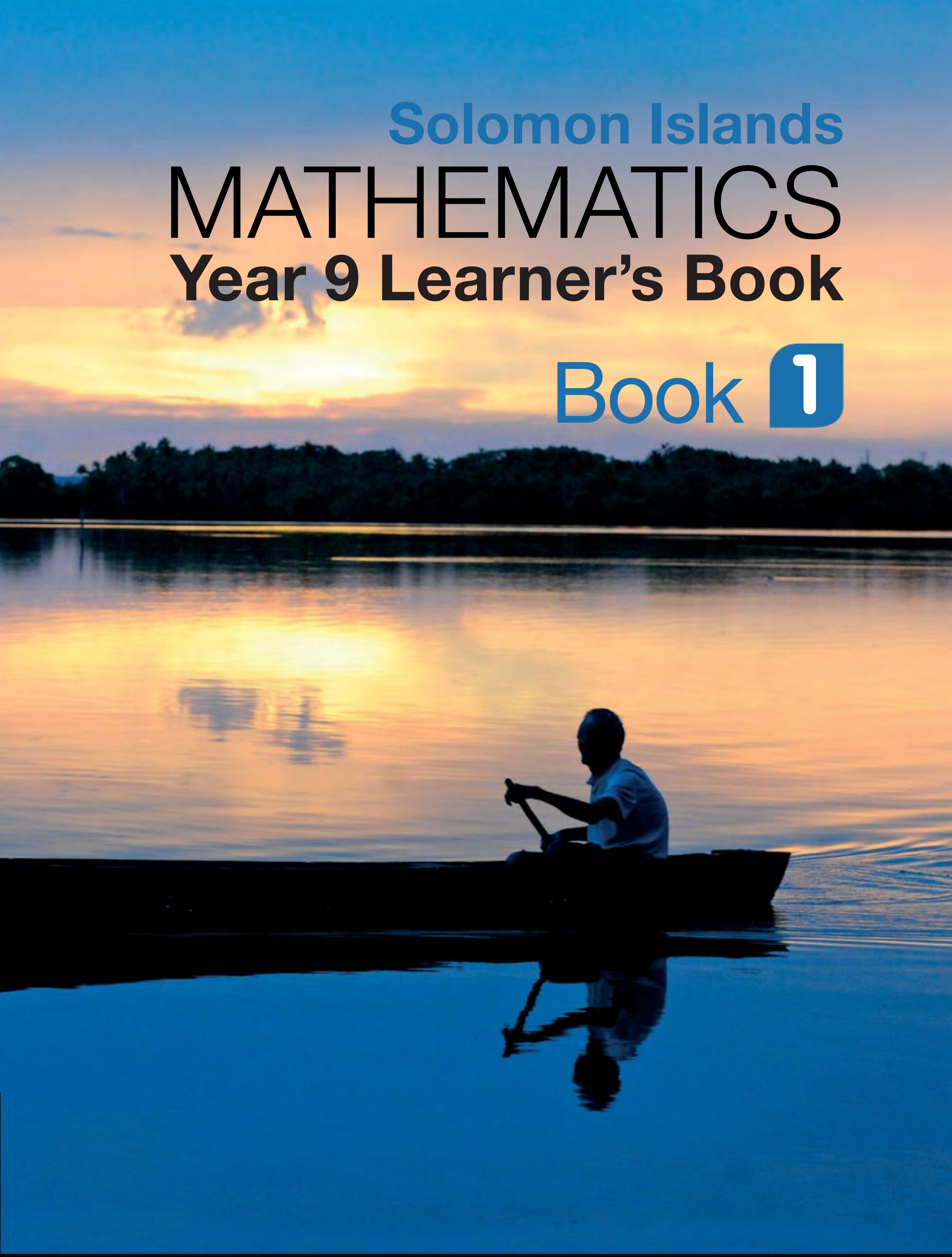
Solomon Islands MATHEMATICS

Year 9 Learner's Book

Book 1



ALWAYS LEARNING

PEARSON

Solomon Islands

MATHEMATICS

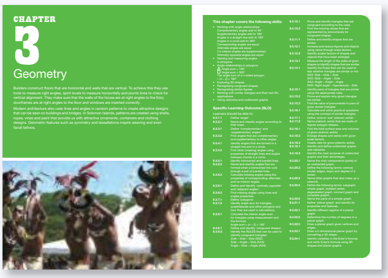
Year 9 Learner's Book

Book 1

CONTENTS

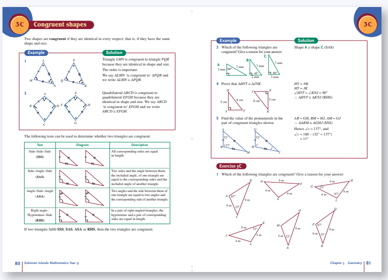
HOW TO USE THIS BOOK

The **Solomon Islands Mathematics** series has been written to cover the General Learning Outcomes of the Solomon Islands Secondary Mathematics Syllabus Years 7 to 9.



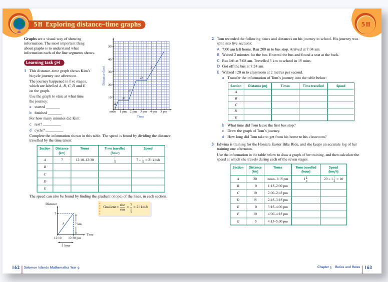
Chapter opening pages

Chapter opening pages include a contemporary or historical context for the content and provide learners with a list of the skills that are covered in the chapter.



Theory and exercise sections

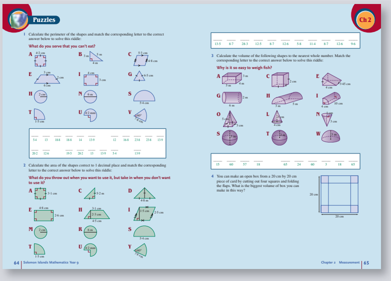
Theory and exercise sections contain explanations, examples and exercises designed to develop understanding of concepts and provide opportunities for students to practise new skills.



Explorations

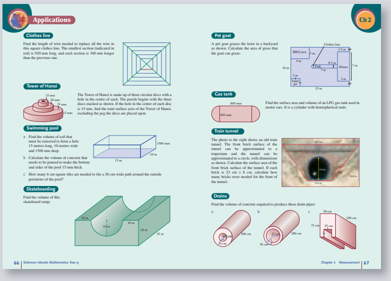
Explorations are scattered throughout the chapters, allowing students to work independently on non-standard problems and construct their own understandings.

These features are found at the end of each chapter



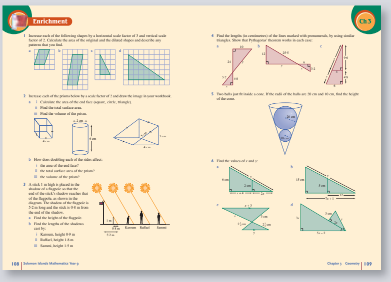
Puzzles

Puzzles are included for extra skills practice.



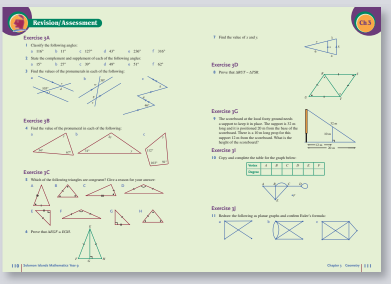
Application

Application sections investigate and apply mathematical ideas in a creative way and provide activities for a range of learner abilities.



Enrichment

Enrichment sections contain challenging tasks for learners to apply and extend their understanding of concepts.



Revision/Assessment

Revision/Assessment sections provide opportunities for learners to consolidate their understanding of concepts.

Solomon Islands Mathematics Year 9 Learner's Book

Introduction

This book is written to help you learn Mathematics by actively participating in a variety of activities. The book has a total of 12 chapters. Each chapter focuses on a particular topic from one of the strands in the Solomon Islands Junior Secondary Mathematics Syllabus. The strands are *Number*, *Measurement*, *Algebra*, *Geometry*, *Trigonometry*, *Statistics and Probability*. We hope that the activities in this book will encourage you to learn Mathematics effectively, and gain enjoyment and enrichment from the topics and contexts involved.

Chapter organisation

The chapter order provides opportunities to revise topics studied in earlier years, to learn new knowledge and skills, and to review and develop your understanding throughout the school year.

The Number strand

The chapters that will further develop your number skills include *Number Systems*, *Ratios and Rates*, *Consumer Maths* and *Indices*. You will continue your understanding of numbers, the ways they are represented, and the quantities for which they stand. You will develop accuracy, efficiency, and confidence in calculating, both mentally, and on paper. You will refine your ability to estimate and to make approximations, and to be alert to the reasonableness of results and measurements. It is important to maintain your competency in the four basic operations, and apply them confidently with whole numbers, directed numbers, decimals and fractions. By extending these skills to the real numbers, ratios and rates, and indices, you will be able to apply mathematics to solving problems in real-life contexts, and especially to Consumer Maths.

The Measurement strand

There is just one chapter to consolidate and build on your measurement skills from earlier years in Year 9. New applications explore curved surfaces and composite shapes and you will develop these skills to apply different formulas to surface area and volume problems. You will continue to use and convert metric units, make sensible estimates and round measurements to appropriate degrees of accuracy. Above all, you will need to be able to use measurements to solve problems in practical contexts.

The Algebra strand

The chapters that will develop your algebra skills in Year 9 are *Algebra Skills*, *Linear Equations and Formulas*, and *Linear Graphs*. You will continue to learn to recognise patterns and relationships in mathematics and the real world, and be able to generalise from them. More importantly, you will develop your ability to think abstractly and to use symbols, graphs and diagrams to represent and communicate mathematical relationships, concepts, and generalisations. You will need to manipulate algebraic expressions confidently to solve practical problems.

The Geometry strand

The Geometry chapter at Year 9 draws on much of the work from earlier years concerned with size, shape, position, and the properties of space, and introduces the key concepts of congruent and similar figures. You will need to visualise shapes in two and three dimensions, recognise and appreciate their occurrence in the environment, and to make use of the geometrical properties of everyday objects. You will also appreciate how to use geometric models as aids to solve practical problems in time and space with an introduction to networks.

The Trigonometry strand

Trigonometry is a new strand that is introduced at Year 9 with a chapter of its own. It draws on key skills from other strands, in particular, the geometry and algebra strands. Trigonometry has many applications, especially those of a practical nature such as engineering, architecture, navigation and surveying where problems can be modelled with triangles. By expanding your knowledge and understanding to finding unknown angles and lengths for triangles in two and three dimensions in Year 9, you will provide yourself with a firm foundation for future Mathematical studies.

The Statistics and Probability strand

There is one chapter for your study of *Statistics* and another for *Probability*. Newspapers are full of statistical information and it is important that you understand statements and graphs to check the accuracy of the conclusions. Your study of Probability will help you describe the chance of various events occurring and the decision-making skills can be applied to many real-life situations.

How to learn Mathematics

As you work through the chapters you will be asked to work on your own, work with a partner or in a group, and sometimes with the whole class. Therefore, you must be willing to participate actively in all the tasks and not rely on the teacher or friends for answers. When you actively participate you will learn a great deal as well.

Making mistakes

Learning Mathematics is a skill, like riding a bicycle. You cannot learn to ride a bicycle by just listening to the teacher telling you how to ride, you can only learn by doing it. Nobody has learnt to ride a bicycle without falling off many times. Making mistakes is part of the learning process and this is also true for Mathematics. The more familiar you are with the topic, the fewer mistakes you are likely to make. Like bicycle riding, Mathematics learning needs lots of practice and the exercises in this book are designed to help you practice until you become confident with each new skill. Homework is a chance to further practice the skills learnt in class, and what you can't do on your own, you can ask your teacher or a friend the next day.

Developing skills

Mathematics is more than a series of facts and rules. It involves understandings and skills that can be applied to new situations. After each lesson it is useful to reflect on your learning and in particular about the problem solving strategies that you used that day. Those same strategies may be useful for other problems in the future. And if you discover a new skill, show it to a friend. Not only will your friend benefit, but it will help you remember it too!

Suggested teaching plan for the Year 9 Learner's Book

Semester 1

Weeks	Sub-strands	Allocated Times
	Number	
1	Chapter 1: Number Systems	3 Weeks
2		
3		
	Measurement	
4	Chapter 2: Measurement	3 Weeks
5		
6		
	Geometry	
7	Chapter 3: Geometry	2 Weeks
8		
	Algebra	
9	Chapter 4: Algebra Skills	2 Weeks
10		
	Number	
11	Chapter 5: Ratios and Rates	3 Weeks
12		
13		
14	Chapter 6: Consumer Maths	3 Weeks
15		
16		
	Statistics and Probabilty	
17	Chapter 7: Statistics	3 Weeks
18		
19		
20	*Mid-year Examinations*	1 Week
Mid-year Holidays		

Semester 2

Weeks	Sub-strands	Allocated Times
	Trigonometry	
21	Chapter 8: Trigonometry	3 Weeks
22		
23		
	Algebra	
24	Chapter 9: Linear Equations and Formulas	2 Weeks
25		
26	Chapter 10: Linear Graphs	3 Weeks
27		
	Statistics and Probabilty	
28	Chapter 11: Probability	3 Weeks
29		
30		
	Number	
31	Chapter 12: Indices	3 Weeks
32		
33	*Final Examinations*	1 Week
End-of-year Holidays		

CHAPTER 1

Number Systems

Pythagoras (569–500 BC) was born on the island of Samos and travelled widely through the Middle East and Egypt. He founded a secret spiritual group, the Pythagorean Brotherhood, who believed that ‘number rules the universe’. They used whole numbers, but when they discovered irrational numbers such as $\sqrt{2}$, which couldn’t be expressed as repeating decimals, they realised that these neverending numbers, which they called *alogon* meaning ‘unutterable’, could cause trouble to their beliefs. Any member of the Brotherhood who dared to mention these numbers to the public was put to death. It was to be 200 years before Eudoxus developed a way to work with these numbers.

This chapter covers the following skills:

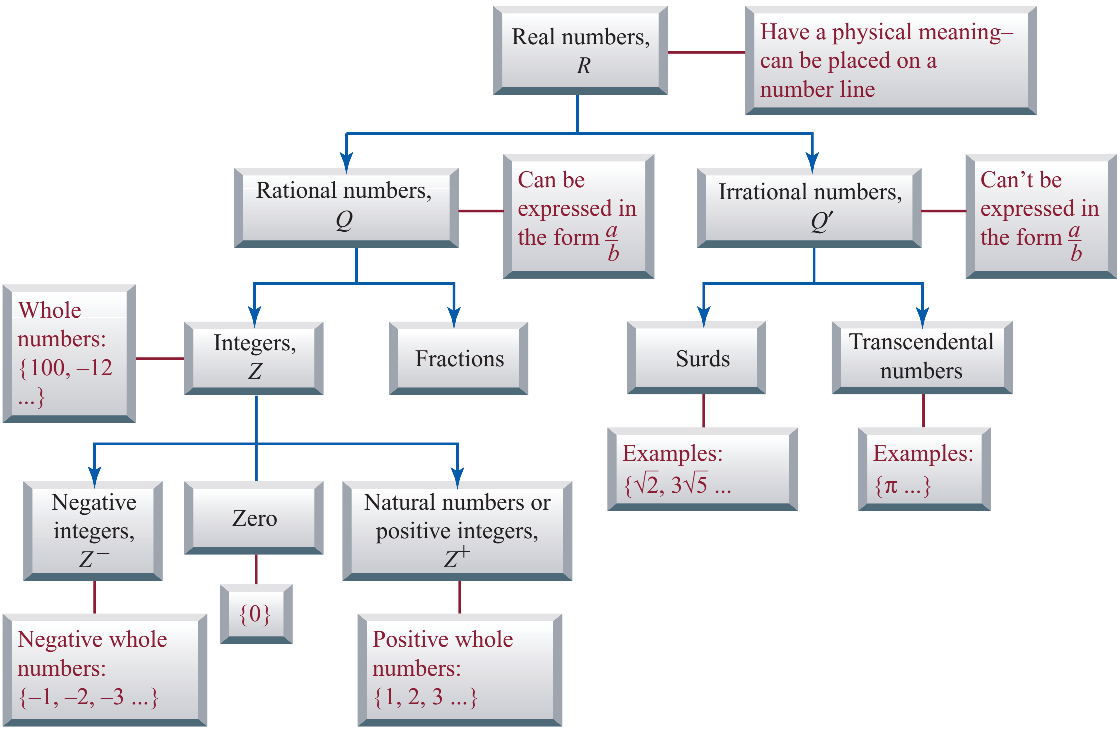
- Working with number systems
  Real numbers can be placed on a number line
  Rational numbers can be expressed in the form
  $\frac{a}{b}$ where $a, b \in Z$
  Irrational numbers cannot be expressed in the form $\frac{a}{b}$ where $a, b \in Z$
  Integers are whole numbers
- Evaluating the squares and square roots of numbers
- Working with and evaluating operations involving the square roots of non-square numbers (surds)
- Estimating the size of surds
- Simplifying surds
  $a\sqrt{b} \times c\sqrt{d} = ac\sqrt{bd}$
- Working with surds using the four operations
- Rationalising the denominators of surds
  $\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$

Specific Learning Outcome (SLO)

Learners should be able to:

9.1.1.1	Identify and define the different types of numbers that make up the Real Number System: *rational numbers*; *irrational numbers*; *integers*; *positive integers* or *natural numbers*; *negative integers*; *surds*; *transcendental numbers*.
9.1.2.1	Define 'natural numbers'.
9.1.2.2	Identify natural numbers and classes of natural numbers.
9.1.3.1	Simplify integers according to the Double Sign Operational Rules: *Same Signs = Positive* *Different Signs = Negative*
9.1.4.1	Evaluate problems that involve combinations of signs using the BODMAS and BIDMAS rules.
9.1.4.2	Find prime factors, common factors and highest common factors of whole numbers.
9.1.5.1	Identify and divide numbers that do **not** have remainders using the Divisibility Rules.
9.1.6.1	Evaluate expressions by substituting in values to replace pronumerals.
9.1.7.1	Define and identify 'rational numbers'.
9.1.8.1	Express numbers as: *proper fractions*; *improper fractions*; *equivalent fractions*; *mixed numbers*.
9.1.8.2	Evaluate expressions involving fractions with pronumerals.
9.1.9.1	Find estimates for rational numbers.
9.1.10.1	Define 'decimal numbers'.
9.1.10.2	Identify decimal numbers that are terminating or repeating.
9.1.10.3	Divide fractions to find decimals that terminate or repeat.
9.1.11.1	Convert fractions to decimals.
9.1.11.2	Evaluate rational numbers that are decimals.
9.1.12.1	Evaluate rational numbers by squaring.
9.1.12.2	Evaluate rational numbers by taking square roots.
9.1.13.1	Define 'surds' an example of an irrational number.
9.1.13.2	Calculate the square roots of numbers to a given number of decimal places.
9.1.14.1	Estimate the size of a surd to the nearest whole number.
9.1.14.2	Indicate the positions of estimated surds on a number line.
9.1.15.1	Simplify surds by splitting the surd into factors and identifying any perfect squares.
9.1.16.1	Express simplified surds as entire surds.
9.1.17.1	Group like surds.
9.1.17.2	Simplify irrational numbers by adding and subtracting like surds.
9.1.18.1	Evaluate surds involving multiplication of numbers outside the square root sign first, followed by the numbers inside the square root sign.
9.1.19.1	Expand and simplify surds involving brackets.
9.1.20.1	Divide surds by writing them as fractions and cancelling common factors.
9.1.21.1	Rationalise the surd denominator of a fraction by multiplying the numerator and the denominator by an appropriate surd.
9.1.22.1	Rationalise the denominators of surd fractions before simplifying.

1A The real number system



Number	Definition	Examples
Rational numbers	Expressed in the form $\frac{a}{b}$ where $a, b \in Z$	$\left\{-1, 2, 0, \frac{2}{3}, 2{\cdot}\dot{5}, -5\frac{3}{4}\right\}$
Irrational numbers	Cannot be expressed in the form $\frac{a}{b}$	$\{\sqrt{2}, -3\sqrt{5}, 2\pi\}$
Integers	Whole numbers	{–2, –11, 5, 12}
Positive integers or natural numbers	Positive whole numbers	{1, 4, 16, 81}
Negative integers	Negative whole numbers	(–2, –5, –12, –78}
Surds	The square root of a non-square number	$\left\{\sqrt{2}, 3\sqrt{7}, \frac{2}{3}\sqrt{5}, \frac{2\sqrt{11}}{9}, \sqrt{2}+1\right\}$
Transcendental numbers	A non-algebraic number	$\left\{2\pi, \frac{\pi}{3}, e, \frac{e}{4}\right\}$

Rational numbers as decimals are either terminating, e.g. $\frac{1}{5} = 0{\cdot}2$, or repeating, e.g. $\frac{2}{3} = 0{\cdot}666666666\ldots$

Note: 0.66666... is usually written as $0{\cdot}\dot{6}$.

Irrational numbers such as $\sqrt{2} = 1{\cdot}414\,213\,562\ldots$ do not repeat or terminate—their decimal parts continue on forever without any repetition. They are expressed in decimal form correct to a number of decimal places, depending on the accuracy of calculations.

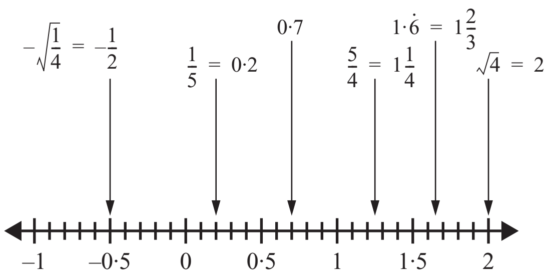
Example

Place the following numbers on a number line:

a 0·7 **b** $\frac{1}{5}$ **c** $\frac{5}{4}$

d $\sqrt{4}$ **e** $1{\cdot}\dot{6}$ **f** $-\sqrt{\frac{1}{4}}$

Solution



Exercise 1A

1 List the first 10:

a positive integers **b** negative integers

2 List the first eight natural numbers.

3 Write five examples of:

a surds **b** transcendental numbers

4 Find the sum of the first 50:

a natural numbers **b** negative integers

5 Examine the truth of the following statements:

a All integers are real numbers but not all real numbers are integers.

b All integers are whole numbers but not all whole numbers are integers.

c All fractions are rational numbers.

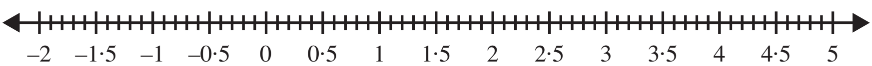
d Zero is not a rational number as it cannot be expressed in the form $\frac{a}{b}$.

e All integers are rational but not all rational numbers are integers.

6 Show the following numbers on the number line and so show them to be real numbers:

a $2\frac{1}{2}$ **b** $-3\frac{3}{4}$ **c** $4\frac{2}{3}$ **d** $\frac{3}{5}$ **e** $-1\frac{1}{4}$

f –1·3 **g** $\frac{2}{3}$ **h** 2·8 **i** 0·05 **j** –4·4



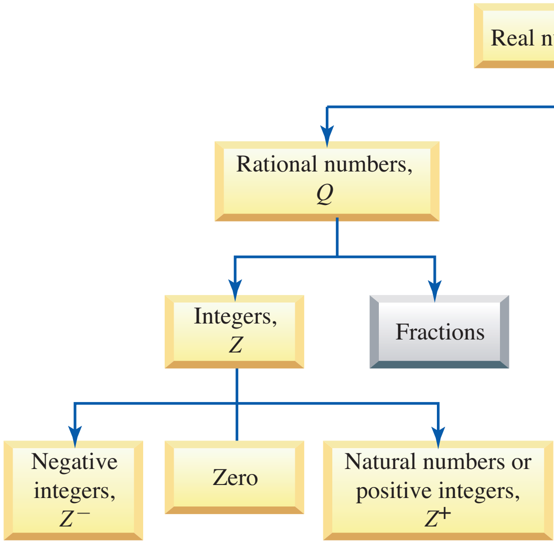
7 Show the following numbers on a number line and so show them to be real numbers:

a $\sqrt{16}$ **b** $\sqrt{\frac{1}{16}}$ **c** $-\sqrt{\frac{36}{4}}$ **d** $\sqrt{\frac{4}{9}}$ **e** $\frac{1}{\sqrt{100}}$

f $-\sqrt{1}$ **g** $-\sqrt{\frac{4}{9}}$ **h** $-\sqrt{4}$ **i** $\sqrt{6\frac{1}{4}}$ **j** $-\sqrt{2\frac{1}{4}}$

1B Natural numbers and integer arithmetic

Natural numbers are positive whole numbers but integers can be positive or negative whole numbers. Arithmetic operations with these numbers can be carried out without using calculators by using algorithms studied to up to and including a Year 9 standard.



Negative integer arithmetic

When working with negative integers and negative signs, certain rules need to be applied.

Double signs can be combined to make a single sign

When the signs are the same, they can be combined to make a positive sign.

$$\left.\begin{matrix} + \; + \\ - \; - \end{matrix}\right\} +$$

When the signs are different, they can be combined to make a negative sign.

$$\left.\begin{matrix} + \; - \\ - \; + \end{matrix}\right\} -$$

Example

1 Simplify:

a $2 - {}^{-}5$

b ${}^{-}3 + {}^{+}8$

c $1 + {}^{-}9$

d ${}^{-}7 - {}^{+}6$

Solution

$2 - {}^{-}5 = 2 + 5 = 7$

${}^{-}3 + {}^{+}8 = {}^{-}3 + 8 = 5$

$1 + {}^{-}9 = 1 - 9 = {}^{-}8$

${}^{-}7 - {}^{+}6 = {}^{-}7 - 6 = {}^{-}13$

Multiplication and division of directed numbers

When the signs are the same, a positive sign results.

$$\left.\begin{matrix} + \times + \\ - \times - \end{matrix}\right\} +$$

When the signs are different, a negative sign results.

$$\left.\begin{matrix} + \times - \\ - \times + \end{matrix}\right\} -$$

Example **Solution**

2 Simplify:

a $^{+}7 \times {}^{+}3$	$^{+}7 \times {}^{+}3 = 21$
b $^{-}6 \times {}^{-}2$	$^{-}6 \times {}^{-}2 = 12$
c $^{+}3 \times {}^{-}8$	$^{+}3 \times {}^{-}8 = {}^{-}24$
d $^{-}5 \times {}^{+}3$	$^{-}5 \times {}^{+}3 = {}^{-}15$

Order of operations

The four operations of addition, subtraction, division and multiplication can be used with these numbers as well as the use of brackets with or without the use of calculators.

The order in which operations are performed is very important—brackets first, then multiplication or division and finally addition or subtraction.

Example **Solution**

3 Calculate the following by hand and check using a calculator:

a $3 \times 16 + 19$

$3 \times 16 + 19$
$= 3 \times 16$
$= 48 + 19$
$= 67$

b $\dfrac{112(3 - 18)}{5}$

$\dfrac{112(3 - 18)}{5}$
$= \dfrac{112(-\cancel{15})^{3}}{\cancel{5}^{1}}$
$= 112(-3)$
$= -336$

c $(1 - 7)^2 - 18 \times 11$

$(1 - 7)^2 - 18 \times 11$
$= (-6)^2 - 198$
$= (-6 \times -6) - 198$
$= 36 - 198$
$= -162$

Other whole number facts are useful to know when working with whole numbers, such as whether whole numbers are prime (have only two factors) or composite (have more than two factors). Knowing the rules for divisibility is also very useful.

For example, the number 60 is not prime and its factors are {1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60}.

Example

4 List the factors of the following numbers and so state as to whether each is a prime or a composite number:

a 5

b 12

5 State the prime factors of 360.

Solution

4a $5 = 5 \times 1$

The number 5 has 2 factors, itself and 1, so it is a prime number.

4b $12 = 1 \times 12 = 2 \times 6 = 3 \times 4$

The factors of 12 are $\{1, 2, 3, 4, 6, 12\}$.

The number 12 has more than two factors, so it is a composite number.

5 $360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$

$= 2^3 \times 3^2 \times 5$

Divisibility rules

Number	Rule
1	All whole numbers are divisible by 1
2	Even numbers are divisible by 2
3	The sum of the digits is a multiple of 3
4	The last two digits are divisible by 4
5	Numbers end with a 5 or zero
6	Even numbers divisible by 3
7	No rule possible
8	The last three digits are a number that is divisible by 8
9	The sum of its digits is divisible by 9
10	Numbers end with zero

Example

6 Show why 6732 is divisible by 6.

7 State the prime factors for 120 and 84 in index form and so find the highest common factor for 120 and 84.

Solution

6 6732 is an even number, so 6732 is divisible by 2.

$6 + 7 + 3 + 2 = 18$, which is divisible by 3, so 6732 is divisible by 3.

As 6732 is divisible by 2 and 3, 6732 must be divisible by 2×3, i.e. it is divisible by 6.

7 $120 = 2^3 \times 3 \times 5$ $\quad$ $84 = 2^2 \times 3 \times 7$

The factors common to both numbers are $2^2 \times 3 = 12$, so 12 is the highest common factor.

Exercise 1B

Calculate these questions by hand and then check answers with a classmate.

1 For the natural numbers, find the sum and the average of the first:

a 3 b 4 c 5

2 For the negative integers, find the sum of the first:

a 5 b 7 c 10

3 The sum of three consecutive numbers is 15. Find the numbers.

4 The sum of four consecutive numbers is 34. Find the numbers.

5 For the natural numbers, pair numbers to find the sum of the first:

a 10 b 20 c 50 d 100 e 1001

6 The average of three of Lucinda's tests is 74. When the fourth mark is included her average is 76. Find the score of her fourth test.

7 The average of 2, 6 and a third number is $^{-}4$. Find the third number.

8 Find the number of positive integers between 1 and 100 inclusive that are divisible by:

a 2 b 3 c 4 d 5 e 6
f 7 g 8 h 9 i 10 j 12

9 For a four-digit number of the form $4a1b$, list all the combinations of a and b such that the number is divisible by 3.

10 Write down three five-digit numbers that are divisible by the following numbers and prove each by showing the appropriate division:

a 3 b 4 c 6 d 8 e 9
f 2 and 5 g 3 and 4 h 3 and 8 i 3 and 5 j 8 and 9

11 List the first three numbers that have:

a 2 factors b 3 factors c 4 factors d 6 factors

12 Find the sum of the first five multiples of:

a 12 b 14 c 20 d 42 e 105

13 State the prime factors of:

a 48 b 216 c 2250 d 900 e 3528

14 Find the highest common factor for:

a 36 and 48 b 32 and 112 c 216 and 120

15 Evaluate the following:

a $^{-}12 - 3 + 5$ b $^{-}2 + {}^{-}6 - 12$ c $5 - 2 + {}^{-}8$
d $^{-}2 \times {}^{-}6$ e $4 \times {}^{-}11$ f $3 \times {}^{-}7 \times 4$
g $^{-}7(4 - 10)$ h $3({}^{-}4 + 7 - 12)$ i $({}^{-}12 + 5 - 7) \div {}^{-}2$
j $^{-}60 \div ({}^{-}3 + 7)$ k $({}^{-}11 - 17) \div {}^{-}7$ l $^{-}5(4 - 8 + {}^{-}3)$

16 Evaluate the following:

a $^{-}2 + ^{-}19 - 11 - ^{-}22$

b $^{-}9 - ^{-}24 + ^{-}12$

c $^{-}14 + ^{-}12 - 20$

d $3(16 - 5) + 2(3 + 6)$

e $3(8 + 9) - 2(17 - 9)$

f $7 + (45 - 1) \div 11$

g $100 - 5(2 + 11)$

h $^{-}2 + 5(2 + 8) \div ^{-}2$

i $^{-}12 \div ^{-}3 + 6 \times 4 - 11$

j $4(8 - 5) + ^{-}2(3 - 12) + 14$

k $^{-}3(2 - 9) + 12 \div (4 - 1) - 10$

l $(11 - 4) \times ^{-}3 + 8 \div ^{-}2 + 11$

m $^{-}6(2 - 5) + (12 + 4) \div ^{-}2$

n $4 + 15(7 - 2 + 11) \div ^{-}8$

17 Emeli Daudau is 30 years old and has two children at the village school.
The age of each child is a factor of Emeli's age.
The sum of their ages is also a factor of Emeli's age. They are not twins.
If Katy is the younger child, and Lencia the elder, how old are they?

18 Evaluate the following:

a $181\,881 \div 7$

b $31\,068 \div 12$

c $123\,837 \div 21$

d $455\,355 \div 15$

e $155\,245 \div 6$

f $638\,277 \div 11$

19 Evaluate the following:

a $\dfrac{3096}{12} - \dfrac{336}{3}$

b $\dfrac{3}{4}(10\,032) + \dfrac{2}{5}(12\,945)$

c $\dfrac{1}{15}(30\,225) - 31 \times 16$

d $\dfrac{2}{3} \times \dfrac{4}{5}(1050)$

e $\dfrac{3}{8}$ of $16\,120 + \dfrac{2}{3}$ of 6315

f $\dfrac{5}{8} \times 470\,184 - \dfrac{4}{9} \times 48\,780$

20 Evaluate the following:

a $^{-}2 \times ^{-}4(12 - 329)$

b $16 \times ^{-}3(23 - 42)$

c $14 - (^{-}12 - 13) \div ^{-}5$

d $^{-}2(12 - 23 - 14) + 55 \div ^{-}11$

e $\dfrac{^{-}12(2 + 13 - 40)}{^{-}60}$

f $(22 - 12 - 3 + 14) \div ^{-}3 + 15$

21 Eggs with an average weight of 67 grams are packed into crates that each hold two gross of eggs (1 gross = 144). If each crate weighs 3512 grams, find the total weight in grams of 12 crates full of eggs.

Substitution with whole numbers 1C

In algebra pronumerals are used to represent numbers. The expression $2a$ means twice the value of the number represented as the letter a. The symbols for half of a are $\frac{1}{2}a$, $\frac{a}{2}$, $a \div 2$, $0{\cdot}5a$ and so on.

Example

If $a = -10$, $b = 4$ and $c = -2$, evaluate the following:

a $2a + 3b - c$

b $2a^2 - 3 + 5bc$

c $\frac{2}{5}a + 4b - c$

Solution

a
$2a + 3b - c$
$= 2 \times -10 + 3 \times 4 - (-2)$
$= -20 + 12 + 2$
$= -6$

b
$2a^2 - 3 + 5bc$
$= 2(-10)^2 - 3 + 5 \times 4 \times -2$
$= 200 - 3 - 40$
$= 157$

c
$\frac{2}{5}a + 4b - c$
$= \frac{2}{5} \times -10 + 4 \times 4 - -2$
$= -4 + 16 + 2$
$= 14$

Exercise 1C

Use substitution to calculate the answers to these questions.

1 If $a = -2$, $b = -3$ and $c = 5$, evaluate:
a $a + b - c$ b $-2a + b + 3c$ c $-a - b + 4c$ d $-9b + 4c$

2 If $a = -1$, $b = 2$ and $c = -6$, evaluate:
a $a - b + c$ b $-3a - 2b + c$ c $4a + 2b - 3c$ d $a + 5b - c$

3 If $a = -1$, $b = 5$ and $c = -3$, evaluate:
a $2(a + b) - c$ b $-2(a - b) + 3c$ c $-3(a + 4c) + 11$ d $-2a(b + c) + 10$

4 If $a = 10$, $b = -4$ and $c = -2$, evaluate:
a $a^2 + b^2 - c^2$ b $-2a^2 + b - c^2$ c $a^2 - b + c^2$ d $a - 2b + 4c^2$

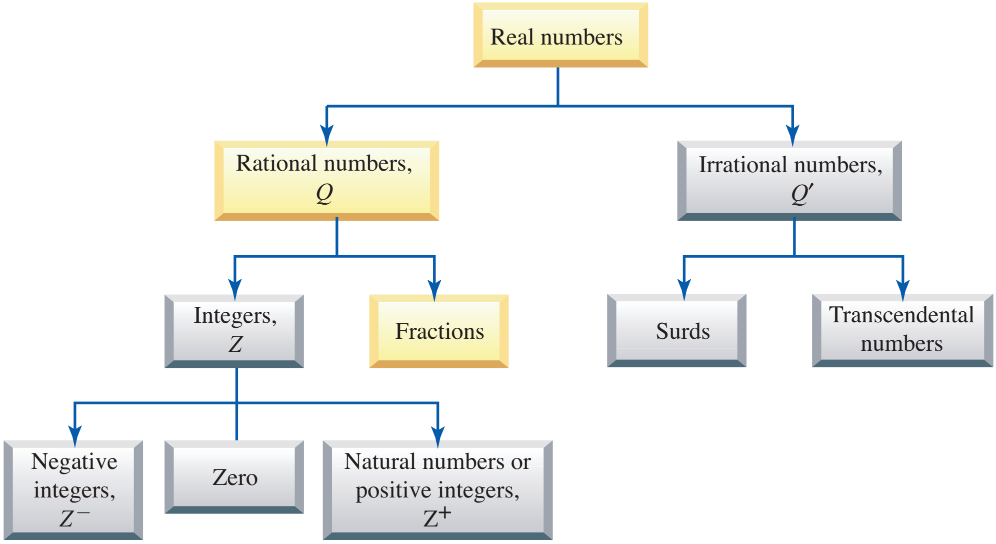
5 If $a = 4$, $b = 5$ and $c = -3$, evaluate:
a $2ab$ b abc c $2abc + ab$ d $-3ac + 2ab - bc$

6 If $a = 2b - b^2$, find the value of a when:
a $b = 2$ b $b = -2$ c $b = 6$ d $b = -8$

7 If $a = 2(1 - 3b^2)$, find without using a calculator, the value of a when:
a $b = 1$ b $b = -5$ c $b = -10$ d $b = 9$

1D Rational number arithmetic: Fractions

Rational numbers are those that can be expressed in the form $\frac{a}{b}$ or in fractional form. It is important to be able to work with them both with and without a calculator.



Example

1 Find the value of a to make the following equivalent fractions:

a $\frac{3}{5} = \frac{a}{20}$

b $\frac{7}{4} = \frac{49}{a}$

2 Write $\frac{118}{3}$ as a mixed fraction.

3 Write $3\frac{3}{8}$ as an improper fraction.

Solution

1a $\frac{3}{5} \times \frac{4}{4} = \frac{12}{20}$

$\therefore a = 12$

1b $\frac{7}{4} \times \frac{7}{7} = \frac{49}{28}$

$\therefore a = 28$

2 $3\overline{)118}$ = 39 Remainder 1

$\therefore 39\frac{1}{3}$

3 $3\frac{3}{8} = \frac{8 \times 3 + 3}{8} = \frac{24 + 3}{8} = \frac{27}{8}$

Example **Solution**

4 Evaluate:

a $\frac{2}{7}+\frac{3}{8}$

$$\frac{2}{7}\times\frac{8}{8}+\frac{3}{8}\times\frac{7}{7}=\frac{16+21}{56}$$
$$=\frac{37}{56}$$

b $\frac{\pi}{4}+\frac{3\pi}{5}$

$$\frac{\pi}{4}\times\frac{5}{5}+\frac{3\pi}{5}\times\frac{4}{4}=\frac{5\pi+12\pi}{20}$$
$$=\frac{17\pi}{20}$$

c $4\frac{1}{2}-\frac{1}{5}-\frac{5}{8}$

$$4\frac{1}{2}-\frac{1}{5}-\frac{5}{8}=\frac{9}{2}-\frac{1}{5}-\frac{5}{8}$$
$$=\frac{180}{40}-\frac{8}{40}-\frac{25}{40}$$
$$=\frac{147}{40}$$
$$=3\frac{27}{40}$$

d $4\frac{1}{4}\times 6\frac{3}{5}\div\frac{2}{5}$

$$4\frac{1}{4}\times 6\frac{3}{5}\div\frac{2}{5}=\frac{17}{4}\times\frac{33}{5}\div\frac{2}{5}$$
$$=\frac{17}{4}\times\frac{33}{5}\times\frac{5}{2}$$
$$=\frac{561}{8}$$
$$=70\frac{1}{8}$$

Estimation

The value of calculations can be estimated by rounding each number to an appropriate whole number. It is important to be able to estimate calculations to check for possible errors.

Example **Solution**

5 Estimate the answer to each of the following and then find the exact answer using a calculator:

a $3\frac{1}{4}(9\frac{3}{4}+1\frac{1}{5})$

$3\frac{1}{4}\approx 3,\ 9\frac{3}{4}\approx 10,\ 1\frac{1}{5}\approx 1$

$$\therefore\ 3\frac{1}{4}(9\frac{3}{4}+1\frac{1}{5})\approx 3(10+1)$$
$$\approx 3\times 11$$
$$\approx 33$$

1D

Example

b $\dfrac{5\frac{2}{3} \times 8\frac{1}{8} - 2\frac{5}{7}}{1\frac{9}{11}}$

Solution

$5\frac{2}{3} \approx 6,\ 8\frac{1}{8} \approx 8,\ 2\frac{5}{7} \approx 3,\ 1\frac{9}{11} \approx 2$

$$\therefore \frac{5\frac{2}{3} \times 8\frac{1}{8} - 2\frac{5}{7}}{1\frac{9}{11}} \approx \frac{6 \times 8 - 3}{2}$$
$$\approx \frac{45}{2}$$
$$\approx 22{\cdot}5$$

Exercise 1D

Evaluate the following questions and check your answers with a classmate.

1 Complete equivalent fractions by finding the missing number, shown as a:

a $\frac{3}{5} = \frac{a}{20}$ b $\frac{2}{3} = \frac{a}{21}$ c $\frac{3}{4} = \frac{a}{24}$ d $\frac{1}{6} = \frac{a}{36}$

e $\frac{3}{7} = \frac{9}{a}$ f $\frac{8}{7} = \frac{24}{a}$ g $\frac{5}{7} = \frac{25}{a}$ h $\frac{7}{3} = \frac{42}{a}$

2 Write the following mixed numbers as improper fractions:

a $3\frac{1}{4}$ b $1\frac{3}{8}$ c $4\frac{2}{7}$ d $12\frac{1}{4}$

e $5\frac{2}{3}$ f $8\frac{5}{7}$ g $3\frac{4}{5}$ h $2\frac{4}{9}$

3 Write the following improper fractions as mixed numbers:

a $\frac{11}{3}$ b $\frac{13}{5}$ c $\frac{41}{3}$ d $\frac{53}{5}$

e $\frac{111}{2}$ f $\frac{35}{4}$ g $\frac{124}{7}$ h $\frac{145}{3}$

4 Evaluate:

a $\frac{1}{3} + \frac{2}{3} + \frac{3}{3}$ b $\frac{1}{4} + \frac{2}{4} + \frac{3}{4} + \frac{4}{4}$ c $\frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5} + \frac{5}{5}$

5 Write the following as a single fraction in simplest form:

a $\frac{1}{2} + \frac{1}{3}$ b $\frac{2}{5} + \frac{1}{4}$ c $\frac{1}{5} + \frac{2}{15}$ d $\frac{1}{7} + 3\frac{2}{3}$

e $1\frac{1}{4} - \frac{2}{3}$ f $3\frac{3}{10} - 1\frac{2}{5}$ g $4\frac{1}{4} + 1\frac{3}{5}$ h $3\frac{2}{3} - \frac{1}{2} - \frac{3}{4}$

6 Complete the following additions:

a $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$ b $\frac{2}{3}+\frac{3}{4}+\frac{4}{5}$ c $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}$ d $\frac{\pi}{2}+\frac{\pi}{5}$

e $\frac{\pi}{2}+\frac{\pi}{3}$ f $\frac{\pi}{4}+\frac{\pi}{3}$ g $\frac{2\pi}{3}+\frac{\pi}{4}$ h $\frac{5\pi}{3}+\frac{\pi}{4}$

7 Complete the following subtractions:

a $\frac{5}{8}-\frac{1}{5}$ b $\frac{2}{3}-\frac{1}{12}$ c $\frac{5}{7}-\frac{1}{2}$ d $\frac{\pi}{2}-\frac{\pi}{4}$

e $\frac{\pi}{3}-\frac{\pi}{4}$ f $\frac{\pi}{4}-\frac{\pi}{5}$ g $\frac{2\pi}{3}-\frac{\pi}{4}$ h $\frac{2\pi}{5}-\frac{\pi}{3}$

8 Estimate the answer and evaluate the following without using a calculator:

a $\frac{2}{3}\times\frac{5}{8}\div\frac{1}{4}$ b $\frac{2}{3}\div\frac{4}{5}\times\frac{9}{11}$ c $2\frac{1}{5}\times 3\frac{3}{4}$ d $5\frac{1}{4}\times\frac{2}{3}\div\frac{4}{7}$

e $3\frac{1}{2}\times 4\frac{2}{5}\div\frac{4}{5}$ f $2\left(1\frac{2}{5}+3\frac{1}{4}\right)$ g $3\frac{1}{3}\left(2\frac{2}{5}-5\frac{3}{4}\right)$ h $\left(1\frac{5}{8}+2\frac{1}{3}\right)\times 5\frac{1}{4}$

9 Complete these grids:

a

$+$		$\frac{1}{4}$	$\frac{1}{5}$	
$\frac{1}{2}$				$\frac{9}{14}$
		$\frac{11}{12}$		
	$1\frac{1}{12}$		$\frac{19}{20}$	
		$\frac{17}{20}$		

b

$\times$		$\frac{3}{4}$	$\frac{4}{5}$	
$\frac{1}{4}$				$\frac{1}{10}$
		$\frac{1}{4}$		
	$\frac{4}{9}$		$\frac{8}{15}$	
		$\frac{3}{5}$		

10 Evaluate:

a $\frac{2}{3}\times\frac{3}{4}$ b $\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}$ c $\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}$

d $\frac{2}{3}\div\frac{4}{9}$ e $\frac{3}{4}\div\frac{1}{8}$ f $\frac{3}{5}\times\frac{2}{3}\div\frac{4}{5}$

g $1\frac{1}{2}\times\frac{4}{5}$ h $2\frac{1}{4}\times\frac{2}{3}\div 1\frac{1}{8}$ i $\dfrac{3\frac{1}{2}\times\frac{1}{14}}{2\frac{3}{4}}$

11 Estimate the answer and evaluate the following without using a calculator:

a $\dfrac{1\frac{1}{2} + 2\frac{2}{3}}{1\frac{1}{4}}$ b $\dfrac{2(2\frac{1}{4} - 1\frac{1}{2})}{(1\frac{1}{6} - \frac{1}{2})}$ c $\dfrac{4(3\frac{3}{4} + \frac{1}{8})}{7\frac{3}{4}}$

12 Simplify:

a $\dfrac{\frac{1}{2} + \frac{2}{3}}{\frac{2}{3} - \frac{1}{2}}$ b $\dfrac{1 - \frac{1}{3}}{1 + \frac{1}{5}}$ c $\dfrac{1}{1 + \frac{1}{1 + \frac{1}{2}}}$

13 Find the fraction which is halfway between:

a $\frac{3}{4}$ and $\frac{1}{2}$ b $\frac{1}{2}$ and $\frac{1}{6}$ c $\frac{2}{3}$ and $\frac{1}{5}$ d $\frac{1}{4}$ and $\frac{4}{5}$

14 If $\frac{1}{a} = \frac{1}{b} + \frac{1}{c}$, find a when $b = \frac{2}{3}$ and $c = 1\frac{1}{4}$.

15 If $\frac{2}{3}$ of my pocket money is \$5, how much pocket money do I get?

16 A tank that is $\frac{4}{5}$ full contains 2000 litres. Find the capacity of the tank.

17 A tank that is initially full loses 400 litres. If it is now $\frac{4}{5}$ full, find the capacity of the tank.

18 A tank when a quarter full contains $10\frac{1}{4}$ litres. How much water will it hold when it is $\frac{2}{3}$ full?

19 Find two numbers with the following properties:

a a sum of $\frac{9}{10}$ and a difference of $\frac{1}{10}$ b a sum of $\frac{9}{10}$ and a difference of $\frac{7}{10}$

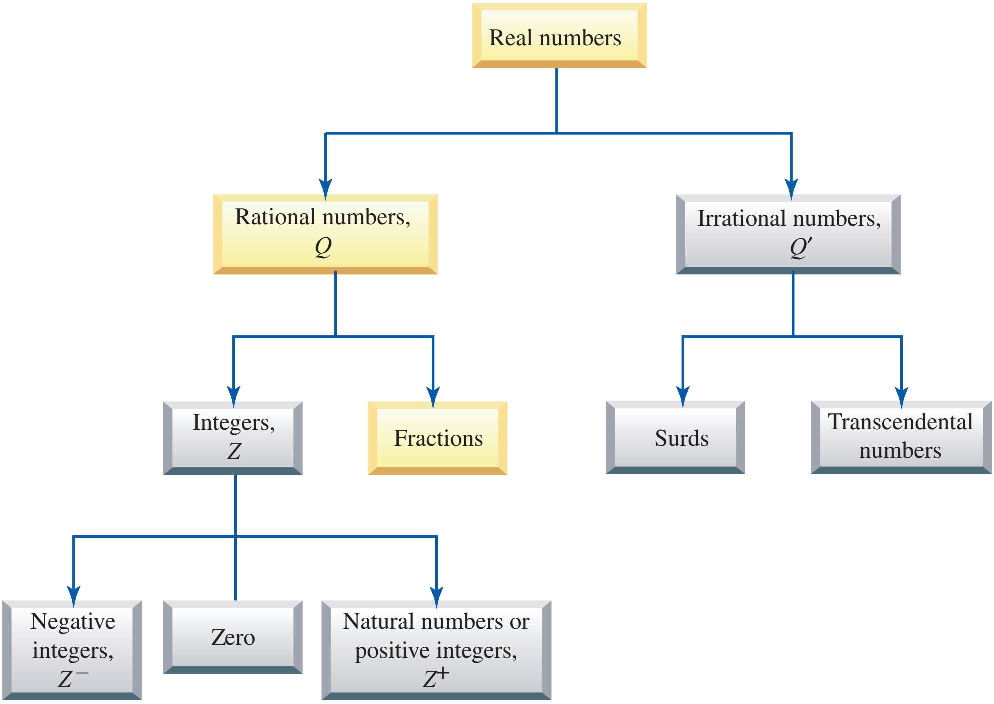
c a sum of 1 and a product of $\frac{2}{9}$ d a sum of $\frac{17}{4}$ and a product of 1

20 Calculate the following without using a calculator:

a $\dfrac{\frac{1}{2} + \frac{3}{4}}{1 - \frac{1}{2} \times \frac{3}{4}}$ b $\dfrac{2 \times 3 \times 5 \times 8}{4 \times 10 \times 12}$ c $\dfrac{\frac{3}{4}}{15}$ d $\dfrac{1}{\sqrt{6 + (\frac{1}{2})^2}}$

Rational number arithmetic: Decimals

Rational numbers in the form $\frac{a}{b}$ can be expressed in decimal form by dividing the numerator by the denominator. Some fractions terminate exactly and others repeat in a pattern. Decimal numbers can be rounded off to a specified number of decimal places.



Example

1 Write $3\frac{6}{11}$ as a decimal number, expressed:

i in recurring form

ii correct to 2 decimal places

Solution

$$3\frac{6}{11} = \frac{39}{11}$$

$$\begin{array}{r} 3\cdot\ 5\ 4\ 5\ 4\ldots \\ 11\,\overline{)\,39\cdot{}^{6}0{}^{5}0{}^{6}0{}^{5}0\ldots} \end{array}$$

$$\begin{aligned} 3\frac{6}{11} &= 3\cdot5454\ldots \\ &= 3\cdot\overline{54} \\ &\approx 3\cdot54 \end{aligned}$$

1E

Example | **Solution**

2 Evaluate each of the following:

i without using a calculator

ii using a calculator

a $4{\cdot}09 + 0{\cdot}214$

$$\begin{array}{r} 4{\cdot}09 \\ +\ 0{\cdot}2\overset{}{1}4 \\ \hline 4{\cdot}304 \\ \hline \end{array}$$

b $3{\cdot}014 \times 0{\cdot}6$

$$\begin{array}{r} 3014 \\ \times\quad 6 \\ \hline 18084 \\ \hline \end{array}$$

$\therefore\ 3{\cdot}014 \times 0{\cdot}6 = 1{\cdot}8084$

c $5{\cdot}472 \div 1{\cdot}2$

$\frac{5{\cdot}472}{1{\cdot}2} = \frac{54{\cdot}72}{12}$

$$\begin{array}{r} 4{\cdot}\ 5\ 6 \\ 12\overline{)54{\cdot}{}^{6}7{}^{7}2} \end{array}$$

$\therefore\ 5{\cdot}472 \div 12 = 4{\cdot}56$

3 Express $0{\cdot}\overline{31}$ in fractional form:

a by hand

b using a calculator

$$\begin{aligned} 0{\cdot}\overline{31} &= 0{\cdot}31313131\ldots \\ 100 \times 0{\cdot}\overline{31} &= 31{\cdot}313131\ldots \\ 99 \times 0{\cdot}\overline{31} &= 31 \\ 0{\cdot}\overline{31} &= \frac{31}{99} \end{aligned}$$

Exercise 1E

1 Write the following fractions as decimals:

a $\frac{5}{8}$ b $\frac{7}{8}$ c $\frac{5}{11}$ d $\frac{5}{12}$ e $\frac{7}{12}$

2 Convert this set of ninths to decimals and comment on your findings: $\left\{\frac{1}{9}, \frac{2}{9} \ldots \frac{9}{9}\right\}$

3 Convert this set of elevenths to decimals and comment on your findings: $\left\{\frac{1}{11}, \frac{2}{11} \ldots \frac{11}{11}\right\}$

4 Express these recurring decimals in fractional form:

a $1{\cdot}\overline{12}$ b $1{\cdot}\overline{23}$ c $3{\cdot}\overline{43}$ d $6{\cdot}\overline{09}$

5 Evaluate the following:

a $90{\cdot}23 \times 0{\cdot}01$ b $0{\cdot}82 \times 0{\cdot}1$ c $345 \times 0{\cdot}001$ d $43{\cdot}02 \times 0{\cdot}010$

e $64{\cdot}981 \times 0{\cdot}1$ f $45{\cdot}38 \times 100$ g $1{\cdot}0334 \div 0{\cdot}01$ h $0{\cdot}0382 \div 0{\cdot}001$

6 Evaluate the following:

a $\dfrac{13\cdot456}{10}$ b $\dfrac{456\cdot2}{100}$ c $\dfrac{0\cdot2563}{1000}$ d $\dfrac{43\cdot46}{20}$

e $\dfrac{9\cdot6}{1\cdot2}$ f $\dfrac{14\cdot4}{1\cdot2}$ g $\dfrac{9\cdot9}{0\cdot11}$ h $\dfrac{0\cdot48}{0\cdot06}$

7 Given that $\dfrac{8}{90} = 0\cdot0\dot{8}$, express the following in fractional form:

a $\dfrac{8}{9}$ b $\dfrac{80}{9}$ c $\dfrac{8}{900}$ d $\dfrac{0\cdot8}{90}$

8 Calculate the following:

a $12 + 34\cdot2 + 3\cdot034$ b $123\cdot045 + 90\cdot360 + 0\cdot002$ c $7\cdot3 + 98\cdot02 + 1\cdot002$

d $86\cdot9404 - 13\cdot94$ e $452\cdot020 - 14\cdot6573$ f $3\cdot9439 - 1\cdot999\,99$

9 Find the average of the following numbers:

a 9·4, 5·12, 2·91 b 67·09, 8·54, 15·9, 32·87 c 98·5, 23·05, 7·23, 0·056, 9·004

10 Fill in the missing numbers in the spaces indicated:

a $\begin{array}{r} 6\cdot4\square76 \\ -\ 2\cdot594\square \\ \hline \square\cdot81\square7 \\ \hline \end{array}$ b $\begin{array}{r} 7\square108\cdot2 \\ -\ \square46\square4\cdot\square \\ \hline 25\square2\square\cdot6 \\ \hline \end{array}$ c $\begin{array}{r} \square64\square0\cdot8 \\ +\ 19\square39\cdot\square \\ \hline 8\square32\square\cdot6 \\ \hline \end{array}$ d $\begin{array}{r} 4\square90\cdot\square4 \\ +\ 43\square1\cdot9\square \\ \hline \square27\square\cdot70 \\ \hline \end{array}$

11 Power poles in a town are exactly 12·04 metres apart along a straight road. Find the total distance between:

a 12 poles b 13 poles

c 42 poles d 100 poles

e 2010 poles

12 A truck averages 21.03 litres of fuel per 100 kilometres. Find the amount of fuel that will be used for the following journeys:

- **i** to the nearest millilitre
- **ii** to the nearest litre

a 400 km	**b** 700 km
c 120 km	**d** 260 km
e 390 km	

13 Given that $21{\cdot}2 \div 1{\cdot}9 = 11{\cdot}157\,897\,74$, correct to 8 decimal places, find:

a $212 \div 1{\cdot}9$	**b** $21{\cdot}2 \div 19$	**c** $2{\cdot}12 \div 1{\cdot}9$	**d** $2{\cdot}12 \div 0{\cdot}19$
e $212 \div 19$	**f** $0{\cdot}212 \div 0{\cdot}19$	**g** $0{\cdot}212 \div 19$	**h** $0{\cdot}212 \div 190$

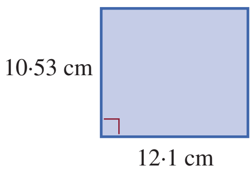
14 Express the following irrational numbers to:

- **i** 2 decimal places
- **ii** 3 decimal places
- **iii** 6 decimal places

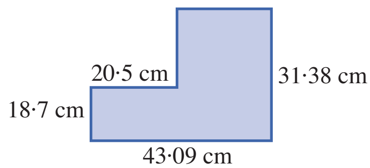
a $\sqrt{15} \cong 3{\cdot}872\,983\,346$ **b** $4\sqrt{3} \cong 6{\cdot}928\,203\,23$ **c** $\dfrac{\sqrt{11}}{5} \cong 0{\cdot}663\,324\,958\,1$

15 Find the perimeter and area of each of the following rectangles, giving the answer correct to the nearest whole number:

a



b



Square numbers and their square roots 1F

A **square number** is made by multiplying two identical numbers together.

Example **Solution**

1 Evaluate the following:

a 3 squared — $3^2 = 3 \times 3 = 9$

b 5 to the power of 2 — $5^2 = 5 \times 5 = 25$

c 4^2 — $4^2 = 4 \times 4 = 16$

d 12^2 — $12^2 = 12 \times 12 = 144$

This also is true for decimals and fractions.

Example **Solution**

2 Evaluate the following:

a $1{\cdot}3^2$ — $1{\cdot}3^2 = 1{\cdot}3 \times 1{\cdot}3 = 1{\cdot}69$

b $0{\cdot}01^2$ — $0{\cdot}01^2 = 0{\cdot}01 \times 0{\cdot}01 = 0{\cdot}0001$

c $\left(\frac{2}{3}\right)^2$ — $\left(\frac{2}{3}\right)^2 = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

d $\left(1\frac{5}{8}\right)^2$ — $\left(1\frac{5}{8}\right)^2 = \left(\frac{13}{8}\right)^2 = \frac{13}{8} \times \frac{13}{8} = \frac{169}{64} = 2\frac{41}{64}$

The opposite process to squaring a number is to take the square root of the number. If a number is a square, its square root is easily found.

Example **Solution**

3 Evaluate the following:

a $\sqrt{144}$ — $\sqrt{144} = 12$ because $12 \times 12 = 144$

b $2\sqrt{49}$ — $2\sqrt{49} = 2 \times 7 = 14$

c $\sqrt{0{\cdot}04}$ — $\sqrt{0{\cdot}04} = 0{\cdot}2$ because $0{\cdot}2 \times 0{\cdot}2 = 0{\cdot}04$

d $\sqrt{2\frac{1}{4}}$ — $\sqrt{2\frac{1}{4}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$ because $\frac{3}{2} \times \frac{3}{2} = \frac{9}{4}$

Exercise 1F

1 Evaluate the following:

a $9^2 =$ ____ b $16^2 =$ ____ c $28^2 =$ ____ d $87^2 =$ ____

e $101^2 =$ ____ f $2{\cdot}1^2 =$ ____ g $0{\cdot}4^2 =$ ____ h $9{\cdot}1^2 =$ ____

i $0{\cdot}13^2 =$ ____ j $12{\cdot}1^2 =$ ____ k $\left(\frac{3}{4}\right)^2 =$ ____ l $\left(\frac{2}{5}\right)^2 =$ ____

m $\left(\frac{1}{11}\right)^2 =$ ____ n $\left(\frac{5}{9}\right)^2 =$ ____ o $\left(\frac{2}{7}\right)^2 =$ ____ p $\left(2\frac{3}{4}\right)^2 =$ ____

q $\left(1\frac{1}{3}\right)^2 =$ ____ r $\left(3\frac{1}{6}\right)^2 =$ ____ s $\left(7\frac{4}{7}\right)^2 =$ ____ t $\left(2\frac{11}{12}\right)^2 =$ ____

2 Find the square root of the following square numbers:

a $\sqrt{16}$ b $\sqrt{9}$ c $\sqrt{64}$ d $\sqrt{49}$ e $\sqrt{100}$
f $\sqrt{121}$ g $\sqrt{169}$ h $\sqrt{196}$ i $\sqrt{225}$ j $\sqrt{256}$
k $\sqrt{441}$ l $\sqrt{625}$ m $\sqrt{1681}$ n $\sqrt{3844}$ o $\sqrt{7921}$
p $\sqrt{1764}$ q $\sqrt{2704}$ r $\sqrt{4624}$ s $\sqrt{9801}$ t $\sqrt{10\,816}$

3 Complete the following:

a $\sqrt{\_\_\_} = 9$ b $\sqrt{\_\_\_} = 4$ c $\sqrt{\_\_\_} = 5$ d $\sqrt{\_\_\_} = 6$
e $\sqrt{\_\_\_} = 10$ f $\sqrt{\_\_\_} = 13$ g $\sqrt{\_\_\_} = 14$ h $\sqrt{\_\_\_} = 27$
i $\sqrt{\_\_\_} = 32$ j $\sqrt{\_\_\_} = 41$ k $\sqrt{\_\_\_} = 65$ l $\sqrt{\_\_\_} = 74$

4 Complete the following:

a $\sqrt{\_\_\_} = 0{\cdot}3$ b $\sqrt{\_\_\_} = 0{\cdot}4$ c $\sqrt{\_\_\_} = 1{\cdot}2$ d $\sqrt{\_\_\_} = 2{\cdot}1$
e $\sqrt{\_\_\_} = 1{\cdot}1$ f $\sqrt{\_\_\_} = 2{\cdot}5$ g $\sqrt{\_\_\_} = 1{\cdot}4$ h $\sqrt{\_\_\_} = 5{\cdot}1$
i $\sqrt{\_\_\_} = 2{\cdot}9$ j $\sqrt{\_\_\_} = 4{\cdot}7$ k $\sqrt{\_\_\_} = 5{\cdot}2$ l $\sqrt{\_\_\_} = 8{\cdot}2$

5 Complete the following:

a $\sqrt{\_\_\_} = \frac{3}{5}$ b $\sqrt{\_\_\_} = \frac{2}{3}$ c $\sqrt{\_\_\_} = \frac{1}{10}$ d $\sqrt{\_\_\_} = \frac{4}{7}$
e $\sqrt{\_\_\_} = \frac{5}{11}$ f $\sqrt{\_\_\_} = 1\frac{1}{2}$ g $\sqrt{\_\_\_} = 1\frac{2}{3}$ h $\sqrt{\_\_\_} = 2\frac{3}{4}$

6 Evaluate the following:

a $\sqrt{16} + \sqrt{36}$ b $\sqrt{1} + \sqrt{144}$ c $\sqrt{9} + \sqrt{25}$ d $\sqrt{81} + \sqrt{49}$
e $\sqrt{1} + \sqrt{324}$ f $\sqrt{16} + \sqrt{225}$ g $\sqrt{36} + \sqrt{64}$ h $\sqrt{225} + \sqrt{1024}$
i $\sqrt{81} + \sqrt{121}$ j $\sqrt{289} + \sqrt{529}$ k $\sqrt{441} + \sqrt{676}$ l $\sqrt{1681} + \sqrt{4}$

7 Evaluate the following:

a $2\sqrt{4} = \_\_\_\_$ b $5\sqrt{9} = \_\_\_\_$ c $7\sqrt{16} = \_\_\_\_$
d $3\sqrt{25} = \_\_\_\_$ e $5\sqrt{49} = \_\_\_\_$ f $2\sqrt{100} = \_\_\_\_$
g $6\sqrt{64} = \_\_\_\_$ h $3\sqrt{81} = \_\_\_\_$ i $10\sqrt{144} = \_\_\_\_$
j $3\sqrt{361} = \_\_\_\_$ k $4\sqrt{289} = \_\_\_\_$ l $15\sqrt{484} = \_\_\_\_$

8 Evaluate the following:

a $(\sqrt{81})^2$ b $(\sqrt{16})^2$ c $(\sqrt{9})^2$ d $(\sqrt{36})^2$
e $(\sqrt{121})^2$ f $(2\sqrt{9})^2$ g $(2\sqrt{4})^2$ h $(3\sqrt{16})^2$
i $(2\sqrt{49})^2$ j $(2\sqrt{144})^2$ k $(2\sqrt{169})^2$ l $(2\sqrt{289})^2$
m $(3\sqrt{625})^2$ n $(3\sqrt{2401})^2$ o $(12\sqrt{17\,161})^2$ p $(7\sqrt{2704})^2$

9 Evaluate the following:

a $\sqrt{16} - \sqrt{9} = \_\_\_\_$ b $\sqrt{25} - \sqrt{4} = \_\_\_\_$ c $\sqrt{144} - \sqrt{1} = \_\_\_\_$
d $\sqrt{256} - \sqrt{121} = \_\_\_\_$ e $\sqrt{676} - \sqrt{144} = \_\_\_\_$ f $\sqrt{484} - \sqrt{144} = \_\_\_\_$
g $4\sqrt{36} - 2\sqrt{81} = \_\_\_\_$ h $4\sqrt{16} - 5\sqrt{9} = \_\_\_\_$ i $9\sqrt{441} - 3\sqrt{3364} = \_\_\_\_$
j $5\sqrt{225} - 3\sqrt{64} = \_\_\_\_$ k $3\sqrt{289} - \sqrt{361} = \_\_\_\_$ l $7\sqrt{676} - 4\sqrt{529} = \_\_\_\_$

Evaluating surds 1G

When a calculator is used to evaluate the square root of a number that is not a perfect square, the answer is a decimal. Such numbers have an infinite number of decimal places with no recurring pattern.

Here is the decimal expression for the square root of 3 written to 14 decimal places: $\sqrt{3} = 1{\cdot}732\,050\,807\,568\,88$

Numbers which are shown in square root form such as $\sqrt{5}$, $\sqrt{6}$ are called **surds**. They are also called irrational numbers because when they are expressed in decimal form they continue in a non-repeating, unpredictable manner.

However, it is possible to use a trial and error method to calculate the decimal value of surds when a calculator is not available.

Example **Solution**

Express the following to the number of decimal places as indicated:

a $\sqrt{2}$ (2 decimal places)

Method: Trial and Error

List all the square numbers between 1 and 100.

$\{1, 4, 9, 16, 25, 36, 49, 64, 81, 100\}$

Note that 2 is between 1 and 4, then $\sqrt{2}$ must lie between $\sqrt{1}$ and $\sqrt{4}$, that is, between 1 and 2.

Next, choose a number between 1 and 2 that can be squared to give the approximate value of 2:

Trial & Error 1: $1{\cdot}5 \times 1{\cdot}5 = 2{\cdot}25$ (Value exceeds 2)

Trial & Error 2: $1{\cdot}4 \times 1{\cdot}4 = 1{\cdot}96$ (Value is close to 2 to 1 decimal place)

Trial & Error 3: $1{\cdot}41 \times 1{\cdot}41 = 1{\cdot}9881 \approx 2$ (Value even closer to 2)

$\therefore \sqrt{2} \approx 1{\cdot}41$ (to 2 decimal places).

b $\sqrt{11}$ (3 decimal places)

Method: Trial and Error

List all the square numbers between 1 and 100.

$\{1, 4, 9, 16, 25, 36, 49, 64, 81, 100\}$

Note that 11 is between 9 and 16 so $\sqrt{11}$ must lie between $\sqrt{9}$ and $\sqrt{16}$, that is, between 3 and 4.

Next, choose a number between 3 and 4 that can be squared to give the approximate value of 11:

Trial & Error 1: $3{\cdot}5 \times 3{\cdot}5 = 12{\cdot}25$ (Value exceeds 11)

Trial & Error 2: $3{\cdot}3 \times 3{\cdot}3 = 10{\cdot}89$ (Value is close to 11 to 1 decimal place)

Trial & Error 3: $3{\cdot}31 \times 3{\cdot}31 = 10{\cdot}9561$ (Value is closer to 11 to 2 decimal places)

Trial & Error 4: $3{\cdot}317 \times 3{\cdot}317 = 11{\cdot}002\,489$ (Value is approximately 11 to 3 decimal places)

$\therefore \sqrt{11} \approx 3{\cdot}317$ (to 3 decimal places)

Example

c $3\sqrt{5}$ (1 decimal place)

Solution

Method: Trial and Error

List all the square numbers between 1 and 100.

{1, 4, 9, 16, 25, 36, 49, 64, 81, 100}

Step 1:

Note that 5 is between 4 and 9 so $\sqrt{5}$ must lie between $\sqrt{4}$ and $\sqrt{9}$, that is, between 2 and 3.

Next choose a number between 2 and 3 that can be squared to give the approximate value of 5

Trial & Error 1: $2{\cdot}5 \times 2{\cdot}5 = 6{\cdot}25$ (Value exceeds 5)

Trial & Error 2: $2{\cdot}2 \times 2{\cdot}2 = 4{\cdot}84$ (Value is approximately 5 to 1 decimal place)

Trial & Error 3: $2{\cdot}23 \times 2{\cdot}23 = 4{\cdot}97$ (Value closer to 5 to 2 decimal places).

$\therefore \sqrt{5} \approx 2{\cdot}23$ (to 2 decimal places).

Step 2: Multiply by 3. $3\sqrt{5} = 3 \times \sqrt{5} = 3 \times 2{\cdot}23 = 6{\cdot}69$ (to 2 decimal places)

Step 3: Reduce answer to 1 decimal place:

$\therefore 3\sqrt{5} \approx 6{\cdot}7$ (to 1 decimal place).

Exercise 1G

1 Evaluate the square root of the following to 1 decimal place:

a $\sqrt{6}$ b $\sqrt{17}$ c $4\sqrt{12}$ d $3\sqrt{15}$ e $15\sqrt{3}$

2 Evaluate the square root of the following to 2 decimal places:

a $\sqrt{18}$ b $\sqrt{13}$ c $10\sqrt{9}$ d $2\sqrt{21}$ e $7\sqrt{45}$

3 Evaluate the square root of the following to 2 decimal places:

a $\sqrt{7}$ b $\sqrt{14}$ c $5\sqrt{12}$ d $13\sqrt{31}$ e $7\sqrt{42}$

4 Given that $\sqrt{7} = 2{\cdot}645\,751\,311$, $\sqrt{8} = 2{\cdot}828\,427\,125$ and $\sqrt{10} = 3{\cdot}162\,277\,66$, evaluate the following to the number of decimal places indicated:

a $\sqrt{7} + \sqrt{8} + \sqrt{10}$ (9 decimal places)
b $\sqrt{10} + \sqrt{8} - \sqrt{7}$ (8 decimal places)
c $\sqrt{10} + \sqrt{7} - \sqrt{8}$ (6 decimal places)
d $\sqrt{7} + \sqrt{8} - \sqrt{10}$ (5 decimal places)
e $2\sqrt{10}$ (3 decimal places)
f $5\sqrt{8}$ (4 decimal places)
g $9\sqrt{7}$ (8 decimal places)
h $2(\sqrt{7} + \sqrt{10})$ (6 decimal places)

5 Given that $\sqrt{5} = 2{\cdot}236\,067\,97$, $\sqrt{11} = 3{\cdot}316\,624\,79$ and $\sqrt{12} = 3{\cdot}464\,101\,615$, evaluate the following to the number of decimal places indicated:

a $\sqrt{5} + \sqrt{11} + \sqrt{12}$ (8 decimal places)
b $\sqrt{12} + \sqrt{11} - \sqrt{5}$ (3 decimal places)
c $\sqrt{12} + \sqrt{5} - \sqrt{11}$ (5 decimal places)
d $\sqrt{5} + \sqrt{11} - \sqrt{12}$ (6 decimal places)
e $2\sqrt{11}$ (4 decimal places)
f $5\sqrt{12}$ (5 decimal places)
g $11\sqrt{5}$ (3 decimal places)
h $3(\sqrt{12} + \sqrt{11})$ (6 decimal places)

Estimating the size of surds 1H

It is important to estimate the size of numbers in calculations to check that the calculator input is correct.

Example **Solution**

1 Estimate the size of the following square root numbers in terms of their size in relation to the nearest whole number:

a $\sqrt{5}$

$\sqrt{5}$ is a little larger than $\sqrt{4}$ which is 2, so $\sqrt{5}$ is a little larger than 2.

b $2\sqrt{24}$

$\sqrt{24}$ is a little less than $\sqrt{25}$ which is 5. So twice this value will be a little less than 2×5 or 10.

c $5\sqrt{17} + 10$

$\sqrt{17}$ is a little larger than $\sqrt{16}$ which is 4. So 5 times this number plus 10 will be a little larger than 30.

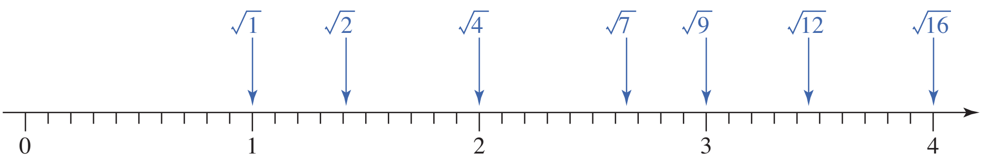
The above result can be shown by using a number line.

