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NEW CENTURY MATHS

MATHEMATICS STANDARD (PATHWAY 2)

YEAR 11

Klaas Bootsma
Sarah Hamper
Margaret Willard
Robert Yen

4TH EDITION

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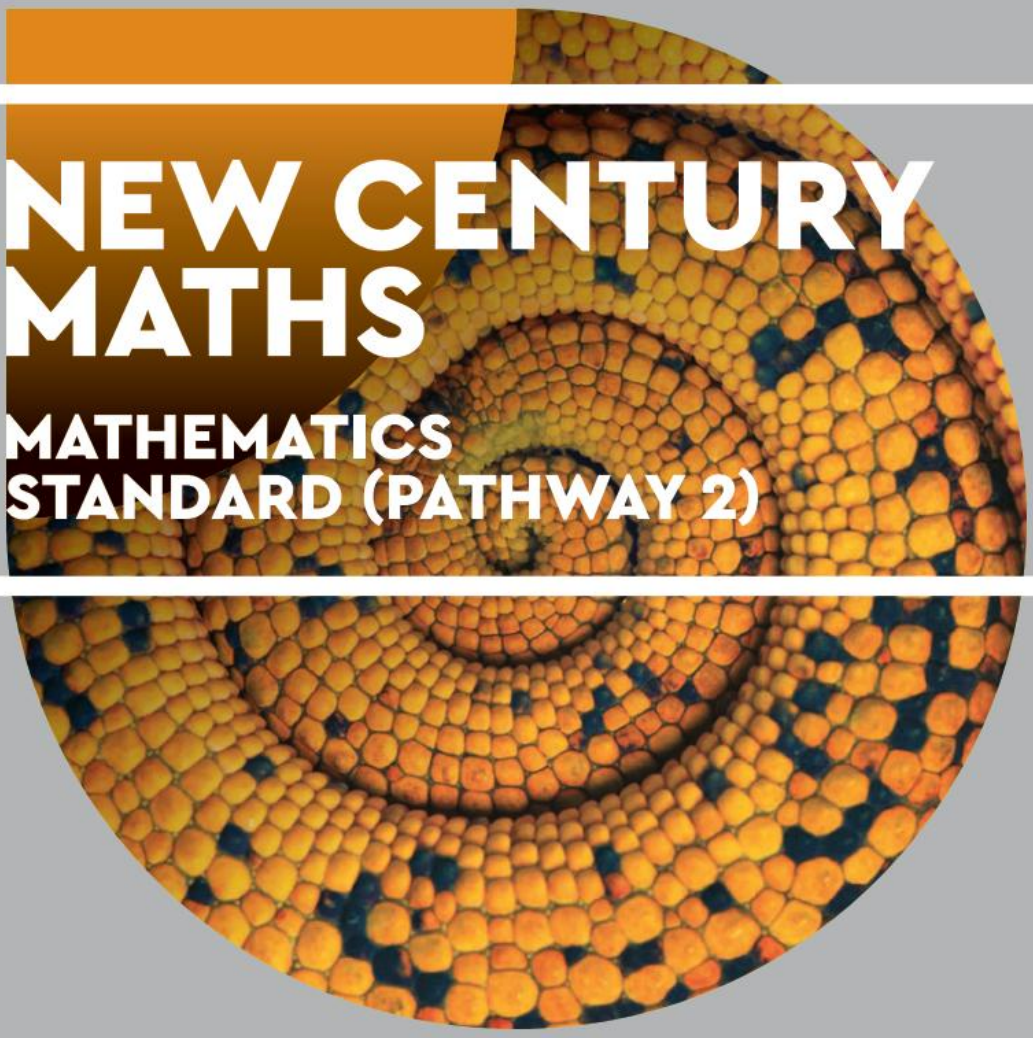

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Series editor: Robert Yen

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Publishing editor: Robert Yen
Project editors: Alan Stewart and Anna Pang
Editor: Marcia Bascombe
Cover design: Chris Starr (MakeWork)
Text design: Nicole Melbourne
Project designer: Aisling Gallagher
Cover image: iStock.com/GlobalP
Permissions researcher: Lyahna Spencer
Production controller: Christine Fotis
Typeset by: Cenveo Publisher Services

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National Library of Australia Cataloguing-in-Publication Data
Bootsma, Klaas, author.
New century maths year 11: mathematics standard pathway 2 / Klaas Bootsma, Sarah Hamper, Margaret Willard, Robert Yen.
4th
9780170413565 (paperback)
For secondary school age.
Mathematics--Problems, exercises, etc.
Mathematics--Textbooks.

Hamper, Sarah, author.
Willard, Margaret, author.
Yen, Robert, author.

Cengage Learning Australia
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South Melbourne, Victoria Australia 3205

Cengage Learning New Zealand
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331 Rosedale Road, Albany, North Shore 0632, NZ

For learning solutions, visit **cengage.com.au**

Printed in China by China Translation & Printing Services.
1 2 3 4 5 6 7 21 20 19 18 17

CONTENTS

PREFACE vi
ABOUT THE AUTHORS vii
SYLLABUS REFERENCE GRID viii
ABOUT THIS BOOK ix
MATHEMATICAL VERBS xiv
SYMBOLS AND ABBREVIATIONS xv

1

COLLECTING AND PRESENTING DATA 2

S1.1 1.01 Interpreting graphs 5
S1.1 1.02 Misleading graphs 12
S1.1 1.03 Types of data 15
S1.1 1.04 Sampling techniques 18
S1.1 1.05 Constructing graphs 26
S1.1 1.06 Frequency histograms and polygons 37
S1.1 1.07 Dot plots and stem-and-leaf plots 45
Study tip: Four practical steps in studying maths 52
Chapter summary 53
Test yourself 1 54

2

FORMULAS AND EQUATIONS 58

A1 2.01 Simplifying algebraic expressions 61
A1 2.02 Expanding algebraic expressions 65
A1 2.03 Formulas 68
A1 2.04 Solving equations 74
A1 2.05 Formulas and equations 78
A1 2.06 Changing the subject of a formula 82
Study tip: Topic summaries and mind maps 84
Chapter summary 85
Test yourself 2 86

3

EARNING MONEY AND TAXATION 88

F1.2 3.01 Wages, salaries and overtime 91
F1.2 3.02 Commission, piecework and royalties 95
F1.2 3.03 Bonuses, allowances and annual leave loading 99
F1.2 3.04 Government allowances and pensions 102
F1.2 3.05 Gross pay and net pay 108
F1.2 3.06 Income tax and Medicare levy 113
F1.2 3.07 PAYG and tax returns 117
F1.1 3.08 GST and VAT 119
Study tip: Showing your working 124
Chapter summary 125
Test yourself 3 126

PRACTICE PAPER 1 128

ISBN 9780170413565

4

PROBABILITY 134

S2 4.01 Probability of simple events 138
S2 4.02 Tables and tree diagrams 144
S2 4.03 Complementary events 150
S2 4.04 Relative frequency 155
S2 4.05 Comparing relative frequency and theoretical probability 162
S2 4.06 Probability tree diagrams 168
Study tip: When and where to study 174
Chapter summary 175
Test yourself 4 176

5

MEASUREMENT 178

M1.1, 1.3 5.01 Metric units 181
M1.1 5.02 Error in measurement 185
M1.1 5.03 Significant figures 190
M1.1 5.04 Scientific notation 193
M1.2 5.05 Perimeters of circular and composite shapes 197
M1.2 5.06 Area 202
M1.2 5.07 Land surveying 210
M1.2 5.08 Volume and surface area of a prism 215
M1.2 5.09 Volume and surface area of a cylinder 222
M1.2 5.10 Volume and surface area of a sphere 227
M1.2 5.11 Volume and surface area of composite solids 231
Study tip: Making lists and managing your time 235
Chapter summary 236
Test yourself 5 237

6

MANAGING A HOME 240

F1.3 6.01 Water usage in the home 243
M1.2, S1.1 6.02 Water tanks and dams ... 247
M1.3, S1.1, F1.3 6.03 Electricity usage in the home 253
M1.3, F1.3 6.04 Energy consumption and the costs of appliances 261
M1.3 6.05 Food and energy consumption 268
F1.3 6.06 Household budgeting 275
Study tip: Organising your notes 279
Chapter summary 280
Test yourself 6 281

PRACTICE PAPER 2 283

7

LINEAR FUNCTIONS 288

A2 7.01 Graphing linear functions 291
A2 7.02 The gradient formula 297
A2 7.03 Linear modelling 300
A2 7.04 Direct linear variation 307
A2 7.05 Conversion graphs 313
Study tip: Attacking your weak areas 318
Chapter summary 319
Test yourself 7 320

ISBN 9780170413565

8

INTERESTING AND DEPRECIATION 322

F1.1 8.01 Percentage increase and decrease 325
F1.1 8.02 Simple interest 328
F1.1 8.03 Simple interest graphs 331
F1.1 8.04 Compound interest 335
F1.1 8.05 Straight-line depreciation 342
Study tip: Useful exam tips 349
Chapter summary 350
Test yourself 8 351

9

OWNING A CAR 354

F1.3, S1.1 9.01 Car insurance 357
F1.3 9.02 Stamp duty 364
F1.3 9.03 On-road costs 369
F1.3 9.04 Running costs 374
F1.3 9.05 Fuel consumption and prices 377
F1.3 9.06 Choosing the best car 384
Study tip: Your study routine 387
Chapter summary 388
Test yourself 9 389

PRACTICE PAPER 3 392

10

ANALYSING DATA 398

S1.2 10.01 The mean, median and mode ..402
S1.2 10.02 Quartiles, deciles and percentiles 417
S1.2 10.03 The range and interquartile range 426
S1.2 10.04 The effect of outliers 429
S1.1 10.05 Cumulative frequency graphs ... 434
S1.2 10.06 Box plots 439
S1.2 10.07 Standard deviation 445
S1.2 10.08 The shape of a distribution 452
Study tip: Looking after yourself 460
Chapter summary 461
Test yourself 10 462

11

WORLD LOCATIONS AND TIMES 466

M2 11.01 Latitude and longitude 469
M2 11.02 Time 477
M2 11.03 Timetables 481
M2 11.04 International time zones 485
Study tip: Before an exam 495
Chapter summary 496
Test yourself 11 497

12

DRIVING SAFELY 500

A1, S1.1 12.01 Blood alcohol content (BAC) 503
S1.1 12.02 Accident statistics 513
A1 12.03 Speed, distance and time 521
A1 12.04 Stopping distance 524
Study tip: More exam tips 534
Chapter summary 536
Test yourself 12 537

PRACTICE PAPER 4 541

ANSWERS 547
GLOSSARY/INDEX 592

ISBN 9780170413565

PREFACE

New Century Maths 11 Mathematics Standard (Pathway 2), first published in 2000, has been rewritten for the new Mathematics Standard syllabus (2017). In this 4th edition of the book, teachers will find those familiar features that have made New Century Maths a leading mathematics series, such as clear worked examples, graded exercises, syllabus codes and references, Investigation, Technology, SkillCheck pre-chapter exercises, chapter summary mind maps, practice papers and a glossary/index.

The syllabus describes two pathways for Mathematics Standard that begin in *Year 11*:

- a vocational pathway that we will call **Pathway 1**, for students heading towards the workforce or further training after school, providing practical mathematical skills for life
- a more traditional and academic pathway that we will call **Pathway 2**, for students heading towards an HSC exam, ATAR and university studies.

This book caters for Pathway 2 in Year 11. Both pathways share a common Year 11 course that splits into the Mathematics Standard 1 and 2 courses in Year 12, but because students taking each pathway have specific learning needs, we have published two levels of texts for *both* Years 11 and 12.

Mathematics Standard 2 is designed for students who will work or study in fields that require a mathematical or statistical background. We have endeavoured to produce a practical text that captures the spirit of the course, providing relevant and meaningful examples of mathematics being used in society and industry.

The *NelsonNet* student and teacher websites contain additional resources such as worksheets, video tutorials and topic tests. We wish all teachers and students using this book every success in embracing the new senior mathematics course.

ISBN 9780170413565

ABOUT THE AUTHORS

Klaas Bootsma was head teacher of mathematics at Ambarvale High School in Campbelltown and has taught at Lurnea and Grantham high schools. He was a senior HSC examiner and has worked on the HSC Advice Line. Klaas has been the lead author of *New Century Maths 9–10* for over 20 years.

Sarah Hamper teaches at Abbotsleigh School in Wahroonga and has taught at Meriden and Tara Anglican schools. Her expertise is in using modelling, problem solving and ICT for the effective learning of mathematics, and she has presented workshops for MANSW and nationally. Sarah is a co-author of the current editions of *New Century Maths 9–10*.

Margaret Willard has extensive experience writing units of work designed for distance education and was Manager at TAFE's distance education unit, OTEN (Open Training and Education Network). She has served on the executive of MANSW (the Mathematical Association of NSW), managed its post-secondary programs and has presented at both MANSW and TAFE conferences.

Robert Yen has taught at Hurlstone Agricultural, Ambarvale and Eagle Vale high schools in southwest Sydney. He co-edited *Reflections*, the MANSW journal, and wrote General Mathematics HSC study guides for *The Sydney Morning Herald*. Robert has been writing for *New Century Maths 7–12* for over 20 years and now works for Nelson Cengage as the mathematics publisher.

CONTRIBUTING AUTHORS

Megan Boltze (Ashcroft HS) and **Kuldip Khehra** (Quakers Hill HS) wrote and edited many of the *NelsonNet* worksheets.

John Drake, **Katie Jackson** and **Joanne Magner** created the video tutorials.

Trisha Goss wrote the topic tests.

Roger Walter wrote the *ExamView* questions.

Tracey Macbeth-Dunn wrote the worked solutions to all exercise sets.

ISBN 9780170413565

SYLLABUS REFERENCE GRID

Topic and subtopic	*New Century Maths 11 Mathematics Standard (Pathway 2)* chapter
Algebra	
MS-A1 Formulae and equations	2 Formulas and equations 12 Driving safely
MS-A2 Linear relationships	2 Formulas and equations 7 Linear functions
Measurement	
MS-M1 Applications of measurement M1.1 Practicalities of measurement M1.2 Perimeter, area and volume M1.3 Units of energy and mass	5 Measurement 6 Managing a home 9 Owning a car 12 Driving safely
MS-M2 Working with time	11 World locations and times
Financial Mathematics	
MS-F1 Money matters F1.1 Interest and depreciation F1.2 Earning and managing money F1.3 Budgeting and household expenses	2 Formulas and equations 3 Earning money and taxation 6 Managing a home 8 Interest and depreciation 9 Owning a car
Statistical Analysis	
MS-S1 Data analysis S1.1 Classifying and representing data S1.2 Exploring and describing data	1 Collecting and presenting data 6 Managing a home 9 Owning a car 10 Analysing data 12 Driving safely
MS-S2 Relative frequency and probability	4 Probability

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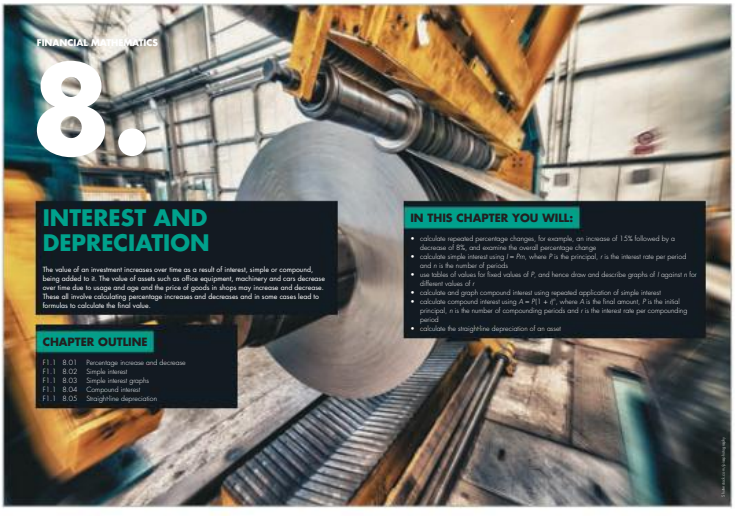
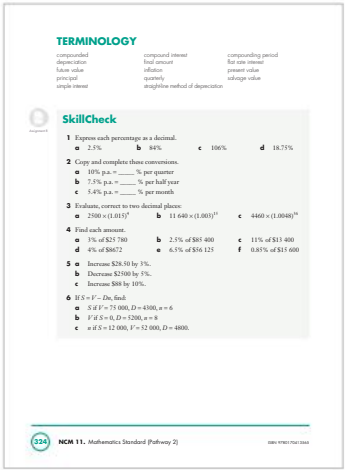
ABOUT THIS BOOK

AT THE BEGINNING OF EACH CHAPTER

- Each chapter begins on a double-page spread showing the **Chapter contents** with syllabus codes, and a list of chapter outcomes



- **Terminology** previews the key words and phrases from within the chapter, while **SkillCheck** reviews important prerequisite skills and knowledge for the chapter


TERMINOLOGY

compounded
depreciation
future value
principal
simple interest
compound interest
final amount
inflation
quarterly
straight-line method of depreciation
compounding period
flat rate interest
present value
salvage value

SkillCheck

1 Express each percentage as a decimal.
a 2.5% b 84% c 106% d 18.75%

2 Copy and complete these conversions.
a 10% p.a. = ____ % per quarter
b 7.5% p.a. = ____ % per half year
c 5.4% p.a. = ____ % per month

3 Evaluate, correct to two decimal places:
a $2500 \times (1.015)^8$ b $11\,640 \times (1.003)^{43}$ c $4460 \times (1.0048)^{16}$

4 Find each amount.
a 3% of $25 780 b 2.5% of $85 400 c 11% of $13 400
d 4% of $8672 e 6.5% of $56 125 f 0.85% of $15 600

5 a Increase $28.50 by 3%.
b Decrease $2500 by 5%.
c Increase $88 by 10%.

6 If $S = V - Dn$, find:
a S if $V = 75\,000$, $D = 4300$, $n = 6$
b V if $S = 0$, $D = 5200$, $n = 8$
c n if $S = 12\,000$, $V = 52\,000$, $D = 4800$.

324 NCM 11. Mathematics Standard (Pathway 2)

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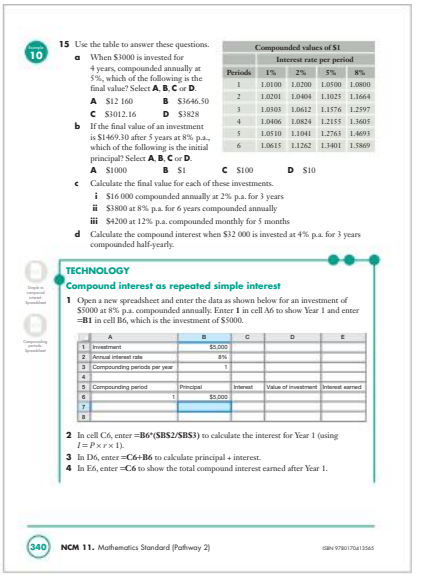
IN EACH CHAPTER

- Important facts and formulas are highlighted in a shaded box.
- Important words and phrases are printed in red and listed in the glossary at the back of the book.
- Graded exercises are linked to worked examples and include multiple-choice questions, exam-style problems and realistic applications.
- Worked solutions to all exercise questions are provided on the *NelsonNet* teacher website.
- **Investigations** explore the syllabus in more detail, through group work, discovery and modelling activities.
- **Technology** promotes ICT in the classroom, using spreadsheets, graphics calculators and the Internet.
- **Did you know?** contains interesting facts and applications of the mathematics learned in the chapter.



Example 10

15 Use the table to answer these questions.

Compounded values of $1				
	Interest rate per period			
Periods	1%	2%	5%	8%
1	1.0100	1.0200	1.0500	1.0800
2	1.0201	1.0404	1.1025	1.1664
3	1.0303	1.0612	1.1576	1.2597
4	1.0406	1.0824	1.2155	1.3605
5	1.0510	1.1041	1.2763	1.4693
6	1.0615	1.1262	1.3401	1.5869

a When $3000 is invested for 4 years, compounded annually at 5%, which of the following is the final value? Select **A**, **B**, **C** or **D**.
A $12 160 **B** $3646.50 **C** $3012.16 **D** $3828

b If the final value of an investment is $1469.30 after 5 years at 8% p.a., which of the following is the initial principal? Select **A**, **B**, **C** or **D**.
A $1000 **B** $1 **C** $100 **D** $10

c Calculate the final value for each of these investments.
i $16 000 compounded annually at 2% p.a. for 3 years
ii $3800 at 8% p.a. for 6 years compounded annually
iii $4200 at 12% p.a. compounded monthly for 5 months

d Calculate the compound interest when $32 000 is invested at 4% p.a. for 3 years compounded half-yearly.

Simple vs compound interest Spreadsheet

Compounding periods Spreadsheet

TECHNOLOGY

Compound interest as repeated simple interest

1 Open a new spreadsheet and enter the data as shown below for an investment of $5000 at 8% p.a. compounded annually. Enter **1** in cell A6 to show Year 1 and enter **=B1** in cell B6, which is the investment of $5000.

	A	B	C	D	E
1	Investment	$5,000			
2	Annual interest rate	8%			
3	Compounding periods per year	1			
4					
5	Compounding period	Principal	Interest	Value of investment	Interest earned
6	1	$5,000			
7					
8					

2 In cell C6, enter **=B6*($B$2/$B$3)** to calculate the interest for Year 1 (using $I = P \times r \times 1$).
3 In D6, enter **=C6+B6** to calculate principal + interest.
4 In E6, enter **=C6** to show the total compound interest earned after Year 1.

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INVESTIGATION

COMPOUND INTEREST GRAPH

Blake invests $20 000 in an account that earns 6% p.a. interest, compounded annually.

1 If you graphed the size of his investment growing over 10 years, what do you think the graph would look like? Give reasons for your answer.

2 Copy and complete this table using the formula $A = P(1 + r)^n$, rounding your values to the nearest whole dollar.

No. of years, n	Amount, A ($)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

3 Use the table to construct a graph that shows the amount of the investment, A, growing over 10 years.

4 What is the vertical intercept of this graph and what does it represent?

5 Comment on what the shape of the graph shows.

6 Use the graph to estimate:
i the value of the investment after $8\frac{1}{2}$ years
ii when the value of the investment reaches $35 200.

Straight-line depreciation formula practice

8.05 Straight-line depreciation

A new vehicle loses around 30%–40% of its value in the first year. Loss in value can occur through age, wear and tear, or simply by becoming outdated when a new model is released.

EXAMPLE 11

a A boat was purchased for $37 800 and decreased in value by 11% in its first year. To the nearest dollar, what was its value after 1 year?

b A new Porsche was purchased for $239 850 and was valued at $175 600 twelve months later. What is the percentage decrease in price, to the nearest unit, in the first year?

Solution

a Purchase price = $37 800

11% decrease in value = 89% of price 100% − 11% = 89%

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DID YOU KNOW?

Car theft

- About 47% of vehicles stolen in Australia are stolen outside the home of the owner or a friend of the owner, so park your car in a garage or in a secure parking area.
- Most vehicles are stolen in the late afternoon or evening, especially on Fridays, Saturdays and Sundays.
- Most cars are stolen for short-term use (such as joyriding, transport) or for resale (either as a whole vehicle or as stripped parts).
- Stolen motorcycles have the lowest rate of recovery with only 43% of thefts recovered.
- About 30% of stolen vehicles are recovered within 10 km from where they were stolen.

Cheaper cars are more likely to be stolen, and account for about 40% of all short-term vehicle thefts. Give a possible reason for this happening.

