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THIRD EDITION

11

MATHEMATICS GENERAL (PATHWAY 2) Preliminary Course

NEW CENTURY MATHS

margaret **WILLARD**
robert **YEN**

series editor robert **YEN**

nelsonnet

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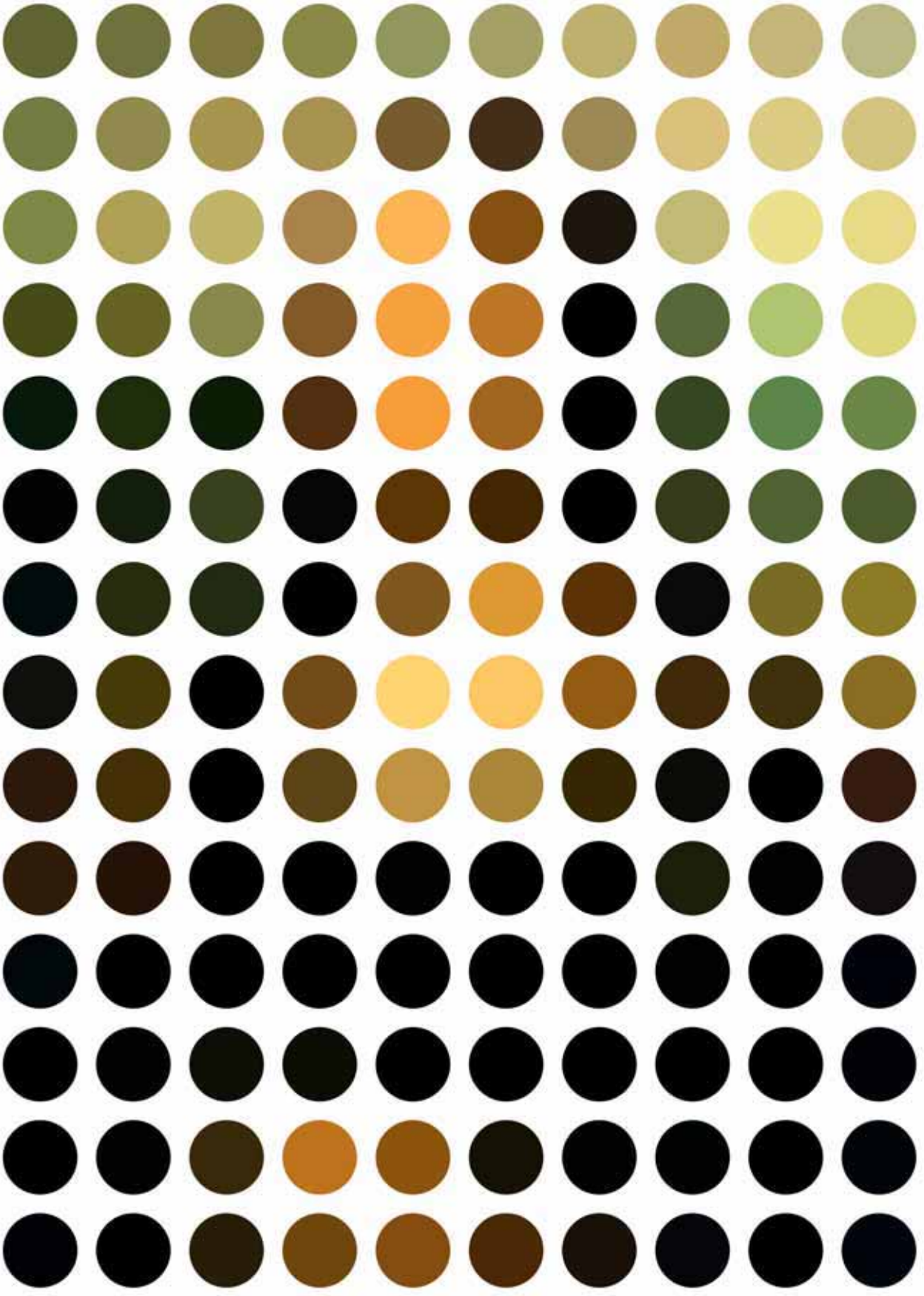

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Paintings throughout history rely on strict mathematical foundations. They paved the way in perspective and grid structures, highlighting the thought process and intricate construction behind art. Clarke's artwork has taken paintings one step further, remixing and reducing da Vinci's *Mona Lisa* to a simple mathematical colour dot equation. Mona Lisa for the Twitter generation.

Left: Mona Lisa remixed, © Gary Andrew Clarke

Right: Mona Lisa, c.1503-6 (oil on panel), Leonardo da Vinci, (1452–1519) The Bridgeman Art Library/ Giraudon/Louvre, Paris

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PREFACE

First published in 2000, *New Century Maths 11 Mathematics General Preliminary course (Pathway 2)* has been rewritten for the new Mathematics General syllabus (2012). In this book, teachers will find those familiar features that have made *New Century Maths* a leading mathematics series, such as clear worked examples, graded exercises, syllabus codes and references, Investigation, Technology, SkillCheck pre-chapter exercise, worked solutions to exercises, Topic overview with mind map, glossary, worksheets and topic tests.

The syllabus describes two pathways for Mathematics General that begin in Year 11:

- one, which we have called Pathway 1, is a vocational pathway, for 'content-endorsed' course that is not HSC-examinable, for students heading towards the workforce or further training after school, providing practical mathematical skills for life
- the other, which we have called Pathway 2, is the more traditional and academic pathway, for students heading towards an HSC exam, an ATAR and university studies.

This book caters for Pathway 2 in Year 11. Even though both pathways share a common Preliminary (Year 11) course, those students taking each pathway have specific learning needs, so we have published two levels of texts for both Years 11 and 12.

The Mathematics General 2 course is designed for students who will work or study in fields that require a good mathematical or statistical background. We have endeavoured to produce a practical text that captures the spirit of the course, providing relevant and meaningful examples of mathematics being used in society and industry. This 3rd edition also heralds the new web-based, interactive NelsonNetBook version of the text, and the NelsonNet student and teacher websites of print and multimedia resources.

We thank our families for their continued support and patience, and Alan Stewart and Anna Pang for swiftly transforming our manuscript into this fine text. Finally, we wish all teachers and students using this book every success in embracing twenty-first century mathematics in the classroom and beyond.

ABOUT THE AUTHORS

Margaret Willard has taught at numerous TAFE colleges and high schools for 35 years, including holding the positions of Senior Head Teacher and Manager at TAFE's distance education unit, OTEN (Open Training and Education Network). Margaret has extensive experience writing and reviewing units of work designed for students involved in distance education. She has served on the executive of MANSW (the Mathematical Association of NSW), managed its post-secondary programs and has presented at both MANSW and TAFE conferences.

Robert Yen has taught for 20 years, at Hurlstone Agricultural, Ambarvale and Eagle Vale high schools in southwest Sydney. He has been writing for *New Century Maths* since 1995, as well as writing and presenting for MANSW and co-editing its journal, *Reflections*. Robert has written the General Mathematics HSC study guides for *The Sydney Morning Herald*, and now works for Cengage Learning as an associate mathematics publisher and series editor.

Contributing authors

Kuldip Khehra wrote and edited many of the NelsonNet print resources (blackline masters) and is head teacher of mathematics at Quakers Hill High School.

Trisha Goss wrote the topic tests.

Tracey MacBeth-Dunn wrote the worked solutions to all exercise sets.

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SYLLABUS REFERENCE GRID

Strand and substrand	New Century Maths 11 Mathematics General (Pathway 2) Preliminary Course Chapter
Financial mathematics	
FM1 Earning and managing money	4 Earning money and taxation
FM2 Investing money	11 Investing money
FM3 Taxation	4 Earning money and taxation
Data and Statistics	
DS1 Statistics and society, data collection and sampling	1 Collecting and presenting data
DS2 Displaying and interpreting single data sets	1 Collecting and presenting data 9 Analysing data
DS3 Summary statistics	9 Analysing data
Measurement	
MM1 Units of measurement and applications	6 Measurement
MM2 Applications of perimeter, area and volume	6 Measurement
MM3 Similarity of two-dimensional figures, right-angled triangles	3 Similar figures and trigonometry
Probability	
PB1 Relative frequency and probability	5 Probability
Algebra and Modelling	
AM1 Algebraic manipulation	2 Algebra and equations
AM2 Interpreting linear relationships	7 Linear functions
Mathematics and Communication	
FSCo1 Mobile phone plans	10 Phone plans and downloading data
FSCo2 Digital download and file storage	10 Phone plans and downloading data
Mathematics and Driving	
FSDr1 Costs of purchase and insurance	8 Buying a car
FSDr2 Running costs and depreciation	8 Buying a car
FSDr3 Safety	12 Driving safely

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USER'S GUIDE

AT THE BEGINNING OF EACH CHAPTER

- Each chapter begins with a **Chapter outline** showing the content of the chapter with syllabus codes, a list of syllabus content descriptions (© NSW Board of Studies 2012), and a **Terminology** word list.
- **SkillCheck** reviews prerequisite skills and knowledge for the chapter.

IN EACH CHAPTER

- Important facts and formulas are highlighted in a **Summary** box.
- Graded exercises are linked to worked examples and include multiple-choice questions, exam-style problems and realistic applications.
- **Worked solutions** to all exercise questions are provided on the NelsonNet student website.
- **Investigations** explore the syllabus in more detail, through group work, discovery and modelling activities.
- **Technology** promotes ICT in the classroom, using spreadsheets, graphics calculators and the Internet.
- **Just for the record** contains interesting facts and applications of the mathematics learned in the chapter.

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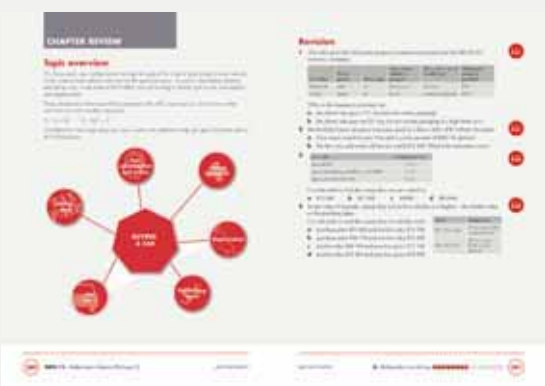

AT THE END OF EACH CHAPTER

- **Sample HSC problem** demonstrates a typical HSC question using content learned from the chapter.
- **Study tip** provides useful advice for senior students in revising coursework and preparing for exams.
- **Topic overview** concludes the chapter and includes a mind map exercise.
- **Revision** is a set of review exercises that are linked to the relevant exercise set.
- **Practice papers** after every three chapters revise the skills and knowledge of those chapters (the NelsonNet website also contains a Practice Yearly Exam and two mini-Yearly Exams).

AT THE END OF THE BOOK

- **Instructional glossary** describes mathematical 'doing' words and HSC-exam-style keywords and verbs.
- **Mathematical glossary** is a comprehensive dictionary of course terminology.
- All terms printed in **red** in this book appear in the glossary.
- **Answers** and **Index** included.

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NELSONNET STUDENT WEBSITE

- Margin icons link to print (PDF) and multimedia resources found on the NelsonNet student website, www.nelsonnet.com.au. These include:

Worksheet Skillsheet Spreadsheet

- **Worksheets** are write-in enabled PDFs
- **Skillsheets** of examples and exercises of prerequisite skills and knowledge
- **Worked solutions** to each exercise set
- **Spreadsheets:** Excel files
- **ExamView quizzes:** interactive and self-marking.

NELSONNET TEACHER WEBSITE

The NelsonNet teacher website, also at www.nelsonnet.com.au, contains:

- **Teaching program**, in Microsoft Word and PDF formats
- **Topic tests**, in Microsoft Word and PDF formats
- **Chapter PDFs** of the textbook
- **ExamView** exam-writing software and questionbanks
- **Resource Finder:** search engine for NelsonNet resources.

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- Chapters can be customised for different groups of students.

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DATA AND STATISTICS

1.

COLLECTING AND PRESENTING DATA

Today, we live in the 'Information Age'. The world economy has progressed from manufacturing products to the complex task of processing data to manage the needs of a growing population. Statistics are used by governments to plan for community facilities, by scientists and economists to predict the future, by businesses to forecast sales and by advertisers to help sell products.

CHAPTER OUTLINE

DS2	1-01	Interpreting graphs
DS2	1-02	Misleading graphs
DS1	1-03	Types of data
DS1	1-04	Statistics in society
DS1	1-05	Sampling techniques
DS2	1-06	Constructing graphs
DS2	1-07	Frequency histograms and polygons
DS2	1-08	Dot plots and stem-and-leaf plots
DS2	1-09	Radar charts

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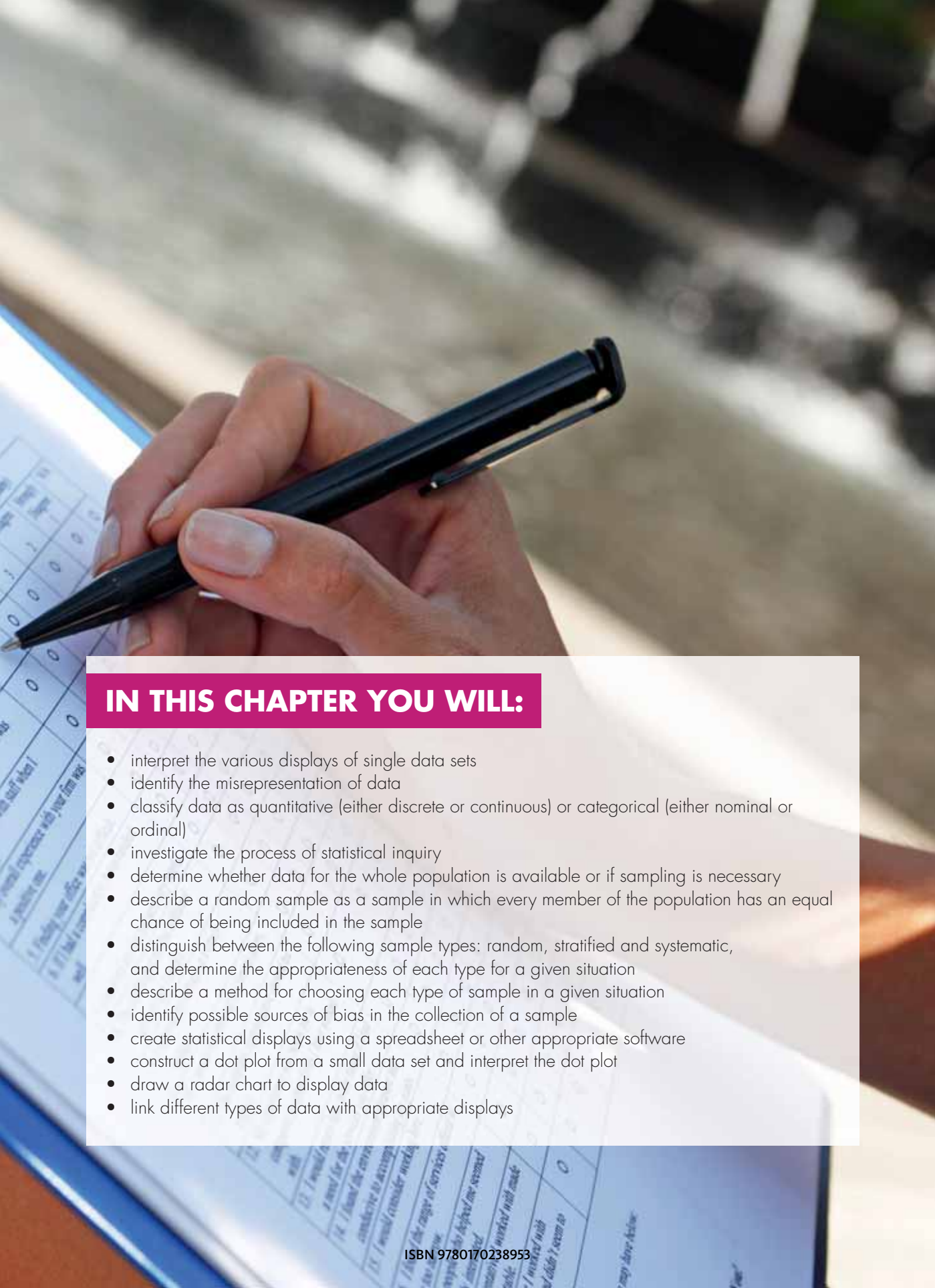

IN THIS CHAPTER YOU WILL:

- interpret the various displays of single data sets
- identify the misrepresentation of data
- classify data as quantitative (either discrete or continuous) or categorical (either nominal or ordinal)
- investigate the process of statistical inquiry
- determine whether data for the whole population is available or if sampling is necessary
- describe a random sample as a sample in which every member of the population has an equal chance of being included in the sample
- distinguish between the following sample types: random, stratified and systematic, and determine the appropriateness of each type for a given situation
- describe a method for choosing each type of sample in a given situation
- identify possible sources of bias in the collection of a sample
- create statistical displays using a spreadsheet or other appropriate software
- construct a dot plot from a small data set and interpret the dot plot
- draw a radar chart to display data
- link different types of data with appropriate displays

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TERMINOLOGY

analysing	bias	categorical data
census	class centre	class interval
continuous data	discrete data	dot plot
frequency histogram	frequency polygon	misleading
nominal data	ordinal data	population
quantitative data	random sample	sample
scale	sector graph	stem-and-leaf plot
stratified sample	systematic sample	unambiguous

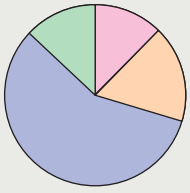
Assignment 1

SkillCheck

1 Match each of the following statistical displays to its correct name from this list:

column graph | divided bar graph | dot plot
line graph | sector graph | stem-and-leaf plot

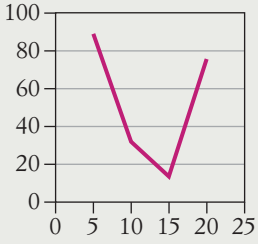
a



b

3	0 4 4
4	3 6 6 6 8
5	2 4 5 7
6	1 3 3

c



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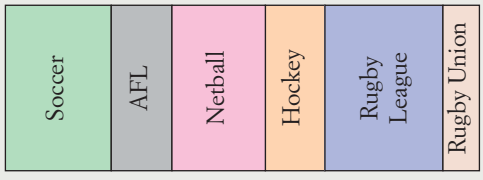
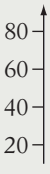


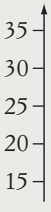
2 Draw the two statistical displays from the list in question **1** that were not illustrated in the question.

3 Write the size of one unit on each of these vertical scales.

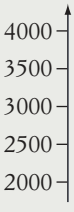
a



b

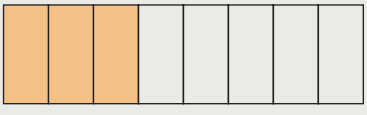


c



4 **a** What fraction of the rectangle is shaded?

b What percentage of the rectangle is shaded?



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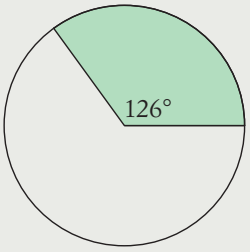
5 **a** What fraction of the circle is shaded?

b What percentage of the circle is shaded?



6 The ages of the patients at a hospital were grouped into the following class intervals: 0–9, 10–19, 20–29, and so on. What is the class centre of the interval 20–29?

1-01 Interpreting graphs

Every picture tells a story

Graphs are used to display statistical information in an attractive and meaningful way. Four simple types of graphs are:

- **column graphs**, also known as **bar graphs**
- **line graphs**
- **sector graphs**, also known as **pie charts**
- **divided bar graphs**.

EXAMPLE 1

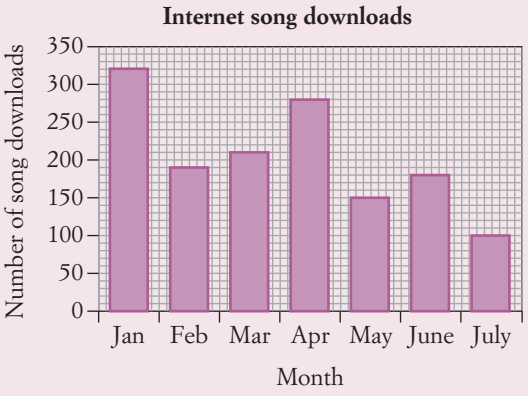
This column graph shows the number of songs downloaded from the Internet over 7 months by a group of teenagers.

a What is the size of one unit on the vertical axis?

b How many songs were downloaded in April?

c In which month were 100 songs downloaded?

d In which month were the highest number of songs downloaded? Suggest a reason why this may be so.



Solution

a 10 songs

b 280 songs

c July

d January. Teenagers may have had more time or money after Christmas to download songs.

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EXAMPLE 2

This sector graph shows the countries where Australian students overseas studied during one year.

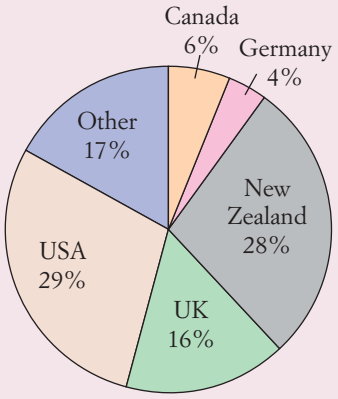
a What was the second most popular overseas country for Australian students to study in?

b What percentage of students chose neither the United Kingdom (UK) nor the United States of America (USA)?

c If a total of 3685 students was surveyed, how many of them went to Canada?

d Calculate, correct to the nearest degree, the angle size of the 'Other countries' sector.

Australian students overseas



Solution

a New Zealand

b 100% – 16% – 29% = 55% **Or 17% + 6% + 4% + 28% = 55%**

c 6% × 3685 = 221.1 ≈ 221

221 students went to Canada.

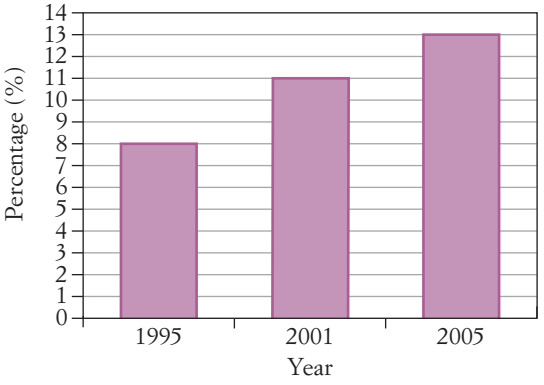
d 17% × 360° = 61.2° ≈ 61° **There are 360° in a circle.**

Example 1

Exercise 1-01 Interpreting graphs

1 The column graph on the right shows the percentage of Australians drinking alcohol at a 'risky' or 'high risk' level in three different national health surveys.

Risky/high risk alcohol consumption



a What percentage of people were drinking at a 'risky/high risk' level in 2005?

b What was the percentage increase in 'risky/high risk' drinking in the 10 years after the 1995 survey? Write your answer to the nearest whole number.

c If 35 650 people were surveyed in 2001, how many were drinking at a 'risky' or 'high risk' level?

d If 42 320 people were surveyed in 2005, how many were not drinking at a 'risky' or 'high risk' level?

e Predict what might happen in the results of the next survey.

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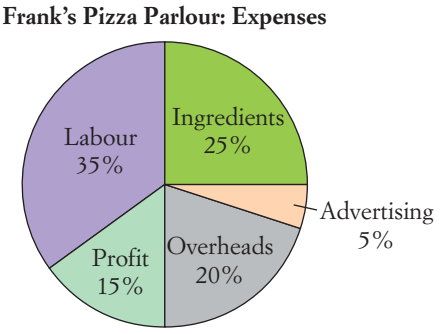
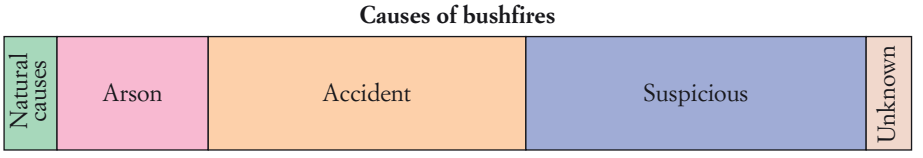
Example 2

2 This sector graph shows how the owner of Frank's Pizza Parlour spends his money from pizza sales.



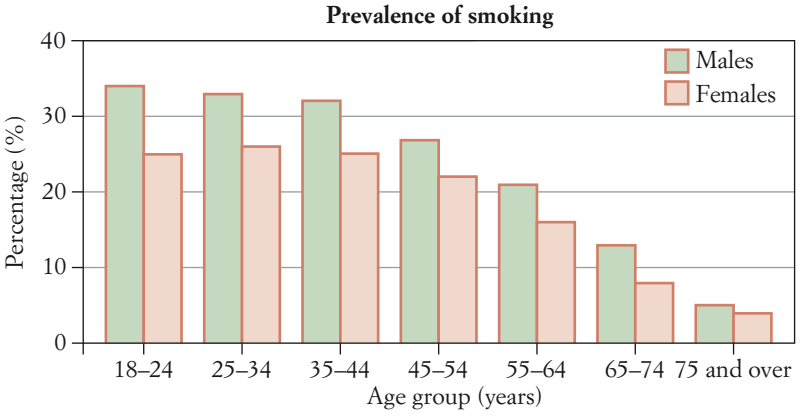
- **a** What fraction of his takings did Frank spend on ingredients?
- **b** What was Frank's largest expense?
- **c** If Frank sells a family-size pizza for $13.50, how much of this is profit?
- **d** If $285 per week is spent on advertising, what are Frank's total weekly sales?
- **e** What is the angle size of the 'Profit' sector on the graph?
- **f** Overheads are business expenses, such as rent. Calculate how much Frank spends on overheads if he sells $5300 worth of pizza in a week.

3 This divided bar graph shows causes of bushfires during one summer.



- **a** What fraction of bushfires were caused by arsonists?
- **b** What percentage of bushfires were accidental?
- **c** Which cause was linked to 37.5% of bushfires?
- **d** If there were 275 bushfires during summer, how many were started by natural causes?

4 This graph is based on data about smoking across age groups and genders.



Source: *National Health Survey*: Summary of Results 2004–05 (ABS cat no. 4364.0)

- **a** Which gender has the higher percentage of smokers?
- **b** Which age group contains the highest percentage of female smokers?

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c What percentage of males aged 35 to 44 are smokers?

d If 14 205 females were surveyed in the 65–74 age group, how many were smokers?

e In which age group were 13% of males smokers?

f What generally happens to the percentage of smokers as age increases?

5 This graph shows the average number of hours Australians spent per week using the Internet and television, over 3 years.

Which of the following statements about weekly Internet usage is true?

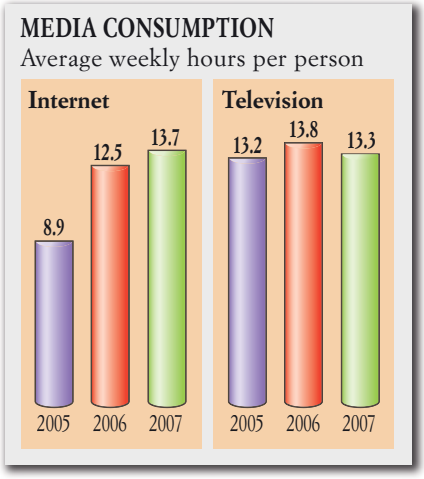
Select **A**, **B**, **C** or **D**.

A It dropped by 0.5 hours in 2007.

B It has increased by 4.8 hours over 3 years.

C It was more popular than watching television in 2006.

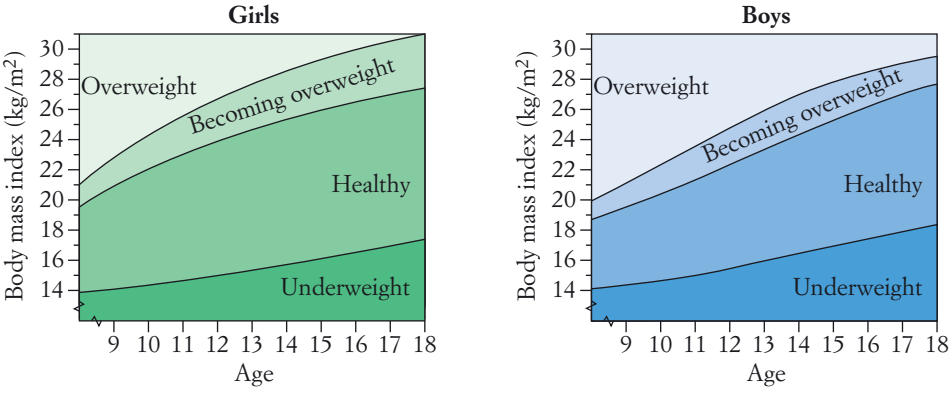
D It will continue to rise.



Source: *Sydney Morning Herald*, 27 March 2008

6 The graphs below illustrate healthy weight ranges, according to gender, age and body mass index (BMI). Your BMI is calculated by dividing your weight (in kilograms) by your height (in metres) squared.

Your healthy weight range



a Is a 16-year-old girl with a BMI of 20 considered healthy?

b Describe the health of a 13-year-old boy with a BMI of 14.

c What is the minimum BMI for a healthy 17-year-old boy?

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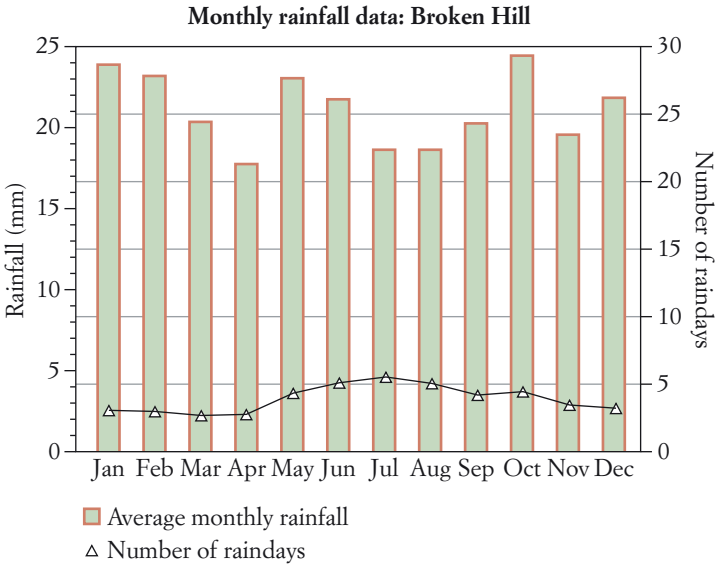
d Jake is aged 15, weighs 78 kg and is 173 cm tall. Calculate his BMI and state what advice you would give him about his health.

e Write the lowest and highest BMI for a healthy 14-year-old girl.

f Calculate your BMI and use the graph to find your state of health.

7 This graph from the Bureau of Meteorology shows the monthly rainfall and number of raindays for Broken Hill.



a What is the size of one unit on the 'Rainfall' axis?

b What is graphed on the line graph?

c What was the amount of rainfall and the number of raindays in May?

d Which month had the highest rainfall? How many millimetres fell in that month?

e Which month had the most raindays? How many days of rain did it have?

f What was the driest month?

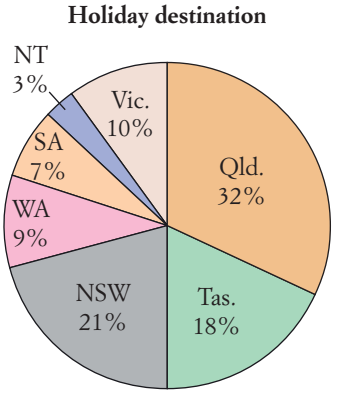
g How many days in summer had rain?

8 A group of Australians was surveyed about their favourite state or territory to visit on holidays. The results are shown on the sector graph.

If 2475 tourists chose Western Australia, which of the following is the total number of people who participated in the survey? Select **A**, **B**, **C** or **D**.

A 27 500 **B** 8800

C 5775 **D** 25 025



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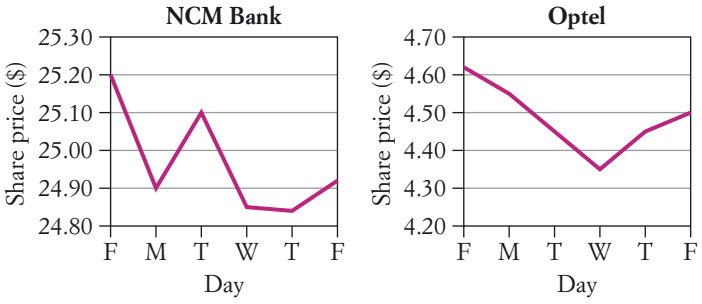
9 **a** What do these graphs illustrate?



b What is the size of one unit on the vertical axis of the NCM Bank graph?

c What was the Optel share price on Monday?

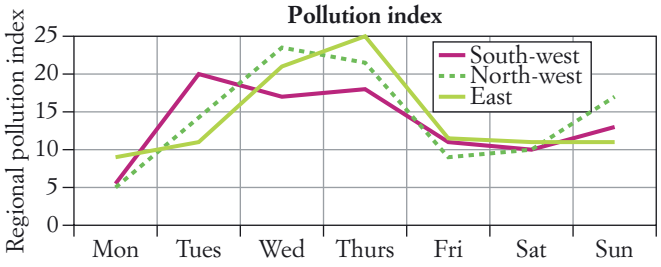
d On which day was the NCM Bank share price $25.10?

e Describe what happened to the Optel share price over the week.

f What would have been the worst day to sell NCM shares? Why?

g Which company's shares showed greater changes in price over the week?

10 These line graphs show the pollution index in three regions over a week.



a What was the south-west's pollution index on Wednesday?

b Which region had the highest index for the week? What was this reading and on which day did it occur?

c Which day showed the lowest index for all three regions? Why do you think it did?

d Which day showed the greatest difference in readings between the south-west and north-west? What was this difference?

e On which day were the readings about the same for all three regions? Why do you think it was?

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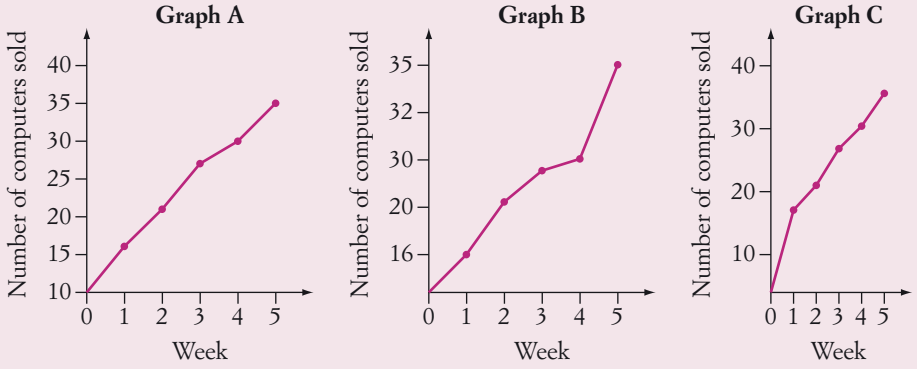
1-02 Misleading graphs

Statistical graphs are sometimes used to misrepresent information, and to falsely support a particular opinion or claim. For example, an advertisement may use a misleading graph to promote a product, or a company may use one to exaggerate its achievements and profits.

EXAMPLE 3

The three misleading graphs below were used by the marketing manager of a computer store to highlight the increase in the sales of notebook computers over five weeks.



The sales figures are shown in this table.

Week	1	2	3	4	5
Computers sold	16	21	27	30	35

a Explain what is incorrect and misleading about each graph.

b Draw the correct graph to represent the sales data.

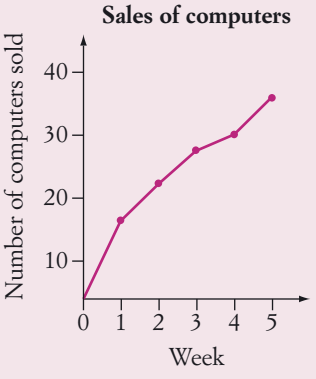
Solution

a In Graph A, the vertical axis begins at 10 instead of 0, so the section of the graph between 10 and 40 has been exaggerated or 'stretched' to give the impression that sales have increased greatly.

In Graph B, the scale on the vertical axis is uneven, which gives the false impression that the increases in sales are greater than they actually are.

In Graph C, the horizontal axis has been compressed or 'shrunk' to make it look like the sales have increased quicker.

b The correct graph has an even scale on both axes with the vertical axis beginning at zero. It correctly shows the sales pattern.



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SUMMARY

A misleading graph can give a wrong impression by:

- not having a scale
- having an uneven scale or showing only part of the scale
- not showing the correct position of zero on the scale.

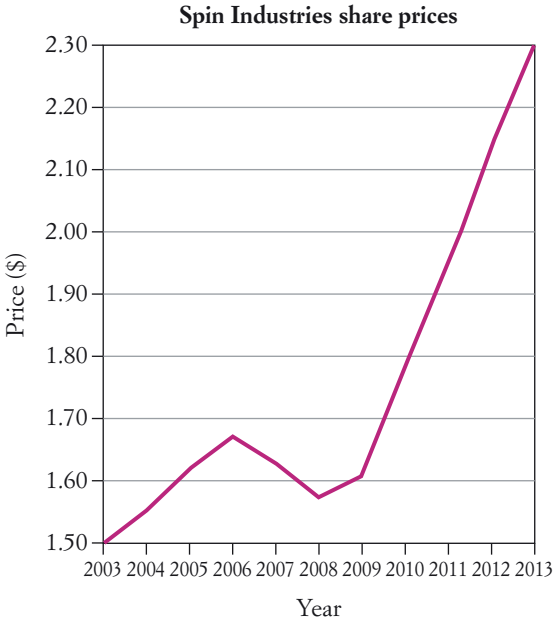
Exercise 1-02 Misleading graphs

Example 3

1 The directors of Spin Industries used the graph on the right to make the following claim to shareholders: 'The company's share price has risen dramatically in the last 10 years'.

a How does the graph mislead the shareholders?

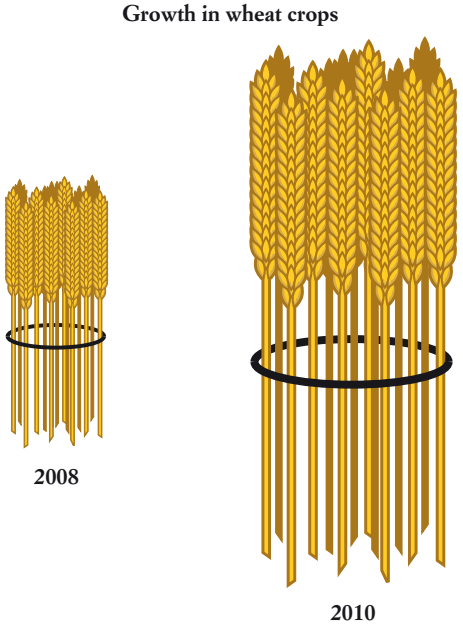
b Redraw the graph correctly.



2 Using pictures in graphs can be misleading if they are badly scaled to exaggerate differences. The graph on the right should show that the wheat crop has doubled in two years, but the 2010 wheat picture is double the length and double the width of the 2008 wheat picture.

a How is the graph misleading?

b Redraw the graph for 2010 correctly.



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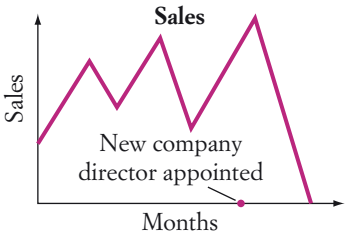
3 The sales graph on the right was presented to a meeting of shareholders by the ex-director of a company.

- **a** What does the graph describe?
- **b** What message is the ex-director trying to convey to the shareholders?
- **c** What information is missing from the graph?

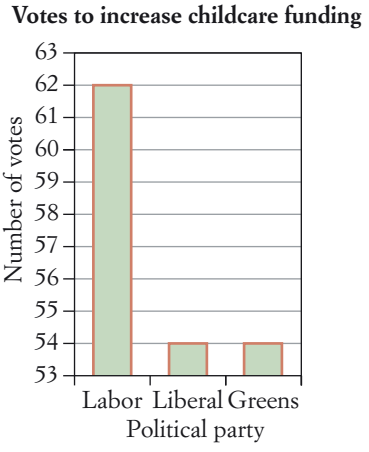


4 The graph on the right depicts the average weekly wage now compared to that of 5 years ago.

- **a** How is the graph misleading to the reader?
- **b** Redraw the graph correctly.

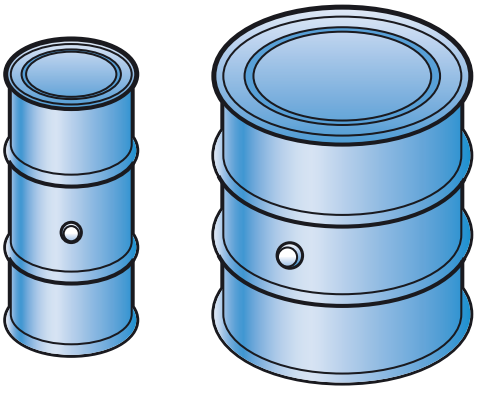
5 The column graph on the right illustrated the newspaper article 'Labor votes overwhelmingly to increase childcare funding.'

- **a** How are Labor's votes exaggerated on this graph?
- **b** Redraw the graph correctly.
- **c** Do you agree with the headline? Justify your answer.



6 Oil prices have increased so much that $100 buys only half as much oil as it did 10 years ago. A news website used the following graph to illustrate this.

- **a** How is this graph misleading?
- **b** Redraw the graph so that it represents the situation accurately.



$100 of oil today **$100 of oil 10 years ago**

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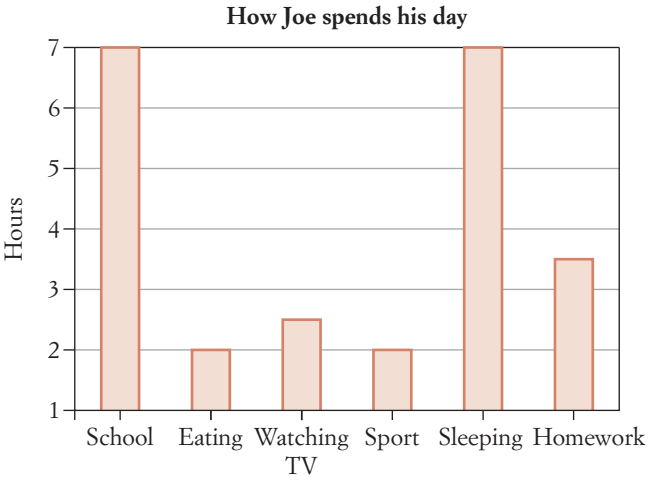
7 Joe drew the bar graph on the right to illustrate how he spends a typical weekday.

a How many hours does Joe spend sleeping?

b Explain what is wrong with this graph.

c What misleading impression does this graph give?

d Redraw this bar graph to show accurately how Joe spends a typical weekday.



Statistical data match-up

1-03 Types of data

There are two main types of statistical data:

- **categorical data** are represented by categories, usually in words or symbols
- **quantitative data** are represented by quantities or numbers

EXAMPLE 4

Classify each of the following types of data as categorical or quantitative:

a number of tries scored by a football team

b colour of hair

c month of birth

d amount of petrol in cars

Solution

a quantitative — **For example 0 or 3.**

b categorical — **For example blonde or brown.**

c categorical — **For example May or September.**

d quantitative — **For example 32.5 L or 40 L.**

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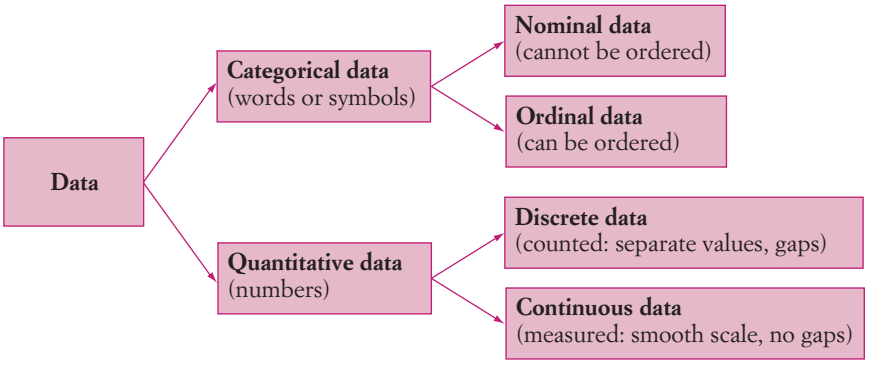
Categorical data can be further divided into **nominal** and **ordinal** data, while **quantitative data** can be further divided into **discrete** and **continuous** data.



Colour of hair is an example of **nominal data** because the categories cannot be ordered, for example, blonde does not come before or after brown.

Month of birth is an example of **ordinal data** because the categories can be ordered, for example, May comes before September.

Number of tries scored is an example of **discrete data** because they are numbers that are obtained by *counting*. Discrete values are separate numbers (in this case, whole numbers) with 'gaps' between the possible values. You cannot have $2\frac{1}{2}$ tries or 1.8 tries scored.

Amount of petrol is an example of **continuous data** because they are numbers that are obtained by *measuring*. Continuous values lie on a smooth scale with no 'gaps' between the possible values. You *can* have 43.1 L, 43.15 L or 43.02 L of petrol.

EXAMPLE 5

Classify each type of categorical data as nominal or ordinal:

a method of travel to work

b report grade

c suburb of home

d fan setting of air conditioner

Solution

a Nominal

b Ordinal **For example, outstanding, satisfactory.**

c Nominal

d Ordinal **For example, low, medium.**

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EXAMPLE 6

Classify each type of quantitative data as discrete or continuous:

a amount of electricity used in homes

b number of DVDs owned

c shoe size

d weekly profit of a business

Solution

a	Continuous	**Smooth scale of values.**
b	Discrete	**Separate values (for example 32 and 58).**
c	Discrete	**Separate values (for example 7 and $8\frac{1}{2}$).**
d	Continuous	**Smooth scale of values.**

Exercise 1-03 Types of data

Example 4

1 Classify each of the following types of data as categorical (C) or quantitative (Q):

a type of religion

b number of computers in a home

c type of driver's licences held

d population of a city

e number of CDs bought online

f marital status

g method of payment for shopping

h movie classifcation

i patient's blood pressure

j swimmer's finishing place in a race

k amount of time spent on the Internet per week

l town or suburb where people live

2 What is the difference between categorical data and quantitative data? Give an example of each type.

Example 5

3 Classify each of the following types of categorical data as nominal (N) or ordinal (O):

a brand of car

b level of manager

c level of satisfaction with the government

d marital status

e favourite football team

f colour of Olympic medal

g size of soft drink sold at a fast food store

h a person's gender

Example 6

4 Classify each of the following types of quantitative data as discrete (D) or continuous (C):

a amount of rainfall each day

b mass of an athlete

c length of a mobile phone call

d running time for a 100 m race

e star rating of a hotel

f height of the world's tallest building

g number of seats on a bus

h loudness of a lawn mower

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5 Give one example of:

- **a** nominal data
- **b** ordinal data
- **c** discrete data
- **d** continuous data

6 Give the correct name for:

- **a** numerical data
- **b** data that are usually described by words
- **c** data that are usually measured on a smooth scale
- **d** categorical data that cannot be sorted in order

7 Which of the following statements about categorical data is true? Select **A**, **B**, **C** or **D**.

- **A** They are sometimes represented by numbers that indicate size or an amount.
- **B** They are sometimes represented by symbols.
- **C** They are represented by numbers that take on separate values.
- **D** They are represented on a smooth scale.

1-04 Statistics in society

The word **statistics** originally meant 'information about the state' (country or nation). Today statistics is the science of collecting, classifying and analysing any kind of information, while the singular word 'statistic' means a numerical value (such as the mean or range) that is calculated from a set of data.

Governments, businesses and other organisations collect and use statistics to plan and make intelligent decisions for the future. For example:

- government ministers want to know how many people use public transport
- a babygoods manufacturer wants to test the safety of a new high chair
- the owner of a chain of coffee shops wants to decide where to open the next shop
- the directors of an international relief agency need to find out which countries require aid most urgently
- the marketing staff of an electronics company want to know when to introduce a new mobile phone.

There are six stages in a statistical investigation, as the following describes.

Stage 1: Posing questions

Before an investigation can begin, the researcher needs to determine clearly the key questions of the study: the main aims, the ideas and the topics of concern.

Stage 2: Collecting data

According to the Australian Bureau of Statistics (ABS), data are 'observations or facts which, when collected, organised and evaluated, become information'. Collecting data usually

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requires a great deal of work and time, when the researcher goes out 'into the field' to find the necessary facts, through surveys, interviews, questionnaires, observations, tests, experiments and existing records.

Stage 3: Organising data

Once the raw data have been collected, they are converted into meaningful information for further study. This involves grouping, ordering or classifying the data in different ways, such as into categories, lists, tables or spreadsheets.

Stage 4: Summarising and displaying data

Any patterns or trends are clearer when displayed graphically, such as on a line graph or stem-and-leaf plot, or summarised by measures such as the mean, median, mode or range. These calculated values are called summary statistics.

Stage 5: Analysing data and drawing conclusions

'Analyse' means to examine in closer detail, and analysing the data is the most important stage of a statistical investigation. This involves comparing statistics and noting any significant results, answering the questions posed in Stage 1 and making important decisions and recommendations.

Stage 6: Writing a report

Finally the researcher needs to write a report about the investigation, summarising clearly the project's aims, methods, findings and conclusions.

EXAMPLE 7

Suppose the state government is investigating the best location to build a new hospital. Describe what might happen at each of the six stages of the investigation.

Solution

Stage 1 **Posing questions:**

- 'Which location is central to most areas?'
- 'Which location has the fastest-growing population?'
- 'Which location has the facilities and road networks to support a new hospital?'

Stage 2 **Collecting data:** The researcher searches for data by looking at existing records or conducting surveys and interviews.

Stage 3 **Organising data:** The researcher groups the data into a table showing the features of each possible location for the hospital.

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Stage 4	**Summarising and displaying data:** The researcher calculates statistics and/or uses graphs to compare the locations and look for a pattern.
Stage 5	**Analysing data and drawing conclusions:** The researcher examines the data and patterns in more detail, aiming to make a definite decision on the best location for the hospital.
Stage 6	**Writing a report:** The researcher reports on the investigation, clearly describing what data about hospital locations were collected, how they were collected, and the patterns, results and recommendations.

Exercise 1-04 Statistics in society

1 In groups, or as a whole class activity, match each of the following organisations to its field of activity or business.

	Organisation		**Activity**
a	CSIRO	**A**	National intelligence and security
b	RTA	**B**	Administration of the rugby league competition
c	UNICEF	**C**	Scientific research
d	GIO	**D**	International aid for underprivileged children
e	BOM	**E**	Weather forecasting and record-keeping
f	ASX	**F**	Administration of motor vehicles and road transport
g	ASIO	**G**	International health authority
h	WHO	**H**	Insurance
i	NRL	**I**	Buying and selling shares

2 Find out what each set of initials used to name an organisation in question **1** stands for.

3 Select four of the organisations from question **1** and, for each one, state:

a an example of the data it collects

b how it uses the data.

Example 7

4 A medical researcher is investigating whether a new drug improves a person's memory. There are fifty patients in the test group who take the memory pill, while the fifty patients in the control group take a placebo (a fake pill that has no effect but the patients think it's the memory pill). Both groups then complete a memory test and their scores are compared. Describe what might happen at each of the following stages in the investigation.

Stage 1: Posing questions

Stage 2: Collecting data

Stage 3: Organising data

Stage 4: Summarising and displaying data

Stage 5: Analysing data and drawing conclusions

Stage 6: Writing a report

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5 At what stage of a statistical investigation does the researcher:

a make a decision or recommendation after the results of the study become clear?

b determine the key goals and issues of the investigation?

c calculate statistics from the data and look for some relationship between them?

d gather the raw data required for study?

6 Visit the Australian Bureau of Statistics website, www.abs.gov.au, to collect the following data from the last Australian census.

a the population of Australia

b the actual date the census was taken

c the number of females in Australia

d the percentage of Australians who were born in New Zealand

e the most popular language spoken at home after English

f the percentage of Australians who are married.

7 Living in the 'Information Age', you should be aware of privacy issues and the need to keep your personal details confidential. What data can be collected about you when:

a you use an ATM?

b you respond to an online survey on the Internet?

c you send an SMS message on a mobile phone?

d you purchase items with a credit card?

e you are photographed by a speed camera while driving?

f you borrow from a library or video store?

8 Identify two dangers of revealing too much information about yourself.

9 In the media, we often see advertisers' claims similar to the following.

- 'Most popular restaurant in the Blue Mountains.'
- 'Lowest prices for electrical appliances!'
- 'The television program that all of Australia is talking about.'
- 'Best-selling book of the year.'
- 'Most effective pain reliever.'
- 'Australia's leading Internet service provider.'

Select two of these claims, explain what they mean and describe what data could be collected to support them.

1-05 Sampling techniques

Sampling vs census

In statistics, the population means all of the items being studied, whether they are people, fish, cars or buildings. A **census** is a survey of the population in which every item is included.

In Australia, a national census takes place every 5 years, in a year ending with a 1 or a 6, such as 2011 or 2016.

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If the whole population is too large or too difficult to survey, a **sample** of items is selected from the population. The results are then used to draw conclusions about the whole population. For example, a market researcher investigating the television viewing habits of Australians will survey a representative sample of Australians.

Sampling	Taking a census
• Surveys a representative group of items from a population	• Surveys all items in a population
• Provides an estimate or approximate information about the population	• Provides exact information about the population
• Simple and not expensive	• Complex and expensive

EXAMPLE 8

Determine whether sampling or taking a census is more appropriate for each of the studies below.

a finding the most popular radio station in Newcastle

b conducting a survey on whether Australians want a new national flag

c finding the exact number of young women living on the South Coast, with the view to building a new maternity hospital

d selecting the politicians to form a new state government

Solution

a sampling **It would be impractical to survey everyone in the population.**

b sampling **It would be impractical to survey everyone in the population.**

c census **An exact number is required.**

d census **This decision is important and requires everyone's vote.**

Types of samples

If a sample is to be truly representative of a population, then each item in the population must have an equal chance of being chosen. Also, the larger the sample, the more accurate the results of the study will be. There are three main types of samples.

- random sample
- systematic sample
- stratified sample

In a **random sample**, each item is chosen completely at random from the population (for example, selecting names out of a box, or setting a computer to create random numbers to choose people or items).

In a **systematic sample**, the first item is chosen at random and all other items are chosen at regular intervals. For example every 20th mobile phone is taken from a factory for testing, or the driver of every 8th car on the road is selected for random breath testing. 'Systematic' means a 'system' is used to select items.

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In a **stratified sample**, the population is divided into strata (layers) according to some characteristic (for example, gender or age group) and a random sample is taken from each layer using representative proportions or percentages. For example, if 28% of all shoppers in Australia are male, then a stratified sample of shoppers must be 28% male. 'Stratified' means 'in layers,' like stratified rock (as shown in the photo on the right).

Shutterstock.com/Dwight Smith

EXAMPLE 9

Which type of sample would you use to investigate each of the following?

a the opinion of Year 11 students on a new senior school uniform

b the quality of Buzz rechargeable batteries

c the amount of time spent on the Internet by Australian primary schoolchildren

d the popularity of the prime minister

Solution

a A random sample will provide a fair representation of Year 11.

b A systematic sample will ensure that batteries produced at different times will be tested.

c A stratified sample will ensure that children of all types and areas are included.

d A stratified sample will ensure that people of all types and areas are included.

EXAMPLE 10

At Westvale High School, there are 126 senior students and 572 junior students. The student council wants to survey 80 students about the school's environmental program. How many senior students should be selected for a stratified sample?

Solution

Total number of students = 126 + 572 = 698

Number of senior students = 126

Sample of students = 80

Number of senior students in sample is $\frac{126}{698} \times 80 = 14.44\ldots \approx 14$

14 senior students should be selected for the sample.

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Bias in sampling

Bias is an unwanted influence that favours a particular section of the population unfairly. A **biased sample** is not truly representative of the population and negatively affects the quality or reliability of a statistical study.

EXAMPLE 11

Explain the bias in each of the following samples.

a A group of Year 12 students sitting together in the playground was surveyed on their preferences for a school formal.

b A random selection of homes was contacted by telephone at midday for a survey about political opinions.

c A current affairs television program asked viewers to participate in an SMS (mobile phone) poll on gun laws.

Solution

a The group members are likely to be friends with similar opinions, so this sample may not be representative of all Year 12 students.

b This is biased against people who were not at home during the day.

c The sample would contain only viewers of the program who have a strong opinion about the subject or who make the effort to vote.

Exercise 1-05 Sampling techniques

Example 8

1 Determine whether sampling or taking a census is more appropriate for each of the studies below.

- **a** counting the number of students who are learning Japanese at a school
- **b** finding the number of people who watch soccer on SBS-TV
- **c** judging the most popular supermarket in town
- **d** finding the number of Australians who were born in Europe
- **e** investigating people's opinions on whether Australia should become a republic
- **f** determining the number of people in NSW who have red hair
- **g** finding the exact number of schoolchildren living in Parramatta
- **h** testing whether a brand of mobile phone battery lasts 50 hours

2 When will the next Australian national census be held?

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Example 9

3 Select an appropriate type of sample (random, systematic or stratified) for each study below. Give a reason for each answer.

a testing running shoes for quality

b finding the most watched TV station on Tuesday nights

c testing the reading abilities of a school's students

d testing a new drug on patients

e investigating people's views on euthanasia

f investigating how many Year 11 students at a school work part-time

g surveying bank customers about their satisfaction levels

h determining the most popular car colour

Example 10

4 A department store has a customer database of 3347 female shoppers and 1504 male shoppers. It wants to introduce a new credit card to a stratified sample of 200 shoppers. How many female shoppers should be included in the sample?

5 A public transport department intends to survey a sample of 600 commuters about the efficiency of a city's rail network. It is known that each region of the city has the following number of commuters:

North	South	East	West	South-west
123 760	149 810	81 060	260 450	215 880

If the sample is to be stratified, how many commuters from the 'West' should be included?

6 A school's gardening program is run by 25 students, five teachers and eight parents. A stratified sample of eight people is selected to attend an environmental conference. Which of the following is the number of teachers who should be in the sample? Select **A**, **B**, **C** or **D**.

A 0 **B** 1 **C** 2 **D** 5

Example 11

7 Explain why each of the following samples is biased.

a A sample of Dolly magazine readers was surveyed to investigate teenagers' views on road safety.

b Diners at a restaurant can give feedback about their meal by filling in a survey and posting it to the restaurant.

c For a survey about canteen food, the school council interviewed the first 20 students to arrive at school one morning.

d At a factory, a sample of executives was surveyed about safety in the workplace.

e A sample of audience members at a film premiere was surveyed about their views on the film.

f Triple J radio listeners were asked to vote for the best song of the year.

8 A sample of shoppers is to be chosen for a survey about customer service at a new shopping centre.

a How would you choose the shoppers to ensure that you have an unbiased sample?

b List two ways of choosing shoppers that would result in a biased sample.

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JUST FOR THE RECORD

How television ratings are measured

Each week, research company OzTAM investigates the television viewing habits of Australians, based on a sample of over 5000 homes. OzTAM, which stands for Australian Television Audience Measurement, investigates which television channels and programs are being watched by a sample of Australian households stratifed according to the ages and number of people living in each home. Participants in the survey have their television sets linked to a black box called a 'peoplemeter' that records the details of all programs being watched each day (including pay television). The results are sent overnight by modem to OzTAM.

During the week ending 26 May 2012, the Australian television networks had the following percentage shares of viewers in Sydney, Melbourne, Brisbane, Adelaide and Perth, from 6:00 a.m. to 12:00 midnight.

Television station	Percentage share
Seven	17.2%
Nine	17.0%
Ten	10.7%
ABC1	7.4%
ABC2	4.3%
Eleven	2.9%
GO!	2.9%
SBS One	2.8%

Source: OzTAM

What types of people would be interested in the television viewing habits of Australians and why?

INVESTIGATION

ESTIMATING POPULATIONS

How many students at your school:

a have red hair?
b have brown eyes?
c are left-handed?
d are vegetarian?
e walk to school?
f have a dog as a pet?

Estimate the answers to these questions by surveying a sample of 50 students from your school and making appropriate calculations. How could you improve on your estimates?

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Student survey form

Census questions

INVESTIGATION

DESIGNING A QUESTIONNAIRE

A survey is often conducted using a questionnaire, which asks for details about a person's lifestyle, beliefs, attitudes and behaviours. The table below describes the features of a good questionnaire, giving examples.

Feature	Bad example	Good example
1 **Simple language** All instructions and questions should be easily understood.	'Describe the frequency with which you consume takeaway food on a weekly basis'.	'How many times per week do you eat takeaway food?'
2 **Unambiguous questions** Questions must not have more than one meaning or interpretation.	'What type of car do you own?' Does the question mean: • the make of car (for example Toyota)? • the style of car (for example hatchback)? • the size of car (for example small)?	'What make of car do you own? Tick one box from the following list: …' (This is followed by a list of different makes of cars.)
3 **Respect for privacy** A survey should be anonymous and not require personal details. Questions that are too personal should be optional.	'What is your email address?' People are concerned about receiving junk email if they give out their email address.	'In case we need to contact you further regarding this survey, please supply your email address. This is optional and your details will not be forwarded to other parties or used for promotional purposes.'
4 **Freedom from bias** Questions should not suggest or influence a person's response.	'What do you think of changing the Australian flag, given that it is part of our history and our soldiers have died for it in the past?' Too emotional, not objective, influencing people to vote against a new flag.	"Do you think the current Australian flag should be changed?'
5 **Consideration of a number of choices** A question may have many different answers, so either make the question very specific or provide a list of answers from which to choose.	'How well did you enjoy your meal?' (from a restaurant survey). Too open-ended. There would be a wide range of possible answers that would be difficult to organise and analyse.	'How satisfied were you with your meal? Choose from one of the following: Very satisfied ☐ Satisfied ☐ Not satisfied ☐ Very unsatisfied ☐ No opinion ☐

1 Explain what is wrong with each survey question and write a better question each time.

a Are politicians paid too much?

b What is your favourite food?

c How often do you smoke?

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d Which mode of transport do you employ to commute to your place of work?

e Are you happy at school?

f How satisfied are you with the level of customer service at the New Century Bank?

2 **a** In groups of two to four, design a questionnaire of 10 to 12 questions about one of the following topics.

- the TV viewing habits of people
- the spending habits of teenagers
- students' use of computers and the Internet
- family and household details
- favourite sports to watch and play

b Decide on the type of facts you wish to collect, then determine suitable wording for each question and the format of each answer. Make sure that your questions reflect the features of an effective questionnaire.

1-06 Constructing graphs

Australian statistics

When displaying a set of data, remember to:

- select the most appropriate graph or display
- make an impact
- show enough detail, including a title, labels, key or scale
- avoid giving a misleading impression.

	Useful for:	Strengths	Weaknesses
Column/bar graph	• categorical or quantitative data • showing differences in amounts.	• Shows exact values • Can graph a large amount of data	• Takes a long time to draw
Line graph	• continuous quantitative data • showing changes over a time period.	• Shows exact values • Can graph a large amount of data • Shows differences and patterns clearly • Can be used to estimate values between points	• Can only be used to graph continuous data
Sector graph and divided bar graph	• categorical data • comparing parts of a whole.	• Shows parts of a whole	• Doesn't show exact values • Doesn't show small proportions or a large number of categories well

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EXAMPLE 12

Here are the 2006 Census results for the religions of Australian people.

Represent this data on:

a a sector graph

b a divided bar graph.

Religion	Frequency
Buddhist	418 749
Christian	12 685 829
Hindu	148 130
Islamic	340 394
Jewish	88 832
Other religion	5 380
No religion	3 706 550
Total	**17 393 864**

Solution

a First calculate the angle of each sector, rounded to the nearest degree.

Buddhist: $\frac{418\,749}{17\,393\,864} \times 360° \approx 9°$

Write each religion as a fraction and multiply by 360°.

Christian: $\frac{12\,685\,829}{17\,393\,864} \times 360° \approx 263°$

Hindu: $\frac{148\,130}{17\,393\,864} \times 360° \approx 3°$

Islamic: $\frac{340\,364}{17\,393\,864} \times 360° \approx 7°$

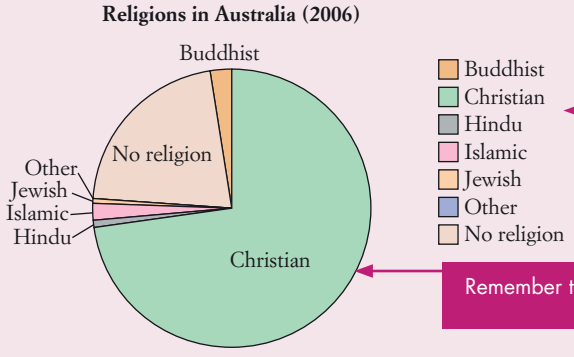
Jewish: $\frac{88\,832}{17\,393\,864} \times 360° \approx 2°$

Other religion: $\frac{5380}{17\,393\,864} \times 360° \approx 0.1°$

'Other religion' will be too small to show as a sector.

No religion: $\frac{3\,706\,550}{17\,393\,864} \times 360° \approx 77°$

Note: Due to rounding, the angle sizes may not add exactly to 360°.



Instead of labelling the sectors, you could provide a 'key' like this.

Remember to add a title and to label each sector.

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b Draw a long rectangle (for example 12 cm long), then calculate the length of the section representing each religion, rounded to the nearest millimetre.

Buddhist: $\frac{418\ 749}{17\ 393\ 864} \times 12\text{ cm} \approx 0.3\text{ cm}$

Write each religion as a fraction and multiply by 12 cm.

Christian: $\frac{12\ 685\ 829}{17\ 393\ 864} \times 12\text{ cm} \approx 8.8\text{ cm}$

Hindu: $\frac{148\ 130}{17\ 393\ 864} \times 12\text{ cm} \approx 0.1\text{ cm}$

Islamic: $\frac{340\ 394}{17\ 393\ 864} \times 12\text{ cm} \approx 0.2\text{ cm}$

Jewish: $\frac{88\ 832}{17\ 393\ 864} \times 12\text{ cm} \approx 0.06\text{ cm}$

Other religion: $\frac{5380}{17\ 393\ 864} \times 12\text{ cm} \approx 0.0037\text{ cm}$

'Jewish' and 'Other religion' will be too small to show as sections.

No religion: $\frac{3\ 706\ 550}{17\ 393\ 864} \times 12\text{ cm} \approx 2.6\text{ cm}$

Note: Due to rounding, the sections may not add exactly to 12 cm.

Remember to add a title and label to each segment.
Instead of labelling the segments, you could provide a 'key'.

Exercise 1-06 Constructing graphs

1 A group of 40 students was surveyed about the number of DVDs they each watched last weekend. The results were:

2	1	1	2	3	3	4	3	1	4	3	3	6	3	2	0	3	4	2	2
3	3	2	1	1	3	0	5	4	4	2	2	2	5	4	0	4	3	1	3

a Organise the data in a frequency table, then present it as a column graph.

b Why is a line graph not appropriate for this data?

c What was the mode (the most common number of DVDs watched)?

d What was the highest number of DVDs watched and how many students watched that many?

e What percentage of students watched more than four DVDs?

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Example 12

2 Georgie won $800 000 on a quiz show and spent it as shown in the table on the right.

Use of money	Amount
Pay home loan	$235 000
New car	$96 000
Home renovations	$144 000
Travel	$125 000
Savings	$200 000

a Why is a sector graph or divided bar graph suitable for this information?

b Draw a sector graph for this information.

c What did Georgie do with one-quarter of her winnings?

d What fraction of Georgie's winnings was spent on home renovations?

e What percentage of Georgie's winnings was spent on a new car?

3 A sample of Sydney households was surveyed about which television network they watched for the evening news. The results are shown in the table on the right.

Network	Number of households
ABC1	3460
SBS1	855
Seven	4140
Nine	4215
Ten	2300

a How many households were surveyed?

b Name the three types of graphs that could be used to display this data.

c Represent the data on a divided bar graph.

d What percentage of households watch the ABC1 news?

e Do the viewers of Seven News and Nine News combined make up more than half of the households surveyed?

f Does this data reflect the news-watching habits of all Australian households? Justify your answer.

4 The teachers at Summer Heights High School were surveyed on the number of cups of coffee they drank in one day.

Number of cups	0	1	2	3	4	5	6
Number of teachers	7	5	11	21	15	9	4

a How many teachers are there at the school?

b What type of graph can be used to illustrate this data? Give a reason for your answer.

c What percentage of the teachers did not drink coffee that day? Answer correct to one decimal place.

d If the recommended healthy coffee intake is no more than three cups of coffee per day, what fraction of teachers had a healthy intake of coffee?

5 Serena wants to graph the results of a survey about people's favourite flavour of ice cream. Which of the following graphs can she use? Select **A**, **B**, **C** or **D**.

A column graph

B sector graph

C divided bar graph

D all of the above

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6 During a two-week period, Australian Customs recorded the number of immigrants arriving at Sydney airport, as shown in the table.

Country of origin	Number of immigrants
United Kingdom	20
New Zealand	35
Middle East	7
India	10
Africa	3
Asia	12
Other regions	6

a Draw a divided bar graph to represent this data.

b 'More than half of the immigrants came from the UK or New Zealand.' True or false?

c What fraction of immigrants came from Asia?

d What percentage (to the nearest whole number) of immigrants came from the Middle East?

e Which region represents 10.75% of the immigrants?

7 This table shows the numbers of motor vehicle and home thefts in the Shoalhaven area over 5 years.

Year	2002	2003	2004	2005	2006
Motor vehicle theft	419	381	379	369	312
Home theft	517	567	467	449	397

a Draw line graphs showing both types of theft on the same axes. Include labels or a key.

b In which year was home theft highest?

c In which year was motor vehicle theft lowest?

d Which type of theft is generally more common?

e Write a sentence to describe what happened to the number of home thefts over the 5 years.

f What do you think happened to motor vehicle theft in 2007? Justify your answer.

JUST FOR THE RECORD

Bias in drafting US soldiers

Bias can affect life-or-death situations, as some young Americans found out during the Vietnam War. In 1970, men aged 19 to 26 years were randomly drafted into the armed forces according to their birthdates (this also occurred in Australia). The 366 dates of the year were written on plastic capsules and put into a large bowl. Capsules were then picked out and the men who had the selected birthdates were drafted. However, it was later discovered that birthdates in November and December had a slightly higher chance of being drawn because the capsules were placed in the bowl in order and not mixed properly, and the ones chosen were often from the top of the pile.

Suggest two ways the bias could have been eliminated during this selection process.

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TECHNOLOGY: DRAWING SECTOR GRAPHS

We can use the Chart Wizard in a spreadsheet to draw different types of graphs.

The members of a sample of 800 commuters were surveyed about the way they travelled to work.

	A	B
1	Train	287
2	Car	233
3	Walk	140
4	Bus	73
5	Cycle	67

Step 1: Enter the data from the survey into a spreadsheet as shown.

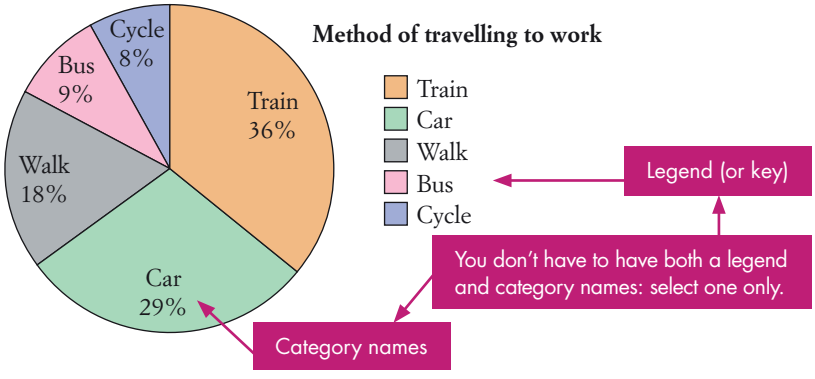
Step 2: Select (highlight) all the cells A1:B5 and open **Chart Wizard** using either the icon from the toolbar or **Chart** from the **Insert** menu.

Step 3: In the Chart Type dialog box: For a sector graph, select **Pie** and click **Next**.

Step 4: In the Chart Source Data dialog box: Select **Series in columns** and click **Next**. (We would select 'rows' if the table data were in two rows.)

Step 5: In the Chart Options dialog box: use the Titles tab. Under **Chart title**, enter the title: 'Method of travelling to work' use the Legend tab, with **Show legend** ticked, to display the legend/key (this is optional) use the Data Labels tab. Tick **Category name** and **Percentage** to display sector names and percentages, and click **Next**.

Step 6: In the Chart Location dialog box: Click Finish to make the graph appear on your spreadsheet. (You can cut and paste this graph into a document if you wish.)



Step 7: Format any part of your graph by clicking on it and resize the graph by dragging the corners of the chart area until you have the size and features you like.

Further investigations

1 Repeat the steps above but experiment with different types of sector graphs and different chart options, such as labels.

2 Graph the same data as a column graph or bar chart (horizontal column graph) by right clicking the graph and changing the Chart Type.

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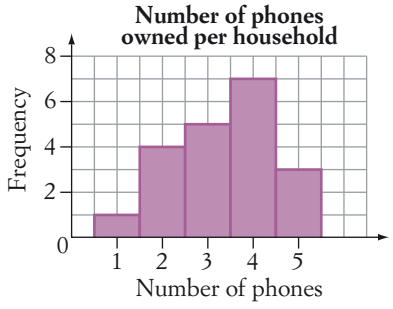
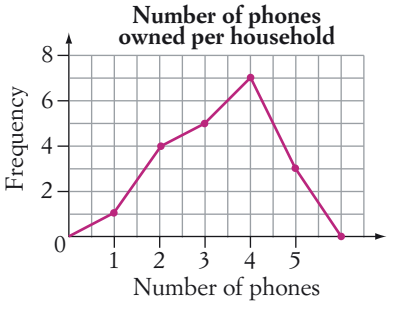
1-07 Frequency histograms and polygons

Frequency tables

A **frequency histogram** or **frequency polygon** graphs quantitative data, including data grouped into class intervals.





Frequency histogram

- no gaps between columns
- has a half-column gap at the start

Frequency polygon

- can be drawn by joining the midpoints of the tops of the columns of a histogram
- starts and ends on the horizontal axis

	Useful for:	Strengths	Weaknesses
Frequency histogram and polygon	• quantitative data • data grouped into class intervals • showing the shape of a set of data; how the scores are spread out	• Easy to read • Shows differences and patterns clearly • Can graph a large amount of data • Shows how the scores are spread out	• Takes a long time to draw

EXAMPLE 13

The ages of the patients at a medical centre in one afternoon were recorded.

37	12	19	65	44	30	1	8	14	36	25	57	21	16
4	30	40	49	38	69	33	55	16	4	8	16	6	28
29	54	43	48	33	18	9	59	21	70	22	33	2	60

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a Copy and complete this frequency distribution table for the data.

Class interval	Class centre	Tally	Frequency
0–9			
10–19			
20–29			
30–39			
40–49			
50–59			
60-69			
70–79			
		Total	

b Construct a frequency histogram for this data.

c How many patients went to the medical centre?

d How many of these patients were aged 50 or over?

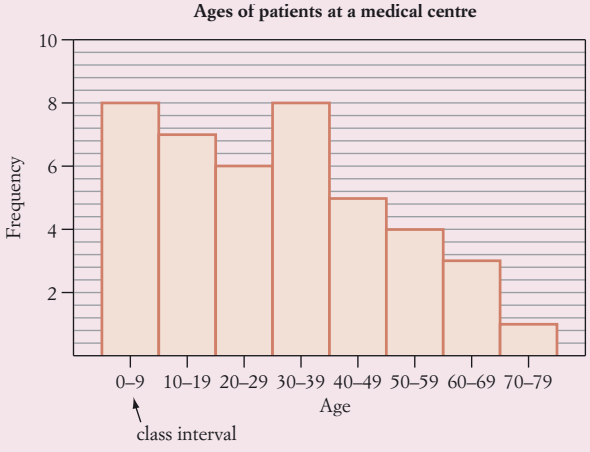
Solution

These are discrete class intervals.

a

Class interval	Class centre	Tally	Frequency
0–9	4.5	卌 |||	8
10–19	14.5	卌 ||	7
20–29	24.5	卌 |	6
30–39	34.5	卌 |||	8
40–49	44.5	卌	5
50–59	54.5	||||	4
60-69	64.5	|||	3
70–79	74.5	|	1
		Total	**42**

b When graphing data that are grouped into discrete class intervals, either the class intervals or class centres are shown on the horizontal axis.



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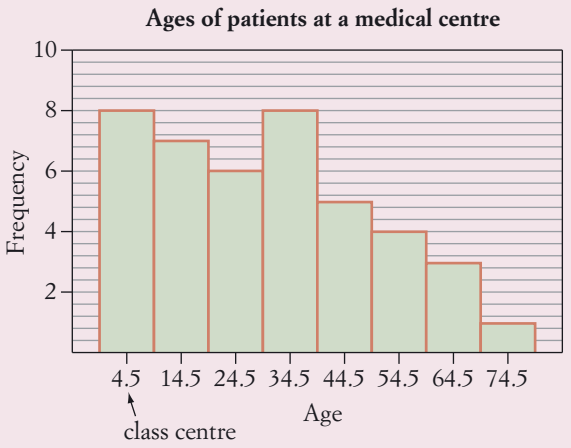

c 42 patients went to the medical centre **Total frequency.**

d Number of patients aged 50 or over = 4 + 3 + 1 = 8

EXAMPLE 14

The monthly call costs for a group of 120 mobile phone users were recorded and grouped into class intervals as shown.

a Construct a frequency histogram and polygon for this data.

b Which call costs were the most common?

c Which call costs were used by exactly 10 people?

d What percentage of phone users had call costs below $60?

Call cost ($)	Class centre	Number of users
0–< 20	10	6
20–< 40	30	8
40–< 60	50	13
60–< 80	70	17
80–< 100	90	23
100–< 120	110	20
120–< 140	130	16
140–< 160	150	10
160–< 180	170	4
180–< 200	190	3
	Total	**120**

These are continuous class intervals.

Solution

a When graphing data grouped into continuous class intervals, either the class boundaries or class centres are shown on the horizontal axis

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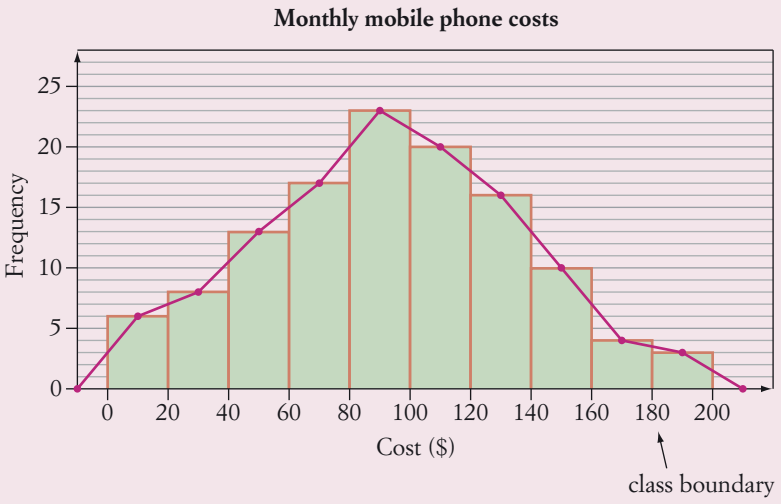
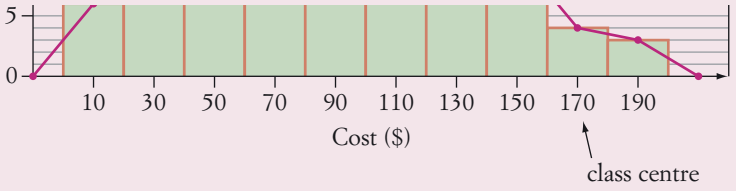


or with this horizontal axis:



b $80–< $100 **Had the highest frequency (23).**

c $140–< $160 **Had a frequency of 10.**

d Number of users with call costs below $60 = 6 + 8 + 13 = 27

Percentage of users with call costs below $60 $= \frac{27}{120} \times 100\% = 22.5\%$

Exercise 1-07 Frequency histograms and polygons

1 Forty people were surveyed about the number of times they accessed the Internet in one day. The results are shown on the right.

2	1	5	1	3	4	3	5
5	4	3	4	5	2	1	0
3	4	1	1	2	3	2	0
1	5	4	5	3	3	1	1
1	4	4	5	2	3	1	5

a Organise this data into a frequency distribution table.

b Construct a frequency histogram for the data.

2 The masses (in grams) of a sample of 48 eggs were measured.

57	58	61	59	62	59	59	56	60	64	58	58
56	59	64	57	60	62	58	60	64	57	61	58
59	57	64	58	59	57	58	64	60	58	60	57
61	64	58	60	61	62	62	58	60	61	57	58

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a Construct a frequency distribution table for this data.

b Draw a frequency polygon for the data.

c If these eggs are labelled as being 60 grams, would you say that the label is misleading? Give reasons for your answer.

3 The table below shows the number of road fatalities according to age group on a section of the Pacific Highway over 6 months.

Example 13

Age group	Road fatalities
0–9	6
10–19	14
20–29	28
30–39	11
40–49	8
50–59	3

a How many people died on this section of the highway during the period?

b Draw a frequency histogram and polygon for this data.

c What observations can you make about road fatalities from the graphs?

4 The number of words in each sentence of a magazine article was counted, with the results shown below.

27	22	15	8	14	7	9	25	15	17	5	24
9	11	22	8	5	15	25	18	10	21	24	13
9	14	18	11	9	23	15	19	10	8	14	17

a Is this data discrete or continuous?

b Using class intervals of 1–5, 6–10, etc., complete a frequency table for the data.

c Construct a frequency polygon for the data.

d What was the modal class (the class with the highest frequency)?

5 Here are the ages of the employees at Burger Heaven.

18	19	18	17	20	20	24	15	24	19
15	40	21	17	20	22	23	21	24	23
34	19	45	20	15	21	24	27	19	33
34	24	16	18	30	21	26	31	16	25
49	21	21	35	16	22	15	25	44	23

a Organise the data into a frequency table using classes 15–19, 20–24, etc.

b How many people work at Burger Heaven?

c Draw a frequency histogram and polygon for the data.

d Comment on the shape of the data.

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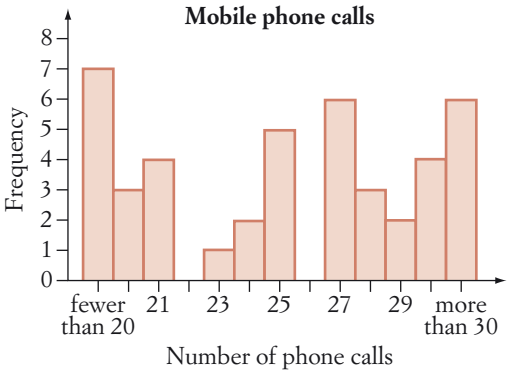
6 A group of students was surveyed on the number of mobile phone calls made in a week. The results are shown in this histogram.



a How many students made 22 calls?

b How many students were surveyed?

c What percentage of students made fewer than 20 calls? Answer correct to one decimal place.

d Can you tell how many students made 32 calls? Why?

Example 14

7 The number of kilograms lost by each person attending Julie's Weight Loss Centre is shown below.

1.8	2.5	4.3	6.5	2.7	4.6	11.0	10.8	0.3	8.2
2.1	3.8	4.4	5.8	1.6	5.9	7.6	9.3	4.8	3.4
12.5	4.6	2.5	6.9	7.5	3.5	4.8	12.2	4.3	3.7
0.0	0.9	2.6	7.8	4.9	7.4	9.8	10.4	2.6	8.2

a Is this data discrete or continuous?

b Complete a frequency table for this data using class intervals 0–< 2, 2–< 4, etc.

c Draw a frequency histogram for this data.

d What is the modal class?

e Use your histogram to comment on the effectiveness of the weight loss program.

8 The heights (in centimetres) of new applicants to a modelling agency are:

168	182	187	185	178	176	174	163	164
165	189	185	165	162	173	164	173	178
172	186	189	175	176	180	170	168	165

a Using class intervals of 160–< 165, 165–< 170, etc., complete a frequency table for this data.

b Draw a frequency polygon for the data.

c What is the modal class interval?

d What is the lowest class interval?

e Which class interval is least common?

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TECHNOLOGY: FREQUENCY HISTOGRAMS AND POLYGONS

We can use the Chart Wizard in a spreadsheet to draw a frequency histogram and polygon.

	A	B
1	Call cost ($)	Number of users
2	**0**	**0**
3	10	6
4	30	8
5	50	13
6	70	17
7	90	23
8	110	20
9	130	16
10	150	10
11	170	4
12	190	3
13	**210**	**0**

Step 1: Enter the data about the cost of mobile phone calls from Example **14** into a spreadsheet but add a first row (0, 0) and a last row (210, 0) as shown. This allows you to draw a frequency polygon which touches the x-axis.

Step 2: Select B2:B13, the values in the 'Number of users' column, as the 'y-values'. (Do not select B1 because we will add titles later.)

Step 3: Open the **Chart Wizard** using either the icon from the toolbar or **Chart** from the **Insert** menu.

Step 4: In the Chart Type dialog box: To draw a histogram, select **Column** and click **Next**. Or, to draw a polygon, select **Line** and click **Next**.

Step 5: In the Chart Source Data dialog box:

- use the Data Range tab. Select **Series in Columns**.
- use the Series tab. For 'Category (X) axis labels', select A2:A13, the values in the 'Call cost ($)' column, as the x-values and click **Next**.

Step 6: In the Chart Options dialog box click on the Titles tab and

- for **Chart title**, enter the title 'Mobile phone call costs'
- for **Category (X) axis**, enter the label 'Call costs ($)'
- for **Value (Y) axis**, enter the label 'Number of users'.

  Click on the Legend tab and remove the tick from **Show legend**, so that the 'Series 1' key is not displayed. Click **Next**.

Step 7: In the Chart Location dialog box, click **Finish** to make the graph appear on your spreadsheet.

Step 8: For a histogram, to remove the gaps, right-click any column of the graph, select **Format Data Series** and click on the Options tab. Set the 'Gap width' to 0 and click **OK**. To draw a frequency polygon, right click on any column, select **Chart Type** then Line and **OK**.

Further investigation

Repeat the above steps but experiment with different chart options.

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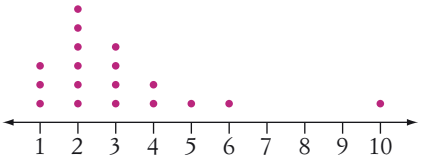
1-08 Dot plots and stem-and-leaf plots



Stem	Leaf
7	9 5 8 1 6 9
8	4 2 5 0 0 8 0 3 2 4 2 8
9	0 8 4 2 2 0 0 8 0 5 6
10	5 4

Dot plot

- a simple type of column graph
- uses a dot, cross or symbol to display each score
- also called a line plot.

Stem-and-leaf plot

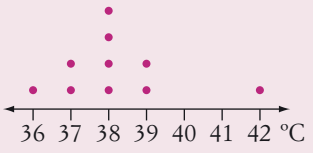
- actual scores are listed, usually in order
- the stem shows the first digit(s)
- the leaf shows the last digit(s)
- also called a stem plot.

	Useful for:	Strengths	Weaknesses
Dot plot and stem-and-leaf plot	• quantitative data • small sets of data • showing gaps and clusters • showing outliers (scores that are much different from the rest)	• Easy to read and draw • Shows differences and patterns clearly • Shows the spread of scores • Stem-and-leaf plot shows actual scores	• Hard to display large sets of data

EXAMPLE 15

The dot plot below shows the temperatures of a sample of hospital patients.

Patients' temperatures



a How many patients are in the sample?

b How many patients have a normal body temperature of 37°C?

c What is the mode (the most common score)?

d Where are the temperatures clustered?

e Find the outlier and describe the patient who has this temperature.

Solution

a 10 — **10 dots in total.**

b 2 patients — **There are two dots at 37°C.**

c 38°C — **The score with the most dots.**

d From 37°C to 39°C.

e The outlier temperature is 42°C and a patient with this temperature would be very hot and ill.

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EXAMPLE 16

This stem-and-leaf plot shows the exam marks of a class of students.

Stem	Leaf
4	3 7
5	0 2 2 4 8
6	1 3 3 3 6 6 9
7	5 7 8
8	4 6
9	2

The stem shows the tens digit, and the leaf shows the units digit: 6 | 1 means 61.

a What is the highest mark?

b How many students are in the class?

c What is the most common mark?

d What percentage of students scored over 60?

Solution

a 92

b 20 **The number of values in the 'Leaf' column.**

c 63 **This score occurs the most, 3 times.**

d Number of students who scored over 60 = 13

Percentage of students who scored over 60 = $\frac{13}{20} \times 100\% = 65\%$

Exercise 1-08 Dot plots and stem-and-leaf plots

Example 15

1 A class was surveyed to find out how many hours each student spent on maths homework each week. The results are shown below.

7	6	8	9	5	10	6	9	9	0	9	8
18	7	5	3	4	9	6	7	8	10	7	8

a Draw a dot plot for this data.

b How many students are in the class?

c What are the outliers? Comment on possible reasons for each of these.

d What are the clusters or gaps?

2 This dot plot shows the shoe sizes of a sample of Year 11 students.

a How many students are there in the sample?

b What is the mode?

c Find the outlier and describe the student that has this outlier shoe size.

d What fraction of students have a shoe size of 8 or 9?

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3 The Bennelong Bears scored the following numbers of goals per game in their first season.

4 0 2 3 7 1 2 4 5 0 3 5

a Display the data as a dot plot.

b What was the lowest score?

c Comment on any clusters or outliers.

4 A survey was conducted to determine the number of people in each car in a sample of cars on a road.

6 3 1 2 3 1 1 3 4 2
5 3 1 1 2 3 4 5 6 4

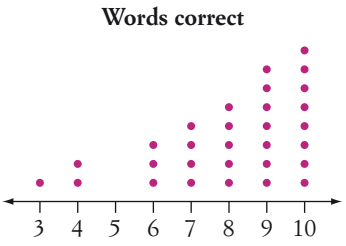
a Display the data as a dot plot.

b How many cars were surveyed?

c What was the highest score?

d Comment on any clusters or outliers.

5 This dot plot shows the scores (out of 10) of students competing in a spelling bee.



a How many students competed in the spelling bee?

b What percentage of students scored more than 8?

c Where were the scores clustered?

d Comment on the general performance of the contestants in the spelling bee.

Example 16

6 This unordered stem-and-leaf plot represents the number of points scored per match by the Sharks in a football season.

Stem	Leaf
4	5 3 9
5	7 2 0 8
6	4 7 8 5 1 2
7	2 9 3 0
8	9 4 2

a Redraw the data as an ordered stem-and-leaf plot.

b How many matches were played in the season?

c What was the Sharks' highest score for a match?

d For what percentage of matches did the Sharks score below 56 points?

7 Jeremy sells sausage sandwiches outside the hardware store on Sundays. The number of sandwiches sold each day over 30 Sundays was recorded as follows.

66	64	28	93	47	110	53	68	117	43
72	68	84	103	59	82	78	61	104	79
51	63	112	81	79	94	42	57	83	100

a Draw a stem-and-leaf plot to represent this data.

b On what percentage of days did Jeremy sell more than 50 sausage sandwiches?

c What is the outlier? Give one possible reason for this outlier.

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8 This plot shows the times (in seconds) in a Slalom ski race.

Stem	Leaf
9	1 5 7 9
10	2 4 5 6 6 8
11	0 2 2 3 4 4 5
12	1 2 3 3 3 7 9
13	2 3 4 5 7 7
14	3 6 9
15	0 1 2

a How many skiers participated in the race?

b What was the winning time?

c If those skiers with times under 110 seconds were of Olympic standard, what percentage of skiers were of this standard?

9 A nurse at Greenacres Hospital took the pulse rates (in heartbeats per minute) of 40 patients.

71	81	63	55	93	52	69	78	84	65
72	80	68	74	85	79	90	84	76	68
58	64	60	97	83	69	74	56	64	89
94	81	63	60	76	72	110	83	90	64

a Draw a stem-and-leaf plot for this data.

b If the average pulse is from 65 to 75 beats per minute inclusive, how many patients had an average pulse?

c Find the outlier and describe the patient that has this pulse.

d A very fit person has a low resting pulse (say, less than 60). How many patients were in this category?

10 This plot shows the prices (in $1000s) of homes listed by a Mudgee real estate agent.

Stem	Leaf
2	04 10 17 18 26 35 37 76 77
3	10 26 42 66 54 76 85 85 85
4	02 53 62 65 73 75 76 85
5	03 10 24 60 68 92
6	14 29 32 34 35 35 54
7	18 52 63 85
8	12 25 46 72 93
9	38 75

a What is the price of the most expensive home?

b How many homes are listed?

c Which price is shared by three different homes?

11 At the school athletics carnival, runners recorded the following times (in seconds) in the 100 metres sprint.

12.1	13.6	11.8	18.1	12.0	15.6	13.9	17.2
14.5	18.7	15.7	14.6	16.3	11.6	17.7	15.6
14.6	15.4	12.4	16.5	17.4	14.6	16.8	14.3

a Represent this data on a stem-and-leaf plot using stems of 11, 12, etc.

b How many runners were there?

c What was:

i the best time recorded?

ii the worst time recorded?

d What percentage of students took more than 15 seconds to run the distance?

Answer correct to one decimal place.

e To qualify for the regional carnival, a runner's time must be less than 13.5 seconds. What fraction of students qualified for the regional carnival?

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JUST FOR THE RECORD

Outliers and global warming

An outlier can often be the most important score in a data set.

Global warming and the effect of the hole in our ozone layer are troubling issues in our world today. In 1976, scientists in Antarctica began detecting low levels of ozone in the Earth's atmosphere, but dismissed these low levels as 'unimportant outliers'. Because of this, the damage to our ozone layer went undetected and untreated for another 9 years.

How does the hole in the ozone layer contribute to global warming?

Radar chart paper

1-09 Radar charts

A **radar chart** is a circular line graph that looks like the radar screen used on ships and aeroplanes for navigation, or like a spider's web. It is useful for graphing data over a cycle, such as a patient's temperature over a day, or the monthly rainfall of a town over a year.

EXAMPLE 17

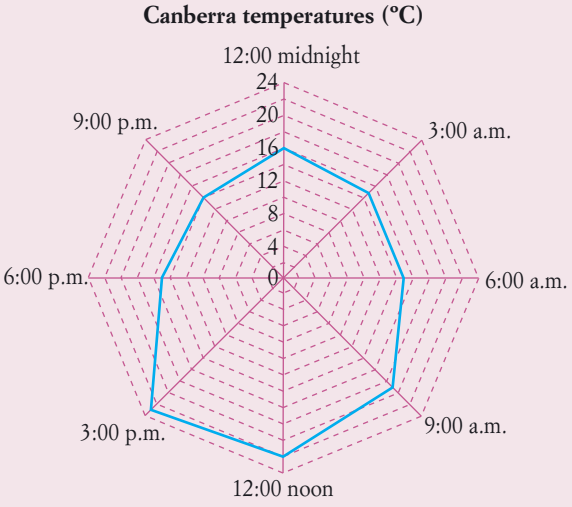
This radar chart shows the temperature (in °C) in Canberra over one day, measured every 3 hours.

a What was the temperature at 6:00 a.m.?

b When was the temperature 16°C?

c What was the highest temperature recorded and when did it occur?

d How are high temperatures shown on a radar chart?



Solution

a 15°C **b** At 12:00 midnight. **c** 23°C at 3:00 p.m.

d High temperatures are shown towards the outer edges of the chart, away from the centre.

	Useful for:	Strengths	Weaknesses
Radar chart	• continuous quantitative data • showing changes over a time cycle	• Small and attractive • Shows exact values • Shows differences and patterns clearly	• Difficult to draw • Scale may be hard to read

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EXAMPLE 18

Mr Harte is a patient at Hollywood Hospital. Every 2 hours, the nurse records his fluid intake (such as water and medicine) and fluid output (such as urine), as shown in this table.

Time	0700	0900	1100	1300	1500	1700	1900	2100	2300	0100	0300	0500
Intake (mL)	280	620	50	210	620	150	300	300	170	430	560	310
Output (mL)	650	200	100	480	270	50	110	540	90	320	180	290

a Plot both sets of data (fluid intake and fluid output) on the same radar chart.

b When was Mr Harte's fluid intake lowest?

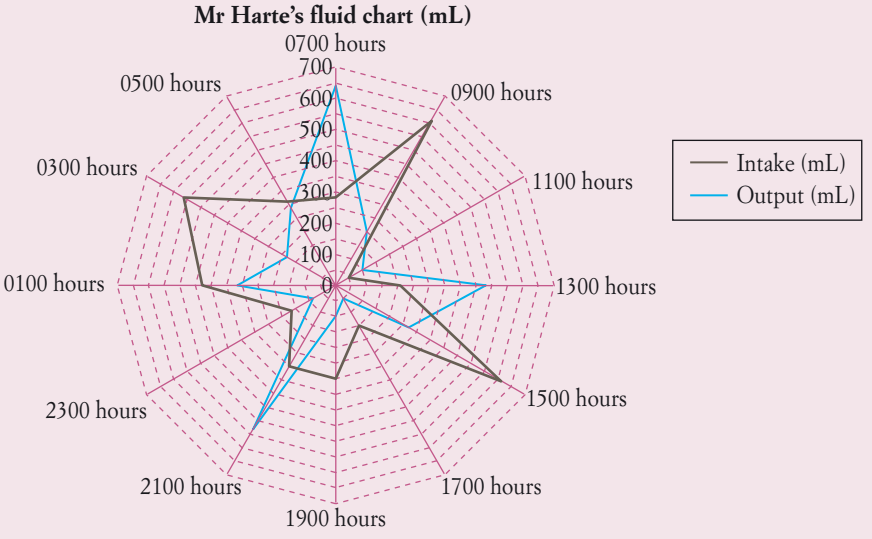
c When was Mr Harte's fluid output highest?

d For which two consecutive times was the amount of Mr Harte's fluid intake the same?

e How many times was his fluid output above 500 mL?

Solution

a



b At 1100 hours. **Where the intake graph is closest to the chart centre.**

c At 0700 hours. **Where the output graph is closest to the outer edge.**

d At 1900 hours and 2100 hours. **Where the intake graph is the same distance from the centre.**

e Two times (at 0700 hours and 2100 hours).

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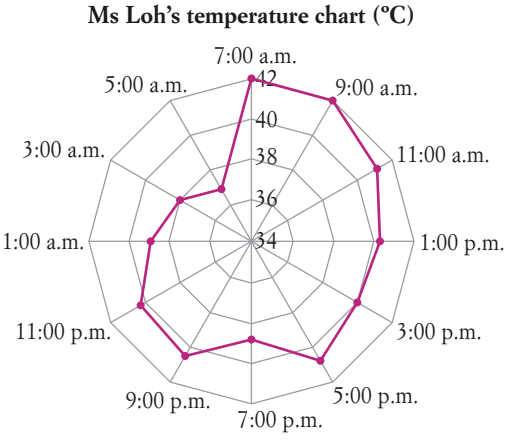
Summary of statistical displays

Display	Useful for:
Column/bar graph	• categorical or quantitative data • showing differences in amounts
Line graph	• continuous quantitative data • showing changes over a time period • estimating values between points
Sector graph and divided bar graph	• categorical data • comparing parts of a whole • a small number of categories
Frequency histogram and polygon	• quantitative data • data grouped into class intervals • showing the shape of a set of data, how the scores are spread out
Dot plot and stem-and-leaf plot	• small sets of quantitative data • showing clusters, gaps and outliers • showing the shape of a set of data, how the scores are spread out
Radar chart	• continuous quantitative data • showing changes over a time cycle

Example 17

Exercise 1-09 Radar charts

1 Ms Loh was admitted to hospital with swine flu. This radar chart shows her temperature readings taken every 2 hours over a 24-hour period, beginning at 7:00 a.m.



a What was Ms Loh's temperature at 11:00 a.m.?

b When was her temperature lowest? What was this temperature?

c How many times was Ms Loh's temperature above 39°C?

d For which two consecutive times was her temperature the same?

e When did Ms Loh's temperature change the most from the previous reading? What was this change?

f Describe what happened between 5:00 p.m. and 9:00 p.m.

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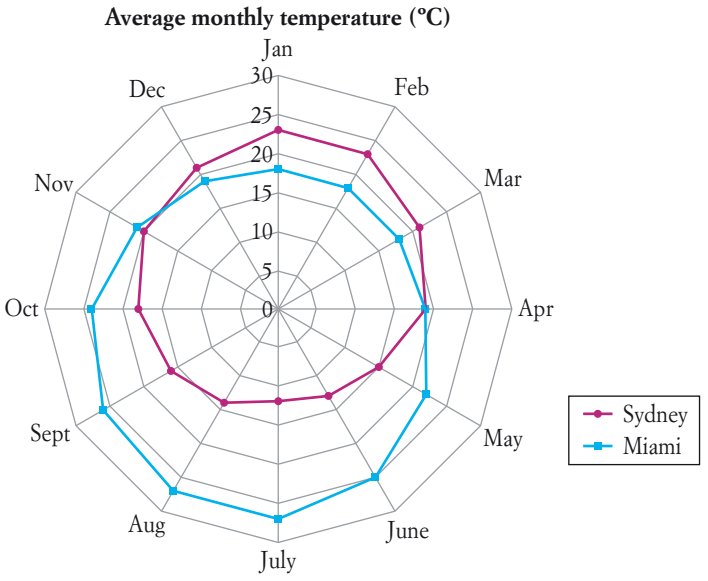
2 This radar chart compares average monthly temperatures (in °C) in Sydney and Miami (USA) over a year.



a Which of the following was the average temperature in Sydney for May? Select **A**, **B**, **C** or **D**.

A 10°C **B** 15°C **C** 17°C **D** 22°C

b In which month was the average temperature 19°C in Miami?

c What was the coldest month in Sydney?

d Which city has the warmer weather overall?

e What is the difference in average temperatures between Sydney and Miami in July?

3 Chrystal was admitted to hospital and her pulse rate was monitored every 3 hours. The results are shown in the table.

Example 18

Time	0600 hours	0900 hours	1200 hours	1500 hours	1800 hours	2100 hours	0000 hours	0300 hours
Pulse rate (beats/min)	65	73	75	70	68	79	72	60

a Plot a radar chart of this data.

b What was Chrystal's lowest pulse rate and when did this occur?

c Why do you think Chrystal's pulse was lowest at this time?

d Between which two times did her pulse increase the most?

e Is it possible to describe what happened to Chrystal's pulse between readings? Give reasons for your answer.

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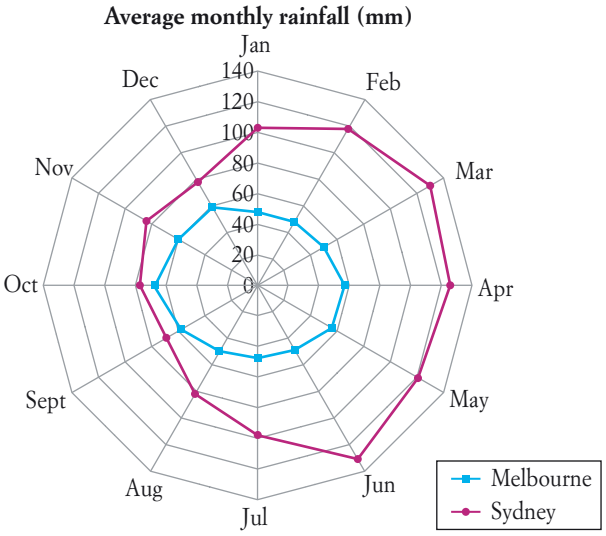
4 This radar chart shows the average monthly rainfall (in millimetres) for Sydney and Melbourne.



a What was the lowest average rainfall for Sydney? When did this happen?

b What was the highest average rainfall for Melbourne? When did this happen?

c Which city has the higher average rainfall?

d Which city's rainfall is more consistent throughout the year?

e Which month showed the greatest difference in rainfall between the two cities?

5 Bill and Ben sell flowerpots to various nurseries. The average number of pots sold by each person over a 12-month period are shown below.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
Bill	1400	1750	2100	2850	4450	4650	4250	3600	4050	2700	1800	1450
Ben	750	1300	1600	2150	3050	5400	4600	2800	3700	1900	2450	1750

a Represent the data on a radar chart.

b In how many months do Bill's sales exceed 3000?

c What is Ben's best month for sales?

d What is the greatest difference in sales between Bill and Ben? When does this difference occur?

e Who is the better salesman? Why?

6 We have examined nine types of statistical displays in this chapter:

column graph	sector graph	divided bar graph
line graph	frequency histogram	frequency polygon
dot plot	stem-and-leaf plot	radar chart

a Which two displays are best for:

i showing parts of a whole, especially for categorical data?

ii displaying small sets of data?

iii displaying large sets of data grouped into class intervals?

iv showing changes over a time period?

b Which display lists the actual data values rather than graphs them?

c Which display is used to show changes occurring over a cycle?

d Which four displays have both horizontal and vertical axes?

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TECHNOLOGY: RADAR CHARTS

We can use the **Chart Wizard** in a spreadsheet program to plot a radar chart.

Step 1: Enter the patient fluid intake and output data from Example **18** into a spreadsheet as shown below. To display 0700 as exactly 0700 and not have the spreadsheet convert it to 700, enter it as '0700 (with a quote mark at the start).

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	**Time**	0700	0900	1100	1300	1500	1700	1900	2100	2300	0100	0300	0500
2	**Intake (mL)**	280	620	50	210	620	150	300	300	170	430	560	310
3	**Output (mL)**	650	200	100	480	270	50	110	540	90	320	180	290

Step 2: Select A2:M3 (the rows of cells showing the Intake and Output) as the '*y*-values'.

Step 3: Open the **Chart Wizard** using either the icon from the toolbar or **Chart** from the **Insert** menu.

Step 4: In the Chart Type dialog box: Select **Radar** and click **Next**.

Step 5: In the Chart Source Data dialog box:

- use the Data Range tab. Select 'Series in Rows' because the data are in rows in the table.
- use the Series tab. For 'Category (X) axis labels', select B1:M1, the Time values in the first row, as the *x*-values and click **Next**.

Step 6: In the Chart Options dialog box:

- use the Titles tab. Under Chart title, enter the title 'Mr Harte's fluid chart'.
- use the Gridlines tab. Tick both **Major gridlines** and **Minor gridlines** to display the extra lines on the chart.
- use the Legend tab. Keep the tick for **Show legend** so that the Intake/Output key is displayed.

Click **Next**.

Step 7: In the Chart Location dialog box: Click **Finish** to make the graph appear on your spreadsheet. (You can cut and paste the chart into another document later.)

Step 8: Format any part of your graph by clicking on it, and resize the graph by dragging the corners of the chart area until you have the size and features you like.

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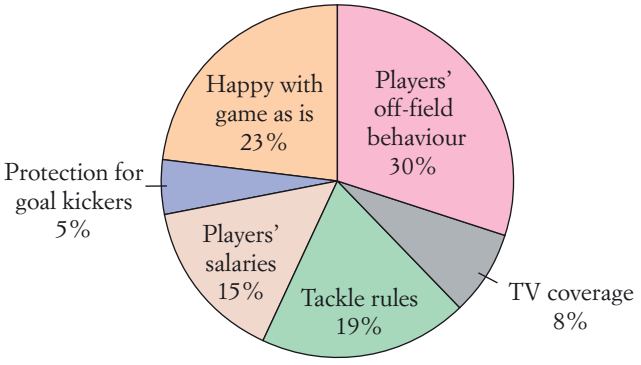
Sample HSC problem

A sample of 3000 Rugby League fans was surveyed on their main concern about the current state of the national competition. The results of the survey are shown by the sector graph below.

Fans concerns about Rugby League



a Which one of the following statements is correct? Select **A**, **B**, **C** or **D**.

- **A** The data shown in the sector graph is categorical.
- **B** More than one-third of fans were concerned about players' off-field behaviour.
- **C** The data shown in the sector graph could also be displayed on a radar chart.
- **D** The least common concern of fans was television coverage.

b Describe a method of selecting a stratified sample of Rugby League fans.

c Calculate, correct to the nearest degree, the angle size of the sector for 'Happy with the game as it is'. Show your working.

d Which concern was shared by 450 fans?

e Write one disadvantage of using a sector graph to illustrate data.

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Study tip

PRACtical steps in studying maths

'A journey of a thousand miles begins with a single step.'

Lao Tzu (c. 570–490 BCE), ancient Chinese philosopher

*Step 1: **P**ractise your maths*

- Do your homework.
- Learning maths is about mastering a collection of skills.
- You become successful at maths by doing it more, through regular practice and training.
- Aim to achieve a high level of understanding.

*Step 2: **R**ewrite your maths*

- Homework and study are not the same thing.
- Study is your private 'revision' work for strengthening your understanding of a subject.
- Take ownership of your maths.
- Rewrite the theory and examples in your own words.
- Summarise each topic to see the 'whole picture' and know it all.

*Step 3: **A**ttack your maths*

- All maths knowledge is interconnected.
- If you don't understand one topic fully, then you may have trouble learning another topic: 'You cannot run until you have learned to walk'.
- Mathematics is not a subject you can learn 'by halves' – you have to know it all!
- Fill in any gaps in your mathematical knowledge to see the 'whole picture'.
- Identify your areas of weakness and work on them.
- Spend most of your study time on the topics with which you have difficulty.

*Step 4: **C**heck your maths*

- Once you have mastered the maths skills, there is no further learning or reading needed.
- Compared to other subjects, the types of questions asked in maths exams are more conventional and predictable.
- Test your understanding with revision exercises, practice papers and past exams.
- Develop your exam technique and problem-solving skills.
- Go back to Steps 1 to 3 to improve your study.

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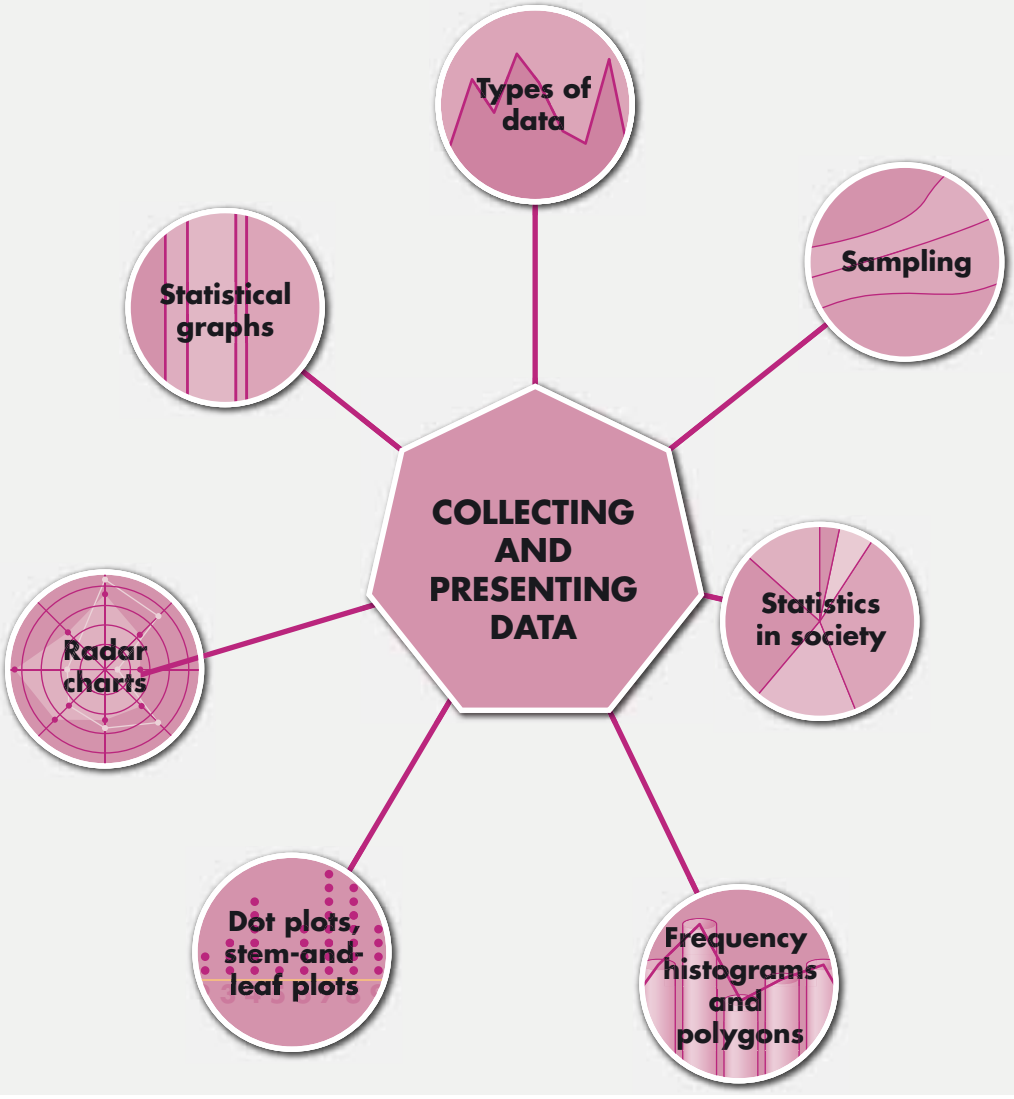
CHAPTER REVIEW

Topic overview

This chapter, 'Collecting and presenting data', looked at the graphing of data, the role of statistics in society, sampling techniques and the methods of data collection. You should be able to interpret and construct various graphs and displays, know the different types of data and samples, understand the stages of a statistical investigation and be familiar with statistical terminology.

Make a summary of this topic. Use the chapter outline at the beginning of this chapter and the mind map below as a guide. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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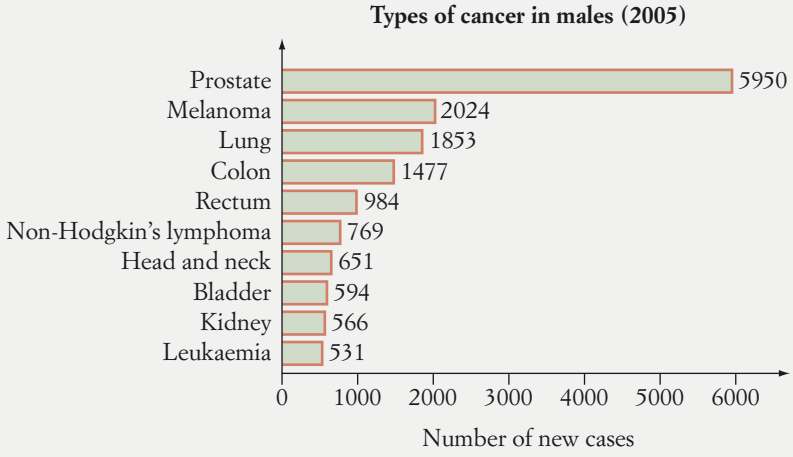
Revision

Exercise 1-01

1 This bar graph shows the ten most common types of cancer for males in 2005 as registered by the Cancer Council of NSW.



a How many cases of colon cancer were registered?

b What was the fifth most common type of cancer?

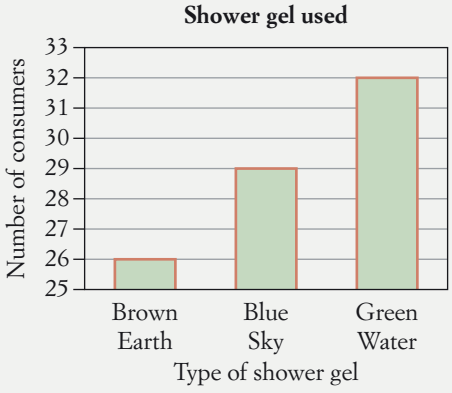
c How many types of cancer recorded fewer than 1000 cases each?

d 'There were more prostate cancers registered than the next three types of cancers put together.' True or false?

e Why would it be unsuitable to graph this data on a sector graph?

Exercise 1-02

2 A sample of consumers in a supermarket was surveyed about the brand of shower gel they used. This graph of the survey results was used by Green Water in their advertising campaign.



a Describe one way in which this graph is misleading and what a correct graph should look like.

b Name another type of graph that could be used to display this data.

Exercise 1-03

3 Classify each of the following types of data as categorical (C) or quantitative (Q).

a the number of people watching a State of Origin football match

b the different brands of surfing magazines

c level of University degree

d the temperatures at an airport

e the number of levels in an office building

f the country of birth of each person living in Gosford

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Exercise 1-03

4 For each of the categorical data identified in question **3**, classify as nominal (N) or ordinal (O).

Exercise 1-03

5 For each of the quantitative data identified in question **3**, classify as discrete (D) or continuous (C).

Exercise 1-04

6 Suppose you need to investigate the best types of music to play at your next school disco. Describe what might happen at each of the six stages of statistical investigation.

Exercise 1-04

7 At what stage of a statistical investigation does a researcher calculate statistics or use graphs to look for a pattern in the data?

Exercise 1-05

8 The manufacturers of Burp Cola want to conduct a quality control study to ensure that each bottle of drink made does, in fact, contain 1.25 litres. Which type of sample would be most appropriate: random, systematic or stratified? Give a reason for your answer.

Exercise 1-05

9 **a** To investigate the types of music to play at the school disco, Tasha surveyed a sample of 50 students visiting the canteen. Why might this sample be biased?

b Suppose Tasha uses a stratified sample instead, based on the following numbers of school students in each Year group.

Year	7	8	9	10	11	12
Number of students	114	120	114	128	105	96

If she surveys a sample of 50 students, how many Year 10 students should be in the sample?

Exercise 1-06

10 Explain what is wrong with each of the following survey questions and write a better question each time.

a How often do you visit the doctor?

b What did you like or dislike about the film?

c Is it time that Australia grew up and became a republic?

Exercise 1-06

11 A sample of senior citizens was surveyed about the number of countries they have visited. The results are shown in the table on the right.

Number of countries	Number of people
0	3
1–3	8
4–10	32
11–20	28
21–30	17
31+	5

a How many people were in the sample?

b Graph this data on a sector graph.

c What is the angle of the sector representing those who have not travelled outside Australia?

d What percentage of the sample have visited 4 to 10 countries? Answer correct to one decimal place.

e 'More than half of the sample have visited 11 or more countries.' True or false?

f Why would it be inappropriate to graph this data on a line graph?

Exercise 1-07

12 The masses (in kilograms) of 40 skydivers were recorded.

58	63	77	82	53	69	65	80	96	105
79	63	52	90	104	85	65	87	68	105
65	87	109	84	62	75	102	78	93	84
68	105	74	59	68	74	88	66	70	62

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a Are people's masses discrete data or continuous data?

b Organise this data into a frequency distribution table with class intervals 50–< 60, 60–< 70, etc.

c Draw a frequency histogram and polygon to represent the data.

d Which class interval had the highest frequency?

e What fraction of skydivers were in the 80–< 90 kg class?

13 The ages of a sample of children at a Wiggles concert are listed below.

Exercise 1-08

6	0	4	1	5	6	4	2	8	6	4	4
3	6	5	5	2	5	3	1	5	6	3	3

a Draw a dot plot for this data.

b How many children were in the sample?

c What fraction of children were over 5 years old?

d Comment on any clusters or outliers.

14 This stem-and-leaf plot shows the ages of visitors entering the Royal Easter Show in a 5-minute period.

Exercise 1-08

Stem	Leaf
0	3 8 9
1	0 2 2 2 5 7 6 9
2	0 2 3 4 6 7
3	1 3 3 4 9
4	3 4 7 8
5	5 5 8

a What was the age of the youngest visitor?

b How many visitors entered the show during the 5-minute period?

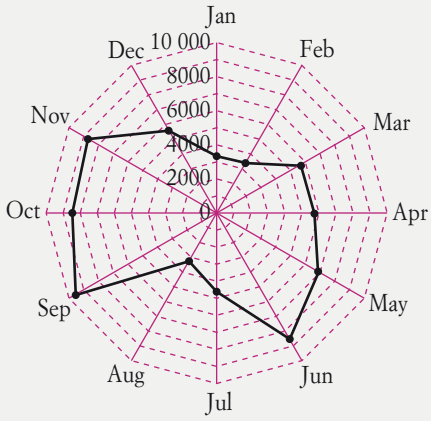
c What was the most common age of the visitors?

d What percentage (correct to the nearest whole number) of visitors were over 30?

15 The monthly earnings of a scaffolding contractor are shown in this radar chart.

Exercise 1-09

Earnings of scaffolding contractor ($)



a Which month showed the second-highest earnings?

b Estimate the contractor's earnings in February.

c In which month did the contractor earn $6800?

d In which month did the contractor's earnings decrease the most from the previous month?

e In how many months did the contractor earn more than $8000?

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ALGEBRA AND MODELLING

2.

ALGEBRA AND EQUATIONS

In 1905, when he was aged only 26, German scientist Albert Einstein proposed a new theory of physics for small particles of matter (such as atoms) moving at very high speeds. He proposed that matter (mass) could be converted to large amounts of energy describing this in his Theory of Special Relativity with the algebraic formula: $E = mc^2$, where E stands for energy, m stands for mass and c stands for the speed of light (300 000 000 m/s). Einstein's theory revolutionised conventional laws of physics and led to the development of nuclear energy.

Algebra is the branch of mathematics that uses symbols and formulas to describe number patterns and relationships occurring in nature, industry and society.

CHAPTER OUTLINE

*Year 12 content

AM1	2-01	Linear number patterns
AM1	2-02	Simplifying algebraic expressions
AM1	2-03	Expanding algebraic expressions
AM1	2-04	Formulas
AM1	2-05	Solving equations
AM3*	2-06	Formulas and equations

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IN THIS CHAPTER YOU WILL:

- add, subtract, multiply and divide algebraic terms
- simplify algebraic expressions involving multiplication and division, for example, $\frac{9y}{4} \times 5y$, $\frac{4m}{5n} \div \frac{m}{20n}$, $\frac{4wn}{b} \times \frac{nb}{2w}$
- expand and simplify algebraic expressions
- substitute numerical values into algebraic expressions
- substitute given values for the other pronumerals in a mathematical formula to find the value of the subject of the formula
- solve linear equations

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TERMINOLOGY

base	equation	evaluate
expand	expression	formula
LHS	like terms	linear
power	RHS	simplify
solve	subject	substitute
term	variable	

Assignment 2

SkillCheck

1 Simplify:

a $\frac{21}{27}$ **b** $\frac{14}{20}$ **c** $\frac{8}{12}$

2 Evaluate without using a calculator:

a $9 \times (-3)$ **b** $9 + (-3)$ **c** $2 - 10$

d $(-5)^2$ **e** $-24 \div 6$ **f** $-2 - (-4)$

3 If $d = 3$, evaluate:

a $8d - 9$ **b** $\frac{d+6}{3}$ **c** $7 + 3d$

d $\sqrt{12d}$ **e** d^5 **f** $2d^2$

4 If $u = -2$, evaluate:

a $u^2 - 4$ **b** $3(u + 1)$ **c** $8 - u$

d u^7 **e** $\frac{-18}{u}$ **f** $5 - 2u$

5 Simplify:

a $4x + 7 - 3x - 10$ **b** $18 - 2a + a + 2$ **c** $5r + 12 - 15 - 5r$

6 Expand:

a $3(2x - 1)$ **b** $7(4a + 5)$ **c** $-2(6p - 2)$

7 Solve:

a $3b = -15$ **b** $w - 8 = 9$ **c** $2r + 4 = 22$

8 If $A = \frac{1}{2}(a+b)h$ and $a = 10$, $b = 15$ and $h = 3$, evaluate A.

Finding the rule

2-01 Linear number patterns

A **linear number pattern** is a list of numbers that increases or decreases by the same amount for each new term.

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EXAMPLE 1

3, 7, 11, 15, … is a linear number pattern.

a What is the pattern for this list of numbers?

b Write the next three terms in the pattern.

c Find a formula for the value of the xth term of the pattern.

d Use the formula to find the 15th term of the pattern.

Solution

a The terms are increasing by 4 each time.

b 19, 23, 27

c Complete a table of values, in which x is the position of the term in the pattern and y is the value of the term.

x	1	2	3	4
y	3	7	11	15

+4 +4 +4

Note that the y values are increasing by 4, so the formula to find the value of y is of the form:

$y = 4x$ ____

When $x = 1$: $y = 4 \times 1 - \mathbf{1} = 3$

After multiplying by 4, we need to subtract 1 each time to fit the pattern.

When $x = 2$: $y = 4 \times 2 - \mathbf{1} = 7$

When $x = 3$: $y = 4 \times 3 - \mathbf{1} = 11$

So the formula for the value of the xth term of the pattern is: $y = 4x - 1$

d Using the formula from part **c**, when $x = 15$:

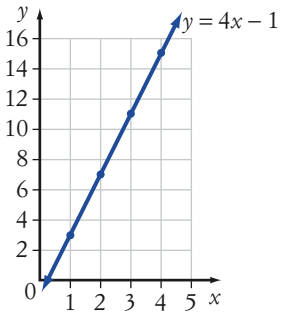
$y = 4 \times 15 - \mathbf{1}$

$= 59$

The 15th term of the pattern is 59.

Check that this is true by extending the pattern: 3, 7, 11, 15, …

If a **linear number pattern** is graphed on a number plane, the graph is a straight line. That is why the pattern is called 'linear'. The graph on the right illustrates the number pattern in Example **1**, using the values from the table in the example.



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EXAMPLE 2

a Which of the following number patterns is linear?

A 2, 4, 8, 16, … **B** 10, 8, 6, 4, … **C** 5, 8, 12, 17, …

b Write the next three terms of the linear pattern.

c Find a formula for the general term of the linear pattern.

d Graph the linear pattern on a number plane.

e Find the 9th term of the linear pattern.

Solution

a Pattern **A** is not increasing by the same amount each time. The terms are being doubled each time.

Pattern **B** is decreasing by 2 each time.

Pattern **C** is not increasing by the same amount each time. It increases by 3, then by 4, then by 5.

So Pattern **B** is a linear pattern.

b 2, 0, –2.

c

x	1	2	3	4
y	10	8	6	4

–2 –2 –2

The y values decreasing by 2, so the formula is of the form:

$y = -2x$ ____

When $x = 1$: $y = -2 \times 1 + 12 = 10$

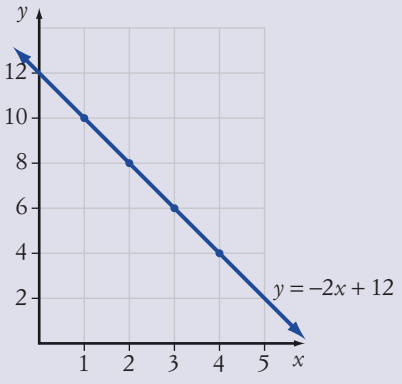
When $x = 2$: $y = -2 \times 2 + 12 = 8$

When $x = 3$: $y = -2 \times 3 + 12 = 6$

After multiplying by –2, we need to add 12 each time to fit the pattern.

So the formula is $y = -2x + 12$.

d



e Using the formula from part **c**, when $x = 9$:

$y = -2 \times 9 + 12$

$= -6$

The 9th term of the pattern is –6.

Check that this is true by extending the pattern: 10, 8, 6, 4, …

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Exercise 2-01 Linear number patterns

1 Which of the following number patterns are linear?

A 5, –2, –9, –16, … **B** 4, 8, 16, 32, … **C** 1, 2, 3, 5, …

D 6, 10, 14, 18, … **E** 3, 13, 23, 33, … **F** 24, 12, 6, 3, …

G 20, 19, 18, 17, … **H** 7, 11, 15, 19, … **I** –3, –0.5, 2, 4.5, …

2 Which of the following is the correct formula for this table of values? Select **A**, **B**, **C** or **D**.

x	1	2	3	4
y	–1	6	13	20

A $y = 5x - 4$ **B** $y = 5x - 6$ **C** $y = 7x - 1$ **D** $y = 7x - 8$

3 Find the formula for each of these tables of values.

a

x	1	2	3	4
y	1	4	7	10

b

x	1	2	3	4
y	7	12	17	22

c

x	1	2	3	4
y	11	9	7	5

d

x	1	2	3	4
y	21	15	9	3

e

x	1	2	3	4
y	9	10	11	12

f

x	1	2	3	4
y	4	8	12	16

g

x	1	2	3	4
y	7	4	1	–2

h

x	1	2	3	4
y	2.5	3	3.5	4

Example 1

4 **a** Explain why 6, 11, 16, 21, … is a linear number pattern.
b Write the next three terms in the pattern.
c Find a formula for the xth term of the pattern.
d Use the formula to find the 20th term of the pattern.

Example 2

5 **a** Write the next three terms in the linear pattern 18, 15, 12, 9, …
b Find a formula for the general term of the pattern.
c Graph the formula on a number plane.
d Find the 11th term of the pattern.

6 **a** Find a formula for the general term of the pattern 11, 22, 33, 44, …
b Find the 16th term of the pattern.

7 **a** Find a formula for the xth term of the pattern 8, 14, 20, 26, …
b Graph the formula on a number plane.
c Find the 100th term of the pattern.

8 **a** Find a formula for the xth term of the pattern 33, 27, 21, 15, …
b Find the 12th term of the pattern.

9 **a** Find a formula for the general term of the pattern $3, 4\frac{1}{2}, 6, 7\frac{1}{2}, \ldots$
b Graph the formula on a number plane.
c Find the 7th term of the pattern.
d Which term of the pattern is 30?

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10 a Find a formula for the xth term of the pattern 12, 11, 10, 9, …

b Find the 15th term of the pattern.

c Which term of the pattern is 0?

JUST FOR THE RECORD

Al Gebra is Arabic

In 825 CE, the Arabic mathematician **al-Khwarizmi** wrote a book called *Hisab al-jabr w'al-muqabala* (or *The Science of Equations*). The Arabic word '*al-jabr*' meant the process of adding the same amount to both sides of an equation but, when it was translated into Latin, it was changed to 'algebra' and became the name of a whole branch of mathematics, not just equation-solving.

From al-Khwarizmi's name, we get the word *algorithm*. What is the meaning of 'algorithm'?

Collecting like terms

Algebra using diagrams

2-02 Simplifying algebraic expressions

An **algebraic expression** is made up of terms involving variables and numerals. For example $3x^2 - x + 10$ has three terms: $3x^2$, $-x$ and 10.

SUMMARY

'Like terms' have exactly the same variables.

Only 'like terms' can be added or subtracted.

EXAMPLE 3

Simplify:

a $2a^2 - a + 5 + 8a$

b $4kr - 6pr - pr + 10kr$

c $2xy + 4 - y + 4yx$

Solution

a $2a^2 - a + 5 + 8a = 2a^2 - a + 8a + 5$ — **$-a$ and $8a$ are like terms.**

$= 2a^2 + 7a + 5$ — **$2a^2$ and $-a$ are not like terms.**

b $4kr - 6pr - pr + 10kr = 4kr + 10kr - 6pr - pr$ — **Grouping pairs of like terms.**

$= 14kr - 7pr$

c $2xy + 4 - y + 4yx = 2xy + 4yx - y + 4$ — **$2xy$ and $4yx$ are like terms.**

$= 6xy - y + 4$

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SUMMARY

When **multiplying terms**, multiply the numbers and variables separately.
When **multiplying terms with powers** that have the same base, add the powers.

EXAMPLE 4

Simplify:

a $-3bc \times 8ab$ **b** $4p^2q \times \frac{3p^3}{2}$ **c** $(-5r)^2$

Solution

a $-3bc \times 8ab = (-3 \times 8) \times (a \times b \times b \times c)$
$= -24ab^2c$

First multiply the numbers. Then multiply the variables in alphabetical order.

b $4p^2q \times \frac{3p^3}{2} = \frac{4 \times 3}{2} \times p^2 \times p^3 \times q$
$= 6p^5q$

$p^2 \times p^3 = p^5$
Add the powers.

c $(-5r)^2 = (-5r) \times (-5r)$
$= -5 \times (-5) \times r \times r$
$= 25r^2$

SUMMARY

When **dividing terms**, divide the numbers and variables separately.
When **dividing terms with powers** that have the same base, subtract the powers.

EXAMPLE 5

Simplify:

a $\frac{8m^3n}{2m^2}$

b $-15w \div 3w^2$

c $\frac{6ak^3}{20ak}$

Solution

a $\frac{8m^3n}{2m^2} = \frac{8}{2} \times \frac{m^3n}{m^2}$
$= 4mn$

$\frac{m^3}{m^2} = m^1 = m$.
Subtract the powers.

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b $-15w \div 3w^2 = \dfrac{-15w}{3w^2}$

$= \dfrac{-15}{3} \times \dfrac{w}{w^2}$

$= -5 \times \dfrac{1}{w}$

$= \dfrac{-5}{w}$

$\dfrac{w}{w^2} = \dfrac{\overset{1}{\cancel{w}}}{w\cancel{w}} = \dfrac{1}{w}$

c $\dfrac{6ak^3}{20ak} = \dfrac{6}{20} \times \dfrac{ak^3}{ak}$

$= \dfrac{3}{10} \times k^2$

$= \dfrac{3k^2}{10}$

EXAMPLE 6

Simplify each expression.

a $\dfrac{4a}{5} \times \dfrac{5h}{12a}$

b $\dfrac{3m}{10n} \div \dfrac{6m}{25n}$

Solution

a $\dfrac{4a}{5} \times \dfrac{5h}{12a} = \dfrac{\overset{1}{\cancel{4}}\,\cancel{a}}{\cancel{5}} \times \dfrac{\cancel{5}h}{\underset{3}{\cancel{12}}\,\cancel{a}}$

$= \dfrac{1 \times h}{3}$

$= \dfrac{h}{3}$

Each fraction cannot be simplified on its own, so simplify the numerator of each fraction with the denominator of the other fraction.

$\dfrac{4}{12} = \dfrac{1}{3}, \ \dfrac{a}{a} = 1, \ \dfrac{5}{5} = 1$

b $\dfrac{3m}{10n} \div \dfrac{6m}{25n} = \dfrac{3m}{10n} \times \dfrac{25n}{6m}$

When dividing by a fraction, multiply by its reciprocal.

$= \dfrac{\overset{1}{\cancel{3}}\,\cancel{m}}{\underset{2}{\cancel{10}}\,\cancel{n}} \times \dfrac{\overset{5}{\cancel{25}}\,\cancel{n}}{\underset{2}{\cancel{6}}\,\cancel{m}}$

$= \dfrac{1 \times 5}{2 \times 2}$

$= \dfrac{5}{4}$

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Exercise 2-02 Simplifying algebraic expressions

Example 3

1 Simplify:

a $2x^2 + 3x + x^2 + 2x$ **b** $2ab + a + 5ab + b$ **c** $10 - 4m^2 + 6m^2 - 5$

d $3r + 4ar - 2ar - 7r$ **e** $y^2 - y + 3y - 5$ **f** $2a^2 - 4 - a^2 + 6$

g $k^2 - k - 3k - 1$ **h** $3de - 6ed + 2d$ **i** $p^2 - p - p^2 - 5p$

j $-5 + 7u - 10 - 3u$ **k** $t^2 + at + t^2 + 2at$ **l** $15x - 5 + 3x + 5$

Example 4

2 Simplify:

a $4ak \times 4am$ **b** $-3d \times 8cd$ **c** $\frac{1}{4}t^2w \times 10tw$

d $(5x^3)^2$ **e** $9n^2 \times \frac{2n^2}{3}$ **f** $7k^3p \times 2k^2p$

g $-6m \times (-6n)$ **h** $5e \times \frac{3e^2}{10}$ **i** $(2xy^2)^2$

j $10ab \times (-2ab)$ **k** $-4n^3 \times \frac{1}{4}n^3$ **l** $(-3k)^2$

Example 5

3 Simplify:

a $\frac{4m}{2m}$ **b** $\frac{4x^2y}{2xy}$ **c** $-15e^2 \div 3e^2$

d $10u^2t^3 \div 5ut^2$ **e** $\frac{3x}{x^3}$ **f** $\frac{27pr}{-9pq}$

g $\frac{12g^2h}{20gh}$ **h** $a^2 \div a^3$ **i** $\frac{15m^4n}{9m^2n^2}$

j $\frac{-4de}{20e}$ **k** $u \div 4u$ **l** $6wx^2 \div 24w^2$

4 Which of the following is the simplifed expression for $\frac{2xy}{5} \times \frac{-3}{10y}$? Select **A**, **B**, **C** or **D**.

A $-\frac{3x}{25}$ **B** $-\frac{3x}{25y^2}$ **C** $-\frac{6x}{50}$ **D** $-\frac{3}{25y}$

Example 6

5 Simplify each expression.

a $\frac{9y}{4} \times 5y$ **b** $\frac{4a}{5} \times \frac{2h}{7}$ **c** $\frac{x}{3} \div \frac{x}{6}$

d $\frac{20r}{6} \div 4r$ **e** $\frac{4nw}{b} \times \frac{bn}{2w}$ **f** $\frac{3m}{6m} \times \frac{10n^2}{4n}$

g $\frac{4a}{5} \div \frac{2h}{7}$ **h** $\frac{2e}{5} \div \frac{3e}{8}$ **i** $\frac{2de}{7p} \times \frac{14p}{4e}$

6 Simplify:

a $2k + 8 - k^2 - 4k$ **b** $ay^2 \times \frac{1}{2}y^2$ **c** $-5t \div (-20t)$

d $\frac{-32f^3g^2}{4f^2g}$ **e** $-3c^2 + 7c - 7c^2 + 6c$ **f** $4z + z + 8 - 4z^2$

g $-3a \times (-2a^2)$ **h** $\frac{24cd}{8de}$ **i** $10 - 4m^2 + 6m^2 - 5$

j $\frac{20n^3x}{10nx^2}$ **k** $4x^2 - 2xy + 3x + 4xy$ **l** $6a^4m^2 \div (-2a^2m^2)$

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INVESTIGATION

NUMBER PATTERNS

10	11	12
17	18	19
24	25	26

1. Choose a month from any calendar and draw a box around any block of nine numbers. See the boxed numbers on the right for example.
2. Add together the numbers in any line of three numbers going through the centre of the box (row, column, or diagonal). For example 10 + 18 + 26.
3. Add another line of three numbers that goes through the centre of the box.
4. There are two more lines that go through the centre of the box. Find their sums as well.
5. What do you notice? Why does this pattern work? *Hint*: Let the number in the top left-hand corner be x.

WS Expanding algebra

2-03 Expanding algebraic expressions

Expanding an expression means removing grouping symbols (brackets) by multiplying each term inside the grouping symbols by the term outside the grouping symbols.

SUMMARY

$$a(b + c) = a \times (b + c) = ab + ac$$

EXAMPLE 7

Expand each of these expressions.

a $5(2t - 3)$ **b** $2k(7 + 3k)$ **c** $-3(d^2 - 3d + 8)$

Solution

a $5(2t - 3) = 5 \times 2t - 5 \times 3$

$= 10t - 15$

Multiply each term inside the grouping symbols by the 5 (outside the grouping symbols).

b $2k(7 + 3k) = 2k \times 7 + 2k \times 3k$

$= 14k + 6k^2$

c $-3(d^2 - 3d + 8) = -3 \times d^2 + (-3) \times (-3d) + (-3) \times 8$

$= -3d^2 + 9d - 24$

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EXAMPLE 8

Expand and simplify:

a $x(x-1)+4(x+1)$ **b** $2m(m-3)-(m-6)$ **c** $5u(2-y)+u(2y-9)$

Solution

a $x(x-1)+4(x+1) = x^2 - x + 4x + 4$

$= x^2 + 3x + 4$ **Simplifying by collecting the like terms –x and 4x.**

b $2m(m-3)-(m-6) = 2m^2 - 6m - m + 6$ **–(m – 6) means –1(m – 6)**

$= 2m^2 - 7m + 6$

c $5u(2-y)+u(2y-9) = 10u - 5uy + 2uy - 9u$

$= u - 3uy$

Exercise 2-03 Expanding algebraic expressions

Example 7

1 Expand each of the following expressions.

a $3(a+2)$ **b** $5(3-2b)$ **c** $-2(2a+1)$

d $-6(b-2)$ **e** $3x(x-2)$ **f** $3p(p-a)$

g $-4(2k+4)$ **h** $2t(3-4t)$ **i** $-d(d-5)$

j $k(7-5k)$ **k** $-9b(b-1)$ **l** $2y(7x+4y)$

2 Expand each of the following expressions.

a $-6n(4-n)$ **b** $5x(rx+2r)$ **c** $-(2a^2-4)$

d $5b(a^2+3b-7)$ **e** $-(x^2-4x+10)$ **f** $3h(h-7e-4eh)$

g $y(2y+3-y^2)$ **h** $de(d^2-2+e^2)$ **i** $-3v(-3av+v-2a)$

Example 8

3 Expand and simplify each of the following expressions.

a $5(x+4)-2(x+3)$ **b** $3(d-4)-2(d+5)$ **c** $6(r+10)-4(r-5)$

d $8(f+2)-(f+7)$ **e** $3(2x-4)-5(3x+4)$ **f** $6x(x+4)-3x(x-1)$

g $3b(b+5)-b(b-8)$ **h** $4w(w-7)-w(w+1)$ **i** $6(k+p)+3(k+2p)$

j $2(a-b)+2(b+a)$ **k** $x(2v+4)-x(v+1)$ **l** $-3(t+w)-2(2t-w)$

m $e(3e+5)-(2e-e^2)$ **n** $-2(a+3)+4(a-3)$ **o** $p(p-q)-q(q-p)$

4 **a** Evaluate 7 – 5 and 5 – 7. How are the two answers related?

b Evaluate 4 – 10 and 10 – 4.

c Is $a-b$ always the same as $-(b-a)$? Can you prove it algebraically?

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INVESTIGATION

MENTAL MULTIPLICATION BY EXPANDING

Expanding is useful for multiplying numbers mentally without using a calculator, especially if one of the numbers is close to 10, 100 or 1000.

1 Study the following examples.

a $35 \times 11 = 35 \times (10 + 1)$ **Think of 11 as 10 + 1**
$= 35 \times 10 + 35 \times 1$ **Expand**
$= 350 + 35$ **Simplify**
$= 385$

b $43 \times 102 = 43 \times (100 + 2)$ **Think of 102 as 100 + 2**
$= 43 \times 100 + 43 \times 2$ **Expand**
$= 4300 + 86$ **Simplify**
$= 4386$

c $16 \times 8 = 16 \times (10 - 2)$ **Think of 8 as 10 – 2**
$= 16 \times 10 - 16 \times 2$ **Expand**
$= 160 - 32$ **Simplify**
$= 128$

2 Use expanding to multiply these numbers mentally.

a 25×12 **b** 18×9 **c** 6×105
d 87×11 **e** 50×99 **f** 45×8

2-04 Formulas

WS Substitution code puzzle

EXAMPLE 9

If $x = 4$, $y = 7$ and $z = -1$, then evaluate each expression.

a $5x^2 + 2z$ **b** $\dfrac{z-8}{10-y}$ **c** $6(yz + 5)$ **d** $\sqrt{3y - x - 1}$

Solution

a $5x^2 + 2z = 5 \times 4^2 + 2 \times (-1)$
$= 78$

b $\dfrac{z-8}{10-y} = \dfrac{-1-8}{10-7}$
$= \dfrac{-9}{3}$
$= -3$

c $6(yz + 5) = 6[7 \times (-1) + 5]$
$= 6 \times (-2)$
$= -12$

d $\sqrt{3y - x - 1} = \sqrt{3 \times 7 - 4 - 1}$
$= \sqrt{16}$
$= 4$

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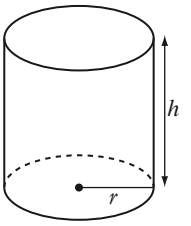
A **formula** is an algebraic rule that describes a mathematical relationship between variables. For example, the volume of a cylinder has the formula $V = \pi r^2 h$, where r is the radius of the cylinder's base and h is its perpendicular height.



Because the formula describes V, with V on the left-hand side of the '=' sign, we say that V is the **subject** of the formula.

EXAMPLE 10

If a principal, \$$P$, is invested at an interest rate of r per annum (where r is written as a decimal) then, after n years, it will grow to \$$A$, where A is given by the compound interest formula $A = P(1 + r)^n$. Use the formula to calculate the final amount of an investment of \$4000 after 6 years if invested at 11% p.a. Express your answer rounded to the nearest cent.

Solution

$P = 4000$, $r = 11\% = 0.11$, $n = 6$

$$\begin{aligned} A &= P(1 + r)^n \\ &= 4000(1 + 0.11)^6 \\ &= 4000(1.11)^6 \\ &= 7481.658\,209 \ldots \\ &\approx \$7481.66 \end{aligned}$$

The final amount is \$7481.66.

EXAMPLE 11

From a height of h metres above sea level, an observer can see a distance of d km to the horizon, where $d = 8\sqrt{\frac{h}{5}}$. What distance, correct to the nearest kilometre, can be seen from the top of Sydney Harbour Bridge, 134 m above sea level?

Solution

$h = 134$

$$\begin{aligned} d &= 8\sqrt{\frac{h}{5}} \\ &= 8\sqrt{\frac{134}{5}} \\ &= 8\sqrt{26.8} \\ &= 41.\,414\,97 \ldots \\ &\approx 41 \text{ km} \end{aligned}$$

A distance of 41 km can be seen from the top of Sydney Harbour Bridge.

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Exercise 2-04 Formulas

Example 9

1 If $a = -3$, $b = 10$ and $c = 6$, then evaluate each expression.

a $b^2 - a^2$ **b** $\frac{b+c}{ac}$ **c** $\sqrt{c+3b}$ **d** $4(3a + 9)$

e $8c + 4a$ **f** $\frac{2b}{5}$ **g** $b(b - 4)$ **h** $\sqrt{\frac{9b}{c+4}}$

i $c^2 + c$ **j** $\frac{2ac}{3}$ **k** $\sqrt{a^2 + 7c - 2}$ **l** $(a - b)^2$

2 Calculate the volume, correct to two decimal places, of a cylinder with a base radius of 4.07 cm and perpendicular height of 11.58 cm, using the formula $V = \pi r^2 h$.

3 The temperature T (in °C) of the water in a kettle t minutes after it is switched on is given by the formula $T = 18t + 28$. Find the temperature of the water:

a 4 minutes after it is turned on

b $1\frac{1}{2}$ minutes after it is turned on

c when the kettle is first turned on.

4 The formula for converting Australian dollars ($A) to US dollars ($US) is US = 0.835A. Convert the following $A amounts to $US, correct to the nearest cent.

a $20.00 **b** $89.50 **c** $4800

5 The angle sum $A°$ of a polygon with n sides is $A = 180(n - 2)$. Find the angle sum of:

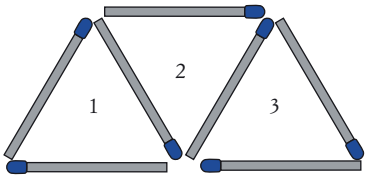
a a quadrilateral **b** a hexagon **c** a decagon.

6 If an object is moving with speed u m/s and acceleration a m/s^2, then its speed v m/s after t seconds is $v = u + at$.

a Which variable is the subject of the formula?

b Calculate the speed of a car after 5 seconds if its speed now is 6 m/s and it is accelerating at 2 m/s^2.

7 The number of matches, m, needed to make this pattern of triangles is $m = 2t + 1$, where t is the number of triangles in the pattern. How many matches are required to make:



a 8 triangles?

b 40 triangles?

c 150 triangles?

8 The volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$. Calculate, correct to one decimal place, the volume of a sphere with radius 14.5 cm.

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9 The time, T seconds, it takes a swing to go back and forth once is $T = 2\pi\sqrt{\frac{l}{g}}$ where l m is the length of the swing and g is the gravitational acceleration. Find T, correct to two decimal places, if $l = 2.35$ and $g = 10$.

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Example 10

10 Use the compound interest formula $A = P(1 + r)^n$ to calculate the amount to which a principal of \$5600 will grow if invested at 9.4% p.a. for 5 years.

11 The maximum distance, d m, a ball travels if thrown with speed v m/s, is $d = \frac{v^2}{g}$. Find correct to one decimal place the maximum distance if a ball is thrown at a speed of 11.5 m/s and $g = 9.8$.

12 The body-mass index (BMI) of an adult is $B = \frac{m}{h^2}$, where m is the mass in kilograms and h is the height in metres. Zoe is 1.7 m tall and weighs 60 kg.

a What is the subject of the formula given above?

b Calculate Zoe's BMI, correct to one decimal place.

c If a BMI between 21 and 25 is an indication of good health, then how can Zoe improve her health?

13 The formula for converting Fahrenheit temperatures (°F) to Celsius (°C) is $C = \frac{5}{9}(F - 32)$.

Convert each of the following temperatures to Celsius, correct to the nearest degree.

a New York 77°F

b Los Angeles 100°F

c Rio de Janeiro 59°F

d normal body temperature 98.4°F

Example 11

14 The formula for converting a speed of k km/h to metres per second (m/s) is $M = \frac{5k}{18}$.

A speed of 80 km/h is closest to which of the following speeds? Select **A**, **B**, **C** or **D**.

A 22 m/s **B** 32 m/s **C** 35 m/s **D** 47 m/s

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15 The distance, d km, an observer can see to the horizon from a height h m above sea level is $d = 8\sqrt{\frac{h}{5}}$. What distance (correct to the nearest kilometre) can a person see from the top of Sydney Tower at 305 m?

16 Liam earns a weekly wage of \$450 plus a commission of 11% of the value of phone cards he sells in excess of \$900. His weekly pay is given by the formula $P = 450 + 0.11(V - 900)$, where V is the total value of the phone cards sold. How much will Liam earn for selling \$2310 worth of phone cards in a week?

17 The braking distance (in metres) of a bicycle travelling at a speed of V m/s is $d = \frac{V(V+1)}{2}$. Calculate the braking distance of a bicycle travelling at a speed of 6 m/s.

18 The surface area of a cylinder of radius r and height h is $S = 2\pi r(r + h)$. Calculate the surface area, correct to two decimal places, of a cylinder with radius 3 cm and height 8 cm.

19 The speed, V m/s, required for a spacecraft to escape the Earth's gravitational pull during take-off is $V = \sqrt{2gr}$ where g is 9.8 m/s^2 and r is the radius of the Earth (6 378 000 m). Which of the following is closest to the escape speed of a spacecraft leaving the Earth's atmosphere? Select **A**, **B**, **C** or **D**.

A 8840 m/s **B** 11 180 m/s **C** 12 500 m/s **D** 35 000 m/s

INVESTIGATION

GETTING THE RIGHT FORMULA

Listed below are 15 commonly-used formulas. As a group or individual activity, select eight formulas and, for each one:

a describe what the formula is used for

b write the subject of the formula, and what it stands for

c describe what the other variables in the formula stand for.

1 $V = \frac{1}{3}Ah$

2 $A = 180(n - 2)$

3 $c^2 = a^2 + b^2$

4 $A = \pi r^2$

5 $S = 2\pi r^2 + 2\pi rh$

6 $I = Prn$

7 $m = \frac{y_2 - y_1}{x_2 - x_1}$

8 $C = 2\pi r = \pi d$

9 $A = \frac{1}{2}xy$

10 $S = \frac{d}{t}$

11 $A = \frac{1}{2}(a + b)h$

12 $A = P(1 + r)^n$

13 $A = s^2$

14 $V = \pi r^2 h$

15 $V = \frac{4}{3}\pi r^3$

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JUST FOR THE RECORD

The body mass index

The **body mass index (BMI)** has been used by the World Health Organisation (WHO) since 1997 as an international standard of health and fitness for adults. The formula for BMI is $B = \frac{m}{h^2}$, where m is a person's mass in kilograms and h is the person's height in metres. This is a convenient measure that represents the health of an adult by a single value, applicable to both males and females, without the need for reading height–weight charts. The table on the right describes a range of BMI scores.

BMI	Health status
Under 18.5	Underweight
18.5 to 24.9	Normal
25.0 to 29.9	Overweight
30 and above	Obese

According to this measure, about 58% of Australian men and 40% of Australian women are overweight or obese, and have a greater risk of heart disease, stroke, diabetes, high blood pressure and high cholesterol.

Because the BMI is an algebraic *model*, the WHO acknowledges that it has some limitations and inaccuracies. It is useful for classifying the general population but does not take into account a person's frame size, muscle mass, bone density or distribution of body fat. For this reason, the BMI should not be used for measuring the health of body builders, athletes, children under 19, pregnant women, the frail and sedentary elderly, and Aboriginal, Pacifc Island and Asian people.

1 **Why is the BMI less accurate for measuring the levels of fitness of the types of people listed above?**

2 **Would each of the types of people listed above score unusually high or unusually low on the BMI scale?**

2-05 Solving equations

An **equation** contains an algebraic expression and an equals (=) sign. For example $3x - 4$ is an **expression**, while $3x - 4 = -13$ is an **equation**. An equation is **solved** when a value of the variable (for example, x) is found that makes the equation true.

Equations code puzzle

Solving equations by balancing

Solving equations by backtracking

Solving equations using diagrams

SUMMARY

To **solve an equation**:

- keep the equation balanced by performing the same operation on both sides
- aim to have the variable (for example x) on one side of the equation and a number on the other side. For example, $x = 4$.

