collect _____ .

8 The _____ (3, 5) can be plotted on the _____ by finding the point 3 across and 5 up. Draw a diagram clearly showing this point plotted in its correct location.

Fluency

1 Write the following situations using algebra. **5.1**

(a) There are 12 biscuits in a packet. How many are in n packets?

(b) Tarin has a packet of p lollies. He eats half, then gives 3 to his brother. How many lollies does he have left?

(c) Multiply p by 7, then subtract 9.

(d) Subtract g from 11, then divide by 5.

2 Write an equation for each of the following situations, taking care to define all the variables you have used. **5.2**

(a) Joe's age in years is four plus three times Beth's age.

(b) Three bottles of juice and five salad rolls cost $18.60.

3 For each of the following rules: **5.3**

(i) draw a flowchart that shows how to get to y from x

(ii) write the rule using algebra

(iii) copy and complete the table of values for that rule.

(a) Add three to x to get y.

x	57	34	12	4	1.1	64
y						

(b) To find y, multiply x by three, then subtract five.

x	4	3	$2\frac{1}{2}$	9	10	12
y						

4 Choose the correct algebraic notation for each rule. **5.3**

(a) y is equal to x plus 6.

A $y = x - 6$ **B** $y = x + 6$ **C** $y = x \times 6$ **D** $x = y + 6$

(b) Subtract five from x, then multiply by three to get y.

A $y = (x - 3) \times 5$ **B** $y = (x + 5) \times 3$ **C** $y = (3x - 5)$ **D** $y = 3x - 5$

5 For each of the following formulas, evaluate p by substituting the given values of a. **5.4**

(a) $p = 6 - 20a$ **(i)** $a = 3$ **(ii)** $a = -1$

(b) $p = 2(a - 4) + 11$ **(i)** $a = 2$ **(ii)** $a = -5$

6 If you substitute $x = 3$ into the formula $y = 5x - 2$, the value of y is: **5.4**

A $y = 3$ **B** $y = 5$ **C** $y = 13$ **D** $y = 17$

7 Use each of the following rules to complete the tables of values. **5.4**

(a) $y = x - 7$

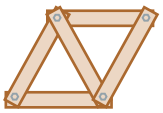
x	7	9	12	20	8.1	107
y						

(b) $y = 10(x - 3)$

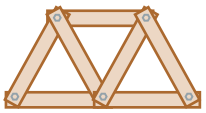
x	4	$5\frac{1}{2}$	13	7	78	54
y						

8 The frame of a bridge is made up of triangular sections. The triangles are made up of girders held together by bolts. **5.5**

1 triangle
3 girders
3 bolts



2 triangles
5 girders
4 bolts



3 triangles
7 girders
5 bolts

(a) Complete the table of values by continuing the pattern.

Number of triangles (t)	1	2	3	4	5
Number of girders (g)	3	5			
Number of bolts (b)	3	4			

(b) Write the rule that relates the number of triangles, t, to the number of girders, g.

(c) Write the rule that relates the number of triangles, t, to the number of bolts, b.

(d) (i) How many girders are needed to create each extra triangular section?

(ii) How is this reflected in the table of values and the rule?

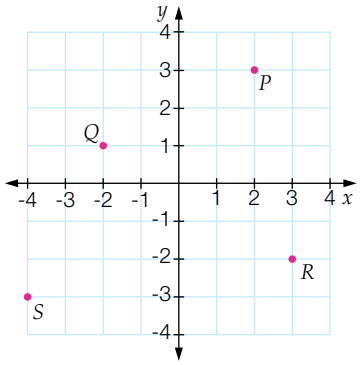
(e) (i) How many bolts are needed for each new triangular section?

(ii) How is this reflected in the table of values and the rule?

9 Simplify each expression by adding or subtracting like terms. **5.6**

(a) $12a - 7a$ (b) $6a + 12b - 7a + 11b$ (c) $x + y + 3x$

10 Write the coordinates of each of the following points shown on the Cartesian plane at right. **5.7**



(a) P (b) Q

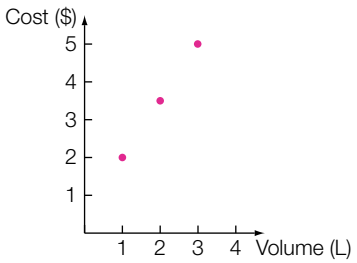
(c) R (d) S

11 (a) Plot the points (-1, -4), (1, 2), (2, 5) and (3, 8) on a number plane and draw a straight line passing through all the points. **5.8**

(b) Summarise the set of points in a table.

(c) Write a rule linking the x-values and y-values.

12 Juice is sold in three sizes: 1 L, 2 L and 3 L. Use the graph opposite to answer the following questions. **5.9**



(a) How much does a 2 L juice cost?

(b) Which size is the best value? Why?

(c) Why is this information given in a point graph?

Understanding

13 Gayle the kindergarten teacher has 8 packets of crayons with n crayons in each packet, plus an extra 3 loose crayons. **5.1**

(a) Write an expression for the total number of crayons that Gayle has.

(b) If all the crayons are shared equally among the 15 children in Gayle's class so that none are left over, write an expression to show how many crayons each child receives.

(c) Find the smallest possible value for n.

14 Draw a flowchart for each of the following rules. **5.3**

(a) $h = 7g$ (b) $d = 9c - 2$ (c) $d = \frac{c}{10} + 13$ (d) $h = 5(g + 8)$

15 Tamsin is working out the number of bottles of soft drink she needs for a party. She is going to allow 1 bottle for every 4 people, plus 5 extra bottles. **5.4**

(a) Write Tamsin's rule using algebra. Let b = number of bottles of drink required and n = number of people at the party.

(b) If Tamsin has 16 people at the party, how many bottles of drink will she need?

16 Simplify if possible: **5.6**

(a) $2a - 7b + 3a + 2$ (b) $5a + 12b - 7 - 3b$ (c) $x + y + xy$

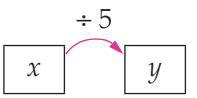
17 The points (0, 1), (2, 5) and (5, 11) are plotted on a number plane and a straight line drawn through the points. Which one of the following points also lies on the line? **5.7**

A (1, 2) **B** (3, 7) **C** (3, 6) **D** (4, 10)

Reasoning

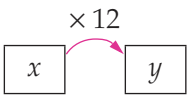
18 The following tables of values have only the y-values filled in. Work backwards along the flowchart to determine the values of x that were used, and complete the tables. **5.3**

(a)



x						
y	2	-7	20	1	0	-6

(b)



x						
y	72	120	36	-12	0	60

19 For each of these tables, find the rule that is being used. **5.5**

(a)

x	76	54	8	28	9	103
y	69	47	1	21	2	96

(b)

x	15	10	2	7.3	9	22
y	34	24	8	18.6	22	48

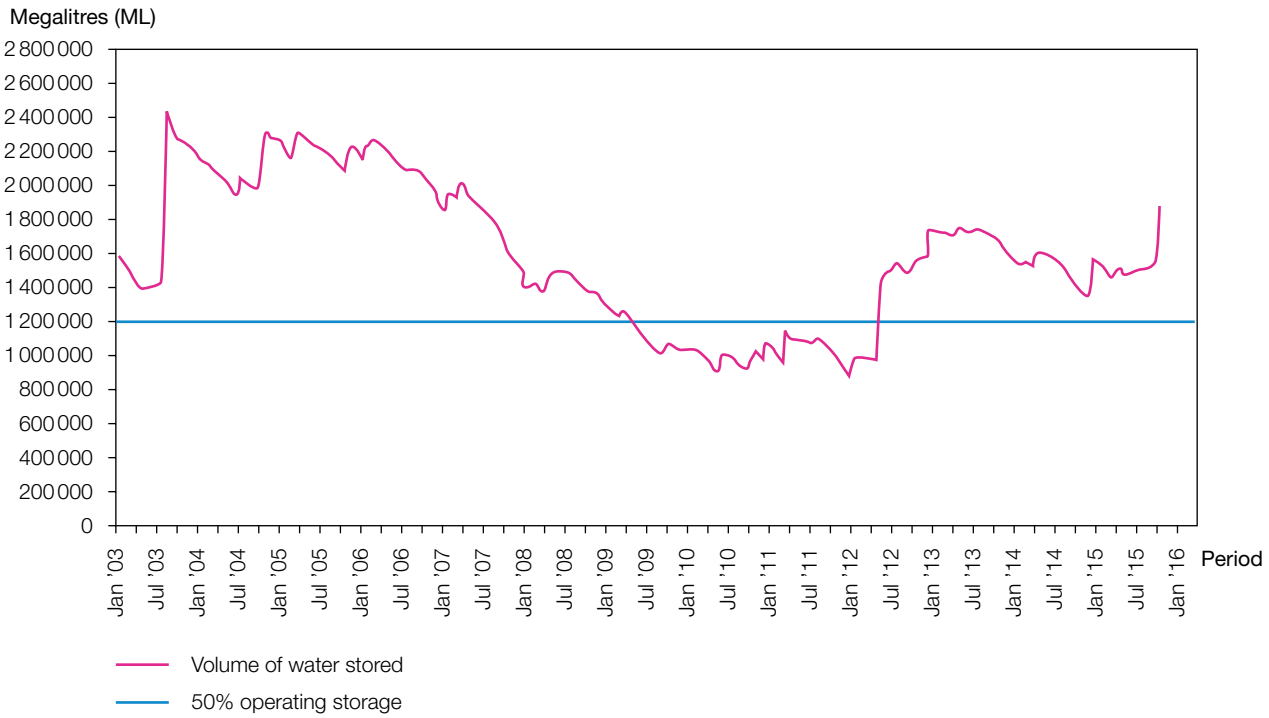
20 Jack has $21.00 to buy some apples and oranges. An apple costs $1.80 and an orange costs $1.40. **5.4**

(a) Using appropriate pronumerals to represent the number of apples and oranges purchased, write an equation to show the situation when Jack spends all of the $21.00.

(b) How many apples and how many oranges could Jack buy using all the money?

(c) If Jack buys 4 apples and 5 oranges, how much change will he receive?

21 Below is a graph of a city's water storage. The volume of water stored is given in megalitres (ML), where 1 ML = 1 000 000 litres. **5.9**



Use the previous graph to answer the following questions.

(a) What was the volume of water (in megalitres) at the start of 2015?

(b) During which month and year were the dams the fullest?

(c) During which month and year were the dams at their lowest capacity?

(d) What does the shape of the graph tell you about the city's rainfall from July 2003 to January 2004?

(e) What does the shape of the graph tell you about the city's rainfall from January 2005 to July 2012?

(f) During what period was the storage level below 1 200 000 megalitres (below 50%)?

Numeracy practice 5

Non-calculator

1 A number is multiplied by eight and then 5 is added. The answer is 61. What is the number?

2 x and y stand for numbers. x and y are connected by a rule.

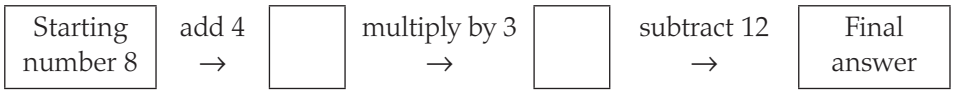
x	2	3	5	8
y	8	13	23	38

What is the rule?

A $y = 4x + 1$ **B** $y = 5x - 2$

C $y = 4x$ **D** $y = 10x - 27$

3 Ting Li followed this rule. She started with 8.



(a) What is the correct final answer using Ting Li's starting number of 8?

(b) Alex followed the same rule using a different starting number. Alex's starting number is -2. What was his final answer?

Calculator allowed

4 Ethan followed a different rule to Ting Li. There were three steps. Ethan started with the number 21 and ended up with a final answer of 30.

Starting number 21	Step 1 →		Step 2 →		Step 3 →	Final answer

(a) Write the numbers from 1 to 3 on the lines below to show an order of steps that Ethan followed.

Step _____ add 8

Step _____ divide by 3

Step _____ double the number

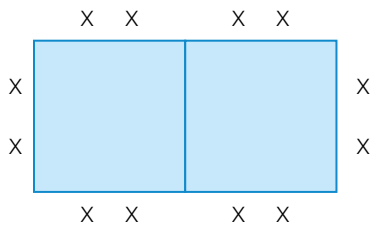
(b) Repeat the procedure in part **(a)** to get a final answer of 22.

5 A pack of 10 candles costs $4.60.
A pack of 6 candles costs $3.20.
You need to buy 22 candles.

What is the least amount you can pay?

6 Yusef investigated the seating plan for a restaurant using this table of values.

Number of tables	2	3	4	5
Number of people seated	12	16	20	?



The diagram shows the seating arrangement for 2 tables.

What rule did Yusef use to work out how many people can be seated at 5 tables?

A (number of people seated) = (number of tables)

B (number of people seated) = (number of tables) + 4

C (number of people seated) = (number of tables) × 6 – 1

D (number of people seated) = (number of tables) × 4 + 4


6
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27
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Measurement

6

Hat trick at Qatar. In 2009, 23-year-old Australian Casey Stoner won the MotoGP in Qatar for the third season in a row.

The 22-lap race was postponed because torrential rain made it impossible to see far enough ahead on the twists and turns, with the floodlights reflecting off the wet track.

Casey won the race with a comfortable 7.771 second lead over Valentino Rossi. Riding his 800 cc motorbike, he reached speeds of up to 324.7 km/h on the 1.068 km main straight.

In 2007, Casey won his first World Motorcycle Grand Prix title, aged just 21. He was the 2008 Young Australian of the Year.

In 2010, Casey won his fourth straight Australian MotoGP at Phillip Island. He led all the way and crossed the line 8.598 seconds clear of Jorge Lorenzo.

Forum

In swimming and running races, times are measured to the nearest 0.01 seconds. In this fraction of time the swimmer or runner can move about 2 cm. How far would a motorcyclist move in this time? Why are times for Grand Prix and MotoGP races measured to the nearest 0.001 seconds?

Casey Stoner rides an 800 cc motorbike. What does the 'cc' stand for?

Do you think a knowledge of maths would help you become a good MotoGP racer?

Why learn this?

Measurement helps us keep records, whether for the height of a growing child or the volume of water in a dam. Good measurement skills are useful in many practical ways. They help a carpenter to work out how much timber is needed to construct a cupboard, enable a painter to calculate the volume of paint needed to paint a room and allow a gardener to determine the area of lawn to be planted or fertilised.

After completing this chapter you will be able to:

- choose appropriate units for measuring and convert between them
- estimate common measurements using standard units
- calculate the perimeter of different shapes
- calculate the area of different shapes including triangles and quadrilaterals
- calculate the volume of rectangular prisms and related objects.

Recall 6

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

1 Calculate the following.

(a) $3.4 + 6.8 + 4.2 + 2.1$ **(b)** $45.6 - 5.28$ **(c)** $54 + 11.3 - 26.04$

(d) 6×4.2 **(e)** 9.2×6.1 **(f)** 18.96×1000

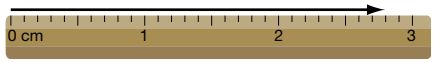
(g) $2780 \div 100$ **(h)** $3 \div 10$ **(i)** $45.9 \div 1000$

2 State the length of the arrow shown on each ruler:

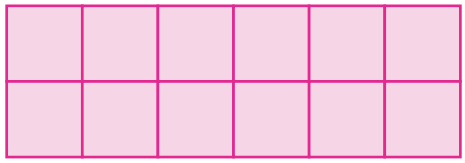
(i) in millimetres **(ii)** in centimetres.

(a)

(b)



3 This rectangle is made up of centimetre squares. Calculate:



(a) the perimeter in cm

(b) the area in cm^2.

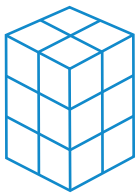
4 Copy and complete the following conversions.

(a) 2 km = _____ m **(b)** 30 mm = _____ cm **(c)** 5 m = _____ cm

(d) 1500 g = _____ kg **(e)** 4 L = _____ mL **(f)** 20 kg = _____ g

(g) 250 mL = _____ L **(h)** 1.75 m = _____ mm **(i)** 500 g = _____ kg

5 How many sugar cubes are in the stack shown here?

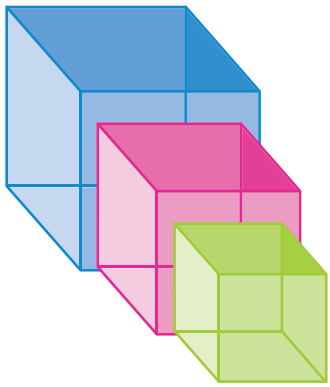


Exploration Task

You can download this activity from the eBook or the Pearson Places website.

What size?

In this activity, you will investigate the possible range of dimensions for a room of a fixed volume. Are all possibilities reasonable?



Units of length

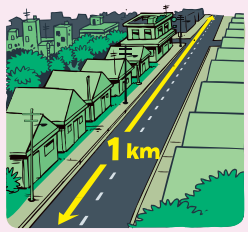
6.1

The system of measurement used in Australia before 1970 was the 'imperial', or 'British' system, which uses units of length such as inches, feet, yards, miles, chains and furlongs. In this system, it can be difficult to convert between different units.

In late 1700s, French scientists instituted a new scientific system of measurement, known as the metric system. All lengths in the metric system come from dividing or multiplying a standard length, the '**metre**', by powers of 10 (such as 10, 100, 1000, …). This makes it easier to convert between different lengths and different units. Over time, it was developed into the International System of Units (SI), which is used by scientists and engineers across the world today.

From 1970, Australia steadily introduced the measurement units of the metric system as the official measurement units to be used. The metric units of length used most often are:

In the 1100s, King Henry I of England decreed that the measure of his own arm would be applied as the standard length for a 'yard'. This unit of length was then used for nearly 900 years!

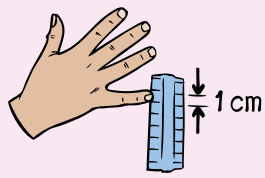


kilometre (km)

1 km = 1000 metres

metre (m)

1 m = 100 centimetres



centimetre (cm)

millimetre (mm)

1 cm = 10 millimetres

When estimating lengths, it helps to be able to picture the size of each unit of length, or compare the object in question to a 'reference' measurement.

Worked example 1

W.E. 1

Use the reference measurement to help you estimate:

(a) the length

(b) the height

of this motor home.

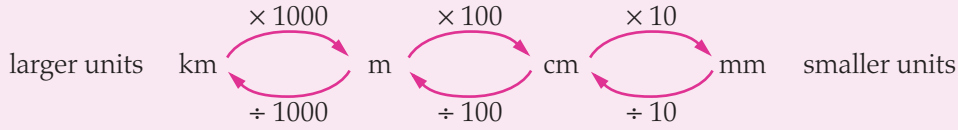
Thinking	Working
(a) Look at the reference measurement. Estimate how many times this length would need to be used to match the length of the motor home.	(a) The motor home is approximately 5 m long.

(b) Again, estimate how many times the length of the reference measurement would need to be used to match the height of the motor home.

(b) The motor home is about 3 m high.

Converting units of length

Easily converting between units is a great advantage of the metric system. Converting between metric units of length is easy because you only ever need to multiply or divide by powers of 10.



To convert from a *larger* unit to a *smaller* unit, you *multiply*.
To convert from a *smaller* unit to a *larger* unit you *divide*.

The conversion table above shows that when converting from kilometres to metres, you multiply by 1000. When converting from centimetres to metres, divide by 100. To convert from kilometres to centimetres you multiply by 1000, then by 100.

Worked example 2

W.E. 2

Copy and complete the following conversions.

(a) 62.4 km = _____ m **(b)** 87.5 cm = _____ m

Thinking	Working
(a) Larger unit to smaller unit, so multiply. There are 1000 m in 1 km, so multiply by 1000.	*(a) 62.4 km* *= 62.4 × 1000 m* *= 62 400 m*
(b) Smaller unit to larger unit, so divide. There are 100 cm in 1 m, so divide by 100.	*(b) 87.5 cm* *= 87.5 ÷ 100 m* *= 0.875 m*

Worked example 3

W.E. 3

Copy and complete the following conversions.

(a) 8.7 m = _____ mm **(b)** 530 cm = _____ km

Thinking	Working
(a) 1 Larger unit to smaller unit, so multiply. First, multiply by 100 to convert m to cm.	*(a) 8.7 m* *= 8.7 × 100 cm* *= 870 cm*
2 Then, multiply by 10 to convert cm to mm.	*= 870 × 10 mm* *= 8700 mm*

(b)	**1**	Smaller unit to larger unit, so divide. First, divide by 100 to convert cm to m.	*(b)* 530 cm = 530 ÷ 100 m = 5.3 m
	2	Then, divide by 1000 to convert m to km.	= 5.3 ÷ 1000 km = 0.0053 km

Notice that in part **(a)** of the example above, you multiply by 100, then by 10. This is equivalent to multiplying by 1000. In part **(b)** you divide by 100, then by 1000. This is equivalent to dividing by 100 000.

6.1 Units of length

Navigator

1, 2 (columns 1–2), 3 (columns 1–2), 4, 5, 6, 7, 8, 9, 10, 13, 14, 16, 17, 18, 19	1, 2 (columns 2–3), 3 (columns 2–3), 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19 (a–b), 20	1, 2 (column 3), 3 (column 3), 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20

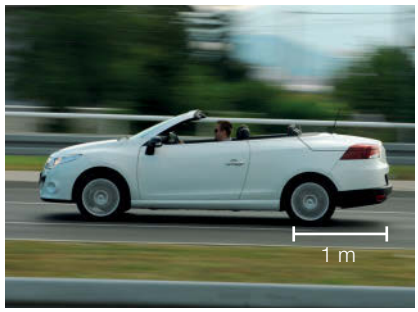
Answers p. 683

Fluency

1 Use the reference measurement to help you estimate the following.

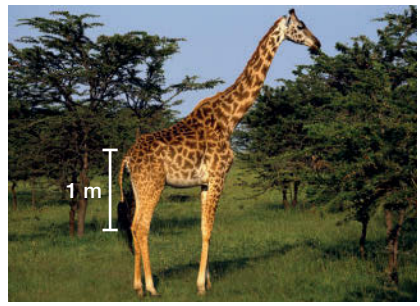
W.E. 1

(a)



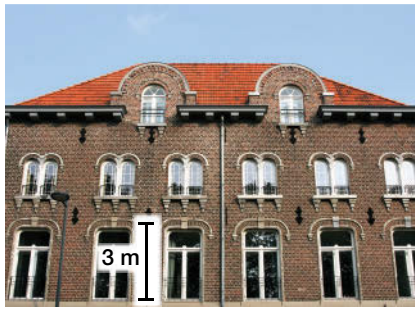
The length of this car.

(b)



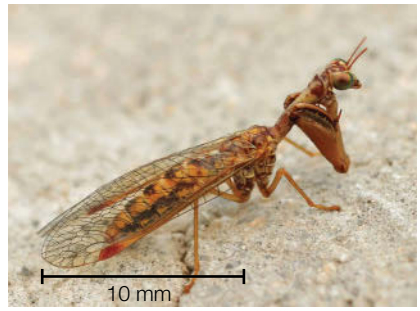
The adult giraffe's height.

(c)



The height of the tallest part of this building.

(d)



The length of this mantis fly.

W.E. 2

2 Copy and complete the following conversions.

(a) 5 km = _____ m **(b)** 3.6 km = _____ m **(c)** 0.008 km = _____ m

(d) 65 m = _____ cm **(e)** 0.55 m = _____ cm **(f)** 1.2 m = _____ cm

(g) 2.9 cm = _____ mm **(h)** 6.1 cm = _____ mm **(i)** 0.35 cm = _____ mm

(j) 90 cm = _____ m **(k)** 0.3 cm = _____ m **(l)** 3750 cm = _____ m

(m) 4200 m = _____ km **(n)** 570 m = _____ km **(o)** 9.2 m = _____ km

(p) 80 mm = _____ cm **(q)** 255 mm = _____ cm **(r)** 1.8 mm = _____ cm

W.E. 3

3 Copy and complete the following conversions.

(a) 3.2 m = _____ mm **(b)** 4.95 m = _____ mm **(c)** 9000 cm = _____ km

(d) 34 500 cm = _____ km **(e)** 0.003 km = _____ cm **(f)** 560 mm = _____ m

(g) 97 mm = _____ m **(h)** 0.342 km = _____ mm **(i)** 7800 mm = _____ km

(j) 1.92 km = _____ cm **(k)** 2400 cm = _____ km **(l)** 0.89 m = _____ mm

4 State which metric unit would be most appropriate for measuring:

(a) the length of your foot **(b)** the distance from Sydney to Melbourne

(c) the distance around an athletics track **(d)** the length of a shoelace

(e) the distance run in a marathon **(f)** the length of your bed.

5 (a) Which of the following gives the best estimate of the distance across a suburban street?

A 50 cm **B** 5 m **C** 50 m **D** 5 km

(b) Which of the following gives the best estimate of the length of a mouse's tail?

A 0.1 cm **B** 1 cm **C** 10 cm **D** 100 cm

(c) The diameter of a 20 cent coin is about:

A 2.8 mm **B** 2.8 cm **C** 2.8 m **D** 2.8 km

(d) The length of a ball-point pen could be:

A 15 mm **B** 50 mm **C** 15 cm **D** 15 m

(e) The height of Mount Everest is closest to:

A 8.9 mm **B** 8.9 cm **C** 8.9 m **D** 8.9 km

6 List the lengths below in order from smallest to largest, by first converting them to a common unit.

(a) 0.4 m, 4000 mm, 4 cm, 0.04 km **(b)** 3.2 km, 360 m, 290 000 cm, 3100 m

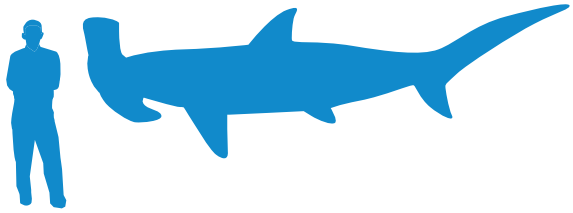
(c) 71.4 m, 0.64 km, 5600 cm, 820 000 mm **(d)** 0.9 cm, 90 mm, 0.095 m, 0.0089 km

Understanding

7 At the Beijing Olympics in 2008, Australia's Steve Hooker won a gold medal and broke the Olympic record in the men's pole vault, clearing the bar at 596 centimetres. How many metres is this?

8 Situated on Heard Island, Big Ben is one of Australia's few active volcanoes. Its summit is 2745 m above sea level. What is its height in kilometres?

9 The average man is 1.8 metres tall. Use this scale diagram to estimate the length of the hammerhead shark.



10 The bird-eating spider of South America has a body length of 89 mm and a leg span of 254 mm. Write these measurements in centimetres.

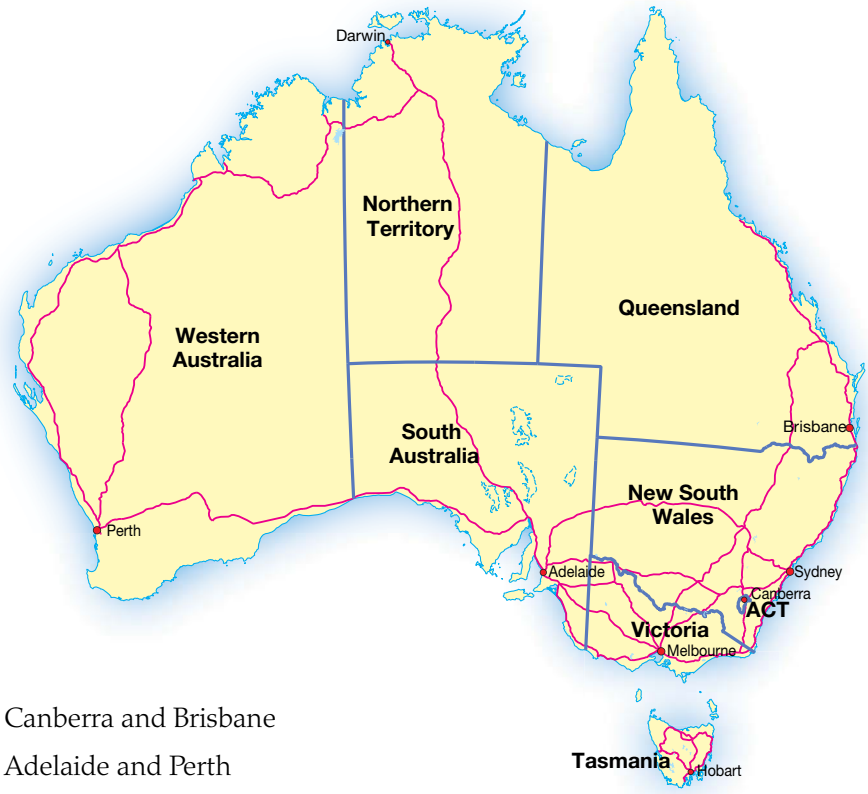
11 A giant jellyfish was once measured at 2290 mm across and 36 000 mm long. Convert these dimensions to metres.

12 Claire is building a desk and she wants it to be wide enough so that she can fit the length of two A4 sheets across it. If the length of an A4 sheet is 298 mm, how wide does Claire's desk have to be in millimetres? What is this length in centimetres?

13 A carpenter cuts a piece of timber so that it is 1.2 m wide and 3.7 m long. Convert the width and length measurements into millimetres.

Reasoning

14 The direct distance 'as the crow flies' between Melbourne and Adelaide is about 800 km. Estimate the direct distance between:



(a) Canberra and Brisbane

(b) Adelaide and Perth

(c) Sydney and Darwin.

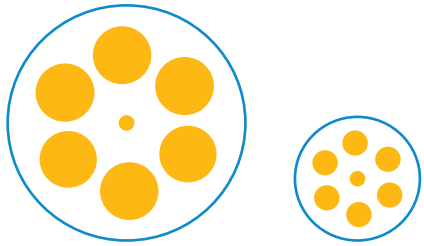
15 One of the fastest growing plants in the world is the bamboo plant. Its stem can grow 300 mm in one day. If, on Monday morning, a bamboo plant was one metre tall, and it grew 300 mm every day, how tall will it be on Saturday morning? Write the answer in metres.

16 An athlete runs 100 m each minute. If she starts running at 10:00 am, how many kilometres will she have run by 10:20 am?

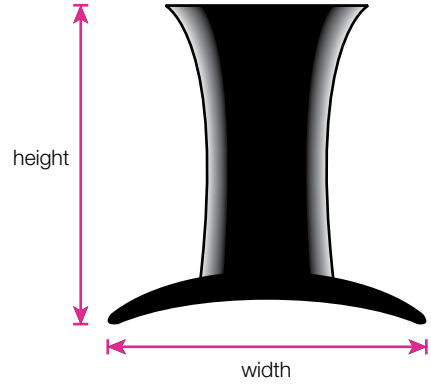
17 **(a)** Guess the answer to each of the following, then use your ruler to help you decide.

(i) Which horizontal line is longer?

(ii) Which diagram has the larger central circle?



(iii) Which is longer, the height of the hat or its width?



(b) Explain each illusion or effect above and why you think it happens.

Open-ended

18 A builder wrote the measurements needed for a door.

height of door	2032 mm
width of door	821 mm

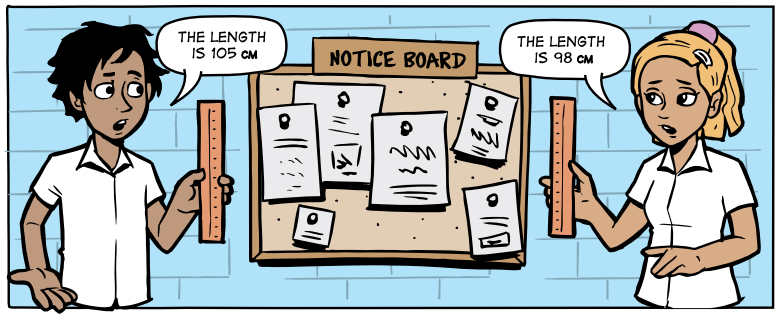
Why did the builder write these measurements using millimetres instead of cm or m?

19 The following are mistakes made by a student when converting units. For each, explain what mistake has been made and write the correct answer.

(a) 27 m = 0.27 cm ✗ (b) 765 mm = 7.65 cm ✗

(c) 3800 m = 3.08 km ✗ (d) 1356 cm = 1.356 m ✗

20 Two students have been asked to measure the length of the school notice board.



Suggest possible reasons for the difference in their answers.

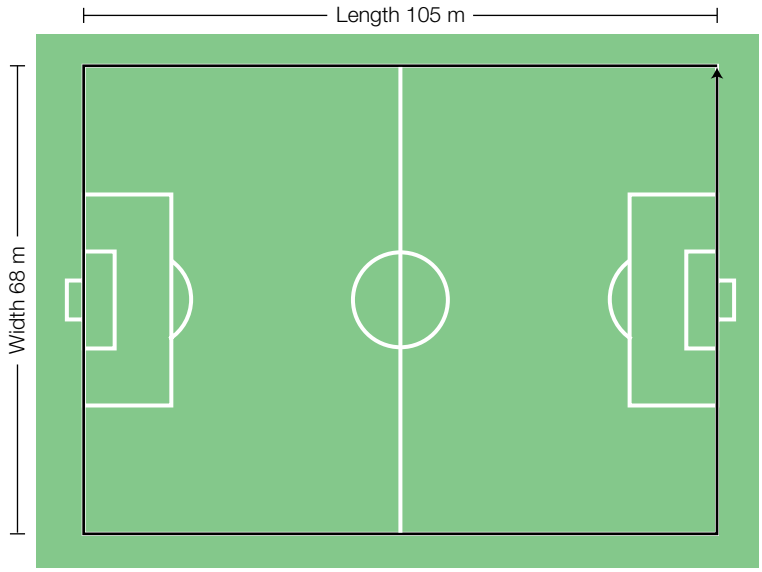
6.2 Perimeter

Perimeter is the distance along the boundary of a shape.

The word perimeter comes from two Greek words: *peri*, meaning 'around' and *metron*, meaning 'measure'.

Here are some examples:

- A frame goes along the perimeter of a picture.
- A fence is built along the perimeter of a property.
- A soccer pitch has a boundary line marked along its perimeter (shown here in red).



To find the perimeter of a shape, you simply add up the lengths of the sides (first making sure they are all in the same units).

For example, the perimeter of this soccer pitch is 105 + 68 + 105 + 68 = 346 m.

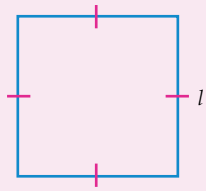
Because the pitch is rectangular in shape, it has 2 pairs of equal sides: 2 lengths and 2 widths.

You can use this to write the perimeter of the pitch as $P = 2 \times 105 + 2 \times 68$ m. If you let the pronumerals l and w represent the length and the width, you can write a formula for the perimeter of a rectangle: $P = 2l + 2w$. This formula can also be written as $P = 2(l + w)$: Add the length and the width, then multiply by 2.

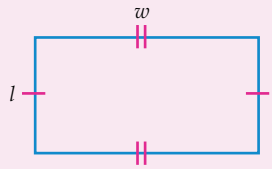
A square has 4 equal sides. If you use l to represent the length of each side, you can write the perimeter as: $P = l + l + l + l$. A shorter way to write this is $P = 4l$.

To find the perimeter of a square or a rectangle:

$P = 4l$



$P = 2l + 2w$
or
$2(l + w)$



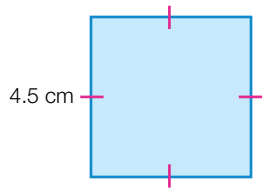
Sides that are marked in the same way (with the same number of small | dashes) have the same length.

Worked example 4

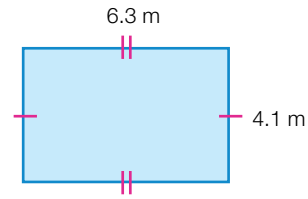
W.E. 4

Find the perimeter of each of these shapes.

(a)



(b)

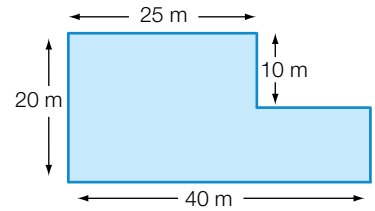


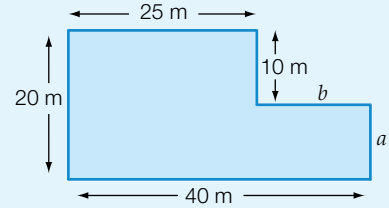
Thinking	Working
(a) 1 Write the formula for the perimeter of a square.	(a) $P = 4l$
2 Substitute the side length into the formula.	$= 4 \times 4.5$
3 Evaluate, writing the answer with the correct units.	$= 18$ cm
(b) 1 Write the formula for the perimeter of a rectangle.	(b) $P = 2l + 2w$
2 Substitute the side lengths into the formula.	$= 2 \times 6.3 + 2 \times 4.1$ $= 12.6 + 8.2$
3 Evaluate, writing the answer with the correct units.	$= 20.8$ m

Worked example 5

W.E. 5

Calculate the perimeter of the shape below.



Thinking	Working
1 First, find the lengths of the unmarked sides. Label them a and b, then use the given lengths to find them.	 $a = 20$ m $- 10$ m $= 10$ m $b = 40$ m $- 25$ m $= 15$ m
2 Add up the lengths of all the sides.	$P = 10 + 25 + 20 + 40 + 10 + 15$ $= 120$ m

6.2 Perimeter

Navigator

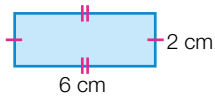
Answers p. 684

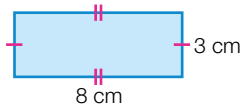
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15	2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15

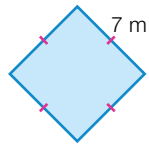
Fluency

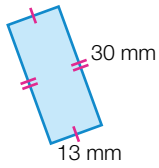
W.E. 4

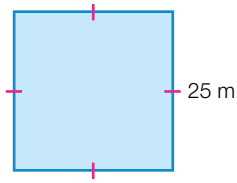
1 Find the perimeter of each of these shapes.

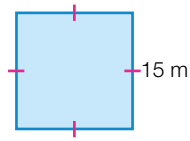
(a) 

(b) 

(c) 

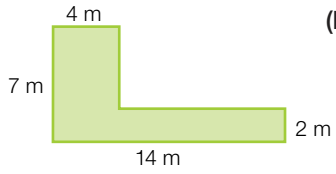
(d) 

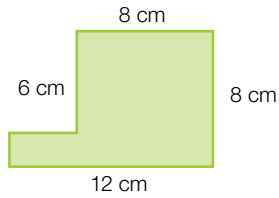
(e) 

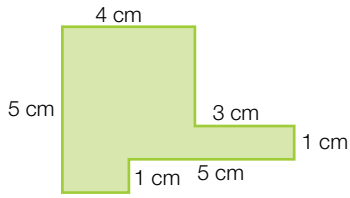
(f) 

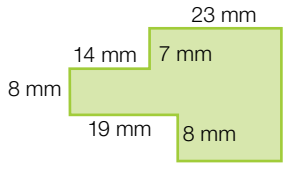
W.E. 5

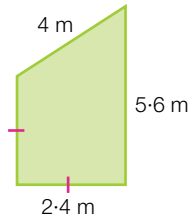
2 Calculate the perimeter of the shapes below.

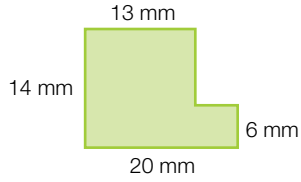
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

3 **(a)** What is the perimeter of a rectangle with a length of 25 cm and a width of 10 cm?

A 35 cm **B** 45 cm **C** 70 cm **D** 250 cm

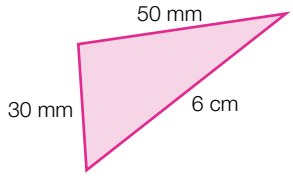
(b) What is the perimeter of a square of side length 460 mm?

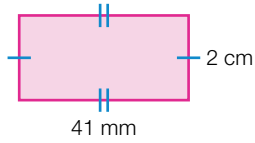
A 18.4 m **B** 9.2 m **C** 1.84 m **D** 0.92 m

(c) What is the perimeter of a square of side length 3 km?

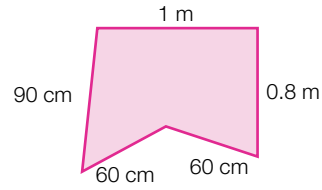
A 3 km **B** 9 km **C** 12 km **D** 6 km

4 Calculate the perimeter of each of the following, first converting side lengths to the smaller unit when necessary.

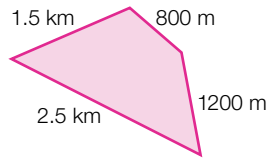
(a) 

(b) 

(c)



(d)



Understanding

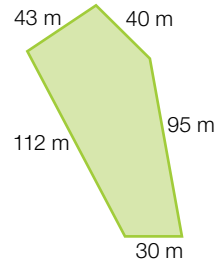
5 A yacht sails three straight legs of a course which form a triangle. The length of the legs are 2.5 km, 1.7 km and 3.9 km. What distance does the yacht sail to complete the course?

6 An interior decorator is making a border to go around the length and width of a rectangular wall in a house. The wall is 3 m high and 4 m long. What is the total length of the border?

7 A card maker is making a glitter border on cards that are 25 cm long and 20 cm wide. If there are 4 cards that each need the glitter border, then what is the total length of the glitter border?

8 A group of students run around the perimeter of their gym, which is rectangular and has dimensions 38 m by 16 m. What distance will they run if they complete 6 laps?

9 Juan wants to run two strands of wire around his property to mark its boundary. A plan of the property is shown at right. If the wire costs 60 cents per metre, how much will the wire for this boundary cost?



10 Estimate the perimeter of each of the following objects.

(a) your desk **(b)** a basketball court

Reasoning

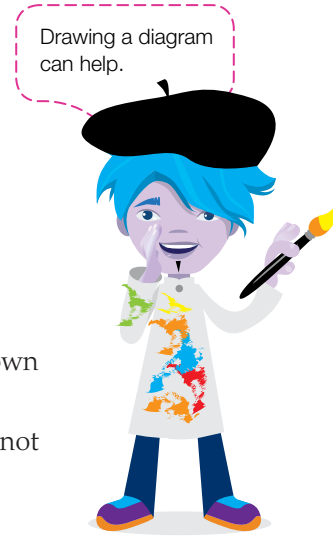
11 If the perimeter of a rectangle is 240 mm and its width is 20 mm, then its length is:

A 100 mm **B** 110 mm **C** 120 mm **D** 220 mm

12 If the perimeter of a square table is 64 cm, then what are its side lengths?

A 8 cm **B** 16 cm **C** 32 cm **D** 6.4 cm

13 The local swimming pool is rectangular, with dimensions of 25 m by 12 m. If it has a 1 m wide rectangular path around its perimeter, what is the distance along the outer edge of the path?



Open-ended

14 The length and width of a rectangle are both whole numbers in centimetres. Write down some possible dimensions if its perimeter is 84 cm.

15 'I am holding a picture of a shape with a perimeter of 16 cm' the teacher said, 'but it is not a rectangle nor a triangle'. Draw two possible examples of the shape this may be.

Problem solving

Ram's rulers

Ram is planning a woodwork model and needs to draw a line 6 cm long. For rulers, he has been using two straight pieces of wood with no markings, but he knows their lengths are 10 cm and 8 cm. How can he use these two pieces of wood to measure a 6 cm line?

Strategy options

- Guess and check.
- Work backwards.
- Make a model.

6.3 Area

Before laying tiles on a bathroom floor, you need to find the **area** of the floor so that you know how many tiles to buy.

Area is the amount of surface inside a plane (flat) shape.

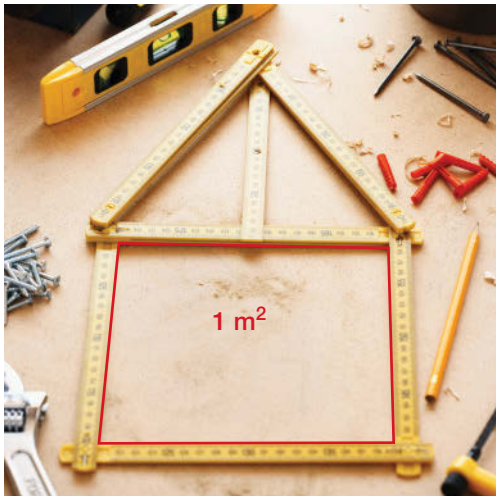
Area is measured in 'square' units.

When you calculate the area of a shape you are finding the number of squares that can fit inside the shape.

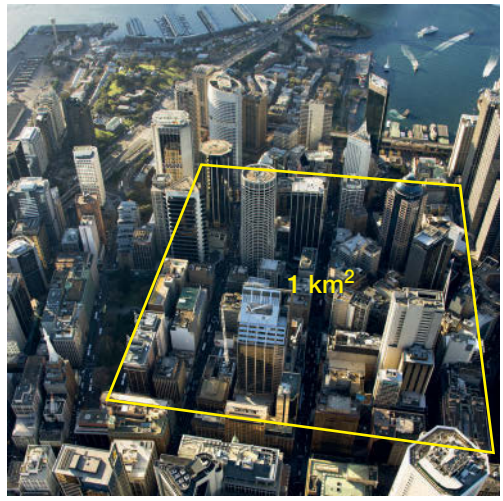
Units of area

One square millimetre, mm^2 (actual size)
(1 mm length × 1 mm width = 1 mm^2)

One square centimetre, cm^2 (actual size)
(1 cm length × 1 cm width = 1 cm^2)



One square metre, m^2
(1 m length × 1 m width = 1 m^2)

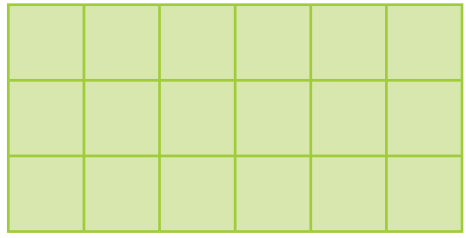


One square kilometre, km^2
(1 km length × 1 km width = 1 km^2)

You can say the names of square units, such as 1 cm^2, as '1 centimetre squared' or as '1 square centimetre'.

Area of a rectangle

Consider this rectangle of length 6 cm and width 3 cm.



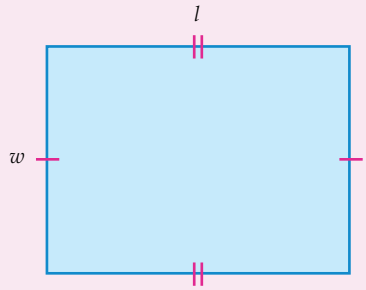
The rectangle contains 3 rows of 6 squares.
Its area is equal to 3 cm × 6 cm = 18 cm^2.

The area (A) of a rectangle is equal to its length (l) multiplied by its width (w), or $A = l \times w$.

This formula can be written as $A = lw$.

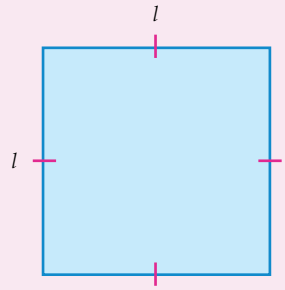
The length and width of a square are equal, so the area of a square can be found using the formula $A = l \times l$, or $A = l^2$.

Area of a rectangle



$A = lw$

Area of a square



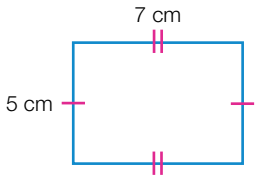
$A = l^2$

Worked example 6

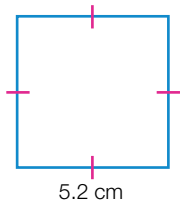
W.E. 6

Calculate the area of the following shapes.

(a)



(b)



Thinking		Working	
(a)	1 Write the formula for the area of a rectangle.	(a)	$A = lw$
	2 Identify l and w, and substitute their values into the formula.		$A = 7 \times 5$
	3 Evaluate, writing the answer with the correct units.		$A = 35\ cm^2$
(b)	1 Write the formula for the area of a square.	(b)	$A = l^2$
	2 Identify l and substitute its value into the formula.		$A = 5.2 \times 5.2$
	3 Evaluate, writing the answer with the correct units.		$A = 27.04\ cm^2$

6.3 Area

Navigator

Answers p. 684

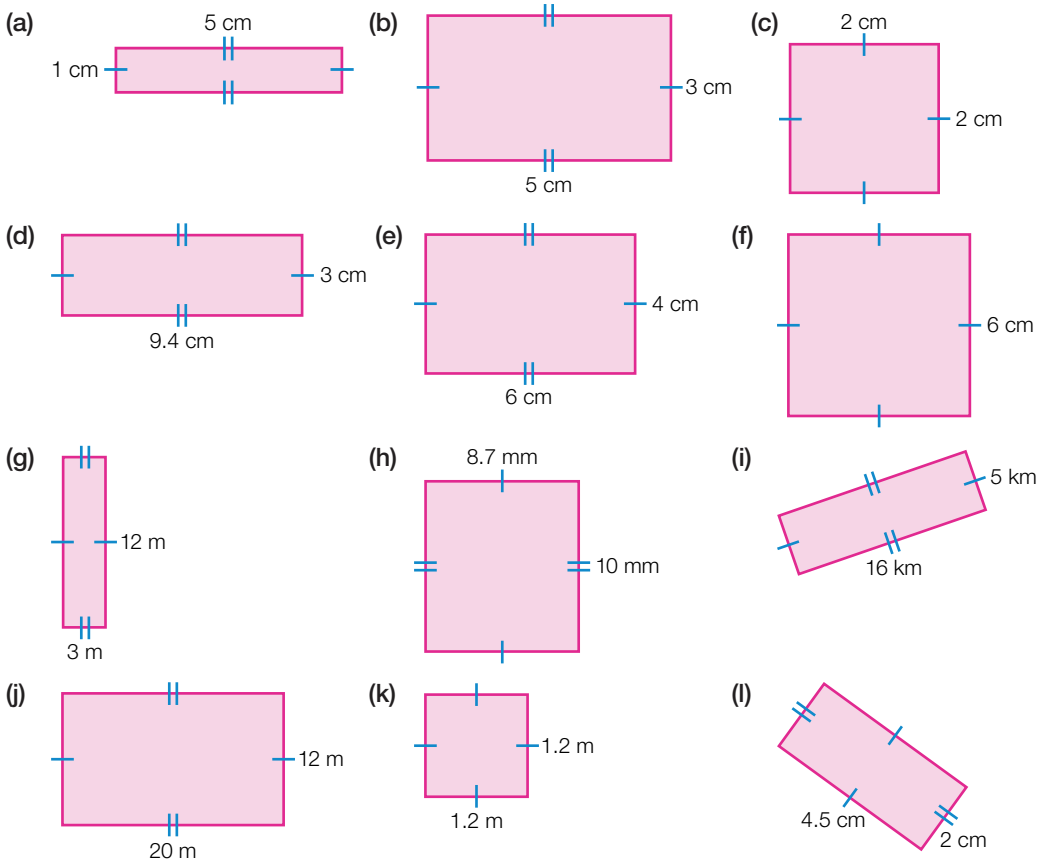
1 (columns 1–2), 2, 3, 4, 5, 6, 7, 8, 10, 12, 14, 16, 17, 18, 22, 23, 24, 25	1 (columns 2–3), 2, 3, 4, 5, 6, 8, 10, 11, 12, 14, 15, 16, 17 (a–b), 18 (a), 20, 21, 22, 23, 24, 25	1 (column 3), 2 (column 1), 3, 4 (b, d), 5, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18 (a), 19, 20, 21, 22, 24, 25

Equipment required: calculator may be used for Questions **12–16**

Fluency

W.E. 6

1 Calculate the area of the following shapes.



2 State which of the units of area—mm^2, cm^2, m^2 or km^2—would be most suitable for measuring the area of:

(a) a sheet of writing paper
(b) your little toenail
(c) a pizza
(d) a house block
(e) a football oval
(f) Tasmania
(g) a small watch face
(h) the floor of a classroom.

3 **(a)** What is the area of a square of side length 3 km?

A $3\ km^2$ **B** $9\ km^2$ **C** $12\ km^2$ **D** $6\ km^2$

(b) A square has an area of 25 cm^2. What is its side length?

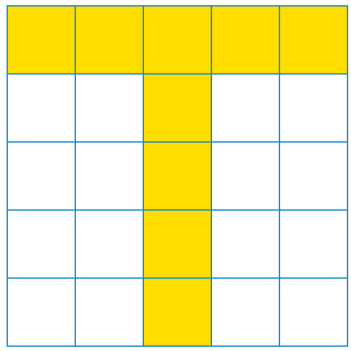
A 5 cm **B** 12.5 cm **C** 2.5 cm **D** 25 cm

(c) If the area of a rectangle is 24 cm^2 and its width is 10 mm, then its length is:

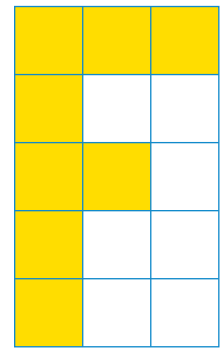
A 2.4 mm **B** 2.4 cm **C** 11 cm **D** 24 cm

4 The following shapes have been drawn on centimetre grid paper. Find **(i)** the perimeter and **(ii)** the area of each one.

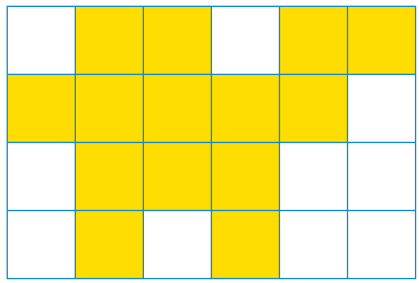
(a)



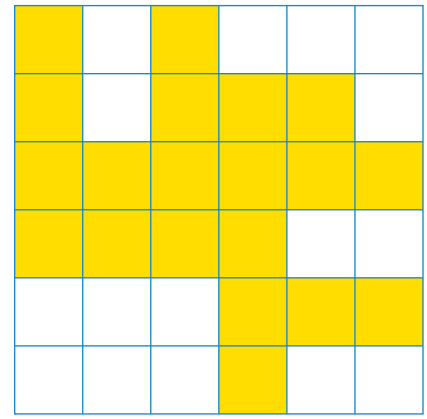
(b)



(c)



(d)



Understanding

5 What is the area in cm^2 of a rectangle of length 40 mm and width 4 cm?

6 Find the area in km^2 of a rectangle of length 3 km and width 800 m.

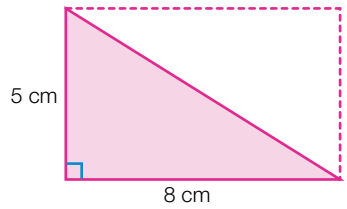
7 What is the area in cm^2 of a rectangle of length 450 mm and width 70 cm?

8 Find the width of a rectangle with length 7 cm and area 28 cm^2.

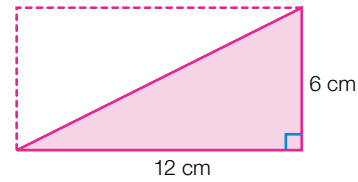
9 Find the length of a rectangle with width 2.6 m and area 13.52 m^2.

10 Find the area of each of the following triangles by first considering a rectangle.

(a)



(b)



11 Find the side length of a square table whose area is 400 cm^2 and perimeter is 80 cm. (Check your answer by calculating the area from the side length.)

12 The Ray family are installing solar power to their house. There will be 6 panels attached to their roof, each with dimensions of 2.4 m × 2.1 m.

(a) What is the area of each solar panel?

(b) What is the total area of the solar panels?

(c) If the solar panels generate a maximum of 6.5 kilowatt-hours (kWh) of energy per square metre each day, how much energy could the Ray family's new solar system produce in a day?

13 Guy wants to plant a lawn in a rectangular section of his yard, which is 8.5 m long and 6.9 m wide.

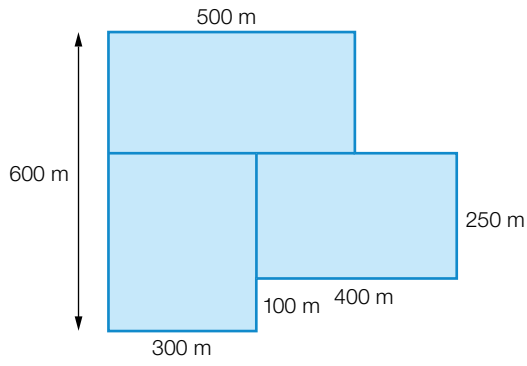
(a) How many grams of lawn seed will he need to use if the recommended seed coverage is 60 grams per square metre?

(b) What length of timber would Guy need to purchase to create a timber border for the section?

14 A builder wants to estimate the number of bricks required to build a wall 4.2 m wide and 2.6 m high. If 1 square metre of brick wall contains 48 bricks, how many bricks are needed?

15 Jasmin decided to make a 45 cm square pillow for her study chair. One piece of fabric will be required for the front of the pillow, and one for the back. Each piece of fabric will need an extra 1 cm on all edges for the front and back to be sewn together. What area of fabric will Jasmin need to make the pillow?

16 A farmer wants to create three separate rectangular paddocks on his farm, as shown in the diagram.



(a) Find the area of the three paddocks using the plan provided.

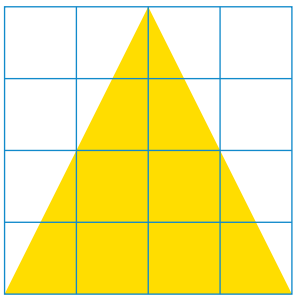
(b) To sow his paddocks, the farmer needs to spread 60 g of seed for every square metre. How many kilograms of seed will he need?

(c) What is the total length of fencing needed to surround each of the three paddocks?

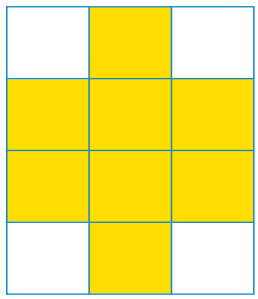
Reasoning

17 The following shapes have been drawn on centimetre grid paper. Find the shaded area in each case.

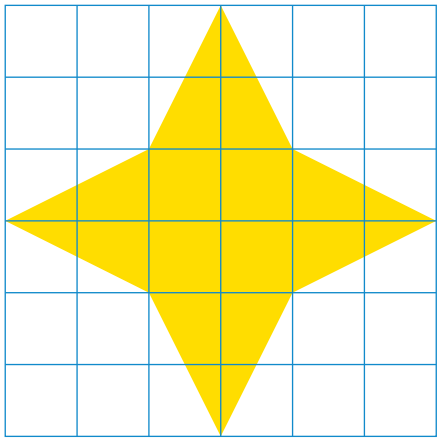
(a)



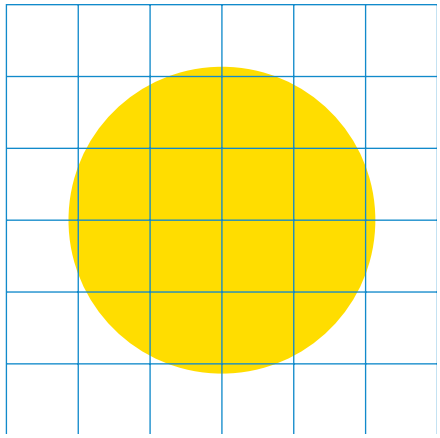
(b)



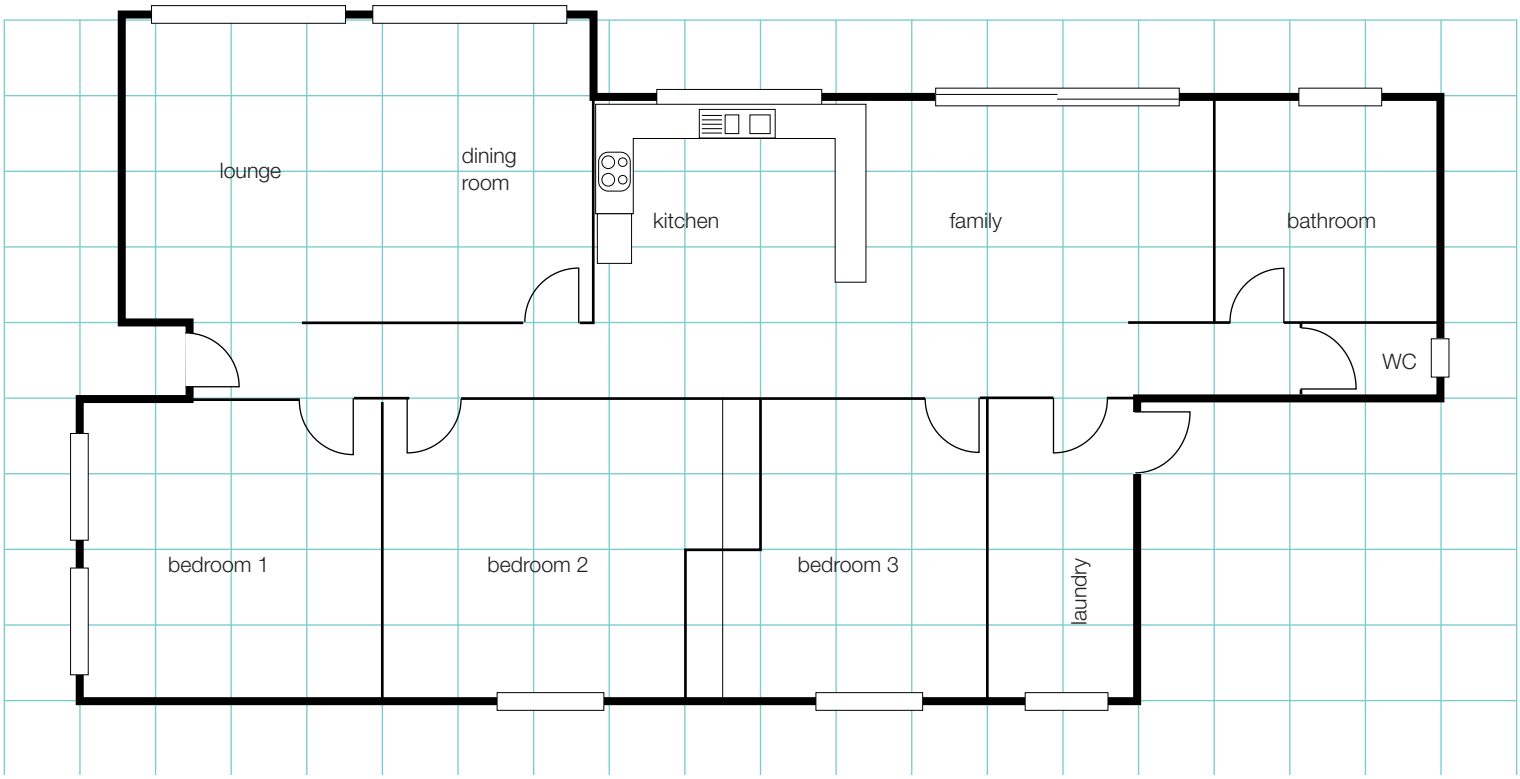
(c)



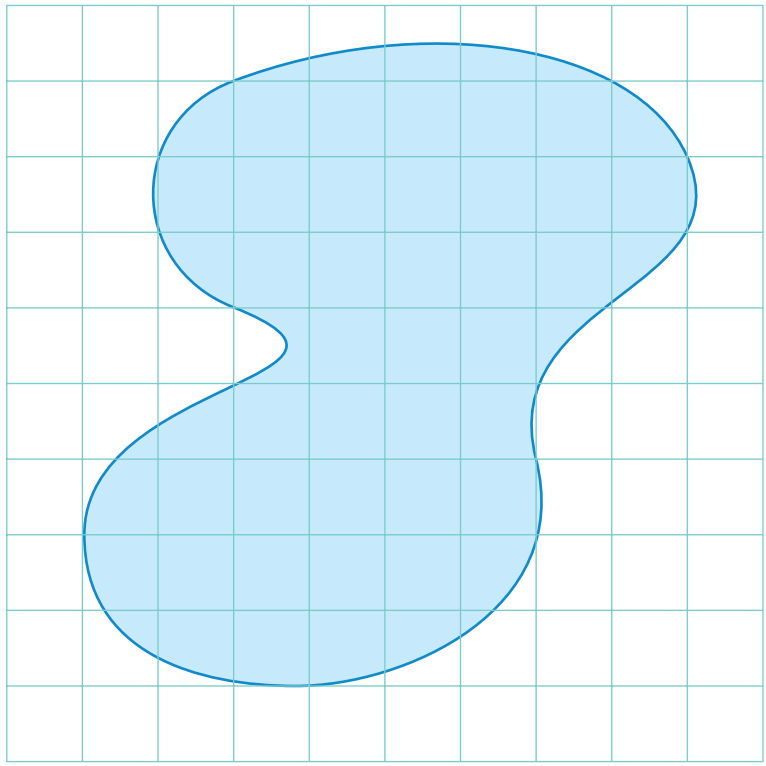
(d)



18 (a) Use the grid to estimate the area covered by the floor plan of this house. The squares on the plan below represent square metres.



(b) Use the grid to estimate the area of the lake. The squares on the grid represent square kilometres.

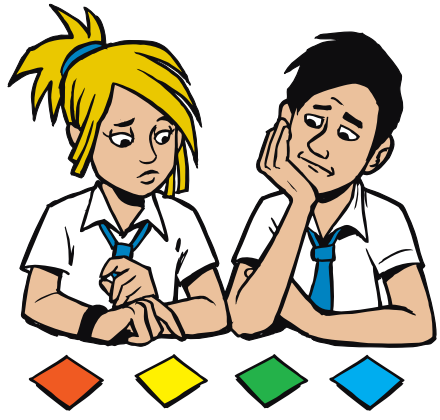


19 Write all the possible whole number dimensions of a rectangle with an area of 20 cm^2.

20 Find the length and width of a rectangular room whose area is 60 m^2 and perimeter is 34 m.

Open-ended

21 Ethan and Magda were asked to each make a shape using four 1 cm squares, and then find the perimeter of their shapes. Ethan said the perimeter of his shape was 10 cm, whereas Magda said her shape had a perimeter of only 8 cm. Could they both be right? Draw what their shapes may have looked like.



22 What method would you use to measure the area of the following objects? Find these areas.

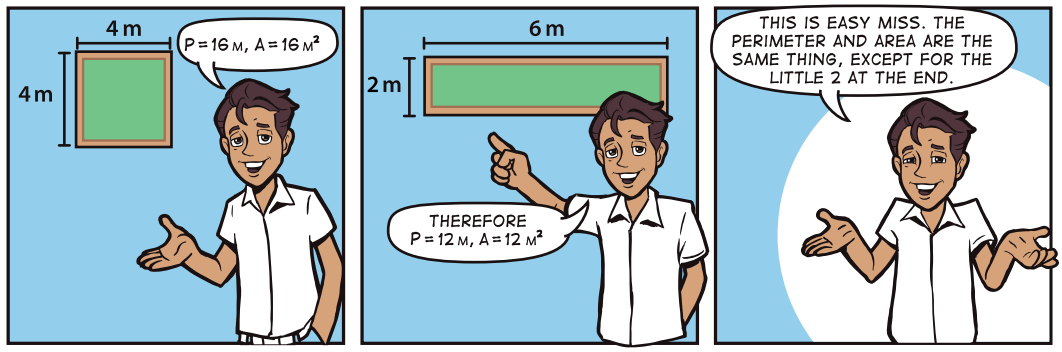
(a) your workbook

(b) your desk

(c) your classroom

23 Trace the shape of your pencil case or phone on centimetre grid paper, then find the area of the shape.

24



(a) One of Sam's calculations is not right. What has he done wrong?

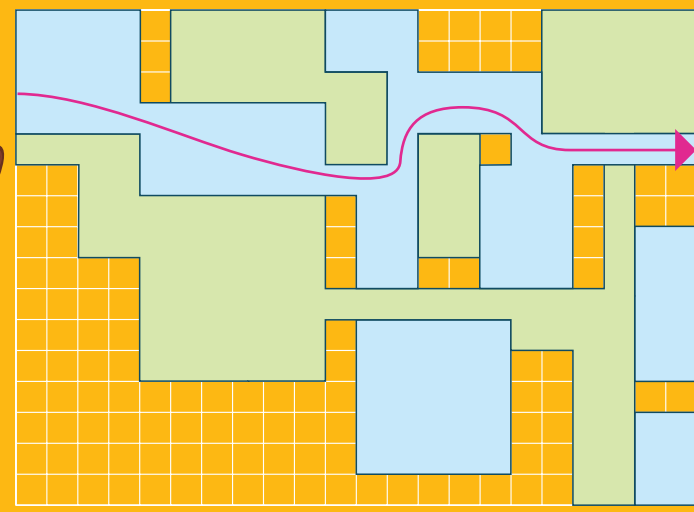
(b) How would you explain the difference between area and perimeter to Sam?

(c) What is meant by the 'little 2' and why must it be written on the answers for area?

25 List three materials that are sold by the square metre.

Game

Jungle pathways



Equipment required:
graph paper or grid paper, 2 dice, 2 different coloured pens or pencils

There are jungles in the world that have never been crossed. The race is on to be the first to find the way through.

How to win:
Make an unbroken path of blocks from one side of the jungle (graph paper) to the other. Your opponent will be trying to cut you off as you both race to finish first.

How to play:

1. Take turns to roll the dice. The two numbers rolled represent the size of the rectangular block you will draw, its length and its width.
2. Choose which die will be used for width and which will represent length, and then draw your block on the graph. The block drawn must be a rectangle or a square.
3. You can draw your block anywhere on the graph paper; however, blocks cannot overlap with another block, or go off the graph. If the numbers rolled cannot make a rectangle or square that will fit on the grid, the player does not draw a block for that turn.
4. The first player to make a path of their own blocks, connected all the way from one side to the other, is the winner!

Investigation

Paddocks

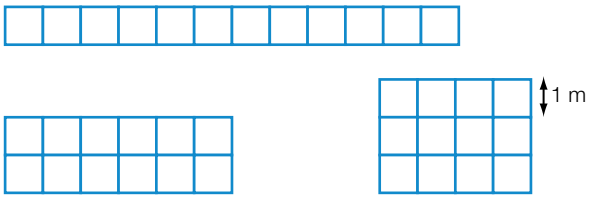
Equipment required:
graph paper

The Big Question

A farmer is planning to fence an area of 400 m^2 for the calves on the farmer's property. What dimensions (length and width) would give the paddock an area that is cheapest to fence?

Engage

1 Shapes with the same area can have different perimeters. Each of the rectangles below has been made using 12 squares. Each square has a side length of 1 m.



(a) Copy and complete the table below for the three rectangles.

Dimensions of shape	Area (m^2)	Perimeter (m)
1×12		
2×6		
3×4		

(b) Which of the rectangles above has the shortest perimeter?

(c) Why would the farmer be interested in having the shortest perimeter?

Explore

2 List all possible pairs of whole number dimensions that would give a paddock with an area of exactly 400 m^2.

Strategy options

- Draw a diagram.
- Test all possible combinations.

3 (a) Select three of the paddocks you think are most suitable for containing the calves. Draw the three paddocks on graph paper, showing their dimensions. To show their relative sizes, use a scale of 1 cm on page = 5 m in real life.

(b) Find the perimeter of each of the three paddocks.

4 Use the following information to complete the table on the following page and calculate the total cost of the fencing for each paddock.

Fencing wire: Each fence has five strands of wire. Each strand goes all the way around the fence. The wire costs 85 cents per metre.

Corner posts: The paddocks will need a large post at each corner, each costing $40.

Standard posts: There must be a post at least every 5 m along a section of fence. If the distance between the corner posts is longer than 5 m, a standard post is put along the fence. Mark where the standard posts will be required on the diagrams of each of your paddocks, and count the number needed for each paddock. Each standard post costs $15.

This table will help you to calculate the total cost of constructing each paddock. Copy the table and complete the columns for each of your three paddocks.

Paddock	Dimensions	Perimeter	Metres of fencing wire needed	Cost of fencing wire	Number of corner posts needed	Cost of corner posts	Number of standard posts needed	Cost of standard posts	Total cost
Paddock 1									
Paddock 2									
Paddock 3									

Explain

5 What happens to the perimeter of a paddock as it gets longer in length and shorter in width? Try to explain why this is. The 12 m^2 rectangles you used in **1** may help.

Elaborate

6 Answer the Big Question by stating which of the three paddocks you think the farmer should choose, giving reasons for your choice.

Evaluate

7 **(a)** What information did you discover from the rectangles in **1** that could be used to determine the dimensions of a larger paddock with the shortest perimeter?

(b) Why did you select the three paddocks that you did?

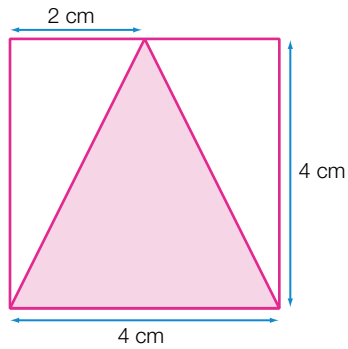
(c) Cost is one consideration when designing paddocks. What are other considerations that the farmer would need to take into account?

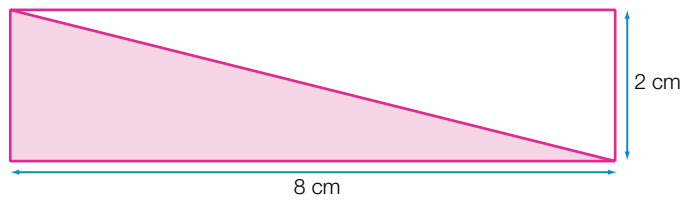
Extend

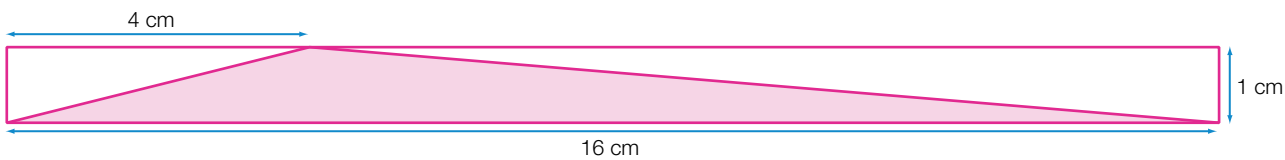
8 Investigate the perimeter and the area of other shapes by looking at the triangles below. Each triangle has an area of 8 cm^2.

(a) Copy them onto graph paper using the actual measurements given in the diagrams. Then measure the perimeter of each triangle with a ruler.

(b) Which of the three triangles below has the shortest perimeter? Is this what you expected? Explain.





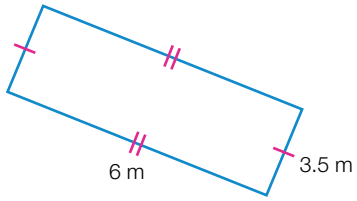


Half-time 6

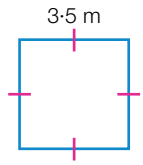
6.2, 6.3

1 Find **(i)** the perimeter and **(ii)** the area of the following shapes.

(a)



(b)



6.1

2 Complete the following conversions.

(a) 45 cm = _____ m **(b)** 0.015 km = _____ m **(c)** 307 m = _____ km

(d) 0.6 km = _____ cm **(e)** 196 000 m = _____ km **(f)** 2640 cm = _____ km

6.2, 6.3

3 A garden bed is 5.2 m long and 3 m wide.

(a) How much fertiliser will be required to cover the garden bed, if the recommended amount is 0.75 kg per square metre?

(b) What length of wooden fencing will be needed to surround the bed?

6.3

4 Which unit of area—mm^2, cm^2, m^2 or km^2—would be the most appropriate for measuring the area of:

(a) a soccer field

(b) a postage stamp

(c) a dinner plate

(d) a national park?

6.3

5 A rectangle has a perimeter of 26 cm and an area of 30 cm^2. What are its dimensions (length and width)?

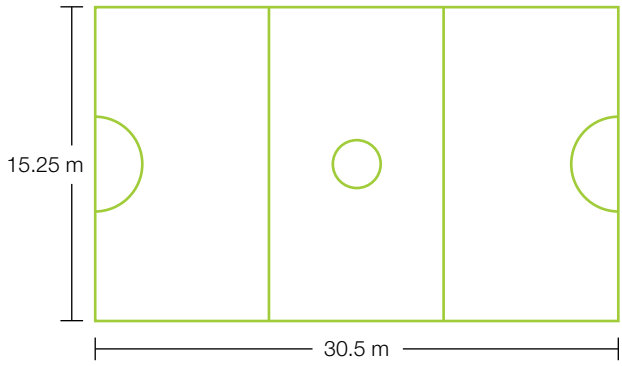
6.2, 6.3

6 A netball court is 30.5 m long and 15.25 m wide.

(a) At training, players complete five laps around the boundary of the court. How many metres is this?

(b) Calculate the area of the court.

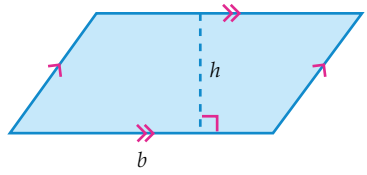
(c) The player in the position of 'goal attack' is allowed to move around in two-thirds of the court's area. Calculate this area, correct to 1 decimal place.



Area of a parallelogram

6.4

A **parallelogram** is a four-sided shape that has two pairs of parallel sides. The pairs of parallel sides are marked by > and >> arrow marks. The parallelogram has a **base** b and a **height** h, which is **perpendicular** to the base.



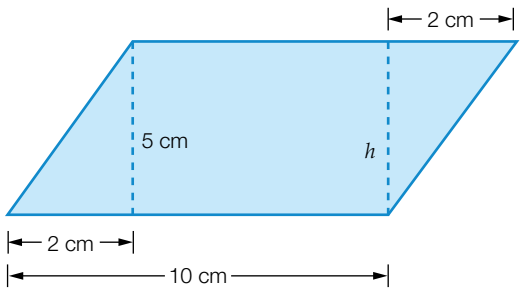
Perpendicular means 'at right angles to'. This is shown by the symbol for the right angle ⌞.

Developing a formula for area

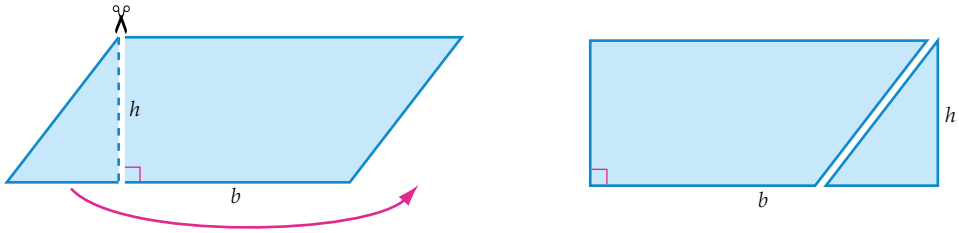
You can find a formula for calculating the area of a parallelogram by cutting and rearranging it into a familiar shape: a rectangle.

Construct a parallelogram by ruling a horizontal line 10 cm long and labelling it b. At one end of the line, and 2 cm in from the other end, measure two perpendicular heights of 5 cm. Mark them with dotted lines and label them h. Use the marked heights to rule a second 10 cm line parallel to the first. Make sure that this line starts 2 cm in from one end, and extends 2 cm past the other end of the first line.

If you have done this accurately, the two shorter, sloping sides of the parallelogram will also be parallel when you rule them in.



Transform your parallelogram into a rectangle by cutting down one of the dotted lines marking the height and rearranging the two pieces formed into a rectangle, as shown below.



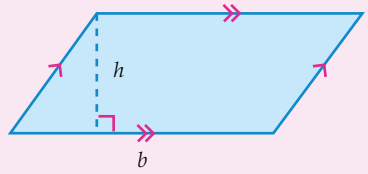
The base and height of the parallelogram become the length and width of a rectangle, and so you can multiply them together to find the area.

Area of a parallelogram

Multiply the base (b) by its perpendicular height (h).

$A = bh$

Parallel sides are marked with the same number of > arrow marks.

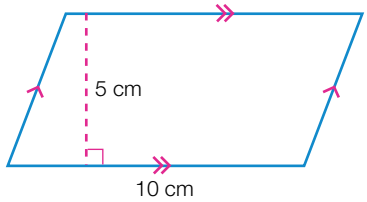


Worked example 7

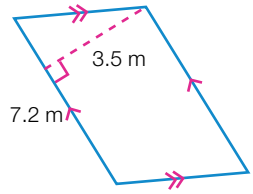
W.E. 7

Find the area of each of the following parallelograms.

(a)



(b)



Thinking		Working
(a) 1	Write the formula for the area of a parallelogram.	(a) $A = bh$
2	Identify b and h, and substitute their values into the formula.	$A = 10 \times 5$
3	Evaluate, writing the answer with the correct units.	$A = 50\ cm^2$
(b) 1	Write the formula for the area of a parallelogram.	(b) $A = bh$
2	Identify b and h, and substitute their values into the formula.	$A = 7.2 \times 3.5$
3	Evaluate, writing the answer with the correct units.	$A = 25.2\ m^2$

6.4 Area of a parallelogram

Navigator

1, 2, 3, 5, 7, 9, 10, 11	1, 2, 3, 4, 5, 7, 8, 9, 10, 11	1 (column 1), 2, 3, 4, 5, 6, 8, 9, 11

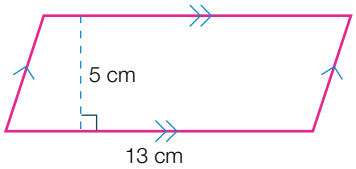
Answers p. 685

Fluency

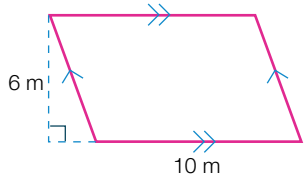
1 Find the area of each of the following parallelograms.

W.E. 7

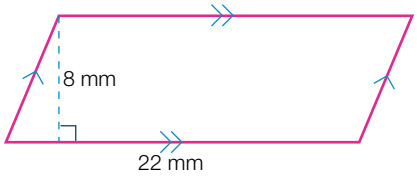
(a)



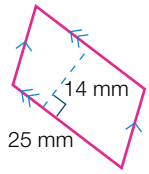
(b)



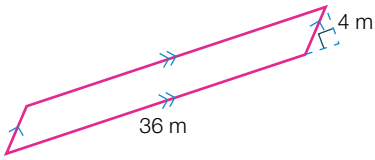
(c)



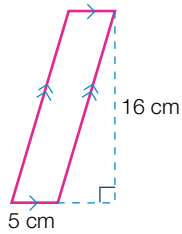
(d)



(e)

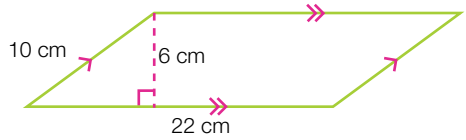


(f)



2 **(a)** Find the area of this parallelogram, in cm^2:

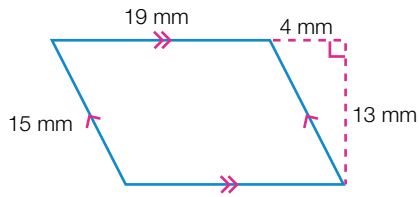
(b) Find the perimeter, in cm.



3 **(a)** The height of this parallelogram is:

A 4 mm **B** 13 mm

C 15 mm **D** 19 mm



(b) The area of a parallelogram with a base length of 3 km and a height of 4 km is:

A 3 km^2 **B** 14 km^2 **C** 12 km^2 **D** 7 km^2

(c) A parallelogram with a base length of 14 m, a sloping side length of 5 m and a height of 4 m has an area of:

A 20 m^2 **B** 28 m^2 **C** 56 m^2 **D** 70 m^2

Understanding

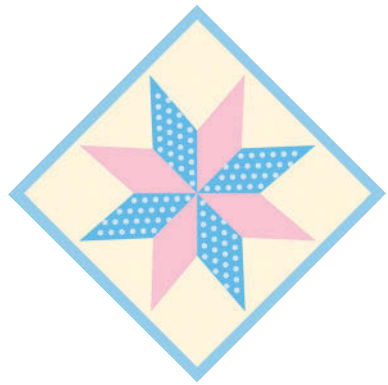
4 A parallelogram with a base length of 12 cm has an area of 90 cm^2. What is the height of the parallelogram?

5 A parallelogram with a height of 6 mm has an area of 24 mm^2. What is the base length of the parallelogram?

6 The picture frame shown has a height of 15 cm and encloses an area of 300 cm^2. What is the base length of the picture frame?

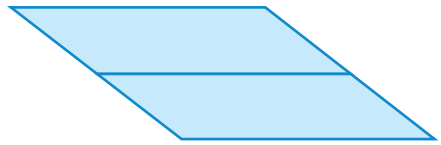
7 **(a)** How many identical parallelograms are shown in the quilt pattern below? (Ignore the different colours.)

(b) If a single parallelogram has a base length of 8.5 cm and a height of 6.5 cm, calculate the area of material required for all of the parallelograms on the quilt.



Reasoning

8 Two parallelogram-shaped shade sails are sewn together to provide shade for a courtyard, as shown. Each parallelogram has a base length of 4 m and a height of 2.5 m.



(a) At midday when the Sun is directly overhead, what area of shade will the shade sails provide?

(b) Show another way that the shade sails could be sewn together. Would this provide the same area of midday shade? Explain.

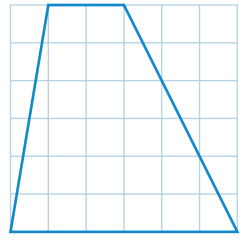
9 The shape shown is made of two parallelograms. The height of each parallelogram is 5 cm and the base length is 8 cm.

(a) Calculate the area of one parallelogram.

(b) What is the total area of the shape?

10 **(a)** Copy the trapezium and show the position of the cuts and the transformations required to form a rectangle.

(b) Calculate the area of the trapezium if one grid square = 1 cm^2.



Open-ended

11 State the dimensions (base and height) of three parallelograms that have an area of 32 cm^2.

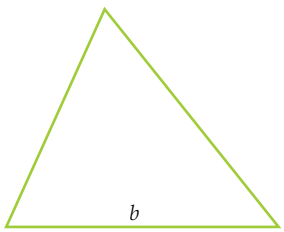
Area of triangles and composite shapes

6.5

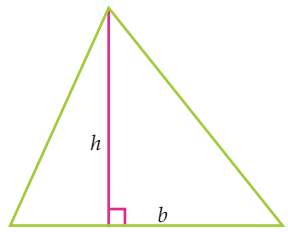
Area of a triangle

How do you find the area of a triangle? When faced with a new problem, you should first try to solve it by using what you already know.

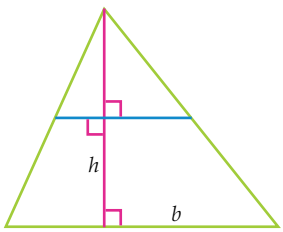
To find the area of a triangle, you can use a shape we already know how to find the area of: the rectangle. Can triangles be 'rearranged' to form rectangles?



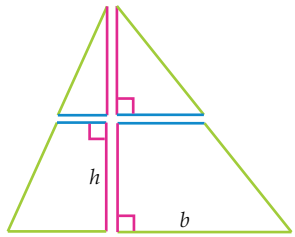
1 Use a ruler and a pencil to draw a triangle. Make the side lengths at least 10 cm long. Label one side as the base, *b*.



2 Rule a line from the apex (the point at the top) to the base. The line should be perpendicular (at a right angle of 90°) to the base. Label this line *h*, the height of the triangle.



3 Mark a point half-way up this line and use it to rule a line parallel to the base.



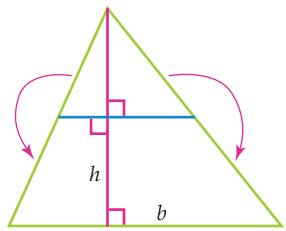
4 Cut along these two inside lines to create four pieces.

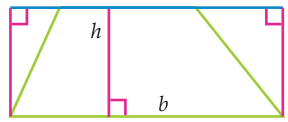
Now:

- Use the four pieces to form a rectangle.
- Form the triangle again.
- Keep moving between the rectangle and the triangle until you are able to do it easily.
- What is the simplest way to move from the triangle to the rectangle? Write the steps involved, then share them with a friend or the class.

Did you find that the simplest way was to rotate the two top pieces until they touched the bottom two?

You can find the area of the rectangle formed by finding the length and the width and multiplying them.





The length of the rectangle is the length of the base of the triangle, b.

The width of the rectangle is half the height of the triangle, $\frac{1}{2}h$, or $\frac{h}{2}$. (Can you see why? Look back at the instructions for step 3.)

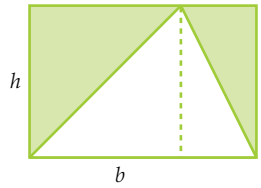
So, instead of writing A = length × width, you can write:

$A = b \times \frac{h}{2}$, or $A = \frac{bh}{2}$ for the area of the rectangle, which is also the area of our original triangle.

This rule will work for all triangles. You can check this by repeating steps **1**–**4** with differently shaped triangles, then rearranging them into rectangles.

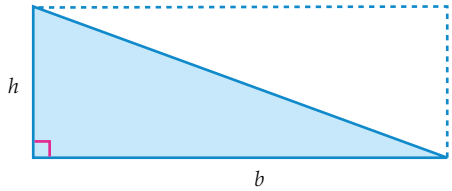
Another way of looking at it

Another way of showing that the area of a triangle is equal to $\frac{bh}{2}$ is to draw a rectangle around the triangle, touching the points of the triangle, like this:

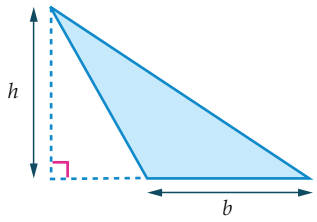


The area of this rectangle is $b \times h$, or bh. You can see by using symmetry that the two smaller, shaded triangles together are equal to the original, unshaded triangle. This means that the original triangle occupies half the area of the rectangle, or $\frac{bh}{2}$.

It is easy to see that the area of a triangle is half the area of the enclosing rectangle if you consider a right-angled triangle.



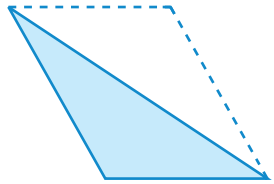
For some triangles, such as this one, the perpendicular height is measured outside of the triangle. You can find the height by extending the base, then drawing a perpendicular line from this extension to the opposite vertex. These triangles also follow the rule $A = \frac{bh}{2}$.



You can draw another identical triangle to form an enclosing parallelogram.

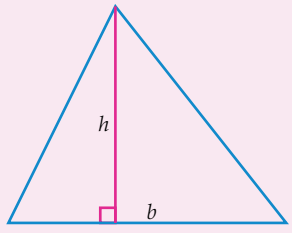
The area of the parallelogram is bh.

One triangle forms half of the parallelogram, so its area is $\frac{bh}{2}$.



The area of a triangle is equal to half of the product of the base and the perpendicular height.

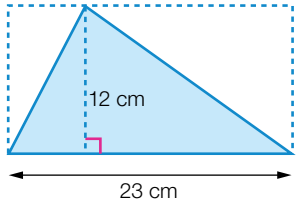
$A = \frac{bh}{2}$



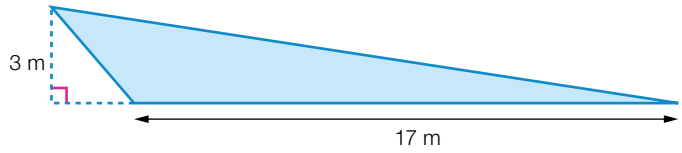
Worked example 8 W.E. 8

Calculate the area of the following triangles.

(a)



(b)



Thinking	Working
(a) 1 Write the formula for the area of a triangle.	(a) $A = \frac{bh}{2}$
2 Identify b and h, and substitute their values into the formula.	$A = \frac{23 \times 12}{2}$
3 Evaluate, writing the answer with the correct units.	$A = 138\ cm^2$
(b) 1 Write the formula for the area of a triangle.	(b) $A = \frac{bh}{2}$
2 Identify b and h, and substitute their values into the formula.	$A = \frac{17 \times 3}{2}$
3 Evaluate, writing the answer with the correct units.	$A = 25.5\ m^2$

Composite shapes

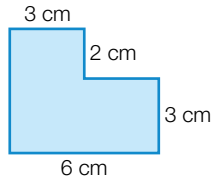
A **composite shape** is a shape that is made by combining two or more shapes, such as rectangles, squares, parallelograms and triangles. Examples of composite shapes are shown below.

To find the area of a composite shape, divide it into simpler shapes that you can find the area of, then add or subtract the areas of each individual shape.

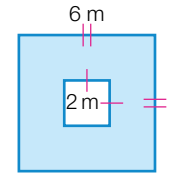
Worked example 9 W.E. 9

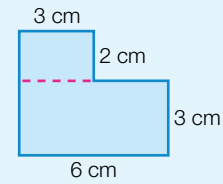
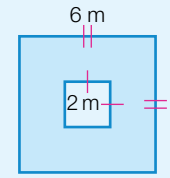
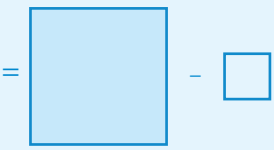
Calculate the shaded area of the following composite shapes.

(a)



(b)



Thinking	Working
(a) 1 Split the composite shape into recognisable shapes. (This may be done in different ways.)	(a) 
2 Write the area of the composite shape as the sum of the areas of the individual shapes.	Total area =  $= L \times W + l \times w$ $= 3 \times 2 + 6 \times 3$
3 Calculate each of the smaller areas.	$= 6 + 18$
4 Do the addition by writing the answer with the appropriate units.	$= 24\ cm^2$
(b) 1 Write the area of the composite shape as the area of the larger shape minus the area of the smaller shape.	(b)  Total area =  $= L^2 - l^2$
2 Calculate each of the smaller areas.	$= 6^2 - 2^2$
3 Do the subtraction, writing the answer with the appropriate units.	$= 36 - 4$ $= 32\ m^2$

Note that when you have two of the same type of shape, you can use capital letters (L, W) for the dimensions of the larger shape and lower-case letters (l, w) for the dimensions of the smaller shape. This is demonstrated in the example above.

6.5 Area of triangles and composite shapes

Navigator

1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 15	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 15, 16	1, 2, 3, 5, 6, 7, 9, 10, 12, 13, 14, 16

Answers p. 685

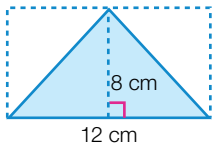
Equipment required: calculator may be used for Questions **2, 4, 6–10**

Fluency

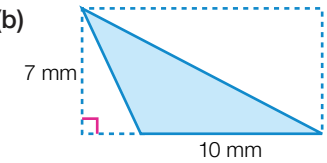
1 Calculate the area of the following triangles.

W.E. 8

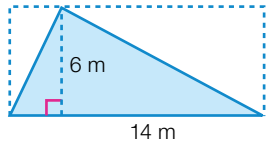
(a)



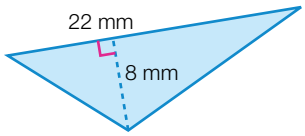
(b)



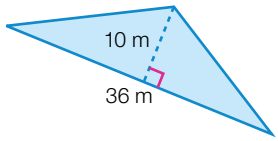
(c)



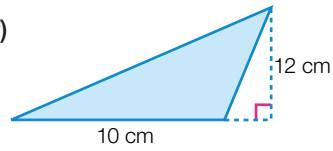
(d)



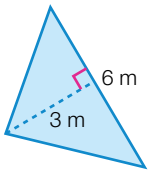
(e)



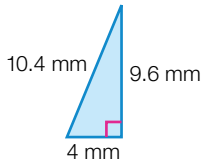
(f)



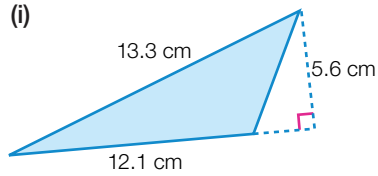
(g)



(h)



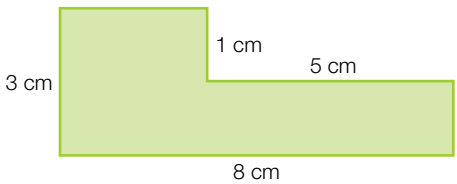
(i)



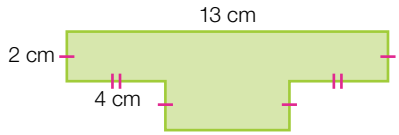
2 Calculate the shaded area of the following composite shapes.

W.E. 9

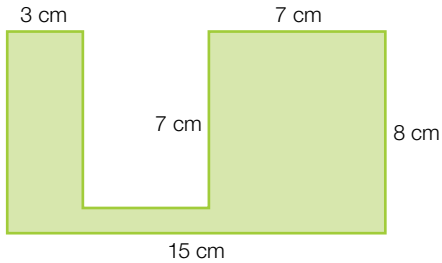
(a)



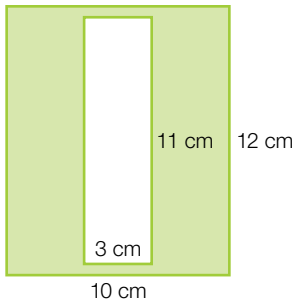
(b)



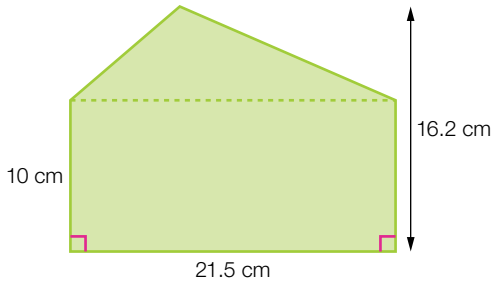
(c)



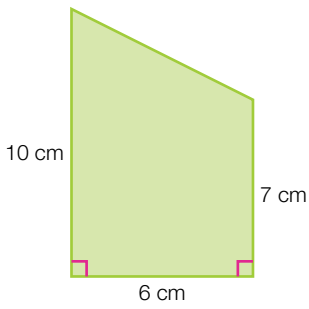
(d)



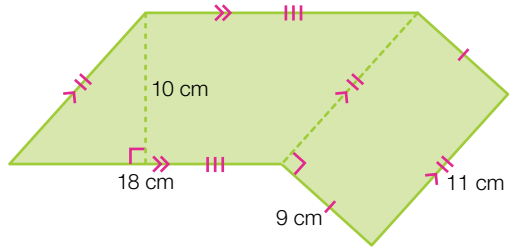
(e)



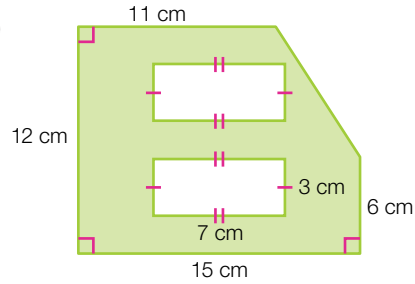
(f)



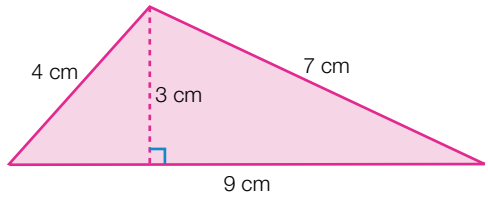
(g)



(h)



3 **(a)** In the diagram, the height of the triangle is:



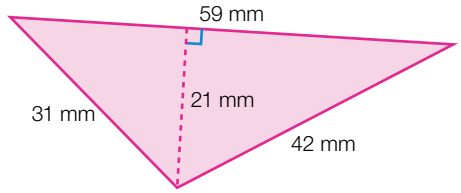
A 3 cm

B 4 cm

C 7 cm

D 23 cm

(b) The base length of this triangle is:



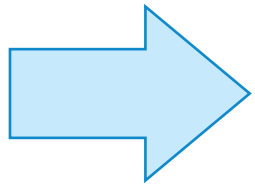
A 21 mm

B 31 mm

C 42 mm

D 59 mm

(c) Which option best describes this composite shape?



A triangle and square

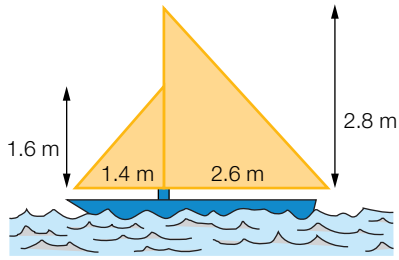
B triangle and rectangle

C heptagon

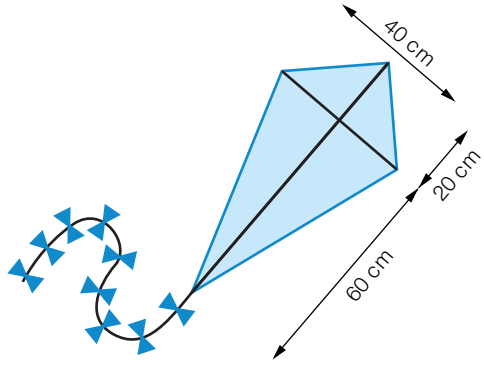
D parallelogram and rectangle

Understanding

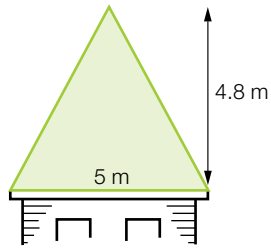
4 Find the total area of the sails on this yacht.



5 Find the area of paper required to build the kite shown.

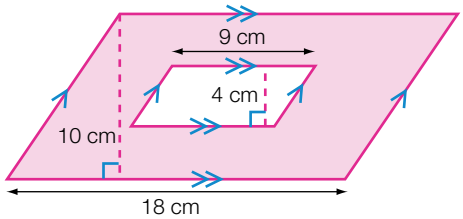


6 The label on a tin of paint claims the contents will cover 4 square metres. How many tins would be needed to paint the triangular roof section pictured?

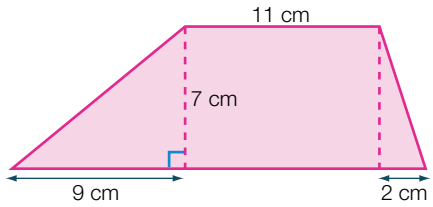


7 Find the area of each of the following composite shapes.

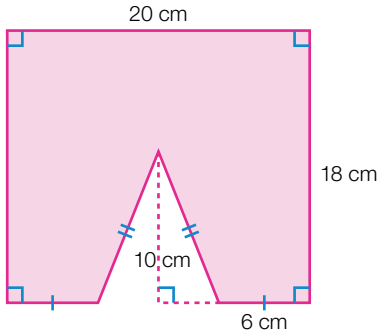
(a)



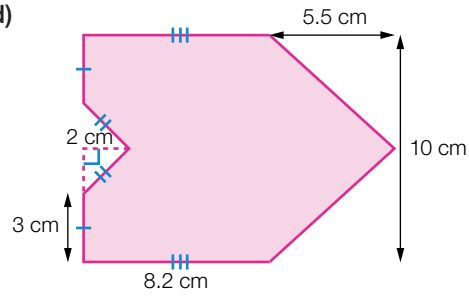
(b)



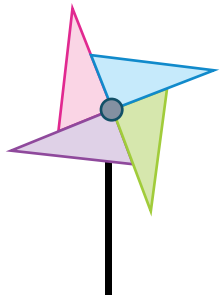
(c)



(d)



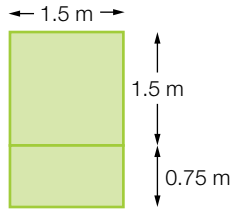
8 Daniel is making a wind spinner as shown. If the spinner shape is made of 4 triangles with $h = 10$ cm and $b = 20$ cm, then what is the total area of the wind spinner?



9 A garden bed area is made of a rectangle and a square joined together as shown. A gardener wants to separate the total area into two equal parts.

(a) What is half of the total area?

(b) How can the total area be represented by two equal triangles?

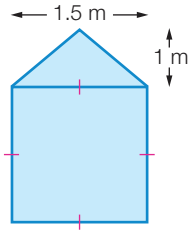


10 Terry is having a doorway put in one of the walls of his house.

(a) If the wall is 5 m long and 2.4 m high, and the doorway measures 0.82 m by 2 m, what area of wall will remain after the doorway is completed?

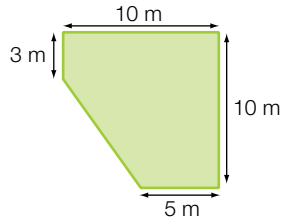
(b) Terry installs a door in the doorway and paints two coats of varnish on both sides of the door. If 1 litre of varnish covers 4 square metres, how many litres of varnish did Terry use?

11 The wall of a house is made of a square with a triangle on top, as shown. Calculate the total area of the wall.

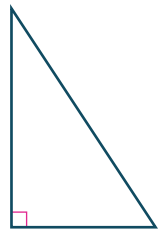


Reasoning

12 Describe a method for finding the area of the composite shape shown without splitting it up into known shapes. Use your method to find the answer.



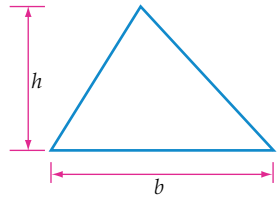
13 This triangle can be cut into two pieces to form a rectangle.



(a) Describe exactly where and how to make the cut.

(b) Use a diagram to show how the two pieces fit together to make a rectangle.

14 This triangle can be cut into four pieces to form a rectangle.



(a) Describe exactly where and how to make the cuts.

(b) Use a diagram to show how the four pieces fit together to make a rectangle.

Open-ended

15 A triangle has an area of 36 cm^2. Give two possible combinations for the triangle's base length and height.

16 The difference in area between a rectangle and a triangle is 12 cm^2. Draw a possible example of the pair, showing their dimensions.

Volume

6.6

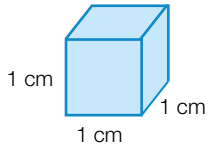
A box is a three-dimensional (3D) object, which means it has length, width and height. The amount of space taken up by a box is its **volume**.

Volume is the amount of space occupied by a 3D object.

Volume is measured in 'cube' units. When you calculate volume, you are finding the number of cubes that could fit inside the object.

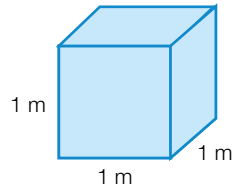
You can measure the volume of an object by counting or calculating the number of cubic centimetres that would fit inside it. A cubic centimetre (cm^3) is a cube with a length, width and height of 1 cm. The volume of larger objects may be measured in cubic metres (m^3), a cube with length, width and height of 1 m.

A cubic centimetre



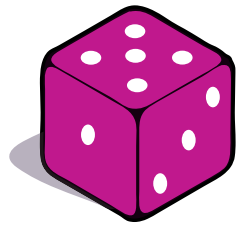
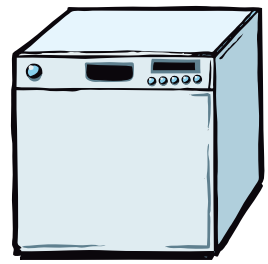
1 cm × 1 cm × 1 cm = 1 cm^3

A cubic metre



1 m × 1 m × 1 m = 1 m^3

Note that this cube is not drawn to scale.

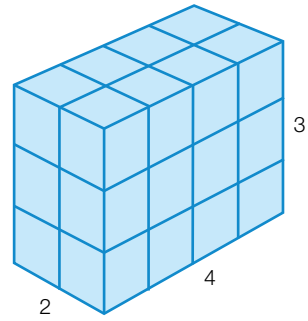
mm^3	cm^3	m^3
		
The volume of the head on a match is about 4 mm^3.	The volume of a die is about 1 cm^3.	The volume of a small dishwasher is about 1 m^3.

The symbol cm^3 can be said as 'cubic centimetres' or 'centimetres cubed'.

A **rectangular prism** is made up of layers of cubes. Each layer has the same rectangular shape, meaning that there is the same number of cubes in each layer.

You can find the volume of a prism by counting the number of cubes on the bottom face (or top face, if that is easier to see) and then multiplying this by how many layers of cubes there are.

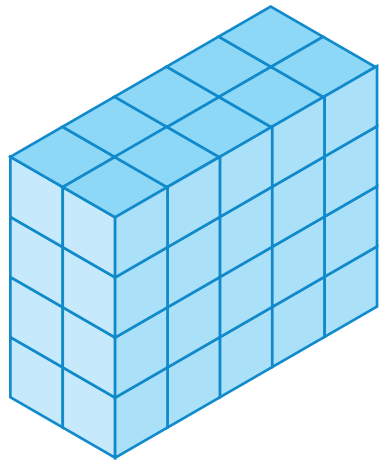
The rectangular prism shown here has 8 cm cubes on its top face, and 3 layers of cubes. Its volume is $8 \times 3 = 24$ cm^3.



Worked example 10

W.E. 10

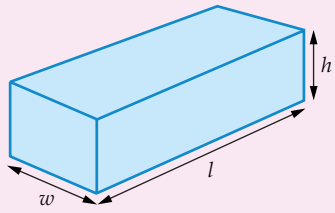
How many cubic centimetres are there in the following rectangular prism? (Each small cube represents 1 cm^3.)



Thinking	Working
1 Find the number of squares on the top face of the prism.	There are 10 squares on the top face.
2 Find the height of the prism.	It is 4 units high.
3 Work out the total number of cubes.	$10 \times 4 = 40$ cubes $= 40\ cm^3$

The number of cubes showing on the top or bottom face of a rectangular prism is the same as the area of that rectangular face. Instead of counting the number of cubes in the top or bottom face, you can multiply the length by the width to get the area of the face ($A = l \times w$), then multiply by the number of layers of cubes. So, to find the volume of a rectangular prism, multiply the length, width and height values together.

$V = l \times w \times h$

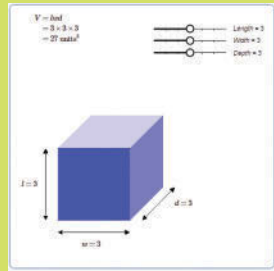


Interactive

Volume

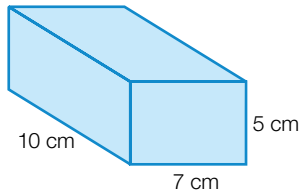
Explore how the volume of a rectangular prism changes as you vary the dimensions.

Go to the eBook or the Pearson Places website to access this interactive.



Worked example 11 W.E. 11

Find the volume of the following rectangular prism in cm^3.



Thinking	Working
1 State the formula for calculating the volume of rectangular prisms.	$V = lwh$
2 Substitute the values for l, w and h into the formula and evaluate.	$V = 10 \times 7 \times 5$ $= 350\ cm^3$

6.6 Volume

Navigator

1, 2, 3, 4, 5, 6, 7 (columns 1–2), 8, 9, 11, 12, 13, 14, 18	1, 2, 3, 4, 5, 6, 7 (columns 2–3), 8, 9, 11, 12, 13, 14, 17, 18	2 (d–f), 3, 4, 5, 6, 7 (column 3), 9, 10, 11, 12, 13, 14, 15, 16, 17, 18

Answers p. 686

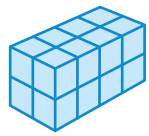
Equipment required: calculator may be used for Questions **8–10, 15, 18**

Fluency

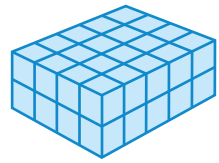
1 How many cubic centimetres are there in the following rectangular prisms? (Each small cube represents 1 cm^3.) W.E. 10

(a)

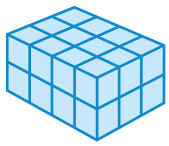
(b)



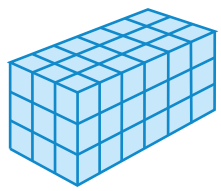
(c)



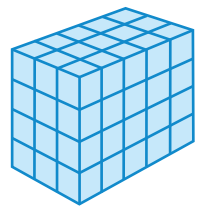
(d)



(e)

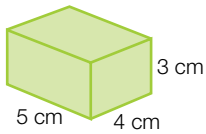


(f)

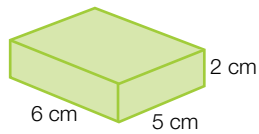


2 Find the volume of the following rectangular prisms in cm^3. W.E. 11

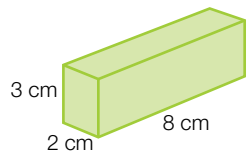
(a)



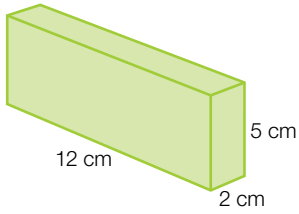
(b)



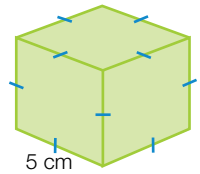
(c)



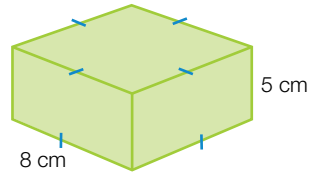
(d)



(e)



(f)



3 A room is 3 m long, 2 m wide and 2 m high. What is its volume in metres cubed?

4 A small case is 6 cm long, 2.5 cm wide and 1 cm high. What is its volume?

5 A pocket dictionary has a cover 12 cm long and 9 cm wide and is 2 cm thick. Find the volume of this book.

6 Choose the correct answer for each of the following rectangular prisms.

(a) Length = 20 cm; width = 10 cm; height = 5 cm. The volume is:

A 35 cm^3 B 100 cm^3 C 700 cm^3 D 1000 cm^3

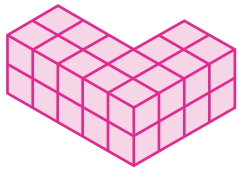
(b) Length = 12.5 m; width = 4 m; height = 3 m. The volume is:

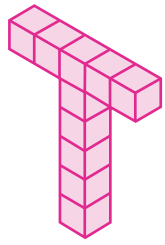
A 150 m^3 B 1500 cm^3 C 800 m^3 D 70 m^3

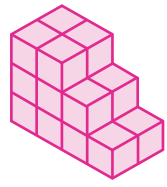
(c) Length = 3 m; width = 75 cm; height = 50 cm. The volume is:

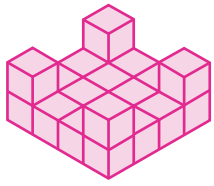
A 11.25 m^3 B 112.5 cm^3 C 11 250 cm^3 D 1 125 000 cm^3

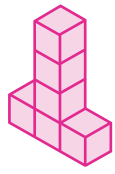
7 Find the volume of each of these compound solids by counting the number of centimetre cubes.

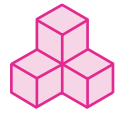
(a) 

(b) 

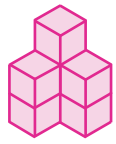
(c) 

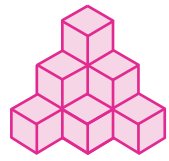
(d) 

(e) 

(f) 

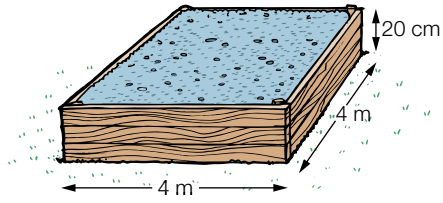
(g) 

(h) 

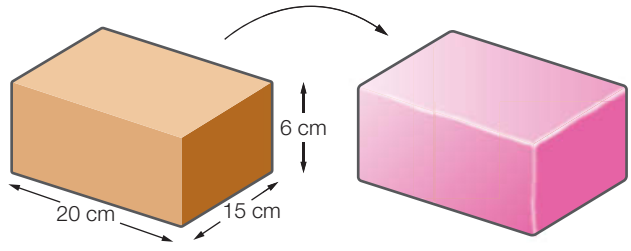
(i) 

Understanding

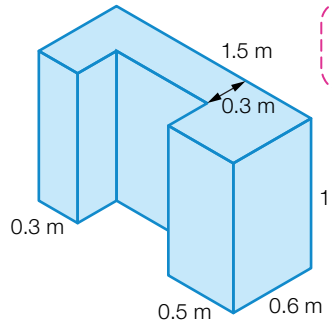
8 A water tank is to be put on a wooden box filled with gravel to keep it level. How much gravel is needed, in m^3, to fill the base shown?



9 A cake in the shape of a rectangular prism is 20 cm long, 15 cm wide and 6 cm high when baked. What will be the volume of the cake after it is iced all over (including the bottom) with a layer of icing 0.5 cm thick?



10 Lachlan has finished the brickwork for his backyard barbecue, as shown. Find the volume of bricks used in building the barbecue.

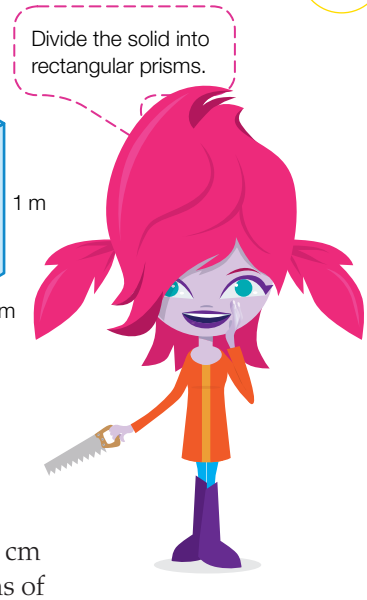


11 Measure the dimensions and calculate the volume of:

(a) a tissue box

(b) an eraser

(c) the room you are in.



Reasoning

12 What is the volume of a box that is 40 cm long, 15 cm wide and 8 cm high? How much will the volume increase if the height is doubled?

13 Eight planks of wood are stored on a shelf. Each plank of wood is 80 cm long, 10 cm wide and 3 cm thick. What is the total volume of all planks of wood in cm^3?

14 A large toy box is completely full of identical wooden blocks which are 10 cm long, 6 cm wide and 4 cm in height. How many blocks can fit inside the toy box, if the dimensions of the toy box are: length 40 cm, width 30 cm and height 20 cm?

15 A rectangular prism has a volume of 216 cm^3. It is twice as long as it is wide, and its height is 3 cm. Find its length.

Open-ended

16 A rectangular prism is made from 30 one-centimetre cubes. Give possible values for the length, width and height of the prism.

17 At a school fair, a game involves guessing the number of jelly beans in a container. The person with the closest guess will win all the jelly beans.

(a) How many jelly beans do you think are in the jar?

(b) How can you make an accurate estimate of the number of jelly beans in the jar?

(c) If the jelly beans were each the same volume but cube-shaped, would more or fewer jelly beans fit in the jar?

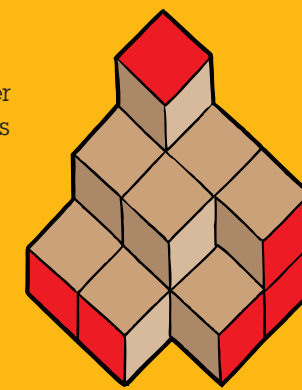
18 A small cube is put inside a large cube of volume 64 cm^3. Give three possible side lengths of the smaller cube and calculate the volume of empty space inside the large cube.

Problem solving

The red cube

An artist is building a large red cube out of smaller cubes, as shown. How many of the smaller cubes shown will need one face painted? Two faces painted? Three faces painted?

In the completed large red cube, how many of the smaller cubes will have one, two and three faces painted?



Strategy options

- Have I seen a similar problem?
- Make a model.

Gamespace

City Mission

Coder X, the evil code-breaking master, is on a spree through the city's streets, decoding top secret information to gain access to all of the city's finances, gold and valuables. Your mission is to answer the questions Coder X has left on electronic devices at locations around the city, starting at the police station. Answering the questions correctly will spell the access code to Coder X's computer files and give you the location of the next clue. The path taken will lead you to Coder X's hideout. Can you break the code before Coder X escapes?

The volume of air that can fit inside this telephone booth is:

- **K** 12 800 m^3 (go to newspaper stand)
- **L** 6400 m^3 (go to library)
- **M** 360 m^3 (go to police station)
- **N** 1.28 m^3 (go to train map)

The length of this single-carriage train could be:

- **Y** 0.07 km (go to police station)
- **Z** 200 m (go to train map)
- **A** 2000 cm (go to library)
- **B** 1200 mm (go to bank)

The length of this train line is:

- **R** 4.6 km (go to train carriage)
- **S** 3.8 km (go to L'Hotel)
- **T** 42 km (go to museum)
- **U** 52 km (go to telephone booth)

0.67 km is equal to:

- **N** 6700 cm (go to Pop's Pizza)
- **O** 670 m (go to telephone booth)
- **P** 6.7 m (go to library)
- **Q** 0.0067 m (go to train carriage)

The length of this police sign is:

- **A** 40 cm (go to library)
- **B** 1600 cm (go to telephone booth)
- **C** 400 cm (go to bank)
- **D** 800 cm (go to train map)

32 metres is equal to:

- **V** 0.32 km (go to post office)
- **W** 0.032 cm (go to bank)
- **X** 320 cm (go to museum)
- **Y** 3200 cm (go to rubbish bin)

The volume of this rubbish bin will give you Coder X's hideout:

- **E** 0.36 m^3 (go to Pig and Whistle Café)
- **F** 3.6 m^3 (go to park bench)
- **G** .72 m^3 (go to Ye Olde Bookstore)
- **H** 7.2 m^3 (go to train platform exit)

The area of this shape is:

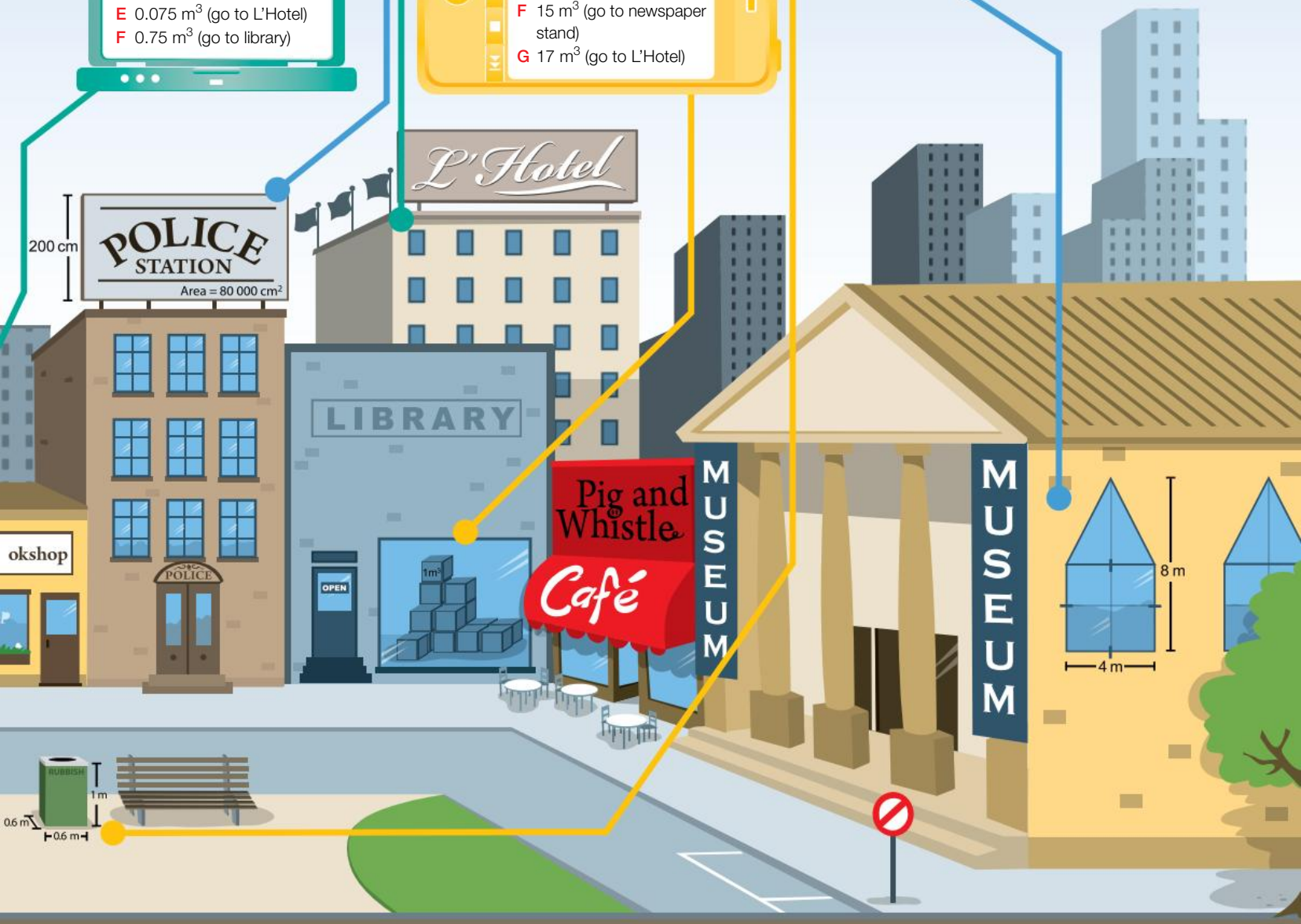
- **P** 36 m^2 (go to L'Hotel)
- **Q** 32 m^2 (go to telephone booth)
- **R** 30 m^2 (go to train carriage)
- **S** 24 m^2 (go to newspaper stand)

The volume of this stack of newspapers is:

- **C** 1.3 m^3 (go to rubbish bin)
- **D** 3 m^3 (go to post office)
- **E** 0.075 m^3 (go to L'Hotel)
- **F** 0.75 m^3 (go to library)

The volume of this stack of boxes is:

- **D** 16 m^3 (go to museum)
- **E** 13 m^3 (go to train map)
- **F** 15 m^3 (go to newspaper stand)
- **G** 17 m^3 (go to L'Hotel)



Maths 4 Real

Landscape Gardening

Landscape gardening involves designing and creating outside spaces, such as the area around a house or building, or public parks. There are many DIY magazines and TV shows transforming the average backyard into a haven of 'outdoor rooms', lights, waterfalls and vegetable gardens.

Before a garden can be transformed, a detailed design is usually drawn on grid or graph paper.

1 **(a)** Why is it helpful to draw the design on a grid?

(b) The scale at the top of a grid states that 1 cm on the page represents 1 m in real life. What real life distance would 1 mm represent?

At the top of the following page is a plan used to transform an Australian backyard.

Refer to the plan for Questions **2**–**8**.

2 What is the total area of the backyard (not including the house)?

3 The lawn will be sown by seed. 30 g of grass seed is needed to cover each square metre of lawn.

(a) What is the area to be covered in grass?

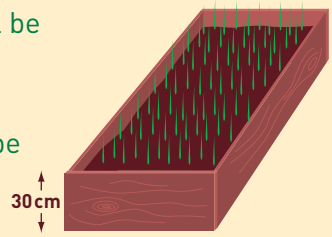
(b) How much grass seed will you need to buy? Give your answer in kilograms.

4 The path that extends from the shed to and around the vegetable garden will be paved with bricks. The base dimensions of one brick are 20 cm × 10 cm. Calculate:

(a) How many bricks will cover each square metre?

(b) What is the total number of bricks required to complete the path?

5 The vegetable garden will be constructed out of timber sleepers with a height of 30 cm. The box needs to be filled with a mixture of soil and compost.

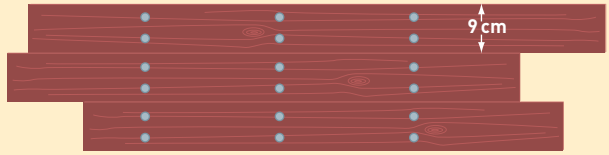


(a) What is the total length of the timber sleepers needed to make the vegetable garden box?

(b) Find the area of the topsoil of the vegetable garden.

(c) What volume of soil mixture will be needed to fill the box to the top?

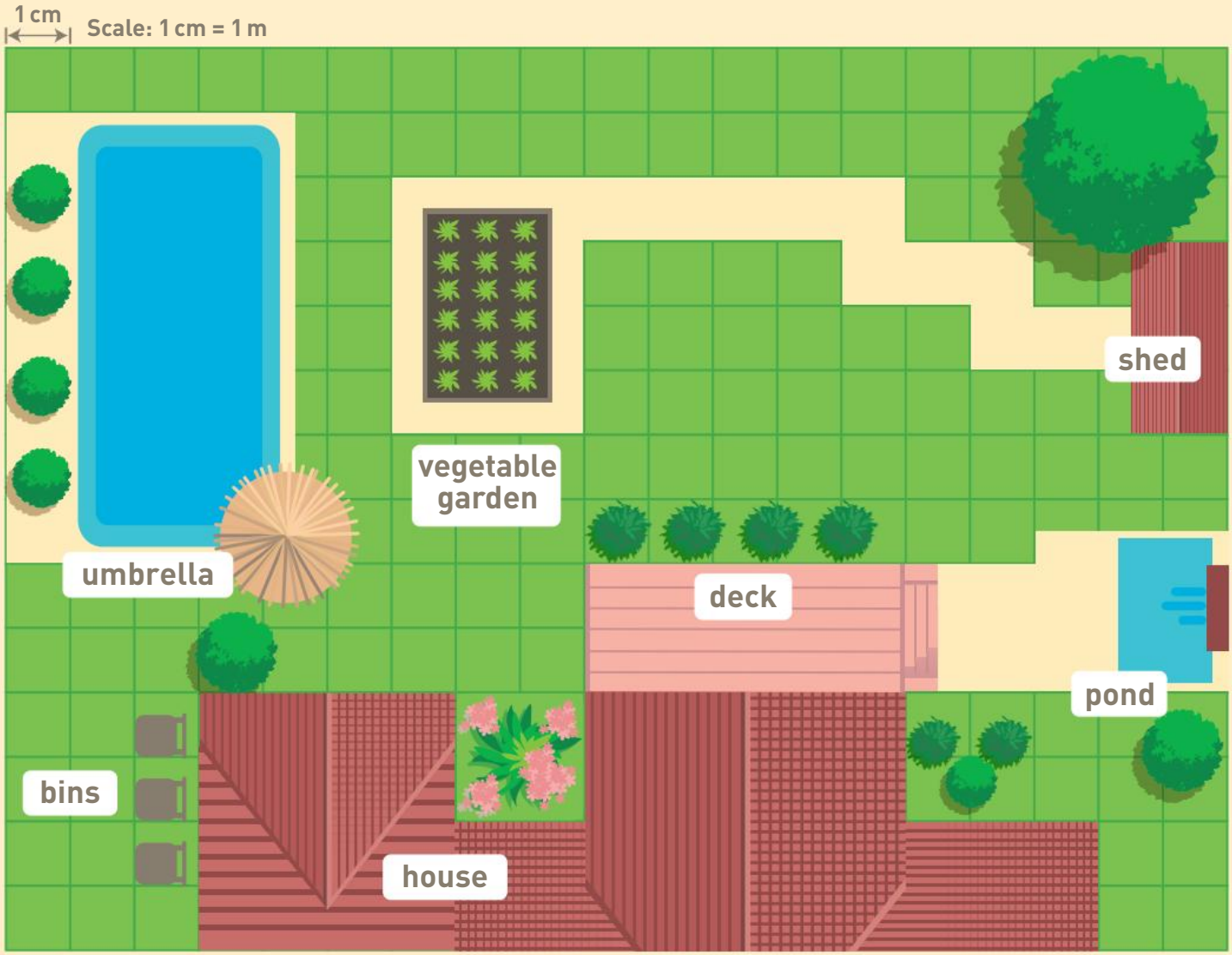
6 The deck will be made using hardwood boards that are 9 cm wide. Decking boards are sold in different lengths and then cut to size.



(a) What area is to be decked? (Include the step in your calculations.)

(b) How many boards are needed to cover a width of 1 metre?

(c) What length of decking will need to be bought to make the deck?



When designing a backyard, it is important to consider the cost. A detailed budget needs to be completed before the project can begin.

7 Using the figures you have calculated in Questions **2–6**, copy and complete the budget below.

Item	Number of items required or area to cover (m^2)	Cost	Total cost of each item
Grass seed		$8.95 per 1 kg box	
Bricks		$0.80 each	
Timber sleepers		$12 per metre	
Soil mixture		$42 per m^3	
Decking		$4.50 per metre	
		Total cost	$

8 What other costs in making this backyard have not been included in this budget?

Research

Design your own 'ideal' backyard. Make a scale drawing of it using similar symbols as above. Consider aspects such as space, cost, slope of the land, water restrictions, pets and your family's needs.

Investigation

Equipment required: centimetre cubes (optional), graph paper or isometric dot paper

Blocks of chocolate

The dimensions of a chocolate bar are shown below.

The surface area of a chocolate bar is the combined area of the top, bottom and all four sides. It is the area that is covered in chocolate.

The smaller the surface area of a chocolate bar, the less packaging will be required to wrap around it.

The Big Question

What is the smallest possible surface area for the chocolate bar, if it has a volume of 36 cm^3?

Engage

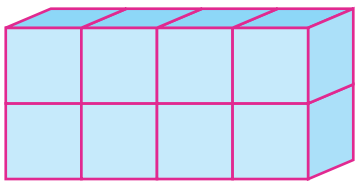
Chocolate comes in blocks that resemble rectangular prisms. The volume of a rectangular prism can be calculated using the formula:

Volume = length × width × height

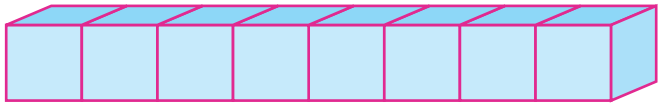
If the volume is known to be 8 cm^3 and we want to find the possible dimensions for the rectangular prism, we need to find three numbers that multiply to give 8:

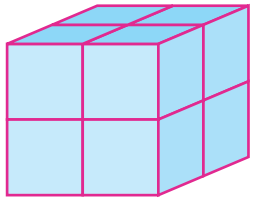
For example: 8 cm^3 = 4 × 2 × 1

These dimensions give the following rectangular prism.

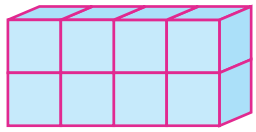
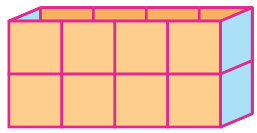
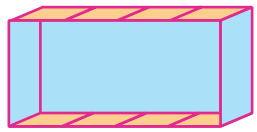
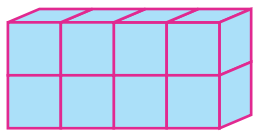


These eight cubes can be arranged to make two other rectangular prisms:





1 The surface area of a rectangular prism can be found by counting or calculating the number of squares on the surface of each face (including the bottom) and adding them together. The surface area of a rectangular prism with length 4 cm, width 1 cm and height 2 cm is shown in the table below.

Dimensions	 4 cm × 2 cm × 1 cm
Area of front and back	 8 + 8 = 16
Area of top and bottom	 4 + 4 = 8
Area of sides	 2 + 2 = 4
Total surface area	 16 + 8 + 4 = 28 cm^2

(a) Calculate the surface area of the other two rectangular prisms with volume 8 cm^3. If you have centimetre cubes, use them to help you.

(b) All three rectangular prisms have a volume of 8 cm^3. What are the dimensions of the one with the smallest surface area?

(c) Draw the chocolate bar as a rectangular prism on graph paper or dot paper.

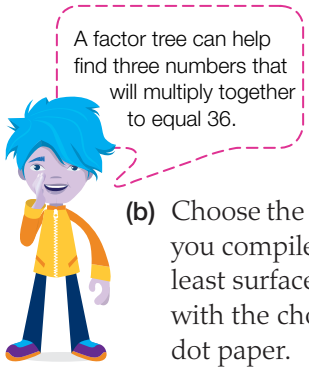
(d) What is the surface area of the chocolate bar?

Explore

2 The volume of the chocolate bar shown is $12 \times 3 \times 1 = 36$ cm^3.

(a) Other whole number dimensions of a rectangular prism will give a volume of 36 cm^3. List all the different combinations of whole numbers that will make this statement true.

36 cm^3 = ____ × ____ × ____



Strategy options

- Make a table.
- Have I seen a similar problem?
- Make a model.

(b) Choose the three sets of dimensions from the list you compiled in **(a)** that you think will have the least surface area. Draw three chocolate bars with the chosen dimensions on graph paper or dot paper.

(c) Copy and complete the table below to find the total surface area of the three prisms.

Dimensions of chocolate bar	Area of front and back	Area of top and bottom	Area of sides	Total surface area

Explain

3 **(a)** Use your results from **2** to answer the Big Question. Include dimensions of the bar and its surface area in your answer.

(b) How much less packaging would this new bar require, compared to the current one?

Elaborate

4 **(a)** Explain how you can predict which dimensions of a rectangular prism will have the smallest surface area.

(b) Write a statement that explains how you would find the dimensions that give the smallest surface area for a rectangular prism.

Evaluate

5 **(a)** Describe, using a step-by-step method, the approach you used to solve this problem. Did you use any of the information you found in **1**?

(b) Can this problem be solved another way? What other methods or shortcuts could be used when approaching problems like this one?

(c) Why do you believe current chocolate bars are the shape they are? Do you think that chocolate companies may consider changing the dimensions of chocolate bars? List some advantages and disadvantages of doing this.

Extend

6 **(a)** Construct a model of your alternative chocolate bar that has the smallest surface area. Design the packaging and then fold it into a rectangular prism.

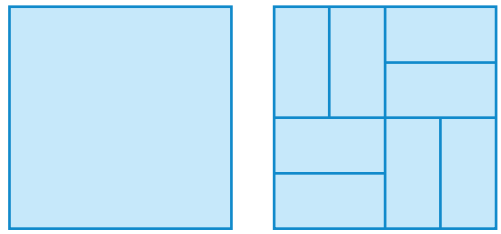
(b) Prepare a report on the size, dimensions and packaging of your new chocolate bar. Your report should include:

- the dimensions and surface area of your newly designed chocolate bar
- environmental reasons for reducing the amount of packaging used for individual chocolate bars
- financial reasons for reducing the amount of packaging used for individual chocolate bars
- an example of what your new chocolate bar will look like (your design).

Challenge 6

1 If a $2 coin is about 2 mm thick, then how much money is in a one-kilometre-high stack of $2 coins?

2 The perimeter of this square paddock is 800 m. It is divided into eight identical paddocks as shown.



The perimeter, in metres, of each of the eight smaller paddocks is:

A 150 **B** 200

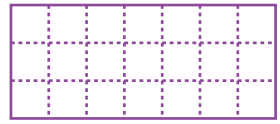
C 300 **D** 400

3 The weight limit for an elevator is 1500 kilograms. The average weight of the people in the lift is 80 kilograms. If the combined weight of the people is 100 kilograms over the limit, how many people are in the lift?

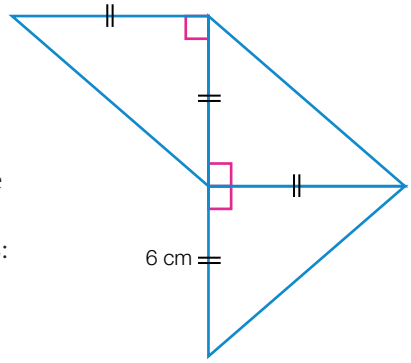
A 13 **B** 16 **C** 17 **D** 20

4 When a square piece of paper is folded in half vertically, the resulting rectangle has a perimeter of 39 cm. Find the area of the original sheet of paper, in square centimetres.

5 A rectangular area on a wall measuring 7 units by 3 units is covered with 7 rectangular tiles measuring 1 unit by 3 units. In how many ways can this be done?



6 The area of the entire figure shown is:



A 36 cm^2 **B** 54 cm^2

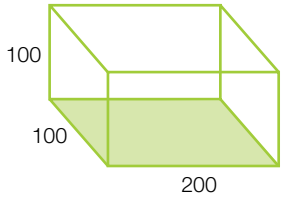
C 72 cm^2 **D** 108 cm^2

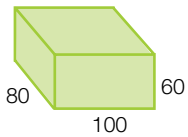
7 A rectangular sign has the dimensions 10 m by 16 m. If a square advertisement to be painted on it must have a border at least 1.5 m wide, the area of the largest square advertisement that can be painted on the sign is:

A 49 m^2 **B** 72.25 m^2

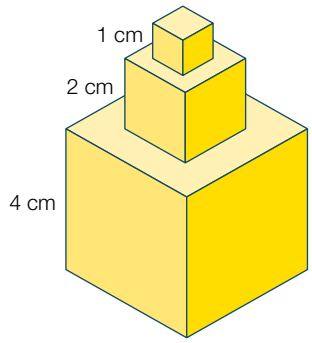
C 91 m^2 **D** 160 m^2

8 A fish tank has a base of 100 cm by 200 cm and a depth of 100 cm. A solid metal rectangular prism with dimensions 80 cm by 100 cm by 60 cm is put in the tank and water is poured into the tank until the water is level with the top of the metal prism. After the prism is carefully removed from the tank, the depth of the water remaining is measured. What are the three possible depths?





9 Three cubes are sitting on top of each other. The bottom cube has a side length of 4 cm, the cube on top of it has a side length of 2 cm and the very top cube has a side length of 1 cm. What is the visible area of the cube stack?



Chapter review 6

Maths literacy

area	height	metric	perpendicular
base	kilometre	millimetre	rectangular prism
centimetre	length	parallelogram	volume
composite shape	metre	perimeter	width

Copy and complete the following using the words and phrases from this list, where appropriate. A word or phrase may be used more than once.

1 _____ is the distance around the outside of a shape.