Exercise 9.01 Car insurance

1 Third Party Property insurance costs $38.17 per month. What is the cost per week? Select **A**, **B**, **C** or **D**.
A $8.80 **B** $9.54 **C** $8.81 **D** $7.63

2 Mia paid her comprehensive car insurance at $150 per month. How much would she save if she paid a lump sum of $1265? Select **A**, **B**, **C** or **D**.
A $685 **B** $1115 **C** $0 **D** $535

Example 1

3 Jack bought a new van for $28 350. His CTP insurance is $626 per year and his comprehensive insurance is $75.78 per month. What is the total cost of insurance per year?

4 Which insurance is the cheapest option? Select **A**, **B** or **C**.
A $985 per year paid in a lump sum
B $83.10 per month
C $38.25 per fortnight

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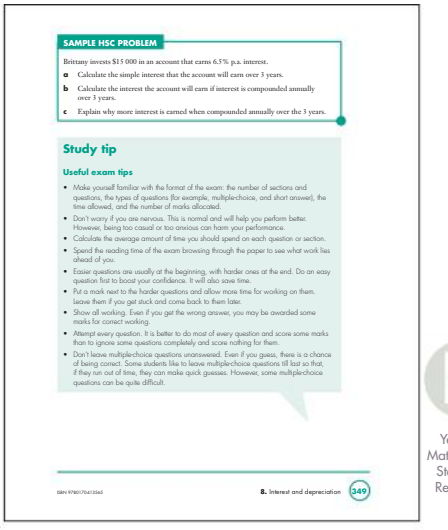
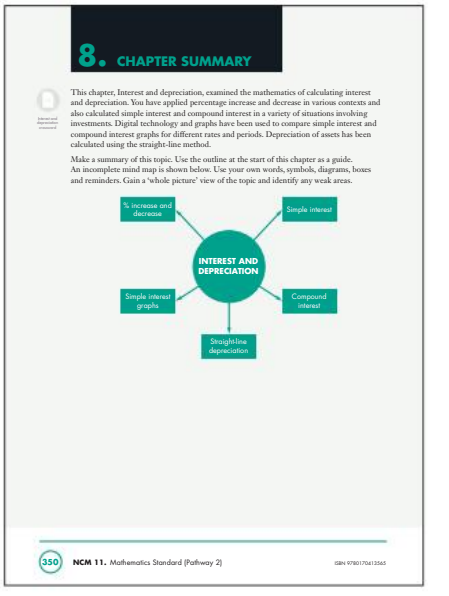
AT THE END OF EACH CHAPTER

- **Sample HSC problem** demonstrates a typical HSC question using content learned from the chapter.
- **Study tip** provides useful advice for senior students in revising coursework and preparing for exams.
- **Chapter summary** concludes the chapter and includes a mind map exercise.
- **Test yourself** contains chapter revision and are linked to to the relevant exercise set.
- **Practice papers** after every three chapters revise the skills and knowledge of those chapters.



AT THE END OF THE BOOK

- **Glossary and index** includes a comprehensive dictionary of course terminology.
- **Answers** (worked solutions are on the teacher website).



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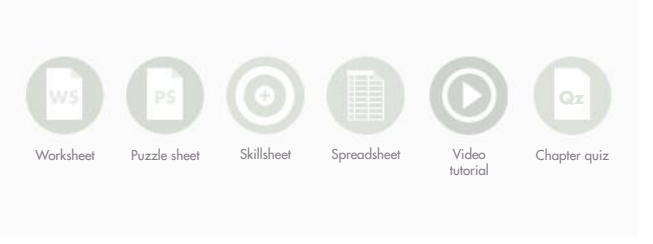

- **Worksheets** and **puzzle sheets** that are write-in enabled PDFs
- **Skillsheets** of examples and exercises of prerequisite skills and knowledge
- **Video tutorials**: worked examples explained online
- **Spreadsheets**: *Excel* files
- **ExamView quizzes**: interactive and self-marking

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NELSONNET TEACHER WEBSITE

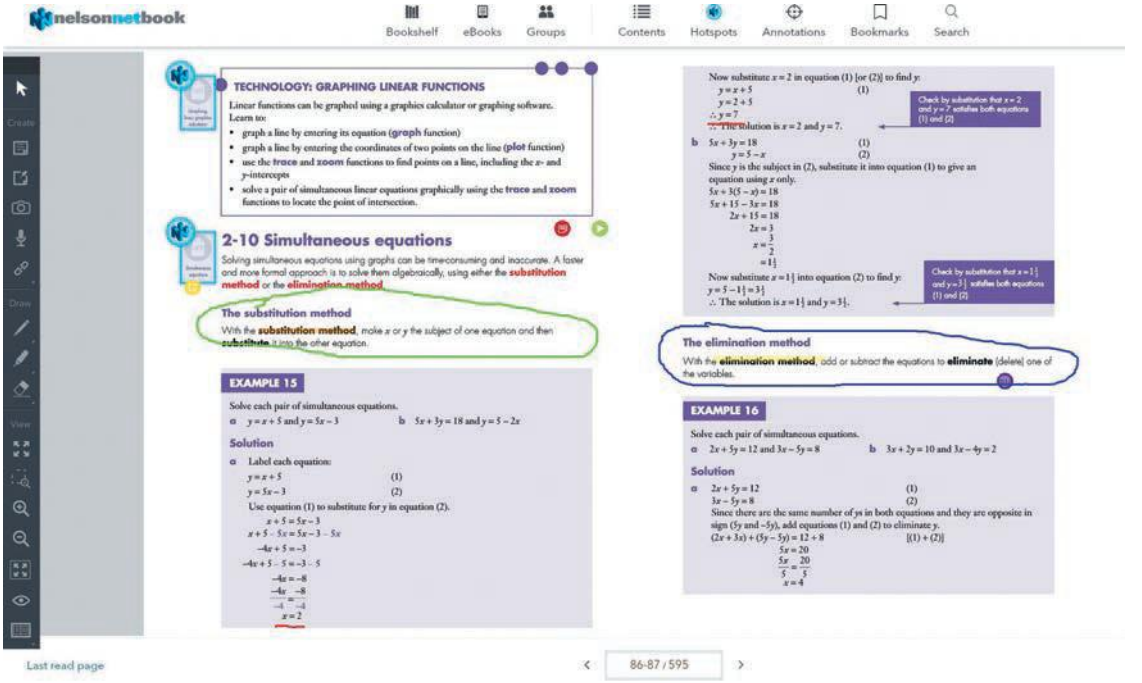
The *NelsonNet* teacher website, also at **www.nelsonnet.com.au**, contains:

- A **teaching program**, in Microsoft Word and PDF formats
- **Topic tests**, in Microsoft Word and PDF formats
- **Worked solutions** to each exercise set
- **Chapter PDFs** of the textbook
- **ExamView** exam-writing software and questionbanks
- **Resource Finder**: search engine for *NelsonNet* resources

NELSONNETBOOK

NelsonNetBook is the web-based interactive version of this book found on *NelsonNet*.

- To each page of *NelsonNetBook* you can add notes, voice and sound bites, highlighting, weblinks and bookmarks
- **Zoom** and **Search** functions
- Chapters can be customised for different groups of students



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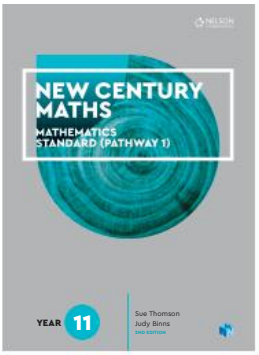

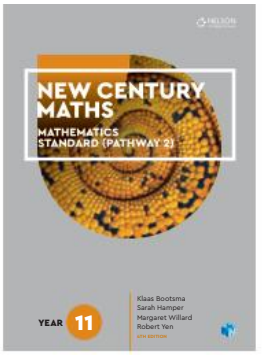

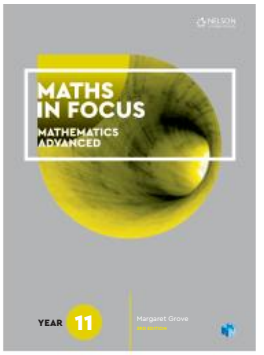

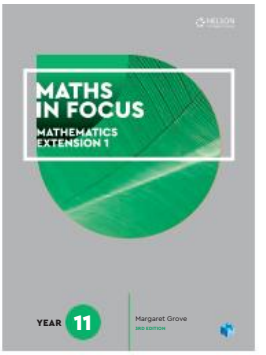

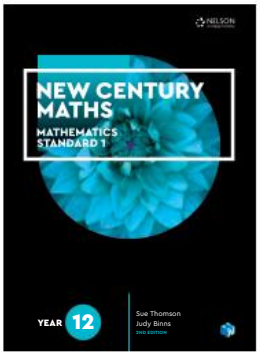

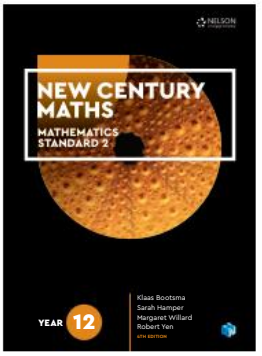

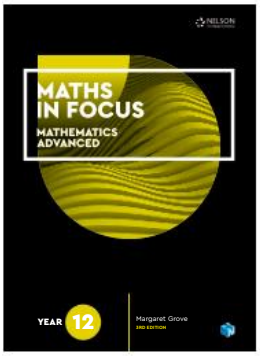

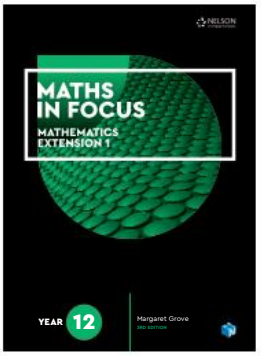

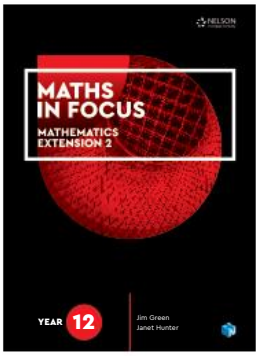

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MATHEMATICAL VERBS

A glossary of 'doing words' commonly found in mathematics problems

analyse: study in detail the parts of a situation

apply: use knowledge or a procedure in a given situation

calculate: *see* **evaluate**

classify, **identify**: state the type, name or feature of an item or situation

comment: express an observation or opinion about a result

complete: fill in detail to make a statement, diagram or table correct or finished

compare: show how two or more things are similar or different

construct: draw an accurate diagram

convert: change from one form to another, for example, from a fraction to a decimal, or from kilograms to grams

decrease: make smaller

describe: state the features of a situation

estimate: make an educated guess for a number, measurement or solution, to find roughly or approximately

evaluate, **calculate**: find the value of an expression, for example, 3×8^2 or $4x + 1$ when $x = 5$

expand: remove brackets in an algebraic expression, for example, expanding $3(2y + 1)$ gives $6y + 3$

explain: describe why or how

give reasons: show the rules or thinking used when solving a problem. *See also* **justify**

graph: display on a number line, number plane or statistical graph

hence find/prove: find an answer or prove a result using previous answers or information supplied

identify: *see* **classify**

increase: make larger

interpret: find meaning in a mathematical result

justify: give reasons or evidence to support your argument or conclusion. *See also* **give reasons**

measure: determine the size of something, for example, use a ruler to find the length of a pen

prove that: *see* **show that**

recall: remember and state

show that, **prove**: (in questions where the answer is given) use calculation, procedure or reasoning to prove that a result is true

simplify: give a result in its most basic, shortest, neatest form, for example, simplifying a ratio or algebraic expression

sketch: draw a rough diagram that shows the general shape or ideas, less accurate than **construct**

solve: find the value(s) of an unknown pronumeral in an equation or inequality

state: *see* **write**

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substitute: replace a variable by a number and evaluate

verify: check that a solution or result is correct, usually by substituting back into the equation or referring back to the problem

write, **state**: give the answer, formula or result without showing any working or explanation (This usually means that the answer can be found mentally, or in one step)

SYMBOLS AND ABBREVIATIONS

$=$	is equal to
$\neq$	is not equal to
$\approx$	is approximately equal to
$<$	is less than
$>$	is greater than
$\leq$	is less than or equal to
$\geq$	is greater than or equal to
( )	parentheses, round brackets
[ ]	(square) brackets
{ }	braces
$\pm$	plus or minus
π	pi = 3.14159…
$0.\dot{1}5\dot{2}$	the recurring decimal 0.152152 …
$^\circ$	degree
$\angle$	angle
Δ	triangle
$\parallel$	is parallel to
$\perp$	is perpendicular to
$\therefore$	therefore
x^2	x squared, $x \times x$
x^3	x cubed, $x \times x \times x$
$\sqrt{\ }$	square root, radical sign
$\sqrt[3]{\ }$	cube root
$P(E)$	the probability of event E occurring
$P(\tilde{E})$	the probability of event E not occurring
LHS	left-hand side
RHS	right-hand side
%	percentage
cos	cosine ratio
sin	sine ratio
tan	tangent ratio
$\overline{x}$	the mean
Σ	the sum of
Q_1	first quartile or lower quartile
Q_2	median (second quartile)
Q_3	third quartile or upper quartile
IQR	interquartile range
α	alpha
θ	theta
μ	micro-, mu

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STATISTICAL ANALYSIS

1

COLLECTING AND PRESENTING DATA

Today, we live in the Information Age. The world economy has progressed from manufacturing products to the complex task of processing data to manage the needs of a growing population. Statistics are used by governments to plan for community facilities, by scientists and economists to predict the future, by businesses to forecast sales and by advertisers to help sell products.

CHAPTER OUTLINE

S1.1 1.01 Interpreting graphs
S1.1 1.02 Misleading graphs
S1.1 1.03 Types of data
S1.1 1.04 Sampling techniques
S1.1 1.05 Constructing graphs
S1.1 1.06 Frequency histograms and polygons
S1.1 1.07 Dot plots and stem-and-leaf plots

IN THIS CHAPTER YOU WILL:

- interpret and construct a variety of statistical graphs and displays: bar charts, sector graphs, divided bar graphs, line graphs, Pareto charts, tables, frequency histograms and polygons, dot plots and stem-and-leaf plots
- identify graphs that mislead and misrepresent
- classify data as numerical (either discrete or continuous) or categorical (either nominal or ordinal)
- distinguish between a sample and a population, and the collection of data for each
- distinguish between random, systematic, stratified and self-selected sampling, and determine the appropriateness of each type for a given situation
- identify bias in sampling

iStockphoto/Pinkypills

TERMINOLOGY

bar chart
categorical data
class interval
discrete data
frequency histogram
nominal data
Pareto chart
sample
self-selected sample
stratified sample
bias
census
cluster
divided bar graph
frequency polygon
numerical data
population
sample size
statistics
survey
biased sample
class centre
continuous data
dot plot
line graph
ordinal data
random sample
sector graph
stem-and-leaf plot
systematic sample

SkillCheck

Assignment 1

1 Match each statistical display to its correct name from this list:

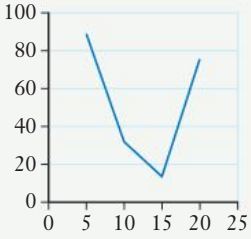
bar chart (column graph) divided bar graph dot plot
line graph sector graph stem-and-leaf plot

a

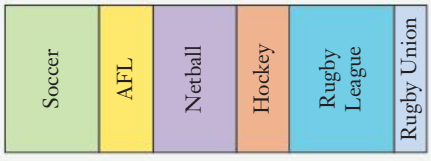
b

3	0 4 4
4	3 6 6 6 8
5	2 4 5 7
6	1 3 3

c



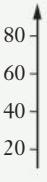
d



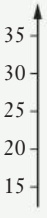
2 Draw the two statistical displays from the list in Question **1** that were not illustrated in the question.

3 Write the size of one unit on each vertical scale.

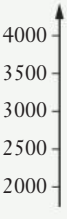
a



b



c



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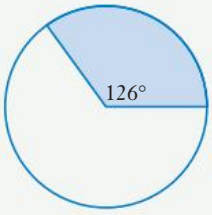
4 **a** What fraction of this rectangle is shaded?

b What percentage of this rectangle is shaded?

5 **a** What fraction of this circle is shaded?

b What percentage of this circle is shaded?



6 The ages of the patients at a hospital were grouped into the class intervals 0–9, 10–19, 20–29 and so on. What is the class centre of the interval 20–29?

1.01 Interpreting graphs

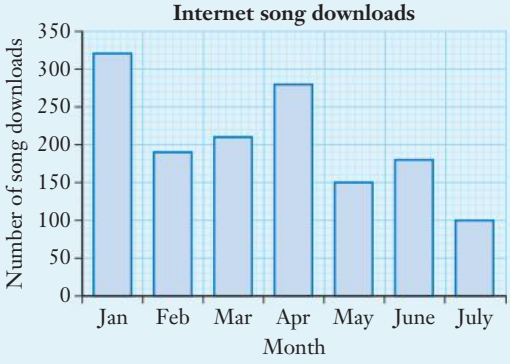
Every picture tells a story

Graphs are used to display statistical information in an attractive and meaningful way. Four simple types of graph are:

- **bar chart,** also known as a **column graph**
- **line graph**
- **sector graph**, also known as a **pie chart**
- **divided bar graph**.

EXAMPLE 1

This bar chart shows the number of songs downloaded from the Internet by a group of teenagers each month over 7 months.



a What is the size of one unit on the vertical axis?

b How many songs were downloaded in April?

c In which month were 100 songs downloaded?

d In which month were the highest number of songs downloaded? Suggest a reason why this may be so.

Solution

a 10 songs **b** 280 songs **c** July

d January. Teenagers may have had more time or money after Christmas to download songs.

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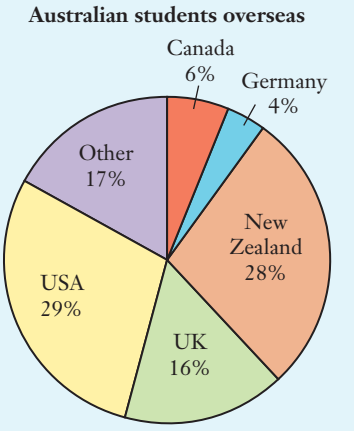
EXAMPLE 2

This sector graph shows the overseas countries where Australian students studied during one year.



a What was the second most popular overseas country for Australian students to study in?

b What percentage of students chose neither the United Kingdom (UK) nor the United States of America (USA)?

c If a total of 3685 students was **surveyed**, how many of them went to Canada?

d Calculate, correct to the nearest degree, the angle size of the 'Other' countries sector.

Solution

a New Zealand

b $100\% - 16\% - 29\% = 55\%$ Or $17\% + 6\% + 4\% + 28\% = 55\%$

c $6\% \times 3685 = 221.1 \approx 221$

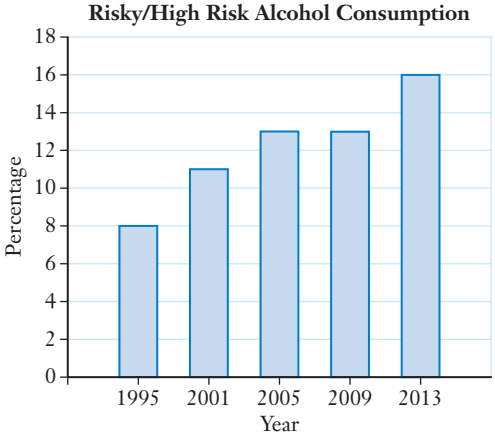
221 students went to Canada.

d $17\% \times 360° = 61.2° \approx 61°$ There are 360° in a circle.

Exercise 1.01 Interpreting graphs

Example 1

1 This bar chart shows the percentage of Australians drinking alcohol at a 'risky' or 'high risk' level in five different years.



a What percentage of people were drinking at a risky/high risk level in 2013?

b What was the percentage increase in risky/high risk drinking in the 4 years after the 2009 survey?

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c If 35 750 people were surveyed in 2009, how many were drinking at a risky/high risk level?

d If 43 320 people were surveyed in 2013, how many were *not* drinking at a risky/high risk level?

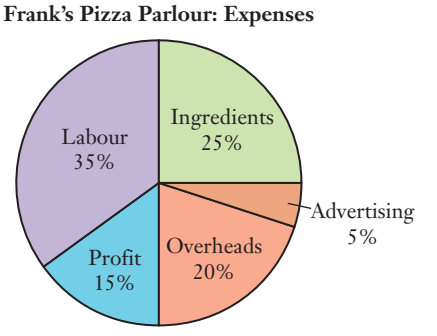
e Comment on the percentage of Australians drinking alcohol at a risky/high risk level between 2005 and 2009.

f Comment on the statement: 'The percentage of Australians drinking at a risky/high risk level has doubled between 1995 and 2013'.

g Predict what might happen in the results of the next survey.

Example 2

2 This sector graph shows how Frank's Pizza Parlour spends its budget.



a What fraction of his budget does Frank spend on ingredients?

b What is the largest expense?

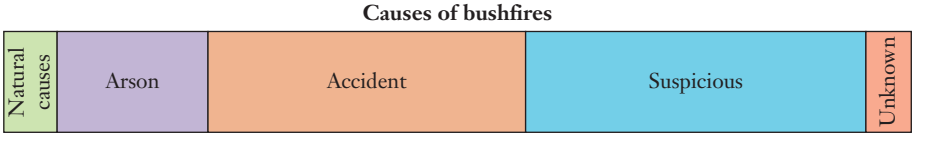
c For each family-size pizza Frank sells for $17.50, how much of this is profit?

d If $325 per week is spent on advertising, what is his total weekly budget?

e What is the angle size of the 'Profit' sector on the graph?

f Overheads are business expenses, such as rent. Calculate how much Frank spends on overheads if his weekly budget is $5300.

3 This divided bar graph shows causes of bushfires during one summer.



a What fraction of bushfires were caused by arsonists?

b What percentage of bushfires were accidental?

c Which cause was linked to 37.5% of bushfires?

d If there were 275 bushfires during summer, how many were started by natural causes?

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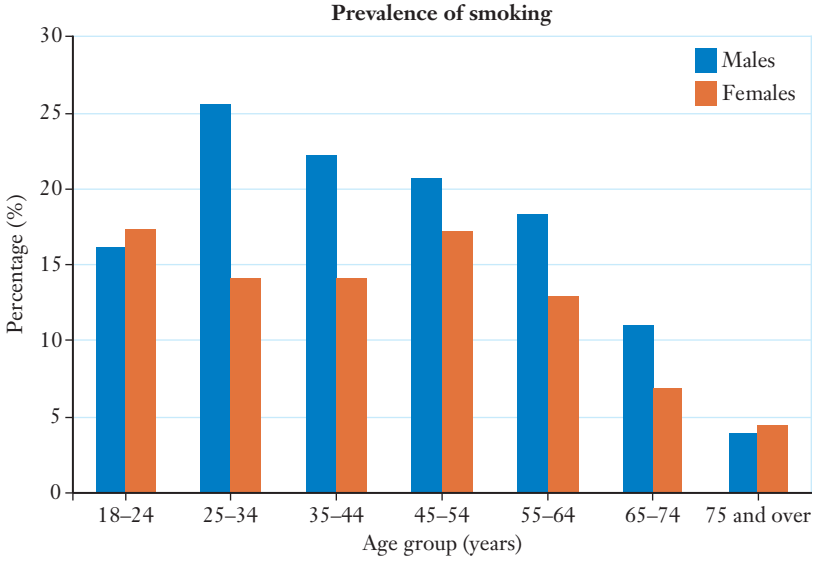
4 This graph shows the percentage of people in each age group and gender who are smokers.



Source: Australian Bureau of Statistics (ABS), National Health Survey: Summary of Results 2014–15

a Which gender has the higher percentage of smokers?

b Which age group contains the highest percentage of male smokers?