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EXAMPLE 12

Solve each of these equations.

a $3x - 4 = -13$ **b** $\frac{b}{2} + 7 = 1$ **c** $\frac{h-10}{4} = 3$

Solution

a

$$3x - 4 = -13$$

$$3x - 4 + 4 = -13 + 4$$ **Adding 4 to both sides.**

$$3x = -9$$

Remember: We are aiming to get x on its own.

$$\frac{3x}{3} = \frac{-9}{3}$$ **Dividing both sides by 3.**

$$x = -3$$

Check by substituting $x = -3$ back into the original equation.

$$\text{LHS} = 3(-3) - 4$$

LHS means 'left-hand side'.

$$= -13$$

$$= \text{RHS}$$

RHS means 'right-hand side'.

b

$$\frac{b}{2} + 7 = 1$$

$$\frac{b}{2} + 7 - 7 = 1 - 7$$ **Subtracting 7 from both sides.**

$$\frac{b}{2} = -6$$

$$\frac{b}{2} \times 2 = -6 \times 2$$ **Multiplying both sides by 2.**

$$b = -12$$

Check by substituting $b = -12$ back into the original equation.

$$\text{LHS} = \frac{-12}{2} + 7$$

$$= 1$$

$$= \text{RHS}$$

c

$$\frac{h-10}{4} = 3$$

$$\frac{h-10}{4} \times 4 = 3 \times 4$$ **Multiplying both sides by 4.**

$$h - 10 = 12$$

$$h - 10 + 10 = 12 + 10$$ **Adding 10 to both sides.**

$$h = 22$$

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EXAMPLE 13

Solve each of these equations.

These equations require more steps to solve.

a $5y + 5 = 2y + 17$

b $\dfrac{2m-7}{3} = 6$

c $4(1 - 2t) = 16$

Solution

a $5y + 5 = 2y + 17$

Remember: We are aiming to get y on its own.

$5y + 5 - 2y = 2y + 17 - 2y$ **Subtracting 2y from both sides to get all the ys on the LHS.**

$3y + 5 = 17$

$3y + 5 - 5 = 17 - 5$ **Subtracting 5 from both sides to get all the numbers on the RHS.**

$3y = 12$

$\dfrac{3y}{3} = \dfrac{12}{3}$ **Dividing both sides by 3.**

$y = 4$

Checking:

LHS $= 5(4) + 5$

$= 25$

RHS $= 2(4) + 17$

$= 25$

$=$ LHS

b $\dfrac{2m-7}{3} = 6$

$\dfrac{2m-7}{3} \times 3 = 6 \times 3$ **Multiplying both sides by 3.**

$2m - 7 = 18$

$2m = 25$ **Adding 7 to both sides.**

$m = \dfrac{25}{2}$ **Dividing both sides by 2.**

$= 12\dfrac{1}{2}$

Checking:

LHS $= \dfrac{2\left(12\frac{1}{2}\right) - 7}{3}$

$= \dfrac{18}{3}$

$= 6$

$=$ RHS

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c $4(1 - 2t) = 16$

$4 - 8t = 16$ **Expanding LHS first.**

$-8t = 12$ **Subtracting 4 from both sides.**

$t = \frac{12}{-8}$ **Dividing both sides by (–8).**

$= -1\frac{1}{2}$

Exercise 2-05 Solving equations

Example 12

1 Solve each of these equations.

a $3d + 2 = 20$ **b** $2p - 3 = 2$ **c** $4u + 6 = 20$

d $5a + 3 = -12$ **e** $12b + 8 = 4$ **f** $3 - 2a = -6$

g $3m = m - 10$ **h** $\frac{3h}{4} = 9$ **i** $\frac{r-1}{6} = 2$

j $-\frac{2x}{5} = 8$ **k** $\frac{y+2}{-3} = 1$ **l** $11 - 4n = 15$

m $5y + 6 = 4y + 11$ **n** $\frac{4c}{10} = 3$ **o** $\frac{z}{3} - 11 = 9$

2 Which of the following is the solution of $2t - 4 = 10 + t$? Select **A**, **B**, **C** or **D**.

A $t = 3$ **B** $t = 6$ **C** $t = 4\frac{2}{3}$ **D** $t = 14$

Example 13

3 Solve each of these equations.

a $5k - 13 = 3k + 9$ **b** $8e = 2(e - 6)$ **c** $\frac{2f+7}{2} = 10$

d $3(x - 2) = 45$ **e** $\frac{w}{5} - 8 = 6$ **f** $4(2d - 9) = -12$

g $\frac{4n+7}{9} = 2$ **h** $7u + 7 = 2u - 10$ **i** $3p + 4 = 4p$

j $\frac{5z+8}{6} = -3$ **k** $\frac{8-2b}{2} = 7$ **l** $6(2 - 3q) = -24$

4 In which line was an error made in solving the following equation? Select **A**, **B** or **C**.

A Line 2 **B** Line 3 **C** Line 4

Line 1: $\frac{c-4}{8} + 2 = 6$

Line 2: $\frac{c-4}{8} = 8$

Line 3: $c - 4 = 64$

Line 4: $c = 68$

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2-06 Formulas and equations

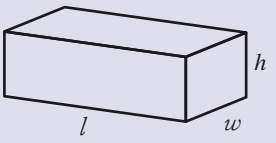
Sometimes, after we substitute values into a formula, the result is an equation that must be solved.

Working with formulas

EXAMPLE 14

The surface area of a rectangular prism of length l, width w and height h is $S = 2lw + 2lh + 2wh$.

If a rectangular prism with surface area 132 cm² has length 7 cm and width 3 cm, find its height.



Solution

$S = 132, l = 7, w = 3$

$S = 2lw + 2lh + 2wh$

$132 = (2 \times 7 \times 3) + (2 \times 7 \times h) + (2 \times 3 \times h)$ **Substituting into formula.**

$132 = 42 + 14h + 6h$

$132 = 42 + 20h$ **Collecting like terms.**

$90 = 20h$ **Subtracting 42 from both sides.**

$h = \frac{90}{20}$ **Dividing both sides by 20.**

$= 4\frac{1}{2}$

The height of the rectangular prism is $4\frac{1}{2}$ cm.

EXAMPLE 15

The formula for converting a Fahrenheit temperature (°F) to Celsius (°C) is $C = \frac{5}{9}(F - 32)$. Convert 100°C to °F.

Solution

When $C = 100$:

$100 = \frac{5}{9}(F - 32)$

$\frac{5}{9}(F - 32) = 100$

$5(F - 32) = 900$ **Multiplying both sides by 9.**

$5F - 160 = 900$ **Expanding the LHS.**

$5F = 1060$ **Adding 160 to both sides.**

$F = \frac{1060}{5}$ **Dividing both sides by 5.**

$= 212$

100°C = 212°F.

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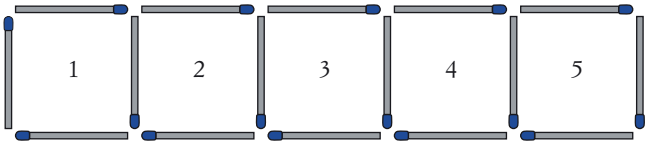
Exercise 2-06 Formulas and equations

Example 14

1 The number of matchsticks, m, needed to make a pattern of s squares is $m = 3s + 1$.



a How many matches are needed to make:

i 4 squares? **ii** 10 squares?

b How many squares can be made from:

i 22 matches? **ii** 55 matches?

2 Find the base length, b cm, of a triangle with area 90 cm^2 and perpendicular height 15 cm, given the area formula $A = \frac{1}{2}bh$.

3 The average speed of a moving object in metres per second is $s = \frac{d}{t}$, where d is the distance travelled in metres and t is the time taken in seconds.

a Find the distance travelled by a car in 20 seconds if its speed is 15 m/s.

b Find the time taken (to the nearest second) for a cyclist to travel 250 m if his speed is 4.2 m/s.

Example 15

4 The formula for the area of a trapezium is $A = \frac{1}{2}(x + y)h$, where x and y are the lengths of the parallel sides, and h is the distance between them. What is the length of one parallel side of a trapezium if the other parallel side is 7 m, the distance between them is 5 m and the area is 22.5 m^2?

5 The mean, M, of three numbers x, y and z is calculated using the formula $M = \frac{x + y + z}{3}$. If three numbers have a mean of 17 and two of the numbers are 10 and 20, find the third number.

6 The circumference of a circle with radius r is $C = 2\pi r$. If a circle has a circumference of 50.27 cm, find its radius to the nearest centimetre.

7 If a principal, \$$P$, is invested at an interest rate of r per annum (where r is written as a decimal) then, after n years, it will grow to \$$A$, where A is given by the compound interest formula $A = P(1 + r)^n$. What principal needs to be invested at 11% p.a. for it to grow to \$6000 in 3 years? Express your answer to the nearest cent.

8 A kettle is boiled and the temperature, T°C, of the water after t minutes is $T = 18t + 28$. After how many minutes is the temperature:

a 64°C? **b** 92.8°C?

9 The formula for converting miles (M) to kilometres (K) is $K = 1.61M$. Convert each of the following distances to miles, correct to two decimal places.

a 5 km **b** 1.5 km

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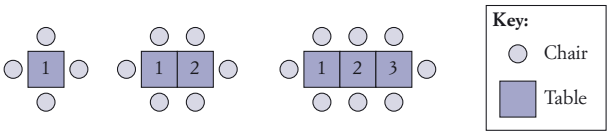
10 The number of chairs, c, that can be seated around t square tables is $c = 2t + 2$.



a How many chairs can be seated around:

i 5 tables? **ii** 12 tables?

b How many square tables are required to seat:

i 24 chairs? **ii** 40 chairs?

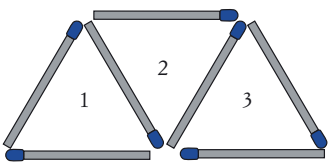
11 Joanne earns a weekly wage of \$540 plus commission of 12% on the value of cosmetics she sells in excess of \$2000. Her total pay, P, is given by $P = 540 + 0.12(V - 2000)$, where V is the value of the cosmetics sold that week. What was the value of the cosmetics Joanne sold if her total pay was \$852?

12 The surface area of a rectangular prism of length l, width w and height h is given by $S = 2lw + 2lh + 2wh$. If a rectangular prism has length 4 cm, width 2 cm and surface area 58 cm^2, which of the following is its height? Select **A**, **B**, **C** or **D**.

A 3.5 cm **B** $4\frac{1}{6}$ cm **C** 5.5 cm **D** 7.25 cm

13 The formula for converting k kilometres to M miles is $M = \frac{5k}{8}$. Convert 12 miles to kilometres.

14 The number of matchsticks, m, needed to make this pattern of triangles is $m = 2t + 1$, where t is the number of triangles in the pattern.



How many triangles can be made with:

a 37 matches? **b** 55 matches?

15 For babysitting, Kirstie charges \$18 for the first hour and \$14 for each hour after that. This charge, \$$C$, can be expressed using the formula $C = 18 + 14(h - 1)$, where h is the number of hours worked. If Kirstie earned \$46 for babysitting one night, for how many hours did she work?

16 Jeff works in a high-rise office building on the 32nd floor. At lunchtime, he travels down in the lift at a speed of two floors per second. This is described by the formula $F = 32 - 2t$ where F is the floor being passed by the lift and t is the time in seconds.

a Which floor is Jeff passing after:

i 8 seconds? **ii** 14 seconds?

b After how many seconds will Jeff pass the:

i 18th floor? **ii** 8th floor?

17 According to one theory, the recommended nightly hours of sleep for a child is $S = 8 + \frac{18 - a}{2}$, where a is the age of the child in years. Which of the following is the age of a child who requires $14\frac{1}{2}$ hours of sleep each night? Select **A**, **B**, **C** or **D**.

A 2.5 years **B** 3 years **C** 3.5 years **D** 5 years

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18 If an object has an initial speed u m/s and acceleration a m/s^2 after t seconds, its final speed (in m/s) is given by the formula $v = u + at$. Find the acceleration of an object if its initial speed is 8 m/s and its final speed after 12 seconds is 44 m/s.

Sample HSC problem

Paula the plumber charges according to the formula $C = 60 + 32(h - 1)$, where \$$C$ is the charge and h is the number of hours worked.

a How much does Paula charge for working $2\frac{1}{2}$ hours?

b Copy and complete this table using the formula $C = 60 + 32(h - 1)$.

h	1	2	3
C			

c Do the values of C in the table above follow a linear pattern? Give a reason for your answer.

d Expand and simplify this expression: $60 + 32(h - 1)$.

e Find the number of hours Paula worked if she charged \$172.

Study tip

Topic summaries and mind maps

Summarise each topic once you have completed it, to create useful study notes for revising the course, especially before exams. Use a small exercise book or folder for listing the important ideas, formulas, terminology and skills of each topic. Educational research shows that effective learning takes place when we rewrite learned knowledge in our own words.

A good topic summary runs for two to four pages. It is a condensed, personalised version of your course notes. This is your interpretation of a topic, so include your own comments, symbols, diagrams, observations and reminders. Highlight important facts using boxes and include a glossary of key words and phrases. (There is a glossary at the back of this book.)

A mind map is a topic summary in graphic form, with boxes, branches and arrows showing the connections between the main ideas of the topic. (See the next page for an example.) The topic name is written at the centre of the map, with branches leading off to key concepts or subheadings. Smaller branches hang off these branches, listing important details and formulas. Mind maps are powerful because they present an overview of a topic on one large sheet of paper. Visual learners absorb and recall information better using mind maps.

When compiling a topic summary, use your class notes and this textbook, especially the front and back sections of each chapter. Ask your teacher for a copy of the course syllabus or the school's teaching program, which lists the knowledge and skills of every topic in dot point form.

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CHAPTER REVIEW

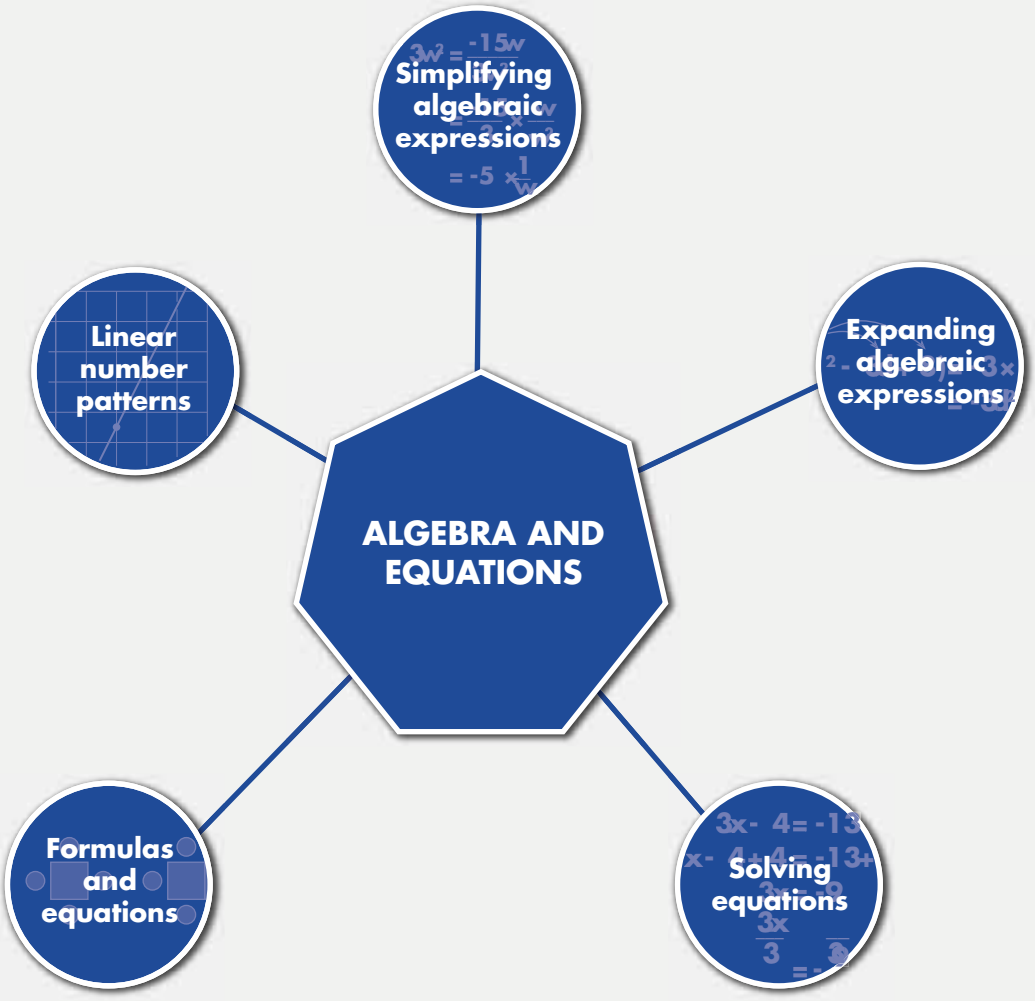
Topic overview

Algebra review

This chapter, 'Algebra and equations', revised basic algebra skills. Make sure you master the algebraic techniques required to:

- recognise linear number patterns
- simplify expressions
- expand expressions
- solve equations
- work with formulas.

Make a summary of this topic. Use the chapter outline at the beginning of this chapter and the mind map below as a guide. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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Revision

Exercise 2-01

1 **a** Which of the following number patterns is linear? Select **A**, **B** or **C**.

A 1, 4, 9, 16, … **B** 3, 6, 12, 24, … **C** 5, 9, 13, 17, …

b Find a formula for the general term of the linear pattern in part **a**.

c Find the 14th term of the linear pattern in part **a**.

Exercise 2-02

2 Simplify each of these expressions.

a $5ut + 2t^2 - t^2 + 4ut$

b $3k^2 \times 5k$

c $-9d \times \frac{2d}{3}$

d $8p^2 \times 4p^3$

e $-1 + 4h + 8 - 10h$

f $\frac{16r^3}{2r}$

g $(-3d^2)^3$

h $\frac{9n^2}{15n^2}$

i $-3v^2w^2 \div 21vw$

j $10x^2 + 7x - 2x^2 + x$

k $\frac{24bc}{8b^2}$

l $\frac{-a}{a^3}$

m $\frac{20r}{6} \div 4r$

n $\frac{4y}{3} \times \frac{5v}{10}$

o $\frac{10x}{18p} \times \frac{3p}{20}$

p $\frac{3y}{2a} \div \frac{9dy}{10d}$

Exercise 2-03

3 Expand each of these expressions.

a $5(2x - 4)$

b $-3(a + 7)$

c $4(12t - y)$

d $-9(r^2 + 2w)$

e $8mn(m - n)$

f $-2d(4d - d^2)$

Exercise 2-03

4 Expand and simplify each of these expressions.

a $3(4x + 1) + 2(x - 2)$

b $2n(n - 1) + (n - 1)$

c $6(2 - d) - 4(d - 3)$

d $p(p + 4) - p(p + 8)$

e $3(4u + 5) - (u + 7)$

f $h(5h - 1) + 3h(h + 9)$

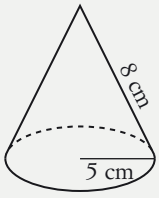
Exercise 2-04

5 If $p = 4$, $q = -5$ and $r = 20$, then evaluate each expression.

a $3p^2 + 4r$ **b** $\frac{7r}{q}$ **c** pqr **d** $\sqrt{p(r-q)}$

Exercise 2-04

6 The surface area of a cone is given by the formula $S = \pi r(r + s)$, where r is the radius of the base and s is the slant height of the cone. Find, correct to two decimal places, the surface area of a cone with base radius 5 cm and slant height 8 cm.



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7 Brett earns a weekly wage of \$480 plus a commission of 10% of the value of the vacuum cleaners he sells in excess of \$1200. His total weekly pay, \$$P$, is given by the formula $P = 480 + \frac{V - 1200}{10}$, where V is the value of the vacuum cleaners sold. Calculate Brett's pay for a week in which he sold \$3400 worth of vacuum cleaners. Exercise 2-04

8 Solve each of these equations. Exercise 2-05

a $5p - 4 = 21$ **b** $-2a + 6 = 8$ **c** $\frac{b-3}{2} = -6$

d $23 - 8r = 19$ **e** $\frac{4n}{5} = 11$ **f** $\frac{r}{3} + 7 = 1$

g $\frac{20 - 4n}{4} = 7$ **h** $3t + 13 = t - 12$ **i** $5(2g - 4) = -30$

9 If an object is travelling with initial speed u m/s, accelerating at a rate of a m/s^2, and covers a distance s m, then its final speed v m/s follows the rule $v^2 = u^2 + 2as$. Calculate the distance travelled by a car whose speed increases from 11 m/s to 28 m/s with an acceleration of 3 m/s^2. Exercise 2-06

10 According to one theory, the formula that links the surrounding air temperature, T°C, to the number of chirps per minute, C, made by a cricket at night during summer is $T = \frac{C}{8} + 3$. How many chirps per minute are made by a cricket when the temperature is 13°C? Exercise 2-06

11 The average blood pressure, P, of a person aged y years, measured in millimetres of mercury (mm Hg), is given by the formula $P = 110 + \frac{y}{2}$. Find the age of a person whose blood pressure is 124 mm Hg. Exercise 2-06

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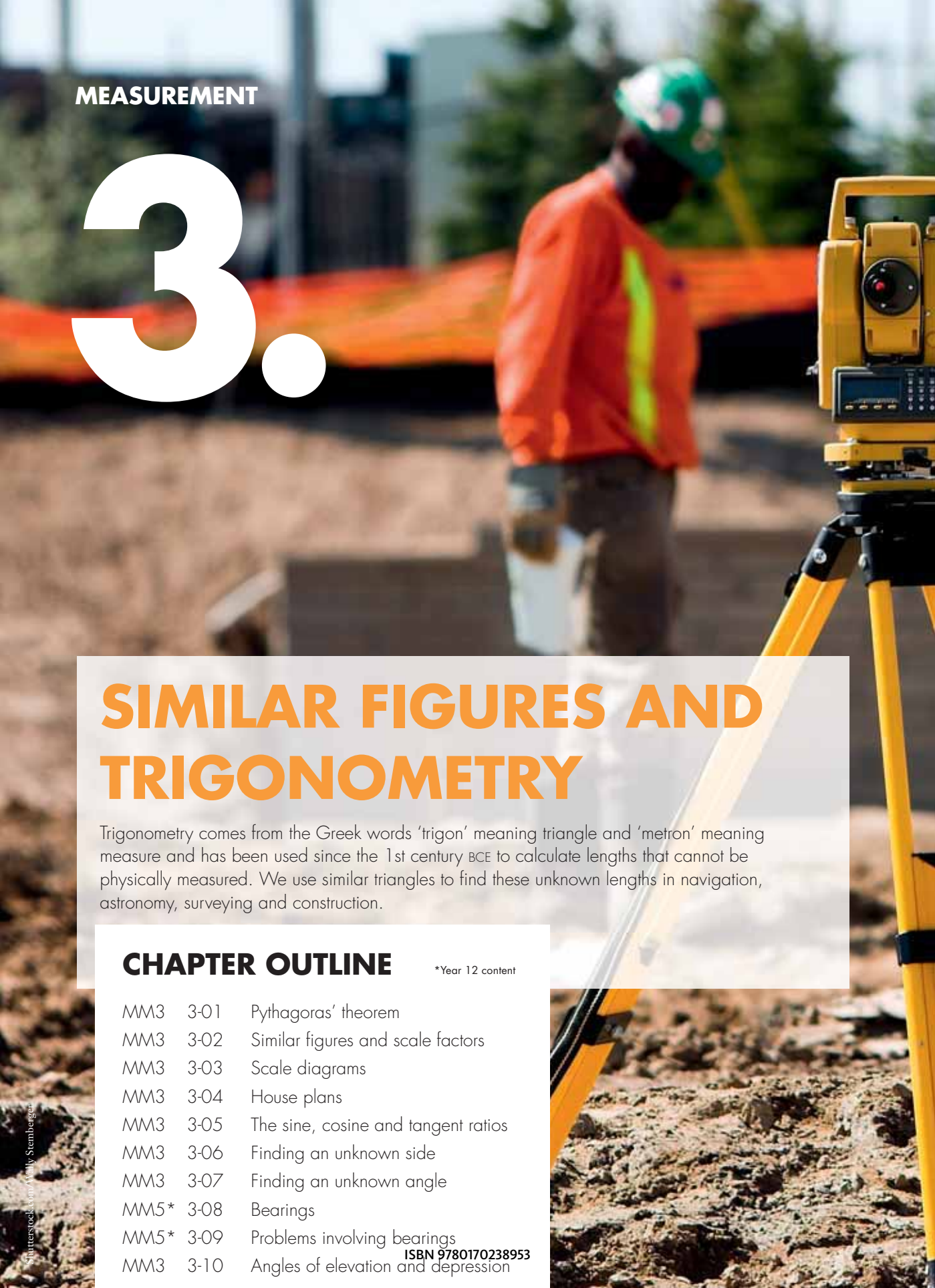

MEASUREMENT

3.

SIMILAR FIGURES AND TRIGONOMETRY

Trigonometry comes from the Greek words 'trigon' meaning triangle and 'metron' meaning measure and has been used since the 1st century BCE to calculate lengths that cannot be physically measured. We use similar triangles to find these unknown lengths in navigation, astronomy, surveying and construction.

CHAPTER OUTLINE

*Year 12 content

MM3	3-01	Pythagoras' theorem
MM3	3-02	Similar figures and scale factors
MM3	3-03	Scale diagrams
MM3	3-04	House plans
MM3	3-05	The sine, cosine and tangent ratios
MM3	3-06	Finding an unknown side
MM3	3-07	Finding an unknown angle
MM5*	3-08	Bearings
MM5*	3-09	Problems involving bearings
MM3	3-10	Angles of elevation and depression

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IN THIS CHAPTER YOU WILL:

- apply Pythagoras' theorem to practical applications
- calculate scale factors for similar figures
- use scale factors to solve problems involving similar figures
- recognise that the ratio of matching sides in similar right-angled triangles is constant for equal angles
- calculate sine, cosine and tangent ratios
- use trigonometric ratios to find the length of an unknown side in a right-angled triangle
- determine whether an answer seems reasonable by using a diagram drawn roughly in proportion
- use trigonometric ratios to find the size of an unknown angle in a right-angled triangle, correct to the nearest degree
- use compass bearings (eight points only) and true bearings (three-figure bearings) in problem solving related to maps and charts
- calculate angles of elevation and depression, given the appropriate diagram
- solve practical problems using scale diagrams and factors, similarity and trigonometry

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TERMINOLOGY

adjacent	angle of depression	angle of elevation
compass bearing	cosine	degree
elevation	enlargement	floor plan
hypotenuse	image	matching
minute	object	opposite
Pythagoras' theorem	ratio	reduction
scale diagram	scale factor	similar figures
sine	tangent	true bearing

Assignment 3

SkillCheck

1 Evaluate, correct to two decimal places:

a $\sqrt{5^2+6^2}$ **b** $\sqrt{9^2-3^2}$

c $\dfrac{54.6}{21.4}$ **d** $\dfrac{5400}{35.7}$

2 Copy and complete each statement.

a 2500 cm = _____ mm **b** 1400 mm = _____ m

c 2.5 m = _____ mm **d** 9 m = _____ cm

e 960 000 mm = _____ m **f** 3600 cm = _____ m

3 Copy and complete each sentence.

a The angle sum of a triangle is _____°.

b One revolution is _____°.

4 Calculate each trigonometric ratio, correct to 4 decimal places where appropriate.

a tan 23° **b** sin 49° **c** cos 17°

d tan 45° **e** cos 89° **f** sin 30°

g cos 60° **h** tan 5.7° **i** cos 35.8°

5 Simplify each expression.

a $\dfrac{18}{30}$ **b** $\dfrac{15}{9}$ **c** 12 : 28

d 27 : 18 **e** $\dfrac{3}{4} : \dfrac{5}{8}$ **f** 15 cm : 40 mm

g 1 m : 10 cm **h** 50 km : 500 m **i** 1 cm : 2 m

6 Solve each equation.

a $\dfrac{h}{2} = \dfrac{24}{5}$ **b** $18 = \dfrac{k}{3}$ **c** $4 = \dfrac{28}{p}$

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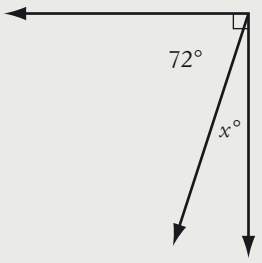
7 Find the value of each pronumeral.

a



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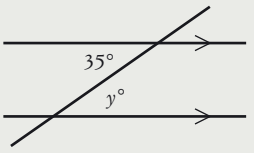

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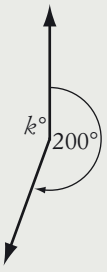
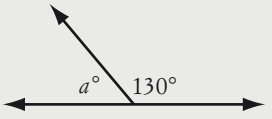


d



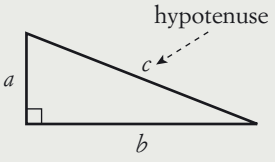
3-01 Pythagoras' theorem

Pythagoras' theorem

SUMMARY

Pythagoras' theorem

In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



The **hypotenuse** is the longest side of a right-angled triangle

As a formula: $c^2 = a^2 + b^2$

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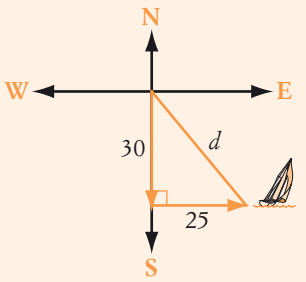
EXAMPLE 1

A boat sailed 30 nautical miles due south, then 25 nautical miles due east. How far, correct to the nearest nautical mile, is it from its starting point?



Solution

Let the boat be d nautical miles from its starting point.

$d^2 = 30^2 + 25^2$

$= 1525$

$d = \sqrt{1525}$

$= 39.0512 \ldots$

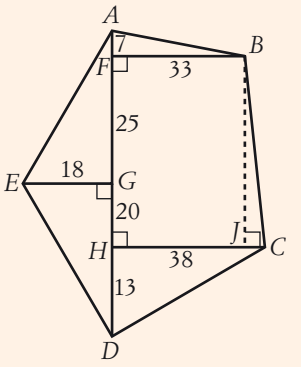
≈ 39

From the diagram, an answer of 39 nautical miles seems reasonable.

The boat is 39 nautical miles from its starting point.

EXAMPLE 2

A surveyor measured different distances (in metres) on the irregular field $ABCDE$, as shown on the diagram on the right. How many metres of fencing, correct to the nearest 0.1 m, are needed to fence the field?



Solution

The amount of fencing is the perimeter of the field.

In $\triangle AGE$: $AE^2 = 18^2 + 32^2$ **$AG = 25 + 7 = 32$**

$= 1348$

$AE = \sqrt{1348}$

$= 36.7151\ldots$

Do not round until the end of this solution.

In $\triangle DGE$: $ED^2 = 18^2 + 33^2$

$= 1413$

$ED = \sqrt{1413}$

$= 37.5898\ldots$

In $\triangle DHC$: $DC^2 = 13^2 + 38^2$

$= 1613$

$DC = \sqrt{1613}$

$= 40.1621\ldots$

In $\triangle BJC$: $BC^2 = 45^2 + 5^2$

$= 2050$

$BC = \sqrt{2050}$

$= 45.2769\ldots$

In $\triangle AFB$: $AB^2 = 7^2 + 33^2$

$= 1139$

$AB = \sqrt{1139}$

$= 33.7342\ldots$

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Perimeter = $AE + ED + DC + BC + AB$

$= 193.4781\ldots$

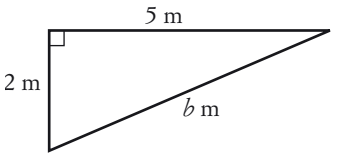
≈ 193.5 ← Round at this step.

The amount of fencing needed is 193.5 m.

Exercise 3-01 Pythagoras' theorem

1 Find the value of the pronumeral in each triangle, correct to two decimal places.

a



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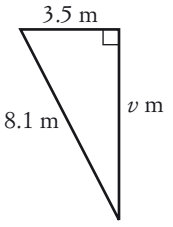
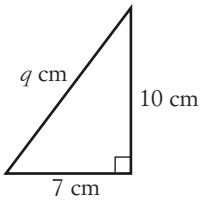
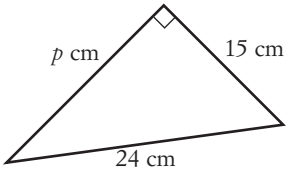


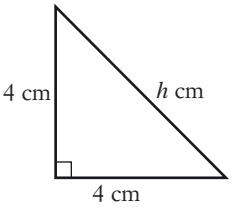
c



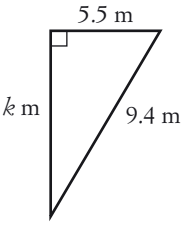
d



e

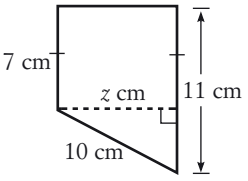


f

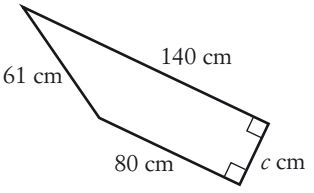


2 Find the value of each pronumeral, correct to one decimal place.

a



b



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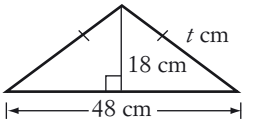

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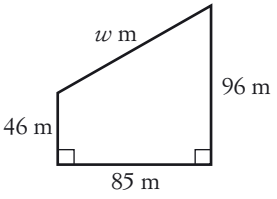
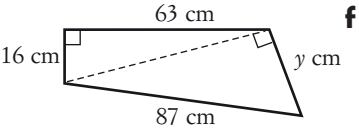
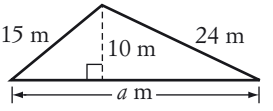


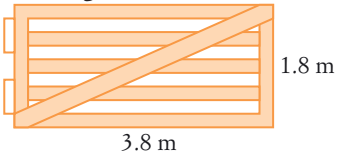
e



f



3 A gate has dimensions 3.8 m by 1.8 m. What is the length of its diagonal brace (to the nearest centimetre)? Example 1



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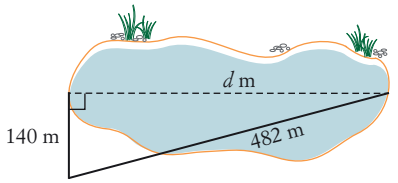
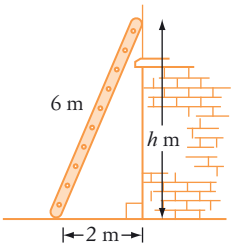
4 Vanessa calculated the distance across the lake by taking the measurements shown on the right. What was the distance across the lake, to the nearest metre?

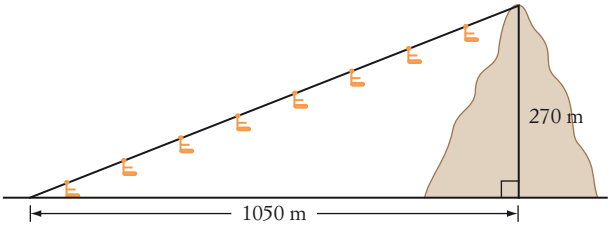


5 A 6 m ladder leans against a house, so that its base is 2 m out from the bottom of the house. How far up the wall of the house does the ladder reach, to the nearest centimetre?



6 A yacht leaves Newcastle and sails 160 nautical miles due east. It turns and sails due north until it is directly 200 nautical miles from Newcastle. How far north did it sail?

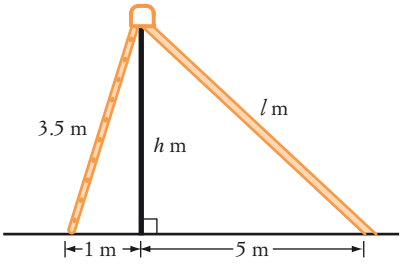
7 A chairlift takes skiers up to the top of a hill 270 m high and 1050 m away. How long is the chairlift's cable, correct to the nearest 10 m?



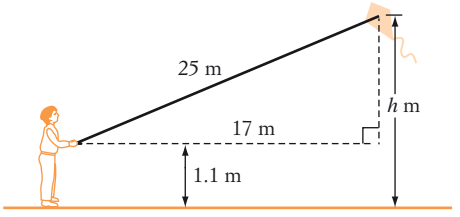
8 A playground slide is made up of two right-angled triangles. Find, correct to the nearest centimetre:

a h metres, the height of the slide

b l metres, the length of the slide.



9 This diagram shows a boy flying a kite. How high is the kite above the ground, correct to one decimal place?



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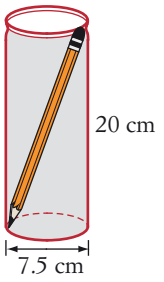
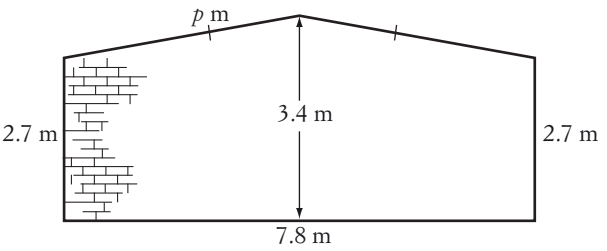
10 Jackie wants to use an old tennis ball container as a pencil case. If the container has a diameter of 7.5 cm and a height of 20 cm, what is the length of the longest pencil that will fit inside the container (to the nearest millimetre)?

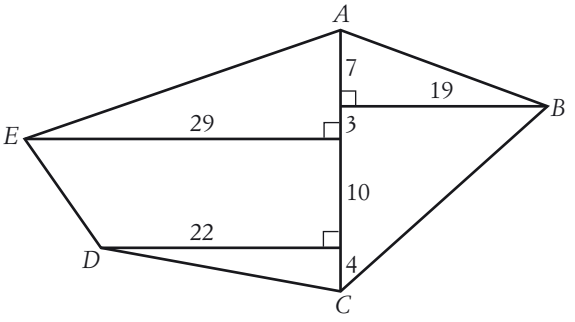


11 Find the length of the pitch line, p m, of the roof of this house (to the nearest centimetre).



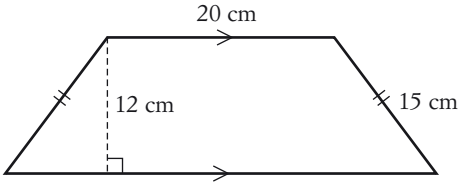
12 Calculate the perimeter of the field *ABCDE* in this diagram, correct to two decimal places. All measurements are in metres.

Example 2

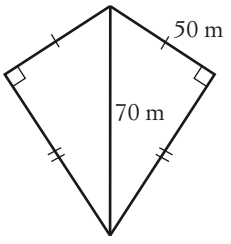


13 Calculate the perimeter of each figure, to the nearest 0.1 m.

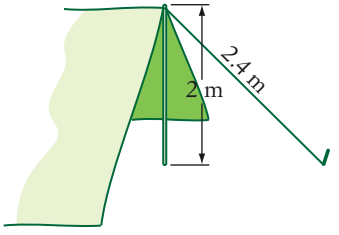
a



b



14 If a tent pole is 2 m high and the rope is 2.4 m long, how far from the base of the pole should the rope be pegged (correct to one decimal place)?



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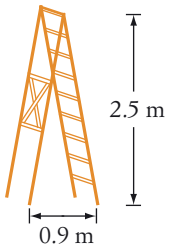
15 When a stepladder is placed upright, its top is 2.5 m above the ground and its legs are 0.9 m apart. How long is the stepladder, correct to two decimal places?



Finding sides in similar figures

Finding sides in similar triangles

3-02 Similar figures and scale factors

Similar figures have the same shape but are not necessarily the same size. They have:

- matching angles equal
- matching sides in the same ratio

The ratio of the matching sides of two similar figures is called the **scale factor**. The original figure is called the **object** and the second figure is called the **image**.

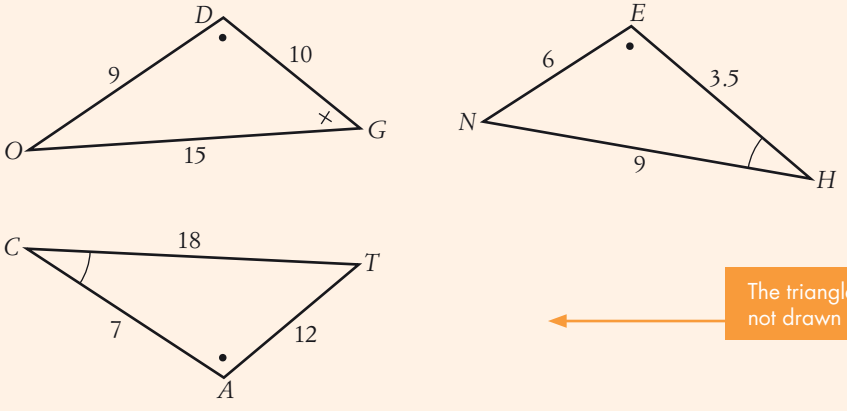
SUMMARY

$$\text{Scale factor} = \frac{\text{image length}}{\text{object length}}$$

The scale factor measures how much the object would be enlarged or reduced by to make it the same size as the image.

EXAMPLE 3

a Which two of the following triangles are similar and why?



The triangles are not drawn to scale.

b What is the scale factor of the similar triangles?

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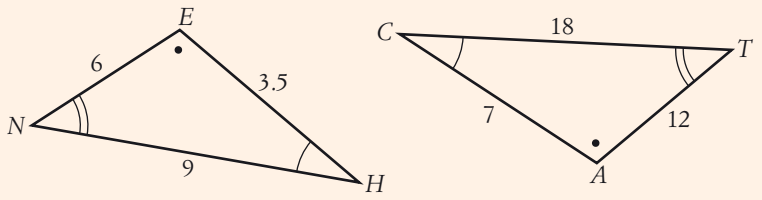
Solution

a $\triangle HEN$ and $\triangle CAT$ have two pairs of matching angles equal, so the third pair must also be equal (because all triangles have an angle sum of 180°). So $\triangle HEN$ and $\triangle CAT$ are similar.



Also, the matching sides are in the same ratio.

$$\frac{CA}{HE} = \frac{7}{3.5} = 2 \qquad \frac{AT}{EN} = \frac{12}{6} = 2 \qquad \frac{CT}{HN} = \frac{18}{9} = 2$$

b Scale factor $= \dfrac{\text{image length}}{\text{object length}} = 2$

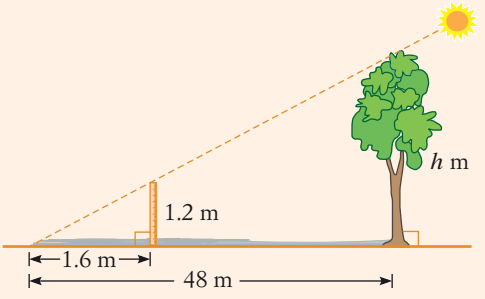
$\triangle CAT$ is the image and is double the size of the object, $\triangle HEN$.

EXAMPLE 4

At the same time of day, a 1.2 m stick throws a shadow of 1.6 m while a tree throws a shadow of 48 m. What is the height, h metres, of the tree?

Solution

The stick and tree are matching sides of similar right-angled triangles.



Scale factor $= \dfrac{48}{1.6} = 30$

$\therefore h = 30 \times 1.2$

$= 36$

The height of the tree is 36 m.

From the diagram, a height of 36 m seems reasonable.

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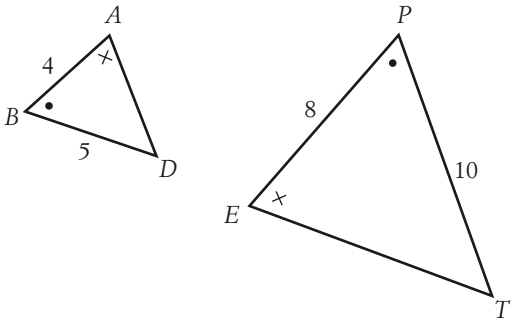
Exercise 3-02 Similar figures and scale factors

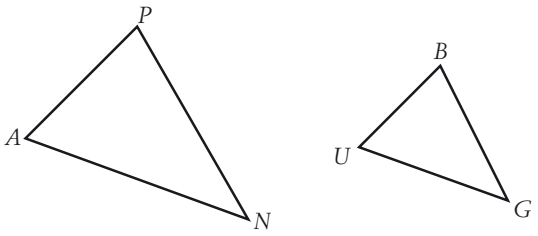
1 If $\triangle BAD$ and $\triangle PET$ are similar, and $\triangle BAD$ is the object, which of the following is the scale factor? Select **A**, **B**, **C** or **D**.



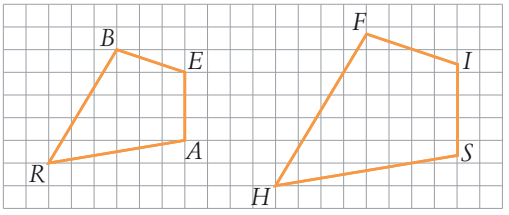
A $\frac{1}{2}$ **B** 3

C 2 **D** $1\frac{1}{2}$

2 By measuring the side lengths, find the scale factor when $\triangle PAN$ is reduced to give $\triangle BUG$.

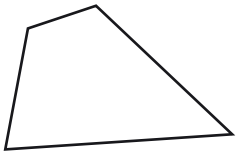


3 Find the scale factor when the quadrilateral *BEAR* is enlarged to give the quadrilateral *FISH*.

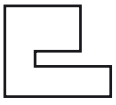


4 Copy each of the following figures onto grid paper and use the given scale factor to draw a similar figure.

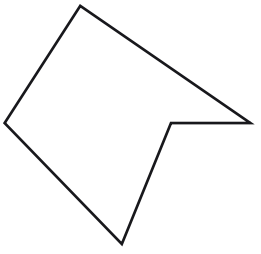
a Scale factor = 2



b Scale factor = 4



c Scale factor = 1.5



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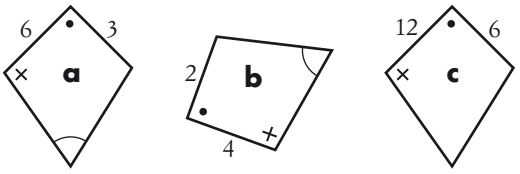
5 If quadrilateral **a** on the right is the object, which of the following statements is true? Select **A**, **B**, **C** or **D**.



Example 3

A **a** and **c** are similar, with scale factor $\frac{1}{2}$.

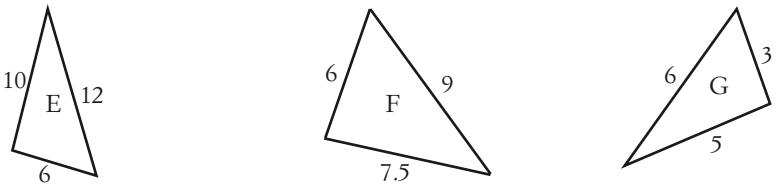
B **a** and **b** are similar, with scale factor $\frac{2}{3}$.

C **a** and **c** are similar, with scale factor 2.

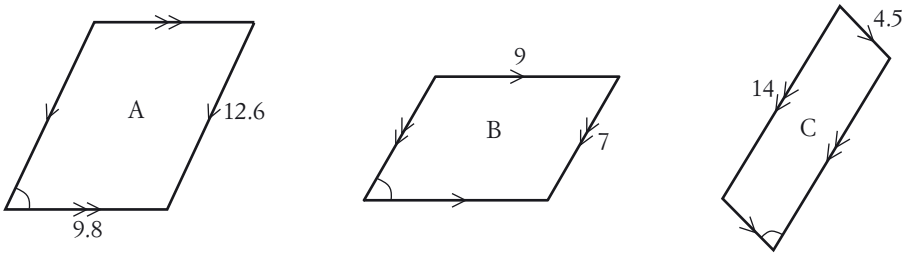
D **a** and **b** are similar, with scale factor $\frac{3}{4}$.

6 For each set of shapes, select the pair of similar figures and calculate the scale factor.

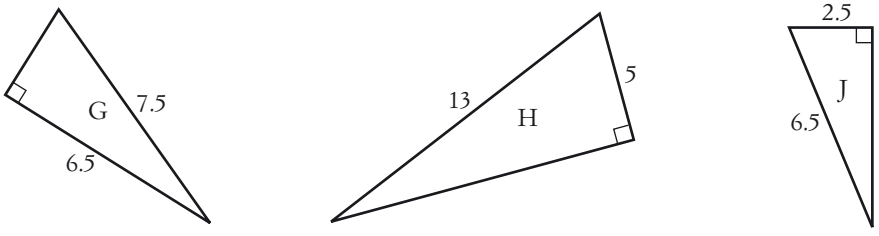
a



b

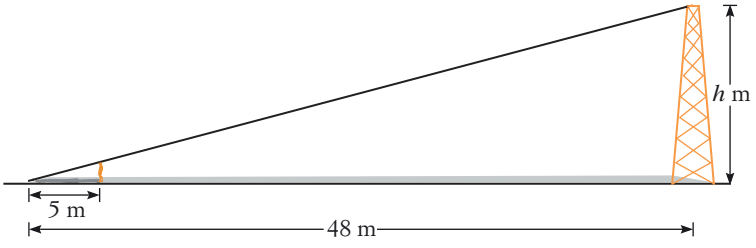


c



7 Robert is 2 m tall and casts a shadow 5 m long. At the same time, a tower casts a shadow 48 m long. How high is the tower? Answer to one decimal place.

Example 4



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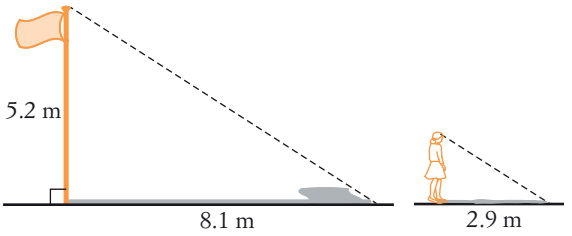
8 A 5.2 m flagpole casts a shadow of 8.1 m, while Arlene casts a shadow of 2.9 m. How tall is Arlene? Answer to the nearest 0.1 m.



9 A school building throws a shadow of 26 m at the same time during the day as a 3 m high tree throws a shadow of 5 m. What is the height of the building?

10 At sunset, a mountain of height 1500 m casts a long shadow of length 6 km, while a television tower throws a shadow of 15 m.

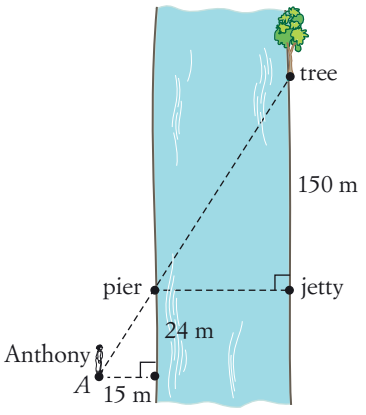
a Why are shadows longer at sunset? **b** How tall is the tower?

11 A 1.5 m high fence has a shadow of length 1.8 m. How long is the shadow cast by a building that is 4.8 m high at the same time?

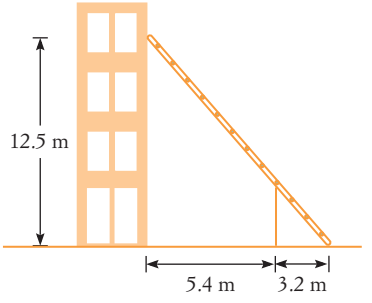
12 Jaani holds a 30 cm ruler and measures its shadow to be 24 cm. At the same time Costa measures the shadow of a lamp post to be 10.5 m long. How tall is the lamp post?

13 A church spire is 9.4 m high and casts a shadow of 18.6 m. At the same time, young Cassius casts a shadow 2.3 m long. How tall is Cassius, correct to two decimal places?

14 Anthony stands at point A, shown on the diagram, so that he is in line with the pier and the tree on the other side of the canal. He then takes measurements of 15 m and 24 m, as shown. The distance of the tree from the jetty is known to be 150 m. Use two similar triangles to find the width of the canal.



15 A fireman's ladder reaches 12.5 m up a building. The ladder is propped up by a support, placed 5.4 m from the building and 3.2 m from the base of the ladder. What is the length of the support, correct to one decimal place?



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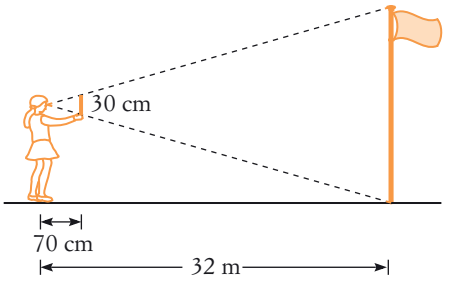
16 Galila stands 32 m from a flagpole. She holds up a 30 cm ruler and moves it until it appears to be the same height as the flagpole. If this occurs when the ruler is held 70 cm from Galila's eye, how tall is the flagpole, correct to one decimal place?



INVESTIGATION

SCALE FACTORS

1 Investigate the scale factor of enlargements obtained using an overhead or data projector. Is there a relationship between the distance of the projector from the screen and the scale factor of the resulting projection?

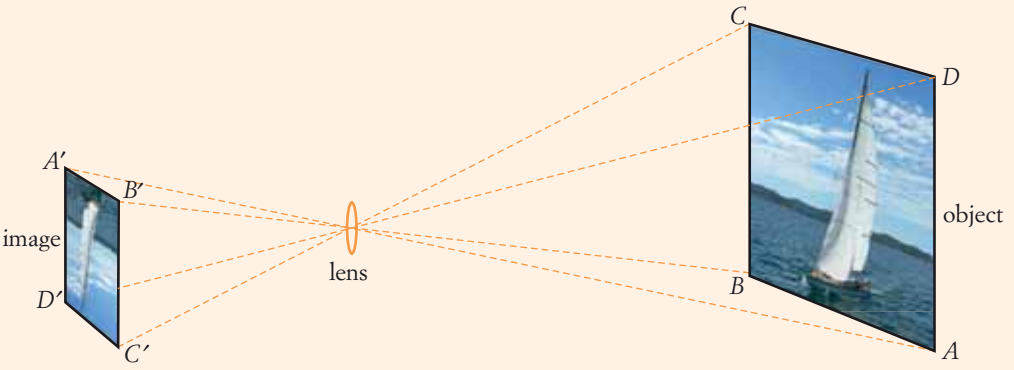
2 Use a photocopier to investigate enlarging and reducing a figure. Select, for example, 120% and 70%, then measure the sides and write the scale factors.

3 Using a shadow-stick method similar to that in Example **4** on page 93, find the height of a flagpole, building, tree, person or tower.

JUST FOR THE RECORD

Photographic images

When you take a photograph of an object, the image produced inside the camera is smaller, upside-down and back-to front. The object and the image are similar figures.



1 A photograph of Jeff shows him to be 3.5 cm tall. If the photograph has a scale factor of 1 : 50, what is Jeff's real height?

2 If Felicity is 168 cm tall, what is her height on a photograph with a scale factor of 1 : 50?

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3-03 Scale diagrams

A **scale diagram** is usually a reduction of a real object but can be an enlargement of a very small object, such as a computer microchip. The scale used in a scale diagram can be represented in several ways.

EXAMPLE 5

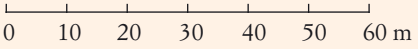
Write each of the scales shown below as a ratio.

a

b 10 mm to 1 m

c 1 cm to 0.4 mm

d 1 cm = 2 m

e 

Solution

a Scale = 1 cm : 5 km — **By measurement, 1 cm represents 5 km on the scale.**

= 1 cm : 5 × 1000 × 100 cm — **Converting 5 km to cm.**

= 1 : 500 000

b Scale = 10 mm : 1 m

= 10 mm : 1000 mm — **Converting 1 m to mm.**

= 1 : 100

c Scale = 1 cm : 0.4 mm

= 10 mm : 0.4 mm — **Convert 1 cm to mm.**

= 10 : 0.4

= 100 : 4 — **Multiplying both terms by 10.**

= 25 : 1 ← This scale indicates an enlargement of the object.

d Scale = 1 cm : 2 m

= 1 cm : 200 cm

= 1 : 200

e Scale = 8 mm : 10 m

= 8 mm : 10 000 mm

= 1 : 1250

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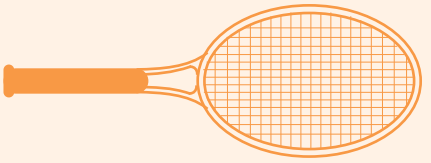
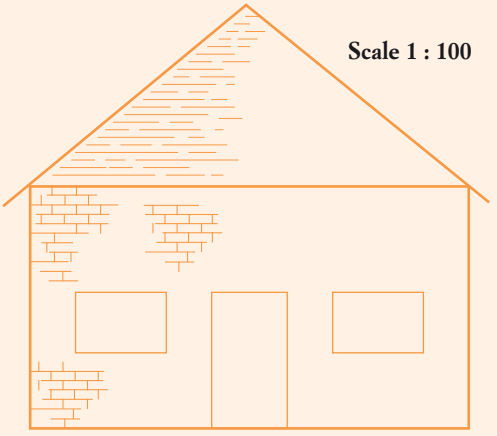
EXAMPLE 6

a By measurement and calculation, find the real length (in centimetres) of the tennis racquet.



Tennis racquet 1 : 12

b This diagram of a house is drawn to a scale of 1 : 100.



i By measurement and calculation, find the actual height of the door in the diagram, in metres.

ii What is the scaled length, in millimetres, of the house if it is actually 9.5 m long?

Solution

a Scaled length of racquet = 5.5 cm **By measurement.**

Real length of racquet = 12 × 5.5 cm

= 66 cm

A scale of 1 : 12 means the racquet is 12 times larger than the scale diagram.

b i Scaled height of door = 18 mm **By measurement.**

Actual height of door = 100 × 18 mm

= 1800 mm

= 1.8 m

ii Actual length of house = 9.5 m

Scaled length of house = 9.5 m ÷ 100 (or 9500 mm ÷ 100)

= 0.095 m

= 95 mm

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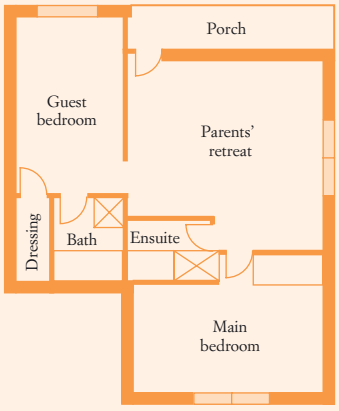
EXAMPLE 7

An architect made this scale diagram of an extension to Natalie and Bill's house.

a What is the length of the porch, in metres?

b What is the cost of tiling the porch if tiles are \$62.50 per square metre laid?

New extension



Scale 1 : 200

Solution

a Scaled length of porch = 27 mm **By measurement.**

Actual length of porch = 200 × 27 mm **Scale is 1 : 200**

= 5400 mm

= 5.4 m

b Scaled width of porch = 6 mm

Actual length of bedroom = 200 × 6 mm

= 1200 mm

= 1.2 m

Area of porch = 5.4 × 1.2

= 6.48 m^2

Cost of tiling porch = 6.48 × \$62.50

= \$405

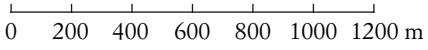
Example 5

Exercise 3-03 Scale diagrams

1 Write each of these scales as a ratio in simplest form.

a

b 2 mm : 1 cm

c 

d 10 cm to 1 m

e 5 m : 2 cm

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2 The scale on a map of Australia is 1 : 6 000 000. If the distance from Perth to Kalgoorlie on the map is 6 cm, which of the following is the actual distance from Perth to Kalgoorlie? Select **A**, **B**, **C** or **D**.

A 100 km **B** 360 km **C** 1000 km **D** 3600 km

3 The distance from Sydney to Penrith is 55 km. What scaled distance would this be on a map with a scale of 1 : 2 000 000?

4 For this map, find

a the scale factor

b the distance (in a straight line) from:

i Sydney to Katoomba

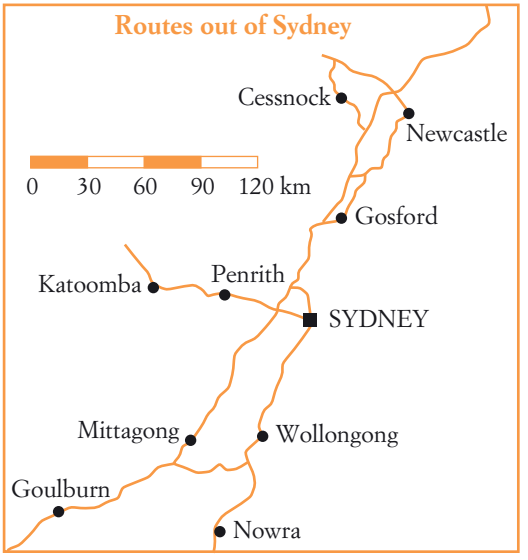
ii Newcastle to Nowra

iii Goulburn to Gosford

iv Sydney to Cessnock

c the town that is 92 km from Sydney

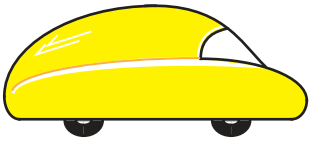
d two towns that are exactly 40 km apart.



Example 6

5 By measurement and calculation, find the real lengths of these objects.

a

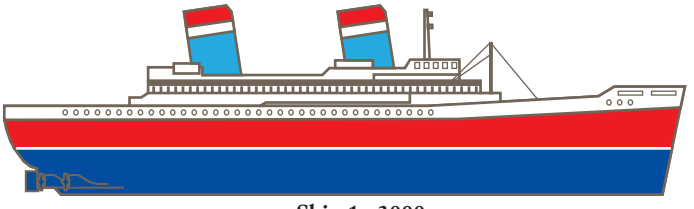


Robocar 1 : 100

b

Ant 4 : 1

c



Ship 1 : 3000

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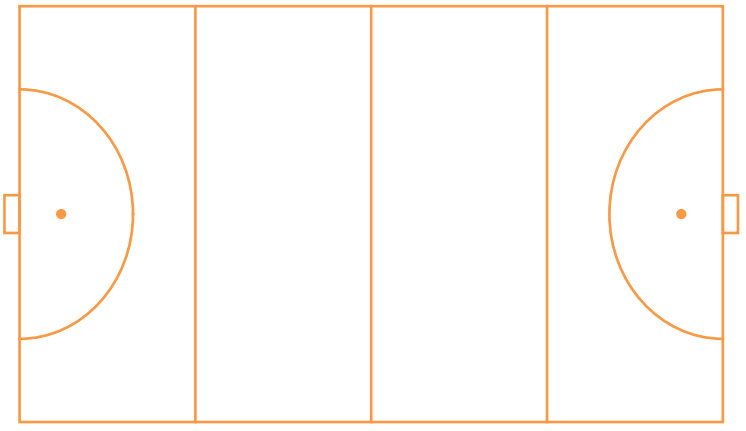
6 This is a scale drawing of a hockey field.



Scale: 1 cm = 10 m

a Measure the scaled length and width of the field.

b What is the actual width of the hockey field?

c What is the actual length of the hockey field?

7 The width of a terrace house is 7.4 m. On a drawing with a scale of 1 : 200, which of the following would be the width of the house? Select **A**, **B**, **C** or **D**.

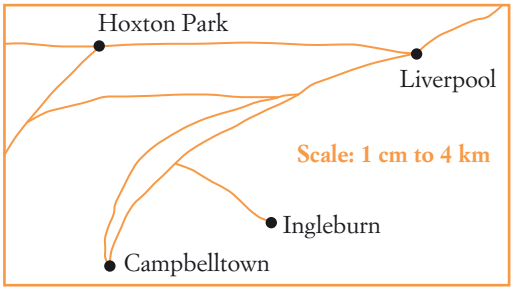
A 3.7 cm **B** 1.48 cm **C** 370 mm **D** 14.8 cm

8 What is the distance (in a direct line) between:

a Liverpool and Campbelltown?

b Hoxton Park and Ingleburn?

c Campbelltown and Hoxton Park?



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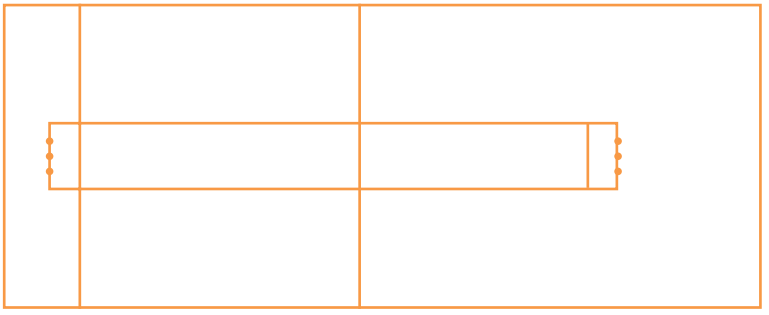
9 Below is a scale drawing of an indoor cricket court, with the cricket pitch shown with the stumps marked by dots.



Scale: 1 cm to 3 m

- **a** What is the actual length of the court?
- **b** What is the actual width of the pitch?
- **c** What is the actual length of the pitch?

10 For the scale diagram in Example **7** on page 100, find:

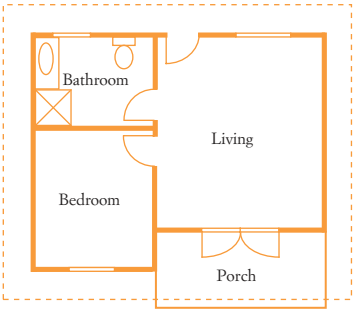
- **a** the length of the main bedroom window
- **b** the area of the guest bedroom.

Example 7

11 This is a scale diagram of a cabin in a national park. The dotted lines indicate how far the roof extends past the walls.

- **a** What are the internal dimensions of the living room?
- **b** What is the total area inside the cabin?
- **c** How wide are the double-doors onto the porch?
- **d** What is the area of the porch?
- **e** How much would it cost to carpet the bedroom if carpet costs \$34 per m^2?

Plan



Scale 1 : 200

3-04 House plans

Interpreting an office plan

An architect drawing diagrams for a new building includes:

- a plan: a diagram of the floor
- the elevations: the views of the front, back and sides of the house.

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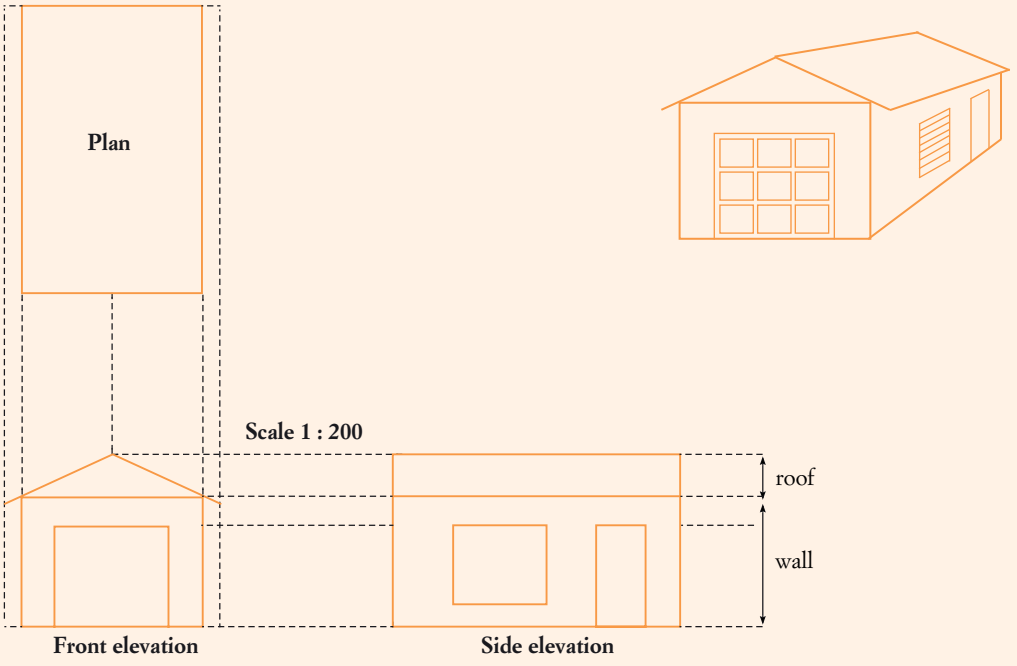
EXAMPLE 8

Here is a drawing of a garage, showing its plan, front and side elevations.



a What is the floor area of the garage in square metres?

b What is height of the highest point of the garage's roof?

c The four interior walls of the garage need to be painted, including the doors but not the window (of dimensions 2.6 m by 2.2 m). How many whole litres of paint would you need to buy to put two coats of paint on the walls if 1 L covers 10 m^2?

Solution

a Scaled width of garage = 2.5 cm — **By measurement.**

Actual width = 2.5 cm × 200 — **Scale factor is 200.**

= 5 m

Scaled length of garage = 4 cm

Actual length = 4 cm × 200

= 8 m

Floor area = 5 m × 8 m

= 40 m^2

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b Scaled height of highest point = 2.4 cm

Actual height = 2.4 cm × 200

= 4.8 m

c Scaled height of ceiling = 1.7 cm

Actual height = 1.7 cm × 200

= 3.4 m

Width of garage = 5 m **From part a above.**

Area of front and back walls = 2 × 5 × 3.4

= 34 m²

Length of garage = 8 m **From part a above.**

Area of side walls = 2 × 8 × 3.4

= 54.4 m²

Area of window = 2.6 × 2.2

= 5.72 m²

Area to be painted = 34 + 54.4 – 5.72

= 82.68 m²

Two coats = 2 × 82.68

= 165.36 m²

Amount of paint needed = 165.36 ÷ 10 L

= 16.536 L

≈ 17 L

Symbols on plans

Plans usually contain symbols and abbreviations showing features such as doors and toilets.

	Hinged door	Ens	Ensuite (bathroom)
or WC	Toilet (water closet)		Bath
	Shower		Vanity
	Stove		Kitchen sink
Ptry	Pantry		Laundry tub
WM	Washing machine		Linen cupboard
Robe	Wardrobe	WIR	Walk-in (ward)robe
FL	Floor level	GL	Ground level
CL	Ceiling level		Window

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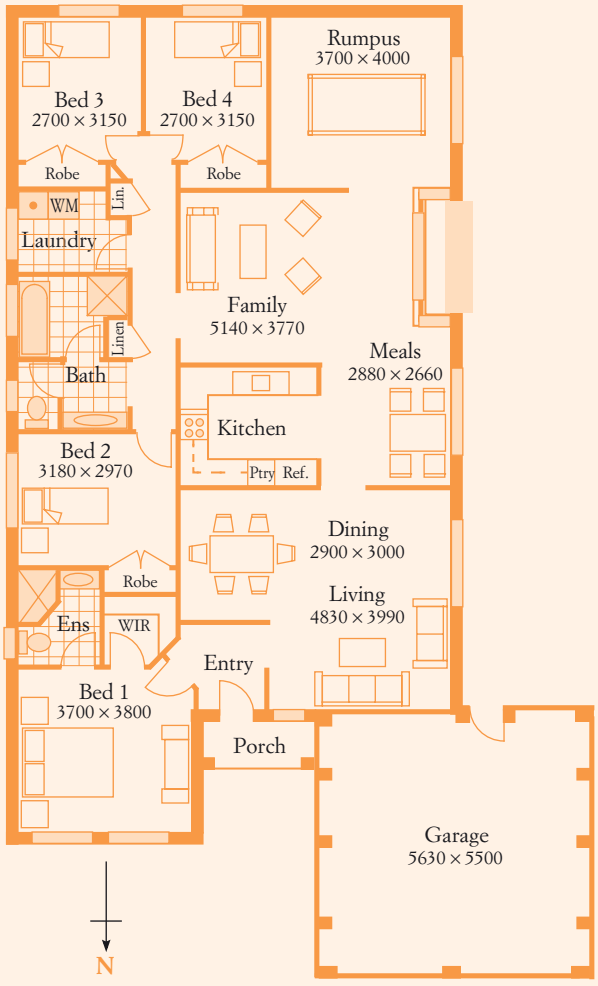
Measurements on plans are always shown in **millimetres** because this avoids the use of decimal points, which can lead to errors in printing and reading.

EXAMPLE 9

This house plan, not to scale, shows measurements in mm.



a Write the symbol for a window and count the number of windows on the plan.

b Count the number of hinged doors on the plan, excluding those on wardrobes or cupboards.

c How many showers are there in this house?

d What is used to indicate the wall cupboards in the kitchen on this plan?

e What is the area of the family room, correct to the nearest m^2?

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Solution

a There are 16 windows.

b There are 10 hinged doors.

c There are two showers.

d Dotted lines.

e Area of the family room = 5.140 m × 3.770 m **Converting the dimensions to metres.**

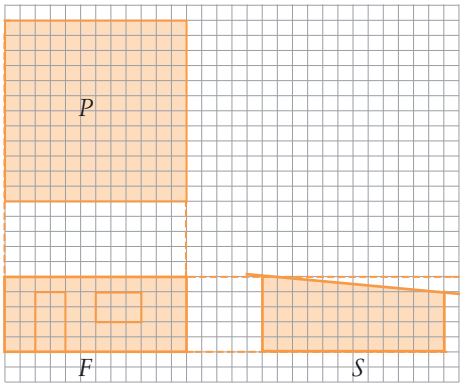
= 19.3778 m²

≈ 19 m²

Exercise 3-04 House plans

Example 8

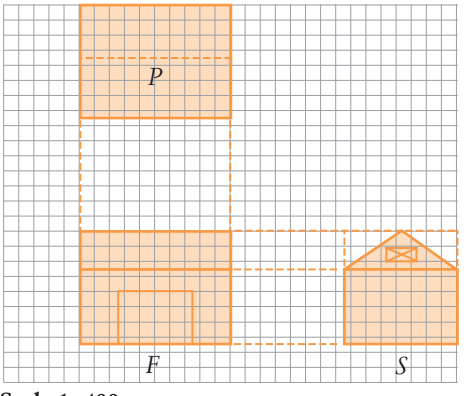
1 The scaled plan for a fishing shack is shown on the right. The floor plan is labelled *P*, the front elevation is labelled *F* and the side elevation is labelled *S*.



Scale 1 cm : 2.5 m

- **a** What is the height of the back wall?
- **b** What is the width of the shack?
- **c** What is the floor area?
- **d** How many whole litres of paint are required to paint the outside walls of the shack, including the door but not the window, if 1 L of paint covers 10 m²?

2 The scaled plan for a barn is shown on the right.



Scale 1 : 400

- **a** What is the height of the highest point of the roof from the ground?
- **b** What is the length of the front of the barn?
- **c** What is the floor area?
- **d** What is the cost of tiling the floor if tiles are $84.50 per square metre laid?

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Example 9

3 Refer to the house plan in Example **9** to answer these questions.

a How many bedrooms are there in the house?

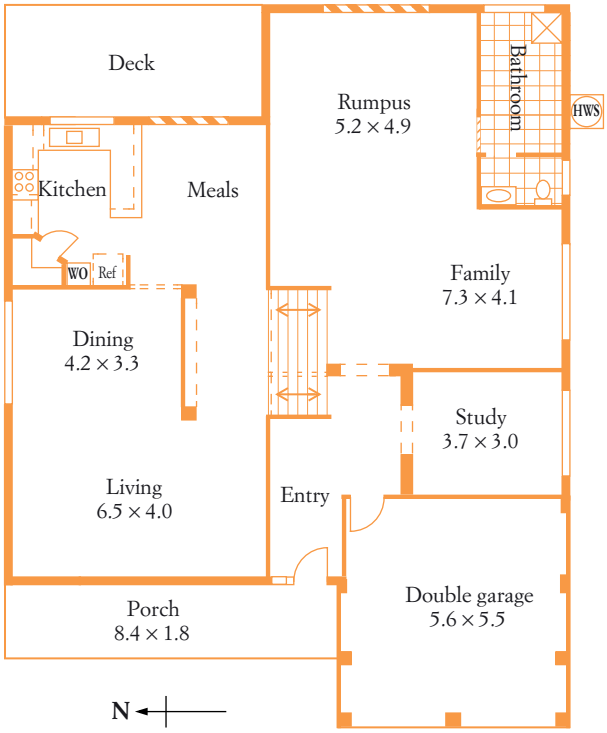
b What does the abbreviation WIR stand for?

c How many baths are in the house?

d In which room is the pantry?

e What is the area of the combined living and dining room, to the nearest m^2?

4 This is the floor plan of a split-level home.



a How many steps do we go down to get from the family room to the dining and living level?

b Draw the symbols used on the plan to represent:

i the hot water service

ii the hotplates

iii the wall oven

iv the front door

v the sliding door onto the deck

c The porch is to be tiled with terracotta tiles. How many square metres of tiles are needed?

d If each tile measures 30 cm by 30 cm, how many whole tiles are needed to tile the porch?

e The dining and living areas are to be carpeted. How much will it cost to carpet the rooms if carpet costs \$68 per m^2 laid?

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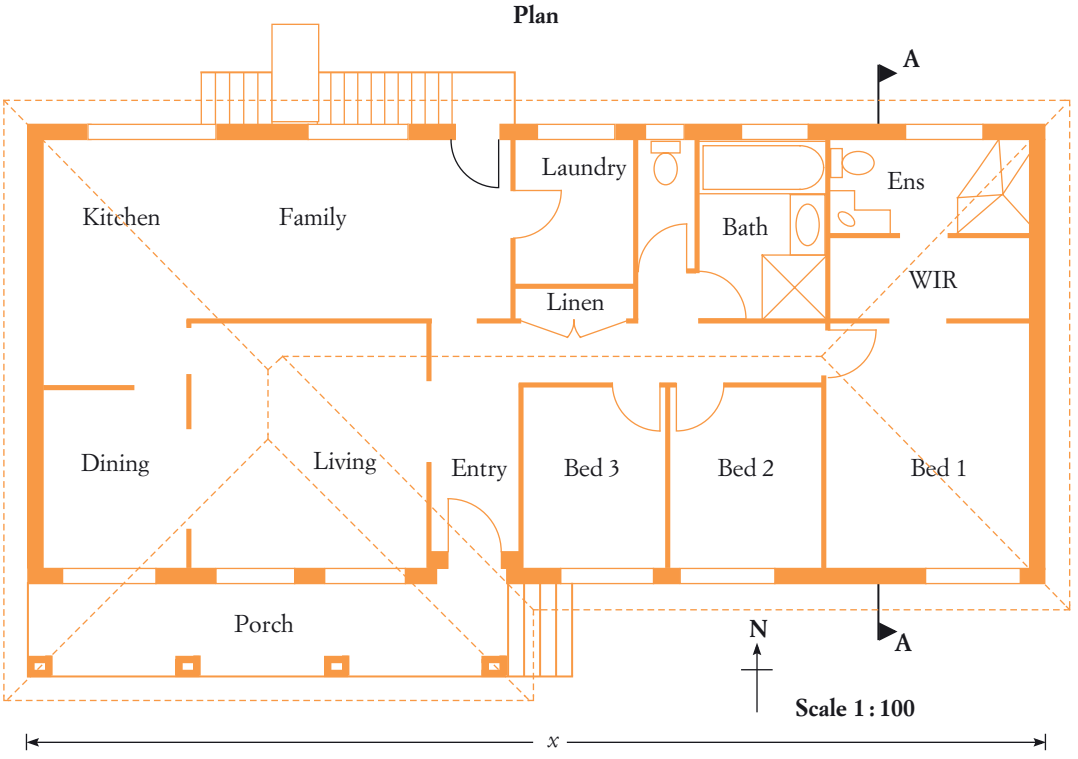
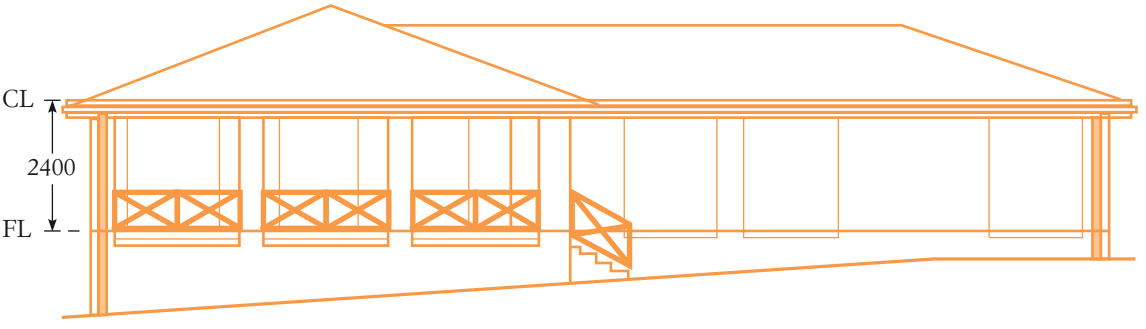
5 Here are the floor plan and south elevation of a home built on a sloping block of land.

Plan



South elevation



a What scale is shown on the plan?

b What is the actual length of the house (denoted by x)?

c How many windows are there in the house?

d How many external doors are there in the house?

e What is the area of the porch? Give your answer to the nearest m^2.

f What is the ceiling height?

g The entry and hallway are to have slate tiles. These cost \$105 per square metre laid. How much will it cost to have the entry and hallway tiled? Give your answer to the nearest \$100.

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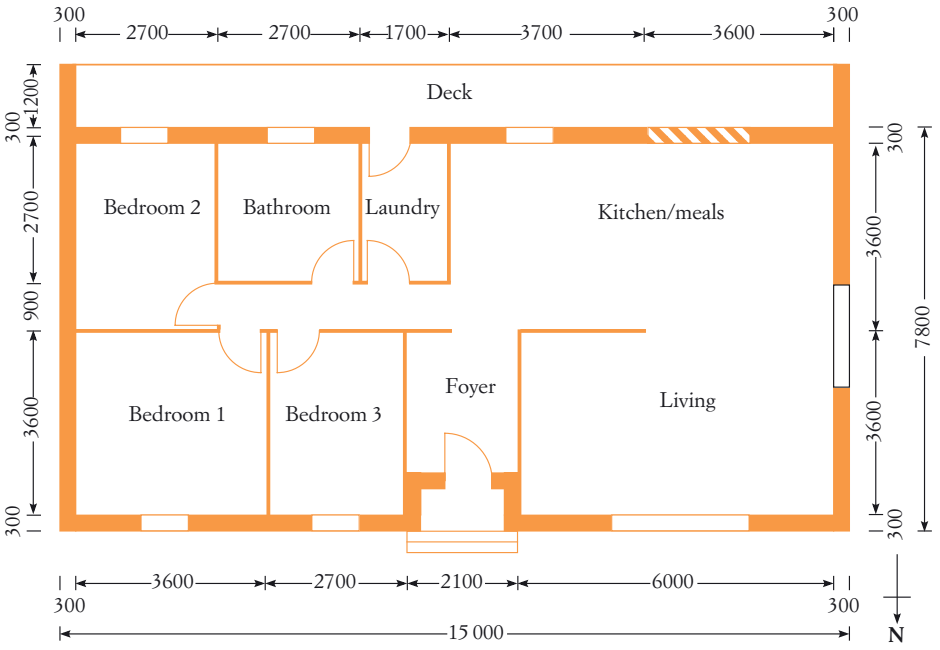
6 This is the plan for a three-bedroom house. The dimensions are in millimetres.



a What are the internal dimensions of bedroom 1?

b A wardrobe, 600 mm wide, is to be built along the eastern wall of bedroom 1.

- **i** What is the maximum length the wardrobe can be?
- **ii** What floor area will be left in the bedroom to carpet?

c What is the area of the deck (to the nearest square metre)?

d What is the overall area of the house (to the nearest square metre)?

e How much will it cost to build this house (including the deck) if the current building rate is $1540 per square metre?

f The bathroom floor is to be tiled with 20 cm square tiles. Find:

- **i** the number of tiles (rounded to the nearest whole number) that will fit along the length of the bathroom floor
- **ii** the number of tiles required.

INVESTIGATION

HOUSE PLANS

1. Collect plans from newspapers, magazines or the Internet. Discuss the various features, the symbols used and the size of rooms.
2. Choose one house plan and say what it is you like about the plan. Is it easy to read? Does it give you a clear idea of what the actual house will look like?
3. Draw a scaled floor plan of your house, apartment or classroom.

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3-05 The sine, cosine and tangent ratios

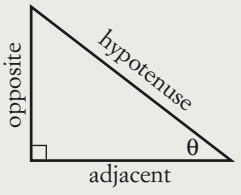
Trigonometric calculations

Trigonometry is useful for calculating lengths and distances in the real world that are difficult or impossible to measure directly. There are three basic ratios in trigonometry for finding the size of unknown sides and angles in right-angled triangles.

SUMMARY

For angle θ in the right-angled triangle:

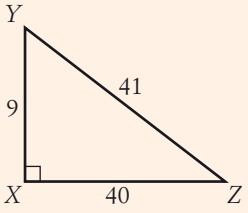
Ratio	Meaning	Initials
sine	$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$	SOH
cosine	$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$	CAH
tangent	$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$	TOA



To learn these trigonometric ratios, some students memorise the initials, SOH-CAH-TOA (pronounced 'so car toe ah'), while others remember a phrase such as '**S**ome **O**ld **H**ens **C**an **A**lways **H**ide **T**heir **O**ld **A**ge'. Find your own mnemonic (memory aid) to remember this.

EXAMPLE 10

Write as fractions the sine, cosine and tangent ratios for $\angle Y$ and $\angle Z$ in $\triangle XYZ$ shown.



Solution

$\sin Y = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{40}{41}$ $\quad \cos Y = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{9}{41}$ $\quad \tan Y = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{40}{9}$

$\sin Z = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{9}{41}$ $\quad \cos Z = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{40}{41}$ $\quad \tan Z = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{9}{40}$

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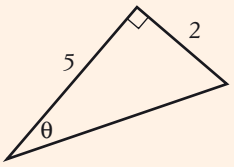
EXAMPLE 11

Find each ratio, correct to 4 decimal places where necessary.

a $\tan\theta$ **b** $\sin\theta$



Solution

a $\tan\theta = \frac{2}{5} = 0.4$

b Let the hypotenuse of the triangle be h. By Pythagoras' theorem:

$$h^2 = 5^2 + 2^2$$
$$h^2 = 29$$
$$h = \sqrt{29}$$
$$\sin\theta = \frac{2}{\sqrt{29}}$$
$$= 0.371\,390\ldots$$
$$\approx 0.3714$$

Degrees, minutes and seconds

Angles are measured in **degrees**. One degree is divided into 60 **minutes** and one minute is divided into 60 **seconds**.

SUMMARY

$1° = 60'$	1 degree = 60 minutes
$1' = 60''$	1 minute = 60 seconds

When rounding an angle size to the nearest degree or minute, use 30 as the halfway mark. The degrees-minutes-seconds key ° ' " on your calculator can be used to enter an angle size or to convert an angle size from decimal degrees format to degrees-minutes-seconds format.

EXAMPLE 12

a Calculate $\cos 64°22'$, correct to four decimal places.

b Find B to the nearest degree if $\sin B = 0.8315$.

c Find θ, to the nearest minute, if $\tan\theta = \frac{2}{5}$.

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Solution

a $\cos 64°22' = 0.432\,610...$ **On a calculator:** cos 64 °'" 22 °'" =

≈ 0.4326

b $\sin B = 0.8315$

$B = 56.2531...°$ **On a calculator:** SHIFT sin 0.8315 =

$\approx 56°$

On some calculators, use 2ndF instead of SHIFT.

c $\tan \theta = \frac{2}{5}$

$\theta = 21.8014...°$ **On a calculator:** SHIFT tan 2 $a^b/_c$ 5 =.

On some calculators, use instead of $a^b/_c$.

$= 21°48'5.07''$ **On a calculator:** °'" **to convert to degrees and minutes.**

$\approx 21°48'$ **Round to the nearest minute.**

5.07″ is less than 30″, so round down

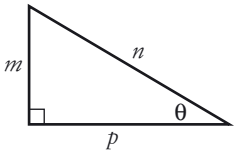
Exercise 3-05 The sine, cosine and tangent ratios

Example 10

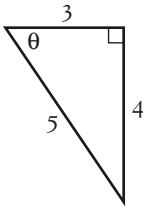
1 For each triangle below, write each ratio as a fraction.

i $\sin \theta$ **ii** $\cos \theta$ **iii** $\tan \theta$

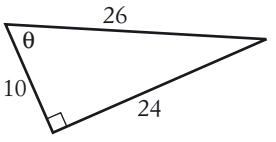
a



b



c

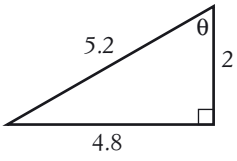


Example 11

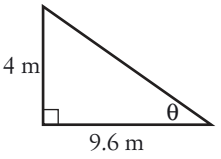
2 For each triangle below, calculate each ratio correct to four decimal places where appropriate.

i $\sin \theta$ **ii** $\cos \theta$ **iii** $\tan \theta$

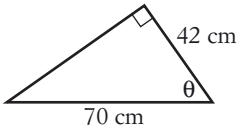
a



b



c



Example 12

3 Use your calculator to evaluate each ratio correct to four decimal places.

a $\tan 80°$ **b** $\cos 31.5°$ **c** $\sin 12°$

d $\tan 76.5°$ **e** $\cos 53°8'$ **f** $\sin 8.9°$

g $\tan 17°55'$ **h** $\sin 54.6°$

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4 Find each of the following unknown angles, correct to the nearest degree.

a $\sin\theta = \frac{8}{19}$ **b** $\tan A = 3.2692$ **c** $\cos\theta = \frac{3}{6}$

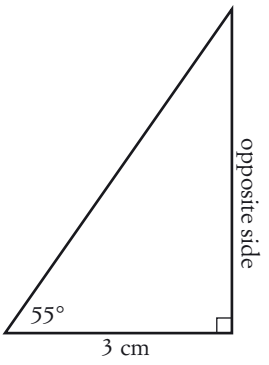
d $\tan R = 0.4129$ **e** $\sin X = 0.9815$

5 Find each of the following unknown angles, correct to the nearest minute.

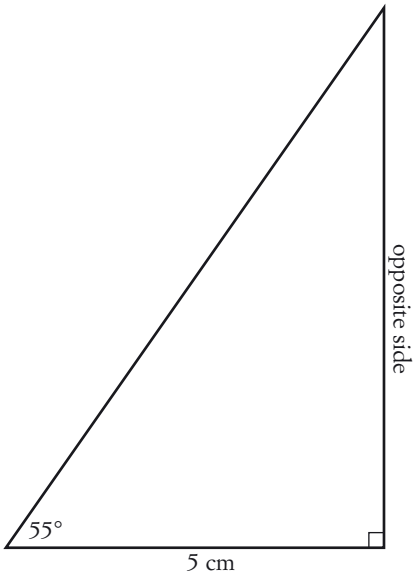
a $\cos\theta = 0.3368$ **b** $\tan Z = \frac{11}{3}$ **c** $\sin Y = 0.5$

6 These right-angled triangles each have one angle that measures 55°.

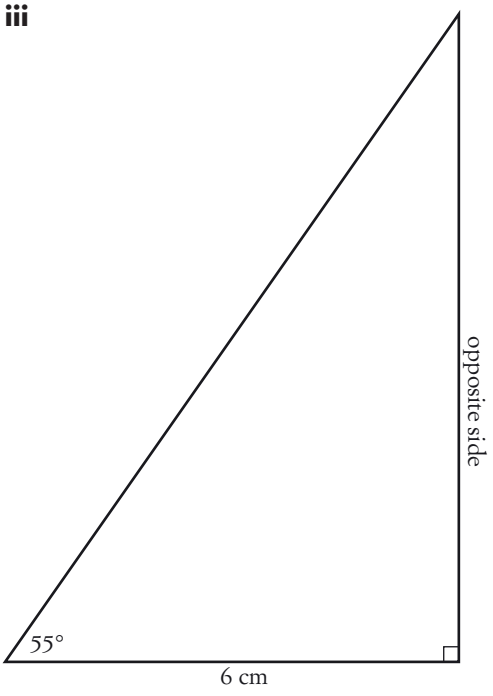
i



ii



iii



a Explain why these triangles are similar.

b By measuring the length of the opposite and adjacent sides, show that tan 55° is constant and does not depend on the size of the triangle.

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INVESTIGATION

MEASURING SINE, COSINE AND TANGENT

1 Use your calculator to find the ratios of the angles in this table, correct to 4 decimal places.

θ	1°	10°	30°	45°	60°	75°	89°
sin θ							
cos θ							
tan θ							

a Look at the sine and cosine ratios for 30° and 60°. Why do we get these values?

b What happens to the sine and cosine ratios as θ increases in size from 0° to 90°?

c What happens to the tan ratio as θ increases in size from 0° to 90°?

2 Draw a right-angled triangle which has one angle measuring 30°.

a What is the size of the third angle?

b Is there only one possible triangle with the angle measures described? Why?

c Measure the sides of your triangle, then calculate the sine, cosine and tangent ratios for 30° and compare these with those in the completed table from question **1**.

d Repeat part **c** for the third angle of the triangle (60°).

3 Draw a right-angled triangle which has one angle measuring 45°.

a What is the size of the third angle?

b Is there only one possible triangle with the angle measures described?

c Measure the sides of your triangle, then calculate the sine, cosine and tangent ratios for 45° and compare these with those in the completed table from question **1**.

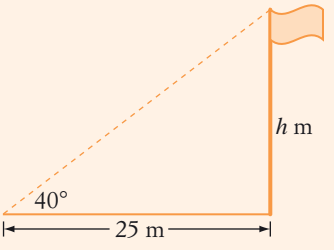
3-06 Finding an unknown side

Trigonometry crossword

We can use a trigonometric ratio to calculate the length of an unknown side in a right-angled triangle, if one angle and one side are known. We need to select the ratio that links the angle to the unknown side and the known side.

EXAMPLE 13

Find the height, h m, of this flagpole correct to one decimal place.



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Solution

For the angle 40°, h m is the opposite side and 25 m is the adjacent side, so we use the tan ratio.

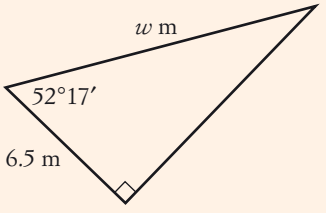
$$\tan 40° = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{h}{25}$$
$$h = 25 \tan 40°$$
$$= 20.9774...$$
$$\approx 21.0$$

The flagpole is 21.0 m high.

For a flagpole, and from the diagram, a height of 21.0 m sounds like a reasonable answer.

EXAMPLE 14

Find w, correct to the nearest whole number.



Solution

For the angle of 52°17′, the 6.5 m side is the adjacent side and the w m side is the hypotenuse, so we use the cos ratio.

$$\cos 52°17' = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{6.5}{w}$$

When the pronumeral is in the denominator, we divide to find the answer.

$$w = \frac{6.5}{\cos 52°17'}$$
$$= 10.6251...$$
$$\approx 11 \text{ m}$$

From the diagram, 11 m looks like a reasonable answer.

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Exercise 3-06 Finding an unknown side

1 Evaluate each expression correct to 2 decimal places.

a $7 \tan 28°$ **b** $\dfrac{25}{\sin 10°}$ **c** $\dfrac{7}{\tan 63°}$

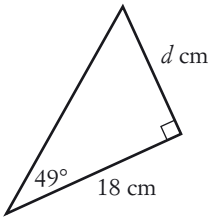
d $8 \cos 12°33'$ **e** $\dfrac{11.2}{\tan 77°}$ **f** $7 \sin 25°16'$

g $6.5 \tan 45°$ **h** $\dfrac{45.8}{\cos 27°19'}$ **i** $12 \sin 41°50'$

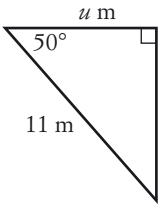
Example 13

2 Find the value of each pronumeral, correct to 2 decimal places.

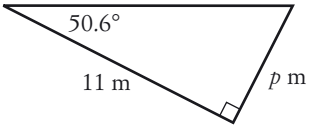
a



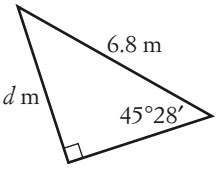
b



c



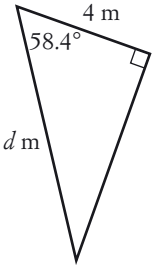
d



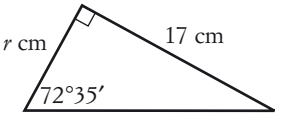
Example 14

3 Find the value of the pronumerals, correct to two decimal places.

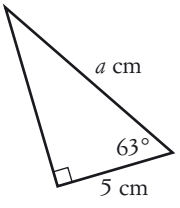
a



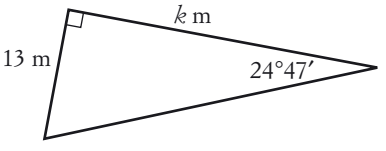
b



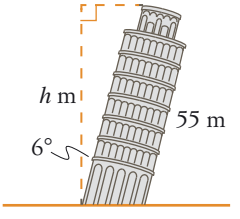
c



d



4 The Leaning Tower of Pisa is 55 m tall but it leans at 6° from the vertical. How high is it actually above the ground, to one decimal place?



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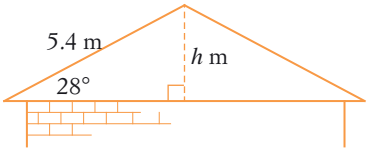
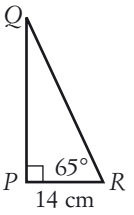
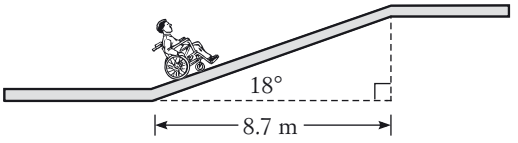
5 The pitch of a roof is 28°. If the peak of the roof is 5.4 m from the gutter, how much higher is the peak than the gutter?



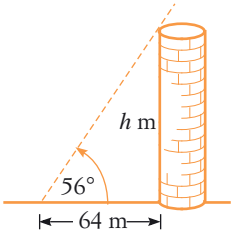
6 A triangle PQR has the following dimensions: $\angle P = 90°$, $\angle R = 65°$ and $PR = 14$ cm. Find the length of side PQ, correct to one decimal place.



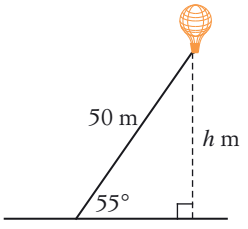
7 A wheelchair ramp is inclined at 18° to the horizontal. How long is the ramp if it links two levels 8.7 m apart?



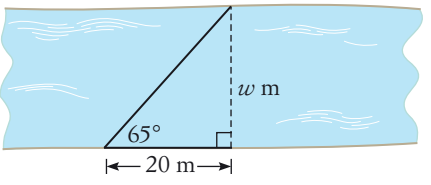
8 The angle of elevation of a chimney from a distance of 64 m from its base is 56°. Calculate the height of the chimney, to the nearest metre.



9 A hot-air balloon is anchored by a 50 m towline, but the wind pushes it off-centre. How high is the balloon above the ground (to the nearest metre) when the towline is at an angle of 55° to the ground?



10 Find the width, w metres, of the river shown in the diagram, correct to one decimal place.



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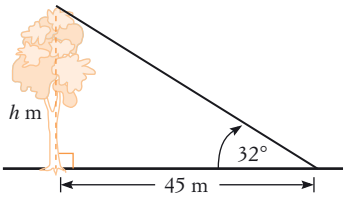
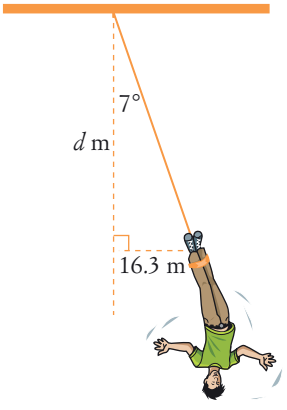
11 In $\triangle HIJ$, $\angle J = 90°$, $\angle I = 26°$ and $HJ = 5.1$ m. Find the length of HI, correct to one decimal place.

12 From a point 45 m from the base of a tree, Julie observed an angle of elevation of 32°. Calculate the height of the tree, correct to one decimal place.



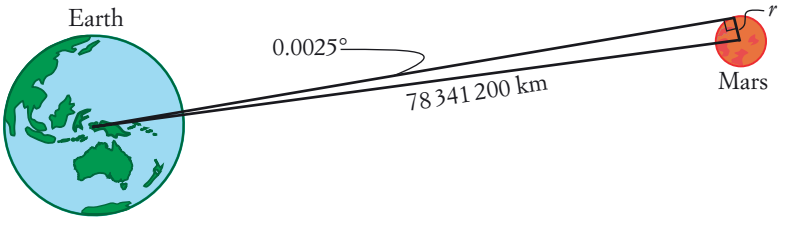
13 The vertical distance between the top of a ramp and the bottom of the ramp is 1.7 m. If the ramp is inclined at 12° to the ground, find the length of the ramp, correct to the nearest centimetre.

14 A bungy jumper leaps off a bridge at an angle of 7° to the vertical. If he is swinging 16.3 m off-centre, what vertical distance has he dropped (correct to one decimal place)?



15 A rectangular field of width 38 m has a diagonal path that makes an angle of 40° with the 38 m side. Find the length of the path correct to the nearest 0.1 m.

16 The distance between Mars and Earth is 78 341 200 km. If the angle between the centre of Mars and its surface, measured from Earth, is 0.0025°, calculate Mars' radius, r, to the nearest kilometre.



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INVESTIGATION

USING A CLINOMETER TO MEASURE HEIGHTS

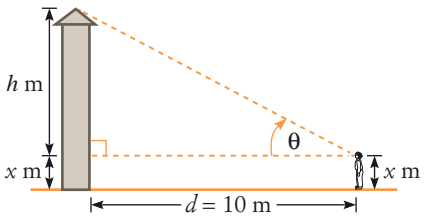
You will need: A clinometer, a trundle wheel, a metre ruler or height chart, a calculator. Choose a tall object outside, such as a tree, a flagpole, a football goalpost, a tower or a building. You will use a clinometer and a trundle wheel to measure the object's height.

1 Copy the following table.

Distance, d	Angle of elevation, θ	Height, $h = d \tan \theta$	Revised height, $h + x$
10 m			
15 m			
20 m			
30 m			
50 m			

2 Use the trundle wheel to measure 10 m from the base of the object and stand at that point.

3 Use the clinometer to view and measure the angle of elevation, θ, to the top of the object. Write this value in the 'Angle of elevation, θ' column in the table.



4 $\tan \theta = \dfrac{h}{d}$, so $h = d \tan \theta$. Find h correct to two decimal places and write that value in the 'Height, $h = d \tan \theta$' column of the table.

5 Measure x, the height of the observer's eye above the ground, in metres, correct to two decimal places.

6 Calculate $h + x$ and write the answer in the 'Revised height, $h + x$' column of the table. This is the height of the object.

7 Repeat this experiment for other values of d, at different distances from the base of your object, using the values given in the table, or some of your own.

8 How do your calculated results compare with each other? They are all meant to be the same, but why might the answers be different?

9 How could we come up with a single accurate value for the height of the object, using these results?

10 Use the clinometer to measure and calculate the heights of other objects around your school.

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3-07 Finding an unknown angle

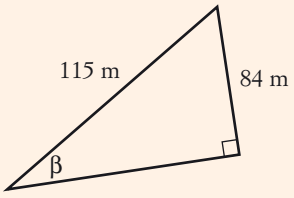
Trigonometry review

We can use a trigonometric ratio to calculate the size of an unknown angle in a right-angled triangle if two sides are known. We need to select the ratio that links the unknown angle with the two known sides.

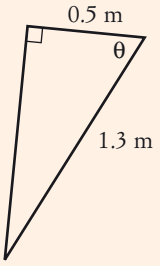
EXAMPLE 15

Find the value of each pronumeral, correct to the nearest degree.

a



b



Solution

a For angle β, the 84 m side is the opposite side and the 115 m side is the hypotenuse, so we use the sine ratio.

$$\sin \beta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$= \frac{84}{115}$$
$$\beta = 46.922...$$
$$\approx 47°$$

b For angle θ, 0.5 m is the adjacent side and 1.3 m is the hypotenuse, so we use the cosine ratio.

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$= \frac{0.5}{1.3}$$
$$\theta = 67.380...$$
$$\approx 67°$$

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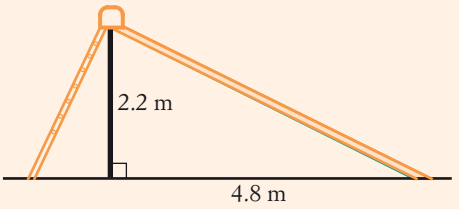
EXAMPLE 16

Find, correct to the nearest minute, the angle this playground slide makes with the ground.



Solution

Let the unknown angle be θ.

$\tan\theta = \dfrac{2.2}{4.8}$

$\theta = 24.623\ldots$

$= 24°37'24.83''$

$\approx 24°37'$ **Rounded down, because the seconds part 24.83″ < 30″.**

The slide makes an angle of 24°37′ with the ground.

Exercise 3-07 Finding an unknown angle

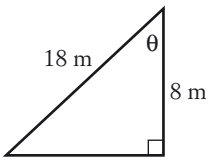
1 If $\sin\theta = \dfrac{2.4}{9.8}$, which of the following angles is closest to the size of θ? Select the correct answer **A**, **B**, **C** or **D**.

A 14° **B** 15° **C** 76° **D** 75°

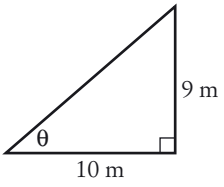
Example 15

2 Find the value of θ, correct to the nearest degree.

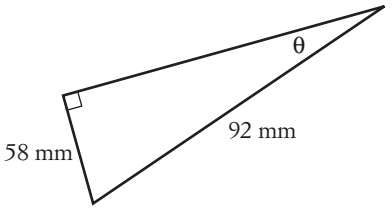
a



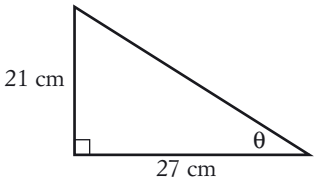
b



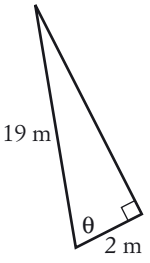
c



d



e



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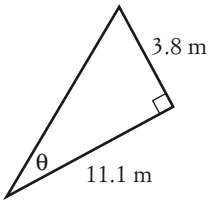
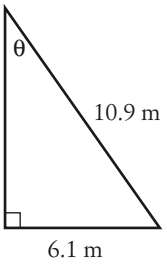
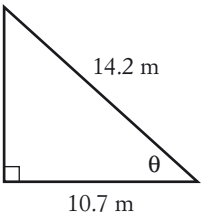


g



h



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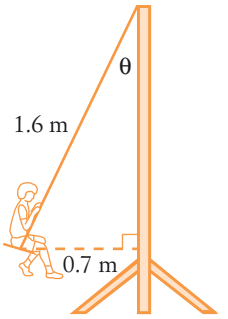
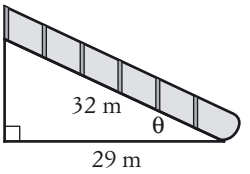
Example 16

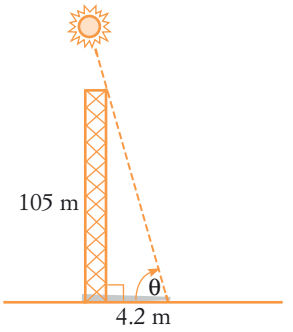
3 Caitlin is on a swing of length 1.6 m. When she is 0.7 m away from the swing's frame, what angle does the swing make with the vertical? Give your answer to the nearest degree.



4 An escalator ramp has a length of 32 m and covers a horizontal distance of 29 m. What is its angle of inclination, θ, to the nearest minute?

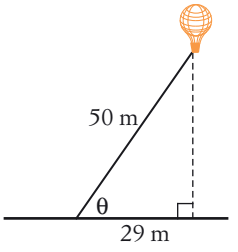


5 What angle do the Sun's rays make with the horizontal when a tower 105 m tall has a shadow of 4.2 m? Give your answer to the nearest minute.

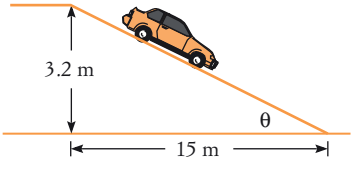


6 A plane reached a height of 171 m after flying 500 m in a straight line. At what angle to the horizontal was the plane flying, to the nearest degree?

7 A hot-air balloon is anchored by a 50 m towline, but the wind pushes it 29 m off-centre. What angle, θ, does the towline make with the ground, to the nearest degree?



8 Find the angle of inclination, θ, of this ramp in a multi-storey car park, to the nearest degree.



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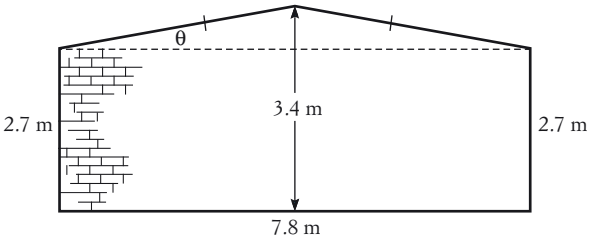
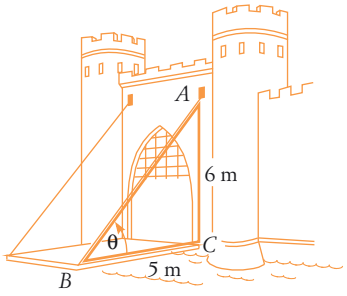
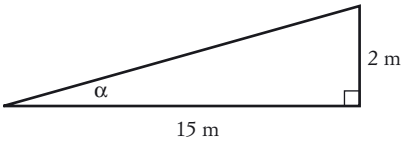
9 Find the pitch, θ, of the roof of this house (to the nearest degree).



10 The drawbridge in front of this castle is 5 m long and is pulled by chains from 6 m above ground level. What angle do the chains make with the drawbridge, to the nearest degree?



11 A hill has a gradient of $\frac{2}{15}$, which means that it rises 2 m for every 15 m horizontally. Find the angle the hill makes with the horizontal, to the nearest minute.



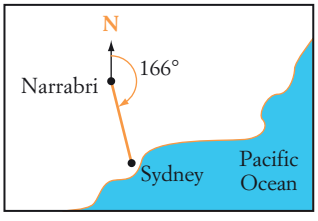
Year 12 3-08 Bearings

Bearings use angles to show the direction of one location from a given point.

For example, the bearing of Sydney from Narrabri is 166°. This means that from Narrabri, the direction of Sydney is 166° from north measured in a clockwise direction.

16 points of the compass

A page of bearings



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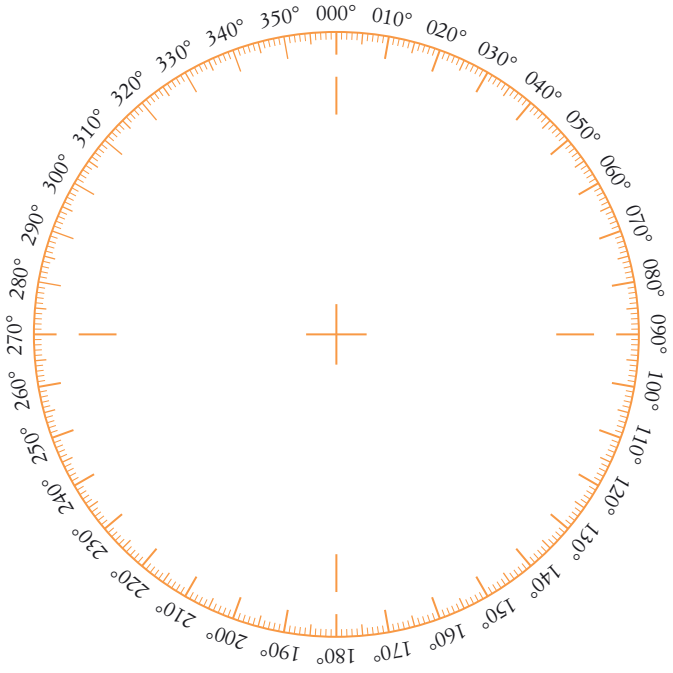
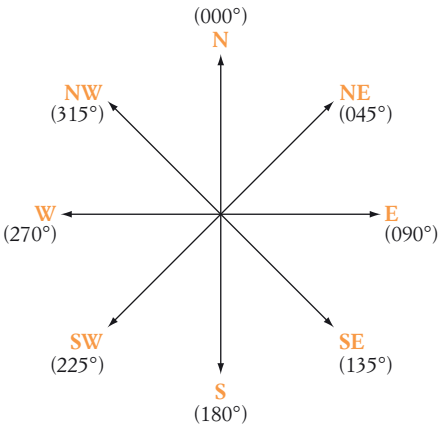
There are two types of bearings.

True bearings or **three-figure bearings** are measured in a clockwise direction from North 000°.



True bearings



Compass bearings

Compass bearings refer to eight points of the compass: north, south, east, west, north-east, north-west, south-east and south-west.

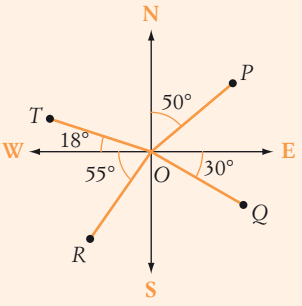
EXAMPLE 17

a Find the true bearing of P, Q, R and T from point O on the diagram on the right.

b Sketch on a compass rose the bearings of X (345°) and Y (200°) from O.

c What is the true bearing of south-west?

d Which compass bearing has a true bearing of 315°?



Solution

a P: 050°

Q: 090° + 30° = 120°

R: 270° – 55° = 215°

T: 270° + 18° = 288°

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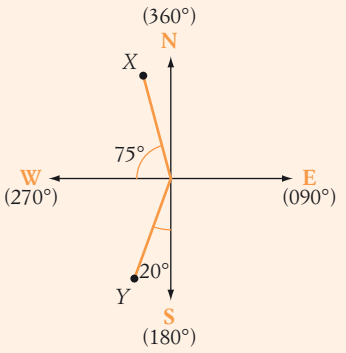
b X is between 270° and 360°, so it is in the NW quadrant, 75° from west (270° + 75° = 345°).

Y is between 180° and 270°, so it is in the SW quadrant, 20° from south (180° + 20° = 100°).

c South-west is 225° (180° + 45°)

d 315° is north-west. (270° + 45°)



SUMMARY

Bearings from 000° to 090° are in the NE quadrant.
Bearings from 090° to 180° are in the SE quadrant.
Bearings from 180° to 270° are in the SW quadrant.
Bearings from 270° to 360° are in the NW quadrant.

Example 17

Exercise 3-08 Bearings

1 Write the true bearings of each of the following.

a south **b** north-east **c** west
d south-west **e** north-west **f** east

2 Write the compass bearing that is the same as each true bearing.

a 270° **b** 045° **c** 315°
d 000° **e** 135° **f** 360°

3 In which quadrant (NE, SE, SW, NW) does each true bearing lie?

a 260° **b** 145° **c** 073°
d 102° **e** 340° **f** 237°

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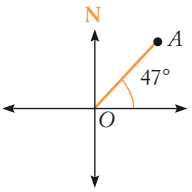
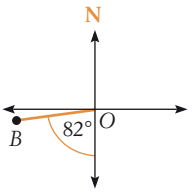
4 Write the true bearing of each point from O.

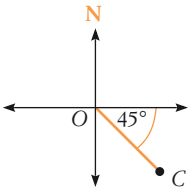
a



b



c



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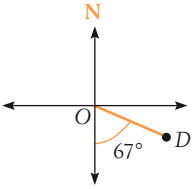

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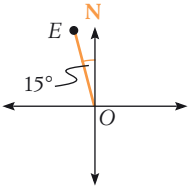

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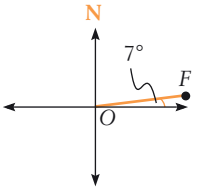

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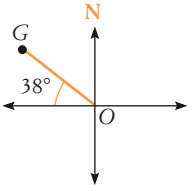

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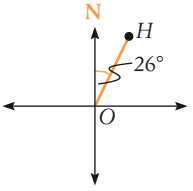
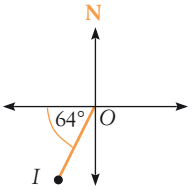


i



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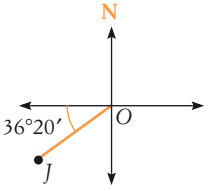

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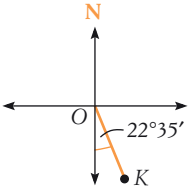

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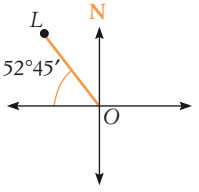


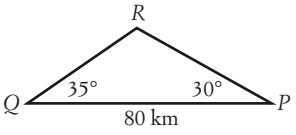
5 Sketch each bearing on a compass rose.

a 352° **b** 250° **c** 166° **d** 078° **e** 196°

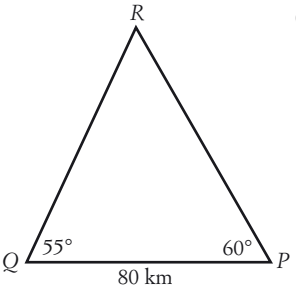
f 303° **g** 147° **h** 270° **i** 025°

6 Petersville (P) is 80 km due east of Queensfield (Q). Rossmore (R) is on a bearing of 055° from Queensfield and 300° from Petersville. Which of the following diagrams represents this? Select **A**, **B** or **C**.

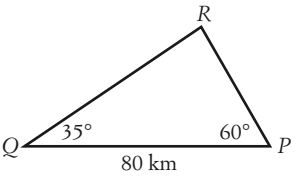
A



B



C



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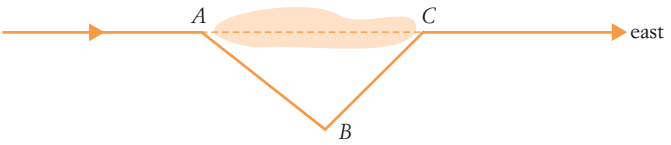
7 A surveyor travelling due east needs to make a detour on a bearing of 128° to avoid a flooded area. After detouring 8 km, he turns and travels NE until he rejoins the original line of travel.



a Copy the diagram above and mark on it all the information given.

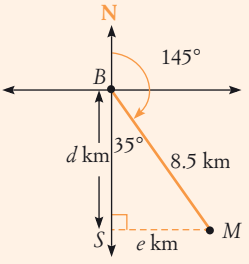
b Find the size of each angle in $\triangle ABC$.

Year 12 3-09 Problems involving bearings

For problems involving bearings, take note where the bearing is *from*. This is the point where you draw your compass rose and *from where* you take the bearing.

EXAMPLE 18

Matthew walked for 8.5 km on a bearing of 145° from base camp. His position is shown by M on the diagram on the right.



a How far south is he from base camp (correct to 1 decimal place)?

b How far east is he from base camp (correct to 1 decimal place)?

c What is the bearing of base camp from Matthew?

Solution

a Let Matthew be d km south of base camp.

In $\triangle BSM$: $\cos 35° = \dfrac{d}{8.5}$

$$d = 8.5 \cos 35°$$
$$= 6.9627...$$
$$\approx 7.0$$

Matthew is 7.0 km south of base camp.

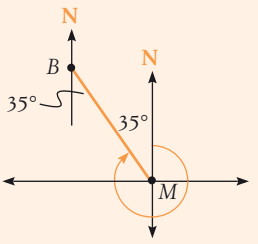
b Let Mathew be e km east of base camp.

In $\triangle BSM$: $\sin 35° = \dfrac{e}{8.5}$

$$e = 8.5 \sin 35°$$
$$= 4.8753...$$
$$\approx 4.9$$

Matthew is 4.9 km east of base camp.

c Draw the compass rose through M and mark in the alternate angle that is 35°. (Alternate angles between parallel lines are equal).



The bearing of the base camp, B, from Matthew, M, is $360° - 35° = 325°$.

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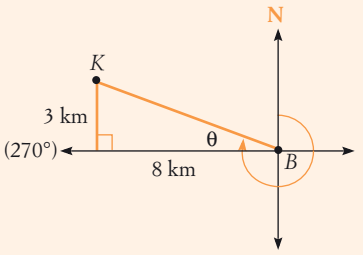
EXAMPLE 19

Kerr's Creek, K, is 8 km west and 3 km north of Bobtown, B. Find the bearing, correct to the nearest degree, of Kerr's Creek from Bobtown.

Solution



Let θ be the angle shown.

$\tan\theta = \frac{3}{8}$

$\theta = 20.556...°$

$\approx 21°$

The bearing of K from B is 270° + 21° = 291°

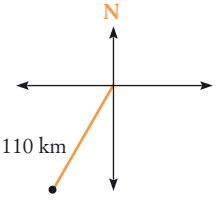
Exercise 3-09 Problems involving bearings

Example 18

1 Amy drives on a bearing of 210° for 110 km. Find, to the nearest kilometre:

a how far west Amy is from her starting point

b how far south Amy is from her starting point.



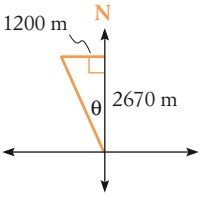
Example 19

2 Selena walks 2670 m north, then turns and walks 1200 m west, as shown in the diagram.

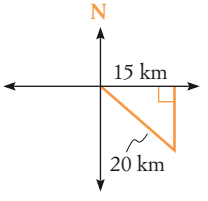
a Find the value of θ, to the nearest minute.

b Calculate Selena's bearing from her starting point.

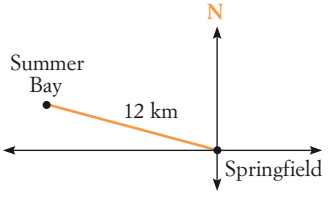
c How far is Selena from her starting point (to the nearest 10 m)?



3 A ship sails in a south-easterly direction for 20 km until it is 15 km east of its starting point. What is its bearing from its starting point, to the nearest degree?



4 Springfield and Summer Bay are 12 km apart and the bearing of Summer Bay from Springfield is 285°. How far is Summer Bay west of Springfield, correct to two decimal places?



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5 Two cars leave from the same place: the first heads due east, the other on a bearing of 165°. After travelling 8 km, the first car is due north of the second car. How far has the second car travelled, to the nearest kilometre?

6 A plane flies NW from the airport for 6 km, then turns and flies due east until it is due north of the airport. How far is it from the airport then, correct to two decimal places?

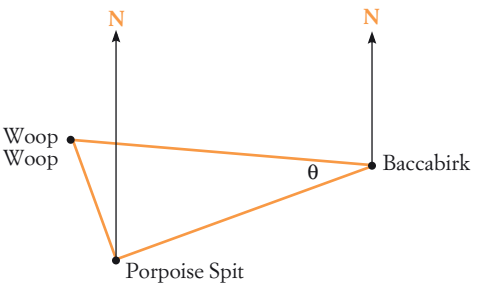
7 A hot air balloon travelled 15 km due east but drifted 1.8 km south. What was its bearing from its starting point, to the nearest minute?

8 A bushwalker leaves camp, hiking 25 km on a bearing of 148°. How far south of camp is he, to the nearest kilometre?

9 The town of Woop Woop is 22 km from the town of Porpoise Spit, on a bearing of 340°. The town of Baccabirk is 48 km from Porpoise Spit, on a bearing of 070°.

a Find θ to the nearest degree.

b Find the bearing of Woop Woop from Baccabirk. (Hint: Use alternate angles.)



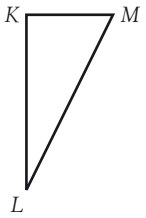
10 Leo, standing at a point *P*, measures the bearing of a television transmission tower, *H*, to be 332°. A big tree, *T*, 4.9 km due west of Leo, is directly south of the tower. Calculate the distance between the tower and the tree, to the nearest kilometre.

11 A yacht sails 8 km on a bearing of 105° from Jervis Bay. How far will it now need to sail along a bearing of 060° so that it is due east of Jervis Bay? Give your answer to the nearest kilometre.

12 Kingsley is 79 km due west of Macksville and Laurieton has a bearing of 207°19′ from Macksville. If Laurieton is due south of Kingsley calculate, to the nearest kilometre, the distance between:

a Kingsley and Laurieton

b Laurieton and Macksville.



13 Two ships, *P* and *Q*, leave port at the same time. Ship *P* sails on a course NE, while ship *Q* sails on a bearing of 124°. After *P* sails 152 km, it is due north of *Q*. Calculate, correct to two decimal places:

a the distance between *P* and *Q*

b how far *Q* has sailed.

14 After flying for 2 hours on a bearing of 306°45′, an aircraft is 600 km north of its starting point. Find the distance travelled and the average speed of the plane. Give your answer correct to two decimal places.

15 A triathlete cycles 20 km on a bearing of 200°, from the end of a swim leg to the finish line.

a How far to the south has the triathlete cycled, to the nearest kilometre?

b What is the bearing of the end of the swim leg from the finish line?

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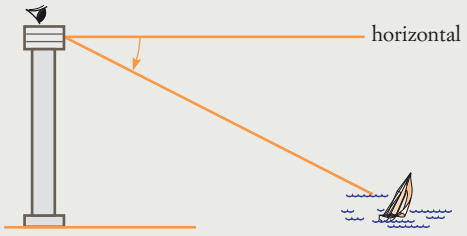
3-10 Angles of elevation and depression

Elevations and bearings

SUMMARY

The **angle of elevation** is the angle of *looking up*, measured from the horizontal. When you feel 'elevated', things are 'looking up'.

The **angle of depression** is the angle of *looking down*, measured from the horizontal. When you feel 'depressed', things are 'looking down'.



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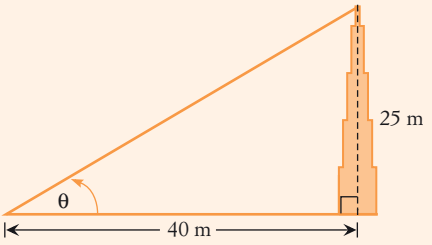
EXAMPLE 20

Imran stands 40 m from the base of a 25 m tower. What is the angle of elevation, to the nearest degree, of the tower from Imran?



Solution

The angle of elevation is θ.

$$\tan\theta = \frac{25}{40}$$

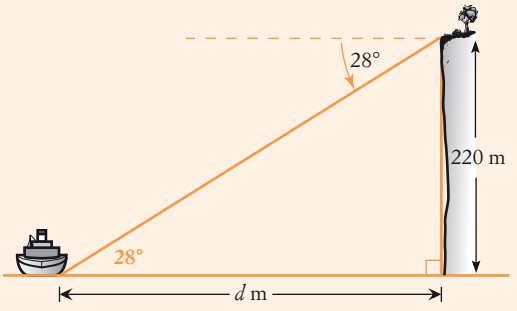
$$\theta = 32.0053\ \ldots$$

$$\approx 32°$$

The angle of elevation of the tower is 32°.

EXAMPLE 21

The angle of depression of a ship from the top of a 220 m vertical cliff is 28°. To the nearest metre, how far is the ship from the base of the cliff?



Solution

The angle of depression of 28° is shown at the top of the cliff, measured downwards from the horizontal (dotted) line.

Let d metres be the distance between the ship and the base of the cliff.

The angle at the ship is also 28° because together with the angle of depression they are alternate angles between parallel lines.

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Method 1

$$\tan 28° = \frac{220}{d}$$

$$d = \frac{220}{\tan 28°}$$

$$d = 413.7598...$$

$$\approx 414$$

The ship is 414 m from the base of the cliff.

Method 2

The third angle in the triangle is 90° – 28° = 62°.

$$\tan 62° = \frac{d}{220}$$

$$d = 220 \tan 62°$$

$$d = 413.7598...$$

$$d \approx 414$$

The ship is 414 m from the base of the cliff.

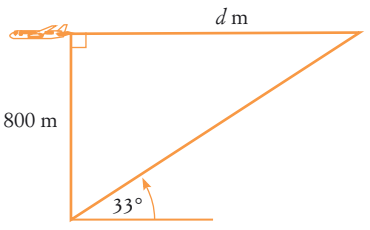
Exercise 3-10 Angles of elevation and depression

Example 20

1 From street level, Sonja sees the top of a 180 m tower at an angle of elevation of 49°. How far, to the nearest metre, is she from the base of the tower?

2 From his ship, Captain Sven observes a lighthouse on top of a 40 m cliff, at an angle of elevation of 16°41′. How far, to the nearest metre, is the ship from the base of the cliff?

3 Ben observed a plane at an angle of elevation of 33°. It flew at a constant height of 800 m until it was directly above him.



a How far was the plane from Ben when he first saw it?

b What was the distance, d metres, travelled by the plane?

Example 21

4 From her apartment, 130 m above ground level, Maddy sights the park at an angle of depression of 50°. How far, to the nearest metre, is the park from the base of Maddy's building?

5 In a concert hall, Liam's seat is 20 m from the stage and 5 m above it. What is his angle of depression to the stage, correct to the nearest degree?

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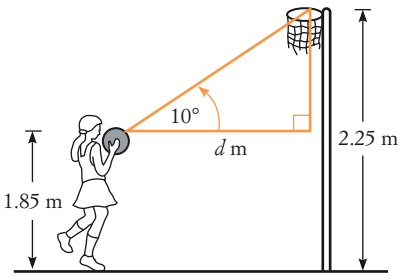
6 Jaz is shooting for goal towards a netball ring at an angle of 10°. Find:

a the distance of the ball from the net

b the distance, d m, between Jaz and the goal post.

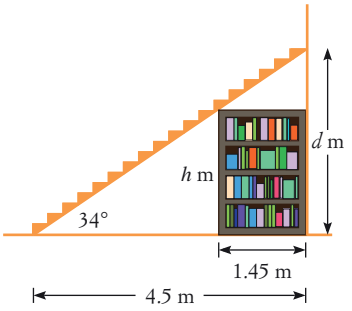


7 A staircase is inclined at 34° and has a horizontal length of 4.5 m. A bookcase, 1.45 m long, is placed under the stairs. Find, correct to two decimal places:

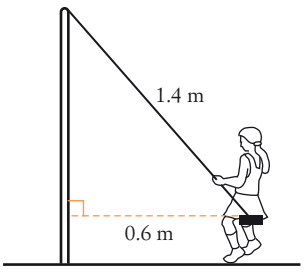
a the vertical height, h metres, of the tallest bookcase that will fit under the stairs

b the distance, d m, of the top of the stairs from the ground

c the length of the staircase.



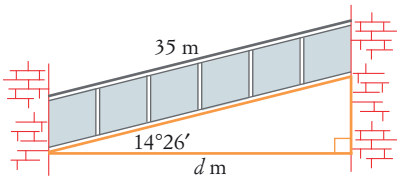
8 Alexis is on a swing of length 1.4 m. She pulls back 0.6 m from the frame to begin swinging. What angle, to the nearest degree, does the swing make with the vertical?



9 A pedestrian skyway, linking two buildings, is 35 m long and inclined at 14°26′ to the horizontal. Calculate correct to 2 decimal places:

a the distance between the two buildings

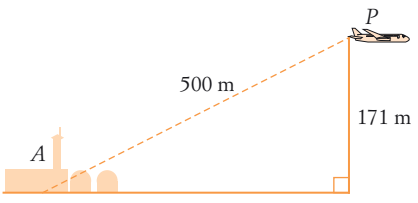
b the vertical rise of the skyway.



10 A plane, P, leaves an airport, A, and flies 500 m to reach a height of 171 m.

a What is the angle of elevation of the plane from the airport?

b What is the horizontal distance between the plane and the airport?



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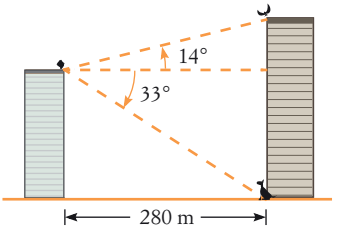
11 Amien stands on the roof of his apartment block and sees a bird on the roof of a neighbouring office building at an angle of elevation of 14°. At the same time he sees a dog at the bottom of the office building at an angle of depression of 33°. If the two buildings are 280 m apart, find to the nearest metre:



a how far is the bird from Amien?

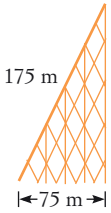
b how far is the bird from the dog?

12 A 2.5 m totem pole casts a shadow of 2 m.

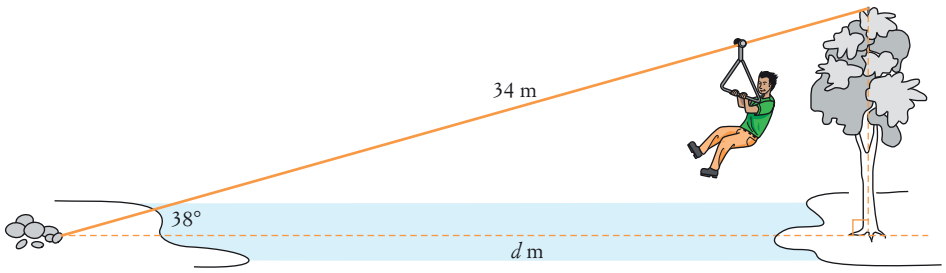
a What is the length of the shadow of a 6.4 m tree at the same time, correct to two decimal places?

b What is the angle of elevation of the Sun, correct to the nearest minute?

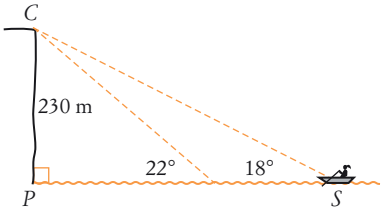
13 A section of a rollercoaster is shown in the diagram. What is the angle of inclination of this section, correct to the nearest minute?



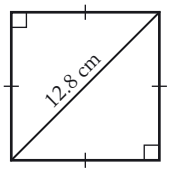
14 A flying fox is constructed between the top of a tree and a large rock. It has a length of 34 m and an angle of inclination of 38°. What is the (horizontal) distance between the tree and the rock, correct to one decimal place?



15 Stephanie, at point S, is rowing towards Point Perpendicular, P, a 230 m vertical cliff. She measures the angle of elevation of the top of the cliff, C, to be 18° and, three minutes later, she measures the angle of elevation as 22°. How far, to the nearest metre, did Stephanie row during this period?



16 The diagonal of a square is 12.8 cm long. Find the length of one side, correct to two decimal places:



a using trigonometry

b using Pythagoras' theorem

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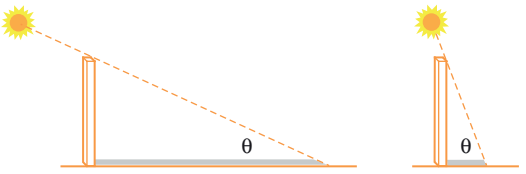
INVESTIGATION

THE ALTITUDE OF THE SUN

You will need: A metre ruler, a tape measure, a watch and a calculator.

The length of an object's shadow depends on the altitude of the Sun, the angle at which the Sun's rays hit the ground. When the Sun is at a low angle (at the start or end of the day), the shadow is long. When the Sun is at a high angle (in the middle of the day), the shadow is short.

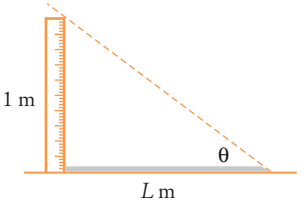


1 Go outside and find a spot to measure the length of a metre ruler's shadow at different times of the day, preferably at hourly intervals. Copy the following table.

Time	Length of shadow, L	Altitude, θ (where $\tan\theta = \frac{1}{L}$)
e.g. 9:00 a.m.	1.12 m	41°46′

2 The altitude of the Sun, θ, can be found using $\tan\theta = \frac{1}{L}$

Use this formula to find θ to the nearest minute. Write your results in the table.



3 What patterns do you notice in your completed table?

4 What happened to the altitude of the Sun over the day?

5 When was the shadow of the ruler:

a longest? **b** shortest?

6 **a** What is the altitude of the Sun when the shadow is the same length as the ruler?

b What is the altitude of the Sun when the ruler has no shadow?

c Estimate the times at which the two events described in parts **a** and **b** happened.

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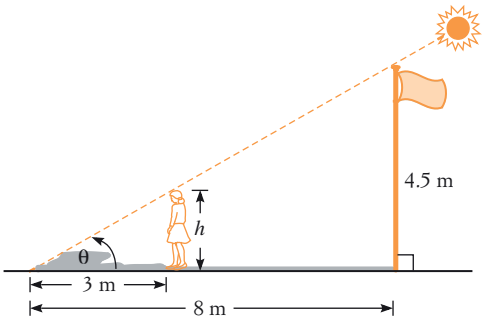
Sample HSC question

At a certain time during the day, Ashleigh noted that her shadow was 3 m long while the shadow cast by a 4.5 m flagpole was 8 m long.



a Find θ, the angle of elevation of the Sun, correct to the nearest minute.

b Calculate Ashleigh's height in metres, correct to four decimal places, using:

- **i** similar triangles
- **ii** angle θ and a trigonometric ratio.

c Which answer in part **b** is more accurate: **i** or **ii**? Give a reason for your answer.

Study tip

Showing your working

Why is it important to show your working when answering a mathematics question?

- To demonstrate that you know your mathematics and the steps required to solve the problem
- So that, in an exam, you can receive 'part marks' for correct working, even if you get the wrong answer
- To show that you really do know how to answer the question and aren't simply guessing or bluffing
- To remind yourself how you solved the problem, especially when studying
- So that you can describe how you achieved your answer (explaining something to someone often reinforces your learning and understanding)

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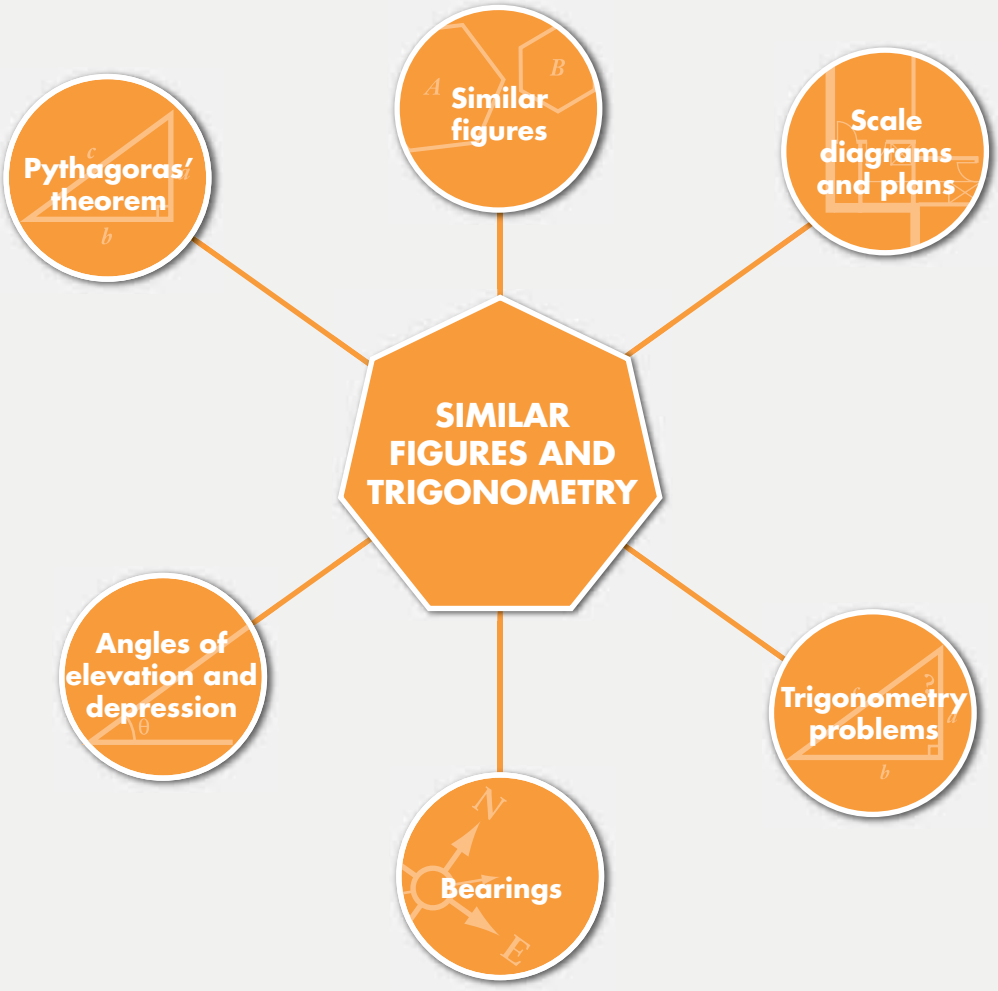
CHAPTER REVIEW

Topic overview

This chapter, 'Similar figures and trigonometry', revised and extended measurement skills in similar figures and trigonometry. You investigated similarity in the real world, interpreted scale factors and scale diagrams, including house plans. You learnt how to find unknown sides and angles in right-angled triangles using the trigonometric ratios and Pythagoras' theorem, and solved problems involving bearings and angles of elevation and depression.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map has been printed below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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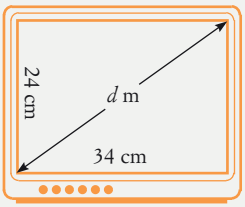
Revision

1 A TV screen is 34 cm long and 24 cm high. What is the length of its diagonal, correct to the nearest centimetre?

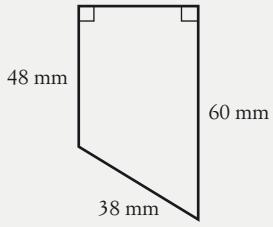


Exercise 3-01

2 Find the perimeter of each figure, correct to two decimal places.

Exercise 3-01

a



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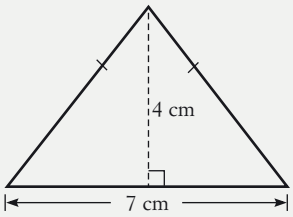
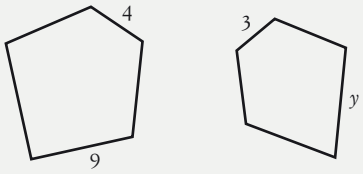


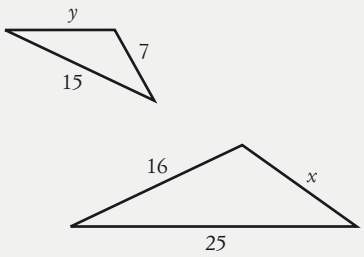
3 Find the value of the pronumeral in each pair of similar figures.

Exercise 3-02

a



b



4 An object is 20 cm from a camera lens, and the image is projected onto a screen that is 3 m from the lens. If the object is 20 mm tall:

Exercise 3-02

a what is the scale factor? **b** how tall is the image?

5 Huynh is 1.4 m tall and casts a shadow of length 2 m while, at the same time, a 9 m pole also casts a shadow. What is the length of the pole's shadow, correct to two decimal places?

Exercise 3-02

6 The plan of a house is drawn to a scale of 1 : 250. If a bedroom measures 18 mm by 15 mm on the plan, what is the actual area of the bedroom, to the nearest m^2?

Exercise 3-03

7 This is a scale diagram of a squash court.

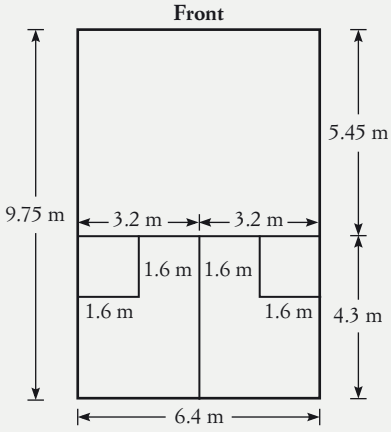
Exercise 3-03

a What is the scale used?

b What is the area of the front-of-court section, to the nearest m^2?

c What is the area of each (small) service square?

d How many square metres of timber flooring are needed for a squash court, to the nearest m^2?



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Exercise 3-04

8 For this question, refer to the house plan in Example **9** on page 106.

a What symbol is used for:

i a washing machine?

ii a stove?

iii a vanity basin?

b What is the combined area, correct to the nearest m^2, of the four bedrooms?

c The family room floor is to be covered with tiles measuring 30 cm by 30 cm. How many whole tiles are needed?

Exercise 3-05

9 Evaluate each ratio, correct to two decimal places.

a sin 39° **b** tan 84°13′ **c** cos 52.7°

Exercise 3-05

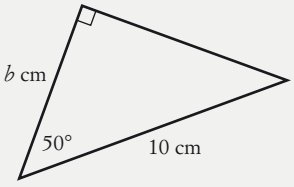
10 Find the size of each angle, correct to the nearest degree.

a $\cos \beta = 0.3674$ **b** $\sin \theta = 0.7354$ **c** $\tan \alpha = 1.2964$

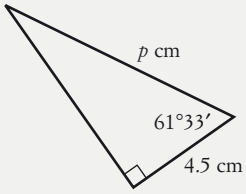
Exercise 3-06

11 Find the value of each pronumeral, correct to two decimal places.

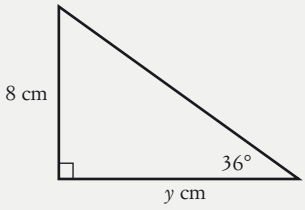
a



b

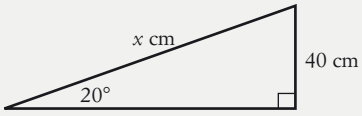


c



Exercise 3-06

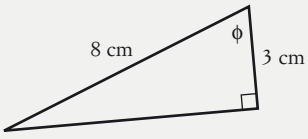
12 A skateboard ramp is 40 cm high and makes an angle of 20° with the ground. How long is the ramp, to the nearest centimetre?



Exercise 3-07

13 Find the value of each pronumeral, correct to the nearest degree.

a



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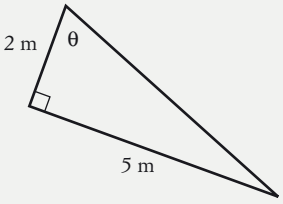
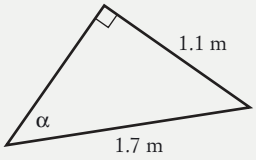
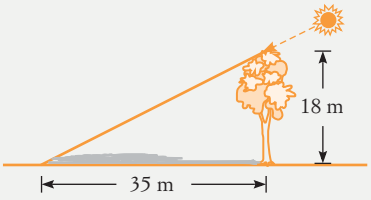


c



Exercise 3-07

14 A tree 18 m high casts a shadow of length 35 m. What angle do the rays of the Sun make with the ground, to the nearest minute?



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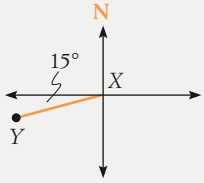
15 A 13 m ladder is placed against a building, so that its foot is 3 m from the base of the building. Exercise 3-07

a How high is the top of the ladder above the ground, to the nearest 0.1 m?

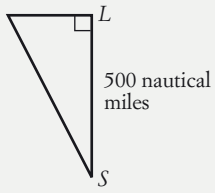
b What angle does the ladder make with the ground, to the nearest minute?

16 What is the bearing of Y from X? Exercise 3-08



17 Sketch a point that has a bearing of 157° on a compass rose. Exercise 3-08

18 A lighthouse is 500 nautical miles due north of a ship. If the ship sails on a course of 333° until it is due west of the lighthouse, how far west is it, correct to two decimal places? Exercise 3-09

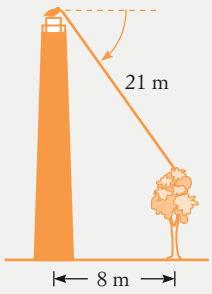


19 A boat sails due east for 18 km, then turns and sails due south for 5 km. Exercise 3-09

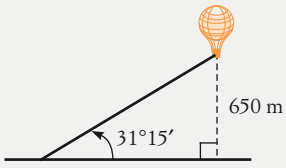
a How far is the boat from its starting point, correct to two decimal places?

b What is the boat's bearing from its starting point, to the nearest degree?

20 A pigeon is perched on top of a beacon. The direct distance between the pigeon and the top of a tree is 21 m. If the tree and the tower are 8 m apart, what is the angle of depression of the top of the tree from the pigeon (to the nearest degree)? Exercise 3-10



21 Jayden saw a hot air balloon flying horizontally at an angle of elevation of 31°15′ and a height of 650 m. How far towards Jayden, to the nearest metre, must the balloon fly before it is directly above Jayden? Exercise 3-10



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Practice paper 1

Recommended time: One hour

Section 1

10 objective-response questions: 1 mark each

For Questions 1 to 6, select the correct answer, A, B, C or D

Exercise 2-02

Question 1

Simplify $\frac{5k^3p}{25k^2}$

(A) $5kp$

(B) $5k^5p$

(C) $\frac{p}{5}$

(D) $\frac{kp}{5}$

Exercise 3-03

Question 2

Argyle Street is 850 m long. Its length on a map with a scale of 1 : 5000 is:

(A) 17 mm

(B) 85 mm

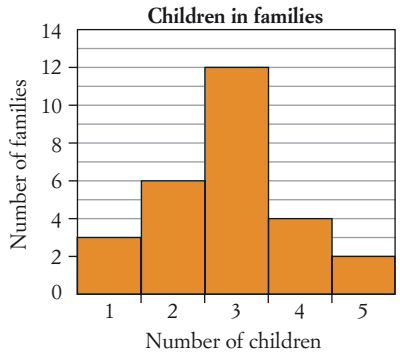
(C) 170 mm

(D) 425 mm

Exercise 1-07

Question 3

This frequency histogram shows the results of a survey about the number of children in each family. How many families were surveyed?



(A) 12

(B) 15

(C) 27

(D) 77

Exercise 2-04

Question 4

Use the formula $v = u + at$ to find the value of v if $u = 8$, $a = 10$ and $t = 5$.

(A) $v = 90$

(B) $v = 58$

(C) $v = 42$

(D) $v = 113$

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Exercise 1-03

Question 5

Which one of these is an example of categorical data?

(A) the capacity of Warragamba Dam

(B) the postcode of a town

(C) the height of a person in your class

(D) the time it took a triathlete to complete her race

Exercise 2-05

Question 6

Solve the equation $3(5y - 4) = 6$.

(A) $y = \frac{2}{5}$

(B) $y = 1\frac{1}{5}$

(C) $y = 3\frac{3}{5}$

(D) $y = -\frac{2}{5}$

For Questions 7 to 10, write the answer only (do not show your working).

Exercise 2-03

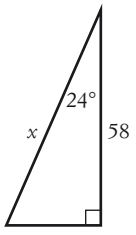
Question 7

Expand and simplify: $3(2p + 5) - 4(p + 3)$.

Exercise 3-06

Question 8

Calculate the value of x in the diagram, correct to two decimal places.



Exercise 3-08

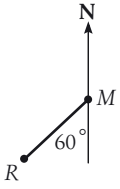
Question 9

If $x = -2$, $y = 6$ and $w = 12$, then evaluate correct to two decimal places the value of $\sqrt{\frac{x+y}{20-w}}$

Exercise 3-08

Question 10

In the diagram on the right, what is the true bearing of Robertson (R) from Margaretville (M)?



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Section 2

3 questions: 10 marks each

Question 11 **Marks**

Exercise 1-05

Exercise 1-08

(a) Explain in words the meaning of '**stratified sample**'. **2**

(b) The waiting times, in minutes, for patients at the Westvale Health Centre are shown in the stem-and-leaf plot on the right.

Stem	Leaf
0	2 3 4 5
1	0 1 3 6 □ 8
2	1 1 3 4 4
3	1 2
4	1
5	1

(i) One entry is missing and is shown as □.
What *waiting time* could be represented by this entry? **1**

(ii) How many patients were surveyed? **1**

(iii) Calculate, correct to one decimal place, the percentage of patients who waited for longer than half an hour. **1**

(iv) What was the shortest waiting time? **1**

Exercise 3-10

(c) From the top of a lighthouse 11.2 m high, the angle of depression of a boat at sea is 12°. How far, to the nearest metre, is the boat from the base of the lighthouse? **2**

Exercise 2-06

(d) The volume of a pyramid has the formula $V = \frac{1}{3}Ah$, where A is the area of the pyramid's base and h is its perpendicular height. If a pyramid 30 m high has a volume of 150 m^3, what is the area of its base? **2**

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Question 12 **Marks**

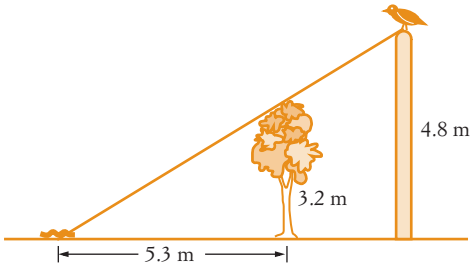
Exercise 1-03

Exercise 3-02

(a) Is the colour of a car an example of nominal data or ordinal data?

Give a reason for your answer. **2**

(b) A bird sitting on top of a pole of height 4.8 m sees a worm on the ground when it looks over the top of a tree of height 3.2 m.