2 To calculate the _____ of a rectangle, you multiply the _____ by the width.

3 The area of a triangle is half the _____ multiplied by the perpendicular _____.

4 The _____ of a rectangular prism is found by multiplying the length, width and height together.

5 The system of measurement used in Australia is called the _____ system.

6 To find the area of a _____ we break it up into simpler shapes.

7 Multiplying the base length by the perpendicular height will find the area of a _____.

Fluency

1 **(a)** The height of an average house ceiling is approximately: **6.1**

A 260 mm **B** 2.6 m **C** 0.26 km **D** 2600 m

(b) The length of a classroom desk is approximately:

A 18 cm **B** 280 mm **C** 180 cm **D** 180 m

2 Copy and complete the following length conversions. **6.1**

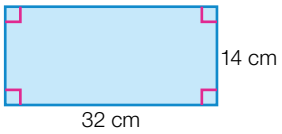
(a) 45.9 km = _____ m **(b)** 58 000 cm = _____ km

(c) 9.2 m = _____ mm **(d)** 42 m = _____ cm

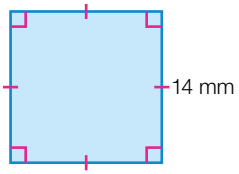
(e) 980 m = _____ km **(f)** 0.006 73 km = _____ mm

3 Find the **(i)** perimeter and **(ii)** area of each of the following shapes. **6.2, 6.3, 6.5**

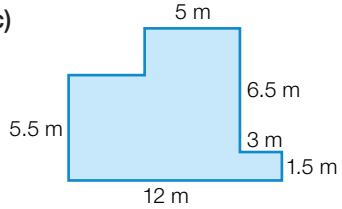
(a)



(b)



(c)



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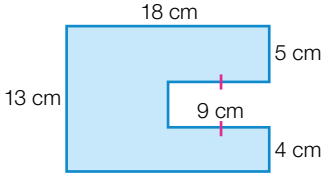
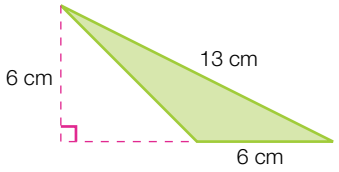


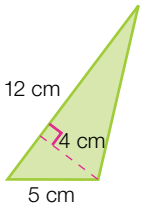
4 Find the perimeter and area of a squash court 9.75 m long and 6.4 m wide. **6.2, 6.3**

5 Find the area of each of the following shapes. **6.3, 6.4**

(a)



(b)



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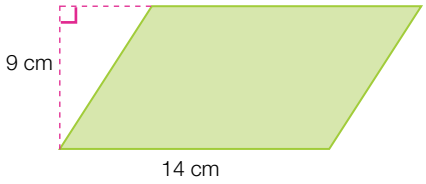
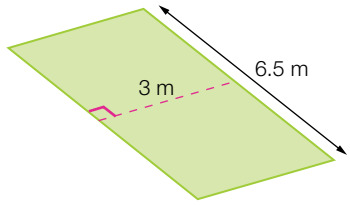
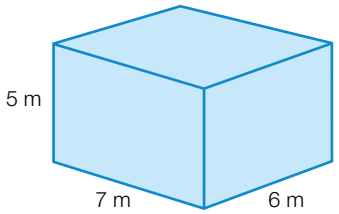


(d)



6 Determine the volume of the following solids. **6.6**

(a)



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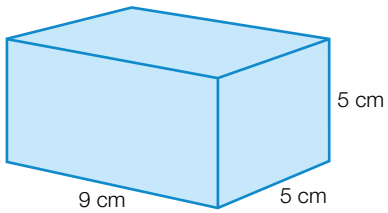
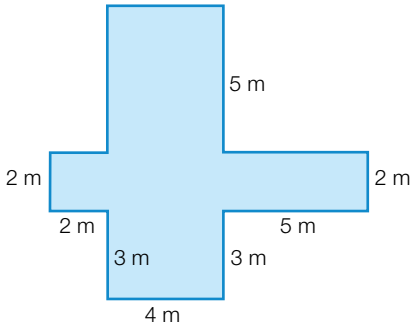


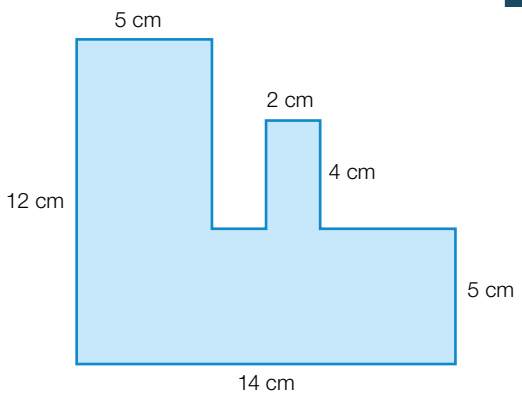
Understanding

7 Find the area of each composite shape below. **6.5**

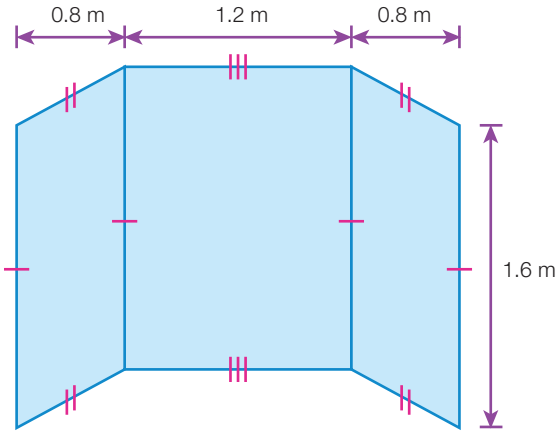
(a)



(b)



(c)



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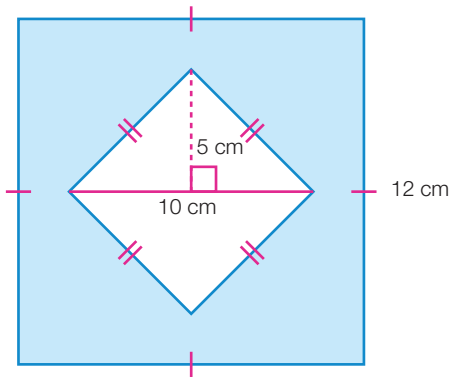


8 A carpenter calculates that the total length of timber she needs to make a bookcase is 4850 mm.

6.1

(a) How many metres is this?

(b) If timber costs $6 per metre at the hardware store, how much will she pay for the exact length she needs?

9 Robbie is tiling the floor of his bathroom, which is a rectangle measuring 3 m by 2.5 m.

6.1, 6.3

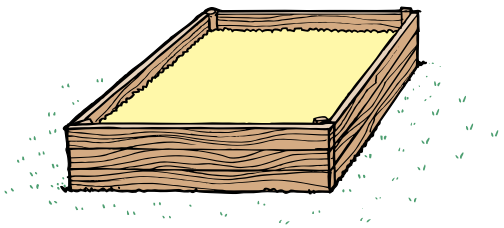
(a) Write these measurements in centimetres, then calculate the area of the floor in square centimetres.

(b) Robbie is using square tiles that have a side length of 25 cm. Calculate the number of tiles needed to cover the floor.

(c) If the tiles cost 80 cents each, calculate how much Robbie will pay for the required number of tiles.

10 Theresa is constructing a sandpit for her children. She has created a rectangular wooden box that is 2 m long, 1.5 m wide and 0.3 m high.

6.6



(a) Calculate the perimeter of the box.

(b) Calculate the volume of the box.

(c) If one cubic metre of sand costs $15, calculate how much it will cost to fill the box to a depth of 25 cm with sand.

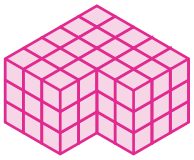
11 Tricia has obtained a quote of $21 per square metre for her driveway to be resurfaced. If her driveway is rectangular and measures 12 m long and 3.2 m wide, what will the job cost her?

6.3

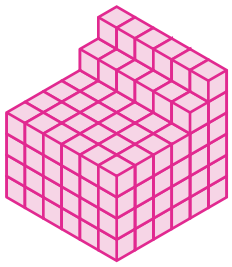
12 Determine the volume of each of the following solids. (The cubes are centimetre cubes.)

6.6

(a)

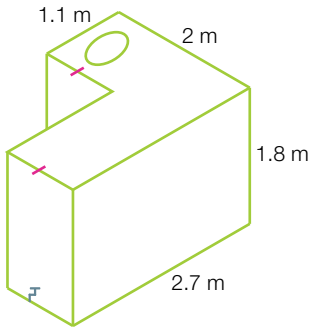


(b)



13 Cameron has had a water tank specially made to fit the side of his house. Calculate the volume of the tank, in cubic metres.

6.6



Reasoning

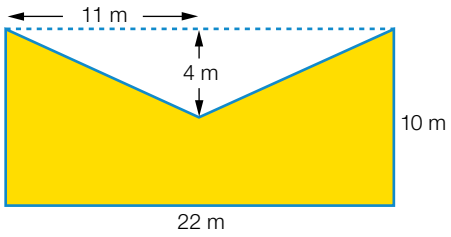
14 What is the perimeter of a square with an area of 25 cm^2?

6.2

15 What is the area of a rectangle with a perimeter of 32 cm and a length of 10 cm?

6.3

16 The diagram on the right shows the available floor space (shaded) in a car showroom. Calculate the area available for a display.



6.5

17 Five blocks of dimensions 3 cm × 5 cm × 6 cm are put in a toy box of length 40 cm, width 20 cm and height 9 cm. Determine if they fit by finding the volume of space left in the box.

6.6

18 Draw a diagram to help you explain:

6.5

(a) how to turn a parallelogram into a rectangle

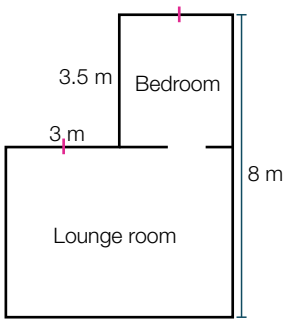
(b) how to turn a triangle into a rectangle.

Numeracy practice 6

Non-calculator

1 Which has the larger area: a rectangle with a length of 20 cm and a width of 15 cm, or a triangle with a base of 20 cm and a height of 15 cm?

2 Li is having new carpet laid in the lounge room and bedroom of her apartment. What area is to be carpeted?



3 What is the perimeter of a square that has an area of 100 cm^2?

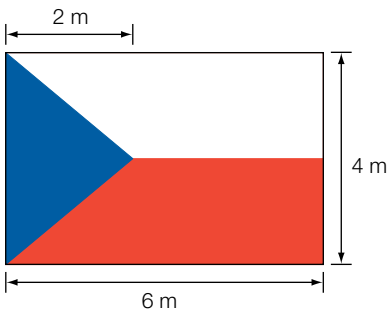
4 A rectangular picture frame has a perimeter of 80 centimetres. The frame has a length of 25 centimetres.

What is the width of the picture frame? _____ cm

Calculator allowed

5 The flag of the Czech Republic is shown at right.

The line where the red and white sections meet is half-way up the flag. If the blue triangle has a height of 2 m, the area of the flag that is red is:

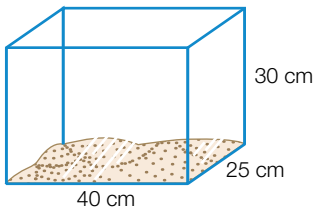


A 8 m^2 **B** 9 m^2

C 10 m^2 **D** 16 m^2

6 A pack of playing cards is 2.6 cm high. If there are 52 cards in the pack, what is the thickness of each card?

7 The Hudson family have bought a new fish tank, requiring a 20 mm deep layer of pebbles on the bottom. Which volume of pebbles should they buy?



A 1000 cm^3 **B** 2000 cm^3

C 20 000 cm^3 **D** 30 000 cm^3

8 Sanjay used identical cubes to build a rectangular prism. There were 18 cubes in its base. She used 54 cubes altogether. Which of these could be the dimensions of Sanjay's prism?

A 6 × 3 × 2 **B** 3 × 2 × 9 **C** 9 × 3 × 3 **D** 6 × 3 × 9

Mixed review C

Fluency

1 Write the following in algebra. **5.1**

(a) Akash has n pencils in his pencil case. He takes 8 out and lends them to a friend. How many pencils are in the case now?

(b) k dollars is divided evenly among 6 people. How much money does each person receive?

(c) There are 50 matches in one box. How many matches are there in h boxes?

2 Arrange the following decimals in order from smallest to largest. **4.1**

(a) 2.3, 2.003, 2.33, 2.323, 2.302 **(b)** 0.089, 0.129, 0.091, 0.0909, 0.0199

3 Write the following as fractions or mixed numbers in simplest form. **3.1**

(a) 6 out of 24 students in the class have black hair.

(b) 5 full dozen egg cartons and 4 eggs in another carton.

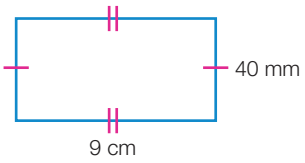
(c) 25 minutes out of an hour.

4 Write each of the following numbers as a product of their prime factors in index form. **2.3**

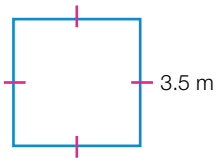
(a) 28 **(b)** 200 **(c)** 990

5 Find **(i)** the perimeter and **(ii)** the area of the following shapes, giving your answer in the larger unit where necessary. **6.2, 6.3, 6.5**

(a)



(b)



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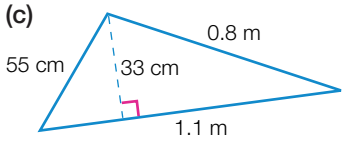


6 Calculate: **1.5**

(a) $7 + 32 \div 4 \times 2$ **(b)** $(33 + 12) \div 3 + 19$ **(c)** $6 \times 9 \div 2 + 4$

7 Calculate the following, writing your answers in simplest form and as a mixed number, if appropriate. **3.5, 3.6**

(a) $\frac{5}{6} \times \frac{4}{9}$ **(b)** $\frac{9}{10} \div \frac{7}{15}$ **(c)** $1\frac{3}{5} \times 2\frac{7}{8}$ **(d)** $4 \div \frac{2}{3}$

8 Copy and complete the following conversions. **6.1**

(a) 6.09 km = _____ m **(b)** 83 cm = _____ m **(c)** 5400 cm = _____ km

9 Calculate the following. **2.5–2.7**

(a) $-2 + 8$ **(b)** $14 - 17$ **(c)** $-15 + 3$ **(d)** $-5 - 6$

(e) $3 + (-9)$ **(f)** $-4 - (-1)$ **(g)** $11 - (-2)$ **(h)** $-7 - 10$

10 Use a mental or written strategy to calculate the following. **1.1, 1.3**

(a) 28×43 **(b)** $990 \div 6$ **(c)** 57×4

11 There are 8 red, 12 blue and 4 black jelly beans in a packet. **4.9**

(a) Write the ratio blue:red:black in simplest form.

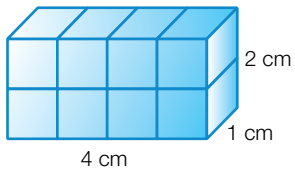
(b) Write the number of red jelly beans as a percentage of the total, rounded to 1 decimal place.

12 Write the following decimals as fractions in simplest form. **4.3**

(a) 0.23 **(b)** 0.06 **(c)** 0.505

13 Find the volume of the following rectangular prisms. **6.6**

(a)



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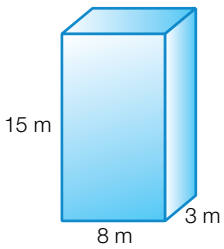

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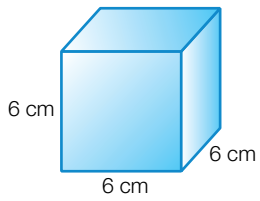


14 Copy and complete the table using the rule $y = 3(x + 2)$. **5.4**

x	0	2	3	6	10
y					

15 Calculate: **4.4**

(a) $4.67 + 8.8$ **(b)** $8.93 + 0.006\,35$ **(c)** $19.87 + 21.075$

16 Calculate: **4.6**

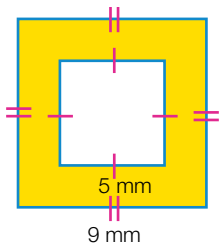
(a) $1.86 \div 600$ **(b)** $0.42 \div 1000$ **(c)** $10.605 \div 0.005$

Understanding

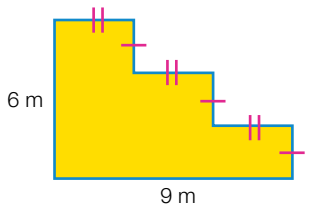
17 Joseph had $578.97 in his bank account. After his pay was deposited, there was $665.49. How much was Joseph's pay? **4.3**

18 Find the area of the shaded regions. **6.3, 6.5**

(a)



(b)



19 Ben surveyed some Year 7 students to find out their favourite Australian animal. Here are his results: **3.2, 4.3, 4.7**

Koala	8 students	Bilby	6 students
Wombat	4 students	Kangaroo	2 students

Write the preference for each animal **(a)** as a fraction in simplest terms, and **(b)** as a percentage of the total number of students.

Reasoning

20 Which is larger: 2^8 or 8^2? **1.2**

21 The Hawksbury Magpies were playing in the U14 football grand final. All their goals were scored by three players. Johan scored $\frac{1}{3}$ of the goals, Rihbi scored $\frac{1}{2}$ of the goals and Kyle scored the rest. If the Magpies scored 24 goals, how many goals were scored by Kyle? **3.4, 3.5**

22 A rectangle has a length 5 cm greater than its width and a perimeter of 42 cm. What are the dimensions of the rectangle? **5.4, 6.3**

23 At a shop I can buy three cans of soft drink for the same price as a hamburger. If a can of soft drink costs $\$c$ and a hamburger costs $\$h$, write an equation for this situation. **5.2**

Exploration STEM

You can download this activity from the eBook or the Pearson Places website.

The grand architect

The population is growing and the government needs a new secondary school. Your task is to plan the new school buildings to meet the government's needs, creating the building designs and layout. This is an opportunity to release your creativity and let your imagination roam free.

Exploration STEM

You can download this activity from the eBook or the Pearson Places website.

Survey and discovery

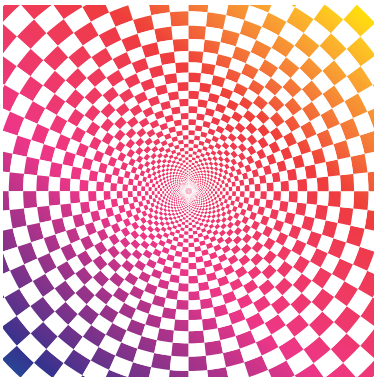
Collecting information from groups can help you to discover and understand many interesting things about the group. But there are different ways to collect and present information. Which ways are appropriate and useful? Your task is to find the best questions to ask and the best ways to analyse and present the information.

Exploration Coding

You can access this activity from the eBook or the Pearson Places website.

Creating geometric patterns

Many complex geometric designs can be made just by repeating small simple steps. Explore algorithms that will create geometric patterns using basic shapes.




7

7 Linear equations

From physical to digital. Why do people learn physical tasks at different rates?

Why do some people seem to have 'natural talent' when others struggle to master the basics? University researchers are studying how people learn various skills. They are measuring the motions involved in tasks as simple as playing racquet sports through to more complex activities, such as flying a fighter jet, in the hope that this research will help people to learn these skills faster and more effectively.

Some research projects have used motion-capture technology, including game console devices such as Microsoft Xbox Kinect and Nintendo Wii, to create equations that represent skills involved in many human activities from sport to surgery. Data from a motion-capture device can be used to measure a range of movements that is then used to create mathematical equations.

One useful aim of research like this is to program robots to teach people the most efficient way to learn new tasks. By bringing robotics and virtual reality together, learning can be easier than trial and error. For example, if you were learning to play tennis, a robotic device could make learning easier, such as a robotic sleeve that prompts you to move your arm in a more correct way.

Forum

Researchers have used commercial game consoles to investigate ways that different movements can be recorded. Why might universities be interested in this? How do you think this could be used in classrooms?

Why learn this?

Whether you are deciding how much material to buy to make curtains, working to a family budget, or calculating whether a business has made a profit, equations can be used to represent a variety of situations. Solving equations can help you to make informed and accurate decisions.

After completing this chapter you will be able to:

- understand how equations work
- write an equation from information given in words
- check solutions to equations by substitution
- solve equations using a variety of methods
- solve worded problems and everyday problems using equations.

Recall 7

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

1 Answer true (T) or false (F) to each of the following number sentences.

(a) $3 + 7 = 5 + 6$ **(b)** $8 \times 3 = 20 + 4$ **(c)** $\frac{24}{8} = 3 \times 5 - 14$

2 **(a)** The term $5x$ means:

A $5 + x$ **B** $5 - x$ **C** $5 \times x$ **D** $x \div 5$

(b) The term $\frac{x}{7}$ means:

A $x - 7$ **B** $x + 7$ **C** $x \times 7$ **D** $x \div 7$

(c) $4(x + 3)$ means:

A $4 + (x + 3)$ **B** $4 \times (x + 3)$ **C** $4 \times x + 3$ **D** $4 - (x + 3)$

3 Rewrite each of the following sentences using mathematical symbols and numbers only.

(a) Five added to seven is equal to twelve.

(b) Subtracting two from the product of four and six is equal to twenty-two.

(c) When the sum of eight and six is divided by seven, the answer is equal to two.

4 What is the final result after applying the following operations to the number in the box?

(a) $\boxed{8}$ $\times 2$ then -4 then $\div 3$ then $+5$ **(b)** $\boxed{10}$ $\div 2$ then $\div 5$ then $+6$

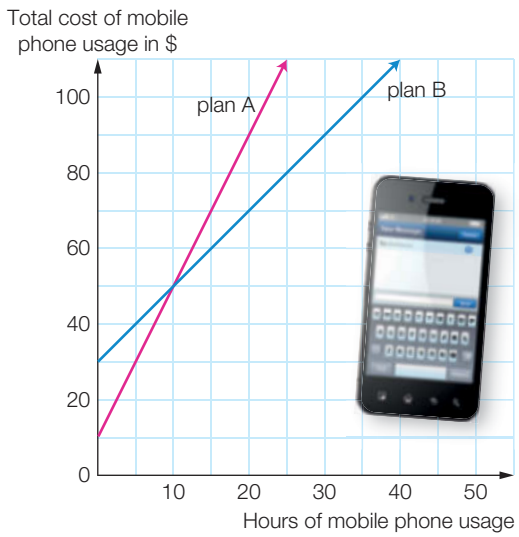
5 **(a)** If $x = 2$, find the value of $4x - 1$. **(b)** If $x = 7$, find the value of $\frac{x - 3}{2}$.

Exploration Task

You can download this activity from the eBook or the Pearson Places website.

Comparing mobile phone plans—which is better?

In this activity, you will explore how linear equations and linear graphs can be used to understand different costs of different mobile phone plans.



Number sentences

7.1

Vlado wants change for a $5 note. His friend Perry gives him two $2 coins and a $1 coin in exchange for the note. This can be written as $5 = 2 + 2 + 1$, which is an example of a **true number sentence**. It is true because the expressions on both sides of the = symbol are equal.

=

$2 \times 4 = 5 + 3$

left-hand side (LHS) | equals symbol | right-hand side (RHS)

This is another example of a true number sentence, as both the left-hand side (LHS) and the right-hand side (RHS) have the same value of 8.

LHS = RHS

$2 \times 4 = 5 + 4$
LHS ≠ RHS

This is an example of a **false number sentence**, because the LHS has a value of 8 and the RHS has a value of 9.

≠ means 'not equal to', < means 'less than', > means 'greater than'

Worked example 1

W.E.1

The following number sentences are not true. Rewrite each number sentence by changing the coloured number so that you have a true number sentence.

(a) $5 + 8 = 15$ **(b)** $2 \times 5 - 1 = 7$

Thinking		Working
(a)	**1** Find the value of the side that has the coloured number.	(a) $5 + 8 = 13$
	2 Compare the answer to the other side of the number sentence.	$13 < 15$
	3 Try different numbers until both sides of the number sentence are equal. If the answer is too high, try a smaller number. If the answer is too low, try a larger number.	$5 + 10 = 15$

(b)	**1**	Find the value of the side that has the coloured number.	(b) $2 \times 5 - 1 = 9$
	2	Compare the answer to the other side of the number sentence.	$9 > 7$
	3	Try different numbers until both sides of the number sentence are equal. If the answer is too high, try a smaller number. If the answer is too low, try a larger number.	$2 \times 4 - 1 = 7$

Worked example 2

W.E. 2

Decide which of the following are true number sentences.

(a) $8 \times 3 + 4 = 28$

(b) $\frac{6 \times 5}{3} = 7 + 2$

Thinking			Working
(a)	**1**	Evaluate the LHS of the number sentence.	(a) LHS $= 8 \times 3 + 4$ $= 28$
	2	Compare the LHS and RHS to check whether they are equal.	LHS = RHS
	3	State your answer.	The sentence is true.
(b)	**1**	Evaluate the LHS of the number sentence.	(b) LHS $= \frac{6 \times 5}{3}$ $= 10$
	2	Evaluate the RHS of the number sentence.	RHS $= 7 + 2$ $= 9$
	3	Compare the LHS and RHS to check whether they are equal.	LHS $\neq$ RHS
	4	State your answer.	The sentence is false.

7.1 Number sentences

Navigator

1, 2 (columns 1–2), 3, 4, 5, 6, 7, 8, 9, 10, 12 (a–e), 14, 16	1, 2 (columns 2–3), 3, 4, 5, 6, 7, 8, 9, 10, 11 (a), 12 (a–f), 13, 14, 15, 16, 17	2 (column 3), 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18

Answers p. 687

Fluency

1 The following number sentences are not true. Rewrite each number sentence by changing the coloured number so that you have a true number sentence. W.E. 1

(a) $3 + 4 = 10$ **(b)** $5 \times 6 = 25$ **(c)** $\frac{24}{4} = 4$

(d) $22 - 7 = 8 + 4$ **(e)** $3 \times 7 + 2 = 8 + 9$ **(f)** $1 + \frac{15}{5} = 6$

2 Decide which of the following are true number sentences by checking if the LHS = RHS. W.E. 2

(a) $\frac{32}{4} + 7 = 16$ **(b)** $4 + 3 \times 2 = 14$ **(c)** $6 + 4 \times 5 = 25$

(d) $30 \div 6 = 4 + 1$ **(e)** $23 - 7 = 12 + 4$ **(f)** $8 + 4 = 6 \times 2$

(g) $7 - 5 = 9 \div 3$ **(h)** $10 \times 8 = 90 - 10$ **(i)** $\frac{6 + 10 + 12}{7} = \frac{20 + 4}{6}$

3 (a) The number missing from the true number sentence $3 \times \_\_\_\_ = 19 + 5$ is:

A 3 **B** 6 **C** 7 **D** 8

(b) The number missing from the true number sentence $4 \times 2 = \_\_\_\_ + 4$ is:

A 0 **B** 2 **C** 4 **D** 8

(c) The number missing from the true number sentence $\frac{24}{8} + \_\_\_\_ = 10 - 2$ is:

A 4 **B** 5 **C** 6 **D** 8

4 Write a true number sentence that is equal to 46, using:

(a) addition **(b)** multiplication and addition

(c) division and subtraction.

Understanding

5 Rewrite each true number sentence using numbers and mathematical symbols only.

(a) Forty-three added to five is equal to forty-eight.

(b) Three multiplied by seven is equal to nineteen plus two.

(c) Fifty divided by five is equal to the product of five and two.

(d) When the sum of the numbers six and eight is subtracted from twenty, the answer is equal to twelve divided by two.

6 Write each of the following as a number sentence, then state whether it is true or false.

(a) Nineteen subtracted from thirteen is equal to six.

(b) Four divided by two is equal to eight.

(c) Eleven subtracted from eight is equal to six minus three.

(d) Twelve divided by four is equal to fifteen divided by five.

7 Jane is 5 years older than Todd. If Todd is 7 years old, then write a true number sentence to show Jane's age.

8 Six pens at 60 cents each cost the same as buying 4 exercise books at 90 cents each. Write this as a true number sentence.

9 The cost of a new shirt is $25 after a discount of $7 is given. Write a true number sentence to show the original price of the shirt.

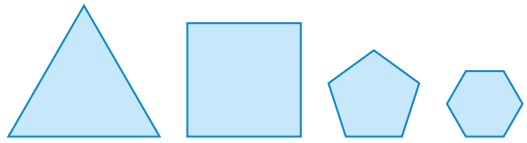
10 A bus can carry 48 passengers. The bus is full, then 11 people leave while another 5 get on at the next stop. Write a true number sentence to show how many passengers are left on the bus.

11 Numbers are consecutive if one follows the other. The numbers 8 and 9 are consecutive numbers that add to 17.

(a) Write a true number sentence that shows two consecutive numbers that add to 33.

(b) Write a true number sentence that shows three consecutive numbers whose sum is 54.

Reasoning



12 A piece of wire 60 cm long is used to make a triangle shape that has all three sides the same length. Write a true number sentence to answer each of the following.

(a) How long is each side of the triangle?

(b) The wire is now used to make a square. How long is each side?

(c) The wire is bent to make a regular pentagon. How long is each side?

(d) The wire is then used to make a regular hexagon. How long is each side?

(e) Write a number sentence to determine the length of the side of a shape that has 20 sides of equal length made from the same piece of wire.

(f) Write a number sentence to determine the length of the side of a shape that has 50 sides of equal length made from a wire 200 cm long.

(g) Use a 100 cm length of wire to make a shape whose sides are of equal length. Find three possible values for the side length the shape can have if the side length is a whole number and shorter than 30 cm.

13 Emma was working out an answer for the question $2 \times (16 + 24) =$ _____.
She wrote the following number sentence:

$(16 + 24) = 40 \times 2 = 80$

Explain why this is a false mathematical statement.

Open-ended

14 Use the numbers 1, 2, 4, 8, the equal sign, =, and as many of the symbols +, –, ×, ÷ as you like to write three different true number sentences whose right-hand side is a number from 1 to 10.

15 A wire is used to make a shape that has eight sides of equal length. Find five different whole number values for the length of the wire, if the length of each side is greater than the number of sides.

16 $\square + \square \times \square - \square = 20$

What might the missing numbers be? Give three different answers.

17 Sam and Ruben have written the following number sentences. Sam wrote $3 + 4 \times 21 = 147$. Ruben wrote $3 + 4 \times 21 = 87$.

Who is correct? Why? Write a sentence of advice so that the person who was incorrect can improve their skills.

18 Two different students were given the following number sentence:

$38 - 16 = \square - 12$

They presented the following solutions.

$38 - 16 = \square - 12$
$38 - 12 - 4 = \square - 12$
$38 - 4 - 12 = \square - 12$
$34 - 12 = \square - 12$
$\square = 34$

$38 - 16 = \square - 12$
$38 - (12 + 4) = \square - 12$
$38 + 4 - 12 = \square - 12$
$42 - 12 = \square - 12$
$\square = 42$

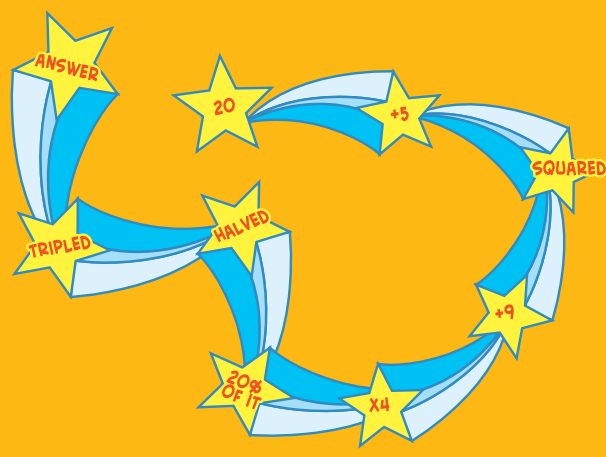
Which of the solutions is correct? Why? What is wrong with the incorrect solution?

Game

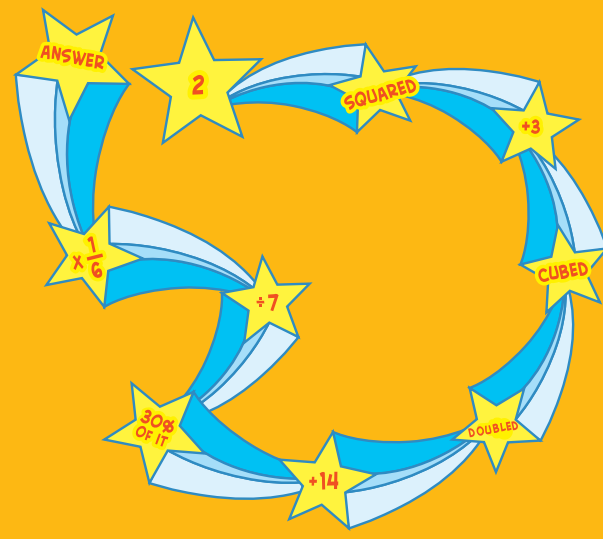
Star quest

Equipment required: 1 watch

(a) Start at the first star and follow the path of shooting stars. Time yourself to see how much time it takes to complete all of the operations.



(b) What about trying something more challenging? Time yourself completing this star quest.



7.2 Introduction to equations

Often, in mathematics, you want to write information in a number sentence but some information is missing. Where a value is unknown, you can use a pronumeral to represent the unknown amount. This number sentence is now called an equation.

$2 \times 4 = 5 + 3$ is a true number sentence. $2 \times 4 = 5 + 4$ is a false number sentence.

$2 \times 4 = 5 + x$ is an equation. Equations always contain an equals sign.

You need to be able to understand and interpret the information given to you in an equation.

The equals symbol was invented in 1557 by the Welsh mathematician Robert Recorde.

Worked example 3 W.E. 3

Write each of the following equations in words.

(a) $5x - 3 = 1$ **(b)** $6 = \frac{x}{5} + 2$

Thinking			Working
(a)	**1**	Call the variable 'a number' and write in words the operation that has been performed on it first, using the correct order of operations.	(a) When a number is multiplied by five
	2	Decide what happens next to the LHS and add this on to what you have already written.	and then three is subtracted
	3	State the final result. This is the value shown on the RHS.	the result is equal to one.
	4	Write the complete equation in words.	When a number is multiplied by five, and then three is subtracted, the result is equal to one.
(b)	**1**	Call the variable 'a number' and write in words the operation that has been performed on it first using the correct order of operations.	(b) When a number is divided by five
	2	Decide what happens next to the RHS and add this on to what you have already written.	and then two is added
	3	State the final result. This is the value shown on the LHS.	the result is equal to six.
	4	Write the complete equation in words.	When a number is divided by five, and then two is added, the result is equal to six.

You would **solve** an equation by finding the unknown value that makes an equation a true number sentence. By substituting $x = 3$ into $2 \times 4 = 5 + x$ we get a true number sentence, so the **solution** to $2 \times 4 = 5 + x$ is $x = 3$.

If x is any other value, you would get a false number sentence.

Equations are useful in solving everyday problems by providing a shorthand way of writing information. There are many different methods used to solve equations.

Solving by inspection

Many equations involve only one operation (+, –, ×, ÷) and are sometimes called one-step equations. For example, $3x = 9$, $\frac{x}{2} = 7$, $a + 3 = 11$, $4 - c = 1$ are one-step equations.

These equations can be solved by simply looking at the equation and working out the value of the variable in your head. When you do this, you need to check that the solution will make a true number sentence. This method is called '**solving by inspection**'.

Here are some more one-step equations and their solutions:

For $k - 4 = 5$, the solution is $k = 9$. Check: $9 - 4 = 5$ is a true number sentence.

For $\frac{a}{3} = 5$, the solution is $a = 15$. Check: $\frac{15}{3} = 5$ is a true number sentence.

Worked example 4

W.E. 4

Solve the following expression by inspection and check the answer.

$5x = 15$

Thinking	Working
1 Use words to describe the equation.	Five times a number is 15
2 In your head consider the possible number that will make the equations true.	$= 5 \times 3$ $= 15$ as $x = 3$
3 Check to see whether this answer is the same as the RHS of the equation.	RHS = 15 Yes, $x = 3$ is the solution.

Solving equations using guess, check and improve

You can use '**guess, check and improve**' to solve equations with more than one step. As with solving by inspection, you look at the equation, then guess a number you think will make it true. Check your solution using substitution to see if the LHS = RHS. If the sides are equal, then your guess is correct. If the sides are not equal, then you will need to improve your guess to try another solution.

Hint: If the LHS and RHS are close, then choose another number close to your original guess. If the sides are not close, then take another look at the equation and guess again.

Worked example 5

W.E. 5

Find the solution to the following equation by using the guess, check and improve method.

$$\frac{6x - 1}{5} = 7$$

Thinking	Working
1 Guess a value for x and substitute it into the LHS of the equation to see if it makes a true number sentence.	Try $x = 5$: $\frac{6 \times 5 - 1}{5} = \frac{30 - 1}{5}$ $= \frac{29}{5}$ $\neq$ RHS
2 Improve your guess by trying another value for x and substituting it into the LHS of the equation.	The original guess made the LHS close to the RHS so try a value close to 5. $x = 6$: $\frac{6 \times 6 - 1}{5}$ $= \frac{36 - 1}{5}$ $= \frac{35}{5}$ $= 7$ = RHS
3 As this value makes a true number sentence, we have a solution. If the LHS still does not equal the RHS, try another value.	$x = 6$ is the solution to the equation.

7.2 Introduction to equations

Navigator

Answers p. 688

1 (column 1), 2 (columns 1–2), 3 (columns 1–2), 4, 5, 6, 7, 8 (a–b), 9, 10, 12, 13	1 (column 2), 2 (columns 2–3), 3 (columns 2–3), 5, 6, 7, 8, 9, 10, 12, 13	1 (column 3), 2 (column 3), 3 (column 3), 4, 6, 7, 8, 9, 10, 11, 12, 13

Fluency

W.E. 3

1 Write each of the following equations in words.

(a) $n + 3 = 10$ **(b)** $4 + p = 7$ **(c)** $a + 5 = 6$

(d) $x - 2 = 4$ **(e)** $k - 7 = 2$ **(f)** $a - 8 = 6$

(g) $5k = 20$ **(h)** $7p = 14$ **(i)** $7a = 21$

(j) $\frac{x}{3} = 4$ **(k)** $\frac{f}{2} = 6$ **(l)** $\frac{y}{6} = 12$

(m) $2 + 3x = 7$ **(n)** $2 + 6x = 10$ **(o)** $1 + 3x = 8$

W.E. 4

2 For the following equations, check whether the value given in the brackets is the solution. (Does the value make the equation true?) Answer yes or no.

(a) $x + 6 = 11$	$(x = 5)$	**(b)** $y + 4 = 10$	$(y = 5)$	**(c)** $m + 2 = 11$	$(m = 9)$
(d) $d - 6 = 15$	$(d = 21)$	**(e)** $k - 3 = 8$	$(k = 5)$	**(f)** $a - 6 = 9$	$(a = 15)$
(g) $3x = 12$	$(x = 3)$	**(h)** $5a = 20$	$(a = 4)$	**(i)** $7m = 14$	$(m = 2)$
(j) $5m + 1 = 9$	$(m = 2)$	**(k)** $2a - 2 = 8$	$(a = 3)$	**(l)** $5s - 4 = 6$	$(s = 2)$
(m) $\frac{x}{3} + 2 = 5$	$(x = 9)$	**(n)** $5 + \frac{b}{2} = 9$	$(b = 8)$	**(o)** $\frac{a}{15} + 2 = 5$	$(a = 30)$

3 Find the solution to each of the following equations by using inspection or the 'guess, check and improve' method. W.E. 5

(a) $2x + 1 = 9$ **(b)** $6 + 5x = 41$ **(c)** $7x + 3 = 52$

(d) $\frac{x-8}{3} = 7$ **(e)** $\frac{x-5}{2} = 6$ **(f)** $\frac{x-4}{5} = 12$

(g) $3(x - 11) = 21$ **(h)** $8(x - 2) = 32$ **(i)** $6(x + 13) = 54$

(j) $\frac{x}{2} - 5 = 33$ **(k)** $\frac{x}{6} - 8 = 14$ **(l)** $\frac{x}{3} + 7 = 61$

4 In words, the equation $2 = 5 - \frac{x}{6}$ can be written as:

A two is equal to five added to a number that is divided by six

B five subtracted from six times a number is equal to two

C five added to two is equal to a number that is divided by six

D two is equal to a number divided by six and the result is subtracted from five.

5 Which equation describes each sentence?

(a) Four added to a number is equal to ten.

A $n + 10 = 4$ **B** $n + 4 = 10n$ **C** $4 + 10 = n$ **D** $n + 4 = 10$

(b) A number multiplied by three is equal to six.

A $x + 3 = 6$ **B** $6x = 3$ **C** $3x = 6$ **D** $\frac{x}{3} = 6$

(c) A number divided by four is equal to three.

A $4x = 3$ **B** $x + 4 = 3$ **C** $4 - x = 3$ **D** $\frac{x}{4} = 3$

6 (a) The solution to the equation $x + 3 = 18$ is:

A $x = 6$ **B** $x = 15$ **C** $x = 16$ **D** $x = 21$

(b) The solution to the equation $\frac{x}{2} = 4$ is:

A $x = 1$ **B** $x = 2$ **C** $x = 6$ **D** $x = 8$

(c) The solution to the equation $4 = x - 11$ is:

A -15 **B** 7 **C** 11 **D** 15

Understanding

7 Which equation best describes each situation?

(a) The cost, c, in dollars, of some grocery items plus a $3 delivery charge totals $55.

A $3c = 55$ **B** $c - 3 = 55$ **C** $c + 3 = 55$ **D** $55 + 3 = c$

(b) Each of 4 people receives the same number of lollies, n, when a bag containing 36 lollies is shared between them.

A $4n = 36$ **B** $4 + n = 36$ **C** $\frac{n}{4} = 36$ **D** $n - 4 = 36$

(c) A school minibus seats a total of 12 passengers when full. The number of passengers p on the bus decreases by 3 at the next stop so that the bus is now half full.

A $p - 3 = 12$ **B** $3p = 6$ **C** $p + 3 = 6$ **D** $p - 3 = 6$

8 Use 'guess, check and improve' to find the solution to the following equations.

(a) $\frac{x}{2} = x - 5$ **(b)** $3x = 5x - 8$ **(c)** $\frac{3x-1}{4} = \frac{2(x+9)}{5}$

9 Daniel is three years older than his sister Nadia.

(a) Define a variable to represent Daniel's age.

(b) Use this variable to write an expression for Nadia's age.

(c) Write an expression to show how old Nadia will be in 15 years' time.

(d) If Nadia will be 32 in 15 years' time, write an equation using your answer to part **(c)** as the LHS of the equation and solve it to find Daniel's age now.

Reasoning

10 Cameron has some money in his wallet. Libby has $5.

(a) Define a variable to represent the amount of money Cameron has in his wallet.

(b) If Cameron spends $14, write an expression to show how much money he has left.

(c) If Libby and Cameron now have $23 between them, write an equation and solve it to find how much money Cameron had to start with.

Open-ended

11 Pete is twice as old as Chee.

(a) Choose a value that is a multiple of 3 as the sum of their ages and write the equation that you would use to find Chee's age.

(b) Choose another value that is also a multiple of 3 as the sum of their ages and write a new equation you would use to find Chee's age.

(c) What age might Chee be if the sum of their ages is less than 30?

12 Write a short story that could be represented by the following equations.

(a) $14 - x = 11$ **(b)** $\frac{x}{5} = 12$ **(c)** $6x = 108$

13 Which method would you use to solve the following equations: solving by inspection or solving by guess, check and improve? Explain why you would choose that method. (Do not solve the equations.)

(a) $\frac{4y + 8}{5} = \frac{4}{y - 2}$ **(b)** $y + 7 = 14$ **(c)** $\frac{y + 6}{2} = y + 1$ **(d)** $18 - y = 15 + y$

Puzzle

The great number mystery

What number am I?

- I am a prime number.
- If you triple me, then I will be made up of two digits.
- If I am multiplied by four and then divided by five, then I will be one smaller than my original value.
- If I am squared, then I will be five less than six times my original value.

Solving equations using backtracking

7.3

Some equations are difficult to solve by inspection or by guess, check and improve because the solution is not a whole number, or because more than one operation is involved.

For example: $x + 3\frac{1}{4} = 7\frac{1}{5}$, $3x = 157$, $x - 3.75 = 21.8$ or $\frac{x}{7} = 53\frac{1}{9}$ are all one-step equations, but their solutions are not easy to see.

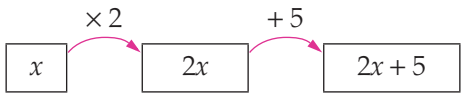
$2x + 5 = 11$, $3x - 4 = 31$, $\frac{5x}{4} = 5$, $\frac{x+6}{8} = 2$ are all two-step equations.

You need to use other methods to solve these equations.

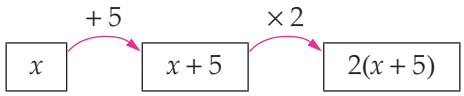
Using flowcharts

You have seen flowcharts used to build expressions. Flowcharts can also be used to help you solve equations. To use a flowchart correctly, you need to apply the order of operations that were used to build the expression.

For the expression $2x + 5$, x is first multiplied by 2 to give $2x$, then 5 is added to give us $2x + 5$, so our flowchart looks like this:



If you wanted to add 5 first before multiplying by 2, the flowchart would look like this:



This expression is $2(x + 5)$, which is quite different from $2x + 5$.

Applying the same operations in a different order produces different expressions.

Inverse operations

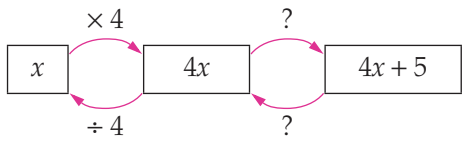
To solve equations using flowcharts, undo operations used to build the equation by applying **inverse operations** (opposite operations) in the reverse order. Move backwards along the flowchart.

Worked example 6

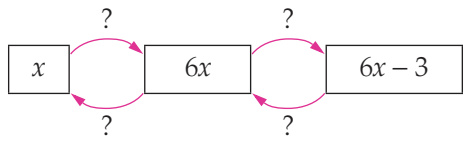
W.E. 6

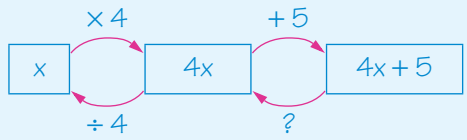
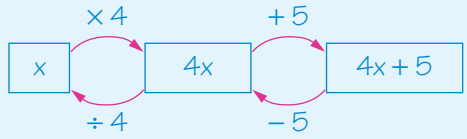
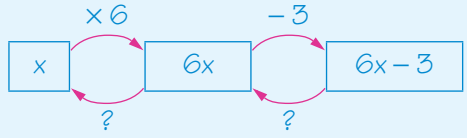
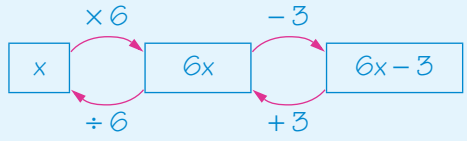
Copy and complete the following simplified flowcharts to show the order of the operations needed to build and undo the expression.

(a)



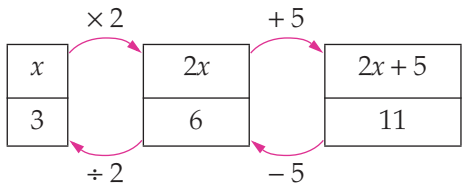
(b)



Thinking	Working
(a) 1 Above the top arrow write the missing operation needed to build the expression.	
2 Below the bottom arrow write the missing operation that will undo this expression.	
(b) 1 Above the top arrows write the missing operation needed to build each expression.	
2 Below the bottom arrows write the missing operation needed to undo this expression.	

Backtracking

To solve $2x + 5 = 11$, set up the following flowchart.



Step 1 Draw a flowchart for the expression on the LHS of the equation.

Step 2 Add boxes underneath the flowchart boxes and arrow signs in the opposite direction.

Step 3 Write the inverse operations on the arrow signs.

Step 4 Write the number on the RHS of the equation in the box on the right-hand side of the flowchart underneath the expression.

Step 5 Complete the flowchart using the inverse operations.

Step 6 The number under x is the solution.

This method of solving equations is called **backtracking**. Backtracking along this flowchart gives the solution $x = 3$ to the equation $2x + 5 = 11$.

When backtracking, undo each operation by using the inverse (opposite) operation.

+ and − are inverse operations

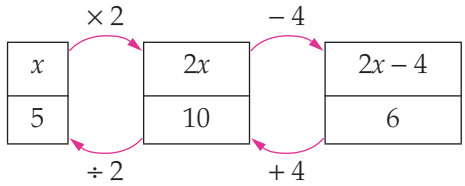
× and ÷ are inverse operations

Worked example 7

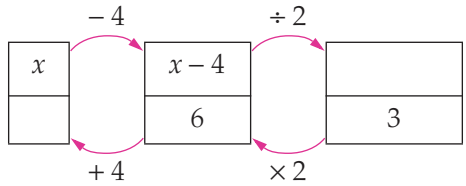
W.E. 7

Write **(i)** the equation to be solved and **(ii)** the solution to the equation shown, for each of the following flowcharts. Complete the flowchart first if necessary.

(a)



(b)



Thinking

(a) (i) The expression in the last box on the right and the number in the box underneath it form the equation.

(ii) The solution is the number under x in the first box on the left of the flowchart.

(b) (i) 1 Complete the missing parts of the flowchart.

2 The expression in the last box on the right and the number in the box underneath it form the equation.

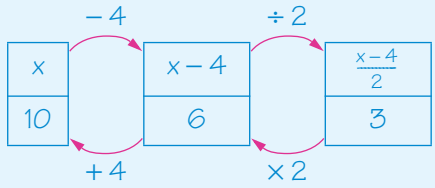
(ii) The solution is the number under x in the first box on the left of the flowchart.

Working

(a) (i) Equation is $2x - 4 = 6$

(ii) Solution is $x = 5$

(b) (i)



$\frac{x-4}{2} = 3$

(ii) $x = 10$

Worked example 8

W.E. 8

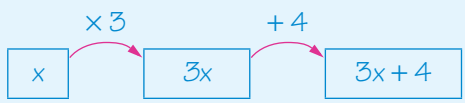
Draw a flowchart and use backtracking to solve the equation: $3x + 4 = 22$.

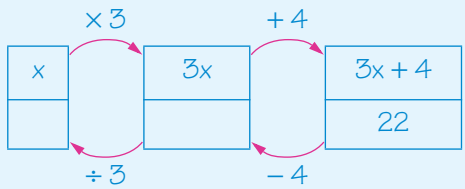
Thinking

1 Build the expression on the LHS of the equation with a flowchart. Start with x and identify the operations needed. (× 3, + 4)

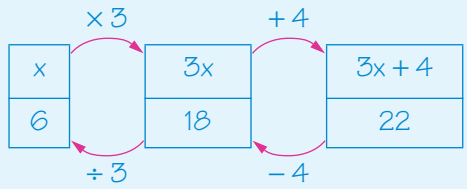
2 Write the value from the RHS of the equation under the last box on the right of the built expression. Identify the operations needed to undo the equation and write them underneath with reverse arrows. (÷ 3, − 4)

Working





3 Perform the required operations to obtain a value for x.



4 State the solution.

$x = 6$

Checking by substitution

You can check that the answer obtained is the solution to the equation by substituting it into the equation. Evaluate both sides of the equation. If the LHS = RHS, the answer is the solution. This process is called **checking by substitution**.

In the previous example, you would check the solution by substituting $x = 3$ into the left-hand side of the equation.

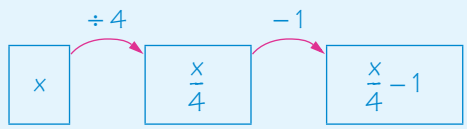
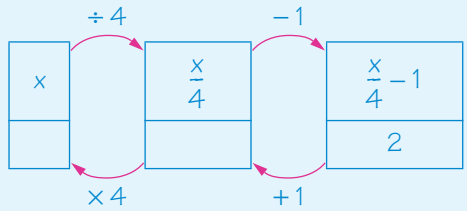
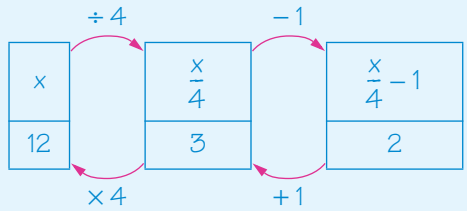
Check: LHS $= 3x + 4$
$= 3 \times 6 + 4$
$= 18 + 4$
$= 22$
$=$ RHS

So, $x = 6$ is the solution.

Worked example 9

W.E. 9

Draw a flowchart and use backtracking to solve the equation $\frac{x}{4} - 1 = 2$. Check your solution by substitution.

Thinking	Working
1 Build the expression on the LHS of the equation with a flowchart. Start with x and identify the operations needed. (÷ 4, − 1)	
2 Write the value from the RHS of the equation under the right-hand box of the built expression. Identify the operations needed to undo the equation and write them underneath with reverse arrows. (× 4, + 1)	
3 Perform the required operations to obtain a value for x.	
4 State the solution.	$x = 12$

5 Check the solution by substituting your answer into the left-hand side of the equation. If the left-hand side equals the right-hand side of the equation, you have found the solution.

Check: $\text{LHS} = \frac{x}{4} - 1$
$= \frac{12}{4} - 1$
$= 3 - 1$
$= 2$
$= \text{RHS}$

7.3 Solving equations using backtracking

Navigator

1, 2, 3, 4, 5, 6, 7 (columns 1–2), 8 (columns 1–2), 9, 10, 12, 13 (a–d), 14	1, 2 (column 1), 3, 4, 5, 6, 7 (columns 2–3), 8 (columns 2–3), 9, 10, 11, 12, 13 (a–e), 14	1 (column 2), 2 (column 2), 3 (d–f), 4 (d–f), 5, 6, 7 (column 3), 8, 9, 10, 11, 12, 13, 14, 15

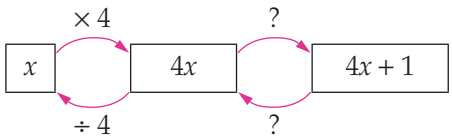
Answers p. 688

Fluency

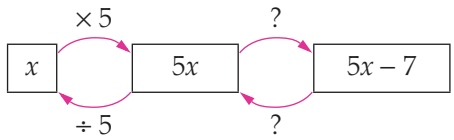
W.E. 6

1 Copy and complete the following simplified flowcharts to show the order of the operations needed to build and undo the expression.

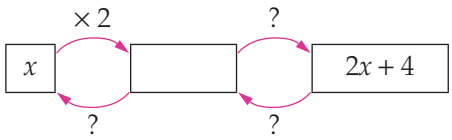
(a)



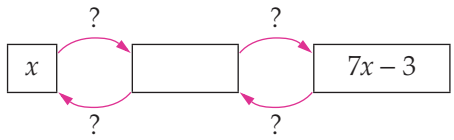
(b)



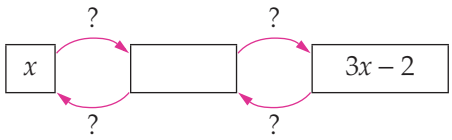
(c)



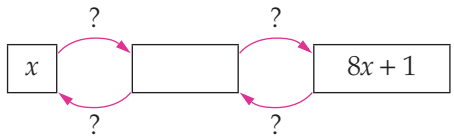
(d)



(e)



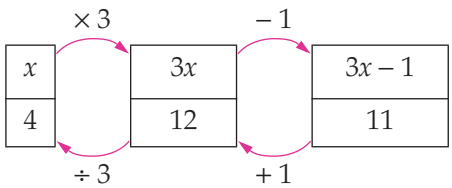
(f)



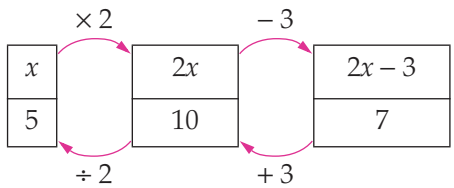
W.E. 7

2 Write **(i)** the equation to be solved and **(ii)** the solution to the equation shown in each of the following flowcharts. Complete the flowchart first if necessary.

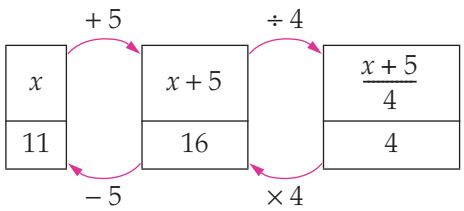
(a)



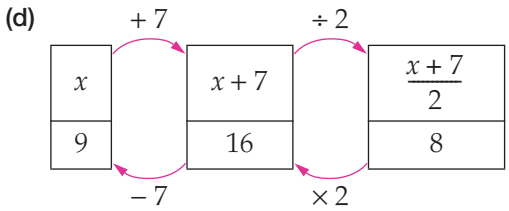
(b)



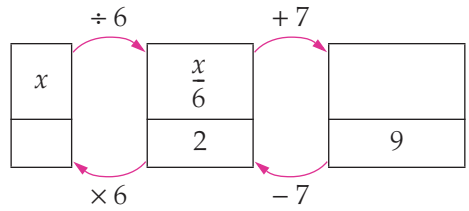
(c)



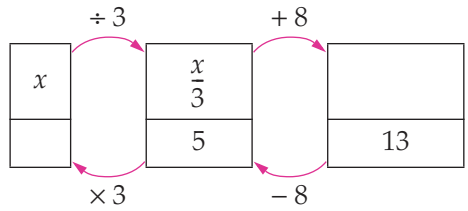
(d)



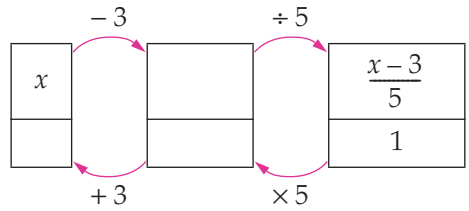
(e)



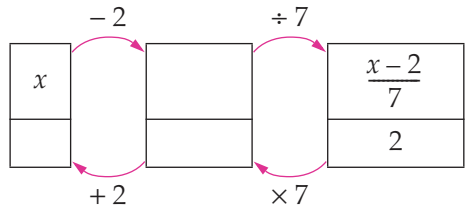
(f)



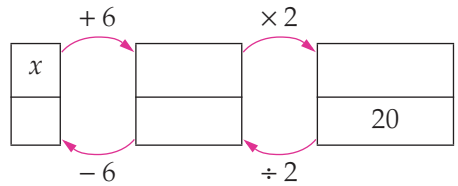
(g)



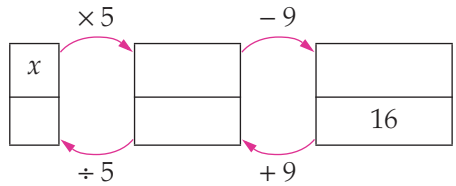
(h)



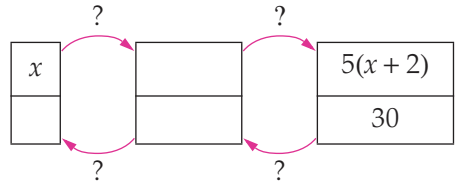
(i)



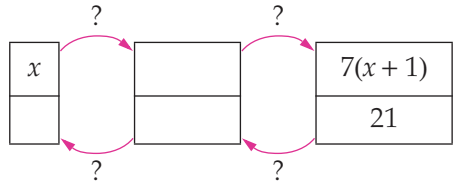
(j)



(k)



(l)



W.E. 8

3 Draw a flowchart and use backtracking to solve each of the following equations.

(a) $5x + 3 = 8$ **(b)** $7x + 3 = 24$ **(c)** $2x + 3 = 11$

(d) $2x - 7 = 3$ **(e)** $3x - 2 = 10$ **(f)** $8x - 11 = 13$

W.E. 9

4 Draw a flowchart and use backtracking to solve each of the following equations. Check your solutions by substitution.

(a) $\frac{x}{4} + 5 = 7$ **(b)** $\frac{x}{2} + 6 = 11$ **(c)** $\frac{x}{7} + 3 = 5$

(d) $\frac{x}{2} - 2 = 1$ **(e)** $\frac{x}{5} - 1 = 2$ **(f)** $\frac{x}{2} - 4 = 2$

5 Use backtracking to solve each of the following equations. Check your solutions by substitution.

(a) $3(x + 1) = 12$ **(b)** $2(x + 3) = 16$ **(c)** $5(x - 6) = 10$ **(d)** $6(x - 4) = 18$

6 Use backtracking to solve each of the following equations. Check your solutions by substitution.

(a) $\frac{x + 8}{3} = 3$ **(b)** $\frac{x + 5}{2} = 4$ **(c)** $\frac{x - 2}{5} = 1$ **(d)** $\frac{x - 3}{4} = 2$

7 Use backtracking to solve each of the following equations. Check your solutions by substitution.

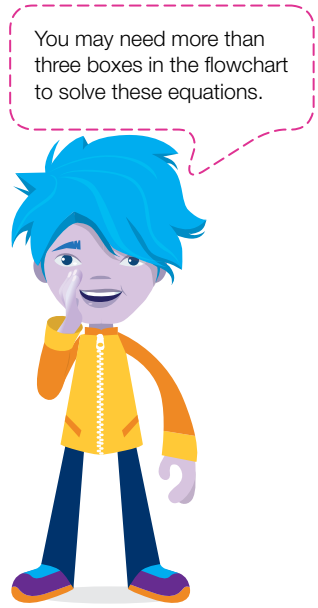
(a) $2x + 3 = 13$ **(b)** $3x + 2 = 11$ **(c)** $5x + 6 = 11$

(d) $5x - 3 = 17$ **(e)** $4x - 1 = 7$ **(f)** $8x - 9 = 7$

(g) $2(x + 7) = 20$ **(h)** $4(x - 3) = 8$ **(i)** $6(x - 1) = 12$

(j) $\frac{x}{2} + 3 = 8$ **(k)** $\frac{x}{7} + 10 = 12$ **(l)** $\frac{x}{3} - 2 = 3$

(m) $\frac{x+2}{4} = 2$ **(n)** $\frac{x-4}{5} = 2$ **(o)** $\frac{x-3}{2} = 2$



8 Solve each of the following three-step equations using backtracking. Check your solutions by substitution.

(a) $\frac{2x+1}{3} = 3$ **(b)** $\frac{3x+7}{2} = 11$ **(c)** $\frac{5x-2}{4} = 7$

(d) $\frac{3x}{5} + 6 = 9$ **(e)** $\frac{2x}{7} + 8 = 12$ **(f)** $\frac{7x}{2} - 10 = 4$

(g) $\frac{2(x+4)}{5} = 6$ **(h)** $\frac{3(x+2)}{4} = 3$ **(i)** $\frac{2(x-3)}{7} = 6$

9 The solution to $\frac{4x}{5} + 6 = 14$ is:

A $x = 1$ **B** $x = 10$ **C** $x = 16$ **D** $x = 25$

Understanding

10 Write an equation using the following information, then solve each equation using backtracking. Use n to represent the unknown number.

(a) A number is doubled, then one is added to the result to give an answer of forty-three.

(b) When four is added to a number and the result is divided by five, the answer is four.

(c) A number is multiplied by seven and the result is divided by three to give an answer of twenty-one.

(d) A number is doubled, then six is added. The result is divided by ten to give an answer of two.

11 Belinda buys 3 packets of pencils. She gives five pencils to her friends. She sells the rest for 10 cents each and makes $6.70. How many pencils were in each packet? Using n as the number of pencils in the packet, write an equation and use backtracking to answer this question.

Reasoning

12 To encourage Lui to save his allowance of $10 per week, his parents would triple the amount Lui saved before subtracting $12 as well. Let d represent the dollar amount Lui saved.

(a) Write an expression to represent how Lui's parents calculate this.

(b) Calculate the amount (if any) that will be given if he saves:

(i) $6 **(ii)** $4 **(iii)** $7.50

(c) What is the maximum amount that Lui's parents would give?

(d) What will happen if Lui saves only $2 in a week?

13 Ying repairs televisions. The expression for the amount of money ($) she charges is $30x + 20$, where x is the number of hours she takes to repair a TV. Aldo also repairs televisions. The expression for the amount of money ($) he charges is $20x + 50$ where x is the number of hours he takes to repair a TV.

(a) Show the flowchart for the expression $30x + 20$.

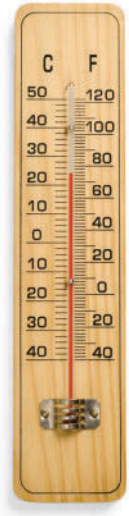
(b) Show the flowchart for the expression $20x + 50$.

(c) Substitute the number $x = 1$ in both flowcharts and write down how much money Ying and Aldo charge for a repair that takes one hour.

(d) Ying charges a customer $95. How long did it take her to repair this TV?

(e) If Aldo charged $95, how long would he have taken?

(f) By trying different values for x, find the number of hours for which the amount charged by Ying and Aldo is the same.



Open-ended

14 If x is the temperature in degrees Celsius, the expression $\frac{9x}{5} + 32$ gives the temperature in degrees Fahrenheit.

(a) Build and complete the flowchart for the expression.

(b) Use your flowchart to find two values for the temperature in degrees Celsius that give a whole number temperature for degrees Fahrenheit.

15 After Ross performs four different operations on a number, he has a result of 10. Write an equation that fits this situation.

Puzzle

Gold digger returns!

Your aim is to find the gold, using the numbered squares to eliminate the possibilities. On the two separate maps, drawn as square grids, there are numbers that mark the squares that have gold nearby. So if a number is written on a grid square on the map it has no gold, but there must be gold in the squares touching it (either horizontally, vertically or diagonally sharing an edge or a corner). If a square is numbered 3, then there must be gold in 3 of the squares that touch it, and so on for other numbers. No square contains more than one piece of gold.

Here are the two maps to solve:

(a)

	2			
	4		1	
4			3	1
				1
	3			2
				1

(b)

3				1	
		4		3	
				3	
0		3		2	

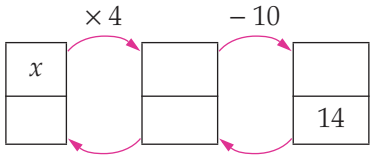
Half-time 7

1 Solve the equation $\frac{x}{12} = 7$ by inspection. **7.2**

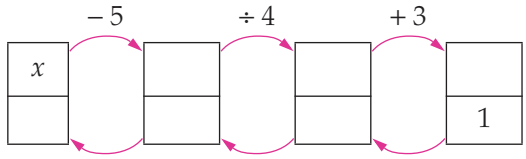
2 The perimeter of a rectangular room is 18 m. If the length of the room is 5 m, what is the width? Form an equation and solve it with backtracking. **7.3**

3 Copy and complete the following flowcharts. Write the equation and use the flowchart to find the solution for x. **7.3**

(a)



(b)



4 Write a true number sentence that shows two consecutive numbers whose sum is 43. **7.1**

5 **(a)** James is 5 years older than his brother Tom who is 7 years old. If James is j years old: **7.2**

(i) write an equation to represent this situation with Tom's age on the RHS

(ii) solve the equation by inspection to find James' age.

(b) Jude is 4 years younger than his sister Amanda who is 16 years old. If Jude is d years old:

(i) write an equation to represent this situation with Amanda's age on the RHS

(ii) solve the equation by inspection to find Jude's age.

6 Use backtracking to solve each of the following equations. **7.3**

(a) $3x + 4 = 11$ **(b)** $\frac{x}{2} + 5 = -3$ **(c)** $3(x - 8) = 15$

7 Write the equation $5x + 3 = 23$ in words. **7.2**

8 The following number sentences are not true. Rewrite them by changing the coloured number so that you have a true number sentence. **7.1**

(a) $3 + 6 = 11$ **(b)** $4 \times 3 = 26 - 6$ **(c)** $4 + \frac{28}{4} = 18$

9 Kate has a piece of cardboard that is x centimetres wide. She joins a 50-centimetre piece of cardboard of the same length to it and then divides the width into 7 strips so she can paint each strip a different colour of the rainbow. Each strip is 10 cm wide. Use backtracking to find the width of the original piece of cardboard. **7.3**

10 State whether the following number sentences are true (T) or false (F). **7.1**

(a) $6 - 4 = 4 - 6$ **(b)** $2(3 + 5) = 2 \times 3 + 2 \times 5$ **(c)** $(8 \times 9) \times 2 = 8 \times (9 \times 2)$

11 Pete is five years older than his brother Sam. The sum of their ages is 23. If Pete is p years old: **7.3**

(a) write an expression to represent Sam's age

(b) write an equation to represent this situation, with 23 on the RHS of the equation

(c) solve the equation using backtracking to find the ages of Pete and Sam.

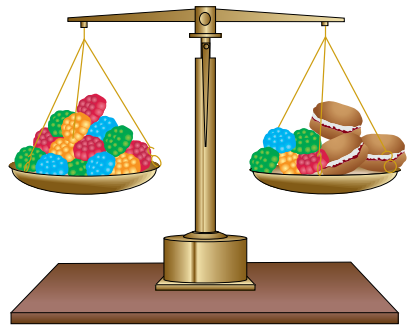
7.4 Solving equations using the balance method

Scales can be used to measure the mass of an object if other masses are available for comparison. To balance the scales, the two sides of the scales must hold equal masses.

Worked example 10

W.E. 10

This set of scales is balanced. The left-hand side has 14 lollies, while the right-hand side has three biscuits and five lollies.



(a) If one lolly is taken from the right-hand side, the scales become unbalanced. Which of the sides is now heavier?

(b) How can the scales be balanced without putting the lolly back?

(c) Starting again, if all of the lollies are taken from the right-hand side, what should be done to the left-hand side to balance the scales?

(d) How many lollies are equivalent to the three biscuits?

(e) How many lollies are equivalent to one biscuit?

Thinking	Working
(a) Look at the diagram and decide which side is heavier after removing the lolly.	(a) The left-hand side is now heavier.
(b) Balance the scales by doing the same to the other side.	(b) Take a lolly from the left-hand side.
(c) Identify how many lollies were on the right-hand side at the start (5). Take the same amount from the left-hand side.	(c) Take five lollies from the left-hand side.
(d) The beam is horizontal. Look at how many lollies are on the left-hand side (9).	(d) Nine lollies = three biscuits.
(e) The beam is balanced, so the biscuits on the right-hand side equal the lollies on the left-hand side.	(e) Three biscuits = nine lollies. One biscuit = three lollies.

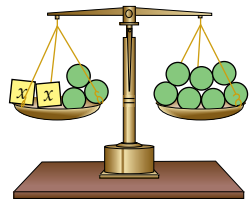
Equivalent equations

Equivalent equations are those that have the same solution. For example, $2x = 4$ is equivalent to $2x + 3 = 7$, because both equations have the same solution of $x = 2$. This can be checked by substituting $x = 2$ into both equations to show they are both true number sentences:

$2x = 4$ $\qquad$ $2x + 3 = 7$

$2 \times 2 = 4$ $\qquad$ $2 \times 2 + 3 = 7$

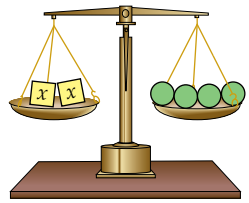
This situation can also be represented with balance scales. The equals symbol can be thought of as sitting in the middle of the two balanced sides. These scales represent $2x + 3 = 7$.



$2x + 3 = 7$

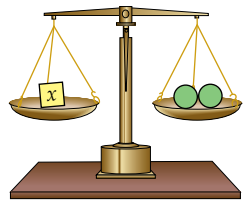
When 3 is taken off both sides, the scales are still balanced and now represent $2x = 4$.

Because the scales are still balanced we say that $2x + 3 = 7$ and $2x = 4$ are equivalent equations.



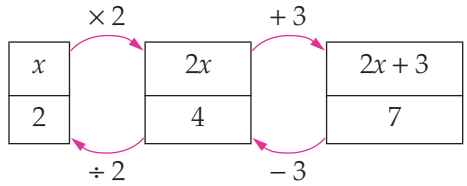
$2x = 4$

As two identical unknown masses equal 4, then one mass equals 2. The scales represent $x = 2$.



$x = 2$

These steps can be shown on a flowchart.



All these equations are equivalent. The simplest equivalent equation is $x = 2$, which is also the solution.

$x = 2$ $\qquad$ $2x = 4$ $\qquad$ $2x + 3 = 7$

You can create equivalent equations by performing the same operation on both sides of the equation.

Worked example 11

W.E. 11

Form an equivalent equation to each of the following by performing the operation given in brackets to both sides of the equation.

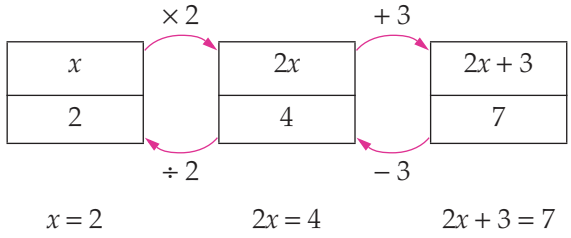
(a) $x - 5 = 7 \quad (+ 6)$ $\qquad$ **(b)** $5x + 3 = 8 \quad (- 2)$ $\qquad$ **(c)** $\frac{x}{2} = 4 \quad (\times 2)$ $\qquad$ **(d)** $8x = 24 \quad (\div 4)$

Thinking			Working	
(a)	**1**	Write the equation.	*(a)*	$x - 5 = 7$
	2	Perform the same operation on both sides (add 6).		$x - 5 + 6 = 7 + 6$
	3	Simplify.		$x + 1 = 13$
(b)	**1**	Write the equation.	*(b)*	$5x + 3 = 8$
	2	Perform the same operation on both sides (subtract 2).		$5x + 3 - 2 = 8 - 2$
	3	Simplify.		$5x + 1 = 6$
(c)	**1**	Write the equation.	*(c)*	$\frac{x}{2} = 4$
	2	Perform the same operation on both sides (multiply by 2).		$\frac{x}{2} \times 2 = 4 \times 2$
	3	Simplify.		$x = 8$
(d)	**1**	Write the equation.	*(d)*	$8x = 24$
	2	Perform the same operation on both sides (divide by 4).		$\frac{8x}{4} = \frac{24}{4}$
	3	Simplify.		$2x = 6$

Solving equations using the balance method

By using a set of scales to represent our equation, you can now solve equations using the **balance method**, or inverse operations. Up until now, you have used flowcharts to show both the order and the operations needed to solve equations. With the balance method, you still need to use the same information, but now you can show it in a different way.

Consider $2x + 3 = 7$.



Notice that the three boxes contain three equivalent equations. We have moved backwards through the flowchart and used inverse (opposite) operations (– 3, ÷ 2) to solve the equation $2x + 3 = 7$. Let's now show the same steps using the balance method. The order of operations in the flowchart tells us how our equation was built. By moving backwards and performing the inverse operation, we will undo our equation to find the solution.

$$2x + 3 = 7$$
$$2x + 3 - 3 = 7 - 3 \quad \text{(subtracting 3 from both sides)}$$
$$2x = 4$$
$$\frac{2x}{2} = \frac{4}{2} \quad \text{(dividing both sides by 2)}$$
$$x = 2$$

Note that the three equivalent equations formed here $2x + 3 = 7$, $2x = 4$, $x = 2$ are the same as the three equations formed from the boxes in the flowchart.

Worked example 12 W.E. 12

Solve the equation $3x - 7 = 11$ using the balance method. Check your solution by substitution.

Thinking	Working
1 Write the equation. Identify the last operation to be performed on the LHS of it. This is the first operation to be undone. (– 7)	$3x - 7 = 11$
2 Use the inverse operation (+ 7) on both sides of the equals sign and simplify your equation.	$3x - 7 + 7 = 11 + 7$ $3x = 18$
3 Identify the next operation to be undone and apply the inverse operation (÷ 3). If one side of the equation is now the variable by itself, you have found the solution. Otherwise, continue the process until you do have the variable by itself.	$\frac{3x}{3} = \frac{18}{3}$ $x = 6$
4 Check the solution by substitution.	Check: LHS $= 3x - 7$ $= 3 \times 6 - 7$ $= 18 - 7$ $= 11$ $=$ RHS

7.4 Solving equations using the balance method

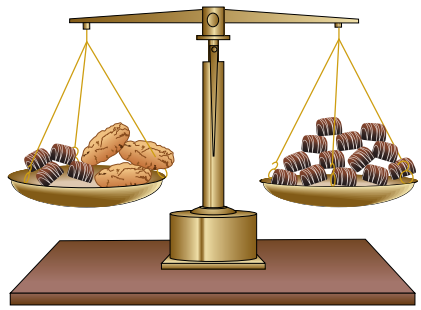
Navigator

1, 2, 3 (columns 1–2), 4, 5, 6, 9, 10	1, 2 (column 1), 3 (column 2), 4, 5, 6, 7, 8, 9, 10	1, 3 (column 3), 4, 5, 6, 7, 8, 9, 10

Answers p. 690

Fluency

1 This set of scales is balanced. The left-hand side has 4 chocolates and 3 biscuits, while the right-hand side has 13 chocolates.



W.E. 10

(a) If one chocolate is taken from the left-hand side, the scales become unbalanced. Which of the sides is now heavier?

(b) How can the scales be balanced without putting the chocolate back?

(c) Starting again, if all of the chocolates are taken from the left-hand side, what should be done to the right-hand side to balance the scales?

(d) How many chocolates are equivalent to the three biscuits?

(e) How many chocolates are equivalent to one biscuit?

W.E. 11

2 Form an equivalent equation to each of the following by performing the operation given in brackets to both sides of the equation.

(a) $x + 5 = 8 \quad (+3)$

(b) $x + 4 = 12 \quad (+5)$

(c) $x + 1 = 7 \quad (+6)$

(d) $2x + 2 = 10 \quad (-1)$

(e) $3x + 5 = 10 \quad (-5)$

(f) $6x + 8 = 14 \quad (-5)$

W.E. 12

3 Solve each of the following equations using the balance method. Check your solutions by substitution.

(a) $3x + 5 = 8$

(b) $5x + 1 = 21$

(c) $7x + 3 = 17$

(d) $2x + 1 = 11$

(e) $2x + 5 = 19$

(f) $2x - 10 = 8$

(g) $3x - 4 = 20$

(h) $5x - 1 = 4$

(i) $6x - 2 = 16$

(j) $4x - 3 = 17$

(k) $2x - 3 = 7$

(l) $4x - 1 = 31$

(m) $\frac{2x}{3} = 8$

(n) $\frac{2x}{7} = 2$

(o) $\frac{5x}{2} = 20$

(p) $\frac{x+5}{3} = 5$

(q) $\frac{x+1}{3} = 7$

(r) $\frac{x+4}{5} = 3$

(s) $2(x - 5) = 14$

(t) $3(x + 2) = 24$

(u) $7(x - 2) = 35$

Understanding

4 This set of scales is balanced with 3 apples and 2 bananas on the left-hand side and 7 apples on the right-hand side.

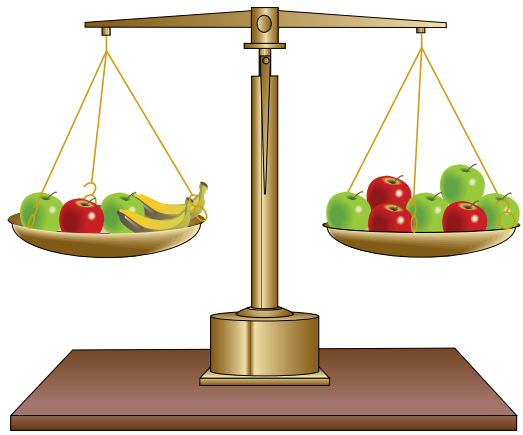
If 4 apples are taken from the right-hand side, the scale can be balanced by:

A taking 4 apples from the left-hand side

B taking 3 apples and 1 banana from the left-hand side

C taking 2 apples and 2 bananas from the left-hand side

D taking 2 bananas from the left-hand side.



5 Trixie's father weighs twice as much as Trixie plus 12 kilograms. If her father weighs 104 kg:

(a) write an equation representing the situation using t to represent Trixie's weight

(b) solve the equation to find out Trixie's weight.

6 Naram has shot a number of 3-point goals as well as 18 other points in a game of basketball. If he shot a total of 30 points in the game:

(a) write this as an equation with p representing the number of 3-point goals shot by Naram

(b) solve your equation to find the number of 3-point goals that he shot.

Reasoning

7 Fatima drives to and from work each day for the five-day working week, and drives an additional 85 kilometres on the weekend. She drives a total of 425 kilometres every week in her car. How far is it from her home to her workplace?

8 George has to take 5 litres of water with him on a boat trip. He has one 1.25 L container and three other containers of equal capacity. Together, the four containers will hold 5 litres. What is the capacity of the other containers?

Open-ended

9 Write three different equations that are equivalent to $5x + 4 = 14$.

10 Vejay was given the equation $\frac{2x+3}{7} = 3$ to solve and presented the following solution, shown at right.

$$\frac{2x+3}{7} = 3$$
$$\frac{2x+3}{7} - 3 = 3 - 3$$
$$\frac{2x}{7} = 0$$
$$\frac{2x}{7} \times 7 = 0 \times 7$$
$$2x = 0$$
$$\frac{2x}{2} = \frac{0}{2}$$
$$x = 0$$

(a) Explain to Vejay why his working out is incorrect.

(b) Show Vejay the process he should have used.

(c) How could Vejay avoid making the same mistake next time he is solving equations?

Problem solving

Paper mountain

Imagine a very, very large sheet of paper that is 0.1 mm thick.

If you fold it in half, it will be 0.2 mm thick.

How thick will it be if you fold it in half again?

Imagine the paper can be folded an unlimited number of times.

How many folds would be needed to create a paper stack that is taller than the tallest mountain in the world, Mount Everest? (Mount Everest is 8848 m high.)

Strategy options

- Act it out.
- Look for a pattern.

Maths 4 Real

Dance party dilemma

Have you ever wondered how to organise a fund-raising event such as a dance party? You need to book a venue, make tickets, create advertising material and provide drinks, as well as a range of other things.

Believe it or not, a lot of the costs to be considered can be usefully expressed as an equation. Many costs are variables that will change as the number of people attending the party changes. There are also fixed costs to consider, which will be the same no matter how many people attend.

If you can write an equation that represents all of the costs, then you can calculate how much the tickets should cost, to be able to make a profit or a loss.

1 The table below represents the total costs for a dance party. It shows that it would cost $50 to organise the party even if no people attend. Why might this be? Explain.

Number of people, *n*	0	10	20	30	40	50
Total cost ($), *C*	50	150	250	350	450	550

2 **(a)** What happens to the cost every time you increase the number of people attending the dance party by 10?

(b) Use your answer to **(a)** to determine the increase in cost for just one extra person.

3 From the table in Question **1**, what equation could you use to calculate the costs of running the dance party? Use ***C*** to represent the total cost and ***n*** to represent the number of people attending.

$C = \_\_ \times n + \_\_$

4 Use the equation that you found in Question **3** to calculate the costs of the dance party for the following number of people attending.

(a) 25 **(b)** 36 **(c)** 83

5 Suppose the following table represents the money charged to attend the dance party. As the organiser of the party, this is your income. What equation could you use to represent this? Use ***I*** for income and ***n*** for the number of people attending.

Number of people, *n*	0	10	20	30	40	50
Income ($), *I*	0	120	240	360	480	600

6 Use the equation that you found in Question **5** to calculate the income from the dance party for the following number of people attending.

(a) 32 **(b)** 53 **(c)** 95

If your income is bigger than your costs you will make a profit. However, if your costs are greater than your income, you will make a loss.

7 Use your equations from Questions **3** and **5** to complete the following table showing all the costs and income associated with this dance party. Make sure that you also calculate the profit or loss (the difference between the cost and the incomes). To show a loss, write a negative sign in front of the number.

Number of people, *n*	0	15	35	45	63	100
Cost ($), *C*	**50**	**200**	**400**	**500**		
Income ($), *I*	**0**	**180**	**420**			
Profit/loss ($), *P*	**-50**	**-20**	**20**			

8 The profit or loss made at the party is found by subtracting the cost from the income. Using ***P*** to represent the profit or loss, write an equation that shows this.

9 What is the smallest number of people you would need to attend the dance party in order to make a profit? Show how you worked this out.

Research

Investigate the costs of organising a dance party for 40 people. You need to find the following:

- cost of a suitable venue
- catering costs
- music costs
- cleaning costs

Present a report that outlines a number of different options for each of the above costs. Which is your preferred option and why? What are the benefits of hiring a venue compared to having a party at home?

Exploration Spreadsheet

Equipment required: Microsoft® Excel or similar spreadsheet software. (For Casio ClassPad CAS or TI-Nspire CAS, you can download instructions from the eBook or the Pearson Places website.)

Algebra using a spreadsheet

Did you know that your computer or calculator can do algebra? A spreadsheet program, such as Excel, uses formulas to do a lot of repetitive substitution. Spreadsheets on a CAS calculator work in the same way.

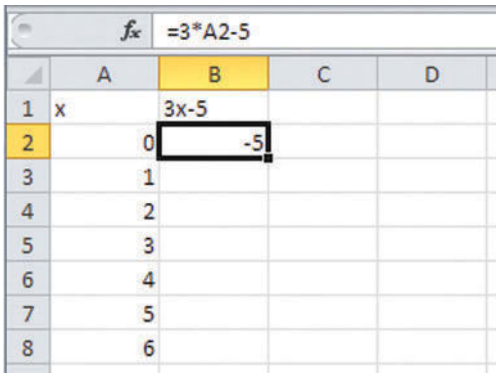
There are many ways to solve algebraic equations. You have been learning some algebraic methods. In this exploration, we are going to use a spreadsheet to find the solution to algebraic equations through a numerical process.

1 In a new spreadsheet, enter x as a heading in cell **A1**.

(a) Enter x-values from 0 to 6 in column **A**, starting at cell **A2**. After you enter 0 in the first cell (cell **A2**), you can enter the formula **=A2+1** into cell **A3** and **Fill Down** this formula to copy it down the column.

(b) Enter the expression $3x - 5$ as a heading in cell **B1**.

(c) Enter the formula **=3*A2-5** into cell **B2**. This will calculate the value for $3x - 5$ using the x-value in cell **A2** (which is 0).



The formula for column **B** must be changed in each row to use the x-value from column **A** in that row. The formula will then calculate each new value for $3x - 5$.

(d) Copy this formula down to cell **B8** by using **Fill Down**. This will change the formula to refer to the correct x-value in each cell.

2 Use your spreadsheet to find the solution for the following:

(a) $3x - 5 = 4$

(b) $3x - 5 = 13$

(c) $3x - 5 = -2$

In Excel, you can highlight these solutions using any fill colour you want. Select the cell that contains your solution, then use the fill colour icon under the **Home** tab to select a colour.

3 **(a)** In column **C**, enter the expression **'-2x+5** as a heading in cell **C1**.

(b) In cell **C2**, enter the formula to calculate a value for the expression $-2x + 5$ for $x = 0$.

(c) Copy this formula down the column, using the same method as before. Make sure that each cell calculates its formula using the x value from the correct row of the **A** column.

4 Use your spreadsheet to find the solution for the following:

(a) $-2x + 5 = 5$

(b) $-2x + 5 = -3$

(c) $-2x + 5 = -5$

Highlight these solutions with a different fill colour.

5 Use your spreadsheet to find the solution to $3x - 5 = -2x + 5$.

Highlight this solution using another fill colour.

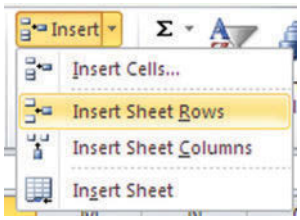
6 To solve $4x - 6 = 30$, first enter **4x–6** as a heading in cell **D1** and enter the formula **=4*A2–6** in cell **D2**.

(a) Copy this formula down the column (**Fill Down**) to **D8**. You can see that none of the values are equal to 30, so the solution is not in the 0 to 6 values for x.

(b) You need to extend the values for x beyond $x = 6$, so extend your x-values down the column to $x = 10$.

(c) Now you can find a solution to $4x - 6 = 30$. Highlight the solution.

7 Sometimes, the solution might be less than 0, so you will need to insert values less than 0 in column **A**. You can insert new rows above row **2** by selecting the row, and then under the **Home** tab selecting the command **Insert** > **Insert Sheet Rows**. You can then copy the formulas up to fill the column.



Now, find the solution to:

(a) $3x - 5 = -11$ (b) $-2x + 5 = 11$

Highlight with the same fill colours you chose for these expressions previously.

8 Solutions to equations are not always integers. For example, try to solve $4x - 6 = 25$. For $x = 7$, $4x - 6 = 22$ and for $x = 8$, $4x - 6 = 26$, so there is no whole number solution for $4x - 6 = 25$.

Because 25 is between 22 and 26, the solution should be between 7 and 8. Insert a row between $x = 7$ and $x = 8$ and enter 7.5. This gives $4x - 6 = 24$. You now know that the solution must be between 7.5 and 8.

	A	B
	fx	=4*A13-6
1	x	4x-6
2	-3	-18
3	-2	-14
4	-1	-10
5	0	-6
6	1	-2
7	2	2
8	3	6
9	4	10
10	5	14
11	6	18
12	7	22
13	7.75	25
14	8	26
15	9	30
16	10	34
17	11	38
18	12	42
19	13	46

(a) You can replace the 7.5 with 7.7 to find $4x - 6 = 24.8$.

(b) You can replace the 7.7 with 7.75, to find the solution $4x - 6 = 25$.

Highlight this solution.

9 Insert new formulas in your spreadsheet to find the solutions for the following:

(a) $5x + 6 = 22$

(b) $8x + 1 = 4$

(c) $10x + 3.3 = 140$

Highlight each of these solutions.

Taking it further

10 Try to find the solution to $3x - 4 = 7$. It may look like the solution to this equation is 3.6, but $3 \times 3.6 - 4 = 6.8$, not 7. So, what has gone wrong?

Select the cells and then use the 'increase decimal places' icon  found under the **Home** tab to show 4 decimal places in the cells in column **A** and in the column used for $3x - 4$ (do not include the heading row). This shows that $x = 3.6666$ gives 6.9998.
Do you think you could find an exact solution to this equation with this spreadsheet? Explain your answer.

11 Give an example of another equation that will have the same problem.

12 'Spreadsheets are very useful for solving equations, but are not as good as algebraic solutions found by hand.' Do you agree with this statement? Give reasons to support your argument.

7.5 Solving problems with equations

Equations can be very useful for solving everyday problems. So far, you have practised solving equations presented to you. Now you will be forming your own equation from information supplied in a worded question. Solving this equation will help you find the unknown value asked for in the question. There may be more than one way to write the equation and solve it.

Worked example 13

W.E. 13

Phil has 70 cents. He buys a pear and has 25 cents left. If x represents the cost of a pear (in cents), form an equation and solve it to find the cost of the pear.

Thinking	Working
1 Define a variable to represent the unknown quantity.	Let x be the cost of a pear.
2 Form an equation using the information given.	$x + 25 = 70$
3 Identify the first operation to undo and then use the inverse operation on both sides of the equals sign to balance the equation.	$x + 25 - 25 = 70 - 25$ $x = 45$
4 Check the solution.	If Phil spent 45 cents out of his 70 cents he would have $70 - 45 = 25$ cents left.
5 State the solution in words.	The pear costs 45 cents.

Worked example 14

W.E. 14

Erica buys two cheap movie tickets and one small bucket of popcorn for a total of $21. If the popcorn costs $5, how much does one movie ticket cost? Using t as the cost of a movie ticket, form an equation and solve it to find the value of t. (Assume both tickets are the same price.)

Thinking	Working
1 Define a variable to represent the unknown quantity.	Let t represent the cost in dollars of one movie ticket.
2 Form an equation using the information given.	$2t + 5 = 21$
3 Identify the first operation to undo and then use the inverse operation on both sides of the equals sign to balance the equation.	$2t + 5 - 5 = 21 - 5$ $2t = 16$
4 Identify the second operation to undo and then use the inverse operation on both sides of the equals sign.	$\frac{2t}{2} = \frac{16}{2}$ $t = 8$
5 Check the solution.	Two \$8 movie tickets is: $2 \times 8 = \$16$ plus a \$5 bucket of popcorn, gives a total of: $16 + 5 = \$21$
6 State the solution in words.	One movie ticket costs \$8.

7.5 Solving problems with equations

Navigator

1, 2, 3, 4, 5, 6, 7, 9, 11, 12 (a–c), 13	1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14	1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14

Answers p. 691

Fluency

1 **(a)** Carmen has 95 cents. She buys a mini ice-cream and has 20 cents left. If x represents the cost of the ice-cream (in cents), form an equation and solve it to find the cost of the ice-cream.

W.E. 13

(b) Matthew can run 100 m in 12.8 seconds. Tony ran a personal best 1.5 seconds faster. If t represents Tony's time, form an equation and solve it to find Tony's time.

2 **(a)** Tran bought two sushi rolls and extra wasabi for a total of \$8. If the extra wasabi costs \$1.60, how much does one sushi roll cost? Using d as the cost of a sushi roll, form an equation and solve it to find the value of d. (Assume both sushi rolls are the same price.)

W.E. 14

(b) Asif bought three apps and an eBook for \$59. If the eBook costs \$21.50, how much does one app cost? Using v as the cost of an app, form an equation and solve it to find the value of v. (Assume all apps are the same price.)

3 A bus has the capacity to seat 45 passengers. Eighteen passengers have boarded the bus and taken seats.

(a) If n represents the number of seats still available, the equation which can be formed is:

A $n - 18 = 45$ **B** $n + 45 = 18$ **C** $18n = 45$ **D** $n + 18 = 45$

(b) Solve the equation formed in part **(a)** to find the number of seats still available on the bus.

4 A roll of fabric is divided evenly between six people. Each person receives 2.5 m of fabric.

(a) If t represents the total length of fabric in the roll (in m), the equation that can be formed is:

A $t = 6 + 2.5$

B $t + 2.5 = 6$

C $\frac{t}{6} = 2.5$

D $2.5t = 6$

(b) Solve the equation formed in part **(a)** to find the total length of the roll of fabric.

Understanding

5 The length of a rectangular vegetable garden is 4 m. The perimeter of wire fencing needed to enclose this garden is 13 m.

(a) If w represents the width (in m) of the vegetable garden, the equation that can be formed is:

A $w + 4 = 13$ **B** $w + 8 = 13$ **C** $2w - 8 = 13$ **D** $2w + 8 = 13$

(b) Solve the equation formed in part **(a)** to find the width of the vegetable garden.

6 Lucas bought a salad sandwich and a container of milk for his lunch.

(a) If the salad sandwich cost $\$x$ and the milk cost \$1.50, write an expression for the total cost of his lunch.

(b) Form an equation and solve it to find the cost of the salad sandwich if Lucas spent \$5.70 on his lunch.

7 **(a)** If x represents the length (in cm) of a square floor tile, write an expression for its perimeter.

(b) Form an equation and solve it to find the length of the floor tile if its perimeter is 84 cm.

8 A restaurant has several tables that seat four people, but only three tables that seat two people.

(a) If x represents the number of four-person tables available, write an expression for the total number of people that can be seated at the restaurant.

(b) A Saturday night booking for 62 people is made. Six of these people are to be seated at the three two-person tables. Form an equation and solve it to find how many four-person tables are needed.

9 At a particular rollerskating rink, it costs \$5 to hire skates and \$3 for each hour spent on the rink. If it costs Natalie \$14, form an equation and solve it to find how many hours she spent rollerskating. Let h represent the number of hours spent rollerskating.

10 The tallest man in reliably recorded history was Robert Wadlow. At age 22, his height was 54 cm less than twice his height at age 5. If his height at age 22 was 272 cm, form an equation and solve it to find his height at age 5.

11 A company hires out electrical tools, charging a non-refundable deposit of \$15 for each item, plus an amount per day according to the type of tool.

Tool	Hire charge per day
Electric drill	\$8
Electric sander	\$13
Electric circular saw	\$24
Electric jackhammer	\$35
Concrete mixer	\$48

Let d = the number of days the tools are hired. Form an equation and solve it to answer each of the following.

(a) If Allie hires an electric drill and pays \$63, for how many days did she hire it?

(b) If Serena hires a concrete mixer and pays \$111, for how many days did she hire it?

(c) Kosta hires an electric sander and an electric saw. For how many days did he hire the tools if he paid \$178?

Reasoning

12 The local council has hired a concreting firm to pave some footpaths. Each footpath is to be one metre wide. A wooden frame is needed before the concrete is poured. This frame is made up of one-metre lengths of wood.

The frames for some different lengths of footpath are shown in the diagrams below.

2 m long footpath 3 m long footpath 4 m long footpath

(a) How many pieces of wood are needed to construct the frame of a footpath of length:

(i) 2 metres **(ii)** 3 metres **(iii)** 4 metres?

(b) Write an expression for the number of pieces of wood needed to construct the frame for a footpath of length x metres.

(c) Form an equation and solve it to find how many pieces of wood are needed to construct the frame for a footpath of length:

(i) 6 metres **(ii)** 10 metres **(iii)** 13 metres.

(d) The concreting workers have only 36 pieces of wood. By using your equation and solving it, find the length of the longest footpath that could be poured at the one time.

Open-ended

13 Gary the gardener is keen to do some landscaping on a Saturday and has allowed for 7 hours work. It costs $30 per hour for him to hire a jackhammer and $22 per hour to hire a mulcher. He has budgeted to spend no more than $180. Keeping in mind that he can only use one piece of equipment at a time, what are three options for using the equipment and what would each option cost him?

14 A man is 5 times the age of his son. Given that the sum of their ages is no greater than 70, what are their possible ages? Assume that the answers are whole numbers.

Problem solving

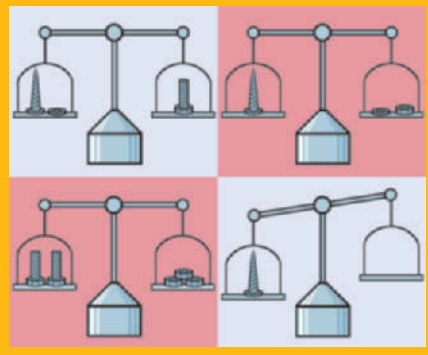
Algebraic puzzles

1 Four numbers are added together and the result is 255. One of the numbers is the square of one of the others. The largest number is double the square number, and the fourth number is 55. Find the numbers.

2 A screw and a washer balance with a bolt. A screw balances with a washer and a nut. Two bolts balance with three nuts. How many washers will balance with a screw?

Strategy options

- Guess and check.
- Break problem into manageable parts.



Investigation

The great escape

Farmer Sue has a paddock where she brings her cows to wait for milking. She also has a large yard for her free-range chickens. Every now and then, some of her chickens escape into the cow paddock.

One day, the gate connecting the chicken yard and the cow paddock was left wide open and all of Farmer Sue's chickens escaped into the cow paddock, where all of her cows were waiting to be milked. Farmer Sue raced over and counted 40 heads and 112 legs.

The Big Question

How many chickens and cows did Farmer Sue own?

Engage

1 Last week, Farmer Sue found 4 chickens and 8 cows in the paddock and had to race around to catch the chickens and return them to their own yard.

(a) How many heads would she have counted?

(b) How many legs would she have counted?

(c) If there had been 8 chickens and 4 cows:

(i) how many heads would she have counted?

(ii) how many legs would she have counted?

Explore

2 If n represents the number of cows in the paddock and c represents the number of chickens that have escaped into the paddock:

(a) write an equation to show 12 heads altogether

(b) what are all the possible combinations of cows and chickens that would give a total of 12 heads? Present your answers in a list or table.

(c) Write an equation to show 40 legs altogether.

(d) Substitute $n = 8$ and $c = 4$ into your equation to make a true statement.

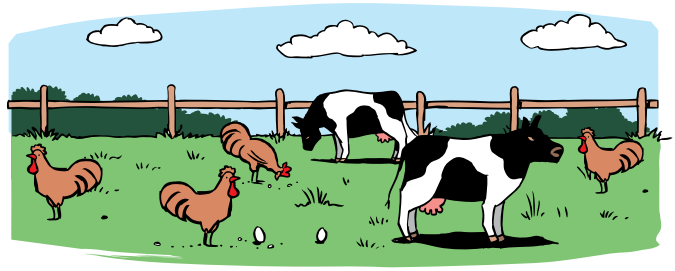
Strategy options

- Guess and check.
- Make a table.
- Make a model.
- Look for a pattern.

3 (a) If Farmer Sue counted 40 heads, choose two possible values for the number of cows and chickens she might own.

(b) For each choice, using an appropriate equation, calculate the number of legs she would have counted.

(c) Can you now find the values for n and c that would give 40 heads and 112 legs?



Explain

4 If the number of heads stays the same but the number of legs changes, explain how the numbers of each animal in the paddock change.

Elaborate

5 (a) State your answer to the Big Question. Explain how it was obtained.

(b) What would be the solution for n and c if the number of heads was still 12 but the number of legs was now only 32? Show how you worked this out.

Evaluate

6 (a) What methods did you use to solve this problem?

(b) Did you use a particular problem-solving strategy or method?

(c) Why did you choose that particular method?

(d) Do you think this was the best method?

(e) Can you think of any other ways to solve this problem?

Extend

7 Deep in the ocean, a scuba diver discovers a collection of 21 crabs and octopuses together in the same small area. The diver counts 198 legs altogether. (A crab has 10 legs and an octopus has 8.) How many crabs and octopuses were there?

Gamespace

The case of the confusing clues

Cara Loft is on a secret mission. The golden calculator has been stolen by the mysterious Dr Equation. Where has he hidden it? All Cara has is a series of clues spread throughout the rather creepy House of Calculus. You need to help Cara by finding your way through the maze. As you move through the maze, you will enter some tricky 'equation chambers'.

When you enter the chamber:

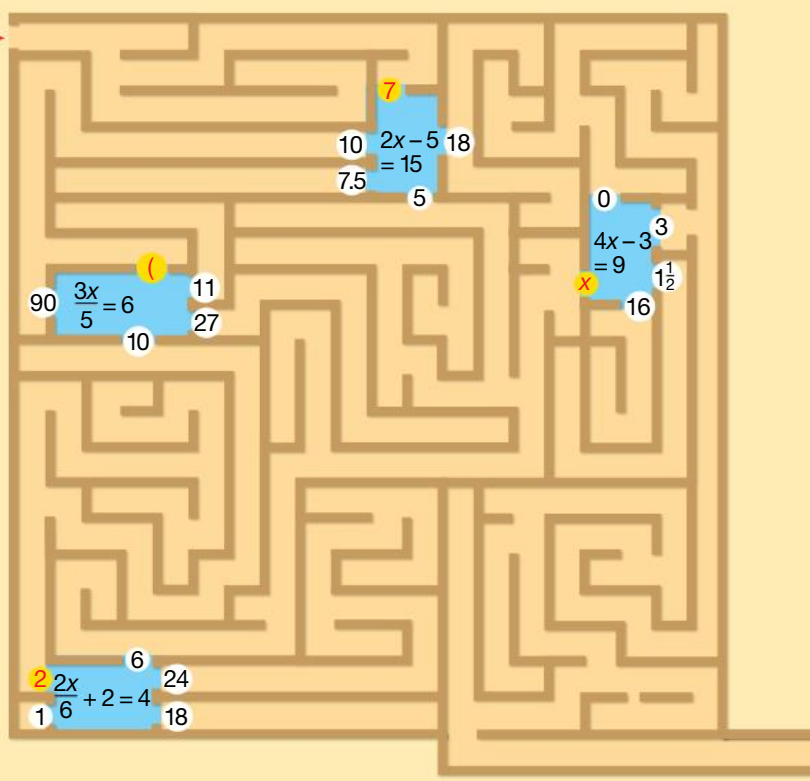
- make sure that you enter through a coloured number or symbol and write it down (they form an equation you will need to solve at the end).
- To exit the equation chamber, solve the equation and move out through the exit showing the correct solution.

When you finally exit the maze, you need to complete the practice session to help you get ready for the next section. Good luck!

Practice session 1

Solve these equations:

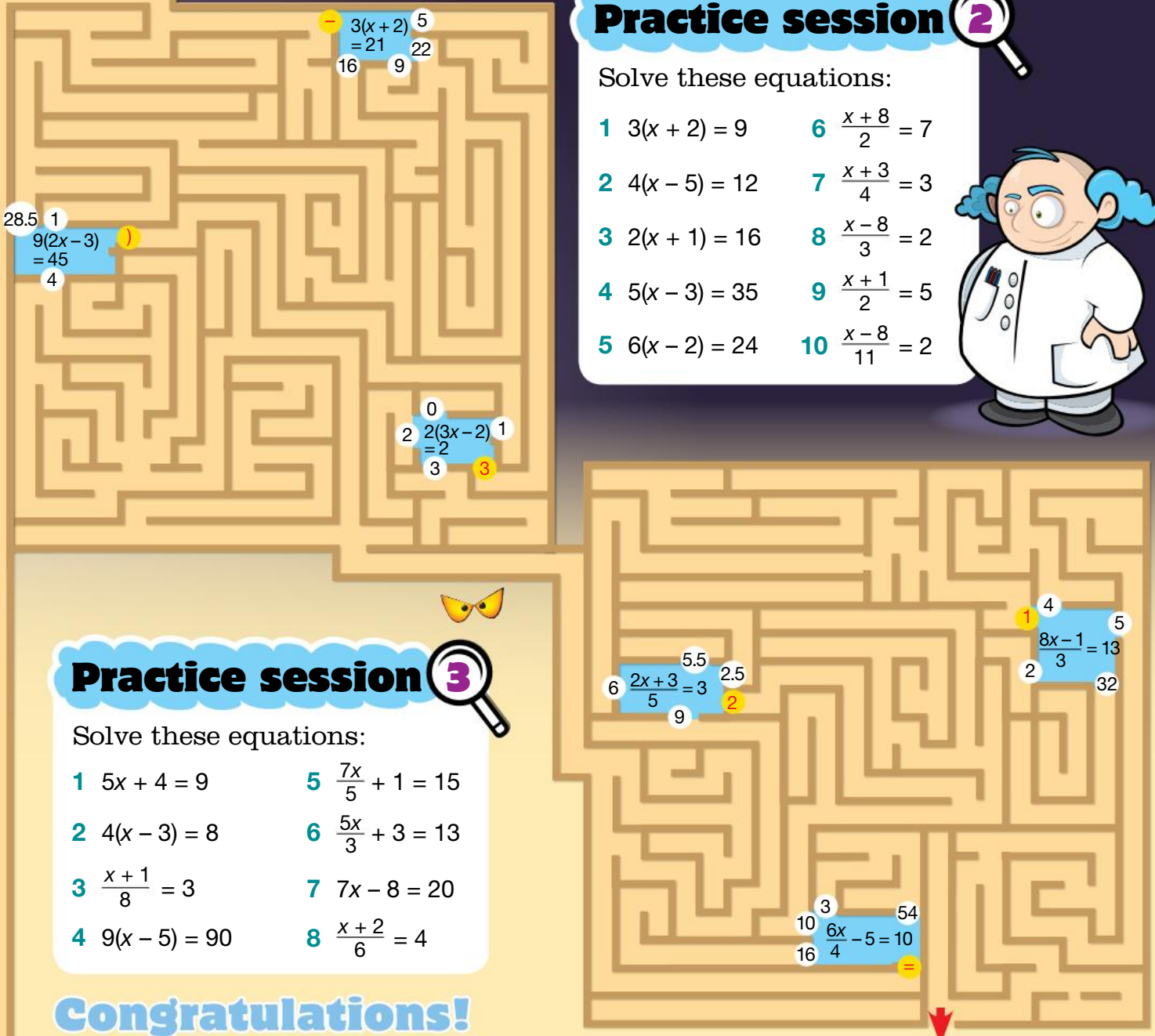
1. $3x + 1 = 10$
2. $4x - 4 = 12$
3. $8x - 1\frac{1}{2} = 6\frac{1}{2}$
4. $7x - 3.5 = 31.5$
5. $5x + \frac{1}{3} = 15\frac{1}{3}$
6. $\frac{2x}{3} + 1 = 5$
7. $\frac{5x}{2} + 1 = 11$
8. $\frac{7x}{5} - 3 = 11$
9. $\frac{8x}{6} - 4 = 0$
10. $\frac{9x}{10} + 1 = 8.2$



Practice session 2

Solve these equations:

1. $3(x + 2) = 9$
2. $4(x - 5) = 12$
3. $2(x + 1) = 16$
4. $5(x - 3) = 35$
5. $6(x - 2) = 24$
6. $\frac{x + 8}{2} = 7$
7. $\frac{x + 3}{4} = 3$
8. $\frac{x - 8}{3} = 2$
9. $\frac{x + 1}{2} = 5$
10. $\frac{x - 8}{11} = 2$



FINISH

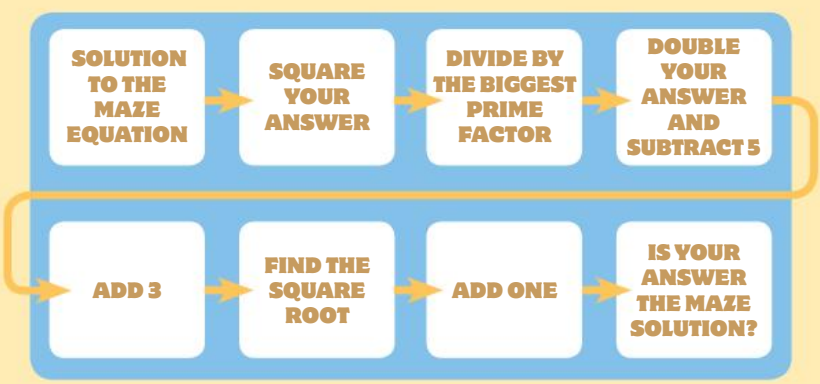
Practice session 3

Solve these equations:

1. $5x + 4 = 9$
2. $4(x - 3) = 8$
3. $\frac{x + 1}{8} = 3$
4. $9(x - 5) = 90$
5. $\frac{7x}{5} + 1 = 15$
6. $\frac{5x}{3} + 3 = 13$
7. $7x - 8 = 20$
8. $\frac{x + 2}{6} = 4$

Congratulations!

You have successfully mastered the House of Calculus. Or have you? Where is the golden calculator? Dr Equation has left you a final challenge. Piece together the clues you have collected in order and then solve the equation that they form. You then need to use your answer to attempt the biggest challenge of all—the sinister 'Steps of Arithmetica'. Successfully meet the challenge and you will find the golden calculator. Fail and you will have to go back to the start!

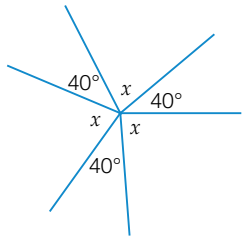


Challenge 7

1 When half of a number is increased by 15 the result is 49. The original number is:

A 68 **B** 34 **C** 32 **D** 17

2 Write an equation and solve it to find the value of x in the diagram.



3 Solve the equation: $\frac{3x-5}{4} = -6$

4 If $59 + x = 73 + y$, then:

A $x = 14 - y$ **B** $x = 14 + y$

C $x = y - 14$ **D** $x = 14y$

5 Using only the digits 1, 2, 3 and 7 and the two mathematical symbols + and = only once, you can create a true number sentence $2^3 = 1 + 7$.

(a) Using the digits 2, 3, 4, 5 and the two mathematical symbols + and = only once and a number as a power, create a true number sentence.

(b) Using any different four digits from 0 to 9 and the two mathematical symbols + and = only once, and using one of the numbers as a power, create five different true number sentences. Do not use $2^3 = 1 + 7$ or your answer to **(a)**. Use 0 and 2 as a power only once.

(c) Use any different four digits from 0 to 9 and the two mathematical symbols – and = only once, and using one of the numbers as a power, create five different true number sentences. Use 0 and 1 as a power only once.

6 The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$. If the reciprocal of $\frac{4x}{5}$ is $\frac{1}{20}$, then the value of x is:

A $\frac{1}{25}$ **B** $\frac{1}{16}$ **C** 16 **D** 25

7 Solve for y:

$$\frac{1}{4} + \frac{1}{5} + \frac{1}{y} = 1$$

8 Vladimir buys an equal number of 55 cent and $1.10 lollies and spends $80.85. How many lollies did he buy altogether? Form an equation and solve it.

9 A lottery winner won $\frac{9}{10}$ of the total prize pool. Shortly after, she spent $\frac{3}{4}$ of her winnings, but still had $2700 left. What was the value of the total prize pool? Form an equation and solve it.

10 If $\frac{1}{3} > \frac{1}{4} > \frac{1}{m}$ and the difference between the first two fractions equals the difference between the last two fractions, find the value of m. (Hint: Write $\frac{1}{m}$ on one side of the equation before you take the inverse.)

11 Two integers, a and b, do not end in zero. If the product ab is a multiple of 10 and $a > b$, then the last digit of $a - b$ cannot be:

A 1 **B** 3 **C** 5 **D** 7

Chapter review 7

Maths literacy

backtracking	false number sentence	solve
balance method	guess, check and improve	solving by inspection
checking by substitution	inverse operations	true number sentence
equivalent equations	solution	

Copy and complete the following using the words and phrases from this list, where appropriate. A word or phrase may be used more than once.

1 An equation can be solved by performing the _____ on both sides of the equation.

2 To _____ an equation is to find the value of the unknown that will make a true number sentence.

3 $4 + 5 = 10$ is a _____.

4 One way of solving an equation is by looking at the equation to see what number might be the answer. This is called _____.

5 Equations that have the same solution are said to be _____.

6 _____ is a method we can use to solve equations. It can be shown by moving backwards along a flowchart.

7 _____ is another method used to solve equations.

Fluency

1 The following number sentences are not true. Rewrite each number sentence by changing the coloured number so that you have a true number sentence. **7.1**

(a) $5 \times 4 = 30 - 2$ **(b)** $2 \times 7 + 6 = 9 + 5$

2 Find the unknown number in each of the following sentences. **7.1**

(a) Six subtracted from a number is equal to nine.

(b) A number divided by four is equal to seven.

3 Find the solution to each of the following equations by inspection. **7.2**

(a) $2x = 18$ **(b)** $x + 7 = 10$ **(c)** $\frac{x}{5} = 3$

(d) $x - 4 = 11$ **(e)** $x + 3 = 15$ **(f)** $x - 4 = 24$

4 Solve the following equations using guess, check and improve. **7.2**

(a) $3x + 25 = 16$ **(b)** $\frac{x}{7} + 51 = 63$ **(c)** $\frac{3x + 5}{4} = 8$

5 Write the inverse (or opposite) operation to each of the following. **7.3**

(a) $+ 5$ **(b)** $\div 3$ **(c)** $\times 8$ **(d)** $- 4$

6 Solve each of the following equations using a flowchart and backtracking. **7.3**

(a) $3x - 7 = 2$ **(b)** $\frac{x+6}{5} = 4$ **(c)** $4(x+1) = 20$ **(d)** $\frac{x}{8} - 2 = 3$

7 Use the balance method to solve the following equations. **7.4**

(a) $4x + 7 = 19$ **(b)** $5(x-2) = 20$ **(c)** $\frac{x}{3} + 5 = 10$

(d) $\frac{x-8}{6} = 3$ **(e)** $\frac{x+4}{5} = 5$ **(f)** $\frac{x-7}{3} = 10$

8 The equation $5x - 6 = 4$ has the solution: **7.4**

A $x = 1$ **B** $x = 2$ **C** $x = 3$ **D** $x = 4$

9 Danni bought 5 tickets to a show. She was given $27.50 change from $200. Use an equation to find how much each ticket cost. Let t be the cost of a ticket. **7.5**

Understanding

10 Which equation describes this situation? **7.2**

A restaurant seats a maximum of 64 customers. On Saturday afternoon, it is fully booked for dinner that evening, until a group booking for 16 is cancelled. If p represents the number of people who are still booked, the equation that can be formed is:

A $p - 16 = 64$ **B** $p = 64 + 16$ **C** $16p = 64$ **D** $p + 16 = 64$

11 Ross buys three dim sims and a bottle of juice. **7.5**

(a) If each dim sim costs x cents and the bottle of juice costs $1.40, write an expression for the total cost C (in cents) of the food and juice.

(b) Use an equation to find the cost of one dim sim if Ross spends a total of $2.75.

12 The air temperature at 4 pm is 28 °C. Over the past hour, the temperature has risen by 5 °C. What was the temperature at 3 pm? (Form an equation to solve by letting t represent the air temperature at 3 pm.) **7.5**

Reasoning

13 A triangle  is made from three matchsticks of equal length. If more matchsticks are added to make the sides of another triangle, this forms the shape  . **7.5**

(a) How many matchstick sides does this shape have?

(b) A third triangle is added to the shape. How many matchstick sides does this shape have?

(c) If n represents the number of triangles joined together in a row, then use the pronumeral n to write the number of matchstick sides that the shape has in total.

(d) **(i)** Use your answer to part **(c)** to write the equation for n triangles joined together that have 97 matchstick sides of equal length.

(ii) Solve the equation found in part **(d) (i)**.

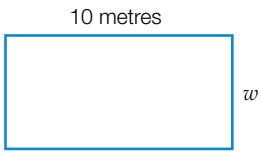
Numeracy practice 7

Non-calculator

1 Jan is baking biscuits. She puts her biscuits in 3 equal rows. When they are baked, she eats one. She has 44 biscuits left. How many biscuits did she have in each row?

A 3 **B** 10 **C** 15 **D** 20

2 A rectangular garden bed has a perimeter of 30 metres and a length of 10 metres. If the width of the garden can be written as w, which equation best represents this situation?



A $w + 10 = 30$ **B** $w + 30 = 10$

C $2w + 10 = 30$ **D** $2w + 20 = 30$

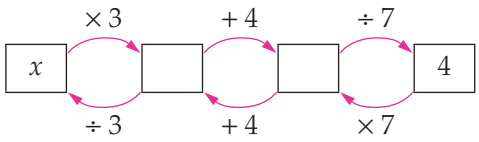
3 What is the missing number that makes this number sentence true?

$3 \times \square = 12 + 3 \times 6$

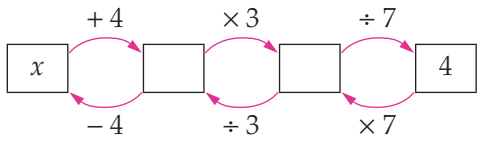
A 1 **B** 7 **C** 9 **D** 10

4 Voula thinks of a number and asks Renza to guess what it is. Voula states that after the number is tripled, four added and the result divided by seven, the answer is four. Which flowchart best represents how Renza might find Voula's mystery number? Let the mystery number = x.

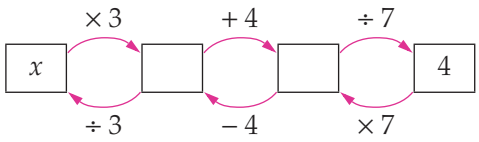
A



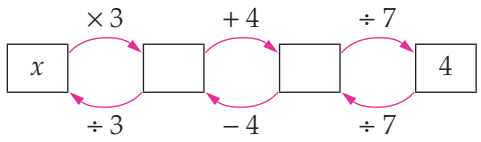
B



C



D



Calculator allowed

5 A pack of 4 donuts costs $3.60. A pack of 6 donuts costs $4.00.
You need to buy 34 donuts.
What is the least amount you can pay?

A $23.60 **B** $26.40 **C** $26.50 **D** $29.90

6 The cooking time for a turkey is 30 minutes plus 20 minutes per kilogram. If M is the mass of the turkey in kilograms, the total cooking time in minutes is:

A $20 + 30M$ **B** $20M + 30$ **C** $20 \times 30M$ **D** $30 - 20M$

7 Julie, Nikki and Romina earned a total of $3400 in the last week. Julie earned twice as much as Nikki whereas Romina earned $200 more than Nikki. How much did Nikki earn in the last week?

A $600 **B** $800 **C** $1000 **D** $1600

8 The solution to $\frac{3(x-4)}{2} = 15$ is:

A $x = 6$ **B** $x = 10$ **C** $x = 14$ **D** $x = 58$


8

8

Angles and shapes

Leaning Tower of Pisa stops leaning!

Sometimes, angles can cause all sorts of problems.

The building of the Leaning Tower of Pisa began in the 12th century in Pisa, Italy. The tower started tipping sideways before building had finished and, for centuries, it kept tipping further and further.

The tower would have eventually fallen over if engineers and architects hadn't found a way to stabilise the base. In 1990, it was leaning at an angle of 5.5° from the vertical. Engineers removed soil from underneath the raised end, which straightened it a little. In 2008, even more soil was removed and now the tower leans at 3.97°. The engineers believe that it is now stabilised and will not get any worse for more than 200 years.

Forum

What problems could be caused if a building starts to lean?

At what angle do you think the Leaning Tower of Pisa would actually fall over?

Discuss how using angles could be important for engineers fixing the Leaning Tower of Pisa.

Why learn this?

You need to get things straight about angles! They are crucial in sport—whether deciding at what angle to kick a ball, choosing the best club to use for a golf shot or trying to pocket a ball on a pool table. Construction workers need to use angles to make sure that buildings and structures are secure and will not fall down. Surveyors use distances and angles to accurately determine positions on the Earth's surface, working in construction and establishing boundary lines between properties.

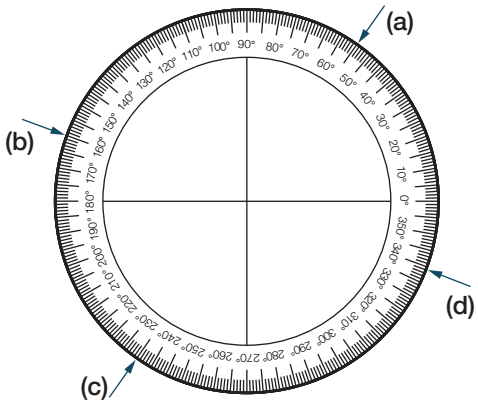
After completing this chapter you will be able to:

- measure, estimate, draw, classify and name angles
- calculate the size of complementary and supplementary angles, vertically opposite angles and angles in a revolution
- understand transversals and identify alternate, corresponding and co-interior (allied) angles
- understand the properties of angles formed by transversals and parallel lines
- define and classify triangles, quadrilaterals and other polygons, identifying their properties
- construct angles and shapes using a compass or a protractor.

Recall 8

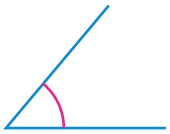
Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

1 State the value indicated by each arrow on the diagram of the protractor.

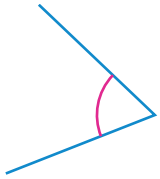


2 Arrange the following angles in order from smallest to largest.

A



B

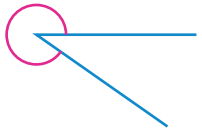


C

D

E

F



3 List the pairs of angles that look to be the same size.

A

B

C

D

E

F

Exploration Task

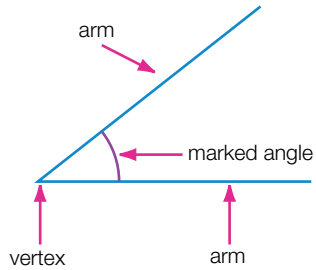
You can download this activity from the eBook or the Pearson Places website.

Are you sure they are parallel?

In this activity, you will explore parallel lines in crossing railway tracks.

8.1 Measuring, estimating and drawing angles

An **angle** is formed when two lines touch or intersect at a point. This point is called the **vertex** and the lines are called the **arms** of the angle.

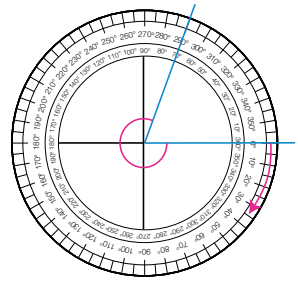


The size of an angle is a measure of the turn from one arm to another about the vertex.

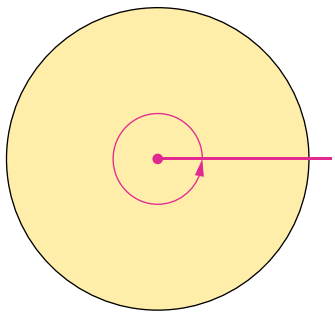
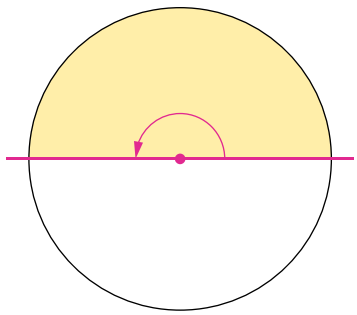
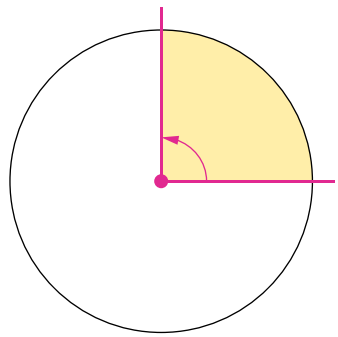
Angles can be measured using **degrees** (°). When an angle is rotated completely around a circle for one full turn, this is called a **revolution**. A revolution is divided into 360 parts, so that the turn through one part is an angle of 1 degree.

Why is a revolution divided into 360 degrees?

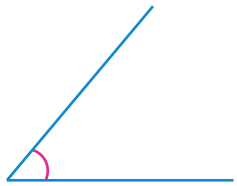
It is thought that the ancient Babylonians, who counted in lots of 60 and who measured their year as 360 days, were responsible for dividing a revolution into 360 parts. The number 360 is convenient for making fractions, so this is one reason why degrees are still used to measure angles today.

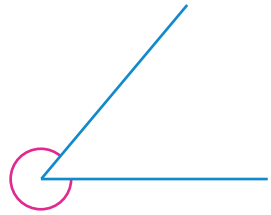


A **protractor** shows a revolution divided into 360° and can be used to draw and measure angles reasonably accurately.

 Full turn	 Half turn	 Quarter turn
360°	180°	90°

Because two angles can be formed between any two arms ('inside' or 'outside' the arms, see below), you need to mark the angle that is being considered.





Estimating and measuring angles

It is often important to be able to estimate angles. It may not always be possible or necessary to measure angles exactly, but you need to be able to understand how big or small an angle is.

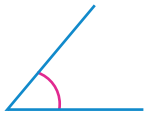
If you need to measure angles, you can use a semicircular protractor or a full-circle protractor.

Worked example 1 W.E.1

Estimate the size of each of the following angles, then use a protractor to measure the size of the angles. Write your answer, to the nearest degree, in a table using the headings as shown. Calculate the difference between your estimate and the measured size.

Estimated angle size	Measured angle size	Difference

(a)



(b)

Thinking

For angles less than 180°

(a) **1** As the angle is small and about half of a right angle (90°), estimate the size of the angle.

2 Using a semicircular protractor, place the centre point of the protractor over the vertex of the angle and align the baseline of the protractor with an arm of the angle. Use the scale on the protractor that starts with 0°. This protractor measures the angle shown as 50°.

3 Complete the table.

For angles larger than 180°

(b) **Method 1: Using a semicircular protractor**

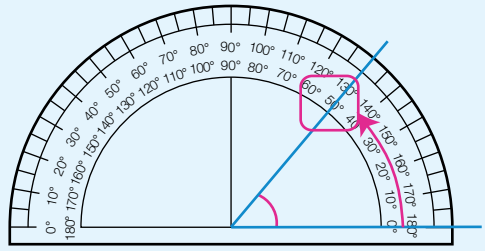
1 As the angle is greater than three right angles (270°), but less than four right angles (360°), estimate the size of the angle.

2 Using a semicircular protractor, measure the smaller angle and subtract this angle from 360°.

The small angle measures 70°, so the marked angle must be 360° – 70° = 290°.

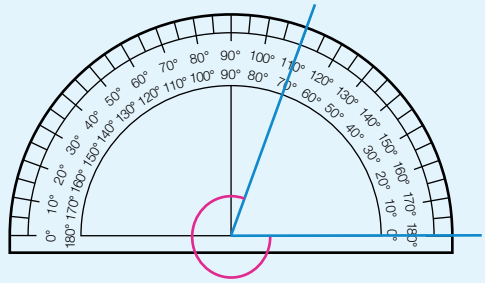
Working

(a) Estimate: 45°



Estimated angle size	Measured angle size	Difference
45°	50°	5°

(b) Estimate: 300°



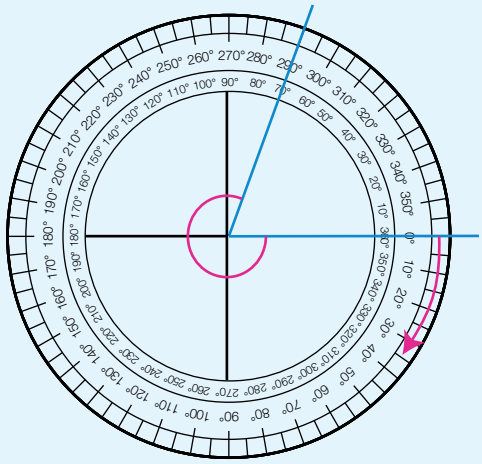
3 Complete the table.

Estimated angle size	Measured angle size	Difference
300°	290°	10°

Method 2: Using a full-circle protractor

1 Using a full-circle protractor, line up the centre point with the vertex and the 0° line along one arm, as with the semicircular protractor. Use whichever scale increases from zero. In this case, the outer scale, going clockwise.

(b)



2 Complete the table.

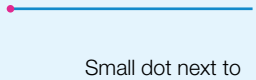
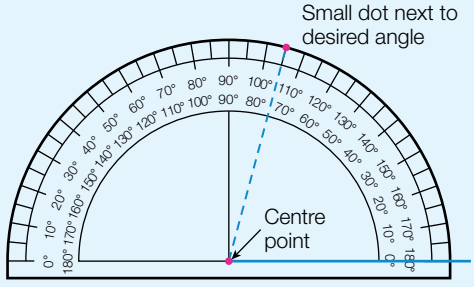
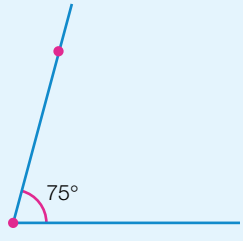
Estimated angle size	Measured angle size	Difference
300°	290°	10°

Drawing angles with a protractor

Worked example 2 W.E. 2

Use a protractor to draw the following angles.

(a) 75° **(b)** 200°

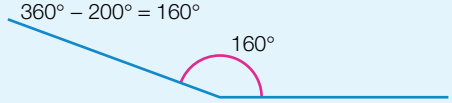
Thinking	Working
For angles less than 180°	(a)
(a) 1 Draw a straight horizontal line.	
2 Place the centre point of the baseline of the protractor on one end of the straight line. Locate the desired angle and mark a small dot as shown (in this case, 75°). (The base line is not the edge of the protractor. Make sure you use the scale starting at the end of your line segment.)	
3 Remove the protractor and join the dot with the end of the line where the centre point was located. Mark the angle.	

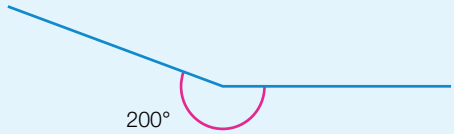
For angles larger than 180°

(b) 1 Subtract the angle from 360° to get the smaller angle needed to make up a full turn. Now, draw this angle. (For example, to draw 200°, you need to first draw 160°.)

2 Label the correct, larger angle.

(b) 360° – 200° = 160°





8.1 Measuring, estimating and drawing angles

Navigator

Answers p. 692

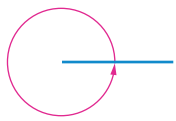
1, 2, 3 (columns 1–2), 4, 5, 6, 7, 8, 10, 11, 12 (column 1), 13, 15, 16	1, 2, 3 (columns 2–3), 4, 5, 6, 7, 8, 9, 10, 11, 12 (column 2), 13, 14, 15, 16	2 (column 1), 3 (column 3), 4, 6, 7, 8, 9, 10, 11, 12 (column 3), 14, 15, 16, 17

Equipment required: protractor

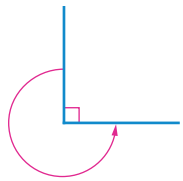
Fluency

1 State the exact size of the following angles.

(a)

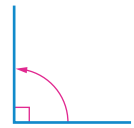


(b)



(c)

(d)



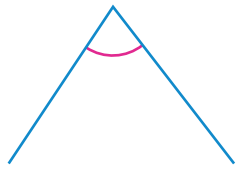
W.E. 1

2 Estimate the size of each of the following angles, then use a protractor to measure the size of the angles. Write your answer, to the nearest degree, in a table using the headings as shown. Calculate the difference between your estimate and the measured size.

Estimated angle size	Measured angle size	Difference

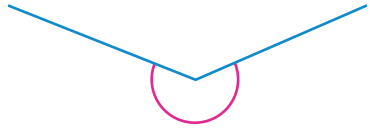
(a)

(b)

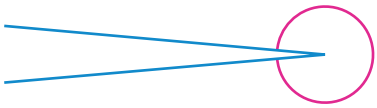


(c)

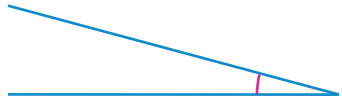
(d)



(e)



(f)



W.E. 2

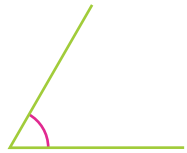
3 Use a protractor to draw the following angles.

(a) 10° **(b)** 70° **(c)** 55°

(d) 100° **(e)** 175° **(f)** 108°

(g) 290° **(h)** 265° **(i)** 318°

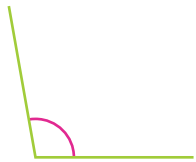
4 (a) The angle shown here is approximately:



A 20° **B** 45°

C 60° **D** 85°

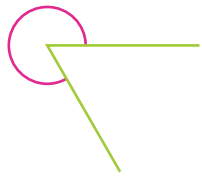
(b) The angle shown here is approximately:



A 80° **B** 100°

C 170° **D** 200°

(c) The angle shown here is approximately:

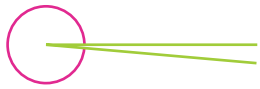


A 60° **B** 180°

C 240° **D** 300°

5 (a) Which of the angles below is about 5°?

A



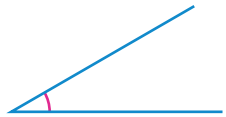
B

C

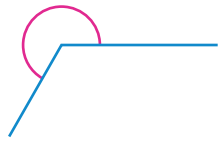
D

(b) Which of the angles below is about 330°?

A

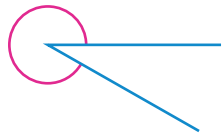


B



C

D



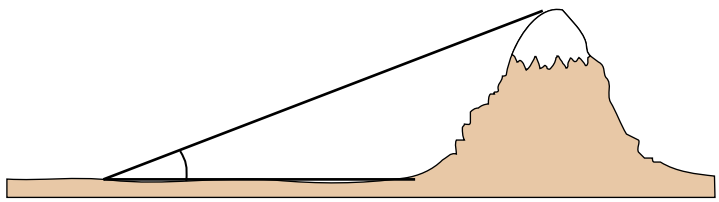
6 To draw a 250° angle with a semicircular protractor, which of the following would you do first?

A Draw a 50° angle. **B** Draw a 90° angle.

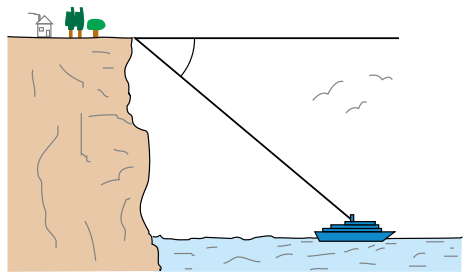
C Draw a 110° angle. **D** Draw a 200° angle.

Understanding

7 In the diagram, estimate the angle shown from ground level to the mountain top and then measure the angle with your protractor.



8 In the diagram, estimate the angle shown from the top of the cliff to the ship and then measure the angle with your protractor.

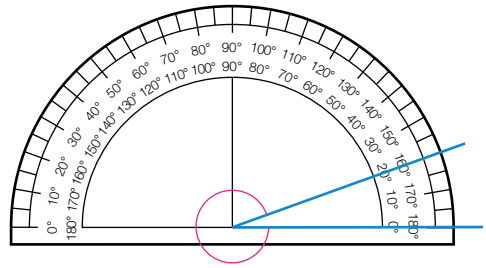


9 Explain how you would draw an angle of 320° with a semicircular protractor.

10 Liz uses a semicircular protractor to measure an angle larger than 180°.

Find the size of the actual angle (larger than 180°) if the smaller angle she has measured is:

(a) 30° **(b)** 115° **(c)** 160°



11 Draw the following using your protractor.

(a) a ramp with a slope of 12°

(b) a ladder leaning against a vertical wall at 62° to the ground

(c) a round birthday cake sliced into 6 equal pieces

Reasoning

12 For each of the following angles:

(a) 35° **(b)** 65° **(c)** 83°

(d) 120° **(e)** 102° **(f)** 152°

(g) 200° **(h)** 265° **(i)** 325°

(i) draw an estimate of the angle without the use of a protractor

(ii) use your protractor to measure the angles you have drawn to find the actual size of your angles

(iii) find the error (the difference between the given angle and your measurement)

(iv) copy and complete the following table.

Estimated angle size	Measured angle size	Difference

(v) Which angles did you draw the most accurately?

(vi) Do you think it is harder to estimate angles greater than 180° than those less than 180°? Why?

Open-ended

13 Draw the following angles, estimate the size of the angle you have drawn and then check the size with a protractor.

(a) any angle smaller than 90° **(b)** any angle between 90° and 180°

(c) any angle between 180° and 360°

14 Your friend Nick has just answered the following question.

> Use a protractor to measure the size of the following angle. Give your answer to the nearest degree.
>
> 

Nick says 'The answers in the back of the book must be wrong! The book says that the answer is 135°, but my protractor says 45°.' Explain how Nick might have found his answer. Give Nick a hint so that he can measure accurately all the time.

15 Without using a protractor, draw the following angles and estimate the size of each angle.

(a) two angles that add up to 90°

(b) three angles that add up to 180°

(c) four angles that add up to 360°

(d) Check your answers to parts **(a)**, **(b)** and **(c)** with a protractor.

16 **(a)** Choose any angle between 40° and 60°, draw it carefully with a protractor and mark the size of your angle on your diagram.

(b) Using a protractor, draw an angle 15° larger than the angle drawn in part **(a)** and mark the size of your angle on your diagram.

(c) Using a protractor, draw an angle 195° larger than the angle drawn in part **(a)** and mark the size of your angle on your diagram.

17 **(a)** Choose any angle less than 90°, draw it carefully with a protractor and mark the size of your angle on your diagram.

(b) Using a protractor, draw an angle twice the size of the angle you chose in part **(a)** and mark the size of your angle on your diagram.

(c) Using a protractor, draw an angle twice the size of the angle in part **(b)** and mark the size of your angle on your diagram.

Game

Angle Master

Equipment required: ruler, protractor

How to win:
The winner is the first player to score 15 points, and is proclaimed the 'Angle Master'.

How to play:
The aim of the game is to guess the size of different angles more accurately than your opponent.

1 Take it in turns to draw a freehand angle.

2 Starting with your opponent, take turns in bidding to see who can guess the size of the angle without going over the angle's measurement. The bidding ends when one player believes the other has overestimated the angle and calls for the angle to be measured with a protractor at this guess.

Scoring
0–1° under = 5 points
2–5° under = 4 points
6–10° under = 2 points
11+° under = 1 point

Correctly claiming your opponent has overestimated the angle = 3 points

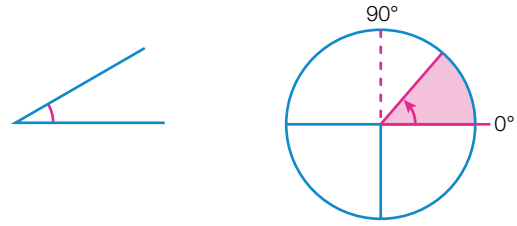
Example game:
Imagine player 1 draws the following angle.

Player 2 starts the bidding at 110°, player 1 ups the bidding to 118°, player 2 ups the bidding to 124°, and player 1 calls for the angle to be measured. The angle is measured at 126°, so player 2 scores 4 points.

8.2 Classifying and naming angles

Classifying angles

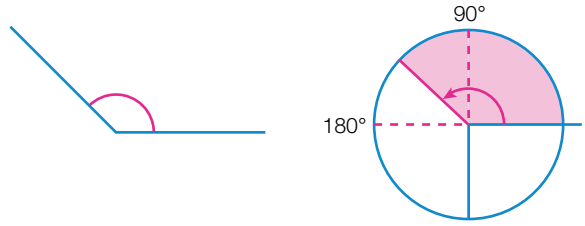
Acute angle: greater than 0° but less than 90°.



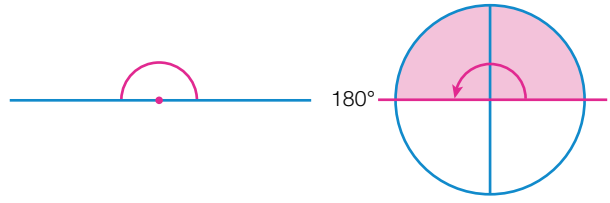
Right angle: exactly 90°, a $\frac{1}{4}$ turn.

A small square drawn in the corner of an angle means it is a right angle.

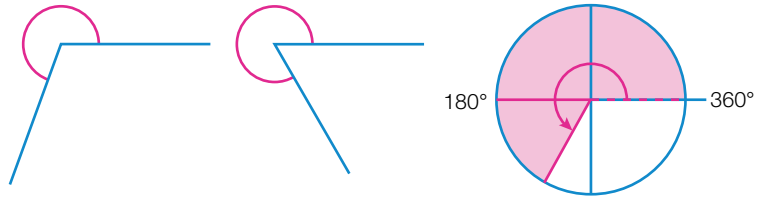
Obtuse angle: more than 90° but less than 180°.



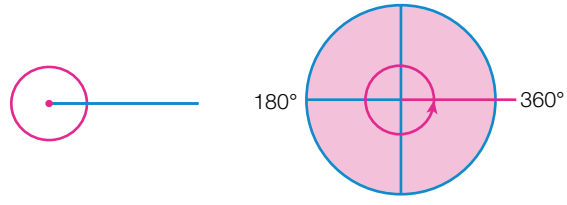
Straight angle: exactly 180°, a straight line angle, a half turn.



Reflex angle: greater than 180° but less than 360°.



Revolution: 360°, a full turn.



Points, lines and angles

A **point** defines a position in space but it has no size. It has no width or length, so a dot you can see or draw is only an approximation of a point. You can represent a point with a dot and label it with capital letters such as A, B and P.

Similarly, a **line** you can draw or see is only an approximation of a line, because a line has no width and no beginning or end, extending infinitely in both directions. In mathematics the word 'line' always means a straight line and does not include curves. You can draw a line through any two points to represent a mathematical line, and use arrowheads on each end to show that it goes on forever in both directions.

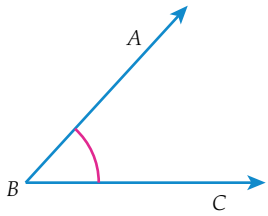
A line that goes through points A and B can be labelled as $\overleftrightarrow{AB}$.

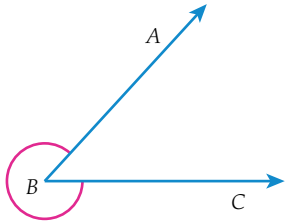
A **ray** is part of a line. Unlike a line, a ray has a beginning. It starts at a point called its vertex and extends infinitely in one direction like a ray of light from a torch. You can draw a ray by starting it at a point and using an arrowhead on the other end.

A ray that starts at point A and passes through B can be labelled as $\overrightarrow{AB}$.

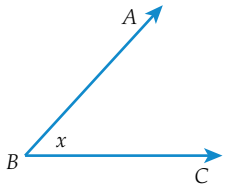
A **line segment** is part of a line that has a beginning and an end. It can also be called an interval. Line segments are often referred to as lines, but this is not strictly correct. A line segment that starts at A and ends at B can be labelled as $\overline{AB}$.