Example

2 Show the approximate positions of these surds, given their decimal equivalents:

$\sqrt{1} = 1,\ \sqrt{2} \approx 1{\cdot}41,\ \sqrt{4} = 2,\ \sqrt{7} \approx 2{\cdot}65,\ \sqrt{9} = 3,\ \sqrt{12} \approx 3{\cdot}46,\ \sqrt{16} = 4$

Solution



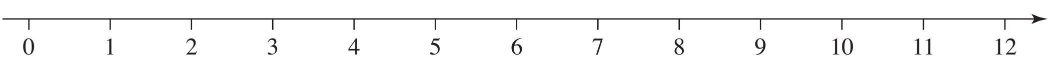
Exercise 1H

1 Estimate the size of the following square root numbers in terms of their size in relation to the nearest whole number, without using a calculator:

a $\sqrt{17}$ b $\sqrt{24}$ c $\sqrt{26}$ d $\sqrt{35}$ e $\sqrt{37}$ f $\sqrt{48}$

g $\sqrt{50}$ h $\sqrt{62}$ i $\sqrt{65}$ j $\sqrt{80}$ k $\sqrt{83}$ l $\sqrt{98}$

2 Place each of the surds in Question 1 onto this number line:



3 Estimate the size of the following to the nearest whole number:

a $3\sqrt{23}$ b $4\sqrt{101}$ c $2\sqrt{26}$ d $8\sqrt{82}$

e $10\sqrt{146}$ f $8\sqrt{10} + 2$ g $16\sqrt{99} - 12$ h $3\sqrt{23} + 11$

i $5\sqrt{18} + 7\sqrt{38}$ j $4\sqrt{62} + 3\sqrt{80}$ k $3\sqrt{401} + 2\sqrt{195}$ l $5\sqrt{101} + \sqrt{785}$

1I Simplifying surds

When surds are expressed as decimals, they continue unpredictably and can't be written exactly. It is much better to leave them in exact surd form. It is possible to write those which contain square factors in simpler form.

The numerical result can be summarised as, for example, $\sqrt{18} = \sqrt{9} \times \sqrt{2} = 3 \times \sqrt{2} = 3\sqrt{2}$.

When simplifying surds in this way, always split the number under the root sign using the largest square factor. This process holds for algebraic expressions.

Example

Simplify the following:

a $\sqrt{320}$

b $\sqrt{a^2b}$

c $\sqrt{8a^2}$

d $\sqrt{27a^4b}$

Solution

$\sqrt{320} = \sqrt{64} \times \sqrt{5} = 8\sqrt{5}$

$\sqrt{a^2b} = \sqrt{a^2} \times \sqrt{b} = a\sqrt{b}$

$\sqrt{8a^2} = \sqrt{4} \times \sqrt{2} \times \sqrt{a^2} = 2 \times \sqrt{2} \times a$
$= 2a\sqrt{2}$

$\sqrt{27a^4b} = \sqrt{9} \times \sqrt{3} \times \sqrt{a^4} \times \sqrt{b}$
$= 3 \times \sqrt{3} \times a^2 \times \sqrt{b} = 3a^2\sqrt{3b}$

Exercise 1I

1 Complete the following:

a $\sqrt{98} = \sqrt{\_\_\_} \times \sqrt{2} = \_\_\_\sqrt{2}$

b $\sqrt{45} = \sqrt{\_\_\_} \times \sqrt{5} = \_\_\_\sqrt{5}$

c $\sqrt{200} = \sqrt{\_\_\_} \times \sqrt{2} = \_\_\_\sqrt{2}$

d $\sqrt{63} = \sqrt{\_\_\_} \times \sqrt{7} = \_\_\_\sqrt{7}$

e $\sqrt{112} = \sqrt{\_\_\_} \times \sqrt{7} = \_\_\_\sqrt{7}$

f $\sqrt{275} = \sqrt{\_\_\_} \times \sqrt{11} = \_\_\_\sqrt{11}$

2 Simplify these surds:

a $\sqrt{48}$ b $\sqrt{63}$ c $\sqrt{20}$ d $\sqrt{27}$ e $\sqrt{128}$ f $\sqrt{200}$

g $\sqrt{220}$ h $\sqrt{120}$ i $\sqrt{198}$ j $\sqrt{162}$ k $\sqrt{243}$ l $\sqrt{140}$

3 Simplify the following:

a $2\sqrt{75}$ b $5\sqrt{300}$ c $2\sqrt{98}$ d $4\sqrt{27}$ e $9\sqrt{192}$ f $2\sqrt{28}$

g $4\sqrt{45}$ h $5\sqrt{24}$ i $6\sqrt{48}$ j $2\sqrt{98}$ k $4\sqrt{80}$ l $4\sqrt{72}$

4 Complete the following:

a $\sqrt{a^2c} = \sqrt{a^2} \times \sqrt{\_\_\_} = a\sqrt{\_\_\_}$

b $\sqrt{b^2d} = \sqrt{b^2} \times \sqrt{\_\_\_} = b\sqrt{\_\_\_}$

c $\sqrt{2a^2f} = \sqrt{2} \times \sqrt{a^2} \times \sqrt{\_\_\_} = \sqrt{2} \times a \times \sqrt{\_\_\_} = a\sqrt{\_\_\_}$

d $\sqrt{3b^2c} = \sqrt{3} \times \sqrt{b^2} \times \sqrt{\_\_\_} = \sqrt{3} \times b \times \sqrt{\_\_\_} = b\sqrt{\_\_\_}$

5 Simplify the following:

a $\sqrt{a^2f}$ b $\sqrt{x^2y}$ c $\sqrt{f^2g}$ d $\sqrt{e^2t}$ e $\sqrt{w^2z}$

f $\sqrt{2a^2d}$ g $\sqrt{3b^2g}$ h $\sqrt{5e^2g}$ i $\sqrt{9d^2e}$ j $\sqrt{4d^2h}$

k $\sqrt{16d^2ef}$ l $\sqrt{36e^2st}$ m $\sqrt{18a^2bc}$ n $\sqrt{8a^2d}$ o $\sqrt{27f^2g}$

p $\sqrt{48x^2y}$ q $\sqrt{108p^2t}$ r $\sqrt{75a^2c}$ s $\sqrt{24x^3y}$ t $\sqrt{72a^5b^7}$

Expressing entire surds 1J

Making entire surds is the opposite process to simplifying them. This process can also be used with algebraic expressions.

Example **Solution**

Express the following as entire surds:

a $3\sqrt{2}$ — $3\sqrt{2} = \sqrt{9} \times \sqrt{2} = \sqrt{9 \times 2} = \sqrt{18}$

b $2\sqrt{3}$ — $2\sqrt{3} = \sqrt{4} \times \sqrt{3} = \sqrt{4 \times 3} = \sqrt{12}$

c $2\sqrt{5a}$ — $2\sqrt{5a} = \sqrt{4} \times \sqrt{5a} = \sqrt{4 \times 5a} = \sqrt{20a}$

d $a\sqrt{3a}$ — $a\sqrt{3a} = \sqrt{a^2} \times \sqrt{3a} = \sqrt{a^2 \times 3a} = \sqrt{3a^3}$

e $2a\sqrt{3a^3}$ — $2a\sqrt{3a^3} = \sqrt{4a^2} \times \sqrt{3a^3} = \sqrt{4a^2 \times 3a^3} = \sqrt{12a^5}$

f $3a^2\sqrt{5ab}$ — $3a^2\sqrt{5ab} = \sqrt{9a^4} \times \sqrt{5ab} = \sqrt{45a^5b}$

Exercise 1J

1 Complete the following:

a $3\sqrt{5} = \sqrt{\_\_\_} \times \sqrt{5} = \sqrt{\_\_\_ \times 5} = \sqrt{\_\_\_}$

b $2\sqrt{7} = \sqrt{\_\_\_} \times \sqrt{7} = \sqrt{\_\_\_ \times 7} = \sqrt{\_\_\_}$

c $4\sqrt{2} = \sqrt{\_\_\_} \times \sqrt{2} = \sqrt{\_\_\_ \times 2} = \sqrt{\_\_\_}$

d $4\sqrt{5} = \sqrt{\_\_\_} \times \sqrt{5} = \sqrt{\_\_\_ \times 5} = \sqrt{\_\_\_}$

e $12\sqrt{3} = \sqrt{\_\_\_} \times \sqrt{3} = \sqrt{\_\_\_ \times 3} = \sqrt{\_\_\_}$

f $4\sqrt{11} = \sqrt{\_\_\_} \times \sqrt{11} = \sqrt{\_\_\_ \times 11} = \sqrt{\_\_\_}$

g $5\sqrt{6} = \sqrt{\_\_\_} \times \sqrt{6} = \sqrt{\_\_\_ \times 6} = \sqrt{\_\_\_}$

h $9\sqrt{2} = \sqrt{\_\_\_} \times \sqrt{2} = \sqrt{\_\_\_ \times 2} = \sqrt{\_\_\_}$

2 Express the following as entire surds:

a $3\sqrt{2}$ b $6\sqrt{5}$ c $10\sqrt{2}$ d $2\sqrt{7}$ e $7\sqrt{11}$ f $9\sqrt{2}$

g $5\sqrt{3}$ h $2\sqrt{6}$ i $12\sqrt{3}$ j $7\sqrt{5}$ k $8\sqrt{7}$ l $4\sqrt{13}$

3 Complete the following:

a $a\sqrt{5a} = \sqrt{\_\_\_} \times \sqrt{5a} = \sqrt{\_\_\_ \times 5a} = \sqrt{\_\_\_}$

b $a\sqrt{6a} = \sqrt{\_\_\_} \times \sqrt{6a} = \sqrt{\_\_\_ \times 6a} = \sqrt{\_\_\_}$

c $a\sqrt{2b} = \sqrt{\_\_\_} \times \sqrt{2b} = \sqrt{\_\_\_ \times 2b} = \sqrt{\_\_\_}$

d $b\sqrt{3ab} = \sqrt{\_\_\_} \times \sqrt{3ab} = \sqrt{\_\_\_ \times 3ab} = \sqrt{\_\_\_}$

e $a\sqrt{abd} = \sqrt{\_\_\_} \times \sqrt{abd} = \sqrt{\_\_\_ \times abd} = \sqrt{\_\_\_}$

f $a\sqrt{2bc} = \sqrt{\_\_\_} \times \sqrt{2bc} = \sqrt{\_\_\_ \times 2bc} = \sqrt{\_\_\_}$

4 Express the following as entire surds:

a $a\sqrt{9a}$ b $b\sqrt{2b}$ c $a\sqrt{6b}$ d $b\sqrt{2ac}$

e $a\sqrt{4ab}$ f $b\sqrt{3ab}$ g $a\sqrt{2ab}$ h $a\sqrt{abc}$

i $d\sqrt{5def}$ j $e\sqrt{7efg}$ k $c\sqrt{abc}$ l $b\sqrt{abc}$

5 Express the following as entire surds:

a $3a\sqrt{a}$ b $4a\sqrt{a}$ c $7b\sqrt{b}$ d $8a\sqrt{a}$

e $7c\sqrt{c}$ f $6ab\sqrt{a}$ g $3ab\sqrt{b}$ h $3ac\sqrt{5c}$

i $5bc\sqrt{abc}$ j $2ad\sqrt{3d}$ k $2a\sqrt{3abc}$ l $5a\sqrt{2abc}$

m $5a\sqrt{a^3}$ n $6ab\sqrt{b}$ o $2a^2b\sqrt{a^3b^5}$ p $3a^2b\sqrt{a^3b}$

1K Adding and subtracting surds

When surds are of the same type (i.e. they are like terms) they can be added or subtracted. This is similar to the way in which like terms can be added or subtracted in algebra.

As $2a + 3a = 5a$ so, for example, $2\sqrt{7} + 3\sqrt{7} = 5\sqrt{7}$.

Just as $3a + 4b$ can't be simplified, $3\sqrt{2} + 4\sqrt{7}$ can't be simplified, as the surds are different.

We need to look at the surds carefully as they may be able to be simplified and then added or subtracted: $\sqrt{2} + \sqrt{8} = \sqrt{2} + 2\sqrt{2} = 3\sqrt{2}$

Example

1 Group like surds together:
$\sqrt{2}, \sqrt{3}, 3\sqrt{2}, 5\sqrt{3}, 4\sqrt{2},$
$7\sqrt{3}, 4\sqrt{2}, 12\sqrt{2}, 6\sqrt{3}$

Solution

There are two groups:

those containing $\sqrt{2}$
$\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, 4\sqrt{2}, 12\sqrt{2}$

those containing $\sqrt{3}$
$\sqrt{3}, 5\sqrt{3}, 7\sqrt{3}, 6\sqrt{3}$

Example

2 Simplify the following where possible:

a $3\sqrt{2} + 5\sqrt{2}$

b $5\sqrt{3} - 3\sqrt{3}$

c $2\sqrt{a} + 4\sqrt{a}$

d $3\sqrt{2} + \sqrt{8}$

Solution

$3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$

$5\sqrt{3} - 3\sqrt{3} = 2\sqrt{3}$

$2\sqrt{a} + 4\sqrt{a} = 6\sqrt{a}$

$3\sqrt{2} + \sqrt{8} = 3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2}$

Exercise 1K

1 Group the like surds together:

a $\sqrt{5}, \sqrt{3}, 3\sqrt{5}, 5\sqrt{5}, ^-4\sqrt{3}$

b $^-\sqrt{11}, ^-3\sqrt{6}, 5\sqrt{11}, ^-4\sqrt{6}$

c $\sqrt{3}, ^-\sqrt{3}, \sqrt{5}, 5\sqrt{3}, ^-2\sqrt{5}$

d $^-\sqrt{2}, \sqrt{7}, ^-8\sqrt{7}, ^-4\sqrt{7}, ^-4\sqrt{2}$

e $^-\sqrt{3}, \sqrt{3}, ^-3\sqrt{6}, 5\sqrt{7}, ^-4\sqrt{3}, \sqrt{7}, \sqrt{6}$

f $2\sqrt{13}, \sqrt{3}, 3\sqrt{13}, ^-2\sqrt{3}, ^-4\sqrt{3}, \sqrt{13}, ^-\sqrt{3}$

2 Complete the following:

a $4\sqrt{5} + 2\sqrt{5}$ = ___ $\sqrt{5}$

b $3\sqrt{2} + \sqrt{2}$ = ___ $\sqrt{2}$

c $\sqrt{11} + 4\sqrt{11}$ = ___ $\sqrt{11}$

d $3\sqrt{6} + \sqrt{6}$ = ___ $\sqrt{6}$

e $2\sqrt{7} + 3\sqrt{7} + \sqrt{7}$ = ___ $\sqrt{7}$

f $\sqrt{5} + \sqrt{5} + 9\sqrt{5}$ = ___ $\sqrt{5}$

3 Complete the following:

a $6\sqrt{5} - 3\sqrt{5}$ = ___ $\sqrt{5}$

b $8\sqrt{2} - \sqrt{2}$ = ___ $\sqrt{2}$

c $14\sqrt{2} - 9\sqrt{2}$ = ___ $\sqrt{2}$

d $5\sqrt{3} - 2\sqrt{3}$ = ___ $\sqrt{3}$

e $9\sqrt{11} - 7\sqrt{11}$ = ___ $\sqrt{11}$

f $12\sqrt{7} - 7\sqrt{7}$ = ___ $\sqrt{7}$

4 Complete the following:

a $3\sqrt{2} + \sqrt{2} + 2\sqrt{3}$ = ___ $\sqrt{2} + 2\sqrt{3}$

b $4\sqrt{5} + \sqrt{5} + 3\sqrt{7}$ = ___ $\sqrt{5} + 3\sqrt{7}$

c $5\sqrt{2} - \sqrt{2} + 3\sqrt{3}$ = ___ $\sqrt{2} + 3\sqrt{3}$

d $4\sqrt{2} - \sqrt{5} + 3\sqrt{2}$ = ___ $\sqrt{2} - \sqrt{5}$

e $4\sqrt{3} + 2\sqrt{2} - 2\sqrt{3}$ = ____________

f $5\sqrt{7} - 3\sqrt{3} - 2\sqrt{7}$ = ____________

g $6\sqrt{7} - \sqrt{7} + 5\sqrt{11}$ = ____________

h $\sqrt{5} + 3\sqrt{3} - \sqrt{5}$ = ____________

Multiplying surds

1L

Surds can be multiplied together and then simplified. Multiply any numbers outside the square root sign first, then multiply the numbers inside the square root sign.

This can be expressed as $a\sqrt{b} \times c\sqrt{d} = ac\sqrt{bd}$.

Example

Complete the following:

a $\sqrt{2} \times \sqrt{5}$

b $3\sqrt{2} \times 5\sqrt{3}$

c $\sqrt{2}(\sqrt{3} + \sqrt{2})$

d $(\sqrt{2} + \sqrt{3})(\sqrt{2} + 7)$

Solution

a $\sqrt{2} \times \sqrt{5} = \sqrt{2 \times 5} = \sqrt{10}$

b $3\sqrt{2} \times 5\sqrt{3} = 3 \times 5 \times \sqrt{2} \times \sqrt{3}$
$= 3 \times 5 \times \sqrt{2 \times 3} = 15\sqrt{6}$

c $\sqrt{2}(\sqrt{3} + \sqrt{2}) = \sqrt{2} \times \sqrt{3} + \sqrt{2} \times \sqrt{2}$
$= \sqrt{6} + \sqrt{4} = \sqrt{6} + 2$

d $(\sqrt{2} + \sqrt{3})(\sqrt{2} + 7)$
$= \sqrt{2} \times \sqrt{2} + 7 \times \sqrt{2} + \sqrt{3} \times \sqrt{2} + 7 \times \sqrt{3}$
$= \sqrt{4} + 7\sqrt{2} + \sqrt{6} + 7\sqrt{3}$
$= 2 + 7\sqrt{2} + \sqrt{6} + 7\sqrt{3}$

Exercise 1L

1 Complete the following:

a $\sqrt{2} \times \sqrt{7} = \sqrt{\_\_\_ \times \_\_\_} = \sqrt{\_\_\_}$

b $\sqrt{3} \times \sqrt{5} = \sqrt{\_\_\_ \times \_\_\_} = \sqrt{\_\_\_}$

c $\sqrt{5} \times \sqrt{6} = \sqrt{\_\_\_ \times \_\_\_} = \sqrt{\_\_\_}$

d $\sqrt{7} \times \sqrt{5} = \sqrt{\_\_\_ \times \_\_\_} = \sqrt{\_\_\_}$

e $\sqrt{11} \times \sqrt{5} = \sqrt{\_\_\_ \times \_\_\_} = \sqrt{\_\_\_}$

f $\sqrt{17} \times \sqrt{2} = \sqrt{\_\_\_ \times \_\_\_} = \sqrt{\_\_\_}$

2 Multiply the following and simplify where possible:

a $\sqrt{2} \times \sqrt{6}$
b $\sqrt{3} \times \sqrt{6}$
c $\sqrt{7} \times \sqrt{14}$
d $4\sqrt{2} \times 3\sqrt{7}$
e $3\sqrt{2} \times 7\sqrt{11}$
f $3\sqrt{6} \times 4\sqrt{7}$
g $3\sqrt{2} \times 4\sqrt{3}$
h $2\sqrt{5} \times 3\sqrt{7}$
i $4\sqrt{6} \times 3\sqrt{2}$
j $2\sqrt{3} \times 3\sqrt{6}$
k $2\sqrt{6} \times 3\sqrt{8}$
l $2\sqrt{8} \times 3\sqrt{10}$

3 Expand the brackets and express in simplest form:

a $3(\sqrt{7} + \sqrt{3})$
b $^-2(\sqrt{5} + 2)$
c $8(3\sqrt{2} - 3)$
d $^-5(\sqrt{2} + \sqrt{11})$
e $\sqrt{2}(\sqrt{5} + \sqrt{3})$
f $\sqrt{3}(\sqrt{2} - \sqrt{7})$
g $\sqrt{11}(\sqrt{2} + \sqrt{5})$
h $\sqrt{6}(2\sqrt{5} - \sqrt{7})$
i $3\sqrt{2}(\sqrt{2} + \sqrt{3})$
j $2\sqrt{5}(\sqrt{5} + \sqrt{3})$
k $4\sqrt{2}(\sqrt{2} - 3\sqrt{5})$
l $4\sqrt{5}(2\sqrt{7} + \sqrt{5})$

4 Expand the brackets and express in simplest form:

a $(\sqrt{5} + \sqrt{2})(\sqrt{3} + 7)$
b $(3 + \sqrt{2})(\sqrt{6} + \sqrt{3})$
c $(\sqrt{2} + 4)(\sqrt{3} + \sqrt{7})$
d $(3 + \sqrt{5})(2 + \sqrt{2})$
e $(2\sqrt{2} + \sqrt{3})(3 + \sqrt{7})$
f $(\sqrt{5} + 3\sqrt{3})(2 + \sqrt{2})$
g $(\sqrt{3} + \sqrt{5})(3\sqrt{2} + 2)$
h $(3 + \sqrt{5})(\sqrt{2} + 1)$
i $(\sqrt{3} + 2)(3\sqrt{5} + 2)$
j $(11\sqrt{2} + 2\sqrt{3})(\sqrt{3} + 2)$
k $(\sqrt{2} + \sqrt{3})(\sqrt{6} + 4)$
l $(\sqrt{5} + 3\sqrt{2})(\sqrt{10} + \sqrt{2})$

1M Dividing surds

Surds can be divided by first writing them as fractions and then cancelling common factors. Simplify the numbers outside the square root sign first, then simplify the numbers inside the square root sign.

Example

Simplify the following:

a $3\sqrt{6} \div \sqrt{6}$

b $\dfrac{10\sqrt{24}}{5\sqrt{3}}$

c $\dfrac{\sqrt{12} \times \sqrt{2}}{\sqrt{3}}$

d $\dfrac{3\sqrt{6} \times 4\sqrt{12}}{2\sqrt{3}}$

Solution

a $\dfrac{3\sqrt{6}}{\sqrt{6}} = \dfrac{3}{1} \times \dfrac{\sqrt{6}}{\sqrt{6}} = 3$

b $\dfrac{10\sqrt{24}}{5\sqrt{3}} = \dfrac{10}{5}\dfrac{\sqrt{24}}{\sqrt{3}} = 2\sqrt{8} = 2 \times \sqrt{4} \times \sqrt{2}$

$= 2 \times 2\sqrt{2} = 4\sqrt{2}$

c $\dfrac{\sqrt{12} \times \sqrt{2}}{\sqrt{3}} = \dfrac{\sqrt{12 \times 2}}{\sqrt{3}} = \dfrac{\sqrt{24}}{\sqrt{3}} = \sqrt{8}$

$= 2\sqrt{2}$

d $\dfrac{3\sqrt{6} \times 4\sqrt{12}}{2\sqrt{3}} = \dfrac{3\sqrt{6} \times 2\sqrt{4}}{1}$

$= 6\sqrt{24}$

$= 12\sqrt{6}$

Exercise 1M

1 Simplify:

a $\sqrt{5} \div \sqrt{5}$ b $\sqrt{2} \div \sqrt{2}$ c $3\sqrt{7} \div \sqrt{7}$ d $7\sqrt{5} \div \sqrt{5}$

e $12\sqrt{3} \div 6\sqrt{3}$ f $4\sqrt{7} \div 3\sqrt{7}$ g $6\sqrt{5} \div 3\sqrt{5}$ h $\sqrt{5} \div 3\sqrt{5}$

i $4\sqrt{11} \div 2\sqrt{11}$ j $16\sqrt{2} \div 3\sqrt{2}$ k $42\sqrt{7} \div 7\sqrt{7}$ l $5\sqrt{11} \div 45\sqrt{11}$

2 Simplify:

a $\sqrt{25} \div \sqrt{5}$ b $\sqrt{27} \div \sqrt{9}$ c $\sqrt{18} \div \sqrt{9}$ d $\sqrt{32} \div \sqrt{8}$

e $\sqrt{50} \div \sqrt{5}$ f $\sqrt{22} \div \sqrt{2}$ g $\sqrt{32} \div \sqrt{2}$ h $\sqrt{48} \div \sqrt{6}$

i $\sqrt{81} \div \sqrt{3}$ j $\sqrt{24} \div \sqrt{2}$ k $\sqrt{54} \div \sqrt{3}$ l $\sqrt{48} \div \sqrt{2}$

3 Simplify the following:

a $2\sqrt{18} \div \sqrt{3}$ b $3\sqrt{30} \div 2\sqrt{6}$ c $5\sqrt{30} \div \sqrt{2}$ d $5\sqrt{50} \div 4\sqrt{5}$

e $15\sqrt{15} \div 5\sqrt{3}$ f $18\sqrt{10} \div 6\sqrt{5}$ g $4\sqrt{60} \div 4\sqrt{6}$ h $25\sqrt{12} \div 4\sqrt{6}$

i $13\sqrt{18} \div 5\sqrt{3}$ j $21\sqrt{20} \div 6\sqrt{2}$ k $5\sqrt{80} \div 3\sqrt{10}$ l $32\sqrt{18} \div 4\sqrt{3}$

4 Simplify the following:

a $\dfrac{3\sqrt{6} \times \sqrt{2}}{3\sqrt{3}}$ b $\dfrac{9\sqrt{3} \times 2\sqrt{12}}{6\sqrt{6}}$ c $\dfrac{3\sqrt{6} \times 2\sqrt{12}}{\sqrt{18}}$ d $\dfrac{9\sqrt{3} \times 2\sqrt{8}}{8\sqrt{2}}$

e $\dfrac{6\sqrt{2} \times 5\sqrt{12}}{4\sqrt{2}}$ f $\dfrac{2\sqrt{3} \times 18\sqrt{48}}{3\sqrt{6}}$ g $\dfrac{\sqrt{8} \times 2\sqrt{27}}{3\sqrt{3}}$ h $\dfrac{\sqrt{2} \times 8\sqrt{18}}{4\sqrt{3}}$

Rationalising denominators 1N

An expression that has a surd in the denominator is not easy to work with. It is possible to turn the denominator into a rational number, or to rationalise its denominator, by multiplying the numerator and denominator of the fraction by the surd.

Example

Solution

1 Rationalise these expressions:

a $\frac{2}{\sqrt{5}}$

$\frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{25}} = \frac{2\sqrt{5}}{5}$

b $\frac{2}{3\sqrt{2}}$

$\frac{2}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{3 \times \sqrt{4}} = \frac{2\sqrt{2}}{3 \times 2} = \frac{\sqrt{2}}{3}$

c $\frac{5\sqrt{2}}{3\sqrt{5}}$

$\frac{5\sqrt{2}}{3\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5 \times \sqrt{10}}{3 \times \sqrt{25}} = \frac{5 \times \sqrt{10}}{3 \times 5} = \frac{\sqrt{10}}{3}$

d $\frac{2 + \sqrt{7}}{\sqrt{3}}$

$\frac{2 + \sqrt{7}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3} + \sqrt{21}}{3}$

2 Rationalise the denominators of these fractions and express as a single fraction:

a $\frac{2}{\sqrt{3}} + \frac{5}{\sqrt{2}}$

$\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} + \frac{5}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{3}}{3} + \frac{5\sqrt{2}}{2}$

$= \frac{2 \times 2\sqrt{3} + 3 \times 5\sqrt{2}}{6} = \frac{4\sqrt{3} + 15\sqrt{2}}{6}$

b $\frac{4}{3\sqrt{3}} - \frac{1}{\sqrt{2}}$

$\frac{4}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} - \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{3}}{9} - \frac{\sqrt{2}}{2}$

$= \frac{2 \times 4\sqrt{3} - 9 \times \sqrt{2}}{18} = \frac{8\sqrt{3} - 9\sqrt{2}}{18}$

Exercise 1N

1 Complete the following to rationalise the denominators:

a $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \_\_$

b $\frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}} \times \_\_ = \_\_$

c $\frac{5}{\sqrt{7}} = \frac{5}{\sqrt{7}} \times \_\_ = \_\_$

d $\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times \_\_ = \_\_$

e $\frac{6}{\sqrt{2}} = \frac{6}{\sqrt{2}} \times \_\_ = \_\_$

f $\frac{7}{\sqrt{2}} = \frac{7}{\sqrt{2}} \times \_\_ = \_\_$

g $\frac{3}{2\sqrt{3}} = \frac{3}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \_\_$

h $\frac{2}{3\sqrt{5}} = \frac{2}{3\sqrt{5}} \times \_\_ = \_\_$

Puzzles

1 Simplify the surds and match the letter to the correct simplified surd expression below to find the next line in the joke:

What baby is born with whiskers?

E	$\sqrt{200}$	**I**	$\sqrt{48}$	**J**	$\sqrt{162}$
K	$\sqrt{147}$	**M**	$\sqrt{128}$	**N**	$\sqrt{125}$
R	$\sqrt{75}$	**S**	$\sqrt{24}$	**T**	$\sqrt{242}$

___	___	___	___	___	___
$7\sqrt{3}$	$4\sqrt{3}$	$11\sqrt{2}$	$11\sqrt{2}$	$10\sqrt{2}$	$5\sqrt{5}$

2 Simplify the surds and match the letter to the correct simplified surd expression below to find the next line in the joke:

What goes all around the earth but stays in one place?

A	$5\sqrt{2}+\sqrt{8}$	**B**	$4\sqrt{2}+\sqrt{8}$
L	$2\sqrt{3}+\sqrt{27}$	**M**	$5\sqrt{3}+\sqrt{27}$
P	$5\sqrt{2}+\sqrt{18}$	**Q**	$\sqrt{2}+\sqrt{18}$
S	$5\sqrt{3}-\sqrt{12}$	**T**	$\sqrt{32}-2\sqrt{2}$

___	___	___	___	___
$3\sqrt{3}$	$2\sqrt{2}$	$7\sqrt{2}$	$8\sqrt{3}$	$8\sqrt{2}$

3 Using the surds $\sqrt{2}$, $\sqrt{8}$, $\sqrt{10}$, $\sqrt{160}$ and $\sqrt{320}$, and the operations ÷ and ×, how many ways can you make 4?
You can use each surd more than once.

4 Simplify the surds and match the letter to the correct simplified surd expression below to find the answer to the riddle:

What has arms, but can't hug and legs but can't walk?

A $\sqrt{5}\times\sqrt{3}$	**B** $2\sqrt{4}\times\sqrt{2}$	**C** $\sqrt{6}\times 3\sqrt{3}$
D $5\sqrt{2}\times\sqrt{6}$	**E** $6\sqrt{3}\times 2\sqrt{8}$	**H** $3\sqrt{5}\times 2\sqrt{10}$
I $\sqrt{30}\div\sqrt{5}$	**L** $3\sqrt{21}\div\sqrt{7}$	**N** $15\sqrt{15}\div 3\sqrt{5}$
R $\dfrac{6\sqrt{2}\times 5\sqrt{12}}{4\sqrt{2}}$	**T** $\dfrac{\sqrt{10}\times 2\sqrt{27}}{3\sqrt{6}}$	**W** $\dfrac{3\sqrt{6}\times 4\sqrt{12}}{2\sqrt{3}}$

___		___	___	___	___	___		___	___	___
$\sqrt{15}$		$9\sqrt{2}$	$30\sqrt{2}$	$\sqrt{15}$	$\sqrt{6}$	$15\sqrt{3}$		$\sqrt{15}$	$5\sqrt{3}$	$10\sqrt{3}$
___		___	___	___	___	___				
$\sqrt{15}$		$2\sqrt{5}$	$\sqrt{15}$	$4\sqrt{2}$	$3\sqrt{3}$	$24\sqrt{6}$				

5 Rationalise the denominator of the surds and match the letter to the correct simplified surd expression below to solve the riddle:

What has a head and a foot but no body, and has a mouth but never smiles?

A $\dfrac{1}{\sqrt{3}}$	**B** $\dfrac{2}{\sqrt{5}}$	**D** $\dfrac{6}{\sqrt{2}}$
E $\dfrac{10}{\sqrt{5}}$	**I** $\dfrac{8}{3\sqrt{6}}$	**N** $\dfrac{7}{\sqrt{2}}$
R $\dfrac{3\sqrt{3}}{2\sqrt{2}}$	**V** $\dfrac{3\sqrt{5}}{5\sqrt{8}}$	**W** $\dfrac{5\sqrt{6}}{8\sqrt{10}}$

___		___	___	___		___	___	___
$\dfrac{\sqrt{3}}{3}$		$\dfrac{2\sqrt{5}}{5}$	$2\sqrt{5}$	$3\sqrt{2}$		$\dfrac{\sqrt{3}}{3}$	$\dfrac{7\sqrt{2}}{2}$	$3\sqrt{2}$
___		___	___	___	___	___		
$\dfrac{\sqrt{3}}{3}$		$\dfrac{3\sqrt{6}}{4}$	$\dfrac{4\sqrt{6}}{9}$	$\dfrac{3\sqrt{10}}{20}$	$2\sqrt{5}$	$\dfrac{3\sqrt{6}}{4}$		

Applications

Simplifying surds

When simplifying or reducing surds, first write the number under the square root sign as a multiple of a square factor. This square factor can then be removed from the square root sign completely. Using the largest square factor makes the process quick. If a smaller square factor is selected, the process may need to be repeated.

Complete each of the following to show that $\sqrt{192}$ can be simplified to $8\sqrt{3}$ by three different paths:

a $\sqrt{192} = \sqrt{4 \times \underline{\quad}}$
$= 2\sqrt{48}$
$= 2\sqrt{4 \times \underline{\quad}}$
$= 4\sqrt{\underline{\quad}}$
$= 4\sqrt{4 \times \underline{\quad}}$
$= 8\sqrt{\underline{\quad}}$

b $\sqrt{192} = \sqrt{16 \times \underline{\quad}}$
$= 4\sqrt{\underline{\quad}}$
$= 4\sqrt{4 \times \underline{\quad}}$
$= 8\sqrt{\underline{\quad}}$

c $\sqrt{192} = \sqrt{64 \times \underline{\quad}}$
$= 8\sqrt{\underline{\quad}}$

Use similar multiple pathways to simplify the following surds:

d $\sqrt{320}$ **e** $\sqrt{448}$ **f** $\sqrt{704}$ **g** $\sqrt{162}$ **h** $\sqrt{405}$ **i** $\sqrt{567}$

Surds obeying the rules of algebra

Surds are entities that have a dual mathematical personality. On the one hand, they are expressions which are made up of numbers and, on the other hand, the addition and subtraction processes work as in algebra.

It can be seen that $\sqrt{3} + \sqrt{3} = 2\sqrt{3}$ in exactly the same way that $x + x = 2x$.

a Write an algebra example that covers these surd processes:

- **i** $\sqrt{2} + 3\sqrt{2} = 4\sqrt{2}$
- **ii** $3\sqrt{5} + 4\sqrt{5} = 7\sqrt{5}$
- **iii** $2\sqrt{10} + 11\sqrt{10} = 13\sqrt{10}$
- **iv** $4\sqrt{2} - \sqrt{2} = 3\sqrt{2}$
- **v** $12\sqrt{5} - 7\sqrt{5} = 5\sqrt{5}$
- **vi** $6\sqrt{6} - 3\sqrt{6} = 3\sqrt{6}$

b Write a surd example that can be used to represent these equations:

- **i** $4x + 3x = 7x$
- **ii** $2x + x = 3x$
- **iii** $6x + 2x = 8x$
- **iv** $6x - 2x = 4x$
- **v** $9x - 3x = 6x$
- **vi** $12x - 7x = 5x$

In algebra $x + y$ cannot be simplified as they are not alike, and unlike terms cannot be added.

c Write five examples of surds showing this.

Sometimes surds can be simplified; for example, $\sqrt{2} + \sqrt{8} = \sqrt{2} + 2\sqrt{2} = 3\sqrt{2}$.

d Write five examples of surds showing this.

Enrichment

1 Express the following in simplest form:

a $\sqrt{18} + 4\sqrt{72} - 3\sqrt{28}$ b $2\sqrt{3} + \sqrt{75} + 10\sqrt{300} - \sqrt{30\,000}$

c $2\sqrt{75} + 4\sqrt{3} - 3\sqrt{27}$ d $2\sqrt{48} + 5\sqrt{300} - 12\sqrt{12}$

2 Rationalise the denominators and complete the following:

a $\dfrac{\sqrt{3}}{\sqrt{6}} + \dfrac{1}{\sqrt{24}}$ b $\dfrac{\sqrt{2}}{2\sqrt{5}} + \dfrac{\sqrt{3}}{\sqrt{45}}$ c $\dfrac{5}{2\sqrt{3}} + \dfrac{\sqrt{2}}{\sqrt{27}}$

d $\dfrac{\sqrt{2}+1}{\sqrt{3}} + \dfrac{\sqrt{5}-1}{\sqrt{5}}$ e $\dfrac{\sqrt{3}+2}{\sqrt{2}} + \dfrac{\sqrt{3}-1}{\sqrt{3}}$ f $\dfrac{2\sqrt{2}-3}{\sqrt{7}} + \dfrac{3\sqrt{2}+2}{\sqrt{2}}$

3 Expand the brackets and express in simplest surd form:

a $(\sqrt{3}+1)(\sqrt{3}+2)$ b $(\sqrt{5}+2)(\sqrt{3}+4)$

c $(\sqrt{7}-3)(\sqrt{3}+7)$ d $(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})$

e $(\sqrt{3}+5)(\sqrt{3}-5)$ f $(3\sqrt{2}+2)(\sqrt{2}-1)$

g $(\sqrt{3}+\sqrt{5})(\sqrt{3}-\sqrt{5})$ h $(2\sqrt{2}+\sqrt{3})(2\sqrt{2}-\sqrt{3})$

i $(5\sqrt{7}+\sqrt{2})(5\sqrt{7}-\sqrt{2})$ j $(\sqrt{3}+\sqrt{5})(\sqrt{6}-\sqrt{8})$

k $(2\sqrt{5}+5\sqrt{2})(\sqrt{10}-\sqrt{2})$ l $(7\sqrt{7}-2\sqrt{5})(3\sqrt{2}-4\sqrt{5})$

4 Complete the following:

a $\dfrac{\sqrt{2}}{\sqrt{5}+2} + \dfrac{1}{\sqrt{2}}$ b $\dfrac{5}{2\sqrt{2}+7} + \dfrac{2}{\sqrt{3}}$ c $\dfrac{2}{2-\sqrt{3}} + \dfrac{3}{\sqrt{5}}$

d $\dfrac{\sqrt{3}}{\sqrt{3}-2} \times \dfrac{1}{\sqrt{3}}$ e $\dfrac{2\sqrt{3}}{5\sqrt{2}} \times \dfrac{10\sqrt{5}}{4\sqrt{3}}$ f $\dfrac{3\sqrt{3}}{15\sqrt{2}} \times \dfrac{5\sqrt{8}}{2\sqrt{27}}$

5 Express the following expressions in simplest form by using $a = \sqrt{2}$, $b = \sqrt{3}+1$ and $c = \sqrt{3}-1$:

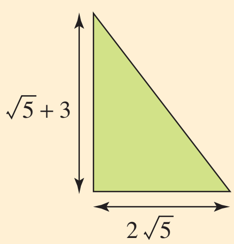
a ab b ac c bc d a^2+1 e b^2

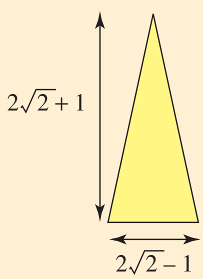
f c^2 g abc h b^2c^2 i $\dfrac{b}{a}$ j $\dfrac{c}{b}$

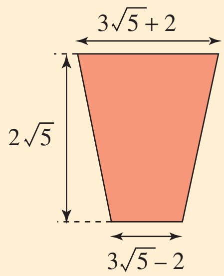
6 The following amounts are to be shared in the ratio $(\sqrt{2}+1):3$. Find the size of each part if the amount to be shared is:

a $\sqrt{18}+12$ b $\sqrt{200}+40$ c $\sqrt{1250}+100$ d $\sqrt{1800}+120$

7 Find the area of the following shapes expressed in exact surd form:

a


b


c


Revision/Assessment

Exercise 1A

1 Write five natural numbers between 5 and 20.

2 Name the following numbers in as many ways as possible:

a $2\frac{2}{3}$ b $\sqrt{4}$ c $\sqrt{11}$ d $^{-}7$ e 13

Exercise 1B

3 Find the sum of the first six multiples of 7.

4 Evaluate the following:

a $^{-}11 + 4 - 13$ b $^{-}11(3 - 4 - 6) + 12$ c $\dfrac{^{-}3 \times 12 + 6}{^{-}2}$

5 Find the sum of the factors of 18.

Exercise 1C

6 If $a = -4$, $b = 3$ and $c = -5$, evaluate the following without using a calculator:

a $a - 2b + 3c$ b $a^2 - 3b - 10c$ c $-4(2a + 3b + 4c)$

Exercise 1D

7 Evaluate the following without using a calculator and then check your answer with a calculator:

a $\dfrac{3\pi}{4} - \dfrac{2\pi}{3}$ b $2\frac{2}{3} \times 1\frac{1}{2}$ c $1\frac{1}{4}\left(2\frac{1}{2} - \frac{1}{3}\right)$

8 Estimate the value of each and then insert > or < to make the following true:

a $5\frac{1}{4} \times 2\frac{2}{3}$ ______ $4\frac{1}{8} + 11\frac{7}{9}$ b $\dfrac{6\frac{1}{4} + 3\frac{7}{8}}{2\frac{1}{11}}$ ______ $3\frac{5}{6} \times 1\frac{2}{3}$

Exercise 1E

9 Convert $\frac{5}{12}$ to decimal form by dividing the denominator into the numerator.

10 Evaluate the following without using a calculator:

a $\dfrac{0{\cdot}63}{0{\cdot}07}$ b $34{\cdot}002 \times 10$ c $23{\cdot}001 + 0{\cdot}03 + 1{\cdot}2$

Exercise 1F

11 Evaluate the following:

a $\left(2\frac{2}{3}\right)^2$ b $1{\cdot}5^2$ c $(3\sqrt{324})^2$ d $(5\sqrt{361})^2$ e $\left(\frac{1}{2}\sqrt{324}\right)^2$

12 Evaluate:

a $\sqrt{784} + \sqrt{36}$ b $\sqrt{1296} + \sqrt{25}$ c $2\sqrt{1089} - 3\sqrt{100}$ d $6\sqrt{36} - 2\sqrt{9}$

Exercise 1I

13 Simplify the following:

a $\sqrt{8}$ b $\sqrt{32}$ c $\sqrt{12}$ d $3\sqrt{245}$ e $3\sqrt{80}$ f $9\sqrt{72}$

14 Simplify the following:

a $\sqrt{a^2b}$ b $\sqrt{b^2c}$ c $\sqrt{4a^2b}$ d $\sqrt{9e^2f}$ e $\sqrt{3y^2z}$ f $\sqrt{5a^2c}$

Exercise 1J

15 Express the following as entire surds:

a $4\sqrt{2}$ b $2\sqrt{7}$ c $3\sqrt{11}$ d $4\sqrt{5}$ e $3\sqrt{2}$ f $5\sqrt{3}$

16 Express the following as entire surds:

a $a\sqrt{4a}$ b $b\sqrt{3b}$ c $a\sqrt{3b}$ d $2a\sqrt{3a}$ e $a\sqrt{4ac}$ f $3a\sqrt{2ab}$

Exercise 1K

17 Complete the following:

a $6\sqrt{5}-2\sqrt{5}$ b $7\sqrt{3}-3\sqrt{3}$ c $11\sqrt{5}+9\sqrt{5}$
d $\sqrt{3}+2\sqrt{5}-\sqrt{3}$ e $11\sqrt{7}+2\sqrt{3}-\sqrt{3}$ f $9\sqrt{2}-3\sqrt{7}+4\sqrt{2}$
g $8\sqrt{b}-2\sqrt{b}$ h $\sqrt{c}+4\sqrt{c}$ i $10\sqrt{a}-3\sqrt{a}$

18 Simplifying the following before adding or subtracting:

a $3\sqrt{2}+\sqrt{8}$ b $9\sqrt{2}-\sqrt{8}$ c $\sqrt{18}+7\sqrt{2}$ d $4\sqrt{3}+\sqrt{27}$

Exercise 1L

19 Multiply the following and simplify where possible:

a $\sqrt{2}\times\sqrt{5}$ b $\sqrt{7}\times\sqrt{3}$ c $\sqrt{3}\times\sqrt{13}$ d $\sqrt{5}\times\sqrt{15}$
e $3\sqrt{3}\times 2\sqrt{5}$ f $5\sqrt{2}\times 3\sqrt{12}$ g $2\sqrt{3}\times 3\sqrt{2}$ h $5\sqrt{5}\times 3\sqrt{3}$

20 Expand the brackets and express in simplest form:

a $\sqrt{3}(\sqrt{5}+\sqrt{2})$ b $\sqrt{5}(\sqrt{2}-\sqrt{7})$ c $\sqrt{3}(\sqrt{2}+\sqrt{5})$
d $(\sqrt{3}+\sqrt{5})(3+\sqrt{7})$ e $(\sqrt{2}+2\sqrt{5})(1+\sqrt{2})$ f $(\sqrt{2}+3)(\sqrt{2}+2)$

Exercise 1M

21 Simplify the following:

a $\dfrac{3\sqrt{6}\times\sqrt{5}}{\sqrt{2}}$ b $\dfrac{\sqrt{2}\times 5\sqrt{12}}{3\sqrt{6}}$ c $\dfrac{3\sqrt{3}\times 6\sqrt{12}}{\sqrt{6}}$ d $\dfrac{4\sqrt{3}\times 3\sqrt{12}}{2\sqrt{6}}$

Exercise 1N

22 Rationalise the denominators of the following:

a $\dfrac{3}{\sqrt{2}}$ b $\dfrac{3}{\sqrt{5}}$ c $\dfrac{2}{\sqrt{7}}$ d $\dfrac{12}{\sqrt{3}}$ e $\dfrac{18}{\sqrt{2}}$

23 Rationalise the denominators of the following:

a $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{3}}$ b $\dfrac{\sqrt{3}+3}{\sqrt{2}}$ c $\dfrac{\sqrt{3}-5}{\sqrt{2}}$ d $\dfrac{2+\sqrt{3}}{\sqrt{5}}$ e $\dfrac{3\sqrt{2}-\sqrt{7}}{\sqrt{3}}$

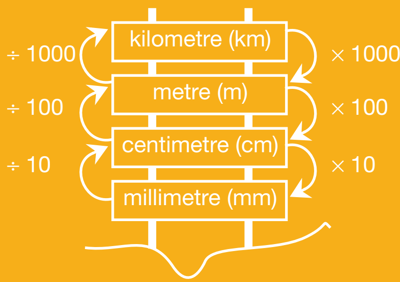
CHAPTER 2

Measurement

Solomon Islands hosted visitors from many Pacific countries for the 11th Festival of Pacific Arts in July 2012. A special village was built in Honiara where the artists could stay, display their crafts and perform cultural dances. Houses were also built for the use of representatives from the different Solomon Island provinces. The carpenters, craftsmen and women used a range of measurements in their work. Some used traditional measurements with ropes, sticks and parts of the body, which have been used since ancient times. Others used recently introduced units such as fathoms, feet and inches, or the metric units, metres and millimetres, for lengths and distances. Some materials required calculations of area or volume, and others for capacity or mass. The measurement of circular objects required the use of a special irrational number, pi, which mathematicians represent with the symbol π.

This chapter covers the following skills:

- Recognising and converting between units of perimeter, area and volume



- Calculating the perimeters of polygons, irregular shapes and circles
  $P = \pi D$ or $P = 2\pi r$
- Calculating arc lengths
  Arc length $= \dfrac{\text{angle}° \times 2\pi r}{360°}$
  Calculating the areas of triangles, quadrilaterals, circles, composite shapes and sectors
  Triangle: $A = \frac{1}{2}bh$
  Rectangle: $A = l \times w$
  Parallelogram: $A = b \times h$
  Trapezium: $A = \frac{1}{2}(a + b)h$
  Circle: $A = \pi r^2$
- Calculating the total surface areas of solids
- Calculating the volumes of solids including prisms, pyramids and spheres
  Prism: $V = A_{base} \times H$
  Pyramid: $V = \frac{1}{3} \times A_{base} \times H$
  Sphere: $V = \frac{4}{3}\pi r^3$
- Calculating capacity and converting between liquid measures
  1 mL = 1 cm^3
  100 mL = 1000 cm^3
  1000 L = 1 kL
  1 000 000 L = 1 ML

Specific Learning Outcome (SLO)

Learners should be able to:

9.2.1.1 Identify metric units that are used to measure the followings: *length*, *area*, *volume*, *capacity*, *temperature*, *time*, *speed* and *mass*.

9.2.2.1 Convert metric units from one to another.

9.2.3.1 Define ‘perimeter’.

9.2.4.1 Calculate the perimeter of given regular and irregular polygons.

9.2.5.1 Define ‘arc length of a circle’.

9.2.5.2 Calculate the arc length of a circle.

9.2.6.1 Identify parts of the circle.

9.2.6.2 Define ‘circumference of a circle’.

9.2.7.1 Find the circumference and arc length of the circle given the diameter and radius.

9.2.7.2 Calculate the perimeter of given compound shapes.

9.2.8.1 Define ‘area’.

9.2.8.2 Identify appropriate units that are used to measure the area of different shapes.

9.2.9.1 Convert metric units of area from one to another.

9.2.10.1 Identify the formulas or rules that are used to calculate the areas of different shapes.

9.2.11.1 Calculate the areas of different shapes: *triangle*, *rectangle*, *square*, *parallelogram*, *rhombus* and *trapezium*

9.2.12.1 Define area of circle.

9.2.12.2 Calculate the area of given circles and sectors given the diameter and radius.

9.2.13.1 Identify different shapes that make up compound shapes.

9.2.14.1 Calculate the area of composite shapes using appropriate formulas.

9.2.15.1 Define ‘prism’.

9.2.15.2 Identify the surface area of a prism.

9.2.16.1 Calculate the total surface area of given prisms by adding the areas of each surface.

9.2.17.1 Define pyramid.

9.2.17.2 Calculate the surface area of various pyramids.

9.2.18.1 Identify different types of solids such as cylinder, cones, and spheres.

9.2.18.2 Calculate the surface areas of cylinders, cones and spheres using formulas.

9.2.18.3 Solve word problems for practical situations related to the areas of cylinders, cones and spheres.

9.2.19.1 Define ‘volume’.

9.2.19.2 Identify the units that are used for volume: mm^3, cm^3 and m^3.

9.2.20.1 Calculate volumes of prisms.

9.2.20.2 Calculate the volumes of solids given the area of the bases (cross sections).

9.2.21.1 Identify the formulas that are used to calculate the volume of pyramids and cones.

9.2.21.2 Calculate the volume of pyramids and cones using the formula.

9.2.22.1 Identify the formula that is used to calculate the volume of sphere.

9.2.22.2 Calculate the volume of spheres.

9.2.23.1 Define ‘capacity’.

9.2.23.2 Identify appropriate units that are used for capacity.

9.2.23.3 Convert units of capacity from one to another. For example: L to cm^2, ml to L, ml to cm^2.

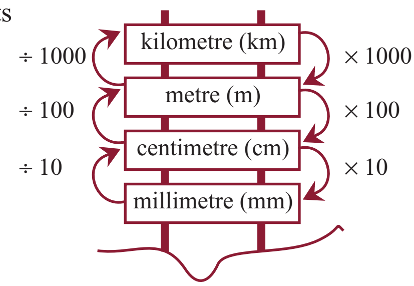
9.2.24.1 Calculate the capacity of various objects that are commonly found in homes. For example: pot, esky, bucket, drum.

2A Conversion of measurement units

There are lots of things that we may need to measure. The most common items would be measured using one of the units listed in the table below. In Science you may also need to measure humidity, air pressure, voltage, current, power and radiation.

Things to measure	Standard International Unit	Other Common Units	Metric conversions
Length	metre (m)	kilometre (km) centimeter (cm) millimetre (mm)	1 km = 1000 m 1 m = 100 cm 1 cm = 10 mm
Mass (Weight)	kilogram (kg)	tone (t) gram (g) milligram (mg)	1 t = 1000 kg 1 kg = 1000 g 1 g = 1000 mg
Area	square metre (m^2)	hectare (ha) square kilometre (km^2) square metre (m^2) square centimetre (cm^2) square millimetre (mm^2)	1 ha = 10 000 m^2 1 km^2 = 100 ha 1 m^2 = 10 000 cm^2 1 cm^2 = 100 mm^2
Volume	cubic metre (m^3)	cubic centimetre (cm^3)	1 m^3 = 1 000 000 cm^3
Capacity	litre (L)	megalitre (ML) millilitre (mL)	1L = 1000 mL
Temperature	Kelvin (K)	degrees Celsius (C)	
Speed	metres per second (m/s)	kilometres per hour (km/h)	1 km/h = 0.28 m/s
Time	seconds (s)	year (yr) month (m) week (wk) day (d) hour (hr) minute (min)	1 yr = 12 m = 52 wks = 365 days 1 wk = 7 days 1 day = 24 hrs 1hr = 60 min 1 min = 60 s

The common metric units of measuring or converting units of length and mass are given below.



Example

Solution

1 Change 2·5 metres to centimetres.

$2{\cdot}5 \text{ m} = 2{\cdot}5 \times 100 \text{ cm}$
$= 250 \text{ cm}$

2 Change 14 600 centimetres to kilometres.

$14\,600 \text{ cm} = 14\,600 \div 100 \text{ m}$
$= 146 \text{ m}$
$= 146 \div 1000 \text{ km}$
$= 0{\cdot}146 \text{ km}$

3 Change 2 kilograms into grams.

$2 \text{ kg} = 2 \times 1000 \text{ g}$
$= 2000 \text{ g}$

Exercise 2A

1 State what unit would be the most appropriate unit to measure the following items?

- **a** length of your table
- **b** length of your pen
- **c** distance to the nearest village
- **d** weight of a biro
- **e** weight of a truck full of coconuts
- **f** length of the blackboard
- **g** your height
- **h** weight of your textbook
- **i** time taken to boil a pot of rice
- **j** weight of a dog

2 Estimate the length of:

- **a** your textbook
- **b** your thumbnail
- **c** a car
- **d** a basketball court
- **e** your arm span
- **f** the distance from your classroom to the Principal's office

3 Choose the best estimation in each of these situations:

a Width of your exercise book:

A 2·1 cm **B** 21 cm **C** 19 mm **D** 0·02 m

b Length of your index finger:

A 200 mm **B** 0·15 m **C** 65 mm **D** 10 cm

c Length of your school soccer field:

A 165 000 mm **B** 14 000 mm **C** 150 cm **D** 1·6 km

d Distance between Savo and Point Cruz:

A 2 000 000 cm **B** 4000 m **C** 32 km **D** 310 000 m

4 Fill in the spaces:

- **a** 450 m = ______ cm = ______ mm
- **b** 0·3 km = ______ m = ______ cm = ______ mm
- **c** ______ kg = ______ g = 1 500 000 mg
- **d** $3\frac{1}{2}$ hrs = ______ min = ______ s
- **e** ______ m = ______ cm = ______ 125 000 mm
- **f** ______ km = 850 m = ______ cm = ______ mm
- **g** 6 t = ______ kg = ______ g = ______ mg
- **h** ______ hrs = ______ min = 7200 sec

5 Convert the following to the units indicated:

- **a** 3500 km = ______ m
- **b** $2\frac{1}{2}$ cm = ______ mm
- **c** 36 min = ______ sec
- **d** 356 g = ______ mg
- **e** 6 hrs = ______ min
- **f** 0·21 km = ______ m
- **g** 2100 cm = ______ mm
- **h** 3·5 kg = ______ mg
- **i** 26 cm = ______ mm
- **j** $6\frac{3}{4}$ m = ______ cm
- **k** 6·4 kg = ______ g
- **l** 56 m = ______ mm

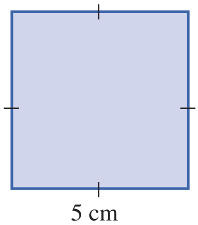
2B Perimeters of shapes

The **perimeter** of a shape is the distance around the outside or boundary of the shape. When finding the perimeter of a shape, ensure that all the lengths are given in the same unit before adding the lengths together.

Example

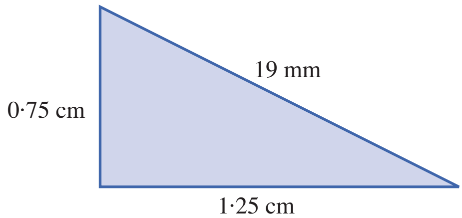
Find the perimeter of the following shapes:

a



All sides are the same length.
Perimeter = 4 × 5 = 20 cm

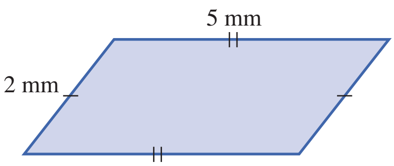
b



Change units to be the same.
19 mm = 1·9 cm

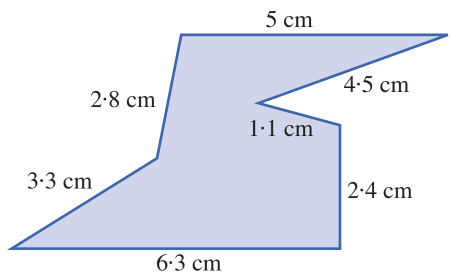
Perimeter = 1·9 + 1·25 + 0·75
= 3·9 cm

c



Perimeter = 2 + 5 + 2 + 5
= 14 mm

d

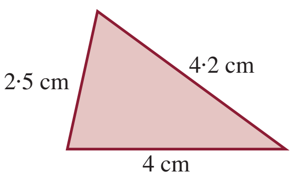


Perimeter
= 5 + 4·5 + 1·1 + 2·4 + 6·3 + 3·3 + 2·8
= 25·4 cm

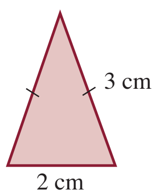
Exercise 2B

1 Find the perimeter of these polygons:

a



b



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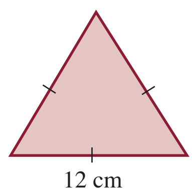
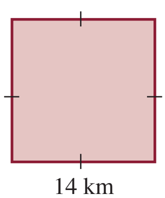
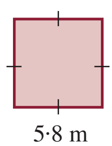


d



e



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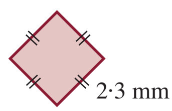
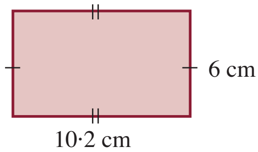
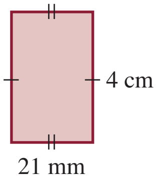


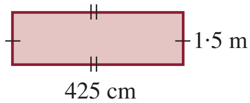
g



h

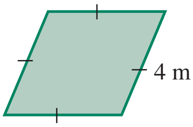


i



2 Find the perimeter of these quadrilaterals:

a



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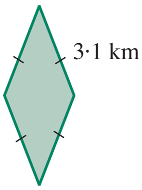

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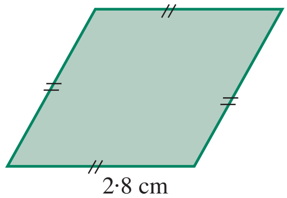
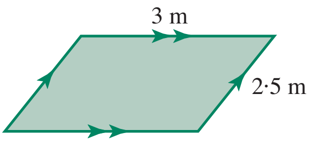
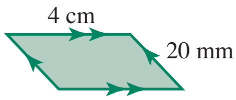


d



e



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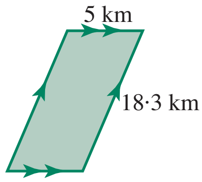
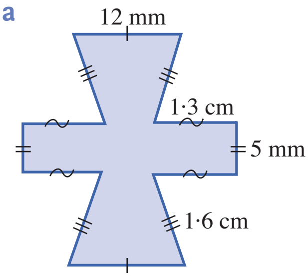


3 Find the perimeter of these irregular shapes:

a



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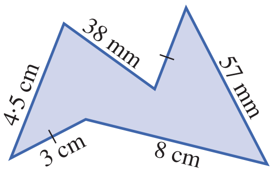

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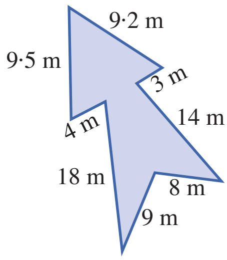

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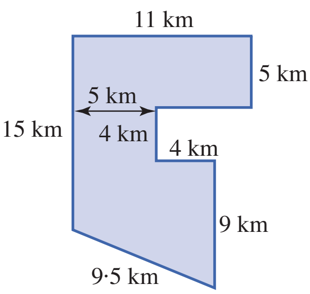

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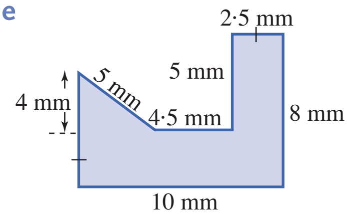

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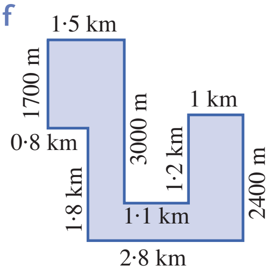
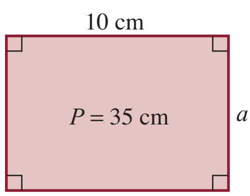


4 Find the value of the pronumeral (unknown) to 1 decimal place in each diagram, given the perimeter:

a



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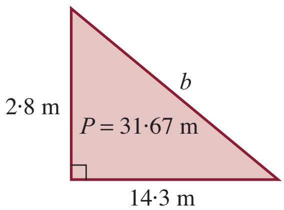
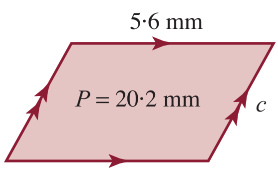


c



2C Exploring the arc lengths of circles

Learning task 2C

Copy the table below into your workbooks:

	1	2	3	4	5	6
Arc	Arc length	Radius (r)	Circumference ($C = 2\pi r \approx 6{\cdot}28 \times r$)	Angle	$\frac{\text{Angle}}{360}$	$\frac{\text{Angle}}{360} \times$ circumference
1						
2						
3						
4						
5						
6						

For each arc below follow the steps and record your results in the table.

Step 1: Measure the arc length shown by the solid line by using a piece of string. Record the result in column 1.

Step 2: Measure the radius. Record the result in column 2.

Step 3: Calculate the circumference of a full circle for that radius. Record it in column 3.

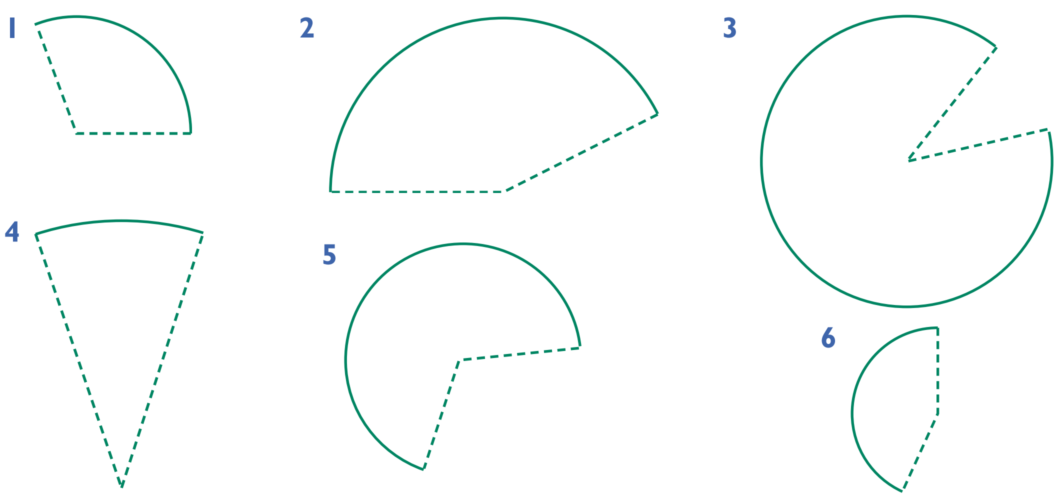
Step 4: Measure the angle and record it in column 4.

Step 5: Calculate the angle ÷ 360. Record the result in column 5.

Step 6: Multiply your answer in column 5 by the circumference. Record the result in column 6.

Step 7: Repeat steps 1 to 6 for each arc.

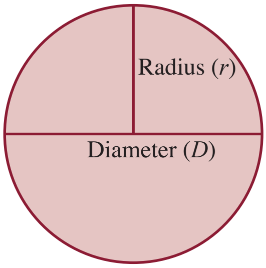
Step 8: Compare column 1 with column 6. Write a sentence to explain what you notice.



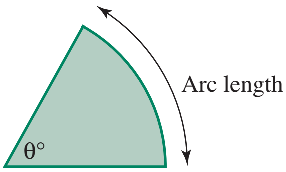
Circumferences of circles and arc lengths

2D

The perimeter of a circle is called the **circumference** and a part of the perimeter of a circle is called the **arc**. To measure the circumference or an arc length you would need to measure the distance with a piece of string. This is not very accurate, so it is better to use a formula, which requires you to know either the diameter or the radius of the circle. The radius and diameter are much easier to measure accurately.



Perimeter or circumference of a circle
$C = \pi D$ or
$C = 2\pi r$
Using the π button on your scientific calculator, π is approximately 3·141 592 654…

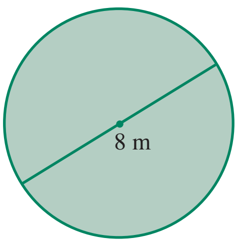


$$\text{Arc length} = \frac{\text{angle in degrees}}{360} \times 2\pi r$$

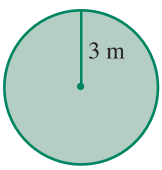
Example

1 Find the circumferences of these circles, expressing your answers to 2 decimal places:

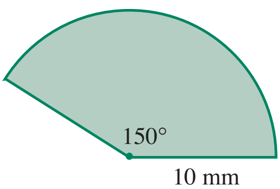
a



b



2 Find the length of the arc:



Solution

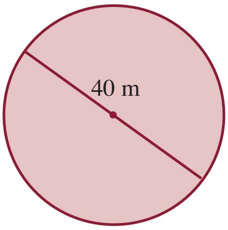
1 a
$$C = \pi D$$
$$= \pi \times 8$$
$$= 25·13 \text{ m}$$

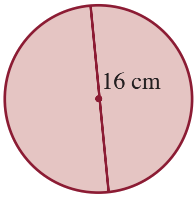
b
$$C = 2\pi r$$
$$= 2 \times \pi \times r$$
$$= 2 \times \pi \times 3$$
$$= 18·85 \text{ m}$$

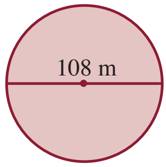
2 Arc length
$$= \frac{\text{angle in degrees}}{360} \times 2\pi r$$
$$= \frac{150}{360} \times 2 \times \pi \times 10$$
$$= 26·18 \text{ m}$$

Exercise 2D

1 Find the circumference of the following circles:

a


b


c


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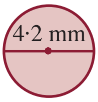
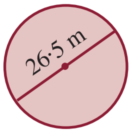


e


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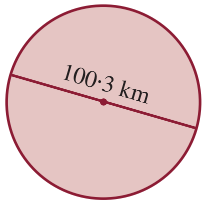
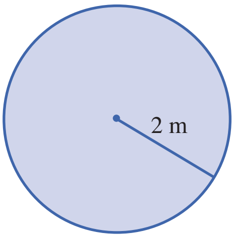


2 Find the circumference of the following circles:

a


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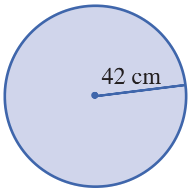
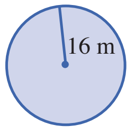
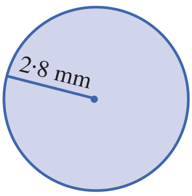


c


d


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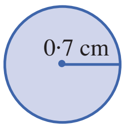

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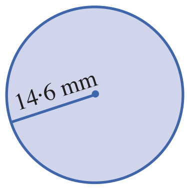
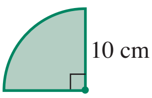


3 Find the arc lengths in the following diagrams, expressing your answers in terms of π:

a


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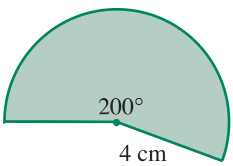

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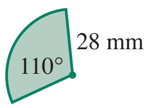

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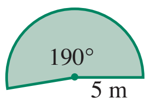
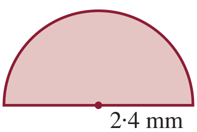
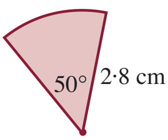
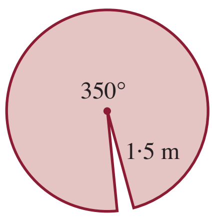


4 Find the perimeter of these shapes correct to 2 decimal places:

a


b


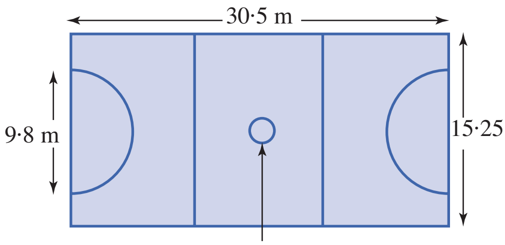
c


5 Different bicycles have wheels of different sizes. How far will each of these wheels travel in one revolution?

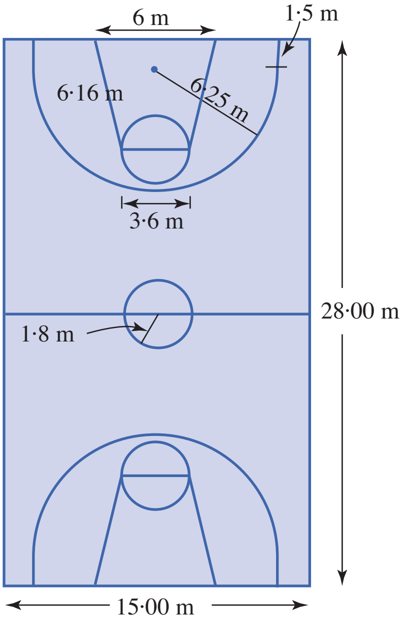
6 What distance will the minute hand of a clock trace between 2:30 pm and 2:55 pm if the minute hand is 10·5 cm in length?

7 Find the total length of lines in the following playing fields:

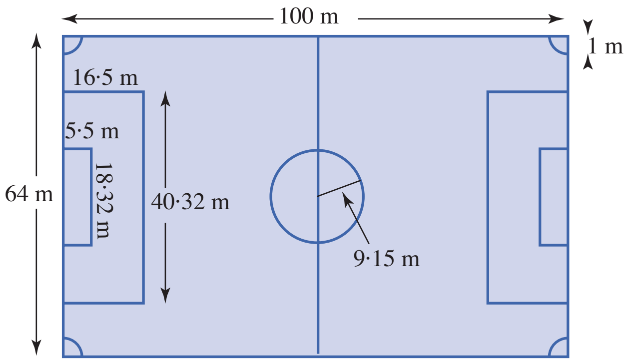
a Netball



b Basketball

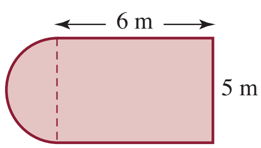


c Soccer

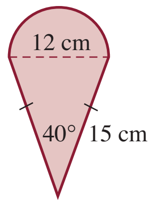


8 Find the perimeter of the following shapes:

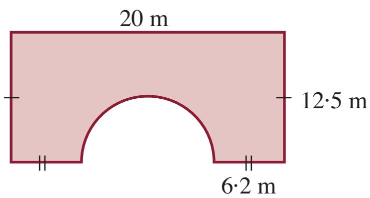
a



b

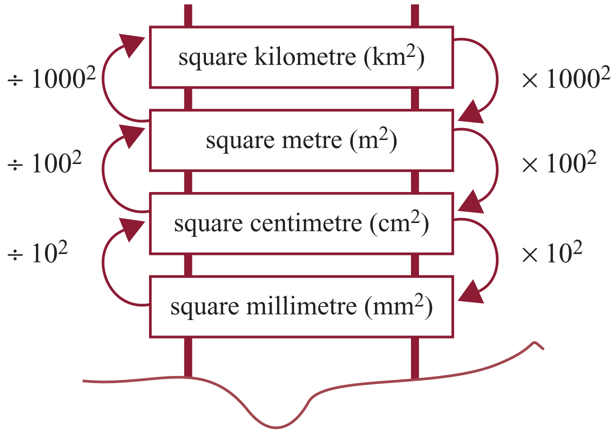


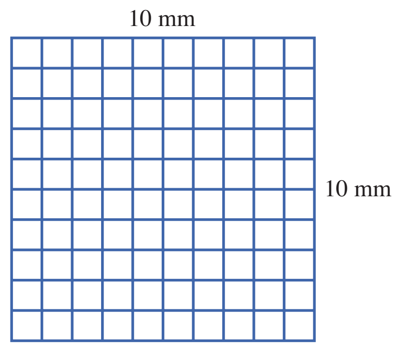
c



2E Area units

The **area** of a shape is the amount of flat space inside the shape. The units used are square millimetres (mm²), square centimetres (cm²), square metres (m²), square kilometres (km²) and hectares (ha). The conversion ladder for area is the same as for length with each of the values squared.





10 mm × 10 mm = 100 mm²   1 cm × 1 cm = 1 cm²

Note: Diagram is not to scale

Also:

- 1 km² = 100 hectares
- 1 hectare = 10 000 m² (i.e. a 100 m by 100 m square)

Exercise 2E

1 State what unit would be the most appropriate to measure the area of:

a your textbook b a CD c your fingernail
d your shadow e the school playing ground f a decimal point
g Honiara h the school farm i Isabel

2 Vangunu Island has an area of about 480 square kilometres. Use the map to estimate the area of the land in:

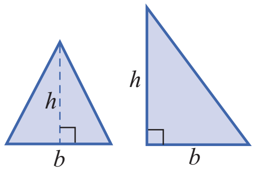
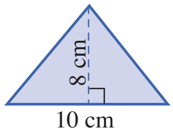
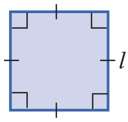
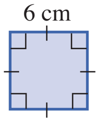
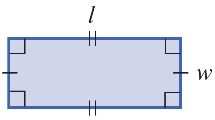
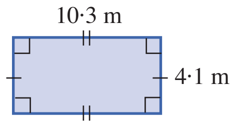
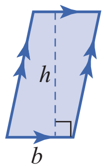
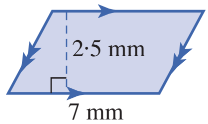
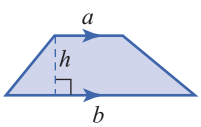
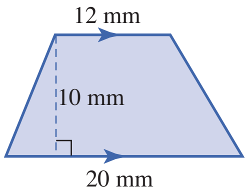
a Santa Isabel
b Guadalcanal
c Makira and Ulawa
d Choiseul

3 Convert the following units:

a 4·2 cm² = _______ mm²
b 0·8 m² = _______ cm²
c 0·067 km² = _______ m²
d 5 m² = _______ mm²
e 0·0005 m² = _______ cm²
f 14 cm² = _______ mm²

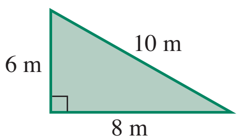
Areas of shapes

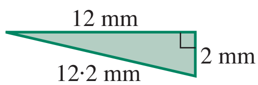
These are the formulas for shapes, which you need to be able to recognise and use.