(i) If the worm is 5.3 m from the base of the tree, how far is it from the base of the pole? **2**

(ii) How far is the bird from the worm, in a direct line correct to one decimal place? **1**

Exercise 2-04

(c) The following formula gives the number of hours it takes for a man's blood alcohol content (BAC) to return to zero after he has been consuming alcohol:

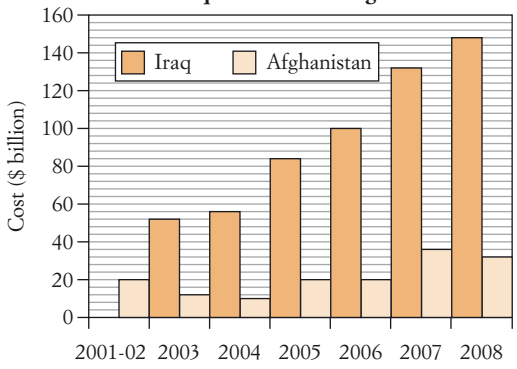
$$\text{Number of hours} = \frac{\text{BAC}}{0.015}$$

If Ben has a BAC of 0.024, how many hours and minutes will it take for his BAC to return to zero? **2**

Exercise 1-01

(d) The graph on the right shows the amount spent by the USA on the Iraq and Afghan wars from 2001 to 2008.

Comparative Department of Defence spending on the Iraq war and the Afghan war



(i) How much was spent by the USA on the Iraq war in 2006? **1**

(ii) Describe the trend in spending for the Iraq war. **1**

(iii) Which year showed the greatest difference in spending between the Iraq and Afghan wars? **1**

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Exercise 1-07

Question 13 **Marks**

(a) The masses, in kilograms, of 20 Year 11 students were recorded:

58	43	62	52	49	48	52	45	68	72
54	41	65	41	48	51	60	66	57	69

(i) Are people's masses discrete data or continuous data? **1**

(ii) Organise the data into a frequency table with classes 40 – < 45, 45 – < 50, and so on. **1**

(iii) Draw a frequency histogram to represent the data. **2**

(iv) What percentage of students were 60 kg or more? **1**

Exercise 3-04

(b) The diagram on the next page shows the floor plan of a house. All measurements are in metres

(i) What is the thickness of the external walls, in centimetres? **1**

(ii) What is the internal perimeter of the garage, in metres? **1**

(iii) Bedroom 1 is to be carpeted. The carpet is 3.6 m wide and is sold at $165 per metre laid. How much will it cost to carpet Bedroom 1? **2**

(iv) How many windows in this house face north? **1**

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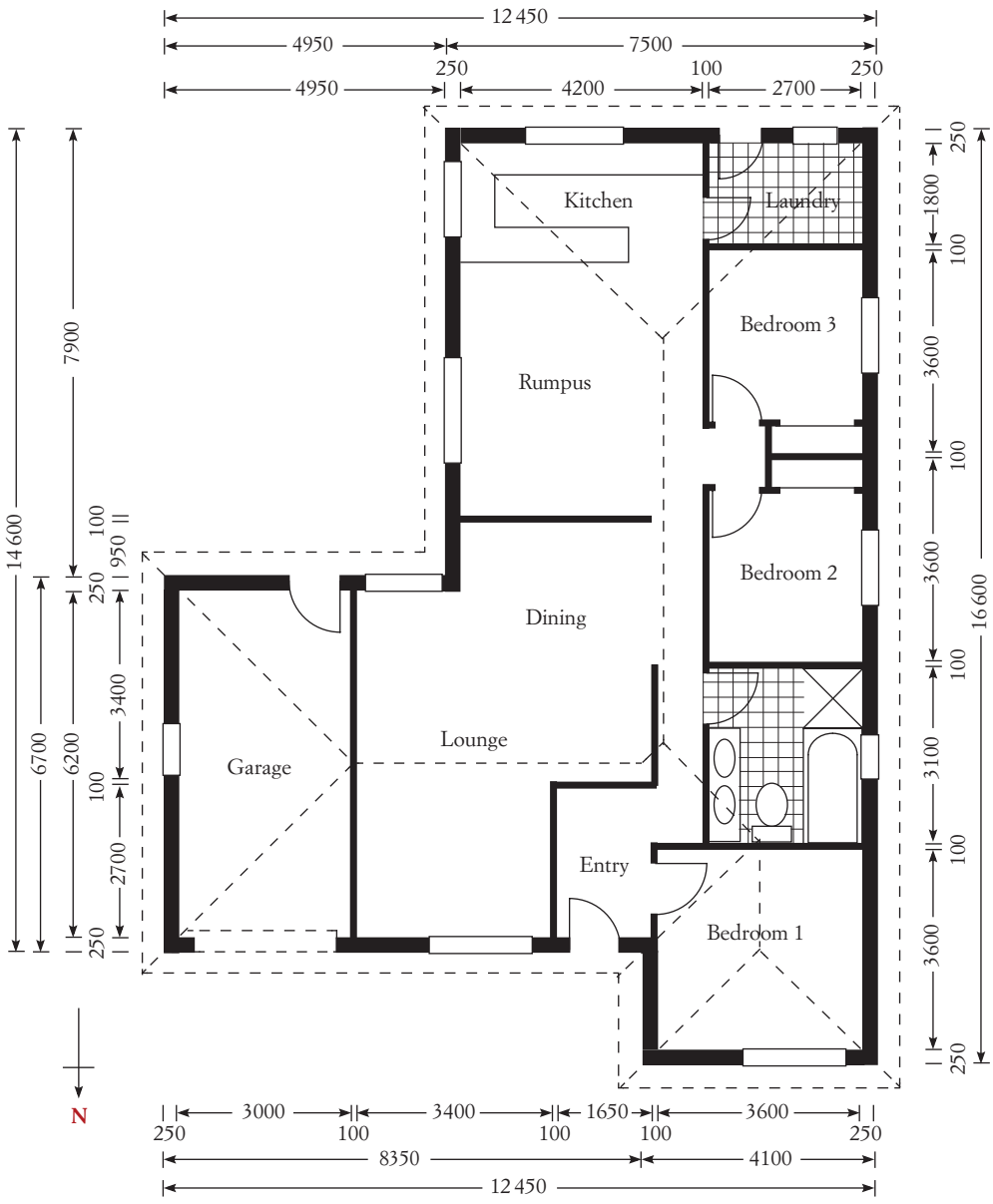

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FINANCIAL MATHEMATICS

4.

EARNING MONEY AND TAXATION

The Australian federal government introduced a goods and services tax (GST) in 2000 which brought us into line with other countries that impose a tax on spending. A tax on spending is a fairer way of collecting revenue because it is a 'user pays' system. Before the introduction of the GST, the government raised most of its revenue from personal income tax, which is a tax on earnings.

CHAPTER OUTLINE

FM1	4-01	Wages, salaries and overtime
FM1	4-02	Commission, piecework and royalties
FM1	4-03	Bonuses, allowances and annual leave loading
FM1	4-04	Government allowances and pensions
FM1	4-05	Gross and net pay
FM1	4-06	Budgeting
FM3	4-07	Income tax and Medicare levy
FM3	4-08	PAYG and tax returns
FM3	4-09	GST and VAT
FM3	4-10	Graphs of tax rates

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IN THIS CHAPTER YOU WILL:

- calculate monthly, fortnightly, weekly, daily and hourly pay rates from a given salary
- calculate wages involving hourly rates and penalty rates, for example, overtime; and special allowances, including allowances for wet work, confined spaces, toxic substances, heat, heights, etc.
- calculate earnings based on commission (including commission based on a sliding scale), piecework and royalties
- calculate annual leave loading
- calculate payments based on government allowances and pensions
- determine deductions from income, for example, tax instalments, superannuation contributions, health-fund instalments, union fees and HECS repayments
- calculate net pay following deductions from gross pay
- prepare a budget for a given income, taking into account fixed and discretionary spending.
- calculate the amount of allowable deductions from gross income, to calculate taxable income
- calculate the Medicare levy (basic levy only) and Pay As You Go (PAYG) tax payable or refund owing, using current tax scales
- calculate the goods and services tax (GST) payable on a range of goods and services
- create and interpret graphs to illustrate and describe different tax rates.

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TERMINOLOGY

allowable deduction	allowance	annual leave loading
bonus	budget	commission
double time	expense	goods and services tax (GST)
gross pay	income	income tax
Medicare levy	net pay	overtime
PAYG tax	piecework	royalties
salary	taxable income	tax refund
time-and-a-half	value-added tax (VAT)	wage

Assignment 4

Percentage calculations

Mental percentages

SkillCheck

1 Find the cost of:

a five calculators @ \$17.60 each

b 1.5 kg carrots @ \$3.10 per kg

c 1400 protractors @ 35c each.

2 Calculate, correct to the nearest cent:

a 34% of \$42 568 **b** \$153 000 ÷ 52.18 **c** 1.5% of \$85 629

3 What percentage is \$2360 of \$36 580, correct to the nearest whole number?

4 Evaluate 58 000 + 0.45 × (250 000 – 180 000).

Wages and salaries

Wages

4-01 Wages, salaries and overtime

Wages and overtime

A person's **wage** is calculated on the number of hours they worked in a given period, with **overtime pay** being paid at a higher rate for working beyond normal hours. Types of overtime are shown in the table below. People who earn a wage include waiters, mechanics and receptionists.

Type of overtime	Meaning
Time-and-a-half	1.5 × normal rate
Double time	2 × normal rate
Double time-and-a-half	2.5 × normal rate
Triple time	3 × normal rate

EXAMPLE 1

This week, Christy worked at a childcare centre for 37 hours at \$16.40 per hour plus 6 hours overtime at time-and-a-half. Calculate her earnings for the week.

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Solution

Normal pay = 37 × \$16.40 = \$606.80

Overtime pay = 6 × 1.5 × \$16.40 = \$147.60

Total earnings = \$606.80 + \$147.60 = \$754.40

EXAMPLE 2

Ian works as a bartender at an RSL club. His timesheet for three days he worked in April is shown below.

Day	Start time	Finish time
Tuesday	3:00 p.m.	10:00 p.m.
Wednesday (Anzac Day holiday)	9:00 a.m.	2:00 p.m.
Thursday	12:00 noon	7:00 p.m.

The rate for bartenders is \$18.17 per hour between 8:30 a.m. and 6:00 p.m., time-and-a-half between 6:00 p.m. and 11:30 p.m., and double time on public holidays. Calculate Ian's pay for the three days he worked.

Solution

Day	Normal hours	Time-and-a-half	Double time
Tuesday	3	4	0
Anzac Day	0	0	5
Thursday	6	1	0
Total hours	**9**	**5**	**5**

Normal pay = 9 × \$18.17 = \$163.53

Time-and-a-half pay = 5 × 1.5 × \$18.17 = \$136.275

Double-time pay = 5 × 2 × \$18.17 = \$181.70

Total pay = \$163.53 + \$136.275 + \$181.70

= \$481.505

≈ \$481.51

Salaries

A **salary** is a fixed amount of annual pay (yearly pay), paid weekly, fortnightly or monthly. People who earn a salary include teachers, website designers and police officers.

To convert a yearly salary to a weekly amount, divide by 52.18 because one year is not exactly 52 weeks.

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Number of weeks in an 'average year' $= 365\frac{1}{4} \div 7$ **1 week = 7 days**

$= 52.1785$

≈ 52.18

Every four years, a leap year has 366 days so an 'average year' has $365\frac{1}{4}$ days.

SUMMARY

1 year = 12 months

1 fortnight = 2 weeks

1 year = 52.18 weeks for salary earners

1 year = 52 weeks for wage earners (because they are paid 52 times in a year)

Note: 1 month ≠ 4 weeks. To convert between weeks and months, convert to years first.

EXAMPLE 3

Siobhan earns a salary of $98 025 as an IT consultant. To the nearest cent, how much is she paid:

a monthly?

b fortnightly?

Solution

a Monthly pay = $98 025 ÷ 12

= $8168.75

b Fortnightly pay = $98 025 ÷ 52.18 × 2 **1 year ≈ 52.18 weeks**

= $3757.1866

≈ $3757.19

Answers to money problems should be rounded to the nearest cent, and not necessarily to the nearest 5 cents. It is only for cash payments that amounts are rounded to the nearest 5 cents. Amounts to the nearest cent can still be paid electronically by cheque or EFTPOS.

Exercise 4-01 Wages, salaries and overtime

1 A gardener works from 6:30 a.m. to 5:00 p.m., Monday to Friday, and earns $18.45 per hour. Calculate the gardener's weekly pay, correct to the nearest cent.

Example 1

2 A computer operator is paid $20.50 per hour. She works 9 hours per day, Monday to Friday, and 5 hours on Saturday, for which she is paid time-and-a-half. Calculate her weekly wage.

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3 If an annual salary of $120 000 is paid weekly, which of the following is the correct weekly amount? Select **A**, **B**, **C** or **D**.

A $2307.69 **B** $2299.73 **C** $2307.70 **D** $2299.75

4 If a labourer earns a weekly wage of $523.30, which of the following is the amount he earns in a year? Select **A**, **B**, **C** or **D**.

A $27 211.60 **B** $27 305.79 **C** $27 211.00 **D** $27 305.80

5 An Internet website designer works a 38-hour week and earns $2115.35 every fortnight. Calculate her earnings:

a per year **b** per week **c** per hour.

6 A trainee nurse works 36 hours at a standard rate of $17.44 and 10 hours at time-and-a-half. Calculate her total pay.

7 A childcare worker earns $682.40 per week. How much will she earn in:

a a year? **b** a month?

8 On a long weekend, a waitress is paid time-and-a-half on Sunday and double time on Monday. How much does she earn for working 9 hours on each of the days if her normal wage rate is $17.54 per hour?

9 A chef earns $23.10 per hour, but is paid time-and-a-half for working after 8:00 p.m. on weekdays and any time on weekends. The timesheet below shows the times he worked last week. Copy and complete the table, then calculate the chef's total pay for the week. Example 2

Day	Start time	Finish time	Normal hours	Overtime hours
Monday	12:00 noon	6:00 p.m.		
Tuesday	12:00 noon	6:00 p.m.		
Wednesday	12:00 noon	7:00 p.m.		
Thursday	12:00 noon	10:00 p.m.		
Friday	12:00 noon	12:00 midnight		
Saturday	4:00 p.m.	12:00 midnight		
Sunday	4:00 p.m.	10:00 p.m.		
		Total hours		

10 A personal assistant earns $18.30 per hour for a 35-hour week, time-and-a-half for the first 4 hours of overtime, and double time thereafter. Calculate her total earnings for a week in which she worked from 8:30 a.m. to 5:30 p.m. from Tuesday to Saturday.

11 A Member of Parliament earns a salary of $145 239. How much would he be paid if he was paid: Example 3

a fortnightly? **b** monthly? **c** weekly?

12 A law clerk earns a salary of $83 215. How much does he earn:

a a month? **b** a fortnight? **c** a weekday?

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13 A computer systems engineer earns a salary of $104 235. If he works a 42-hour week, calculate how much he earns:

a per hour

b per fortnight

c per month.

14 The table below shows the hours worked by a hotel porter. If he earns $16.28 per hour from 0600 hours to 1900 hours (in 24-hour time) and time-and-a-half otherwise, calculate his weekly pay.

Day	Start time	Finish time
Tuesday	0630	1400
Wednesday	0800	1530
Friday	1500	2330
Sunday	0500	1330

15 A newspaper printer worked for 33 hours at $16.72 per hour during the week before Good Friday, then for 6 hours at double-time-and-a-half on Good Friday. Calculate the printer's total pay.

16 A survey field-hand earns $22.18 per hour, working from 7:00 a.m. to 4:30 p.m. every weekday. Calculate how much he is paid:

a a day

b a week

c a month.

17 Which of the following people earns the most? Select **A**, **B**, **C** or **D**.

A Ali: $18.70 per hour for a 35-hour week

B Boun: $580 per week

C Carlos: $3460 per month

D Dimitri: $31 600 salary

18 A baker is paid $163.22 each day for working from 4:30 a.m. to 1:00 p.m. If he works 6 days per week, how much does he earn:

a per hour?

b per week?

c per year?

19 Each day, a bricklayer earns $28.10 per hour for 9 hours and time-and-a-half for any extra hours. Last week, he worked the following number of hours each day: 9, 8, 10, 14 and 12.5. Calculate his total pay for the week.

20 Alex worked 32 hours at the normal rate and 4 hours at time-and-a-half, earning a total of $719.34. Find her normal rate of pay per hour.

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TECHNOLOGY: PAYROLL CALCULATIONS

1 Enter the following data into a spreadsheet and use appropriate formulas to calculate the normal and overtime pays.

	A	B	C	D
1		**Number of hours**	**Pay rate per hour**	**Pay**
2	Normal	37	$18.65	
3	Time-and-a-half	3.5		
4	Double time	1		
5	**Total hours**		**Total pay**	

2 Use your spreadsheet to check the answers for questions **8**, **9**, **10**, **14**, **19** and **20** in Exercise 4-01.

4-02 Commission, piecework and royalties

Some workers are not paid by the amount of time they work, but by the number of items they make or process.

- **Commission** is income based on the value of items sold. A real estate agent earns commission on each house sold.
- **Piecework** is income based on the number of items processed. Fruit pickers and garment makers earn money this way.
- **Royalties** is income based on the number of copies sold. Musicians and authors earn royalties from sales of their music and books.

EXAMPLE 4

Josh is paid 45c for each shirt folded and boxed. Last week he processed 1440 shirts. How much did he earn?

iStockphoto.com/zoomstudio

Solution

Earnings = 1440 × $0.45 = $648

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Percentage calculations

Mental percentages

EXAMPLE 5

Ally earns a **retainer** of \$415 per week plus a commission of 8.5% on the value of Wonder mops sold.

A retainer is a fixed amount earned that does not depend on the number of items sold.

a To the nearest cent, how much does Ally earn for selling \$1595 worth of mops in one week?

b If Ally earned \$700 in a week, what was the value of mops sold that week, to the nearest cent?

Solution

a Earnings $= \$415 + 8.5\% \times \$1595$

$= \$550.575$

$\approx \$550.58$

b Let: $\$V =$ value of mops sold

Earnings $=$ retainer $+ 8.5\% \times V$

$700 = 415 + 0.085V$

$285 = 0.085V$

$V = \dfrac{285}{0.085}$

$= 3352.9411...$

≈ 3353.94

The value of the mops sold is \$3353.94.

Example 4

Exercise 4-02 Commission, piecework and royalties

1 Brett is paid \$3.30 for each bucket of cherries he picks. If he picked 31 buckets of cherries, which of the following is the amount he earned? Select **A**, **B**, **C** or **D**.

A \$102.30 **B** \$93.00 **C** \$102.00 **D** \$99.00

2 Danny is paid 6.5c for each envelope he folds and addresses. If he can process 14 envelopes in 5 minutes:

a how many envelopes can he process in 1 hour?

b how much does he earn in $2\frac{1}{2}$ hours?

3 Leila is paid 75c for each garment she irons. If a basket contains 18 items and she irons five basketfuls, which of the following is the amount she earns? Select **A**, **B**, **C** or **D**.

A \$13.50 **B** \$37.50 **C** \$90 **D** \$67.50

4 A stockbroker receives a commission of 2% of the selling price of shares.

a If he sold 350 shares at \$4.30 each, calculate his commission.

b What was the value of the shares he sold if he earned \$135 commission?

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5 A car salesman's commission is 4% for the first \$18 000 worth of sales and 2.5% for the rest. If he is also paid a retainer of \$320, calculate his total earnings in a week in which he sold \$270 000 worth of cars.

6 A real estate agent charges the following commission for selling properties, based on this sliding scale of property prices:

- 5% for the first \$85 000
- 3% for the next \$60 000
- 2.5% for the remaining amount.

What commission will the real estate agent earn for selling:

a an apartment for \$242 400?

b a house for \$310 000?

7 A songwriter earns a royalty of 47c every time her song is played on the radio. If, in one week, her song was played 434 times on radio stations, calculate her royalty payment for that week.

8 An inventor is paid 1.8% of the total sales of a can-opener that uses his original design. How much does the inventor earn if 8400 can-openers are sold at \$8.95 each?

9 Missy earns 1.5% of the sales of her CD album. Last year, 1 245 000 copies of her album sold at \$39.50 each.

a Calculate the total value of the sales of Missy's CDs.

b Calculate Missy's royalties from these sales.

10 Truong earns 11 cents for every advertising brochure he delivers to letterboxes. Example 5

a How much does Truong earn in a day in which he delivers 1329 brochures?

b If Truong earned \$90.31 one day, how many brochures did he deliver?

11 Nick earns \$5.24 for every car he vacuums. He vacuums an average of $2\frac{1}{2}$ cars per hour. How much will he earn in 6 hours?

12 Libby earns 8% commission for selling cosmetics door-to-door. What value of cosmetics did she sell this week if she earned \$62.72?

13 An author earns 13% in royalties on the sale of her novel. If 34 700 copies of her novel sold for \$32.95 each, calculate the author's royalty payment.

14 Pranathi earns \$12.40 for every dress she sews. If she can make 7.5 dresses per day, calculate her earnings for a 5-day week.

15 Debbie earns 5% commission on all the plastic containers she sells. If she earned \$83.90 commission this week, what was the total value of the plastic containers she sold?

16 An agent earned, as his commission, \$2024 from the actor's fee in a commercial. What percentage commission does this represent if the actor's fee was \$23 000?

17 Grant is paid 15.5 cents for every newspaper he delivers. How many newspapers did he deliver if he was paid \$214.21?

18 At Christmas time, Shane earns \$1.05 for every item he giftwraps.

a Calculate Shane's earnings for wrapping 114 gifts in one day.

b How many gifts did Shane wrap on the next day if his earnings were \$223.65?

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19 Salma earns \$8.40 each time she taste-tests a new brand of food. How many tastings must she perform to earn over \$500?

20 Travis is paid the following commission rates for selling mobile phone plans:

- 4.8% for the frst \$3000
- 6% for the next \$3000
- 7.5% for the remaining amount.

Calculate Travis' commission for selling plans amounting to \$8758 in value.

4-03 Bonuses, allowances and annual leave loading

Bonuses, allowances and annual leave loading are examples of extra pay.

A **bonus** is paid to employees who produce work of a high standard or volume, or for meeting an important quota, goal or deadline. The owners of some businesses give their employees a Christmas bonus to reward them for a productive year's work.

An **allowance** is paid to employees for work-related expenses (such as travelling, tools or laundry), or who work under difficult or dangerous conditions (such as in wet weather, in isolated areas, or great heights).

Annual leave loading (also called **holiday loading**) is extra pay given during annual leave. It is usually paid at a rate of 17.5% of 4 weeks normal pay.

EXAMPLE 6

A construction worker in an isolated area earns a weekly wage of \$1080.60 plus a site allowance of \$104.80 per week, a meal allowance of \$16.20 per day and a height allowance of \$5.60 per hour. If she works 9 hours a day for 5 days, calculate her total weekly pay.

Solution

Write all incomes as weekly amounts.

$$\begin{aligned}
\text{Wage} &= \$1080.60 \\
\text{Site allowance} &= \$104.80 \\
\text{Meal allowance} &= 5 \times \$16.20 \\
&= \$81.00 \\
\text{Height allowance} &= 5 \times 9 \times \$5.60 \\
&= \$252.00 \\
\text{Total weekly pay} &= \$1080.60 + \$104.80 + \$81.00 + \$252.00 \\
&= \$1518.40
\end{aligned}$$

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EXAMPLE 7

Hayden earns a yearly salary of $46 598. Calculate his holiday loading, which is 17.5% of the normal pay for 4 weeks, correct to the nearest cent.

Solution

4 weeks pay = $46 598 ÷ 52.18 × 4

= $3572.096…

Do not round your answer. Leave it on the calculator display.

Holiday loading = 17.5% × $3572.096…

= $625.116…

≈ $625.12

Holiday pay

Exercise 4-03 Bonuses, allowances and annual leave loading

1 Name an occupation that may require working in each situation.

a wet weather
b isolated areas
c cramped conditions
d hot temperatures
e cold temperatures
f high places
g dangerous conditions
h dirty conditions.

2 A painter on the Sydney Harbour Bridge earns $24.15 per hour, as well as a height allowance of $25.10 per day. If he works 10 hours per day for 5 days, calculate his total earnings. Example 6

3 A police officer earns a salary of $68 275. Calculate her annual leave loading if it is 17.5% of 4 weeks normal pay. Example 7

4 At the end of the year, the owners of a small business gave their employees a Christmas bonus of 60% of a normal week's pay. Calculate Lisa's total pay for that week if her usual pay is $419.70.

5 A miner earns a wage of $28.42 per hour, plus an allowance of $9.97 per hour for working underground in confined spaces. If the miner works from 8:30 a.m. to 4:00 p.m. each day from Monday to Friday, calculate:

a his total weekly income
b his yearly income.

6 Scientists working at the South Pole earn a salary of $83 204 plus a weekly allowance of $177.35 for working under extreme and isolated conditions. Calculate their fortnightly pay.

7 For meeting a target one week ahead of schedule, the owners of a firm gave their staff a bonus equal to 15% of the normal pay for 1 month. What is Amanda's bonus if her salary is $53 045?

8 A window cleaner earns $17.90 per hour and a multi-storey allowance of $23.40 per day. If he works 10 hours each weekday, calculate his total weekly earnings.

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9 Each day, an electrician is paid $124.30 plus the following allowances: Site allowance $12.25, meal allowance $8.40 and laundry allowance $3.21.

a If the electrician works 6 days per week, calculate his total weekly earnings.

b Calculate the electrician's annual leave loading if it is 17.5% of the normal pay for 4 weeks (without allowances).

10 An oil rig worker earns a wage of $32.56 per hour, plus a dirt allowance of $7.49 per hour. If she works $9\frac{1}{2}$ hours a day, calculate her total weekly earnings for a 5-day week.

11 The owners of a biscuit factory gave their casual staff a bonus of $272 at Christmas, while the permanent staff received a holiday loading of 17.5% of the pay for 4 weeks. If the normal weekly pay for the permanent staff was $632.58, what was the difference between their holiday loading and the bonus given to the casual staff?

12 Brock works at a chemical factory and is paid $18.14 per hour, plus an additional allowance of $5.10 per hour for working with toxic substances. What is his total weekly pay if he works from 7:00 a.m. to 3:00 p.m. for 5 days a week?

13 When it rains, road workers are paid an extra $4.60 per hour. Last week, Paul worked 9 hours a day for 5 days at $15.23 per hour. If it rained continuously on two of those days, calculate Paul's total earnings for the week.

Shutterstock.com/viappy

14 Each week, a real estate agent earns a retainer of $510, a commission of 3.5% of the value of properties sold, and a car allowance of $128.70. Calculate her total earnings for a week in which she sold $943 658 worth of properties.

15 A telephone interviewer employed by a market research company earns $18.20 per hour for a 35-hour week. Calculate her holiday loading if it is 17.5% of the normal pay for 4 weeks.

16 One year, Kirsten received a holiday loading of $403.12. What was her normal weekly wage?

4-04 Government allowances and pensions

WS
Earning money

Centrelink is an Australian government agency which supports people in need and assists them to become self-sufficient. Various allowances and benefits are available for the aged, the unemployed, students, parents and the disabled.

Age pension

The age pension is a fortnightly payment to seniors, aged 65 or over, who are unable to support themselves in their retirement. To be eligible, a person needs to have low income and assets. The payment and income test tables are shown on the opposite page.

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Payment per fortnight

Marital status	Full age pension payment
Single	$569.80
A couple	$475.90 each

Income test

Marital status	To get full payment, earnings per fortnight* can be up to:
Single	$138, plus $24.60 per dependent child
A couple (combined income)	$240, plus $24.60 per dependent child

*Income over these amounts reduces the rate of pension payable by 40 cents in the dollar (single), and 20 cents in the dollar each (for couples).

Source: www.centrelink.gov.au, 2009

EXAMPLE 8

Use the tables above to calculate the fortnightly age pension payment for each of the following seniors.

a Beryl, aged 72 and single, who receives $400 per year dividend from her shares

b Dulcie (aged 65) and Bert (aged 69), who are legal guardians of their grand-daughter and have a combined income of $156 per week doing odd jobs

c Stanley, aged 63, who has no income and lives alone

Solution

a Fortnightly income = $400 ÷ 52.18 × 2

= $15.331…

$15.331 < $138

Beryl is single and earns less than $138 per fortnight, so she receives the full payment of $569.80.

b Combined fortnightly income = $156 × 2

= $312

To receive the full payment, the couple must earn less than:

$240 + $24.60 = $264.60 **$24.60 for the grand-daughter.**

The payment is reduced by 40c for each dollar earned over $264.60.

Combined pension = (2 × $475.90) – 0.4 × ($312 – $264.60)

= $932.84

Dulcie and Bert receive a combined pension of $932.84 per fortnight.

c Stanley is not eligible for the age pension because he is under 65 years old.

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Youth and study allowances

There are several allowances available for young people or students. Some examples are given in the table below.

Allowance	Conditions for eligibility
Youth allowance	Aged 16–20 years, studying full-time, undertaking an apprenticeship or looking for work Aged 21–24 years, studying full-time or undertaking an apprenticeship
Austudy	Aged over 25 years and studying full-time or undertaking an apprenticeship
ABSTUDY	Indigenous students or apprentices

EXAMPLE 9

The tables below show the maximum fortnightly amounts for youth allowance and Austudy.

Youth allowance

Conditions for eligibility	Maximum fortnightly payment
Single with no children, under 18 years and living at home	$203.30
Single with no children, under 18 years and not living at home	$371.40
Single with no children, 18 years and over and living at home	$244.40
Single with no children, 18 years and over and not living at home	$371.40
Single with children	$486.60
Partnered with no children	$371.40
Partnered with children	$407.80

Austudy (for people aged over 25)

Conditions for eligibility	Maximum fortnightly payment
Single	$371.40
Single, with children	$486.60
Partnered, no children	$371.40
Partnered, with children	$407.80

Source: www.centrelink.gov.au, 2009

Calculate the fortnightly youth allowance or Austudy payment for each of the following people.

Remember to check the eligibility criteria as well.

a Pablo, aged 17, a first-year apprentice plumber living with his parent.

b Cameron, aged 27, studying his HSC full-time at TAFE while raising his son alone.

c Sally, aged 32, studying part-time and living with her husband and two children.

d Greg, aged 16, has left school, is homeless, and is looking for full-time employment.

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Solution

a Pablo is under 18 and is living at home, so he receives a youth allowance of $203.30.

b Cameron is over 25 and a single parent, so he receives an Austudy payment of $486.60.

c Sally is not eligible for Austudy, because she is not studying full-time.

d Greg is under 18 and is not living at home, so he receives a youth allowance of $371.40

EXAMPLE 10

The table below shows fortnightly ABSTUDY allowances for independent Indigenous students.

ABSTUDY

Conditions for eligibility	Allowance per fortnight
Single, no children	
16–20 years, not at home	$371.40
Under 16, at home	$203.30
16–17 years, at home	$203.30
18–20 years, at home	$244.40
21 years and over	$453.30
Partnered, no children	
16–20 years	$371.40
21 years and over	$409.00
Single with dependent child	
16–20 years	$486.60
21 years and over	$490.40
Partnered with dependent child	
16–20 years	$407.80
21 years and over	$409.00

Source: www.centrelink.gov.au, 2009

Calculate the fortnightly ABSTUDY payment for each of the following Indigenous students.

a Wilf, aged 18, who is living with his partner and her two-year-old son.

b Cecily, aged 14, who is living at home with her parents.

c Gracie, aged 27, who is living with her husband and has no children.

Solution

a Wilf is 16–20 and is partnered, with a dependent child, so he receives $407.80.

b Cecily is under 16 and is living at home so she receives $203.30.

c Gracie is over 21 and is partnered, with no children, so she receives $409.00.

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Exercise 4-04 Government allowances and pensions

Use the tables from Examples 8, 9 and 10 on the last three pages, to answer the questions in this exercise.

Example 8

1 If the following people all satisfy the Centrelink assets test, calculate their fortnightly age pension payments.

a Elsie, aged 89 and single, receives $850 per year from her investments.

b George, aged 71, and Beryl, aged 75, care for their disabled daughter full-time and have a combined income of $144 per week from doing odd jobs.

c Dell, aged 62 years, has no savings, no job and lives alone.

2 Why do you think that age pensioners receive less if they are partnered rather than single?

3 The Newstart allowance is paid to unemployed people aged 21 years or over. The maximum fortnightly rates are shown in the table on the right.

Status	Dependants	Newstart allowance
Single	No children	$453.30
Single	Children	$490.40
Partnered		$409.00

a Hani is unemployed, and is single with no children. How much Newstart allowance would she receive if it was paid:

i fortnightly? **ii** yearly? **iii** monthly?

b Jay is married, unemployed and has two children. How much does he receive over 6 months?

c Does a single unemployed person receive more, less or the same as one with a partner? Is this fair? Give a reason for your answer.

Example 9

4 Carrie is 18 years old and is studying full-time at a senior high school. She lives with her parents and has a two-year-old daughter. Which of the following is her youth allowance? Select **A**, **B**, **C** or **D**.

A $371.40 **B** $244.40 **C** $486.60 **D** $407.80

5 The parenting payment is paid to parents of children aged 18 years or younger and is not dependent on the number of children they have. The maximum fortnightly rates are shown in the table on the right.

Status	Parenting payment
Partnered	$409.00
Partnered but separated due to illness, respite or prison	$490.40
Single	$569.80

a The Taylor family has three children: Daniel 15, Julianne 12 and Liam 8. How much does the Taylor family receive as a parenting payment per fortnight?

b The Lovejoys have ten children under 18 at home and two children living away from home. Mr Lovejoy is currently in prison. What parenting payment does Mrs Lovejoy receive per fortnight?

c Karen is separated from her partner and has a boy aged 9 years and a girl aged 7 years. What is her fortnightly parenting payment?

d Nash's wife is in hospital for the next 12 months. He is looking after their twin 11-year-old boys. How much parenting payment will he receive for:

i 1 year? **ii** 1 week?

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Example 10

6 Dean is a 19-year-old Indigenous student living at home with his parents. Which of the following is his fortnightly ABSTUDY allowance? Select **A**, **B**, **C** or **D**.

A $244.40 **B** $371.40 **C** $407.80 **D** $486.60

7 Use the ABSTUDY table from Example **10** to determine the fortnightly payment for each Indigenous student described below.

a Jill is 17 years old, and is living with her partner.

b Brett is 28 years old, and is living at home with his wife and two children under 10 years.

c Maude is 21 years old, and is living away from her parents' home with her boyfriend.

d Dee is 15 years old, and is living at home with her grandparents.

INVESTIGATION

VISIT CENTRELINK

1 Investigate the different allowances by visiting a Centrelink branch or www.humanservices.gov.au.

2 Research one type of allowance, looking at the eligibility conditions, residential qualifications, and income and assets tests.

3 Write a report and present this to the class.

4-05 Gross and net pay

WS Pay day

Gross pay is the amount a person earns, while **net pay** is the amount remaining after income tax and other deductions have been taken from gross pay.

SUMMARY

Net pay = gross pay – tax – other deductions

Other deductions include:

- superannuation contribution (retirement fund)
- private health fund instalment
- union fees
- direct debits for bills
- savings.

EXAMPLE 11

Diane earns $629.60 per week. Deductions of $185.92 for tax, $42.17 for superannuation, $26.34 for a health fund contribution and $17.50 for gym fees are taken from her gross wage.

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Calculate her:

a net pay

b income tax as a percentage of her gross pay, correct to one decimal place.

Solution

a Net pay = \$629.60 – \$185.92 – \$42.17 – \$26.34 – \$17.50

= \$357.67

Diane's net pay is \$357.67.

b Percentage tax $= \dfrac{\text{tax}}{\text{gross pay}} \times 100\%$

$= \dfrac{\$185.92}{\$629.60} \times 100$

$= 29.5298...$

$\approx 29.5\%$

Diane's income tax is 29.5% of her gross pay.

EXAMPLE 12

Aidan earns a salary of \$67 510, pays 34% tax and has \$28.70 per fortnight deducted for union fees. Correct to the nearest cent, calculate:

a how much tax Aidan pays per fortnight

b Aidan's gross fortnightly pay

c Aidan's net fortnightly pay.

Solution

a Yearly tax = 34% × \$67 510

= \$22 953.40

Fortnightly tax = \$22 953.40 ÷ 52.18 × 2

= \$879.7776…

≈ \$879.78

Aidan pays \$879.78 tax per fortnight.

b Gross fortnightly pay = \$67 510 ÷ 52.18 × 2

= \$2587.5814…

≈ \$2587.58

Aidan's gross fortnightly pay is \$2587.58.

c Net fortnightly pay = \$2587.58 – \$879.78 – \$28.70

= \$1679.10

Aidan's net fortnightly pay is \$1679.10.

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Exercise 4-05 Gross and net pay

1 Alice earns $827.25 per week and has the following deductions from her gross wage: Tax $244.04, superannuation $64.83, union fees $18.20 and health fund $26.15. Which of the following is Alice's net pay? Select **A**, **B**, **C** or **D**.

A $827.25 **B** $583.21 **C** $474.03 **D** $718.07

2 Melanie earns $945.30 per week and has deductions of $235.90 for tax, $55.17 for superannuation, $24.78 for a health fund contribution and $17.50 for union fees. Calculate her: Example 11

a net pay

b income tax as a percentage of her gross pay, correct to one decimal place.

3 Zhi earns a salary of $45 674. Each week, he pays 28% tax, $30.76 to his superannuation fund and $86.11 in other deductions. Calculate Zhi's: Example 12

a gross weekly pay **b** tax **c** net weekly pay.

4 Fatima earns a gross weekly wage of $635.25, from which is deducted $26.11 for superannuation and $7.90 for union fees.

a If Fatima's net pay is $429.07, how much tax does she pay?

b What percentage of Fatima's gross wage is paid in tax (correct to two decimal places)?

c What percentage of Fatima's gross wage is paid in superannuation (to two decimal places)?

5 Jeff pays 27.5% of his gross pay in tax and 8% in superannuation. Each week, he also has $7.10 deducted for union fees and $14.84 for life insurance premiums. If Jeff's gross weekly pay is $638.20, calculate his:

a tax **b** weekly superannuation payment

c total weekly deductions **d** net weekly pay.

6 Lindy pays 22% of her gross pay in tax and 10% in superannuation. She also contributes $4.70 towards a staff social fund every week.

a If Lindy pays $135.52 tax each week, find her gross weekly pay.

b Calculate Lindy's weekly superannuation payment.

c Find Lindy's net weekly pay.

7 **a** Fady earns a salary of $54 251. Calculate his fortnightly pay.

b Each fortnight, Fady's deductions from his pay are $307.26 tax, $145.60 for a car loan repayment and $27.14 to a health fund. What is Fady's net pay?

c What percentage of Fady's gross pay is paid in tax, correct to one decimal place?

8 Melinda earns $19.38 per hour for a 39-hour week and pays 25% tax. Each week, she has $17.40 deducted from her pay for accident insurance. Calculate her:

a gross weekly pay **b** tax **c** net weekly pay.

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9 Matthew earns a salary of $108 275. He is paid monthly but, each payday, $4330.00 is deducted in tax, $1080.50 is deducted for superannuation and $107.54 is deducted for medical insurance.

a Calculate Matthew's monthly gross and net incomes.

b What percentage of Matthew's gross pay is paid in tax?

c What percentage of Matthew's gross pay is his net pay?

10 Copy and complete the pay statement below, using a calculator or spreadsheet.

Springfield Nuclear Power				
Employee: H. Simpson		Employee number: 5521716		
Pay period: 5.11.14–11.11.14		Hourly rate: $21.48		
	Hours	**Pay**	**Deductions**	
Normal	35		Tax (25%)	
Time-and-a-half	4		Superannuation	$60.65
Double time	1		Union	$18.15
	Gross wage		Health fund	$24.80
	Total deductions			
	Net wage			

11 Copy and complete the employee payslip below.

State Education Board			
Employee: E. Krabappel		Serial number: 8805285	
Pay period: 14.3.11–27.3.11		Salary: $87 026	
Other deductions		**Pay for this fortnightly period**	
Superannuation	$71.60	**Gross pay**	
Union	$16.22	Tax (31%)	
Loan repayment	$43.55	Other deductions	
Car insurance	$20.84	**Net pay**	
Total deductions			

4-06 Budgeting

Budget grid

A **budget** is a plan for managing your income wisely. Whether you are running a household, a business or just managing your pocket money, it is essential that you do not spend more than you earn. A budget is divided into **income** and **expenses**, and a balanced budget is one in which the total expenses equal the total income.

Budgeting scenarios

EXAMPLE 13

Lucy was saving for an overseas trip and accepted some extra casual work. She set up a weekly budget, as shown in the table on the next page, and aimed to save $150 per week.

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Income		Expenses	
Wages	$681	Rent	$265
Casual wage	_____	Groceries	$143
		Fares	$42
		Car loan	$90
		Petrol/car maintenance	$63
		Entertainment	$185
		Savings	_____
Total:	$884	**Total:**	$ _____

a How much did Lucy earn from her casual job?

b What should Lucy's total expenses be?

c To balance her budget, how much can Lucy save per week?

d Suggest a way Lucy could increase her savings to $150 per week.

Solution

a Casual wage = $884 – $681
= $203

b Total expenses = $884 **Equal to total income.**

c Savings = $884 – ($265 + $143 + $42 + $90 + $63 + $185)
= $96

d Suggestions: cut down on entertainment, rent with a friend, sell the car, or look for weekly grocery specials.

Exercise 4-06 Budgeting

1 Categorise each of the following items as either an income or an expense.

a a car insurance payment

b an inheritance from your grandmother

c a birthday present for a friend

d the cost of a plane ticket to London

e a second prize win in the lottery

f the monthly rental of a computer

g a rent payment to a real estate agent

h the admission price to the cinema

i the rent from an investment unit

j pay to an electrician for installing new power points

k your weekly wage

l the cost of a gym class

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m pocket money

n a Christmas present from your aunt

o the cost of a Friday night pizza

Example 13

2 Mariella's budget allowed her to save $60 each week. Her budget is shown below.

Income		Expenses	
Youth allowance	$185.70	Rent	$160
Casual job	_____	Fares	$56
		Groceries	$124
		Clothes	$25
		Entertainment	____
		Bills	$58
		Savings	$60
Total:	$537.00	**Total:**	$___

a How much did Mariella earn for her casual job?

b What amount did Mariella spend on entertainment?

c Suggest two ways that Mariella could increase her weekly savings to $80.

3 Jess wants to save $2200 for a trip in one year's time. She earns $615.40 net a week and has the following weekly expenses: meals $40, rent $138, groceries $52, fares $36, tennis $20, entertainment $60, car loan $85, bills $65 and magazines $8.

a Calculate the amount Jess has left for savings each week.

b Will Jess have enough saved to pay for her trip in one year?

c If the trip price increases by 20%, would Jess have enough saved in one year to pay the new price?

4 The Rogers family has the following net weekly incomes: wages $1901.77, part-time job wages $289.53, and parenting allowance $197.20. The family also has the following weekly expenses: bills $105.30, school fees $85.80, entertainment costs $295, health fund payment $53.40, clothes $132, home maintenance $184, groceries $210.50, petrol $85.40, Friday night takeaway $46, newspapers and magazines $26, home loan repayments $545.60, and car loan repayment $184.22.

a Use a table or spreadsheet to display this data as a budget, adding an entry for savings.

b What amount does the Rogers family save per week?

5 Zac shares an apartment and pays $110 a week towards the rent. He spends $64 on food and $46 on public transport. His shares of the quarterly electricity bill and two-monthly phone bill are $45.10 and $12.80 respectively. He earns $551.40 net per week, spends about $45 per week on entertainment and visits a chiropractor once a month at a cost of $42 a visit. Convert Zac's expenses to weekly amounts and draw up a weekly budget, including an amount for savings.

a What is Zac's weekly income?

b What is the total of Zac's weekly expenses?

c How much can Zac save per week?

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INVESTIGATION

PREPARING A BUDGET

Imagine that you are 25 years old with a net monthly income of $3340.

	Amount	Per	Weekly amount
Income			
	$3340.00	month	
Expenses			
Rent	$185.00	week	
Mobile phone		month	
Gym membership	$484.00	year	
Internet	$49.95	month	
Water	$114.80	quarter	
Electricity	$122.50	quarter	
Health fund	$16.40	week	
Groceries		week	
Petrol/car maintenance		week	
Fares		week	
Clothes		week	
Entertainment		week	
Sport		week	
Gifts/donations		week	
Savings		week	
Other		week	

a Use a calculator or spreadsheet to copy this weekly budget, and complete it after converting the fixed expenses to weekly amounts.

b Choose the amount you wish to spend per week on each variable expense and enter these amounts.

c Suggest three ways that you could save more per week.

4-07 Income tax and Medicare levy

Taxable income and allowable deductions

Anyone who earns an income pays **income tax** to the government. The government uses this money to fund public services such as schools, hospitals, roads, trains, the defence services and the police force. Not all of a person's income is taxed, however. If we use some of our income for work-related expenses or to donate money to charities, these amounts are called **allowable (tax) deductions** and are not taxed. The remaining part of our income that is taxed is called **taxable income**.

Income tax tables

Income tax

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SUMMARY

Taxable income = gross income – allowable (tax) deductions

Here are some examples of allowable deductions:

- work-related expenses, such as tools of trade, uniforms, car-related expenses, trade journals, subscriptions to professional organisations and travel to conferences
- donations to charities
- depreciation of home office equipment that is used to earn income
- bank fees on investments for which interest is earned.

Income tax

Income tax is calculated on our taxable income (rounded down to the nearest dollar) for a financial year (1 July of one year to 30 June of the next year) and operates on a sliding scale so that the more we earn, the higher the rate of tax we pay.

Income tax rates for Australian residents	
Taxable income	**Tax on this income**
\$0 – \$18 200	Nil
\$18 201 – \$37 000	19c for each \$1 over \$18 200
\$37 001 – \$80 000	\$3572 plus 32.5c for each \$1 over \$37 000
\$80 001 – \$180 000	\$17 547 plus 37c for each \$1 over \$80 000
\$180,001 and over	\$54 547 plus 45c for each \$1 over \$180 000

Source: Australian Taxation Office, 2012–13

EXAMPLE 14

Cyndi earned \$56 375 and claimed \$326.20 in work-related expenses and \$125 in donations to charities. Find the amount of income tax Cyndi should pay, using the table above.

Solution

Taxable income = \$56 375 – \$326.20 – \$125

= \$55 923.80

≈ \$55 923 **Rounded down to the nearest dollar.**

Cyndi's income \$55 923 is in the 3rd tax bracket (\$37 001–\$80 000) for which the tax is '\$3572 plus 32.5c for each \$1 over \$37 000'.

Income tax = \$3572 + 0.325 × (\$55 923 – \$37 000)

= \$9721.975

≈ \$9721.98

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Medicare levy

The **Medicare levy** is a tax to fund Medicare, our public healthcare system. It is 1.5% of taxable income, although some low income earners pay a reduced levy or no levy.

EXAMPLE 15

Calculate the Medicare levy for Suyen, who has a taxable income of $62 379.

Solution

Medicare levy = 1.5% × $62 379
= $935.685
≈ $935.69

Exercise 4-07 Income tax and Medicare levy

1 Find the amount of income tax payable for each of the following people. Example 14

- **a** Kamel who earned $31 425 and claimed $285 in work-related expenses.
- **b** Susie who earned $131 412 and claimed $1036 in travel expenses and $643 depreciation of her home office equipment.
- **c** Toula who received $14 615 for the year.
- **d** Jude who received $9240 Austudy allowance for the year and claimed $356 for study materials.

2 Jenni earns a salary of $91 262 and claims a tax deduction of $1810.15 for work-related expenses. Example 15

- **a** Calculate Jenni's taxable income.
- **b** Calculate Jenni's amount of tax payable.
- **c** Calculate the amount of Jenni's Medicare levy.

3 Catriona earns a salary of $87 210 and claims these deductions:

Travel and entertainment expenses	$650.25
Depreciation of home computer	$314.80
Membership of a professional association	$120.50

- **a** Calculate Catriona's taxable income.
- **b** Calculate Catriona's income tax payable.
- **c** What is the amount of Catriona's Medicare levy?

4 Luke earns $952.80 per week as a teacher. Last financial year, he also earned $286.10 bank interest, $2050.96 share dividends and $892.51 book royalties. He is able to make tax deductions of $241.60 for union fees, $345.80 for car expenses, $175.80 for professional journals and $843.50 for expenses related to the writing of his book. For the financial year, calculate Luke's:

a taxable income **b** income tax payable **c** Medicare levy.

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5 The table below shows the tax rates for Australian citizens who live in another country.

Income tax rates for Australian non-residents	
Taxable income	**Tax on this income**
\$0–\$80 000	32.5c for each \$1
\$80 001–\$180 000	\$26 000 + 37c for each \$1 over \$80 000
\$180 001 and over	\$63 000 + 45c for each \$1 over \$180 000
Non-residents are not required to pay the Medicare levy.	

Source: Australian Taxation Office, 2012–13

Find the income tax payable by each of the following Australian citizens.

a Claude, who lives in France and has a taxable income of \$37 850.

b Marie, who has a taxable income of \$102 670 and lives in Canada.

c Benny, who lives in Sweden and has a taxable income of \$203 500.

TECHNOLOGY: INCOME TAX CALCULATOR

Create and complete the spreadsheet below for calculating the income tax payable when you enter the taxable income in the correct tax bracket. Two formulas have already been given.

	A	B	C	D
1	Tax bracket			
2	From	To	Taxable income	Income tax
3	\$0	\$18200		\$0
4	\$18201	\$37000		=(C4-B3)*0.19
5	\$37001	\$80000		=3572+(C5-B4)*0.325
6	\$80001	\$180000		
7	\$180001	and over		

Create a similar spreadsheet to calculate the Medicare levy as well.

JUST FOR THE RECORD

Income tax in other countries

Denmark has the highest income tax rates, ranging from 38% to 59%, while the United Arab Emirates (UAE) has no income tax at all. The website www.worldwide-tax.com gives the current tax rates around the world. Here are the tax rates for six countries:

- Russia 13%
- Hong Kong 2% to 17%

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- Saudi Arabia 20%
- USA 15% to 35%
- Spain 24% to 45%
- Sweden 0% to 57%

Find the current tax rates in five more countries.

4-08 PAYG and tax returns

To avoid having to pay income tax as a huge lump sum at the end of a financial year, tax is deducted every payday from a person's gross pay and is called **PAYG (Pay As You Go) tax**. The employer estimates how much tax to take out of each pay and, at the end of the financial year, the actual income tax payable is calculated.

Every income earner must submit a **tax return** to the Australian Tax Office (ATO) at the end of the financial year. A PAYG Payment Summary, showing total gross earnings and PAYG tax paid, is issued by the employer and used by the employee to complete the tax return, including any allowable deductions. The ATO then calculates the income tax and Medicare levy payable. Any earner who has paid more PAYG tax than necessary will receive a **tax refund**. However, anyone who has not paid enough PAYG tax will have a **tax debt** and will need to pay the difference owing.

SUMMARY

Tax refund = PAYG tax paid – (income tax payable + Medicare levy)

Tax debt = (income tax payable + Medicare levy) – PAYG tax paid

EXAMPLE 16

Mia earns a salary of $115 412.70 and claims tax deductions of $1743. Last year she paid a total of $37 412 PAYG tax. Calculate her:

a taxable income

b income tax payable (by referring to the table on page 172)

c Medicare levy

d tax refund or tax debt.

Solution

a Taxable income = $115 412.70 – $1743

= $113 669.70

≈ $113 669 **Rounding down to the nearest dollar.**

Mia's taxable income is $113 669.

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b Tax payable = \$17 547 + 0.37 × (\$113 669 –\$80 000) **From the table on page 172.**

= \$30 004.53

Mia's income tax is \$30 004.53.

c Medicare levy = 1.5% × \$113 669

= \$1705.035

≈ \$1705.04

Mia's Medicare levy is \$1705.04.

d Total taxes = \$30 004.53 + \$1705.04 = \$31 709.57

Amount of PAYG tax paid = \$37 412

Tax refund = \$37 412 – \$31 709.57 **Refund because**

= \$5702.43 **PAYG tax paid > total taxes**

Mia's tax refund will be \$5702.43.

Exercise 4-08 PAYG and tax returns

1 Jasmine paid \$314.68 PAYG tax out of her weekly wage of \$1014.65.

a How much PAYG tax did Jasmine pay for the year?

b How much did Jasmine earn for the year, after tax?

2 Hope earned a salary of \$73 489 per year. She received a weekly pay packet, after PAYG tax of \$475.15 and \$26.80 health insurance were taken out.

a How much PAYG tax did Hope pay in a year?

b How much was in Hope's weekly pay packet?

Use the table of tax rates on page 172 to answer questions 3 to 7.

Example 16

3 Gabby is an Australian resident who earned a yearly salary of \$204 560. She claimed tax deductions of \$10 534 and paid a total of \$72 156 PAYG tax. Calculate her:

a taxable income

b income tax payable

c Medicare levy

d tax debt or tax refund.

4 Anand had a taxable income of \$278 639 and paid \$104 568 in PAYG tax over the financial year. Find his:

a income tax payable **b** Medicare levy **c** tax debt or tax refund.

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5 Liam earned a wage of $589.60 per week and paid PAYG tax of $136.45. He was able to claim $38.25 per week in allowable deductions and $150 for the year in donations. Find his:

a taxable income

b income tax payable

c Medicare levy

d tax debt or tax refund.

6 Dave paid PAYG tax of $7836 and had a taxable income of $34 589. Which of the following is Dave's tax refund? Select **A**, **B**, **C** or **D**.

A $2675.30 **B** $2788.20 **C** $3113.72 **D** $4670.79

7 Kim had a taxable income of $381 459 and paid $138 639 in PAYG tax. Which of the following is Kim's tax debt? Select **A**, **B**, **C** or **D**.

A $6564.55 **B** $3301.76 **C** $9642.17 **D** $8281.60

8 Use the tax table for Australian non-residents on page 174 to find the tax refund or tax debt for each of the following Australian non-residents.

a Cecily, who has a taxable income of $93 457 and paid $734 per fortnight PAYG tax.

b Alice, who paid $5279 PAYG tax every quarter and earned a taxable income of $98 534.

c Simon, who paid $37 460 tax per month and earned a taxable income of $1.2 million.

INVESTIGATION

TAX RETURNS

1 Choose the profile of a person with a particular occupation and salary or wage. Decide on what PAYG tax that person pays each week, fortnight or quarter.

2 Research the allowable deductions for this occupation. These may include travel expenses, special clothing, tools of trade, books and journals, attendance at conferences and seminars, home office expenses and self-education expenses.

3 Add any other income that you wish, such as bank interest or money from a part-time job. If you do this, there may be other allowable deductions, such as bank fees or travel between jobs, which are claimable.

4 Use the form from a Tax Pack to complete a tax return for the person.

5 Calculate the person's Medicare levy.

6 Calculate the person's tax payable and any tax refund or tax debt.

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4-09 GST and VAT

While income tax is a tax on earnings, the GST and VAT are taxes on spending or consuming.

The **goods and services tax (GST)** in Australia is 10% of the cost of the goods or services. New Zealand, Canada and Singapore also have a GST. In other countries, it is called the **value-added tax (VAT)**.

- Prices in Australia are usually printed with the GST included.
- Prices in other countries are often printed 'before tax', with the tax added when the consumer pays.
- The GST is not charged on essential items such as basic foods, childcare or renting a home.
- The consumer pays GST for goods and services, such as electrical appliances, lease of premises, plumbing work, IT support.

The GST/VAT rate in some countries (correct in 2012) is shown in the table on the right.

GST/VAT rates	
Australia	10%
Canada	7%
China	17%
France	21.2%
New Zealand	15%
Singapore	7%
South Africa	14%
Sweden	25%
Taiwan	5%
Thailand	7%

EXAMPLE 17

a A camera costs \$568 plus 10% GST. What is its selling price?

b A 3D television costs \$1995, inclusive of GST. What was the original price of the television, to the nearest 5c, before GST was added?

Solution

a Selling price = 110% × \$568 — **Increasing the price by 10%.**

= \$624.80

The selling price of the camera is \$624.80.

b 110% of original price = \$1995

1% of original price = \$1995 ÷ 110 — **Using the unitary method: Find 1% first.**

= \$18.136 36…

Original price = \$18.136 36… × 100 — **To find 100%.**

= \$1813.636

≈ \$1813.65 — **Rounded to the nearest 5 cents.**

The original price of the television was \$1813.65.

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EXAMPLE 18

A handbag in Norway costs 425 kroner before VAT and 531.25 kroner after. What is the rate of VAT charged?

Solution

VAT = 531.25 – 425 = 106.25 kroner

$$\text{VAT rate} = \frac{106.25}{425} \times 100\%$$

$$= 25\%$$

The rate of VAT charged is 25%.

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Exercise 4-09 GST and VAT

Example 17

1 A hamburger in Sydney costs $5.50 plus 10% GST. Which of the following is the hamburger's selling price? Select **A**, **B**, **C** or **D**.

A $5.60 **B** $5.00 **C** $6.05 **D** $6.50

2 A computer was purchased in Ireland for €1500 (Euro). What is the computer's selling price when 21% VAT is added?

3 A watch in Thailand costs 3724 baht plus 7% VAT. What is the watch's selling price? Answer to the nearest whole baht.

4 A car costs $42 000 including 10% GST. Which of the following is the car's price before GST, correct to the nearest dollar? Select **A**, **B**, **C** or **D**.

A $38 182 **B** $37 800 **C** $38 181 **D** $46 200

5 A train ticket from Sydney to Nowra costs $18.50 including 10% GST. What is the price of the ticket before tax?

6 A night's accommodation in London is £126.90 (pounds) including 17.5% VAT. What was the price before tax?

7 An overseas telephone call from South Africa costs 53.01 rand after a VAT of 14% is imposed. What was its price before VAT, rounded to two decimal places?

8 A box of chocolates from the Netherlands costs €6.50, plus €1.24 VAT. What percentage VAT, to the nearest whole number, has been imposed?

Example 18

9 The cost of a laundry service in Greece was €9.88 after a VAT of €1.58 was added.

a What was the original price of the laundry service?

b What was the rate of VAT charged, to the nearest percentage?

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10 In New Zealand, a 15% GST of $4.62 was added to the original price of a set of steak knives. What was:

a the original price of the knives? **b** the final price of the knives?

11 A Korean camera costs 142 900 won before VAT, and 157 190 won after VAT.

a What amount of VAT was imposed?

b What percentage VAT was imposed?

12 An Italian suit costs €220 before VAT and €264 after VAT.

a What VAT is charged? **b** What percentage VAT is charged?

13 A television set costs $1245 plus $224.10 GST. What percentage rate of GST has been imposed?

14 A painting was bought in Luxembourg and a 15% GST of 31.17 euros was added to the original price. Find the selling price of the painting.

15 A roof tiler in New Zealand charges $152.72 for a day of work after a GST of 15% is added. What was the charge before GST?

16 If an Australian Akubra hat has a 10% GST of $2.83 included in its selling price, find its selling price.

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17 A necklace in Thailand costs 4419 baht after a VAT of 7% has been added. What was its original price, to the nearest whole baht?

18 If a video store charges $6.50 rental before GST, and $6.93 after GST, what percentage rate of GST is imposed? Answer to two decimal places.

19 At a Canadian cafe, $2.40 GST was added to the cost of a dinner for two. If this represents 5% of the original cost of the meal, calculate the cost of the meal before and after GST.

20 A Belgian key-cutter charges €3, plus €0.57 in VAT, for every key cut. What percentage VAT is being charged?

INVESTIGATION

SAME DIFFERENCE, MORE-OR-LESS?

1 Why can we find the GST paid on an item by simply dividing its advertised price by 11?

2 The Great Gals electrical store adds 10% GST to the prices of its goods. If a washing machine had an original price of $1200, what is its advertised price with GST included?

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3 At the Great Gals end-of-year sale, customers get a 10% discount on all goods.

a Will the discounted price be more than, less than or the same as the original price before GST was added?

b What is the discounted price of the washing machine in question **2** at the end-of-year sale?

c How does the discounted price of the washing machine compare with the original price?

d Would it make a difference to the discounted price of the washing machine if the 10% discount was applied first before the 10% GST was added?

4 What can you say generally about an item that has 10% added then 10% subtracted?

5 Calculate the overall percentage change when 10% is added to an amount then 10% is subtracted.

4-10 Graphs of tax rates

Goods and services tax amounts can be shown using a **linear graph**. Income tax is represented by a combination of line segments ('pieces') of different gradients, called a **piecewise linear graph**. These segments reflect the different rates of tax.

EXAMPLE 19

The GST in New Zealand is 15%.

a Construct a conversion graph showing the GST payable in New Zealand for prices in the range \$0 to \$100.

b Use your graph to estimate:

i the GST payable on a \$70 handbag

ii the price of a bus tour before GST of \$9.50 is added

iii the price of a bowl after GST of \$4 is added.

Solution

a GST = 0.15 × price

Price (\$)	0	20	40	60	80	100
GST (\$)	0	3	6	9	12	15

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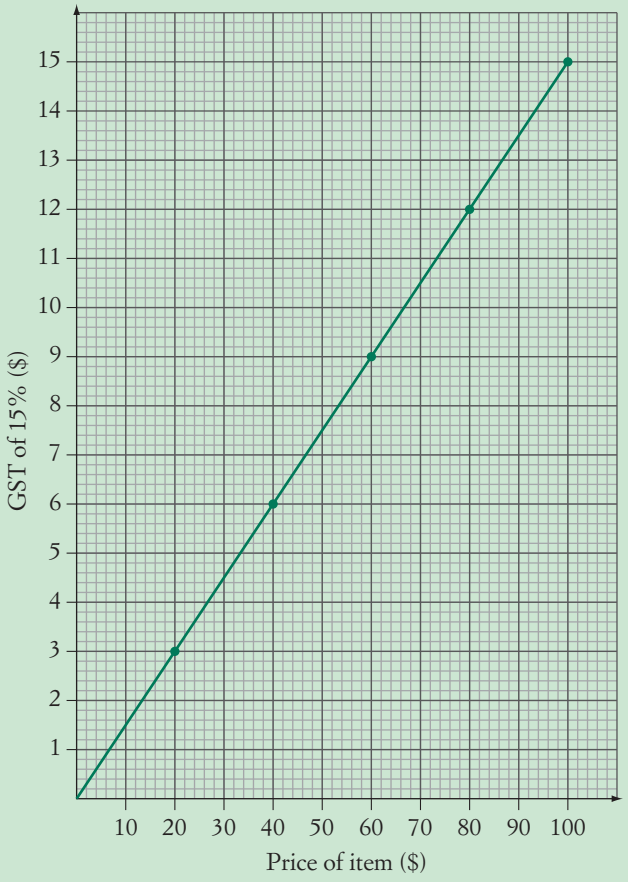

We can also create this graph using a graphics calculator, spreadsheet program or graphing software by entering the equation of the line ($y = 0.15x$) or the table of values.

b Reading from the graph:

- **i** when the price is \$70, the GST is \$10.50
- **ii** when the GST is \$9.50, the price is \$64
- **iii** when the GST is \$4, the price is \$26, so the selling price is \$26 + \$4 = \$32.

EXAMPLE 20

Income tax is calculated on a sliding scale as we saw earlier in this chapter. The table of tax rates for taxable incomes up to \$80 000 is shown below.

Income tax rates for Australian residents	
Taxable income	**Tax on this income**
\$0–\$18 200	Nil
\$18 201–\$37 000	19c for each \$1 over \$18 200
\$37 001–\$80 000	\$3572 plus 32.5c for each \$1 over \$37 000

Source: Australian Taxation Office, 2012–13

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a Construct a piecewise linear graph, showing tax payable on incomes from \$0 to \$80 000.

b Use your graph to estimate the tax payable, to the nearest \$100, on an income of:

i \$30 000 **ii** \$56 000.

c Estimate the income earned if the tax payable is:

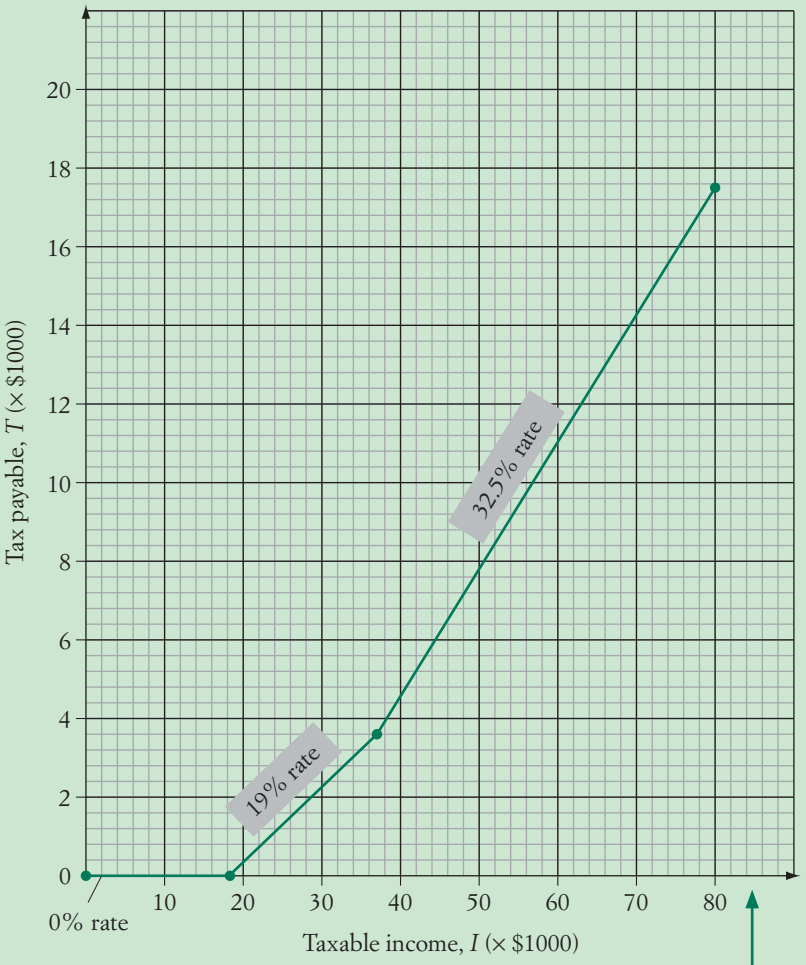
i \$8400 **ii** \$12 800.

Solution

a Construct a table of values using the endpoints of the tax brackets and use this to draw the graph.

I (\$)	0	18 200	37 000	80 000
T (\$)	0	0	3572	17 547

Tax payable on income from \$0 to \$80 000



The gradients of the three line segments are the percentage tax rates: 0, 0.19 and 0.325.

b **i** \$2200 **ii** \$9800

c **i** \$52 000 **ii** \$65 000

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Exercise 4-10 Graphs of tax rates

Example 19

1 Use the graph in Example **19** on the previous page to estimate the 15% GST on each New Zealand price.

a $25 **b** $72 **c** $90 **d** $16

2 Use the graph in Example **19** on p. 182 to estimate the New Zealand prices which have the following amounts of GST imposed:

a $7 **b** $3 **c** $1.60 **d** $11.20

Example 20

3 Use the graph in Example **20** on p. 183 to estimate the income tax payable on each of the following taxable incomes:

a $32 000 **b** $17 000 **c** $55 000 **d** $80 000

4 Use the graph in Example **20** to estimate the taxable income if the tax payable is:

a $12 000 **b** $2400 **c** $14 000 **d** $17 000

5 **a** Construct a conversion graph showing the 7% GST payable on goods in Singapore from $10 to $100.

b Use your graph to estimate:

i the GST payable on a tennis racquet marked at $95

ii the original price of a watch when the GST is $6.50

iii the selling price of a pair of shoes with a GST of $3.40 included.

6 **a** Construct a conversion graph showing the 20% VAT payable on goods in England ranging in price from £50 to £500.

b Use your graph to estimate:

i the VAT payable on a hat marked at £65

ii the original price of a pair of sunglasses when the VAT is £18

iii the selling price of tickets to a cricket match including a VAT of £64.

7 The table below shows the income tax rates in New Zealand.

Taxable income (NZ$)	Tax on this income
Up to $14 000	10.5 cents for each dollar
$14 001 to $48 000	$1470 plus 17.5c for each $1 over $14 000
$48 001 to $70 000	$7420 plus 30c for each $1 over $48 000
$70 001 and over	$14 020 plus 33c for each $1 over $70 000

Source: New Zealand Inland Revenue, 2012–13

a Find the tax payable in New Zealand on each of these taxable incomes.

i $14 000 **ii** $48 000 **iii** $70 000 **iv** $90 000

b Use your answers from part **a** to help you draw a piecewise linear graph showing tax payable in New Zealand on taxable incomes from $0 to $90 000.

c Use your graph to estimate the tax payable, to the nearest $1000, on an income of:

i $54 000 **ii** $82 000

d Estimate the income earned (to the nearest $1000) if the tax payable is:

i $5300 **ii** $16 700

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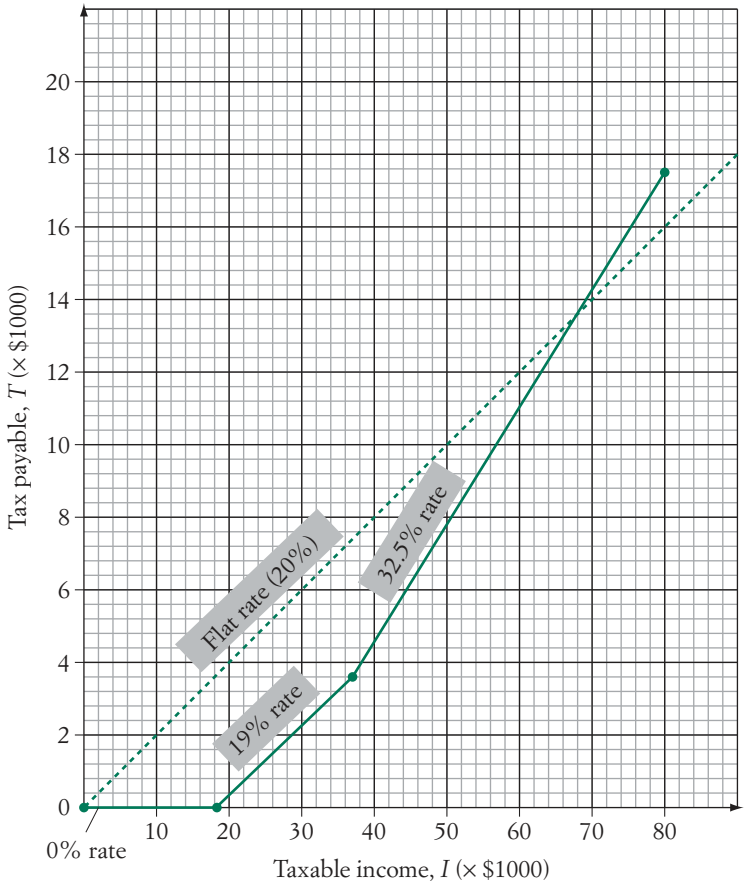
INVESTIGATION

A FLAT RATE OF TAX?

Suppose the progressive tax rate was replaced by a simple flat rate of 20%. The graph shown below illustrates the flat rate alongside the current progressive rate.

Flat rate of tax compared to progressive rate of tax



1. Who would benefit from this flat rate system?
2. Who would suffer the most?
3. Are there any incomes for which the tax would be the same under both systems?
4. What are the advantages of having a flat-rate income tax system?
5. Do you think a flat rate would be fairer? Why or why not?
6. Have a class discussion and/or write a report on your conclusions.

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Sample HSC problem

Nathan paid PAYG tax of $237.80 per week from his weekly wage of $789.40.

a How much PAYG tax did Nathan pay in the year?

b What was Nathan's gross income for the year?

c If he had $986.50 in allowable deductions, what was Nathan's taxable income?

d Use the income tax table on page 172 to calculate Nathan's income tax and Medicare levy.

e Will Nathan receive a tax refund or will he need to pay a tax debt? Calculate the amount.

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Study tip

When and where to study

Are you an early bird or a night owl? Do you work best in the morning, in the afternoon or at night? Identify your **peak performance period** and study during this time each day, so that it becomes a habit. There will be times when you can't study due to other commitments but it's important to develop a **regular routine** so that your family and friends know when you should not be interrupted.

Students often ask, 'How many hours should I study each night?', but it is the **quality** of study that is more important than the **quantity**. It is a question of the effort and commitment required, not the number of hours. The amount of time you should study is the amount of time necessary for you to fulfil all of your study tasks and demands. Some students like to do the same amount each day, some do more or less on weekends, or have one day that is study-free. Find out what works best for you, and stick to it.

Students often study by themselves in their rooms or at the library. Some prefer company and use the dining table, because they like the noise and space. Others like to sit outside in the fresh air. You can choose different places for different types of homework.

A good study place has:

- plenty of space to spread out work, such as on a big desk
- minimal noise, few distractions and interruptions
- good lighting and ventilation, and is neither too hot nor too cold
- comfortable and supportive seating.

Study is a serious business. You will be concentrating for a while, so choose a place where you won't be easily distracted. Use a good desk lamp and open the window to prevent sore eyes and drowsiness.

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CHAPTER REVIEW

Topic overview

This chapter, 'Earning money and taxation', looked at the different ways people earn or receive incomes and pay taxes based on earning and spending. You should be able to calculate incomes earned in a variety of ways, including commission, annual leave loading, allowances and overtime. You have learnt to plan a budget, calculate income tax and the Medicare levy, and calculate the tax refund or debt after PAYG tax has been deducted. Finally, you looked at GST and VAT, and constructed and read tax graphs.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map is shown below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.

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Revision

Earning and taxation crossword

1 Convert each of the following to a yearly amount, correct to the nearest cent. Exercise 4-01

- **a** a weekly wage of $631.15
- **b** a fortnightly wage of $1548
- **c** a monthly wage of $3786

2 For each of the following annual salaries state the amount paid for the pay period given, correct to the nearest cent. Exercise 4-01

- **a** $138 611 paid weekly
- **b** $98 503 paid fortnightly
- **c** $263 764 paid monthly

3 Marley works three days a week as a part-time carer. She is paid $25.70 per hour up to 5:00 p.m. and time-and-a-half after that. Use the roster below to calculate how much she earned this week. Exercise 4-01

Marley's weekly roster		
Day	**Start time**	**Finish time**
Monday	1000	1900
Wednesday	0900	1300
Friday	1600	2200

4 Huon picked 28 buckets of grapes at $3.65 per bucket. Exercise 4-02

- **a** How much did Huon earn?
- **b** How many buckets would Huon need to pick to earn $390.55?

5 Maggie made cupcakes and sold them to a shop at $4.80 per dozen, or $38 for 100. How much more per cupcake did she earn by selling them in lots of 100 rather than by the dozen? Exercise 4-02

6 Goran earned a 10% royalty on each of his books sold. If his book sells for $45.95, how much will he earn if 2537 books are sold? Exercise 4-02

7 Byron works in a mine and earns an hourly rate of $39.65. He also gets a below ground allowance of $9.84 per hour, double his hourly rate on weekends and a meal allowance of $27.40 per day if he works six or more hours. Exercise 4-03

Calculate Byron's total income in a week where he worked the following shifts.

Monday:	12:00 noon–5:00 p.m. above ground
Wednesday:	2:00 p.m.–7:00 p.m. below ground
Friday:	9:00 a.m.–12:00 noon above ground, and 2:00 p.m.–6:00 p.m. below ground
Saturday:	10:00 a.m.–2:00 p.m. below ground, and 3:00 p.m.–5:00 p.m. above ground

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Exercise 4-03

8 The Great Gals electrical store pays a holiday loading of 17.5% of four weeks pay to each employee. How much holiday loading is paid to Nirmal, the manager, who earns a salary of $86 430?

Exercise 4-04

9 Use the tables from Examples **9** and **10**, on pages 162–63 to help you find the type and amount of the allowance to which each of the following students is entitled.

a Richard, aged 18, who is a second-year apprentice tiler living with his parents.

b Ben, aged 24, who is a single Indigenous student living away from home.

c Alison, aged 29, who is studying full-time at college while raising her daughter alone.

Exercise 4-05

10 Jack earns a salary of $94 762, pays $594.10 PAYG tax per week and has $28.70 per week deducted for union fees. Correct to the nearest cent:

a how much PAYG tax does Jack pay per year?

b how much PAYG tax does Jack pay per fortnight?

c calculate Jack's gross fortnightly pay

d calculate Jack's net pay per fortnight.

Exercise 4-06

11 Chloe earns $1654 per fortnight as a graphic designer as well as collecting $1270 monthly rent from an apartment she owns.

a Calculate Chloe's total weekly income.

b Each week, Chloe has the expenses shown below. Prepare a balanced weekly budget for Chloe, including appropriate amounts for entertainment, clothes and savings.

Groceries	$142
Home loan repayment	$235
Health fund	$28.20
Household bills	$43
Newspapers and magazines	$18
Takeaway meals	$32
Petrol	$70
Fares	$56.70
Entertainment	$?
Clothes	$?
Savings	$?

c Name one way Chloe could save $40 more per week.

Exercise 4-07

Exercise 4-08

12 Melissa earned a yearly salary of $118 764. She paid $1424.30 PAYG tax per fortnight and claimed tax deductions of $13 102.20 for the year. Calculate her:

a taxable income (rounded up to the nearest dollar)

b income tax payable (using the table on page 172)

c Medicare levy

d tax debt or tax refund.

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13 A dishwasher has a selling price in Australia of $1246 (including GST). If GST is 10% of the original price, how much GST is included in this price? Exercise 4-09

Use the following table to answer questions 14 and 15.

Country	Currency	VAT
Hungary	forint	20%
Mexico	peso	15%
Romania	new lei	19%
South Africa	rand	14%

14 Find the VAT to be added on goods priced at: Exercise 4-09

- **a** 1262 pesos
- **b** 235 new lei
- **c** 26 950 rand

15 Calculate the price before VAT was added to these items: Exercise 4-09

- **a** a car purchased for 200 000 pesos
- **b** a television bought for 171 400 forint.

16 a Construct a graph showing the 7% VAT payable on Thai goods ranging in price from 0 baht to 500 baht. Exercise 4-10

- **b** Use your graph to estimate:
  - **i** the VAT payable on a basket marked at 250 baht
  - **ii** the original price of a statue before VAT of 28 baht is added
  - **iii** the price of a Thai meal when VAT of 16 baht has been added.

17 Use the graph in Example **20** on page 183 to find: Exercise 4-10

- **a** the tax payable on an income of $72 000
- **b** the taxable income if the tax is $14 000.

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PROBABILITY

5.

PROBABILITY

Insurance originated in ancient Babylon and China around 3000 BCE, when voyagers and traders formed financial societies to insure their ships against damage or loss. In 600 CE, the Greeks and Romans organised similar schemes to help families during times of illness and to pay funeral expenses upon death. In 1693, the English astronomer and mathematician Edmund Halley, who discovered Halley's Comet, calculated the chances of people dying at different ages. Modern insurance companies use statisticians called actuaries to study the chances of losses and deaths occurring, such as:

- dying in a plane crash — 1 chance in 720 000
- being struck by lightning — 1 chance in 200 000
- dying in a car crash — 1 chance in 5000
- being in a house fire — 1 chance in 800
- going to prison — Male: 1 chance in 800. Female: 1 chance in 24 000.

CHAPTER OUTLINE

PB1 5-01 Probability of simple events
PB1 5-02 Multi-stage events
PB1 5-03 The multiplication principle for counting
PB1 5-04 Complementary events
PB1 5-05 Experimental probability
PB1 5-06 Comparing experimental and theoretical probabilities

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IN THIS CHAPTER YOU WILL:

- identify events with equally likely outcomes
- use the following definition of the theoretical probability of an event where outcomes are equally likely:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

- express probabilities as fractions, decimals and percentages
- recognise that $0 \le P(\text{event}) \le 1$
- determine the number of outcomes for a multi-stage experiment by multiplying the number of choices at each stage
- verify the total number of outcomes for simple multi-stage experiments by systematic listing
- comment critically on the validity of simple probability statements
- calculate the probability of the complement of an event using the relationship $P(\text{an event does not occur}) = 1 - P(\text{the event does occur})$
- perform simple experiments and use recorded results to obtain relative frequencies
- estimate the relative frequencies of events from recorded data
- use relative frequencies to obtain approximate probabilities
- compare theoretical probabilities with experimental estimates
- illustrate the results of experiments using statistical graphs and displays

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TERMINOLOGY

at least	calculated probability	certain
chance	complementary event	equally likely
even chance	event	experimental probability
frequency	impossible	likely
multiplication principle	multi-stage event	outcome
probability	random	relative frequency
sample space	tree diagram	unlikely

Assignment 5

Simplifying fractions

SkillCheck

1 Simplify each fraction.

a $\frac{28}{100}$ **b** $\frac{7}{77}$ **c** $\frac{13}{52}$ **d** $\frac{24}{36}$

2 Evaluate:

a $1-\frac{13}{40}$ **b** $1-\frac{22}{25}$ **c** $1-\frac{9}{72}$ **d** $1-\frac{16}{36}$

3 Convert each fraction to a decimal.

a $\frac{17}{20}$ **b** $\frac{9}{36}$ **c** $\frac{25}{40}$ **d** $\frac{4}{15}$

4 Convert each fraction to a percentage.

a $\frac{10}{25}$ **b** $\frac{45}{72}$ **c** $\frac{4}{52}$ **d** $\frac{16}{48}$

5 How many:

a cards in a normal deck of playing cards? **b** numbers on a die?

c months in a year? **d** letters in the alphabet?

e Aces in a deck of playing cards? **f** vowels in the alphabet?

6 A sock drawer contains 8 white socks, 2 black socks, 4 grey socks and 6 blue socks.

a What fraction of the socks are grey?

b What percentage of the socks are blue?

c What fraction of the socks are not white?

7 A sample of students was surveyed on the number of cars owned by their families and the results are shown in the table on the right.

a Illustrate this data on a frequency histogram.

b What fraction of the sample owned no cars?

c What percentage of the sample owned three cars?

d What percentage of the sample owned at least one car?

e What was the most frequent number of cars?

Number of cars	Frequency
0	4
1	16
2	11
3	0
4	1

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8 Rate each of the following events as being very unlikely (VU), unlikely (U), likely (L) or almost certain (AC).

a It will rain in your area tomorrow.

b A car number plate contains numbers.

c A mother has triplets.

d There is a traffic jam in Sydney on Friday morning.

e You will live to be 100 years old.

f You arrive to school on time on Monday.

g There is a hailstorm in your area this week.

h You will send an email today.

i A double six appears when a pair of dice is rolled.

j An adult is married.

The language of chance

5-01 Probability of simple events

Probability or chance can be measured on a 'likelihood scale' between 'impossible' and 'certain', or on a numerical scale between 0 and 1.

Games of chance

0		$\frac{1}{2}$		1
impossible	unlikely	even chance	likely	certain

SUMMARY

An **outcome** is the result of an experiment or game. When a die is rolled, there are six possible outcomes: 1, 2, 3, 4, 5 or 6.

The **sample space** is the set of all possible outcomes, for example {1, 2, 3, 4, 5, 6} when a die is rolled.

An **event** is a group of one or more outcomes, for example the event of rolling an even number on a die is {2, 4, 6}.

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Sample space

EXAMPLE 1

List the sample space and write the number of outcomes for each situation described below.

a tossing a coin

b the colour shown on a traffic light

c the last digit of a phone number

Solution

a {head, tail}: two outcomes

b {red, amber, green}: three outcomes

c {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}: 10 outcomes

Equally likely outcomes

We assume that all outcomes of an experiment or game are **equally likely** (have an equal chance). However, sometimes this is not the case:

- a biased coin may show heads more often
- loaded dice show some numbers more often
- in a competition some players or teams are more skilled.

Can you think of another situation where outcomes are not equally likely?

EXAMPLE 2

A letter of the alphabet is chosen at random from the page of a magazine.

'At random' means that every letter on the page has an equal chance of being chosen.

a How many outcomes are there in the sample space?

b Are all the outcomes equally likely?

Solution

a There are 26 outcomes, because there are 26 letters in the alphabet.

b The outcomes are not equally likely, because some letters appear in words more often (for example 'e').

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Theoretical probability

The probability of an event occurring, where all outcomes in the sample space are equally likely, is given by the formula:

SUMMARY

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

Probability can be expressed as a fraction, a decimal or a percentage.

EXAMPLE 3

A bag of jelly lollies contains 6 red lollies, 7 green lollies, 6 orange lollies, 5 yellow lollies and 4 white lollies. If one lolly is chosen at random, what is the probability that it is:

a red?

b orange or yellow?

c blue?

d not white?

Solution

Total number of outcomes $= 6 + 7 + 6 + 5 + 4$

$= 28$

There are 28 lollies and, therefore, 28 possible outcomes in the sample space.

a $P(\text{red}) = \frac{6}{28} = \frac{3}{14}$ **Six red lollies.**

b $P(\text{orange or yellow}) = \frac{6+5}{28} = \frac{11}{28}$

c $P(\text{blue}) = \frac{0}{28} = 0$ **No blue lollies.**

d $P(\text{not white}) = \frac{6+7+6+5}{28} = \frac{24}{28} = \frac{6}{7}$ **24 non-white lollies.**

Exercise 5-01 Probability of simple events

Example 1

1 List the sample space and write the number of outcomes for each of the following situations.

a selecting a day of the week to go shopping

b choosing a letter from the word 'MATHEMATICS'

c the classification rating of a movie

d the gender of the next baby born at a hospital

e the result of a driving test

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2 Count the number of outcomes in the sample space for each of these:

a the consonants from the alphabet

b choosing the month to set a wedding date

c choosing a four-digit PIN for accessing an ATM

d the Year group of a primary school student

e rolling a die in the shape of a triangle-based pyramid

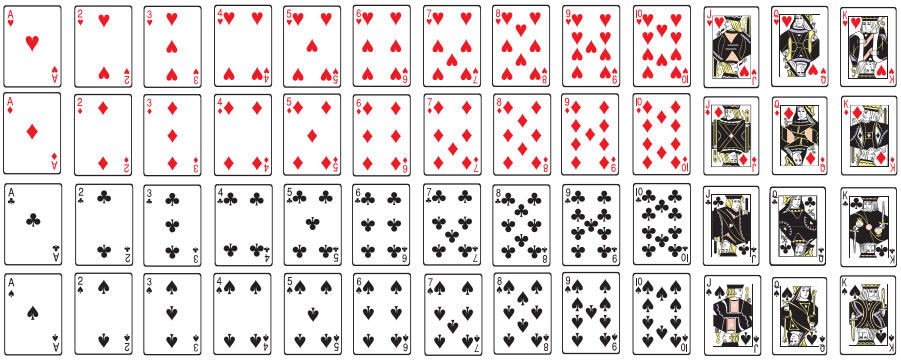
3 A normal deck of playing cards contains 52 cards. There are four suits of 13 cards each.

Hearts:

Diamonds:

Clubs:

Spades:



One card is drawn at random from a deck. Count the number of possible outcomes for each event described.

a a red card **b** a 10 **c** an even number

d a black Ace **e** a picture card **f** the Queen of hearts.

Example 2

4 If England plays Brazil in a soccer match:

a what are the three possible outcomes in the sample space for the result of the game?

b are all three outcomes equally likely?

5 Explain what is wrong with the following statement: 'Since the weather is either fine or raining each day, the probability of a fine day is $\frac{1}{2}$.'

Example 3

6 A bag contains 6 blue marbles and 3 white marbles. If one marble is selected without looking, what is the probability that it is:

a blue? **b** white? **c** red? **d** blue or white?

7 The numbers 1 to 15 are written on separate cards. A card is selected at random. What is the probability that this number:

a is even? **b** is odd? **c** is 9? **d** contains the digit 1?

8 A letter is chosen at random from the words 'NEW CENTURY'. Express as a decimal the probability that the chosen letter is:

a E **b** a vowel **c** W or Y **d** a letter after N in the alphabet.

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9 A multiple choice question has answers labelled A, B, C and D. If you know the answer is not B, what is the probability of guessing the correct answer?

10 Adriana's purse has four \$2 coins, one \$1 coin, two 50-cent coins, six 20-cent coins, eight 10-cent coins and three 5-cent coins. If she takes out a coin at random, what is the percentage probability that it is:

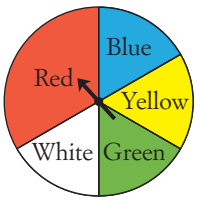
a a 10-cent coin? **b** a \$1 coin? **c** a silver coin?

d not a 20-cent coin? **e** a \$2 coin or a 50-cent coin?

11 Ken, Lydia, Melanie, Nathan, Olivia and Paula wrote their names on separate cards. Which of the following is the probability that a card chosen at random will have a girl's name on it? Select **A**, **B**, **C** or **D**.

A 25% **B** $33\frac{1}{3}\%$ **C** 50% **D** $66\frac{2}{3}\%$

12 This spinner has a red sector that is twice as large as each sector of the other colours. If the spinner is spun, find the probability that the arrow points to:



a red **b** white

c red or green **d** not blue.

13 One card is selected at random from a deck of playing cards. What is the probability that it is:

a a red card? **b** a black picture card? **c** a King?

d a number card? **e** a red 4? **f** the Ace of spades?

g the Queen of hearts or the Jack of clubs? **h** a diamond?

14 A total of 1200 raffle tickets were sold at the school fete. Ramy bought eight tickets. What is the probability that he wins first prize?

15 What is the probability that a person chosen at random was born in a month beginning with A? Express the answer as a recurring decimal.

16 Ryan says that, because there are 16 teams in the NRL football competition, his favourite team, the Eels, has a chance of $\frac{1}{16}$ of winning the competition this year. Why is he incorrect in saying this?

17 A number from 1 to 20 is chosen at random by a computer. Find the percentage probability that it is:

a greater than 8 **b** a square number **c** divisible by 3.

18 A loaded die is biased so that it lands on 6 twice as often as it lands on any other number. Find the probability that, when it is rolled, it lands on:

a 6 **b** an even number **c** 5

d a number greater than 2 **e** a number other than 6 **f** 7

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TECHNOLOGY: TOSSING A COIN

A spreadsheet can be used to simulate tossing a coin 20 times, by making the spreadsheet generate, at random, either the number 0 (for tails) or 1 (for heads).

Step 1: Type in cell A1 of a new spreadsheet the bold heading 'Tossing a coin 20 times', and in cell A2 type '0 = tails, 1 = heads'.

Step 2: In cell B4, enter the formula '=INT(RAND()*2)' to generate a random number 0 or 1 in that cell.

For an explanation of the INT and RAND functions on a spreadsheet, see 'Using technology: Generating random numbers' on page 221.

Step 3: **Fill right** to copy the formula across the row to cell F4 and simulate four more tosses of the coin.

Step 4: **Fill down** to copy the formulas from rows 4 to 7, for a total of 20 tosses. This should give a random series of 0s and 1s, similar to the spreadsheet shown below.

	A	B	C	D	E	F
1	**Tossing a coin 20 times**					
2	0 = tails, 1 = heads					
3						
4		1	1	0	0	1
5		1	0	0	0	1
6		1	0	0	1	0
7		0	0	0	1	0

Step 5: Decide how many times you would you expect heads (1) to come up in 20 tosses of a coin. Are there close to this many 1s in your 20 tosses?

Step 6: To simulate the toss of the coin another 20 times, press the **F9** key to 'recalculate' another set of random numbers. Count how many 'heads' there are this time. (Is the number close to 10?)

Step 7: Press the **F9** key each time you want to toss the coin another 20 times. Count the number of heads each time.

Step 8: We can make the spreadsheet count the number of heads by adding all 20 random values together, since heads = 1, tails = 0. In cell B9, type 'Number of heads' and in cell C9, type '=sum(B4:F7)'.

Step 9: Press the **F9** key a few more times to see how many heads appear in every 20 tosses of the coin.

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5-02 Multi-stage events

A **multi-stage experiment** has two or more stages occurring, for example:

- rolling two dice (two stages)
- tossing a coin three times (three stages)
- observing the weather for each day of a long weekend (three stages)
- drawing out five winning raffle tickets from a barrel. (five stages)

A **multi-stage event** consists of two or more events occurring together, such as rolling a sum of 10 on a pair of dice, or getting rain three days in a row. The sample space of a multi-stage experiment can be found using a **list**, a **table** or a **tree diagram**.

EXAMPLE 4

A coin is tossed twice.

a Show all the outcomes in the sample space using:

i a list **ii** a table **iii** a tree diagram.

b What is the probability of tossing:

i two heads? **ii** at least one tail?

Solution

a Let H = heads, and T = tails.

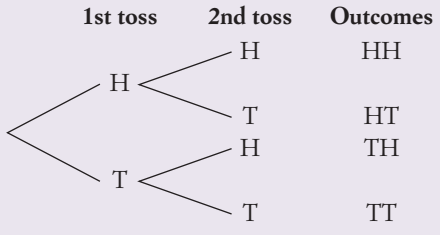
i Using a list:

Sample space = {HH, HT, TH, TT}

ii Using a table:

		1st toss	
		H	T
2nd toss	H	HH	TH
	T	HT	TT

iii Using a tree diagram:



A tree diagram shows all the different alternative pathways for outcomes at each stage.

For the 1st toss, you could get heads or tails. If you get heads, then for the 2nd toss, you could get heads or tails. If you get tails for the first toss then, for the 2nd toss, you could get heads or tails.

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b **i** $P(2 \text{ heads}) = P(\text{HH})$

$= \frac{1}{4}$

ii $P(\text{at least one tail}) = P(\text{HT or TH or TT})$

$= \frac{3}{4}$

'At least one' means 1 or more (that is, not none).

Tables and tree diagrams

Tables and tree diagrams are effective because they ensure that all possible arrangements are included.

A **table** can list the possible outcomes of a *two-stage* experiment.

A **tree diagram** lists the possible outcomes of an experiment with *two or more stages*. Branches stretch out to show the possible pathways of outcomes at each stage. An outcomes column at the end of the diagram lists the sample space.

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EXAMPLE 5

There are eight people in a dance class: three men (Grant, Rick, Stefan) and five women (Bethany, Tania, Elise, Chloë, Felicity).

a List all possible male-female dancing couples, using:

i a table **ii** a tree diagram

b How many dancing couples are possible?

c If one of these dancing couples is selected at random, what is the probability that:

i it is Rick and either Tania or Chloë?

ii it does not include Grant or Felicity?

iii it includes Stefan but not Bethany?

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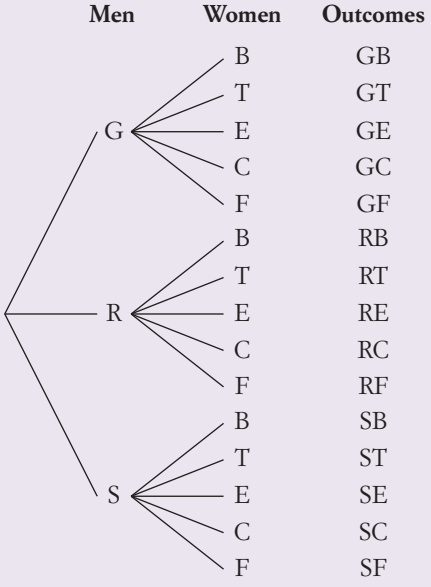
Solution

a Let: G = Grant R = Rick
S = Stefan B = Bethany
T = Tania E = Elise
C = Chloë F = Felicity

i Using a table:

		Men		
		G	**R**	**S**
Women	**B**	GB	RB	SB
	T	GT	RT	ST
	E	GE	RE	SE
	C	GC	RC	SC
	F	GF	RF	SF

ii Using a tree diagram:



b There are 15 possible dancing couples.

Note that, when there are 3 men and 5 women, there are 3 × 5 = 15 possible couples.

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c **i** $P(\text{Rick, Tania/Chloë}) = P(\text{RT or RC})$

$$= \frac{2}{15}$$

ii $P(\text{not Grant, not Felicity}) = P(\text{all outcomes without a G or F})$

$$= \frac{8}{15}$$

iii $P(\text{Stefan, not Bethany}) = P(\text{ST or SE or SC or SF})$

$$= \frac{4}{15}$$

Exercise 5-02 Multi-stage events

1 Match each description (**a** to **i**) below with the correct set of *all* counting numbers that meet that description (**A** to **F**).

a	at least 3	**b**	from 1 to 4	**c**	greater than 3
d	3 or more	**e**	between 1 and 4	**f**	at most 3
g	3 or less	**h**	less than 3	**i**	fewer than 5
A	{2, 3}	**B**	{0, 1, 2, 3}	**C**	{1, 2, 3, 4}
D	{3, 4, 5, …}	**E**	{4, 5, 6, …}	**F**	{1, 2}

Example 4

2 **a** How many outcomes are possible when:

i tossing a coin? **ii** tossing a die?

b A coin and a die are tossed together. Show all outcomes in the sample space, using:

i a list **ii** a table **iii** a tree diagram.

c How many outcomes in the sample space?

d Find the probability of tossing:

i heads and an even number

ii tails and a number less than 3

iii heads and not 1.

3 The digits 4, 5, 7 and 8 are written on separate cards and placed in a box. Two cards are drawn out at random to form a two-digit number. Note that the same digit cannot be used twice, so '75' is allowed but '77' is not.

a Use a tree diagram to list all possible two-digit numbers.

b How many two-digit numbers are possible?

c Find, as a percentage, the probability that the number drawn:

i has a first digit of 7 **ii** is odd

iii is less than 60 **iv** is divisible by 5.

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4 A coin is tossed three times.

a Show all the possible outcomes of heads (H) and tails (T) using:

i a list

ii a tree diagram with three stages.

b How many outcomes are there in the sample space?

c Find, as a decimal, the probability of tossing:

i exactly one tail

ii at least two heads

iii three heads

iv more tails than heads.

5 How many possible outcomes are there when a coin is tossed:

a once? **b** two times?

c four times? **d** 10 times?

6 Two dice are rolled together and their sum is calculated.

Example 5

a Copy and complete this table of possible sums.

		1st die					
	+	1	2	3	4	5	6
2nd die	1						
	2						
	3						
	4						
	5						
	6						

b What is the total number of possible outcomes?

c Find the probability of rolling a sum:

i of 10 **ii** of 6

iii less than 4 **iv** of 1

v that is odd **vi** of 7 or 11.

d Find the probability of rolling:

i a 'double one' (1 and 1)

ii a 'double' anything (the same number twice).

e Which sum(s) is:

i most likely? **ii** least likely?

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7 For breakfast, when staying at a hotel, Simone can choose one item from each course of a set menu.

1st course	2nd course
Cereal (C)	Bacon and eggs (B)
Raisin toast (R)	Pancakes (P)
Watermelon (W)	Sausages and hash browns (S)
Yoghurt (Y)	

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a Use a table to list all the different two-course breakfasts available.

b How many two-course breakfasts are possible?

c If one of the combinations is chosen at random, find the probability that it includes:

i cereal or raisin toast **ii** watermelon but not pancakes

iii yoghurt or pancakes **iv** yoghurt and pancakes.

8 Harry, Rita, Alison, Kirstie and Thuc are on the formal committee. They need to select a chairperson and a secretary.

a Use a table to determine all the possible pairings of chairperson and secretary. (Remember that a person cannot hold both positions.)

b If each person is equally likely to be selected, find the decimal probability that:

i Kirstie is chairperson and Thuc is secretary

ii Rita is either chairperson or secretary

iii Alison is secretary

iv Harry and Thuc fill the positions (in any order).

9 Consider a family with three children.

a Use a tree diagram to list the possible order of boys and girls for the children in the family.

b Which of the following is the number of possible outcomes? Select **A**, **B**, **C** or **D**.

A 8 **B** 16 **C** 7 **D** 6

c Find the probability of having:

i no boys

ii exactly two boys

iii at least one girl

iv at most one girl

v the eldest and youngest both being boys

vi more than one girl.

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10 Suppose that the weather forecast for each day of the week is sunny, cloudy or raining, each being equally likely.

a Use a tree diagram to show all the possible outcomes for the weather for Saturday and Sunday, the two days of the weekend.

b Which of the following is the number of stages in this experiment?
Select **A**, **B**, **C** or **D**.

A 2 **B** 3 **C** 4 **D** 6

c How many outcomes are possible?

d Find the probability that:

i it rains on both days

ii the weather is the same on both days

iii it doesn't rain on both days

iv it is sunny on at least one of the days

v it is cloudy on one of the days and sunny on the other.

5-03 The multiplication principle for counting

For multi-stage experiments involving many stages, it is impractical to use a tree diagram to list the entire sample space. Instead, we can use a multiplication principle for counting all of the possible outcomes.

Review the following examples from the previous section.

Number of possible outcomes when:	Number of possible outcomes when:	Number of possible outcomes when:
selecting from three male dancers = 3	selecting from five female dancers = 5	selecting male–female dance couples = $3 \times 5 = 15$
tossing a coin = 2	rolling a die = 6	tossing a coin and die together = $2 \times 6 = 12$
predicting the gender of a child = 2	predicting the gender of another child = 2	predicting the possible outcomes for the genders of three children = $2 \times 2 \times 2 = 8$

SUMMARY

To count the number of possible outcomes in a multi-stage experiment, multiply together the number of possible outcomes at each stage.

- If Stage A has a possible outcomes and Stage B has b possible outcomes, then A and B together have $a \times b$ possible outcomes.
- More generally, if Stage A has a possible outcomes, Stage B has b possible outcomes, Stage C has c possible outcomes, and so on, then A, B, C, ... together have $a \times b \times c \times \ldots$ possible outcomes.

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EXAMPLE 6

a How many outcomes are possible when three dice are rolled?

b What is the probability that all three dice show 2?

c What is the probability that all three dice show the same number?

Solution

a The first die has six possible outcomes, the second die has six possible outcomes, and the third die has six possible outcomes.

$\therefore$ Total possible outcomes $= 6 \times 6 \times 6 = 216$.

b $P(2, 2, 2) = \frac{1}{216}$ **There is only one way of getting the outcome (2, 2, 2).**

c $P(3 \text{ dice show same number}) = P(\text{all 1s, or all 2s, or all 3s, or all 4s, or all 5s, or all 6s})$

$$= \frac{6}{216}$$

$$= \frac{1}{36}$$

EXAMPLE 7

New NSW car number plates have two letters, then two numbers, then two letters, for example, RY 28 AU.

a In how many ways can you choose the first letter?

b In how many ways can you choose the first number?

c How many different number plates are possible?

d How many different number plates are possible if the first letter must be N?

Solution

a 26 ways A to Z

b 10 ways 0 to 9

c Possible number plates $= 26 \times 26 \times 10 \times 10 \times 26 \times 26$ **Two letters, two numbers, two letters.**

$= 45\ 697\ 600$

d Possible number plates $= 1 \times 26 \times 10 \times 10 \times 26 \times 26$ **Only one possibility for the first letter.**

$= 1\ 757\ 600$

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EXAMPLE 8

The digits 1, 3, 5, 7 and 8 are written on separate cards and placed in a box. Two cards are drawn out at random to form a two-digit number.

a How many different numbers are possible? List them.

b How many numbers over 50 are possible?

c How many even numbers are possible?

Solution

a Number of ways of choosing first digit = 5

Number of ways of choosing second digit = 4 **Four digits left after the first digit is chosen.**

$\therefore$ Number of possible two-digit numbers = 5 × 4

= 20

They are 13, 15, 17, 18, 31, 35, 37, 38, 51, 53, 57, 58, 71, 73, 75, 78, 81, 83, 85 and 87.

b 12 numbers over 50. **51, 53, 57, 58, 71, 73, 75, 78, 81, 83, 85 and 87.**

c Four even numbers (all ending in 8). **18, 38, 58 and 78.**

Exercise 5-03 The multiplication principle for counting

Example 6

1 A coin and two dice are tossed together.

a How many possible outcomes are there?

b What is the probability that:

i the coin shows tails and both dice show 5?

ii the two dice show 6 and 2 (in any order)?

2 Which of the following is the number of outcomes possible when two coins and one die are tossed together? Select **A**, **B**, **C** or **D**.

A 10 **B** 24 **C** 48 **D** 72

3 In the game of Two-up, three coins are tossed together.

a How many outcomes are there in the sample space? List them.

b Find the percentage probability of:

i three heads or three tails **ii** two heads

iii at least one tail **iv** no tails

v at most one head **vi** at least two heads.

4 New South Wales AM radio station call signs have the number 2 followed by two letters, for example 2UE, 2SM or 2WG. How many different call signs are possible?

Example 7

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5 Guidi's Italian restaurant has a special set menu where customers choose a three-course meal from the following menu.

Entrée	Main course	Dessert
Rocket salad	Whole fish	Passionfruit brulee
Antipasto	Ravioli	Gelato
Garlic prawns	Veal parmigiana	Venetian torte
Bruschetta	Seafood pizza	Tiramisu
	Chicken fettucine	

a How many different three-course meals can be chosen from this menu?

b How many possible meals are there if:

i you always have gelato?

ii you don't eat veal?

iii you are allergic to seafood?

6 If a car number plate is made up of three letters and three numbers, which of the following is the number of possible different plates? Select **A**, **B**, **C** or **D**.

A 1 560 000 **B** 308 915 776 **C** 2 340 000 **D** 17 576 000

7 Four-digit postcodes beginning with the number 2 represent places in NSW.

a How many different NSW postcodes are possible?

b What could be done when there are more towns and suburbs than there are possible postcodes?

8 A bicycle's combination lock has four dials and each dial has the digits 0 to 7 on it. How many four-digit codes are possible?

9 A new make of car has the following options.

Colour: red, white, blue, green or silver

Transmission: manual or automatic

Number of doors: 3, 4 or 5

Engine size: 1.3 litres, 1.6 litres or 2 litres

a How many different arrangements of these options are possible?

b How many options are possible if:

i the transmission must be manual and the engine size must not be 1.3 L?

ii the colour must not be blue or green and the car must have four doors?

iii the colour must be white and the transmission must be automatic?

10 Darren has 8 shirts, 5 pairs of pants and 4 pairs of shoes. In how many different ways can he dress himself?

11 In a football-tipping competition, tipsters attempt to pick the winners of eight matches. In each match, two teams play and one wins (there are no draws). How many possible arrangements of eight winners are there?

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12 Telephone numbers in Batemans Bay have eight digits, but the first two digits must be 44. How many possible telephone numbers are there for Batemans Bay?

13 A password is made up of six characters, alphabetic or numeric, for example CHR9S2.

a How many passwords are possible?

b If a password is selected at random, what is the probability that:

i the first character is 'P'?

ii the last character is '4'?

iii the first character is a letter and the last character is a number?

14 In the game of 'Yahtzee', five dice are rolled together. Which of the following is the number of different outcomes possible? Select **A**, **B**, **C** or **D**.

A 30 **B** 32 **C** 7776 **D** 15 625

Example 8

15 The digits 2, 5, 6 and 9 are written on separate cards and placed in a box. Two cards are drawn from the box, at random, to form a two-digit number.

a How many different numbers are possible? List them.

b How many numbers below 60 are possible?

c If one of the two-digit numbers is selected at random, what is the decimal probability that it is divisible by 5?

d If three cards are drawn from the box at random, how many different three-digit numbers are possible?

e If two cards are drawn from the box at random, but the first card is returned to the box so that it could be drawn out again (for example to form 22), how many different two-digit numbers are possible?

16 A school captain and vice-captain are elected from a field of nine candidates. How many possible pairings of school captain and vice-captain are there?

17 Juanita has five different Christmas cards to put into five different-coloured envelopes. In how many different ways can she place the cards into the envelopes (one card per envelope)?

18 Messages were once transmitted over long distances in Morse code, using either sound (a series of long and short beeps) or light (a series of long and short flashes). Letters and other characters were represented by long dashes (–) and/or short dots (•). The total number of dashes and dots that represented one character ranged from 1 to 4. For example, E was • while Y was – • – –.

a How many different letter codes are possible using:

i one dash or dot? **ii** two dashes or dots?

iii three dashes or dots? **iv** four dashes or dots?

b How many letter codes are possible in total?

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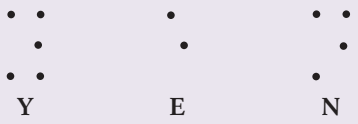
JUST FOR THE RECORD

The Braille alphabet

The Braille alphabet is a system of raised dots that allows blind people to read by touch. Letters and other symbols are represented by a cell of 1 to 6 dots printed in two columns. For example, the cells for the letters Y, E and N are:



There are also Chinese, Russian, Vietnamese and Korean versions of Braille. Frenchman Louis Braille invented the Braille alphabet in 1821. He became blind at 4 years of age after poking his eye accidentally with a saddler's awl (a tool for making holes in leather).

How many different cells are possible using the Braille system?
(Note: A completely blank cell is allowed.)

5-04 Complementary events

The range of probability

Because probability is defined as a fraction, as shown below, its value ranges from 0 to 1 (or, as a percentage, from 0% to 100%):

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

- A **certain** event must occur and has a probability of 1.
- An **impossible** event cannot occur and has a probability of 0.

EXAMPLE 9

A die is rolled. Find:

a $P(\text{rolling a 3}) + P(\text{not rolling a 3})$

b $P(\text{rolling a number less than 5}) + P(\text{rolling 5 or more})$

c $P(\text{rolling an odd number}) + P(\text{rolling an even number})$

Solution

a $P(3) + P(\text{not } 3) = P(3) + P(1, 2, 4, 5, 6) = \frac{1}{6} + \frac{5}{6} = 1$

b $P(< 5) + P(\geq 5) = P(1, 2, 3, 4) + P(5, 6) = \frac{2}{3} + \frac{1}{3} = 1$

c $P(\text{odd}) + P(\text{even}) = P(1, 3, 5) + P(2, 4, 6) = \frac{3}{6} + \frac{3}{6} = 1$

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The probability of an event not occurring

'Rolling 3 on a die' and 'not rolling 3 on a die' are examples of **complementary events** (or one event and its 'opposite'). 'Complement' means something that completes or matches with something to make a whole, and complementary events cover all the possibilities of an experiment. Because of this, the probabilities of complementary events must add to 1. Other examples of complementary events are:

- 'rolling a number less than 5', and 'rolling a number 5 or more'
- 'rolling an odd number', and 'rolling an even number'

The 'complement of an event E' means all those outcomes that are not E, written as $\overline{E}$, and $P(\overline{E})$ is the probability of E *not occurring*.

SUMMARY

$P(\text{event does occur}) + P(\text{event does not occur}) = 1$

$P(\text{event does not occur}) = 1 - P(\text{event does occur})$

$P(\overline{E}) = 1 - P(E)$

If the probability is in percentage form, then $P(\overline{E}) = 100\% - P(E)$

EXAMPLE 10

A ball is selected at random from a bag containing five green, seven red and two yellow balls. Find the probability that the ball selected is:

a yellow **b** not yellow **c** blue

d not blue **e** not red **f** not red or not green.

Solution

Total number of balls $= 5 + 7 + 2 = 14$

a $P(\text{yellow}) = \frac{2}{14} = \frac{1}{7}$

b $P(\text{not yellow}) = 1 - P(\text{yellow})$

$= 1 - \frac{1}{7}$

$= \frac{6}{7}$

c $P(\text{blue}) = \frac{0}{14} = 0$ **There are no blue balls. This is an impossible event.**

d $P(\text{not blue}) = 1 - P(\text{blue})$

$= 1 - 0$

$= 1$ **All balls are not blue. This is a certain event.**

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e $P(\text{not red}) = 1 - P(\text{red})$

$$= 1 - \frac{7}{14}$$

$$= \frac{7}{14}$$

$$= \frac{1}{2}$$

f $P(\text{not red or green}) = 1 - P(\text{red or green})$

$$= 1 - \frac{7+5}{14}$$

$$= \frac{1}{7}$$

Why is this answer the same as the answer for part **a**?

Exercise 5-04 Complementary events

1 Write the complementary event for each of the following events.

a getting a tail when a coin is tossed

b selecting a heart card from a deck of playing cards

c failing a driving test

d having over three children in your family

e selecting a black sock from a drawer of black socks and white socks

f the weather today being rainy

g winning a swimming race

h a traffic light showing green

Example 9

2 A die is rolled. What is the probability that the result is:

a not a 6? **b** not less than 3? **c** not a multiple of 3?

3 Two coins are tossed together. List the sample space, then find the probability that the result is:

a two tails **b** not two tails **c** one tail

d not one tail **e** no tails **f** at least one tail.

4 Repeat question **3** but with three coins tossed together.

Example 10

5 A computer randomly selects a number from 1 to 20. Find the decimal probability that it is:

a not less than 10 **b** not divisible by 3 **c** not a two-digit number

d not containing a 7 **e** not divisible by 5 **f** not a factor of 20.

6 Samantha bought a ticket in a raffle in which 1000 tickets were sold. If there are eight prizes, which of the following is the probability that Samantha doesn't win a prize? Select **A**, **B**, **C** or **D**.

A $\frac{124}{125}$ **B** $\frac{7}{8}$ **C** $\frac{1007}{1008}$ **D** $\frac{999}{1000}$

7 Each of the letters in the word 'PROBABILITY' is written on a piece of paper and put into a box. What is the probability that a letter drawn out at random is:

a not 'P'? **b** not 'B'? **c** not a vowel? **d** not 'A' or 'B'?

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8 What is the probability that a baby chosen at random from a maternity hospital was not born on a Saturday or Sunday?

9 The ratio of green to yellow to white lollies in a jar is 4 : 3: 2. One lolly is selected at random. Find the probability that it is:

a not green **b** yellow **c** not white

d not yellow **e** not red **f** not green or yellow.

10 In the dice game 'Craps', players lose if they roll a sum of 2 or 12 on a pair of dice. Which of the following is the probability of not losing? Select **A**, **B**, **C** or **D**.

A $\frac{5}{6}$ **B** $\frac{17}{18}$ **C** $\frac{35}{36}$ **D** $\frac{8}{9}$

11 The probability of rain this weekend is 23%. What is the probability that it will not rain?

12 A deck of cards is shuffled and one card is selected at random. Find:

a P(not a 10) **b** P(not a picture card) **c** P(not a club)

d P(not a black card) **e** P(not a red Ace) **f** P(not a black 7, 8, 9 or 10)

13 A traffic light is red for 63 seconds, green for 99 seconds and amber for 2 seconds. Express, correct to three decimal places, the probability that a car faces a traffic light that is:

a red

b not red

c not amber

d not green

e not red or amber

f not red, green or amber.

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14 What is the probability that a student chosen, at random, from your class was not born on 3 June?

15 What is the percentage probability that a mobile phone number selected at random does not end in 0 or 9?

16 In a football match, the Knights have a 42% chance of beating the Dragons, while the Dragons have a 51% chance of beating the Knights.

a What other outcome is possible?

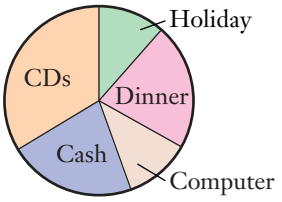
b What is the probability of this outcome?

17 The probability that Westvale High will win the debate is $\frac{13}{20}$. What is the probability that Westvale High will not win?

18 Lisa spins the wheel shown to determine her prize. What is the probability that she:

a does not win the holiday?

b does not win the cash or computer?



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A page of spinners

Coins probability

Dice probability

5-05 Experimental probability

Sometimes, we can only calculate probabilities based on past statistics (for example the probabilities of a rainy day, of having heart disease, of winning a football match or of a car accident occurring at a particular time of day).

The **experimental probability** of an event is a probability based on repeated trials of the same experiment or situation. It is the event's **relative frequency**, which is the number of times the event occurred as a fraction of the total number of trials.

SUMMARY

$$\text{Experimental probability of an event} = \text{relative frequency of the event}$$
$$= \frac{\text{frequency of the event}}{\text{total frequency}}$$

EXAMPLE 11

This table shows the times of day that car accidents occurred in Uptown last month.

Time of accident	Frequency
midnight–4:00 a.m.	4
4:00 a.m.–8:00 a.m.	13
8:00 a.m.–noon	17
noon–4:00 p.m.	20
4:00 p.m.–8:00 p.m.	23
8:00 p.m.–midnight	7

Based on these results, what is the experimental probability of a car accident in Uptown occurring:

a between midnight and 4:00 a.m.?

b between 8:00 a.m. and 4:00 p.m.?

c after midday?

d earlier than 8:00 p.m.?

Solution

Total frequency $= 4 + 13 + 17 + 20 + 23 + 7 = 84$

There were 84 road accidents that month.

a $P(\text{midnight–4:00 a.m.}) = \frac{4}{84} = \frac{1}{21}$

b $P(\text{8:00 a.m.–4:00 p.m.}) = \frac{17 + 20}{84} = \frac{37}{84}$

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c $P(\text{after midday}) = \dfrac{20+23+7}{84} = \dfrac{50}{84} = \dfrac{25}{42}$

d $P(\text{earlier than 8:00 p.m.}) = 1 - P(\text{after 8:00 p.m.})$

$$= 1 - \frac{7}{84}$$

$$= 1 - \frac{1}{12} = \frac{11}{12}$$

EXAMPLE 12

A pair of dice was rolled 50 times and their sum calculated each time. The results are shown in this table.

Sum	2	3	4	5	6	7	8	9	10	11	12
Frequency	0	2	4	6	5	5	9	6	8	3	2

a Draw a dot plot to illustrate this data.

b Which sum is:

i most likely? **ii** least likely?

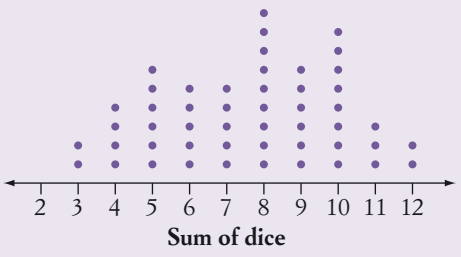
c Calculate, as a decimal, the relative frequency of rolling a sum:

i of 7 or 11 **ii** that is an odd number

iii greater than 9.

Solution

a



b **i** 8 is most likely. **It has the highest frequency (9).**

ii 2 is least likely. **It has the lowest frequency (0).**

c **i** $P(7 \text{ or } 11) = \dfrac{5+3}{50} = \dfrac{8}{50} = 0.16$

ii $P(\text{odd}) = \dfrac{2+6+5+6+3}{50} = \dfrac{22}{50} = 0.44$

iii $P(> 9) = \dfrac{8+3+2}{50} = \dfrac{13}{50} = 0.26$

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Example 11

Exercise 5-05 Experimental probability

1 A sample of matchboxes was taken and the number of matches in each box was counted. The results are shown in this table.

Number of matches	Frequency
48	2
49	15
50	70
51	56
52	6
> 52	1

a How many matchboxes were in the sample?

b What is the experimental probability of a matchbox containing:

i 51 matches?

ii more than 50 matches?

iii 49 or fewer matches?

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2 A die was rolled 80 times, with the results shown below.

Outcome	1	2	3	4	5	6
Frequency	11	13	9	13	12	22

a Are the outcomes equally likely?

b Do you think this die is loaded (biased)? Give a reason for your answer.

c Express as a percentage the relative frequency of rolling 6 on this die.

d If this die was rolled 100 times, how many 6s would you expect?

3 Thanh surveyed the number of children in each house in her street, and found the following results.

Number of children	0	1	2	3	4	5
Frequency	5	10	14	7	3	1

a Which number of children is second-most likely?

b If a house is selected at random from Thanh's street, what is the decimal probability that it contains:

i two children?

ii no children?

iii at least one child?

iv more than one child?

v three or more children?

vi at most three children?

c How many four-child families would you expect to find in a similar sample of 150 houses?

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4 In all of their home games, the Dragons won 16 matches and lost 12 matches. Which of the following is their experimental probability of losing a home game? Select **A**, **B**, **C** or **D**.

A 25% **B** $42\frac{6}{7}\%$ **C** $57\frac{1}{7}\%$ **D** 75%

5 The daily weather in Springfield during the month of June is shown in the table on the right.

Weather	Number of days
Fine	13
Rain	10
Cloudy	7

a Are the outcomes equally likely?

b Find the relative frequency of the Springfield weather in June being:

i rainy

ii not cloudy.

6 In 2006, Australia's population was 19 855 288, including 10 056 038 females.

a What is the probability that a person chosen at random from the Australian population in 2006 was female? Express your answer as a percentage, correct to two decimal places.

b Is the person chosen in part **a** more likely to be male or to be female?

c If Australia's population now is 22 400 000, how many males would you expect there to be?

7 Copy the following table and use it to record the results for throwing a drawing pin 30 times and observing whether it lands point up or point down. Example 12

Outcome	Tally	Frequency
Point up		
Point down		

a Are the outcomes equally likely?

b Graph the results on a sector graph.

c What is the experimental probability that a drawing pin lands point up?

8 Copy this table and complete it by noting the first letter of the surname of every student in your class.

Surname initial	Tally	Frequency
A–E		
F–J		
K–O		
P–T		
U–Z		

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a Which class interval of initials is:

i least likely? **ii** most likely?

b What is the relative frequency of a student selected at random from the class having a surname beginning with a letter:

i from A to E? **ii** after O in the alphabet? **iii** not from K to O?

c If there are 120 students in Year 11, estimate how many students have a surname beginning with a letter from F to J.

9 Toss three coins together 40 times. Copy and complete this table by recording the number of tails that come up each time.

Alternatively, you could simulate tossing three coins together using a calculator or spreadsheet. (See 'Using technology: Generating random numbers' on the next page.)

Number of tails	Tally	Frequency
0		
1		
2		
3		

a Are the outcomes equally likely? If not, which outcome is least likely?

b Express as a decimal the experimental probability of tossing:

i no tails **ii** fewer than three tails **iii** two heads.

c If three coins were tossed 100 times, estimate how many times two tails would result.

10 Draw a card at random from a deck of playing cards and record its suit in a table similar to the one below. Do this 40 times, returning the card to the deck each time and reshuffling.

Suit	Tally	Frequency
Clubs		
Spades		
Hearts		
Diamonds		

a If a card is selected at random 40 times from a deck, how often should a hearts card be selected?

b Represent the data from your completed table on a sector graph.

c Find the relative frequency of selecting a card that is:

i a spade **ii** not hearts **iii** red.

11 Copy and complete this table by recording the current ages of the students in your class.

Age	Tally	Frequency
15		
16		
17		
18		
19+		

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a Represent this data on a dot plot.

b Which age is most frequent?

c Find the experimental probability of a student from your class:

i being age 16 **ii** being age below 18 **iii** not being age 17.

d If there are 125 students in Year 11, estimate how many of them are 17 years old.

TECHNOLOGY: GENERATING RANDOM NUMBERS

Heads and tails

Random numbers can be generated on a calculator or spreadsheet using the RAN# or =RAND() functions. This function outputs a random decimal between 0 and 1 but we can use it to create a random number of any range.

To generate a random whole number from 0 to *n*

Multiply the random decimal generated (between 0 and 1) by (n + 1), then ignore the decimal part of the number (for example read 1.092 as 1).

For example, to create a random number from 0 to 4:

- on a scientific calculator, type RAN# × 5 =. Press = repeatedly for more random numbers.
- on a graphics calculator, enter 'int(ran#×5)' from the probability functions and press EXE. Press EXE repeatedly for more random numbers.
- on a spreadsheet, type into a cell '=INT(RND()*5)' and press the Enter key. Note that, unlike other spreadsheet functions, =RAND() does not require a cell address inside its brackets.

The INT function outputs the integer (whole) part of a decimal only.

To generate a random whole number from 1 to *n*

Multiply the random decimal generated (between 0 and 1) by n, then add 1, and ignore the decimal part of the number.

For example, to create a random number from 1 to 6:

- on a scientific calculator, type RAN# × 6 + 1. Press = repeatedly for more random numbers.
- on a graphics calculator, enter 'int(ran#×6+1)' from the probability functions and press EXE. Press EXE repeatedly for more random numbers.
- on a spreadsheet, type into a cell '=INT(RND()*6 + 1)' and press the Enter key.

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TECHNOLOGY: ROLLING A PAIR OF DICE

A spreadsheet can be used to simulate the rolling of two dice and calculate their sum.

	A	B	C	D
1	**Rolling a pair of dice**			
2				
3	Trial	Die 1	Die 2	Sum
4	1			
5				
6				
7				
8				
9				
10				
11				

Step 1: Enter the labels on the right into a blank spreadsheet.

Step 2: We will simulate the rolling of two dice 10 times so, in the 'Trial' column, we need the numbers 1 to 10. In cell A5, enter '=A4 +1' to get 2, then use **Fill down** to copy that formula to cell A13 to generate the rest of the Trial numbers.

Step 3: In cell B4, enter '=INT(RAND()*6+1)' to generate a random integer from 1 to 6 and simulate the rolling of the first die. Copy or use **Fill right** to simulate the rolling of the second die and check that you have two numbers from 1 to 6 for each trial.

Step 4: In cell D4, enter an appropriate formula to calculate the sum for the pair of dice.

Step 5: **Fill down** the formulas in columns B, C and D to row 13 so that there are 10 trials for the rolling of two dice.

Step 6: To generate another 10 trials, press the **F9** key to 'recalculate' another set of random dice numbers. Do these numbers seem reasonable for the rolling of dice?

Step 7: Keep pressing the **F9** key to simulate rolling the die another 10 times and observe the sums that are generated. Does 7 come up more often than other numbers?

Step 8: Extend this spreadsheet to simulate 100 trials of rolling a pair of dice.

Step 9: Modify this spreadsheet to simulate the tossing of two coins (tails = 0, heads = 1) and change the 'Sum' column so it calculates the 'Number of heads'.

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5-06 Comparing experimental and calculated probabilities

Probability of $\frac{1}{2}$

How is the experimental probability of an event related to its theoretical or calculated probability?

EXAMPLE 13

A die is rolled 60 times and the results are as shown.

Outcome	1	2	3	4	5	6
Frequency	6	13	12	8	9	12

a Find the experimental probability of rolling a 4:

i as a fraction **ii** as a decimal, correct to three decimal places.

b Find the calculated probability of rolling a 4:

i as a fraction **ii** as a decimal, correct to three decimal places.

c Are the experimental and calculated probabilities similar?

d If a die is rolled 60 times, what is the expected number of times a 4 will be rolled?

e The die is rolled 300 times, with the following results.

Outcome	1	2	3	4	5	6
Frequency	42	45	44	52	50	57

Calculate, to three decimal places, the experimental probability of rolling a 4.

f What do you notice about the experimental and calculated probabilities now?

g If a die is rolled 300 times, what is the expected number of times a 4 will be rolled?

Solution

a Experimental probability:

i $P(4) = \frac{8}{60} = \frac{2}{15}$ **ii** $P(4) = 0.1333\ldots \approx 0.133$

b Calculated probability:

i $P(4) = \frac{1}{6}$ **ii** $P(4) = 0.1666\ldots \approx 0.167$

c By examining the decimal values, the experimental and calculated probabilities are close.

d Expected number of 4s $= \frac{1}{6} \times 60 = 10$

e Experimental $P(4) = \frac{52}{300} = 0.1733\ldots \approx 0.173$

f The experimental probability is now closer to the calculated probability (0.167).

g Expected number of 4s $= \frac{1}{6} \times 300 = 50$

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SUMMARY

As the number of trials increases, the experimental probability becomes closer to the calculated probability. This is sometimes called 'the law of averages'.

Exercise 5-06 Comparing experimental and calculated probabilities

This exercise may be completed as a group activity.

Example 13

1 Ethan tossed three coins together.

a List all the possible outcomes in the sample space.

b Write, as a percentage, the probability of tossing:

i two heads **ii** one or three heads.

c If Ethan tossed three coins 88 times, how many times should he get:

i two heads? **ii** one or three heads?

d Ethan actually had the following results.

Number of heads	0	1	2	3
Frequency	11	35	30	12

Write, as a percentage correct to two decimal places, the experimental probability of getting:

i two heads **ii** one or three heads.

e How do the experimental probabilities compare with the calculated probabilities?

f If three coins are tossed together 200 times, what is the expected number of times that:

i two heads will be tossed? **ii** one or three heads will be tossed?

g Stephanie tossed three coins 200 times with the following results.

Number of heads	0	1	2	3
Frequency	23	72	80	25

Write, as a percentage, the experimental probability of tossing:

i two heads **ii** one or three heads.

h Do the experimental probabilities become closer to the calculated probabilities with more repeated trials?

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2 Roll a die 150 times and record your results in a table similar to the one below.

Alternatively, you could simulate rolling a die using a calculator or spreadsheet. (See 'Using technology: Generating random numbers' on page 221.)

Outcome	Tally	Frequency
1		
2		
3		
4		
5		
6		

a Express as a decimal, correct to three decimal places:

i the experimental probability of rolling a 1

ii the calculated probability of rolling a 1

iii the experimental probability of rolling an even number

iv the calculated probability of rolling an even number.

b What is the expected number of times a 1 should appear when a die is rolled 150 times?

c By pooling your results with those of other students or groups in your class, repeat part **a** to see whether, with more trials, your experimental probabilities become closer to the calculated probabilities.

3 **a** Copy and complete this table of the possible sums when a pair of dice are rolled together.

		1st die					
	+	1	2	3	4	5	6
2nd die	1						
	2						
	3						
	4						
	5						
	6						

b As a decimal, correct to four decimal places, what is the calculated probability of rolling a sum of:

i 4? **ii** 2 or 12? **iii** 7 or 11?

c Copy the table below, then roll a pair of dice 120 times and record your results in the table.

Sum	Tally	Frequency
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

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d If a pair of dice is rolled 120 times, what is the expected number of:

i 4s? **ii** 2s or 12s? **iii** 7s or 11s?

e As a decimal, correct to four decimal places, what is the experimental probability of rolling a sum of:

i 4? **ii** 2 or 12? **iii** 7 or 11?

4 A card is randomly drawn from a deck of cards to see if it is a picture card (Jack, Queen or King). This is done 78 times, with each card being replaced in the deck before the next draw. The results are shown in the table on the right.

Outcome	Frequency
Picture card	20
Not a picture card	58

a Express as a decimal, correct to three decimal places:

i the calculated probability of selecting a picture card

ii the experimental probability of selecting a picture card.

b If a card is randomly drawn from a deck of cards 78 times, which of the following is the expected number of times a picture card should be drawn? Select **A**, **B**, **C** or **D**.

A 18 **B** 21 **C** 24 **D** 26

c Use a deck of cards to perform this experiment 78 times, and check whether your experimental probability is close to the calculated probability.

5 **a** How many outcomes are there in the sample space when a coin and a die are tossed together?

b Tahlia tossed a coin and a die together 48 times and found the frequencies shown on the right.

		Die number					
		1	**2**	**3**	**4**	**5**	**6**
Coin	**Heads**	4	5	3	2	4	5
	Tails	3	6	5	4	4	3

Calculate as a decimal, correct to three decimal places:

i the calculated probability of tossing a tail and a 3

ii the experimental probability of tossing a tail and a 3

iii the calculated probability of tossing a head and an even number

iv the experimental probability of tossing a head and an even number

v the calculated probability of tossing a tail and any number but 4

vi the experimental probability of tossing a tail and any number but 4.

c If a coin and a die are tossed together 48 times, how many times should each of the following events occur?

i a tail and a 3

ii a head and an even number

iii a tail and any number but 4.

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6 A coin is tossed repeatedly, and it lands with heads uppermost eight times in a row. On the next toss, is a head more likely to result again, or a tail? Or does each outcome still have an equal chance?

7 Zeinab uses a computer program to simulate tossing a coin but she suspects that the simulated coin is biased when she notices the pattern of results recorded in the table below.

Total number of tosses	50	140	200	540
Number of heads	23	55	84	226

a Calculate as a percentage, correct to one decimal place, the experimental probability of the computer simulation 'tossing heads' when the computer tosses the coin:

i 50 times

ii 140 times

iii 200 times

iv 540 times

b Is Zeinab justified in believing that the computer program's coin is biased? Give a reason for your answer.

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INVESTIGATION

ARE THE LOTTO NUMBERS EQUALLY LIKELY?

Lotto is a gambling game run by NSW Lotteries, in which 45 numbered balls are placed in a barrel and eight numbers are selected at random from the barrel: six main numbers plus two supplementary numbers. Between 2001 and 2008, there was a total of 3376 Lotto numbers drawn for the Saturday game (eight numbers each week). The frequency of drawing each number (from 1 to 45) was as follows.

Number	Frequency	Number	Frequency	Number	Frequency
1	77	**16**	58	**31**	70
2	74	**17**	69	**32**	85
3	82	**18**	70	**33**	78
4	72	**19**	67	**34**	78
5	82	**20**	68	**35**	65
6	91	**21**	97	**36**	83
7	94	**22**	86	**37**	75
8	81	**23**	77	**38**	72
9	68	**24**	71	**39**	76
10	73	**25**	71	**40**	95
11	85	**26**	72	**41**	79
12	81	**27**	71	**42**	75
13	58	**28**	75	**43**	68
14	68	**29**	67	**44**	63
15	76	**30**	58	**45**	75

Source: www.nswlotteries.com.au

1 If, over the eight years, 3376 numbers were drawn, and each number from 1 to 45 was equally likely, how often would you expect each number to have been drawn?

2 How many of the frequencies from the above table are equal to the expected number you suggested in question **1**?

3 Which Lotto number came up:

i most often? **ii** least often?

4 Do you think the Lotto numbers are equally likely, or is the Lotto draw biased? Give a reason for your answer.

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Sample HSC problem

Two dice are rolled together and the difference between the larger and smaller number is calculated.

a Copy and complete this table of possible differences.

		1st die					
	–	1	2	3	4	5	6
2nd die	1						
	2				2		
	3						
	4						
	5	4					
	6						0

b Which difference is most likely?

c Find the probability of rolling a difference:

i of 2

ii that is an odd number

iii less than 5.

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Study tip

Making lists and managing your time

You cannot just make up your study routine as you go along. You need to have a plan, even if it is a rough one. Make two lists of the things you want to achieve:

- an 'A list' of urgent tasks, such as homework and reading
- a 'B list' of less pressing demands, such as research assignments and revising for an exam.

Write in your diary what you want to accomplish each day or each week, and include all important dates, deadlines and reminders.

Make sure your 'A list' is realistic, taking into account the time and effort required for each task. Remember that your time is both precious and limited, so it is not possible to do everything. Make the most of the time that you have and don't be afraid to cut corners. Set realistic goals and spend time only on what is important. Start studying straight away and avoid timewasters, such as cleaning your room, decorating title pages and even devising elaborate study timetables (if you do these, then you are really putting things off).

From your 'A list', choose the easiest task to begin with, so that you can ease yourself into the routine. Alternatively, some students like to start with a difficult task while their mind is fresh and alert. Once you have completed a significant amount of work, give yourself a break and a small reward.

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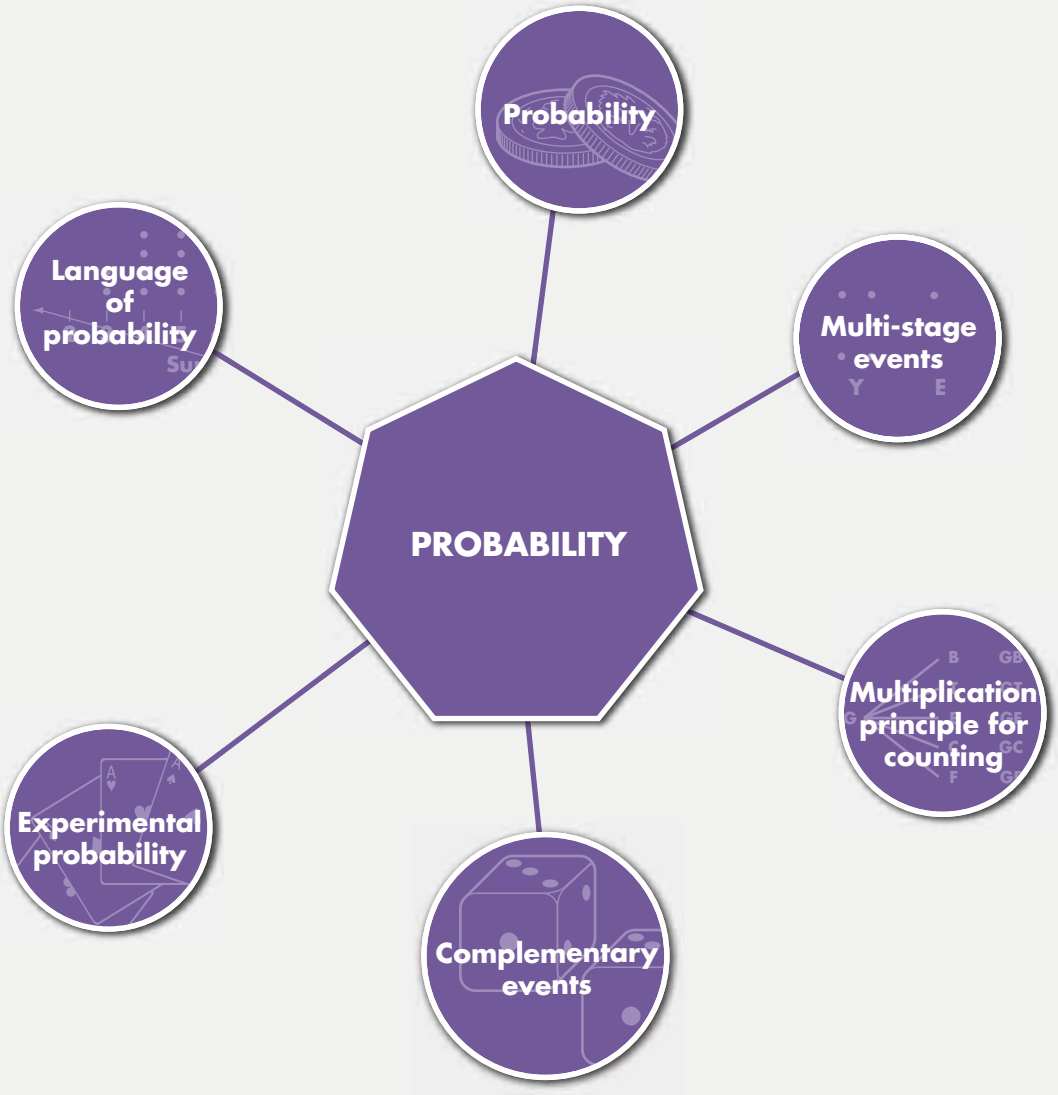
CHAPTER REVIEW

Topic overview

This chapter, 'Probability', looked at the probabilities of simple and multi-stage events. You should have a good understanding of probability concepts, including complementary events, multi-stage events and determining sample spaces using lists, tables, tree diagrams and the multiplication principle. You should be able to find and compare calculated and experimental probabilities, using given data or through repeated trials of an experiment.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map has been printed below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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Matching probabilities

Probability review

Probability crossword

Revision

Exercise 5-01

1 A letter is selected at random from the alphabet. Find the probability that it is:

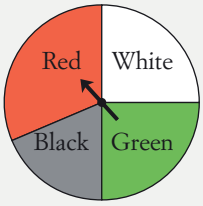
a P

b a vowel or Y

c not W, X, Y or Z

Exercise 5-01

2 A spinner is divided into sectors as shown, with 'Red' taking up one-third of the circle. What is the percentage probability that the pointer points to:



a green?

b black?

c red or green?

d a sector that is not red?

Exercise 5-01

3 Explain what is wrong with the following statement: 'Since a person can be either married or single, the probability of a person being married is 50%'.

Exercise 5-02

4 **a** Use a tree diagram to list the sample space of the different possible combinations of boys (B) and girls (G) in a family of three children.

b If a three-child family is selected at random, what is the probability that it contains:

i all girls? **ii** exactly one boy?

iii more boys than girls? **iv** at least one girl?

v at most one girl? **vi** a boy as the middle child?

Exercise 5-02

5 Two dice are rolled together. What is the probability that the two dice:

a show the same number? **b** show a sum of 8?

Exercise 5-03

6 Computers use a code of seven binary digits (0s and 1s) to represent characters. For example 101 0000 means 'P' while 011 0100 means '4'. How many different characters can be represented this way?

Exercise 5-03

7 Three dice are rolled together. Which of the following is the number of different outcomes possible? Select **A**, **B**, **C** or **D**.

A 36 **B** 729 **C** 216 **D** 18

Exercise 5-03

8 Six-digit phone numbers beginning with the numbers '13' can be accessed from anywhere in Australia for the cost of a local call.

a How many possible '13' phone numbers are there?

b If a '13' number is selected at random, what is the percentage probability that it:

i is 13 13 13?

ii ends in 0?

iii contains only odd numbers?

Exercise 5-04

9 If Santi has a 68% chance of missing a basketball shot, what is the probability that she scores with the shot?

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10 In a batch of 240 instant (scratch) lottery tickets, 54 contain a prize. Which of the following is the decimal probability of not winning a prize from one of these tickets? Select **A**, **B**, **C** or **D**.

Exercise 5-04

A 0.225 **B** 0.443 **C** 0.46 **D** 0.775

11 A sample of students was surveyed about how they travelled to school. The results are shown in the table on the right.

Exercise 5-04

Outcome	Frequency
Walk	43
Car	30
Bicycle	18
Bus	28
Train	21

What is the probability that a student, chosen at random, travels by:

a car?

b public transport?

c a method other than bicycle?

12 A factory tests a batch of batteries and finds that 77 of them are good while three are faulty.

Exercise 5-05

a Express as a percentage the probability that a battery selected at random is good.

b If the factory makes 5000 batteries, how many would you expect to be faulty?

13 Roll a die 48 times and record in a table, similar to the one shown, the number of times the die lands with '6' uppermost.

Exercise 5-05

Exercise 5-06

Outcome	Tally	Frequency
6		
Not 6		

a Represent this data on a divided bar graph.

b Find the experimental probability of rolling a 6 on this die.

c What is the actual probability of rolling a 6 on a die?

d What is the expected number of 6s when a die is rolled 48 times?

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6.

MEASUREMENT

One gigalitre is 10^9 litres, or one billion litres. There are approximately 500 gigalitres of water in Sydney Harbour. The entire Sydney urban area uses the equivalent of four Sydney Harbours of water each year, while the whole of Australia uses the equivalent of fifty Sydney Harbours per year.

CHAPTER OUTLINE

MM1	6-01	Metric units
MM1	6-02	Error in measurement
MM1	6-03	Significant figures
MM1	6-04	Scientific notation
MM1	6-05	Ratio problems
MM1	6-06	Dividing a quantity in a given ratio
MM1	6-07	Rate problems
MM1	6-08	Converting rates
MM1	6-09	Percentage change
MM2	6-10	Perimeters of composite shapes
MM1,2	6-11	Area
MM2	6-12	Offset surveys
MM2	6-13	Volume
MM1,2	6-14	Volume and capacity

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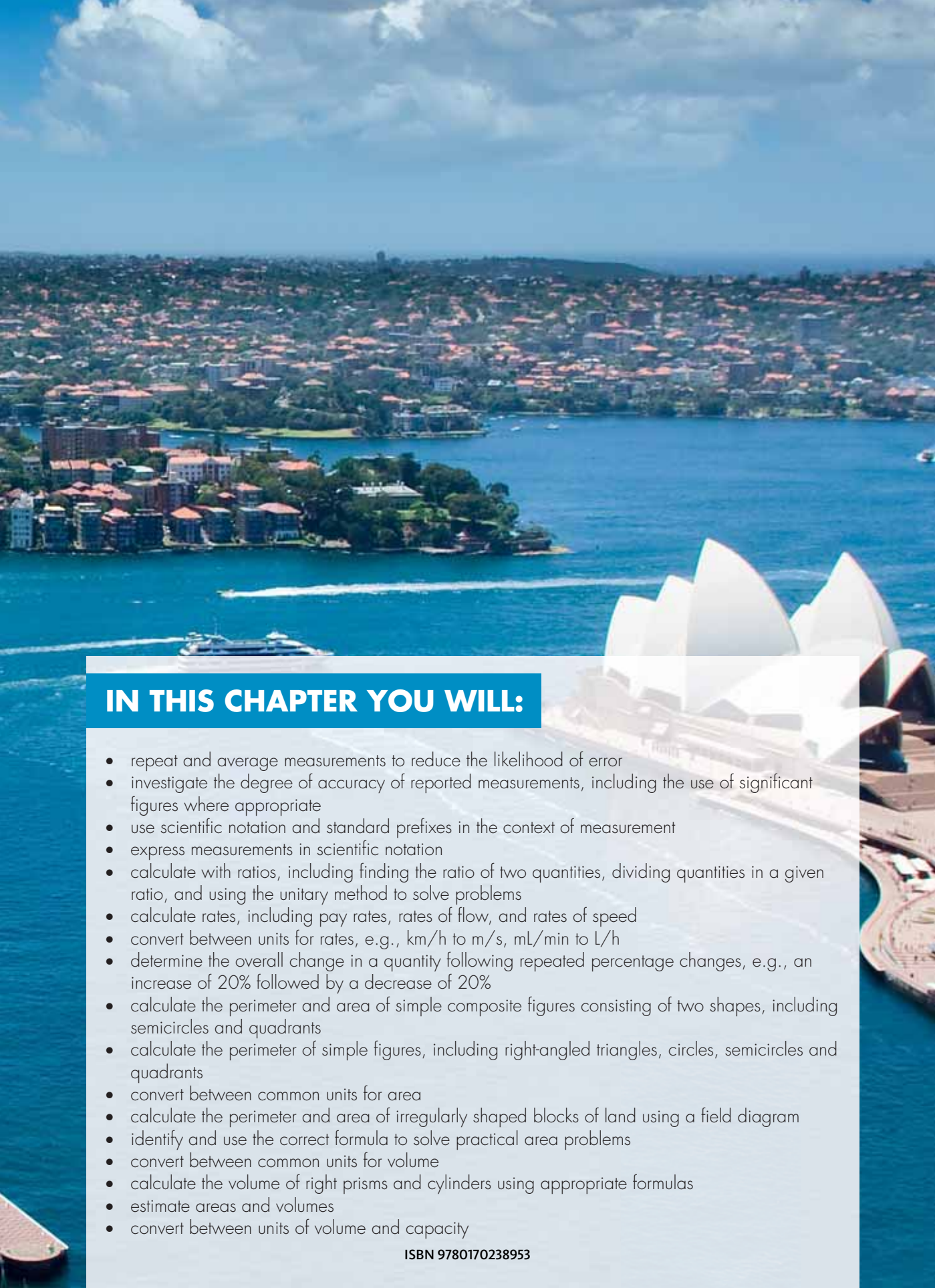

IN THIS CHAPTER YOU WILL:

- repeat and average measurements to reduce the likelihood of error
- investigate the degree of accuracy of reported measurements, including the use of significant figures where appropriate
- use scientific notation and standard prefixes in the context of measurement
- express measurements in scientific notation
- calculate with ratios, including finding the ratio of two quantities, dividing quantities in a given ratio, and using the unitary method to solve problems
- calculate rates, including pay rates, rates of flow, and rates of speed
- convert between units for rates, e.g., km/h to m/s, mL/min to L/h
- determine the overall change in a quantity following repeated percentage changes, e.g., an increase of 20% followed by a decrease of 20%
- calculate the perimeter and area of simple composite figures consisting of two shapes, including semicircles and quadrants
- calculate the perimeter of simple figures, including right-angled triangles, circles, semicircles and quadrants
- convert between common units for area
- calculate the perimeter and area of irregularly shaped blocks of land using a field diagram
- identify and use the correct formula to solve practical area problems
- convert between common units for volume
- calculate the volume of right prisms and cylinders using appropriate formulas
- estimate areas and volumes
- convert between units of volume and capacity

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TERMINOLOGY

absolute error
concentration
hectare
offset survey
prefix
rate
significant figures
accuracy
cross-section
megalitre (ML)
percentage error
prism
ratio
unitary method
capacity
cylinder
notebook entry
perpendicular height
pyramid
scientific notation

Assignment 6

SkillCheck

1 Evaluate without using a calculator.

a 20.83×1000 **b** $970.2 \div 10$ **c** $6.59 \times 10\,000$

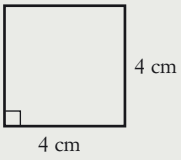
d $72.5 \div 100$ **e** $10.4 \div 1000$ **f** 0.0735×10

2 Evaluate:

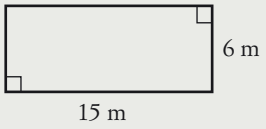
a 10^7 **b** 10^4 **c** 10^{-2}

3 Find the area of each shape. Express the answer to part **c** to two decimal places.

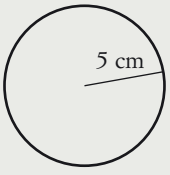
a



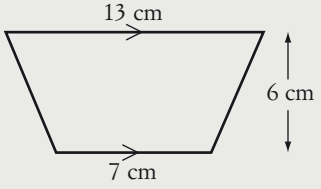
b



c

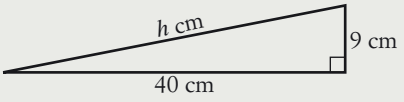


d



4 **a** Find the area of this triangle.

b Find the value of h.



5 **a** If bananas cost \$5.10 per kilogram, how much would you pay for 250 grams?

b If potatoes cost \$4.15 for 5 kg, how much would 3 kg of potatoes cost?

6 Convert 210 cm to:

a metres **b** millimetres **c** kilometres.

7 Convert 8.1 L to:

a kilolitres **b** millilitres.

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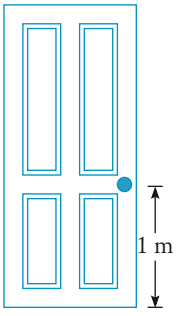
6-01 Metric units

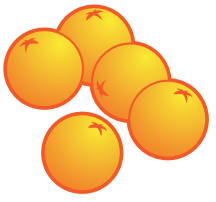
This table shows some common metric units for length, mass, time and capacity.

Unit	Relationships
Length	
micrometre (μm)	
millimetre (mm)	
centimetre (cm)	1 cm = 10 mm
metre (m)	1 m = 100 cm = 1000 mm = 1 000 000 μm
kilometre (km)	1 km = 1000 m
Mass	
milligram (mg)	
gram (g)	1 g = 1000 mg
kilogram (kg)	1 kg = 1000 g
tonne (t)	1 t = 1000 kg
megatonne (Mt)	1 Mt = 1 000 000 t
Time	
second (s)	
minute (min)	1 min = 60 s
hour (h)	1 h = 60 min = 3600 s
day (day)	1 day = 24 h
Capacity	
millilitre (mL)	
litre (L)	1 L = 1000 mL
kilolitre (kL)	1 kL = 1000 L
megalitre (ML)	1 ML = 1 000 000 L

Alamy/GOIM-



A **metre** is about the height of a door handle.



A **kilogram** is the mass of about five oranges.

A **litre** is the capacity of a tall carton of milk.

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SUMMARY

Metric prefixes

- micro- means 'one-millionth' or $\frac{1}{1\,000\,000}$
- milli- means 'one-thousandth' or $\frac{1}{1000}$
- centi- means 'one-tenth' or $\frac{1}{10}$
- kilo- means 'one thousand' or 1000
- mega- means 'one million' or 1 000 000

EXAMPLE 1

Convert:

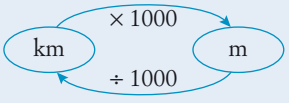
a 23.5 km to metres

b 450 L to kilolitres

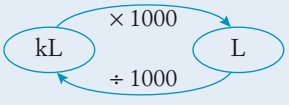
c 84 000 mg to kilograms

Solution

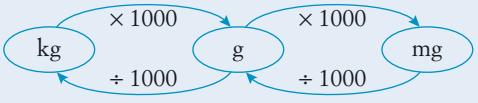
a 23.5 km = 23.5 × 1000 m
= 23 500 m



b 450 L = (450 ÷ 1000) kL
= 0.45 kL



c 84 000 mg = (84 000 ÷ 1000 ÷ 1000) kg
= 0.084 kg



SUMMARY

- To convert a larger unit to a smaller one, we *multiply* to get more of the smaller units.
- To convert a smaller unit to a larger one, we *divide* to get fewer larger units.

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Exercise 6-01 Metric units

Example 1

1 Convert:

a 6.3 cm to millimetres **b** 4.36 m to centimetres **c** 7200 mm to metres
d 285 g to kilograms **e** 6.9 mm to micrometres **f** 58 000 mL to litres
g 5.32 kg to grams **h** 3400 kg to tonnes **i** 4720 L to kilolitres
j 6000 mg to kilograms **k** 7.5 h to seconds **l** 9.4 km to centimetres.

2 Which of the following is the number of grams in 100 kg? Select **A**, **B**, **C** or **D**.

A 100 000 **B** 1000 **C** 1 000 000 **D** 10 000

3 Blake's height is 169 cm. Convert this to:

a millimetres **b** metres.

4 Mai weighs 57.5 kg. What is her mass in grams?

5 Rhys swam in a 1500 m race. How many kilometres did he swim?

6 Erin left home at 7:41 a.m. and arrived at work at 9:18 a.m. What was her travelling time in:

a minutes? **b** seconds? **c** hours (correct to one decimal place)?

7 Yusef's water consumption for July was 59 kL. How many litres of water did he use?

8 What metric unit would you use to measure each of the following?

a your mass **b** your height
c the length of your bedroom **d** the distance from Perth to Sydney
e the capacity of your kitchen sink **f** the mass of an ant
g the width of a sheet of paper **h** the amount of water in an Olympic-sized pool
i the mass of a train **j** the thickness of a strand of hair

9 Sara's pet mouse weighs 102 g and is 5.2 cm long. Write its:

a mass, in kilograms **b** length, in metres.

10 The Great Pyramid of Cheops (or Khufu) at Giza in Egypt contains 2 000 000 blocks, each weighing 2.45 tonnes. What is the total mass of the pyramid in megatonnes?

11 Madison competed in a triathlon. She took 12 min for the 5400 m bike ride, 38 min for the 3.8 km swim and 58 min for the 9.5 km run.

a What was the total distance Madison covered, in kilometres?

b What was Madison's total time, in hours and minutes?