If two rays have a common vertex, two angles are formed, so when you draw an angle you need to mark clearly the angle that is meant.





Angles are usually drawn using line segments. You can label the angles that are formed by $\overline{AB}$ and $\overline{BC}$ as $\angle ABC$ and as $\angle CBA$, where the vertex B is the middle of the three points. You can use the words acute, obtuse or reflex to indicate which angle is meant.



An easier and clearer way to name an angle is to use a lowercase letter such as x or z in the angle space.

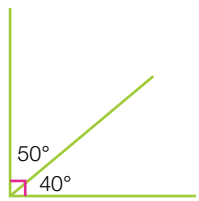
Greek letters such as θ (theta) and α (alpha) are often used to represent unknown angles.

Complementary and supplementary angles

Complementary angles add to 90° (a right angle).

40° and 50° are complementary angles because they add to 90°.

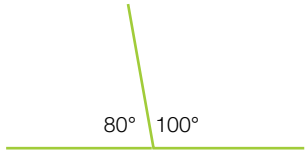
If two angles are complementary, we say one is the complement of the other. For example, 40° is the complement of 50°.



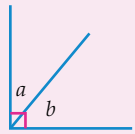
Supplementary angles add to 180° (a straight angle).

100° and 80° are supplementary angles because they add to 180°.

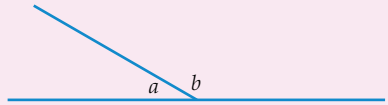
If two angles are supplementary, we say one is the supplement of the other. For example, 100° is the supplement of 80°.



Complementary angles add to 90°:
$a + b = 90°$

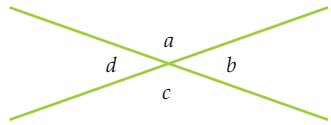


Supplementary angles add to 180°:
$a + b = 180°$



Vertically opposite angles

Whenever two lines intersect, four angles are formed. In the diagram opposite, these angles have been labelled as a, b, c and d. Pairs of angles such as a and c are given a special name—they are called **vertically opposite angles**.



There is another pair of vertically opposite angles in the diagram: b and d.

If you measured a and c with your protractor you would find they are the same size. The same is true for b and d. Check both of these pairs of angles for yourself.

Vertically opposite angles are equal.

8.2 Classifying and naming angles

Navigator

Answers p. 693

1 (columns 1–2), 2, 3, 4, 5, 6, 8, 9, 10, 12, 13, 14 (a–b), 15, 16	1 (columns 2–3), 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14 (a–c), 15, 16, 17	1 (column 3), 2 (columns 2–3), 3, 4, 5, 6, 7, 8 (a–c), 9, 10, 11, 13, 14, 16, 17

Equipment required: protractor

Fluency

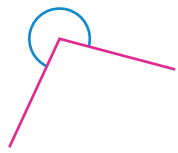
1 State the type of angle shown in each case.

(a)

(b)

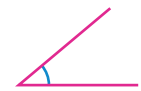
(c)

(d)



(e)

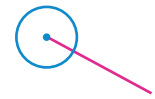
(f)



(g)

(h)

(i)

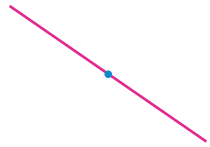


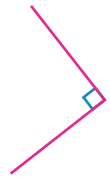
(j) 

(k) 

(l) 

(m) 

(n) 

(o) 

(p) 

(q) 

(r) 

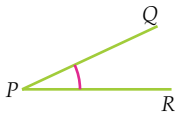
2 Classify the following angles (as acute, obtuse, right and so on).

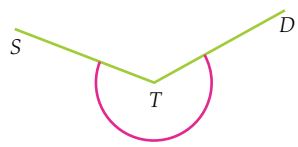
(a) 23° **(b)** 117° **(c)** 275°

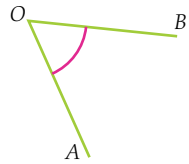
(d) 360° **(e)** 180° **(f)** 75°

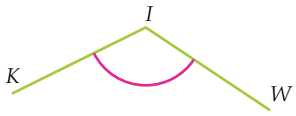
(g) 90° **(h)** 165° **(i)** 341°

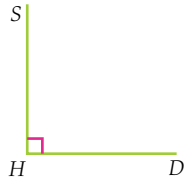
3 Name the following angles.

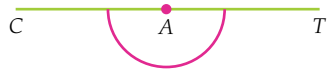
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

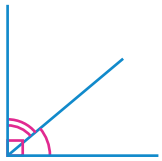
4 **(a)** Which one of the following is a pair of complementary angles?

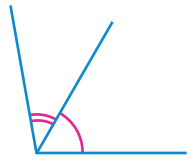
A 20° and 40° **B** 330° and 30° **C** 30° and 150° **D** 15° and 75°

(b) Which one of the following is a pair of supplementary angles?

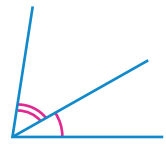
A 0° and 90° **B** 45° and 55° **C** 90° and 90° **D** 180° and 20°

(c) Which diagram shows supplementary angles?

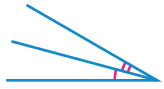
A 

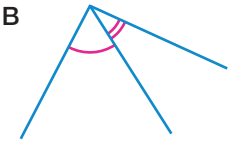
B 

C 

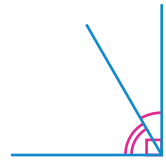
D 

(d) Which diagram shows complementary angles?

A 

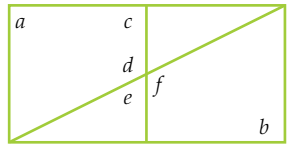
B 

C 

D 

It might help you to remember that the 'c' of complementary stands for a corner (right angle) and the 's' of supplementary stands for a straight angle.

5 Which two angles are vertically opposite?



A a and b **B** a and c

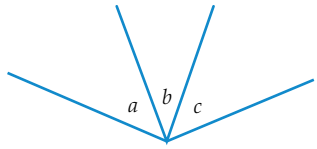
C d and e **D** d and f

Understanding

6 Explain why the angles 85° and 5° are complementary angles.

7 Explain why 160° and 72° are not supplementary angles.

8 The angles a, b and c in this diagram have been drawn as adjacent angles. They have a common vertex and common arms.



For each of the following:

(a) 100°, 210°, 50° **(b)** 32°, 40°, 18° **(c)** 40°, 40°, 100°

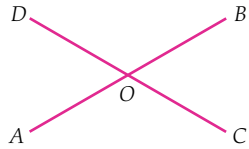
(d) 32°, 161°, 85°, 82° **(e)** 44°, 46° **(f)** 39°, 141°

(i) draw the angles as adjacent angles

(ii) find the angle sum

(iii) describe the angle you have formed.

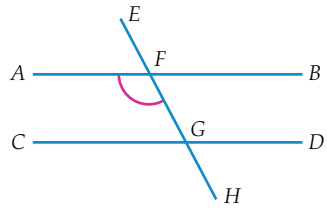
9 In the diagram opposite:



(a) name a pair of supplementary angles

(b) name a pair of vertically opposite angles.

10 In the diagram, what should the marked angle be named?



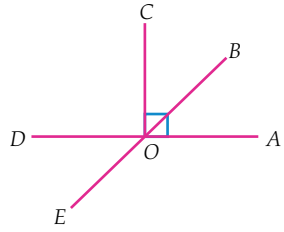
A $\angle F$

B $\angle AFE$

C $\angle GFA$

D $\angle BFG$

11 Using the diagram opposite, find, using their letter names:



(a) an acute angle **(b)** an obtuse angle

(c) a reflex angle **(d)** a right angle

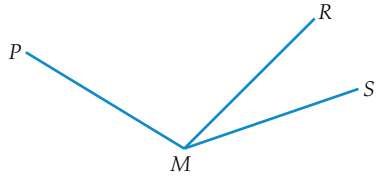
(e) a straight angle **(f)** one pair of complementary angles

(g) one pair of supplementary angles

(h) one pair of vertically opposite angles.

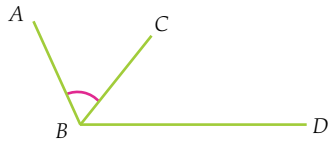
Reasoning

12 **(a)** Measure the reflex angle $\angle RMS$ in the diagram.



(b) Why is it necessary to specify 'reflex' in part **(a)**?

13 **(a)** Why shouldn't you refer to the marked angle as $\angle B$?



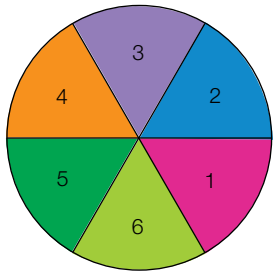
(b) Name two acute angles that have B as a vertex.

(c) Name an obtuse angle that has B as a vertex.

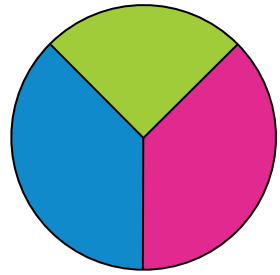
(d) Name a reflex angle that has B as the vertex.

14 Use a protractor to draw accurate copies of the following.

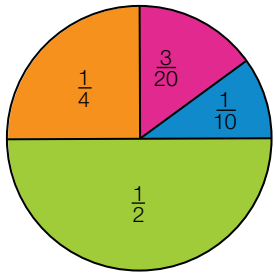
(a) A raffle wheel divided into six equal parts.



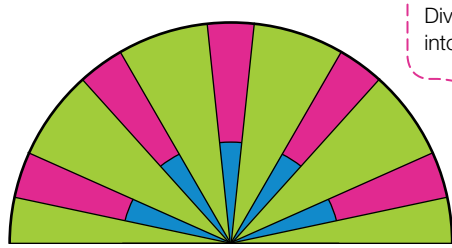
(b) This sign.



(c) This pie chart.



(d) This semicircular stained glass window.



Open-ended

15 Draw two examples of:

(a) an acute angle **(b)** a reflex angle **(c)** an obtuse angle.

16 **(a)** Write two examples of pairs of complementary angles.

(b) Write two examples of pairs of supplementary angles.

17 Write two examples of angles that add to make a revolution.

Problem solving

Count the angles

Equipment required: 1 protractor

1 How many different acute angles can you count in the diagram? Measure each one.

2 How many different angles can you count above the horizontal line? Measure the size of each one.

3 How many different angles can you count in the diagram? Measure the size of each one.

Strategy options

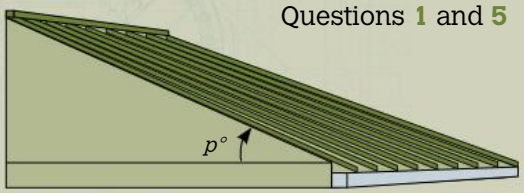
- Test all possible combinations.
- Break problem into manageable parts.

Maths 4 Real

Constructing

The 'pitch' of a roof is the angle that the roof makes with the horizontal. The pitch can depend on a number of factors including visual preference, style of house, the roof material, or the weather conditions in the area where the house is built.

Equipment required: 2 rulers and a protractor for Questions 1 and 5



The most common reason that houses have a pitched roof is to redirect rainwater. A pitch of between 20° and 30° is common.

1 Measure the angle p of the pitched roof above to determine a common pitch angle for Australian house roofs.

2 Imagine you are building a house in an area that regularly receives a lot of heavy rain. What problems might this cause if the roof is:

(a) too flat (pitch angle is too small), or

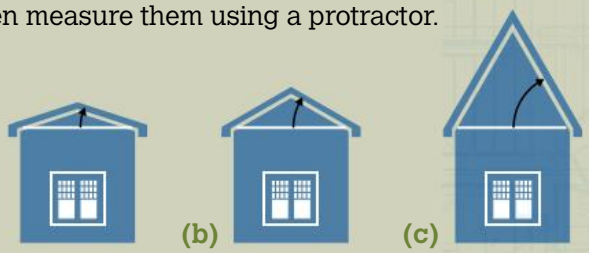
(b) too steep (pitch angle is too big)?

5 To measure an angle, builders use a pitch angle and level finder for construction.

Construct your own pitch angle and level finder out of two rulers and a protractor. Use it to measure the pitch of the roof of a building in your area, such as a school building or your house. You can do this by standing on the ground and holding your pitch angle and level finder in line with the roof of the building.

angles

3 Estimate the pitches of the roofs on these houses, then measure them using a protractor.



4 In places where there is a lot of snow, the pitch of the roof must be at least 30°. Why do you think this is the case?

Research

6 (a) Find out some of the common pitch angles used by builders. Does there seem to be a minimum or maximum value for the pitch angle? What problems might be created if the angle is too steep?

(b) Research the difference in pitch required by different materials, such as a thatched roof, tiled roof or corrugated iron roof.

8.3 Calculating angles

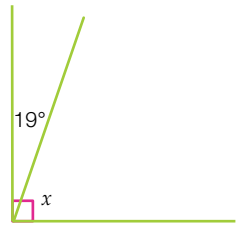
Complementary and supplementary angles

The sum of complementary angles is 90°, so you can subtract a known angle from 90° to find the unknown complementary angle. In a similar way, you can subtract a known angle from 180° to find the unknown supplementary angle.

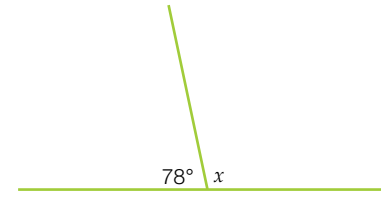
Worked example 3 W.E. 3

Find the size of the angle x in each diagram.

(a)



(b)



Thinking		Working
(a) 1	Identify the angles as complementary angles.	(a) x and 19° are complementary angles.
2	Write an equation with the RHS equal to 90°.	$x + 19 = 90$
3	Solve the equation to find the angle.	$x + 19 - 19 = 90 - 19$ $x = 71°$
(b) 1	Identify the angles as supplementary angles.	(b) x and 78° are supplementary angles.
2	Write an equation with the RHS equal to 180°.	$x + 78 = 180$
3	Solve the equation to find the angle.	$x + 78 - 78 = 180 - 78$ $x = 102°$

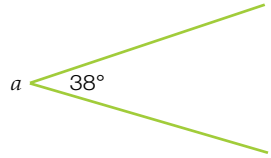
Angles in a revolution

Angles in a revolution add to 360°, so you can subtract a known angle from 360° to find the unknown angle.

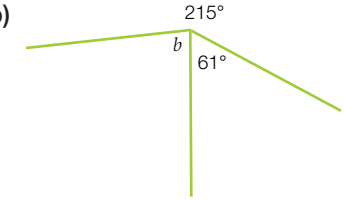
Worked example 4 W.E. 4

Find the value of the pronumeral in each diagram.

(a)



(b)



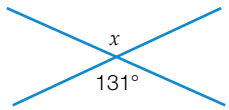
Thinking	Working
(a) 1 Add all the angles in the revolution, to give 360°.	(a) $a + 38 = 360$
2 Find the value of a.	$a + 38 - 38 = 360 - 38$ $a = 322°$
(b) 1 Add all the angles in the revolution, to give 360°.	(b) $215 + 61 + b = 360$ $276 + b = 360$
2 Find the value of b.	$276 - 276 + b = 360 - 276$ $b = 84°$

Vertically opposite angles

Vertically opposite angles are equal, so if you know one angle then you know that the vertically opposite angle is the same size.

Worked example 5 W.E. 5

Find the value of x in the diagram.



Thinking	Working
These are vertically opposite angles, which means they are equal.	$x = 131°$

8.3 Calculating angles

Navigator

1 (columns 1–2), 2 (columns 1–2), 3, 4, 5, 6, 10 (columns 1–2), 12, 13	1 (columns 2–3), 2 (columns 2–3), 3 (a–d), 4, 5, 6, 7, 8, 9 (a–b), 10, 11, 12, 13	1 (column 3), 2 (column 3), 3 (a–c), 4, 5, 6, 7, 8, 9, 10, 11, 12, 13

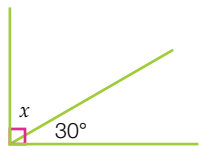
Answers p. 694

Equipment required: protractor

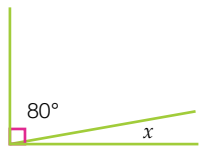
Fluency

1 Find the size of angle x in each diagram. W.E. 3

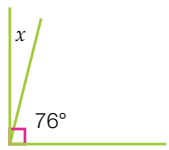
(a)



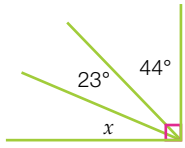
(b)



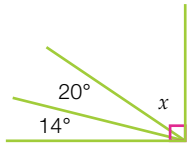
(c)



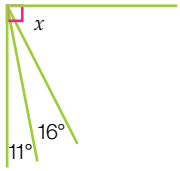
(d)

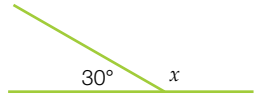


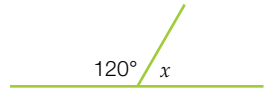
(e)



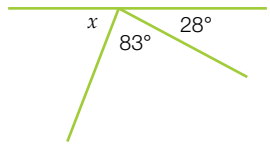
(f)

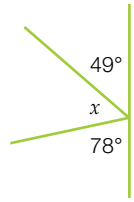


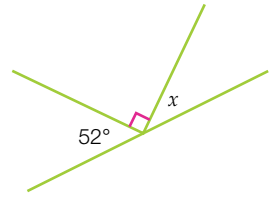
(g) 

(h) 

(i) 

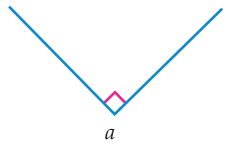
(j) 

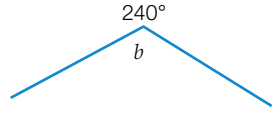
(k) 

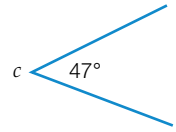
(l) 

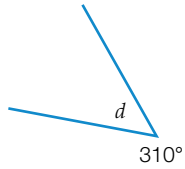
W.E. 4

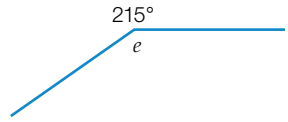
2 Find the value of the unknown angle represented by the pronumeral in each diagram.

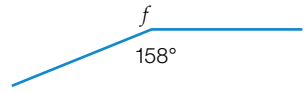
(a) 

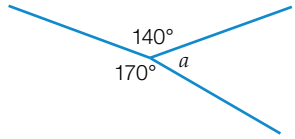
(b) 

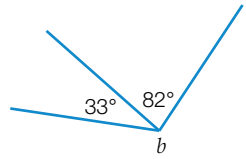
(c) 

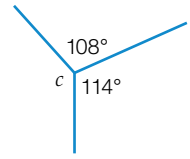
(d) 

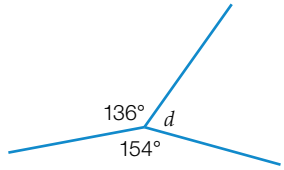
(e) 

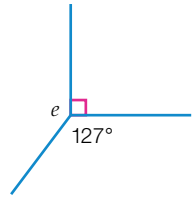
(f) 

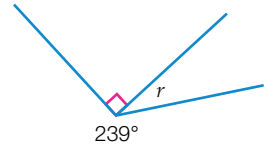
(g) 

(h) 

(i) 

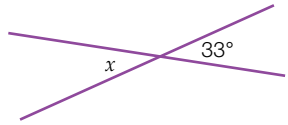
(j) 

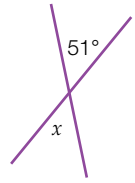
(k) 

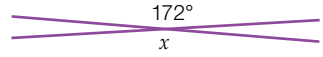
(l) 

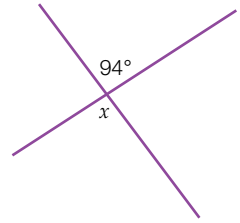
W.E. 5

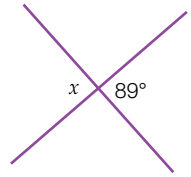
3 Find the value of x in each diagram.

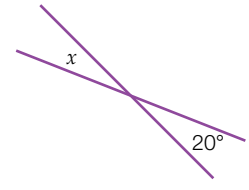
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

4 Find the complement of the following angles.

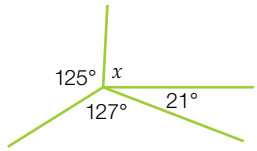
(a) 27° **(b)** 45° **(c)** 68° **(d)** 15°

5 Find the supplement of the following angles.

(a) 32° **(b)** 90° **(c)** 124° **(d)** 176°

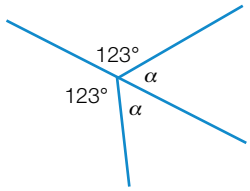
Understanding

6 Find the value of x in the diagram.

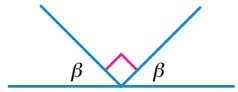


7 Find the value of the pronumerals in each diagram.

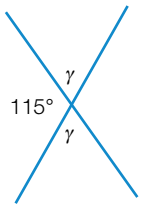
(a)



(b)



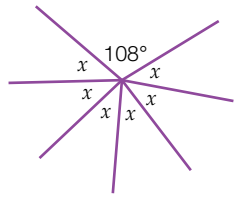
(c)



8 The value of x in the opposite diagram is:

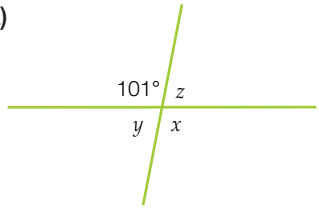
A 42° **B** 60°

C 108° **D** 360°

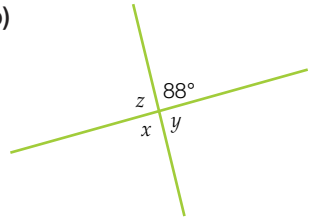


9 Find the value of the pronumerals in each diagram.

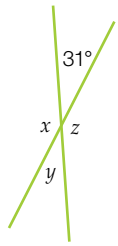
(a)



(b)



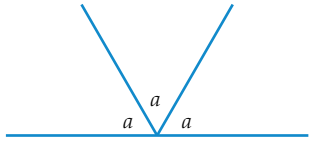
(c)



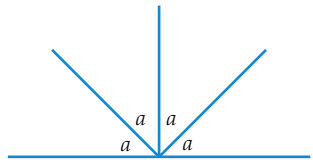
Reasoning

10 Determine the size of angle a in each diagram. Give reasons for your answer.

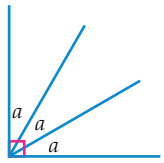
(a)



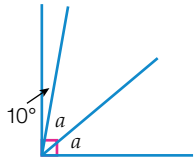
(b)



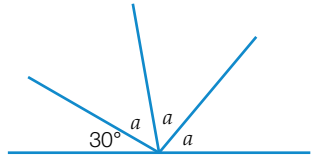
(c)



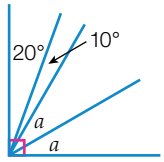
(d)



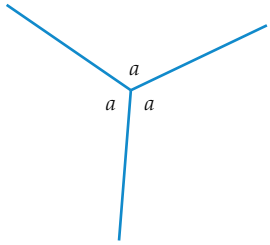
(e)



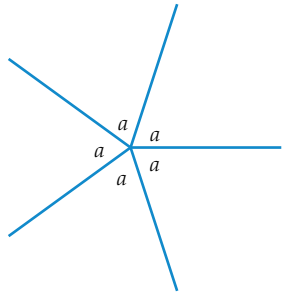
(f)



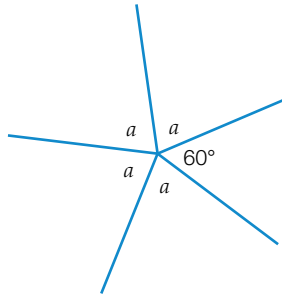
(g)



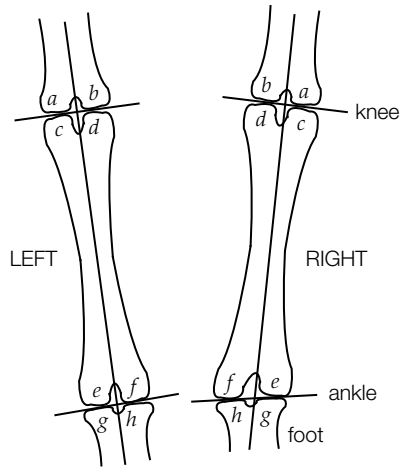
(h)



(i)



11 After breaking her leg in an accident, Kira had difficulty walking. She took her X-ray to a specialist, who marked in the following lines. He followed a procedure to decide whether she needed an operation. (You will need a protractor to measure some of the angles.)



(a) Measure all the angles and fill in the table below.

	a	b	c	d	e	f	g	h
Left leg								
Right leg								

(b) All angles in Kira's left and right legs and ankles should be approximately equal. Which leg do you think Kira broke? Why?

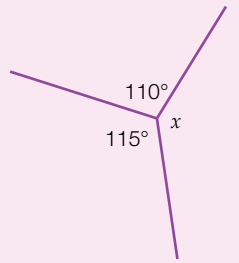
(c) The differences between the angles b and f, and c and g, must be no more than 8°. If they are more, Kira needs an operation. Does she need an operation? Why?

Open-ended

12 Sabine was answering the question on the right.

Using her protractor, Sabine measured the angle x and found that it was 140°. This was incorrect. Explain where Sabine made a mistake.

Find the value of the pronumeral.



13 **(a)** Draw two lines that intersect each other.

(b) Measure the two supplementary angles you have created.

(c) Use your answers to part **(b)** to find the two angles you did not measure.

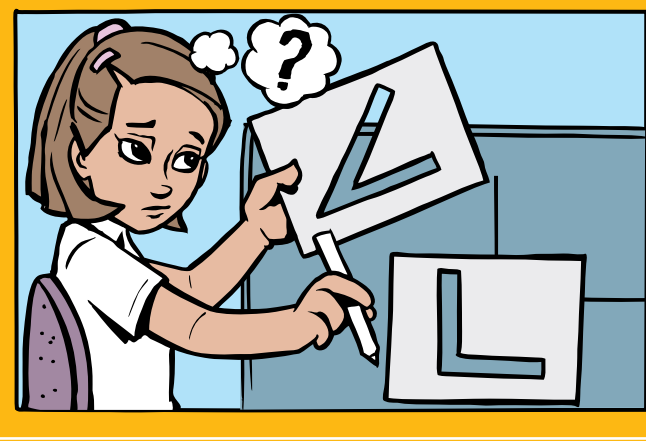
Problem solving

Ella's angles

Ella needs to draw a 60° angle, but she has lost her protractor. She has a stencil that can draw 90° and 40° angles. How can she use these two angle sizes to draw a 60° angle?

Strategy options

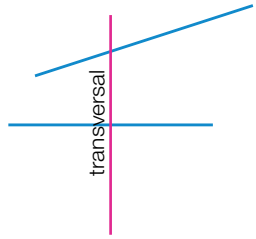
- Draw a diagram.
- Guess and check.



Angles and transversals

8.4

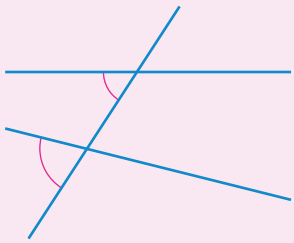
A **transversal** is a line that intersects (crosses or transverses) two or more other lines, as shown.



Angles formed when a transversal cuts two lines

When a transversal intersects two other lines, pairs of angles are formed. These angles are given special names.

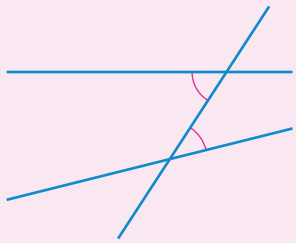
Corresponding angles



These angles are either above or below the two lines cut by the transversal, on the same side of it.

'Corresponding' means 'matching', so corresponding angles are in matching positions. There are four pairs of corresponding angles formed when a transversal cuts two lines.

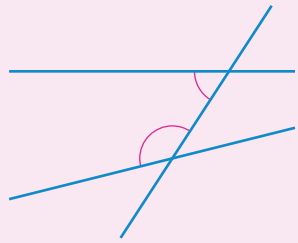
Alternate angles



These angles are between the two lines cut by the transversal, but on opposite sides of it.

'Alternate' can mean 'opposite', so alternate angles swap sides of the transversal. There are two pairs of alternate angles formed when a transversal cuts two lines.

Co-interior (allied) angles

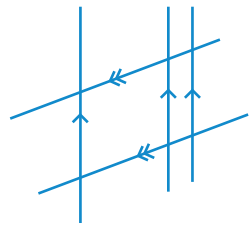


These angles are between the lines cut by the transversal, on the same side of it.

'Co' means 'with' and 'interior' means 'inside', so co-interior angles are inside and on the same side with the transversal.

There are two pairs of co-interior angles formed when a transversal cuts two lines.

Parallel lines are lines that lie in the same **plane** (same flat surface) and are always the same distance apart. The lines on a page of lined paper are parallel.

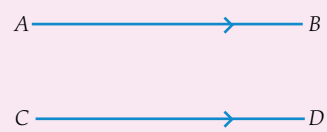


Lines that are parallel are marked with an arrow pointing in the same direction.

If more than one set of parallel lines appears in a diagram, then you can use more than one arrow to distinguish the different sets.

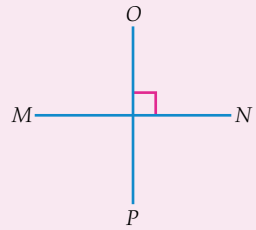
Perpendicular lines intersect at right angles.

|| means 'is parallel to'



$AB \parallel CD$

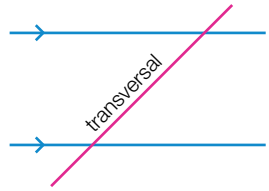
$\perp$ means 'is perpendicular to'



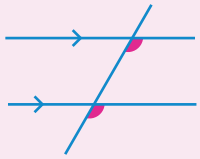
$OP \perp MN$

Properties of angles formed when a transversal cuts parallel lines

When parallel lines are crossed by a transversal, the pairs of angles on parallel lines described have special properties.

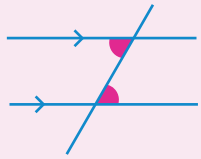


Corresponding angles



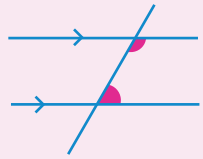
Corresponding angles on parallel lines are *equal*.

Alternate angles



Alternate angles on parallel lines are *equal*.

Co-interior (allied) angles



Co-interior angles on parallel lines are *supplementary*. They add to 180°.

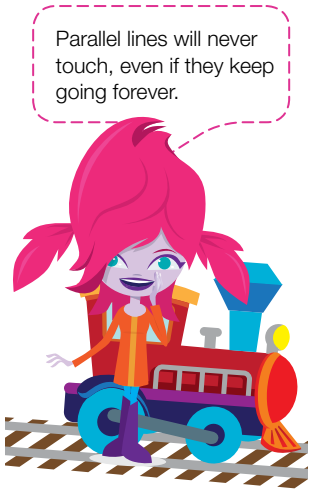
The opposite of the above is also true. If corresponding or alternate angles are equal or if co-interior angles are supplementary when two lines are cut by a transversal, then the two lines must be parallel.

8.4 Angles and transversals

Navigator

Answers p. 694

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 14 (columns 1–2), 15	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14 (columns 2–3), 15	1, 2 (columns 2–3), 3, 4, 5, 6, 7 (columns 2–3), 8, 9 (columns 2–3), 11, 12, 13, 14, 15, 16

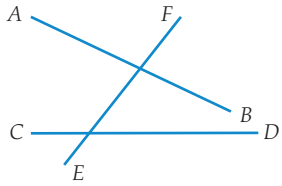


Equipment required: protractor for Question **7**

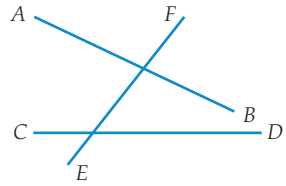
Fluency

1 Answer true (T) or false (F) for each of the following.

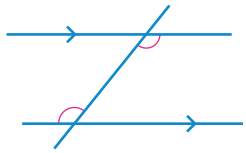
(a) The line AB is the transversal.



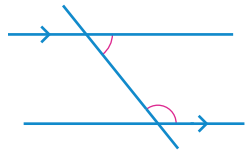
(b) AB is a line and its transversal is CD



(c) The two angles shown are a pair of alternate angles and they are equal.

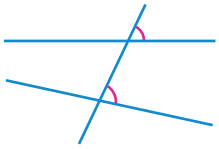


(d) The two angles shown are not equal, but they are co-interior and sum to 180°.

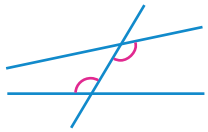


2 Identify each of the following pairs of angles as corresponding, alternate or co-interior angles.

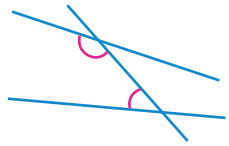
(a)



(b)

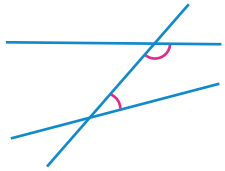


(c)



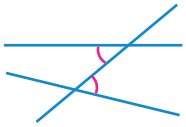
(d)

(e)

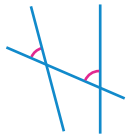


(f)

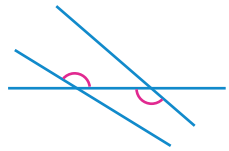
(g)



(h)

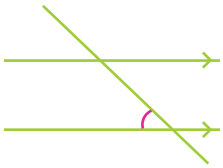


(i)

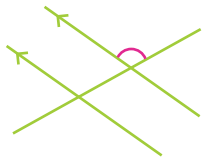


3 Copy each of the following and mark in an angle corresponding to the one shown.

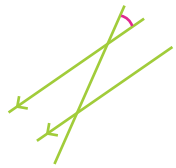
(a)



(b)

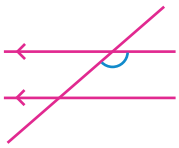


(c)

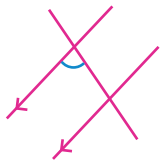


4 Copy each of the following and mark in an angle alternate to the one shown.

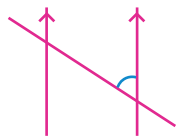
(a)



(b)

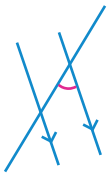


(c)

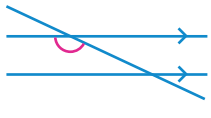


5 Copy each of the following and mark in an angle that is co-interior with the one shown.

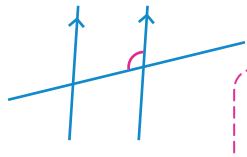
(a)



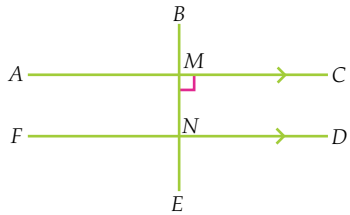
(b)



(c)



6 Which of the following statements is not true about the diagram shown?

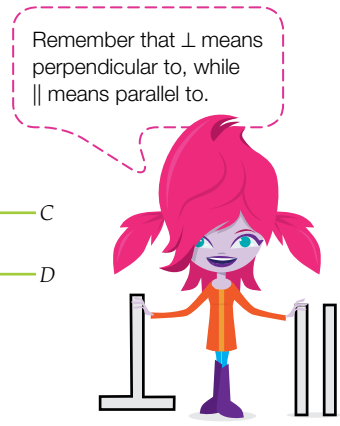


A $\angle AMB$ is corresponding to $\angle FNB$

B $\angle BNF$ is alternate to $\angle EMC$

C $\angle AMN$ and $\angle CMN$ are co-interior angles

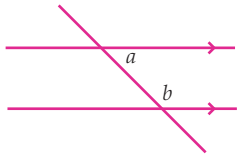
D BE is $\perp$ to FD



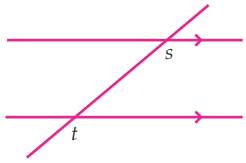
Understanding

7 **(a)** Use a protractor to measure the labelled angles in each diagram below. State whether each pair of angles is corresponding, alternate or co-interior.

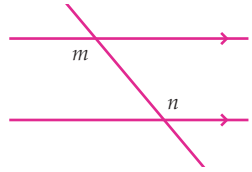
(i)



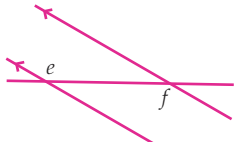
(ii)



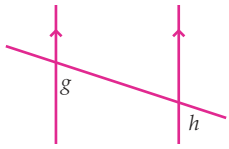
(iii)



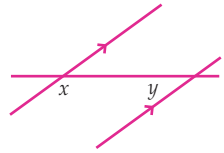
(iv)



(v)



(vi)



(b) State which pairs of angles in part **(a)** are equal. Which pairs of angles add to 180°?

8 State which angle in the diagram is:

(a) corresponding to u

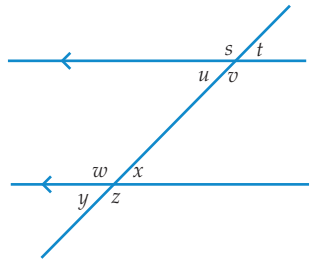
(b) alternate to v

(c) co-interior with w

(d) co-interior with x

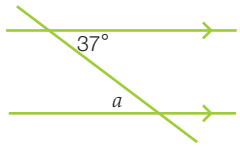
(e) corresponding to w

(f) alternate to u.

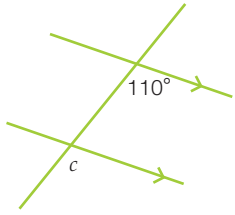


9 Find the value of the pronumerals in each case, and give a reason for your answer (e.g. corresponding angles).

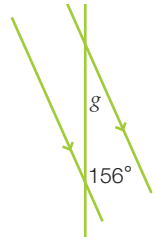
(a)



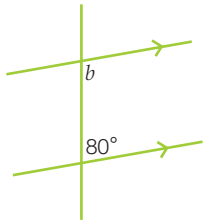
(b)



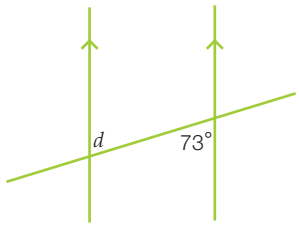
(c)



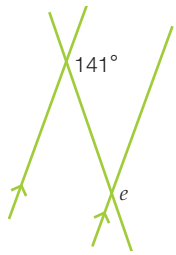
(d)



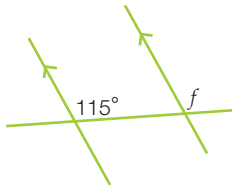
(e)



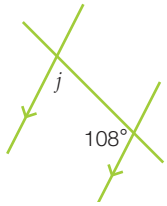
(f)



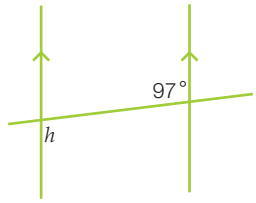
(g)



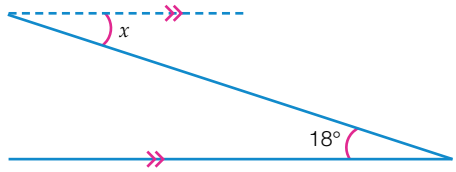
(h)



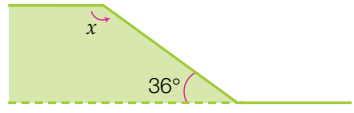
(i)



10 A pilot of a plane flying parallel to the ground can see the runway as it is coming in to land. The angle of approach to the runway is 18°. What angle must the pilot turn the plane down through to start the approach?

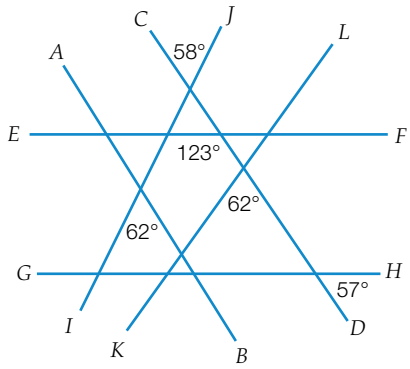


11 In an underground carpark a ramp is being constructed. The ramp will ascend at an angle of 36°. At what angle must the supports be positioned at the top of the ramp?



Reasoning

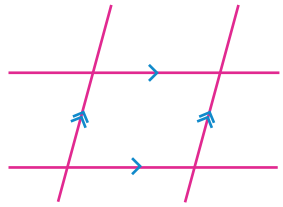
12 Which two lines are parallel? Why?



13 When a pair of parallel lines is cut by two parallel transversals, how many pairs of angles are formed that are:

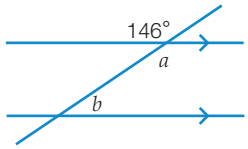
(a) corresponding **(b)** alternate

(c) co-interior?

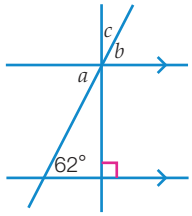


14 Find the value of each pronumeral below, giving a reason for your answer.

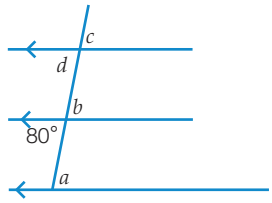
(a)



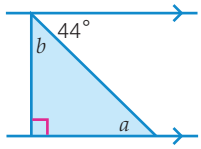
(b)



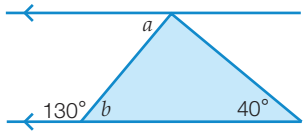
(c)



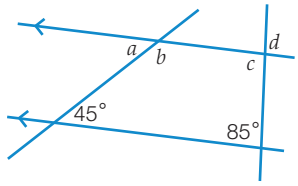
(d)



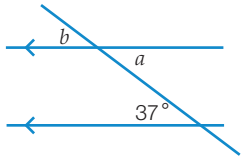
(e)



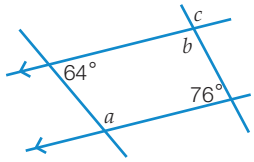
(f)



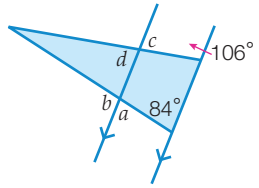
(g)



(h)

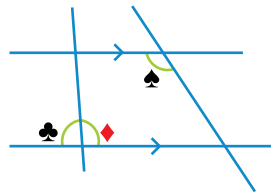


(i)



Open-ended

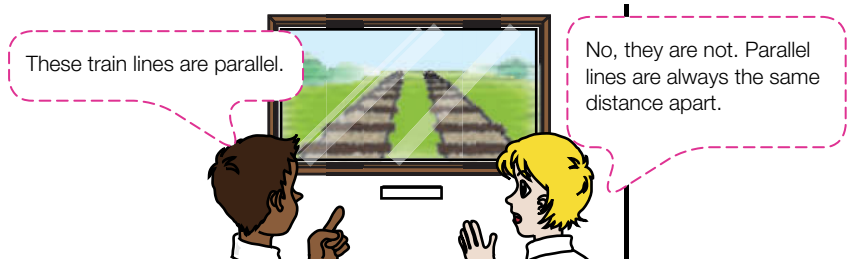
15 Aileen has drawn a pair of parallel lines with two transversals. She has marked three angles using different symbols.



(a) Copy the diagram into your book. Mark another three angles using the same symbols as Aileen so that there is a pair of corresponding angles, alternate angles and co-interior angles.

(b) In how many different ways can the three other angles be arranged, with no pair of angles sharing an angle with another pair? Draw a diagram for each combination.

16 How would you explain to the students whether the train lines are parallel or not?



Equipment required: TI-Nspire CAS or Casio ClassPad CAS

Investigating angles on parallel lines

You can use CAS technology to explore different angles on parallel lines. By doing this, you will become more familiar with how to use your CAS to help study and understand the special relationships between parallel lines and angles.

Creating a pair of parallel lines and a transversal

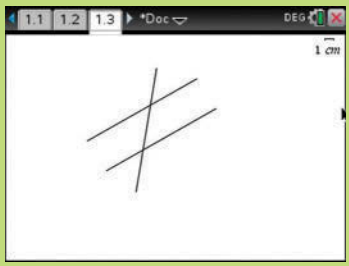
Parallel lines are lines that are equidistant from each other (the distance between the lines is the same all the way along them). A transversal is a line that cuts two or more other lines. (The lines cut by a transversal do not have to be parallel.)

Using TI-Nspire CAS

Add **Geometry** to your document. To create a line segment, select **menu** > **Points & Lines** > **Segment** and select the start and end points of the line segment.

To create a parallel line segment, select **menu** > **Transformation** > **Translation** and select the line segment, then select an end point of the line segment. You can now move the cursor and select to place a parallel copy of the original line segment.

To draw a transversal (crossing line) across the two parallel lines, again select **menu** > **Points & Lines** > **Segment** to draw a new line segment..

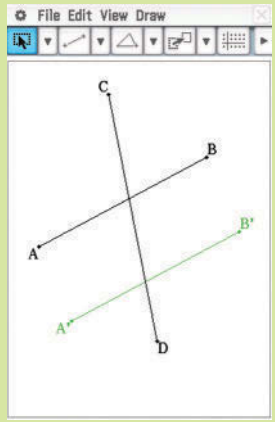


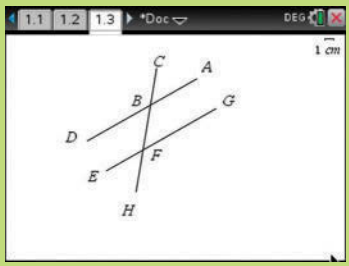
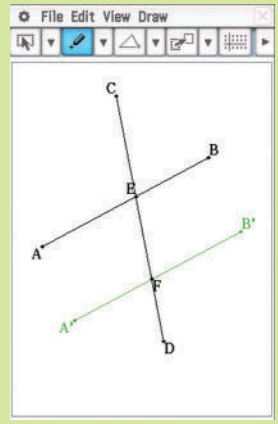
Using Casio ClassPad CAS

From the menu select **Geometry**. To create a line segment, select **Draw** > **Basic Object** > **Line Segment** and select the start and end points of the line segment.

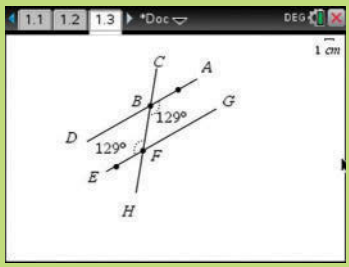
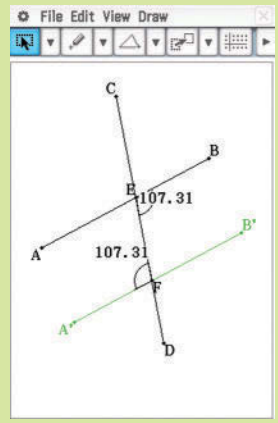
To create a parallel line segment, select **Draw** > **Construct** > **Translation** and enter numbers to indicate a direction, which is where a parallel copy of the original line segment will appear. The top number you enter is the horizontal direction, while the bottom number is the vertical direction.

To draw a transversal (crossing line) across the two parallel lines, again select **Draw** > **Basic Object** > **Line Segment** to draw a new line segment.



Using TI-Nspire CAS	Using Casio ClassPad CAS
To label the end points of the line segments and the points of intersection, you can create a text box for each point. Select **menu** > **Actions** > **Text** to make text box labels *A* to *H* for the 8 points, as shown below. 	To label the points of intersection, you can select **Draw** > **Basic Object** > **Point** and select the intersection points, until all points are labelled, as shown. 

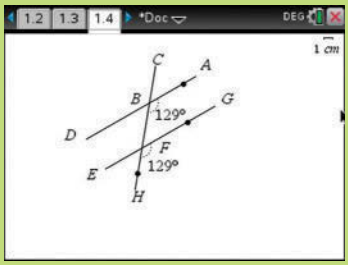
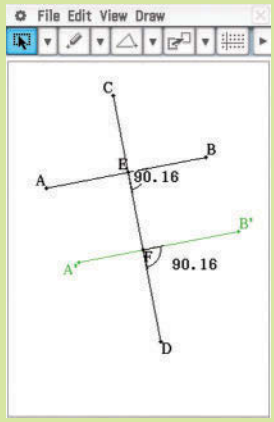
Alternate angles and parallel lines

Using TI-Nspire CAS	Using Casio ClassPad CAS
With the points labelled as shown, there are a pair of alternate angles $\angle ABF$ and $\angle BFE$. Alternate angles sit on opposite sides of a transversal but are both between the parallel lines. To measure each angle, select **menu** > **Measurement** > **Angle** and then select the three points that form one of the angles. The size of the angle will appear. Do this for all alternate angles, then select **esc** to stop measuring angles.  Alternate angles sit on opposite sides of a transversal but are both between the parallel lines.	With the points labelled as shown, there are a pair of alternate angles $\angle BEF$ and $\angle EFA'$. (Note that the vertex of the angle is the middle letter.) Alternate angles sit on opposite sides of a transversal but are both between the parallel lines. To measure each angle, use the cursor to select the line *AB* and the line *CD*, then select **Draw** > **Attached Angle**. The size of the angle will appear. If it is the wrong angle, then select it and drag it to move this measurement to the correct angle. Create a measurement for each of the alternate angles. Alternate angles sit on opposite sides of a transversal but are both between the parallel lines. 

You can select the points and move them around to see how the angles change.

1. You have just marked two pairs of alternate angles between a pair of parallel lines. What do you notice as you move the lines around?
2. Copy and complete the statement: 'Alternate angles between parallel lines are _____'.

Corresponding angles and parallel lines

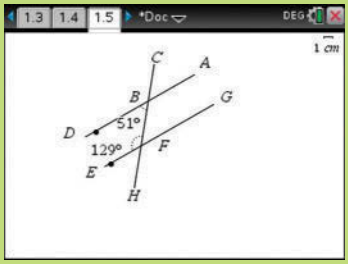
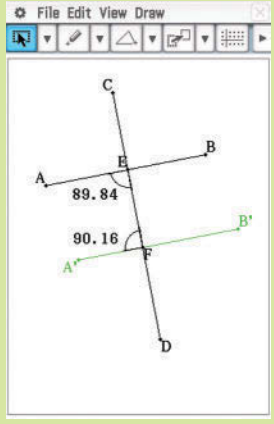
Using TI-Nspire CAS	Using Casio ClassPad CAS
Before measuring more angles, you should first delete the existing measurements. You can select each measurement and then select **ctrl** > **clear**.	Before measuring more angles, you should first delete the existing measurements. You can select each measurement and then select **Edit** > **Delete**.
With the points labelled as shown, there are a pair of corresponding angles $\angle ABF$ and $\angle GFH$. Corresponding angles sit in the same position in a group of four angles. Measure these angles in the same way as you measured the other angles before. 	With the points labelled as shown, there are a pair of corresponding angles $\angle BEF$ and $\angle B'FD$. Corresponding angles sit in the same position in a group of four angles. Measure these angles in the same way as you measured the other angles before. 

You can select the points and move them around to see how the angles change.

3 You have just marked two pairs of corresponding angles on a pair of parallel lines. What do you notice as you move the lines around?

4 Copy and complete the statement: 'Corresponding angles on parallel lines are _____'.

Co-interior angles and parallel lines

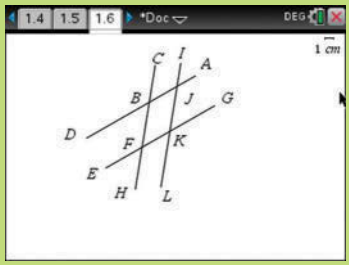
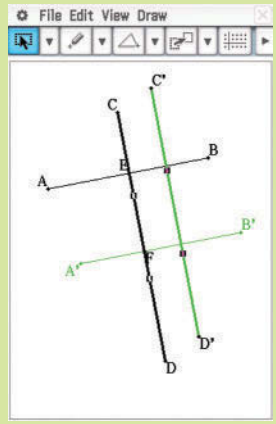
Using TI-Nspire CAS	Using Casio ClassPad CAS
With the points labelled as shown, there are a pair of co-interior angles $\angle DBF$ and $\angle BFE$. Co-interior angles (also called allied angles) sit on the same side of the transversal and between the parallel lines. Measure these angles in the same way as you measured the other angles before. (Before measuring more angles, you should first delete the existing measurements.) 	With the points labelled as shown, there are a pair of co-interior angles $\angle AEF$ and $\angle EFA'$. Co-interior angles (also called allied angles) sit on the same side of the transversal and between the parallel lines. Measure these angles in the same way as you measured the other angles before. (Before measuring more angles, you should first delete the existing measurements.) 

You can select the points and move them around to see how the angles change.

5 You have just marked two pairs of co-interior angles between a pair of parallel lines. What do you notice as you move the lines around?

6 Copy and complete the statement: 'Co-interior angles between parallel lines are _____'.

Taking it further

Using TI-Nspire CAS	Using Casio ClassPad CAS
First, delete the existing measurements. You should now create a new line segment that is parallel to the transversal. (The transversal is labelled *CH* below.) To create a parallel line, follow the same steps as at the start of this activity. Then create new text boxes to label the new points *I*, *J*, K, *L* as shown. 	First, delete the existing measurements. You should now create a new line segment that is parallel to the transversal. (The transversal is labelled *CD* shown right.) To create a parallel line, follow the same steps as at the start of this activity. 
Measure the angle $\angle BJK$, using the same method as before. Now find two angles that are alternate to this angle. Measure them to confirm your understanding of alternate angles. Make sure you use correct terminology to name the angles.	Measure the angle $\angle BEF$, using the same method as before. Now find two angles that are alternate to this angle. Measure them to confirm your understanding of alternate angles. Make sure you use correct terminology to name the angles.
Now find two angles that are corresponding to the angle $\angle BJK$, and measure them to confirm your understanding.	Now find two angles that are corresponding to the angle $\angle BEF$, and measure them to confirm your understanding.
Then find two angles that are co-interior to the angle $\angle BJK$, and measure them to confirm your understanding.	Then find two angles that are co-interior with the angle $\angle BEF$, and measure them to confirm your understanding.

What else can you say about these angles?

7 **(a)** Name as many pairs of vertically opposite angles as you can see in your diagram.

(b) Name any pairs of adjacent supplementary angles that you can see in your diagram.

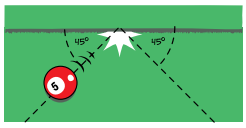
Investigation

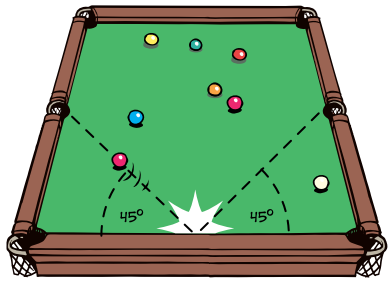
Billiard ball bounces

Equipment required: centimetre grid paper, protractor, ruler

The game of billiards is played on a large rectangular table that is covered in felt and surrounded by a cushion along the edges. Players use a long stick, called a cue, to hit balls into pockets around the edges of the table. Snooker and pool are variations of the game that have become extremely popular over time. The size of the table can vary from 12 feet (3.7 m) along the larger side to 9 feet (2.7 m), 8 feet (2.4 m) or 7 feet (2.1 m) in length.

When planning their shots, good billiards players use the fact that a ball will bounce off the side of a table at the same angle at which it hits, as shown here:





Here, the ball approaches the cushion at an angle of 45° and rebounds at an angle of 45°.

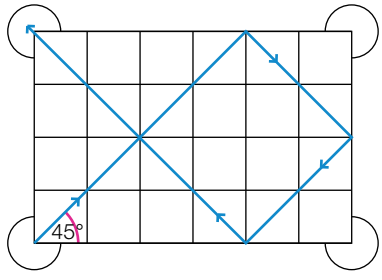
The ball may bounce off the sides several times before it either stops rolling or goes into a pocket. For this investigation, you should assume that the ball always keeps rolling until it falls into a pocket.

The Big Question

Can you predict how many times a billiard ball will bounce on tables of different sizes?

Can you predict which pocket the ball will fall into?

Engage



Here, the ball is hit from the bottom left corner at an angle of 45° and bounces off the sides of the table three times before falling into the opposite pocket along the short side.

1 **(a)** To see what would happen if you made the table bigger, use your grid paper to draw a similar 'table' to the one on the previous page, but make it 6 cm × 8 cm. Starting in the same corner (bottom left), trace the path of a ball hit at an angle of 45°, bouncing it off the sides until the path ends in a pocket. You should find the ball bounces five times and falls into the opposite pocket on the long side.

(b) What would happen if the table was square?

Explore

2 **(a)** Draw up tables of the following dimensions on your grid paper (you might like to share this task with a partner).

1×2, 1×4, 2×3, 2×4, 2×8, 3×4, 3×6, 3×8, 3×12, 4×5, 4×6, 4×7, 4×8, 5×6, 5×7, 5×8, 6×7, 6×8, 6×9, 7×9, 8×10, 9×12

(b) Starting in the same corner each time, trace the path of the ball hit at an angle of 45°. Count the number of bounces, and also note which pocket the ball falls into.

Strategy options

- Make a table.
- Look for a pattern.

Explain

3 **(a)** Collect all of the drawings you have made of the tables and their ball paths. To help you see any patterns or connections between them, group together the tables that have something in common. Some of the groups you could make are:

- tables where the pattern traced by the path of the ball is identical
- tables where the ball passes through every square on the grid.

(b) Once you have made these groups, compare the length, width and 'bounce' numbers for each table in the group. Are they connected in some way? Compare the starting and finishing pockets of the balls in each group. Is there a pattern here?

Elaborate

4 **(a)** Write a couple of sentences that answer the Big Question about predicting the number of bounces for tables of different sizes.

(b) Write a couple of sentences that answer the Big Question about predicting which pocket the ball will fall into.

Evaluate

5 **(a)** Consider how you worked on this task and the methods that you used. How did you organise or keep track of your results? Could you have done this better?

(b) Did grouping your results help you to spot patterns and connections between them? Which groups were useful?

Extend

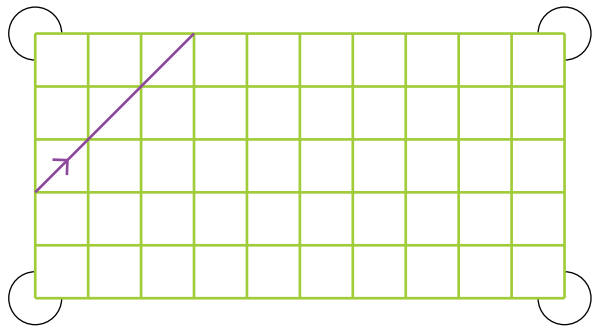
6 **(a)** Predict the number of bounces and which pocket the ball will fall into for the following table sizes.

(i) 18×27 **(ii)** 17×19

In which of these tables will the ball path pass through every square on the grid?

Draw these tables and check your prediction.

(b) Draw a 5×10 table on grid paper and trace the path of the ball hit from this position.



What do you notice? Try starting at different positions and comment on what you find.

(c) A billiard ball bouncing at the same angle at which it hits is an example of 'the law of reflection'. Investigate this law and other examples of where it can be seen.

Half-time 8

Equipment required: protractor for Questions **3, 5**

8.1

1 Draw the following angles by estimating their size. (Do not use a protractor.)

(a) 90° **(b)** 180° **(c)** 360°

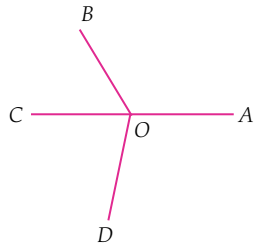
(d) 34° **(e)** 75° **(f)** 120°

8.2

2 **(a)** Name the two acute angles in the diagram opposite.

(b) Name the three obtuse angles in the diagram opposite.

(c) Give another name for $\angle AOB$.



8.1

3 Draw the following angles using a protractor.

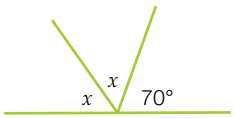
(a) 90° **(b)** 180° **(c)** 360°

(d) 34° **(e)** 75° **(f)** 120°

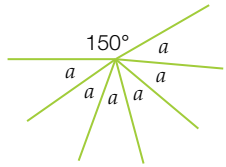
8.3

4 Find the value of the pronumerals in each case and give reasons for your answer.

(a)

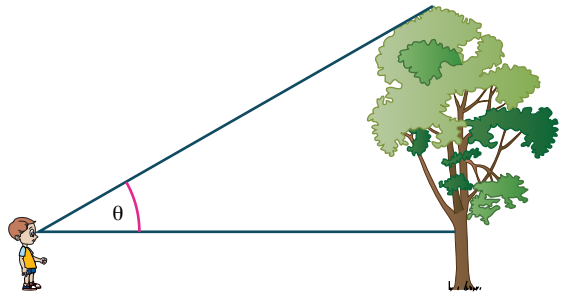


(b)



8.1

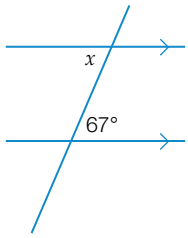
5 Measure the angle marked in the diagram opposite.



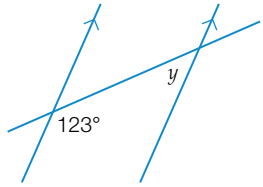
8.4

6 Find the value of the pronumerals in each case and give reasons for your answer.

(a)



(b)



8.3, 8.4

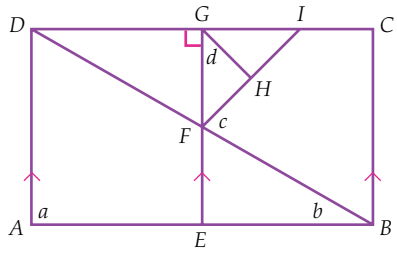
7 Using letter names such as $\angle ABC$:

(a) name an angle corresponding to a

(b) name an angle alternate to b

(c) name an angle supplementary to c

(d) name an angle complementary to d.



Polygons

8.5

You can draw many shapes on a flat surface called a plane. These are called two-dimensional or **plane shapes**. Plane shapes whose sides are all straight lines are called **polygons**.

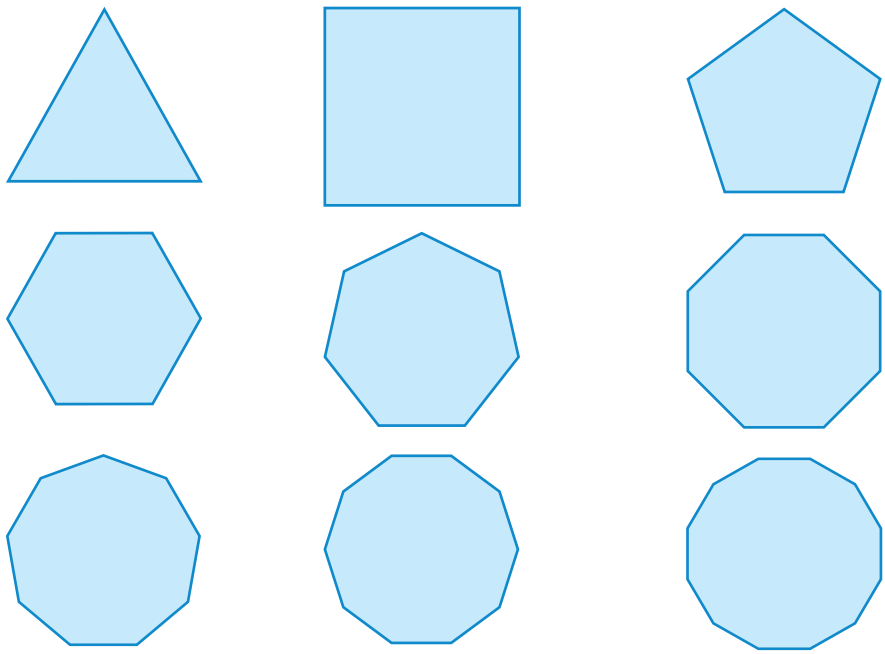
The word polygon is made up of two Greek words: *poly* (meaning many) and *gon* (meaning angle), so a polygon is a many-angled shape. As you can see in the following diagrams of polygons, the number of sides in a polygon is also equal to its number of angles.

The table opposite shows the names given to the first 10 polygons.

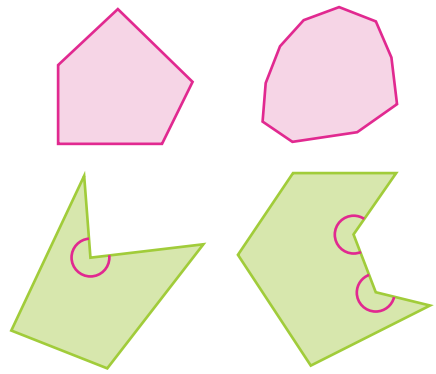
Number of sides	Polygon name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
11	Undecagon
12	Dodecagon

A **regular polygon** has all sides of equal length and all angles of equal size. The number of sides gives the name of the polygon.

You are probably familiar with the regular polygons shown below.



If the sides are *not* all equal, they are called **irregular** and the polygon is called an irregular polygon.



Concave and convex polygons

All of the polygons shown so far have been **convex** polygons, as they contain no interior angles greater than 180°. The shapes at right are examples of **concave** polygons, which contain at least one internal angle greater than 180°. (The sides 'cave in'.)

8.5 Polygons

Navigator

Answers p. 696

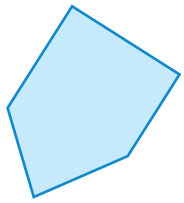
1 (columns 1–2), 2, 3, 4, 5, 6, 7, 8	1 (columns 2–3), 2, 3, 4, 5, 6, 7, 8, 9	1 (column 3), 3, 4, 5, 6, 7, 8, 9

Equipment required: ruler for Questions **7**, **9**

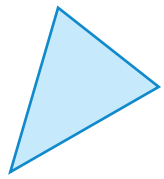
Fluency

1 Name each of the polygons below. State whether each one is concave or convex.

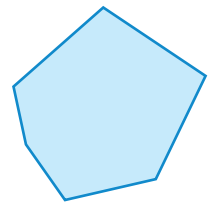
(a)



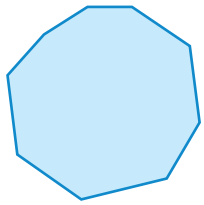
(b)



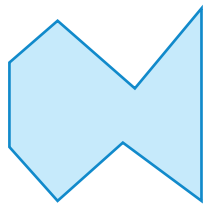
(c)



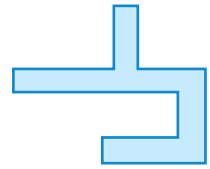
(d)



(e)



(f)



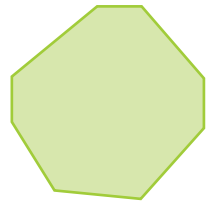
(g)

(h)

(i)

2 Choose the correct answer.

(a) The shape opposite is:



A a hexagon

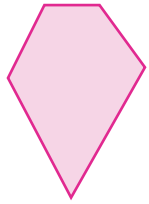
B a heptagon

C an octagon

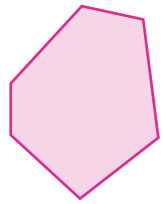
D a nonagon

(b) Which of the shapes below is a heptagon?

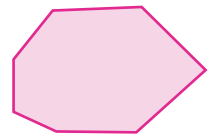
A



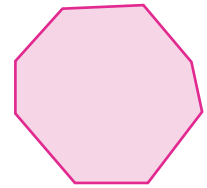
B



C



D



Understanding

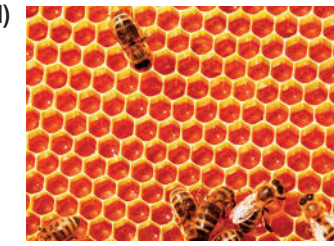
3 Name the type of polygon in each image.

(a)

(b)

(c)

(d)



Reasoning

4 What is the greatest number of right angles that a pentagon can have? Demonstrate your answer.

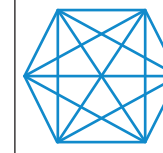
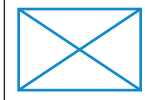
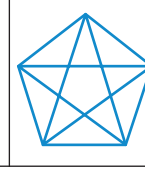
5 **(a)** If a quadrilateral can have two sets of parallel sides, how many sets of parallel sides can a hexagon have?

(b) Based on your results from part **(a)**, how many pairs of parallel sides can an octagon and a decagon have? Explain any pattern you have found.

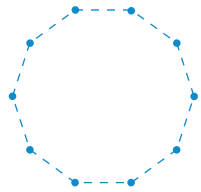
(c) Can this pattern be applied to a pentagon or to a heptagon? Explain why or why not.

6 A diagonal is a line drawn inside a polygon that connects any two vertices. The table below shows a series of polygons with diagonals included so that each vertex is directly connected to every other vertex.

(a) Can you find a pattern and predict the number of lines and diagonals contained by a decagon?

Name of shape	Shape	Number of sides and diagonals	Name of shape	Shape	Number of sides and diagonals
Triangle		3	Hexagon		15
Quadrilateral		6	Heptagon		
Pentagon		10	Octagon		

(b) Ten people meet at a party. Standing in a circle they shake hands with the person on either side of them. How many handshakes are needed so that each person shakes hands with everyone else once?



Open-ended

7 Using a ruler, draw any:

(a) concave quadrilateral **(b)** concave hexagon **(c)** concave octagon.

8 Name some common objects that contain regular polygons.

9 Using a ruler, draw the following shapes:

(a) a pentagon with two pairs of parallel sides

(b) a heptagon with three pairs of parallel sides.

Problem solving

The hex is gone

Equipment required: scissors

This is a regular hexagon.

On loose paper, draw at least six regular hexagons that are each about a quarter of the size of an A4 page. Cut out the hexagons and use them to help answer the following questions.

(a) How could a regular hexagon be cut into two pieces which, when put together, make a parallelogram?

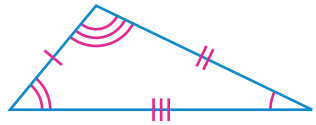
(b) How could a regular hexagon be cut into three pieces which, when put together, make a rhombus?

(c) How could a regular hexagon be cut into four pieces which, when put together, make two equilateral triangles?

Triangles

8.6

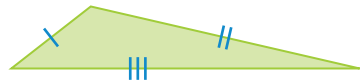
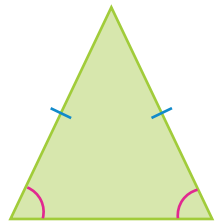
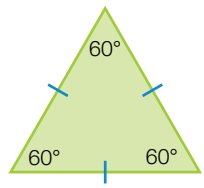
A triangle is the simplest polygon, with three straight sides and three angles.



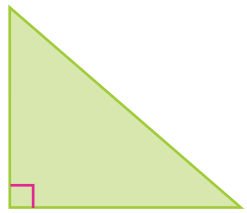
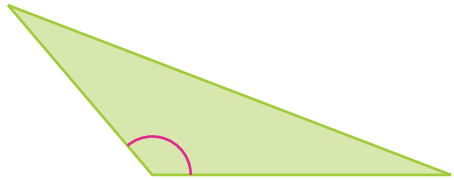
Classifying triangles

Triangles are classified according to their sides or angles.

When using the side lengths to classify a triangle, there are three main types:

Scalene triangle—no equal sides and no equal angles.	
Isosceles triangle—two equal sides and two equal base angles, which are opposite the two equal sides.	
Equilateral triangle—three equal sides and three equal angles of 60° each.	

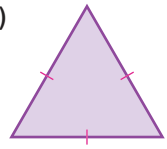
When using angles to classify a triangle, the three main types are:

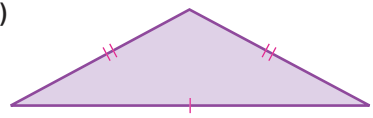
Acute-angled triangle—all three angles are acute (less than 90°).	
Right-angled triangle—one angle is a right angle (equal to 90°).	
Obtuse-angled triangle—one angle is obtuse (greater than 90° but less than 180°).	

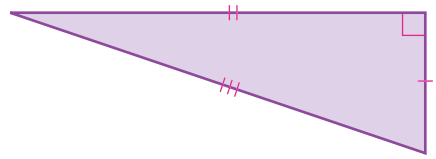
When the angles or lines of a shape are marked in a particular way, it means that all other angles or sides with that same marking are equal in size.

Worked example 6 W.E. 6

Give each of the following triangles **(i)** its side name **(ii)** its angle name and **(iii)** describe the triangle with an angle and a side name.

(a) 

(b) 

(c) 

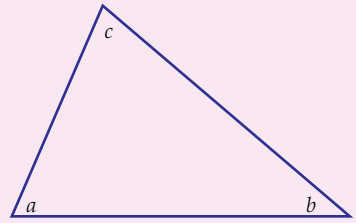
Thinking	Working
(a) (i) How many sides are the same length? (three)	(a) (i) equilateral triangle
(ii) What type of angle is the largest angle? (acute)	(ii) acute-angled triangle
(iii) Combine the side and angle name.	(iii) an acute-angled equilateral triangle
(b) (i) How many sides are the same length? (two equal sides)	(b) (i) isosceles triangle
(ii) What type of angle is the largest angle? (obtuse)	(ii) obtuse-angled triangle
(iii) Combine the side and angle name.	(iii) an obtuse-angled isosceles triangle
(c) (i) How many sides are the same length? (none)	(c) (i) scalene triangle
(ii) What type of angle is the largest angle? (one right angle)	(ii) right-angled triangle
(iii) Combine the side and angle name.	(iii) a right-angled scalene triangle

(*Note*: In **(a)**, it is unnecessary to describe an equilateral triangle with an angle name, as all angles must be 60° and therefore acute angles.)

Angle sum in a triangle

In any triangle, the three angles have a sum of 180°.

$a + b + c = 180°$

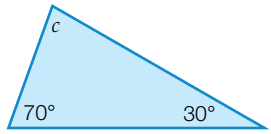


Worked example 7

W.E. 7

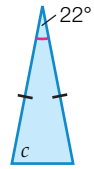
Find the size of the angle labelled c in each of the following triangles.

(a)



(b)

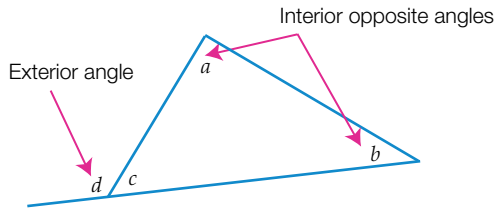
(c)



Thinking			Working
(a)	1	The three angles add to 180°. Write an equation that shows this.	(a) $c + 30 + 70 = 180$
	2	Simplify the equation by adding the given angles.	$c + 100 = 180$
	3	Solve the equation (subtract 100 from both sides).	$c = 80°$
(b)	1	The three angles add to 180°. Write an equation that shows this. (Remember that the small square in the triangle indicates a 90° angle.)	(b) $c + 90 + 16 = 180$
	2	Simplify the equation by adding the given angles.	$c + 106 = 180$
	3	Solve the equation (subtract 106 from both sides).	$c = 74°$
(c)	1	The three angles add to 180°. Write an equation that shows this. (Because this is an isosceles triangle, the two angles opposite the equal sides are equal.)	(c) $c + c + 22 = 180$
	2	Simplify the equation by adding like terms.	$2c + 22 = 180$
	3	Solve the equation (subtract 22 from both sides, then divide by 2).	$2c = 158$ $c = 79°$

Exterior angle of a triangle

An exterior angle of a triangle is formed on the outside of a triangle when one of the sides is extended.



In the diagram, $\angle d$ is the exterior angle.

There can be six different exterior angles formed on any triangle. Can you find them all? What relationship does the exterior angle have to the interior angles of a triangle?

Note that $\angle a + \angle b + \angle c = 180°$ (angles of a triangle add to 180°)

$\angle a + \angle b + \angle c - \angle c = 180° - \angle c$ (subtracting $\angle c$ from both sides)

$\angle a + \angle b = 180° - \angle c$

$\angle c + \angle d = 180°$ (angles on a straight line add to 180°)

$\angle c - \angle c + \angle d = 180° - \angle c$ (subtracting $\angle c$ from both sides)

$\angle d = 180° - \angle c$

Therefore, $\angle a + \angle b = \angle d$ (both equal to $180° - \angle c$)

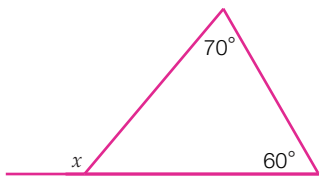
The exterior angle of a triangle is equal to the sum of the two interior opposite angles in the triangle.

Worked example 8

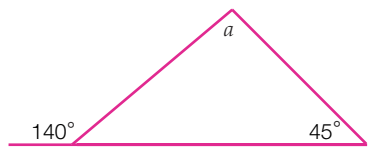
W.E. 8

Calculate the value of the pronumeral in each of the following.

(a)



(b)



Thinking	Working
(a) As the exterior angle is equal to the sum of the two interior opposite angles, substitute the known angles into the rule and calculate the exterior angle.	(a) $x = 60 + 70$ $x = 130°$
(b) As the exterior angle is equal to the sum of the two interior opposite angles, substitute the known angles into the rule and calculate the unknown angle.	(b) $140 = a + 45$ $a = 140 - 45$ $a = 95°$

8.6 Triangles

Navigator

1 (columns 1–2), 2, 3 (columns 1–2), 4 (columns 1–2), 5 (columns 1–2), 6, 7, 8, 9, 10, 12, 13, 15, 16 (a–b), 18, 19, 20	1 (columns 2–3), 2, 3 (columns 2–3), 4 (columns 2–3), 5 (column 2), 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20	1 (column 3), 2, 3 (column 3), 4 (column 3), 5 (column 3), 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20

Answers p. 696

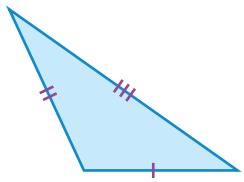
Equipment required: protractor and ruler for Questions **5**, **18**, **19**, **20**

Fluency

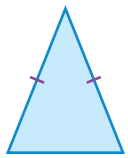
1 Give each of the following triangles **(i)** its side name **(ii)** its angle name and **(iii)** describe the triangle with an angle and a side name.

W.E. 6

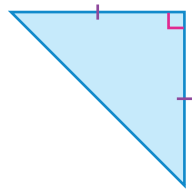
(a)



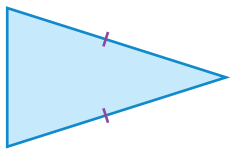
(b)



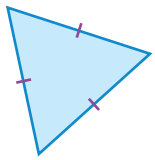
(c)



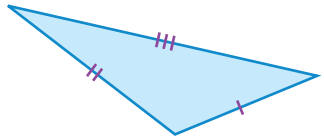
(d)



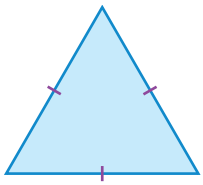
(e)



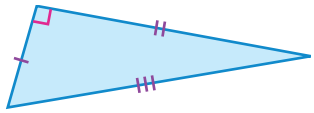
(f)



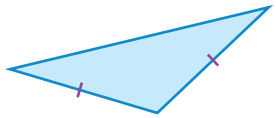
(g)



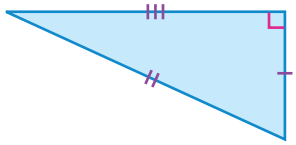
(h)



(i)

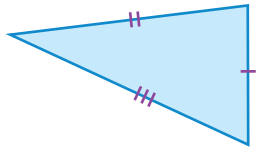


(j)



(k)

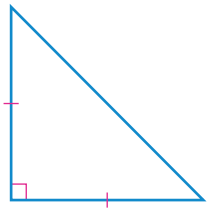
(l)



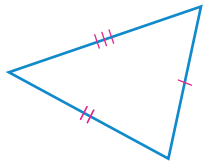
2 Select the correct statements from the list below for each of the following triangles.

A It is a polygon.
B It is a regular polygon.
C It is an irregular polygon.
D It is an isosceles triangle.
E It is a right-angled triangle.
F It is a scalene triangle.
G It has an obtuse angle.

(a)



(b)

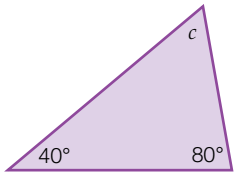


(c)

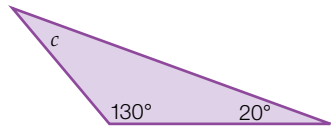
W.E. 7

3 Find the size of the angle labelled c in each of the following triangles.

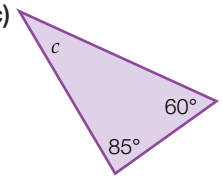
(a)



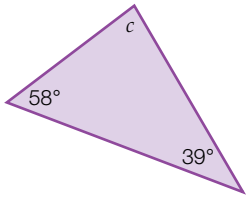
(b)



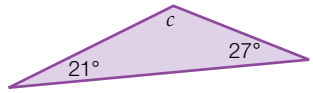
(c)



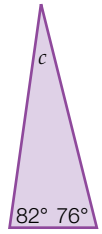
(d)



(e)

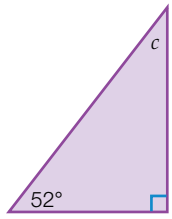


(f)

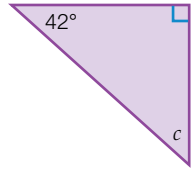


(g)

(h)



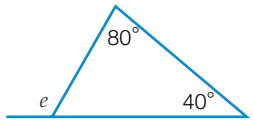
(i)



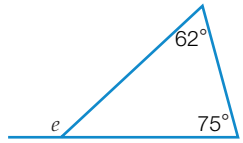
W.E. 8

4 Calculate the value of the pronumeral in each of the following.

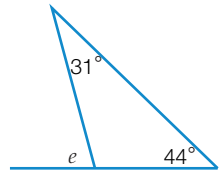
(a) **(i)**



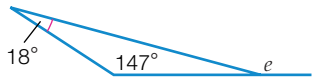
(ii)



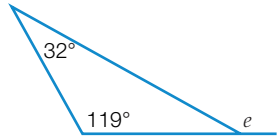
(iii)



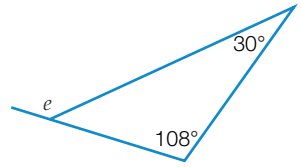
(iv)



(v)



(vi)



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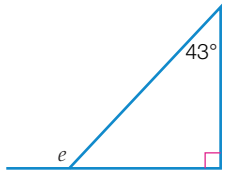
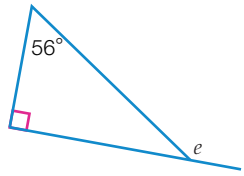
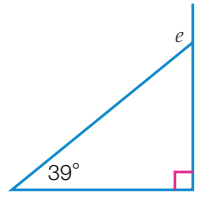


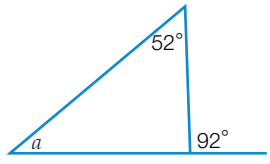
(viii)



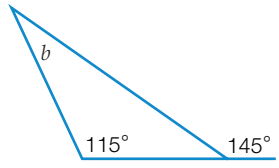
(ix)



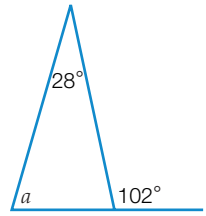
(b) **(i)**



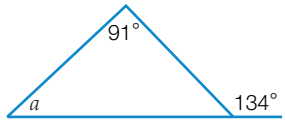
(ii)



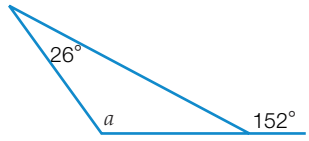
(iii)



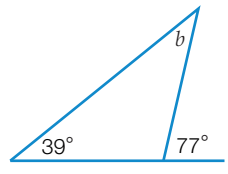
(iv)



(v)



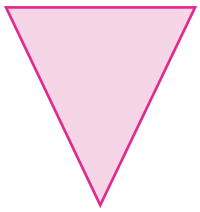
(vi)



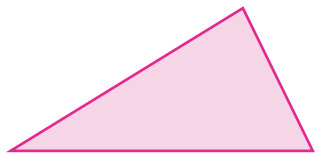
Understanding

5 Use a ruler to measure the side lengths of the following triangles, and hence give a side name for each one. Give the angle names as well, using a protractor if necessary to check the angle types.

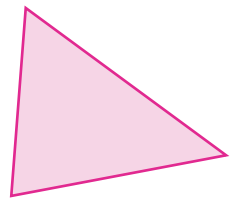
(a)



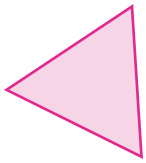
(b)



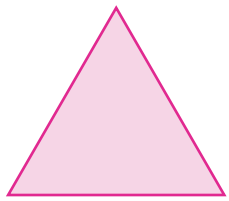
(c)



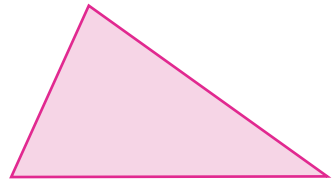
(d)



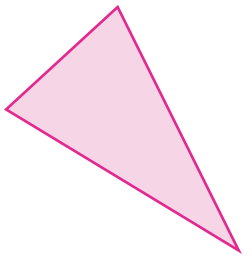
(e)



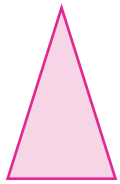
(f)



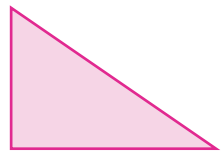
(g)



(h)



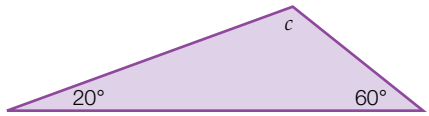
(i)



6 (a) The size of angle c is:

A 20° **B** 60°

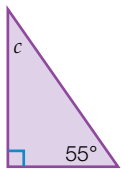
C 80° **D** 100°



(b) The size of angle c is:

A 35° **B** 45°

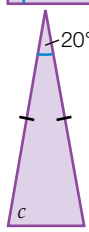
C 55° **D** 125°



(c) The size of angle c is:

A 20° **B** 80°

C 90° **D** 160°



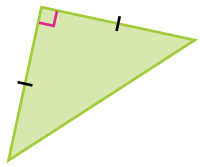
7 The triangle shown is best described as:

A isosceles, acute-angled

B isosceles, right-angled

C scalene, acute-angled

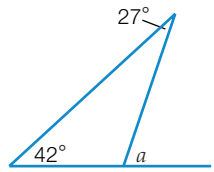
D scalene, right-angled



8 **(a)** The value of the pronumeral is:

A 15° **B** 63°

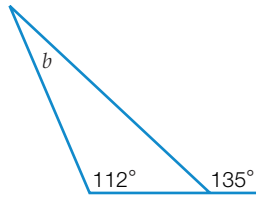
C 69° **D** 111°



(b) The value of the pronumeral is:

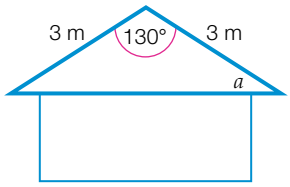
A 23° **B** 36°

C 42° **D** 68°

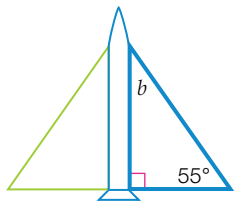


9 Name the type of triangle shown outlined in each diagram below. (Side or angle names may be used.) Find the value of any unknown angles marked with a pronumeral.

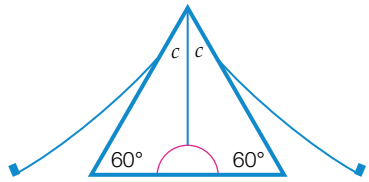
(a)



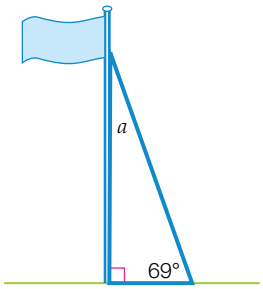
(b)



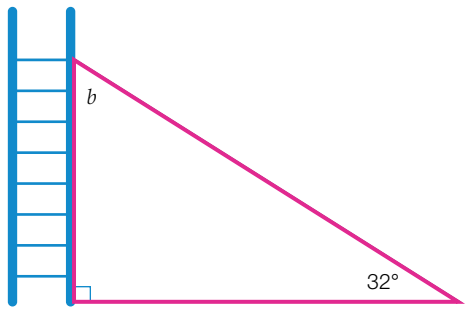
(c)



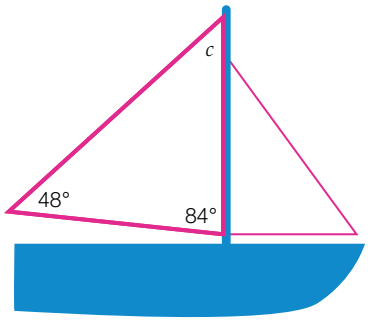
(d)



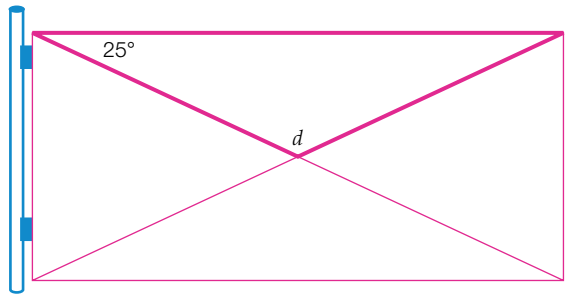
(e)



(f)



(g)



10 The diagonal of a square cuts the square into two triangles. Give the side and angle name of the triangles formed.

11 Two equilateral triangles are placed next to each other so that two sides are joined along their whole length to form a quadrilateral.

(a) What can you say about the side lengths of the quadrilateral (four-sided shape) formed?

(b) What will be the size of each of the angles in the quadrilateral?

Reasoning

12 Is it possible to have a triangle with more than one obtuse angle? Explain by drawing a diagram.

13 What is the minimum number of acute angles a triangle must have?

14 Give the side and angle names of each of the following triangles, with reasons why.

(a) a triangle with one angle measuring 36° and another angle measuring 54°

(b) a triangle with one angle measuring 28° and another angle measuring 124°

(c) a triangle with two angles measuring 60° each

(d) a triangle with one angle measuring 12° and another angle measuring 12°

15 Explain why the following triangles are not possible.

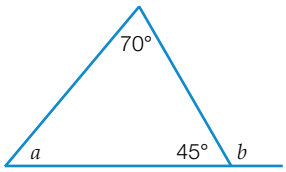
(a) a triangle with angles 55°, 65° and 70°

(b) an equilateral triangle with three 55° angles

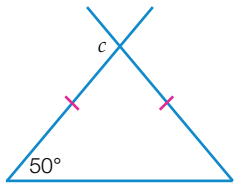
(c) isosceles triangle with 45°, 55° and 80° angles

16 Find the unknown angle or angles (represented by the pronumeral) in each of the following diagrams. You may need to use your knowledge of angle properties.

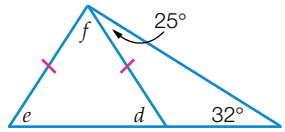
(a)



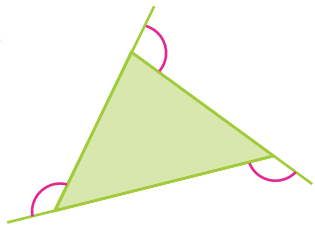
(b)



(c)



17 Find the sum of the marked angles in the diagram on the right without measuring the angles.



Open-ended

18 Draw three different obtuse-angled scalene triangles.

19 Draw an example of:

(a) an acute-angled scalene triangle

(b) an acute-angled isosceles triangle.

20 Draw three different triangles with an exterior angle between 80° and 100°.

Puzzle

Eggs, spoons and turtles

1 Jia, Sabah, Julian, Bianca and Marwen all took part in their school egg-and-spoon race. Find the order in which they finished the race.

- Bianca finished the race in between Jia and Julian.
- The winner has five letters in their name.
- Jia finished faster than Marwen and slower than Julian.

2 Four students and their turtles took part in the annual School Turtle Championships. Match the students to their turtles, and find the order in which they finished.

- Chloe's turtle is Sandy, who didn't finish in last place.
- Percy finished immediately in front of Vicky's turtle.
- Myrtle finished behind Ernest, who is Vicky's turtle.
- Ethan was celebrating as his turtle finished two places higher than Warren's.

Investigation

Angle sum of polygons

Equipment required: paper, ruler, protractor

The Big Question

Is there a relationship between the number of sides of a polygon and the angle sum of that polygon?

Engage

1 **(a)** On a blank sheet of paper, use a ruler to draw a triangle of reasonable size (about a quarter of a page).

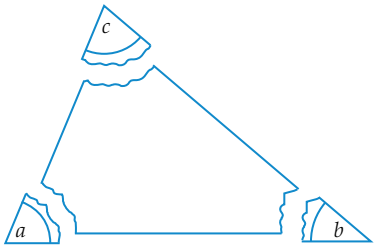
(b) Use a protractor to measure each angle.

(c) Add these angles together. Did you get 180°?

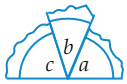
(d) Repeat this with a second triangle.

2 **(a)** Draw a third triangle on a separate piece of paper and cut it out carefully.

(b) Tear off the angles as shown.



(c) Place all the vertices together with arms touching. (Did they form a straight line?)



(d) Paste these angles together on your page.

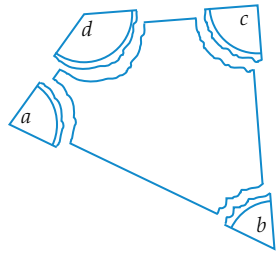
3 **(a)** On your first sheet of paper, use a ruler to draw a quadrilateral.

(b) Measure each angle.

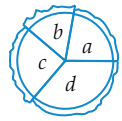
(c) Add these together. Did you get 360°?

(d) Draw another quadrilateral and cut it out carefully.

(e) Tear off the angles as shown.



(f) Place all the vertices together with arms touching. (Did they form a revolution?)



(g) Paste these angles together on your first page.

(h) Draw a second quadrilateral and divide it into two triangles.

(i) Calculate the angle sum of the quadrilateral.

Explore

4 On a blank sheet of paper, use a ruler to draw a pentagon, a hexagon, a heptagon and an octagon. Make sure that each polygon is a reasonable size (about a quarter of a page).

5 Use your protractor to measure the angles of each shape.

6 Find the angle sum for each shape. Did you get a multiple of 180° for each shape?

7 Use straight lines from vertex to vertex to divide each shape into as many triangles as you can, but do not allow any of the lines to cross. Count the number of triangles in each shape.

8 Multiply the number of triangles by 180° for each polygon.

9 Copy the following table and enter your results.

Polygon	Number of angles	Number of triangles	Angle sum (°)	180° × number of triangles (°)
Triangle				
Quadrilateral				
Pentagon				
Hexagon				
Heptagon				
Octagon				

Strategy options

- Make a table.
- Make a model.
- Look for a pattern.

Explain

10 Explain how you found a relationship between the number of angles (column 1) and the number of triangles (column 2) for each shape.

11 Explain how you found a relationship between the number of angles (or sides) of a polygon and the angle sum of that polygon.

Elaborate

12 Answer the Big Question by writing the rule you have found, describing carefully any of the variables you have used.

13 Use your rule to find the angle sum of a nonagon, a decagon and a dodecagon.

14 Draw each of these and measure each angle.

15 Find the sum of the angles for each shape and compare your answers with your answers to **13**.

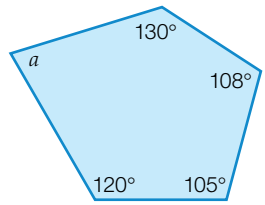
Evaluate

16 Did you get exactly the same answer when you measured your angles and added them as you did when you multiplied the number of triangles by 180°. Why could they be different? Which is more accurate?

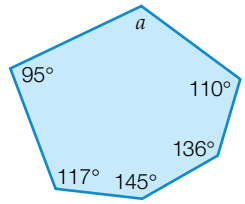
Extend

17 Calculate the size of the angle a in each of the following polygons.

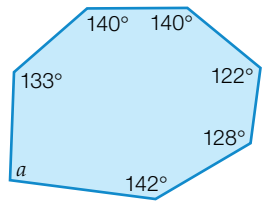
(a)



(b)



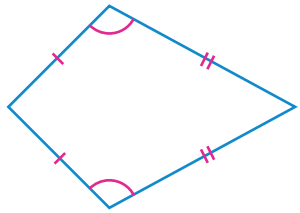
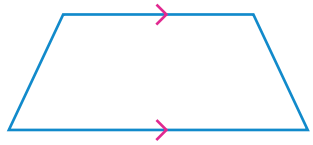
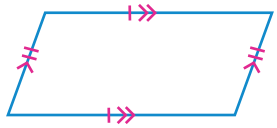
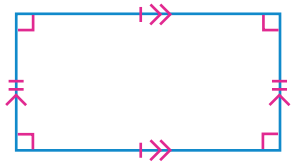
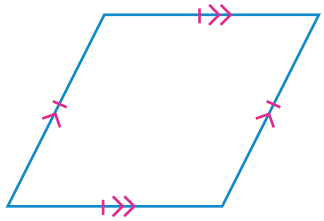
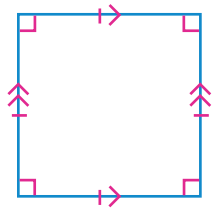
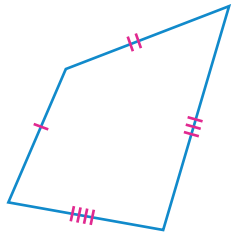
(c)



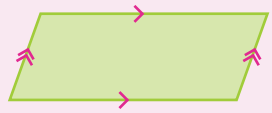
8.7 Quadrilaterals

A plane shape with four straight sides is called a **quadrilateral**. You can name quadrilaterals by looking at the properties of their sides and angles.

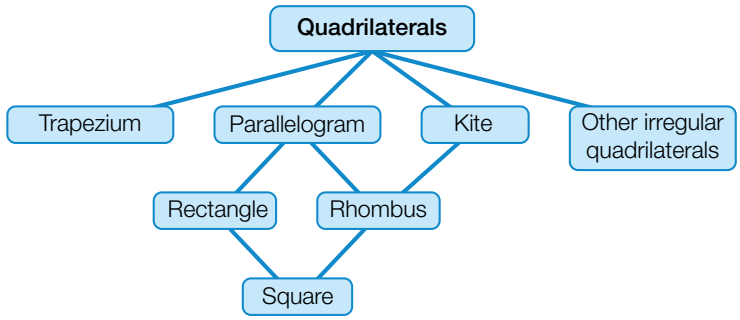
Quadrilaterals

Quadrilateral	Description
Kite 	Two pairs of equal adjacent sides (sides next to each other) and one pair of equal opposite angles.
Trapezium 	Only one pair of parallel sides.
Parallelogram 	Two pairs of parallel sides. Opposite sides and angles are equal.
Rectangle 	Two pairs of opposite sides equal and all internal angles are right angles (equal to 90°).
Rhombus 	All four sides equal; opposite angles are equal. (This can be thought of as a parallelogram with all sides equal or a 'pushed over' square.)
Square 	All sides equal and all internal angles are right angles (equal to 90°).
Irregular quadrilateral 	A four-sided plane shape that has no special qualities.

Parallel side pairs are indicated by marking the side with the same number of arrows in the same direction, as shown.



A quadrilateral may have more than one correct name. For example, a square may also be called a rectangle, because it has all the angle and side properties of a rectangle (two pairs of parallel sides and 90° angles). It could also be called a rhombus, because its pairs of opposite sides are equal and its opposite angles are equal.



Angles in a quadrilateral

The sum of the angles in a quadrilateral is 360°.

Angle properties of special quadrilaterals

The diagonally opposite angles of a parallelogram are equal.

The two interior opposite angles of a kite where unequal sides meet are equal.

8.7 Quadrilaterals

Navigator

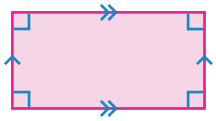
1, 2, 3, 4 (columns 1–2), 5, 6 (columns 1–2), 7 (a–b), 8, 9, 11, 12, 13, 15, 16, 17, 18, 19	1, 2, 3, 4 (columns 2–3), 5, 6 (columns 2–3), 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20	1, 2, 3, 4 (column 3), 6 (column 3), 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20

Answers p. 697

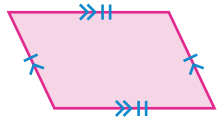
Fluency

1 Give the most accurate name for each shape below.

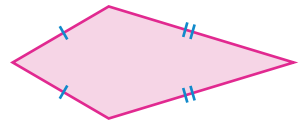
(a)



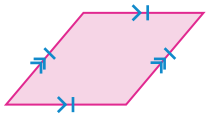
(b)



(c)

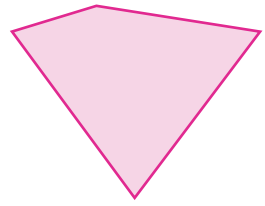


(d)



(e)

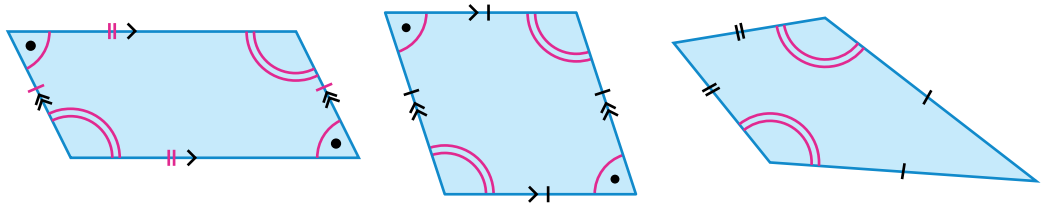
(f)



2 A quadrilateral has both pairs of opposite sides parallel and equal in length. You can be sure this quadrilateral is a:

A parallelogram **B** rhombus **C** rectangle **D** square

3 State true (T) or false (F) for each of the following, referring to the three quadrilaterals below.



(a) The opposite angles in a parallelogram are equal.

(b) All four angles in a parallelogram are equal.

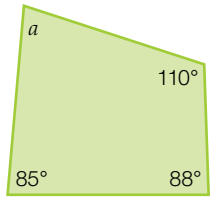
(c) The opposite angles in a rhombus are equal.

(d) All four angles in a rhombus are equal.

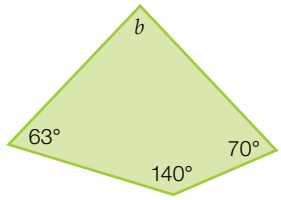
(e) One pair of opposite angles in a kite are equal.

4 Find the size of the unknown angle represented by a letter in each of the following quadrilaterals.

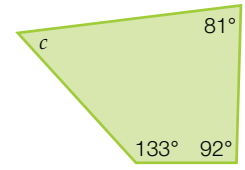
(a)



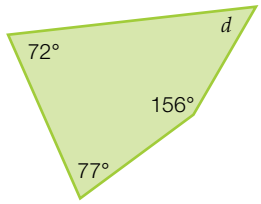
(b)



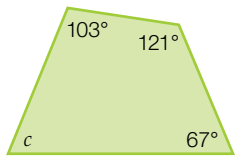
(c)



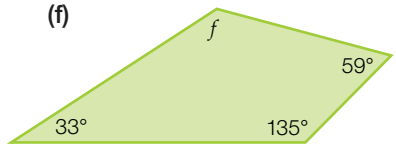
(d)



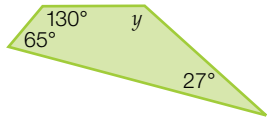
(e)



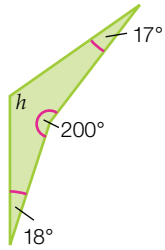
(f)



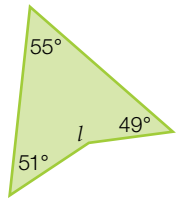
(g)



(h)



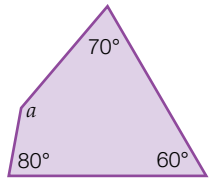
(i)



5 (a) The size of angle a shown is:

A 70° **B** 90°

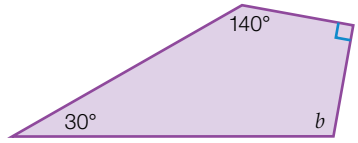
C 100° **D** 150°



(b) The size of angle b shown is:

A 10° **B** 90°

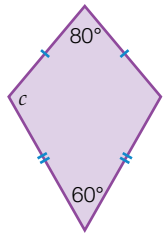
C 100° **D** 170°



(c) The size of angle c shown is:

A 55° **B** 80°

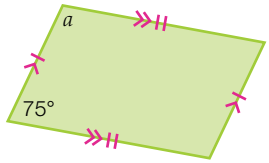
C 110° **D** 140°



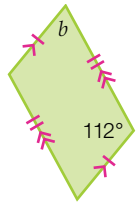
6 Find the size of the angle labelled with a letter in each quadrilateral.

(a)

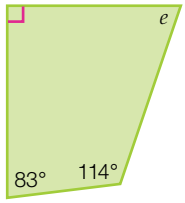
(b)



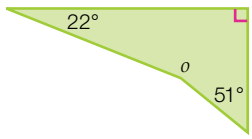
(c)



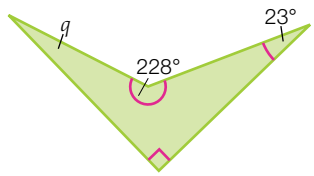
(d)



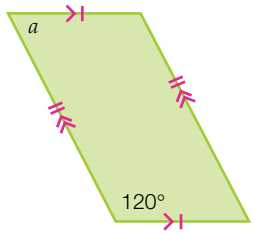
(e)



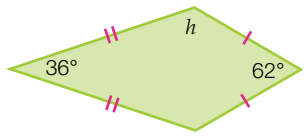
(f)



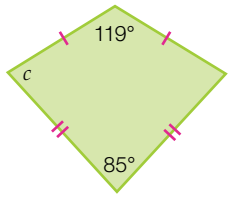
(g)



(h)



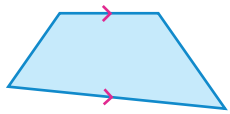
(i)



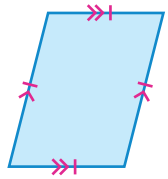
Understanding

7 The diagrams below were created by a trainee illustrator. What type of shape do you think the illustrator meant to draw in each case? Draw a more accurate version of each shape, given that the markings on the lines are correct.

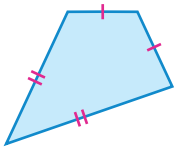
(a)



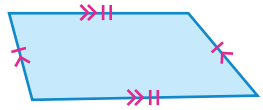
(b)



(c)



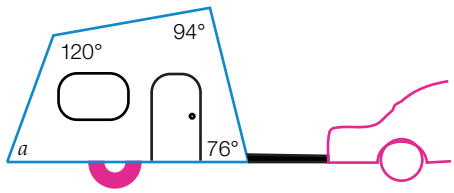
(d)



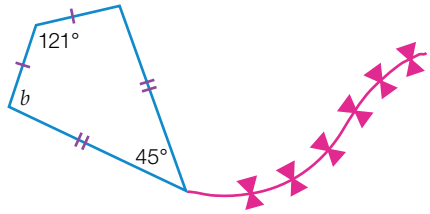
For Question **7**, make sure you do the markings for things such as equal sides, parallel sides and right angles.

8 Find the angle represented by the pronumeral in each quadrilateral below. Give reasons to justify your answer.

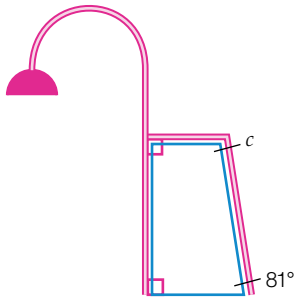
(a)



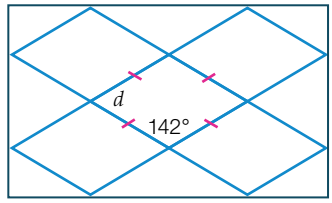
(b)



(c)



(d)



9 An angle in a parallelogram measures 40°. Draw a diagram and mark in the size of each of the other three angles.

10 A quadrilateral has one 45° angle and three unknown angles that are equal to each other. What is the size of each of the three unknown angles?

Reasoning

11 All rhombuses are parallelograms, but not all parallelograms are rhombuses.

(a) What makes the rhombus a special type of parallelogram?

(b) Why is a trapezium not a parallelogram?

12 All rectangles are parallelograms, but not all parallelograms are rectangles. Is this true? Explain why or why not.

13 What is the maximum number of:

(a) obtuse angles a quadrilateral can have? Confirm by drawing a diagram.

(b) reflex angles a quadrilateral can have? Confirm by drawing a diagram.

14 A quadrilateral has two angles of 70° and 34°. The other two angles are equal to each other.

(a) What is the size of the two unknown angles?

(b) What type of quadrilateral could this shape be?

15 Can a quadrilateral have the angles 35°, 85°, 130° and 120°? Explain your answer.

16 What properties of a square make it also classifiable as:

(a) a rectangle **(b)** a rhombus **(c)** a parallelogram?

17 Sam is looking at a kite, which has two given angles of 50° and 120°, but it is unclear which angle is which and where they are located on the kite. Sam thinks that the best way to solve a problem is to draw a diagram. Help Sam by drawing three different kites that could contain the two known angles of 50° and 120°.

Open-ended

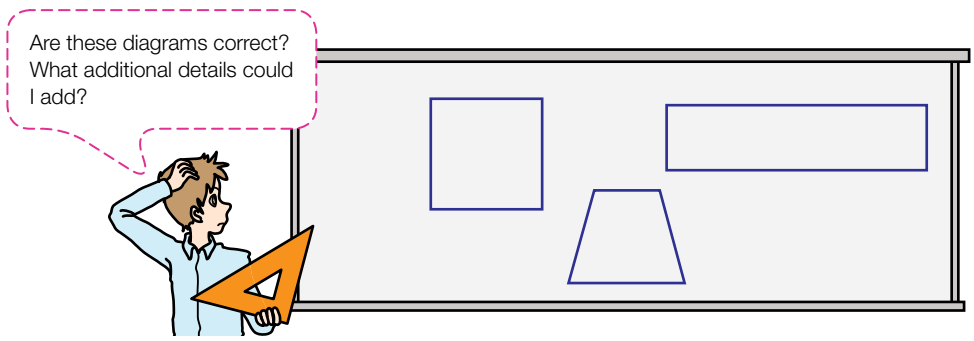
18 A quadrilateral has two angles equal to 80°.

(a) Name one type of quadrilateral that this could be and draw it, marking in the 80° angles.

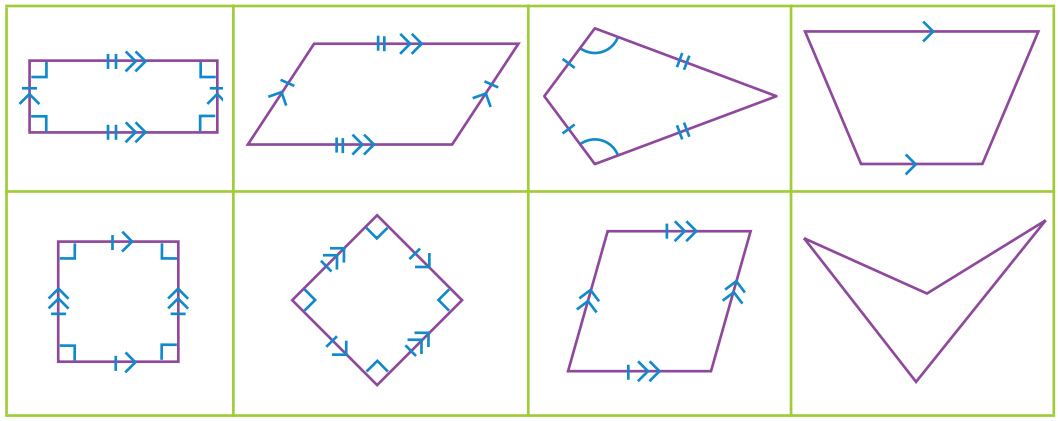
(b) How many different types of quadrilaterals can have two angles equal to 80°? List the different types.

(c) Draw an example of each, marking in the 80° angles.

19 Jack is asked by his teacher to draw a square, a rectangle and a trapezium. Do Jack's diagrams include enough information to clearly depict a square, a rectangle and a trapezium? Redraw his diagrams, adding any missing details.



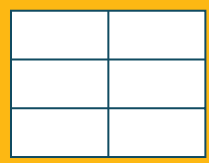
20 Divide these quadrilaterals into two groups. Explain the reason(s) you used to organise them into each group.



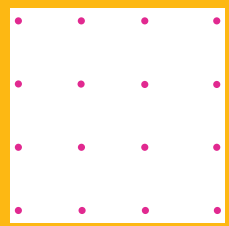
Problem solving

Quadrilateral quandaries

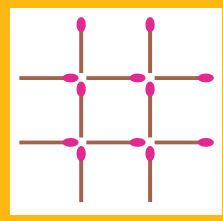
1 How many rectangles can you count here altogether? (Include rectangles that are the same size even if they are drawn in different positions.)



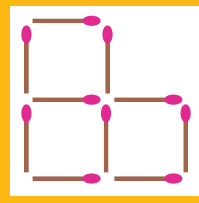
2 Find how many different size squares can be drawn on the grid shown.



3 Move three matchsticks in the arrangement shown to create three squares that touch each other.



4 Take away one match and move three others to form one square and two parallelograms.



Strategy options

- Draw a diagram.
- Look for a pattern.

PLANET POLYGON

While exploring the universe, you and your co-pilot Violet enter the atmosphere of the strange planet Polygon. Your ship malfunctions, and to avoid certain death you both eject out of the ship. On landing you are knocked unconscious. When you wake, you look around—where is Violet?

You find a note on your PCD (Personal Communication Device)…

Message #1: 2:15 pm Alien code

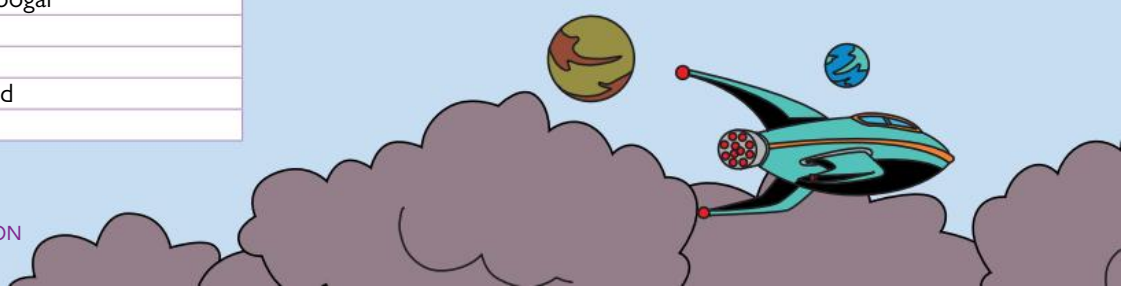
I couldn't find you when I woke. I've met a group of aliens who I can only call Polygons. They don't have a verbal language, but communicate instead with shapes. Through our conversation I think I have managed to decode some of their language.

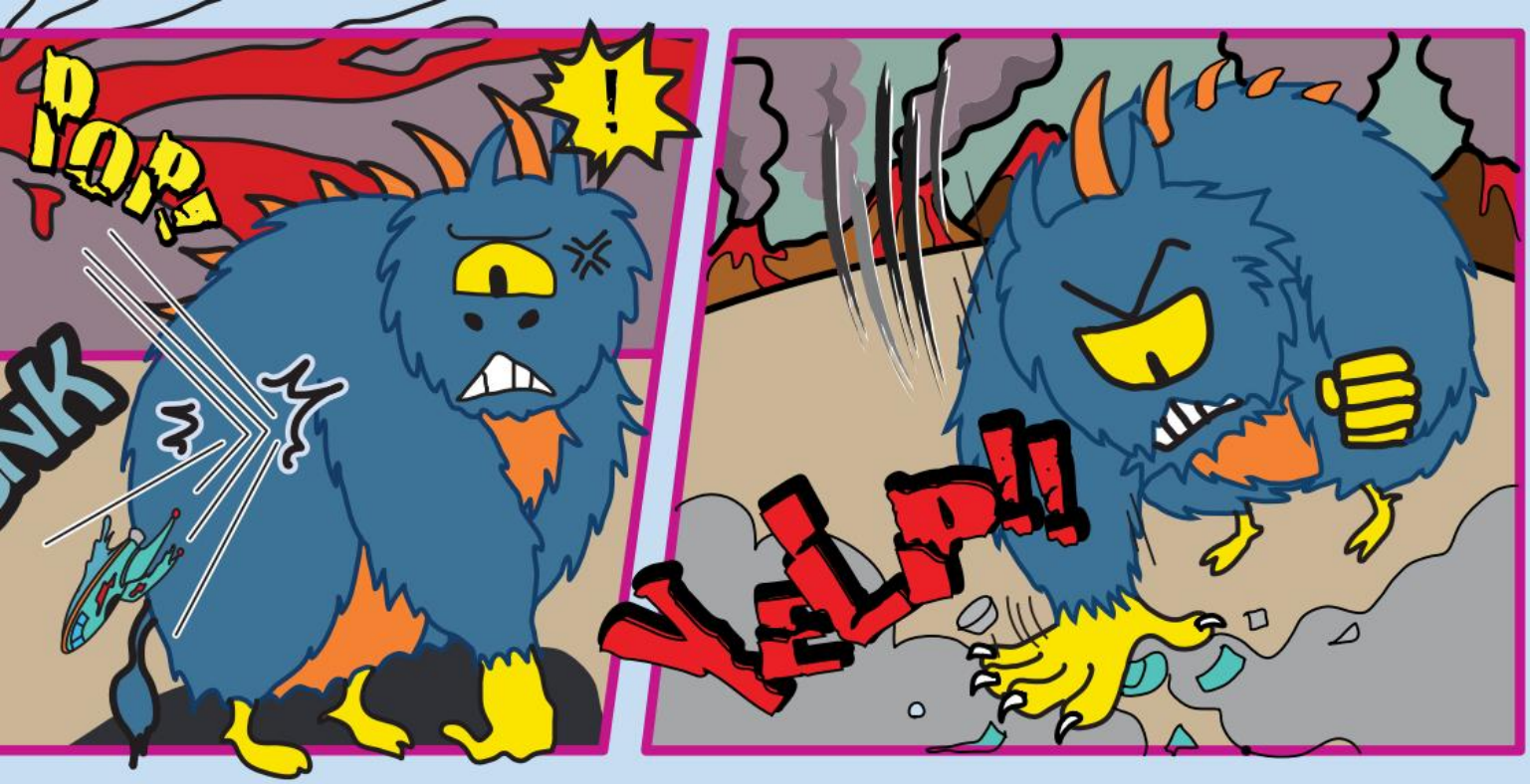
Shape	Possible meaning
square	catapulted
kite	flying thing
isosceles triangle	volcano
rhombus	large moogal
rectangle	paw
trapezium	into
polygon	squashed
scalene triangle	hit

I asked them what happened to our ship, they answered, and I have described the shapes. Here is approximately what they said.

- quadrilateral with 2 pairs of equal sides, 1 pair of equal angles only
- quadrilateral with 1 pair of parallel sides only
- 3-sided shape with 2 equal angles
- special rectangle with equal side lengths
- 3-sided shape with 3 different side lengths
- parallelogram that has all equal sides, but is not a square
- 2 pairs of parallel sides, with 90° angles (but is not a square)
- hexagonal-looking shape
- quadrilateral with 2 pairs of equal sides, 1 pair of equal angles only

1 Decode what the aliens said happened to the ship.

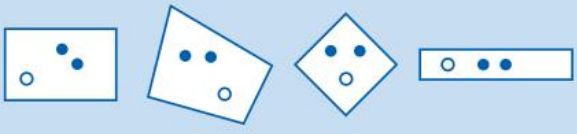




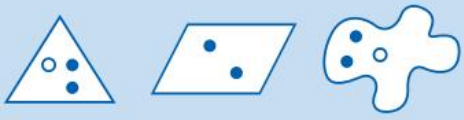
Message #2: 3:15 pm
Alien intelligence test

The Polygons are willing to give us the materials to fix our spaceship, but we both have to pass their test. Good luck!

2 These are Kwoogals:

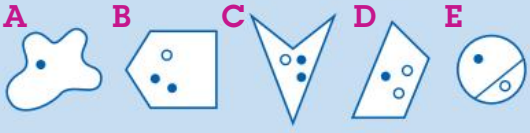


These are not Kwoogals:



(a) Which one of these is a Kwoogal?

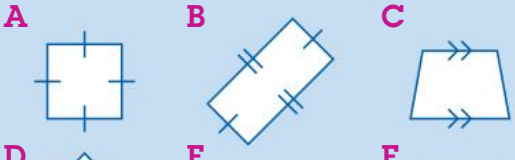
A B C D E



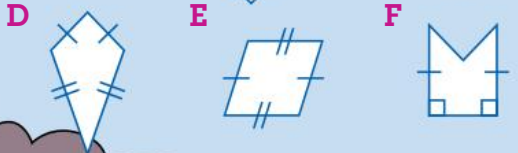
(b) Write the definition of a Kwoogal.

3 Which of these are parallelograms? (More than one answer is okay.)

A B C



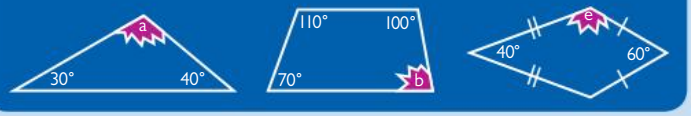
D E F



Message #3: 4:18 pm
Spaceship blueprints

I've found the plans to fix our spaceship but some of the angle values have been burnt in the crash. Can you calculate what the missing angles should be?

Fuel cells



Neutron absorber

Black hole combustion unit

What is a moogal?

Use your answers for the spaceship blueprint message to solve the code below.

110°		70°	100°	110°	25°	25°	155°		
80°	130°	110°	70°	335°					

8.8 Compass constructions

Using a ruler, a pencil and a compass, you can construct many angles and shapes without measuring any angles with a protractor. Constructions can also be made using CAS technology or computer geometry software. This can be useful for investigating geometrical ideas and mathematical proofs.

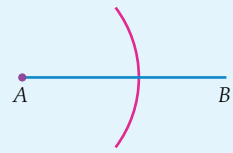
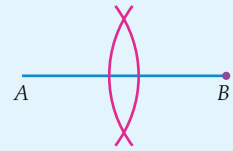
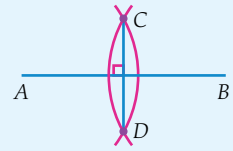
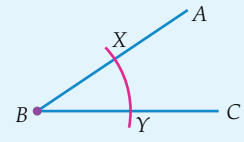
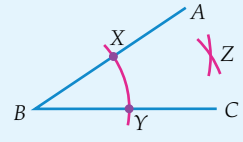
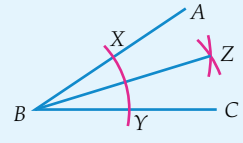
Most constructions are based on two basic methods: bisecting a line and bisecting an angle. (Bisect means to divide into halves.) A **perpendicular bisector** cuts through the midpoint of a line at right angles. You should use a sharp pencil for drawing lines and for use in our compass, and leave all construction lines on the diagrams.

Unless otherwise instructed, lines and compass openings should be between 5 cm and 8 cm long and angles between 40° and 50°.

Worked example 9 W.E. 9

Using only a compass, a ruler and a pencil:

(a) bisect a line **(b)** bisect an angle.

Thinking	Working
(a) 1 Use your ruler to draw a line $\overline{AB}$. Place the point of the compass on point A so that the compass opening is more than half the length of the line and draw an arc above and below the line.	(a) 
2 Keeping the compass at the same opening, place your compass point on point B and draw an arc to cut the first arc above and below the line.	
3 With your ruler join the points CD where the arcs cross. As this line crosses AB at right angles, it is the perpendicular bisector of the line.	
(b) 1 Draw an acute angle. Place the point of the compass on the vertex of $\angle ABC$ and draw an arc to cut both arms of your angle at X and Y.	(b) 
2 Keeping the compass at the same opening, place your compass point on point X and draw an arc. Repeat this using point Y so that the arcs cross.	
3 Using a ruler, draw a line connecting the vertex of $\angle ABC$ with Z, the **point of intersection** of the arcs. This line BZ bisects the angle (cuts the angle in half).	

An equilateral triangle not only has three equal sides but also has three equal angles of 60°. A square has four equal sides and all angles are right angles, so you can use our previous constructions to help us construct a square.

Worked example 10 W.E. 10

Using only a compass, a ruler and a pencil:

(a) construct an equilateral triangle **(b)** construct a square.

Thinking

(a) 1 Use your ruler to draw a line $\overline{AB}$. Open your compass to the length of line AB. Place the point of the compass on point A and draw an arc above the line. Place the point of the compass on point B and draw another arc above the line so that the arcs cross at point C.

2 Using a ruler, join points A and B to point C. ΔABC is an equilateral triangle. Mark all sides as equal and all angles as 60°.

(b) 1 Construct a right angle on a horizontal line AB by constructing the perpendicular bisector of the line.

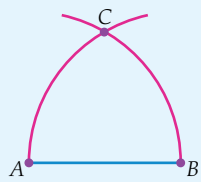
2 Place your compass point on point X, the point of intersection of AB and the perpendicular bisector. With the compass opened to length BX, cut the perpendicular bisector at point C.

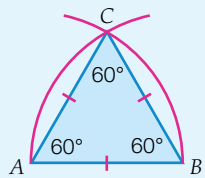
3 Keeping the compass opening the same, place the compass point on point B and make an arc above the line. Place the compass point on point C and make an arc to cut the last arc at D.

4 Join B to D and the point C to point D. $BXCD$ is a square.

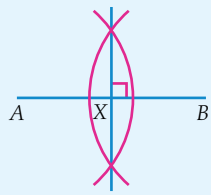
Working

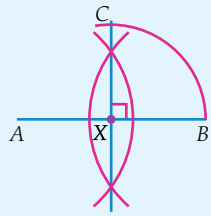
(a)

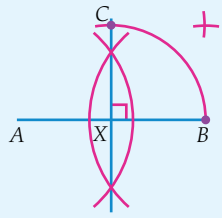


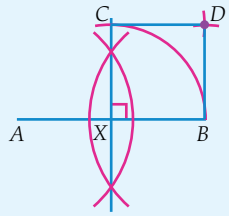


(b)









Worked example 11

W.E. 11

Using only a compass, a ruler and a pencil:

(a) construct an angle equal to another angle **(b)** construct a pair of parallel lines.

Thinking

(a) **1** Draw an acute $\angle ABC$.

2 Use a compass and, with the compass point on point B, draw an arc through point C.

3 Draw a line segment and label one end point Y. Without changing the opening on your compass and, with the compass point on point Y, draw an arc to cut the line segment at Z.

4 Place the compass point on point C and alter the compass opening so that it measures the length $\overline{CA}$. Place the compass point on point Z and draw an arc above $\overline{YZ}$.

5 Place the compass point on point B and alter the compass opening so that it measures the length $\overline{BA}$. Place the compass point on point Y and draw an arc to cut the previous arc at X. Join $\overline{XY}$.

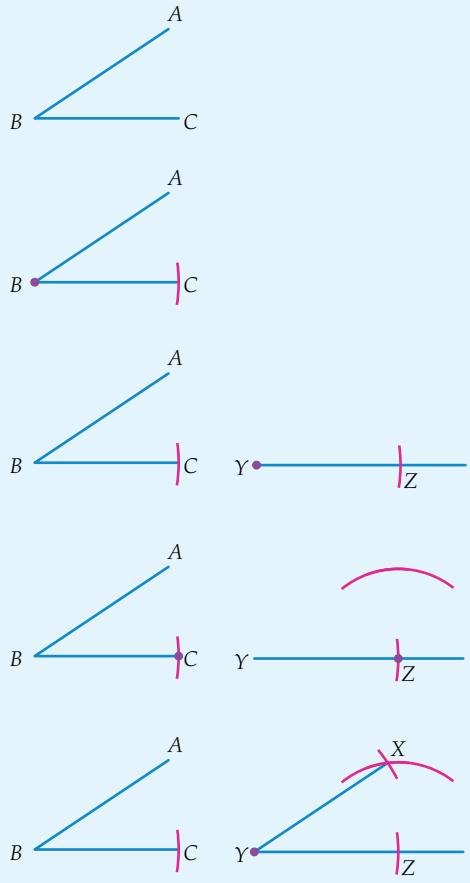
(b) **1** Draw a line segment $\overline{AB}$. Draw another line $\overline{EH}$ to intersect with $\overline{AB}$. Label the point of intersection G. Choose a point F between E and G. Using the instructions in part **(a)**, construct acute $\angle YFG$ equal to acute $\angle BGH$.

2 Draw a line segment through $\overline{YF}$ and extend to a point X.

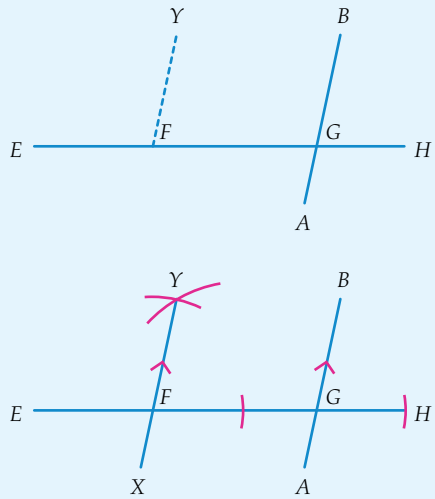
$\overline{XY}$ will be parallel to $\overline{AB}$, as you have constructed equal corresponding angles.

Working

(a)



(b)



8.8 Compass constructions

Navigator

1, 2, 3, 4, 5, 6, 8, 9, 11, 12, 14, 15	1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15

Answers p. 698

Equipment required: compass, ruler, pencil

Fluency

1 Using only a compass, a ruler and a pencil:

(a) bisect a line

(b) bisect an angle

(c) construct an equilateral triangle

(d) construct a square

(e) construct an angle equal to another angle

(f) construct a pair of parallel lines.

W.E. 9
W.E. 10
W.E. 11

Understanding

2 Using only a compass, a ruler and a pencil:

(a) construct a 90° angle

(b) bisect this angle to make a 45° angle

(c) construct a 60° angle

(d) make a 30° angle from the previous angle

(e) bisect a line 10 cm long

(f) construct a parallelogram.

Reasoning

3 Construct a triangle with one angle equal to 30° and another angle equal to 45° and the arm between these two angles equal to 8 cm.

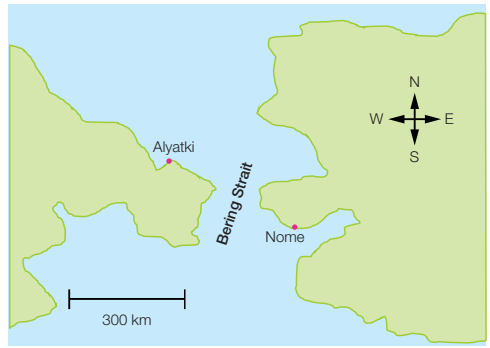
4 Construct the following angles: **(a)** 75° **(b)** 15° **(c)** 135°

5 Construct an isosceles right-angled triangle where the two equal sides are 5 cm long.

6 Construct an equilateral triangle on each side of a square.

7 Use a compass to draw a circle. Select a point on the circle. Using the same compass opening, divide the circle perimeter into 6 equal pieces. Join the points of intersection in order to make a hexagon. Explain why this construction works.

8 Stanislav is the navigator on a fishing vessel. He is completely lost but, using his radio navigation equipment, he knows he is 300 km from Alyatki and 300 km from Nome. Exactly where on the map shown could Stanislav's fishing vessel be located?



9 Use a compass to draw a circle. Construct an octagon by dividing the perimeter of the circle into 8 equal pieces. Explain how your knowledge of constructing a 90° angle and bisecting an angle helped you.

Open-ended

10 Using only your compass, construct the following pattern.

11 Choose any two polygon shapes and use your construction knowledge to use them to make a design.

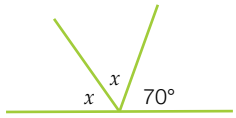
Challenge 8

1 The smaller angle between the hands of a clock at half past two is:

A 90° **B** 105° **C** 120° **D** 135°

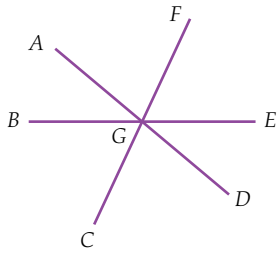
2 Two angles are complementary and one angle is 46° more than the other. What is the size of the larger angle?

3 Find the value of the pronumerals in the following.



4 In the diagram, $\angle BGF = 115°$ and $\angle AGE = 140°$.

What is the size of $\angle CGD$?



5 Two angles are supplementary and one angle is 54° more than the other. What is the size of the smaller angle?

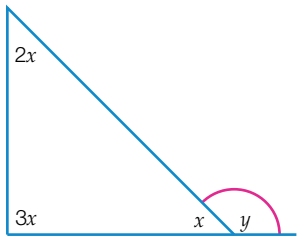
6 In the diagram, the value of y is:

A 30°

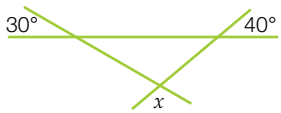
B 60°

C 90°

D 150°



7 In the diagram, the value of x is:



A 110°

B 70°

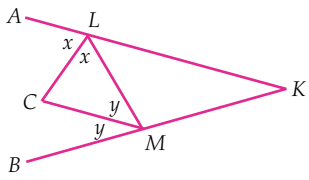
C 40°

D 30°

8 **(a)** Shortly after 12 o'clock, the hour and minute hands of a clock form an angle of 55°. What is the time?

(b) What is the acute angle between the hands 40 minutes later?

9 In the diagram, ALK and BMK are straight lines. The bisector of $\angle ALM$ meets the bisector of $\angle BML$ at C. If $\angle LCM$ is 70°, then $\angle LKM$, in degrees, is:



A 10°

B 20°

C 40°

D 110°

Chapter review 8

Maths literacy

acute angle	equilateral triangle	plane	rhombus
acute-angled triangle	intersect	plane shapes	right angle
alternate angles	irregular	point of intersection	right-angled triangle
angle	isosceles triangle	polygon	scalene triangle
arms	kite	protractor	square
co-interior (allied) angles	line segment	quadrilateral	straight angle
complementary angles	obtuse angle	ray	supplementary angles
concave	obtuse-angled triangle	rectangle	transversal
convex	parallel lines	reflex angle	trapezium
corresponding angles	perpendicular bisector	regular polygon	vertex
degree	perpendicular lines	revolution	vertically opposite angles

Copy and complete the following using the words and phrases from this list, where appropriate. A word or phrase may be used more than once.

1 To measure an angle, the centre point of the _____ must match with the _____ of the angle. The lines must match with the _____ of the angle.

2 40° and 50° are _____.

3 _____ add to 180 _____s.

4 Lines drawn at right angles to each other are called _____.

5 The angle names listed in order from largest to smallest are_____, _____, _____, _____, _____ , and _____.

6 The most general name for a four-sided figure is a _____.

7 Lines in the same plane that never meet are called _____.

8 An _____ has two sides equal in length and one unequal side.

9 A closed shape with only straight line sides is called a _____.

10 A square is a _____ with right angles.

11 A _____ has three sides all of different lengths.

12 A line crossing other lines is called a _____.

13 A _____ is a quadrilateral with one pair of parallel sides.

14 Angles that are between parallel lines and on the same side of the transversal are _____ and are _____.

Equipment required: protractor for Questions **1**, **3**, **14**; and a compass for Question **13**

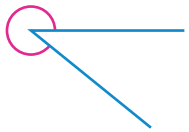
Fluency

1 Use a protractor to measure each of the following angles. **8.1**

(a) 

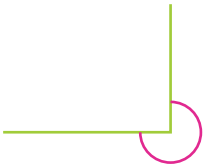
(b) 

(c) 

(d) 

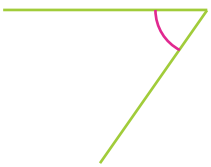
2 Match the following angle sizes with the drawn angles. Do not use a protractor. **8.1**

A 20° **B** 55° **C** 120° **D** 210° **E** 270° **F** 310°

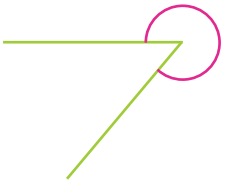
(a) 

(b) 

(c) 

(d) 

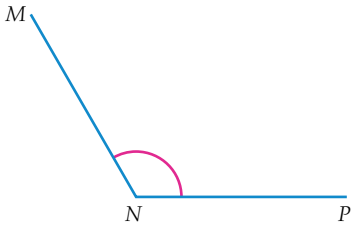
(e) 

(f) 

3 Use a protractor and a ruler to draw an angle of each size stated. **8.1**

(a) 56° **(b)** 162° **(c)** 257° **(d)** 304°

4 Which of the following is not true of the angle shown? **8.2**



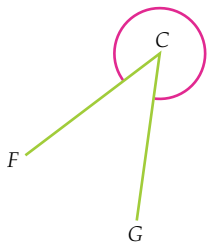
A It is named $\angle NPM$.

B It is named $\angle MNP$.

C It is named $\angle PNM$.

D It is an obtuse angle.

5 Which of the following is true of the angle shown? **8.2**



A It is obtuse.

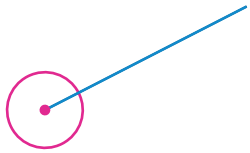
B It is reflex.

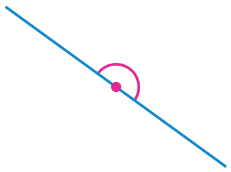
C It is less than 180°.

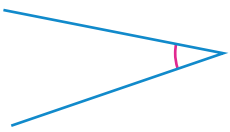
D It is called $\angle CFG$.

6 State the type of each angle below. **8.2**

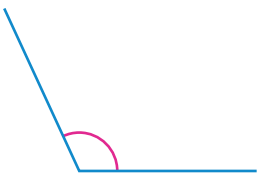
(a) 

(b) 

(c) 

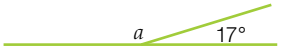
(d) 

(e) 

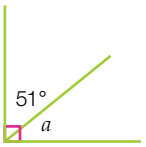
(f) 

7 Find the value of a in each of the following diagrams. **8.3**

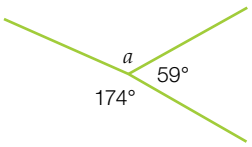
(a)



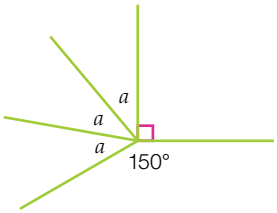
(b)



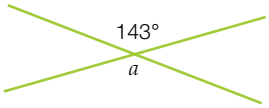
(c)



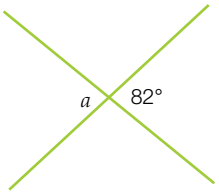
(d)



(e)

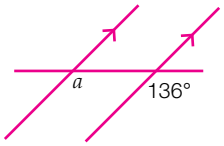


(f)

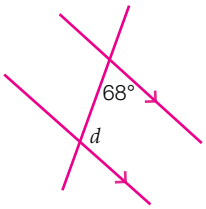


8 Find the value of the pronumeral in the following diagrams. **8.4**

(a)



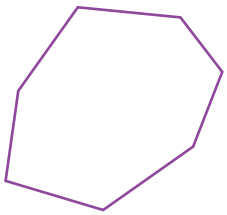
(b)



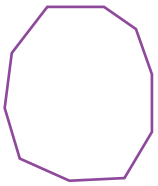
9 Name the following polygons, also stating whether they are regular or irregular, convex or concave. **8.5**

(a)

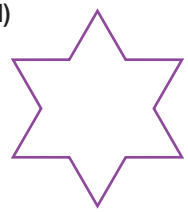
(b)



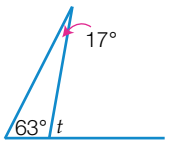
(c)



(d)

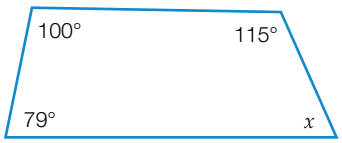


10 Find the value of the pronumeral in the diagram. **8.6**

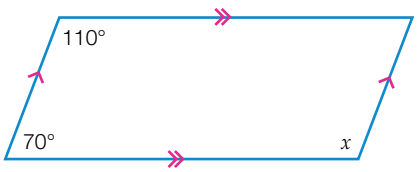


11 Find the unknown angle in each of the following quadrilaterals. Make sure you support your answer with reason(s). **8.7**

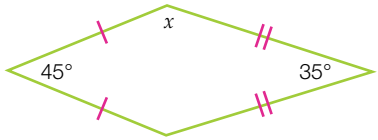
(a)



(b)



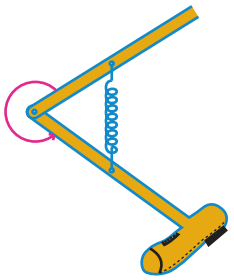
12 Find the value of x in the following quadrilateral. Make sure you support your answer with reason(s). **8.7**



13 Use a compass to construct a square of side length 4 cm. **8.8**

Understanding

14 Eloise has constructed a mechanical leg to kick a football as part of her investigation into kicking techniques. During one trial, the initial leg position was as shown. Find the size of the reflex angle of the leg.



8.1

15 One angle in a right-angled triangle is 45°.

(a) What is the value of the other two angles?

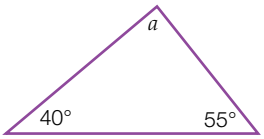
(b) The triangle described in **(a)** was named a right-angled triangle. Give another name for this triangle.

8.1, 8.2

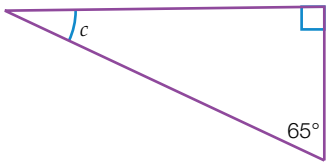
16 Calculate the missing angle in each of the following triangles. Make sure you support your answer with reason(s).

8.1, 8.2

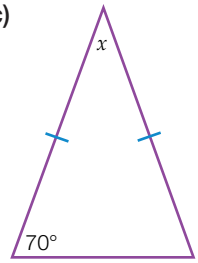
(a)



(b)

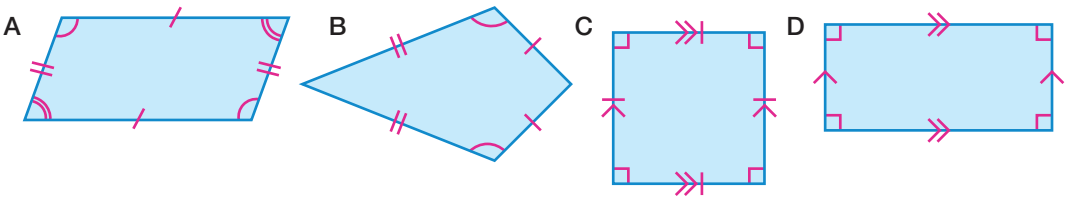


(c)



17 Referring to the shapes below, answer true (T) or false (F) to the following statements.

8.7



(a) All four shapes are quadrilaterals.