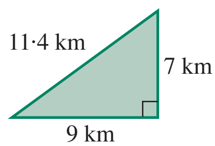
Triangle $A = \frac{1}{2}bh$			$A = \frac{1}{2}bh$ $= \frac{1}{2} \times 10 \times 8$ $= 40\ \text{cm}^2$
Square $A = l^2$			$A = l^2$ $= 6 \times 6$ $= 36\ \text{cm}^2$
Rectangle $A = l \times w$			$A = l \times w$ $= 10{\cdot}3 \times 4{\cdot}1$ $= 42{\cdot}23\ \text{m}^2$
Parallelogram and rhombus $A = bh$			$A = b \times h$ $= 2{\cdot}5 \times 7$ $= 17{\cdot}5\ \text{mm}^2$
Trapezium $A = \frac{1}{2}(a + b)h$			$A = \frac{1}{2}(a + b)h$ $= \frac{1}{2}(12 + 20) \times 10$ $= \frac{1}{2}(32) \times 10$ $= 16 \times 10$ $= 160\ \text{mm}^2$

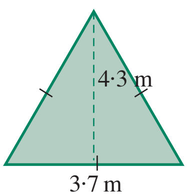
Exercise 2F

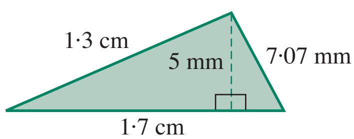
1 Calculate the area of each of the following shapes:

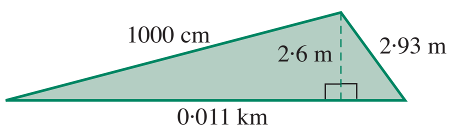
a 

b 

c 

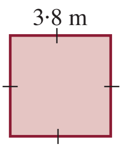
d 

e 

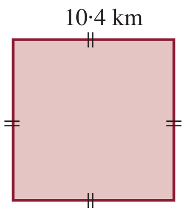
f 

2 Calculate the area of each of the following shapes:

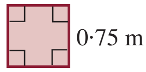
a



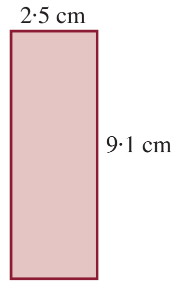
b



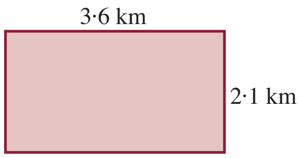
c



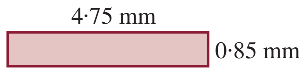
d



e

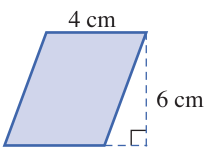


f

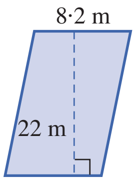


3 Calculate the area of each of the following shapes:

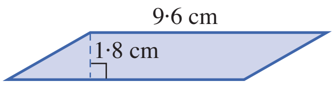
a



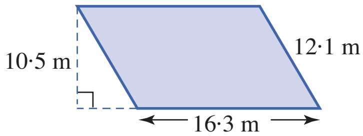
b



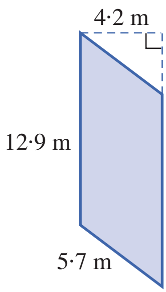
c



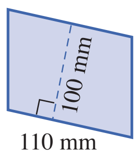
d



e

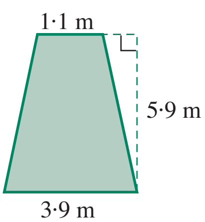


f

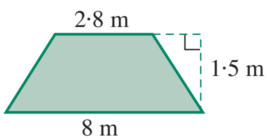


4 Calculate the area of each of the following shapes:

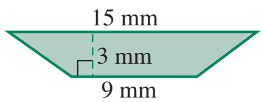
a



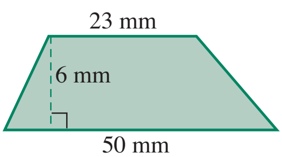
b



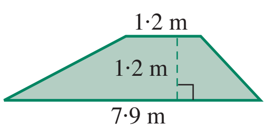
c



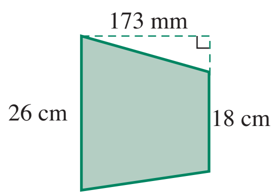
d



e

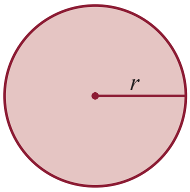


f



Areas of circles and sectors

The area of a circle is the amount of space inside the circle.

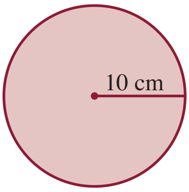


Area = πr^2

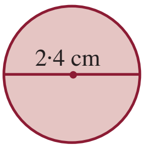
Example

1 Find the area of these circles:

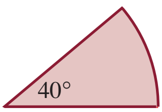
a



b



2 Find the area of this sector:



8 mm

Solution

1 a
$$\text{Area} = \pi r^2 = \pi \times 10 \times 10 = 314{\cdot}16 \text{ cm}^2$$

b
$D = 2{\cdot}4$ cm

$r = 1{\cdot}2$ cm

$$\text{Area} = \pi r^2 = \pi \times 1{\cdot}2^2 = 4{\cdot}52 \text{ cm}^2$$

2 Area of a sector

$$= \frac{\text{angle}}{360} \times \pi r^2$$

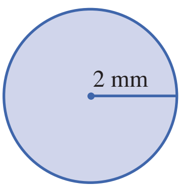
$$= \frac{40}{360} \times \pi \times 8 \times 8$$

$$= 22{\cdot}34 \text{ mm}^2$$

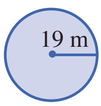
Exercise 2G

1 Find the area of the following circles:

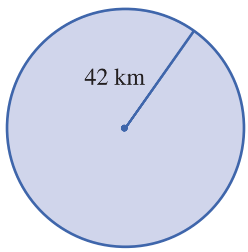
a



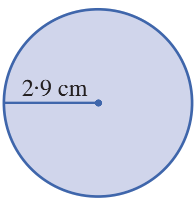
b



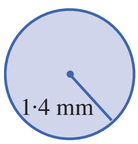
c



d



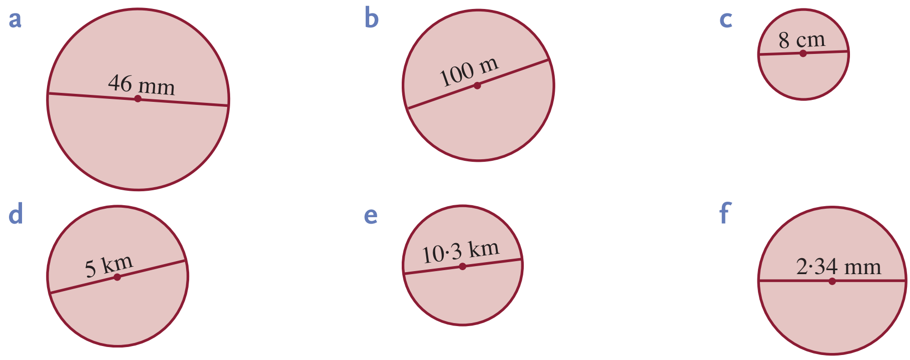
e



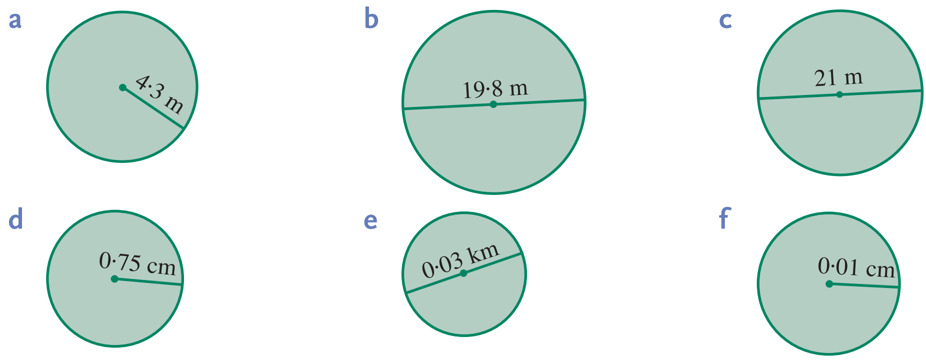
f

2G

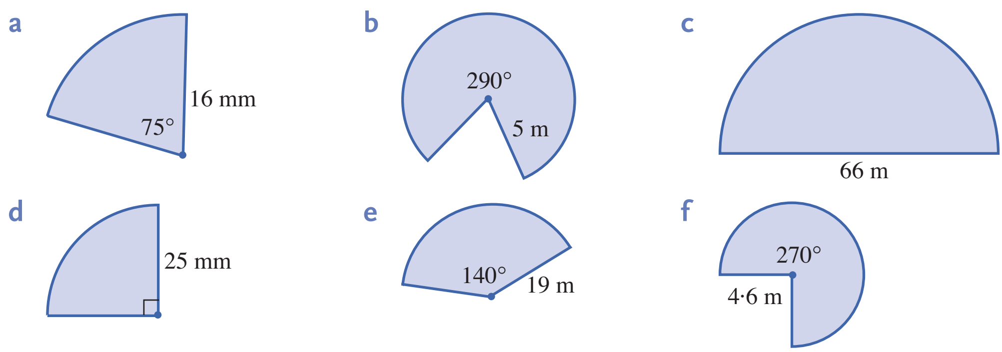
2 Find the area of the following circles:



3 Find the area of the following circles:



4 Find the area of the following sectors:



5 Sarie and Henry shows the cross-sectional cut of a watermelon. Each cross-sectional half is cut into eight equal pieces. Find the area of each piece of watermelon for the following sizes:

Big melon: diameter = 50 cm

Medium melon diameter = 26 cm

Small melon diameter = 16 cm

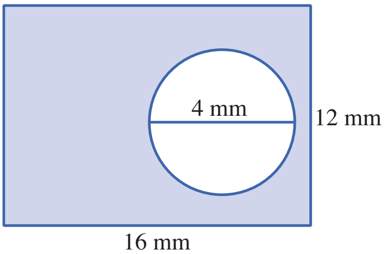
6 Find the area that the minute hand on a clock sweeps out as it moves from 2 pm to 2:25 pm, if the length of the minute hand is 8·5 cm.

Composite shapes

Composite shapes are made up of a number of different shapes. The area of a composite shape is found by adding or subtracting the area of each different shape.

Example

Find the shaded area:



Solution

Area of rectangle $= l \times w$

$= 12 \times 16$

$= 192\ \text{mm}^2$

Area of circle $= \pi r^2$

$= \pi \times 2 \times 2$

$= 12{\cdot}57\ \text{mm}^2$

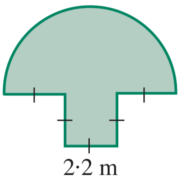
Shaded area $= 192 - 12{\cdot}57$

$= 179{\cdot}43\ \text{mm}^2$

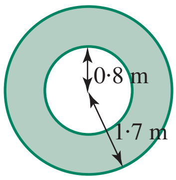
Exercise 2H

Find the area of the following shapes or shaded regions:

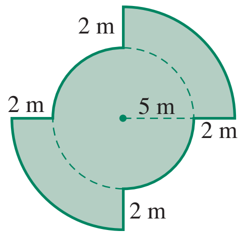
a



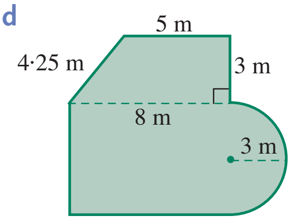
b



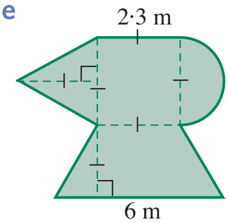
c



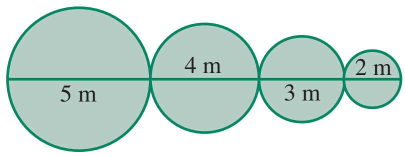
d



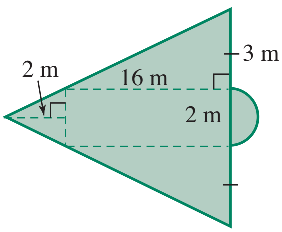
e



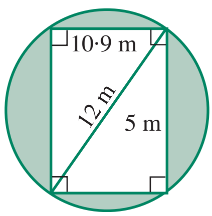
f



g



h



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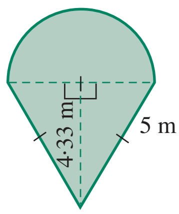
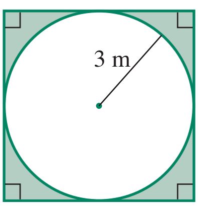
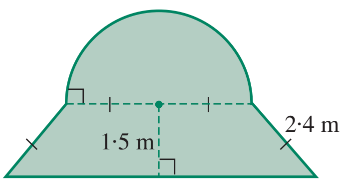


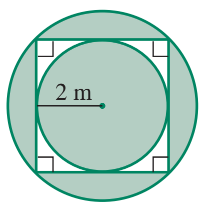
j



k



l



2I Total surface areas of prisms

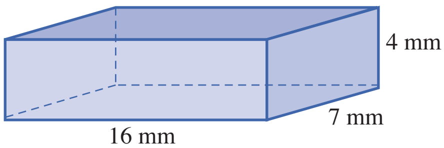
The total surface area of a three-dimensional shape is found by adding together the area of each surface of the shape.

A **prism** is a solid figure which has two faces that are the same size and shape and the corresponding sides are parallel. The faces must be polygons.

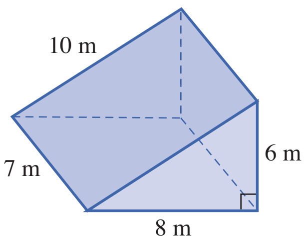
Example

Find the total surface area of these 3D shapes:

a

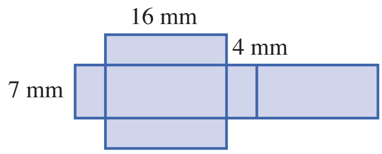


b



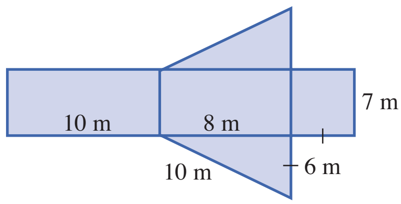
Solution

a



$$\begin{aligned}\text{Area} &= 2 \times 16 \times 7 + 2 \times 7 \times 4 + 2 \times 16 \times 4\\ &= 224 + 56 + 128\\ &= 408 \text{ mm}^2\end{aligned}$$

b

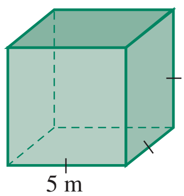


$$\begin{aligned}\text{Area} &= \tfrac{1}{2}(6 \times 8) \times 2 + 10 \times 7 + 8 \times 7 + 6 \times 7\\ &= 48 + 70 + 56 + 42\\ &= 216 \text{ m}^2\end{aligned}$$

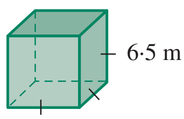
Exercise 2I

1 Find the total surface area of the following prisms:

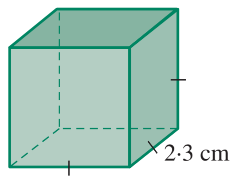
a



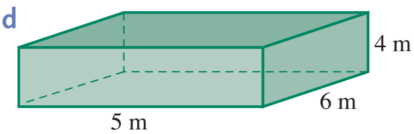
b



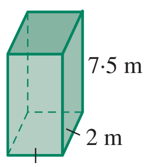
c



d



e



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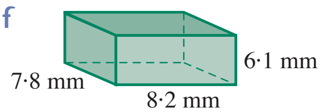
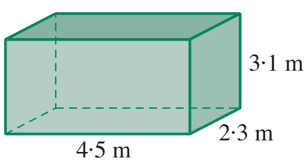
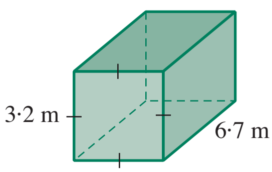


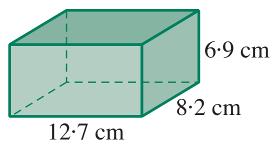
g



h

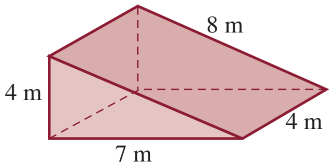


i

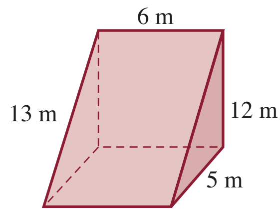


2 Find the total surface area of these triangular prisms:

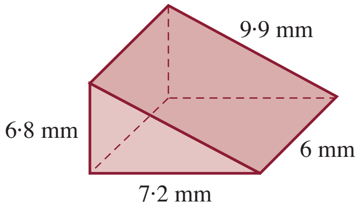
a



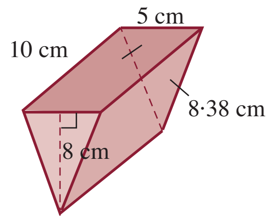
b



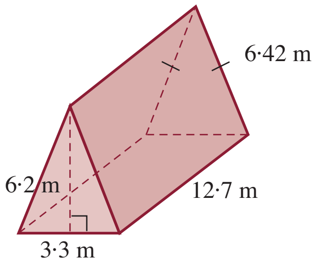
c



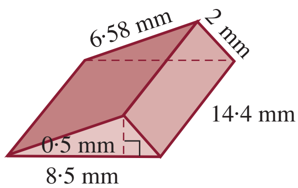
d



e

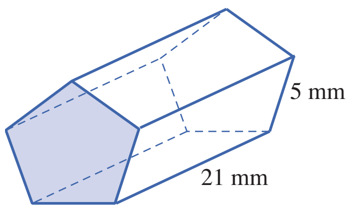


f



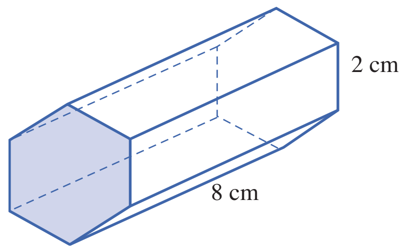
3 Find the total surface area of these regular prisms:

a



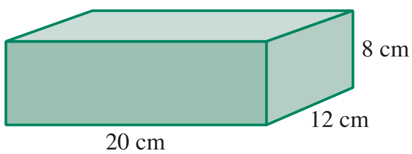
Area of end = 43 mm^2

b

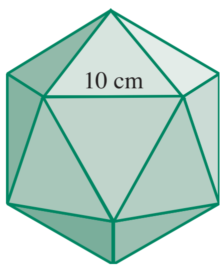


Area of end = 10·39 cm^2

4 Find the total surface area of this box, then find the dimensions of a piece of wrapping paper used to wrap the box, if an overlap of 3 cm is needed on the top and at the ends of the box.



5 This solid is called an icosahedron and has 20 faces, each of which is an equilateral triangle. The height of each triangular face is 8·67 cm. How much paint is needed to paint the icosahedron, if 1 litre covers 10 m^2?

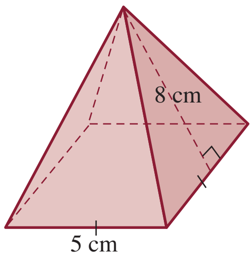


2J Surface areas of pyramids

A **right pyramid** is a three-dimensional shape with a polygon base. All other sides are triangles forming an apex or point above the centre of the base. The horizontal cross-section is similar in shape to the base; however, its size decreases as you move towards the apex.

Example

Find the total surface area (TSA) of this pyramid:



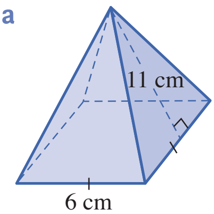
Solution

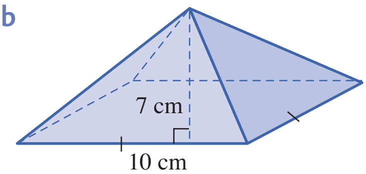
Total surface area = area of square base + area of four triangular faces

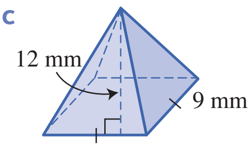
$$TSA = 5 \times 5 + 4 \times \tfrac{1}{2}(5 \times 8)$$
$$= 25 + 80$$
$$= 105\ \text{cm}^2$$

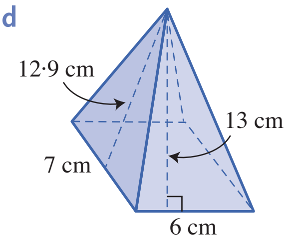
Exercise 2J

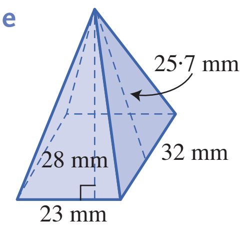
1 Find the total surface area of these pyramids:

a


b


c


d


e


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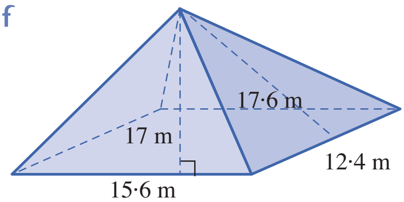

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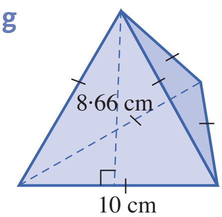

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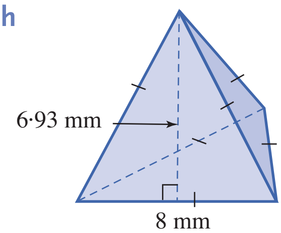
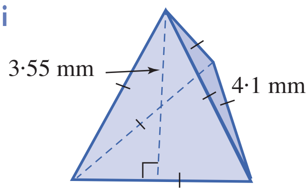


i


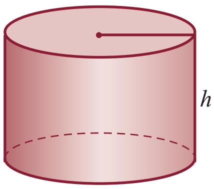
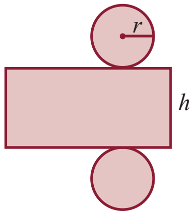
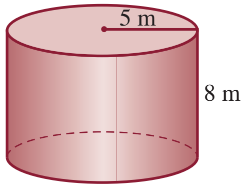
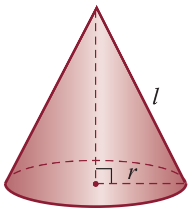
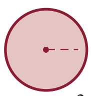
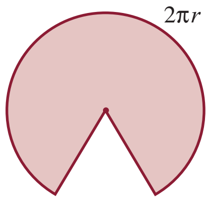
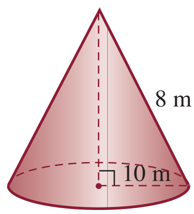
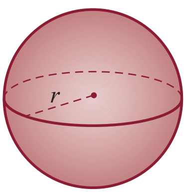
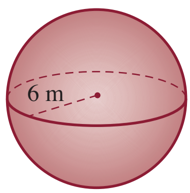
2 Find the amount of paper needed to cover:

a a pyramid with a square base of 8·5 cm and a perpendicular height on the triangular faces of 12·3 cm

b a triangular pyramid of side length 9·2 cm and perpendicular height on the triangular faces of 7·97 cm

Cylinders, cones and spheres

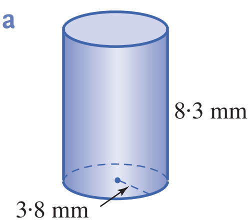
The surface area of cylinders, cones and spheres can be found using the formulas shown in the table:

Cylinder  	 $TSA = 2\pi r^2 + 2\pi rh$ $= 2\pi r(r + h)$	 $TSA = 2\pi r^2 + 2\pi rh$ $= 2\pi r(r + h)$ $= 2\pi \times 5^2 + 2\pi \times 5 \times 8$ $= 50\pi + 80\pi$ $= 130\pi \text{ m}^2$
Cone  	 $A = \pi r^2$  $A = \frac{2\pi r}{2\pi l} \times \pi l^2$ $= \pi rl$ $TSA = \pi r^2 + \pi rl = \pi r(r + l)$	 $TSA = \pi r^2 + \pi rl$ $= \pi \times 10^2 + \pi \times 10 \times 8$ $= 100\pi + 80\pi$ $= 180\pi \text{ m}^2$
Sphere  	$TSA = 4\pi r^2$	 $TSA = 4\pi r^2$ $= 4\pi \times 6^2$ $= 144\pi \text{ m}^2$

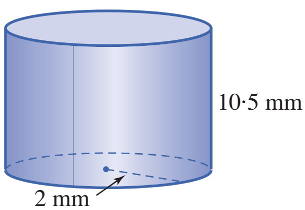
Exercise 2K

1 Find the total surface area of these solids:

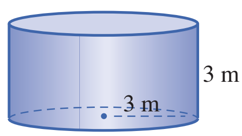
a

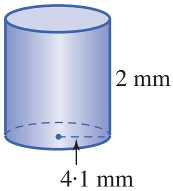


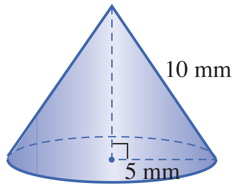
b

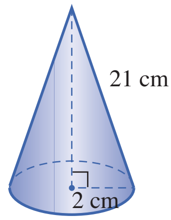


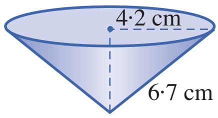
c

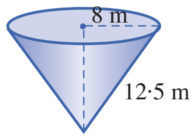


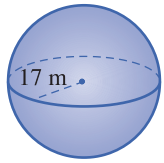
d 

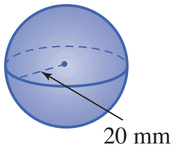
e 

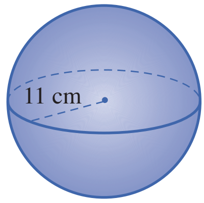
f 

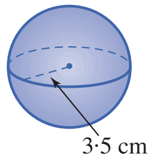
g 

h 

i 

j 

k 

l 

2 Find the area of steel needed to make a 425 g can with diameter of 75 mm and a height of 11 cm. (The can is a cylinder.)

3 **a** Find the area of leather needed to make a ball of diameter 20 cm.
b Find the area of furry material needed to make a tennis ball of diameter 67 mm.
c Find the area of metal needed to make a 4 m length of water pipe with a diameter of 100 mm.

4 Calculate the area of paper needed to wrap a cone-shaped ice cream of diameter 6 cm and side length of 15 cm.

5 Find the surface area of the Earth, which has a radius of approximately 6400 kilometres.

6 Find the surface area of the following:

a 
A soccer ball (sphere) with a radius of 12 cm.

b 
A floater, which is a sphere with a radius of 26 cm.

c 
A soccer cone with a base radius 10 cm and side length 30 cm.

d 
A 200 litre drum of petrol with a radius of 26 cm and height 96 cm.

Volume of prisms

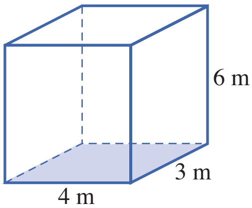
The **volume** of a solid is the amount of space inside the solid. The units used for solids in this exercise are either mm^3, cm^3 and m^3.

> The volume of prisms and other solids with a constant cross-section
> = area of base × height

Example

Find the volume of the following prisms:

a

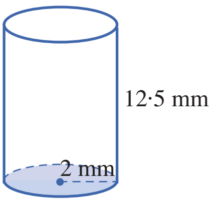


Solution

Area of base $= 3 \times 4$
$= 12 \text{ m}^2$

Volume $= 12 \times 6$
$= 72 \text{ m}^3$

b



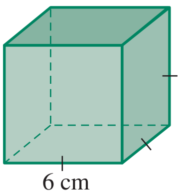
Area of base $= \pi r^2$
$= \pi \times 2 \times 2$
$= 12{\cdot}57 \text{ mm}^2$

Volume $= 12{\cdot}57 \times 12{\cdot}5$
$= 157{\cdot}1 \text{ mm}^3$

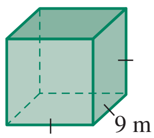
Exercise 2L

1 Find the volume of the following solids:

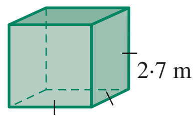
a



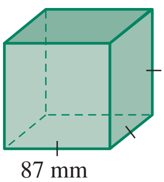
b



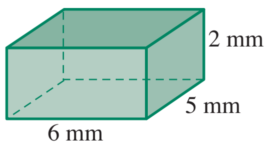
c



d



e



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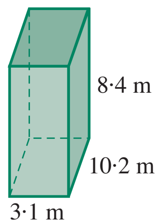
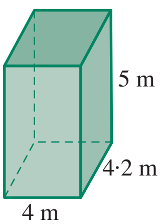
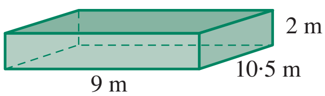


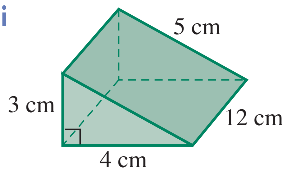
g



h



i



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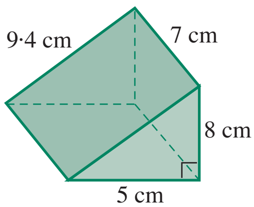

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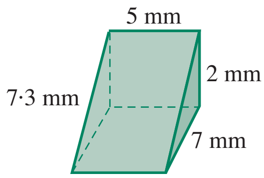

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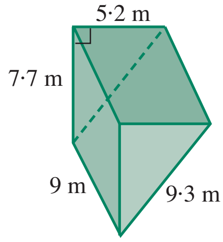

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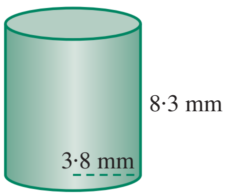

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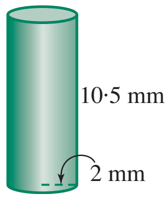

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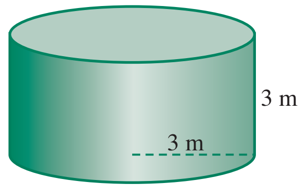
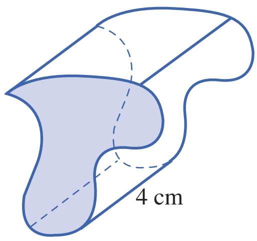
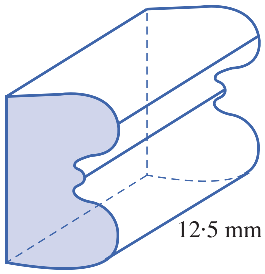


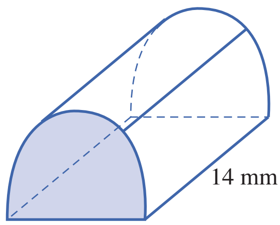
2 Find the volume of these solids; the area of the base has been calculated for you:

a


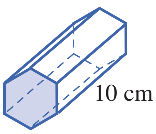
Area = 12·2 cm^2

b


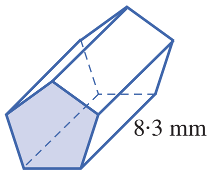
Area = 32·7 mm^2

c


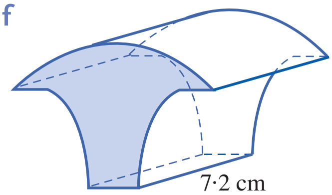
Area = 124 mm^2

d


Area = 82·5 cm^2

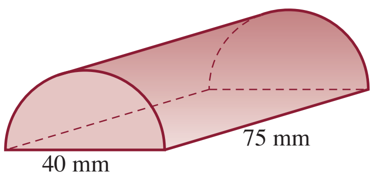
e


Area = 46 mm^2

f


Area = 17·5 mm^2

3 Find the volume of these solids:

a


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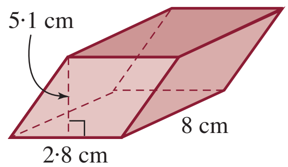

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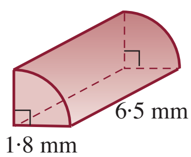
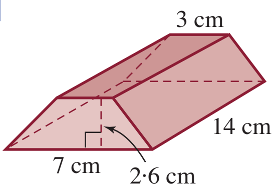
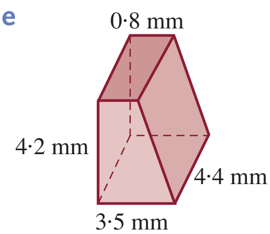


d


e


Volumes of pyramids and cones

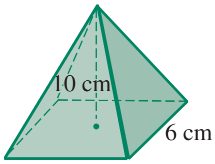
2M

- Volume of a pyramid or cone $= \frac{1}{3} \times$ area of base $\times$ height

Example

Find the volume of these solids:

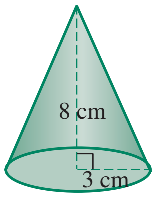
a



Solution

$$\begin{aligned} \text{Volume} &= \tfrac{1}{3} \times \text{area of base} \times \text{height} \\ &= \tfrac{1}{3} \times 5 \times 6 \times 10 \\ &= 100\ \text{cm}^3 \end{aligned}$$

b

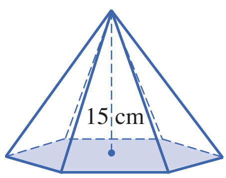


$$\begin{aligned} \text{Volume} &= \tfrac{1}{3} \times \pi r^2 \times h \\ &= \tfrac{1}{3} \times \pi \times 3^2 \times 8 \\ &= 24\pi\ \text{cm}^3 \\ &= 75{\cdot}4\ \text{cm}^3 \end{aligned}$$

Exercise 2M

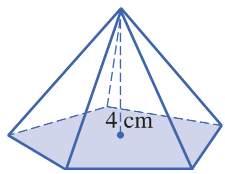
1 Find the volume of these two solid pyramids:

a



Area of base = 40 cm^2

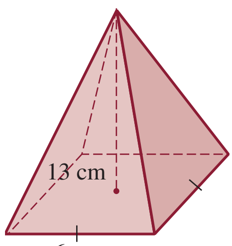
b



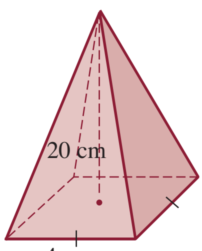
Area of base = 27·5 cm^2

2 Find the volume of the following solid pyramids:

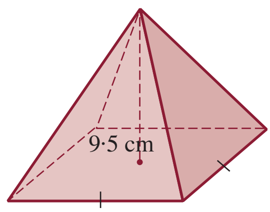
a



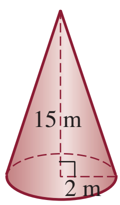
b



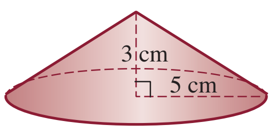
c



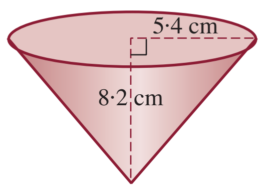
d



e



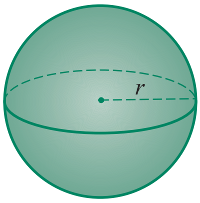
f



3 Find the volume of a solid conical funnel with a radius of 10 cm and a height of 8 cm.

2N Volume of a sphere

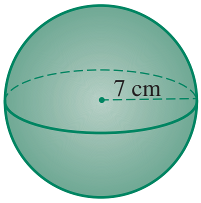
The formula used to find the volume of a sphere is shown below:



Volume of a sphere $= \frac{4}{3}\pi r^3$

Example

Find the volume of this sphere:



Solution

$$\text{Volume} = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} \times \pi \times 7 \times 7 \times 7$$
$$= 1436{\cdot}76 \text{ cm}^3$$

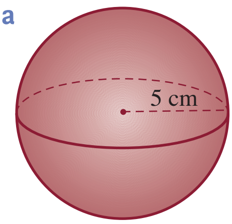
Exercise 2N

1 Find the volume of solid spheres with the following radii:

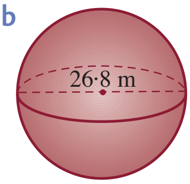
a 16 m **b** 0·4 cm **c** 1·1 mm **d** 2·4 cm

2 Find the volume of the following solid spheres:

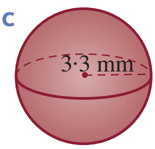
a



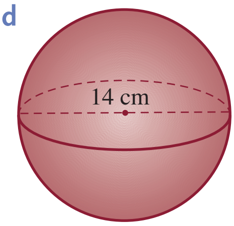
b



c



d



3 **a** Find the volume of the Earth if its radius is approximately 6400 km.

b Find the volume of each of these planets with the following radii:

i Venus 6052 km **ii** Mercury 2436 km **iii** Mars 3398 km

iv Jupiter 71 434 km **v** Saturn 59 826 km **vi** Uranus 25 700 km

vii Neptune 25 193 km **viii** Pluto 1750 km

4 Find the volume of solid hemispheres with radii of:

a 15 cm **b** 9 cm **c** 2·8 cm **d** 4·5 m

5 Find the volume of these spheres:

a Basketball of radius 30 cm

b Beach ball of radius 35·4 cm

c Tennis ball of radius 6·7 cm

Capacity 20

Capacity is a measure of how much a container can hold, and is usually used to refer to liquids.

Common units are millilitre (mL), litre (L), kilolitre (kL) and megalitre (ML).

1 mL = 1 cm^3 1000 mL = 1 L = 1000 cm^3 1000 L = 1 kL 1 000 000 L = 1 ML

Exercise 2O

1 Which units would you use to measure the:

a water in Lake Rennell
b capacity of a bottle of Szeba
c drink of Milo in a cup
d dose of cough medicine
e milk needed for a cake recipe
f water in a swimming pool
g soft drink in large bottle
h petrol in a full measuring cylinder

2 Convert the following units:

a 2·5 L = ______ cm^3
b 20 mL = ______ cm^3
c 3200 mL = ______ L
d 5750 L = ______ kL
e 4500 cm^3 = ______ L
f 6·6 cm^3 = ______ mL
g 4·2 ML = ______ L
h 0·016 kL = ______ cm^3

3 Calculate the capacity in litres of the following items:

a Saucepan of radius 10·2 cm and height 14·5 cm

b Fish tank in the shape of half a sphere of radius 10·4 cm

c Esky in the shape of a rectangular prism, height 40 cm, width 25 cm length 35 cm

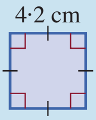
d Thermos in the shape of a cylinder, height 20 cm and radius 5 cm

Puzzles

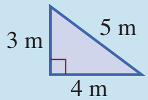
1 Calculate the perimeter of the shapes and match the corresponding letter to the correct answer below to solve this riddle:

What do you serve that you can't eat?

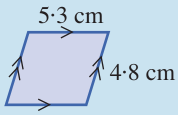
A



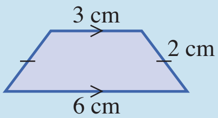
B



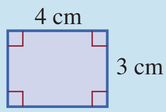
C



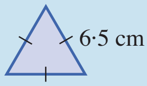
E



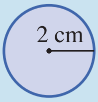
I



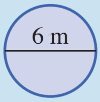
G



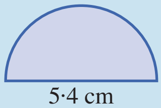
H



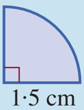
N



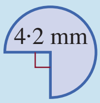
S



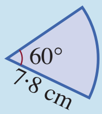
T



U



V

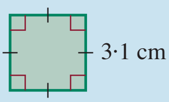


___	___	___	___	___	___		___	___	___	___	___
5·4	13	18·8	18·8	14	13·9		12	16·8	23·8	23·8	13·9
___	___		___	___	___	___	___		___		
20·2	12·6		19·5	28·2	13	13·9	5·4		13·9		

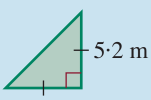
2 Calculate the area of the shapes correct to 1 decimal place and match the corresponding letter to the correct answer below to solve this riddle:

What do you throw out when you want to use it, but take in when you don't want to use it?

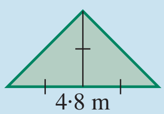
A



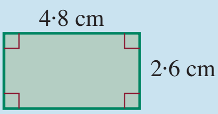
C



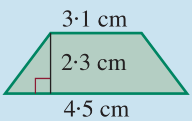
D



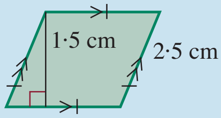
E



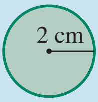
H



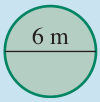
I



M



R



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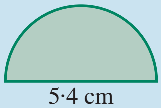
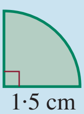
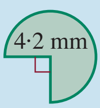


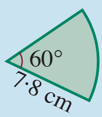
T



U



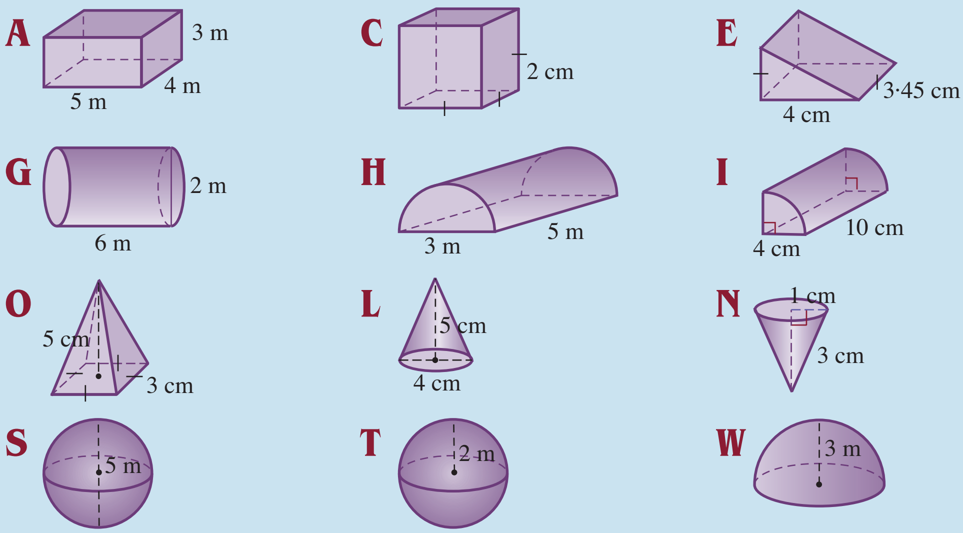
Y



___	___	___	___	___	___	___	___	___	___	___
13·5	8·7	28·3	12·5	8·7	12·6	5·8	11·4	8·7	12·6	9·6

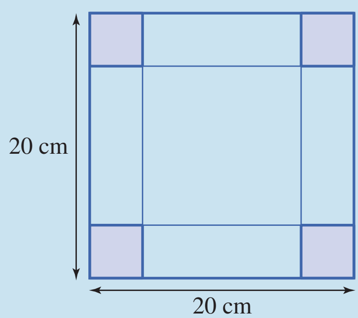
3 Calculate the volume of the following shapes to the nearest whole number. Match the corresponding letter to the correct answer below to solve this riddle:

Why is it so easy to weigh fish?



___	___	___	___		___	___	___	___	___	___
15	60	57	18		65	24	60	3	18	65

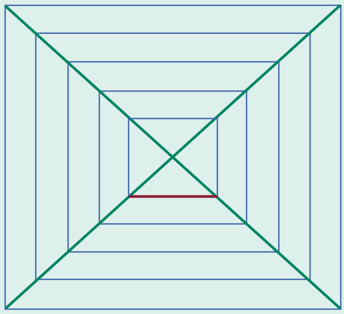
4 You can make an open box from a 20 cm by 20 cm piece of card by cutting out four squares and folding the flaps. What is the biggest volume of box you can make in this way?



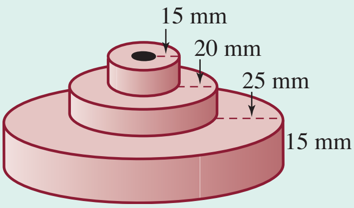
Applications

Clothes line

Find the length of wire needed to replace all the wire in this square clothes line. The smallest section (indicated in red) is 920 mm long, and each section is 340 mm longer than the previous one.



Tower of Hanoi

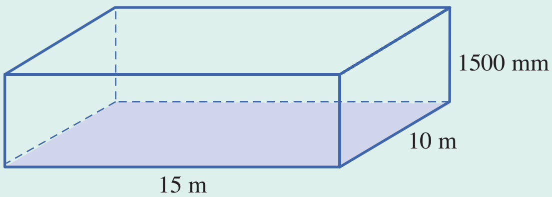


The Tower of Hanoi is made up of three circular discs with a hole in the centre of each. The puzzle begins with the three discs stacked as shown. If the hole in the centre of each disc is 15 mm, find the total surface area of the Tower of Hanoi, excluding the peg the discs are placed upon.

Swimming pool

a Find the volume of soil that must be removed to form a hole 15 metres long, 10 metres wide and 1500 mm deep.

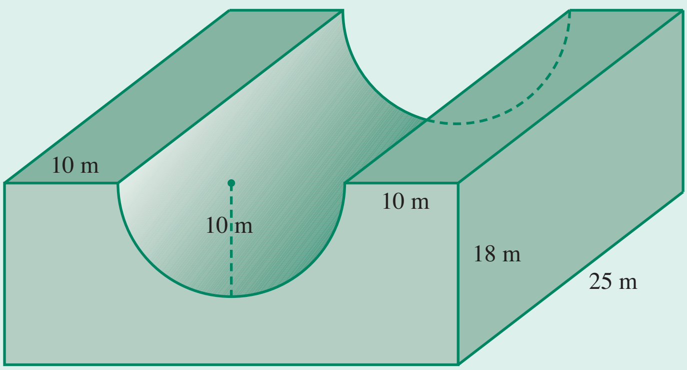
b Calculate the volume of concrete that needs to be poured to make the bottom and sides of the pool 15 mm thick.



c How many 6 cm square tiles are needed to tile a 30-cm-wide path around the outside perimeter of the pool?

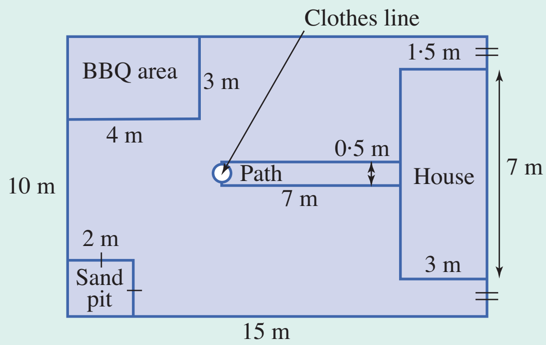
Skateboarding

Find the volume of this skateboard ramp:

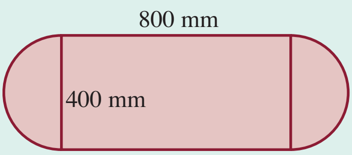


Pet goat

A pet goat grazes the lawn in a backyard as shown. Calculate the area of grass that the goat can graze.



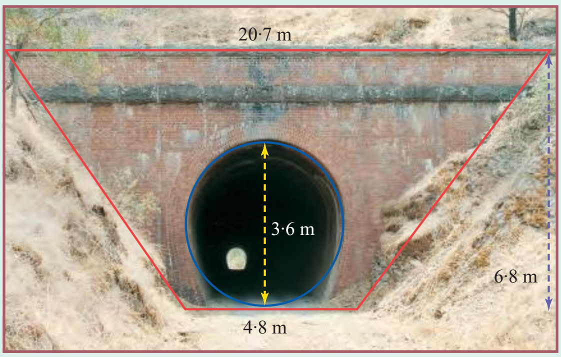
Gas tank



Find the surface area and volume of an LPG gas tank used in motor cars. It is a cylinder with hemispherical ends.

Train tunnel

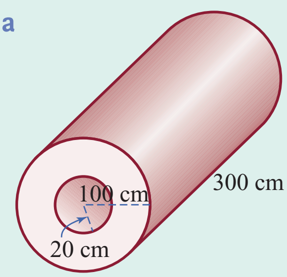
The photo to the right shows an old train tunnel. The front brick surface of the tunnel can be approximated to a trapezium and the tunnel can be approximated to a circle, with dimensions as shown. Calculate the surface area of the front brick surface of the tunnel. If each brick is 23 cm × 8 cm, calculate how many bricks were needed for the front of the tunnel.



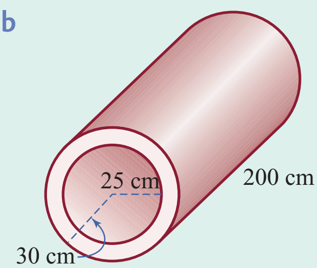
Drains

Find the volume of concrete required to produce these drain pipes:

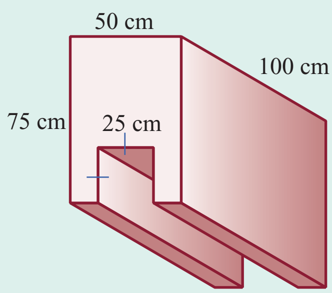
a



b



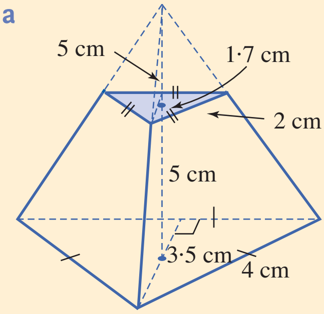
c



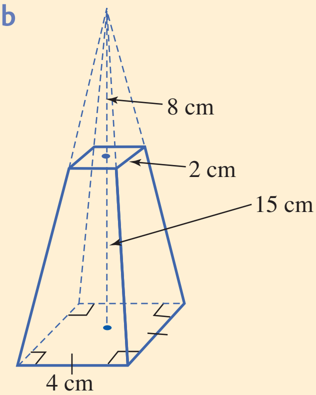
Enrichment

1 Find the volume of the following truncated shapes (shapes which have had their tops or points cut off):

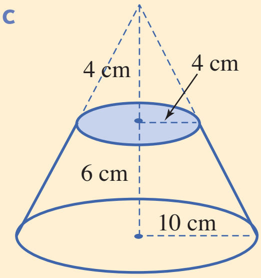
a



b

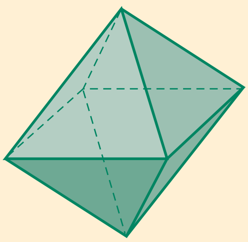


c

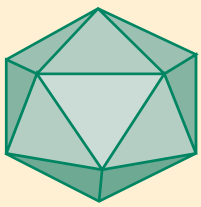


2 Draw a 5 cm equilateral triangle. Measure the perpendicular height of the triangle. Use this information to find the total surface area of these regular polyhedrons or platonic solids with sides of 5 cm:

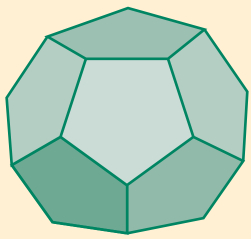
a Octahedron



b Icosahedron

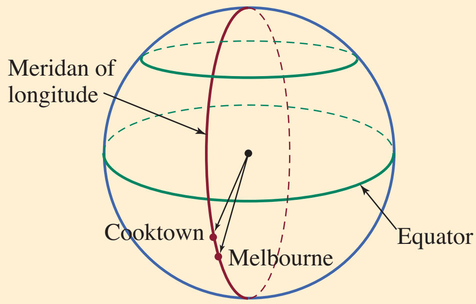


c Dodecahedron

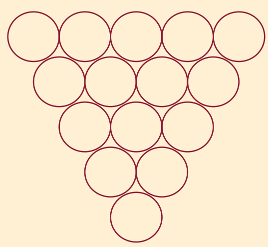


3 Meridians of longitude pass through the north and south poles, and have a radius of 6400 kilometres. Using the arc length formula, calculate the shortest distance between the following cities:

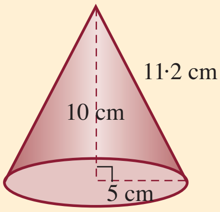
a Melbourne 38°S and Cooktown 16°S on the 145°E meridian
b San Antonio, USA, 29°N and Winnipeg, Canada, 50°N on the 97°W meridian
c Tokyo, Japan, 35°N and Mt Isa, Queensland, 20°S on the 139°E meridian
d Cape Town, South Africa, 34°S and Sarajevo, Yugoslavia, 43°N on the 18°E meridian

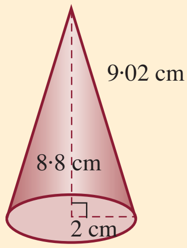


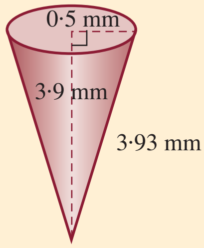
4 At the start of each game of billiards, the 15 balls are arranged in a triangle as shown. If the diameter of each ball is 5·2 centimetres, find the inside perimeter of the smallest triangle possible to fit around all the balls. Find the volume of space inside the triangle when filled with the balls. (Hint: Volume of triangular prism minus volume of all the balls.)



5 Find the surface area and volume of these cones:

a


b


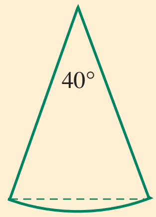
c


6 A child's swing of length 2·5 metres moves through an angle of 40°.

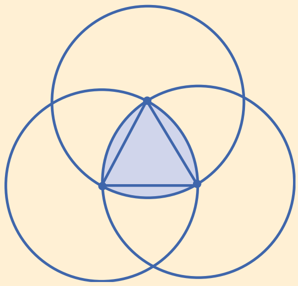
a Find the length of the arc that the swing traces out.

b Find the area that the swing sweeps out.

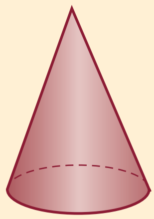
c Draw a scale diagram to find the shortest distance between the end points of the swing.



7 Draw an equilateral triangle with side lengths of 5 cm. Using a vertex of the triangle as the centre of a circle, draw a circle that passes through each of the other two vertices. Repeat this step at each of the other vertices. Find the area of the overlap of the three circles.



8 When the top is sliced off a cone, parallel to the base, a truncated cone or frustum is formed. The top of the frustum is circular. If the top is sliced off at an angle to the base, then the top of the truncated cone is an ellipse or oval (a squashed circle shown below).



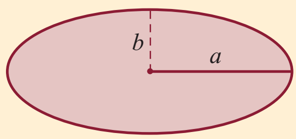
The values of a and b represent the values of the major and minor axes. These values can be used to calculate the area of the ellipse by using the formula:

$A = \pi ab$

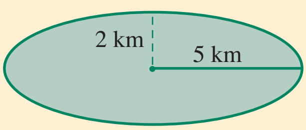
If $a = 6$ and $b = 4$, the area $= \pi \times 6 \times 4$

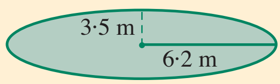
$= 24\pi$

$= 75{\cdot}4$ sq units



Find the area of the following ellipses:

a


b


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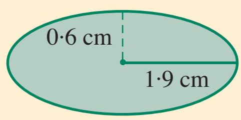

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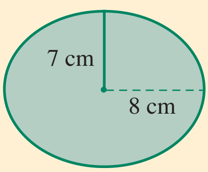
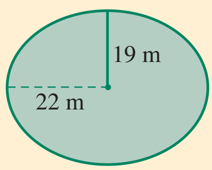


e


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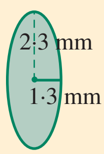


Revision/Assessment

Exercise 2A

1 Convert the following units to metres:

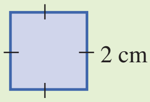
a 400 cm b 1·4 km c 160 000 mm

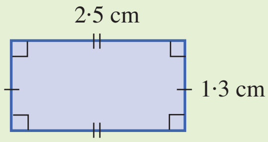
2 Convert the following units to centimetres:

a 4300 mm b 0·4 m c 2 km

Exercises 2B and 2D

3 Find the perimeter of the following shapes:

a


b


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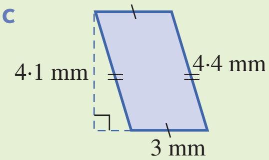
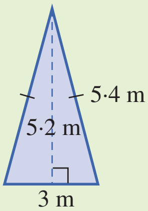


d


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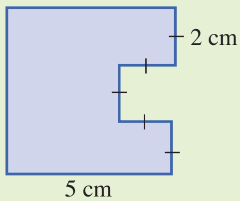

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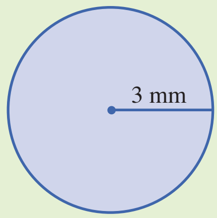
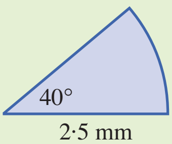


g


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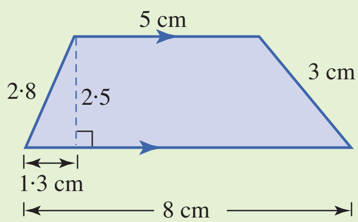


Exercises 2E

4 Convert the following units:

a $4000\ \text{mm}^2 =$ _______ cm^2 b $600\ \text{ha} =$ _______ km^2

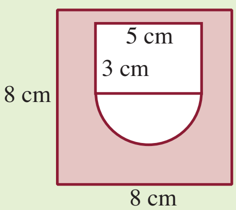
c $56\ \text{m}^2 =$ _______ cm^2 d $5{\cdot}2\ \text{km}^2 =$ _______ ha

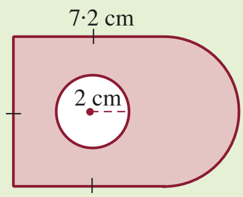
Exercises 2F

5 Find the area of the shapes in Question 3.

Exercises 2G and 2H

6 Find the area of the shaded regions:

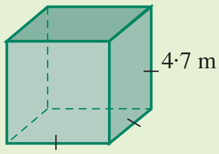
a


b


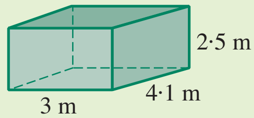
Exercises 2I and 2L

7 Find the volume and total surface area of the following prisms:

a



b



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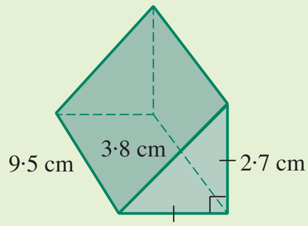

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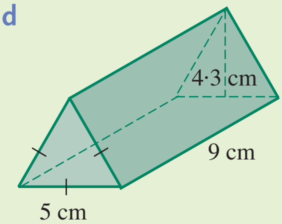

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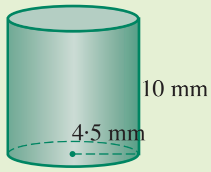

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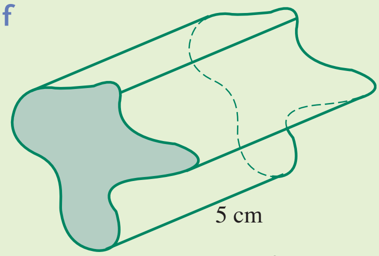
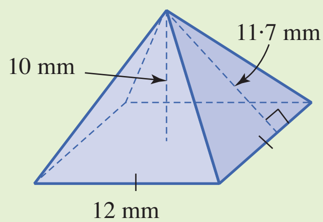


Area of the end = 20 cm^2

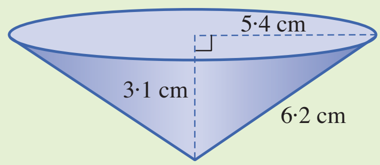
Exercises 2J, 2K and 2M

8 Find the total surface area and volume of these pyramids:

a



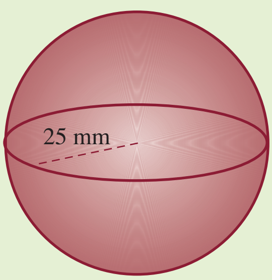
b



Exercises 2K and 2N

9 Find the total surface area and volume of these spheres:

a



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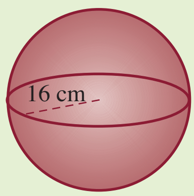

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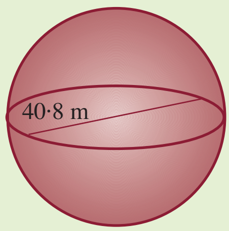


Exercise 2O

10 Find the capacity in litres of a cylindrical can with height 12 cm and radius 5 cm, correct to 2 decimal places.

11 Convert the following units:

a 0·025 kL = ______ cm^3

b 4·1 cm^3 = ______ mL

CHAPTER

3

Geometry

Builders construct floors that are horizontal and walls that are vertical. To achieve this they use tools to measure right angles, spirit levels to measure horizontally and plumb lines to check for vertical alignment. They make sure that the walls of the house are at right angles to the floor, doorframes are at right angles to the floor and windows are inserted correctly.

Modern architecture also uses lines and angles in random patterns to create attractive designs that can be seen on buildings and bridges. In Solomon Islands, patterns are created using shells, ropes, vines and paint that provide us with attractive ornaments, containers and clothing designs. Geometric features such as symmetry and tessellations inspire weaving and even facial tattoos.

This chapter covers the following skills:

- Working with angle relationships
  Complementary angles add to 90°
  Supplementary angles add to 180°
  Angles in a straight line add to 180°
  Angles in a circle add to 360°
  Corresponding angles are equal
  Alternate angles are equal
  Co-interior angles are supplementary
  Vertically opposite angles are equal
- Naming and measuring angles in polygons
- Angle relationships in polygons
  △ Angle sum = 180°
  □ Angle sum = 360°
  The angle sum of a n-sided polygon $= (n - 2) \times 180°$
- Exploring 3D shapes
- Recognising congruent shapes
- Recognising similar figures
- Working with similar triangles and their real-life applications
- Using networks and undirected graphs

Specific Learning Outcome (SLO)

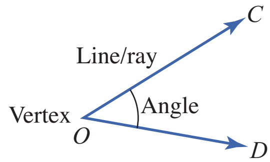
Learners should be able to:

9.3.1.1	Define 'angle'.
9.3.2.1	Name and classify angles according to their sizes.
9.3.3.1	Define 'complementary' and 'supplementary angles'.
9.3.3.2	Find angles that are complementary and supplementary to other angles.
9.3.4.1	Identify angles that are formed in a straight line and in a circle.
9.3.4.2	Find other (missing) angles using properties of straight lines and angles between chords in a circle.
9.3.5.1	Identify transversal and parallel lines.
9.3.5.2	Name and identify angles that are formed when a transversal line cuts through a pair of parallel lines.
9.3.5.3	Calculate missing angles using the properties of corresponding, alternate and co-interior angles.
9.3.6.1	Define and identify 'vertically opposite' and 'adjacent angles'.
9.3.6.2	Find missing angles using lines and angles properties.
9.3.7.1	Define 'polygons'.
9.3.7.2	Identify angle-sum for triangles, quadrilaterals and other polygons and how they are used in calculations.
9.3.8.1	Calculate the interior angle–sum for triangles using measurement and the formula: Angle sum $= (n - 2) \times 180°$
9.3.9.1	Define and identify 'congruent shapes'.
9.3.9.2	Identify the RULES that can be used to identify congruent triangles: *Side – Side – Side (SSS)* *Side – Angle – Side (SAS)* *Angle – Side – Angle (ASA)*
9.3.10.1	Prove and identify triangles that are congruent according to the rules.
9.3.10.2	Find the missing values that are represented by pronumerals for congruent shapes.
9.3.11.1	Define and identify shapes that are similar.
9.3.12.1	Increase and reduce figures and objects using ratios through scale factors.
9.3.12.2	Identify scales factors of shapes and objects that have been enlarged.
9.3.13.1	Measure the length of the sides of given shapes to identify shapes that are similar.
9.3.14.1	Identify the Rules that can be used to test whether triangles are similar or not. *SSS: Side – Side – Side* *SAS: Side – Angle – Side* *AAA: Angle – Angle – Angle* *RHS: Right angle – Hypotenuse – Side*
9.3.15.1	Identify pairs of triangles that are similar using the appropriate rules.
9.3.15.2	Prove and explain why given triangles are similar.
9.3.15.3	Find the value of pronumerals in pairs of given similar triangles.
9.3.16.1	Calculate and solve practical questions using the concept of similar triangles.
9.3.17.1	Define 'prisms' and 'platonic solids'.
9.3.17.2	Name platonic solids that are made of regular polygon shapes.
9.3.18.1	Find the total surface area and volumes of given plutonic solids.
9.3.18.2	Enlarge shapes and solids with given scale factors.
9.3.18.3	Create nets for given platonic solids.
9.3.19.1	Identify and define undirected graphs and networks.
9.3.19.2	Identify the main purpose of undirected graphs and their advantages.
9.3.20.1	Name the main components (parts) of an undirected graph.
9.3.20.2	Define the following terms: *vertices (node)*, *edges*, *loops* and *degree of a vertex*.
9.3.20.3	Name other graphs that also make up a network.
9.3.20.4	Define the following terms: *subgraph*, *simple graph*, *isolated vertex*, *degenerated graph*, *connect graph* and *complete graphs*.
9.3.20.5	Name the parts of a simple graph.
9.3.21.1	Define 'planar graph' and identify its properties and features.
9.3.22.1	Identify different regions of a planar graph.
9.3.22.2	Determine the number of degrees in a planar graph.
9.3.22.3	Draw a planar graph given vertices and edges.
9.3.23.1	Draw a 2-dimensional planar graph by converting a 3D shape.
9.3.24.1	Identify variables in the Euler's formula and verify Euler's formula using 3D shapes and planar graphs.

3A Angle review

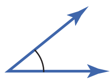
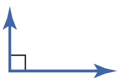
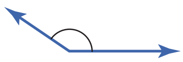
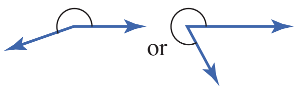
Naming angles

An **angle** is formed at the point, or vertex, where two lines, or rays, meet. When naming angles we make sure that the letter at the vertex is placed in the middle and we use the symbol ∠ instead of the word 'angle'. The angle *COD* shown could be named $\angle COD$, $\angle DOC$, $\angle O$, $C\hat{O}D$ or $D\hat{O}C$.

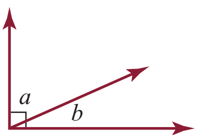


Types of angles

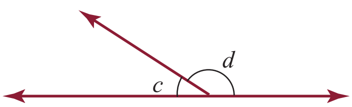
Angles may be classified according to their size:

Type of angle	Diagram	Angle size
Acute angle		An angle between 0° and 90°
Right angle		90° or one quarter turn indicated by the little square
Obtuse angle		An angle between 90° and 180°
Straight angle		180° or one half turn or two quarter turns
Reflex angle		An angle between 180° and 360°
Perigon or full circle		360° or one complete turn

Complementary and supplementary angles

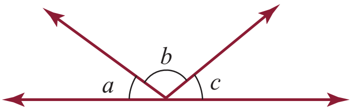


Two angles are **complementary** if their angle sum is 90°:
$\angle a + \angle b = 90°$



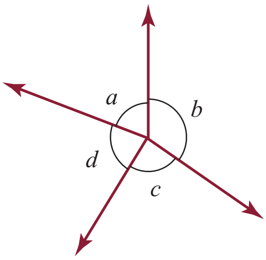
Two angles are **supplementary** if their angle sum is 180°:
$\angle c + \angle d = 180°$

Angles in a straight line



Angles which form a straight line add up to 180°:
$\angle a + \angle b + \angle c = 180^\circ$

Angles in a circle



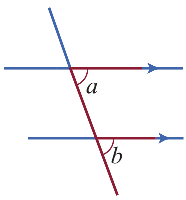
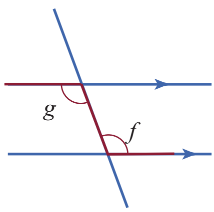
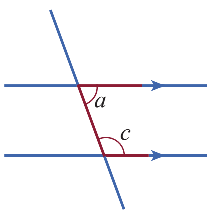
Several smaller angles can be added to give 360°, or full circle:
$\angle a + \angle b + \angle c + \angle d = 360^\circ$

A full circle, 360°, can also be termed a revolution or a perigon.

Angles and parallel lines

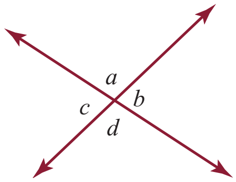
Parallel lines never meet. The distance between them is always the same. Parallel lines are marked by identical arrowheads.

A straight line that cuts one or more parallel lines is called a **transversal**. The angles created when a transversal intersects a pair of parallel lines are shown below:

Type of angle	Diagram	Description
Corresponding (F)		Corresponding angles are equal $\therefore \angle a = \angle b$ Remember the **F** shape.
Alternate (Z)		Alternate angles are equal $\therefore \angle g = \angle f$ Remember the **Z** shape.
Co-interior (C)		Co-interior angles are supplementary, i.e. add up to 180° $\therefore \angle a + \angle c = 180^\circ$ Remember the **C** shape.

3A

Vertically opposite angles

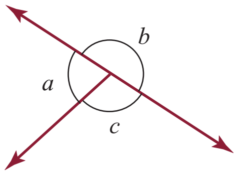


When two lines intersect they form two pairs of **vertically opposite** angles. Vertically opposite angles are equal:

$\angle a = \angle d$ (vertically opposite)

$\angle b = \angle c$ (vertically opposite)

Adjacent angles



Adjacent angles share the same vertex:

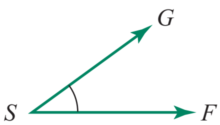
$\angle a$ and $\angle b$ are adjacent

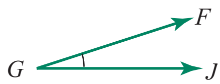
$\angle b$ and $\angle c$ are adjacent

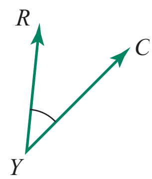
$\angle a$ and $\angle c$ are adjacent

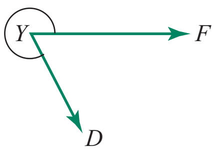
Exercise 3A

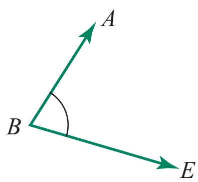
1 Name each of the following angles:

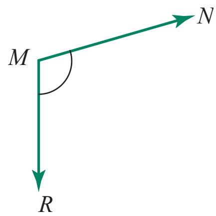
a


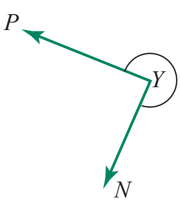
b


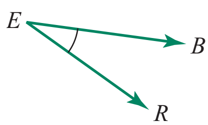
c


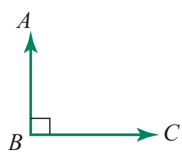
d


e


f


g


h


i


2 Classify each of the following angles according to their size:

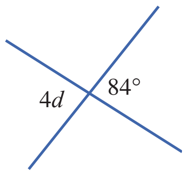
a 113°	b 21°	c 237°	d 45°
e 256°	f 336°	g 115°	h 227°
i 32°	j 410°	k 112·5°	l 126°
m 113°	n 92°	o 173·4°	p 42·2°
q 272·5	r 279°	s 119·1°	t 229·7°
u 339°	v 69°	w 147°	x 163°

3 Write down the complement and the supplement of each of the following angles:

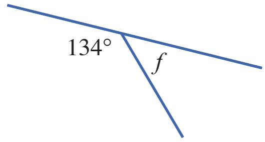
a 12°	**b** 22°	**c** 37°	**d** 45°
e 56°	**f** 63°	**g** 62.5°	**h** 66°
i 71°	**j** 87°	**k** 82°	**l** 80°
m 13·3°	**n** 14·2°	**o** 15·7°	**p** 16·2°
q $n°$	**r** $(17{\cdot}9 - n)°$	**s** $(n - 1)°$	**t** $(n + 13)°$
u $(53 + n)°$	**v** $(40 - n)°$	**w** $(90 - n)°$	**x** $(90 + n)°$

4 Find the value of the pronumerals in each of the following:

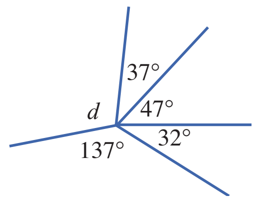
a



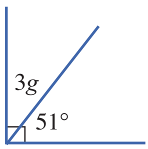
b



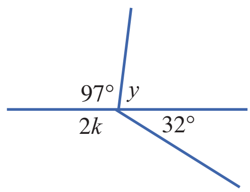
c



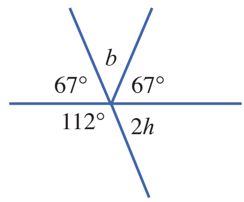
d



e

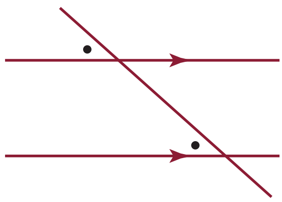


f

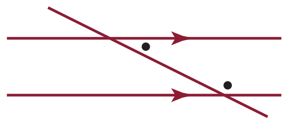


5 Are the following pairs of angles corresponding, alternate or co-interior?

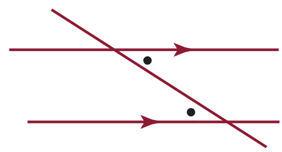
a



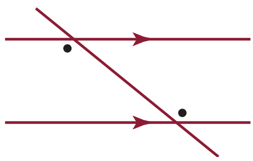
b



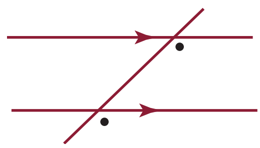
c



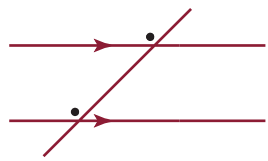
d



e

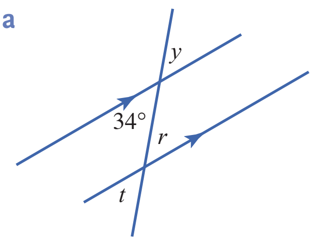


f

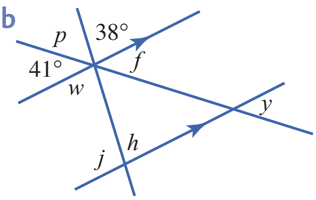


6 Find the value of the pronumerals in each of the following:

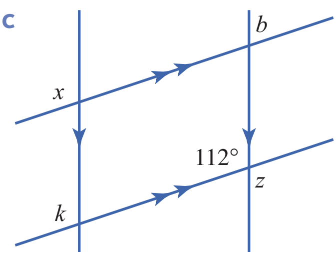
a



b



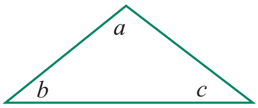
c



3B Polygon facts

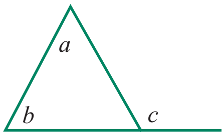
Polygons are enclosed figures with three or more straight sides. In a regular polygon all the sides are equal in length and all the angles are equal in size.

Angle sum in a triangle



The sum of the interior angles in any triangle is 180°:
$\angle a + \angle b + \angle c = 180°$

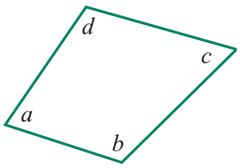
Exterior angle properties of a triangle



Angle c is known as an **exterior angle** in the triangle shown.

The sum of the two opposite interior angles, $\angle a$ and $\angle b$, is equal to the size of the exterior angle ($\angle c$):
$\angle a + \angle b = \angle c$

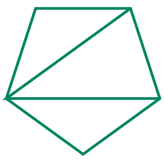
Angle sum in a quadrilateral



The shape is a quadrilateral.