12 Sarah's swimming pool holds 8560 L of water. How many kilolitres is this?

13 Which is the most likely measurement for each of the following? Select **A**, **B**, **C** or **D** each time.

a the height of a 10-storey building

A 400 m **B** 40 m **C** 4 m **D** 4000 m

b the capacity of a backyard swimming pool

A 30 L **B** 300 L **C** 3000 L **D** 30 000 L

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c the mass of an orange

A 3 mg **B** 3 g **C** 300 g **D** 3000 g

d the distance from Sydney to Brisbane

A 100 km **B** 500 km **C** 1000 km **D** 5000 km

e the diameter of a DVD

A 12 cm **B** 12 mm **C** 120 m **D** 1.2 cm

f the capacity of a soft drink can

A 380 mL **B** 3800 mL **C** 3.8 mL **D** 38 mL

14 On a 2.5 km walk, John covers 80 cm with each step, while his daughter Anna covers 55 cm with each of her steps. How many more steps does Anna take than her father during the walk?

15 How many seconds are there in one day?

JUST FOR THE RECORD

Metric prefixes

Prefix	Abbreviation	Meaning	Example
atto	a	10^{-18}	
femto	f	10^{-15}	
pico	p	10^{-12}	
nano	n	10^{-9}	nanosecond
micro	µ	$10^{-6} = \frac{1}{1\,000\,000}$	microgram
milli	m	$10^{-3} = \frac{1}{1000}$	millibar
centi	c	$10^{-2} = \frac{1}{100}$	centilitre
deci	d	$10^{-1} = \frac{1}{10}$	decibel
deca	da	$10^{1} = 10$	decametre
hecto	h	$10^{2} = 100$	hectopascal
kilo	k	$10^{3} = 1000$	kilojoule
mega	M	$10^{6} = 1000\,000$	megahertz
giga	G	10^{9}	gigawatt
tera	T	10^{12}	terabyte
peta	P	10^{15}	
exa	E	10^{18}	

Choose four of the units from the 'Example' column in this table and find out what they are used to measure.

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6-02 Error in measurement

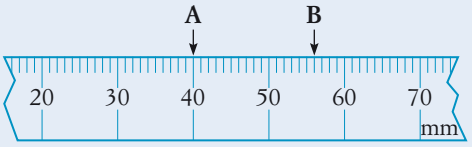
Absolute error

Any measurement should only be recorded correct to the smallest unit on the scale of the measuring device. This unit is called the **precision** of the device.

The **absolute error** of a measurement is the greatest possible error or the 'furthest by which you can be wrong'. It is equal to $\pm\frac{1}{2}$ of the smallest unit marked on the scale.

EXAMPLE 2

This diagram shows a section of a builders' tape.



a What is the smallest unit marked on the tape?

b How would you record the measurements **A** and **B**? Why?

c What is the absolute error of measurement **A**?

Solution

a 1 mm

b **A** = 40 mm and **B** = 56 mm. Measurements can only be recorded correct to the nearest millimetre (the smallest unit marked on the scale).

We should not estimate or write fractions of a millimetre when measuring with this ruler.

c Absolute error $= \pm\frac{1}{2} \times 1$ mm $= \pm 0.5$ mm

A measured length recorded as 40 mm means that the true length could be anywhere in the range (40 ± 0.5) mm, which means 39.5 mm to 40.5 mm.

(40 ± 0.5) mm means '40 millimetres, give or take half a millimetre'.

EXAMPLE 3

Copy and complete the table below.

Measurement	Smallest unit	Absolute error	Limits
40 cm	1 cm	±0.5 cm	39.5 cm to 40.5 cm
56 s			
3.4 mL			
600 kg (to the nearest kilogram)			
600 kg (to the nearest 10 kg)			

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Solution

Measurement	Smallest unit	Absolute error	Limits
40 cm	1 cm	±0.5 cm	39.5 cm to 40.5 cm
56 s	1 s	±0.5 s	55.5 s to 56.5 s
3.4 mL	0.1 mL	±0.05 mL	3.35 mL to 3.45 mL
600 kg (to the nearest kilogram)	1 kg	±0.5 kg	599.5 kg to 600.5 kg
600 kg (to the nearest 10 kg)	10 kg	±5 kg	595 kg to 605 kg

Year 12 Percentage error

The **percentage error** of a measurement is the size of the absolute error as a percentage of the measurement. For example an error of 0.5 m in 1000 m is relatively small compared to an error of 0.5 m in 10 m.

SUMMARY

$$\text{Percentage error} = \frac{\text{absolute error}}{\text{measurement}} \times 100\%$$

EXAMPLE 4

The measured length of the Sydney Harbour Bridge is 503 m, and the measured length of a basketball court is 26 m.

a Find the percentage error of each measurement given above, correct to three decimal places.

b Which of the two measurements is more accurate?

Solution

Both measurements are measured to the nearest metre, so they have the same absolute error of ±0.5 m.

a Sydney Harbour Bridge:

$$\text{Percentage error} = \frac{\pm 0.5}{503} \times 100\%$$
$$= \pm 0.099\,40\ldots\%$$
$$\approx \pm 0.099\%$$

Basketball court:

$$\text{Percentage error} = \frac{\pm 0.5}{26} \times 100\%$$
$$= \pm 1.923\,07\ldots\%$$
$$\approx \pm 1.923\%$$

b The Harbour Bridge measurement has a smaller percentage error, so it is more accurate.

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Exercise 6-02 Error in measurement

Example 2

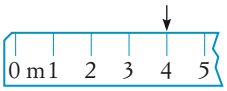
1 For each measuring device shown below, write:

i the size of the smallest unit on the scale

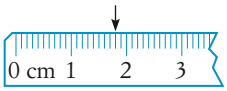
ii the measurement shown

iii the absolute error of the measurement.

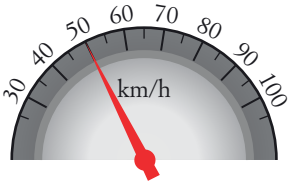
a



b



c



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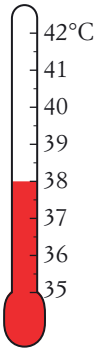
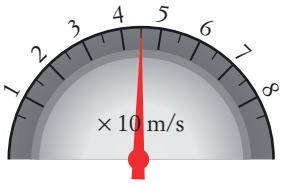
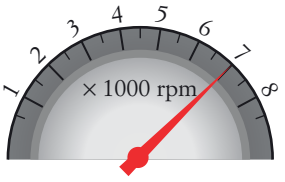


e



f



2 When using a jug marked in millilitres, which of the following is the absolute error of a measurement? Select **A**, **B**, **C** or **D**.

A ±1 mL **B** ±0.5 mm **C** ±0.5 mL **D** ±1 L

3 Which of the following lengths could be measured using a measuring tape with a precision of 1 cm? Select all correct answers.

A 37 cm **B** 24.6 cm **C** 125 mm

D 9 cm **E** 8 mm **F** 16.45 cm

4 Which of the following angle sizes are incorrectly recorded if using a protractor marked in degrees? Select all correct answers.

A 32° **B** 145° **C** 103.5°

D $64\frac{1}{2}°$ **E** 170° **F** 88.4°

5 State the absolute error for each measurement below.

a 25 mL **b** 5560 mm

c 6.5 g **d** 78.1 s

e 500 g (to the nearest gram) **f** 24 m

g 600 mL (to the nearest millilitre) **h** 500 g (to the nearest 10 grams)

i 8.6 cm **j** 5000 g (to the nearest 100 grams)

Example 3

6 To minimise error, Samantha measured the length of her bedroom four times. She recorded the results 2964 mm, 3021 mm, 2938 mm and 2899 mm and calculated the average correct to the nearest millimetre. What was her answer?

Example 4

7 The width of an A4 sheet of paper is 21 cm.

a What is the absolute error of this measurement?

b Between what values does the true measurement lie?

c What is the percentage error, correct to two decimal places, of this measurement?

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8 A truck weighs 2.3 t. What is the percentage error (correct to two decimal places) of this measurement?

9 The Earth has a calculated diameter of 12 683 km. Find the percentage error of this measurement, to four decimal places.

10 By comparing their percentage errors, determine which of the following measurements is the most accurate.

a 15.2 mg **b** 800 g (to nearest 10 g) **c** 0.15 g

11 A Swans' AFL match at the SCG had a crowd of 52 350. Why can't we find a percentage error for this figure?

12 Explain the difference between 38 L of petrol and 38.0 L of petrol.

INVESTIGATION

REPEATING AND AVERAGING MEASUREMENTS

You will need a roll of builders tape, paper, pen and calculator.

1 Choose a netball court, classroom or part of a building to measure. Work with a partner and measure the length correct to the nearest cm.

2 Repeat the measurement 4 times.

3 Find the mean (average) of your 5 measurements.

4 What was your range of measurements?

5 Which of your 5 measurements:

a was closest to the mean? **b** was furthest from the mean?

6 Why is repeating and averaging measurements a good way of determining a length?

7 Jessie says the length of the classroom is 45 rulers and Ted says it is 44 rulers. List as many reasons as you can to explain the different results.

WS Significant figures

6-03 Significant figures

Numbers can be rounded to decimal places or significant figures to show their level of accuracy .

EXAMPLE 5

Round:

a 3812.4 correct to two significant figures

b 1005 correct to three significant figures

c 0.003 572 correct to two significant figures.

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Solution

a 38**1**2.4

The first two significant figures are 38 and the next digit is **1**, so we round down to **8** and use two zeros as placeholders.

3812.4 ≈ 3800 (correct to two significant figures)

b 100**5**

The first three significant figures are 100 and the next digit is **5**, so we round up to **1** and use a zero as a placeholder.

1005 ≈ 1010 (correct to three significant figures)

c 0.003 5**7**2

For decimals, ignore any zeros at the front of the number. The first two significant figures are 35 and the next digit is **7**, so we round up to **6**.

0.003 572 ≈ 0.0036 (correct to two significant figures)

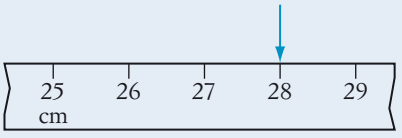
SUMMARY

- The first significant figure in a number is the first non-zero digit.
- When rounding to significant figures, start counting from the first digit that is not 0.
- If it is a large number, you may need to insert 0s at the end as placeholders.
- Zeros at the end of a whole number or at the beginning of a decimal are not significant.

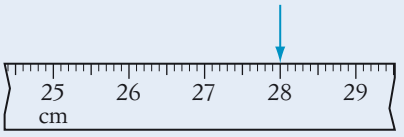
Significant figures in measurement

As we saw in the previous section, we can only record a measurement correct to the smallest unit marked on the measuring device. This means that we must only record a measurement using the appropriate number of significant figures.

EXAMPLE 6



Tape A



Tape B

Sections of two tape measures are shown. Tape A has a scale with 1 cm units, while tape B has a scale with 0.1 cm units.

a Write the measurements shown by the arrows on Tapes A and B and state how many significant figures each measurement has.

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b Which of the two measurements you wrote in part **a** is more accurate?

c Thuong recorded a measurement of 57.4 cm using Tape A. What should he have written instead? Why?

d Juliet recorded a measurement of 19 cm using Tape B. What should she have written instead? Why?

Solution

a Tape A: 28 cm, two significant figures

Tape B: 28.0 cm, three significant figures

b 28.0 cm is more accurate than 28 cm because it is correct to the nearest 0.1 cm.

c 57 cm; we can only record to the nearest centimetre, using two significant figures.

d 19.0 cm; we can only record to the nearest 0.1 cm, using three significant figures.

Exercise 6-03 Significant figures

Example 5

1 Which of the following is 305.725 rounded to two significant figures? Select **A**, **B**, **C** or **D**.

A 306 **B** 310 **C** 300 **D** 305.73

2 Write each number correct to two significant figures.

a 3812.4 **b** 2068 **c** 0.006 102

d 245 698 **e** 14 569 235 **f** 0.000 467

3 Write each number correct to three significant figures.

a 129.805 **b** 4982 **c** 0.010 609

d 1 359 048 **e** 25 430 082 **f** 0.000 680 32

4 Write each number correct to one significant figure.

a 2914.23 **b** 3.2548 **c** 11 950

d 0.005 134 **e** 20.46 **f** 0.6517

Example 6

5 Measurements made using a rain gauge marked in millimetres should be recorded correct to one significant figure for rainfall less than 10 mm, and to two significant figures for rainfall from 10 mm to 99 mm.

State whether each of the following rainfall measurements have been recorded correctly.

a 9 mm **b** 7.23 mm

c 24 mm **d** 6.5 mm

e 31 mm **f** 10 mm

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6 A micrometre is one-thousandth of a millimetre and is used for very small measurements. If each of the following measurements should be recorded correct to two significant figures, which measurements have been recorded incorrectly?

a 23 μm **b** 104 μm **c** 19.6 μm
d 832 μm **e** 28.6 μm **f** 3 μm

7 Evaluate each expression, correct to two significant figures.

a $0.2 \div 0.3$ **b** $11 \div 1990$ **c** $16 \div 12$
d $\sqrt{0.0075}$ **e** $9\,300\,000 \times 0.085$ **f** 2.7^2
g $\sqrt{560}$ **h** $\sqrt{5.6}$ **i** 3.4×9.9

8 At the time of the 2011 census, the population of New South Wales was 7 247 777. Round this number to three significant figures.

9 Express each of the following measurements correct to three significant figures.

a mass of an aeroplane, 351 540 kg
b average depth of the Pacific Ocean, 4188 m
c speed of a greyhound, 67.14 km/h
d capacity of a tablespoon, 14.79 mL
e distance from the Earth to the Sun, 149 573 881 km

6-04 Scientific notation

WS Scientific notation puzzle

Scientific notation is a shorthand way of writing very large or very small numbers, using powers of 10.

SUMMARY

Scientific notation has the form:

$a \times 10^x$ where a is a decimal between 1 and 10 and x is an integer.

EXAMPLE 7

Write each number in scientific notation.

a 23 500 000 **b** 0.004 762

Solution

a *Step 1:* 2.3 500 000 **Move the decimal point to make a decimal between 1 and 10.**

Step 2: $a = 2.35$ **Simplify the decimal.**

Step 3: $x = 7$ **To move the decimal point back to where it was, we go right seven places.**

Or count the number of places after the first significant figure (which is the figure 2).

$23\,500\,000 = 2.35 \times 10^7$

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b *Step 1:* 0004.762 **Move the decimal point to make a decimal between 1 and 10.**

Step 2: $a = 4.762$ **Simplify the decimal.**

Step 3: $x = -3$ **To move the decimal point back to where it was, we go left three places.**

Or count the number of zeros before the first significant figure (which is the figure 4)

$0.004\,762 = 4.762 \times 10^{-3}$

EXAMPLE 8

Write 354 780 in scientific notation, correct to two significant figures.

Solution

$354\,780 = 3.5478 \times 10^5$ **Written in scientific notation.**

$\approx 3.5 \times 10^5$ **Rounded to two significant figures.**

We could also round to two significant figures first, then write in scientific notation.

EXAMPLE 9

Write each number in normal decimal form.

a 6.92×10^4 **b** 3.716×10^{-5}

Solution

a $6.92 \times 10^4 = 69\,200$ **Moving the decimal point four places to the right (or making four places after the 6).**

b $3.716 \times 10^{-5} = 0.000\,037\,16$ **Moving the decimal point five places to the left (or inserting four zeros before the 3).**

Scientific notation on a calculator

To enter a number in scientific notation on a calculator, use the $\times 10^x$ or EXP key.

EXAMPLE 10

Evaluate:

a $(6.8 \times 10^5) \times (7.4 \times 10^{-2})$

b $(3.2 \times 10^{-4}) \div (9.1 \times 10^6)$, correct to two significant figures.

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Solution

a $(6.8 \times 10^5) \times (7.4 \times 10^{-2}) = 50\ 320$

Calculator steps: 6.8 [×10^x] 5 [×] 7.4 [×10^x] [(–)] 2 [=]

b $(3.2 \times 10^{-4}) \div (9.1 \times 10^6) = 3.516\ 483\ldots \times 10^{-11}$

$\approx 3.5 \times 10^{-11}$ **Rounded to two significant figures.**

Calculator steps: 3.2 [×10^x] [(–)] 4 [÷] 9.1 [×10^x] 6 [=]

Exercise 6-04 Scientific notation

Example 7

1 Write each number in scientific notation.

a 42 130 000 **b** 0.0181 **c** 3400
d 20 000 **e** 0.0035 **f** 0.0002
g 0.33 **h** 0.004 **i** 230
j 0.000 072 3 **k** 610 000 000 **l** 0.000 000 08

Example 8

2 Write each number in scientific notation, correct to two significant figures.

a 53 467 892 **b** 146 089 **c** 2453
d 0.000 457 3 **e** 0.002 652 **f** 0.102 05
g 0.000 033 38 **h** 0.444 44 **i** 6.5207

3 What is 356 587 239 in scientific notation correct to three significant figures? Select **A**, **B**, **C** or **D**.

A 357 000 000 **B** 3.57×10^8 **C** 3.6×10^8 **D** 3.56×10^8

Example 9

4 Write each number in normal decimal form.

a 7.4×10^5 **b** 3.12×10^{-1} **c** 1.85×10^3
d 6.6×10^{-4} **e** 2.54×10^{-3} **f** 4.751×10^8
g 9.8×10^{-2} **h** 3×10^2 **i** 5.497×10^{-2}
j 1.216×10^4 **k** 8.02×10^{-1} **l** 6.309×10^3

5 Write each measurement in scientific notation.

a The age of the universe is about 13 700 000 000 years.

b One micrometre (1 μm) is $\frac{1}{1\ 000\ 000}$ m.

6 Write each measurement in normal decimal notation.

a The diameter of an atom is 3×10^{-8} mm.

b One light year, the distance light travels in one year, is equal to 9.461×10^9 m.

c The diameter of the flu virus is 2×10^{-6} mm.

d The distance of the Earth from the Sun is 1.526×10^8 km.

7 The world population at a certain second in August 2012 was 7 034 307 183. Write this number in scientific notation, correct to three significant figures.

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Example 10

8 Evaluate each expression using scientific notation.

a $(6.7 \times 10^4) \times (3.2 \times 10^2)$

b $(1.92 \times 10^{10}) \div (6 \times 10^4)$

c $(3.75 \times 10^4) - (2.5 \times 10^3)$

d $(4.8 \times 10^{-3})^2$

e $(8.1 \times 10^4)^3$

f $\dfrac{9.4 \times 10^{-2}}{2.5 \times 10^{-3}}$

g $(4.2 \times 10^5) \times (3 \times 10^{-2})$

h $\sqrt{5.29 \times 10^{10}}$

9 Evaluate each expression in scientific notation, correct to two significant figures.

a $(5.1 \times 10^3) + (2.5 \times 10^4)$

b $(7.8 \times 10^{-5}) - (3.7 \times 10^{-3})$

c $(9.6 \times 10^8) \times (4.3 \times 10^3)$

d $(3.2 \times 10^{-3})^3$

e $\dfrac{3.2 \times 10^2}{5.4 \times 10^{-3}}$

f $\sqrt[3]{3.8 \times 10^{-5}}$

g $\sqrt{8.5 \times 10^5}$

h $\dfrac{6 \times 10^3}{5.1 \times 10^7}$

10 Evaluate each expression, expressing the answer in normal decimal form.

a $96 \div 20\,000$

b $480\,000 \times 91\,000$

c $\dfrac{0.235}{25}$

d $(13\,700)^2$

e $(4.4 \times 10^4) \times (3.1 \times 10^2)$

f $(1.9 \times 10^7) + (6.3 \times 10^4)$

g $\sqrt{2.704 \times 10^{-9}}$

h $\dfrac{7.7 \times 10^9}{2.2 \times 10^4}$

11 a What is the largest number you can enter into your calculator in scientific notation?

b What is the smallest number you can enter into your calculator using scientific notation?

Simplifying fractions

Ratios

6-05 Ratio problems

A **ratio** compares parts or shares of quantities of the same type. The numbers in a ratio are called its **terms** and a **simplified ratio** has terms that have no common factors.

EXAMPLE 11

Simplify each ratio.

a $8 : 12 : 28$

b $\dfrac{3}{4} : \dfrac{2}{5}$

c $1\frac{1}{2}$ h compared to 35 min

Solution

a $8 : 12 : 28 = \dfrac{8}{4} : \dfrac{12}{4} : \dfrac{28}{4}$

$= 2 : 3 : 7$

Dividing each term by the common factor, 4.

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b $\frac{3}{4} : \frac{2}{5} = \frac{3}{4} \times 20 : \frac{2}{5} \times 20$ **Multiplying each term by 20 to make them whole.**

$= 15 : 8$

c $1\frac{1}{2}$ h to 35 min = 90 min to 35 min **Converting to the same units (minutes).**

$= \frac{90}{5} : \frac{35}{5}$ **Dividing each term by 5.**

$= 18 : 7$

EXAMPLE 12

Nabilah and Hannah invested money in an apartment in the ratio 3 : 5. If Hannah invested \$235 000, find:

a the total cost of the apartment

b the amount Nabilah invested.

Solution

a Use the unitary method.

Nabilah's share : Hannah's share = 3 : 5

5 parts (Hannah) = \$235 000

1 part = \$235 000 ÷ 5 **Dividing by 5 to find one part.**

= \$47 000

Total cost = 8 × \$47 000 **The total number of shares is 3 + 5 = 8.**

= \$376 000

b Nabilah's share (3 parts) = 3 × \$47 000

= \$141 000

Check: \$235 000 + \$141 000 = \$376 000 **The two shares add to equal the whole.**

Exercise 6-05 Ratio problems

Example 11

1 Simplify:

a 5 : 15

b 4 : 2 : 10

c 14 : 14

d 105 : 75

e 17 : 68

f 12 : 36 : 9

g 0.04 : 0.4

h $\frac{4}{5} : \frac{2}{3}$

i $\frac{3}{8} : 1\frac{1}{4}$

j 250 mL compared to 2 L

k 3 days compared to 2 weeks

l \$4.20 compared to 95c

m 45 cm compared to 3 m

n \$2000 compared to \$1 000 000

o $\frac{1}{2}$ cup compared to $2\frac{1}{2}$ cups

p 1 fortnight compared to 3 weeks compared to 1 year

q 2 ha compared to 100 m^2

r 350 mL compared to 2.4 L

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2 Concrete is made up of sand, cement and lime in the ratio 2.5 : 0.5 : 0.25. Express this ratio in simplest form.

3 Which of the following is the simplified form of 27 : 99? Select **A**, **B**, **C** or **D**.

A 9 : 33 **B** 11 : 3 **C** 3 : 11 **D** 1 : 3.6

4 Which of the following is the simplified form of 0.5 kL : 1.4 L? Select **A**, **B**, **C** or **D**.

A 500 : 1.4 **B** 1 : 2.8 **C** 7 : 2500 **D** 2500 : 7

5 Fifteen out of 130 female smokers are likely to develop lung cancer. Express this statistic as a simplified ratio.

6 Half of Robert's class walk to school, one-third take the bus and the rest ride a bike. Find the ratio of:

a bike riders to walkers

b bus travellers to the whole class

c walkers to bus travellers.

7 What is the ratio of a square's side length to its perimeter?

Example 12

8 Brodie cuts a length of ribbon in three pieces in the ratio 1 : 3 : 5. If the largest piece is 75 cm long:

a how long are the other two pieces?

b how long was the entire length of ribbon?

9 At Point Perpendicular High School, the ratio of boys to girls is 23 : 27. If there are 529 boys, what is the total school population?

10 A large jar of jellybeans contains only yellow, white and green jellybeans, in the ratio 12 : 11 : 22. If there are 600 g of yellow jellybeans, what is the mass of:

a the white jellybeans?

b all of the jellybeans?

11 The ratio of teachers to students in Westvale TAFE college is 1 : 15. If Westvale TAFE has 1200 students, how many teachers does it have?

12 Lawnmower fuel is made up of three parts petrol to two parts oil. Which of the following is the amount of petrol that needs to be added to 3 L of oil to make the fuel? Select **A**, **B**, **C** or **D**.

A 3.5 L **B** 4 L **C** 4.5 L **D** 7.5 L

13 Jesse's soccer team won eight games for every five games they lost. If the team lost 15 games, how many games did they win?

14 The ratio of fiction to non-fiction books in the library is 3 : 7. If there are 13 980 fiction books, how many books are in the library?

15 A concrete mix contains cement, sand and gravel in the ratio 1 : 4 : 6.

a How many shovelfuls of sand are needed in a mix containing three shovelfuls of cement?

b If 2.2 t of gravel are used to make some concrete mix, how many *kilograms* of sand are used? Answer to the nearest kilogram.

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16 Rachel made chocolate crackles containing copha, coconut, rice bubbles and cocoa in the ratio 2 : 2 : 5 : 1. If she used 400 g of copha, then find the mass of the:

a cocoa

b rice bubbles

c chocolate crackles

17 a Joshua is making a fruit cordial drink for a party, mixing two parts of concentrate to 13 parts of water. How much concentrate does Joshua need to add to 2.6 L of water?

b If Hani makes a similar drink to Joshua, with the same ratio of concentrate to water, how much water does she need to add to 600 mL of concentrate?

18 Thelma and Louise share a company in the ratio 7 : 8. If Louise invests $384 000, how much does Thelma invest?

6-06 Dividing a quantity in a given ratio

WS Ratio problems

SUMMARY

To divide a quantity in a given ratio, use the unitary method.

Step 1: Find the total number of parts (by adding).

Step 2: Find the size of one part (by dividing).

Step 3: Find the size of each term of the ratio (by multiplying).

Check that your answers add to the original quanity.

EXAMPLE 13

Dana's estate of $1 200 000 was divided between her children, Kieran and Nicola, in the ratio 3 : 2. How much did each receive?

Solution

Total number of parts = 3 + 2 = 5

1 part = $1 200 000 ÷ 5

= $240 000

Kieran's share (3 parts) = 3 × $240 000

= $720 000

Nicola's share (2 parts) = 2 × $240 000

= $480 000

Check: $720 000 + $480 000 = $1 200 000

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Example 13

Exercise 6-06 Dividing a quantity in a given ratio

1 A cash prize of \$1200 is divided between Vila and Kim in the ratio 2 : 3. Which of the following is Vila's share? Select **A**, **B**, **C** or **D**.

A \$240 **B** \$480 **C** \$600 **D** \$400

2 Divide each of the following quantities in the given ratio.

a \$31 500 in the ratio 4 : 5

b 3750 kg in the ratio 13 : 2

c 34 000 m^2 in the ratio 6 : 7 (to the nearest m^2)

d $2\frac{1}{2}$ cups in the ratio 1 : 4

e 7500 L in the ratio 2 : 3

f 10 ha in the ratio 5 : 3

3 The ratio of boys to girls at Westvale Public School is 4 : 5. How many girls attend the school if the total enrolment is 846 students?

4 An alloy contains copper and nickel in the ratio 7 : 4. How much of each metal (correct to the nearest kilogram) is in 500 kg of the alloy?

5 Tan and Brad own a business with shares in the ratio 5 : 3. How should they share a profit of \$53 600?

6 Max won \$200 000 in Lotto and divided it between himself and his three grandchildren in the ratio 3 : 2 : 2 : 1. How much did each person receive?

7 Two-stroke fuel for an outboard motor is made up of oil and petrol in the ratio 1 : 49. How much oil and petrol are needed to make 20 L of fuel?

8 550 kg of concrete is made by mixing cement, sand and aggregate in the ratio 1 : 4 : 6. How many kilograms of each component are needed?

9 Punch is made up of fruit juice, lemonade and ginger ale in the ratio 2 : 2 : 1. How many millilitres of each ingredient are needed to make 6 L of punch?

10 Rose, Lisa and Ally share the weekly rent of a house in the ratio 3 : 2 : 1. If the rent is \$450, how much does each person pay?

11 Baby formula is made up of 1 tablespoon of powdered formula to 14 tablespoons of water. How many millilitres of water are needed to make up 300 mL of formula?

12 A weed spray is made up of wetting agent and a herbicide in the ratio 1 : 100. How much wetting agent (to the nearest millilitre) do you need to make 85 L of weed spray?

13 Tim pays 30% of his weekly income of \$1080 in tax. The remaining income is divided into savings and living expenses in the ratio 2 : 5.

a How much does Tim save?

b How much does Tim use for living expenses?

14 Tasha's daily diet consists of protein, fat and carbohydrate in the ratio 3 : 0.5 : 9.5.

a Simplify this ratio.

b How much protein does Tasha have in one day if she consumes 3.5 kg of food? Answer to the nearest gram.

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c How much would Tasha eat in a day if her diet contained 20 g of fat?

d How much carbohydrate is in Tasha's 507 g bowl of pasta if it contains the same ratio of components as her whole diet?

6-07 Rate problems

Rate problems

Speed problems

A **rate** is a measurement that compares two different quantities. For example, a heart rate compares the number of heartbeats to a period of time, and is measured in beats per minute, written as beats/min.

EXAMPLE 14

a A hose delivers 3840 litres of water in one hour. What is the flow rate in litres per minute?

b The cost of 35 L of petrol is \$44.45. Express this cost as a rate in cents per litre.

Solution

a $\text{Flow rate} = \dfrac{3840\text{ L}}{1\text{ h}}$

$= \dfrac{3840\text{ L}}{60\text{ min}}$ **Converting 1 h to 60 min.**

$= 64\text{ L/min}$

b $\text{Petrol cost} = \dfrac{\$44.45}{35\text{ L}}$

$= \dfrac{4445\text{ c}}{35\text{ L}}$ **Converting \$44.45 to 4445c.**

$= 127\text{ c/L}$

EXAMPLE 15

Craig's heart beats 72 times per minute.

a How long will Craig's heart take to beat 2052 times?

b How many times will Craig's heart beat in one hour?

Solution

a Time taken for 2052 beats $= 2052 \div 72$

$= 28.5\text{ min}$

b Beats in one hour $= 60 \times 72$ **60 min = 1 hour**

$= 4320$

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Rate problems are solved by either multiplying or dividing. One simple rule is to write the units of the rate as a fraction.

In the heartbeat problem from Example **15**:

$$\text{beats/minute} = \frac{\text{beats}}{\text{minutes}}$$

a To find the number of *minutes*, we **divide** by the rate: $2052 \div 72$.
b To find the number of *beats*, we **multiply** by the rate: 60×72.

Concentrations

When mixing a solution of medicine, pharmacists and nurses need to make sure that the solution contains the right amount of the drug. When mixing a batch of pesticide, a farmer needs to make sure it is of the correct strength. The **concentration** of a substance is the amount of the substance compared to the whole mixture or solution. Concentration can be expressed as a rate or percentage, in the following forms:

- weight per weight (w/w), for solids mixed with solids (for example g/kg for fertiliser)
- weight/volume (w/v), for solids dissolved in liquids (for example mg/mL for drugs)
- volume/volume (v/v), for liquids mixed with liquids (for example mL/L for fuels)

EXAMPLE 16

A pesticide contains a chemical in the concentration of 8 g/L.

a How much of the chemical is required to make 4.5 L of the pesticide?

b Express the concentration of the chemical as a percentage w/v.

Solution

a Amount of chemical $= 4.5 \times 8$ g
$= 36$ g

b First express both quantities in the same units.

Concentration $= 8$ g/L
≈ 8 mL/L

When comparing weight to volume:
1 g ≈ 1 mL
1 kg ≈ 1 L

$= \dfrac{8 \text{ mL/L}}{1000 \text{ mL}} \times 100$
$= 0.8\%$ w/v.

Remember : w/v means 'weight per volume'.

The concentration is 0.8% w/v.

Exercise 6-07 Rate problems

1 What units are used to express each of the following rates?

a the cost of a mobile phone call
b a wage rate
c typing speed
d the postage rate for a parcel
e the cost of potatoes
f the speed of an athlete

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2 Name a rate that uses each of the following units.

a persons/km^2 **b** cents/word

c litres/100 km **d** megabytes/second

e births/1000 persons **f** \$/m^2

Example 14

3 A tree grew 31.2 cm in height in a quarter of a year. Calculate its growth rate in centimetres/week.

4 Troy scored 42 runs in 8 overs of a cricket match. Calculate his run rate in runs/over.

5 A town's population grew by 16 500 over 6 years. What was its growth rate in persons/year?

6 Steak costs \$4.55 for 160 g. Calculate the cost as a rate in \$/kg.

7 A computer downloaded a 6.5 megabyte file in 20 seconds. Calculate the download rate in MB/s.

8 Australia's population is 23 194 000 and its area is 7 682 300 km^2. Calculate its population density in persons/km^2, correct to two decimal places.

Example 15

9 A bus travels at a speed of 96 km/h.

a How far will the bus travel in $3\frac{1}{2}$ hours?

b How long will the bus take to travel 480 km?

10 Jake types 64 words per minute. How long will it take him to type a 1500-word essay? Give your answer to the nearest minute.

11 A breakfast cereal contains 3 mg of fat in every 60 g serve.

a Express the fat content as a rate in mg/g.

b How much fat would be in a 100 g serving?

12 Hayden's swimming pool is being filled at a rate of 50 L/min. How long would it take to fill if it has a capacity of 38 kL? Give your answer in hours and minutes.

13 A cricket team had a run rate of 4.9 runs/over. How many runs did the team score in 30 overs?

14 While travelling in the USA, Mikayla exchanged A\$750 for US\$474.50.

a How much was A\$1 worth in US dollars?

b Convert US\$200 to Australian dollars, correct to the nearest cent.

15 The recommended daily dose for a child's medicine is 80 mg for every kilogram of body weight. Which of the following is Elizabeth's daily dose in grams, if she weighs 30 kg? Select **A**, **B**, **C** or **D**.

A 2.4 **B** 3.75 **C** 2400 **D** 375

16 A tap is dripping water at a rate of 40 drops per minute. Each drop contains 1.3 mL of water. How many litres will drip from the tap in a day?

17 Australia's death rate last year was 5.2 per 1000. If Australia's population was 21 435 680 at the start of the year, how many people died during the year? Give your answer to the nearest whole number.

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18 A lawn fertiliser is spread at the rate of 5 kg per 100 m^2 of lawn.

a How much fertiliser is needed for a rectangular lawn measuring 11 m by 7.5 m? Give your answer to the nearest 0.1 kg.

b How many times can the whole lawn described in part **a** be treated if a 25 kg bag of fertiliser is purchased?

Example 16

19 A 400 mL quantity of disinfectant contains 240 mL of alcohol. Express the concentration in:

a mL/L **b** % v/v.

20 A stock solution contains 125 mg of the drug cortisone per every 5 mL. If a sheep needs to be injected with 45 mg of cortisone, calculate the volume of solution required for the injection.

21 An antibiotic in a syrup has a strength of 25 mg/mL. Calculate the volume of syrup that gives a dosage of 120 mg.

22 A solution contains 75 mg of a sedative per every 10 mL. If a patient needs 975 mg of the sedative per day, how much of the solution should the nurse prepare?

23 A 5 L bucket of cleaning solution contains 125 mL of detergent.

a Express the concentration as a percentage v/v.

b How much detergent is there in 2.4 L of the cleaning solution?

24 A patient is prescribed 3 mg of pethidine. Capsules of the drug contain a concentration of 20 mg/mL of pethidine. How many millilitres of the capsules should be given to the patient?

25 For a cat's daily vitamin supplement, a vet recommends a concentration of 0.6 g supplement in a solution of 300 mL. What is this concentration:

a in mg/mL? **b** as a percentage w/v?

26 A vet has recommended a vitamin supplement for Carmen's sick guinea pigs. It is to be mixed at the rate of 10 g/L in their drinking water.

a How much of the vitamin supplement is required for the guinea pigs' 500 mL water bottle?

b Carmen obtained the vitamin supplement in solution in a concentration of 240 g/300 mL from the chemist. How many millilitres of this solution should Carmen put in the 500 mL water bottle?

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6-08 Converting rates

EXAMPLE 17

Convert:

a 125 L/h to L/day **b** 24c/min to \$/h **c** 80 km/h to m/s

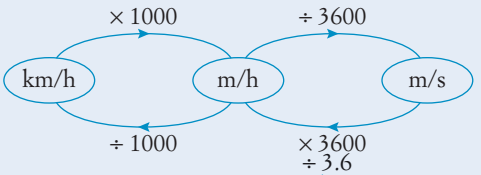
Solution

a 125 L/h = (125 × 24) L/day **Converting from 'per hour' to 'per day'.**
= 3000 L/day

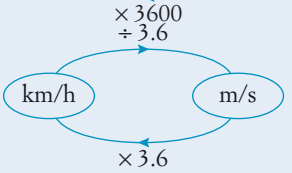
b 24c/min = \$0.24/min **Converting cents to dollars.**
= (\$0.24 × 60)/h **Converting minutes to hours.**
= \$14.40/h

c 80 km/h = (80 × 1000) m/h **Converting kilometres to metres.**
= 80 000 m/h
= (80 000 ÷ 3600) m/s **Converting from 'per hour' to 'per second'.**
$= 22\frac{2}{9}$ m/s

For km/h ↔ m/s conversions, you may find this diagram useful.



Some students prefer to combine the two operations by simply dividing or multiplying by 3.6.



Exercise 6-08 Converting rates

Example 17

1 Convert:

a \$46 212/year to \$/month **b** 75 km/h to km/min
c 5 L/h to L/day **d** 3 m/s to m/min
e 0.8 cm/min to cm/h **f** 78 words/min to words/s
g 8.2 L/100 km to L/km **h** 1750 mL/s to mL/min.

2 Convert:

a 24c/min to \$/hour **b** 8 m/s to km/h
c 15 mL/s to L/h **d** \$10.80/h to c/min
e 60 km/h to m/s **f** 3 t/day to kg/min.

3 The world's fastest cricket bowler is Pakistan's Shoaib Akhtar. In 2003, he bowled a ball 20.12 m in 0.45 seconds. Calculate the speed of the ball he bowled, to the nearest km/h.

4 The speed of sound, called Mach 1, is 1224 km/h. Which of the following is this speed in m/s? Select **A**, **B**, **C** or **D**.

A 34 **B** 44 **C** 340 **D** 441

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5 It takes 20 seconds to fill a 10 L bucket with water. What is the rate for filling the bucket, in litres per hour?

6 Jamaican athlete Usain Bolt is the world's fastest man, running 100 metres in 9.58 seconds in 2009. What was his speed in km/h, correct to one decimal place?

7 The lift in Sydney Tower travels 298 m in 42 seconds. What is its average speed in km/h, correct to two decimal places?

8 A world record for water speed, 250.7 knots (nautical miles per hour), was achieved in 1977 by Ken Warby in his hydroplane *Spirit of Australia* at Blowering Lake Dam in Tumut, NSW.

a Convert 250.7 knots to km/h (correct to one decimal place) given that 1 nautical mile = 1.852 km.

b At that speed, how far (to the nearest metre) did the hydroplane travel in:

i one minute? **ii** one second?

9 A car is travelling at 75 km/h. How far, to the nearest 0.1 metre, will it travel in the 5 seconds it takes you to cross the road?

10 If the reaction time of a car driver is 0.9 seconds, how far will the car travel in this time if its speed is:

a 60 km/h? **b** 84 km/h? **c** 100 km/h?

6-09 Percentage change

Percentage increase and decrease

EXAMPLE 18

a Jamie's weekly wage of \$720 increases by 6%. What is his new wage?

b A trampoline with a marked price of \$280 is discounted by 15%. What is the sale price?

Solution

a Increasing an amount by 6% is the same as finding 106% of that amount, because

$100\% + 6\% = 106\%$.

$106\% \times \$720 = 1.06 \times \$720$

$= \$763.20$

Jamie's new wage is \$763.20.

b Decreasing an amount by 15% is the same as finding 85% of it, because

$100\% - 15\% = 85\%$

$85\% \times \$280 = 0.85 \times \$280$

$= \$238$

The sale price of the trampoline is \$238.

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Repeated percentage change

A retail store increases its prices by 10% at the start of the year, but has a '10% off' clearance sale at the end of the year. Do the increase and decrease cancel each other out, or will there be a combined percentage change?

Suppose the original price was \$100.

Increase \$100 by 10%: $110\% \times \$100 = 1.1 \times \$100 = \$110$

Decrease \$110 by 10%: $90\% \times \$110 = 0.9 \times \$110 = \$99$

The overall change is a decrease of \$1, or 1%.

EXAMPLE 19

J-mart is having a '15% off' sale this week. Because Sarah works there, she receives a further 5% staff discount on the sale price. What percentage discount is equivalent to the combined discounts Sarah receives?

Solution

Method 1: Using numbers

Let the original price be \$100.

Decrease \$100 by 15%:

$$\begin{aligned} 85\% \times \$100 &= 0.85 \times \$100 \\ &= \$85 \end{aligned}$$

Decrease \$85 by 5%:

$$\begin{aligned} 95\% \times \$85 &= 0.95 \times \$85 \\ &= \$80.75 \end{aligned}$$

Combined discount = \$100 – \$80.75 = \$19.25

Overall percentage discount = 19.25%

Method 2: Using percentages only

'Decrease by 15%' means 85%.

'Decrease by 5%' means 95%.

$$\begin{aligned} \text{Overall change} &= 0.85 \times 0.95 \\ &= 0.8075 \\ &= 80.75\% \end{aligned}$$

Overall percentage discount = 100% – 80.75% = 19.25%

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EXAMPLE 20

The value of Grant's shares decreased by 8% one year and then increased by 20% the following year. What percentage change is equivalent to a decrease of 8% followed by an increase of 20%?

Solution

Method 1: Using numbers

Decrease \$100 by 8%:

$92\% \times \$100 = 0.92 \times \$100 = \$92$

Increase \$92 by 20%:

$120\% \times \$92 = 1.2 \times \$92 = \$110.40$

Combined change = \$110.40 – \$100 = \$10.40 increase

Overall percentage change = 10.4% increase

Method 2: Using percentages only

'Decrease by 8%' means 92%.

'Increase by 20%' means 120%.

Overall change $= 0.92 \times 1.2$

$= 1.104$

$= 110.4\%$

Overall percentage change = 110.4% – 100% = 10.4%

Exercise 6-09 Percentage change

Example 18

1 A school purchased 150 calculators at \$19 each and was given a discount of 12%. Find the total paid for the calculators.

2 If Australia's population at the start of last year was 21 435 680, what is the population this year if it has increased by 1.2%? Answer to the nearest whole number.

3 Phuong usually earns \$680.50 per week but, this week, she received an 8% bonus. How much did she receive altogether this week?

4 Hardly Normal Discounts is having a '15% off' sale on all items. How much will you pay for a television set priced at \$460?

5 The value of Lisa's computer depreciated (decreased) by 9% this year. If its value last year was \$2410, calculate its current value.

6 A restaurant adds a 12% surcharge to all bills on public holidays. Calculate the total to be paid if the bill for a dinner for two on Australia Day comes to \$74.50.

Example 19

7 Clara's toy shop increased the price of all toys by 8% due to a manufacturer's price rise. Clara had a sale later that year in which she offered a discount of 5% on all items. For a doll's house originally marked at \$285, calculate:

a the price after the 8% rise **b** the sale price after discount

c the overall change in price and the percentage change on the original price.

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8 A town's population of 2000 increased by 10% one year and then decreased by 4% the following year. Which of the following was the town's new population figure? Select **A**, **B**, **C** or **D**.

A 2120 **B** 2112 **C** 1720 **D** 2060

Example **20**

9 Which of the following overall percentage increases is equivalent to a percentage increase of 5% followed by another increase of 3%? Select **A**, **B**, **C** or **D**.

A 8% **B** 1.5% **C** 1.85% **D** 8.15%

10 Dean's investment of $1000 increased by 8%, 8% and 7% over three years.

a How much, correct to the nearest cent, was Dean's investment worth at the end of three years?

b What was the overall percentage change in Dean's investment, to the nearest percentage?

11 What percentage change is equivalent to:

a an increase of 20%, followed by a decrease of 20%?

b a decrease of 20%, followed by an increase of 20%?

c an increase of 8%, followed by another increase of 8%?

d an increase of 10%, followed by a decrease of 15%?

e a decrease of 4%, followed by a decrease of 6%?

f a decrease of 5%, followed by an increase of 2%?

12 An iceberg of mass 3.5 t lost 5% of its mass in the first year, 4% of its mass in the second year and 2% of its mass in the third year.

a What was the mass of the iceberg after 3 years, correct to two decimal places?

b What is the percentage decrease (correct to one decimal place) of the iceberg from its original mass?

6-10 Perimeters of composite shapes

SUMMARY

The **perimeter** of a shape is the distance around the shape.

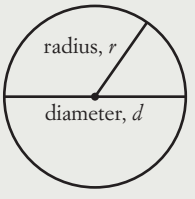
It is the sum of the lengths of the sides of the shape.

The perimeter of a **circle** is called its **circumference**.

The **circumference of a circle** with diameter, d, and radius, r, is given by the formula:

$$C = \pi d$$

or $C = 2\pi r$



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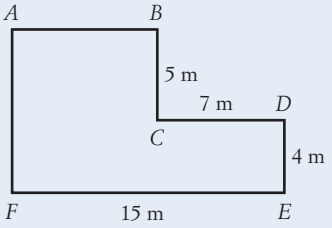
EXAMPLE 21

Find the perimeter of each composite shape.

a



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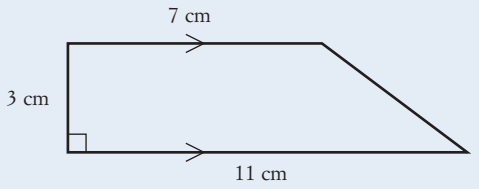


Solution

a Find the lengths of the unknown sides first.

$AB = 15 - 7$ $\qquad$ $AF = 5 + 4$

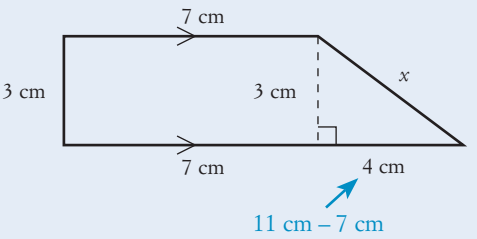
$= 8$ m $\qquad$ $= 9$ m

Perimeter $= 8 + 5 + 7 + 4 + 15 + 9$

$= 48$ m

Note: It is also possible to make this shape into one big rectangle by moving the 5 m side to the right and the 7 m side upwards. This rectangle has a length of 15 m and a width of 9 m, so its perimeter ($15 + 15 + 9 + 9 = 48$ m) should be the same as the perimeter of the composite shape.

b Let x be the length of the unknown slanted side, which can be found using Pythagoras' theorem.



$x^2 = 3^2 + 4^2$

$= 25$

$x = \sqrt{25}$

$= 5$ cm

Perimeter $= 7 + 5 + 11 + 3$

$= 26$ cm

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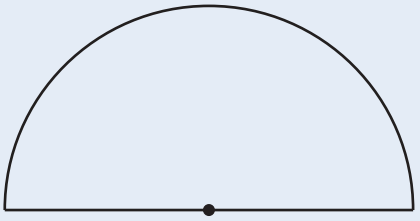
EXAMPLE 22

Find, correct to one decimal place, the perimeter of each shape.

a



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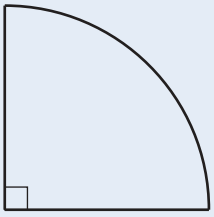


Solution

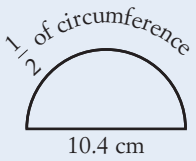
a Perimeter of semicircle $= \frac{1}{2} \times$ circumference $+ 10.4$

$= \frac{1}{2} \times \pi \times 10.4 + 10.4$ $\quad$ $C = \pi d$

$= 26.73628 \ldots$

$= 26.7$ cm

From the diagram, a perimeter of 26.7 cm looks reasonable

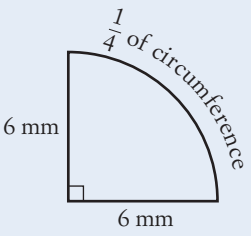


b Perimeter of quadrant $= \frac{1}{4} \times$ circumference $+ 6 + 6$

$= \frac{1}{4} \times 2 \times \pi \times 6 + 12$ $\quad$ $C = 2\pi r$

$= 21.424777 \ldots$

$= 21.4$ mm



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Example 21

Exercise 6-10 Perimeters of composite shapes

1 Find the perimeter of each composite shape.

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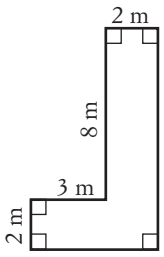

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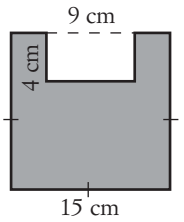

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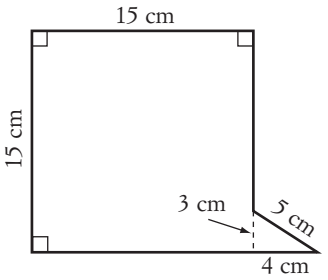

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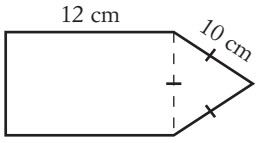

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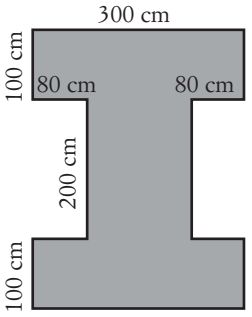

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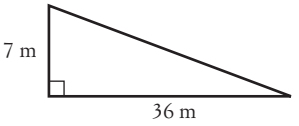

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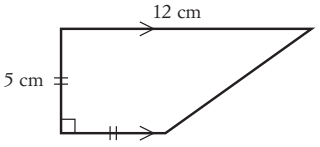

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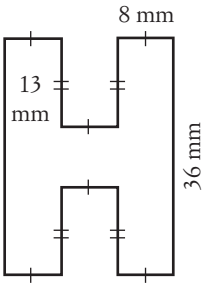

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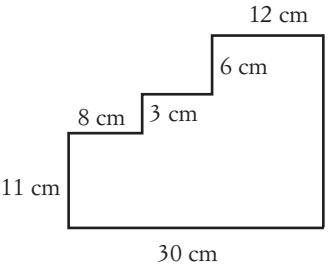

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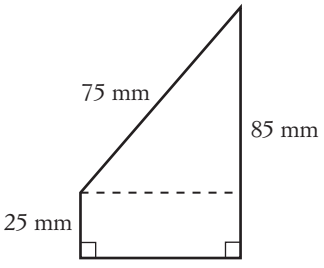

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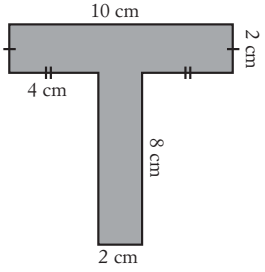

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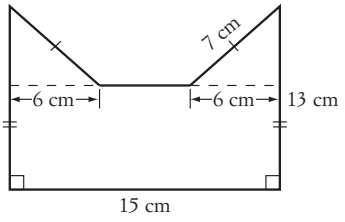

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2 Find, correct to one decimal place, the perimeter of each shape.

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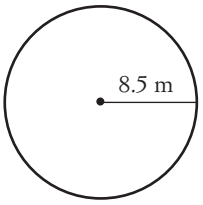

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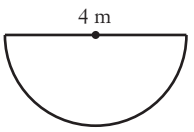

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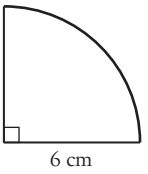

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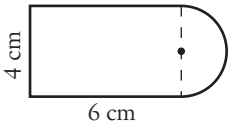

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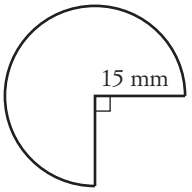

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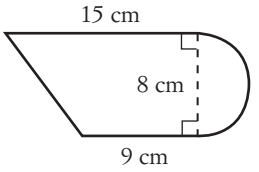

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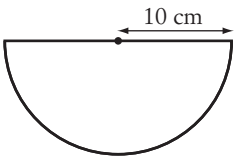

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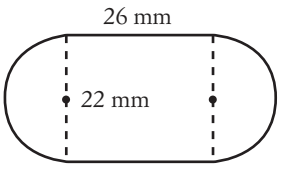

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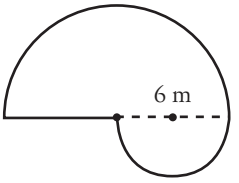

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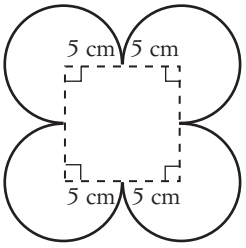

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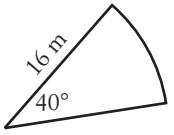

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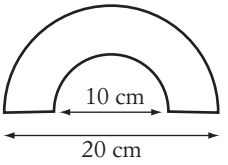
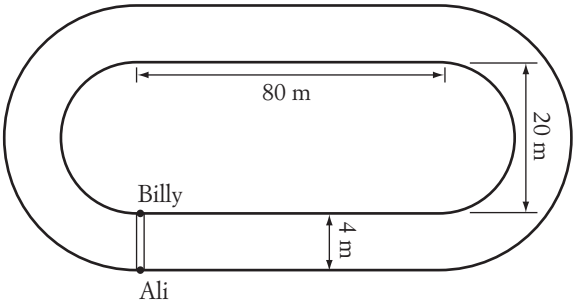


3 Ali and Billy ran around the racetrack shown below on different lanes. Calculate how much further Ali ran than Billy in one lap of the track. Answer correct to the nearest 0.1 metre.



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6-11 Area

The **area** of a plane (flat) shape is the amount of surface it occupies. This is measured in **square units**.

Metric units for area

Unit	Relationship	Size
square millimetre (mm^2)	$1\ mm^2 = 1\ mm \times 1\ mm$	The area of this 1 mm square.
square centimetre (cm^2)	$1\ cm^2 = 1\ cm \times 1\ cm$ $= 10\ mm \times 10\ mm$ $= 100\ mm^2$	The area of this 1 cm square.
square metre (m^2)	$1\ m^2 = 1\ m \times 1\ m$ $= 100\ cm \times 100\ cm$ $= 10\ 000\ cm^2$	The area of a 1 m square (about the area of the floor of a large shower recess)
hectare (ha)	$1\ ha = 100\ m \times 100\ m$ $= 10\ 000\ m^2$	The area of a 100 m square (about the area of two football fields)
square kilometre (km^2)	$1\ km^2 = 1\ km \times 1\ km$ $= 1000\ m \times 1000\ m$ $= 1\ 000\ 000\ m^2$	The area of a 1 km square (about the area of a large theme park)

When converting units of area, we need to multiply or divide by two lots of units.
For example, if 1 m = 100 cm, then $1\ m^2 = 100 \times 100\ cm^2 = 10\ 000\ cm^2$.

EXAMPLE 23

Convert:

a $55\ m^2$ to cm^2 **b** $2350\ m^2$ to hectares **c** $0.8\ km^2$ to hectares

Solution

a $55\ m^2 = (55 \times 100 \times 100)\ cm^2$
$= 550\ 000\ cm^2$

Because $1\ m^2 = 100\ cm \times 100\ cm$.

b $2350\ m^2 = (2350 \div 10\ 000)\ ha$
$= 0.235\ ha$

Because $1\ ha = 10\ 000\ m^2$.

c $0.8\ km^2 = (0.8 \times 1000 \times 1000)\ m^2$
$= 800\ 000\ m^2$
$= (800\ 000 \div 10\ 000)\ ha$
$= 80\ ha$

Because $1\ km^2 = 1000\ m \times 1000\ m$.
Converting to m^2 first.
Because $1\ ha = 10\ 000\ m^2$.

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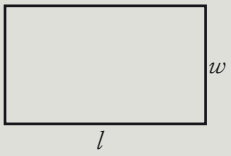
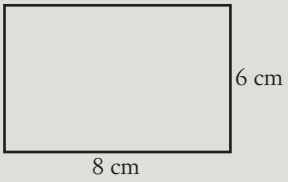
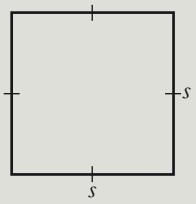
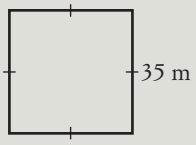
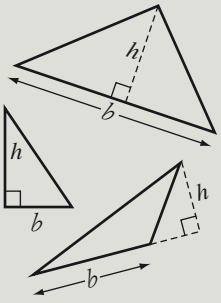
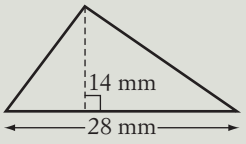
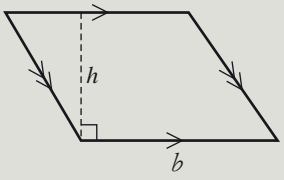
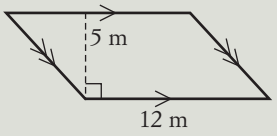
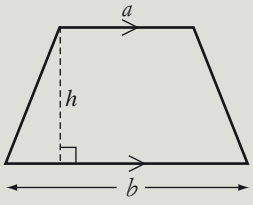
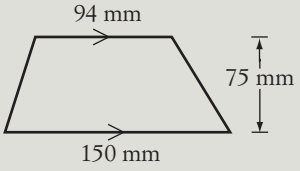
Area formulas

The table below lists the formulas for the areas of plane shapes that you should know from Years 7–10.

Areas of composite shapes

Shape	Area formula	Example
Rectangle 	A = length × width $A = lw$	 $A = 8 \times 6 = 48\ \text{cm}^2$
Square 	A = side × side $= (\text{side})^2$ $A = s^2$	 $A = 35^2 = 1225\ \text{m}^2$
Triangle 	$A = \frac{1}{2} \times$ base × perpendicular height $A = \frac{1}{2}bh$	 $A = \frac{1}{2} \times 28 \times 14 = 196\ \text{mm}^2$
Parallelogram 	A = base × perpendicular height $A = bh$	 $A = 12 \times 5 = 60\ \text{m}^2$
Trapezium 	$A = \frac{1}{2} \times$ sum of parallel sides × perpendicular distance between sides $A = \frac{1}{2}(a + b)h$	 $A = \frac{1}{2} \times (94 + 150) \times 75$ $= 9150\ \text{mm}^2$

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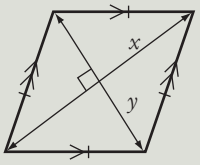
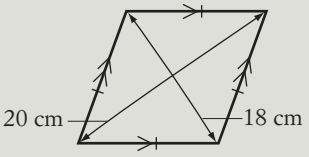
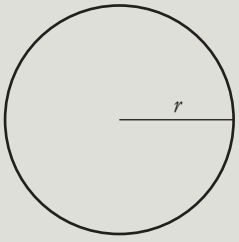
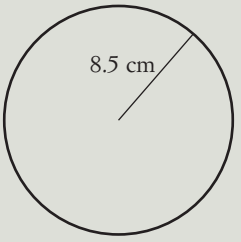
Shape	Area formula	Example
Rhombus or kite 	$A = \frac{1}{2} \times$ product of diagonals $A = \frac{1}{2}xy$	 $A = \frac{1}{2} \times 20 \times 18 = 180$ cm^2
Circle 	$A = \pi \times (\text{radius})^2$ $A = \pi r^2$	 $A = \pi \times 8.5^2 \approx 227$ cm^2

Exercise 6-11 Area

1 How many square centimetres in 300 m^2? Select **A**, **B**, **C** or **D**.

A 3 **B** 3000 **C** 3 000 000 **D** 300

2 How many hectares in 5 km^2? Select **A**, **B**, **C** or **D**.

A 500 **B** 50 **C** 50 000 **D** 0.5

3 Convert:

a 5 m^2 to cm^2 **b** 2500 cm^2 to mm^2

c 72 000 m^2 to hectares **d** 6800 cm^2 to m^2

e 3.09 km^2 to m^2 **f** 3.6 km^2 to hectares

g 4.73 m^2 to mm^2 **h** 540 ha to km^2

Example 23

4 The area of NSW is 801 600 km^2. How many hectares is this in scientific notation?

5 Builders measure lengths in millimetres, and carpet layers quote per square metre. How many square metres of carpet are needed in a room that measures 2300 mm by 1830 mm? Answer this question using the following two methods.

a Calculate the carpet area in mm^2, and then convert to m^2.

b Convert the lengths to metres first and then calculate the carpet area in m^2.

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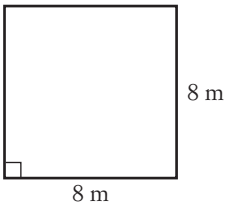
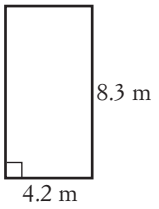
6 Find the area of each shape, correct to one decimal place where appropriate.
(*Note*: Some questions require the use of Pythagoras' theorem first.)

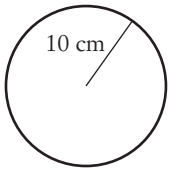
a



b



c



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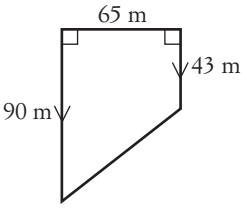

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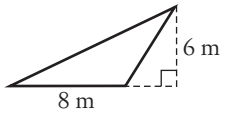

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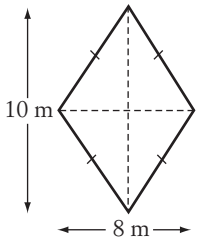
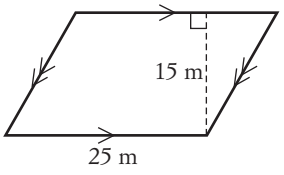


g



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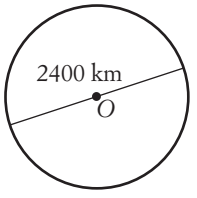

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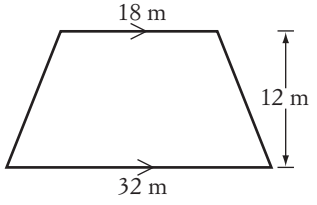

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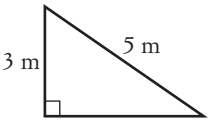

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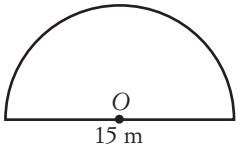
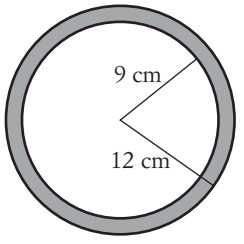
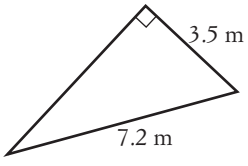


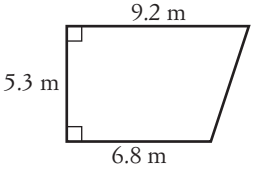
l Find the area of the shaded annulus.



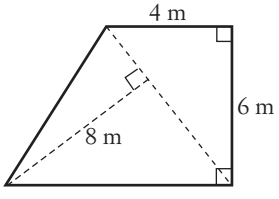
m



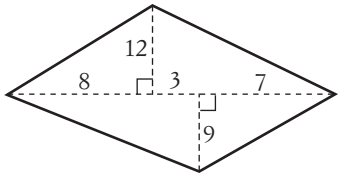
n



o



p Measurements are in metres.



7 A DVD has a diameter of 12 cm.

a How many significant figures are there in the measurement of 12 cm?

b Calculate the area of the DVD, correct to the same number of significant figures as your answer for part **a**.

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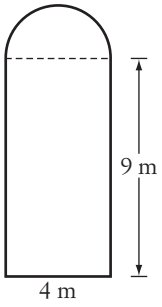
8 A farmer owns a large rectangular piece of land, measuring 2 km by 450 m. How many hectares does he own?

9 Find, correct to the nearest 0.1 m^2, the area of the glass in this window.

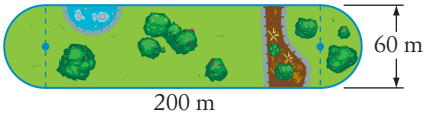


10 A factory worker cuts circular discs from square sheets of metal.

a What is the area of the largest disc that can be cut from a square of side 20 cm? Give your answer correct to two decimal places.

b What is the area of the remaining metal after the disc is cut from the square?

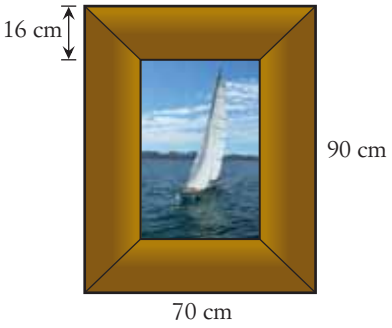
11 Find the area of this garden, correct to two significant figures.



12 A wooden picture frame is specifically made for the picture shown. The cost is \$135 per square metre. Calculate:

a the area of the frame, in square metres

b the cost of the frame, to the nearest cent.



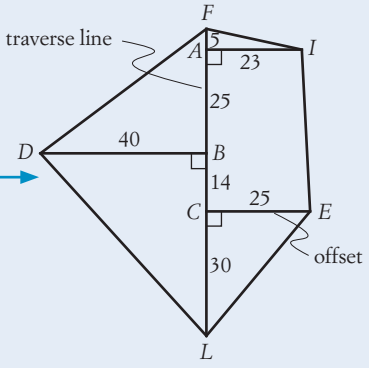
6-12 Offset surveys

Offset surveys

One method that surveyors use to measure the area of an irregular block of land is an **offset survey**, also called a **traverse survey**. The surveyor chooses a diagonal of the field to be the **traverse line** (traverse means 'to cross'), then measures the offsets (perpendicular distances) to each corner.

EXAMPLE 24

An offset survey was conducted on an irregular block of land, as shown on the **field diagram**, using *LF* as the traverse line. All measurements are in metres. Find the area of the block.



As most measurements in the diagram are correct to two significant figures, we should write the final answer correct to two significant figures.

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Solution

The block is made up of one big triangle, two small triangles and a trapezium.

Area of $\triangle FLD = \frac{1}{2} \times 74 \times 40$ **$LF = 5 + 25 + 14 + 30 = 74$**

$= 1480 \text{ m}^2$

Area of $\triangle FIA = \frac{1}{2} \times 23 \times 5$

$= 57.5 \text{ m}^2$

Area of trapezium $\triangle IECA = \frac{1}{2} \times (23 + 25) \times 39$ **$AC = 25 + 14 = 39$**

$= 936 \text{ m}^2$

Area of $\triangle CEL = \frac{1}{2} \times 25 \times 30$

$= 375 \text{ m}^2$

Total area of block $= 1480 + 57.5 + 936 + 375$

$= 2848.5$

$\approx 2800 \text{ m}^2$ **to 2 significant figures**

Notebook entries

When conducting an offset survey, the surveyor marks the measurements in a notebook first. The notebook entries for the field in Example **2** look like those shown on the right.

The surveyor uses a tape measure to measure the increasing distances along the traverse line from *L* to *F*. For example, 74 m is the **entire** distance from *L* to *F*, whereas the distance $CB = 44 - 30 = 14$ m.

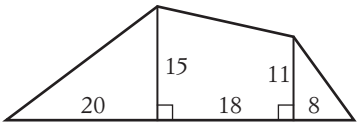
		F	
		74	
		69	23 *I*
D	40	44	
		30	25 *E*
		0	
		L	

Exercise 6-12 Offset surveys

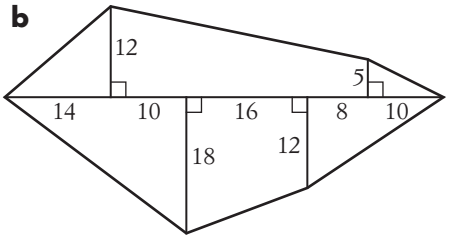
In this exercise, all measurements are in metres.

1 Find the area of each field shown below, correct to two significant figures.

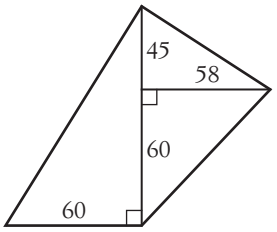
a



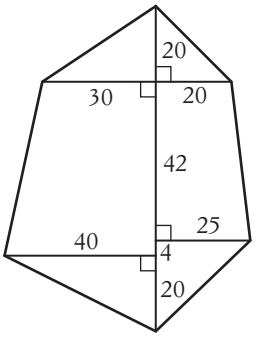
b



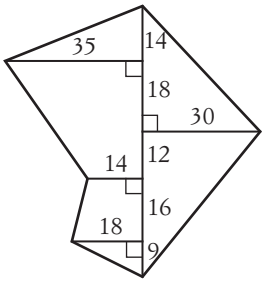
c



d



e



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Example 24

2 For each of the surveyor's notebook entries shown below, sketch a field diagram and then calculate its area correct to the nearest square metre.

a

	P	
	0	
	8	16 Q
S 7	18	
	23	
	R	

b

	F	
	52	
	34	36 G
	20	15 H
J 40	12	
	0	
	I	

c

	V	
	38	
	26	28 W
	20	36 X
Z 26	0	
	Y	

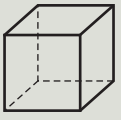
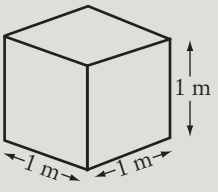
WS

Estimating area and volume

6-13 Volume

Metric units for volume

The **volume** of a solid is the amount of space it occupies. This is measured in **cubic units**.

Unit	Relationship	Size
cubic millimetre (mm^3)	$1\ mm^3 = 1\ mm \times 1\ mm \times 1\ mm$	The volume of a 1 mm cube (about the volume of a grain of raw sugar)
cubic centimetre (cm^3)	$1\ cm^3 = 1\ cm \times 1\ cm \times 1\ cm$ $= 10\ mm \times 10\ mm \times 10\ mm$ $= 1000\ mm^3$	The volume of a 1 cm cube (about the volume of a peanut) 
cubic metre (m^3)	$1\ m^3 = 1\ m \times 1\ m \times 1\ m$ $= 100\ cm \times 100\ cm \times 100\ cm$ $= 1\ 000\ 000\ cm^3$	The volume of a 1 m cube (about the volume of two washing machines) 

When converting units of volume, we need to multiply or divide by three lots of units.

For example if: $1\ m = 100\ cm$

then: $1\ m^3 = 100 \times 100 \times 100\ cm^3 = 1\ 000\ 000\ cm^3$

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EXAMPLE 25

Convert:

a 74 m^3 to cm^3

b 4600 mm^3 to cm^3.

Solution

a $74\ m^3 = (74 \times 100 \times 100 \times 100)\ cm^3$ **Because 1 m^3 = 100 cm × 100 cm × 100 cm.**

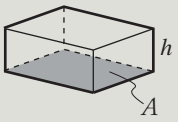
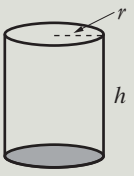
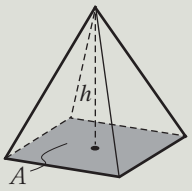
$= 74\ 000\ 000\ cm^3$

b $4600\ mm^3 = (4600 \div 10 \div 10 \div 10)\ cm^3$ **Because 1 cm^3 = 10 mm × 10 mm × 10 mm.**

$= 4.6\ cm^3$

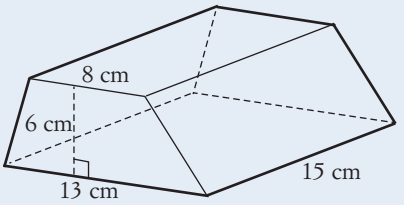
Volume formulas

This table lists the formulas we should learn for the volumes of solids.

Shape	Volume formula
Prism 	V = area of base × perpendicular height $V = Ah$
Cylinder 	$V = \pi \times (\text{radius})^2 \times$ perpendicular height $V = Ah = \pi r^2 h$
Pyramid 	$V = \frac{1}{3} \times$ area of base × perpendicular height $V = \frac{1}{3}Ah$

EXAMPLE 26

Find the volume of this trapezoidal prism.



Solution

Volume $= Ah$ **A = area of trapezium and h = 15 cm.**

$= \frac{1}{2} \times (8 + 13) \times 6 \times 15$

$= 945\ cm^3$

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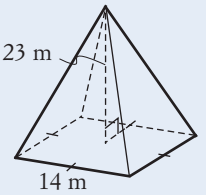
EXAMPLE 27

Find the volume of this square pyramid, correct to two significant figures.



Solution

$$\text{Volume} = \frac{1}{3}Ah$$

A = area of square and h = 23 m

$$= \frac{1}{3} \times 14^2 \times 23$$

$$= 1502.666\ldots$$

$$\approx 1500\ \text{m}^3$$

Exercise 6-13 Volume

Example 25

1 Convert:

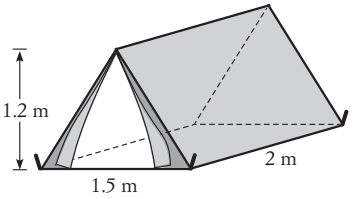
a 7 m³ to cm³ **b** 50 cm³ to mm³ **c** 89 000 cm³ to m³

d 0.468 m³ to cm³ **e** 2400 mm³ to cm³ **f** 5 600 000 cm³ to m³

g 9 100 000 mm³ to cm³ **h** 12 m³ to cm³

Example 26

2 Find the volume of this tent.



3 Which of the following is the volume of a cube of length 2.5 m? Select **A**, **B**, **C** or **D**.

A 6.25 mm³ **B** 15.625 m³ **C** 30 m³ **D** 37.5 m³

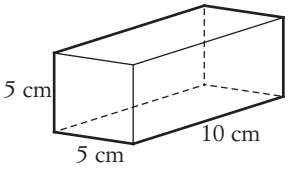
Example 27

4 Which of the following is closest to the volume of a square pyramid with base length 35 cm and perpendicular height 67 cm? Select **A**, **B**, **C** or **D**.

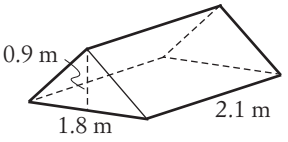
A 82 075 cm³ **B** 27 358 cm³ **C** 1173 cm³ **D** 782 cm³

5 Find the volume of each solid. (Round the answer to part **c** to one decimal place.)

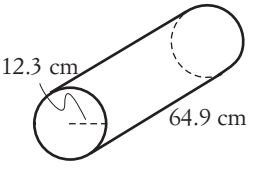
a



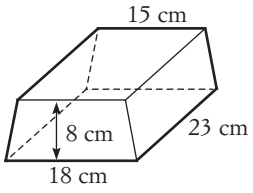
b



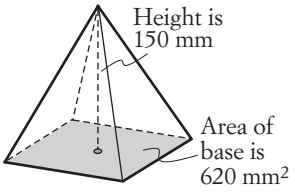
c



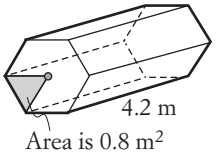
d



e



f



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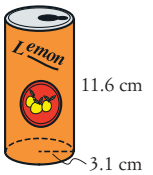
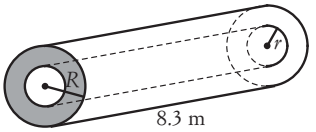
6 **a** What is the volume of this can, in cubic centimetres?

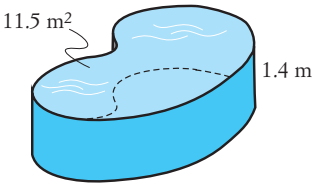
b If the can holds 350 mL of drink, what can you say about the relationship between the volume in cubic centimetres and the capacity in millilitres?



7 What is the volume of this piece of pipe if the outer radius, R, is 54 cm and the inner radius, r, is 32 cm? Give your answer in cubic metres, correct to two decimal places.



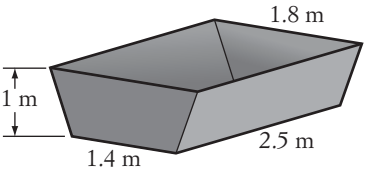
8 The cross-section of a kidney-shaped swimming pool has an area of 11.5 m^2. The pool has a constant depth of 1.4 m. What volume of water does it hold?



9 The largest pyramid in Egypt is the Great Pyramid of Cheops (or Khufu). It has a square base of side length 230 m and a perpendicular height of 137 m. What is its volume, correct to three significant figures?

10 **a** This rubbish skip in the shape of a trapezoidal prism is delivered to a building site. What volume of rubbish will it hold?

b If building rubbish costs \$16.50 per cubic metre to dump at the local tip, how much will it cost to dump four full skips?



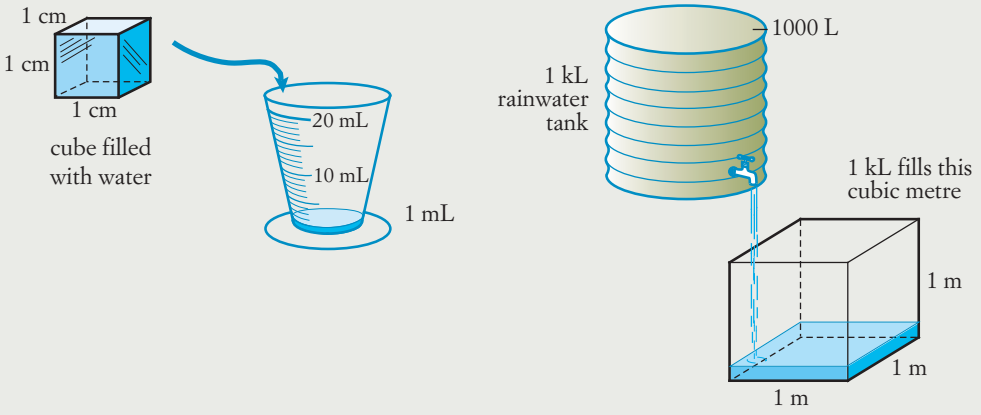
6-14 Volume and capacity

Volume and capacity are related. While volume is the amount of space taken up by a solid, **capacity** is the amount of material (usually liquid) that a solid or container can hold.

SUMMARY

1 cm^3 = 1 mL

1 m^3 = 1 kL = 1000 L



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EXAMPLE 28

Convert:

a 46 L to cm^3 **b** 10 400 cm^3 to L **c** 3500 m^3 to ML.

Solution

a 46 L = 46 000 mL **1 L = 1000 mL**

= 46 000 cm^3 **1 cm^3 = 1 mL**

b 10 400 cm^3 = 10 400 mL **1 cm^3 = 1 mL**

= 10.4 L

c 3500 m^3 = 3500 kL **1 m^3 = 1 kL**

= 3 500 000 L **1 kL = 1000 L**

= 3.5 ML **1 ML = 1 million litres**

Example 28

Exercise 6-14 Volume and capacity

1 Convert:

a 680 cm^3 to millilitres **b** 8500 cm^3 to litres **c** 22 m^3 to litres

d 8000 L to m^3 **e** 3.5 m^3 to millilitres **f** 690 L to cm^3

g 55 m^3 to litres **h** 4300 m^3 to kilolitres **i** 9500 L to m^3

j 8.5×10^4 cm^3 to litres **k** 4.3×10^{-3} kL to cm^3 **l** 10^6 m^3 to megalitres.

2 Which of the following is the capacity of a box with a volume of 2 m^3?
Select **A**, **B**, **C** or **D**.

A 2 L **B** 2000 L **C** 20 000 L **D** 200 L

3 A fish tank is 1.2 m long, 80 cm wide and 90 cm high.

a What is the volume of the tank, in cubic metres?

b What is the capacity of the tank, in litres?

4 A circular swimming pool is 8 m in diameter and has a constant depth of 1.5 m. To three significant figures, find the capacity of the pool in:

a litres **b** kilolitres.

5 Engine capacity is the total capacity of the cylinders of the engine. The engine of a V8 Holden car has eight cylinders, each with a diameter of 8 cm and a height of 10 cm.

a What is the capacity of the engine (the eight cylinders) in litres, correct to three significant figures?

b If the cylinders are bored out to a diameter of 8.1 cm, what is the increase in capacity of the engine in millilitres?

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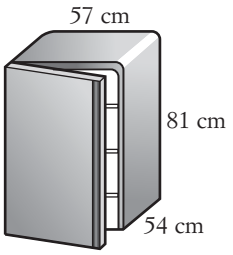
6 What is the capacity of this bar fridge, correct to the nearest litre ?



7 Which of the following is the volume of a tank with a capacity of 5 kL? Select **A**, **B**, **C** or **D**.

A 5 m^3 **B** 50 m^3 **C** 500 m^3 **D** 5000 m^3

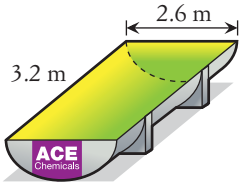
8 Warragamba Dam supplies Sydney with water and has a capacity of 2.031×10^6 ML. Use scientific notation to express the dam's capacity:

a in litres **b** in cubic metres.

9 A cylindrical rainwater tank has a base radius of 1 m and a height of 3.4 m. How much water does it hold to the nearest litre?

10 Sydney Harbour contains 5×10^{11} L of water. What is this volume in cubic kilometres?

11 This semicircular chemical vat is filled to capacity.
How many litres of acid will it contain, to the nearest litre?



JUST FOR THE RECORD

Mass and capacity

One litre of water has a mass of one kilogram. For water, 1 L = 1 kg. What is the mass of:

a one cubic metre of water?

b one cubic centimetre of water?

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Sample HSC problem

Police investigating a car crash concluded that the car travelled 20.7 metres in 1.2 seconds.

a How many significant figures are there in the distance measurement of 20.7 metres?

b Calculate, correct to two decimal places, the percentage error of the distance measurement.

c Calculate, correct to three significant figures, the average speed of the car:

i in m/s **ii** in km/h

Study tip

Organising your notes

One feature of effective study is an organised collection of class notes. This allows you to quickly and easily locate your work on any topic. The key to maintaining a neat set of notes is to label everything: the names of the topics, the theory, the class examples, the exercises, your folders.

Your maths class follows a program of topics, which may correspond roughly to the chapters of this textbook. Ask your teacher for a copy of the program or syllabus and organise your folder according to these topics. Use coloured dividers or title pages to mark the start of each new topic.

Start each maths lesson with the title of the lesson and the date. Pay attention to your teacher's explanations and copy the class examples into your book. Make sure you write the question as well as each step in the solution. You can add any personal notes, comments or reminders to help you later, when revising the work. If you don't understand something you should ask your teacher to explain again.

When using a loose-leaf folder, make sure your notes are filed in the correct order. Number the pages of each topic so that you can refile if they get out of order. If your folder becomes too bulky to manage, you should start a new one or remove some of the older topics and file them at home. Realise that you do not need to bring to school all your notes for the entire maths course, just what you are currently working on in class.

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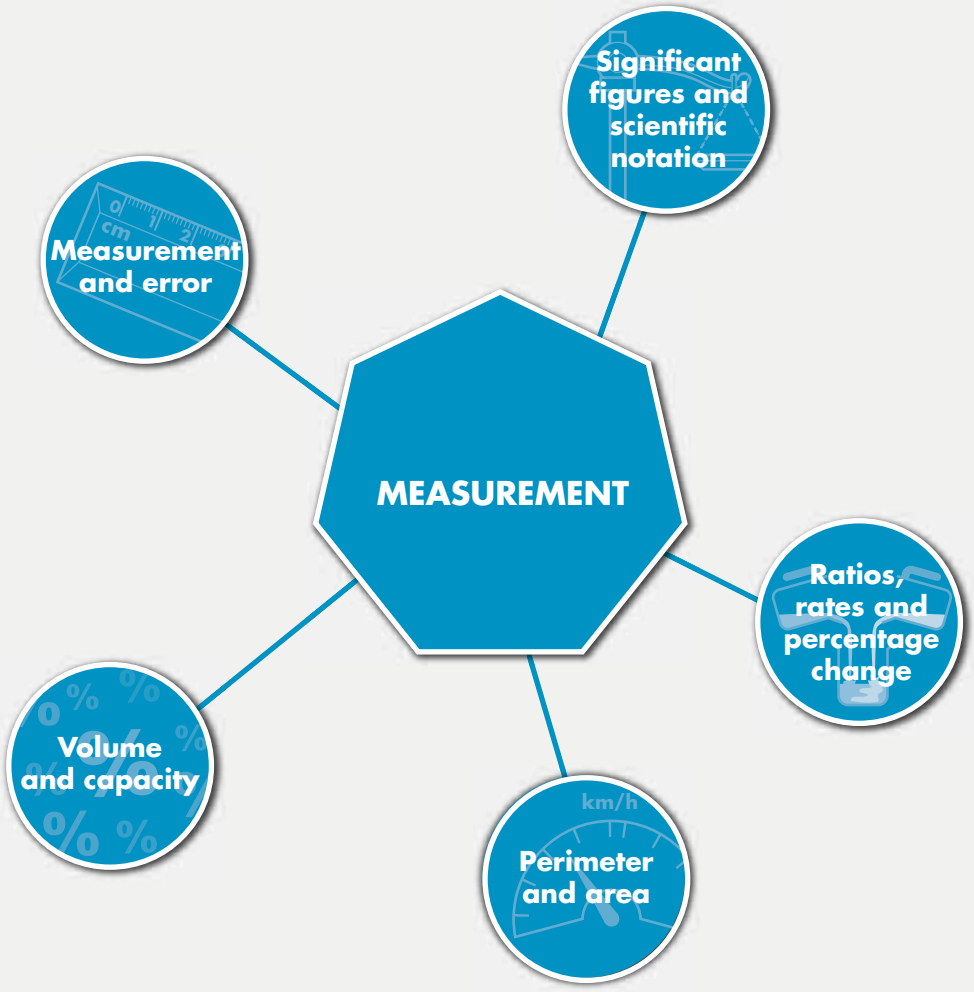
CHAPTER REVIEW

Topic overview

This chapter, 'Measurement', revised and extended concepts in measurement, including metric units, percentage change, ratios, rates, perimeter, area, volume and capacity. You should have a good understanding of percentage error, significant figures and scientific notation, and the facts and formulas behind the metric system, circumference of a circle, and areas and volumes of shapes. You should be able to apply your skills to solve practical problems involving ratios, rates, perimeter, area, offset surveys, volume and capacity.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map is shown below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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Revision

Exercise 6-01

1 Convert:

a 28.5 km to m **b** 6.4 t to kg **c** 340 mL to L

Exercise 6-01

2 Farmer Joe's dam holds 43 ML of water. How many kilolitres does it hold?

Exercise 6-02

3 The width of a room is measured as 3.66 m. Find:

a the absolute error of this measurement

b the percentage error, correct to two decimal places.

Exercise 6-03

4 Write each value correct to two significant figures.

a 38.915 **b** 1036 **c** 0.008 72 **d** 6 587 200

Exercise 6-04

5 A bank has \$8 350 000 000 in its vault. Write this value in scientific notation.

Exercise 6-04

6 A cell is 4.6×10^{-6} mm in diameter. Write this in normal decimal notation.

Exercise 6-05

7 Simplify each ratio.

a 15 : 60 **b** $1\frac{1}{2} : 3\frac{1}{4}$ **c** 5 min : 100 s **d** 50 kg : 1 t

Exercise 6-05

8 A milkshake contains 10 parts milk, 2 parts flavouring and 4 parts ice cream.

a How many millilitres of flavouring are there in half a litre of milkshake?

b How much milkshake would contain 110 mL of flavouring?

Exercise 6-06

9 Divide \$44 380 between Phoebe and Patrick in the ratio 3 : 2.

Exercise 6-07

10 A hose fills a 10 L bucket in 25 seconds. Calculate, to the nearest minute, the time required to fill a 40 kL pool.

Exercise 6-8

11 Australian swimmer Eamon Sullivan became the world's fastest swimmer in 2008 when he swam 50 metres in 21.28 seconds.

a Calculate, correct to four significant figures, his average speed:

i in m/s **ii** in km/h.

b If Eamon could maintain this speed, how long would it take him to swim 1 km?

Exercise 6-9

12 Melanie owned \$2400 of shares in XYZ Mining. After the stock market crash the shares dropped 18% in value but, in the new year, they gained 5%.

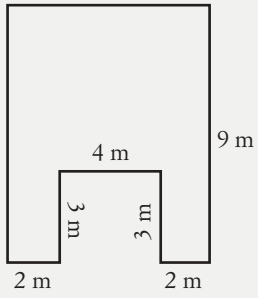
a What was the share price after these two percentage changes?

b What was the overall percentage change?

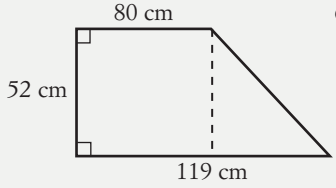
Exercise 6-10

13 Calculate the perimeter of each shape (correct to two decimal places for shape **c**).

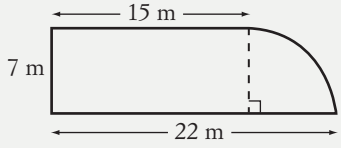
a



b



c



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14 Convert:

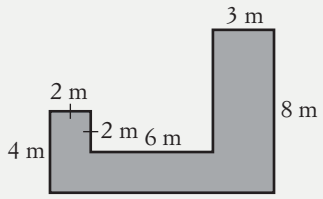
a 8400 mm² to cm² **b** 5.6 ha to m².

Exercise 6-11

15 Calculate the area of each shape (correct to two significant figures for shape **c**).

Exercise 6-11

a



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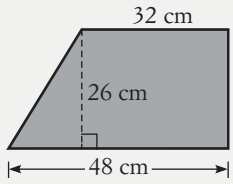

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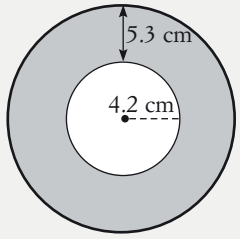


Exercise 6-12

16 Felix completed an offset survey of a field, *PQRST*, and recorded the entries on the right in his notebook. Draw a field diagram and calculate its area:

a in m² **b** in ha.

		S		
R	16	52		
Q	10	32		
		24	15	*T*
		0		
		P		

Exercise 6-13

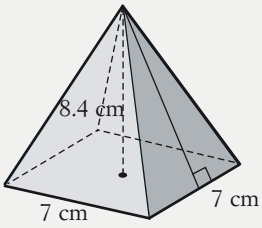
17 Convert:

a 20.7 cm³ to mm³ **b** 1 650 000 cm³ to m³.

Exercise 6-13

18 Find the volume of each solid.

a



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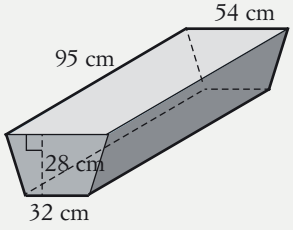

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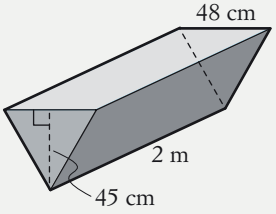


Exercise 6-14

19 **a** How many millilitres will a container of volume 894 cm³ hold?

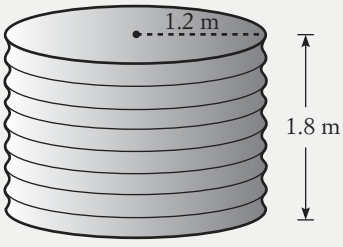
b How many litres will a container of volume 6.5 m³ hold?

Exercise 6-14

20 A water tank has the shape of a cylinder with a base radius of 1.2 m. If the tank is 1.8 m high find, correct to two significant figures:

a its volume in m³

b its capacity in litres.



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Practice paper 2

Recommended time: One hour

Section 1

10 objective-response questions: 1 mark each
For Questions 1 to 6, select the correct answer, A, B, C or D

Exercise 6-04

Question 1

The value of $\sqrt{\dfrac{45.8}{3.8 \times 10^{-5}}}$ in scientific notation, correct to three significant figures, is:

(A) 1.09×10^{3} (B) 1.09×10^{-2} (C) 1.10×10^{-2} (D) 1.10×10^{3}

Exercise 5-04

Question 2

If there is a 30% chance of rain today, what is the chance that it does not rain today?

(A) 60% (B) 70% (C) 15% (D) 20%

Exercise 6-05

Question 3

A pancake recipe, serving three people, requires the following ingredients:

1 cup of flour, 5 tablespoons of milk, a pinch of salt, 2 teaspoons of butter.

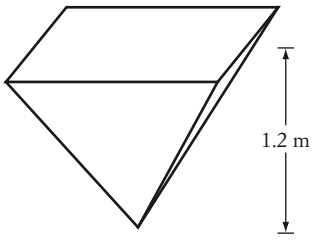
If the recipe is varied to serve 14 people, approximately how many teaspoons of butter would be needed?

(A) 9 (B) 12 (C) 21 (D) 28

Exercise 6-13

Question 4

This grain hopper is in the shape of a square pyramid. If it has a volume of 3.5 m^3 and a depth of 1.2 m, what is the area of its square top?



(A) 0.97 m^2 (B) 2.92 m^2
(C) 2.96 m^2 (D) 8.75 m^2

Exercise 5-03

Question 5

In NSW, some white-on-black number plates have two letters followed by four digits (for example JB1234). How many such number plates are possible?

(A) 10 000 (B) 520 000 (C) 4 435 236 (D) 6 760 000

Exercise 6-06

Question 6

Karin, Tiana and Nuala share a Lotto prize of \$13 122 in the ratio 2 : 3 : 4. What is Karin's share?

(A) \$1458 (B) \$2916 (C) \$4374 (D) \$5832

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For Questions 7 to 10, write the answer only (do not show your working).

Question 7

A camera has a price of $137.50 after a GST of 10% is added. What was the camera's price before GST?

Exercise 4-09

Question 8

Jarryd earns 2% commission from each house he sells. How much would he earn in a month in which he sells three houses for $810 000, $675 000 and $408 600?

Exercise 4-02

Question 9

Bianca measures a length to be 420 mm with her builders' tape which is marked in millimetres. What is the percentage error, to two decimal places, of her measurement?

Exercise 6-02

Question 10

Express as a simplified fraction the probability of randomly drawing an Ace from a normal deck of cards.

Exercise 5-01

Section 2

3 questions: 10 marks each

Question 11 — Marks

Exercise 5-02

(a) A biased die has sides 1, 2, 3, 3, 5 and 6. It is rolled twice and the sum of the numbers is recorded.

(i) The table below can be used to list all possible outcomes. Copy and complete the table. **1**

		Second roll					
		1	**2**	**3**	**3**	**5**	**6**
First roll	**1**	2	3	4	4	6	7
	2	3	4	5	5	7	8
	3	4	5	6	6	8	9
	3	4	5	6	6	8	9
	5						
	6						

(ii) How many possible outcomes are there? **1**

(iii) What is the probability of rolling a sum of 6? **1**

(iv) What is the probability of rolling a sum that is an even number? **1**

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Exercise 6-07

(b) A citrus tree is sprayed with a pesticide solution to prevent white mould from forming on it. 15 L of solution contains 180 mL of pesticide.

(i) Express the concentration of pesticide as a rate in mL/L. **1**

(ii) How much pesticide is in 100 L of solution? **1**

Exercise 4-01

(c) The table below shows the weekly pay of four employees at Wonka's chocolate factory. Overtime is paid at time-and-a-half. Complete the table by calculating the missing values at (i), (ii), (iii) and (iv). **4**

Name	Pay rate (per hour)	Normal hours	Overtime hours	Weekly pay
Kovi	$17.00	38	2	(i)
Mia	$13.50	12	(ii)	$182.25
Jeeva	(iii)	38	4	$677.60
Charlie	$18.00	(iv)	3	$567.00

Exercise 5-03

Question 12 **Marks**

(a) At the Greasy Spoon takeaway, customers can purchase a special meal deal of a main meal, a side dish of potatoes and a drink, from the following menu:

Main	Potatoes	Drink
Hamburger	Chips	Soft drink
Vegieburger	Wedges	Juice
Fish	Whole potato	Tea
Sausage sandwich	Mashed potato	Coffee
Fried chicken		
Supreme pizza		

(i) How many meal deal combinations are possible? **1**

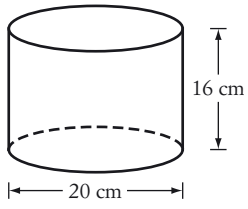
(ii) How many meal deal combinations are possible if the customer only drinks coffee and does not eat fish or mashed potato? **1**

Exercise 6-07

(b) A tap drips 0.8 mL of water per second. Calculate how many litres of water will drip from the tap in one day. **2**

Exercise 6-14

(c) A can has a diameter of 20 cm and a height of 16 cm. Find its capacity in litres, correct to two significant figures. **2**



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(d) Last financial year, Richard earned a wage of $804.10 each week.

(i) How much did Richard earn over the year? **1**

(ii) On his tax return, Richard claimed deductions of $341.90 for work-related expenses and $144.00 for donations to charities. What was his taxable income, rounded down to the nearest dollar? **1**

(iii) Calculate Richard's total tax payable, rounded up to the nearest cent, given the following rates: **2**

Income tax: $3572 plus 32.5c for each $1 over $37 000

Medicare levy: 1.5% of taxable income

Exercise 4-08

Question 13 — Marks

(a) A packet of lollies contains 10 red, 8 yellow, 4 purple and 2 green lollies. Jessica takes one lolly out of the packet at random. What is the probability that it is:

(i) purple? **1**

(ii) red or green? **1**

(iii) not green? **1**

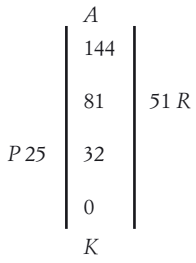
Exercise 5-01

(b) What percentage discount is equivalent to a discount of 6% followed by a further discount of 2%? **2**

Exercise 6-09

(c) An offset survey on a field, *PARK*, was conducted and the following notebook entries were made by the surveyor. All measurements are in metres.

Exercise 6-12



(i) Draw a diagram of the field, showing all measurements. **2**

(ii) Calculate the area of the field. **2**

(d) A building site manager earns a salary of $83 561. Calculate correct to the nearest cent his annual leave loading if it is 17.5% of 4 weeks of normal pay. **1**

Exercise 4-03

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ALGEBRA AND MODELLING

7.

LINEAR FUNCTIONS

A criminologist discovered that, in a big city, if the number of police on street patrol was increased, the number of crimes committed decreased. She noticed the following pattern.

Number of police	50	150	200	250	300
Number of crimes (per month)	3100	2800	2650	2500	2350

Is it possible for the criminologist to find an algebraic formula to fit this pattern? Algebraic modelling is the study of relationships and the formulation of a mathematical rule or model to describe such relationships.

CHAPTER OUTLINE

*Year 12 content

AM2 7-01 Graphing linear functions
AM2 7-02 The gradient formula
AM2 7-03 Linear modelling
AM4* 7-04 Direct linear variation
AM2 7-05 Conversion graphs
AM2 7-06 Intersection of lines
AM2 7-07 Stepwise linear functions

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iStockphoto/Holger Mette

IN THIS CHAPTER YOU WILL:

- generate tables of values from a linear equation
- graph linear functions with pencil and paper, and with technology, given an equation or a table of values
- calculate the gradient of a straight line from a graph
- determine the y-intercept for a given graph
- identify independent and dependent variables in practical contexts
- establish a meaning for the intercept on the vertical axis in a given context
- use linear equations to model practical situations
- describe the limitations of linear models in practical contexts.
- sketch graphs of linear functions expressed in the form $y = mx + b$ without the use of tables
- use graphs to make conversions
- sketch the graphs of a pair of linear equations to find the point of intersection
- find the solution of a pair of simultaneous linear equations from a given graph
- solve practical problems using graphs of simultaneous linear equations
- use stepwise linear functions to model and interpret practical situations

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TERMINOLOGY

break-even point
dependent variable
independent variable
linear model
rate of change
simultaneous equations
varies as
constant of variation
direct linear variation
limitation
point of intersection
rise
step graph
vertical intercept
conversion graph
gradient
linear function
proportional to
run
stepwise linear function
y-intercept

Assignment 7

SkillCheck

1 Graph this table of values on a number plane and rule a line through the points.

x	–1	0	1	2
y	–4	–1	2	5

2 Copy and complete each table of values using the given formula.

a $y = 2x + 7$

x	–1	0	1	2
y				

b $y = -x - 4$

x	–1	0	1	2
y				

3 If $y = mx + b$, find the value of b if $x = 2$, $y = 5$ and $m = 3$.

4 If $y = kx$:

a find the value of y if $k = 0.7$ and $x = 4$

b find the value of k if $x = 2.5$ and $y = 10$.

5 Given the formula $V = -450t + 2575$:

a find the value of V if:

i $t = 3$

ii $t = 4.2$

b find the value of t if:

i $V = 1450$

ii $V = 10$

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7-01 Graphing linear functions

The formula or equation $y = 3x - 2$ is called a **linear function** because its graph is a straight line. **Linear** means 'of a line', while a **function** is an algebraic rule similar to a 'number machine', as shown below, that changes an input value, x, into an output value, y.

Function: $y = 3x - 2$

$x = 5 \xrightarrow{\text{input}} \boxed{\text{process}} \xrightarrow{\text{output}} y = 13$

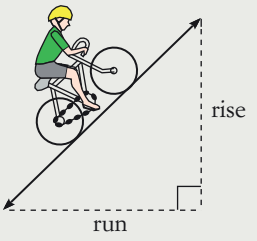
A line has a **gradient** (the measure of its steepness) and a ***y*-intercept** (the value where the line crosses the y-axis).

SUMMARY

The gradient of a line is given by the formula:

$$m = \frac{\text{rise } (\uparrow)}{\text{run } (\rightarrow)}$$

- The **rise** is the vertical change in position (going up).
- The **run** is the horizontal change in position (going across to the right).



EXAMPLE 1

a Copy and complete this table of values for the linear function $y = 3x - 2$.

x	–1	0	1	2	3
y					

b What pattern do you notice in the y values in the bottom row of the completed table?

c Graph $y = 3x - 2$ on a number plane.

d Find the gradient and y-intercept of the line you drew in part **c**.

Solution

a

x	–1	0	1	2	3
y	–5	–2	1	4	7

b The y values increase by 3 each time.

Drawing gradients

Graphing linear functions

A page of number planes

Graphing lines: Graphics calculator

Graphing linear equations

Gradient and y-intercept

$y = mx + b$

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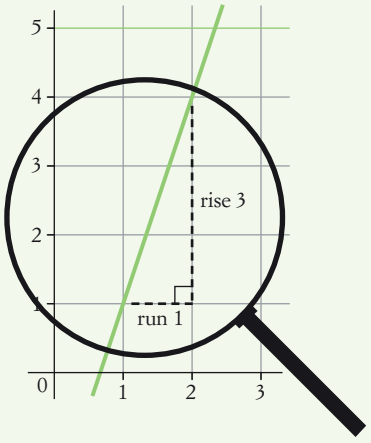
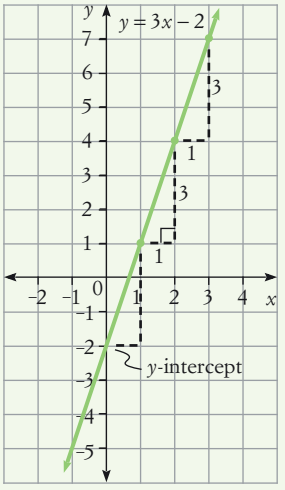
c Use the (x, y) ordered pairs from the table to graph the points: $(-1, -5)$, $(0, -2)$, $(1, 1)$, $(2, 4)$, $(3, 7)$. Then rule and label the line through the points.

d Choose two points on the line, say $(1, 1)$ and $(2, 4)$.





Gradient $(m) = \dfrac{\text{rise}}{\text{run}} = \dfrac{3}{1} = 3$

The y-intercept is -2.

SUMMARY

A linear function has the form $y = mx + b$, where m is the gradient and b is the y-intercept.

$y = mx + b$ — gradient; y-intercept

$y = 3x - 2$ — gradient 3; y-intercept -2

EXAMPLE 2

a Complete this table of values for the linear function $y = -2x + 4$.

x	–1	0	1	2	3
y					

b What pattern do you notice in the y values in the completed table?

c Graph $y = -2x + 4$ on a number plane.

d Find the gradient and y-intercept of the line you drew in part **c**.

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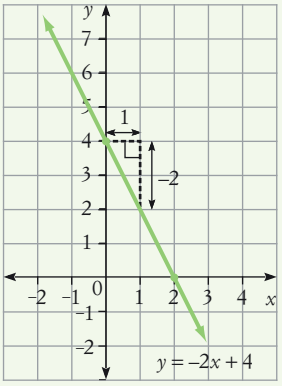
Solution

a

x	–1	0	1	2	3
y	6	4	2	0	–2

b The y values decrease by 2 each time.

c



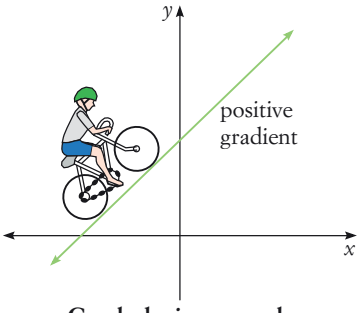
d Using points (0, 4) and (1, 2):

$$\text{Gradient } (m) = \frac{\text{rise}}{\text{run}} = \frac{-2}{1} = -2$$

The y-intercept is 4.

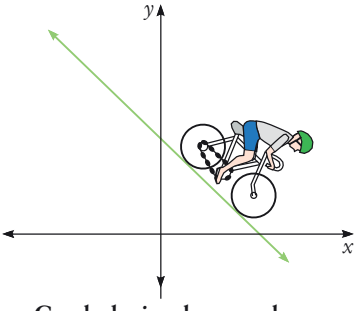
Positive and negative gradients

A line with a **positive gradient** slopes upwards (from left to right) because its y values are *increasing*. A gradient of 3 means that, for each 1 unit we move across, the line goes up 3 units or, as the x values increase by 1, the y values *increase* by 3.



Graph sloping upwards

A line with a **negative gradient** slopes downwards (from left to right) because its y values are *decreasing*. A negative gradient has a 'negative rise', or a 'drop'. A gradient of –2 means that, for each unit we move across, the line goes down 2 units, or as the x values increase by 1, the y values *decrease* by 2.



Graph sloping downwards

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EXAMPLE 3

Graph each of these linear functions after finding the gradient and y-intercept.

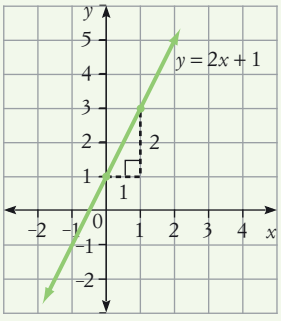
a $y = 2x + 1$ **b** $y = -4x$ **c** $y = \frac{2}{3}x - 4$

Solution

a Gradient = 2, y-intercept = 1

To graph the line:

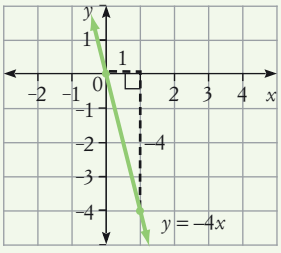
- plot the y-intercept, 1, on the y-axis
- indicate a gradient of 2 from this point, by moving across 1 unit and up 2 units and marking that point
- rule a line through both points you marked
- draw arrows at the ends and label the line with its equation.



b Gradient = –4, y-intercept = 0

To graph the line:

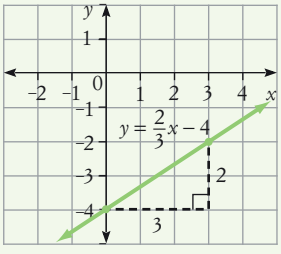
- plot the y-intercept, 0, at the origin
- indicate a gradient of –4 from this point, by moving across 1 unit and down 4 units and marking that point
- rule a line through both points you marked, add arrows and label.



c Gradient = $\frac{2}{3}$, y-intercept = –4

To graph the line:

- plot the y-intercept –4 on the y-axis
- indicate a gradient of 3 from this point, by moving across 3 units and up 2 units and marking that point
- rule a line through both points you marked, add arrows and label.



Exercise 7-01 Graphing linear functions

A page of number planes

Number plane grid paper

1 Write the equation of the line with:

a gradient 3, y-intercept 7

b gradient –2, y-intercept 1

c gradient 1, y-intercept –1

d gradient $\frac{1}{3}$, y-intercept $\frac{1}{2}$

e gradient $-\frac{5}{4}$, y-intercept 0

f gradient 0, y-intercept 5

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Example 1

Example 2

Example 3

2 Graph each of these linear functions.

a $y = 4x - 3$ **b** $y = \frac{1}{2}x + 2$ **c** $y = x - 2$

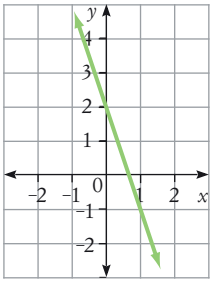
d $y = -2x$ **e** $y = 2x - 5$ **f** $y = \frac{3}{5}x - 1$

g $y = -x + 4$ **h** $y = \frac{4}{3}x + 1$ **i** $y = -\frac{1}{4}x + 3$

3 Which of the following is the gradient of this line?
Select **A**, **B**, **C** or **D**.

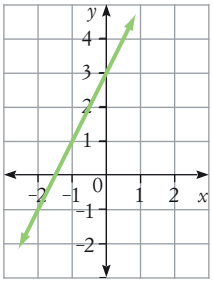
A $\frac{1}{3}$ **B** $-\frac{1}{3}$

C 3 **D** -3

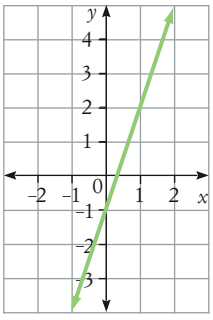


4 Find the gradient and y-intercept of each line below, and write its equation.

a



b



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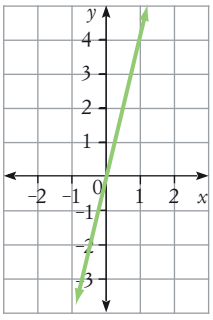

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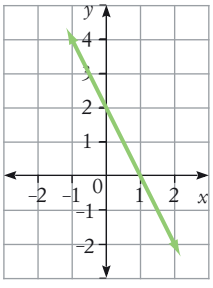

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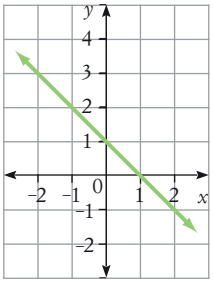

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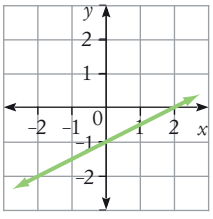

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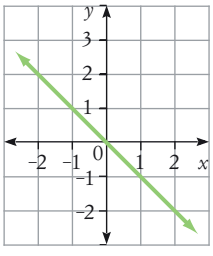

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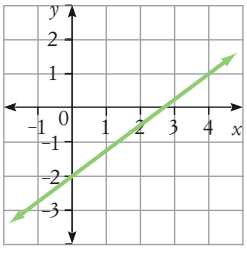

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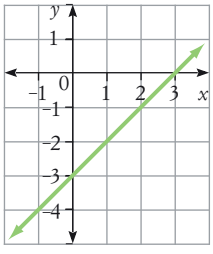

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INVESTIGATION

GRADIENTS

a What does a line with a gradient of 0 look like?

b What does a line with a gradient of 1 look like?

c What does a line with a gradient of –1 look like?

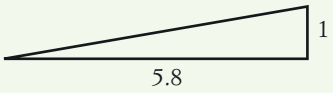
d What is the highest gradient a line can have?

JUST FOR THE RECORD

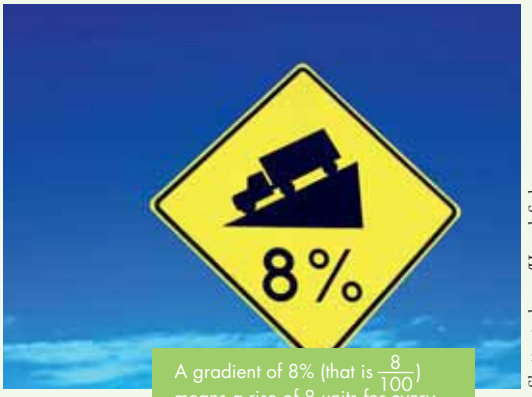
Steep roads and hills

Australia is a fairly flat continent but, in Europe, road signs often display the steepness of a road for drivers (or the steepness of a hill for hikers).

Consider this example:



For this example, the steepness is $\frac{1}{5.8}$, also written as '1 : 5.8' or '1 in 5.8'.



A gradient of 8% (that is $\frac{8}{100}$) means a rise of 8 units for every 100 units of run.

Shutterstock.com/Henryk Sadura

Bulli Pass, north of Wollongong, has a gradient of $\frac{1}{6}$ while Victoria Pass, east of Lithgow, has a gradient of $\frac{1}{8}$. Which pass is steeper?

7-02 The gradient formula

There is a formula for the gradient of a line which is useful if:

- the coordinates of two points on the line are given, or
- a table of values is given.

SUMMARY

$$m = \frac{\text{rise } (\uparrow)}{\text{run } (\rightarrow)} = \frac{\text{change in } y}{\text{change in } x}$$

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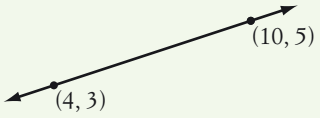
EXAMPLE 4

Find the gradient of each line.

a



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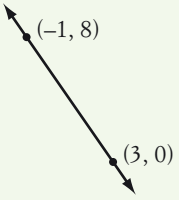

c

Solution

a $m = \dfrac{\text{change in } y}{\text{change in } x}$

$= \dfrac{5-3}{10-4}$

$= \dfrac{2}{6}$

$= \dfrac{1}{3}$ **A gradient of $\frac{1}{3}$ is not steep.**

b $m = \dfrac{0-8}{3-(-1)}$

$= \dfrac{-8}{4}$

$= -2$ **Negative gradient, the line slopes downward.**

c $m = \dfrac{2-2}{8-3}$

$= \dfrac{0}{5}$

$= 0$ **Zero gradient, line is flat.**

EXAMPLE 5

Find the gradient of the line represented by each table of values.

a

x	0	4	8	20
y	5	11	17	35

b

x	2	6	9	12
y	4	–8	–17	–26

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Solution

a Choose any two points from the table, say (0, 5) and (4, 11):

$$m = \frac{\text{change in } y}{\text{change in } x}$$
$$= \frac{11 \quad 5}{4 \quad 0}$$
$$= \frac{6}{4}$$
$$= \frac{3}{2}$$

The gradient is $\frac{3}{2}$.

b Using (2, 4) and (6, –8) from the table:

$$m = \frac{-8-4}{6-2}$$
$$= \frac{-12}{4}$$
$$= -3$$

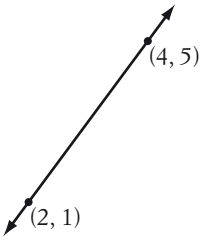
Negative gradient (*y* decreases as *x* increases).

Example 4

Exercise 7-02 The gradient formula

1 Find the gradient of each line.

a



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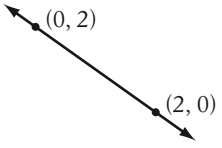

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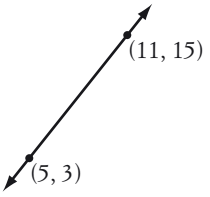

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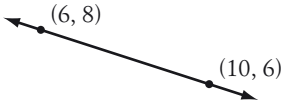

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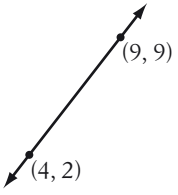

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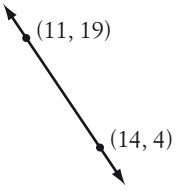

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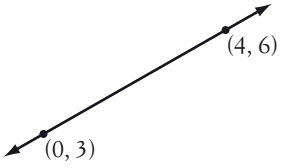

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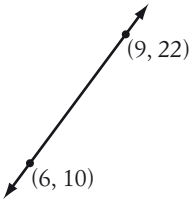

i

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Example 5

2 Find the gradient of the line represented by each of the following tables of values.

a

x	0	4	7	10
y	–1	11	20	29

b

x	1	5	7	12
y	1	21	31	56

c

x	0	8	12	20
y	3	5	6	8

d

x	1	3	6	8
y	7	3	–3	–7

e

x	0	10	25	40
y	–4	2	11	20

f

x	2	8	14	16
y	4	1	–2	–3

7-03 Linear modelling

WS Finding the equation of a line

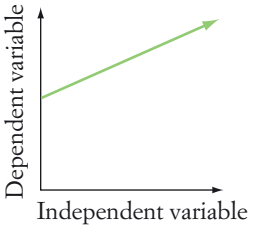
WS Linear functions code puzzle

Scientists and researchers observing patterns in nature and society often use a mathematical formula to represent a real-life situation. This is called **algebraic modelling** and, if the relationship uses the linear function $y = mx + b$, it is called **linear modelling**.

Dependent variables and independent variables

Because the value of y depends on the value of x, y is called the **dependent variable** and x is called the **independent variable**.

When a function is graphed, the independent variable is shown on the horizontal axis, while the dependent variable is shown on the vertical axis.



Understanding the gradient and the y-intercept

The **gradient** measures the steepness of a line, but it also shows how quickly the y-values are changing.

Look at this table of values for $y = 5x - 4$:

+1 +1 +1 +1

x	–1	0	1	2	3
y	–9	–4	1	6	11

+5 +5 +5 +5

As the x values increase by 1, the y values increase by 5. If this linear function is graphed, for every 'run' of 1 unit, there is a 'rise' of 5 units, giving a gradient of $\frac{5}{1} = 5$.

SUMMARY

- The **gradient** of a linear function is the **rate of change** of y.
- The higher the gradient, the steeper the line, and the faster y increases relative to x.
- The **y-intercept** is the value of y when $x = 0$.

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EXAMPLE 6

Brett is organising a 21st birthday party. The total cost of the party will include \$200 for hiring the hall, \$120 for the music and \$14 per person for catering. The table below shows the total cost, \$$C$, for different numbers of guests, n, attending the party.

Number of guests, n	80	100	120	130	150
Party cost, \$$C$	1440	1720	2000	2140	2420

a Which variable, n or C, is the dependent variable?

b Find the gradient of the linear relationship between n and C.

c Find the vertical intercept of this linear relationship.

'Vertical intercept' is the more general name for the y-intercept, because linear functions can use variables other than x and y.

d Write the linear function for C in terms of n.

e If this function was graphed, which variable would be shown on the horizontal axis?

f What does the vertical intercept of this function represent?

g Find the cost of the party if there were 95 guests.

Solution

a C, because C depends on n.

b Choosing the ordered pairs (80, 1440) and (100, 1720) from the table:

$$m = \frac{1720 - 1440}{100 - 80} = \frac{280}{20} = 14$$

The gradient is 14.

c The linear function is: $C = mn + b$

The gradient is: $m = 14$ **From the answer to part b above.**

So: $C = 14n + b$

To find the value of b, the vertical intercept, substitute an ordered pair into the function.

Substitute (100, 1720):

$1720 = 14(100) + b$

$1720 = 1400 + b$

$b = 320$

The vertical intercept is 320.

d The function is $C = 14n + 320$.

This formula should work for the other ordered pairs in the table.

e n, because it is the independent variable.

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f The vertical intercept, 320, is the value of C when $n = 0$, which is the cost of the party if there are no guests attending. This makes sense because it is the total fixed cost of hiring the hall and the music ($200 + $120).

g Substitute $n = 95$ into the function:

$$\begin{aligned} C &= 14n + 320 \\ &= 14(95) + 320 \\ &= 1650. \end{aligned}$$

The cost for the party with 95 guests is $1650.

EXAMPLE 7

A criminologist discovered that the number of crimes committed per month, C, in a big city decreased as the number of police officers, P, patrolling the city increased. After graphing her data on a number plane, she found the linear relationship to be $C = -3P + 3250$.

a What is the independent variable in this relationship?

b Copy and complete this table for the equation $C = -3P + 3250$.

Number of police, P	50	150	200	250	300
Crimes per month, C					

c Graph $C = -3P + 3250$ on the number plane.

d What is the gradient of the graph you drew in part **c** and what does it represent?

e What is the vertical intercept of the graph you drew in part **c** and what does it represent?

f Calculate how many crimes are committed when 100 police officers are on patrol.

g Calculate how many police officers are required to reduce the number of crimes to 1900.

Solution

a P, because C depends on P.

b

Number of police, P	50	150	200	250	300
Crimes per month, C	3100	2800	2650	2500	2350

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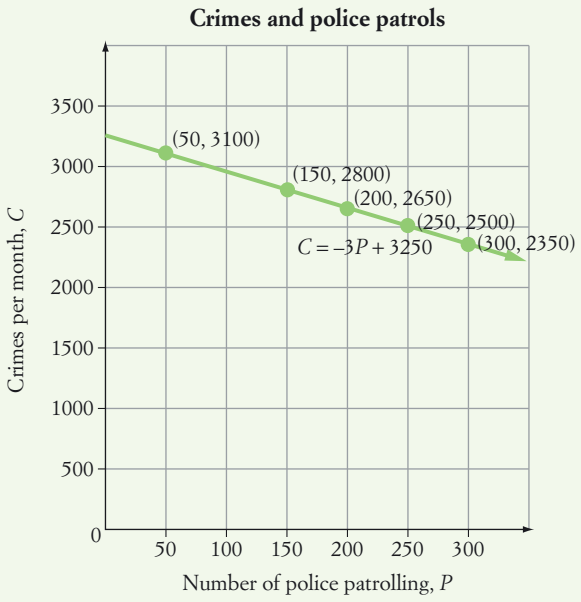
c



d From the equation, the gradient is –3 and it represents the rate of change in crime as the number of police increases. As the number of police increases by 1, the number of crimes decreases by 3.

e The vertical intercept is 3250, and this represents the number of crimes if no police officers were on patrol. It is the value of C when $P = 0$.

f Substitute $P = 100$:

$C = -3P + 3250$

$C = -3(100) + 3250$

$= 2950$

2950 crimes are committed when 100 police are on patrol.

g Substitute $C = 1900$:

$C = -3P + 3250$

$1900 = -3P + 3250$

$-3P = -1350$

$P = \dfrac{-1350}{-3}$

$= 450$

450 police officers are required to reduce the number of crimes to 1900.

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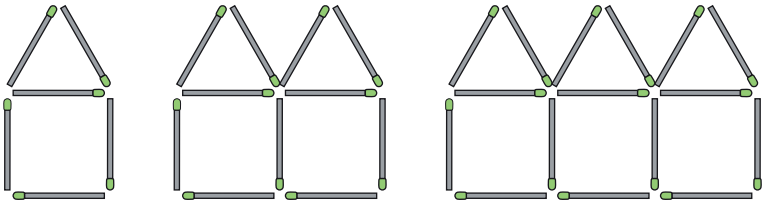
Exercise 7-03 Linear modelling

Example 6

1 This is a matchstick pattern of houses.



One house uses 6 matches. Two houses use 11 matches. Three houses use 16 matches.

a Copy and complete this table for the pattern above.

Number of houses, h	1	2	3	4	5	6
Number of matches, N						

b What is the dependent variable?

c Find the linear relationship in the form $N = mh + b$.

d How many matches are required to make 20 houses?

e How many houses can be made from 81 matches?

f Graph the linear function on a number plane.

g Write the gradient and the vertical intercept of the graphed line.

2 This table shows the cost, C cents, of mobile phone calls under the Oz-Zone Budget Plan, for calls of different lengths, t minutes.

Length of call, t (min)	1	2	5	10	15
Cost, C (cents)	102	182	422	822	1222

a Find the linear relationship in the form $C = mt + b$.

b If this function was graphed on a number plane, which variable would be shown on the vertical axis?

c Use the relationship you found in part **a** to calculate the cost of an 18-minute call.

d What is the vertical intercept of this function and what does it represent?

e If a phone call is extended by 3 minutes, by how much would its cost increase?

f How long is a phone call under this plan if it cost \$5.82?

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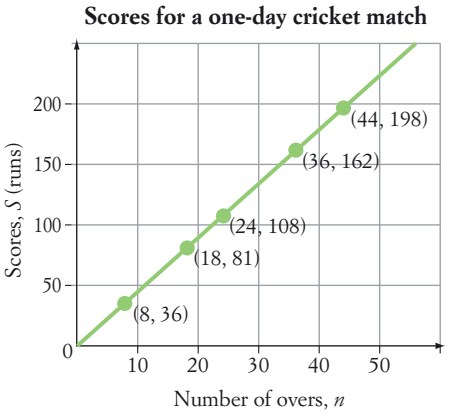
3 A cricket team's progressive score during a one-day cricket match can be approximated by the linear function graphed on the right.



A one-day match has 50 overs, where an over is a set of 6 balls bowled by the same bowler. The variable, n, represents the number of overs bowled, while S represents the total number of runs scored by the batting team.

a Is S the dependent variable or the independent variable?

b Find the formula for S in terms of n.

c The gradient of this linear function is also the team's run rate. What is the gradient of the function and what units is this run rate measured in?

d What is the vertical intercept and what does it represent?

e What was the score after:

i the 21st over? **ii** the 50th over?

f At the end of which over had the score reached:

i 54 runs? **ii** 180 runs?

g The graph for a real cricket match would not be a straight line but would normally flatten out later in the innings. Why?

Example 7

4 The value of a notebook computer depreciates according to the formula $V = -420t + 1900$, where V is the value in dollars and t is the time in years.

a Copy and complete this table for the formula $V = -420t + 1900$.

Time, t (years)	1	2	3	4
Value of computer, V ($)				

b Graph this linear relationship on a number plane.

c What does the gradient of this linear function represent?

d What was the value of the computer after $2\frac{1}{2}$ years?

e What was the original value of the computer?

f This linear model does not work when $t = 5$ and beyond. Why not?

g Find, correct to one decimal place, the time when the computer has zero value.

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5 Jessica works for a pizza shop and, each day, she earns a base pay of \$50, plus \$3 for every pizza she delivers.

a If n is the number of pizzas Jessica delivers in a day and P is her total pay in dollars, write a formula for P in terms of n.

b If this linear function was graphed, which variable would be represented on the horizontal axis?

c What is the vertical intercept of this function and what does it represent?

d How much will Jessica earn for delivering 28 pizzas in a day?

e If Jessica earned \$98 today, how many pizzas did she deliver?

6 This table shows the linear relationship between distances measured in miles and distances measured in kilometres.

Miles, M	15	25	30	45
Kilometres, K	24	40	48	72

a Graph this linear relationship on a number plane.

b Find the equation of the line.

c What can you say about the value of the vertical intercept? Why?

d What is the gradient and what does it represent?

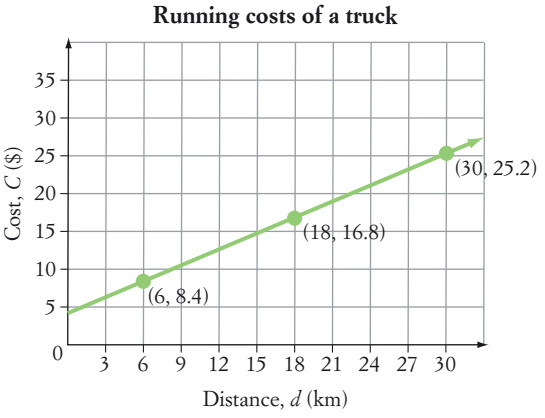
e Use your equation from part **b** to convert:

i 100 miles to kilometres **ii** 100 km to miles.

f Use your graph from part **a** to convert:

i 12 miles to kilometres **ii** 20 km to miles.

7 This graph shows the linear relationship between the distance, d km, travelled by a truck and the running costs, \$$C$, for the trip.



a What is the dependent variable?

b Find the gradient and the vertical intercept of this linear relationship.

c Write the formula for this linear relationship.

d If the length of a trip is extended by 5 km, by how much will the charge increase?

e Calculate the running costs for a trip of length:

i 20 km **ii** 0 km

f Calculate the distance travelled if the running costs were \$37.80.

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8 During summer, crickets chirp faster at night if the temperature is higher. There is a linear relationship between the temperature and a cricket's chirping rate, shown in the table below.

Temperature, T (°C)	12	15	19	22	28
Chirp rate, n (chirps/min)	72	96	128	152	200

a Is T the dependent variable or the independent variable?

b Find the linear function for n in terms of T.

c Graph the linear function you found in part **b**.

d If the temperature increases by 2°C, what happens to the crickets' chirp rate?

e Find the chirp rate of a cricket when the temperature is 26°C.

f At what temperature does a cricket chirp 144 times per minute?

g What is the vertical intercept of this function? Why doesn't the linear model work for this value?

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Year 12 7-04 Direct linear variation

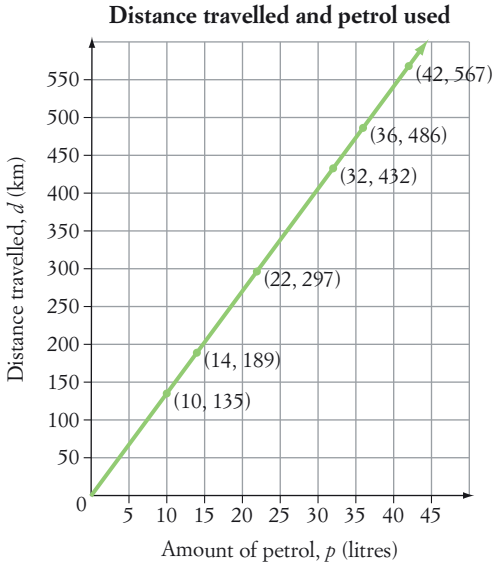
Ann-Marie noticed that, with her new car, she could drive a distance of 567 km on a full tank of petrol (42 litres). She also discovered the results shown in the table below.

Amount of petrol used, p (L)	10	14	22	32	36	42
Distrance travelled, d (km)	135	189	297	432	486	567

When she graphed these values, Anne-Marie found a linear relationship.

This relationship must be of the form $d = mp + b$, but the vertical intercept is 0 because, when $p = 0$, $d = 0$ (no petrol, no distance). So the linear function is $d = mp$.

To find the value of m, substitute any point from the table into the equation, for example (10, 135).



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$$d = mp$$
$$135 = m(10)$$
$$m = \frac{135}{10}$$
$$= 13.5$$

$\therefore$ The linear function is $d = 13.5p$.

Choose another point from the table to check that this formula is correct.

Because $d = 13.5p$, the distance, d, is found by multiplying the amount of petrol, p, by a constant amount, 13.5. This is an example of **direct linear variation**, and we can say that 'd varies directly as p', or that 'd is directly proportional to p'.

SUMMARY

If y varies as x, or y is directly proportional to x, then $y = kx$, where k is a constant. k is called the **constant of variation** or **constant of proportionality**.

A direct linear relationship exists between x and y. If x increases (or decreases), y increases (or decreases). If x is doubled (or halved), y is doubled (or halved).

EXAMPLE 8

The mass, M (in kilograms), of a metal varies directly as its volume, V (in cubic centimetres).

Volume, V (cm³)	0	48	60	80	116	140
Mass, M (kg)	0	14.4	18	24	34.8	42

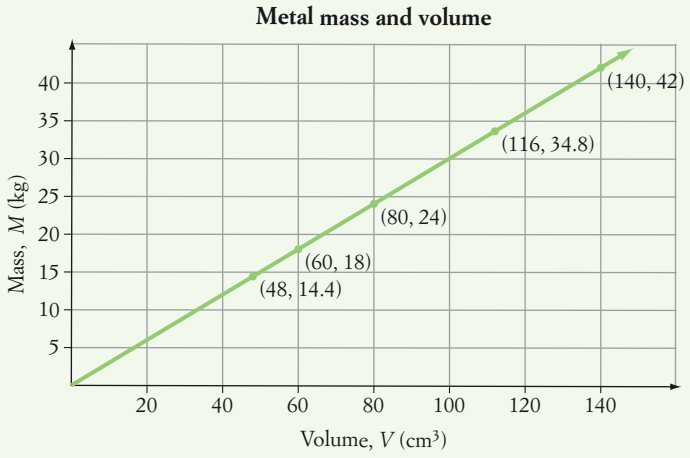
a Graph the relationship between M and V.

b Find the equation for M in terms of V.

c Find the mass of 212 cm³ of the metal.

Solution

a



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b $M = kV$

Substituting (60, 18) from the table to find k:

$18 = k(60)$

$k = \frac{18}{60}$

$= \frac{3}{10}$

$\therefore M = \frac{3V}{10}$ (or $M = 0.3V$)

Choose another point from the table to check that this formula is correct.

c Substitute $V = 212$ into the formula.

$M = \frac{3(212)}{10}$

$= 63.6$

The mass of the metal is 63.6 kg.

EXAMPLE 9

The stretch, S (in centimetres), of a spring varies as the mass, M (in kilograms), of the load pulling it. A load of 24 kg causes the spring to stretch 15 cm.

a Find the variation equation relating S to M.

b What is the stretch caused by a load of 8 kg?

c What is the mass of the load that will cause a stretch of 25 cm?

Solution

a $S = kM$

Substitute $M = 24$ and $S = 15$ to find k.

$15 = k(24)$

$k = \frac{15}{24}$

$= \frac{5}{8}$ (or 0.625)

$\therefore S = \frac{5M}{8}$ (or $S = 0.625M$)

b When $M = 8$:

$S = \frac{5(8)}{8}$

$= 5$

A stretch of 5 cm is caused by a load of 8 kg.

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c When $S = 25$:

$$25 = \frac{5M}{8}$$

$$5M = 200$$

$$M = 40$$

A 40 kg load will cause a stretch of 25 cm.

SUMMARY

To solve a linear variation problem:

Step 1: Identify the two variables (say x and y) and form a variation equation, $y = kx$.

Step 2: Substitute values for x and y to find k, the constant of variation.

Step 3: Rewrite $y = kx$ using the value of k.

Step 4: Substitute a value for x or y into $y = kx$ to solve the problem.

Exercise 7-04 Direct linear variation

Example 8

Example 9

1 Copy and complete this table after finding the value of k in $y = kx$.

x	1	4	6	10	11	15
y		14	21			52.5

2 The distance travelled by a bicycle varies directly with the number of revolutions made by the pedals.

a Form a variation equation and find the constant of variation, given that the bicycle travels 55 metres for 20 revolutions of the pedals.

b Calculate the distance travelled for 33 revolutions of the pedals.

c Calculate the number of revolutions of the pedals required for the bike to travel 99 m.

3 The increase in pressure experienced by a scuba diver is directly proportional to her depth under the water. The increase in pressure at 25 m is 147 kilopascals (kPa).

a Form a variation equation.

b What increase in pressure is experienced at 40 m?

c At what depth is the increase in pressure 833 kPa?

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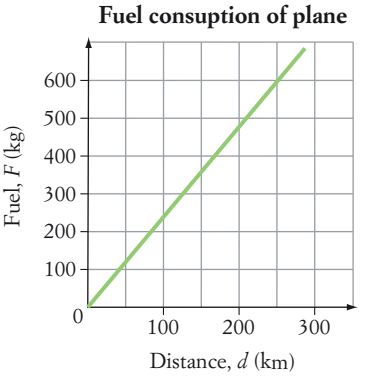
4 The graph on the right illustrates the fact that the mass of fuel an aeroplane requires varies directly with the distance it flies.

a Form a variation equation, using a point on the graph to find the constant of variation.

b Use the equation to calculate the amount of fuel required to fly 3250 km.



iStockphoto/thebroker

5 A speed expressed in km/h is directly proportional to the same speed expressed in miles/h.

Speed in miles/h, x	30	36	45	50	62.5	74
Speed in km/h, y	48	57.6	72	80	100	118.4

a Find the equation for this table of values.

b Convert 60 miles/h to km/h.

c Convert 120 km/h to miles/h.

6 For an object that is cooling, the drop in temperature varies directly with time. If the temperature drops 8°C in 5 minutes, which of the following is the amount of time it would take for the temperature to drop 10°C? Select **A**, **B**, **C** or **D**.

A 6.25 min **B** 7 min **C** 12.8 min **D** 16 min

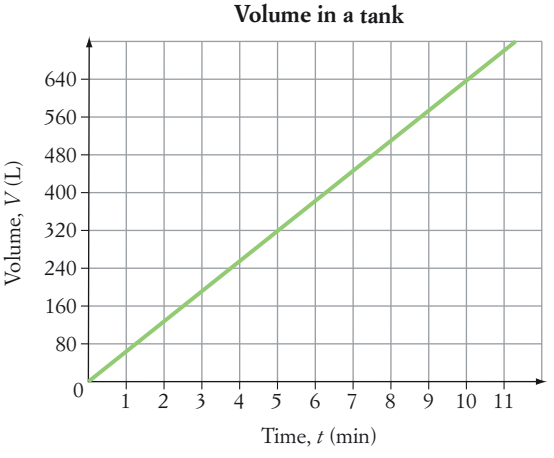
7 The volume, V litres, of water in a tank as it is filled is proportional to the time taken, t minutes, as shown on the graph on the right.

a Find the constant of variation.

b What does the constant you found in part **a** represent?

c Calculate the amount of water in the tank after 8 minutes.

d Calculate how long it takes to pump 960 L of water into the tank.



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8 The distance travelled by a marathon runner varies directly with time. If he covers 9.75 km in 45 minutes, calculate how long it will take him to run 26 km.

9 The weight of an astronaut on Mars is proportional to his weight on Earth. A 72 kg astronaut weighs 27.4 kg on Mars.

a Calculate how much a 60 kg astronaut weighs on Mars, correct to one decimal place.

b If an astronaut weighs 32 kg on Mars, calculate his weight on Earth, correct to one decimal place.

10 Nick noticed that, during his road trip, the amount of petrol used by his car varied with the amount of time he drove. It consumed 45 L of petrol in 4 hours. How long, to the nearest minute, will Nick's car take to consume 100 L of petrol?

11 The download time of a computer file is directly proportional to the size of the file. If a file of 1800 kilobytes requires 36 seconds to download, calculate:

a how long it will take to download a 3000 kilobyte file

b the size of a file that requires 80 seconds to download.

12 The compression of a car spring is proportional to the force applied. A force of 200 N (newtons) causes a compression of 1.6 cm. Which of the following amounts of compression is caused by a 5000 N force? Select **A**, **B**, **C** or **D**.

A 25 cm **B** 40 cm **C** 62.5 cm **D** 200 cm

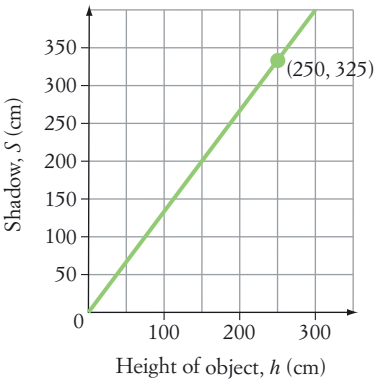
13 At 3:30 p.m. one day, the shadows of objects of different heights were measured, then graphed.

a Find the equation of the line.

b Calculate the length of the shadow of a lamppost of height 244 cm.

c What is the height of a letterbox, correct to the nearest centimetre, if its shadow is 120 cm long?

Shadows of objects of different heights



14 When an object is dropped under gravity, its speed varies with time. If its speed after 5 seconds is 49 m/s, calculate:

a its speed after 12 seconds

b the time taken to reach a speed of 175 m/s (to the nearest second).

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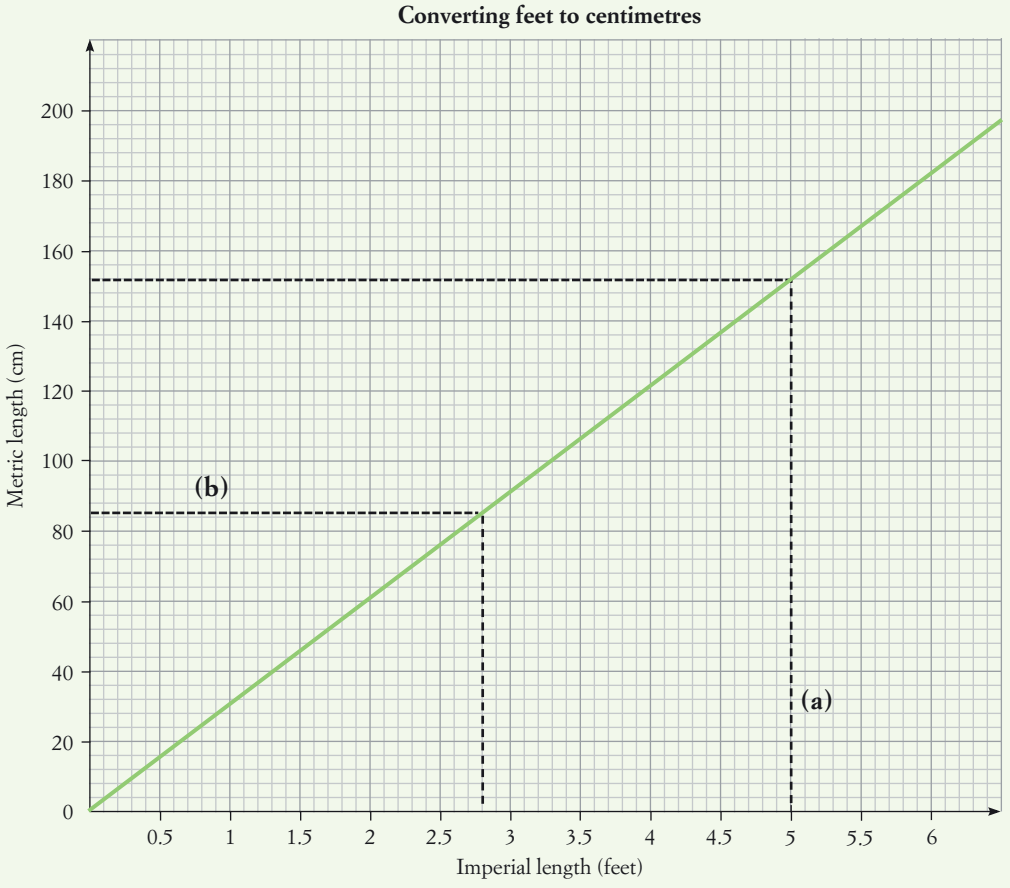
7-05 Conversion graphs

Currency conversion graph

A **conversion graph** is used to convert between different units, such as between metric and imperial units, or between currencies in foreign currency exchange.

EXAMPLE 10

The conversion graph below converts between feet (an imperial unit) and centimetres (a metric unit).



a Kylie is 5 feet tall. What is her height in centimetres?

b A doorway is 85 cm wide. Convert this to feet, correct to one decimal place.

c The gradient of the line in the conversion graph is 30.5. What does this value represent?

Solution

a Reading from the graph: 5 feet = 152 cm

b Reading from the graph: 85 cm = 2.8 feet

c The gradient represents the rate of change of centimetres per foot: 30.5 cm = 1 foot

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Exercise 7-05 Conversion graphs

Example **10**

1 Use the conversion graph from Example **10** to answer the following questions.

a A bookshelf is 3.5 feet tall. What is its height in centimetres?

b Michael is 180 cm tall. Convert this height to feet, correct to one decimal place.

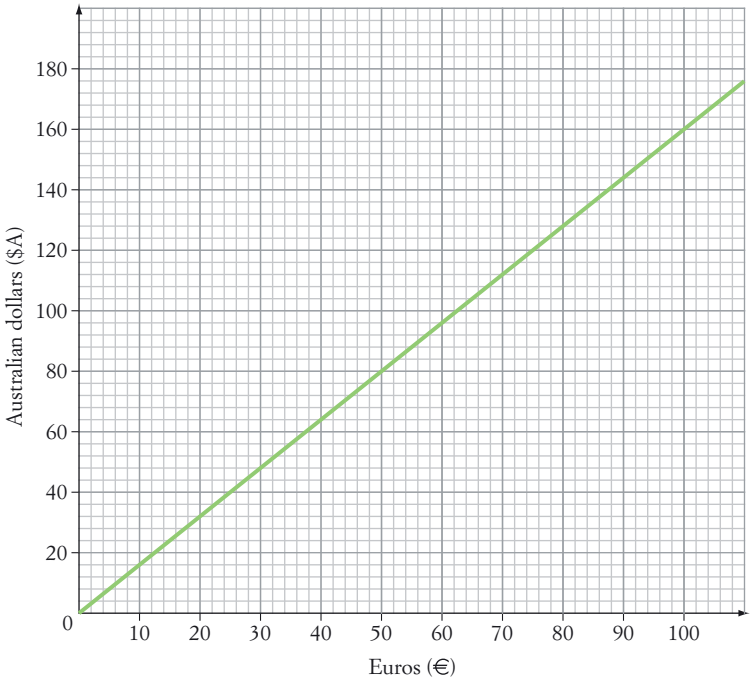
c Convert each of the following lengths to centimetres.

i 2 feet **ii** 2.5 feet **iii** 6 feet

d Convert your own height to feet and inches, given that 1 foot = 12 inches.

2 This currency conversion graph converts between the euro (€) and the Australian dollar ($A).

Converting between Australian dollars and Euros



Convert each Euro price to Australian dollars.

a	a watch	€88	**b**	theme park entry	€20
c	a mobile phone	€76	**d**	a camera	€52
e	a city tour	€25	**f**	car hire per day	€42

3 The conversion graph from question **2** can also be used to convert small amounts. For example, from the graph: €46 = A$88. So it is also true that €0.46 = A$0.88.
Use the graph to help you convert each of the following prices to Australian dollars.

a	a can of drink	€0.92	**b**	posting a letter	€0.40
c	30 min of Internet use	€0.66	**d**	a newspaper	€0.75

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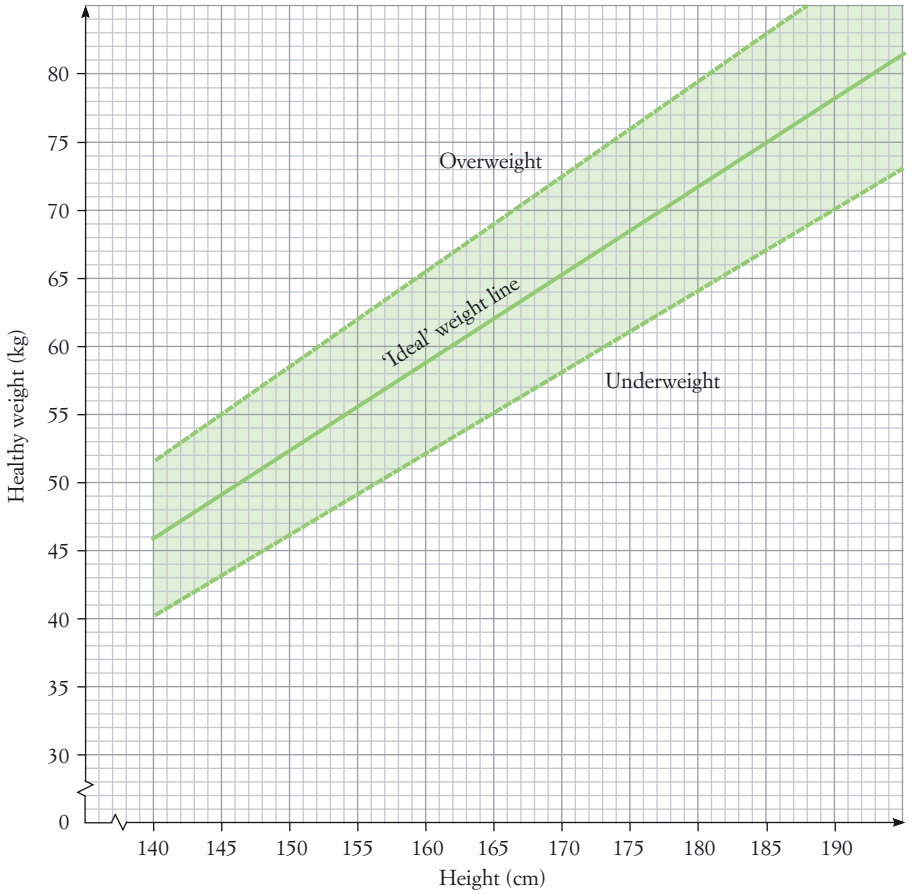
4 Use the conversion graph from question **2** to convert each of the following Australian prices to euros.

a	a movie DVD	A$24	**b**	a cordless phone	A$76
c	a soccer ball	A$32	**d**	a computer desk	A$130
e	an exercise bike	A$108	**f**	fish and chips	A$5.20

5 Calculate the gradient of the currency conversion graph in question **2**, correct to two decimal places. What does this value represent?

6 This height–weight graph shows the healthy weight range (shaded) for Australian adults of different heights.

Height–weight graph



a Find the 'ideal' weight of a person whose height is:

i 180 cm **ii** 155 cm **iii** 192 cm.

b Find the height of a person with an 'ideal' weight of:

i 75 kg **ii** 58 kg **iii** 80 kg.

c Calculate the gradient of the 'ideal' weight line. What rate does this gradient value represent?

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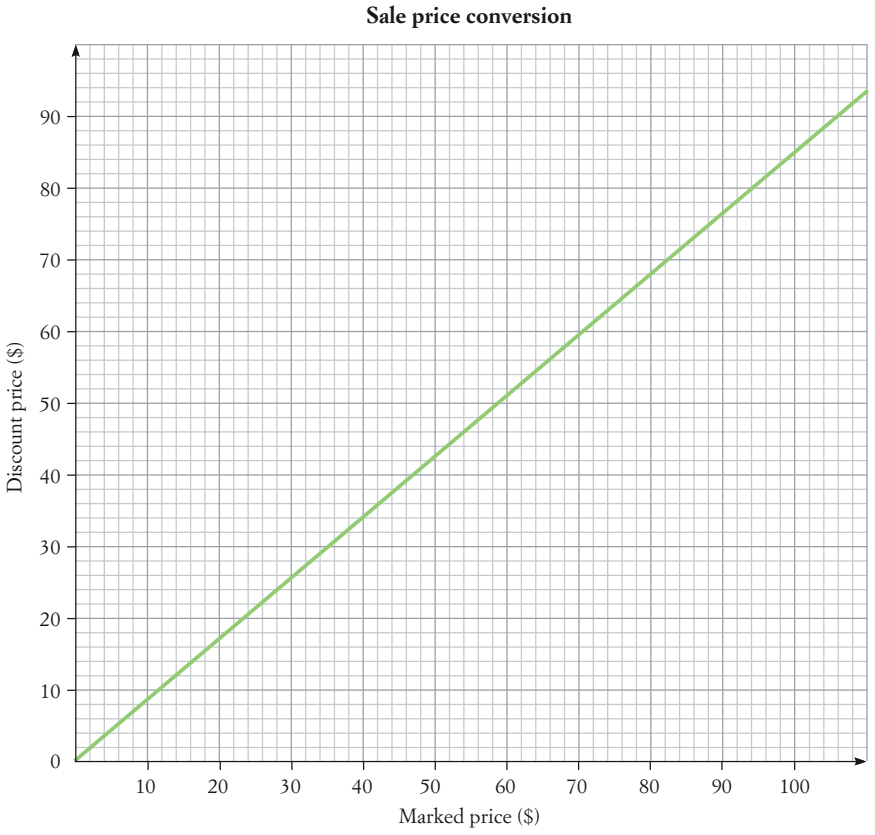
7 This graph converts marked prices to discount prices for a sale at Angry Andy's Bargain Basement.



a Find the discount price of an item with a marked price of:

i \$60 **ii** \$28 **iii** \$82

b Find the marked price of an item with a discount price of:

i \$25 **ii** \$80 **iii** \$54

c Calculate the gradient of this graph to two decimal places. What does the gradient value represent?

d Which of the following is the percentage discount given at Angry Andy's Bargain Basement? Select **A**, **B**, **C** or **D**.

A 5% **B** 10% **C** 15% **D** 85%

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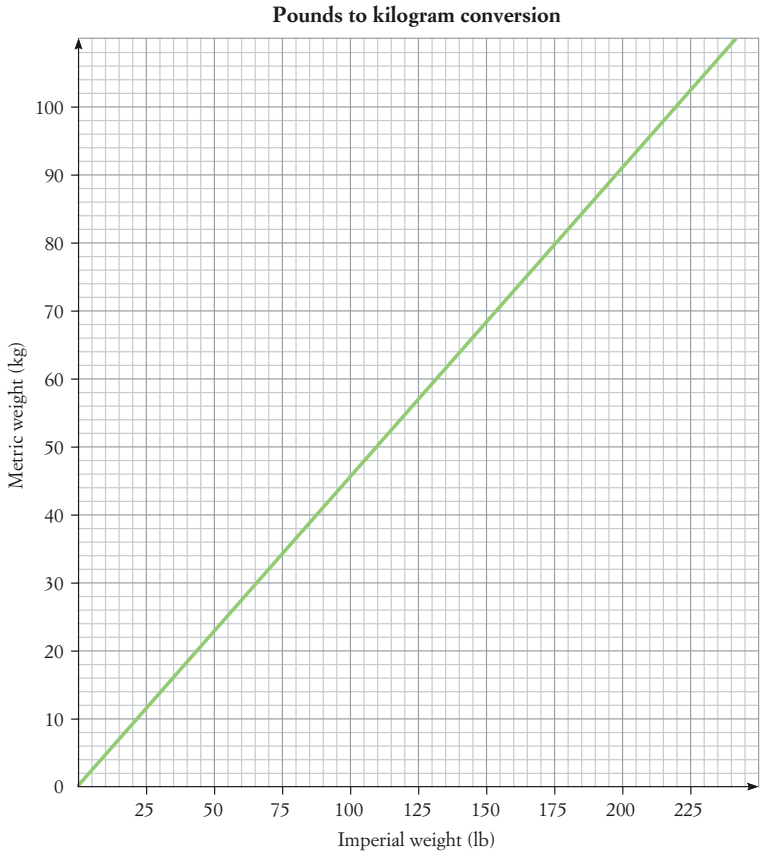
8 The graph below converts between pounds (an imperial unit of mass, with abbreviation 'lb') to kilograms (the metric unit).



a Convert each of these masses to kilograms.

i 90 lb **ii** 135 lb **iii** 160 lb

b Convert each of these masses to pounds.

i 40 kg **ii** 64 kg **iii** 90 kg

c What is the conversion rate in kg/lb? Write your answer correct to two decimal places.