(b) A and B are kites.

(c) A, C and D could be classified as parallelograms.

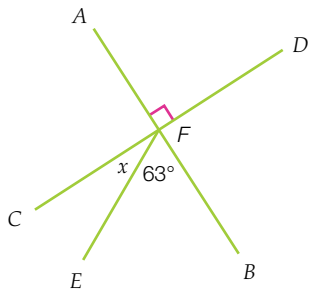
(d) A is a rhombus.

(e) C is a rectangle.

Reasoning

18 Find the value of the angle *x*, giving reasons for each step.

8.3



19 Use your knowledge of angles to answer the following questions:

8.2

(a) What type of angle would result from adding an acute angle and a right angle?

(b) What type of angle would result from adding a straight angle and an obtuse angle?

(c) What types of angle could result from subtracting a reflex angle from a revolution?

Numeracy practice 8

Non-calculator

1 Which is the largest angle?

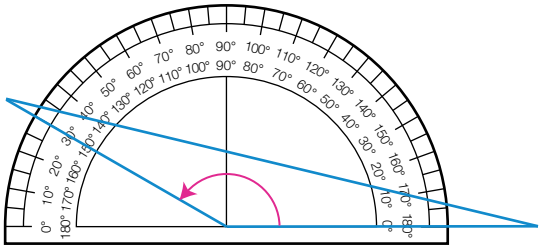
A

B

C

D

2 What is the size of the angle in the triangle marked by the arrow?



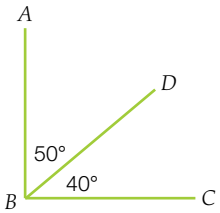
3 $\angle ABD$ and $\angle DBC$ are best described as:

A vertically opposite angles

B supplementary angles

C angles at a point

D complementary angles.

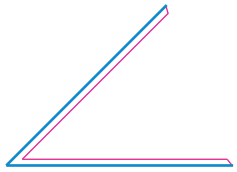


Calculator allowed

4 Which diagram shows a book opened to about 30°?

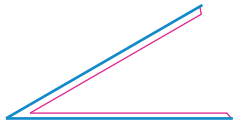
A

B



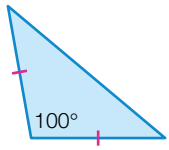
C

D

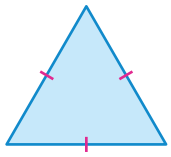


5 Which one of these is a right-angled isosceles triangle?

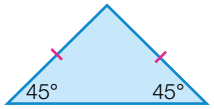
A



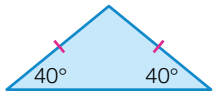
B



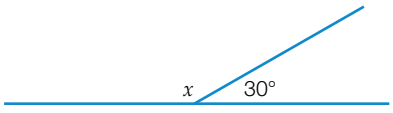
C



D



6 What is the value of the angle x in this diagram?



A 30° **B** 60° **C** 120° **D** 150°

Mixed review D

Equipment required: protractor for Question **15 (c)**

Fluency

1 Simplify: **5.6**

(a) $2a + 3b + 4a - 7b$ **(b)** $6a + 3b - 2c - b + 5c$

2 Write factor trees for each of the following numbers, then express each number as the product of its prime factors in index form. **2.3**

(a) 45 **(b)** 24 **(c)** 630

3 Write the following percentages as fractions in simplest form. **4.7**

(a) 75% **(b)** 50% **(c)** 30% **(d)** 4%

4 Evaluate without using a calculator: **4.4, 4.5**

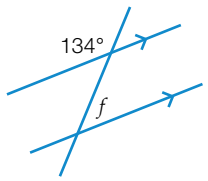
(a) 1.1×0.23 **(b)** $0.24 + 3.7 - 1.503$

5 Solve the following equations using algebra. **7.4**

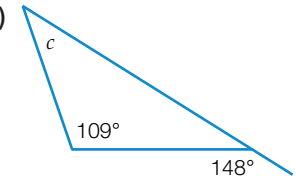
(a) $3x - 4 = -7$ **(b)** $\frac{x+5}{3} = -12$

6 Find the value of the pronumeral in the following diagrams. **8.4, 8.6**

(a)



(b)

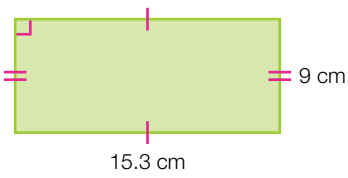


7 Evaluate without a calculator: **3.4–3.6**

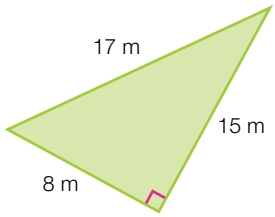
(a) $3\frac{1}{5} + 2\frac{1}{2} - 1\frac{3}{4}$ **(b)** $2\frac{1}{4} \times 1\frac{1}{3} \div 3\frac{1}{5}$

8 Find the area of the following shapes. **6.4, 6.5**

(a)



(b)



Understanding

9 The table on the right shows the minimum and maximum temperatures at Frosty Valley for 1 week in winter. **1.2**

(a) On which day was the lowest maximum recorded?

(b) On which day was the highest minimum recorded?

(c) Which day had the greatest difference between minimum and maximum?

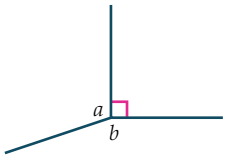
Day	Minimum (°C)	Maximum (°C)
Monday	-4	5
Tuesday	-8	0
Wednesday	-2	1
Thursday	-5	-1
Friday	-6	6
Saturday	-4	4
Sunday	-9	2

10 Which of the following is the correct answer to $(4 + 3) \times 10 - 4 \div 2$? **1.6**

A 68 **B** 35 **C** 33 **D** 21

Reasoning

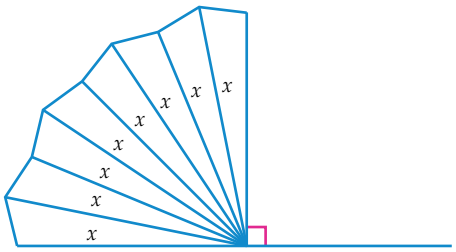
11 The angles in this diagram can be called: **8.1**



A vertically opposite **B** complementary

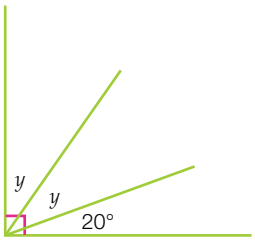
C supplementary **D** angles at a point.

12 A fan, when opened out, looks like this. What is the size of the angle x between each fold of the fan? **8.3**

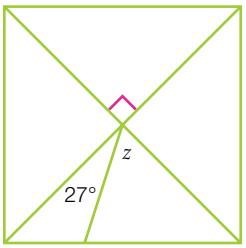


13 Find the value of the pronumeral. **8.3**

(a)

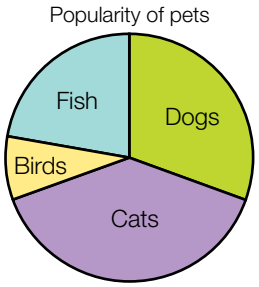


(b)



14 What two angles are formed by the two hands on a clock face at 5:00 pm? **8.2**

15 John has drawn this pie chart, showing the popularity of pets. **8.1**



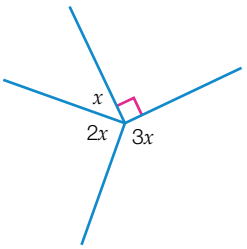
(a) Estimate the angle at the centre of each wedge.

(b) Without using a protractor, how could you check to see how reasonable the estimates you made in **(a)** are?

(c) Using a protractor, measure the size of each wedge.

16 You enter an elevator on the fourth floor. Once inside, you are trapped because the elevator software malfunctions. First, you go up five floors, then fall six floors, then fall another two floors before finally going up one floor and coming to rest. The doors open, much to your relief. What floor are you on? **2.6**

17 Find the size of each angle in the diagram by first finding the value of x. **7.5, 8.3**

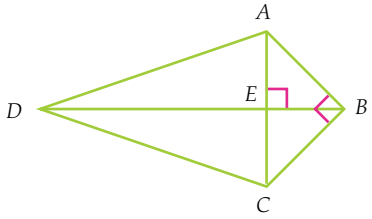


18 **(a)** Draw and label an acute angle $\angle ABC$. **8.2**

(b) There is another angle in your diagram, apart from the acute angle. Classify it.

(c) Name the angle in **(b)**.

19 Name as many pairs of complementary angles, supplementary angles, vertically opposite angles and angles in a revolution as you can see in the figure. **8.2**




9

Statistics and probability

9

Caution required when exercising.

Everyone knows that exercise is good for you, but you also need to be careful, especially when using gym equipment.

Hospital records indicate that treadmill-related injuries are on the rise. Out of the total number of treadmill-related injuries that occurred in a five-year period, it was found that 38% of the injuries were reported in the fifth year. The most worrying statistic was that children and teenagers are most at risk. The age range 0–14 years accounted for 60% of all injuries, with the 0–4 age group accounting for 38% of the injuries. Of the 12% of injuries that required hospital admission, most were in this youngest age group. The high proportion of young children with these injuries is related to the fact that 70% of the injuries happened in the home.

Forum

What sport do you think would result in the most injuries? How would you research this issue? Is the sport with the highest number of reported injuries necessarily the most dangerous?

Why learn this?

In the media, we are often presented with some sort of statistics or probability, such as the average temperature for the month or claims by advertisers about the worth of their products. Understanding probability and statistics can help you to make sense of this information and to be aware when someone might be trying to mislead you.

After completing this chapter you will be able to:

- organise, classify, present and interpret different kinds of data
- determine the mean, median, mode and range of a data set using various techniques
- present data appropriately using dot plots, stem-and-leaf plots, bar graphs, frequency tables, frequency column graphs, histograms, sector graphs and line graphs
- analyse univariate and simple bivariate data
- construct sample spaces for single-step experiments and use them to assign probabilities.

Recall 9

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

1 **(a)** List the numbers you get if you count by fives from 0 to 30.

(b) List the numbers you get if you count by twos from 16 to 28.

(c) List the numbers you get if you count by hundreds from 1300 to 1700.

(d) List the numbers you get if you count by twenties from 60 to 200.

2 **(a)** List the multiples of 50 from 50 to 250.

(b) List the multiples of 25 from 100 to 200.

(c) List the multiples of 200 from 1400 to 2000.

3 Write the following lists of numbers in order from smallest to largest.

(a) 12, 22, 14, 20, 10, 16, 24, 6

(b) 15, 0, 50, 20, 55, 100, 25, 5

(c) 5, $1\frac{1}{2}$, $6\frac{1}{2}$, 1, $\frac{1}{2}$, 0, $3\frac{1}{2}$, $5\frac{1}{2}$, 2, 3, $2\frac{1}{2}$, 4

4 Copy and complete the pattern by adding the same number each time.

(a) 12, ____, ____, 21, ____, 27

(b) 120, ____, ____, ____, ____, 220

(c) 225, ____, ____, 300, ____, 350

5 Write the following fractions in simplest form.

(a) $\frac{2}{6}$ **(b)** $\frac{6}{8}$

Exploration Task

You can download this activity from the eBook or the Pearson Places website.

Improving the odds

In this activity, you will explore how adding a single dot to a standard six-sided die can affect the chances of winning a dice game.

Collecting data

9.1

The study of statistics is the collection, interpretation and presentation of **data**. Data can be any sort of information that can be recorded and analysed, such as the colour of a car or the number of days of rain in a month. It is important to understand how to collect data, present it in useful forms, and interpret it.

Primary data is data that you collect yourself. You can collect data specific to your question, but you must make sure that this data is reliable and unbiased by collecting it in a reliable way.

Secondary data is data that you find from another source, such as from the internet or a news article. You must make sure that this data is appropriate to answer your questions. You must also make sure the data is reliable by checking that it comes from a source that can be trusted, such as the Australian Bureau of Statistics.

Types of data

You can collect two main types of data.

The first main type is called **categorical data** because the data values are information about categories, or groups of the same type. An example of categorical data could be the different eye colours of a group of students (blue eyes, brown eyes, green eyes and so on). Another example could be people's favourite fruits: apples, bananas, oranges etc.

The second main type of data is called **numerical data** because the data values are numbers. It is also useful to consider numerical data as two different types:

Discrete numerical data are numbers that can only be particular separate values. For example, the number of runs scored by a cricketer over a season or the number of points scored by a basketball team are **discrete data**. They are discrete because they cannot be any number; they can only be whole number values. (It is impossible to score a fraction or decimal value such as 3.24 points.).

Continuous numerical data are numbers that are measured, and can be any number. For example, the weights of babies at a hospital or volume of water in a dam each day are **continuous data**. The accuracy of measurements depends on the measuring equipment used.

Suppose you asked pet owners the following questions about their pets.

1 What type of pet do you own?

2 How many pets do you own?

3 What does each of your pets weigh?

The first question will collect categorical data because the answers will be cat, dog, rat, tortoise, rabbit etc.

The second question will collect discrete data because the answers will be counted and given as 1, 2, 3, 4 etc.

The third question will collect continuous data from measurements such as 5.3 kg, 34.7 kg, 24.3 kg, 2.6 kg etc.

Worked example 1 W.E. 1

For each of the following, state whether the data from a survey asking these questions is categorical, discrete or continuous.

(a) How many televisions are in your house?

(b) What is the height of the tallest person in your family?

(c) What is your favourite dessert?

Thinking	Working
(a) Are you counting, measuring or putting things in categories? (Here, you are counting the number of televisions.)	(a) Discrete data
(b) Are you counting, measuring or putting things in categories? (Here, you are measuring height.)	(b) Continuous data
(c) Are you counting, measuring or putting things in categories? (Here, you are putting things in categories.)	(c) Categorical data

Frequency tables

Any data that has repeated values can be summarised in a **frequency table**. In a frequency table, each different data value is listed. Each time that data value occurs, it is recorded in the **tally** column. You then count the data in the tally column and write this number in the **frequency** column. In this way, the 'frequency' is the number of times that each data value occurs.

Worked example 2

W.E. 2

A Year 7 class of 25 students was surveyed about the number of brothers and sisters they have. The results were: 2, 3, 5, 0, 1, 1, 2, 0, 3, 1, 4, 0, 1, 1, 3, 2, 2, 2, 1, 1, 0, 1, 1, 2, 4

(a) Is this data categorical, discrete or continuous?

(b) Construct a frequency table showing this information.

Thinking	Working
(a) Is the data categorical, discrete or continuous?	(a) It is discrete data.
(b) **1** Construct a table with three columns that shows the item being surveyed, the tally and the frequency. **2** Enter the data into the tally column. Start at the left of the list of numbers and work to the right. (Don't count all the 1s, then all the 2s etc.) *Note:* 𝍸 is used to represent 5. **3** Enter these tallies as numbers in the frequency column. Add the numbers in this column to check that you have tallied correctly.	(b) Frequency table

Number of brothers and sisters	Tally	Frequency
0	||||	4
1	𝍸 ||||	9
2	𝍸 |	6
3	|||	3
4	||	2
5	|	1
	Total	25

Range

The spread of data values from maximum to minimum is called the **range**. The range is a useful statistic for understanding the spread of a set of data values.

Range = (largest data value) – (smallest data value)

Grouped data

When there is a large amount of numerical data spread out over many values, data is often grouped into sets called 'classes'. This means that statistics can be calculated from each class, instead of using every individual value.

Data is usually grouped in intervals of 2, 5, 10 etc., so that you get between five and ten groups when you divide your chosen interval into the range. The spread of the values in each group gives the **class interval**. The class intervals are then used as the groups in the first column of the frequency table.

The class intervals for discrete data are presented in discrete groups written, for example, as 0–9, 10–19, 20–29. The class intervals for continuous data are presented in continuous groups written, for example, as 0–<10, 10–<20, 20–<30. Both of these examples have a class interval of 10, because the difference between the beginning of one class and the beginning of the next class is 10 in each case.

Worked example 3 W.E. 3

The Exceptional Felines basketball team keeps track of the points they score on the way to the championship. The point totals are:

73, 84, 68, 45, 52, 44, 45, 52, 66, 42, 43, 40, 53, 47, 82, 76, 42, 57, 65, 81,
80, 40, 56, 72, 74, 83, 41, 66, 76, 75, 68, 81, 82, 79, 58, 81, 78, 80, 78, 76

(a) Is the data discrete or continuous?

(b) Group the results and then construct a frequency table showing this information.

Thinking	Working
(a) Are the values counted (discrete values) or measured (continuous values)?	(a) The data is discrete.
(b) **1** Decide on how to group the data by subtracting the smallest score from the highest to find the range. Then divide this number by 2, 5, 10 etc. until you get an interval size that gives between five and 10 groups. (If you get a decimal number, round to the nearest whole number.)	(b) Range = 84 – 40 = 44 44 ÷ 5 = 8.8 Number of groups = $\frac{44}{5}$ = 8.8 So, we will group with an interval size of 5 and have 9 groups.

2 Construct a table with three columns. Write the grouped values in the first column. Be careful with your groupings so that you don't miss values or include them twice. (In this example, the groupings are 40–44, 45–49, 50–54 and so on.)

3 Use the tally column to help with the count. Remember to go through the data list systematically from the first piece of data to the last, tallying as you go.

4 Enter the tallies as numbers in the frequency column. Add the numbers in this column to check that you have tallied correctly.

Number of points scored	Tally	Frequency
40–44	𝍸 ||	7
45–49	|||	3
50–54	|||	3
55–59	|||	3
60–64		0
65–69	𝍸	5
70–74	|||	3
75–79	𝍸 ||	7
80–84	𝍸 ||||	9
	Total	40

You must always be very careful when collecting and interpreting data from any source.

Primary data can be misleading if you ask questions that are not specific enough, if you don't look for all the relevant information, or if you record data from sources that are biased.

Secondary data can be misleading if it presents information in a distorted or incomplete way, or if it is not correct. For example, news articles sometimes talk about the most dramatic piece of data without considering the full data set.

9.1 Collecting data

Navigator

Answers p. 700

1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 14, 15	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15	1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15

Fluency

W.E. 1

1 For each of the following, state whether the data from a survey asking these questions is categorical, discrete or continuous.

(a) How do you travel to school?

(b) What colour hair do you have?

(c) How much time did you spend at the beach last summer?

(d) What is your height?

(e) How many cars passed by the school in a day?

(f) What is your favourite book?

(g) How many trees are in your front yard?

W.E. 2

2 A Year 7 class of 25 students was surveyed about the number of pets they own. The results were:

0, 0, 2, 1, 1, 1, 0, 6, 9, 4, 3, 0, 1, 3, 2, 1, 1, 4, 5, 4, 3, 2, 1, 4, 7

(a) Is this data categorical, discrete or continuous?

(b) Construct a frequency table showing this information.

3 The number of calls per day to the help department of an automobile association was recorded for the month of June. The results were:

W.E. 3

120	100	75	60	66	100
110	197	55	80	155	179
154	43	75	98	172	130
25	55	110	99	72	90
113	65	87	94	112	18

(a) Is the data discrete or continuous?

(b) Group the results in classes of size 20, starting at 0, and then construct a frequency table showing this information.

4 A class of students were surveyed about their favourite takeaway food. The results were:

Emma—pizza
Whitney—hamburgers
Morgan—Chinese
Kimiko—pizza
Liam—hamburgers
Olivia—chicken
David—never eats takeaway
Malcolm—pizza
Gary—hamburgers
Dieter—chicken
Yang—pizza
Brent—fish & chips
Ruby—pizza
Romano—fish & chips
Noah—chicken
Fleur—never eats takeaway
Jonty—pizza
Carol—Chinese
Ava—hamburgers
Marcia—pizza
Ethan—pizza
Yusef—fish & chips

(a) Is the data categorical, discrete or continuous?

(b) Copy and complete the frequency table below to show this information.

Type of takeaway	Tally	Frequency
Pizza		
Fish & chips		
?		
?		
?		
None		

Understanding

5 A group of people were surveyed about how much television they watched on a normal weeknight. The results were recorded in hours rounded to the nearest half-hour, as follows:

$3, 2, 1\frac{1}{2}, \frac{1}{2}, 1, 2\frac{1}{2}, 1, 0, 4\frac{1}{2}, 2\frac{1}{2}, 2, 2\frac{1}{2}, 2, 0, 4, 2, 1\frac{1}{2}, 2\frac{1}{2}, 3, \frac{1}{2}, 0, \frac{1}{2}, 1, 2\frac{1}{2}, 2, 2, \frac{1}{2}$

(a) Is the data discrete or continuous?

(b) Construct a frequency table showing this information. (Show the frequency for each half-hour time period.)

6 A Year 7 class was surveyed about how much money they had in their pockets at that time. The results were:

$1.20	$0.75	$1.00	$2.55	$1.50	$4.00	$0.00	$0.00
$0.80	$2.40	$1.25	$0.00	$3.60	$4.40	$4.30	$2.90
$1.80	$2.35	$2.20	$0.50	$0.40	$2.50	$1.50	$1.75

Group the results and construct a frequency table showing this information. Group the data using intervals that are 50 cents apart.

7 Look at the following data list.

2	8	9	15	22	24	17	16	28	31	40
15	52	16	51	42	39	27	26	42	55	46
17	22	18	5	47	46	33	21	19	21	17

Group the data using 0–9, 10–19, 20–29 etc. as the groups.

(a) The frequency for the 10–19 group is:

A 6 **B** 8 **C** 9 **D** 10

(b) The frequency for the last group is:

A 2 **B** 3 **C** 4 **D** 5

8 Tuna comes in tins labelled with contents of 95 g. Thirty-eight tins were weighed with results rounded down to the nearest gram:

123	120	126	124	124	127	127	123	131	128
126	121	119	121	126	123	121	125	123	122
123	126	121	125	124	123	125	126	118	125
125	121	123	123	122	125	124	123		

(a) To group the data into 5 to 10 class intervals, an appropriate first group would be:

A 116–120 **B** 115–<120 **C** 117–118 **D** 118–<120

(b) Using these intervals the number of groups would be:

A 6 **B** 7 **C** 8 **D** 9

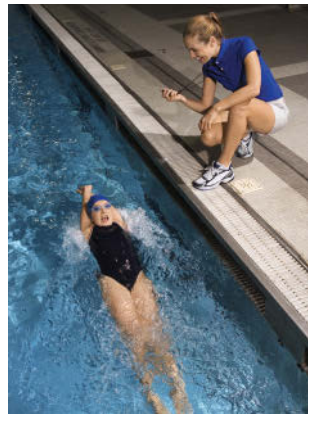
(c) The number of tins in the most frequently-occurring class would be:

A 9 **B** 10 **C** 11 **D** 12

Reasoning

9 A class was surveyed about the time spent exercising over a weekend. The results were recorded in hours, rounded to the nearest hour:

1, 3, 2, 1, 1, 2, 0, 2, 0, 1, 1, 2, 6, 3, 4, 2, 1, 0, 2, 8, 3, 5, 1, 2, 0, 2



(a) How many students were surveyed?

(b) Construct a frequency table of this data.

(c) How many students did not exercise at all?

(d) Complete this sentence with the smallest number possible:

'The majority of students spent no more than … hours exercising.'

(e) One student hoped to qualify for the next Olympics, one student wanted to be drafted into the AFL and another student's netball team was in the finals. Which data values are mostly likely to belong to these three students? Explain your answer.

10 The cast and crew of a school musical were to be given T-shirts advertising the musical. There were five sizes available, XS (extra small), S (small), M (medium), L (large) and XL (extra large). The orders are:

S, M, S, XL, L, M, L, S, XS, M, L, S, M, L, L, M, XL, S, M, M, XL, L, M, S, XS, L, S, XL, M, L

Five students, all in Year 7, forgot to order a T-shirt.

(a) How many students were in the cast and crew of the musical?

(b) Construct a frequency table for this data.

(c) What type of data is this?

(d) What was the most popular T-shirt size ordered?

(e) How many T-shirts are smaller than the M size?

(f) The music teacher, Ms Campbell, could not decide whether to order one extra T-shirt in each size or five more M size T-shirts for the students who had not submitted an order. What would you advise her to do? Explain your advice.

11 Look at the following data set.

20	32	42	53	63	21	33	42	53	65	22	34	43	57	61
24	34	48	50	61	24	39	40	50	62	29	31	41	52	62

(a) Construct a frequency table using class intervals of 20–29, 30–39 etc.

(b) Describe what you find in your frequency table.

(c) Construct a frequency table using class intervals of 20–24, 25–29, 30–34 etc.

(d) Describe what you find in your frequency table.

(e) What does this show?

12 Sixty 13-year-old students were tested to find their pulse rate (heartbeat) when resting. The following beats per minute were measured.

70	68	68	76	79	68	76	55	55	60	60	94	72	65	64
93	71	62	67	82	76	65	77	82	81	59	74	74	67	68
78	76	63	82	81	82	74	70	66	63	84	81	69	84	79
71	70	54	68	64	78	58	84	61	75	72	73	71	91	66

(a) Construct a frequency table using class intervals of 51–60, 61–70 etc.

(b) Construct a frequency table using class intervals of 51–55, 56–60, 61–65 etc.

(c) Which table shows the spread of results more clearly? Give a brief reason to support your answer.

Open-ended

13 The following is a list of birth weights of 30 babies. The weights are in grams, rounded to the nearest whole number.

2900	2805	2925	2010	2720	3125	2670
2555	2963	2972	3151	2515	3529	3098
2126	2417	3000	3254	2997	2986	2719
2842	3519	3509	3218	3002	2437	2222
2019	2113					

Construct a frequency table for this data using a suitable class interval. Briefly explain why you chose this class interval.

14 Find out how much television each of the students in your class watches on a normal weeknight by surveying your class. Construct a frequency table of your results. Record your data in hours, rounded to the nearest half-hour.

(a) How many students watch no television on a normal weeknight?

(b) How many students watch less than 1 hour of television on a normal weeknight?

(c) How many students watch 3 or more hours of television on a normal weeknight?

(d) How much television do the majority of the students of your class watch on a normal weeknight?

(e) Do you watch more or less television than the majority of the students of your class?

15 Find out the favourite school subject of each of the students in your class by surveying your class. Construct a frequency table of your results.

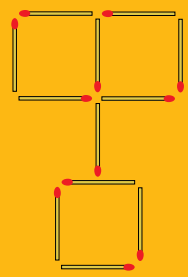
(a) What is the favourite subject of the members of your class?

(b) What is the second favourite subject of the members of your class?

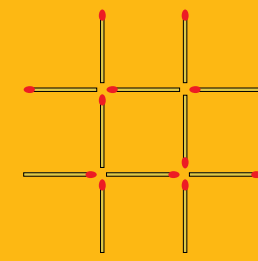
(c) Did any student have no favourite subjects?

Puzzle

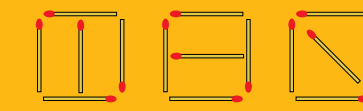
Matchstick puzzles



Move four matchsticks to make exactly five squares.



Move three matchsticks to make exactly three squares.



Remove six of the fifteen matchsticks to leave ten.

Measures of centre and spread

9.2

In everyday life, people talk about **averages**, such as 'the average temperature' or 'the average number of points scored'. When people say 'average', they are usually referring to the **mean** of a set of numbers, which is a statistical value that can be used to represent the 'centre' of the data.

The mean is not the only measure of the centre that you can use. The **median** ('middle') and **mode** ('most common') are two other useful ways of finding a central value of a group of data values.

The mean, median and mode are all **measures of centre** for numerical data (sometimes called **measures of location**), but each is calculated in a different way and gives different information about the data.

Mean

The mean is the sum of all the values in the data set divided by the number of values in that data set.

$$\text{Mean} = \frac{\text{sum of data values}}{\text{number of data values}}$$

Median

The median is the *middle* value of an ordered data set.

To calculate the median, arrange the data set in order from lowest to highest, then count the number of data values.

If the number of values is odd, then the median is the middle value.

If the number of values is even, then find the two middle values. Add them together and divide by 2 to calculate the median.

Mode

The mode is the value that occurs *most often* in a data set.

Sometimes, a set of results can have more than one result that occurs most frequently. For example, consider the following: 1, 1, 3, 3, 3, 4, 5, 5, 6, 7, 7, 7, 8, 8, 9

Here, both 3 and 7 occur three times. When there are two values that occur most frequently, you say that the results are **bi-modal**. If there are three or more values that each have the maximum frequency, you say that the data is **multimodal**. The mode is not very useful in a multimodal set of data, so multimodal data sets are sometimes said to have no mode.

Range

Range = (largest data value) – (smallest data value)

The range is a measure of the spread of data values from maximum to minimum. The range is a useful statistic for understanding the spread of a set of data values.

Worked example 4 W.E. 4

Find the **(i)** mean, **(ii)** median, **(iii)** mode and **(iv)** range of each of the following sets of test results.

(a) 9, 4, 5, 7, 8, 7, 2

(b) 10, 7, 3, 4, 8, 2, 6, 5

Thinking			Working
(a)	**(i)** **1**	Write the rule for finding the mean.	(a) (i) Mean $= \frac{\text{sum of data values}}{\text{number of data values}}$
	2	Substitute data into the rule.	$= \frac{9+4+5+7+8+7+2}{7}$
	3	Simplify and evaluate.	$= \frac{42}{7}$
	4	Write the mean.	Mean $= 6$
	(ii) **5**	Order the values from smallest to largest. Count the number of values. If the number is odd, select the middle value.	(ii) 2, 4, 5, (7), 7, 8, 9 ↑ 7
	6	Write the median.	Median $= 7$
	(iii) **7**	Does any value appear more often than any other? This is the mode. (Yes)	(iii)
	8	Write the mode.	Mode $= 7$
	(iv) **9**	Range is (highest data value) – (lowest data value).	(iv) Range $= 9 - 2$ $= 7$
(b)	**(i)** **1**	Write the rule for finding the mean.	(b) (i) Mean $= \frac{\text{sum of data values}}{\text{number of data values}}$
	2	Substitute data into the rule.	$= \frac{10+7+3+4+8+2+6+5}{8}$ $= \frac{45}{8}$
	3	Simplify and evaluate.	$= 5.625$
	4	Write the mean.	Mean $= 5.625$
	(ii) **5**	Order the values from smallest to largest. Count the number of values. If the number is even, select the two middle values.	(ii) 2, 3, 4, 5, 6, 7, 8, 10 2, 3, 4, (5, 6), 7, 8, 10
	6	Find the mean of these two numbers to find the median.	Median $= \frac{5+6}{2}$
	7	Write the median.	Median $= 5.5$

(iii)	8	Does any value appear more often than any other? This is the mode. (No)	(iii)
	9	Write that there is no mode.	There is no mode.
(iv)	10	Range is (highest data value) – (lowest data value).	(iv) Range = 10 – 2 = 8

9.2 Measures of centre and spread

Navigator

1, 2, 3, 4, 5, 6, 7, 9, 10	1, 2 (a–d), 3, 4, 5, 6, 7, 8, 9, 10	1 (c–f), 2, 4, 5, 6, 7, 8, 9, 10, 11

Answers p. 702

Equipment required: calculator for Question **2**

Fluency

W.E. 4

1 Find the **(i)** mean, **(ii)** median, **(iii)** mode and **(iv)** range of each of the following sets of test results.

(a) 7, 1, 7 **(b)** 6, 6, 6, 6, 6 **(c)** 7, 6, 7, 5, 7, 9, 7, 8

(d) 8, 7, 9, 5, 8, 2, 3 **(e)** 4, 3, 6, 4, 8, 2, 5, 7, 6, 4 **(f)** 1, 6, 2, 7, 9, 8, 9

2 Find the **(i)** mean, **(ii)** median, **(iii)** mode and **(iv)** range of each of the following sets of data. Round your answers to 2 decimal places if necessary.

(a) 29, 36, 29, 23, 32, 25

(b) 67, 73, 72, 82, 50, 45, 76

(c) 56, 72, 39, 47, 89, 81, 63, 81

(d) 12.8, 6.5, 19.3, 14.2, 4.7, 6.8, 6.5, 11.3

(e) 16.3, 15.8, 11.4, 15.2, 12.3, 11.4, 13.5

(f) 11.01, 11.001, 10.1, 10.11, 10.01, 1.11, 11.11, 1.01, 11

Understanding

3 *Without doing any calculations*, in each case, choose which one of the alternatives could be the mean.

(a) 9, 7, 8, 7, 7, 8, 6, 6, 9

A 3 **B** 5 **C** 7 **D** 10

(b) 6, 6, 6, 7, 6, 7, 4, 7, 2, 7

A 2 **B** 4 **C** 6 **D** 8

(c) 12, 19, 14, 18, 11, 17

A 10 **B** 12 **C** 16 **D** 19

(d) 86, 74, 85, 68, 91, 76, 80, 90

A 74 **B** 81 **C** 89 **D** 91

4 The Korea–Japan 2002 World Cup in soccer was played in more stadiums than ever before. This record was not beaten in Germany in 2006 (12 stadiums), in South Africa in 2010 (10 stadiums) or in Brazil in 2014 (12 stadiums). The following table shows the city and capacity of the 20 different stadiums.

Japan	Capacity
Sapporo	42 000
Miyagi	49 000
Niigata	42 300
Ibaraki	42 000
Saitama	63 000
Shizuoka	50 600
Kobe	42 000
Osaka	50 000
Oita	43 000
Yokohama	70 000

Korea	Capacity
Seoul	63 961
Incheon	52 179
Suwon	43 188
Daejeon	40 407
Daegu	68 014
Jeonju	42 391
Gwangju	42 880
Ulsan	43 550
Busan	55 982
Seogwipo	42 256

(a) Find the **(i)** mean, **(ii)** median and **(iii)** range for the stadium capacities in Korea.

(b) Which stadium has closest to the mean capacity in Korea?

(c) Find the **(i)** mean, **(ii)** median and **(iii)** range for the stadium capacities in Japan.

(d) Which stadium has closest to the mean capacity in Japan?

(e) Use the mean and median to compare the sizes of the stadiums in the two countries.

(f) Find the **(i)** mean, **(ii)** median and **(iii)** range for all 20 stadium capacities.

(g) Which stadium is closest to the mean capacity for all 20 stadiums?

Reasoning

5 Consider the following test results: 7, 5, 6, 8, 3

(a) Do you expect the mean to be an actual data value? Explain your answer.

(b) Find the mean of these results. Is the mean an actual data value?

(c) Do you expect the median to be an actual data value? Explain your answer.

(d) Find the median for the test results above.

6 **(a)** Explain what is meant by the statement 'The average family has 2.3 children'.

(b) Explain why the median of a group of whole-number values must always either end in .5 or be a whole number.

(c) If the data values are all whole numbers, can the mode ever be a decimal? Explain why or why not.

7 A small company has a manager, an assistant manager, two office workers and 10 factory workers. The manager is paid $125 000 per year, the assistant manager $85 000, the office workers $55 000 each, and the factory workers $50 000 each.

(a) Find the **(i)** mean, **(ii)** median and **(iii)** mode for the annual income of all the people in the company, rounding your answer to the nearest cent.

(b) Which measure of centre gives you the best idea about the salaries paid by this company?

(c) Omit the manager's salary and recalculate the **(i)** mean, **(ii)** median and **(iii)** mode.

(d) Comment on what you found.

Open-ended

8 Fiona scored 6, 8, 7, 9 and 7 in the first five spelling tests of the term. If all the tests are out of 10, what scores could she get in the last five tests to average 8 out of 10 overall? (The 'average' here is the mean.)

9 As well as giving a set of results, make sure you include a short explanation of how you found your answers for each of the following.

(a) Find five test results for which the mean, median and mode are all equal to 6 if the test results are not all the same.

(b) Find five test results for which the mean and the median are 7 and the mode is 9.

(c) Find five test results for which the mean is 7 and the median and mode are 6.

10 Three students are finding the median of the following data values.
20, 17, 28, 23, 19, 21, 22, 20

Sam says 'The median is the fourth value, so is 23.'

Veran says 'No, you need to put the data values in order first, so the median is 20.'

Lakme says 'No, you are both wrong.'

Help Lakme explain to Sam and Veran why they are incorrect.

11 Nine friends went tree planting at their local creek. The median number of trees planted was 5, the mean was 6 and the mode was 4. Find a data set for the number of trees each person planted to match the given statistics.

Problem solving

Running the race

1 In a sprint, the first three runners had times of 12.1, 12.1 and 10.9 seconds. Find the mean, median and mode for the three times.

2 In a different race, the first three runners had a mean time of 11.7 seconds, a median time of 10.8 seconds and a modal time of 10.8 seconds. Find the times for the three runners.

3 In a third race, the first three runners had a mean time of 11.8 seconds and a median time of 11.9 seconds. If all three times are different what can you say about the first and third place getters?

Give at least three pairs of possible times for these two runners, using one decimal place accuracy in your solutions. Explain how you found your answers.

Strategy options

- Work backwards.
- Test all possible combinations.

Gamespace

Crack the code

You have been captured by Dr Code and find yourself in a locked room with the following message written on the door above three symbols: a red square, a yellow circle and a blue triangle. You know that this coded message tells you the order in which to press the symbols to open the door, but how do you crack the code?

Code #1

UWJXX YMJ DJQQTB HNWHQJ YMJS YMJ WJI
XVZFWJ FSI KNSFQQD YMJ GQZJ YWNFSLQJ

You decide to try a letter-shift code, knowing that E is the most frequently used letter in the English language. (In a letter-shift code, each letter is moved forwards or backwards by the same amount. For example, if the shift is 3, then A would be written as D, B as E, C as F, … X as A, Y as B and Z as C.)

1 (a) Which letter in the code appears the most often?

(b) What are the two possible shifts that would produce this match?

(c) Try the simpler of the two shifts. What does the message say?

Now you can escape the locked room!

So far, so good, but you now find yourself facing a much more difficult code to escape the next room. You know this because somebody has written, above the new code, 'Aha—not just a letter shift this time!'

You suspect that this time the letter substitution is random, not a shift. To find which letters have been substituted for each other, you will need to know the frequency that different letters are used in English. This frequency of different letters is shown in the following. This is based on the number of times each letter appears for every one hundred letters used in English.

Frequency of letters used in English per 100 letters

A	B	C	D	E	F	G	H
8.17	1.49	2.78	4.25	12.70	2.23	2.02	6.09

I	J	K	L	M	N	O	P
6.97	0.15	0.77	4.03	2.41	6.75	7.51	1.93

Q	R	S	T	U	V	W	X
0.10	5.99	6.33	9.06	2.76	0.98	2.36	0.15

Y	Z
1.97	0.07

Again, the code requires you to press some buttons that look the same as the last ones.

Code #2

Aha—not just a letter shift this time!

NRCD NCJG EYGDD NRG QOSG NYCFXUOG
NRGX NRG VGOOWT ACYAOG NTCAG FXP
BCXFOOV NRG YGP DUSFYG BWSY NCJGD

2 (a) Looking at the table of English letter frequencies, what are the top five most frequently used letters?

(b) Draw a frequency table for Code #2. Which letter is used most frequently? Put this against E in an alphabet chart. You may like to label the second-most frequently used letter too.

(c) Using your knowledge from Code #1 about the words Dr Code often uses in his messages, see if you can guess a few of the words. Use these to solve more of the letter substitutions in the code.

(d) What is the button sequence this time?

If you have broken the code you can escape.

9.3 Graphing univariate data

Information presented in tables is useful, but it is often much easier to interpret this information if it is displayed in a graph. There are many types of graphs you can use for displaying data. In this section you will explore graphs of **univariate data**, which is data that consists of only one variable. In statistics, a 'variable' is data that can vary, by having different values. Univariate data is data that is only about one thing: for example, the shoe sizes of students in a class.

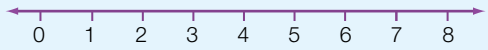
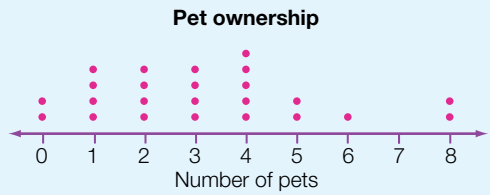
Dot plots

A **dot plot** is a simple statistical graph where each data value is represented by a dot, placed on a scale. When drawing a dot plot it is important to space the dots evenly to give an accurate picture of the data set. A dot plot can be used for categorical data such as car colour, or discrete numerical data such as the number of people who live in each of the houses on a street.

Worked example 5 W.E. 5

Draw a dot plot to illustrate the following data that represents the number of pets owned by each student in a class.

2, 5, 8, 6, 2, 4, 3, 4, 2, 1, 8, 3, 4, 0, 3, 1, 0, 3, 5, 1, 4, 2, 1, 4

Thinking	Working
1 Identify the lowest and the highest values and make these the end points of a scale.	
2 Complete the plot by marking a dot for each of the data values, being careful to space them evenly.	
3 Add the axis label and a brief title.	

Stem-and-leaf plots

When there is a large amount of numerical data, it is often convenient to combine several pieces of data into a group. **Stem-and-leaf plots** (sometimes just called stem plots) are useful because they can group data while still showing all the individual data values.

A stem-and-leaf plot consists of two parts, a stem and a leaf. Each data value must be divided into two parts so it can be plotted like this. For example, a score of 23 could be written as:

STEM	LEAF
2	3

The smallest place value digit of the number is the leaf, while the remaining part of the number is the stem. The leaf must only contain one digit from 0 to 9 to represent each data value. For example, 34.7 could be written as:

STEM	LEAF
34	7

Note: the decimal point is not shown on the plot.

A stem-and-leaf plot must always include a key to show the stem-and-leaf numbering.

For example: Key: 34 | 7 = 34.7

As another example, 536 could be written as:

STEM	LEAF
53	6

Key: 53 | 6 = 536

Before a stem-and-leaf plot is used to calculate statistics, you should arrange the numbers in the leaf section in ascending order to construct an ordered stem-and-leaf plot.

Worked example 6

W.E. 6

A class obtained the following percentage results (rounded to the nearest whole number) in a science test.

Express the following data as an ordered stem-and-leaf plot with a class interval of 10.

69, 54, 91, 42, 73, 75, 81, 96, 88, 76, 65, 83, 74, 76, 82, 61, 57, 64, 58, 79, 83, 90, 73, 60, 51

Thinking

1. Decide upon the stem you will use for your stem-and-leaf plot. This will be the key for your stem-and-leaf plot.
2. List the stems down the page in ascending order. In this case, the tens digit is the stem (4, 5, 6, 7, 8, 9). Write the key you are using beside the stem-and-leaf plot.
3. Work through the data in the order given. List the leaf of each individual result in the appropriate row. The leaf is the units digit.
4. Order the data in the leaf part of the stem-and-leaf plot, putting the lowest value closest to the stem. Add a title.

Working

STEM	LEAF
4	2
5	4 7 8 1
6	9 5 1 4 0
7	3 5 6 4 6 9 3
8	1 8 3 2 3
9	1 6 0

Key: 5 | 4 = 54

Science test percentages

STEM	LEAF
4	2
5	1 4 7 8
6	0 1 4 5 9
7	3 3 4 5 6 6 9
8	1 2 3 3 8
9	0 1 6

Key: 5 | 4 = 54

It is important that there are enough stems to spread the data into useful groups. When data values are very close together, each division of the stem may be divided into two parts, a lower and an upper section. The lower part has the digits 0 to 4 in the leaf and the upper part has the digits 5 to 9 in the leaf. For example, consider the following data set.

21, 32, 41, 22, 25, 29, 33, 42, 33, 44, 21, 28, 37, 40, 39, 36, 24, 32, 44, 30, 34

If you used the tens value as the stem you would have only three stems (2, 3 and 4). To spread out the data we can use 2_L for the lower values 20, 21, 22, 23, 24 and 2_U for the upper values 25, 26, 27, 28, 29. The stem-and-leaf plot then looks like this:

STEM	LEAF
2_L	1 1 2 4
2_U	5 8 9
3_L	0 2 2 3 3 4
3_U	6 7 9
4_L	0 1 2 4 4

Key: 3 | 2 = 32

Outliers

Sometimes in a data set, there may be values that are quite different from the other data values. These extreme values may be **outliers**. Outliers can be errors made in recording or measuring data incorrectly, or the result of unusual circumstances, or simply the result of natural variation in the data. Outlier values are often not included when calculating the statistics, because they can distort the statistics in ways that are misleading.

Values can often be more easily identified as possible outliers by looking at the visual data in dot plots or stem-and-leaf plots.

Using dot plots and stem-and-leaf plots

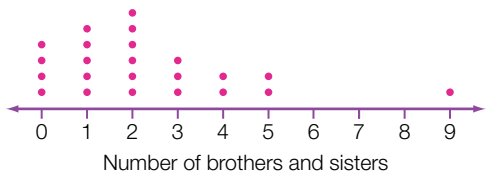
Dot plots and ordered stem-and-leaf plots can both be used to find the mean, median and range of the data set, because all the data values are included in both types of plots.

Worked example 7

W.E. 7

Use the graphs to calculate the mean, median and range for each of the following. Round the mean to 1 decimal place. Comment on whether there are any values that may be outlier in the data, and what effect these have on the statistics.

(a) Siblings for our class



(b) Museum's new display daily attendance

STEM	LEAF
15	6
16	5 5 6 9
17	0 3 7 7 7
18	4 6 8 9
19	0 2 5 9
20	0
21	
22	
23	5

Key: 15 | 5 = 155

Thinking	Working
(a) 1 Find the total of the data by multiplying the number of dots by the data value and adding the products. Find the number of data by counting all the dots. Find the mean using the formula: $\text{Mean} = \frac{\text{total of data}}{\text{number of data}}$	(a) Total of data: $0 \times 4 + 1 \times 5 + 2 \times 6 + 3 \times 3 + 4 \times 2 + 5 \times 2 + 9 \times 1$ $= 0 + 5 + 12 + 9 + 8 + 10 + 9$ $= 53$ Number of data: $4 + 5 + 6 + 3 + 2 + 2 + 1 = 23$ $\text{Mean} = \frac{53}{23}$ $= 2.304\ldots$ ≈ 2.3
2 Identify the middle value to find the median. (As there are 23 values here, the middle value is the 12th value.)	Median = 2
3 Find the range by subtracting the lowest value from the highest value.	Range: $9 - 0 = 9$
4 Comment on any possible outliers and their effect on the values of the mean, median and range.	9 is a possible outlier. The median is not affected. The mean has been pulled to the right. The range is larger than it would have been.
(b) 1 Read each data value (multiplying if the value appears more than once) and add to find the total of the data. Find the number of data by counting the leaves. Find the mean using the formula: $\text{Mean} = \frac{\text{total of data}}{\text{number of data}}$	(b) Total of data: $156 + 165 \times 2 + 166 + 169 + 170 + 173 + 177 \times 3 + 184 + 186 + 188 + 189 + 190 + 192 + 195 + 199 + 200 + 235$ $= 3653$ Number of data: $1 + 4 + 5 + 4 + 4 + 1 + 1 = 20$ $\text{Mean} = \frac{3653}{20}$ $= 182.65$ ≈ 182.7
2 Identify the middle values and use them to find the median.	$\text{Median} = \frac{177 + 184}{2}$ $= 180.5$
3 Find the range by subtracting the lowest value from the highest value.	Range: $235 - 156 = 79$
4 Comment on any possible outliers and their effect on the values of the mean, median and range.	235 is a possible outlier. The median is not affected. The mean is larger than it would have been. The range is larger than it would have been.

Column graphs and bar graphs

Column graphs present data in vertical columns, with a space between each column.
Bar graphs are the same as column graphs except the bars are horizontal. Column or bar graphs should usually be used for categorical data.

Drawing a column graph

All column graphs should include:

1. a title
2. categories with equally spaced intervals (gaps) between the numbers on the horizontal axis
3. the frequency starting from zero and going up in regular intervals on the vertical axis
4. a label for the horizontal axis that explains the variable being represented
5. a label for the vertical axis that shows the frequency
6. a small gap between each column (usually half a column width) to show that this is categorical or discrete (counted) data.

Worked example 8 W.E. 8

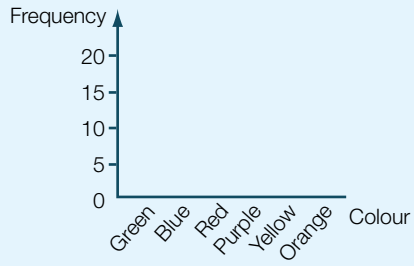
The frequency table below shows the responses from a group of students who were asked 'What is your favourite colour?' Only one response per student was recorded. Draw a column graph to represent this information.

Colour	Green	Blue	Red	Purple	Yellow	Orange
Frequency	12	15	19	11	5	13

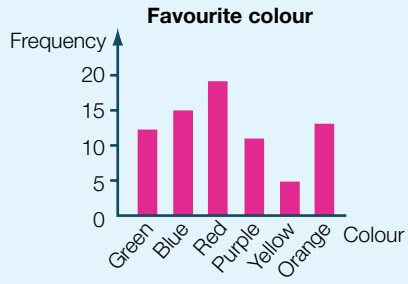
Thinking

1 Draw axes. Scale and label the vertical axis appropriately. Space the categories evenly across the horizontal axis. Label this axis.

Working



2 Use the frequency values to draw a rectangular column for each category. (Here, each rectangle represents a different colour.) The length of the rectangle indicates the frequency. Ensure that each rectangle has the same width on the horizontal axis. Insert a small space between each rectangle. Add a title.



Histograms

Histograms present data in vertical columns like column graphs. The columns have no space between them and each column includes a range of values rather than a single value. Histograms should be used for continuous numerical data.

Drawing a histogram

All histograms should include:

1. a title
2. a scale, with regular intervals on the horizontal axis, for the variable being represented
3. a label for the horizontal axis that explains the variable that is being represented
4. frequency starting from zero and going up in regular intervals on the vertical axis
5. a label for the vertical axis that shows the frequency
6. no gap between each column (except where there is a frequency of zero)
7. a half-column width before the first column at the start, even if the first set of data starts at zero.

Worked example 9

W.E. 9

100 Year 7 students were surveyed about the time they spent watching TV over the weekend. The data is presented below.

Time (h)	0–<1	1–<2	2–<3	3–<4	4–<5	5–<6
Number of students	8	17	27	32	10	6

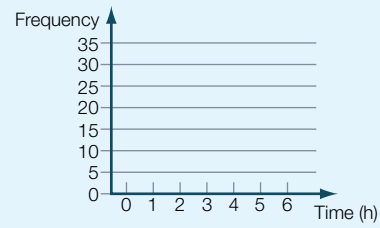
Draw a histogram to represent this data.

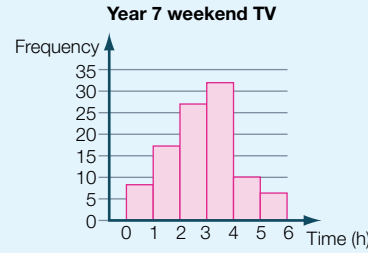
Thinking

1 Draw axes; scale and label the axes appropriately.

2 Ensure rectangles each have the same width with no gaps (unless there is a frequency of zero). Add a title.

Working





9.3 Graphing univariate data

Navigator

Answers p. 703

1, 2, 3, 4, 5, 6, 8, 9, 11, 13 (a–c), 14, 15, 16	1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 13 (a–e), 14, 15, 16	1, 2, 3, 4, 5 (b), 6 (b), 7, 8, 10, 11, 12, 13, 14, 15, 16

Fluency

W.E. 5

1 Draw a dot plot to illustrate the following data that represents the number of birds counted by each member of a school group on an excursion to a wetlands area.

12, 18, 12, 16, 19, 13, 19, 20, 17, 15, 20, 21, 18, 15, 20, 18, 21, 21, 17, 12, 17, 19, 12, 21

W.E. 6

2 Express each of the following data as an ordered stem-and-leaf plot with a class interval of 10.

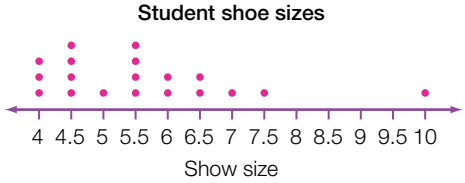
(a) The number of cinema visits in a year by a group of students:
57, 48, 42, 41, 31, 22, 37, 30, 23, 28, 34, 52

(b) The number of books read so far this year by the library monitors:
25, 44, 15, 29, 37, 26, 33, 35, 14, 8, 2, 27, 30, 42, 34, 21, 18, 3

W.E. 7

3 Calculate the mean, median and range for each of the following graphs. Round the mean values to 1 decimal place. Comment on whether there may be any outliers and what effect this has on the mean, median and range.

(a)



(b) AFL opening round points

STEM	LEAF
5	5 6 9
6	7
7	5 7
8	8 8 8 9
9	9
10	0
11	0 2 5
12	1

Key: 8 \| 8 = 88

W.E. 8

4 The frequency table below shows the colour of cars passing the school in a particular timed period. Draw a column graph to represent this information.

Colour	White	Red	Green	Blue	Silver	Other
Frequency	42	6	5	12	18	17

W.E. 9

5 The owner of a car dealership wants a graph of the number of cars sold plotted against the value of the cars sold in the last year. The data is presented below.

Value of car	Under $10 000	$10 000– < $20 000	$20 000– < $30 000	$30 000– < $40 000	$40 000– < $50 000	$50 000– < $60 000
Number of cars sold	25	73	132	43	18	9

Draw a histogram to represent this data.

6 Express the following data as ordered stem-and-leaf plots with class intervals of 5.

(a) 23, 28, 34, 27, 38, 42, 31, 21, 22, 27, 20

(b) 48, 57, 56, 53, 68, 64, 47, 58, 52, 51, 61, 42, 47

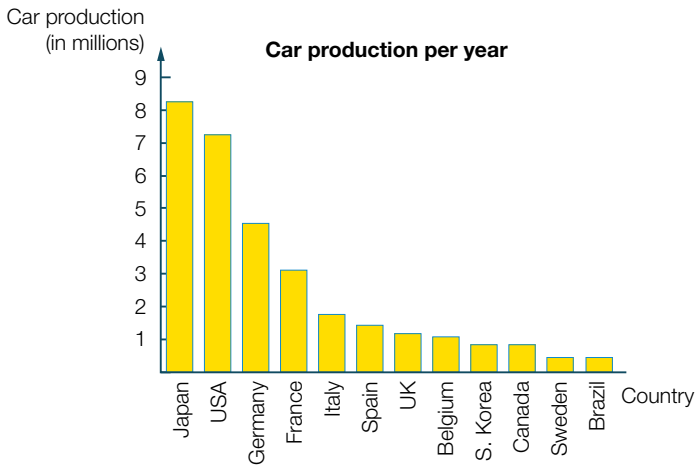
Understanding

7 The number of chocolate bars produced at a factory every hour over a 20-hour period is 146, 128, 133, 138, 140, 142, 125, 129, 136, 130, 133, 142, 126, 129, 150, 134, 126, 121, 128, 143.

(a) Draw an ordered stem-and-leaf plot of the data if it is expressed in intervals of 10. Explain why using intervals of 10 is not ideal for this data set.

(b) Using class intervals of 5, produce an ordered stem-and-leaf plot of the data.

8 This column graph shows the numbers of cars produced in a year by the world's major car-producing countries.



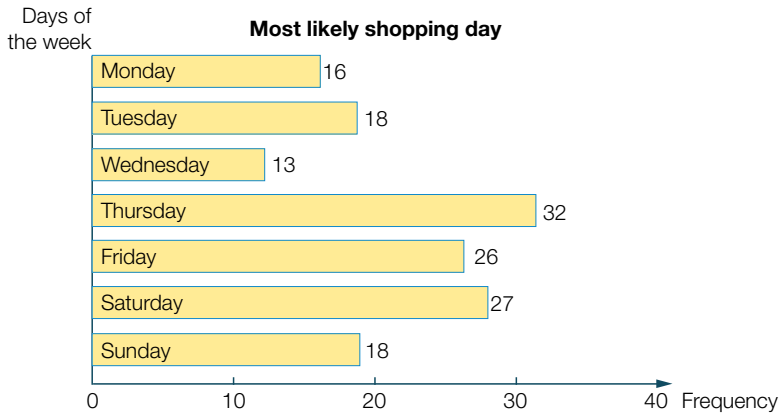
(a) Which country produces the most cars?

A USA **B** Germany **C** Japan **D** Brazil

(b) The number of cars produced by Germany is:

A 4.5 **B** 450 000 **C** 4 500 000 **D** 45 000 000

9 The bar graph below shows the results for a survey of 150 people as to which day they were most likely to do their supermarket shopping. The actual numbers are included on the graph to make it easier for you to calculate your answers.

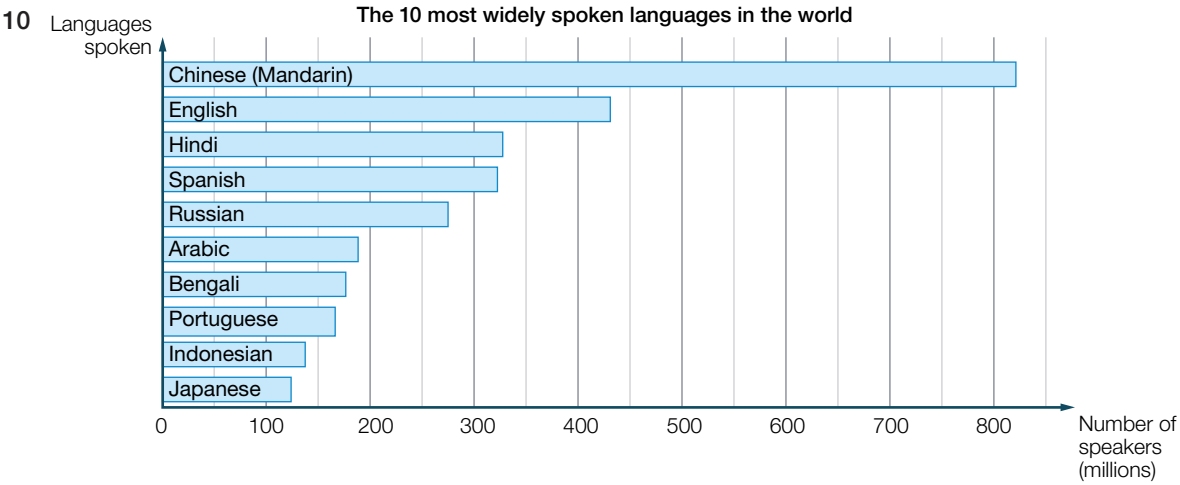


(a) Which day was most popular?

(b) How many people chose the least popular day?

(c) How many people chose the weekend?

10



The horizontal bar graph above shows the 10 most widely spoken languages in the world.

(a) What are the differences between this and a column graph?

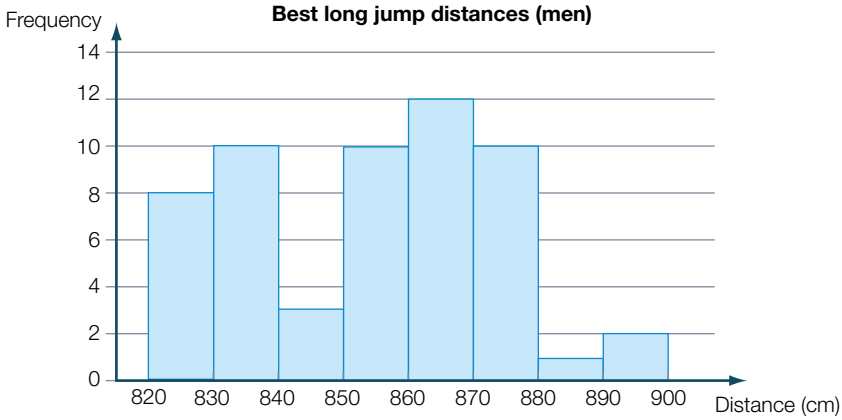
(b) Approximately how many people in the world speak Mandarin?

(c) Approximately how many people in the world speak English?

(d) What is the fourth most-spoken language in the world?

Reasoning

11



The histogram above shows the best long jump distances each year for men recorded from 1960 to 2015.

(a) What is the class interval?

(b) What is the range of the recorded distances? Why is this value just an estimate?

(c) Which class intervals had ten recorded jumps?

(d) How many jumps over 870 cm were recorded in this time period?

(e) How many jumps are given?

12 A class of Year 7 students were asked how long it took them to get to school each day.

The times, rounded up to the nearest minute, are:
34, 13, 26, 38, 23, 15, 19, 30, 28, 40, 36, 25, 14, 7, 10, 11, 8, 24, 32, 69, 29, 32, 44, 21, 18, 22

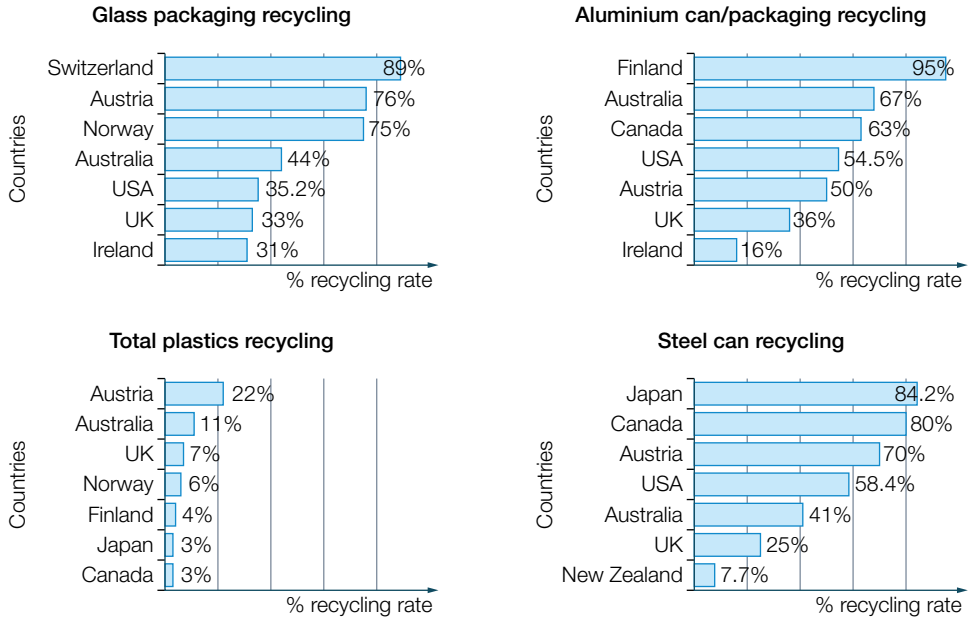
(a) Is this discrete or continuous data?

(b) Group the data with appropriate class intervals.

(c) Draw an appropriate graph to represent the data.

(d) Is there a possible outlier? Why or why not?

13 Compare the four recycling graphs to answer the following questions.



(a) Does Australia or Canada have a better recycling rate for aluminium?

(b) Does Australia or Canada have a better recycling rate for steel cans?

(c) Australia, the UK and Austria appear in all four graphs. Which country seems to be the worst recycler of the three?

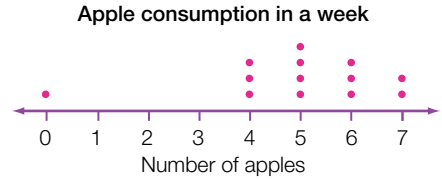
(d) Which kind of recycling does Australia seem to be best at?

(e) Which kind of recycling does Australia seem to be worst at?

(f) Compare Australia's recycling rate in all four areas with the best recycling countries on the graphs given.

(g) Compare Australia's recycling rate in all four areas with the worst recycling countries on the graphs given.

14 The dot plot shows the number of apples eaten in a week by a group of 13 students.



(a) Which measure of centre would you expect to be higher, the mean or the median? Calculate both to confirm your answer. Round the mean correct to 1 decimal place.

(b) Suggest a reason for any possible outliers.

(c) If the extreme value was replaced with the nearest data value, what happens (if anything) to the values of the mean, median and range? Give the value of any changes.

Open-ended

15 Complete the following.

(a) Determine the heights in centimetres of every student in your class, rounded to the nearest whole number. Organise this data into a stem-and-leaf plot with class intervals of 10 cm. If there are fewer than 5 groups, redo the display using class intervals of 5 cm.

(b) Collect some discrete data from your class, such as number of brothers and sisters or number of pets. Display the data as a dot plot.

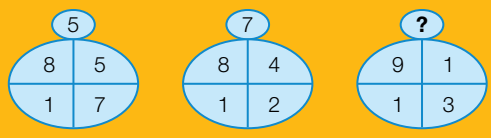
(c) Collect some categorical data from your class, such as main exterior house colour. Display the data as a column graph.

16 Collect some **(a)** categorical, **(b)** discrete and **(c)** continuous data from the students in your class and draw an appropriate graph to represent each.

Puzzle

Number puzzles

1 Find the **?** number that completes the sequence of diagrams:



2 Find as many solutions as you can for the following, where A, B and C are all different and are all positive whole numbers less than ten. Each question can have new values for A, B and C.

$A \times A = B \times C$ $\quad$ $A \times A = B + C$

$A \times A = B \div C$ $\quad$ $A + A = B \times C$

Comparing parts of a whole with graphs

9.4

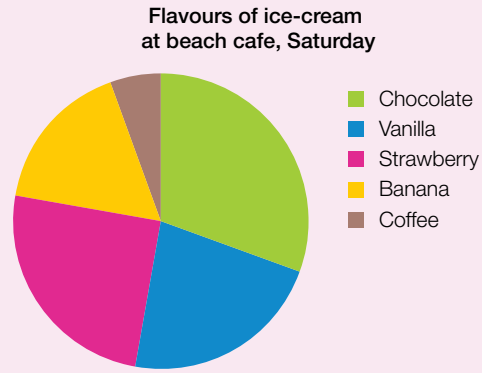
Often, data is given that is about the various parts of a whole. For example, a budget shows how much out of a total (limited) amount is spent on individual items. The two types of graphs used to display this type of data are sector graphs (often called pie graphs or pie charts) and divided bar graphs. Both of these are used to show a whole divided into parts.

Sector graphs (pie graphs)

A sector graph is often called a pie graph because it looks like a pie divided into pieces from the centre of the circle.

Drawing sector graphs

1. Add the parts (the data) to find the total.
2. Divide each part by the total to find it as a fractional part of the whole.
3. Multiply each fractional part by 360° to find the part's sector angle.
4. Use a protractor to divide a circle into the various parts and draw your graph.
5. Sector graphs should have labels or a key that identifies each of the sectors. It should also have a title.



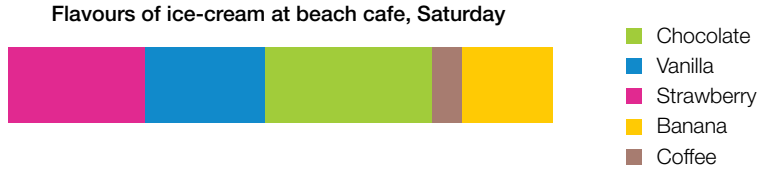
Divided bar graphs

Another way of showing parts of a whole is to divide a rectangle according to the size of the pieces.

Drawing divided bar or column graphs

1. Draw a rectangle, making the length equal to the total of each item.
2. To work out the total (as well as the length of each piece), multiply each data value by a common multiple.

The sector graph above can be represented by a divided bar graph as shown below.



Worked example 10

W.E. 10

All Year 7 students at a certain school were surveyed about their favourite type of pet and the responses were recorded in the table opposite.

Type of pet	Number recorded
Dog	78
Cat	65
Bird	39
Rabbit	26
Other	52

For the data in the table:

(a) draw a sector graph

(b) draw a divided bar graph.

Thinking

(a) 1 Add the number of responses to find a total.

2 Find the fraction of the whole by dividing the number recorded for each type by the total and record your results in a table.

3 Multiply the fraction for each type by 360° and round your answer to the nearest degree. Check that the sum of your angles is 360°. (It may be slightly more or less after rounding.)

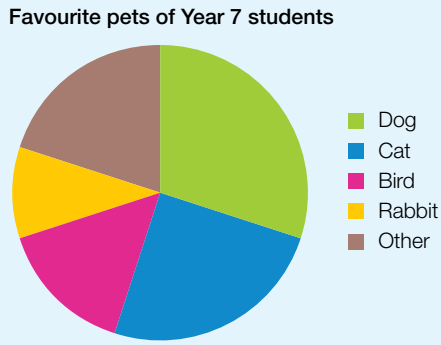
Working

(a) 260

Type of pet	Number recorded	Fraction of a whole
Dog	78	$\frac{78}{260}$
Cat	65	$\frac{65}{260}$
Bird	39	$\frac{39}{260}$
Rabbit	26	$\frac{26}{260}$
Other	52	$\frac{52}{260}$
Total	260	1

Type of pet	Number recorded	Fraction of a whole	Sector angle (°)
Dog	78	$\frac{78}{260}$	108
Cat	65	$\frac{65}{260}$	90
Bird	39	$\frac{39}{260}$	54
Rabbit	26	$\frac{26}{260}$	36
Other	52	$\frac{52}{260}$	72
Total	260	1	360

4 Rule a line from the centre to the edge of the circle. Use this line and a protractor to measure the sector angles and rule each sector. Give your graph an informative title and provide a key for each of the sectors.



(b) 1 Add the number of responses to find a total.

(b) 260

2 Choose a length that is a multiple or a factor of the total. (As 130 mm is half of 260, draw a line 13 cm long.) Multiply by the same factor or multiple to find the length of each section of the graph. (Multiply each recorded response by $\frac{1}{2}$.)

Type of pet	Number recorded	Length of line (mm)
Dog	78	39
Cat	65	32.5
Bird	39	19.5
Rabbit	26	13
Other	52	26
Total	260	130

3 Draw a divided bar graph using a ruler to measure the lengths of each section.

9.4 Comparing parts of a whole with graphs

Navigator

Answers p. 704

1, 2, 3, 4, 5, 6 (a–d), 7, 8 (a–b), 9, 11, 12	1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12	1, 3, 4, 6, 7, 8, 9, 10, 11, 12

Equipment required: protractor for Questions **1**, **2**, **7**

Fluency

W.E. 10

1 For the data in each of the following tables:

(i) draw a sector graph

(ii) draw a divided bar graph.

(a) Belinda picked some vegetables from her garden and recorded the number of each type of vegetable.

Vegetable	Number recorded
Tomatoes	20
Carrots	12
Cucumbers	8
Lettuce	10

(b) A group of tourists was asked their nationality and their responses were recorded.

Nationality	Number recorded
Italian	22
French	18
Japanese	12
Indonesian	8

(c) The number of pizzas purchased one evening at Abracadabra Pizza Parlour was recorded.

Pizzas	Number recorded
Super Supreme	6
Capricciosa	8
Hawaiian	4
Italiana	3
Australiana	2

(d) The number of different cars in the staff car park at a small secondary school was recorded.

Car make	Number recorded
Ford	4
Holden	5
Mazda	4
Toyota	3
Mitsubishi	2
Other	5

(e) Fifty songs played on the radio were categorised by genre.

Music style	Rock	Hip-hop	Dance	R&B	Punk	Pop
Number	10	9	6	3	5	17

(f) Seventy-five people were asked their favourite vegetable.

Vegetable	Potato	Carrot	Pumpkin	Cabbage	Capsicum	Cauliflower
Number	33	22	9	4	4	3

(g) Justin has been working in a full-time job for 6 months. He was asked how he spent his money. He gave the following estimates.

Item	Rent	Food	Entertainment	Clothes	Other
Percentage	47	30	10	10	3

2 As part of a project to tackle climate change at a local level, a Victorian local council conducted a survey to see what percentage of greenhouse gas emissions was produced by each of its facilities and services. The following data was recorded: buildings 28%, streetlights 47%, waste 1%, vehicle fleet 24%.

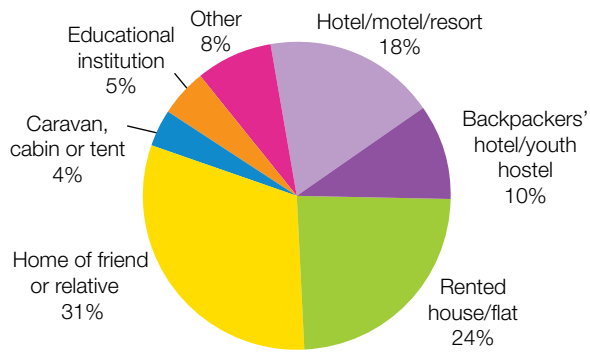
(a) Create a sector graph to display the data.

(b) Create a divided bar graph to display the data.

(c) Which graph do you find easier to understand?

Understanding

3 The sector graph shown represents information about the type of accommodation used by overseas visitors to Australia.



(a) One of the sectors uses an angle of 64.8°. Which one is it?

A Hotel/motel/resort

B Backpackers' hotel/youth hostel

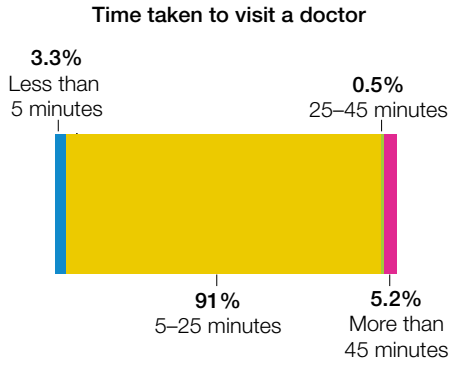
C Rented house/flat

D Other

(b) The 'Educational institution' sector uses an angle closest to:

A 5° **B** 10° **C** 13° **D** 18°

4 The following graph shows how much time people take when they visit a doctor.



(a) What percentage of people spend less than 5 minutes when they visit a doctor?

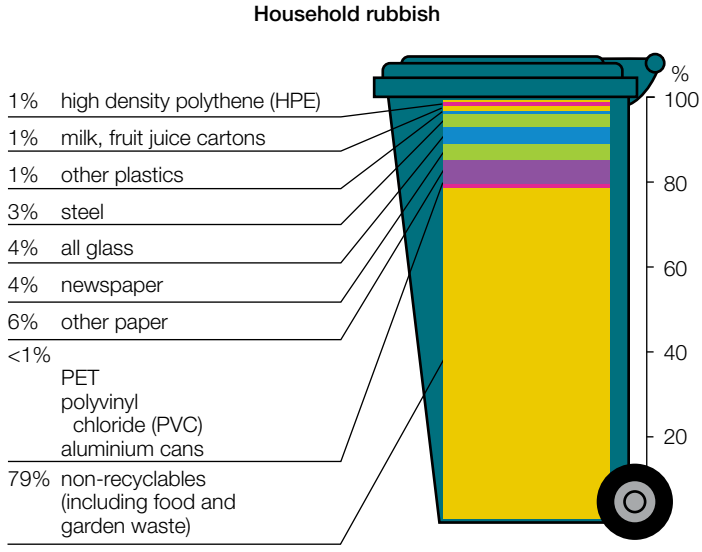
(b) What percentage of people spend more than 45 minutes when they visit a doctor?

(c) How long do the majority of people take when they visit the doctor?

(d) What percentage of people spend less than 25 minutes when they visit the doctor?

(e) The graph displays numerical data. Why is the data not suitable for display as a histogram?

5 Look at the following divided bar graph, which shows how much of different types of rubbish Australians throw out.



(a) What percentage of our rubbish is all glass?

(b) What percentage of our rubbish is milk and fruit juice cartons?

(c) What percentage of our rubbish is newspaper?

(d) **(i)** What is the largest category of rubbish?

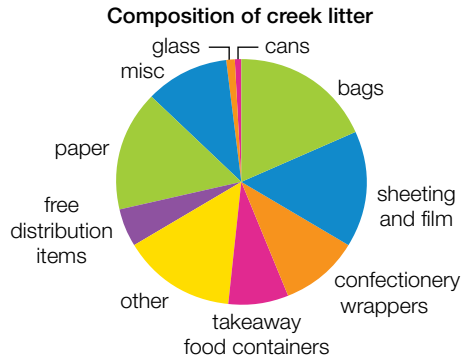
(ii) What percentage of rubbish is in this category?

(e) Do we throw out more high density polythene or more paper?

(f) Do we throw out more steel or more glass?

(g) Is the picture of the rubbish bin actually part of the graph?

6 Look at the following sector graph showing the type of litter in a particular creek.



(a) What was the most common type of litter?

(b) Was there more cans or more sheeting and film?

(c) Was there more takeaway food containers or more confectionery wrappers?

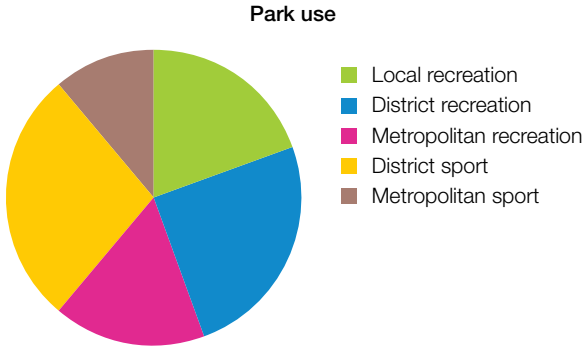
(d) When you compare this sector graph with the one in Question **3**, what is missing?

(e) 'Misc' stands for miscellaneous. What kinds of rubbish would be classified as miscellaneous?

(f) The graph also tells how many litter items were collected in the survey. Is this important? Why or why not?

(g) Why are sector graphs like divided bar graphs?

7 The following sector graph shows park use in a local council area of an Australian city.



(a) Use a protractor to find the angle size of each sector.

(b) Use your answer from part **(a)** to find the percentage of the park that is used for metropolitan recreation.

(c) Use your answer from part **(a)** to find the percentage of the park that is used for district sport, to the nearest whole number.

(d) Use your answer from part **(a)** to find the percentage of the park that is used for sport, to the nearest whole number.

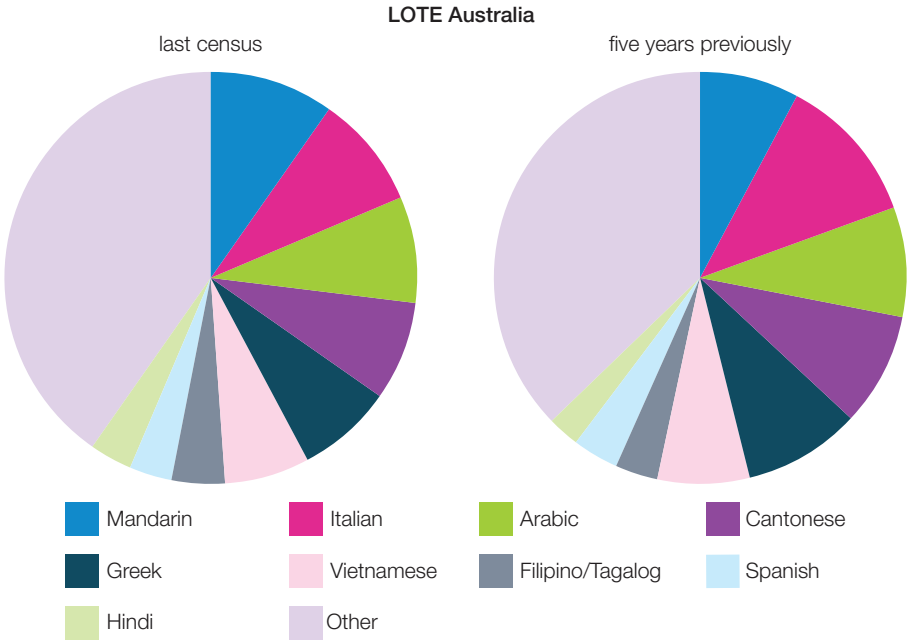
(e) Is more parkland used for sport or for recreation? State which is larger and by how much.

(f) What type of park use is represented by the smallest sector?

(g) Draw a divided bar graph of the data.

Reasoning

8 A census showed that 18% of Australians used languages other than English (LOTE) as their main language at home. The first graph shows the number of people currently using each of these languages, while the second graph shows the number of people using those languages five years previously.



(a) The portions for 'other' languages included languages with fewer people than the smallest shown with its own sector. What does this say about these 'other' languages?

(b) Which of the nine specified languages has had the biggest increase in proportional representation? Which decreased the most?

(c) If the number of people who speak Mandarin at home was 336 178 in the latest census, how many people speak Greek at home? Round your estimate to the nearest thousand.

9 The table shows the new car sales in Australia for July 2015 by the country where each car was made.

Place or origin	Number
Australia	7 819
Japan	26 293
Thailand	18 906
Korea	11 056
Germany	7 299
USA	4 543
England	2 809
South Africa	1 979
Spain	1 750
India	1 711
Other	8 143
Total	92 308

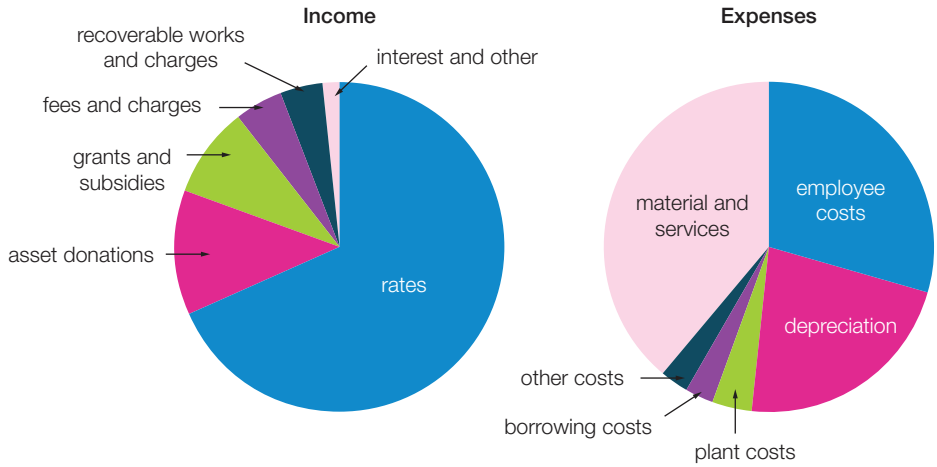
(a) Describe the order of the countries in the list. Explain why this order was used.

(b) If you draw a divided bar graph 92 mm long, how long will the section representing Australia be?

(c) Construct the divided bar graph with the countries in the same order as the table.

(d) From your graph (and confirmed from the table) complete the sentence: 'More than 60% of the new cars sold in Australia in July 2015 were imported from _____, _____ or _____.'

10 The graphs give the income and expenses of Logan City Council over a recent financial year.



(a) Is it possible to tell from these graphs whether or not the income from rates is enough to pay for 'material and services' expenses as well as employee costs? Explain why or why not.

(b) If the income from rates is $353.3m (million), estimate the total income. Write your answer to the nearest $1m.

(c) If the employee costs are $123.5m (million), estimate the total expenses. Write your answer to the nearest $1m.

(d) Budget surplus is the difference between income and expenses, so long as income is higher than expenses. Find Logan city council's budget surplus for the financial year.

Open-ended

11 Draw a sector graph to represent one characteristic of the students in your class, such as 'type of transport to get to school' or 'hair colour'.

12 Draw a divided bar graph to represent a different characteristic of the students in your class, such as 'favourite holiday' destinations.

Problem solving

Missing pieces of the pie

1 A sector graph is drawn and labelled, but there is a 60° gap in it. This needs to be divided into two pieces, *A* and *B*, so that *A* is twice the size of *B*. How big will the two pieces be? (What will be their angles?) Give your answers correct to 1 decimal place if necessary.

2 This time, a 60° piece is to be divided into three pieces, *A*, *B* and *C*, where *A* is twice as big as *B* and *B* is twice as big as *C*. How big will the three pieces be? Give your answers correct to 1 decimal place if necessary.

3 This time, a 60° piece is to be divided into four pieces, *A*, *B*, *C*, and *D*, where *A* is twice as big as *B*, *B* is twice as big as *C* and *C* is twice as big as *D*. How big will the four pieces be? Give your answers correct to 1 decimal place if necessary.

Strategy options

- Guess and check.
- Look for a pattern.

Half-time 9

9.2

1 Find the **(i)** mean (correct to 1 decimal place), **(ii)** median, **(iii)** mode and **(iv)** range for each of the following sets of test results.

(a) 3, 2, 4, 8, 6, 2, 4, 6, 3, 2, 1 **(b)** 4, 6, 3, 4, 3, 6, 5, 9, 8, 7, 6, 3, 2, 4, 5, 6, 4

9.4

2 Using the table of data:

(a) draw a sector graph **(b)** draw a divided bar graph.

Victoria's greenhouse gas emissions (end use allocation of emissions)	
Residential	17%
Commercial	16%
Manufacturing	32%
Passenger transport	14%
Primary production net	12%
Freight transport	5%
Waste	3%
Other	1%

9.1

3 The height (in cm) of each of the 25 students in a class was recorded.

142	147	151	142	146	151	157	138	141
140	129	135	142	151	150	160	141	152
148	157	130	143	147	149	154		

(a) Why are class intervals of 125–129, 130–134 etc. appropriate?

(b) Using these class intervals, draw a frequency table showing this information.

9.1

4 The number of students absent from class was recorded for each of the 21 school days in a month.

3	0	1	2	4	1	0	1	0	2	0
1	0	0	2	1	3	0	1	2	0	

Construct a frequency table showing this information.

9.3

5 Draw a dot plot to illustrate the following data that represents the number of times the students in a class have travelled interstate.

6	4	0	1	2	3	1	2	2	0	0	0
1	2	4	5	3	2	5	3	0	1	1	2

9.3

6 The number of marks taken by the top 12 players in the AFL after round 15 of a season are given below.

86 74 76 91 132 84 67 80 82 62 85 90

(a) Draw a stem-and-leaf plot to represent this data.

(b) Find the median for this data set.

(c) Can you identify any possible outliers? Justify your answer.

Exploration Spreadsheet

Equipment required: Microsoft® Excel or similar spreadsheet software. (For Casio ClassPad CAS or TI-Nspire CAS, you can download instructions from the eBook or the Pearson Places website.)

Finding statistics using technology

When you have a very large data set it is best to use some form of technology to calculate statistics such as mean, mode, median and range. A spreadsheet is a common useful tool to do this.

The following table shows the height, in cm, of the 47 players on the list for an AFL team. A spreadsheet can be used to find the mean, median, mode and range for this data.

182	187	187	200	195	175	179	176	191	184
176	191	187	178	187	184	192	197	182	185
183	195	189	183	189	189	197	192	188	189
195	182	202	193	189	192	182	206	177	182
193	179	180	198	198	189	180			

1. In a new spreadsheet, enter the data in column **A**.
2. In column **B**, enter the labels for the statistics to be found: the Mean in cell **B1**, Median in **B2** and Mode in **B3**, as shown.
3. The range is the difference between the highest (maximum) value and the lowest (minimum) value. Enter the labels Max in cell **B4**, Min in **B5** and Range in **B6**.
4. You can now use the spreadsheet's built-in formulas to calculate these statistical values. In cell **C1**, enter the formula **=AVERAGE(A1:A47)** to find the mean.
5. Enter the formulas to find the median, mode, maximum and minimum in the corresponding cells (in column **C**), next to their labels. In Excel, under the **Formulas** tab select **Insert Function**, then find each appropriate function to use from the list. Then you can again select the cells **A1:A47** as the values for the function.
6. To find the range, write a formula to find the difference between the maximum and minimum values.

If you have trouble with these spreadsheet formulas, check the formulas shown in the image below. If you are using spreadsheet software other than Excel, the formulas may be different.

	A	B	C
1	182	Mean	187.7872
2	187	Median	188
3	187	Mode	189
4	200	Max	206
5	195	Min	175
6	175	Range	31
7	179		
8	176		
9	191		
10	184		
11	176		
12	191		
13	187		
14	178		
15	187		
16	184		
17	192		
18	197		
19	182		
20	185		
21	183		
22	195		
23	189		
24	183		
25	189		
26	189		
27	197		
28	192		
29	188		
30	189		
31	195		
32	182		
33	202		
34	193		
35	189		
36	192		
37	182		
38	206		
39	177		
40	182		
41	193		
42	179		
43	180		
44	198		
45	198		
46	189		
47	180		

B	C
Mean	=AVERAGE(A1:A47)
Median	=MEDIAN(A1:A47)
Mode	=MODE.SNGL(A1:A47)
Max	=MAX(A1:A47)
Min	=MIN(A1:A47)
Range	=C4-C5

Taking it further

7. The weights, in kg, of the 47 players on the list for an AFL football team are shown below.

   A spreadsheet can be used in a similar way to find the mean, median, mode and range of the weights.

88	84	88	102	85	77	88	77	89	72
84	87	82	81	91	85	88	101	83	100
84	95	87	88	85	92	99	87	79	92
99	70	108	102	93	94	86	105	80	73
92	68	80	94	104	88	80			

9.5 Graphing bivariate data

Univariate data consists of one variable. **Bivariate data** consists of two variables: an independent variable and a dependent variable. The dependent variable changes as it depends on changes to the independent variable.

For example, the distance travelled at a constant speed in a given time period is bivariate data: distance and time are the variables, the time is the independent variable and the distance travelled is the dependent variable (because the distance travelled depends on the time).

Line graphs

A line graph is used when you have data that shows changes over time. Line graphs are useful if you are trying to identify a **trend** in the data. That means you are looking for a general pattern that the data follows, which you could use to predict future values in the data set.

Line graphs are used for continuous data, such as mass, height, profits and temperatures. These are all data that can be measured over time.

Drawing a line graph

All line graphs should include:

1. a title
2. the independent variable on the horizontal axis
3. a scale across the horizontal axis that uses equally spaced intervals and is clearly labelled, including any relevant unit of measurement
4. the dependent variable on the vertical axis
5. a scale on the vertical axis that also uses equally spaced intervals and is clearly labelled, including any relevant unit of measurement.

When drawing line graphs you need to choose a suitable scale. The following data set is a useful example of this.

Worked example 11

W.E. 11

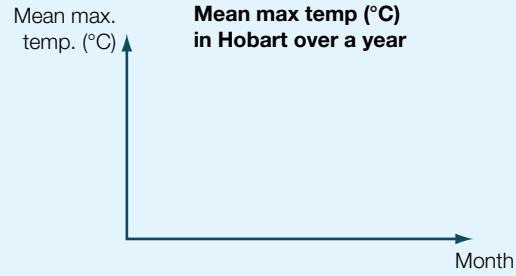
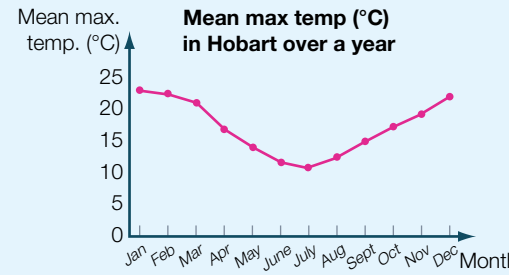
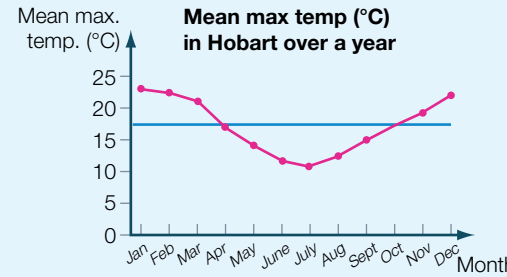
The mean daily maximum temperature (°C) for each month in Hobart (Botanical Gardens) is shown in the table below:

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Mean max. temp. (°C)	23.1	22.2	20.7	17.0	13.9	11.4	10.5	12.2	14.9	17.4	19.4	21.9

Bureau of Meteorology

(a) Draw a line graph to represent this data.

(b) For which months was the mean maximum temperature below 17 °C?

Thinking	Working
(a) 1 Draw the axes, label them and give the graph a title.	*(a)* 
2 Look for the highest value to be graphed. In this case, going up in intervals of 5 seems reasonable.	*Highest value = 23.1 °C*
3 Scale both axes and plot the pairs of values. (The first one to plot would be January, 23.1. Choose Jan on the horizontal axis and 23.1 on the vertical axis and plot the point.) Join each point to the next with a straight line.	
(b) 1 Draw a horizontal line across from the vertical axis for the given value (17 °C).	*(b)* 
2 Find the points on the line graph under this line and read off the values on the horizontal axis that correspond to these points.	*May, June, July, August, September*

The points in the line graph of the example above are joined by lines, which help you visualise the trend. However, it is important to note that these lines only show the trend from point to point, so you cannot read new data values from the lines between these points.

In some line graphs, the values between plotted points can be used to estimate data values in-between. The accuracy of estimates will depend on the type of data and the strength of the trend.

9.5 Graphing bivariate data

Navigator

Answers p. 706

1, 2, 3, 5, 7, 9, 10, 11 (a) | 1, 2, 3, 4, 5, 7, 9, 10, 11 | 1, 4, 5, 6, 7, 8, 9, 10, 11

Fluency

W.E. 11

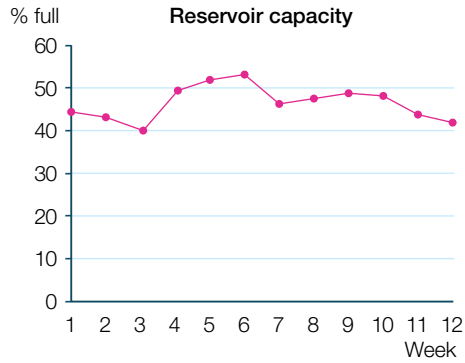
1 At an athletics competition, the wind speed is measured so that new records can be accepted. The following table shows the wind speed (in metres per second) at 10-minute intervals for the hour over which the 100 m sprints were run.

Time	1:00 pm	1:10 pm	1:20 pm	1:30 pm	1:40 pm	1:50 pm	2:00 pm
Wind speed (metres per second)	0.9	0.95	1.3	1.2	1.15	1.1	0.95

(a) Draw a line graph to represent this data.

(b) When was the recorded wind speed greater than 0.95 metres per second?

2 Each week, the amount of water in a reservoir is recorded as a percentage of its maximum capacity. The graph on the right shows the results for a number of weeks.



(a) How full was the reservoir when measured in week 4?

A 40% **B** 45% **C** 50% **D** 55%

(b) There was heavy rain during a particular week. The measurement for which week shows the result of this rain?

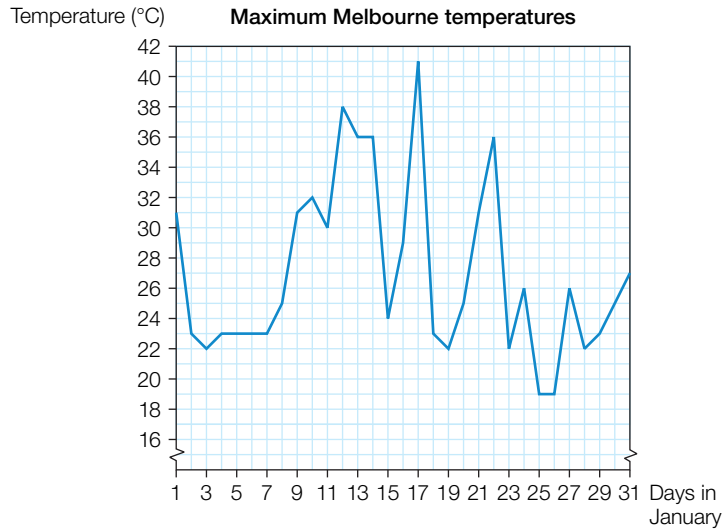
A 2 **B** 4 **C** 5 **D** 6

(c) A large quantity of water was released from the reservoir to flush out the downstream areas. The measurement for which week most likely shows the result of this action?

A 2 **B** 3 **C** 7 **D** 10

Understanding

3 The line graph opposite shows the maximum temperatures in Melbourne for January of a particular year.



(a) **(i)** What was the highest maximum temperature reached?

(ii) On which date was this temperature reached?

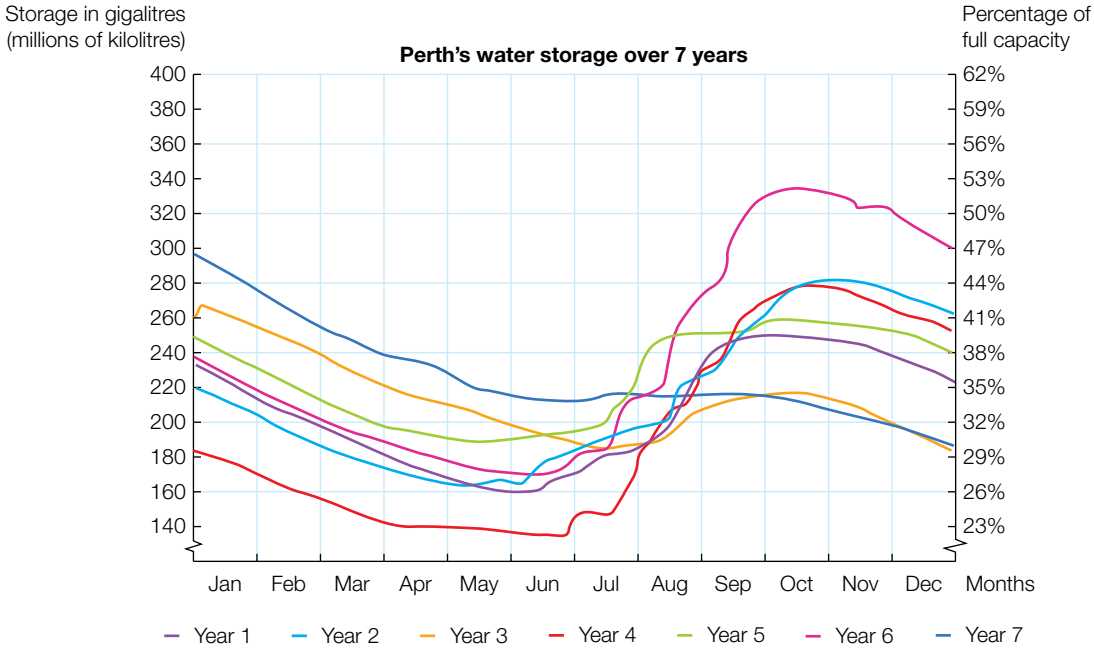
(b) **(i)** What was the lowest maximum temperature reached?

(ii) On which date(s) was this temperature reached?

(c) Up until 15 January, the maximum temperature was always between which two values?

(d) After 23 January, the maximum temperature was always between which two values?

4 Below is a graph of Perth's water storage over a 7-year period. Use this graph to answer the following questions.



(a) What was the storage percentage at the start and end of year 5?

(b) In which year was there almost no change in storage levels from the beginning of June to the end of September?

(c) Which 6-month period over the 7 years had the lowest water storage? What does this tell you about the rainfall in that period?

(d) By what percentage did the water storage levels decrease in December of year 6? Check the percentage drop in water storage levels for other years in December. What does this tell you about the rainfall in December in Perth?

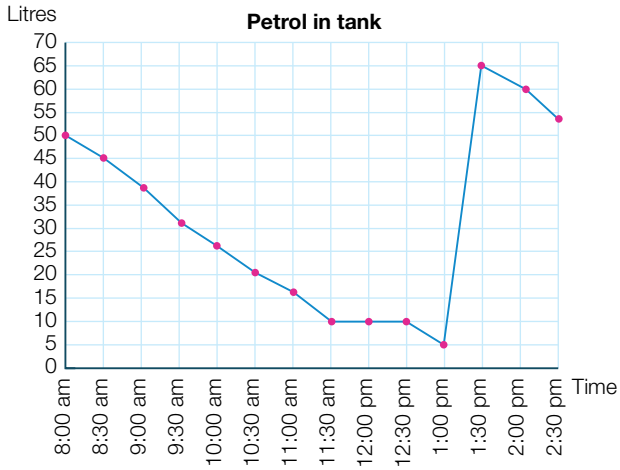
(e) Did the water storage levels increase or decrease during year 6? Find the increase or decrease percentage.

(f) From the data shown here, in which month is Perth likely to be the wettest?

(g) From the data shown here, in which month is Perth likely to be the driest?

Reasoning

5 The graph on the right shows the number of litres of petrol in a car at various times during the day.



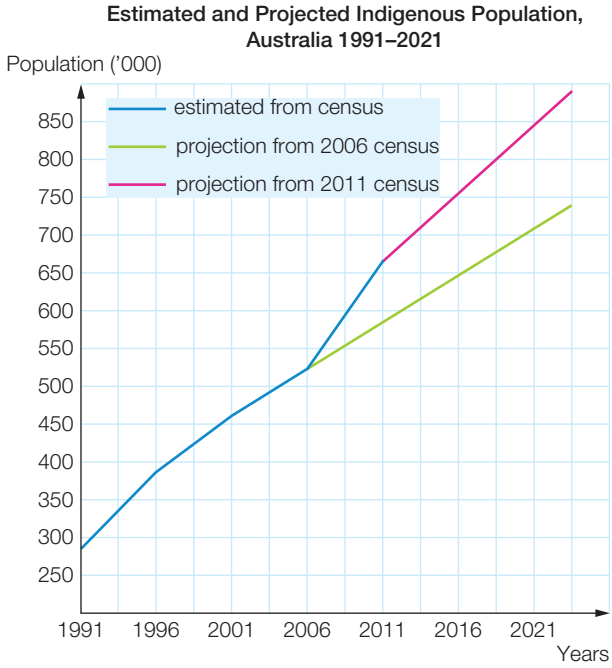
(a) How many litres were in the tank at 8 am when the journey began?

(b) When did the driver visit the petrol station? How much time did this take?

(c) What was the number of litres of petrol purchased?

(d) Between which times is it likely the driver had lunch?

6 This graph shows the Indigenous population of Australia since 1991, including estimates and projections based on Australian census data.



(a) Why are the years given as 1991, 1996, 2001 etc. rather than 1990, 1995, 2000 etc.?

(b) Why do you think the graph from 1991 to 2011 is an estimated graph?

(c) What was the estimated Indigenous population of Australia in 1991?

(d) What was the estimated Indigenous population of Australia 20 years later?

(e) Why do you think the value for 2011 projected from the 2006 census differed so much from the value estimated from the 2011 census?

(f) Give another reason why you think the Indigenous population may increase so much in the 30 years from 1991 to 2021.

7 Some estimated and projected figures for the population of Australia are given in the table below.

2001	2011	2021	2031	2041	2051	2061
19 275 000	22 340 000	26 452 000	30 501 000	34 295 000	37 956 000	41 513 000

(a) Draw a line graph by hand or using technology.

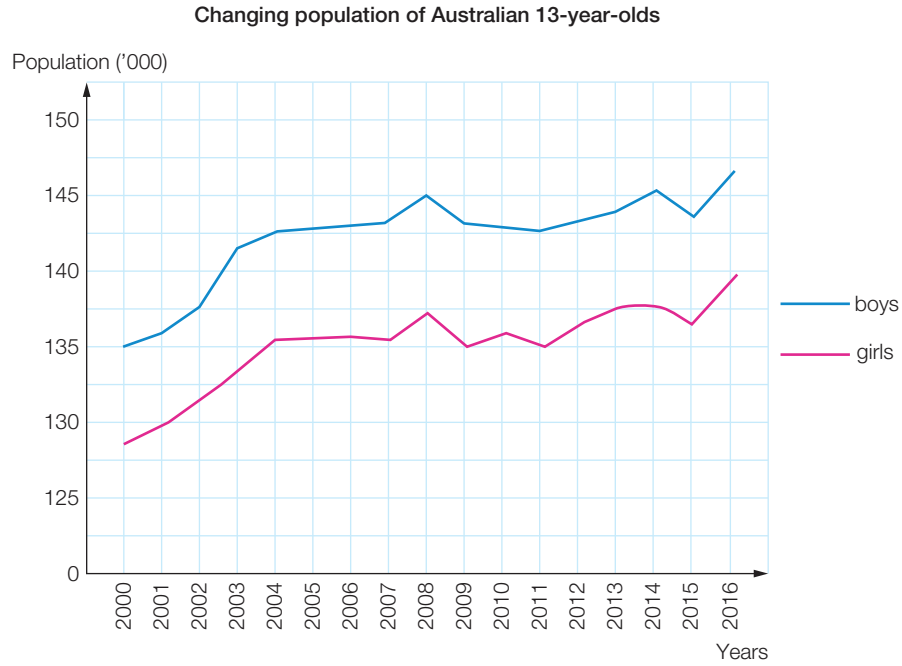
(b) Use your graph to predict the population in 2025.

(c) What is the predicted increase in the population over the entire period shown on the graph?

(d) What factors contribute to population increase? What factors contribute to population decrease?

(e) What needs to happen if the population of Australia is to stay stable (the same year after year)?

8 The graph shows the number of 13-year-old boys and girls living in Australia during the first part of this century.

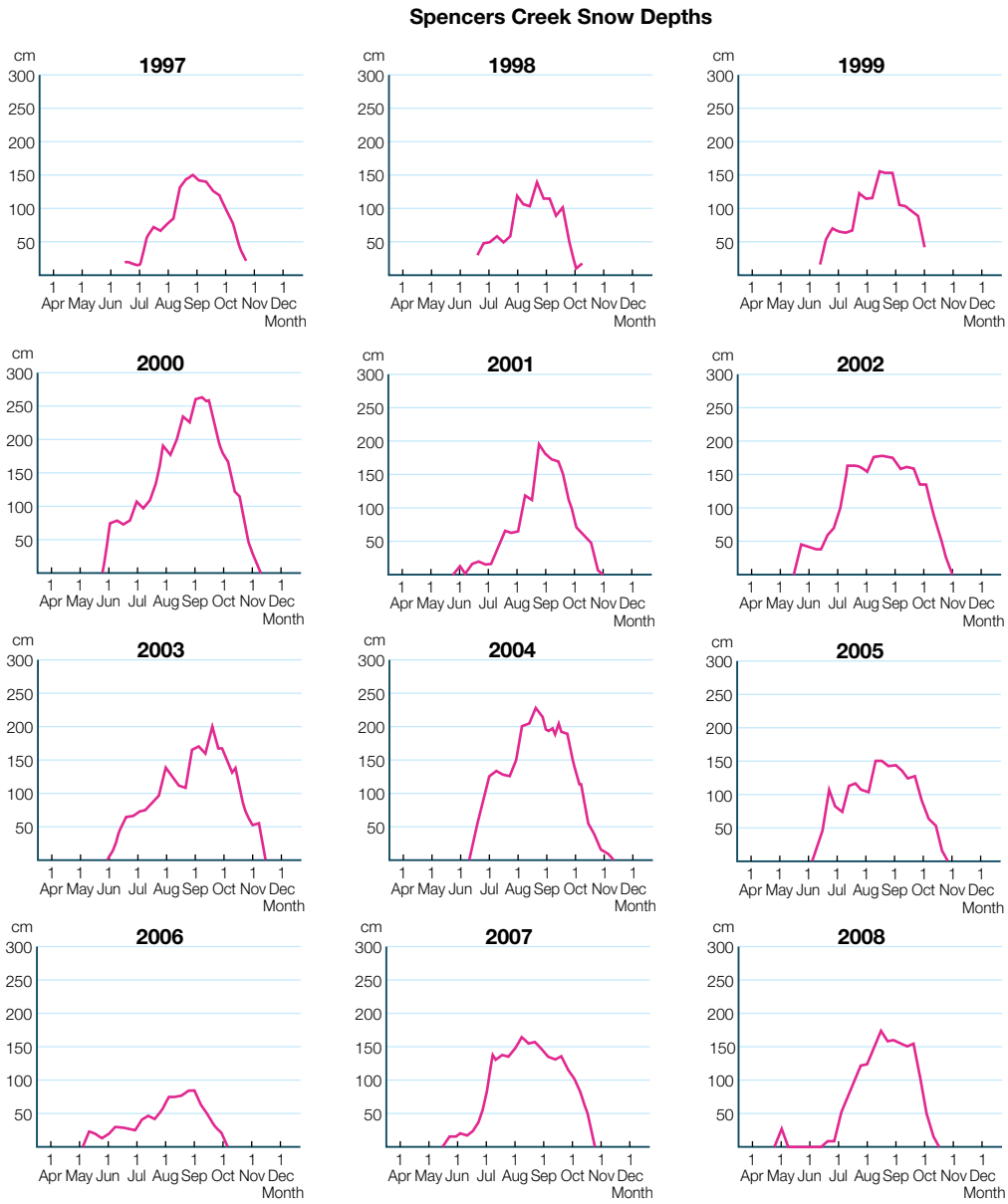


(a) Compare the shape and position of the two graph lines and say what this means about the number of boys compared to the number of girls.

(b) For overall numbers of 13-year-olds in Australia, compare the trend at the beginning of the century with the trend later on. Think of a possible explanation for the change.

(c) For overall numbers of 13-year-olds in Australia, what is the average annual percentage increase over the 16 years? Give your answer correct to one decimal place.

9 Look at the following line graphs that show the average snow depths (in centimetres) at Spencers Creek, Mt Kosciuszko, from 1997 to 2008. (The deeper the snow, the better it is for skiing.)



(a) Did the snow depth reach 2.5 metres at any time during the 12 years? If so, which year(s) did this happen?

(b) What was the greatest snow depth reached in 1997?

(c) What was the greatest snow depth reached in 2008?

(d) Which year had the most snow in June?

(e) Which year had the most snow through the whole of July?

(f) Which year had the worst October skiing conditions?

(g) Which year had the best October skiing conditions?

(h) Which year would have probably had the best November skiing conditions?

(i) Which year had the worst skiing conditions overall?

(j) Which month usually has the best skiing conditions?

(k) Which month is usually the second best skiing month?

(l) There is evidence (from other sources) that New South Wales' overall temperature has been rising due to climate change. Is there any evidence in these graphs to support this?

(m) Why use line graphs rather than bar graphs to show this information?

Open-ended

10 When do you think you feel hungriest during the day? Draw a line graph that shows your level of hunger during the day, from the time you wake up until the time you go to sleep. Do you think the shape of the graph would be similar for most people? Explain any differences you can think of.

11 The Australian Bureau of Statistics (ABS) produces a 'population pyramid' data set, available from their website, that gives estimated and projected values for the population of Australia. Produce a line graph from a portion of this data and give a summary of the trends, similarities and differences that you find, to answer the following questions.

(a) Compare the number of male and female 60-year-olds over the first part of this century. How does this compare with the population of male and female 13-year-olds?

(b) What happens to the numbers in the population to a group as they age? For example, look at the numbers of 13-year-olds in one year and 14-year-olds in the next year. Can you explain the changes from year to year?

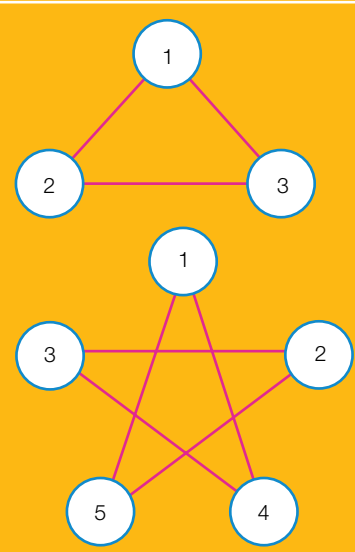
Problem solving

Add it up

A game board consists of the numbers 1, 2 and 3 arranged in a triangle as shown. The first player puts a counter on any of the three numbers and takes that number as their score. The other player then moves the counter to one of the other numbers and takes that as their score. The first player moves the counter and adds the value to their total. Continue in this way until a player gets a total of 12. If you go over 12 you lose.

Explain the strategy you would follow if you were playing this game. You might like to play the game with a friend to help you come up with a strategy.

Now, make up a strategy for the second game board shown. What would be a suitable target number for this board?



Maths 4 Real

The 1-watt initiative

To reduce our 'greenhouse gas emissions footprint' we should reduce the amount of electricity we use in our households. The unit that measures how much electrical power is used is called the watt and has the symbol W.

When an electronic device such as a television is off, it is often not completely off. It is usually in what is called standby mode, which still uses some power, so that it can be turned on again more quickly. It has been estimated that standby power is responsible for 5–10% of total electricity use in most homes.

1 How many devices can you think of in your home that would be in standby mode when not being used? Make a list of them now.

How big an impact does this have?

As a simple example, you might have a microwave oven in your house that has a digital clock built into it. If you used the microwave for only an average of 10 minutes each day to cook food, then about 21% of its total energy consumption will be just the energy used to display the clock!

The following table shows how much power some electronic devices use per hour.

Appliance	Off (W)	Passive standby (W)	Active standby (W)	Delay start (W)
air conditioner	0.2	1.7		3.0
clothes dryer	0.2		2.4	3.8
dishwasher	0.6		2.7	3.0
front-loading washing machine	0.9		3.5	4.0
top-loading washing machine	1.2		3.0	3.5
microwave oven		3.0		
laptop computer	1.4			
computer monitor	0.9	1.9		
inkjet printer	0.8	3.8		
LCD TV	0.5	1.6		
DVD recorder		7.3	21.5	
set-top box		8.4	13.5	

'Off' means the device is switched off at the device but not at the wall. Some appliances continue to use electricity even when you think they are off!

'Passive standby' means the device is not performing its main function but is ready to be switched on quickly, or is performing a secondary function, like a clock on a microwave oven.

'Active standby' means the device is on but not performing its main function. For example, a DVD recorder may be on but is not playing or recording.

'Delay start' means the device has been programmed to start at a later time.

2 As you read across the table, the energy use increases. Suggest a reason for this.

How much does all of this cost us?

As an example, consider an air conditioner in passive standby mode. According to the table this uses 1.7 W of power. This means that it uses an amount of energy every hour, which is 1.7 W h (watt hours). You can multiply this by 24 to get a daily use (energy used in 24 hours). This can then be multiplied by 365 to get a yearly use:

$1.7 \times 24 \times 365 = 14\,892$ W h per year

To avoid using large numbers, you can use kilowatt hours (kW h). 1000 W h = 1 kW h.

If you assume electricity costs 12.2 cents per kW h, then the air conditioner uses 14.892 kW h, which costs $14.892 \times 12.2 = 181.6824$ cents per year.

This is an annual cost of approximately $1.82. This may not seem like much, but it is only one device, and one of the more efficient ones. Imagine how much this would be for every air conditioner in Australia!

3 Use the information in the table to calculate the yearly costs for the following.

(a) a clothes dryer on active standby

(b) an inkjet printer on passive standby

(c) a computer monitor switched off

(d) the difference between a front-loading washing machine and a top-loading washing machine, both switched off

(e) the total cost of an LCD TV connected to a DVD recorder and a set-top box, all in passive standby mode

4 In 2006 the Australian government introduced a policy stating that by 2012 all new appliances with a standby mode should use no more than 1 watt of electricity for the standby component. What is the maximum annual standby cost per machine after this 1-watt initiative was implemented?

Research

Find out more about how you can help reduce your greenhouse gas emissions by making small changes to your behavior.

See how many electronic devices you have in your house that are on standby. Think of some energy-saving suggestions for your home.

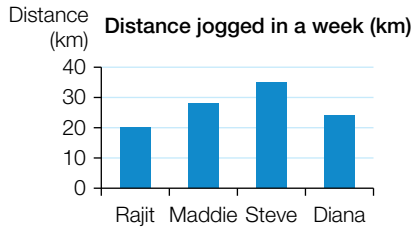
9.6 Comparing data sets

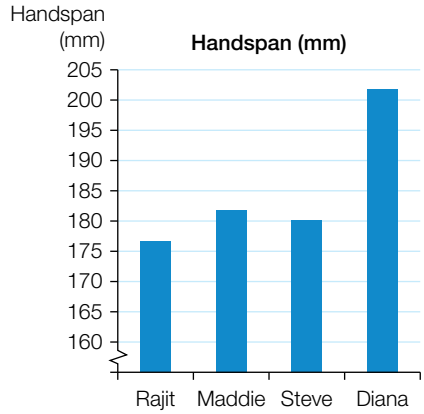
Comparing column graphs

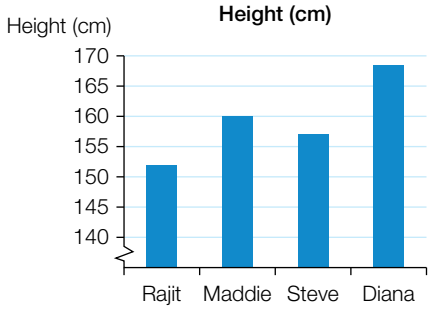
You can use column graphs to compare different variables. If the graph shapes are similar, this shows that there may be a connection between the variables. If the graph shapes are very different, this shows that there may be no connection.

Worked example 12 W.E. 12

Rajit, Maddie, Steve and Diana are keen joggers. The column graphs below show some data about them. Comment on the likelihood that there is a connection between any of the variables.





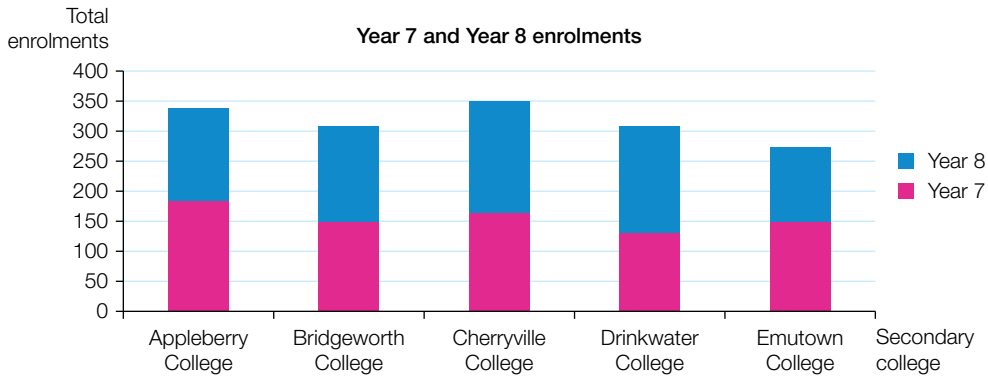


Thinking	Working
Look at the shape of the graphs and make a comment about similarities and differences.	The shape of the graph of distance jogged each week is quite different from the shape of the other two graphs. There seems to be no connection between the distance they jog and either their height or their handspan. The shapes of the graphs showing the height of the students and their handspan are very similar, so there could be a connection between these variables. The graphs for handspan and height have a similar shape, so you might assume that the taller the person, the bigger their handspan will be.

Comparing divided column graphs

You can use several divided column graphs together to compare data.

Worked example 13 W.E. 13



The graph above shows the Year 7 and Year 8 enrolments in five secondary colleges. Use the graph to answer the following questions.

(a) Which school has the highest enrolment and what is its total enrolment?

(b) Which part of the graph displays the number of Year 7 students and which school has the highest number of Year 7 students enrolled?

(c) Which school has the lowest number of Year 8 students enrolled?

(d) Which school has almost the same number of Year 8 as Year 7 students?

(e) Which schools have about the same total number of students enrolled?

Thinking	Working
(a) Identify the variable that has the longest column and find the number that corresponds to the length of the column.	(a) Cherryville College 350
(b) Identify which part of the column corresponds to the data required and find the column in which this part of the column is longest.	(b) The lower part of each column corresponds to Year 7 students. Appleberry College has the highest number of Year 7 students enrolled.
(c) Identify which part of the column corresponds to the data required and find the column in which this part of the column is shortest.	(c) Emutown College has the lowest number of Year 8 students enrolled.
(d) Identify a column where the lengths of each part of the column are about the same.	(d) Bridgeworth College has about the same number of Year 7 and Year 8 students enrolled.
(e) Identify all the columns that have the same length.	(e) Bridgeworth College and Drinkwater College have the same number of students.

Statistical measures of centre (mean, median and mode) give information about data sets. However, they do not give the complete picture. Consider the following data sets.

Set A: 11, 12, 14, 14, 14, 16, 17 Set B: 1, 2, 14, 14, 14, 26, 27

In both of these sets, the mean, median and mode are all equal to each other, even though they are clearly different data sets. To help you get a better picture of the data, you need to also consider the range. The range is defined as the difference between the highest and the lowest values in the data set.

$$\text{For Set A: range} = 17 - 11 = 6$$

$$\text{For Set B: range} = 27 - 1 = 26$$

As the range is larger in Set B, we can see that the values are spread out much more than in Set A.

Using tables of information

Worked example 14 W.E. 14

The following table gives the monthly mean maximum temperatures, measured in °C, for Adelaide (Kent Town) and Brisbane (Brisbane Aero).

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Adelaide	29.1	29.3	26.3	22.6	19.0	16.1	15.3	16.7	19.0	21.8	25.1	27.0
Brisbane	29.0	29.1	28.0	26.0	23.6	21.3	20.9	21.8	24.2	25.2	26.8	28.3

Compare the monthly mean maximum temperatures for the two cities using measures of centre, extreme values and the range.

Thinking

1 Calculate the mean, median and range (correct to 1 decimal place) in the normal way.

Working

	Mean	Median	Minimum	Maximum	Range
Adelaide	22.3	22.2	15.3	29.3	14.0
Brisbane	24.4	25.6	20.9	29.1	8.2

2 Compare the means and medians for the two cities and use them to describe any differences or similarities. Also, compare the ranges to explain the pattern of temperatures in each city over the year.

Brisbane is hotter on average according to both the mean and the median temperatures, although both cities have a very similar highest mean maximum temperature.
The range of temperatures is much greater in Adelaide, which means there is a much greater change in the mean maximum temperature from summer to winter than in Brisbane.

9.6 Comparing data sets

Navigator

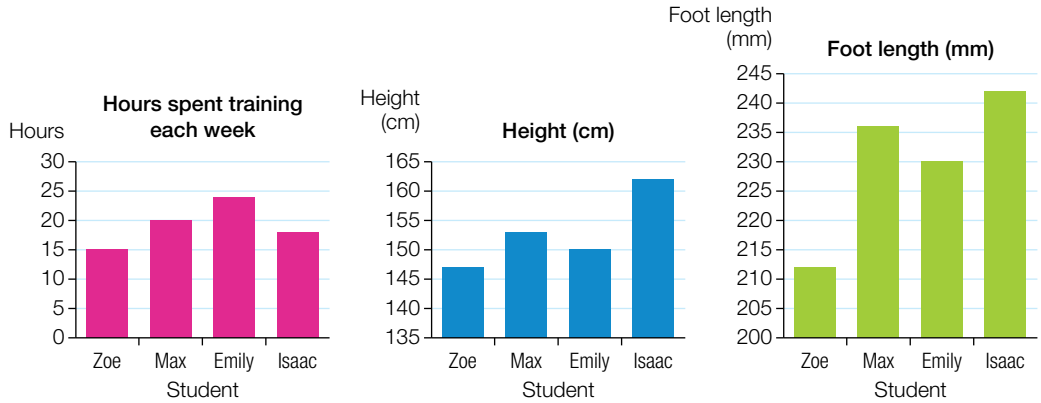
1, 2, 3, 4, 5, 6, 7, 8, 11, 12	1, 2, 3, 4, 5, 7, 8, 9, 11, 12, 13	1, 2, 3, 4 (a), 5, 7, 8, 9, 10, 11, 12, 13

Answers p. 707

Fluency

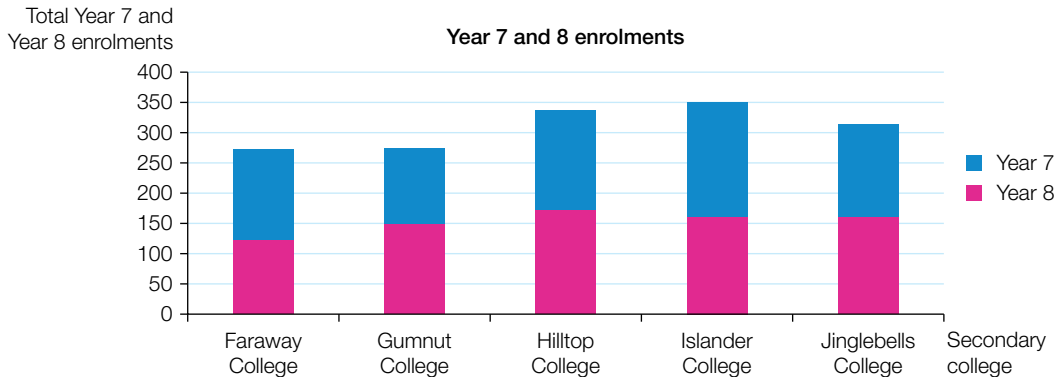
1 Zoe, Max, Emily and Isaac are keen swimmers. The column graphs below show some data on the four students. Comment on the likelihood that there is a connection between any of the variables.

W.E. 12



2 The graph below shows the Year 7 and Year 8 enrolment in five secondary colleges. Use the graph to answer the following questions.

W.E. 13



(a) Which school has the highest enrolment and what is its total enrolment?

(b) Which part of the graph displays the number of Year 8 students and which school has the highest number of Year 8 students enrolled?

(c) Which school has the lowest number of Year 7 students enrolled?

(d) Which school has almost the same number of Years 7 students as Year 8 students?

(e) Which schools have about the same number of students enrolled?

W.E. 14

3 The following table gives the monthly mean minimum temperatures, measured in °C, for Adelaide (Kent Town) and Brisbane (Brisbane Aero).

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Adelaide	17.0	17.1	15.2	12.4	10.2	8.1	7.5	8.2	9.7	11.4	14.0	15.5
Brisbane	21.1	21.1	19.3	16.2	12.8	10.5	8.8	9.6	12.7	15.7	18.0	20.1

Compare the monthly mean minimum temperatures for the two cities using measures of centre, extreme values and the range. Where necessary, round your answers to 1 decimal place.

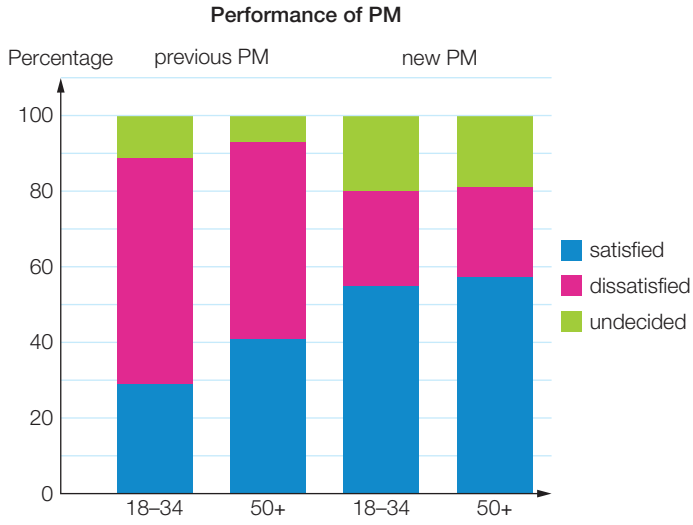
4 Compare the following data sets using mean, median, mode and range.

(a) Set A: 1, 2, 4, 5, 6, 7, 7, 7, 8, 10 Set B: 2, 2, 2, 2, 4, 5, 8, 9, 9, 9, 9

(b) Set A: 2, 4, 1, 5, 6, 2, 3, 7, 8, 2, 4, 1 Set B: 5, 2, 7, 4, 7, 1, 8, 3, 9, 8, 9, 6

Understanding

5 Surveys on voter satisfaction with the performance of the prime minister (PM) are carried out on a regular basis. The results of two such surveys, one taken just before and one taken just after a leadership change, are summarised below for the youngest and oldest categories of eligible voters.



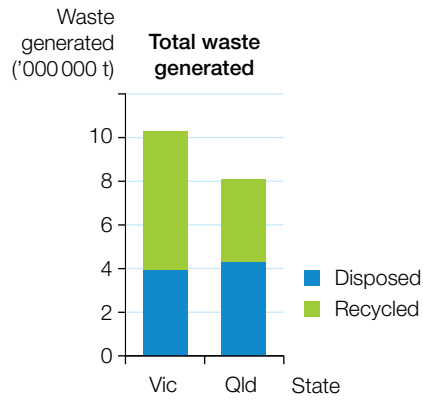
(a) What happened to the satisfaction rating when the leadership changed?

(b) Did the two age categories agree about the performance of the previous PM or the new PM?

(c) What happened to the portion of people who were 'undecided'? Explain why this might make sense.

6 April went to the market and purchased 5 boxes of bananas at $40 a box, 3 boxes of apples at $55 a box and a box of mangoes at $65 a box. Her friend Izzy bought 3 boxes of bananas at $45 a box, 3 boxes of apples at $50 a box and a box of mangoes for $70. Compare the overall prices per box between the two friends, using mean and range.

7 The graph shows the estimated total waste generated in Victoria and Queensland over one year. The divisions show the amount of waste disposed (sent to landfill) and the amount of waste recycled. Use the graph to answer the following questions.



(a) Which of the two states generated more waste?

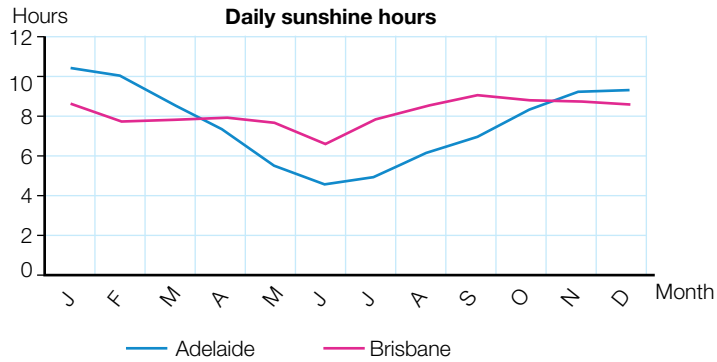
(b) How much waste did Queensland generate?

(c) Which of the two states recycled more than they disposed?

(d) How much more waste did Victoria recycle than Queensland?

Reasoning

8 The graph shows the monthly mean hours of sunshine for Adelaide and Brisbane.

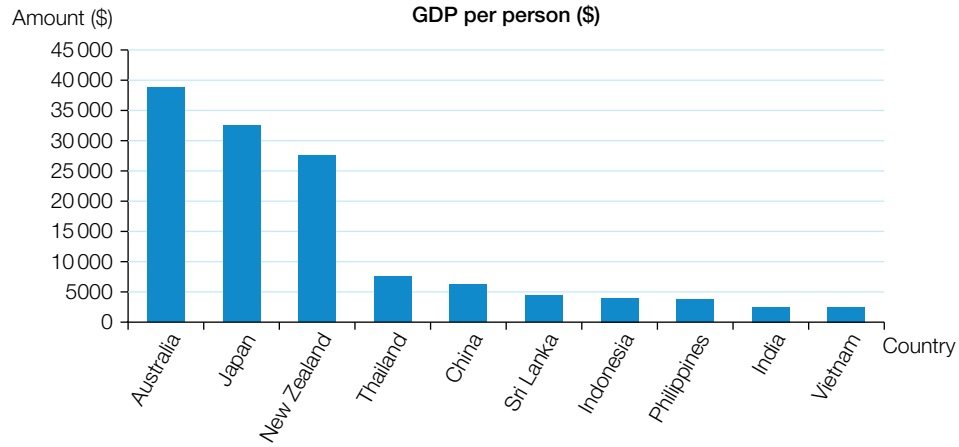


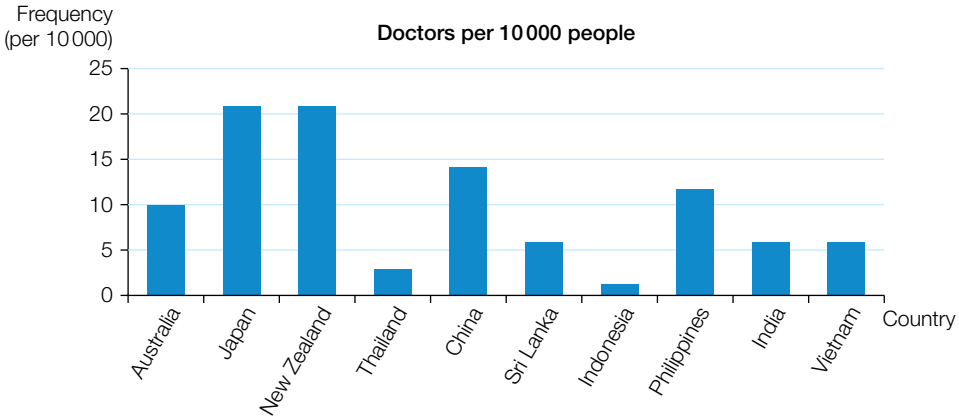
(a) Without doing any calculations, compare the amount of sunshine for the two capitals in terms of the mean values and the spread.

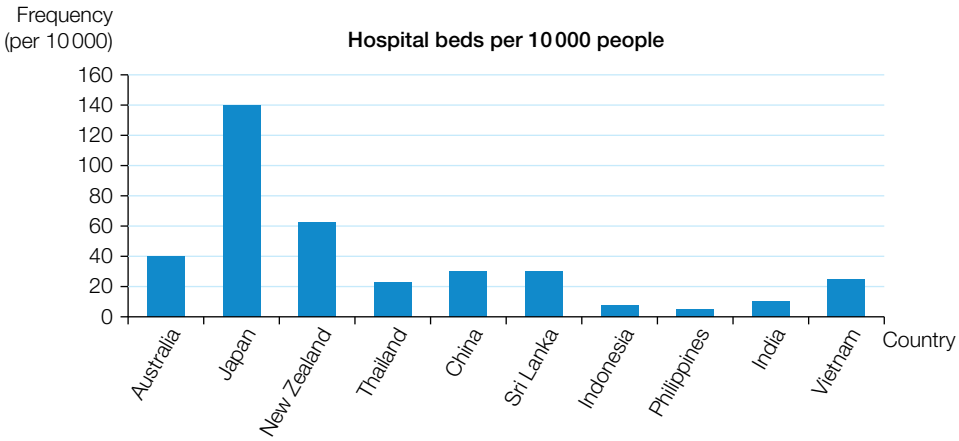
(b) Use the extreme values for each capital to calculate the range. Would you change your answer to the previous question?

(c) Read the monthly values from the graph and use them to calculate the mean and median for each capital. What additional comment can you make now?

9







The Gross Domestic Product (GDP) of a country is one measure of a country's wealth. The number of doctors and hospital beds are measures of the healthcare facilities of a country. Use the graphs above to answer the following questions.

(a) Do any of the graphs have a similar shape?

(b) Which country appears to have the best healthcare facilities? Give a reason for your answer.

(c) Compare the healthcare facilities of this country with its GDP per person.

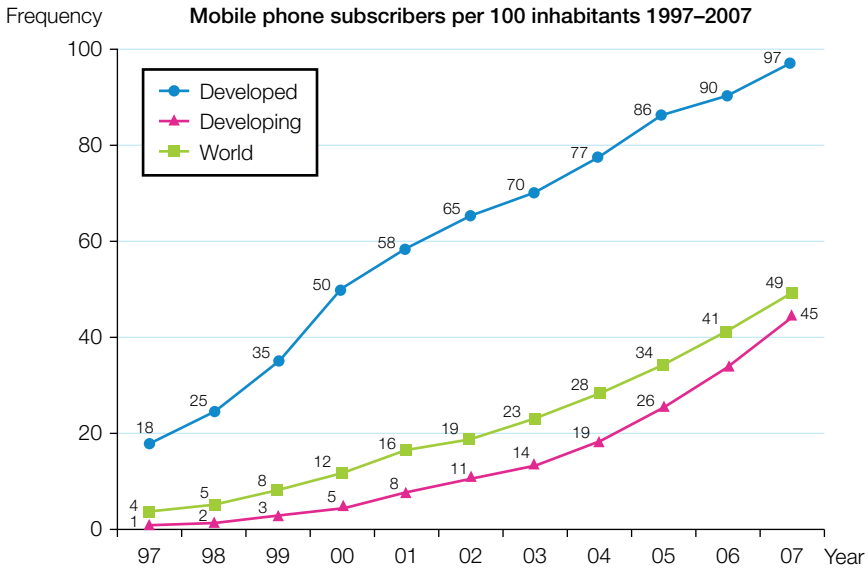
(d) Which country appears to have the worst healthcare facilities?

(e) Compare the healthcare facilities of this country to its GDP per person.

(f) Comment on Austalia's healthcare facilities compared with its GDP per person.

(g) Comment on Vietnam's healthcare facilities compared with its GDP per person.

10 The graph below shows the number of people out of every hundred who had a mobile phone, during the 10 years after mobile phones were first made widely available. The 'developed' countries include wealthier countries such as the United States, Australia, Japan and most of Europe. The 'developing' countries include poorer countries, including many in Africa and Asia.



(a) Compare the number of people who owned a mobile phone in developed and developing countries in the year 2000.

(b) What happened in the developed world over the years 2000–2007 to the number of people who own a mobile phone?

(c) What happened in the developing world in the years 2000–2007 to the number of people who own a mobile phone?

(d) Why is the world graph closer in shape to the developing world graph than the developed world graph?

(e) In which year did the biggest jump in mobile phone ownership in the developed world occur?

(f) If the trend of mobile phone ownership in the developing world shown by the graph has continued over the years since 2007, what do you think is the number of people per 100 who own a mobile phone now?

11 The following table shows the number of members for 16 AFL teams for two different years, and figures for two teams who joined the league more recently. Adelaide's figures for 2015 are an estimate only. (The V in brackets shows Victorian-based teams.)

Club	2005	2015
Adelaide	43 256	52 920
Brisbane Lions	28 913	25 408
Carlton (V)	33 534	47 305
Collingwood (V)	38 612	75 037
Essendon (V)	32 734	60 818
Fremantle	34 178	51 433
Geelong (V)	30 821	44 312
Hawthorn (V)	29 261	72 924
Melbourne (V)	24 805	35 953
North Melbourne (V)	24 154	41 012
Port Adelaide	36 834	54 057
Richmond (V)	28 029	70 809
St Kilda (V)	32 043	32 746
Sydney Swans	24 955	48 836
West Coast	42 406	60 221
Western Bulldogs (V)	21 974	35 222
Gold Coast		13 643
GWS		13 480

(a) Which club has been most successful at increasing its membership base?

(b) Use mean, median and range to compare the membership numbers for the Victorian-based teams against the non-Victorian-based teams for 2015 only. On the basis of this comparison, what comment could you make about the popularity of AFL clubs?

(c) Use mean, median and range to compare the club memberships for the two different years. What comment could you make about the number of people who are members of AFL clubs? Restrict your analysis to the 16 clubs who were in the league for both of the years being compared.

Open-ended

12 Here is a list of animals. Add more animals to the list if you like (only one word for each animal). Then, from these names choose a group of 10 animals you like, and choose another group of 10 animals you don't like.

bear	crocodile	lion	elephant	snake	giraffe	meerkat
cat	dog	rhinoceros	cheetah	chimpanzee	otter	vulture
zebra	hyena	tortoise	penguin	frog	baboon	bat
wolf	skunk	wombat	chicken	duck	armadillo	mongoose
emu	goat	cow	horse	koala	llama	lemming

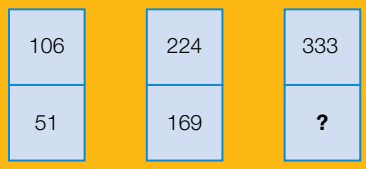
Count the number of letters in each word and analyse this data by finding the mean, median and mode. Does there seem to be a connection between the number of letters in an animal's name and how much you like it?

13 Eric is comparing two sets of data. He has written that the mean of Set A is 1.5 greater than the mean of Set B, but the median for Set A is 2 less than the median of Set B. The mode and the range are the same for both sets. Construct two data sets that meet these descriptions, with at least 15 results in each data set.

Puzzle

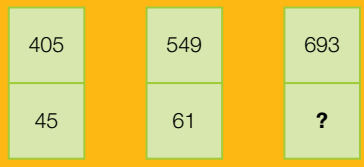
Number puzzles

1 In each of the following sets of three diagrams, one of the numbers is missing. To find it you will need to work out the connection between the numbers in the other two parts of the diagram. As an example:

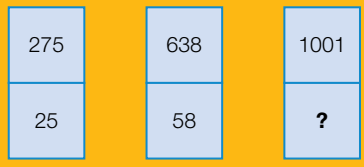


The '**?**' here would be replaced by 278.
106 – 51 = 55 and 224 – 169 = 55, so the missing number would need to follow the same pattern: 333 – 55, which is 278.

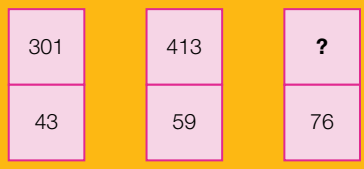
(a)



(b)

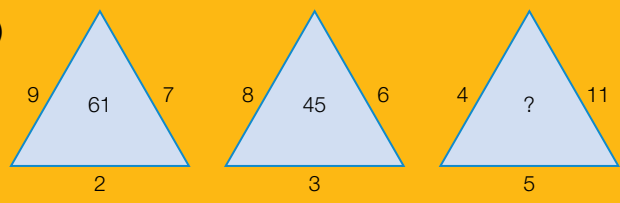


(c)

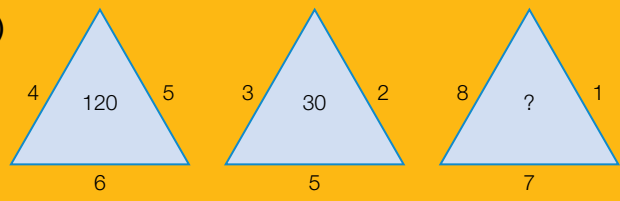


2 On each of the following sets of three triangles there is a number missing. To find it you will need to work out the connection between the outside numbers and the inside number in each of the first two triangles.

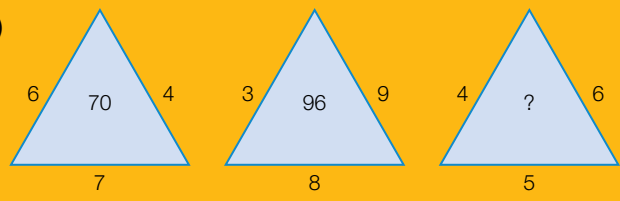
(a)



(b)



(c)



9.7 Probability and sample space

In everyday language, the words **probability** and **chance** mean the same thing. People make statements that involve probability all the time.

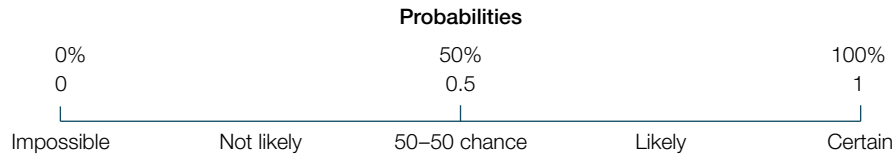
'Rain', 'passing a test', 'winning a one-day series' and 'playing in the finals' are examples of events. An event is something that may happen. The **outcome** of an event is the result that happens.

The probability of an event happening is the chance of that event happening.

Probabilities can be written as fractions or as decimals from 0 to 1, or as percentages from 0% to 100%.