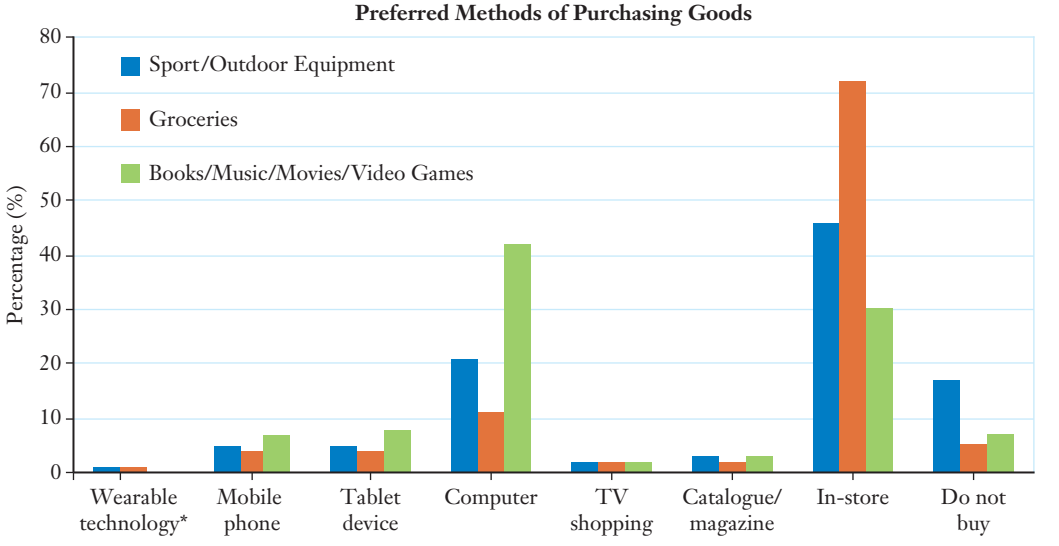
c What percentage of females aged 35 to 44 are smokers?

d If 1 705 000 females were surveyed in the 25–34 age group, how many were smokers?

e In which age group were 11% of men smokers?

f What generally happens to the percentage of smokers as age increases?

5 This **clustered bar chart** shows the different ways people shop for sports/outdoor equipment, groceries and home entertainment.



* e.g. watch, glasses

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a Name the two most popular methods of purchasing goods.

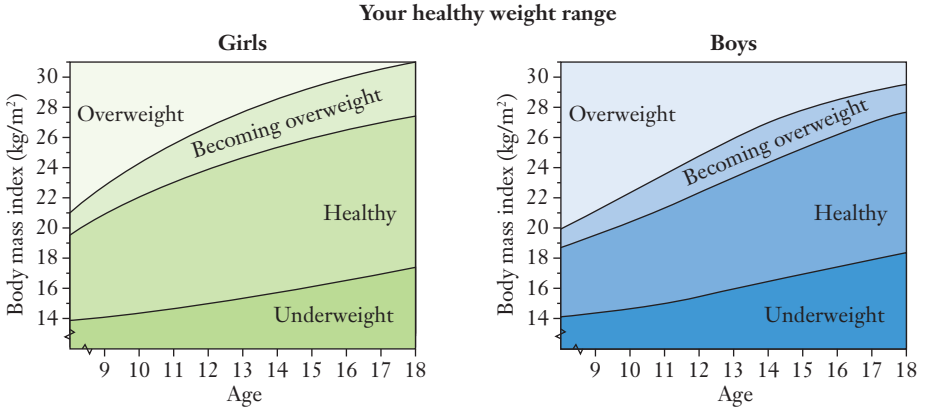
b What is the most popular method for purchasing:

- **i** groceries?
- **ii** sports/outdoor equipment?
- **iii** home entertainment?

c What percentage of people:

- **i** do not buy sports/outdoor equipment at all?
- **ii** buy home entertainment items using their tablets?
- **iii** buy groceries using their computers?

6 The graphs below illustrate healthy weight ranges, according to gender, age and body mass index (BMI). Your BMI is calculated by dividing your weight (in kilograms) by your height (in metres) squared.



a Is a 16-year-old girl with a BMI of 20 considered healthy?

b Describe the health of a 13-year-old boy with a BMI of 14.

c What is the minimum BMI for a healthy 17-year-old boy?

d Jake is aged 15, weighs 78 kg and is 173 cm tall. Calculate his BMI and state what advice you would give him about his health.

e Write the lowest and highest BMI for a healthy 14-year-old girl.

f Calculate your BMI and use the graph to find your state of health.

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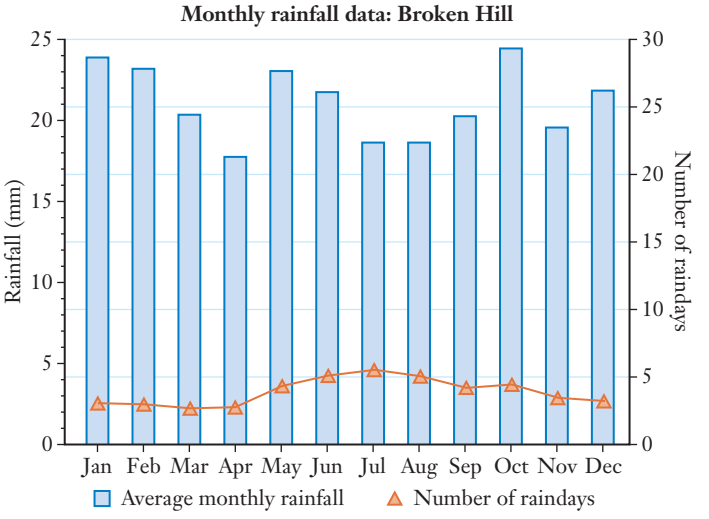
7 This graph from the Bureau of Meteorology shows the monthly **rainfall** and number of raindays for Broken Hill.



© Copyright Commonwealth of Australia 2017, Bureau of Meteorology.
Attribution 3.0 Australia (CC BY 3.0 AU).

a What is the size of one unit on the 'Rainfall' axis?

b What is graphed on the line graph?

c What was the amount of rainfall and the number of raindays in May?

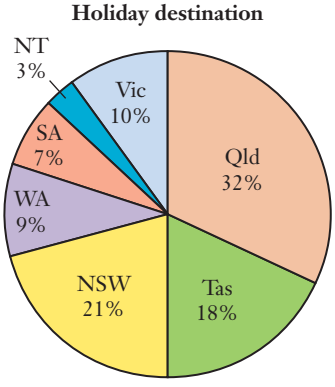
d Which month had the highest rainfall? How many millimetres fell in that month?

e Which month had the most raindays? How many days of rain did it have?

f What was the driest month?

g How many days in summer had rain?

8 A group of people was surveyed about their favourite Australian state or territory to visit. The results are shown on the sector graph.



If 2475 tourists chose Western Australia, which of the following is the total number of people who participated in the survey? Select **A**, **B**, **C** or **D**.

A 27 500 **B** 8800

C 5775 **D** 25 025

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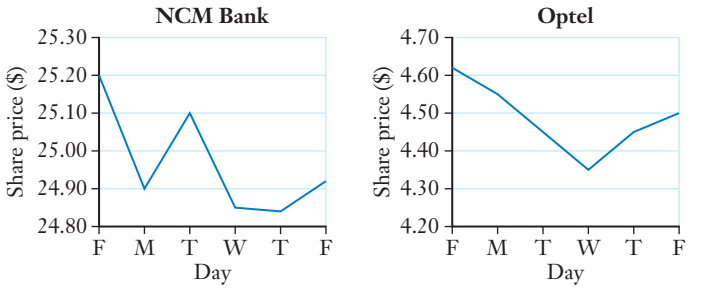
9 a What do these graphs illustrate?



b What is the size of one unit on the vertical axis of the NCM Bank graph?

c What was the Optel share price on Monday?

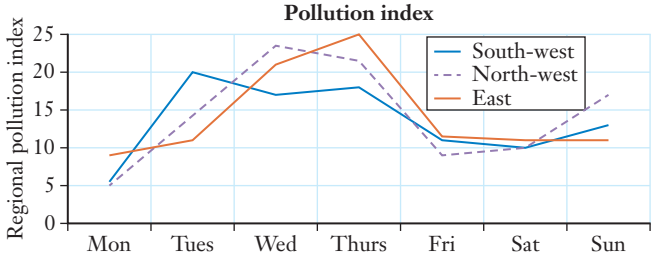
d On which day was the NCM Bank share price $25.10?

e Describe what happened to the Optel share price over the week.

f What would have been the worst day to sell NCM Bank shares? Why?

g Which company's shares showed more changes in price over the week?

10 These line graphs show the pollution index in three Sydney regions over a week.



a What was the south-west's pollution index on Wednesday?

b Which region had the highest index for the week? What was this reading and on which day did it occur?

c Which day showed the lowest index for all three regions? Why do you think it did?

d Which day showed the greatest difference in readings between the south-west and north-west? What was this difference?

e On which day were the readings about the same for all three regions? Why do you think it was?

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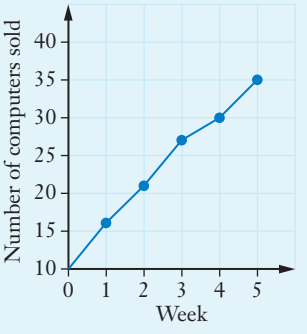
1.02 Misleading graphs

Statistical graphs are sometimes used to misrepresent information, and to falsely support a particular opinion or claim. For example, an advertisement may use a **misleading graph** to promote a product, or a company may use one to exaggerate its achievements and profits.

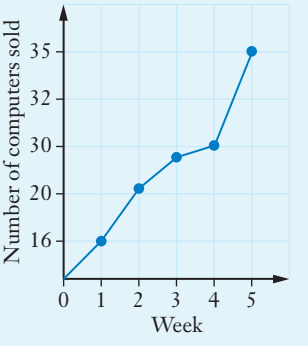
EXAMPLE 3

Three misleading graphs are shown below, used by the marketing manager of a computer store to highlight the increase in the sales of notebook computers over five weeks.

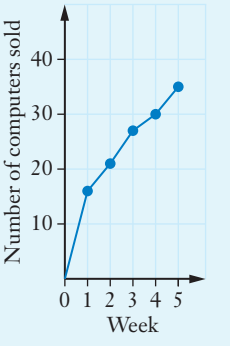
Graph A



Graph B



Graph C



The sales figures are shown in this table:

Week	1	2	3	4	5
Computers sold	16	21	27	30	35

a Explain what is incorrect and misleading about each graph.

b Draw the correct graph to represent the sales data.

Solution

a In Graph A, the vertical axis begins at 10 instead of 0, so the section of the graph between 10 and 40 has been exaggerated or 'stretched up' to give the impression that sales have increased greatly.

In Graph B, the scale on the vertical axis is not even, which gives the false impression that the increases in sales are greater than they actually are.

In Graph C, the horizontal axis has been compressed or 'shrunk' to make it look like the sales have increased quicker.

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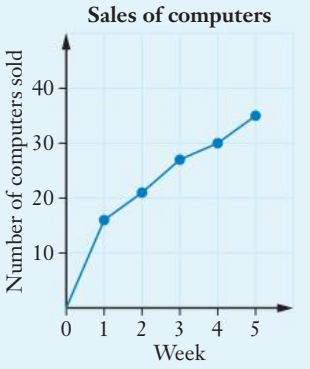
b The correct graph has an even scale on both axes with the vertical axis beginning at zero. It correctly shows the sales pattern.



Misleading graphs

A misleading graph can give a wrong impression by:

- not having a scale
- having an uneven scale or showing only part of the scale
- not showing the correct position of zero on the scale.

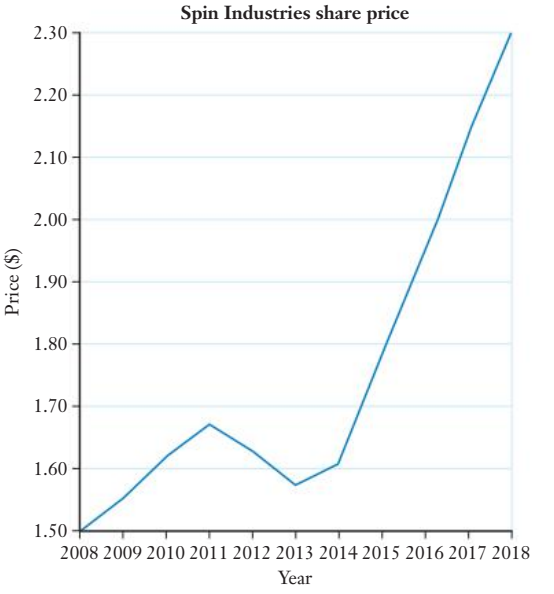
Exercise 1.02 Misleading graphs

1 Spin Industries used this graph to make the following claim to shareholders: 'The company's share price has risen dramatically in the last 10 years'.

a How does the graph mislead the shareholders?

b Redraw the graph correctly.

Example 3



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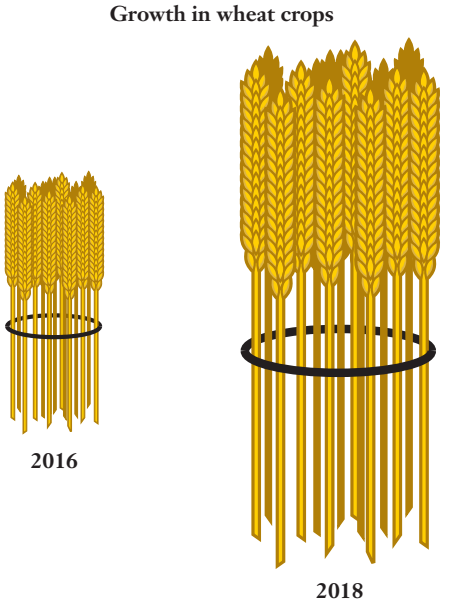
2 Using pictures in graphs can be misleading if they are badly scaled to exaggerate differences. This graph should show that the wheat crop has doubled in two years, but the 2018 wheat picture is double the length and double the width of the 2016 wheat picture.

a How is the graph misleading?

b Redraw the graph for 2018 correctly.

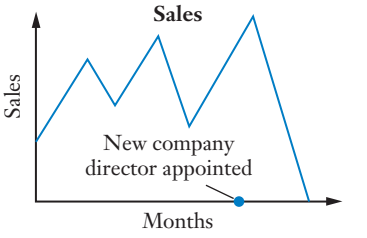


3 This sales graph was presented to a meeting of shareholders by the ex-director of a company.

a What does the graph describe?

b What message is the ex-director trying to convey to the shareholders?

c What information is missing from the graph?



4 This graph depicts the average weekly wage now compared to that of 5 years ago.

a How is the graph misleading to the reader?

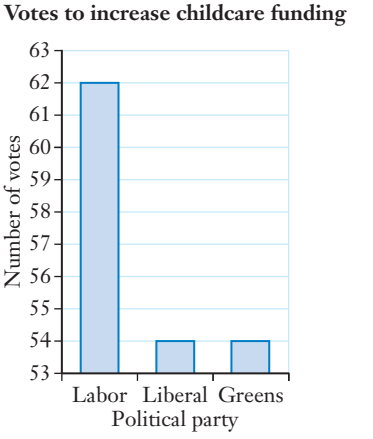
b Redraw the graph correctly.

5 This bar chart appeared in a news article with the headline 'Labor votes overwhelmingly to increase childcare funding.'

a How are Labor's votes exaggerated on this graph?

b Redraw the graph correctly.

c Do you agree with the headline? Justify your answer.



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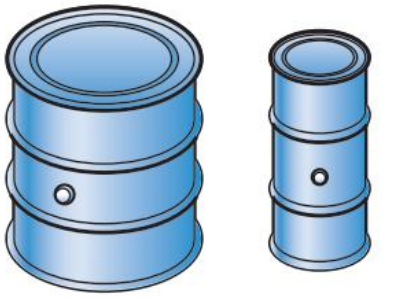
6 Oil prices have increased so much that $100 buys only half as much oil as it did 10 years ago. A newspaper article used the following graph to illustrate this.

a How is this graph misleading?

b Redraw the graph so that it represents the situation accurately.



$100 of oil 10 years ago **$100 of oil today**

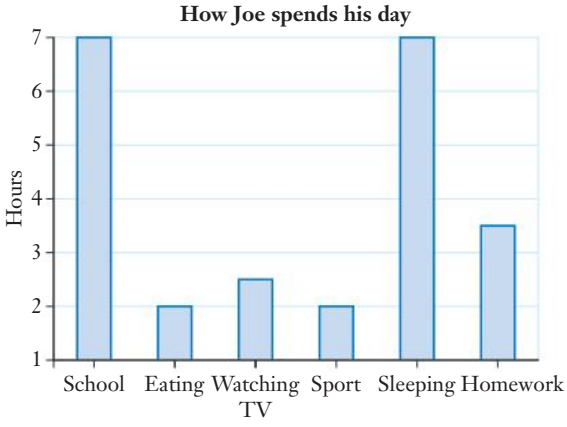
7 Joe drew this bar chart on the right to illustrate how he spends a typical weekday.

a How many hours does Joe spend sleeping?

b Explain what is wrong with this graph.

c What misleading impression does this graph give?

d Redraw this graph to show accurately how Joe spends a typical weekday.



1.03 Types of data

Statistical data match-up

There are two main types of statistical **data**:

- **categorical data** are represented by categories, usually in words or symbols
- **numerical data** are represented by quantities or numbers.

EXAMPLE 4

Classify each type of data as categorical or numerical.

a number of tries scored by a football team

b colour of hair **c** month of birth **d** amount of petrol in cars.

Solution

a	numerical	For example, 0 or 3
b	categorical	For example, blonde or brown
c	categorical	For example, May or September
d	numerical	For example, 32.5 L or 40 L

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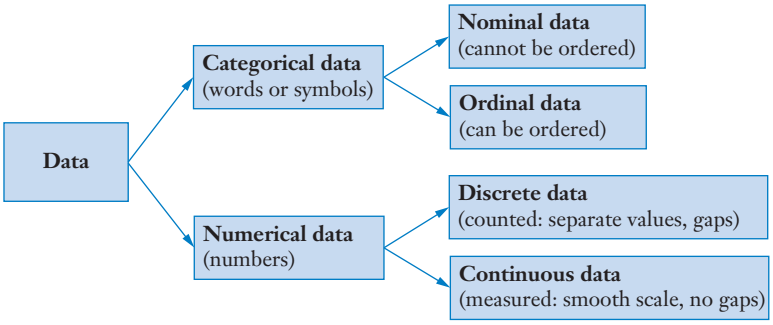
Categorical data can be further divided into **nominal** and **ordinal data**, while **numerical data** can be further divided into **discrete** and **continuous data**.



Colour of hair is an example of **nominal data** because the categories cannot be ordered, for example, blonde does not come before or after brown.

Month of birth is an example of **ordinal data** because the categories can be ordered, for example, May comes before September.

Number of tries scored is an example of **discrete data** because they are numbers that are obtained by *counting*. Discrete values are separate numbers (in this case, whole numbers) with 'gaps' between the possible values. You cannot have $2\frac{1}{2}$ tries or 1.8 tries scored.

Amount of petrol is an example of **continuous data** because they are numbers that are obtained by *measuring*. Continuous values lie on a smooth scale with no 'gaps' between the possible values. You *can* have 43.1 L, 43.15 L or 43.02 L of petrol.

EXAMPLE 5

Classify each type of categorical data as nominal or ordinal.

a method of travel to work

b report grade

c suburb of home

d fan setting of air conditioner

Solution

a nominal

b ordinal — For example, outstanding, satisfactory

c nominal

d ordinal — For example, low, medium

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EXAMPLE 6

Classify each type of numerical data as discrete or continuous.

a amount of electricity used in homes

b numbers of DVDs owned

c shoe size

d weekly profits of a business

Solution

a	continuous	Smooth scale of values
b	discrete	Separate values (for example, 32 and 58)
c	discrete	Separate values (for example, 7 and $8\frac{1}{2}$)
d	continuous	Smooth scale of values

Exercise 1.03 Types of data

1 Classify each type of data as categorical (C) or numerical (N). Example 4

a types of religion

b numbers of computers in homes

c types of driver's licences held

d populations of a city

e numbers of apps bought online

f marital status

g method of payment for shopping

h movie classification

i patients' blood pressure

j swimmers' finishing place in a race

k amount of time spent on the Internet per week

l town or suburb where people live

2 What is the difference between categorical data and numerical data? Give an example of each type.

3 Classify each type of categorical data as nominal (N) or ordinal (O). Example 5

a brand of car

b level of manager

c level of satisfaction with the government

d marital status

e favourite football team

f colour of Olympic medal

g size of soft drink sold at a fast food store

h a person's gender

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Example 6

4 Classify each type of numerical data as discrete (D) or continuous (C).

a amount of rainfall each day
b mass of an athlete
c length of a mobile phone call
d running time for a 100 m race
e star rating of a hotel
f height of the world's tallest building
g number of seats on a bus
h loudness of a lawn mower

5 Give one example of:

a nominal data
b ordinal data
c discrete data
d continuous data.

6 Give the correct name for:

a data that are usually described by numbers
b data that are usually described by words
c data that are usually measured on a smooth scale
d categorical data that cannot be sorted in order.

7 Which statement about categorical data is true? Select **A**, **B**, **C** or **D**.

A They are sometimes represented by numbers that indicate size or an amount.
B They are sometimes represented by symbols.
C They are represented by numbers that take on separate values.
D They are represented on a smooth scale.

1.04 Sampling techniques

Sampling versus census

In **statistics**, a **population** means *all* of the items being studied, whether they are people, fish, cars or buildings. A **census** is a survey of a population in which every item is included.

In Australia, a national census takes place every 5 years, in a year ending with a 1 or a 6, such as 2016 or 2021.

If the whole population is too large or too difficult to survey, a **sample** of items is selected from the population. The results are then used to draw conclusions about the whole population. For example, a market researcher investigating the TV viewing habits of Australians will survey a representative sample of Australians.

Sampling	Taking a census
Surveys a representative group of items from a population	Surveys all items in a population
Provides an estimate or approximate information about the population	Provides exact information about the population
Simple and not expensive	Complex and expensive

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EXAMPLE 7

Determine whether sampling or taking a census is more appropriate for each study below.

a finding the most popular radio station in Newcastle

b conducting a survey on whether Australians want a new national flag

c finding the exact number of young women living on the South Coast, with the view to building a new maternity hospital

d selecting the politicians to form a new state government

Solution

a	sampling	It would be impractical to survey everyone in the population.
b	sampling	It would be impractical to survey everyone in the population.
c	census	An exact number is required.
d	census	This decision is important and requires everyone eligible to vote.

Types of samples

If a sample is to be truly representative of a population, then each item in the population must have an equal chance of being chosen. Also, the **sample size** is important. The larger the sample, the more accurate the results of the study will be. There are four types of sample, described below.

In a **random sample**, each item is chosen completely at random from the population (for example, selecting names out of a box, or setting a computer to create random numbers to choose people or items).

In a **systematic sample**, the first item is chosen at random and all other items are chosen at regular intervals. For example, every 20th mobile phone is taken from a factory for testing, or the driver of every 8th car on the road is selected for random breath testing. 'Systematic' means a 'system' is used to select items.

In a **stratified sample**, the population is divided into strata (layers) according to some characteristic (for example, gender or age group) and a random sample is taken from each layer using representative proportions or percentages. For example, if 28% of all shoppers in Australia are male, then a stratified sample of shoppers must be 28% male. 'Stratified' means 'in layers,' like stratified rock (as shown in the photo).