7-06 Intersection of lines

Intersection of lines

Simultaneous equations

A pair of equations, such as $y = x + 2$ and $y = 2x - 5$, can be solved together to find the values of x and y that satisfy *both* equations. Because they are solved at the same time, they are called **simultaneous equations**.

If simultaneous equations are graphed on the number plane, their solution is the coordinates of the point where their graphs intersect.

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EXAMPLE 11

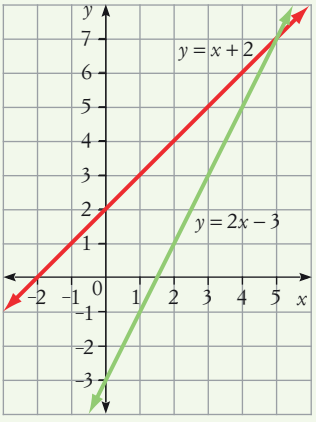
a Graph the equations $y = x + 2$ and $y = 2x - 3$ on the same axes.

b Find the coordinates of the point of intersection of the two lines.

c Write the solution of the simultaneous equations $y = x + 2$ and $y = 2x - 3$.

Solution

a



b The point of intersection is (5, 7).

c The solution to the simultaneous equations is: $x = 5$, $y = 7$.

Note: We can check this solution by calculating y when $x = 5$.

For $y = x + 2$:
$y = 5 + 2$
$= 7$ ✓

For $y = 2x - 3$:
$y = 2(5) - 3$
$= 7$ ✓

Applying simultaneous equations

Simultaneous equations can be used to solve problems involving linear modelling.

EXAMPLE 12

Jeanette is comparing the plans of two telephone companies providing national calls. OzExpress One has a monthly access fee of \$14 and charges \$1.30 per call, while Optel Easy has a monthly access fee of \$25 and charges \$0.80 per call. The costs, \$$C$, of these plans can be represented by the following formulas, where n is the number of calls made per month.

OzExpress One: $C = 1.3n + 14$ Optel Easy: $C = 0.8n + 25$

a Graph both formulas on the same axes, for values of n from 0 to 50.

b If Jeanette makes around 18 national calls per month, which is the cheaper plan for her?

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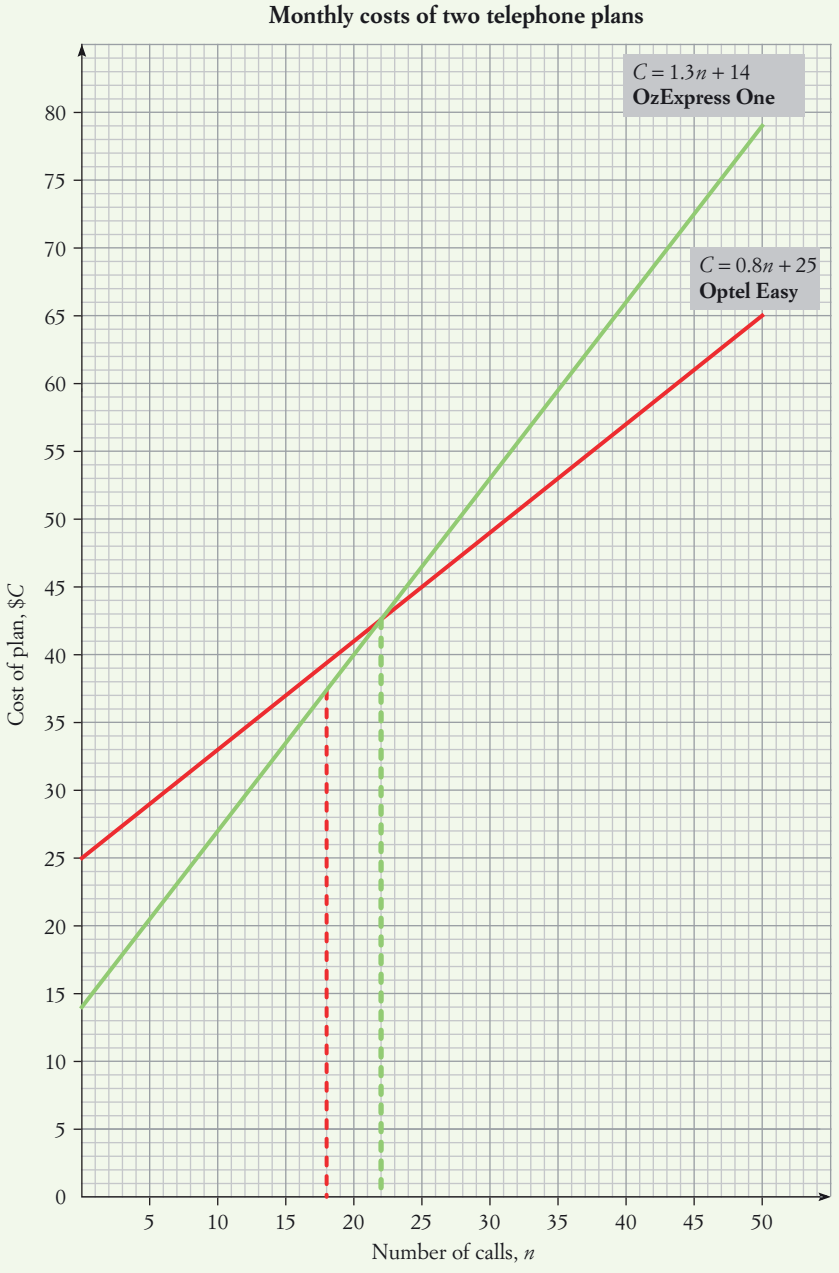
c For what number of calls per month do both plans charge the same cost?

d To what type of caller would you recommend:

i the OzExpress One plan? **ii** the Optel Easy plan?

Solution

a



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b When $n = 18$, OzExpress One Plan is cheaper (marked by the red dotted line on the graph).

c 22 calls ($n = 22$ where the two lines intersect, marked by the green dotted line on the graph).

We can check this by substituting $n = 22$ into both formulas.
OzExpress One: $C = 1.3(22) + 14 = 42.6$ Cost = \$42.60
Optel Easy: $C = 0.8(22) + 25 = 42.6$ Cost = \$42.60

d **i** OzExpress One is cheaper for callers who make fewer than 22 calls per month.

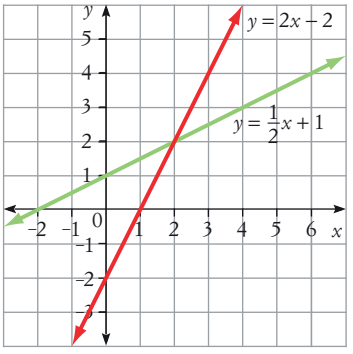
ii Optel Easy is cheaper for callers who make more than 22 calls per month.

Exercise 7-06 Intersection of lines

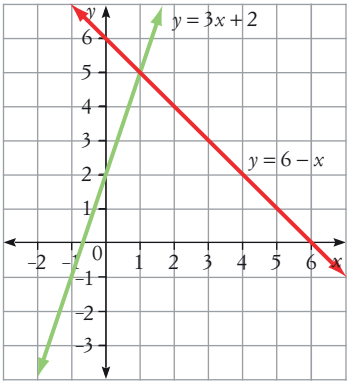
Example 11

1 Solve each pair of simultaneous equations given their graphs below.

a $y = \frac{1}{2}x + 1$
$y = 2x - 2$



b $y = 3x + 2$
$y = 6 - x$



2 **a** Graph the equations $y = 2x + 2$ and $y = x + 5$ on the same axes.

b Use the graphs you constructed in part **a** to solve the simultaneous equations $y = 2x + 2$ and $y = x + 5$.

Example 12

3 Megan and Jane both sell kitchenware and earn commissions. Megan's commission is \$240 plus 5% of her sales, while Jane's commission is 20% of her sales (with no retainer). These rates can be expressed by the formulas below.

Megan's commission: $C = 0.05x + 240$

Jane's commission: $C = 0.2x$

where \$$x$ is the total value of sales and \$$C$ is the commission.

a Graph both commission formulas on the same axes, for values of x from \$0 to \$2400.

b If each person sells \$1200 worth of kitchenware, who earns more?

c For what value of sales does Megan earn more than Jane?

d For what value of sales does Megan earn the same as Jane?

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4 The cost, $\$C$, of making n yo-yos is given by the formula $C = 3n + 90$, while the revenue, $\$R$, from selling n yo-yos is given by the formula $R = 5n$.

Revenue means income or money received.

a Graph both formulas on the same axes, for values of n from 0 to 50.

b What is the cost of making 20 yo-yos?

c What is the revenue from selling 20 yo-yos?

d What is the loss from making 20 yo-yos?

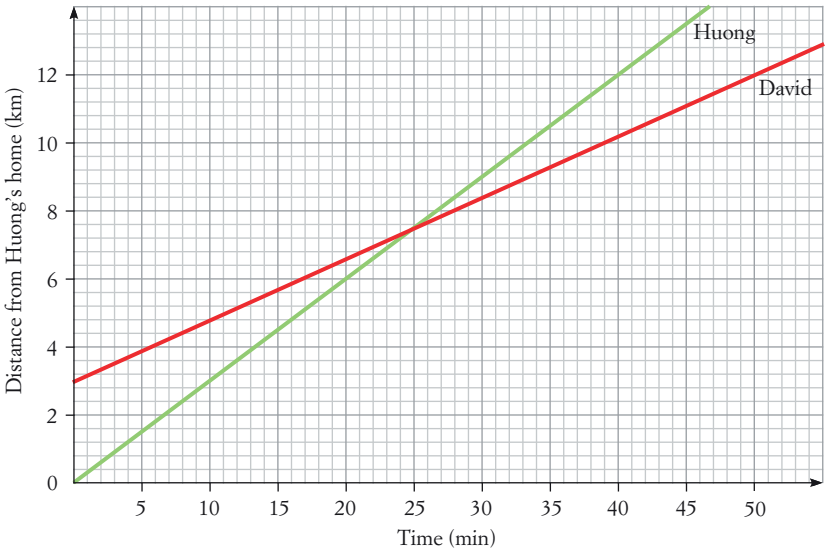
e How much does one yo-yo sell for?

f For what value of n does the revenue equal the cost?

g The value of n in part **f** is called the break-even point. Why would it be unwise to sell fewer than that number of yo-yos?

5 David jogged to the park while Huong rode her bike. David was already 3 km ahead of Huong when she started from home, and jogged at 10.8 km/h, while Huong rode at a speed of 18 km/h. The travel graph below shows their journeys, with the distances from Huong's home shown.

David and Huong's journey



a Did Huong catch up to David? If so, when?

b If the park is 12 km from Huong's home, who got there first and by how many minutes?

6 Steve is investigating two Internet plans. BigNet has a \$15 monthly access fee and a usage charge of \$1.20 per hour, while QuickByte has a \$30 monthly access fee and a usage charge of \$0.45 per hour.

a Graph the following formulas on the same axes, for values of t from 0 to 60, where t represents the monthly number of hours of Internet usage and $\$C$ represents the cost.

BigNet: $C = 1.2t + 15$

QuickByte: $C = 0.45t + 30$

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b If Steve uses the Internet for an average of 30 hours per month, which plan is better for him?

c Calculate the cost of 30 hours of Internet use for each plan.

d For what number of hours is the cost the same for both plans?

e To what type of Internet user would you recommend:

i the BigNet plan? **ii** the QuickByte plan?

f Which plan would you choose for yourself? Why?

7 Geoff wants to hire a car over the three days of a long weekend. There are two rental companies he can choose from.

Shifty: \$45 per day plus \$1.85 per kilometre

Megahertz: \$80 per day plus \$1.55 per kilometre

For three days, these rates can be represented by the following formulas.

Shifty: $C = 1.85d + 135$

Megahertz: $C = 1.55d + 240$

where d is the distance travelled, in kilometres, and C is the cost in dollars.

a Graph both formulas for values of d from 0 km to 500 km.

b Where do the two graphs intersect? What does this represent?

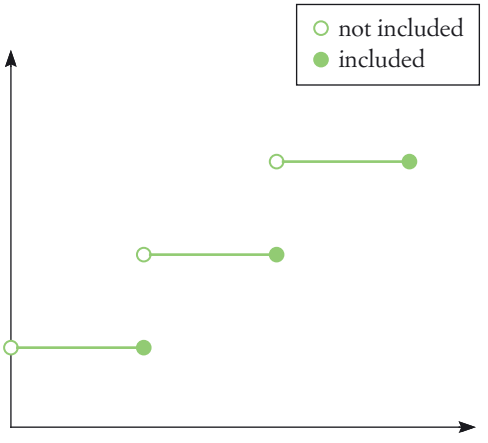
c If Geoff plans to drive an average of 100 km each day of the long weekend, which plan would be better for him and what would be his cost?

7-07 Stepwise linear functions

Stepwise linear functions

A **stepwise linear function** is a function that does not change gradually at a constant rate but in distinct 'jumps'. It is represented by a **step graph**, which is made up of broken 'flat steps'.

The edges of each step of a step graph are labelled with circles. A blank circle means that the value is not included, while a shaded circle means that the value is included.



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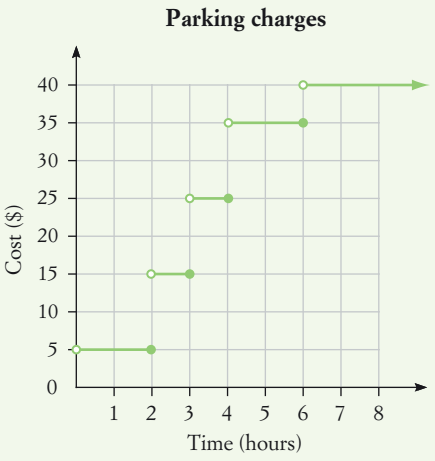
EXAMPLE 13

This step graph shows the daily parking charges at a car park.



a Find the charge for parking for:

i 3 hours **ii** 55 minutes

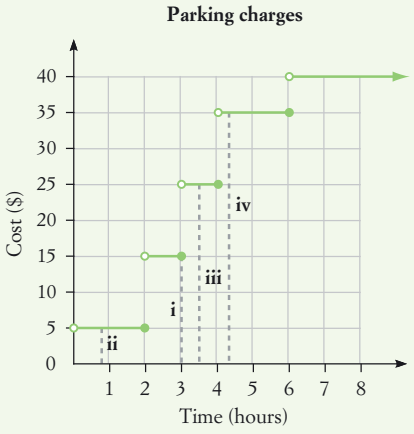
iii $3\frac{1}{2}$ hours **iv** 4 hours 1 minute.

b For what range of times can a driver park for $25?

c What does the arrow on the $40 step mean?

Solution

a



Reading from the graph as indicated:

i 3 hours costs $15

ii 55 minutes costs $5

iii $3\frac{1}{2}$ hours costs $25

iv 4 hours 1 minute costs $35.

b More than 3 hours but less than or equal to 4 hours.

c After 6 hours, the parking charge remains constant at $40. The maximum daily charge is $40.

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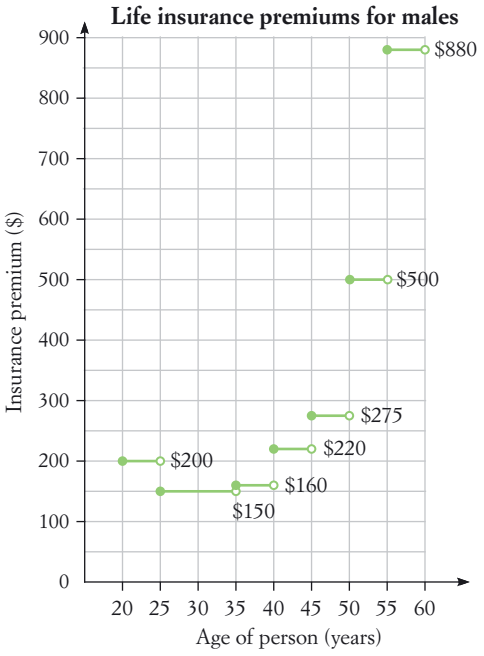
Exercise 7-07 Stepwise linear functions

Example 13

1 This step graph shows the annual premiums (costs) of life insurance policies for males of different ages.



a What is life insurance?

b What is the annual premium for a man aged:

i 35 years?

ii 48 years?

iii 22 years?

c Why do you think the cost of life insurance is lowest for men aged 25 to 34?

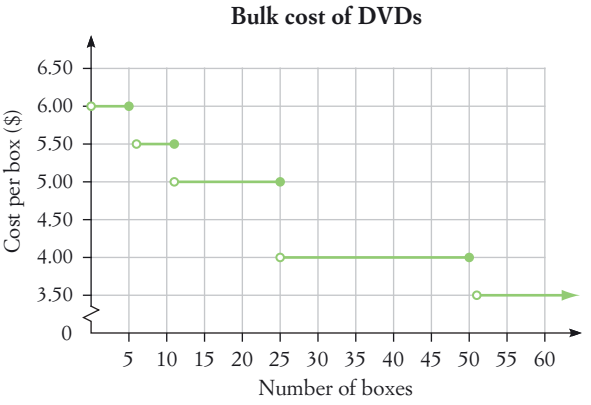
d As the age of a man increases, what generally happens to the insurance premium? Why?

e For what age group is the annual premium $200?

f What is the age of the youngest man who can pay a premium of:

i $220? **ii** $500?

2 This step graph shows the cost per box of blank DVDs when they are bought in bulk.



a What happens to the cost per box as the number of boxes increases? Why?

b Find the cost per box when purchasing:

i 40 boxes

ii 8 boxes

iii 50 boxes

iv 1 box.

c What is the maximum number of boxes that can be bought for $4 each?

d What does the arrow on the $3.50 step mean?

3 The size of a speeding fine imposed on a driver depends upon the number of km/h by which the speed limit was exceeded, as shown in the table on the right. Graph this data using a step graph.

Excess over speed limit (km/h)	Fine ($)
10 or less	84
Over 10 and up to 20	197
Over 20 and up to 30	338
Over 30 and up to 45	647
Over 45	1744

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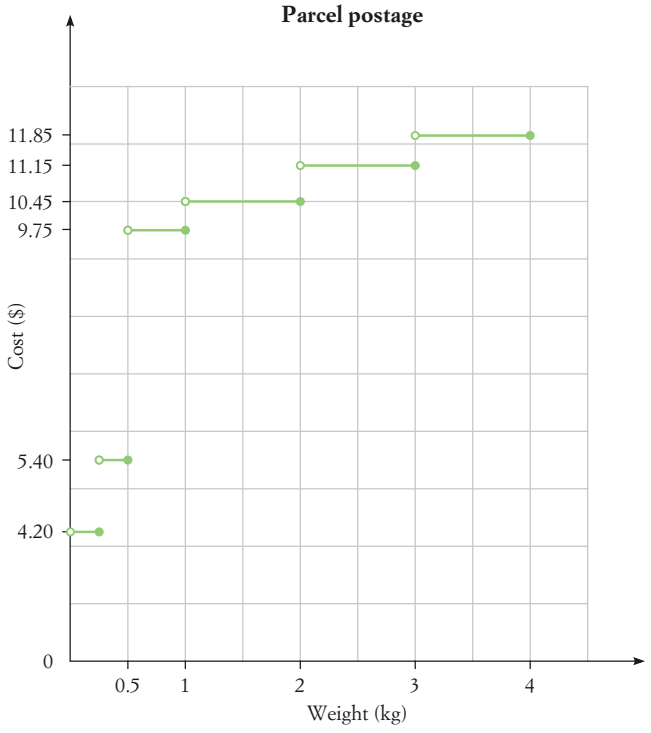
4 This step graph shows the charges for posting parcels using surface mail, from Sydney to Melbourne, according to the weight of the parcel.



a Find the cost of posting a parcel that has a mass of:

i 3.5 kg

ii 0.5 kg

iii 2.3 kg.

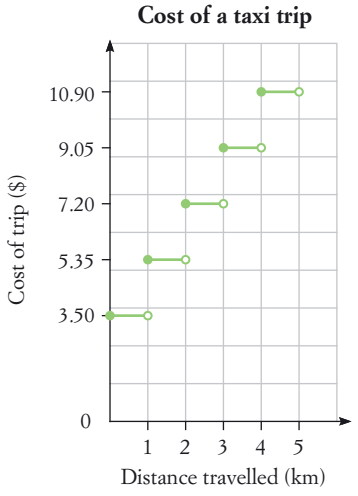
b Describe in your own words the relationship between mass and cost of postage.

c For what range of masses can we post a parcel for $5.40?

d Find the mass of the heaviest parcel that can be posted for:

i $9.75 **ii** $4.20 **iii** $11.15.

5 The cost of a trip in a taxi depends on the distance travelled, as shown by this step graph.



a Find the cost of travelling a distance of:

i 4 km **ii** 2.7 km **iii** 1.1 km.

b For what range of trip lengths is the cost $9.05?

c What is the 'flag fall' (the charge for starting a trip)?

d Which of the following is the amount charged, after flag fall, for every additional kilometre? Select **A**, **B**, **C** or **D**.

A $1.85 **B** $2.68

C $3.50 **D** $3.60

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Sample HSC problem

Mr and Mrs Dillon give their children weekly pocket money according to a function based on the age of the child, as shown in the table below.

Age, n years	7	9	10	13	15	18
Pocket money, \$$P$	4	12	16	28	36	48

a Find the formula for P in terms of n.

b What is the gradient of this function and what does it stand for?

c What is the pocket money for a child aged 16?

d What is the age of the child who receives \$20 per week?

e According to Mr and Mrs Dillon, this linear model holds true only for values of n from 7 to 18. Why do you think the formula is not valid for values of n:

i below 7? **ii** above 18?

Study tip

Useful exam tips

- Make yourself familiar with the format of the exam: the number of sections and questions, the types of questions (for example, multiple-choice, and short answer), the time allowed, and the number of marks allocated.
- Don't worry if you are nervous. This is normal and will help you perform better. However, being too casual or too anxious can harm your performance.
- Calculate the average amount of time you should spend on each question or section.
- Spend the reading time of the exam browsing through the paper to see what work lies ahead of you.
- Easier questions are usually at the beginning, with harder ones at the end. Do an easy question first to boost your confidence. It will also save time.
- Put a mark next to the harder questions and allow more time for working on them. Leave them if you get stuck and come back to them later.
- Show all working. Even if you get the wrong answer, you may be awarded some marks for correct working.
- Attempt every question. It is better to do most of every question and score some marks for them than to ignore some questions completely and score nothing for them.
- Don't leave multiple-choice questions unanswered. Even if you guess, there is a chance of being correct. Some students like to leave multiple-choice questions till last so that, if they run out of time, they can make quick guesses. However, some multiple-choice questions can be quite difficult.

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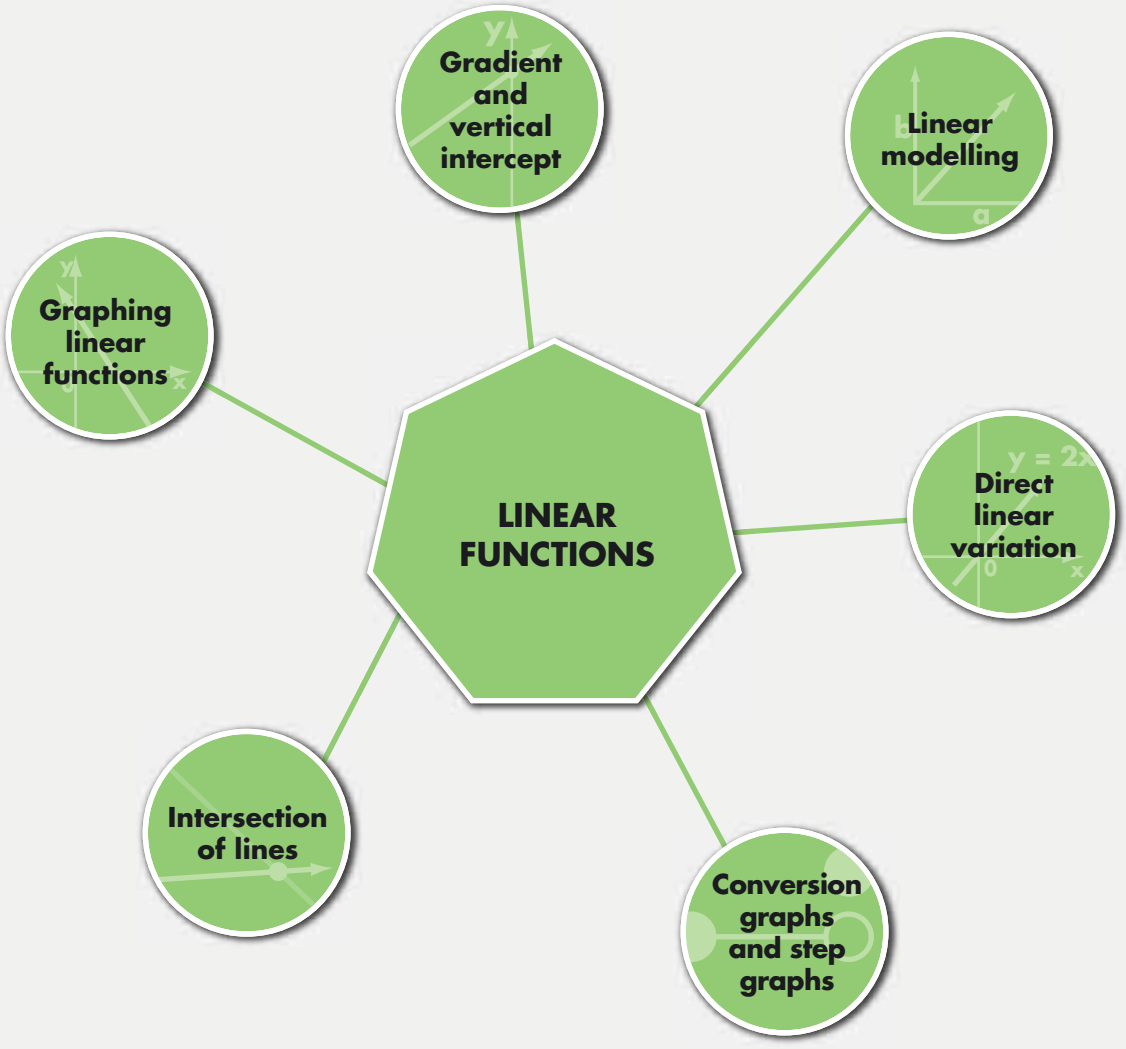
CHAPTER REVIEW

Topic overview

This chapter, Linear functions, introduced the concept of the linear function and its graph and applications. Learn the meanings of the gradient and vertical intercept of a linear function and practise solving problems involving linear modelling, variation and intersection of lines (simultaneous equations). Revise how to read conversion graphs and step graphs.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map is shown below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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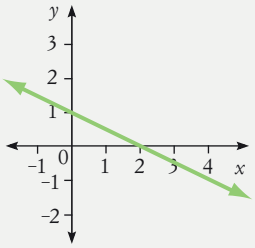
Revision

1 Graph each of these linear functions on a number plane. Exercise 7-01

a $y = -2x + 1$ **b** $y = 4x$

2 Find the equation of each of these lines. Exercise 7-01

a



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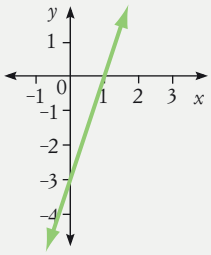
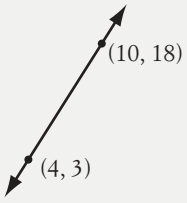


3 Find the gradient of each of these lines. Exercise 7-02

a



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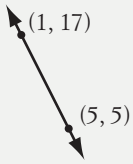

4 Find the gradient of the function for each of these tables of values. Exercise 7-02

a

x	0	4	6	12
y	12	14	15	18

b

x	2	5	9	11
y	42	30	14	6

5 The length, L inches, of a shoe has a linear relationship with the size, S, of the shoe. Exercise 7-03

Shoe size, S	2	5	7	8	12
Length, L (inches)	9	10	$10\frac{2}{3}$	11	$12\frac{1}{3}$

a Which is the independent variable?

b Graph this linear relationship.

c What is the gradient, and what does this represent?

d How long is a shoe of size 0? How is this shown on the graph?

e What is the formula for L in terms of S?

f What is the length of a shoe of size $7\frac{1}{2}$?

g What size is a shoe of length 13 inches?

6 The distance travelled by a car is directly proportional to the number of rotations of its tyres. If 950 metres are travelled after 540 rotations, calculate how much distance, to the nearest kilometre, is covered after 10 000 rotations. Exercise 7-04

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Exercise 7-05

7 This currency conversion graph converts between Thai baht (the currency of Thailand) and Australian dollars (A$).

a Convert each of these Thai prices to Australian dollars (to the nearest dollar).

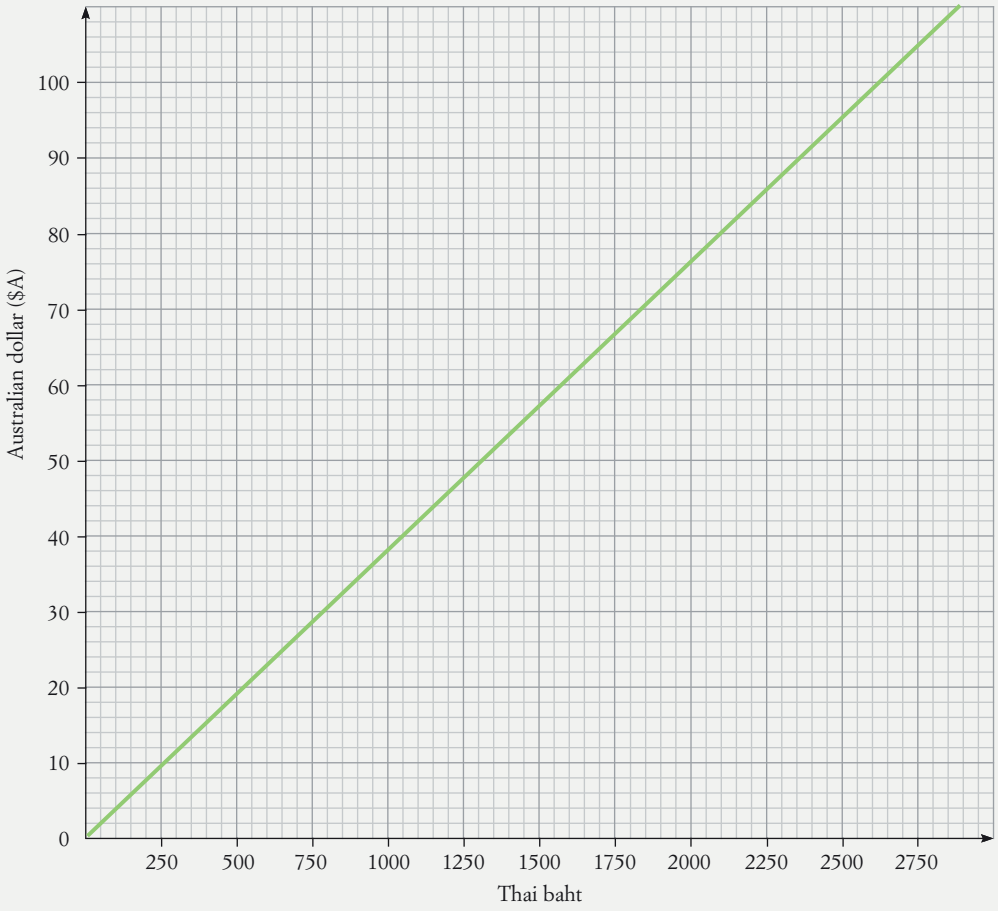
i 800 baht **ii** 2450 baht

b Convert each of these Australian prices to Thai baht (to the nearest 50 baht).

i A$65 **ii** A$82

c Calculate the gradient of this graph correct to two decimal places. What does this value represent?

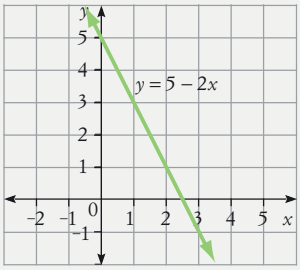
Converting Thai baht to Australian dollars



Exercise 7-06

8 **a** Copy the graph and graph the equation $y = x - 1$ on the same axes.

b Use your graph from part **a** to solve the simultaneous equations $y = 5 - 2x$ and $y = x - 1$.



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Exercise 7-06

9 Salma is organising an end-of-year outdoor lunch for her staff. The cost of hiring a tent is \$133 and the food is \$7.50 per guest. To cover these costs, Salma is charging each guest \$11. The cost and revenue functions may be represented by the following linear functions:

Cost $C = 7.5n + 133$

Revenue $R = 11n$

where C and R are in dollars and n represents the number of guests.

a Graph both the C and R functions on the same axes, for values of n from 0 to 50.

b Use the graph to find the cost and revenue for running the lunch for 32 guests.

c Why would it be unwise for Salma to run a lunch for 32 guests?

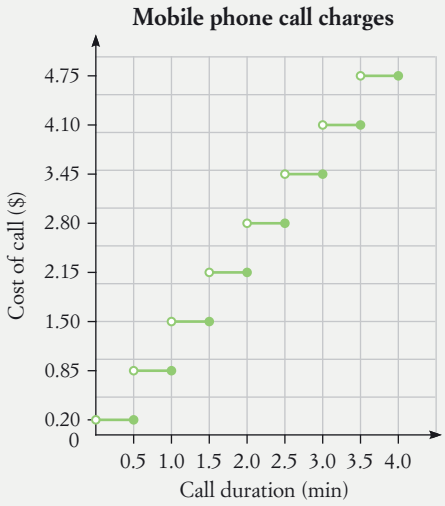
d For what value of n does the revenue from the lunch equal the cost?

e Why is the value you found in part **d** called the 'break-even point'?

f What is the profit made when the lunch caters for 50 guests?

Exercise 7-07

10 This step graph shows the cost of mobile phone calls, for call durations from 0 minutes to 4 minutes.



a What is the cost of a mobile phone call that lasts:

i 3 min 10 s?

ii 15 s?

iii 1 min 59 s?

b What is the longest call that can be made for \$2.15?

c For which of the following ranges of call durations is the cost \$4.10?
Select **A**, **B**, **C** or **D**.

A Between 3 and 3.5 minutes

B Over 3 minutes and up to and including 3.5 minutes

C 3 minutes and up to and including 3.5 minutes

D 3 minutes or more but less than 3.5 minutes

d By what rate does the call cost increase?

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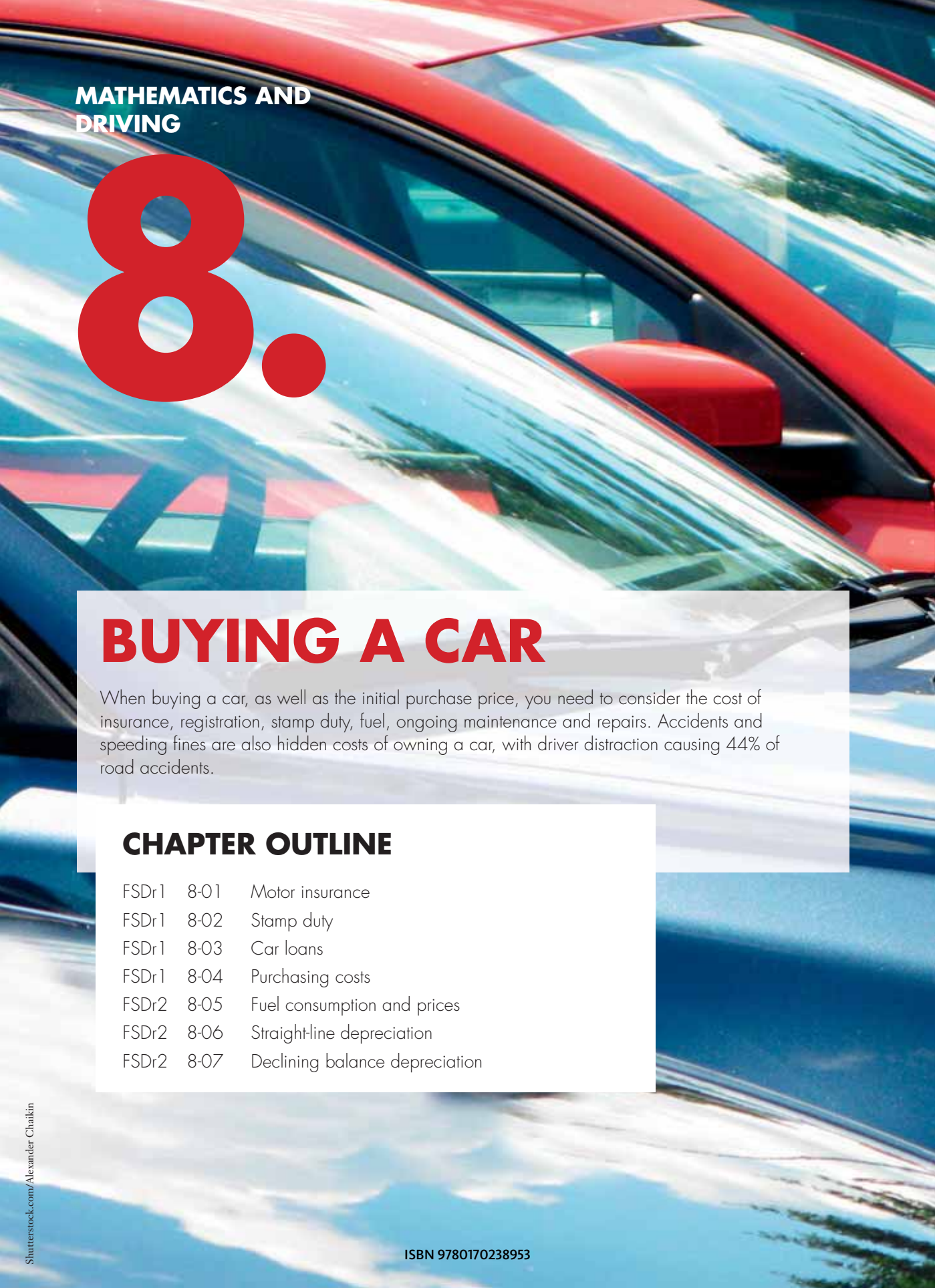

MATHEMATICS AND DRIVING

8.

BUYING A CAR

When buying a car, as well as the initial purchase price, you need to consider the cost of insurance, registration, stamp duty, fuel, ongoing maintenance and repairs. Accidents and speeding fines are also hidden costs of owning a car, with driver distraction causing 44% of road accidents.

CHAPTER OUTLINE

FSDr1	8-01	Motor insurance
FSDr1	8-02	Stamp duty
FSDr1	8-03	Car loans
FSDr1	8-04	Purchasing costs
FSDr2	8-05	Fuel consumption and prices
FSDr2	8-06	Straight-line depreciation
FSDr2	8-07	Declining balance depreciation

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IN THIS CHAPTER YOU WILL:

- describe the different types of motor insurance available, including compulsory and non-compulsory third-party insurance, and comprehensive insurance
- compare regional theft statistics and the related cost of insurance
- analyse theft and accident statistics in relation to insurance costs
- calculate the cost of stamp duty payable using current rates
- create a line graph showing the stamp duty payable on vehicles of various prices
- determine the monthly repayments on a reducing balance personal loan using tables or an online calculator
- compare the sale price of a car and the total amount repaid over the full period of a loan
- compare the cost of purchase of different motor vehicles (cars and motorcycles only), including finance, transfer of registration, and insurance
- identify fuel consumption measures as rates
- compare fuel consumption statistics for various vehicles
- calculate the amount of fuel used on a trip
- compare the amount of fuel needed and the costs associated for various sizes, makes and models of vehicles, over various distances
- collect and present data on the price of fuel over time to identify trends
- calculate the percentage decrease in the price of a new vehicle after one year
- calculate the depreciation of a vehicle using the straight-line method and the declining balance method
- create a depreciation graph based on the straight-line method of depreciation (graphs to be produced from formulas and tables)
- use prepared graphs and tables of straight-line depreciation and declining balance depreciation to solve problems

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TERMINOLOGY

comprehensive insurance	CTP insurance	declining balance method
depreciation	flat rate loan	green slip
period	premium	reducing balance loan
salvage value	stamp duty	straight-line method
third party insurance	trend	vehicle registration

Assignment 8

SkillCheck

1 Copy and complete.

a \$148 per month = ... per year

b \$350 per fortnight = ... per year

c \$276 per week = ... per year

d \$2754 per year =

i ..... per week **ii** ..... per fortnight **iii** ..... per month

e \$182 per month = ... per week [Hint: find amount per year first]

2 A loan repayment is \$128 per fortnight. To the nearest cent, how much is this:

a per week? **b** per year? **c** per month?

3 Find each amount:

a 3% of \$25 680 **b** 2.5% of \$64 320 **c** 11% of \$12 700

d 4% of \$2564 **e** 6.5% of \$68 530

4 If $y = mx + b$ find:

a y if $m = 4$, $b = 6$ and $x = 15$

b y if $m = 30$, $b = -20$ and $x = 8$

c x if $m = 150$, $b = 400$ and $y = 0$

5 If $S = V - Dn$ find:

a S if $V = 7500$, $D = 4300$, $n = 6$ **b** V if $S = 0$, $D = 5200$, $n = 8$

c n if $S = 12\ 000$, $V = 6000$, $D = 3800$

6 If a car is stolen in Australia every 10 minutes, how many cars are stolen:

a per day? **b** per week? **c** per month?

7 A Range Rover 4WD has a 150 L fuel tank. What would it cost, to the nearest cent, to fill the car with fuel at \$1.53 per litre?

8 If the buyer of a \$65 000 car received a 10% discount for paying cash then a further 3% discount in the sale, how much did he pay for the car?

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8-01 Motor Insurance

Investigating car insurance

Insurance is a financial scheme to protect you in the event of an accident or theft. In return for paying a regular fee, called the **premium**, the insurance company agrees to pay you a large amount to cover the costs of repair or replacement if the unfortunate accident occurs.

Types of insurance

There are three main types of motor insurance:

Compulsory Third Party (CTP) personal insurance protects vehicle owners and drivers from legal liability for personal injury or death to any other person ('third party') including drivers, passengers, cyclists and pedestrians. It is also called 'green slip' insurance because of the colour of the insurance document and it must be obtained before a vehicle can be registered.

Third Party Property insurance is optional and covers damage to other vehicles and property in an accident where you are at fault. It does not cover damage to your vehicle.

Comprehensive insurance is optional and covers damage to all vehicles and property, including your own. It also protects against theft, attempted theft, accidental loss, fire, vandalism or hail damage to your vehicle.

EXAMPLE 1

Mia bought a second-hand utility ('ute') for her farm. Her comprehensive insurance was \$631.08 upfront or \$61.75 per month. How much extra would Mia pay per year if she chose to pay her insurance per month?

Solution

Cost when paid upfront = \$631.08

Cost when paid monthly = \$61.75 × 12

= \$741.00

Difference = \$741.00 – \$631.08

= \$109.92

Mia pays an extra \$109.92 if she pays per month.

Alamy/Bill Bachman

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Insurance premiums and discounts

A motor insurance **premium** depends on factors such as vehicle type, driver age and driving record, and home suburb. Some insurance companies give a **loyalty discount** if you have been with them for several years, a **multi-policy discount** for having more than one type of insurance policy with them, and a **no claim discount** if you have not made a claim for several years.

If you have an accident and it is your fault, an assessor will view the damage and assess the costs involved. You will be asked to complete a **claim form** and pay an **excess**, that is, some money towards the costs before the company pays the rest. Some customers agree to pay a larger excess in return for a cheaper policy.

EXAMPLE 2

This table shows 5 examples of comprehensive car insurance premiums with PICKME insurance.

Driver gender	Driver age	Home suburb	Car make	Car year	Basic premium
male	55	Goulburn	Ford	1999	$508
female	33	Wollongong	Mazda	2011	$657
female	24	Dubbo	Mitsubishi	2005	$1339
male	21	Newcastle	Holden	2010	$1161
female	45	Bathurst	Hyundai	2012	$432

Calculate the premium for:

a the woman from Bathurst who gets a 10% loyalty discount

b the Ford driver who pays a 5% surcharge for an older car

c the owner of the Mitsubishi who pays a 13% levy as the car is financed

d the Mazda driver who gets a discount of 2.5% for a good driving record as well as a further discount of 1.7% for loyalty

Solution

a

$$\begin{aligned} \text{Basic premium} &= \$432 \\ \text{Discount} &= 10\% \\ \text{Discounted premium} &= 90\% \times \$432 \\ &= \$388.80 \end{aligned}$$

b

$$\begin{aligned} \text{Basic premium} &= \$508 \\ \text{Surcharge} &= 5\% \\ \text{Actual premium} &= 105\% \times \$508 \\ &= \$533.40 \end{aligned}$$

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c

$$\begin{aligned} \text{Basic premium} &= \$1339 \\ \text{Levy} &= 13\% \\ \text{Actual premium} &= 113\% \times \$1339 \\ &= \$1513.07 \end{aligned}$$

d

$$\begin{aligned} \text{Basic premium} &= \$657 \\ \text{Discount} &= 2.5\% \text{ followed by } 1.7\% \\ \text{Discounted premium} &= \$657 \times 97.5\% \times 98.3\% \\ &= \$629.6852... \\ &\approx \$629.69 \end{aligned}$$

[100 – 2.5 = 97.5, 100 – 1.7 = 98.3]

Theft and accident statistics

The National Motor Vehicle Theft Reduction Council (NMVTRC) says that a car is stolen somewhere in Australia every 6 minutes. Each year, insurance companies charge $1 billion more nationally for insurance premiums due to car theft. Older cars are less secure than newer ones and attract more thieves. Insurance costs more in areas that have a higher theft and/or accident rate.

EXAMPLE 3

The percentage probabilities for vehicle theft and recovery in 5 states and territories each year are shown in the table.

State	Chance of theft (%)	Chance of recovery (%)
ACT	0.2	78
NT	0.15	75
Tas.	0.13	92
Vic.	0.11	67
Qld	0.09	70

Source: National Motor Vehicle Theft Reduction Council (NMVTRC)

a Which state has the highest rate of vehicle theft?

b If there are 186 000 cars in the Northern Territory, how many are expected to be stolen each year?

c If 412 vehicles were recovered in Victoria:

i how many were stolen in Victoria?

ii how many vehicles in Victoria?

Solution

a ACT, since the chance of theft is greatest at 0.2%.

b

$$\begin{aligned} \text{Expected no. stolen cars} &= 0.15\% \times 186\,000 \\ &= 0.0015 \times 186\,000 \\ &= 279 \end{aligned}$$

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c **i** 67% of stolen vehicles are recovered.

$$67\% = 412 \text{ vehicles}$$

$$1\% = \frac{412}{67}$$

$$100\% = \frac{412}{67} \times 100$$

$$= 614.92537...$$

$$\approx 615$$

615 vehicles were stolen in Victoria.

ii 615 vehicles is 0.11% of all vehicles in Victoria.

$$0.11\% = 614.92537... \quad \textbf{from i}$$

$$1\% = \frac{614.92537...}{0.11}$$

$$100\% = \frac{614.92537...}{0.11} \times 100$$

$$= 559\,023.0665$$

$$\approx 559\,023$$

There are 559 023 vehicles in Victoria.

Exercise 8-01 Motor insurance

1 Third party property insurance costs $38.17 per month. What is the cost per week, correct to the nearest cent? Select the correct answer **A**, **B**, **C** or **D**.

A $8.78 **B** $9.54 **C** $16.54 **D** $7.63

Example 1

2 Bill paid his comprehensive car insurance at $150 per month. How much would he save if he paid a lump sum of $1265? Select **A**, **B**, **C** or **D**.

A $685 **B** $1115 **C** $0 **D** $535

3 Jack bought a new van for $28 350. His CTP insurance is $426 per year and his comprehensive insurance is $75.78 per month. What is the total cost of insurance per year?

4 Which insurance is the cheapest option? Select **A**, **B** or **C**.

A $985 per year paid in a lump sum

B $83.10 per month

C $38.25 per fortnight

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5 The table gives the third party property insurance premiums for the RIGHT4U insurance company.

Example 2

Car make	Driver gender	Driver age	Town where vehicle is garaged	Place where car is usually kept	Third party property premium
Suzuki	male	19	Campbelltown	on street	$345
Honda	female	56	Campbelltown	carport	$198
Mitsubishi	male	42	Parramatta	driveway	$297
Holden	female	23	Parramatta	locked garage	$243
Toyota	male	83	Bondi	on street	$312
Nissan	female	65	Bondi	security compound	$302

What is the third party property insurance premium, to the nearest cent, for:

a the woman from Campbelltown who gets a 12% no claim discount?

b the driver who has the car in a locked garage and gets a 3.5% discount?

c the driver under 20 years old who must pay an 11.6% surcharge because he parks on the street?

d the Toyota driver who pays a 24% age levy but then gets a 4.2% loyalty discount on the increased premium?

6 Colin bought a motor scooter. His annual CTP green slip was $238 and his third party property insurance was $19.35 per month. He also agreed to pay the first 8.5% of a claim if he had an accident that was his fault.

a What was his yearly insurance cost?

b He had an accident that was his fault and the damage bill to the other car was $7320. How much did he contribute to the claim?

c In the same accident, his scooter suffered $1360 worth of damage. How much did Colin pay towards its repair?

7 The NO FRILLS insurance company advertises a fixed insurance premium of $500 for a car garaged in a metropolitan area and $550 otherwise. The premium is adjusted for age and gender of the driver as shown in the table.

NO FRILLS insurance		
Driver gender	**Driver age**	**% adjustment**
male	< 29	+14.6%
female	< 29	+ 13.5%
male	30 – 69	– 4.6%
female	30 – 69	– 6.3%
male	> 70	+ 17.4%
female	> 70	+ 9.8%

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Calculate the insurance premium for each driver.

	Driver gender	Driver age	Vehicle garaged
a	male	24	metropolitan area
b	female	76	country town
c	male	32	on a farm
d	female	47	metropolitan area
e	male	28	metropolitan area
f	female	18	country town

Example 3

8 Refer to the table in Example **3** on page 335 to answer these questions.

a If there are 135 000 vehicles in Tasmania:

i how many would you expect to be stolen?

ii how many would you expect to be recovered?

b 76 stolen vehicles were never recovered in Queensland.

i How many were stolen?

ii How many vehicles are there in Queensland?

9 Ellie bought a small second hand car for $12 350. Her compulsory third party insurance was $286 and she took out comprehensive insurance at $12.45 per week.

a What was her yearly insurance cost?

b She ran into the back of a car that had stopped at the traffic lights and did $6320 damage to the car as well as $4280 to her own car. If Ellie had to pay an excess of $901 towards the repair costs, how much did the insurance company pay?

c Ellie's comprehensive insurance premium will increase by 13.5% next year. How much, to the nearest cent, will the annual premium be?

10 The top six stolen passenger vehicles in NSW one year are listed.

Car type	No. Thefts	% of thefts recovered
Holden Commodore 1990	317	86.8
Holden Commodore 1989	272	83.5
Holden Commodore 1991	264	83.3
Ford Falcon 1989	214	54.7
Ford Falcon 1990	189	52.9
Toyota Camry 1989	182	83.0

Source: National Motor Vehicle Theft Reduction Council (NMVTRC)

a How many Holden Commodores were stolen?

b How many Ford Falcons were recovered?

c How many Toyota Camrys were not recovered?

d Why do think older cars are more likely to be stolen?

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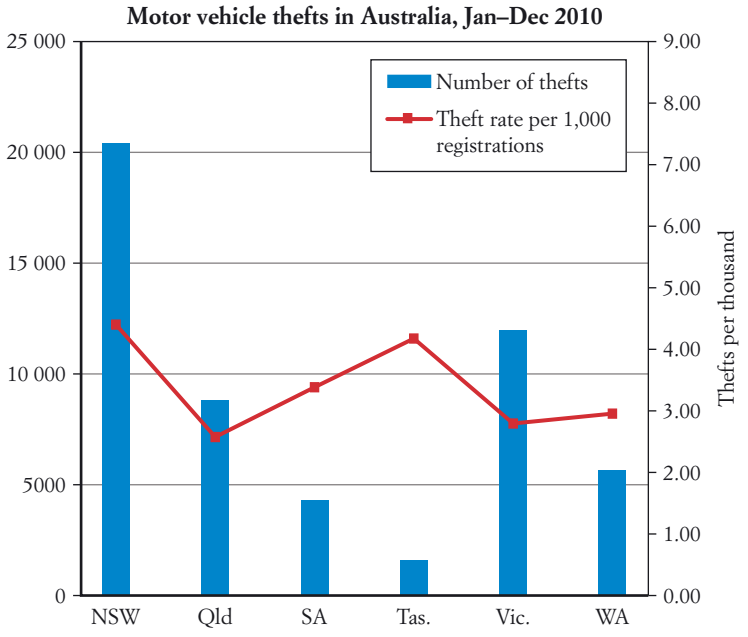
11 The table shows the number and rate of motor vehicle thefts in each state.



Source: National Motor Vehicle Theft Reduction Council

The state in which you live influences the insurance premiums of all car owners in that state.

a Which state had the most car thefts?

b Which state had the lowest number of thefts per 1000 vehicles?

c In which state would you pay a higher car insurance premium? Why?

d Which state had 3 thefts per 1000 vehicles?

e In which state would you expect the car insurance premiums to be lowest? Why?

f Use the graph to write a sentence about the car theft situation in Tasmania.

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INVESTIGATION

FACTORS AFFECTING INSURANCE PREMIUMS

Here are some of the insurance companies that provide motor insurance:

AAMI, Allianz, Budget, GIO, NRMA, QBE

Insurance costs depend on several factors, including:

- vehicle type, age and engine capacity (cc)
- postcode where vehicle is garaged
- owner's traffic record of 'at fault' accidents and moving traffic offences
- driver's age, gender

1 Visit the website of an insurance company and obtain an 'online quote' for third party property insurance and/or comprehensive insurance by completing the details of a car of your choice with one owner/driver aged 21 who has a clean driving record.

2 Redo the quote 5 times by changing just one factor at a time.

- **a** vehicle type and age
- **b** postcode where vehicle is kept (choose a city or country one)
- **c** traffic record (add 2 'at fault' accidents for the owner/driver)
- **d** owner/driver age
- **e** owner/driver gender

Compare the 6 quotes and write the factors that affect the original quote.

3 Is comprehensive insurance a good idea for young drivers? Why?

TECHNOLOGY: CTP INSURANCE ONLINE CALCULATORS

The NSW Motor Accidents Authority (MAA) has a comparative guide for green slip prices on its website at www.maa.nsw.gov.

For example, in 2012, the cost of a CTP green slip for a 2-year-old car garaged in the metropolitan area with a 20-year-old driver was between $733 and $743.

Use the website to find:

- **a** the cheapest green slip today for a motor car with a 20-year-old driver who has no traffic offences
- **b** the most expensive green slip for the car of your choice
- **c** the difference in price for an 18-year-old male or female driver of a financed vehicle
- **d** the difference in price for garaging the same vehicle in Sydney and Broken Hill.

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8-02 Stamp duty

WS
Stamp duty

Stamp duty is a tax paid to the Office of State Revenue when you buy a new or used vehicle. It is calculated on the market value of the vehicle or the price actually paid, whichever is the greater. Stamp duty is usually included in the 'on-road' price of a new vehicle.

EXAMPLE 4

Lexie purchased a vehicle for $50 000. The stamp duty is 3% of the purchase price up to $40 000 plus 5% for every dollar over $40 000.

a How much stamp duty did Lexie pay?

b Complete the table below.

Vehicle value ($)	0	10 000	20 000	30 000	40 000	50 000	60 000	70 000	80 000
Stamp duty ($)		300			1200		2200		3200

c Use the completed table of values to construct a piecewise linear graph.

d Use the graph to find, to the nearest $100:

i the stamp duty on a $55 000 vehicle

ii the value of a vehicle if the stamp duty is $750.

Solution

a Purchase price = $50 000 = $40 000 + $10 000

$$\begin{aligned}\text{Stamp duty on first \$40 000} &= 3\% \times \$40\,000\\ &= \$1200\\ \text{Stamp duty on next \$10 000} &= 5\% \times \$10\,000\\ &= \$500\\ \text{Total} &= \$1200 + \$500\\ &= \$1700\end{aligned}$$

Lexie paid $1700 stamp duty.

b

Vehicle value ($)	0	10 000	20 000	30 000	40 000	50 000	60 000	70 000	80 000
Stamp duty ($)	0	300	600	900	1200	1700	2200	2700	3200

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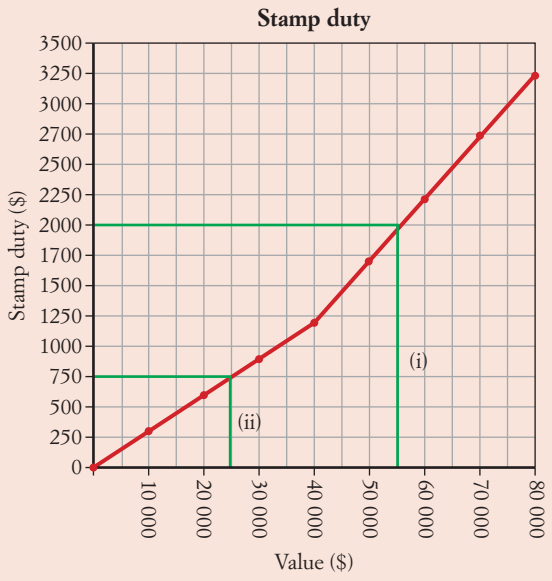
c



We also used piecewise linear graphs when we graphed income tax in Chapter 4, *Earning money and taxation*.

d **i** $2000 **ii** $25 000

Exercise 8-02 Stamp duty

1 Stamp duty is 3% of the purchase price of a small car. Find, to the nearest dollar, the stamp duty on a small Hyundai with purchase price of $22 120. Select the correct answer **A**, **B**, **C** or **D**.

A $660 **B** $664 **C** $662 **D** $663

2 Stamp duty on a vehicle is 3% for the first $45 000 and 5% on the remainder. Which of the following amounts is the stamp duty on a $25 000 ute? Select **A**, **B**, **C** or **D**.

A $1350 **B** $1250 **C** $1750 **D** $750

Example 4

3 Vehicle stamp duty is 3% of the purchase price up to $30 000, plus 11% for every dollar between $30 000 and $40 000, plus 4% for every dollar over $40 000.

a Find the stamp duty on a vehicle costing:

i $30 000 **ii** $40 000 **iii** $60 000

b Construct a piecewise linear graph showing the stamp duty for cars from $0 to $80 000. [Use a scale of 1 cm = $10 000 on the horizontal axis and 1 cm = $500 on the vertical axis.]

c Use your graph to find, to the nearest $100:

i the stamp duty on a $75 000 car

ii the cost of a car if stamp duty is $3000.

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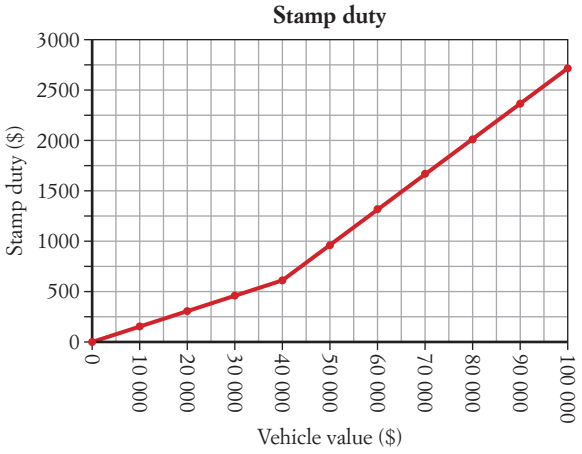
4 If stamp duty on a new vehicle is 2.75% of the purchase price up to $20 000, plus 5.75% for every dollar between $20 000 and $45 000, plus 6.5% for every dollar over $45 000, how much stamp duty will you pay on a car worth:

a $20 000? **b** $45 000? **c** $78 500?

5 Stamp duty charges on vehicles in one state are shown in the graph.



a To the nearest $100, find the stamp duty on a vehicle worth:

i $33 000 **ii** $66 000 **iii** $85 000

b To the nearest $1000, find the value of a vehicle if the stamp duty is:

i $2300 **ii** $1700 **iii** $750

6 In one state, there is no stamp duty on a car valued at less than $6000. The stamp duty on a car valued at $6000 or more attracts a stamp duty charge of $10 for every $200 or part thereof of its value. To the nearest dollar, what is the stamp duty on a car valued at:

a $6350 **b** $68 980 **c** $146 000?

7 If stamp duty on a vehicle is $60 for the first $3000 of the purchase price plus 4% for every dollar over $3000, calculate the stamp duty payable on a vehicle with purchase price:

a $8000 **b** $12 500 **c** $ 69 750

8 Stamp duty in one state is based on the market value of the vehicle, regardless of the price paid. The rates are:

2.5% of the market value up to $57 009

5% of the market value where this is over $57 009

a Calculate the stamp duty on the following vehicles.

i market value of $57 100

ii market value of $22 500

iii market value of $132 400

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b Construct a piecewise linear graph showing the stamp duty on vehicles with a market value from \$0 to \$140 000. [Use a scale of 1 cm = \$10 000 on the horizontal axis and 1 cm = \$500 on the vertical axis.]

Read the stamp duty values for the vehicles in part **a** and compare with the calculated values.

9 The stamp duty on wrecked cars is calculated using the following table.

Car value	Stamp duty
\$1000 or less	1% with a minimum of \$5
\$1001 to \$2000	\$10 + 2% on the amount over \$1000
\$2001 to \$3000	\$30 + 3% on the amount over \$2000
\$3001 or more	\$60 + 4% on the amount over \$3000

Find the stamp duty, to the nearest dollar, on wrecked cars worth:

a \$980 **b** \$2850 **c** \$3200

10 Stamp duty charges based on the value of a car are listed in the table.

Car values	Stamp duty
\$600 or less	\$18
\$601 to \$35 000	3%
\$35 001 to \$40 000	\$1 050 + 11% for the amount over \$35 000
\$40 001 or over	4%

Calculate the stamp duty charges, to the nearest cent, for cars valued at:

a \$585 **b** \$34 800 **c** \$105 500 **d** \$890 **e** \$38 750

INVESTIGATION

CAR VALUES AND STAMP DUTY RATES

1. Choose 5 new cars with a current market value (including GST) of \$100 000 or less.
2. Find the current stamp duty rates for new cars in your state.
3. Use these rates to draw a graph showing the stamp duty on vehicles from \$0 to \$100 000.
4. Use your graph to read off the stamp duty for your 5 chosen cars.

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TECHNOLOGY: STAMP DUTY ONLINE CALCULATOR

Use an Internet search engine to find a 'stamp duty calculator'.

This calculator shows that the stamp duty in NSW for a vehicle with a purchase price of $40 000 is $1200.

State	NSW
Purchase Price	40000
NSW Stamp Duty: $1,200.00	

1 For a 6 cylinder car with a purchase price of $40 000, find:

- **a** which state has the largest stamp duty
- **b** which state has the lowest fee to transfer the car registration to a new owner
- **c** the total payable in Victoria
- **d** the range of stamp duties
- **e** the average stamp duty for the 6 states and the Northern Territory.

2 Find the stamp duty payable on:

- **a** a car worth $22 000 in NSW
- **b** a van with a value of $32 500 in Tasmania
- **c** a hybrid car in Queensland valued at $31 480
- **d** a 4WD in Western Australia worth $102 450.

8-03 Car loans

Where can you buy a car?

- car dealership
- at auction
- from a website or classified advertisement
- at a car market
- private sale

How can you pay for a car?

- cash
- finance through the car dealership
- loan from a bank or lending institution
- personal loan from parents or rich relative.

If you need to arrange finance to purchase a car, there are two types of loans available:

- flat interest rate loan
- reducing balance loan.

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Flat rate loans

A **flat rate loan** charges simple interest, calculated on the total amount borrowed.

The interest rate, term of the loan, and repayment amount are usually fixed.

SUMMARY

The **simple interest formula** is:

$I = Prn$ where

I = interest charged

P = principal (amount borrowed)

r = interest rate per period, expressed as a decimal

n = number of periods in the term

EXAMPLE 5

To buy a van for his business, Brad borrowed \$12 500 from Conn Bank at a flat rate of 9.5% p.a. over 6 years. He also paid a loan application fee of \$250 in cash.

a How much did he actually pay for the van?

b What was his monthly repayment, correct to the nearest cent?

Solution

a First find the interest charged on the loan.

$P = \$12\ 500 \qquad r = 9.5\% = 0.095 \qquad n = 6$

$I = Prn$

$= \$12\ 500 \times 0.095 \times 6$

$= \$7125$

Amount actually paid:

$\$12\ 500 + \$7125 + \$250 = \$19\ 875$ **Loan + interest + application fee**

b Total to repay on loan $= \$12\ 500 + \$7125$ **Application fee not included as it was paid in cash.**

$= \$19\ 625$

Monthly repayment $= \dfrac{19\,625}{72}$ **6 years = 72 months**

$= \$272.5694\ldots$

$\approx \$272.57$ **Always round up for repayments.**

His monthly repayment is \$272.57.

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Reducing balance loans

The interest on a **reducing balance loan** is calculated on the **balance owing**, not on the original principal borrowed. You pay less interest on a reducing balance loan because the interest decreases as the balance owing decreases.

Reducing balance loan grid

Reducing balance loan

EXAMPLE 6

Shelley borrowed $18 000 at 11% p.a. reducible interest to buy a car and made monthly payments of $270.

a Draw up a table to show the progress of her loan for the first 6 months.

b How much interest did she pay in the first 6 months?

c How much has she paid off the loan at the end of the 6 months?

Solution

a Amount owing (principal) at start of 1st month = $18 000

Interest for 1st month = $\$18\,000 \times \frac{0.11}{12} = \$165$

Amount owing = $18 000 + $165 = $18 165 **Amount owing + interest**

Amount owing after repayment = $18 165 – $270 = $17 895

Principal at start of 2nd month = $17 895

Continue in this way to complete the table below.

Shelley's car loan				
Loan amount	$18 000			
Interest rate p.a.	11%			
Monthly repayment (R)	$270			
Month	**Principal (P)**	**Interest (I)**	**Amount owing ($P + I$)**	**Amount owing after repayment ($P + I - R$)**
1	$18 000.00	$165.00	$18 165.00	$17 895.00
2	$17 895.00	$164.04	$18 059.04	$17 789.04
3	$17 789.04	$163.07	$17 952.11	$17 682.11
4	$17 682.11	$162.09	$17 844.20	$17 574.20
5	$17 574.20	$161.10	$17 735.30	$17 465.30
6	$17 465.30	$160.10	$17 625.40	$17 355.40

Note that the interest decreases because the principal decreases

b Total interest = $975.40 **Adding amounts in Interest column**

c After 6 months, Shelley still owes $17 355.40.

Amount paid off loan = $18 000 – 17 355.40

= $644.60

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Exercise 8-03 Car loans

1 Calculate the interest payable on a flat rate loan of $5000 at 9.4% p.a. over 6 months. Select the correct answer **A**, **B**, **C** or **D**.

A $2820 **B** $2350 **C** $235 **D** $282 000

2 What is the monthly repayment on a flat rate loan of $3000 at 10% p.a. over 2 years? Select **A**, **B**, **C** or **D**.

A $150 **B** $600 **C** $137.50 **D** $25

Example 5

3 Farmer Brown bought a tractor for his farm priced at $24 500. He paid $4500 cash and borrowed the other $20 000 from Cows R Us finance company at a flat rate of 8.5% p.a. over 3 years. He also paid the application fee of $320 in cash.

a How much did he actually pay for the tractor?

b What was his monthly repayment rounded up to the nearest cent?

4 The table shows interest rates and fees for flat rate car loans with 6 lending institutions.

Institution	Fixed flat interest rate (% p.a.)					Loan amount ($)		Application fee ($)
	1 yr	2 yr	3yr	4yr	5yr	Min	Max	
Buzz Bank	10.99	10.99	10.99	10.99	10.99	3000	80 000	195
ABC Credit Union (new car loan)			14.29	12.73	11.84	7500		250
ABC Credit Union (used car loan)			16.41	13.62	12.84	7500	35 000	250
Upfront Finance	13.04	13.04	13.04	13.04	13.04	500		150
BAN Bank			16.29	14.73	13.84	7500		250
Lowloan Credit			14.84	10.23	9.85	5000	50 000	nil

Note: application fee is to be paid in cash

For each loan listed below, calculate:

i the interest payable

ii the total amount repaid over the period of the loan

iii the monthly repayment (correct to the nearest cent) for a:

a $23 000 loan over 2 years with Upfront Finance

b $49 500 loan over 3 years with Lowloan Credit

c $17 000 new car loan over 4 years with ABC Credit Union

d $37 450 used car loan over 5 years with BAN Bank

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5 Usain borrowed $100 000 to buy three vehicles to expand his personal training business. He took out a reducing balance loan at 10% p.a. reducible interest with fortnightly repayments of $540.

Example 6

a Copy and complete the loan table.

Fortnight	Principal (P)	Interest (I)	Amount owing ($P + I$)	Balance ($P + I - R$)
1	$100 000	$384.62	$100 384.62	$99 844.62
2	$99 844.62	$384.02	$100 228.64	$99 688.64
3				
4				
5				
6				

b How much has he paid off the loan after 6 repayments?

c How much interest did he pay in the first 3 fortnights?

d How much interest did he pay in the next 3 fortnights?

6 **a** Copy and complete the table for a loan of $3000 at 13.5% reducible interest with a monthly repayment of $375 for 8 months, followed by one final payment to pay off the loan.

Loan amount	$3000			
Interest rate p.a.	13.50%			
Monthly repayment (R)	$375			
Month	**Principal (P)**	**Interest (I)**	**Amount owing ($P + I$)**	**Amount owing after repayment ($P + I - R$)**
1	$3000.00	$33.75	$3033.75	$2658.75
2	$2658.75	$29.91	$2688.66	$2313.66
3	$2313.66	$26.03	$2339.69	$1964.69
4				
5				
6				
7				
8				

b Find the total interest paid.

c How much is still owing after 8 months?

d Calculate the total repayments made.

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7 Emma bought a Vespa and borrowed $3600 at 12.4% reducible interest with a monthly repayment of $500.

- **a** Draw up a loan repayment table and calculate values until the loan is paid off.
- **b** How much is her last payment?
- **c** How much interest did she pay?
- **d** How much did she repay over the period of the loan?

8 George borrowed $2500 at 14.6% reducible interest to buy a trailer and made a fortnightly repayment of $480.

- **a** Draw up a loan repayment table and calculate values until the loan is paid off.
- **b** What was the amount of the last payment?
- **c** How many weeks did it take to pay off the loan?
- **d** How much interest was charged?
- **e** How much did he repay over the period of the loan?

Reducing balance loan

TECHNOLOGY: REDUCING BALANCE LOANS

Use a spreadsheet to construct the table for Shelley's car loan in Example **6**. The formulas for some cells are shown. Provided that you use the correct formulas, you can alter the values in B1, B2 and B3 and the spreadsheet will recalculate cell values.

Note: Use a $ sign for absolute cell references when you **do not** want the cell name and value to change when a formula is copied, such as the interest rate in B2 and the monthly repayment in B3. You do not need to retype a formula in each cell. Put formulas in Row 7 and drag the cells in each column to fill down.

	A	B	C	D	E
1	Loan amount	$18000			
2	Interest rate p.a.	0.11			
3	Monthly repayment R	$270			
4					
5	Months	Principal ($P)	Interest (I)	Amt owing (P+I)	Amt owing after repayment (P+I-R)
6	1	=$B$1	=B6*$B$2/12	=B6+C6	=D6-$B$3
7	=A6+1	=E6	=B7*$B$2/12	=B7+C7	=D7-$B$3
8	=A7+1	=E7	=B8*$B$2/12	=B8+C8	=D8-$B$3
9	=A8+1	=E8			
10	=A9+1				
11	=A10+1				
12					
13					

Note that your spreadsheet's values will be slightly different to those in Example **6** because a spreadsheet does not always round up interest values like we do and it also uses many decimal places in its calculations, not just two.

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1 Complete the spreadsheet for up to 12 months and use it to find:
   - **a** the total interest paid in the 1st year
   - **b** the amount owing at the end of the 1st year
   - **c** the amount paid off the principal in the 1st year
2 Drag the cells in each column and fill down until the loan is paid off, then find:
   - **a** the value of the last payment
   - **b** how long it takes to pay off the loan
   - **c** the total repaid
3 Change the loan amount to $20 000, the interest rate to 12.5% and the monthly repayment to $285 and recalculate the spreadsheet. Fill down and give:
   - **a** the total interest paid
   - **b** the value of the last payment
   - **c** the number of months it takes to pay off the loan
   - **d** the total repaid

TECHNOLOGY: ONLINE LOAN CALCULATORS

Use a search engine on the Internet to find an online loan calculator, similar to the one shown below.

*Loan amount: *Term (length) of loan: *Interest rate:

*Repayment Frequency:

◉ Monthly

○ Fortnightly

○ Weekly

CALCULATE

By entering a loan amount, term, interest rate and payment frequency, you will be able to determine the payment required each period.

- **a** Choose a loan amount to buy a vehicle of your choice. Enter the current interest rate for a car loan and choose a repayment frequency and loan term. Calculate the repayment.
- **b** Could you afford this repayment?
- **c** If the answer to **b** is no, continue to enter data until you find a loan amount you can afford.

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8-04 Purchasing costs

You have your licence, you have saved money from your part-time job and now it is time to buy your first car. So what exactly are the costs of purchasing a car?

- Purchase price
- Stamp duty
- **Vehicle registration**: yearly process of paying and obtaining permission for using a vehicle on public roads
- CTP (compulsory third party) insurance – a 'green slip'
- Third party property or comprehensive insurance (optional but advisable)
- Loan interest and fees

The cost of registration and CTP insurance is often included in the purchase price of a car, also stamp duty if the car is new. These are called 'on-road costs.'

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EXAMPLE 7

Caine purchased a van for $26 850 to use in his landscaping business. He set up a table of costs associated with his purchase.

Purchase price	$26 850
GST	10% of purchase price
Stamp duty	3% of GST price
Registration	$425
CTP insurance (green slip)	$269
Comprehensive insurance	$963

What was the total cost of purchasing this vehicle?

Solution

		Costs
GST price	$26 850 + 10% × $26 850	$29 535
Stamp duty	3% × $29 535	$886.05
Registration		$425
CTP insurance		$269
Comprehensive insurance		$963
Total cost		**$32 078.05**

The total cost was $32 078.05.

EXAMPLE 8

Suppose Caine paid 35% of the total cost of the van from Example **7** in cash and borrowed the rest at 11% p.a. flat interest over 5 years. How much does this add to the cost of his van?

Solution

The extra he pays is the interest on the loan.

Amount borrowed = 65% × $32 078.05 **100% − 35% = 65%**

= $20 850.7325

≈ $20 850.73

Interest = 11% × $20 850.73 × 5

= $11 467.9015

≈ $11 467. 90

He pays an extra $11 467.90.

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Exercise 8-04 Purchasing costs

1 Angela paid cash for a second-hand motor scooter with the following upfront costs.

Purchase price (including GST)	$15 000
Stamp duty	2% of purchase price
Transfer of registration	$200
CTP insurance	$150
Third party insurance	$240

What was the total cost? Select the correct answer **A**, **B**, **C** or **D**.

A $15 590 **B** $20 090 **C** $15 620 **D** $15 890

2 Naazmi bought a new 4WD with the upfront costs as shown. Calculate the total amount paid. Select **A**, **B**, **C** or **D**.

Purchase price	$45 000
GST	10%
Stamp duty	5% of GST price
Registration	$860
Green slip	$420

A $50 780 **B** $53 255

C $48 530 **D** $68 780

Example 7

3 Libby bought a sports car for $32 600 plus costs as shown in the table.

Purchase price	$32 600
GST	10% of purchase price
Stamp duty on GST price	2% for first $25 000 3% for each $ over $25 000
Registration	$615
CTP insurance	$419
Comprehensive insurance	$1325

What was the total cost of her sports car?

Example 8

4 If Libby paid 40% of the total cost of her car (from question **3**) in cash and borrowed the rest at 13.5% flat interest over 4 years:

a how much did she borrow, rounded up to the nearest dollar?

b what effect did borrowing have on the purchase price?

5 Charles bought his second-hand station wagon with the following upfront costs.

Purchase price	$31 280
Stamp duty	1.5% of purchase price
Transfer of registration	$173
Green slip	$427
Third party property insurance	$317

How much did he pay for his vehicle?

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6 Charles borrowed 75% of the total cost of his wagon (from question **5**) at 12.4% p.a. flat interest over 3 years.

a How much did he borrow, rounded up to the nearest dollar?

b What was the total interest paid?

c How much per month did he repay?

7 A car dealer is offering a 15% discount on the marked price of all vehicles. The costs of GST, stamp duty, registration and CTP insurance are then added to the discounted price to give the total cost.

Type of car	Registration	CTP insurance (compulsory)	Third party property insurance (optional)	Comprehensive insurance (optional)
Holden	$325	$614	$250	$1268
Ford	$318	$598	$264	$1095
Toyota	$336	$437	$206	$999
Nissan	$384	$503	$295	$1134
Honda	$396	$602	$301	$1003
Subaru	$401	$599	$278	$1236

GST: 10% of discounted price
Stamp duty: 2% for first $25 000 plus 3% for each dollar over $25 000.
Dealer special: No stamp duty on GST prices of $6500 or less.

Use the table to find the total cost of:

a a Holden with a marked price of $31 468 and third party property insurance

b a Ford with a marked price of $6500 and no optional insurance

c a Toyota with a marked price of $65 489 and comprehensive insurance

d a Nissan with a marked price of $27 650 and no optional insurance

e a Honda with a marked price of $18 639 and comprehensive insurance

f a Subaru with a marked price of $58 790 and third party property insurance

8 Sascha bought a 4WD minivan with the following upfront costs.

Purchase price	$43 860
Stamp duty	1.5% for first $25 000 plus 3% for each $ over $25 000
Registration	$726
CTP green slip	$854
Comprehensive insurance	$1352

What was the total cost of his van?

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9 The extra costs involved with a car purchase at Wally's Wheels are listed in the table.

GST	10% of purchase price
Stamp duty (on GST price)	3% of the price inclusive of GST up to \$45 000 5% for every dollar over \$45 000
Registration	\$743 + transfer fee of \$50
Green slip	\$837
Third party property insurance	\$$P$ + 5% extra for drivers 25 yrs and under
Comprehensive insurance	\$$C$ + 10% extra for drivers under 30 yrs

Note: \$$P$ and \$$C$ are basic amounts depending only on vehicle type. Add age amounts if required.

Find the total cost for purchasing each vehicle, correct to the nearest dollar:

a \$56 780 sports car bought by 29-year-old Josh with comprehensive basic \$1034.

b \$36 204 sedan bought by 40-year-old Natalie with third party property basic \$328.

c 4WD bought by 26-year-old Heath for \$38 759 with third party property basic \$407.

d Truck bought by 50-year-old 'Highway Harry' for \$128 000 with comprehensive basic \$6945.

e \$15 320 car bought by 24-year-old Jasmine with third party property basic \$269.

f \$21 648 van bought by 29-year-old 'Bill the Baker' with comprehensive basic \$747.

INVESTIGATION

CAINE'S LOAN

Set up a spreadsheet for Caine's loan in Example **7**. Use this to show that:

1 the time it takes Caine to pay off his loan is 22 months

2 the value of his last payment is \$21.16

3 the total interest paid is \$2270.16

4 the total paid for his van is \$33 548.

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JUST FOR THE RECORD

Best-selling car brands

The table shows the ten best-selling car brands in Australia in 2001 and 2011.

	2001	2011
1	Holden Commodore	Mazda 3
2	Ford Falcon	Holden Commodore
3	Toyota Corolla	Holden Cruze
4	Holden Astra	Toyota Corolla
5	Mitsubishi Magna V6	Toyota Hilux
6	Hyundai Accent	Hyundai i30
7	Nissan Pulsar	Mazda 2
8	Toyota Camry (4 cyl)	Ford Falcon
9	Mitsubishi Lancer	Ford Territory
10	Toyota Hilux	Toyota Camry

Source: fleetcare.com.au

Very little changed over the ten years. The new cars on the 2011 list tended to be similar to the cars they replaced (Holden Cruze instead of Holden Astra, Hyundai i30 instead of Hyundai Accent). The most remarkable change was the meteoric rise of the Mazda brand.

Find out what the list looks like this year.

TECHNOLOGY: PURCHASING YOUR FIRST CAR

The Office of Fair Trading has a website www.fairtrading.nsw.gov.au. It provides information on purchasing items, including the booklet '*The car buyer's handbook*'. Use the booklet or website to answer these questions.

1. What things should be on your vehicle checklist before you buy a car?
2. What are the 7 steps to follow when you buy a car privately?
3. What are RTA and REVS and how can they help you to buy a car?
4. Why should you avoid buying a car advertised on the roadside?
5. What is meant by 'rebirthing'? How does this affect insurance premiums?

Investigate other websites that may assist you in the purchase of a vehicle.

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8-05 Fuel consumption and prices

How much petrol?

The types of fuel used to power vehicles include petrol, diesel, liquefied petroleum gas (LPG), ethanol and electricity. Petrol is the most common fuel and most cars use unleaded petrol (ULP).

The **fuel consumption** is the rate at which a car uses fuel and is usually measured in litres per 100 km (L/100 km). For example, a large car may use 12.3 L of fuel to travel 100 km whereas a small car may use only 5.8 L. The small car has a better fuel consumption and is more fuel efficient. Listed below are the fuel consumption rates for some of Australia's best-selling cars.

	L/100 km
Ford Focus	6.2
Holden Commodore	8.9
Holden Cruze	6.4
Hyundai i30	6.5
Mazda 2	6.4
Mazda 3	6.1
Mitsubishi Lancer	8.8
Nissan Navara	8.5
Nissan X-trail	8.4
Toyota Camry Hybrid	5.2
Toyota Corolla	7.3
Toyota Hilux	13.3
Toyota Prado	11.5
Toyota Yaris	6.1
Volkswagen Golf	7.7

Source: www.greenvehicleguide.gov.au

The actual fuel consumption of a vehicle will depend on several factors. You use more fuel if:

- stop-start driving in traffic (urban areas)
- driving on unsealed roads
- using a badly-tuned engine
- using a driving style involving hard acceleration and/or braking
- using under-inflated tyres
- towing a trailer or caravan
- using an air conditioner

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EXAMPLE 9

A Mini Cooper has an advertised fuel consumption of 5.8 L/100 km.

a How much fuel, to 2 decimal places, does it use for a trip of 1530 km?

b If fuel is \$2.04 per litre, what is the fuel cost , to the nearest dollar, for this trip?

c How far, to the nearest kilometre, can the Mini travel on a full tank of 35 L?

d If the Mini used 25 L to go 400 km on an outback trip, how far could it go on 42 L at the same rate?

Solution

a Fuel for 100 km $= 5.8$ L

Fuel for 1 km $= \frac{5.8}{100}$ L **Using the unitary method**

Fuel for 1530 km $= 1530 \times \frac{5.8}{100}$ L

$= 88.74$ L

b Fuel cost $= 88.74 \times \$2.04$

$= \$181.0296$

$\approx \$181$

c 5.8 L is used for 100 km

1 L is used for $\frac{100}{5.8}$ km

35 L is used for $\frac{100}{5.8} \times 35$ km $= 603.44\,827\ldots$ km

≈ 603 km

The Mini can travel 603 km (to nearest km) on a full tank of fuel.

d 25 L takes the car 400 km

1 L takes the car $\frac{400}{25}$ km

42 L takes the car $\frac{400}{25} \times 42$ km $= 672$ km

The Mini can travel 672 km on 42 L.

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Graph of petrol prices

EXAMPLE 10

This table shows the highest fuel price in state capitals over a 12-month period.

	Petrol (c/L)	Diesel (c/L)	LPG (c/L)
Brisbane	162.8	178.7	72.3
Sydney	171.3	188.6	72.6
Melbourne	169.2	185.8	70.1
Adelaide	172.7	188.8	76.4
Perth	160.0	189.7	73.2
Hobart	165.4	190.6	87.0

a Which capital has the highest fuel price for:

i petrol? **ii** diesel? **iii** LPG?

b What is the average highest price, in dollars and cents, for petrol in the capitals?

c What is the range of LPG prices?

d If a motorist filled up her 60 L tank with petrol in Sydney at the price shown:

i how much did she pay?

ii how far could she travel on this tank if her car used 8 L/100 km?

Solution

a **i** Adelaide **ii** Hobart **iii** Hobart

b $\text{Average} = \dfrac{162.8 + 171.3 + 169.2 + 172.7 + 160.0 + 165.4}{6}$

$= 166.9 \text{ cents}$

$\approx \$1.67$

c Range = 87.0 – 70.1 = 16.9

d **i** Cost of fuel = 60 × 171.3 cents

= 10 278 cents

= $102.78

ii 8 L takes her 100 km

1 L takes her $\dfrac{100}{8}$ km

60 L takes her $\dfrac{100}{8} \times 60 \text{ km} = 750 \text{ km}$

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Exercise 8-05 Fuel consumption and prices

1 A motorcycle has a fuel consumption of 3.2 L/100 km. How far can it travel on 1L of fuel? Select the correct answer **A**, **B**, **C** or **D**.

A 32 km **B** 16.1 km **C** 31.25 km **D** 3.2 km

2 A car travels 160 km on 14 L petrol. What is its fuel consumption in L/100 km? Select **A**, **B**, **C** or **D**.

A 11.4 **B** 8.7 **C** 22.4 **D** 8.8

Example 9

3 A Mazda has an urban fuel consumption of 11.5 L/100 km.

a What is the fuel consumption in L/km?

b How much fuel, to the nearest litre, does it use to travel 980 km?

c How far could it travel on a 55 L tank of fuel? Answer to the nearest kilometre.

d If the Mazda used 48 L to go 695 km in the country, how far (to the nearest km) could it go on 102 L at the same rate?

4 A Ford Falcon Ute has a fuel consumption of 11.7 L/100 km. How much fuel, in litres correct to 2 decimal places, does it use to travel:

a 10 km? **b** 2347 km?

5 If a car uses 8 L of fuel to travel 100 km, how much will it use to go:

a 1 km **b** 150 km

6 Refer to the table of fuel consumption of Australia's best-selling cars on page 358.

a Which car has the best petrol consumption?

b How far (to the nearest kilometre) can this car travel on a 45L tank of fuel?

c How much petrol does the Mazda 2 use for a 2500 km trip at the rate shown?

d If petrol was $2.23 per litre, what is the cost of a 1348 km trip in the Toyota Hilux?

Use the table in Example **10** on page 360 to answer questions **7** and **8**.

Example 10

7 **a** What is the average highest price, in dollars and cents, for diesel in the capitals?

b What is the range of petrol prices?

c Tyson filled up his 160 L Hummer fuel tank with petrol in Melbourne at the price shown.

i How much did he pay?

ii If his fuel consumption is13.2 L/100 km, how many whole kilometres would he go before running out of petrol?

8 **a** What is the average highest price paid for diesel in the east coast capitals (Sydney, Melbourne and Brisbane)?

b What is the range of diesel prices in the east coast capitals?

c What does it cost Angela to fill her 50 L Diesel Golf fuel tank at the price shown in Hobart?

d 'Alice' the VW beetle, travelled 440 km on a 40 L tank of fuel. What was her fuel consumption in L/100 km correct to two decimal places?

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9 The table shows the price of petrol in one week.

Petrol (c/L)	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Brisbane	143.4	140.4	139.2	150.7	150.6	148.3	146.2
Sydney	149.7	147.6	146.6	159.5	158.0	154.7	151.8
Melbourne	151.1	149.0	148.0	158.9	157.4	155.2	153.0
Adelaide	150.0	147.8	148.0	160.9	158.5	155.9	152.7
Perth	146.7	146.6	146.3	145.9	146.3	146.5	146.6
Hobart	154.1	154.1	154.0	154.0	154.0	154.0	153.9

a What was the average petrol price in Brisbane this week, correct to two decimal places?

b What was the average petrol price on Monday for the cities?

c What was the range of prices on Saturday for the cities?

d What was the range of prices for the week in Perth?

e Which city had the lowest average weekly petrol price?

f On which day was the average petrol price greatest?

g Which city had the greatest range of prices over the week?

h Which day showed the smallest range in prices for the cities?

10 Lachlan, Brodie and Mitchell each did a 3-day trip and recorded their fuel usage.

Brodie	Distance travelled (km)	Fuel used (L)
Day 1	497	47
Day 2	364	43
Day 3	579	56

Lachlan	Distance travelled (km)	Fuel used (L)
Day 1	543	68
Day 2	297	43
Day 3	410	56

Mitchell	Distance travelled (km)	Fuel used (L)
Day 1	624	56
Day 2	208	39
Day 3	524	45

Who had the best fuel consumption overall?

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11 Copy and complete this table.

Distance travelled (km)	Fuel used (L)	Fuel consumption (L/100 km)
863	62	
	43	11.2
1304		8.4
	86	9.7
2356		12.9
1730	167	

JUST FOR THE RECORD

Ten ways of saving fuel

1. Drive at a moderate speed: not too slow, not too fast. Driving at 110 km/h and beyond uses much more fuel than driving at 80–100 km/h.
2. Service your car regularly and check the tyre pressure.
3. Avoid the stop-start traffic during peak times and on busy roads.
4. Avoid excessive braking. Allow the car to come to a natural stop.
5. If waiting in traffic for a long time, switch off the engine.
6. Close the windows when driving to reduce drag.
7. Switch off the air conditioner.
8. Reduce the load on your car by removing heavy objects from the boot and removing roof racks if they are not being used.
9. Plan to do all your small jobs in one trip rather than lots of short trips.
10. Use public transport, walk or ride a bike.

Choose two of the ways and explain how or why they save fuel.

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JUST FOR THE RECORD

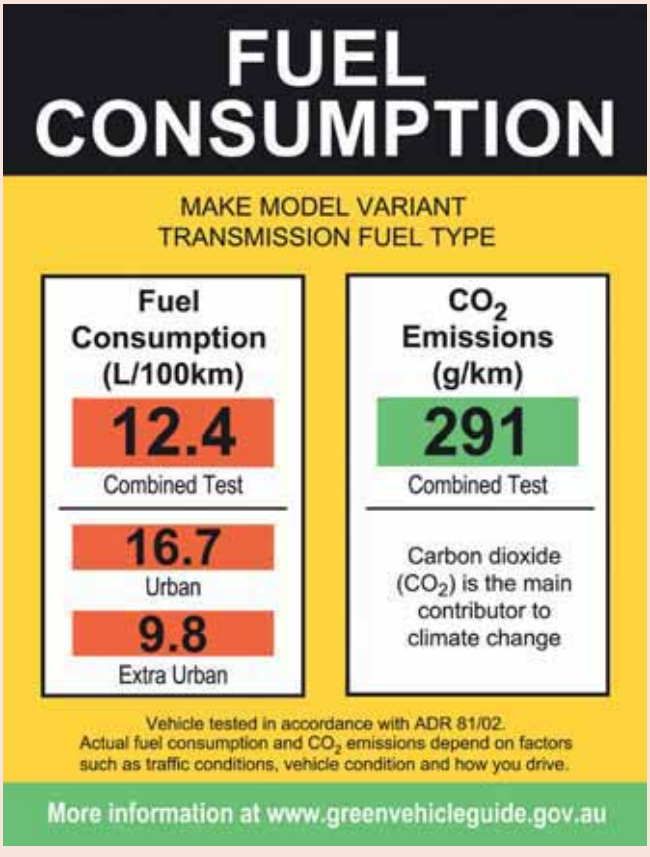
Fuel consumption labels

All new light vehicles must display this fuel consumption label on the windscreen. It shows the combined fuel consumption rate as well as the specific rates for city (Urban) and highway (Extra-Urban) travel. It also shows the carbon dioxide (CO_2) emissions of the car in g/km.

Insurance companies are increasingly aware of climate change and give discounts to people who drive cars with low CO_2 emissions and high fuel efficiency.

In 2012, the Toyota Prius Hybrid was the most fuel-efficient car in Australia. It had a 1.8 L four cylinder engine and used 3.9 L/100 km.

What is the most fuel efficient car today and what is its fuel consumption?



AAP Image

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TECHNOLOGY: CALCULATING FUEL CONSUMPTION AND COSTS

1 Use an Internet search engine to find a fuel consumption calculator similar to this one.

Fuel spent litres (L):	60
Kilometres (km):	725
Select units type:	Metric units (L, km)
Rounding options:	2 digits after decimal point

SUBMIT

Calculate the fuel consumption in L/100km for the following vehicles.

i A Jeep that travels 3200 km and uses 380 L fuel.

ii A Volkswagen that travels 1850 km and uses 160 L fuel.

iii A Smart car that travels 790 km and uses 60 L fuel.

2 Use a search engine to locate a calculator which finds annual fuel costs, such as this one.

Calculate your annual fuel costs					
	Fuel price ($/litre)	Fuel consumption value (L/100 km)	Annual distance travelled in km	Annual fuel cost ($)	
	CAR 1				
	1.95	8.28	15000	CALCULATE	

a Calculate the annual fuel costs for Bob who travels 550 km each week when fuel costs $1.95 per litre. His car can travel 600 km on a 50 L tank of fuel.

b Survey 5 friends or family members to see how far each one travels per year and the fuel consumption of their car in L/100 km, then calculate their annual fuel cost at the current petrol price.

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8-06 Straight-line depreciation

Straight-line depreciation formula practice

Percentage decrease

A new vehicle loses around 30% of its value in the first year. Loss in value can occur through age, wear and tear, or simply by becoming outdated when a new or more efficient model is released.

EXAMPLE 11

a A Skoda was purchased for \$25 000 and decreased in value by 28% in its first year. To the nearest dollar, what was its value after 1 year?

b A new Porsche was purchased for \$115 600 and was valued at \$102 580 twelve months later. What is the percentage decrease in price, to the nearest whole number, in the first year?

Solution

a Purchase price = \$25 000

% decrease in value after 1 year = 28%

Value after 1 year = 72% × \$25 000

= \$18 000

b Decrease in value = \$115 600 – \$102 580

= \$13 020

$$\text{Percentage decrease} = \frac{13\,020}{115\,600} \times 100\%$$

$$= 11.2629\ldots\ \%$$

$$\approx 11\%$$

Straight-line depreciation

Depreciation is the loss in value of an asset over time and is the biggest cost of car ownership. According to the results of a survey by an insurance company, a new car depreciates around 50% in the first 2 or 3 years of its life, and depreciation accounts for about 37% of the costs of running a small car, and about 49% of the costs of running a large four-wheel-drive.

There are two methods for depreciating an asset.

- **Straight-line method of depreciation**, where the asset's value decreases by the same amount each period
- **Declining balance method of depreciation**, where the asset's value decreases by the same percentage each period

The formula for the straight-line method of depreciation is a **linear function** (of the form $y = mx + b$) and its graph is a straight line.

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SUMMARY

Straight-line formula for depreciation

$S = V_0 - Dn$,

where S is the salvage (current) value of the asset after n periods,
V_0 is the initial value of the asset,
D is the amount of depreciation per period,
and n is the number of periods.

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EXAMPLE 12

A motorcycle is purchased for \$16 000 and depreciates by \$2400 each year.

a Use the straight-line depreciation formula to find the value of the motorcycle after 3 years.

b When, to the nearest month, will the motorcycle be worth half its purchase price?

c After how many months, to the nearest month, will the motorcycle have a \$0 salvage value?

Solution

a $V_0 = 16\,000$, $D = 2400$, $n = 3$, $S = ?$

$$\begin{aligned} S &= V_0 - Dn \\ &= 16\,000 - 2400 \times 3 \\ &= 8800 \end{aligned}$$

The value after 3 years is \$8800.

b $V_0 = 16\,000$, $S = 8000$, $D = 2400$, $n = ?$

$$\begin{aligned} S &= V_0 - Dn \\ 8000 &= 16\,000 - 2400 \times n \\ 2400n &= 16\,000 - 8000 \\ 2400n &= 8000 \\ n &= \frac{8000}{2400} \\ &= 3.3333\ldots \text{ years} \qquad \textbf{or } 3\tfrac{1}{3} \textbf{ years} \\ &= 3 \text{ years and } 4 \text{ months} \end{aligned}$$

The motorcycle will be worth half its purchase price after 3 years and 4 months.

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c $S = V_0 - Dn$

$0 = 16\,000 - 2400 \times n$ **Salvage value is \$0**

$2400n = 16\,000$

$n = \dfrac{16\,000}{2400}$

$= 6.666\ldots$ years **or $6\frac{2}{3}$ years**

$= 6$ years and 8 months

The motorcycle will have a \$0 salvage value after 6 years and 8 months.

EXAMPLE 13

A truck was purchased for \$35 000 and depreciated by \$2700 per year. The depreciation schedule shows the accumulated depreciation and value for the first 8 years of its life.

Year	Accumulated depreciation (\$)	Value at end of year (\$)
0	0	35 000
1	2700	32 300
2	5400	29 600
3	8100	26 900
4	10 800	24 200
5	13 500	21 500
6	16 200	18 800
7	18 900	16 100
8	21 600	13 400

$V_0 = 35\,000$
$D = 2700$
$n = 8$

a Use the table to find:

i by how much the truck has depreciated after

A 3 years **B** 7 years

ii the salvage value at the end of:

A 3 years **B** 7 years

iii the truck's value after 10 years by continuing the table for a further 2 years.

b Construct a straight line graph with year on the horizontal axis and value on the vertical axis.

c Use the graph to find:

i the value, to the nearest \$1000, after $4\frac{1}{2}$ years

ii when the value is \$12 000

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Solution

a **i** **A** \$8100 **B** \$18 900

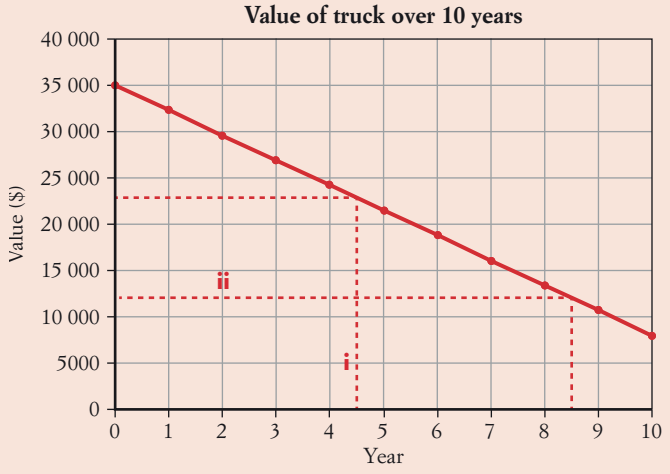
ii **A** \$26 900 **B** \$16 100

iii Continue the table for 2 more years as shown.

Value after 10 years is \$8000.

9	24 300	10 700
10	27 000	8000

b



c **i** \$23 000 **ii** $8\frac{1}{2}$ years = 102 months

Exercise 8-06 Straight-line depreciation

Example 11

1 An Audi was bought for \$64 000 and decreased in value by 35% in the first year. What was its value at the end of the first year? Select the correct answer **A**, **B**, **C** or **D**.

A \$41 600 **B** \$21 000 **C** \$22 400 **D** \$19 200

2 A vintage car purchased for \$58 600 increased in value by 12% in the first year. What was its value at the end of the first year? Select **A**, **B**, **C** or **D**.

A \$7032 **B** \$64 460 **C** \$65 632 **D** \$51 568

Example 12

3 A motorbike was purchased for \$11 700. It was expected to last 6 years and be worthless at the end of this time.

a What is the yearly depreciation?

b What is the value after:

i 1 year? **ii** 3 years?

4 A second-hand car was purchased for \$12 200 and depreciated to \$5000 at the end of 5 years.

a What was the annual depreciation?

b What was the value after:

i 2 years? **ii** 3 years?

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5 Penny's Plumbing purchased a van for \$16 000. After 3 years it depreciated to half its purchase price. What was the value after 6 years?

6 New machinery was purchased for \$110 000. It had an expected life of 15 years with a final scrap value of \$5000.

a What is the yearly depreciation?

b What is the value after 5 years?

Example 13

7 A baker's van cost \$18 000 and is estimated to be worth \$5000 after 5 years.

a What is the annual depreciation?

b Complete the table of values.

No. of years, n	0	1	2	3	4	5
Value, \$$V$	18 000					5000

c Draw a straight line graph showing the depreciation over 5 years.

d On your graph, what is the vertical intercept and what does it represent?

e Use the graph to find the value of the van after $2\frac{1}{2}$ years.

f After how many years is the value \$8900?

8 This is the depreciation schedule for an electric scooter.

Year	Accumulated depreciation (\$)	Value at end of period (\$$S$)
0	0	5600
1	850	4750
2	1700	3900
3	2550	3050
4	3400	2200

Use the table to find:

a the purchase price

b the depreciation in the:

i 2nd year **ii** in the first two years.

c the salvage value at the end of:

i 2 years **ii** 4 years.

d the value after 6 years by continuing the table for a further 2 years.

9 For the scooter in question **8** above:

a draw a straight-line graph to represent the depreciation over 6 years.

b what is the vertical intercept and what does it represent?

c when does the value fall below \$1000?

d when is the value of the scooter \$3500?

e what is the value after 18 months?

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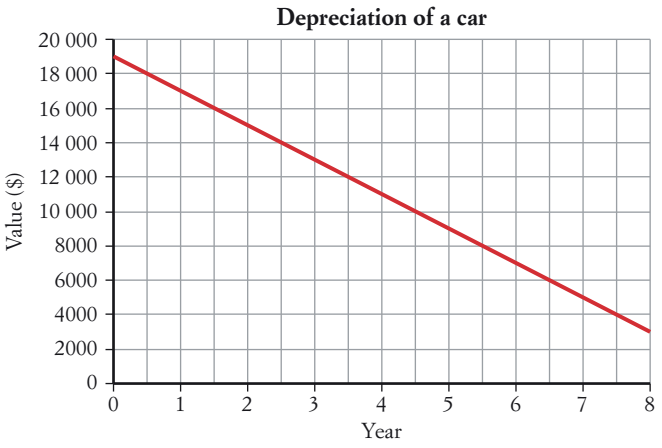
10



The graph shows the depreciation of a car over its useful life.

a What was the purchase price?

b What is the car worth after:

i 3 years? **ii** 8 years?

c What is the yearly depreciation?

TECHNOLOGY: STRAIGHT-LINE DEPRECIATION

1 Complete this spreadsheet for the first 10 years of the truck's life in Example **13**.

	A	B	C	D	E	F
1	Year	Accumulated depreciation ($)	Value at end of year ($)			
2	0	0	=$F$4			
3	=A2+1	=$F$5	=C2-$F$5			
4	=A3+1	=B3+$F$5	=C3-$F$5		V0 =	35000
5					D =	2700
6						
7						

2 Use the chart option to draw the line graph as shown in Example **13**.

3 Change the purchase price in cell F4 to 120000 and the depreciation amount in F5 to 6350 and the spreadsheet will recalculate cell values. Draw the appropriate line graph.

4 What does the vertical intercept on the graph represent?

5 Compare the table and the graph and give two reasons why:

a the graph is a better mathematical representation of depreciation

b the table is a better mathematical representation of depreciation.

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8-07 Declining balance depreciation

Declining balance depreciation formula practice

Declining balance depreciation: Spreadsheet

Declining balance depreciation

A vehicle generally depreciates more in the first few years of its life rather than by the same amount each year. The **declining balance method of depreciation** takes this into account and reduces the value of the asset each period by a constant *percentage* rather than a constant amount. This percentage is called the **declining balance rate**.

EXAMPLE 14

A Ferrari is purchased for $120 000 and depreciates at 15% p.a.

NEIL ROY JOHNSON /Shutterstock.com

a Calculate its depreciation in the 1st year.

b What is the value of the Ferrari at the end of the 1st year?

c What is the value at the end of the 3rd year?

d Construct a table showing the depreciation and value for the first 5 years.

e Graph the depreciating value of the Ferrari over 5 years.

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Solution

a Depreciation in first year = 15% × \$120 000

= \$18 000

b Value at the end of 1st year = \$120 000 – \$18 000

= \$102 000

Or: Value at the end of 1st year = \$120 000 × 85%

= \$120 000 × 0.85

= \$102 000

c Value at the end of 2nd year = value at end of 1st year × 85%

= (\$120 000 × 0.85) × 0.85

= \$120 000 × $(0.85)^2$

= \$86 700

Value at end of 3rd year = value at end of 2nd year × 85%

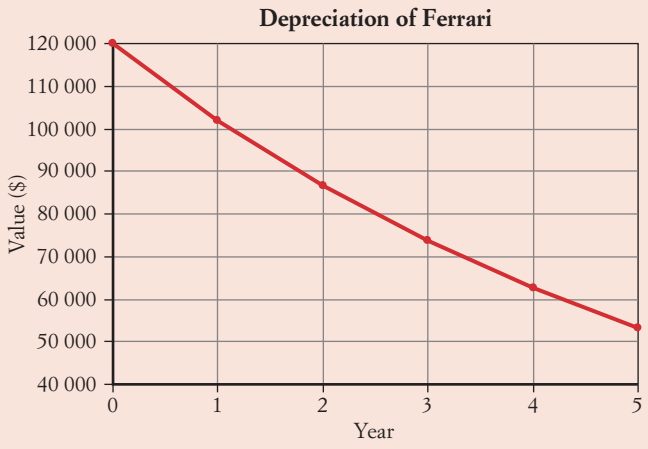
= \$120 000 × $(0.85)^3$

= \$73 695

d

Year	Depreciation (\$)	Value at end of year (\$)
0	0	120 000
1	18 000	102 000
2	15 300	86 700
3	13 005	73 695
4	11 054	62 641
5	9396	53 245

e



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SUMMARY

Declining balance formula for depreciation

$S = V_0(1 - r)^n$,

where S is the salvage (current) value of the asset after n periods,
V_0 is the initial value of the asset,
r is the depreciation (declining balance) rate per period, expressed as a decimal,
and n is the number of periods.

This formula is the compound interest formula with a negative interest rate.

- If you invest $\$P$ at an interest rate of r per period for n periods, the value of your investment **increases** to $\$A$ according to the formula $A = P(1 + r)^n$.
- If you purchase an item for $\$V_0$ and it depreciates at a rate of r per period for n periods, its value **decreases** to $\$S$ according to the formula $S = V_0(1 - r)^n$.

EXAMPLE 15

Kevin bought a camper trailer for \$34 500 with an expected depreciation rate of 18% p.a.

Newspix/MICHAEL PERINI

a Use the declining balance formula to find its value, correct to the nearest dollar:

i after 1 year **ii** after 5 years

b What is the total depreciation in the first 5 years?

c After how many years, to 1 decimal place, will the camper trailer be worth \$1000? [Hint: use a guess-and-check method.]

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Solution

a $S = ?$, $V_0 = 34\ 500$, $r = 0.18$

i When $n = 1$

$$\begin{aligned} S &= V_0(1 - r)^n \\ &= 34\ 500(1 - 0.18)^1 \\ &= 34\ 500\ (0.82)^1 \\ &= 28\ 290 \end{aligned}$$

Value after 1 year is \$28 290.

ii When $n = 5$

$$\begin{aligned} S &= V_0(1 - r)^n \\ &= 34\ 500(1 - 0.18)^5 \\ &= 34\ 500\ (0.82)^5 \\ &= 12\ 790.52459 \\ &\approx 12\ 791 \end{aligned}$$

to the nearest dollar

Value after 5 years is \$12 791.

b Total depreciation in first 5 years = purchase price – value after 5 years

= \$34 500 – \$12 791

= \$21 709

c Want $S = 1000$

Try $n = 17$:

$S = 34\ 500(0.82)^{17} = \$1182.1010...$ **too big**

Try $n = 18$:

$S = 34\ 500(0.82)^{18} = \$969.3228...$ **too small**

Try $n = 17.7$: $S = \$1028.7841...$

Try $n = 17.8$: $S = \$1008.5691...$

Try $n = 17.9$: $S = \$988.7512...$

The camper trailer will be worth less than \$1000 after 17.9 years.

Exercise 8-07 Declining balance depreciation

Example 15

1 A motorcyle costs \$3240 and depreciates at 18% p.a. What is its value three years after purchase? Select the correct answer **A**, **B**, **C** or **D**.

A \$1490 **B** \$1786 **C** \$2980 **D** \$0

2 An industrial machine in a car factory is expected to last 8 years. It cost \$9800 and depreciates at 12% p.a.

a Find, correct to the nearest dollar, its value after:

i 1 year **ii** 3 years **iii** 8 years

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b What is the total depreciation for the first:

i 3 years? **ii** 8 years?

c After how many years, to nearest year, will the machine be worth $1000? [Use a guess-and-check method.]

3 Quong bought a Holden for $28 000 with an annual depreciation rate of 22.5%.

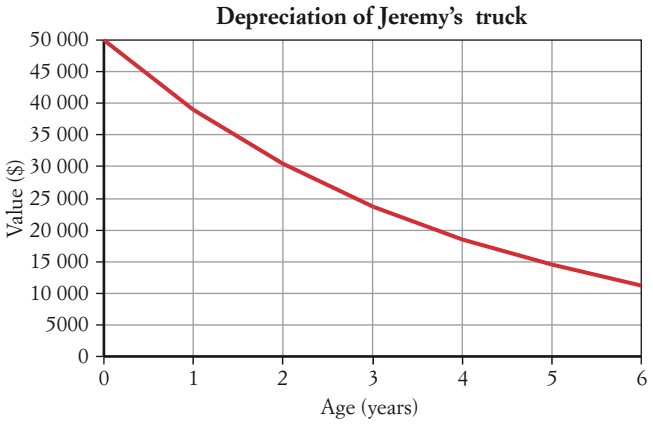
a How much would it be worth after 1 year?

b What is its expected trade-in value after 5 years? Answer to the nearest dollar.

c After how many years, to 1 decimal place, will it be worth half its purchase price?

4 Ron bought a second-hand Rolls Royce for $95 000 and was told it would only depreciate at 12% p.a. What is its expected resale value (to the nearest dollar) after 3 years?

5 The graph shows the value of Jeremy's truck over 6 years. The truck depreciates at 22% p.a.



a How much did Jeremy pay for his truck?

b What is its value after:

i 3 years? **ii** 6 years?

c What is the depreciation in the:

i 2nd year? **ii** 4th year?

d Use the declining balance formula to find the value of the truck after 8 years.

6 Baldwin Enterprises purchased a tractor for $110 000. Using a declining balance rate of 20% p.a., find (to the nearest dollar):

a the value of the tractor after:

i 3 years **ii** 8 years

b the depreciation in the:

i 3rd year **ii** 8th year

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7 The table shows the value of a bus over 5 years.

Age of bus (years)	Value of bus ($)
0	169 500
1	127 125
2	95 344
3	71 508
4	53 631
5	40 223

a How much is the bus worth after 3 years?

b What is the rate of depreciation of the bus?

c In what year will its value fall below $20 000? [Hint: use a guess-and-check method.]

d Plot the values and draw the depreciation graph for the bus over 5 years.

e What is the vertical intercept of the graph and what does it represent?

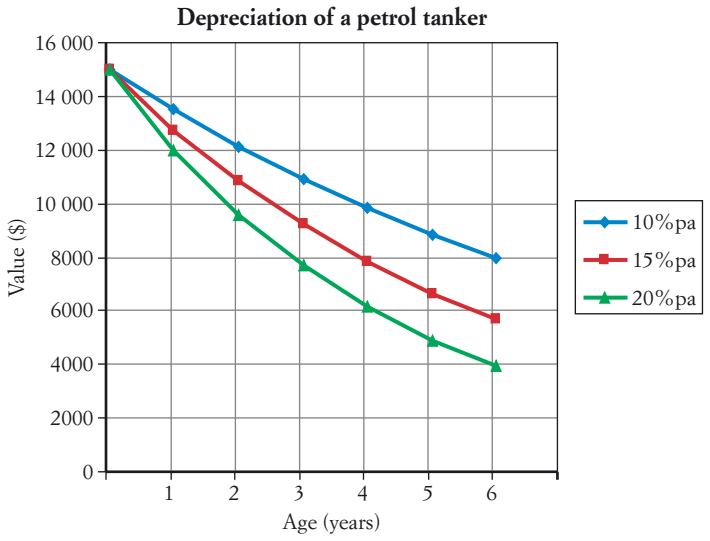
8 Robyn Removals bought a new van and after 3 years it was valued at $24 000. If depreciation is 24% p.a.:

a how much did Robyn pay for the van?

b what is the expected value of the van after 5 years?

9 Doug's Diner purchased a 3-year-old car for its chef for $21 350. What would the new price of the car have been if it depreciated at 18% p.a.?

10



The graph shows the decreasing value of a petrol tanker at different rates of depreciation.

a What is the initial cost of the tanker?

b What was the value (to nearest $1000) of the tanker after 4 years at:

i 10% p.a. depreciation? **ii** 20% p.a. depreciation?

c What was the depreciation in the 2nd year at 15% depreciation?

d Use the declining balance formula and a depreciation rate of 20% p.a. to calculate the value of the tanker after 10 years. Answer to the nearest dollar.

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Declining balance depreciation

Declining balance depreciation: Spreadsheet

TECHNOLOGY: DECLINING BALANCE DEPRECIATION

Declining balance depreciation can be calculated on a graphics calculator using the compound interest function with a 'negative' interest rate.

Choose the TVM mode (for more information about TVM mode, see page 492) and using the values for the camper trailer in Example **15**:

For part **a i**:

$n = 1$

$I\% = -18$

PV = 34 500

PMT = 0, FV = 0

P/Y = 1, C/Y = 1

Press *FV* (future value) to get $28 290.

For part **a ii**:

Enter $n = 5$ and press FV to get $12 791 (to the nearest dollar).

For part **c**:

Enter values of n until you reach a FV of $1000.

Sample HSC problem

Max bought a new tractor for $130 000. He wants to claim its depreciation as a tax deduction and must choose a depreciation method.

1 If his tractor was depreciated at $13 000 p.a., use the straight-line depreciation formula to find:
- **a** the value of the tractor after 10 years
- **b** the depreciation in the 10th year.

2 If the tractor was depreciated at 25% p.a., use the declining balance depreciation formula to find (correct to the nearest dollar):
- **a** the value of the tractor after 10 years
- **b** the depreciation in the 10th year.

3 With which method can Max claim more depreciation over the ten years?

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Study tip

Attacking your weak areas

Most of your study time should be spent on attacking your weak areas to fill in any gaps in your Maths knowledge. Don't spend too much time on work you already know well, unless you need a confidence boost! Ask your teacher, use study guides or other textbooks to improve the understanding of your weak areas and to practise Maths skills. Use your topic summaries for *general revision*, but spend longer study periods on overcoming any difficulties in your mastery of the course.

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CHAPTER REVIEW

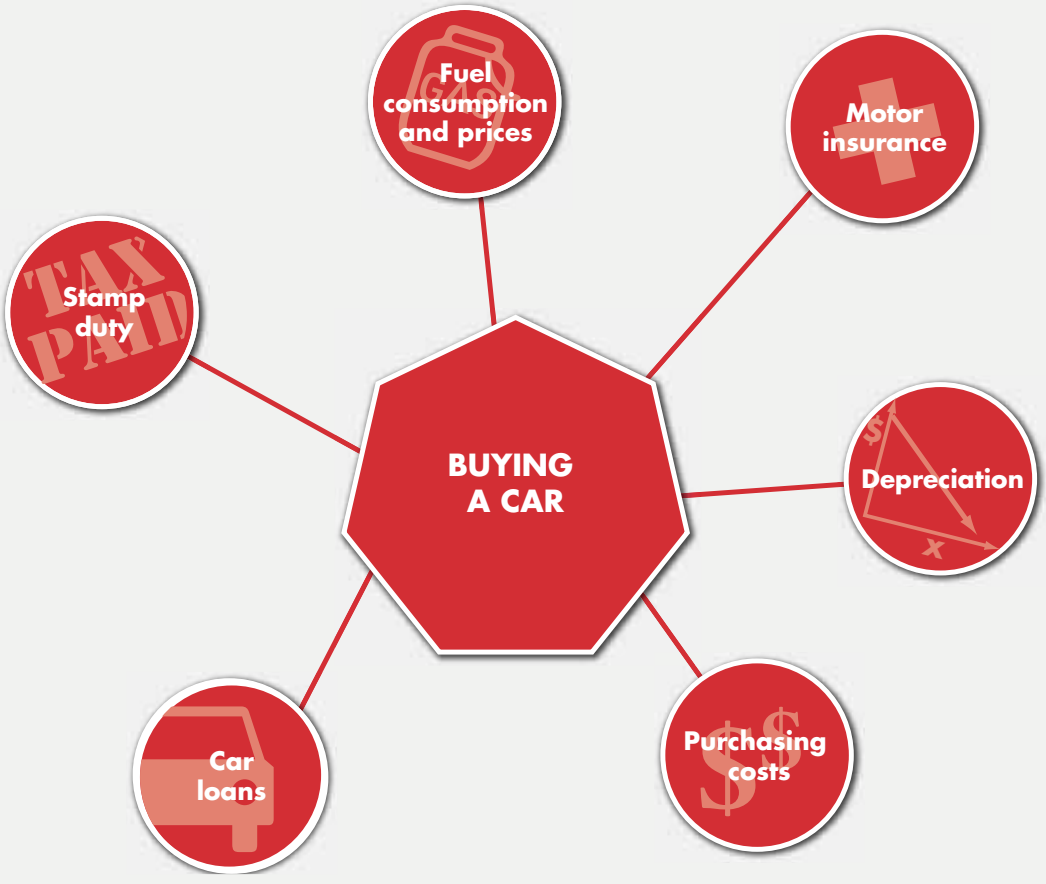
Topic overview

This focus study uses mathematical concepts to explore the costs of purchasing a motor vehicle. These costs include upfront costs such as the purchase price, insurance, registration, finance and stamp duty. It also looks at the hidden costs of owning a vehicle such as fuel consumption and depreciation.

These depreciation formulas will be provided in the HSC exam but you should know what each formula and variable represents.

$S = V_0 - Dn \qquad S = V_0(1 - r)^n$

Complete the mind map using your own words and symbols to help you get a complete picture of this focus topic.



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Revision

1 The table gives the third party property insurance premiums for the RIGHT4U insurance company. Exercise 8-01

Car make	Driver gender	Driver age	Town where vehicle is garaged	Place where car is usually kept	Third party property premium
Mitsubishi	male	42	Parramatta	driveway	$297
Nissan	female	65	Bondi	security compound	$302

What is the insurance premium for:

a the driver who gets a 5% discount for secure garaging?

b the driver who pays an 8% levy for non-secure garaging in a high theft area?

2 Ben had third party property insurance, paid by a direct debit of $15.80 per fortnight. Exercise 8-01

a How much would he save if he paid a yearly amount of $402.50 upfront?

b He hit a tree and wrote off his car, worth $15 600. What is his insurance cover?

3 Exercise 8-02

Car value	Stamp duty rate
up to $5000	1.5%
plus each $ between $5001 and $35 000	2.5%
plus each $ over $35 000	5.0%

Use this table to find the stamp duty on cars valued at:

a $23 600 **b** $57 000 **c** $4800 **d** $84 600

4 In one state of Australia, stamp duty is based on whichever is higher – the market value or the purchase price. Exercise 8-02

Use this table to find the stamp duty on vehicles with:

a purchase price $23 600 and market value $24 500

b purchase price $36 750 and market value $32 400

c market value $69 300 and purchase price $71 260

d market value $59 800 and purchase price $58 000.

Value	Stamp duty
$54 500 or less	$5 for every $200 or part thereof
Over $54 500	$10 for every $200 or part thereof

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Example 8-03

5 The table shows flat interest rates and fees for car loans with 4 lending institutions.

Institution	Fixed flat interest rate (%pa)					Loan amount ($)		Application fee ($)
	1 yr	2 yr	3 yr	4 yr	5 yr	Min	Max	
Buzz Bank	10.99	10.99	10.99	10.99	10.99	3000	80 000	195
Upfront Finance	13.04	13.04	13.04	13.04	13.04	500		150
BAN Bank			16.29	14.73	13.84	7500		250
Lowloan Credit			14.84	10.23	9.85	5000	50 000	nil

Note: Application fees to be paid in cash.

Calculate for each loan below:

i the interest payable

ii the total amount repaid over the period of the loan

iii the monthly repayment

a $34 000 loan over 3 years with Lowloan Credit

b $49 000 four year loan with Upfront Finance

c $57 000 new car loan over 5 years with Buzz Bank

Example 8-04

6 Penelope bought her new car with the following upfront costs.

Purchase price	$24 999
Stamp duty	1.5% up to $5000 plus 2.5% for each dollar between $5000 and $20 000 plus 4.5% for each dollar over $20 000
Registration	$328
CTP green slip	$503
Comprehensive insurance	$1204

What was the total upfront cost of her car?

Example 8-05

7 The fuel consumption of a Toyota Landcruiser was calculated under various conditions and recorded in the table.

Conditions	Fuel consumption (L/100 km)	Trip distance (km)	Fuel used (L)
City driving non-peak hour	14.5	35	
City driving in peak hour	12.7	26	
Highways	10.8	460	
Outback dirt roads	8.4	2635	
Towing a caravan	7.3	9870	

a What is the average fuel consumption (in L/100 km) for these 5 conditions?

b Complete the table by calculating, correct to 1 decimal place, the fuel used for the trip distances for each condition.

c What are 3 ways to improve fuel consumption?

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8 The table shows the fuel consumption for some vehicles.

Example 8-05

Car type	Fuel consumption (L/100 km)
Ford Falcon	11.7
Holden Commodore	11.3
Honda Jazz	5.7
Hyundai Sonata SLX	8.0
Lexus GS300	9.8
Mini Cooper	5.8
Mitsubishi Colt	5.6
Nissan Patrol	17.2
Peugot 308	6.7
Toyota Aurion	9.9

a What is the range of fuel consumption for these 10 cars?

b What is the average fuel consumption for these 10 cars?

c What is the rate in L/km for:

i Peugot 308? **ii** Honda Jazz? **iii** Holden Commodore?

d How many kilometres can the Ford Falcon travel on a 70 L tank of fuel?

e How much fuel does the Lexus use for a trip of 2500 km?

9 A new van cost $24 000, including registration and insurance. It depreciates by the same amount each year and after 6 years is expected to be worth $3000 as a trade-in on another van.

Example 8-06

a What is the yearly depreciation of the van?

b Draw up a depreciation schedule showing the value of the van for the first 6 years and the accumulated depreciation.

c What will be the trade-in value of the van after 5 years?

d What is the linear function that represents the value of the van after n years?

10 Mandy bought a new 4WD ute for $23 990 with an annual depreciation rate of 25.8%.

Example 8-07

a How much will it be worth after 1 year?

b What is its expected trade-in value after 4 years?

c After how many years, to 1 decimal place, will it be worth half its purchase price?

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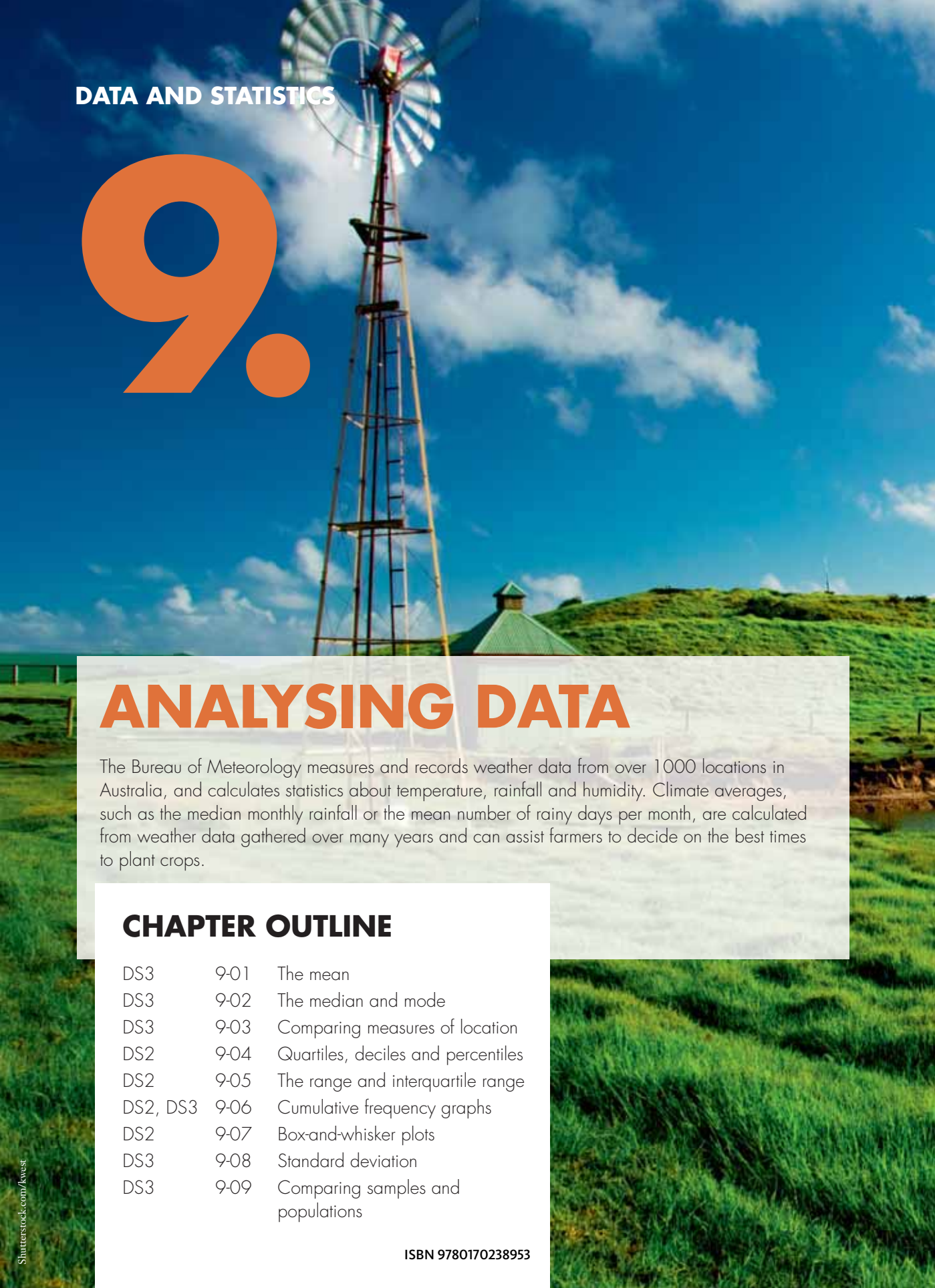

DATA AND STATISTICS

9.

ANALYSING DATA

The Bureau of Meteorology measures and records weather data from over 1000 locations in Australia, and calculates statistics about temperature, rainfall and humidity. Climate averages, such as the median monthly rainfall or the mean number of rainy days per month, are calculated from weather data gathered over many years and can assist farmers to decide on the best times to plant crops.

CHAPTER OUTLINE

DS3	9-01	The mean
DS3	9-02	The median and mode
DS3	9-03	Comparing measures of location
DS2	9-04	Quartiles, deciles and percentiles
DS2	9-05	The range and interquartile range
DS2, DS3	9-06	Cumulative frequency graphs
DS2	9-07	Box-and-whisker plots
DS3	9-08	Standard deviation
DS3	9-09	Comparing samples and populations

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IN THIS CHAPTER YOU WILL:

- calculate the measures of location – mean, mode and median – for grouped data presented in table or graphical form
- determine the mean for larger data sets of either ungrouped or grouped data using the statistical functions of a calculator
- calculate the median, including from stem-and-leaf plots and cumulative frequency polygons
- select and use the appropriate statistic (mean, median or mode) to describe features of a data set, for example median house price or modal shirt size
- assess the effect of outlying values on summary statistics for small data sets
- divide large sets of data into deciles, quartiles and percentiles and interpret displays
- calculate and interpret the range and interquartile range as measures of the spread of a data set
- construct frequency tables for grouped data from cumulative frequency graphs (histograms and polygons)
- estimate the median and upper and lower quartiles of a data set from a cumulative frequency polygon for grouped data
- establish a five-number summary for a data set (lower extreme, lower quartile, median, upper quartile and upper extreme)
- develop a box-and-whisker plot from a five-number summary
- calculate (population) standard deviation using a calculator
- compare summary statistics of various samples from the same population

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TERMINOLOGY

box-and-whisker plot
five-number summary
lower extreme
measure of location
median class
ogive
quartile
summary statistics
cumulative frequency
grouped data
lower quartile
measure of spread
modal class
outlier
range
upper extreme
decile
interquartile range
mean
median
mode
percentile
standard deviation
upper quartile

Assignment 9

SkillCheck

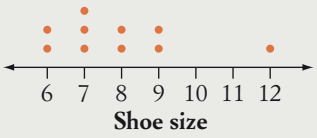
1 This stem-and-leaf plot shows the ages of visitors entering the Royal Easter Show in a five-minute period.

Stem	Leaf
0	3 8 9
1	0 2 2 2 5 6 7 9
2	0 2 3 4 6 7
3	1 3 3 4 9
4	3 4 7 8
5	5 5 8

- **a** How many visitors entered the show during the five-minute period?
- **b** What was the age of the oldest visitor?
- **c** What was the most common age?
- **d** How many visitors were under 16 years old?
- **e** What was the middle age?

2 Is a frequency histogram a line graph or a column graph?

3 The dot plot on the right shows the shoe sizes of a sample of Year 11 students.



- **a** How many students in the sample?
- **b** What is the most common shoe size for these students?
- **c** Find the outlier and describe the student that has this outlier.
- **d** How many students had a shoe size of 10?
- **e** What percentage of students had a shoe size larger than 8?

4 A sample of students was surveyed about the number of cars owned by each of their families. The results are shown in the table on the right.

Number of cars	Frequency
0	4
1	16
2	11
3	0
4	1

- **a** How many families did not own a car?
- **b** What was the most common number of cars owned?
- **c** What was the highest number of cars owned?
- **d** How many students were surveyed?
- **e** What was the total number of cars owned?

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5 The masses (in kilograms) of 40 skydivers were recorded. The results are shown below.

58	63	77	82	53	69	65	80	96	105
79	63	52	90	104	85	65	87	68	105
65	87	109	84	62	75	102	78	93	84
68	105	74	59	68	74	88	66	70	62

Copy and complete the frequency table below using the data about the mass of the skydivers.

Mass (kg)	Class centre	Frequency
50–<60		
60–<70		
70–<80		
80–<90		
90–<100		
100–<110		

9-01 The mean

The **mean**, **median** and **mode** are three summary statistics that represent the centre or average of a set of data. They are called **measures of location**.

The **mean** (or average) has the symbol $\bar{x}$, and is the sum of all of the scores divided by the number of scores.

Mean, median and mode

Statistical measures

SUMMARY

$$\text{Mean, } \bar{x} = \frac{\text{sum of scores}}{\text{number of scores}}$$

$$= \frac{\Sigma x}{n}$$

Σ means 'the sum of'.
x represents a score.
n is the total number of scores.

EXAMPLE 1

Calculate, correct to one decimal place, the mean of each set of data below.

a The maximum daily temperature (in °C) in Mudgee for the first two weeks in January.

30 28 26 31 34 35 32 33 21 25 28 32 32 35

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b A stem-and-leaf plot of the marks (out of 100) in a maths test for a class of students.

Stem	Leaf
4	4 7 7 8
5	2 6 8 9 9
6	1 3 5 5 7 8
7	0 2 3 4 5
8	3 7 8
9	2 8

Solution

a Sum of scores $(\Sigma x) = 30 + 28 + 26 + 31 + 34 + 35 + 32 + 33 + 21 + 25 + 28 + 32 + 32 + 35$

$= 422$

$$\text{Mean, } \bar{x} = \frac{\text{sum of scores}}{\text{number of scores}}$$

$$= \frac{422}{14}$$

$= 30.142\,85\ldots$

≈ 30.1

Note that the mean temperature of 30.1°C is at the centre of all 14 temperatures.

b $\text{Mean, } \bar{x} = \dfrac{1671}{25}$

The sum of the 25 marks is 1671.

$= 66.84$

≈ 66.8

The statistics mode on a calculator

WS Statistics mode: Graphics calculator

Scientific and graphics calculators have a statistics mode (SD or STAT). Follow the instructions in the table below to calculate the mean of the temperatures from Example **1a** using your calculator's statistics mode.

Operation	Casio scientific	Sharp scientific
Start statistics mode.	MODE STAT 1-VAR	MODE STAT =
Clear the statistical memory.	SHIFT 1 Edit, Del-A	2ndF DEL
Enter data.	SHIFT 1 Data to get table 30 = 28 =, etc. to enter in column AC to leave table	30 M+ 28 M+, etc.
Calculate the mean. ($\bar{x} = 30.142\,85\ldots$)	SHIFT 1 Var $\bar{x}$ =	RCL $\bar{x}$
Check the number of scores. ($n = 14$)	SHIFT 1 Var n =	RCL n
Return to normal (COMP) mode.	MODE COMP	MODE 0

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Operation	Casio graphics	Texas Instruments graphics
Start statistics mode.	MENU STAT for Lists table	Y= and delete any function by highlighting it and pressing CLEAR STAT EDIT
Clear the statistical memory.	With cursor in List 1 column EXIT F6 DEL-A Yes	With cursor on L1 CLEAR ENTER
Enter data.	30 EXE 28 EXE, etc. to enter in List 1 column	30 ENTER 28 ENTER, etc. to enter in List 1 column
Calculate the mean. ($\bar{x} = 30.142\ 85\ldots$) Calculate the sum of scores. ($\Sigma x = 422$) Check the number of scores. ($n = 14$)	F6 CALC SET to make these settings (if different): 1Var XList: List 1 1Var Freq: 1 EXE 1VAR to calculate many statistics (scroll down for more)	STAT CALC 1-Var Stats ENTER to calculate many statistics (scroll down for more)

The mean from a frequency table

If the scores in a data set are presented in a frequency distribution table then, by adding an '*fx*' column, the mean can be calculated using the formula shown below.

SUMMARY

$$\text{Mean, } \bar{x} = \frac{\text{sum of } fx}{\text{sum of } f} = \frac{\Sigma fx}{\Sigma x}$$

EXAMPLE 2

The scores for the players in a nine-hole golf competition were sorted into the frequency table below.

Score (x)	Frequency (f)
37	2
38	4
39	7
40	4
41	1

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a How many players were there?

b Calculate the mean of the scores, correct to one decimal place.

Solution

a 18 players **Sum of $f = 18$**

b

Score (x)	Frequency (f)	fx
37	2	74
38	4	152
39	7	273
40	4	160
41	1	41
Totals	$\Sigma x = 18$	$\Sigma fx = 700$

fx means '$f \times x$'

$37 \times 2 = 74$

$38 \times 4 = 152$, etc.

This means that there were two scores of 37, four scores of 38, etc. The 'fx' column groups equal scores and adds them together.

$$\text{Mean, } \bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{700}{18}$$

The sum of all 18 scores = 700.

$$= 38.888\,8\ldots$$

$$\approx 38.9$$

Follow the instructions in the table below to use a calculator's statistics mode to calculate the mean of the golf scores from Example **2**.

Operation	Casio scientific	Sharp scientific
Start statistics mode.	MODE STAT 1-VAR SHIFT MODE scroll down to STAT Frequency? ON	MODE STAT =
Clear the statistical memory.	SHIFT 1 Edit, Del-A	2ndF DEL
Enter data.	SHIFT 1 Data to get table 37 = 38 =, etc. to enter in x column 2 = 4 =, etc. to enter in FREQ column AC to leave table	37 2ndF STO 2 M+ 38 2ndF STO 4 M+ etc.
Calculate the mean. ($\bar{x} = 38.888\,8\ldots$)	SHIFT 1 Var $\bar{x}$ =	RCL $\bar{x}$
Check the number of scores. ($n = 18$)	SHIFT 1 Var n =	RCL n
Return to normal (COMP) mode.	MODE COMP	MODE 0

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Operation	Casio graphics	Texas Instruments graphics
Start statistics mode.	MENU STAT for Lists table	Y= and delete any function by highlighting it and pressing CLEAR STAT EDIT
Clear the statistical memory.	With cursor in List 1 column EXIT F6 DEL-A Yes Repeat for List 2	With cursor on L1 CLEAR ENTER Repeat for L2
Enter data.	37 EXE 38 EXE, etc. to enter in List 1 column 2 EXE 4 EXE, etc. to enter in List 2 column	37 ENTER 38 ENTER, etc. to enter in List 1 column 2 ENTER 4 ENTER, etc. to enter in List 2 column
Calculate the mean. ($\bar{x} = 38.888\ 8\ldots$) Calculate the sum of scores. ($\Sigma x = 700$) Check the number of scores. ($n = 18$)	F6 CALC SET to make these settings (if different): 1Var XList: List 1 1Var Freq: List 2 EXE 1VAR to calculate many statistics (scroll down for more)	STAT CALC 1-Var Stats and type 'L1, L2' by pressing 2nd 1 , 2nd 2 ENTER to calculate many statistics (scroll down for more)

The mean of grouped data

For data grouped into **class intervals**, an **estimate** of the mean can be calculated using the **class centres**. It is only an estimate because, with class intervals, we do not know the exact value of every score.

EXAMPLE 3

The ages of the patients at a medical centre in one afternoon were recorded and grouped into the frequency table below.

Age	Frequency
0–9	8
10–19	7
20–29	6
30–39	8
40–49	5
50–59	4
60–69	3
70–79	1

a Calculate, correct to one decimal place, the estimated mean age of the patients.

b How many patients went to the medical centre?

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Solution

a

Age	Class centre, x	Frequency, f	fx
0–9	4.5	8	36
10–19	14.5	7	101.5
20–29	24.5	6	147
30–39	34.5	8	276
40–49	44.5	5	22.5
50–59	54.5	4	218
60–69	64.5	3	193.5
70–79	74.5	1	74.5
	Totals	$\Sigma f = 42$	$\Sigma fx = 1269$

Estimate of the mean, $\bar{x} = \dfrac{\Sigma fx}{\Sigma f}$

$= \dfrac{1269}{42}$

$= 30.214\,2\ldots$

≈ 30.2

Note that the estimated mean age of 30.2 is at the centre of the data set.

b 42 patients. $\Sigma f = 42$.

Exercise 9-01 The mean

Example 1

1 Find the mean for each set of data below.

a 1 1 2 5 5 7 9 10

b 37 31 35 39 31 32 34 32 35 38

c 28 40 38 42 45 29 31 41 30

d 5 8 14 9 10 7 11 15 8 7 5

2 The stem-and-leaf plot on the right represents the number of points scored by the Sharks in every match of the football season.

Stem	Leaf
4	3 5 9
5	0 2 7 8
6	1 2 4 5 7 8
7	0 2 3 9
8	2 4 9

a How many matches were played in the season?

b Calculate the mean score.

3 Ngaire is training for a triathlon. She swam the following times, in minutes, in her last 10 races.

28 34 22 24 25 24 26 26 24 27

a Which of the following is Ngaire's mean swim time? Select **A**, **B**, **C** or **D**.

A 24 min **B** 25 min **C** 25.5 min **D** 26 min

b If the outlier score of 34 was removed from the set of data, would the mean go up or down? Give a reason for your answer.

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Example 2

4 Students were surveyed about the number of DVDs they purchased in the last six months. The results are shown in the frequency table on the right.

Number of DVDs (x)	Frequency (f)
0	6
1	7
2	8
3	10
4	9
5	5
6	5

a How many students were surveyed?

b Use an 'fx' column to calculate the mean number of DVDs purchased, correct to one decimal place.

5 The stem-and-leaf plot on the right shows the times, in seconds, for the competitors in a slalom ski race.

Stem	Leaf
9	1 7 9
10	4 5 6 6 8
11	0 2 3 4 4 5
12	1 2 3 3 3 9
13	2 3 4 7 7
14	6 9
15	0 1

a How many skiers participated in the race?

b Calculate the mean time for the race, correct to one decimal place.

6 'Average contents 50' is printed on each box of Meg's Matches. A quality controller counted the contents of a sample of 160 matchboxes from the production line and tabulated the results, as shown on the right.

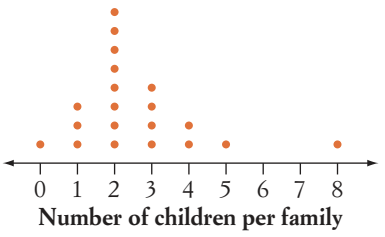
Number of matches (x)	Frequency (f)
48	10
49	45
50	52
51	39
52	9
53	5

a Use an 'fx' column, or your calculator's statistics mode, to calculate the mean number of matches per box, correct to one decimal place.

b Is the claim 'Average contents 50' justifed?

Give a reason for your answer.

7 This dot plot shows the number of children in each household on Willard Crescent.



a How many families live on Willard Crescent?

b Use a frequency table, or your calculator's statistics mode, to calculate the mean number of children per family.

c What is the outlier?

d If the outlier is removed from the data set, how is the mean affected?

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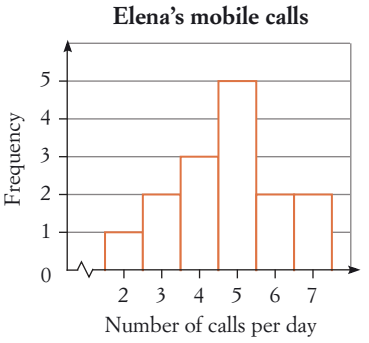
8 This frequency histogram shows the number of mobile phone calls made by Elena each day over a number of days.



- **a** Draw a frequency table for this data, including an '*fx*' column.
- **b** Over how many days was the number of calls Elena made recorded?
- **c** Calculate the mean number of phone calls made by Elena per day, correct to one decimal place.

Example 3

9 The police used radar to check the speeds of motor vehicles driving in a 40 km/h zone outside a local primary school one morning. They recorded the results in the table on the right.

Speed (km/h)	Number of cars, *f*
36–40	64
41–45	36
46–50	18
51–55	15
56–60	11
61–65	5

- **a** Add a column of class centres to the table and calculate an estimate for the mean speed of the vehicles, correct to two decimal places.
- **b** How many motor vehicles had their speeds checked?

10 The weekly wages of the staff at Yen's restaurant are shown in the frequency table on the right.

Wage ($)	Number of employees
100–<200	5
200–<300	11
300–<400	20
400–<500	4
500–<600	3
600–<700	1

- **a** How many employees are there?
- **b** Calculate an estimate for the mean wage, correct to the nearest dollar.
- **c** What could account for the large number of employees earning under $400 per week?

INVESTIGATION

JOKING MATHEMATICALLY

Mitchell told this joke: 'If a football player moved from Sydney to Brisbane, the average IQ of both cities would go up!' (IQ is an intelligence rating).

What does this joke mean?

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9-02 The median and mode

Mean, median and mode

Statistical measures

SUMMARY

When the scores are ordered from lowest to highest, the **median** is:

- the middle score, for an odd number of scores
- the average of the two middle scores, for an even number of scores.

'Median' sounds like 'medium', which is halfway between small and large. Half of the scores are below the median, the other half are above the median.

Mode = 'most often'

The **mode** is the most common score or category.
A set of data can have more than one mode, or no mode at all.

EXAMPLE 4

The maximum daily temperatures (in °C) in Mudgee for the first two weeks in January were:

30	28	26	31	34	35	32
33	21	25	28	32	32	35

For this data, find:

a the median **b** the mode.

Solution

a Placing the scores in order:

21 25 26 28 28 30 31 32 32 32 33 34 35 35

$$\text{Median} = \frac{31+32}{2}$$
$$= 31.5$$

For 14 scores, the middle scores are the 7th and 8th scores.

b Mode = 32

The most common score (it occurred three times).

Note that the mean (30.1), median (31.5) and mode (32) are all around the same central value. (We found the mean for this data in Example 1.)

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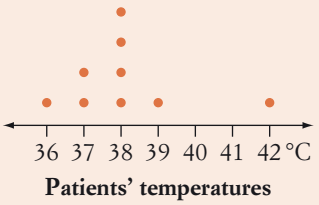
EXAMPLE 5

This dot plot shows the body temperatures (in °C) of a sample of hospital patients.



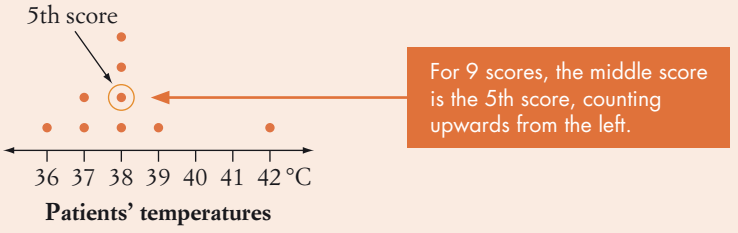
For this data, find:

a the median

b the mode.

Solution

a



The median is 38.

b Mode = 38 **This temperature has the highest frequency (four instances).**

The median and mode from a frequency table

EXAMPLE 6

The scores for the players in a nine-hole golf competition are shown below.

Score (x)	Frequency (f)
37	2
38	4
39	7
40	4
41	1

For this data, find:

a the mode

b the median.

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Solution

a Mode = 39 **39 has the highest frequency, 7.**

b To the table, add a cumulative frequency column which keeps a running total of the frequencies.

Score (x)	Frequency (f)	Cumulative frequency
37	2	2
38	4	6
39	7	13
40	4	17
41	1	18

2 + 4 = 6

6 + 7 = 13, etc.

Because there are 18 scores, the two middle scores are the 9th and 10th scores. Reading from the cumulative frequency column, the 6th score is 38, and the 13th score is 39, so the 9th and 10th scores must both be 39.

$$\text{Median} = \frac{39 + 39}{2}$$
$$= 39$$

Note that the mean (38.9), median (39) and mode (39) are all at the centre of the data set. (We found the mean for this data in Example **2**.)

The median class and modal class of grouped data

SUMMARY

For grouped data:

- the **median class** is the class interval that contains the median score
- the **modal class** is the most common class interval(s).

EXAMPLE 7

The monthly call costs of a sample of mobile phone users were grouped as shown in the cumulative frequency table on the right.

For this data, find:

a the median class

b the modal class.

Call cost ($)	Frequency	Cumulative frequency
0–<20	6	6
20–<40	8	14
40–<60	13	27
60–<80	17	44
80–<100	23	67
100–<120	20	87
120–<140	16	103
140–<160	10	113
160–<180	4	117
180–<200	3	120

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Solution

a There are 120 scores. The two middle scores are the 60th and 61st scores.
From the cumulative frequency column, the 60th and 61st scores are in the 80–<100 class.
The median class is 80–<100.

b The modal class is 80–<100. **This class has the highest frequency, 23.**

Exercise 9-02 The median and mode

Example 4

1 For each data set below, find:

i the median **ii** the mode.

a 1 1 2 5 5 7 9 10

b 37 31 35 39 31 32 34 32 35 38

c 28 40 38 42 45 29 31 41 30

d 5 8 14 9 10 7 11 15 8 7 5

Example 5

2 The maths test marks obtained by a class of students are shown in the ordered stem-and-leaf plot on the right.

For this data, find:

a the median

b the modes.

Stem	Leaf
4	4 7 7 8
5	2 6 8 9 9
6	1 3 5 5 7 8
7	0 2 3 4 5
8	3 7 8
9	2 8

3 Ngaire swam the following times (in minutes) in 10 races.

28 34 22 24 25 24 26 26 24 27

a Which of the following was her median swim time in minutes? Select **A**, **B**, **C** or **D**.

A 24 **B** 25 **C** 25.5 **D** 26

b Which of the following was Ngaire's modal swim time for the 10 races? Select **A**, **B**, **C** or **D**.

A 24 **B** 25 **C** 25.5 **D** 26

Example 6

4 Some students were surveyed about the number of DVDs they had purchased in the last six months. The results are shown in the frequency table on the right.

a Find the mode.

b Find the median.

Numberof DVDs (x)	Frequency (f)
0	6
1	7
2	8
3	10
4	9
5	5
6	5

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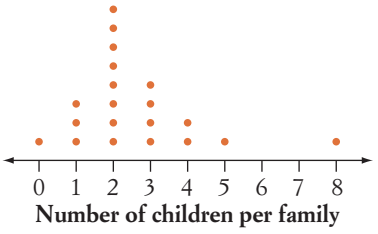
5 This dot plot shows the number of children in each family living on Willard Crescent.



a What is the median?

b What is the mode?

c What is the outlier?

d If the outlier is removed from the data set, how does this affect:

i the median?

ii the mode?

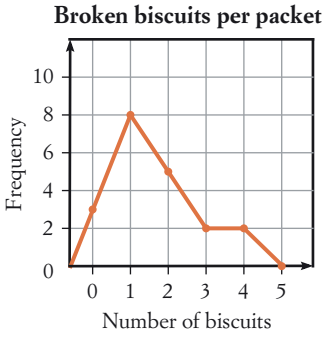
6 This stem-and-leaf plot shows the times, in seconds, of the competitors in a slalom ski race.

Stem	Leaf
9	1 7 9
10	4 5 6 6 8
11	0 2 3 4 4 5
12	1 2 3 3 3 9
13	2 3 4 7 7
14	6 9
15	0 1

a Find the median time of the skiers in the slalom race.

b Find the modal time.

7 A quality-control test was conducted to calculate the average number of broken biscuits in a packet of biscuits. A random sample was taken, and the number of broken biscuits per packet is shown on the frequency polygon on the right.



a Construct a frequency table for this data.

b How many packets of biscuits were tested?

c Find the mode of this data.

d Find the median of this data.

e Calculate the mean of the data.

8 The heights of young trees in a section of a nursery were measured before planting. The results are shown in the table on the right.

Example 7

Height (cm)	Number of trees
20–29	28
30–39	45
40–49	74
50–59	63
60–69	24

For this data, find:

a the median class

b the modal class.

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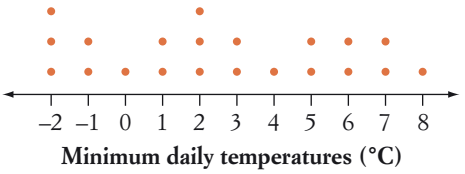
9 This dot plot shows the minimum daily temperatures (in °C) in Camden over a 3-week period.



a What is the mode?

b What is the median?

c Calculate the mean, correct to one decimal place.

10 The weekly wages of the staff at Yen's restaurant are shown in the frequency table below.

Wage ($)	Number of employees
100–<200	5
200–<300	11
300–<400	20
400–<500	4
500–<600	3
600–<700	1

iStockphoto/fototrav

a What is the modal class for the wages?

b What is the median class for the wages?

TECHNOLOGY: CALCULATING MEASURES OF LOCATION

Step 1: Open a blank spreadsheet to enter the following temperature data about Mudgee from Example **4**.

	A	B	C	D	E	F	G
1	**Maximum daily temperatures in Mudgee**						
2	30	28	26	31	34	35	32
3	33	21	25	28	32	32	35
4							
5				Mean			
6				Median			
7				Mode			
8							

Step 2: In cell E5, enter the formula **=AVERAGE(A2:G3)** to calculate the mean (30.142 85 …).

Step 3: In cell E6, enter the formula **=MEDIAN(A2:G3)** to calculate the median (31.5).

Step 4: In cell E7, enter the formula **=MODE(A2:G3)** to calculate the mode (32).

If there is more than one mode in a data set, the spreadsheet displays *only one* of the modes.

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9-03 Comparing measures of location

Statistical match-up

The following table summarises the three measures of location.

Measure of location	Features	When is it most appropriate?
Mean $\bar{x} = \dfrac{\text{sum of scores}}{\text{number of scores}}$ $\bar{x} = \dfrac{\Sigma x}{n}$ $\bar{x} = \dfrac{\Sigma fx}{\Sigma x}$	• Depends on all scores in the data set. • Is affected by outliers.	When the data set does not have many outliers
Median Middle score or average of two middle scores	• Not affected by outliers.	When the data set has many outliers, for example house prices, salaries
Mode Most popular score(s)	• Not affected by outliers.	When the most common score or category is needed (for example dress size); also useful for categorical data

EXAMPLE 8

Which measure of location is most appropriate for describing each of the following averages?

a the average price of a new car

b the most common number of bedrooms in a house

c a cricket player's batting average

d average weekly income

Solution

a Median, because there would be many outliers (the prices of expensive cars).

b Mode, because the most frequent score is needed.

c Mean, because all scores are required in the calculation.

d Median, because there would be many outliers (the incomes of very rich people).

EXAMPLE 9

Ten houses were sold this week at Westvale Lakes for the following prices.

\$376 000	\$1 200 000	\$270 000	\$308 000	\$372 000
\$409 000	\$387 000	\$582 000	\$460 000	\$238 000

a Calculate the mean house price.

b Calculate the median house price.

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c Which measure of location is higher, the mean or the median?

d Which measure of location is more appropriate for describing the average house price?

Shutterstock.com/Berni

Solution

a Mean, $\bar{x} = \dfrac{4\ 602\ 000}{10}$

$= \$460\ 200$

b Prices in order:

\$238 000	\$270 000	\$308 000	\$372 000	\$376 000
\$387 000	\$409 000	\$460 000	\$582 000	\$1 200 000

Median $= \dfrac{\$376\ 000 + \$387\ 000}{2}$

$= \$381\ 500$

Note that eight of the ten house prices are below the mean ($460 200).

c The mean is higher.

d The median, because it is not distorted by the outlier of $1 200 000.