- Events that have no chance of happening (impossible) have a probability of 0 or 0%.
- Events that are certain to happen have a probability of 1 or 100%.
- Events that have an equal (50–50) chance of happening or not have a probability of $\frac{1}{2}$ or 0.5 or 50%.

Probabilities can be any value between 0 and 1. You can think of probability as a number line, where the closer you get to 0 the less likely an event is to occur, while the closer you get to 1 the more likely an event is to occur.



Sample space

To determine the probability of a particular event occurring, it is useful to list the **sample space**. The sample space is the complete list of all the possible outcomes. For example, if you toss a coin, there are two possible outcomes: heads or tails.

When the outcomes of an experiment are equally likely, you can use the sample space to find the **theoretical probability** of a particular event occurring. A 'favourable' or 'successful' outcome is one that you are calculating the probability of.

Pr is often written as an abbreviation for probability. The event is then written in brackets. So, if you were rolling two dice, you could write Pr(odd) to stand for the probability that the dice roll will show an odd number.

When the outcomes are equally likely, you can calculate the theoretical probability:

$$\text{Pr(event)} = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

Worked example 15 W.E. 15

Find the probability of:

(a) rolling an odd number with a standard six-sided die

(b) selecting a vowel (a, e, i, o, u) at random from the 26 different letters of the alphabet.

Thinking	Working
(a) **1** List the sample space and count the number in the sample space.	(a) Sample space: 1, 2, 3, 4, 5, 6 There are 6 outcomes in the sample space.
2 Identify the favourable outcomes and count the number of favourable outcomes.	Favourable outcomes are: 1, 3 and 5. There are 3 favourable outcomes.
3 Find the probability using the rule Pr(event) $= \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$ and simplify the fraction if possible.	Pr(odd number) $= \frac{3}{6}$ $= \frac{1}{2}$

(b) 1	List the sample space and count the number in the sample space.	(b) Sample space: a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z There are 26 outcomes in the sample space.
2	Identify the favourable outcomes and count the number of favourable outcomes.	Favourable outcomes are: a, e, i, o, u There are 5 favourable outcomes.
3	Find the probability using the rule $\Pr(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$ and simplify the fraction if possible.	$\Pr(\text{vowel}) = \frac{5}{26}$

Estimating probability

You can often determine a theoretical probability by listing the sample space and investigating the situation mathematically, especially if the different possible outcomes are all equally likely.

You can also often determine an **experimental probability** for an event, through experiments or by collecting long-term data, to see how often an event occurs in real life. This lets you find an estimate of the probability, based on the history of prior events. Estimated probabilities are usually written as decimals rounded to 2 decimal places, or whole number percentages.

Worked example 16

W.E. 16

A young couple are planning a wedding in May. They know that in their region, the month of May has on average 16 days of rain. What is the probability that they will have no rain on their wedding day?

Thinking	Working
1 Write a formula comparing favourable outcomes with total outcomes.	$\Pr(\text{no rain}) = \frac{\text{number of days with no rain in May}}{\text{number of days in May}}$
2 Substitute values and simplify the fraction if possible.	$= \frac{31 - 16}{31}$ $= \frac{15}{31}$
3 Convert the fraction to a decimal rounded correct to 2 decimal places.	$= 0.483\ldots$ ≈ 0.48
4 Write the answer in words using percentages.	There is a 48% change of a day with no rain for the wedding.

Probability experiments

When historical data is not available, you can sometimes estimate probability by conducting your own experiments. Each part of an **experiment** that gives a result is called a **trial**. If you toss a coin 50 times and record each result as 'heads' or 'tails', then you have run an experiment of 50 trials.

Worked example 17 W.E. 17

Ken bought a packet of seeds. Each seed will grow a flower that is either red (R) or white (W). He plants some of the seeds and his flower garden produces the following results:

W W W R W W R W W W R R W W W W R R R W W W W W W

What is the probability that the next seed from this packet will grow a red flower?

Thinking	Working
1 Write a formula comparing favourable outcomes with total outcomes.	$Pr(R) = \frac{\text{number of red flowers}}{\text{total number of flowers}}$
2 Find the number of red flowers and the total number of flowers. Substitute into the fraction and simplify if possible.	$= \frac{7}{25}$
3 Convert the fraction to a decimal rounded correct to 2 decimal places.	$= 0.28$
4 Write the answer in words using percentages.	There is a 28% chance that the next seed from this packet will grow a red flower.

Fair games

There are many sorts of games. Some rely mainly on skill to win, such as chess. Some require a mixture of skill and luck, such as poker. Some rely purely on luck, such as snakes-and-ladders.

When you play a game you expect it to be fair. In probability, 'fair' means that each person playing has the same chance of winning. Rules are usually written to make games fair.

You can look at a game's sample space to help decide whether a game is fair. For example, if the game consists of rolling a single die, then the sample space is the numbers 1, 2, 3, 4, 5, 6, as these are the possible outcomes when the die is rolled. If one player wins when an even number is rolled and the other player wins when a multiple of 3 is rolled, then the first player has three chances of winning (2, 4 and 6) out of six, while the second player has only two chances (3 and 6) out of six. The rules mean that the first player has a better chance of winning, so the game is not fair.

9.7 Probability and sample space

Navigator

Answers p. 708

1, 2, 3, 4, 5, 6, 7, 8, 9 (a), 11, 13	1, 2, 3, 4, 5, 6, 7, 8, 9 (a–b), 11, 12, 13, 14	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14

Note: A standard pack of 52 playing cards is made up of 4 suits: hearts ♥, diamonds ♦, clubs ♣ and spades ♠. Hearts and diamonds are red cards. Clubs and spades are black cards. Each suit includes 13 cards: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King.

Fluency

W.E. 15

1 Find the probability of:

(a) rolling an even number with a normal die

(b) choosing an 'a' from the letters of the word *mathematics*

(c) getting a 4 on a roll of a normal die

(d) drawing a card from a standard pack and getting a diamond

(e) drawing a card from a standard pack and getting a king

(f) drawing a card from a standard pack and getting a black 4

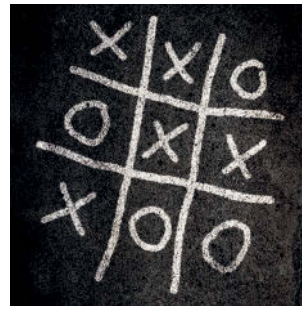
(g) drawing a card from a standard pack and getting a red 7

(h) drawing a card from a standard pack and getting the 9 of spades.

W.E. 16

2 In the game of noughts and crosses, players take turns to write an X (for one player) or an O (for the other player) in a 3-by-3 grid. The first player to get three Xs or three Os in a row, column or diagonal is the winner.

Jing has played Peter at noughts and crosses many times. They have each won six games, while 20 games ended in a draw. What is the probability that their next game is a draw? Write your answer as a percentage to the nearest whole number.



W.E. 17

3 Elle suspects that her die is biased and rolls a 6 more often than usual (that is, more often than $\frac{1}{6}$ of the time). She decides to conduct an experiment to test this. Here are the results of her 60 trials:

3	4	6	6	4	2	6	5	6	4	6	1
2	6	6	6	3	2	4	1	3	1	6	2
6	5	3	6	4	4	2	6	5	5	3	3
5	6	5	5	2	6	4	2	4	2	5	6
4	2	2	6	2	6	6	4	6	4	6	4

(a) Find the experimental probability of rolling a 6 with this die. Write your answer as a percentage, to the nearest whole number.

(b) Do these results suggest that the die is biased? Explain your answer.

4 Decide whether you would describe the following events as 'impossible', 'not likely', 'likely' or 'certain'.

(a) Tomorrow will have a maximum temperature of -20 °C.

(b) Christmas Day will be on 25 December.

(c) Sri Lanka will win the next cricket World Cup.

(d) The Sun will set tonight.

(e) If you toss an ordinary coin, it will show either heads or tails.

(f) If you roll a standard six-sided die, the result will be 1.

(g) If you roll a standard six-sided die, the result will be 7.

(h) If you take a card from a standard pack of 52 playing cards, it will be the Ace of clubs.

(i) If you take a card from a standard pack of 52 playing cards, it will be either red or black.

5 Choose the value that you think is closest to the correct answer.

(a) The probability of winning first prize in an Australia-wide lottery is:

A 0 **B** 0.5

C 0.6 **D** 0.75

(b) The probability of Australia winning a gold medal in swimming at the next Olympics is:

A 0.3 **B** 0.4

C 0.9 **D** 1

(c) The probability of rain in a coastal town at some time during the next two months is:

A 0 **B** 0.2

C 0.5 **D** 1

(d) The probability of tossing a coin and getting tails is:

A 0.2 **B** 0.3 **C** 0.5 **D** 0.7

(e) The probability of drawing a heart from a standard pack of 52 playing cards:

A 0.0001 **B** 0.25 **C** 0.5 **D** 1

6 Write the sample space for each of the following.

(a) A ten-sided die, with its sides labelled from 1 to 10, is rolled once.

(b) A letter is chosen at random from the letters of the word *probability*.

(c) Two coins are tossed at the same time.

7 A bag of jelly beans contains 1 black, 2 purple, 3 green, 4 blue and 5 red jelly beans.

(a) Rachel puts her hand in the bag and selects one jelly bean without looking. What is the probability of the jelly bean being:

(i) red **(ii)** green **(iii)** blue or purple?

(b) Rachel pulls out one red jelly bean and eats it. She selects another one from the bag without looking. Recalculate the three probabilities you calculated in **(a)**.

Understanding

8 For each of the following, say whether or not the two events are equally likely. Write the probability of each.

(a) An unbiased twelve-sided die, with sides labelled 1 to 12, is rolled once. The two events are 'rolling a 1' and 'rolling a 10'.

(b) A letter is chosen at random from the letters of the word *probability*. The two events are 'choosing *p*' and 'choosing *i*'.

(c) A coin is tossed two times. The two events are 'heads appears twice' and 'heads appears once, tails appears once'.

9 If each person playing a game has the same chance of winning from the start, then the game is said to be 'fair'. Determine whether the following games are fair by listing the sample space and the successful outcomes for each player.

(a) **(i)** Joe and Henry roll a die. Joe wins if a 1, 2 or 3 turns up, and Henry wins if a 4, 5 or 6 turns up. Is this a fair game?

(ii) If Joe and Henry play their game 100 times, how many wins would you expect them to have each?

(b) **(i)** Edward and Irene roll a die. Edward wins if a 3 turns up, and Irina wins if a 3 doesn't turn up. Is this a fair game?

(ii) If Edward and Irene play their game a large number of times, and Edward wins 40 times, how many times would Irene probably have won?

(c) **(i)** Ravi and Simon roll a die. Ravi wins if a multiple of 3 turns up, and Simon wins if a multiple of 3 doesn't turn up. Is this a fair game?

(ii) Ravi and Simon score a point each time one of them wins. Suppose the game is the first one to 60 points. Simon wins, but a week later they argue about what the final score was. Simon says he won 60 : 31, while Ravi says Simon only won 60 : 55. Who do you think was probably right?

10 Viv and Ben play a set of tennis every week. Viv has won 20 out of 37 sets so far. How many sets would Viv expect to win in one year (52 weeks)?

Reasoning

11 The following is a list of words and phrases you may have heard associated with probability.

even chance	very likely	virtually impossible	often
sure thing	most unlikely	almost always	quite often
likely	impossible	sometimes	usually
quite frequently	almost never	maybe	more often than not

Put these words in order of probability, from impossible to certain. Can you explain and justify your order?

12 A drawer contains six identical blue socks and two identical brown socks. How many socks do you have to take from the drawer to be certain of having:

(a) a pair of blue socks

(b) a pair of brown socks

(c) a pair of socks

(d) two odd socks?

Explain your answers in each case.

Open-ended

13 **(a)** Write two events that you think are impossible.

(b) Write two events that you think are not likely.

(c) Write two events that you think are likely.

(d) Write two events that you think are certain.

14 Isaac has 20 counters in a bag, numbered from 1 to 20. He draws a counter from the bag.

For each of the following possible events in this situation, describe another event that would have the same probability.

(a) drawing an even number

(b) drawing a number from 5 to 16

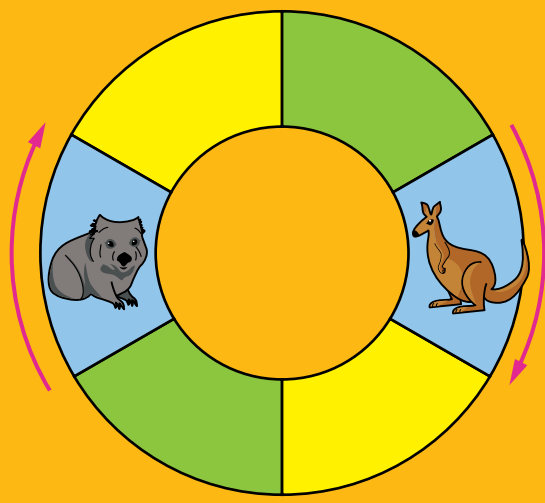
(c) drawing a multiple of 3 that is less than 12.

Game

Catch me if you can

Equipment required: 2 dice, 2 unique counters

How to win:
Land on the segment that has the other player's counter on it.



How to play:

Decide which player is the 'kangaroo' and which is the 'wombat' and place your counter on the appropriate start position. Take turns rolling the dice and move your counter one space, given the following rules.

The wombat moves if the sum of the dice is equal to: 2, 3, 4, 5, 10, 11, 12.

The kangaroo moves if the sum of the dice is equal to: 6, 7, 8 or 9.

Continue rolling the two dice until one of the animals catches the other.

After you have played a few times, decide whether you think the game is fair. Explain your reasons. If you don't think it is fair, see if you can change the rules to make it fair. Play the new game a few times to test it.

Investigation

Experimental probability

Equipment required: calculator, drawing pin, coin

The Big Question

Can you use probability to decide whether a game of chance is fair?

Engage

1 A dropped drawing pin can land two possible ways, as shown. Try dropping a drawing pin a few times and see if it lands point up or point down.

point up

point down

Do you think you need to be careful in the way you drop the pin? Should you give it a 'flip' to help it fall in a particular way? Why do you think it is different to use a pin instead of a coin or a die?

Explore

Don and Jacob play a game that involves dropping a drawing pin.

If the drawing pin falls point up, Don scores 1 point.
If the drawing pin falls point down, Jacob scores 1 point.
The first player to get to 50 points wins.

2 (a) Do you think this is a fair game? Who do you think will win?

(b) Predict the final score. Is it harder to make this sort of prediction than the predictions in the previous exercise?

(c) Play the game with a partner. Decide who scores 1 point for 'point up' and who scores 1 point for 'point down'. Tally your results in a table like this:

Point up	𝍸 𝍸 |||
Point down	𝍸 𝍸 𝍸 |||

How close were your predictions?

(d) Compare your results with other games by constructing a table like this:

	Score	
	Point up	Point down
Game 1	32	50
Game 2	50	44
Game 3		

(e) Do you notice any pattern in the results of the games?

(f) (i) Find the point up total by adding all the numbers in the point up column.

(ii) Find the point down total by adding up all the numbers in the point down column.

(iii) Then, find the total number of tosses by adding the point up and point down totals.

(g) Find the probability that a drawing pin will fall point up by calculating

$$\frac{\text{point up total}}{\text{total number of tosses}}$$ as the following:

(i) as a fraction

(ii) as a decimal

(iii) as a percentage.

(iv) Could the game still be fair if the percentage is not exactly 50%?

(h) Find the probability that a drawing pin will fall point down by calculating

$$\frac{\text{point down total}}{\text{total number of tosses}}$$ as the following:

(i) as a fraction

(ii) as a decimal

(iii) as a percentage.

(iv) Could the game still be fair if the percentage is not exactly 50%?

(v) Look at the two probabilities you have found in parts **(g) (ii)** and **(h) (ii)**. Add them together. What do you notice?

(vi) If the game is fair, what do you think each of the two probabilities should be equal to? Is the game fair? Is it close to being a fair game?

Strategy options

- Make a table.
- Act it out.
- Look for a pattern.

Explain

3 How could you change the rules for the drawing pin game to make it fair?

Elaborate

4 Omar and Michelle take it in turns to toss a coin four times between them. If four heads come up, or if two heads and two tails come up, Omar scores a point. If four tails come up, or mostly tails, Michelle scores a point. If three heads come up then they both score a point. The first one to 50 points is the winner.

(a) Do you think this is a fair game? Who do you think will win?

(b) Make a prediction about what you think the final score will be.

(c) Play the game with a partner. Decide who is Omar and who is Michelle.

Tally your results in a table. How close were your predictions?

(d) Compare your results with other pairs of students by constructing a table.

(e) Do you notice any pattern in the results of the games?

(f) **(i)** Find Omar's total by adding all the numbers in the Omar column.

(ii) Find Michelle's total by adding all the numbers in the Michelle column.

(iii) Then, find the total number of tosses by adding Omar's and Michelle's totals.

(g) **(i)** Find the fraction of points that Omar gets by calculating:

$$\frac{\text{Omar's total}}{\text{total number of tosses}}$$

(ii) Change this to a decimal. Write the probability that Omar will win as Pr(Omar wins) = …

(iii) Write the percentage of times Omar will win.

(h) **(i)** Find the fraction of points that Michelle gets by calculating:

$$\frac{\text{Michelle's total}}{\text{total number of tosses}}$$

(ii) Change this to a decimal. Write the probability that Michelle will win as Pr(Michelle wins) = …

(iii) Write the percentage of times Michelle will win.

(iv) Find Pr(Omar wins) + Pr(Michelle wins).

(v) By looking at your values for Pr(Omar wins) and Pr(Michelle wins), would you say that the game is fair? Is it close to being fair?

Evaluate

5 How could you change the rules for the coin game to make it fair?

Extend

6 What other sorts of probability games do you think you could play? Try to come up with some probability games that do not involve coins or dice.

7 **(a)** Did you affect the result by the way you dropped the pin or tossed the coin?

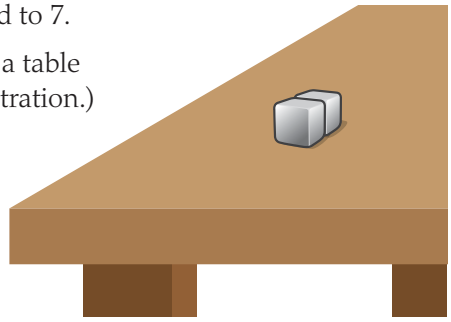
(b) Do you think the tasks in this investigation have helped you understand probability? Explain why or why not.

(c) Do you think it is important for you to know whether a game is fair? Why?

Challenge 9

1 The dots on the opposite faces of a standard die add to 7.

Two standard dice are placed next to each other on a table as shown. (The dice dots are not shown in this illustration.)



Kenny walks around the table to look at all the visible faces on the dice.

What is greatest possible number of dots that Kenny can see, adding together all the visible faces of the dice?

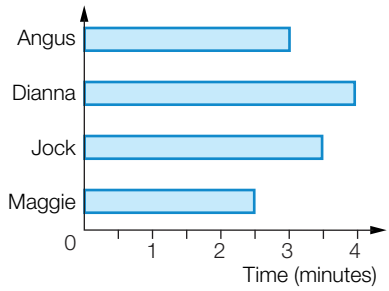
A 30 **B** 34 **C** 36 **D** 42

2 You meet three people at a meeting: Ash Black, Jade Green and Blanche White. They are each wearing a tie that is black, green or white. Ash Black says, 'Did you notice that the colours of our ties are different to the colours of our names?' The person in the green tie says, 'Yes, you are correct!' What colour tie is each person wearing?

3 A die is constructed by labelling the faces of a wooden cube with the numbers 1, 1, 2, 3, 3 and 4. When it is rolled, each face has an equal chance of coming up. If this die is rolled once, what is the probability of rolling an odd number?

A $\frac{1}{6}$ **B** $\frac{1}{3}$ **C** $\frac{1}{2}$ **D** $\frac{2}{3}$

4 Four Jack Russell terriers had a race, swimming across a pond. The graph shows the time that each dog took to complete the race. Which dog won the race?



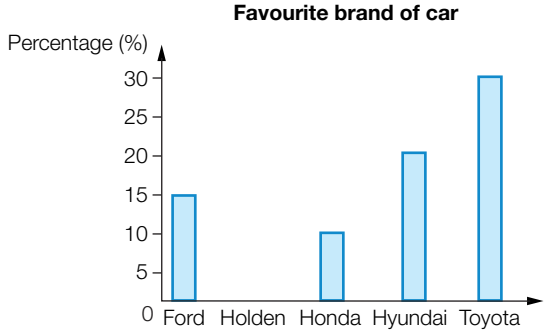
A Angus **B** Dianna

C Jock **D** Maggie

5 Vijay has a bag that contains 3 blue marbles, 6 green marbles, 2 red marbles and 6 yellow marbles. Vijay adds a number of white marbles to the bag and tells Samantha that if he now draws a marble from the bag, the probability of it being blue or green is $\frac{3}{7}$. The number of white marbles that Vijay added to the bag is:

A 3 **B** 4 **C** 5 **D** 6

6 In a survey, 1000 people were asked 'What is your favourite brand of car?' Their responses were recorded and then graphed as percentages in a column graph. The column representing Holden has been deleted. What percentage of the people selected a Holden as their favourite make of car?



A 25 **B** 20

C 15 **D** 5

7 Jade has played 20 games of tennis and has won 95% of the games. How many more games in a row must she win to have then won 96% of the games?

A 1 **B** 4 **C** 5 **D** 10

Chapter review 9

Maths literacy

average	continuous data	measures of centre	range
bar graph	data	median	sample space
bi-modal	discrete data	mode	secondary data
bivariate data	dot plot	multimodal	stem-and-leaf plot
categorical data	frequency	outcomes	tally
chance	frequency table	outlier	theoretical probability
class interval	histogram	primary data	trend
column graph	mean	probability	univariate data

Copy and complete the following using the words and phrases from this list, where appropriate. A word or phrase may be used more than once.

1 The graphs in this chapter are all ways of presenting _____.

2 The listing of all the outcomes possible in a probability experiment is called the _____.

3 Of the three measures of central tendency considered, the one that represents the value occurring most frequently is called the _____.

4 The probability of an event occurring is another way of saying what _____ there is of the event happening.

5 In a frequency table for grouped data, each group is called a _____.

6 The _____ is the sum of all the values in the data set divided by the number of values in that data set.

Equipment required: calculator for Questions **2**, **16**, protractor for Question **12**

Fluency

1 A Year 7 class was surveyed about how many smartphones their families had at home. The results were: **9.1**

1, 1, 1, 1, 2, 1, 1, 3, 2, 5, 2, 1, 1, 0, 1, 2, 0, 4, 1, 2, 0, 3, 1

Construct a frequency table for this information.

2 Find the mean, median, mode and range for the following sets of data. Use a calculator when necessary, and give your answers correct to 2 decimal places. **9.2**

(a) 2, 4, 2, 7, 3, 5, 4, 2, 3, 6, 1 **(b)** 40, 20, 60, 30, 50, 10

(c) 3.3, 4.1, 3.9, 2.9, 4.4, 4.1, 3.8, 3.3, 4.4, 3.2, 3.9, 4.1, 2.9

3 Draw a bar graph to show the following data that represents the favourite style of TV program for a sample of 100 people. **9.3**

Style	Number of people
Comedy	35
Drama	21
Reality	17
Documentary	8
Game	19

4 **(a)** Draw a dot plot to represent the number of customers in a cafe recorded in 15-minute intervals over the course of 8 hours. **9.3**

4	4	7	3	6	4	5	9	4	8	8	5	7	9	10	12
11	8	9	8	6	2	4	4	5	5	2	5	3	0	1	1

(b) Calculate the median of the data set.

(c) Do there appear to be any possible outliers?

5 **(a)** Draw an ordered stem-and-leaf plot to represent the number of runs scored by each of the players at a local cricket club in one round of the season. **9.3**

46	64	31	0	50	42	11	7	4	0	5
21	17	115	0	42	61	15	18	5	3	1
40	0	52	56	77	50	41	20	7	2	2
88	17	58	63	14	0	18	19	17	6	2

(b) Calculate the median of the data set.

(c) Do there appear to be any possible outliers? If so, which values are they?

6 The sector graph at right represents information about the money spent by overseas visitors to Australia. **9.4**

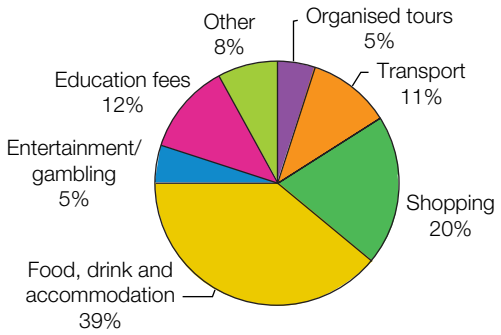
(a) One of the sectors uses an angle of 28.8°. Which one is it?

A organised tours

B transport

C entertainment/gambling

D other



(b) The 'Food, drink and accommodation' sector uses an angle closest to:

A 95° **B** 110° **C** 130° **D** 140°

7 The following are sales figures for computer games sold by a particular company from 2011 to 2016. **9.5**

Year	2011	2012	2013	2014	2015	2016
Units sold	10 000	25 000	35 000	30 000	40 000	45 000

Draw a line graph for this information.

8 The following table gives the mean rainfall, measured in mm, for Cairns (Aero) and Brisbane (Aero). **9.6**

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Cairns	396.8	452.5	426.1	198.2	91.4	46.1	29.3	27.0	33.4	39.7	93.4	178.5
Brisbane	109.3	121.6	76.1	63.5	122.8	70.6	27.1	35.2	32.7	65.2	102.0	120.9

Compare the mean rainfall for the two cities using as many statistics as you can.

9 The following table shows the handspan, measured in cm, of twenty 16-year-old students, divided into boys and girls. **9.6**

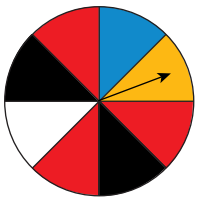
Boys	19.5	21.5	21	22.5	19	19.5	21	21.5	20.5	20
Girls	18	18.5	17.5	17	17	17.5	18	18.5	17	19.5

Draw a parallel dot plot and use it to help you describe the data set.

10 **(a)** When a normal die is rolled, the probability of getting a number less than 5 is equal to: **9.7**

A $\frac{1}{5}$ **B** $\frac{2}{3}$ **C** $\frac{5}{6}$ **D** 1

(b) If the spinner shown on the right is spun once, then what is the probability of the result being red?



A $\frac{1}{4}$ **B** $\frac{3}{8}$ **C** $\frac{1}{2}$ **D** $\frac{3}{5}$

Understanding

11 A number of Year 7 students were surveyed about their shoe size. The results were: **9.1, 9.2**

$3, 5, 5\frac{1}{2}, 5, 4\frac{1}{2}, 4, 2\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}, 6, 2, 5\frac{1}{2}, 1\frac{1}{2}, 1, 3\frac{1}{2}, 4\frac{1}{2}, 3, 4, 5, 5\frac{1}{2}, 2\frac{1}{2}, 2, 3\frac{1}{2}, 4\frac{1}{2}, 3, 1, 3\frac{1}{2}, 1\frac{1}{2}, 4, 3\frac{1}{2}$

(a) Construct a frequency table to show this information.

(b) Which shoe size occurred most frequently?

(c) Which shoe size occurred least frequently?

12 Look at the following information. **9.5**

How much Australians spend on pets each year ($ million)				
Expenditure type	**Dogs**	**Cats**	**Other**	**Total**
Food	560	431	90	1081
Vet charges and prescriptions	230	148	26	404
Pet care products/equipment	153	85	12	250
Pet services	116	35	–	151
Other expenses	109	20	12	141
Total	1168	719	140	2027

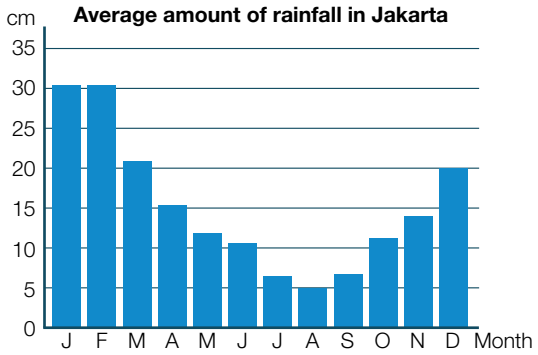
(a) Draw a bar chart showing the various expenditure types for dogs.

(b) Draw a divided bar chart to show the total expenditure for the three categories of pets.

(c) Draw a sector graph to show the various expenditure types for cats.

(d) Explain why a line graph could not be used for any of these sets of data.

13 Look at the following graph, which shows the average amount of rainfall (cm) in the city of Jakarta over 1 year. **9.3**



(a) What type of graph is this?

(b) What is Jakarta's second driest month?

(c) What is the highest average monthly rainfall?

(d) Which 4 months form the rainy season?

14 Shelley and Douglas play a game in which they toss two coins. Shelley wins if both coins show heads, and Douglas wins if both coins show tails. If one coin shows heads and the other shows tails, then they toss *one* of the coins again. If the re-tossed coin shows heads, Shelley wins, but if the re-tossed coin shows tails, Douglas wins. Do you think this is a fair game? **9.7**

15 **(a)** Estimate the probability that you will be struck by lightning in the next minute. **9.7**

(b) Estimate the probability that it will rain at some time in the next six months.

(c) If something has a probability of $\frac{1}{2}$ and something else has a probability of $\frac{3}{4}$, which one of the two is more likely to occur?

(d) If someone tells you that something has a probability of 1.2, how do you know that they have made a mistake?

Reasoning

16 Twenty-five Year 7 students and 25 Year 8 students had their height measured to the nearest cm. The results are as follows. **9.1, 9.2, 9.3**

Year 7:	140	143	144	142	148	148	152	140	146	144
	154	151	145	140	141	147	153	150	149	140
	147	152	148	151	149					
Year 8:	142	151	157	138	145	148	143	141	146	154
	156	154	150	143	150	144	148	144	157	156
	146	157	154	145	152					

(a) Construct an ungrouped frequency table for the Year 7 students.

(b) Find the **(i)** mode **(ii)** median and **(iii)** mean height for Year 7 students.

(c) Construct an ungrouped frequency table for Year 8 students.

(d) Find the **(i)** mode **(ii)** median and **(iii)** mean height for the Year 8 students.

(e) Describe the similarities and differences between the heights of the Year 7 students and the Year 8 students.

(f) Draw a combined ungrouped frequency table for Year 7 students and Year 8 students.

(g) Find the **(i)** mode **(ii)** median and **(iii)** mean height for all the students.

(h) Describe the heights of the students.

17 A game is played using 12 counters, three dice and a copy of the numbered grid shown. Players put all 12 of their counters on the numbered squares (more than one counter can be on a square). The three dice are rolled and the results added to create one 'total number'. If there is a counter on that 'total number', it is removed from the grid. Dice are rolled repeatedly until the board is clear. **9.7**

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20

(a) Which numbers should a player never put any counters on? Explain why.

(b) Which numbers should a player put most counters on? Explain why.

(c) After choosing where to place the counters, will every game take the same number of dice rolls to clear the board? Explain your answer.

Numeracy practice 9

Non-calculator

1 Some students were surveyed about the number of hours spent using a computer over the last weekend. The following results were recorded: 4, 7, 3, 2, 4, 6, 8, 5, 9.

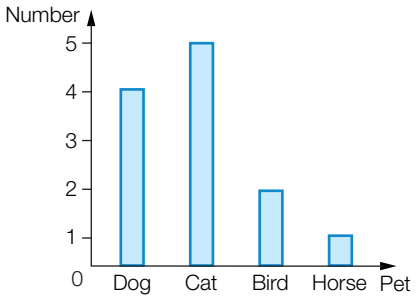
The median result is:

A 4 **B** 5 **C** 6 **D** 9

2 A jar contains 16 red, 12 black, 9 yellow and 8 green lollies. Without looking, Jen takes one lolly from the jar. What is the chance that the lolly is yellow?

A $\frac{1}{2}$ **B** $\frac{1}{3}$ **C** $\frac{1}{4}$ **D** $\frac{1}{5}$

3 Ricky surveyed the members of his cricket team to find out what pets they had. The results are shown in the graph on the right.

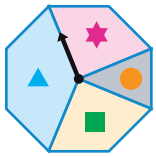


How many pets does the team have altogether?

A 4 **B** 5

C 12 **D** 15

4 Kenny spins the spinner 80 times.



Which table is *most* likely to show his results?

A

Shape section	Number of spins
★	10
●	30
■	20
▲	20

B

Shape section	Number of spins
★	20
●	30
■	10
▲	20

C

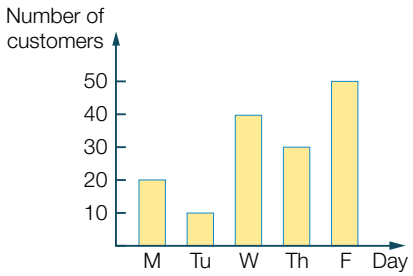
Shape section	Number of spins
★	20
●	10
■	20
▲	30

D

Shape section	Number of spins
★	20
●	20
■	10
▲	30

Calculator allowed

5 A business recorded the number of customers each day for a working week.



(a) Which day was the busiest?

(b) What was the mean number of customers per day?

(c) On which day was the number of customers closest to the mean number?

6 The goal attack for the under-13 Panthers netball team scored the following number of goals in the first six games of the season: 14, 19, 23, 20, 28, 19.

The mean number of goals scored is closest to:

A 6 **B** 19.5 **C** 20.5 **D** 126


10

Transformation and visualisation

10

Why is 'The Pentagon' a pentagon?

In 1941, a radical new design for a government building impressed the President of the USA.

Located near Washington, D.C., the building known as 'The Pentagon' was built during World War II to house the War Department, now the United States Department of Defense. It is one of the largest buildings in the world, despite being only five storeys high. (During World War II, the steel required for extra storeys was needed for warships and weapons.)

The five-sided shape was chosen because it was the best fit to the shape of the land at the original location chosen. President Roosevelt changed the location, but kept the shape.

The Pentagon is a regular pentagon. Each of the five sides and the five angles formed where the sides meet are equal. Because it is a regular shape, the Pentagon has a high degree of reflectional and rotational symmetry.

Forum

How do you think the builders of the Pentagon ensured that the five sides and angles were equal?

Looking at the Pentagon from above:

How many shapes can you identify within the main pentagon shape?

How many straight lines can you draw that divide the shape into two identical halves?

Many buildings and structures that are considered 'beautiful' are symmetrical. Do you think humans have a natural feeling for symmetry?

Why learn this?

Every day you see and work with geometrical figures. You get food and drink from cylindrical cans, pack things into prism-shaped boxes, and play sport on rectangular courts with spherical balls. Each of these objects has been designed so that its shape suits its purpose. Many careers involve design, from architecture (designing buildings), to industrial design (designing products to be sold) to graphic design (designing the images used in advertising, books, magazines and websites). Knowledge of geometry and appreciation of symmetry is important in these and many other careers, as well as in everyday life.

After completing this chapter you will be able to:

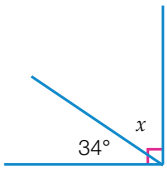
- translate, reflect and rotate shapes
- describe the process and compare the results of different combinations of transformations
- identify lines of symmetry and recognise different kinds of symmetry in design and in nature
- identify the order of reflectional symmetry and the order of rotational symmetry
- visualise and draw 3D shapes using isometric perspective
- draw elevations and plan views of 3D shapes.

Recall 10

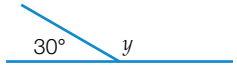
Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

1 Find the size of the unknown angle in each of the following.

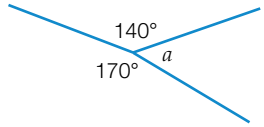
(a)



(b)

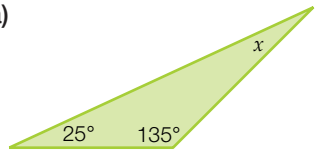


(c)

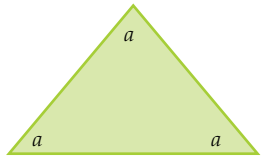


2 Find the size of the unknown angle in each of the following.

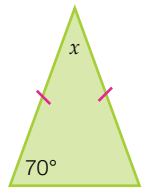
(a)



(b)

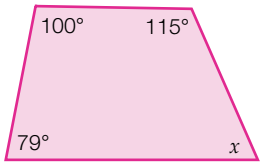


(c)

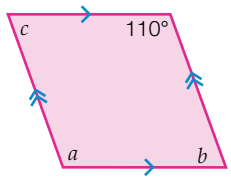


3 Find the size of the unknown angle in each of the following.

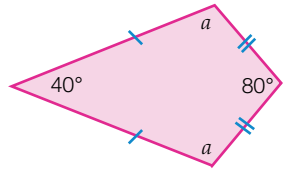
(a)



(b)



(c)



4 Draw an example of:

(a) a rhombus **(b)** a trapezium **(c)** an equilateral triangle

(d) a kite **(e)** an isosceles triangle **(f)** a parallelogram.

Mark all sides that are equal and/or parallel. Use letter names to name your figures.

Exploration Task

You can download this activity from the eBook or the Pearson Places website.

Shape shifting

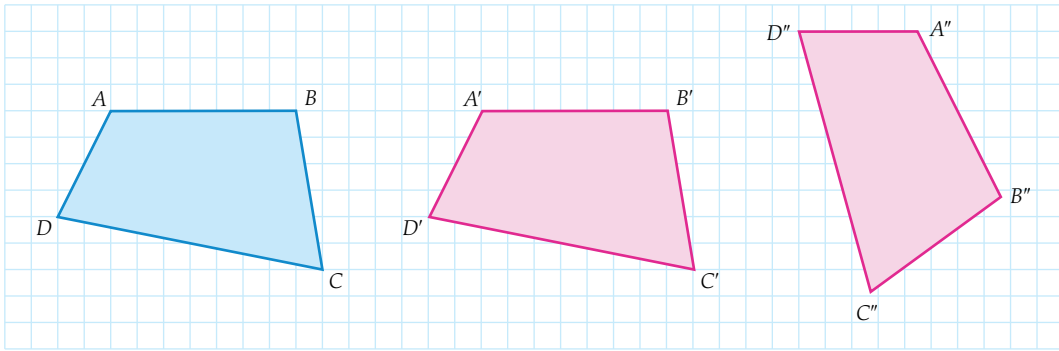
In this activity, you will investigate what combination of translations, reflections and rotations you can use to create a rectangle from a set of points. Can it be done in one move?

Translations

10.1

A **transformation** changes a particular shape (or figure) or set of points according to a defined rule. A transformation may move, turn, flip or change the shape of a figure.

The transformations you will deal with here are those that move a shape but retain its original shape and size. The first figure is called the **original** and the transformed shape is called the **image** of the transformation. When the original shape is given letter names such as *A*, *B*, *C*, *D*, then the image letter names are usually marked *A′*, *B′*, *C′*, *D′*. Transformations may change the position only, or they may involve a change in orientation such as a turn or a reflection.



Original

Image 1
Change in position only

Image 2
Change in position and orientation

There are three types of transformations in which the shape does not change:

- translation
- reflection
- rotation.

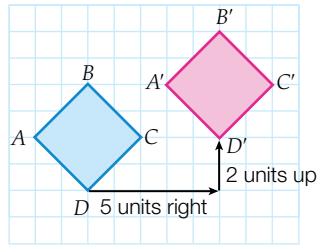
Each type of transformation moves a figure in a particular way. Transformations can be combined to make a complex move and can be reversed to move the image back to its original position and orientation.

- Vertices of a transformed image are denoted by a dash, e.g. *A* prime. This is said as *A* dash or *A* prime.
- To show the difference between the original figure and the transformed image, you can use a different colour or broken lines.

Translations

A **translation** is when a figure is shifted or slid across to change its position. When a figure is translated, the image is moved without turning. It is simply moved a number of units left or right, or a number of units up or down, or a combination of both.

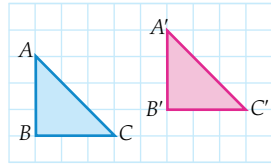
The example at right shows a figure that has been translated 5 units to the right and 2 units up. This translation can be written as [5, 2].



- Translations can be reversed by doing the opposite horizontal and vertical moves. For example, the reverse of a translation of 4 units left and 3 units down is 4 units right and 3 units up.

Worked example 1 W.E. 1

Describe the translation shown in the diagram.

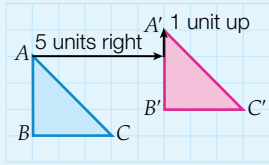


Thinking

1 Start by selecting one of the vertices and count the units in the horizontal and vertical directions to get to the corresponding translated vertex.

2 Write the translation in words or as an ordered pair.

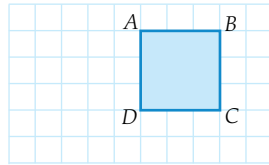
Working



5 units right and 1 unit up or [5, 1].

Worked example 2 W.E. 2

Copy the following onto grid paper and draw the resulting image after the translation: 3 units left and 4 units down.



Thinking

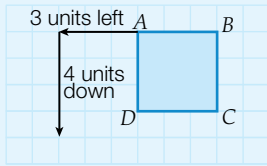
1 Start by selecting one of the vertices (A) and move according to the given translation.

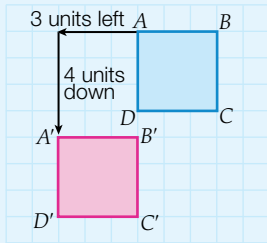
2 Label this vertex with a letter using image notation (A').

3 Repeat with the remaining vertices and then draw the resulting image in a different colour.

4 Label all vertices of the image using image notation.

Working





If your shape is drawn on the Cartesian plane, you can identify the coordinates of the vertices and then find the coordinates of the transformed shape after a given transformation. In this section, you will identify the coordinates of a shape after a given translation.

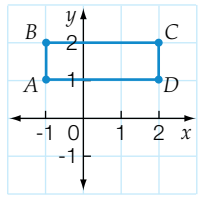
Worked example 3 W.E. 3

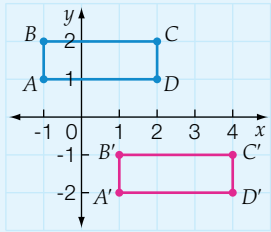
(a) Write the coordinates of each vertex on the given shape.

(b) Copy the shape onto grid paper and draw the resulting image after the translation of 2 units to the right and 3 units down.

(c) Write the coordinates of the translated vertices.

(d) Explain how the coordinates of the image could be found without drawing the shape.



Thinking	Working
(a) Use the x- and y-axes to identify the coordinates of each of the vertices.	(a) $A = (-1, 1)$, $B = (-1, 2)$, $C = (2, 2)$, $D = (2, 1)$
(b) Translate each vertex by the given translation, labelling each vertex with image notation. (Each point has been moved 2 units to the right and 3 units down. This is a translation of [2, -3].)	(b) 
(c) Identify the coordinates of each vertex on the resulting image.	(c) $A' = (1, -2)$, $B' = (1, -1)$, $C' = (4, -1)$, $D' = (4, -2)$
(d) Look for the connection between the original coordinates, the transformation and the image coordinates.	(d) Adding 2 to the x-coordinate of the original and subtracting 3 from the y-coordinate of the original will give the image coordinates.

10.1 Translations

Navigator

1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 18, 19	1, 2 (column 1), 3, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 18, 19, 20	1 (c–d), 2 (column 2), 3, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20

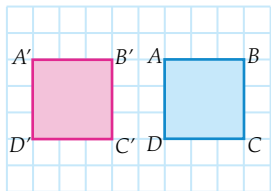
Answers p. 711

Equipment required: grid paper or graph paper

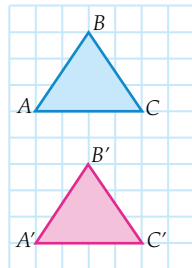
Fluency

1 Describe the translation shown in each of the following diagrams.

(a)

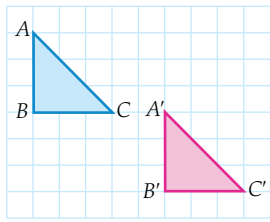


(b)

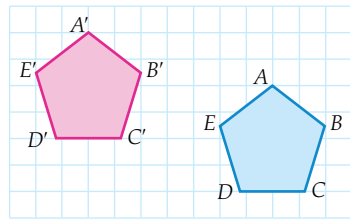


W.E. 1

(c)



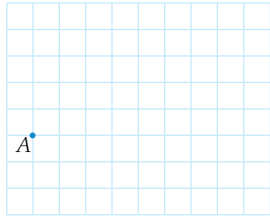
(d)



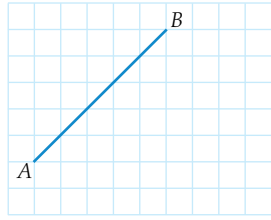
W.E. 2

2 Copy each of the following onto grid paper and draw the resulting image after the translation.

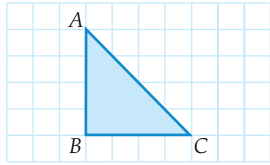
(a) 6 units right and 2 units up



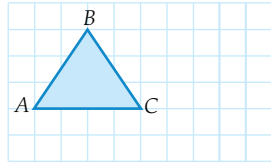
(b) 6 units left and 6 units down



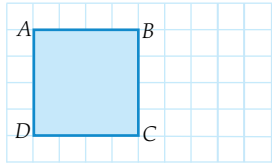
(c) 6 units left and 1 unit down



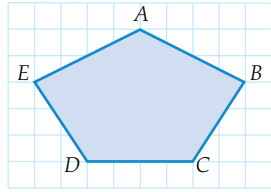
(d) 1 unit right and 6 units down



(e) [7, -5]



(f) [-6, 4]



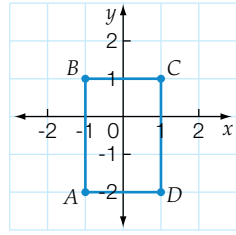
W.E. 3

3 **(a)** Write the coordinates of each vertex on the given shape.

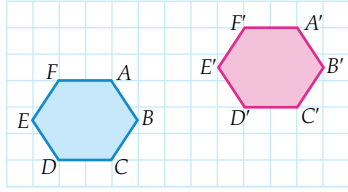
(b) Copy the shape onto grid paper and draw the resulting image after the translation of 1 unit to the left and 2 units up.

(c) Write the coordinates of the translated vertices.

(d) Explain how the coordinates of the image could be found without drawing the shape.



4 How has the hexagon in the diagram been translated?



A 7 units up and 2 units right

B 7 units right and 2 units up

C 7 units left and 2 units down

D 6 units right and 2 units up

5 What would be the reverse translation of 4 units left and 8 units down?

A 4 units up and 8 units right

B 4 units left and 8 units up

C 4 units left and 8 units down

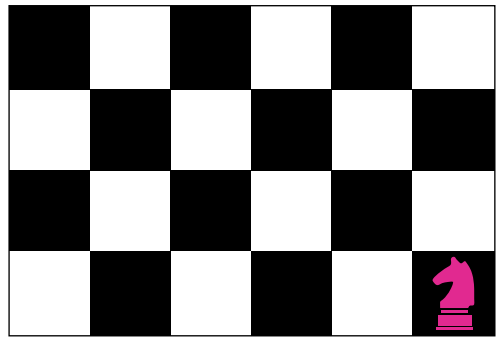
D 4 units right and 8 units up

6 Write the translation [-2, 6] using direction instructions (right, left, up, down).

7 Write the translation of 3 units left and 4 units down using square brackets.

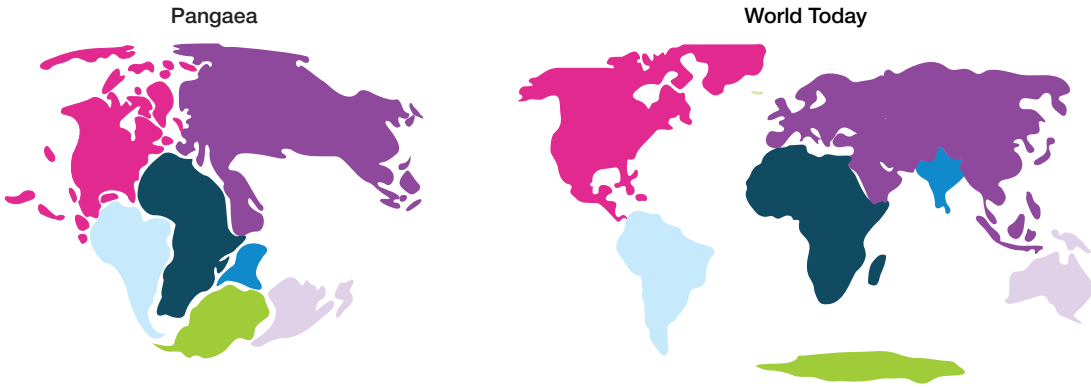
8 Write the reverse translation of 10 units right and 4 units up using square brackets.

9 In the game of chess, the knight (sometimes called the horse) can move in an L-shaped path in any direction. This means it can move to a square that is two squares horizontally and one square vertically away, or two squares vertically and one square horizontally away, from its starting position on the chessboard.



Describe the two ways that a knight can move if it is in the corner of the chessboard, as shown.

10 Pangaea was a supercontinent that existed 300 million years ago, before it drifted apart into the continents of the world we know today. Based on the maps below, describe how each continent (with a different colour) moved around as translations from Pangaea to the world today.



Understanding

11 **(a)** Plot the points (-1, 4), (2, 3) and (0, 2) on a Cartesian plane and label them *A*, *B* and *C*.

(b) Join the points to form triangle *ABC*, then translate the shape 2 units right and 1 unit down, labelling the image with image notation.

(c) Write the coordinates of the translated points.

12 A figure is translated 4 units left, 3 units down, 6 units left and 7 units up. What would be the final position of the image compared to that of the original figure?

13 What single translation would be equivalent to 12 units down, 5 units right, 7 units left, 8 units up and 2 units right?

14 What single translation would be equivalent to a translation of [2, 3] followed by a translation of [-2, 6]?

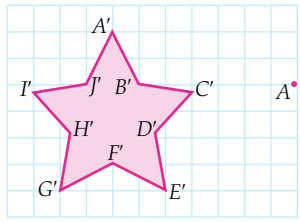
Reasoning

15 **(a)** Without plotting points, write the coordinates of the image after the line joining $A(4, 2)$ and $B(6, -1)$ is translated 2 units left and 2 units up.

(b) Plot the line AB and its image $A'B'$ on a Cartesian plane to check your answer to part **(a)**.

16 If a particular translation was 2 units right, what would be the single translation that would be equivalent to repeating 2 units right a hundred times?

17 The following diagram is of a translated image and a single point of the original figure.



(a) What translation has occurred to the original figure?

(b) What is the reverse translation?

(c) Use this to reverse the translation and draw the original figure.

Open-ended

18 Plot three points on a Cartesian plane and write their coordinates. Decide upon a particular translation and use this translation to translate your shape. Draw the image after the translation and write down the coordinates of the translated points using image notation.

19 On grid paper, draw a figure that has 4 vertices. Then, translate that figure in any direction and identify the reverse translation required to get the image back to the original figure.

20 Provide an explanation either for or against the following statement:

'If a figure undergoes more than one translation, the order in which you translate a figure does not affect the final position of the image.'

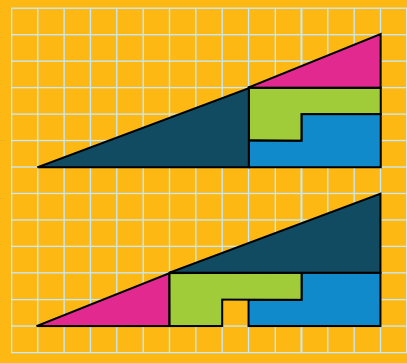
Puzzle

Jigsaw paradox

This jigsaw paradox was made famous by Martin Gardner, but is claimed to have been invented by New York magician Paul Curry, and may have originated much earlier.

The puzzle shows two arrangements of four identical shapes to create two right-angled triangles, but one triangle has a 1×1 hole in it. Both triangles appear to have a base of 13 units long and a height of 5 units.

How can this be true? Can you explain what has happened to the 'missing' 1×1 square?



Reflections

10.2

A **reflection** is a transformation that creates an image of the original figure in the same way that a mirror creates an image when you look into it. A reflection is therefore often called a mirror image. It is a reversed or 'flipped over' version of the original about a **line of reflection**.

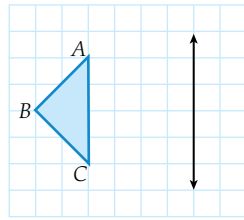
A reflected image has the same shape and size as the original but its position has changed and its orientation is reversed.

Each point on a reflected image is the same distance from the line of reflection as the original, but on the opposite side. The distance used is the **perpendicular distance**, which is measured by a line that makes a right angle with the line of reflection. To reflect a figure, reflect each vertex on the figure and then join the points in order.

Worked example 4

W.E. 4

Copy the following figure onto grid paper and draw the resulting image when the triangle is reflected in the line of reflection shown.



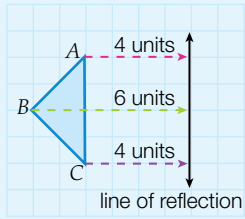
Thinking

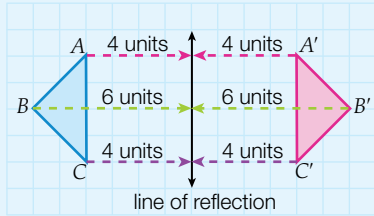
1 Measure the perpendicular distance between each vertex and the line of reflection.

2 Use the distance between the vertices on the line of reflection to plot the points on the opposite side of the line of reflection and reproduce the original figure.

3 Label the image vertices with image notation.

Working





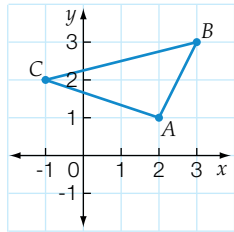
- A reflection always flips the original, so the order in which the vertices are labelled in the image will be opposite to that in the original figure.
- A reflection is as far from one side of the line of reflection as the original figure is from the other side of it.
- When reflected, the properties of the shape will remain the same; for example, the size of the side lengths or angles will stay the same.

You can find the image of a shape drawn on a Cartesian plane after it is reflected in either the x-axis or y-axis.

Worked example 5

W.E. 5

(a) **(i)** Write the coordinates of each vertex on the given shape.

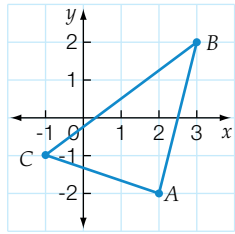


(ii) Copy the shape onto grid paper and draw the resulting image after a reflection in the x-axis, labelling each vertex with image notation.

(iii) Write the coordinates of the reflected vertices.

(iv) Explain how the coordinates of the image could be found without drawing the shape.

(b) **(i)** Write the coordinates of each vertex on the given shape.



(ii) Copy the shape onto grid paper and draw the resulting image after a reflection in the y-axis, labelling each vertex with image notation.

(iii) Write the coordinates of the reflected vertices.

(iv) Explain how the coordinates of the image could be found without drawing the shape.

Thinking

(a) **(i)** Use the x- and y-axes to identify the Cartesian coordinates of the points.

(ii) Reflect each point in the x-axis and join them in order to form the reflected shape, labelling each vertex with image notation.

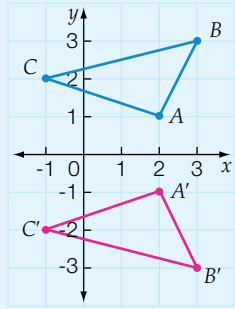
(iii) Identify the coordinates of each vertex on the resulting image.

(iv) Look for the connection between the coordinates of the original and the image.

Working

(a) (i) $A = (2, 1)$, $B = (3, 3)$, $C = (-1, 2)$

(ii)



(iii) $A' = (2, -1)$, $B' = (3, -3)$, $C' = (-1, -2)$

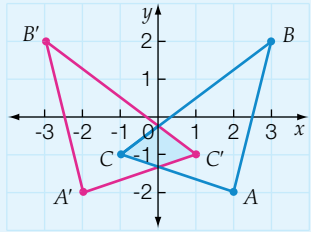
(iv) The x-coordinate of the image stays the same as that of the original. The y-coordinate of the image is the negative of that of the original.

(b) (i) Use the x- and y-axes to identify the Cartesian coordinates of the points.

(b) (i) $A = (2, -2)$, $B = (3, 2)$, $C = (-1, -1)$

(ii) Reflect each point in the y-axis and join them in order to form the reflected shape, labelling each vertex with image notation.

(ii)



(iii) Identify the coordinates of each vertex on the resulting image.

(iii) $A' = (-2, -2)$, $B' = (-3, 2)$, $C' = (1, -1)$

(iv) Look for the connection between the coordinates of the original and the image.

(iv) The x-coordinate of the image is the negative of that of the original. The y-coordinate of the image stays the same as that of the original.

10.2 Reflections

Navigator

1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 14	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 16	1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16

Answers p. 713

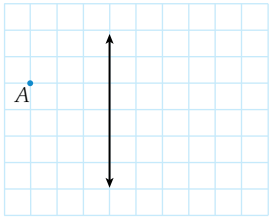
Equipment required: grid paper or graph paper

Fluency

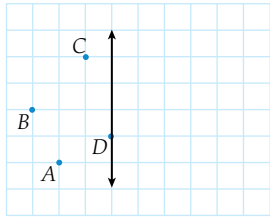
1 Copy the following figures onto grid paper and draw the resulting image when each is reflected in the line of reflection shown.

W.E. 4

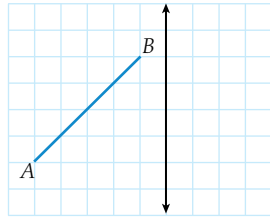
(a)



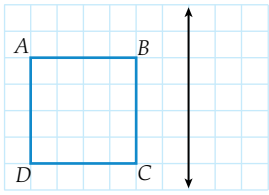
(b)



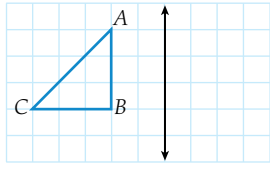
(c)



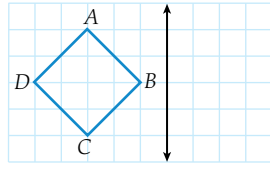
(d)



(e)

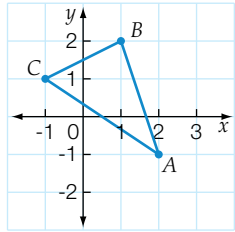


(f)



2 (a) Write the coordinates of each vertex on the given shape.

W.E. 5

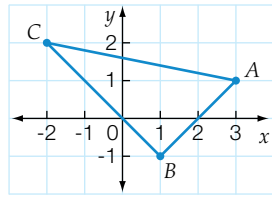


(b) Copy the shape onto grid paper and draw the resulting image after a reflection in the x-axis, labelling each vertex with image notation.

(c) Write the coordinates of the reflected vertices.

(d) Explain how the coordinates of the image could be found without drawing the shape.

3 **(a)** Write the coordinates of each vertex on the given shape.



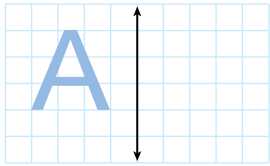
(b) Copy the shape onto grid paper and draw the resulting image after a reflection in the y-axis, labelling each vertex with image notation.

(c) Write the coordinates of the reflected vertices.

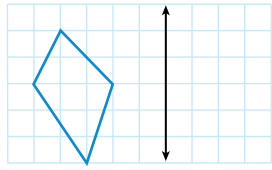
(d) Explain how the coordinates of the image could be found without drawing the shape.

4 Copy each of the following figures onto grid paper, then draw the reflection of each in the line of reflection shown.

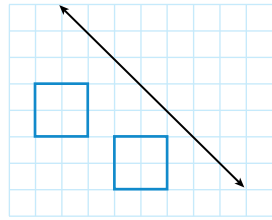
(a)



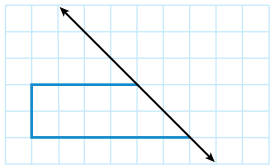
(b)



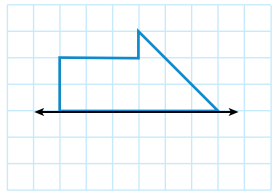
(c)



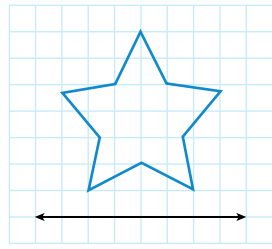
(d)



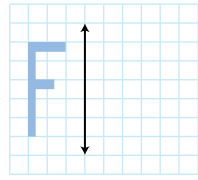
(e)



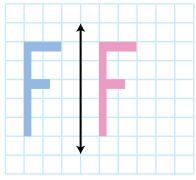
(f)



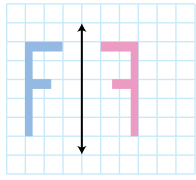
5 Which of the following options is the correct reflection of the capital F in the vertical line of reflection?



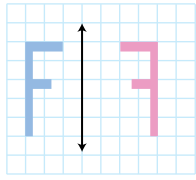
A



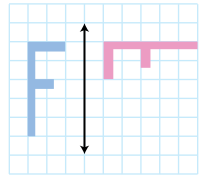
B



C



D



6 When the uppercase letter **A** is placed to the left of the axis of reflection, its reflection on the right of the axis looks identical. See the letters below and list any five other letters with the same identical reflection.

A B C D E F G H I J K L M
N O P Q R S T U V W X Y Z

Understanding

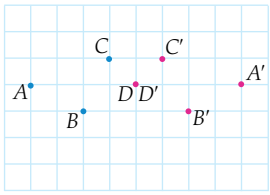
7 **(a)** Plot the points (-1, 5), (3, 1) and (2, -1) on a Cartesian plane and label them A, B and C.

(b) Join the points to form triangle ABC, then reflect the shape first in the x-axis and then in the y-axis.

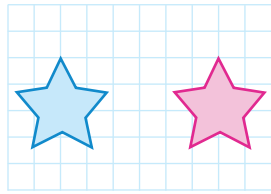
(c) Write the coordinates of the points after the second reflection.

8 Copy each of the following onto grid paper and indicate where the line of reflection should be placed to produce the given image.

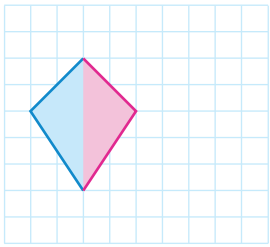
(a)



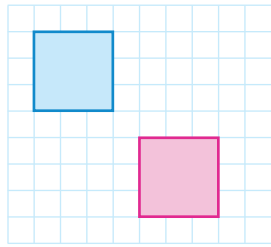
(b)



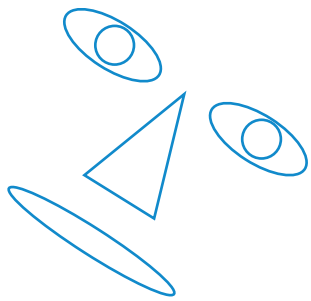
(c)



(d)

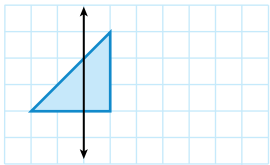


9 Copy the 'face' on the right and draw the axis of reflection for the face.

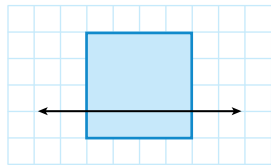


10 Copy each of the following onto grid paper and draw the reflected images. The image may be on both sides of the line of reflection.

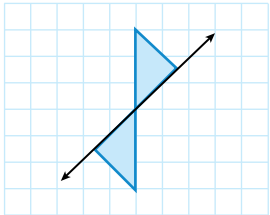
(a)



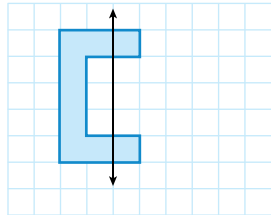
(b)



(c)



(d)



Reasoning

11 **(a)** Without plotting points, write the coordinates of the image after the line joining $A(-2, 1)$ and $B(4, -3)$ is reflected in the x-axis.

(b) Plot the line AB and its image $A'B'$ on a Cartesian plane to check your answer to part **(a)**.

(c) Without plotting points, write the coordinates of the image after the line joining $A(-2, 1)$ and $B(4, -3)$ is reflected in the y-axis.

(d) Plot the line AB and its image $A'B'$ on a Cartesian plane to check your answer to part **(c)**.

12 Bruno has a digital clock on his bedside table. At the time 8:30, he looked at the clock and noticed that this time was reflected perfectly on the shiny table. Thinking this to be interesting, he decided to find out the following.

(a) List the number of minutes past the hour when the minute part of the time reflects the correct time (include 00).

(b) For 12-hour time, list the hours when the hour part of the time reflects the correct time.

(c) How many minutes will the reflection on the table show the correct time in 12-hour time, over a period of 12 hours?

(d) How many minutes will the reflection on the table show the correct time in 24-hour time, over a period of 24 hours?

(e) Is your answer to part **(d)** double your answer to part **(c)**? Explain your answer.

Open-ended

13 Plot three points on a Cartesian plane and write their coordinates. Reflect this shape in either the x-axis or y-axis. Draw the image after the reflection and write the coordinates of the reflected points using image notation.

14 On grid paper, draw a figure with four vertices. Reflect this figure in:

(a) a horizontal line of reflection

(b) a vertical line of reflection.

15 Repeat the previous question using a figure with five, six or seven vertices.

16 On grid paper, draw a figure with four vertices.

(a) Reflect this figure in a horizontal line of reflection and then in a vertical line of reflection.

(b) Reflect the same figure in the vertical line of reflection and then in the horizontal line of reflection.

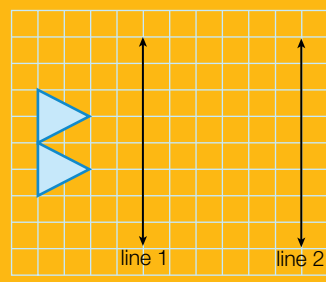
(c) Comment on the images found in parts **(a)** and **(b)**.

Problem solving

Mirror, mirror on the wall, does it matter where I stand at all?

Equipment required: grid paper

The diagram below consists of two blue triangles and two vertical lines of reflection.



To begin, reflect the two triangles in the line of reflection labelled 'line 1' and then in the line of reflection labelled 'line 2'. Note the final location of the image. Next, switch the order in which the two triangles are reflected so that they are reflected in line 2 first, and then in line 1, and note the final location of the image.

1. Does the order in which these triangles are reflected in either line affect the location and orientation of the final image?
2. What did you notice about the final image?
3. Can this transformation be described another way?

To justify your answer, experiment with different situations, such as drawing the two triangles originally on the right-hand side of line 2 or in between the lines. You can also change the position of the lines of reflection. For example, line 1 and line 2 could be two horizontal lines, or one line could be horizontal and the other vertical.

Strategy options

- Draw a diagram.
- Test all possible combinations.

10.3 Rotations

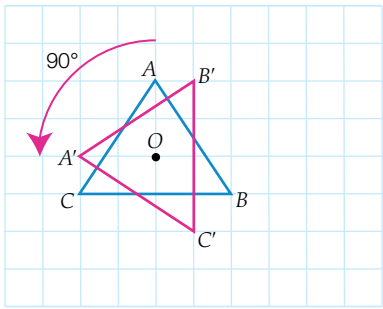
You also transform a figure when you **rotate** it around a fixed point. This is called a **rotation**. The fixed point is called the **centre of rotation** and is labelled with the letter *O*.

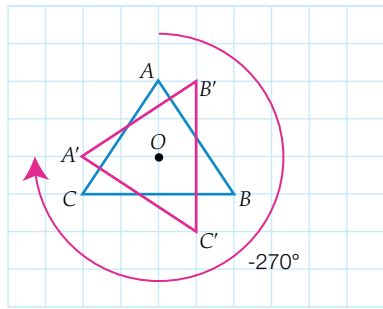
When rotating any figure you need to know:

- the location of the centre of rotation
- the size of the angle of rotation
- the direction of the rotation (clockwise ↷ or anticlockwise ↶).

The centre of rotation can be located inside or outside the figure and is the only point that does not rotate. The size of the angle of rotation is generally in multiples of 30° or 45°. Common angles of rotation are 30°, 45°, 60°, 90°, 180° and 270°. A rotation of 360° will rotate the image of the figure through a full revolution about the centre of rotation and will return the figure to its original position.

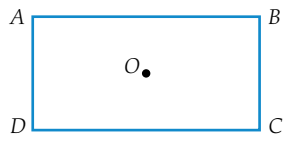
- The vertices of a rotated image are labelled in the same order as the original figures.
- Rotations of 360°, 720° and other multiples of 360° result in the image in the same position as the original figure.
- Two rotations that result in an image in the same position will always add to 360°. For example, 90° anticlockwise is equivalent to 270° clockwise as 90° + 270° = 360°.
- A rotation in an anticlockwise direction is a positive rotation.
- A rotation in a clockwise direction is a negative rotation.

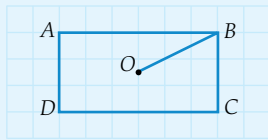




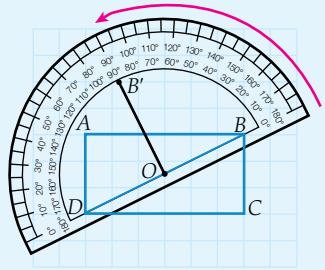
Worked example 6 W.E. 6

Copy the following figure onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you. Rotate 90° in an anticlockwise direction (90°) about *O*.

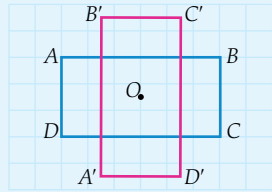


Thinking	Working
1 Select a key point, such as a vertex, and join this point to the centre of rotation using a straight line.	

2 Place the centre of a protractor on the centre of rotation, with the chosen vertex on the base line. Measure the given angle of rotation in the given direction from this vertex. Plot the resulting point. The new point must be the same distance from O as the original point.



3 Continue the process with the other vertices and connect the resulting points to produce the original figure in its rotated position.

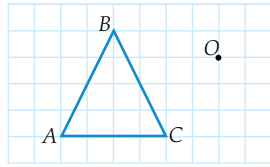


The same steps are used when the centre of rotation is outside the shape. The distance between the vertices and the centre of rotation is the same before and after the rotation, similar to reflection of an object in a line of reflection. Measuring this distance and checking that it is the same before and after the rotation is an easy way to check that the rotation is correct.

Worked example 7

W.E. 7

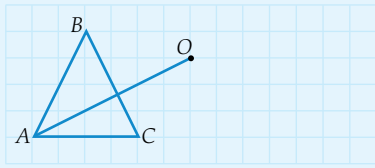
Copy the following figure onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you. Rotate 180° in a clockwise direction (-180°) about O.



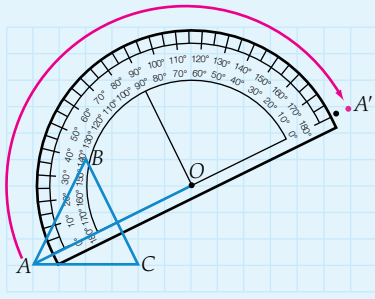
Thinking

Working

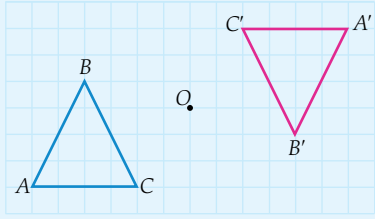
1 Select a key point, such as a vertex, and join this point to the centre of rotation using a straight line.



2 Place the centre of a protractor on the centre of rotation, with the chosen vertex on the base line. Measure the given angle of rotation in the given direction from this vertex and plot a point. Extend AO through this point and mark the point A' so that OA' is the same length as OA.



3 Continue the process with the other vertices and connect the resulting points to reproduce the original figure in its rotated position.

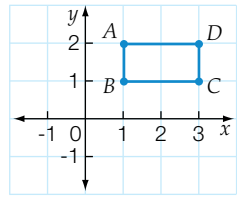


You can find the coordinates of an image after a rotation of 90°, 180° or 270° in a clockwise or anticlockwise direction about the origin.

Worked example 8

W.E. 8

(a) Write the coordinates of each vertex on the given shape and copy the shape onto grid paper.



(b) (i) Draw the resulting image after the original is rotated 90° in an anticlockwise direction about the origin.

(ii) Write the coordinates of the rotated vertices.

(iii) Explain how the coordinates of the image could be found without drawing the shape.

(c) (i) Draw the resulting image after the original is rotated 180° in an anticlockwise direction about the origin.

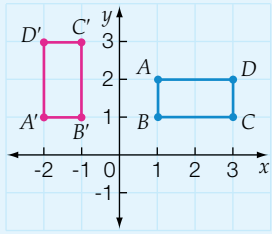
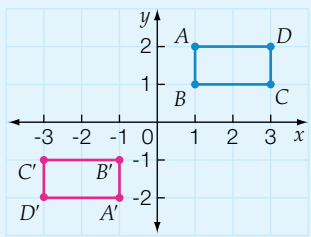
(ii) Write the coordinates of the rotated vertices.

(iii) Explain how the coordinates of the image could be found without drawing the shape.

(d) (i) Draw the resulting image after the original is rotated 270° in an anticlockwise direction.

(ii) Write the coordinates of the rotated vertices.

(iii) Explain how the coordinates of the image could be found without drawing the shape.

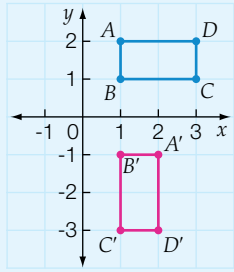
Thinking	Working
(a) Use the x- and y-axes to identify the coordinates of each of the vertices.	(a) $A = (1, 2)$, $B = (1, 1)$, $C = (3, 1)$, $D = (3, 2)$
(b) (i) Rotate each vertex around the origin by the given rotation, labelling each vertex with image notation. (Each point has been rotated by 90° in an anticlockwise direction about the origin.)	(b) (i) 
(ii) Identify the coordinates of each vertex on the resulting image.	(ii) $A' = (-2, 1)$, $B' = (-1, 1)$, $C' = (-1, 3)$, $D' = (-2, 3)$
(iii) Look for the connection between the original coordinates, the transformation and the image coordinates.	(iii) The x-coordinates of the image vertices are the negative of the y-coordinates of the original vertices and the y-coordinates of the image vertices are the x-coordinates of the original vertices.
(c) (i) Rotate each vertex around the origin by the given rotation, labelling each vertex with image notation. (Each point has been rotated by 180° in an anticlockwise direction about the origin.)	(c) (i) 
(ii) Identify the coordinates of each vertex on the resulting image.	(ii) $A' = (-1, -2)$, $B' = (-1, -1)$, $C' = (-3, -1)$, $D' = (-3, -2)$

(iii) Look for the connection between the original coordinates, the transformation and the image coordinates.

(iii) *All the coordinates of the image vertices are the negative of the original coordinates of the vertices.*

(d) (i) Rotate each vertex around the origin by the given rotation, labelling each vertex with image notation. (Each point has been rotated in an anticlockwise direction by 270° or -90°.)

(d) (i)



(ii) Identify the coordinates of each vertex on the resulting image.

(ii) $A' = (2, -1)$, $B' = (1, -1)$, $C' = (1, -3)$, $D' = (2, -3)$

(iii) Look for the connection between the original coordinates, the transformation and the image coordinates.

(iii) *The x-coordinates of the image vertices are the y-coordinates of the original vertices and the y-coordinates of the image vertices are the negative of the x-coordinates of the original vertices.*

10.3 Rotations

Navigator

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 (a), 13, 14, 15 (a–c), 17	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18	1, 2 (column 2), 3 (column 2), 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19

Answers p. 715

Equipment required: protractor and grid paper

Fluency

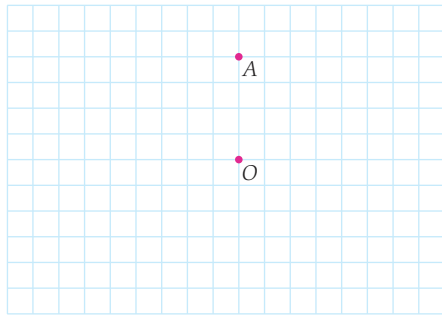
1 Copy the following 2 points (labelled *A* and *O*) onto grid paper and draw the resulting image of *A* after the given rotation. Use a protractor to help you.

(a) Rotate the point *A* 45° in a clockwise direction about *O*.

(b) Rotate the point *A* 90° in a clockwise direction about *O*.

(c) Rotate the point *A* 180° in a clockwise direction about *O*.

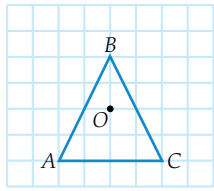
(d) Rotate the point *A* 360° in a clockwise direction about *O*.



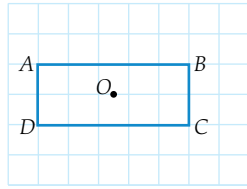
W.E. 6

2 Copy each of the following figures onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you.

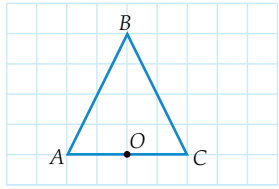
(a) Rotate 180° in an anticlockwise direction (180°) about *O*.



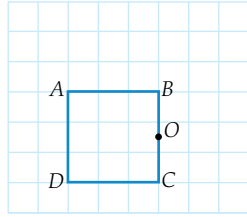
(b) Rotate 270° in an anticlockwise direction (270°) about *O*.



(c) Rotate 180° in a clockwise direction (-180°) about *O*.



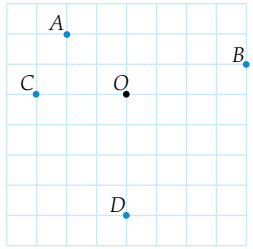
(d) Rotate 90° in a clockwise direction (-90°) about *O*.



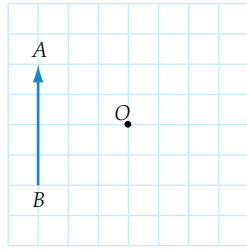
W.E. 7

3 Copy each of the following onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you.

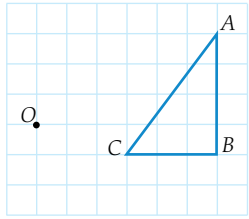
(a) Rotate each of the points 90° in a clockwise direction (-90°) about *O*.



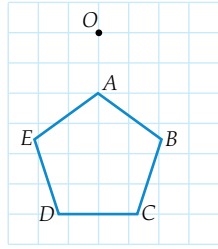
(b) Rotate 270° in an anticlockwise direction (270°) about *O*.



(c) Rotate 180° in a clockwise direction (-180°) about *O*.

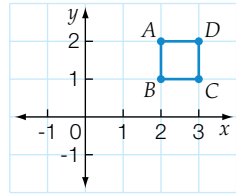


(d) Rotate 90° in an anticlockwise direction (90°) about *O*.



W.E. 8

4 **(a)** Write the coordinates of each vertex on the given shape and copy the shape onto grid paper.



(b) **(i)** Draw the resulting image after the original is rotated 90° in an anticlockwise direction about the origin.

(ii) Write the coordinates of the rotated vertices.

(iii) Explain how the coordinates of the image could be found without drawing the shape.

(c) **(i)** Draw the resulting image after the original is rotated 180° in an anticlockwise direction about the origin.

(ii) Write the coordinates of the rotated vertices.

(iii) Explain how the coordinates of the image could be found without drawing the shape.

(d) **(i)** Draw the resulting image after the original is rotated 270° in an anticlockwise direction about the origin.

(ii) Write the coordinates of the rotated vertices.

(iii) Explain how the coordinates of the image could be found without drawing the shape.

5 Which of the following statements about rotation is incorrect?

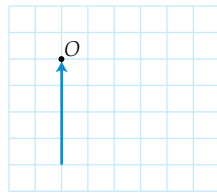
A Rotation can be in either a clockwise or an anticlockwise direction.

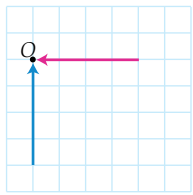
B A rotation of 360° will result in the image located in the same position as the original figure.

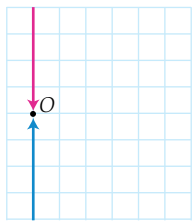
C A rotation will always produce an image that is a perfect replica and in the same orientation as the original figure.

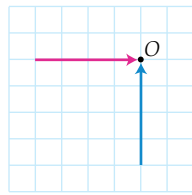
D A rotation of 90° in a clockwise direction is equivalent to a rotation of 270° in an anticlockwise direction about the same centre of rotation.

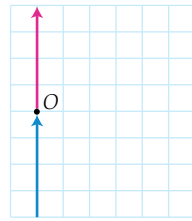
6 This arrow turns about the centre of rotation *O*. What does the shape look like after a turn of 180° anticlockwise?



A


B


C


D


Understanding

7 **(a)** Plot the points (1, 4), (2, 4), (2, 2) and (1, 2) on a Cartesian plane and label them *A*, *B*, *C* and *D*.

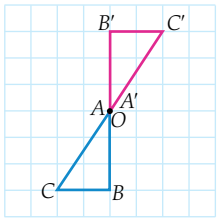
(b) Join the points to form a figure *ABCD*, then rotate the figure 90° in a clockwise direction about point *A*.

(c) Write the coordinates of the rotated points using image notation.

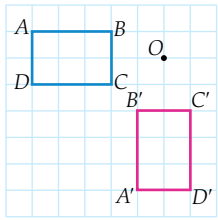
8 A clockwise rotation of 120° is the same as an anticlockwise rotation of how many degrees?

9 Find the size and direction of each rotation that has been done to produce the following images. Identify other rotations that could produce the same result.

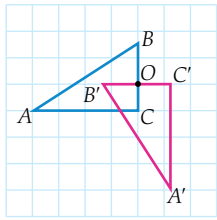
(a)



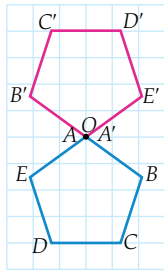
(b)



(c)



(d)



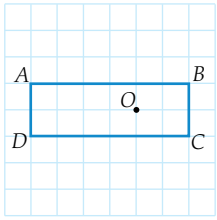
Reasoning

10 **(a)** Without plotting points, write the coordinates of the image after the line joining $A(4, 2)$ and $B(3, -2)$ is rotated 180° clockwise about the origin.

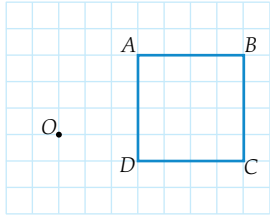
(b) Plot the line AB and its image $A'B'$ on a Cartesian plane to check your answer to part **(a)**.

11 Copy each of the following figures onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you.

(a) Rotate 90° in an anticlockwise direction about O.



(b) Rotate 90° in a clockwise direction about O.



12 The following are the capital letters of the alphabet.

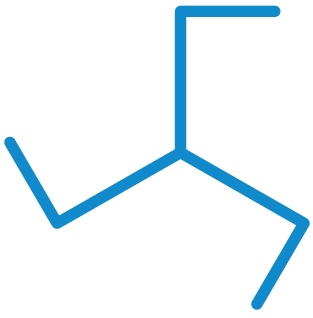
A B C D E F G H I J K L M
N O P Q R S T U V W X Y Z

(a) Which letters of the alphabet are able to be rotated about a centre of rotation in either direction and at an angle of less than 360° to produce an identical image in the same orientation as the original letter?

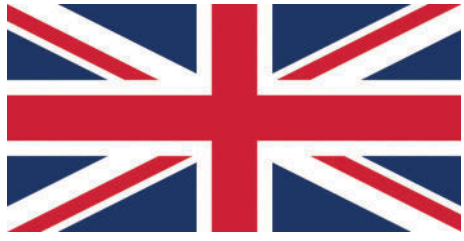
(b) Four of these letters have something else in common with each other. Can you identify these four letters? Describe the similarity you have noticed.

13 Estimate the smallest angle through which the following figures need to be rotated so the resulting image will match the original figure.

(a)



(b)



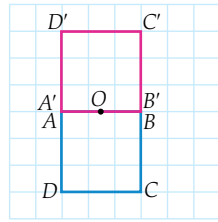
(c) 

(d) 

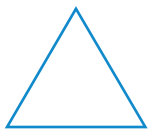
(e) 

(f) 

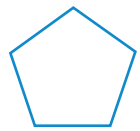
14 Is the following transformation a rotation of 180° around point O, a reflection along the line AB or both? Provide an explanation.

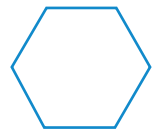


15 What is the smallest angle about which the following polygons need to be rotated, so that the resulting image will match the original figure? The centre of rotation is at the centre of each of the polygons.

(a) 
equilateral triangle

(b) 
square

(c) 
regular pentagon

(d) 
regular hexagon

16 What formula gives the smallest angle of rotation for any regular polygon to be rotated about its centre, so that the image is the same as the original figure? Use your answer to the previous question to find this.

Open-ended

17 Plot four points on a Cartesian plane and write their coordinates. Decide upon a particular rotation and use this rotation to rotate your shape. Draw the image after the rotation and write down the coordinates of the rotated points using image notation.

18 Explain whether you agree or disagree with the following statement.

'A rotation of 180° in either direction and about any point of rotation can be equivalent to reflecting the same figure in a line of reflection through that point.'

19 On grid paper, draw two figures, a regular and an irregular polygon with the same number of sides. Decide upon a centre of rotation for each polygon that is on a vertex of that polygon.

(a) Rotate each of the figures by 90° about the centre of rotation in a clockwise direction and draw the image.

(b) Repeat this four times.

(c) Rotate the figures 45° about the centre of rotation.

(d) Repeat this eight times.

(e) Comment on the final images in parts **(b)** and **(d)**.

Problem solving

Tangram magic

Equipment required: 2 pieces of grid paper, blank paper, ruler, pens, scissors

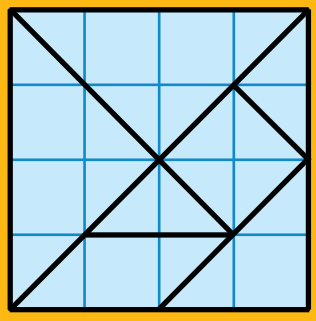
Tangram is an ancient Chinese puzzle. The object of the puzzle is to rearrange the pieces of a square (the puzzle pieces) to form as many different shapes as you can using all seven pieces.

Before you start this puzzle you will need to make the seven pieces to form the shapes.

Step 1 On a piece of grid paper, rule a square. The larger the square, the larger the pieces will be to work with. An 8 cm × 8 cm square is recommended.

Step 2 Use the tangram template shown below to divide your square into seven pieces as shown.

Step 3 Cut out your pieces carefully.



The tangram pieces will be:

- one square
- two small identical isosceles triangles
- two large identical isosceles triangles
- one medium isosceles triangle
- one parallelogram.

After the tangram pieces are cut out, you can try to use all the pieces to form the 11 shapes given below. You can translate, reflect and rotate the pieces, but in each shape you must use all seven pieces.

When you have formed a shape, trace it carefully on blank paper, showing how the pieces fit together. Keep a list of the shapes you have made.

Runner

Dancer

House

Tree

Kangaroo

Boat

Shirt

Swan

Double-headed arrow

Letter C

Cat

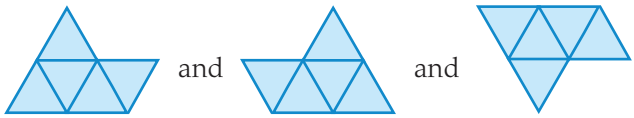
Investigation

Equipment required: isometric grid paper, isometric dot paper
(A computer drawing program or other software may also be used to produce the required shapes.)

Diamonds are forever!

A *frieze* is a decorative horizontal strip that shows a repeating pattern. The repeating pattern is formed by combining shapes with translations, reflections and rotations, so a frieze has a high degree of symmetry.

When you look for different ways to connect your shapes, remember that mirror reflections or rotations are not considered to be different. For example,



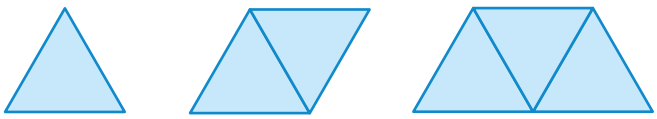
are not different ways to connect five equilateral triangles.

The Big Question

Can you make a frieze with translations, reflections and rotations or at least five different shapes made from equilateral triangles, diamonds and trapeziums?

Engage

If an equilateral triangle or 'moniamond' is copied and reflected along an edge, then it becomes a diamond (rhombus). If the triangular shape is copied again reflected along another edge, then it becomes a triamond (trapezium).

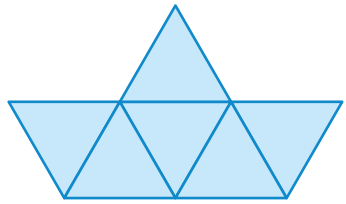


1 On isometric grid paper, draw the three different ways that four equilateral triangles can be connected to make a new shape. These are tetriamonds.

2 On isometric grid paper, draw the four different ways that five equilateral triangles can be connected to make a new shape. These are pentiamonds.

Explore

3 Six equilateral triangles can be connected to make a new shape. These are hexiamonds. There are 12 different hexiamond shapes that can be made. Here is an example:



On isometric grid paper, copy the hexiamond on the right and find the 11 other hexiamond shapes.

The 12 names given below might help you find the shape. Can you match each name to a shape?

- bar (parallelogram)
- crook (club)
- crown
- sphinx
- snake
- yacht
- chevron (bat)
- signpost (pistol)
- lobster
- hook (shoe)
- hexagon
- butterfly

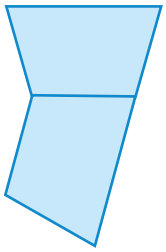
If a diamond shape (rhombus) is translated by a full side length and then joined to the original shape, it makes a shape we can call a straight bidiamond. If the diamond shape is translated again to make three diamond shapes joined together in a row, the result is a shape we can call a straight tridiamond.

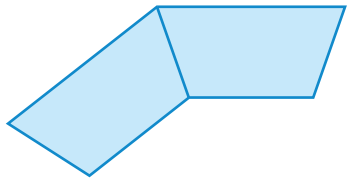
Three diamonds can be connected to make tridiamonds of different shapes, including the straight tridiamond given above.

4 Nine of the 12 hexiamonds found in **3** can be made by reflecting, translating and rotating diamonds to make these tridiamonds. On isometric dot paper, draw the nine tridiamonds showing the diamonds that have been used to form them.

5 One of the hexiamond shapes when reflected along one of its sides will form a six-pointed star. Find the shape, reflect it to form the star and then translate the shape to make a row of three six-pointed stars. Draw your pattern on isometric dot paper.

6 There are nine ways that two triamonds (trapeziums made from three equilateral triangles) can be joined exactly edge to edge to make different shapes. On isometric dot paper, draw the nine triamonds (including the two given below) showing the triamonds that have been used to form them.





7 In how many ways can three triamonds be joined together? On isometric dot paper, draw at least three different shapes showing the triamonds that have been used to form them.

8 In how many ways can four triamonds be joined together? On isometric dot paper, draw at least three different shapes showing the triamonds that have been used to form them.

Strategy options

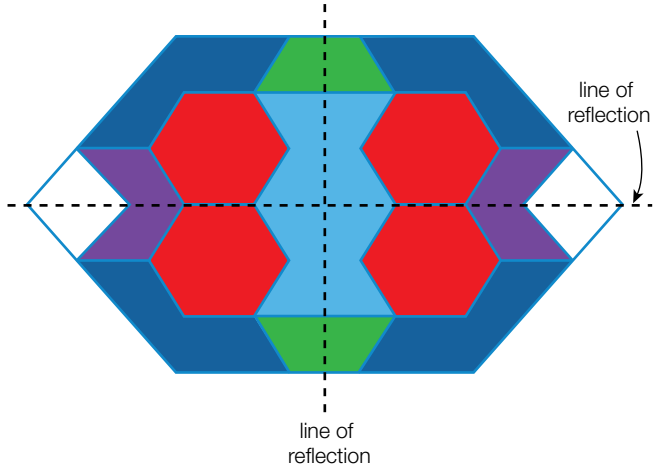
- Draw a diagram.
- Guess and check.
- Look for a pattern.

Explain

9 For two hexiamonds found in **3**, two tridiamonds found in **4** and two shapes found in **7**, explain how they were formed in terms of translation, reflection and rotation.

Elaborate

10 Answer the Big Question by making a frieze with at least five different shapes made from equilateral triangles, diamonds and trapeziums that includes translations, reflections and rotations. Your frieze pattern needs to be at least four triangles high and nine triangle lengths long. An example is given below. Indicate any lines of reflection.



Evaluate

11 Did you find it hard to find all the different shapes? What made it difficult?

12 Did the names help you to find some of the shapes in **3**?

Extend

13 There are at least 40 ways of using the 12 different shapes found in **3** to cover a rhombus of side length 6 units. Can you find one?

Combined transformations

10.4

You can combine translations, reflections and rotational transformations into one transformation of a figure.

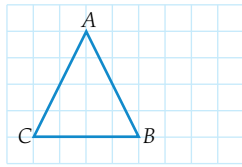
When combining transformations, the transformations are performed in sequence (one after the other) and, after each transformation, the vertices are labelled with an additional dash. For example, the vertex A''' has been transformed by three different transformations.

- Translations, reflections and rotations can be combined into a sequence of transformations.
- A side length can act as the line of reflection during a combined transformation.
- Different colours can be used to distinguish between several transformations.
- The properties of the three different types of transformations still hold true for combined transformations.

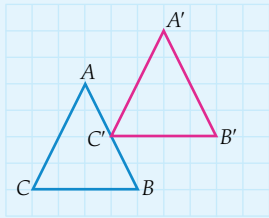
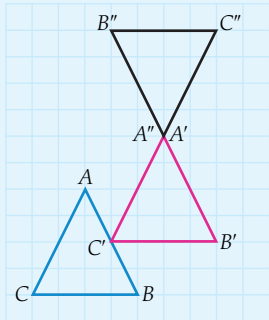
Worked example 9

W.E. 9

Copy the following figure onto grid paper and draw the resulting image after the combined transformation has been carried out. Use a protractor to help you. Label each vertex appropriately after each transformation.



A translation of 3 units right and 2 units up, followed by a rotation of 180° in a clockwise direction about the vertex A'.

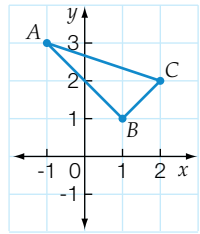
Thinking	Working
1 Perform the first identified transformation in the instructions. Make sure you use the correct notation with the transformed vertices.	
2 Perform each subsequent transformation on the next transformed image. You may want to distinguish each transformation by using a different colour.	

If a figure is drawn on a Cartesian plane, you can perform more than one type of transformation on it. For example, you can perform a rotation, then a translation. You could then reflect the figure in the x- or y-axis. You can then identify the coordinates of the final image.

Worked example 10

W.E. 10

For the figure given opposite:



(a) write the coordinates of each vertex on the given shape and copy the shape onto grid paper

(b) reflect the shape in the x-axis

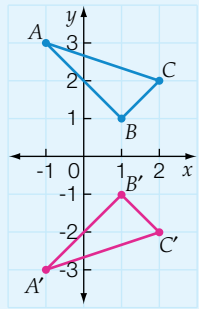
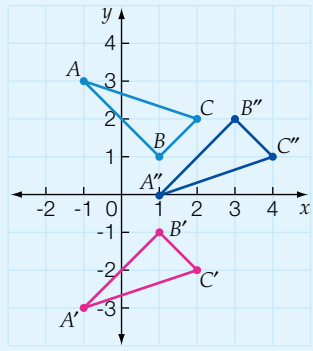
(c) write the coordinates of the reflected vertices

(d) translate this image 2 units right and 3 units up

(e) write the coordinates of the translated vertices

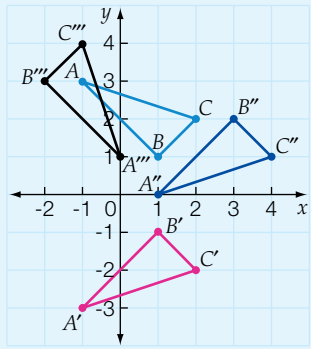
(f) rotate this translated image 90° in an anticlockwise direction about the origin

(g) write the coordinates of the vertices of the final image.

Thinking	Working
(a) Use the x- and y-axes to identify the coordinates of each of the vertices.	(a) $A = (-1, 3)$, $B = (1, 1)$, $C = (2, 2)$
(b) Reflect each vertex in the x-axis.	(b) 
(c) Identify the coordinates of each vertex on the resulting image.	(c) $A' = (-1, -3)$, $B' = (1, -1)$, $C' = (2, -2)$
(d) Translate each vertex with the given translation. (Each vertex has been translated 2 units to the right and 3 units up.)	(d) 
(e) Identify the coordinates of each vertex on the resulting image.	(e) $A'' = (1, 0)$, $B'' = (3, 2)$, $C'' = (4, 1)$

(f) Rotate each vertex around the origin by the given rotation, labelling each vertex with image notation. (Each point has been rotated in an anticlockwise direction by 90°.)

(f)



(g) Identify the coordinates of each vertex on the resulting image.

(g) $A''' = (0, 1)$, $B''' = (-2, 3)$, $C''' = (-1, 4)$

10.4 Combined transformations

Navigator

1, 2, 3, 4, 5, 6, 7 (a–b), 8 (a–d), 10, 11, 12, 13, 14, 16, 17	1, 2, 3, 4, 5, 6, 7, 8, 9 (a–c), 10, 11, 12, 13, 14, 16, 17	1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18

Answers p. 717

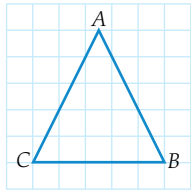
Equipment required: protractor and grid paper

Fluency

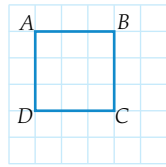
1 Copy each of the following figures onto grid paper and draw the resulting image after the combined transformation has been carried out. Use a protractor to help you. Label each vertex appropriately after each transformation.

W.E. 9

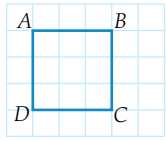
(a) A translation of 3 units left and 1 unit down, followed by a reflection in the line $B'C'$.



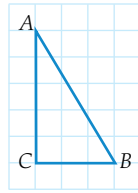
(b) A translation of 5 units right and 2 units down, followed by a reflection in the line $D'C'$.



(c) A translation of 5 units right, followed by a rotation of 90° about B' in a clockwise direction.

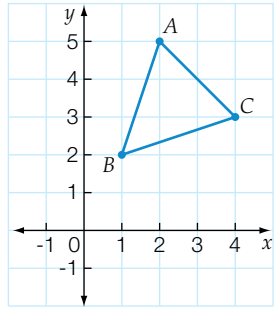


(d) A translation of 3 units left and 4 units down, followed by a rotation of 180° about C' in a clockwise direction.



W.E. 10

2 For the figure given opposite:



(a) write the coordinates of each vertex on the given shape and copy the shape onto grid paper

(b) reflect the shape in the x-axis

(c) write the coordinates of the reflected vertices

(d) translate this image 5 units left and 3 units up

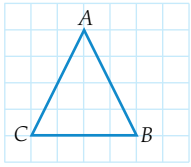
(e) write the coordinates of the translated vertices

(f) rotate this translated image 180° in a clockwise direction about the origin

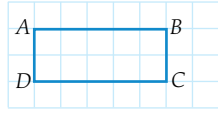
(g) write the coordinates of the vertices of the final image.

3 Copy each of the following figures onto grid paper and draw the resulting image after the combined transformation has been carried out. Use a protractor to help you. Label each of the vertices appropriately after each transformation (for parts **(e)**–**(h)**, point O is a point on or inside the figure and is reflected with the shape).

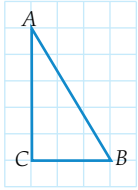
(a) Reflect along the line CB and translate 6 units right and 2 units up.



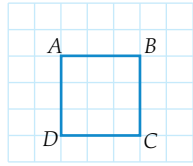
(b) Reflect along the line BC and translate 1 unit left and 3 units down.



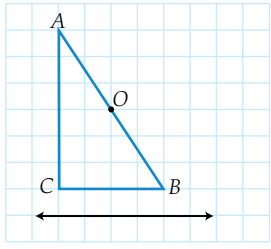
(c) Rotate 90° in a clockwise direction about vertex A and translate 8 units down.



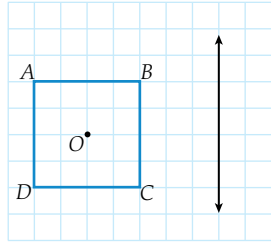
(d) Rotate 180° in an anticlockwise direction about vertex D and move 2 units left and 6 units up.



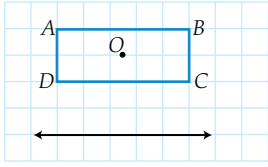
(e) Reflect along the line of reflection and then rotate 90° in an anticlockwise direction about the centre of rotation O'.



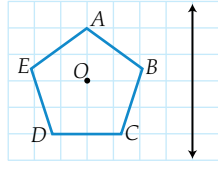
(f) Reflect along the line of reflection and then rotate 90° in a clockwise direction about the centre of rotation O'.



(g) Rotate 90° in an anticlockwise direction about the centre of rotation O and then reflect in the line of reflection.



(h) Rotate 180° in a clockwise direction about the centre of rotation O and then reflect in the line of reflection.



4 Which transformation reverses the order in which the vertices are labelled?

A rotation and translation **B** reflection

C translation **D** rotation

5 Which transformation can result in an image in the same position and orientation as the original figure?

A rotation and translation **B** translation

C reflection **D** rotation

Understanding

6 **(a)** Plot the points (-1, 2), (2, 3) and (3, -1) on a Cartesian plane and label them *A*, *B* and *C*.

(b) Join the points to form a triangle *ABC*, then translate the shape 3 units right and 1 unit down.

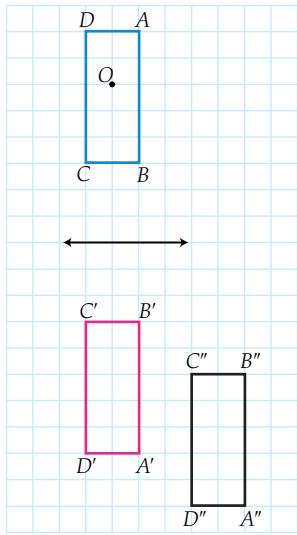
(c) Now, rotate the figure 90° in a clockwise direction about the origin.

(d) Write the coordinates of the vertices of the final image using image notation.

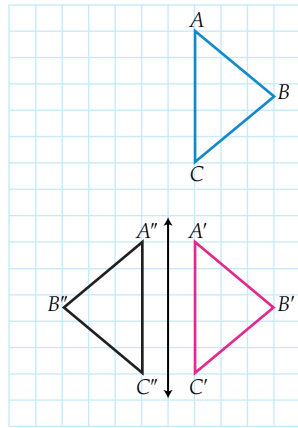
7 For the combined transformations in each part of Question **1**, identify the necessary combined transformation that would move the image back to the position and orientation of the original figure.

8 Describe the transformations that have taken place to move the original figure in blue to the position in pink, then to its final position in black. List the relevant names of each transformation.

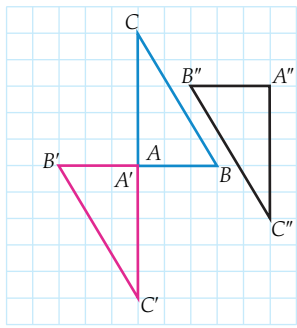
(a)



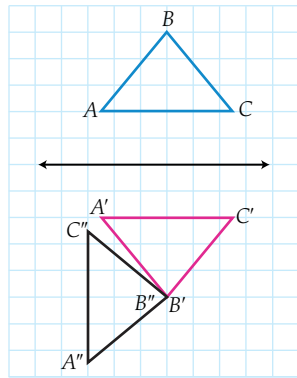
(b)



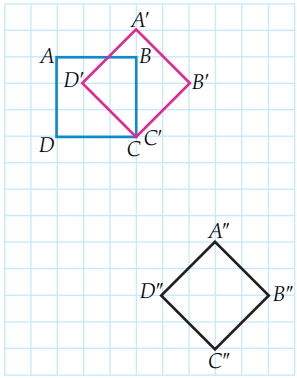
(c)



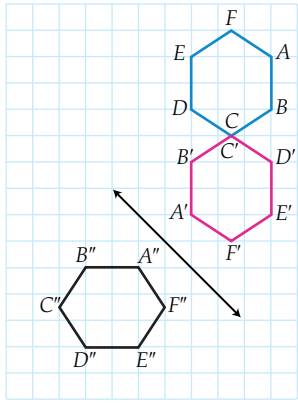
(d)



(e)



(f)



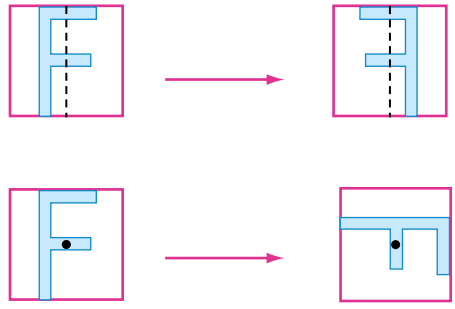
9 For the combined transformations in each part of Question **8**, identify the transformations required to move the final image back to the original position and orientation of the figure.

Reasoning

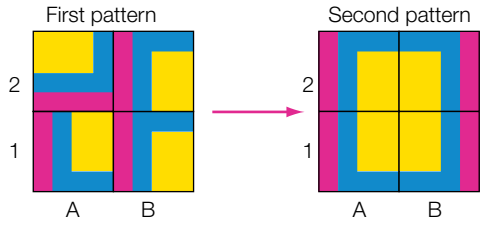
10 **(a)** Without plotting points, write the coordinates of the image after the line joining $A(-2, 3)$ and $B(4, -1)$ is translated 1 unit right and 5 units down, then reflected in the y-axis.

(b) Plot the line AB, the first image $A'B'$ and the final image $A''B''$ on a Cartesian plane to check your answer to part **(a)**.

11 **(a)** Sarah has written a computer program to transform pictures of tiles. There are only two instructions in her program, shown graphically at right. Describe these two instructions in words.



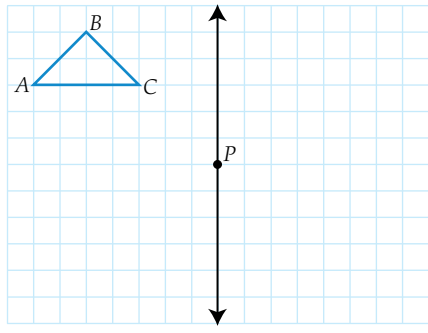
(b) Sarah wants to transform the first pattern to the second pattern. Using only the two instructions from part **(a)**, describe the transformations that must be done to each of the pattern parts A1, A2, B1 and B2.



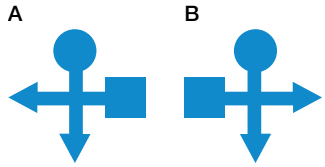
12 The following combined transformation is performed on triangle ABC.

- A reflection in the given line of reflection.
- A rotation of 180° about point P.
- A reflection in the given line of reflection.

Find a single transformation that will move the image $A'''B'''C'''$ back to the original position of the figure. Draw a diagram to show each required transformation.

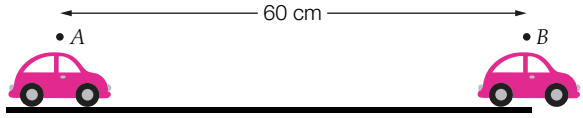


13 Is it possible to form figure B (as shown) by only translating and rotating figure A?

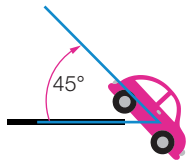


14 A cart travels along a ramp from the point *A* to the point *B*.

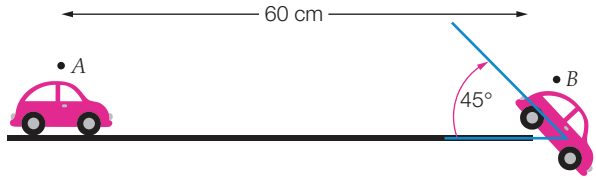
(a) Describe its movement as a translation.



(b) The car tips over the edge of the ramp. Describe its movement as a rotation.

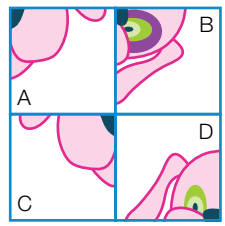


(c) Describe the movement of the car in parts **(a)** and **(b)** as a transformation.



15 An illustration of a dog's face is divided into four equal parts. Some panels have been rotated while others have been translated to produce the figure shown.

Describe which parts need to be rotated or translated, and by how much, to restore the original image of the dog.



Open-ended

16 Plot three points on a Cartesian plane and write their coordinates. Decide upon two particular transformations and use them to transform your figure. Draw the image after both transformations and write down the coordinates of the transformed vertices using image notation.

17 Select one of the following capital letters: *B, C, D, E, F, G*. Draw the letter on a grid.

(a) Make a translation and then a reflection in a vertical line of reflection. Draw the resulting images on grid paper and label each image appropriately.

(b) Find a single transformation that will move the final image back to the original position. Show this transformation on your diagram.

(c) Use your answer for part **(b)** to find a single transformation that would produce the same result as the two transformations in part **(a)**.

18 A shape is rotated through 180° about a point *O* and then its image is reflected in a horizontal line of reflection passing through *O*.

(a) Choose any shape and do the transformations described above (on grid paper).

(b) Find a single transformation that would have the same results as the two transformations described above.

Game

Conway's soldiers

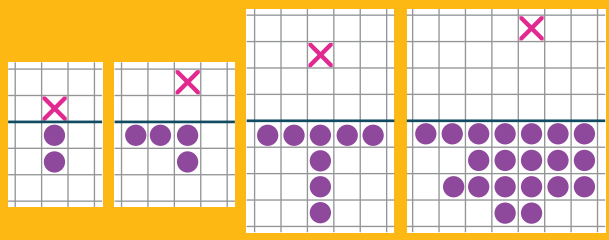
Equipment required: grid paper, approximately 40 small counters

Conway's soldiers is a one-person puzzle created by mathematician John Horton Conway. It uses a grid of squares divided by a horizontal line. Above the line are empty cells and below the line are a random number of game pieces, called the 'soldiers'.

How to play:

You play by jumping any piece over another piece that is next to it, into an empty square, horizontally or vertically (not diagonally).

The jumped-over piece is then removed.



How to win:

The aim is to place a piece as far above the horizontal line as possible. There are mathematical limits to how far this can be, depending on the original arrangement of pieces. In the diagrams shown, the X marks the maximum distance that a piece can reach for the given arrangement.

Questions

1. Create your own grid arrangements with the horizontal line drawn. Begin by investigating how far four counters can be moved beyond the horizontal line. Try at least three different arrangements of the four counters. Is it possible to move a counter past the second row?
2. Continue to investigate with 4, 5, 6, 7, 8, 9, 10, 15 and 20 counters, to see how far past the horizontal line the counters can be moved.
3. Use your observations to predict if it is possible to move a counter to the fifth row above the horizontal line. Explain your answers.

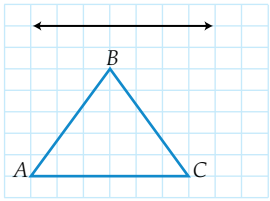
Strategy options

- Act it out.
- Test all possible combinations.

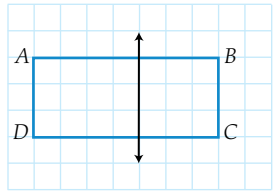
Half-time 10

1 Reflect each of the following figures in the line of reflection shown. **10.2**

(a)

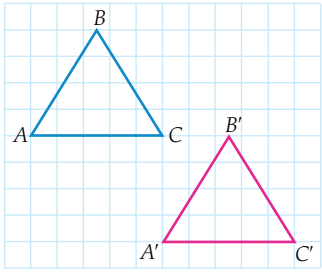


(b)

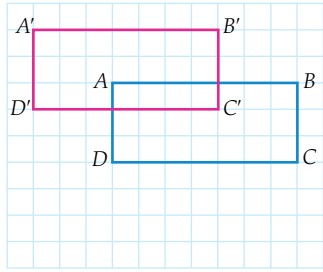


2 Describe the translation shown in each of the following diagrams. **10.1**

(a)



(b)

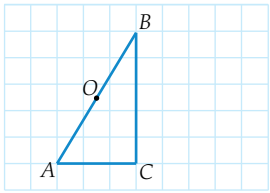


3 Show the image of each of the following figures after it is rotated about the given centre of rotation. **10.3**

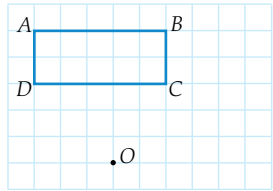
Rotate 180° in an anticlockwise direction about *O*.

Rotate 90° in a clockwise direction about *O*.

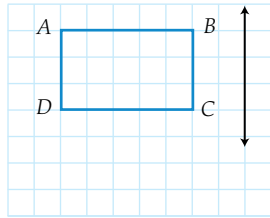
(a)



(b)



4 Reflect the following shape in the line of reflection shown, then translate it 2 units to the right and 1 unit down. **10.4**



5 **(a)** Plot the points (1, 2), (3, 4) and (5, -2) on a Cartesian plane and join them to form a triangle. **10.4**

(b) Draw the image after a translation of 6 units left and 1 unit up.

(c) Write the coordinates of the vertices of the translated points.

(d) Draw the image after the image drawn in part **(b)** is reflected in the x-axis.

(e) Write the coordinates of the vertices of this new image.

(f) Finally, rotate the image drawn in part **(d)** 180° clockwise about the origin.

(g) Write the coordinates of your final image.

Exploration CAS

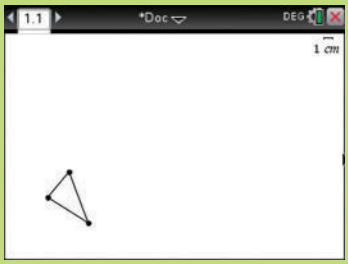
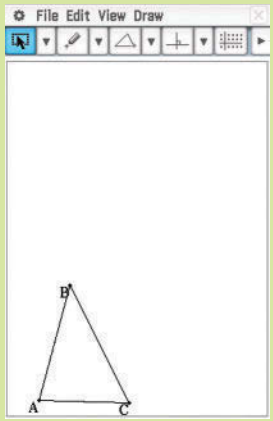
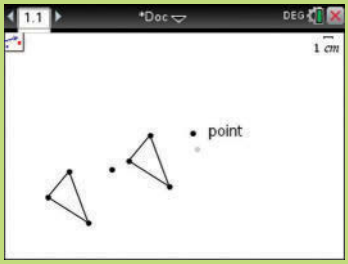
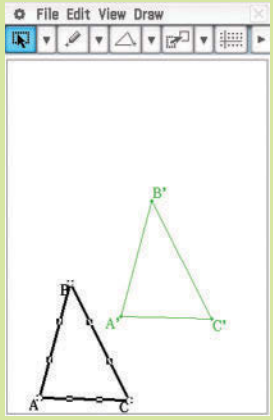
Equipment required: TI-Nspire CAS or Casio ClassPad CAS

Transformations with CAS

You can use CAS technology to perform different transformations on different shapes. By doing this, you will become more familiar with how to use your CAS to help study and understand transformations and symmetry.

Translating an object

A translation is a slide. You can use CAS technology to construct a triangle and translate it.

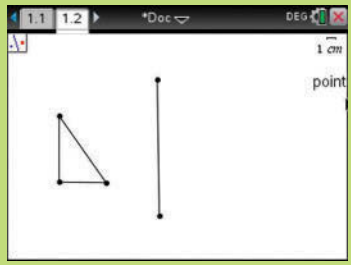
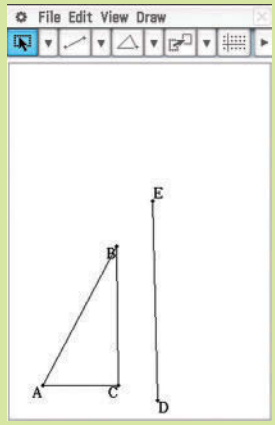
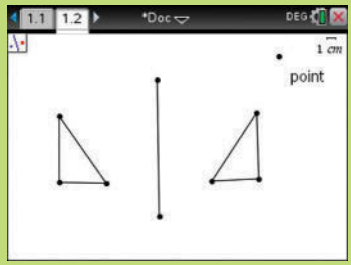
Using TI-Nspire CAS	Using Casio ClassPad CAS
Add **Geometry** to your document. Select **menu** > **Shapes** > **Triangle**, then move the cursor to select the points where you want each vertex of the triangle to appear. To make later transformations easier, it will be useful to make a small triangle in the corner of the screen. Press **esc** to stop drawing. 	From the menu select **Geometry**. Select the scalene triangle icon (in the third sub-menu from the left) and move the cursor to choose where you want the shape to appear. To make later transformations easier, it will be useful to make a small triangle in the corner of the screen. You can select and move the vertices to change the dimensions of the shape. 
To make a translation of the triangle, select **menu** > **Transformation** > **Translation**. You can then select your triangle and a point to make a translated copy. 	To make a translation of the triangle, select **Draw** > **Construct** > **Translation**. Enter the horizontal distance to be translated as the top number (positive numbers translate right, negative translates left). Enter the vertical distance to be translated as the bottom number (positive numbers translate up, negative translates down). This will make a translated copy of your triangle. 

You can select one of the vertices of your original triangle and move it around to see how this changes the translated copy.

What is the relationship between the points on the original triangle and the translated points on the image?

Reflecting an object in a line

A reflection is a flip. You can now construct a right-angled triangle and reflect it in a line.

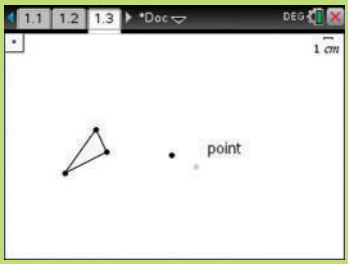
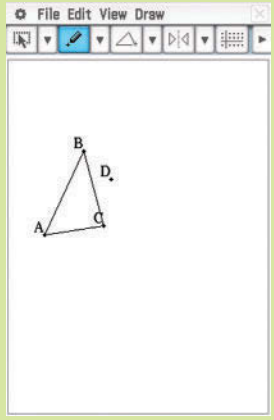
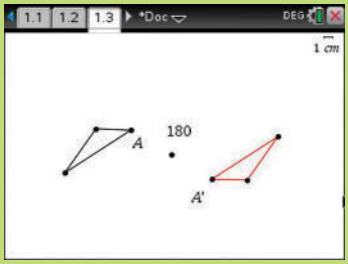
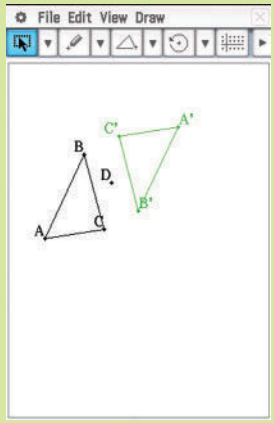
Using TI-Nspire CAS	Using Casio ClassPad CAS
Add **Geometry** to your document. Select **menu** > **Actions** > **Delete all** to clear the screen if necessary. Select **menu** > **Shapes** > **Triangle**, then move the cursor to select the points where you want each vertex of the triangle to appear. Try to make this a right-angled triangle, or as close as possible to right angled. Next, select **menu** > **Points & Lines** > **Segment** to draw a vertical line segment next to the triangle, as shown. 	From the menu select **Geometry**. Select **Edit** > **Clear All** to clear the screen if necessary. Select the scalene triangle icon (in the third sub-menu from the left) and move the cursor to choose where you want the shape to appear. Select and move the vertices to try to make this a right-angled triangle, or as close as possible to right angled.  Next, select **Draw** > **Basic Object** > **Line Segment** to draw a vertical line segment next to the triangle, as shown.
To make a reflection of the triangle in the vertical line segment, select **menu** > **Transformation** > **Reflection**. You can then select your triangle and select the vertical line. A reflected copy should appear. 	To make a reflection of the triangle, select **Draw** > **Construct** > **Reflection** and then select the vertical line segment. A reflected copy of your triangle should appear. 

You can select one of the vertices of your original triangle and move it around to see how this changes the reflected copy.

What is the relationship between the points on the original triangle and the reflected points on the image?

Rotating an object

A rotation is a turn. You can now construct a triangle and rotate it by 180° in a clockwise direction.

Using TI-Nspire CAS	Using Casio ClassPad CAS
Add **Geometry** to your document. Select **menu** > **Actions** > **Delete all** to clear the screen if necessary. Select **menu** > **Shapes** > **Triangle**, then move the cursor to select the points where you want each vertex of the triangle to appear. To make later transformations easier, it will be useful to make a small triangle near the middle of the screen. Next, select **menu** > **Points & Lines** > **Point** to draw a point next to the triangle, as shown. 	From the menu select **Geometry**. Select **Edit** > **Clear All** to clear the screen if necessary. Select the scalene triangle icon and move the cursor to choose where you want the shape to appear. To make later transformations easier, it will be useful to make a small triangle near the middle of the screen.  Next, select **Draw** > **Basic Object** > **Point** to draw a point next to the triangle, as shown.
To make a rotation of the triangle by 180° about the point, select **menu** > **Transformation** > **Rotation**. You can then select your triangle, select the point, and enter 180 to create the rotated copy. 	To make a rotation of the triangle by 180° about the point, select **Draw** > **Construct** > **Rotation** and then select the point. Enter 180 to create the rotated copy. 

You can select one of the vertices of your original triangle and move it around to see how this changes the rotated copy.

What is the relationship between the points on the original triangle and the rotated points on the image?

Rotate the original triangle again by following the same steps above, but this time using different angle sizes, 90° and 270°.

You should now have four triangles visible.

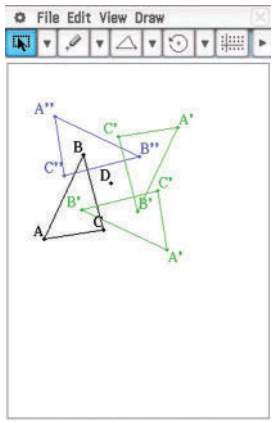
Using TI-Nspire CAS	Using Casio ClassPad CAS
To better understand how the points of the original triangle are related to the rotated images, you can now draw a circle on the screen. Select **menu** > **Shapes** > **Circle** and select the centre point to be the point used as the centre of rotation. Then select a vertex on the original triangle to be the circle's circumference point.	To better understand how the points of the original triangle are related to the rotated images, you can now draw a circle on the screen. Select **Draw** > **Basic Object** > **Circle** and select the centre point to be the point used as the centre of rotation. Then select a vertex on the original triangle to be the circle's circumference point.

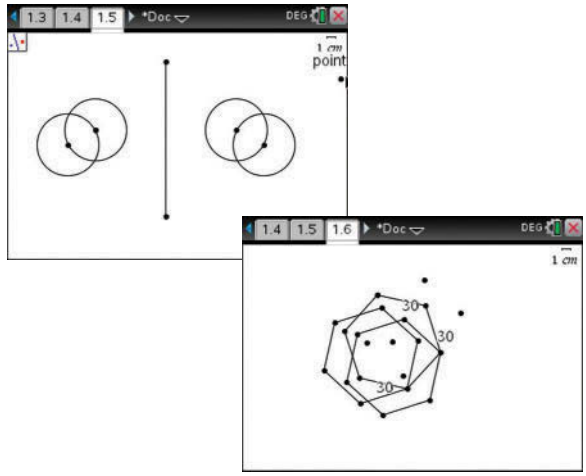
What do you notice about the circle and the triangles?

Now draw another two circles, using the other two vertices of the original triangle to be each circle's circumference point. What do you notice about all three circles and the triangles?

Taking it further

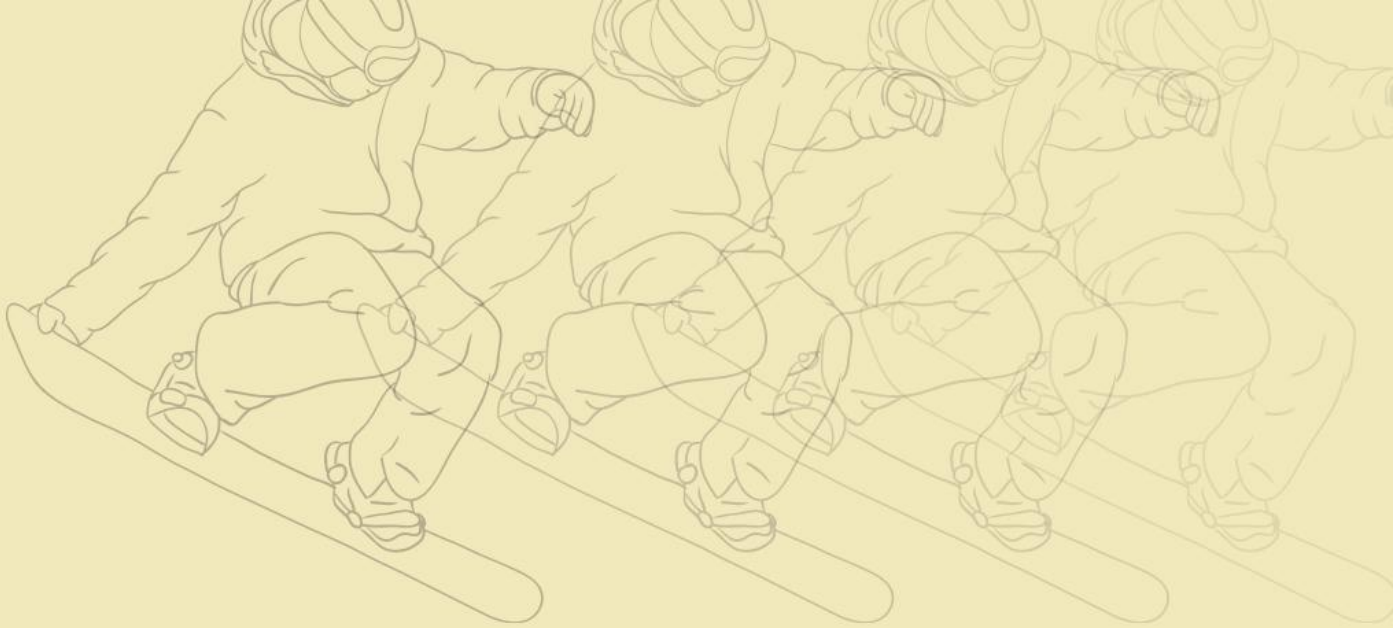
Perform reflections, translations and rotations with various other shapes such as those shown. Be creative—see if you can create a pattern by performing a series of transformations on a shape, or combining it with other shapes.





Maths 4 Real

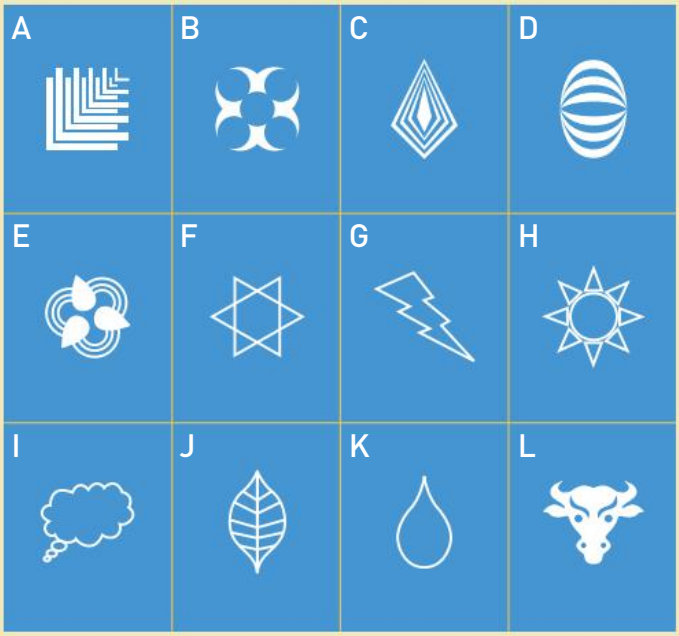
SNOW LOGO



Symmetry is used in art, architecture, company logos and even in the design of national flags. Humans seem to recognise and like symmetrical objects, and designers and artists take advantage of our preference for symmetry.

Imagine you are a young marketing executive for a company that sells snowboards. You know that many companies succeed or fail depending on how well customers recognise their brand. The easiest method for a recognisable brand is to include elements of symmetry in a company logo. Your manager, who loves symmetry, has given you the task of selecting the most symmetrical logo (by reflectional and rotational symmetry) from a collection of sample logos designed by your company's graphic artists.

Sample logos



1 Use the following table as a template to record the orders of reflectional and rotational symmetry for each of the sample logos.

Logo	Order of reflectional symmetry	Order of rotational symmetry
A		
B		
C		

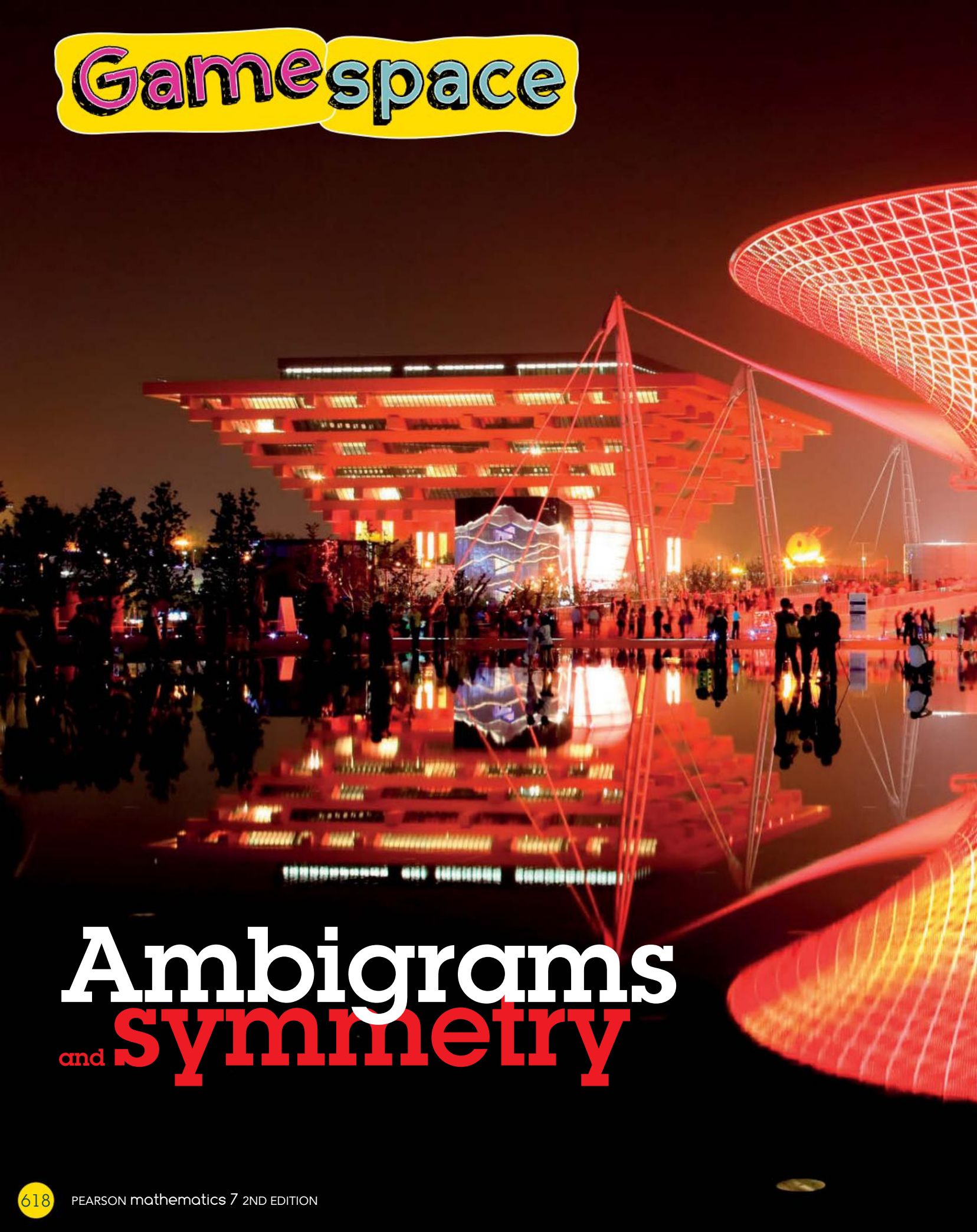
2 After analysing these logos, you think that you may be able to develop a better-looking, more symmetrical logo for your company. Your task is to design your own new logo and identify all elements of its symmetry to show your manager.

3 Aside from the symmetry elements, write two or three other things you might consider when deciding which is the best logo for a company.

Research

- Find as many different examples as you can of well-known company logos with reflectional and/or rotational symmetry. (Car companies are a good place to start.) Also find some examples that are asymmetric (have no symmetry).
- Large companies spend a lot of money on getting the right design for their logo—the symbol by which the company is identified. Why do you think such importance is given to the logo design?
- Which logo do you think is the most recognised throughout the world? Try to find out.

Gamespace



Ambigrams and symmetry

Equipment required: a small mirror

Letter ambigrams

An ambigram is a word that you can read in more than one direction or orientation. The word 'ambigram' comes from *ambi* (meaning 'both' or 'on both sides') and *gram* (meaning 'something written or drawn').

Ambigrams can have either reflectional symmetry or rotational symmetry.

Vertical reflectional symmetry

fantasy

Use a mirror to check

Horizontal reflectional symmetry

COOKBOOK

Rotational symmetry

dollop

Rotate this page to check the rotational symmetry

1 For each of the following ambigrams, complete the word (if needed) and state whether it has rotational, vertical reflectional, or horizontal reflectional symmetry. (Use a mirror to check.)

(a) SWIMS

(b) NOPE

(c) MUM

(d) BID

(e) bid

(f) su____

2 **(a)** Some pairs of letters, such as 'b' and 'q', have rotational symmetry when used together. (Rotate 'b q' or 'q b' to see how this works.) What other pairs of letters have rotational symmetry when used together?

(b) What single letters have rotational symmetry when used by themselves?

(c) Create your own ambigram with rotational symmetry or vertical reflectional symmetry.

Number ambigrams

Some numbers have symmetrical properties. For instance, the number 88 has vertical reflectional and horizontal reflectional symmetry, as well as rotational symmetry.

3 Find some number ambigrams that have the following symmetries.

(a) vertical reflectional symmetry

(b) horizontal reflectional symmetry

(c) rotational symmetry

4 Digital clocks often display numbers using a digital number grid. This means that some digital numbers look the same upside down.

0123456789

(a) Which of these digital numbers have rotational symmetry?

(b) Which digital numbers between 1 and 100 are ambigrams? What types of symmetry do they have?

(c) What is the largest 4-digit ambigram you can make that:

(i) uses only one of the digits 0 to 9

(ii) uses two of the digits 0 to 9?

(d) What is the largest 5-digit ambigram you can make that:

(i) uses only one of the digits 0 to 9

(ii) uses three of the digits 0 to 9?

Grid symmetry

Draw a Cartesian plane that goes from 0 to 4 on the horizontal axis, and from 0 to 3 on the vertical axis.

Using the clues below, place the letters R, C, K, H, S, U, Y, M, and B at their correct places on the Cartesian plane.

Clues

- The letter at (1, 2) is not symmetrical in any way.
- The letters at (0, 0), (1, 1), (2, 2) and (3, 3) all have horizontal reflectional symmetry.
- The letters at (0, 2) and (3, 3) have rotational symmetry.
- The letters at (3, 0), (3, 1) and (3, 2) all have vertical reflectional symmetry.
- The letters at (1, 1) and (3, 1) contain only curved lines.
- The letters at (2, 2) and (3, 2) contain only straight lines.
- The letters at (3, 0), (3, 1) and (3, 2), in that order, spell a word that means 'tasty'.

What word is spelled by the letters at (1, 2), (3, 1), (0, 0) and (3, 0), in that order?

10.5 Symmetry

Reflectional symmetry

You have seen how a line of reflection can produce a reversed or mirror image of an object, but many objects in our world, from flowers and insects in nature to company logos, flags and letters of the alphabet, have **reflectional symmetry** or mirror symmetry by themselves. Reflectional symmetry occurs when a line can be drawn through an object to divide it into two identical but reversed parts. Each part is the mirror image of the other. This line of reflection is called the **axis of symmetry**. In the photo below, a line can be drawn to divide the building into two identical mirror-image parts.

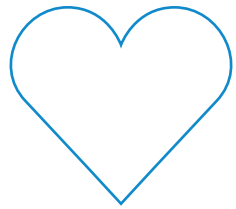
Some objects have multiple axes of symmetry. The **order of reflectional symmetry** is the number of axes of reflectional symmetry that can be drawn through a shape. A pentagon has five axes of symmetry, so it has symmetry of order 5. Objects with no line of symmetry are called **asymmetrical** (or 'not symmetrical'). The letter R has no axes of symmetry, so it is asymmetrical.

Is a human face symmetrical? Although faces often seem symmetrical, you can see by reflecting each side of a face separately that the two halves are slightly different. The face you see in a mirror is the mirror image or reversed view of your actual face.

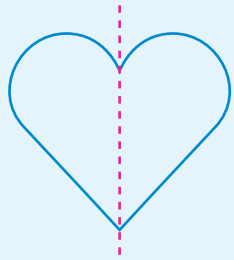
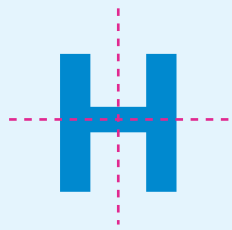
Worked example 11 W.E. 11

Copy each of the following figures and determine **(i)** whether it has reflectional symmetry, by showing every possible axis of symmetry, and **(ii)** the order of reflectional symmetry, if so.

(a)

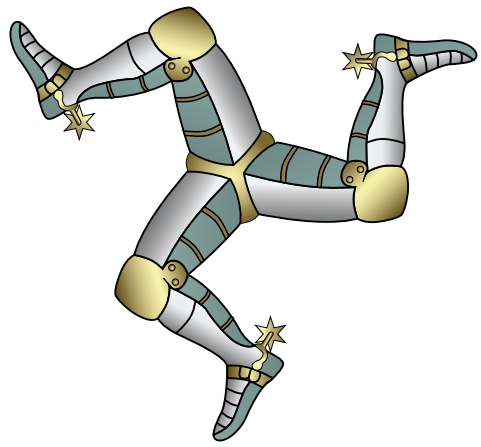


(b)

Thinking	Working
(a) 1 Draw as many axes of symmetry as possible so that the object is divided into two identical but reversed halves.	*(a)* (i) 
2 Count the number of axes of symmetry. This gives us the order of reflectional symmetry.	(ii) *Order of reflectional symmetry = 1.*
(b) 1 Draw as many axes of symmetry as possible so that the object is divided into two identical but reversed halves.	*(b)* (i) 
2 Count the number of axes of symmetry. This gives us the order of reflectional symmetry.	(ii) *Order of reflectional symmetry = 2.*

Rotational symmetry

An object can also have **rotational symmetry**. If an object can be turned about a point by less than 360° and still look like the original object, then that object has rotational symmetry. The point about which the object is rotated is called the centre of rotation.

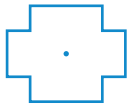


An object with rotational symmetry is the 'triskelion' symbol of three joined legs, which is a traditional symbol of the Isle of Man. The triskelion has three rotational symmetries, called 'three-fold symmetry', because every full turn will produce the same image three times. In other words, any rotation of 120° produces the original image.

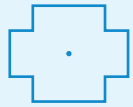
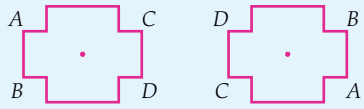
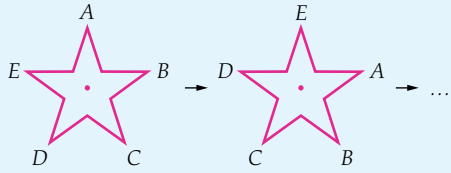
The **order of rotational symmetry** is the maximum number of times in a complete 360° rotation that the original orientation of the shape appears. For a pentagon, every rotation of 72° about the centre of the pentagon produces the same image as the original. This occurs five times in a complete 360° rotation, so a pentagon has an order of rotational symmetry of 5. A pentagon is therefore an interesting shape, as it has an order of 5 in both reflectional and rotational symmetry.

Worked example 12 W.E. 12

Determine **(i)** which of the following figures has rotational symmetry, **(ii)** the order of rotational symmetry and **(iii)** the size of the angle of rotation required to produce an identical image.

(a) 

(b) 

Thinking	Working
(a) Find the centre of rotation and place your pen/pencil on it.	(a) 
Rotate the image around this point to determine whether the original image is reproduced.	
Count the number of times in a complete 360° rotation that an identical image would be produced. This is the order of rotational symmetry. Divide this number into 360°. This gives the angle of rotation.	(i) Yes, it has rotational symmetry. (ii) Order of rotational symmetry = 2. (iii) $\frac{360}{2}$ = 180°
(b) Find the centre of rotation and place your pen/pencil on it.	(b) 
Rotate the image around this point, to determine whether the original image is reproduced.	
Count the number of times in a complete 360° rotation that an identical image would be produced. This is the order of rotational symmetry. Divide this number into 360°. This gives the angle of rotation.	(i) Yes, it has rotational symmetry. (ii) Order of rotational symmetry = 5 (iii) $\frac{360}{5}$ = 72°

- An axis of symmetry is a mirror line that can be drawn through a shape to divide it into two identical but reversed halves.
- The order of reflectional symmetry is related to the number of lines of reflectional symmetry.
- An asymmetrical object has no reflectional symmetry.
- If a figure can be turned less than 360° so that it matches the original figure, it has rotational symmetry.
- The order of rotational symmetry is the number of times in a complete 360° rotation that an identical image of the original would be produced.

10.5 Symmetry

Navigator

1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 18, 19	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 (columns 1–2), 13, 14, 16, 18, 19, 20	1, 2, 3, 7, 8, 9, 10, 11, 12 (columns 2–3), 13 (b–c), 14, 16, 17, 18, 19, 20

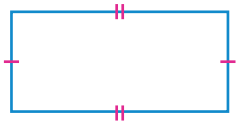
Answers p. 721

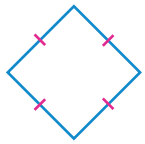
Equipment required: grid paper for Question **13**

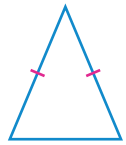
Fluency

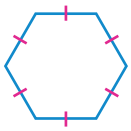
1 Copy each of the following figures and determine **(i)** whether it has reflectional symmetry by showing every possible axis of symmetry and **(ii)** the order of reflectional symmetry, if present.