Shutterstock.com/Dwight Smith

In a **self-selected sample**, people choose to participate in a survey, so the sample is not randomly chosen nor necessarily representative of the population.

For example, people who phone in to vote in a poll about changing the national anthem could form an unrepresentative or biased sample if only people with strong opinions either way participated. People may be biased towards a particular view being promoted by a newspaper, radio station, news channel or social media website.

EXAMPLE 8

Which type of sample would you use to investigate each issue?

a the opinion of Year 11 students on a new senior school uniform

b the quality of Buzz rechargeable batteries

c the amount of time spent on the Internet by Australian primary schoolchildren

d the support for a new government tax from the listeners of a radio talkback show

e the popularity of the prime minister of Australia

Solution

a A random sample will provide a fair representation of Year 11.

b A systematic sample will ensure that batteries produced at different times will be tested.

c A stratified sample will ensure that children of all types and areas are included.

d A self-selected sample is required as listeners have to call in to express their views.

e A stratified sample will ensure that people of all types and areas are included.

EXAMPLE 9

At Newcent High School, there are 126 senior students and 572 junior students. The student council wants to survey 80 students about the school's environmental program. How many senior students should be selected for a stratified sample?

Solution

Total number of students = 126 + 572 = 698

Number of senior students = 126

Sample of students = 80

Number of senior students in sample is $\frac{126}{698} \times 80 = 14.44\ldots \approx 14$

14 senior students should be selected for the sample.

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Capture-recapture technique (Syllabus update, 2019): This extra resource is online: download it from NelsonNet.

Capture-recapture technique

Bias in sampling

Bias is an unwanted influence that favours a particular section of the population unfairly. A **biased sample** is not truly representative of the population and negatively affects the quality or reliability of a statistical study.

EXAMPLE 10

Explain the bias in each sample.

a A group of Year 12 students sitting together in the playground was surveyed on their preferences for a school formal venue.

b A random selection of homes was contacted by telephone at midday for a survey on political opinions.

c A current affairs TV program asked viewers to participate in an SMS (mobile phone) poll on gun laws.

Solution

a The group members are likely to be friends with similar opinions, so this sample may not be representative of all Year 12 students.

b This is biased against people who are not at home during the day.

c The sample would contain only (self-selected) viewers of the program who have a strong opinion about the subject or who make the effort to vote.

Exercise 1.04 Sampling techniques

Example 7

1 Determine whether sampling or taking a census is more appropriate for each study below.

a counting the number of students learning Japanese at a school

b finding the number of people who watch soccer on SBS TV

c judging the most popular supermarket in town

d finding the number of Australians who were born in Europe

e investigating people's opinions on whether Australia should become a republic

f determining the number of people in NSW who have red hair

g finding the exact number of schoolchildren living in Parramatta

h testing whether a brand of mobile phone battery lasts 50 hours

2 When will the next Australian national census be held?

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Example 8

3 Select an appropriate type of sample (random, systematic, stratified or self-selected) for each study below. Give a reason for each answer.

- **a** testing running shoes for quality
- **b** finding the most watched television network on Tuesday nights
- **c** voting for the best contestant on a TV talent show
- **d** testing the reading abilities of a school's students
- **e** testing a new drug on patients
- **f** investigating people's views on renewable energy
- **g** investigating how many Year 11 students at your school work part-time
- **h** surveying bank customers about their satisfaction levels
- **i** determining the most popular car colour
- **j** a Facebook poll on where a social group should hold their next party

Example 9

4 A department store has a customer database of 3347 female shoppers and 1504 male shoppers. It wants to introduce a new credit card to a stratified sample of 200 shoppers. How many female shoppers should be included in the sample?

5 A public transport department surveys a sample of 600 commuters about the efficiency of the city's rail network. It is known that each region of the city has the following number of commuters:

North	South	East	West	Southwest
123 760	149 810	81 060	260 450	215 880

If the sample is to be stratified, how many commuters from the West should be included?

6 A school's gardening program is run by 25 students, 5 teachers and 8 parents. A stratified sample of 8 people is selected to attend an environmental conference. Which of the following is the number of teachers who should be in the sample? Select **A**, **B**, **C** or **D**.

A 0 **B** 1 **C** 2 **D** 5

Example 10

7 Explain why each sample is biased.

- **a** A sample of *Girlfriend* magazine readers was surveyed to investigate teenagers' views on road safety.
- **b** Diners at a restaurant can give feedback about their meal by filling in a survey and posting it to the restaurant.
- **c** For a survey about canteen food, the school council interviewed the first 20 students to arrive at school this morning.

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d At a factory, a sample of executives was surveyed about safety in the workplace.

e A sample of audience members at a film premiere was surveyed about their views on the film.

f Triple J radio listeners were asked to vote for the best song of the year.

g After a plane flight, passengers were emailed a customer experience survey to complete.

8 By referring to Question **7**, determine which samples are self-selected.

9 A sample of shoppers is to be chosen for a survey about customer service at a new shopping centre.

a How would you choose the shoppers to ensure that you have an unbiased sample?

b List two ways of choosing shoppers that would result in a biased sample.

DID YOU KNOW?

How TV ratings are measured

Each week, research company OzTAM investigates the TV viewing habits of Australians, based on a sample of over 5000 homes. OzTAM, which stands for Australian Television Audience Measurement, investigates which TV channels and programs are being watched by a sample of Australian households stratified according to the ages and number of people living in each home. Participants in the survey have their television sets linked to a black box called a 'peoplemeter' that records the details of all programs being watched each day (including pay television). The results are sent overnight by modem to OzTAM.

Television network	Percentage share
Seven	15.0%
Nine	15.0%
Ten	8.8%
ABC	8.4%
ABC2	4.7%
SBS	3.1%
7TWO	2.3%
7mate	2.3%
ONE	2.3%
GO!	2.2%
Eleven	2.1%

Source: OzTAM

During one week in August 2017, the Australian television networks had the following percentage shares of viewers in Sydney, Melbourne, Brisbane, Adelaide and Perth, from 6.00 a.m. to 12.00 midnight.

What types of people would be interested in the television viewing habits of Australians and why?

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INVESTIGATION

ESTIMATING POPULATIONS

How many students at your school:

a have red hair? **b** have brown eyes? **c** are left-handed?

d are vegetarian? **e** walk to school? **f** have a dog as a pet?

A survey is often conducted using a questionnaire, which asks for details about a person's lifestyle, beliefs, attitudes and behaviours. The table below describes the features of a good questionnaire, giving examples.

Estimate the answers to these questions by surveying a sample of 50 students from your school and making appropriate calculations. How could you improve on your estimates?

INVESTIGATION

DESIGNING A QUESTIONNAIRE

Student survey form

Census questions

A survey is often conducted using a questionnaire, which asks for details about a person's lifestyle, beliefs, attitudes and behaviours. The table below describes the features of a good questionnaire, giving examples.

Feature	Bad example	Good example
1 Simple language All instructions and questions should be easily understood.	'Describe the frequency with which you consume takeaway food on a weekly basis'.	'How many times per week do you eat takeaway food?'
2 Unambiguous questions Questions must not have **more than** one meaning or interpretation.	'What type of car do you own?' Does the question mean: • the make of car (for example, Toyota)? • the style of car (for example, hatchback)? • the size of car (for example, small)?	'What make of car do you own? Tick one box from the following list: …' (This is followed by a list of different makes of cars.)
3 Respect for privacy A survey should be anonymous and not require personal details. Questions that are too personal should be optional.	'What is your email address?' People are concerned about receiving junk email if they give out their email address.	'In case we need to contact you further regarding this survey, please supply your email address. This is optional and your details will not be forwarded to other parties or used for promotional purposes.'

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4 Freedom from bias Questions should not suggest or influence a person's response.	'What do you think of changing the Australian flag, given that it is part of our history and our soldiers have died for it in the past?' Too emotional, not objective, influencing people to vote against a new flag.	'Do you think the current Australian flag should be changed?'
5 Consideration of a number of choices A question may have many different answers, so either make the question very specific or provide a list of answers from which to choose.	'How well did you enjoy your meal?' (from a restaurant survey). Too open ended. There would be a wide range of possible answers that would be difficult to organise and analyse.	'How satisfied were you with your meal? Choose from one of the following: Very satisfied ☐ Satisfied ☐ Not satisfied ☐ Very unsatisfied ☐ No opinion ☐

1 Explain what is wrong with each survey question and write a better question each time.

a Are politicians paid too much?

b What is your favourite food?

c How often do you smoke?

d Which mode of transport do you employ to commute to your place of work?

e Are you happy at school?

f How satisfied are you with the level of customer service at the New Century Bank?

2 a In groups of 2 to 4 students, design a questionnaire of 10 to 12 questions about one of the following topics:

- TV viewing habits of people
- spending habits of teenagers
- students' use of smartphones and/or tablets
- family and household details
- favourite sports to watch and play.

b Decide on the type of facts you wish to collect, then determine suitable wording for each question and the format of each answer. Make sure that your questions reflect the features of an effective questionnaire.

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Australian statistics

Statistical graphs

1.05 Constructing graphs

When displaying a set of data, remember to:

- select the most appropriate graph or display
- catch the reader's attention
- show enough detail, including a title, labels, key or scale
- avoid giving a misleading impression.

	Useful for:	Strengths	Weaknesses
Bar chart	• Categorical or numerical data • Showing differences in amounts	• Shows exact values • Can graph a large amount of data	• Takes a long time to draw
Line graph	• Continuous numerical data • Showing changes over a time period	• Shows exact values • Can graph a large amount of data • Shows differences and patterns clearly • Can be used to estimate values between points	• Can only be used to graph continuous data
Sector graph and divided bar graph	• Categorical data • Comparing parts of a whole	• Shows parts of a whole	• Doesn't show exact values • Doesn't show small proportions or a large number of categories well

EXAMPLE 11

The 2016 Census results for the religions of Australian people are shown in the table.

Represent this data on:

a a sector graph

b a divided bar graph.

Religion	Frequency
Buddhist	563 674
Christian	12 201 600
Hindu	440 300
Islamic	604 240
Jewish	91 022
Other religion	2 567 348
No religion	6 933 708
Total	**23 401 892**

Source: Australian Bureau of Statistics (ABS), 2016 Census

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Solution

a First calculate the angle of each sector, rounded to the nearest degree.

Buddhist: $\frac{563\,674}{23\,401\,892} \times 360° \approx 9°$

Write each religion as a fraction and multiply by 360°.

Christian: $\frac{12\,201\,600}{23\,401\,892} \times 360° \approx 188°$

Hindu: $\frac{440\,300}{23\,401\,892} \times 360° \approx 7°$

Islamic: $\frac{604\,240}{23\,401\,892} \times 360° \approx 9°$

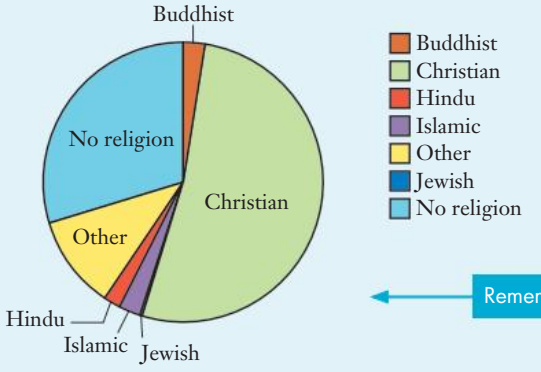
Note: Due to rounding, the sector angles may not add exactly to 360°.

Jewish: $\frac{91\,022}{23\,401\,892} \times 360° \approx 1°$

Other religion: $\frac{2\,567\,348}{23\,401\,892} \times 360° \approx 39°$

No religion: $\frac{6\,933\,708}{23\,401\,892} \times 360° \approx 107°$

Religions in Australia



Instead of labelling the sectors, you could provide a 'key' like this.

Remember to add a title and to label each sector.

b Draw a long rectangle (for example, 12 cm long), then calculate the length of the section representing each religion, rounded to the nearest millimetre.

Buddhist: $\frac{563\,674}{23\,401\,892} \times 12 \text{ cm} \approx 0.3 \text{ cm}$

Write each religion as a fraction and multiply by 12 cm.

Christian: $\frac{12\,201\,600}{23\,401\,892} \times 12 \text{ cm} \approx 6.3 \text{ cm}$

Hindu: $\frac{440\,300}{23\,401\,892} \times 12 \text{ cm} \approx 0.2 \text{ cm}$

Islamic: $\frac{604\,240}{23\,401\,892} \times 12 \text{ cm} \approx 0.3 \text{ cm}$

Jewish: $\frac{91\,022}{23\,401\,892} \times 12 \text{ cm} \approx 0.05 \text{ cm}$

'Jewish' will be too small to show as a section.

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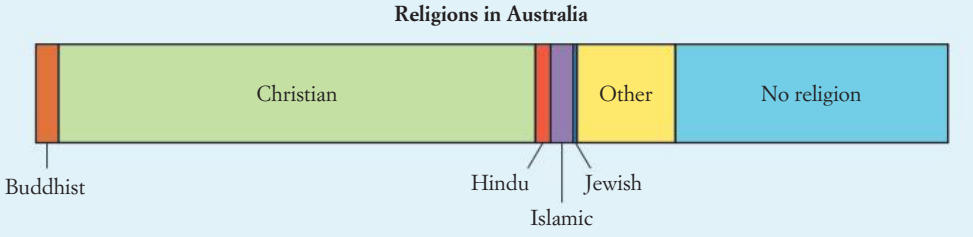
Other religion: $\frac{2\,567\,348}{23\,401\,892} \times 12 \text{ cm} \approx 1.3 \text{ cm}$

No religion: $\frac{6\,933\,708}{23\,401\,892} \times 12 \text{ cm} \approx 3.6 \text{ cm}$

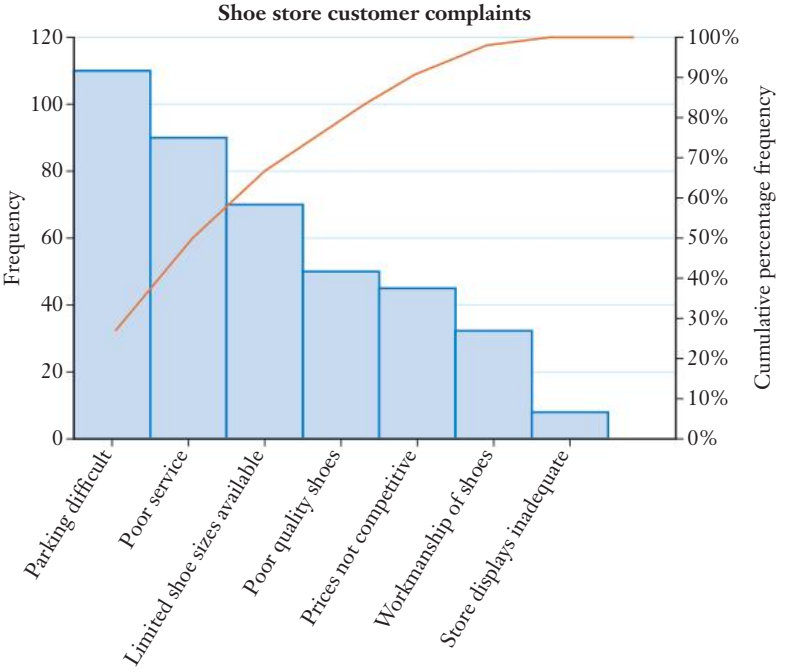
Note: Due to rounding, the sections may not add exactly to 12 cm.

Remember to add a title and to label each section.
Instead of labelling the sections, you could provide a 'key'.



Pareto charts

A **Pareto chart** contains a bar chart and a line graph combined. Individual values are represented in **descending order** with bars. The cumulative total (or cumulative frequency) of the data is represented by the line. For example, the Pareto chart below explains the decrease in profits at a shoe store after surveying customers.



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Originally, the manager assumed that the decrease in sales was because this season's shoes were not very appealing to customers, and she blamed the shoe supplier for this problem. After the customer survey results were displayed on a Pareto chart, the manager recognised the real reasons for her poor profits. The customer car park, sales staff attitudes and limited shoe sizes were the main issues.

Pareto charts are often used in quality control for businesses. The taller columns are drawn on the left and highlight the key problems or issues.

Exercise 1.05 Constructing graphs

1 A group of 40 students was surveyed about the number of DVDs they each watched last weekend. The results were:

2	1	1	2	3	3	4	3	1	4
3	3	6	3	2	0	3	4	2	2
3	3	2	1	1	3	0	5	4	4
2	2	2	5	4	0	4	3	1	3

a Organise the data in a frequency table, then present it as a bar chart.

b Why is a line graph not appropriate for this data?

c What was the mode (the most common number of DVDs watched)?

d What was the highest number of DVDs watched and how many students watched that many?

e What percentage of students watched more than four DVDs?

2 Georgie won $800 000 on a quiz show and spent it as shown in the table.

Example 11

Use of money	Amount
Pay home loan	$235 000
New car	$96 000
Home renovations	$144 000
Travel	$125 000
Savings	$200 000

a Why is a sector graph or divided bar graph suitable for this information?

b Draw a sector graph for this information.

c What did Georgie do with one-quarter of her winnings?

d What fraction of Georgie's winnings was spent on home renovations?

e What percentage of Georgie's winnings was spent on a new car?

3 A sample of Sydney households was surveyed about which TV network they watched for the evening news. The results are shown in the table.

Network	Number of households
ABC	3460
SBS	855
Seven	4140
Nine	4215
Ten	2300

a How many households were surveyed?

b Name the three types of graphs that could be used to display this data.

c Represent the data on a divided bar graph.

d What percentage of households watch the ABC news?

e Do the viewers of the Seven News and Nine News combined make up more than half of the households surveyed?

f Does this data reflect the news-watching habits of all Australian households? Justify your answer.

4 The teachers at Summer Heights High School were surveyed on the number of cups of coffee they drank yesterday.

Cups of coffee	0	1	2	3	4	5	6
Frequency	7	5	11	21	15	9	4

a How many teachers are there at the school?

b What type of graph can be used to illustrate this data? Give a reason for your answer.

c What percentage of the teachers did not drink coffee that day? Answer correct to one decimal place.

d If the recommended healthy coffee intake is no more than three cups of coffee per day, what fraction of teachers had a healthy intake of coffee?

5 Serena wants to graph the results of a survey about people's favourite ice cream flavour. Which of the following graphs can she use? Select **A**, **B**, **C** or **D**.

A bar chart

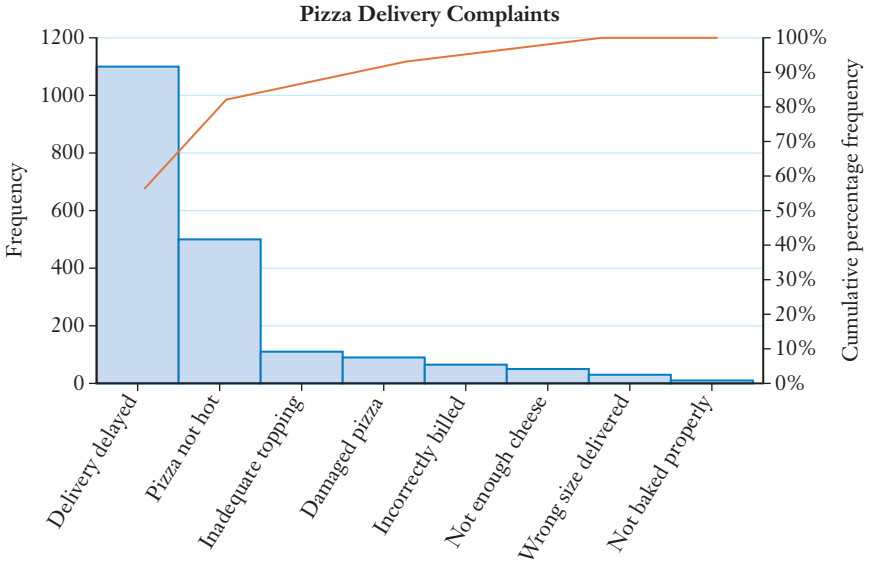
B sector graph

C divided bar graph

D all of the above

ISBN 9780170413565

6 This Pareto chart shows complaints about the delivery of pizzas from a new chain of pizza stores.



Use the graph to answer these questions:

a What are the two most common complaints?

b How many customers complained about late delivery of pizzas?

c What percentage of customers complained about late delivery of pizzas?

d 'Customers are more likely to complain that they have been incorrectly billed for their order than they are likely to complain that their pizza has not been baked properly'. True or false?

e Where should the owners of this business direct their efforts to improve their delivery service? Name at least two ways they could improve.

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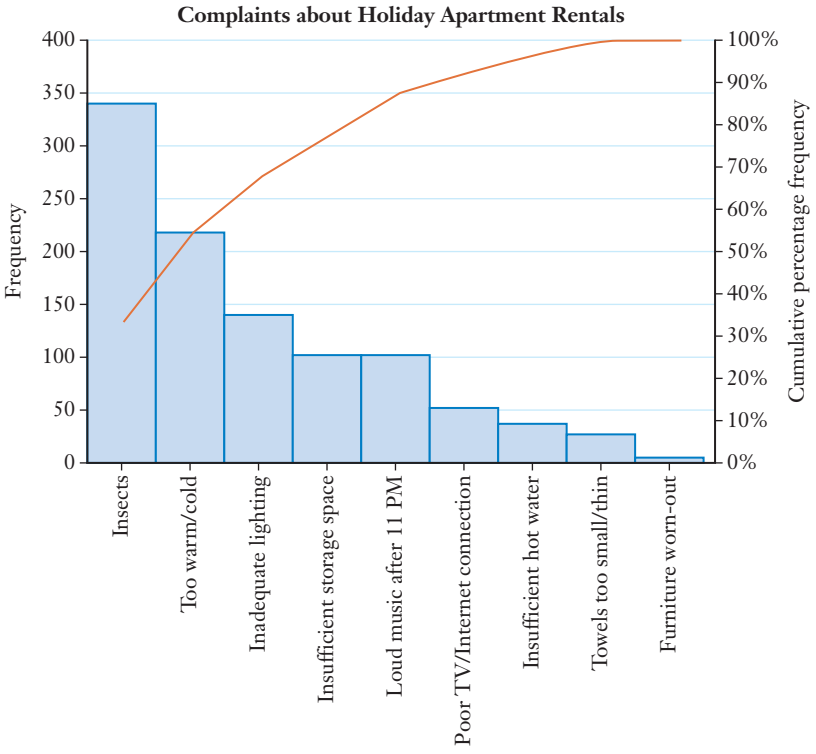
7 The Pareto chart below shows complaints about apartments that are rented out to holiday-makers.



a What is the most common complaint?

b There are two areas that received the same number of complaints. What are they?

c How many holiday-makers complained about loud music after 11 p.m.?

d What percentage of holiday-makers complained about loud music after 11 p.m?

e Are these statements true or false?

- **i** Most holiday renters are satisfied with the quality of TV/Internet connection.
- **ii** Worn-out furniture is a major concern of holiday renters.
- **iii** Holiday renters are more concerned about the heating/cooling of the apartments than they are about inadequate lighting and insufficient hot water combined.

f Name two significant ways that the owners of the apartments could improve the apartments.

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8 The table lists the concerns of customers who purchase flat pack furniture from the specialist furniture store 'I-AKE'. Flat pack furniture requires the customer to assemble each item themselves.

Pareto chart template

Flat pack furniture defects	Number of complaints
Damaged parts	90
Dirty	55
Missing instructions	45
Missing parts	15
Unpainted	10
Poor quality	5

a Download the 'Pareto chart template' worksheet from NelsonNet to draw a Pareto chart for the data shown.

b What are the two most common complaints?

c Name two ways I-AKE could improve the quality of their furniture.