Exercise 9-03 Comparing measures of location

1 Decide which *M* (mean, median or mode) is correct for each statement.

a This *M* takes all scores in the data set into account.

b This *M* is one of the scores if there is an odd number of scores.

c Half of the scores are above this *M*, the other half are below.

d There can be more than one *M* in a set of data.

e This *M* often needs to be rounded to decimal places.

f This *M* can also be used for categorical data.

g This *M* can be distorted by many outliers.

h This *M* must be one of the scores in the data set.

Example 8

2 Which measure of location is most appropriate for describing each average?

a the average exam mark for the class

b the average shirt size for teenage girls

c the average rent paid for a house in Sydney

d the average screen size of a notebook computer

e the average mass of football players in a team

f the average brand of mobile phone

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Example 9

3 A small business employs staff with the following salaries.

- General Manager $158 300
- three factory hands $64 300 each
- supervisor $85 600
- two clerical offcers $68 500 each

a How many people are there on staff?

b Calculate the mean salary of the staff, correct to the nearest $100.

c Calculate the median salary of the staff.

d Which measure of location is higher, mean or median? Why?

e Which measure of location best describes the average salary at this business?

4 The ages of the maths teachers at Westvale Christian College are:

49 32 37 32 25 41 39 50

a For this data, find:

i the mean **ii** the median **iii** the mode.

b The 39-year-old teacher is replaced by a new teacher, aged 22. Describe how this will affect:

i the mean **ii** the median **iii** the mode.

5 The colours of the new cars sold last week at Huxley Motors were recorded. The results are shown in the table below.

Colour	Black	Blue	Red	Silver	White
Frequency	4	7	7	9	12

a How many new cars were sold?

b What is the mode for this data?

c Why isn't it possible to find the median or mean colour?

6 The weekly mortgage repayments (in dollars) of 11 home owners are:

370 628 299 417 354 1027 585 435 509 652 481

a For this data, find:

i the mean, correct to the nearest dollar

ii the median

iii the mode.

b Why isn't the mean or mode an appropriate measure of location for this set of data?

c If the outlier is removed from the data, check whether the new mean will be closer to the new median than the mean was to the median for the original set of data.

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7 The dot plot on the right shows the shoe sizes of a sample of Year 11 students.

a For this data, find:

i the mean

ii the median

iii the mode.

b If the outlier is removed, state what will happen to:

i the mean **ii** the mode.

c A shoe store needs to buy more shoes for a back-to-school sale. Which measure of location is the most appropriate for the store to use in this situation?

8 The numbers of accidents per month in a motor vehicle factory over a year were:

3 0 0 1 2 1 6 0 0 2 1 0

a For this data, find:

i the mean, correct to one decimal place

ii the median

iii the mode.

b If the workplace union representatives wanted to complain about the lack of safety at the factory, which statistic would they choose?

c If the factory managers wanted to claim that there were very few accidents at the factory, which statistic would they choose?

d A safety researcher chose the median as the average. Why did he do this?

9 The heights, in centimetres, of a team of basketball players are:

204 195 184 189 193 178 199

a For this data, find:

i the mean, correct to one decimal place **ii** the median.

b The 184 cm player is replaced by a 200 cm player. Describe how this will affect:

i the mean **ii** the median.

10 The stem-and-leaf plot on the right shows the maximum daily temperatures (in °C) in Port Macquarie for the last two weeks in December.

Stem	Leaf
2	2 4 4 5 6 6 7 7 7 8 8 9
3	1 4

Source: Bureau of Meteorology

a For this data, find:

i the mean

ii the median

iii the mode.

b Which measure of location is the most appropriate for describing the average maximum daily temperature?

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9-04 Quartiles, deciles and percentiles

The median and quartiles

SUMMARY

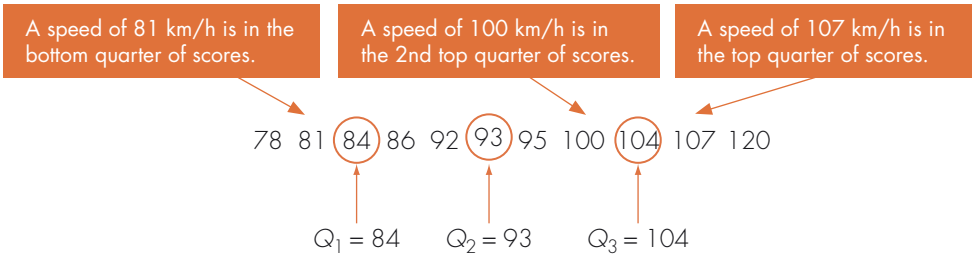
The three **quartiles** of a data set are those values that separate the data into quarters.

- The **lower quartile**, Q_1 or Q_L, separates the bottom quarter (25%) of scores from the rest of the scores.
- The **upper quartile**, Q_3 or Q_U, separates the top quarter (25%) of scores from the rest of the scores.
- The **middle quartile**, Q_2, is the median, and separates the two middle quarters.

These speeds (in km/h) were recorded for 11 cars driving along a major country road:

104 86 95 100 81 120 84 78 93 92 107

When we sort the scores, in ascending order, we can find the quartiles:



SUMMARY

To find the quartiles of a data set:

Step 1: sort the scores in order, find the median and call it Q_2

Step 2: find the median of the bottom half of scores and call it Q_1

Step 3: find the median of the top half of scores and call it Q_3.

EXAMPLE 10

Find the quartiles for each data set below.

a The marks obtained by a class of students for an art project.

51 41 60 38 46 57 39 61 43 64

b The scores obtained by a golfer for the first nine holes of a golf course.

4 3 5 6 4 3 8 6 6

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Solution

a First, sort the marks and place them in order.

38 39 41 43 46 51 57 60 61 64

$Q_2 = \frac{46 + 51}{2} = 48.5$

$Q_1 = 41$ $Q_3 = 60$

b 3 3 4 4 5 6 6 6 8

$Q_1 = \frac{3 + 4}{2} = 3.5$ $Q_3 = \frac{6 + 6}{2} = 6$

$Q_2 = 5$

Deciles

Quartiles (Q_1, Q_2 and Q_3) separate data into quarters.

Deciles (D_1, D_2, D_3, D_4, D_5, D_6, D_7, D_8 and D_9) separate data into tenths. 'Deci-' means one-tenth.

For example:

- D_1 cuts off the lowest 10% of scores.
- D_4 cuts off the lowest 40% of scores.
- D_9 cuts off the lowest 90% of scores (or the top 10% of scores).

EXAMPLE 11

The lengths (in centimetres) of 20 new-born infants at a hospital were recorded.

51 49 52 49 47 56 48 48 52 50

55 49 48 51 44 52 50 50 53 45

a What is the 3rd decile for this data?

b What is the 5th decile for this data?

c What is another name for the 5th decile?

d Find the value that separates the bottom 70% of lengths from the top 30%.

e If the length of new-born baby James is in the top 10% of infant lengths, what value must it be greater than?

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Solution

Place the values in order first.

D_1	D_2	D_3	D_4	D_5
44 45 47	48 48	48 49	49 49	50
50 50 51	51 52	52 52	53 55	56
D_6	D_7	D_8	D_9	

a $D_3 = \dfrac{48+49}{2} = 48.5$

b $D_5 = \dfrac{50+50}{2} = 50$

c The median, because it cuts off the lowest 50% of scores.

d $D_7 = \dfrac{51+52}{2} = 51.5$

e James' length must be greater than $D_9 = \dfrac{53+55}{2} = 54\text{ cm}$

Percentiles

Percentiles (P_1, P_2, P_3, …, P_{99}) separate data into hundredths.

For example:

- P_{24} cuts off the lowest 24% of scores
- P_{60} cuts off the lowest 60% of scores
- P_{87} cuts off the lowest 87% of scores (or the top 13% of scores)

Deciles and percentiles are only meaningful when analysing large sets of data.

EXAMPLE 12

The following information is based on population data for the heights of girls aged 16 years.

- The median is 163 cm
- The 3rd quartile $Q_3 = 167$ cm
- The 9th decile $D_9 = 171$ cm
- The 5th percentile $P_5 = 152$ cm
- The 97th percentile $P_{97} = 175$ cm

In the following questions, all of the girls mentioned are aged 16.

a Holly's height is 175 cm. Is she tall for her age and what percentage of 16-year-old girls are taller than her?

b Olga is taller than 90% of girls her age. What is her height?

c If $\frac{1}{4}$ of girls her age are taller than Verity, how tall is she?

d What height separates the bottom 5% of heights from the top 95%?

e What percentile is a height of 163 cm?

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Solution

a Yes, $P_{97} = 175$ cm, which means Holly is taller than 97% of girls her age. So only 3% of girls aged 16 are taller than her.

b Olga's height $= P_{90} = D_9 = 171$ cm.

c Verity is taller than $\frac{3}{4}$ of girls her age, so her height is $P_{75} = Q_3 = 167$ cm

d $P_5 = 152$ cm

e 163 cm is the median, so it is also the 50th percentile P_{50} (the height that cuts off the lowest 50% of scores).

The median is the 2nd quartile Q_2, the 5th decile D_5 and the 50th percentile P_{50}.

Example 10

Exercise 9-04 Quartiles, deciles and percentiles

1 Find the quartiles Q_1, Q_2 and Q_3 for each data set below.

a The times, in seconds, to run 100 metres.

8.7 9.1 11.0 13.5 10.6 8.9 10.1 9.6
12.3 9.9 9.0 10.8 9.2 13.1 10.6

b The number of matches in a box.

49 50 52 48 50 51 49 50 52 51 50 50

c The prices, in dollars, of a bag of potatoes.

3.50 3.20 3.50 4.10 3.00 3.50 3.90 2.80 3.40 3.00

d The weekly rainfall, in millimetres, over three months.

16 24 18 26 21 27 5 7 17
21 9 0 22

2 The stem-and-leaf plot on the right shows the game scores of a group of ten-pin bowlers.

For this data, find:

a the median

b the lower quartile

c the upper quartile.

Stem	Leaf
8	2 7 8
9	0 3 4 6 9
10	4 4 5 8 8 8
11	2 3 4 6 7 9 9
12	0 0 5 6 6 8
13	1 1 4 7 9

3 The dot plot on the right shows the number of vehicles driving past Westvale High School each minute in a 20-minute period. Which of the following is the upper quartile Q_3?

Select **A**, **B**, **C** or **D**.

A 7.5 **B** 7

C 8.5 **D** 8

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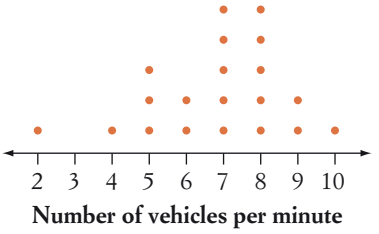
4 The percentage scores of a class of 30 students in a science test are shown below.

61	75	46	78	81	95
67	61	50	74	100	57
83	64	69	95	85	89
66	45	71	87	84	80
63	92	64	75	97	60



Example 11

a What is the 8th decile?

b What is the 3rd decile?

c What is the 40th percentile?

d Find the value that cuts off the lower 20% of scores from the upper 80%.

e What percentage of students scored higher than 79?

5 For the data shown in the dot plot in question **3**, find:

a the 1st decile

b the 5th decile

c the value that cuts off the lower 70% of scores

d the value that cuts off the top 60% of scores

e the 90th percentile

6 The following information is based on population data, for the body-mass indices (BMI kg/m^2) of boys aged 16 years.

Example 12

- The 1st quartile $Q_1 = 18.8$
- The 1st decile $D_1 = 17.6$
- The 9th decile $D_9 = 25.4$
- The 50th percentile $P_{50} = 20.6$
- The 97th percentile $P_{97} = 29.4$

In the questions below, all of the boys mentioned are aged 16.

a Sanjay has the median BMI for boys his age. What is his BMI?

b Michael has a BMI of 18.8. Is this high for his age? What percentage of boys aged 16 have a BMI lower than him?

c 10% of boys aged 16 have a higher BMI than Harley. What is his BMI?

d Adrian has a BMI of 29.4. Is this high for his age? What percentage of boys aged 16 have a BMI higher than him?

e What percentile is a BMI of 17.6?

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7 The information below is based on weather records kept by the Bureau of Meteorology, for the maximum daily temperatures in November for Newcastle.

- The mean is 23.5°C
- The highest temperature on record was 41.0°C (on 19 November 1968)
- The lowest temperature on record was 15.6°C (on 19 November 1986)
- The 1st decile $D_1 = 18.9$°C
- The 9th decile $D_9 = 28.6$°C

Source: Bureau of Meteorology

a What is the range of temperatures?

b What percentage of temperatures were higher than 28.6°C?

c To what value would you expect the median to be close (but not necessarily equal)?

d What value is higher than 10% of all temperatures recorded?

e What is the size of the 9th decile band (the difference between the highest temperature and the 9th decile)?

8 True or false?

a $P_{75} = Q_1$ **b** $P_{60} = D_6$ **c** $P_{50} = \text{Median}$

d $Q_3 = P_{75}$ **e** $D_8 = P_{20}$ **f** $Q_2 = D_5$

9 This table was published by the University Admissions Centre (UAC) giving the percentiles of different UAIs (University Admission Indices) for the 2008 HSC.

UAI	50	60	70	80	90	95	99
Percentile	27.3	29.2	52.7	67.7	83.6	91.7	98.4

Source: UAC

a What percentage of HSC students scored a UAI:

i below 60? **ii** below 80? **iii** above 90? **iv** between 50 and 60?

b 91.7% of students scored a UAI of below what value?

c Only 1.6% of students scored a UAI above what value?

d Write an estimate of the median UAI.

e What should be the percentile of a UAI of 100?

10 This table shows the percentiles for the heights (in cm) of girls aged 2 to 5 years, according to the child growth standards of the World Health Organisation (WHO).

Age (years)	P_5	P_{25}	P_{50}	P_{75}	P_{85}	P_{99}
2	80.4	83.5	85.7	87.9	89.1	93.2
2.5	84.9	88.3	90.7	93.1	94.3	98.9
3	88.8	92.5	95.1	97.6	99.0	103.9
3.5	92.4	96.3	99.0	101.8	103.3	108.5
4	95.6	99.8	102.7	105.6	107.2	112.8
4.5	98.7	103.1	106.2	109.2	110.9	116.7
5	100.5	106.2	109.4	112.6	114.4	120.5

Source: WHO

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a What is the median height of a 4-year-old girl?

b Libby is aged 2.5 and is 88.3 cm tall. Is she tall for her age? What percentage of girls her age are shorter than her?

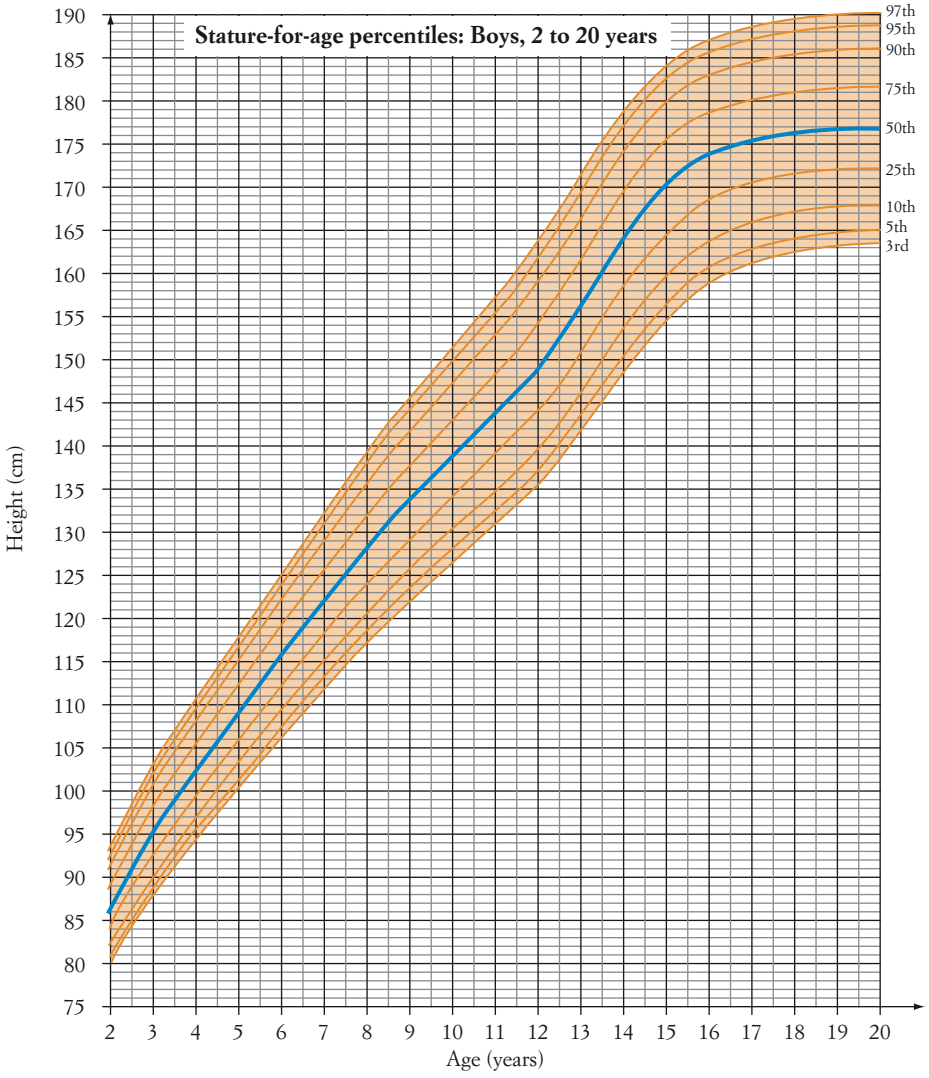
c What is Libby's expected height when she turns 5 years old?

d Only 15% of girls Renee's age are taller than her. How tall is she if she is 3.5 years old?

e Mikayla is 2 years old and 93.2 cm tall. Is she short for her age? What percentage of girls her age are taller than her?

f Mia is aged 3 and her height is at the 3rd quartile. What is her height now and in 18 months time?

11 This stature-for-age percentiles chart shows the range of heights for boys aged 2 to 20 years.



Source: National Center for Health Statistics (US) and National Center for Chronic Disease Prevention and Health Promotion (CDC)

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a Adam is aged 9 and 129 cm tall. What percentage of boys his age are shorter than him?

b Justin is 11 years old and 155 cm tall. What percentage of boys his age are shorter than him?

c How tall should Justin be when he turns 18?

d Liong is 103 cm tall, which is at the 1st decile for boys his age. How old is Liong?

e Asam is 16 and his height is at the 3rd quartile.

- **i** What is Asam's height now?
- **ii** What will Asam's height be when he turns 20 years old?

JUST FOR THE RECORD

Healthy growth charts for children

In 2006, the World Health Organisation (WHO) developed growth charts based on good health standards rather than the general population. They selected 8440 children who grew up in optimal healthy environments, from six countries: Brazil, Ghana, India, Norway, Oman and USA. These children were chosen because they were well-fed, breastfed as infants, not obese, their mothers did not smoke, and they had access to good health care where infections were controlled and prevented.

Selecting children from six different countries to represent the world's children is an example of stratified sampling. Why do you think the WHO chose those particular countries?

TECHNOLOGY: CALCULATING QUARTILES AND PERCENTILES

A spreadsheet can be used to calculate the quartiles, deciles and percentiles of a set of data.

Step 1: Open a blank spreadsheet to enter data in rows 2 and 3 as shown using the infant lengths from Example **11**.

	A	B	C	D	E	F	G	H	I	J	K
1	LENGTHS OF INFANTS (CM)										
2		44	45	47	48	48	48	49	49	49	50
3		50	50	51	51	52	52	52	53	55	56
4											
5				1st quartile Q1		48					
6				3rd quartile Q3		52					
7				2nd decile D2		48					
8				7th decile D2		51.3					
9				32nd percentile P32		49					
10				95th percentile P95		55.05					

Note: A spreadsheet calculates quartiles using a slightly different method to the method we have described, so its answers may not be exactly the same as ours, but they should be close.

Step 2: In cell F5, enter the formula **=QUARTILE(B2:K3,1)** to calculate $Q_1 = 48$.

Step 3: In cell F6, enter **=QUARTILE(B2:K3,3)** to calculate $Q_3 = 52$.

Step 4: In cell F7, enter **=PERCENTILE(B2:K3,0.2)** to calculate $P_{20} = D_2 = 48$.

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Step 5: In cell F8, enter **=PERCENTILE(B2:K3,0.7)** to calculate $P_{70} = D_7 = 51.3$.
Step 6: In cell F9, enter **=PERCENTILE(B2:K3,0.32)** to calculate $P_{32} = 49$.
Step 7: In cell F10, enter **=PERCENTILE(B2:K3,0.95)** to calculate $P_{95} = 55.05$.

9-05 The range and interquartile range

Interquartile range

While the mean, median and mode describe the centre of a data set, there are three summary statistics that describe the **spread** of data: the **range**, the **interquartile range** and the **standard deviation**. These are called **measures of spread**.

SUMMARY

Range = highest score – lowest score
Interquartile range (IQR) = upper quartile – lower quartile = $Q_3 - Q_1$

Standard deviation will be explained in Section 9-08, beginning on page 423.

EXAMPLE 13

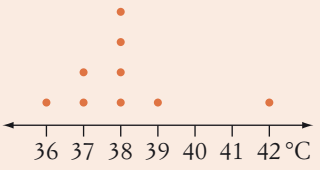
For each data set below, find:

i the range **ii** the interquartile range.

a The maximum daily temperature (in °C) in Mudgee for the first two weeks in January.

30 28 26 31 34 35 32 33 21 25 28 32 32 35

b The body temperatures (in °C) of a sample of hospital patients, as shown in the dot plot on the right.



Patients' temperatures

Solution

a **i** Range = 35 – 21 = 14

ii Placing the scores in order:

21 25 26 28 28 30 31 32 32 32 33 34 35 35

$Q_1 = 28$ Q_2 $Q_3 = 33$

IQR = $Q_3 - Q_1$
= 33 – 28
= 5

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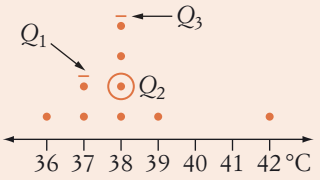
b **i** Range = 42 – 36 = 6

ii

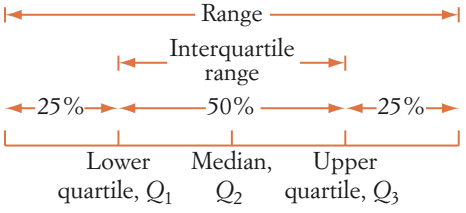


Patients' temperatures

Of 9 scores, the median, Q_2, is the 5th score, counting upwards from the left.

$$Q_1 = \frac{37 + 37}{2} = 37, \quad Q_3 = \frac{38 + 39}{2} = 38.5 \quad \text{IQR} = Q_3 - Q_1$$
$$= 38.5 - 37$$
$$= 1.5$$

The **range** represents the total spread of scores but it is not a good measure if there are outliers. The **interquartile range** is not affected by outliers, because it measures the range of the middle two quarters only.



Exercise 9-05 The range and interquartile range

Example 13

1 Calculate the range of each data set below.

a Number of accidents per month in a factory.

3 0 0 1 2 1 6 0 0 2 1 0

b A golfer's scores for the first nine holes of a golf course.

4 3 5 6 4 3 8 6 6

c Weekly mortgage repayments, in dollars.

370 628 299 417 354 1027 585 435 509 652 481

d Times, in minutes, for the swim-leg of a triathlon.

28 34 22 24 25 24 26 26 24 27

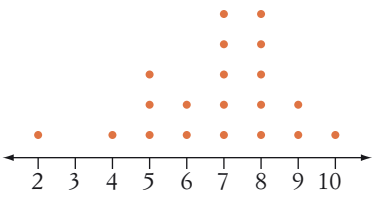
2 Calculate the interquartile range of each data set in question **1**.

3 The dot plot on the right shows the number of vehicles driving past Westvale High School each minute in a 20-minute period.

Which of the following is the interquartile range of this data set? Select **A**, **B**, **C** or **D**.

A 2.5 **B** 3

C 5 **D** 8



Number of vehicles per minute

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4 This stem-and-leaf plot shows the marks out of 100 for a class of students in a maths test.

For this data, find:

a the range

b the interquartile range.

Stem	Leaf
4	4 7 7 8
5	2 6 8 9 9
6	1 3 5 5 7 8
7	0 2 3 4 5
8	3 7 8
9	2 8

5 Fifteen job applicants took a short general knowledge multiple-choice quiz. Their times (in seconds) to complete this quiz were:

45 37 46 34 26 15 35 61
43 48 52 38 30 44 37

a What was the range of times?

b What was the interquartile range?

c Give a possible reason for the outlier.

6 The stem-and-leaf plot on the right represents the number of points per match scored by the Sharks in a football season.

Which of the following is the interquartile range of this data set? Select **A**, **B**, **C** or **D**.

A 46 **B** 18 **C** 21 **D** 20

Stem	Leaf
4	3 5 9
5	0 2 7 8
6	1 2 4 5 7 8
7	0 2 3 9
8	2 4 9

7 Calculate the range of the data set from question **6**.

9-06 Cumulative frequency graphs

Cumulative frequency graphs

A **cumulative frequency histogram** is a column graph of cumulative frequency.

A **cumulative frequency polygon**, also called an **ogive** (pronounced 'oh-jive') is drawn by joining the **top right-hand corner** of each column in a cumulative frequency histogram.

EXAMPLE 14

The maximum daily temperatures (in °C) in Campbelltown in June were recorded and grouped into the frequency table shown.

a Draw a cumulative frequency histogram and polygon for the data.

b Use the frequency polygon to find the median and calculate the interquartile range.

Temperature (°C)	Frequency	Cumulative frequency
12	1	1
13	2	3
14	6	9
15	2	11
16	6	17
17	3	20
18	6	26
19	1	27
20	2	29
21	1	30

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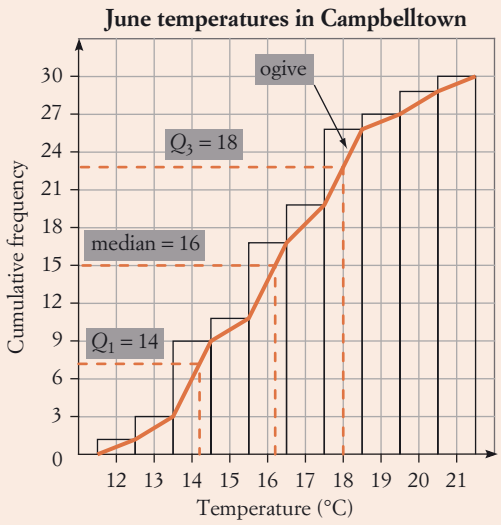
Solution

a



The ogive (polygon) is always inside the columns.

b Draw a horizontal line from the halfway mark (15) on the cumulative frequency axis to where it meets the ogive. The median is the corresponding value on the 'Temperature' axis.

Median = 16

To find Q_1, draw a horizontal line from the quarter mark ($\frac{1}{4} \times 30 = 7.5$) on the cumulative frequency axis to where it meets the ogive, then read the temperature value.

$Q_1 = 14$

To find Q_3, draw a horizontal line from the three-quarter mark ($\frac{3}{4} \times 30 = 22.5$) on the cumulative frequency axis.

$Q_3 = 18$

$$\begin{aligned}\text{Interquartile range} &= Q_3 - Q_1 \\ &= 18 - 14 \\ &= 4\end{aligned}$$

EXAMPLE 15

Use the cumulative frequency graph from Example **14** to answer the following questions.

a Find:

i the 4th decile, D_4

ii the 7th decile, D_7.

b What value cuts off the top 20% of temperatures?

c Between which two deciles would you find a temperature of 14°C?

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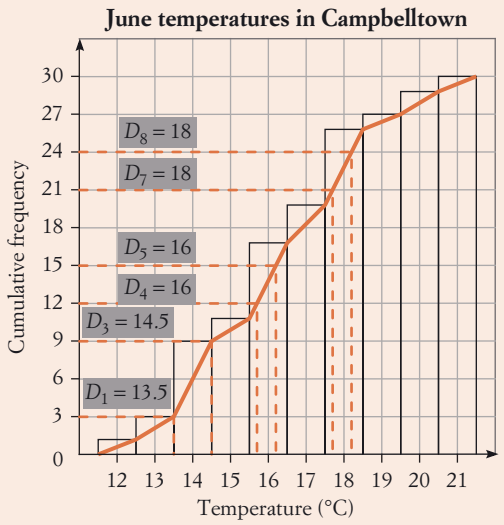
Solution

a



The deciles are marked at intervals of three units on the cumulative frequency axis.

i $D_4 = 16$

ii $D_7 = 18$

b D_8 cuts off the top 20% of temperatures, so the value is 18.

c Between D_1 and D_3.

EXAMPLE 16

The number of cases of ovarian cancer in women from various age groups is shown below.

Age (years)	Class centre	Frequency	Cumulative frequency
35–<45	40	28	28
45–<55	50	61	89
55–<65	60	65	154
65–<75	70	92	246
75–<85	80	74	320

Draw an ogive for this data and use it to find an estimate for:

a the median

b the 3rd quartile

c the 9th decile

d the interquartile range.

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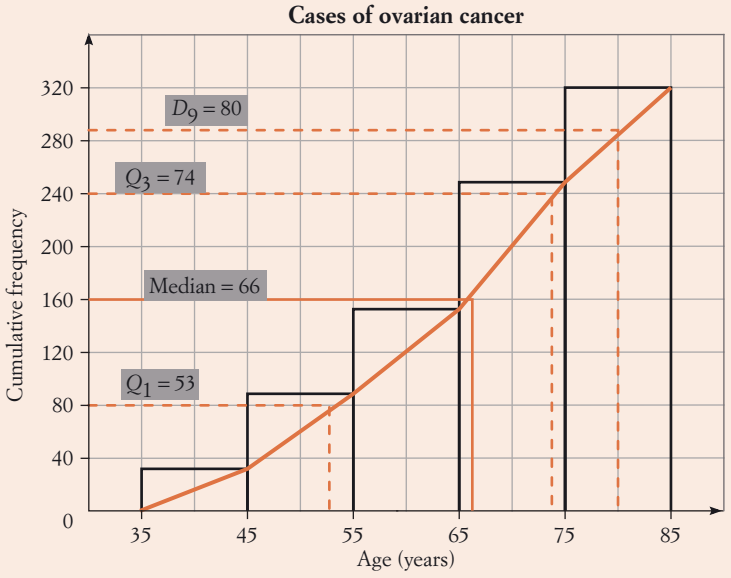
Solution



a Halfway point on the 'Cumulative frequency' axis = 160

Median ≈ 66 **Estimating from the 'Age' axis.**

All these values are estimates because the data has been grouped into class intervals.

b The three-quarter point on the 'Cumulative frequency' axis $= \frac{3}{4} \times 320 = 240$

$Q_3 \approx 74$

c 90% point on the 'Cumulative frequency' axis $= 0.9 \times 320$

$= 288$

$D_9 \approx 80$

d Quarter point on the 'Cumulative frequency' axis $= \frac{1}{4} \times 320$

$= 80$

$Q_1 \approx 53$

Interquartile range $= Q_3 - Q_1 = 74 - 53$

$= 21$

Exercise 9-06 Cumulative frequency graphs

Example 14

1 A sample of households was surveyed on the number of television sets owned. The results are shown in the table below.

Number of TVs	Frequency	Cumulative frequency
1	1	
2	7	
3	9	
4	6	
5	0	
6	1	

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a Copy the table and complete the cumulative frequency column. Then use the completed table to find the median.

b Construct a cumulative frequency histogram and polygon.

c Use the graphs you drew in part **b** to find:

i the median

ii the interquartile range.

Example **15**

2 This ogive shows the speeds of motor vehicles travelling along the main street of a town.

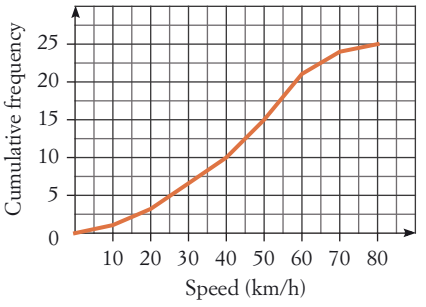
a How many vehicles were in the survey?

b Estimate the median speed of the vehicles

c Estimate the interquartile range.

d Estimate the 9th decile.

Speed of motor vehicles on main street



Fairfaxphotos/Gary Medlicott

3 A packet of jelly beans is labelled 'Contents 30' but a quality control check found the results shown in the table.

Number of jelly beans	Frequency	Cumulative frequency
28	6	
29	34	
30	56	
31	28	
32	5	
33	1	

a Copy the table and complete the cumulative frequency column.

b Construct an ogive and use it to find an estimate of:

i the median

ii the interquartile range

iii the 4th decile.

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Example 16

4 The heights of 50 students were measured and grouped into class intervals. The results are shown in the table below.

Height (cm)	Class centre	Frequency	Cumulative frequency
134–<141		2	
141–<148		3	
148–<155		4	
155–<162		13	
162–<169		15	
169–<176		11	
176–<183		2	

a Copy and complete the table.

b What is the modal class?

c What is the median class?

d Construct an ogive and use it to estimate:

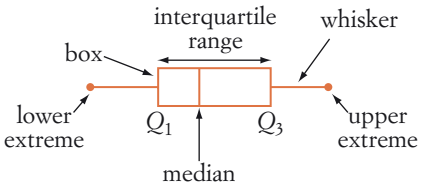
i the median

ii the interquartile range

iii the 7th decile.

9-07 Box-and-whisker plots

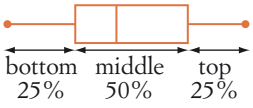
A **box-and-whisker** plot displays the quartiles of a set of data and the lowest and highest scores. The 'box' represents the middle 50% of scores and the interquartile range, while the 'whiskers' represent the lowest and highest 25% of scores.



WS Box-and-whisker plots: Graphics calculator

A box-and-whisker plot gives a **five-number summary** of a data set:

- the lower extreme (lowest score)
- the lower quartile, Q_1
- the median, Q_2
- the upper quartile, Q_3
- the upper extreme (highest score).



EXAMPLE 17

The ages of 10 people at a park were:

21 13 64 75 35 83 7 71 18 29

a Find a five-number summary for this data.

b Represent this data on a box-and-whisker plot.

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Solution

a In order:

7 13 18 21 29 35 64 71 75 83

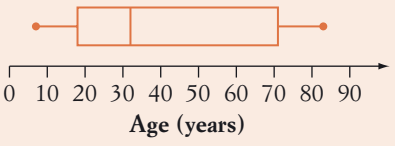
Q_1 Q_2 Q_3

Lower extreme = 7 Lower quartile = 18

Median $= \dfrac{29+35}{2} = 32$

Upper quartile = 71 Upper extreme = 83

b

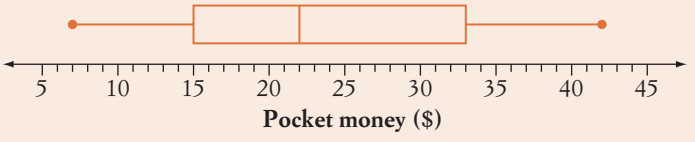


This box-and-whisker plot shows that, roughly:

- the bottom 25% of scores lie from 7 to 18
- the next 25% of scores lie from 18 to 32
- the median is 32
- the top 25% of scores lie from 71 to 83.

EXAMPLE 18

This box-and-whisker plot represents the amount of pocket money in dollars earned by a sample of 48 children.



a Find the median.

b Find the range.

c How many children earned between:

i \$33 and \$42? **ii** \$15 and \$42?

d Find the interquartile range.

Solution

a Median = \$22

b Range = \$42 – \$7 = \$35

c **i** $\frac{1}{4} \times 48$ children = 12 children **Top 25%.**

ii $\frac{3}{4} \times 48$ children = 36 children **Top 75%.**

d Interquartile range = \$33 – \$15 = \$18

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Exercise 9-07 Box-and-whisker plots

Example 17

1 Tom's scores for the 18 holes of a golf course were:

3	4	6	8	7	9	5	9	11
5	7	4	5	8	6	9	10	5

a Find a five-number summary for this data.

b Represent this data on a box-and-whisker plot.

2 Fifteen job applicants took a short general knowledge multiple-choice quiz. Their times, in seconds, to complete this quiz are shown below. Graph this data on a box-and-whisker plot.

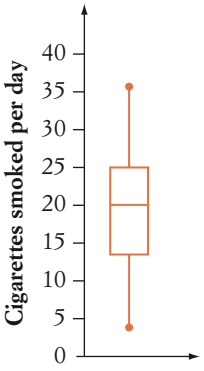
45	37	46	34	26	15	35	61
43	48	52	38	30	44	37	

3 Find a five-number summary for the data in this stem-and-leaf plot of ages of people at the cinema, then draw a box-and-whisker plot for them.

Stem	Leaf
1	4 7 7 8
2	6 8 9 9
3	1 3 5 5 7 8
4	0 2 2 4 5
5	3 7 8
6	2 9

Example 18

4 This box-and-whisker plot illustrates the number of cigarettes smoked per day by a sample of 60 smokers who are trying to quit.



a What is the median number of cigarettes smoked per day?

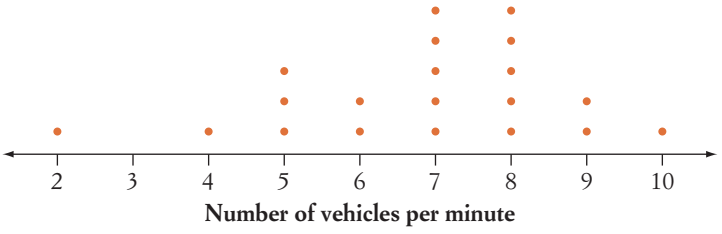
b What is the interquartile range?

c What is the lower extreme?

d How many people smoked between 20 and 25 cigarettes per day?

e How many people smoked fewer than 20 cigarettes per day?

5 This dot plot shows the number of vehicles driving past Westvale High School per minute in a 20-minute period.



a Find the five-number summary for this data and draw a box-and-whisker plot.

b Compare the box-and-whisker plot you drew in part **a** with the original dot plot. Which one do you prefer? Why?

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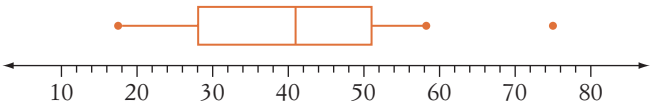
6 This box-and-whisker plot represents the annual wages (× \$1000) of the administration staff at a TAFE college.

Annual wages of TAFE administrative staff



Wages (× \$1000)

a One of the wages is an outlier which was not included in the box-and-whisker plot. What is the outlier?

b What is the median wage?

c Excluding the outlier, what is the range of wages?

d Including the outlier, what is the range of wages?

e Between what two amounts are the middle 50% of staff wages?

f What percentage of the staff earn less than \$28 000?

9-08 Standard deviation

Statistical calculations

Standard deviation is a better measure of spread than the range and interquartile range because, like the mean, its value depends on every score in the data set. Standard deviation measures how different each score in a data set is from the mean.

The symbol for standard deviation is σ_n or σ, which is the Greek letter for 's', pronounced 'sigma'.

Standard deviation can be calculated using the calculator's statistics mode.

The formula for calculating standard deviation is quite complicated, and does not need to be memorised. There are also two types of standard deviation, but in this course, we will use only the *population* standard deviation (not the sample standard deviation).

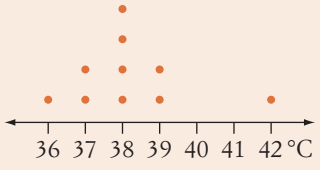
EXAMPLE 19

Use the calculator's statistics mode to find, correct to one decimal place, the standard deviation of each data set below.

a The maximum daily temperature (in °C) in Mudgee for the first two weeks in January.

30	28	26	31	34	35	32
33	21	25	28	32	32	35

b The body temperatures (in °C) of a sample of hospital patients.



Patients' temperatures

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Solution

a $\sigma_n = 3.9434... \approx 3.9$

Operation	Casio scientific	Sharp scientific
Refer to page 388 of this textbook to enter the data.		
Calculate the population standard deviation ($\sigma_n = 3.9434...$)	SHIFT 1 Var $x\sigma_n$ =	RCL σx

b $\sigma_n = 1.5362...$

≈ 1.5

Refer to the table of calculator instructions on page 390 to enter data presented in a frequency table, then follow the instructions from part **a** above.

Exercise 9-08 Standard deviation

Example 19

1 The number of monthly accidents at a construction site over 8 months was:

3 0 4 2 3 0 2 2

a Calculate the mean number of accidents per month.

b Find the standard deviation for the data, correct to one decimal place.

2 An express train from Central Station was late in arriving at Homebush by the following times (in minutes):

6 0 3 –2 5 –1 0 3 –1 6 7 1

a Find the mean, $\bar{x}$.

b Calculate, correct to two decimal places, the standard deviation, σ_n.

c Evaluate $\bar{x} + \sigma_n$ and $\bar{x} - \sigma_n$, the values that are, respectively, one standard deviation below and one standard deviation above the mean.

d How many of the given scores lie within one standard deviation of the mean, that is, between the two values you calculated in part **c**?

e What percentage, correct to one decimal place, of scores are within one standard deviation of the mean?

3 A sample of mobile phone batteries was tested for charge life (in hours). The results are shown below.

60 73 65 84 77 64 66 73 88 90 79 81

Calculate:

a the mean

b the standard deviation (to two decimal places).

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4 Blake's weekly commissions, in dollars, for selling Internet plans were:

540 510 1100 1350 780 650 920 590 1080

Calculate for this data, correct to the nearest dollar:

a the mean

b the standard deviation.

5 Students were surveyed on the number of movies they had downloaded in the last six months, with the results shown in the frequency table on the right.

Score (x)	Frequency (f)
0	6
1	7
2	8
3	10
4	9
5	5
6	5

a For this data, find the mean, $\bar{x}$.

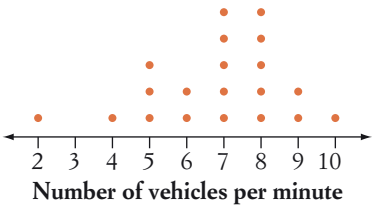
b Calculate, correct to one decimal place, the standard deviation.

c How many scores were within one standard deviation of the mean?

d What percentage of scores were within one standard deviation of the mean?

For many large sets of data, approximately 68% (slightly more than $\frac{2}{3}$) of the scores lie within one standard deviation of the mean.

6 This dot plot shows the number of vehicles driving past Westvale High School every minute for a 20-minute period.



a Find the mean.

b Calculate, correct to two decimal places, the standard deviation.

c How many scores are within one standard deviation of the mean?

d What percentage of scores are within one standard deviation of the mean?

7 The table on the right shows the weekly wages of employees at Great Gals electrical store, grouped in classes of \$100.

Weekly wage (\$)	Class centre	Frequency
\$500–<\$600		7
\$600–<\$700		20
\$700–<\$800		36
\$800–<\$900		17
\$900–<\$1000		11
\$1000–<\$1100		3

a Copy and complete the table.

b Find, to the nearest cent, an estimate for:

i the mean

ii the standard deviation.

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TECHNOLOGY: CALCULATING MEASURES OF SPREAD

Step 1: Open a blank spreadsheet and enter the temperature data about Mudgee from Example **19**.

	A	B	C	D	E	F	G
1			**Maximum daily temperatures in Mudgee**				
2	30	28	26	31	34	35	32
3	33	21	25	28	32	32	35
4							
5			Highest score				
6			Lowest score				
7			Upper quartile, Q_3				
8			Lower quartile, Q_1				
9							
10			Range				
11			Interquartile range				
12			Standard deviation				
13							

Step 2: In cell E5, enter the formula **=MAX(A2:G3)** to calculate the highest score (35).

Step 3: In cell E6, enter **=MIN(A2:G3)** to calculate the lowest score (21).

Step 4: In cell E7, enter **=QUARTILE(A2:G3,3)** to calculate the upper quartile, Q_3 (32.75).

Note: A spreadsheet calculates quartiles using a slightly different method to the method we have described, so its answers may not be exactly the same as ours, but they should be close.

Step 5: In cell E8, enter **=QUARTILE(A2:G3,1)** to calculate the lower quartile, Q_1 (28).

Step 6: In cell E10, enter **=E5-E6** to calculate the range (14).

Step 7: In cell E11, enter **=E7-E8** to calculate the interquartile range (4.75).

Step 8: In cell E12, enter **=STDEV.P(A2:G3)** to calculate the population standard deviation

9-09 Comparing samples and populations

Governments and businesses do not base important decisions on just one sample. They generally take a number of samples from a population and calculate the statistics of each sample. The averages of these sample statistics are then used to estimate the statistics for the entire population. Also, the larger the samples are, the better the estimates.

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EXAMPLE 20

The ages of 100 people working at Burger Haven this year are:

18	19	18	17	20	20	24	15	24	19
15	35	15	24	22	19	15	17	23	29
15	40	21	17	20	22	23	21	24	23
22	16	36	15	16	24	16	15	19	15
34	19	45	20	15	21	24	27	19	33
18	27	15	30	15	34	17	29	25	17
24	24	16	18	25	21	26	21	16	25
18	18	16	22	25	22	15	38	34	43
29	21	21	18	16	22	15	25	18	23
17	24	18	18	20	22	28	22	16	25

a Randomly select five samples of 10 ages from this population of employees and, for each sample, calculate the mean, standard deviation, median and interquartile range.

b Estimate the mean, standard deviation, median and interquartile range of the population from your five samples.

Solution

a Randomly select five samples of 10 ages from the list above. For example:

Sample 1:	21	16	15	15	16	30	19	30	35	24
Sample 2:	15	23	15	25	17	43	20	24	40	24
Sample 3:	22	19	24	24	22	24	19	15	23	15
Sample 4:	20	24	15	18	18	18	24	24	23	22
Sample 5:	25	25	16	24	29	16	15	18	24	23

Then calculate the statistics for each sample.

	Mean	Standard deviation	Median	Interquartile range
Sample 1	22.1	6.93	20	14
Sample 2	24.6	9.18	23.5	8
Sample 3	20.7	3.35	22	5
Sample 4	20.6	3.07	21	6
Sample 5	21.5	4.59	23.5	9

b Estimates of the population statistics by taking averages:

$$\text{Mean} = \frac{22.1 + 24.6 + 20.7 + 20.6 + 21.5}{5} = 21.9$$

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$$\text{Standard deviation} = \frac{6.93 + 9.18 + 3.35 + 3.07 + 4.59}{5} = 5.424$$

$$\text{Median} = \frac{20 + 23.5 + 22 + 21 + 23.5}{5} = 22$$

$$\text{Interquartile range} = \frac{14 + 8 + 5 + 6 + 9}{5} = 8.4$$

Example 20

Exercise 9-09 Comparing samples and populations

1 **a** As in Example **20**, randomly select five samples of 10 ages from the population of Burger Heaven employees and, for each sample, calculate and record in a table the mean, sample standard deviation, median and interquartile range.

b Estimate the mean, standard deviation, median and interquartile range of the population from your five samples.

2 **a** For the entire population of 100 ages given in Example **20**, calculate the actual mean, standard deviation, median and interquartile range.

b How do the statistics of each of your samples compare to the population statistics?

c How do the estimated statistics from question **1b** compare to the population statistics?

3 **a** Copy this table.

Sample size	Mean	Sample standard deviation	Mean	Interquartile range
$n = 5$				
$n = 10$				
$n = 20$				
$n = 30$				
$n = 40$				

b Select a random sample of only five ages from the population given in Example **20** and calculate and record in the table the mean, standard deviation, median and interquartile range for this sample.

c Repeat part **b** for random samples of 10 ages, 20 ages, 30 ages and 40 ages, recording your results in the table.

d How do your results compare to the actual population statistics calculated in question **2a**? As the sample size increases, do the sample statistics become more accurate and closer to the population statistics?

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Sample HSC problem

The ages, in years, of a sample of patients at a hospital are shown in the stem-and-leaf plot on the right.

a Find the mean age of the patients.

b Find the median age of the patients.

c Is the mean or median more appropriate for describing the average age of the patients? Give a reason for your answer.

d Find the interquartile range of the patients' ages.

e Represent this data set on a box-and-whisker plot.

Stem	Leaf
1	2 2 3 4 6
2	1 2
3	0 0 0 3
4	4 7 8
5	1 1
6	
7	5 7 8
8	1

Study tip

Your study routine

- Your study routine will be more effective if you maintain a healthy lifestyle: eat, sleep, rest, exercise, socialise and generally be happy!
- Try to study at the same time and place each day.
- When studying, alternate between easy tasks and hard tasks.
- Stop and reflect on what you have achieved before moving on.
- Use your time wisely and take regular breaks.
- Ask yourself: 'Am I really studying effectively or am I wasting time? Do I feel a sense of achievement?'
- Take a 1-minute break after completing each study or homework task.
- Take a longer break after working for more than 45 minutes continuously.
- Reward yourself when you feel you have completed a significant amount of work: watch a bit of television, have a snack, play some sport or a game, or go for a walk.

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CHAPTER REVIEW

Topic overview

This chapter, Analysing data, examined the statistical measures of location (mean, median mode) and spread (range, interquartile range, standard deviation). You should be competent at making statistical calculations on sets of quantitative data, including those represented in frequency tables, class intervals (grouped data), dot plots and stem-and-leaf plots. Make sure you know how to use the statistical functions on your calculator. You should understand the new concepts of quartiles, deciles and percentiles, and be able to interpret cumulative frequency graphs and box-and-whisker plots.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map has been printed below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.

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Revision

Statistics review

Statistics crossword

Exercise 9-01

1 The heights (in centimetres) of a group of ballet dancers are listed below. Calculate the mean correct to one decimal place.

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165	183	170	168	175	179	168	170
181	168	172	177	171	170	175	179

Exercise 9-01

2 Motor vehicles were clocked, by police radar, travelling at the following speeds (in km/h).

78	95	64	77	81	84	77	89	90	78
79	80	82	84	80	79	95	86	84	70
78	65	82	91	89	60	85	81	78	68
90	84	69	70	80	91	85	84	80	76
68	65	85	76	79	83	82	91	84	80

a Sort the data in a frequency table using classes of 60–<70, 70–<80, and so on, and include a column of class centres.

b Calculate an estimate for the mean speed.

Exercise 9-02

3 Find the median and mode of the ballet dancers' heights from question **1**.

Exercise 9-02

4 This dot plot represents the sums of two dice rolled 20 times. Find the median and mode of this data.

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Exercise 9-03

5 The house prices realised at auction one Saturday in Vincentia were:

\$342 000	\$264 000	\$268 000	\$517 000
\$1 044 000	\$420 000	\$348 000	\$297 000

a Calculate the mean price.

b Calculate the median price.

c Is the mean or the median the better measure to use as the average price of a house in Vincentia? Why?

Exercise 9-03

6 Which measure of location is the most appropriate for describing each average below? Give a reason for each answer.

a the average men's shoe size

b the average height of Year 11 students

c the average starting salary of an Australian worker

Exercise 9-04

7 In a national mathematics test, Simone scored 84.

a This score was above the 7th decile, D_7. Approximately what percentage of students taking the test scored lower than her?

b More specifically, Simone's score was at the 78th percentile, P_{78}. What percentage of students scored higher than her?

Exercise 9-05

8 **a** What is the meaning of 'interquartile range'?

b A random sample of 15 packets of chips had the following masses, in grams. Find the range and interquartile range of these masses.

52	51	50	49	50	50	48	51
51	50	49	53	50	49	51	

Exercise 9-05

9 This stem-and-leaf plot represents the number of points per match scored by the Sharks in a football season.

Stem	Leaf
4	3 5 5
5	0 2 7 8
6	1 2 4 5 7 8
7	0 2 2 3
8	2 4 9

For this data, find:

a the median

b the mode

c the range

d the interquartile range.

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10 Students were surveyed about the number of pairs of shoes they owned, and the results are shown in the table below. Exercise 9-06

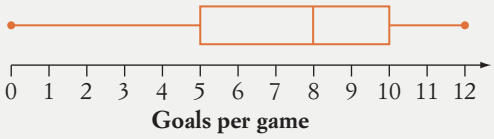
Pairs of shoes	Frequency
5	8
6	10
7	9
8	5
9	5

a Copy the table, adding a cumulative frequency column, then draw a cumulative frequency histogram and polygon.

b Use your polygon to calculate:

i the median **ii** the interquartile range **iii** the 3rd decile.

11 This box-and-whisker plot represents the number of goals scored per game by a hockey team over a season. Exercise 9-07



a What was the lowest score?

b Find the interquartile range.

c In what fraction of games were over 8 goals scored?

d In what percentage of games were less than 5 goals scored?

12 a Create a five-number summary for the chip packet masses in question **8b**. Exercise 9-07

b Represent the mass data on a box-and-whisker plot.

13 For quality testing, a manufacturer takes a random sample of 10 screws, each designed to have a length of 2 cm. The actual lengths of the screws, in centimetres, are: Exercise 9-08

2.00 1.99 1.98 2.01 2.01 1.97 2.03 1.98 2.01 2.00

a Find the mean screw length.

b Find the standard deviation, correct to two decimal places.

14 For the shoe data from question **10**, calculate correct to one decimal place: Exercise 9-08

a the mean **b** the standard deviation.

15 When taking samples from a population to estimate the statistics of the population, what are two ways of improving the estimates? Exercise 9-09

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Practice paper 3

Recommended time: One hour

Section 1

10 objective-response questions: 1 mark each
For Questions 1 to 6, select the correct answer, A, B, C or D

Exercise 7-01

Question 1

Which one of these is not a linear function?

(A) $xy = 3$

(B) $y = \frac{1}{2}x + 3$

(C) $y = 4 - x$

(D) $y = x$

Exercise 8-05

Question 2

A Subaru car can travel 465 km on a 50 L tank of petrol. What is its fuel consumption in L/100 km?

(A) 0.9

(B) 1.1

(C) 9.3

(D) 10.8

Exercise 8-01

Question 3

Belinda pays for third party property motor insurance by a direct debit of \$19.10 per fortnight. How much would she save by paying a yearly amount of \$399 instead?

(A) nothing

(B) \$97.60

(C) \$169.80

(D) \$195.20

Exercise 9-02

Question 4

Which statement about the set of scores {0, 1, 2, 3, 4, 4, 7} is true?

(A) The mean and mode are equal.

(B) The mean and median are equal.

(C) The median and mode are equal.

(D) The mean, median and mode are not equal.

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Exercise 7-04

Question 5

T is directly proportional to M. When $M = 4$, $T = 10$. What is the value of T when $M = 5$?

(A) 20

(B) 12

(C) 12.5

(D) 2.5

Exercise 8-03

Question 6

To buy a car, Jesse borrowed \$7500 from NCM Bank at a flat rate of 7.5% p.a. over 4 years. What is his monthly repayment?

(A) \$46.88

(B) \$117.19

(C) \$187.50

(D) \$203.13

For Questions 7 to 10, write the answer only (do not show your working).

Exercise 8-01

Question 7

The stamp duty for purchasing a car is 3% of the purchase price up to \$45 000 plus 5% for every dollar over \$45 000. How much stamp duty did Tayla pay when buying a car for \$39 450?

Exercise 7-02

Question 8

Find the gradient of the line that goes through the points (1, 8) and (5, 0).

Exercise 8-05

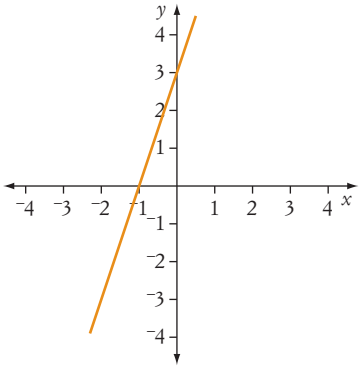
Question 9

A Holden ute has a fuel consumption of 9.8 L/100 km. How much fuel does it need to travel 1250 km?

Exercise 7-03

Question 10

Find the equation of the line.



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Section 2

3 questions: 10 marks each

Question 11 **Marks**

Exercise 7-03

(a) (i) What is the gradient and y-intercept of the line $y = -\frac{1}{2}x + 4$? **2**

Exercise 9-08

(ii) Graph the line on a number plane. **2**

(b) The number of accidents per month on a building site were:

9 4 3 0 1 7 5 6 5 3

For this data, find:

(i) the mean **1**

(ii) the median **1**

(iii) the interquartile range **1**

(iv) the standard deviation, correct to two decimal places **1**

Exercise 9-04

(c) The following information is based on population data for the body-mass indices (BMI kg/m^2) of boys aged 16 years.

- The 1st quartile $Q_1 = 18.8$
- The 5th decile $D_5 = 20.6$
- The 9th decile $D_9 = 25.4$
- The 10th percentile $P_{10} = 17.6$
- The 97th percentile $P_{97} = 29.4$

(i) Dylan is 16 and has a BMI of 25.4. What percentage of boys aged 16 have a BMI higher than him? **1**

(ii) Santosh has the median BMI for 16-year-old boys. What is his BMI? **1**

Question 12

Exercise 7-03

(a) The table below shows the volume of water (in kilolitres) in a pool as it is drained.

Time, t (hours)	2	3	6	8	9
Volume, V (kL)	73	67.5	51	40	34.5

(i) Find the linear function for V in terms of t. **1**

(ii) What does the gradient of this function represent? **1**

(iii) What does the vertical intercept of this function represent? **1**

(iv) After how many hours, to one decimal place, is the pool drained completely? **1**

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Marks

Exercise 9-06

(b) A sample of households was surveyed on the number of motor vehicles owned.

No. of vehicles	Frequency	Cumulative frequency
0	1	
1	14	
2	26	
3	7	
4	2	

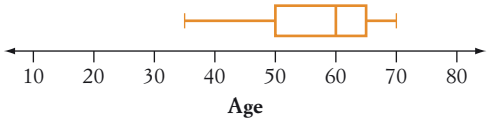
(i) Copy and complete the cumulative frequency column of the table. **1**

(ii) Construct a cumulative frequency histogram and polygon for the data. **1**

(iii) Use the cumulative frequency polygon to find the interquartile range. **1**

Exercise 9-07

(c) This box-and-whisker plot shows the ages of members at a lawn bowls club.



(i) What is the age of the oldest member? **1**

(ii) Calculate the interquartile range of ages. **1**

(iii) If there are 124 members in the club, how many of them are aged between 50 and 60? **1**

Question 13

Exercise 8-04

(a) A car is purchased for \$32 000 and depreciates by \$3500 each year.

(i) Calculate its value after $3\frac{1}{2}$ years. **1**

(ii) Find when the car's value will be \$4000. **1**

Exercise 7-06

(b) Graph $y = 5 - x$ and $y = 2x - 4$ on the same number plane and write the solution to the simultaneous equations $y = 5 - x$ and $y = 2x - 4$. **3**

Exercise 8-02

(c) The stamp duty for purchasing a car is 3% of the purchase price up to \$45 000 plus 5% for every dollar over \$45 000.

(i) Copy and complete the table below. **2**

Price (\$)	0	10 000	20 000	30 000	40 000	45 000	50 000	60 000
Stamp duty (\$)	0	300						

(ii) Use the completed table of values from part **(i)** to construct a piecewise linear graph for stamp duty. **2**

(iii) Use the graph to estimate the stamp duty on a vehicle purchased for \$35 000. **1**

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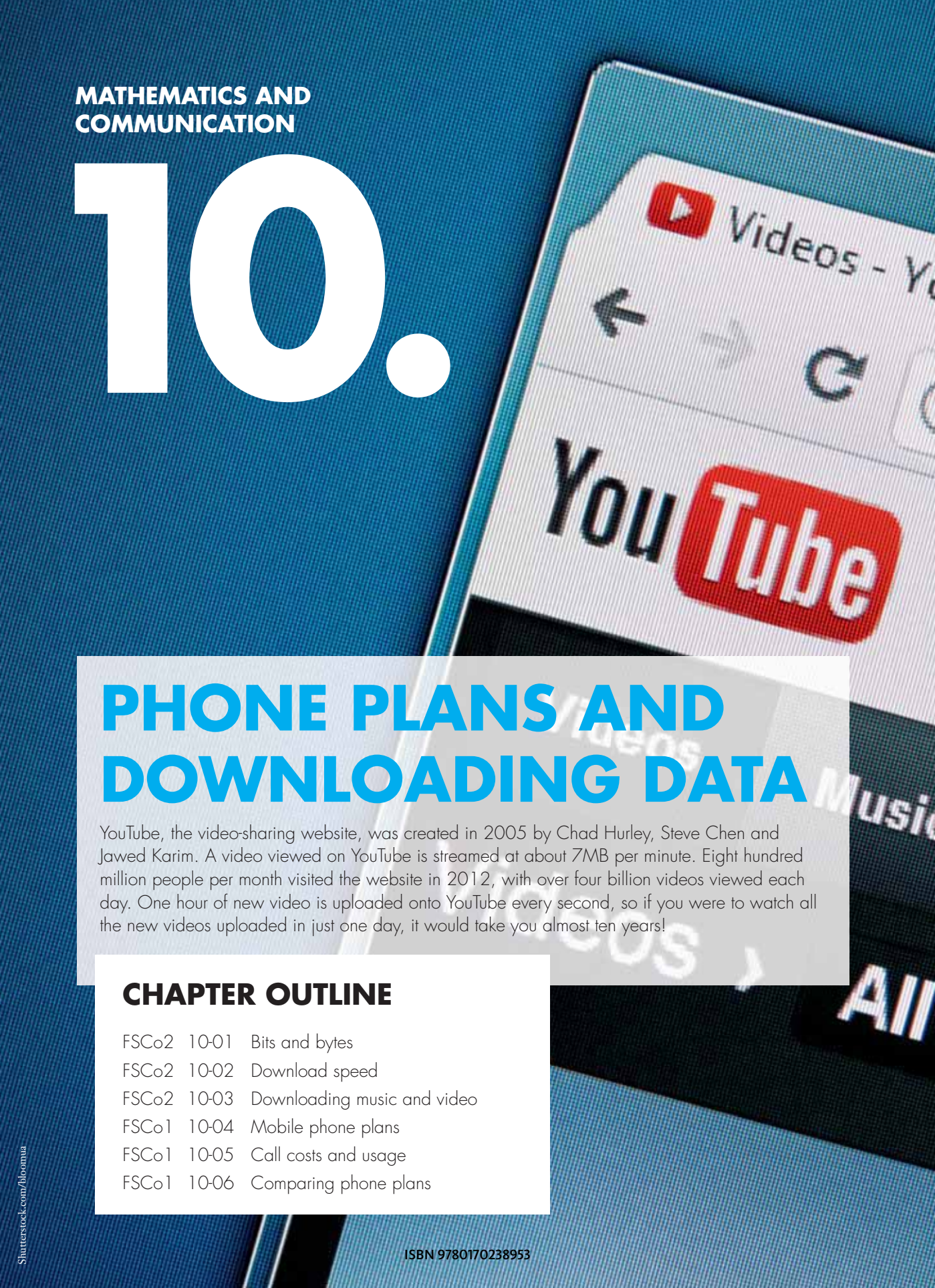
MATHEMATICS AND COMMUNICATION

10.



PHONE PLANS AND DOWNLOADING DATA

YouTube, the video-sharing website, was created in 2005 by Chad Hurley, Steve Chen and Jawed Karim. A video viewed on YouTube is streamed at about 7MB per minute. Eight hundred million people per month visited the website in 2012, with over four billion videos viewed each day. One hour of new video is uploaded onto YouTube every second, so if you were to watch all the new videos uploaded in just one day, it would take you almost ten years!

CHAPTER OUTLINE

FSCo2	10-01	Bits and bytes
FSCo2	10-02	Download speed
FSCo2	10-03	Downloading music and video
FSCo1	10-04	Mobile phone plans
FSCo1	10-05	Call costs and usage
FSCo1	10-06	Comparing phone plans

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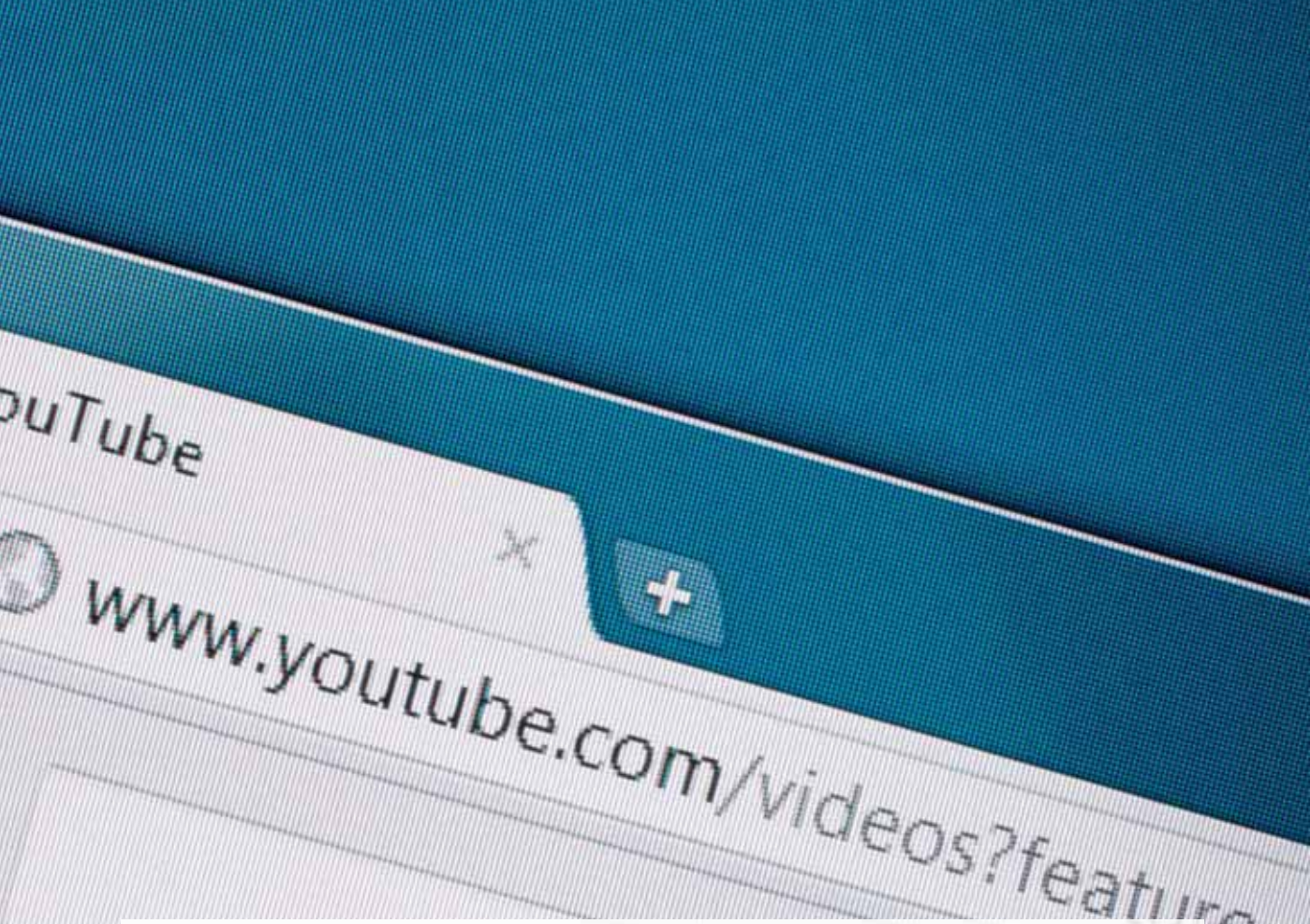

IN THIS CHAPTER YOU WILL:

- use prefixes to describe the size of units of storage, for example, mega-, giga-, tera-
- convert units of storage from bits to bytes, and vice versa
- convert between units for measuring memory size, file size and secondary storage on devices such as USB drives and external hard drives
- calculate the time to download or upload a file, given a download or upload speed in bits per second, or kilobits per second, where a kilobit is defined as 1000 bits
- collect, display and analyse statistics on the downloading of music and videos
- calculate the probabilities of songs being played using the random selection mode
- interpret statistics related to the effect of downloading audio files and video files, legally and illegally, on the sales of media companies
- read and interpret mobile phone bills
- read and interpret mobile phone plans
- calculate the cost of calls, given the time and duration, based on different mobile phone plans
- calculate the cost of sending and receiving messages, given the details of the mobile phone plan
- calculate the cost of data usage, given the details of the mobile phone plan
- construct and interpret tables and graphs of mobile phone usage and the cost of making mobile phone calls (including graphs of step-wise linear functions)
- identify patterns of usage for a given mobile phone bill
- determine a suitable mobile phone plan using calculations based on a typical usage pattern

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TERMINOLOGY

additional
byte
data usage
included credit
megabyte (MB)
post-paid plan
upload
bit
cap plan
download
kilobits per second (kbps)
memory size
SMS text
bits per second (bps)
connection fee
gigabyte (GB)
kilobyte (kB)
prepaid plan
terabyte (TB)

WS
Assignment 10

SkillCheck

1 Convert:

a 56 kg to g **b** 128 ML to L **c** 3500 m to km

2 By how much is:

a 1024 greater than 1000? **b** $344.20 greater than $250?

3 A car is travelling at a speed of 85 km/h.

a How far will it travel in 4 hours?

b How long will it take to travel 765 km?

c How long (correct to the nearest second) will it take to travel 500 metres?

4 If a bottle of drink costs $2.10, how many bottles can you buy for $15?

5 A plumber charges a $30 callout fee plus $26 per hour of working or part thereof (meaning that a part of an hour is counted as one whole hour). How much will she charge for a job taking 2 hours and 15 minutes?

6 Convert:

a 12.49 minutes to minutes and seconds

b 4 min 25s to seconds

c 2458 seconds to minutes and seconds

d 693 seconds to minutes and seconds

7 This step graph shows the cost of an international phone call that lasts under 5 minutes.

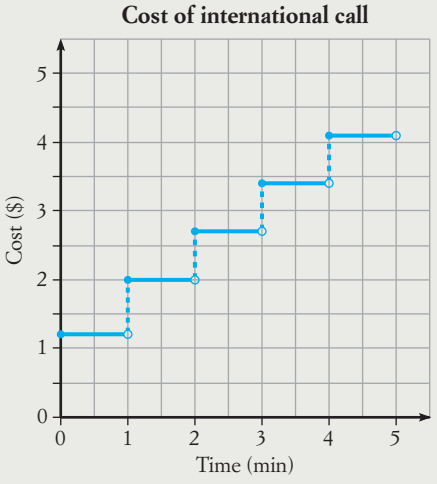
a What is the cost of a call lasting:

i 4 min 20 s? **ii** 2 min?

b What is the minimum time you can talk for $3.40?

c What is the maximum time you can talk for $3.40?

d You called a friend overseas, the phone was busy and you hung up. How much were you charged?



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8 A phone call costs 74c per minute.

a Find the cost of a 7-minute call.

b If Jessie's call cost $9.62, how long was the call?

10-01 Bits and bytes

Memory size cards

Amrita bought a computer with a 64-bit processor, 3 gigabytes of RAM and 1 terabyte of hard disk space. What does this mean?

In technology, digital memory size is measured in **bits** or **bytes**.

Bits

At its most basic level, a computer represents all information as a series of 0s and 1s, representing the electrical values of OFF or ON respectively. These **binary digits**, or **bits**, give their name to the smallest unit of computer memory, abbreviated **b**. The table below shows units of memory involving bits.

SUMMARY

Unit	Relationship	Metric prefix
kilobit (kb)	1 kb = 1000 b	kilo- means 'one thousand' 1 kb = 1000 b = 10^3 b
megabit (Mb)	1 Mb = 1000 kb	mega- means 'one million' 1 Mb = 1 000 000 b = 10^6 b
gigabit (Gb)	1 Gb = 1000 Mb	giga- means 'one billion' 1 Gb = 1 000 000 000 b = 10^9 b
terabit (Tb)	1 Tb = 1000 Gb	tera- means 'one trillion' 1 Tb = 1 000 000 000 000 b = 10^{12} b

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EXAMPLE 1

Convert:

a 250 Mb to b **b** 5600 b to kb **c** 34 Gb to kb

Give answers in scientific notation where necessary.

Solution

a $250 \text{ Mb} = 250 \times 1\,000\,000 \text{ b}$ **1 Mb = 1 million bits**

$= 250\,000\,000 \text{ b}$

$= 2.5 \times 10^8 \text{ b}$

b $5600 \text{ b} = 5600 \div 1000 \text{ kb}$ **1 kb = 1000 bits**

$= 5.6 \text{ kb}$

c $34 \text{ Gb} = 34 \times 1\,000 \text{ Mb}$ **1 Gb = 1000 Mb**

$= 34 \times 1\,000 \times 1000 \text{ kb}$ **1 Mb = 1000 kb**

$= 34\,000\,000 \text{ kb}$

$= 3.4 \times 10^7 \text{ kb}$

Bytes

Computer memory is usually measured in **bytes**, where one byte is equal to 8 bits. A byte, abbreviated **B**, is the amount of memory needed to store one character, such as @, g, #, 7 or ?. For example, the letter 'g' is represented by the byte 01100111.

The table below shows units of **memory** involving bytes.

SUMMARY

Unit	Relationship	Number of bytes
byte (B)	1 B = 8 b	1 B
kilobyte (kB)	1 kB = 1024 B	1024 B or 2^{10} B (approx. 1000)
megabyte (MB)	1 MB = 1024 kB	$1024 \times 1024 = 1\,048\,576$ B or 2^{20} B (approx. one million)
gigabyte (GB)	1 GB = 1024 MB	$(1024)^3 = 1\,073\,741\,824$ B or 2^{30} B (approx. one billion)
terabyte (TB)	1 TB = 1024 GB	$(1024)^4 = 1\,099\,511\,627\,776$ B or 2^{40} B (approx. one trillion)

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Note that although 'kilo' means thousand, one kilobyte is only *approximately* equal to 1000 bytes. It is exactly equal to 1024 bytes, where $1024 = 2^{10}$.

- A **kilobyte** stores about half a page of text.
- A **megabyte** is about the size of a novel.
- A **gigabyte** stores about 600 photos or 7.5 hours of video.
- A **terabyte** is about the size of all of the books in a large library.

Some examples of digital memory sizes:

- E-mail (without attachment): 50 kB
- YouTube video: 7 MB per minute
- Web page: 400 kB
- CD: 750 MB
- Photo: 800 kB
- DVD: 8 GB
- Music file: 6 MB
- Blu-ray disc: 27 GB

EXAMPLE 2

Convert:

a 16 MB to:

i kB **ii** bytes **iii** kilobits

b 256 kilobits to:

i bits **ii** megabytes (correct to three significant figures)

Solution

a **i** $16 \text{ MB} = 16 \times 1024 \text{ kB}$ **Changing MB to kB**

$= 16\,384 \text{ kB}$

ii $16 \text{ MB} = 16 \times 1024 \times 1024 \text{ B}$ **Changing MB to B: or 16×2^{20}**

$= 16\,777\,216 \text{ B}$

iii $16 \text{ MB} = 16\,777\,216 \times 8 \text{ b}$ **Changing B to b (bits)**

$= 134\,217\,728 \text{ b}$

$= 134\,217\,728 \div 1000 \text{ kb}$ **Changing b to kb**

$= 134\,217.728 \text{ kb}$

b **i** $256 \text{ kb} = 256 \times 1000 \text{ b}$ **Changing kb to b**

$= 256\,000 \text{ b}$

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ii 256 kb = 256 000 ÷ 8 B **Changing b to B**

= 32 000 B

= 32 000 ÷ 1024 ÷ 1024 MB **Changing B to MB: or 32 000 ÷ 2^{20}**

= 0.030517… MB

≈ 0.0305 MB

EXAMPLE 3

Sue wants to take photos during her trip to Africa. If each photo requires 800 kB of memory, how many photos can she take if her camera has a 2GB memory card?

Solution

2 GB = 2 × 1024 × 1024 kB **Converting GB to kB: or 2 × 2^{20}**

= 2 097 152 kB

Number of photos = 2 097 152 ÷ 800 **800 kB per photo**

= 2621.44

≈ 2621

Exercise 10-01 Bits and bytes

Example 1

1 Convert:

a 15 600 Mb to b **b** 2700 b to kb

c 480 Gb to b **d** 2.9×10^3 Tb to kb

Give answers in scientific notation to 2 significant figures where necessary.

Example 2

2 Convert 500 megabytes to:

a kilobytes **b** bytes **c** bits **d** kilobits

3 If 9.6 GB is converted to bits, what is the answer? Select the correct answer **A**, **B**, **C** or **D**.

A 1.03×10^{10} bits **B** 9.6×10^9 bits **C** 8.25×10^{10} bits **D** 8.59×10^9 bits

Example 3

4 Harrison said he could fit 1200 images on a 2GB USB drive. If each image took up 360 kB of memory, is he correct? Justify your answer.

5 Taylor used 100 MB of data per week on her mobile phone. Convert this to:

a bytes **b** kilobits

6 Callan had a computer with 3 GB of RAM (working memory). Convert this to:

a kilobytes **b** kilobits

7 Zoe pays $35 per month for 620 MB of data and Nikki pays $50 per month for 1 GB of data. Who has the better deal and why?

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8 Calculate the number of MP3 files of average size 3.7 MB that can be stored on a 16 GB MP3 player.

9 If 50 videos fill an 8 GB external hard drive, what is the average size of each video in MB?

10 How many files of size 2.8 kB will fit on a 4 GB USB drive?

11 A bitmap image uses 560 000 bits of memory. What is this size in kilobytes? Select the correct answer **A**, **B**, **C** or **D**.

A 56.0 kB **B** 54.7 kB **C** 70 MB **D** 68.4 MB

12 Here are the sizes of 10 document files.

3.18 MB	207 kB	3.18 MB	869 kB	1.02 MB
4.02 MB	3.15 MB	963 kB	5.68 MB	753 kB

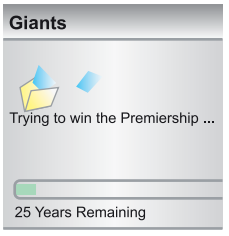
a What is the average file size in MB, correct to 3 significant figures?

b How many kilobytes of space are needed to store all files?

13 Mala made a 3-minute phone call to her friend in Queensland using the Internet application Skype at 1 MB per minute. How many kB did she use?

14 Tyson filled his 2 terabyte hard drive with 240 movies. What was the average size of each movie in GB correct to two decimal places?

10-02 Download speed



Downloading means receiving a file from the Internet or a larger computer/server for copying to a smaller computer or device, while **uploading** means sending a file from a computer or device to the Internet or a larger computer/server.

Download speeds are usually measured in **bits per second** (bps or b/s) or **kilobits per second** (kbps or kb/s).
The table shows the download speeds for 7 different types of Internet connection.

Connection	Download speed (or transfer rate)
dial-up	56.6 kbps
satellite	100 kbps
wireless	125 kbps
next G	7200 kbps
broadband 1	14 000 kbps
broadband 2	24 000 kbps
NBN cable	30 000 kbps

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The speed formula also works for download speed if we replace 'distance' with 'data'.

SUMMARY

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{Download speed} = \frac{\text{data}}{\text{time}}$$

EXAMPLE 4

It took 2 minutes and 4 seconds to download a 13.2 MB video file. Calculate the download speed correct to the nearest kbps.

Solution

As the file size is in megabytes and the download speed is in kilobits/s, we need to first convert 13.2 MB to kilobits.

$13.2 \text{ MB} = 13.2 \times 1024 \times 1024 \text{ B}$ **Converting to bytes first: 1 MB = 1024 × 1024 B**

$= 13\,841\,203.2 \text{ B}$

$= 13\,841\,203.2 \times 8 \text{ b}$ **Converting to bits: 1 B = 8 b**

$= 110\,729\,625.6 \text{ b}$

$= 110\,729\,625.6 \div 1000 \text{ kb}$ **Converting to kilobits: 1 kb = 1000 b**

$= 110\,729.6256 \text{ kb}$

$$\text{Download speed} = \frac{\text{data}}{\text{time}}$$

$$= \frac{110\,729.6256 \text{ kb}}{124 \text{ s}}$$ **2 min 4 s = 124 s**

$= 892.9808\ldots \text{ kb/s}$

$\approx 893 \text{ kbps}$

EXAMPLE 5

Find the time taken (in minutes and seconds to the nearest second) to download:

a a 378 kilobit JPEG image using dial-up at 56 600 bps

b a 4.2 MB song using satellite at 100 kbps

c a 2.5 GB video using NBN cable at 30 000 kpbs

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Solution

a Data = 378 kb, speed = 56 600 bps

Data = 378 × 1000 b = 378 000 b **Changing data to bits first**

$$\text{Download speed} = \frac{\text{data}}{\text{time}}$$

$$56\,600 = \frac{378\,000}{\text{time}}$$

$$\text{Time} = \frac{378\,000}{56\,600}$$ **To find time, divide by the speed**

$$= 6.678\,44...$$

$$\approx 7 \text{ s}$$

b Data = 4.2 MB, speed = 100 kbps

Need to change data to kilobits:

Data = 4.2 × 1024 × 1024 B **Changing data to bytes first**

= 4 404 019.2 B

= 4 404 019.2 × 8 b **Changing to bits**

= 35 232 153.6 b

= 35 232.1536 kb **Changing to kb**

$$\text{Time} = \frac{35\,232.1536}{100}$$

= 352.321 536 s

= 352.321 536 ÷ 60 min **Changing to min**

= 5.872 0256 min

≈ 5 min 52 s **Press** $\boxed{\circ\,'\,''}$ **on the calculator**

c Data = 2.5 GB, speed = 30 000 kbps

Need to change data to kilobits:

Data = 2.5 × 1024 × 1024 × 1024 B **Changing data to bytes first**

= 2 684 354 560 B

= 2 684 354 560 × 8 b **Changing to bits**

$= 2.147\,483\,648 \times 10^{10}$ b

$= 2.147\,483\,648 \times 10^{10} \div 1000$ kb **Changing to kb**

= 21 474 836.48 kb

$$\text{Time} = \frac{21\,474\,836.48}{30\,000}$$

= 715.827 8827 s

= 715.827 8827 ÷ 60 min

= 11.930 46… min

≈ 11 min 56 s **Press** $\boxed{\circ\,'\,''}$ **on the calculator**

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Exercise 10-02 Download speed

1 Which speed is equivalent to 256 kbps? Select the correct answer **A**, **B**, **C** or **D**.

A 32 kB/s **B** 256 kB/s **C** 31.25 kB/s **D** 2048 kB/s

2 True or false?

a a byte is made up of 8 bits

b a bit is either 0 or 1

c a download speed of 3 Mbps means 3 megabits per second

d a kilobyte is equal to 1000 bytes

e an upload speed of 56 bps means 56 bytes per second

f a kilobit is equal to 1000 bits

Example 4

3 Silas downloaded 226 megabits in 1 minute, 8 seconds. What was the download speed in kb/s correct to one decimal place?

4 Bree took 45 minutes to upload a 2 GB video. What was the upload speed to the nearest bps?

5 A 6 MB music file was downloaded in 8 minutes. What was the download speed in kbps?

Example 5

6 If your download speed is 56 600 bits per second, how long correct to two decimal places would it take to download a 12 kb image?

7 How many minutes and seconds would it take to download an 8.3 MB file if the download speed is 125 kbps?

8 A broadband download speed is 14 000 kbps. How long will it take to download a 4.2 GB file? Select the correct answer **A**, **B**, **C** or **D**.

A 42 min **B** 5 min **C** 43 min **D** 2577 min

9 Harper took 2 min 21s to upload an 18 Mb JPEG image to a SharePhoto website. What was the upload speed to the nearest kbps?

10 How long does it take to upload a 800 kB picture if the upload speed is 128 kilobits per second?

11 Suri downloaded a 71.5 MB application (app) at a transfer rate of 1.41 MB/s. How many seconds correct to one decimal place did it take to download?

12 Mia's internet connection has a download speed of 1200 kilobits per second while Leckie's has a download speed of 900 kilobits per second.

a How long does it take each of them to download 0.8 MB correct to two decimal places?

b How many GB can Mia download in 1 minute correct to one significant figure?

c How many kB can Leckie download in 30 seconds correct to the nearest whole number?

13 The table shows the 4 Internet plans offered by one provider.

	Upload speed	Download speed
GoFast	768 kbps	1 Mbps
DropBear	1.5 Mbps	3 Mbps
ZipIt	4 Mbps	7 Mbps
FreeLoad	10 Mbps	15 Mbps

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a How many images of average size 320 kb can you upload on GoFast in 1 hour?

b How long does it take to upload a 4.2 MB file on ZipIt correct to the nearest 0.1 s?

c How long does it take to download a 2.7 MB file on DropBear correct to the nearest 0.1 s?

d How many whole MP3 files of size 600 kB can you download on FreeLoad in 5 minutes?

14 Josh's normal upload speed is 1.75 Mbps. He took 2 m 12 s to upload 30 MB of music to his iPod. Was the upload speed slower than normal? Justify your answer.

15 Huy downloaded a 1 GB video in 1 h 13 m 20 s. What was the download speed to the nearest kbps?

16 Sam downloaded and stored 9.6 MB of songs on her computer. How many seconds does it take if the download speed is 512 kbps?

17 Zina took 14 min 35 s to upload a 3.5 GB video to YouTube. What was the upload speed (correct to two decimal places) in:

a megabytes per second? **b** megabits per second?

18 Reece played video games on his computer, using 150 MB per hour.

a if he played for two and a half hours, how much memory did he use?

b if he can only use 1 GB data per month for games, for how many hours per month can he play? Answer to the nearest hour.

TECHNOLOGY: ONLINE CALCULATORS

There are online calculators for calculating download time and testing the speed of your computer's Internet connection.

1 Use a search engine to find an online calculator that calculates download time.

2 Show that an 8.3 MB file downloaded at 512 kbps would take 2 min 15.99 s to download. Verify this by doing the mathematical calculations.

3 A broadband speed test allows you to check the download and upload speeds of your broadband connection. The upload speed becomes important if you quickly want to upload a photo or video onto, for example, email, Facebook or YouTube. Use a search engine to find a website that calculates your computer's download and upload speed.

4 Write down your computer's download and upload speed, then calculate the time it takes your computer to:

a download a 2.64 MB file **b** upload 878 kB of data

5 Find two reasons why the speed of your computer may vary during a download or upload.

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10-03 Downloading music and video

Downloading songs and videos

EXAMPLE 6

Finn downloaded his 10 favourite songs on his MP3 player. They are shown in the table with their memory size and track time.

Song	Size (MB)	Time (min:s)
Get on up	3.67	3:59
Do da Waggle	4.72	5:07
Late for my party	4.24	4:35
In my life	3.18	3:25
My name is Buggy	1.01	1:04
Who's in charge here?	3.06	3:18
Feels like chocolate	1.57	1:41
Push my care button	2.98	3:14
Makes me want to fly	1.18	1:16
Not gonna say goodbye	3.76	4:04

a Which song is the largest in memory size and longest in time?

b Find the average time of the songs.

c Finn uses the random selection mode on his MP3 player to play one of his songs. What is the probability that the song chosen is under 3 MB in size?

d In how many different ways could all ten songs be played in order, with no song being repeated?

Solution

a Do da Waggle is the largest and longest song. **At 4.72 MB and 5 min 7 s**

b Sum of times = 28 min 220 s **Adding min and s of every song separately**

$= 28 \times 60 + 220$ s **Converting to s**

$= 1900$ s

Average time $= \dfrac{1900\text{s}}{10}$ $\dfrac{\textbf{sum of times}}{\textbf{no. of songs}}$

$= 190$ s

$= 3$ min 10 s

c There are 4 songs in the list that are under 3 MB in size.

$P(\text{chosen song} < 3\text{ MB}) = \dfrac{4}{10} = \dfrac{2}{5}$

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d The first song chosen has 10 possible outcomes.

The second song chosen has 9 possible outcomes, if the first song cannot be repeated.

The third song chosen has 8 possible outcomes, if the first and second song cannot be repeated.

Continuing this way:

Number of ways all ten songs can be played without repetition

$= 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

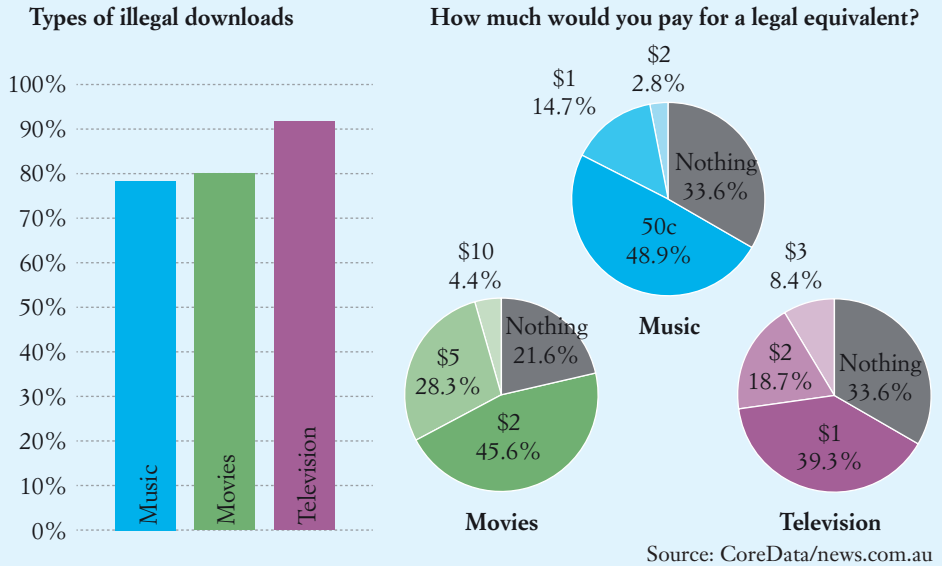
$= 3\ 628\ 800$

Illegal downloading

An estimated 2.8 million Australians download music illegally each year via file sharing networks. Each person illegally downloads approximately 30 songs a month, which adds up to a billion songs per year. Taking music illegally is sometimes referred to as 'music piracy' and the worst offenders are aged 14 to 17. When a song, video or movie is downloaded illegally, its creators are missing out on income for their work.

EXAMPLE 7

In 2010, 7000 people were surveyed on the types of illegal downloads they obtained, as well as how much they would pay for a legal equivalent. The results are shown in the graphs below.



Source: CoreData/news.com.au

a How many of those surveyed illegally downloaded movies?

b How many of these said they would pay $2 for a legal copy of a movie?

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c What percentage of those surveyed said they would not pay for a legal copy of a TV show?

d How many people who downloaded music were prepared to pay something for a legal copy?

Solution

a Number of people illegally downloading movies = 80% × 7000 = 5600

b Number of people who would pay $2 for a movie = 45.6% × 7000 = 3192

c Percentage who would not pay for downloading a TV show = 33.6%

d Percentage of people willing to pay for music

= 48.9% + 14.7% + 2.8% **or 100% – 33.6%**

= 66.4%

Number of people willing to pay for music = 66.4% × 7000 = 4648

Example 6

Exercise 10-03 Downloading music and video

1 A 1-hour video uses 420 MB of memory. How many of these videos will fit on a 32 GB digital tablet?

2 Morris has a library of 2000 songs with an average size of 3 MB. Can he fit all the songs on his 8 GB USB drive?

Shutterstock.com/Olga Sapegina

3 Ritha has an MP3 player that holds 16 GB of data. How many tunes of average size 3.8 MB can she store on it?

4 Dean was having a party and downloaded 20 songs for a playlist. The sizes are shown.

3.67 MB	2.98 MB	5.02 MB	4.72 MB	3.62 MB
3.70 MB	1.18 MB	4.72 MB	3.18 MB	1.46 MB
2.04 MB	4.24 MB	2.64 MB	2.89 MB	3.76 MB
1.57 MB	3.18 MB	3.06 MB	1.01 MB	4.00 MB

a Find the total storage needed for the playlist.

b Calculate the mean song size correct to two decimal places.

c Find the median song size.

d Calculate the range of song sizes.

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e Calculate the total cost of the downloaded songs if a song less than 2 MB is $1.69 and one 2 MB or more is $2.19.

f If one of these songs was randomly selected, what is the chance that it is more than 3.5 MB in size?

g If the random selection mode plays 5 songs from the list, in how many different ways can it choose 5 songs from the list:

i if any song could not be repeated? **ii** if any song could be repeated?

5 A new hard drive can store 2 terabytes of data.

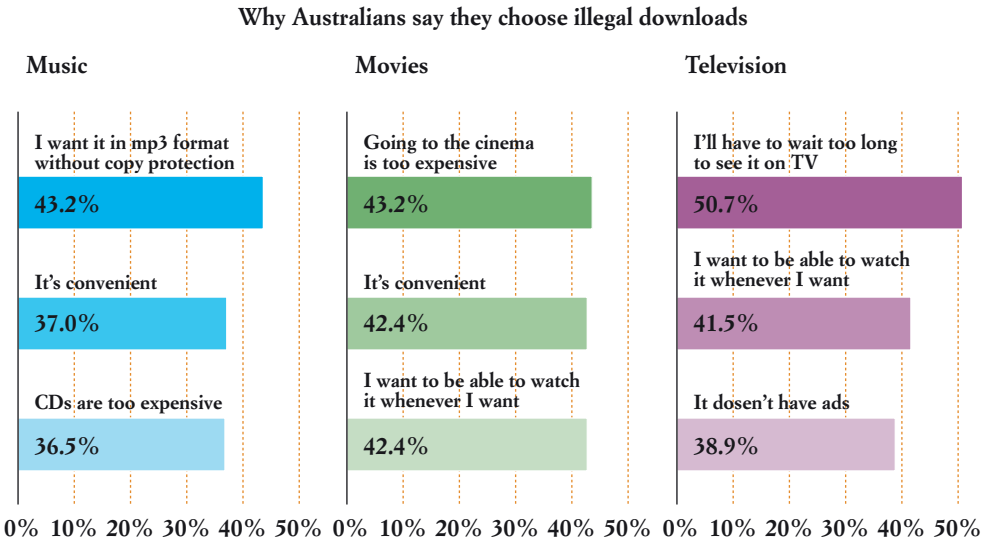
a How many songs with an average size of 4 MB can be stored on this drive?

b If an average song lasts 3 minutes, how many hours of music could you listen to?

c If an average movie needs 5 GB of memory, could you fit 1000 movies on this drive? Justify your answer.

Example 7

6 A sample of 7000 Australians was surveyed on the reasons they illegally downloaded music, movies and TV shows. The results are shown in the graphs below.



Source: CoreData/news.com.au

a How many in the sample download music illegally because it's convenient?

b Why do you think the percentages for each category add up to more than 100%?

c How many people think it is too expensive to go to the movies?

d List two advantages of watching a movie at a cinema rather than downloading it illegally.

e What reason was given by 2723 Australians for downloading TV shows illegally?

f What reason was given by more than half of the sample? How could this affect what we watch on TV in the future?

7 If 20 blu-ray discs fill a 1 terabyte hard drive, what is the average size of each blu-ray disc? Answer to the nearest 0.1 GB.

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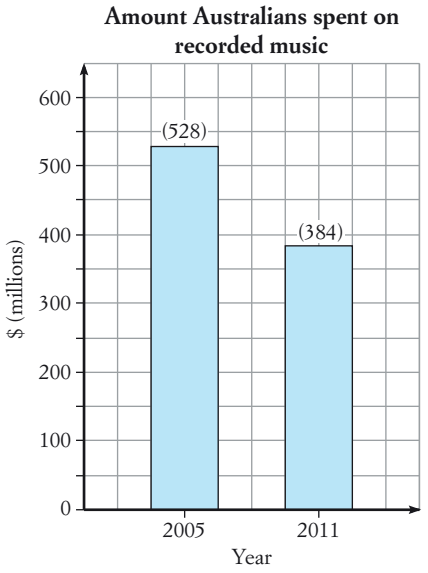
8 These ARIA (Australian Recording Industry Association) figures show how the amount Australians spend on recorded music has fallen between 2005 and 2011.



a If the approximate population of Australia was 20.5 million in 2005 and 21.5 million in 2011, find the average amount spent by Australians on recorded music:

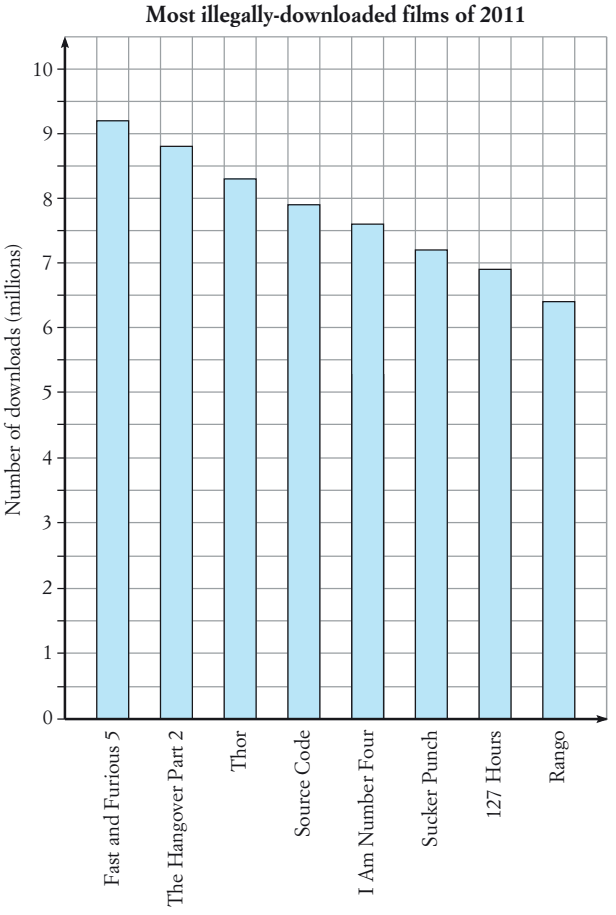
i in 2005 **ii** in 2011

b What was the percentage decrease (correct to one decimal place) in sales from 2005 to 2011?

c Why do you think there was such a difference between 2005 and 2011 given that Australia had one million more people in 2011?

9 A flash drive has a storage capacity of 64 GB. How many songs with an average size of 4 MB will fit on this drive?

10 This column graph shows the 8 most illegally-downloaded films of 2011.



a Which film was downloaded illegally 7 200 000 times?

b How many times was *Rango* downloaded illegally?

c What is the average number of downloads for the 8 movies?

d If the cost to filmmakers is $5 per illegal download, how much money did filmmakers lose in revenue for *The Hangover Part 2*?

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11 Suppose a legal movie download costs an average of $20 and the wholesaler receives 24%, the distributor 68% and the royalties to artists form the remainder. How much do the artists receive for each movie download?

12 The table shows the age groups of people who illegally download music and videos from the Internet and their percentages of all illegal downloaders, according to a recent survey.

Age	16–24	25–29	30–39	40–49
%	39	17	26	18

a Draw a sector graph to represent the data.

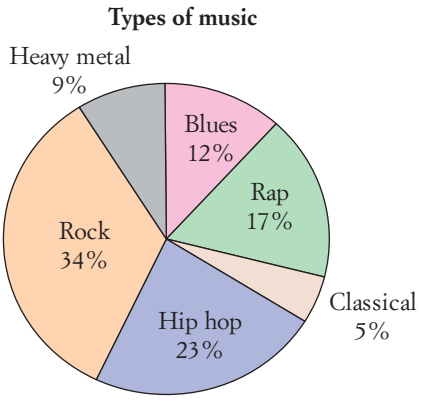
b If 2350 people from the survey who downloaded illegally were in the 30–39 age group:

i how many people participated in this survey?

ii how many people downloaded illegally in the 25–29 age group?

c In a group of 2000 of the people who illegally download music and videos, how many would you expect to be in the 16–24 age group?

13 The sector graph shows the types of music downloaded by a sample of 500 students.



a What was the least downloaded type of music?

b What percentage of students downloaded hip hop?

c What is the sector angle for hip hop?

d How many students in the sample downloaded rock music?

e Was the most downloaded type of music preferred by more than one-third of the sample?

14 This table shows the number and value of different types of music and videos sold in 2010 and 2011.

2011 ARIA Yearly Statistics Net Wholesale Sales of Sound Recordings & Music Videos						
	2011		2010		Percentage change	
Configuration	Units	Dollar Value	Units	Dollar Value	Units	Dollar Value
CD Singles	47 472	151 402	29 174	159 286	62.72%	–4.95%
CD Albums	20 539 253	222 698 401	23 521 928	252 747 622	–12.68%	–11.89%
Music Video/DVD	1 856 507	17 812 847	2 542 386	25 512 729	–26.98%	–30.18%
Digital Track	68 473 070	79 552 930	49 180 480	56 797 421	39.23%	40.06%
Digital Album	4 817 546	46 029 988	3 301 366	31 667 505	45.93%	45.35%

a How many CD albums were sold in 2010?

b What was the dollar value of the digital tracks sold in 2011?

c Which configuration had the largest percentage decrease in units sold from 2010 to 2011?

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d Which configuration had the largest percentage increase in dollar value from 2010 to 2011?

e Show how 45.93% was obtained for the increase in sales of digital albums.

f Was more money made through digital sales or through the sales of CDs and DVDs? Justify your answer.

15 It is claimed that 95% of all music downloaded from the Internet is illegal, and that an average iPod contains illegally-downloaded songs with a total value of $800. Most people who download music illegally rarely or never go on to purchase the music legitimately. Half of them question why they should have to pay for music when they can download it for free via file-sharing networks. What is your opinion on this issue?

INVESTIGATION

DOWNLOADING MUSIC

1 Find the size of 10 of your favourite songs and calculate the average size. Using this average, decide how many songs will fit on:

a a 16 GB digital tablet

b a notebook computer with 500 GB storage

c a 32 GB USB drive

d a 1 TB external hard drive

2 Investigate the sales of recorded music in the last 5 years. Display the amounts on a graph and discuss possible reasons for any trends. Suggest effects any trends may have on the sales of recorded music in the future.

WS Investigating phone plans

10-04 Mobile phone plans

A **mobile phone plan** is a contract between you and a phone company for using its network. One type of contract is a **cap plan** where the monthly access fee is fixed or 'capped', for example, a '$49 Cap Plan' that costs $49 per month and includes $450 worth of credit. If you exceed the credit limit, you pay the extra amount.

WS Calculating a phone bill

EXAMPLE 8

The Optel $49 Cap Plan has the following features.

- Monthly access fee: $49
- Included credit: $450
- Call rate: 40c connection fee, then 98c per minute
- Optel FreeTime: From 8 p.m. to 12 midnight, calls to other Optel mobile phones in Australia are free for the first 20 minutes, then normal rates apply
- Text: 30c per SMS

Connection fee (also called 'flagfall') is the initial charge of making a call

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- Included data: 1 GB
- Additional data cost: 25c per MB

a What is the cost of an 8-minute phone call at normal rates?

b What is the cost of a 30-minute call to another Optel mobile phone:

i at 9 p.m.? **ii** at 9 a.m.?

c If Shaun sent 87 SMS texts last month, what was the cost?

d If Tori makes $540 worth of calls and texts this month, how much does she pay for them?

e If Billy used 1400 MB worth of data, how much does he pay for them?

Solution

a Cost of call = $0.40 + $0.98 × 8 **Connection fee + 8 minutes**

= $8.24

b **i** 9 p.m. is during Optel FreeTime, when the first 20 minutes are free.

Additional minutes = 30 – 20 = 10

Cost of call = $0.98 × 10 **No connection fee + 10 minutes**

= $9.80

ii 9 a.m. is outside Optel FreeTime, so normal rates apply.

Cost of call = $0.40 + $0.98 × 30

= $29.80

c Cost of texts = $0.30 × 87

= $26.10

d Because $540 is above $450, Tori must pay for the extra amount.

Cost = $540 – $450 = $90

e Because 1400 MB is above 1 GB (= 1024 MB), Billy must pay for the additional data.

Additional data = 1400 MB – 1024 MB

= 376 MB

Cost = $0.25 × 376 = $94

Using more than your included credit is expensive!

Mobile phone bills

Each phone company posts or e-mails a monthly bill to each customer, detailing calls, SMS texts and data usage. The 'Amount due' is clearly displayed on the bill, as well as the 'Date due' when the amount due must be paid.

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An example of a mobile phone bill is shown below.

OPTEL MOBILE

Date of issue
8 Oct 2014

MR PETER TALK
14 DOWN ST
CLARKESVILLE NSW 2038

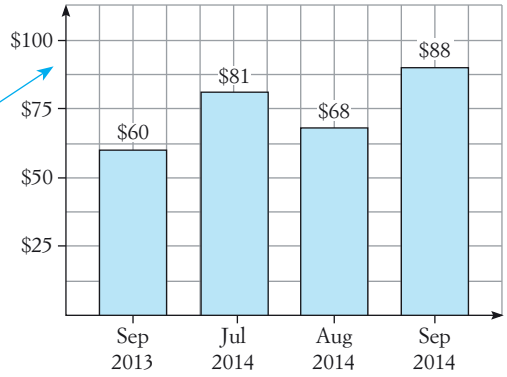
1 Account number
041 354 658

Invoice number
976 543 210

2

Last bill	What you paid	Adjustments	New charges
$68.44	$68.44	$0.00	$87.70

3



Amount due	$87.70
Date due	25 Oct 2014

4

New charges 25 Aug 2014 to 25 Sep 2014

5 Service summary	$71.20
6 Handset repayment	$9.00
7 Insurance	$5.00
8 Credit card surcharge	$0.50
9 Paper invoice charge	$2.00
Total	$87.70

10

Service summary Optel $49 Cap

	Included	Used	Excess	Cost
$49 Cap 24 months				$49.00
Calls and text	$450	$472.20	$22.20	$22.20
Data	1024 MB	726 MB	0 MB	$0.00
Total				$71.20

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Features

1 Account number, Invoice number: your ID number and the number of this bill.

2 Last bill, What you paid: the amount charged on last month's bill and the amount you paid.

3 Column graph of bill amounts: the amounts of the current bill, the two previous bills, and the bill this time last year.

4 Amount due, Date due: the amount of this bill and when you have to pay it by.

5 Service summary: the total charges for phone use this month, shown in more detail in the table in **10** below.

6 Handset repayment: monthly amount to pay off your phone.

7 Insurance: Optional insurance to replace the phone if it is stolen or misplaced.

8 Credit card surcharge: fee for paying your bill by credit card, to encourage customers to use direct debit from their bank accounts instead.

9 Paper invoice charge: fee for having the bill printed and posted to you, to encourage customers to have their bills e-mailed instead.

10 Included, Used, Excess: the amount of included credit in the plan, the amount actually used this month, the additional amount (if more than the included credit has been used).

EXAMPLE 9

The following questions refer to the mobile phone bill on the opposite page.

a Has the amount for this bill increased or decreased since last month's bill?

b What will be the total cost of the phone handset if the repayments are to be made over 24 months?

c What was the total value of the phone calls and texts made this month?

d Explain why the cost of the calls and texts is only \$22.20.

Solution

a This month's bill = \$87.70, last month's bill = \$68.44, so the amount has increased.

b Total cost of handset = \$9.00 × 24 = \$216.00

c Total value of phone calls and text = \$472.20

d This plan has an included credit of \$450, so you only need to pay for the amount over \$450.

Cost = \$472.20 – \$450

= \$22.20

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Example 8

Exercise 10-04 Mobile phone plans

1 Refer to the Optel $49 Cap Plan from Example **8** on page 456 to answer the following questions.

- **a** Robert makes a 6-minute call at the normal rate. How much does it cost?
- **b** Melanie makes a 10-minute call to another Optel mobile phone at 9:15 p.m., then a 25-minute call at 10:05 p.m. How much does each call cost?
- **c** Vila sent 105 SMS messages in January. How much did they cost him?
- **d** Hannah used 1650 MB of data. How much did it cost her?
- **e** Nawi makes $265 worth of phone calls and texts in a month. How much does he pay for them?
- **f** Filomena makes $505 worth of phone calls and texts in a month. How much does she pay for them?
- **g** Lachie's phone call cost $61.16. If this was charged at the normal rate, for how many minutes did the call last?
- **h** Hong makes 5 calls per day at an average of 4 minutes each call. Calculate the total cost of her calls over a week.
- **i** Show that the minimum cost of the Optel $49 Cap Plan is $1176 for the 24-month contract.

2 Luka signs onto the Optel $49 Cap Plan for a 24-month contract but after 18 months he decides to end the contract and switch to TelNet, another phone company. Optel requires him to pay the monthly access fee for the remaining months of the contract. How much must Luka pay?

3 TelNet charges a 32c connection fee for each call, then 31c per 30 seconds or part thereof.

part thereof here means part of 30 seconds is counted as a full 30 seconds, for example, 12 min 40 s is rounded up to 13 min

Calculate the cost of a TelNet phone call that lasts:

- **a** 7 minutes
- **b** 14 minutes
- **c** 10.5 minutes
- **d** 5.5 minutes
- **e** 4 min 9 seconds
- **f** 2 min 38 seconds

4 Maxine's QuickChat phone plan charges a 40c connection fee plus 74c per minute for phone calls and 25c per SMS text. Maxine had a 'text conversation' with her best friend Georgia in which she sent 35 texts. Investigate whether it would have been cheaper for her to have a 10-minute phone call with Georgia instead.

Example 9

Questions **5** to **7** refer to the Optel mobile phone bill from Example **9**.

5
- **a** What is the amount due for this bill and when is it due?
- **b** Between which dates is the period for which this bill covers?
- **c** In the past three months, which month's bill had the lowest amount?
- **d** Did Peter spend more or less than this time last year?
- **e** What was the amount paid in the last bill? Was this amount paid in full?
- **f** What is the invoice number for this bill?

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6 How much does Peter pay for making $472.20 worth of calls and texts this month? Select the correct answer **A**, **B**, **C** or **D**.

A $22.20 **B** $49.00 **C** $71.20 **D** $450

7 **a** How much data was used this month and how much of it was free?

b What is the annual cost of phone insurance?

c What is the charge for receiving a paper bill?

d Why do you think Optel would prefer to email you the bill instead?

e What is the cost of Peter's phone handset if the repayments will stop after 24 months?

f One way of avoiding the credit card surcharge is to direct debit from your bank account. Why do you think Optel prefers its customers to use direct debit?

8 Name four ways you can save money when using a mobile phone.

9 Find a mobile phone plan, either one that you would like to have or the one that you currently have. Use the Internet or visit a mobile phone store for a brochure.

a What is the name of the plan?

b What is the monthly access fee and included credit?

c For phone calls, what is the connection fee (or flagfall) and call rate?

d When are free or discounted calls available?

e What is the included data amount?

f What is the cost of additional data?

g What is the cost of each SMS text?

h Is the term of the contract 12 or 24 months and what is the minimum cost over this term?

i Are there any special offers with this plan, such as a free phone or unlimited calls?

INVESTIGATION

Mobile phone usage

- Estimate how many mobile phone calls you make in a month.
- How many SMS texts would you send in a month?
- Do you know how much data you download and upload?

Compare your answers to the values in the table below.

	Low user	Average user	High user	Very high user
Phone calls	50	150	300	500
SMS text	20	50	250	700
Data	100 MB	250 MB	1 GB	3 GB

What type of mobile phone user are you?

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INVESTIGATION

SCAM-BUSTING

How expensive is downloading a ring tone, wallpaper or app on your phone, or voting on a reality TV program, or subscribing to a premium SMS service? Don't be fooled into paying money into scams. Choose one and analyse all of the charges and conditions.

1. What is the minimum cost?
2. What are the hidden risks and costs?
3. Will you be charged regularly?
4. Will you have to pay a fee to unsubscribe?
5. Will you be charged for *receiving* SMS texts?

10-05 Call costs and usage

Graphing call costs

Investigating call times

EXAMPLE 10

The cost of a call on the Modalfone \$79 Plan is 30c connection fee plus 84c per minute.

a Complete this table.

Length of call, n min	0	3	5	8	10	15	20
Cost of call, \$$C$							

b Find the linear function relating C to n.

c What is the gradient of this function and what does it represent?

d Graph the linear function and use it to estimate:

i the cost of a 12-minute call

ii the length of the call that costs \$15.40.

Solution

a For 0 mins, $C = 0.30$ **The connection fee**

For 3 mins, $C = 0.30 + 0.84 \times 3$

$= 2.82$

Continuing in the same way for the rest of the table, we have:

Length of call, n min	0	3	5	8	10	15	20
Cost of call, \$$C$	0.30	2.82	4.50	7.02	8.70	12.90	17.10

b Following the pattern from part **a**:

$C = 0.30 + 0.84 \times n$

$= 0.3 + 0.84n$

c The gradient is 0.84 and represents the cost of a call in dollars per minute.

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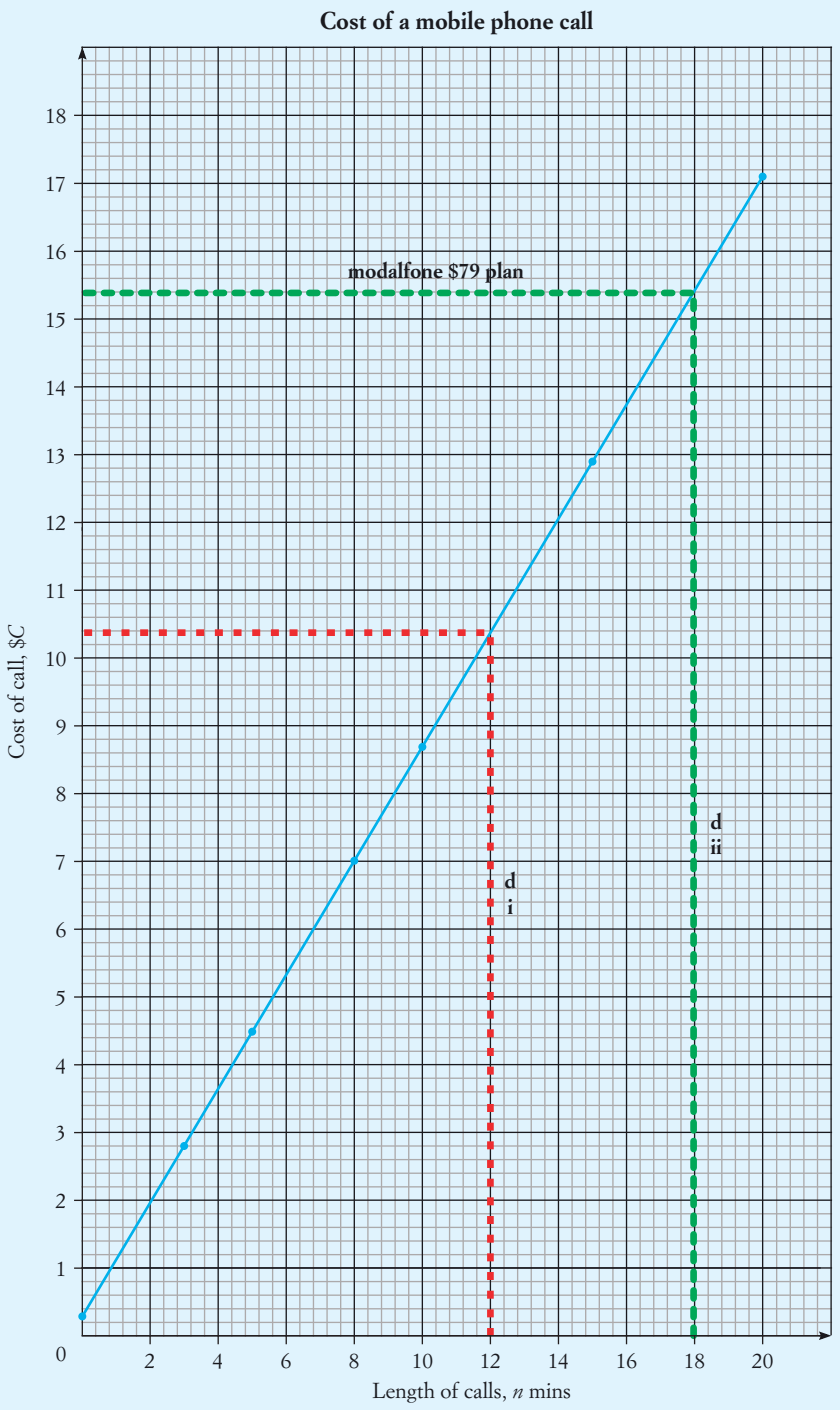
d



From graph:

i Cost of a 12-minute call ≈ $10.40

ii Length of the call that costs $15.40 ≈ 18 min

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EXAMPLE 11

Eddie's QuickChat bill lists all of his mobile phone calls made in a month. There are three types of calls.

- National: calls to a landline (non-mobile) phone in Australia
- QuickChat: calls to another mobile phone on a QuickChat plan
- Other mobile: calls to another mobile phone (not QuickChat)

Date	Time	Type	min:sec	Amount ($)
21 Dec	10:27pm	National	01:34	1.77
23 Dec	08:09am	National	02:49	2.50
23 Dec	01:01pm	Other mobile	09:36	7.59
23 Dec	06:47pm	National	08:48	6.86
24 Dec	11:22am	Other mobile	00:42	1.05
24 Dec	08:59pm	National	01:16	1.41
24 Dec	09:02pm	QuickChat	00:17	0.00
24 Dec	09:40pm	National	00:19	0.68
27 Dec	11:25am	Other mobile	00:33	1.05
27 Dec	12:03pm	Other mobile	01:06	1.41
29 Dec	05:09pm	National	04:10	3.59
29 Dec	09:11pm	National	05:20	4.32
30 Dec	02:30pm	QuickChat	02:54	0.00
30 Dec	03:29pm	QuickChat	04:40	0.00
30 Dec	04:25pm	National	00:38	1.05
30 Dec	06:11pm	National	00:40	1.05
30 Dec	06:31pm	QuickChat	00:28	0.00
31 Dec	05:30pm	National	00:24	0.68
31 Dec	05:38pm	National	04:32	3.95
02 Jan	11:27am	Other mobile	09:11	7.23
02 Jan	04:09pm	National	00:42	1.05
02 Jan	04:14pm	QuickChat	00:35	0.00
03 Jan	12:19pm	QuickChat	04:55	0.00
03 Jan	06:07pm	National	01:13	1.41
03 Jan	06:10pm	Other mobile	01:46	1.77
04 Jan	04:59pm	Other mobile	00:37	1.05
05 Jan	05:45pm	National	02:35	2.50
06 Jan	08:53am	National	01:42	1.77

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Date	Time	Type	min:sec	Amount ($)
06 Jan	02:09pm	National	00:41	1.05
06 Jan	05:14pm	National	00:43	1.05
06 Jan	06:09pm	National	00:38	1.05
06 Jan	08:14pm	QuickChat	00:52	0.00
06 Jan	09:54pm	Other mobile	01:04	1.41
07 Jan	10:42am	QuickChat	00:55	0.00
07 Jan	02:37pm	Other mobile	00:28	0.68
07 Jan	03:50pm	National	05:17	4.32
07 Jan	09:05pm	QuickChat	01:01	0.00
08 Jan	03:27pm	National	06:35	5.41
09 Jan	10:16am	QuickChat	01:40	0.00
10 Jan	04:12pm	QuickChat	00:44	0.00
12 Jan	06:00pm	National	00:23	0.68
13 Jan	02:19pm	Other mobile	08:02	6.50
13 Jan	05:31pm	Other mobile	00:16	0.68
13 Jan	06:15pm	Other mobile	03:40	3.23
13 Jan	08:48pm	QuickChat	01:23	0.00
14 Jan	10:09am	QuickChat	02:06	0.00
14 Jan	12:32pm	National	00:51	1.05
14 Jan	05:33pm	National	00:51	1.05
14 Jan	05:34pm	National	00:40	1.05
14 Jan	06:24pm	National	03:36	3.23
14 Jan	06:36pm	QuickChat	01:24	0.00
14 Jan	08:41pm	Other mobile	02:09	2.14
14 Jan	09:03pm	Other mobile	02:03	2.14
15 Jan	09:30am	Other mobile	00:57	1.05
15 Jan	09:31am	National	00:12	0.68
15 Jan	09:47am	National	13:37	10.50
15 Jan	11:05am	National	00:39	1.05
15 Jan	09:26pm	National	06:07	5.04
16 Jan	06:16pm	National	01:05	1.41
17 Jan	05:34pm	Other mobile	03:29	2.86
17 Jan	05:41pm	National	00:24	0.68
18 Jan	01:37pm	QuickChat	00:49	0.00
19 Jan	11:04am	National	02:15	2.14
20 Jan	02:28pm	QuickChat	02:07	0.00

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a How long was the longest phone call made and what was its cost?

b Complete this frequency table of call times.

Call time	Frequency
6 a.m. up to 8 a.m.	
8 a.m. up to 10 a.m.	
10 a.m. up to 12 p.m.	
12 p.m. up to 2 p.m.	
2 p.m. up to 4 p.m.	
4 p.m. up to 6 p.m.	
6 p.m. up to 8 p.m.	
8 p.m. up to 10 p.m.	
10 p.m. up to 12 a.m.	

c What is the modal class?

d How many calls were made during the monthly period?

e What percentage of these calls were to Quickchat mobile phones?

f Graph the data from the frequency table on a histogram.

Solution

a Longest phone call lasted 13 min 37 s (on 15 Jan) and cost \$10.50.

b By counting the number of calls in each timeslot:

Call time	Frequency
6 a.m. up to 8 a.m.	0
8 a.m. up to 10 a.m.	5
10 a.m. up to 12 p.m.	8
12 p.m. up to 2 p.m.	5
2 p.m. up to 4 p.m.	8
4 p.m. up to 6 p.m.	15
6 p.m. up to 8 p.m.	11
8 p.m. up to 10 p.m.	11
10 p.m. up to 12 a.m.	1

c Modal class is 4 p.m. up to 6 p.m. **Has the highest frequency, 15**

d By adding the values in the frequency column of the table in part **b**:

Total number of calls $= 0 + 5 + 8 + 5 + 8 + 15 + 11 + 11 + 1$

$= 64$

e From Eddie's bill, we can count 16 calls to QuickChat phones over the month.

Percentage of calls to QuickChat phones $= \frac{16}{64} \times 100\% = 25\%$

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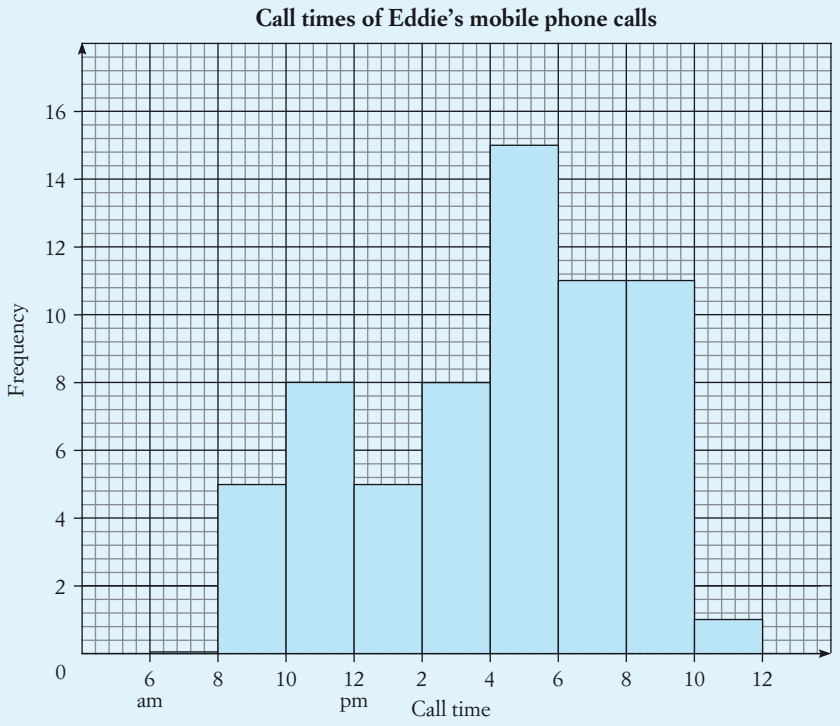
f



Exercise 10-05 Call costs and usage

The graphs in this exercise may be created using a spreadsheet or other graphing software.

Example **10**

1 Use the graph from Example **10** on page 463 to estimate:

a the cost of a 16-minute call

b the length of a call that costs \$5.30.

2 The cost of a call using the Modalfone plan from Example **10** can be more accurately represented by a stepwise linear function because it does not increase gradually at a constant rate between each minute, but in distinct 'jumps' as shown by a step graph.

a Complete this table.

b Find the cost of a call that lasts:

i 3 min

ii 3 min 1 s

iii 5 min 41 s

c Draw a step graph relating C to values of n from 0 to 7.

Length of call, n min	Cost of call, \$$C$
0 up to 1	
Over 1 and up to 2	
Over 2 and up to 3	
Over 3 and up to 4	
Over 4 and up to 5	
Over 5 and up to 6	
Over 6 and up to 7	

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3 The cost of a mobile phone call on the TelNet $109 Plan is 30c flagfall followed by 35c per 30 seconds.

a Complete this table.

Length of call, n min	0	2	3.5	5	6	8.5	10.5
Cost of call, $\$C$							

b Find the linear function relating C to n.

c Graph the linear function and use it to estimate:

i the cost of a call that lasts 4 minutes

ii the length of the call that costs $5.20

d Use the linear function to calculate the cost of an 8-minute call.

e What is the gradient of the function and what does it represent?

f What is the vertical intercept of the function and what does it represent?

g Draw a step graph relating C to values of n from 0 to 3.

Example **11**

Questions **4** to **7** refer to Eddie's QuickChat bill from Example **11** on page 464.

4 **a** How long was Eddie's shortest phone call and what was its cost?

b On what day and time was the earliest phone call made?

c How many calls were made:

i on 5 January? **ii** to landline phones? **iii** after 9 p.m.?

d What percentage (correct to one decimal place) of calls cost over $5?

e What do all calls made to QuickChat mobile phones have in common? Why do you think this is so?

5 **a** Complete this frequency table of call times.

Call time	Frequency
6 a.m. up to 9 a.m.	
9 a.m. up to 12 p.m.	
12 p.m. up to 3 p.m.	
3 p.m. up to 6 p.m.	
6 p.m. up to 9 p.m.	
9 p.m. up to 12 a.m.	
12 a.m. up to 3 a.m.	
3 a.m. up to 6 a.m.	

b What is the modal class? Suggest a reason this period is the mode.

c Graph the data on a radar chart.

d Describe the pattern shown in the radar chart over the 24-hour period.

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6 **a** Complete this frequency table of call types.

Call time	Frequency
National	
QuickChat mobile	
Other mobile	

b What percentage (correct to one decimal place) of calls were to mobile phones that are not QuickChat?

c What is the mode?

d Graph the data on a sector graph.

e Describe the pattern shown in the sector graph.

7 **a** Complete this frequency table of call lengths.

Call lengths (min)	Class centre	Frequency
0 up to 3		
3 up to 6		
6 up to 9		
9 up to 12		
12 up to 15		

b Which class interval was least common? Suggest a reason.

c What is the modal class?

d Graph the data on a frequency histogram.

e Calculate, correct to two decimal places, an estimate for the mean call length.

f What is the probability that Eddie makes a phone call that lasts under six minutes?

INVESTIGATION

PATTERNS OF PHONE USAGE

Find a mobile phone bill, either one of your own or a sample one from a phone company website.

1. What is the monthly access fee and included credit?
2. Did the user use more than the included credit?
3. Between which dates is the period for which this bill covers?
4. What is the amount due and the due date?
5. Name the different types of phone calls listed.
6. How many SMS texts were sent and how much did they all cost?
7. Name three different ways you can pay this bill.
8. How much data was used and how much did it cost?
9. What is the difference between the amounts for this bill and the last bill? Has it increased or decreased?
10. How long was the longest phone call, when was it made and how much did it cost?

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11 Create and complete a frequency table for one or two of the following features.

- time of call
- day of week of call
- call length (min:sec)
- call cost or daily call cost
- number of calls made per day
- number of SMS texts sent per day

12 Choose an appropriate graph to represent your data, such as a column graph, sector graph, radar chart, divided bar graph, line graph, histogram or polygon.

13 Write 2 to 3 sentences to describe any patterns you find in phone usage.

WS Investigating phone plans

10-06 Comparing phone plans

EXAMPLE 12

This table shows the different mobile phone plans available from Optel.

	$29 Cap	$49 Cap	$79 Cap	$99 Cap	$129 Cap
Monthly fee	$29	$49	$79	$99	$129
Included credit	$200	$450	$850	$1200	unlimited
Call connection fee	47c	40c	35c	35c	na
Call rate per min	100c	98c	98c	85c	na
Cost per SMS text	30c	30c	25c	25c	22c
Included data	200 MB	1 GB	2 GB	3 GB	4 GB
Additional data cost	50c per MB	25c per MB			
Calls to other Optel mobiles	Normal rates	Free between 8 p.m. and 12 midnight for first 20 mins		Free anytime	
Minimum cost over 24 months	$696	$1176	$1896	$2376	$3096

a Calculate the cost of a 5-minute call under the:

i $29 Cap plan **ii** $79 Cap Plan

b What happens to the call rate when you choose a more expensive cap plan?

c Calculate the cost of a 25-minute call to another Optel mobile phone at 10:45 p.m. under the:

i $29 Cap plan **ii** $99 Cap plan

d Calculate the cost of $1000 worth of calls and texts in a month under the:

i $79 Cap plan **ii** $129 Cap plan

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Solution

a **i** \$29 plan

Cost = \$0.47 + \$1.00 × 5

= \$5.47

ii \$79 Plan

Cost = \$0.35 + \$0.98 × 5

= \$5.25

b The call rate decreases when you choose a more expensive cap plan.

c **i** \$29 plan

Cost = \$0.47 + \$1.00 × 25

= \$25.47

ii \$99 plan

Cost = \$0.00 **Free anytime**

d **i** \$79 plan has included credit of \$850

Cost = \$1000 – \$850

= \$150

ii \$129 plan has unlimited credit

Cost = \$0

Prepaid phone plans

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Phone plans requiring the user to pay their monthly phone bill at the end of each month are called **postpaid phone plans** because you 'pay after' usage.

A **prepaid phone plan** is one where you 'pay before' usage. You purchase credit in advance to avoid overspending. As each credit purchase has a limited time period, such as 30 days, you must 'recharge' or 'top up' before the expiry date to keep your credit and account going. Instead of receiving a bill, you check on your remaining credit by phone or website.

EXAMPLE 13

This table shows the different prepaid mobile phone plans available from Optel.

Cost of plan	\$19	\$29	\$49	\$79	\$99
Credit for calls and SMS texts	\$140	\$300	\$1000	\$1300	\$1500
Expiry time	30 days	30 days	30 days	60 days	60 days
Call connection fee	48c	48c	40c	40c	38c
Call rate per min	112c	102c	95c	90c	88c
Cost per SMS text	30c	30c	25c	25c	22c
Included data	140 MB	500 MB	1 GB	2 GB	4 GB
Additional data cost	\$2 per MB	\$1 per MB			
Calls to other Optel mobiles	Normal rates	Free for the first 10 mins			

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a Under the \$19 Optel prepaid plan, calculate the cost of:

i a 3-minute phone call **ii** sending 125 SMS texts in a month

b What happens to the amount of included data usage when you choose a more expensive prepaid plan?

c Under the \$49 Optel prepaid plan, calculate the cost of using:

i 1200 MB of data before 30 days **ii** 950 MB of data before 30 days

d Under the \$99 Optel prepaid plan, calculate the cost of a 16-minute call to another Optel mobile phone.

Solution

a **i** 3-minute phone call = \$0.48 + \$1.12 × 3

= 3.84

ii 125 texts = \$0.30 × 125

= \$37.50

b The amount of included data usage increases when you choose a more expensive prepaid plan.

c **i** Included data = 1 GB = 1024 MB

Additional data = 1200 MB – 1024 MB = 176 MB

Cost = \$1 × 176 = \$176

ii 950 MB < 1 GB so the cost is \$0.

d The cost of a call to another Optel mobile phone is free for the first 10 minutes.

Additional minutes = 16 – 10 = 6

Cost = \$0.88 × 6 = \$5.28

Exercise 10-06 Comparing phone plans

Questions **1** to **4** refer to the table of Optel postpaid phone plans from Example **12** on page 470.

Example **12**

1 **a** Calculate the cost of a 9-minute call under the:

i \$49 Cap plan **ii** \$99 Cap plan **iii** \$129 Cap plan

b What happens to the call connection fee when you choose a more expensive cap plan?

c In the \$129 Cap column of the table, there are two cells labelled 'na', which stands for 'not applicable'. Why?

d Calculate the cost of using 520 MB of data under the:

i \$29 Cap plan **ii** \$79 Cap plan

e Why is it important to avoid using more than your included amount of data?

f Calculate the cost of a 41-minute call to another Optel mobile phone at 8:30 p.m. under the:

i \$29 Cap plan **ii** \$49 Cap plan **iii** \$99 Cap plan

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g What is the minimum cost of using the $79 plan over 2 years? Explain how this figure was calculated.

h What happens to the cost of SMS texts when you choose a more expensive cap plan?

2 Felicity uses about $8 worth of phone calls and texts per day.

a In 31 days, about how much in calls and texts would she use?

b Which cap plan would be best for her?

3 Why is it important to select a plan with an amount of included credit that you won't exceed regularly?

4 Choose a suitable postpaid plan for each person described, giving reasons for your answer.

a Hayley spent the following amounts on phone calls in the past six months: $73, $70, $66, $85, $82, $89.

b Max has a pet-washing business and his monthly calls for 3 months were $240, $300 and $375.

c Shane uses his phone mainly to keep in touch with his parents, who both own Optel phones.

d Jarryd, an electrician, makes about $26 worth of calls each day from his van.

e You.

5 **a** Under the $79 Optel prepaid plan shown in Example **13** on page 471, calculate the cost of:

Example **13**

i a 4-minute call **ii** sending 51 SMS texts **iii** using 200 MB of data

b What happens to the call rate when you choose a more expensive prepaid plan?

c Are the call rates of prepaid phone plans generally higher or lower than those of the postpaid plans?

d What happens to the credit expiry period when you choose a more expensive plan?

e Calculate the cost of a 13-minute call to another Optel mobile phone under the:

i $99 plan **ii** $49 plan **iii** $19 plan

f What is the minimum cost of using the $79 plan over 360 days?

6 Name two advantages and two disadvantages of having:

a a prepaid mobile phone plan

b a postpaid mobile phone plan.

7 Which type of plan – prepaid or postpaid – is described by each phrase below?

a easier to quit the plan

b more suitable for low users of mobile phones

c prevents the risks of overspending

d involves a monthly bill

e has higher costs for additional data usage

f account needs to be recharged regularly

8 Choose a suitable prepaid plan from Example **13**, page 471, for each person mentioned in question **4**, giving reasons for your answer.

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TECHNOLOGY: PHONE PLAN COMPARISON WEBSITES

Use a search engine to find a mobile phone plan comparison website such as Phonechoice. Use it to select a suitable phone plan for yourself.

1. What is the name of the plan?
2. What is the monthly access fee and included credit?
3. What is the connection fee for each call and the call rate?
4. What is the minimum cost of the contract?
5. For how many months are you locked in?
6. How can you save money using this plan? Is there any 'free time' or 'off-peak time'?
7. What is the cost of each SMS text?
8. What is the included data amount?
9. What is the cost of additional data?
10. Are there any special offers with this plan, such as free SMS texts or unlimited calls?
11. Is there a discount for 'bundling' your plan with your home phone or Internet plan as well?
12. Does this plan sound too good to be true? Can you find any hidden catches or risks in the small print?

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Sample HSC problem

This table shows a range of QuickChat's prepaid mobile phone plans.

Cost of plan	$19	$30	$45	$75	$120
Credit for calls and SMS texts	$90	$180	$400	$800	$1250
Expiry time	30 days	30 days	30 days	60 days	180 days
Call connection fee	45c	45c	30c	30c	24c
Call rate per min	105c	95c	80c	72c	66c
Cost per SMS text	30c	28c	25c	22c	20c
Included data	150 MB	400 MB	1.5 GB	3 GB	6 GB
Additional data cost	$1 per MB	$0.50 per MB		$0.30 per MB	
Calls to other Optel mobiles	Normal rates	Free for the first 20 mins between 8 PM and 8 AM		Free anytime	

a Other than the increase in credit for calls and SMS texts, name one advantage of choosing the $75 plan over the $45 plan.

b Calculate the cost of an 8-minute call if using the $75 plan:

i to another Optel mobile **ii** to a landline phone

c Under the $30 plan, Daniel used 740 MB of data. How much did this cost him?

d If Daniel uses 740 MB regularly each month, which is the best plan for him to use? Give reasons for your answer.

e Under the $120 plan, Bella spent $18.20 on sending SMS texts. How many texts did she send?

Study tip

Looking after yourself

- While studying, don't forget to keep it all in perspective.
- Remember to have your own life outside school.
- Look after your physical and mental health.
- Eat properly and have enough sleep.
- Exercise regularly, play sport and go out.
- Plan to do nothing occasionally.
- Relax and rest regularly.
- Talk to your family, visit your friends.
- Be positive and sensible.
- Have confidence in yourself and don't stress.
- Don't worry, be happy.

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CHAPTER REVIEW

Topic overview

This chapter, 'Phone plans and downloading data', examined the measurement and statistics behind using mobile phones and downloading data from the Internet. You investigated units of data storage memory such as kilobits and megabytes, calculated download times and interpreted graphs and tables involving downloading music and video. You analysed the features and costs of different types of mobile phone plans and identified patterns of phone usage in order to make wise decisions for selecting a suitable plan.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map has been printed below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.

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Revision

1 Convert:

a 3.4 kb to b **b** 5.6×10^{11} b to kB **c** 68 MB to B

Exercise 10-01

2 Tegan said that she could store 5000 songs on her new 16 GB MP3 player. Is this true, if an average song is 3.5 MB?

Exercise 10-01

3 How many songs with an average size of 4 MB will fit on a 1 TB external hard drive?

Exercise 10-01

4 A 500 MB file was downloaded in 3 m 15 s. What was the download speed, to the nearest kbps?

Exercise 10-02

5 If a 3.8 MB file was downloaded at 256 kbps, how long would it take? Answer to the nearest second.

Exercise 10-02

6 This column graph shows the five most illegally-downloaded films of 2010.

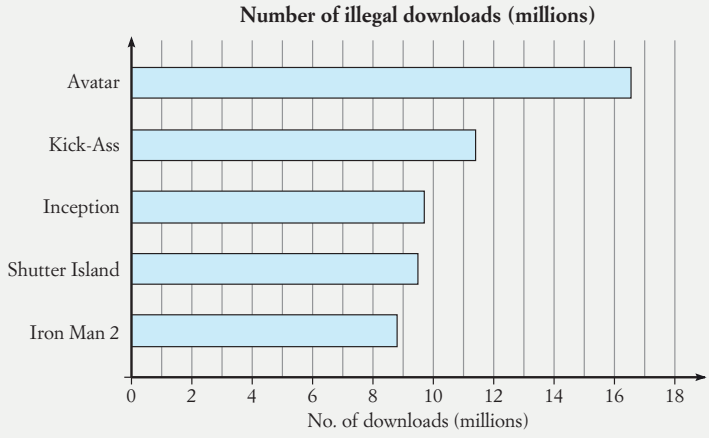
Exercise 10-03

a How many illegal downloads of *Iron Man 2* were there?

b Which film had 16.6 million downloads?

c If each illegally downloaded film amounts to lost income of $8.40 to filmmakers, what is the total lost revenue for the film *Kick-Ass*?

d What is the main reason people download films illegally?



7 This sector graph shows the types of items illegally downloaded over a month.

Exercise 10-03

a What was the third-most downloaded item?

b Which two items together make up just under half of all illegal downloads?

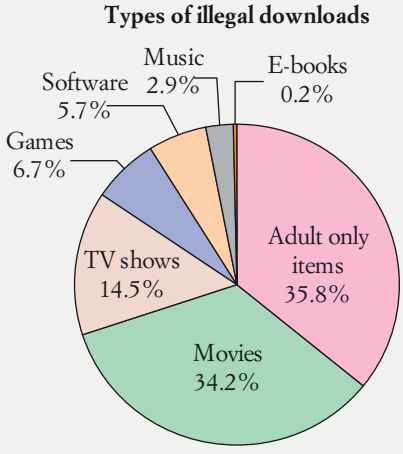
c What percentage of illegal downloads were of games?

d If there were 2 million downloads in the month, how many of them were:

i movies?

ii TV shows?

iii e-books or software?



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Exercise 10-04

8 The TelNet $59 Cap Plan has the following features.

Monthly access fee: $59
Included credit: $520 of calls
Call rate: 40c connection fee, then 37c per 30 seconds or part thereof
Discounted call rate: 24c per 30 seconds or part thereof between 10 p.m. and 8 a.m.
Text: 26c per SMS
Included data: 1.5 GB, then 25c per additional MB
Minimum cost: $708

a Petra made a 4 min 12 s call to her father at 11:30 p.m. How much did it cost?

b Greta sent 236 SMS texts. What was the total cost?

c Zaiba used 1080 MB of data. How much did it cost her?

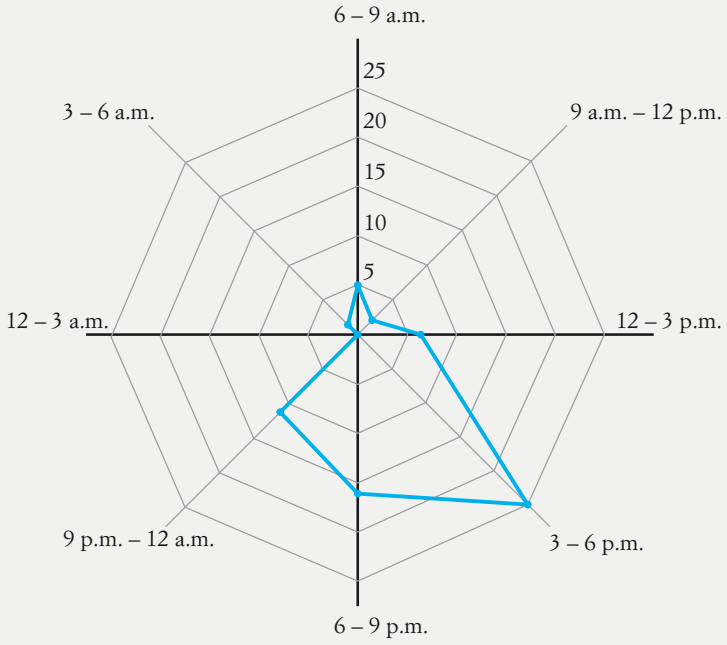
d Armand makes $546 worth of phone calls and texts one month. What is the total cost?

e Is the term of this plan 12 or 24 months? Use calculations to justify your answer.

f Jules makes 6 calls per day at an average of 3 minutes per call. Calculate the total cost of calls over a week.

Exercise 10-05

9 This radar chart shows the number of mobile phone calls made by a sample of students over a 24-hour period, in 3-hour blocks.



a How many phone calls were made between 9 p.m. and 12 midnight?

b What is the mode? Why do you think this is so?

c When were 16 calls made?

d When were no calls made?

e In one or two sentences, describe the pattern of phone calls over the 24-hour period.

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10 Use the table of Optel postpaid mobile phone plans from Example **12** on page 470 to answer the following questions.

a Calculate the cost of a 11-minute call to another Optel mobile phone at 10:30 a.m. under the:

i $49 Cap plan **ii** $99 Cap plan

b What happens to the included data allowance when you choose a more expensive cap plan?

c The $129 Cap plan is an 'unlimited' plan. What does that mean?

d Why is it important to not go over your included data allowance or credit?

e Calculate the extra cost of $904 of calls and text in a month under the $79 plan.

f Show the calculation for finding the minimum cost of the $129 Cap plan.

g Each day, Harry sends about 20 SMS texts and makes 4 phone calls costing around $7 each. Select an appropriate plan for him.

h Harry's grandmother uses her mobile phone to call her family twice a week. She does not send SMS texts and rarely makes any other calls. Select an appropriate plan for her.

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FINANCIAL MATHEMATICS

11.

INVESTING MONEY

People once kept their savings under the mattress or in an old tin, before they started using banks. Today's investors are more informed about financial planning and more likely to invest in stocks and shares, a superannuation fund or property rather than leave money in a low-interest savings account. More than half of all Australians own shares, often through a superannuation fund.

CHAPTER OUTLINE

FM2	11-01	Simple interest
FM2	11-02	Simple interest graphs
FM2	11-03	Compound interest
FM2	11-04	Compound interest tables
FM2	11-05	Account fees and charges
FM2	11-06	Inflation and appreciation
FM2	11-07	Investing in shares
FM2	11-08	Share tables and graphs

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IN THIS CHAPTER YOU WILL:

- calculate monthly, quarterly and six-monthly interest rates based on quoted rates per annum (p.a.)
- calculate simple interest using $I = Prn$, where P is the principal, r is the interest rate per period and n is the number of periods
- use tables of values for fixed values of P, and hence draw and describe graphs of I against n for different values of r
- compare costs associated with maintaining accounts with financial institutions
- calculate the final amount, interest and principal using the compound interest formula $A = P(1 + r)^n$ where A (amount) represents the final amount, P (principal) represents the initial amount, n is the number of compounding periods and r is the interest rate per compounding period
- calculate and compare the final amount, interest and principal using a table of compounded values of one dollar
- compare different investment strategies
- calculate the price of goods following inflation and investigate the effect of inflation on prices
- calculate the appreciated value of items, for example, stamp collections and other memorabilia
- record and graph the price of a share over time
- calculate the dividend paid on a shareholding and the dividend yield (excluding franked dividends)

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TERMINOLOGY

appreciation	brokerage	compound interest
compounded	dividend	dividend yield
final amount	inflation	market value
overdrawn account	principal	quarterly
shareholder	simple interest	stockbroker term

Assignment 11

SkillCheck

1 Copy and complete:

a 10% p.a. = ___% per quarter

b 6% p.a. = ___ % per half year

c 1% per month = ___ % p.a.

2 Evaluate, correct to two decimal places:

a $3600 \times (1.03)^3$ **b** $1268 \times (1.065)^6$ **c** $2600 \times (1.002)^{24}$

3 Larni received a 35c dividend (interest) for each of her 1200 shares.

a How much did Larni receive?

b If each share is worth \$6.50, express the 35c dividend as a percentage of the share value, correct to one decimal place.

4 **a** Increase \$3500 by 2%.

b Decrease \$1200 by 5%.

c Increase \$2700 by 1.5%.

Simple interest

11-01 Simple interest

Interest is money earned on an investment with a bank, credit union or other financial institution. **Simple interest** is interest calculated as a percentage of the **principal** (the amount of money invested).

SUMMARY

The **simple interest formula** is: $I = Prn$,

where I = interest

P = principal or initial amount

r = interest rate per period, expressed as a decimal

n = number of periods

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EXAMPLE 1

Julia invested her inheritance of \$12 520 at 6.35% p.a. for 20 months.

Calculate the simple interest Julia earned.

p.a. means 'per annum' (or per year).

Solution

$P = \$12\ 520 \qquad r = \dfrac{0.0635}{12}$ per month $\qquad n = 20$ months

r and n are expressed in the same units (months).

$$\begin{aligned} I &= Prn \\ &= \$12\ 520 \times \frac{0.0635}{12} \times 20 \\ &= \$1325.0333\ldots \\ &\approx \$1325.03 \end{aligned}$$

Interest earned is always rounded down to the nearest cent.

EXAMPLE 2

Carmel invested \$3500 in a savings account. After $4\frac{1}{2}$ years, her investment grew to \$4185.12. What simple interest rate, as a percentage to two decimal places, did Carmel receive on her investment?

Solution

Interest, $I = \$4185.12 - \$3500$ **Final amount – principal**

$= \$685.12$

$P = \$3500 \qquad n = 4.5$ years

$$\begin{aligned} I &= Prn \\ \$685.12 &= \$3500 \times r \times 4.5 \\ \$685.12 &= \$15\ 750r \\ r &= \frac{\$685.12}{\$15\ 750} \\ &= 0.043\ 49\ldots \\ &= 4.349\ldots\% \\ &\approx 4.35\% \end{aligned}$$

Simple interest rate is 4.35%.

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Exercise 11-01 Simple interest

Example 1

1 Which of the following is the simple interest earned when \$7400 is invested at 5% p.a. for 4 years? Select **A**, **B**, **C** or **D**.

A \$148 **B** \$14.80 **C** \$1480 **D** \$14 800

2 Calculate the simple interest earned from each of the following investments.

a \$2136 invested at 6% p.a. for $2\frac{1}{2}$ years

b \$16 200 invested at 7.65% p.a. for 3 years

c \$12 874 invested at 0.61% per month for 6 months

d \$4510 invested at 0.0301% per day for 31 days

e \$20 016 invested at $7\frac{1}{2}$% p.a. for 5 months

f \$8250 invested at 10.5% p.a. for 240 days.

3 Kalena earned \$262.44 in simple interest from investing an amount for 3 years at 5.4% p.a. What was the amount Kalena invested?

Example 2

4 Alan earned \$5409.25 simple interest from an investment of \$9835 over 5 years. What was the interest rate per annum?

5 For how long must a principal of \$15 750 be invested at 9.8% p.a. simple interest for it to grow to \$18 837?

6 Zoran earned \$675 interest from \$7500 invested for 9 months. What was the interest rate per annum?

7 What principal would earn \$3729.60 in interest if invested for 3 years at 16.8% p.a.?

8 OzExpress Credit Union has the term deposit accounts shown in the table on the right, in which the principal must be invested for a fixed period. Calculate the simple interest earned on the following investments.

a \$6300 for 3 years **b** \$13 750 for 8 months

c \$7800 for 5 years **d** \$14 240 for 2 months

Term	Interest rate
1–6 months	8.75% p.a.
7–11 months	9.25% p.a.
1–3 years	10.5% p.a.
4–5 years	12% p.a.
Minimum investment: \$1000	

9 Kylie earned \$80.58 interest from an investment of \$2530 over 91 days. What was the interest rate per day, correct to two significant figures?

10 For how long would \$4720 need to be invested at 0.67% per month to earn \$474.36 in simple interest?

11 An amount of \$9020 was invested for 2 years with simple interest and grew to \$10 751.84. Calculate the monthly interest rate.

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INVESTIGATION

INTEREST RATES

Choose a bank and collect information about the different types of accounts available, and the interest rates and features of each.

Examples: savings account, investment account, fixed term deposit, cheque account, Christmas club account and young saver account.

1 Which type of account would you choose for saving your money?

2 Compare and contrast the different types of account and suggest the type of person each account would suit.

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11-02 Simple interest graphs

WS 2 mm grid paper

Simple interest is represented by the linear function $I = Prn$. If the principal, P, and the interest rate, r, are known, then the interest, I, is the dependent variable and the number of periods, n is the independent variable.

EXAMPLE 3

Rachel invests \$20 000 in an account that earns 6% p.a. simple interest.

a Construct a graph that shows the simple interest, I, earned in dollars for n years, for values of n from 0 to 8.

b What does the gradient of this graph represent?

c Use the graph to estimate:

i the simple interest earned after $4\frac{1}{2}$ years

ii when \$8000 in interest will be earned.

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Solution

a $I = Prn$ where $P = 20\ 000$ and $r = 0.06$

$\therefore I = 20\ 000 \times 0.06 \times n$

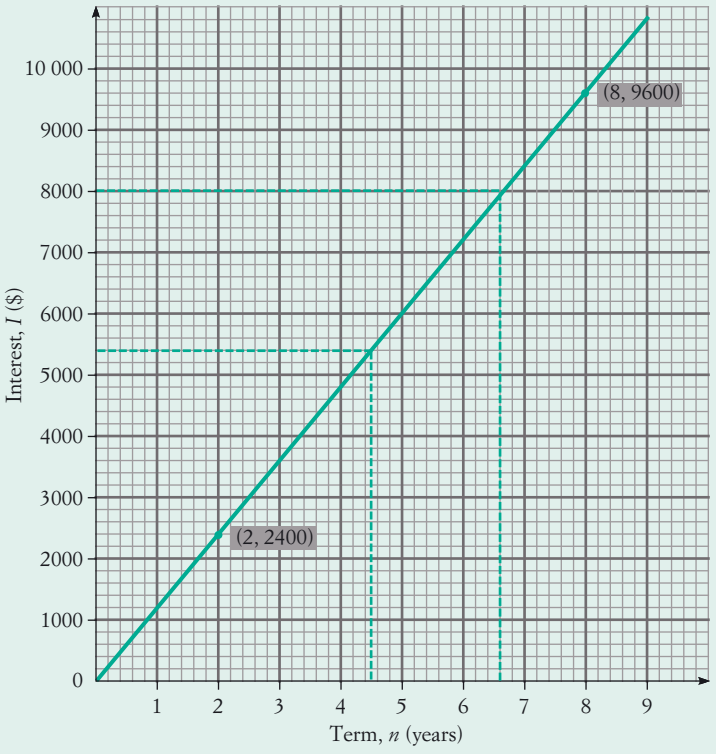
$I = 1200n$

This is a linear function with gradient 1200 and vertical intercept 0.