The sum of the interior angles in any quadrilateral is 360°:
$\angle a + \angle b + \angle c + \angle d = 360°$

Angle sum in any polygon



In an n-sided polygon there are $n - 2$ triangles.
So the interior angle sum in any polygon is $(n - 2) \times 180°$, where n is equal to the number of sides of the polygon.

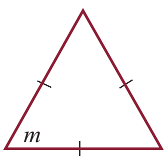
In the pentagon shown, the angle sum
$= (5 - 2)$ triangles $\times 180°$
$= 3 \times 180°$
$= 540°$

The sum of the exterior angles in any polygon is 360°.

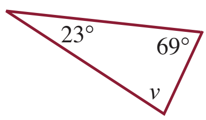
Exercise 3B

1 Find the value of the pronumerals:

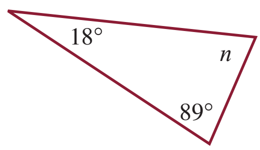
a



b

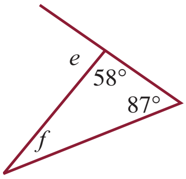


c

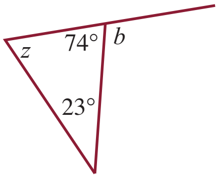


2 Find the value of the pronumerals in each of the following:

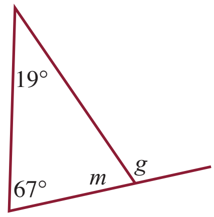
a



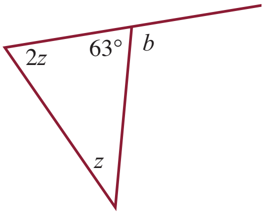
b



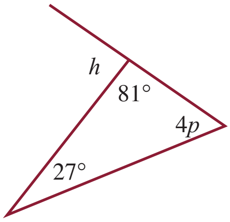
c



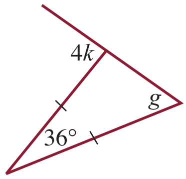
d



e

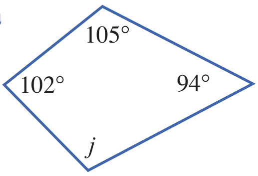


f

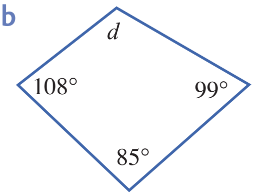


3 Find the value of the pronumeral in each of the following, giving your answers correct to 2 decimal places:

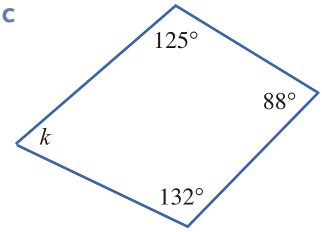
a



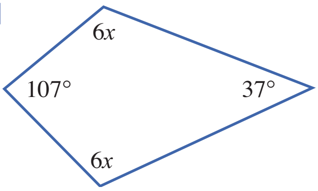
b



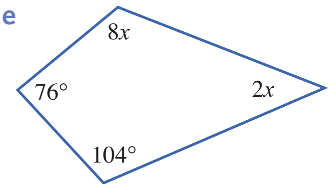
c



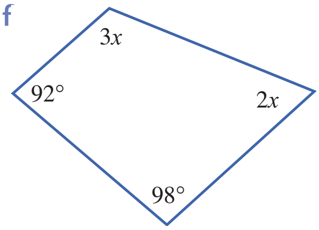
d



e



f



4 What is the angle sum of the following polygons?

a hexagon **b** octagon **c** 17-agon

d 100-agon **e** 120-agon **f** heptagon

5 Find the size, correct to 2 decimal places, of each of the interior angles in a regular:

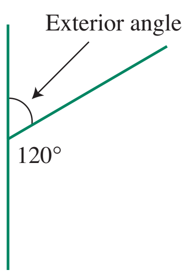
a 7-sided shape **b** 9-sided shape **c** 10-sided shape

d 15-sided shape **e** 20-sided shape **f** 13-sided shape

6 A regular polygon has an interior angle of 120°.

a Name the polygon.

b Find the size of its exterior angle.

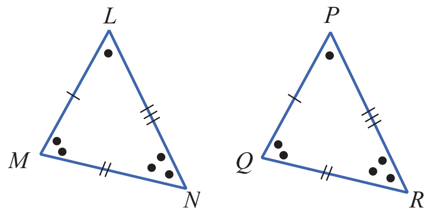


3C Congruent shapes

Two shapes are **congruent** if they are identical in every respect; that is, if they have the same shape and size.

Example

1



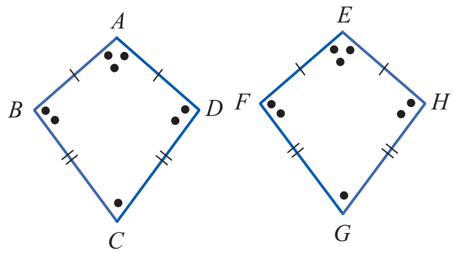
Solution

Triangle *LMN* is congruent to triangle *PQR* because they are identical in shape and size.

The order is important.

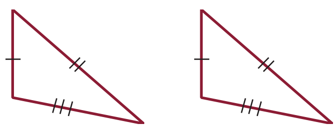
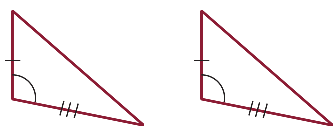
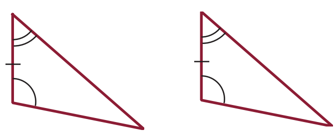
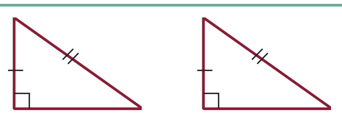
We say ΔLMN 'is congruent to' ΔPQR and we write $\Delta LMN \cong \Delta PQR$.

2



Quadrilateral *ABCD* is congruent to quadrilateral *EFGH* because they are identical in shape and size. We say *ABCD* 'is congruent to' *EFGH* and we write $ABCD \cong EFGH$.

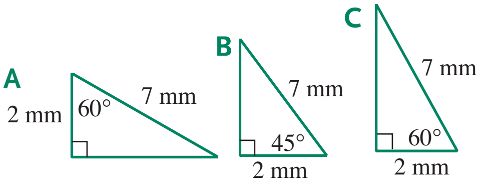
The following tests can be used to determine whether two triangles are congruent:

Test	Diagram	Description
Side–Side–Side **(SSS)**		All corresponding sides are equal in length.
Side–Angle–Side **(SAS)**		Two sides and the angle between them, the included angle, of one triangle are equal to the corresponding sides and the included angle of another triangle.
Angle–Side–Angle **(ASA)**		Two angles and the side between them of one triangle are equal to two angles and the corresponding side of another triangle.
Right angle–Hypotenuse–Side **(RHS)**		In a pair of right-angled triangles, the hypotenuse and a pair of corresponding sides are equal in length.

If two triangles fulfil **SSS**, **SAS**, **ASA** or **RHS**, then the two triangles are congruent.

Example

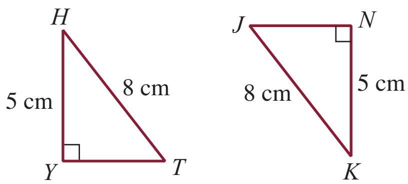
3 Which of the following triangles are congruent? Give a reason for your answer:



Solution

Shape **A** ≅ shape **C** (SAS)

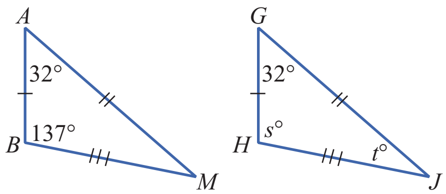
4 Prove that $\Delta HYT \cong \Delta JNK$.



Solution

$HY = NK$
$HT = JK$
$\angle HYT = \angle KNJ = 90°$
$\therefore\ \Delta HYT \cong \Delta KNJ$ (RHS)

5 Find the value of the pronumerals in the pair of congruent triangles shown:



Solution

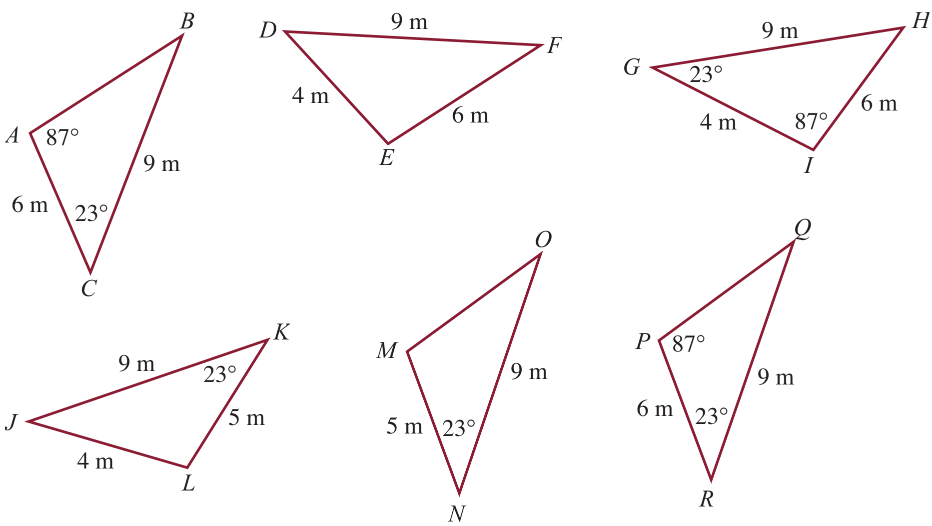
$AB = GH$, $BM = HJ$, $AM = GJ$
$\therefore\ \Delta ABM \cong \Delta GHJ$ (SSS)

Hence $\angle s = 137°$, and

$\angle t = 180 - (32° + 137°)$
$= 11°$

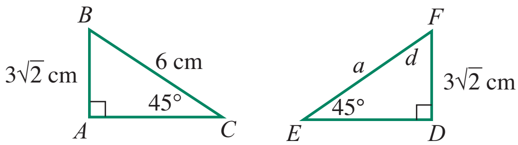
Exercise 3C

1 Which of the following triangles are congruent? Give a reason for your answer:

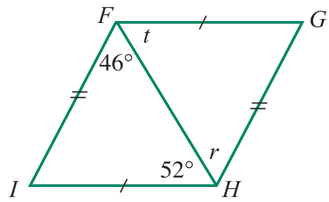


2 Find the value of the pronumerals in each of the following pairs of congruent triangles. Give reasons for your answers:

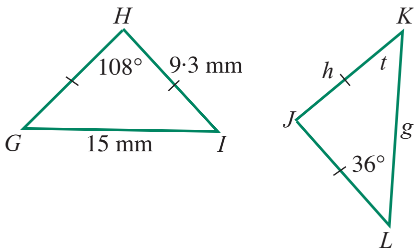
a



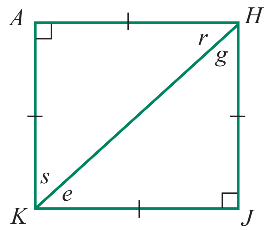
b



c

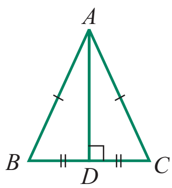


d

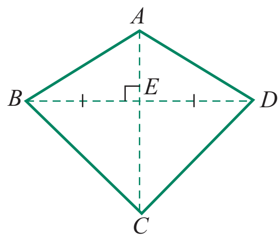


3 For each of the following, prove that:

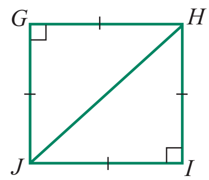
a $\Delta ABD \cong \Delta ACD$



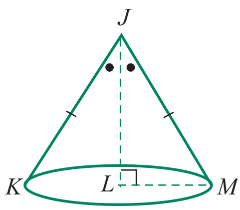
b $\Delta AEB \cong \Delta AED$



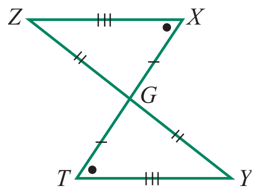
c $\Delta GHJ \cong \Delta IJH$



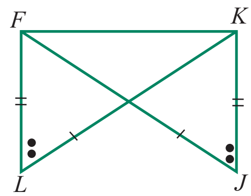
d $\Delta JKL \cong \Delta JML$



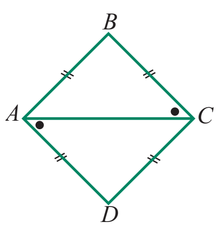
e $\Delta ZXG \cong \Delta YTG$



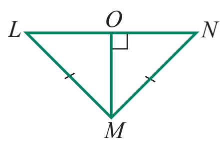
f $\Delta FJK \cong \Delta KLF$



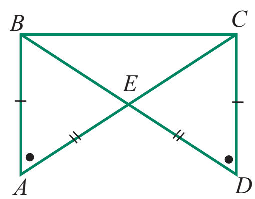
g $\Delta ABC \cong \Delta CDA$



h $\Delta LMO \cong \Delta NMO$



i $\Delta AEB \cong \Delta DEC$



Two objects that have the same shape but are different in size are said to be **similar**.

These soccer players are the same, only their size is different.

The heights of players in A are twice that of players in B (scale factor 2), or the height of players in B is half that of players in A (scale factor $\frac{1}{2}$).

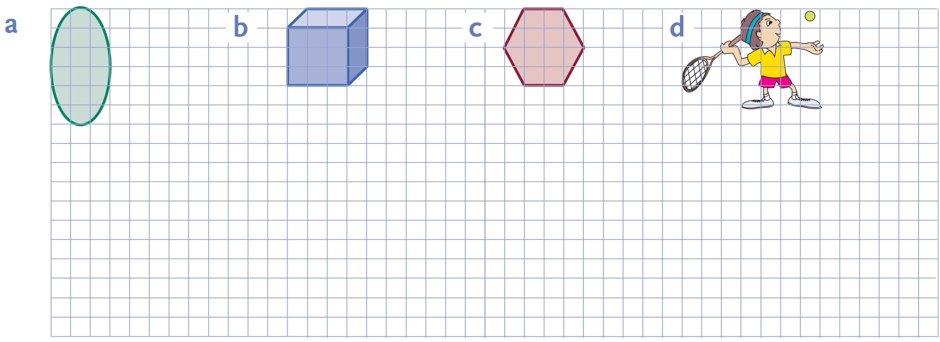
The ratio of the size of players in A compared to players in B is 2:1, and the ratio of players in B compared to players in A is 1:2.

A

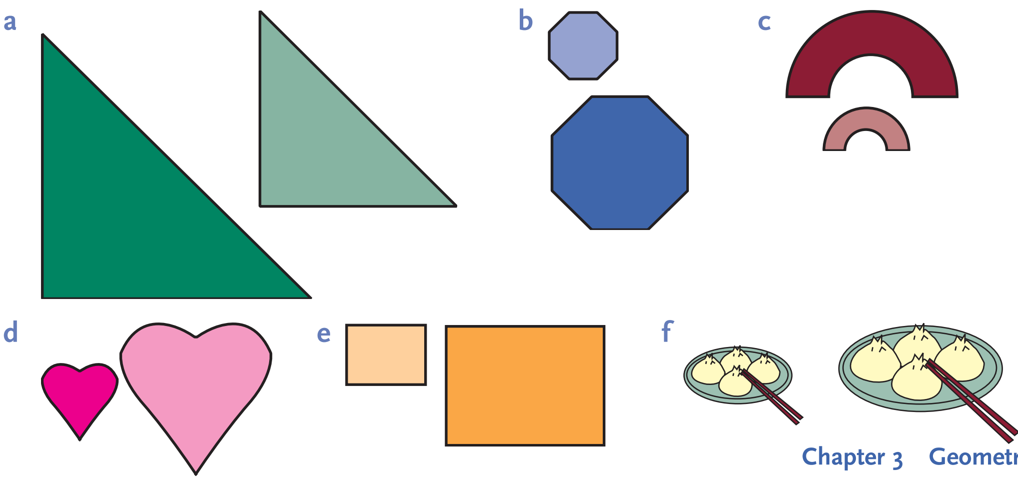
B

Exercise 3D

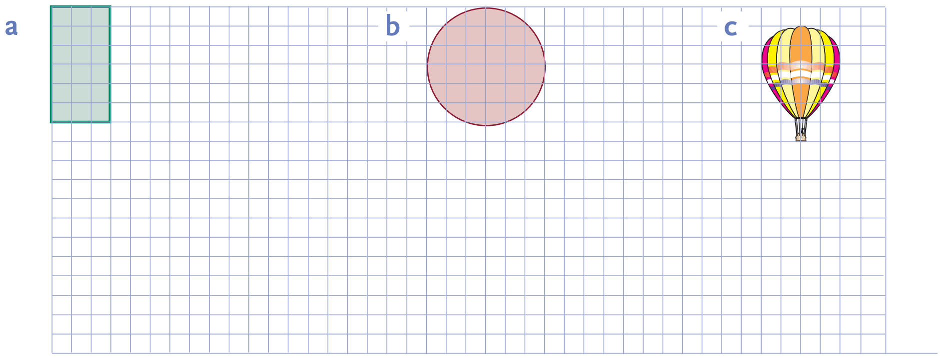
1 Draw up gridlines in your exercise book. Produce figures similar to those below by enlarging each by a scale factor of 2:



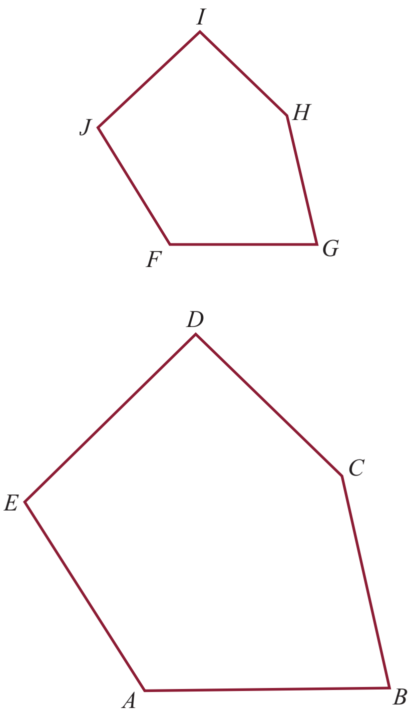
2 Measure these similar figures and determine the scale factor used to enlarge the smaller figure to make the larger figure:



3 Draw up gridlines in your exercise book. Increase these figures in the ratio 3 : 4 and state what fraction the new area is of the original area:



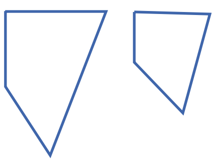
4 Measure the lengths of the lines in these shapes in millimetres, record them in the table and determine their ratios:



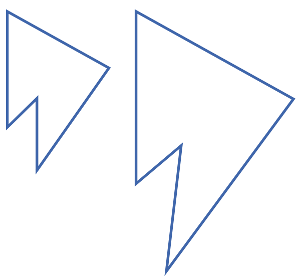
Length	Length	Ratio
$AB =$	$FG =$	$\frac{AB}{FG} =$
$BC =$	$GH =$	$\frac{BC}{GH} =$
$CD =$	$HI =$	$\frac{CD}{HI} =$
$DE =$	$IJ =$	$\frac{DE}{IJ} =$
$AE =$	$FJ =$	$\frac{AE}{FJ} =$

5 Measure the lengths of the lines and show whether or not these shapes are similar:

a



b



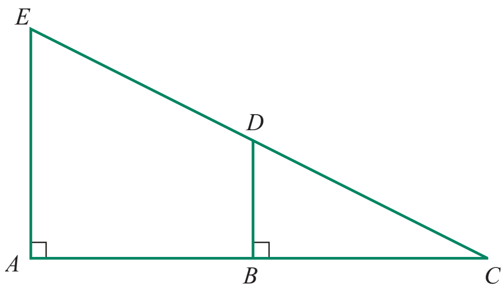
Exploring similar triangles 3E

Similar triangles have the same shape; only their sizes are different.

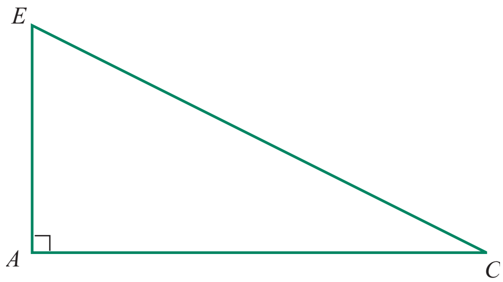
Learning task 3E

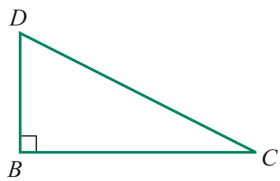
1 a Measure the following angles and lengths, correct to 1 decimal place, and record them in this table:

Angle	Size (°)	Line	Length (cm)
BCD		AC	
ACE		BC	
CDB		AE	
CEA		BD	
		CD	
		CE	



b Here the two triangles have been drawn separately. Mark the angles and lengths onto the triangles:





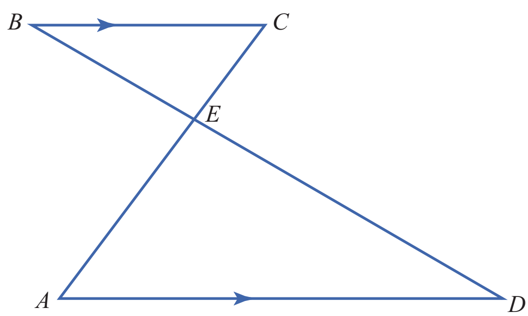
c Calculate the following ratios:

i $\frac{AC}{BC}$ ii $\frac{AE}{BD}$ iii $\frac{CE}{CD}$

d Explain why the two triangles are similar.

2 a Measure the following angles and lengths and record them in this table:

Angle	Size (°)	Line	Length (cm)
BCE		CE	
CBE		BE	
BEC		BC	
ADE		AE	
DAE		DE	
AED		AD	



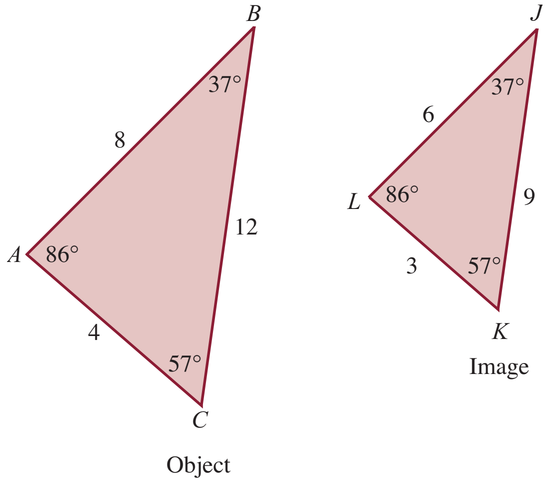
b Calculate the following ratios, correct to 1 decimal place:

i $\frac{BC}{AD}$ ii $\frac{CE}{AE}$ iii $\frac{BE}{DE}$

3F Similar triangles

A triangle is the simplest polygon that we can use in calculations to determine unknown lengths. It follows that two shapes can be similar if their corresponding angles are the same but their side lengths are different. A reduction or an enlargement of the original object will produce an image that is **similar**.

The following triangles are similar. Their angles are the same but one triangle is a reduction of the other—it is the same shape but smaller.

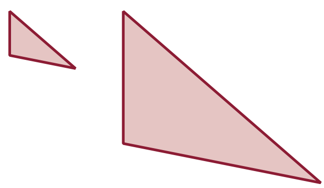
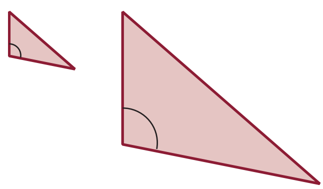
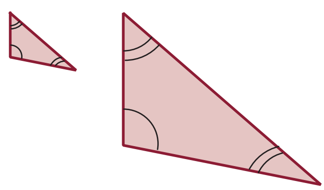
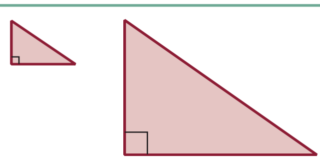


The **corresponding angles** in similar figures are **equal** in size; that is, $\angle BAC = \angle JLK$, $\angle ACB = \angle LKJ$ and $\angle ABC = \angle LJK$.

The **corresponding sides** in similar triangles are **in the same ratio**; that is, $\frac{LJ}{AB} = \frac{LK}{AC} = \frac{JK}{BC}$.

We use the sign ~ to denote similarity, so $\Delta LJK \sim \Delta ABC$.

The following tests can be used to determine whether two triangles are similar.

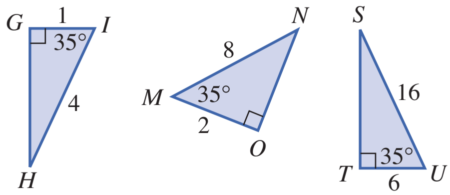
Test	Diagram	Description
Side–Side–Side (**SSS**)		All corresponding sides are in the same ratio.
Side–Angle–Side (**SAS**)		Two pairs of corresponding sides are in the same ratio and the angle between them, the included angle, is equal in size.
Angle–Angle–Angle (**AAA**)		All corresponding angles are equal in size.
Right angle–Hypotenuse–Side (**RHS**)		In a pair of right-angled triangles, the hypotenuse and a pair of corresponding sides are in the same ratio.

If two triangles fulfil **SSS**, **SAS**, **AAA** or **RHS**, then the two triangles are similar.

Example

Solution

1 Which of the following triangles are similar? All measurements are in millimetres:



As all angles are the same size, we need to establish a common ratio between the pairs of corresponding sides.

For triangles *GHI* and *MNO*:

$$\frac{MN}{HI} = \frac{8}{4} = 2$$

and $\frac{MO}{GI} = \frac{2}{1} = 2$

$\therefore \Delta GHI \sim \Delta ONM$ **(SAS)**

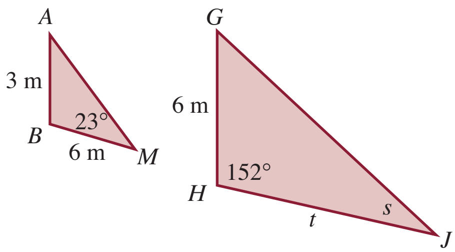
For triangles *MNO* and *STU*:

$$\frac{SU}{MN} = \frac{16}{8} = 2$$

and $\frac{TU}{MO} = \frac{6}{2} = 3$

$\therefore \Delta MNO$ and ΔSTU are not similar.

2 Find the value of the pronumerals in the following pair of similar triangles:



$\Delta ABM \sim \Delta GHJ$ (given)

$\therefore s = 23°$

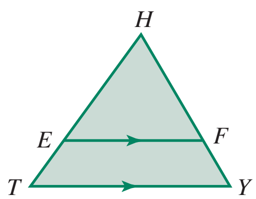
$$\text{Scale factor} = \frac{GH}{AB} = \frac{6}{3} = 2$$

$\therefore t = \text{scale factor} \times BM$

$= 2 \times 6$

$= 12$ m

3 Prove that $\Delta HTY \sim \Delta HEF$:



$\angle EHF = \angle THY$ (same angle)

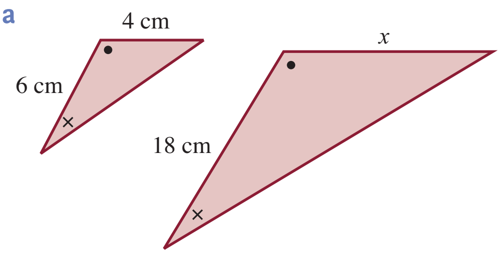
$\angle HEF = \angle HTY$ (corresponding)

$\angle HFE = HYT$ (corresponding)

$\therefore \Delta HEF \sim \Delta HTY$ **(AAA)**

4 Find the value of *x* if the following triangles are similar:

a



Method 1

$$\frac{x}{4} = \frac{18}{6}$$

$\therefore 6x = 4 \times 18$

$$x = \frac{4 \times 18}{6}$$

$= 12$ cm

or

Method 2

$$\text{Scale factor} = \frac{18}{6}$$

$= 3$

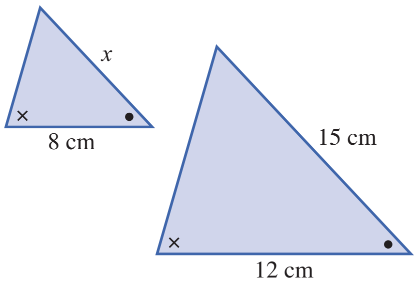
$\therefore x = \text{SF} \times 4$

$= 3 \times 4$

$= 12$ cm

Example

b



Solution

Method 1

$$\frac{x}{15} = \frac{8}{12}$$

$$\therefore\ 12x = 8 \times 15$$

$$x = \frac{8 \times 15}{12}$$

$$= 10 \text{ cm}$$

or

Method 2

$$\text{Scale factor} = \frac{8}{12}$$

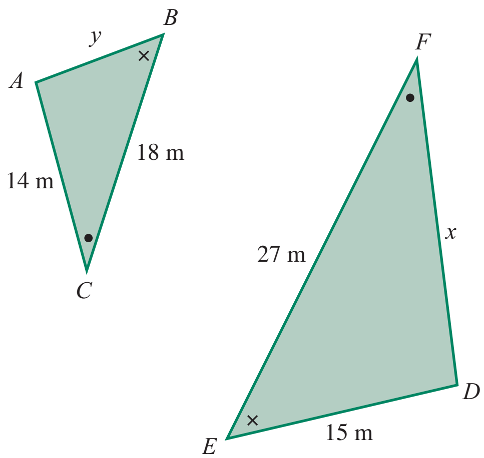
$$= \frac{2}{3}$$

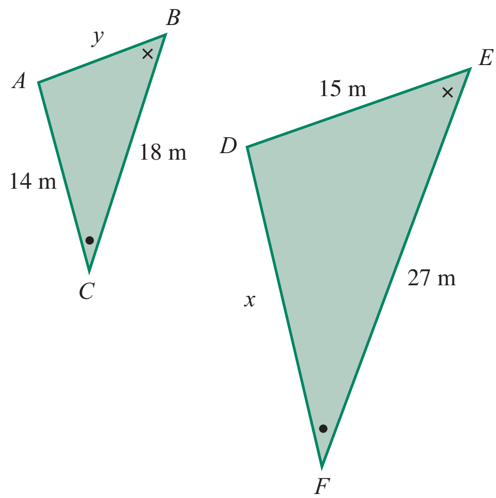
$$\therefore\ x = \text{SF} \times 15$$

$$= \frac{2}{3} \times 15$$

$$= 10 \text{ cm}$$

5 ΔABC and ΔDEF are similar. Turn ΔDEF around to face the same way as ΔABC, match the corresponding sides and find the unknown lengths:





$$\frac{x}{14} = \frac{27}{18}$$

$$\therefore\ 18x = 27 \times 14$$

$$x = \frac{27 \times 14}{18}$$

$$= 21 \text{ m}$$

$$\frac{y}{15} = \frac{18}{27}$$

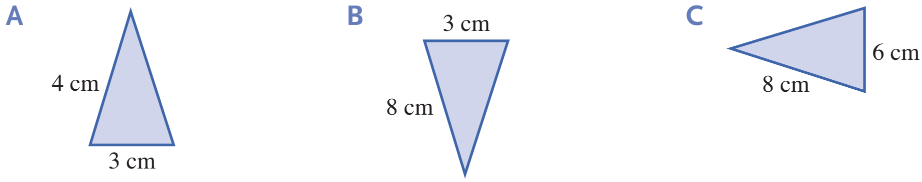
$$\therefore\ 27x = 18 \times 15$$

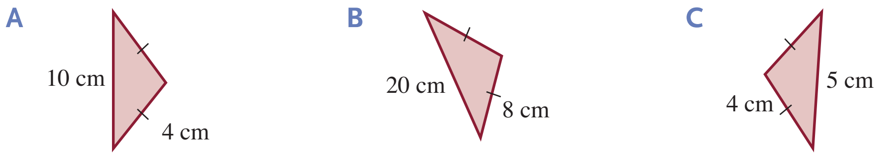
$$y = \frac{18 \times 15}{27}$$

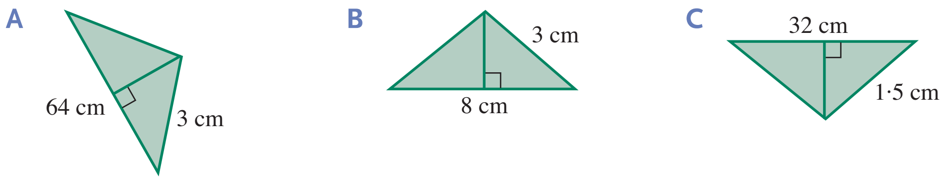
$$= 10 \text{ m}$$

Exercise 3F

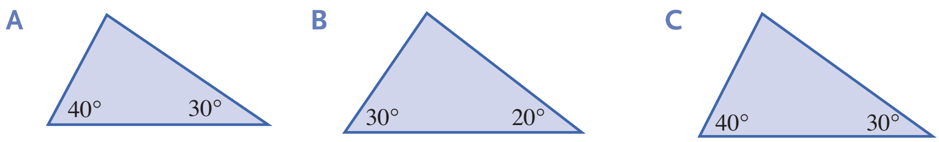
1 Which of the following triangles are similar? Give a reason for your answer:

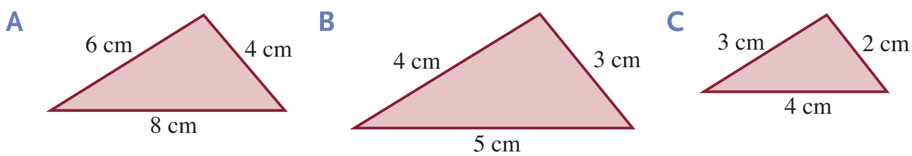
a


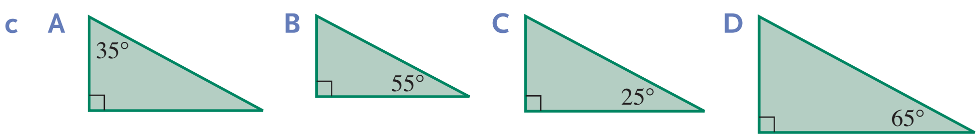
b


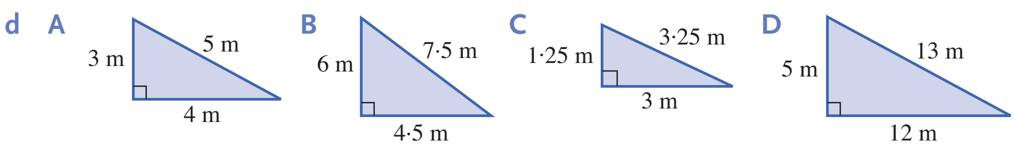
c


2 Choose the pairs of similar triangles, giving reasons for your answer:

a


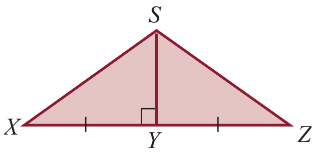
b


c


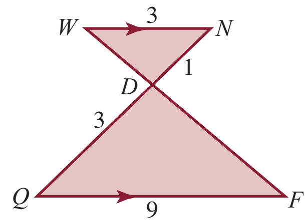
d


3 For each of the following, prove that:

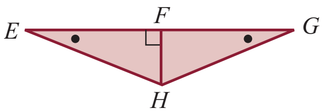
a $\Delta SXY \sim \Delta SZY$



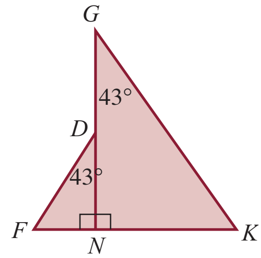
b $\Delta WND \sim \Delta QFD$



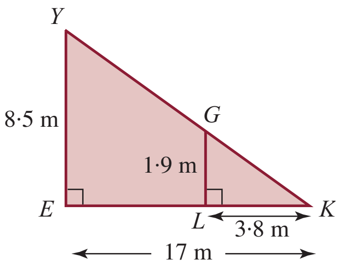
c $\Delta EFH \sim \Delta GFH$



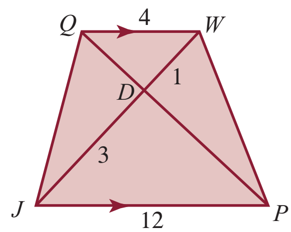
d $\Delta GNK \sim \Delta DNF$



e $\Delta YKE \sim \Delta GKL$

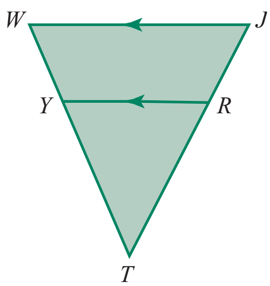


f $\Delta WQD \sim \Delta JPD$

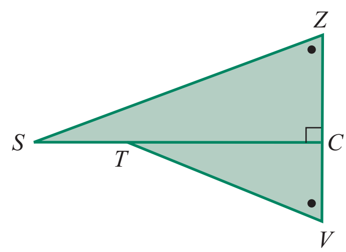


4 Find two similar triangles in each of the following:

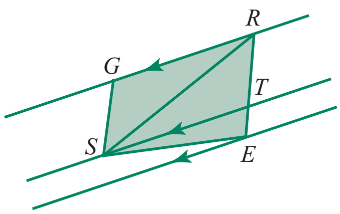
a



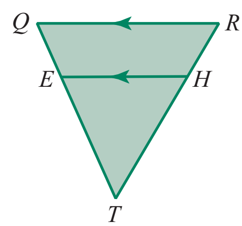
b



c

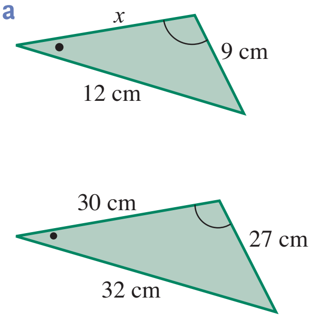


d

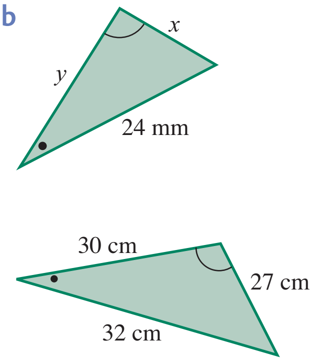


5 Find the value of the pronumerals in the following pairs of similar triangles:

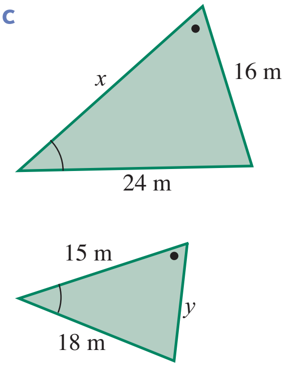
a



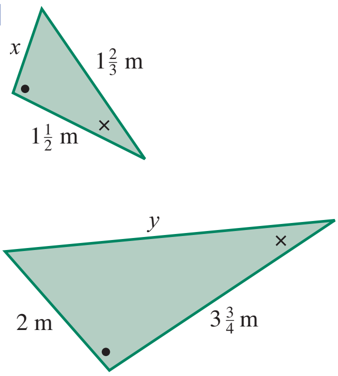
b



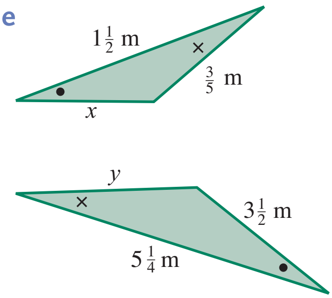
c



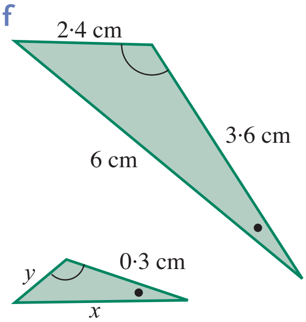
d



e

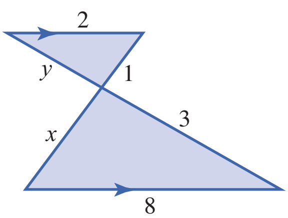


f

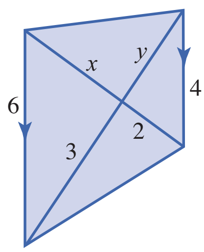


6 Find the value of the pronumerals in the following (all lengths are given in centimetres):

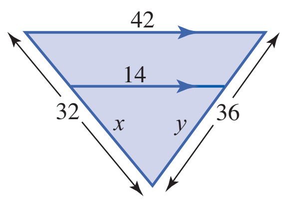
a



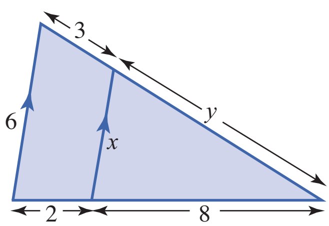
b



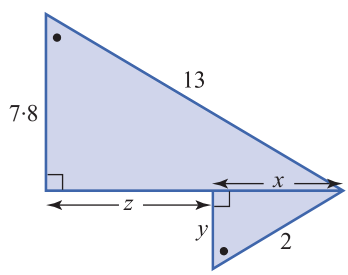
c



d



e

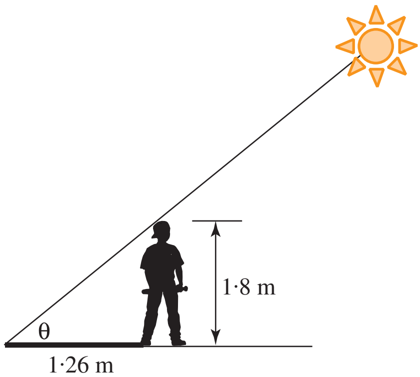


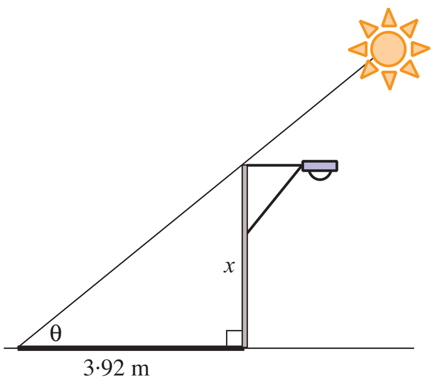
3G Applying similar triangles

Similar triangles and similarity in general can be applied to real practical situations.

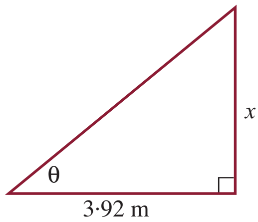
Example

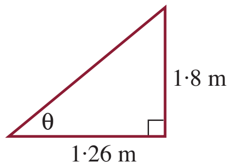
Hao is 1·8 m tall and casts a shadow 1·26 m long. At the same time of day a light pole casts a shadow 3·92 m long, find the height of the pole.





Solution





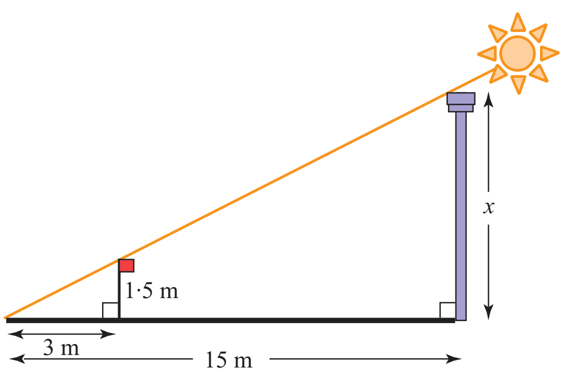
$$\frac{x}{1.8} = \frac{3.92}{1.26}$$

$$\therefore x = \frac{1.8 \times 3.92}{1.26}$$

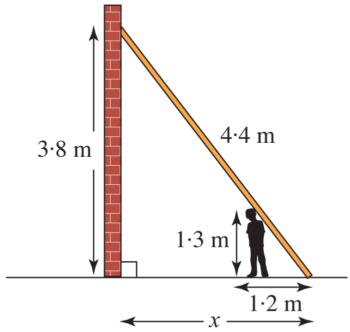
$$\therefore x = 5.6 \text{ m}$$

Exercise 3G

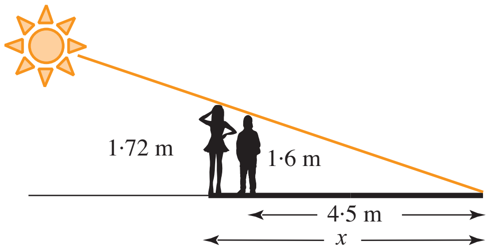
1 A flagpole 1·5 metres tall casts a shadow 3 metres in length. Find the height of a tower that casts a shadow 15 metres long at the same time.



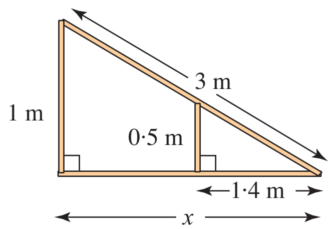
2 A ladder 4·4 m long rests against a wall. Daniel, who is 1·3 m tall, stands under the ladder 1·2 m from the foot of the ladder, so that his head just touches the ladder. If the ladder reaches 3·8 m up the wall, how far is the foot of the ladder from the wall?



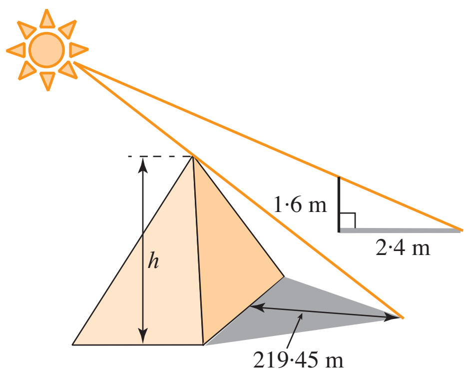
3 Ethan who is 1·6 m tall casts a shadow 4·5 m long at a certain time of the day. If Jacinta is 1·72 metres tall, how long would her shadow be?



4 A metal brace 3 m in length touches the top of the first wooden strut at a height of 0·5 m, and the top of another wooden strut 1 m in length. If the first wooden strut is 1·4 m from the base of the metal brace, how far is the second wooden strut from the base of the metal brace?



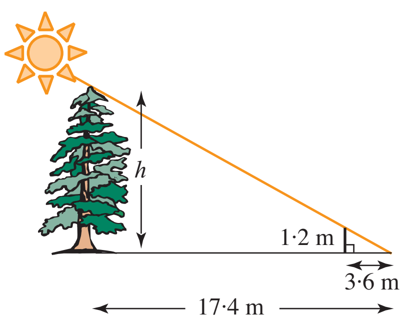
5 The great pyramid at Giza, in Egypt, casts a shadow which is 219·45 metres as measured from the centre of its side. If at the same time of day a stick 1·6 m tall casts a shadow 2·4 m long, find the height of the pyramid.



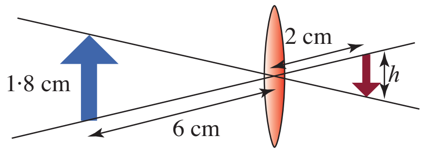
6 A sapling, which is 1·2 metres tall, is planted near a tree. If the sapling's shadow is 3·6 metres long and the tree's shadow is 17·4 metres long, find:

a the height of the tree

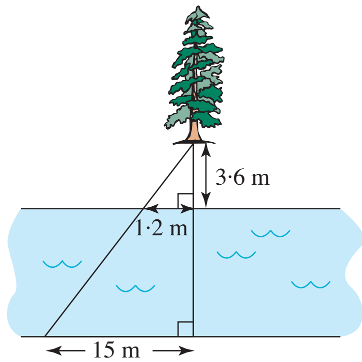
b the length of the shadow that the sapling will cast later in the day when the tree's shadow is 11·2 m long



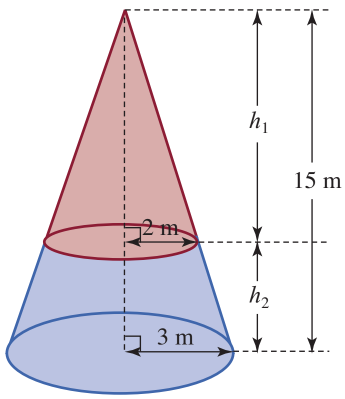
7 A slide of a blue arrow placed in front of a lens casts a red image of the arrow behind the lens as shown here. Find the height (h) of the red arrow.



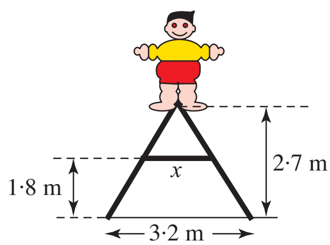
8 A tree that is 3·6 metres from one side of a river bank is sighted from the other side of the river. Other measurements are taken and are shown on this diagram. Find the width of the river.



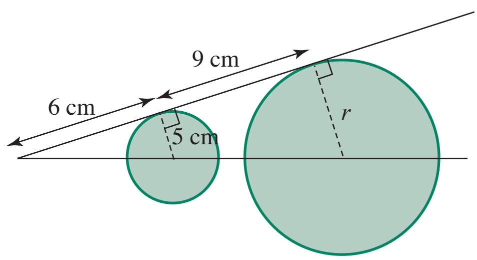
9 A large right cone is 15 metres tall and has a radius of 3 metres. The top part of the cone is coloured red and the bottom part of the cone (frustum) is coloured blue. If the radius of the red cone section is 2 metres find the heights of the red cone (h_1) and the blue frustum (h_2).



10 A large inflatable figure is to be used in the grand final of the local netball team. Lengths of wood painted black support the figure. The 'feet' of the supports are 3·2 m apart. Find the length of the horizontal strut between the supports.



11 The component parts of a quartz crystal clock are placed as shown in this diagram. Find the radius of the larger circle (r).

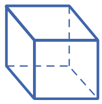
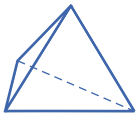
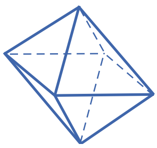
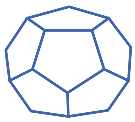
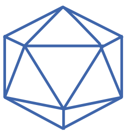


Exploring 3D shapes 3H

To represent three-dimensional shapes on a two-dimensional surface, we can draw the visible edges using a solid line and the hidden edges with a broken, or dotted, line.

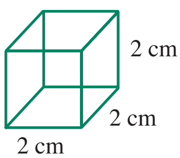
A **prism** has two faces which are parallel and the same shape and size. They have at least three parallel edges, or sides, and a regular cross-section.

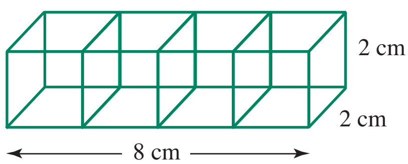
Platonic solids, shown in the table below, are those with a regular polygon for each face.

Polyhedron	Diagram	Number of faces	Shape of each face
Cube		6	Square
Tetrahedron		4	Equilateral triangle
Octahedron		8	Equilateral triangle
Dodecahedron		12	Regular pentagon
Icosahedron		20	Equilateral triangle

Learning task 3H

1 i Find the total surface area and volume of the following shapes.
 ii Enlarge each shape by a scale factor of 2 and find the total surface area and volume of the enlarged shape.
 iii How much bigger, in total surface area and volume, is the enlarged shape?
 iv By what scale factor have the total surface area and the volume changed?

a


b


2 If the shapes in Question 1 were enlarged by a scale factor of 3, by what factor would the:
 i surface area increase?
 ii volume increase?

3 Create a net for each of the platonic solids shown in Question 1. Cut the nets from coloured cardboard and assemble, then hang them in your classroom.

31 Undirected graphs and networks

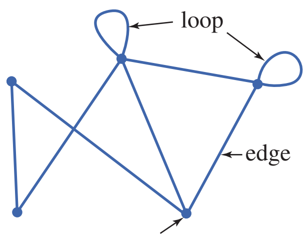
When a telephone or Internet company is trying to decide where to lay cables, they want to minimise the amount of materials used. Similarly, when the council is planning the collection of rubbish for recycling they want to find the shortest or quickest route.

In these situations we use **undirected graphs** to minimise cost and maximise efficiency. These types of graphs are not graphs with axes and coordinates, they are graphs with **vertices (nodes)**, **edges** and **loops**. These graphs can also be called **networks**.

Representing a network

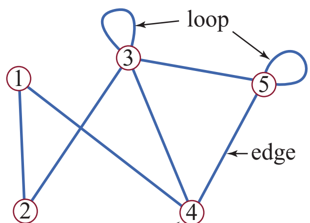
Networks can be represented in a variety of ways. A graph, or network, is a two-dimensional collection of vertices and edges.

In this figure there are 5 vertices and 8 edges. Two of these edges are loops.



Original graph

We label the vertices with numbers: 1, 2, 3, 4 and 5. There are five vertices.



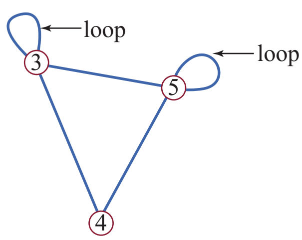
The degree of a vertex

Each vertex has a number of edges connecting it to the rest of the network. This number is called the degree. To determine the degree of a vertex we count the number of edges leaving it.

Vertex	1	2	3	4	5
Degree	2	2	5	3	4

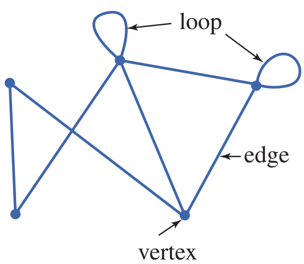
Loops

A loop is an edge that connects a vertex to itself and contributes 2 towards the degree of that vertex.

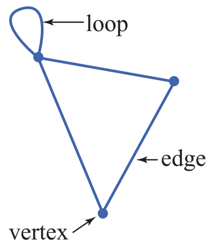


Subgraph

In some situations only part of a network needs to be represented. This is called a subgraph of the network. A subgraph contains some of the vertices and edges of the original graph.



Original graph

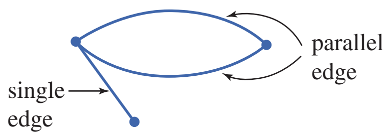


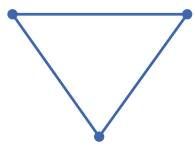
Subgraph

Simple graph

If two or more edges connect the same pair of vertices they are called parallel edges (or multiple edges; see diagram below) and all these edges count towards the degree of the vertex. If there is only one edge between two vertices the connection is called a **simple**, or **single**, connection. Simple graphs contain no multiple edges.

The sum of the degrees of the vertices in a simple graph is always even.

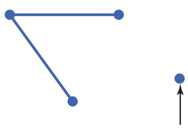




simple graph

Isolated vertex

A vertex that is not connected to any other vertex is called an isolated vertex and has degree 0.

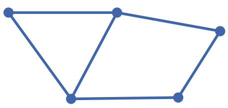


isolated vertex

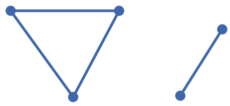
A **degenerate graph** is one that is only made up of isolated vertices. Note: All the vertices of a degenerate graph have degree zero.

Connected graph

If it is possible to follow all the edges from any vertex to any other vertex then the graph is a connected graph.



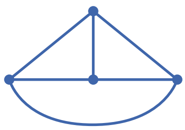
Connected graph



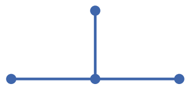
Graph is not connected.

Complete graphs

In a complete graph, every vertex is connected by an edge to every other vertex.



Complete graph

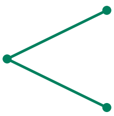


Graph not complete because each vertex is not connected to every other vertex.

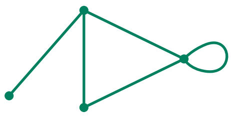
Exercise 3I

1 Count the number of vertices and edges in the following networks:

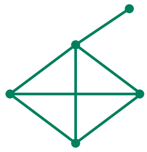
a



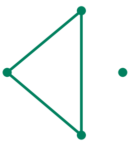
b



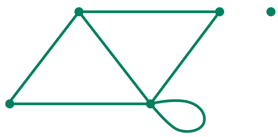
c



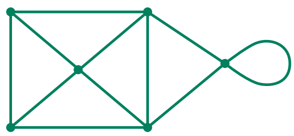
d



e

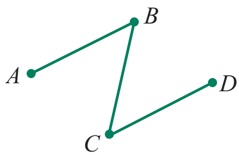


f

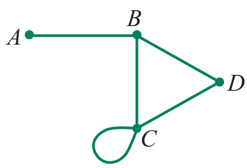


2 Determine the degree of each of the vertices in each diagram:

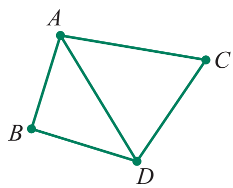
a



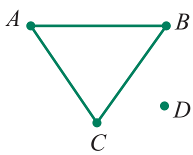
b



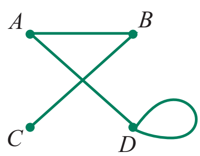
c



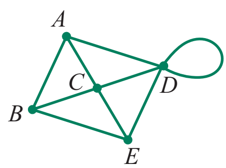
d



e

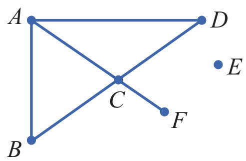


f



3 Which of the networks in Question 2 has an isolated node?

4



a Copy and complete the table for the graph shown:

Vertex	*A*	*B*	*C*	*D*	*E*	*F*
Degree						

b What is the sum of the degrees of the vertices?

5 Consider a network of three vertices in which each vertex is connected to each of the other two vertices with a single edge. (That is there are no loops, isolated vertices or parallel edges.)

a List the vertices and edges.

b Construct a diagram of the network.

c List the degree of each vertex.

6 Repeat Question 5 for a network of:

i 5 vertices

ii 7 vertices

7 Construct a network that represents the following family tree. Use a single node to represent each married couple.

- Pete and Jodie had 3 children, Will, Max and Abby.
- Luke married Fiona and had 2 children, Dylan and Niamh.
- Richard married Marylyn and had 1 child, Helen.
- Stephen married Karen and had 4 children, Daniel, Jessie, Emily and Jamie.

3J Planar graphs and Euler's formula

The additional properties of planar graphs will allow us to map two-dimensional and even three-dimensional objects onto graphs.

A **planar graph** is one that has no edges (paths) that cross.

Consider the following graphs.

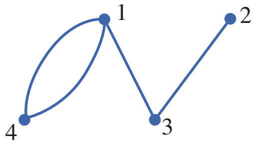


Figure a

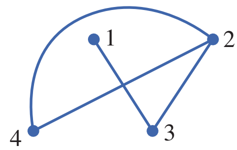
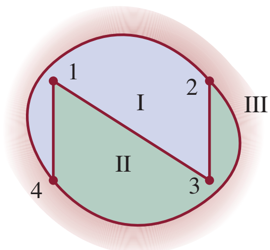


Figure b

Figure **a** is a planar graph because none of the paths linking vertices 1, 2, 3, and 4 cross each other.

Figure **b** is apparently not a planar graph because path (1, 3) crosses path (2, 4).

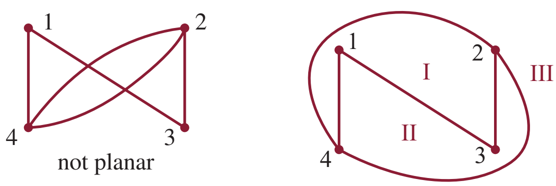
The regions of a planar graph



A planar graph divides a plane into regions. This planar graph can be divided into 3 **regions**: regions I, II and III. Regions I and II are finite. Region III is the area outside the graph and is infinite. These regions are also called **faces**.

Although it may *appear* that a graph is not planar, by drawing the graph in another way it may become clearly planar. If we redraw a graph by changing the position of an edge or vertex, we produce a graph that is equivalent or **isomorphic** to the original.

For example, the graph below has been redrawn so that it is a planar graph and the faces (regions) of the planar graph have been indicated.



The *degree* of each face is equal to the number of edges defining that region. In the planar graph above:

- face I is defined by edges (4, 2), (2, 3), (3, 1) and ((1, 4), so it is of degree 4
- face II is defined by edges (2, 4), (4, 1), (1, 3) and (3, 2) so it is of degree 4
- face III is defined by edges (4, 2) and (2, 4) so it is of degree 2.

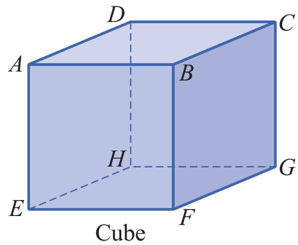
Converting three-dimensional solids to planar graphs

An application of planar graphs is the conversion of the graph representing a three-dimensional solid (with flat faces) to a planar graph.

The figure below shows a cube with vertices, $V = \{A, B, C, D, E, F, G, H\}$.

To convert this to a planar graph:

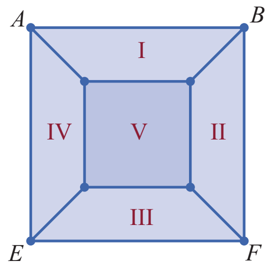
- List all 12 edges.
  $E = \{(A, B), (A, D), (A, E), (B, C), (B, F), (C, D), (C, G), (D, H), (E, F), (E, H), (F, G), (G, H)\}$
- Imagine the three-dimensional cube 'collapsing' to a two-dimensional graph, then try collapsing the face $A–B–C–D$ into the face $E–F–G–H$.



Cube

There are some interesting features of this planar graph:

1. The planar graph is a two-dimensional representation of the original cube.
2. The original 'front' of the cube ($A–B–E–F$) has become the *infinite* region of the planar graph.



Euler's formula

There is a mathematical relationship between the vertices, edges and faces of planar graphs.

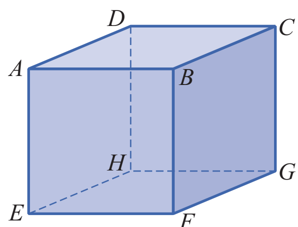
Vertices = edges – faces + 2
$V = E - F + 2$

Verifying Euler's formula for the cube:

1. List the vertices: $V = \{A, B, C, D, E, F, G, H\}$
   $\therefore V = 8$
2. List the edges:
   $E = \{(A, B), (A, D), (A, E), (B, C), (B, F), (C, D), (C, G), (D, H), (E, F), (E, H), (F, G), (G, H)\}$
   $\therefore E = 12$
3. Define the regions (faces):
   There are six regions in all: {I, II, III, IV, V, VI}
   $\therefore F = 6$
4. Confirm Euler's formula:

$$V = 8, E = 12, F = 6$$
$$V = E - F + 2$$
$$8 = 12 - 6 + 2$$
$$= 8$$

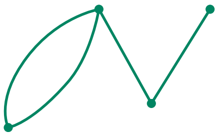
LHS = RHS
Therefore, Euler's formula is verified.

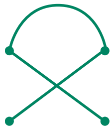


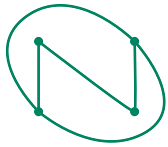
Cube

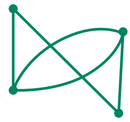
Exercise 3J

1 Identify which graphs are planar graphs:

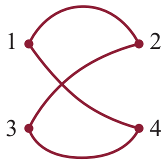
A


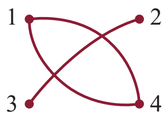
B


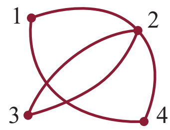
C


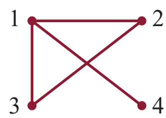
D


2 Modify the following graphs so that their representations are planar:

a


b


c


d


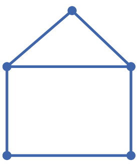
3 Draw the following networks as planar graphs:

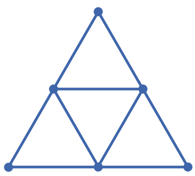
a $V = \{1, 2, 3, 4\}$, $E = \{(1, 2), (1, 3), (1, 4)\ (2, 3)\}$

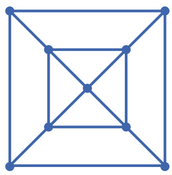
b $V = \{1, 2, 3, 4, 5\}$, $E = \{(1, 2), (2, 3), (3, 4)\ (2, 5)\}$

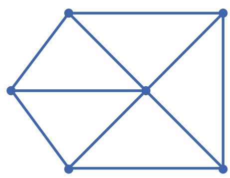
c $V = \{1, 2, 3, 4, 5\}$, $E = \{(1, 3), (2, 4), (3, 4)\ (5, 2), (2, 2), (1, 4)\}$

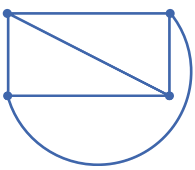
4 Copy and complete the table to verify Euler's formula:

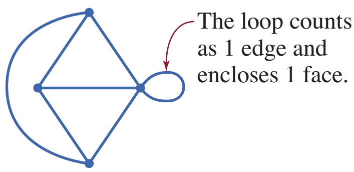
A


B


C


D


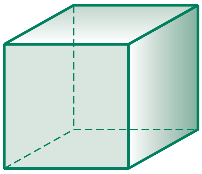
E


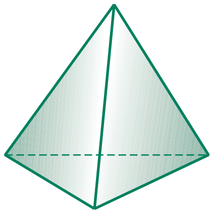
F


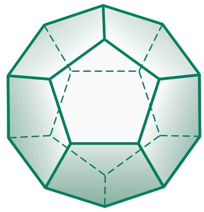
Graph	Number of vertices (V)	Number of faces (F)	Number of edges (E)	$V + F - E$
Example	5	6	9	$5 + 6 - 9 = 2$
A				
B				
C				
D				
E				
F				

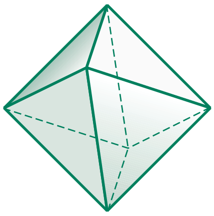
5 Which of the graphs in Question 4 are planar?

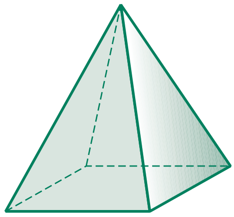
6 Draw the following shapes as planar graphs.

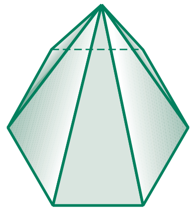
a

Cube

b

Tetrahedron

c

Dodecahedron

d

Octahedron

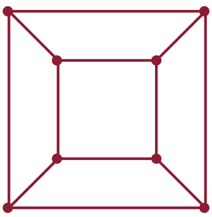
e

Square-based pyramid

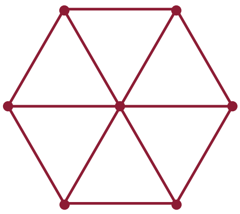
f

Hexagonal-based pyramid

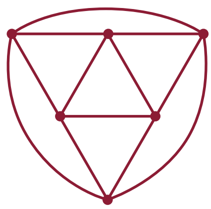
7 Verify Euler's formulas for the graphs in Question 6.

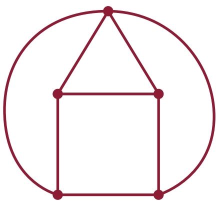
8 In almost all cases, each region will have a degree of *at least* 3. Why? Can you think of exceptions?

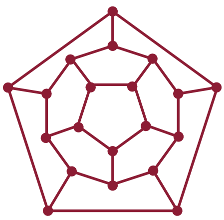
9 The following planar graphs correspond to 3D shapes.

A


B


C


D


E


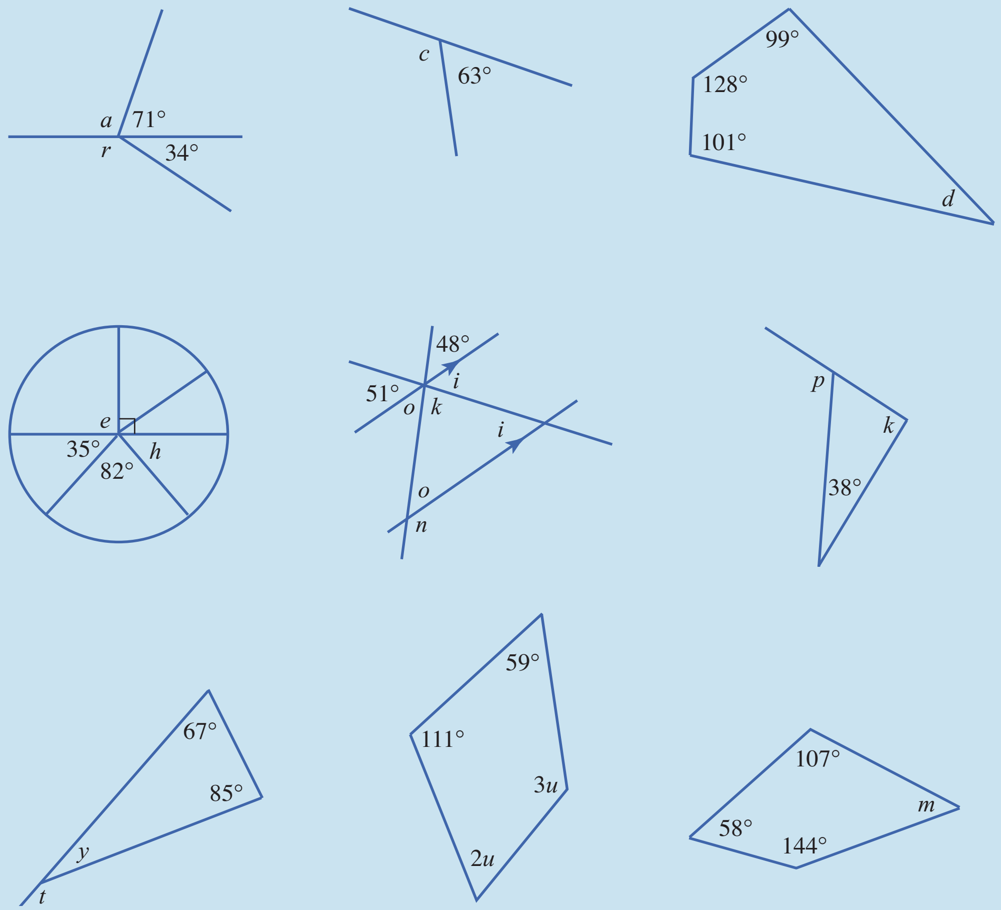
a Which planar graph could represent a tetrahedron?
b Which planar graph could represent a cube?
c Which planar graph could represent a square-based pyramid?
d Which planar graph could represent a octahedron?
e Which planar graph could represent a dodecahedron?
f Which planar graph could represent a hexagonal-based pyramid?

10 Verify that Euler's formula works for the planar graphs in Question 9.

Puzzles

1 Find the size of the unknowns in the following shapes. Match each of the corresponding letters to the correct answers below to solve the riddle:

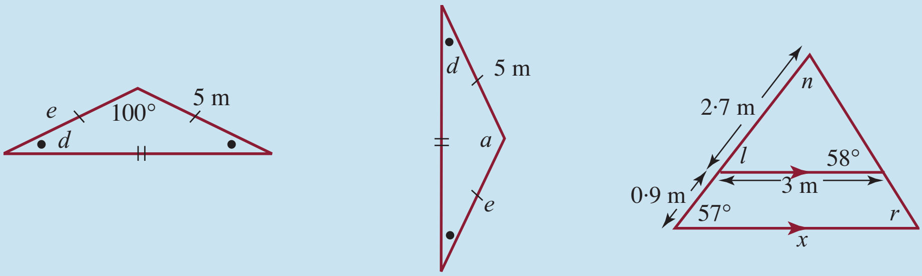
Why can't you take a photo of someone with crutches?



28°	48°	38°		132°	90°	90°	32°		109°		
117°	109°	51°	90°	146°	109°		152°	48°			
152°	109°	81°	90°		109°		119°	63°	48°	152°	48°

2 Find the value of the pronumerals in each of the congruent and similar triangles shown below. Match the corresponding letter to the correct value to solve the riddle:

Alexander's mother had three children named April, May and ...?

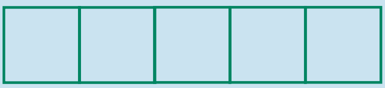


____	____	____	____	____	____	____	____	____
100°	57°	5 m	4 m	100°	65°	40°	5 m	58°

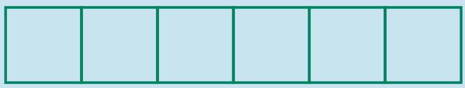
3 The shapes below are each made up of 1 cm × 1 cm squares. The shapes are to be enlarged by the scale factors given. Calculate the area of the enlarged shapes. Write the areas and corresponding letters in ascending order to find:

The name of the set of values where a function exists.

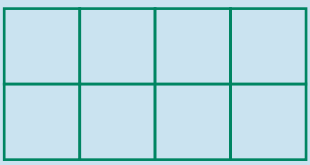
I Scale factor of 4



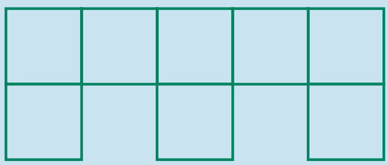
D Scale factor of 2



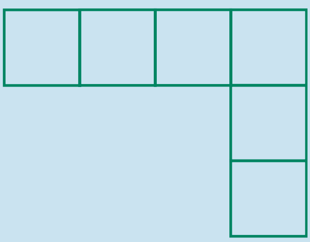
N Scale factor of 3



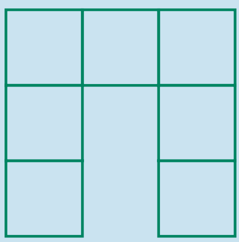
M Scale factor of 2



A Scale factor of 3



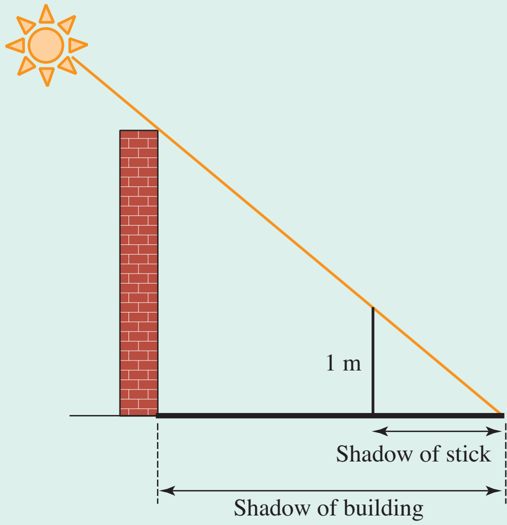
O Scale factor of 2



____ ____ ____ ____ ____ ____

Applications

Finding the height of tall objects



The heights of tall buildings or trees are difficult to measure directly. It is likely that the height of the Great Pyramid at Giza, in Egypt, for example, was determined by comparing the length of its shadow with that cast by a stick of known height.

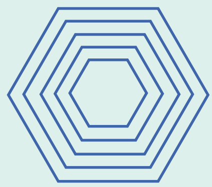
Choose a tall vertical object which has space around it so that its shadow can easily be measured. Place a stick of known height (1 metre for simplicity) in the shadow of the vertical object as shown here.

Use similar triangles to determine the height of the object. Discuss what errors could occur in such an approach, and give an upper and lower estimate of the height of the object.