9 In a two-week period, Australian Customs recorded the number of immigrants arriving at Sydney airport, as shown in the table.

Country of origin	Number of immigrants
United Kingdom	20
New Zealand	35
Middle East	7
India	10
Africa	3
Asia	12
Other regions	6

a Draw a divided bar graph to represent this data.

b 'More than half of the immigrants came from the UK or New Zealand.' True or false?

c What fraction of immigrants came from Asia?

d What percentage (to the nearest whole number) of immigrants came from the Middle East?

e Which region represents 10.75% of the immigrants?

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10 This table shows the numbers of motor vehicle and home thefts in the Shoalhaven area over 5 years.

Year	2011	2012	2013	2014	2015
Motor vehicle theft	571	702	562	484	483
Home theft	589	880	600	704	707

a Draw line graphs showing both types of theft on the same axes. Include labels or a key.

b In which year was home theft highest?

c In which year was motor vehicle theft lowest?

d Which type of theft is generally more common?

e Write a sentence to describe what happened to the number of home thefts over the 5 years.

f What do you think happened to motor vehicle theft in 2016? Justify your answer.

DID YOU KNOW?

Bias in drafting US soldiers

Bias can affect life-or-death situations, as some young Americans found out during the Vietnam War. In 1970, men aged 19 to 26 were randomly drafted into the armed forces according to their birthdates (this also occurred in Australia). The 366 dates of the year were written on plastic capsules and put into a large bowl. Capsules were then picked out and the men who had the selected birthdates were drafted. However, it was later discovered that birthdates in November and December had a slightly higher chance of being drawn because the capsules were placed in the bowl in order and not mixed properly, and the ones chosen were often from the top of the pile.

Suggest two ways the bias could have been eliminated during this selection process.

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TECHNOLOGY

Drawing sector graphs

We can use **Insert** and **Chart** in a spreadsheet to draw different types of graphs. A sample of 800 commuters was surveyed about the way they travelled to work.

Step 1: Enter the data from the survey into a spreadsheet as shown.

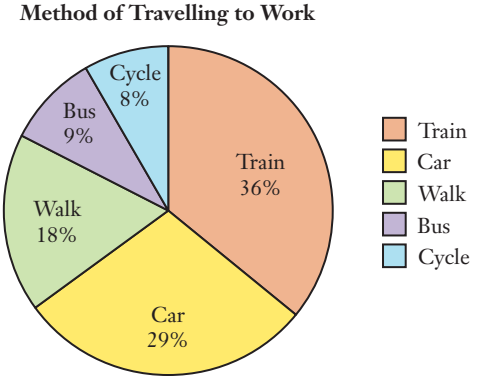
	A	B
1	Train	287
2	Car	233
3	Walk	140
4	Bus	73
5	Cycle	67

Step 2: Select (highlight) all the cells A1:B5 and click **Insert** and **Pie** or **Doughnut Chart**.

Step 3: Click **Quick Layout**

from the **Design** menu and choose **Layout 6** which shows a **Legend** and **Chart Title**. Percentages and sector names are also now shown on the sector graph.

Step 4: Click on 'Chart Title' and replace with the title given below.



Step 5: Format any features of your graph, including colour and design, by clicking on it and using **Format** or **Design** from the Toolbar. You may also resize the graph.

Further investigations

- Repeat the steps above for other data, creating different sector graphs and layout options.
- Graph the same data as a bar chart by clicking **Design** and **Change Chart Type**.

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TECHNOLOGY

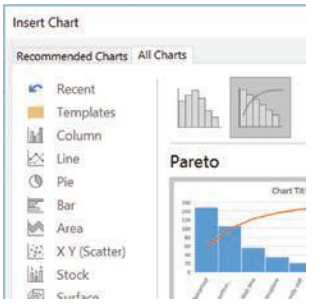
Drawing Pareto charts

A spreadsheet can be used to easily draw a Pareto chart.

	A	B
1	Wait time	56
2	No atmosphere	34
3	Unfriendly staff	21
4	Small portion sizes	105
5	Too noisy	12
6	Not clean	18
7	Food not fresh	12
8	Overpriced	148

Step 1: In a spreadsheet, enter the data showing customer complaints at a restaurant as shown.

Step 2: Highlight the data from cell A1:B8. Click **Insert** and **Chart** in a spreadsheet. Then choose **All Charts** then **Histogram** and **Pareto Chart**.



Note: The data does not need to be reordered from largest complaints to smallest, as the spreadsheet will account for this automatically when the graph is created.

Step 3: If necessary, resize the chart so all labels can be read and then click **Chart Title** and give the Pareto chart an appropriate title for this data.

Step 4: Write a brief report advising the restaurant of the main areas of complaint and make some suggestions as to how they might improve their customer service and/or restaurant.

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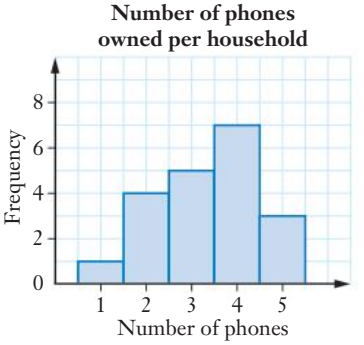
1.06 Frequency histograms and polygons

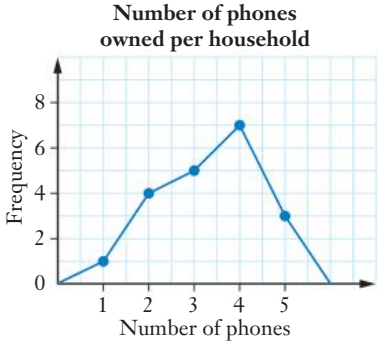
Frequency tables

A **frequency histogram** or **frequency polygon** graphs numerical data, including data grouped into **class intervals**.

Class intervals and centres

- In statistics, when there are many data scores, they may be grouped into **class intervals**. For example, ages of people may be grouped into class intervals of 1–10, 11–20, 21–30 and so on.
- A **class centre** is the centre of a class interval. For example, the class centre of the class interval 10–19 is 14.5.





Frequency histogram

- no gaps between columns
- has a half-column gap at the start.

Frequency polygon

- can be drawn by joining the midpoints of the tops of the columns of a histogram
- starts and ends on the horizontal axis.

	Useful for	Strengths	Weaknesses
Frequency histogram and polygon	• numerical (**quantitative**) data • data grouped into class intervals • showing the shape of a set of data; how the scores are spread out	• easy to read • shows differences and patterns clearly • can graph a large amount of data • shows how the scores are spread out	• takes a long time to draw

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EXAMPLE 12

The ages of the patients at a medical centre in one afternoon were recorded:

37	12	19	65	44	30	1	8	14	36	25	57	21	16
4	30	40	49	38	69	33	55	16	4	8	16	6	28
29	54	43	48	33	18	9	59	21	70	22	33	2	60

a Copy and complete this frequency distribution table for the data.

Class interval	Class centre	Tally	Frequency
0–9			
10–19			
20–29			
30–39			
40–49			
50–59			
60-69			
70–79			
		Total	

b Construct a frequency histogram for this data.

c How many patients went to the medical centre?

d How many of these patients were aged 50 or over?

Solution

a These are discrete class intervals.

Class interval	Class centre	Tally	Frequency
0–9	4.5	𝍸 |||	8
10–19	14.5	𝍸 ||	7
20–29	24.5	𝍸 |	6
30–39	34.5	𝍸 |||	8
40–49	44.5	𝍸	5
50–59	54.5	||||	4
60-69	64.5	|||	3
70–79	74.5	|	1
		Total	42

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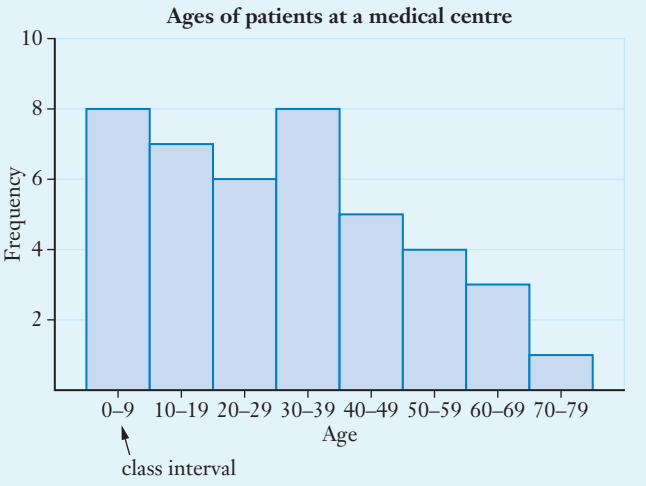
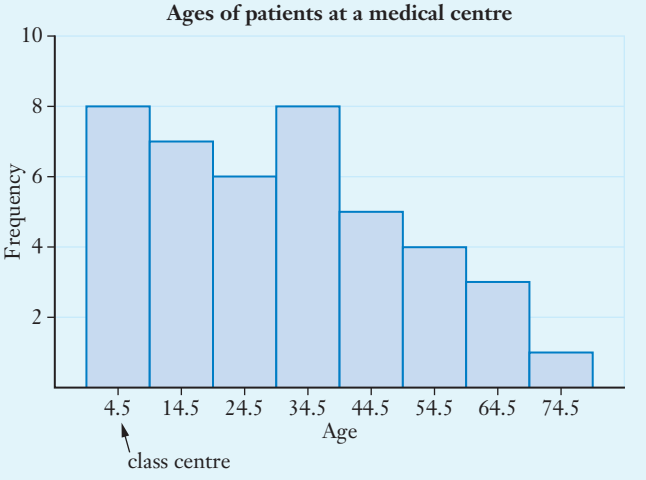
b When graphing data that are grouped into discrete class intervals, either the **class intervals** or **class centres** are shown on the **horizontal axis**.



OR:



c 42 patients went to the medical centre. Total frequency

d Number of patients aged 50 or over = 4 + 3 + 1 = 8

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EXAMPLE 13

The monthly call costs for a group of 120 mobile phone users were recorded and grouped into class intervals as shown.

Call cost ($)	Class centre	Number of users
0–< 20	10	6
20–< 40	30	8
40–< 60	50	13
60–< 80	70	17
80–< 100	90	23
100–< 120	110	20
120–< 140	130	16
140–< 160	150	10
160–< 180	170	4
180–< 200	190	3
	Total	120

These are continuous class intervals.

a Construct a frequency histogram and polygon for this data.

b Which call costs were the most common?

c Which call costs were used by exactly 10 people?

d What percentage of phone users had call costs below $60?

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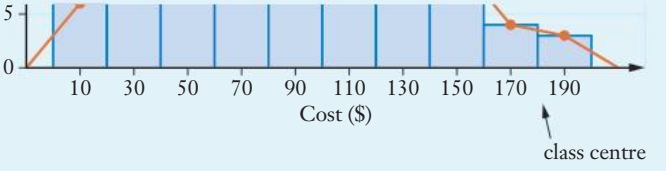
Solution

a When graphing data grouped into continuous class intervals, either the class boundaries or class centres are shown on the horizontal axis, like this:

or like this:



b $80–< $100 Had the highest frequency (23)

c $140–< $160 Had a frequency of 10

d Number of users with call costs below $60 = 6 + 8 + 13 = 27

Percentage of users with call costs below $\$60 = \frac{27}{120} \times 100\% = 22.5\%$

Exercise 1.06 Frequency histograms and polygons

1 Forty people were surveyed about the number of times they posted photos online in one day. The results are shown below.

2	1	5	1	3	4	3	5	5	4
3	4	5	2	1	0	3	4	1	1
2	3	2	0	1	5	4	5	3	3
1	1	1	4	4	5	2	3	1	5

a Organise this data into a frequency distribution table.

b Construct a frequency histogram for the data.

2 The masses (in grams) of a sample of 48 eggs were measured and recorded.

57	58	61	59	62	59	59	56	60	64	58	58
56	59	64	57	60	62	58	60	64	57	61	58
59	57	64	58	59	57	58	64	60	58	60	57
61	64	58	60	61	62	62	58	60	61	57	58

a Construct a frequency distribution table for this data.

b Draw a frequency polygon for the data.

c If these eggs are labelled as being 60 grams, would you say that the label is misleading? Give reasons for your answer.

Example 12

3 The table shows the number of road fatalities according to age group on a section of the Pacific Highway over 6 months.

Age group	Road fatalities
0–9	6
10–19	14
20–29	28
30–39	11
40–49	8
50–59	3

a How many people died on this section of the highway during the period?

b Draw a frequency histogram and polygon for this data.

c What observations can you make about road fatalities from the graphs?

4 The number of words in each sentence of a magazine article was counted, with the results shown below.

27	22	15	8	14	7	9	25	15	17	5	24
9	11	22	8	5	15	25	18	10	21	24	13
9	14	18	11	9	23	15	19	10	8	14	17

a Is this data discrete or continuous?

b Using class intervals of 1–5, 6–10 etc., complete a frequency table for the data.

c Construct a frequency polygon for the data.

d What was the modal class (the class with the highest frequency)?

5 Here are the ages of the employees at Burger Heaven.

18	19	18	17	20	20	24	15	24	19
15	40	21	17	20	22	23	21	24	23
34	19	45	20	15	21	24	27	19	33
34	24	16	18	30	21	26	31	16	25
49	21	21	35	16	22	15	25	44	23

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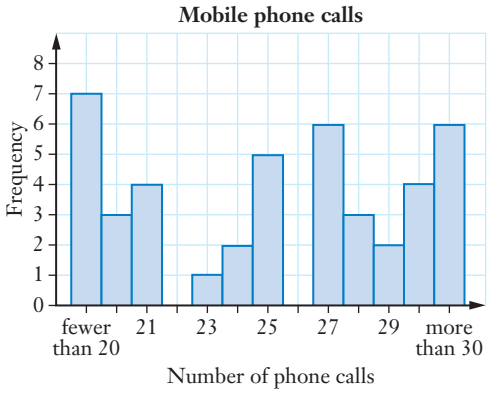
a Organise the data into a frequency table using classes 15–19, 20–24 etc.

b How many people work at Burger Heaven?

c Draw a frequency histogram and polygon for the data.

d Comment on the shape of the data.

6 A group of students was surveyed on the number of mobile phone calls made in a week. The results are shown in this histogram.



a How many students made 22 calls?

b How many students were surveyed?

c What percentage of students made fewer than 20 calls? Answer correct to one decimal place.

d Can you tell how many students made 32 calls? Why?

7 The number of kilograms lost by each person attending Julie's Weight Loss Centre is shown below.

Example 13

1.8	2.5	4.3	6.5	2.7	4.6	11.0	10.8	0.3	8.2
2.1	3.8	4.4	5.8	1.6	5.9	7.6	9.3	4.8	3.4
12.5	4.6	2.5	6.9	7.5	3.5	4.8	12.2	4.3	3.7
0.0	0.9	2.6	7.8	4.9	7.4	9.8	10.4	2.6	8.2

a Is this data discrete or continuous?

b Complete a frequency table for this data using class intervals $0 - < 2$, $2 - < 4$ etc.

c Draw a frequency histogram for this data.

d What is the modal class?

e Use your histogram to comment on the effectiveness of the weight loss program.

8 The heights (in centimetres) of new applicants to a modelling agency are:

168	182	187	185	178	176	174	163	164
165	189	185	165	162	173	164	173	178
172	186	189	175	176	180	170	168	165

a Using class intervals of $160 - < 165$, $165 - < 170$ etc., complete a frequency table for this data.

b Draw a frequency polygon for the data.

c What is the modal class interval?

d What is the lowest class interval?

e Which class interval is least common?

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TECHNOLOGY

Drawing frequency histograms and polygons

We can use **Insert** and **Chart** in a spreadsheet to draw a frequency histogram and polygon.

	A	B
1	Call cost ($)	Number of users
2	10	6
3	30	8
4	50	13
5	70	17
6	90	23
7	110	20
8	130	16
9	150	10
10	170	4
11	190	3
12	210	0

Step 1: Enter the data about the cost of mobile phone calls from Example **13** into a spreadsheet but add a last row (210, 0) as shown.

This allows you to draw a frequency polygon which touches the x-axis.

Step 2: Select B2:B13 the values in the 'Number of users' column, as the 'y-values'. (Do not select B1 because we will add the titles later.)

Step 3: Click **Insert** then click **Insert Column** or **Bar Chart** and **Clustered Column**.

Step 4: Right-click on the graph and click **Select Data**. Click **Horizontal (Category) Axis Labels** then **Edit** and highlight A2:A13.

Step 5: Click **Add Chart Element**

and **Axes Titles Primary Horizontal and Primary Vertical** and enter appropriate axes titles.

Step 6: Right-click on the bar chart and choose **Format Data Series**. Change **Gap Width** to 0%.

Step 7: Click on **Chart Title** on the graph and enter an appropriate title for the data in the frequency histogram.

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Step 8: To make a Polygon select B2:B13 the values in the 'Number of users' column, as the '*y*-values'. (Do not select B1 because we will add the titles later.)

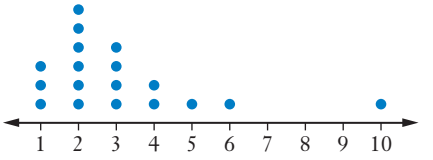
Step 9: Click **Insert** then click **Insert Line** or **Area Chart** and click **Line with Markers**.

Step 10: Right-click on the graph and click **Select Data**. Click **Horizontal (Category) Axis Labels** then **Edit** and highlight A2:A13.

Step 11: Click **Add Chart Element**

and **Axes Titles Primary Horizontal** and **Primary Vertical** and enter appropriate axes titles. Then click **Chart Title** and enter an appropriate title for the polygon.

1.07 Dot plots and stem-and-leaf plots



Stem	Leaf
7	9 5 8 1 6 9
8	4 2 5 0 0 8 0 3 2 4 2 8
9	0 8 4 2 2 0 0 8 0 5 6
10	5 4

Dot plot

- a simple type of bar chart
- uses a dot, cross or symbol to display each score
- also called a line plot.

Stem-and-leaf plot

- actual scores are listed, usually in order
- the stem shows the first digit(s)
- the leaf shows the last digit(s)
- also called a stem plot.

	Useful for:	Strengths	Weaknesses
Dot plot and stem-and-leaf plot	• numerical (quantitative) data • small sets of data • showing gaps and **clusters** • showing outliers (scores that are much different from the rest)	• easy to read and draw • shows differences and patterns clearly • shows the spread of scores • stem-and-leaf plot shows actual scores	• hard to display large sets of data

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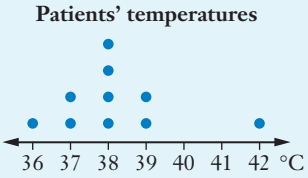
EXAMPLE 14

This dot plot shows the temperatures of a sample of hospital patients.



a How many patients are in the sample?

b How many patients have a normal body temperature of 37°C?

c What is the mode (the most common score)?

d Where are the temperatures clustered?

e Find the outlier and describe the patient who has this temperature.

Solution

a 10 — 10 dots in total

b 2 patients — There are two dots at 37°C

c 38°C — The score with the most dots

d From 37°C to 39°C.

e The outlier temperature is 42°C and a patient with this temperature would be very hot and ill.

EXAMPLE 15

This stem-and-leaf plot shows the exam marks of a class of students.

Stem	Leaf
4	3 7
5	0 2 2 4 8
6	1 3 3 3 6 6 9
7	5 7 8
8	4 6
9	2

The stem shows the tens digit, and the leaf shows the units digit: 6 | 1 means 61.

a What is the highest mark?

b How many students are in the class?

c What is the most common mark?

d What percentage of students scored over 60?

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Solution

a 92

b 20 The number of values in the 'Leaf' column

c 63 This score occurs the most, 3 times

d Number of students who scored over 60 = 13

Percentage of students who scored over $60 = \frac{13}{20} \times 100\% = 65\%$

Display	Useful for:
Bar chart	• categorical or numerical data • showing differences in amounts
Line graph	• continuous numerical data • showing changes over a time period • estimating values between points
Pareto chart	• categorical data • showing differences in amounts • sorting categories in order of frequency/importance
Sector graph and divided bar graph	• categorical data • comparing parts of a whole • a small number of categories
Frequency histogram and polygon	• numerical data • data grouped into class intervals • showing the shape of a set of data, how the scores are spread out
Dot plot and stem-and-leaf plot	• small sets of numerical data • showing clusters, gaps and outliers • showing the shape of a set of data, how the scores are spread out

Exercise 1.07 Dot plots and stem-and-leaf plots

Example 14

1 A class was surveyed to find out how many hours each student spent on maths homework each week. The results are shown below.

7	6	8	9	5	10	6	9	9	0	9	8
18	7	5	3	4	9	6	7	8	10	7	8

a Draw a dot plot for this data.

b How many students are in the class?

c What are the outliers?

d What are the clusters or gaps?

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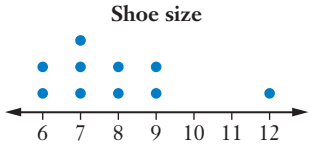
2 This dot plot shows the shoe sizes of a sample of Year 11 students.



- **a** How many students are there in the sample?
- **b** What is the mode?
- **c** Find the outlier and describe the student that has this outlier shoe size.
- **d** What fraction of students have a shoe size of 8 or 9?

3 The Bennelong Bears scored the following numbers of goals per match in their first season.

4 0 2 3 7 1 2 4 5 0 3 5

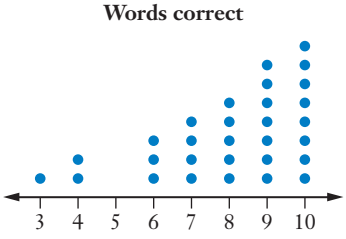
- **a** Display the data as a dot plot.
- **b** What was the lowest score?
- **c** Comment on any clusters or outliers.

4 A survey was conducted to determine the number of people in each car in a sample of cars on a road. The results are shown below.

6 3 1 2 3 1 1 3 4 2
5 3 1 1 2 3 4 5 6 4

- **a** Display the data as a dot plot.
- **b** How many cars were surveyed?
- **c** What was the highest score?
- **d** Comment on any clusters or outliers.

5 This dot plot shows the scores (out of 10) of students competing in a spelling bee.



- **a** How many students competed in the spelling bee?
- **b** What percentage of students scored more than 8?
- **c** Where were the scores clustered?
- **d** Comment on the general performance of the contestants in the spelling bee.

Example 15

6 This unordered stem-and-leaf plot represents the number of points scored per match by the Sharks in a football season.

Stem	Leaf
4	5 3 9
5	7 2 0 8
6	4 7 8 5 1 2
7	2 9 3 0
8	9 4 2

- **a** Redraw the data as an ordered stem-and-leaf plot.
- **b** How many matches were played in the season?
- **c** What was the Sharks' highest score for a match?
- **d** For what percentage of matches did the Sharks score below 56 points?

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7 Jeremy sells sausage sandwiches outside the hardware store on Sundays. The number of sandwiches sold each day over 30 Sundays was recorded as follows.

66	64	28	93	47	110	53	68	117	43
72	68	84	103	59	82	78	61	104	79
51	63	112	81	79	94	42	57	83	100

a Draw a stem-and-leaf plot to represent this data.

b On what percentage of days did Jeremy sell more than 50 sausage sandwiches?

c What is the outlier? Give one possible reason for this outlier.

8 This plot shows the times (in seconds) in a slalom ski race.

Stem	Leaf
9	1 5 7 9
10	2 4 5 6 6 8
11	0 2 2 3 4 4 5
12	1 2 3 3 3 7 9
13	2 3 4 5 7 7
14	3 6 9
15	0 1 2

a How many skiers participated in the race?

b What was the winning time?

c If those skiers with times under 110 seconds were of Olympic standard, what percentage of skiers were of this standard?