Construct a table of values to graph the line:

No. of years, n	0	2	8
Interest, I ($)	0	2400	9600

Simple interest earned from an investment of $20 000 at 6% p.a.



b The gradient, 1200, represents the simple interest earned per year.

c **i** $5400 interest **Reading from the graph at $n = 4\frac{1}{2}$**

ii 6.6 years **Reading from the graph at $I = 8000$**

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Exercise 11-02 Simple interest graphs

Questions **1** to **3** refer to the simple interest graph from Example **3** on page 486.

Example 1

1 According to the graph, what is the interest earned on an investment of \$20 000 over 4 years at 6% pa? Select the correct answer **A**, **B**, **C** or **D**.

A \$3150 **B** \$3300 **C** \$4800 **D** \$4900

2 Use the graph to estimate the simple interest earned when \$20 000 is invested at 6% p.a. for:

a 9 years **b** $2\frac{1}{2}$ years **c** 7.6 years.

3 If \$20 000 is invested at 6% p.a, how long will it take for each of the following amounts of simple interest to be earned?

a \$10 000 **b** \$4200 **c** \$8500.

4 Graph on the same axes the simple interest, I, earned for n years, for values of n from 0 to 10, when \$4000 is invested in an account earning:

a 4% p.a. **b** 12% p.a.

5 **a** What is the gradient of the line you drew in question **4b** and what does it represent?

b Use your graphs from question **4** to estimate the simple interest earned from an investment of \$4000:

i over 6 years at 4% p.a. **ii** over 8 years at 12% p.a.

c How do the two different interest rates appear on the graphs?

6 The graph on the right shows the simple interest earned on a principal of \$20 000 for up to 5 years at three different interest rates per annum.

According to this graph, how long will it take a principal of \$20 000 to earn \$2400 if the interest rate is 8% p.a.? Select the correct answer **A**, **B**, **C** or **D**.

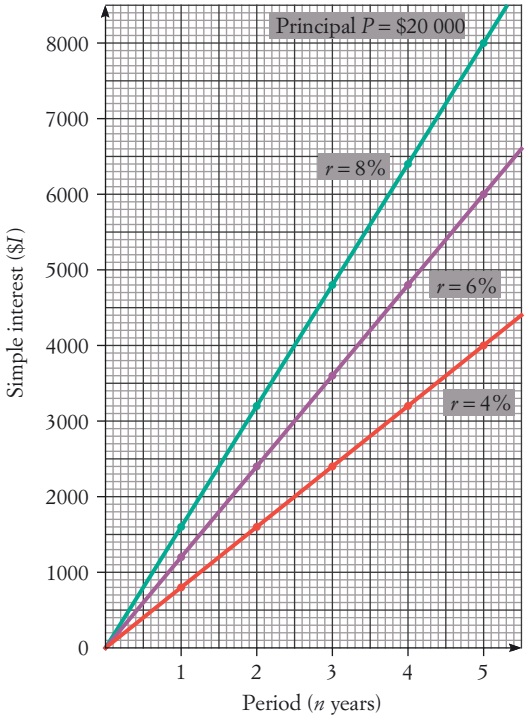
A $1\frac{1}{2}$ years

B 2 years

C 2 years 7 months

D 3 years

Interest earned on a principal of \$20 000



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7 Use the graph from question **6** to estimate:

- **a** the interest earned at 6% p.a. after $3\frac{1}{2}$ years
- **b** the interest rate when \$3000 interest is earned after $2\frac{1}{2}$ years
- **c** the number of months it takes to earn \$4500 interest at 6% p.a.
- **d** the time it takes to earn \$7400 in interest at 8% p.a.
- **e** the interest earned after 21 months at 8% p.a.
- **f** the interest rate if \$1800 interest is earned after 27 months.

8 The simple interest graph is a linear graph passing through the origin. Explain why the vertical intercept of this graph is 0.

11-03 Compound interest

Compound interest

Simple vs compound interest

Simple vs compound interest: Spreadsheet

Compounding periods: Spreadsheet

Most savings accounts pay **compound interest**, rather than simple interest. Compound interest is interest that is added to the principal and reinvested. The principal plus interest becomes the new principal on which interest is calculated. In other words, we earn 'interest on our interest' as well as on the original principal.

EXAMPLE 4

A principal of \$3000 is invested at 6% p.a., compounded yearly.

- **a** Find the amount of the investment at the end of 3 years.
- **b** Calculate the compound interest earned after 3 years.
- **c** Find the amount of the investment after 5 years.
- **d** Find a formula for the amount of the investment after n years.

Solution

a At the end of the 1st year:

$\text{Amount} = \$3000 + 6\% \times \$3000$ **Principal + interest**

$= \$3000 \times 1.06$ **Increasing \$3000 by 6%.**

At the end of the 2nd year:

$\text{Amount} = (\$3000 \times 1.06) \times 1.06$ **Increasing (\$3000 × 1.06) by another 6%.**

$= \$3000 \times 1.06^2$

At the end of the 3rd year:

$\text{Amount} = (\$3000 \times 1.06^2) \times 1.06$ **This is the same as increasing (\$3000 × 1.06²) by another 6%.**

$= \$3000 \times 1.06^3$

$= \$3573.048$

$\approx \$3573.04$ **Rounding down.**

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b Compound interest earned = final amount – principal

$= \$3573.04 - \$3000$

$= \$573.04$

c Increasing \$3000 by 6% five times:

Amount $= \$3000 \times 1.06^5$

$= \$4014.676\ 733$

$\approx \$4014.67$

d Increasing \$3000 by 6% n times:

Amount $= 3000(1.06)^n$

SUMMARY

The compound interest formula is $A = P(1 + r)^n$,

where: A = final amount

P = principal or initial amount

r = interest rate per compounding period, expressed as a decimal

n = number of compounding periods

The compound interest (I) earned can be found from the formula $I = A - P$.

In the world of finance, the compound interest formula $A = P(1 + r)^n$ is written as: $FV = PV(1 + r)^n$ where FV is called **future value** and PV is called **present value**.

EXAMPLE 5

If \$12 500 is invested at 4.8% p.a. with the interest compounded monthly, calculate the final value and total interest earned over 2 years.

Solution

Because interest is compounded monthly, r and n must be expressed in months.

$P = \$12\ 500 \qquad r = \dfrac{0.048}{12} = 0.004$ per month $\qquad n = 2 \times 12 = 24$ months

Final value $= \$12\ 500(1 + 0.004)^{24}$

$= \$12\ 500(1.004)^{24}$

$= \$13\ 756.8537\ldots$

$\approx \$13\ 756.85$

Compound interest $= \$13\ 756.85 - \$12\ 500$

$= \$1256.85$

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EXAMPLE 6

What present value must be invested at 4.5% p.a. for 8 years so that it grows to \$10 000?

Solution

$A = \$10\,000 \qquad r = 0.045 \qquad n = 8$

$$A = P(1 + r)^n$$

$$\$10\,000 = P(1 + 0.045)^8$$

$$= P(1.045)^8$$

$$P = \frac{\$10\,000}{(1.045)^8}$$

$$= \$7031.8512\ldots$$ **Round the present value up to give the required final amount.**

$$= \$7031.86$$

A present value of \$7031.86 is needed.

Exercise 11-03 Compound interest

Example 4

1 Calculate the final amount of each of the following investments.

- **a** \$7400 invested at 5% p.a. compound interest for 4 years
- **b** \$2840 invested at 6.5% p.a. compound interest for 5 years
- **c** \$4500 invested at 4.9% p.a. compound interest for 2 years
- **d** \$17 000 invested at 0.5% per month compound interest for 10 months
- **e** \$9250 invested at 0.82% per month compound interest for 6 months
- **f** \$9000 invested at 8.4% p.a. for 8 months, compounded monthly

Example 5

2 Calculate the final amount of each of the following investments and, hence, the amount of compound interest earned.

- **a** \$12 900 invested at 10.8% p.a. for 1 year, compounded monthly
- **b** \$13 800 invested at 7.5% p.a. for 2 years, compounded half-yearly
- **c** \$13 800 invested at 7.5% p.a. for 2 years, compounded quarterly
- **d** \$6920 invested at 9% p.a. for 240 days, compounded daily ← 1 year = 365 days

Example 6

3 Paul wants to invest some money so that it will grow to \$24 000 in 5 years time, when he will travel through Europe. If the interest rate is 5.5% p.a., what present value should Paul invest, to the nearest cent?

4 A sum of \$8500 is invested at 7% p.a. for 5 years.

- **a** Calculate the total interest earned if the interest is:
  - **i** calculated at a flat rate (simple interest)
  - **ii** compounded yearly.
- **b** **i** Why does one type of interest result in a greater amount than the other?
  - **ii** By how much is it greater?

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5 A principal of $21 000 is invested at 5% p.a. Calculate the future value of the investment after 2 years if the interest is compounded:

a yearly **b** half-yearly **c** quarterly **d** monthly **e** daily.

6 Judging from your results for question **5**, what happens to the amount of interest earned as the frequency of compounding increases? Why?

7 Zara has $4000 in an account earning 4% p.a. interest, compounded yearly. By guessing and checking, determine how long it will take Zara to double her money. Give your answer to the nearest year.

8 How long, to the nearest year, will an investment of $2400 take to grow to $3265 at 8% p.a. interest, compounded yearly?

9 After 8 years, the value of Corrina's investment grew to $6260.14. What was the initial amount of Corrina's investment if the interest rate was 8.75% p.a.?

10 Determine the sum to be deposited if $10 000 is required in 6 years time and terms of 3.75% p.a. (compounded quarterly) are available.

11 A principal of $10 000 is to be invested for 3 years. Which of the following is the best investment option? Select **A**, **B**, **C** or **D**.

A 6% p.a. simple interest **B** 5.9% p.a. compounded annually

C 5.85% p.a. compounded half-yearly **D** 5% p.a. compounded monthly

12 Jack won $1 000 000 in the lottery. If he invests all his winnings in a term deposit, earning 7.4% p.a. interest compounded annually, how many years will it take for him to double his money?

INVESTIGATION

DOUBLE YOUR MONEY USING THE RULE OF 72

How long will it take for money invested at a particular interest rate to double in size? We can estimate the time it takes to double our money by dividing 72 by the interest rate per compounding period.

For example:

a How long will it take a principal of $20 000 to double if interest is compounded annually at 4% p.a.?

$72 \div 4 = 18$ years

b What if the interest is 6% p.a.?

$72 \div 6 = 12$ years

1 Use the compound interest formula, $A = P(1 + r)^n$, to check that $20 000 does in fact double with the above interest rates and terms.

2 Use the 'rule of 72' to calculate how long it will take $20 000 to double if invested at an interest rate of:

i 7.5% pa **ii** 5%

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3 Check that your answers to question **2** are correct using the compound interest formula.

4 Did you notice that you don't actually use '$20 000' in your calculations? That means that the time required to 'double your money' does not depend on the amount. Choose another principal amount and check that your answers to question **2** are correct for it as well.

TECHNOLOGY: COMPOUND INTEREST ON A GRAPHICS CALCULATOR

Graphics calculators have a financial mode for compound interest calculations. For Casio, press MENU and select **TVM** (time value of money), then **Compound Interest**. For Texas Instruments, press FINANCE, then **TVM Solver**. You should see a screen of seven variables:

n = number of compounding periods

I% = interest rate as a percentage

PV = present value (principal; enter as negative value)

PMT = payment per period (for regular payments such as superannuation)

FV = future value (final amount)

P/Y = number of periods per year

C/Y = number of compounding periods per year

If the values for six of the variables are entered, the calculator finds the value of the seventh variable.

1 In Example **4** (on page 488), a principal of $3000 was invested at 6% p.a. for 3 years. Enter the following values, pressing EXE or ENTER each time.

$n = 3$

$I\% = 6$

PV = –3000	Enter the principal as a negative value.
PMT = 0	For compound interest, there are no regular payments, just the principal.
P/Y = 1, C/Y = 1	Interest is compounded once per year.

For Casio calculators, select FV. For Texas Instruments calculators, move the cursor to 'FV=' and press SOLVE. This should calculate the future value (final amount): $3573.048

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2 If Grace needs \$10 000 for university in 6 years time, find the principal she needs to invest now at 5% compounded quarterly. Enter:

$n = 24$	Number of quarters $= 6 \times 4$
$I\% = 5$	Interest rate per quarter.
PMT = 0	
FV = 10 000	
P/Y = 4, C/Y = 4	Interest compounded four times per year.

Select PV to get \$7421.98 (rounded up to the nearest cent).

3 Use your calculator to check the answers you calculated in Exercise 10-03.

INVESTIGATION

COMPOUND INTEREST GRAPH (EXTENSION)

Blake invests \$20 000 in an account that earns 6% p.a. interest, compounded annually.

1 If you graphed the size of his investment growing over 10 years, what do you think the graph would look like? Give reasons for your answer.

2 Copy and complete this table using the formula $A = P(1 + r)^n$, rounding your values to the nearest whole dollar.

No. of years, n	Amount, A (\$)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

3 Use the table to construct a graph that shows the amount of the investment, A, growing over 10 years.

4 What is the vertical intercept of this graph and what does it represent?

5 Comment on what the shape of the graph shows.

6 Use the graph to estimate:

i the value of the investment after $8\frac{1}{2}$ years

ii when the value of the investment reaches \$30 000.

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Compound interest table

11-04 Compound interest tables

The following table shows the final balance when $1 is invested at various interest rates and terms. Such tables are used by accountants and financial advisers to quickly calculate the future value of a principal that is earning compound interest.

Compound interest table $(1 + r)^n$ Final amount of investment (per dollar)						
Number of periods (n)	Interest rate per compounding period (r)					
	0.01 1%	0.05 5%	0.08 8%	0.1 10%	0.15 15%	0.2 20%
1	1.010	1.050	1.080	1.100	1.150	1.200
2	1.020	1.103	1.166	1.210	1.323	1.440
3	1.030	1.158	1.260	1.331	1.521	1.728
4	1.041	1.216	1.360	1.464	1.749	2.074
5	1.051	1.276	**1.469**	1.611	2.011	2.488
6	1.062	1.340	1.587	1.772	2.313	2.986
7	1.072	1.407	1.714	1.949	2.660	3.583
8	1.083	**1.477**	1.851	2.144	3.059	4.300
9	1.094	1.551	1.999	2.358	3.518	5.160
10	**1.105**	1.629	2.159	2.594	4.046	6.192

EXAMPLE 7

Use the table above to calculate:

a the final balance when $2000 is invested at 8% p.a. for 5 years

b the final balance when $11 000 is invested for 8 months at 5% per month

c the compound interest when $4780 is invested at 12% p.a., compounded monthly, for 10 months.

Solution

a For a principal of $1, when $r = 0.08$ and $n = 5$, $A = \$1.469$ (from the table).

So, for a principal of $2000, $A = 2000 \times \$1.469 = \$2938$.

b For a principal of $1, when $r = 0.05$ and $n = 8$, $A = \$1.477$.

So, for $11 000, $A = 11\,000 \times \$1.477 = \$16\,247$.

c For a principal of $1, when $r = \frac{0.12}{12} = 0.01$ per month and $n = 10$ months, $A = \$1.105$.

r and *n* are in months

So, for $4780, $A = 4780 \times \$1.105 = \$5281.90$.

Compound interest $= \$5281.90 - \$4780 = \$501.90$

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Exercise 11-04 Compound interest tables

Example 7

Use the compound interest table on the previous page to answer these questions.

1 When $3000 is invested for 4 years, compounded annually at 5%, which of the following is the final value? Select **A**, **B**, **C**, or **D**.

A $12 160 **B** $3648

C $3012.16 **D** $3828

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2 If the final value of an investment is $2159 after 10 years at 8% p.a., which of the following is the principal? Select **A**, **B**, **C**, or **D**.

A $1000 **B** $1

C $100 **D** $10

3 Calculate the final value and amount of compound interest earned for each of these investments.

	Principal	Interest rate	Period
a	$16 000	10% p.a.	4 years
b	$3185	20% p.a.	8 years
c	$1	15% p.a.	10 years
d	$8790	8% p.a.	2 years
e	$11 450	1% p.a.	6 years
f	$7252	8% per month	3 months
g	$18 200	5% per month	6 months
h	$32 000	10% per month	7 months
i	$4200	12% p.a. compounded monthly	8 months
j	$5300	16% p.a. compounded half-yearly	2 years

4 How long, to the nearest year, will it take a principal to double when invested at 15% p.a. compound interest? Explain how you found your answer.

5 If a principal took approximately 9 years to double in value, at what interest rate was it invested?

6 $1000 invested at 20% p.a. was worth triple the amount after how many years?

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Simple vs compound interest: Spreadsheet

TECHNOLOGY: SIMPLE AND COMPOUND INTEREST

Spreadsheet 1: Simple versus compound interest

Let us compare the simple interest and compound interest that would be earned on an investment of $1000 at 6% p.a. over 10 years. You can compare interest earned for any principal and rate by adjusting the values in cells D1 and D2.

	A	B	C	D	E
1		Investment amount		$1,000	
2		Interest rate p.a.		0.06	
3					
4		Simple interest		Compound interest	
5	Year	Interest per year	Investment value	Interest per year	Investment value
6	1	$60	$1,060	$60	$1,060.00
7	2	$60	$1,120	$63.60	$1,123.60
8	3	$60	$1,180	$67.42	$1,191.02
9	4	$60	$1,240	$71.46	$1,262.48
10	5	$60	$1,300	$75.75	$1,338.23
11	6	$60	$1,360	$80.29	$1,418.52
12	7	$60	$1,420	$85.11	$1,503.63
13	8	$60	$1,480	$90.22	$1,593.85
14	9	$60	$1,540	$95.63	$1,689.48
15	10	$60	$1,600	$101.37	$1,790.85
16	Total	$600		$790.85	

Here are the necessary formulas for Years 1 and 2 (rows 6–7). Once you enter these, use Fill down from row 7 to copy the formulas down the columns.

5	Year	Interest per year	Investment value	Interest per year	Investment value
6	1	=$D$1*$D$2	=$D$1+B6	=$D$1*$D$2	=$D$1+D6
7	2	=$D$1*$D$2	=C6+B7	=$D$2*E6	=E6+D7

1 Modify your spreadsheet to show that:

a the difference between the simple interest and compound interest earned on a principal of $5000, invested at 4.5% p.a. over 10 years, is $514.85

b the value of a principal of $3500 invested at 7% p.a., compounded half-yearly for 3 years, is $4302.39.

Compounding periods: Spreadsheet

Spreadsheet 2: Compounding periods

As we saw in Spreadsheet 1 above, we earn more money with compound interest than we do with simple interest. Create the following spreadsheet for a principal of $1000 invested at 6% p.a. over 10 years for various compounding periods. Enter appropriate formulas to find the given values. Let 1 year = 365 days.

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	A	B	C	D	E
1	Principal	$1,000			
2	Interest rate (% p.a.)	0.06			
3	Term (years)	10			
4					
5		**Interest rate per period (%)**	**Number of periods**	**Final amount**	**Total interest earned**
6	**Simple interest**	0.06	10	$1,600.00	$600.00
7	**Compound interest**				
8	Compounded yearly	0.06	10	$1,790.85	$790.85
9	Compounded half-yearly	0.03	20	$1,806.11	$806.11
10	Compounded quarterly	0.015	40	$1,814.02	$814.02
11	Compounded monthly	0.005	120	$1,819.40	$819.40
12	Compounded daily	0.000164384	3650	$1,822.03	$822.03

2 **a** Show that the difference between the interest compounded yearly and the interest compounded daily on a principal of $1000 at 6% p.a. is $31.18.

b Show that the interest earned on $10 000 at 4.3% p.a., compounded daily for 4 years, is $1876.66.

c Why is it better for an investor to earn interest compounded daily rather than interest compounded monthly?

11-05 Account fees and charges

Banks, credit unions and other financial institutions have many types of accounts such as:

- savings accounts
- online accounts (e-banking)
- young saver accounts
- incentive saver accounts.

All accounts have associated fees and charges, for situations such as:

- excessive withdrawals per month
- ATM withdrawals made at another bank's ATM
- overdrawn accounts, when we withdraw more money than is available in our account
- when cheques given to us cannot be paid due to insufficient funds in the cheque writer's account
- when a personal cheque is not honoured by our bank due to insufficient funds in our account
- not paying the minimum amount, or exceeding the credit limit, on a credit card.

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EXAMPLE 8

OziBank's CashCard savings account has the associated fees and charges shown in the table below.

First eight withdrawals per month (not including withdrawals at another bank's ATM, direct debit or transfers to another bank account: see below)*	Free
Each subsequent withdrawal	
ATM/EFTPOS/cheque withdrawals	$1.00
Phone/Internet banking (for paying bills and transfers)	$0.60
Withdrawal at bank	$3.00
* Withdrawal at another bank's ATM	$2.00
* Direct debit	Free
Monthly account-keeping fee (for balances below $500)	$5.00
Overdrawn account	$20.00
Cheque/direct-debit dishonour fee	$30.00
Card replacement	$20.00
New chequebook	$10.00
* Transfers to another bank account (made at bank)	$5.00
Yearly dormancy fee (if no transactions made in a year)	$10.00

Manuel has $1420.50 in his CashCard account.

a Does Manuel need to pay the monthly account-keeping fee?

b During the month, Manuel makes 10 withdrawals from his account, the last two being EFTPOS and a cash withdrawal at the bank. Calculate the charges incurred.

c Manuel also wrote a cheque which was dishonoured. What is the fee charged?

d Manuel's deposits in the month total $616.20 and his withdrawals total $502.55. If Manuel also buys a new chequebook, calculate his final balance after all charges have been applied.

Solution

a No, Manuel does not need to pay the monthly account-keeping fee because his account balance is above $500.

b EFTPOS charge is $1.00. Bank withdrawal is $3.00.

Total charges = $1.00 + $3.00 = $4.00

The first eight withdrawals are free.

c The cheque dishonour fee is $30.

d Old balance = $1420.50

Total deposits = $616.20

Total withdrawals = $502.55

Total charges = $4.00 + $30.00 + $10.00 = $44.00

Final balance = $1420.50 + $616.20 – $502.55 – $44.00

= $1490.15

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Exercise 11-05 Account fees and charges

Questions 1 to 8 refer to OziBank's CashCard savings account in Example 8.

Example 8

1 Jeremy has already made eight withdrawals from his OziBank CashCard account this month. What will it cost him now to withdraw some money from an OziBank ATM?

2 **a** After eight withdrawals have been made from a CashCard account, which method of withdrawing money incurs the lowest charge? Why?

b Which method of withdrawing money incurs the highest charge? Why?

3 Jess has $287.50 in her account and has lost her card.

a What is the cost of acquiring a new CashCard?

b Does Jess need to pay the monthly account-keeping fee? Give a reason for your answer.

c During the month, Jess makes 11 withdrawals from her account, all at her bank's ATM. How much will she be charged for this?

d If Jess's deposits for the month total $612.40 and her withdrawals total $540, calculate her final balance after all fees have been applied.

4 OziBank imposes a fee if no transactions are made on the CashCard account during the year.

a What is this fee called and how much is it?

b Why do you think a bank would impose such a fee?

5 Claire wrote a cheque for $340 when she had only $289.26 in her account.

a What type of fee will Claire be charged?

b How much is this fee?

6 Samir had $8061 in his account and made nine withdrawals, the last one being the payment of his electricity bill using Internet banking. He went to his bank branch to transfer $2400 to an account at another bank, and made a withdrawal of $120 from another bank's ATM. All of this happened in the same month.

a Calculate the total charges incurred on the account during the month.

b If Samir made no deposits and his first nine withdrawals totalled $831.90, calculate his final balance after all withdrawals and charges had been deducted.

7 Why do you think OziBank imposes:

a a charge for more than eight withdrawals made in the month?

b a monthly account-keeping fee for balances below $500?

c a cheque/direct-debit dishonour fee?

8 OziBank published a brochure for its customers listing ways of minimising account charges. Explain how each of the following suggestions will reduce account charges.

a If you use EFTPOS, ask the retailer if you can also withdraw extra cash.

b If you currently withdraw small amounts nearly every day, try withdrawing larger amounts once or twice a week instead.

c Pay by credit or direct debit rather than by withdrawing by ATM, EFTPOS or cheque.

d Combine all of your separate accounts into one account.

e Bank via the telephone or the Internet.

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9 Why do you think banks prefer you to make a withdrawal at one of their ATMs rather than at another bank's ATM?

10 Banks now encourage customers to use online banking and they have websites designed to answer 'frequently asked questions'. Give three reasons why banks discourage customers from having the older types of savings accounts involving frequent bank visits.

INVESTIGATION

ACCOUNT-KEEPING FEES

1 Obtain a brochure from one or more of the banks or credit unions in your local area, or from a bank's website, and compare the fees and charges of the different types of savings accounts available. Examine a monthly statement of your own account (or your family's) and see how and when these fees and charges are applied.

2 Prepare a report comparing the main fees and charges of two different savings accounts, either within the same bank or between two banks or credit unions.

11-06 Inflation and appreciation

Inflation

Inflation is an increase in the prices of all goods and services. The yearly percentage increase is called the **annual inflation rate** and this is usually accompanied by increases in workers' wages and salaries to 'keep up with inflation'. The annual inflation rate in Australia is generally around 3%.

Calculating the new prices of goods and services after inflation is an application of increasing a quantity by a percentage repeatedly, so we can use the compound interest formula $A = P(1 + r)^n$.

EXAMPLE 9

A new Holden car costs \$32 460 today. If inflation remains constant at 3.4% p.a. for the next 4 years, what will be the price of a new Holden in 4 years time?

Solution

$P = \$32\ 460 \qquad r = 0.034 \qquad n = 4$

$A = P(1 + r)^n$

$= \$32\ 460(1.034)^4$

$= \$37\ 104.849\ldots$

$\approx \$37\ 105$

Price in 4 years $\approx$ \$37 105

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EXAMPLE 10

The current price of a magazine is \$7.20. Calculate its price 10 years ago if the inflation rate was a constant 2.8% p.a. during this time. Round your answer up to the nearest 5c.

Solution

$A = \$7.20 \qquad r = 0.028 \qquad n = 10$

$$A = P(1 + r)^n$$

$$\$7.20 = P(1.028)^{10}$$

$$P = \frac{\$7.20}{(1.028)^{10}}$$

$$= \$5.4626\ldots$$

$$\approx \$5.50$$

Price 10 years ago ≈ \$5.50

Appreciation

Appreciation is the increase in value of an item (such as artwork, gold, a prestige car, land or a house) over time. It is the opposite of **depreciation**, which is a decrease in value over time of items such as cars, furniture and electrical appliances.

Calculating the value of items after appreciation is another application of repeated percentage increase, so we can use the compound interest formula again.

EXAMPLE 11

Arno bought a block of land for \$142 500 and it appreciated at a rate of 6.5% p.a. Calculate its value, to the nearest dollar, 3 years later.

Solution

$P = \$142\ 500 \qquad r = 0.65 \qquad n = 3$

$$A = P(1 + r)^n$$

$$= \$142\ 500(1.065)^3$$

$$= \$172\ 132.8216$$

$$\approx \$172\ 133$$

Value after 3 years ≈ \$172 133

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Exercise 11-06 Inflation and appreciation

Example 9

1 A car costs $20 000 today. If inflation remains constant at 3% p.a. for the next 6 years, which of the following will be the price of the car in 6 years time? Select **A**, **B**, **C** or **D**.

A $23 880 **B** $21 800 **C** $23 820 **D** $23 600

Example 10

2 A textbook today costs $54.00. What was its price 8 years ago if the inflation rate over this period was 4% p.a.?

Example 11

3 A house cost $500 000 and appreciated in value at a rate of 5% p.a. Which of the following would be the value of the house 6 years after it was purchased? Select **A**, **B**, **C** or **D**.

A $669 113 **B** $670 048 **C** $650 000 **D** $530 000

4 A loaf of bread currently costs $4.00. Calculate its price in 5 years if the inflation rate is 4.7% p.a.

5 A 15-day holiday package costs $8300. Calculate its cost in 2 years time if the inflation rate is 6.1% p.a.

6 If gold appreciates at 9.3% p.a. and its current value is $840 per ounce, calculate the value of gold in 5 years time.

7 A collection of Beatles memorabilia appreciates at 4.8% p.a. and is currently valued at $12 400. What will be its value in 9 years?

8 **a** Name three items that appreciate in value over time.

b Name three items that depreciate in value over time.

9 A house increased in value this year from $547 000 to $603 000. What was the rate of appreciation per annum, as a percentage to one decimal point?

10 An accounting executive's salary increases with the annual inflation rate of 3.1%. Calculate her salary in 3 years if it is currently $72 020.

11 A pair of jeans costs $95 today. What was the value of a pair of jeans 10 years ago if the inflation rate was constant at 3.5% p.a.?

12 How much will $50 be worth by today's standards in 5 years time if the inflation rate is 2.8% p.a.? (*Hint*: Final value = $50.)

13 Calculate the cost of a $1200 refrigerator in 6 years if the inflation rate is 4% p.a.

14 An antique lamp has a current value of $290. What was its value 15 years ago, to the nearest dollar, if it has appreciated at 6.3% p.a.?

15 Katrina has a pearl necklace worth $2100. What will be its value after 12 years if it appreciates by 9% annually?

16 A car costs $28 500. What was the value of a similar earlier model 10 years ago if the inflation rate has been 3.4% p.a.? Answer to the nearest dollar.

17 An artwork appreciates by 10% each year. Calculate its value in 20 years if its current value is $20 000.

18 By 'guessing-and-checking', find how long it will take prices to double if the annual inflation rate is 4%. Give your answer to the nearest year.

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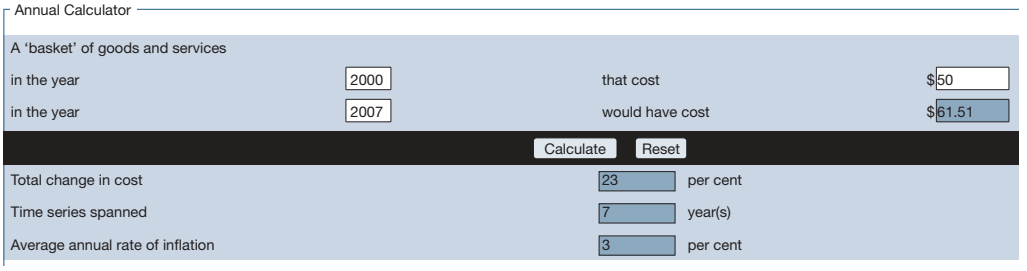
TECHNOLOGY: ONLINE INFLATION CALCULATOR

The Reserve Bank of Australia website, www.rba.gov.au/calculator/, has an inflation calculator to find an average annual inflation rate.



Experiment with the inflation calculator to find how much a car would have cost in 1990 if its price today is $25 000. What is the average annual inflation rate?

JUST FOR THE RECORD

The Consumer Price Index

The Consumer Price Index (CPI) is a statistic that measures the average price of consumer goods and services. The percentage change in the CPI is a measure of inflation.

$$\text{Inflation rate} = \frac{\text{New CPI} - \text{Old CPI}}{\text{Old CPI}} \times 100\%$$

This table shows the percentage change in prices of goods and services from March 2011 to March 2012.

Goods and services	Percentage change
Food and non-alcoholic beverages	–2.5
Alcohol and tobacco	3.5
Clothing and footwear	1.5
Housing	3.4
Furnishings, household equipment and services	0.7
Health	4.2
Transport	2.5
Communication	1.6
Recreation and culture	–0.9
Education	6.0
Insurance and financial services	4.0
All groups	1.6

Source: Australian Bureau of Statistics

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1. If a weekly train ticket cost \$34.50 in March 2011, how much did it cost in March 2012?
2. Find the CPI for this year and for the same time last year, and hence calculate the current inflation rate.

11-07 Investing in shares

Another way of investing money is to buy shares in a company, such as Woolworths, BHP Billiton, National Australia Bank (NAB) or Qantas. The **market price** of shares changes daily due to many factors, such as demand for the shares, the state of the company and the economic situation at the time.

Dividend and dividend yield

When we own shares in a company we are **shareholders** and earn a **dividend** (a type of interest) on our investment. The dividend can be collected by the investor or can be reinvested in more shares to earn a higher dividend next time. The **dividend yield** is the dividend as a percentage of the market price of the share (similar to an interest rate).

SUMMARY

$$\text{Dividend yield} = \frac{\text{dividend per share}}{\text{market price of share}} \times 100\%$$

EXAMPLE 12

Alyssia had 400 NAB shares valued at \$7740 and received a dividend of 32c per share. What was the dividend yield, correct to two decimal places?

Solution

Market price of each share = \$7740 ÷ 400 = \$19.35

Dividend per share = \$0.32

$$\text{Dividend yield} = \frac{\text{dividend per share}}{\text{market price of share}} \times 100\%$$

$$= \frac{\$0.32}{\$19.35} \times 100\%$$

$$= 1.6537\ldots\%$$

$$\approx 1.65\%$$

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Purchasing shares

A **stockbroker** buys and sells shares at the current market price, charging a commission called the **brokerage**. This cost should be taken into account when calculating our earnings.

EXAMPLE 13

Kalan bought 500 shares in Woolworths at \$24.60 per share with a dividend yield of 4.2%. One year later he sold the parcel of shares for \$31.80 per share. His stockbroker charges brokerage of 1.8% for buying and 1% for selling a parcel of shares.

Shutterstock.com/bloomua

a What was the total cost to Kalan of purchasing his shares?

b Calculate Kalan's total earnings over the year, after costs.

Solution

a Purchase price = 500 × \$24.60
= \$12 300

Brokerage for buying = 1.8% × \$12 300
= \$221.40

Total cost = \$12 300 + \$221.40
= \$12 521.40

The total cost of the shares to Kalan was \$12 521.40.

b Dividend earned = 4.2% × \$12 300
= \$516.60

Selling price = 500 × \$31.80
= \$15 900

Brokerage for selling = 1% × \$15 900
= \$159

Total earnings = selling price + dividend – brokerage – total cost
= \$15 900 + \$516.60 – \$159 – \$12 521.40
= \$3736.20

Kalan's total earnings were \$3736.20.

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Exercise 11-07 Investing in shares

Example 12

1 Kell bought 100 shares at $4 each and received a dividend of 5c per share. Which of the following is the dividend yield? Select **A**, **B**, **C** or **D**.

A 1.25% **B** $5 **C** 12.5% **D** $20

2 Adrian wants to purchase 2500 shares in ANZ Banking. The market price of the shares is $14.30.

a Calculate the total cost of the shares.

b Adrian's stockbroker charges a commission of 2% of the cost of the shares. Calculate the total brokerage.

c A dividend of 47c was paid when the market price of the share was $16.50. Calculate the dividend yield, correct to two decimal places.

3 Terry bought 4000 shares for $8920, with a dividend yield of 12%. Calculate:

a the market price of one share

b the total dividend earned.

4 Eliza bought 200 shares in Fairfax for $286. Calculate:

a the market price of one share

b the dividend per share, to the nearest 0.1 cent, if the dividend yield is 6.2%.

Example 13

5 Zeli bought 40 000 shares in Austar at $0.75 each. For buying or selling, her stockbroker charged 2.5% brokerage for the first $15 000 of shares and 0.75% thereafter.

a Calculate the total purchase cost of Zeli's shares.

b If Zeli collects a dividend yield of 4.8% and then she sells all her shares for $1.09 each, calculate her total earnings after costs.

6 Vicky bought 8200 shares in the Ten Network at $1.02 each.

a Calculate the value of her shares.

b Additional costs were:

- Broker's service fee: $5.50
- Brokerage: 2.5% of the share value up to $5000
  2% of the next $10 000
  1.5% of the next $35 000
  1% of the remainder.

Calculate the total cost of buying the shares.

c If the company pays a dividend of 28c per share, calculate Vicki's dividend.

d Calculate the dividend yield, correct to two decimal places.

7 Village Roadshow shares have a market value of $1.05 with a dividend yield of 8.2%.

a What is the total market value of 300 shares?

b What is the dividend per share?

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8 Karl manages the share portfolio described in the table below.

Karl's portfolio			
Number of shares	**Company**	**Market value**	**Dividend yield**
1000	Pacifc Mining	$0.25	3.3%
600	QBE Insurance	$23.20	6%
800	Tabcorp	$6.80	5.1%
1700	C-C Amatil	$9.64	5.2%

a Calculate the total value of Karl's shares.

b Calculate the total dividend earned.

9 George owns 2500 AGL energy shares. The dividend per share is 42c and the market price is $14.05.

a What dividend will George receive?

b Calculate the dividend yield, correct to one decimal place.

c How many extra shares could George purchase if he reinvested his dividend?

10 Caltex shares have a market value of $7.63. If the dividend is 35% of the market value, calculate the dividend earned from 4600 Caltex shares.

11 Paula bought 5500 shares in Qantas Air at $2.47 per share, with 1.7% brokerage.

a Calculate the total cost of her shares.

b If the company pays a dividend of 14c per share, calculate Paula's dividend.

c When the market price of the shares is $2.80, calculate:

i the value of Paula's shares

ii the dividend yield.

12 Sandy bought 3500 shares in Fosters at $5.24 with a dividend yield of 7%. For buying shares, brokerage is 2.5% and for selling shares, brokerage is 1%.

a Calculate the total cost of purchasing the shares.

b If Sandy sells her shares 1 year later at $7.14 each, find her total profit after costs.

11-08 Share tables and graphs

Share tables

Daily information about share prices can be found in the finance section of a newspaper, listed in a table similar to the one on the next page. All decimal values refer to dollars and cents. The table has been put together as an example and does not contain factual information on the status of the company shares.

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52 week							
High	Low	Company	Last sale	Move	Buy	Sell	Dividend yield (%)
15.84	10.20	AGL Energy	14.05	–31	14.03	14.05	3.77
40.65	15.00	Wesfarmers	16.23	+10	16.25	16.23	12.32
5.04	2.35	David Jones	2.71	–4	2.70	2.74	9.96
4.92	2.10	Qantas	2.47	+11	2.46	2.47	14.17
13.00	5.01	Seven Network	5.78	+11	5.77	5.82	5.88
14.16	6.68	Billabong	7.68	+22	7.66	7.68	7.23
14.84	6.06	Tabcorp	6.75	+7	6.75	6.80	6.96
27.22	14.59	Westpac Bank	16.11	–5	16.10	16.14	8.81

52 week high/low: The maximum and minimum price in the past 52 weeks
Last sale: The market price of the share at close of day
Move: The change in cents from the previous trading day
Buy: The highest buying price offered that day
Sell: The lowest selling price offered that day

EXAMPLE 14

Use the share table above to find:

a the market price of Qantas shares
b the price of Wesfarmers shares the day before
c the share that was 31c lower than the day before
d the highest price paid for Westpac Bank shares in the past 52 weeks
e the lowest price paid for David Jones shares that day
f the Billabong dividend, if it was paid on this day, to the nearest 0.1 cent.

Solution

a \$2.47 **Last sale.**

b \$16.23 – 10c = \$16.13 **Market price was 10c higher than the day before.**

c AGL Energy was 31c lower on this day than on the day before. **See 'Move' column.**

d The highest price paid for Westpac Bank shares was \$27.22. **52-week high.**

e The lowest price paid for David Jones shares on this day is \$2.70. **Buy column.**

f $7.23\% \times \$7.68 = \$0.555\ 264$
≈ 55.5 cents

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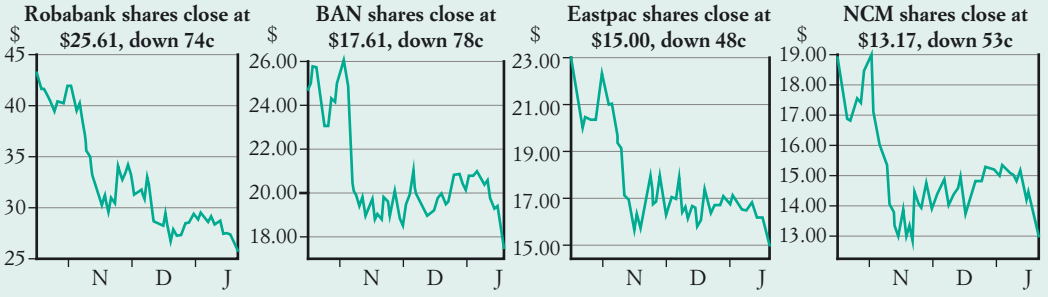
Share graphs

Newspapers use graphs to chart the changing prices of shares over a period of time.

EXAMPLE 15

These graphs show the movement in the share prices of four banks from mid-October to mid-January.



a What is the highest price paid for BAN shares?

b What is the lowest price reached by Robabank shares?

c What was the range of prices of NCM shares, to the nearest dollar?

d Describe the performance of Robabank shares over the period.

e What are two similarities between the four graphs?

f Predict what is likely to happen to the share prices of the four banks in February. Justify your answer.

Solution

a $26.00

b $25.61 **From the title of the graph.**

c The range of prices of the NCM shares is: $19.00 – $12.00 = $7.00

d Apart from a couple of small rises, the Robabank share price continued its downward trend.

e Some similarities are:

- the share prices for all banks peaked around the beginning of November
- the prices dropped rapidly in the first half of November
- the lowest share price for all banks was in mid-January

f It looks like the prices may fall even further.

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Exercise 11-08 Share tables and graphs

Use the share table used for Example 14 (page 508) to answer questions 1 to 9.

1 Which of the following was the market value of AGL Energy shares on the previous day? Select **A**, **B**, **C** or **D**.

A $14.36 **B** $14.34 **C** $13.74 **D** $13.72

2 Which of the following was the range in Seven Network share prices over the year? Select **A**, **B**, **C** or **D**.

A 4 cents **B** $7.99 **C** 5 cents **D** $8

Example 14

3 For Billabong shares:

a what is the market price?

b what was the market price the previous day?

c what was the lowest price in the past 52 weeks?

d what was the lowest buying price in the past 24 hours?

4 **a** Which share in the share table showed the smallest change in price from the previous day?

b What was the change in price referred to in part **a**?

5 **a** Which share in the share table showed the greatest change in price from yesterday?

b What was the change in price referred to in part **a**?

6 **a** Which share has the highest market price?

b Which share has the lowest market price?

7 For Westpac shares:

a what was the lowest price in the past 52 weeks?

b what was the dividend, if paid on the day shown by the table, to the nearest 0.1 cent?

c what was the highest buying price in the past 24 hours?

8 Which share in the share table had the lowest market price in the past 52 weeks?

9 **a** What is Tabcorp's current share price in the table?

b What was Tabcorp's price the previous day?

c What would be the Tabcorp dividend per share if it was paid on the day shown by the table?

Example 15

10 For the graphs of the bank share prices in Example **15** on page 509, find:

a the market price of NCM in mid-November

b the lowest Eastpac price over the period

c the number of months covered by the period

d the number of times the price of NCM shares rose above $15.00

e the price of BAN shares in mid-December

f an estimate of the price of BAN shares on 1 January.

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11 The market value of a share was noted every Friday for 6 weeks. The results are shown in the table on the right.

Week 1	Week 2	Week 3	Week 4	Week 5	Week 6
$11.25	$14.50	$18.60	$12.36	$13.20	$10.40

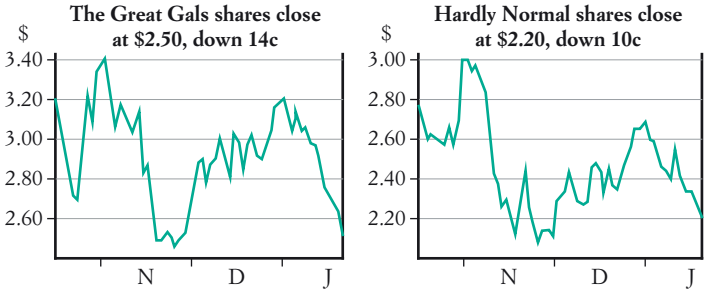
a What was the range of share prices?

b Draw a line graph showing the share price in the 6-week period.

c Describe the trend over the 6 weeks.

d Is it possible to predict what will happen in the next 6 weeks?

12 These graphs show the performance of the shares of two retailers over a 3-month period.



a What was the highest price for The Great Gals over this period?

b What was the lowest price for Hardly Normal over this period?

c How many times was the price for The Great Gals shares $2.60?

d What was the range in price for Hardly Normal shares?

e Describe the performance of The Great Gals shares in this 3-month period.

f Estimate what the Hardly Normal share prices were on:

i 1 December **ii** 31 October

g Would January be a good time to buy shares in Hardly Normal ? Why?

h If an investor bought shares in The Great Gals around 4 November, would you suggest selling them in mid-January? Why?

INVESTIGATION

AUSTRALIAN SECURITIES EXCHANGE (ASX)

The Australian Stock Exchange was established in 1987 in Sydney, after the amalgamation of the six state stock exchanges, and is now one of the world's top ten exchanges. Since 2006, it has been known as the Australian Securities Exchange (ASX). Over 1200 Australian companies and 600 overseas companies are listed on the ASX. The All Ordinaries Index (abbreviated to All Ords) is a statistic that measures the value of 500 of our largest companies. The S&P/ASX 200 is a newer index that summarises the value of 200 shares.

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1. Visit the ASX website www.asx.com.au and play the Schools Sharemarket Game.
2. Find the name of eight companies listed on the ASX and list the market value of their shares and the dividend yield. Compile a list of values, over a week (or longer), then graph the results and discuss your findings. Decide which company you would invest in based on their dividend yields.
3. Visit the stock exchange in Sydney, if possible, to see how share prices are displayed.
4. Find out which country's stock exchanges are associated with these indices:
   - **a** the Dow Jones
   - **b** the Nikkei
   - **c** the FT100
   - **d** the Hang Seng.

Sample HSC problem

Brittany invests $5000 in an account that earns 6.5% p.a. interest, compounded annually.

a How much interest will the account earn in 4 years?

b How much more interest would the account earn if the interest was compounded monthly?

c Explain why more interest is earned when the interest is compounded monthly.

Study tip

Before an exam

- Make a study plan early – don't leave it to the last minute.
- Read and memorise your topic summaries.
- Work on your weak areas – learn from your mistakes.
- Don't spend too much time studying work you know already.
- Use revision exercises and past exams/assignments for practice.
- Vary the way you study so that you don't become bored: have someone quiz you, voice-record your summary, design a poster or mind map, explain the work to someone.
- Anticipate the exam:
  - How many questions?
  - What type of questions: multiple-choice, short answer, long answer, problem-solving?
  - Which topics will be tested?
  - How many marks for each section?
  - How long is the exam?
  - How much time should be spent on each question/section?

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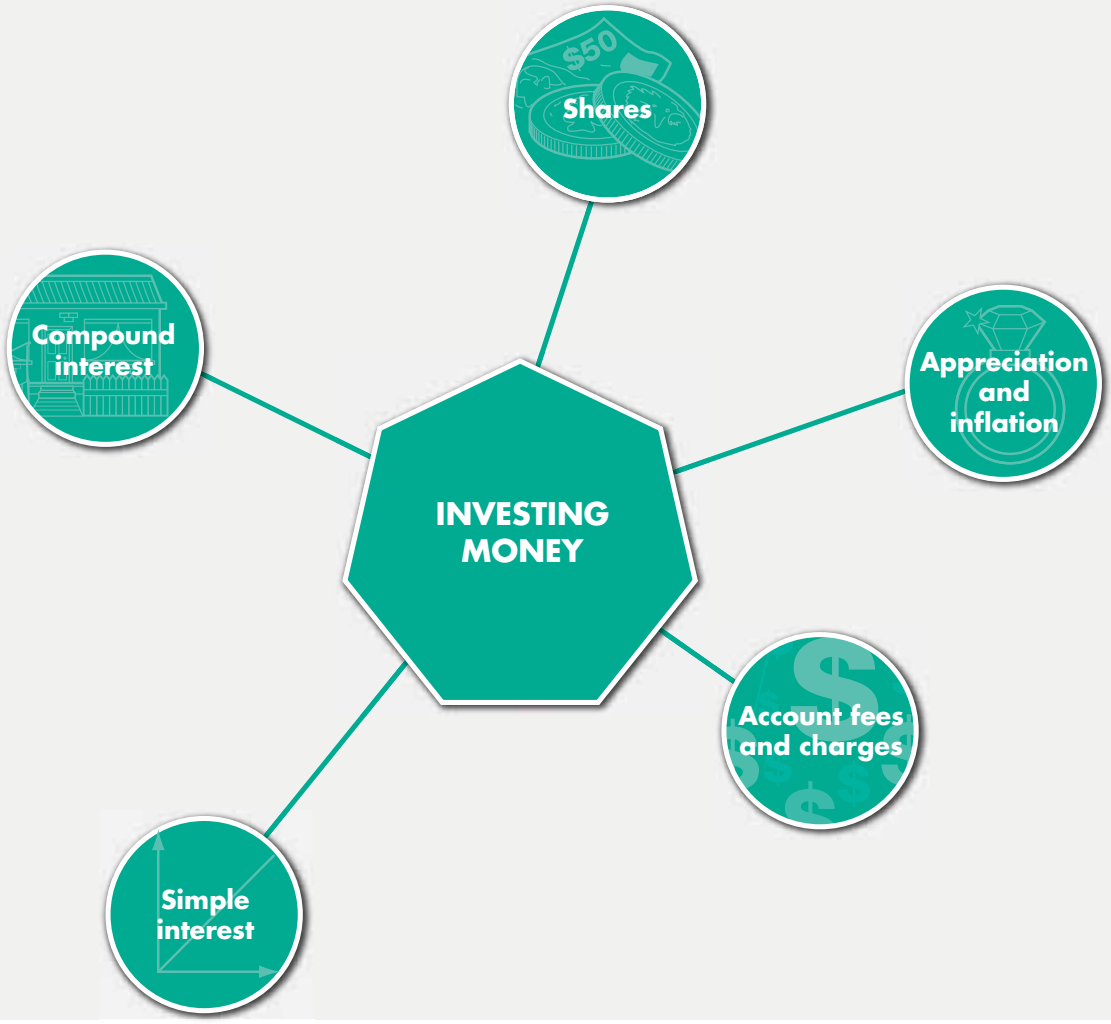
CHAPTER REVIEW

Topic overview

This chapter, 'Investing money', examined the mathematics of saving and investing. You applied the simple interest and compound interest formulas in a variety of situations, such as investments, appreciation and inflation, and learnt that you earn more money if interest is compound interest rather than simple interest. You looked at the fees and charges associated with financial institution accounts. You investigated the share market, including share prices displayed in tables and graphs, and learnt new terminology such as 'market price', 'brokerage' and 'dividend yield'.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map is shown below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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Revision

WS Investing money crossword

Example 11-01

1 Calculate the simple interest earned from each of the following investments.

a \$7200 invested at 6.25% p.a. for 3 years

b \$4050 invested at 3% p.a. for 16 months

c \$10 300 invested at 0.75% per month for 2 years.

Example 11-01

2 An investment of \$8900 earned \$2002.50 in simple interest after 5 years. What was the annual interest rate?

Example 11-01

3 For how long must a principal of \$4000 be invested at 6.6% p.a. for it to earn \$2376 in simple interest?

Example 11-02

4 **a** Construct a graph that shows the simple interest, I, earned in dollars on a principal of \$4800 invested at 3% p.a. for n years, for values of n from 0 to 6.

b What is the gradient of this graph and what does it represent?

c Use the graph to estimate when \$500 in interest will be earned.

Example 11-03

5 Calculate the final amounts of the following investments and the amount of compound interest earned.

a \$7200 at 6.25% p.a. for 3 years

b \$4050 at 3% p.a. for 16 months, compounded monthly.

c \$10 300 at 0.75% per month for 2 years, compounded monthly

Example 11-03

6 What principal must be invested at 5.8% p.a., compounded annually, for it to grow to \$15 000 in 8 years? (Give your answer to the nearest cent.)

Example 11-04

7 Use the table below to calculate the final amount and compound interest earned when \$900 is invested at 10% p.a. for 2 years, compounded half-yearly.

Compounded values of \$1			
	Interest rate per period		
Periods	1%	5%	10%
1	1.010	1.050	1.100
2	1.020	1.103	1.210
3	1.030	1.158	1.331
4	1.041	1.216	1.461
5	1/051	1.276	1.611

Example 11-05

8 OziBank's Incentive Saver account charges a monthly account-keeping fee of \$5 if the balance falls below \$800. There is no charge for the first five withdrawals in a month, but any further withdrawals incur a fee of 80c each. Last month, Charlotte made eight withdrawals from her account, leaving her with a balance of \$685.81. Calculate the total fees for the month.

Example 11-06

9 A \$5000 diamond ring appreciates in value at 4.5% p.a. Calculate its value to the nearest dollar in 25 years.

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10 Raymond is keeping his first pay cheque of $519.50 and framing it. If he decides to cash this cheque in 10 years time, what will it be worth in today's terms if the annual inflation rate over this period is 2.2%? (*Hint*: Let the final value be $519.50.) Example 11-06

11 Maddison bought 4000 shares at $3.80 with a dividend yield of 6.5%. Brokerage costs are 1.8% of the purchase price for both buying and selling shares. Example 11-07

a Calculate the total cost of purchasing the shares.

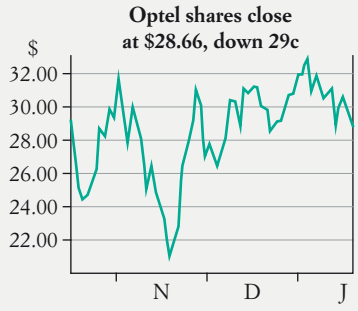
b One year later, Maddison sold her shares for $4.80 each after collecting her dividends. Calculate her total earnings for the year, after costs.

12 QBE Insurance shares have a market value of $22.60 and pay a dividend of 14% of the market value. Example 11-07

a Parker bought 350 QBE shares. How much did they cost him, if he paid 0.5% brokerage?

b How much dividend was Parker paid?

13 The graph on the right shows the performance of Optel shares over a period. Example 11-08



a How long is the period covered by this graph?

b What was the lowest share price over this period, and in which month did it occur?

c Describe the performance of the Optel shares over this period.

d How many times did the price reach $26?

e Estimate the value of the shares on 22 December.

f Would 20 October be a good time to buy or sell? Give reasons for your answer.

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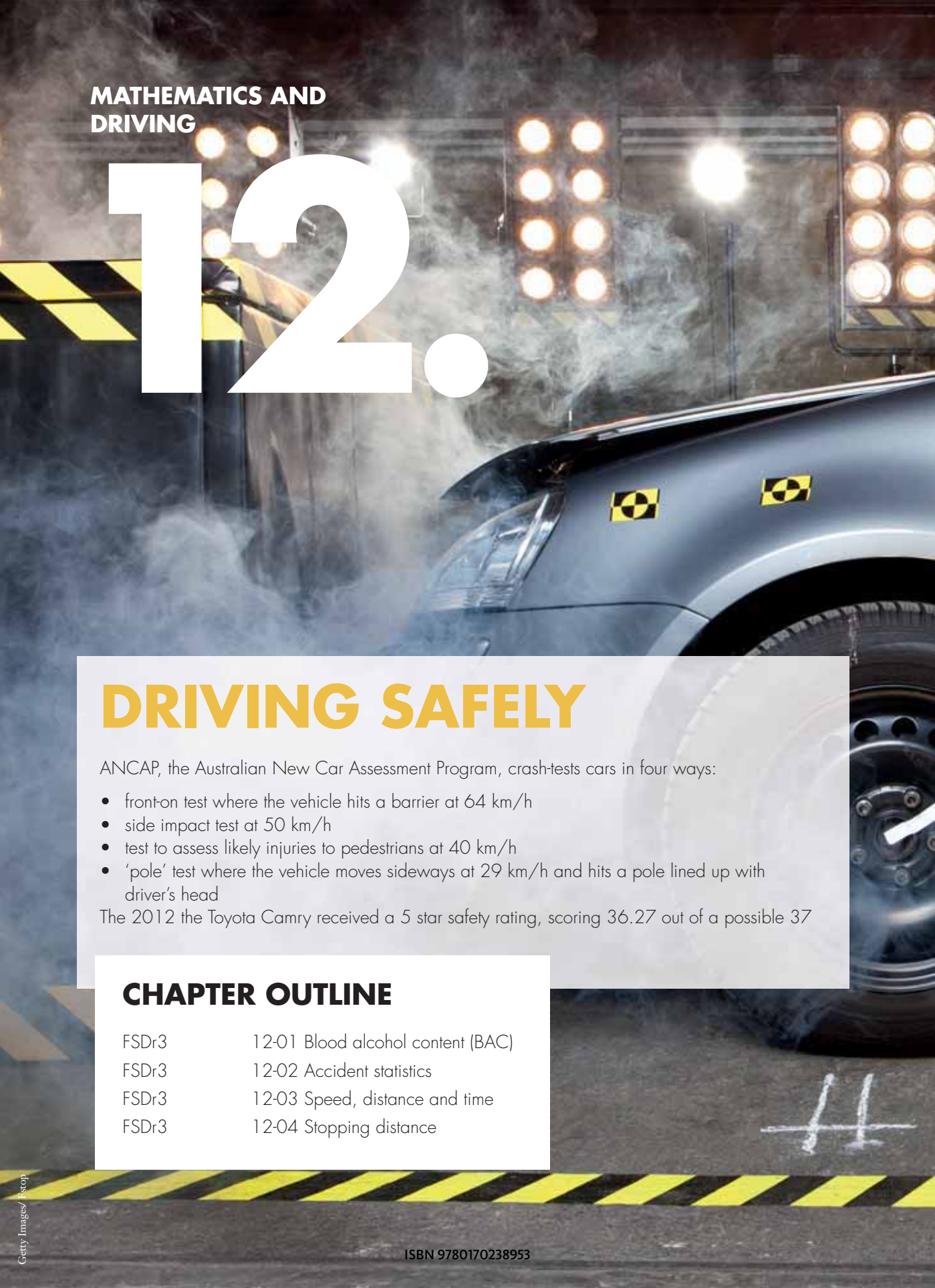

MATHEMATICS AND DRIVING

12.

DRIVING SAFELY

ANCAP, the Australian New Car Assessment Program, crash-tests cars in four ways:

- front-on test where the vehicle hits a barrier at 64 km/h
- side impact test at 50 km/h
- test to assess likely injuries to pedestrians at 40 km/h
- 'pole' test where the vehicle moves sideways at 29 km/h and hits a pole lined up with driver's head

The 2012 the Toyota Camry received a 5 star safety rating, scoring 36.27 out of a possible 37

CHAPTER OUTLINE

FSDr3	12-01 Blood alcohol content (BAC)
FSDr3	12-02 Accident statistics
FSDr3	12-03 Speed, distance and time
FSDr3	12-04 Stopping distance

Getty Images/Istop

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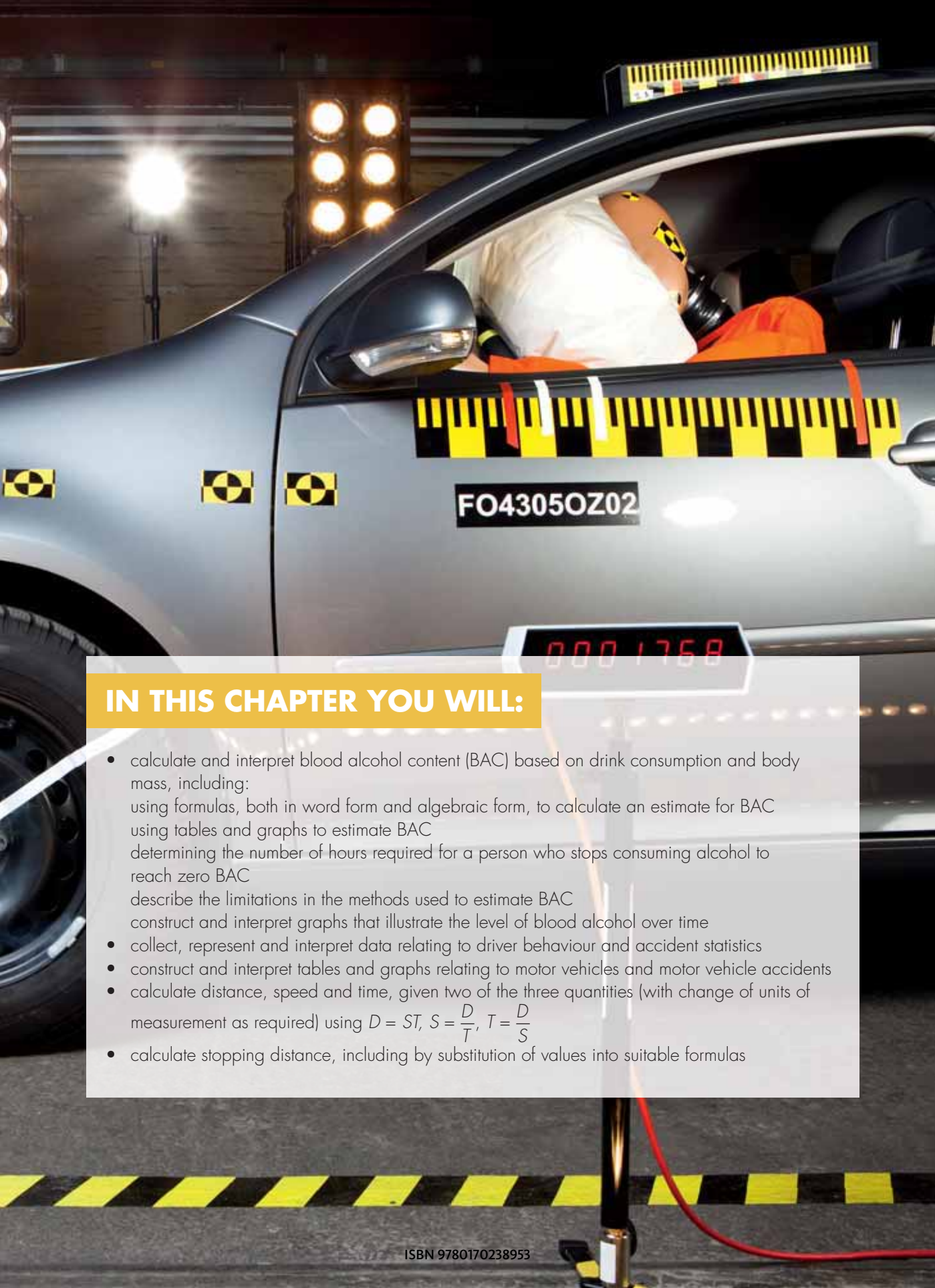

IN THIS CHAPTER YOU WILL:

- calculate and interpret blood alcohol content (BAC) based on drink consumption and body mass, including:
  using formulas, both in word form and algebraic form, to calculate an estimate for BAC
  using tables and graphs to estimate BAC
  determining the number of hours required for a person who stops consuming alcohol to reach zero BAC
  describe the limitations in the methods used to estimate BAC
  construct and interpret graphs that illustrate the level of blood alcohol over time
- collect, represent and interpret data relating to driver behaviour and accident statistics
- construct and interpret tables and graphs relating to motor vehicles and motor vehicle accidents
- calculate distance, speed and time, given two of the three quantities (with change of units of measurement as required) using $D = ST$, $S = \frac{D}{T}$, $T = \frac{D}{S}$
- calculate stopping distance, including by substitution of values into suitable formulas

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TERMINOLOGY

alcohol consumption
average speed
blood alcohol content (BAC)
braking distance
drink driving
fatality
fatigue
reaction distance
reaction time
standard drink
stopping distance

WS
Assignment 12

SkillCheck

1 Find, correct to 2 decimal places:

a $\frac{0.03}{0.015}$ **b** $\frac{0.04}{0.023}$ **c** $\frac{0.1}{0.034}$

d $\frac{26-15}{5.5\times45}$ **e** $\frac{10\times7-5\times7.5}{6.8\times64}$ **f** $\frac{2\times10-18}{5.5\times83}$

2 **a** Copy and complete this table for the linear function $y = 4 - 0.3x$.

x	0	1	2	3	4	5
y						

b If this function was graphed, what would be the gradient of the line?

3 For each set of data, find the:

i mean **ii** median **iii** mode **iv** range **v** interquartile range.

Answer correct to 1 decimal place where necessary.

a 5 7 8 9 10 4 5 2 4 1

b 10 12 14 9 0 2 8 12 1

4 Evaluate correct to one decimal place the subject of each formula.

a $v = u + at$ when $u = 2.3$, $a = 8.7$, $t = 3$

b $s = kv(kv + b)$ when $k = 4.1$, $v = 42.3$, $b = 7.2$

c $s = \frac{d}{t}$ when $d = 128$, $t = 3.4$

5 Convert:

a 25 km to m **b** 2463 m to km **c** 3 h 45 min to hours

d 5.2 h to hours and minutes **e** $7.\dot{3}$ h to hours and minutes

f 20 km/h to m/s **g** 110 km/h to m/s **h** 3.5 m/s to km/h

6 A plane moves at an average speed of 840 km/h for 1 day. How many km does it travel?

7 A car travels 245 km in 3 h 45 min. What is its average speed in km/h?

8 Find correct to the nearest cent:

a 14 % of \$35.80 **b** 9.5% of \$26 580 **c** 12.5% of \$298.60

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12-01 Blood alcohol content (BAC)

BAC formula practice

Graphing BAC

Your **blood alcohol content (BAC)** is the amount of alcohol in your blood. A person with a BAC of 0.05 has 0.05 grams (or 50 mg) of alcohol per 100 mL of blood.

When alcohol is consumed, safe driving skills become impaired and a driver will have difficulty concentrating on multiple tasks at a time. For example, a driver may be able to monitor their speed but be unable to stay within their lane.

Getty Images/ Fstop

Alcohol affects a driver's:

- **judgment**: ability to reason and respond appropriately
- **concentration**: ability to focus
- **comprehension**: ability to understand a situation quickly
- **reaction time**: ability to respond to a situation quickly

Calculating BAC

A **standard drink** contains 10 grams of alcohol, for example a middy of beer or a small glass of wine.

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The formulas used to determine a person's BAC depends on four factors:

- the sex of the person
- number of alcoholic drinks consumed
- how quickly the drinks are consumed
- the mass of the person.

SUMMARY

Formulas for calculating blood alcohol content

$$\text{BAC}_{\text{male}} = \frac{10N - 7.5H}{6.8M} \quad \text{and} \quad \text{BAC}_{\text{female}} = \frac{10N - 7.5H}{5.5M}$$

where

N = number of standard drinks consumed

H = number of hours drinking

M = mass in kg

Blood accounts for about 7% of your weight so an 80 kg **male driver** will have 5.6 L of blood whereas a 60 kg **female driver** has only 4.2 L of blood. The male has a greater mass to 'soak up' the alcohol and will not get drunk as quickly. *Note*: assume 1 kg ≈ 1 L

EXAMPLE 1

Justin and Britney were at a party for 5 hours. Justin, who weighs 75 kg, had 6 standard drinks and Britney, who weighs 58 kg, had 5 standard drinks.

a Who had the lower blood alcohol content (BAC)?

b If 7% of Justin's mass is blood, what is his blood volume?

Solution

a For Justin: $N = 6,\ H = 5,\ M = 75$

$$\begin{aligned}\text{BAC}_{\text{male}} &= \frac{10N - 7.5H}{6.8M} \\ &= \frac{10\times6 - 7.5\times5}{6.8\times75} \\ &= 0.0441\ldots \\ &\approx 0.044\end{aligned}$$

For Britney: $N = 5,\ H = 5,\ M = 58$

$$\begin{aligned}\text{BAC}_{\text{female}} &= \frac{10N - 7.5H}{5.5M} \\ &= \frac{10\times5 - 7.5\times5}{5.5\times58} \\ &= 0.0391\ldots \\ &\approx 0.039\end{aligned}$$

Britney had the lower BAC.

b Justin's blood volume = 7% × 75 = 5.25 L.

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Getting back to a BAC of zero

A healthy liver can only break down about one standard drink (10 g alcohol) per hour. Not even black coffee or cold showers will speed up the rate at which your body gets rid of alcohol. Once a person stops consuming alcohol and is at a certain BAC level, a normal body can only reduce the BAC by between 0.015 and 0.02 per hour.

SUMMARY

For a body that reduces BAC by 0.015 per hour, the number of hours before zero BAC is reached is:

$$\text{Number of hours} = \frac{\text{BAC}}{0.015}$$

For a body that reduces BAC by 0.02 per hour, the number of hours before zero BAC is reached is:

$$\text{Number of hours} = \frac{\text{BAC}}{0.02}$$

EXAMPLE 2

Justin and Britney from Example **1** are both on provisional licences (P plates). They stopped drinking at 11 p.m. and waited until one of them had a BAC of zero and could legally drive home. Who drove home and at what time? [Assume BAC is reduced by 0.015 per hour]

Solution

For Justin : BAC = 0.044

$$\text{No.hours} = \frac{\text{BAC}}{0.015} = \frac{0.044}{0.015} = 2.9333\ldots\ \text{h} = 2\ \text{h}\ 56\ \text{min}$$

For Britney: BAC = 0.039

$$\text{No. hours} = \frac{\text{BAC}}{0.015} = \frac{0.039}{0.015} = 2.6\ \text{h} = 2\ \text{h}\ 36\ \text{min}$$

'Press SHIFT o' " to change decimal hours to hours and minutes.

Britney drove home 2 h 36 min after 11 p.m., that is, at 1.36 a.m.

Limitations of the formulas

The formulas used here only give an approximate BAC as this is affected by your health and the time taken by your liver to process alcohol. Many factors affect your BAC, such as:

- amount of alcohol consumed (size and type of drink)
- how fast you drink (the faster you drink, the higher your BAC)
- body weight (heavier people have more blood and water in their bodies to dilute alcohol)

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- food in stomach (alcohol is absorbed more slowly into the bloodstream if food is present)
- type of mixer used (water and juice slow absorption whereas carbonated mixers speed it up)
- temperature of drink (warm alcohol is absorbed quicker)
- gender (females usually reach a higher BAC faster)

Graphing BAC

EXAMPLE 3

After drinking 5 standard drinks, Basim's BAC was 0.09. His BAC decreases over time according to the linear function $B = 0.09 - 0.014H$.

a Use the formula to complete this table of values, correct to 3 decimal places.

Time after drinking, *H* hours	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Blood alcohol content, *B*	0.09		0.076					0.041			

b Graph the linear function on a number plane.

c What is the gradient of the line and what does it represent?

d Use your graph to find:

i Basim's BAC after 3h 15 min. **ii** when his BAC reaches 0.05.

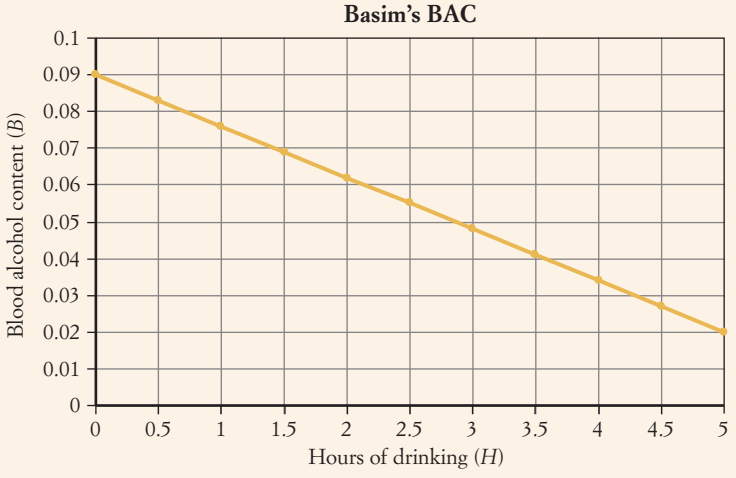
e Use the equation to find when Basim's BAC reaches zero.

Solution

a

Time after drinking, *H* hours	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Blood alcohol content, *B*	0.09	0.083	0.076	0.069	0.062	0.055	0.048	0.041	0.034	0.027	0.02

b



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c Gradient $= \dfrac{0.02 - 0.09}{5 - 0} = -0.14.$

The gradient is the rate at which Barry's BAC is decreasing per hour.

d **i** BAC after 3.25 h ≈ 0.045 **from graph**

ii BAC reaches 0.05 after about 2.9 h (or 2 h 51 min)

e When $B = 0$, $0 = 0.09 - 0.014H$

$$0.014H = 0.09$$
$$H = \frac{0.09}{0.014}$$
$$= 6.428...$$
$$\approx 6\text{ h } 26\text{ min}$$

Barry's BAC reaches 0 after 6 h 26 min.

JUST FOR THE RECORD

Princess Diana's chauffeur

Henri Paul was the driver of the Mercedes S280 that crashed in Paris in 1997, killing Princess Diana (mother of Princes William and Harry), Dodi Fayed as well as himself. A blood test revealed that he had a BAC of 0.187. This is more than 3 times the legal limit of 0.05 and is equivalent to drinking nine shots of whisky in rapid succession.

If Henri Paul's BAC dropped by 0.017 per hour, how long would it have taken for his BAC to return to zero?

Exercise 12-01 Blood alcohol content (BAC)

1 What is the main factor affecting your blood alcohol content? Select **A**, **B**, **C** or **D**.

A age **B** number of drinks consumed

C time of day **D** heart function

2 Christine weighs 61 kg, Carla weighs 52 kg and Connie weighs 55 kg. If each drink 8 standard drinks over 6 hours, who has the highest BAC? Select **A**, **B**, **C** or **D**.

A Christine **B** Carla

C Connie **D** BAC is the same as they are all female

3 Mabel and Madge drank wine at an 80th birthday party over 4 hours. Mabel, who weighs 94 kg, had 4 standard drinks and Madge, who weighed 87 kg, had 3 standard drinks. Example 1

a Who had the higher blood alcohol concentration?

b If 7% of Mabel's weight was blood, how much blood did she have?

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4 Who has the lowest blood alcohol concentration (BAC)? Select **A**, **B**, **C** or **D**.

A a 72 kg male who drank 8 drinks over 4 hours

B a 65 kg female who drank 9 drinks over 5 hours

C an 82 kg female who drank 7 drinks over 6 hours

D a 93 kg male who drank 10 drinks over 7 hours

5 This table shows the BAC (to 2 decimal places) for various body weights and number of drinks.

	BAC									
Body weight (kg)	**1 drink**	**2 drinks**	**3 drinks**	**4 drinks**	**5 drinks**	**6 drinks**	**7 drinks**	**8 drinks**	**9 drinks**	**10 drinks**
50	0.03	0.07	0.10	0.14	0.17	0.21	0.24	0.28	0.31	0.34
55	0.03	0.06	0.09	0.13	0.16	0.19	0.22	0.25	0.28	0.31
73	0.02	0.05	0.07	0.09	0.12	0.14	0.16	0.19	0.21	0.23
82	0.02	0.04	0.06	0.08	0.10	0.13	0.15	0.17	0.19	0.21
95	0.02	0.04	0.05	0.07	0.09	0.11	0.13	0.14	0.16	0.18
100	0.02	0.03	0.05	0.07	0.09	0.10	0.12	0.14	0.15	0.17

a How many drinks does a 73 kg person need to drink to reach the legal limit of 0.05?

b How many drinks does a 100 kg person need to drink to reach the legal limit of 0.05?

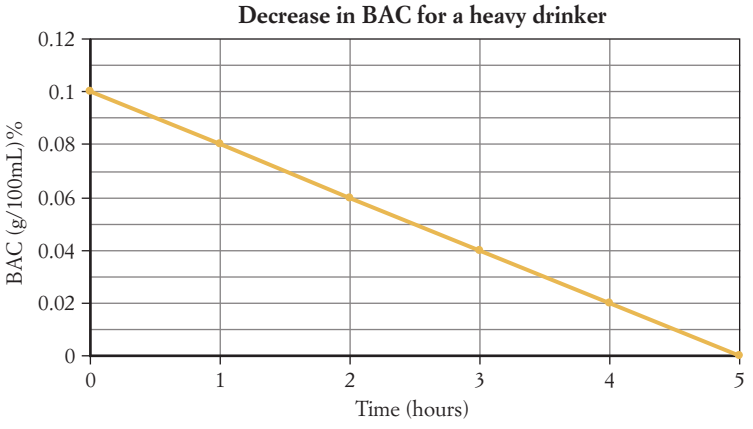
c What is the weight of a person who has a BAC of 0.16 after 7 drinks?

d What is the BAC of a 55 kg person after 8 drinks?

e Draw a straight-line graph showing the BAC of a 50 kg person for 1 to 10 drinks.

Example 3

6 A heavy drinker consumes a large quantity of alcohol most days of the week whereas a moderate drinker consumes less alcohol less frequently. The two graphs show the difference in the decrease of the BAC for a heavy drinker and a moderate drinker over time.



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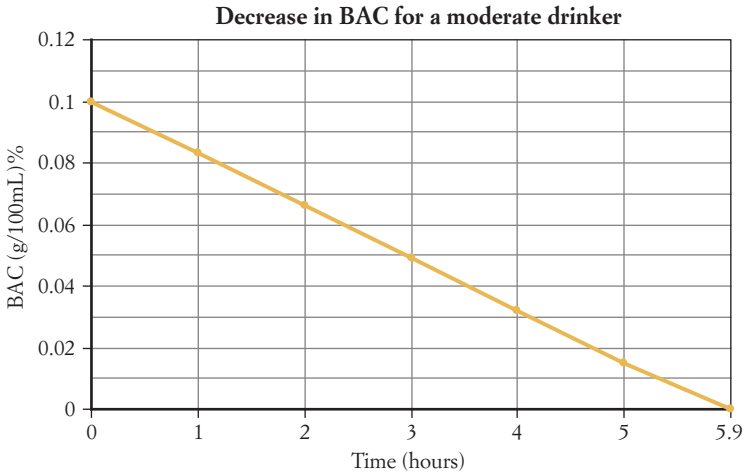

a Which drinker's BAC returns to zero quicker?

b What is the rate of decrease (correct to three decimal places) of BAC for a:

i heavy drinker **ii** moderate drinker?

c What is the BAC of a heavy drinker after 1h 30 min?

d What is the BAC of a moderate drinker after 4h 15min?

e What is the difference in the BAC for a heavy and a moderate drinker after 3 hours?

f Write a sentence or two describing the difference in the rate of BAC decrease for each type of drinker and possible reasons for the difference.

Example 2

7 Mark is 19, weighs 72 kg and holds a provisional licence with a zero alcohol limit. He started drinking at 6 p.m. and had 10 schooners of full strength beer (15 standard drinks) over 6 hours. He stopped drinking at midnight.

a What was his BAC, to 2 decimal places, at midnight?

b How long did it take before his BAC was back to zero if his BAC reduced by 0.02 per hour?

c At what time could he legally drive his car?

8 Maria is 17, weighs 61 kg and holds a provisional licence with a zero alcohol limit. She started drinking at 10 p.m. and had 6 mixer (9 standard) drinks over 4 hours.

a What was her BAC, to 2 decimal places, at 2 a.m. when she stopped drinking?

b How long did it take before her BAC was back to zero if her BAC reduced by 0.015 per hour?

c At what time could she legally drive her car?

9 Hugh had a BAC of 0.06 when he stopped drinking and it took 5 hours for his BAC to return to zero. At what rate per hour was his BAC reducing?

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10 Chloe has a BAC of 0.08 and burns off alcohol at the rate of 0.016 per hour.

a Complete this table of values showing the decline in her BAC over 5 hours.

Time (hours)	BAC
0	0.08
0.5	0.072
1	
1.5	
2	
2.5	
3	
3.5	
4	
4.5	
5	

b Use the table of values to draw a straight line graph representing this information.

c If Chloe stopped drinking at midnight, estimate to the nearest 15 minutes, when Chloe's BAC is equal to:

i 0.05 **ii** 0?

d What is the gradient of the line and what does it represent?

JUST FOR THE RECORD

BAC limits for NSW drivers

The BAC limits (NSW, 2012) are:

0	for all learner (L) and provisional (P1 and P2) drivers
under 0.02	for drivers of buses, taxis, dangerous goods vehicles and large trucks
under 0.05	for all other drivers

The risk of an accident with a BAC of:

0.05 is double

0.08 is 7 times

0.15 is 20 times

How many drinks can a provisional driver have if they plan to drive? Justify your answer.

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INVESTIGATION

BAC FEELINGS AND EFFECTS

BAC	Person's feelings	Actual effects
0.03 to 0.05	relaxed talkative	worse concentration and coordination less able to make a judgement
0.05 to 0.1	blunted feelings lack of self-control	bad reflexes unable to perceive depth reasoning skills decreased
0.1 to 0.2	need to be over-expressive or act in a loud and disruptive manner	decrease in reaction time loss of physical control slurred speech
0.2 to 0.3	can't remember simple things in a daze	loss of understanding loss of sensations possible loss of consciousness
Over 0.3	depressed think they can do anything	can't read signs can't respond to traffic signals impaired breathing loss of bladder control loss of consciousness or even death

For example, a person in a nightclub with a BAC of 0.15 might talk loudly, disrupt other patrons and express themselves in a way that is not typical of their personality. Their speech may be slurred and they may not be able to walk or dance properly.

a How does a person feel with a BAC of 0.24?

b State 3 effects of drinking to a BAC of 0.06.

c What is the legal BAC for:

i a learner driver? **ii** a P-plate driver?

d Investigate the penalties for driving with a BAC of:

i 0.05 **ii** 0.1 **iii** 0.4

e Does a BAC below the legal limit mean that it is safe to drive? Explain.

12-02 Accident statistics

Alcohol, excessive speed, driver fatigue and distraction are the main causes of road accidents.

WS Investigating drivers in road fatalities

Shutterstock.com/ Brad Remy

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The risk of an accident resulting in death or serious injury is doubled for every 5 km/h you travel over 60 km/h. For example, a car travelling at 65 km/h is twice as likely to crash and a car travelling at 70 km/h is four times as likely to crash!

Fatal accidents

In a fatal crash, one or more persons die but there may be others who suffer serious injuries or permanent disability.

Fatal means 'resulting in death'.

Here are some statistics regarding fatal accidents in NSW.

- 72% occur in country areas
- 20% involve alcohol
- 20% of motorcyclists involved in fatal accidents have a BAC over 0.05
- 88% of drink drivers involved in fatal accidents are male

EXAMPLE 4

The following table gives the number of road fatalities in NSW from 2002 to 2011.

Year	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
No. fatalities	561	539	510	508	496	435	374	453	405	394

Source: Centre for Road Safety, Transport for NSW

a For the 10-year period, find the:

- **i** mean number of fatalities per year
- **ii** range
- **iii** median
- **iv** interquartile range

b Draw a column graph representing the data.

c Write a sentence discussing the trend in fatalities from 2002 to 2011. Suggest reasons for these trends.

Solution

a **i**

$$\text{Mean} = \frac{\text{sum of fatalities}}{\text{number of years}}$$

$$= \frac{4675}{10}$$

$$= 467.5$$

ii Put the values in ascending order:

374 394 405 435 453 496 508 510 539 561

lowest score | lower quartile | median | upper quartile | highest score

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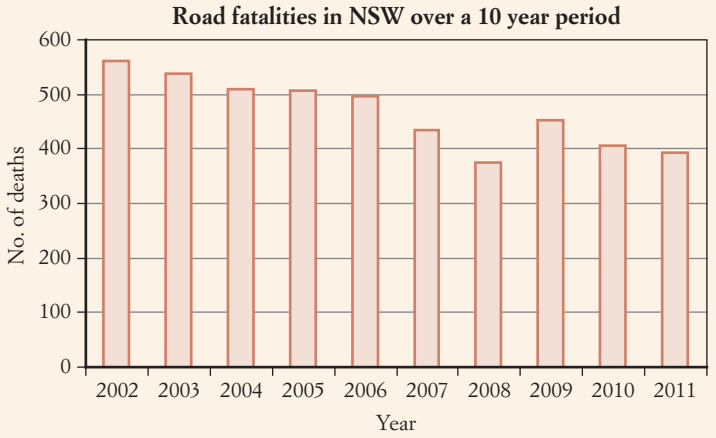
$$\text{Range} = \text{highest score} - \text{lowest score}$$
$$= 561 - 374$$
$$= 187$$

iii $\text{Median} = \dfrac{453 + 496}{2} = 474.5$

iv $\text{Interquartile range} = \text{upper quartile} - \text{lower quartile}$
$$= 510 - 405$$
$$= 105$$

b



c The main trend is the decline over time of the number of fatalities. This decline is possibly due to better road safety awareness programs, more police patrols and lower speed limits in school and roadworks zones.

Fatigue-related accidents

Fatigue is mental or physical tiredness. It occurs through lack of sleep and is more likely to affect drivers during 2 p.m.–4 p.m. and midnight–6 a.m. Fatigue affects about 20% of head-on collisions where a vehicle crosses to the wrong side of the road due to a driver's slower reaction time.

Colourbox/© Andreas Altenburger

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Here are some statistics on drivers in head-on crashes involving fatigue.

- 77.1% were driving cars
- 72.6% were male
- 21.7% were young people between 17 and 24 years
- 22.3% were aged over 60 years

Alamy/© BSIP SA

EXAMPLE 5

The table shows the fatalities for 3 types of crashes in 2010 in NSW, as compiled by the Centre for Road Safety, Transport for NSW.

Alcohol involved	Fatal crash	Injury crash	Non-injury crash	Total
Yes	58	A	681	1597
No	307	B	C	40 702
Total	365	18 971	22 963	D

Source: Centre for Road Safety, Transport for NSW

a Find the values of A, B, C, D.

b In what percentage (correct to one decimal place) of fatal crashes was alcohol involved?

c What percentage, to 1 decimal place, of the total crashes had no alcohol involved?

d What percentage, to 1 decimal place, of the total crashes involved an injury or a fatality?

e If 14% of accidents involving alcohol were head-on crashes, how many was this?

Solution

a A = 1597 – (58 + 681) = 858

B = 18 971 – 858 = 18 113

C = 22 963 – 681 = 22 282

D = 1597 + 40 702 = 42 299

There is more than one way to find these values.

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Alcohol involved	Fatal crash	Injury crash	Non-injury crash	Total
Yes	58	858	681	1597
No	307	18 113	22 282	40 702
Total	365	18 971	22 963	42 299

b Percentage of fatal crashes involving alcohol $= \frac{58}{365} \times 100\%$

$= 15.8904...\%$

$\approx 15.9\%$

c Percentage of all crashes not involving alcohol $= \frac{40\,702}{42\,299} \times 100\%$

$= 96.2244...\%$

$\approx 96.2\%$

d Percentage of all crashes involving an injury or fatality $= \frac{365 + 18\,971}{42\,299} \times 100\%$

$= 45.7126...\%$

$\approx 45.7\%$

e No. of head-on crashes $= 14\% \times 1597$

$= 223.58$

≈ 224

Exercise 12-02 Accident statistics

1 In one year, 440 driver deaths on Australian roads were attributed to alcohol. 224 of these drivers had a BAC of over 0.05 and 273 were aged 15–34.

For road accidents attributed to alcohol:

a what fraction of drivers who died had a BAC over 0.05?

b what percentage (to the nearest whole number) of the drivers who died were aged 15–34?

c what percentage of drivers who died were aged 15–34 and had a BAC over 0.05?

2 The table gives the driver deaths in each state and territory of Australia for 2 calendar years.

Year	NSW	Vic.	Qld	SA	WA	Tas.	NT	ACT
2010	185	130	114	57	99	16	24	10
2011	190	121	108	40	86	14	17	3

Source: Bureau of Infrastructure and Regional Economics (BITRE)

a How many driver deaths were there in Australia in:

i 2010? **ii** 2011?

b Find the average annual number of driver deaths in Australia over the two-year period.

c Which state(s) had fewer driver deaths in 2011 than they did in 2010?

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d Write a sentence discussing the similarities and/or differences in the data for the years 2010 and 2011. Suggest possible reasons for these.

e Draw a divided bar graph representing the data for the year 2011.

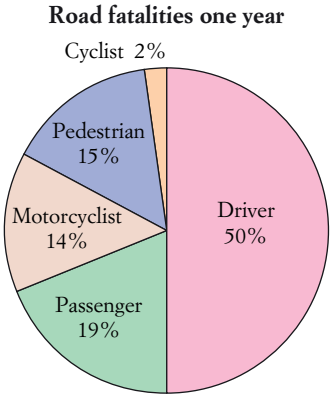
3 A survey revealed that for fatigued drivers in 18 764 single vehicle crashes, 70.2% were driving cars, 75.5% were male, 35% were between 17 and 24 years and 9.6% were over 60 years old.

a How many drivers were aged 24 to 60 years?

b How many crashes did not involve cars?

c How many drivers were female?

4 The sector graph shows the category of person killed in road accidents one year.



a To the nearest degree, what is the sector angle in the graph for:

i drivers

ii passengers

iii cyclists

b If there were 443 fatalities, how many persons killed were:

i passengers

ii motorcyclists?

c If there were 68 pedestrians killed, how many drivers were killed?

Example 5

5 This table describes Australian drivers under 40 years who died in road accidents in 2008–2011.

	Year			
Age of driver	**2008**	**2009**	**2010**	**2011**
Under 17	88	105	74	93
17–25	376	360	336	283
26–39	345	356	305	278

Source: BITRE

a Find the number of drivers aged 25 or less who were killed over the 4 years.

b Calculate the percentage (correct to one decimal place) of drivers killed in 2011 who were 17–25 years old.

c What fraction of drivers killed in 2009 were 26–39 years old?

d Represent the data for 2011 in a divided bar graph.

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Example 4

6 This table shows the number of fatal road accidents involving articulated trucks in each state and territory for the years 2006–2011.

Year	NSW	VIC	QLD	SA	WA	TAS	NT	ACT	Australia
2006	57	26	34	9	12	5	2	0	145
2007	53	30	38	6	14	4	2	0	147
2008	47	22	35	9	8	6	3	0	130
2009	33	17	38	9	13	10	2	2	124
2010	41	31	25	7	12	3	1	1	121
2011	47	21	32	12	10	2	2	0	126

Source: BITRE

a Which year and state had the highest number of crashes?

b How many crashes were there in Australia over the six years?

c What is the mean number of crashes (correct to one decimal place) in NSW over the 6 years?

d What is the range of crashes in Tasmania over the 6 years?

e What is the median number of crashes in Western Australia over the 6 years?

f What is the interquartile range of crashes for Victoria over the 6 years?

g Represent the data for your state on a column graph.

7 This table shows the number of fatal crashes in each state and territory during the period 2007–2011.

Year	NSW	VIC	QLD	SA	WA	TAS	NT	ACT	Australia
2007	405	289	338	107	214	39	47	14	1453
2008	353	278	294	87	185	37	67	14	1315
2009	408	268	296	104	176	52	30	11	1345
2010	365	260	236	105	176	29	46	16	1233
2011	349	259	227	95	168	24	37	6	1165

Source: BITRE

a Calculate for Victoria:

- **i** the mean number of crashes over the 5 years
- **ii** the median
- **iii** the range
- **iv** the interquartile range

b Construct a line graph to represent the data for the NT with years on the horizontal axis and number of crashes on the vertical axis, with the scale starting at 30.

c Looking at the data for the whole of Australia, write a sentence noting any similarities or differences over the 5 years and possible reasons for these.

d Why do you think there are a lot more crashes in NSW than in the Northern Territory?

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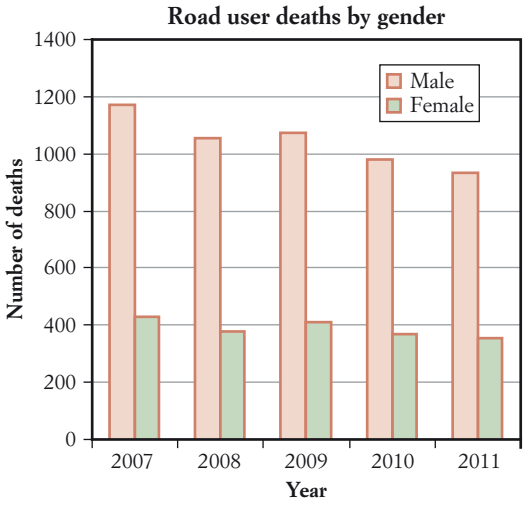
8 The graph represents the male and female road user deaths from 2007 to 2011



Source: BITRE

a How many male deaths, to the nearest 100, occurred in 2009?

b How many female deaths, to the nearest 100, occurred in 2010?

c How many road user deaths, to the nearest 100, occurred in 2011?

d Write a sentence giving any similarities and differences between male and female deaths, stating reasons for your comments.

9 This table shows the number of road fatalities each month in 2002 and 2011.

a Copy and complete the last two rows showing totals and means.

b Which month in 2011 had:

i the lowest number of deaths?

ii the highest number of deaths?

c For the year 2011, find:

i the range

ii the interquartile range

iii the median

d **i** Write a sentence saying in which month a road death is most likely, stating reasons for your answer.

ii Do you think this will still be the case in 2020? Give your reasons.

Road fatalities		
Month	**2002**	**2011**
January	113	84
February	122	94
March	137	101
April	127	104
May	149	101
June	125	96
July	117	92
August	136	99
September	120	89
October	127	84
November	133	113
December	120	110
Total		
Mean		

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10 This table shows the categories of people killed in road accidents in 2011.

	NSW	Australia
Drivers	190	579
Passengers	73	286
Pedestrians	52	189
Motorcycle riders and passengers	52	201
Cyclists	10	35
All road users	**377**	**1291**

Source: BITRE

a What percentage of Australian road deaths occurred in NSW? Answer to 3 significant figures.

b What percentage, to 2 decimal places, of road deaths in NSW were pedestrians?

c How many road deaths in Australia were not drivers or passengers?

d What percentage of NSW fatalities were cyclists? Answer correct to one decimal place.

12-03 Speed, distance and time

Speed formula practice

Speed stories

Speed limits are imposed to ensure drivers travel at a safe speed for the road and surrounding environment. Some common speed limits in NSW are:

School zone	40 km/h
Residential area	50 km/h
City street	60 km/h
Highway	100 km/h
Motorway	110 km/h

Average speed

When you go on a trip, your speed varies depending on where you are. The average speed for a journey is calculated using the formula:

SUMMARY

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

or $s = \dfrac{d}{t}$

This formula can also be written as $d = st$ or $t = \dfrac{d}{s}$.

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EXAMPLE 6

Dean left home at 6 a.m. to travel from Sydney to Brisbane, a distance of 1027 km. He stopped for meal breaks and fuel and arrived in Brisbane at 8.30 p.m.

a What was his average speed for the trip, correct to the nearest km/h?

b How far, to nearest kilometre, could he travel at this speed in 50 minutes?

c How long, in hours and minutes, would it take him to drive 864 km at this speed?

Solution

a $d = 1027\text{ km};\ \ t = 14.5\text{ h}$

6 a.m to 8.30 p.m.

$$\begin{aligned} s &= \frac{d}{t} \\ &= \frac{1027}{14.5} \\ &= 70.8275... \\ &\approx 71\text{ km/h} \end{aligned}$$

Average speed was 71 km/h.

b $s = 71\text{ km/h};\ \ d = ?;\ \ t = 50\text{ min} = \frac{50}{60}\text{ h} = \frac{5}{6}\text{ h}$

$$\begin{aligned} d &= s \times t \\ &= 71 \times \frac{5}{6} \\ &= 59.166\ldots \\ &\approx 59\text{ km} \end{aligned}$$

He could travel 59 km.

c

$$\begin{aligned} t &= \frac{d}{s} \\ &= \frac{864}{71} \\ &= 12.169... \\ &\approx 12\text{ h } 10\text{ min } 8.45\text{ s} \end{aligned}$$

Press SHIFT o' " to convert decimal hours to hours, minutes and seconds.

It would take him 12 h 10 min.

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Exercise 12-03 Speed, distance and time

1 Eva drove 93 km from Wollongong to Nowra in 1 hr 30 min. What was the average speed in km/h? Select **A**, **B**, **C** or **D**.

A 46.5 **B** 31 **C** 62 **D** 60

2 Les travelled 160 km at an average speed of 76 km/h. How long was his journey in hours and minutes? Select **A**, **B**, **C** or **D**.

A 2 h 10 min **B** 2 h 6 min **C** 2.2 h **D** 2.1 h

Example 6

3 Larni drove 742 km from home to her farm in 9 h 30 min.

a What was her average speed, correct to the nearest km/h?

b How long, in hours and minutes, would it take her to drive 1026 km at this rate?

c How far, to nearest kilometre, could she travel at this rate in 3 h 10 min?

4 Cathy drove at an average speed of 57 km/h. At this rate, how long, in hours and minutes, will it take her to drive:

a 507 km **b** 160 km **c** 440 km

5 Tan cycled 24 km to Scott's place at an average speed of 16 km/h. As Scott was not home, Tan cycled back at 12 km/h.

Find:

a the total time for Tan's complete trip

b Tan's average speed, to nearest km/h, for the trip.

6 Justine's car trip took 9 h 45 min at an average speed of 9.5 m/s. What is her:

a average speed in km/h?

b distance travelled, correct to the nearest metre?

7 How far, in metres to 1 decimal place, will a vehicle travel in 0.1 seconds when travelling at:

a 40 km/h **b** 60 km/h **c** 80 km/h **d** 100 km/h

8 Courtney drove a dragster for 402 m at 530 km/h.

a How long, to nearest second, did it take?

b How far correct to the nearest metre could she travel in 10 seconds at this speed?

9 Luis drove his motorbike 460 m in 6.011 sec.

a What was his speed in m/s correct to one decimal place?

b How long, in minutes to 1 decimal place, would it take to go 58 km at this rate?

10 a Casey's average speed was calculated at 208.33 km/h for a 60.72 km motorbike race. How long, in minutes to 1 decimal place, did he take?

b If he took the same time to complete a 72.4 km race, what was his average speed for this race correct to the nearest km/h?

11 Shellie and Roxy set off for the beach 11.5 km away. Shellie rides her motor scooter at 32 km/h and Roxy cycles at 11 km/h. How many minutes head start should Shellie give Roxy so they arrive at the beach at the same time?

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12 A car is travelling at 75 km/h. How far, to the nearest 0.1 metre, will it travel in the 5 seconds it takes you to cross the road?

13 Police investigating a car crash determined that the car covered 20.7 m in 1.2s. What speed was the car doing in km/h?

12-04 Stopping distance

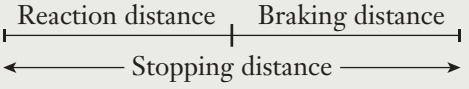
See, think, brake

Imagine you are driving along a road and see a child run out in front of you. You hit the brakes to avoid an accident. What is the time and distance you have travelled from when you sense the problem to when you actually apply the brakes and then come to a stop?

Shutterstock.com/Stephen Aaron

SUMMARY

Reaction time is the time you take from sensing a situation until you apply the brakes.
Reaction distance is the distance travelled during your reaction time.
Braking distance is the distance travelled from when you apply the brakes until you stop.
Stopping distance = Reaction distance + braking distance



Braking distance

In dry conditions, a car travelling at 60 km/h can stop 20 m before one travelling at 80 km/h.

Drivers tend to underestimate the distance needed to stop as the time taken to stop depends on many factors:

- the speed at which you are travelling
- your reaction time
- the condition of your brakes and tyres
- the rate at which you decelerate
- the road surface and slope
- wind speed and weather
- weight of vehicle
- driver visibility, whether it is night or day

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SUMMARY

Braking distance formula

The approximate braking distance (d m) can be found using the formula $d = kv^2$, where v km/h is the initial speed of the car and k is a constant.

EXAMPLE 7

Ben drives a Holden car and has a reaction time of 1.5 s. He was travelling at 60 km/h when he saw a tree had fallen across the road. He applied his brakes and stopped 17.4 m later.

a What was his reaction distance, to the nearest metre?

b What was his stopping distance?

c Use the braking distance formula $d = kv^2$ and the given values to find:

- **i** the value of k, correct to 2 signficant figures
- **ii** the braking distance, to nearest metre, when travelling at 84 km/h
- **iii** the stopping distance, to nearest metre, when travelling at 84 km/h
- **iv** the speed of a vehicle with a braking distance of 100 m. Answer to nearest km/h.

Solution

a Reaction distance is the distance the car travels in 1.5 s at 60 km/h.

$$60 \text{ km/h} = 60\,000 \text{ m/h}$$ **Converting km to m**

$$= \frac{60\,000}{3600} \text{ m/s}$$ **Converting h to s**

$$= 16.666... \text{ m/s}$$

$$d = s \times t$$

$$= 16.666... \times 1.5$$

$$= 25 \text{ m}$$

Reaction distance is 25 m.

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b Stopping distance = reaction distance + braking distance

$= (25 + 17.4)$ m

$= 42.4$ m

c **i** Given $d = 17.4$ m and $v = 60$ km/h,

$$d = kv^2$$
$$17.4 = k \times 60^2$$
$$k = \frac{17.4}{60^2}$$
$$= 0.004\,8333\ldots$$
$$\approx 0.0048$$

correct to 2 significant figures

ii Given $v = 84$ km/h and $k = 0.0048$,

$$d = kv^2$$
$$= 0.0048 \times 84^2$$
$$= 33.8688$$
$$\approx 34 \text{ m}$$

Braking distance is 34 m.

iii Stopping distance = reaction distance + braking distance

$= (25 + 34)$ m

$= 59$ m

iv Given $d = 100$ m and $k = 0.0048$,

$$d = kv^2$$
$$100 = 0.0048 \times v^2$$
$$v^2 = \frac{100}{0.0048}$$
$$= 20\,833.333\ldots$$
$$v = \sqrt{20\,833.333\ldots}$$
$$= 144.3375\ldots$$
$$\approx 144 \text{ km/h}$$

Speed is 144 km/h.

Speed and road conditions

The braking distance changes with the condition of the road. It will take longer to stop on a wet road and even longer to stop on an icy road.

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EXAMPLE 8

This table shows the different stopping distances for 4 different speeds and two different road conditions for a driver whose reaction time is 2 s.

Stopping distances for various speeds and road conditions						
Speed (km/h)	Road	Reaction time (s)	Reaction distance (m)	Braking distance (m)	Stopping distance (m)	Comparisons
40	dry	2	22.2	7.9	30.1	
40	wet	2	22.2	12.6	34.8	
50	dry	2	27.8	12.3	40.1	8 cars end to end
50	wet	2	27.8	19.7	47.5	4 buses
60	dry	2	33.3	17.7	51.0	olympic swimming pool
60	wet	2	33.3	28.3	61.6	3 cricket pitches
100	dry	2	55.6	49.2	104.8	length of a football field
100	wet	2	55.6	78.7	134.3	width of 10 houses

a What do you notice about the reaction distances?

b Discuss the difference in braking distance for a car travelling at:

- **i** 40 km/h on a wet road and 50 km/h on a dry road
- **ii** 50 km/h on a wet road and 100 km/h on a wet road.

c Discuss the difference in stopping distance for a car travelling at:

- **i** 40 km/h on a dry road and 40 km/h on a wet road
- **ii** 60 km/h on a dry road and 100 km/h on a dry road.

d Why should you reduce speed when travelling on a wet road?

e Draw a clustered column graph to represent the data and write why this is a good representation of the data.

Solution

a The reaction distances increase with speed but do not depend on the road condition.

b **i** There is a 10 km/h difference in speed but the braking distances are about the same: 12.6 m and 12.3 m.

ii The speed is doubled but the braking distance is roughly 4 times greater.

c **i** At 40 km/h there is a 4.7 m difference in the stopping distance between a dry and wet road and this difference is significant.

ii There is a 40 km/h difference in speed but the stopping distance is doubled: 51.0 m and 104.8 m.

d You take longer to stop on a wet road so you should reduce your speed to avoid any incidents.

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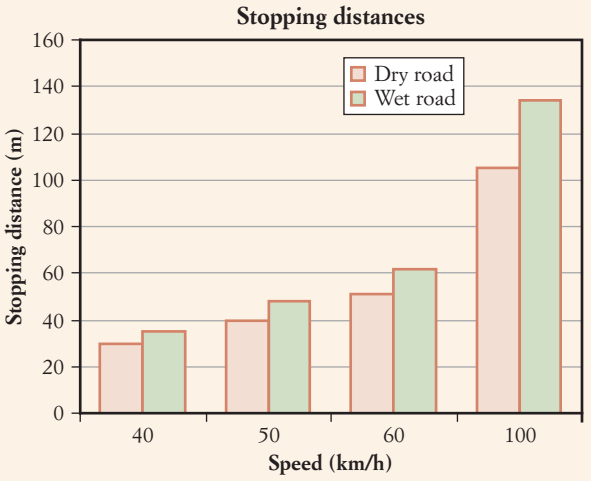
e



The height of the columns shows clearly that as the speed increases, the difference between the stopping distances on dry and wet roads is greater.

Exercise 12-04 Stopping distance

1 Darley saw a stationary car 80 m in front of him. He just stopped in time after travelling 51 m while braking. What was his reaction distance? Select **A**, **B**, **C** or **D**.

A 29 m **B** 80 m **C** 51 m **D** 30 m

Example 7

2 A van moving at 110 km/h travelled a distance of 118.4 m on a wet road before its driver applied the brakes and finally stopped another 79.3 m later. What was the van's stopping distance?

3 A motorcyclist with a quick reaction time stopped in 15.6 m after braking for a distance of 9.8 m. How far did he travel during the time it took to react to the crisis?

4 The table shows the reaction, braking and stopping distances for a driver with a reaction time of one second on a wet road.

Speed (km/h)	Reaction time (s)	Reaction distance (R m)	Braking distance (B m)	Stopping distance ($R + B$)m
40	1	11.1	B	23.7
50	1	A	19.7	33.6
60	1	16.7	C	45.0
100	1	27.8	78.7	D

a Find the values of **A**, **B**, **C** and **D**.

b When the speed is doubled from 50 km/h to 100 km/h, find to the nearest 1% the percentage increase in:

i braking distance **ii** stopping distance?

c If a driver with a reaction time of 1 s is travelling on a wet road at 100 km/h and sees an obstacle 110 m ahead, would he stop in time? Give reasons for your answer.

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Example 8

5 Two cars were driving side by side along a dry road in a 40 km/h school zone when they each saw children crossing 50 m ahead. Fran applied her brakes within 1 second but Hanna took 2 seconds. Use the table of stopping distances from Example **8** on page 541 to find:

a how far Hanna travelled under brakes

b how far Fran travelled before she applied her brakes

c if either would stop in time. Why?

6 Two truck drivers were travelling through a town on a dry road. Mark was doing 60 km/h and took 2 s to react to an accident up ahead while Sanjay was travelling at 40 km/h but took 4 s to react. Use the table of stopping distances from Example **8** on page 541 to find:

a how far Sanjay travelled before the brakes were applied

b how far Mark travelled under brakes

c which truck driver had the greater stopping distance?

7 A driver has a reaction time of 1.6 s. When travelling at 80 km/h, he applied his brakes and travelled 62 m before stopping. Use the braking distance formula $d = kv^2$ to find:

a the constant k correct to 4 decimal places

b the braking distance, to the nearest 0.1 metre, when travelling at 105 km/h

c the speed (to the nearest km/h) of the car that travelled 75 m under brakes before stopping

d the stopping distance, to the nearest 0.1 m, from a speed of:

i 80 km/h **ii** 105 km/h

8 A car travelling at 115 km/h travelled 89.6 m under brakes. Use the formula $d = \frac{v^2}{2a}$ where d is the distance travelled under brakes, a is a constant and v is the initial speed of the car to find:

a the constant a correct to 3 significant figures

b the braking distance, to 1 decimal place, if the initial speed is 95 km/h

c the stopping distance for a driver with a reaction time of 2 s who is travelling at 115 km/h.

9 Use the formula $d = 0.02754v^2$ where v km/h is the speed of a car and d m is the distance the car travels under brakes, to find, correct to 3 significant figures, the braking distance of a vehicle travelling at:

a 35 km/h **b** 61 km/h **c** 112 km/h **d** 93 km/h

10 A car cruising at 80 km/h travelled 41.8 m on an icy road during the time it took for the driver to see a stop sign and start braking. It then travelled 57.2 m under brakes before it stopped.

a What was the driver's reaction time in seconds to 2 decimal places?

b What was the stopping distance?

11 Use the formula $d = 0.00435v^2$, where d m is the distance travelled under brakes and v km/h is the speed as brakes are applied, to find the speed of a racecar to the nearest km/h with a braking distance of:

a 142 m **b** 91.2 m **c** 101.4 m **d** 68.9 m

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12 A car travelled 84.3 m under brakes when initially travelling at 106.3 km/h. Use the braking formula $d = kv^2$ to find:

a the constant k correct to 3 significant figures

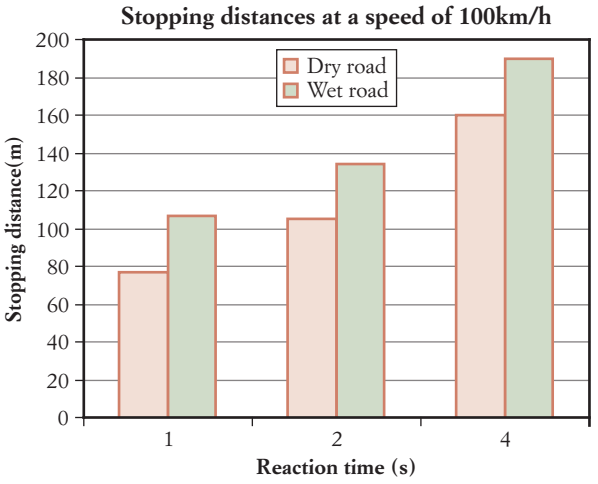
b the braking distance, to 1 decimal place, for a car applying brakes from a speed of 110 km/h

c the stopping distance, to 1 decimal place, of a car travelling at 110 km/h if the driver's reaction time is 2 s.

13 The formula $d = \frac{1}{2}x^2 + 5x$, where x km/h is the speed of a vehicle and d m is the distance travelled, is only used for very fast cars. How many metres can a racing car travel at a speed of:

a 100 km/h? **b** 250 km/h? **c** 300 km/h?

14 The graph shows the stopping distances on wet and dry roads with various reaction times.



a A driver on a dry road has a reaction time of 4 s. How far will he travel before he stops?

b How many metres does it take for a car to stop when travelling at 100 km/h on a wet road if the driver has a reaction time of:

i 1 s **ii** 2 s **iii** 4 s?

c Use the graph to organise and present the data in a table.

Road condition	Reaction time	Stopping distance

d Discuss the benefits of data in a table versus data represented on a graph.

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INVESTIGATION

STOPPING DISTANCES

1 This table of values, to 1 decimal place, was found using an online calculator.

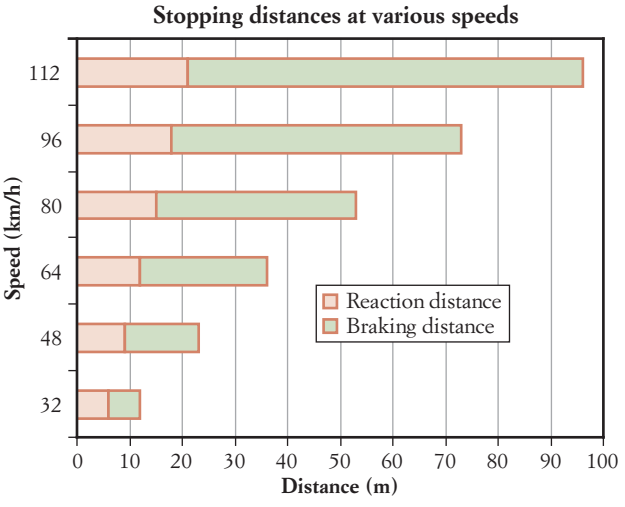
Distances for a reaction time of 1.5 seconds				
Road condition	**Speed (km/h)**	**Reaction distance (m)**	**Braking distance (m)**	**Stopping distance (m)**
Dry	60	25.0	17.4	42.4
	100	41.7	48.2	89.9
Wet	60	25.0	27.8	52.8
	100	41.7	77.2	118.8
Icy	60	25.0	138.9	163.9
	100	41.7	358.8	427.5

a Use the table to investigate the relationship between braking distance, speed and road conditions.

b Find an online calculator and use it to determine the stopping distance for:

- **i** a car travelling at 60 km/h on an icy road if the driver has a reaction time of 1.6 s
- **ii** the difference in stopping distances at 80 km/h for reaction times of 0.9 s and 1.4 s
- **iii** a car travelling on a dry or wet road at 110 km/h, if reaction time is 1.5 s.

2



The graph shows the reaction and braking distances for cars at various speeds. Use the graph to investigate the effect on the stopping distances at various speeds.

Give one trend that you have discovered by looking at stopping distances with road conditions and speed.

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INVESTIGATION

REACTION TIME TESTS

1 Use an Internet search engine to find a **Reaction Time Test** where the times you take to click the button when the light comes on are recorded and the average reaction time calculated.

Work with 4 friends/relatives and decide who has the best average reaction time.

2 Work in pairs.

- Student A holds the top of a 30 cm ruler between thumb and forefinger.
- Student B sits on a chair and holds thumb and forefinger each side of the bottom of the ruler.
- Student B says 'ready'.
- Student A drops the ruler within 5 seconds.
- Student B catches the ruler and records the 'reaction distance'.
- Repeat 5 times and find your average reaction distance.
- Swap places and repeat for student A.

If this is done in a classroom, you could find the person with the fastest and slowest reaction times as well as the class average.

3 Name 3 activities when a fast reaction time is an advantage.

Alamy/Don Hover

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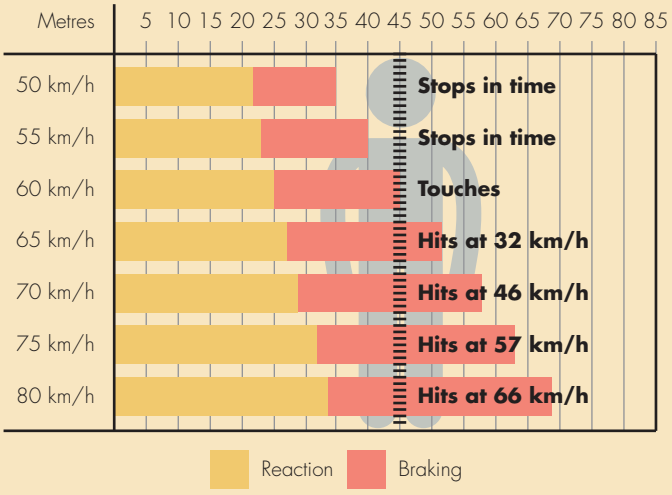
JUST FOR THE RECORD

Will you stop in time?

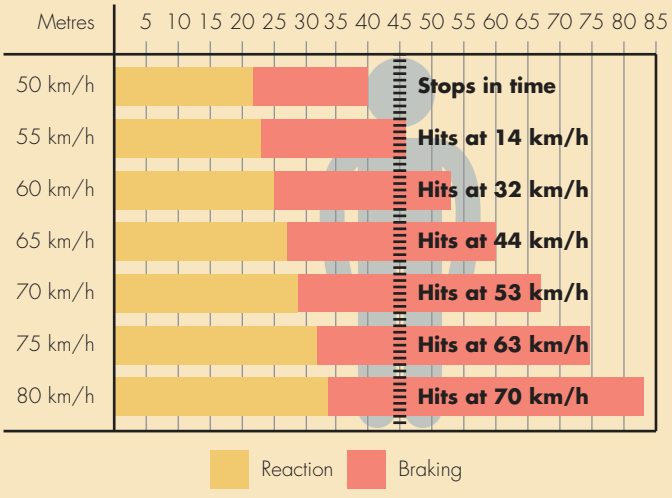
Dry Conditions

The road is dry, you have a modern vehicle with good brakes and tyres. A child runs onto the road 45 m ahead of you while you are travelling in a 60 km/h zone. You brake hard. **Will you stop in time?**



Wet Conditions

The road is wet, you have a modern vehicle with good brakes and tyres. A child runs onto the road 45 m ahead of you while you are travelling in a 60 km/h zone. You brake hard. **Will you stop in time?**



Source: Australian Federal Police, ACT

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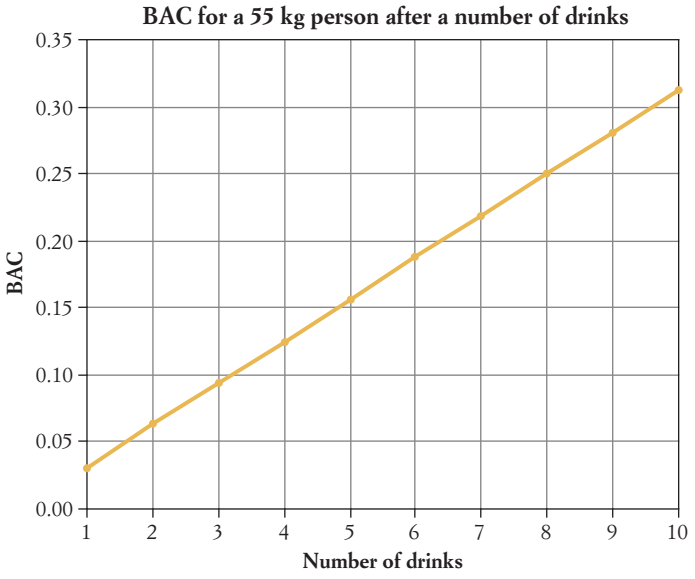
Sample HSC problem

This graph shows the blood alcohol content of a 55 kg woman according to the number of drinks she has had.



a Estimate the woman's BAC after 5 drinks.

b How many drinks will cause her to reach a BAC beyond 0.1?

c When her BAC reaches 0.3, the woman will lose consciousness. After how many drinks will this occur?

d What is the gradient of the line in the graph and what does it represent?

e How many hours and minutes would it take the woman to return to a BAC of zero after 8 drinks? [Use $h = \frac{\text{BAC}}{0.015}$]

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Study tip

More exam tips

1. Read all instructions carefully. Don't rush.
2. Calculate the average time you should spend on each question.
3. If one answer is taking too long, stop and ask yourself: 'Am I on the wrong track?' Don't get bogged down. You may need to retrace your steps, start again or come back later.
4. When you have finished a question, make sure you have actually answered it. Do you need to write the answer in a sentence? Put a circle or box around the answer to highlight it.
5. Make sure that your answer is reasonable, especially if it involves money or measurement. Did you round the answer correctly and include the correct units?
6. If you have some time left at the end of the exam, double-check your answers, especially those for the more difficult questions or ones that you are unsure about.

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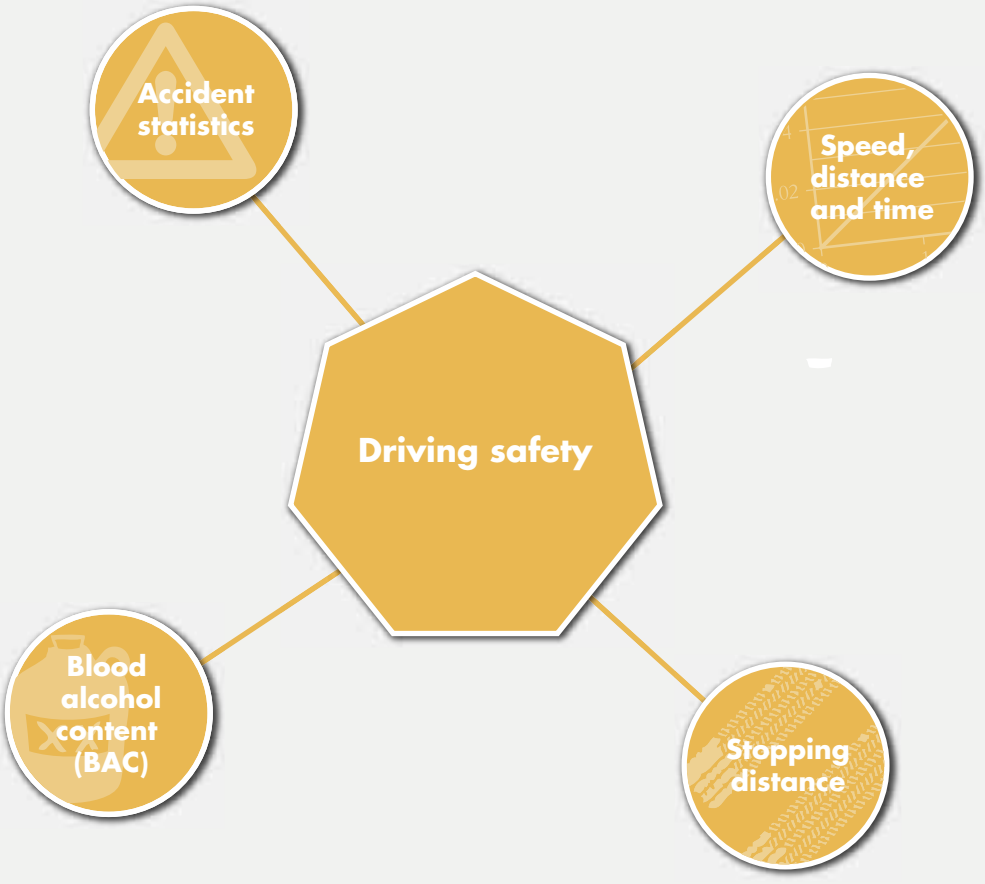
CHAPTER REVIEW

Topic overview

This chapter, Driving safety, looked at the mathematics behind safe driving, covering the areas of measurement (blood alcohol, speed and stopping distance), algebra (formulas) and data analysis (road accident statistics). You should be familiar with the terminology, formulas, graphs and calculations involved with BAC, road accidents, speed and stopping distance.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map has been printed below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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Revision

Exercise 12-01

1 Jennifer weighs 56 kg and drank 3 glasses of wine at a party over 3 hours. She thought she was safe to drive but when pulled over and tested, she was over the legal limit of 0.05. She later discovered that each wine glass held $1\frac{1}{2}$ standard drinks. Use the formulas $BAC_{female} = \frac{10N - 7.5H}{5.5M}$; Number of hours $= \frac{BAC}{0.015}$ to answer the following questions.

a What was her BAC, to 3 decimal places, when she was tested?

b How long, in hours and minutes, did it take before her BAC was back to zero?

c What would her BAC be, to 3 decimal places, after 3 standard drinks in 3 hours?

Exercise 12-01

2 This table shows the BAC for women of different weights and the number of drinks consumed in an hour.

	Body weight (kg)						
Drinks per hour	45	55	64	73	82	91	100
1	0.05	0.04	0.03	0.03	0.03	0.02	0.02
2	0.09	0.08	0.07	0.06	0.05	0.05	0.04
3	0.14	0.11	0.10	0.09	0.08	0.07	0.06

A 64 kg woman had no alcohol before she went to a party. She consumed 2 drinks in the first hour and 3 drinks in the second hour of the party.

a What was her BAC at the start of the party?

b What was her BAC at the end of the first hour?

c What was her BAC at the end of the second hour?

d If she had no more drinks, how many hours would it have taken for her BAC to return to zero? Assume that her body reduces her BAC at the rate of 0.018 per hour.

Exercise 12-01

3 **a** Why is a heavier person less affected by alcohol?

b Why are females more affected by alcohol?

Exercise 12-02

4

Road Fatalities in Australia for years 2007–2011				
	Drivers		Passengers	
Years	**Male**	**Female**	**Male**	**Female**
2007	616	169	178	158
2008	496	175	173	128
2009	522	186	179	147
2010	469	166	154	129
2011	432	147	156	127

Source: BITRE

a How many drivers were killed between 2007 to 2011?

b How many passengers were killed between 2007 to 2011?

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c What is the mean number of male drivers killed per year?

d What is the range of female passengers killed over the 5 years?

e What is the median number of female drivers killed over the 5 years?

f What is the interquartile range of male passengers killed over the 5 years?

Exercise 12-02

5

Driver/passenger deaths as a result of a car crash		
Years	**Male**	**Female**
2008	669	303
2009	701	333
2010	623	295
2011	588	274
TOTAL		

Source: BITRE

a What was the total number of driver and passenger deaths over the 4 years?

b What is the mean number of male drivers killed per year?

c What is the interquartile range in the numbers of females killed over the 4 years?

d Write a sentence suggesting why there are significantly less females killed in car crashes.

e Draw a sector graph to illustrate the number killed in 2010.

Exercise 12-03

6 Scott took $3\frac{1}{4}$ h to drive to his holiday destination 210 km away.

a What was his average speed, correct to the nearest km/h?

b How long, in hours and minutes, would it take him to drive 350 km at this rate?

c How far, to nearest kilometre, could he travel at this rate in 4 h 20 min?

Exercise 12-04

7 A truck driver was travelling in the Northern Territory at 150 km/h on a road with no speed limits. He began braking 3 seconds after he saw a sign to a truck stop. If he travelled 810 m under brakes before coming to a stop, how far did he travel after seeing the sign?

Exercise 12-04

8 George is travelling on a dry road at 65 km/h and sees a cow in the middle of the road about 70 m ahead. He takes 1.2 s to apply the brakes, then travels 24 m under brakes before coming to a stop. Did he hit the cow? Give reasons for your answer.

Exercise 12-04

9

Stopping distances in a 40 km/h zone					
Speed (km/h)	**Road**	**Reaction time (s)**	**Reaction distance (R m)**	**Braking distance (B m)**	**Stopping distance (R + B m)**
40	wet	1	11.1	12.6	A
40	wet	2	22.2	12.6	B
40	wet	4	44.4	12.6	C

a What do you notice about the braking distances?

b What do you notice about the reaction distances?

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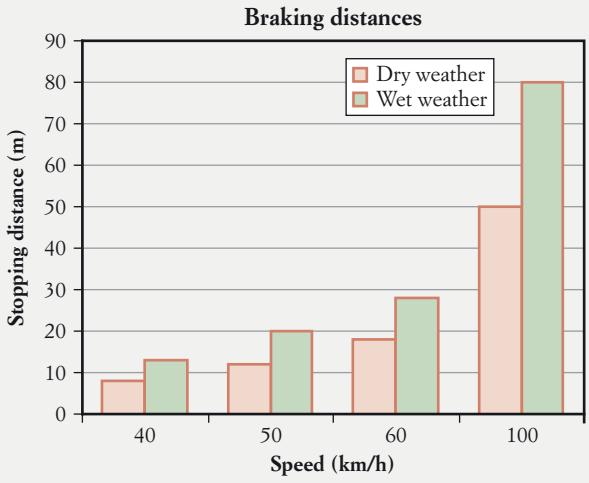
c Find the values of A, B and C.

d If an 85-year-old driver in a school zone has a slow reaction time of 4 s, would he stop in time if he sees a child crossing the road 50 m ahead?

e Suggest two strategies for this driver that would help him to avoid any incidents when driving.

Exercise 12-04

10 The graph gives the braking distance for a car travelling on a dry or wet road at various speeds.



a What is the difference in braking distance on a dry road for two cars travelling at 50 km/h and 100 km/h respectively?

b How much further will a car travel under brakes on a wet road at 60 km/h than on a dry road at the same speed?

c If a car travels 50 m under brakes on a dry road, what is its approximate speed?

d What is the approximate speed of a car on a dry road that travels 12 m after the brakes are applied?

e Write a sentence noting any similarities or differences between braking distances on wet and dry roads, giving reasons for your answer.

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Practice paper 4

Recommended time: One hour

Section 1

10 objective-response questions: 1 mark each
For Questions 1 to 6, select the correct answer, A, B, C or D

Exercise 10-01

Question 1

How many kilobits in 4 megabits?

(A) 4000

(B) 4096

(C) 4 000 000

(D) 4 194 304

Exercise 11-01

Question 2

A principal of $25 000 earns 3.5% simple interest p.a. What is the interest earned in 7 months?

(A) $875

(B) $510.42

(C) $150.94

(D) $125

Exercise 11-06

Question 3

Sofia bought a pink diamond for $5000. If it appreciates at a rate of 5.6% p.a., what will be its value in 7 years, correct to the nearest dollar?

(A) $6960

(B) $7322

(C) $19 600

(D) $36 960

Exercise 10-01

Question 4

How many 800 kB photos can be stored on a 750 MB CD?

(A) 937

(B) 960

(C) 1041

(D) 1066

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Exercise 10-01

Question 5

A mobile phone plan has a monthly fee of $39 and handset monthly repayments of $6 for 24 months. Calculate the minimum cost of this plan over 2 years.

(A) $222

(B) $942

(C) $1080

(D) $1830

Exercise 12-01

Question 6

A blood alcohol concentration (BAC) of 0.2 means the amount of alcohol in blood is:

(A) 0.2 g/L

(B) 0.2 mg/L

(C) 0.2 g/100 mL

(D) 0.2 mg/100 mL

For Questions 7 to 10, write the answer only (do not show your working).

Exercise 10-01

Question 7

How many bytes in 2.5 megabytes?

Exercise 12-03

Question 8

Joe left Sydney at 8 a.m. to travel to Melbourne, a distance of 1000 km. During the day, he had rest breaks so he arrived in Melbourne at 9 p.m. What was his average speed for the journey, correct to the nearest km/h?

Exercise 11-07

Question 9

Ristin has 12 600 shares in Lickity Lick gelato company. The dividend per share is 7.4 cents. What dividend will she receive?

Exercise 11-03

Question 10

What principal must be invested at 5.7% p.a. compounded yearly for 6 years so that it grows to $24 000? Give your answer rounded up to the nearest cent.

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Section 2

3 questions: 10 marks each

Question 11 **Marks**

Exercise 12-04

(a) Jayden was travelling at 70 km/h when he saw a car stopped ahead of him. He took 1.2 s to react and apply his brakes, then stopped 14 m later.

(i) What was his reaction distance, to the nearest metre? **1**

(ii) Use the braking distance formula $d = kv^2$ to find the value of k correct to two significant figures. **1**

(iii) Find Jayden's braking distance correct to one decimal place if he was travelling at 90 km/h. **1**

Exercise 11-06

(b) Kristin bought a house for $720 000. Use the table below to calculate its value in 5 years time if it appreciates at 10% p.a. **2**

Compounded values of $1				
Number of periods	Interest rate per period			
	1%	5%	10%	12%
1	1.010	1.050	1.100	1.120
2	1.020	1.103	1.210	1.254
3	1.030	1.158	1.331	1.405
4	1.041	1.216	1.461	1.574
5	1.051	1.276	1.611	1.762

Exercise 11-04

(c) Use the compound interest table above to find:

(i) the compound interest earned on $8420 invested at 5% p.a. compounded yearly for 3 years. **1**

(ii) the final amount when $12 600 is invested at 12% p.a. compounded monthly for 5 months. **1**

Exercise 12-02

(d) This table shows the number of road fatalities per week on a major highway over a 6-week period.

Week	1	2	3	4	5	6
No. fatalities	4	3	7	6	8	10

(i) Find correct to two decimal places the mean number of fatalities over the period. **1**

(ii) Find the median. **1**

(iii) Write a sentence discussing the trend in fatalities. Suggest reasons for the trend. **1**

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Question 12 **Marks**

Exercise 10-04

(a) With Asam's mobile phone plan, he pays \$50 per month for \$650 of credit in calls and SMS texts and 3 GB of included data per month. Excess data usage is charged at \$1 per MB.

(i) If Asam used \$720 in calls and SMS texts in one month, how much in total must he pay that month? **1**

(ii) If Asam used 3.2 GB of data in one month, how much extra must he pay? **1**

Exercise 11-07

(b) Sandy bought 3500 shares in Fosters at \$6.10 with a dividend yield of 7%. Her stockbroker charges 2.5% brokerage for buying shares and 1% brokerage for selling shares.

(i) Calculate the total cost to Sandy of buying the shares. **1**

(ii) If Sandy sells her shares at \$8.54 each twelve months after buying them, calculate her total profit after costs. **2**

Exercise 12-01

(c) Tegan weighs 56 kg and has had 5 standard alcoholic drinks over 4 hours.

(i) Calculate correct to three decimal places her blood alcohol content using the formula:

$$\text{BAC}_{\text{female}} = \frac{10N - 7.5H}{5.5M}$$

where

N = number of standard drinks consumed

H = number of hours drinking

M = mass in kg **1**

(ii) If Tegan drank the same number of standard drinks over a longer time, how would her blood alcohol content have been affected? **1**

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Exercise 10-06

(d) This table shows a range of Optel's prepaid mobile phone plans.

Cost of plan	$15	$30	$45	$75	$120
Credit for calls and SMS texts	$90	$180	$400	$800	$1250
Expiry time	30 days	30 days	30 days	60 days	180 days
Call connection fee	45c	45c	30c	30c	24c
Call rate per min	105c	95c	80c	72c	66c
Cost per SMS text	30c	28c	25c	22c	20c
Included data	150 MB	400 MB	1.5 GB	3 GB	6 GB
Additional data cost	$1 per MB	$0.50 per MB		$0.30 per MB	
Calls to other Optel mobiles	Normal rates	Free for the first 20 mins between 8 PM and 8 AM		Free anytime	

Marks

(i) Annabel sent 450 SMS texts under the $30 plan. How much did they cost (ignoring the $180 credit)? **1**

(ii) Calculate the cost of 5-minute call to a mobile phone not on the Optel network under the $120 plan. **1**

(iii) How is the call rate affected by the cost of the plan? **1**

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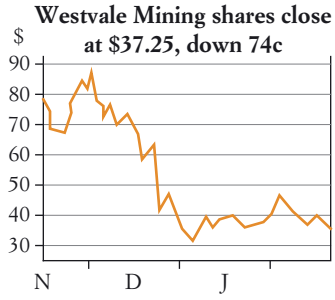
Question 13

Marks

Exercise 11-08

(a) This graph shows the price of Westvale Mining shares over a period.



(i) What length of time does the graph cover? **1**

(ii) What was the highest price reached? **1**

(iii) How many times was the share price $40? **1**

Exercise 10-03

(b) The table shows the number of movie downloads in one year.

Movie	No. downloads in millions
Avatar	21
The Dark Knight	18.5
Transformers	18.5
Inception	17
The Hangover	16
Star Trek	14.5
Kick-Ass	12.5
The Incredible Hulk	10

(i) Draw a horizontal column graph to represent the data. **1**

(ii) Find the average number of downloads for the 8 movies. **1**

(iii) Suppose each download costs an average of $20 and the wholesaler receives 24%, the distributor 68% and the royalties to artists the remainder. How much will the artists receive for the total downloads of *Star Trek*? **2**

Exercise 10-02

(c) Cooper has an MP3 player that holds 8 GB of data.

(i) How many songs of average size 3.8 MB can he store? **1**

(ii) If he downloads a 3.8 MB song at a speed of 1.75 Mbps, how long will it take? Answer to the nearest second. **2**

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GLOSSARY

Mathematical 'doing' words

calculate To find the value of a numerical expression. See also **evaluate**.

classify To sort into categories or types.

complete To fill in detail that makes a statement or diagram correct or finished.

compare To show how things are similar or different.

construct To draw a geometrical figure accurately.

convert To change from one form to another. For example, convert a fraction to a decimal, or convert dollars to cents.

decrease To make smaller.

describe To state characteristics or features.

estimate To make an educated guess for a number or measurement; to find roughly or approximately.

evaluate To find the value of an expression. For example, evaluate 3×8^2, or evaluate $4x + 1$ when $x = 5$.

expand To remove the grouping symbols (brackets) in an algebraic expression. Expanding $3(2y + 1)$ gives $6y + 3$.

explain To describe why or how.

give reasons When solving a problem, to show the mathematical rules or thinking used.

graph To display on a number line, number plane or statistical graph.

hence find/prove To find an answer or prove a result using answers found previously.

increase To make larger.

interpret To find meaning in the mathematical result.

justify To support your argument or conclusion.

measure To determine the size of something. For example, to determine length using a ruler.

prove/show that In questions where the answer is given, to use mathematical methods or reasoning to prove that the answer is true.

simplify To give the answer in its simplest, shortest, neatest form.

sketch To draw a rough diagram. Less accurate than to **construct**.

solve To find the value of the unknown pronumeral in an equation or inequality.

substitute To replace a variable with a number and evaluate.

verify To show that a statement or result is correct, usually by substituting back.

write/state To write an answer, formula or result without showing any working or explanation. (This usually means that the answer can be found mentally, or in one step.)

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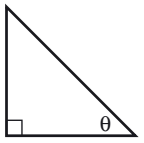
+	plus	$\sqrt{\ }$	the square root of
−	minus	$\sqrt[3]{\ }$	the cube root of
×	multiplied by (times)	$P(E)$	the probability of event E occurring
÷	divided by	$P(\bar{E})$	the probability of event E not occurring
%	per cent	$\angle ABC$	angle ABC
( )	parentheses, round brackets	$\triangle ABC$	triangle ABC
[ ]	(square) brackets	−3	negative 3
{ }	braces	$0.\dot{6}$	the recurring decimal 0.666 666…
=	is equal to	3(4)	3 multiplied by 4, 3×4
≈	is approximately equal to	$3x$	3 multiplied by x, $3 \times x$
≠	is not equal to	$\frac{x}{2}$	x divided by 2, $x \div 2$
<	is less than	x^2	x squared, $x \times x$
>	is greater than	x^3	x cubed, $x \times x \times x$
⩽	is less than or equal to	$\bar{x}$	the mean (average)
⩾	is greater than or equal to	π	pi (3.141 59…)
∴	therefore	θ	theta
		α	alpha
		μ	micro- (one millionth)

A

absolute error The maximum possible error for a measurement, equal to $\pm\frac{1}{2}$ of the smallest unit marked on the measurement scale.

adjacent side In a right-angled triangle, the side 'next to' the given angle, leading to the right angle.



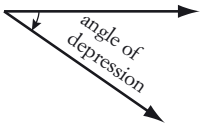
adjacent side (adjacent to θ)

allowable (tax) deduction A part of a person's yearly income that is not taxed, such as work-related expenses or donations to charities. All deductions are subtracted from yearly income to determine **taxable income**.

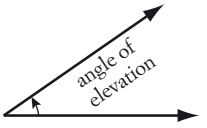
allowance, government Money paid by the government to support individuals for specific purposes (for example to support the aged, unemployed, disabled, students and parents).

allowance, worker's Money paid to a worker for expenses incurred as part of his or her job (for example, for travel, for special clothing, or for working in isolated or dangerous areas).

angle of depression The angle between the horizontal and the line of sight, looking down.



angle of elevation The angle between the horizontal and the line of sight, looking up.



annual leave loading Extra payment to a worker based on a percentage (usually 17.5%) of 4 weeks annual leave.

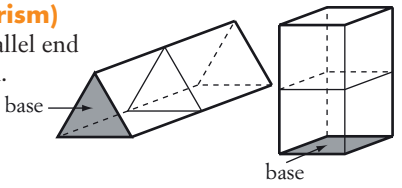
appreciation Increase in value of an object (for example, a work of art, a piece of jewellery or antique furniture) over a period

at least A number equal to or greater than the given value (for example 'at least 2' means 2 or more).

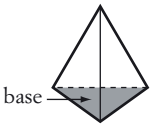
B

base (in index notation) A number being raised to a power. For example, in 2^5, the base is 2.

base (of a prism) One of the parallel end faces of a prism.



base (of a pyramid) The plane figure on which a pyramid rests.



bearing See **compass bearing** and **true bearing**.

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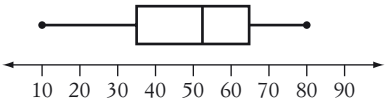
bias In statistics, an unwanted influence that stops a sample from being representative of a population.

bit (symbol b) The smallest unit of digital (computer) memory, can store a 0 or 1, short for **b**inary dig**it**.

blood alcohol content (BAC) The concentration of alcohol in a person's blood, measured in g/100 mL.

bonus Extra pay for achieving a high quality or volume of work, such as meeting an important quota, goal or deadline.

box-and-whisker plot (or boxplot) A diagram that displays the quartiles of a set of data as a box and the extremes as whiskers.



braking distance The distance travelled between when a driver applies the brakes and when the vehicle stops completely. See also **reaction distance** and **stopping distance**.

break-even point The point or value at which a business stops making a loss and starts making a profit.

brokerage Commission paid to a stockbroker for buying or selling a parcel of shares for a client.

budget A plan for managing money.

byte A unit of digital memory equal to 8 bits, which can store a character of information.

C

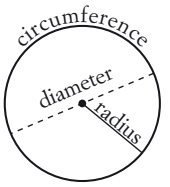
cap plan A type of phone plan in which the monthly fee is fixed or 'capped', unless you spend more than the **included credit** or use more than the **included data**.

capacity Maximum volume of liquid that can be held by a container, usually measured in millilitres (mL), litres (L) or kilolitres (kL).

categorical data Information or data represented as a category rather than as a number (for example the makes of cars, or the colours of eyes). Differs from **quantitative data**.

census Collection of information about every member of a population.

circumference The perimeter of a circle. $C = \pi d$ or $C = 2\pi r$, where C is the circumference, π is pi (3.141 59 …), d is the **diameter** and r is the **radius**.

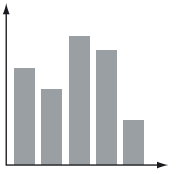


class centre The centre of a class interval. For example, the class centre of the class interval 10–19 is 14.5.

class interval In statistics, when there are many data scores, they may be grouped into class intervals. For example, ages of people may be grouped into class intervals of 1–10, 11–20, 21–30, and so on.

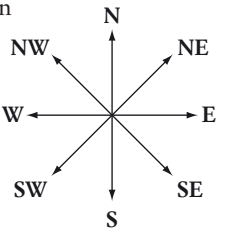
cluster A group of data scores that are bunched or close together.

column graph A graph consisting of vertical bars of equal width.



commission The earnings of a sales person or agent; usually a percentage of the value of items sold.

compass bearing A direction shown by the points on a compass, such as north (N), south (S), east (E), west (W), north-east (NE), north-west (NW), south-east (SE), south-west (SW).



complementary event All the outcomes that are not the event. For example, the complementary event to rolling a 1 on a die is rolling a number that is not 1.

compound interest Interest paid on the principal invested as well as on any accumulated interest. Differs from **simple interest**.

compounding period How often interest is calculated (for example monthly, quarterly or yearly).

comprehensive motor insurance Insurance that covers all damage to vehicles and property, including your own, in an accident in which you are at fault. See also **CTP motor insurance** or **third party property motor insurance**.

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concentration The amount of liquid or solid in another liquid or solid expressed as a rate or percentage, for example the concentration of salt in water in g/mL.

connection fee The initial charge of making a mobile phone call, also called 'flag fall'.

constant A value that does not change. See also **variable**.

constant of variation (or constant of proportionality) The constant in a variation equation. For example, if y varies as x, the equation is $y = kx$ and the constant of variation is k.

continuous data Quantitative (numerical) data that can be measured on a smooth scale of values (without 'gaps'), such as the heights of people.

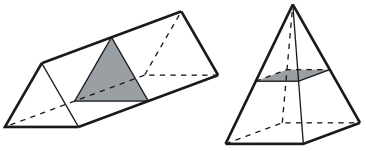
conversion graph A graph that is used to convert between different units, such as between metric and imperial units of measurement, or between currencies in foreign currency exchange.

cosine A ratio in a right-angled triangle:

$$\cos\theta = \frac{\text{side adjacent to } \theta}{\text{hypotenuse}}$$

where θ is an angle. See also **sine** and **tangent**.

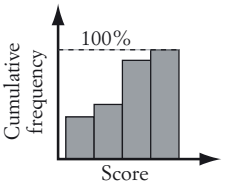
cross-section of a solid A plane shape formed when a solid is cut parallel to an end.



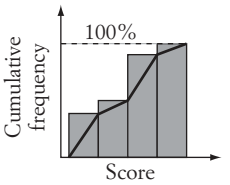
CTP motor insurance Compulsory Third Party insurance that covers personal injury or death to another person ('third party') in an accident in which you are at fault. Also called 'green slip' because the insurance certificate is green. See also **comprehensive motor insurance** or **third party property motor insurance**.

cumulative frequency A running total of frequencies.

cumulative frequency histogram A histogram in which the height of each column represents the cumulative frequency at each score.



cumulative frequency polygon A line graph formed by joining the top right hand corners of the columns of the cumulative frequency histogram. Also called an **ogive**.



D

data Observations or facts which, when collected, organised and evaluated, become information.

data allowance See **included data**.

deciles Values that divide a data set into 10 equal parts when the scores are arranged in order. See also **percentiles** and **quartiles**.

declining balance method of depreciation Method of depreciation in which an item's value decreases by the same percentage each period. Differs from **straight-line method of depreciation**.

deduction (tax) See **allowable (tax) deduction**.

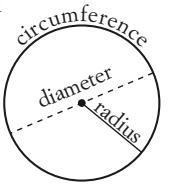
deductions (from pay) Amounts taken out of a person's gross pay (for example union fees, superannuation, and income tax).

denominator The number on the bottom of a fraction. For example, in $\frac{3}{5}$ the denominator is 5.

dependent variable The variable in a function that depends on another variable for its value. For example, if $y = 3x + 5$, y is the dependent variable because its value depends on the value of x. Differs from an **independent variable**.

depreciation Loss in value of an item or asset over time.

diameter The length of the interval passing through the centre of a circle and joining two points on the **circumference** of the circle. The diameter is double the **radius**.



direct variation (or direct proportion) The relationship between two variables (say x and y) by an equation of the form $y = kx$, where k is the constant of variation.

discrete data Quantitative (numerical) data that can be counted and whose values are separate and distinct, such as the numbers of pets owned, or the numbers of people in families.

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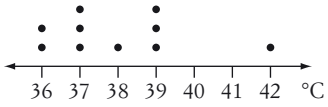
divided bar graph A graph representing the parts of a whole by a rectangle divided into proportionately-sized sections. Displays the same kind of data as a **sector graph**.

dividend The share of profits in a company paid to each shareholder.

dividend yield The percentage return on an investment.

$$\text{dividend yield} = \frac{\text{dividend per share}}{\text{market price of share}} \times 100\%$$

dot plot A graph that uses dots to show frequencies of data scores (for example the temperatures, in °C, of 10 hospital patients).



Different from a **stem-and-leaf plot**.

double time Wages paid at twice the normal rate (for example for working on a Sunday or a public holiday).

download Transfer of data from a source (such as the Internet or a larger computer/server) to a smaller computer or device (such as a tablet or phone).

download speed The rate at which data is downloaded, measured in bits per second (bps or b/s) or kilobits per second (kbps or kb/s).

E

equally likely Having an equal chance of occurring.

equation A mathematical statement that two quantities are equal. An equation contains an equals sign (for example $4x - 5 = 11$).

event In probability, a result involving one or more outcomes. For example, when rolling a die, the event 'rolling an even number' contains the three outcomes $\{2, 4, 6\}$.

expense The cost of spending.

experimental probability An estimate of the theoretical or calculated probability; the relative frequency of an event in repeated trials of an experiment.

extremes (upper and lower) The highest and lowest scores in a set of data.

F

fatality An accident resulting in death, or a person who dies in an accident.

five-number summary Lower extreme, lower quartile, median, upper quartile and upper extreme of a data set. These five numbers are used to draw a **box-and-whisker plot**.

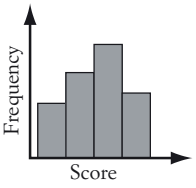
flagfall See **connection fee**.

flat rate loan A loan in which simple interest is charged on the principal borrowed for the term of the loan. See also **reducing balance loan**.

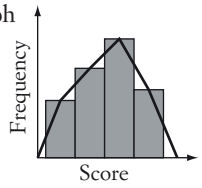
formula A rule written as an algebraic equation, using variables. For example the formula for the area of a triangle is $A = \frac{1}{2}bh$.

frequency The number of times a score or group of scores occurs in a data set.

frequency histogram A histogram in which the height of each column represents the frequency of a single score or group of scores.



frequency polygon A line graph formed by joining the midpoints of the tops of the columns of a frequency histogram.



fuel consumption The rate at which fuel is used by a vehicle, usually measured in L/100 km.

function The relationship between two variables (for example $y = 3x - 5$).

G

gigabyte (GB) 1024 megabytes or $(1024)^3$ bytes.

goods and services tax (GST) The tax a consumer pays on a purchased item or service (for example buying a car, or hiring a painter).

gradient (symbol *m*) Slope of a line.

$$\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$$

green slip See **CTP insurance**.

gross pay (or gross wage) A person's pay before tax is deducted. Different from net pay.

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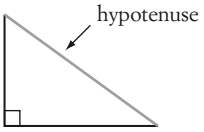
H

hectare A land measure equal to the area of a square 100 m by 100 m. 1 ha = 10 000 m^2.

histogram See **frequency histogram**.

hypotenuse The longest side of a right-angled triangle; the side opposite the right angle.



I

included credit The amount of dollar credit in phone calls and SMS texts given to the user of a phone plan when the user pays a monthly access fee or a prepaid amount.

included data The amount of data usage (upload and download) given to the user of a mobile phone plan when the user pays a monthly access fee or a prepaid amount.

income Money that is earned or gained (usually regularly).

income tax A tax on a person's income, paid to the government.

independent variable A variable in a function whose values do not depend on any other variable. For example, if $A = \pi r^2$, r is the independent variable. This differs from the **dependent variable**.

inflation The increase in the prices of goods and services over time.

insurance A scheme in which you pay an insurance company regularly in return for being paid a larger amount if an accident occurs.

interest Money earned on an investment, or money paid to a financial institution for borrowing.

interquartile range The difference between the upper quartile and lower quartile of a data set ($Q_3 - Q_1$). It is a measure of the spread of the data.

K

kilobit (kb) 1000 bits.

kilobyte (kB) 1024 bytes.

L

LHS Left-hand side (of an equation).

likely Probably will happen; having a probability above $\frac{1}{2}$.

linear A word used to describe something to do with a line.

linear function A function of the form $y = mx + b$, whose graph is a straight line.

linear modelling Using a linear function to approximate a real-life situation.

M

market price (or market value) Current sale value of a share or item.

mean The average of a set of scores.

$$\text{mean (or } \bar{x}) = \frac{\text{sum of scores}}{\text{number of scores}}$$

measure of location A statistical value, such as the mean, median or mode, which describes the centre or average of a set of data.

measure of spread A statistical value, such as the range, interquartile range or standard deviation, which describes the spread of a set of data.

median The middle score of a data set when scores are arranged in ascending order. If there are two middle scores, the median is the average of the two.

median class The class interval that contains the median score.

Medicare levy A tax to cover the costs of the public health system, calculated as a percentage (usually 1.5%) of a person's income.

mega- (symbol M) A prefix meaning 'one million'. For example a megalitre is one million litres.

megabit (Mb) 1000 kilobits or 1 000 000 bits.

megabyte (MB) 1024 kilobytes or 1024 × 1024 bytes.

micro- (symbol μ) A prefix meaning 'one millionth'. For example, a micrometre is one-millionth of a metre.

minute (angle measure) A measure of angle size. $\frac{1}{60}$ of a degree. $1° = 60'$.

minute (time) A measure of time. $\frac{1}{60}$ of an hour. 1 h = 60 min.

modal class The class interval with the highest frequency.

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mode The most common or frequent score(s) in a set of data.

modelling Using mathematics to describe a real-life pattern or relationship.

more than A number greater than the given value. For example 'more than 2' means any number above 2 (not including 2).

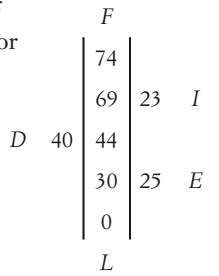
multi-stage event An event that involves two or more outcomes occurring together. For example a two-stage event that involves tossing a coin and rolling a die together could be tossing tails and rolling an odd number.

N

net pay (or net wage) A person's pay after tax has been deducted. Differs from **gross pay**.

nominal data Categorical data that cannot be ordered, for example, colour of eyes.

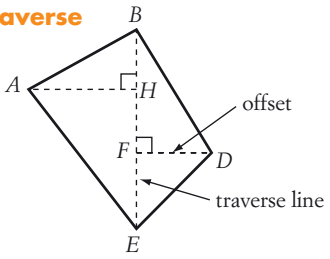
notebook entry A diagram of measurements drawn by a surveyor in an **offset survey**.



numerical data See **quantitative data**.

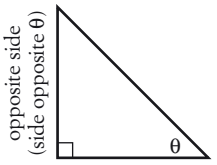
O

offset survey (or traverse survey) A method of measuring lengths in an irregularly-shaped field, using offsets to each corner of the field from a traverse line.



ogive See **cumulative frequency polygon**.

opposite side In a right-angled triangle, the side directly facing the given angle.



ordinal data Categorical data that can be ordered, for example, level of swimming class.

outcome In probability, the result of a situation or experiment. For example, when rolling a die, one possible outcome is 4.

outlier An extreme (high or low) score in a data set that is much different from the other scores.

overdrawn account A bank account that has had more money withdrawn from it than there is available.

overtime Time worked beyond usual working hours, usually paid at a higher rate.

P

PAYG tax 'Pay As You Go' tax; income tax deducted from your pay in instalments each payday by your employer.

per annum (p.a.) Per year.

percentage error The maximum possible error for a measurement, expressed as a percentage.

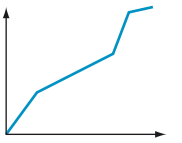
$$\text{percentage error} = \frac{\text{absolute error}}{\text{measurement}} \times 100\%$$

percentiles Values that divide a data set into 100 equal parts when the scores are arranged in order. See also **deciles** and **quartiles**.

period Amount of time (for example month, week, year).

pie chart See **sector graph**.

piecewise linear graph A line graph made up of line segments or 'pieces' with different gradients.



piecework Earnings based on the number of items processed, made or delivered, paid at a rate per item (rather than per hour).

population All of the items under investigation. See also **sample**.

postpaid phone plan A plan in which you pay regularly for using a phone network *after* usage, for example, at the end of each month. See also **prepaid phone plan**.

power (index) The number of times a base is multiplied by itself. In 2^5, the power is 5.

precision The smallest unit on the scale of a measuring device. For example the precision of a ruler may be one millimetre.

premium The regular cost of an insurance policy.

prepaid phone plan A plan in which you pay for using a phone network *before* usage, for example, paying for 60 days' usage and credit in advance. See also **postpaid phone plan**.