Calculate the tolerance in your estimate and the percentage error in your answer.

Regular polygons

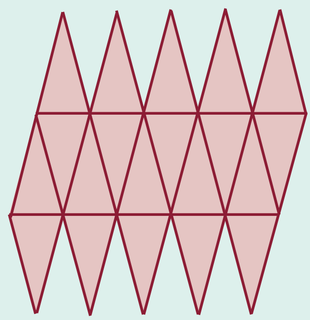
How was the following picture created? Create a similar one using regular pentagons.



Design a mobile for your classroom based on your creation.

Parallelogram puzzle

How many parallelograms can you see?



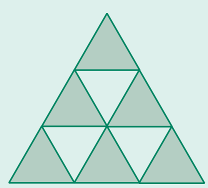
Design a poster for your classroom based on parallelograms.

Shapes in advertising

Look through a newpaper or magazine. Cut out five pictures that use geometric shapes to sell their product. Paste them in your workbook. Below each picture write down the geometric shape that is featured in the picture and how it is used to sell the product.

Equilateral triangles

How many equilateral triangles can you see?



Shapes in design

Look at the photo. What geometrical shape has been used to construct this dome? Why has the designer chosen this shape? What is its purpose? Name three other objects that have been built using a geometric shape in their design.

Constructing polygons

Regular polygons may be constructed within a circle.

1
- Using a compass, draw a circle with a radius of 5 cm.
- Draw in the radius OJ.
- Using a protractor, draw in $\angle JOK = 120°$ (note: $\frac{1}{3}$ of $360° = 120°$).
- Using a protractor, draw in $\angle KOL = 120°$.
- Using a coloured pencil and your ruler, draw in the straight lines $\overline{JK}$, $\overline{KL}$, $\overline{LJ}$.
- Measure the lengths of the lines $\overline{JK}$, $\overline{KL}$, $\overline{LJ}$.
- Measure the size of the angles $\angle JKL$, $\angle KLJ$, $\angle LJK$.
- What is the name of the regular polygon you have constructed?

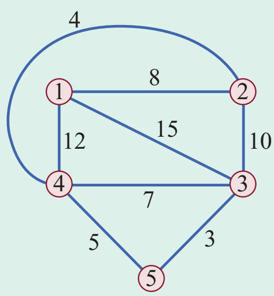
2
- Using a compass, draw a circle with a radius of 5 cm.
- Draw in the radius OJ.
- Using a protractor, draw in $\angle JOK = 90°$ (note: $\frac{1}{4}$ of $360° = 90°$).
- Using a protractor, draw in $\angle KOL = 90°$.
- Using a protractor, draw in $\angle LOM = 90°$.
- Using a coloured pencil and your ruler, draw in the straight lines $\overline{JK}$, $\overline{KL}$, $\overline{LJ}$.
- Measure the lengths of the lines $\overline{JK}$, $\overline{KL}$, $\overline{LM}$, $\overline{MJ}$.
- Measure the size of the angles $\angle JKL$, $\angle KLM$, $\angle LMJ$, $\angle MJK$.
- What is the name of the regular polygon you have constructed?

3 By using a construction approach similar to the one used in questions 1 and 2, write down the instructions for constructing the following:

a a pentagon **b** a hexagon **c** an octagon **d** a decagon

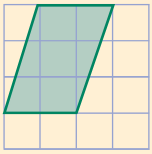
Speedy delivery

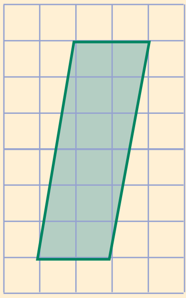
A delivery person has to make deliveries to the occupants of houses situated at 1, 2, 3, 4 and 5. The time taken, in minutes, between delivery points is indicated on the graph. Identify the path the delivery person would need to take to minimise the time it would take to make their deliveries.

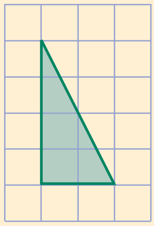


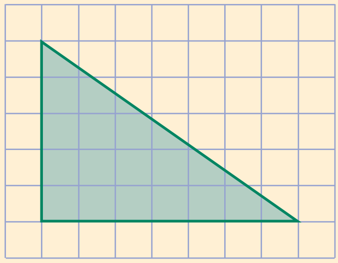
Enrichment

1 Increase each of the following shapes by a horizontal scale factor of 3 and vertical scale factor of 2. Calculate the area of the original and the dilated shapes and describe any patterns that you find.

a 

b 

c 

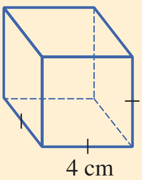
d 

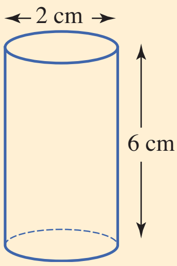
2 Increase each of the prisms below by a scale factor of 2 and draw the image in your workbook.

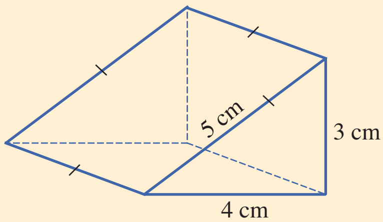
a i Calculate the area of the end face (square, circle, triangle).

ii Find the total surface area.

iii Find the volume of the prism.







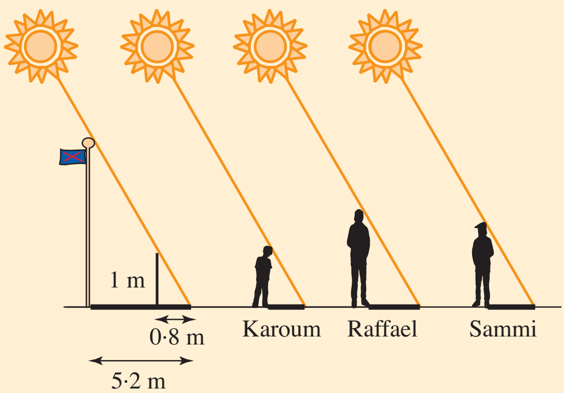
b How does doubling each of the sides affect:

i the area of the end face?

ii the total surface area of the prism?

iii the volume of the prism?

3 A stick 1 m high is placed in the shadow of a flagpole so that the end of the stick's shadow reaches that of the flagpole, as shown in the diagram. The shadow of the flagpole is 5·2 m long and the stick is 0·8 m from the end of the shadow.



a Find the height of the flagpole.

b Find the lengths of the shadows cast by:

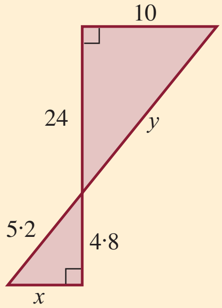
i Karoum, height 0·9 m

ii Raffael, height 1·8 m

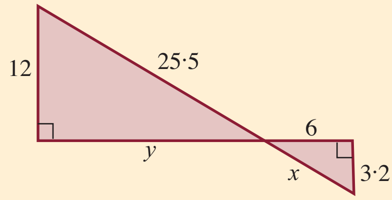
iii Sammi, height 1·5 m

4 Find the lengths (in centimetres) of the lines marked with pronumerals, by using similar triangles. Show that Pythagoras' theorem works in each case:

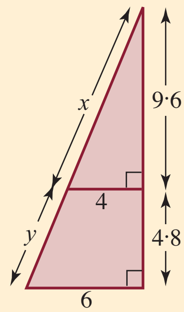
a



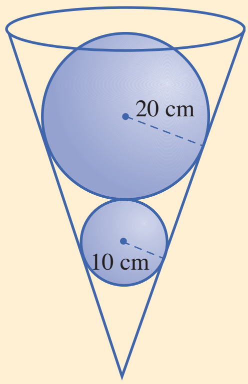
b



c

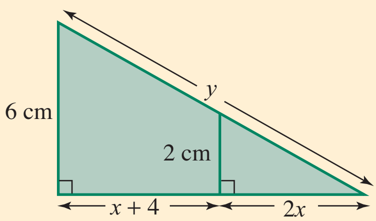


5 Two balls just fit inside a cone. If the radii of the balls are 20 cm and 10 cm, find the height of the cone.

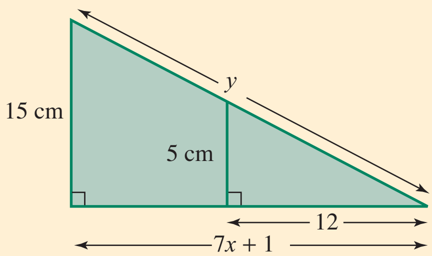


6 Find the values of x and y:

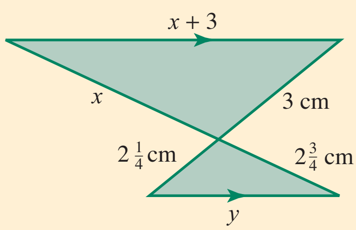
a



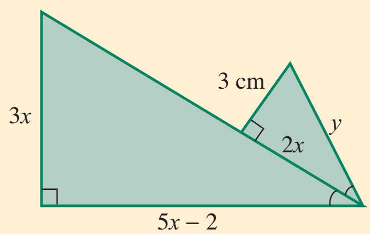
b



c



d



Revision/Assessment

Exercise 3A

1 Classify the following angles:

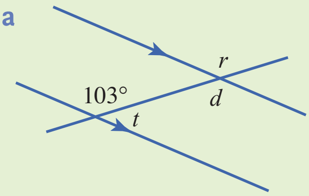
a 116° **b** 11° **c** 127° **d** 43° **e** 236° **f** 316°

2 State the complement and supplement of each of the following angles:

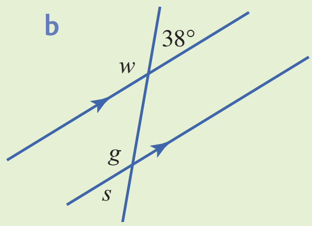
a 15° **b** 27° **c** 39° **d** 49° **e** 51° **f** 62°

3 Find the values of the pronumerals in each of the following:

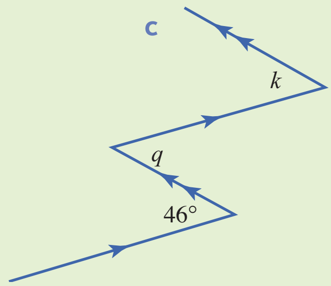
a



b



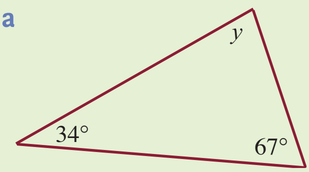
c



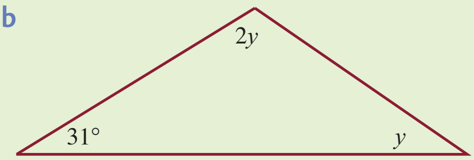
Exercise 3B

4 Find the value of the pronumeral in each of the following:

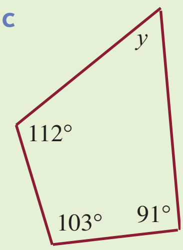
a



b

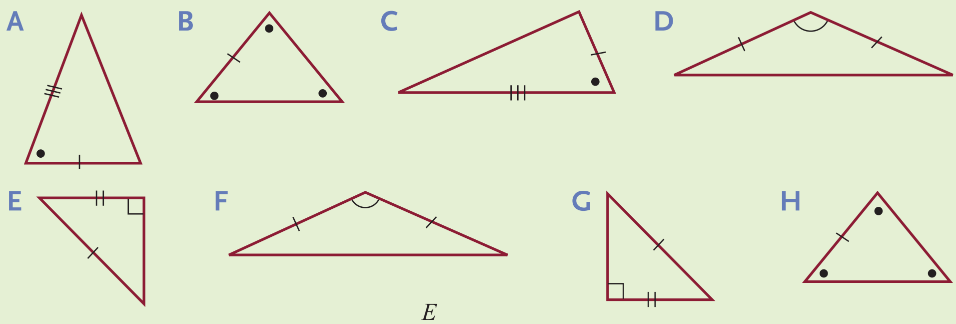


c

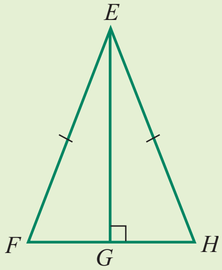


Exercise 3C

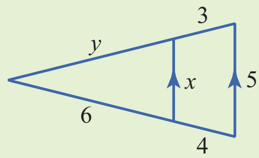
5 Which of the following triangles are congruent? Give a reason for your answer:



6 Prove that $\Delta EGF \cong EGH$.

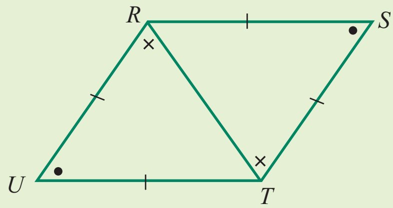


7 Find the value of x and y.



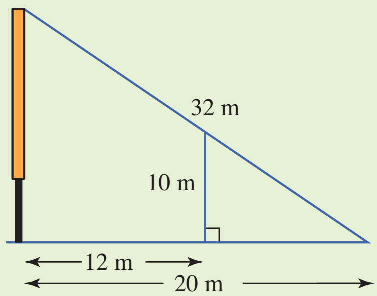
Exercise 3D

8 Prove that $\Delta RUT \sim \Delta TSR$.



Exercise 3G

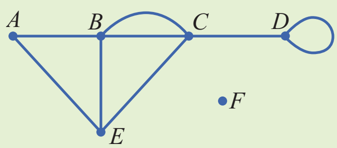
9 The scoreboard at the local soccer ground needs a support to keep it in place. The support is 32 m long and it is positioned 20 m from the base of the scoreboard. There is a 10 m long prop for this support 12 m from the scoreboard. What is the height of the scoreboard?



Exercise 3I

10 Copy and complete the table for the graph below:

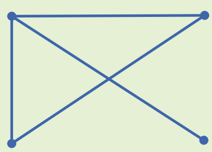
Vertex	A	B	C	D	E	F
Degree						



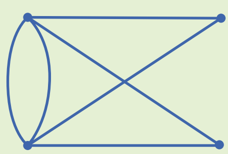
Exercise 3J

11 Redraw the following as planar graphs and confirm Euler's formula:

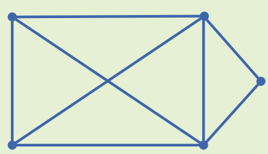
a



b



c



CHAPTER 4

Algebra Skills

Algebra is the part of mathematics that focuses on 'rules of operations and relationships' of two or more quantities. It gives a symbolic way to describe and generalise patterns that occur in the real world. This enables us to identify patterns and make predictions from these patterns.

This chapter will enable learners to continue to use algebraic techniques to manipulate algebraic expressions into different forms and to be able to compare these patterns. It will provide learners with skills to devise new rules for non-standard situations and use pronumerals to write their own equations and use algebra to solve practical application. Understanding and interpreting these equations help us investigate the past, explain the present and predict the future.

This chapter covers the following skills:

- Variables and expressions
- Recognise like terms
- Expanding brackets with:
  - Single bracket
  - Two brackets
  - Pairs of brackets using difference of two squares and perfect squares
- Factorise expressions and equations
  - Highest Common Factor (HCF)
  - Difference of two squares
  - Perfect squares identity
- Substitution into formulae and evaluate
- Find missing values using Pythagoras' Theorem
- Re-arrange formulas to make one quantity the subject
- Solve application questions

Specific Learning Outcomes (SLO):

9.4.1.1 Define 'algebra'.
9.4.1.2 Identify and define terms and words that are used in algebra.
9.4.2.1 Identify the algebraic terms such as *pronumeral*, *variable*, *term*, *expression*, *coefficient*, *constant* and *equation*.
9.4.3.1 Identify and differentiate between 'like' and 'unlike' terms.
9.4.4.1 Simplify expressions with different terms by grouping like terms.
9.4.5.1 Expand brackets using **distributive law**.
9.4.5.2 Expand brackets that are in an expression and equation using distributive law and simplify them by collecting like terms.
9.4.6.1 Define and differentiate 'difference of two squares' and 'perfect squares'.
9.4.7.1 Expand and simplify pairs of brackets with a negative and positive signs using difference of two squares and perfect squares.
9.4.8.1 Define 'factorisation'.
9.4.8.2 Identify the highest common factor.
9.4.9.1 Factorise expressions by taking the highest common factor.
9.4.9.2 Factorise expressions by grouping those terms that are the same.
9.4.10.1 Factorise expressions using difference of two squares.
9.4.11.1 Factorise expressions using perfect squares.
9.4.12.1 Substitute variables in expressions and formulas.
9.4.12.2 Evaluate expressions and formulas by replacing variables with values.
9.4.13.1 Define Pythagoras' theorem.
9.4.13.2 Find the hypotenuse side using Pythagoras' theorem.
9.4.13.3 Find the other two shorter sides using Pythagoras' theorem.
9.4.14.1 Re-arrange formulas and equations to make one quantity the subject.
9.4.15.1 Evaluate formulas and equations by replacing unknown numbers in the equation and formulas.
9.4.16.1 Convert mathematical statements in words to algebraic statements in symbols.
9.4.17.1 Solve word problems algebraically by using logical steps provided.

4A Variables and expressions

The language of algebra

To be able to use algebra to solve problems, you need to understand the language.

In Year 8 you were introduced to many new words and conventions, but it is important to go through them again as a recap to assist you to effectively participate in the year course.

A **pronumeral** is a letter that represents an unknown number. For example, x might represent the number of school days in a year.

A **variable** describes an unknown amount in an algebraic expression or equation. Variables are represented by a letter or a symbol. For example, the average amount of water used by a Year 9 student per day is a variable that can be represented by symbols such as a or x. These symbols can also be referred to as pronumerals.

A **term** can be:

- a number or a variable by itself: 7, x
- a number multiplied by a variable: $12c$, $5k$
- different variables multiplied together: mn, abc
- a variable multiplied by itself a number of times: a^3, b^2
- the product of a number and several variables: $3xy$, $4a^2b$.

An **expression** is formed when terms are added or subtracted; for example, $3x + 4y$.

A **coefficient** of x is a number by which the variable is multiplied. The coefficient is written in front of any variable and includes any negative sign in front of the number. 8 is the coefficient of $8x$ and –6 is the coefficient of $-6y$. Note that the coefficient of x is 1.

A **constant** is a term that does not contain any variable factors. It is a number written by itself. 6 is the constant in $3x + 6 = 2x - 4$.

Example | **Solution**

1 For $6a + 3b - 4 = 2ab - c + 7$:

Example	Solution
a Identify whether this is an equation or an expression.	It is an equation because it has an equals sign.
b Write down the coefficient of b.	3 is the coefficient.
c Write down all the terms in the equation or expression.	$6a$, $3b$, –4, $2ab$, $-c$ and 7.
d List the variables.	a, b and c are variables.
e Write down any constants.	–4 and 7 are constants.

2 Pete has b blue pens and r red pens. Sue has three times as many blue pens as Pete. She has 2 fewer red ones than Pete.

Example	Solution
a Write an expression for the number of pens Pete has altogether.	$b + r$
b If Pete has 16 pens to start with, write an equation to show this information.	$b + r = 16$

Example	Solution
c Write an expression for the number of blue pens Sue has.	$3b$
d Write an expression for the number of red pens Sue has.	$r - 2$
e If Sue has 28 pens altogether, write an equation to show this information.	$3b + r - 2 = 28$
f If Pete loses 4 of his pens, write an expression in terms of b and r to show how many pens Pete has now.	$b + r - 4$

Exercise 4A

1 For each of the following:

i $7t + 8r - 5bt - 9 + 3b$ ii $4g - 2r + 7rf = 9 - 5b$ iii $b + n - 5bn + 10$

a Identify whether this is an equation or an expression.

b Write down the coefficient of b.

c Write down all the terms in the equation or expression.

d List the variables.

e Write down any constants.

2 Agnes has w watches and r rings. Donna has two times as many watches and 2 fewer rings than Agnes.

a Write an expression for the number of watches and rings Agnes has altogether.

b If Agnes has 8 watches and rings to start with, write an equation to show this information.

c Write an expression for the number of watches Donna has.

d Write an expression for the number of rings Donna has.

e If Donna has 14 watches and rings altogether, write an equation to show this information.

f If Agnes loses 2 of her rings, write an expression in terms of w and r to show how many watches and rings Agnes has now.

3 Find the coefficient of the following:

a The coefficient of x in $6y + 7xy + 5x$.

b The coefficient of xy in $5 + 8xy - 2y + 4x$.

c The coefficient of a in $5b + 2ab + 6$.

d The coefficient of x in $4xy - 6y + x + 8$.

4 Find the constant of the following:

a $x^2y - 3 + 5xy$

b $3ef + 7efg + 12 + 11ef + 4e$

4B Recognise like terms

Like terms

Algebraic terms that have exactly the same variables are called **like terms**.

We can simplify algebraic expressions by collecting like terms. We collect like terms by adding their coefficients. $3x$ and $5x$ are like terms and can be simplified by adding the coefficients of x. $3x + 5x = 8x$.

Unlike terms are terms that have different variables. $15x$ and $23y$ and $-3a^2h^2$ are examples of unlike terms. Unlike terms cannot be simplified because they are all different.

Pronumerals are the same as variables, which represent unknown numbers or values.

Expressions can be simplified by collecting like terms.

Example

Simplify $3x + 2y - 4x - 5$.

Solution

$3x + 2y - 4x - 5$
$= 2y - x - 5$

Exercise 4B

1 List the groups of like terms for each of the following:

a $4x, -2y, 4x, 7y, -8x$
b $a, -ab, 6ab, 5ba, -6a, 4ab, -5b$
c $5p^2q, -2pq, pq^2, 6pq, -qp$
d $x^2, -x, 5xy, -x^2, -x^2y, 6yx, -3yx^2$
e $2c^2d^2, c^2d, 8d^2c, -c^2d^2$
f $x, y^2, 2x, x^2, 2y, 2y^2, -y, 2xy$

2 Simplify the following expressions by combining like terms:

a $m + m + m + n + 2$
b $x + y + x + y + 4 + x + x + x + 6$
c $3 + x + x - y + y - y - y$
d $4 - b + a - a - b + a + 5$
e $cd + cd - dc + 2cd + dc$
f $ef - 2ef + ef - ef - fe + fe + ef$

3 Simplify the following expressions by combining like terms:

a $x + 5x$
b $5x + 10x$
c $12x + 7x$
d $7x - 12x$
e $15a - 7a$
f $7a - 15a$
g $-2d - 3d$
h $-3x - 4x$
i $-8m - 12m$
j $-x - x + 7x$
k $-2y - y + 10y$
l $-4y - 3y + 7y$

4 Simplify the following expressions by combining like terms:

a $6xy - 4xy$
b $12xy - 9yx$
c $6ab - 5ab$
d $9ab - 6ba + ba$
e $4pq - 8pq + 9pq$
f $7pq + 2pq - 4qp$
g $8mn + 2mn - 16mn$
h $15mn - 17nm - 5nm$
i $19xy - 15xy - 6xy$
j $7xy - 3yx - xy$
k $7wx - 12wx + 5wx$
l $-6rs + 2rs + 4rs$

5 Simplify the following expressions by combining like terms:

a $3x + 5x - y + 6y$
b $5m + 2m + 3n - 6n$
c $6xy + 2xy + zy - 2zy$
d $6pq - q - 2qp + 5q$
e $8x - 3y - 4x + 5y$
f $3x + 2 - 4x - 5$
g $a - 5 - 2a + 2$
h $4 + 3x - 2 - 5x$
i $6x + 3y - 2 - 4x - 5y + 1$
j $3d - 2e + 3e - 4d$
k $8x - 2y + 7 - 9x + y - 6$
l $3g + 6h - 4g - 6h$
m $4x^2 + 5x - 2x^2 - 8x$
n $2x^2 + x - 5x^2 - 7x$
o $26a + 18a - 10a$
p $7rk - rk + 5r^4$
q $15t^2 - 4t + 3t$
r $4l - l^3 + 15l$
s $4r^2 + 5r - r^2 - 3r$
t $3wt + 13t^2 - t^2 - wt$
u $6p^3 + 8p^3 - 6p + p^3$

Expanding brackets 4C

Brackets are used to group terms together and to indicate multiplication. Terms that are in and outside in a one or two bracket can be multiplied using **distributive law**. Also, pairs of brackets can be expanded using difference of two squares and perfect squares.

Expansion is the process of removing brackets.

Factorisation is the process in which we restore the brackets in an expression.

$(x + 7)(x - 4)$

factorisation ↑ ↓ expansion

$x^2 + 3x - 28$

To expand the brackets we multiply the terms in an expression by using the **distributive law**.

$a(x + b)$ $= a \times x + a \times b$ $= ax + ab$	$(x + y)(a + b)$ $= x(a + b) + y(a + b)$ $= x \times a + x \times b + y \times a + y \times b$ $= ax + bx + ay + by$

When we expand an expression that contains brackets, we use the following steps:
Step 1: Write down the expression we wish to expand.
Step 2: Work from left to right to expand each bracket.
Step 3: Collect together like terms.

Expanding and collecting like terms

After an expression is expanded using the distributive law, it may contain like terms. These can be collected and the expression simplified.

Example

Expand and simplify:

a $5(x + 7)$

b $-3(y + 2)$

c $3(m + 3) - 2(m - 1)$

d $2 + 3(x + 2)$

e $(x + 3)(x + 5)$

Solution

a $5(x + 7)$
$= 5 \times x + 5 \times 7$
$= 5x + 35$

b $-3(y + 2)$
$= -3 \times y + -3 \times 2$
$= -3y - 6$

c $3(m + 3) - 2(m - 1)$
$= 3 \times m + 3 \times 3 - 2 \times m - 2 \times -1$
$= 3m + 9 - 2m + 2$
$= m + 11$

d $2 + 3(x + 2)$
$= 2 + 3 \times x + 3 \times 2$
$= 2 + 3x + 6$
$= 3x + 8$

e $(x + 3)(x + 5)$
$= x(x + 5) + 3(x + 5)$
$= x \times x + x \times 5 + 3 \times x + 3 \times 5$
$= x^2 + 5x + 3x + 15$
$= x^2 + 8x + 15$

Example

f $3(2x+9)(x-2)$

Solution

$3(2x+9)(x-2)$
$= 3[2x(x-2)+9(x-2)]$
$= 3[2x \times x - 2x \times 2 + 9 \times x - 9 \times 2]$
$= 3[2x^2 - 4x + 9x - 18]$
$= 3[2x^2 + 5x - 18]$
$= 6x^2 + 15x - 54$

Exercise 4C

1 Expand and simplify the following by filling in the spaces:

a $2(m+2) = 2 \times \_\_\_ + 2 \times \_\_\_$
$= \_\_\_\_\_ + \_\_\_\_\_$

b $3(m+9) = 3 \times \_\_\_ + 3 \times \_\_\_$
$= \_\_\_\_\_ + \_\_\_\_\_$

c $11(h-5) = 11 \times \_\_\_ - 11 \times \_\_\_$
$= \_\_\_\_\_ - \_\_\_\_\_$

d $8(h-12) = 8 \times \_\_\_ - 8 \times \_\_\_$
$= \_\_\_\_\_ - \_\_\_\_\_$

e $-7(g-6) = \_\_\_ \times g + \_\_\_ \times 6$
$= \_\_\_\_\_ + \_\_\_\_\_$

f $-8(g-4) = \_\_\_ \times g + \_\_\_ \times 4$
$= \_\_\_\_\_ + \_\_\_\_\_$

2 Expand and simplify the following:

a $5(x+3)$ b $3(x+4)$ c $2(c+5)$ d $7(c+6)$
e $5(g+4)$ f $2(g+7)$ g $7(3b+5)$ h $3(2b+9)$
i $8(4h-3)$ j $6(2p-7)$ k $5(2w-5)$ l $8(3y-1)$
m $-2(y+5)$ n $-5(3y+2)$ o $-b(b+3)$ p $-k(3k+2)$
q $-2k(k-5)$ r $-3t(t-7)$ s $-12(5+p)$ t $-15(3+2p)$
u $-9(8+9p)$ v $-k(3k-7t)$ w $-2k(k-5t)$ x $-2g(g-11p)$

3 Expand and simplify the following:

a $2(x+2)+3(x+1)$ b $4(x+3)+2(x+4)$ c $3(y+1)+2(y+5)$
d $2(y-5)+5(y+2)$ e $4(7+t)+5(t-8)$ f $2(5t+4)+7(3t-2)$
g $6p(6p+3)+9(1+p)$ h $3p(p-5)+2(p-4)$ i $5m(3m-2)-4m(4m-6)$

4 Expand and simplify the following:

a $2+3(x+1)$ b $6+2(x+4)$ c $3+2(y+5)$
d $5+5(y-2)$ e $4+5(t-8)$ f $3+4(3t-2)$
g $4p-2(p+3)$ h $3p-2(p+4)$ i $5m-4m(4m+1)$
j $8m-3m(2m-1)$ k $2c-c(3-c)$ l $4d-3d(5-2d)$

5 Expand and simplify the following:

a $(x+4)(x+3)$ b $(x+5)(x+7)$ c $(w+2)(w-7)$
d $(w+1)(w-9)$ e $(r-8)(r+2)$ f $(r-3)(r+8)$
g $(d-3)(d-5)$ h $(d-9)(d-2)$ i $(3+x)(x-6)$
j $(x-3)(3x-2)$ k $(2y+1)(y+7)$ l $(2y+5)(y+4)$
m $(5x+1)(2x-3)$ n $(6x+1)(2x-9)$ o $(8w-3)(2w+5)$
p $2(4w-3)(3w+7)$ q $3(2+7q)(8-q)$ r $4(q-3)(q+1)$
s $-3(t+2)(5+2t)$ t $-4(m-1)(2+3m)$ u $-2(2m-5)(5-3m)$

More on expanding brackets 4D

The difference of two squares and a perfect square

The **difference of two squares** identity involves two brackets. The first bracket is the difference of the two terms and the second is the sum of the two terms. Expanding the two brackets results in the difference of the squares of the two terms, as the middle terms cancel.

$$\begin{aligned}(a-b)(a+b) &= a(a+b)-b(a+b)\\ &= a^2+ab-ba-b^2\\ &= a^2-b^2\end{aligned}$$

Square of the first term   Square of the second term   Middle terms cancel

Difference of two squares

$(a-b)(a+b)=a^2-b^2$

Example

Solution

1 Expand and simplify the following by using the difference of two squares identity:

a $(x-a)(x+a)$

$(x-a)(x+a)=x^2-a^2$

b $(p-3)(p+3)$

$$\begin{aligned}(p-3)(p+3)&=p^2-3^2\\ &=p^2-9\end{aligned}$$

c $(2k-5)(2k+5)$

$$\begin{aligned}(2k-5)(2k+5)&=(2k)^2-5^2\\ &=4k^2-25\end{aligned}$$

A **perfect square** involves two brackets with the same terms, which can be expanded to give an expression with three terms called a **trinomial**. Expanding a perfect square gives the square of the first term, then twice the product of the two terms followed by the square of the second term.

$$\begin{aligned}(x+y)^2&=(x+y)(x+y)\\ &=x(x+y)+y(x+y)\\ &=x^2+xy+xy+y^2\\ &=x^2+2xy+y^2\end{aligned}$$

Square of the first term   Twice the product of the two terms   Square of the second term

$$\begin{aligned}(x-y)^2&=(x-y)(x-y)\\ &=x(x-y)-y(x-y)\\ &=x^2-xy-xy+y^2\\ &=x^2-2xy+y^2\end{aligned}$$

Square of the first term   Twice the product of the two terms   Square of the second term

Perfect squares

$(x+y)^2=x^2+2xy+y^2$ and $(x-y)^2=x^2-2xy+y^2$

Example

Solution

2 Expand and simplify the following by using the perfect square identity:

a $(a + 3)^2$

$(a + 3)^2$
$= a^2 + 2 \times a \times 3 + 3^2$
$= a^2 + 6a + 9$

b $(5 - p)^2$

$(5 - p)^2$
$= 5^2 - 2 \times 5 \times p + p^2$
$= 25 - 10p + p^2$

c $(1 - 7m)^2$

$(1 - 7m)^2$
$= 1^2 - 2 \times 1 \times 7m + (7m)^2$
$= 1 - 14m + 49m^2$

d $2(5a - 3b)^2$

$2(5a - 3b)^2$
$= 2((5a)^2 - 2 \times 5a \times 3b + (3b)^2)$
$= 2(25a^2 - 30ab + 9b^2)$
$= 50a^2 - 60ab + 18b^2$

Exercise 4D

1 Expand and simplify these expressions by using the difference of two squares identity:

a $(a - 4)(a + 4)$ **b** $(a - 7)(a + 7)$ **c** $(a - 3)(a + 3)$
d $(m + 1)(m - 1)$ **e** $(m + 5)(m - 5)$ **f** $(m + 6)(m - 6)$
g $(9 - x)(9 + x)$ **h** $(3 - x)(3 + x)$ **i** $(8 - y)(8 + y)$
j $(2y - 1)(2y + 1)$ **k** $(3y - 2)(3y + 2)$ **l** $(5y - 7)(5y + 7)$
m $(5 + 2m)(5 - 2m)$ **n** $(10 - 11n)(10 + 11n)$ **o** $(15 - 9n)(15 + 9n)$

2 Expand and simplify the following by using the perfect square identity:

a $(x + 2)^2$ **b** $(x + 6)^2$ **c** $(x + 5)^2$ **d** $(x + 3)^2$
e $(f + 4)^2$ **f** $(f + 9)^2$ **g** $(f + 7)^2$ **h** $(f + 1)^2$
i $(m + 8)^2$ **j** $(m + 11)^2$ **k** $(m + 12)^2$ **l** $(m + 20)^2$
m $(9 + t)^2$ **n** $(8 + t)^2$ **o** $(5 + t)^2$ **p** $(10 + t)^2$

3 Expand and simplify the following by using the perfect square identity:

a $(x - 1)^2$ **b** $(x - 5)^2$ **c** $(x - 2)^2$ **d** $(x - 8)^2$
e $(t - 4)^2$ **f** $(t - 3)^2$ **g** $(t - 6)^2$ **h** $(t - 7)^2$
i $(w - 11)^2$ **j** $(w - 9)^2$ **k** $(w - 12)^2$ **l** $(w - 15)^2$
m $(5 - r)^2$ **n** $(3 - r)^2$ **o** $(10 - r)^2$ **p** $(11 - r)^2$

4 Expand and simplify:

a $(3x - 2)^2$ **b** $(5x - 3)^2$ **c** $(7x - 5)^2$ **d** $3(2x - 7)^2$
e $3(6t - 1)^2$ **f** $2(4t - 9)^2$ **g** $4(1 - 5t)^2$ **h** $5(2 - 7t)^2$
i $-(5x - 11b)^2$ **j** $-2(5x - 2b)^2$ **k** $-3(2x - b)^2$ **l** $-4(x - 2b)^2$
m $-2(2m - g)^2$ **n** $-3(3m - 2g)^2$ **o** $-5(4m - 3g)^2$ **p** $-7(5m - 3g)^2$

Factorisation: Highest common factors 4E

Factorisation is the reverse process of expansion. It involves restoring the brackets in an expression. The first step in factorising is to take out common factors from all terms.

Example **Solution**

1 Factorise the following:

a $4x + 24$

The highest common factor is 4, so:

$4x + 24 = 4 \times x + 4 \times 6$

$= 4(x + 6)$ fully factorised

b $y^2 - 5y$

The highest common factor is y, so:

$y^2 - 5y = y \times y - y \times 5$

$= y(y - 5)$

Expressions that have four terms may be factorised by grouping:

Example **Solution**

2 Factorise by grouping:

a $3b(a + 2) + 2(a + 2)$

$= 3b(a + 2) + 2(a + 2)$

$= (a + 2)(3b + 2)$

b $7xy + 14y - 5x - 10$

$7xy + 14y - 5x - 10$

$= 7y(x + 2) - 5(x + 2)$

$= (x + 2)(7y - 5)$

Exercise 4E

1 Factorise the following by removing the highest common factor:

a $6a + 12$ **b** $26a + 13$ **c** $18 + 9a$ **d** $9 - 3b$

e $25 - 5b$ **f** $49 - 7b$ **g** $x^2 + 6x$ **h** $x^2 + 15x$

i $x^2 + 13x$ **j** $x^2 - 4x$ **k** $x^2 - 20x$ **l** $x^2 - 32x$

m $2y^2 + 10y$ **n** $3y^2 + 45y$ **o** $5y^2 + 50y$ **p** $4y^2 - 24y$

2 Factorise the following by removing the highest common factor:

a $3b + 6b^2 + 9ab$ **b** $10b^2 + 100b - 1000by$ **c** $-6v - 18v^2 - 36v^3$

d $-35v - 49v^2 + 56v^3$ **e** $a^2b - ab + 7ab^2$ **f** $3x^2y - 3xy + 12x$

g $x^4 - x^3 + 7x^5$ **h** $6y^2 + 12xy - 24x^2y$ **i** $24am^2 + 36a^2m + 56am$

j $5y^3 - y^2 + y^4$ **k** $2m^2 + 4m^4 + 2tm^3$ **l** $15ab^2 + 2b^2a + 7ab$

3 Factorise by grouping:

a $b(a + 2) + 3(a + 2)$ **b** $c(b - 7) - 2(b - 7)$ **c** $m(x + 5) + 4(x + 5)$

d $2x(t + 2) - 3(t + 2)$ **e** $5m(y - 5) + 1(y - 5)$ **f** $7a(b + 4) - 3(b + 4)$

g $xy + 5y + x + 5$ **h** $ab + b + 7a + 7$ **i** $ab + 2b + 5a + 10$

j $2y + xy + 7x + 14$ **k** $3t + 6 + st + 2s$ **l** $x^2 + xy + 2x + 2y$

m $2a - 2 + ab - b$ **n** $3b - 12 + ab - 4a$ **o** $rs + 2r - 3s - 6$

4F Factorisation: Difference of two squares

Another method of factorisation uses the **difference of two squares** identity.

When the two brackets below are expanded the middle terms cancel, leaving the square of the first term minus the square of the second term.

$$\begin{aligned}(a-b)(a+b) &= a(a+b) - b(a+b) \\ &= a^2 + ab - ba - b^2 \\ &= a^2 - b^2\end{aligned}$$

Factorising is the reverse process. The difference of two squares can be factorised to give the **first** term **plus** the **second** term **multiplied** by the **first** term **minus** the **second** term.

Difference of two squares

$a^2 - b^2 = (a-b)(a+b)$

Example

Use the difference of two squares identity to factorise the following expressions:

a $q^2 - 7^2$

b $128x^2 - 18$

c $(b+3)^2 - 25$

Solution

a $q^2 - 7^2 = (q-7)(q+7)$

b Take out the common factor first:
$128x^2 - 18$
$= 2(64x^2 - 9)$
$= 2[(8x)^2 - 3^2]$
$= 2(8x-3)(8x+3)$

c $(b+3)^2 - 25$
$= (b+3)^2 - 5^2$
$= [(b+3) - 5)][(b+3) + 5]$
$= (b-2)(b+8)$

Exercise 4F

1 Use the difference of two squares identity to factorise the following:

a $b^2 - 5^2$	**b** $a^2 - 64$	**c** $h^2 - 49$	**d** $9 - x^2$
e $16t^2 - n^2$	**f** $36s^2 - y^2$	**g** $49x^2 - y^2$	**h** $81n^2 - f^2$
i $100 - s^2$	**j** $1 - 9y^2$	**k** $49 - 16g^2$	**l** $144 - 81j^2$
m $128a^2 - 2b^2$	**n** $32 - 50y^2$	**o** $75 - 48m^2$	**p** $\frac{1}{a^2} - 25$

2 Use the difference of two squares identity to factorise the following:

a $(b+1)^2 - 1$	**b** $(x+2)^2 - 9$	**c** $(k-3)^2 - 100$
d $2(b+4)^2 - 2$	**e** $3(x+2)^2 - 27$	**f** $2(k+3)^2 - 128$
g $81 - (b+1)^2$	**h** $25 - (x-2)^2$	**i** $100 - (k-1)^2$
j $18 - 2(b+2)^2$	**k** $50 - 2(x+2)^2$	**l** $200 - 2(k+2)^2$
m $(b+2)^2 - (b-2)^2$	**n** $(x+1)^2 - (x+3)^2$	**o** $(k-2)^2 - (k+3)^2$
p $3 - 3(x+1)^2$	**q** $18 - 2(b+2)^2$	**r** $200 - 2(k-3)^2$

Factorisation: The perfect square identity 4G

Factorisation can also be carried out by using the perfect square identity.

When the brackets below are expanded the middle terms are the same. The result is the square of the first term plus twice the product of the first and second terms, plus the square of the second term.

$$(x+y)^2 = x^2 + 2 \times x \times y + y^2 = x^2 + 2xy + y^2 \quad \text{and} \quad (x-y)^2 = x^2 - 2 \times x \times y + y^2 = x^2 - 2xy + y^2$$

Factorising is the reverse process. The perfect square identity is shown below:

Perfect square identity

$x^2 + 2xy + y^2 = (x+y)^2$ and $x^2 - 2xy + y^2 = (x-y)^2$

Example

Express these trinomials as perfect squares:

a $a^2 + 2ab + b^2$

b $c^2 - 6cn + 9n^2$

c $4x^2 + 28xy + 49y^2$

Solution

a $a^2 + 2ab + b^2 = a^2 + 2(a)(b) + (b)^2 = (a+b)^2$

b $c^2 - 6cn + 9n^2 = c^2 - 2(c)(3n) + (3n)^2 = (c-3n)^2$

c $4x^2 + 28xy + 49y^2 = (2x)^2 + 2(2x)(7y) + (7y)^2 = (2x+7y)^2$

Exercise 4G

1 Factorise the following trinomials using the perfect square identity:

a $h^2 + 2hy + y^2$ b $k^2 + 2kt + t^2$ c $x^2 + 2xy + y^2$
d $h^2 - 2hm + m^2$ e $j^2 - 4jy + 4y^2$ f $x^2 - 2xy + y^2$
g $b^2 + 8bg + 16g^2$ h $d^2 + 6de + 9e^2$ i $e^2 + 10eg + 25g^2$
j $c^2 - 12cd + 36d^2$ k $w^2 - 10wp + 25p^2$ l $t^2 - 20st + 100s^2$
m $h^2 + 16hy + 64y^2$ n $k^2 + 14kt + 49t^2$ o $m^2 + 4mp + 4p^2$

2 Factorise the following trinomials using the perfect square identity:

a $9x^2 + 6xy + y^2$ b $4b^2 + 4bc + c^2$ c $64x^2 + 16xy + y^2$
d $25h^2 - 10hm + m^2$ e $16j^2 - 8jy + y^2$ f $49g^2 - 14gh + h^2$
g $16b^2 + 8bg + g^2$ h $81d^2 + 18de + e^2$ i $25e^2 + 10eg + g^2$
j $y^2 - 18yt + 81t^2$ k $f^2 - 14fh + 49h^2$ l $m^2 - 16mp + 64p^2$
m $k^2 + 20kq + 100q^2$ n $n^2 + 24nr + 144r^2$ o $p^2 + 22ps + 121s^2$
p $2c^2 - 4cd + 2d^2$ q $5w^2 - 10wp + 5p^2$ r $6t^2 - 24st + 24s^2$
s $2x^2 + 12xy + 18y^2$ t $3b^2 + 18bc + 27c^2$ u $20b^2 + 20bm + 5m^2$

4H Substitution for variables

Variables in an expression have unknown values. If we want to know the value for an expression when the variables have certain values, we can replace the variables in the expression with those values. This process is called **substitution**. In the expression $3x + 2y$, x and y are variables. When $x = 5$ and $y = 1$, the expression can be evaluated by substituting 5 for x and 1 for y so: $3x + 2y = 3 \times (5) + 2 \times (1) = 17$.

Because the variables can vary, different values for the variable will give different values for the expression. When $x = –3$ and $y = 7$, the expression can be evaluated by substituting –3 for x and 7 for y so: $3x + 2y = 3 \times (–3) + 2 \times (7) = 5$.

Formulas with variables or pronumerals can also use substitution methods to calculate solutions.

Example **Solution**

A formula for calculating the interest of a principal, p, of money invested in a bank at a rate, r, for time, t, is $I = \frac{(p \times r \times t)}{100}$

a Find I if $p = 2500$, $r = 5$, $t = 10$

Substituting the values in the formula:

$$I = \frac{(2500 \times 5 \times 10)}{100} = 1250$$

b Find I if $p = 10\,000$, $r = k$, $t = 2k + 1$

Substituting and simplifying:

$$I = \frac{(10\,000 \times k \times (2k + 1))}{100} = 100k(2k + 1) = 200k^2 + 100k$$

Exercise 4H

1 A formula for finding the perimeter of a triangle is $P = a + b + c$. Find P if:

a $a = 5, b = 7, c = 9$
b $a = 10, b = 6, c = 8$
c $a = 10, b = 12, c = 12$
d $a = b = 5, c = 6$
e $a = \mathrm{b} = c = 17$
f $a = 6, b = a + 1, c = 8$
g $a = 1{\cdot}1, b = 2{\cdot}4, c = 3{\cdot}1$
h $a = 0{\cdot}07, b = 0{\cdot}15, c = 0{\cdot}1$
i $a = b – 1, b = 4{\cdot}5, c = b + 1$
j $a = k + 1, b = c = k = 8$

2 A formula for finding the perimeter of a rectangle is $P = 2x + 2y$. Find P if:

a $x = 7, y = 10$
b $x = 14, y = 16$
c $x = 13, y = 9$
d $x = y = 7$
e $x = 3, y = x + 1$
f $x = 1{\cdot}8, y = 2{\cdot}1$
g $x = 0{\cdot}22, y = 0{\cdot}05$
h $x = y – 1, y = 3{\cdot}7$
i $x = k, y = k – 1$
j $x = 2k – 3, y = 3k + 1$

3 Evaluate the following expressions when $a = 3$ and $b = 6$.

a $a + 2b$
b $3a + 2b$
c $2a + 5b$
d ab
e $3ab$
f $10ab – a$
g $8b – 16a$
h $10ab – b$
i $6b – ab$
j $12a^2$
k $2b^2$
l $4a^3$
m $a^2 – b^3$
n $3a^2 – 2b^2$
o $2a^5 – 2b^3$
p $\frac{2a^2}{b}$
q $\frac{a^3}{b^2}$
r $\frac{144a^2}{3b^3}$

4 Evaluate these expressions when $d = -2$ and $e = 5$.

a $-3de - 6$ b $-5de + 100$ c $-8de + 15$ d $\frac{10}{e}$

e $\frac{40}{d}$ f $\frac{e}{d}$ g $\frac{2}{d} - \frac{5}{e}$ h $\frac{12}{d} - \frac{30}{e}$

i $\frac{15}{e} - \frac{18}{d}$ j $e^3 - 3d^2$ k $e^2 - 3d^2$ l $2e^3 - 3d^3$

5 A formula for finding the surface area of a cube is $S = 6a^2$. Remember to use BIDMAS and find S if:

a $a = 2$ b $a = 9$ c $a = 11$ d $a = 22$ e $a = 2{\cdot}5$

f $a = 7{\cdot}2$ g $a = 0{\cdot}3$ h $a = 10{\cdot}1$ i $a = 3{\cdot}7$

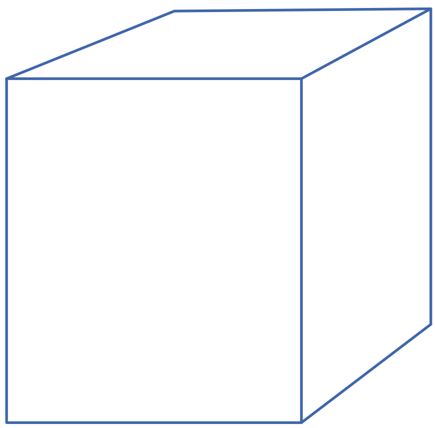
6 The dimensions of a box are given as length L, width W and height H.

a Find a formula for the volume, V, of the box.

b Show how to use your formula to calculate the volume of a box of length 25 cm, width 3 cm and height 35 cm.

c Find a formula for the surface area, S, of the box in terms of L, W and H.

d Show how to use your formula to calculate the surface area of the box with the same dimensions as in part b.

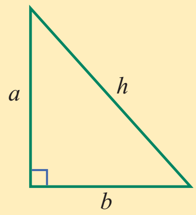


7 Copy and complete the table below if the values for a, b, c and d remain the same for each row:

a	b	c	d	$a+b$	$b+2c$	$3c+2d$	$4(a+c)$	$(d-b)^2$	$10-c^2$
1	2	3	4						
2	5	6	10						
3	6	7	8						
2	5	5	11						
6	13	17	20						
5	15	25	40						
80	100	120	150						
45	65	25	145						
33	44	67	86						
1·5	2·5	7	10						
	0·25	0·25	0·5	0·75					
	0·2	0·8	0·9	0·3					
	0·4	0·4	0·4	0·8					
	15·9	15·8	15·7	31·9					
	100	400	500	400					
		7·2	9·9	7·6	15·7				
		399	502	728	1180				
		201	344	779	977				
			345	1356	1279	2057			
			999	777	1332	2997			
				0	5	8	–20	1	1-
					19	39	28	16	–39
					–4	–3	12	9	9
					0	0	0	0	10

Pythagoras' theorem

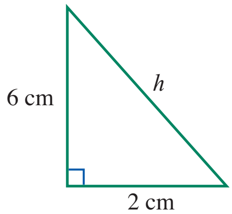
Pythagoras' theorem applies to right-angled triangles and states that 'the square on the hypotenuse equals the sum of the squares on the other two sides'.



$h^2 = a^2 + b^2$

Example

1 Find the value of the hypotenuse:

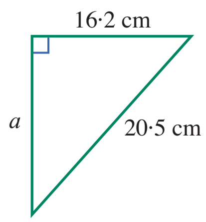


Solution

$$\begin{aligned} h^2 &= a^2 + b^2 \\ &= 6^2 + 2^2 \\ &= 36 + 4 \\ &= 40 \end{aligned}$$

$$h = \sqrt{40} = \sqrt{4 \times 10} = 2\sqrt{10}$$
$$\approx 6{\cdot}32 \text{ cm}$$

2 Find the value of the unknown side:

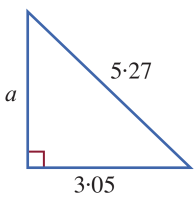


$$h^2 = a^2 + b^2$$
$$(20{\cdot}5)^2 = (16{\cdot}2)^2 + a^2$$
$$420{\cdot}25 = 262{\cdot}44 + a^2$$
$$a^2 = 420{\cdot}25 - 262{\cdot}44$$
$$a^2 = 157{\cdot}81$$
$$a = \sqrt{157{\cdot}81}$$
$$a = 12{\cdot}56 \text{ cm}$$

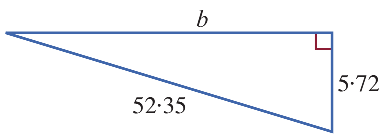
Exercise 4I

1 Find the value of the unknown side in each triangle, expressing your answer to 2 decimal places where necessary (all measurements are in centimetres):

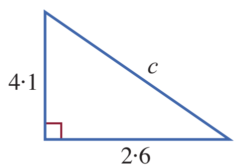
a



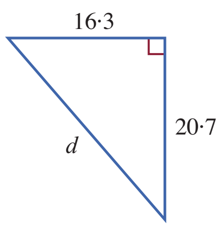
b



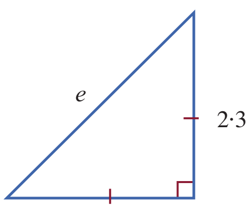
c



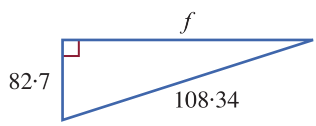
d

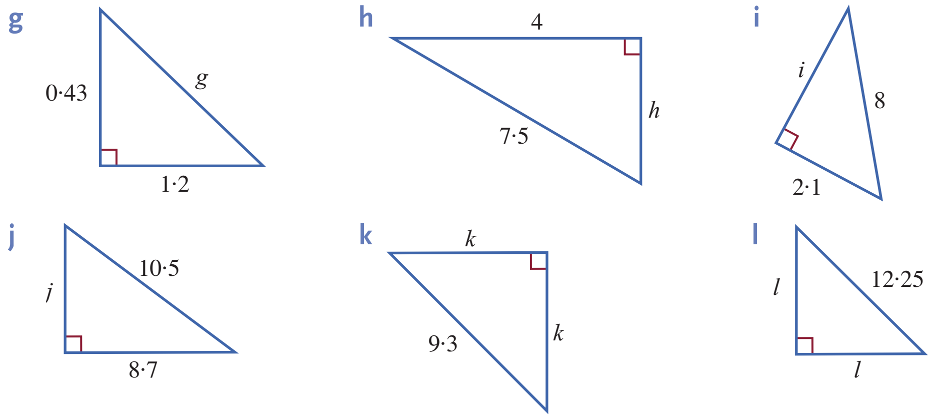


e



f





2 A power pole has a cable attached 15 metres up to provide support. The cable is attached to the ground 6 metres from the base of the pole. Find the length of the cable.

3 A 6·5-metre ladder is placed with its base 1·3 metres from a wall.

a How high up the wall will the ladder reach?

b If the foot of the ladder is moved 0·2 metres closer to the wall, how high up the wall does the ladder now reach?

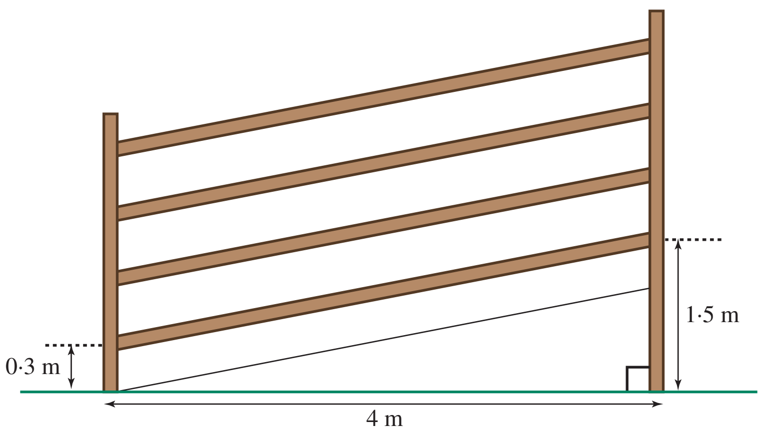
c How much higher up the wall does the ladder reach?

4 A farmer wishes to put four rails on his new cattle ramp. Give your answers to the nearest millimetre.

a Find the length of one rail.

b Find the total length of the rails for one side of the ramp.

c Find the total length of rails for both sides of the ramp.



5 The school council needs to have ramps built over the steps at each of the building exits, to accommodate a student in a wheelchair.

a Draw a diagram and calculate the length of the ramp in each case:

i The junior school building is 35 cm off the ground and has steps that reach out 50 cm.

ii The art and woodwork building is 80 cm off the ground and the last step is 1 m from the doorway.

b Draw a diagram and calculate the distance of the ramp from the building:

i The western entrance is 1·8 metres high and the ramp would need to be 6·95 m long.

ii The eastern entrance is 1·15 metres high and the ramp would need to be 5·5 m long.

4J Rearranging formulas

A **formula** is an **equation** showing the relationship between two or more variables or unknowns. It is written with one variable on the left-hand side of the equation and all other variables on the right-hand side. The variable on the left-hand side is called the **subject** of the formula.

When the variable required is not the subject of the formula, we need to rearrange the equation.

We need to use the correct order of operations to rearrange the formula correctly.

Example | **Solution**

1 The formula for finding the velocity, v, of an object after accelerating over a distance d is $v^2 = u^2 + 2ad$.

a Rearrange the formula to make a the subject.

$$v^2 = u^2 + 2ad$$
$$v^2 - u^2 = u^2 + 2ad - u^2$$
$$v^2 - u^2 = 2ad$$
$$\frac{v^2 - u^2}{2d} = \frac{2ad}{2d}$$
$$\frac{v^2 - u^2}{2d} = a$$
$$a = \frac{v^2 - u^2}{2d}$$

b Find a when $v = 5$, $u = 1$ and $d = 2$.

Substitute the known values into the rearranged formula.

$$a = \frac{v^2 - u^2}{2d}$$
$$a = \frac{5^2 - 1^2}{2 \times 2}$$
$$a = \frac{25 - 1}{4}$$
$$a = \frac{24}{4}$$
$$a = 6$$

2 Rearrange the formula in each of the following to make the variable in brackets the subject of the formula.

a $A = \pi r^2 \quad (r)$

$$A = \pi r^2$$
$$\frac{A}{\pi} = r^2$$
$$r = \sqrt{\frac{A}{\pi}}$$

b $T = 2\pi\sqrt{\frac{l}{g}} \quad (l)$

$$T = 2\pi\sqrt{\frac{l}{g}}$$
$$\frac{T}{2\pi} = \sqrt{\frac{l}{g}}$$
$$\frac{T^2}{(2\pi)^2} = \frac{l}{g}$$
$$l = \frac{T^2 g}{4\pi^2}$$

Exercise 4J

1 a i Rearrange the formula to make k the subject if $P = mk$.
 ii Find k when $P = 32$ and $m = -4$.
 b i Rearrange the formula to make b the subject if $y = mx + b$.
 ii Find b when $y = 9$, $m = 5$ and $x = 2$.
 c i Rearrange the formula to make h the subject if $C = ah - g$.
 ii Find h when $C = 19$, $a = -24$ and $g = -7$.
 d i Rearrange the formula to make x the subject if $a = mx + c$.
 ii Find x when $a = -21$, $m = 4$ and $c = 3$.
 e i Rearrange the formula to make p the subject if $k = \dfrac{mp - 2}{n}$.
 ii Find p when $k = 42$, $m = -4$ and $n = 5$.
 f i Rearrange the formula to make w the subject if $u = \dfrac{w}{v} - xy$.
 ii Find w when $u = 6$, $x = -3$, $y = 7$ and $v = -2$.

2 Rearrange the formula in each of the following to make the variable in brackets the subject of the formula. (Assume all variables are positive.)

a $V = \pi r^2 h$ (r) b $E = \dfrac{1}{2}mv^2$ (v) c $F = \dfrac{dh^2}{k}$ (h)

d $k = 7\sqrt{\dfrac{m}{n}}$ (m) e $d = 2\sqrt{\dfrac{b}{c}}$ (b) f $m = \mathrm{x}\sqrt{\dfrac{y}{z}}$ (y)

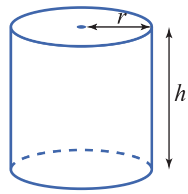
3 Rearrange each of the following to make x the subject.

a $x + y = z$ b $x - 3a = 2b$ c $kx - m = n$
d $dx + c = f$ e $rt - x = p$ f $vy - x = {}^{-}w$
g $\dfrac{2x + y}{p} = k$ h $\dfrac{t + rx}{w} = m$ i $\dfrac{mp + px}{m} = n$
j $cdx + e = gh$ k $axp - k = mn$ l $s - tux = vw$
m $b + \dfrac{xd}{y} = f$ n $\dfrac{mx}{n} + kp = t$ o $ab - \dfrac{dx}{c} - f = o$

4 The formula for finding the velocity, v m/s of an object starting with a velocity of u m/s and undergoing a constant acceleration of a m/s^2 over time t s is $v = u + at$.
 a Rearrange the formula to make t the subject.
 b Find the value of t when:
 i $a = 14$ m/s^2, $u = 20$ m/s and $v = 76$ m/s
 ii $a = 2{\cdot}4$ m/s^2, $u = 5{\cdot}7$ m/s and $v = 12{\cdot}9$ m/s

5 The circumference (C) of a circle is related to the radius (r) by the formula $C = 2\pi r$. Use the calculator value for π and give your answer correct to two decimal places.
 a Rearrange the formula to make r the subject.
 b Find the value of r when:
 i $C = 628$ ii $C = 16{\cdot}4$

6 The formula for the volume (V) of a cylinder is $V = \pi r^2 h$. Rearrange the formula to find h when $r = 5$ and $V = 157$. Use the calculator value for π and give your answer correct to two decimal places.



7 The perimeter of a rectangle is given by $P = 2(l + w)$, where l and w represent the length and width of the rectangle, respectively.

Rearrange the formula to make w the subject. Find the width of the rectangle if the length is 23 m and the perimeter is 84 m.

8 The area of a trapezium is given by the formula $A = \frac{(a+b)h}{2}$.

- **a** Rearrange the formula to make b the subject.
- **b** Use this rearranged formula to find the value of b if $a = 9$, $h = 6$ and $A = 66$.
- **c** We could substitute these values for a, h and A into the original formula and solve the equation $66 = \frac{(9+b)\times 6}{2}$ to find the value of b. Explain why would we rearrange the formula to find the value of b.

9 Ruth has been rearranging the formula for the area of a trapezium: $A = \frac{(a+b)h}{2}$.

She has produced the following working while trying to make a the subject.

$$A = \frac{h(a+b)}{2}$$
$$2A = h(a+b)$$
$$\frac{2A}{h} = a + b$$
$$a = \frac{2A}{h} - b$$

Where has she made a mistake? Now rearrange the formula to make b the subject.

Using algebra to solve word problems 4K

A useful strategy to solving word problems is:

1. Read the question carefully to identify what type of answer is required
2. Identify suitable pronumerals for unknown values
3. Write the problem using the pronumerals in expressions or equations
4. Solve the equation(s)
5. Interpret the solution in the context of the problem.

Example / **Solution**

1 Find two consecutive even numbers that add to 74.

Examples of consecutive even numbers are 4 and 6, 22 and 24 or 148 and 130.
Let the numbers be p and q.
The algebraic problem is:
Find p and q when $p + q = 74$.
But $q = p + 2$ (consecutive even numbers)
Substituting the second equation in the first gives $p + p + 2 = 74$
Solving: $2p + 2 = 74$
$2p + 2 - 2 = 74 - 2$
$2p = 72$
$p = 36$
Since $q = p + 2$, then $q = 36 + 2 = 38$.
So the consecutive even numbers that add to 74 are 36 and 38.

2 I'm thinking of a whole number between 1 and 100.
When the number is doubled and 16 is subtracted, the first expression is $2t - 16$.
The answer is the same as when the number is the second expression is $\frac{t}{2} + 26$ halved and 26 is added.
What is the number?

Let the number be t.
The expressions are the same when:
$2t - 16 = \frac{t}{2} + 26$
$2t = \frac{t}{2} + 42$
$\frac{3t}{2} = 42$
$t = \frac{84}{3} = 28$
The original number must be 28.

3 The product of two consecutive even numbers is 288. Find the numbers.

Let the first number $= n$
and the second number $= n + 2$.
$n(n + 2) = 288$
$n^2 + 2n - 288 = 0$
$(n - 16)(n + 18) = 0$
$n - 16 = 0$ or $n + 18 = 0$
$n = 16$ or $n = -18$
If $n = 16$, the two numbers are 16 and 18.
If $n = -18$, the two numbers are -18 and -16.

Example

4 A road sign is 5 cm longer than it is wide. If its area is 750 cm^2, find the dimensions of the sign.

Solution

Let length = x and width = $x - 5$.

$x(x - 5) = 750$
$x^2 - 5x - 750 = 0$
$(x - 30)(x + 25) = 0$
$x - 30 = 0$ or $x + 25 = 0$
$x = 30$ or $x = -25$

The length cannot be a negative number, so the dimensions are:

Length = 30 cm
Width = 30 – 5 = 25 cm

Exercise 4K

1 Use algebra to solve these word problems:

- **a** Find two consecutive whole numbers that add to 17
- **b** Find two consecutive whole numbers that sum to 83
- **c** Find three consecutive numbers that add to 57
- **d** Find four consecutive numbers that add to 258

2 Use algebra to solve these word problems:

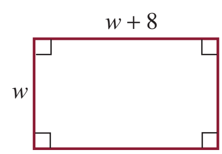
- **a** Find two consecutive even numbers that add to 70
- **b** Find three consecutive even numbers that sum to 204
- **c** Find three consecutive odd numbers whose sum is 477
- **d** Find ten consecutive numbers that total 55.

Use quadratic equations to help you solve the following problems.

3 The product of two consecutive numbers is 240. Find the numbers.

4 The product of two consecutive odd numbers is 323. Find the numbers.

5 The dimensions of a rectangular label are such that its length is 8 centimetres more than its width. If the area of the label is 20 square centimetres, find the dimensions of the label.



6 The length of a rectangle is 5 m longer than its width. If the area of the rectangle is 104 m^2, find its width and length.

7 Imagine that you have a piece of string. How many pieces will you have if you cut the string once? If you cut one of the pieces of string again, how many pieces will you have? Check your answer using some string and scissors, and then complete the following table:

Number of cuts	0	1	2	3	4
Total number of pieces	1	2			

Use pronumerals and algebra to calculate how many cuts would result in a total of 23 pieces of string.

8 Imagine you have a piece of string and this time you fold it in half before you cut it. Repeat the process for the next cut for one of the pieces. Check your answer with string and scissors and complete the following table:

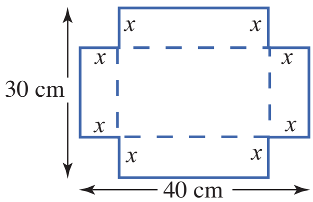
Number of cuts	0	1	2	3	4
Total number of pieces	1	3			

Use pronumerals and algebra to calculate how many cuts would result in a total of 53 pieces of string.

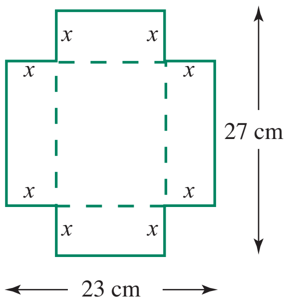
9 The greatest number of pieces a pizza can be cut with one straight cut is 2. Draw diagrams to show that the greatest number of pieces a pizza can be cut with two straight cuts is 4, and for three straight cuts is 7 pieces. Jemima cuts a pizza into 22 pieces using only straight cuts. How many straight cuts did Jemima make to produce 22 pieces?

10 A group of Year 9 learners from different schools attend a chess competition. Before they start playing chess they are asked to shake hands with each other. A teacher counts the total number of handshakes as 66. How many chess players attended the competition? Complete a table and use the pattern to solve this problem.

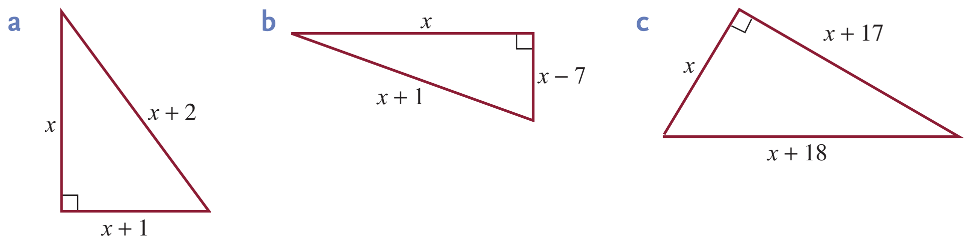
11 Find the dimensions of the box of base area 600 cm^2 which is formed by folding along the dotted lines in the diagram:



12 Find the dimensions of the box of base area 480 cm^2 which is formed by folding along the dotted lines in the diagram:



13 Find the value of x in the following triangles:



Puzzles

1 Expand the expressions below, then match the letter to the number of the corresponding expanded expression to find the title of the puzzle:

A	$(x+1)^2$	**1**	$-18x^2 - 24x + 24$
B	$(x-3)^2$	**2**	$x^2 - 25$
C	$(x+5)(x-5)$	**3**	$2x^2 - 10x + 12$
D	$(1-x)(1+x)$	**4**	$x^2 + 2x + 1$
E	$(4x-3)^2$	**5**	$-x^2 - 5x + 6$
G	$(x-2)^2 - 5$	**6**	$x^2 - 6x + 9$
L	$(2x-1)(3x+5)$	**7**	$6x^2 + 7x - 5$
M	$-(x+6)(x-1)$	**8**	$16x^2 - 24x + 9$
R	$2(x-3)(x-2)$	**9**	$1 - x^2$
S	$-3(2x+4)(3x-2)$	**10**	$x^2 - 4x - 1$

___	___	___	___	___	___	___	___	___
1	2	3	4	5	6	7	8	9
___	___	___	___					
8	10	10	1					

GESG

2 Factorise the expressions below, then match the letter to the number of the corresponding expanded expression to find the title of the puzzle:

B	$x^2 + 7x + 12$	**1**	$(x+1)(x+2)$
D	$x^2 - 7x + 12$	**2**	$(x-3)(x-2)$
E	$x^2 + x - 6$	**3**	$(x-4)(x+2)$
L	$x^2 + 3x - 10$	**4**	$(x-2)(x+3)$
I	$x^2 - 5x + 6$	**5**	$(x-3)(x-4)$
M	$x^2 + 3x + 2$	**6**	$(x-5)(x+2)$
O	$x^2 - 3x + 10$	**7**	$(x+5)(x+2)$
S	$x^2 + 6x + 8$	**8**	$(x+3)(x+4)$
U	$x^2 + 7x + 10$	**9**	$(x+5)(x-2)$
X	$x^2 - 2x - 8$	**10**	$(x+4)(x+2)$

OBULEDS
DBOULES
ULDSEBO
SLOBUED

___	___	___	___	___		
1	2	3	4	5		
___	___	___	___	___	___	___
5	6	7	8	9	4	10

3 Solve the following equations, then match the letter to the number of the correct solution to find the title of the puzzle below:

MORE
IT IT
THANI

A $x(x + 1) = 0$	**E** $x(x + 5) = 0$	**H** $x(x + 2) = 0$
I $x(x - 5) = 0$	**N** $x^2 - 3x = 0$	**M** $x^2 - x = 0$
O $x^2 = 2x$	**R** $x^2 = 6x$	**T** $x^2 - 9 = 0$
S $x^2 - 1 = 0$	**Y** $x^2 = 25$	

0, 1	0, 2	0, 6	0, –5		±3	0, 2		0, 5	±3
±3	0, –2	0, –1	0, 3		0, 1	0, –5	0, –5	±3	±1
±3	0, –2	0, –5		0, –5	–5, 5	0, –5			

4 Solve the following equations, then match the letter to the number of the correct solution to find the title of the puzzle below:

HOLIDAY
CCCCC

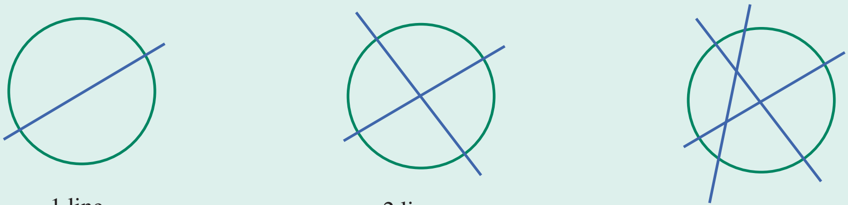
A $x^2 + 3x + 2 = 0$	**D** $x^2 - 5x - 6 = 0$	**E** $x^2 - 4x + 3 = 0$
H $x^2 - 3x + 2 = 0$	**I** $x^2 + 3x = 4$	**L** $x^2 - 3x = 4$
O $x^2 - 3x = 10$	**R** $x^2 + 3x = 10$	**S** $x^2 - 7x + 12 = 0$
V $x^2 + 7x + 12 = 0$	**Y** $x^2 - 7x + 10 = 0$	

1, 2	–2, 5	–1, 4	–4, 1	–6, 1	–1, –2	2, 5	
–2, 5	–3, –4	3, 1	2, –5	3, 4	3, 1	–1, –2	3, 4

Applications

Circle regions

a Investigate the relationship between the number of lines (n) drawn through a circle and the maximum number of regions (R) created.



1 line, 2 regions — 2 lines, 4 regions — 3 lines, 7 regions

b Record your observations in a table like this one.

c Can you find a relationship between the number of lines (n) and the number of regions (R) created? Write down the relationship you have discovered.

d Use technology to sketch an accurate graph of this relationship. What do you notice about the shape of the graph?

Number of lines (n)	Regions (R)
0	
1	
2	
3	
4	
5	
6	

Handshakes

There are 15 learners in Emily's Physics class. At the beginning of the school year the teacher asks each learner to shake the hand of each of the other learners in the class.

a Investigate the relationship between the number of learners in the class (n) and the total number of handshakes (H).

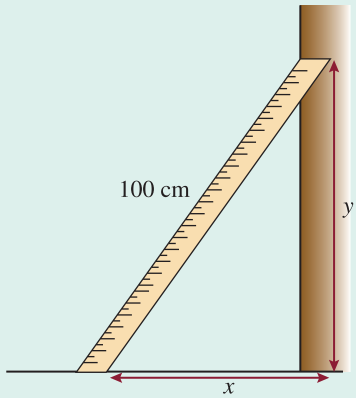
b Record your results in a table and see if you can establish a mathematical relationship between the two.

c Use technology to sketch a graph of this relationship.

Number of learners (n)	Handshakes (H)
0	
1	
2	
3	
4	
5	
6	

Investigating Pythagoras

A ruler of length 100 cm rests against a vertical wall, with its foot x cm out from the wall and the top reaching y cm up the wall. The formula relating x to y is $x^2 + y^2 = 100^2$.



a Draw a careful diagram showing the situation with lengths x and y clearly marked.

b Rearrange the formula to make y the subject.

c Complete the following table, which gives y for various values of x.

x	0	10	20	30	40	50	60	70	80	90	100
y											

d On a suitably scaled set of axes of your own, plot y against x, joining the points with a smooth curve. Plot x horizontally and y vertically. Don't forget to label and scale your axes.

e What shape graph do you get in part d?

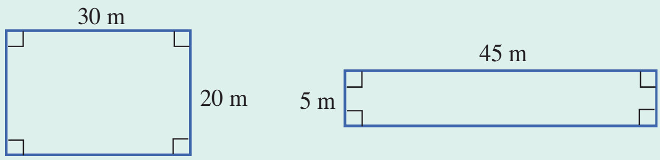
f Finally rearrange the formula to make x the subject.

Optimisation

A farmer has 100 m of fencing to build an enclosure for a family of pigs he is raising.

He wants to build an enclosure that has the largest area possible. To do this he uses a spreadsheet to substitute some values.

Here are two of the enclosures that are possible:



a Draw two more possible enclosures with side lengths that are whole numbers.

Generalise the problem:

If l = length of the enclosure (m)
w = width of the enclosure (m)
$P = 100$ m
$100 = 2l + 2w$
$50 = l + w$
$w = 50 - l$
Area $= l \times w$
$A = l(50 - l)$

Length	Width	Area	Perimeter
1	=50–A2	=A2*B2	=2(A2+B2)

Length	Width	Area	Perimeter
1	49	49	100
2	48	96	100
3	47	141	100
4	46	184	100

b Set up a table or a spreadsheet using the formulas shown to find the possible areas for the enclosure, using whole numbers.

c What is the maximum area possible?

d Investigate what happens if you increase the length in 0·1 cm increments. Does this change the maximum area?

e Draw a graph of the data, with area on the y-axis and length on the x-axis. Describe the shape of the curve.

f Write a report of your findings and make a recommendation to the farmer.