9 A nurse at Greenacres Hospital took the following pulse rates (in heartbeats per minute) of 40 patients.

71	81	63	55	93	52	69	78	84	65
72	80	68	74	85	79	90	84	76	68
58	64	60	97	83	69	74	56	64	89
94	81	63	60	76	72	110	83	90	64

a Draw a stem-and-leaf plot for this data.

b If the average pulse is from 65 to 75 beats per minute inclusive, how many patients had an average pulse?

c Find the outlier and describe the patient that has this pulse.

d A very fit person has a low resting pulse (say, less than 60). How many patients were in this category?

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10 This plot shows the prices (in $1000s) of homes listed by a Mudgee real estate agent.

Stem	Leaf
2	04 10 17 18 26 35 37 76 77
3	10 26 42 66 54 76 85 85 85
4	02 53 62 65 73 75 76 85
5	03 10 24 60 68 92
6	14 29 32 34 35 35 54
7	18 52 63 85
8	12 25 46 72 93
9	38 75

a What is the price of the most expensive home?

b How many homes are listed?

c Which price is shared by three different homes?

11 At the school athletics carnival, runners recorded the following times (in seconds) in the 100-metre sprint.

12.1 13.6 11.8 18.1 12.0 15.6 13.9 17.2
14.5 18.7 15.7 14.6 16.3 11.6 17.7 15.6
14.6 15.4 12.4 16.5 17.4 14.6 16.8 14.3

a Represent this data on a stem-and-leaf plot using stems of 11, 12 etc.

b How many runners were there?

c What was:

i the best time recorded?

ii the worst time recorded?

d What percentage of students took more than 15 seconds to run the distance? Answer correct to one decimal place.

e To qualify for the regional carnival, a runner's time must be less than 13.5 seconds. What fraction of students qualified for the regional carnival?

DID YOU KNOW?

Outliers and global warming

An outlier can often be the most important score in a **data set**.

Global warming and the effect of the hole in Earth's ozone layer are troubling issues in our world today. In 1976, scientists in Antarctica began detecting low levels of ozone in Earth's atmosphere, but dismissed these low levels as 'unimportant outliers'. Because of this, the damage to our ozone layer went undetected and untreated for another 9 years.

How does the hole in the ozone layer contribute to global warming?

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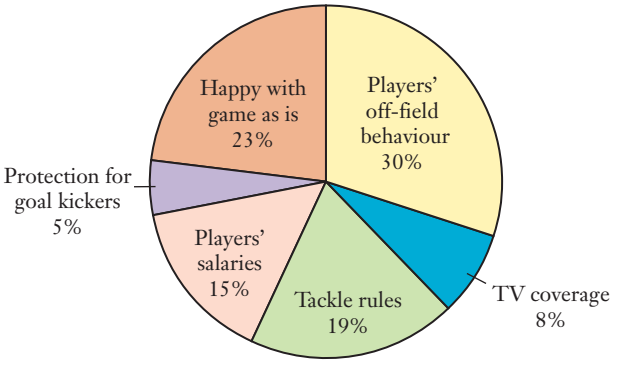
SAMPLE HSC PROBLEM

A sample of 3000 rugby league fans was surveyed on their main concerns about the national competition. The results of the survey are shown by the sector graph below.

Fans concerns about rugby league



a Which one of the following statements is correct? Select **A**, **B**, **C** or **D**.

A The data shown in the sector graph is categorical.

B More than one-third of fans were concerned about players' off-field behaviour.

C The data shown in the sector graph could also be displayed on a line graph.

D The least common concern of fans was TV coverage.

b Describe a method of selecting a stratified sample of rugby league fans.

c Calculate, correct to the nearest degree, the angle size of the sector for 'Happy with the game as it is'. Show your working.

d Which concern was shared by 450 fans?

e Write one disadvantage of using a sector graph to illustrate data.

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Study tip

Four practical steps in studying maths

'A journey of a thousand miles begins with a single step.'

Lao Tzu (c. 570–490 BCE), ancient Chinese philosopher

Step 1: Practise your maths

- Do your homework.
- Learning maths is about mastering a collection of skills.
- You become successful at maths by doing it more, through regular practice and training.
- Aim to achieve a high level of understanding.

Step 2: Rewrite your maths

- Homework and study are not the same thing.
- Study is your private 'revision' work for strengthening your understanding of a subject.
- Take ownership of your maths.
- Rewrite the theory and examples in your own words.
- Summarise each topic to see the 'whole picture' and know it all.

Step 3: Attack your maths

- All maths knowledge is interconnected.
- If you don't understand one topic fully, then you may have trouble learning another topic: 'You cannot run until you have learned to walk'.
- Mathematics is not a subject you can learn 'by halves' – you have to know it all!
- Fill in any gaps in your mathematical knowledge to see the 'whole picture'.
- Identify your areas of weakness and work on them.
- Spend most of your study time on the topics which you find difficult.

Step 4: Check your maths

- Once you have mastered the maths skills, there is no further learning or reading needed.
- Compared to other subjects, the types of questions asked in maths exams are more conventional and predictable.
- Test your understanding with revision exercises, practice papers and past exams.
- Develop your exam technique and problem-solving skills.
- Go back to Steps 1 to 3 to improve your study.

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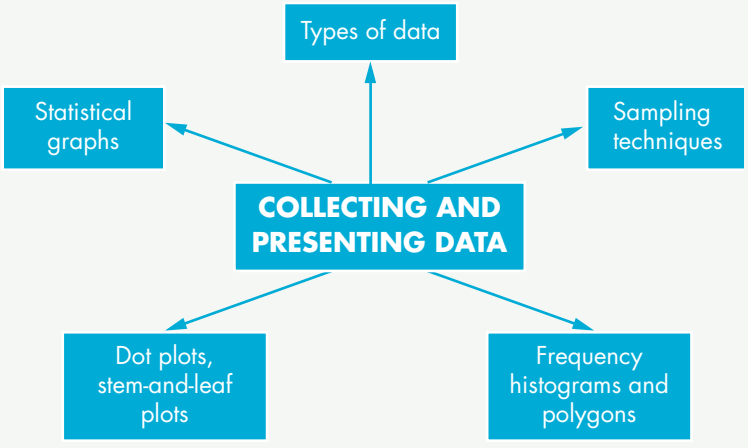
1. CHAPTER SUMMARY

This chapter, Collecting and presenting data, looked at the graphing of data, sampling techniques and the methods of data collection. You should be able to interpret and construct various graphs and displays, know the different types of data and samples, and be familiar with statistical terminology.

Make a summary of this topic. Use the chapter outline at the beginning of this chapter and the mind map below as a guide. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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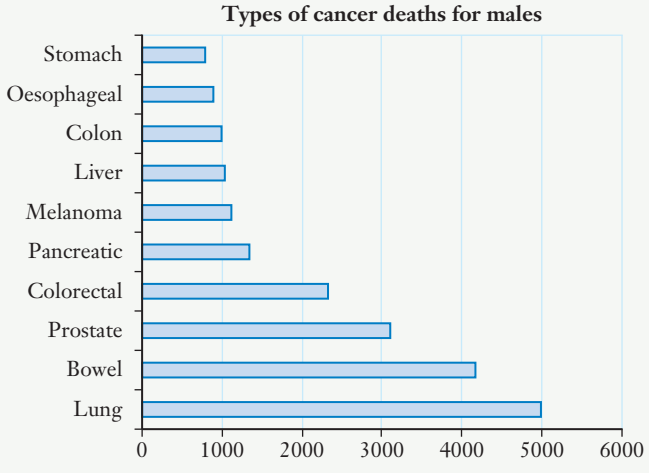
1. TEST YOURSELF

Exercise 1.01

1 This bar graph shows the ten most fatal types of cancer for males in Australia in 2013.



Source: Australian Institute of Health and Welfare 2013

a How many fatal cases of bowel cancer were registered?

b What was the fifth most fatal type of cancer?

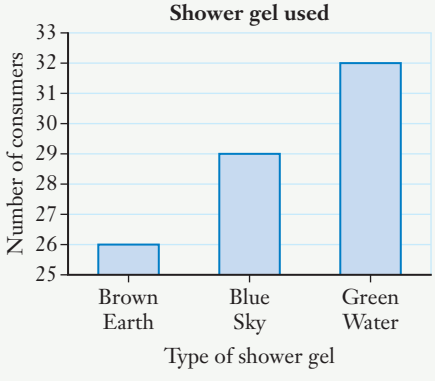
c How many types of cancer recorded fewer than 2000 cases each?

d 'There were more lung cancer deaths recorded than bowel and prostate cancer deaths combined.' True or false?

e Why would it be unsuitable to graph this data on a sector graph?

Exercise 1.02

2 A sample of consumers was surveyed about the brand of shower gel they used. This graph of the survey results was used by Green Water in their advertising campaign.



a Describe one way in which this graph is misleading and what a correct graph should look like.

b Name another type of graph that could be used to display this data.

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3 Classify each of the following types of data as categorical (C) or numerical (N). Exercise 1.03

- **a** the number of people watching a State of Origin football match
- **b** the different brands of surfing magazines
- **c** level of university degree
- **d** the temperatures at an airport
- **e** the number of levels in an office building
- **f** the country of birth of each person living in Gosford.

4 For each of the categorical data identified in Question **3**, classify as nominal (N) or ordinal (O). Exercise 1.03

5 For each of the numerical data identified in Question **3**, classify as discrete (D) or continuous (C). Exercise 1.03

6 The manufacturers of Burp Cola want to conduct a quality control study to ensure that each bottle of drink made does, in fact, contain 1.25 litres. Which type of sample would be most appropriate: random, systematic, stratified or self-selected? Give a reason for your answer. Exercise 1.04

7 **a** To investigate the types of music to play at the school disco, Tasha surveyed a sample of 50 students visiting the canteen. Why might this sample be biased? Exercise 1.04

b Suppose Tasha uses a stratified sample instead, based on the following numbers of school students in each Year group:

Year	7	8	9	10	11	12
Number of students	114	120	114	128	105	96

If she surveys a sample of 50 students, how many Year 10 students should be in the sample?

8 Explain what is wrong with each survey question below and write a better question each time. Exercise 1.04

- **a** How often do you visit the doctor?
- **b** What did you like or dislike about the film?
- **c** Is it time that Australia grew up and became a republic?

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Exercise 1.05

9 A sample of senior citizens was surveyed about the number of countries they have visited. The results are shown in the table.

Number of countries	Number of people
0	3
1–3	8
4–10	32
11–20	28
21–30	17
31+	5

a How many people were in the sample?

b Graph this data on a sector graph.

c What is the angle of the sector representing those who have not travelled outside Australia?

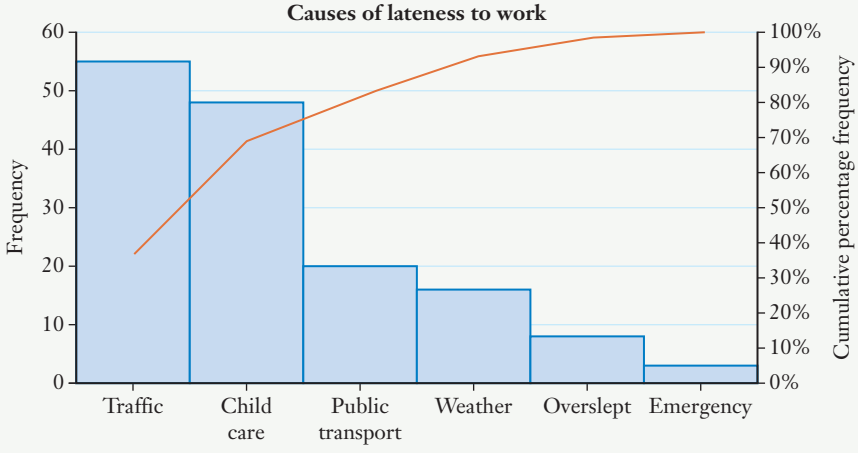
d What percentage of the sample have visited 4 to 10 countries? Answer correct to one decimal place.

e 'More than half of the sample have visited 11 or more countries.' True or false?

f Why would it be inappropriate to graph this data on a line graph?

Exercise 1.05

10 A company surveyed its employees on reasons for arriving late to work, to help assist people to be more punctual. The results are shown on the Pareto chart.



a What are the two main reasons employees give for being late to work?

b What is the least common reason for being late to work?

c How many employees are late due to using public transport?

d What percentage of employees are late due to using public transport?

e What are two possible ways this company might assist its employees to improve their ability to arrive at work on time?

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11 The masses (in kilograms) of 40 skydivers were recorded. The results are shown below.

Exercise 1.06

58	63	77	82	53	69	65	80	96	105
79	63	52	90	104	85	65	87	68	105
65	87	109	84	62	75	102	78	93	84
68	105	74	59	68	74	88	66	70	62

a Are people's masses discrete data or continuous data?

b Organise this data into a frequency distribution table with class intervals 50–< 60, 60–< 70 etc.

c Draw a frequency histogram and polygon to represent the data.

d Which class interval had the highest frequency?

e What fraction of skydivers were in the 80–< 90 kg class?

12 The ages of a sample of children at a Wiggles concert are listed below.

Exercise 1.07

6	0	4	1	5	6	4	2	8	6	4	4
3	6	5	5	2	5	3	1	5	6	3	3

a Draw a dot plot for this data.

b How many children were in the sample?

c What fraction of children were over 5 years old?

d Comment on any clusters.

13 This stem-and-leaf plot shows the ages of visitors entering the Royal Easter Show in a 5-minute period.

Exercise 1.07

Stem	Leaf
0	3 8 9
1	0 2 2 2 5 7 6 9
2	0 2 3 4 6 7
3	1 3 3 4 9
4	3 4 7 8
5	5 5 8

a What was the age of the youngest visitor?

b How many visitors entered the show during the 5-minute period?

c What was the most common age of the visitors?

d What percentage (correct to the nearest whole number) of visitors were over 30 years old?

Qz Chapter quiz

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ALGEBRA

2.

FORMULAS AND EQUATIONS

German scientist Albert Einstein was only 26 years old when he proposed a new theory of physics for small particles of matter (such as atoms) moving at very high speeds. In 1905, he proposed that matter (mass) could be converted to large amounts of energy, describing this relationship with the formula $E = mc^2$, where E stands for energy, m stands for mass and c stands for the speed of light. Einstein's Theory of Special Relativity revolutionised conventional laws of physics and led to the development of nuclear energy.

CHAPTER OUTLINE

A1 2.01 Simplifying algebraic expressions
A1 2.02 Expanding algebraic expressions
A1 2.03 Formulas
A1 2.04 Solving equations
A1 2.05 Formulas and equations
A1 2.06 Changing the subject of a formula

IN THIS CHAPTER YOU WILL:

- expand and simplify algebraic expressions
- substitute values into algebraic expressions and formulas
- solve linear equations, including after substituting into a formula
- change the subject of a formula

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TERMINOLOGY

base
equation
expand
formula
LHS
like terms
power
RHS
rule
simplify
subject
substitute
variable

SkillCheck

Assignment 2

1 Evaluate each expression.

a $9 \times (-3)$ **b** $9 + (-3)$ **c** $2 - 10$

d $(-5)^2$ **e** $-24 \div 6$ **f** $-2 - (-4)$

2 If $d = 3$, evaluate each expression.

a $8d - 9$ **b** $\frac{d+6}{3}$ **c** $7 + 3d$

d $\sqrt{12d}$ **e** d^5 **f** $2d^2$

3 If $u = -2$, evaluate each expression.

a $u^2 - 4$ **b** $3(u + 1)$ **c** $8 - u$

d u^7 **e** $\frac{-18}{u}$ **f** $5 - 2u$

4 Simplify each expression.

a $4x + 7 - 3x - 10$ **b** $18 - 2a + a + 2$ **c** $5r + 12 - 15 - 5r$

5 Expand each expression.

a $3(2x - 1)$ **b** $7(4a + 5)$ **c** $-2(6p - 2)$

6 Solve each equation.

a $3b = -15$ **b** $w - 8 = 9$ **c** $2r + 4 = 22$

7 Evaluate $A = \frac{1}{2}(a+b)h$ if $a = 10$, $b = 15$ and $h = 3$.

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2.01 Simplifying algebraic expressions

An **algebraic expression** is made up of **terms** involving **variables** and numerals.
For example, $3x^2 - x + 10$ has three terms: $3x^2$, $-x$ and 10.

Collecting like terms

Algebra using diagrams

Working with like terms

- **Like terms** have exactly the same variables.
- Only like terms can be added or subtracted.

EXAMPLE 1

Simplify each expression.

a $2a^2 - a + 5 + 8a$ **b** $4kr - 6pr - pr + 10kr$ **c** $2xy + 4 - y + 4yx$

Solution

a $2a^2 - a + 5 + 8a = 2a^2 - a + 8a + 5$ — $-a$ and $8a$ are like terms

$= 2a^2 + 7a + 5$ — $2a^2$ and $-a$ are not like terms

b $4kr - 6pr - pr + 10kr = 4kr + 10kr - 6pr - pr$ — Grouping pairs of like terms

$= 14kr - 7pr$

c $2xy + 4 - y + 4yx = 2xy + 4yx - y + 4$ — $2xy$ and $4yx$ are like terms

$= 6xy - y + 4$

Multiplying terms

- When multiplying terms, multiply the numbers and variables separately.
- When multiplying terms with **powers** that have the same **base**, add the powers.

EXAMPLE 2

Simplify each expression.

a $-3bc \times 8ab$ **b** $4p^2q \times \frac{3p^3}{2}$ **c** $(-5r)^2$

Solution

a $-3bc \times 8ab = (-3 \times 8) \times (a \times b \times b \times c)$

$= -24ab^2c$

To **simplify**, first multiply the numbers. Then multiply the variables in alphabetical order.

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b $4p^2q \times \frac{3p^3}{2} = \frac{4 \times 3}{2} \times p^2 \times p^3 \times q$

$= 6p^5q$

$p^2 \times p^3 = p^5$
Add the powers.

c $(-5r)^2 = (-5r) \times (-5r)$

$= -5 \times (-5) \times r \times r$

$= 25r^2$

Dividing terms

- When dividing terms, divide the numbers and variables separately.
- When dividing terms with powers that have the same base, subtract the powers.

EXAMPLE 3

Simplify each expression.

a $\frac{8m^3n}{2m^2}$ **b** $-15w \div 3w^2$ **c** $\frac{6ak^3}{20ak}$

Solution

a $\frac{8m^3n}{2m^2} = \frac{8}{2} \times \frac{m^3n}{m^2}$

$= 4mn$

$\frac{m^3}{m^2} = m^1 = m.$
Subtract the powers.

b $-15w \div 3w^2 = \frac{-15w}{3w^2}$

$= \frac{-15}{3} \times \frac{w}{w^2}$

$= -5 \times \frac{1}{w}$

$= \frac{-5}{w}$

$\frac{w}{w^2} = \frac{1}{ww} = \frac{1}{w}$

c $\frac{6ak^3}{20ak} = \frac{6}{20} \times \frac{ak^3}{ak}$

$= \frac{3}{10} \times k^2$

$= \frac{3k^2}{10}$

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EXAMPLE 4

Simplify each expression.

a $\frac{4a}{5} \times \frac{5h}{12a}$ **b** $\frac{3m}{10n} \div \frac{6m}{25n}$

Solution

a $\frac{4a}{5} \times \frac{5h}{12a} = \frac{\overset{1}{\cancel{4}\,\cancel{a}}}{\cancel{5}} \times \frac{\cancel{5}h}{\underset{3}{\cancel{12}\,\cancel{a}}}$

$= \frac{1 \times h}{3}$

$= \frac{h}{3}$

Each fraction cannot be simplified on its own, so simplify the numerator of each fraction with the **denominator** of the other fraction.

$\frac{4}{12} = \frac{1}{3}$, $\frac{a}{a} = 1$, $\frac{5}{5} = 1$

b $\frac{3m}{10n} \div \frac{6m}{25n} = \frac{3m}{10n} \times \frac{25n}{6m}$

When dividing by a fraction, multiply by its reciprocal.

$= \frac{\overset{1}{\cancel{3}\,\cancel{m}}}{\underset{2}{\cancel{10}\,\cancel{n}}} \times \frac{\overset{5}{\cancel{25}\,\cancel{n}}}{\underset{2}{\cancel{6}\,\cancel{m}}}$

$= \frac{1 \times 5}{2 \times 2}$

$= \frac{5}{4}$

Exercise 2.01 Simplifying algebraic expressions

1 Simplify each expression. Example 1

a $2x^2 + 3x + x^2 + 2x$ **b** $2ab + a + 5ab + b$ **c** $10 - 4m^2 + 6m^2 - 5$

d $3r + 4ar - 2ar - 7r$ **e** $y^2 - y + 3y - 5$ **f** $2a^2 - 4 - a^2 + 6$

g $k^2 - k - 3k - 1$ **h** $3de - 6ed + 2d$ **i** $p^2 - p - p^2 - 5p$

j $-5 + 7u - 10 - 3u$ **k** $t^2 + at + t^2 + 2at$ **l** $15x - 5 + 3x + 5$

2 Simplify each expression. Example 2

a $4ak \times 4am$ **b** $-3d \times 8cd$ **c** $\frac{1}{4}t^2w \times 10tw$

d $(5x^3)^2$ **e** $9n^2 \times \frac{2n^2}{3}$ **f** $7k^3p \times 2k^2p$

g $-6m \times (-6n)$ **h** $5e \times \frac{3e^2}{10}$ **i** $(2xy^2)^2$

j $10ab \times (-2ab)$ **k** $-4n^3 \times \frac{1}{4}n^3$ **l** $(-3k)^2$

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Example 3

3 Simplify each expression.

a $\frac{4m}{2m}$ **b** $\frac{14x^2y}{2xy}$ **c** $-15e^2 \div 3e^2$

d $10u^2t^3 \div 5ut^2$ **e** $\frac{3x}{x^3}$ **f** $\frac{27pr}{-9pq}$

g $\frac{12g^2h}{20gh}$ **h** $a^2 \div a^3$ **i** $\frac{15m^4n}{9m^2n^2}$

j $\frac{-4de}{20e}$ **k** $u \div 4u$ **l** $6wx^2 \div 24w^2$

4 Which of the following is the simplified expression for $\frac{2xy}{5} \times \frac{-3}{10y}$? Select **A**, **B**, **C** or **D**.

A $-\frac{3x}{25}$ **B** $-\frac{3x}{25y^2}$ **C** $-\frac{6x}{50}$ **D** $-\frac{3}{25y}$

Example 4

5 Simplify each expression.

a $\frac{9y}{4} \times 5y$ **b** $\frac{4a}{5} \times \frac{2h}{7}$ **c** $\frac{x}{3} \div \frac{x}{6}$

d $\frac{20r}{6} \div 4r$ **e** $\frac{4nw}{b} \times \frac{bn}{2w}$ **f** $\frac{3m}{6m} \times \frac{10n^2}{4n}$

g $\frac{4a}{5} \div \frac{2h}{7}$ **h** $\frac{2e}{5} \div \frac{3e}{8}$ **i** $\frac{2de}{7p} \times \frac{14p}{4e}$

6 Simplify each expression.

a $2k + 8 - k^2 - 4k$ **b** $ay^2 \times \frac{1}{2}y^2$ **c** $-5t \div (-20t)$

d $\frac{-32f^3g^2}{4f^2g}$ **e** $-3c^2 + 7c - 7c^2 + 6c$ **f** $4z + z + 8 - 4z^2$

g $-3a \times (-2a^2)$ **h** $\frac{24cd}{8de}$ **i** $2 - 4m^2 - 6m^2 - 2$

j $\frac{20n^3x}{10nx^2}$ **k** $4x^2 - 2xy + 3x + 4xy$ **l** $6a^4m^2 \div (-2a^2m^2)$

DID YOU KNOW?