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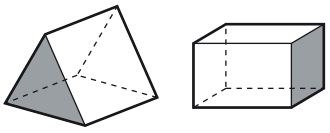
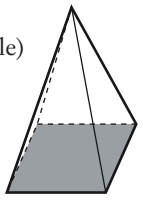
principal The original amount of money invested or borrowed.

prism A solid with flat faces and a uniform cross-section.

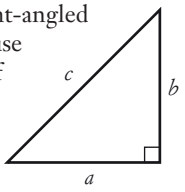


probability A measure of how likely an event E is to occur, written $P(\text{E})$. Its value ranges from 0 to 1.

pyramid A solid with a polygon as a base (for example a rectangle, or a triangle) and triangular side faces that meet at a point (called the apex).



Pythagoras' theorem In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the two shorter sides. $c^2 = a^2 + b^2$



Q

quantitative data (or numerical data) Data that involve numbers, such as heights, or the number of DVDs owned.

quartiles Values that divide a set of data into four equal parts when the scores are arranged in order. The 1st quartile (Q_1) is the lower quartile, the 2nd quartile (Q_2) is the median and the upper quartile (Q_3) is the upper quartile. See also **deciles** and **percentiles**.

quartiles (upper and lower) The upper quartile is the 3rd quartile (Q_3) that cuts off the top 25% of scores in a data set, and the lower quartile is the 1st quartile (Q_1) that cuts off the bottom 25% of scores.

R

radar chart A display of data on a circular grid (like a spider's web) that shows changes over a certain period or cycle.

radius (plural: radii) The length of the interval joining the centre of a circle to the circumference. The radius is half of the **diameter**.

random sample A sample for which every member of a population has an equal chance of selection.

range The difference between the highest score and the lowest score in a set of data.

rate A relationship between two quantities expressed in different units. For example a tree's growth rate can be measured in cm/year.

ratio A relationship between quantities that are measured in the same units. For example 200 cm : 40 cm = 5 : 1.

reaction distance The distance travelled during a driver's **reaction time**. See also **braking distance** and **stopping distance**.

reaction time The time between when a driver senses he needs to stop and when he applies the brakes.

reducing balance loan A loan in which interest is charged on the amount owing each period and not on the principal borrowed. See also **flat rate loan**.

registration See **vehicle registration**.

relative frequency The number of times an event or score occurs, written as a fraction of the total number of events or scores.

retainer A fixed amount paid to a salesperson before commission is added.

RHS Right-hand side (of an equation).

rise The vertical change in position between two points on a line; the number of units going up. Used with the **run** to calculate **gradient**.

royalties Income earned by recording artists and authors, based on the number of copies of their work that are sold.

run The horizontal change in position between two points on a line; the number of units going to the right. Used with the **rise** to calculate **gradient**.

S

salary Fixed earnings quoted as a yearly amount, but paid weekly, fortnightly or monthly. See also **wage**.

salvage value The current value of an item that is depreciating in value.

sample A group of items selected from a population.

sample size The number of items in a **sample**.

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sample space In probability, the set of all the possible outcomes of a situation or experiment.

scale drawing A drawing of an object, usually smaller, whose lengths are in the same ratio as the actual lengths of the object.

scale factor The ratio $\frac{\text{image length}}{\text{object length}}$ for a pair of similar figures. For example, a scale factor of 3 means every length of the image is 3 times the corresponding length of the object.

scientific notation A way of writing very large or very small numbers. For example $98\,000\,000 = 9.8 \times 10^7$.

second (angle size) A measure of angle size. $\frac{1}{60}$ of a minute. $1' = 60''$.

second (time) A measure of time. $\frac{1}{60}$ of a minute. 1 min = 60 s.

sector graph (or pie chart) A graph representing the parts of a whole population using a circle divided into proportionately sized sectors. Displays the same kind of data as a **divided bar graph**.

shareholder A person who owns shares in a company.

shares (or stock) A portion of a company owned by a shareholder.

significant figures Meaningful digits in a numeral that tell 'how many'. For example 98 000 000 has two significant figures: 9 and 8.

similar figures Figures that have the same shape but not necessarily the same size. Matching sides are in the same ratio.

simple interest (or flat interest) Interest earned or charged only on the original amount of money (principal) invested or borrowed. Differs from **compound interest**.

simultaneous equations Two or more equations that must be solved together so that the solution satisfies all equations. For example the simultaneous equations $y = 3x - 5$ and $y = 2x$ have the solution $x = 5$, $y = 10$.

sine A ratio in a right-angled triangle:

$$\sin\theta = \frac{\text{side opposite to } \theta}{\text{hypotenuse}}$$

where θ is an angle. See also **cosine** and **tangent**.

speed A rate that compares distance travelled with time taken. Speed is often measured in kilometres per hour (km/h) or metres per second (m/s).

$$\text{average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

stamp duty A tax paid to the Office of State Revenue, for example, when buying a vehicle, house or shares.

standard deviation (symbol σ_n) A statistical measure of the spread of a set of scores.

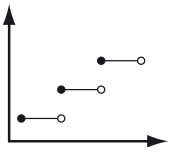
standard drink A drink that contains 10 grams of alcohol, such as a small glass of beer or wine.

statistics Collection, tabulation and displaying of data for the purpose of analysis.

stem-and-leaf plot A 'number graph' that lists all the data scores, in groups. This stem-and-leaf plot shows 12 test scores, from 42 to 82. Different from a **dot plot**.

Stem	Leaf
4	2 5
5	0 2 8
6	6 7
7	3 5 7 7
8	2

stepwise linear function A function whose graph is comprised of two or more horizontal line segments forming 'steps' (for example postage rates).



stockbroker An agent who buys and sells shares.

stopping distance The distance travelled between when a driver senses he needs to stop and when the vehicle stops completely, equal to the sum of the **reaction distance** and **braking distance**.

straight-line method of depreciation Method of depreciation in which an item's value decreases by the same amount each period. Differs from **declining balance method of depreciation**.

stratified sample A sample consisting of a percentage of items from each 'strata' or 'layer' of a population. For example a stratified sample from a population of 35% children and 65% adults should contain 35% children and 65% adults.

summary statistic A calculated value that represents or summarises a set of data (for example, the mean or the standard deviation).

superannuation A retirement fund for employees. Every payday, an employee and his/her employer invest a part of the employee's pay into a fund in order to provide income for the employee during retirement.

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survey To gather information for statistical purposes.

systematic sample A sample chosen by using a set pattern (for example choosing every 10th number in a phone book).

T

tangent A ratio in a right-angled triangle:

$$\tan\theta = \frac{\text{side opposite to } \theta}{\text{side adjacent to } \theta}$$

where θ is an angle. See also **cosine** and **sine**.

tax debt The amount by which the amount of **PAYG tax** already paid is below the amount of tax due. This is owed by the taxpayer to the Australian Taxation Office.

tax deduction See **allowable (tax) deduction**.

tax refund The amount by which the amount of **PAYG tax** already paid is above the amount of tax due; given back to the taxpayer.

tax return A form completed at the end of a financial year, to account for income earned, allowable deductions and tax already paid. Used to calculate a **tax refund** or **tax debt**.

taxable income The part of a person's income that is taxed, equal to yearly income minus allowable deductions.

terabyte (TB) 1024 gigabytes or $(1024)^4$ bytes.

term of a loan The amount of time it takes for a loan to be completely paid off.

third party property motor insurance Insurance that covers damage to another person's ('third party') vehicle or property in an accident in which you are at fault. It does not cover damage to your vehicle. See also **comprehensive motor insurance** or **CTP motor insurance**.

three-figure bearing See **true bearing**.

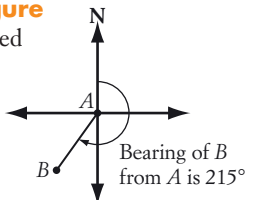
time-and-a-half Wages paid at 1.5 times the normal rate (for example for working on a Saturday).

traverse survey See **offset survey**.

tree diagram A diagram for listing all the possible outcomes of a multi-stage experiment such as tossing three coins together.

trigonometry The branch of mathematics dealing with the relationships between the sides and angles of triangles.

true bearing (or three-figure bearing) A direction measured as an angle clockwise from due north, from 000° to 360°.



U

unbiased sample A sample that is truly representative of the population.

unitary method A way of calculating a value by first finding one unit then multiplying.

upload Sending data from a computer or device to the Internet or larger computer/server.

upload speed The rate at which data is uploaded, measured in bits per second (bps or b/s) or kilobits per second (kbps or kb/s).

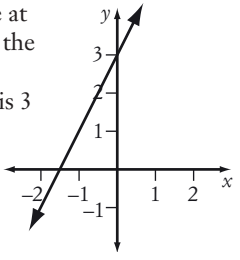
V

value-added tax (VAT) A tax on goods or services used in some countries; similar to the GST in Australia.

variable A pronumeral that can take a range of values (that is, it is not a **constant**).

vehicle registration The yearly process of paying for and obtaining permission for using a vehicle on public roads.

vertical intercept The value at which a straight line graph cuts the vertical axis. For example the vertical intercept of this graph is 3 (also called the ***y*-intercept**).



volume The amount of space occupied by a solid, measured in cubic units.

W

wage The amount earned by an employee for a set number of working hours, usually paid weekly. See also **overtime** and **salary**.

Y

***y*-intercept** See **vertical intercept**.

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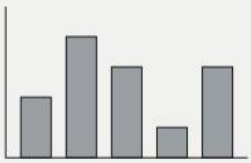
ANSWERS

Chapter 1

SkillCheck

1 **a** sector graph **b** stem-and-leaf plot
c line graph **d** divided bar graph

2



column graph

dot plot

3 **a** 20 **b** 5 **c** 500

4 **a** $\frac{3}{8}$ **b** $37\frac{1}{2}\%$

5 **a** $\frac{7}{20}$ **b** 35%

6 24.5

Exercise 1-01

1 **a** 13.4% **b** 5%
c about 3850 **d** about 36 650
e The percentage of people drinking at a risky level will be higher.

2 **a** $\frac{1}{4}$ **b** labour **c** \$2.03
d \$5700 **e** 54° **f** \$1060

3 **a** $\frac{1}{6}$ **b** 35% **c** suspicious
d 16

4 **a** male **b** 25–34 **c** 32%
d 1136 **e** 65–74 **f** decreases

5 B

6 **a** Yes **b** underweight **c** 17.5
d 26; lose weight **e** 16 to 25

7 **a** 1 mm **b** number of rain days
c 23 mm; 4 **d** October; 24.5 mm
e July; 6 **f** April (lowest rainfall)
g 9

8 A

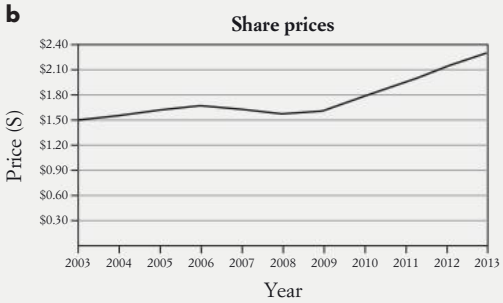
9 **a** share prices over 6 days **b** 25c
c \$4.55 **d** Tuesday
e It fell, then rose.
f Thursday; lowest price
g NCM Bank

10 **a** 17 **b** East; 25 on Thursday
c Monday; first working day
d Wednesday; 6 **e** Saturday; no industry

Exercise 1-02

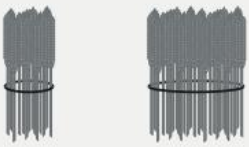
1 **a** The vertical axis starts at \$1.50 so increases in share price appear steeper than they actually are.

b



2 **a** Because the 2010 picture is double the length and double the width, it is actually four times the size of the 2008 picture instead of two times the size.

b



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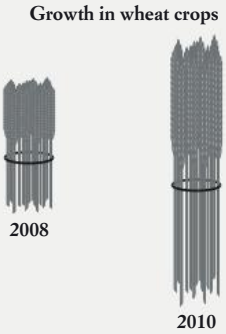

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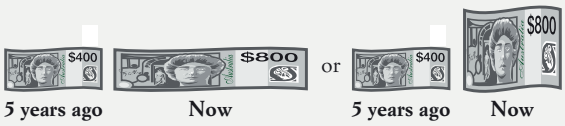
3 **a** a company's sales

b The new director has made sales go down.

c sales figures and months

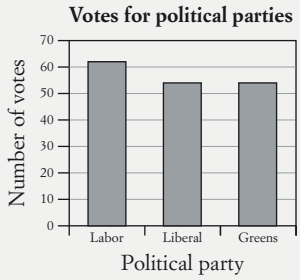
4 **a** The $800 note is double the length and double the width of the $400 note and this makes it four times the size of the $400 note instead of two times the size.

b



5 **a** By starting the scale on the vertical axis at 53.

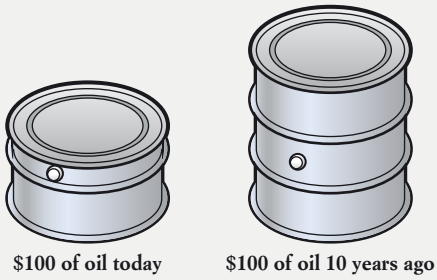
b



c The Labor votes are not overwhelmingly higher. They are just 8 more.

6 **a** The picture for '$100 of oil today' is actually about $\frac{1}{3}$ the size of the picture for '10 years ago'.

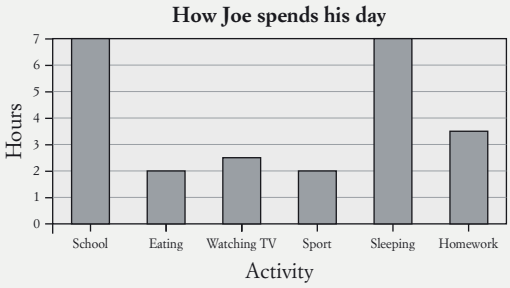
b



7 **a** 7 **b** The hours scale does not begin at 0.

c Big difference between school or sleeping and other activities.

d



Exercise 1-03

1 **a** C **b** Q **c** C **d** Q
e Q **f** C **g** C **h** C
I Q **j** Q **k** Q **l** C

2 Categorical data uses words/symbols, for example, type of pet. Quantitative data uses quantities/numbers, for example, person's height.

3 **a** N **b** O **c** O **d** N
e N **f** O **g** O **h** N

4 **a** C **b** C **c** C **d** C
e D **f** C **g** D **h** C
i C

5 **a** HSC subjects

b size of hot chips at a fast food store

c number of children in a family

d heights of boys in year 11

6 **a** quantitative **b** categorical
c continuous **d** nominal

7 B

Exercise 1-04

1 **a** C **b** F **c** D
d H **e** E **f** I
g A **h** G **i** B

2 **a** Commonwealth Scientific and Industrial Research Organisation

b Road Traffic Authority

c United Nations Children's Fund

d Government Insurance Office

e Bureau of Meteorology

f Australian Securities Exchange

g Australian Security Intelligence Organisation

h World Health Organisation

i National Rugby League

3–4 Teacher to check.

5 **a** stage 5 **b** stage 1
c stage 4 **d** stage 2

6 Teacher to check.

7 **a** bank account details, amount of transaction, time and date

b personal details, email address, interests and preferences

c your location, time and date of message

d amount of spending, your location

e your location and speed, time and date of driving

f type of book/video borrowed, time and date, your location

8 theft of identity; money stolen from accounts

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Exercise 1-05

1 **a** C **b** S **c** S
d C **e** S **f** S
g C **h** S

2 2016, 2021

3 Teacher to check reasons.
a systematic **b** random **c** stratified
d random **e** random **f** random
g random **h** random

4 138

5 188

6 B

7 **a** Not all teens read magazines.
b Only those diners interested/volunteering to complete the survey are sampled.
c Need to select more students and not only those who arrive early.
d Need to sample workers as well, not only management who may have a biased view.
e People at a premiere are not the usual moviegoers.
f It is restricted to one radio station.

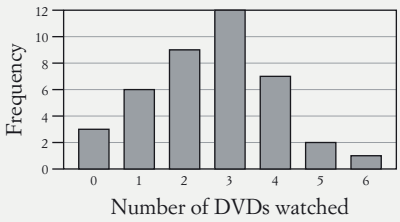
8 **a** systematic sample
b shoppers in 1 store only; only females or males

Exercise 1-06

1 **a**

Score	Frequency
0	3
1	6
2	9
3	12
4	7
5	2
6	1
	40

DVDs watched

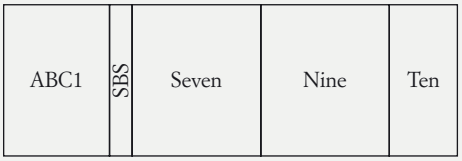


b data is discrete (not continuous)
c 3 **d** 6; 1 student **e** 7.5%

2 **a** shows parts of the whole amount
b **How Georgie spent $800 000**

c saved it **d** $\frac{9}{50}$ **e** 12%

3 **a** 14 970 **b** sector; divided bar; column
c



TV evening news watched in Sydney

d 23% **e** yes
f No, since the survey only included Sydney households.

4 **a** 72 **b** column graph; discrete data
c 9.7% **d** $\frac{11}{18}$

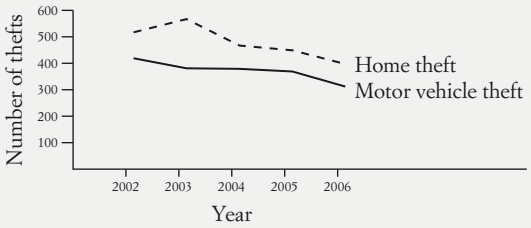
5 D

6 **a**

Country of origin of immigrants

b T **c** $\frac{4}{31}$ **d** 8% **e** India

7 **a** Number of motor vehicle and house thefts



b 2003 **c** 2006 **d** home theft
e Home thefts rose from 2002 to 2003, then declined steadily.
f Decreased; table and graph show a steady decline.

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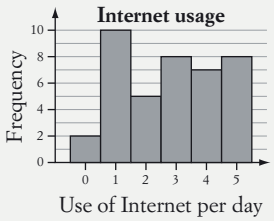
Exercise 1-07

1 **a**

Internet access/day	Number of people
0	2
1	10
2	5
3	8
4	7
5	8
	40

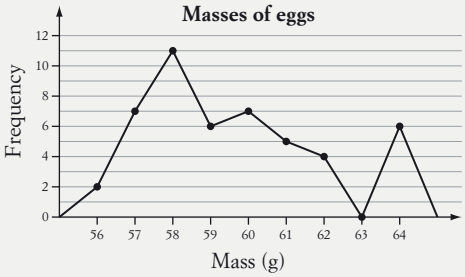
b



2 **a**

Mass (g)	Number of eggs
56	2
57	7
58	11
59	6
60	7
61	5
62	4
63	0
64	6
	48

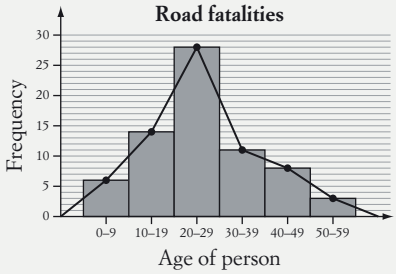
b



c Yes, since 26 our of 48 (about 54%) eggs are less than 60 g.

3 **a** 70

b



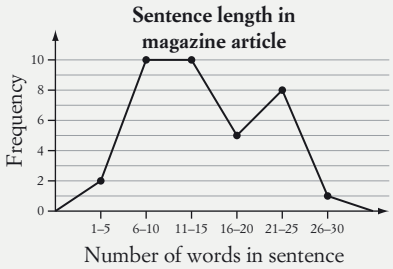
c Most road deaths occur in the (10–39) age group.

4 **a** discrete

b

Number of words in sentence	Class centre	Tally	Number of sentences
1–5	3	II	2
6–10	8	卌 卌	10
11–15	13	卌 卌	10
16–20	18	卌	5
21–25	23	卌 III	8
26–30	28	I	1
			36

c



d (6–10) and (11–15) are the two modal classes.

5 **a**

Class	Class centre	Tally	Frequency
15–19	17	卌 卌 卌 I	16
20–24	22	卌 卌 卌 卌	20
25–29	27	IIII	4
30–34	32	卌	5
35–39	37	I	1
40–44	42	II	2
45–49	47	II	2
			50

b 50

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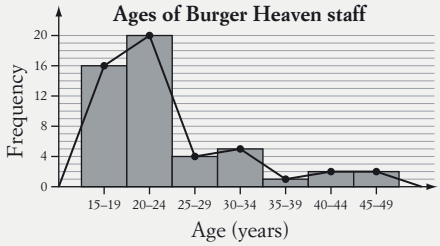
c



d The data is skewed towards younger workers; most of the workers are aged < 25 years.

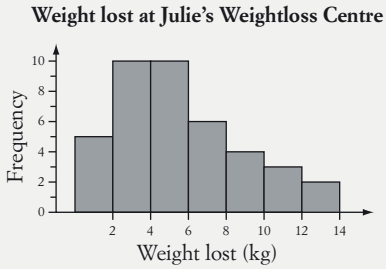
6 **a** none **b** 43 **c** 16.3%
d No, since the last column only tells those who made more than 30 calls.

7 **a** continuous

b

Class	Class centre	Frequency
0 – < 2	1	5
2 – < 4	3	10
4 – < 6	5	10
6 – < 8	7	6
8 – < 10	9	4
10 – < 12	11	3
12 – < 14	13	2
		40

c



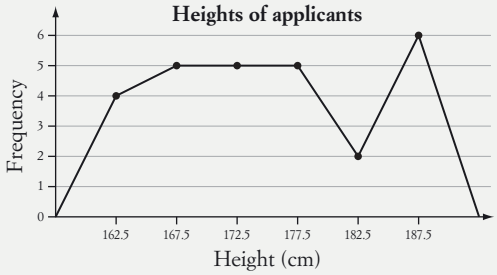
d Modal classes are (2 – < 4) and (4 – < 6).

e The program seems effective, because most people lost weight.

8 **a**

Height (cm)	Class centre	Tally	Number of applicants
160 – <165	162.5	IIII	4
165 – <170	167.5	~~IIII~~	5
170 – <175	172.5	~~IIII~~	5
175 – <180	177.5	~~IIII~~	5
180 – <185	182.5	II	2
185 – <190	187.5	~~IIII~~ I	6
			27

b

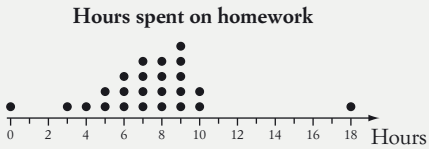


c Modal class interval is 185 – < 190

d (160 – < 165) **e** (180 – < 185)

Exercise 1-08

1 **a**



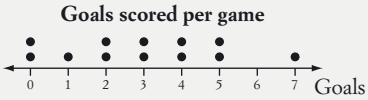
b 24 **c** 0 and 18

d cluster 6–9; gaps at 1–2, 11–17 hours

2 **a** 10 **b** 7

c 12; a student with big feet **d** $\frac{2}{5}$

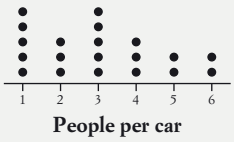
3 **a**



b 0

c Goals per game ranged from 0 to 7, so the team is not consistent.

4 **a**



b 20 **c** 6 **d** no real clusters or outliers

5 **a** 30 **b** 50% **c** around 6–10

d Mostly good results but 3 quite low scores.

6 **a**

Stem	Leaf
4	3 5 9
5	0 2 7 8
6	1 2 4 5 7 8
7	0 2 3 9
8	2 4 9

b 20 **c** 89 **d** 25%

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7 **a**

Stem	Leaf
2	8
3	
4	2 3 7
5	1 3 7 9
6	1 3 4 6 8 8
7	2 8 9 9
8	1 2 3 4
9	3 4
10	0 3 4
11	0 2 7

b 87% **c** 28; rainy day and few customers

8 **a** 36 **b** 91 seconds **c** about 28%

9 **a**

Stem	Leaf
5	2 5 6 8
6	0 0 3 3 4 4 4 5 8 8 9 9
7	1 2 2 4 4 6 6 8 9
8	0 1 1 3 3 4 4 5 9
9	0 0 3 4 7
10	
11	0

b 10 **c** 110, heart is beating quickly **d** 4

10 **a** $975 000 **b** 50 **c** $385 000

11 **a**

Stem	Leaf
11	6 8
12	0 1 4
13	6 9
14	3 5 6 6 6
15	4 6 6 7
16	3 5 8
17	2 4 7
18	1 7

b 24

c **i** 11.6 s **ii** 18.7 s

d 50.0% **e** $\frac{5}{24}$

Exercise 1-09

1 **a** 41°C **b** 5.00 a.m.; 37.5°C

c 8 **d** 7.00 a.m. and 9.00 a.m.

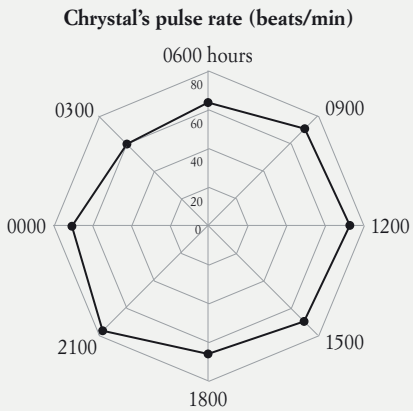
e 7.00 a.m., 5°C

f temperature dropped 2°C then increased 2°C

2 **a** B **b** April and December

c July **d** Miami **e** 15°C

3 **a**



b 60 at 3.00 a.m. **c** She was sleeping.

d from 6.00 p.m. to 9.00 p.m.

e No, only possible to know the actual readings.

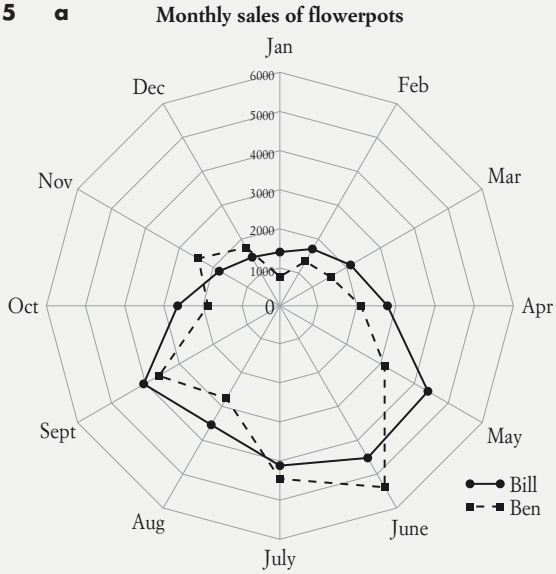
4 **a** about 70 mm; September

b about 70 mm; October

c Sydney **d** Melbourne

e March and June are about the same.

5 **a**



b 5 **c** June **d** 1400; May

e Bill; he sells more plants.

6 **a** **i** sector graph; divided bar graph

ii dot plots; stem-and-leaf plots

iii frequency histogram and polygon

iv line graph; radar chart

b stem-and-leaf plot

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c radar chart

d column graph, line graph, frequency histogram, frequency polygon

Sample HSC problem

a A

b Select some fans from each team's home ground.

c 83° **d** players' salaries

e Doesn't show exact values; it is difficult to compare similar values.

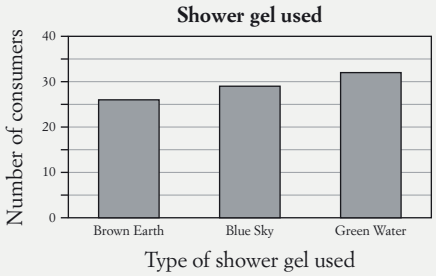
Revision

1 **a** 1477 **b** rectum

c 6 **d** T

e These cancers do not represent parts of a whole.

2 **a** The vertical scale should start at zero.



b sector graph, divided bar graph

3 **a** Q **b** C **c** C

d Q **e** Q **f** C

4 **b** N **c** O **f** N

5 **a** D **d** C **e** D

6 Teacher to check.

7 Stage 4: Summarising and displaying data.

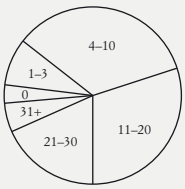
8 Systematic, for example, take every 10th bottle and test.

9 **a** The sample may not necessarily take students from each Year into account.

b 9

11 **a** 93

b Number of countries visited



c 12° **d** 34.4% **e** T

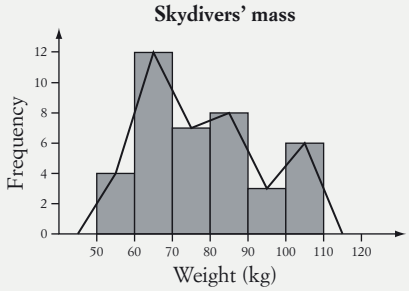
f The data is not continuous; the class intervals are not equal in size.

12 **a** continuous

b

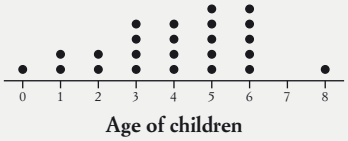
Mass (kg)	Number of skydivers
50 – <60	4
60 – <70	12
70 – <80	7
80 – <90	8
90 – <100	3
100 – <110	6
	40

c



d 60 – <70 **e** $\frac{1}{5}$

13 **a**



b 24 **c** $\frac{1}{4}$

d The 8-year-old is an outlier; scores are clustered around 3–6 years.

14 **a** 3 years **b** 29

c 12 **d** 41%

15 **a** November **b** about $3300

c May **d** June to July **e** 4

Chapter 2

SkillCheck

1 **a** $\frac{7}{9}$ **b** $\frac{7}{10}$ **c** $\frac{2}{3}$

2 **a** –27 **b** 6 **c** –8

d 25 **e** –4 **f** 2

3 **a** 15 **b** 3 **c** 16

d 6 **e** 243 **f** 18

4 **a** 0 **b** –3 **c** 10

d –128 **e** 9 **f** 9

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5 **a** $x - 3$ **b** $20 - a$ **c** -3

6 **a** $6x - 3$ **b** $28a + 35$ **c** $-12p + 4$

7 **a** $b = -5$ **b** $w = 17$ **c** $r = 9$

8 37.5

Exercise 2-01

1 A, D, E, G, H, I

2 D

3 **a** $y = 3x - 2$ **b** $y = 5x + 2$

c $y = -2x + 13$ **d** $y = -6x + 27$

e $y = x + 8$ **f** $y = 4x$

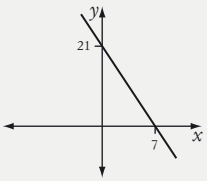
g $y = -3x + 10$ **h** $y = \frac{1}{2}x + 2$ or $y = 0.5x + 2$

4 **a** The terms are increasing by the same amount, 5, each time

b 26, 31, 36 **c** $y = 5x + 1$ **d** 101

5 **a** 6, 3, 0 **b** $y = -3x + 21$

c

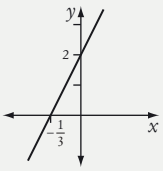


d -12

6 **a** $y = 11x$ **b** 176

7 **a** $y = 6x + 2$

b

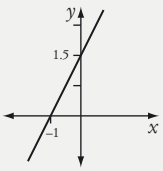


c 602

8 **a** $y = -6x + 39$ **b** -33

9 **a** $y = \frac{3}{2}x + \frac{3}{2}$ or $y = 1.5x + 1.5$

b



c 12 **d** 19th

10 **a** $y = -x + 13$ **b** -2 **c** 13th

Exercise 2-02

1 **a** $3x^2 + 5x$ **b** $7ab + a + b$ **c** $2m^2 + 5$

d $-4r + 2ar$ **e** $y^2 + 2y - 5$ **f** $a^2 + 2$

g $k^2 - 4k - 1$ **h** $-3de + 2d$ **i** $-6p$

j $4u - 15$ **k** $2t^2 + 3at$ **l** $18x$

2 **a** $16a^2km$ **b** $-24cd^2$ **c** $\frac{5}{2}t^3w^2$

d $25x^6$ **e** $6n^4$ **f** $14k^5p^2$

g $36mn$ **h** $\frac{3}{2}e^3$ **i** $4x^2y^4$

j $-20a^2b^2$ **k** $-n^6$ **l** $9k^2$

3 **a** 2 **b** $2x$ **c** -5

d $2ut$ **e** $\frac{3}{x^2}$ **f** $\frac{-3r}{q}$

g $\frac{3g}{5}$ **h** $\frac{1}{a}$ **i** $\frac{5m^2}{3n}$

j $\frac{-d}{5}$ **k** $\frac{1}{4}$ **l** $\frac{x^2}{4w}$

4 A

5 **a** $\frac{45y^2}{4}$ **b** $\frac{8ah}{35}$ **c** 2

d $\frac{5}{6}$ **e** $2n^2$ **f** $\frac{5n}{4}$

g $\frac{14a}{5h}$ **h** $\frac{16}{15} = 1\frac{1}{15}$ **i** d

6 **a** $-k^2 - 2k + 8$ **b** $\frac{1}{2}ay^4$ **c** $\frac{1}{4}$

d $-8fg$ **e** $-10c^2 + 13c$ **f** $-4z^2 + 5z + 8$

g $6a^3$ **h** $\frac{3c}{e}$ **i** $2m^2 + 5$

j $\frac{2n^2}{x}$ **k** $4x^2 + 2xy + 3x$ **l** $-3a^2$

Exercise 2-03

1 **a** $3a + 6$ **b** $15 - 10b$ **c** $-4a - 2$

d $-6b + 12$ **e** $3x^2 - 6x$ **f** $3p^2 - 3ap$

g $-8k - 16$ **h** $6t - 8t^2$ **i** $-d^2 + 5d$

j $7k - 5k^2$ **k** $-9b^2 + 9b$ **l** $14xy + 8y^2$

2 **a** $-24n + 6n^2$ **b** $5rx^2 + 10rx$ **c** $-2a^2 + 4$

d $5a^2b + 15b^2 - 35b$ **e** $-x^2 + 4x - 10$

f $3h^2 - 21eh - 12eh^2$ **g** $2y^2 + 3y - y^3$

h $d^3e - 2de + de^3$ **i** $9av^2 - 3v^2 + 6av$

3 **a** $3x + 14$ **b** $d - 22$ **c** $2r + 80$

d $7f + 9$ **e** $-9x - 32$ **f** $3x^2 + 27x$

g $2b^2 + 23b$ **h** $3w^2 - 29w$ **i** $9k + 12p$

j $4a$ **k** $vx + 3x$ **l** $-7t - w$

m $4e^2 + 3e$ **n** $2a - 18$ **o** $p^2 - q^2$

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4 **a** 2 and –2; opposites **b** –6 and 6
c yes; $-(b-a) = -b + a$
$= a - b$

Exercise 2-04

1 **a** 91 **b** $\frac{-8}{9}$ **c** 6
d 0 **e** 36 **f** 4
g 60 **h** 3 **i** 42
j –12 **k** 7 **l** 169
2 602.63 cm^3
3 **a** 100°C **b** 55°C **c** 28°C
4 **a** \$16.70 **b** \$74.73 **c** \$4008.00
5 **a** 360° **b** 720° **c** 1440°
6 **a** v **b** 16 m/s
7 **a** 17 **b** 81 **c** 301
8 12 770.1 cm^3
9 3.05 s
10 \$8775.56
11 13.5 m
12 **a** B **b** 20.8
c Increase her weight to around 65 kg.
13 **a** 25°C **b** 38°C **c** 15°C **d** 37°C
14 A
15 62 km
16 \$605.10
17 21 m
18 207.35 cm^2
19 B

Exercise 2-05

1 **a** $d = 6$ **b** $p = 2\frac{1}{2}$ **c** $u = 3\frac{1}{2}$
d $a = -3$ **e** $b = -\frac{1}{3}$ **f** $a = 4\frac{1}{2}$
g $m = -5$ **h** $h = 12$ **i** $r = 13$
j $x = -20$ **k** $y = -5$ **l** $n = -1$
m $y = 5$ **n** $c = 7\frac{1}{2}$ **o** $z = 60$
2 D
3 **a** $k = 11$ **b** $e = -2$ **c** $f = 6\frac{1}{2}$
d $x = 17$ **e** $w = 70$ **f** $d = 3$
g $n = 2\frac{3}{4}$ **h** $w = -3\frac{2}{5}$
i $p = 4$ **j** $z = -5\frac{1}{5}$
k $b = -3$ **l** $q = 2$
4 A

Exercise 2-06

1 **a** **i** 13 **ii** 31
b **i** 7 **ii** 18
2 12 cm
3 **a** 300 m **b** 60 s
4 2 m
5 21
6 8 cm
7 \$4387.15
8 **a** 2 min **b** 3.6 min
9 **a** 3.11 miles **b** 0.93 miles
10 **a** **i** 12 **ii** 26
b **i** 11 **ii** 19
11 \$4600
12 A
13 19.2 km
14 **a** 18 **b** 27
15 3 h
16 **a** **i** 16th **ii** 4th
b **i** 7 s **ii** 12 s
17 D
18 3 m/s^2

Sample HSC problem

a \$108 **b**

h	1	2	3
C	60	92	124

c yes; increasing by 32 each time
d $28 + 32h$ **e** $4\frac{1}{2}$ hours

Revision

1 **a** C **b** $y = 4x + 1$ **c** 57
2 **a** $t^2 + 9ut$ **b** $15k^3$ **c** $-6d^2$
d $32p^5$ **e** $-6h + 7$ **f** $8r^2$
g $-27d^6$ **h** $\frac{3}{5}$ **i** $\frac{-vw}{7}$
j $8x^2 + 8x$ **k** $\frac{3c}{b}$ or $3cb^{-1}$ **l** $\frac{-1}{a^2}$ or $-a^{-2}$
m $\frac{5}{6}$ **n** $\frac{2vy}{3}$ **o** $\frac{x}{12}$ **p** $\frac{5}{3a}$
3 **a** $10x - 20$ **b** $-3a - 21$ **c** $48t - 4y$
d $-9r^2 - 18w$ **e** $8m^2n - 8mn^2$ **f** $-8d^2 + 2d^3$
4 **a** $14x - 1$ **b** $2n^2 - n - 1$ **c** $24 - 10d$
d $-4p$ **e** $11u + 8$ **f** $8h^2 + 26h$
5 **a** 128 **b** –28 **c** –400 **d** 10
6 204.20 cm^2
7 \$700

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8 **a** $p = 5$ **b** $a = -1$ **c** $b = -9$
d $r = \frac{1}{2}$ **e** $n = 13\frac{3}{4}$ **f** $r = -18$
g $n = -2$ **h** $t = -12\frac{1}{2}$ **i** $g = -1$
9 110.5 m
10 80
11 28 years

Chapter 3

SkillCheck

1 **a** 7.81 **b** 8.49
c 2.55 **d** 151.26
2 **a** 25 000 **b** 1.4 **c** 2500
d 900 **e** 960 **f** 36
3 **a** 180 **b** 360
4 **a** 0.4245 **b** 0.7547 **c** 0.9563
d 1 **e** 0.0175 **f** 0.5
g 0.5 **h** 0.0998 **i** 0.8111
5 **a** $\frac{3}{5}$ **b** $1\frac{2}{3}$ **c** 3 : 7
d 3 : 2 **e** 6 : 5 **f** 15 : 4
g 10 : 1 **h** 100 : 1 **i** 1 : 200
6 **a** $h = 9.6$ or $9\frac{3}{5}$ **b** $k = 54$ **c** $p = 7$
7 **a** $x = 18$ **b** $y = 35$
c $k = 160$ **d** $a = 50$

Exercise 3-01

1 **a** 5.39 m **b** 7.30 m **c** 12.21 cm
d 18.73 cm **e** 5.66 cm **f** 7.62 m
2 **a** 9.2 cm **b** 11.0 cm **c** 30.0 cm
d 98.6 m **e** 57.8 cm **f** 33.0 m
3 4.20 m
4 461 m
5 5.66 m
6 120 nautical miles
7 1080 m
8 **a** 3.35 m **b** 6.02 m
9 19.4 m
10 21.4 cm
11 3.96 m
12 110.99 m
13 **a** 0.9 m **b** 198.0 m
14 1.3 m
15 2.54 m

Exercise 3-02

1 C
2 $\frac{2}{3}$
3 $\frac{4}{3}$
4 Teacher to check.
5 B
6 **a** E and G; $\frac{1}{2}$ **b** A and B; $\frac{7}{9}$ **c** H and J; $\frac{1}{2}$
7 19.2 m
8 1.9 m
9 15.6 m
10 **a** The Sun is lower. **b** 3.75 m
11 5.76 m
12 13.125 m
13 1.16 m
14 93.75 m
15 4.7 m
16 13.7 m

Exercise 3-03

1 **a** 1 : 100 000 **b** 1 : 5 **c** 1 : 25 000
d 1 : 10 **e** 250 : 1
2 B
3 2.75 cm
4 **a** 1 : 4 000 000
b **i** 84 km **ii** 244 km
iii 216 km **iv** 120 km
c Mittagong
d Katoomba and Penrith; Mittagong and Wollongong
5 **a** 4 m **b** 5 mm **c** 270 m
6 **a** Length = 9.2 cm, width = 5.5 cm
b 55 m **c** 92 m
7 A
8 **a** 20 km **b** 13.2 km **c** 11.6 km
9 **a** 30 m **b** 2.7 m **c** 22.5 m
10 **a** 2 m **b** 4.8 m by 2.8 m = 13.44 m^2
11 **a** 4.2 m by 4.8 m **b** 37.56 m^2
c 2 m **d** 8.4 m^2 **e** \$346.80

Exercise 3-04

1 **a** 2 m **b** 6 m
c 36 m^2 **d** 52.5 m^2 to paint; 6 L needed
2 **a** 6 m **b** 8 m
c 48 m^2 **d** \$4056

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3 **a** 4 **b** walk-in robe
c 1 **d** kitchen **e** 28 m²

4 **a** 7
b **i**

ii **iii**

iv **v**
c 15.12 m² **d** 168 **e** \$2710.48

5 **a** 1 : 100 **b** 13.5 m
c 12 windows **d** 2 doors **e** 8 m²
f 2400 mm (2.4 m) **g** \$700

6 **a** 3.6 m × 3.6 m
b **i** 3.6 m **ii** 10.8 m²
c 17 m² **d** 135 m² **e** \$207 900
f **i** 14 **ii** 196

Exercise 3-05

1 **a** **i** $\frac{m}{n}$ **ii** $\frac{p}{n}$ **iii** $\frac{m}{p}$
b **i** $\frac{4}{5}$ **ii** $\frac{3}{5}$ **iii** $\frac{4}{3}$
c **i** $\frac{24}{26} = \frac{12}{13}$ **ii** $\frac{10}{26} = \frac{5}{13}$
iii $\frac{24}{10} = \frac{12}{5}$

2 **a** **i** 0.9231 **ii** 0.3846 **iii** 2.4
b **i** 0.3846 **ii** 0.9231 **iii** 0.4167
c **i** 0.8 **ii** 0.6 **iii** 1.3333

3 **a** 5.6713 **b** 0.8526 **c** 0.2079
d 4.1653 **e** 0.6000 **f** 0.1547
g 0.3233 **h** 0.8151

4 **a** 25° **b** 73° **c** 60°
d 22° **e** 79°

5 **a** 70°19′ **b** 74°45′ **c** 30°

6 **a** Three sets of matching angles equal.
b $\tan 55° = \frac{\text{opposite}}{\text{adjacent}} \approx 1.43$ for all triangles

Exercise 3-06

1 **a** 3.72 **b** 143.97 **c** 3.57
d 7.81 **e** 2.59 **f** 2.99
g 6.5 **h** 51.55 **i** 8.00

2 **a** 20.71 **b** 7.07 **c** 13.39 **d** 4.85

3 **a** 7.63 **b** 5.33 **c** 11.01 **d** 28.16

4 54.7 m **5** 2.5 m **6** 30.0 cm

7 9.1 m **8** 95 m **9** 41 m
10 42.9 m **11** 11.6 m **12** 28.1 m
13 8.18 m
14 132.8 m **15** 49.6 m **16** 3418 km

Exercise 3-07

1 A

2 **a** 64° **b** 42° **c** 39° **d** 38°
e 84° **f** 19° **g** 34° **h** 41°

3 26° **4** 25°0′ **5** 87°43′
6 20° **7** 55° **8** 12°
9 10° **10** 50° **11** 7°36′

Exercise 3-08

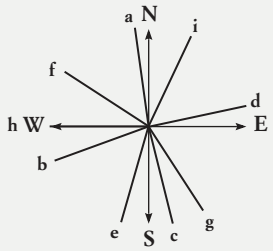
1 **a** 180° **b** 045° **c** 270°
d 225° **e** 315° **f** 090°

2 **a** W **b** NE **c** NW
d N **e** SE **f** N

3 **a** SW **b** SE **c** NE
d SE **e** NW **f** SW

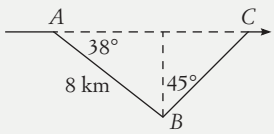
4 **a** 043° **b** 262° **c** 135°
d 113° **e** 345° **f** 083°
g 308° **h** 026° **i** 206°
j 233°40′ **k** 157°25′ **l** 322°45′

5 Rough diagram only: teacher to check.



6 A

7 **a**



b $\angle A = 38°$ $\angle B = 97°$ $\angle C = 45°$

Exercise 3-09

1 **a** 55 km **b** 95 km

2 **a** 24°12′ **b** 335°48′ **c** 2930 m

3 131° **4** 11.59 km **5** 31 km

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6 4.24 km **7** 96°51′ **8** 21 km
9 **a** 25° **b** 275°
10 9 km **11** 4 km
12 **a** 153 km **b** 172 km
13 **a** 179.98 km **b** 129.64 km
14 1002.80 km; 501.40 km/h
15 **a** 19 km **b** 020°

Exercise 3-10

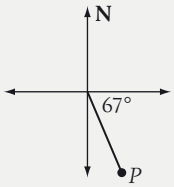
1 156 m **2** 133 m
3 **a** 1469 m **b** 1232 m
4 109 m **5** 14°
6 **a** 2.30 m **b** 2.27 m
7 **a** 2.05 m **b** 3.04 m **c** 5.43 m
8 25°
9 **a** 33.90 m **b** 8.72 m
10 **a** 20° **b** 470 m
11 **a** 289 m **b** 252 m
12 **a** 5.12 m **b** 51°20′
13 64°37′ **14** 26.8 m **15** 139 m
16 **a**, **b** 9.05 cm

Sample HSC problem

a 29°21′ **b** **i** 1.6875 m **ii** 1.6870 m
c **i** Similar triangles because we use the measurements given rather than the 'calculated value' of θ.

Revision

1 42 cm
2 **a** 182.06 mm **b** 17.63 cm
3 **a** $y = 6\frac{3}{4}$ **b** $x = 11\frac{2}{3}$, $y = 9\frac{3}{5}$
4 **a** 15 : 1 **b** 300 mm
5 12.86 m **6** 17 m²
7 **a** 1 : 200 **b** 35 m² **c** 2.56 m² **d** 62 m²
8 **a** **i** **ii** **iii**
b 41 m² **c** 216
9 **a** 0.63 **b** 9.87 **c** 0.61
10 **a** 68° **b** 47° **c** 52°
11 **a** 6.43 **b** 9.45 **c** 11.01
12 117 cm
13 **a** 68° **b** 68° **c** 40°
14 27°13′
15 **a** 12.6 m **b** 76°39′
16 255°
17



18 254.76 nautical miles
19 **a** 18.68 km **b** 106°
20 68° **21** 1071 m

Practice paper 1

1 D **2** C **3** C **4** B
5 B **6** B **7** $2p + 3$
8 63.49 **9** 0.71 **10** 240°
11 **a** A sample in which the proportions of items accurately represent the proportions of those items in the population.
b **i** 16 min or 17 min or 18 min **ii** 19
iii 21.1% **iv** 2 min
c 53 m **d** 15 m²
12 **a** Nominal data, because colours are not ordered.
b **i** 7.95 m **ii** 9.3 m
c 1 h 36 min
d **i** $100 billion **ii** increasing steadily
iii 2008
13 **a** **i** continuous
ii

Masses (kg)	Frequency
40–< 45	3
45–< 50	4
50–< 55	4
55–< 60	2
60–< 65	2
65–< 70	4
70–< 75	1

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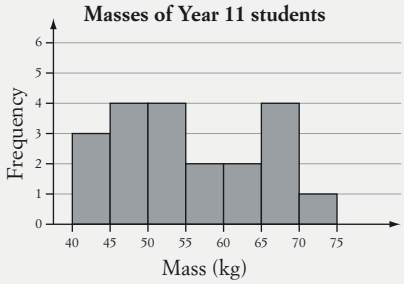
iii



iv 35%

b **i** 25 cm **ii** 18.4 m **iii** $594 **iv** 2

Chapter 4

SkillCheck

1 **a** $88 **b** $4.65 **c** $490
2 **a** $14 473.12 **b** $2932.16 **c** $1284.44
3 6% **4** 89 500

Exercise 4-01

1 $968.63 **2** $1076.25 **3** B **4** A
5 **a** $54 999.10 **b** $1057.68 **c** $27.83
6 $889.44
7 **a** $35 484.80 **b** $2957.07
8 $552.51 **9** $1501.50 **10** $969.90
11 **a** $5566.85 **b** $12 103.25 **c** $2783.42
12 **a** $6934.58 **b** $3189.54 **c** $318.95
13 **a** $47.56 **b** $3995.21 **c** $8686.25
14 $565.73 **15** $802.56
16 **a** $210.71 **b** $1053.55 **c** $4565.38
17 C
18 **a** $19.20 **b** $979.32 **c** $50 924.64
19 $1636.83 **20** $18.93

Exercise 4-02

1 A
2 **a** 168 **b** $27.30
3 D
4 **a** $30.10 **b** $6750
5 $7340
6 **a** $8485 **b** $10 175
7 $203.98 **8** $1353.24
9 **a** $49 177 500 **b** $737 662.50
10 **a** $146.19 **b** 821
11 $78.60 **12** $784 **13** $148 637.45
14 $465 **15** $1678 **16** 8.8% **17** 1382
18 **a** $119.70 **b** 213
19 60 **20** $530.85

Exercise 4-03

1 **a** roof tiler **b** nurse
c airline steward **d** steelworker
e scientist in Antarctica **f** scaffolder
g army officer **h** miner
2 $1333 **3** $915.92 **4** $671.52
5 **a** $1439.63 **b** $74 860.50
6 $3543.81 **7** $663.06 **8** $1012
9 **a** $888.96 **b** $522.06
10 $1902.38 **11** $170.81 **12** $929.60
13 $768.15 **14** $33 666.73 **15** $445.90
16 $575.89

Exercise 4-04

1 **a** $569.80 **b** $942.44 **c** $0 (< 65 years)
2 Because a couple live in the same house and the cost of living is shared.
3 **a** **i** $453.30 **ii** $11 785.80 **iii** $982.15
b $5317
c More. This is fair because they have no partner to share costs.
4 C
5 **a** $409 **b** $490.40 **c** $569.80
d **i** $12 750.40 **ii** $245.20
6 A
7 **a** $371.40 **b** $409
c $409 **d** $203.30

Exercise 4-05

1 C
2 **a** $611.95 **b** 25.0%
3 **a** $875.32 **b** $245.09 **c** $513.36
4 **a** $172.17 **b** 27.10% **c** 4.11%
5 **a** $175.51 **b** $51.06
c $248.51 **d** $389.69
6 **a** $616 **b** $61.60 **c** $414.18
7 **a** $2079.38 **b** $1599.38 **c** 14.8%
8 **a** $755.82 **b** $188.96 **c** $549.46
9 **a** $9022.92; $3504.88 **b** 48% **c** 38.8%

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10

	Pay	Deductions	
	$751.80	Tax (25%)	$230.91
	$128.88	Superannuation	$60.65
	$42.96	Union	$18.15
Gross wages	$923.64	Health fund	$24.60
Total deductions	$334.51		
Net wage	$589.13		

11

Gross pay	$3335.61
Tax (31%)	$1034.04
Other deductions	$152.21
Net pay	$2149.36

Total deductions = $151.21

Exercise 4-06

1 **a** expense **b** income **c** expense
d expense **e** income **f** expense
g expense **h** expense **i** income
j expense **k** income **l** expense
m income **n** income **o** expense

2 **a** $351.30 **b** $54
c Spend less on clothes, look for grocery specials.

3 **a** $111.40 **b** yes **c** yes

4 **a** Teacher to check. **b** $435.28

5 **a** $551.40 **b** $279.64 **c** $271.76

Exercise 4-07

1 **a** $2458.60 **b** $35 948.21
c $0 **d** $0

2 **a** $89 451 **b** $21 043.87 **c** $1341.77

3 **a** $86 124 **b** $19 812.88 **c** $1291.86

4 **a** $51 168 **b** $8176.60 **c** $767.52

5 **a** $12 301.25 **b** $34 387.90 **c** $73 575

Exercise 4-08

1 **a** $16 363.36 **b** $36 398.44

2 **a** $24 707.80 **b** $906.42

3 **a** $194 026 **b** $60 858.70
c $2910.39 **d** refund of $8386.91

4 **a** $98 934.55 **b** $4179.59
c refund of $1453.86

5 **a** $28 520 **b** $1960.80
c $427.80 **d** refund of $4706.80

6 D **7** A

8 **a** debt of $11 895.09
b debt of $11 741.58
c debt of $72 480

Exercise 4-09

1 C **2** €1815 **3** 3985 baht
4 A **5** $16.82 **6** £108
7 46.50 rand **8** 19%
9 **a** €8.30 **b** 19%
10 **a** $30.80 **b** $35.42
11 **a** 14 290 won **b** 10%
12 **a** €44 **b** 20%
13 18% **14** €238.97 **15** $132.80
16 $31.13 **17** 4130 baht **18** 6.62%
19 $48; $50.40 **20** 19%

Exercise 4-10

1 **a** $3.80 **b** $10.80
c $13.50 **d** $2.40

2 **a** $47 **b** $20
c $11 **d** $75

3 **a** $2600 **b** $0
c $9600 **d** $17 500

4 **a** $63 000 **b** $31 000
c $69 000 **d** $78 000

5 **a**

b **i** $6.65 **ii** $93 **iii** $48 + $3.40 = $51.40

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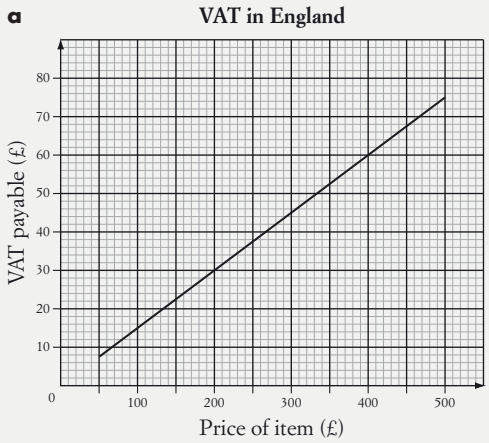
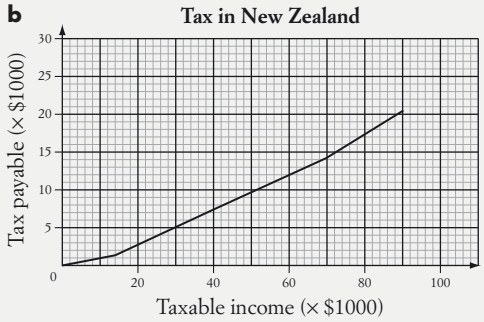
6 **a**



b **i** £13 **ii** £90 **iii** £30 + £64 = £384

7 **a** **i** $1470 **ii** $7420 **iii** $14 020 **iv** $20 620

b



c **i** $9 000 **ii** $18 000

d **i** $36 000 **ii** $78 000

Sample HSC problem

a $12 365.60 **b** $41 048.80 **c** $40 062

d income tax $4567.15, Medicare levy $600.93

e tax refund of $7197.52

Revision

1 **a** $32 819.80 **b** $40 248 **c** $45 432

2 **a** $2656.40 **b** $3775.51 **c** $21 980.33

3 $578.25

4 **a** $102.20 **b** 107

5 Maggie earns 2c less, not 2c more.

6 $11 657.52 **7** $1371.93 **8** $1159.47

9 **a** $244.40 **b** $453.30 **c** $486.60

10 **a** $30 893.20 **b** $1188.20

c $3632.12 **d** $2386.52

11 **a** $1120

c cut down on takeaway meals, newspapers and magazines, for example

12 **a** $105 662 **b** $27 041.94

c $1584.93 **d** refund of $9989.86

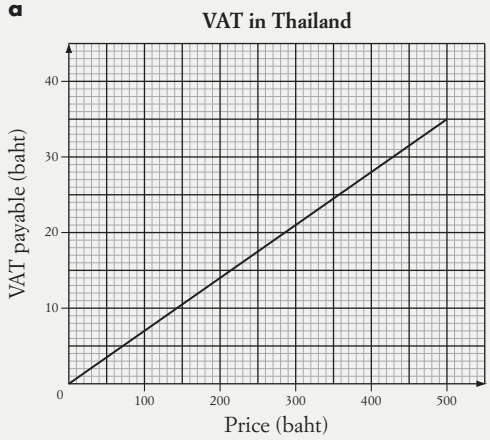
13 $113.27

14 **a** 189.3 pesos **b** 44.65 new lei

c 3773 rand

15 **a** 173 913.04 pesos **b** 142 833.33 forint

16 **a**



b **i** 17.5 baht **ii** 400 baht

iii 230 + 16 = 246 baht

17 **a** $15 000 **b** $69 000

Chapter 5

SkillCheck

1 **a** $\frac{7}{25}$ **b** $\frac{1}{11}$ **c** $\frac{1}{4}$ **d** $\frac{2}{3}$

2 **a** $\frac{27}{40}$ **b** $\frac{3}{25}$ **c** $\frac{7}{8}$ **d** $\frac{5}{9}$

3 **a** 0.85 **b** 0.25 **c** 0.625 **d** $0.2\dot{6}$

4 **a** 40% **b** 62.5% **c** $7\frac{9}{13}$% **d** $33\frac{1}{3}$%

5 **a** 52 **b** 6 **c** 12 **d** 26

e 4 **f** 5

6 **a** $\frac{1}{5}$ **b** 30% **c** $\frac{3}{5}$

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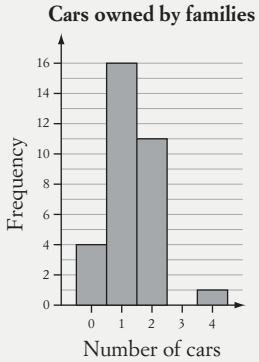
7 **a**



b $\frac{1}{8}$ **c** 0% **d** 87.5% **e** 1

8 **a** Teacher to check. **b** AC **c** VU
d AC **e** VU **f** Teacher to check.
g VU **h** Teacher to check. **i** U
j L

Exercise 5-01

1 **a** {Mon, Tues, Wed, Thurs, Fri, Sat, Sun}; 7
b {M, A, T, H, E, I, C, S}; 8
c {G, PG, M, MA, R}; 5
d {boy, girl}; 2 **e** {pass, fail}; 2

2 **a** 21 **b** 12 **c** 10 000 **d** 7
e 4

3 **a** 26 **b** 4 **c** 20 **d** 2
e 12 **f** 1

4 **a** England wins, Brazil wins, draw **b** No

5 Each event (fine, rain) is not equally likely.

6 **a** $\frac{2}{3}$ **b** $\frac{1}{3}$ **c** 0 **d** 1

7 **a** $\frac{7}{15}$ **b** $\frac{8}{15}$ **c** $\frac{1}{15}$ **d** $\frac{7}{15}$

8 **a** $\frac{1}{5}$ **b** $\frac{3}{10}$ **c** $\frac{1}{5}$ **d** $\frac{1}{2}$

9 $\frac{1}{3}$

10 **a** $33\frac{1}{3}$% **b** $4\frac{1}{6}$% **c** $79\frac{1}{6}$% **d** 75%
e 25%

11 D

12 **a** $\frac{1}{3}$ **b** $\frac{1}{6}$ **c** $\frac{1}{2}$ **d** $\frac{5}{6}$

13 **a** $\frac{1}{2}$ **b** $\frac{3}{26}$ **c** $\frac{1}{13}$ **d** $\frac{9}{13}$
e $\frac{1}{26}$ **f** $\frac{1}{52}$ **g** $\frac{1}{26}$ **h** $\frac{1}{4}$

14 $\frac{1}{150}$ **15** $0.1\dot{6}$

16 Each of the 16 teams is not equally likely to win the competition, the levels of ability/success are different.

17 **a** 60% **b** 20% **c** 30%

18 **a** $\frac{2}{7}$ **b** $\frac{4}{7}$ **c** $\frac{1}{7}$
d $\frac{5}{7}$ **e** $\frac{5}{7}$ **f** 0

Exercise 5-02

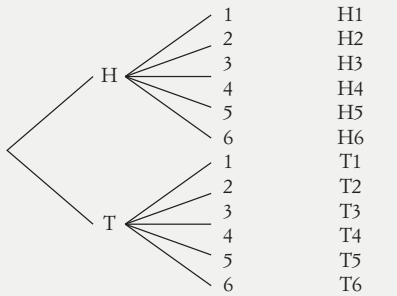
1 **a** D **b** C **c** E **d** D
e A **f** B **g** B **h** F
i C

2 **a** **i** 2 **ii** 6
b **i** {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6} where H = head, T = tail

ii

Die \ Coin	H	T
1	H1	T1
2	H2	T2
3	H3	T3
4	H4	T4
5	H5	T5
6	H6	T6

iii



c 12 **d** **i** $\frac{1}{4}$ **ii** $\frac{1}{6}$ **iii** $\frac{5}{12}$

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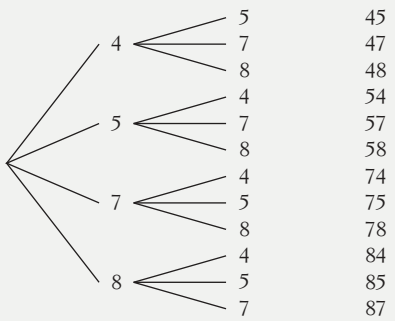
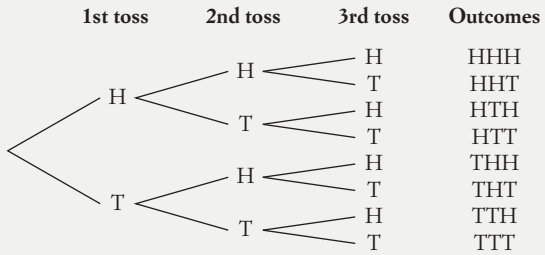
3 **a**



b 12 **c** **i** 25% **ii** 50% **iii** 50%
iv 25%

4 **a** **i** {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

ii



b 8

c **i** 0.375 **ii** 0.5 **iii** 0.125 **iv** 0.5

5 **a** 2 **b** 4 **c** 16 **d** 1024

6 **a**

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b 36

c **i** $\frac{1}{12}$ **ii** $\frac{5}{36}$ **iii** $\frac{1}{12}$

iv 0 **v** $\frac{1}{2}$ **vi** $\frac{2}{9}$

d **i** $\frac{1}{36}$ **ii** $\frac{1}{6}$

e **i** 7 **ii** 2, 12

7 **a**

2nd course \ 1st course	C	R	W	Y
B	CB	RB	WB	YB
P	CP	RP	WP	YP
S	CS	RS	WS	YS

b 12 **c** **i** $\frac{1}{2}$ **ii** $\frac{1}{6}$ **iii** $\frac{1}{2}$

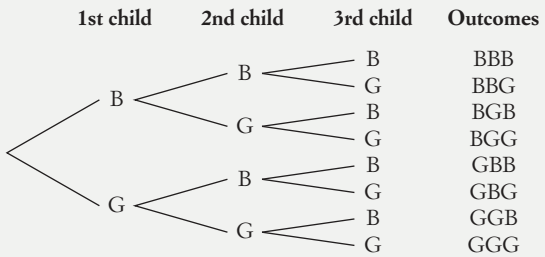
iv $\frac{1}{12}$

8 **a**

Secretary \ Chairperson	H	R	A	K	T	
H	–	RH	AH	KH	TH	H = Harry
R	HR	–	AR	KR	TR	R = Rita
A	HA	RA	–	KA	TA	A = Alison
K	HK	RK	AK	–	TK	K = Kirstie
T	HT	RT	AT	KT	–	T = Thuc

b **i** 0.05 **ii** 0.4 **iii** 0.2 **iv** 0.1

9 **a**

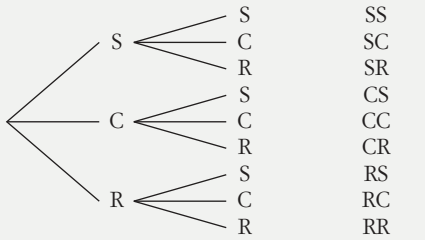


b A

c **i** $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{7}{8}$

iv $\frac{1}{2}$ **v** $\frac{1}{4}$ **vi** $\frac{1}{2}$

10 **a**



b A **c** 9

d **i** $\frac{1}{9}$ **ii** $\frac{1}{3}$ **iii** $\frac{8}{9}$ **iv** $\frac{5}{9}$ **v** $\frac{2}{9}$

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Exercise 5-03

1 **a** 72 **b** **i** $\frac{1}{72}$ **ii** $\frac{1}{18}$

2 B

3 **a** 8, {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

b **i** 25% **ii** 37.5% **iii** 87.5%

iv 12.5% **v** 50% **vi** 50%

4 676

5 **a** 80 **b** **i** 20 **ii** 64 **iii** 36

6 D

7 **a** 1000 **b** 5-digit postcodes

8 4096

9 **a** 90 **b** **i** 30 **ii** 18 **iii** 9

10 160 **11** 256 **12** 1 000 000

13 **a** 2 176 782 336

b **i** $\frac{1}{36}$ **ii** $\frac{1}{36}$ **iii** $\frac{65}{324}$

14 C

15 **a** 12 {25, 26, 29, 52, 56, 59, 62, 65, 69, 92, 95, 96}

b 6 **c** 0.25 **d** 24 **e** 16

16 72 **17** 120

18 **a** **i** 2 **ii** 4 **iii** 8 **iv** 16

b 30

Exercise 5-04

1 **a** getting a head

b selecting a diamonds, clubs or spades card

c passing a driving test

d having 3 or fewer children

e selecting a white sock

f the weather today not being rainy

g not winning the race

h a traffic light showing red or amber

2 **a** $\frac{5}{6}$ **b** $\frac{2}{3}$ **c** $\frac{2}{3}$

3 {HH, HT, TH, TT}

a $\frac{1}{4}$ **b** $\frac{3}{4}$ **c** $\frac{1}{2}$

d $\frac{1}{2}$ **e** $\frac{1}{4}$ **f** $\frac{3}{4}$

4 {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

a $\frac{3}{8}$ **b** $\frac{5}{8}$ **c** $\frac{3}{8}$

d $\frac{5}{8}$ **e** $\frac{1}{8}$ **f** $\frac{7}{8}$

5 **a** 0.55 **b** 0.7 **c** 0.45

d 0.9 **e** 0.8 **f** 0.7

6 A

7 **a** $\frac{10}{11}$ **b** $\frac{9}{11}$ **c** $\frac{7}{11}$ **d** $\frac{8}{11}$

8 $\frac{5}{7}$

9 **a** $\frac{5}{9}$ **b** $\frac{1}{3}$ **c** $\frac{7}{9}$ **d** $\frac{2}{3}$

e 1 **f** $\frac{2}{9}$

10 B **11** 77%

12 **a** $\frac{12}{13}$ **b** $\frac{10}{13}$ **c** $\frac{3}{4}$

d $\frac{1}{2}$ **e** $\frac{25}{26}$ **f** $\frac{11}{13}$

13 **a** 0.384 **b** 0.616 **c** 0.988

d 0.396 **e** 0.604 **f** 0

14 $\frac{364}{365}$ **15** 80%

16 **a** draw **b** 7%

17 $\frac{7}{20}$

18 **a** $\frac{8}{9}$ **b** $\frac{2}{3}$

Exercise 5-05

1 **a** 150

b **i** $\frac{28}{75}$ **ii** $\frac{21}{50}$ **iii** $\frac{17}{150}$

2 **a** no **b** yes, biased towards 6

c 27.5% **d** 28

3 **a** 1

b **i** 0.35 **ii** 0.125 **iii** 0.875

iv 0.625 **v** 0.275 **vi** 0.9

c 11

4 B

5 **a** no **b** **i** $\frac{1}{3}$ **ii** $\frac{23}{30}$

6 **a** 50.65% **b** female **c** 11 055 151

7–11 Teacher to check.

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Exercise 5-06

1 **a** {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

b **i** $37\frac{1}{2}\%$ **ii** 50%

c **i** 33 **ii** 44

d **i** 34.09% **ii** 53.41%

e They are close.

f **i** 75 **ii** 100

g **i** 40% **ii** 48.5%

h yes

2 **a** **i, iii** Teacher to check.

ii 0.167 **iv** 0.5

b 25

3 **a**

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b **i** 0.0833 **ii** 0.0556 **iii** 0.2222

d **i** 10 **ii** 7 **iii** 27

4 **a** **i** 0.231 **ii** 0.256 **b** A

5 **a** 12

b **i** 0.083 **ii** 0.104 **iii** 0.25

iv 0.25 **v** 0.417 **vi** 0.438

c **i** 4 **ii** 12 **iii** 20

6 Each outcome has an equal chance.

7 **a** **i** 46% **ii** 39.3% **iii** 42%

iv 41.9%

b Yes, the coin seems to be coming up heads around 42% of the time instead of 50%.

Sample HSC problem

a

–	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

b 1 **c** **i** $\frac{2}{9}$ **ii** $\frac{1}{2}$ **iii** $\frac{17}{18}$

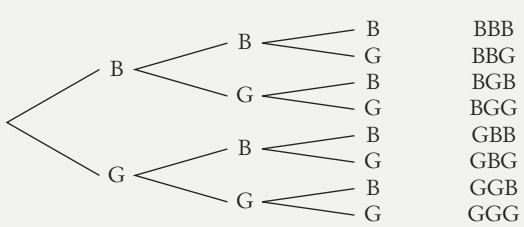
Revision

1 **a** $\frac{1}{26}$ **b** $\frac{3}{13}$ **c** $\frac{11}{13}$

2 **a** 25% **b** $16\frac{2}{3}\%$ **c** $58\frac{1}{3}\%$ **d** $66\frac{2}{3}\%$

3 Being married and being single are not equally likely outcomes.

4 **a**



b **i** $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{1}{2}$ **iv** $\frac{7}{8}$ **v** $\frac{1}{2}$ **vi** $\frac{1}{2}$

5 **a** $\frac{1}{6}$ **b** $\frac{5}{36}$

6 128

7 C

8 **a** 10 000 **b** **i** 0.01% **ii** 0.1% **iii** 6.25%

9 32%

10 D

11 **a** $\frac{3}{14}$ **b** $\frac{7}{20}$ **c** $\frac{61}{70}$

12 **a** 96.25% **b** 188

13 **a** Teacher to check. **b** Teacher to check.

c $\frac{1}{6}$ **d** 8

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Chapter 6

SkillCheck

1 **a** 20 830 **b** 97.02
c 65 900 **d** 0.725
e 0.0104 **f** 0.735
2 **a** 10 000 000 **b** 10 000
c 0.01
3 **a** 16 cm^2 **b** 90 m^2
c 78.54 m^2 **d** 60 cm^2
4 **a** 180 cm^2 **b** 41
5 **a** \$1.28 **b** \$2.49
6 **a** 2.1 m **b** 2100 mm
c 0.0021 km
7 **a** 0.0081 kL **b** 8100 mL

Exercise 6-01

1 **a** 63 mm **b** 436 cm
c 7.2 m **d** 0.285 kg
e 6900 µm **f** 58 L
g 5320 g **h** 3.4 t
i 4.72 kL **j** 0.006 kg
k 27 000 s **l** 940 000 cm
2 A
3 **a** 1690 mm **b** 1.69 m
4 57 500 g **5** 1.5 km
6 **a** 97 min **b** 5820 s **c** 1.6 h
7 59 000 L
8 **a** kg **b** cm **c** m **d** km
e L **f** mg **g** cm **h** kL
i t **j** mm or µm
9 **a** 0.102 kg **b** 0.052 m
10 4.9 Mt
11 **a** 18.7 km **b** 1 h 48 min
12 8.56 kL
13 **a** B **b** C **c** C
d C **e** A **f** A
14 about 1420 **15** 86 400

Exercise 6-02

1 **a** **i** 1 m **ii** 4 m **iii** ±0.5 m
b **i** 0.1 cm or 1 mm
ii 1.8 cm or 18 mm
iii ±0.5 mm
c **i** 5 km/h **ii** 50 km/h
iii ±2.5 km/h
d **i** 0.5°C **ii** 38°C
iii ±0.25°C
e **i** 5 m/s **ii** 45 m/s
iii ±2.5 m/s
f **i** 500 rpm **ii** 7000 rpm
iii ±250 rpm
2 C **3** A, D **4** C, D, F
5 **a** ±0.5 mL **b** ±0.5 mm
c ±0.05 g **d** ±0.05 s
e ±0.5 g **f** ±0.5 m
g ±0.5 mL **h** ±5 g
i ±0.05 cm **j** ±50 g
6 2956 mm
7 **a** ±0.5 cm **b** 20.5 cm and 21.5 cm
c ±2.38%
8 ±2.17% **9** ±0.0039%
10 **a** ±0.33% **b** ±0.63% **c** ±3.33%
15.2 mg is the most accurate because it has the smallest percentage error.
11 We don't know the absolute error; the figure could be to the nearest whole number, ten or fifty.
12 38 L is written correct to the nearest 1 L, 38.0 L is correct to 0.1 L. So 38.0 L is more accurate.

Exercise 6-03

1 B
2 **a** 3800 **b** 2100
c 0.0061 **d** 250 000
e 15 000 000 **f** 0.000 47
3 **a** 130 **b** 4980
c 0.0106 **d** 1 360 000
e 25 400 000 **f** 0.000 680
4 **a** 3000 **b** 3 **c** 10 000
d 0.005 **e** 20 **f** 0.7
5 **a** Y **b** N **c** Y
d N **e** Y **f** Y
6 b, c, d, e, f
7 **a** 0.67 **b** 0.0055 **c** 1.3
d 0.087 **e** 790 000 **f** 7.3
g 24 **h** 2.4 **i** 34

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8 7 250 000

9 **a** 352 000 kg **b** 4190 m **c** 67.1 km/h
d 14.8 mL **e** 150 000 000 km

Exercise 6-04

1 **a** 4.213×10^{7} **b** 1.81×10^{-2}
c 3.4×10^{3} **d** 2×10^{4}
e 3.5×10^{-3} **f** 2×10^{-4}
g 3.3×10^{-1} **h** 4×10^{-3}
i 2.3×10^{2} **j** 7.23×10^{-5}
k 6.1×10^{8} **l** 8×10^{-8}

2 **a** 5.3×10^{7} **b** 1.5×10^{5}
c 2.5×10^{3} **d** 4.6×10^{-4}
e 2.7×10^{-3} **f** 1.0×10^{-1}
g 3.3×10^{-5} **h** 4.4×10^{-1}
i 6.5×10^{0}

3 B

4 **a** 740 000 **b** 0.312
c 1850 **d** 0.000 66
e 0.002 54 **f** 475 100 000
g 0.098 **h** 300
i 0.054 97 **j** 12 160
k 0.802 **l** 6309

5 **a** 1.37×10^{10} **b** 1×10^{-6}

6 **a** 0.000 000 03 **b** 9 461 000 000
c 0.000 002 **d** 152 600 000

7 7.03×10^{9}

8 **a** 2.144×10^{7} **b** 3.2×10^{5}
c 3.5×10^{4} **d** 2.304×10^{-5}
e $5.314\,41 \times 10^{14}$ **f** 3.76×10^{1}
g 1.26×10^{4} **h** 2.3×10^{5}

9 **a** 3.0×10^{4} **b** -3.6×10^{-3}
c 4.1×10^{12} **d** 3.3×10^{-8}
e 5.9×10^{4} **f** 3.4×10^{-2}
g 9.2×10^{2} **h** 1.2×10^{-4}

10 **a** 0.0048 **b** 43 680 000 000
c 0.0094 **d** 187 690 000
e 13 640 000 **f** 19 063 000
g 0.000 052 **h** 350 000

11 Answers will vary: teacher to check.

Exercise 6-05

1 **a** 1 : 3 **b** 2 : 1 : 5 **c** 1 : 1
d 7 : 5 **e** 1 : 4 **f** 4 : 12 : 3
g 1 : 10 **h** 6 : 5 **i** 3 : 10
j 1 : 8 **k** 3 : 14 **l** 84 : 19
m 3 : 20 **n** 1 : 500 **o** 1 : 5
p 2 : 3 : 52 **q** 200 : 1 **r** 7 : 48

2 10 : 2 : 1 **3** C **4** D **5** 3 : 26

6 **a** 1 : 3 **b** 1 : 3 **c** 3 : 2

7 1 : 4

8 **a** 15 cm; 45 cm **b** 135 cm

9 1150

10 **a** 550 g **b** 2250 g

11 80 teachers **12** C

13 24 **14** 46 600 books

15 **a** 12 shovelfuls **b** 1467 kg

16 **a** 200 g **b** 1000 g or 1 kg
c 2000 g or 2 kg

17 **a** 400 mL or 0.4 L **b** 3.9 L

18 $336 000

Exercise 6-06

1 B

2 **a** $14 000; $17 500 **b** 3250 kg; 500 kg
c 15 692 m^2; 18 308 m^2 **d** $\frac{1}{2}$ cup; 2 cup
e 3000 L; 4500 L
f 62 500 m^2; 37 500 m^2 or 6.25 ha; 3.75 ha

3 470 girls **4** 318 kg; 182 kg

5 Tan $33 500; Brad $20 100

6 $75 000; $50 000; $50 000; $25 000

7 0.4 L of oil, 19.6 L of petrol

8 50 kg of cement; 200 kg of sand; 300 kg of aggregate

9 2400 mL of juice; 2400 mL of lemonade; 1200 mL of ginger ale

10 Rose $225; Lisa $150; Ally $75

11 280 mL **12** 842 mL

13 **a** $216 **b** $540

14 **a** 6 : 1 : 19 **b** 808 g
c 520 g **d** 370.5 g

Exercise 6-07

1 **a** $1/min **b** $/hour **c** words/min
d $/g **e** $/kg **f** m/s

2 **a** population density
b cost of newspaper classified ad
c fuel consumption

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d internet download speed
e birth rate **f** cost of carpeting

3 2.4 cm/week **4** 5.25 runs/over
5 2750 persons/year **6** \$28.44 kg
7 0.325 MB/s **8** 3.02 persons/km^2
9 **a** 336 km **b** 5 h
10 23 min
11 **a** 0.05 mg/g **b** 5 mg
12 760 min or 12 h 40 min **13** 147
14 **a** US\$0.63 **b** A\$316.12
15 A **16** 74.88 L **17** 111 466
18 **a** 4.1 kg **b** 6 times
19 **a** 600 mL/L **b** 60% v/v
20 1.8 mL **21** 4.8 mL **22** 130 mL
23 **a** 2.5% v/v **b** 60 mL
24 0.15 mL
25 **a** 2 mg/mL **b** 0.2%
26 **a** 5 g **b** 6.25 mL

Exercise 6-08

1 **a** \$3851/month **b** 1.25 km/min
c 120 L/day **d** 180 m/min
e 48 cm/h **f** 1.3 words/s
g 0.082 L/km **h** 105 000 mL/min
2 **a** \$14.4/h **b** 28.8 km/h
c 54 L/h **d** 18 c/min
e $16\frac{2}{3}$ m/s **f** 2.08 kg/min
3 161 km/h **4** C **5** 1800 L/h
6 37.0 km/h **7** 25.54 km/h
8 **a** 464.3 km/h
b **i** 7738 m **ii** 129 m
9 104.2 m
10 **a** 15 m **b** 21 m **c** 25 m

Exercise 6-09

1 \$2508 **2** 21 692 908
3 \$734.94 **4** \$391
5 \$2193.10 **6** \$83.44
7 **a** \$307.80 **b** \$292.41 **c** \$7.41; 2.6%
8 B **9** D
10 **a** \$1248.05 **b** 25% increase
11 **a** 4% decrease **b** 4% decrease
c 16.64% increase **d** 6.5% increase
e \$7.41; 2.6% **f** 3.1% decrease
12 **a** 3.13 t **b** 10.6%

Exercise 6-10

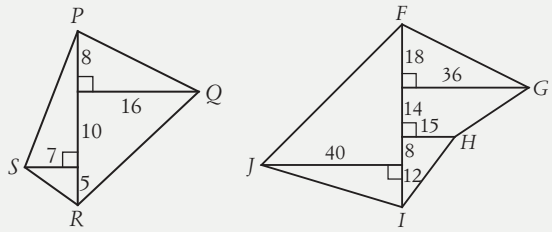
1 **a** 30 m **b** 68 cm **c** 66 cm
d 54 cm **e** 1720 cm or 17.2 m
f 79.67 m **g** 30.6 cm **h** 172 mm
i 100 cm **j** 230 mm = 23 cm
k 40 cm **l** 58 cm
2 **a** 53.4 m **b** 10.3 m **c** 21.4 cm
d 22.3 cm **e** 100.7 mm **f** 46.6 cm
g 51.4 cm **h** 121.1 mm **i** 34.3 m
j 94.2 cm **k** 43.2 m **l** 57.1 cm
3 247.96 – 222.83 = 25.1 m

Exercise 6-11

1 C **2** A
3 **a** 50 000 **b** 25 **c** 7.2
d 0.68 **e** 3 090 000 **f** 360
g 4 730 000 **h** 5.4
4 8.016×10^7
5 **a** 4 209 000 mm^2, 4.209 m^2
b $2.3 \times 1.83 = 4.209$ m^2
6 **a** 64 m^2 **b** 34.9 m^2 **c** 314.2 cm^2
d 4322.5 m^2 **e** 24 m^2 **f** 40 m^2
g 375 m^2 **h** 4 523 893.4 km^2
i 300 m^2 **j** 6 m^2 **k** 88.4 m^2
l 197.9 cm^2 **m** 11.0 m^2 **n** 42.4 m^2
o 40.8 m^2 **p** 189 m^2
7 **a** 2 **b** 110 cm^2
8 90 **9** 42.3 m^2
10 **a** 314.16 cm^2 **b** 85.84 cm^2
11 15 000 m^2
12 **a** 0.4096 **b** \$55.30

Exercise 6-12

1 **a** 430 m^2 **b** 960 m^2 **c** 6200 m^2
d 3800 m^2 **e** 2400 m^2
2 **a** 265 m^2 **b** 1871 m^2



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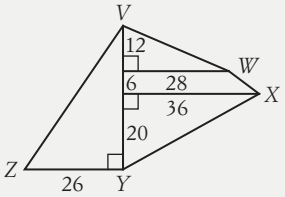
c 1214 m²



Exercise 6-13

1 **a** 7 000 000 cm³ **b** 50 000 mm³
c 0.089 m³ **d** 468 000 cm³
e 2.4 cm³ **f** 5.6 m³
g 9100 cm³ **h** 12 000 000 cm³
2 1.8 m³ **3** B **4** B
5 **a** 250 cm³ **b** 1.701 m³
c 30 846.4 cm³ **d** 3036 cm³
e 31 000 mm³ **f** 20.16 m³
6 **a** 350.21 cm³ **b** 1 cm³ = 1 mL
7 4.93 cm³ **8** 16.1 m³ or 16 100 L
9 2 420 000 m³
10 **a** 4 m³ **b** $264

Exercise 6-14

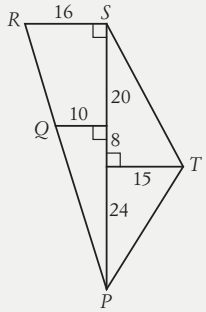
1 **a** 680 mL **b** 8.5 L
c 22 000 L **d** 8 m³
e 3.5×10^6 mL **f** 690 000 cm³
g 55 000 L **h** 4300 kL
i 9.5 m³ **j** 85 L
k 4300 cm³ **l** 1000 ML
2 B
3 **a** 0.864 m³ **b** 864 L
4 **a** 75 400 L **b** 75.4 kL
5 **a** 4.02 L **b** 101 mL
6 249 L **7** A
8 **a** 2.031×10^{12} L **b** 2.031×10^9 m³
9 10 681 L **10** 0.5 km³ **11** 8495 L

Sample HSC problem

a 3 **b** 0.24% **c** **i** 17.3 m/s **ii** 62.1 km/h

Revision

1 **a** 28 500 m **b** 6400 kg **c** 0.34 L
2 43 000 kL
3 **a** 0.005 **b** 0.14%
4 **a** 39 **b** 1000
c 0.0087 **d** 6 600 000
5 8.35×10^9 **6** 0.000 004 6
7 **a** 1 : 4 **b** 6 : 13 **c** 3 : 1 **d** 1 : 20
8 **a** 62.5 mL **b** 880 mL
9 Phoebe $26 628, Patrick $17 752
10 1667 min
11 **a** **i** 2.350 m/s **ii** 8.459 km/h
b 7.093 min
12 **a** $2066.40 **b** decrease of 13.9%
13 **a** 40 m **b** 316 cm **c** 55.00 cm
14 **a** 84 cm² **b** 56 000 m²
15 **a** 44 m² **b** 1040 cm² **c** 230 cm²
16



a 810 m² **b** 0.810 ha
17 **a** 20 700 mm³ **b** 1.65 mm³
18 **a** 137.2 cm³ **b** 114 380 cm³
c 216 000 cm³ or 0.216 m³
19 **a** 894 mL **b** 6500 L
20 **a** 8.1 m³ **b** 8100 L

Practice paper 2

1 D **2** B
3 A **4** D
5 D **6** B **7** $125
8 $37 872 **9** 0.12% **10** $\frac{1}{13}$
11 **a** **i**

6	7	8	8	10	11
7	8	9	9	11	12

ii 36 **iii** $\frac{1}{6}$ **iv** $\frac{5}{9}$
b **i** 12 mL/L **ii** 1200 mL
c **i** $697 **ii** 1
iii $15.40 **iv** 27
12 **a** **i** 96 **ii** 15
b 69.12 L **c** 5.0 L
d **i** $41 813.20 **ii** $41 327 **iii** $5598.18

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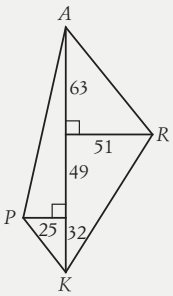
13 **a** **i** $\frac{1}{6}$ **ii** $\frac{1}{2}$ **iii** $\frac{11}{12}$

b 7.88%

c **i**



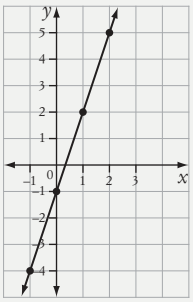
ii 5472 m²

d \$1120.98

Chapter 7

SkillCheck

1



2 **a** y: 5, 7, 9, 11 **b** y: −3, −4, −5, −6

3 $b = -1$

4 **a** $y = 2.8$ **b** $k = 4$

5 **a** **i** $V = 1225$ **ii** $V = 685$

b **i** $t = 2.5$ **ii** $t = 5.7$

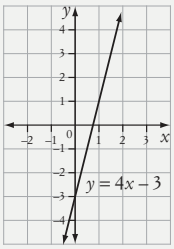
Exercise 7-01

1 **a** $y = 3x + 7$ **b** $y = -2x + 1$

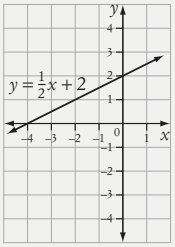
c $y = x - 1$ **d** $y = \frac{1}{3}x - \frac{1}{2}$

e $y = -\frac{5}{4}x$ **f** $y = 5$

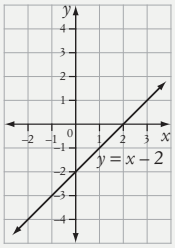
2 **a**



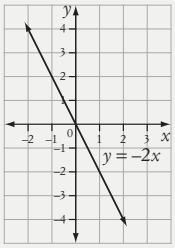
b



c



d



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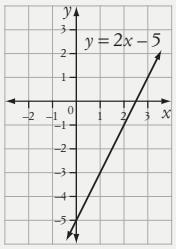
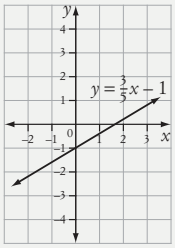
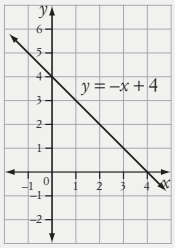


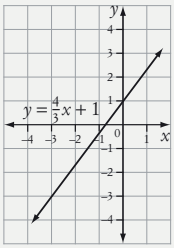
f



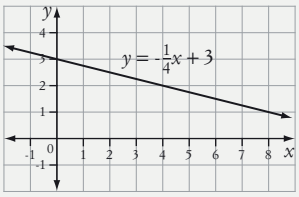
g



h



i



3 D

4 **a** $y = 2x + 3$ **b** $y = 3x - 1$

c $y = 4x$ **d** $y = -2x + 2$

e $y = -x + 1$ **f** $y = \frac{1}{2}x - 1$

g $y = -x$ **h** $y = -\frac{3}{4}x - 2$

i $y = x - 3$

Exercise 7-02

1 **a** 2 **b** −1 **c** 2

d $-\frac{1}{2}$ **e** $\frac{7}{5}$ **f** −5

g $\frac{3}{4}$ **h** 4 **i** $\frac{2}{9}$

2 **a** 3 **b** 5 **c** $\frac{1}{4}$

d −2 **e** $\frac{3}{5}$ **f** $-\frac{1}{2}$

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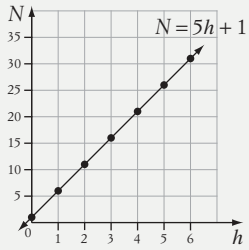
Exercise 7-03

1 **a**

h	1	2	3	4	5	6
N	6	11	16	21	26	31

b N **c** $N = 5h + 1$
d 101 **e** 16
f



g gradient 5, vertical intercept 1

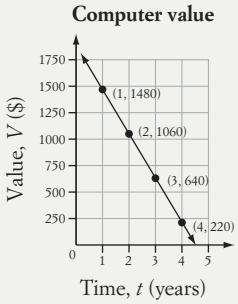
2 **a** $C = 80t + 22$ **b** C
c \$14.62
d 22, starting cost of call (at 0 min) in cents
e \$2.40 **f** 7 min

3 **a** dependent **b** $S = 4.5n$
c 4.5 runs/over
d 0, the number of runs scored after 0 overs
e **i** 95 **ii** 225
f **i** 12 **ii** 40
g Weaker batters bat later, with lower run rate.

4 **a**

t	1	2	3	4
V	1480	1060	640	220

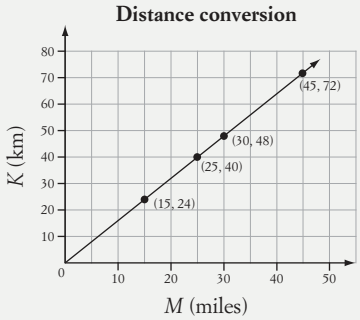
b



c Rate of depreciation in dollars per year.
d \$850 **e** \$1900
f V becomes negative. **g** 4.5 years

5 **a** $P = 50 + 3n$ **b** n
c 50, the base pay
d \$134 **e** 16

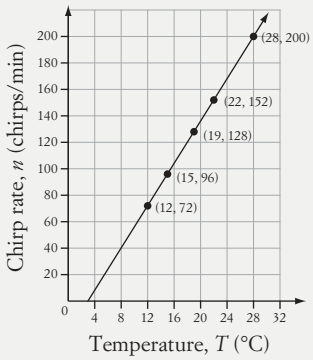
6 **a**



b $K = 1.6M$ **c** 0, 0 miles = 0 km
d 1.6, the number of kilometres in 1 mile.
e **i** 160 km **ii** 62.5 miles
f **i** 19 km **ii** 13 miles

7 **a** C
b Gradient 0.7, vertical intercept 4.2.
c $C = 0.7d + 4.2$ **d** \$3.50
e **i** \$18.20 **ii** \$4.20
f 48 km

8 **a** independent **b** $n = 8T - 24$
c



d Increases by 16 chirps/min.
e 184 chirps/min **f** 21°C
g –24, the temperature would not be –24°C.

Exercise 7-04

1 $k = 3.5$, $y = 3.5$, 35, 38.5
2 **a** $d = 2.75r$ **b** 90.75 m **c** 36 revolutions
3 **a** $p = 5.88d$ **b** 235.2 kPa **c** $141\frac{2}{3}$ m
4 **a** $F = 2.4d$ **b** 7800 kg
5 **a** $y = 1.6x$ **b** 96 km/h **c** 75 miles/h
6 A
7 **a** 64
b increase in water volume per minute
c 512 L **d** 15 min
8 120 min or 2 hours

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9 **a** 22.8 kg **b** 84.1 kg

10 8 h 53 min

11 **a** 60 seconds or 1 min **b** 4000 kilobytes

12 B

13 **a** $S = 1.3h$ **b** 317.2 cm **c** 92 cm

14 **a** 117.6 m/s **b** 18 s

Exercise 7-05

1 **a** 107 cm **b** 5.9 ft

c **i** 61 cm **ii** 76 cm **iii** 182 cm

2 **a** \$141 **b** \$32 **c** \$122

d \$84 **e** \$40 **f** \$68

3 **a** \$1.48 **b** \$0.64

c \$1.06 **d** \$1.20

4 **a** €15 **b** €47 **c** €20

d €82 **e** €67 **f** €3.20

5 1.60, the no. of A\$ per €, what €1 equals in A\$.

6 **a** **i** 72 kg **ii** 56 kg **iii** 80 kg

b **i** 185 cm **ii** 159 cm **iii** 193 cm

c 0.65, rate of change of weight per height in kg/cm.

7 **a** **i** \$51 **ii** \$24 **iii** \$70

b **i** \$29 **ii** \$94 **iii** \$63

c 0.85, the discount price as a proportion of the marked price.

d C

8 **a** **i** 41 kg **ii** 62 kg **iii** 73 kg

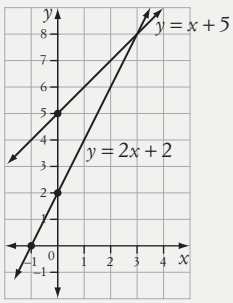
b **i** 88 lb **ii** 140 lb **iii** 198 lb

c 0.46 kg/lb

Exercise 7-06

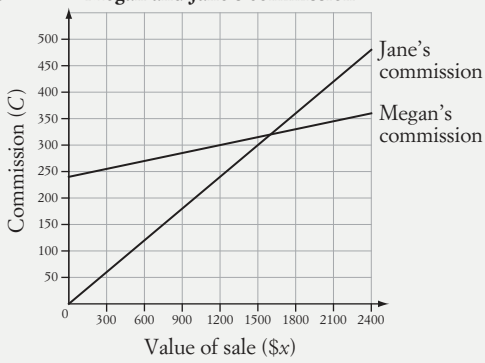
1 **a** $x = 2, y = 2$ **b** $x = 1, y = 5$

2 **a**



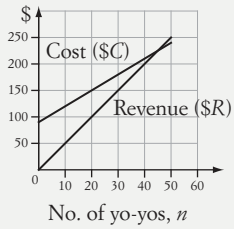
b $x = 3, y = 8$

3 **a**



b Megan **c** less than \$1600 **d** \$1600

4 **a**



b \$150 **c** \$100 **d** \$50

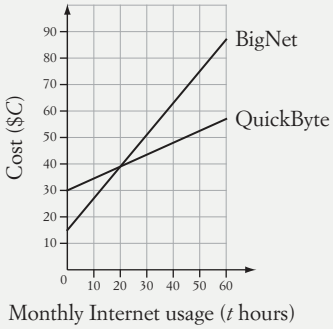
e \$5 **f** 45

g Because the revenue would be less than the cost, meaning the business would make a loss.

5 **a** Yes, after 25 minutes.

b Huong, by 10 minutes.

6 **a**



b QuickByte

c BigNet \$51, QuickByte \$43.50

d 20 hours

e **i** Infrequent users, less than 20 hours per month.

ii Users who access more than 20 hours per month,

f Teacher to check.

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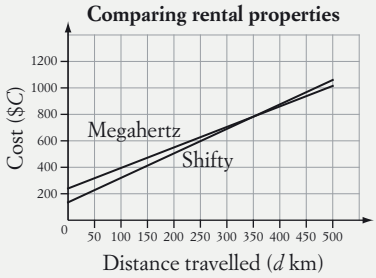
7 **a**



b (350, 782.5), the point where the costs are the same for the same distance travelled (350 km, $782.50)

c For $d = 3 \times 100 = 300$ km, Shifty is the cheaper plan, at $690.

Exercise 7-07

1 **a** Payment to family/spouse if a person dies.

b **i** $160 **ii** $275 **iii** $200

c Least likely to die in that age group.

d Increases, more likely to die.

e 20 to < 25 years

f **i** 40 years **ii** 50 years

2 **a** Cost decreases, cheaper to buy in bulk, less packaging and handling.

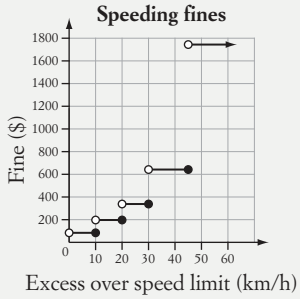
b **i** $4.00 **ii** $5.50

iii $4.00 **iv** $6

c 50

d Cost stays constant after 50 boxes, minimum cost is $3.50.

3



4 **a** **i** $11.85 **ii** $5.40 **iii** $11.15

b As mass increases, cost of postage increases.

c Over 0.25 kg and up to 0.5 kg.

d **i** 1 kg **ii** 0.25 kg **iii** 3 kg

5 **a** **i** $10.90 **ii** $7.20 **iii** $5.35

b 3 – <4 min **c** $3.50 **d** A

Sample HSC problem

a $P = 4n - 24$

b 4, increase in pocket money in dollars per year of age.

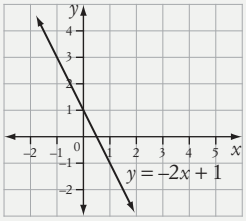
c $40 **d** 11

e **i** Values of P will be 0 or less.

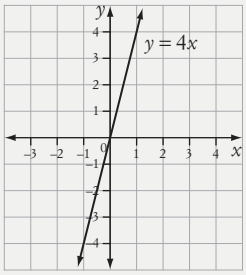
ii After 18 years, children become adults and either receive no pocket money or an amount based on a different formula.

Revision

1 **a**



b



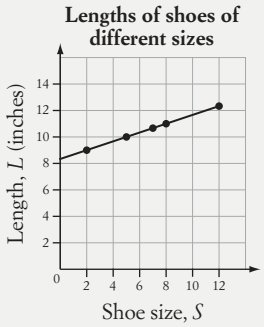
2 **a** $y = -\frac{1}{2}x + 1$ **b** $y = 3x - 3$

3 **a** $2\frac{1}{2}$ **b** -3

4 **a** $\frac{1}{2}$ **b** -4

5 **a** S

b



c $\frac{1}{3}$, the increase in length per shoe size, in inches/size

d $8\frac{1}{3}$ inches, the vertical intercept

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e $L = \frac{1}{3}S + 8\frac{1}{3}$ **f** $10\frac{5}{6}$ **g** 14

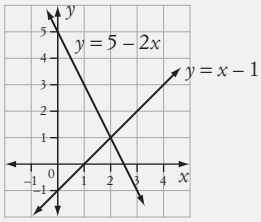
6 18 km

7 **a** **i** \$31 **ii** \$93

b **i** 1700 baht **ii** 2150 baht

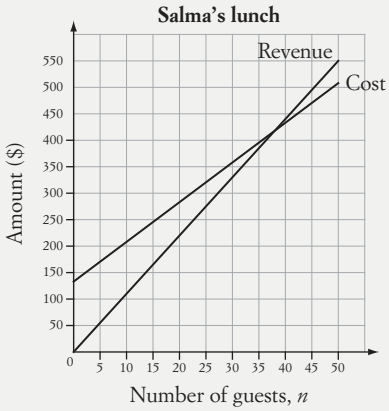
c 0.04, number of A\$ per baht

8 **a**



b $x = 2, y = 1$

9 **a**



b cost \$370, revenue \$350

c She will make a loss (cost > revenue).

d 38

e It is the number of guests (38) at which Salma will stop making a loss because the revenue will equal the cost rather than be below it. For higher numbers of guests, Salma will start making a proft.

f \$42

10 **a** **i** \$4.10 **ii** \$0.20 **iii** \$2.15

b 2 min **c** B

d 65c/30 s or 65c/$\frac{1}{2}$ min

Chapter 8

SkillCheck

1 **a** \$1776

b \$9100

c \$14 352

d **i** \$52.78 **ii** \$105.56 **iii** \$229.50

e \$41.86

2 **a** \$64 **b** \$3328 **c** \$277.33

3 **a** \$770.40 **b** \$1608 **c** \$1397

d \$102.56 **e** \$4454.45

4 **a** $y = 66$ **b** $y = 220$ **c** $x = -2\frac{2}{3}$

5 **a** $S = -18\ 300$ **b** $V = 41\ 600$

c $n = -1\frac{11}{19}$

6 **a** 144 **b** 1008 **c** 4368

7 \$229.50 **8** \$56 745

Exercise 8-01

1 A **2** D

3 \$1335.36

4 A

5 **a** \$174.24 **b** \$234.50

c \$385.02 **d** \$370.63

6 **a** \$470.20 **b** \$622.20

d \$1360 (he did not have comprehensive insurance to cover himself in the accident)

7 **a** \$573 **b** \$603.90 **c** \$524.70

d \$468.50 **e** \$573 **f** \$624.25

8 **a** **i** $175.5 \approx 176$

ii $92\% \times 175.5 \approx 161$

b **i** 253 **ii** 281 481

9 **a** \$933.40 **b** \$9699 **c** \$734.80

10 **a** 853 **b** 217

c 31 **d** easier to break into

11 **a** NSW **b** Qld

c NSW, high theft rates **d** WA

e Qld, theft rate lowest

f Tasmania has the lowest number of thefts but one of the highest theft rates per thousand.

Exercise 8-02

1 B **2** D

3 **a** **i** \$900 **ii** \$2000 **iii** \$2800

b graph, goes through (0, 0), (30 000, 900), (40 000, 2000), (80 000, 3600)

c **i** \$3400 **ii** \$65 000

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4 **a** $550 **b** $1987.50 **c** $4165

5 **a** **i** $500 **ii** $1500 **iii** $2200

b **i** $90 000 **ii** $70 000 **iii** $45 000

6 **a** $320 **b** $3450 **c** $7300

7 **a** $260 **b** $440 **c** $2730

8 **a** **i** $2855 **ii** $562.50 **iii** $6620

9 **a** $9.80 **b** $55.50 **c** $68

10 **a** $18 **b** $1044 **c** $4220

d $26.70 **e** $1462.50

Exercise 8-03

1 C **2** A

3 **a** $29 920 **b** $697.23 (round up)

4 **a** $5998.40 $28 998.40 $1208.27

b $22 037.40 $71 537.40 $1987.15

c $8656.40 $25 656.40 $534.51

d $25 915.40 $63 365.40 $1056.09

5 **a**

Fortnights	Principal (P)	Interest (I)	Amount owing ($P + I$)	Balance ($P + I - R$)
1	$100 000	$384.62	$100 384.62	$99 844.62
2	$99 844.62	$384.02	$100 228.64	$99 688.64
3	$99 688.64	$383.42	$100 072.06	$99 532.06
4	$99 532.06	$382.82	$99 914.88	$99 374.88
5	$99 374.88	$382.21	$99 757.09	$99 217.09
6	$99 217.09	$381.60	$99 598.69	$99 058.69

b $941.31 **c** $1152.06 **d** $1146.63

6 **a**

Months	Principal (P)	Interest (I)	Amt owing ($P + I$)	Amt owing after repayment ($P + I - R$)
1	$3000.00	$33.75	$3033.75	$2658.75
2	$2658.75	$29.91	$2688.66	$2313.66
3	$2313.66	$26.03	$2339.69	$1964.69
4	$1964.69	$22.10	$1986.79	$1611.79
5	$1611.79	$18.13	$1629.92	$1254.92
6	$1254.92	$14.12	$1269.04	$894.04
7	$894.04	$10.06	$904.10	$529.10
8	$529.10	$5.95	$535.05	$160.05

b $160.05 **c** $160.05 **d** $3160.05

7 **a**

Months	Principal (P)	Interest (I)	Amt owing + interest ($P + I$)	Amt owing after repayment ($P + I - R$)
1	$3600	$37.20	$3637.20	$3137.20
2	$3137.20	$32.42	$3169.62	$2669.62
3	$2669.62	$27.59	$2697.21	$2197.21
4	$2197.21	$22.70	$2219.91	$1719.91
5	$1719.91	$17.77	$1737.68	$1237.68
6	$1237.68	$12.79	$1250.47	$750.47
7	$750.47	$7.75	$758.22	$258.22
8	$258.22	$2.67	$260.89	$0.00

b $260.89 **c** $160.89 **d** $3760.89

8 **a**

Fortnights	Principal (P)	Interest (I)	Amt owing + interest ($P + I$)	Amt owing after repayment ($P + I - R$)
1	$2500	$14.04	$2514.04	$2034.04
2	$2034.04	$11.42	$2045.46	$1565.46
3	$1565.46	$8.79	$1574.25	$1094.25
4	$1094.25	$6.14	$1100.39	$620.39
5	$620.39	$3.48	$623.87	$143.87
6	$143.87	$0.81	$144.68	$0.00

b $144.68

c 6 fortnights = 12 weeks

d $44.68

e $2544.68

Exercise 8-04

1 D

2 B

3 $39 044.80

4 **a** $23 427

b It added $12 651 to the cost. This was the interest on the amount borrowed.

5 $32 666.20

6 **a** $24 500 **b** $9114 **c** $933.72

7 **a** $31 164 **b** $6994 **c** $64 424

d $27 210 **e** $19 745 **f** $57 496

8 $47 732.80

9 **a** $67 448 **b** $42 977 **c** $45 951

d $155 512 **e** $19 270 **f** $26 979

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Exercise 8-05

1 C **2** A

3 **a** 0.115 L/km **b** 113 L
c 478 km **d** 1477 km

4 **a** 1.17 L **b** 274.60 L

5 **a** 0.08 L **b** 12 L

6 **a** Toyota Camry Hybrid **b** 865 km
c 160 L **d** $399.80

7 **a** $187.03 **b** 12.7
c **i** $270.72 **ii** 1212 km

8 **a** 187.0 c/L **b** 9.9
c $95.30 **d** 9.09 L/100 km

9 **a** 145.54 c/L **b** 149.17 c/L
c 9.4 c/L **d** 0.8 c/L
e Brisbane (145.54) **f** Thursday (154.98)
g Adelaide (13.1) **h** Sunday (7.7)

10 Brodies (10.14 L/100 km)

11 Fill in the missing amounts, correct to 1 decimal place.

Distance travelled (km)	Fuel used (L)	Fuel consumption (L/100 km)
863	62	**7.2**
384	43	11.2
1304	**110**	8.4
887	86	9.7
2356	**304**	12.9
1730	167	**9.7**

Exercise 8-06

1 A **2** C

3 **a** $1950
b **i** $9750 **ii** $5850

4 **a** $1440
b **i** $9320 **ii** $7880

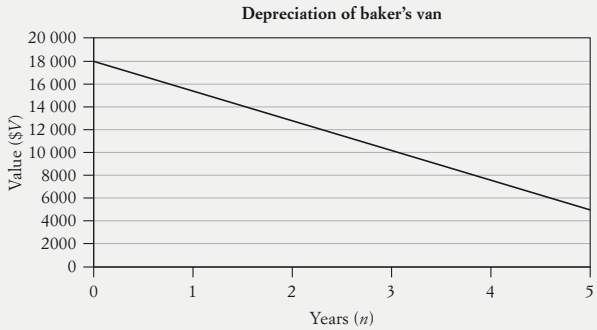
5 $0

6 **a** $7000 **b** $75 000

7 **a** $2600
b

No. of years n	0	1	2	3	4	5
Value $\$V$	18 000	15 400	12 800	10 200	7600	5000

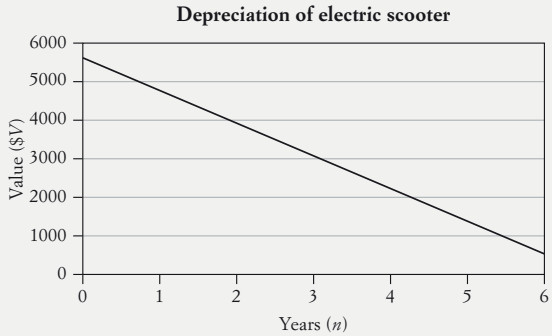
c



d 1800, The initial value of the van.
e $11 500 **f** $3\frac{1}{2}$

8 **a** $5600
b **i** $850 **ii** $1700
c **i** $3900 **ii** $2200
d $500

9 **a**



b $5600, the purchase price **c** $5\frac{1}{2}$ years
d $2\frac{1}{2}$ years **e** $4300

10 **a** $19 000
b **i** $13 000 **ii** $3000
c $2000

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Exercise 8-07

1 B

2 **a** **i** \$8624 **ii** \$6678 **iii** \$3524

b **I** \$3122 **ii** \$6276

c about 18 years [actual value is 17.86 years]

3 **a** \$21 700 **b** \$7828 **c** 2.7 years

4 \$64 740

5 **a** \$50 000

b **i** \$24 000 **ii** \$11 000

c **i** about \$9000 **ii** about \$5000

d \$6850.57

6 **a** **i** \$56 320 **ii** \$18 455

b **i** \$14 080 **ii** \$4614

7 **a** \$71 508 **b** 25%

c after 8 years **e** \$169 500, purchase price

8 **a** \$54 673 **b** \$13 862

9 \$38 722

10 **a** \$15 000

b **i** \$10 000 **ii** \$6000

c \$2000 **d** \$1611

Sample HSC problem

1 **a** \$0 **b** \$13 000

2 **a** \$7321 **b** \$2440

3 The straight-line method.

Revision

1 **a** \$286.90 **b** \$320.76

2 **a** \$8.30

b Nothing. He was only insured for damage to other people's property. He needs comprehensive insurance to insure his own property.

3 **a** \$540 **b** \$1925 **c** \$72 **d** \$3305

4 **a** \$615 **b** \$920 **c** \$3570 **d** \$2990

5 **a** **i** \$15 136.80 **ii** \$49 136.80 **iii** \$1364.91

b **i** \$25 558.40 **ii** \$74 558.40 **iii** \$1553.30

c **i** \$31 321.50 **ii** \$88 321.50 **iii** \$1472.03

6 \$27 709

7 **a** 10.74 L/100 km

b

Fuel used (L)
5.1
3.3
49.7
221.3
720.5

c Don't speed; accelerate slowly; walk or cycle short distances.

8 **a** 11.6 L/100 km **b** 9.17 L/100 km

c **i** 0.067 **ii** 0.057 **iii** 0.113

d 598 km **e** 245 L

9 **a** \$3500

b

Year	Dep	Accum dep	End year value
1	3500	3500	20 500
2	3500	7000	17 000
3	3500	10 500	13 500
4	3500	14 000	10 000
5	3500	17 500	6500
6	3500	21 000	3000

c \$6500 **d** $S = 24000 - 3500n$

10 **a** \$17 801 **b** \$7272 **c** 2.4

Chapter 9

SkillCheck

1 **a** 29 **b** 58 **c** 12 **d** 8 **e** 24

2 column graph

3 **a** 10 **b** 7

c 12, wears large shoes, has big feet

d 0 **e** 30%

4 **a** 4 **b** 1 **c** 4 **d** 32 **e** 42

5 **a**

50 – < 60	55	4
60 – < 70	65	12
70 – < 80	75	7
80 – < 90	85	8
90 – < 100	95	3
100 – < 110	105	6

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Exercise 9-01

1 **a** 5 **b** 34.4 **c** 36 **d** 9

2 **a** 20 **b** 64.5

3 **a** D

b Down, the high outlier increases the mean so its removal would decrease the mean.

4 **a** 50 **b** 2.9

5 **a** 29 **b** 120.8 s

6 **a** 50.0 **b** Yes, the mean is 50.0.

7 **a** 20 **b** 2.6 **c** 8 **d** decreases

8 **a**

Number of calls, x	Frequency f	fx
2	1	2
3	2	6
4	3	12
5	5	25
6	2	12
7	2	14
	15	71

b 15 **c** 4.7

9 **a** 38, 43, 48, 53, 58, 63, $\bar{x} \approx 44.24$ **b** 149

10 **a** 44 **b** \$332

c part-time or casual staff

Exercise 9-02

1 **a** **i** 5 **ii** 1, 5

b **i** 34.5 **ii** 31, 32, 35

c **i** 38 **ii** no mode

d **i** 8 **ii** 5, 7, 8

2 **a** 65 **b** 47, 59, 65

3 **a** C **b** A

4 **a** 3 **b** 3

5 **a** 2 **b** 2 **c** 8

d **i** no effect **ii** no effect

6 **a** 121 s **b** 123 s

7 **a**

Number of biscuits	Frequency
0	3
1	8
2	5
3	2
4	2

b 20 **c** 1 **d** 1 **e** 1.6

8 **a** 40–49 **b** 40–49

9 **a** –2, 2 **b** 2 **c** 2.6

10 **a** 300 – < 400 **b** 300 – < 400

Exercise 9-03

1 **a** mean **b** median **c** median **d** mode

e mean **f** mode **g** mean **h** mode

2 **a** mean **b** mode **c** median

d mode or mean **e** mean **f** mode

3 **a** 7 **b** \$82 000 **c** \$68 500

d Mean, the high salary of the general manager pushes it up.

e median

4 **a** **i** 38.125 **ii** 38 **iii** 32

b **i** decrease **ii** decrease **iii** no effect

5 **a** 39 **b** white

c Data is categorical, not quantitative.

6 **a** **i** \$523 **ii** \$481 **iii** no mode

b Mean distorted by outlier of \$1027, there is no mode.

c Mean \$473, median \$458, yes.

7 **a** **i** 7.9 **ii** 7.5 **iii** 7

b **i** decrease **ii** no effect

c mode

8 **a** **i** 1.3 **ii** 1 **iii** 0

b mean **c** mode

d Not affected by the outlier of 6.

9 **a** **i** 191.7 cm **ii** 193 cm

b **i** increase **ii** increase

10 **a** **i** 27°C **ii** 27°C **iii** 27°C

b mean

Exercise 9-04

1 **a** 9.1, 10.1, 11.0 **b** 49.5, 50, 51

c 3.00, 3.45, 3.50 **d** 8, 18, 23

2 **a** 113.5 **b** 101.5 **c** 125.5

3 D

4 **a** 88 **b** 64 **c** 68

d 61 **e** 40%

5 **a** 4.5 **b** 7 **c** 8

d 7 **e** 9

6 **a** 20.6 **b** No, 25% **c** 25.4

d Yes, 3% **e** 10th

7 **a** 25.4° **b** 10% **c** 23.5°

d 18.9° **e** 12.4°

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8 **a** F **b** T **c** T
d T **e** F **f** T

9 **a** **i** 29.2% **ii** 67.7%
iii 16.4% **iv** 1.9%
b 95 **c** 99 **d** 69 **e** 100%

10 **a** 102.7 cm **b** No. 25% **c** 106.2 cm
d 103.3 cm **e** No. 1% **f** 97.6 cm, 109.2 cm

11 **a** 25% **b** 95% **c** 188 cm **d** 5 years
e **i** 178 or 179 cm **ii** 182 cm

Exercise 9-05

1 **a** 6 **b** 5 **c** 728 **d** 12

2 **a** 2 **b** 2.5 **c** 258 **d** 3

3 A

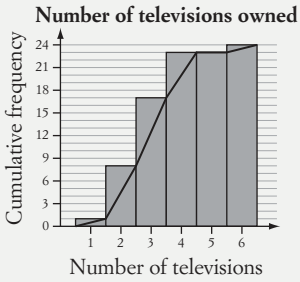
4 **a** 54 **b** 17.5

5 **a** 46 **b** 12
c The person who took 15 s to complete the test may be very smart or may have guessed the answers.

6 B **7** 46

Exercise 9-06

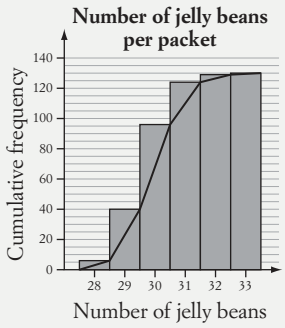
1 **a** 1, 8, 17, 23, 23, 24, median = 3
b



c **i** 3 **ii** 2

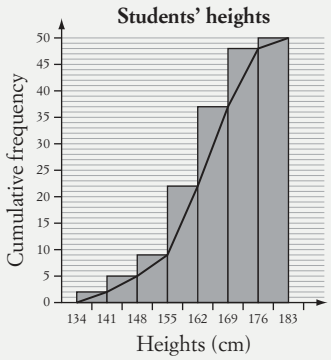
2 **a** 25 **b** 45
c 56 – 27 = 29 **d** 65

3 **a** 6, 40, 96, 124, 129, 130
b



i 30 **ii** 31 – 29 = 2 **iii** 30

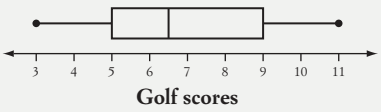
4 **a** Class centres: 137.5, 144.5, 151.5, 158.5, 165.5, 172.5,179.5;
Cumulative frequencies: 2, 5, 9, 22, 37, 48, 50
b 162 – < 169 **c** 162 – < 169
d



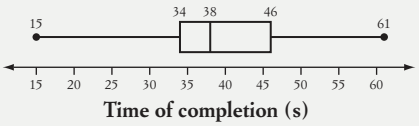
i 164 **ii** 171 – 158 = 13
iii 168

Exercise 9-07

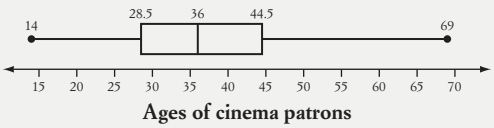
1 **a** 3, 5, 6.5, 9, 11
b



2



3



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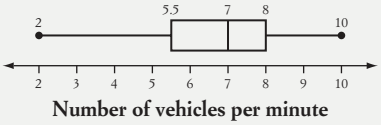
4 **a** 20 **b** 12 **c** 3
d 15 **e** 30

5 **a**



b Different answers possible. The dot plot shows individual scores and any clusters and outliers, while the box-and-whisker plot identifies the quartiles, including the median.

6 **a** \$75 000 **b** \$41 000
c \$41 000 **d** \$57 000
e \$28 000, \$51 000 **f** 25%

Exercise 9-08

1 **a** 2 **b** 1.3
2 **a** 2.25 **b** 3.03 **c** 5.28, –0.78
d 6 **e** 50%
3 **a** 75 **b** 9.43
4 **a** \$836 **b** \$277
5 **a** 2.88 **b** 1.8 **c** 27 **d** 54%
6 **a** 6.8 **b** 1.86 **c** 15 **d** 75%
7 **a** 550, 650, 750, 850, 950, 1050
b **i** \$764.89 **ii** \$119.36

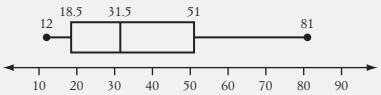
Exercise 9-09

Teacher to check.

2 **a** mean 21.97, SD 6.47, median 21, IQR 7

Sample HSC problem

a 39.25 **b** 31.5
c Mean, involves every score and there are no outliers.
d 51 – 18.5 = 32.5
e



Revision

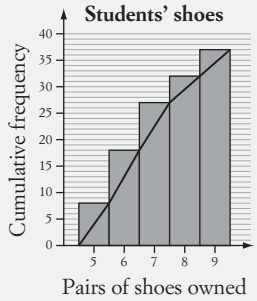
1 173.2

2 **a**

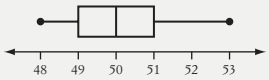
Speed (km/h)	Class centre	Frequency
60–<70	65	7
70–<80	75	13
80–<90	85	23
90–<100	95	7
		50

b 81 km/h

3 Median 171.5; modes 168, 170
4 Median 7, mode 7
5 **a** \$437 500 **b** \$345 000
c Median, outlier of \$1 044 000 does not affect its value as it does the mean.
6 **a** mode **b** mean **c** median
7 **a** 70% **b** 22%
8 **a** Range of the middle 50% of scores.
b Range 53 – 48 = 5, interquartile range 51 – 49 = 2
9 **a** 64.5 **b** 45, 72
c 46 **d** 72 – 54.5 = 17.5
10 **a** Cumulative frequency 8, 18, 27, 32, 37



b **i** 7 **ii** 8 – 6 = 2 **iii** 6
11 **a** 0 **b** 5 **c** $\frac{1}{2}$ **d** 25%
12 **a** 48, 49, 50, 51, 53
b



13 **a** 1.998 cm **b** 0.02 cm
14 **a** 6.7 **b** 1.3
15 Take large samples, take more samples.

Practice paper 3

1 A **2** D **3** B
4 B **5** C **6** D
7 \$1183.50 **8** –2 **9** 122.5 L
10 $y = 3x + 3$

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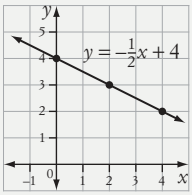
11 **a** **i** gradient $-\frac{1}{2}$, y–intercept 4

ii



b **i** 4.3 **ii** 4.5

iii 3 **iv** 2.57

c **i** 10% **ii** 20.6

12 **a** **i** $V = -5.5t + 84$

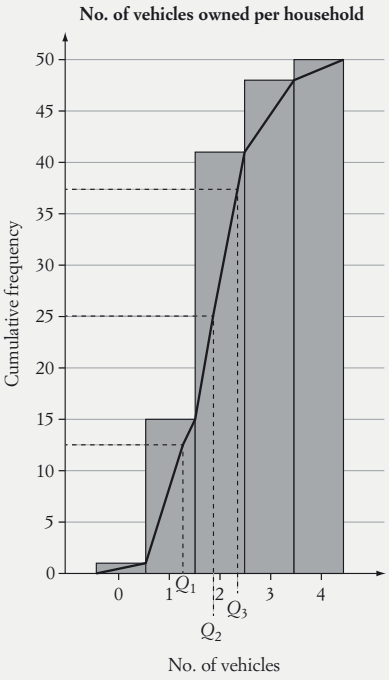
ii The decrease in water volume in kilolitres per hour.

iii The amount of water in the pool before you start draining it.

iv 15.3 h

b **i** 1, 15, 41, 48, 50

ii

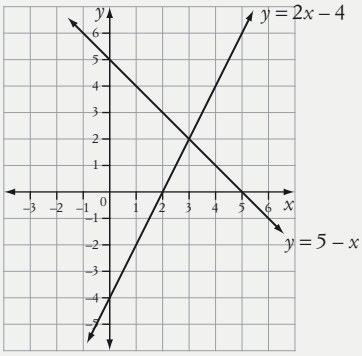


iii Interquartile range = 2 – 1 = 1

c **i** 70 **ii** 65 – 50 = 15 **iii** 31

13 **a** **i** \$19 750 **ii** 8 years

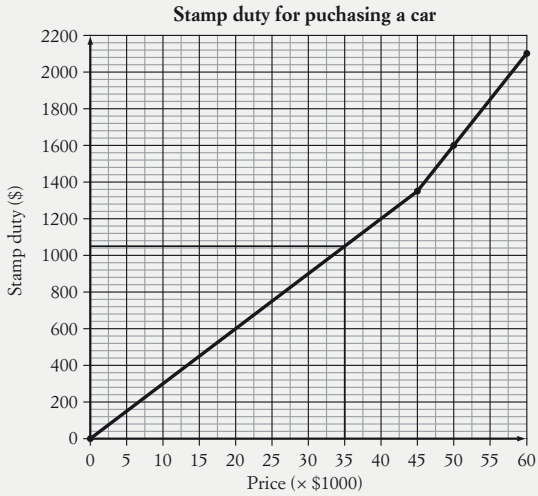
b



$x = 3, y = 5$

c **i** 600, 900, 1200, 1350, 1600, 2100

ii



iii \$1050

Chapter 10

SkillCheck

1 **a** 56 000 g **b** 128 000 000 L **c** 3.5 km

2 **a** 24 **b** \$94.20

3 **a** 340 km **b** 9 h **c** 21 s

4 7 **5** \$108

6 **a** 12 min 29s **b** 265 s

c 40 min 58 s **d** 11 min 33 s

7 **a** **i** \$4.10 **ii** \$2.70

b 3 min **c** 3 min 59 s **d** 41.20

8 **a** \$5.18 **b** 13 min

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Exercise 10-01

1 **a** 1.56×10^{10} b **b** 2.7 kb
c 4.8×10^{11} b **d** 2.9×10^{12} kb

2 **a** 512 000 kB **b** 524 288 000 B
c 4 194 304 000 b **d** 4 194 304 kb

3 C

4 Yes. 2GB = 2 097 152 kB and the images only need 432 000 kB of space.

5 **a** 104 857 600 B **b** 838 860.8 kb

6 **a** 3 145 728 kB **b** 25 769 803.78 kb

7 Nikki who pays $0.049 per 1 MB whereas Zoe pays $0.056 per 1 MB.

8 4428 files **9** 163.84 MB

10 1 497 965 files **11** D

12 **a** 2.30 MB **b** 23 507.52 kB

13 3072 kB **14** 8.53 GB

Exercise 10-02

1 C **2** T, T, T, F, F, T

3 3323.5 kb/s **4** 6 362 914 bps

5 105 kbps **6** 0.21 s

7 9 min 17 s **8** C

9 128 kbps **10** 51.2 s

11 50.7 s

12 **a** M 5.59s L 7.46s
b 0.008 GB **c** 3296 kB

13 **a** 8640 **b** 8.8 s
c 7.5 s **d** 915 whole files

14 No, it was faster at 1.91 Mbps.

15 1952 kbps **16** 157 s

17 **a** 4.10 MB/s **b** 34.36 Mbps

18 **a** 375 MB **b** 7 hours

Exercise 10-03

1 78

2 Yes, the mp3 player holds 8192 MB and he only needs 6000 MB of space.

3 4311 whole tunes.

4 **a** 62.64 MB **b** 3.13 MB **c** 3.18 MB
d 4.01 **e** $41.80
f $\frac{9}{20}$ or 0.45 or 45%
g **i** 1 860 480 **ii** 3 200 000

5 **a** 524 288 **b** 26 214.4 hours
c No, you can only fit 409 whole movies.

6 **a** 2590
b Some people are included in more than one category.
c 3024
d Bigger screen, can see the movie before it is released on DVD.
e It doesn't have ads.
f They have to wait too long to see a TV show. If people download a show illegally and don't wait to see it on TV, advertisers will not sponsor shows and fewer TV shows will be produced.

7 51.2 GB

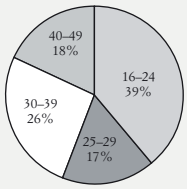
8 **a** **i** $25.76 **ii** $17.86
b 27.3%
c People bought fewer physical CDs and more recorded music online; more people illegally downloading.

9 16 384

10 **a** Sucker Punch **b** about 6 400 000
c about 8 million **d** about $44 million

11 $1.60

12 **a**



b **i** 9038 **ii** 1537
c 780

13 **a** classical **b** 23% **c** 82.8°
d 170 **e** rock

14 **a** 23 521 928 **b** $79 552 930
c music video/DVD **d** digital albums
e $\frac{4\,817\,546 - 3\,301\,366}{3\,301\,366} \times 100\% = 45.9258...\%$
$\approx 45.93\%$
f CDs and DVDs sold $519 082 287 in the two years while digital sales were $214 047 844.

15 Teacher to check.

Exercise 10-04

1 **a** $6.28 **b** $0, $4.90 **c** $31.50
d $156.50 **e** $0 **f** $55
g 62 min
h $151.20 **i** 24 × $49 = $1176

2 6 × $49 = $294

3 **a** $4.66 **b** $9 **c** $6.83
d $3.73 **e** $3.11 **f** $2.18

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4 It is cheaper to have a phone call – \$7.80 for a call but \$8.75 for texts.

5 **a** \$87.70; 25 October 2014

b 24 August to 25 September 2014

c August \$68 **d** more

e \$68.44; yes **f** 976 543 210

6 \$22.20

7 **a** 726 MB; all of it as 1024 MB of data is included.

b \$60 **c** \$2

d It saves printing and postage costs. **e** \$216

f A direct debit ensures that Optel is paid on time each month and there is no need for costly chasing of money owed.

8 Don't go over your call and text limit; don't use more data than included in your plan; pay by direct debit; don't get a paper bill; make short calls rather than have 'text conversations'; choose a cap that suits your usage; keep an eye on usage several times a month to ensure you are not going over your quota.

Exercise 10-05

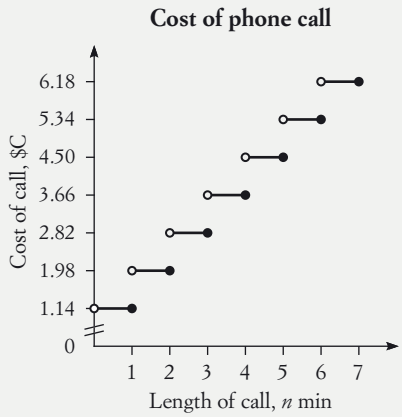
1 **a** \$13.70 **b** 6 min

2 **a**

Length of call, n min	Cost of call, \$$C$
0 up to 1	1.14
Over 1 and up to 2	1.98
Over 2 and up to 3	2.82
Over 3 and up to 4	3.66
Over 4 and up to 5	4.50
Over 5 and up to 6	5.34
Over 6 and up to 7	6.18

b **i** \$2.82 **ii** \$3.66 **iii** \$5.34

c

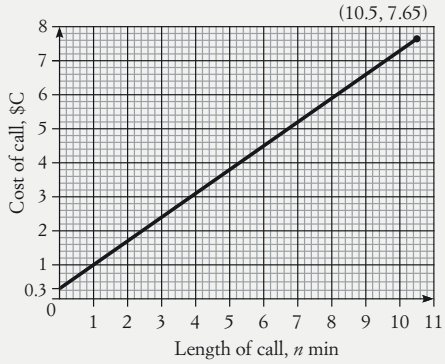


3 **a**

Length of call, n min	0	2	3.5	5	6	8.5	10.5
Cost of call, \$$C$	0.3	1.70	2.75	3.80	4.50	6.25	7.65

b $C = 0.7n + 0.3$

c



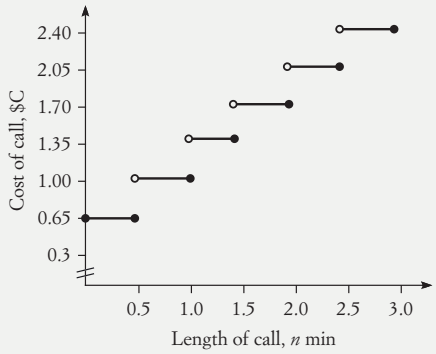
i \$3.10 **ii** 7 min

d \$5.90

e 0.7, the cost per minute of a call.

f 0.3; the connection fee of 30c.

g



4 **a** 12 s at 9.31 a.m. on 15 January cost 68c.

b 8.09 a.m. on 23 December cost \$2.50

c **i** 1 **ii** 32 **iii** 8

d 10.9%

e They are free; to encourage people to use the QuickChat network.

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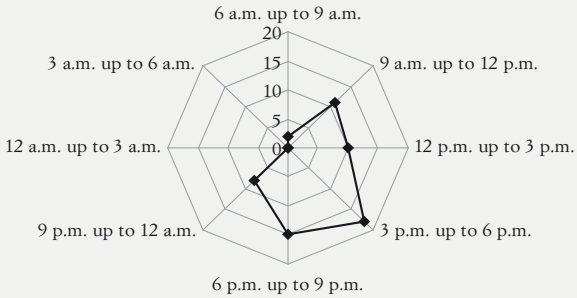
5 **a**

Call time	Frequency
6 a.m. up to 9 a.m.	2
9 a.m. up to 12 p.m.	11
12 p.m. up to 3 p.m.	10
3 p.m. up to 6 p.m.	18
6 p.m. up to 9 p.m.	15
9 p.m. up to 12 a.m.	8
12 a.m. up to 3 a.m.	0
3 a.m. up to 6 a.m.	0

b 3–6 p.m.; just after people finish school or work.

c No. of calls per 3-hour period.



d More calls are made in the afternoon and night, fewer in the morning, none between midnight and 6 a.m.

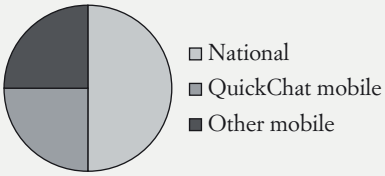
6 **a**

Call type	Frequency
National	32
QuickChat mobile	16
Other mobile	16

b 25%

c National (landline)

d Types of calls made



e Half of all calls were to landlines and the number of calls to each type of mobile were equal.

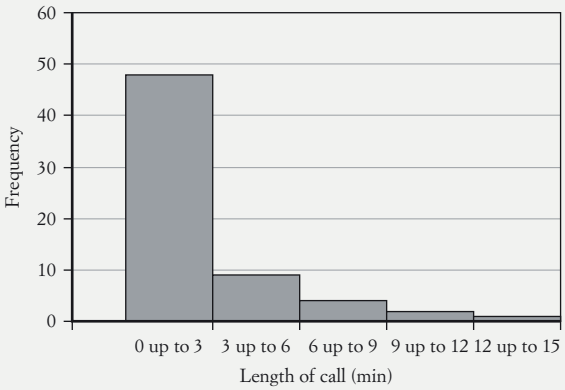
7 **a**

Call lengths (min)	Class centre	Frequency
0 up to 3	1.5	48
3 up to 6	4.5	9
6 up to 9	7.5	4
9 up to 12	10.5	2
12 up to 15	13.5	1

b 12 up to 15 min; longer call time costs more.

c 0 up to 3 min.

d Lengths of calls made.



e 2.77 min **f** $\frac{57}{64}$

Exercise 10-06

1 **a** **i** \$9.22 **ii** \$8 **iii** \$0

b decreases

c no charge for calls (credit is unlimited)

d **i** \$160 **ii** \$0

e very expensive

f **i** \$41.47 **ii** \$20.98 **iii** \$0

g \$1896, Monthly fee $\$79 \times 12 \times 2$ years = \$1896.

f cost decreases

2 **a** \$248 **b** \$49 cap plan

3 Extra usage over the included credit is very expensive.

4 **a** \$29. She uses less than \$100 per month.

b \$49. He uses between \$200 and less than \$450 per month.

c \$29 assuming he uses less than \$200 credit OR \$49 if he rings between 8 p.m. and midnight for less than 20 min as the calls are free.

d \$99. He makes about \$800 worth of calls per month.

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5 **a** **i** \$4 **ii** \$12.75
iii \$0 (less than the 2GB included data)
b decreases **c** higher **d** increases
e **i** \$3.02 **ii** \$3.25 **iii** \$15.04
f \$474

6 **a** Prepaid: you are not locked into a plan, you know how much credit and data you can use to avoid overspending but the costs are higher and extra usage is very expensive.
b Postpaid: you don't need to keep recharging for more credit, the costs are lower but there is a minimum cost over 24 months and you are locked into the cap plan for 24 months.

7 **a** Prepaid **b** Prepaid
c Postpaid **d** Postpaid
e Prepaid **f** Prepaid

8 **a** \$19 **b** \$49 **c** \$49 **d** \$49

Sample HSC Question

a Call rate lower; SMS cost lower; more data included; lower cost for excess data usage; free anytime calls to other Optel mobiles.
b **i** \$0 **ii** \$6.06
c \$170
d \$45 plan so he does not pay for excess data usage.
e 91

Revision

1 **a** 3400 b **b** 68 359 375 kB
c 71 303 168 B

2 No, she needs 17.09 GB of space.

3 262 144 **4** 21 509 kbps **5** 16 s

6 **a** about 8.9 million **b** Avatar
c about 9.6 million dollars
d To save money; get the film before it is in the cinemas.

7 **a** TV shows **b** movies and TV shows
c 6.7%
d **i** 684 000 **ii** 290 000 **iii** 118 000

8 **a** \$2.56 **b** \$61.36
c \$0 as it was below her quota of 1536 MB (1.5 GB)
d \$26
e 12 months; $\$59 \times 12 = \$708$ which is the minimum cost for the period of the contract.
f \$110.04

9 **a** 12
b 3–6 p.m.; people using their phone after school or work.
c Between 6 p.m. and 9 p.m.
d Between midnight and 3 a.m.
e There were a small number of calls from midnight to 3 p.m., the frequency of calls was greatest between about 3 p.m. and midnight, then it decreased again to 3 p.m. the next day.

10 **a** **i** \$11.18 **ii** \$0
b It increases.
c You can make as many phone calls and send as many text messages as you like – only the amount of data included is capped.
d It will cost you a lot of money.
e \$54
f $\$129 \times 24 = \$3096$
g \$99 cap plan (his cost is about \$1000 and this plan has \$1200 credit).
h \$29 cap plan.

Chapter 11

SkillCheck

1 **a** 2.5 **b** 3 **c** 12
2 **a** 3933.82 **b** 1850.19 **c** 2727.71
3 **a** \$420 **b** 5.4%
4 **a** \$3570 **b** \$1140 **c** \$2740.50

Exercise 11-01

1 C
2 **a** \$320.40 **b** \$3717.90 **c** \$471.18
d \$42.08 **e** \$625.50 **f** \$569.58
3 \$1620 **4** 11%
5 2 years **6** 12%
7 \$7400
8 **a** \$1984.50 **b** \$847.91
c \$4680 **d** \$207.66
9 0.035% **10** 15 months **11** 0.8%

Exercise 11-02

1 C
2 **a** \$10 800 **b** \$3000 **c** \$9100
3 **a** 8.3 years **b** $3\frac{1}{2}$ years **c** 7.1 years

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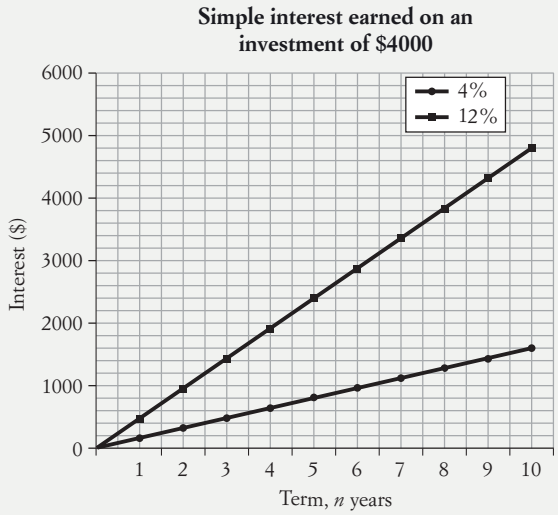
4



5 **a** 480, the interest ($) earned per year.

b **i** $1000 **ii** $3800

c *y*-values on 12% graph are 3 times those of the 4% graph each year, the gradient of its graph is steeper.

6 A

7 **a** $4200 **b** 6% **c** 45 months

d 4.6 years **e** $2800 **f** 4%

8 You earn zero interest if money is invested for zero years.

Exercise 11-03

1 **a** $8994.74 **b** $3891.04 **c** $4951.80

d $17 869.38 **e** $9714.53 **f** $9516.52

2 **a** $14 364.27, $1464.27

b $15 989.37, $2189.37

c $16 011.05, $2211.05

d $7341.81, $421.81

3 $18 363.23

4 **a** **i** $2975 **ii** $3421.68

b **i** Compound interest is greater because interest is calculated on accumulated interest as well.

ii $446.68

5 **a** $23 152.50 **b** $23 180.07 **c** $23 194.20

d $23 203.76 **e** $23 208.43

6 The amount of interest increases because you get interest on your interest more often.

7 18 years **8** 4 years **9** $3200

10 $7993.54 **11** C **12** 10 years

Exercise 11-04

1 B **2** A

3 **a** $A = \$23\ 424$; $I = \$7424$

b $A = \$13\ 695.50$; $I = \$10\ 510.50$

c $A = \$4.04$; $I = \$3.04$

d $A = \$10\ 249.14$; $I = \$1459.14$

e $A = \$12\ 159.90$; $I = \$709.90$

f $A = \$9137.52$; $I = \$1885.52$

g $A = \$24\ 388$; $I = \$6188$

h $A = \$62\ 368$; $I = \$30\ 368$

i $A = \$4548.60$; $I = \$348.60$

j $A = \$7208$; $I = \$1908$

4 5 years, first value in table column past 2.

5 8% p.a. **6** 7 years

Exercise 11-05

1 $1.00

2 **a** phone/Internet banking, automated and cashless

b at bank, requires bank staff

3 **a** $20.00

b Yes, her balance is under $500.

c $3.00 **d** $331.90

4 **a** dormancy fee, $10.00

b To encourage customers to use their accounts, for example.

5 **a** cheque dishonour fee

b $30.00

6 **a** $7.60 **b** $4701.50

7 **a** To discourage too many withdrawals.

b To encourage higher balances, for example.

c Bad cheques create more work for the bank.

8 **a** Counts as one withdrawal, to stay within monthly limit of free withdrawals.

b Minimises number of withdrawals.

c Do not attract fees.

d Minimises fees for separate accounts, balance has more chance of staying above $500.

e Attracts lower fees.

9 The other bank charges your bank when a withdrawal is made at its ATM.

10 Requires bank staff and more paperwork, creates customer queues at bank branches.

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Exercise 11-06

1 A **2** $39.46
3 B **4** $5.03
5 $9343.48 **6** $1310.33 per ounce
7 $18 909.20
8 **a** land, antiques, jewellery
b cars, computers, cameras
9 10.2% **10** $78 927.64
11 $67.35 **12** $43.55
13 $1518.38 **14** $116
15 $5906.60 **16** $20 400
17 $134 550 **18** 18 years

Exercise 11-07

1 A
2 **a** $35 750 **b** $715 **c** 2.85%
3 **a** $2.23 **b** $1070.40
4 **a** $1.43 **b** 8.9c
5 **a** $30 487.50 **b** $13 963
6 **a** $8364 **b** $8561.78
c $2296 **d** 27.45%
7 **a** $315 **b** 8.61c
8 **a** $35 998 **b** $1973.06
9 **a** $1050 **b** 3.0% **c** 74
10 $12 284.30
11 **a** $13 815.95 **b** $770
c **i** $15 400 **ii** 5%
12 **a** $18 798.50 **b** $7725.20

Exercise 11-08

1 A **2** B
3 **a** $7.68 **b** $7.46 **c** $6.68 **d** $7.66
4 **a** David Jones **b** down 4c
5 **a** AGL Energy **b** down 31c
6 **a** Wesfarmers **b** Qantas
7 **a** $14.59 **b** $1.419 **c** $16.10
8 Qantas
9 **a** $6.75 **b** $6.68 **c** 46.98c
10 **a** $14.00 **b** $15.00 **c** 3 months
d 4 **e** $19.00 **f** $20.00

11 **a** $8.20
b

c Price rose to a high at week 3, then dropped to a low at week 6.
d No
12 **a** $3.40 **b** $2.05 **c** 3
d 75 cents
e Dropped near the end of October. High at the start of November, then dropped to lowest point towards end of November, rose steadily until end of December, then fell steadily during first half of January.
f **i** $2.10 **ii** $3.00
g Yes, price is low. **h** No, price is low.

Sample HSC problem

a $1432.33 **b** $47.77
c When interest is calculated more often, there is more interest on interest.

Revision

1 **a** $1350 **b** $162 **c** $1854
2 4.5%
3 9 years
4 **a** Teacher to check.
b 144, the simple interest earned per year.
c 3.5 years
5 **a** $8636.13, $1436.13 **b** $4215.07, $165.07
c $12 323.05, $2023.05
6 $9554.46 **7** $1094.40, $194.40
8 $7.40 **9** $15 027 **10** $417.90
11 **a** $15 473.60 **b** $4368.80
12 **a** $7949.55 **b** $1107.40

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13 **a** 3 months

b $20.50, mid-November

c Share price has been variable, moving up and down, lowest in mid-November and highest at the start of January.

d 6 **e** $30

f Buy, because the price is low.

Chapter 12

SkillCheck

1 **a** 2 **b** 1.74 **c** 2.94

d 0.04 **e** 0.07 **f** 0

2 **a**

x	0	1	2	3	4	5
y	4	3.7	3.4	3.1	2.8	2.5

b –0.3

3

	i	ii	iii	iv	v
a	5.5	5	4,5	9	4
b	7.6	9	12	14	10.5

4 **a** 28.4 **b** 31 326.7 **c** 37.6

5 **a** 25 000 m **b** 2.463 km **c** 3.75 h

d 5h 12 min **e** 7 h 18 min

f 5.555... m/s **g** 30.555... m/s

h 12.6 km/h

6 20 160 km **7** 65.3 km/h

8 **a** $5.01 **b** $2525.10 **c** $37.33

Exercise 12-01

1 B **2** B

3 **a** Mabel **b** 6.58 L

4 D

5 **a** 2 **b** 3 **c** 73 kg

d 0.25 **e** graph

6 **a** heavy

b **i** 0.02 **ii** 0.017

c 0.07 **d** about 0.028 **e** about 0.01

f A heavy person gets rid of the alcohol in their blood system more quickly.

7 **a** 0.21 **b** 10 h 30 min

c 10.30 a.m. the next day.

8 **a** 0.18 **b** 12 h

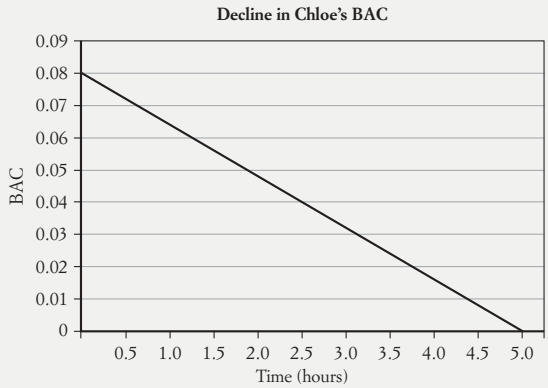
c 2 p.m. the next day.

9 0.012 per hour.

10 **a**

0	0.08
0.5	0.072
1	0.064
1.5	0.056
2	0.048
2.5	0.04
3	0.032
3.5	0.024
4	0.016
4.5	0.008
5	0

b



c **i** about 1:45 a.m. **ii** 5 a.m.

d 0.016, rate at which BAC is decreasing per hour.

Exercise 12-02

1 **a** $\frac{28}{55}$ **b** 62%

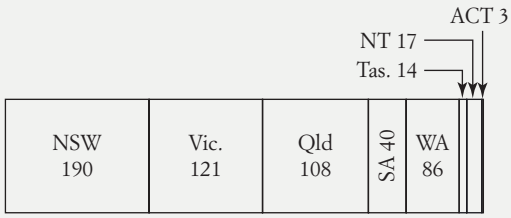
c 57 drivers = 13%

2 **a** **i** 635 **ii** 579

b 607 **c** All except NSW.

d NSW had just 5 more deaths so perhaps the road safety campaigns warning drivers of potential accident risks were working.

e



Driver deaths, 2011

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3 **a** $55.4\% \times 18764 \approx 10395$
b 5592 **c** 4597

4 **a** **i** 180° **ii** 68° **iii** 7°
b **i** 84 **ii** 62
c 227

5 **a** 1715 **b** 43.3% **c** $\frac{356}{821}$
d

Under 17 93	17–25 283	26–39 278

Driver deaths, 2011: age of driver

6 **a** NSW in 2006 **b** 793
c 46.3 **d** 8 **e** 12
f 9 **g** Teacher to check graph.

7 **a** **i** 270.8 **ii** 268
iii 30 **iv** 24
b Teacher to check graph.
c Decline in number of crashes perhaps due to road safety campaigns, more police presence, large fines for speeding, more cameras.
d More cars in NSW than in NT so greater the chance of a crash.

8 **a** about 1080 **b** 400
c about 1300
d Much more male deaths, possibly due to more males on roads, more males speeding or driving recklessly.

9 **a**

Total	1526	1167
Mean	127.17	97.25

b **i** January and October
ii November
c **i** 29 **ii** 12 **ii** 97.5
d Road death is more likely in November. Possibly due to driver's speeding to get ready for Christmas or heading off on holidays or heading to school or work functions.
e Yes. The trend is for deaths to be lower in 2011 but still November is the month where most deaths occurred in each year.

10 **a** 29.2% **b** 13.79%
c 425 **d** 2.7%

Exercise 12-03

1 C **2** B

3 **a** 78 km/h **b** 13 h 9 min **c** 247 km

4 **a** 8 h 54 min **b** 2 h 48 min **c** 7 h 43 min

5 **a** $3\frac{1}{2}$ h **b** 14 km/h

6 **a** 34.2 km/h **b** 333 450 m

7 **a** 1.1 m **b** 1.7 m **c** 2.2 m **d** 2.8 m

8 **a** 3 s **b** 1472 m

9 **a** 76.5 m/s **b** 12.6 min

10 **a** 17.5 min **b** 248 km/h

11 41 min **12** 104.2 m **13** 62.1 km/h

Exercise 12-04

1 A **2** 197.7 m **3** 5.8 m

4 **a** A = 13.9 B = 12.6 C = 28.3 D = 106.5
b **i** 299% **ii** 217%
c Yes. It is 110 m ahead, but only takes 106.5 m to stop.

5 **a** 7.9 m **b** 11.1 m
c Both would stop in time. Hanna in 30.1 m and Fran in 19 m.

6 **a** 44.4 m **b** 17.7 m
c Sanjay (52.3 m)

7 **a** $k = 0.0097$ **b** 106.9 m **c** 88 km/h
d **i** 97.6 m **ii** 153.6 m

8 **a** 73.8 **b** 61.1 m **c** 153.5 m

9 **a** 33.7 m **b** 102 m
c 345 m **d** 238 m

10 **a** 1.88 s **b** 99 m

11 **a** 181 km/h **b** 145 km/h
c 153 km/h **d** 126 km/h

12 **a** 0.007 46 **b** 90.3 m **c** 151.4 m

13 **a** 5500 m **b** 32 500 m **c** 46 500 m

14 **a** 160 m
b **i** 110 m **ii** 130 m **iii** 190 m
c

Road	Reaction time	Stopping distance
dry	1	78
dry	2	105
dry	4	160
wet	1	108
wet	2	135
wet	4	190

d You can see values in a table but a graph gives a good visual overall picture of the situation.

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HSC Problem

a 0.16 **b** 4 **c** 10

d 0.03, rate at which BAC is increasing with each drink consumed.

e 16 h 40 min

Revision

1 **a** 0.073 **b** 4 h 52 min **c** 0.024

2 **a** 0 **b** 0.07 **c** 0.17

d 9 h 27 min

3 **a** Heavier people have more blood and water in their bodies to dilute alcohol.

b Females tend to be lighter than males.

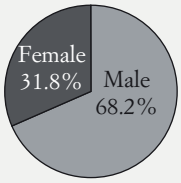
4 **a** 3378 **b** 1529 **c** 507

d 31 **e** 169 **f** 23.5

5 **a** 3786 **b** 645.25 **c** 33.5

d Females drive with more care to avoid accidents and/or there may be less female drivers.

e



6 **a** 65 km/h **b** 5h 23 min

c 280 km

7 935 m

8 No, he travels 21.7 m before he applies the brakes so his stopping distance is 21.7 m + 24 m = 45.7 m.

9 **a** same

b increases as reaction time increases

c A = 23.7 B = 34.8 C = 57

d No, he needs 57m to stop.

e Drive slower than the speed limit, get someone else to drive you.

10 **a** about 38 m **b** 10 m

c 100 km/h **d** 50 km/h

e It takes much longer to stop on a wet road. If the speed is doubled, the distance is much more than doubled. For example, at 100 km/h on a wet road it takes 4 times as long to stop as it does at 50 km/h.

Practice paper 4

1 A **2** B **3** B

4 B **5** C **6** C

7 2 621 440 **8** 77 km/h **9** $932.40

10 $17 209.23

11 **a** **i** 23 m **ii** $k = 0.0029$

iii 23.1 m

b $1 159 920

c **i** $1330.36 **ii** $13 242.60

d **i** 6.33 **ii** 6.5

iii Gradually increasing, more traffic, busy season such as Christmas.

12 **a** **i** $120 **ii** $204.80

b **i** $21 883.75 **ii** $9201.85

c **i** 0.065

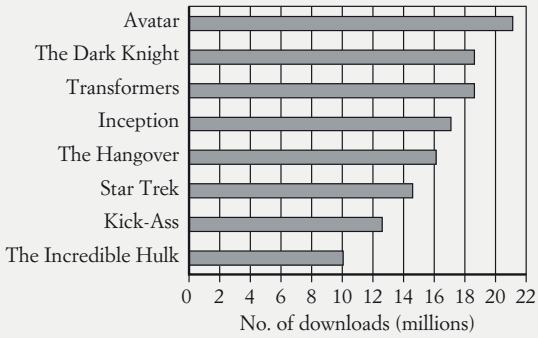
ii BAC would have been lower.

d **i** $126 **ii** $3.54

iii Call rate decreases as the cost of the plan increases.

13 **a** **i** 3 months **ii** about 88c **iii** 6 or 7

b **i**



ii 16 000 000 **iii** $23 200 000

c **i** 2155 **ii** 18 s

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