Enrichment

Factorising trinomials with a coefficient of x^2 greater than 1

The examples dealt with so far have had a coefficient of 1 for x^2, for example, $x^2 + 9x + 20$. We should also consider expressions with coefficients other than 1, such as $2x^2 - 7x - 15$.

Example

1 Factorise $2x^2 - 7x - 15$.

Solution

$2x^2 - 7x - 15$

What multiplies to give $2x^2$?
$2x$ and x

What are the factors of -15?
$+3$ and -5

$(2x \quad +3)$
$(x \quad -5)$

Cross multiply to check
$-10x + 3x = -7x$
So, $2x^2 - 7x - 15 = (2x - 5)(x + 3)$

1 Factorise:

a $2x^2 + 5x + 2$
b $3x^2 + 4x + 1$
c $4x^2 + 8x + 3$
d $2x^2 + 13x + 20$
e $2x^2 + 9x + 7$
f $3x^2 - 11x + 6$
g $2x^2 - 7x + 3$
h $5x^2 - 11x + 2$
i $4x^2 - 7x - 2$

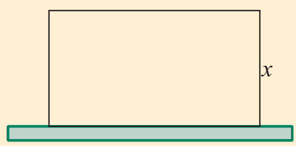
2 The height above the ground (in metres) of a model rocket after t seconds, is given by the rule $h = -16t^2 + 96t - 2$. Find the time(s) when the height of the rocket is 126 m.

3 The height above the ground (in metres) of a cricket ball after t seconds, is given by the rule $h = 80t - 5t^2$.

a Find the time(s) when the ball is on the ground.
b Find the time(s) when the height is 320 m.

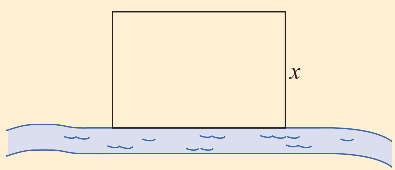
4 The temperature T°C in a greenhouse t hours afte dusk is given by the rule $T = 0{\cdot}25t^2 - 5t + 30$. Find the time when the temperature first falls to 21°C.

5 A gardener has 50 m of fencing to enclose a rectangular garden plot on three sides. The fourth side of the plot is enclosed by a hedge.



a Find the rule for the area of the plot.
b Find the dimensions of the plot if the area is 300 m^2.

6 A paddock that runs alongside a straight stretch of river is to be enclosed on the other three sides by 120 m of wire fencing.



a Find the rule for the area of the paddock.
b Find the dimensions of the paddock if the area is 1800 m^2.

Adding and subtracting algebraic fractions

When adding and subtracting algebraic fractions, find the common denominator.

Example

2 Simplify:

a $\frac{x+2}{3}+\frac{x}{4}$

b $\frac{x+4}{4}-\frac{3}{x-1}$

Solution

a
$$\begin{aligned}\frac{x+2}{3}+\frac{x}{4} &= \frac{4(x+2)}{4(3)}+\frac{3(x)}{3(4)}\\ &= \frac{4x+8}{12}+\frac{3x}{12}\\ &= \frac{7x+8}{12}\end{aligned}$$

b
$$\begin{aligned}\frac{x+4}{4}-\frac{3}{x-1} &= \frac{(x+4)(x-1)}{4(x-1)}-\frac{4(3)}{4(x-1)}\\ &= \frac{x^2+3x-1}{4x-4}-\frac{12}{4x-4}\\ &= \frac{x^2+3x-13}{4x-4}\end{aligned}$$

7 Simplify:

a $\frac{x+1}{2}+\frac{2}{3}$ b $\frac{x-1}{3}+\frac{1}{4}$ c $\frac{x+3}{5}-\frac{1}{2}$

d $\frac{x+1}{2}+\frac{x-1}{3}$ e $\frac{x+2}{3}-\frac{x-2}{4}$ f $\frac{x+2}{2}-\frac{x-4}{5}$

g $\frac{2(x+1)}{3}+\frac{1}{2x}$ h $\frac{7}{3x}+\frac{5(x+2)}{(x+3)}$ i $\frac{10}{4x}+\frac{x+1}{3(x-2)}$

8 Simplify:

a $\frac{x+1}{2}+\frac{2}{x+3}$ b $\frac{x-1}{3}+\frac{1}{x-2}$ c $\frac{x+3}{5}-\frac{6}{x+4}$

d $\frac{x+1}{2}+\frac{x+1}{x-1}$ e $\frac{x-2}{x+3}+\frac{x-3}{4}$ f $\frac{x+2}{2}-\frac{x-4}{x-2}$

g $\frac{x+3}{4}+\frac{x}{2}+\frac{1}{3}$ h $\frac{x+1}{2}+\frac{x}{3}+\frac{x}{5}$ i $\frac{x-3}{5}-\frac{2-x}{4}+2$

When multiplying algebraic fractions we try to factorise and cancel first.

9 Multiply:

a $\frac{x^2-4}{x-2}\times\frac{(x+3)}{(x+2)}$ b $\frac{9-b^2}{3+b}\times\frac{8-b}{3-b}$ c $\frac{144-x^2}{12-x}\times\frac{12+x}{8-x}$

d $\frac{x+3}{x-2}\times\frac{2x-4}{x^2-9}$ e $\frac{4x^2+12x+9}{x-5}\times\frac{x^2-25}{2x+3}$ f $\frac{64-m^2}{3x-2y}\times\frac{9x^2-12xy+4y^2}{8-m}$

Exercise 4A

1 For each of the following:

i $3b + 22t = -5t + 10r + 7$ ii $12j - 9b + 34k - 11j + 7g$

a identify whether this is an equation or an expression
b write down the coefficient of b
c write down all the terms in the equation or expression
d list the variables
e write down any constants.

2 Fred has c cassava and p potatoes. Rita has four times as many cassava and 3 fewer potatoes than Fred.

a Write an expression for the number of cassava and potatoes Fred has altogether.
b If Fred has 8 cassava and potatoes to start with, write an equation to show this information.
c Write an expression for the number of cassava Rita has.
d Write an expression for the number of potatoes Rita has.
e If Rita has 14 cassava and rings altogether, write an equation to show this information.
f If Fred loses 2 of his potatoes, write an expression in terms of c and p to show how many cassava and potatoes Fred has now.

Exercise 4B

3 Simplify:

a $4r + 5r$ b $5t + 13t$ c $6a + 18a$
d $14m + 5m + 3m$ e $15y + 13y + y$ f $8m + 3b + 7m$

Exercise 4C

4 Expand and simplify the following expressions:

a $2(x + 4)$ b $3(b - 2)$ c $3(k - 5)$
d $-5(m + 2)$ e $-2(m + 3)$ f $-2(b - 4)$

5 Expand and simplify the following expressions:

a $2(r + 7) + 13(r + 5)$ b $5(x + 3) + 2(x + 1)$ c $9(q + 6) - 3(q + 10)$
d $3(r - 5) + 3(r + 7)$ e $12(r + 10) - 3(r - 9)$ f $-3(c + 2) - (c - 3)$

6 Expand and simplify the following expressions:

a $(q + 5)(q + 7)$ b $(g + 1)(g + 9)$ c $(m + 3)(m - 8)$
d $(w + 2)(w - 4)$ e $(t - 10)(t + 2)$ f $(y - 8)(y - 2)$

Exercise 4D

7 Expand and simplify the following expressions by using the difference of two squares identity:

a $(d + 12)(d - 12)$ b $(f + 13)(f - 13)$ c $(m - 15)(m + 15)$
d $(2c + 3)(2c - 3)$ e $(6g - 9)(6g + 9)$ f $(25k + 2)(25k - 2)$

8 Expand and simplify the following expressions by using the perfect squares identity:

a $(d-3)^2$ b $(2+h)^2$ c $(7h-3)^2$

d $(5+2w)^2$ e $(4r-2)^2$ f $(3+7t)^2$

Exercise 4E

9 Factorise the following expressions by first taking out the highest common factor:

a $2x-12$ b $3x^2-5x$

c $13d^3+169d^2-39d$ d $5abc-25a^2bc+625ab^2c$

e $2m^2+14m+24$ f $10m^2-40m+30$

Exercise 4F

10 Factorise the following expressions by using the difference of two squares identity:

a b^2-15^2 b a^2-49 c $64-x^2$

d $36-n^2$ e $100x^2-y^2$ f $1-81y^2$

Exercise 4G

11 Factorise the following expressions by using the perfect squares identity:

a $s^2+2sy+y^2$ b $p^2-2pm+m^2$ c $w^2-6wy+9y^2$

d $q^2-8qh+16h^2$ e $4a^2+20at+25t^2$ f $16w^2+96wp+144p^2$

Exercise 4H

12 Evaluate the following expressions when $a=3$ and $b=6$:

a $a+2b$ b $2a+5b$ c $10ab-a$

d $12a^2$ e $2b^3$ f $2a^3-2b^3$

g $\dfrac{144a^2}{3b^3}$ h $a^3+2b^3-\dfrac{b}{2}$ i $3b^2+\dfrac{a^5}{2}-5$

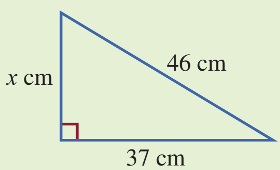
13 Evaluate these expressions when $d=-2$ and $e=5$:

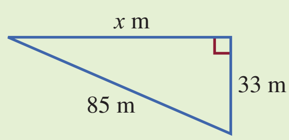
a $-3de-6$ b $-5de+100$ c $-8de+15$

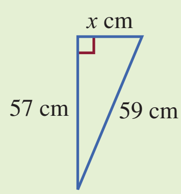
d $\dfrac{10}{e}$ e $\dfrac{40}{d}$ f $\dfrac{e}{d}$

Exercise 4I

14 Use Pythagoras' theorem to find the value of x. Write your answer correct to two decimal places if necessary:

a


b


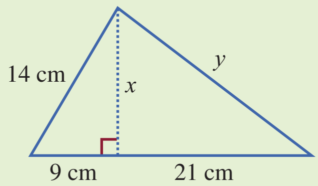
c


15 Find the value of c in each of the following Pythagorean triples:

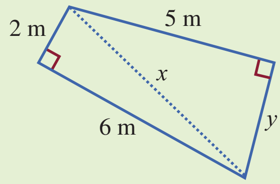
a $a=12, b=16, c=?$ b $a=45, b=200, c=?$ c $a=132, b=385, c=?$

16 Find the exact value of the variables in the following diagrams:

a



b



Exercise 4J

17 The area of a trapezium is given by the formula $A = \dfrac{(a+b)h}{2}$.

- **a** Rearrange the formula to make b the subject.
- **b** Use this rearranged formula to find the value of b if $a = 9$, $h = 6$ and $A = 66$.
- **c** We could substitute these values for a, h and A into the original formula and solve the equation $66 = \dfrac{(9+b)\times 6}{2}$ to find the value of b. Explain why we would rearrange the formula to find the value of b.

Exercise 4K

18
- **a** A rectangular lawn is such that its length is 10 metres more than its width. If the area of the lawn is 200 square metres, find its dimensions.
- **b** A right-angled triangle has sides of lengths x, $x + 1$ and $x - 31$. Find the dimensions of the triangle and calculate its area.

CHAPTER 5

Ratios and Rates

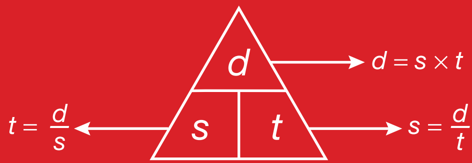
A ratio helps us relate two or more quantities. Air is about 21% oxygen and 78% nitrogen (plus very small amounts of other chemicals). So the ratio of oxygen to nitrogen is 21 parts to 78 parts. We write this as 21:78. Here we are thinking of the volume of each gas. Since the capacity of an average woman's lungs is 4.2 litres, she is likely to breathe 0.88 litres of oxygen and 3.28 litres of nitrogen when filling her lungs with air.

Sometimes we relate two amounts by comparing weights. An example of a weight ratio for sea water is: a kilogram of sea water holds about 35 grams of salt in 965 grams of fresh water, so the ratio of salt to freshwater is 35:965. We can make this ratio simpler by dividing each number by 5 to get 7:193, which is the same ratio in smaller numbers. This tells us that we can also think of seawater as made up of 7 parts of salt to 193 parts of fresh water.

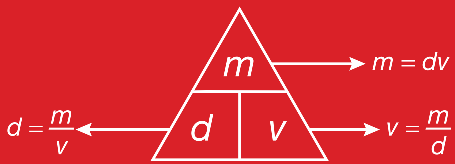
This chapter covers the following skills:

- Simplifying ratios
- Using ratios to find quantities
- Increasing and decreasing in a given ratio
- Applying ratios and rates

  Distance, time and speed



  Density



- Calculating rates of change
- Exploring distance–time graphs

Specific Learning Outcome (SLO)

Learners should be able to:

9.5.1.1 Define 'ratio'.

9.5.1.2 Interpret the use of the symbol for ratio " : ".

9.5.1.3 Express quantities as a ratio or as a fraction.

9.5.2.1 Express quantities as a ratio in its simplest form.

9.5.3.1 Find missing numbers or quantities for given ratios.

9.5.4.1 Share quantities using given ratios.

9.5.4.2 Solve practical word problems involving ratios.

9.5.5.1 Increase or decrease quantities by multiplying them with given ratios expressed as fractions.

9.5.6.1 Define 'rate'.

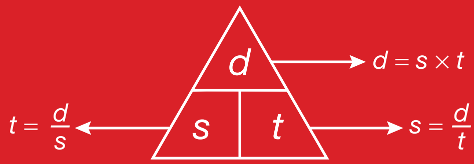
9.5.6.2 Express rates in fraction form and then simplify them.

9.5.6.3 Calculate quantities per unit.

9.5.7.1 Calculate quantities using given rates.

9.5.8.1 Convert units of rates from one unit to another.

9.5.9.1 Explain different formulas that can be used to find the rates of: *speed*, *time* and *distance*.

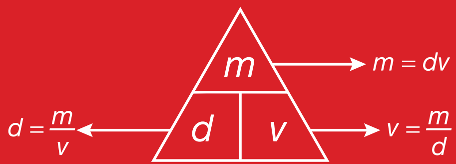


9.5.9.2 Identify appropriate units for speed rates.

9.5.9.3 Apply speed, time and distance formulas to practical problems.

9.5.10.1 Define 'density'.

9.5.10.2 Calculate density, mass and volume of various objects and materials using the appropriate formulas.



9.5.11.1 Name and label the features of a distance–time graph.

9.5.12.1 Label the axes with of a distance–time graph with suitable scales.

9.5.12.2 Plot distance–time graphs and use them to calculate speed, distance and time for given measurements.

9.5.13.1 Identify the gradient (slope) of a linear graph.

9.5.13.2 Identify the component parts for a gradient:

$$\text{Gradient} = \frac{\text{Rise}}{\text{Run}}$$

9.5.13.3 Calculate the speed of an object from a graph by measuring the gradient.

9.5.14.1 Calculate rates of change from a distance–time graph, and identify when the speed is increasing or decreasing.

9.5.15.1 Draw tangents to curved line graphs at given points on the curved line to determine rates of change.

9.5.15.2 Define 'Average Rate of Change'.

9.5.15.3 Define 'instantaneous'.

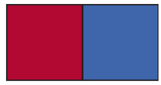
9.5.15.4 Calculate rates of change for given measurements.

5A Fractions, ratios and simplifying

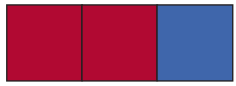
A **ratio** is a comparison of two or more quantities in which we look at how two quantities are related to each other. Ratios are usually written in the form $a:b$ but can also be written as $\frac{a}{b}$.

All the parts of the ratio need to be the same or expressed in the same unit so that they can be compared.

A ratio expressed as 1:1 indicates that the quantities being compared are of equal size. Here the ratio of the red area to the blue area is 1:1.



A ratio expressed as 2:1 indicates that the one part is twice the size of the other. Here the ratio of the red area to the blue area is 2:1.



Example / **Solution**

1 A bathroom is tiled with 45 red tiles and 315 blue tiles.

a Express this as a ratio using the smallest numbers possible (simplest terms).

$\div 45$ 45 : 315 $\div 45$
1 : 7

b If five such bathrooms were to be tiled, how many of each tile would be needed?

$45 \times 5 = 225$ red tiles
$315 \times 5 = 1575$ blue tiles

2 a Express the ratio 15:95 as a fraction and simplify it.

$\frac{15}{95} = \frac{3}{19}$

b Write the fraction as a ratio.

$\frac{3}{19}$ is the same as 3:19

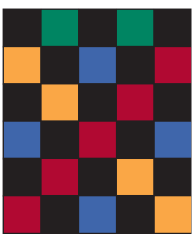
3 The weight of one coconut in a 50-kilogram bag of coconuts is 350 grams. Express this as a ratio in simplest form.

1 kg = 1000 g
$\frac{350}{50\,000} = \frac{35}{5000} = \frac{7}{1000}$
Ratio is 7:1000

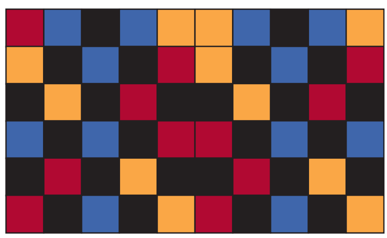
Exercise 5A

1 Express as a ratio the number of squares of each colour to the total number of squares in the following:

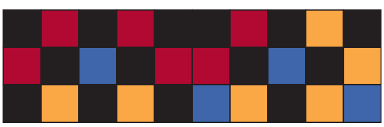
a



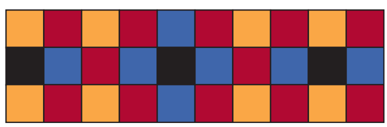
b



c



d



e

2 Simplify these ratios:

a 20:25	b 15:65	c 24:42	d 12:15	e 81:27
f 120:15	g 24:18	h 123:12	i 75:15	j 72:18

3 Express the following as the simplest ratios. Remember to convert units first:

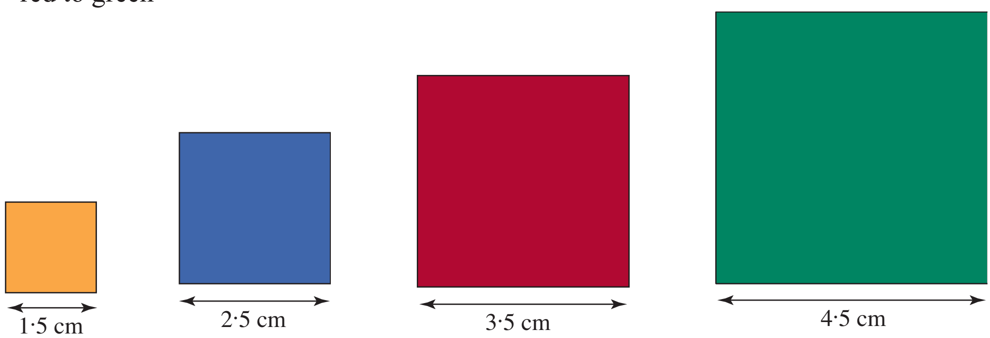
a 120 cm:1 m	b 28 mm:10 cm	c 132 m:11 cm
d 380 m:2·5 km	e 4 kg:250 g	f 720 g:1·2 kg
g 8·6 t:3450 kg	h 0·05 t:2000 g	i 5 min:120 s
j 480 s:2 min	k 3 h:20 min	l 5 days:60 h

4 Express the ratio of the areas of the following squares in simplest form:

a yellow to i blue ii red iii green

b blue to i red ii green

c red to green

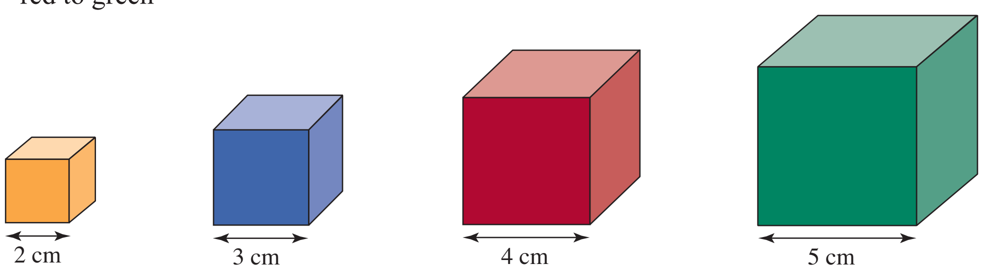


5 Express the ratio of the volumes of these cubes in simplest form:

a yellow to i blue ii red iii green

b blue to i red ii green

c red to green



5B Using ratios to find quantities

If we know the ratio between two or more quantities and the size of one quantity, it allows us to find the size of the other quantities.

Example

1 Find the missing number in this statement:

$2:x = 10:50$

Solution

$\div 5$

$2:\boxed{10} = 10:50 \qquad \therefore x = 10$

$\div 5$

2 Find the value of x to make the ratios the same, using algebra.

a $x:4{\cdot}5 = 2:3$

Solution

$$\frac{x}{4{\cdot}5} = \frac{2}{3}$$

$$\therefore 3x = 2 \times 4{\cdot}5$$

$$\therefore x = \frac{2 \times 4{\cdot}5}{3} = 3$$

b $3:x = 24:16$

Solution

$$\frac{3}{x} = \frac{24}{16}$$

$$\therefore 24x = 3 \times 16$$

$$\therefore x = \frac{3 \times 16}{24} = 2$$

3 Lollies are to be shared in the ratio of 3 : 2 : 1 between Carol, Sue and Jenny according to their ages. If Sue is 12 years old and receives 98 lollies, find:

a the ages of the other sisters

Solution

For Sue: $12 \div 2 = 6$
Carol: $3 \times 6 = 18$ years
Jenny: $1 \times 6 = 6$ years

b the number of lollies that each of the other sisters receive

Solution

For Sue: $98 \div 2 = 49$
Carol: $3 \times 49 = 147$ lollies
Jenny: $1 \times 49 = 49$ lollies

Exercise 5B

1 Find the missing numbers in these ratios:

a $5:\square = 15:24$ **b** $3:\square = 12:16$ **c** $\square:7 = 15:14$

d $5:7 = \square:28$ **e** $\square:12 = 48:96$ **f** $4:\square = 44:66$

g $1{\cdot}4:\square = 0{\cdot}84:1{\cdot}92$ **h** $2{\cdot}6:\square = 2{\cdot}34:2{\cdot}88$ **i** $5{\cdot}6:3{\cdot}8 = \square:15{\cdot}2$

2 Find the value of x to make the ratios the same, using algebra:

a $x:24 = 6:36$ **b** $x:9 = 3:27$ **c** $x:2{\cdot}5 = 12:15$

d $6:x = 18:21$ **e** $3:x = 12:20$ **f** $9:x = 0{\cdot}84:1{\cdot}92$

g $7:8 = x:24$ **h** $2:11 = x:99$ **i** $7{\cdot}5:8{\cdot}5 = x:1{\cdot}7$

j $18:5 = 6:x$ **k** $22:7 = 4:x$ **l** $2{\cdot}1:1 = 3:x$

3 The ratio of dogs to cats in a city pound is 2:3. If there are 36 cats, then find:

a the number of dogs

b the total number of animals in the pound

4 A cordial drink is made up by mixing syrup to water in the ratio 2:5. If 450 mL of water is used, find:

a the volume of syrup used **b** the total volume of the drink

5 The ratio of sand to cement when making mortar is 5:1. What weight of sand should be mixed with 1·5 kg of cement?

6 The winnings in the lottery are to be shared in the ratio in which Agatha, Bree and Christie contribute to the ticket. Agatha puts in \$3·80, Bree puts in \$4·80 and Christie puts in the rest to buy the \$12 ticket. How much will each receive if they win \$10 680?

7 The profits a company makes are distributed in the ratio of the number of hours that each partner works in a week. For a particular week Duy works for 65 hours, Effie for 55 hours and Gary for 45 hours. If the company makes a \$40 590 profit, how much will each partner receive?

8 A property is to be divided between three children in the ratio 1:2:3. If the area of the property is 73·8 hectares find the area that each will receive.

9 Young Will is trying to pour water into a bottle. For every 30 mL he pours, he spills 10 mL.

a Find the amount of water he will spill while filling a 600 mL container.

b Find the total amount of water that he needs to pour to fill an 800 mL container.

10 For every \$8 I earn I have to pay \$2·50 in taxes. If I earn \$132 160 in a part-time job, find:

a the amount of tax that I need to pay **b** the amount that I can keep

11 The ratio of staff to learners on an Outdoor Activities Program is 2:7. Find the maximum number of students who can be taken if 10 teachers go.

12 A two-stroke fuel is made by mixing petrol with oil in the ratio 10:1. If 5·5 litres of mixture is made, find the volume of petrol and oil used.

13 The money from petrol sales is shared between the government, refiners/wholesalers, oil producers and service station owners in the ratio 9:4:2:1. How much does each receive in a year if:

a the government receives \$1 130 670 from the operation of a Giza service station?

b a total of \$15 497 184 of petrol is sold in Auki?

14 In the search for a super pizza topping, 9 g of olives, 11 g of tomato sauce and 15 g of cheese are used to make a small trial pizza. How many grams of which ingredient needs to be added to the recipe so that the ratio of olives to tomato to cheese is 3:4:5?

5C Changing quantities in a given ratio

When a quantity is multiplied by a number that is greater than 1, the quantity will be increased.

When a quantity is multiplied by a number that is less than 1, the quantity will be reduced.

Example

1 Increase 40 metres in the ratio 3:2.

Solution

Increase by 3:2

Factor $= \frac{3}{2}$

$\therefore\ 40 \times \frac{3}{2} = 60$

2 Decrease $50 in the ratio 3:4.

Decrease by 3:4

Factor $= \frac{3}{4}$

$\therefore\ 50 \times \frac{3}{4} = \$37{\cdot}50$

3 If a 700 mL can of drink is decreased by one-fifth, find the new volume of the drink.

New fraction of drink $= 1 - \frac{1}{5} = \frac{4}{5}$

New volume of drink $= \frac{4}{5} \times 700 = 560$ mL

4 If Wally's wage of $240 per week is increased by three-eighths, find his new wage.

New fraction of wage $= 1 + \frac{3}{8} = \frac{11}{8}$

New wage $= \frac{11}{8} \times 240 = \$330$

Exercise 5C

1 If each of the following is increased in the ratio of 5:2, find the new value:

a 60 metres
b 80 centimetres
c 45 grams
d 12 kilograms
e $456
f 48 minutes
g 24 hours
h 1000 millimetres
i 98 tonnes
j $25·20
k 78 seconds
l 234 metres
m 905 tonnes
n 67 kilometres
o 46 grams

2 If each of the following is decreased in the ratio 1:3, find the new value:

a 120 metres
b 981 millimetres
c $23·10
d 471 kilograms
e 48 hours
f 63 tonnes
g $78·30
h 1005 grams
i 2556 minutes
j 37·74 centimetres
k $1758
l 75 hours
m 42 kilometres
n 291 tonnes
o 288 metres

3 What fraction remains when quantities are reduced by the following amounts?

a one-fifth
b two-thirds
c five-eighths
d four-sevenths
e three-quarters
f two-elevenths

4 What fraction of the original quantity is there if it is increased by the following amounts?

a one-seventh b one-eighth c two-thirds
d five-ninths e two-fifths f three-quarters

5 The following blocks of cheese are decreased by two-thirds. Find their new weights:

a 120 g b 567 g c 981 g d 1002 g e 465 g

6 In order to attract more members, the committee of a tennis club reduces subscriptions by one-seventh. Find the new subscriptions payable by the following club members:

	Old subscription	New subscription
President	\$1610	
Senior	\$1820	
Committee member	\$1680	
Junior	\$910	

7 A local real-estate agent claims that the property prices in Flemington have increased by one-sixth. Use this estimate to calculate the value of the following properties:

	Old value	New value
Two-storey villa unit	\$5 340 000	
Flat	\$1 410 000	
Apartment	\$4 680 000	
Single-storey house	\$5 940 000	
Double-storey house	\$3 375 000	
Shop front	\$1 335 000	

8 A soft-drink manufacturer decides to decrease the size of its products by one-tenth. Find the new volumes of drinks if the old volumes were:

a 1200 mL b 1500 mL c 2600 mL d 5 litres e 20 litres

9 The lengths of the rides at the fabulous Platinum Coast theme park are to be increased by two-sevenths. Find the new lengths of the following rides:

	Old length	New length
Bully Dipper	413 m	
Spin ’n’ Scream	441 m	
Water Spout of Terror	378 m	
Cannon Shot	448 m	

5D Rates of change

A **rate** is a measure of the way in way one quantity changes with respect to another. Rates are stated using units in fraction form. For example, speed is the rate of change of distance with respect to time measured in km/h or m/s. Other familiar rates might the price of fish in $ per kilogram, or the rate at which a pot of cassava heats up in °C per minute.

Example

1 Express the following as rates:

a William grew 42 cm in 2 years.

b 45 litres of petrol cost $553·50.

c 17 canoes carried 714 people.

2 A pot of rice is heated up at a constant rate of 8°C per minute. If it starts at 16°C, how long will it take to reach:

a 24°C?

b 20°C?

c 38°C?

3 If 9 kilograms of gas costs $112·50, find the cost of:

a 5 kg

b 12·2 kg

Solution

1

a $\frac{42 \text{ cm}}{2 \text{ years}} = 21$ cm/year

b $\frac{\$553{\cdot}50}{45 \text{ L}} = \$12{\cdot}30$/litre

c $\frac{714}{17} = 42$ people per canoe

2

a $24 - 16 = 8$°C increase; $t = 8/8 = 1$ minute

b $20 - 16 = 4$°C increase; $t = 4/8 = \frac{1}{2}$ minute

c $38 - 16 = 22$°C increase

$t = \frac{22}{8} = 2\frac{3}{4}$ minutes

3

a $5 \times \frac{112{\cdot}50}{9} = \$62{\cdot}50$

b $12{\cdot}2 \times \frac{112{\cdot}50}{9} = \$152{\cdot}50$

Exercise 5D

1 What units would you use to measure the following rates?

a The cost of a mobile phone call (____/minute).

b The price of fish sold at the Fishing Village market ($/____).

c The rise of temperature in a boiled kettle (____/____).

d The scale used on a map (____/____).

e Bill's pay when he works at the Solomon Telekom (____/____).

2 Give an example for each of the following rates:

a points per game

b dollars per kilogram

c kilogram per square metre

d cubic centimetres per hour

e cents per litre

f fouls per quarter

3 Express each of the following situations using a rate in simplest form:

- **a** The rateable value of a 1800-square-metre block was \$140 400 (\$____ /m²).
- **b** Thuy was able to type 288 words in 6 minutes (____ words/min).
- **c** Four dozen rolls cost \$4·80 (\$____ roll).
- **d** A soccer team had 336 kicks in general play to score 12 goals (____ kicks/goal).
- **e** A crowd of 2540 people paid a total of \$65 024 to see a concert (\$____ /ticket).
- **f** A swimming pool with capacity of 20 000 litres was filled in 25 hours (____ L/h).
- **g** The cost of 2·4 cubic metres of tan bark is \$46·80 (\$____ /cubic metre).
- **h** The temperature rose from 6°C to 18°C between 6 am and noon (____ °C/h).
- **i** A bus used 120 litres of diesel fuel to travel 1020 km (____ km/L).
- **j** An oil spill spread 600 m^2 in 12 minutes (____ m^2/min).
- **k** 12 480 cows are carried on a 4160-acre dairy farm (____ cows/acre).
- **l** It takes 90 minutes to mow a lawn of area 2700 m^2 (____ m^2/min).

4 To win, Western United FC needs to accumulate 24 points in its 8 soccer games.

- **a** Express this as a simple rate (3 points per game).
- **b** At this rate, how many points should they accumulate after winning the following number of games?
  - **i** 2 **ii** 3 **iii** 5 **iv** 7

5 If the cost of the fuel is \$12.50 per litre, what value of fuel is consume by outboard motor that uses:

- **a** 15 litres **b** 45 litres
- **c** 60 litres **d** 250 litres

6 My pancake recipe indicates that 200 mL of water is to be added to 250 g of self-raising flour and two eggs.
Express the rate at which water is added to flour in simplest form and then find the volume of water needed if the following weights of flour are used:

- **a** 1200 g **b** 2200 g **c** 3·75 kg **d** 12 kg **e** 22·5 kg

7 The public bus travels 8 km and uses 1·6 litres of petrol. Express this as a simple rate and find the amount of petrol used on these journeys:
(Rate: 5 km per litre)

- **a** White River School to St John School: 6 km.
- **b** St John's School to Mbokona School: 3·8 km.
- **c** Mbokona School to Honiara High School: 4·2 km.
- **d** Honiara High School to Florence Young School: 3·2 km.
- **e** Florence Young School to KGVI School: 5·6 km.
- **f** KGVI School to Betikama School: 4·9 km.

8 Water is delivered from a high-pressure hose so that 360 litres of water is delivered in 4 minutes. Express this as a simple rate and use it to find the amount of water delivered in:

- **a** 1·2 minutes **b** 2·9 minutes **c** 18 minutes **d** 12 minutes 15 seconds

5E Rates and units

Rates can be expressed in different units. In order to convert rates, individual unit conversions need to be known. For example, 1 L = 1000 mL, 1 km = 1000 m, 1 h = 3600 s and so on.

Example

1 A new racing car is able to drive at a top speed of 360 km/h. Express the speed of the car in:

a m/h
b m/min
c m/s

2 A tap delivers water at the rate of 40 mL/min. Express this in the units of L/h.

Solution

a $360 \text{ km/h} \times 1000 = 360\,000 \text{ m/h}$

b $360 \text{ km/h} \times 1000 \div 60 = 6000 \text{ m/min}$

c $360 \text{ km/h} \times 1000 \div 60 \div 60 = 100 \text{ m/s}$

2 $40 \text{ mL/min} = 40 \div 1000 \times 60 = \frac{2400}{1000}$

$= 2{\cdot}4 \text{ L/h}$

Exercise 5E

1 Convert the following speeds to metres per second:

a 48 km/h　b 72 km/h　c 1800 km/h　d 32 km/h　e 24 km/h
f 2 km/h　g $18\frac{1}{3}$ km/h　h 0·2 km/h　i 120 km/h　j $12\frac{1}{2}$ km/h

2 Convert the following speeds to km/h:

a 60 m/s　b 36 m/s　c 90 m/s　d 20 m/s　e 108 m/s
f 18 m/s　g 14·4 m/s　h 288 m/s　i 5 m/s　j 7·2 m/s

3 Convert the following units to the units indicated:

a 3600 L/min = _______ L/h　b \$8/min = \$______/h
c 1200 g/min = _______ kg/min　d 9 m/s = _______ m/min

4 Fill in the spaces:

a 2400 mL/min = ________ mL/h　b 3600 g/min = ________ kg/min
c 2000 kg/min = ________ t/min　d 2400 cm/s = ________ cm/min
e 420 L/min = ________ L/h　f 8·4 L/h = _______ L/day
g 1200 \$/day = \$_________/min　h 280 mm/min = __________ cm/h
i 450 kg/h = __________ g/min　j 380 g/s = __________ kg/h

5 Watering systems use water at different flow rates. Express the following rates in mL/min:

a slow feed: 6 L/h　b normal use: 36 L/h　c express use: 3000 L/h

6 The cheetah is the fastest land mammal in the world. Express its top speed of 150 kilometres per hour in the following units:

a km/min　b km/s　c m/h　d m/min　e m/s

7 The fastest wind speed ever recorded is 530 km/h, which was reached when tornadoes hit Oklahoma in the US. Express this speed in the following units:

a km/min　b km/s　c m/h　d m/min　e m/s

8 The cockroach can reach speeds of 30 cm/s and the centipede can sprint at a maximum speed of 50 cm/s. Express these speeds in the following units:

a m/min **b** m/s **c** m/h **d** m/min **e** km/h

9 Tectonic plates carry the continents, and when the plates move into each other earthquakes and volcanic action can result. The fastest tectonic movement on Earth has been recorded at 24 cm/year. Express this in the following units:

a cm/month **b** m/year **c** mm/day **d** m/month
e mm/month **f** cm/day **g** km/h **h** m/s

10 When exercising, these people have the following pulse rates:

Fasi: 120 beats/min Sale: 150 beats/min Kwaimani: 180 beats/min

Express each person's pulse rate in:

a beats/hour **b** beats/second **c** beats/day

11 A pumpkin was weighed at regular intervals and it was found to grow at the constant rate of 190 g/day over a 4-week period.

a If its weight at the start of the trial was 1·2 kg, find its weight at the end of each week of the trial.

b Express the rate of the increase of its weight in:

i g/h **ii** g/min **iii** g/s

12 A petrol bowser is able to deliver petrol at the rate of 20 L/min.

a Express this rate in:

i L/h **ii** mL/min **iii** mL/h

b Find how long it will take in hours and in minutes to fill a car with a petrol tank that holds:

i 60 litres **ii** 45 litres **iii** 25 litres **iv** 120 litres

13 A pie-eating competition is hosted by a football team as a fundraiser. The average rate at which pies are eaten in this competition is 120 pies/min.

a Express the rate in:

i pies/s **ii** pies/h **iii** pies/day

b At this rate, find how long it will take participants to eat:

i 540 pies **ii** 1020 pies **iii** 1470 pies
iv 2220 pies **v** 2370 pies

14 Paint is sold in a number of different-sized cans.

a If a 20-litre can of paint costs $2400, express this as a ratio in dollars : volume.

b Use the ratio to find the cost of the following sizes of cans:

i 10 litres **ii** 50 litres **iii** 250 mL
iv 500 mL **v** 2500 mL

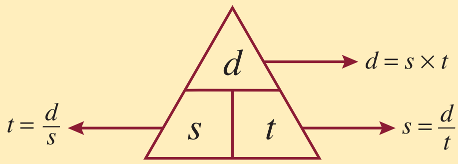
c Find the volume of paint that is sold for:

i $48 **ii** $108 **iii** $114
iv $15 **v** $51

Using rates: Distance, time, speed

When using an outboard motor to sail between distant islands in the Western Solomons, it is important for safety to estimate the time that a journey will take. The time can be estimated by knowing the outboard motor's speed and the distance to be travelled. **Speed** is the rate at which distance changes with respect to time. It is measured in units of distance divided by time, such as metres per second or kilometres per hour.

The formulas involving speed, distance and time are shown in these diagrams.



d is distance
s is speed
t is time

To use these formulas the units must be the same. If the units are not the same convert them before starting a solution.

Example

1 Find the average speed of a motorcycle that travels 630 km in 9 hours.

2 How far does a car travelling at an average speed of 72 km/h travel in 3 hours 20 minutes?

3 How long does it take for a aeroplane to travel 650 km if it is travelling at 120 km per hour?

4 An elephant's speed is recorded as 40 km/h.

a Express this speed in m/s.

b At this rate how far will it run in 10 seconds?

Solution

$s = \frac{d}{t} = \frac{630}{9}$
$= 70$ km/h

$t = 3$ h 20 min
$= 3\frac{20}{60}$ h $= 3\frac{1}{3}$ h

$d = s \times t = 72 \times 3\frac{1}{3}$
$= 240$ km

$t = \frac{d}{s} = \frac{650}{120}$
$= 5\frac{5}{12}$ h $= 5$ h 25 min

$\frac{40 \times 1000}{1 \times 60 \times 60} = 11\frac{1}{9}$ m/s

$d = s \times t$
$= 11\frac{1}{9} \times 10 = 111\frac{1}{9}$ m

Exercise 5F

1 Select the units of speed from km/h, m/s, cm/s or mm/s that would best be used to state the speed of:

a an ant crawling along a bench
b a stone being thrown off a building
c a slug moving across a lawn
d a rocket taking off
e a ball being rolled along a floor
f the pendulum of a clock swinging
g travelling on a steep water slide
h moving a chess piece across a chessboard

2 Give two examples of moving things whose speed is best measured using each of the following units:

a km/h **b** m/s **c** cm/s **d** mm/s

3 Calculate the speed in kilometres per hour (km/h) of:

a a racing car that travels 913·5 km in 4·5 h

b an aeroplane that travels 7752 km in 17 h

c a bus that travels 340 km in 4 h

d a helicopter that travels 1800 km in 4 hours 30 minutes

e a jogger who runs 16 km in $2\frac{1}{2}$ h

f an elephant that travels 16·25 km in $3\frac{1}{4}$ h

4 Calculate the speed in metres per second (m/s) of:

a an arrow that travels 240 metres in 0·8 second

b a jogger who runs 2 km in 20 minutes

c a bird that flies 30 metres in a minute

d a stone that falls 120 centimetres in 1·5 seconds

e an insect that runs 6780 mm in 5 minutes

5 In the ocean the bottlenosed dolphin travels at 64 km/h, the killer whale at 48 km/h, the mako shark at 32 km/h and the penguin at 24 km/h. The fastest marine animal is the blue fin tuna, which swims at 88 km/h.

a Find the distances that each of these animals would swim in:

i 10 minutes

ii 25 minutes

iii three-quarters of an hour

iv one and a half hours

b Express the speeds of these animals in metres per second, and find the distance to the nearest metre that each can travel in:

i 5 seconds **ii** half a minute **iii** 2 minutes **iv** $3\frac{1}{2}$ minutes

6 The fastest speeds of the following insects are recorded in km per hour:

Bumble bee: 11 Hoverfly: 14 Hawkmoth: 50 Dragonfly: 75

a Express the speed of each insect in metres per second.

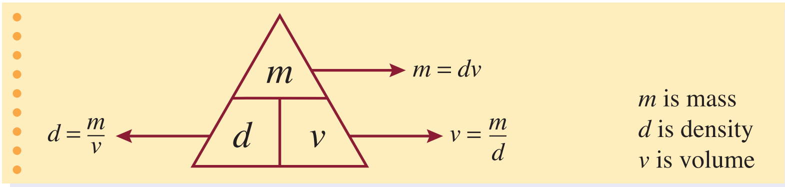
b The insects take part in a 100-metre race. Find the time to the nearest second that each insect will take to complete the race.

7 The top speed of a duck is 85 km/h, that of a teal is 120 km/h and that of a crow is 40 km/h. Find the time taken for each bird to fly 100 metres:

a as a fraction of an hour **b** to the nearest tenth of a second

5G Density

Some substances are heavier than others per unit volume. One cubic centimetre of water has a mass of 1 gram at 4°C, while 1 cubic centimetre of lead has a mass of 11·23 grams. The density of a material is the ratio of its mass to a standard volume. The units of density are usually stated in grams per cubic centimetre (g/cm^3) or kilograms per cubic metre (kg/m^3). The density of gases and liquids changes with temperature, so the temperature needs to be quoted for these figures.



Example

1 Find the densities in grams per cubic centimetre of the following correct to 1 decimal place:

Solution

$d = \frac{m}{v}$

a a 12 cubic centimetre sample of zinc of mass 85·2 grams

$d = 85·2 \div 12$
$= 7·1$ g/cm^3

b 117 grams of steel which occupies 15 cubic centimetres

$d = 117 \div 15$
$= 7·8$ g/cm^3

c an ice cube with a volume of 5 cubic centimetres and mass of 4·6 grams

$d = 4·6 \div 5$
$= 0·9$ g/cm^3

2 A block of brass has a volume of 14 cubic centimetres. Find its mass if the density of brass is 8·6 g/cm^3.

$m = dv$
$= 14 \times 8·6$
$= 120·4$ g

Exercise 5G

1 Find the densities of the following, in grams per cubic centimetre:

a 150 cm^3 of granite of mass 405 g
b 230 cm^3 of copper of mass 2047 g
c 167 cm^3 of water of mass 167 g
d 23 cm^3 of glass of mass 57·5 g

2 Find the mass of the following samples of granite if the density of granite is 2·7 g/cm^3:

a 12 cm^3 **b** 18 cm^3 **c** 29 cm^3 **d** 190 cm^3 **e** 2000 cm^3

3 Find the mass of the following samples of iron if the density of iron is 7·9 g/cm^3:

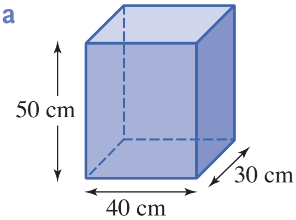
a 14 cm^3 **b** 98 cm^3 **c** 39 cm^3 **d** 120 cm^3 **e** 3000 cm^3

4 Find the density of each material in the units of g/cm^3 if:

a 200 cm^3 of polystyrene has a mass of 3·2 g
b 750 cm^3 of wood has a mass of 825 g
c 600 cm^3 of lead has a mass of 6·84 kg
d 400 cm^3 of aluminium has a mass of 1·08 kg

5 The density of water at 4°C is 1 g/cm^3. Find the mass of water in these tanks if they are filled to the top:

a



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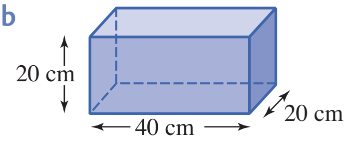
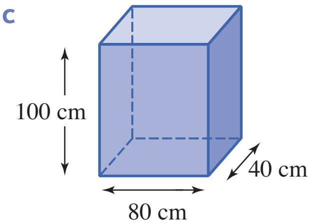
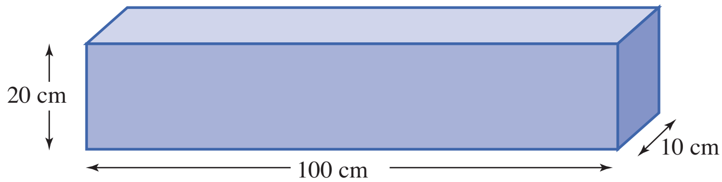


c

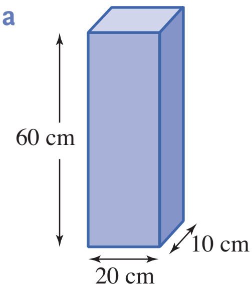


6 The density of iron is 7·9 g/cm^3. Find the mass of this block of iron:

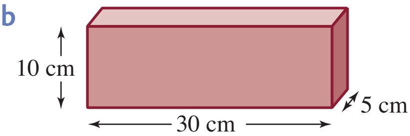


7 The density of plastic A is 1·5 g/cm^3 and the density of plastic B is 0·9 g/cm^3. Find the mass of the following blocks of each type of plastic and state the difference in mass between them:

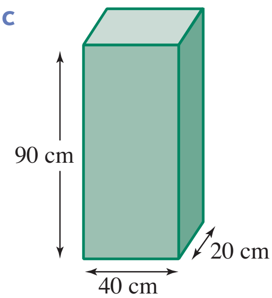
a



b



c

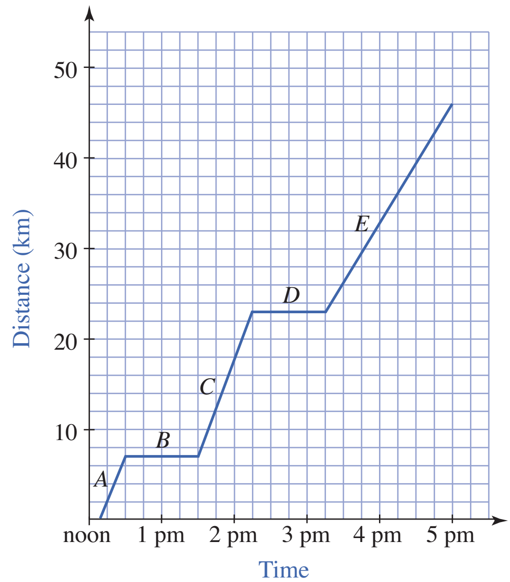


8 The densities, expressed in the units of kilograms per cubic metre (kg/m^3) and grams per cubic centimetre (g/cm^3), of some materials are listed in the following table. Complete the table:

Material	Density (kg/m^3)	Density (g/cm^3)
Polystyrene (a plastic)	16	
Polypropylene (a plastic)		0·9
Nylon (a plastic)		1·14
White gum (wood)	1100	
Lead (metal)		11·4
Wet sand	1230	

5H Exploring distance–time graphs

Graphs are a visual way of showing information. The most important thing about graphs is to understand what information each of the line segments shows.



Learning task 5H

1 This distance–time graph shows Kim's bicycle journey one afternoon.

The journey happened in five stages, which are labelled *A*, *B*, *C*, *D* and *E* on the graph.

Use the graph to state at what time the journey:

a started ________

b finished ________

For how many minutes did Kim:

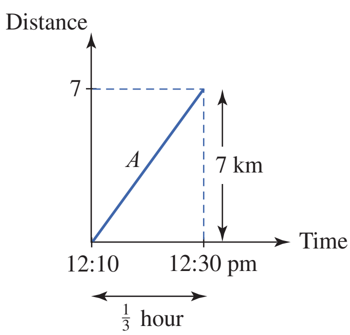
c rest? __________

d cycle? __________

Complete the information shown in this table. The speed is found by dividing the distance travelled by the time taken:

Section	Distance (km)	Times	Time travelled (hour)	Speed
A	7	12:10–12:30	$\frac{1}{3}$	$7 \div \frac{1}{3} = 21$ km/h
B				
C				
D				
E				

The speed can also be found by finding the gradient (slope) of the lines, in each section.



$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{7}{\frac{1}{3}} = 21 \text{ km/h}$$

2 Tom recorded the following times and distances on his journey to school. His journey was split into five sections:

A 7:00 am left home. Ran 200 m to bus stop. Arrived at 7:04 am.

B Waited 2 minutes for the bus. Entered the bus and found a seat at the back.

C Bus left at 7:08 am. Travelled 3 km to school in 15 mins.

D Got off the bus at 7:24 am.

E Walked 120 m to classroom at 2 metres per second.

a Transfer the information of Tom's journey into the table below:

Section	Distance (m)	Times	Time travelled	Speed
A				
B				
C				
D				
E				

b What time did Tom leave the first bus stop?

c Draw the graph of Tom's journey.

d How long did Tom take to get from his home to his classroom?

3 Edwina is training for the Honiara Easter Bike Ride, and she keeps an accurate log of her training one afternoon.

Use the information in the table below to draw a graph of her training, and then calculate the speed at which she travels during each of the seven stages.

Section	Distance (km)	Times	Time travelled (hour)	Speed (km/h)
A	20	noon–1:15 pm	$1\frac{1}{4}$	$20 \div 1\frac{1}{4} = 16$
B	0	1:15–2:00 pm		
C	10	2:00–2:45 pm		
D	15	2:45–3:15 pm		
E	0	3:15–4:00 pm		
F	10	4:00–4:15 pm		
G	5	4:15–5:00 pm		

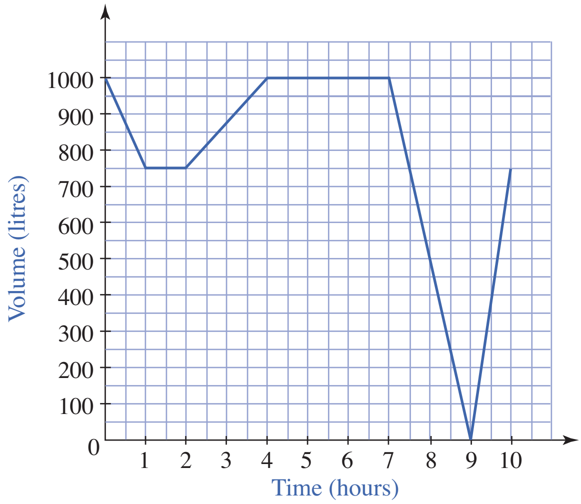
51 Rates and graphs

The way in which quantities change (i.e. either increasing or decreasing) and the rate at which they change can be measured from a graph. These rates are represented as the gradients of the line segments under investigation.

Example

1 The volume of a small water tank is monitored over a 10-hour period from noon. The results are shown on the graph.

Solution



2 **a** Between what times did the volume of the tank increase?

Increased between 2 pm and 4 pm and between 9 pm and 10 pm.

b Use the graph to estimate the rate of increase of water in litres per hour during this time.

2 pm to 4 pm:
1000 – 750 = 250 litres in 2 hours

$\text{Rate} = \frac{250}{2} = 125 \text{ L/h}$

9 pm to 10 pm:
750 – 0 = 750 litres in 1 hour

$\text{Rate} = \frac{750}{1} = 750 \text{ L/h}$

3 **a** Between what times did the volume of the tank decrease?

Decreased between noon and 1 pm and between 7 pm and 9 pm.

b Use the graph to estimate the rate of decrease of water in litres per hour during this time.

Noon to 1 pm:
1000 – 750 = 250 litres in 1 hour

$\text{Rate} = \frac{250}{1} = 250 \text{ L/h}$

7 pm to 9 pm:
1000 – 0 = 1000 litres in 2 hours

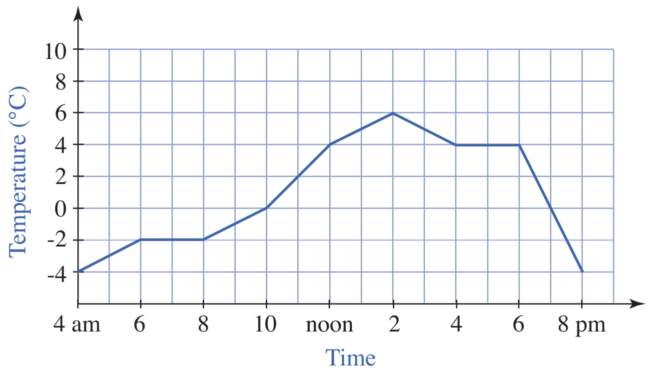
$\text{Rate} = \frac{1000}{2} = 500 \text{ L/h}$

Exercise 5I

1 Temperature was recorded on a 2-hourly basis and the temperature–time graph was drawn.

a Complete the following table:

Time	Temperature (°C)
4:00 am	

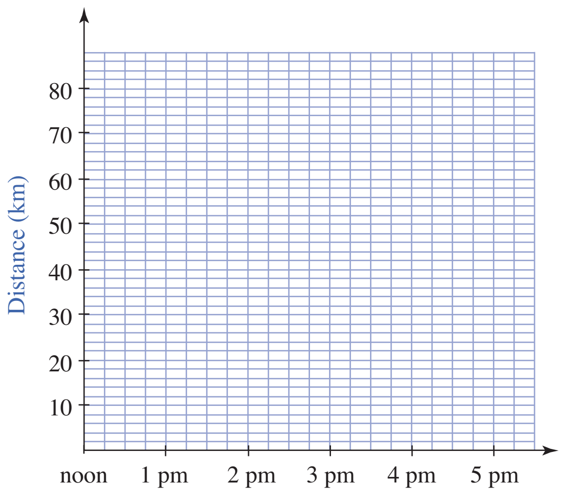


b What was the average temperature change in degrees/hour between each of the readings?

2 William keeps a record of his bike ride training.

a His records for one Saturday afternoon are shown in the table. Draw William's progress on the graph below and label each section from *A* to *G*.

Section	Distance (km)	Times
A	18	noon–1:00 pm
B	0	1:00–1:30 pm
C	25	1:30–2:15 pm
D	0	2:15–2:45 pm
E	22	2:45–3:45 pm
F	0	3:45–4:15 pm
G	15	4:15–5:00 pm



b State at what time the training ride:

i started __________ ii finished __________

For how many minutes did William:

iii rest? __________ iv cycle? __________

c William cycled for four sections of the trip. Find the average speed at which he travelled for each of these sections.

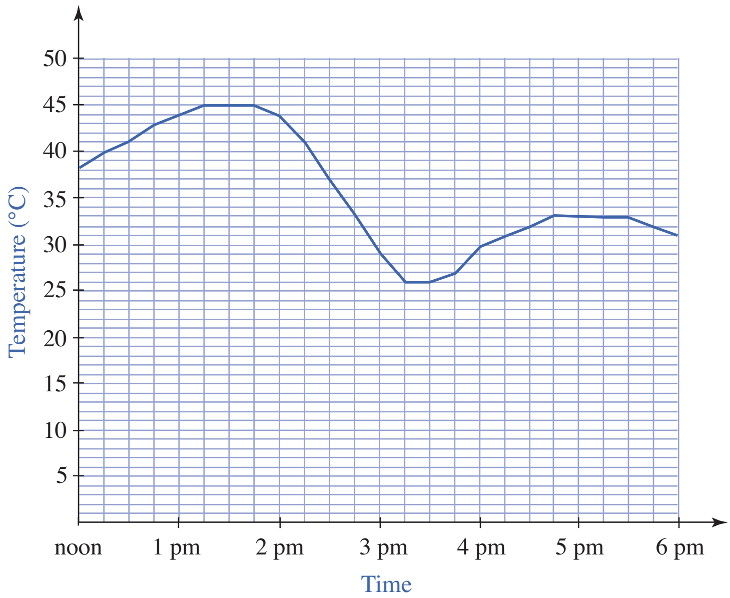
5J Exploring rates of change graphs

It has been shown that the rate at which a quantity changes can be found by dividing the height (rise) by the base (run) of the right-angled triangles that are drawn between the points being compared.

In practical situations graphs have curved sections and triangles are only drawn to give us an average figure for the rates being found.

Learning task 5J

The temperature was recorded every quarter hour for 6 hours starting at noon on a summer's day. The results are shown on this graph:



1 From the graph, read off the temperatures to the nearest degree at 15-minute intervals between noon and 6 pm and record the values in this table:

Time	Temperature (°C)	Time	Temperature (°C)
noon	38	3:15	
12:15 pm		3:30	26
12:30	41	3:45	
12:45		4:00	30
1:00	44	4:15	
1:15		4:30	
1:30		4:45	
1:45		5:00	33
2:00	44	5:15	
2:15		5:30	
2:30		5:45	
2:45		6:00	31
3:00	29		

The **average rate of change** of temperature looks at the final and initial values only and not at the values in between, and gives an overall indication of the way quantities have changed.

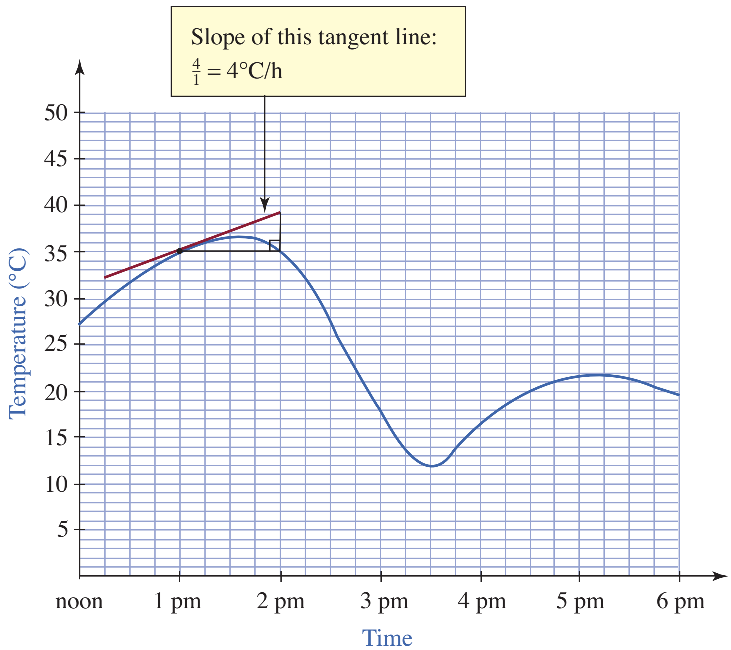
2 Using the appropriate values, find the average temperature change for the half-hour intervals shown in the table:

Time interval	Rate of change
noon–12:30 pm	$41 - 38 = 3°C$ in half an hour $\therefore$ rate of temperature change $= \frac{3}{\frac{1}{2}} = 3 \times 2 = 6°C/h$ Temperature increases at 6°C/h
1:00–1:30 pm	
2:00–2:30 pm	
3:00–3:30 pm	
4:00–4:30 pm	
5:00–5:30 pm	

The **instantaneous rate of change** of temperature is a measure of the gradient of the tangent line at that point. A tangent is a line that touches the curve at that point.

Example

Estimate the rate of temperature change at 1:00 pm by drawing a tangent line to the curve at this time. The gradient of the tangent is the instantaneous rate of change of temperature.



3 From the graph in the example, estimate the rate of change of temperature for the following times:

12:30 pm, 1:30 pm, 2:00 pm, 2:30 pm, 3:00 pm, 3:30 pm,
4:00 pm, 4:30 pm, 5:00 pm, 5:30 pm and 6:00 pm

Puzzles

1 Find the value of the unknown in the following ratios and match the letter to the correct number to solve the puzzle:

R|E|A|D|I|N|G

A:4 = 6:8	**B**:2 = 16:8	**D**:3 = 18:9	**E**:5 = 4:20
2:**G** = 10:25	7:**H** = 21:27	15:**I** = 45:90	20:**L** = 40:50
1:3 = **N**:30	2:3 = **R**:36	2:3 = **S**:18	1:2 = **T**:30
4:7 = 16:**W**			

___ ___ ___ ___ ___ ___ ___
24 1 3 6 30 10 5

___ ___ ___ ___ ___ ___ ___   ___ ___ ___
4 1 15 28 1 1 10   15 9 1

___ ___ ___ ___ ___
25 30 10 1 12

2 Increase the numbers below in the ratio 2:3 then match the letter to the correct number to solve the puzzle:

0
PhD
BSc
MA

B 4	**D** 10	**E** 8
G 6	**H** 20	**L** 12
O 14	**R** 18	**S** 3
T 5	**W** 15	**Z** 9

___ ___ ___ ___ ___   ___ ___ ___ ___ ___ ___ ___
7·5 30 27 12 12   15 12 9 27 12 12 4·5

___ ___ ___ ___ ___   ___ ___ ___ ___
6 12 18 21 22·5   13·5 12 27 21

3 Calculate the speeds in kilometres per hour for each distance and time. Match the letter to the correct answer to solve the puzzle:

Your i
Your i
Your i
Your i

only

E	100 km 2 h	**F**	75 km 45 min
L	50 km 15 min	**N**	500 m 3 min
O	100 km 4 h	**R**	70 m 2 min
S	2585 k 55 h	**U**	500 m 10 min
Y	10 m 1 s		

___	___	___		___	___	___	___	
100	25	2·1		36	25	3	2·1	
___	___	___	___		___	___	___	___
50	36	50	47		25	10	200	36

4 Calculate the density in grams per cubic centimetre for each mass and volume given below. Match the letter to the correct answer to solve the puzzle:

E C N A L G

A	6 g	2 cm^3	**B**	20 g	4 cm^3	**C**	44·5 g	5 cm^3
D	12·6 g	2·1 cm^3	**E**	5 kg	250 cm^3	**G**	10·4 kg	1000 cm^3
K	13·75 g	2·5 cm^3	**L**	2·7 g	0·03 cm^3	**N**	6·5 g	0·5 cm^3
R	7·8 g	1·2 cm^3	**S**	1·25 g	2·5 cm^3	**W**	22·1 g	17 cm^3

___	___	___	___	___	___			
10·4	90	3	13	8·9	20			
___	___	___	___	___	___	___	___	___
5	3	8·9	5·5	1·3	3	6·5	6	0·5

Applications

Heart rates and health

Heart rate is the number of times our hearts beat per minute. Resting heart rate averages between 60 and 80 beats per minute. In some unfit adults the resting rate can exceed 100 beats per minute. In highly trained athletes resting rates in the range of 28–40 beats per minute have been reported.

Your resting heart rate (HR) typically decreases with age and environmental factors such as temperature and altitude. A mathematical formula to predict your maximum HR is as follows:

Female: Max HR = 226 − your age   Male: Max HR = 220 − your age

These formulas are more accurate for adults and there is an error of least ±10–15 beats per minute.

1 Use the formula to calculate the heart rate of a:
   a 46-year-old woman   b 68-year-old man   c 57-year-old woman

2 Use the heart rate of a 40-year-old woman to calculate how often her heart would beat:
   a in a year   b in 20 years   c in a lifetime of 70 years

3 Use the heart rate of a 50-year-old man to calculate how often his heart would beat:
   a in a year   b in 20 years   c in a lifetime of 70 years

4 Research your library or the Internet to find how much blood is pumped, on average, for each heart beat. Calculate the volume of blood in litres that your heart pumps in:
   a a day   b 20 years   c in a lifetime of 70 years

Studies have shown that early signs of disease can be found in adolescents who smoke. Smoking decreases a young person's physical fitness in terms of both performance and endurance.

The resting heart rates of young adult smokers are two to three beats per minute faster than those of non-smokers.

5 Research the connection between heart rates and the health risks of smoking.

Navigational speed

A nautical mile is a unit of length used in navigation. There are 360 degrees in a circle and 60 minutes in 1 degree. One nautical mile is defined as the length spanned by the angle of 1 minute at the equator.

a Find the number of minutes in a circle.

b If the length of the equator is 40 000 kilometres find the length of 1 nautical mile to the closest metre.

A knot is the speed of 1 nautical mile per hour.

c State the speed of 1 knot in kilometres per hour.

d A boat travels down a river. Express the following speeds in metres per hour:
   i 5 knots   ii 8 knots   iii 12 knots   iv 20 knots   v 30 knots

Scale model of the planets

The following table gives the approximate diameter of the planets and their distance from the Sun:

Planet	Diameter (km)	Distance from the Sun (km)
Mercury	4 878	57 910 000
Venus	12 103	108 200 000
Earth	12 756	149 600 000
Mars	6 786	227 940 000
Jupiter	138 346	778 330 000
Saturn	114 632	1 426 980 000
Uranus	50 532	2 870 990 000
Neptune	49 064	4 497 070 000
Pluto	2 284	5 913 520 000

Using the Earth as a starting point, calculate the relative sizes of the other planets. Construct a set of models using balloons, paper maché or a range of balls (basketballs, tennis balls, table tennis balls) that could represent the relative sizes of the planets.

Choose a suitable scale and arrange your planet models along the classroom wall, using the distances from the Sun given in the table above.

Paper sizes

White paper is generally sized according to the international A system. This system is based on a standard size called A0—a rectangular sheet of paper 1189 mm by 841 mm.

To make the next sheet in the series, the sheet of A0 paper is folded in half so that it measures 594 mm by 841 mm. The process is continued in this way to give paper sizes of A0, A1, A2, A3, A4, A5 and A6.

a Find the dimensions of paper sizes A2, A3, A4, A5 and A6 and determine the ratios of length to width for each paper size.

b Find the area of an A0 sheet of paper and explain why this is chosen as being the first sheet in the series. Find the area of each paper size and find the ratio of its area to that of an A0 sheet.

c Find the length of the diagonals of each paper size to the nearest centimetre.

d Find the ratio of the diagonals of consecutive paper sizes such as A0 : A1, A1 : A2, A2 : A3, A3 : A4, etc. and comment on the results.

Enrichment

1 Water is poured into a $1\frac{1}{2}$-litre container. It is known that the ratio of water spilt to that successfully delivered into the container is 3 : 7.

a How much water was spilt?

b What was the total volume of water needed to fill the container?

2 The distance from Island X to Island Y was 300 km. Bilikiki flies from Island X at an average speed of 76 km/h. He maintains this speed for 2 hours before increasing it by 4 km/h for the rest of the journey to Island Y.

a How long did Bilikiki take to complete the whole journey?

b What was his average speed from Island X to Island Y?

3 a An antelope, a zebra and an ostrich are entered in a 100-metre race. If their top speeds are 95 km/h, 65 km/h and 50 km/h respectively, how many seconds, to the nearest hundredth of a second, after the antelope will the zebra and ostrich complete the race?

b A race of 100 metres is held between a sailfish (top speed 110 km/h), a penguin (40 km/h) and a dolphin (64 km/h). How long, to the nearest hundredth of a second, after the sailfish will the penguin and dolphin complete the race?

4 Bluey and Ruby decided to paddle between Savo and Vislae Point. Ruby starts at Savo and finishes at Visale Point, while Bluey starts at Visale Point and finishes to Savo.

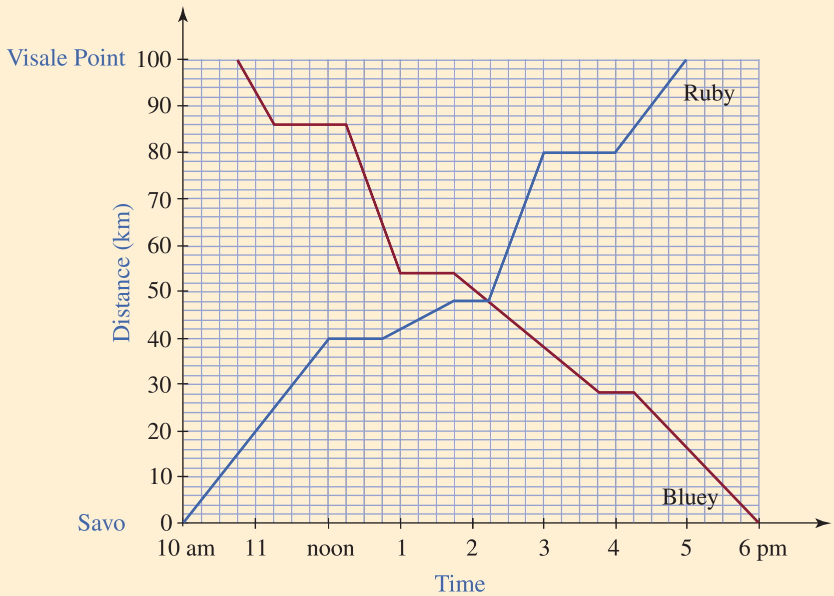
a From the graph below find the time that each took to complete their journey, and find each person's average speed.

b At what time did they meet and what did each of them doing when they met?

c For how long did each of them rest?

d List the speeds each of them achieved for the sections that they paddled, giving the answers to the nearest km/h.

e Find the actual average speed to the nearest km/h for each of them, considering only the sections in which they are paddling (that is ignoring their rest stops).



5 Thuy goes on a bicycle ride and his journey is shown on this graph. He stopped twice on the ride.

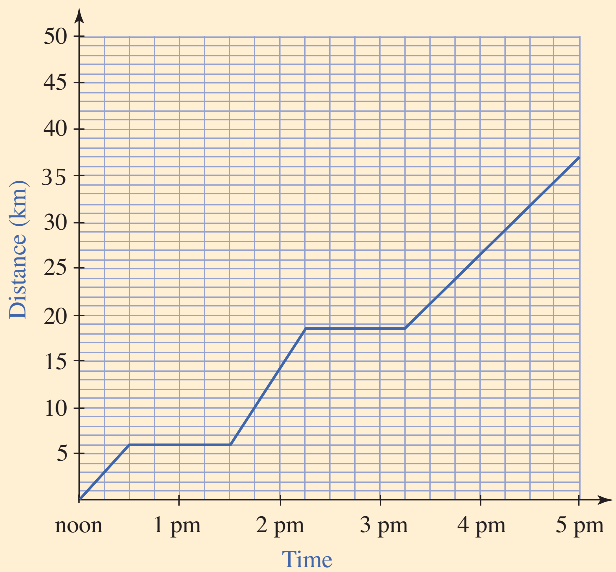
a Label the various sections of the graph *A*–*E*.

b Between what times did Thuy stop for a rest?

c Between what times did he travel?

d Complete the table:

Time	Distance travelled (km)
noon–12:30 pm	6
12:30–1:30 pm	0
1:30–	



e The graph shows that Thuy cycled for three sections of the journey. Find his speed on the three sections to the nearest km/h.

f What was the maximum speed for the journey?

g What was the average speed over the first 2 hours?

h What was his average speed over the 5 hours?

i What was the instantaneous speed after 2 hours?

j What was his instantaneous speed when 12 km from the start?

6 Two containers *A* and *B* sit side by side on the kitchen bench. Container *A* holds 1 litre of water and container *B* holds 1 litre of cordial. First, 100 cm^3 of water is carefully poured from container *A* into container *B* and the mixture is stirred thoroughly. Then 100 cm^3 of the mixture in container *B* is returned to container *A*.

Which is the larger ratio, the ratio water : cordial in container *A*, or the ratio cordial : water in container *B*?

7 Two quantities *A* and *B* are in the ratio 2 : 3.

a If *B* is decreased by 5 units the ratio becomes 4 : 5. Find *A* and *B*.

b Find how much needs to be added to *A* so that the ratio becomes:

i 1 : 1 **ii** 2 : 1 **iii** 3 : 1 **iv** 4 : 1 **v** $x : 1$

8 Find the ratio of $z : y$ if:

a $3x = 2y$ and $5z = 7x$

b $5x = 0{\cdot}001y$ and $3z = 2000x$

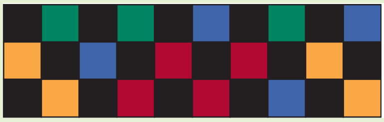
c $y = \dfrac{x-6}{3}$ and $2z - x + 6 = 0$

d $x^2y + 1 = xy(x+1)$ and $z + \dfrac{(1-y)(1+y)}{y} = x$

Revision/Assessment

Exercise 5A

1 Express the ratio of coloured squares to black squares for each colour.



2 Simplify these ratios:

a 28 : 32 b 35 : 65 c 24 : 36 d 18 : 15

3 Express the following as simplest ratios:

a 5 cm : 1·2 m b 360 s : 4 min c 3 kg : 200 g d 21 days : 2 weeks

Exercise 5B

4 Find the value of x to make the ratios the same, using algebra:

a $x : 24 = 6 : 36$ b $18 : x = 3 : 27$ c $30 : 25 = x : 15$ d $36 : 16 = 8 : x$

5 The ratio of shirts to jeans in a clothes shop is 7 : 3. If there are 133 shirts, find:

a the number of jeans

b the total number of shirts and jeans in the shop

6 A cordial drink is made by mixing syrup with water in the ratio 3 : 25. If 268·8 litres of cordial is made, find the amount of syrup and water used.

Exercise 5C

7 Increase the following in the ratio of 5 : 4:

a 30 metres b 120 centimetres c 46 grams

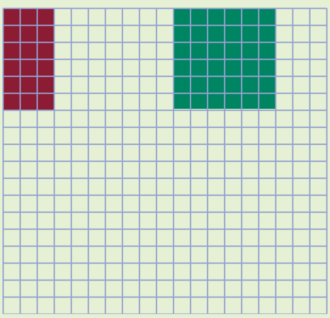
d 18 kilograms e $130 f 92 minutes

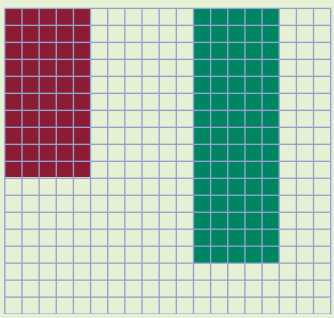
8 Decrease the following in the ratio 1 : 5:

a 120 grams b 895 millimetres c $45·10

d 768·5 kilograms e 45 hours f 10·5 hours

9 a Increase these shapes in the ratio 3 : 2. b Decrease these shapes in the ratio 1 : 5.





Exercise 5D

10 Express each of the following situations using a rate in simplest form:

a 25 litres of petrol was used in travelling 280 km.

b The car travelled 52 kilometres in three-quarters of an hour.

c The oven heated up from 10°C to 260°C in 5 minutes.

d A bowler bowled 69 overs to take 6 wickets.