Al Gebra is Arabic

In 825 CE, the Persian mathematician **al-Khwarizmi** wrote a book called *Hisab al-jabr w'al-muqabala* (or *The Science of Equations*). The Arabic word '*al-jabr*' meant the process of adding the same amount to both sides of an equation but, when it was translated into Latin, it was changed to 'algebra' and became the name of a whole branch of mathematics, not just equation-solving.

From al-Khwarizmi's name, we get the word *algorithm*. What is the meaning of *algorithm*?

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INVESTIGATION

NUMBER PATTERNS

10	11	12
17	18	19
24	25	26

1 Choose a month from any calendar and draw a box around any block of nine numbers. See the boxed numbers on the right for example.

2 Add together the numbers in any line of three numbers going through the centre of the box (row, column, or diagonal). For example, $10 + 18 + 26$.

3 Add another line of three numbers that goes through the centre of the box.

4 There are two more lines that go through the centre of the box. Find their sums as well.

5 What do you notice? Why does this pattern work? *Hint:* let the number in the top left-hand corner be x.

2.02 Expanding algebraic expressions

Expanding an **expression** means removing grouping symbols (brackets) by multiplying each term inside the grouping symbols by the term outside the grouping symbols.

WS Expanding algebra

Expanding algebraic expressions

$$a(b + c) = a \times (b + c) = ab + ac$$

EXAMPLE 5

Expand each expression.

a $5(2t - 3)$ **b** $2k(7 + 3k)$ **c** $-3(d^2 - 3d + 8)$

Solution

a $5(2t - 3) = 5 \times 2t - 5 \times 3$
$= 10t - 15$

Multiply each term inside the grouping symbols by the 5 (outside the grouping symbols).

b $2k(7 + 3k) = 2k \times 7 + 2k \times 3k$
$= 14k + 6k^2$

c $-3(d^2 - 3d + 8) = -3 \times d^2 + (-3) \times (-3d) + (-3) \times 8$
$= -3d^2 + 9d - 24$

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EXAMPLE 6

Expand and simplify each expression.

a $x(x-1)+4(x+1)$ **b** $2m(m-3)-(m-6)$ **c** $5u(2-y)+u(2y-9)$

Solution

a $x(x-1)+4(x+1) = x^2 - x + 4x + 4$
$= x^2 + 3x + 4$

Simplify by collecting the like terms $-x$ and $4x$

b $2m(m-3)-(m-6) = 2m^2 - 6m - m + 6$
$= 2m^2 - 7m + 6$

$-(m-6)$ means $-1(m-6)$

c $5u(2-y)+u(2y-9) = 10u - 5uy + 2uy - 9u$
$= u - 3uy$

Exercise 2.02 Expanding algebraic expressions

Example 5

1 Expand each expression.

a	$3(a+2)$	**b**	$5(3-2b)$	**c**	$-2(2a+1)$
d	$-6(b-2)$	**e**	$3x(x-2)$	**f**	$3p(p-a)$
g	$-4(2k+4)$	**h**	$2t(3-4t)$	**i**	$-d(d-5)$
j	$k(7-5k)$	**k**	$-9b(b-1)$	**l**	$2y(7x+4y)$

2 Expand each expression.

a	$-6n(4-n)$	**b**	$5x(rx+2r)$	**c**	$-(2a^2-4)$
d	$5b(a^2+3b-7)$	**e**	$-(x^2-4x+10)$	**f**	$3h(h-7e-4eh)$
g	$y(2y+3-y^2)$	**h**	$de(d^2-2+e^2)$	**i**	$-3v(-3av+v-2a)$

Example 6

3 Expand and simplify each expression.

a	$5(x+4)-2(x+3)$	**b**	$3(d-4)-2(d+5)$	**c**	$6(r+10)-4(r-5)$
d	$8(f+2)-(f+7)$	**e**	$3(2x-4)-5(3x+4)$	**f**	$6x(x+4)-3x(x-1)$
g	$3b(b+5)-b(b-8)$	**h**	$4w(w-7)-w(w+1)$	**i**	$6(k+p)+3(k+2p)$
j	$2(a-b)+2(b+a)$	**k**	$x(2v+4)-x(v+1)$	**l**	$-3(t+w)-2(2t-w)$
m	$e(3e+5)-(2e-e^2)$	**n**	$-2(a+3)+4(a-3)$	**o**	$p(p-q)-q(q-p)$

4 **a** Evaluate $7-5$ and $5-7$. How are the two answers related?
b Evaluate $4-10$ and $10-4$.
c Is $a-b$ always the same as $-(b-a)$? Can you prove it algebraically?

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INVESTIGATION

MENTAL MULTIPLICATION BY EXPANDING

Expanding is useful for multiplying numbers mentally without using a calculator, especially if one of the numbers is close to 10, 100 or 1000.

1 Study the following examples.

a $35 \times 11 = 35 \times (10 + 1)$ Think of 11 as 10 + 1

$= 35 \times 10 + 35 \times 1$ Expand

$= 350 + 35$ Simplify

$= 385$

b $43 \times 102 = 43 \times (100 + 2)$ Think of 102 as 100 + 2

$= 43 \times 100 + 43 \times 2$ Expand

$= 4300 + 86$ Simplify

$= 4386$

c $16 \times 8 = 16 \times (10 - 2)$ Think of 8 as 10 – 2

$= 16 \times 10 - 16 \times 2$ Expand

$= 160 - 32$ Simplify

$= 128$

2 Use expanding to multiply each pair of numbers mentally.

a 25×12 **b** 18×9 **c** 6×105

d 87×11 **e** 50×99 **f** 45×8

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2.03 Formulas

Substitution code puzzle

EXAMPLE 7

If $x = 4$, $y = 7$ and $z = -1$, evaluate each expression.

a $5x^2 + 2z$ **b** $\dfrac{z-8}{10-y}$ **c** $6(yz+5)$ **d** $\sqrt{3y-x-1}$

Solution

a $5x^2 + 2z = 5 \times 4^2 + 2 \times (-1)$
$= 78$

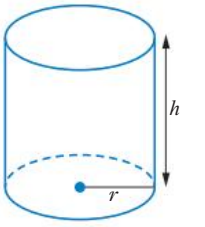
b $\dfrac{z-8}{10-y} = \dfrac{-1-8}{10-7}$
$= -3$

c $6(yz+5) = 6(7 \times (-1) + 5)$
$= 6 \times (-2)$
$= -12$

d $\sqrt{3y-x-1} = \sqrt{3 \times 7 - 4 - 1}$
$= \sqrt{16}$
$= 4$

Formulas

A **formula** is an algebraic **rule** that describes a mathematical relationship between variables. For example, the volume of a cylinder has the formula $V = \pi r^2 h$, where r is the radius of the cylinder's base and h is its perpendicular height.



Because the formula describes V, with V on the left-hand side of the '=' sign, we say that V is the **subject** of the formula.

EXAMPLE 8

If a principal, \$$P$, is invested at an interest rate of r **per annum** (where r is written as a decimal) then, after n years, it will grow to \$$A$, where A is given by the compound interest formula $A = P(1+r)^n$. Use the formula to calculate (to the nearest cent) the final amount of an investment of \$4000 after 6 years at 11% p.a.

Solution

$P = 4000$, $r = 11\% = 0.11$, $n = 6$

$A = P(1+r)^n$

$= 4000(1 + 0.11)^6$

$= 7481.658\,209\ldots$

≈ 7481.66

The final amount is \$7481.66.

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EXAMPLE 9

The correct **dosage** of a medicine for infants (babies up to 2 years) depends on the child's age or weight. There are different formulas for calculating this.

Fried's rule: $\text{Child dosage} = \dfrac{\text{age in months}}{150} \times \text{adult dosage}$

Young's rule: $\text{Child dosage} = \dfrac{\text{age in years}}{\text{age in years} + 12} \times \text{adult dosage}$

Ryan is 18 months old and weighs 13 kg. The adult dosage of a drug prescribed for him is 20 g. Calculate Ryan's dosage using:

a Fried's rule **b** Young's rule.

Solution

a $\text{Ryan's dosage} = \dfrac{18}{150} \times 20 \text{ g}$

$= 2.4 \text{ g}$

b $\text{Ryan's dosage} = \dfrac{1.5}{1.5 + 12} \times 20 \text{ g}$ Age = 18 months = 1.5 years

$= 2.2222\ldots \text{ g}$

$\approx 2.2 \text{ g}$

EXAMPLE 10

From a height of h metres above sea level, an observer can see a distance of d km to the horizon, where $d = 8\sqrt{\dfrac{h}{5}}$. What distance, correct to the nearest kilometre, can be seen from the top of Sydney Harbour Bridge, 134 m above sea level?

Solution

$h = 134$

$d = 8\sqrt{\dfrac{h}{5}}$

$= 8\sqrt{\dfrac{134}{5}}$

$= 41.414\,97\ldots$

≈ 41 km

A distance of 41 km can be seen from the top of Sydney Harbour Bridge.

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Exercise 2.03 Formulas

Example 7

1 If $a = -3$, $b = 10$ and $c = 6$, evaluate each expression.

a $b^2 - a^2$ **b** $\frac{b+c}{ac}$ **c** $\sqrt{c+3b}$ **d** $4(3a+9)$

e $8c + 4a$ **f** $\frac{2b}{5}$ **g** $b(b-4)$ **h** $\sqrt{\frac{9b}{c+4}}$

i $c^2 + c$ **j** $\frac{2ac}{3}$ **k** $\sqrt{a^2 + 7c - 2}$ **l** $(a-b)^2$

2 Calculate the volume, correct to two decimal places, of a cylinder with a base radius of 4.07 cm and perpendicular height of 11.58 cm, using the formula $V = \pi r^2 h$.

3 The temperature T (in °C) of the water in a kettle t minutes after it is switched on is given by the formula $T = 18t + 28$. Find the temperature of the water:

a 4 minutes after it is turned on

b $1\frac{1}{2}$ minutes after it is turned on

c when the kettle is first turned on.

4 The formula for converting Australian dollars ($A) to US dollars ($US) is US = 0.722A. Convert the following $A amounts to $US, correct to the nearest cent.

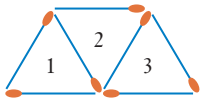
a $20.00 **b** $89.50 **c** $4800

5 If an object is moving with speed u m/s and acceleration a m/s^2, then its speed v m/s after t seconds is $v = u + at$.

a Which variable is the subject of the formula?

b Calculate the speed of a car after 5 seconds if its speed now is 6 m/s and it is accelerating at 2 m/s^2.

6 The number of matches, m, needed to make this pattern of triangles is $m = 2t + 1$, where t is the number of triangles in the pattern. How many matches are required to make:



a 8 triangles?

b 40 triangles?

c 150 triangles?

7 The volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$. Calculate, correct to one decimal place, the volume of a sphere with radius 14.5 cm.

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8 The time, T seconds, it takes a swing to go back and forth once is $T = 2\pi\sqrt{\frac{l}{g}}$ where l m is the length of the swing and g is the gravitational acceleration. Find T, correct to two decimal places, if $l = 2.35$ and $g = 10$.

Shutterstock.com/Stephen Rees

9 Use the compound interest formula $A = P(1 + r)^n$ to calculate the amount to which a principal of \$5600 will grow if invested at 9.4% p.a. for 5 years.

Example 8

10 The maximum distance, d m, a ball travels if thrown with speed v m/s, is $d = \frac{v^2}{g}$. Find the maximum distance if a ball is thrown at a speed of 11.5 m/s and $g = 9.8$.

11 The body mass index (BMI) of an adult is $B = \frac{m}{h^2}$, where m is the mass in kilograms and h is the height in metres. Zoe is 1.7 m tall and weighs 60 kg.

a What is the subject of the formula given above?

b Calculate Zoe's BMI, correct to one decimal place.

c If a BMI between 21 and 25 is an indication of good health, then how can Zoe improve her health?

12 The formula for converting Fahrenheit temperatures (°F) to Celsius (°C) is $C = \frac{5}{9}(F - 32)$. Convert each of the following temperatures to Celsius, correct to the nearest degree.

a New York 77°F

b Los Angeles 100°F

c Rio de Janeiro 59°F

d Normal body temperature 98.6°F

13 The formula for converting a speed of k km/h to metres per second (m/s) is $M = \frac{5k}{18}$.

A speed of 80 km/h is closest to which of the following speeds? Select **A**, **B**, **C** or **D**.

A 22 m/s **B** 32 m/s **C** 35 m/s **D** 47 m/s

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Example 9

14 Mia is two years old and weighs 11 kg. A nurse calculates her medicine dosage given the adult dosage is 900 mg per day. Calculate Mia's daily dosage to the nearest mg using:

a Fried's rule **b** Young's rule.

15 Clark's rule for children 2 years and over is:

$$\text{Child dosage} = \frac{\text{weight in kg}}{70} \times \text{adult dosage}$$

Ashleigh is 16 years old and weighs 50 kg. She takes a drug with a recommended adult dose of 1680 mg each day. Using Clark's rule, what would be Ashleigh's daily dosage? Select **A**, **B**, **C** or **D**.

A 1200 mg **B** 1800 mg **C** 300 mg **D** 336 mg

16 Cowling's rule for calculating a child's dosage is:

$$\text{Child dosage} = \frac{\text{age in years} + 1}{24} \times \text{adult dosage}$$

If the adult daily dosage of an antibiotic is 100 mg, use Cowling's rule to find the daily dosage, correct to one decimal place, for:

a Brittany, aged 2 **b** Patrick, aged 6 **c** Mercedes, aged $10\frac{1}{2}$.

17 Elena is $1\frac{1}{2}$ years old. If the adult dosage of a drug is 1000 mg per day, find, correct to the nearest mg, Elena's appropriate daily dosage using:

a Young's rule **b** Fried's rule.

Example 10

18 The distance, d km, an observer can see to the horizon from a height, h m, above sea level is $d = 8\sqrt{\frac{h}{5}}$. What distance (correct to the nearest kilometre) can a person see from the top of Sydney Tower at 305 m?

19 Liam earns a weekly wage of \$450 plus commission of 11% of the value of phone cards he sells in excess of \$900. His weekly pay is given by the formula $P = 450 + 0.11(V - 900)$, where V is the total value of the phone cards sold. How much will Liam earn for selling \$2310 worth of phone cards in a week?

20 The braking distance (in metres) of a bicycle travelling at a speed of V m/s is $d = \frac{V(V+1)}{2}$. Calculate the braking distance of a bicycle travelling at a speed of 6 m/s.

21 The surface area of a cylinder of radius, r, and height, h, is $S = 2\pi r(r + h)$. Calculate the surface area, correct to two decimal places, of a cylinder with radius 3 cm and height 8 cm.

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22 The speed, V m/s, required for a spacecraft to escape Earth's gravitational pull during take-off is $V = \sqrt{2gr}$ where g is 9.8 m/s^2 and r is the radius of Earth (6 378 000 m). Which of the following is closest to the escape speed of a spacecraft leaving Earth's atmosphere? Select **A**, **B**, **C** or **D**.

A 8840 m/s **B** 11 180 m/s **C** 12 500 m/s **D** 35 000 m/s

DID YOU KNOW?

The body mass index

The **body mass index (BMI)** is used by the World Health Organization (WHO) as an international standard of health and fitness for adults. The formula for BMI is $B = \frac{m}{h^2}$, where m is a person's mass in kilograms and h is the person's height in metres. This is a convenient measure that represents the health of an adult by a single value. The table describes a range of BMI scores.

BMI	Health status
Under 18.5	Underweight
18.5 to 24.9	Normal
25.0 to 29.9	Overweight
30 and above	Obese

According to this model, about 38% of Australian adults are overweight and 28% of Australian adults are obese, having a greater risk of heart disease, stroke, diabetes, high blood pressure and high cholesterol.

Because the BMI is an algebraic *model*, the WHO acknowledges that it has some limitations and inaccuracies. It does not take into account a person's frame size, muscle mass, bone density or distribution of body fat. For this reason, the BMI should not be applied to bodybuilders, athletes, children under 19, pregnant women, the frail and sedentary elderly, and Aboriginal, Pacific Island and Asian people.

- **Why is the BMI less accurate for measuring the levels of fitness of the types of people listed above?**
- **Would each of the types of people listed above score unusually high or unusually low on the BMI scale?**

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INVESTIGATION

GETTING THE RIGHT FORMULA

Listed below are 15 commonly used formulas. As a group or individual activity, select eight formulas and, for each one:

a describe what the formula is used for

b write the subject of the formula, and what it stands for

c describe what the other variables in the formula stand for.

1 $V = \frac{1}{3}Ah$

2 $A = 180(n-2)$

3 $c^2 = a^2 + b^2$

4 $A = \pi r^2$

5 $S = 2\pi r^2 + 2\pi rh$

6 $I = Prn$

7 $m = \frac{y_2 - y_1}{x_2 - x_1}$

8 $C = 2\pi r = \pi d$

9 $A = \frac{1}{2}xy$

10 $S = \frac{d}{t}$

11 $A = \frac{1}{2}(a+b)h$

12 $A = P(1+r)^n$

13 $A = s^2$

14 $V = \pi r^2 h$

15 $V = \frac{4}{3}\pi r^3$

Equations code puzzle

Solving equations by balancing

Solving equations by backtracking

Solving equations using diagrams

2.04 Solving equations

An **equation** contains an algebraic expression and an equals (=) sign. For example, $3x - 4$ is an **expression,** while $3x - 4 = -13$ is an **equation**. An equation is **solved** when the value of the variable (for example, x) is found that makes the equation true.

Solving equations

- Keep the equation balanced by doing the same operation on both sides.
- Aim to have the variable (for example, x) on one side of the equation and a number on the other side. For example, $x = 4$.

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EXAMPLE 11

Solve each equation.

a $3x - 4 = -13$ **b** $\frac{b}{2} + 7 = 1$ **c** $\frac{h-10}{4} = 3$

Solution

a

$3x - 4 = -13$

$3x - 4 + \mathbf{4} = -13 + \mathbf{4}$ Adding 4 to both sides

$3x = -9$ ← Remember: We are aiming to get x on its own.

$\frac{3x}{3} = \frac{-9}{3}$ Dividing both sides by 3

$x = -3$

Check by substituting $x = -3$ back into the original equation:

LHS $= 3(-3) - 4$ ← LHS means 'left-hand side'.

$= -13$

$=$ **RHS** ← RHS means 'right-hand side'.

b

$\frac{b}{2} + 7 = 1$

$\frac{b}{2} + 7 - \mathbf{7} = 1 - \mathbf{7}$ Subtracting 7 from both sides

$\frac{b}{2} = -6$

$\frac{b}{2} \times \mathbf{2} = -6 \times \mathbf{2}$ Multiplying both sides by 2

$b = -12$

Check by substituting $b = -12$ back into the original equation:

$\text{LHS} = \frac{-12}{2} + 7$

$= 1$

$= \text{RHS}$

c

$\frac{h-10}{4} = 3$

$\frac{h-10}{4} \times 4 = 3 \times 4$ Multiplying both sides by 4

$h - 10 = 12$

$h - 10 + \mathbf{10} = 12 + \mathbf{10}$ Adding 10 to both sides

$h = 22$

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EXAMPLE 12

Solve each equation. ← These equations need more steps to solve.

a $5y + 5 = 2y + 17$

b $\frac{2m-7}{3} = 6$

c $4(1 - 2t) = 16$

Solution

a $5y + 5 = 2y + 17$ ← Remember: We are aiming to get y on its own.

$5y + 5 - \mathbf{2y} = 2y + 17 - \mathbf{2y}$ Subtracting $2y$ from both sides to get all the y terms on the LHS

$3y + 5 = 17$

$3y + 5 - \mathbf{5} = 17 - \mathbf{5}$ Subtracting 5 from both sides to get all the numbers on the RHS

$3y = 12$

$\frac{3y}{3} = \frac{12}{3}$ Dividing both sides by 3

$y = 4$

Checking:

LHS $= 5(4) + 5$

$= 25$

RHS $= 2(4) + 17$

$= 25$

$=$ LHS

b $\frac{2m-7}{3} = 6$

$\frac{2m-7}{3} \times 3 = 6 \times 3$ Multiplying both sides by 3

$2m - 7 = 18$

$2m = 25$ Adding 7 to both sides

$m = \frac{25}{2}$ Dividing both sides by 2

$= 12\frac{1}{2}$

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Checking:

$$\text{LHS} = \frac{2\left(12\frac{1}{2}\right)-7}{3}$$
$$= \frac{18}{3}$$
$$= 6$$
$$= \text{RHS}$$

c $4(1-2t) = 16$

$4 - 8t = 16$ Expanding LHS first

$-8t = 12$ Subtracting 4 from both sides

$t = \frac{12}{-8}$ Dividing both sides by (–8)

$= -1\frac{1}{2}$

Exercise 2.04 Solving equations

1 Solve each equation. Example 11

a	$3d + 2 = 20$	**b**	$2p - 3 = 2$	**c**	$4u + 6 = 20$
d	$5a + 3 = -12$	**e**	$12b + 8 = 4$	**f**	$3 - 2a = -6$
g	$3m = m - 10$	**h**	$\frac{3h}{4} = 9$	**i**	$\frac{r-1}{6} = 2$
j	$-\frac{2x}{5} = 8$	**k**	$\frac{y+2}{-3} = 1$	**l**	$11 - 4n = 15$
m	$5y + 6 = 4y + 11$	**n**	$\frac{4c}{10} = 3$	**o**	$\frac{z}{3} - 11 = 9$

2 What is the solution to $2t - 4 = 10 + t$? Select **A**, **B**, **C** or **D**.

A $t = 3$ **B** $t = 6$ **C** $t = 4\frac{2}{3}$ **D** $t = 14$

3 Solve each equation. Example 12

a	$5k - 13 = 3k + 9$	**b**	$8e = 2(e - 6)$	**c**	$\frac{2f+7}{2} = 10$
d	$3(x - 2) = 45$	**e**	$\frac{w}{5} - 8 = 6$	**f**	$4(2d - 9) = -12$
g	$\frac{4n+7}{9} = 2$	**h**	$7u + 7 = 2u - 10$	**i**	$3p + 4 = 4p$
j	$\frac{5z+8}{6} = -3$	**k**	$\frac{8-2b}{2} = 7$	**l**	$6(2 - 3q) = -24$

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4 In which line was an error made in solving the following equation? Select **A**, **B** or **C**.

A Line 2 **B** Line 3 **C** Line 4

Line 1: $\frac{c-4}{8} + 2 = 6$

Line 2: $\frac{c-4}{8} = 8$

Line 3: $c - 4 = 64$

Line 4: $c = 68$

2.05 Formulas and equations

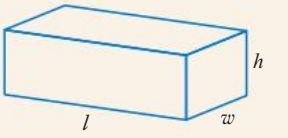
Working with formulas

Sometimes, after we **substitute** values into a formula, the result is an equation that must be solved.

EXAMPLE 13

The surface area of a rectangular prism of length l, width w and height h is $S = 2lw + 2lh + 2wh$.

If a rectangular prism with surface area 132 cm^2 has length 7 cm and width 3 cm, find its height.



Solution

$S = 132, l = 7, w = 3$

$S = 2lw + 2lh + 2wh$

$132 = (2 \times 7 \times 3) + (2 \times 7 \times h) + (2 \times 3 \times h)$ Substituting into formula

$132 = 42 + 14h + 6h$

$132 = 42 + 20h$ Collecting like terms

$90 = 20h$ Subtracting 42 from both sides

$h = \frac{90}{20}$ Dividing both sides by 20

$= 4\frac{1}{2}$

The height of the rectangular prism is $4\frac{1}{2}$ cm.