W.E. 11

(a) 

(b) 

(c) 

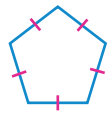
(d) 

(e) 

(f) 

(g) 

(h) 

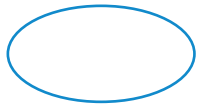
(i) 

2 Determine **(i)** which of the figures from Question **1** has rotational symmetry, **(ii)** the order of rotational symmetry and **(iii)** the size of the angle of rotation required to produce an identical image.

W.E. 12

3 How many axes of symmetry do the following objects have?

(a) 

(b) 

(c) 

For Questions **4**, **5** and **6** refer to the letters of the word M A T H S.

4 Which letter does not have an axis of symmetry?

A M **B** A **C** H **D** S

5 How many of the letters have one axis of symmetry?

A 5 **B** 2 **C** 3 **D** 4

6 Which letter has two axes of symmetry?

A H **B** M **C** A **D** S

Understanding

7 Copy the lowercase letters of the alphabet shown below.

a b c d e f g h i j k l m
n o p q r s t u v w x y z

(a) Which lowercase letters are asymmetrical?

(b) Which lowercase letters have one axis of symmetry?

(c) Which lowercase letters have two or more axes of symmetry?

8 Copy the uppercase letters of the alphabet shown below.

A B C D E F G H I J K L M
N O P Q R S T U V W X Y Z

(a) Which uppercase letters are asymmetrical?

(b) Which uppercase letters have one axis of symmetry?

(c) Which uppercase letters have two or more axes of symmetry?

9 Do any of the uppercase letters from Question **8** have rotational symmetry? If so, which ones are they?

Reasoning

10 Symmetry is common in nature. For each of the following images, determine whether it has almost perfect reflectional and/or rotational symmetry and state the order of each.

(a)

(b)

(c)

(d)

(e)

(f)

11 The original image of a dog's face below is not symmetrical. But the next two images are symmetrical, and were made from the original image. A vertical line through the centre of those next two images will form an axis of symmetry.

Explain how image A and image B were obtained from the original.

original

image A

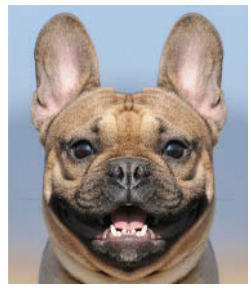
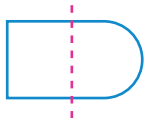


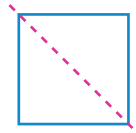
image B

12 For each of the following shapes, state whether or not the dotted line is an axis of symmetry.

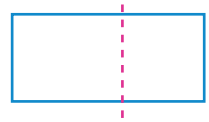
(a)



(b)

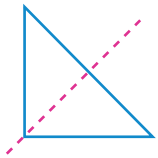


(c)

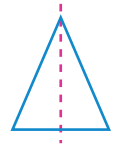


(d)

(e)

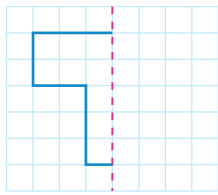


(f)

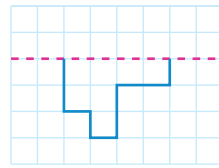


13 For each of the following diagrams, use the dotted line as an axis of symmetry to complete the diagram using grid paper.

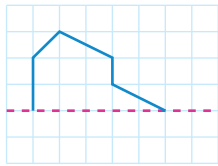
(a)



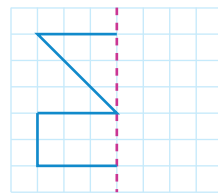
(b)



(c)



(d)



14 How many axes of symmetry do the following regular polygons have?

(a) equilateral triangle

(b) square

(c) regular pentagon

(d) regular hexagon

15 Which regular polygon has exactly three lines of reflectional symmetry?

Open-ended

16 WOW is a symmetrical word. Locate the axis of symmetry and find three more words that are also symmetrical.

17 Some year numbers have reflectional and/or rotational symmetry. An example of a year that has rotational symmetry is 1961 and a year that has reflectional symmetry is 1881.

Identify a year number that has:

(a) rotational symmetry

(b) reflectional symmetry

(c) both reflectional and rotational symmetry.

18 List nine numbers between 100 000 and 200 000 that have reflectional symmetry. (For example, 100 001.)

19 Draw any two figures that have both rotational and reflectional symmetry.

20 Describe when the following statement is true and when it is false.

'This parallelogram has reflectional symmetry.'

Problem solving

Symmetry challenge

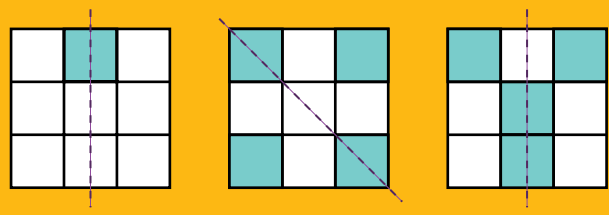
How many symmetrical designs of shaded squares can you make in a blank 3-by-3 square grid?

Hint: Before you shade any squares, can you identify the lines of symmetry in a 3-by-3 grid, like the one shown at left? What symmetries are possible if you shade one square only?

Strategy options

- Draw a diagram.
- Look for a pattern.
- Break problem into manageable parts.

Here are three examples of symmetrical designs. Note the position of the line of symmetry.



When counting the total number of symmetrical designs, don't include rotations. For example,

 is considered the same as  .

Drawing and visualising 3D shapes

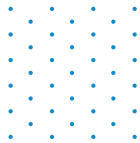
10.6

Objects such as buildings, machinery, car parts, rocks and people have three dimensions: length, width and height. But it is often necessary to draw three-dimensional (3D) objects on a two-dimensional (2D) plane, such as on a piece of paper or a computer screen.

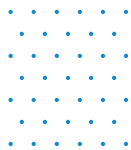
The 2D drawing of the 3D house above is an isometric drawing used by an architect.

Triangular dot paper, also called **isometric** paper, can help to make it easier to draw 3D objects on a 2D plane. Objects represented on triangular dot paper are also known as isometric drawings.

You should use isometric dot paper in this orientation:



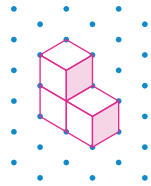
This alternative orientation is more difficult to use (not recommended).

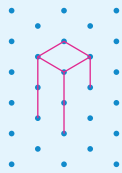
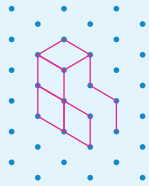
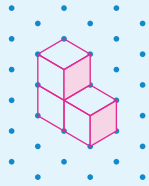


In Greek, 'iso' means 'equal' and 'metric' refers to 'measuring'. In isometric dot paper, the distances between each dot and the dots surrounding it are equal, diagonally and vertically.

Worked example 13 W.E. 13

Use isometric dot paper to copy the following shape.



Thinking	Working
1 Join the points of a parallelogram that will represent the top face. Draw in all the necessary vertical lines from the corners of the face.	
2 Draw in all the lines from the bottom of the verticals you have just drawn in step **1** that represent horizontal lines. Draw in all additional required vertical lines from the ends of the lines you have just drawn.	
3 Join the ends of the lines you have just drawn. To create the illusion of 3D, choose one direction (front, side or top) and shade in all faces you would see from that direction.	

10.6 Drawing and visualising 3D shapes

Navigator

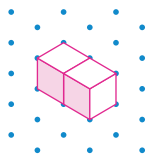
Answers p. 722

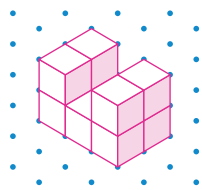
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 (a), 12, 14	1, 2 (columns 1–2), 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15	1, 2 (columns 2–3), 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15

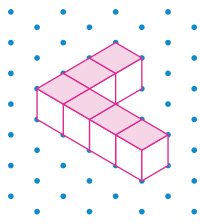
Equipment required: isometric dot paper

Fluency

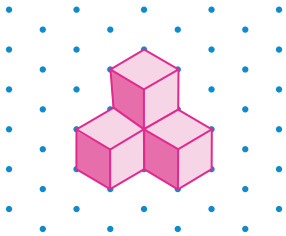
1 Use isometric dot paper to copy the following shapes.

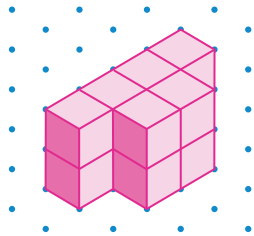
(a) 

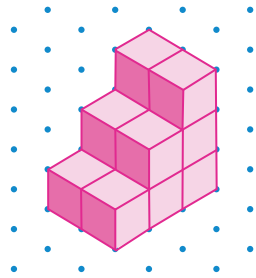
(b) 

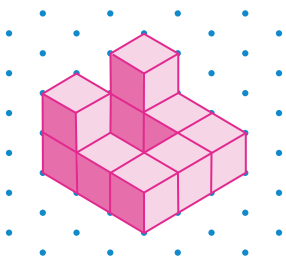
(c) 

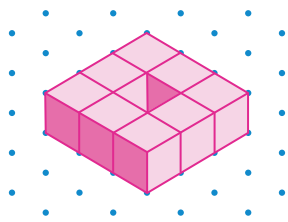
2 How many cubes would be required to build these solids? (Assume that there are no cubes missing at the back of the solids where you cannot see.)

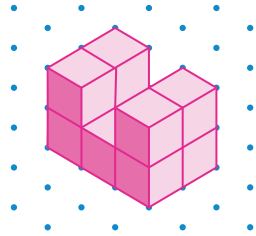
(a) 

(b) 

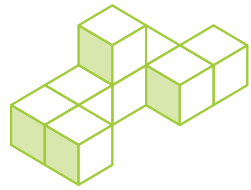
(c) 

(d) 

(e) 

(f) 

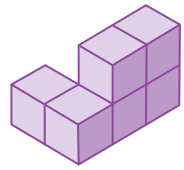
3 The number of cubes required to build this solid is:

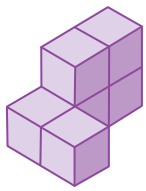


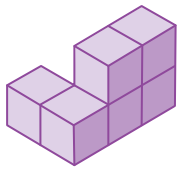
A 7 **B** 8

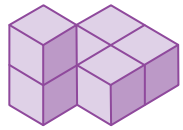
C 9 **D** 10

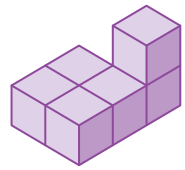
4 Which of the shapes below is *not* a rotation of the one shown?



A 

B 

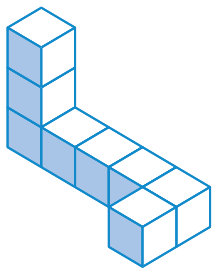
C 

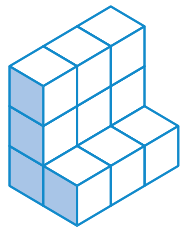
D 

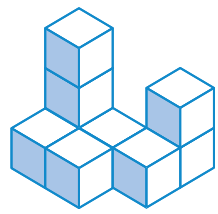
Understanding

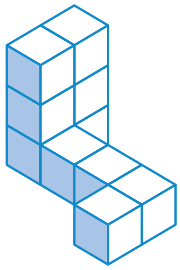
5 Using isometric dot paper, draw a 3D sketch of a Rubik's cube. (A Rubik's cube is made of 27 cubes stacked three layers high, three layers wide and three layers deep.)

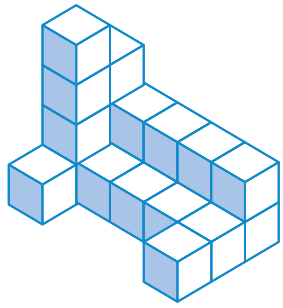
6 How many cubes are in each solid?

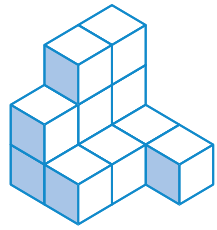
(a) 

(b) 

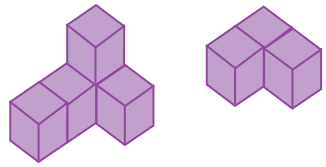
(c) 

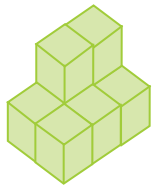
(d) 

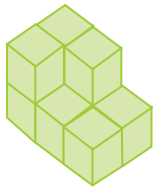
(e) 

(f) 

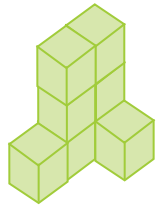
7 Which of the following shapes (A, B, C, D) could result from joining the two shapes on the right?



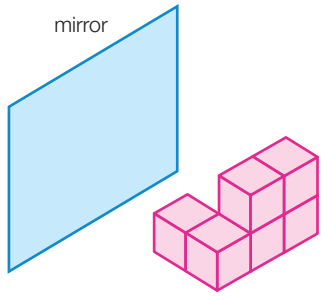
A 

B 

C 

D 

8 The shape shown is placed in front of a mirror. Use isometric paper to draw the image seen in the mirror.



Reasoning

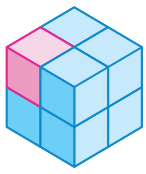
9 Using the aerial photograph of the Pentagon on the opening page of this chapter:

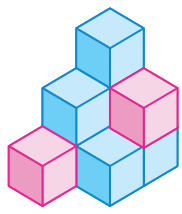
(a) draw a top view of the Pentagon

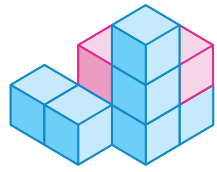
(b) draw a side view of the Pentagon.

(c) Does it matter which side you choose? Explain.

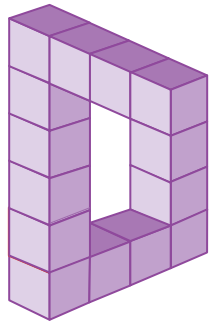
10 Draw each solid after the pink shaded blocks have been removed.

(a) 

(b) 

(c) 

11 This shape is known as the impossible rectangle.



(a) Why is this shape known as the *impossible* rectangle? (You may like to try to make it with blocks.)

(b) Copy the impossible rectangle onto isometric dot paper.

(c) Draw a similar rectangle that is possible. Colour your drawing using three colours.

12 Draw the top view of the isometric house shown on page 627 of this section.

Open-ended

13 A sculptor is designing a sculpture based on an arrangement of 24 cubes. Each cube has a width of 50 cm and the maximum height of the sculpture is to be 2.5 m. The sculptor wants to have a maximum of six cubes at the front and a maximum of four cubes in depth. Each cube must have at least one whole face touching another cube. Use isometric dot paper to design some sculptures that meet all these requirements.

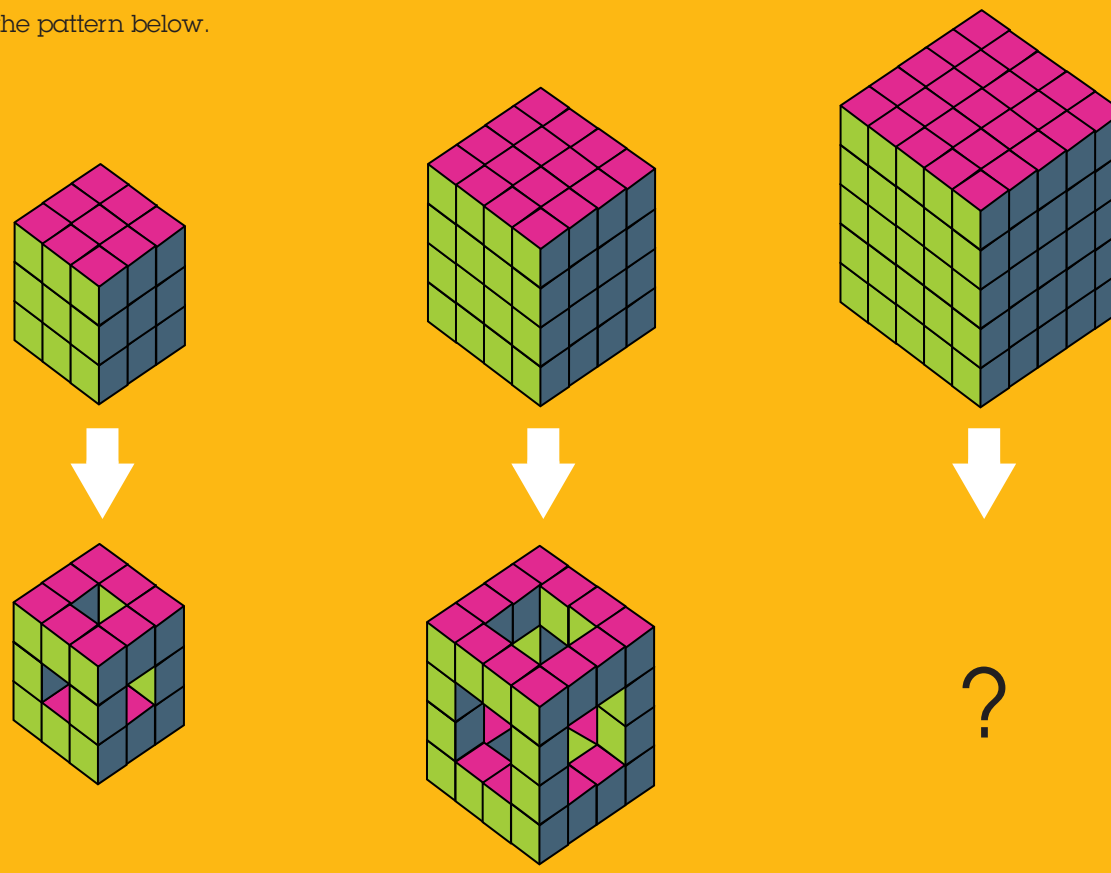
14 Using isometric dot paper, draw three different solids that can be built from 10 cubes that have whole faces touching.

15 Draw three shapes on isometric dot paper that, when joined together, form a $3 \times 3 \times 3$ cube.

Problem solving

What comes next?

Look at the pattern below.



A solid $3 \times 3 \times 3$ cube is made using 27 blocks. When one block is removed from the centre of each of the six faces, and one block is removed from the centre of the cube, a 'frame' of the same cube remains. The 'frame' uses only 20 blocks.

- Draw the next shape in the pattern.
- How many blocks are needed to make the frame of a $5 \times 5 \times 5$ cube?
- How many blocks are needed to make the frame of a $6 \times 6 \times 6$ cube?

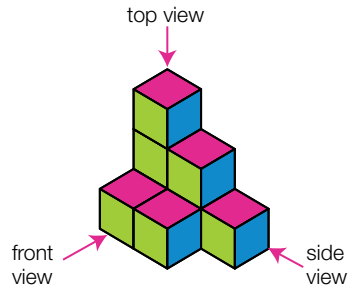
Strategy options

- Make a table.
- Look for a pattern.

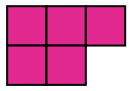
10.7 Plan views and elevations

Three-dimensional objects can be viewed from various positions, or views. Objects looked at from the top, front and side can be drawn in two dimensions, and are known as **plan views**, or orthogonal projections.

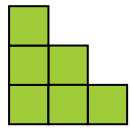
A 3D shape can look different when viewed from different points of view. A top view, or 'bird's eye view' is what you see when looking directly down on the shape. A side view is what you see when looking directly at the side. The front view is what you see when looking at the front.



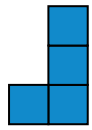
top view



front view



side view



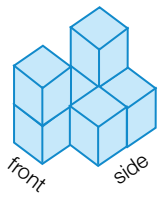
Worked example 14 W.E. 14

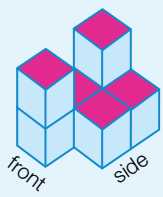
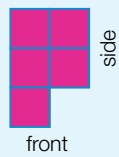
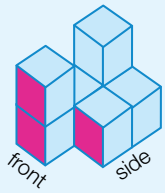
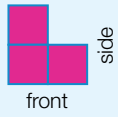
For the following solid, draw the:

(a) top view

(b) front view

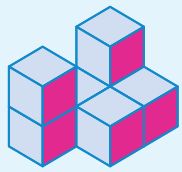
(c) side view.



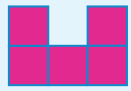
Thinking	Working
(a) 1 Highlight the faces seen from the top.	*(a)* 
2 Draw squares to make the shape you would see if you were looking down on the solid.	
(b) 1 Highlight the faces seen from the front. Do not highlight front faces that are behind another block.	*(b)* 
2 Draw squares to make the shape you would see if you were standing in front of the solid.	

(c) 1 Highlight the faces seen from the side.

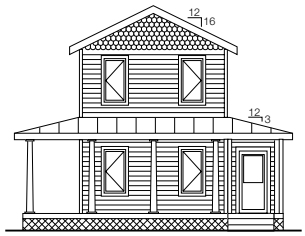
(c)



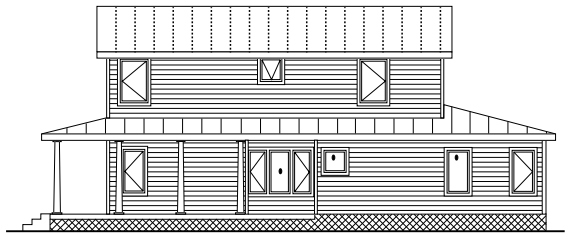
2 Draw squares to make the shape you would see if you were standing at the side of the solid.



Drawings of houses and buildings viewed from a side are called **elevations**. Elevations may be referred to as front, rear and side elevations, or referred to by compass direction, such as an east elevation.



Front elevation



Side elevation

10.7 Plan views and elevations

Navigator

1, 2, 3, 4 (a–c), 6 (a–b), 7, 8, 9, 10, 12	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12

Answers p. 722

Equipment required: isometric dot paper for Questions **6**, **8–12**

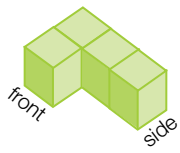
Fluency

1 For both of the following solids, draw the:

W.E. 14

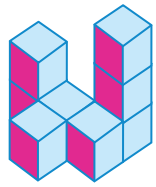
(i) top view **(ii)** front view **(iii)** side view.

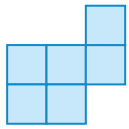
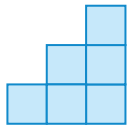
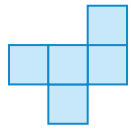
(a)



(b)

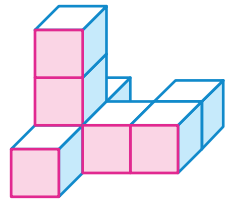
2 The front view of the solid shown has been shaded. The top view of this solid looks like:



A 
B 
C 
D 

Understanding

3 The front view of the following 3D structure is coloured red, the side view is coloured blue and the top view is coloured white. Draw the:

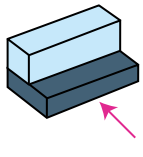


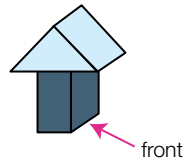
(a) front view

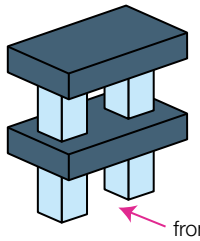
(b) side view

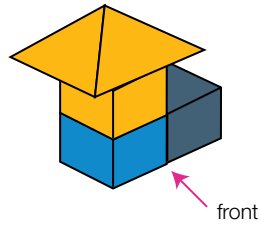
(c) top view.

4 The following structures are made with coloured blocks. Draw the front elevation of each structure by imagining you are standing looking from the position of the arrow, then move 90° to the right and draw the side elevation.

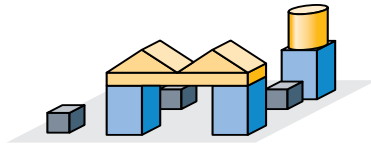
(a) 

(b) 

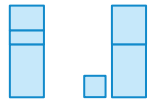
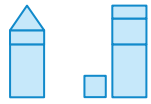
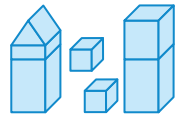
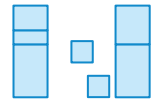
(c) 

(d) 

5 See the figure below to answer the following.



(a) Which diagram below shows a possible side elevation of the blocks above?

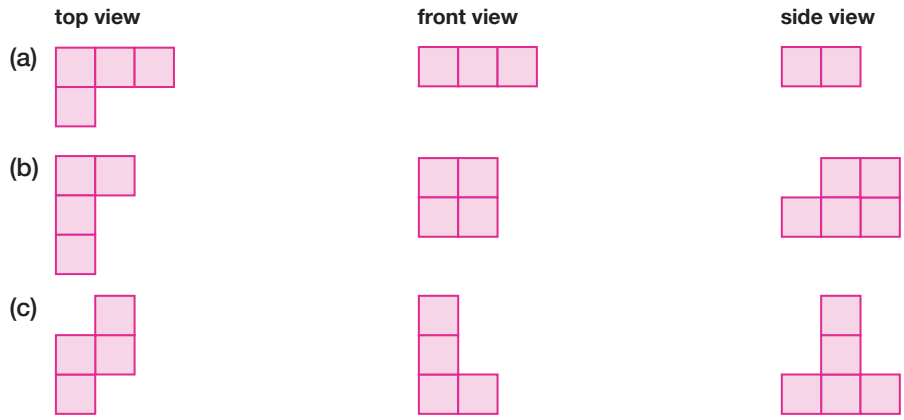
A 
B 
C 
D 

(b) Which elevation of the block structure above is shown opposite?

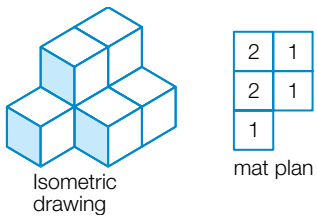
A front **B** rear

C side **D** top

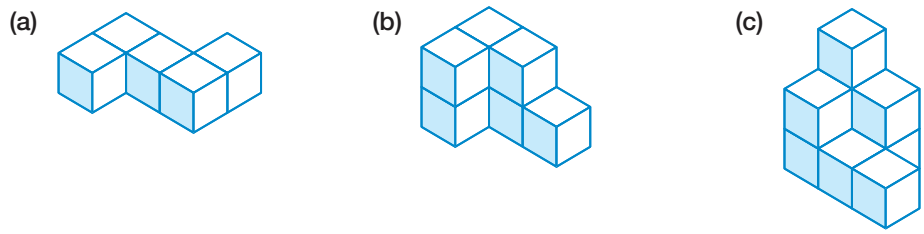
6 For each set of the plan views below, draw the 3D solid on isometric dot paper.



7 A *mat plan* shows the top view of a shape and the number of cubes in each position.

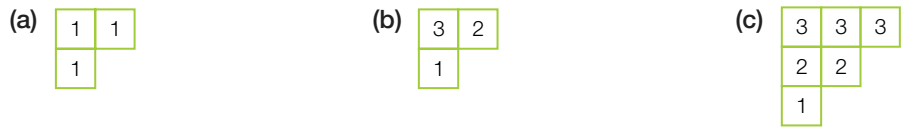


Draw the mat plan for each of the shapes below.

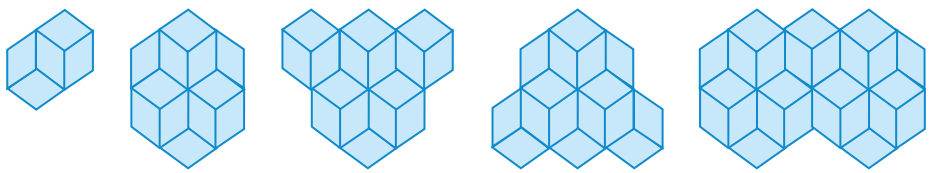


Reasoning

8 Draw each of the following mat plans as 3D shapes on isometric dot paper.



9 **(a)** Copy the following diagrams onto isometric dot paper.



(b) On your drawings, use three colours to shade the different faces of the cubes.

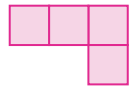
(c) How many cubes are in each of the diagrams?

(d) There is more than one way to colour the diagrams. If the cubes were coloured differently, would each diagram still look like it has the same number of cubes?

(e) Draw the top view of each shape.

Open-ended

10 A shape made with small cubes looks like this from above.


top view

Draw three possible 3D drawings on isometric dot paper, stating the number of cubes in each diagram.

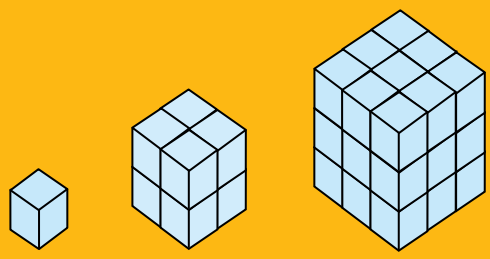
11 On isometric dot paper, draw four possible 3D drawings for the object that has the top view shown below, if the object is made of five cubes total and it is 2 cubes high.

12 Select a three-dimensional object and draw the front, rear and side elevations.

Problem solving

Bird's eye squares

Ezra glued some 1 cm cube blocks together to form different-sized cubes. He made four cubes of side length 2 cm and made four cubes of side length 3 cm, with four 1 cm cube blocks left over.



Ezra made a solid using some of his different-sized cubes. The solid's top view was in the shape of a square.

(a) What is the minimum number of cubes Ezra would need to build this solid?

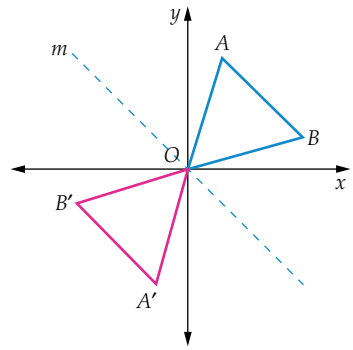
(b) How many different solids can Ezra make, using more than one cube, that have a top view of a square? Draw the top view of each possible solid.

Strategy options

- Draw a diagram.
- Make a model.

Challenge 10

1 The equilateral triangle OAB has been transformed into the equilateral triangle $OA'B'$.



(a) Which single transformation changes OAB into $OA'B'$?

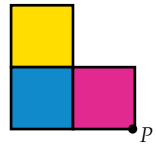
(b) Which two different transformations, applied one after the other, would change OAB into $OA'B'$?

(c) OAB is reflected in the line m. What is the name of the new position of A?

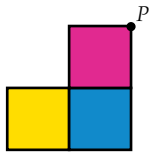
2 Jemima looks in a mirror and sees the clock behind her. The reflected clock hands appear to show 2:45. What will the time appear to be in 15 minutes?

3 A horizontal line CD is copied three times, and each copy appears on top of the original. Using the terms 'rotation' and 'translation', describe how each copy must be moved in order to create a square. C —— D

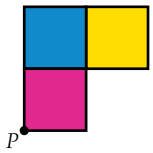
4 If the shape on the right is rotated 90° clockwise about the point P, then the resulting figure is:



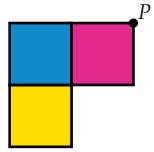
A



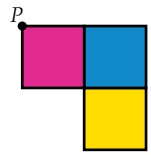
B



C

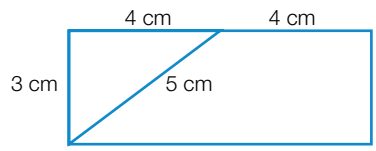


D



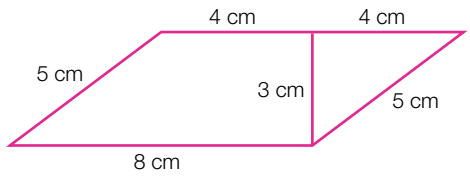
5 A secret message has to be read by looking in a mirror. Decode the following secret mirror message: 'TAƎB OT DRAH SI ƎGƎ ƎLIOB A'. To make sense of the message, what did you have to do?

6 A line is drawn on the 8 cm by 3 cm rectangle shown to make a triangle and a trapezium.

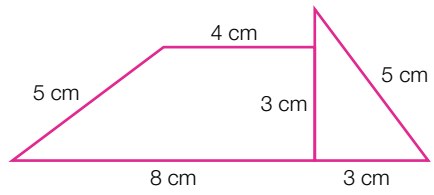


What transformation(s) need to be done to the triangle to create each of the following shapes?

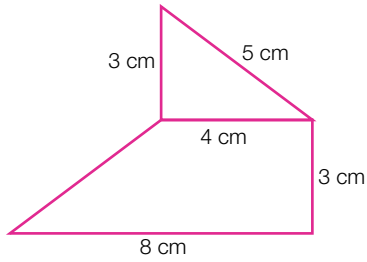
(a)



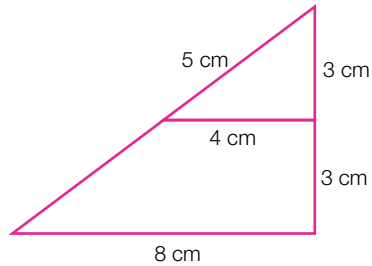
(b)



(c)



(d)



Chapter review 10

Maths literacy

asymmetrical	isometric	perpendicular distance	rotation
axis of symmetry	line of reflection	plan views	rotational symmetry
centre of rotation	order of reflectional symmetry	reflection	transformation
elevations	order of rotational symmetry	reflectional symmetry	translation
image	original	rotate	

Copy and complete the following using the words and phrases from this list, where appropriate. A word or phrase may be used more than once.

1 Reflections, rotations and translations are three types of _____s.

2 When an object is turned about a point, a _____ has been performed.

3 When a figure is moved in a horizontal and vertical direction, a _____ has been performed.

4 When a figure is a reversed image of the original, a _____ has been performed.

5 The transformed version of a shape or diagram is called an _____.

6 A _____ acts as a mirror to reflect an object.

7 A point about which a figure is turned is called the _____.

8 The _____ indicates how many times the image is identical to the original in a complete 360° rotation.

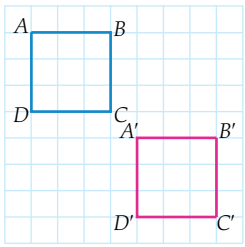
9 A 2D representation of a 3D solid can be drawn on _____ paper.

Equipment required: grid paper for Questions **2**, **4**, **5**, **6**, **7**, **9**, **23**, **28**; isometric dot paper for Questions **14**, **16**, **17**; protractor for Questions **7**, **9**, **23**, **28**

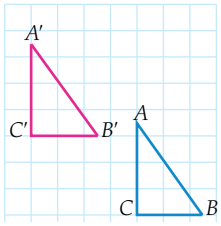
Fluency

1 Describe the translation shown in each of the following diagrams. **10.1**

(a)

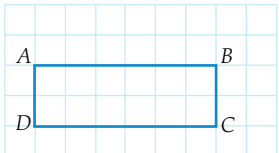


(b)

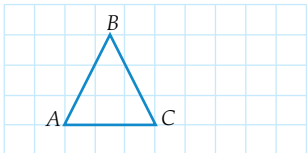


2 Copy each figure below and draw the resulting image after the translation required. **10.1**

(a) 3 units right and 1 unit up



(b) 3 units left and 3 units up



3 What would be the reverse translation of 7 units right and 4 units down? **10.1**

A [7, 4] **B** [-7, 4]

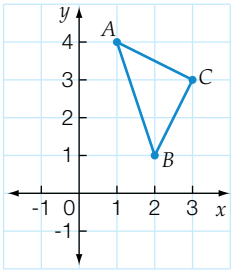
C [-7, -4] **D** [-4, 7]

4 For the figure in the diagram opposite: **10.1**

(a) write the coordinates of the vertices

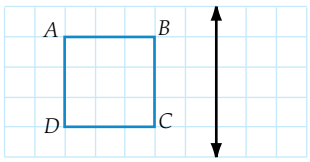
(b) copy the figure and translate it 4 units left and 3 units down, marking the image vertices with image notation

(c) write the coordinates of the vertices of the image.

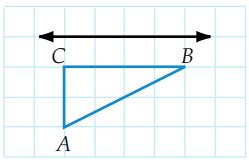


5 Copy each of the following figures onto grid paper and draw the resulting image when the figure is reflected in the line shown. **10.2**

(a)



(b)



6 For the figure in the diagram opposite: **10.2**

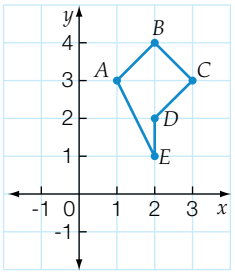
(a) write the coordinates of the vertices

(b) copy the figure and reflect it in the x-axis, marking the image vertices with image notation

(c) write the coordinates of the vertices of this image

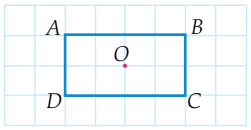
(d) copy and reflect the original figure in the y-axis, marking the image vertices with image notation

(e) write the coordinates of the vertices of this image.

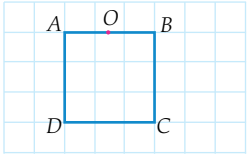


7 Copy each of the following figures onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you. **10.3**

(a) Rotate 90° in a clockwise direction about O.



(b) Rotate 180° in an anticlockwise direction about O.



8 A clockwise rotation of 90° is equivalent to an anticlockwise rotation of how many degrees? **10.3**

A 180° **B** 360° **C** 90° **D** 270°

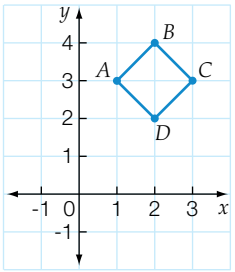
9 For the figure in the diagram opposite: **10.3**

(a) write the coordinates of the vertices

(b) copy the figure and rotate it 90° in an anticlockwise direction about the origin, marking the image vertices with image notation

(c) write the coordinates of the vertices of this image

(d) copy and rotate the original figure 180° in an anticlockwise direction about the origin, marking the image vertices with image notation



(e) write the coordinates of the vertices of this image

(f) copy and rotate the original figure 270° in an anticlockwise direction about the origin, marking the image vertices with image notation

(g) write the coordinates of the vertices of this image.

10 What are the reverse transformations for a translation of 5 units right and 8 units down followed by a clockwise rotation of 90°? **10.4**

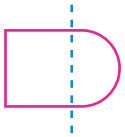
A an anticlockwise rotation of 90° and a translation of 5 units right and 8 units down

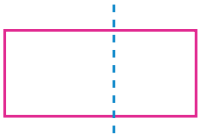
B a clockwise rotation of 180° and a translation of 5 units right and 8 units down

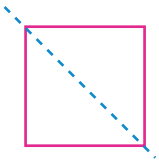
C an anticlockwise rotation of 90° and a translation of 5 units left and 8 units up

D a clockwise rotation of 90° and a translation of 5 units left and 8 units down

11 In which of the following shapes is the dotted line an axis of symmetry? **10.5**

A 

B 

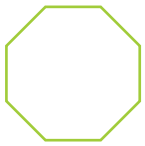
C 

D 

12 Copy the following shapes and draw lines of symmetry. Some shapes may have more than one line of symmetry. **10.5**

(a) 

(b) 

(c) 

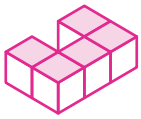
(d) 

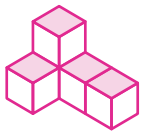
(e) 

(f) 

13 Identify the shapes from the previous question that have rotational symmetry. **10.5**

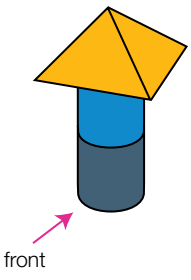
14 Two solids have been made using wooden cubes, as shown. **10.6**

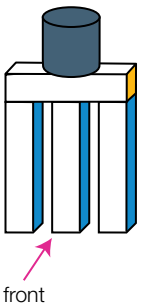
(a) 

(b) 

Use isometric dot paper to copy the shapes.

15 For each of the three-dimensional structures below, draw the front and side elevations. **10.7**

(a) 

(b) 

Understanding

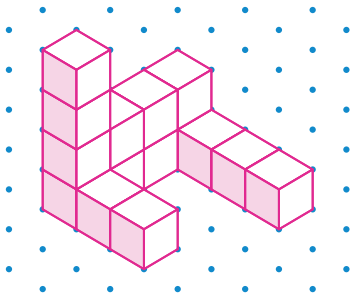
16 **(a)** Use isometric dot paper to draw the shape shown. **10.6, 10.7**

(b) Use the shape shown to draw:

(i) a front view

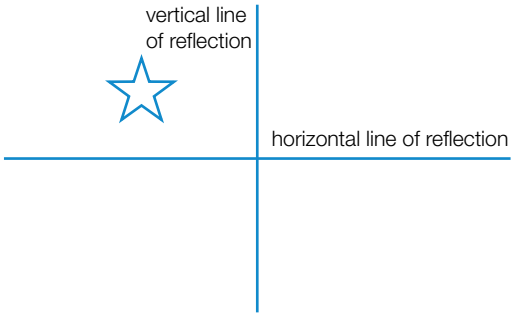
(ii) a top view

(iii) a side view.



17 What single translation would be equivalent to 6 units down, 7 units right, 2 units up, 8 units left, 4 units up and 5 units right? **10.4**

18 **(a)** Using the diagram below, reflect the star in the vertical and then the horizontal lines of reflection. **10.2**



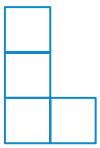
(b) Using the same starting position, reflect the star in the horizontal and then the vertical lines of reflection. Does it result in the same outcome as in part **(a)**?

19 What rotation is equivalent to reflecting an object along horizontal, then vertical, lines of reflection? **10.4**

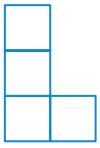
20 How many letters of the word MATHEMATICS do not have any reflectional or rotational symmetry? **10.5**

21 The plan views of a three-dimensional shape made with cubes are shown. Draw the solid on isometric dot paper. **10.6**

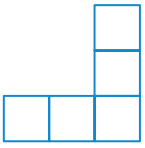
top view



front view



side view



22 On grid paper: **10.4**

(a) draw the line joining the points $A(-1, 3)$ and $B(2, 5)$

(b) translate this line 3 units right and 2 units down

(c) reflect this image in the x-axis to form a new image

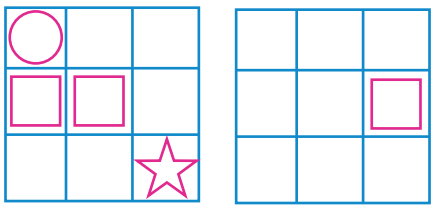
(d) rotate this image 90° in a clockwise direction (about the origin) to form a final image

(e) write the coordinates of the final image.

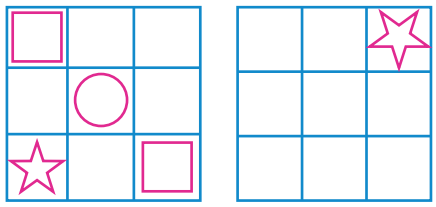
Reasoning

23 The following pairs of 3-by-3 grids have identical symbols in the squares, but each grid has been reflected horizontally, vertically or along the diagonal and is missing three symbols. Identify and draw the missing three symbols in the correct grid squares. **10.2**

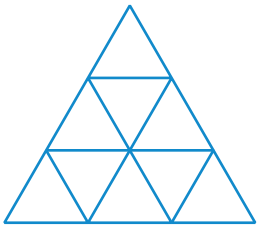
(a)



(b)



24 The following equilateral triangle is made of nine smaller equilateral triangles. **10.5**

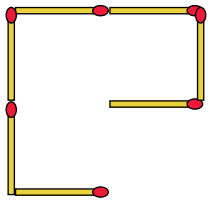


By shading the smaller equilateral triangles, create three different designs that have:

(a) one line of symmetry

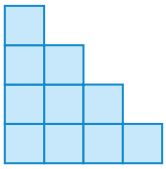
(b) three lines of symmetry.

25 The diagram at right shows a pattern made by arranging seven matchsticks of equal length. What is the smallest number of matchsticks that need to be added so the resulting image has at least one line of reflectional symmetry? **10.5**

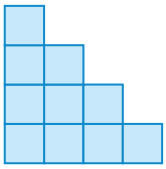


26 A solid is built using cube blocks. The front, side and top views of the solid are shown. What is the minimum number of blocks that this solid can have? **10.6, 10.7**

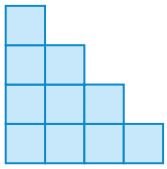
top view



front view



side view



27 Matt is asked to reflect a particular figure in the y-axis. This image is then rotated 180° in a clockwise direction. **10.4**

Matt believes he can replace these two combined transformations with one transformation.

Choose a shape of your own and perform the two transformations on it that Matt was asked to do. Can one transformation replace the two transformations? If so, write the new transformation.

Numeracy practice 10

Non-calculator

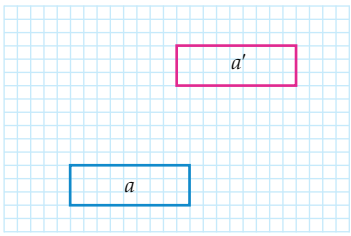
1 The translation shown is:

A 6 units right, 8 units up

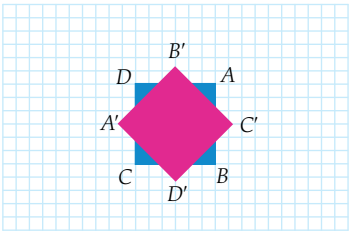
B 8 units right, 9 units up

C 8 units down, 6 units left

D 6 units down, 8 units left



2 The rotation needed to transform the original shape (blue) to the new rotated image (pink) is:



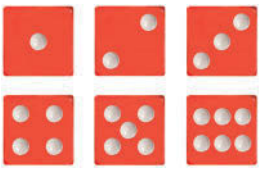
A 45° clockwise

B 135° anticlockwise

C 90° clockwise

D 90° anticlockwise

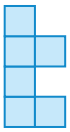
3 How many of the six faces of a standard die have fewer than three lines of symmetry?



A 1 **B** 2

C 3 **D** 4

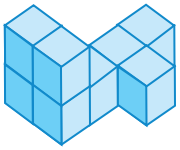
4 The top view of a solid is shown at right.



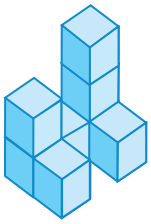
Which solid has this top view?

A

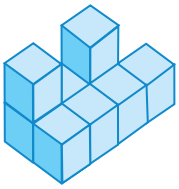
B



C



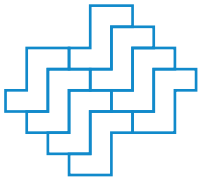
D



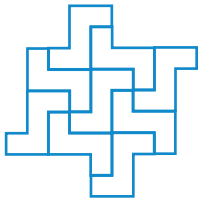
Calculator allowed

5 Mia has these S-shaped tiles.
They are blue on one side and red on the other side.
Mia wants to make a pattern with all the tiles showing their blue side. Which one of the patterns below is not possible to make?

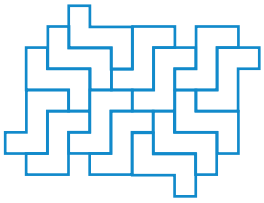
A



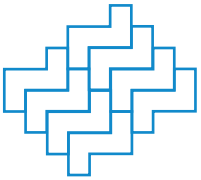
B



C



D

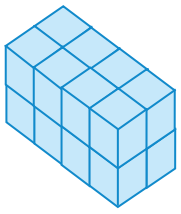


6 Which letter in the word SAME does not have an axis of reflectional symmetry?

A S **B** A **C** M **D** E

7 Janita glued small wooden cubes together to form the solid shown.
She then painted the solid on all outer surfaces.

How many of the small wooden cubes are painted on just one face?



A 0 **B** 2 **C** 4 **D** 8

Mixed review E

Equipment required: grid paper and protractor for Questions **11**, **17**, **18**

Fluency

1 Simplify: **1.5**

(a) $44 - 8 \times 4$ **(b)** $8 \times 6 \div 3 \times 4$ **(c)** $28 \div 4 + 3 \times 6$

2 Simplify: **1.2**

(a) $2^4 - 3^2$ **(b)** $3^3 \times 10^2$ **(c)** $10^5 - 10^3$

3 Calculate: **4.4–4.6**

(a) $15.006 + 2.45 + 0.059$ **(b)** 0.2×0.036 **(c)** $0.597 \div 0.3$

4 Use the following rules to complete the tables. **5.3**

(a) $n = 3m$

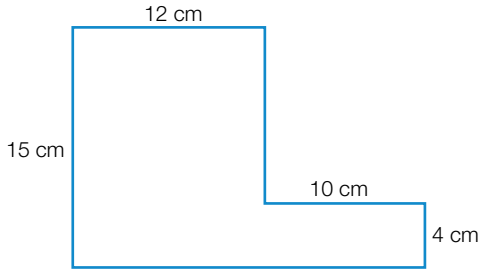
m	4	0	11	25	15
n					

(b) $t = s - 12$

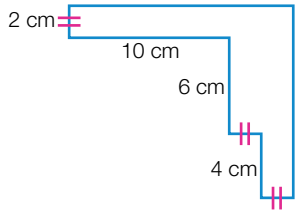
s	18	100	12	22	312
t					

5 Find the area of each of the following shapes. (All angles are right angles.) **6.5**

(a)



(b)



6 Find the following probabilities: **9.7**

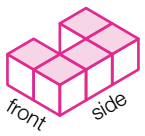
(a) getting a 6 on one roll of a standard six-sided die

(b) getting a red King on a single draw from a standard pack of 52 playing cards

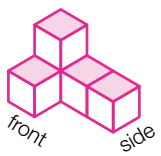
(c) getting two heads when tossing two coins.

7 Two shapes have been made using wooden cubes, as shown. **10.7**

(a)



(b)



For each shape, draw: **(i)** the front elevation **(ii)** the top elevation.

8 Solve each of the following equations using the balance method. Check your answer by substitution. **7.4**

(a) $3(x - 1) = 6$ **(b)** $\frac{x}{4} + 1 = 7$ **(c)** $2(x + 3) = 12$

9 Find the supplementary angles for each of the following. **8.3**

(a) 35° **(b)** 127° **(c)** 90°

10 The following numbers represent the heights of students in a class, to the nearest centimetre. **9.1**

148, 163, 158, 162, 151, 153, 170, 167, 165, 162, 172, 173, 157, 162, 158, 155, 163, 159, 160, 161

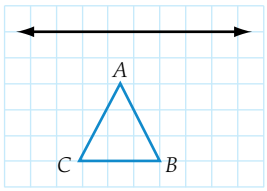
Draw the frequency table showing the information. You will need to group the results. Use 145–149, 150–154, 155–159 etc.

11 Show the image of each of the following figures after each of the following combined transformations. Use a protractor to help you. **10.4**

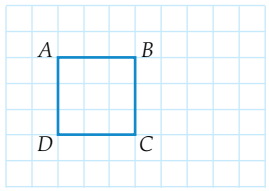
Reflect in the horizontal line of reflection and translate 4 units left and 3 units down.

Reflect in the line BC and rotate 90° in a clockwise direction about point C'.

(a)



(b)



12 For each of the following, identify the number of axes of symmetry. **10.5**

(a) isosceles triangle **(b)** square **(c)** rhombus

13 Find the mean, mode, median and range of the following data, noting any possible outliers. **9.2**

23, 34, 45, 26, 35, 42, 6, 34, 22, 36, 21, 29, 32, 34, 41, 39, 25, 27, 31, 30

Understanding

14 A figure is translated 7 units left, 4 units down, 8 units right and 3 units up. What would be the final position of the image compared to that of the original figure? **10.4**

15 **(a)** Find the prime factors of 24. **2.2**

(b) Find the prime factors of 30.

(c) Use the prime factors to find the highest common factor of 24 and 30.

(d) Use the prime factors to find the lowest common multiple of 24 and 30.

16 **(a)** What is the size of each angle in an equilateral triangle? **8.6, 8.7**

(b) A parallelogram has one angle that measures 126°. What are the sizes of the three remaining angles?

17 Which letters of the alphabet have reflectional and rotational symmetry as upper-case (capital) letters and also as lower-case letters? **10.5**

Reasoning

18 A point (1, 1) is translated 2 units right and 1 unit up. It is then reflected in the y-axis and then reflected in the x-axis. It is then translated 2 units right and 1 unit up. Finally, it is rotated 180° in a clockwise direction about the origin. What are the coordinates of the final image? **10.4**

19 **(a)** Find a data set with at least six different values that has a median of 6 and a mean of 8. **9.2**

(b) Describe the process you followed to find these values.

(c) Why might it be easier to use an odd number of values in a question of this type?

20 A game is played in which three different coins are tossed. It costs 50 cents to play this game. You win $1 (plus your original 50 cents) if more than one of the coins shows heads, or else you lose your money. **9.7**

(a) Write the set of results that are possible.

(b) Is the game fair? Explain your reasoning.

Answers

Worked solutions and answers to *all* activities appear in the Teacher Companion.

Chapter 1

Recall 1

1 (a) B (b) D

2 (a) 0, 12, 74, 567, 602, 4500, 6008, 11 100

(b) 2400, 2196, 1200, 1010, 987, 240, 204, 95

3 (a) 56 895 (b) 7 025 073

4 (a) (i) 1250 (ii) 1200 (iii) 1000

(b) (i) 8980 (ii) 9000 (iii) 9000

5 (a) 18 (b) 90 (c) 8 (d) 10 000

6 (a) 512 (b) 2063 (c) 1698

7 (a) 175 (b) 627 (c) 1157

8 (a) 315 (b) 670 (c) 3230

9 (a) 211 (b) 412 (c) 128 rem 1

Exercise 1.1

1 (a) 42 (b) 38 (c) 64 (d) 90 (e) 70 (f) 180

(g) 73 (h) 107 (i) 199 (j) 60 (k) 120 (l) 420

(m) 210 (n) 120 (o) 280 (p) 120 (q) 180 (r) 380

(s) 228 (t) 178 (u) 216

2 (a) 26 (b) 48 (c) 86 (d) 39 (e) 126 (f) 72

(g) 153 (h) 152 (i) 294 (j) 186 (k) 364 (l) 215

(m) 891 (n) 231 (o) 456 (p) 154 (q) 195 (r) 192

(s) 808 (t) 565 (u) 1364

3 (a) 44 (b) 180 (c) 240 (d) 140 (e) 255 (f) 995

(g) 121 (h) 503 (i) 165 (j) 360 (k) 98 (l) 189

(m) 618 (n) 242 (o) 582 (p) 360 (q) 266 (r) 420

4 (a) T, commutative law

(b) F

(c) T, associative law

(d) F

(e) T, commutative law

(f) F

(g) T, distributive law

(h) T, distributive law

(i) F

5 C

6 (a) 245 km (b) 235 km

7 1000

8 (a) \$136 (b) \$53

9 (a) 720

(b) May be about to begin or end a busy time (e.g. peak hour). A bottleneck, accident or other traffic hold-up may occur.

10 \$390

11 (a) \$7.84 (b) \$2.31 (c) \$7.08 (d) \$4.10

12 (a) Multiplying 7 by 20 gives one lot less of 7 than needed, not one lot less of 21. Need to multiply 7 by 20, then add 7 to get 147.

(b) Doubling twice is the same as multiplying by 4, not by 3. You can double 35 to get 70, then add another 35 to get 105.

(c) The remaining 9 should have been subtracted to get 191, not added.

13 (a) Alex and Khalid's methods are correct.

(b) They both round up the multiplier (Alex rounds 9 to 10 and Khalid 29 to 30), multiply, then subtract 1 lot of the number they multiplied by (Alex subtracts 1 lot of 29, as he had 10 lots instead of 9, and Khalid subtracts 1 lot of 9, as he had 30 lots instead of 29).

(c) The other two students round correctly, but then subtract the wrong number.

14 Using the distributive law:
$23 \times (70 + 5)$
$= 23 \times 70 + 23 \times 5$
$= 1610 + 115$
$= 1725$

The third row of Rosa's solution is 115, found by calculating 23×5.

The fourth row of Rosa's solution is 1610, found by calculating 23×70.

This is done by first finding $23 \times 7 = 161$, then writing a zero digit at the end of 161 to make 1610, which is 161×10, so that $23 \times 7 \times 10 = 23 \times 70$.

The fifth row of Rosa's solution is 1725, found by adding the third and fourth rows (adding 1610 and 115).

Rosa's solution works by doing the same calculations as the distributive law.

Open-ended—Sample answers

15 5 picture hooks at 28 cents each = \$1.40;
12 curtain rings at 15 cents each = $12 \times 10 + 12 \times 5$ = \$1.80.
Total: \$(1.40 + 1.80 + 6) = \$9.20.
Tranh should have enough money.

16 Do the calculations $9 \times 23 + 2 \times 23$.

17 Group the numbers and then use multiplication and addition. There are 7 1s (worth $7 \times 1 = 7$), 9 2s (worth $9 \times 2 = 18$) and so on. When the multiplications are finished, add the results to get 50.

Exercise 1.2

1 **(a)** F **(b)** T **(c)** T **(d)** T **(e)** T **(f)** T
(g) F **(h)** F **(i)** T

2 **(a)** $4 \times 4 = 16$ **(b)** $10 \times 10 = 100$
(c) $10 \times 10 \times 10 = 1000$ **(d)** $3 \times 3 = 9$
(e) $1 \times 1 = 1$ **(f)** $5 \times 5 \times 5 = 125$
(g) $2 \times 2 \times 2 = 8$ **(h)** $2 \times 2 \times 2 \times 2 = 16$
(i) $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$
(j) $1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 = 1$
(k) $0 \times 0 \times 0 \times 0 \times 0 \times 0 \times 0 = 0$
(l) $10 \times 10 \times 10 = 1000$ **(m)** $6 \times 6 \times 6 \times 6 = 1296$
(n) $5 \times 5 \times 5 \times 5 \times 5 = 3125$
(o) $8 \times 8 \times 8 \times 8 \times 8 \times 8 = 262\,144$
(p) $11 \times 11 \times 11 = 1331$
(q) $12 \times 12 \times 12 \times 12 = 20\,736$
(r) $14 \times 14 \times 14 \times 14 \times 14 = 537\,824$

3 **(a)** 8^3 **(b)** 4^6 **(c)** 12^5 **(d)** 16^9
(e) 17^3 **(f)** 19^2 **(g)** 8^4 **(h)** 13^7
(i) 11^7 **(j)** 9^6

4 **(a)** 16 384 **(b)** 46 656 **(c)** 6561 **(d)** 512
(e) 16 807 **(f)** 3375 **(g)** 289 **(h)** 125 000
(i) 64 **(j)** 500 **(k)** 4000 **(l)** 2000
(m) 1024 **(n)** 19 683 **(o)** 1

5 **(a)** five squared **(b)** thirty-one squared
(c) three cubed **(d)** twenty-seven cubed
(e) four to the power of five **(f)** nine to the power of six
(g) one to the power of four
(h) seven to the power of seven

6 25, 36, 49 **7** 64, 125, 216

8 **(a)** 5 **(b)** 7 **(c)** 10 **(d)** 11
(e) 1 **(f)** 0 **(g)** 70 **(h)** 20
(i) 2 **(j)** 4 **(k)** 10 **(l)** 1
(m) 0 **(n)** 5 **(o)** 20 **(p)** 30

9 **(a)** A **(b)** C **(c)** A

10 **(a)** 5 **(b)** 7 **(c)** 65 **(d)** 32
(e) 64 **(f)** 243 **(g)** 21 **(h)** 45
(i) 51

11 **(a)** T **(b)** T **(c)** F **(d)** T
(e) F **(f)** T **(g)** T

12 **(a)** 1^{200}, 5^4, 10^3, 4^5, 5^5, 4^6
(b) 10^5, 100^2, 3^2, 2^3, 1^{1000}, 0^{100}

13 **(a)** 16, 36 **(b)** 49, 81

14 **(a)** 0 **(b)** 190 000 **(c)** 45 000 **(d)** 2 472 768
(e) 537 824 **(f)** 51 998 079 **(g)** 266 240 **(h)** 79 785
(i) 439 291

15 **(a)** 3, 4 **(b)** 2, 3 **(c)** 4, 5 **(d)** 7, 8
(e) 9, 10 **(f)** 1, 2 **(g)** 8, 9 **(h)** 10, 11

16 **(a)** $\sqrt[3]{64}$, 5, $\sqrt{49}$, 2^4 **(b)** $\sqrt[3]{125}$, $\sqrt{60}$, 9, 7^2
(c) $\sqrt[3]{8}$, $\sqrt{90}$, 10, 4^2 **(d)** 8, $\sqrt{80}$, $\sqrt{105}$, 3^3

17 **(a)**

Index form	Expanded form	Value
10^1	10	10
10^2	10×10	100
10^3	$10 \times 10 \times 10$	1 000
10^4	$10 \times 10 \times 10 \times 10$	10 000
10^5	$10 \times 10 \times 10 \times 10 \times 10$	100 000

(b) The index number in the first column is the same as the number of zeros after the 1 in the third column.

18 **(a)** 4 **(b)** 2 **(c)** 3 **(d)** 5 **(e)** 1
(f) 6 **(g)** 4 **(h)** 2 **(i)** 4

19 **(a)** $5^2 \times 5^3 = 25 \times 125 = 3125$; $5 \times 5 \times 5 \times 5 \times 5 = 3125$
(b) $2^3 \times 2^6 = 8 \times 64 = 512$; $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 512$
(c) The sum of the powers is the number of times the base is multiplied with itself to obtain the answer.

20 **(a)** 4 **(b)** 10 **(c)** 16

21 **(a)** **(i)** T **(ii)** T **(iii)** T **(iv)** F
(b) **(i)** F **(ii)** T

22 **(a)** **(i)** one hundred **(ii)** one hundred
(b) **(i)** 10^{100} **(ii)** 10^{100}, more than a lifetime

23 **(a)** 121; 12 321; 1 234 321
(b) 123 454 321; 12 345 654 321; 1 234 567 654 321

Open-ended—Sample answers

24 **(a)** $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$
(b) $729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$
(c) The number 729 is a perfect square and also a perfect cube since $729 = 27^2$ and also $729 = 9^3$.
(d) $4096 = 4^6$
$4096 = 64^2$
$4096 = 16^3$
(e) 5^6, 6^6, 7^6 or any other whole number raised to the 6th power

25 $12^3 = 1728$; $31^2 = 961$; $3^{21} = 10\,460\,353\,203$

26 **(a)** 5, 6, 7, 8 or 9 **(b)** 10, 11, ... 21

27 **(a)** Mina is partly correct. The 'little 2' does mean two 35s, but they are multiplied together, not added. A better explanation for Mina to give Jo would be: 'The 2 tells you how many 35s you need to multiply together.'

(b) 2^2 is a special case. $2^2 = 2 \times 2$, which gives the same answer as 2 + 2 or 'double 2'. However, this is not true for any other number, except 0.

Exercise 1.3

1 (a) 100 (b) 210 (c) 360 (d) 250 (e) 25
(f) 2500 (g) 108 (h) 246 (i) 496 (j) 150
(k) 17 (l) 54 (m) 315 (n) 65 (o) 1050
(p) 180 (q) 18 (r) 15

2 (a) 121 (b) 231 (c) 276 (d) 1050 (e) 1470
(f) 4536 (g) 492 (h) 442 (i) 1292 (j) 1403
(k) 1519 (l) 2184 (m) 1974 (n) 2597 (o) 3025
(p) 5002 (q) 5544 (r) 4896

3 (a) 80 (b) 12 000 (c) 1500 (d) 12 000
(e) 2 800 000 (f) 30 000 000 (g) 4 (h) 300
(i) 200 (j) 2100 (k) 2000 (l) 26 000
(m) 18 (n) 27 (o) 16 (p) 480 000
(q) 12 800 000 (r) 140 000 000 (s) 600 (t) 500
(u) 2100

4 (a) 9 (b) 5 (c) 32 (d) 760
(e) 1060 (f) 1836 (g) 150 (h) 43
(i) 150 (j) 1440 (k) 4248 (l) 2450

5 (a) $248 (b) 14 000 km (c) $250 (d) $60 000

6 48 000 m

7 divide by 10 and double the result: 26

8 $1248 **9** $250 **10** $648 000

11 92 streets, some left over

12 total sales are $754 000 000; profit is $254 000 000

13 7 000 000 000

14 (a) $66 (b) $55 (c) $55 is gained

15 (a)

	100	60	2	
200	20 000	12 000	400	32 400
40	4000	2400	80	6480
6	600	360	12	972
				39 852

(b)

	300	50	6	
400	120 000	20 000	2400	142 400
10	3000	500	60	3560
2	600	100	12	712
				146 672

(c)

	100	0	7	
500	50 000	0	3500	53 500
60	6000	0	420	6420
0	0	0	0	0
				59 920

16 B

Open-ended—Sample answers

17 (a) $2 \times 810, 4 \times 405, 5 \times 324, 10 \times 162, 20 \times 81, 30 \times 54, 6 \times 270, 12 \times 135, 18 \times 90$

(b) $18 \times 90, 30 \times 54, 20 \times 81$

18 (a) $9 \times 24 = 216$
$12 \times 43 = 516$

(b) Kim has forgotten that the first digit in the numbers she is splitting up (24 and 43) is in the 'tens' place value column. She is multiplying them as though they were units.

(c) Write the number being split as tens and units:
$9 \times 24 = 9 \times (20 + 4)$
$12 \times 43 = 12 \times (40 + 3)$

Half-time 1

1 (a) 340 (b) 690 (c) 185
(d) 760 (e) 239 (f) 261

2 (a) (i) 216 (ii) 3125 (iii) 10 (iv) 49
(v) 7 (vi) 2 (vii) 10 (viii) 4
(b) (i) 49 and 4 (ii) 216

3 (a) 171 (b) 328 (c) 190 (d) 374
(e) 441 (f) 636 (g) 4757 (h) 7728

4 (a) 34 (b) 29 (c) 116 (d) 47
(e) 16 (f) 39 (g) 18 (h) 56

5 (a) 3^6 (b) 6^5 (c) 4^7 (d) 15^2
(e) 21^3 (f) 41^3

6 (a) 4200 (b) 3000 (c) 120 000
(d) 200 000 (e) 90 (f) 30
(g) 5 (h) 620 (i) 150

7 $676 000

Exercise 1.4

1 (a) 20 (b) 60 (c) 80 (d) 100
(e) 70 (f) 70 (g) 10 (h) 6
(i) 600 (j) 700 (k) 500 (l) 1000
(m) 5000 (n) 4000 (o) 10 000 (p) 10 000
(q) 80 000 (r) 700 000 (s) 800 000 (t) 2 000 000

2 (a) 1200 (b) 1500 (c) 10 000 (d) 3
(e) 54 000 (f) 30 000 (g) 28 000 (h) 40 000
(i) 48 000 (j) 60 (k) 50 (l) 25
(m) 80 000 (n) 900 000 (o) 2 100 000 (p) 63 000
(q) 300 000 (r) 1000 (s) 50 (t) 60
(u) 90 000 000

3 (a) $40 \times 25 = 1000$ (b) $120 \times 50 = 6000$
(c) $200 \times 15 = 3000$ (d) $400 \div 40 = 10$
(e) $400 \div 20 = 20$ (f) $250 \div 50 = 5$
(g) $1600 \times 20 = 32\,000$ (h) $34\,000 \times 20 = 680\,000$

(i) $4000 \times 250 = 1\,000\,000$ (j) $3500 \div 50 = 70$

(k) $11\,000 \div 100 = 110$ (l) $45\,000 \div 15 = 3000$

(m) $24\,000 \div 400 = 60$ (n) $15\,000 \div 2500 = 6$

(o) $5000 \times 2500 = 12\,500\,000$

4 (a) C

(b) (i) C (ii) B

5 (a) 4800 (b) 3 (c) 3 200 000

(d) 68 000 (e) 15 (f) 3250

6 (a) $1200 + 300 = 1500$ (b) $10\,300 + 200 = 10\,500$

(c) $15\,000 + 3000 = 18\,000$ (d) $820 - 80 = 740$

(e) $1200 - 200 = 1000$ (f) $500 - 300 = 200$

7 (a) 24 000, actual answer will be lower

(b) 800 000, actual answer will be higher

(c) 4000, actual answer will be lower

(d) 100, actual answer will be lower

(e) 10, actual answer will be higher

(f) 6, actual answer will be lower

8 (a) D (b) B (c) A (d) D

9 (a) 300 000 (b) $6 million

10 (a) $760 (b) $9000

11 (a) $7 000 000, underestimate

(b) 15, underestimate

12 (a) A (b) $16 \times 15 \div 2 = 120$

13 79×5003 will give the closer answer because the numbers are much closer to the rounded ones (80 and 5000). 5488 is rounded a long way down and so gives an estimated answer that is a lot less than the actual one.

14 (a) $1344 \div 21 \approx 1000 \div 20 = 50$

(b) $1344 \div 21 \approx 1340 \div 20 = 67$

(c) Answer **(b)** will give a closer estimate to the answer because the initial numbers have been rounded less.

(d) $1344 \div 21 = 64$, much closer to answer **(b)**

15 If she estimated by rounding down, she would run the risk of not having enough seats on the buses for 428 people; e.g. rounding to the first digit gives $400 \div 50 = 8$ buses, but eight 52–seater buses have 416 seats—too few.

Open-ended—Sample answers

16 (a) Natalie's rounded-down total outweighs her rounded-up total. Natalie rounded the T-shirt up by $3 and the lip gloss up by $1 = $4. She rounded the earrings down by $4 and the necklace down by $3 = $7. She is $3 short.

(b) A 'safer' method is to round up all purchases to a convenient multiple of 5 or 10.

17 (a) 39 and 82 (b) 12 and 19

(c) 214 and 487 (d) 72 and 573

Exercise 1.5

1 (a) 3 (b) 50 (c) 3 (d) 11

(e) 180 (f) 29 (g) 168 (h) 13

2 (a) 25 (b) 11 (c) 3 (d) 9

(e) 9 (f) 5 (g) 21 (h) 88

3 (a) 7 (b) 0 (c) 17 (d) 12

(e) 50 (f) 84 (g) 14 (h) 41

4 (a) F (b) T (c) F (d) T

(e) F (f) T

5 (a) C (b) D

6 (a) (i) 2×6, then 2×2 (ii) $15 \div 5$, then 6×3

(b) (i) 6 (ii) 21

7 (a) 19 (b) $(2 + 3) \times 4 + 5 = 25$
$2 + 3 \times (4 + 5) = 29$
$(2 + 3) \times (4 + 5) = 45$

8 (a) $(6 + 6) \times 3 = 36$ (b) $(10 - 4) \times 5 = 30$

(c) $(9 - 8) \times 6 + 4 = 10$ (d) $12 + 6 \div (7 - 4) = 14$

(e) $6 \div (3 + 3) \times 5 = 5$ (f) $3 \times 6 \div (8 - 4 + 5) = 2$

(g) $3 \times (10 - 7) \div 9 + 12 = 13$ (h) $18 \div 3 \times (5 - 3) + 2 = 14$

(i) $(7 + 3) \div (4 + 1) = 2$ (j) $(5 - 3) \times (8 - 6) \div 2 = 2$

9 B and C

10 (a) $73 \times 5 + 29 + 50 \times 8 = 794$, $7.94

(b) $2000 - (5 \times 73 + 29 + 8 \times 50) = 1206$, $12.06

11 (a) $6 \times 90 + 5 \times 70$
$= 540 + 350$
$= 890$ cents
$= \$8.90$

(b) $6 \times 90 + 6 \times 70 + 5 \times 50$
$= 540 + 420 + 250$
$= 1210$ cents
$= \$12.10$

(c) $5.30

12 (a) $2 + 21 \div 3 = 9$ (b) $15 - 6 \div 2 = 12$

(c) $14 - 8 - 6 = 0$ (d) $7 \times 5 - 6 = 29$

(e) $14 + 3 - 2 = 15$ (f) $(24 + 6) \div 10 = 3$

(g) $8 + 5 \times 2 - 6 = 12$ (h) $12 \div 2 + 1 \times 9 = 15$

13 (a) $6 \times (4 \div 2) \times 3 = (6 \times 4) \div 2 \times 3$

(b) $(1 + 4) \times 20 \div 5 > 1 + (4 \times 20) \div 5$

(c) $100 + 10 \div 10 > (100 + 10) \div 10$

(d) $36 \div 6 \times (3 - 3) < 36 \div 6 \times 3 - 3$

14 (a) Both pairs of brackets are unnecessary.

(b) Both pairs of brackets are unnecessary.

(c) The first pair of brackets is unnecessary.

(d) The second pair of brackets is unnecessary.

15 (a) 26 is the correct answer.

(b) The calculator follows order of operations, but the app does every operation as it is entered.

Open-ended—Sample answers

16 $(8 + 4) \times 6 \div 2 - 1 = 35$; $8 + (4 \times 6) \div 2 - 1 = 19$;
$8 + 4 \times (6 \div 2) - 1 = 19$; $8 + 4 \times 6 \div (2 - 1) = 32$;
$(8 + 4 \times 6) \div 2 - 1 = 15$; $8 + (4 \times 6 \div 2) - 1 = 19$;
$8 + 4 \times (6 \div 2 - 1) = 16$

17 (a) Answers may vary.
$1 = (3 - 1) \div 2$
$2 = 3 + 1 - 2$
$3 = 3 \times (2 - 1)$
$4 = 2 \times (3 - 1)$
$5 = (3 + 2) \times 1$
$6 = 3 \times 2 \times 1$
$7 = 2^3 - 1$
$8 = 2^3 \times 1$
$9 = 3^2 \times 1$
$10 = 3^2 + 1$

(b) $2^{31} = 2\,147\,483\,648$

(c) Students' own answers.

18 Correct answer is 75. Step 2 is incorrect: needed to do $30 \div 2$, first, then multiply by 5.

Exercise 1.6

1 (a) 724 **(b)** $9225 **(c)** $351 **(d)** 31 pieces

2 30 minutes **3** 72 laps **4** 366 days

5 (a) 15. A way to find this total using a 'make easy numbers' strategy is:

$3 + 4 + 5 + 3$
$= 3 + 3 + 4 + 5$
$= 6 + 4 + 5$
$= 10 + 5$
$= 15$

(b) $120

6 50 400 min

7 (a) 5998 m **(b)** 4643 m

8 Shoes $68 + 34 = \$102$
T-shirts $19 + 9 = \$28$
Earrings = $25
Total = $155

9 $192 - 18 = 174$ buns sold

10 (a) 222 km **(b)** 3931 km **(c)** 3709 km

11 73 cm **12** $2646

13 (a) 200 seconds

(b) 1 light year $= 299\,792 \times 365 \times 24 \times 60 \times 60$
$\approx 300\,000 \times 400 \times 20 \times 60 \times 60$
$\approx 8\,640\,000\,000\,000$ km

14 (a) $\$10^9$ **(b)** $\$10^{12}$

15 (a) $192

(b) 336 servings each day, with 15 bread rolls left over; can reduce expenses by buying 1 fewer 'baker's dozen' of bread rolls

16 42

17 (a) 55

(b) 1, 4, 9, 16, 25, 36, 49, 64, 81, 100

$1 + 9 + 4 + 16 + 25 + 36 + 64 + 49 + 81 + 100$
$= 10 + 20 + 25 + 100 + 130 + 100$
$= 385$

18 (a) 3 trips there and back (12 boxes), plus 1 more trip to new house (with last box) = 7 times total

(b) 49 km

19 (a) 11 weeks, 3 days

(b) 2 months, 2 weeks, 6 days

(c) 2 months, 3 weeks, 3 days

(d) In **(b)**, 80 was divided by 30 to find the number of months. Dividing by a larger number gives a smaller remainder to split into weeks and days. In **(c)**, 80 was divided by 28 (4 weeks).

20 Vet + Jason + 2 Scruffys: $91 + 56 = 147$ kg
Scruffy's weight $= 147 - 138 = 9$ kg

Open-ended—Sample answers

21 (a) The two numbers need to be different by at least 30; e.g. 25 and 66.

(b) The two numbers need to be single-digit numbers; e.g. 4 and 7, 5 and 6, 9 and 3.

22 (a) 20×300 mL bottles: 6 kg
20×600 mL bottles: 12 kg
20×1000 mL bottles: 20 kg

(b) Numbers of each box that give a total mass of 1000 kg:
1 L: 50 boxes
600 mL: 83 boxes
300 mL: 166 boxes

(c) 23×1 L boxes + 30×600 mL boxes + 30×300 mL boxes
26×1 L boxes + 26×600 mL boxes + 28×300 mL boxes

Challenge 1

1 D
Number must end in 6 or 8 to be even, smallest number needs biggest digit at end, so ends in 8. Largest remaining digit needs to be in the tens place.

2 C
List the possibilities, $2 \times 2 = 4$, $2 \times 4 = 8$, $2 \times 6 = 12$, $2 \times 8 = 16$, Note: 16 is repeated. $4 \times 4 = 16$, $4 \times 6 = 24$, $4 \times 8 = 32$, $6 \times 6 = 36$, $6 \times 8 = 48$, $8 \times 8 = 64$.

3 B
Three-digit number has to be as small as possible, two-digit number as large as possible: $356 - 87 = 269$.

4 $10 \times \$5 = \$50$, Kim has $50 more, whereas Brian has $10 less than the amount they each started with.

By trial and error: If they both start with $50, Kim will now have $100 and Brian will have $40. But $\$40 \times 3 = \$120$. Too much, so try a smaller amount. If they both start with $40, Kim will now have $90 and Brian will have $30. $\$30 \times 3 = \$90$. They both started with $40.

5 Find the factors of 15 181. It helps to realise that the only pairs of numbers that multiply to give a number ending in 1 are: 1 and 1, 3 and 7, 9 and 9. Hence, possible teenage children could be 11, 13, 17 or 19. See which of these numbers are factors of 15 181, then work out the remaining age. The ages are 17, 19 and 47, so the father is 47 years old.

6 Numbers are consecutive odd numbers.

Sum of first row = 1, sum of second row = 3 + 5 = 8, sum of third row = 7 + 9 + 11 = 27

$1 = 1^3$, $8 = 2^3$, $27 = 3^3$. Sum of row is row number cubed.

Sum of 7th row = $7^3 = 343$.
Sum of 100th row = $100^3 = 1\,000\,000$.

7 $\sqrt{8} \approx 2.8$, $\sqrt{8008} \approx 89.5$, so the squares of 3, 4, 5, … 89 are between 8 and 8008. 89 – 2 = 87 numbers.

8 After the first two terms, each term is the sum of the previous two terms: 18 + 29 = 47, 29 + 47 = 76, 47 + 76 = 123. Terms are 47, 76, 123.

9 1001 ÷ 6 = 166.83, so 1001 will appear in the 167th row. The odd-numbered rows run from left to right and the row would start with 997, as row 166 ends with 996 (166 × 6) in column A. Counting across gives 1001 in column E.

10 (a) 0, 1, 4, 9, 16, 25, 36, 49, 64, 81

(b) 0, 1, 4, 5, 6, 9

(c) 2, 3, 7, 8

(d) A, C, E

(e) 961: factor ends in 1 or 9.
$900 = 30^2$, so try 31^2.

Chapter review 1

1 (a) 142 (b) 138 (c) 225 (d) 120
(e) 315 (f) 437 (g) 180 (h) 544

2 (a) 144 (b) 125 (c) 100 000 (d) 8 (e) 30
(f) 3 (g) 129 (h) 3853 (i) 72

3 (a) 7^5 (b) 10^3 (c) 5^2 (d) 12^8

4 (a) 1400 (b) 170 000 (c) 5299 (d) 3038
(e) 22 (f) 1593 (g) 153 (h) 108

5 (a) 8000 (b) 60 000 (c) 50 (d) 300
(e) 60 000 (f) 3 000 000 (g) 8 (h) 300

6 (a) 550 ÷ 10 = 55 (b) 2000 ÷ 20 = 100
(c) 15 600 ÷ 200 = 78 (d) 30 × 600 = 18 000
(e) 160 × 400 = 64 000 (f) 500 × 2000 = 1 000 000
(g) 54 000 ÷ 900 = 60 (h) 12 800 ÷ 200 = 64

7 (a) 1 (b) 13 (c) 11 (d) 6 (e) 5 (f) 41

8 $66

9 720

10 (a) 80 000, close to (b) 90 000, close to
(c) 600, lower (d) 5000, higher

11 (a) 23 (b) 7

12 B

13 (a) 4 × (2 + 3) ÷ 5 – 1 = 3 (b) (5 + 1) ÷ 6 + 4 + 2 = 7

14 (a) $1200

(b) Melissa's weekly salary will be less than $1200.

15 (a) 3 and 4 (b) 5 and 6
(c) 9 and 10 (d) 12 and 13

16 Jessica $300, Amy $275, Aaron $225, Nikkita $150, Josef $350; Total $1300

17 (a) 9 + 7 × 3 = 30 (b) 16 – 4 × 2 ÷ 2 = 12

18 (a) 6 + 3 × 5 (b) $19 change

19 (a) 129 (b) 3853 (c) 36

20 (a) $2^4 = 16$, $2^5 = 32$, $2^6 = 64$

(b) $2^{10} = 1024$; larger than 1000. The difference is 24.

21 4, $\sqrt{81}$, $\sqrt{121}$, 2^4, 3^3, 10^2

22 $21

23 224 ÷ 42 = 5 rem 14, so 6 buses will be needed. Rounding to the first digit gives 200 ÷ 40 = 5, meaning the 14 remaining passengers would be stranded—this is because 224 was rounded down.

24 8 numbers: 17, 18, 19, 20, 21, 22, 23, 24

25 A

26 $3^2 = 3 \times 3$, which is not equal to 3 × 2.

27 Less than, because both of the numbers being rounded will be rounded up.

Numeracy practice 1

1 C 2 C 3 D 4 D
5 D 6 $28.40 7 180 8 C

9 the Farah family

Chapter 2

Recall 2

1 (a) 42, 36, 24, 66, 48 (b) 77, 49, 35, 14, 21
(c) 56, 48, 32, 80, 64 (d) 108, 27, 45, 99, 72
(e) 84, 72, 144, 108, 132

2 (a) 0, 2, 4, 6, 8 (b) 1, 3, 5, 7, 9

3 (a) 10 > 7 (b) 3 < 6 (c) 2 > 0 (d) 0 < 5

4 (a) 23 (b) 34 (c) 65 (d) 5
(e) 47 (f) 76

5 (a) -11 °C, -4 °C, 0 °C, 7 °C, 15 °C, 21 °C

(b) -25 °C, -14 °C, -3 °C, 5 °C, 10 °C, 32 °C

6 (a) 7 × 7 = 49

(b) 3 × 3 × 3 × 3 = 81

(c) 2 × 2 × 2 × 2 × 2 × 2 = 64

(d) 1 × 1 × 1 × 1 × 1 × 1 × 1 × 1 × 1 = 1

7 (a) 225 (b) 8 (c) 100 (d) 32

Exercise 2.1

1 **(a)** 1, 2, 3, 6, 9, 18 **(b)** 1, 2, 4, 8, 16

(c) 1, 23 **(d)** 1, 2, 3, 4, 6, 8, 12, 24

(e) 1, 2, 4, 5, 10, 20 **(f)** 1, 5, 7, 35

(g) 1, 2, 3, 4, 6, 9, 12, 18, 36 **(h)** 1, 2, 3, 6, 7, 14, 21, 42

(i) 1, 53

(j) 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

(k) 1, 7, 11, 77

(l) 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84

2 **(a)** The number is an even number.

(b) The last digit is 0.

(c) The sum of the digits is divisible by 3.

(d) The number is even and also divisible by 3.

(e) The number formed by the last two digits is divisible by 4.

(f) The last digit is 0 or 5.

3 **(a)** 108 **(b)** 92, 108, 3100

(c) 245, 3100 **(d)** none

(e) 108

4 **(a)** 10 **(b)** 9 **(c)** 25 **(d)** 30 **(e)** 28

(f) 24 **(g)** 63 **(h)** 60 **(i)** 66 **(j)** 36

(k) 100 **(l)** 56 **(m)** 60 **(n)** 50 **(o)** 300

5 **(a)** 10: 1, 2, 5, 10
15: 1, 3, 5, 15
HCF = 5

(b) 8: 1, 2, 4, 8
24: 1, 2, 3, 4, 6, 8, 12, 24
HCF = 8

(c) 5: 1, 5
12: 1, 2, 3, 4, 6, 12
HCF = 1

(d) 26: 1, 2, 13, 26
36: 1, 2, 3, 4, 6, 9, 12, 18, 36
HCF = 2

(e) 11: 1, 11
33: 1, 3, 11, 33
HCF = 11

(f) 28: 1, 2, 4, 7, 14, 28
70: 1, 2, 5, 7, 10, 14, 35, 70
HCF = 14

(g) 44: 1, 2, 4, 11, 22, 44
22: 1, 2, 11, 22
HCF = 22

(h) 10: 1, 2, 5, 10
30: 1, 2, 3, 5, 6, 10, 15, 30
HCF = 10

(i) 40: 1, 2, 4, 5, 8, 10, 20, 40
70: 1, 2, 5, 7, 10, 14, 35, 70
HCF = 10

(j) 32: 1, 2, 4, 8, 16, 32
60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
HCF = 4

(k) 35: 1, 5, 7, 35
70: 1, 2, 5, 7, 10, 14, 35, 70
HCF = 35

(l) 42: 1, 2, 3, 6, 7, 14, 21, 42
48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
HCF = 6

6 **(a)** A **(b)** C

7 **(a)** C **(b)** D

8 C

9 **(a)** A **(b)** D

10 D

11 **(a)** F **(b)** F **(c)** T **(d)** T **(e)** F **(f)** F

12 **(a)** **(i)** 60 **(ii)** 2

(b) **(i)** 24 **(ii)** 2

(c) **(i)** 48 **(ii)** 4

(d) **(i)** 200 **(ii)** 5

13 **(a)** divisible **(b)** factor **(c)** multiple **(d)** divisible

14 **(a)** 2 bags with 12 lollies in each bag; 3 bags of 8; 4 bags of 6; 6 bags of 4; 8 bags of 3; 12 bags of 2; 24 bags of 1

(b) 2 bags with 18 lollies in each bag; 3 bags of 12; 4 bags of 9; 6 bags of 6; 9 bags of 4; 12 bags of 3; 18 bags of 2; 36 bags of 1

15 **(a)** 1 row of 96, 2 rows of 48, 3 rows of 32, 4 rows of 24, 6 rows of 16, 8 rows of 12, 12 rows of 8, 16 rows of 6, 24 rows of 4, 32 rows of 3, 48 rows of 2, 96 rows of 1

(b) 8 rows of 12 or 12 rows of 8

16 5 groups of 4 and 2 groups of 3; 2 groups of 4 and 6 groups of 3

17 D

18 5 min, 10 min, 15 min

19 **(a)** 56

(b) 110

(c) 112

(d) 210

20 **(a)** 15 times

(b) 7 times

(c) 4, 7, 10, 13, 16, 19, 22

21 **(a)**

202 008	(2)	(3)	(4)	5	(6)	7	(8)	9	10
12 121 212	(2)	(3)	(4)	5	(6)	7	8	9	10
300 300 300	(2)	(3)	(4)	(5)	(6)	7	8	(9)	(10)
7 500	(2)	(3)	(4)	(5)	(6)	7	8	9	(10)
900 090	(2)	(3)	4	(5)	(6)	7	8	(9)	(10)
123 456 789	2	(3)	4	5	6	7	8	(9)	10

(b) **(i)** 2 **(ii)** 3

(c) 2 is a factor of 4, so if a number is divisible by 4, it is also divisible by 2.

3 is a factor of 9, so if a number is divisible by 9, it is also divisible by 3.

22 6 is the LCM of 2 and 3.

23 **(a)** 28 **(b)** 496

24 18, 20

25 **(a)** Multiply the two numbers together.

(b) If the HCF of the numbers is 1, this will be the LCM.

Open-ended—Sample answers

26 **(a)** 4 rows of 15; 5 rows of 12; 6 rows of 10

(b) Boxes that are closer to a square shape are usually more practical and require less packaging to make, so 5×12 or 6×10 are more likely.

27 Sam's age must be a multiple of 3 and 5, i.e. a multiple of 15: 15, 30, 45, 60, 75.

28 No. Multiplying any common multiple by a whole number greater than 1 gives a higher common multiple.

Exercise 2.2

1 2, 3, 5, 7, 11, 13, 17, 19

2 Four: 2, 3, 5, 7

3 23, 29, 31, 37, 41, 43, 47, 53, 59

4 **(a)** B **(b)** D

5 **(a)** 61 **(b)** 62 **(c)** 9, 15 **(d)** 47

6 **(a)** F **(b)** T **(c)** T **(d)** F **(e)** T **(f)** T **(g)** T **(h)** F

7 **(a)** divisible by 10, 5 and 2 **(b)** divisible by 3 and 9
(c) divisible by 3 and 9 **(d)** divisible by 3 and 9
(e) divisible by 2 and 4 **(f)** divisible by 5

8 **(a)** yes, HCF = 1 **(b)** yes, HCF = 1
(c) no, HCF = 13 **(d)** no, HCF = 9

9 The only common factors of any two prime numbers is 1.

10 All other even numbers have 2 as a factor.

11 It is even, so 2 is a factor.

12 There are no more even prime numbers.

13 1

14 **(a)** 99 and 101 **(b)** 35 and 37

15 No, a prime number will only be co-prime with any whole number that is not a multiple of the original prime number.

16 The smaller of the two numbers will be a common factor.

Open-ended—Sample answers

17 The first 10 even composite numbers:

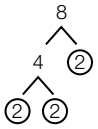
$4 = 2 + 2$
$6 = 3 + 3$
$8 = 3 + 5$
$10 = 5 + 5$
$12 = 5 + 7$
$14 = 7 + 7$
$16 = 11 + 5$
$18 = 11 + 7$
$20 = 13 + 7$
$22 = 11 + 11$

18 3 and 7 $(3 \times 2 + 1 = 7)$
2 and 5 $(2 \times 2 + 1 = 5)$
5 and 11 $(2 \times 5 + 1 = 11)$

Exercise 2.3

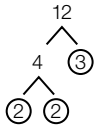
1 Students' factor trees may vary, but the factors at the end must be the same.

(a)



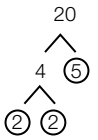
$8 = 2 \times 2 \times 2$
$= 2^3$

(b)



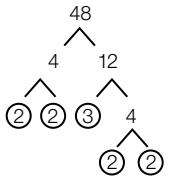
$12 = 2 \times 2 \times 3$
$= 2^2 \times 3$

(c)



$20 = 2 \times 2 \times 5$
$= 2^2 \times 5$

(d)



$48 = 2 \times 2 \times 2 \times 2 \times 3$
$= 2^4 \times 3$

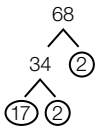
(e)

$14 = 2 \times 7$

(f)

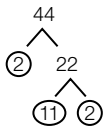
$26 = 2 \times 13$

(g)



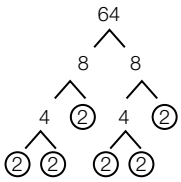
$68 = 2 \times 2 \times 17 = 2^2 \times 17$

(h)



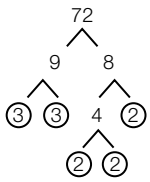
$44 = 2 \times 2 \times 11 = 2^2 \times 11$

(i)



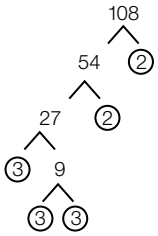
$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 2^6$

(j)



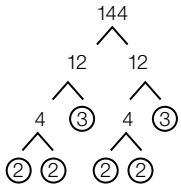
$72 = 2 \times 2 \times 2 \times 3 \times 3$
$= 2^3 \times 3^2$

(k)



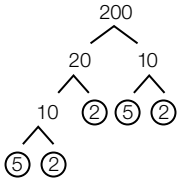
$108 = 2 \times 2 \times 3 \times 3 \times 3$
$= 2^2 \times 3^3$

(l)



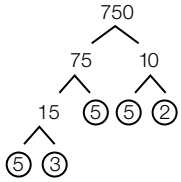
$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$
$= 2^4 \times 3^2$

(m)

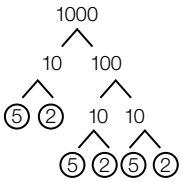


$200 = 2 \times 2 \times 2 \times 5 \times 5$
$= 2^3 \times 5^2$

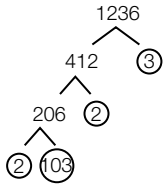
(n)



$750 = 2 \times 3 \times 5 \times 5 \times 5$
$= 2 \times 3 \times 5^3$

(o) 

$1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$
$= 2^3 \times 5^3$

(p) 

$1236 = 2 \times 2 \times 3 \times 103$
$= 2^2 \times 3 \times 103$

2 (a) $8 = 2^3$ (b) $12 = 2^2 \times 3$

(c) $28 = 2^2 \times 7$ (d) $36 = 2^2 \times 3^2$

(e) $39 = 3 \times 13$ (f) $77 = 7 \times 11$

(g) $51 = 3 \times 17$ (h) $38 = 2 \times 19$

(i) $30 = 2 \times 3 \times 5$ (j) $63 = 3^2 \times 7$

(k) $96 = 2^5 \times 3$ (l) $132 = 2^2 \times 3 \times 11$

(m) $168 = 2^3 \times 3 \times 7$ (n) $198 = 2 \times 3^2 \times 11$

(o) $288 = 2^5 \times 3^2$ (p) $212 = 2^2 \times 53$

3 (a) $8 = 2 \times 2 \times 2$
$20 = 2 \times 2 \times 5$
HCF $= 2 \times 2$
$= 4$

(b) $12 = 2 \times 2 \times 3$
$18 = 2 \times 3 \times 3$
HCF $= 2 \times 3$
$= 6$

(c) $12 = 2 \times 2 \times 3$
$36 = 2 \times 2 \times 3 \times 3$
HCF $= 2 \times 2 \times 3$
$= 12$

(d) $36 = 2 \times 2 \times 3 \times 3$
$48 = 2 \times 2 \times 2 \times 2 \times 3$
HCF $= 2 \times 2 \times 3$
$= 12$

(e) $24 = 2 \times 2 \times 2 \times 3$
$56 = 2 \times 2 \times 2 \times 7$
HCF $= 2 \times 2 \times 2$
$= 8$

(f) $28 = 2 \times 2 \times 7$
$84 = 2 \times 2 \times 3 \times 7$
HCF $= 2 \times 2 \times 7$
$= 28$

(g) $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$
HCF $= 2 \times 2 \times 2 \times 2 \times 2$
$= 32$

(h) $70 = 2 \times 5 \times 7$
$98 = 2 \times 7 \times 7$
HCF $= 2 \times 7$
$= 14$

(i) $60 = 2 \times 2 \times 3 \times 5$
$105 = 3 \times 5 \times 7$
HCF $= 3 \times 5$
$= 15$

(j) $66 = 2 \times 3 \times 11$
$110 = 2 \times 5 \times 11$
HCF $= 2 \times 11$
$= 22$

(k) $80 = 2 \times 2 \times 2 \times 2 \times 5$
$128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
HCF $= 2 \times 2 \times 2 \times 2$
$= 16$

(l) $130 = 2 \times 5 \times 13$
$156 = 2 \times 2 \times 3 \times 13$
HCF $= 2 \times 13$
$= 26$

4 (a) D (b) A

5 (a) 72 (b) 300 (c) 54 000

(d) 2205 (e) 15 400 (f) 7744

6 (a) $2^4 \times 3$ (b) $2^5 \times 3$ (c) $2^4 \times 3^2$

7 (a) $2 \times 3 \times 5 \times 11$ (b) $2^2 \times 3 \times 5 \times 11$ (c) $2 \times 3^2 \times 5 \times 11$

8 The factors of a number are all numbers that can evenly divide a number, so for 24 there are 8 factors. The prime factors are all factors that are also prime numbers. Twenty-four has 2 prime factors: 2 and 3.

9 2, 3, 7

10 (a) Prime factors of 36: $2 \times 2 \times 3 \times 3$
Prime factors of 100: $2 \times 2 \times 5 \times 5$
Highest common factor (HCF) of 36 and 100: $2 \times 2 = 4$
Factors of 36 not used to find the HCF: $3 \times 3 = 9$
Factors of 100 not used to find the HCF: $5 \times 5 = 25$
HCF × unused factors of 36 × unused factors of 100:
$4 \times 3 \times 3 \times 5 \times 5 = 900$

(b) (i) 72 (ii) 36 (iii) 40 (iv) 72 (v) 260
(vi) 800 (vii) 1000 (viii) 800 (ix) 4000

11 (a) 6 (b) 14 red balloons and 9 blue balloons

12 17; 510

13 (a) (i) 6 (ii) 30 (iii) 210

(b) (i) 4 (ii) 8 (iii) 16

14 (a) Yes, 105 is a winning number.

(b) There would be four winning tickets: 15, 105, 165 and 195.

Open-ended—Sample answers

15 Trial and error; e.g. try $2 \times 2 \times 3 \times 5$. If it is too small, try a bigger prime. $2 \times 2 \times 3 \times 11 = 132$; $2 \times 2 \times 5 \times 7 = 140$

16 (a) 2, 4, 5, 10 (and 20)

(b) A factor need not be prime. For example, 6 is a factor of 18, but 6 is a composite number.

(c) If a factor is composite, Ray should find its factors.

Exercise 2.4

1 (a) +300 (b) -50 000 (c) -8 (d) +4

(e) +36 (f) +45 (g) -20 (h) +12

(i) -2 (j) -7 (k) +28 (l) -40 000

(m) +225 (n) -9

2 (a) $+6 > -3$ (b) $+1 > -3$ (c) $-7 < +5$

(d) $-2 < +6$ (e) $-12 < -4$ (f) $-7 < -1$

(g) $-2 > -11$ (h) $-9 < 0$ (i) $0 > -2$

(j) $-35 < +7$ (k) $-4 < +67$ (l) $-11 < +75$

3 (a) -12, -4, -3, -1, 6 (b) -11, -10, 2, 9, 10

(c) -6, -4, 1, 2, 9 (d) -8, -6, -5, 1, 7

(e) -12, -8, -7, -4, -1 (f) -79, -56, 0, 6, 23

4 (a) 8, 7, 3, -5, -6 (b) 10, 7, 1, -8, -12

(c) 1, -4, -6, -8, -9 (d) 7, 0, -5, -11, -12

(e) -2, -5, -9, -11, -12 (f) 79, 4, 0, -9, -54

5 (a) east 300 km (b) 3 days early

(c) 7 °C above zero (d) subtract 5

(e) add 22 (f) right 3 m

(g) +2 (h) -16 (i) +350

6 (a) A (b) D

7 (a) T (b) F (c) F (d) T
(e) F (f) F (g) T (h) T

8 (a) -3, -2, -1, 0, 1, 2 (b) -1, 0, 1
(c) -8, -7, -6, -5 (d) -4, -3, -2, -1
(e) -38, -39, -40, -41 (f) -119, -118, -117, -116

9 (a) -12, -10, -8 (b) -5, 0, 5
(c) 0, -3, -6 (d) -24, -30, -36
(e) 0, -10, -20 (f) -3, -8, -13

10 (a) Aran (b) Orlando

11 (a) Tuesday (b) Friday (c) Tuesday

12 (a) 156 m (b) 154 m (c) 310 m

13 (a) 4 km west (b) 4 km east

14 (a) 4 floors up (b) 4 floors down

15 (a) Sam has more money at the end of the week.

(b) Withdrawals \$80 + \$120 + \$50 = \$250
Deposits \$350 + \$40 = \$390
\$140 more in the account

Open-ended—Sample answers

16 any three of -7, -6, -5, -4, -3, -2, -1

17 (a) -1

(b) -5, 6, 6, 1, -5, 1 1, 1, -5, 2, 3, 2 4, 4, -5, 3, 3, -5

18 Students' own answers.

Half-time 2

1 (a) 45 (b) 24

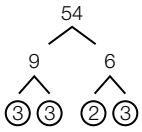
2 B

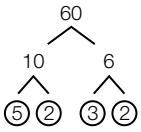
3 (a) +85 (b) -7 (c) +3 (d) +28 000
(e) -40 (f) +11

4 31, 37, 41, 43, 47

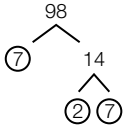
5 (a) $6 < 10$ (b) $-4 < 7$ (c) $3 > -12$ (d) $-8 > -15$

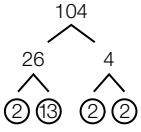
6 (a) $54 = 2 \times 3^3$ (b) $60 = 2^2 \times 3 \times 5$





(c) $98 = 2 \times 7^2$ (d) $104 = 2^3 \times 13$





7 (a) 76 905 is divisible by both 3 and 5, so it is divisible by 15.

(b) 4734 is divisible by 2, as it is even. Sum of digits is 18, which is divisible by 3. Therefore, as 4734 is divisible by both 2 and 3, it is divisible by 6.

8 (a) -37, -30, -3, 0, 3, 7

(b) -40, -20, -5, -1, 54

9 24 **10** D

11 (a) 150 seconds

(b) Honda 6 laps,
Holden 5 laps,
Ford 3 laps

(c) 12 laps

Exercise 2.5

1 (a) 11 (b) 8 (c) 11 (d) 8
(e) -2 (f) -5 (g) -5 (h) -6
(i) 6 (j) 12 (k) 25 (l) 20
(m) -10 (n) -4 (o) -20 (p) -31

2 (a) 2 (b) 6 (c) 5 (d) 0
(e) -6 (f) -11 (g) -8 (h) -4
(i) -10 (j) -8 (k) -20 (l) -12
(m) -12 (n) -30 (o) -69 (p) -124

3 (a) B (b) D

4 (a) B (b) A

5 (a) 12 (b) 11 (c) 12 (d) 7
(e) -18 (f) -22 (g) -5 (h) -10
(i) -8 (j) 13 (k) -1 (l) 1

6 +8 °C **7** the 11th floor

8 34 metres below the surface

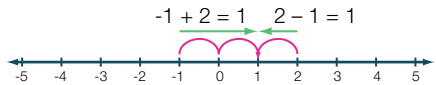
9 5 m **10** \$13

11 (a) positive five subtract positive seven equals negative two

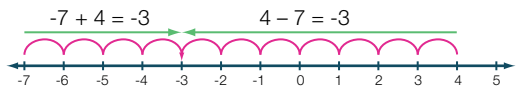
(b) negative eight add positive nine equals positive one

(c) negative four subtract positive three equals negative seven

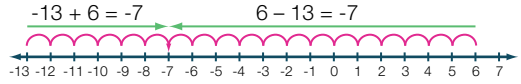
12 (a) (i)



(ii)



(iii)



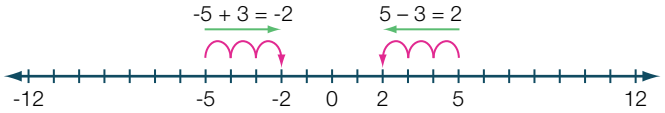
(b) The finishing point of the number line journeys for these pairs of additions and subtractions is the same. The pairs are equivalent—each addition can be rewritten as a subtraction. $-1 + 2 = 2 - 1$.

(c) (i) $-37 + 86 = 86 - 37 = 49$

(ii) $-72 + 95 = 95 - 72 = 23$

(iii) $-104 + 119 = 119 - 104 = 15$

13 (a)



(b) **(i)** -8 + 6 = -2 **(ii)** -5 + 4 = -1

(iii) 10 – 6 = 4 **(iv)** 12 – 9 = 3

(c) **(i)** -6 **(ii)** -21

(iii) -25 **(iv)** -19

Open-ended—Sample answers

14 Assuming that the temperature rises fairly gradually after sunrise, a possible answer might be:

6 am	7 am	8 am	9 am	10 am	11 am	12 pm
-4 °C	0 °C	7 °C	13 °C	17 °C	21 °C	25 °C

15 Any three positive integers less than 13.

Exercise 2.6

1 (a) 1 **(b)** 1 **(c)** 9 **(d)** 6

(e) -4 **(f)** -5 **(g)** -7 **(h)** -11

(i) -9 **(j)** -11 **(k)** -20 **(l)** -34

(m) -10 **(n)** -90 **(o)** -29 **(p)** -133

2 (a) 11 **(b)** 6 **(c)** 10 **(d)** 12

(e) 8 **(f)** 3 **(g)** 16 **(h)** -6

(i) 12 **(j)** -1 **(k)** 8 **(l)** -6

(m) -10 **(n)** -10 **(o)** -26 **(p)** -28

3 (a) A **(b)** B

4 (a) 0 **(b)** -6 **(c)** -7 **(d)** -10

(e) -5 **(f)** 6 **(g)** 5 **(h)** 9

(i) -3 **(j)** -2 **(k)** -8 **(l)** 22

5 (a) C **(b)** A

6 -4 °C **7** -3 (3 floors below ground level)

8 $3500 owing

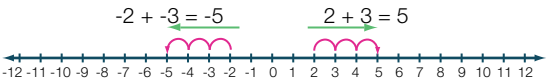
9 9 m **10** 3 m **11** 31 m

12 (a) negative five add negative six equals negative eleven

(b) positive seven subtract negative eight equals positive fifteen

(c) negative nine subtract negative fourteen equals positive five

13 (a)



(b) **(i)** -1 – 6 = -7 **(ii)** -4 – 7 = -11

(iii) 3 + 8 = 11 **(iv)** 5 + 4 = 9

(c) **(i)** -36 **(ii)** -55 **(iii)** -77 **(iv)** -94

14 (a) **(i)** 6 **(ii)** 18 **(iii)** -9 **(iv)** 3

(b) smaller; larger

Open-ended—Sample answers

15 Must be -22 or less; e.g. -25, -35, -28.

16 (a) -9

(b) Max has moved in the wrong direction on the number line. To add a negative number, he needs to face right (to add) but then move backwards (in the negative direction).

Exercise 2.7

1 (a) 7 **(b)** 3 **(c)** 11 **(d)** -2

(e) 2 **(f)** -12 **(g)** -11 **(h)** -10

(i) 9 **(j)** 6 **(k)** -2 **(l)** -4

(m) -1 **(n)** 6 **(o)** 12 **(p)** 2

(q) -10 **(r)** -12 **(s)** 5 **(t)** 4

2 (a) F **(b)** T **(c)** T **(d)** F

3 (a) -12 **(b)** -11 **(c)** 4 **(d)** 3

(e) -5 **(f)** -8 **(g)** -13 **(h)** -12

(i) -30 **(j)** -31 **(k)** 57 **(l)** 65

4 (a) C **(b)** B

5 (a) 6 **(b)** 7 **(c)** -7 **(d)** -4

(e) -6 **(f)** -8 **(g)** 4 **(h)** 1

(i) –12 **(j)** -11 **(k)** -9 **(l)** -9

6 -$150

7 (a) the 10th floor

(b) the 4th floor below the ground floor

(c) the 8th floor

8 -$350, a loss of $350

9 (a)

+	-6	+20	-14	31
+4	-2	+24	-10	35
-7	-13	+13	-21	24
+8	+2	+28	-6	39
-9	-15	+11	-23	22

(b)

–	8	-11	19	-25
3	5	-14	16	-28
-7	15	-4	26	-18
+5	3	-16	14	-30
-10	18	-1	29	-15

10 (a) 6

(b)

1	6	-1
0	2	4
5	-2	3

11 (a)

-3	2	-5
-4	-2	0
1	-6	-1

(b) -6

Each of the three digits in the row, column or diagonal has been reduced by 4. This means that the magic sum of the digits will be $3 \times 4 = 12$ less than the original magic sum.

Open-ended—Sample answers

12 (a)

Hole number	1	2	3	4	5	6	7	8	9	Total
Par for the hole	4	4	5	3	4	5	4	3	4	36
Shots taken	4	3	4	5	5	4	3	3	4	35
Score compared to par	0	-1	-1	+2	+1	-1	-1	0	0	-1

(b)

Hole number	1	2	3	4	5	6	7	8	9	Total
Par for the hole	4	4	5	3	4	5	4	3	4	36
Shots taken	3	6	4	4	5	3	3	4	3	35
Score compared to par	-1	+2	-1	+1	+1	-2	-1	+1	-1	-1

13 Ignoring the signs, the numbers will add to 31; e.g. -14 and 17, -10 and 21, 18 and -13

Challenge 2

1 (a) 1, -3, -7 **(b)** -7, -1, 5

2 The twins are the youngest and have the same age. Look for three factors of 72 in which the smallest two are the same: $1 \times 1 \times 72$, $2 \times 2 \times 18$, $3 \times 3 \times 8$; $3 + 3 + 8 = 14$. ($6 \times 6 \times 2$ gives sum of 14, but twins are not the youngest.) The brothers are 3, 3 and 8 years old.

3 $172 \div 2 = 86$ (average of two numbers is the number in the middle) so the middle number is 86. Numbers are 85, 86, 87.

4 Consider factors of 150. $10 \times 15 = 150$,
$(10 - 4) \times (15 + 10) = 6 \times 25 = 150$, $(10 + 10) \times (15 - 4) = 220$;
$6 \times 25 = 150$, $(6 + 10) \times (25 - 4) = 336$, $(6 - 4) \times (25 + 10) = 70$.
Originally, there were 10 rows of 15 chairs.

5 11×0, $(10 + 10 + 1) \times 1$, $(10 + 10) \times$ (the other digits)
Sum $= 0 + 21 + 20 \times (2 + 3 + 4 + 5 + 6 + 7 + 8 + 9)$
$= 21 + 20 \times 44$
$= 901$

6 The amount that each person raised has been counted twice.
$(\$130 + \$150 + \$100 + \$70 + \$90) \div 2 = \$540 \div 2 = \$270$

7 Numbers that give a remainder of 2 when divided by 4 are: 6, 10, 14, 18, 22, 26, 30, 34, 38, 42, 46, 50, 54, 58, 62, ...

Numbers that give a remainder of 3 when divided by 5 are: 8, 13, 18, 23, 28, 33, 38, 43, 48, 53, 58, 63, ...

Numbers that give a remainder of 4 when divided by 6 are: 10, 16, 22, 28, 34, 40, 46, 52, 58, 64, ...

Looking at the patterns, the first number to appear in all three is 58.

8 B

Prime: $11 \to 11$, $13 \to 31$, $17 \to 71$, $31 \to 13$, $37 \to 73$, $71 \to 17$, $73 \to 37$, $79 \to 97$, $97 \to 79$

Not prime: $19 \to 91$

Any primes in the 20s, 40s, 60s and 80s will give even numbers when reversed, not prime.

Any primes in the 50s will give a number ending in 5 when reversed, not prime.

9 (a) B **(b)** C **(c)** D **(d)** D

Chapter review 2

1 (a) 18 **(b)** 36 **(c)** 30

2 (a) 1, 2, 3, 4, 6, 9, 12, 18, 36

(b) 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

(c) 1, 3, 17, 51

(d) 1, 2, 4, 5, 10, 20, 25, 50, 100

3 (a) 8 **(b)** 18 **(c)** 5

4

5301	2	(3)	4	5	6	(9)	10
10 000	(2)	3	(4)	(5)	6	9	(10)
333 333	2	(3)	4	5	6	(9)	10
31 700	(2)	3	(4)	(5)	6	9	(10)
43 521 820	(2)	3	(4)	(5)	6	9	(10)

5 (a) Prime, because it has exactly two factors: 1 and 5.

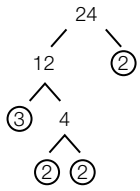
(b) Composite, because it has more than two factors: 1, 2, 4, 8, 16.

(c) Composite, because it has more than two factors: 1, 7, 11, 77.

(d) Composite, because it has more than two factors: e.g. 1, 10, 27 635, 276 350.

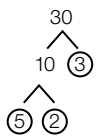
6 Students' factor trees may vary, but the factors at the end must be the same.

(a)



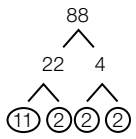
$24 = 2 \times 2 \times 2 \times 3$

(b)



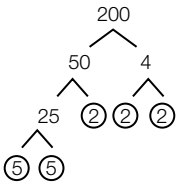
$30 = 2 \times 3 \times 5$

(c)



$88 = 2 \times 2 \times 2 \times 11$

(d)



$200 = 2 \times 2 \times 2 \times 5 \times 5$

7 (a) $27 = 3 \times 3 \times 3$
$36 = 2 \times 2 \times 3 \times 3$
HCF $= 3 \times 3$
$= 9$

(b) $72 = 2 \times 2 \times 2 \times 3 \times 3$
$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$
HCF $= 2 \times 2 \times 2 \times 3$
$= 24$

(c) $108 = 2 \times 2 \times 3 \times 3 \times 3$
$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$
HCF $= 2 \times 2 \times 3$
$= 12$

8 (a) -14 (b) +200

9 (a) south 5 km (b) subtracting 27

10 (a) $-52 < 25$ (b) $19 > -20$

11 (a) -9, -7, 0, 7, 12 (b) -4000, 4, 40, 400

12 (a) 18 (b) 15 (c) -10 (d) 6
(e) -4 (f) 8 (g) -5 (h) -32

13 (a) -3 (b) 3 (c) -7 (d) 0
(e) -7 (f) 2 (g) -3 (h) -12

14 (a) -2 (b) 7 (c) 11 (d) -4
(e) 37 (f) 14 (g) -11 (h) -3

15 (a) divisible, multiple
(b) factor, multiple
(c) divisible, multiple

16 96 (1 packet), 48 (2 packets), 32 (3 packets), 24 (4 packets), 16 (6 packets), 12 (8 packets), 8 (12 packets), 6 (16 packets), 4 (24 packets), 3 (32 packets), 2 (48 packets), 1 (96 packets)

17 (a) -12 (b) -81 (c) -60 (d) -70

18 (a) Start at +3 and walk 8 steps to the left, finishing at -5.
(b) Start at -6 and walk 5 steps to the left, finishing at -11.

19 (a) co-prime (b) not co-prime, HCF = 3
(c) not co-prime, HCF = 7 (d) co-prime

20 -2 runs

21 (a) HCF = 12, so 12 tables are required
(b) 9 senior club members, 7 junior club members

22 504

23 (a) 71 (b) 72

24 (a) decreased (b) $61

25 150 seconds

26 -900 + (-250) = -1150 m

27 (a) = (b) < (c) = (d) <

28 2 and 4 **29** 910 and 13

Numeracy practice 2

1 C **2** B **3** 8th floor (-3 + 11 = 8)

4 B **5** A **6** D

7 $265 **8** B **9** -5

Mixed review A

1 -48, -25, -10, 0, 8, 17, 32

2 (a) 9^2 (b) 7^4 (c) 4^3

3 24, 48, 72, 96

4 (a) -570 (b) +5

5 (a) $-27 < 14$ (b) $0 > -35$

6 (a) 40 (b) 300 (c) 500
(d) 3600 (e) 600 000 (f) 10 000 000

7 (a) 340 (b) 403 (c) 798
(d) 396 (e) 240 (f) 960

8 (a) 72 (b) 47 (c) 29 (d) 47 (e) 0 (f) 80

9 (a) -2 (b) -3 (c) 4 (d) -17

10 (a) 1800 (b) 15 000 (c) 60 000 (d) 100
(e) 40 (f) 2

11 (a) -15, -5, 0, 5, 10 (b) -300, -30, 0, 3, 3000

12 (a) 24 (b) 48

13 (a) 9 (b) 8

14 (a) 1, 2, 3, 6, 9, 18 composite
(b) 1, 23 prime
(c) 1, 2, 4, 11, 22, 44 composite
(d) 1, 79 prime

15 (a) $63 = 3^2 \times 7$ (b) $48 = 2^4 \times 3$
(c) $72 = 2^3 \times 3^2$ (d) $120 = 2^3 \times 3 \times 5$

16 (a) $-6 - 7 = -13$ (b) $4 + 11 = 15$ (c) $-2 + 7 = 5$
(d) $22 - 9 = 13$ (e) $-5 + 10 = 5$ (f) $18 + 3 = 21$

17 -180 m

18 (a) 3, 7, 11, 19, 43, 13, 2 (b) 9, 16, 25
(c) 16, 32, 8, 2

19 216

20 (a) 12 000, actual answer will be lower
(b) 20 000, actual answer will be higher
(c) 100, actual answer will be higher

21 (a) 9000 (b) $108 000

22 (a) more (b) $5

23 B

Chapter 3

Recall 3

1 A, B

2 (a) $\frac{1}{5}$ (b) $\frac{3}{8}$ (c) $2\frac{2}{5}$ or $\frac{12}{5}$ (d) $\frac{1}{10}$
(e) $\frac{5}{8}$ (f) $\frac{2}{3}$ (g) $\frac{12}{5}$ (h) $\frac{7}{16}$

3 (a) $2, 1\frac{1}{4}, 1, \frac{3}{4}, 0$ (b) $0, \frac{1}{8}, \frac{3}{8}, \frac{7}{8}, 1, \frac{9}{8}, \frac{11}{8}$

4 (a) $\frac{5}{7}$ (b) $\frac{6}{11}$ (c) $\frac{4}{3}$ or $1\frac{1}{3}$

5 (a) 8, 16, 24, 32, 40 (b) 12, 24, 36, 48, 60

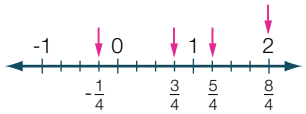
(c) 24 (d) 1, 2, 3, 4, 6, 8, 12, 24

(e) 1, 2, 3, 4, 6, 9, 12, 18, 36 (f) 12

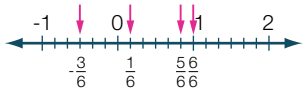
Exercise 3.1

1 (a) 3 (b) 6

2 (a)



(b)



3 (a) (i) $\frac{10}{2}$ (ii) $\frac{35}{7}$ (iii) $\frac{55}{11}$ (iv) $\frac{25}{5}$ (v) $\frac{5}{1}$

(b) (i) $\frac{26}{2}$ (ii) $\frac{65}{5}$ (iii) $\frac{104}{8}$ (iv) $\frac{169}{13}$ (v) $\frac{13}{1}$

4 (a) (i) $\frac{1}{2}$ (ii) $\frac{2}{3}$ (iii) $\frac{6}{5}$ or $1\frac{1}{5}$ (iv) $\frac{10}{7}$ or $1\frac{3}{7}$

(b) (iii) and (iv)

5 (a) 4 (b) 12 (c) 2 (d) 10

6 (a) $\frac{5}{8}$ (b) $\frac{4}{9}$

7 (a) $\frac{1}{2}$ (b) $-\frac{2}{9}$ (c) $\frac{11}{4}$ or $2\frac{3}{4}$

(d) $-\frac{2}{5}$ (e) $\frac{3}{14}$ (f) $\frac{25}{8}$ or $3\frac{1}{8}$

(g) $\frac{7}{6}$ or $1\frac{1}{6}$ (h) $\frac{5}{3}$ or $1\frac{2}{3}$ (i) $-\frac{2}{7}$

8 (a) $\frac{8}{17}$ (b) $\frac{15}{4}$ (c) $\frac{9}{20}$ (d) $2\frac{2}{3}$ (e) $3\frac{7}{24}$

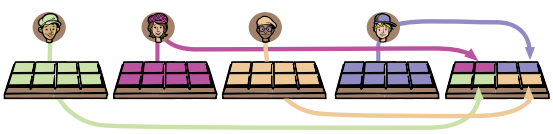
9 (a) $\frac{421}{500}$ (b) $\frac{157}{375}$ (c) $\frac{1}{60}$ (d) $\frac{17}{60}$

(e) $\frac{2}{7}$ (f) $\frac{1}{24}$

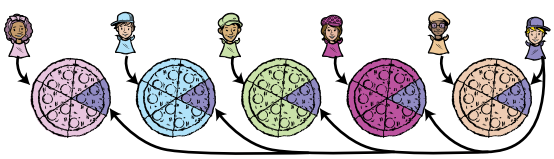
10 (a) $\frac{7}{14}$ or $\frac{1}{2}$ (b) $\frac{11}{14}$ (c) $\frac{10}{14}$ or $\frac{5}{7}$

11 Students' own diagrams may vary.

(a)



(b)



12 (a) $\frac{149}{185}$ (b) $\frac{185}{149}$ or $1\frac{36}{149}$

13 Adults: $\frac{4}{5}$ of a pizza each. Teenagers: $\frac{5}{4}$ of a pizza each $(1\frac{1}{4})$. Teenagers each receive more pizza.

14 (a) $\frac{1}{4}$

(b) not possible, not divided into equal parts and unable to create equal parts by further dividing the shape

(c) $\frac{1}{2}$ (d) $\frac{3}{8}$

(e) not possible, same reason as (b) (f) $\frac{1}{2}$

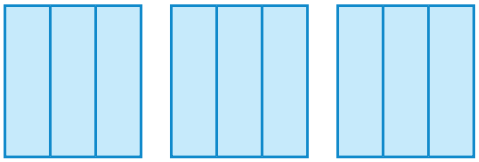
15 D **16** C

Open-ended—Sample answers

17 (a) $\frac{3}{8}$ of the shape is shaded

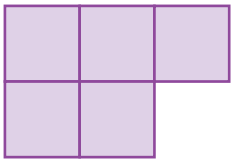
(b) $\frac{5}{6}$ of the collection of stars is shaded.

(c) $\frac{9}{3}$

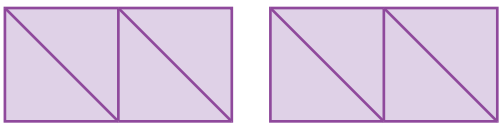


18 (a)

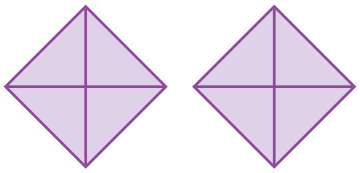
or



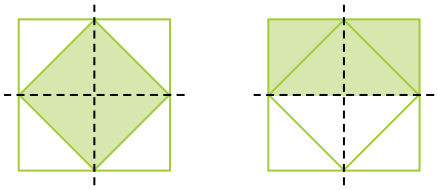
(b)



or



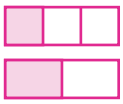
19



Drawing in a vertical and a horizontal line divides the square into quarters and shows that half of each quarter is shaded.

20 (a) Oliver's shaded areas (representing the fractions $\frac{1}{2}$ and $\frac{1}{3}$) look the same because he is not comparing fractions of the same-sized whole.

(b)



$\frac{1}{3}$

$\frac{1}{2}$

Exercise 3.2

1 (a) 15 **(b)** 21 **(c)** 26 **(d)** 40

(e) 2 **(f)** 3 **(g)** 10 **(h)** 63

(i) 36 **(j)** 3 **(k)** 6 **(l)** 18

(m) 5 **(n)** 6 **(o)** 36 **(p)** 16

2 (a) $\frac{1}{2}$ **(b)** $\frac{1}{3}$ **(c)** $\frac{1}{4}$

(d) $\frac{1}{3}$ **(e)** $\frac{2}{5}$ **(f)** $\frac{4}{11}$

(g) $\frac{5}{8}$ **(h)** $1\frac{1}{2}$ or $\frac{3}{2}$ **(i)** $2\frac{2}{3}$ or $\frac{8}{3}$

(j) $3\frac{1}{6}$ or $\frac{19}{6}$ **(k)** $1\frac{3}{10}$ or $\frac{13}{10}$ **(l)** $12\frac{4}{5}$ or $\frac{64}{5}$

(m) $6\frac{4}{9}$ or $\frac{58}{9}$ **(n)** $3\frac{1}{4}$ or $\frac{13}{4}$ **(o)** $2\frac{5}{6}$ or $\frac{17}{6}$

(p) $100\frac{1}{7}$ or $\frac{701}{7}$

3 (a) $1\frac{2}{5}$ **(b)** $2\frac{1}{6}$ **(c)** $5\frac{3}{4}$ **(d)** $2\frac{1}{7}$ **(e)** $3\frac{7}{10}$

(f) $9\frac{3}{5}$ **(g)** $6\frac{2}{7}$ **(h)** $5\frac{9}{10}$ **(i)** $1\frac{7}{100}$ **(j)** $2\frac{17}{40}$

(k) $11\frac{2}{3}$ **(l)** $9\frac{7}{8}$

4 (a) $\frac{7}{4}$ **(b)** $\frac{16}{5}$ **(c)** $\frac{20}{3}$ **(d)** $\frac{59}{10}$ **(e)** $\frac{35}{8}$

(f) $\frac{72}{7}$ **(g)** $\frac{74}{11}$ **(h)** $\frac{309}{100}$ **(i)** $\frac{62}{9}$ **(j)** $\frac{18}{7}$

(k) $\frac{43}{6}$ **(l)** $\frac{139}{15}$

5 (a) $\frac{5}{8}$ **(b)** 40 squares, $\frac{5}{8} = \frac{40}{64}$

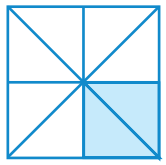
6 (a) C **(b)** D **(c)** C **(d)** B

7 (a) $\frac{6}{10}$ **(b)** $\frac{1}{4}$ and $\frac{3}{12}$

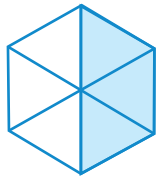
(c) $\frac{10}{12}$ **(d)** $\frac{3}{4}$ and $\frac{6}{8}$

8 (a) $\frac{1}{4}$, $\frac{1}{2}$

(b)



$\frac{1}{4} = \frac{2}{8}$ and



$\frac{1}{2} = \frac{3}{6}$

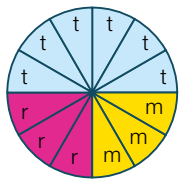
9 (a) They occupy the same position on the number line.

(b)

$-\frac{3}{3}$ and $-\frac{9}{9}$, $-\frac{2}{3}$ and $-\frac{6}{9}$, $-\frac{1}{3}$ and $-\frac{3}{9}$, $\frac{1}{3}$ and $\frac{3}{9}$, $\frac{2}{3}$ and $\frac{6}{9}$,

$\frac{3}{3}$ and $\frac{9}{9}$, $\frac{4}{3}$ and $\frac{12}{9}$, $\frac{5}{3}$ and $\frac{15}{9}$, $\frac{6}{3}$ and $\frac{18}{9}$

10 Monica 3, Michelle 3, Travis 6



11 14 squares

12 (a) $\frac{1}{30}$ **(b)** $\frac{1}{3}$ **(c)** $\frac{1}{5}$ **(d)** $\frac{3}{20}$

13 $\frac{1}{2} = 30$ minutes, $\frac{1}{3} = 20$ minutes, $\frac{1}{4} = 15$ minutes,

$\frac{1}{5} = 12$ minutes, $\frac{1}{6} = 10$ minutes, $\frac{1}{7} = 8\frac{4}{7}$ minutes,

$\frac{1}{8} = 7\frac{1}{2}$ minutes, $\frac{1}{9} = 6\frac{2}{3}$ minutes, $\frac{1}{10} = 6$ minutes,

$\frac{1}{11} = 5\frac{5}{11}$ minutes, $\frac{1}{12} = 5$ minutes

14 (a) 35 pieces **(b)** $3\frac{5}{8}$ pizzas

15 4 whole numbers: 2, 3, 4, 5

16 C

17 $\frac{1}{7}$ and $\frac{1}{11}$ have no equivalent fractions on the wall. This is because their denominators are prime numbers with no multiples in the other denominators on the fraction wall.

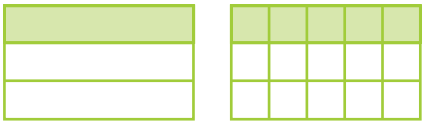
18 (a) $\frac{1}{2}$ **(b)** $\frac{1}{2}$ **(c)** $\frac{3}{8}$ **(d)** $\frac{13}{20}$ **(e)** 0 **(f)** $\frac{9}{22}$

Open-ended—Sample answers

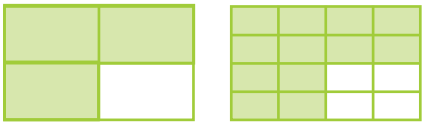
19 $\frac{8}{10}$, $\frac{12}{15}$, $\frac{16}{20}$ etc.

20 $\frac{5}{3}, \frac{15}{9}, \frac{10}{6}$ etc.

21 (a)



(b)



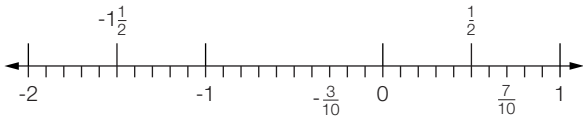
22 $\frac{3}{7}, \frac{7}{9}, \frac{5}{11}$

Exercise 3.3

1 (a)

$-\frac{3}{4}, \frac{2}{3}, 1\frac{1}{3}, \frac{6}{4}$

(b)



$-1\frac{1}{2}, -\frac{3}{10}, \frac{1}{2}, \frac{7}{10}$

2 (a) 1 (b) -1 (c) 1 (d) 7
(e) -2 (f) 0 (g) 1

3 (a) $\frac{3}{5}$ (b) $\frac{5}{4}$ (c) $\frac{7}{8}$ (d) $\frac{7}{12}$
(e) $\frac{5}{7}$ (f) $\frac{2}{9}$ (g) $\frac{5}{12}$ (h) $\frac{11}{6}$
(i) $1\frac{3}{4}$ (j) $\frac{5}{2}$ (k) $\frac{16}{7}$ (l) $2\frac{4}{9}$

4 (a) $\frac{3}{7}$ (b) $\frac{7}{8}$ (c) $\frac{5}{6}$ (d) $\frac{2}{5}$
(e) equivalent fractions (f) $\frac{5}{11}$ (g) $\frac{7}{8}$
(h) $\frac{7}{11}$

5 (a) $\frac{2}{7} < \frac{5}{7}$ (b) $\frac{4}{5} > \frac{4}{7}$ (c) $\frac{1}{2} < \frac{5}{8}$ (d) $\frac{8}{11} < \frac{9}{10}$
(e) $\frac{2}{4} = \frac{3}{6}$ (f) $\frac{6}{12} < \frac{7}{10}$ (g) $\frac{3}{7} < \frac{5}{8}$ (h) $\frac{5}{9} > \frac{2}{5}$
(i) $\frac{5}{6} < \frac{7}{8}$ (j) $\frac{7}{11} < \frac{8}{10}$ (k) $\frac{6}{10} = \frac{3}{5}$ (l) $\frac{4}{5} > \frac{6}{9}$
(m) $\frac{2}{4} < \frac{4}{2}$ (n) $\frac{24}{6} = \frac{8}{2}$ (o) $\frac{18}{11} < \frac{9}{5}$ (p) $\frac{9}{7} > \frac{11}{9}$

6 $\frac{7}{8} > \frac{10}{12}$
Ali is the more accurate shooter.

7 $\frac{5}{16} < \frac{23}{64}$
The larger box has the greater fraction of nut chocolates.

8 (a) $\frac{10}{11}$ (b) $\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{7}{11}$ and $\frac{8}{12}$
(c) $\frac{2}{5}, \frac{3}{7}, \frac{4}{10}$ and $\frac{5}{12}$ (d) $\frac{1}{5}, \frac{2}{10}, \frac{2}{9}, \frac{1}{4}, \frac{2}{8}$ and $\frac{3}{12}$

9 (a) B (b) C

10 (a) C (b) C

11 (a) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{10}{11}$
All of the fractions have a numerator that is 1 less than the denominator—they are '1 piece' less than 1 whole. Higher denominator numbers mean that the whole is being cut into more, smaller-sized pieces. This means that the 'missing pieces' that make the fraction less than 1 are getting smaller.

(b) $0, \frac{4}{7}, \frac{6}{7}, \frac{7}{8}, 1, \frac{7}{4}$
$\frac{7}{4}$ is the only improper fraction and so is greater than 1. All of the other fractions are less than 1, but greater than 0. $\frac{4}{7}$ and $\frac{6}{7}$ have the same denominator, so it is clear that $\frac{6}{7}$ is larger. $\frac{7}{8}$ is greater than $\frac{6}{7}$ for the same reasons described in **(a)**. (Alternatively, they can be converted to fractions with an LCD of 56: $\frac{49}{56} > \frac{48}{56}$.)

12 (a) $1, \frac{45}{63}, \frac{2}{3}, \frac{4}{7}, \frac{11}{21}, \frac{2}{9}$
All of the fractions are proper fractions, so they are all less than 1. They also all have an LCD of 63. Converting them to equivalent fractions with this LCD gives:
$1 > \frac{45}{63} > \frac{42}{63} > \frac{36}{63} > \frac{33}{63} > \frac{14}{63}$.

(b) $1\frac{7}{20}, \frac{21}{16}, 1\frac{3}{10}, \frac{5}{4}, 1\frac{3}{80}, \frac{39}{40}$
Converting all to equivalent fractions with an LCD of 80 gives:
$\frac{108}{80} > \frac{105}{80} > \frac{104}{80} > \frac{100}{80} > \frac{83}{80} > \frac{78}{80}$.

13 (a) approximately $\frac{3}{4}$
(b) Queensland takes up approximately $\frac{1}{5}$ of the area of Australia.
(c) approximately $\frac{2}{3}$

14 (a) same (b) same (c) B
(d) A (e) B (f) same

15 (a) Angus has drunk more water.
(b) Gianni has drunk a greater fraction, $\frac{5}{6}$ to $\frac{4}{5}$.

Open-ended—Sample answers

16 (a) $\frac{4}{5}, \frac{5}{6}$ (b) $\frac{5}{2}, \frac{11}{4}$

17 $\frac{7}{10}, \frac{3}{4}, \frac{5}{7}$

18 Jason is comparing fractions of different wholes (different numbers of marks). To do this, he needs to rewrite them out of the same total by finding equivalent fractions with the LCD. $\frac{16}{20} = \frac{80}{100}$, $\frac{43}{50} = \frac{86}{100}$, so $\frac{43}{50}$ is his better result.

Exercise 3.4

1 (a) $\frac{3}{5}$ (b) $\frac{9}{7}$ (c) $\frac{4}{5}$ (d) 8

2 (a) $\frac{7}{6}$ or $1\frac{1}{6}$ (b) $\frac{1}{20}$ (c) $\frac{9}{20}$

(d) $\frac{14}{55}$ (e) $\frac{1}{2}$ (f) $\frac{50}{49}$ or $1\frac{1}{49}$

(g) $\frac{5}{4}$ or $1\frac{1}{4}$ (h) $\frac{19}{20}$ (i) $\frac{5}{12}$

(j) $\frac{47}{36}$ or $1\frac{11}{36}$ (k) $\frac{25}{42}$ (l) $\frac{13}{12}$ or $1\frac{1}{12}$

(m) $\frac{5}{12}$ (n) $\frac{73}{60}$ or $1\frac{13}{60}$ (o) $\frac{87}{70}$ or $1\frac{17}{70}$

(p) $\frac{3}{10}$

3 (a) $1\frac{1}{6}$ (b) $\frac{25}{28}$

4 (a) $7\frac{3}{7}$ (b) $13\frac{2}{3}$ (c) $9\frac{5}{8}$ (d) $7\frac{9}{10}$

(e) $3\frac{1}{4}$ (f) $6\frac{1}{10}$ (g) $7\frac{1}{4}$ (h) $9\frac{1}{4}$

(i) $12\frac{3}{10}$ (j) $11\frac{1}{18}$ (k) $4\frac{42}{55}$ (l) $3\frac{31}{100}$

5 (a) $1\frac{4}{7}$ (b) $\frac{2}{3}$ (c) $4\frac{4}{9}$ (d) $1\frac{2}{3}$

(e) $1\frac{1}{4}$ (f) $2\frac{1}{2}$ (g) $1\frac{3}{10}$ (h) $1\frac{8}{9}$

(i) $\frac{17}{18}$ (j) $\frac{17}{20}$ (k) $\frac{23}{24}$ (l) $2\frac{11}{24}$

6 (a) $\frac{7}{8}$ (b) $\frac{11}{12}$ (c) 1

(d) $\frac{21}{5}$ or $4\frac{1}{5}$ (e) $\frac{401}{60}$ or $6\frac{41}{60}$ (f) $\frac{127}{60}$ or $2\frac{7}{60}$

7 (a) B (b) D

8 (a) $\frac{1}{2}$ (b) $\frac{3}{4}$

9 $\frac{3}{4}$ of the pizza **10** $\frac{1}{4}$ **11** $10\frac{1}{12}$ hours

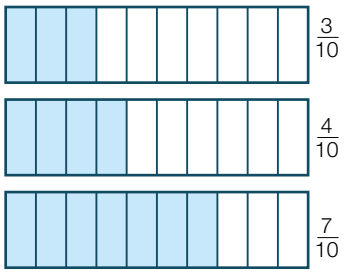
12 $\frac{3}{4}$ of a packet **13** B **14** D

Open-ended—Sample answers

15 The correct answer is 2, as both fractions are slightly less than 1. Students answering 19 or 21 have just added the numerators or denominators.

16 $\frac{11}{40}$ and $\frac{19}{40}$

17 You add the numerators because you are 'counting' how many equal-sized parts you have. Adding the denominators would change the part size.



18 Teachers use fraction notation just as a way of keeping track of marks for different sections of the test. The 'whole' is the total number of marks, and the number of correct answers is written as a fraction of that whole.

Half-time 3

1 (a) $\frac{7}{15}$ (b) 6

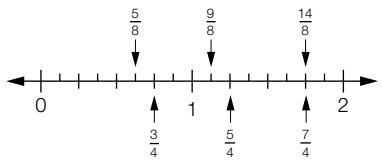
2 (a) $\frac{3}{8}$ (b) $\frac{1}{4}$

(c) $3\frac{3}{4}$ or $\frac{15}{4}$ (d) $7\frac{7}{10}$ or $\frac{77}{10}$

3 (a) $6\frac{3}{4}$ (b) 5 (c) 5 (d) $5\frac{2}{7}$

4 (a) $\frac{5}{8} < \frac{2}{3}$ (b) $\frac{4}{9} < \frac{15}{27}$ (c) $\frac{48}{56} = \frac{6}{7}$ (d) $\frac{11}{5} < \frac{9}{4}$

5 (a)

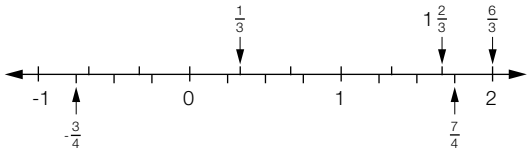


(b) $\frac{1}{4}$ and $\frac{2}{8}$; $\frac{2}{4}$ and $\frac{4}{8}$; $\frac{3}{4}$ and $\frac{6}{8}$; $\frac{5}{4}$ and $\frac{10}{8}$; $\frac{6}{4}$ and $\frac{12}{8}$

(c) (i) $\frac{5}{4}$ (ii) $\frac{7}{4}$

6 (a) $\frac{9}{8}$ or $1\frac{1}{8}$ (b) $\frac{1}{6}$ (c) $\frac{29}{18}$ or $1\frac{11}{18}$ (d) $\frac{7}{36}$

7



In ascending order: $-\frac{3}{4}, \frac{1}{3}, 1\frac{2}{3}, \frac{7}{4}, \frac{6}{3}$

8 (a) 29 slices (b) 21 slices

9 (a) $\frac{281}{60}$ or $4\frac{41}{60}$ (b) $\frac{429}{100}$ or $4\frac{29}{100}$

(c) $\frac{81}{50}$ or $1\frac{31}{50}$ (d) $\frac{79}{50}$ or $1\frac{29}{50}$

10 (a) $\frac{5}{2}$ of $2\frac{1}{2}$ L (b) $1\frac{3}{4}$ L

Exercise 3.5

1 (a) 6 (b) 3 (c) 3 (d) 4

(e) 21 (f) 24 (g) 48 (h) 27

(i) 30 (j) 120 (k) 55 (l) 75

2 (a) $\frac{1}{13}$ (b) $\frac{1}{9}$ (c) $\frac{4}{9}$ (d) $\frac{3}{13}$

(e) $\frac{1}{14}$ (f) $\frac{6}{11}$ (g) $\frac{5}{14}$ (h) $\frac{5}{66}$

(i) $\frac{9}{28}$ (j) $\frac{3}{4}$ (k) $\frac{8}{21}$ (l) $\frac{27}{250}$

3 (a) $\frac{2}{3}$ (b) 6 (c) $1\frac{1}{4}$ (d) $1\frac{1}{4}$

(e) $11\frac{1}{2}$ (f) 14 (g) 27 (h) $40\frac{2}{3}$

(i) 2 (j) $11\frac{1}{5}$ (k) 18 (l) 4

4 (a) $\frac{3}{20}$ (b) $\frac{2}{45}$ (c) $\frac{5}{14}$

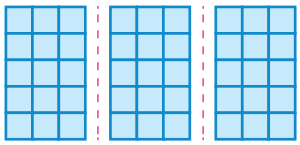
(d) $\frac{10}{13}$ (e) $\frac{7}{6}$ or $1\frac{1}{6}$ (f) $\frac{4}{3}$ or $1\frac{1}{3}$

5 (a) C (b) A

6 (a) $\frac{1}{7}$ (b) \$310

7 Michael \$10 000, Marcus \$4000, Lily \$18 000, Julie \$16 000

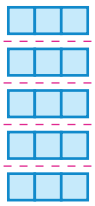
8 (a) (i)



(ii) $\frac{1}{3}$ (iii) 15 (iv) $\frac{1}{3} \times \frac{45}{1} = 15$

(b) (i) $\frac{1}{5}$ (ii) 3 (iii) $\frac{1}{15}$

(iv)



(v) $\frac{1}{5} \times \frac{1}{3} = \frac{1}{15}$

9 $\frac{1}{8}$ each

10 (a) $\frac{3}{20}$ (b) $\frac{1}{5} \times \frac{3}{4}$ or $\frac{3}{4} \times \frac{1}{5}$

11 (a) 13 L (b) 39 L

12 (a) 50 students (b) 40 students

13 (a) $2\frac{1}{2}$

(b) 450 g butter, $1\frac{1}{4}$ cups icing sugar, $3\frac{1}{8}$ cups self-raising flour, $1\frac{2}{3}$ cup custard powder

14 $1\frac{1}{2} \times 12$

15 D

16 (a) (i) 4×3

(ii) Multiplying a number by a fraction less than 1 gives a smaller number.

(b) (i) $2\frac{1}{2} \times 7$

(ii) Multiplying a number by a fraction less than 1 gives a smaller number.

(c) (i) $1\frac{1}{2} \times 12$

(ii) Both 12 and 9 are being multiplied by the same mixed number, so multiplying by 12 will give a larger number because 12 is greater than 9.

17 D

Open-ended—Sample answers

18 $\frac{1}{2} \times \frac{1}{3}$

19 The six possible answers are $\frac{1}{2} \times 30 = 15$, $\frac{1}{2} \times 32 = 16$, $\frac{1}{3} \times 30 = 10$, $\frac{1}{3} \times 33 = 11$, $\frac{1}{3} \times 36 = 12$, $\frac{1}{3} \times 39 = 13$.