11 If 56·4 metres of fencing wire costs $183·30, find the cost per metre of the wire. At this rate find the cost of these lengths of wire:

a 46 metres b 144 metres c 89·7 metres

12 An advertisement containing 38 words costs $68·40. Find the cost per word and then find the cost of an advertisement with the following number of words:

a 30 b 42 c 84 d 546

Exercise 5E

13 Convert the following speeds to metres per second:

a 36 km/h b 42 km/h c 180 km/h d 62 km/h e 86 km/h

14 Convert the following units to the units indicated:

a 4200 L/min = _________ L/h b $12/min = $_________/h

c 45°C/min = _________°C/s d 52 kg/m = _________ g/cm

15 Fill in the missing values:

a 144 $/day = $_________/min b 350 mm/min = _________ cm/h

c 320 kg/h = _________ g/min d 120 g/s = _________ kg/h

16 A petrol bowser is able to deliver petrol at the rate of 30 L/min. Express this rate in:

a L/h b mL/min c mL/h

Exercise 5F

17 Find the speed of the following expressed in the units of km/h and m/s:

a A racing car travels 107 km in half an hour.

b An antelope runs 200 m in 20 seconds.

c A billycart travels 400 m in 10 minutes.

d An arrow travels 120 m in 0·5 seconds.

Exercise 5G

18 Find the densities of the following in grams per cubic centimetre:

a 150 cm^3 of granite of mass 405 g

b 230 cm^3 of copper of mass 2·047 kg

19 Find the mass of the following samples of nylon if the density of nylon is 1·14 g/cm^3:

a 12 cm^3 b 36 cm^3 c 52 cm^3 d 180 cm^3 e 5000 cm^3

CHAPTER 6

Consumer Maths

The government collects tax from nearly everyone who earns money in the country and redistributes it to communities for services such as health, education and infrastructure.
It is required by law that eligible persons must pay tax to the government. Everyone pays Goods and Sales taxes whenever they purchase items from shops or service from service providers.
All employees have to pay PAYE tax out of their wages or salaries.

In Solomon Islands, all forms of income are taxed, including:

- Wages and salaries
- Capital growth and income from property
- Income from business
- Logging and mining companies.

This chapter covers the following skills:

- Converting between fractions, decimals and percentages
- Finding percentages of quantities
- Expressing quantities as percentages
- Finding 100%
- Applying percentages to situations of increase, decrease, appreciation, depreciation, mark-up and discount

  Percentage increase $= \frac{\text{increase} \times 100}{\text{original cost}}\%$

  Percentage decrease $= \frac{\text{decrease} \times 100}{\text{original cost}}\%$

  Percentage profit $= \frac{\text{profit made}}{\text{original cost}} \times 100\%$

  Percentage loss $= \frac{\text{loss made}}{\text{original cost}} \times 100\%$
- Comparing travel and holiday options
- Comparing take-away and home-cooked meals
- Calculating pay rates and scales
- Calculating simple interest $I = \frac{PRT}{100}$

Specific Learning Outcome (SLO)

Learners should be able to:

9.6.1.1 Calculate costs, sales and other business transactions with money.

9.6.2.1 Calculate and compare costs of goods and services to determine the best buy: cheaper goods and services with high quality.

9.6.3.1 Define the term 'percent' and identify the symbol that is used for percent: "%".

9.6.3.2 Find the percentage of shaded diagrams using grid, pie charts and other objects.

9.6.4.1 Convert decimals and fractions to percentages.

9.6.4.2 Convert percentages to decimals and fractions.

9.6.5.1 Define the term 'interest'.

9.6.5.2 Calculate the interest for a sum of money invested at a given rate for a given time.

9.6.6.1 Solve word problems applied to interest transactions for sums of money.

9.6.7.1 Calculate the principal, interest rate or interest in a variety of percentage problems.

9.6.8.1 Define the term 'discount'.

9.6.8.2 Calculate percentage increase or decrease on items.

percentage increase $= \frac{\text{increase}}{\text{original}} \times \frac{100\%}{1}$

percentage decrease $= \frac{\text{decrease}}{\text{original}} \times \frac{100\%}{1}$

9.6.8.3 Calculate discounts to the cost of items.

9.6.9.1 Define the following terms: *profit*, *loss*, *income* and *expenditure*.

9.6.9.2 Calculate the profit or loss to the sale of items.

9.6.10.1 Calculate the percentage profit and loss on items:

percentage profit $= \frac{\text{profit made}}{\text{original cost}} \times \frac{100\%}{1}$

percentage loss $= \frac{\text{loss made}}{\text{original cost}} \times \frac{100\%}{1}$

9.6.11.1 Define the term 'commission'.

9.6.12.1 Calculate the commission on sales and total earnings.

9.6.13.1 Define the term 'tax'.

9.6.13.2 Calculate how much tax is to be deducted from people who earn income.

9.6.14.4 Define the term 'simple interest'.

9.6.14.2 Identify the formula that is used to calculate simple interest:

9.6.14.3 Identify the terms that are used in the simple interest formula: *principal*, *rate* and *time*.

9.6.15.1 Calculate the interest on investments and loans using the simple interest formula.

6A Calculations in business

Businesses often use calculators or computer spreadsheets to add and subtract large sets of numbers.

Exercise 6A

1 The table below shows the number of items sold in a retail store for one calendar year:

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
Number	450	346	231	120	67	45	12	45	78	90	127	136

a Find the total number of sales for the year.

b There has been an error and the number of sales for April should be 92. Find the total for the year using the correct value for April.

Explain an easy method for finding the new total.

c What might the store sell?

2 The table below shows the amount Sam spends each year:

Costs	Food	Clothing	Rent	Entertain-ment	Car	Holidays	Insurance	Tax
Amount $	7800	5500	9200	4500	3400	1800	1200	10 000

a Find the total amount Sam spends each year.

b If he earns $50 000 a year, how much does Sam save each year?

c How much could Sam save on rent by sharing with a friend and paying $80 per week rent?

d If Sam shared with a friend:

i what other area could save him money? ii what costs might increase?

3 The table below shows the amount Sonya spends each year:

Costs	Food	Clothing	Rent	Entertain-ment	Car	Holidays	Insurance	Tax
Amount $	5600	7500	4800	2700	6400	3800	1200	8000

a Find the total amount she spends each year.

b If Sonya earns $50 000 a year, how much does she save each year?

4 The table below shows Wasi's shopping list for buying produce at the Honiara Main Market. Calculate the total cost of the fruits and vegetables that Wasi bought.

Fruits & Vegetables	Number of Hips	Cost per Hip	Cost
Banana	6	$8·00	
Pineapple	4	$12·00	
Eggplant	4	$5·00	
Potato	8	$10·00	
Capsicum	5	$5·00	
		Total	

Learning task 6B

Option 1: Takeaway fish and chips

1 The price for fish and chips at Sarah Habu's Leaf Hut, Honiara, is shown in the table below:

Fish & Chips	$25·00
Fish only	$5·00 each

a Calculate the cost of buying four packets of fish and chips and four extra pieces of fish.

b What costs are not considered when buying takeaway food?

Option 2: Slippery Cabbage (Ba'era) Soup

2 The costs of buying fresh ingredients from the Honiara Main Market to cook Slippery Cabbage with Noodles and Taiyo (Tin) are shown in the table below.

Slippery Cabbage (Ba'era)	$5·00 per parcel
Coconut	$1·00 each
Mamee Noodles	$3·50 each
Tuna Taiyo (tin)—Medium	$10·00 each
Water—1 Litre	$30·00

a Calculate the total cost of four parcels of Slippery cabbage (bae'ra), four Mamee noodles and two tins of taiyo?

b Estimate the costs of preparing coconut cream for four people.

c Compare the likely costs of cooking Ba'era Soup for four people with a wood fire, gas stove and an electric stove.

Option 3: Miscellaneous bargains

3 Which is the best value for money?
Extra White toothpaste (100 g) from Panatina Plaza that costs $14·50 or Best Smile toothpaste (150 g) from NPF Plaza that costs $19·50?

4 The prices of some tuna products that are sold at Point Cruz, Honiara, are shown in the table below:

Taiyo	Small (85 g)	Medium (185 g)	Family (425 g)
Solomon Blue	$4·50	$5·50	$11·00
Waioka Tuna	$4·50	$6·00	$13·00

a How much would four medium tins of Taiyo cost?

b Which product is the most expensive to buy?

c A recipe requires 1 kg of tinned tuna. Which products and sizes would give the best value for my money?

5 Reef fish are sold at the $18 per pound (lb.) at the Central Market, Honiara.

a How much would 25 pounds cost?

b How many pounds of fish would you buy for $1 008?

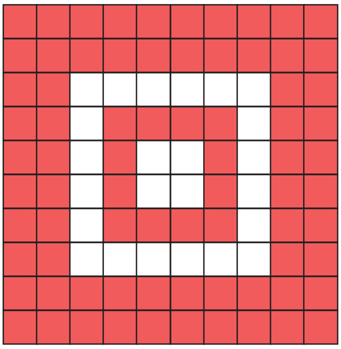
6 Which passenger ship offers the cheapest fare to go from Honiara to Auki?
MV Renbel: $24 000 for a full load of 200 passengers or MV Bikoi: $20 700 for a full load of 230 passengers?

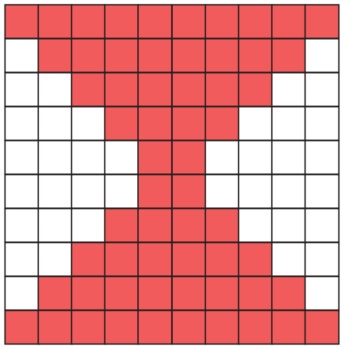
6C Exploring percentages

Percentages are a useful way of comparing quantities. Per cent means 'per hundred' and the symbol is %.

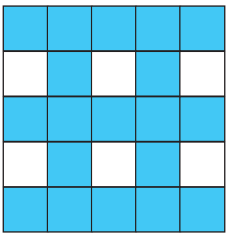
Learning task 6C

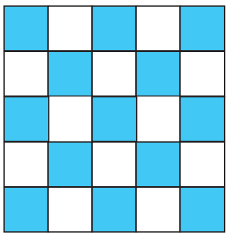
1 Count the number of red squares below. This is the percentage shaded, as there are 100 squares:

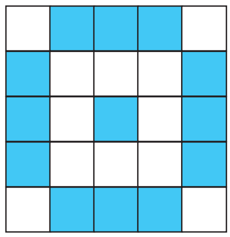
a


b


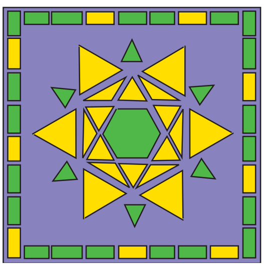
2 Find the percentage of each of these grids which is shaded blue:

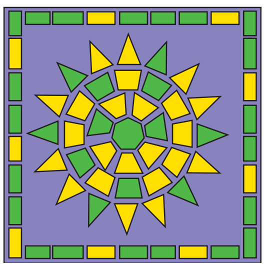
a


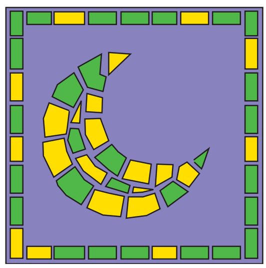
b


c


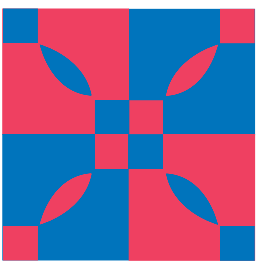
3 The tiles below show a geometric design with green, yellow and purple shapes. Estimate what percentage of each tile is shaded yellow:

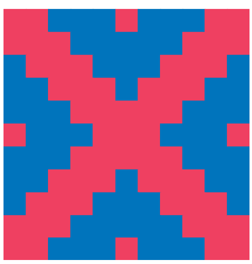
a


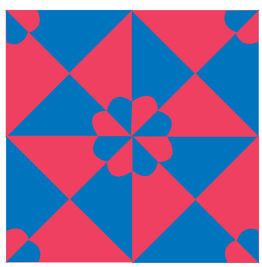
b


c


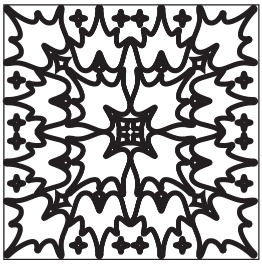
4 The tiles below show a geometric design with red and blue shapes. Estimate what percentage of each tile is shaded red:

a


b


c


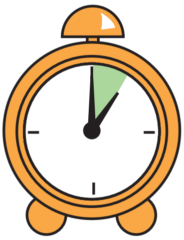
5 The tiles below show a geometric design in black and white. Estimate what percentage of each tile is coloured black:

a 

b 

c 

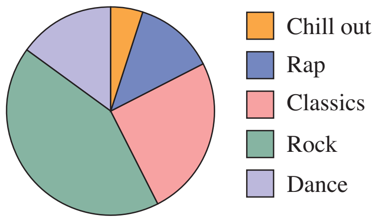
6 Estimate what percentage of each clock face below is shaded green:

a 

b 

c 

7 The pie chart shows the results of a survey on the music tastes of a Year 9 class.

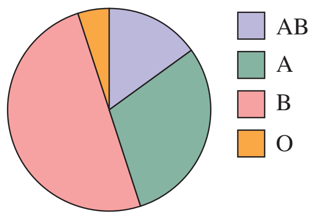


a Which type of music was preferred by the following percentage of learners?

i 25% **ii** 15% **iii** 5%

b List the types of music from most to least favourite.

8 The pie chart shows the number of litres of blood used in the emergency ward of a hospital during one day.



a Which type of blood corresponds to the percentages used below?

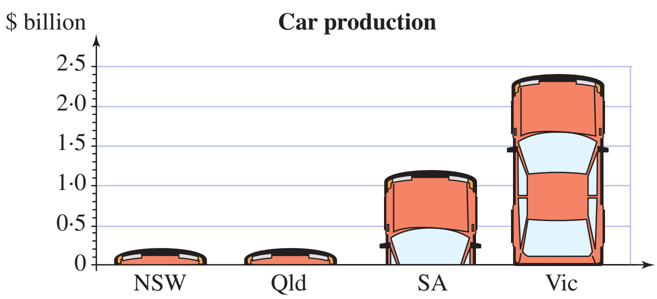
i 50% **ii** 15% **iii** 5%

b List the blood types in order from most to least used.

9 The diagram shows the amount of revenue coming from car production, in the Australian states of NSW, Qld, SA and Vic.

a Draw a table to show your estimates of the percentage that goes to each state. Make sure that the percentages add up to 100%.

b Assuming the total car production is worth $4 billion, add a column to the table to show how much money, in billions of dollars, comes from each state.



6D Converting percentages

Per cent (%) means 'per hundred'. To convert a fraction or decimal to a percentage, express it as a fraction of 100 or simply multiply it by 100.

$$\frac{1}{2} = \frac{50}{100} = 50 \text{ per } 100 = 50\%$$

Example / **Solution**

1 Convert the fraction to a percentage:

a $\frac{7}{10}$ — $\frac{7}{10} \times \frac{10}{10} = \frac{70}{100} = 70\%$

b $4\frac{1}{2}$ — $4\frac{1}{2} = \frac{9}{2} \times \frac{50}{50} = \frac{450}{100} = 450\%$

2 Convert 0·22 to a percentage. — $0{\cdot}22 \times 100 = 22\%$

To convert a percentage to a fraction or a decimal, divide by 100.

Example / **Solution**

3 Convert to a fraction:

a 14% — $14\% = \frac{14}{100} = \frac{7}{50}$

b $12\frac{1}{4}\%$ — $12\frac{1}{4}\% = \frac{12{\cdot}25}{100} = \frac{49}{400}$

4 Convert 87% to a decimal. — $87\% = \frac{87}{100} = 0{\cdot}87$

Exercise 6D

1 Change the following fractions to percentages:

a $\frac{3}{10}$ **b** $\frac{2}{25}$ **c** $\frac{1}{4}$ **d** $\frac{7}{20}$ **e** $\frac{9}{50}$ **f** $\frac{19}{20}$

g $\frac{7}{10}$ **h** $\frac{17}{25}$ **i** $\frac{4}{5}$ **j** $\frac{125}{200}$ **k** $\frac{3}{4}$ **l** $\frac{3}{2}$

2 Change the following decimals to percentages:

a 0·45 **b** 0·74 **c** 0·34 **d** 0·23 **e** 0·19 **f** 0·55

g 0·643 **h** 0·512 **i** 0·09 **j** 0·87 **k** 0·94 **l** 0·07

3 Change the following percentages to fractions:

a 25% **b** 96% **c** 37% **d** 80% **e** 49% **f** 33%

g 50% **h** 44% **i** 16% **j** 21% **k** 76% **l** 65%

4 Change the following percentages to decimals:

a 75% **b** 84% **c** 21% **d** 16% **e** 10% **f** 35%

g 51% **h** 64% **i** 8% **j** 47% **k** 92% **l** 59%

Finding percentages of quantities 6E

When you put money in the bank, you are paid for the use of your money. This is called **interest**.

If you borrow money, the bank will charge you for the privilege of borrowing it. This charge is also called **interest**.

Example

Fiona has $1200 in the bank. If the bank pays Fiona 3% interest, find how much interest she will receive.

Solution

$\frac{3}{100} \times 1200$

$= \$36$

Exercise 6E

1 Find the following:

a 8% of $350 b 2% of $500 c 7% of $200 d 1% of $1000

e 5% of $4500 f 14% of $12 000 g $5\frac{1}{2}$% of $7000 h $\frac{1}{2}$% of $6725

2 Find how much interest is paid on the following amounts at the given interest rate:

a $50 000 at 12% b $35 000 at 8% c $42 500 at 9%

d $82 900 at 11% e $110 000 at $5\frac{1}{2}$% f $155 000 at $5\frac{3}{4}$%

g $96 000 at $6\frac{1}{4}$% h $100 000 at $14\frac{1}{2}$% i $122 000 at $8\frac{1}{4}$%

3 The Walande Women's Community Group (WWCG) owes $86 520 on a loan. Their bank charges them 6% interest per annum (a year).

a Calculate how much the interest payment would be per year.

b How much interest would the Marmalades pay per month?

4 Mrs Tatalo owes $2902·40 on her bankcard.

a If the interest rate is 15·5%, how much interest will she pay per year?

b Interest is calculated daily at 0·042 46% and charged monthly. How much will Mrs Tatalo pay for a 30-day month?

c If she pays $500 off her bankcard bill on 15 July, how much interest will she pay for that month?

5 Brad's bank charges 15·9% per annum and 0·043 56% per day on his credit card debt.

a How much interest per year will he pay if he owes $4250?

b What is the interest charged on February's account in a non-leap year?

c Brad can pay $1000 off his bill on either 5 April or 19 April after receiving his wages. How much money will he save by paying the credit card on 5 April?

6 Some children's bank accounts offer bonus or better interest rates if some money is deposited each month without withdrawing any money in that month. Emily's bank account pays 0·1% interest per annum or 2·5% as a bonus interest rate per annum.

a Convert these rates to daily rates.

b If Emily has $220 in the bank at the start of October and makes no deposits, how much interest will she earn?

6F Expressing quantities as a percentage

Example

1 a Find \$420 as a percentage of \$21 000.

b Find 16 centimetres as a percentage of 10 metres.

Solution

$\frac{420}{21\,000} \times \frac{100}{1} = 2\%$

$\frac{16}{1000} \times \frac{100}{1} = 1{\cdot}6\%$

If we know the percentage of a quantity, we can find the original amount.

Example

2 Johan earned \$48 interest on his bank account. If the interest rate is 3%, how much money does he have in the bank?

Solution

Method 1

3% is \$48
1% is \$48 ÷ 3
= \$16
100% is \$16 × 100
= \$1600

Method 2

$3\% \text{ of } x = 48$

$\frac{3}{100} \times x = 48$

$x = 48 \times \frac{100}{3}$

$x = \$1600$

Exercise 6F

1 Find the first quantity as a percentage of the second:

a 8 of 16 b 24 of 96 c 15 of 75 d 20 of 200
e 14 of 2800 f 1 of 100 g 8 of 48 h 7 of 84

2 Find the first number as a percentage of the second number:

a 20 cm of 100 cm b 2·5 kg of 25 kg c 15 min of 90 min d 8 L of 96 L
e 45 m of 900 m f 5° of 360° g \$50 of \$2000 h 8c of 1200c

3 Find 100% if:

a 6% is 30 b 36% is 1800 c 10% is 64 d 88% is 308
e 65% is 546 f 25% is 1502 g 24% is 2064 h 2% is 1·70

4 Find the bank balance if:

a 4% is \$320 b 8% is \$36 c 5% is \$25 d 3% is \$30·15
e 9% is \$5580 f $2\frac{1}{4}$% is \$11·25 g $3\frac{1}{2}$% is \$4375 h 4·75% is \$298·30

5 John Bakeua earns \$9·70 in interest on a bank balance of \$2112·50. What is the interest rate?

6 The Siosi family pays \$548 interest on their home loan balance of \$94 320 during the month of August.

a What is the interest rate per month on their home loan?
b What is the annual interest rate on their home loan?

Percentage change 6G

Discounts are offered by businesses for many reasons. Sometimes it is an incentive for people to pay their bills early; at other times businesses offer discounts to sell old stock or to encourage people to come into their stores.

$$\text{Percentage increase} = \frac{\text{increase}}{\text{original}} \times \frac{100}{1}\%$$

$$\text{Percentage decrease} = \frac{\text{decrease}}{\text{original}} \times \frac{100}{1}\%$$

Example

a The Card Company increases its number of sales outlets from 300 to 330. What is the percentage increase?

b It offers a discount of 5% for accounts that are paid within 14 days. Mrs Smith's Corner Store buys $6000 worth of cards. How much can she save by paying within 14 days?

Solution

a Increase = 330 − 300 = 30

$$\frac{30}{300} \times \frac{100}{1} = 10\%$$

b
$$\frac{5}{100} \times \frac{6000}{1} = \$30$$

Exercise 6G

1 Find the percentage increase when 60 changes to:

a 90 **b** 105 **c** 75 **d** 80 **e** 70 **f** 100

2 Find the percentage decrease when 80 changes to:

a 60 **b** 50 **c** 10 **d** 12 **e** 45 **f** 70

3 A department store offers its staff a 10% discount on all purchases.

a How much will Ethan pay for jeans priced at $850?

b Nicole purchases perfume for $540 for her mother's birthday; how much will she pay?

c Goran purchases a refrigerator with a price tag of $8750; how much will he pay?

4 The health insurance premiums for the Nayan family have risen from $8700 to $12 600. Find the percentage increase in the premiums.

5 Soley's shares have risen from $56·00 to $70·50 in the last 12 months.

a Find the percentage increase.

b Find the percentage increase if Soley purchased the shares for $49·80.

6 A department store offers 15% discount on all CDs.

a How much will a $320 CD cost with the discount?

b Find the sale price for an $189·50 CD.

c A double disk set is normally $549·50; what will be its sale price?

7 A shop is having a sale on fabric. Find the sale price of the following fabrics:

a Denim was $89·50 and has been reduced by 15%.

b Gingham has a 23% discount on its retail price of $29·50.

c Poly-cotton prints have been reduced by 28% from $35·00.

6H Profit and loss

At regular intervals, all businesses compare their **income** (the money that has come into the business), with their **expenditure** (the money that has been paid out by the business). A company needs to make a profit to stay in business.

- If the income is greater than the expenditure then a **profit** has been made.
- If the income is less than the expenditure then a **loss** has been made.

Example

Is a company with income of $462 980 and expenditure of $478 960 making a profit or loss?

Solution

$462 980 < $478 960

income < expenditure

→ loss

The profit or loss can be placed over the original cost to give a percentage profit or loss.

$$\text{Percentage profit} = \frac{\text{profit made}}{\text{original cost}} \times 100\% \qquad \text{Percentage loss} = \frac{\text{loss made}}{\text{original cost}} \times 100\%$$

Exercise 6H

1 Calculate the profit on each of the following:

a A bike costs $1090 and is sold for $1590.

b Honey costs $90 per tub and is sold for $132·90 per tub.

c An orchid plant costs $43·00 and is sold for $69·90.

d A toaster that retails for $790 was purchased for $420.

2 Calculate the loss on each of the following items:

a Jeans which cost $650 are sold for $500.

b A paperback that cost $270 is sold for $229·50.

c Mangoes purchased at the supermarket for $20 per kilogram are sold for $15 per kilogram.

d A jumper that was originally priced at $480 and is now for sale at $420.

3 The owner of a nursery buys 60 orchids for $2394 and sells them for $69·90 each.

a Find the profit made on each orchid plant.

b How much profit is made if all the orchid plants are sold at full price?

4 The owner of a hardware store makes a $250 profit on each toolbox sold. If a toolbox is sold for $1890, find the price paid for it by the store owner.

5 A car was bought for $189 990 and sold for $147 590. State the loss that was made.

6 Find the percentage profit or loss in the following situations:

a Buy $4000, sell $6500

b Buy $1500, sell $2000

c Buy $7500, sell $6000

d Buy $2200, sell $1400

7 A car is bought for $35 000 and sold for $74 000; find the percentage profit made.

8 A gift shop purchases 50 ornaments for $32·50 each and sells them for $55·00.

a Find the profit on each ornament.

b Calculate the total profit.

c If 10% of the ornaments are sold at half price due to damage, calculate the profit.

Commission 6I

Commission is a form of payment for sales achievements, which is based upon a percentage of the value of items or goods sold. Some people receive a base wage plus commission

Example

A car salesperson receives 5% commission on all car sales. If she sells cars to the value of $64 800 this month, what will her wage be?

Solution

$\frac{5}{100} \times 64\,800 = \$3240$

Exercise 6I

1 Calculate the commission that is paid in each of these situations:

- **a** 5% on $23 600
- **b** 3% on $870 000
- **c** 9% on $12 400
- **d** 15% on $3200
- **e** $12\frac{1}{4}$% on $550·90
- **f** 12% on $100·50
- **g** 18% on $390
- **h** 7·8% on $1450
- **i** 5·6% on $5200

2 Jenly sells children's toys to earn extra money. She earns 6% commission on her sales. Calculate how much she earns each month if she sells toys to the value of:

- **a** $8000
- **b** $12 500
- **c** $38 070
- **d** $56 300

3 A real-estate agent is paid a base wage of $1500 per week and receives 3% commission on all his sales. Calculate his weekly wage, if he makes the following sales over a four-week period:

- **a** $1 890 000
- **b** $7 975 000
- **c** $4 507 890
- **d** $6 539 000

4 A publishing company pays a writer a royalty of 10% of all sales.

- **a** Calculate the income paid to the writer if the book is priced at $350 and the publishing company sells the following number of copies per month:
  - **i** 90
  - **ii** 157
  - **iii** 489
  - **iv** 893
- **b** If the price of the book is increased to $400, calculate the increased income for the writer:
  - **i** per book sold
  - **ii** per 100 books sold
  - **iii** per 500 books sold

5 Mostyn sells insurance on commission at a rate of $12\frac{1}{2}$%.

- **a** Last week Mostyn made sales to the value of $14 600. How much was he paid?
- **b** If Mostyn is hoping to make sales to the value of $23 000 this week, how much will his wage be?
- **c** Mostyn is aiming to earn at least $15 000 each week; what value of insurance must he sell to reach his target?

6 Samson is employed by a real-estate business. He has the option of two payment methods. He can be paid a fixed salary of $6500 per week or a base wage plus commission. The base wage is $1500 per week plus $\frac{1}{5}$% commission on sales up to and including $10 000 000 in a week, and 1% commission on all sales over $10 000 000 in a week.

If Samson's projected sales over the next 8 weeks are as follows, which payment method should he choose?

Week 1	$2 500 000	Week 5	$400 000
Week 2	$1 200 000	Week 6	$1 357 000
Week 3	$4 000 000	Week 7	$12 million
Week 4	$8 700 000	Week 8	$7 689 000

6J Wages and tax scales

Most full-time workers are paid a salary, which is based upon a yearly amount, and paid weekly, fortnightly or monthly. People with part-time or casual positions are paid according to the number of hours they work and the hourly rate.

The government decided that from 2012, Income Tax would be deducted according to the following tax rates:

- The first $15 080 of income would be exempt from tax.
- The balance of income above $15 080 is then taxed at the rates in this table:

Taxable income	Tax rate
$1–$15 000	11%
$15 001–$30 000	23%
$30 001–$60 000	35%
$60 000 and over	40%

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Inland Revenue Division Solomon Islands

Example

Calculate the total tax payable by a person who earns an annual salary of $50 000.

Solution

Gross Annual Salary	$50 000
Less personal exemption	$15 080
Income chargeable to tax	$34 920
The first $15 000 is taxed at 11%	$1650
The second $15 000 is taxed at 23%	$3450
The balance of $4920 is taxed at 35%	$1722
Total tax payable	$6822

Exercise 6J

1 Commins earns $36 400 per year; calculate how much he earns each:

a month **b** fortnight **c** week

2 Snyder earns $12·45 per hour and works 40 hours each week. Calculate how much Snyder earns each:

a week **b** fortnight **c** year

3 **a** Claire earns $38 950 as a consultant. How much tax must she pay?

b Alexander earns $18 534 working part-time while at university.

i If he has paid $2187 in tax, will he get a refund or have to pay more tax?

ii How much refund or tax will be paid?

c Elijah works approximately 6 hours per week at a local café clearing tables and doing dishes. He is paid $8·45 per hour.

i How much will he earn for the year? **ii** How much tax must he pay?

d Aliya earns $72 480 as an area manager for her company. How much tax must she pay?

e If Soria earns $103 489 per year, how much tax must she pay?

4 Hypolite works for 6 months as a labourer and earns $16 320. He works in the supermarket for the remainder of the year and is paid $11 520. Find how much tax he must pay:

a as a labourer **b** for working in the supermarket **c** in total

Simple interest

When you borrow money, you have to pay for the privilege of being able to use money that belongs to another person, a bank or a lending institution. One method of calculating how much you have to pay for the privilege is called **simple interest**. When you borrow money, called the **principal**, you are charged a fee, called the **interest**. The amount of interest is determined by the **rate** (R) in per cent per year and the **term** or **time** (T) in years.

> Simple interest = principal × rate × term
>
> $$I = \frac{PRT}{100}$$

Example

1 Calculate the simple interest charged on a loan of $25 000 at 9% per annum for 5 years.

2 Calculate the simple interest charged on a loan of $25 000 at 9% per annum for 6 months.

Solution

1

$$I = \frac{PRT}{100}$$
$$= 25\,000 \times \frac{9}{100} \times 5$$
$$= \$11\,250$$

$P = \$25\,000$
$R = 9\%$ p.a.
$T = 5$ years

2

$$I = \frac{PRT}{100}$$
$$= 25\,000 \times \frac{9}{100} \times 0{\cdot}5$$
$$= \$1125$$

$P = \$25\,000$
$R = 9\%$ p.a.
$T = 6$ months
$= 0{\cdot}5$ year

Exercise 6K

1 Find the simple interest paid when (p.a. means per annum):

- **a** $P = \$1000$, $R = 5\%$ p.a., $T = 2$ years
- **b** $P = \$5600$, $R = 2{\cdot}4\%$ p.a., $T = 5$ years
- **c** $P = \$20\,000$, $R = 1{\cdot}82\%$ p.a., $T = 6$ months
- **d** $P = \$400$, $R = 6{\cdot}2\%$ p.a., $T = 3$ months
- **e** $P = \$16\,840$, $R = 12{\cdot}5\%$ p.a., $T = 5$ years
- **f** $P = \$11\,340$, $R = 8{\cdot}75\%$ p.a., $T = 1\frac{1}{2}$ years

2 Find the simple interest on a principal of $5000 at 6% per annum for 3 years.

3 Calculate the simple interest on a principal of $12 400 at 7% per annum for 6 years.

4 Calculate the simple interest on a principal of $30 000 at $5\frac{1}{2}\%$ per annum for 10·5 years.

5 How much simple interest would you pay on a principal of $2000 at 8% per annum borrowed for 2 months?

6 Calculate the simple interest would you pay on a principal of $16 500 for 5 years if the interest rate is:

- **a** 3% p.a.
- **b** 5% p.a.
- **c** 11% p.a.
- **d** 18% p.a.

7 How much simple interest is paid on the following investments?

- **a** $140 000 at 8% p.a. for 10 years
- **b** $28 500 at 5% p.a. for 12 years
- **c** $8900 at 6·5% p.a. for 6 years
- **d** $96 000 at 7·2% p.a. for 8 years
- **e** $47 500 at 3·25% p.a. for 5 years
- **f** $5750 at 5·8% p.a. for 3 years

Puzzles

1 Find the following percentages, then match the letter to the correct answer below to find the name of the puzzle:

They associate me with a radio station.

A 10% of $150 **B** 20% of 160 **E** 12% of $450

F 5% of $400 **G** 10% of $370 **H** 35% of $100

I 2% of $450 **N** 2% of $200 **O** 8% of $150

R 25% of $204 **S** 75% of $48 **T** 20% of $50

Y 37·5% of $80 **V** 2·5% of $1000 **W** 1·5% of $200

___	___	___	___	
$35	$15	$25	$30	
___	___	___	___	___
$9	$36	$32	$54	$36

2 Calculate the percentage increase from the first number to the second, then match the letter to the correct answer below to find the name of the puzzle:

YOU CANT HAVE IT

TI ƎVAH TNAↃ UOY

Letter	First number	Second number	Letter	First number	Second number
A	100	150	**T**	300	309
B	10	12	**S**	100	105
C	20	25	**O**	50	65
E	50	52	**U**	75	120
H	36	63	**V**	10	20
I	80	88	**W**	500	530
N	500	510	**Y**	200	202

___ ___ ___   ___ ___ ___ ___   ___ ___ ___ ___

1% 30% 60%   25% 50% 2% 3%   75% 50% 100% 4%

___ ___   ___ ___ ___ ___   ___ ___ ___ ___

10% 3%   20% 30% 3% 75%   6% 50% 1% 5%

Applications

Budgeting

A budget is important to ensure that you don't spend more than you earn.

The Tully family has a weekly net income of \$1117. The family's expenses are listed in the following table.

Home loan repayment	\$600 per fortnight	Insurance	\$1200 per year
Electricity	\$155 per month	Food	\$150 per week
Telephone	\$154 per month	Petrol	\$110 per week
Car loan repayments	\$165 every two months	Car repairs	\$60 per month
Rates	\$898 per year	Miscellaneous expenses	\$80 per week
Mobile phone	\$30 per month	Credit card payment	\$200 per month

a Set up a spreadsheet showing their income, expenditure and profit or loss for the month.

b How much money can they save each month?

c What percentage of their income is:

i spent repaying their home loan? **ii** spent on food? **iii** saved?

If they want to go on a holiday, they must save 7·5% of their income each month.

d How much will they have to spend on a holiday at the end of the year?

e Develop a budget for your family for a week.

Fundraising for a cause

The Year 9 learners decide to sponsor a child for \$33 per month.

a How much money will they need to raise for the year?

b If 80% of the money goes to the child, calculate how much money goes overseas per month and per year.

c List ways in which your year level could raise money for the sponsorship of a child.

d It is decided to hold a casual clothes day to raise money for the sponsorship. Find out how many learners are in your year level and how much money you would raise if:

- **i** all learners wear casual clothes and pay \$1
- **ii** 90% of learners wear casual clothes and pay \$1
- **iii** 85% of learners wear casual clothes and pay \$2

Five Year 9 classes raise all of the sponsorship money of \$33 per month.

e How much money did each class raise for the year?

f Year 9B decide to wash teachers' cars at lunchtime to raise the money for the term. If they charge \$3 to wash a sedan and \$5 to wash a four-wheel drive, how many of each type of car must they wash to raise the money? Write down all the possible options. Which option is best and why?

Enrichment

1 One method of calculating how much interest you will be paid on a term deposit is called **simple interest**. The simple interest is determined by the amount of money you invest, which is called the **principal**, the **rate**, and the length of time for which you invest the money, called the **term** or **time**.

$$\text{Simple interest} = \text{principal} \times \text{rate} \times \text{term}$$

$$I = \frac{PRT}{100}$$

Example

Calculate the simple interest paid on a deposit of \$35 000 at 9% per annum for 5 years.

Solution

$$I = \frac{PRT}{100}$$
$$= 35\,000 \times \frac{9}{100} \times 5$$
$$= \$15\,750$$

a Find the simple interest paid on a principal of \$5000 at 6% per annum for 3 years.

b Calculate the simple interest paid on a principal of \$12 400 at 7% per annum for 6 years.

c Calculate the simple interest paid on a principal of \$30 000 at $5\frac{1}{2}$% per annum for 10 years.

2 **Compound interest** can also be paid. In this case the interest is added to the principal, therefore increasing the balance invested. This increases the value of the next interest calculation if the principal is not reduced. Compound interest is usually calculated more than once per year.

Principal after compounding: $A = P\left(1 + \frac{r}{100}\right)^n$

where P = principal
r = rate for the time period
n = the number of time periods
A = the principal plus interest.

Example

Calculate the interest paid on a principal of \$5000 if the rate is 10% compounded quarterly for 5 years.

Solution

$$A = 5000\left(1 + \frac{2{\cdot}5}{100}\right)^{20}$$
$$= 5000(1{\cdot}025)^{20}$$
$$= \$8193{\cdot}08$$

$$\text{Interest paid} = 8193{\cdot}08 - 5000$$
$$= \$3193{\cdot}08$$

a Calculate the interest paid on \$3000 invested at 12% compounded twice yearly over 5 years.

b Calculate the interest paid on \$10 500 invested at 10% compounded twice yearly over 2 years.

c Calculate the interest paid on a \$7000 loan compounded each month at 10% p.a. over 5 years.

d Calculate the interest paid on a \$7000 loan compounded each month at 10% p.a. over 3 years.

e How much more interest is paid on a \$7000 loan when it is taken for 5 years instead of 3 years?

Revision/Assessment

Exercise 6A

1 The table below shows the amount Santo spends each year.

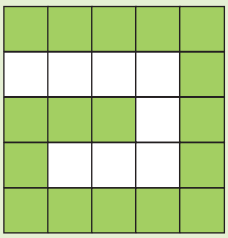
Costs	Food	Clothing	Rent	Entertain-ment	Car	Holidays	Insurance	Tax
Amount $	7600	2500	6400	3200	4560	540	3200	10 000

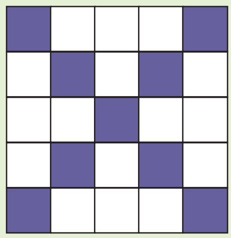
a Find the total amount he spends each year.

b If Santo earns $60 000 a year, how much does he save each year?

Learning task 6C

2 Calculate the percentage of each grid below that is shaded:

a 

b 

Exercise 6D

3 Convert these decimals to percentages:

a 0·5 b 2·5 c 0·1 d 0·05 e 0·444 f 2·1919

g 5·66 h 0·97 i 0·003 j 0·7 k 2·1 l 2·405

4 Convert these fractions to percentages:

a $7\frac{1}{2}$ b $5\frac{1}{4}$ c $2\frac{3}{4}$ d $4\frac{4}{5}$

e $\frac{3}{8}$ f $\frac{5}{6}$ g $\frac{1}{3}$ h $\frac{5}{12}$

5 Pick the odd one out:

a $\frac{1}{4}$, 0·5, 50% b $\frac{1}{3}$, 0·3, $33\frac{1}{3}$% c 10%, 0·1, $\frac{3}{20}$

d 0·8, $\frac{4}{5}$, 8% e 0·25, $\frac{1}{5}$, 25% f 0·5, 5%, $\frac{1}{20}$

Exercise 6E

6 Find the following:

a 10% of $350 b 2% of $600 c 6% of $300

d 1% of $10 000 e 5% of $1500 f 5% of $12

g $5\frac{1}{2}$% of $200 h $\frac{1}{2}$% of $5000 i $\frac{1}{4}$% of $850

7 The following learners gained these marks on a Year 9 test. If the test was out of 60 marks, calculate each learner's percentage:

a Mary-Jane 42 marks
b Cameron 33 marks
c Adrian 45 marks
d Fergus 55 marks
e Janelle 58 marks
f Susie 28 marks

8 If Emerson gained 78% on a test out of 90 marks, how many marks did he gain? (Give your answer to the nearest whole number.)

Exercise 6F

9 Find the first as a percentage of the second:

a 8 of 40
b 1 of 50
c 15 of 500
d 1·2 of 6·4

10 Sheu gained 43 marks on his test, which gave him 64%. How many marks were on the test?

11 Find 100% if:

a 5% is 20
b 10% is 15
c 75% is 64
d 12·5% is 5

Exercise 6G

12 If Clay's bank account of \$904·20 grew to \$1356·30, calculate the percentage increase.

Exercise 6H

13 Ricky bought a car for \$40 110 and sold it for \$33 425. Calculate the percentage loss.

Exercise 6I

14 Calculate the commission that is paid in each of these situations:

a 2% on \$23 500
b 1% on \$570 000
c 10% on \$12 400
d 25% on \$224
e 5% on \$650
f 12% on \$1000

Exercise 6J

15 Helen works at a department store for 10 hours every week and earns \$12·80 per hour.

a Calculate Helen's weekly income.
b Calculate Helen's annual income.
c How much tax does Helen pay for the year?
d How much tax would be taken out of her wage each week?
e How much money does Helen have to spend each week?

Exercise 6K

16 Find the simple interest on a loan of \$5000 at 2% per annum for 5 years.

17 Calculate the simple interest on a principal of \$12 800 at 5% per annum for 10 years.

Answers

These are selected answers only.

Chapter 1

Exercise 1A

1 **a** 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
b $^-1$, $^-2$, $^-3$, $^-4$, $^-5$, $^-6$, $^-7$, $^-8$, $^-9$, $^-10$

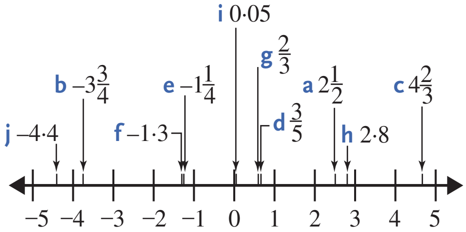
2 1, 2, 3, 4, 5, 6, 7, 8

3 **a** Examples: $\sqrt{2}$, $\sqrt{3}$, $2\sqrt{2}$, $\sqrt{5}$, $\sqrt{7}$
b Examples: π, e, $\frac{\pi}{2}$, $2e$, 5π

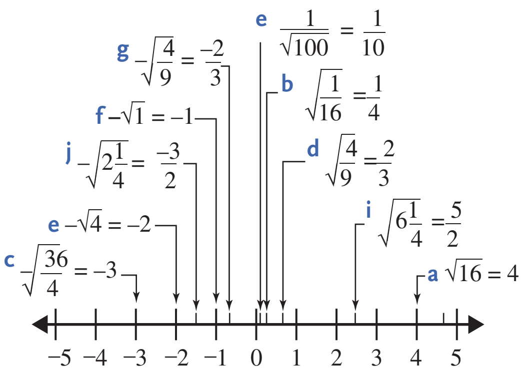
4 **a** 1275 **b** $^-1275$

5 **a** True **b** False **c** True
d False **e** True

6



7



Exercise 1B

1 **a** 6, 2 **b** 10, $\frac{5}{2}$ **c** 15, 3

2 **a** $^-15$ **b** $^-28$ **c** $^-55$

3 4, 5, 6 **4** 7, 8, 9, 10

5 **a** 55 **b** 210 **c** 1275
d 5050 **e** 501 501

6 82 **7** $^-20$

8 **a** 50 **b** 33 **c** 25 **d** 20 **e** 16
f 14 **g** 12 **h** 11 **i** 10 **j** 8

10 **a** 33 333, 66 666, 12 321
b 11 144, 12 364, 91 796
c 66 666, 12 342, 48 138
d 33 128, 54 152, 77 448
e 99 999, 32 112, 94 554
f 61 210, 78 430, 96 190
g 22 464, 98 232, 17 628
h 63 216, 41 784, 27 936
i 33 315, 71 820, 53 640
j 34 128, 47 664, 72 576

11 **a** 2, 3, 5 **b** 4, 9, 25 **c** 6, 8, 10 **d** 12, 18, 20

12 **a** 180 **b** 210 **c** 300 **d** 630 **e** 1575

13 **a** $2^4 \times 3$ **b** $2^3 \times 3^3$ **c** $2 \times 3^2 \times 5^3$
d $2^2 \times 3^2 \times 5^2$ **e** $2^3 \times 3^2 \times 7^2$

14 **a** 12 **b** 16 **c** 24

15 **a** $^-10$ **b** $^-20$ **c** $^-5$ **d** 12
e $^-44$ **f** $^-84$ **g** 42 **h** $^-27$
i 7 **j** $^-15$ **k** 4 **l** 35

16 **a** $^-10$ **b** 45 **c** $^-46$ **d** 51 **e** 35
f 11 **g** 35 **h** $^-27$ **i** 17 **j** 44
k 15 **l** $^-14$ **m** 10 **n** $^-26$

17 Katy is 5 and Lencia is 10 years old.

18 **a** 25 983 **b** 2589 **c** 5897
d 30 357 **e** $25\,874\frac{1}{6}$ **f** $58\,025\frac{2}{11}$

19 **a** 146 **b** 12 702 **c** 1519
d 560 **e** 10 255 **f** 272 185

20 **a** $^-2536$ **b** 912 **c** 9
d 45 **e** $^-5$ **f** 8

21 273 696 grams

Exercise 1C

1 **a** $^-10$ **b** 16 **c** 25 **d** 47

2 **a** $^-9$ **b** $^-7$ **c** 18 **d** 15

3 **a** 11 **b** 3 **c** 50 **d** 14

4 **a** 112 **b** $^-208$ **c** 108 **d** 34

5 **a** 40 **b** $^-60$ **c** $^-100$ **d** 91

6 **a** 0 **b** $^-8$ **c** $^-24$ **d** $^-80$

7 **a** $^-4$ **b** $^-148$ **c** $^-598$ **d** $^-484$

Exercise 1D

1 **a** 12 **b** 14 **c** 18 **d** 6
e 21 **f** 21 **g** 35 **h** 18

2 **a** $\frac{13}{4}$ **b** $\frac{11}{8}$ **c** $\frac{30}{7}$ **d** $\frac{49}{4}$
e $\frac{17}{3}$ **f** $\frac{61}{7}$ **g** $\frac{19}{5}$ **h** $\frac{22}{9}$

3 a $3\frac{2}{3}$ b $2\frac{3}{5}$ c $13\frac{2}{3}$ d $10\frac{3}{5}$
e $55\frac{1}{2}$ f $8\frac{3}{4}$ g $17\frac{5}{7}$ h $48\frac{1}{3}$

4 a 2 b $2\frac{1}{2}$ c 3

5 a $\frac{5}{6}$ b $\frac{13}{20}$ c $\frac{1}{3}$ d $3\frac{17}{21}$
e $\frac{7}{12}$ f $1\frac{9}{10}$ g $5\frac{17}{20}$ h $2\frac{5}{12}$

6 a $1\frac{1}{12}$ b $2\frac{13}{60}$ c $1\frac{11}{12}$ d $\frac{7\pi}{10}$
e $\frac{5\pi}{6}$ f $\frac{7\pi}{12}$ g $\frac{11\pi}{12}$ h $\frac{23\pi}{12}$

7 a $\frac{17}{40}$ b $\frac{7}{12}$ c $\frac{3}{14}$ d $\frac{\pi}{4}$
e $\frac{\pi}{12}$ f $\frac{\pi}{20}$ g $\frac{5\pi}{12}$ h $\frac{\pi}{15}$

8 a $1\frac{2}{3}$ b $\frac{3}{22}$ c $8\frac{1}{4}$ d $6\frac{1}{8}$
e $19\frac{1}{4}$ f $9\frac{3}{10}$ g $^{-}11\frac{1}{6}$ h $20\frac{25}{32}$

9 a

+	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{7}$
$\frac{1}{2}$	$\frac{5}{6}$	$\frac{3}{4}$	$\frac{7}{10}$	$\frac{9}{14}$
$\frac{2}{3}$	1	$\frac{11}{12}$	$\frac{13}{15}$	$\frac{17}{21}$
$\frac{3}{4}$	$1\frac{1}{12}$	1	$\frac{19}{20}$	$\frac{25}{28}$
$\frac{3}{5}$	$\frac{14}{15}$	$\frac{17}{20}$	$\frac{4}{5}$	$\frac{26}{35}$

b

×	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{2}{5}$
$\frac{1}{4}$	$\frac{1}{6}$	$\frac{3}{16}$	$\frac{1}{5}$	$\frac{1}{10}$
$\frac{1}{3}$	$\frac{2}{9}$	$\frac{1}{4}$	$\frac{4}{15}$	$\frac{2}{15}$
$\frac{2}{3}$	$\frac{4}{9}$	$\frac{1}{2}$	$\frac{8}{15}$	$\frac{4}{15}$
$\frac{4}{5}$	$\frac{8}{15}$	$\frac{3}{5}$	$\frac{16}{25}$	$\frac{8}{25}$

10 a $\frac{1}{2}$ b $\frac{2}{5}$ c $\frac{1}{5}$ d $1\frac{1}{2}$ e 6
f $\frac{1}{2}$ g $1\frac{1}{5}$ h $1\frac{1}{3}$ i $\frac{1}{11}$

11 a $3\frac{1}{3}$ b $2\frac{1}{4}$ c 2

12 a 7 b $\frac{5}{9}$ c $\frac{3}{5}$

13 a $\frac{5}{8}$ b $\frac{1}{3}$ c $\frac{13}{30}$ d $\frac{21}{40}$

14 $a = \frac{10}{23}$

15 \$7·50

16 2500 litres

17 2000 litres

18 $27\frac{1}{3}$ litres

19 a $\frac{1}{2}, \frac{2}{5}$ b $\frac{4}{5}, \frac{1}{10}$ c $\frac{1}{3}, \frac{2}{3}$ d $\frac{1}{4}, 4$

20 a 2 b $\frac{1}{2}$ c $\frac{1}{20}$ d $\frac{2}{5}$

Exercise 1E

1 a 0·625 b 0·875 c $0{\cdot}\overline{45}$
d $0{\cdot}41\dot{6}$ e $0{\cdot}58\dot{3}$

2 $\{0{\cdot}\dot{1}, 0{\cdot}\dot{2}, 0{\cdot}\dot{3}, 0{\cdot}\dot{4}, 0{\cdot}\dot{5}, 0{\cdot}\dot{6}, 0{\cdot}\dot{7}, 0{\cdot}\dot{8}, 1\}$

3 $\{0{\cdot}\overline{09}, 0{\cdot}\overline{18}, 0{\cdot}\overline{27}, 0{\cdot}\overline{36}, 0{\cdot}\overline{45}, 0{\cdot}\overline{54}, 0{\cdot}\overline{63}, 0{\cdot}\overline{72}, 0{\cdot}\overline{81}, 0{\cdot}\overline{90}, 1\}$

4 a $\frac{111}{99}$ b $\frac{122}{99}$ c $\frac{340}{99}$ d $\frac{67}{11}$

5 a 0·9023 b 0·082 c 0·345 d 0·4302
e 649·81 f 0·4538 g 103·34 h 38·2

6 a 1·3456 b 4·562 c 0·000 256 3
d 2·173 e 8 f 12
g 90 h 8

7 a $0{\cdot}\dot{8}$ b $8{\cdot}\dot{8}$ c $0{\cdot}00\dot{8}$ d $0{\cdot}00\dot{8}$

8 a 49·234 b 213·407 c 106·322
d 73·0004 e 437·3627 f 1·943 91

9 a 5·81 b 31·1 c 27·568

10 a
$$\begin{array}{r} 6{\cdot}4076 \\ -\ 2{\cdot}5949 \\ \hline 3{\cdot}8127 \\ \hline \end{array}$$
b
$$\begin{array}{r} 70\,108{\cdot}2 \\ -\ 44\,684{\cdot}6 \\ \hline 25\,423{\cdot}6 \\ \hline \end{array}$$
c
$$\begin{array}{r} 66\,480{\cdot}8 \\ +\ 19\,839{\cdot}8 \\ \hline 86\,320{\cdot}6 \\ \hline \end{array}$$
d
$$\begin{array}{r} 4890{\cdot}74 \\ +\ 4381{\cdot}96 \\ \hline 9272{\cdot}70 \\ \hline \end{array}$$

11 a 132·44 m b 144·48 m c 493·64 m
d 1191·96 m e 24 188·36 m

12 a i 84 120 mL ii 84 L
b i 147 210 mL ii 147 L
c i 25 236 mL ii 25 L
d i 54 678 mL ii 55 L
e i 82 017 mL ii 82 L

13 a 111·578 947 37 b 1·115 789 47
c 1·115 789 47 d 11·157 894 74

e 11·157 894 74 f 1·115 789 47
g 0·011 157 89 h 0·001 115 79

14 a i 3·87 ii 3·873 iii 3·872 983
b i 6·93 ii 6·928 iii 6·928 203
c i 0·66 ii 0·663 iii 0·663 325

15 a 45 cm, 127 cm^2 b 149 cm, 1092 cm^2

Exercise 1F

1 a 81 b 256 c 784 d 7569
e 10 201 f 4·41 g 0·16 h 82·81
i 0·0169 j 146·41 k $\frac{9}{16}$ l $\frac{4}{25}$
m $\frac{1}{121}$ n $\frac{25}{81}$ o $\frac{4}{49}$ p $7\frac{9}{16}$
q $1\frac{7}{9}$ r $10\frac{1}{36}$ s $57\frac{16}{49}$ t $8\frac{73}{144}$

2 a 4 b 3 c 8 d 7 e 10
f 11 g 13 h 14 i 15 j 16
k 21 l 25 m 41 n 62 o 89
p 42 q 52 r 68 s 99 t 104

3 a 81 b 16 c 25 d 36
e 100 f 169 g 196 h 729
i 1024 j 1681 k 4225 l 5476

4 a 0·09 b 0·16 c 1·44 d 4·41
e 1·21 f 6·25 g 1·96 h 26·01
i 8·41 j 22·09 k 27·04 l 67·24

5 a $\frac{9}{25}$ b $\frac{4}{9}$ c $\frac{1}{100}$ d $\frac{16}{49}$
e $\frac{25}{121}$ f $\frac{9}{4}$ g $\frac{25}{9}$ h $\frac{121}{16}$

6 a 10 b 13 c 8 d 16
e 19 f 19 g 14 h 47
i 20 j 40 k 47 l 43

7 a 4 b 15 c 28 d 15
e 35 f 20 g 48 h 27
i 120 j 57 k 68 l 330

8 a 81 b 16 c 9 d 36
e 121 f 36 g 16 h 144
i 196 j 576 k 676 l 1156
m 5625 n 21 609
o 2 471 184 p 132 496

9 a 1 b 3 c 11
d 5 e 14 f 10
g 6 h 1 i 15
j 51 k 32 l 90

Exercise 1G

1 a 2·5 b 4·1
c 13·9 d 11·6
e 26·0

2 a 4·24 b 3·61
c 30·00 d 9·17
e 46·96

3 a 2·65 b 3·74
c 17·32 d 72·38
e 45·37

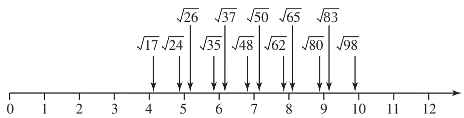
4 a 8·636 456 096 b 3·344 953 47
c 2·979 602 d 2·311 90
e 6·325 f 14·1421
g 23·811 761 80 h 11·616 058

5 a 9·016 794 38 b 4·545
c 2·383 54 d 2·088 591
e 6·6332 f 17·320 51
g 24·597 h 20·342 179

Exercise 1H

1 a A little more than 4 b A little less than 5
c A little more than 5 d A little less than 6
e A little more than 6 f A little less than 7
g A little more than 7 h A little less than 8
i A little more than 8 j A little less than 9
k A little more than 9 l A little less than 10

2



3 a 15 b 40 c 10 d 72
e 120 f 26 g 148 h 26
i 62 j 59 k 88 l 78

Exercise 1I

1 a $7\sqrt{2}$ b $3\sqrt{5}$ c $10\sqrt{2}$
d $3\sqrt{7}$ e $4\sqrt{7}$ f $5\sqrt{11}$

2 a $4\sqrt{3}$ b $3\sqrt{7}$ c $2\sqrt{5}$
d $3\sqrt{3}$ e $8\sqrt{2}$ f $10\sqrt{2}$
g $2\sqrt{55}$ h $2\sqrt{30}$ i $3\sqrt{22}$
j $9\sqrt{2}$ k $9\sqrt{3}$ l $2\sqrt{35}$

3 a $10\sqrt{3}$ b $50\sqrt{3}$ c $14\sqrt{2}$
d $12\sqrt{3}$ e $72\sqrt{3}$ f $4\sqrt{7}$

g $12\sqrt{5}$ h $10\sqrt{6}$ i $24\sqrt{3}$

j $14\sqrt{2}$ k $16\sqrt{5}$ l $24\sqrt{2}$

4 a $a\sqrt{c}$ b $b\sqrt{d}$ c $a\sqrt{2f}$ d $b\sqrt{3c}$

5 a $a\sqrt{f}$ b $x\sqrt{y}$ c $f\sqrt{g}$

d $e\sqrt{t}$ e $w\sqrt{z}$ f $a\sqrt{2d}$

g $b\sqrt{3g}$ h $e\sqrt{5g}$ i $3d\sqrt{e}$

j $2d\sqrt{h}$ k $4d\sqrt{ef}$ l $6e\sqrt{st}$

m $3a\sqrt{2bc}$ n $2a\sqrt{2d}$ o $3f\sqrt{3g}$

p $4x\sqrt{3y}$ q $6p\sqrt{3t}$ r $5a\sqrt{3c}$

s $2x\sqrt{6xy}$ t $6a^2b^3\sqrt{2ab}$

Exercise 1J

1 a $\sqrt{45}$ b $\sqrt{28}$ c $\sqrt{32}$ d $\sqrt{80}$

e $\sqrt{432}$ f $\sqrt{176}$ g $\sqrt{150}$ h $\sqrt{162}$

2 a $\sqrt{18}$ b $\sqrt{180}$ c $\sqrt{200}$ d $\sqrt{28}$

e $\sqrt{539}$ f $\sqrt{162}$ g $\sqrt{75}$ h $\sqrt{24}$

i $\sqrt{432}$ j $\sqrt{245}$ k $\sqrt{448}$ l $\sqrt{208}$

3 a $\sqrt{5a^3}$ b $\sqrt{6a^3}$ c $\sqrt{2a^2b}$

d $\sqrt{3ab^3}$ e $\sqrt{a^3bd}$ f $\sqrt{2a^2bc}$

4 a $\sqrt{9a^3}$ b $\sqrt{2b^3}$ c $\sqrt{6a^2b}$ d $\sqrt{2ab^2c}$

e $\sqrt{4a^3b}$ f $\sqrt{3ab^3}$ g $\sqrt{2a^3b}$ h $\sqrt{a^3bc}$

i $\sqrt{5d^3ef}$ j $\sqrt{7e^3fg}$ k $\sqrt{abc^3}$ l $\sqrt{ab^3c}$

5 a $\sqrt{9a^3}$ b $\sqrt{16a^3}$ c $\sqrt{49b^3}$

d $\sqrt{64a^3}$ e $\sqrt{49c^3}$ f $\sqrt{36a^3b^2}$

g $\sqrt{9a^2b^3}$ h $\sqrt{45a^2c^3}$ i $\sqrt{25ab^3c^3}$

j $\sqrt{12a^2d^3}$ k $\sqrt{12a^3bc}$ l $\sqrt{50a^3bc}$

m $\sqrt{25a^5}$ n $\sqrt{36a^2b^3}$

o $\sqrt{4a^7b^7}$ p $\sqrt{9a^7b^3}$

Exercise 1K

1 a $\sqrt{5}, 3\sqrt{5}, 5\sqrt{5};\ \sqrt{3}, -4\sqrt{3}$

b $\sqrt{6}, -3\sqrt{6}, -4\sqrt{6};\ -\sqrt{11}, 5\sqrt{11}$

c $\sqrt{3}, -\sqrt{3}, 5\sqrt{3};\ \sqrt{5}, -2\sqrt{5}$

d $\sqrt{7}, -8\sqrt{7}, -4\sqrt{7};\ -\sqrt{2}, -4\sqrt{2}$

e $-\sqrt{3}, \sqrt{3}, -4\sqrt{3};\ \sqrt{7}, 5\sqrt{7};\ \sqrt{6}, -3\sqrt{6}$

f $\sqrt{13}, 2\sqrt{13}, 3\sqrt{13};\ \sqrt{3}, -2\sqrt{3}, -4\sqrt{3}, -\sqrt{3}$

2 a $6\sqrt{5}$ b $4\sqrt{2}$ c $5\sqrt{11}$

d $4\sqrt{6}$ e $6\sqrt{7}$ f $11\sqrt{5}$

3 a $3\sqrt{5}$ b $7\sqrt{2}$ c $5\sqrt{2}$

d $3\sqrt{3}$ e $2\sqrt{11}$ f $5\sqrt{7}$

4 a $4\sqrt{2}+2\sqrt{3}$ b $5\sqrt{5}+3\sqrt{7}$

c $4\sqrt{2}+3\sqrt{3}$ d $7\sqrt{2}-\sqrt{5}$

e $2\sqrt{3}+2\sqrt{2}$ f $3\sqrt{7}-3\sqrt{3}$

g $5\sqrt{7}+5\sqrt{11}$ h $3\sqrt{3}$

Exercise 1L

1 a $\sqrt{14}$ b $\sqrt{15}$ c $\sqrt{30}$

d $\sqrt{35}$ e $\sqrt{55}$ f $\sqrt{34}$

2 a $2\sqrt{3}$ b $3\sqrt{2}$ c $7\sqrt{2}$

d $12\sqrt{14}$ e $21\sqrt{22}$ f $12\sqrt{42}$

g $12\sqrt{6}$ h $6\sqrt{35}$ i $24\sqrt{3}$

j $18\sqrt{2}$ k $24\sqrt{3}$ l $24\sqrt{5}$

3 a $3\sqrt{7}+3\sqrt{3}$ b $-2\sqrt{5}-4$

c $24\sqrt{2}-24$ d $-5\sqrt{2}-5\sqrt{11}$

e $\sqrt{10}+\sqrt{6}$ f $\sqrt{6}-\sqrt{21}$

g $\sqrt{22}+\sqrt{55}$ h $2\sqrt{30}-\sqrt{42}$

i $6+3\sqrt{6}$ j $10+2\sqrt{15}$

k $8-12\sqrt{10}$ l $8\sqrt{35}+20$

4 a $\sqrt{15}+7\sqrt{5}+\sqrt{6}+7\sqrt{2}$

b $4\sqrt{6}+5\sqrt{3}$

c $\sqrt{6}+\sqrt{14}+4\sqrt{3}+4\sqrt{7}$

d $6+3\sqrt{2}+2\sqrt{5}+\sqrt{10}$

e $6\sqrt{2}+2\sqrt{14}+3\sqrt{3}+\sqrt{21}$

f $2\sqrt{5}+\sqrt{10}+6\sqrt{3}+3\sqrt{6}$

g $3\sqrt{6}+2\sqrt{3}+3\sqrt{10}+2\sqrt{5}$

h $3\sqrt{2}+3+\sqrt{10}+\sqrt{5}$

i $3\sqrt{15}+2\sqrt{3}+6\sqrt{5}+4$

j $11\sqrt{6}+22\sqrt{2}+6+4\sqrt{3}$

k $7\sqrt{2}+6\sqrt{3}$

l $5\sqrt{2}+\sqrt{10}+6\sqrt{5}+6$

Exercise 1M

1 **a** 1 **b** 1 **c** 3 **d** 7
e 2 **f** $\frac{4}{3}$ **g** 2 **h** $\frac{1}{3}$
i 2 **j** $\frac{16}{3}$ **k** 6 **l** $\frac{1}{9}$

2 **a** $\sqrt{5}$ **b** $\sqrt{3}$ **c** $\sqrt{2}$ **d** 2
e $\sqrt{10}$ **f** $\sqrt{11}$ **g** 4 **h** $2\sqrt{2}$
i $3\sqrt{3}$ **j** $2\sqrt{3}$ **k** $3\sqrt{2}$ **l** $2\sqrt{6}$

3 **a** $2\sqrt{6}$ **b** $\frac{3}{2}\sqrt{5}$ **c** $5\sqrt{15}$ **d** $\frac{5}{4}\sqrt{10}$
e $3\sqrt{5}$ **f** $3\sqrt{2}$ **g** $\frac{4\sqrt{3}}{3}$ **h** $\frac{25}{4}\sqrt{2}$
i $\frac{13}{5}\sqrt{6}$ **j** $\frac{7}{2}\sqrt{10}$ **k** $\frac{10}{3}\sqrt{2}$ **l** $8\sqrt{6}$

4 **a** 2 **b** $3\sqrt{6}$ **c** 12 **d** $\frac{9\sqrt{3}}{2}$
e $15\sqrt{3}$ **f** $24\sqrt{6}$ **g** $4\sqrt{2}$ **h** $4\sqrt{3}$

Exercise 1N

1 **a** $\frac{\sqrt{3}}{3}$ **b** $\frac{2\sqrt{5}}{5}$ **c** $\frac{5\sqrt{7}}{7}$ **d** $\frac{2\sqrt{3}}{3}$
e $3\sqrt{2}$ **f** $\frac{7\sqrt{2}}{2}$ **g** $\frac{\sqrt{3}}{2}$ **h** $\frac{2\sqrt{5}}{15}$

Applications

Simplifying surds

a $8\sqrt{3}$ **b** $8\sqrt{3}$ **c** $8\sqrt{3}$
d $8\sqrt{5}$ **e** $8\sqrt{7}$ **f** $8\sqrt{11}$
g $9\sqrt{2}$ **h** $9\sqrt{5}$ **i** $9\sqrt{7}$

Surds obeying the rules of algebra

a **i** $x+3x=4x$ **ii** $3x+4x=7x$
iii $2x+11x=13x$ **iv** $4x-x=3x$
v $12x-7x=5x$ **vi** $6x-3x=3x$

b **i** $4\sqrt{2}+3\sqrt{2}=7\sqrt{2}$ **ii** $2\sqrt{3}+\sqrt{3}=3\sqrt{3}$
iii $6\sqrt{7}+2\sqrt{7}=8\sqrt{7}$ **iv** $6\sqrt{5}-2\sqrt{5}=4\sqrt{5}$
v $9\sqrt{2}-3\sqrt{2}=6\sqrt{2}$ **vi** $12\sqrt{3}-7\sqrt{3}=5\sqrt{3}$

Enrichment

1 **a** $27\sqrt{2}-6\sqrt{7}$ **b** $7\sqrt{3}$
c $5\sqrt{3}$ **d** $34\sqrt{3}$

2 **a** $\frac{6\sqrt{2}+\sqrt{6}}{12}$
b $\frac{3\sqrt{10}+2\sqrt{15}}{30}$
c $\frac{15\sqrt{3}+2\sqrt{6}}{18}$
d $\frac{5\sqrt{6}+5\sqrt{3}+15-3\sqrt{5}}{15}$
e $\frac{3\sqrt{6}+6\sqrt{2}+6-2\sqrt{3}}{6}$
f $\frac{2\sqrt{14}-3\sqrt{7}+21+7\sqrt{2}}{7}$

3 **a** $5+3\sqrt{3}$
b $\sqrt{15}+4\sqrt{5}+2\sqrt{3}+8$
c $\sqrt{21}+7\sqrt{7}-3\sqrt{3}-21$
d $^{-}1$ **e** $^{-}22$ **f** $4-\sqrt{2}$
g $^{-}2$ **h** 5 **i** 173
j $3\sqrt{2}-2\sqrt{6}+\sqrt{30}-2\sqrt{10}$
k $10\sqrt{2}-2\sqrt{10}+10\sqrt{5}-10$
l $21\sqrt{14}-28\sqrt{35}-6\sqrt{10}+40$

4 **a** $\frac{2\sqrt{10}-3\sqrt{2}}{2}$
b $\frac{^{-}30\sqrt{10}+105+82\sqrt{3}}{123}$
c $\frac{20+10\sqrt{3}+3\sqrt{5}}{5}$ **d** $^{-}\sqrt{3}-2$
e $\frac{\sqrt{10}}{2}$ **f** $\frac{1}{3}$

5 **a** $\sqrt{6}+\sqrt{2}$ **b** $\sqrt{6}-\sqrt{2}$ **c** 2
d 3 **e** $4+2\sqrt{3}$ **f** $4-2\sqrt{3}$
g $2\sqrt{2}$ **h** 4 **i** $\frac{\sqrt{6}+\sqrt{2}}{2}$
j $2-\sqrt{3}$

6 **a** $(3\sqrt{2}+3):9$ **b** $(10\sqrt{2}+10):30$
c $(25\sqrt{2}+25):75$ **d** $(30\sqrt{2}+30):90$

7 **a** $5+3\sqrt{5}$ **b** 3·5 **c** 30

Revision/Assessment

3 147

4 a $^-20$ b 89 c 15

5 39

6 a $^-25$ b 57 c 76

7 a $\frac{\pi}{12}$ b 4 c $\frac{65}{24}$

8 a $15\frac{65}{72}$ b $6\frac{7}{18}$

9 $0{\cdot}41\dot{6}$

10 a 9 b 340·02 c 24·231

11 a $7\frac{1}{9}$ b 2·25 c 2916

d 9025 e 81

12 a 34 b 41

c 36 d 30

13 a $2\sqrt{2}$ b $4\sqrt{2}$ c $2\sqrt{3}$

d $21\sqrt{5}$ e $12\sqrt{5}$ f $54\sqrt{2}$

14 a $a\sqrt{b}$ b $b\sqrt{c}$ c $2a\sqrt{b}$

d $3e\sqrt{f}$ e $y\sqrt{3z}$ f $a\sqrt{5c}$

15 a $\sqrt{32}$ b $\sqrt{28}$ c $\sqrt{99}$

d $\sqrt{80}$ e $\sqrt{18}$ f $\sqrt{75}$

16 a $\sqrt{4a^3}$ b $\sqrt{3b^3}$ c $\sqrt{3a^2b}$

d $\sqrt{12a^3}$ e $\sqrt{4a^3c}$ f $\sqrt{18a^3b}$

17 a $4\sqrt{5}$ b $4\sqrt{3}$ c $20\sqrt{5}$

d $2\sqrt{5}$ e $11\sqrt{7}+\sqrt{3}$

f $13\sqrt{2}-3\sqrt{7}$ g $6\sqrt{b}$

h $5\sqrt{c}$ i $7\sqrt{a}$

18 a $5\sqrt{2}$ b $7\sqrt{2}$ c $10\sqrt{2}$ d $7\sqrt{3}$

19 a $\sqrt{10}$ b $\sqrt{21}$ c $\sqrt{39}$

d $5\sqrt{3}$ e $6\sqrt{15}$ f $30\sqrt{6}$

g $6\sqrt{6}$ h $15\sqrt{15}$

20 a $\sqrt{15}+\sqrt{6}$

b $\sqrt{10}-\sqrt{35}$

c $\sqrt{6}+\sqrt{15}$

d $3\sqrt{3}+\sqrt{21}+3\sqrt{5}+\sqrt{35}$

e $\sqrt{2}+2+2\sqrt{5}+2\sqrt{10}$

f $5\sqrt{2}+8$

21 a $3\sqrt{15}$ b $\frac{10}{3}$ c $18\sqrt{6}$ d $8\sqrt{6}$

22 a $\frac{3\sqrt{2}}{2}$ b $\frac{3\sqrt{5}}{5}$ c $\frac{2\sqrt{7}}{7}$

d $4\sqrt{3}$ e $9\sqrt{2}$

23 a $\frac{\sqrt{15}+3}{3}$ b $\frac{\sqrt{6}+3\sqrt{2}}{2}$ c $\frac{\sqrt{6}-5\sqrt{2}}{2}$

d $\frac{2\sqrt{5}+\sqrt{15}}{5}$ e $\frac{3\sqrt{6}-\sqrt{21}}{3}$

Chapter 2

Exercise 2A

1 a cm b mm

c km d g

e t f m

g m h kg

i min j kg

2 a 25 cm b 10 mm

c 5 m d 28 m

e Learner's own answers. f Answers may vary.

3 a B b C

c A d A

4 a 45 000 cm = 450 000 mm

b 300 m = 30 000 cm = 300 000 mm

c 1·5 kg = 1500 g

d 210 min = 12 600 s

e 125 m = 12 500 cm

f 0·85 km = 85 000 cm = 850 000 mm

g 6000 kg = 6 000 000 g = 6 000 000 000 mg

h 2 h = 120 min

5 a 3·5 m b 25 mm

c 2160 sec d 356 000 mg

e 360 min f 210 m

g 210 000 mm h 3 500 000 mg

i 260 mm j 675 cm

k 6400 g l 56 000 mm

Exercise 2B

1 a 10·7 cm b 8 cm c 36 cm

d 56 km e 23·2 m f 9·2 mm

g 32·4 cm h 12·2 cm i 11·5 m

2 a 16 m b 12·4 km c 11·2 cm

d 11 m e 12 cm f 46·6 km

3 a 15 cm b 28 cm c 74·7 m

d 63·5 km e 41·5 mm f 17·3 km

4 a $a = 7{\cdot}5$ cm b $b = 14{\cdot}57$ m c $c = 4{\cdot}5$ mm

Exercise 2D

1 a 125·66 m b 50·27 cm c 339·29 m
d 13·19 mm e 83·25 m f 315·10 km

2 a 12·57 m b 263·89 cm c 100·53 m
d 17·59 mm e 4·4 cm f 91·73 mm

3 a 5π cm b $\frac{40\pi}{9}$ cm
c $\frac{154\pi}{9}$ mm d $\frac{95\pi}{18}$ m

4 a 12·34 mm b 8·04 cm c 12·16 m

5 Learner's own answers.

6 27·5 cm

7 a 125·12 m b 192·4 m c 661 m

8 a 24·85 m b 48·85 cm c 69·34 m

Exercise 2E

1 a square centimetre b square centimetre
c square millimetre d square metre
e square kilometre or hectare
f square millimetre
g square kilometre or hectare
h square kilometre or hectare
i square kilometre or hectare

2 a 4000 square kilometres
b 5000 square kilometres
c 3000 square kilometres
d 3000 square kilometres

3 a 420 mm^2 b 8000 cm^2
c 67 000 m^2 d 5 000 000 mm^2
e 5 cm^2 f 1400 mm^2

Exercise 2F

1 a 24 m^2 b 12 mm^2 c 31·5 km^2
d 7·96 m^2 e 42·5 mm^2 f 14·3 m^2

2 a 14·44 m^2 b 108·16 km^2 c 0·56 m^2
d 22·75 cm^2 e 7·56 km^2 f 4·04 mm^2

3 a 24 cm^2 b 180·4 m^2 c 17·28 cm^2
d 171·15 m^2 e 54·18 m^2 f 11 000 mm^2

4 a 14·75 m^2 b 8·1 m^2 c 36 mm^2
d 219 mm^2 e 5·46 m^2 f 380·6 cm^2

Exercise 2G

1 a 12·57 mm^2 b 1134·11 m^2 c 5541·77 km^2
d 26·42 cm^2 e 6·16 mm^2 f 0·13 km^2

2 a 1661·91 mm^2 b 7853·98 m^2
c 50·27 cm^2 d 19·63 km^2
e 83·32 km^2 f 4·30 mm^2

3 a 58·09 m^2 b 307·91 m^2 c 346·36 m^2
d 1·77 cm^2 e 0·000 71 km^2 f 0·000 31 cm^2

4 a 167·55 mm^2 b 63·27 m^2 c 1710·60 m^2
d 490·87 mm^2 e 441·04 m^2 f 49·86 m^2

5 Big: 245 square centimetres
Medium: 66 square centimetres
Small: 25 square centimetres

6 94.5 square centimetres

Exercise 2H

a 21·95 m^2 b 7·07 m^2 c 116·24 m^2
d 81·64 m^2 e 19·56 m^2 f 42·41 m^2
g 73·57 m^2 h 58·60 m^2 i 20·64 m^2
j 7·73 m^2 k 19·06 m^2 l 21·70 m^2

Exercise 2I

1 a 150 m^2 b 253·5 m^2 c 31·74 cm^2
d 148 m^2 e 68 m^2 f 323·12 mm^2
g 62·86 m^2 h 106·24 m^2 i 496·7 cm^2

2 a 104 m^2 b 240 m^2 c 192·36 mm^2
d 257·6 cm^2 e 225·44 m^2 f 250·2 mm^2

3 a 611 mm^2 b 116·78 cm^2

4 992 cm^2, 43 cm × 42 cm

5 8·67 mL

Exercise 2J

1 a 168 cm^2 b 240 cm^2 c 297 mm^2
d 210·3 cm^2 e 2202·4 mm^2 f 676·88 m^2
g 173·2 cm^2 h 110·88 mm^2 i 29·11 mm^2

2 a 281·35 cm^2 b 146·65 cm^2

Exercise 2K

1 a 288·90 mm^2 b 157·08 mm^2
c 113·10 m^2 d 157·14 mm^2
e 235·62 mm^2 f 144·51 cm^2
g 143·822 cm^2 h 515·22 m^2
i 3631·68 m^2 j 5026·55 mm^2
k 1520·53 cm^2 l 153·94 cm^2

2 347·54 cm^2

3 a 1256·64 cm^2 b 141·03 cm^2 c 1·26 m^2

4 169·65 cm^2 **5** $5·15 \times 10^8$ km^2

6 a 1809 square centimetres
b 8491 square centimetres
c 1256 square centimetres
d 203 773 square centimetres

Exercise 2L

1 a 216 cm^3 b 729 m^3 c 19·68 m^3
d 658 503 mm^3 e 60 mm^3 f 265·61 m^3

g 84 m^3 h 189 m^3 i 72 cm^3
j 140 cm^3 k 35 mm^3 l 180·18 m^3
m 376·53 mm^3 n 131·95 mm^3 o 84·82 m^3

2 a 48·8 cm^3 b 408·75 mm^3
c 1736 mm^3 d 825 cm^3
e 381·8 mm^3 f 1260 mm^3

3 a 47 123·89 mm^3 b 114·24 cm^3
c 16·54 mm^3 d 182 cm^3
e 39·732 mm^3

Exercise 2M

1 a 200 cm^3 b 36·67 cm^3

2 a 156 cm^3 b 106·67 cm^3 c 121·73 cm^3
d 62·83 m^3 e 78·54 cm^3 f 250·40 cm^3

3 837·76 cm^3

Exercise 2N

1 a 17 157·28 m^3 b 0·27 cm^3
c 5·58 mm^3 d 57·91 cm^3

2 a 523·60 cm^3 b 10 078·66 m^3
c 150·53 mm^3 d 1436·76 cm^3

3 a $1·1 \times 10^{12}$ km^3
b i $9·29 \times 10^{11}$ km^3 ii $6·06 \times 10^{10}$ km^3
iii $1·64 \times 10^{11}$ km^3 iv $1·53 \times 10^{15}$ km^3
v $8·97 \times 10^{14}$ km^3 vi $7·11 \times 10^{13}$ km^3
vii $6·70 \times 10^{13}$ km^3 viii $2·24 \times 10^{10}$ km^3

4 a 7068·58 cm^3 b 1526·81 cm^3
c 45·98 cm^3 d 190·85 m^3

5 a 113 097·34 cm^3 b 185 822·54 cm^3
c 1259·83 cm^3

Exercise 2O

1 a ML b L
c L d mL
e L f kL
g L h L

2 a 2500 cm^3 b 20 cm^3
c 3·2 L d 5·75 kL
e 4·5 L f 6·6 mL
g 4 200 000 L h 16 000 cm^3

3 a 4·7 L b 2·4 L c 35 L d 1·6 L

Applications

Clothes line

Total length = 32 m

Tower of Hanoi

8475π mm^2

Swimming pool

a 225 m^3 b 3·36 m^3 c 4267 tiles

Skateboarding

14 073 m^3

Pet goat

109·4 m^2

Gas tank

TSA = 15 080 cm^2
V = 134 041 cm^3

Train tunnel

About 4158 bricks are needed if all are placed lengthwise.

Drains

a 9·05 m^3 b 0·173 m^3 c 0·313 m^3

Enrichment

1 a 20·5 cm^3 b 112 cm^3 c 980·18 cm^3

2 a 86·6 cm^2 b 216·51 cm^2 c 516 cm^2

3 a 2457·42 km b 2345·72 km
c 6144 km d 8601 km

4 895·67 cm^3

5 a *TSA* = 254·16 cm^2, *V* = 261·8 cm^3 b *TSA* = 69·12 cm^2, *V* = 36·86 cm^3
c *TSA* = 6·96 mm^2, *V* = 1·02 mm^3

6 a 1·75 m b 2·18 m^2 c 1·71 m

7 17·62 cm^2

8 a 31·42 km^2 b 68·17 m^2 c 3·58 cm^2
d 175·93 cm^2 e 1313·19 m^2 f 9·39 mm^2

Revision/Assessment

1 a 4 m b 1400 m c 160 m

2 a 430 cm b 40 cm c 200 000 cm

3 a *P* = 8 cm b *P* = 7·6 cm
c *P* = 14·8 mm d *P* = 13·8 m
e *P* = 26 cm f *P* = 18·85 mm
g *P* = 6·75 mm h *P* = 18·8 cm

4 a 40 cm^2 b 6 km^2
c 560 000 cm^2 d 520 ha

5 a *A* = 4 cm^2 b *A* = 3·25 cm^2
c *A* = 12·3 mm^2 d *A* = 7·8 m^2
e *A* = 21 cm^2 f *A* = 28·27 mm^2
g *A* = 2·18 mm^2 h *A* = 16·25 cm^2

6 a 39·18 cm^2 b 59·63 cm^2

7 a *TSA* = 132·54 m^2, *V* = 103·82 m^3 b *TSA* = 60·1 m^2, *V* = 30·75 m^3
c *TSA* = 94·69 cm^2, *V* = 34·63 cm^3 d *TSA* = 156·5 cm^2, *V* = 96·75 cm^3
e *TSA* = 409·98 mm^2, *V* = 636·17 mm^3 f *TSA* = N/A, *V* = 100 cm^3

8 a $TSA = 424{\cdot}8\text{ mm}^2$, $V = 480\text{ mm}^3$ b $TSA = 196{\cdot}79\text{ cm}^2$, $V = 94{\cdot}66\text{ cm}^3$

9 a $TSA = 7853{\cdot}98\text{ mm}^2$, $V = 65\,449{\cdot}85\text{ mm}^3$ b $TSA = 3216{\cdot}99\text{ cm}^2$, $V = 17\,157{\cdot}28\text{ cm}^3$
c $TSA = 5229{\cdot}62\text{ m}^2$, $V = 35\,561{\cdot}42\text{ m}^3$

10 0·94 L

11 a $25\,000\text{ cm}^3$ b 4·1 mL

Chapter 3

Exercise 3A

1 a $\angle GSF$ or $\angle FSG$ or $\angle S$ or $G\hat{S}F$ or $F\hat{S}G$
b $\angle FGJ$ or $\angle JGF$ c $\angle RYC$ or $\angle CYR$
d $\angle FYD$ or $\angle DYF$ e $\angle ABE$ or $\angle EBA$
f $\angle NMR$ or $\angle RMN$ g $\angle PYN$ or $\angle NYP$
h $\angle BER$ or $\angle REB$ i $\angle ABC$ or $\angle CBA$

2 a Obtuse b Acute c Reflex
d Acute e Reflex f Reflex
g Obtuse h Reflex i Acute
j Acute k Obtuse l Obtuse
m Obtuse n Obtuse o Obtuse
p Acute q Reflex r Reflex
s Obtuse t Reflex u Reflex
v Acute w Obtuse x Obtuse

3 a 78°, 168° b 68°, 158°
c 53°, 143° d 45°, 135°
e 34°, 124° f 27°, 117°
g 27·5°, 117·5° h 24°, 114°
i 19°, 109° j 3°, 93°
k 8°, 98° l 10°, 100°
m 76·7°, 166·7° n 75·8°, 165·8°
o 74·3°, 164·3° p 73·8°, 163·8°
q $(90 - n)°, (180 - n)°$ r $(72{\cdot}1 + n)°, (162{\cdot}1 + n)°$
s $(91 - n)°, (187 - n)°$ t $(77° - n)°, (167 - n)°$
u $(37 - n)°, (127 - n)°$ v $(50° + n)°, (140 + n)°$
w $n°, (n + 90)°$ x $^{-}n°, (90 - n)°$

4 a $d = 21°$ b $f = 46°$
c $d = 107°$ d $g = 13°$
e $y = 83°, k = 74°$ f $h = 34°, b = 46°$

5 a Corresponding b Co-interior
c Alternate d Alternate
e Corresponding f Corresponding

6 a $r = 34°, t = 34°, y = 34°$
b $f = 41°, h = 38°, j = 142°, p = 101°, w = 38°, y = 41°$
c $b = 68°, k = 112°, x = 112°, z = 112°$

Exercise 3B

1 a 60° b 88° c 73°

2 a $e = 122°, f = 35°$ b $b = 106°, z = 73°$
c $g = 86°, m = 94°$ d $b = 117°, z = 39°$
e $h = 99°, p = 18°$ f $g = 72°, k = 27°$

3 a 59° b 68° c 15°
d 18° e 18° f 34°

4 a 720° b 1080° c 2700°
d 17 640° e 21 240° f 900°

5 a 128·57° b 140° c 144°
d 156° e 162° f 152·31°

6 a Hexagon b 60°

Exercise 3C

1 $\Delta BAC \cong \Delta QPR$ (SAS)
$\Delta JKL \cong \Delta ONM$ (SAS)
$\Delta DEF \cong GIH$ (SSS)

2 a $d = 45°$, $a = 6$ cm b $r = 46°$, $t = 52°$ c $t = 36°$, $h = 9{\cdot}3$ mm, $g = 15$ mm
d $e = g = r = s = 45°$

3 a $\Delta ABD \cong \Delta ACD$ (SSS)
b $\Delta AEB \cong \Delta AED$ (SAS)
c $\Delta GHJ \cong \Delta IJH$ (SAS)
d $\Delta JKL \cong \Delta JML$ (SAS)
e $\Delta ZXG \cong \Delta YTG$ (SSS)
f $\Delta FJK \cong KLF$ (SAS)
g $\Delta ABC \cong \Delta CDA$ (SSS)
h $\Delta LMO = \Delta NMO$ (RHS)
i $\Delta AEB = \Delta DEC$ (SAS)

Exercise 3D

2 a 1·4 b 2·25 c 2 d 2 e 2 f 1·6

3 $\frac{16}{9}$

4

Length	Length	Ratio (to 2 dp)
$AB = 32$ mm	$FG = 19$ mm	$\frac{AB}{FG} = 1{\cdot}7$
$BC = 30$ mm	$GH = 18$ mm	$\frac{BC}{GH} = 1{\cdot}7$
$CD = 27$ mm	$HI = 16$ mm	$\frac{CD}{HI} = 1{\cdot}7$
$DE = 32$ mm	$IJ = 19$ mm	$\frac{DE}{IJ} = 1{\cdot}7$
$AE = 30$ mm	$FJ = 18$ mm	$\frac{AE}{FJ} = 1{\cdot}7$

5 a No b Yes

Learning task 3E

1 a

Angle	Size (°)	Line	Length (cm)
BCD	27	*AC*	6
ACE	27	*BC*	3
CDB	63	*AE*	3
CEA	63	*BD*	1·5
		CD	3·4
		CE	6·8

c i 2 ii 2 iii 2

d The two triangles are similar because the ratios of their corresponding lengths are constant in all cases.

2 a

Angle	Size (°)	Line	Length (cm)
BCE	55	*CE*	1·5
CBE	30	*BE*	2·5
BEC	95	*BC*	3
ADE	30	*AE*	3
DAE	55	*DE*	5
AED	95	*AD*	6

b i 0·5 ii 0·5 iii 0·5

Exercise 3F

1 a A and C are similar; C is bigger than A by a factor of 2.
b A and B are similar; B is bigger than A by a factor of 2.
c A and C are similar; A is bigger than C by a factor of 2.

2 a A and C are similar; same corresponding angles.
b A and C are similar; A is larger by a factor of 2.
c A and B are similar; same corresponding angles.
d C and D are similar; D is larger by a factor of 4.

3 a $XY = ZY$
$SY = SY$
$\angle SYX = \angle SYZ$
$\therefore \Delta SYX \cong \Delta SYZ$ (SAS)

b $\angle WND = \angle DQF$ (alternate)
$3DN = DQ$
$\angle WDN = \angle QDF$ (vertically opposite)
$\therefore \Delta WND \cong \Delta QFD$ (ASA)

c $\angle EFH = \angle GFH$
$FH = FH$
$\angle FHE = \angle FHG$
$\therefore \Delta EFH \cong \Delta GFH$ (ASA)

d $\angle FDN = \angle KGN$
$\angle DNF = \angle GNK$
$\therefore$ the third angle in the triangles must be the same
$\therefore \Delta GNK \cong \Delta DNF$ (AAA)

e $YE = 4{\cdot}47GL$
$EK = 4{\cdot}47LK$
$\angle YEK = \angle GLK$
$\therefore \Delta YKE \cong \Delta GKL$ (SAS)

f $PJ = 3WQ$
$DJ = 3WD$
$\angle QWD = \angle PJD$ (alternate)
$\therefore \Delta WQD \cong \Delta JPD$ (SAS)

4 a $\Delta WTJ \sim \Delta YTR$ b $\Delta ZSC \sim \Delta VTC$
c $\Delta RTS \sim \Delta SGR$ d $\Delta RQT \sim \Delta EHT$

5 a $x = 8$ cm, $y = 13{\cdot}5$ cm
b $x = 20{\cdot}25$ mm, $y = 22{\cdot}5$ mm
c $x = 20$ m, $y = 12$ m
d $x = \frac{4}{5}$ m, $y = 4\frac{1}{6}$ m
e $x = 1$ m, $y = 2\frac{1}{10}$ m
f $x = 0{\cdot}5$ cm, $y = 0{\cdot}2$ cm

6 a $x = 4, y = \frac{3}{4}$ b $x = 3, y = 2$
c $x = 3\frac{2}{3}, y = 12$ d $x = 4{\cdot}8, y = 12$
e $x = 1{\cdot}6, y = 1{\cdot}2, z = 8{\cdot}8$

Exercise 3G

1 7·5 m 2 3·5 m 3 4·8 m
4 8 m 5 146·3 m
6 a 5·8 m b 2·3 m
7 0·6 m 8 41·4 m
9 $h_1 = 10$ m, $h_2 = 5$ m
10 1·07 m 11 12·5 cm

Learning task 3H

1 a i $TSA = 24$ cm^2, $V = 8$ cm^3
ii $TSA = 96$ cm^2, $V = 64$ cm^3
iii *TSA* of the enlarged shape is 4 times that of original shape.
Volume of enlarged shape is 8 times that of original shape.
iv Scale factor of *TSA*: 4
Scale factor of *V*: 8

b i $TSA = 72$ cm^2, $V = 32$ cm^3
ii $TSA = 288$ cm^2, $V = 256$ cm^3
iii *TSA* of the enlarged shape is 4 times that of original shape.
Volume of enlarged shape is 8 times that of original shape.
iv Scale factor of *TSA*: 4
Scale factor of *V*: 8

Exercise 3I

1 a 3, 2 b 4, 5 c 5, 7
d 4, 3 e 5, 6 f 6, 11

2

	A	*B*	*C*	*D*	*E*
a	1	2	2	1	–
b	1	3	4	2	–
c	3	2	2	3	–
d	2	2	2	0	–
e	2	2	1	3	–
f	3	3	4	5	3

3 d only, node *D* is an isolated node

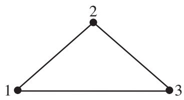
4 a

Vertex	*A*	*B*	*C*	*D*	*E*	*F*
Degree	3	2	4	2	0	1

b 12

5 a Vertices: 1, 2, 3 Edges: (1, 2), (1, 3), (2, 3)

b

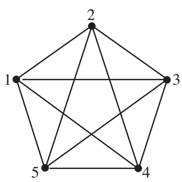


c

Vertex	1	2	3
Degree	2	2	2

6 i a Vertices: 1, 2, 3, 4, 5
Edges: (1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (3, 4), (3, 5), (4, 5)

b

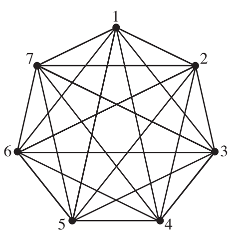


c

Vertex	1	2	3	4	5
Degree	4	4	4	4	4

ii a Vertices: 1, 2, 3, 4, 5, 6, 7
Edges: (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 4), (2, 5), (2, 6), (2, 7), (3, 4), (3, 5), (3, 6), (3, 7), (4, 5), (4, 6), (4, 7), (5, 6), (5, 7), (6, 7)

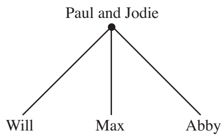
b



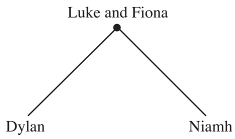
c

Vertex	1	2	3	4	5	6	7
Degree	6	6	6	6	6	6	6

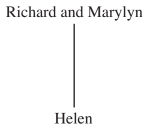
7 a



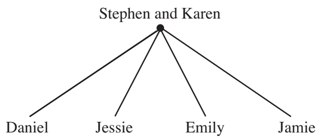
b



c



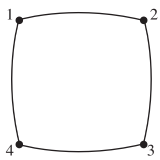
d



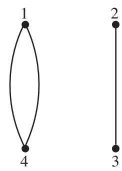
Exercise 3J

1 Planar graphs: A, B, C, D

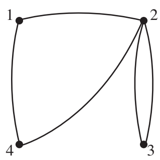
2 a



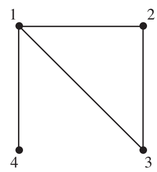
b



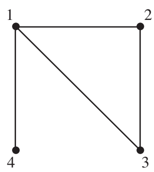
c



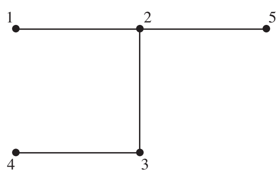
d



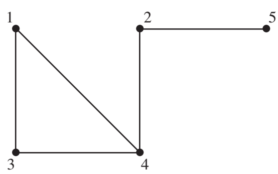
3 a



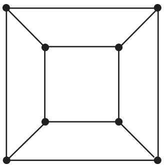
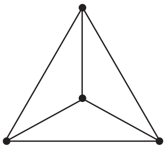
b



c



5 All graphs in Question 4 are planar.

6 **a**

b

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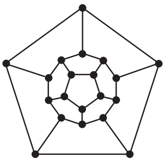
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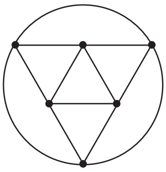
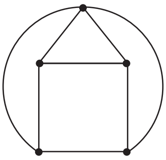
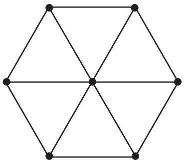

e

f


8 They are 3-dimensional figures.

9 **a** None of them **b** A **c** D
d C **e** E **f** B

Enrichment

1 **a** Original area = 1·92 cm^2,
Final area = 11·52 cm^2 (original × 6)
b Original area = 0·95 cm^2,
Final area = 5·7 cm^2 (original × 6)
c Original area = 3·24 cm^2,
Final area = 19·44 cm^2 (original × 6)
d Original area = 4·38 cm^2,
Final area = 26·25 cm^2 (original × 6)

2 **a** **i** Cube: 16 cm^2
Cylinder: π cm^2
Prism: 6 cm^2
ii Cube: 96 cm^2
Cylinder: 14π cm^2
Prism: 72 cm^2
iii Cube: 64 cm^3
Cylinder: 6π cm^3
Prism: 30 cm^3
b **i** Area of end increases by a factor of 4.
ii Total surface area increases by a factor of 4.
iii Volume increases by a factor of 8.

3 **a** 6·5 m
b Karoum = 0·72 m, Raffael = 1·44 m, Sammi = 1·2 m

4 **a** $x = 2$ cm, $y = 26$ cm
b $x = 6{\cdot}8$ cm, $y = 22{\cdot}5$ cm
c $x = 10{\cdot}4$ cm, $y = 5{\cdot}2$ cm

5 80 cm

6 **a** $x = 1\frac{1}{3}$ cm, $y = 10$ cm **b** $x = 5$ cm, $y = 39$ cm
c $x = 3\frac{2}{3}$ cm, $y = 5$ cm **d** $x = 2$ cm, $y = 5$ cm

Revision/Assessment

1 **a** Obtuse **b** Acute **c** Obtuse
d Acute **e** Reflex **f** Reflex

2 **a** 75°, 165° **b** 63°, 153° **c** 51°, 141°
d 41°, 131° **e** 39°, 129° **f** 28°, 118°

3 **a** $d = 103°$, $r = 103°$, $t = 77°$
b $s = 38°$, $w = 142°$, $g = 142°$
c $k = 46°$, $q = 46°$

4 **a** $y = 79°$ **b** $y = 49{\cdot}67°$ **c** $y = 54°$

5 A ≅ C (SAS)
B ≅ H (ASA)
D ≅ F (SAS)
E ≅ G (RHS)

6 $EF = EH$ (given)
$EG = EG$ (same side)
$\angle EGF = \angle EGH = 90°$
$\therefore \Delta EGF \cong \Delta EGH$ (RHS)

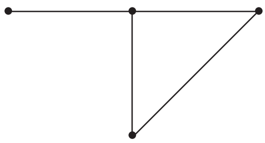
7 $x = 3$, $y = 4{\cdot}5$

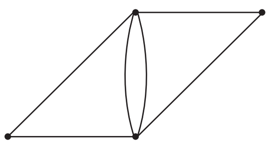
8 $RU = RS$
$UT = ST$
$RT = RT$
$\therefore \Delta RUT \cong \Delta TSR$ (SSS)

9 25 m

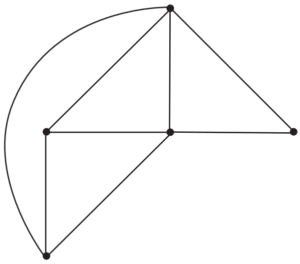
10

Vertex	*A*	*B*	*C*	*D*	*E*	*F*
Degree	2	4	4	3	3	0

11 **a**

$V = 4$, $E = 4$, $F = 2$
$V = E - F + 2$
$4 = 4 - 2 + 2$
$= 4$
Euler's formula confirmed.

b

$V = 4$, $E = 6$, $F = 4$
$V = E - F + 2$
$4 = 6 - 4 + 2$
$= 4$
Euler's formula confirmed.

c



$V = 5, E = 8, F = 5$
$V = E - F + 2$
$5 = 8 - 5 + 2$
$= 5$
Euler's formula confirmed.

Chapter 4

Exercise 4A

1 a i expression ii equation iii expression
b i 3 ii –5 iii 1
c i $7t, 8r, -5bt, -9, 3b$ ii $4g, -2r, 7rf, 9, -5b$
iii $b, n, -5bn, 10$
d i t, r, b ii g, r, b iii b, n
e i –9 ii 9 iii 10

2 a $w + r$ b $w + r = 8$ c $2w$
d $r - 2$ e $2w + r - 2 = 14$ f $w + r - 2$

3 a 5 b 8 c 0 d 1

4 a –3 b 12

Exercise 4B

1 a $(4x, 4x, -8x)$
$(-2y, 7y)$
b $(a, -6a)$
$(-ab, 6ab, 5ba, 4ab)$
$(-5b)$
c $(5p^2q)$
$(-2pq, 6pq, -qp)$
$(5q^2)$
d $(x^2, -x^2)$
$(-x)$
$(5xy, 6yx)$
$(-x^2y, -3yx^2)$
e $(2c^2d^2, -c^2d^2)$
(c^2d)
$(8d^2c)$
f $(x, 2x)$
$(y^2, 2y^2)$
(x^2)
$(2y, -y)$
$(2xy)$

2 a $3m + n + 2$ b $5x + 2y + 10$
c $3 + 2x - 2y$ d $9 - 2b + a$
e $4cd$ f 0

3 a $6x$ b $15x$ c $19x$ d $-5x$
e $8a$ f $-8a$ g $-5d$ h $-7x$
i $-20m$ j $5x$ k $7y$ l 0

4 a $2xy$ b $3xy$ c ab
d $4ab$ e $5pq$ f $5pq$
g $-6mn$ h $-7nm$ i $-2xy$
j $3xy$ k 0 l 0

5 a $8x + 5y$ b $7m - 3n$
c $8xy - zy$ d $4pq + 4q$
e $4x + 2y$ f $-x - 3$
g $-a - 3$ h $2 - 2x$
i $2x - 2y - 1$ j $-d + e$
k $-x - y + 1$ l $-g$
m $2x^2 - 3x$ n $-3x^2 - 6x$
o $34a$ p $6rk + 5r^4$
q $15t^2 - t$ r $19l - l^3$
s $3r^2 + 2r$ t $2wt + 12t^2$
u $15p^3 - 6p$

Exercise 4C

1 a $2m + 4$ b $3m + 27$ c $11h - 55$
d $8h - 96$ e $-7g + 42$ f $-8g + 32$

2 a $5x + 15$ b $3x + 12$
c $2c + 10$ d $7c + 42$
e $5g + 20$ f $2g + 14$
g $21b + 35$ h $6b + 27$
i $32h - 24$ j $12p - 42$
k $10w - 25$ l $24y - 8$
m $-2y - 10$ n $-15y - 10$
o $-b^2 - 3b$ p $-3k^2 - 2k$
q $-2k^2 + 10k$ r $-3t^2 + 21t$
s $-60 - 12p$ t $-45 - 30p$
u $-72 - 81p$ v $-3k^2 + 7kt$
w $-2k^2 + 10kt$ x $-2g^2 + 22gp$

3 a $5x + 7$ b $6x + 20$
c $5y + 13$ d $7y$
e $9t - 12$ f $31t - 6$
g $36p^2 + 27p + 9$ h $3p^2 - 13p - 8$
i $-m^2 + 14m$

4 a $5 + 3x$ b $14 + 2x$
c $2y + 13$ d $5y - 5$
e $5t - 36$ f $12t - 5$
g $2p - 6$ h $p - 8$
i $m - 16m^2$ j $11m - 6m^2$
k $-c + c^2$ l $-11d + 6d^2$

5 a $x^2 + 7x + 12$ b $x^2 + 12x + 35$
c $w^2 - 5w - 14$ d $w^2 - 8w - 9$
e $r^2 - 6r - 16$ f $r^2 + 5r - 24$
g $d^2 - 8d + 15$ h $d^2 - 11d + 18$
i $x^2 - 3x - 18$ j $3x^2 - 11x + 6$
k $2y^2 + 15y + 7$ l $2y^2 + 13y + 20$
m $10x^2 - 13x - 3$ n $12x^2 - 52x - 9$
o $16w^2 + 34w - 15$ p $24w^2 + 38w - 42$
q $-21q^2 + 162q + 48$ r $4q^2 - 8q - 12$

s $-6t^2 - 27t - 30$ t $-12m^2 + 4m + 8$
u $12m^2 - 50m + 50$

Exercise 4D

1 a $a^2 - 16$ b $a^2 - 49$
c $a^2 - 9$ d $m^2 - 1$
e $m^2 - 25$ f $m^2 - 36$
g $81 - x^2$ h $9 - x^2$
i $64 - y^2$ j $4y^2 - 1$
k $9y^2 - 4$ l $25y^2 - 49$
m $25 - 4m^2$ n $100 - 121n^2$
o $225 - 81n^2$

2 a $x^2 + 4x + 4$ b $x^2 + 12x + 36$
c $x^2 + 10x + 25$ d $x^2 + 6x + 9$
e $f^2 + 8f + 16$ f $f^2 + 18f + 81$
g $f^2 + 14f + 49$ h $f^2 + 2f + 1$
i $m^2 + 16m + 64$ j $m^2 + 22m + 121$
k $m^2 + 24m + 144$ l $m^2 + 40m + 400$
m $81 + 18t + t^2$ n $64 + 16t + t^2$
o $25 + 10t + t^2$ p $100 + 20t + t^2$

3 a $x^2 - 2x + 1$ b $x^2 - 10x + 25$
c $x^2 - 4x + 4$ d $x^2 - 16x + 64$
e $t^2 - 8t + 16$ f $t^2 - 6t + 9$
g $t^2 - 12t + 36$ h $t^2 - 14t + 49$
i $w^2 - 22w + 121$ j $w^2 - 18w + 81$
k $w^2 - 24w + 144$ l $w^2 - 30w + 225$
m $25 - 10r + r^2$ n $9 - 6r + r^2$
o $100 - 20r + r^2$ p $121 - 22r + r^2$

4 a $9x^2 - 12x + 4$ b $25x^2 - 30x + 9$
c $49x^2 - 70x + 25$ d $12x^2 - 84x + 147$
e $108t^2 - 36t + 3$ f $32t^2 - 144t + 162$
g $4 - 40t + 100t^2$ h $20 - 140t + 245t^2$
i $-25x^2 + 110xb - 121b^2$ j $-50x^2 + 40xb - 8b^2$
k $-12x^2 + 12xb - 3b^2$ l $-4x^2 + 16xb - 16b^2$
m $-8m^2 + 8mg - 2g^2$ n $-27m^2 + 36mg - 12g^2$
o $-80m^2 + 120mg - 45g^2$ p $-175m^2 + 210mg - 63g^2$

Exercise 4E

1 a $6(a + 2)$ b $13(2a + 1)$
c $9(2 + a)$ d $3(3 - b)$
e $5(5 - b)$ f $7(7 - b)$
g $x(x + 6)$ h $x(x + 15)$
i $x(x + 13)$ j $x(x - 4)$
k $x(x - 20)$ l $x(x - 32)$
m $2y(y + 5)$ n $3y(y + 15)$
o $5y(y + 10)$ p $4y(y - 6)$

2 a $3b(1 + 2b + 3a)$ b $10b(b + 10 - 100y)$
c $-6v(1 + 3v + 6v^2)$ d $-7v(5 + 7v - 8v^2)$
e $ab(a - 1 + 7b)$ f $3x(xy - y + 4)$
g $x^3(x - 1 + 7x^2)$ h $6y(y + 2x - 4x^2)$
i $4am(6m + 9a + 14)$ j $y^2(5y - 1 + y^2)$
k $2m^2(1 + 2m^2 + tm)$ l $ab(17b + 7)$

3 a $(a + 2)(b + 3)$ b $(b - 7)(c - 2)$
c $(x + 5)(m + 4)$ d $(t + 2)(2x - 3)$
e $(y - 5)(5m + 1)$ f $(b + 4)(7a - 3)$
g $(y + 1)(x + 5)$ h $(b + 7)(a + 1)$
i $(b + 5)(a + 2)$ j $(y + 7)(x + 2)$
k $(s + 3)(t + 2)$ l $(x + 2)(x + y)$
m $(b + 2)(a - 1)$ n $(a + 3)(b - 4)$
o $(r - 3)(s + 2)$

Exercise 4F

1 a $(b - 5)(b + 5)$ b $(a - 8)(a + 8)$
c $(h - 7)(h + 7)$ d $(3 - x)(3 + x)$
e $(4t - n)(4t + n)$ f $(6s - y)(6s + y)$
g $(7x - y)(7x + y)$ h $(9n - f)(9n + f)$
i $(10 - s)(10 + s)$ j $(1 + 3y)(1 - 3y)$
k $(7 - 4g)(7 + 4g)$ l $(12 - 9j)(12 + 9j)$
m $2(8a - b)(8a + b)$ n $2(4 - 5y)(4 + 5y)$
o $3(5 - 4m)(5 + 4m)$ p $\left(\frac{1}{a} - 5\right)\left(\frac{1}{a} + 5\right)$

2 a $b(b + 2)$ b $(x - 1)(x + 5)$
c $(k - 13)(k + 7)$ d $2(b + 5)(b + 3)$
e $3(x + 5)(x - 1)$ f $2(k + 11)(k - 5)$
g $(8 - b)(10 + b)$ h $(7 - x)(3 + x)$
i $(11 - k)(9 + k)$ j $2(5 + b)(1 - b)$
k $2(3 - x)(7 + x)$ l $2(8 - k)(12 + k)$
m $8b$ n $-4(x + 2)$
o $-5(2k + 1)$ p $-3x(x + 2)$
q $2(1 - b)(5 + b)$ r $2(13 - k)(7 + k)$

Exercise 4G

1 a $(h + y)^2$ b $(k + t)^2$
c $(x + y)^2$ d $(h - m)^2$
e $(j - 2y)^2$ f $(x - y)^2$
g $(b + 4g)^2$ h $(d + 3e)^2$
i $(e + 5g)^2$ j $(c - 6d)^2$
k $(w - 5p)^2$ l $(t - 10s)^2$
m $(h + 8y)^2$ n $(k + 7t)^2$
o $(m + 2p)^2$

2 a $(3x + y)^2$ b $(2b + c)^2$
c $(8x + y)^2$ d $(5h - m)^2$
e $(4j - y)^2$ f $(7g - h)^2$

g $(4b+g)^2$ **h** $(9d+e)^2$
i $(5e+g)^2$ **j** $(y-9t)^2$
k $(f-7h)^2$ **l** $(m-8p)^2$
m $(k+10q)^2$ **n** $(n+12r)^2$
o $(p+11s)^2$ **p** $2(c-d)^2$
q $5(w-p)^2$ **r** $6(t-2s)^2$
s $2(x+3y)^2$ **t** $3(b+3c)^2$
u $5(2b+m)^2$

Exercise 4H

1 **a** 21 **b** 24 **c** 34 **d** 16 **e** 51
f 21 **g** 6·6 **h** 0·32 **i** 13·5 **j** 25

2 **a** 34 **b** 60 **c** 44 **d** 28 **e** 14
f 7·8 **g** 0·54 **h** 9·1 **i** $4k-2$ **j** $10k-4$

3 **a** 15 **b** 21 **c** 36 **d** 18 **e** 54
f 177 **g** 0 **h** 174 **i** 18 **j** 108
k 72 **l** 108 **m** –207 **n** –45 **o** –189
p 3 **q** 0·75 **r** 2

4 **a** –36 **b** 150 **c** 95 **d** 2 **e** –20
f 2·5 **g** –2 **h** –12 **i** 12 **j** 113
k 13 **l** 274

5 **a** 24 **b** 486 **c** 726 **d** 2904 **e** 37·5
f 311·04 **g** 0·54 **h** 612·06 **i** 82·14

6 **a** $V = L \times W \times H$ **b** 2625 cm^3
c $S = 2(L \times W + W \times H + L \times H)$ **d** 2110 cm^2

See answer for Exercise 4H Question 7 below.

Exercise 4I

1 **a** 4·30 cm **b** 52·04 cm **c** 4·85 cm
d 26·35 cm **e** 3·25 cm **f** 69·99 cm
g 1·27 cm **h** 6·34 cm **i** 7·72 cm
j 5·88 cm **k** 6·58 cm **l** 8·66 cm

2 16·16 m

3 **a** 6·37 m **b** 6·41 m **c** 4 cm

4 **a** 4·176 m **b** 16·704 m **c** 33·409 m

5 **a** **i** 61·03 cm **ii** 128·06 cm
b **i** 6·71 m **ii** 5·38 m

7

a	b	c	d	$a+b$	$b+2c$	$3c+2d$	$4(a+c)$	$(d-b)^2$	$10-c^2$
1	2	3	4	3	8	17	16	4	1
2	5	6	10	7	17	38	32	25	–26
3	6	7	8	9	20	37	40	4	–39
2	5	5	11	7	15	37	28	36	–15
6	13	17	20	19	47	91	92	49	–279
5	15	25	40	20	65	155	120	625	–615
80	100	120	150	180	340	660	800	2500	–14 390
45	65	25	145	110	115	365	280	6400	–615
33	44	67	86	77	178	373	400	1764	–4479
1·5	2·5	7	10	4	16·5	41	34	56·25	–39
0·5	0·25	0·25	0·5	0·75	0·75	1·75	3	0·0625	9·9375
0·1	0·2	0·8	0·9	0·3	1·8	4·2	3·6	0·49	9·36
0·4	0·4	0·4	0·4	0·8	1·2	2	3·2	0	9·84
16	15·9	15·8	15·7	31·9	47·5	78·8	127·2	0·04	–239·64
300	100	400	500	400	900	2200	2800	160 000	–159 990
6·3	1·3	7·2	9·9	7·6	15·7	41·4	54	73·96	–41·84
346	382	399	502	728	1180	2201	2980	14 400	–159 191
204	575	201	344	779	977	1291	1620	53 361	–40 391
988·3333	367·6667	455·6667	345	1356	1279	2057	5776	513·7778	–207 622
111	666	333	999	777	1332	2997	1776	110 889	–110 879
–5	5	0	4	0	5	8	–20	1	10
0	5	7	9	5	19	39	28	16	–39
2	–6	1	–3	–4	–4	–3	12	9	9
0	0	0	0	0	0	0	0	0	10

Exercise 4J

1 a i $k = \frac{P}{m}$ ii -8

b i $b = y - mx$ ii -1

c i $h = \frac{C+g}{a}$ ii $-0{\cdot}5$

d i $x = \frac{a-c}{m}$ ii -6

e i $p = \frac{nk+2}{m}$ ii $210{\cdot}5$

f i $w = v(u + xy)$

2 a $r = \sqrt{\frac{V}{\pi h}}$ b $v = \sqrt{\frac{2E}{m}}$ c $h = \sqrt{\frac{Fk}{d}}$

d $m = \frac{nk^2}{49}$ e $b = \frac{cd^2}{4}$ f $y = \frac{zm^2}{x}$

3 a $x = z - y$ b $x = 2b + 3a$ c $x = \frac{n+m}{k}$

d $x = \frac{f-c}{d}$ e $x = rt - p$ f $x = vy + w$

g $x = \frac{pk-y}{2}$ h $x = \frac{wm-t}{r}$ i $x = \frac{mn-mp}{p}$

j $x = \frac{gh-e}{cd}$ k $x = \frac{mn+k}{ap}$ l $x = \frac{s-vw}{tu}$

m $x = \frac{yf-b}{d}$ n $x = \frac{n(t-kp)}{m}$

o $x = \frac{c(ab-f-o)}{d}$

4 a $t = \frac{v-u}{a}$ b i 4 s ii 10·525 s

5 a $r = \frac{C}{2\pi}$ b i 99·95 ii 2·61

6 $h = \frac{V}{\pi r^2}$ $h = 3{\cdot}14$

7 $w = \frac{P}{2} - l$ $w = 19$ m

8 a $b = \frac{2A-ah}{h}$ b $b = 13$

c to make b the subject of the equation

9 $b = \frac{2A}{h} - a$

Exercise 4K

1 a 8 and 9 b 41 and 42

c 18, 19 and 20 d 63, 64, 65 and 66

2 a 34 and 36 b 66, 68 and 70

c 157, 159 and 161 d 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

3 –16 and –15 or 15 and 16

4 17 and 19 or –19 and –17

5 Width 2 cm and length 10 cm

6 Width 8 cm and length 13 cm

7

Number of cuts	0	1	2	3	4
Total number of pieces	1	2	3	4	5

22 cuts for 23 pieces of string

8

Number of cuts	0	1	2	3	4
Total number of pieces	1	3	7	9	11

26 cuts for 53 pieces of string

9

22 pieces requires 6 cuts

10

Number of chess players	2	3	4	5	6	7
Number of handshakes	1	3	6	10	15	21

12 chess players for 66 handshakes

11 5 cm × 20 cm × 30 cm

12 1·5 cm × 20 cm × 24 cm

13 a 3 b 12 c 7

Applications

Circle regions

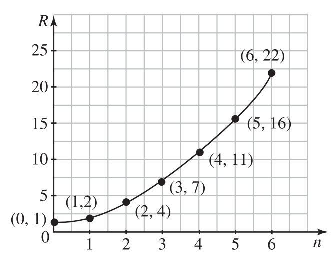
a If there are n lines then the number of regions is the previous number of regions plus n.

b

Number of lines (n)	Regions (R)
0	1
1	2
2	4
3	7
4	11
5	16
6	22

c $R = \frac{n(n+1)}{2} + 1$

d It is in the shape of a curve.



Handshakes

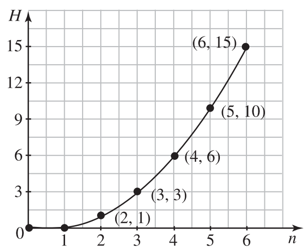
a The number of handshakes increases by $n - 1$ each time.

b

Number of learners (n)	Handshakes (H)
0	0
1	0
2	1
3	3
4	6
5	10
6	15

$H = \frac{n(n-1)}{2}$, $n > 0$ (when $n = 0$, or $1H = 0$)

c



Investigating Pythagoras

b $y = \sqrt{100^2 - x^2}$

e A quarter of a circle

f $x = \sqrt{100 - y^2}$

Optimisation

c The maximum area is 25 m × 25 m = 625 m^2.

d This does not change the maximum area.

e The curve is an inverted parabola with maximum at (25, 625).

f The enclosure area increases as the lengths of the sides become equal. For maximum area the lengths should be equal and the enclosure a square with side length 25 m.

Enrichment

1 a $(2x + 1)(x + 2)$ b $(3x + 1)(x + 1)$
c $(2x + 1)(2x + 3)$ d $(2x + 5)(x + 4)$
e $(2x + 7)(x + 1)$ f $(3x - 2)(x - 3)$
g $(2x - 1)(x - 3)$ h $(5x - 1)(x - 2)$
i $(4x + 1)(x - 2)$

2 2 s and 4 s

3 a 0 and 16 s b 8 s

4 2 h after dusk

5 a Area = $x(50 - 2x)$
b 15 m × 20 m or 10 m × 30 m

6 a Area = $x(120 - 2x)$ b 30 m × 60 m

7 a $\frac{3x+7}{6}$ b $\frac{4x-1}{12}$
c $\frac{2x+1}{10}$ d $\frac{5x+1}{6}$
e $\frac{7x+2}{12}$ f $\frac{3x+18}{10}$
g $\frac{8x^2+4x+3}{6x}$ h $\frac{15x^2+37x+21}{3x^2+9x}$
i $\frac{2x^2+17x-30}{6x^2-12x}$

8 a $\frac{x^2+4x+8}{2(x+3)}$ b $\frac{x^2-3x+5}{3(x-2)}$
c $\frac{x^2+7x-18}{5(x+4)}$ d $\frac{x^2+2x+1}{2(x-1)}$
e $\frac{x^2+4x-17}{4(x+3)}$ f $\frac{x^2-2x+4}{2(x-2)}$
g $\frac{9x+13}{12}$ h $\frac{31x+15}{30}$
i $\frac{9x+18}{20}$

9 a $x + 3$ b $8 - b$
c $\frac{(12+x)^2}{8-x}$ d $\frac{2}{x-3}$
e $(2x + 3)(x + 5)$ f $(8 + m)(3x - 2y)$

Revision/Assessment

1 **a** i equation ii expression
b i 3 ii –9
c i $3b$, $22t$, $-5t$, $10r$, 7
ii $12j$, $-9b$, $34k$, $-11j$, $7g$
d i b, t, r ii j, b, k, g
e i 7 ii 0

2 **a** $c + p$ **b** $c + p = 8$ **c** $4c$
d $p - 3$ **e** $4c + p - 3 = 14$ **f** $c + p - 2$

3 a $9r$ b $18t$ c $24a$
d $22m$ e $29y$ f $15m + 3b$

4 a $2x + 8$ b $3b - 6$ c $3k - 15$
d $-5m - 10$ e $-2m - 6$ f $-2b + 8$

5 a $15r + 79$ b $7x + 17$ c $6q + 24$
d $6r + 6$ e $9r + 147$ f $-4c - 3$

6 a $q^2 + 12q + 35$ b $g^2 + 10g + 9$
c $m^2 - 5m - 24$ d $w^2 - 2w - 8$
e $t^2 - 8t - 20$ f $y^2 - 10y + 16$

7 a $d^2 - 144$ b $f^2 - 169$
c $m^2 - 225$ d $4c^2 - 9$
e $36g^2 - 81$ f $625k^2 - 4$

8 a $d^2 - 6d + 9$ b $h^2 + 4h + 4$
c $49h^2 - 42d + 9$ d $4w^2 + 20w + 25$
e $16r^2 - 16r + 4$ f $49t^2 + 42t + 9$

9 a $2(x - 6)$ b $x(3x - 5)$
c $13d(d^2 + 13d - 3)$ d $5abc(1 - 5a + 125b)$
e $(2m + 6)(m + 4)$ f $(5m - 5)(2m - 6)$

10 a $(b - \sqrt{15})(b + \sqrt{15})$ b $(a - 7)(a + 7)$
c $(8 - x)(8 + x)$ d $(6 - n)(6 + n)$
e $(10x - y)(10x + y)$ f $(1 - 9y)(1 + 9y)$

11 a $(s + y)^2$ b $(p - m)^2$
c $(w - 3y)^2$ d $(q - 4h)^2$
e $(2a + 5t)^2$ f $(4w + 12p)^2$

12 a 15 b 36 c 177 d 108 e 432
f –378 g 2 h 456 i 3910

13 a 24 b 150 c 95 d 2 e –20 f –2·5

14 a 27·33 b 78·33 c 15·23

15 a 20 b 205 c 407

16 a $x = 10{\cdot}72$ cm, $y = 23{\cdot}58$ cm
b $x = 6{\cdot}32$ m, $y = 3{\cdot}87$ m

17 a $b = \frac{2A}{h} - a$ b $b = 2$
c to make B the subject of the formula

18 a 10 m by 20 m
b $x = 6{\cdot}83$, so sides are 6·83, 3·83 and 7·83.
Area = 13·01 units^2

Chapter 5

Exercise 5A

1 a Black 1:2 Red 1:6
Blue 2:15 Green 1:15
Yellow 2:15
b Black 13:30 Red 11:60
Blue 1:5 Yellow 11:60
c Black 7:15 Red 1:5
Blue 2:15 Yellow 1:5
d Black 1:10 Red 2:5
Blue 7:30 Yellow 4:15
e Black 59:102 Red 7:51
Blue 2:17 Green 1:51
Yellow 5:34

2 a 4:5 b 3:13 c 4:7 d 4:5 e 3:1
f 8:1 g 4:3 h 41:4 i 5:1 j 4:1

3 a 6:5 b 7:25 c 1200:1 d 19:125
e 16:1 f 3:5 g 172:69 h 25:1
i 5:2 j 4:1 k 9:1 l 2:1

4 a i 9:25 ii 9:49 iii 1:9
b i 25:49 ii 25:81
c 49:81

5 a i 8:27 ii 1:8 iii 8:125
b i 27:64 ii 27:125
c 64:125

Exercise 5B

1 a 8 b 4 c 7·5 d 20 e 6
f 6 g 3·2 h 3·2 i 22·4

2 a 4 b 1 c 2 d 7
e 5 f $20\frac{4}{7}$ g 21 h 18
i 1·5 j $1\frac{2}{3}$ k $1\frac{3}{11}$ l $1\frac{3}{7}$

3 a 24 b 60

4 a 180 mL b 630 mL

5 7·5 kg

6 Agatha = \$3382
Bree = \$4272
Christie = \$3026

7 Duy = \$1599
Effie = \$1353
Gary = \$1107

8 12·3 ha, 24·6 ha and 36·9 ha

9 a 200 mL b $1066\frac{2}{3}$ mL

10 a \$4130 in tax b \$9086

11 35

12 5 litres of petrol and 0·5 litres of oil

13 a refiners/wholesalers: \$502 520
oil producers: \$251 260
service station owners: \$125 630
b government: \$717 166
refiners/wholesalers: \$3 874 296
oil producers: \$1 937 148
service station owners: \$968 574

14 1 g of tomato sauce

Exercise 5C

1 a 150 m b 200 cm c 112·5 g
d 30 kg e \$1140 f 120 min
g 60 h h 2500 mm i 245 t
j \$63 k 195 s l 585 m
m 2262·5 t n 167·5 km o 115 g

2 a 40 m b 327 mm c \$7·70
d 157 kg e 16 h f 21 t
g \$26·10 h 335 g i 852 min
j 12·58 cm k \$586 l 25 h
m 14 km n 97 t o 96 m

3 a Four-fifths b One-third
c Three-eighths d Three-sevenths
e One-quarter f Nine-elevenths

4 a Eight-sevenths b Nine-eighths
c Five-thirds d Fourteen-ninths
e Seven-fifths f Seven-quarters

5 a 40 g b 189 g c 327 g
d 334 g e 155 g

6

President	$1380
Senior	$1560
Committee member	$1440
Junior	$780

7

Two-storey villa unit	$6 230 000
Flat	$1 645 000
Apartment	$5 460 000
Single-storey house	$6 930 000
Double-storey house	$3 937 500
Shop front	$1 557 500

8 a 1080 mL b 1350 mL c 2340 mL
d 4·5 L e 18 L

9

Bully Dipper	531 m
Spin 'n' Scream	567 m
Water Spout of Terror	486 m
Cannon Shot	576 m

Exercise 5D

1 a $ b kg
c degrees C/min
d varies e.g. scale 1 : 500 000 is 1 unit to 500 000 units.
e $/h

2 Answers will vary. Examples include:
a rugby scoring b copra sales
c sowing seeds d water flow from a tap
e price of milk f basketball

3 a $78 b 48 c $0·10 d 28
e $25·60 f 800 g $19·50 h 2
i 8·5 j 50 k 3 l 30

4 a 3 points/game
b i 6 points ii 9 points
iii 15 points iv 21 points

5 a $187·50 b $562·50
c $750 d $3125

6 4 mL water/5 g flour
a 960 mL b 1760 mL c 3 L
d 9·6 L e 18 L

7 Rate: 5 km/litre
a 1·2 litres b 0·76 litres
c 0·84 litres d 0·64 litres
e 1·12 litres f 0·98 litres

8 90 L/min
a 108 L b 261 L c 1620 L d 1102·5 L

Exercise 5E

1 a 13·33 m/s b 20 m/s
c 500 m/s d 8·89 m/s
e 6·67 m/s f 0·56 m/s
g 5·09 m/s h 0·06 m/s
i 33·33 m/s j 3·47 m/s

2 a 216 km/h b 129·6 km/h
c 324 km/h d 72 km/h
e 388·8 km/h f 64·8 km/h
g 51·84 km/h h 1036·8 km/h
i 18 km/h j 25·92 km/h

3 a 216 000 L/h b $480/h
c 1·2 kg/min d 540 m/min

4 a 144 000 mL/h b 3·6 kg/min
c 2 t/min d 144 000 cm/min
e 25 200 L/h f 201·6 L/day
g $0·83/min h 1680 cm/h
i 7500 g/min j 1368 kg/h

5 a 100 mL/min b 600 mL/min
c 50 000 mL/min

6 a 2·5 km/min b 0·04 km/s
c 150 000 m/h d 2500 m/min
e 41·7 m/s

7 a 8·83 km/min b 0·15 km/s
c 530 000 m/h d 8833 m/min
e 147·2 m/s

8 a 18 m/min and 30 m/min
b 0·3 m/s and 0·5 m/s
c 1080 m/h and 1800 m/h
d 18 m/min and 30 m/min
e 1·08 km/h and 1·8 km/h

9 a 2 cm/month b 0·24 m/year
c 0·66 mm/day d 0·02 m/month
e 20 mm/month f 0·066 cm/day
g 0·000 000 03 km/h h 0·000 000 007 6 m/s

10

	Fasi	Sale	Kwaimani
a	7200	9000	10 800
b	2	2·5	3
c	172 800	216 000	259 200

11 a Week 1: 2·53 kg, Week 2: 3·86 kg,
Week 3: 5·19 kg, Week 4: 6·52 kg
b i 7·92 g/h ii 0·13 g/min iii 0·0022 g/s

12 a i 1200 L/h
ii 20 000 mL/min
iii 1 200 000 mL/h

b i 0·05 h, 3 min
ii 0·038 h, 2·25 min
iii 0·021 h, 1·25 min
iv 0·1 h, 6 min

13 a i 2 pies/s ii 7200 pies/h
iii 172 800 pies/day
b i 4·5 min ii 8·5 min
iii 12·25 min iv 18·5 min
v 19·75 min

14 a \$120 : 20 L = \$6 : 1 L
b i \$60 ii \$300 iii \$1·50
iv \$3 v \$15
c i 8 L ii 18 L iii 19 L
iv 2·5 L v 8·5 L

Exercise 5F

1 a cm/s b m/s c m/h
d m/s or km/h e m/s f cm/s
g m/s h cm/s

2 Examples might be:
a a racing car, bird in flight
b arrow, missile in flight
c wave breaking on a beach, ball rolling down a hill
d ant moving across a table, bread rising in an oven

3 a 203 km/h b 456 km/h c 85 km/h
d 400 km/h e 6·4 km/h f 5 km/h

4 a 300 m/s b 1·67 m/s c 0·5 m/s
d 0·8 m/s e 0·02 m/s

5

		Dolphin	Whale	Shark
a	i	10·67 km	8 km	5·3 km
	ii	26·68 km	20 km	13·25 km
	iii	48 km	36 km	24 km
	iv	96 km	72 km	48 km
b		17·8 m/s	13·3 m/s	8·9 m/s
	i	89 m	67 m	44 m
	ii	533 m	399 m	267 m
	iii	2133 m	1600 m	1067 m
	iv	3733 m	2800 m	1867 m

		Penguin	Tuna
a	i	4 km	14·7 km
	ii	10 km	36·67 km
	iii	18 km	66 km
	iv	36 km	132 km
b		6·7 m/s	24·4 m/s
	i	33 m	122 m
	ii	200 m	733 m
	iii	800 m	2933 m
	iv	1400 m	5133 m

6

	Bee	Hoverfly	Hawkmoth	Dragonfly
a	3·06 m/s	3·89 m/s	13·89 m/s	20·83 m/s
b	33 s	26 s	7 s	5 s

7

	Duck	Teal	Crow
a	$\frac{1}{850}$ h	$\frac{1}{1200}$ h	$\frac{1}{400}$ h
b	4·2 s	3·0 s	9·0 s

Exercise 5G

1 a 2·7 g/cm^3 b 8·9 g/cm^3
c 1 g/cm^3 d 2·5 g/cm^3

2 a 32·4 g b 48·6 g c 78·3 g
d 513 g e 5400 g

3 a 110·6 g b 774·2 g c 308·1 g
d 948 g e 23 700 g

4 a 0·016 g/cm^3 b 1·1 g/cm^3 c 11·4 g/cm^3
d 2·7 g/cm^3

5 a 60 000 g b 16 000 g c 320 000 g

6 158 kg

7 a 18 kg, 10·8 kg, difference = 7·2 kg
b 2·25 kg, 1·35 kg, difference = 0·9 kg
c 108 kg, 64·8 kg, difference = 43·2 kg

8

Material	Density (kg/m^3)	Density (g/cm^3)
Polystyrene (plastic)	16	0·016
Polypropylene (plastic)	900	0·9
Nylon (plastic)	1140	1·14
White gum (wood)	1100	1·1
Lead (metal)	11 400	11·4
Wet sand	1230	1·23

Learning task 5H

1 **a** 12:10 pm **b** 5:00 pm

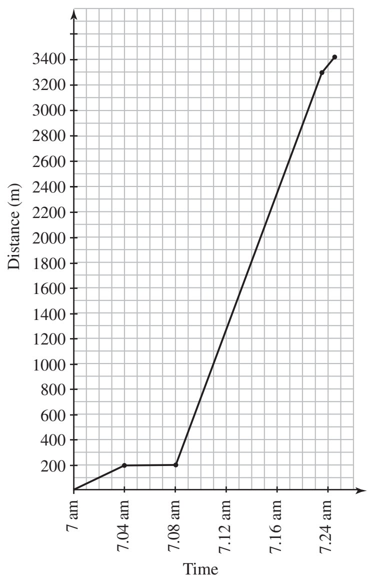
c 120 min **d** 170 min

Section	Distance (km)	Times	Time travelled (hour)	Speed (km/h)
A	7	12:10–12:30	$\frac{1}{3}$	$7 \div \frac{1}{3} = 21$
B	0	12:30–1:30	1	0
C	16	1:30–2:15	$\frac{3}{4}$	21·33
D	0	2:15–3:15	1	0
E	23	3:15–5:00	$1\frac{3}{4}$	13·14

2 **a**

Section	Distance (km)	Times	Time travelled (hour)	Speed (km/h)
A	200	7:00–7:04	4 mins	50 m/min or 0.83 m/s or 5/6 m/s
B	0	7:04–7:08	4 mins	0 m/s
C	3000	7:08–7:23	15 mins	12 km/h
D	0	7:23–7:24	1 min	0 m/s
E	120	7:24–7:25	1 min	2 m/s

c



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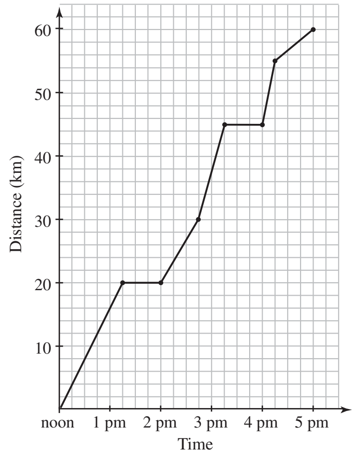


Section	Distance (km)	Times	Time travelled (hour)	Speed (km/h)
A	20	noon–1:15 pm	$1\frac{1}{4}$	$20 \div 1\frac{1}{4} = 16$
B	0	1:15–2:00 pm	$\frac{3}{4}$	0
C	10	2:00–2:45 pm	$\frac{3}{4}$	$\frac{40}{3} = 13{\cdot}33$
D	15	2:45–3:15 pm	$\frac{1}{2}$	30
E	0	3:15–4:00 pm	$\frac{3}{4}$	0
F	10	4:00–4:15 pm	$\frac{1}{4}$	40
G	5	4:15–5:00 pm	$\frac{3}{4}$	$\frac{20}{3} = 6{\cdot}67$

Exercise 5I

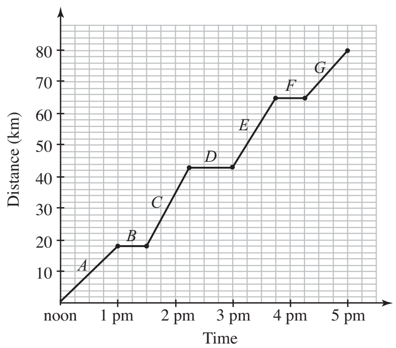
1 **a**

Time	Temperature (°C)
4:00 am	⁻4
6:00 am	⁻2
8:00 am	⁻2
10:00 am	0
noon	4
2:00 pm	6
4:00 pm	4
6:00 pm	4
8:00 pm	⁻4

b

Times	Average temp. change
4:00–6:00 am	$^{+}1$°C
6:00–8:00 am	0°C
8:00–10:00 am	$^{+}1$°C
10:00–noon	$^{+}2$°C
noon–2:00 pm	$^{+}1$°C
2:00–4:00 pm	$^{-}1$°C
4:00–6:00 pm	0°C
6:00–8:00 pm	$^{-}4$°C

2 a



b i noon ii 5:00 pm

iii 90 min iv 210 min

c Section A: 18 km/h, Section C: 33·3 km/h, Section E: 22 km/h, Section G: 20 km/h

Learning task 5J

1

Time	Temperature (°C)	Time	Temperature (°C)
noon	38	3:15	26
12:15	40	3:30	26
12:30	41	3:45	27
12:45	43	4:00	30
1:00	44	4:15	31
1:15	45	4:30	32
1:30	45	4:45	33
1:45	45	5:00	33
2:00	44	5:15	33
2:15	41	5:30	33
2:30	37	5:45	32
2:45	33	6:00	31
3:00	29		

2

Time interval	Rate of change
noon–12:30 pm	increases at 6°C/h
1:00–1:30 pm	increases at 2°C/h
2:00–2:30 pm	decreases at 14°C/h
3:00–3:30 pm	decreases at 6°C/h
4:00–4:30 pm	increases at 4°C/h
5:00–5:30 pm	0°C (temperature does not change)

3

12:30 pm	$^{+}4$°C/h
1:30 pm	$^{+}0{\cdot}5$°C/h
2:00 pm	$^{-}14$°C/h
2:30 pm	$^{-}36$°C/h
3:00 pm	$^{-}35$°C/h
3:30 pm	0°C
4:00 pm	$^{+}20$°C/h
4:30 pm	$^{+}10$°C/h
5:00 pm	$^{+}4$°C/h
5:30 pm	$^{-}5$°C/h
6:00 pm	$^{-}8$°C/h

Applications

Heart rates and health

1 a 180 bpm b 152 bpm c 169 bpm

2 a 97 828 560 beats

b 1 956 571 200 beats

c 6 847 999 200 beats

3 a 89 413 200 beats

b 1 788 264 000 beats

c 6 258 924 000 beats

Navigational speed

a 21 600

b 1·851 85 km ≈ 1852 m

c 1·85 km/h

d i 9260 m/h ii 14 816 m/h

iii 22 224 m/h iv 37 040 m/h

v 55 560 m/h

Scale model of the planets

Mercury 0·38 times the diameter of Earth.
Venus 0·95
Mars 0·53
Jupiter 10·85
Saturn 8·99
Uranus 3·96
Neptune 3·85
Pluto 0·18

Paper sizes

a A0: 1189 mm by 841 mm; 41 : 29
A1: 594 mm by 841 mm; 841 : 594
A2: 594 mm by 420 mm; 99 : 70
A3: 297 mm by 420 mm; 99 : 140
A4: 297 mm by 210 mm; 99 : 70

A5: 149 mm by 210 mm; 149 : 210
A6: 149 mm by 105 mm; 149 : 105

b A0 is used as the first standard size as its area is approximately 1 m^2.

Area	Area ratio	
A0 = 999 949 $mm^2 \approx 1\ m^2$		
A1 = 499 554 mm^2	A1 : A0	1 : 2
A2 = 249 480 mm^2	A2 : A0	1 : 4
A3 = 124 740 mm^2	A3 : A0	1 : 8
A4 = 62 370 mm^2	A4 : A0	1 : 16
A5 = 22 050 mm^2	A5 : A0	1 : 32
A6 = 15 645 mm^2	A6 : A0	1 : 64

c **Length of diagonal**
A0 = 1456 mm
A1 = 1030 mm
A2 = 727 mm
A3 = 514 mm
A4 = 364 mm
A5 = 257 mm
A6 = 182 mm

d A0 : A1 1 : 4 : 1
A1 : A2 1 : 4 : 1
A2 : A3 1 : 4 : 1
A3 : A4 1 : 4 : 1
A4 : A5 1 : 4 : 1
A5 : A6 1 : 4 : 1
The ratios are the same.

Enrichment

1 **a** $\frac{9}{20} \times$ litres
b $2\frac{1}{7}$ litres

2 **a** 3 hours 51 min **b** 77·92 km/h

3 **a** The zebra will finish 1·75 s later and the ostrich will finish 3·41 s later.
b The penguin will finish 5·73 s later and the dolphin will finish 2·36 s later.

4 **a** Bluey: 13·8 km/h
Ruby: 14·3 km/h
b They meet at 2:15 pm when Ruby has just finished resting and Bluey is still paddling.
c Bluey: 2 h and 15 min
Ruby: 2 h and 15 min
d Bluey: 28 km/h, 43 km/h, 13 km/h, 17 km/h
Ruby: 20 km/h, 8 km/h, 43 km/h, 20 km/h
e Bluey: 20 km/h
Ruby: 21 km/h

5 **b** Rests: 12:30 pm–1:30 pm, 2:15 pm–3:15 pm
c Travelled: noon–12:30 pm, 1:30 pm–2:15 pm, 3:15 pm–5:00 pm

d

Time	Distance travelled (km)
noon–12:30	6
12:30–1:30	0
1:30–2:15	12·5
2:15–3:15	0
3:15–5:00	18

e 12 km/h, 17 km/h and 11 km/h
f 17 km/h **g** 7 km/h **h** 7·3 km/h
i 17 km/h **j** 17 km/h

6 The ratios are the same.

7 **a** $A = 20$ $B = 30$
b **i** $\frac{1}{2}A$ **ii** $2A$ **iii** $\frac{7A}{2}$
iv $5A$ **v** $\frac{A(3x-2)}{2}$

8 **a** 14 : 15 **b** 2 : 15 **c** 3 : 2 **d** 1 : 1

Revision/Assessment

1 Yellow:black 4 : 15
Red:black 4 : 15
Blue:black 4 : 15
Green:black 1 : 5

2 **a** 7 : 8 **b** 7 : 13 **c** 2 : 3 **d** 6 : 5

3 **a** 1 : 24 **b** 3 : 2 **c** 15 : 1 **d** 3 : 2

4 **a** 4 **b** 162 **c** 18 **d** $3\frac{5}{9}$

5 **a** 57 jeans **b** 190

6 28·8 L of syrups 240 L of water

7 **a** 37·5 m **b** 150 cms
c 57·5 g **d** 22·5 kg
e $162·50 **f** 115 min

8 **a** 24 g **b** 179 mm
c $9·02 **d** 153·7 kg
e 9 h **f** 2·1 h

10 **a** 11·2 km/L **b** $69\frac{1}{3}$ km/h
c 50°C/minute **d** 11·5 overs/wicket

11 $3·25/metre
a $149·50 **b** $468 **c** $291·53

12 $1·80/word
a $54 **b** $75·60 **c** $151·20 **d** $982·80

13 **a** 10 m/s **b** 11·67 m/s **c** 50 m/s
d 17·22 m/s **e** 23·89 m/s

14 **a** 252 000 L/h **b** $720/h
c 0·75°C/s **d** 520 g/cm

15 **a** $0·10/min **b** 2100 cm/h
c 5333·33 g/min **d** 432 kg/h

16 a 1800 L/h b 30 000 mL/min
c 1 800 000 mL/h

17 a 214 km/h, 59·4 m/s b 36 km/h, 10 m/s
c 2·4 km/h, 0·67 m/s d 864 km/h, 240 m/s

18 a 2·7 g/cm^3 b 8·9 g/cm^3

19 a 13·68 g b 41·04 g c 59·28 g
d 205·2 g e 5700 g

Chapter 6

Exercise 6A

1 a 1747 b 1719
c The store might sell something such as beach wear or fans, as sales were highest in the summer months.

2 a $43 400 b $6600 c $5040
d i Food, insurance ii Car

3 a $40 000 b $10 000

4

Fruits & Vegetables	Number of Hips	Cost per Hip	Cost
Banana	6	$8·00	$48·00
Pineapple	4	$12·00	$48·00
Eggplant	4	$5·00	$20·00
Potato	8	$10·00	$80·00
Capsicum	5	$5·00	$25·00
		Total	$221·00

Learning task 6B

1 a $120·00
b The inconvenience and the cost of travel to the takeaway food store should be considered.
There is no guarantee of freshness and quality when buying take-away food.
The cost of packaging is built into the price of take-away food.

2 a $54·00 b Learner's own answers.
c Learner's own answers.

3 Best Smile toothpaste

4 a Solomon Blue $22·00, Waioka Tuna $24·00
b Family Waioka Tuna
c 2 family tins of Solomon Blue plus 2 small tins of either Solomon Blue or Waioka Tuna

5 a $450·00 b 56 lb

6 MV Bikoi

Learning task 6C

1 a 76% b 60%

2 a 76% b 52% c 52%

3 a 30% b 20% c 10%

4 a 50% b 50% c 50%

5 a 10% b 50% c 80%

6 a 8·3% b 41·7% c 75%

7 a i Classics ii Dance iii Chill out
b Rock, classics, dance, rap, chill out

8 a i B ii AB iii O
b B, A, AB, O

9

	a	b
NSW	$\frac{0{\cdot}2}{4} \times 100 = 5\%$	$0·2 billion
Qld	$\frac{0{\cdot}2}{4} \times 100 = 5\%$	$0·2 billion
SA	$\frac{1{\cdot}2}{4} \times 100 = 30\%$	$1·2 billion
Vic	$\frac{2{\cdot}4}{4} \times 100 = 60\%$	$2·4 billion

Exercise 6D

1 a 30% b 8% c 25% d 35%
e 18% f 95% g 70% h 68%
i 80% j 62·5% k 75% l 150%

2 a 45% b 74% c 34% d 23%
e 19% f 55% g 64·3% h 51·2%
i 9% j 87% k 94% l 7%

3 a $\frac{1}{4}$ b $\frac{24}{25}$ c $\frac{37}{100}$ d $\frac{4}{5}$
e $\frac{49}{100}$ f $\frac{33}{100}$ g $\frac{1}{2}$ h $\frac{11}{25}$
i $\frac{4}{25}$ j $\frac{21}{100}$ k $\frac{19}{25}$ l $\frac{13}{20}$

4 a 0·75 b 0·84 c 0·21 d 0·16
e 0·1 f 0·35 g 0·51 h 0·64
i 0·08 j 0·47 k 0·92 l 0·59

Exercise 6E

1 a $28 b $10 c $14 d $10
e $225 f $1680 g $385 h $33·63

2 a $6000 b $2800 c $3825
d $9119 e $6050 f $8912·5
g $6000 h $14 500 i $10 065

3 a $5191·20 b $432·60

4 a $449·87 b $36·97 c $34·81

5 a $675·75 b $51·84 c $6·10

6 a 0·000 27% 0·0068% b $0·02 c $0.64

Exercise 6F

1 a 50% b 25% c 20% d 10%
e 0·5% f 1% g 16·7% h 8·3%

2 a 20% b 10% c 16·7% d 8·3%
e 5% f 1·39% g 2·5% h 0·67%

3 a 500 b 5000 c 640 d 350
e 840 f 6008 g 8600 h 85

4 a $8000 b $450 c $500
d $1005 e $62 000 f $500
g $125 000 h $6280 i $5600

5 0·46%

6 a 0·581% b 6·97%

Exercise 6G

1 a 50% b 75% c 25%
d 33% e 16·67% f 66·67%

2 a 25% b 37·5% c 87·5%
d 85% e 43·75% f 12·5%

3 a $765·00 b $486·00 c $7875·00

4 44·8%

5 a 25·9% b 41·6%

6 a $272·00 b $161·10 c $467·10

7 a $76·10 b $22·70 c $25·20

Exercise 6H

1 a $500·00 b $42·90 c $26·90 d $370·00

2 a –$150·00 b –$40·50
c –$5·00 d –$60·00

3 a $30·00 b $1800·00

4 $1640·00 5 –$42 400·00

6 a 62·5% b 33·3% c –20% d –36·4%

7 111·4%

8 a $22·50 b $1125·00 c $987·50

Exercise 6I

1 a $1180 b $26 100 c $1116
d $480 e $67·49 f $12·06
g $70·20 h $113·10 i $291·20

2 a $480·00 b $750·00
c $2284·20 d $3378·00

3 a $15 675 b $61 312·50
c $35 309·18 d $50 542·50

4 a i $3150·00 ii $5495·00
iii $17 115·00 iv $31 255·00
b i $5·00 ii $500·00 iii $2500·00

5 a $1825·00 b $2875·00 c $120 000·00

6 Salary: $52 000; Commission: $183 692·00; Base wage plus commission

Exercise 6J

1 a $3033·33 b $1400 c $700

2 a $498 b $996 c $25 896

3 a $7545
b i $1880.10 ∴ gets refund ii $306·90
c i $2636·40 ii $0
d $18 741·60 e $32 189·83

4 a $2826 b $1386 c $4212

Exercise 6K

1 a $100 b $672 c $182
d $6·20 e $10 525 f $1488·38

2 $900 3 $5208

4 $17 325 5 $26·67

6 a $2475 b $4125 c $9075 d $14 850

7 a $112 000 b $17 100 c $3471
d $55 296 e $7718·75 f $1000·50

Applications

Budgeting

a

	Inc	Exp	Balance $
Home loan	4468	1200	3268·00
Electricity		155	3113·00
Telephone		154	2959·00
Car loan		82·5	2876·50
Rates		74·83	2801·67
Mobile phone		30	2771·67
Insurance		100	2671·67
Food		600	2071·67
Petrol		440	1631·67
Car repairs		60	1571·67
Miscellaneous		320	1251·67
Credit card		200	1051·67

b $1051·67

c i 26·86% ii 13·43% iii 23·54%

d $4021·20

Fundraising for a cause

a $396

b $26·40 per month, $316·80 per year

e $79·20

f Possible options are:

Cars	4 × 4	Money raised
7	0	$21
5	1	$20
4	2	$22
2	3	$21
0	4	$20

Enrichment

1 a $900 b $5208 c $16 500

2 a $2372·54 b $2262·82 c $4517·16 d $2437·27 e $2079·89

Revision/Assessment

1 a $38 000 b $22 000

2 a 68% b 36%

3 a 50% b 250% c 10% d 5% e 44·4% f 219·19% g 566% h 97% i 0·3% j 70% k 210% l 240·5%

4 a 750% b 525% c 275% d 480% e 37·5% f 83·3% g 33·3% h 41·7%

5 a $\frac{1}{4}$ b 0·3 c $\frac{3}{20}$ d 8% e $\frac{1}{5}$ f 0·5

6 a $35 b $12 c $18 d $100 e $75 f $0·60 g $11 h $25 i $2·13

7 a 70% b 55% c 75% d 91·7% e 96·7% f 46·7%

8 70 marks

9 a 20% b 2% c 3% d 18·75%

10 67

11 a 400 b 150 c 85·3 d 40

12 50%

13 –16·7%

14 a $470 b $5700 c $1240 d $56 e $32·50 f $120

15 a $128 b $6656 c $998·40 d $19.20 e $108.80

16 $500

17 $6400

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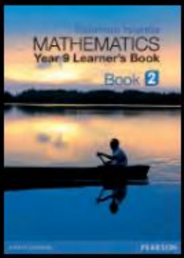

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