20 Asher has cancelled out common factors between the two numerators. Common factors can only be cancelled between numerators and denominators. Correct working:

$$\frac{3}{1\cancel{5}} \times \frac{^{12}\cancel{60}}{1} = \frac{36}{1} = 36$$

Exercise 3.6

1 (a) 24 (b) 12 (c) 21

(d) 36 (e) 18 (f) 5

(g) $\frac{35}{3}$ or $11\frac{2}{3}$ (h) $\frac{63}{4}$ or $15\frac{3}{4}$ (i) $\frac{16}{3}$ or $5\frac{1}{3}$

(j) 3 (k) $\frac{15}{8}$ or $1\frac{7}{8}$ (l) $\frac{4}{3}$ or $1\frac{1}{3}$

2 (a) 2 (b) $\frac{3}{2}$ or $1\frac{1}{2}$ (c) $\frac{9}{8}$ or $1\frac{1}{8}$

(d) $\frac{7}{5}$ or $1\frac{2}{5}$ (e) $\frac{1}{6}$ (f) $\frac{1}{20}$

(g) $\frac{1}{4}$ (h) $\frac{3}{20}$ (i) $\frac{9}{16}$

(j) $\frac{14}{25}$ (k) $\frac{36}{85}$ (l) $\frac{27}{80}$

(m) 2 (n) $\frac{44}{43}$ or $1\frac{1}{43}$ (o) $\frac{15}{4}$ or $3\frac{3}{4}$

(p) $1\frac{5}{7}$

3 (a) C (b) D

4 (a) 2, 2 (b) 6, 6 (c) 4, 4 (d) 8, 8

5 D

6 (a) 9 (b) $4\frac{1}{2}$

7 $2\frac{2}{3}$ days

8 6 students

9 12 whole shifts

10 30 batches of scones

11 5 drinks stations

12 5 complete screenings

13 C

14 $\frac{4}{5} \div \frac{1}{3}$

15 (a) As the denominator number gets larger, the part size gets smaller, so you can fit more of the fraction into the number being divided.

(b) $13\frac{1}{3}$

Open-ended—Sample answers

16 $4 \div \frac{2}{5} = 10$, $4 \div \frac{2}{6} = 12$, $4 \div \frac{2}{7} = 14$, $4 \div \frac{2}{8} = 16$, $4 \div \frac{2}{9} = 18$

17 Gabi has not multiplied by the inverse. 12 is a reasonable answer because she is dividing by a number less than 1.

Exercise 3.7

1 (a) $\frac{9}{4}$ or $2\frac{1}{4}$ (b) $\frac{39}{8}$ or $4\frac{7}{8}$ (c) $\frac{5}{99}$

(d) $\frac{3}{16}$ (e) $\frac{5}{16}$ (f) $\frac{11}{18}$

(g) $3\frac{3}{8}$ or $\frac{27}{8}$ (h) $4\frac{3}{4}$ or $\frac{19}{4}$ (i) $2\frac{13}{20}$ or $\frac{53}{20}$

(j) 1 (k) 4 (l) $\frac{2}{3}$

2 (a) A (b) A

3 (a) $\frac{41}{12}$ or $3\frac{5}{12}$ (b) $\frac{53}{30}$ or $1\frac{23}{30}$ (c) $\frac{17}{5}$ or $3\frac{2}{5}$ (d) 30

4 (a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{3}{8}$ (d) $\frac{5}{8}$

(e) 90 (f) 36

5 (a) $\frac{1}{75}$ (b) 75 (c) $\frac{2}{25}$ (d) 150 m (e) 1125 m

6 $\frac{97}{12}$ or $8\frac{1}{12}$ laps

7 (a) $\frac{11}{8}$ or $1\frac{3}{8}$ km (b) $\frac{95}{24}$ or $3\frac{23}{24}$ km (c) $\frac{32}{3}$ or $10\frac{2}{3}$ km

8 Bryce

9 cutlery set

10 $\frac{63}{64}$

11 (a) $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{8}$ (b) $13 \times 8 = 104$

Open-ended—Sample answers

12 $\frac{5}{8}, \frac{3}{4}, \frac{2}{3}$

13 (a) $\frac{2+1}{2-1} = \frac{3}{1} = 3$, $\frac{3+2}{3-2} = \frac{5}{1} = 5$, $\frac{4+1}{4-1} = \frac{5}{3}$ or $1\frac{2}{3}$ etc.

(b) 17 (when $A = 9$ and $B = 8$)

14 (a) $\frac{7}{3} + \frac{7}{4} = \frac{7}{3} \times \frac{7}{4} = \frac{49}{12}$ or $4\frac{1}{12}$

(b) $\frac{5}{3} + \frac{5}{2} = \frac{5}{3} \times \frac{5}{2} = \frac{25}{6}$ or $4\frac{1}{6}$

(c) The numerators are the sum of the denominators.

(d) Anything that obeys the statement in (c);
e.g. $\frac{9}{4}$ and $\frac{9}{5}$, $\frac{11}{5}$ and $\frac{11}{6}$

Challenge 3

1 C
Find number first, then find required fraction.

2 720 L
Fraction given equals volume given. Dividing by the fraction gives total volume.

3 A
Write fractions with lowest common denominator.

4 C
List all the possible fractions, remembering that they must already be in simplest form. Subtract the smallest fraction from the second list from the largest fraction from the first list and simplify.

5 B
Write fractions with lowest common denominator, then simplify.

6 1:20 pm
Divide each time into 60 minutes; they both croak on the hour every hour. By listing or otherwise, find times after the hour that the frogs croak. This will give the first time they both croak together.

7 C
Rewrite the fraction after doing the addition in the denominator. Then do the division, then add.

8 $\frac{33}{109}$
Simplify each fraction step by step, starting with the $3 + \frac{1}{3}$ fraction. Divide, then repeat the process of adding and dividing until a single fraction remains.

9 36
Subtract $\frac{1}{3}$ of the number from 2 times the number.
This gives the fraction of the number equal to the amount.
Divide this fraction into the amount to find the whole.

10 $\frac{13}{96}, \frac{7}{48}, \frac{5}{32}$
Find the middle number by writing fractions with lowest common denominator. Then find the middle number between this number and the given numbers by finding equivalent fractions.

11 $100

Cancelling common factors:

$$\frac{\cancel{1}}{\cancel{2}_1}\times\frac{\cancel{2}^1}{\cancel{3}_1}\times\frac{\cancel{3}^1}{\cancel{4}_1}\times\frac{\cancel{4}^1}{\cancel{5}_1}\times\frac{\cancel{5}^1}{\cancel{6}_1}\times\frac{\cancel{6}^1}{\cancel{7}_1}\times\frac{\cancel{7}^1}{\cancel{8}_1}\times\frac{\cancel{8}^1}{\cancel{9}_1}\times\frac{\cancel{9}^1}{10}\times\$1000$$

$$=\frac{1}{10}\times\$1000$$

$$=\$100$$

Chapter review 3

1 (a) 16 (b) $\frac{5}{12}$

2 $-\frac{1}{4}, \frac{1}{4}$

3 (a) (i) $\frac{18}{3}$ (ii) $\frac{60}{10}$ (iii) $\frac{6}{1}$

(b) (i) $\frac{33}{3}$ (ii) $\frac{110}{10}$ (iii) $\frac{11}{1}$

4 (a) $\frac{3}{5}$ (b) $1\frac{3}{4}$

5 (a) 42 (b) 4 (c) 90 (d) 30

6 (a) $\frac{4}{5}$ (b) $\frac{12}{5}$ or $2\frac{2}{5}$

(c) $3\frac{5}{6}$ or $\frac{23}{6}$ (d) $1\frac{1}{5}$ or $\frac{6}{5}$

7 (a) $\frac{23}{7}$ (b) $5\frac{5}{9}$

8 (a) < (b) = (c) > (d) <

9 (a) $\frac{31}{24}$ or $1\frac{7}{24}$ (b) $\frac{1}{6}$

(c) $\frac{8}{21}$ (d) $\frac{13}{15}$

10 (a) $1\frac{13}{20}$ (b) $8\frac{1}{6}$

(c) $\frac{37}{7}$ or $5\frac{2}{7}$ (d) $\frac{1}{6}$

11 (a) $12 (b) $\frac{1}{6}$

12 (a) $\frac{5}{3}$ or $1\frac{2}{3}$ (b) $\frac{110}{9}$ or $12\frac{2}{9}$ (c) $\frac{3}{7}$

13 (a) 12 (b) $6\frac{9}{23}$ (c) $\frac{9}{16}$ (d) $\frac{39}{40}$

14 (a) $5\frac{17}{18}$ (b) $4\frac{9}{10}$ (c) $3\frac{9}{32}$

15 (a) $\frac{1}{2}+\frac{2}{5}$ (b) $\frac{9}{10}$

16 (a) $\frac{1}{3}$ (b) $2\frac{1}{6}$ or $\frac{13}{6}$ (c) $1\frac{1}{3}$ or $\frac{4}{3}$ (d) $\frac{3}{8}$

17 (a) $\frac{5}{9}$ (b) $\frac{2}{9}$ (c) $\frac{19}{27}$

18 (a) $\frac{1}{3}, \frac{1}{2}, \frac{3}{5}, \frac{3}{4}, 1$ (b) $\frac{30}{50}, \frac{5}{4}, 1\frac{2}{5}, 2, \frac{13}{5}$

19 Jamal

20 (a) $\frac{4}{15}$

(b) $\frac{16}{15}$ or $1\frac{1}{15}$ = more than a whole block of chocolate

21 $\frac{5}{6}$

22 (a) 30 000 L (b) 50 000 L (c) 12 000 L

23 45 g butter, $\frac{1}{6}$ cup caster sugar, 1 egg, $\frac{7}{16}$ cup self-raising flour, $\frac{1}{2}$ tablespoon cocoa, $\frac{1}{8}$ cup orange juice

24 (a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{3}{8}$

25 (a) Katya (b) Mabok

26 A

Numeracy practice 3

1 D **2** B **3** A **4** B

5 B **6** C **7** D **8** D

Chapter 4

Recall 4

1 (a) 0.1 > 0.01 (b) 2 < 2.2 (c) 0.3 > 0.1

(d) 3.2 < 4.1 (e) 0.008 < 0.09 (f) 0.7 > 0.07

2 (a) seven tens and two ones

(b) six hundreds and three ones

(c) nine thousands, two hundreds, five tens and one one

(d) one ten-thousand, one thousand and eight tens

3 (a) (i) seven tenths (ii) 0.7

(b) (i) eight thousandths (ii) 0.008

(c) (i) three hundredths (ii) 0.03

(d) (i) one and nine tenths (ii) 1.9

4 (a) 110 (b) 1535 (c) 479 (d) 6499

5 (a) 49 (b) 447 (c) 7905 (d) 1648

6 (a) 34 (b) 2400 (c) 40 800

7 (a) 181 (b) 1051 (c) 28

8 (a) 7000 (b) 120 000 (c) 42.7 (d) 5.8

(e) 9.01 (f) 0.0762

9 (a) $\frac{1}{2}$ (b) $\frac{1}{10}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$ (e) $\frac{1}{100}$ (f) $\frac{3}{5}$

Exercise 4.1

1 (a) (i)

Tens 10	Ones 1	.	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$
	3	.	1	2	6

(ii) 3.126

(b) (i)

Tens 10	Ones 1	.	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$
1	7	.	8	0	3

(ii) 17.803

(c) (i)

Tens 10	Ones 1	.	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$
	0	.	0	2	8

(ii) 0.028

2 (a) (i)

Hundreds 100	Tens 10	Ones 1	.	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$
3	1	2	.	7	6	

(ii) $312 + \frac{7}{10} + \frac{6}{100}$

(b) (i)

Hundreds 100	Tens 10	Ones 1	.	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$
		2	.	0	9	3

(ii) $2 + \frac{9}{100} + \frac{3}{1000}$

(c) (i)

Hundreds 100	Tens 10	Ones 1	.	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$
		0	.	6	7	1

(ii) $\frac{6}{10} + \frac{7}{100} + \frac{1}{1000}$

3 (a) (i) 45.462 (ii) 12.519 372 (iii) 3.7985

(iv) 1.008 332 (v) 0.786 06 (vi) 7.033 047

(b) (i) $6 + \frac{6}{10} + \frac{3}{100}$

(ii) $\frac{9}{10} + \frac{2}{100} + \frac{1}{1000}$

(iii) $\frac{7}{10} + \frac{3}{100} + \frac{4}{1000} + \frac{5}{10\,000}$

(iv) $7 + \frac{8}{10} + \frac{2}{100} + \frac{6}{1000}$

(v) $23 + \frac{9}{10} + \frac{1}{100} + \frac{3}{1000} + \frac{4}{100\,000}$

(vi) $45 + \frac{4}{1000} + \frac{5}{10\,000} + \frac{8}{100\,000} + \frac{9}{1\,000\,000}$

4 (a) (i) 6.5 (ii) 0.97 (iii) 0.273

(iv) 37.4201 (v) 14.957 623 (vi) 700.004 59

(b) (i) five ones and two tenths

(ii) four ones and nine tenths

(iii) three tens, four ones, one tenth and seven hundredths

(iv) six tenths and one hundredth

(v) two ones, seven tenths, nine hundredths and four thousandths

(vi) seven ones, five tenths, nine thousandths and two ten-thousandths

(vii) three tens, five ones, eight tenths, six hundredths, five thousandths, four ten-thousandths and three hundred-thousandths

(viii) eight tenths, two hundredths, two hundred-thousandths and seven millionths

5 (a) (i) $\frac{2}{1000}$ (ii) $\frac{2}{100\,000}$ (iii) $\frac{2}{10}$ (iv) 2

(b) (i) seven tenths (ii) seven thousandths

(iii) seven ten-thousandths (iv) seven millionths

6 (a) $2.4 > 0.42$ (b) $2.32 > 1.955$

(c) $0.65 > 0.57$ (d) $0.3003 < 0.333$

(e) $4.7038 < 4.7312$ (f) $8.251 > 8.2501$

(g) $7.02 > 7.002$ (h) $4.7367 < 4.7376$

(i) $0.927 < 0.927\,34$ (j) $6.013 > 6.01$

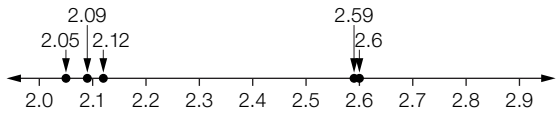
(k) $3.406 < 3.4063$ (l) $0.9995 > 0.9986$

7 (a) F (b) F (c) F (d) F

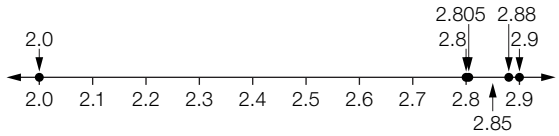
(e) T (f) F (g) T (h) F

8 B 9 B

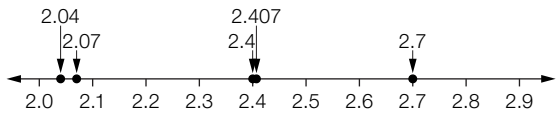
10 (a)



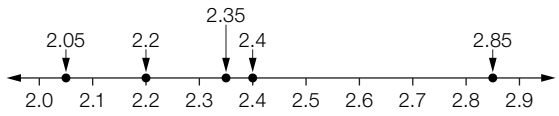
(b)



(c)



(d)



11 (a) 2.03, 2.13, 2.3 (b) 8.007, 8.67, 8.7

(c) 6.64, 6.6403, 6.646 (d) 0.0095, 0.0509, 0.0905

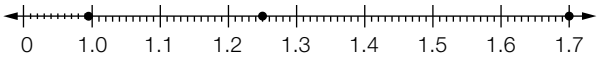
(e) 5.003 821, 5.3281, 5.38 (f) 3.116, 3.616, 3.661

(g) 0.092, 0.29, 0.92 (h) 0.086, 0.815, 0.85

12 37.901

13 (a) $\frac{4}{10} + \frac{3}{100} + \frac{7}{1000}$ (b) $\frac{437}{1000}$

14 (a)


(b)


(c)


(d)


15 (a) 13.08 seconds (b) 13.95 seconds

16 25.6 m

17 (a) (i) 20.2 (ii) 21 (iii) 21.5

(b) 3.75 (c) 1.25 L (d) 2.60

18 (a) The 2 is in the thousandths place, not in the hundredths place.

(b) Should be 409.67. Al has written the 6 in the hundredths place and the 7 in the thousandths place.

(c) Max has put the 7 in the tenths place instead of in the ones place. Answer should be 7.0809.

(d) $\frac{9}{10} + \frac{5}{100} + \frac{6}{1000}$ means 0.956. Polly has not accounted for the 0 in the hundredths column. 0.9056 is the same as $\frac{9}{10} + \frac{5}{1000} + \frac{6}{10\,000}$.

Open-ended—Sample answers

19 (a), (b) 4.51, 4.52, 4.53, 4.6, 4.63, 4.67

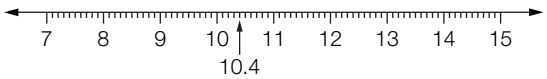
20 (a) $2.13 < 5.6$, $3.52 < 5.1$, $4.69 < 5.3$

(b) 6, 7, 8, 9 could not be in the ones box.

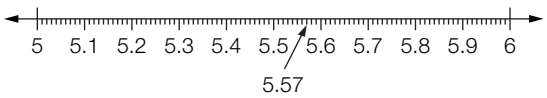
21 (a) 2.57, 5.27, 7.25, 2.75, 5.72, 7.52, six different decimals

(b) 25.7, 27.5, 5.72, 52.7, 57.2, 7.25, 7.52, 72.5, 75.2, nine different decimals

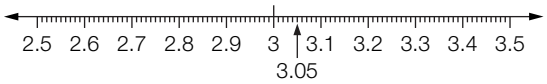
Exercise 4.2

1 (a) (i)


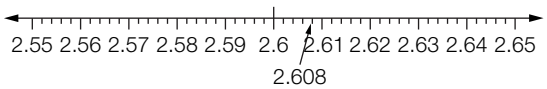
(ii) 10

(b) (i)


(ii) 5.6

(c) (i)


(ii) 3.1

(d) (i)


(ii) 2.61

2 (a) 7.4 (b) 13.85 (c) 0.30 (d) 4.46

(e) 2.764 (f) 0.088

3 (a) 4.9 (b) 6.7 (c) 7.64 (d) 4.55

(e) 0.642 (f) 3.28 (g) 0.3144 (h) 11.83

(i) 2.92 (j) 18.500 (k) 90.9000 (l) 19.000

(m) 23.41 (n) 57.080 (o) 259.9000

4 (a) \$4.50 (b) \$2.75 (c) \$11.05

(d) \$23.90 (e) \$176.35 (f) \$542.05

(g) \$789.55 (h) \$56.75 (i) \$6775.00

(j) \$1149.95 (k) \$8990.00 (l) \$10 000.00

5 D **6** 9.6 seconds

7 \$53.65

8 (a) 99.9 (b) 99.94

9 42 °C **10** \$11.95

11 352.2 metres per second

12 1.27 m or 127 cm

13 (a) 12.138, 12.143, 12.149, 12.152, 12.157

(b) 12.14 s, 12.14 s, 12.15 s, 12.15 s, 12.16 s

(c) Yes—1st and 2nd, and 3rd and 4th, are now tied.

14 (a) \$20.35 (b) \$20.40—5 cents extra

Open-ended—Sample answers

15 3.792, 3.790 56, 3.787

16 (a) \$100.02, \$99.98 (b) \$99.60, \$100.48

17 2.548; rounding at the first step creates errors at the second step.

18 The coins have no real 'purchasing' power any more, as prices get higher and higher. They were not worth the cost of producing them.

Exercise 4.3

1 (a) $\frac{1}{50}$ (b) $\frac{3}{5}$ (c) $2\frac{1}{25}$ (d) $6\frac{1}{5000}$

(e) $\frac{13}{50}$ (f) $\frac{7}{200}$

2 (a) $4\frac{1}{10}$ (b) $6\frac{1}{2}$ (c) $8\frac{3}{5}$ (d) $9\frac{2}{5}$

(e) $4\frac{71}{100}$ (f) $2\frac{37}{100}$ (g) $\frac{7}{20}$ (h) $2\frac{12}{25}$

(i) $5\frac{9}{1000}$ (j) $\frac{221}{250}$ (k) $6\frac{16}{125}$ (l) $3\frac{43}{250}$

(m) $7\frac{9}{200}$ (n) $\frac{11}{125}$ (o) $6\frac{3}{2000}$ (p) $1\frac{3}{400}$

3 (a) 0.7 (b) 0.09 (c) 0.17 (d) 0.123

(e) 0.2 (f) 0.12 (g) 0.26 (h) 1.75

(i) 0.875 (j) 0.6875 (k) 0.575 (l) $0.78\dot{3}$

(m) $0.\dot{1}$ (n) $0.58\dot{3}$ (o) $2.8\dot{3}$ (p) $1.0\dot{6}$

4 D **5** D

6 $\frac{12}{25}$

7 $\frac{127}{50}$ or $2\frac{27}{50}$

8 $1\frac{7}{20}$

9 $\frac{3}{125\,000}$

10 0.000 005

11 $5\frac{17}{20}$%; this is higher than $5\frac{3}{4}$% ($5\frac{15}{20}$%)

12 $4\frac{1}{5}$, 4.375, $4\frac{1}{2}$, 4.865, $4\frac{9}{10}$

Open-ended—Sample answers

13 (a) $3\frac{44}{100}$, $3\frac{45}{100}$, $3\frac{46}{100}$ (b) $3\frac{45}{99}$, $3\frac{44}{98}$, $3\frac{46}{97}$

14 $5\frac{62}{100}$, $5\frac{59}{100}$

Exercise 4.4

1 (a) (i) 4 (ii) 3.9
(b) (i) 15 (ii) 15.7
(c) (i) 2 (ii) 1.89
(d) (i) 9 (ii) 8.71
(e) (i) 4 (ii) 3.64
(f) (i) 8 (ii) 7.65

2 (a) 16.2 (b) 12.94 (c) 0.914 (d) 25.648
(e) 26.418 (f) 39.811 (g) 22.827 (h) 10.259
(i) 8.858 (j) 23.917 (k) 17.595 (l) 11.5834

3 (a) 1.9 (b) 4.12 (c) 12.247 (d) 6.963
(e) 3.184 (f) 3.838 (g) 7.26 (h) 7.45
(i) 2.769 (j) 0.56 (k) 0.08 (l) 92.302

4 D 5 12.70 mm

6 (a) $19.87 (b) $19.85

7 $295.51 8 40.5 9 267.9 km

10 196.96 km 11 $63.65 12 0.375 litres

13 $165.90. The fourth item ($18.48) was added twice.

14 $330.65

Open-ended—Sample answers

15 (a) 0.33 + 2.541 (b) 1.213 + 2.532 + 0.115
(c) The last digits of the numbers have to add up to 10 or 20.

16 (a) 4.1 + 3.85 = 7.95, 4.3 + 2.85 = 7.15, 4.6 + 2.85 = 7.45
(b) The last digit in the 2nd number must be a 5.

17 (a) 1.234 and 9.946 (b) 2.324 and 11.034
(c) The last digits must be the same.

18 53.47, 53.43

Exercise 4.5

1 (a) 30.4 (b) 37.1 (c) 58.5 (d) 23.10
(e) 2.16 (f) 73.98 (g) 64.376 (h) 4.333
(i) 54.927 (j) 287.04 (k) 1219 (l) 1186.88
(m) 0.0008 (n) 0.0035 (o) 3.0018

2 (a) 27.5 (b) 0.73 (c) 176.5 (d) 47
(e) 134.5 (f) 709.9

3 (a) 144 (b) 228.9 (c) 119.64 (d) 114
(e) 2170 (f) 2650 (g) 27 546 (h) 27 434
(i) 29 202 (j) 38 828 (k) 67 201.5 (l) 411 576

4 (a) (i) 12 (ii) 13.86
(b) (i) 2 (ii) 1.824
(c) (i) 7 (ii) 3.876

5 (a) 0.48 (b) 0.24 (c) 0.36 (d) 6.524
(e) 1.236 (f) 4.208 (g) 2.3994 (h) 2.0812
(i) 0.6768 (j) 0.0024 (k) 0.009 24 (l) 0.0036

6 C 7 C 8 C

9 $12.60 10 $18.20 11 10.619 grams

12 (a) 3 500 000 (b) 4 260 000 000
(c) 1 300 000 000 000 (d) 57 080 000
(e) 10 200 000 000 000 (f) 20 045 000 000

13 0.984 metres 14 $117.98 15 $516.00

16 (a) $94.50 (b) $12.83

17 1950 kilograms 18 $80.60 19 37.5 litres

20 292.25 kilojoules

21 (a) 24.1 million
(b) The 'exact' value is not accurate, as births and deaths cause it to change daily. It is not necessary or helpful, for most purposes, to give a more precise number.

22 (a) 6.75 metres (b) 2, 0.25 m

23 (a) $4.80 (b) $0.85
(c) Purchasing 3 multi-bags ($11.85) is slightly cheaper than purchasing individual bars ($12).

24 1.6 kilograms

25 (a) 76.3 kilowatt-hours (b) $9.92

Open-ended—Sample answers

26 (a) He forgot the place value of the digits.
Estimation: 2 × 90 = 180 m.
(b) He could have multiplied 89 by 1, 0.6 and 0.03 and added the answers together.
(c) Students' own answers.

27 (a) 20 and 3.42; 30 and 2.28
(b) 12.5 and 5.472; 15 and 4.56

Half-time 4

1 (a) $\frac{1}{4}$ (b) $\frac{3}{10}$ (c) $3\frac{27}{50}$ (d) $9\frac{69}{200}$

2 0.03, 0.321, 9.003 24, 9.01, 19.12

3 (a) 20.4 (b) 132.919 (c) 2.82 (d) 6.02
(e) 83.861 (f) 1.1

4 (a) 4.877 (b) 13.004 (c) 2.350 (d) 23.032

5 (a) 30.4 (b) 141.5 (c) 12.33 (d) 1356
(e) 520 (f) 3390 (g) 7.31 (h) 613.228
(i) 0.0006

6 (a) \$100.55 (b) \$48.45

7 (a) 23.4907 (b) 14.235 (c) 293.415 (d) 90.6207

8 (a) \$1.95 (b) \$35.10 (c) \$57.45 (d) \$190.00

9 (a) 0.8 (b) 0.85 (c) $0.\dot{6}$ (d) $0.58\dot{3}$

10 59.023, 59.105, 59.125, 59.23

11 \$122.46

Exercise 4.6

1 (a) 2.12 (b) 4.32 (c) 1.58 (d) 0.55
(e) 0.343 (f) 0.567 (g) 6.57 (h) 1.514
(i) 0.052 (j) 0.07 (k) 0.407 (l) 2.637
(m) 30.866 (n) 10.546 (o) 49.644

2 (a) 0.123 (b) 0.154 (c) 5.068 (d) 0.813
(e) 0.000 09 (f) 0.0432

3 (a) 0.09 (b) 0.09 (c) 0.604 (d) 0.483
(e) 0.006 (f) 0.039 (g) 0.078 (h) 0.005
(i) 0.003 (j) 0.57 (k) 0.029 (l) 0.007

4 (a) 25.7 (b) 10.3 (c) 8.5 (d) 387
(e) 100.2 (f) 24.8 (g) 397 (h) 44.4
(i) 24.5 (j) 700 (k) 315 (l) 145
(m) 8700 (n) 11 500 (o) 16 700

5 D **6** C **7** 0.289 kg

8 \$1.965 million

9 \$93.99 or \$94 per guest

10 12 times **11** 259 necklaces

12 12.43 km **13** \$0.22 per 100 mL

14 \$4.71 **15** 1933 worms

16 6.5 minutes **17** \$4.75 per kilometre

18 Angie's team 1.60 m, Jessica's team 1.59 m.
Angie's team is taller on average by 0.01 m.

19 Brazilian Roast at \$2.25 per 100 grams

20 33 5-cent coins, 23 10-cent coins, 26 20-cent coins, 15 50-cent coins

21 217 km

Open-ended—Sample answers

22 2.002, 3.006, 3.666

23 (a) 20.1 ÷ 1 (multiply by 2); 40.2 ÷ 2 (multiply by 4)
(b) 180.2 ÷ 0.1 (multiply by 5); 720.8 ÷ 0.4 (multiply by 20)

24 10 m and 0.845 m, perimeter 21.69 m;
5 m and 1.69 m, perimeter 13.38 m

25 (a) He didn't consider the place value of the digits to the RHS of the decimal point.
(b) To do decimal division more accurately, Tran should perform the division, ignoring the decimal point, and then place a decimal point in the answer so it lines up with the one in the dividend.

Exercise 4.7

1 (a) $\frac{3}{20}$ (b) $\frac{6}{25}$ (c) $\frac{9}{50}$ (d) $\frac{16}{25}$
(e) $\frac{11}{10}$ (f) $\frac{61}{50}$ (g) $\frac{7}{5}$ (h) $\frac{99}{50}$

2 (a) 0.34 (b) 0.13 (c) 0.78 (d) 0.92
(e) 1.2 (f) 1.23 (g) 2.54 (h) 3.2

3 (a) $\frac{3}{5} \times \frac{20}{20} = \frac{60}{100} = 60\%$ (b) $\frac{7}{10} \times \frac{10}{10} = \frac{70}{100} = 70\%$
(c) $\frac{1}{4} \times \frac{25}{25} = \frac{25}{100} = 25\%$ (d) $\frac{2}{3} \times \frac{100}{1} = \frac{200}{3} = 66.\dot{6}\%$
(e) $\frac{5}{8} \times \frac{100}{1} = \frac{500}{8} = 62.5\%$ (f) $\frac{7}{15} \times \frac{100}{1} = \frac{700}{15} = 46.\dot{6}\%$

4 (a) 40% (b) 75% (c) 30% (d) 35%
(e) 110% (f) 150% (g) 112% (h) 120%
(i) 37.5% (j) $33.\dot{3}\%$ (k) $83.\dot{3}\%$ (l) $44.\dot{4}\%$
(m) $86.\dot{6}\%$ (n) $63.\dot{3}\%$ (o) 82.5% (p) $73.\dot{3}\%$

5 (a) 87% (b) 34% (c) 65% (d) 96%
(e) 134% (f) 358% (g) 532% (h) 121%

6

Percentage	Fraction (simplest form)	Decimal
40%	$\frac{2}{5}$	0.4
70%	$\frac{7}{10}$	0.7
5%	$\frac{1}{20}$	0.05
2%	$\frac{1}{50}$	0.02
75%	$\frac{3}{4}$	0.75
125%	$\frac{5}{4}$	1.25

7 (a) 0.25, 0.5, 0.1, 0.2 (b) 25%, 50%, 10%, 20%

8 $\frac{19}{20}$ **9** 62.5%

10 **(a)** 0.03, $\frac{7}{50}$, 57%, $\frac{3}{5}$, 0.8, 82%

(b) 0.05, $\frac{3}{10}$, 43%, $\frac{1}{2}$, 73%, 1.05

11 **(a)** $\frac{3}{5}$ **(b)** 60%

12 63%

13 **(a)** **(i)** $\frac{1}{25}$ **(ii)** 4% **(iii)** 0.04

(b) The percentage, as most people would find the percentage easier to comprehend and visualise.

(c) Answers may vary. '4% of us believe in ghosts'

14 $\frac{1}{4} = 25\%$ and $\frac{3}{4} = 75\%$, $75 = 3 \times 25$

15 **(a)** 75% **(b)** 0.75 **(c)** 25% **(d)** 0.25

Open-ended—Sample answers

16 $\frac{1}{3}, \frac{1}{4}, \frac{3}{10}$

17 34.1%, 34.2%, 34.3%

18 Students' own answers.

Exercise 4.8

1 **(a)** \$18 **(b)** 45 m **(c)** 168 L **(d)** 96 m
(e) 450 kg **(f)** \$168 **(g)** 9.36 kg **(h)** \$95.58
(i) 183.96 m **(j)** 64.75 m **(k)** 71.536 L **(l)** \$1.31

2 **(a)** 85% **(b)** 22.7% **(c)** 40% **(d)** 80%
(e) 76% **(f)** 77.1% **(g)** 55.6% **(h)** 75%
(i) 82.4%

3 **(a)** \$12.50 **(b)** 7.5 kg **(c)** 15 km **(d)** 8 L
(e) 3.6 m **(f)** \$6 **(g)** 210 kg **(h)** 600 mL
(i) \$140

4 B **5** C **6** 10 students

7 57.1% are cream biscuits **8** 0.7 L

9 $83.\dot{3}\%$ **10** Boys: 42.3%; Girls: 57.7%

11 **(a)** Riley 70%, Franco 68.75% **(b)** Riley

12 33.1% of the score

13 **(a)** **(i)** \$8 **(ii)** \$31.25 **(iii)** \$32.50
(iv) \$23.90 **(v)** \$319.60 **(vi)** \$64.95
(b) **(i)** \$32 **(ii)** \$93.75 **(iii)** \$32.50
(iv) \$215.10 **(v)** \$479.40 **(vi)** \$1234.05

14 92%

15 Chelsea—Game 1: 68%, Game 2: 70.4%
Sonia—Game 1: 75%, Game 2: 73.9%

Chelsea increased her percentage, so she improved her accuracy, whereas Sonia's accuracy dropped slightly.

16 **(a)** 22% of 180 = 39.6; 34% of 180 = 61.2; 7% of 180 = 12.6

(b) Each of the answers is a decimal number, which is not possible when working with numbers of people (cannot have 0.6 of a student).

(c) Actual number with blue eyes could be either 39 or 40 (both give a percentage that rounds to 22 as the nearest whole number), actual number speaking two or more languages could be 61 or 62 (both give a percentage of 34 when rounded) and the actual number of left-handed students could be 12 or 13 (both give a percentage of 7 when rounded).

17 **(a)** 2700 kJ **(b)** 73 g of fat

Open-ended—Sample answers

18 Savings look more attractive and are more easily interpreted (e.g. 50% off is easily recognised as half price).

19 50%, \$10; 25%, \$20; 10%, \$50

20 Issa's working is correct. Chiara has confused the method for finding a percentage with the method for writing one amount as a percentage of another. The question asked here has the percentage sign appearing in the question, which means that the first method (multiplying by the per cent) should be used. If the question contains the words 'out of', then the second method (writing one amount as a percentage of another) should be used.

Exercise 4.9

1 **(a)** 5:3 **(b)** 4:7 **(c)** 3:2 **(d)** 8:3
(e) 1:2:3 **(f)** 2:9:2 **(g)** 3:4:7 **(h)** 3:2:1

2 **(a)** 2:3 **(b)** 3:5 **(c)** $\frac{3}{5}$ **(d)** 40%

3 **(a)** 2:6 **(b)** 2:16 **(c)** 2:22 **(d)** 2:66
(e) 12:15 **(f)** 12:28 **(g)** 2:3 **(h)** 2:3
(i) 5:4 **(j)** 2:1 **(k)** 56:35 **(l)** 60:55
(m) 8:3 **(n)** 100:30 **(o)** 20:18 **(p)** 75:65

4 **(a)** 1:5 **(b)** 1:4 **(c)** 2:9 **(d)** 2:7
(e) 3:2 **(f)** 8:5 **(g)** 8:5 **(h)** 4:3
(i) 27:32 **(j)** 14:17 **(k)** 8:9 **(l)** 2:5
(m) 3:2 **(n)** 3:2 **(o)** 7:4 **(p)** 40:11

5 **(a)** B **(b)** C

6 **(a)** 12:7:16 **(b)** 35 **(c)** $\frac{12}{35}$ **(d)** 20%

7 **(a)** 15:7:2 **(b)** $\frac{7}{24}$ **(c)** 29%

8 **(a)** 5:4:3 **(b)** $\frac{5}{12}$ **(c)** 42%

9 **(a)** 6:1 **(b)** 6:1 = 24:6, 6 instructors required
(c) 30 students

10 3:2:1 = 12:8:4, 8 buckets of sand, 12 buckets of gravel

11 **(a)** 10% **(b)** 2 g

12 **(a)** 25% **(b)** 75:25 = 3:1

13 **(a)** B **(b)** F **(c)** C **(d)** E

14 **(a)** 4 teachers are required. 3 would only be enough for 60 students (1:20 = 3:60).

(b) 4:68 = 1:17

15 **(a)** Chloe: 1:5
Hannah: 2:5

(b) Hannah is correct. Looking at the simplified ratios, Hannah has 1 extra part of concentrate for the same number of parts of water.

16 **(a)** $\frac{5}{8}$ **(b)** $\frac{3}{8}$ **(c)** $\frac{4}{9}$

(d) 2 cats and 1 dog

Open-ended—Sample answers

17 **(a)** Answer will depend on the class.

(b) Unlikely, unless it is an all-boys or all-girls school.

(c) No; there would be a big variation in all coeducational schools; some schools are mixed, others are all boys or all girls.

18 8:14, 12:21, 16:28

19 The ratio should be 7:12; the student forgot to count the apples as fruit.

Exercise 4.10

1 **(a)** \$5, \$25 **(b)** ÷ 5, × 2 **(c)** ÷ 5, \$6, × 3, \$18

2 **(a)** \$5.60 **(b)** 104 cans **(c)** \$1.20

(d) \$8500 **(e)** 25 kg **(f)** 525 pages

3 **(a)** 600 g: \$0.68 per 100 g, 250 g: \$1.07 per 100 g;
600 g is the better value

(b) 500 g: \$0.56 per 100 g, 750 g: \$0.46 per 100 g;
750 g is the better value

(c) 375 g: \$0.34 per 100 g, 500 g: \$0.65 per 100 g;
375 g is the better value

(d) 200 g: \$1.61 per 100 g, 375 g: \$1.31 per 100 g;
375 g is the better value

(e) 500 mL: \$0.54 per 100 mL, 920 mL: \$0.47 per 100 mL;
920 mL is the better value

(f) 450 g: \$0.84 per 100 g, 1 kg: \$0.88 per 100 g;
450 g is the better value

4 A **5** D **6** 19.2, or 20 loaves

7 **(a)** the 400 g jar

(b) Yes, the 280 g jar is now slightly better value for money, with both jars approximately \$1.60 per 100 g.

8 \$6 **9** 7.1 runs per over

10 **(a)** **(i)** packet of 6: \$0.81 per bar, packet of 10: \$0.75 per bar; packet of 10 is the better value

(ii) 12-pack: \$1.18 per can, 18-pack: \$1.12 per can, 30-pack: \$0.92 per can; 30-pack is the better value

(iii) box of 50: \$0.06 per bag, box of 200: \$0.04 per bag, box of 200 is the better value

(b) Multiply the cost of the box of 50 by 4 to get the price of 200, or multiply the cost of the box of 50 by 2 and divide the cost of the box of 200 by 2, to get the prices per 100 bags.

11 **(a)** \$1.45 **(b)** \$17.40

12 **(a)** 3.6 L **(b)** 25.2 L

13 **(a)** 3.25 g **(b)** 13 g **(c)** 16.25 g

14 **(a)** \$6.00 **(b)** \$0.60 **(c)** \$7.80

15 Individual cost of each bun in the packet = \$1.20
4 × \$1.20 = \$4.80
\$7.20 (1 packet) + \$4.80 (4 loose) = \$12.00

16 **(a)** 4 kg is better value:
\$15.97 ≈ \$16
≈ \$4/kg

(b) 1 L carton is better:
1 L = 4 × 250 mL
4 × \$1.25 = 2 × \$2.50 = \$5

(c) packet ham is better value:
500 g of ham at \$13/kg
= \$6.50

(d) 400 g jar is better value:
\$7.99 ≈ \$8
≈ \$2/100 g, or \$6 for 300 g

(e) The product may be Australian made, the packaging may be appealing, or you may prefer a particular brand.

17 Post-paid plan: \$4.10 is cheaper, pre-paid: \$4.25 is more expensive

18 **(a)** 2516.13 kL/day

(b) 918 387.45 kL

(c) No; cannot assume that the rainfall in January will be the same for the other months of the year. January is part of the wet season, so other months should be drier.

19 **(a)** The 500 g block is two times bigger than the 250 g block.

(b) 500 g block: \$9.00; 750 g block: \$13.50

(c) The packaging of larger items is cheaper; 'bulk' buys are made more attractive so that consumers will purchase more of the product.

Open-ended—Sample answers

20 sauce, soft drink, fruit juice, yoghurt

21 **(a)** 9 × 500 g; 4 × 1 kg + 1 × 500 g; 2 × 2 kg + 1 × 500 g

(b) 2 × 2 kg + 1 × 500 g is the cheapest (\$10.28 compared to \$11.16 and \$10.64)

Challenge 4

1 B

2 8 × 0.753

3 0.01 − 0.001 = 0.009

4 C \$$\frac{3}{4}$ = \$0.75, \$$\frac{3}{10}$ = \$0.30. Add the amounts.

5 C $200 \div 2.5 = 80$, so $20 \div 2.5 = 8$ and $20 \div 0.25 = 80$

6 B Amount is $49\,095 - 49.95 = 49\,045.05$ to be subtracted.

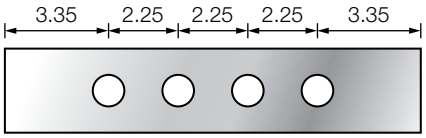
7 **(a)** The last digit of both of the decimal numbers must not multiply together to give a number ending in 0, e.g. $3.21 \times 4.547 = 14.595\,87$.

(b) The last digit of both of the decimal numbers must multiply together to give a number ending in 0, e.g. $1.25 \times 3.766 = 4.7075$.

(c) the last two digits of both of the decimal numbers must multiply together to give a number ending in 00, e.g. $7.25 \times 8.324 = 60.349$.

8 Scores a total of $6 \times 7.5 = 45$ goals. Least number of goals scored is 4, so she scores $45 - 4 = 41$ goals in 5 other games. $41 = 4 \times 8 + 9$, so she must score at least 9 in one game. $41 = 7 + 3 \times 8 + 10$, so she could score more than 9. The lowest value for the highest number of goals she scores is 9.

9 Length of strip of metal $= 2 \times 3.35 + 3 \times 2.25 = 13.45$ cm



10 $0.01 \div 0.002 = 10 \div 2 = 5$

11 C $0.9 \times \$30\,000 = \$27\,000$; $1.1 \times \$27\,000 = \$29\,700$

12 A **13** B

Chapter review 4

1 **(a)** $\frac{9}{10} + \frac{6}{100} + \frac{8}{1000}$ **(b)** $5 + \frac{7}{100} + \frac{2}{10\,000}$

(c) $6 + \frac{5}{1000}$

2 **(a)** 0.6524 **(b)** 0.080 093

3 **(a)** 4.3806 **(b)** 57.032 09

4 **(a)** one one, eight tenths, five hundredths, three thousandths and one ten-thousandth

(b) seven hundredths and six hundred-thousandths

(c) six tens, one one and nine ten-thousandths

5 **(a)** **(i)** $\frac{9}{100}$ **(ii)** nine hundredths

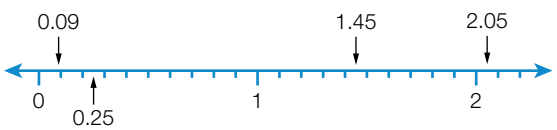
(b) **(i)** $\frac{9}{10\,000}$ **(ii)** nine ten-thousandths

(c) **(i)** $\frac{9}{100}$ **(ii)** nine hundredths

6 **(a)** $3.0427 > 3.0274$ **(b)** $0.009\,95 < 0.01$

7 **(a)** 0.5506, 0.6055, 0.607 **(b)** 0.071, 0.701, 0.71

8



9 **(a)** 4.4 **(b)** 34.65 **(c)** 23.1 **(d)** 102.47

10 **(a)** \$41.20 **(b)** \$10.05 **(c)** \$79.95 **(d)** \$99.00

11 **(a)** $3\frac{9}{10}$ **(b)** $\frac{31}{50}$ **(c)** $2\frac{9}{20}$ **(d)** $\frac{9}{5000}$

12 **(a)** 0.8 **(b)** 0.95 **(c)** $0.\dot{2}$ **(d)** $0.8\dot{3}$

13 **(a)** 28.39 **(b)** 15.4585 **(c)** 41.433
(d) 7.717 **(e)** 1.688 **(f)** 4.7802

14 **(a)** 13.504 **(b)** 104.65 **(c)** 200.97
(d) 47 200 **(e)** 164.4 **(f)** 3.45
(g) 0.54 **(h)** 0.000 36 **(i)** 19.76

15 **(a)** 2.58 **(b)** 1.004 **(c)** 0.469
(d) 0.240 **(e)** 0.016 **(f)** 0.182
(g) 12 **(h)** 2.4 **(i)** 5950

16 **(a)** 70% **(b)** 125% **(c)** 12.5% **(d)** $66.\dot{6}\%$

17 **(a)** 38% **(b)** 96% **(c)** 355% **(d)** 9%
(e) 323.4% **(f)** 54.32%

18 **(a)** \$24 **(b)** 12.75 L **(c)** 166 m **(d)** \$75

19 **(a)** 65% **(b)** 50% **(c)** 35% **(d)** 93.3%

20 **(a)** 8:7 **(b)** $\frac{7}{15}$ **(c)** 53%

21 **(a)** \$7.05 **(b)** \$3.14

22 **(a)** 1 381 000 000 **(b)** 1 320 000 000
(c) 259 900 000 **(d)** 4 560 000

23 **(a)** \$3023.60 **(b)** \$7559.00

24 **(a)** T-shirt discount \$3, Jeans discount \$24

(b) T-shirt \$27, Jeans \$36

25 **(a)** The 400 g jar is better value.

(b) Yes, the 250 g jar is now better value than the 400 g jar.

(c) People might only require a small amount of a product, a larger amount might be wasted, some amounts are in a more convenient size (e.g. travel packs)

26 **(a)** 75 mL **(b)** No; 60:40 = 3:2

27 **(a)** 7.8 **(b)** 23.4 **(c)** 63.18 Yes

28 **(a)** \$5.20 **(b)** \$0.40 **(c)** \$0.80

(d) Buy six 6-packs and four individual cans (total cost of \$32.28).

29 5 supervisors will be needed; 4 supervisors could only manage 48 children at the required ratio.

30 Store A discount = \$16.25, sale price = \$48.75

Store B discount = \$22.50, sale price = \$52.50

Sally should buy her shoes at Store A to pay the lowest price.

Numeracy practice 4

1 D **2** B **3** D **4** B

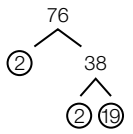
5 \$11 **6** D **7** C **8** \$178.92

Mixed review B

1 **(a)** $\frac{3}{10}$ **(b)** $\frac{3}{10}$ **(c)** $\frac{3}{4}$ **(d)** $\frac{4}{13}$

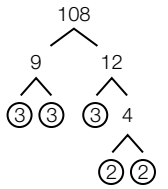
2 **(a)** -74 **(b)** +3 **(c)** -215 **(d)** +8

3 **(a)** **(i)**



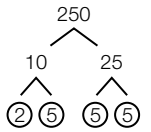
(ii) $2^2 \times 19$

(b) **(i)**



(ii) $2^2 \times 3^3$

(c) **(i)**



(ii) 2×5^3

4 **(a)** 4800 **(b)** 3300 **(c)** 12 **(d)** 4

5 **(a)** > **(b)** < **(c)** <

6 **(a)** 0.27 **(b)** 15.3 **(c)** 435

7 **(a)** -6 **(b)** 14 **(c)** -10 **(d)** -8 **(e)** 6 **(f)** -9

8 **(a)** $\frac{8}{25}$ **(b)** $1\frac{6}{25}$ **(c)** $3\frac{237}{1000}$

9 **(a)** 7^4 **(b)** 4^2 **(c)** $2^2 \times 3^3$ **(d)** 11^3

10 **(a)** 30 **(b)** 6 **(c)** 30

11 **(a)** < **(b)** > **(c)** <

12 **(a)** 6 **(b)** 8 **(c)** 12

13 **(a)** $\frac{19}{20}$ **(b)** $4\frac{1}{24}$ **(c)** $\frac{11}{24}$ **(d)** $\frac{3}{4}$

14 **(a)** 203 **(b)** 484 **(c)** 195 **(d)** 65 **(e)** 280 **(f)** 55

15 **(a)** 4 m **(b)** \$3

16 **(a)** **(i)** $\frac{3}{10}$ **(ii)** 30%

(b) **(i)** $\frac{3}{13}$ **(ii)** 23.1%

17 **(a)** 14 g **(b)** 1.4 g **(c)** 4.2 g

18 **(a)** 29 slices **(b)** 13 slices

19 **(a)** 6400 kJ **(b)** 2400 kJ **(c)** $\frac{1}{12}$

20 $3^2 = 3 \times 3$, not 3×2

21 Twice: if the cycles start on the hour, they will flash at 24 and 48 past the hour.

22 **(a)** $\frac{5}{18}$

(b) Eamon (30 pieces compared to Chloe's 9 pieces).

23 **(a)** 84.5

(b) It is a decimal number, which is impossible when dealing with numbers of students.

(c) The actual number of students was either 84 or 85. Both numbers give a percentage of 65 when the answer is rounded to the nearest whole number.

24 8 **25** 69 cards

Chapter 5

Recall 5

1 **(a)** 5 **(b)** 2 **(c)** 1 **(d)** 4 **(e)** 2 **(f)** ±4

2 **(a)** **(i)** 41 **(ii)** 107 **(iii)** 2197

(b) **(i)** 10 **(ii)** 94 **(iii)** 58

3 **(a)** $11 + 17 = 28$ **(b)** $9 - 7 = 2$

(c) $3 \times 4 = 12$ **(d)** $16 \div 8 = 2$

4 **(a)** $6 \times 2 - 4$; 8 **(b)** $(10 + 5) \div 5$; 3

(c) $(6 + 5) \times 3$; 33 **(d)** $(23 - 21) \times 12$; 24

5 **(a)** peregrine falcon **(b)** kodiak bear

Exercise 5.1

1 **(a)** $p - 3$ **(b)** $k + 9$ **(c)** $12v$

(d) $\frac{a}{4}$ **(e)** $d + e$

2 **(a)** $x + 3$ **(b)** $x - 6$ **(c)** $z + p$

(d) $x - 4$ **(e)** $a - b$ **(f)** $7t$

(g) $5g$ **(h)** $\frac{f}{5}$ **(i)** $\frac{y}{6} + 5$

(j) $\frac{r + s + t}{3} + 11$ **(k)** $y^2 + 20$ **(l)** $8x^2$

3 **(a)** $c + 5$ **(b)** $x + 2$ **(c)** $m - 6$ **(d)** $a - 3$

(e) $7p$ **(f)** $26e$ **(g)** $\frac{p}{5}$

4 B **5** D **6** A

7 **(a)** $5(n + 1)$ **(b)** $\frac{2 + d}{9}$ **(c)** $12(a + b)$ **(d)** $mn - 3$

8 **(a)** $\frac{x}{2}$ **(b)** $\frac{x}{2} + 3$

9 **(a)** \$6d **(b)** \$2.5t

10 \$$(3d + 50)$ **11** \$$(55x + 300)$ **12** \$$bv$

13 **(a)** \$$12t$ **(b)** \$$24s$ **(c)** \$$(12t + 24s)$

14 $j + 2w + 50c$

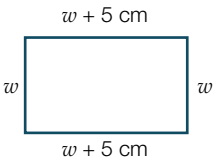
15 **(a)** $5n$ **(b)** $10r$ **(c)** $5n + 10r$

16 **(a)** **(i)** $m + n + q + 1$ **(ii)** $2m + 2n + 2q + 2$

(b) pens in large packs: $2m + 2m$,
pens in small packs: $m + m + m$
total pens = $3m + 4m$ or $7m$

17 (a) (i) $(w + 5)$ cm

(ii)



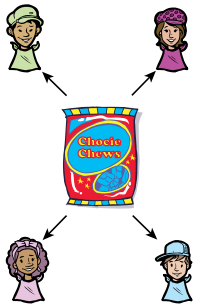
(b) $w + w + 5 + w + w + 5 = w + w + w + w + 10$ or $4w + 10$

Open-ended—Sample answers

18 Students' own diagrams may vary.

(a)

(b) A packet of lollies shared between 4 people:



19 (a) Grandma's age is $2(r + d)$.

(b) Possible ages are: Rory 10, dad 30, grandma 80; Rory 5, dad 25, grandma 60.

Exercise 5.2

1 (a) B (b) A (c) C

2 (a) (i) expression (ii) 2 (iii) x, y, z
(iv) -7 (v) $2x, 3y, -7, 4z$

(b) (i) equation (ii) -4 (iii) x, y, z
(iv) 32 (v) $32, -4x, 3xy, 4z$

(c) (i) equation (ii) -23 (iii) x, y
(iv) 9 (v) $2xy, -3x^2, 9, -23x$

(d) (i) expression (ii) -1 (iii) x, y
(iv) 6 (v) $6, -x, 8y, -4xy$

3 (a) (i) number of girls, number of boys
(ii) Let x = number of boys; let y = number of girls
(iii) $x = 3y$

(b) (i) Fred's age; James' age
(ii) Let g = Fred's age; let h = James' age
(iii) $g = 3h - 1$

(c) (i) cost of an apple, cost of a pear
(ii) Let m = cost of an apple; let n = cost of a pear
(iii) $2m = 3n$

(d) (i) cost of a kilogram of potatoes, cost of a watermelon
(ii) Let a = cost of a kg of potatoes; let b = cost of a watermelon
(iii) $3a + b = \$5.20$

(e) (i) distance to Ranko's house, distance to Skye's house
(ii) Let r = distance to Ranko's house; let s = distance to Skye's house
(iii) $s + 5 = r$

(f) (i) number of plants, number of pavers
(ii) Let v = number of plants, let w = number of pavers
(iii) $2v + 6 = w$

4 (a) T (b) F (c) F (d) T (e) F
(f) T (g) T (h) F (i) T (j) T

5 (a) B (b) D (c) B (d) C (e) C (f) C

6 (a) (i) Let m = mass of 1 tablet
(ii) $25m + 120 = 195$
(iii) 3 grams

(b) (i) Let a = Mei Lee's age now
(ii) $\frac{a}{2} = a - 8$ (iii) 16 years old

(c) (i) Let B = Beths' age; let D = dad's age
(ii) $B + 10 = \frac{D}{2}$

(d) (i) Let number of goals = g;
let number of penalty goals = p
(ii) $8g + p = 100$

7 (a) Let d = distance travelled on first day.

(b) $2d$ (c) $d + 480$ (d) $d + 480 = 3d$

Open-ended—Sample answers

8 (a) Number of goals scored is g and number of behinds is b.

(b) $6g + b = 114$

(c) An example is 12 goals and 42 points.

9 (a) variables are flour (f), sugar (s), eggs (e) and water (w)

(b) $c = 300f + 150s + e + 50w$

(c) For 3 kg of flour you will need 1.5 kg of sugar, 10 eggs and 500 mL of water.

Exercise 5.3

1 (a) $x \rightarrow \boxed{+2} \rightarrow x + 2$ or y

(b) $x \rightarrow \boxed{+3} \rightarrow x + 3 \boxed{\div 2} \rightarrow \frac{x+3}{2}$ or y

(c) $z \rightarrow \boxed{+3} \rightarrow z + 3$ or x

(d) $x \rightarrow \boxed{\times x} \rightarrow x^2$ or y

(e) $x \rightarrow \boxed{-5} \rightarrow x - 5$ or y

(f) $a \rightarrow \boxed{+3} \rightarrow a + 3 \boxed{\div 9} \rightarrow \frac{a+3}{9}$ or c

(g) $x \rightarrow \boxed{\div 2} \rightarrow x \div 2$ or y

(h) $u \rightarrow \boxed{\times u} \rightarrow u^2 \boxed{+2} \rightarrow u^2 + 2$ or w

2 (a) $x \rightarrow \boxed{+2} \rightarrow y$

$y = x + 2$

x	13	11	7	28	1
y	15	13	9	30	3

(b) $x \rightarrow \boxed{-5} \rightarrow y$

$y = x - 5$

x	6	18	9	85	5
y	1	13	4	80	0

(c) $x \rightarrow \boxed{\times 2} \rightarrow y$

$y = 2x$

x	2	3.4	10	11	101
y	4	6.8	20	22	202

(d) $x \rightarrow \boxed{\div 3} \rightarrow y$

$y = \frac{x}{3}$

x	18	12	30	9	0
y	6	4	10	3	0

(e) $x \rightarrow \boxed{\times 5} \rightarrow \boxed{-3} \rightarrow y$

$y = 5x - 3$

x	4	2	0	5	20
y	17	7	-3	22	97

(f) $x \rightarrow \boxed{+5} \rightarrow \boxed{\div 10} \rightarrow y$

$y = \frac{x + 5}{10}$

x	5	25	45	10	33
y	1	3	5	1.5	3.8

(g) $x \rightarrow \boxed{\div 2} \rightarrow \boxed{-1} \rightarrow y$

$y = \frac{x}{2} - 1$

x	10	6	24	9	15
y	4	2	11	3.5	6.5

(h) $x \rightarrow \boxed{\times x} \rightarrow y$

$y = x^2$

x	3	11	7	6	10
y	9	121	49	36	100

3 (a) A **(b)** A **(c)** D

4 (a) $y = 4x - 9$ **(b)** $q = p^2 + 8$

(c) $b = \frac{a}{2} - 7$ **(d)** $b = \frac{a + 3}{6}$

(e) $y = 5(x + 11)$ or $y = (x + 11) \times 5$

(f) $q = \sqrt{p}$

5 (a) $y = x - 18$ **(b)** $y = 60x$

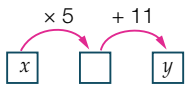
(c) $y = \frac{x}{7}$ **(d)** $y = 20(x + 43)$ or $y = (x + 43) \times 20$

(e) $y = 100x - 50$ **(f)** $y = \frac{x}{16} + 13$

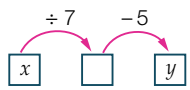
(g) $y = x^2$ **(h)** $y = \frac{x - 12}{9}$

(i) $y = x^2 - 37$

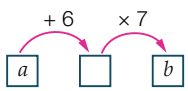
6 (a)



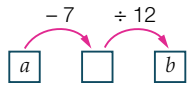
(b)



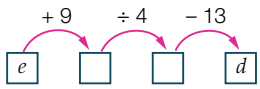
(c)



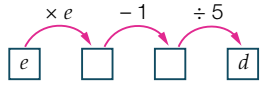
(d)



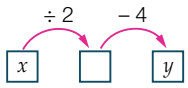
(e)



(f)

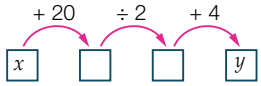


7 (a)



(b) $y = \frac{x}{2} - 4$ **(c)** $y = \frac{20}{2} - 4 = 6$ lollies left

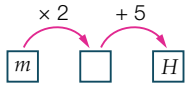
8 (a)



(b) $y = \frac{x + 20}{2} + 4$ **(c)** $y = \frac{12 + 20}{2} + 4 = 16 + 4 = 20$

Anita spent $20.

9 (a)



(b) $H = 2m + 5$ **(c)** $H = 2 \times 12 + 5 = 24 + 5 = 29$

10 (a)

Number of pairs of jeans, n	10	50	150	200
Cost to produce the jeans, C	\$315	\$595	\$1295	\$1645

(b) $C = 7n + 245$

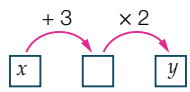
11 (a)

x	2	5	7	4	10
y	5	11	15	9	21

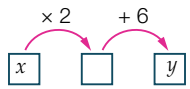
(b)

x	10	1	2	4	18
y	52	16	20	28	84

12 (a) **(i)**



(ii)



(b) **(i)** $y = 2(x + 3)$ or $y = (x + 3) \times 2$

(ii) $y = 2x + 6$

(c) **(i)**

x	2	5	3	4	7
y	10	16	12	14	20

(ii)

x	2	5	3	4	7
y	10	16	12	14	20

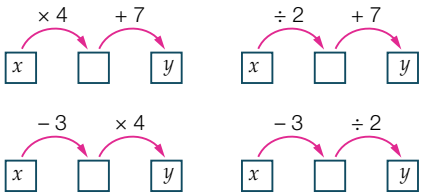
(d) The two tables of values are the same, which means the two rules are equivalent. In the first rule, both x and 3 are doubled in the second step, which gives $2x + 6$.

Open-ended—Sample answers

13 One possible table for the rule $y = 3 + \frac{x}{5}$ is:

x	5	10	15	20	25
y	4	5	6	7	8

14 (a) Some possible two-step flowcharts are:



(b) The rules that correspond to the above flowcharts are:

$y = 4x + 7$ $\quad$ $y = 4(x - 3)$

$y = \frac{x}{2} + 7$ $\quad$ $y = \frac{x - 3}{2}$

15 Kim's statement that they are 'different ways of writing the same thing' is incorrect. Both have the correct operations of ÷ 2 and + 3 in their flowcharts, but the order is important. The rule states that x is first divided by 2, then 3 is added, so Jai's flowchart is correct.

Exercise 5.4

1 (a) 7 **(b)** $5\frac{1}{2}$ **(c)** -1 **(d)** 23

(e) 2.6 **(f)** 25 **(g)** 4 **(h)** 28

(i) 2 **(j)** 32 **(k)** 18 **(l)** 120

(m) 4 **(n)** $\frac{1}{4}$ **(o)** 44 **(p)** 84

2 (a) \$270 **(b)** loss of \$5 **(c)** 22.15

3 (a) T **(b)** T **(c)** F **(d)** T **(e)** F **(f)** T

4 (a)

a	11	20	5	9	50
b	44	80	20	36	200

(b)

x	6	4	10	20	101
y	42	28	70	140	707

(c)

m	1	2	10	6	5
n	5	8	32	20	17

(d)

j	2	5	11	10	100
k	15	27	51	47	407

(e)

p	11	15	10	20	100
q	12	20	10	30	190

(f)

r	1	2	3	0	200
s	6	14	22	-2	1598

(g)

u	5	3	1	201	6
v	16	8	0	800	20

(h)

m	5	10	11	102	52
n	9	24	27	300	150

5 (a) D **(b)** C **(c)** C

6 (a) $m = 7h + 3$ **(b)** $b = 11h$ **(c)** 147

(d) $P = 2M - 154$ **(e)** \$140

7 (a) $\$350 + 10n = p$ **(b)** \$500 **(c)** \$550

8 (a) $D = 3l + 2.6$

(b) $D = 3 \times 3.25 + 2.6$
$= 9.75 + 2.60$
$= \$12.35$

9 (a) $s = 2n + 10$ **(b)** $b = \frac{n}{8}$

(c) Jasmine will need 90 sausages and 5 bowls of salad.

10 Substituting $a = 4$ into $b = 3(a + 5)$ gives $b = 27$.
Substituting $a = 4$ into $b = 3a + 5$ gives $b = 17$.
In the first formula, the order of operations is add 5, then multiply by 3. In the second formula, the order of operations is multiply by 3, then add 5.

Open-ended—Sample answers

11 One possible formula is: $y = 3x + 5$

x	7	20	13	101
y	26	65	44	308

Another possible formula is: $y = 2(x + 4)$

x	7	20	13	101
y	22	48	34	210

12 Tania has substituted correctly. Suri has forgotten that $7x$ means $7 \times x$. In the future, Suri could include this in the first step of her working.

Exercise 5.5

1 (a)

Number of squares (s)	1	2	3	4	5	6	7	8
Number of matches (m)	4	7	10	13	16	24	28	32

(b) Let s = number of squares, m = number of matches.
$m = 4s$

(c) 400 matches are needed to make 100 squares.

2 (a)

B	1	2	3	4	5
S	4	7	10	13	16

(b) $S = 3B + 1$ **(c)** $B = 21$ gives $S = 64$ sleepers needed

3 (a)

T	1	2	3	4	5
P	3	5	7	9	11

(b) $P = 2T + 1$

(c) $T = 203$ gives $P = 407$
Clarence would need 407 pieces of wood.

4 (a)

Number of houses (h)	1	2	3	4	5	6	7	8
Number of matches (m)	6	11	16	21	26	31	36	41

(b) $m = 5h + 1$

(c) If $h = 20$

$m = 5 \times 20 + 1$
$= 101$

101 matches are needed to build 20 houses.

5 (a)

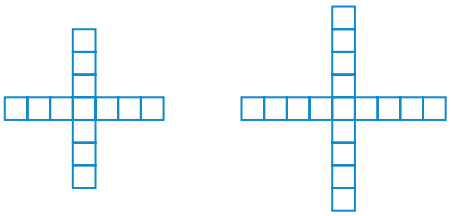
Height of the L (H)	2	3	4	5	6
Number of globes (G)	3	5	7	9	11

(b) $G = 2H - 1$

(c) $H = 120$ gives $G = 239$. So, 239 globes would be needed.

6 B

7 (a)



(b)

a	1	2	3	4
b	5	9	13	17

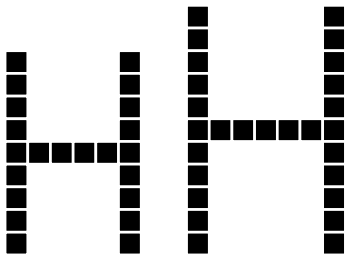
(c)

a	1	2	3	4	5	6
b	5	9	13	17	21	25

(d) $b = 4a + 1$

(e) $a = 52$ gives $b = 209$
209 bricks would be needed.

8 (a)

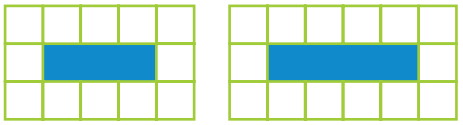


(b)

Width of the H (w)	3	4	5	6	7
Number of tiles (t)	7	12	17	22	27

(c) $t = 5w - 8$ **(d)** For $w = 12$, $t = 52$ tiles needed.

9 (a)

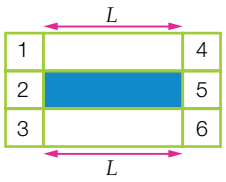


(b)

l	1	2	3	4
p	8	10	12	14

(c) $p = 2l + 6$

(d)



The constant is for the 3 paving blocks at each end of the pool. The coefficient of the pronumeral '2' is the number of paving blocks needed to increase the length each time.

(e) $p = 2(l + 3)$

(f) $l = 345$ gives $p = 696$
696 paving blocks are needed.

(g) The spa is 17 paving blocks long.

10 (a) **(i)** $m = 3s$ **(ii)** $m = 4s$

(iii) $m = 2s + 1$ **(iv)** $m = 3s + 1$

(b) In patterns **(i)** and **(ii)**, the number of matchsticks that are required to make each new shape is the same as the number of sides of the shape (3 for triangles and 4 for squares). In patterns **(iii)** and **(iv)** 1 less matchstick is required, because the new triangle or square joins on to an existing shape. This is shown in the formula of $2s + 1$ and $3s + 1$ instead of $3s$ and $4s$.

11 (a) 16

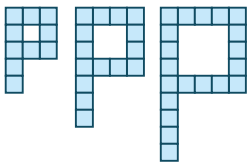
(b) 25

(c)

Large triangle	1	2	3	4	5	6
Number of triangles	1	4	9	16	25	36

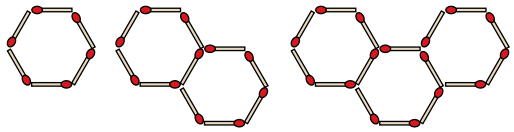
Open-ended—Sample answers

12

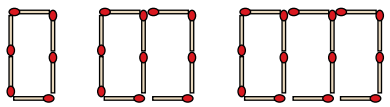


Width of P = w
Number of squares = s
$s = 5w - 5$

13 (a)



(b)



Half-time 5

1 B

2 (a) 2 **(b)** -40 **(c)** 81

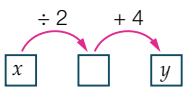
3 A

4 (a) $p + 7 = 28$

(b) $7x - 4 = 20$

(c) $w + g - 4 = 14$

5 (a)



(b) $y = 4 + \frac{x}{2}$

(c)

x	2	8	10	5	13
y	5	8	9	6.5	10.5

6 $6n + 11$ **7** D

8 (a) $C = 35m + 50$ (cost in cents)

(b) 260 cents = \$2.60

(c) For $n = 20$, $C = 750$ (\$7.50), so yes, he has enough money.

Exercise 5.6

1 (a) yes **(b)** no **(c)** no **(d)** yes **(e)** no
(f) yes **(g)** yes **(h)** yes **(i)** no **(j)** yes
(k) no **(l)** no

2 (a) $7x$ **(b)** $16y$ **(c)** $13a$ **(d)** $4y$
(e) $4m$ **(f)** $-11x$ **(g)** $5mn$ **(h)** $5xy$
(i) $z + 4$ **(j)** $7x + 4y$ **(k)** $14w + 3z$ **(l)** $12a + 2b$

3 (a) $10e - 2f$ **(b)** $8p - 4q$ **(c)** $-8t + 12s$
(d) $9x - 4y + y^2$ **(e)** $15w - 2z - 3z^2$ **(f)** $10m + 8n - 5m^2$
(g) $7a + 9$ **(h)** $8x + 5$ **(i)** $20y - 53$

4 (a) A **(b)** D

5 (a) $13e + 14f + 6$ **(b)** $18x + 2y + 2$
(c) $1 + 6x + 5y$ **(d)** $2x - 3y + 4$
(e) $9m - 2n + 3mn$ **(f)** $6a + 11b - 5c - 7$

6 (a) (i) Rule 1: $4x + 6x$
Rule 2: $10x$

(ii) Rule 1:

x	2	3	4
$4x + 6x$	20	30	40

Rule 2:

x	2	3	4
$10x$	20	30	40

(iii) yes, same result

(b) (i) Rule 1: $15y - 10y$
Rule 2: $5y$

(ii) Rule 1:

y	2	3	4
$15y - 10y$	10	15	20

Rule 2:

y	2	3	4
$5y$	10	15	20

(iii) yes, same result

7 (a) (i) $2x$ **(ii)** $4x$ **(iii)** $32x$

(b) $x + 2x + 4x + 8x + 16x + 32x = 63x$

(c) $\$63xv$

(d) $63 \times 6 = 378$. $x = 3$, $v = 2$

8 (a) $\$27d$ **(b)** $d + 3d + 9d + 27d = \$40d$

(c) \$320 **(d)** $81d = 567$, $d = 7$. She received \$7.

9 C

Open-ended—Sample answers

10 (a) $6x + 11y$ **(b)** $15x + 25y$

(c) x and y are either both even or both odd; e.g. $x = 1$, $y = 3$: $15x + 25y = 90$; $x = 2$, $y = 4$: $15x + 25y = 130$.

(d) Students' own answers.

11 (a) $N = 4L$ **(b)** $N = 2L + 2W$

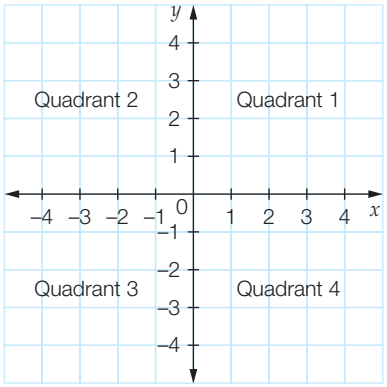
(c) Square garden bed of length 4 sleepers. Rectangular garden bed length 8 sleepers, width 7 sleepers.

(d) $L = 2W$, $N = 2 \times 2W + 2W = 6W$

(e) 10×5, 8×4, 6×3

Exercise 5.7

1



2 (a) $A(4, 3)$, $B(1, -3)$, $C(-4, 5)$, $D(-4, -4)$, $E(-5, 0)$, $F(0, 0)$, $G(2, 0)$, $H(0, 4)$

(b) A quadrant 1, C quadrant 2, D quadrant 3, B quadrant 4, E–H not in a quadrant

3 (a) (i) quadrant 2 **(ii)** quadrant 4
(iii) quadrant 3 **(iv)** quadrant 1
(v) quadrant 2 **(vi)** quadrant 4
(vii) quadrant 3 **(viii)** quadrant 2

(b) (i) y-axis **(ii)** y-axis
(iii) both **(iv)** x-axis
(v) x-axis

4 (3, -3), (-2, -2), (-3, 0), (1, -1), (3, 1), (-1, 1), (-4, 3), (2, 4), (5, 2)

5 (a) C **(b)** A **(c)** B

6 (a) D **(b)** D **(c)** C

7 (a) A **(b)** C **(c)** B

8 (a) C **(b)** D

9 The result is a butterfly.

10 The result is a castle.

11 (a) C **(b)** A

12 **(a)** (4, 6) **(b)** (2, 5)

(c) (2, 1) **(d)** (3, 3)

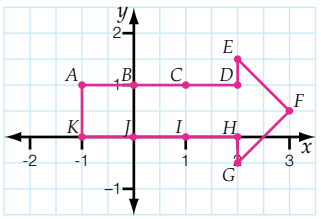
(e) (5, 4) and (6, 4) **(f)** (6, 1), (6, 2), (7, 2), (7, 1)

(g) (1, 4) **(h)** (7, 6)

13 Join (7, 3) (1, 6) (-5, 3) (7, 3) STOP
Join (-4, 3) (-4, -3) (6, -3) (6, 3) STOP
Join (0, -3) (0, -1) (1, -1) (1, -3) STOP
Join (-3, 2) (-1, 2) (-1, 1) (-3, 1) (-3, 2) STOP
Join (-2, 1) (-2, 2) STOP
Join (4, 1) (4, 2) (2, 2) (2, 1) (4, 1) STOP
Join (3, 1) (3, 2) STOP
Join (4, -2) (4, -1) (2, -1) (2, -2) (4, -2) STOP
Join (3, -2) (3, -1) STOP
Join (-1, -2) (-1, -1) (-3, -1) (-3, -2) (-1, -2) STOP
Join (-2, -2) (-2, -1) STOP

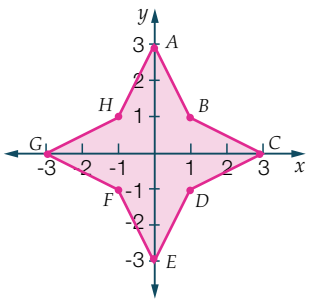
Open-ended—Sample answers

14



A (-1, 1), *B* (0, 1), *C* (1, 1), *D* (2, 1), *E* (2, 1.5), *F* (3, 0.5), *G* (2, -0.5), *H* (2, 0), *I* (1, 0), *J* (0 ,0), *K* (-1, 0)

15 **(a)**

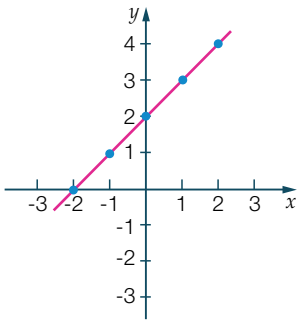


(b) Join the following points with straight lines.
A(0, 3), B(1, 1), *C*(3, 0), *D*(1, -1), *E*(0, -3), *F*(-1, -1), *G*(-3, 0), *H*(-1, 1)

(c) Colour the inside of the shape. Student to give to another student.

Exercise 5.8

1 **(a)–(b)**



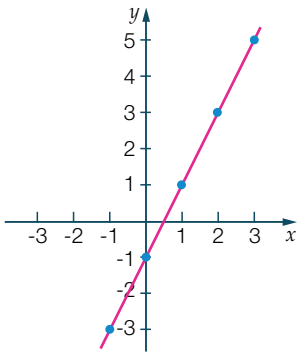
(c)

x	-2	-1	0	1	2
y	0	1	2	3	4

(d) $y = x + 2$

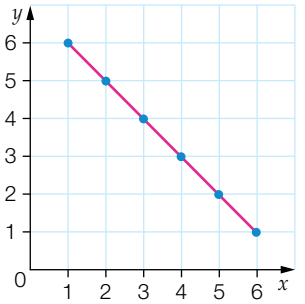
2 **(a)** (-1, -3), (0, -1), (1, 1), (2, 3), (3, 5)

(b)–(c)



(d) $y = 2x - 1$

3 **(a)–(b)**



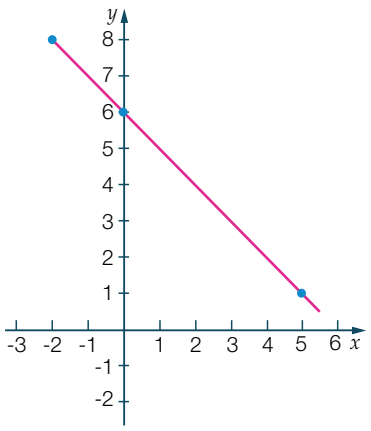
(c)

x	1	2	3	4	5	6
y	6	5	4	3	2	1

(d) $y = 7 - x$

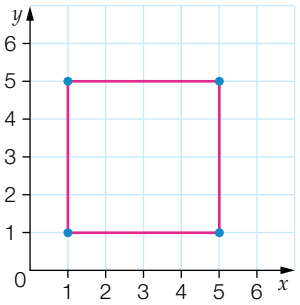
4 C

5 **(a)–(b)**



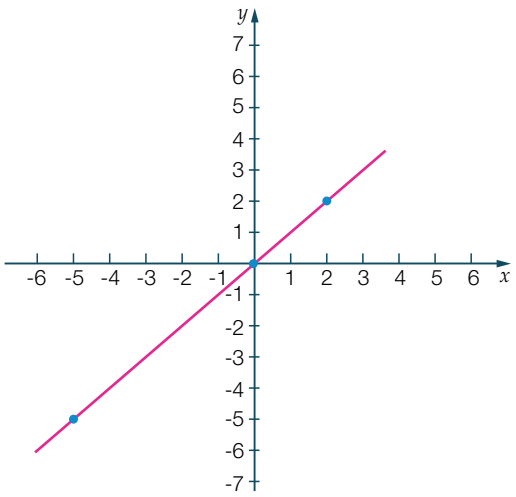
(c) $y = 7$ **(d)** $x = 1$

6 **(a)**



(b) square

7 (a)–(b)



(c) Any three of: (1, 1), (3, 3), (-1, -1), (-2, -2), (-3, -3), (-4, -4),

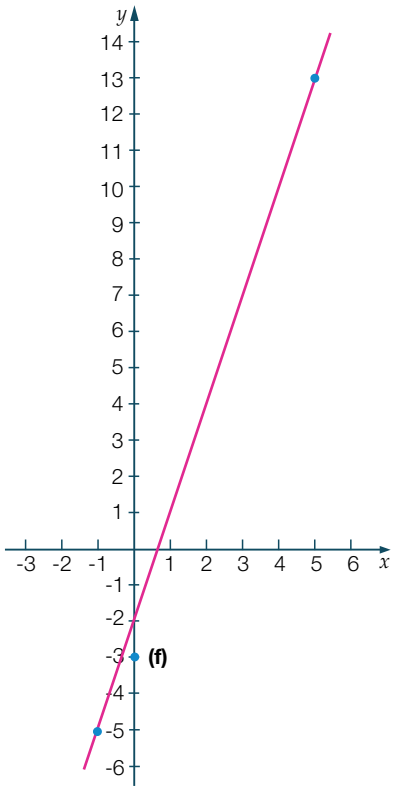
(d)

x	-5	-4	-3	-2	-1	0	1	2	3
y	-5	-4	-3	-2	-1	0	1	2	3

(e) $y = x$

(f) Where $x = -3$, $y = -3$, therefore (-3, -3) does lie on the line.

8 (a)



(b) (2, 4), (3, 7), (4, 10)

(c)

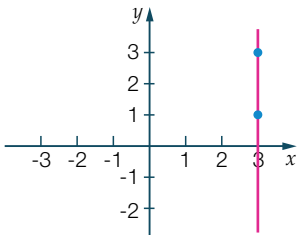
x	-1	2	3	4	5
y	-5	4	7	10	13

(d) $y = 3x - 2$

(e) Where $x = 0$, $y = -2 \neq -3$, so (0, -3) is not on the line.

Open-ended—Sample answers

9 (a)–(b)

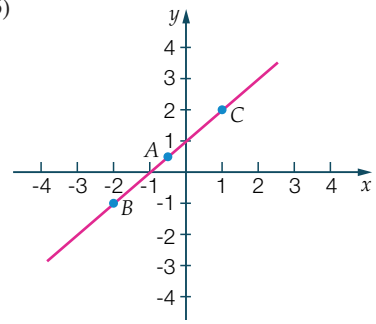


(c) (3, -1) and (3, 2)

(d) $x = 3$

(e) (0, -3), (1, -1) and (2, 1); $y = 2x - 3$

10 (a) (-0.5, 0.5)



(b) (-2, -1) and (1, 2)

(c) $y = x + 1$

Exercise 5.9

1 (a) F **(b)** T **(c)** T **(d)** T

2 (a) (0, 0) **(b)** 12 km **(c)** $\frac{1}{2}$ h **(d)** 1 h

(e) 6 km **(f)** $\frac{1}{2}$ h

(g) They were away for 3 hours.

(h) They rode the fastest from Jim to Terry's house.

3 (a) Yiannis

(b) Meg

(c) Yiannis and Alex take the same shoe size.

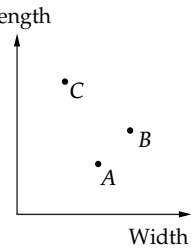
(d) Meg and Alex are the same age.

4 *A*: elephant; *B*: giraffe; *C*: kangaroo

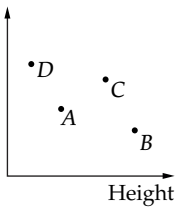
5 D

6 (a) *A*: salmon; *B*: John Dory; *C*: pike

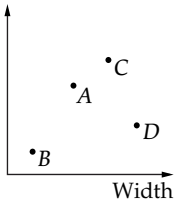
(b)



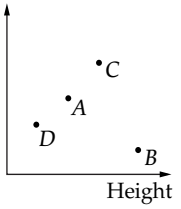
7 (a) Width



(b) Capacity



(c) Capacity



8 (a) 57%

(b) heavy rains

(c) It rains more in winter and autumn, which fall in the second half of the year in Melbourne.

(d) Melbourne's rainfall was well below average—very little rain.

(e) June–July 2007, ≈ 28%

9 (a) Yes, although the start of 2009 was a bit lower than the others. Highest ≈ 39%, lowest ≈ 35%, so difference ≈ 4%.

(b) 54 – 38 = 16% increase

(c) Oct–Dec 2010

(d) Jun–Aug 2009

10 (a) Yes, the covers reduce water temperature, so evaporation would also be reduced.

(b) floating

(c) during the warmer days

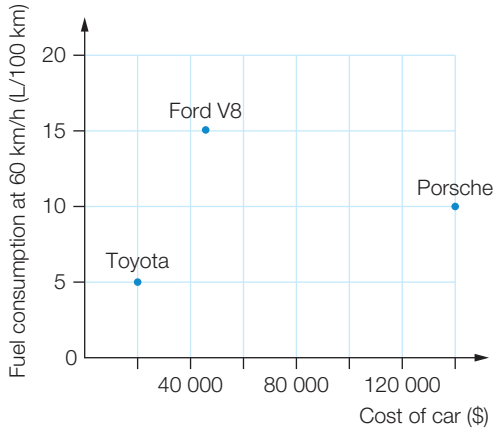
(d) They reduce water temperature by about 10 °C.

11 (a) height and shoe size

(b) The greater the height, the larger the shoe size.

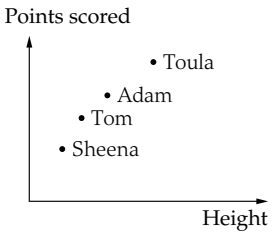
12 (a) no, the positions of the points do not appear to be visually related in any way

(b)



Open-ended—Sample answers

13 Students' own answers, for example:



14 Students' own answers.

Challenge 5

1 George has four 50c pieces.

By trial and error: as 8 + 7 = 15, start with these numbers as they are nearly the same.

8 × 50c = $4, 7 × 20c = $1.40; Value = $5.40

Reverse numbers:
7 × 50c = $3.50, 8 × 20c = $1.60; Value = $5.10

This is 30c less, so fewer 50c pieces are needed.

6 × 50c = $3, 9 × 20c = $1.80; Value = $4.80

Reverse numbers:
9 × 50c = $4.50, 6 × 20c = $1.20; Value = $5.70

This is 90c more, so fewer 50c pieces are needed.

4 × 50c = $2, 11 × 20c = $2.20; Value = $4.20

Reverse numbers:
11 × 50c = $5.50, 4 × 20c = 80c; Value = $6.30

This is $2.10 more, as required.

2 $2 ✹ 8 = 3(2 + 8) = 3 × 10 = 30$

$5 ✹ (2 ✹ 8) = 5 ✹ 30 = 3(5 + 30) = 3 × 35 = 105$

3 The house numbers are 21 and 23.

As house numbers go up by 2 and are even on one side of the street and odd on the other, need to find two numbers that differ by two and multiply to give 483. As the product is odd, then each of the numbers is odd. As 400 = 20 × 20, try the next two odd numbers: 21 × 23 = 483.

4 $\frac{4}{x+1}, \frac{4}{x}, \frac{x}{4}, \frac{x+1}{4}$

If $x > 4$, then $\frac{x}{4} > 1$ and $\frac{x+1}{4} > 1$

As $x + 1 > x$, then $1 < \frac{x}{4} < \frac{x+1}{4}$.

The reverse applies with x in the denominator. If $x > 4$, then $\frac{4}{x} < 1$ and $\frac{4}{x+1} < 1$. As $x + 1 > x$, then the fraction with denominator x is larger than the fraction with denominator $x + 1$, so $\frac{4}{x+1} < \frac{4}{x} < 1$.

Hence, $\frac{4}{x+1} < \frac{4}{x} < 1 < \frac{x}{4} < \frac{x+1}{4}$.

5 B If M = 1, then L = N and K = P, which is not possible as the letters are different numbers.

If L = 1, then M = N, not possible.

N can't be 1 as none of the products of the pairs of numbers ends in 1.

P > 1 as K × M > 1. Hence, K = 1.

L, M, and N can't equal 5. If L = 5 or M = 5, then N = 0 or 5. Hence, P = 5.

Using 2, 3 and 4, the only multiplication is $3 \times 4 = 12$.

Hence, N = 2 and P = 5 = 1 + 4, so M = 4, L = 3.

6 $a = 1$; b can be any number.
(*Note:* Zero is neither positive nor negative.)

7 **(a)** 15 correct means 15 wrong. Marks = $15 \times 9 - 15 \times 5 = 60$

(b) 14 correct means 16 wrong, Marks = $14 \times 9 - 16 \times 5 = 46$ is the most.

(c) It is not possible to score zero on the test.

By trial and error, getting more wrong than correct:

12 correct, 18 wrong:
Marks = $12 \times 9 - 18 \times 5 = 108 - 90 = 18$

11 correct, 19 wrong:
Marks = $11 \times 9 - 19 \times 5 = 99 - 95 = 4$

10 correct, 20 wrong:
Marks = $10 \times 9 - 20 \times 5 = 90 - 100$ = less than zero (-10)

Or, algebraically:

x correct means $(30 - x)$ wrong, so:
Marks = $9 \times x - 5 \times (30 - x) = 14x - 150$

As 14 is not a factor of 150 you can never score 0.

8 The numbers 1 and 8 have only one number next to them, so they should be put in the middle two squares.

	5	3	
7	1	8	2
	4	6	

9 If half of x is 24, then x is 48, so twice x is 96.

10 B $90 \div 3 = 30$, use consecutive integers either side.

11 64 and 15 625.

This can be based on the result $2 \times 5 = 10$, so one of the numbers has only factors of 2 and the other only factors of 5. As there are six zeros, the numbers are 2^6 and 5^6.

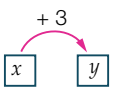
Chapter review 5

1 **(a)** $12n$ **(b)** $\frac{p}{2} - 3$

(c) $7p - 9$ **(d)** $\frac{11 - g}{5}$

2 **(a)** Joe's age is represented as J; Beth's age is represented as B; $J = 3B + 4$

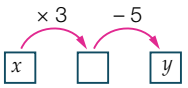
(b) Cost of a bottle of juice = j(\$); cost of a salad roll = r(\$).
$3j + 5r = 18.60$

3 **(a)** **(i)** 

(ii) $y = x + 3$

(iii)

x	57	34	12	4	1.1	64
y	60	37	15	7	4.1	67

(b) **(i)** 

(ii) $y = 3x - 5$

(iii)

x	4	3	$2\frac{1}{2}$	9	10	12
y	7	4	$2\frac{1}{2}$	22	25	31

4 **(a)** B **(b)** C

5 **(a)** **(i)** -54 **(ii)** 26

(b) **(i)** 7 **(ii)** -7

6 C

7 **(a)**

x	7	9	12	20	8.1	107
y	0	2	5	13	1.1	100

(b)

x	4	$5\frac{1}{2}$	13	7	78	54
y	10	25	100	40	750	510

8 **(a)**

Number triangles (t)	1	2	3	4	5
Number girders (g)	3	5	7	9	11
Number bolts (b)	3	4	5	6	7

(b) $g = 2t + 1$

(c) $b = t + 2$

(d) **(i)** 2

(ii) In the table of values, as the number of triangles increases by 1, the number of girders increases by 2. In the rule, 2 is the coefficient of t.

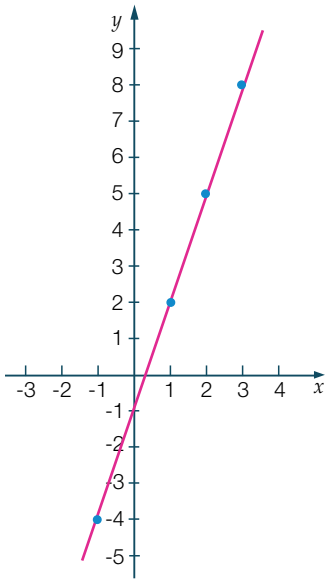
(e) **(i)** 1

(ii) In the table of values, as the number of triangles increases by 1, so does the number of bolts. In the rule, the coefficient of t is 1.

9 (a) $5a$ (b) $-a + 23b$ (c) $4x + y$

10 (a) (2, 3) (b) (-2, 1) (c) (3, -2) (d) (-4, -3)

11 (a)



(b)

x	-1	1	2	3
y	-4	2	5	8

(c) $y = 3x - 1$

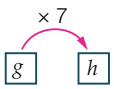
12 (a) \$3.50

(b) 3 L is the best value because it costs less per litre than the other sizes (\$1.67 compared to \$1.75 and \$2.00).

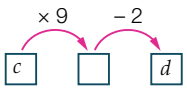
(c) This information is given as a point graph to enable comparison of prices of different sizes, because only these point values are sold. (A straight line would imply that any volume can be bought for a corresponding cost, which is not true.)

13 (a) $8n + 3$ (b) $\frac{8n+3}{15}$ (c) $n = 9$

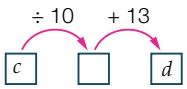
14 (a)



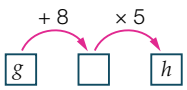
(b)



(c)



(d)



15 (a) $b = \frac{n}{4} + 5$ (b) 9

16 (a) $5a - 7b + 2$ (b) $5a + 9b - 7$ (c) $x + y + xy$

17 B

18 (a)

x	10	-35	100	5	0	-30
y	2	-7	20	1	0	-6

(b)

x	6	10	3	-1	0	5
y	72	120	36	-12	0	60

19 (a) $y = x - 7$ (b) $y = 2x + 4$

20 (a) $1.4c + 1.8d = 21$ (b) 7 apples and 6 oranges

(c) \$6.80

21 (a) 1 500 000 ML (b) August '03

(c) Jan '12 (d) It rained heavily.

(e) There was very little rain—well below average.

(f) from April '09 to April '12

Numeracy practice 5

1 7 **2** B

3 (a) 24 (b) -6

4 (a) 2, 1, 3 (b) 3, 1, 2 or 2, 3, 1

5 \$11.00 **6** D

Chapter 6

Recall 6

1 (a) 16.5 (b) 40.32 (c) 39.26

(d) 25.2 (e) 56.12 (f) 18 960

(g) 27.8 (h) 0.3 (i) 0.0459

2 (a) (i) 13 mm (ii) 1.3 cm

(b) (i) 28 mm (ii) 2.8 cm

3 (a) 16 cm (b) 12 cm^2

4 (a) 2000 m (b) 3 cm (c) 500 cm

(d) 1.5 kg (e) 4000 mL (f) 20 000 g

(g) 0.25 L (h) 1750 mm (i) 0.5 kg

5 12 cubes

Exercise 6.1

1 (a) 4 m (b) 5–6 m (c) 8–10 m (d) 20 mm

2 (a) 5000 m (b) 3600 m (c) 8 m

(d) 6500 cm (e) 55 cm (f) 120 cm

(g) 29 mm (h) 61 mm (i) 3.5 mm

(j) 0.9 m (k) 0.003 m (l) 37.5 m

(m) 4.2 km (n) 0.57 km (o) 0.0092 km

(p) 8 cm (q) 25.5 cm (r) 0.18 cm

3 (a) 3200 mm (b) 4950 mm (c) 0.09 km

(d) 0.345 km (e) 300 cm (f) 0.56 m

(g) 0.097 m (h) 342 000 mm (i) 0.0078 km

(j) 192 000 cm (k) 0.024 km (l) 890 mm

4 (a) cm (b) km (c) m

(d) cm (e) km (f) m

5 (a) B (b) C (c) B (d) C (e) D

6 **(a)** 4 cm, 0.4 m, 4000 mm, 0.04 km

(b) 360 m, 290 000 cm, 3100 m, 3.2 km

(c) 5600 cm, 71.4 m, 0.64 km, 820 000 mm

(d) 0.9 cm, 90 mm, 0.095 m, 0.0089 km

7 5.96 m **8** 2.745 km **9** about 4.7 m

10 length = 8.9 cm, leg span = 25.4 cm

11 2.29 m across, 36 m long

12 596 mm, 59.6 cm

13 width = 1200 mm, length = 3700 mm

14 **(a)** 1100 km **(b)** 2500 km **(c)** 3700 km

15 2.5 m **16** 2 km

17 **(a)** **(i)** The lines are the same length.

(ii) The central circles are the same size.

(iii) The hat is as wide as it is tall.

(b) **(i)** The external arrows make the lines appear longer.

(ii) Being surrounded by bigger dots makes the central dot appear smaller.

(iii) The vertical distance appears bigger by contrast.

Open-ended—Sample answers

18 Decimal points are not needed and conversions are not necessary.

19 **(a)** used division instead of multiplication; 2700 cm

(b) divided by 100 instead of 10; 76.5 cm

(c) zero inserted between 3 and 8; 3.8 km

(d) divided by 1000 instead of 100; 13.56 m

20 Inaccurate measuring with the ruler. The student with the longer length could be including the ends of the ruler before 0 and after 30.

Exercise 6.2

1 **(a)** 16 cm **(b)** 22 cm **(c)** 28 m

(d) 86 mm **(e)** 100 m **(f)** 60 m

2 **(a)** 42 m **(b)** 40 cm **(c)** 24 cm

(d) 120 mm **(e)** 14.4 m **(f)** 68 mm

3 **(a)** C **(b)** C **(c)** C

4 **(a)** 140 mm **(b)** 122 mm

(c) 390 cm **(d)** 6000 m or 6 km

5 8.1 km **6** 14 m **7** 360 cm **8** 648 m

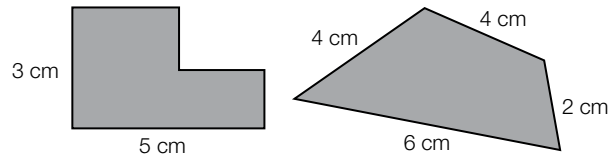
9 $384 **10** Students' own answers.

11 A **12** B **13** 82 m

Open-ended—Sample answers

14 21 cm and 21 cm; 22 cm and 20 cm; 30 cm and 12 cm

15



Exercise 6.3

1 **(a)** 5 cm^2 **(b)** 15 cm^2 **(c)** 4 cm^2

(d) 28.2 cm^2 **(e)** 24 cm^2 **(f)** 36 cm^2

(g) 36 m^2 **(h)** 87 mm^2 **(i)** 80 km^2

(j) 240 m^2 **(k)** 1.44 m^2 **(l)** 9 cm^2

2 **(a)** cm^2 **(b)** mm^2 **(c)** cm^2 **(d)** m^2

(e) m^2 **(f)** km^2 **(g)** mm^2 **(h)** m^2

3 **(a)** B **(b)** A **(c)** D

4 **(a)** **(i)** 20 cm **(ii)** 9 cm^2

(b) **(i)** 18 cm **(ii)** 8 cm^2

(c) **(i)** 24 cm **(ii)** 14 cm^2

(d) **(i)** 32 cm **(ii)** 20 cm^2

5 16 cm^2 **6** 2.4 km^2 **7** 3150 cm^2

8 4 cm **9** 5.2 m

10 **(a)** 20 cm^2 **(b)** 36 cm^2

11 20 cm

12 **(a)** 5.04 m^2 **(b)** 30.24 m^2 **(c)** 196.56 kWh

13 **(a)** 3519 grams **(b)** 30.8 m

14 525 bricks

15 4418 cm^2

16 **(a)** 330 000 m^2 **(b)** 19 800 kg **(c)** 3350 m

17 **(a)** 8 cm^2 **(b)** 8 cm^2

(c) 12 cm^2 **(d)** 14 cm^2 (approx.)

18 Approximate answers are:
(a) 131 m^2 **(b)** 45 km^2

19 Possible answers: 5 cm by 4 cm, 10 cm by 2 cm; 20 cm by 1 cm

20 12 m long, 5 m wide

Open-ended—Sample answers

21 Ethan's shape Magda's shape

22 Students' own answers, for example:

(a) measure with a ruler

(b) measure with a long ruler or a measuring tape

(c) measure with a long measuring tape or a measurement wheel

23 Students' own answers.

24 **(a)** Panel 2: rectangle's perimeter is 16 m.

(b) Perimeter is the length around a shape; area is the space inside a flat shape.

(c) The 'little 2' means 'squared'. Area is measured in square units.

25 wood, carpet, turf (lawn)

Half-time 6

1 **(a)** **(i)** 19 m **(ii)** 21 m^2

(b) **(i)** 14 m **(ii)** 12.25 m^2

2 **(a)** 0.45 m **(b)** 15 m **(c)** 0.307 km

(d) 60 000 cm **(e)** 196 km **(f)** 0.0264 km

3 **(a)** 11.7 kg **(b)** 16.4 m

4 **(a)** m^2 **(b)** mm^2 **(c)** cm^2 **(d)** km^2

5 length 10 cm, width 3 cm

6 **(a)** 457.5 m **(b)** 465.125 m^2 **(c)** 310.1 m^2

Exercise 6.4

1 **(a)** 65 cm^2 **(b)** 60 m^2 **(c)** 176 mm^2

(d) 350 mm^2 **(e)** 144 m^2 **(f)** 80 cm^2

2 **(a)** 132 cm^2 **(b)** 64 cm

3 **(a)** B **(b)** C **(c)** C

4 7.5 cm

5 4 mm

6 20 cm

7 **(a)** 8 **(b)** 442 cm^2

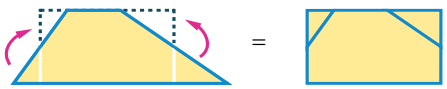
8 **(a)** 20 m^2

(b) The parallelograms can be sewn together as the shape shown in Question **9** and it will provide the same amount of shade.

9 **(a)** 40 cm^2

(b) 80 cm^2

10 **(a)**



(b) 15 cm^2

Open-ended—Sample answers

11 Possible dimensions: base 8 cm, height 4 cm; base 16 cm, height 2 cm; base 1 cm, height 32 cm

Exercise 6.5

1 **(a)** 48 cm^2 **(b)** 35 mm^2 **(c)** 42 m^2

(d) 88 mm^2 **(e)** 180 m^2 **(f)** 60 cm^2

(g) 9 m^2 **(h)** 19.2 mm^2 **(i)** 33.88 cm^2

2 **(a)** 19 cm^2 **(b)** 36 cm^2 **(c)** 85 cm^2

(d) 87 cm^2 **(e)** 281.65 cm^2 **(f)** 51 cm^2

(g) 279 cm^2 **(h)** 126 cm^2

3 **(a)** A **(b)** D **(c)** B

4 4.76 m^2

5 1600 cm^2

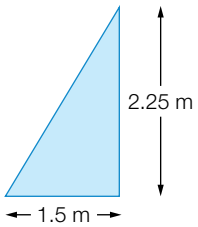
6 3

7 **(a)** 144 cm^2 **(b)** 115.5 cm^2 **(c)** 320 cm^2 **(d)** 105.5 cm^2

8 400 cm^2

9 **(a)** 1.6875 m^2

(b)



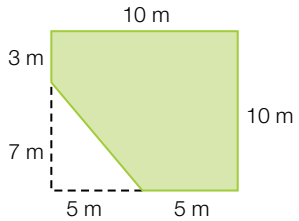
10 **(a)** 10.36 m^2

(b) 1.64 L

11 3 m^2

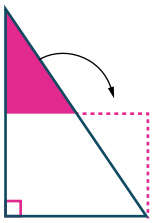
12 Find the area of the big square and subtract the area of the small triangle.

$A = (10 \times 10) - (\frac{1}{2} \times 5 \times 7)$
$= 100 - 17.5$
$= 82.5\ m^2$



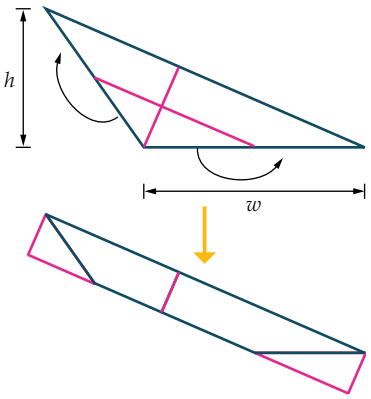
13 **(a)** The cut should be made exactly half-way up the height, parallel to the base or exactly half-way along the base, parallel to the height.

(b)



14 **(a)** Rule a line from the apex to the bottom side that is perpendicular (at right angles) to the bottom side. Half-way along this line, rule a second line parallel to the bottom line. Cut along these two lines.

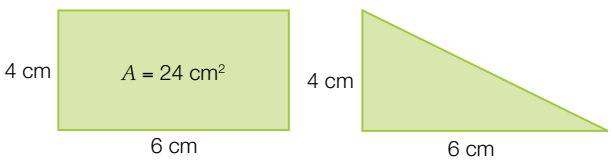
(b)



Open-ended—Sample answers

15 9 cm and 8 cm; 6 cm and 12 cm

16



Exercise 6.6

1 **(a)** 4 cm^3 **(b)** 16 cm^3 **(c)** 40 cm^3
(d) 24 cm^3 **(e)** 54 cm^3 **(f)** 60 cm^3

2 **(a)** 60 cm^3 **(b)** 60 cm^3 **(c)** 48 cm^3
(d) 120 cm^3 **(e)** 125 cm^3 **(f)** 320 cm^3

3 12 m^3 **4** 15 cm^3 **5** 216 cm^3

6 **(a)** D **(b)** A **(c)** D

7 **(a)** 28 cm^3 **(b)** 10 cm^3 **(c)** 18 cm^3
(d) 20 cm^3 **(e)** 6 cm^3 **(f)** 4 cm^3
(g) 4 cm^3 **(h)** 7 cm^3 **(i)** 10 cm^3

8 3.2 m^3

9 2352 cm^3

10 0.69 m^3

11 Students' own answers.

12 4800 cm^3, the volume will also double.

13 19 200 cm^3

14 100

15 12 cm

Open-ended—Sample answers

16 3 cm × 2 cm × 5 cm; 6 cm × 5 cm × 1 cm

17 **(a)** 900 jelly beans

(b) Calculate the dimensions of the jar (length, width and height) in terms of jelly beans, and then calculate the volume in terms of jelly beans.

(c) More jelly beans would be able to fit into the jar, as they would pack more closely together with less space between them.

18 The side length of the large cube must be 4 cm, to give a volume of 64 cm^3. The 3 possible smaller cubes must have side lengths of 1 cm, 2 cm and 3 cm, giving volumes of 1 cm^3, 8 cm^3 and 27 cm^3 respectively.

Volume of smaller cube	Volume of empty space in large cube
1 cm^3	63 cm^3
8 cm^3	56 cm^3
27 cm^3	37 cm^3

Challenge 6

1 $1 000 000

2 C Sides of square are 200 m each. Each rectangle is 100 m by 50 m. Perimeter = 2 × (100 + 50) m

3 D

4 Let square have sides of $2x$ cm. Rectangle has sides $2x$ cm and x cm. Perimeter of rectangle = $6x$ cm.

$6x = 39$, $x = 6.5$ cm. Area of paper = 13 × 13 = 169 cm^2

5 There are 9 different ways in total.

(Colours are used below only to make it easy to see the tiles.)

The first pattern can only occur in 1 way. The second pattern can occur in 5 ways because the horizontal tiles can be in 5 different places.

The third pattern can occur in 3 ways because the vertical tiles can be in 3 places.

6 B **7** A

8 36 cm, 56 cm, 76 cm

60 cm edge vertical,
volume of water = 200 × 100 × 60 − 100 × 80 × 60 = 720 000 cm^3

After block removed,
depth of water = 720 000 ÷ (200 × 100) = 36 cm

80 cm edge vertical,
volume of water = 200 × 100 × 80 − 100 × 80 × 60 = 1 120 000 cm^3

After block removed,
depth of water = 1 120 000 ÷ (200 × 100) = 56 cm

100 cm edge vertical,
volume of water = 200 × 100 × 100 − 100 × 80 × 60 = 1 520 000 cm^3

After block removed,
depth of water = 1 520 000 ÷ (200 × 100) = 76 cm

9 100 cm^2

Area of each face of bottom cube = 4 × 4 = 16 cm^2

Area of each face of middle cube = 2 × 2 = 4 cm^2

Area of each face of top cube = 1 × 1 = 1 cm^2

Visible area = 5 × 16 − 4 + 5 × 4 − 1 + 5 × 1
= 76 + 19 + 5
= 100 cm^2

Chapter review 6

1 **(a)** B **(b)** C

2 **(a)** 45 900 m **(b)** 0.58 km **(c)** 9200 mm
(d) 4200 cm **(e)** 0.98 km **(f)** 6730 mm

3 **(a)** **(i)** 92 cm **(ii)** 448 cm^2
(b) **(i)** 56 mm **(ii)** 196 mm^2
(c) **(i)** 40 m **(ii)** 66.5 m^2
(d) **(i)** 80 cm **(ii)** 198 cm^2

4 Perimeter = 32.3 m, Area = 62.4 m^2

5 **(a)** 18 cm^2 **(b)** 24 cm^2 **(c)** 126 cm^2 **(d)** 19.5 m^2

6 **(a)** 210 m^3 **(b)** 225 cm^3

7 (a) 54 m^2 (b) 113 cm^2 (c) 4.48 m^2 (d) 94 cm^2

8 (a) 4.85 m (b) $29.10

9 (a) 300 cm by 250 cm, area = 75 000 cm^2

(b) 120 tiles needed (c) $96

10 (a) 7 m (b) 0.9 m^3 (c) $11.25

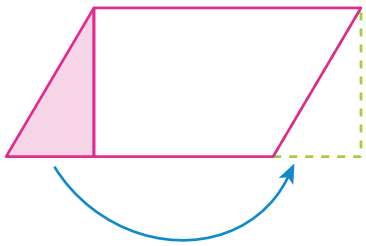
11 $806.40

12 (a) 63 cm^3 (b) 162 cm^3

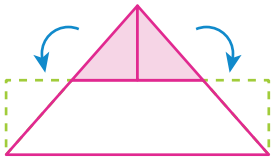
13 6.84 m^3 **14** 20 cm **15** 60 cm^2

16 176 m^2 **17** 6750 cm^3

18 (a)



(b)



Numeracy practice 6

1 rectangle **2** 37.5 m^2 **3** 40 cm **4** 15 cm

5 C **6** 0.05 cm **7** B **8** B

Mixed review C

1 (a) $n - 8$ (b) $\frac{k}{6}$ (c) $50h$

2 (a) 2.003, 2.3, 2.302, 2.323, 2.33

(b) 0.0199, 0.089, 0.0909, 0.091, 0.129

3 (a) $\frac{1}{4}$ (b) $5\frac{1}{3}$ (c) $\frac{5}{12}$

4 (a) $2^2 \times 7$ (b) $2^3 \times 5^2$ (c) $2 \times 3^2 \times 5 \times 11$

5 (a) (i) 26 cm (ii) 36 cm^2

(b) (i) 14 m (ii) 12.25 m^2

(c) (i) 2.45 m (ii) 0.1815 m^2

6 (a) 23 (b) 34 (c) 31

7 (a) $\frac{10}{27}$ (b) $1\frac{13}{14}$ (c) $4\frac{3}{5}$ (d) 6

8 (a) 6090 m (b) 0.83 m (c) 0.054 km

9 (a) 6 (b) -3 (c) -12 (d) -11

(e) -6 (f) -3 (g) 13 (h) -17

10 (a) 1204 (b) 165 (c) 228

11 (a) 3:2:1 (b) 33.3%

12 (a) $\frac{23}{100}$ (b) $\frac{3}{50}$ (c) $\frac{101}{200}$

13 (a) 8 cm^3 (b) 360 m^3 (c) 216 cm^3

14

x	0	2	3	6	10
y	6	12	15	24	36

15 (a) 13.47 (b) 8.936 35 (c) 40.945

16 (a) 0.0031 (b) 0.000 42 (c) 2121

17 $86.52

18 (a) 56 mm^2 (b) 36 m^2

19 (a) Koala $\frac{2}{5}$, Bilby $\frac{3}{10}$, Wombat $\frac{1}{5}$, Kangaroo $\frac{1}{10}$

(b) Koala 40%, Bilby 30%, Wombat 20%, Kangaroo 10%

20 2^8 **21** 4 goals

22 8 cm by 13 cm **23** $3c = h$

Chapter 7

Recall 7

1 (a) F (b) T (c) F

2 (a) C (b) D (c) B

3 (a) $7 + 5 = 12$ (b) $4 \times 6 - 2 = 22$ (c) $\frac{8+6}{7} = 2$

4 (a) 9 (b) 7

5 (a) 7 (b) 2

Exercise 7.1

1 (a) $3 + 7 = 10$ (b) $5 \times 5 = 25$

(c) $\frac{16}{4} = 4$ (d) $22 - 10 = 8 + 4$

(e) $3 \times 5 + 2 = 8 + 9$ (f) $1 + \frac{25}{5} = 6$

2 (a) false (b) false (c) false (d) true (e) true

(f) true (g) false (h) true (i) true

3 (a) D (b) C (c) B

4 (a) Students' own answers, for example $13 + 17 + 16 = 46$

(b) Students' own answers, for example $(20 \times 2) + 6 = 46$

(c) Students' own answers, for example $(100 \div 2) - 4 = 46$

5 (a) $5 + 43 = 48$ (b) $3 \times 7 = 19 + 2$

(c) $\frac{50}{5} = 5 \times 2$ (d) $20 - (6 + 8) = \frac{12}{2}$

6 (a) $13 - 19 = 6$ false (b) $\frac{4}{2} = 8$ false

(c) $8 - 11 = 6 - 3$ false (d) $\frac{12}{4} = \frac{15}{5}$ true

7 $5 + 7 = 12$ **8** $6 \times 60 = 4 \times 90$

9 $25 + $7 = $32 **10** $48 - 11 + 5 = 42$

11 (a) $16 + 17 = 33$ (b) $17 + 18 + 19 = 54$

12 (a) 20 cm (b) 15 cm

(c) 12 cm (d) 10 cm

(e) $60 \div 20 = 3$ (f) $200 \div 50 = 4$

(g) Three from: 5, 10, 20, 25, 50 cm

13 $(16 + 24) = 40 \times 2 = 80$
The equals symbol means that the LHS is equal to the RHS, but $(16 + 24) \neq 80$.

Open-ended—Sample answers

14 $8 + 1 + 2 - 4 = 7$, $8 \times 1 - 4 + 2 = 6$, $8 - \frac{4}{2} + 1 = 7$

15 72, 80, 88, 96, 104

16 $5 + 6 \times 3 - 3 = 20$; $1 + 3 \times 8 - 5 = 20$; $6 + 4 \times 5 - 6 = 20$

17 Ruben is correct. Sam added four before multiplying; which is incorrect. The order of operations should always be followed.

18 Differences need to be kept the same on both sides. Because 12 is 4 less than 16, the missing number must also be 4 less than 38. Therefore, the first solution is correct.

Exercise 7.2

1 **(a)** Three added to a number is equal to ten.

(b) A number added to four is equal to seven.

(c) Five added to a number is equal to six.

(d) Two subtracted from a number is equal to four.

(e) Seven subtracted from a number is equal to two.

(f) Eight subtracted from a number is equal to six.

(g) A number multiplied by five is equal to twenty.

(h) A number multiplied by seven is equal to fourteen.

(i) A number multiplied by seven is equal to twenty-one.

(j) A number divided by three is equal to four.

(k) A number divided by two is equal to six.

(l) A number divided by six is equal to twelve.

(m) Three times a number added to two is equal to seven.

(n) Six times a number added to two is equal to ten.

(o) Three times a number added to one is equal to eight.

2 **(a)** yes **(b)** no **(c)** yes **(d)** yes **(e)** no
(f) yes **(g)** no **(h)** yes **(i)** yes **(j)** no
(k) no **(l)** yes **(m)** yes **(n)** yes **(o)** no

3 **(a)** $x = 4$ **(b)** $x = 7$ **(c)** $x = 7$ **(d)** $x = 29$
(e) $x = 17$ **(f)** $x = 64$ **(g)** $x = 18$ **(h)** $x = 6$
(i) $x = -4$ **(j)** $x = 76$ **(k)** $x = 132$ **(l)** $x = 162$

4 D

5 **(a)** D **(b)** C **(c)** D

6 **(a)** B **(b)** D **(c)** D

7 **(a)** C **(b)** A **(c)** D

8 **(a)** $x = 10$ **(b)** $x = 4$ **(c)** $x = 11$

9 **(a)** D is Daniel's age.

(b) $D - 3$ **(c)** $D + 12$

(d) $D + 12 = 32$, Daniel is 20 years old.

10 **(a)** C is the amount of money Cameron has in his wallet.

(b) $C - 14$

(c) $C - 14 + 5 = 23$, Cameron had \$32 to start with.

Open-ended—Sample answers

11 **(a)** $3c = 36$

(b) $3c = 27$

(c) 9 years old

12 **(a)** Sean started with 14 marbles. He gave a number to his friend Sasha and was left with 11 marbles.

(b) Tali and 4 of her friends shared a packet of lollies equally between them. They received 12 lollies each.

(c) The number of leaves on a tree multiplied by 6 to give 108 leaves in the early days of spring.

13 **(a)** Solve by using guess, check and improve because this equation involves many steps to solve.

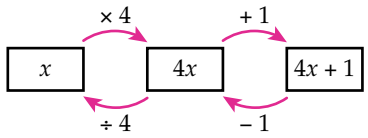
(b) Solve by inspection because it is a one-step equation.

(c) Solve by using guess, check and improve because this equation involves many steps to solve.

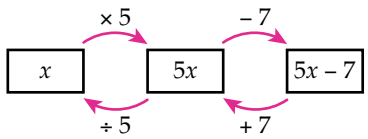
(d) Solve by inspection because it is a one-step equation.

Exercise 7.3

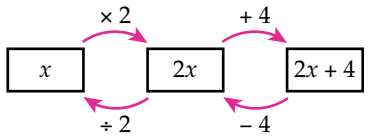
1 **(a)**



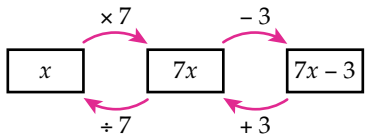
(b)



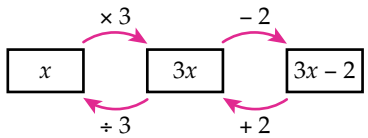
(c)



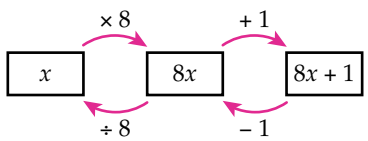
(d)



(e)



(f)



2 **(a)** (i) $3x - 1 = 11$ (ii) $x = 4$

(b) (i) $2x - 3 = 7$ (ii) $x = 5$

(c) (i) $\frac{x+5}{4} = 4$ (ii) $x = 11$

(d) (i) $\frac{x+7}{2} = 8$ (ii) $x = 9$

(e) (i) $\frac{x}{6} + 7 = 9$ (ii) $x = 12$

(f) (i) $\frac{x}{3} + 8 = 13$ (ii) $x = 15$

(g) (i) $\frac{x-3}{5} = 1$ (ii) $x = 8$

(h) (i) $\frac{x-2}{7} = 2$ (ii) $x = 16$

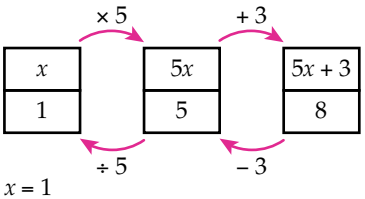
(i) (i) $2(x + 6) = 20$ (ii) $x = 4$

(j) (i) $5x - 9 = 16$ (ii) $x = 5$

(k) (i) $5(x + 2) = 30$ (ii) $x = 4$

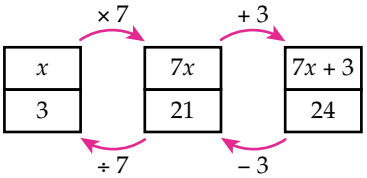
(l) (i) $7(x + 1) = 21$ (ii) $x = 2$

3 **(a)**



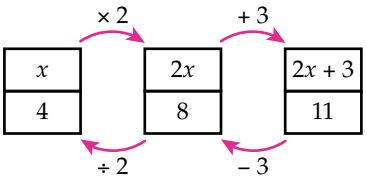
$x = 1$

(b)



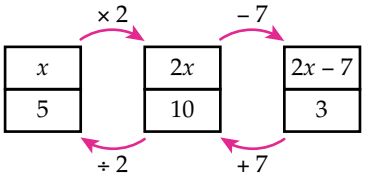
$x = 3$

(c)



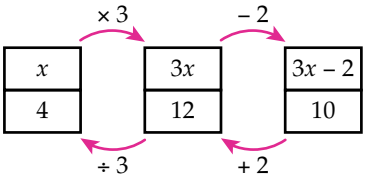
$x = 4$

(d)



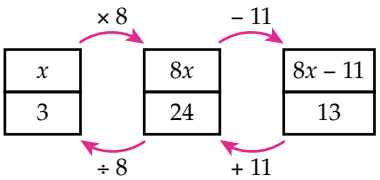
$x = 5$

(e)



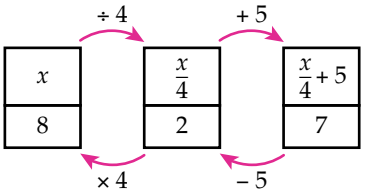
$x = 4$

(f)



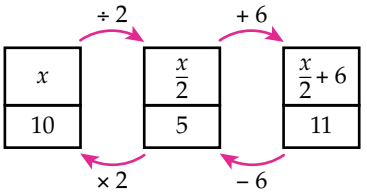
$x = 3$

4 **(a)**



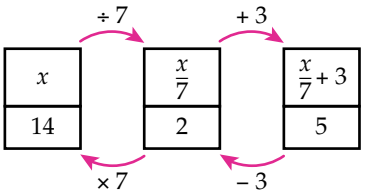
$x = 8$

(b)



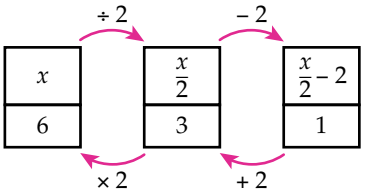
$x = 10$

(c)



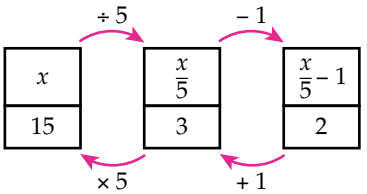
$x = 14$

(d)



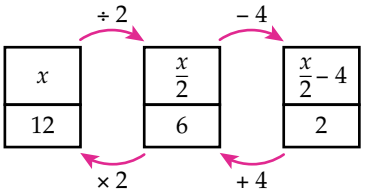
$x = 6$

(e)



$x = 15$

(f)



$x = 12$

5 **(a)** $x = 3$ **(b)** $x = 5$ **(c)** $x = 8$ **(d)** $x = 6$

6 **(a)** $x = 1$ **(b)** $x = 3$ **(c)** $x = 7$ **(d)** $x = 11$

7 **(a)** $x = 5$ **(b)** $x = 3$ **(c)** $x = 1$ **(d)** $x = 4$

(e) $x = 2$ **(f)** $x = 2$ **(g)** $x = 3$ **(h)** $x = 5$

(i) $x = 3$ **(j)** $x = 10$ **(k)** $x = 14$ **(l)** $x = 15$

(m) $x = 6$ **(n)** $x = 14$ **(o)** $x = 7$

8 (a) $x = 4$ (b) $x = 5$ (c) $x = 6$ (d) $x = 5$

(e) $x = 14$ (f) $x = 4$ (g) $x = 11$ (h) $x = 2$

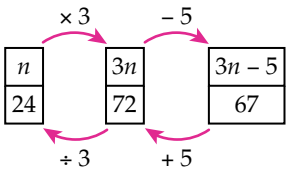
(i) $x = 24$

9 B

10 (a) $2n + 1 = 43$, $n = 21$ (b) $\frac{n+4}{5} = 4$, $n = 16$

(c) $\frac{7n}{3} = 21$, $n = 9$ (d) $\frac{2n+6}{10} = 2$, $n = 7$

11 $3n - 5 = 67$



There were 24 pencils in each packet.

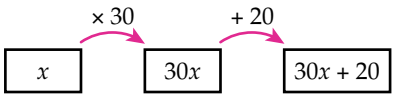
12 (a) $3d - 12$

(b) (i) \$6 (ii) \$0 (iii) \$10.50

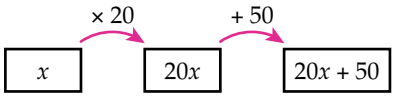
(c) \$18.00

(d) Lui would receive nothing from his parents.

13 (a)



(b)

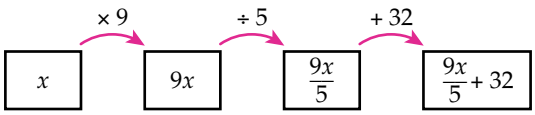


(c) Ying: \$50 Aldo: \$70 (d) $2\frac{1}{2}$ hours

(e) $2\frac{1}{4}$ hours (f) 3 hours

Open ended—Sample answers

14 (a)



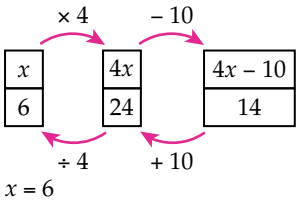
(b) Values include: 5, 10, 15, 20, …

15 $\frac{2x+4}{6} + 6 = 10$

Half-time 7

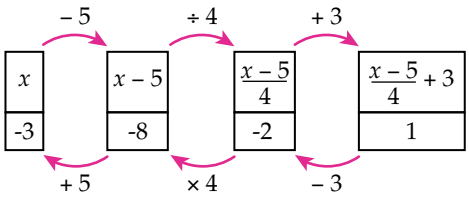
1 $x = 84$ **2** 4 metres

3 (a)



$x = 6$

(b)



$x = -3$

4 $21 + 22 = 43$

5 (a) (i) $j - 5 = 7$ (ii) 12

(b) (i) $d + 4 = 16$ (ii) 12

6 (a) $x = \frac{7}{3}$ (b) $x = -16$ (c) $x = 13$

7 A number is multiplied by 5 and has 3 added to it. The result is 23.

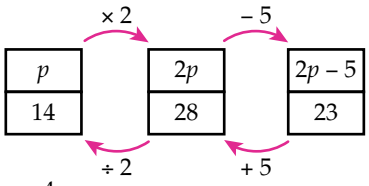
8 (a) $3 + 8 = 11$ (b) $4 \times 5 = 26 - 6$ (c) $4 + \frac{28}{2} = 18$

9 20 cm

10 (a) F (b) T (c) T

11 (a) $p - 5$ (b) $2p - 5 = 23$

(c)



$p = 4$

Pete's age = 14
Sam's age = 9

Exercise 7.4

1 (a) right-hand side (RHS)

(b) Take one chocolate from the RHS.

(c) Take 4 chocolates from the RHS.

(d) 9 chocolates (e) 3 chocolates

2 (a) $x + 8 = 11$ (b) $x + 9 = 17$ (c) $x + 7 = 13$

(d) $2x + 1 = 9$ (e) $3x = 5$ (f) $6x + 3 = 9$

3 (a) $x = 1$ (b) $x = 4$ (c) $x = 2$ (d) $x = 5$

(e) $x = 7$ (f) $x = 9$ (g) $x = 8$ (h) $x = 1$

(i) $x = 3$ (j) $x = 5$ (k) $x = 5$ (l) $x = 8$

(m) $x = 12$ (n) $x = 7$ (o) $x = 8$ (p) $x = 10$

(q) $x = 20$ (r) $x = 11$ (s) $x = 12$ (t) $x = 6$

(u) $x = 7$

4 D

5 (a) $2t + 12 = 104$ (b) 46 kg

6 (a) $3p + 18 = 30$ (b) $p = 4$

7 34 km **8** 1.25 L

Open-ended—Sample answers

9 $5x = 10$, $5x + 5 = 15$, $5x + 9 = 19$

10 (a) Vejay subtracted the 3 incorrectly. He should have done it after multiplying by 7. The order was wrong.

(b) $\frac{2x+3}{7} = 3$

$\frac{2x+3}{7} \times 7 = 3 \times 7$

$2x + 3 = 21$

$2x + 3 - 3 = 21 - 3$

$2x = 18$

$\frac{2x}{2} = \frac{18}{2}$

$x = 9$

(c) He could use a flowchart to work out the order of operations used to 'build up' the equation. He should then work backwards along the flowchart, using the inverse operations, to solve the equation.

Exercise 7.5

1 (a) $x + 20 = 95$, the ice-cream costs 75 cents.

(b) $t + 1.5 = 12.8$, Tony ran the 100 m in 11.3 seconds.

2 (a) $2d + 1.6 = 8$, a sushi roll costs \$3.20.

(b) $3v + 21.5 = 59$, an app costs \$12.50.

3 (a) D **(b)** There are 27 seats available.

4 (a) C **(b)** The total length of the roll of fabric is 15 m.

5 (a) D **(b)** The width of the vegetable garden is 2.5 m.

6 (a) $x + 1.5$

(b) $x + 1.5 = 5.7$; $x = 4.2$; The salad sandwich costs \$4.20.

7 (a) $4x$

(b) $4x = 84$; $x = 21$; The length of the floor tile is 21 cm.

8 (a) $4x + 6$

(b) $4x + 6 = 62$; $x = 14$; Fourteen tables that seat four people each are needed.

9 $3h + 5 = 14$; $h = 3$; Natalie rollerskated on the rink for three hours.

10 $2x - 54 = 272$; $x = 163$; at age 5, Robert Wadlow's height was 163 cm.

11 (a) $8d + 15 = 63$
$d = 6$ days

(b) $48d + 15 = 111$
$d = 2$ days

(c) $13d + 15 + 24d + 15 = 178$
$d = 4$ days

12 (a) **(i)** 6 **(ii)** 8 **(iii)** 10

(b) A footpath of length x metres would need $(2 \times x + 2)$ or $(2x + 2)$ pieces of wood.

(c) **(i)** 14 **(ii)** 22 **(iii)** 28

(d) $2x + 2 = 36$; $x = 17$ m

Open-ended—Sample answers

13 Example: 1 hour jackhammer (\$30) plus 6 hours mulcher (\$132) = \$162

14 $x + 5x \leq 70$
The son is 10 and the father is 50.

Challenge 7

1 A $49 - 15 = 34$
$34 \times 2 = 68$

2 $x + 40 + x + 40 + x + 40 = 360$; $x = 80$

3 $\frac{3x-5}{4} = -6$, $3x - 5 = -24$, $3x = -19$, $x = -\frac{19}{3}$

4 B $x = 73 + y - 59$, $x = 14 + y$

5 (a) $3^2 = 4 + 5$

(b) $4^2 = 7 + 9$, $2^4 = 7 + 9$, $2^3 + 1 = 9$, $3^2 = 1 + 8$, $3^2 = 0 + 9$

(c) $2^3 = 9 - 1$, $3^2 = 9 - 0$, $3^2 - 5 = 4$, $3^2 - 4 = 5$, $4^2 - 9 = 7$

6 D Reciprocal of $\frac{1}{20}$ is 20. $\frac{4x}{5} = 20$, $x = 25$

7 $\frac{5+4}{20} + \frac{1}{y} = 1$, $\frac{1}{y} = 1 - \frac{9}{20}$, $\frac{1}{y} = \frac{11}{20}$, $y = \frac{20}{11}$

8 Let him buy x of each lolly.
$0.55x + 1.1x = 80.85$
$1.65x = 80.85$, $x = 80.85 \div 1.65 = 49$
He bought 98 lollies.

9 $\frac{1}{4} \times \frac{9}{10}x = 2700$, $x = \$12\,000$

10 $\frac{1}{3} - \frac{1}{4} = \frac{1}{4} - \frac{1}{m}$, $\frac{1}{m} = \frac{1}{4} + \frac{1}{4} - \frac{1}{3}$, $\frac{1}{m} = \frac{1}{2} - \frac{1}{3}$, $\frac{1}{m} = \frac{1}{6}$, $m = 6$

11 C The integers end in 2 and 5; 4 and 5; or 8 and 5. As $a > b$, consider $12 - 5 = 7$, $5 - 2 = 3$, $14 - 5 = 9$, $5 - 4 = 1$, $15 - 8 = 7$, $8 - 5 = 3$. So $a - b$ ends in 1, 3, 7 or 9.

Chapter review 7

1 (a) $7 \times 4 = 30 - 2$ **(b)** $2 \times 4 + 6 = 9 + 5$

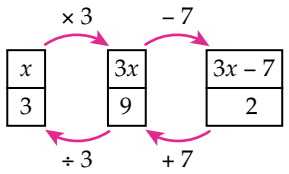
2 (a) 15 **(b)** 28

3 (a) $x = 9$ **(b)** $x = 3$ **(c)** $x = 15$ **(d)** $x = 15$
(e) $x = 12$ **(f)** $x = 28$

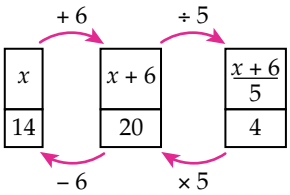
4 (a) $x = -3$ **(b)** $x = 84$ **(c)** $x = 9$

5 (a) -5 **(b)** $\times 3$ **(c)** $\div 8$ **(d)** $+4$

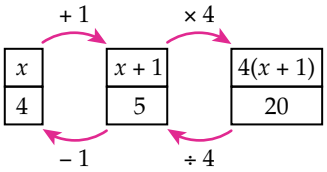
6 (a) $x = 3$



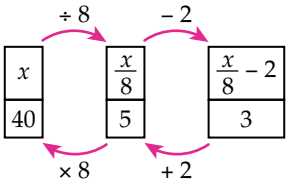
(b) $x = 14$



(c) $x = 4$



(d) $x = 40$



7 (a) $4x + 7 - 7 = 19 - 7$
$4x = 12$
$4x \div 4 = 12 \div 4$
$x = 3$

(b) $5(x - 2) \div 5 = 20 \div 5$
$x - 2 = 4$
$x - 2 + 2 = 4 + 2$
$x = 6$

(c) $\frac{x}{3} + 5 - 5 = 10 - 5$
$\frac{x}{3} = 5$
$\frac{x}{3} \times 3 = 5 \times 3$
$x = 15$

(d) $x - \frac{8}{6} \times 6 = 3 \times 6$
$x - 8 = 18$
$x - 8 + 8 = 18 + 8$
$x = 26$

(e) $x + \frac{4}{5} \times 5 = 5 \times 5$
$x + 4 = 25$
$x + 4 - 4 = 25 - 4$
$x = 21$

(d) $x - \frac{7}{3} \times 3 = 10 \times 3$
$x - 7 = 30$
$x - 7 + 7 = 30 + 7$
$x = 37$

8 B

9 $5t + 27.50 = 200$, $t = \$34.50$

10 D

11 (a) $C = 3x + 140$

(b) $275 = 3x + 140$, $x = 45$ cents

12 $t + 5 = 28$
$t = 23\,°C$

The temperature at 3 pm was 23 °C.

13 (a) 5 (b) 7 (c) $2n + 1$

(d) (i) $2n + 1 = 97$ (ii) $n = 48$

Numeracy practice 7

1 C **2** D **3** D **4** C

5 A **6** B **7** B **8** C

Chapter 8

Recall 8

1 (a) 55° (b) 160° (c) 235° (d) 339°

2 D, A, B, E, C, F

3 A and E; B and F; C and D

Exercise 8.1

1 (a) 360° (b) 270° (c) 180° (d) 90°

2 (a) 102° (b) 71° (c) 129° (d) 225°

(e) 350° (f) 15°

3 Students' own answers.

4 (a) C (b) B (c) D

5 (a) B (b) D

6 C **7** 21° **8** 40°

9 Draw a 40° angle and mark the reflex angle as 320°.

10 (a) 330° (b) 245° (c) 200°

11 Students' own answers.

12 Students' own answers.

Open-ended—Sample answers

13 Students' own answers.

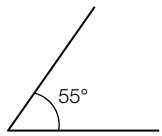
14 Nick has used the incorrect scale on his protractor.
He should start from 0 on one arm and follow around the scale to the second arm.

15 (a) 20°, 70°

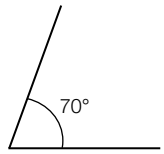
(b) 30°, 100°, 50°

(c) 110°, 120°, 80°, 50°

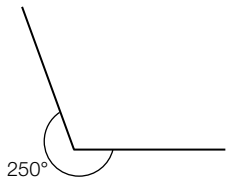
16 (a)



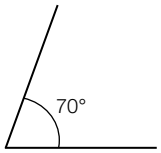
(b)



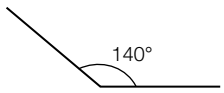
(c)



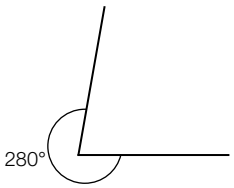
17 **(a)**



(b)



(c)



Exercise 8.2

1 **(a)** straight **(b)** reflex **(c)** obtuse
(d) reflex **(e)** acute **(f)** acute
(g) acute **(h)** obtuse **(i)** revolution
(j) revolution **(k)** right **(l)** straight
(m) obtuse **(n)** straight **(o)** right
(p) right **(q)** revolution **(r)** reflex

2 **(a)** acute **(b)** obtuse
(c) reflex **(d)** revolution
(e) straight **(f)** acute
(g) right **(h)** obtuse
(i) reflex

3 (*Note:* Other forms of angle notation may be acceptable.)
(a) $\angle QPR$ or $\angle RPQ$ **(b)** reflex angle $\angle DTS$ or $\angle STD$
(c) $\angle BOA$ or $\angle AOB$ **(d)** $\angle WIK$ or $\angle KIW$
(e) $\angle SHD$ or $\angle DHS$ **(f)** $\angle TAC$ or $\angle CAT$

4 **(a)** D **(b)** C **(c)** C **(d)** D

5 D

6 85° and 5° sum to 90°; hence, they are a pair of complementary angles.

7 160° and 72° don't add up to 180°; hence, they are not a pair of supplementary angles.

8 **(a)** **(i)** Students' own answers.
(ii) 360° **(iii)** revolution
(b) **(i)** Students' own answers.
(ii) 90° **(iii)** right angle
(c) **(i)** Students' own answers.
(ii) 180° **(iii)** straight angle
(d) **(i)** Students' own answers.
(ii) 360° **(iii)** revolution
(e) **(i)** Students' own answers.
(ii) 90° **(iii)** right angle
(f) **(i)** Students' own answers.
(ii) 180° **(iii)** straight angle

9 **(a)** $\angle AOD$ and $\angle AOC$ or $\angle AOC$ and $\angle BOC$ or $\angle AOD$ and $\angle BOD$ or $\angle BOC$ and $\angle BOD$
(b) $\angle AOD$ and $\angle COB$ or $\angle AOC$ and $\angle DOB$

10 C

11 **(a)** $\angle AOB$, $\angle BOC$, $\angle DOE$
(b) $\angle AOE$, $\angle BOD$, $\angle COE$
(c) $\angle AOB$ (reflex), $\angle AOC$ (reflex), $\angle AOE$ (reflex), $\angle BOC$ (reflex), $\angle BOD$ (reflex), $\angle COD$ (reflex), $\angle COE$ (reflex), $\angle DOE$ (reflex)
(d) $\angle AOC$, $\angle COD$
(e) $\angle AOD$, $\angle BOE$
(f) $\angle AOB$ and $\angle BOC$, $\angle BOC$ and $\angle DOE$
(g) $\angle AOB$ and $\angle BOD$, $\angle AOC$ and $\angle COD$, $\angle AOE$ and $\angle DOE$, $\angle BOC$ and $\angle COE$, $\angle BOD$ and $\angle DOE$
(h) $\angle AOB$ and $\angle DOE$, $\angle AOE$ and $\angle BOD$

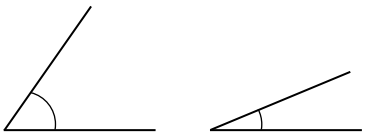
12 **(a)** 334°
(b) $\angle RMS$ could be an acute angle.

13 **(a)** $\angle B$ could refer to a number of angles.
(b) $\angle ABC$, $\angle CBD$
(c) $\angle ABD$
(d) $\angle ABD$ (reflex) or $\angle ABC$ (reflex) or $\angle CBD$ (reflex)

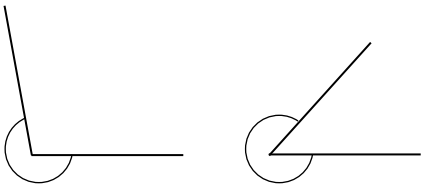
14 Students to provide accurate copies of the figures.

Open-ended—Sample answers

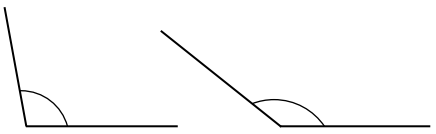
15 **(a)**



(b)



(c)



16 **(a)** 25° and 65°; 12° and 78°
(b) 112° and 68°; 92° and 88°

17 The angles given must add to 360°.
45°, 150° and 165°; 205°, 60° and 95°

Exercise 8.3

1 **(a)** 60° **(b)** 10° **(c)** 14° **(d)** 23°
(e) 56° **(f)** 63° **(g)** 150° **(h)** 60°
(i) 175° **(j)** 69° **(k)** 53° **(l)** 38°

2 **(a)** 270° **(b)** 120° **(c)** 313° **(d)** 50°
(e) 145° **(f)** 202° **(g)** 50° **(h)** 245°
(i) 138° **(j)** 70° **(k)** 143° **(l)** 31°

3 **(a)** 33° **(b)** 51° **(c)** 172° **(d)** 94°
(e) 89° **(f)** 20°

4 **(a)** 63° **(b)** 45° **(c)** 22° **(d)** 75°

5 **(a)** 148° **(b)** 90° **(c)** 56° **(d)** 4°

6 87°

7 **(a)** 57° **(b)** 45° **(c)** 65°

8 A

9 **(a)** $x° = 101°$ (vertically opposite to 101°)
$y° = z° = 79°$ (supplementary to 101°)

(b) $x° = 88°$ (vertically opposite to 88°)
$y° = z° = 92°$ (supplementary to 88°)

(c) $x° = z° = 149°$ (supplementary to 31°)
$y° = 31°$ (vertically opposite to 31°)

10 **(a)** 60° (a straight line is 180°)

(b) 45° (a straight line is 180°)

(c) 30° (a right angle is 90°)

(d) 40° (a right angle is 90°)

(e) 50° (a straight line is 180°)

(f) 30° (a right angle is 90°)

(g) 120° (a revolution is 360°)

(h) 72° (a revolution is 360°)

(i) 75° (a revolution is 360°)

11 **(a)**

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
Left leg	90°	90°	90°	90°	92°	88°	88°	92°
Right leg	91°	89°	89°	91°	80°	100°	100°	80°

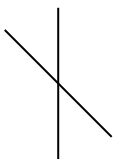
(b) right leg

(c) Yes, the differences are 11°.

Open-ended—Sample answers

12 Sabine did not use the information given in the question. She should have used the property that angles at a point add to 360°, but instead used her protractor. The diagram is not drawn accurately.

13 **(a)**



(b) 45° and 135°

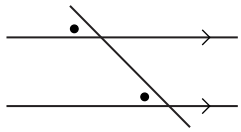
(c) 45° and 135°

Exercise 8.4

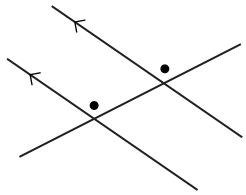
1 **(a)** F **(b)** F
(c) T **(d)** T

2 **(a)** corresponding **(b)** alternate
(c) co-interior **(d)** co-interior
(e) co-interior **(f)** corresponding
(g) alternate **(h)** corresponding
(i) alternate

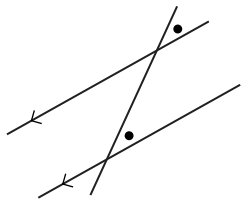
3 **(a)**



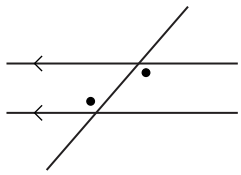
(b)



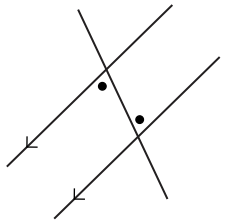
(c)



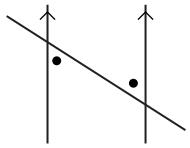
4 **(a)**



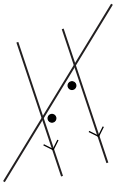
(b)



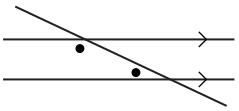
(c)



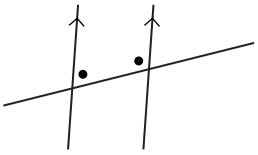
5 **(a)**



(b)



(c)



6 C

7 **(a)** **(i)** $b = 135°$, $a = 45°$ (co-interior)

(ii) $s = 140°$, $t = 140°$ (corresponding)

(iii) $m = 130°$, $n = 130°$ (alternate)

(iv) $e = 151°$, $f = 151°$ (alternate)

(v) $g = 72°$, $h = 72°$ (corresponding)

(vi) $x = 144°$, $y = 36°$ (co-interior)

(b) **(ii)**, **(iii)**, **(iv)** and **(v)** have pairs that are equal; **(i)** and **(vi)** have pairs that add to 180°.

8 **(a)** y **(b)** w **(c)** u **(d)** v **(e)** s **(f)** x

9 **(a)** $a = 37°$ (alternate angles)

(b) $c = 110°$ (corresponding angles)

(c) $g = 24°$ (co-interior angles)

(d) $b = 100°$ (co-interior angles)

(e) $d = 73°$ (alternate angles)

(f) $e = 141°$ (corresponding angles)

(g) $f = 115°$ (corresponding angles)

(h) $j = 72°$ (co-interior angles)

(i) $h = 97°$ (alternate angles)

10 18° **11** 144°

12 EF and GH. Co-interior angles are supplementary.

13 **(a)** 16 **(b)** 8 **(c)** 8

14 **(a)** $a = 146°$ (vertically opposite angle), $b = 34°$ (co-interior angle)

(b) $a = 62°$ (alternate angle), $b = 62°$ (vertically opposite angle), $c = 28°$ (angles in a right angle)

(c) $a = 80°$ (alternate angle), $b = 80°$ (corresponding angle), $c = 80°$ (corresponding angle), $d = 80°$ (vertically opposite angle)

(d) $a = 44°$ (alternate angle), $b = 46°$ (angles in a right angle)

(e) $a = 50°$ (co-interior angle), $b = 50°$ (angles in a straight line)

(f) $a = 45°$ (alternate angle), $b = 135°$ (co-interior angle), $c = 95°$ (co-interior angle), $d = 95°$ (vertically opposite angle)

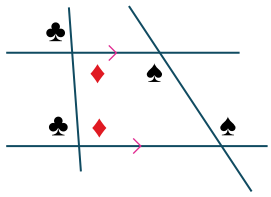
(g) $a = 37°$ (alternate angle), $b = 37°$ (vertically opposite angle)

(h) $a = 116°$ (co-interior angle), $b = 104°$ (co-interior angle), $c = 104°$ (vertically opposite angle)

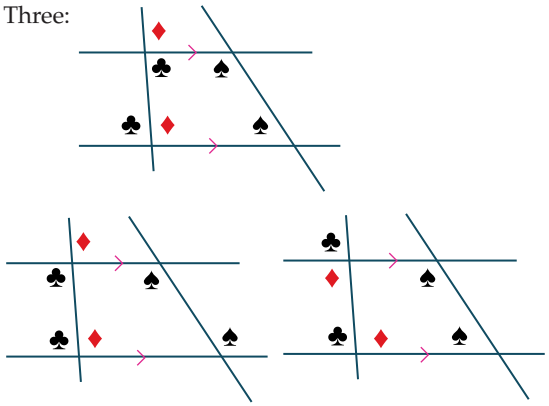
(i) $a = 84°$ (alternate angle), $b = 96°$ (angles in a straight line), $c = 74°$ (co-interior angle), $d = 74°$ (vertically opposite angle)

Open-ended—Sample answers

15 **(a)**



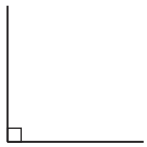
(b) Three:



16 Perspective: in a 2D representation of a 3D situation, the parallel lines appear to converge on the horizon to give the impression of depth.

Half-time 8

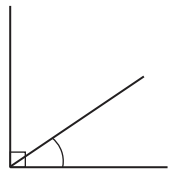
1 **(a)**



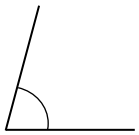
(b)

(c)

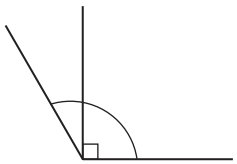
(d) About 30° is about one-third of a right angle.



(e) 75° is five-sixths of a right angle.



(f) 120° is 90° + 30°.



2 (a) $\angle BOC$ (or $\angle COB$), $\angle DOC$ (or $\angle COD$)

(b) $\angle BOA$ (or $\angle AOB$), $\angle AOD$ (or $\angle DOA$), $\angle BOD$ (or $\angle DOB$)

(c) $\angle BOA$

3 Students' own diagrams.

4 (a) $x = 55°$ (angles in a straight line add to 180°)

(b) $x = 35°$ (angles in a revolution add to 360°)

5 30°

6 (a) $a = 67°$ (alternate angles are equal)

(b) $y = 57°$ (co-interior angles add to 180°)

7 (a) $\angle GEB$ (b) $\angle GDF$ (c) $\angle DFI$ (d) $\angle HGI$

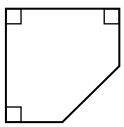
Exercise 8.5

1 (a) pentagon (convex) (b) triangle (convex)
(c) hexagon (convex) (d) nonagon (convex)
(e) octagon (concave) (f) dodecagon (concave)
(g) decagon (concave) (h) hexagon (concave)
(i) hexagon (concave)

2 (a) C (b) C

3 (a) octagon (b) dodecagon
(c) pentagon (d) hexagon

4 3



5 (a) 3

(b) 4 and 5. The pattern is: divide the number of sides by 2.

(c) No, a pentagon and a heptagon have an odd number of sides, so they cannot be paired.

6 (a) 45 (b) 45 handshakes

Open-ended—Sample answers

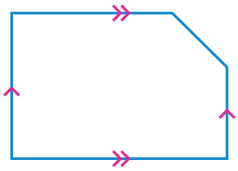
7 (a)

(b)

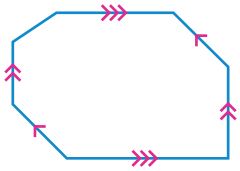
(c)

8 50-cent piece (dodecagon), stop sign (octagon), soccer ball (pentagons & hexagons)

9 (a)



(b)



Exercise 8.6

1 (a) (i) scalene (ii) obtuse
(iii) obtuse-angled scalene triangle

(b) (i) isosceles (ii) acute
(iii) acute-angled isosceles triangle

(c) (i) isosceles (ii) right
(iii) right-angled isosceles triangle

(d) (i) isosceles (ii) acute
(iii) acute-angled isosceles triangle

(e) (i) equilateral (ii) acute
(iii) acute-angled equilateral triangle

(f) (i) scalene (ii) obtuse
(iii) obtuse-angled scalene triangle

(g) (i) equilateral (ii) acute
(iii) acute-angled equilateral triangle

(h) (i) scalene (ii) right
(iii) right-angled scalene triangle

(i) (i) isosceles (ii) obtuse
(iii) obtuse-angled isosceles triangle

(j) (i) scalene (ii) right
(iii) right-angled scalene triangle

(k) (i) isosceles (ii) obtuse
(iii) obtuse-angled isosceles triangle

(l) (i) scalene (ii) acute
(iii) acute-angled scalene triangle

2 (a) A, C, D, E (b) A, C, F (c) A, C, F, G

3 (a) 60° (b) 30° (c) 35°
(d) 83° (e) 132° (f) 22°
(g) 14° (h) 38° (i) 48°

4 (a) (i) 120° (ii) 137° (iii) 75° (iv) 165° (v) 151°
(vi) 138° (vii) 133° (viii) 146° (ix) 129°

(b) (i) 40° (ii) 30° (iii) 74° (iv) 43° (v) 126°
(vi) 38°

5 (a) isosceles, acute-angled (b) scalene, acute-angled
(c) scalene, acute-angled (d) equilateral, acute-angled
(e) equilateral, acute-angled (f) scalene, acute-angled
(g) isosceles, acute-angled (h) isosceles, acute-angled
(i) scalene, right-angled

6 **(a)** D **(b)** A **(c)** B

7 B

8 **(a)** C **(b)** A

9 **(a)** isosceles, obtuse-angled, $a = 25°$

(b) scalene, right-angled, $b = 35°$

(c) equilateral, $c = 30°$

(d) scalene, right-angled, $a = 21°$

(e) scalene, right-angled, $b = 58°$

(f) isosceles, acute-angled, $c = 48°$

(g) isosceles, obtuse-angled, $d = 130°$

10 isosceles, right-angled

11 **(a)** all sides equal length

(b) Two angles will be 60°, the other two will be 120°

12 No, the three sides won't be able to meet.

13 two

14 **(a)** right-angled, scalene (third angle 90°)

(b) obtuse-angled, isosceles (third angle 28°)

(c) acute-angled, equilateral (third angle 60°)

(d) obtuse-angled, isosceles (third angle 156°)

15 **(a)** Angles add to more than 180° (i.e. 190°).

(b) Angles add to less than 180° (i.e. 165°).

(c) Isosceles triangle must have 2 equal angles.

16 **(a)** $a = 65°$, $b = 135°$

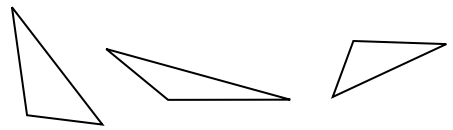
(b) $c = 100°$

(c) $d = 57°$, $e = 57°$, $f = 66°$

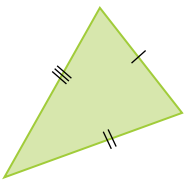
17 360°

Open-ended—Sample answers

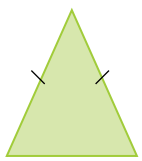
18



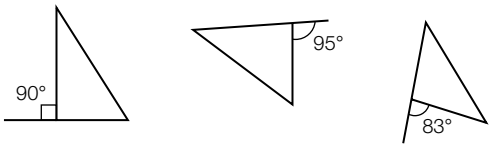
19 **(a)**



(b)



20



Exercise 8.7

1 **(a)** rectangle **(b)** parallelogram

(c) kite **(d)** rhombus

(e) trapezium **(f)** quadrilateral

2 A

3 **(a)** T **(b)** F **(c)** T **(d)** F **(e)** T

4 **(a)** 77° **(b)** 87° **(c)** 54° **(d)** 55° **(e)** 69°

(f) 133° **(g)** 138° **(h)** 125° **(i)** 205°

5 **(a)** D **(b)** C **(c)** C

6 **(a)** 40° **(b)** 105° **(c)** 68° **(d)** 73° **(e)** 197°

(f) 19° **(g)** 60° **(h)** 131° **(i)** 78°

7 **(a)** trapezium **(b)** rhombus

(c) kite **(d)** parallelogram

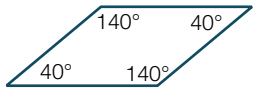
8 **(a)** $a = 70°$, angles in a quadrilateral add to 360°

(b) $b = 97°$, one pair of opposite angles in a kite are equal and angles add to 360°

(c) $c = 99°$, co-interior angle

(d) $d = 38°$, co-interior angle

9



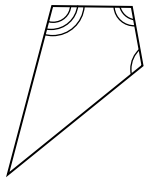
10 105°

11 **(a)** Parallelograms have opposite sides parallel and equal in length, which is also the case for a rhombus. However, a rhombus has all sides equal, and that is not necessarily true for all parallelograms.

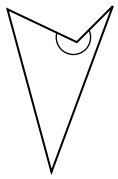
(b) A a parallelogram must have two pairs of parallel sides whereas a trapezium must only have one pair of parallel sides.

12 Yes, rectangles have all the properties of a parallelogram and more. Most parallelograms do not have 90° angles.

13 **(a)** three



(b) one



14 **(a)** 128°

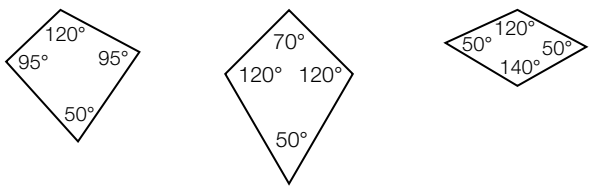
(b) kite or irregular quadrilateral

15 No. Sum of angles in a quadrilateral = 360°, not 370°.

16 **(a)** 4 right angles **(b)** 4 equal sides

(c) opposite sides are parallel

17

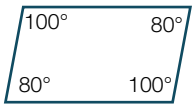


Open-ended—Sample answers

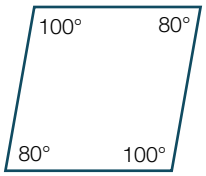
18 (a) Parallelogram, rhombus, kite, trapezium, or irregular quadrilateral.

(b) Five—**(i)** parallelogram, **(ii)** rhombus, **(iii)** kite, **(iv)** trapezium, or **(v)** irregular quadrilateral.

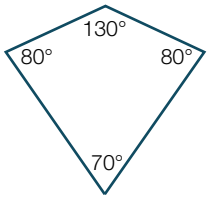
(c) (i)



(ii)

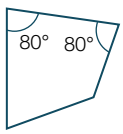


(iii)

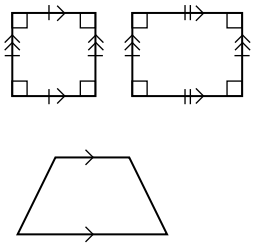


(iv)

(v)



19 Jack's diagrams lack important angle symbols, equal length markings and parallel line markings.



20 Shapes in the table's first row, from left to right, are rectangle, parallelogram, kite, trapezium. Shapes in the second row, from left to right, are square, diamond, rhombus, irregular quadrilateral. These can all be organised into parallelograms and non-parallelograms as follows.

Parallelograms: rectangle, square, diamond, rhombus, parallelogram

Non-parallelograms: irregular quadrilateral, trapezium, kite

Exercise 8.8

1 Follow the worked example instructions to bisect a line and an angle, and to construct an equilateral triangle, square, angle equal to another angle, and a pair of parallel lines.

2 (a) Follow the worked example instructions for bisecting a line to construct a 90° angle.

(b) Bisect the 90° angle to make a 45° angle.

(c) Follow the worked example instructions for constructing an equilateral triangle to construct a 60° angle.

(d) Bisect the 60° angle to make a 30° angle.

(e) Begin with a 10 cm line and follow the worked example instructions for bisecting a line.

(f) Construct pairs of parallel lines.

3 Begin with an 8 cm line. Construct angles of 60° and 90° at either end. Bisect both angles and join the bisectors to find the 3rd vertex of the triangle.

4 (a) Construct angles of 60° and 90° at the same end of the same line. Bisect the angle between the 60° and 90° angles to form a 75° angle with the initial line.

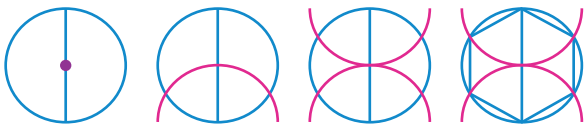
(b) Construct a 60° angle. Bisect this to give a 30° angle. Bisect the 30° angle to give a 15° angle.

(c) Construct a right angle. Bisect the adjacent right angle to give a 45° angle. The right angle and the 45° angle together form a 135° angle.

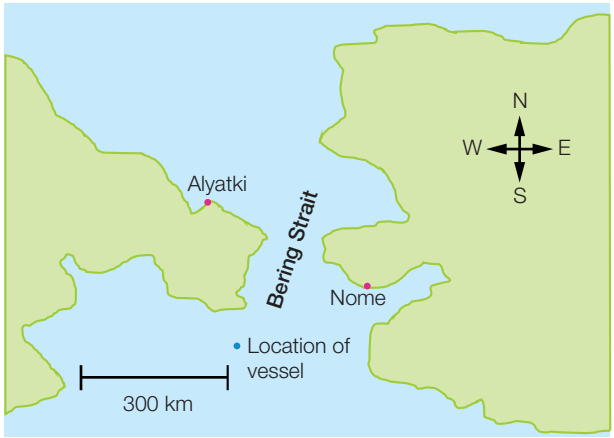
5 Using any base line of length less than 10 cm, open the compass 5 cm to make arcs from either end. Join the intersection of the arcs with the ends of the base line to construct your triangle.

6 Construct a square. Keep the compass at the same side length as the square, and use the sides of the square to construct an equilateral triangle on each side.

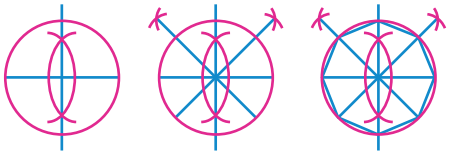
7 This construction works because it has constructed 6 equilateral triangles in which all the angles equal 60°.



8



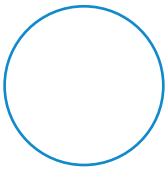
9 This construction works because it has constructed a 90° angle, then bisected it to make a 45° angle. This has now divided the circle into 8 equal parts.



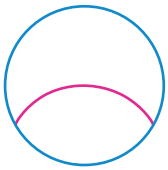
Open-ended—Sample answers

10 Starting from a random location on the circle's circumference with the spread of the compass equal to the circle's radius, draw an arc from the circumference of the circle, through the circle's centre across to the circumference on the other side.

Step 1

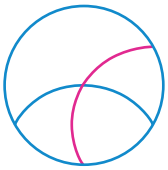


Step 2



Repeat with the compass needle being at one of the intersection points between the arc just constructed and the circle's circumference.

Step 3



Step 4

Step 5

Step 6

Step 7

11 Use the base and sides of a polygon to start the construction of a new polygon.

Challenge 8

1 B At 2:30, the hour hand is half-way between 2 and 3. Total angle is $90° + (30° \div 2) = 105°$.

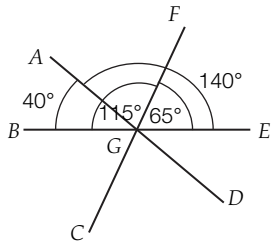
2 $90° - 46° = 44°$. Small angle $= 44° \div 2 = 22°$, large angle $= 46° + 22° = 68°$.

Or: $90° \div 2 = 45°$, $46° \div 2 = 23°$. Small angle $= 45° - 23° = 22°$, large angle $= 45° + 23° = 68°$.

3 $180° - 70° = 110°$. $x + x = 110°$, so $x = 55°$.

4 $\angle CGD = 75°$ (vertically opposite $\angle AGF$)

From the diagram, $\angle AGF = 75°$



5 $180° - 54° = 126°$. Small angle $= 126° \div 2 = 63°$, large angle $= 54° + 63° = 117°$.

6 D $6x° = 180°$, $x = 30°$; $y = 180° - 30° = 150°$

7 A $x = 180° - (30° + 40°) = 180° - 70° = 110°$

8 **(a)** At 12:10, the minute hand makes an angle of 60° with the vertical and the hour hand makes an angle of $\frac{1}{6} \times 30°$ or 5°. The angle between them is $60 - 5° = 55°$.

(b) 40 minutes later, the time is 12:50. The minute hand makes an acute angle of 60° with the vertical and the hour hand makes an angle of $\frac{5}{6} \times 30°$ or 25°. The angle between them is $60° + 25° = 85°$.

9 C $x + y = 110°$, $\angle KLM = 180° - 2x$, $\angle KML = 180 - 2y$
$\angle KLM + \angle KML = 360° - 2(x + y) = 360° - 220° = 140°$,
$\angle LKM = 40°$

Chapter review 8

1 **(a)** 42° **(b)** 196° **(c)** 157° **(d)** 321°

2 **(a)** E **(b)** A **(c)** D **(d)** B **(e)** C **(f)** F

3 Students' own diagrams.

4 A **5** B

6 **(a)** reflex **(b)** revolution **(c)** straight
(d) acute **(e)** right **(f)** obtuse

7 **(a)** 163° **(b)** 39° **(c)** 127°
(d) 40° **(e)** 143° **(f)** 82°

8 **(a)** 136° **(b)** 112°

9 **(a)** irregular concave pentagon
(b) irregular convex heptagon
(c) irregular convex decagon
(d) regular concave dodecagon

10 80°

11 **(a)** 66° (angles in a quadrilateral add to 360°)
(b) 110° (opposite angles in a parallelogram are equal)

12 140°, missing angles are the equal angles in a kite.

13 Students' own diagrams.

14 292°

15 **(a)** 45° and 90° **(b)** isosceles triangle

16 **(a)** 85° **(b)** 25° **(c)** 40°

17 **(a)** T **(b)** F **(c)** T **(d)** F **(e)** T

18 $\angle CFB = 90°$, vertically opposite angles are equal. $x = 27°$, complementary angles add to 90°.

19 **(a)** obtuse angle **(b)** reflex angle

(c) acute angle, right angle or obtuse angle

Numeracy practice 8

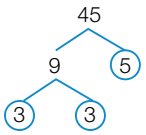
1 B **2** 150° **3** D **4** D

5 C **6** D

Mixed review D

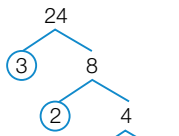
1 **(a)** $6a - 4b$ **(b)** $6a + 2b + 3c$

2 **(a)**



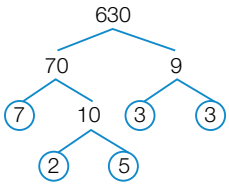
$3^2 \times 5$

(b)



$2^3 \times 3$

(c)



$2 \times 3^2 \times 5 \times 7$

3 **(a)** $\frac{3}{4}$ **(b)** $\frac{1}{2}$ **(c)** $\frac{3}{10}$ **(d)** $\frac{1}{25}$

4 **(a)** 0.253 **(b)** 2.437

5 **(a)** $x = -1$ **(b)** $x = -41$

6 **(a)** 46° **(b)** 39°

7 **(a)** $3\frac{19}{20}$ **(b)** $\frac{15}{16}$

8 **(a)** 137.7 cm^2 **(b)** 60 m^2

9 **(a)** Thursday **(b)** Wednesday **(c)** Friday

10 A **11** D **12** 11.25°

13 **(a)** 35° **(b)** 63°

14 150°, 210°

15 **(a)** Students' own answers.

(b) If they add to 360° then they are more likely to be correct.

(c) Dogs 110°, Cats 140°, Birds 30°, Fish 80°

16 second floor **17** $x = 45°$, $2x = 90°$, $3x = 135°$

18 **(a)** Students' own answers.

(b) reflex angle

(c) $\angle ABC$ (reflex)

19 Complementary angles: $\angle ABE$ and $\angle CBE$ (also $\angle ABE$ and $\angle BAE$, $\angle EBC$ and $\angle ECB$, $\angle BAE$ and $\angle BCE$, $\angle ECD$ and $\angle EDC$, $\angle EAD$ and $\angle EDA$).
Supplementary angles: $\angle AEB$ and $\angle CEB$, $\angle AED$ and $\angle CED$, $\angle AEB$ and $\angle AED$, $\angle CED$ and $\angle CEB$.
Vertically opposite angles: $\angle CEB$ and $\angle AED$, $\angle AEB$ and $\angle CED$.
Angles at a point: $\angle AEB$, $\angle CEB$, $\angle AED$ and $\angle CED$; $\angle BAE$, $\angle EAD$ and $\angle BAD$ (reflex), there are more around the edge of the kite.

Chapter 9

Recall 9

1 **(a)** 0, 5, 10, 15, 20, 25, 30

(b) 16, 18, 20, 22, 24, 26, 28

(c) 1300, 1400, 1500, 1600, 1700

(d) 60, 80, 100, 120, 140, 160, 180, 200

2 **(a)** 50, 100, 150, 200, 250

(b) 100, 125, 150, 175, 200

(c) 1400, 1600, 1800, 2000

3 **(a)** 6, 10, 12, 14, 16, 20, 22, 24

(b) 0, 5, 15, 20, 25, 50, 55, 100

(c) 0, $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4, 5, $5\frac{1}{2}$, $6\frac{1}{2}$

4 **(a)** 15, 18, 24 **(b)** 140, 160, 180, 200

(c) 250, 275, 325

5 **(a)** $\frac{1}{3}$ **(b)** $\frac{3}{4}$

Exercise 9.1

1 **(a)** categorical **(b)** categorical **(c)** continuous

(d) continuous **(e)** discrete **(f)** categorical

(g) discrete

2 **(a)** discrete

(b)

Number of pets owned	Tally	Frequency
0	| | | |	4
1	~~++++~~ | |	7
2	| | |	3
3	| | |	3
4	| | | |	4
5	|	1
6	|	1
7	|	1
8		0
9	|	1
		25

3 (a) discrete

(b)

Number of calls	Tally	Frequency
0–19	|	1
20–39	|	1
40–59	| | |	3
60–79	𝍸 |	6
80–99	𝍸 |	6
100–119	𝍸 |	6
120–139	| |	2
140–159	| |	2
160–179	| |	2
180–199	|	1
		30

4 (a) categorical

(b)

Type of takeaway	Tally	Frequency
Pizza	𝍸 | | |	8
Fish & chips	| | |	3
Hamburgers	| | | |	4
Chicken	| | |	3
Chinese	| |	2
None	| |	2
		22

5 (a) discrete

(b)

Hours of television watched	Tally	Frequency
0	| | |	3
$\frac{1}{2}$	| | | |	4
1	| | |	3
$1\frac{1}{2}$	| |	2
2	𝍸 |	6
$2\frac{1}{2}$	| | | |	5
3	| |	2
$3\frac{1}{2}$		0
4	|	1
$4\frac{1}{2}$	|	1
		27

6

Amount of money	Tally	Frequency
\$0.00–\$0.45	| | | |	4
\$0.50–\$0.95	| | |	3
\$1.00–\$1.45	| | |	3
\$1.50–\$1.95	| | | |	4
\$2.00–\$2.45	| | |	3
\$2.50–\$2.95	| | |	3
\$3.00–\$3.45		0
\$3.50–\$3.95	|	1
\$4.00–\$4.45	| | |	3
		24

7 (a) C **(b)** B

8 (a) D **(b)** B **(c)** C

9 (a) 26

(b)

Number of hours spent exercising	Tally	Frequency
0	| | | |	4
1	𝍸 | |	7
2	𝍸 | | |	8
3	| | |	3
4	|	1
5	|	1
6	|	1
7		0
8	|	1
		26

(c) 4

(d) 2

(e) These students would be expected to exercise significantly more than other students, so they are most likely the largest data values (8, 6 and 5 hours).

10 (a) 35

(b)

Size	Tally	Frequency
XS	| |	2
S	𝍸 | |	7
M	𝍸 | | | |	9
L	𝍸 | | |	8
XL	| | | |	4
Did not order	| | | |	5

(c) categorical **(d)** M **(e)** 9

(f) Because the students who forgot to order were all in Year 7, she should probably order 5 M size T-shirts, as the L and XL would be too big for most Year 7 students.

11 (a)

Class interval	Frequency
20–29	6
30–39	6
40–49	6
50–59	6
60–69	6

(b) Each of the intervals contains six values.

(c)

Class interval	Frequency
20–24	5
25–29	1
30–34	5
35–39	1
40–44	5
45–49	1
50–54	5
55–59	1
60–64	5
65–69	1

(d) The low-half class intervals have 5 values whereas the high-half class intervals have just 1 value. The values tend to appear at the lower end of the 10 groups.

(e) The size of the class interval can have an effect on the way we see the data.

12 (a)

Pulse rate (beats per minute)	Frequency
51–60	7
61–70	21
71–80	19
81–90	10
91–100	3

(b)

Pulse rate (beats per minute)	Frequency
51–55	3
56–60	4
61–65	8
66–70	13
71–75	10
76–80	9
81–85	10
86–90	0
91–95	3

(c) Probably **(b)**. This table shows that the vast majority of students have a pulse rate between 61 and 85 beats per minute. When looking at table **(a)**, the vast majority of students have a pulse rate between 61 and 90 beats per minute, but the high end of 90 might be a cause for medical concern.

Open-ended—Sample answers

13 The highest weight is 3529 g and the lowest is 2010 g. This is a difference of 1519 g. If using 250 g groups, there would be 7 groups. If using 200 g groups, there would be 8 groups. If using 125 g groups, there would be 14 groups —this is too many. Could choose either of the others. The table below is for groups of 250 g. It starts at 2000 g as this is a multiple of the difference being used.

Birth weight (g)	Frequency
2000–2250	5
2250–2500	2
2500–2750	5
2750–3000	8
3000–3250	6
3250–3500	1
3500–3750	3

14 Students' own answers.

15 Students' own answers.

Exercise 9.2

1 (a) (i) 5 (ii) 7 (iii) 7 (iv) 6

(b) (i) 6 (ii) 6 (iii) 6 (iv) 0

(c) (i) 7 (ii) 7 (iii) 7 (iv) 4

(d) (i) 6 (ii) 7 (iii) 8 (iv) 7

(e) (i) 4.9 (ii) 4.5 (iii) 4 (iv) 6

(f) (i) 6 (ii) 7 (iii) 9 (iv) 8

2 (a) (i) 29 (ii) 29 (iii) 29 (iv) 13

(b) (i) 66.43 (ii) 72 (iii) no mode (iv) 37

(c) (i) 66 (ii) 67.5 (iii) 81 (iv) 50

(d) (i) 10.26 (ii) 9.05 (iii) 6.5 (iv) 14.6

(e) (i) 13.7 (ii) 13.5 (iii) 11.4 (iv) 4.9

(f) (i) 8.50 (ii) 10.11 (iii) no mode (iv) 10.1

3 (a) C; only one within the range of scores

(b) C; 8 too high and 4 too low

(c) C; only one of each score, so mean will be close to median

(d) B; close to median as single results relatively evenly spread

4 (a) (i) 49 480.8 (ii) 43 369 (iii) 27 607

(b) Incheon

(c) (i) 49 390 (ii) 46 000 (iii) 48 000

(d) Miyagi

(e) The mean capacity is slightly higher in Korea (49 480.8 compared to 49 390), but the median capacity is higher in Japan (46 000 compared to 43 369). Half of the stadiums in each country have a capacity less than 45 000 (i.e. smaller than the mean value).

(f) (i) 49 435.4 (ii) 43 369 (iii) 29 593

(g) Miyagi (again). (It is interesting to note there are two stadiums in Japan quite close to the mean—Miyagi and Osaka—but none of the Korean stadiums are very close at all.)

5 (a) No, because the mean is the sum of all the data values divided by the number of values and so it will not necessarily be a data value.

(b) 5.8, no it is not a data value.

(c) Yes, because there is an odd number of values and so the median will be the middle value.

(d) 6

6 (a) This statement means that the average number of children of all families is 2.3. Most families have 2 children, but because many families have more than 2 children, the average of all these numbers of children is 2.3.

(b) The middle number will always be a whole number. If there are two middle numbers, then the only decimal you can get by dividing them by 2 is one that ends in .5.

(c) No, as the mode is the value that occurs most often in a data set, and the data set only contains whole numbers.

7 (a) **(i)** $58 571.43 **(ii)** $50 000 **(iii)** $50 000

(b) In this case, the mode does a good job as there are so many of this category of worker—well over half. However, the mean does take account of the other workers as well.

(c) **(i)** $53 461.54 **(ii)** $50 000 **(iii)** $50 000

(d) The mean is the only value that changes, and it changes by a large amount (over $5000).

Open-ended—Sample answers

8 Possible scores: 8, 8, 9, 9, 9 or 10, 10, 10, 10, 3 etc. The total must be 43.

9 (a) 5, 6, 6, 6, 7. As the median is 6, this is the middle number and at least one more 6 is needed to make it the mode. The total of the five results is 30, so there is 18 left, which can be included in any way as long as it doesn't move the median number.

(b) 4, 6, 7, 9, 9. As the median is 7, this has to be the middle number and two 9s are needed after that. This adds to 25 and the total required is 35, so there is 10 left to be included.

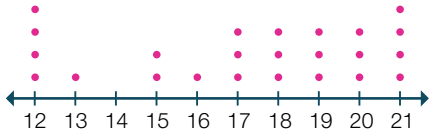
(c) 4, 5, 6, 6, 14. The middle number is 6 and one more is 6. The total is 35, so there is 23 left to be included.

10 While Veran is correct to say the values must be ordered first when there is an even number of values, the median is found exactly midway between two of the values. The ordered list is: 17, 19, 20, 20, 21, 22, 23, 28; so the median is midway between 20 and 21, which is 20.5.

11 As the median is 5, this must be in the middle; as the mean is 6, the values must add up to 54, and 4 must be the most frequent. Sample answer: 1, 4, 4, 4, 5, 6, 6, 7, 17.

Exercise 9.3

1



2 (a)

STEM	LEAF
2	2 3 8
3	0 1 4 7
4	1 2 8
5	2 7

Key: 2 | 3 = 23

(b)

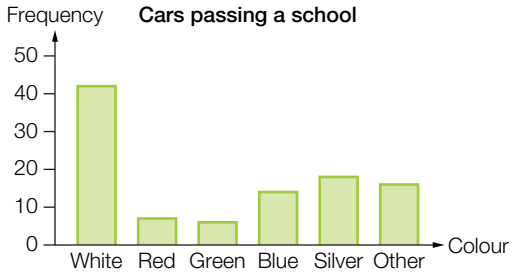
STEM	LEAF
0	2 3 8
1	4 5 8
2	1 5 6 7 9
3	0 3 4 5 7
4	2 4

Key: 1 | 4 = 14

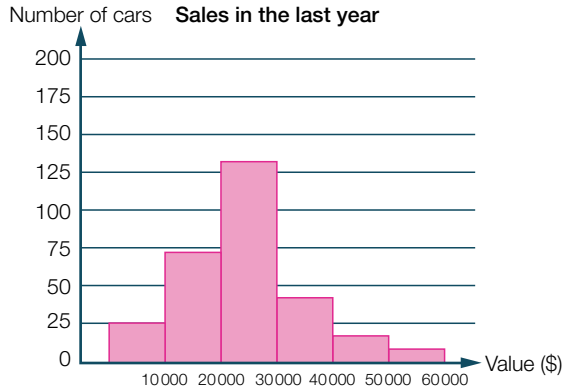
3 (a) The mean is 5.6, the median = 5.5, the range = 6, and 10 is a possible outlier. It has not affected the median. It has pulled the mean to the right and made the range larger.

(b) The mean is 87.4 points, the median is 88 points, the range is 66 points, and there are no outliers.

4



5



6 (a)

STEM	LEAF
2_L	0 1 2 3
2_U	7 7 8
3_L	1 4
3_U	8
4_L	2

Key: 2 | 1 = 21

(b)

STEM	LEAF
4_L	2
4_U	7 7 8
5_L	1 2 3
5_U	6 7 8
6_L	1 4
6_U	8

Key: 5 | 2 = 52

7 (a) Chocolate bar hourly production

STEM	LEAF
12	1 5 6 6 8 8 9 9
13	0 3 3 4 6 8
14	0 2 2 3 6
15	0

Key: 13 | 0 = 130

There are not enough intervals. There should be between 5 to 10 groups.

(b) Chocolate bar hourly production

STEM	LEAF
12_L	1
12_U	5 6 6 8 8 9 9
13_L	0 3 3 4
13_U	6 8
14_L	0 2 2 3
14_U	6
15_L	0

Key: 12 | 6 = 126

8 (a) C (b) C

9 (a) Thursday (b) 13 (c) 45

10 (a) The bars are horizontal; the scale is at the bottom.

(b) 820 million (c) 430 million (d) Spanish

11 (a) 10 cm

(b) 80 cm; this assumes that the values in the first and last groups go to the ends of the intervals. They may not. The range from the raw data could be up to 20 cm less.

(c) 830–<840 cm, 850–<860 cm, 870–<880 cm

(d) 13

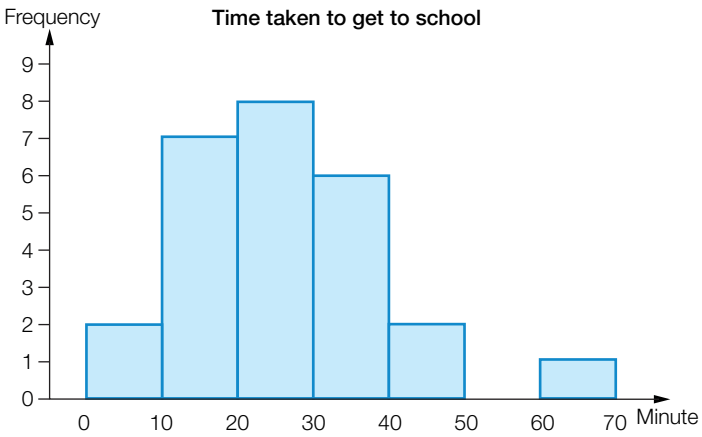
(e) 56

12 (a) continuous

(b)

Distance	Tally	Frequency
0–<10	| |	2
10–<20	𝍸 | |	7
20–<30	𝍸 | | |	8
30–<40	𝍸 |	6
40–<50	| |	2
50–<60		0
60–<70	|	1

(c)



(d) Most students seem to live locally except for one student who travels for 69 minutes. This could be an outlier.

13 (a) Australia (b) Canada

(c) UK (d) Aluminium cans

(e) Plastics

(f) Australia's recycling rate for glass is $\approx \frac{1}{2}$ of Switzerland's rate.
Australia's recycling rate for plastics is $\approx \frac{1}{2}$ of Austria's rate.
Australia's recycling rate for steel cans is a little under $\frac{1}{2}$ of Japan's rate.
Australia's recycling rate for aluminium cans/packaging is $\approx \frac{2}{3}$ of Finland's rate.

(g) Australia's recycling rate for glass is $\approx \frac{4}{3}$ Ireland's rate.
Australia's recycling rate for plastics is ≈ 4 times Canada's rate.
Australia's recycling rate for steel cans is over 5 times New Zealand's rate.
Australia's recycling rate for aluminium cans/packaging is over 4 times Ireland's rate.

14 (a) Expect the median to be higher because the mean value will be pulled to the left by the outlier. The mean is 4.9 apples and the median is 5 apples.

(b) The student might not like apples, or no apples were bought in this week's groceries.

(c) The mean goes up by 0.3 apples. The median does not change. The range goes down by 4 apples.

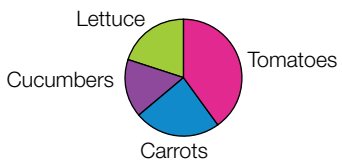
Open-ended—Sample answers

15 Students' own answers.

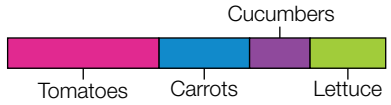
16 Students' own answers.

Exercise 9.4

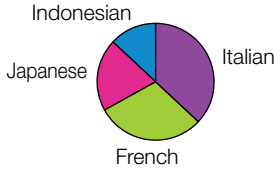
1 (a) (i)



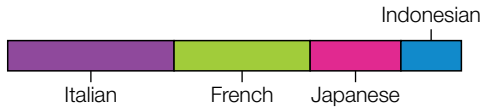
(ii)



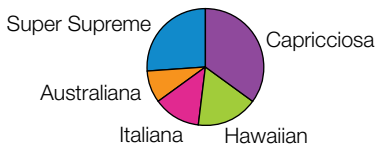
(b) (i)



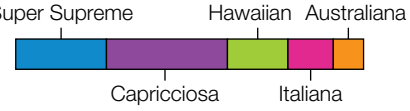
(ii)



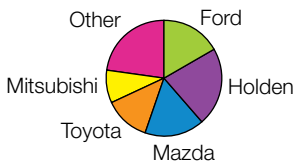
(c) (i)



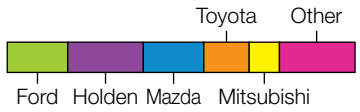
(ii)



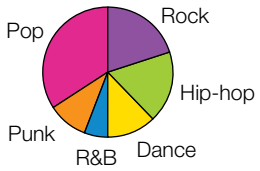
(d) **(i)**



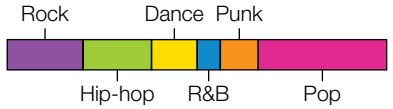
(ii)



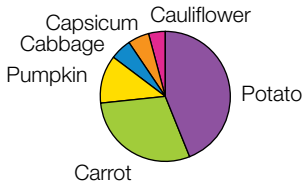
(e) **(i)**



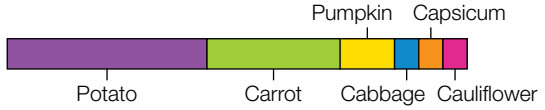
(ii)



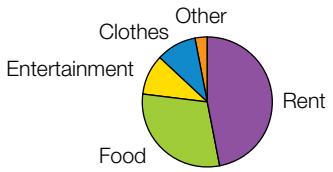
(f) **(i)**



(ii)

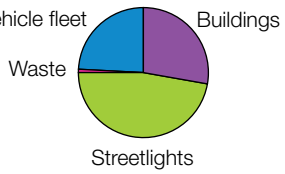


(g) **(i)**



(ii)

2 **(a)**



(b)

3 **(a)** A **(b)** D

4 **(a)** 3.3% **(b)** 5.2%

(c) 5–25 minutes **(d)** 94.3%

(e) The class intervals are not the same size.

5 **(a)** 4% **(b)** 1% **(c)** 4%

(d) **(i)** non-recyclables **(ii)** 79%

(e) paper **(f)** glass **(g)** No

6 **(a)** bags **(b)** sheeting and film

(c) confectionery wrappers **(d)** percentages

(e) all items not included in the other categories

(f) It is important because the survey would not be of much value if only a small number of items had been collected.

(g) They divide something into sections.

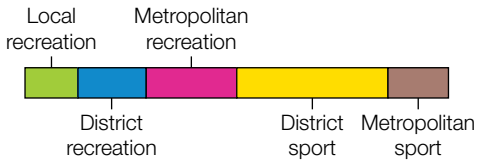
7 **(a)** Local recreation 70°, District recreation 90°, Metropolitan recreation 60°, District sport 100°, Metropolitan sport 40°

(b) 17% **(c)** 28% **(d)** 39%

(e) Recreation, 22% more

(f) Metropolitan sport

(g)



8 **(a)** There are a lot of other languages.

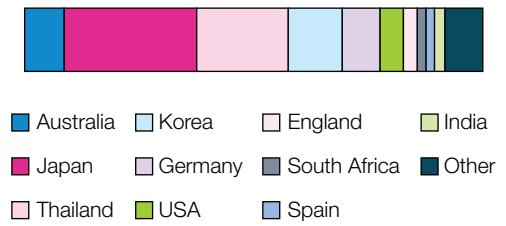
(b) Mandarin increased the most. Italian decreased the most.

(c) 252 000

9 **(a)** Generally the order is largest to smallest, to see the most important ones together and what portion they make up. The exception is to put Australia first so this features the 'home' country. 'Other' goes last as this would be made up of many countries, each with a smaller contribution than the smallest on the list.

(b) 8 mm (to the nearest mm)

(c) **Car sales Australia, July 2015 by place of origin**



(d) More than 60% of the new cars sold in Australia in July 2015 were imported from Japan, Thailand or Korea.

10 **(a)** only if you know that total income = total expenses

(b) $517m **(c)** $416m **(d)** $101m

Open ended—Sample answers

11 Students' own answers.

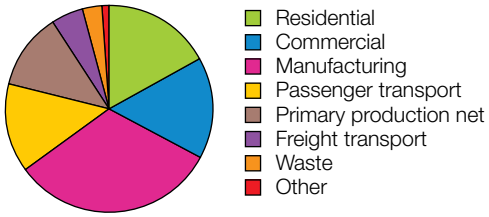
12 Students' own answers.

Half-time 9

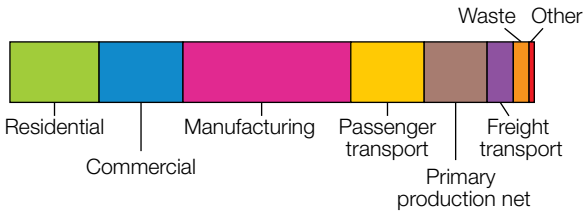
1 (a) (i) 3.7 (ii) 3 (iii) 2 (iv) 7

(b) (i) 5 (ii) 5 (iii) 4 and 6 (iv) 7

2 (a) **Victoria's greenhouse gas emissions**



(b)



3 (a) This gives eight groups, which is within the 5–10 range preferred.

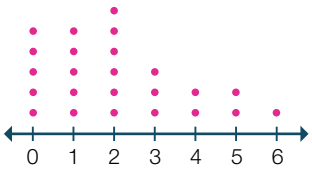
(b)

Height range	Frequency
125–129	1
130–134	1
135–139	2
140–144	7
145–149	5
150–154	6
155–159	2
160–164	1

4

Number absent	Frequency
0	8
1	6
2	4
3	2
4	1

5



6 (a)

STEM	LEAF
6	2 7
7	4 6
8	0 2 4 5 6
9	0 1
10	
11	
12	
13	2

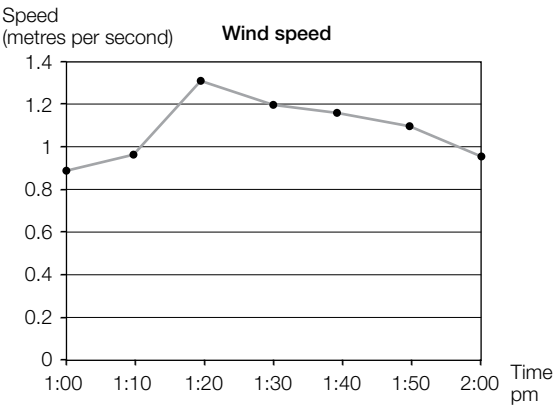
Key: 6 | 2 = 62

(b) 83

(c) 132 appears to be an outlier. The rest of the data in the set is bunched together.

Exercise 9.5

1 (a)



(b) between 1:10 pm and 2:00 pm (not inclusive)

2 (a) C (b) B (c) C

3 (a) (i) 41 °C (ii) 17 January

(b) (i) 19 °C (ii) 25, 26 January

(c) 22 °C and 38 °C (d) 19 °C and 27 °C

4 (a) start Year 5 40%, end Year 5 38%

(b) Year 7 (c) Feb–July Year 4

(d) 3% decrease. As the storage levels drop about 3% at this time each year, there is not a lot of rain in December in Perth.

(e) increase of 9.5%

(f) August

(g) January

5 (a) 50 L

(b) any time between 1:00 pm and 1:30 pm

(c) at least 60 L

(d) 11:30 am to at least 12:30 pm

6 (a) These are the census years.

(b) Actual numbers were unknown because many Indigenous people live in very remote communities and many move from place to place; they were not all able to be given census forms, so they were not all recorded in the census.

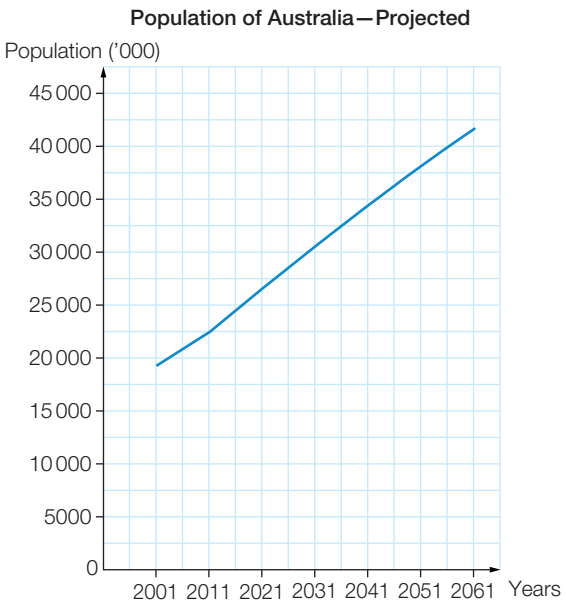
(c) 285 000 (approximately)

(d) 670 000 (approximately)

(e) More people with an Indigenous background identified themselves as being Indigenous.

(f) Better healthcare means more people living for a longer time.

7 (a)



(b) 28 072 000 **(c)** 22 238 000

(d) Factors that increase population: births and immigration (to Australia). Factors that decrease population: death and emigration (from Australia).

(e) (Number of births) + (number of immigrants) = (number of deaths) + (number of emigrants)

8 (a) The lines are the same shape. When there is an increase in one there is an increase in the other. There are consistently more boys than girls.

(b) At the start of the century the rate of increase was higher for about four years. After that there were individual years with big increases but generally the growth has been slower.

(c) 0.5% per year

9 (a) yes, 2000 **(b)** 150 cm **(c)** 175 cm

(d) 2000 **(e)** 2002 **(f)** 1999

(g) 2000 or 2003 **(h)** 2003 **(i)** 2006

(j) August **(k)** September

(l) Perhaps—the last 4 years have seen less snow than the previous 6 years, but this is not conclusive.

(m) Line graphs are more appropriate to show this information as the depth is changing over time, and a large number of bars would be needed to show the daily change.

Open-ended—Sample answers

10 Students' own answers.

11 (a) Students' own answers; should notice that the gap between male and female remains, but this time there are more women.

(b) Students' own answers; increases are caused by higher birth rate and immigration. Decreases are caused by death (more as the group gets older) and emigration.

Exercise 9.6

1 The shape of the graph of hours spent training each week is quite different from the shape of the other two graphs. There seems to be no connection between the hours spent training and either the height or the foot length.

The shapes of the height graph and the foot length graph are very similar, so there could be a connection between these two variables.

2 (a) Islander College, 350 students

(b) Bottom part; Hilltop College

(c) Gumnut College **(d)** Jinglebells College

(e) Faraway and Gumnut Colleges

3

	Mean	Median	Minimum	Range	Maximum
Adelaide	12.2	11.9	7.5	9.6	17.1
Brisbane	15.5	16.0	8.8	12.3	21.1

Brisbane has a much greater range in temperature. But both the mean and median indicate that Brisbane is, on average, a warmer place.

4 (a)

	Mean	Median	Mode	Range
Set A	5.7	6.5	7	9
Set B	5.55	5	2 & 9	7

The numbers in Set A are, on average, slightly larger and more spread out. The mode does not really give any useful information.

(b)

	Mean	Median	Mode	Range
Set A	3.75	3.5	2	7
Set B	5.75	6.5	7, 8 & 9	8

Although the range of both sets is very close, the numbers in Set A appear, on average, smaller than those in Set B. Again, the mode gives no useful information.

5 (a) The portion of voters who were satisfied with the performance of the PM increased dramatically, especially among the younger voters.

(b) new PM

(c) A much larger proportion is 'undecided' after the change of leadership, especially in the older category. This makes sense because people are less likely to have decided on their opinion about someone who is new.

6 April paid a mean price of $47.78 for her fruit, whereas Izzy paid $50.71. April bought 9 cases compared to 7 for Izzy with the range of prices paid by both friends being the same. Due to the small numbers, neither the median nor the mode will give useful information.

7 (a) Victoria

(b) 8 000 000 t

(c) Victoria

(d) almost 3 million tonnes

8 (a) Adelaide seems to average slightly less sunshine than Brisbane. Adelaide has a much greater variety in monthly hours of sunshine, i.e. more in the summer months and less in the winter months than Brisbane.

(b) Adelaide range = 6 h, Brisbane range = 2.2 h. Comment on the average is the same. For variation, can now say that Adelaide has more than double the range of hours of sunshine per month compared to Brisbane.

(c) Adelaide mean ≈ 7.5 h and median ≈ 8.2 h; Brisbane mean ≈ 8.3 h and median ≈ 8.2 h. The median alone suggests the hours of sunshine each month are the same. The considerably less sunshine in Adelaide in the winter has reduced the total so that Adelaide's mean number of hours per month is almost an hour less than Brisbane's.

9 (a) no

(b) Japan, because they have many more hospital beds per 10 000 people than the other countries.

(c) Because Japan has one of the highest GDPs, the amount it spends on health facilities is expected to be more.

(d) Indonesia

(e) Because Indonesia has one of the lower GDPs, the amount it spends on health facilities is expected to be lower.

(f) Australia's healthcare facilities do not seem to be as good as they should be given its GDP per person, which is the highest of all the countries shown.

(g) Vietnam's healthcare facilities seem to be very good given its GDP per person, which is the lowest of all the countries shown.

10 (a) The number of mobile phones in developed countries is 10 times that of developing countries. Half the population of developed countries has a mobile phone.

(b) The number of mobile phones doubled to nearly 100%.

(c) The number has increased 9 times from 2000 to 2007 to almost 50%.

(d) The population in the developing world is much greater than the population of the developed world.

(e) 1999–2000

(f) Students' own answers.

11 (a) Hawthorn

(b) The range for both groups is very high, so there is little consistency with the figures between clubs. The mean and median conflict as to which group has higher membership on average. For the Victorian teams the mean is pulled higher than the median by some very large memberships. For the non-Victorian teams the mean is pulled lower than the median by some very low memberships. Some AFL clubs are very popular both in Victoria and outside the state, but some clubs struggle to attract members (within and outside Victoria).

	Mean	Median	Range
Victorian	51 613.8	45 808.5	42 291
non-Victorian	39 999.75	50 134.5	46 741

(c) The following table summarises the results.

	Mean	Median	Range
2005	31 656.81	31 432	21 282
2015	46 452	48 070.5	61 557

Mean and median agree that average membership of these clubs has gone up by about 15 000–17 000 or increased by about 50% over 10 years. The range has more than doubled, indicating that some clubs are now doing much better than others.

Open-ended—Sample answers

12 Students' own answers.

13 Set A: 13, 10, 13, 21, 5, 12, 19, 12, 25, 20, 16, 3, 10, 10, 18, 17
Set B: 4, 2, 10, 19, 1, 23, 18, 17, 10, 3, 16, 18, 10, 14, 18, 17

Set A: median = 13, mode = 10, mean = 14, range = 22
Set B: median = 15, mode = 10, mean = 12.5, range = 22

Exercise 9.7

1 (a) $\frac{1}{2}$ (b) $\frac{2}{11}$ (c) $\frac{1}{6}$ (d) $\frac{1}{4}$

(e) $\frac{1}{13}$ (f) $\frac{1}{26}$ (g) $\frac{1}{26}$ (h) $\frac{1}{52}$

2 63%

3 (a) 33%

(b) Yes, results suggest bias, because the experimental probability 33% is quite different to the theoretical probability 16.67% for rolling a 6 on the die.

4 (a) impossible (b) certain

(c) not likely (d) certain

(e) certain (f) not likely

(g) impossible (h) not likely

(i) certain

5 (a) A (b) C (c) D (d) C (e) B

6 (a) {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

(b) {*a*, *b*, *i*, *l*, *o*, *p*, *r*, *t*, *y*}

(c) {HH, HT, TH, TT} where H = heads, T = tails

7 (a) (i) $\frac{1}{3}$ (ii) $\frac{1}{5}$ (iii) $\frac{2}{5}$

(b) (i) $\frac{2}{7}$ (ii) $\frac{3}{14}$ (iii) $\frac{3}{7}$

8 (a) Equally likely. $\Pr(1) = \Pr(10) = \frac{1}{12}$

(b) Not equally likely. $\Pr(p) = \frac{1}{11}$, $\Pr(i) = \frac{2}{11}$

(c) Not equally likely. $\Pr(\text{H twice}) = \frac{1}{4}$,

$\Pr(\text{H once, T once}) = \frac{1}{2}$

9 (a) **(i)** yes **(ii)** 50

(b) **(i)** no **(ii)** 200

(c) **(i)** no **(ii)** Simon

10 28

11 Sample answer: impossible, virtually impossible, almost never, most unlikely, sometimes, maybe, even chance, quite frequently, more often than not, quite often, often, likely, usually, very likely, almost always, sure thing

12 (a) 4 (possible: brown, brown, blue, blue)

(b) 8 (possible: blue, blue, blue, blue, blue, blue, brown, brown)

(c) 3 (possible: blue, brown, blue)

(d) 7 (all 6 blues could come out first, then brown)

Open-ended—Sample answers

13 (a) Monday will follow Tuesday. Humans will run 100 m in under 5 seconds.

(b) It will reach 50 °C in Australia next year. Australia will win the most gold medals at the next Olympics.

(c) It will rain in Sydney in the next month. Australia will win a swimming event medal at the next Olympics.

(d) The Sun will rise tomorrow. A thrown ball will fall to the ground if it is not caught.

14 (a) Possible solutions: drawing an odd number; drawing a number less than 11.

(b) Possible solutions: drawing a number from 6 to 17, drawing a number less than 13.

(c) Possible solutions: drawing a multiple of 5 that is less than 20; drawing a number greater than 17.

Challenge 9

1 C If the dice are arranged so that the one-dot and the two-dot faces can't be seen, then the greatest number of dots will be viewable; that is, 42 – 6 = 36.

Opposite dots on a die total 7
i.e. 1 + 6, 2 + 5, 3 + 4, 4 + 3, 5 + 2, 6 + 1;
there are 6 lots, $6 \times 7 = 42$.

Or: $2 \times (6 + 5 + 4 + 3) = 36$

2 Ash Black is wearing the white tie and not the green tie because Ash Black is talking to the person wearing the green tie. Therefore, Ash Black must be wearing the white tie. Jade Green cannot be wearing the white tie so she must be wearing the black tie. Therefore, Blanche White is wearing the green tie.

3 D 4 of the six faces will contain odd numbers.

4 D The shortest time wins the race, so the shortest bar is Maggie.

5 B 3 blue + 6 green + 2 red + 6 yellow = 17 marbles. As the probability is $\frac{3}{7}$, the number of marbles in the bag must be a multiple of 7, so at least 21. There are 9 blue or green marbles, $9 = 3 \times 3$, so need 21 marbles in the bag. Vijay adds 4 white marbles to the bag.

6 A 15 + 10 + 20 + 30 = 75; 100 – 75 = 25

7 C Has won 19 of the 20 games, needs to win 24 out of 25 games to reach 96%, so the next 5 games.

Chapter review 9

1

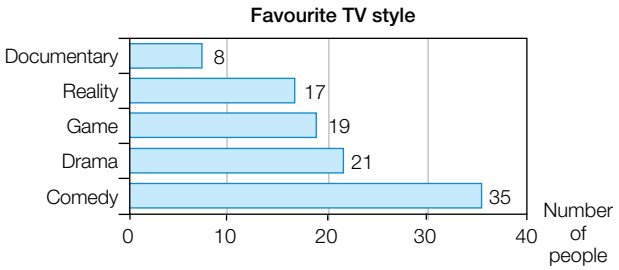
Number of smartphones owned	Tally	Frequency
0	|||	3
1	𝍸 𝍸 |	11
2	𝍸	5
3	||	2
4	|	1
5	|	1
		23

2 (a) mean = 3.55, median = 3, mode = 2, range = 6

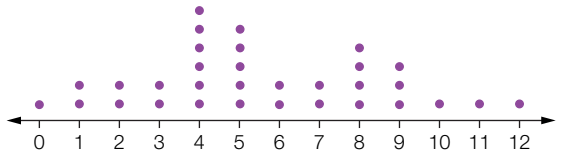
(b) mean = 35, median = 35, there is no mode, range = 50

(c) mean = 3.72, median = 3.9, mode = 4.1, range = 1.5

3



4 (a)



(b) 5 **(c)** No, the data set seems well spread out.

5 (a)

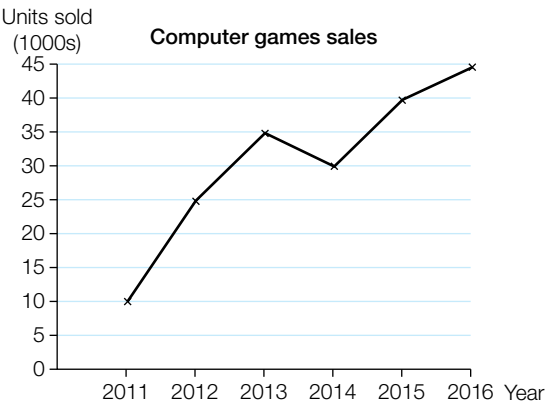
STEM	LEAF
0	0 0 0 0 0 1 2 2 2 3 4 5 5 6 7 7
1	1 4 5 7 7 7 8 8 9
2	0 1
3	1
4	0 1 2 2 6
5	0 0 2 6 8
6	1 3 4
7	7
8	8
9	
10	
11	5

Key: 2 \| 0 = 20

(b) 17.5 **(c)** The score of 115 appears to be an outlier.

6 **(a)** D **(b)** D

7

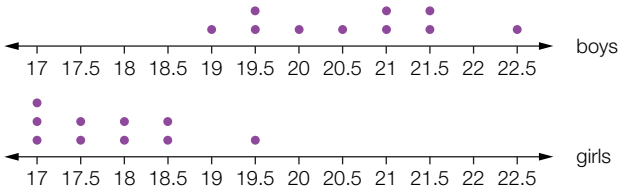


8

	Mean	Median	Range
Cairns	167.7	92.4	425.5
Brisbane	78.9	73.35	95.7

These two cities both receive a lot of rain, although Cairns clearly receives more rain than Brisbane. The wettest month for Cairns is much wetter than the wettest month for Brisbane, as reflected in the mean and range results. The driest months are almost identical. Cairns has a wet summer and a dry winter, whereas Brisbane follows a similar pattern but has a wetter period in May and June. The median results are not that far apart, which reflects the similar patterns, but still shows Cairns to be wetter.

9



The boys clearly have larger handspans. The median for boys is 20.75 and for girls it is 17.75. The range for boys is 3.5 compared to 2.5 for girls, so the data for boys are slightly more spread out. Only one girl has a handspan larger than the boy with the smallest handspan.

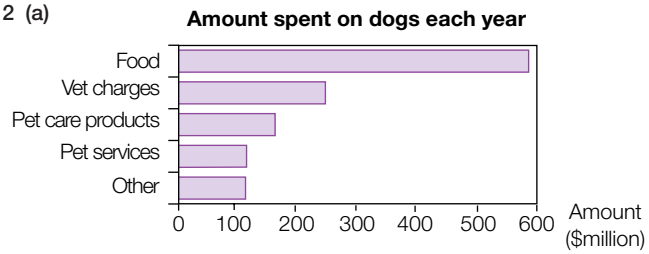
10 **(a)** B **(b)** B

11 **(a)**

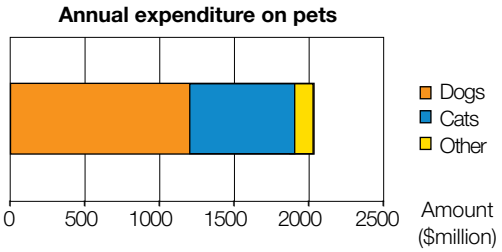
Shoe size	Frequency
1	2
$1\frac{1}{2}$	2
2	2
$2\frac{1}{2}$	3
3	3
$3\frac{1}{2}$	5
4	3
$4\frac{1}{2}$	3
5	3
$5\frac{1}{2}$	3
6	1

(b) $3\frac{1}{2}$ **(c)** 6

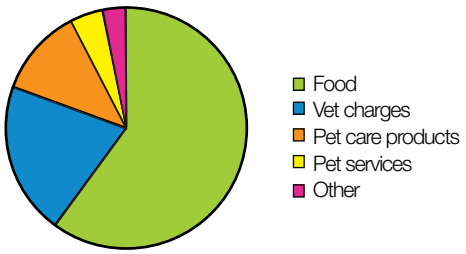
12 **(a)**



(b)



(c)



(d) The data sets were not compared over a period of time.

13 **(a)** column graph

(b) July

(c) 30 cm

(d) December, January, February, March

14 Yes, it is fair.

15 **(a)** 0

(b) 1

(c) the one with the probability of $\frac{3}{4}$

(d) Probabilities can't be greater than 1.

16 **(a)**

Height (cm)	Frequency
140	4
141	1
142	1
143	1
144	2
145	1
146	1
147	2
148	3
149	2
150	1
151	2
152	2
153	1
154	1

(b) **(i)** 140 **(ii)** 147 **(iii)** 146.6

(c)

Height (cm)	Frequency
138	1
141	1
142	1
143	2
144	2
145	2
146	2
148	2
150	2
151	1
152	1
154	3
156	2
157	3

(d) (i) 154, 157 (ii) 148 (iii) 148.8

(e) On all measures of centre, the Year 8 students are taller than the Year 7 students. The shortest student, however, is in Year 8, as is the tallest. The Year 8 heights are clustered more at the top of the range of values.

(f)

Height (cm)	Frequency
138	1
140	4
141	2
142	2
143	3
144	4
145	3
146	3
147	2
148	5
149	2
150	3
151	3
152	3
153	1
154	4
156	2
157	3

(g) (i) 148 (ii) 148 (iii) 147.7

(h) The three measures for the combined group are very close to each other. The average student is about 148 cm tall, the shortest is 138 cm and the tallest is 157 cm.

17 (a) Don't place any counters on 1, 2, 19 or 20 as these totals will never appear as the sum of the three dice.

(b) 10 and 11 should appear more often than any other total.

(c) The dice rolls for any particular game could be quite different from the next game.

Numeracy practice 9

1 B **2** D **3** C **4** C

5 (a) Friday (b) 30 (c) Thursday

6 C

Chapter 10

Recall 10

1 (a) 56° (b) 150° (c) 50°

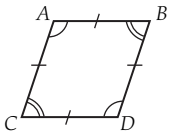
2 (a) 20° (b) 60° (c) 40°

3 (a) 66° (b) $a = 110°$, $b = 70°$, $c = 70°$

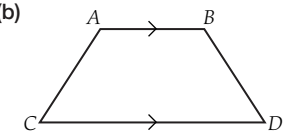
(c) 120°

4 Students' diagrams may vary.

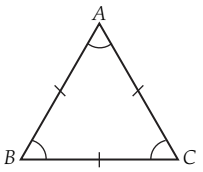
(a)



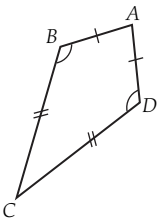
(b)



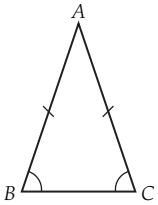
(c)



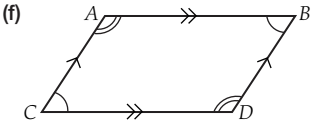
(d)



(e)



(f)



Exercise 10.1

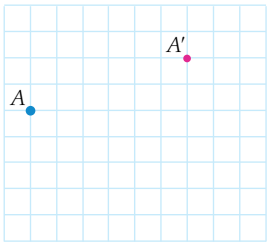
1 (a) 5 units left

(b) 5 units down

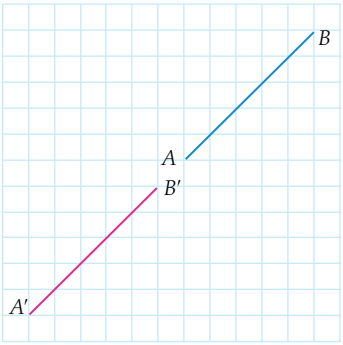
(c) 5 units right and 3 units down

(d) 7 units left and 2 units up

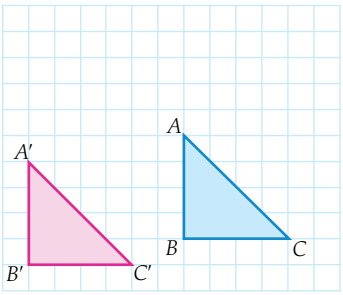
2 (a)



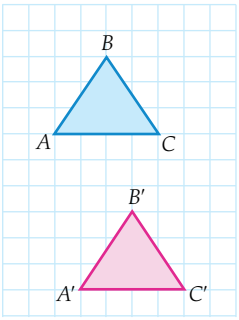
(b)



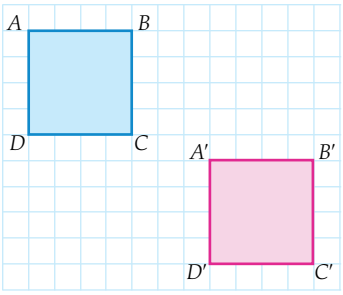
(c)



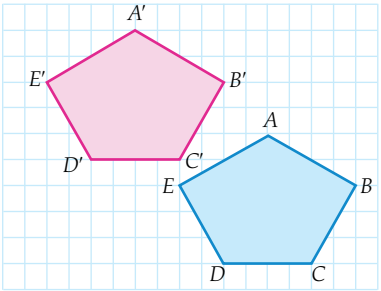
(d)



(e)

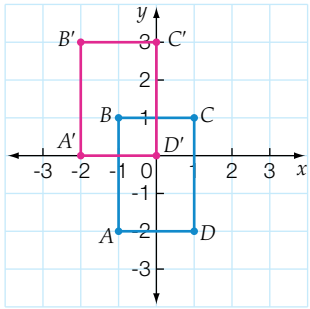


(f)



3 (a) $A(-1, -2)$, $B(-1, 1)$, $C(1, 1)$, $D(1, -2)$

(b)



(c) $A'(-2, 0)$, $B'(-2, 3)$, $C'(0, 3)$, $D'(0, 0)$

(d) Subtracting 1 from the x-coordinates of the original and adding 2 to the y-coordinates of the original will give the image coordinates.

4 B **5** D

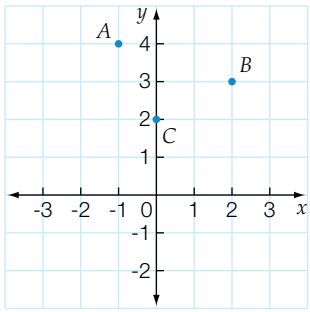
6 2 units left and 6 units up

7 [-3, -4] **8** [-10, -4]

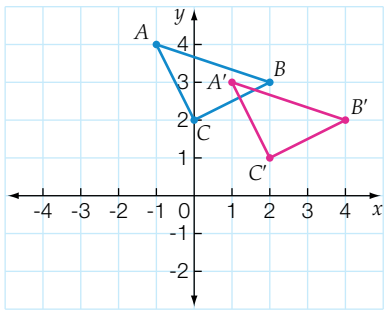
9 'left 2 up 1' or 'up 1 left 2': [-2,+1]
'left 1 up 2' or 'up 2 left 1': [-1,+2]

10 North America (pink) and South America (light blue) move left.
Africa (dark blue) moves slightly up and right.
Antarctica (green) moves down.
Eurasia (dark purple) and India (blue) move up and to the right.
Australia (light purple) moves to the right.

11 (a)



(b)



(c) $A'(1, 3)$, $B'(4, 2)$, $C'(2, 1)$

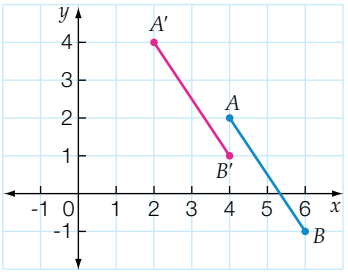
12 10 units left and 4 units up

13 4 units down

14 [0, 9]

15 **(a)** $A'(2, 4)$, $B'(4, 1)$

(b)

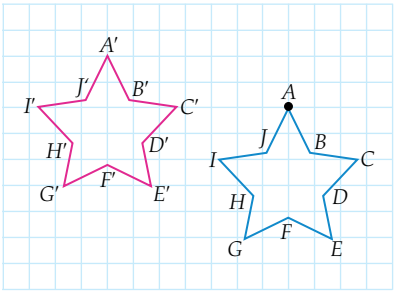


16 200 units right

17 **(a)** 7 units left and 2 units up

(b) 7 units right and 2 units down

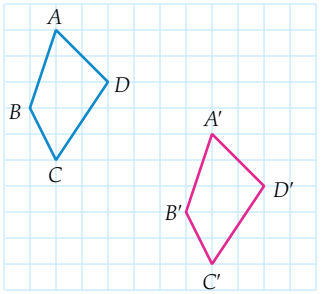
(c)



Open-ended—Sample answers

18 Students' own answers.

19

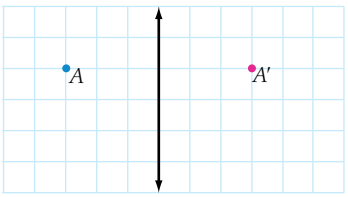


The reverse translation is 6 units left and 4 units up.

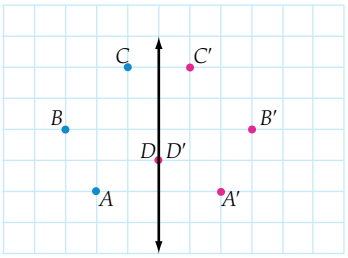
20 The statement is correct, because the order in which you do translations does not affect the result of the final image. This is because when translating a shape you add or subtract units, and adding or subtracting can be done in any order.

Exercise 10.2

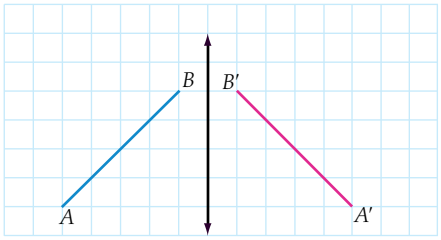
1 **(a)**



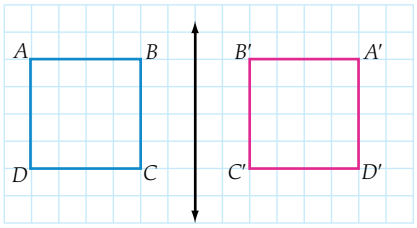
(b)



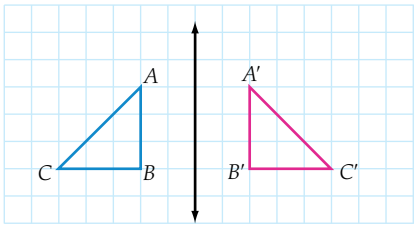
(c)



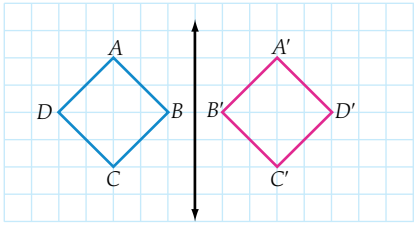
(d)



(e)

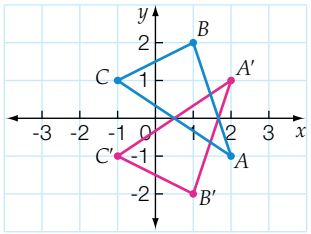


(f)



2 **(a)** $A(2, -1)$, $B(1, 2)$, $C(-1, 1)$

(b)

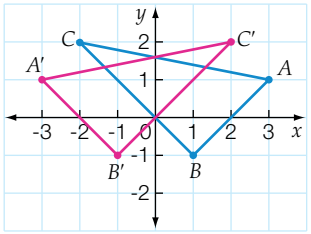


(c) $A'(2, 1)$, $B'(1, -2)$, $C'(-1, -1)$

(d) The x-coordinates of the image are the same as the x-coordinates of the original but the y-coordinates of the image are the negative of the y-coordinates of the original.

3 **(a)** $A(3, 1)$, $B(1, -1)$, $C(-2, 2)$

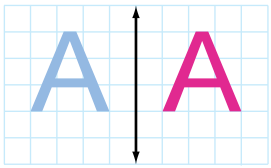
(b)



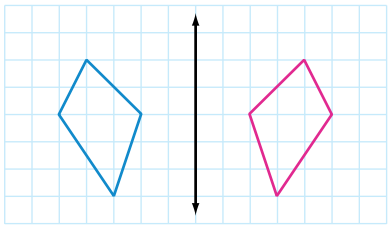
(c) $A'(-3, 1)$, $B'(-1, -1)$, $C'(2, 2)$

(d) The x-coordinates of the image are the negative of the x-coordinates of the original but the y-coordinates of the image are the same as the y-coordinates of the original.

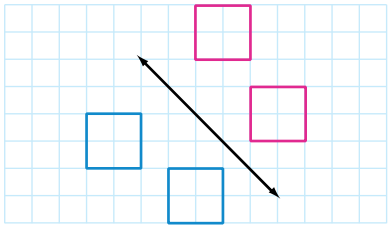
4 (a)



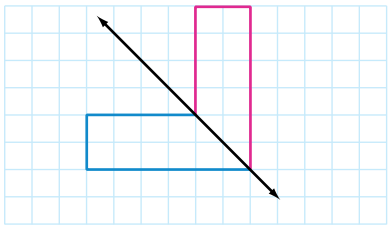
(b)



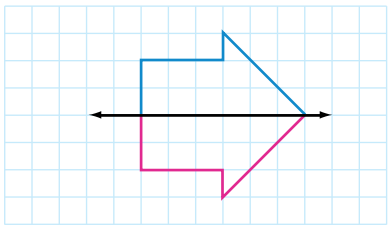
(c)



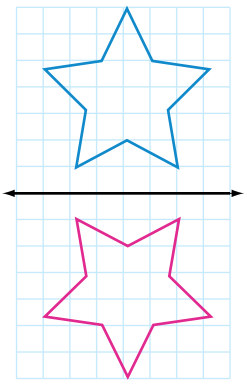
(d)



(e)



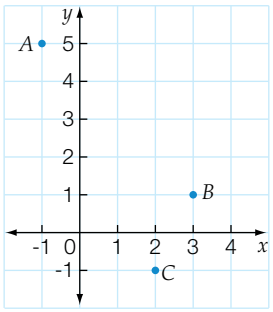
(f)



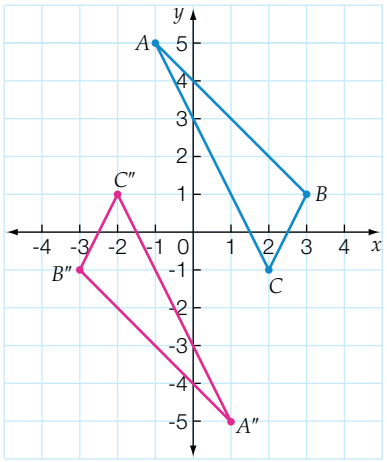
5 B

6 any five of the following letters: H, I, M, O, T, U, V, W, X, Y

7 (a)

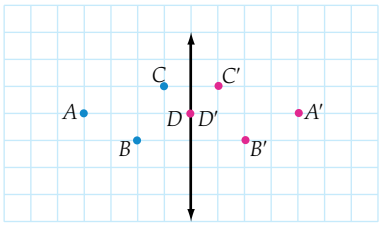


(b)

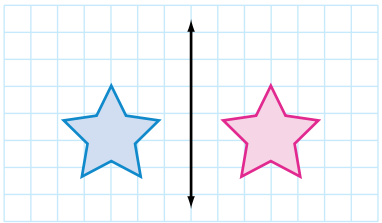


(c) $A''(1, -5)$, $B''(-3, -1)$, $C''(-2, 1)$

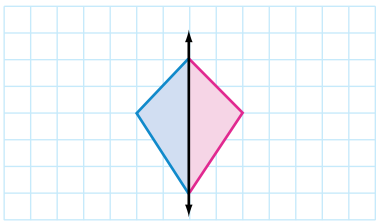
8 (a)



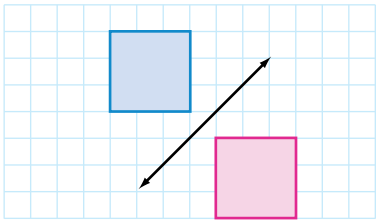
(b)



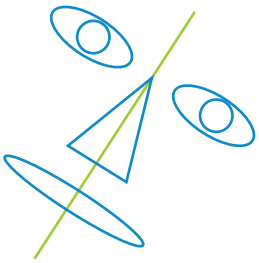
(c)



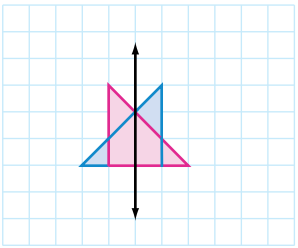
(d)



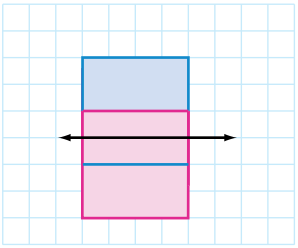
9



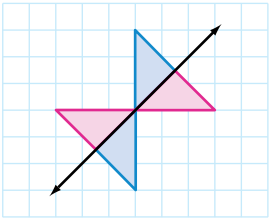
10 (a)



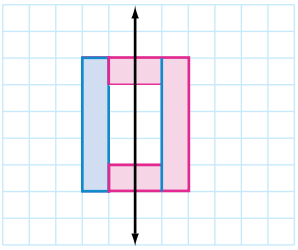
(b)



(c)

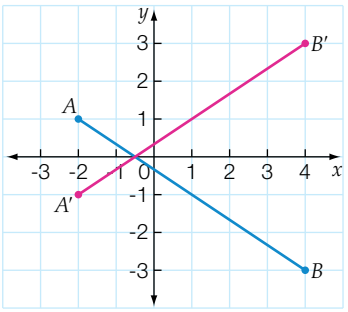


(d)



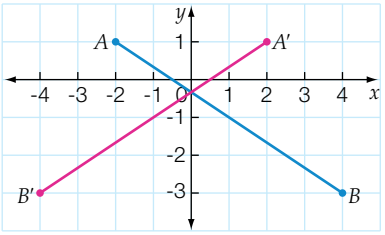
11 (a) $A'(-2, -1)$, $B'(4, 3)$

(b)



(c) $A'(2, 1)$, $B'(-4, -3)$

(d)



12 (a) 00, 01, 03, 08, 10, 11, 13, 18, 30, 31, 33, 38

(b) 1, 3, 8, 10, 11

(c) 72

(d) 96

(e) No. In a 24-hour time, the time is correctly reflected for only two more hours (1300 and 1800) than in 12-hour time.

Open-ended—Sample answers

13 Students' own answers.

14 Students' own answers.

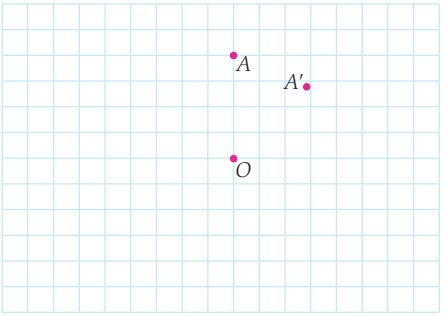
15 Students' own answers.

16 (a)–(b) Students' own diagrams.

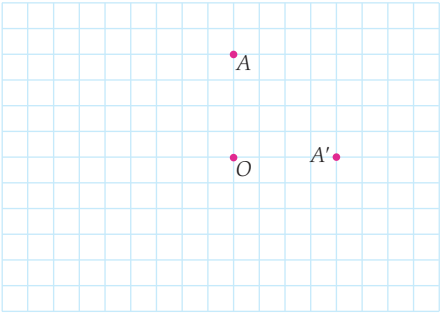
(c) The images found in parts (a) and (b) are the same.

Exercise 10.3

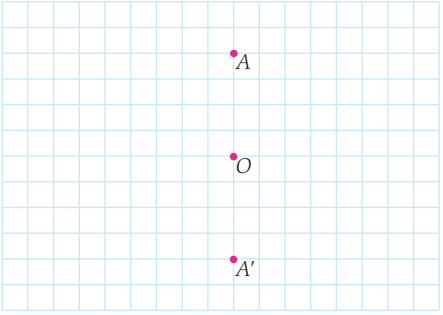
1 (a)



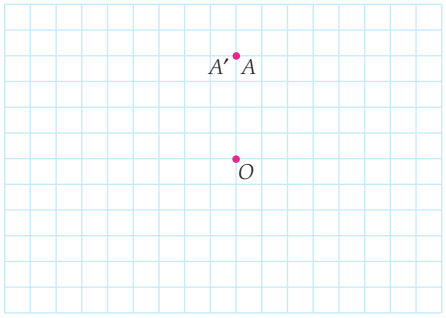
(b)



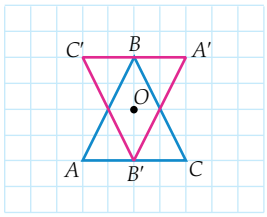
(c)



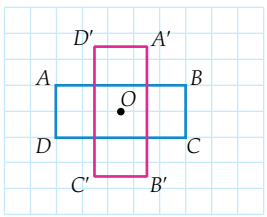
(d)



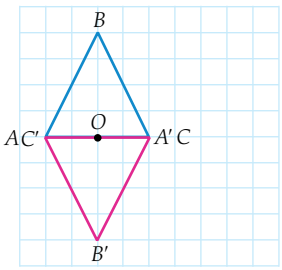
2 (a)



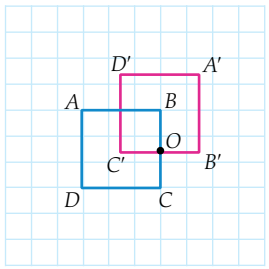
(b)



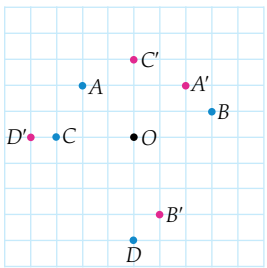
(c)



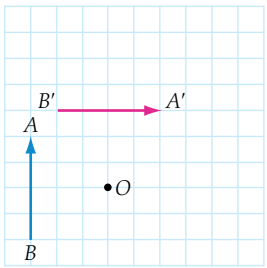
(d)



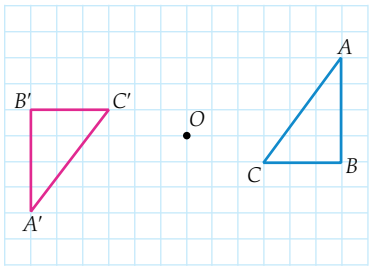
3 (a)



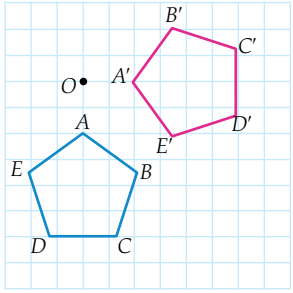
(b)



(c)

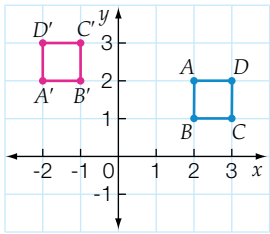


(d)



4 (a) $A(2, 2)$, $B(2, 1)$, $C(3, 1)$, $D(3, 2)$

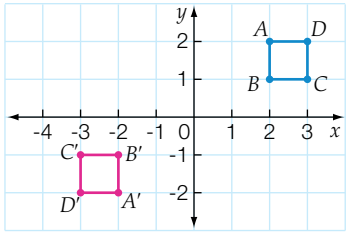
(b) (i)



(ii) $A'(-2, 2)$, $B'(-1, 2)$, $C'(-1, 3)$, $D'(-2, 3)$

(iii) The x-coordinates of the image are the negative of the y-coordinates of the original vertices and the y-coordinates of the image are the x-coordinates of the original vertices.

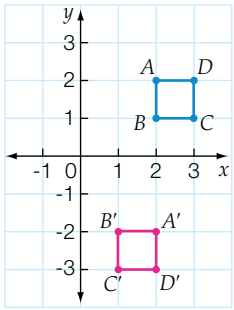
(c) (i)



(ii) $A'(-2, -2)$, $B'(-2, -1)$, $C'(-3, -1)$, $D'(-3, -2)$

(iii) All the coordinates of the image vertices are the negative of the original coordinates of the vertices.

(d) (i)



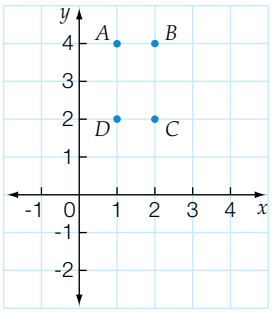
(ii) $A'(2, -2)$, $B'(1, -2)$, $C'(1, -3)$, $D'(2, -3)$

(iii) The x-coordinates of the image vertices are the y-coordinates of the original vertices and the y-coordinates of the image vertices are the negative of the x-coordinates of the original vertices.

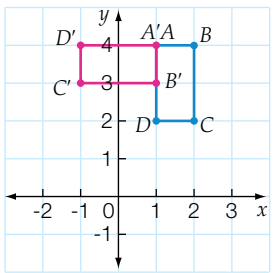
5 C

6 B

7 (a)



(b)



(c) $A'(1, 4)$, $B'(1, 3)$, $C'(-1, 3)$, $D'(-1, 4)$

8 240°

9 (a) rotated 180° about O

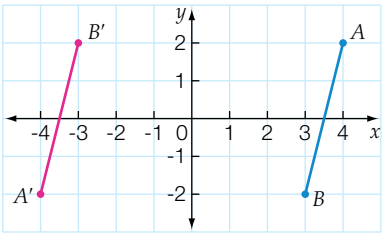
(b) rotated 90° in an anticlockwise direction or 270° in a clockwise direction about O

(c) rotated 90° in an anticlockwise direction or 270° in a clockwise direction about O

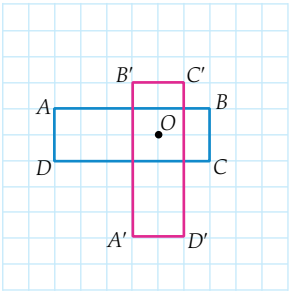
(d) rotated 180° about O

10 (a) $A'(-4, -2)$, $B'(-3, 2)$

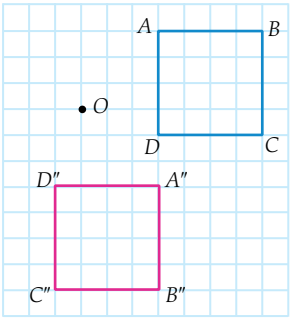
(b)



11 (a)



(b)



12 (a) H, I, N, O, S, X, Z

(b) All of these letters are perfectly symmetrical.

H, I, O, X

13 (a) 120° **(b)** 180° **(c)** 90°

(d) 72° **(e)** 22.5° **(f)** 36°

14 This is a reflection, because the orientation of the red square's vertices is completed in a manner that indicates a direct reflection. If it was rotated, the order of the vertices would also be rotated.

15 (a) 120° **(b)** 90° **(c)** 72° **(d)** 60°

16 angle of rotation $= \frac{360°}{n}$, where n is the number of sides in the regular polygon

Open-ended—Sample answers

17 Students' own answers.

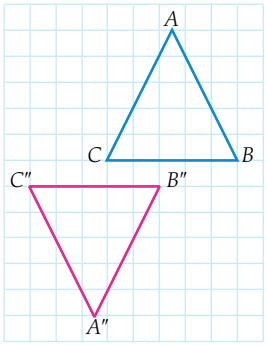
18 Disagree, because the order of the vertices of a figure is shifting in the same direction that the rotation is taking place. In a reflection, the vertices stay in a similar orientation, each an equivalent distance from the line of reflection.

19 (a)–(d) Students' own diagrams.

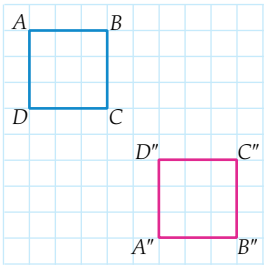
(e) The images in parts **(b)** and **(d)** are the same as the original polygons.

Exercise 10.4

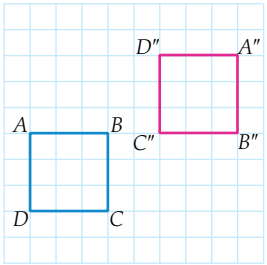
1 (a)



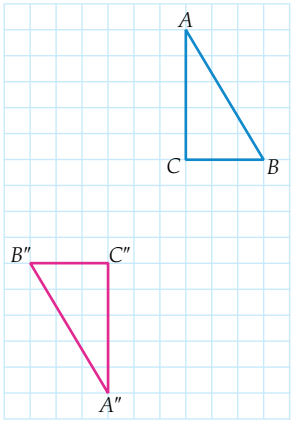
(b)



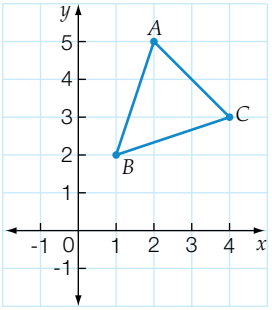
(c)



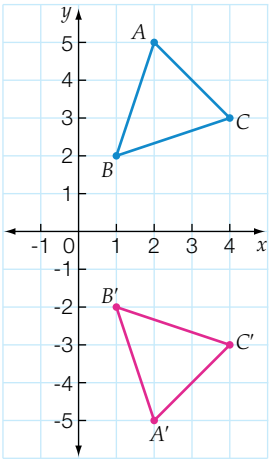
(d)



2 (a) $A(2, 5)$, $B(1, 2)$, $C(4, 3)$

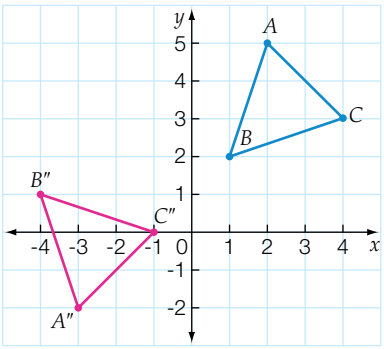


(b)



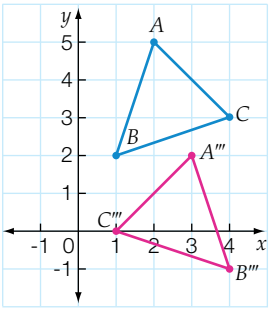
(c) $A'(2, -5)$, $B'(1, -2)$, $C'(4, -3)$

(d)



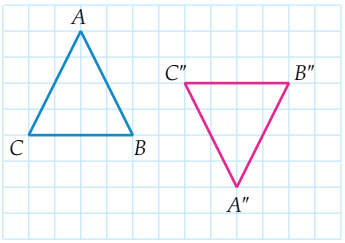
(e) $A''(-3, -2)$, $B''(-4, 1)$, $C''(-1, 0)$

(f)

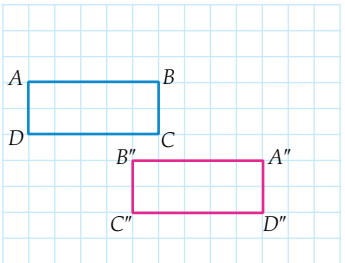


(g) $A'''(3, 2)$, $B'''(4, -1)$, $C'''(1, 0)$

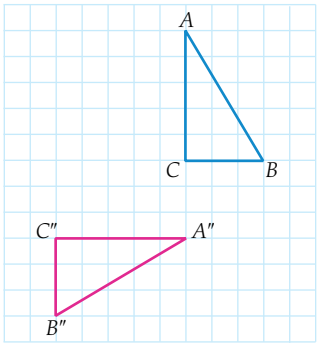
3 (a)



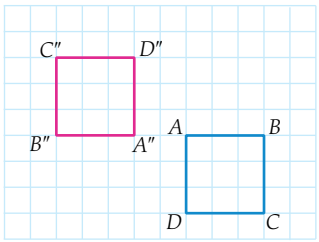
(b)



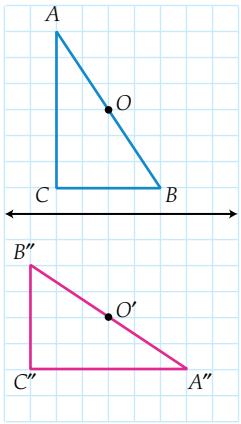
(c)



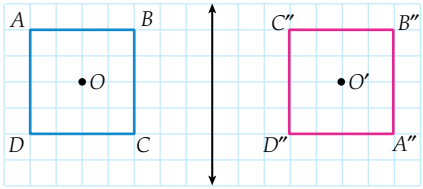
(d)



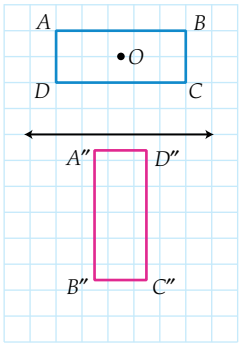
(e)



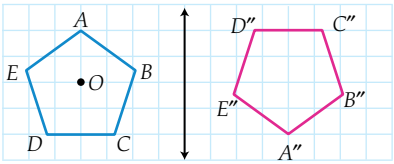
(f)



(g)

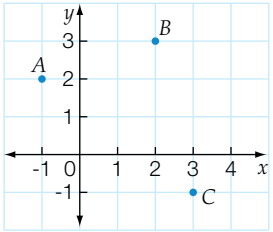


(h)

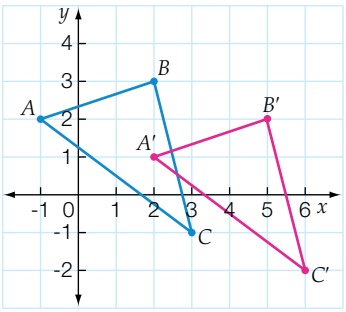


4 B **5** D

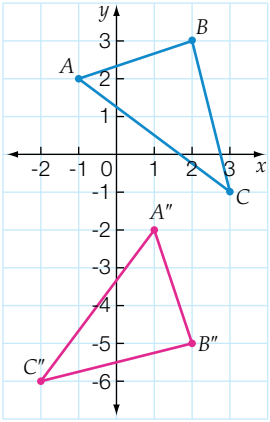
6 (a)



(b)



(c)



(d) $A''(1, -2)$, $B''(2, -5)$, $C''(-2, -6)$

7 (a) Reflect along the line $C''B''$ and translate 3 units right and 1 unit up.

(b) Reflect along the line $D''C''$ and translate 2 units up and 5 units left.

(c) Rotate 90° in an anticlockwise direction about B'' and translate 5 units left.

(d) Rotate 180° about C'' in either direction and translate 4 units up and 3 units right.

8 (a) Reflected along the axis of reflection and then translated 4 units right and 2 units down.

(b) Translated 8 units down and then reflected along the vertical line of reflection.

(c) Rotated 180° about point A and then translated 5 units right and 3 units up.

(d) Reflected along the axis of reflection and then rotated 90° in an anticlockwise direction about point B'.

(e) Rotated 45° in a clockwise direction about point C and then translated 8 units down and 3 units right.

(f) Rotated 180° about point C and reflected along the axis of reflection.

9 (a) Translate 4 units left and 2 units up and reflect along the axis of reflection.

(b) Reflect along the axis of reflection and translate 8 units up.

(c) Translate 3 units down and 5 units left and rotate 180° about point A'.

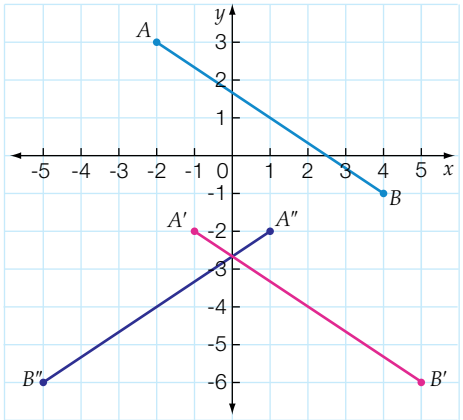
(d) Rotate 90° about point B'' in a clockwise direction and reflect in the axis of reflection.

(e) Translate 8 units up and 3 units left and rotate 45° in an anticlockwise direction about point C'.

(f) Reflect along the axis of reflection and rotate 180° about point C'.

10 (a) $A''(1, -2)$, $B''(-5, -6)$

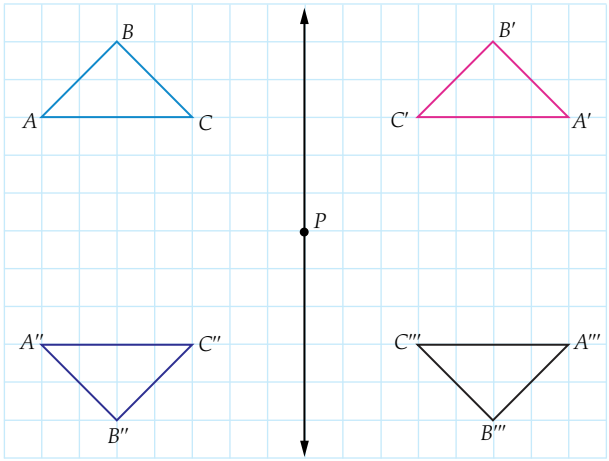
(b)



11 (a) Reflect vertically in the given line of reflection. Rotate 90° clockwise around the given centre of rotation.

(b) A1 Tile is in the correct position.
A2 Reflect vertically, then rotate 90° clockwise.
B1 Rotate 90° clockwise.
B2 Reflect vertically.

12



Rotation 180° about point P.

13 No, it is not possible.

14 (a) translation 60 cm to the right

(b) rotation of 45° anticlockwise direction

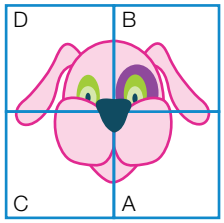
(c) translation 60 cm to the left, rotation of 45 in anticlockwise direction

15 Part D must be translated one up and one to the left, or one to the left and one up.

Part B must be rotated 90° anticlockwise.

Part A must be translated one to the right and one down, or one down and one to the right.

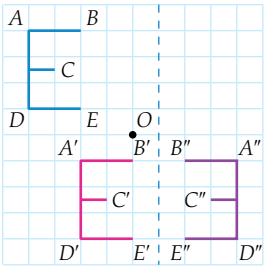
Part C remains the same.



Open-ended—Sample answers

16 Students' own answers.

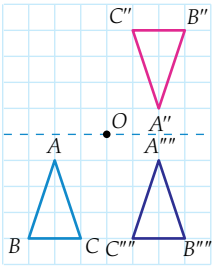
17 (a)



(b) Rotation 180° clockwise about O.

(c) Rotation 180° clockwise about O.

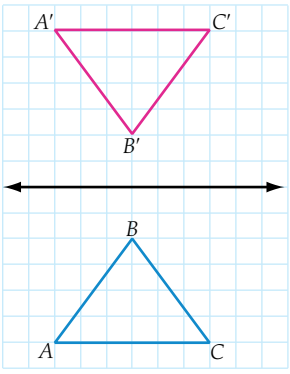
18 (a)



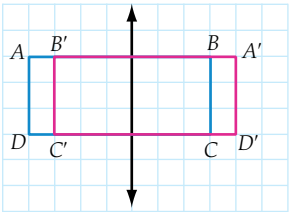
(b) A reflection in a vertical line of reflection passing through O.

Half-time 10

1 (a)



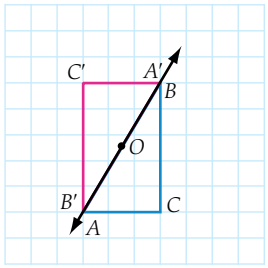
(b)



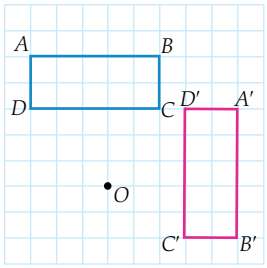
2 (a) Translated 5 units right and 4 units down.

(b) Translated 3 units left and 2 units up.

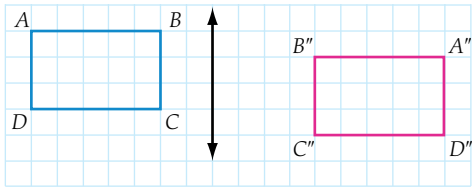
3 (a)



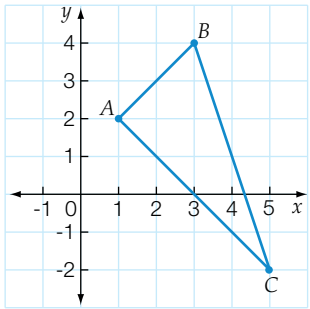
(b)



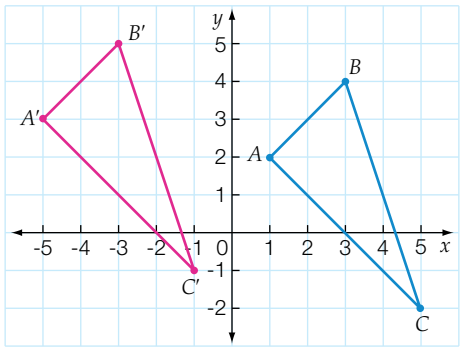
4



5 (a)

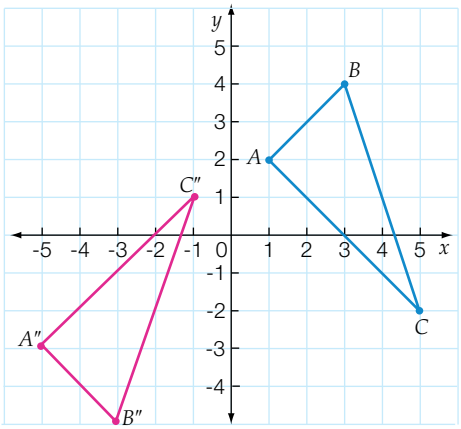


(b)



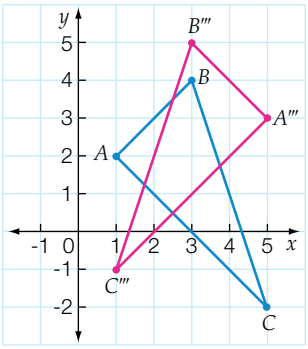
(c) $A'(-5, 3)$, $B'(-3, 5)$, $C'(-1, -1)$

(d)



(e) $A''(-5, -3)$, $B''(-3, -5)$, $C''(-1, 1)$

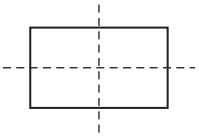
(f)



(g) $A'''(5, 3)$, $B'''(3, 5)$, $C'''(1, -1)$

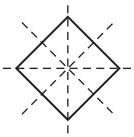
Exercise 10.5

1 (a) **(i)** yes



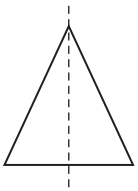
(ii) order = 2

(b) **(i)** yes



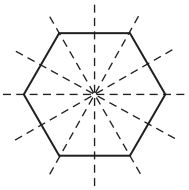
(ii) order = 4

(c) **(i)** yes



(ii) order = 1

(d) **(i)** yes



(ii) order = 6

(e) **(i)** yes

(ii) order = 1

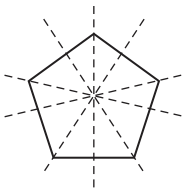
(f) no

(g) no

(h) **(i)** yes

(ii) order = 2

(i) **(i)** yes



(ii) order = 5

2 (a) **(i)** yes **(ii)** 2 **(iii)** 180°

(b) **(i)** yes **(ii)** 4 **(iii)** 90°

(c) **(i)** no It does not have rotational symmetry.

(d) **(i)** yes **(ii)** 6 **(iii)** 60°

(e) **(i)** no It does not have rotational symmetry.

(f) **(i)** no It does not have rotational symmetry.

(g) **(i)** no It does not have rotational symmetry.

(h) **(i)** yes **(ii)** 2 **(iii)** 180°

(i) **(i)** yes **(ii)** 5 **(iii)** 72°

3 (a) 4 **(b)** 2 **(c)** 1

4 D **5** C **6** A

7 (a) a, b, d, e, f, g, h, j, k, m, n, p, q, r, s, t, u, y, z

(b) c, i, v, w

(c) l, o, x

8 (a) F, G, J, K, L, N, P, Q, R, S, Z

(b) A, B, C, D, E, M, T, U, V, W, Y

(c) H, I, O, X

9 yes: H, I, N, O, S, X, Z

10 (a) order 1 reflectional symmetry and no rotational symmetry

(b) order 1 reflectional symmetry and no rotational symmetry

(c) order 5 reflectional symmetry and order 5 rotational symmetry

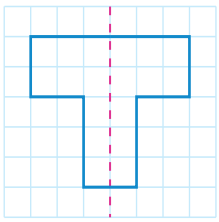
(d) order 5 reflectional symmetry and order 5 rotational symmetry

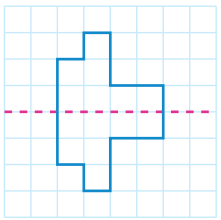
(e) order 1 reflectional symmetry and no rotational symmetry

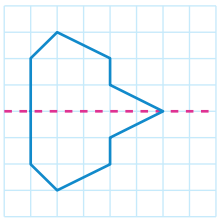
(f) order 1 reflectional symmetry and no rotational symmetry

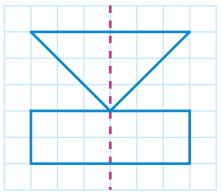
11 Image A is the left half of the original, combined with its mirror image. Image B is the right half of the original, combined with its mirror image.

12 (a) no **(b)** yes **(c)** no **(d)** no **(e)** yes **(f)** yes

13 (a) 

(b) 

(c) 

(d) 

14 (a) 3 (b) 4 (c) 5 (d) 6

15 equilateral triangle

Open-ended—Sample answers

16 O is the axis of symmetry. Other words are TOOT, MUM, MOM, HUH.

17 (a) rotation: 1001, 1111, 1691, 1881, 1961

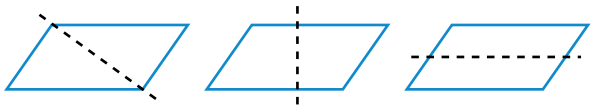
(b) reflection: 1001, 1111, 1331, 1881

(c) rotation and reflection: 1001, 1111, 1881

18 100 001, 101 101, 108 801, 110 011, 111 111, 118 811, 180 081, 181 181, 188 881

19 regular 5-point star; regular pentagon

20 A parallelogram has reflectional symmetry if it is a rhombus, a rectangle or a square. Otherwise, it has no reflectional symmetry.



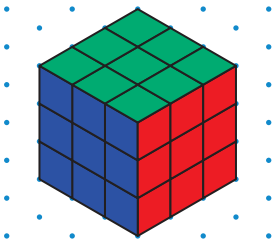
Exercise 10.6

1 Students' own drawings should be checked carefully.

2 (a) 4 (b) 12 (c) 12 (d) 9 (e) 8 (f) 10

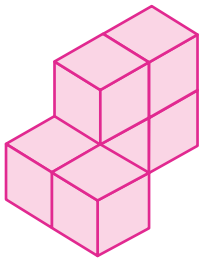
3 B

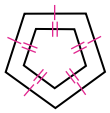
4 A

5 

6 (a) 8 (b) 12 (c) 9 (d) 10 (e) 21 (f) 12

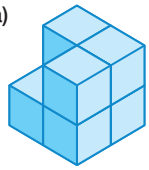
7 D

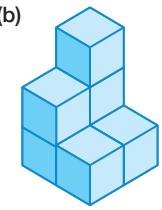
8 

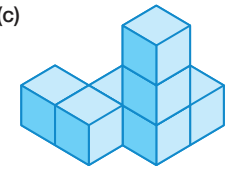
9 (a) 

(b) 

(c) No, each side is a rectangle with the same length and height.

10 (a) 

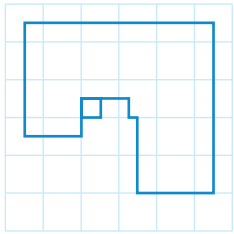
(b) 

(c) 

11 (a) It is not a rectangle, as one side has 6 blocks and another has four blocks. It is not possible to make this shape.

(b) Students' own answers. (c) Students' own answers.

12 Top view

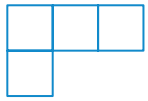


Open-ended—Sample answers

13 Students' own answers. 14 Students' own answers.

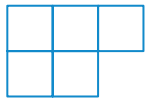
15 Students' own answers.

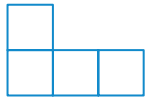
Exercise 10.7

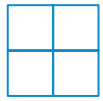
1 (a) (i) 

(ii) 

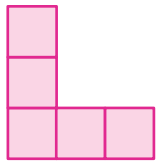
(iii) 

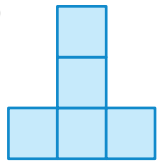
(b) (i) 

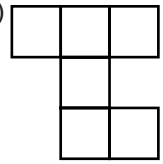
(ii) 

(iii) 

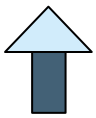
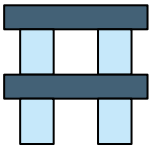
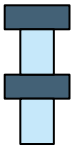
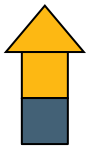
2 D

3 (a) 

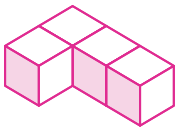
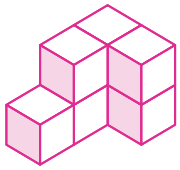
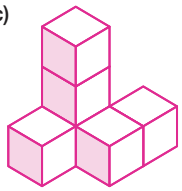
(b) 

(c) 

4

	Front	Side
(a)		
(b)		
(c)		
(d)		

5 (a) A (b) B

6 (a)  (b)  (c) 

7 (a)

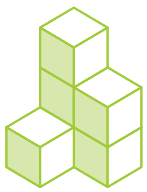
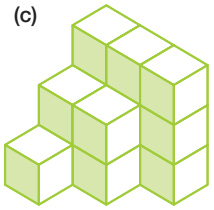
		1
1	1	1
1		

(b)

2	2	1
2		

(c)

3	2	
2	1	1

8 (a)  (b)  (c) 

9 (a)–(b) Students' own drawings.

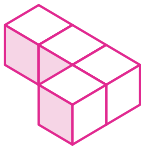
(c) 1; 3; 3 or 5; 3 or 5; 6 or 7

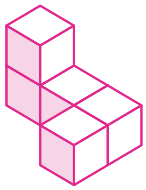
(d) Depending on the way they are shaded, the orientation of the cubes can appear differently. Some cubes may look like concave spaces. The answer to (c) gives the alternative numbers of cubes.

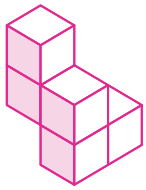
(e) Answers may vary.

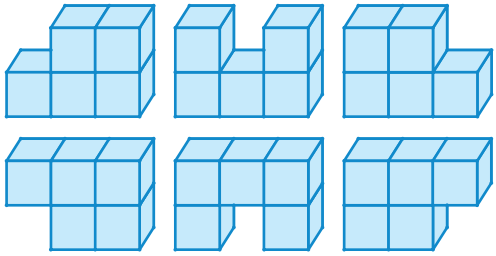
Open-ended—Sample answers

10 Possible answers include:


4 cubes


5 cubes


6 cubes

11 

12 Students' own answers.

Challenge 10

1 (a) Rotation through 180° in either direction.

(b) Reflection in the y-axis followed by reflection in the x-axis, or reflection in the x-axis followed by reflection in the y-axis.

(c) B'

2 2:30

3 One copy is fixed at C and D is rotated anticlockwise through 90°. Another copy is fixed at D and C is rotated clockwise through 90°. The third copy is translated up through a distance equal to the length of CD.

4 B

5 'A BOILED EGG IS HARD TO BEAT'. The whole message needs to be reflected in a line.

6 (a) Translate 8 cm to the right.

(b) Rotate 90° anticlockwise about the bottom corner, then translate 11 cm to the right.

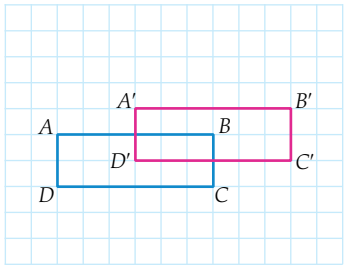
(c) Reflect in the 4 cm side, then translate 4 cm to the right.

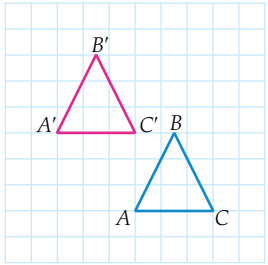
(d) Rotate 180° about the vertex located between the 4 cm and 5 cm sides.

Chapter review 10

1 (a) Translated 4 units right and 4 units down.

(b) Translated 4 units left and 3 units up.

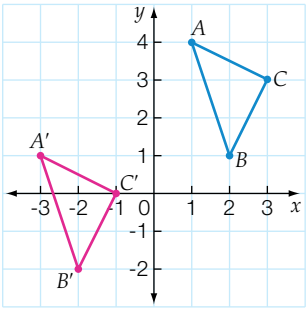
2 (a) 

(b) 

3 B

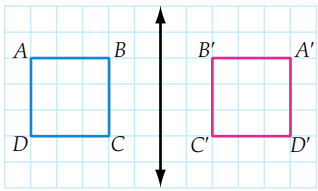
4 **(a)** $A(1, 4)$, $B(2, 1)$, $C(3, 3)$

(b)

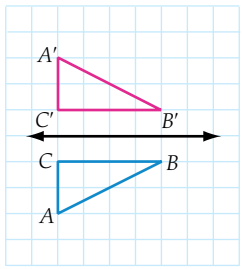


(c) $A'(-3, 1)$, $B'(-2, -2)$, $C'(-1, 0)$

5 **(a)**

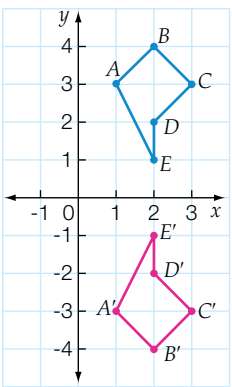


(b)



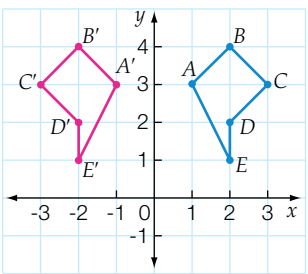
6 **(a)** $A(1, 3)$, $B(2, 4)$, $C(3, 3)$, $D(2, 2)$, $E(2, 1)$

(b)



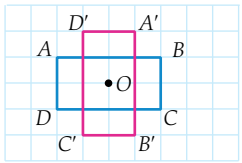
(c) $A'(1, -3)$, $B'(2, -4)$, $C'(3, -3)$, $D'(2, -2)$, $E'(2, -1)$

(d)

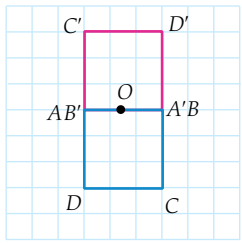


(e) $A'(-1, 3)$, $B'(-2, 4)$, $C'(-3, 3)$, $D'(-2, 2)$, $E'(-2, 1)$

7 **(a)**



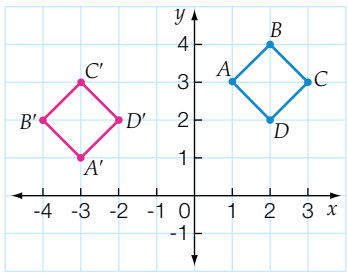
(b)



8 D

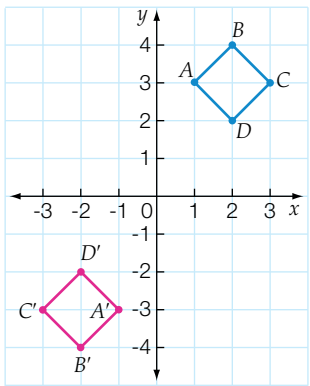
9 **(a)** $A(1, 3)$, $B(2, 4)$, $C(3, 3)$, $D(2, 2)$

(b)



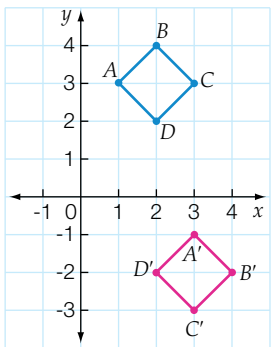
(c) $A'(-3, 1)$, $B'(-4, 2)$, $C'(-3, 3)$, $D'(-2, 2)$

(d)



(e) $A'(-1, -3)$, $B'(-2, -4)$, $C'(-3, -3)$, $D'(-2, -2)$

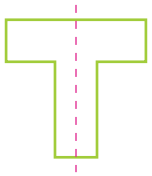
(f)



(g) $A'(3, -1)$, $B'(4, -2)$, $C'(3, -3)$, $D'(2, -2)$

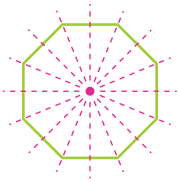
10 C **11** C

12 (a)

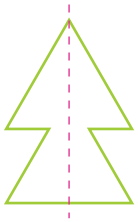


(b)

(c)

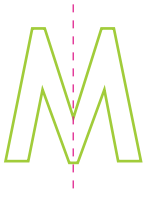


(d)



(e)

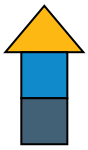
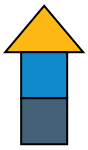
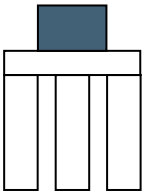
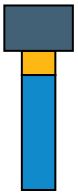
(f)



13 Shapes in (c) and (e) have rotational symmetry.

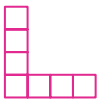
14 Students' own drawings should be checked carefully.

15

	Front elevation	Side elevation
(a)		
(b)		

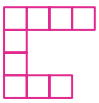
16 (a) Students' own drawings should be checked carefully.

(b) (i)



front

(ii)



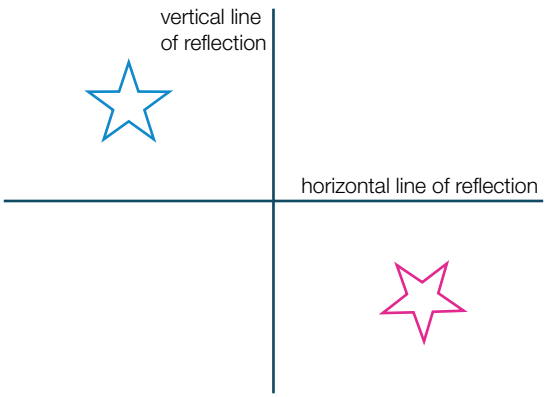
top

(iii)

side

17 4 units right

18 (a)

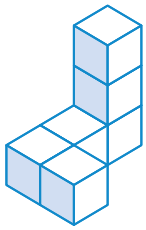


(b) yes

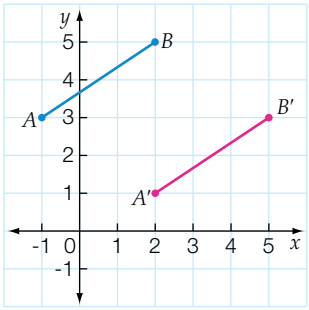
19 rotating 180° in either clockwise or anticlockwise direction about the intersection point of the axes

20 none

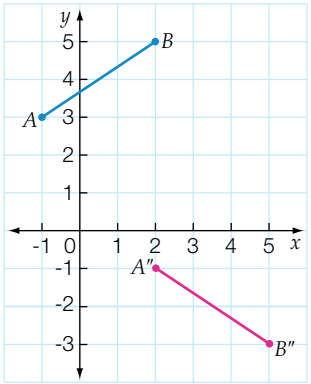
21



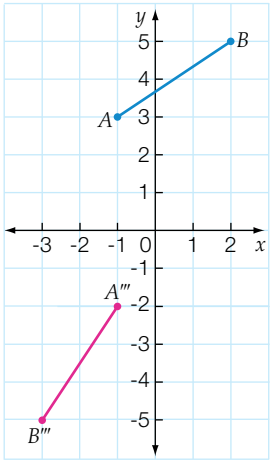
22 (a)–(b)



(c)

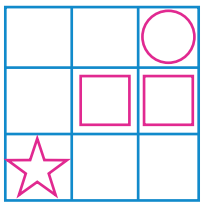


(d)

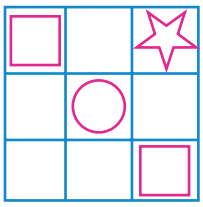


(e) $A'''(-1, -2)$, $B'''(-3, -5)$

23 (a)



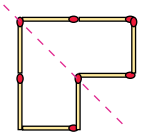
(b)



24 (a)

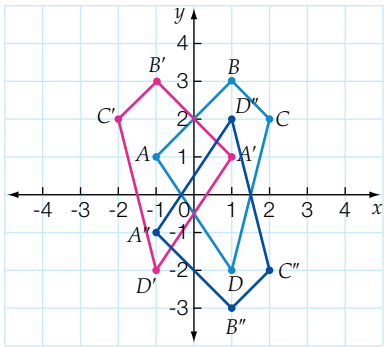
(b)

25 one match



26 19

27



Yes, Matt is correct. These two transformations could be replaced by a reflection in the x-axis.

Numeracy practice 10

1 B **2** B **3** C **4** A

5 D **6** A **7** A

Mixed review E

1 (a) 12 (b) 64 (c) 25

2 (a) 7 (b) 2700 (c) 99 000

3 (a) 17.515 (b) 0.0072 (c) 1.99

4 (a)

m	4	0	11	25	15
n	12	0	33	75	45

(b)

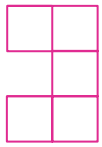
s	18	100	12	22	312
t	6	88	0	10	300

5 (a) 220 cm^2 (b) 60 cm^2

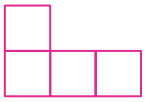
6 (a) $\frac{1}{6}$ (b) $\frac{1}{26}$ (c) $\frac{1}{4}$

7 (a) (i)

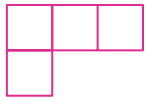
(ii)



(b) (i)



(ii)



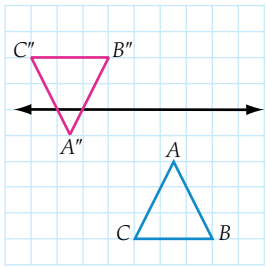
8 (a) $x = 3$ (b) $x = 24$ (c) $x = 3$

9 (a) 145° (b) 53° (c) 90°

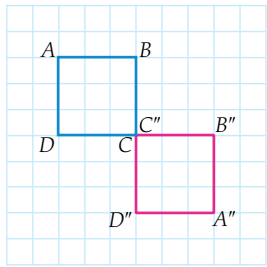
10

Height (cm)	Frequency
145–149	1
150–154	2
155–159	5
160–164	7
165–169	2
170–174	3
Total	20

11 (a)



(b)



12 (a) 1 (b) 4 (c) 2

13 mean = 30.6, median = 31.5, mode = 34, range = 39, possible outlier = 6

14 1 unit right and 1 unit down

15 (a) $24 = 2 \times 2 \times 2 \times 3$ (b) $30 = 2 \times 3 \times 5$

(c) 6 (d) 120

16 (a) 60° (b) 126°, 54°, 54°

17 O, X **18** (1, 1)

19 (a) Students' own answers.

(b) Work out what the numbers need to total to make the mean, then write a list with empty spaces and with 6 in the middle to make it the median. The other numbers can then be filled in around this to make the required total.

(c) It is easier to use an odd number of values because the median is then one of the data values.

20 (a) There are eight possible results: HHH, HHT, HTH, HTT, THH, THT, TTH, TTT (where H = heads, T = tails).

(b) You win with half of the results, and win more than you lose on the other results. So, the game is not fair, but it is better than fair for the player's benefit.

Glossary and index

acute angle an angle greater than 0° but less than 90° (p. 440)

acute-angled triangle a triangle in which all angles are acute (less than 90°) (p. 469)

alternate angles angles found between two lines but on opposite sides of a transversal; alternate angles between parallel lines are equal (pp. 453, 454)

angle the amount of turn between two lines that start at the same point (p. 433)

area the amount of surface within a boundary (p. 344)

arms (of an angle) two straight lines that meet at a vertex, with an angle between them (p. 433)

associative law the order in which numbers are added or multiplied can be swapped without changing the result; in algebra, $a + (b + c) = (a + b) + c$ and also $a \times (b \times c) = (a \times b) \times c$ (p. 5)

asymmetrical not symmetrical, having no symmetry (p. 620)

average *see* mean (p. 511)

axis (*plural* **axes**) number lines used to plot values of a variable; on a Cartesian plane, the x-axis runs in the horizontal direction and the y-axis runs in the vertical direction (p. 302)

axis of symmetry a line that divides a figure into two identical but flipped (mirror-image) parts (p. 620)

backtracking working backwards along a flowchart (p. 400)

balance method making equivalent equations by performing the same operation on both sides of the = symbol at each step keeping the sides balanced (p. 410)

bar graph a graph in which amounts or frequencies of data are shown as horizontal bars (p. 522)

base

1. the number that is raised to an index or power (p. 12)
2. the side of a shape that is perpendicular to the height (p. 355)

bi-modal a data set in which exactly two of the data values have the highest frequency (p. 511)

bivariate data data relating to two different variables (p. 540)

cancelling the process of dividing the numbers in a fraction by a common factor to produce a simpler fraction (p. 121)

Cartesian plane a grid marked with a horizontal x-axis number line and a vertical y-axis number line, so that any point in the plane can be described by its coordinates (x, y) from the origin (0, 0) (p. 302)

categorical data data that is collected according to descriptions or categories, not numbers; e.g. eye colour (p. 503)

centimetre one hundredth of a metre (p. 333)

centre of rotation the point about which a figure is rotated (p. 592)

chance the likelihood of an event, another word for probability (p. 560)

checking by substitution the process of checking an answer by substituting the solution into the equation (p. 402)

class interval the range of data values used to group data into sets for statistics (p. 505)

coefficient a number written before a pronumeral; e.g. in $9x$, the coefficient of x is 9 (p. 266)

co-interior (allied) angles angles found between two lines but on the same side of a transversal; co-interior angles on parallel lines are supplementary (pp. 453, 454)

column graph a graph in which amounts or frequencies of data are shown as vertical columns (p. 522)

common factor a number that is a factor of all the numbers being considered (p. 60)

common multiple a number that is a multiple of all the numbers being considered (p. 60)

commutative law the order in which two numbers are added or multiplied can be swapped without changing the result; in algebra, $a + b = b + a$ and also $a \times b = b \times a$ (p. 5)

complementary angles a pair of angles that add to 90° (p. 441)

composite number a number that has at least two factors other than 1 (p. 65)

composite shape a shape made up of more than one basic shape (p. 361)

concave having a surface or boundary that curves inwards (p. 465)

constant a number by itself; e.g. 2, 7, 9.5 (p. 266)

continuous data numerical data that is collected by measuring, so that the values may be any real numbers within a range; e.g. height (p. 503)

convex having a surface or boundary that curves outwards (p. 465)

coordinates a pair of numbers (x, y) that describes the position of a point on a number plane (p. 302)

co-prime two or more numbers that have only 1 as common factors (p. 65)

corresponding angles angles found either both above or both below two lines on the same side of a transversal; corresponding angles on parallel lines are equal (pp. 453, 454)

cube root a number that is cubed to produce a given number (p. 12)

data raw information (p. 503)

decimal places the number of digits after the decimal point of a number (p. 181)

decimal point a point that separates the whole number part of a number from the decimal part (p. 181)

define to identify a variable and represent it with a pronumeral (p. 267)

degree

1. a unit used to measure the size of an angle's turn, so that a full turn is equal to 360 degrees (p. 433)
2. a unit used to measure temperature, usually either as degrees Celsius (°C) or as degrees Fahrenheit (°F)

denominator the number below the line in a fraction (p. 111)

deposit placing an amount of money into a bank account (p. 77)

digit any numeral from 0 to 9 (p. 181)

discrete data data that is counted; e.g. the number of cans of soup sold by a supermarket over a period of time (p. 503)

distributive law a large number can be multiplied by splitting it into two smaller numbers, which can be multiplied individually, and the products added; in algebra, $a(b + c) = ab + ac$ (p. 6)

dividend in a division calculation, the number being divided; e.g. for $169 \div 8$, the dividend is 169 (p. 22)

divisible able to be divided by a number so that the answer is a whole number with no remainder (p. 58)

divisor in a division calculation, the number being divided by; e.g. for $169 \div 8$, the divisor is 8 (p. 22)

dot plot a graph showing the frequency of each data value as a dot (p. 518)

elevations 2D drawings of a structure from either the front, rear or side (p. 633)

equation two expressions either side of an equals symbol (p. 266)

equilateral triangle a triangle with all sides equal (p. 469)

equivalent equations equations that have the same solution (p. 409)

equivalent fractions fractions that represent the same quantity but are written using different denominators; e.g. $\frac{1}{2} = \frac{2}{4}$ (p. 120)

equivalent ratios ratios that compare amounts in the same proportion, but have different numbers so that one ratio is a multiple of the other (p. 240)

estimate a sensible guess that approximates an answer or value (p. 30)

evaluate to calculate the value of a formula or expression (p. 278)

expanded form a base raised to an index written as a multiplication of the base times itself (p. 12)

experimental probability a probability value that is taken from real-life experimental data of an event (p. 562)

expression a collection of terms added or subtracted together (p. 266)

factor a number that divides exactly into another number, with no remainder (p. 57)

false number sentence a sentence written with numbers that is not true (p. 389)

formula (*plural* **formulas** *or* **formulae**) a rule that is expressed using pronumerals and an equals sign (p. 278)

fraction a number that is not a whole number represented with a numerator and a denominator (p. 111)

frequency the number of times that a data value occurs in a data set (p. 504)

frequency table a table that shows the frequency of different values in a data set (p. 504)

guess, check and improve solving equations by choosing a number, substituting it into the equation and repeating this process until both sides are equal (p. 395)

height the distance from the base to the top of a shape, perpendicular to the base (p. 355)

highest common factor (HCF) the largest number that occurs in each of the sets of factors of a given group of numbers (p. 60)

histogram a frequency column graph for continuous data (p. 523)

image a figure produced by a transformation of the original (p. 579)

improper fraction a fraction with a numerator greater than or equal to the denominator (p. 112)

index (*plural* **indices**) a number, usually written smaller and above another number (the base), that tells how many times the base is multiplied by itself (p. 12)

index form a number multiplied by itself, written as a base raised to an index (p. 12)

integers all of the positive whole numbers 1, 2, 3, …, the negative whole numbers -1, -2, -3, … and zero (p. 77)

intersection *see* point of intersection (p. 488)

inverse of a fraction, when the numerator and denominator are swapped; e.g. the inverse of $\frac{2}{3}$ is $\frac{3}{2}$, the inverse of 4 is $\frac{1}{4}$ (p. 159)

inverse operation an 'opposite' operation; for example, + and – are inverse operations of each other; also × and ÷ are inverse operations of each other (p. 399)

irregular an irregular shape has side lengths and angle sizes that are not the same (p. 465)

isometric a triangular grid used to draw 3D solids on a 2D plane (p. 627)

isosceles triangle a triangle with two equal sides; the angles opposite the equal sides are equal angles (p. 469)

kilometre 1000 metres (p. 333)

kite a quadrilateral with two pairs of adjacent sides equal in length (p. 480)

like terms terms that have the same pronumeral part, but may have different coefficients; e.g. $3ab$ and $5ba$ are like terms, but $2a$ and $3a^2$ are not (p. 294)

line a straight length that extends in both directions without any beginning or end (p. 441)

line graph a graph that shows the relationship between two variables and can consist of several straight line sections (p. 317)

line of reflection a line drawn through an object or image that divides it into two identical (mirror-image) parts (p. 585)

line segment part of a line that has both a beginning and an end (p. 441)

linear graph a graph in which a straight line can be drawn through all the points (p. 310)

loss when an item's selling price is less than its cost price, the difference between these prices (p. 77)

lowest common denominator (LCD) the lowest common multiple of a set of denominators (p. 129)

lowest common multiple (LCM) the smallest number that occurs in each set of multiples of a given group of numbers (p. 60)

mathematical conventions rules that mathematicians agree to use when writing mathematics (p. 38)

mean the average value of a data set (that is, the sum of the data values divided by the number of data values) (p. 511)

measure of centre a single value that represents key information about a data set (p. 511)

median the middle value when a set of data values are put in order (p. 511)

metre the international standard unit of length (p. 333)

millimetre one thousandth of a metre (p. 333)

mixed number a number that is made up of a whole number part and a proper fraction (p. 112)

mode the most frequently occurring data value in a data set (p. 511)

multimodal a data set in which more than two values have the highest frequency (p. 511)

multiple a number that may be divided exactly by a given number (with no remainder) is a multiple of that given number (p. 57)

negative used to describe numbers less than zero on a number line; associated with losses or decreases (p. 76)

not like terms or **non-like terms** terms that have different pronumeral parts (p. 294)

number plane *see* Cartesian plane (p. 302)

numerator the number above the line in a common fraction (p. 111)

numerical data data that is collected according to numbers (p. 503)

obtuse angle an angle greater than 90° but less than 180° (p. 440)

obtuse-angled triangle a triangle in which one angle is obtuse (the other two angles are acute) (p. 469)

order of operations the convention that when calculating you must do brackets first, then indices, then multiplication and division from left to right, then addition and subtraction from left to right (p. 38)

order of reflectional symmetry the number of axes of symmetry that can be drawn through a shape (p. 620)

order of rotational symmetry the number of times in a complete 360° rotation that an identical image of the original would be produced (p. 621)

ordered pair a pair of numbers (coordinates) that uniquely determine the location of a point (usually given as an x-value followed by a y-value) (p. 302)

origin the point of intersection of the vertical and horizontal axes on a number plane; it has the coordinates (0, 0) (p. 302)

original in a transformation, the figure that is transformed to produce the image (p. 579)

outcome the result of a probability experiment or event (p. 560)

outlier an unusual extreme data value, compared to other values in the data set (p. 520)

parallel lines lines in the same plane that are always the same distance apart and will never intersect (p. 453)

parallelogram quadrilateral with both pairs of opposite sides parallel (p. 355)

part:part ratio a ratio that compares two or more parts of a whole (p. 239)

part:whole ratio a ratio that compares a part (or parts) to the whole (p. 239)

per cent literally 'for each hundred' or 'out of one hundred' (p. 224)

percentage a value expressed as an amount out of 100 (p. 224)

perfect cube the result of multiplying a whole number by itself three times (p. 11)

perfect square the result of multiplying a whole number by itself (p. 11)

perimeter the distance around the outside of a shape (p. 340)

perpendicular at right angles to (p. 355)

perpendicular bisector a line at right angles to another line, that cuts this line in half (p. 488)

perpendicular distance in a reflection, the distance measured along a line perpendicular to the line of reflection, from a point on the original to a corresponding point on the image (p. 585)

perpendicular lines lines that meet or intersect at right angles to each other (p. 454)

place value the value of a digit in a number according to its place; in the decimal number system, the place value of a digit is 10 times the place value of the next digit to the right (p. 181)

plan view technical drawing of an object from the top, front or side (p. 632)

plane a flat two-dimensional surface (p. 453)

plane shape a two-dimensional shape drawn on a plane (p. 465)

point a position in space, usually represented as a dot (p. 441)

point graph graph showing information relating to two variables, where the points are not joined (p. 316)

point of intersection the point where two or more lines meet or cross (p. 488)

polygon a many-sided (or many-angled) plane shape (p. 465)

positive used to describe numbers greater than zero on a number line; associated with gains or increases (p. 76)

power *see* index (p. 12)

primary data data collected directly by an observer (p. 503)

prime factor a factor that is a prime number (p. 70)

prime number a number that has only two factors, itself and 1 (p. 65)

probability the likelihood of a specific event, expressed as a number from 0 to 1 (p. 560)

product the result when two or more numbers are multiplied together (p. 6)

profit when a product's selling price is more than its cost price, the difference between these prices (p. 77)

pronumeral a letter or symbol that represents a number (p. 261)

proper fraction a fraction with a numerator less than the denominator (p. 112)

protractor a device for measuring angles (p. 433)

quadrant one of the four parts of a Cartesian plane, when the plane is divided vertically and horizontally by the y-axis and x-axis (p. 302)

quadrilateral a four-sided plane shape (p. 480)

quotient the result of a division calculation (p. 22)

range the difference between the largest value and smallest value in a data set (p. 505)

rate a way of comparing two amounts of different quantities, in terms of how much of one quantity corresponds to how much of the other quantity; usually written using the word 'per' (p. 245)

ratio a comparison of two or more different amounts in relation to each other, usually written using a : symbol (p. 238)

ray part of a line which has a beginning but no end (p. 441)

rectangle a plane shape with two pairs of opposite sides equal and all internal angles 90° (p. 480)

rectangular prism a prism with a rectangular base; sometimes called a cuboid (p. 367)

recurring decimal a decimal number with a pattern of digits that repeats (p. 198)

reflection a transformation of a shape found after drawing the mirror image of it (p. 585)

reflectional symmetry when one or more lines of reflection can be drawn through an object or image (p. 620)

reflex angle an angle greater than 180° but less than 360° (p. 440)

regular polygon a polygon with all sides of equal length and all angles of equal size (p. 465)

relationship in algebra, a connection between two variables so that changing the value of one variable affects the value of the other variable (p. 271)

remainder the number 'left over' at the end of a division calculation, usually indicated by 'rem' or 'r' (p. 22)

revolution a full turn, 360° (p. 433)

rhombus a plane shape with all four sides equal and opposite angles of equal size (p. 480)

right angle an angle of exactly 90° (p. 440)

right-angled triangle a triangle with one right (90°) angle (p. 469)

rotate turn (p. 592)

rotation a transformation of a shape by turning on the plane about a point (p. 592)

rotational symmetry when an object or image can be rotated to produce an identical image in less than a full turn (p. 621)

rounding approximating a number to a certain number of digits or to a certain place value (p. 30)

sample space a listing of all the possible outcomes for a particular situation (p. 561)

scalene triangle a triangle with no equal sides or angles (p. 469)

secondary data data not collected directly, instead taken from another source (p. 503)

simplest form when the numbers in a ratio or fraction have no common factors (p. 121)

simplify to write in simplest form, by cancelling common factors (for fractions and ratios) or collecting like terms (p. 121)

solution the value of a pronumeral that makes the equation true; the answer (p. 395)

solve find the solution to (p. 395)

solving by inspection a method of solving equations by looking at the equation and working out the solution in your head (p. 395)

square

1. to multiply a number by itself
2. a quadrilateral with all sides equal and all internal angles 90° (p. 480)

square root a number that is squared to become a given number (p. 12)

stem-and-leaf plot or **stem plot** a display of data in which the leaf value represents the units digit for each data value, while the stem value represents the other digit; leaf values are ordered from smallest to largest, usually on the right-hand side of the corresponding stem values (p. 518)

straight angle an angle of exactly 180° (p. 440)

substitute to replace a pronumeral with a given number (p. 278)

supplementary angles a pair of angles that add to 180° (p. 441)

table of values a grid that displays the corresponding x-values and y-values of a rule in algebra (p. 272)

tally the count of the frequency of different data values; tally marks are vertical lines, with every group of 5 shown as 4 vertical lines with 1 horizontal line through them (p. 504)

term a single item that is added or subtracted in an expression; for example, a number or a pronumeral by itself (7, x), a number multiplied by pronumerals ($12c$, $5k$, $3xy$, $4a^2$, $5b^2c$), pronumerals multiplied together (mn, abc, a^3, b^2) (p. 266)

terminating decimal a decimal number that has a limited (not infinite) number of digits (p. 198)

theoretical probability a probability value that is found by mathematically considering the sample space; if all outcomes are equally likely, then this is the number of successful outcomes divided by the total number of possible outcomes (p. 561)

transformation any change that can be made to a graph or figure using rotation, dilation, reflection or translation (p. 579)

translation a transformation of a graph or figure that keeps the shape, size and orientation exactly the same, but changes the location (by moving it in a straight line) (p. 579)

transversal a line that crosses (transverses) other lines (p. 453)

trapezium a quadrilateral with only one pair of opposite sides parallel (p. 480)

trend a general pattern that can be used to predict other values in a data set (p. 540)

true number sentence a sentence written with numbers that is true (p. 389)

unit fraction a fraction that has 1 as the numerator (p. 152)

unit price the price per unit of quantity (p. 246)

unitary method when a rate is expressed as an amount of one quantity corresponding to 1 unit of the other quantity (p. 245)

univariate data data relating to one variable only (p. 518)

unknown in algebra, a number that is not known (p. 261)

variable in algebra, an unknown amount that may vary, represented by a pronumeral (p. 261)

vertex (*plural* **vertices**) a point where lines or edges meet (p. 433)

vertically opposite angles pairs of angles that are opposite each other, formed where two lines intersect (p. 442)

volume the amount of space occupied by a three-dimensional object (p. 367)

withdrawal taking an amount of money out of a bank account (p. 77)

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