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EXAMPLE 14

The formula for converting Fahrenheit temperature (°F) to Celsius (°C) is $C = \frac{5}{9}(F - 32)$. Convert 100°C to °F.

Solution

When $C = 100$:

$$100 = \frac{5}{9}(F - 32)$$

$$\frac{5}{9}(F - 32) = 100$$

$5(F - 32) = 900$ Multiplying both sides by 9

$5F - 160 = 900$ Expanding the LHS

$5F = 1060$ Adding 160 to both sides

$F = \frac{1060}{5}$ Dividing both sides by 5

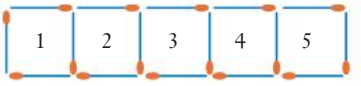
$= 212$

100°C = 212°F.

Exercise 2.05 Formulas and equations

Example 13

1 The number of matchsticks, m, needed to make a pattern of s squares is $m = 3s + 1$.



a How many matches are needed to make:
i 4 squares? **ii** 10 squares?

b How many squares can be made from:
i 22 matches? **ii** 55 matches?

2 Find the base length, b, of a triangle with area 90 cm^2 and perpendicular height 15 cm, given the area formula $A = \frac{1}{2}bh$.

3 The average speed of a moving object in metres per second is $s = \frac{d}{t}$, where d is the distance travelled in metres and t is the time taken in seconds.

a Find the distance travelled by a car in 20 seconds if its speed is 15 m/s.

b Find the time taken (to the nearest second) for a cyclist to travel 250 m if his speed is 4.2 m/s.

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Example 14

4 The formula for the area of a trapezium is $A = \frac{h}{2}(a + b)$, where a and b are the lengths of the parallel sides, and h is the distance between them. What is the length of one parallel side of a trapezium if the other parallel side is 7 m, the distance between them is 5 m and the area is 22.5 m^2?

5 The mean, M, of three numbers, x, y and z, is calculated using the formula $M = \frac{x + y + z}{3}$. If three numbers have a mean of 17 and two of the numbers are 10 and 20, find the third number.

6 The circumference of a circle with radius r is $C = 2\pi r$. If a circle has a circumference of 50.27 cm, find its radius, correct to the nearest centimetre.

7 If a principal, \$$P$, is invested at an interest rate of r per annum (where r is written as a decimal) then, after n years, it will grow to \$$A$, where A is given by the compound interest formula $A = P(1 + r)^n$. What principal needs to be invested at 11% p.a. for it to grow to \$6000 in 3 years? Express your answer to the nearest cent.

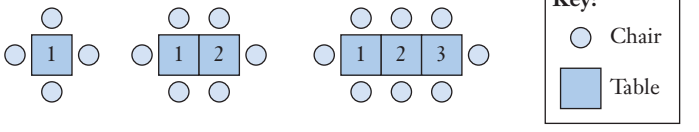
8 A kettle is boiled and the temperature, T°C, of the water after t minutes is $T = 18t + 28$. After how many minutes is the temperature:

a 64°C? **b** 92.8°C?

9 The formula for converting miles (M) to kilometres (K) is $K = 1.61M$.
Convert each of the following distances to miles, correct to two decimal places.

a 5 km **b** 1.5 km

10 The number of chairs, c, that can be seated around t square tables is $c = 2t + 2$.



a How many chairs can be seated around:
i 5 tables? **ii** 12 tables?

b How many square tables are required to seat:
i 24 chairs? **ii** 40 chairs?

11 Rhianna earns a weekly wage of \$540 plus commission of 12% on the value of cosmetics she sells in excess of \$2000. Her total pay, P, is given by the formula $P = 540 + 0.12\,(V - 2000)$, where V is the value of the cosmetics sold that week. What was the value of the cosmetics Rhianna sold if her total pay was \$852?

12 The surface area of a rectangular prism of length l, width w and height h is $S = 2lw + 2lh + 2wh$. If a rectangular prism has length 4 cm, width 2 cm and surface area 58 cm^2, what is its height? Select **A**, **B**, **C** or **D**.

A 3.5 cm **B** $4\frac{1}{6}$ cm **C** 5.5 cm **D** 7.25 cm

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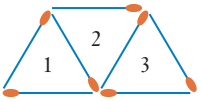
13 The formula for converting k kilometres to M miles is $M = \frac{5k}{8}$. Convert 12 miles to kilometres.

14 The number of matchsticks, m, needed to make this pattern of triangles is $m = 2t + 1$, where t is the number of triangles in the pattern.



How many triangles can be made with:

a 37 matches? **b** 55 matches?

15 For babysitting, Kirstie charges \$18 for the first hour and \$14 for each hour after that. This charge, \$$C$, can be expressed using the formula $C = 18 + 14(h - 1)$, where h is the number of hours worked. If Kirstie earned \$46 for babysitting one night, for how many hours did she work?

16 Mohammed works in a high-rise office building on the 32nd floor. At lunchtime, he travels down in the lift at a speed of two floors per second. This is described by the formula $F = 32 - 2t$ where F is the floor being passed by the lift and t is the time in seconds.

a Which floor is Mohammed passing after:

i 8 seconds? **ii** 14 seconds?

b After how many seconds will Mohammed pass the:

i 18th floor? **ii** 8th floor?

17 According to one theory, the recommended nightly hours of sleep for a child is $S = 8 + \frac{18 - a}{2}$, where a is the age of the child in years. What is the age of a child who requires $14\frac{1}{2}$ hours of sleep each night? Select **A**, **B**, **C** or **D**.

A 2.5 years **B** 3 years **C** 3.5 years **D** 5 years

18 If an object has an initial speed u m/s and acceleration a m/s^2 after t seconds, its final speed (in m/s) is given by the formula $v = u + at$. Find the acceleration of an object if its initial speed is 8 m/s and its final speed after 12 seconds is 44 m/s.

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Changing the subject of a formula

2.06 Changing the subject of a formula

In the formula $v = u + at$, v is called the **subject** of the formula because it is on the LHS. To change the subject of a formula to another variable, use the same rules as for solving an equation. The answer is not a number but an algebraic equation (another formula).

EXAMPLE 15

If a moving object has initial speed u m/s and acceleration a m/s^2, its final speed v m/s is given by the formula $v = u + at$. Change the subject of this formula to:

a u **b** t.

Solution

a $v = u + at$

$u + at = v$ Swap sides so new subject u appears on LHS

$u = v - at$ Subtract at from both sides

b $v = u + at$

$u + at = v$ Swap sides so new subject t appears on LHS

$at = v - u$ Subtract u from both sides

$t = \dfrac{v-u}{a}$ Divide both sides by a

EXAMPLE 16

The volume of a cylinder is given by the formula $V = \pi r^2 h$, where r is the radius of the circular base and h is the height. Make r the subject of the formula.

Solution

$V = \dfrac{1}{3}\pi r^2 h$

$\dfrac{1}{3}\pi r^2 h = V$ Swap sides so r appears on the LHS.

$\pi r^2 h = 3V$ Multiply both sides by 3.

$r^2 = \dfrac{3V}{\pi h}$ Divide both sides by πh.

$r = \pm\sqrt{\dfrac{3V}{\pi h}}$ Take the square root of both sides.

$= \sqrt{\dfrac{3V}{\pi h}}$ Positive square root because radius, r, is positive.

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Exercise 2.06 Changing the subject of a formula

1 Make x the subject of each formula.

Example 15

a $y = 2x + 4$ **b** $T = 3x - 7$ **c** $d = \frac{x+1}{3}$

d $p = 18 - x$ **e** $k = 4x + r$ **f** $C = n + \frac{x}{2}$

g $S = 10bx$ **h** $V = \frac{x-5}{6}$ **i** $z = 12 - ax$

2 If an object travels a distance, d, in time, t, its average speed, S, is given by the formula $S = \frac{d}{t}$. Change the subject of the formula to:

a d **b** t.

3 The angle sum of a shape with n sides is $A°$, where $A = 180(n - 2)$.

a Use the formula to find the angle sum of a shape with 12 sides.

b Make n the subject of the formula.

c If the angle sum of a polygon is 1260°, how many sides does it have?

4 The volume of a **pyramid** has the formula $V = \frac{1}{3}Ah$, where A is the area of the base and h is the perpendicular height. Which of the following is the correct formula for A? Select **A**, **B**, **C** or **D**.

A $A = \frac{1}{3}Vh$ **B** $A = \frac{3}{Vh}$ **C** $A = 3Vh$ **D** $A = \frac{3V}{h}$

5 The area of a trapezium has the formula $A = \frac{h}{2}(a + b)$. Change the subject to:

a h **b** b.

6 Change the subject of each formula to the variable shown in brackets.

Example 16

a $A = \pi r^2$ $[r]$ **b** $y = mx + c$ $[m]$ **c** $v^2 = u^2 + 2as$ $[s]$

d $z = \frac{x-m}{s}$ $[x]$ **e** $E = mc^2$ $[c]$ **f** $K = \frac{1}{2}mv^2$ $[v]$

g $A = P(1 + r)^n$ $[P]$ **h** $x = \sqrt{6y + 3}$ $[y]$ **i** $A = \frac{1}{2}bh$ $[h]$

j $T = 2\pi\sqrt{\frac{l}{g}}$ $[l]$ **k** $V = IR - E$ $[R]$ **l** $e = ir + \frac{Q}{C}$ $[Q]$

m $s = ut + \frac{1}{2}at^2$ $[a]$ **n** $K = \frac{m+d}{w}$ $[d]$

7 The body mass index of an adult is given by the formula $B = \frac{m}{h^2}$, where m is mass in kilograms and h is in metres. Change the subject of the formula to h and hence find, to 2 decimal places, the height of a person with a body mass index of 25 and a mass of 60 kg.

8 The volume of a cone is $V = \frac{1}{3}\pi r^2 h$, where r is the radius of the base and h is the height. Show that this formula can be rewritten as $h = \frac{3V}{\pi r^2}$ and hence find (to the nearest centimetre) the height of a cone with volume 3539.53 cm^3 and base radius 13 cm.

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9 Make F the subject of the formula $C = \frac{5}{9}(F - 32)$ and hence find F when:

a $C = 40$ **b** $C = 100$.

10 Make P the subject of the formula $M = \frac{10}{\pi hP}$, then find the value of P, correct to two significant figures, if $M = 9.8$ and $h = 0.27$.

SAMPLE HSC PROBLEM

Paula the plumber charges according to the formula $C = 60 + 32(h - 1)$, where \$$C$ is the charge and h is the number of hours worked.

a How much does Paula charge for working $2\frac{1}{2}$ hours?

b Make h the subject of the formula.

c Find the number of hours Paula worked if she charged \$172.

Study tip

Topic summaries and mind maps

Summarise each topic once you have completed it, to create useful study notes for revising the course, especially before exams. Use a notebook or folder for listing the important ideas, formulas, terminology and skills of each topic. Educational research shows that effective learning takes place when we rewrite learned knowledge in our own words.

A good topic summary runs for two to four pages. It is a condensed, personalised version of your course notes. This is your interpretation of a topic, so include your own comments, symbols, diagrams, observations and reminders. Highlight important facts using boxes and include a glossary of key words and phrases. (There is a glossary at the back of this book.)

A mind map is a topic summary in graphic form, with boxes, branches and arrows showing the connections between the main ideas of the topic. (See the next page for an example.) The topic name is written at the centre of the map, with branches leading off to key concepts or subheadings. Smaller branches hang off these branches, listing important details and formulas. Mind maps are powerful because they present an overview of a topic on one large sheet of paper. Visual learners absorb and recall information better using mind maps.

When compiling a topic summary, use your class notes and this textbook, especially the front and back sections of each chapter. Ask your teacher for a copy of the course syllabus or the school's teaching program, which lists the knowledge and skills of every topic in dot point form.

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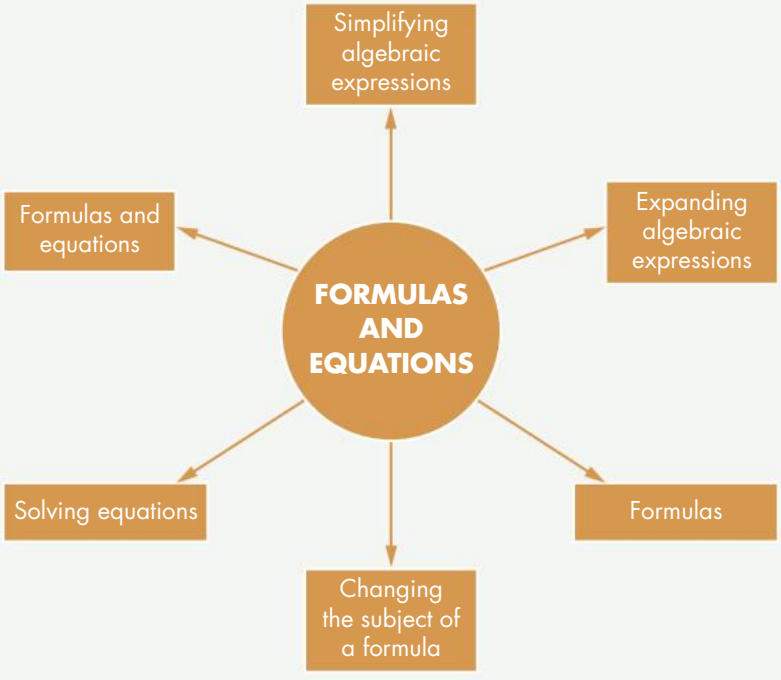
2. CHAPTER SUMMARY

This chapter, Formulas and equations, revised basic and extended algebra skills. Make sure you master the algebraic techniques required to:

Algebra review

- simplify expressions
- expand expressions
- solve equations
- work with formulas
- change the subject of a formula.

Make a summary of this topic. Use the chapter outline at the beginning of this chapter and the mind map below as a guide. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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2. TEST YOURSELF

Exercise 2.01

1 Simplify each expression.

a $5ut + 2t^2 - t^2 + 4ut$

b $3k^2 \times 5k$

c $-9d \times \frac{2d}{3}$

d $8p^2 \times 4p^3$

e $-1 + 4h + 8 - 10h$

f $\frac{16r^3}{2r}$

g $(-3d^2)^3$

h $\frac{9n^2}{15n^2}$

i $-3v^2w^2 \div 21vw$

j $10x^2 + 7x - 2x^2 + x$

k $\frac{24bc}{8b^2}$

l $\frac{-a}{a^3}$

m $\frac{48r}{6} \div 4r$

n $\frac{4y}{3} \times \frac{5v}{10}$

o $\frac{10x}{18p} \times \frac{3p}{20}$

p $\frac{3y}{2a} \div \frac{9dy}{10d}$

Exercise 2.02

2 Expand each expression.

a $5(2x - 4)$

b $-3(a + 7)$

c $4(12t - y)$

d $-9(r^2 + 2w)$

e $8mn(m - n)$

f $-2d(4d - d^2)$

Exercise 2.02

3 Expand and simplify each expression.

a $3(4x + 1) + 2(x - 2)$

b $2n(n - 1) + (n - 1)$

c $6(2 - d) - 4(d - 3)$

d $p(p + 4) - p(p + 8)$

e $3(4u + 5) - (u + 7)$

f $h(5h - 1) + 3h(h + 9)$

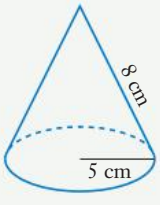
Exercise 2.03

4 If $p = 4$, $q = -5$ and $r = 20$, then evaluate each expression.

a $3p^2 + 4r$ **b** $\frac{7r}{q}$ **c** pqr **d** $\sqrt{p(r-q)}$

Exercise 2.03

5 The surface area of a cone is given by the formula $S = \pi r(r + s)$, where r is the radius of the base and s is the slant height of the cone. Find, correct to two decimal places, the surface area of a cone with base radius 5 cm and slant height 8 cm.



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6 Brett earns a weekly wage of \$480 plus a commission of 10% of the value of the insurance plans he sells in excess of \$1200. His total weekly pay, \$$P$, is given by the formula $P = 480 + \frac{V - 1200}{10}$, where V is the value of the insurance plans sold. Calculate Brett's pay for a week in which he sold \$3400 worth of insurance plans.

Exercise 2.03

7 Solve each equation.

a $5p - 4 = 21$ **b** $-2a + 6 = 8$ **c** $\frac{b-3}{2} = -6$

d $23 - 8r = 19$ **e** $\frac{4n}{5} = 11$ **f** $\frac{r}{3} + 7 = 1$

g $\frac{20 - 4n}{4} = 7$ **h** $3t + 13 = t - 12$ **i** $5(2g - 4) = -30$

Exercise 2.04

8 If an object is travelling with initial speed u m/s, accelerating at a rate of a m/s^2, and covers a distance s m, then its final speed v m/s follows the rule $v^2 = u^2 + 2as$. Calculate the distance travelled by a car whose speed increases from 11 m/s to 28 m/s with an acceleration of 3 m/s^2.

Exercise 2.05

9 According to one theory, the formula that links the surrounding air temperature, T°C, to the number of chirps per minute, C, made by a cricket at night during summer is $T = \frac{C}{8} + 3$. How many chirps per minute are made by the cricket when the temperature is 13°C?

Exercise 2.05

10 The average blood pressure, P, of a person aged y years, measured in millimetres of mercury (mmHg), is given by the formula $P = 110 + \frac{y}{2}$. Make y the subject of this formula and use it to find the age of a person whose blood pressure is 124 mmHg.

Exercise 2.06

Qz Chapter quiz

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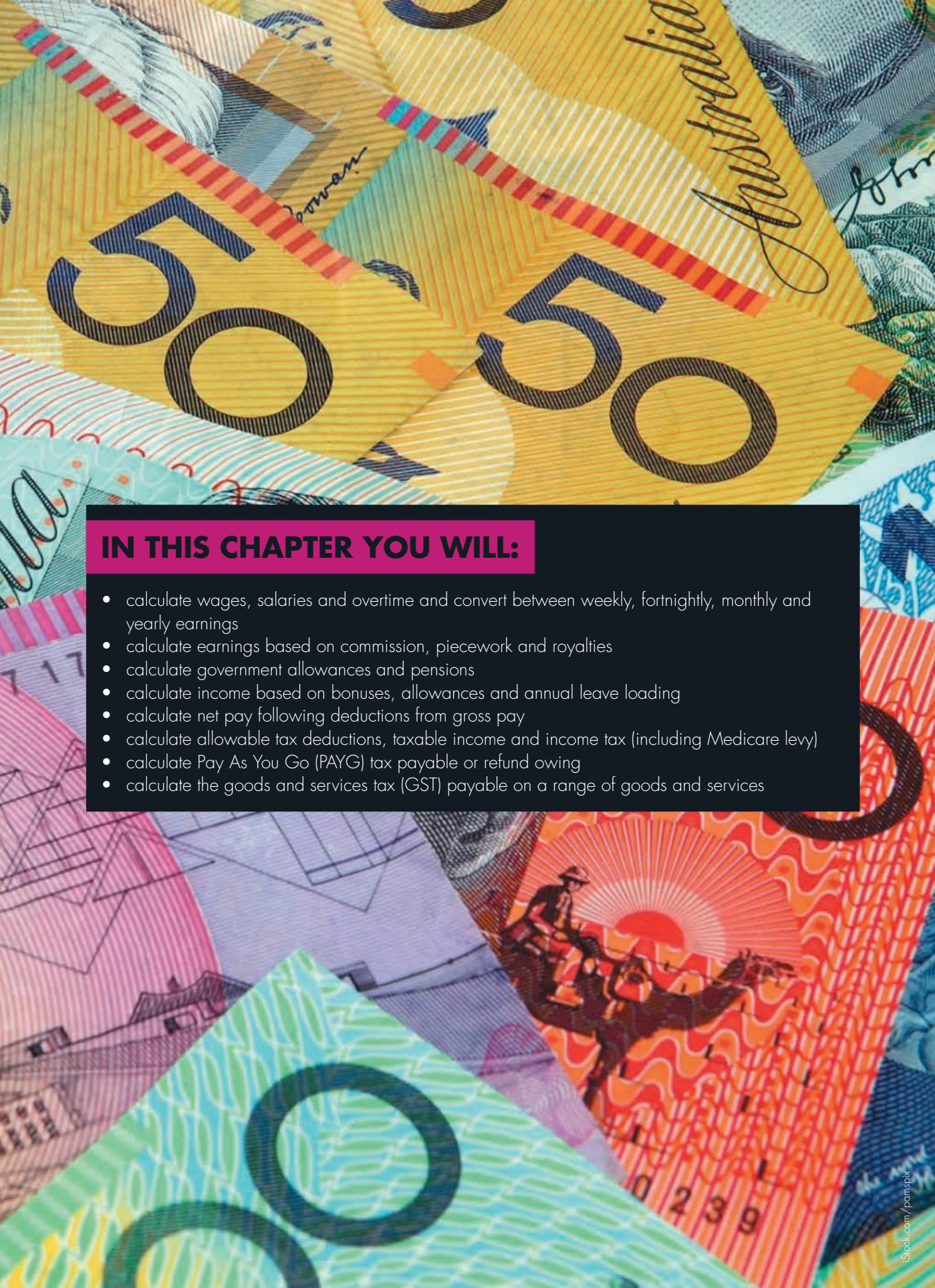
3.

EARNING MONEY AND TAXATION

The Australian government introduced the goods and services tax (GST) in 2000, which brought us into line with other countries that impose a tax on spending. A tax on spending is a fairer way of collecting revenue because it is a 'user pays' system. Before the introduction of the GST, the government raised most of its revenue from personal income tax, which is a tax on earnings.

CHAPTER OUTLINE

F1.2 3.01 Wages, salaries and overtime
F1.2 3.02 Commission, piecework and royalties
F1.2 3.03 Bonuses, allowances and annual leave loading
F1.2 3.04 Government allowances and pensions
F1.2 3.05 Gross pay and net pay
F1.2 3.06 Income tax and Medicare levy
F1.2 3.07 PAYG and tax returns
F1.1 3.08 GST and VAT



IN THIS CHAPTER YOU WILL:

- calculate wages, salaries and overtime and convert between weekly, fortnightly, monthly and yearly earnings
- calculate earnings based on commission, piecework and royalties
- calculate government allowances and pensions
- calculate income based on bonuses, allowances and annual leave loading
- calculate net pay following deductions from gross pay
- calculate allowable tax deductions, taxable income and income tax (including Medicare levy)
- calculate Pay As You Go (PAYG) tax payable or refund owing
- calculate the goods and services tax (GST) payable on a range of goods and services

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TERMINOLOGY

allowable tax deduction
allowance
annual leave loading
bonus
commission
double time
expense
goods and services tax (GST)
gross income
gross pay
income
income tax
Medicare levy
net pay
overtime
Pay As You Go (PAYG) tax
piecework
royalties
salary
superannuation
tax refund
taxable income
time-and-a-half
value-added tax (VAT)
wage

SkillCheck

Assignment 3

Percentage calculations

Mental percentages

1 Find the cost of:
- **a** 5 calculators @ $17.60 each
- **b** 1.5 kg carrots @ $3.10 per kg
- **c** 1400 protractors @ $1.15 each.

2 Calculate, correct to the nearest cent:
- **a** 34% of $42 568
- **b** $153 000 ÷ 52.18
- **c** 1.5% of $85 629

3 To the nearest whole number, what percentage is $2360 of $36 580?

4 Evaluate:
- **a** $36 \times 22.5 + 4 \times 22.5 + 3 \times 22.5 \times 2$
- **b** $58\,000 + 0.45 \times (250\,000 - 180\,000)$

5 How many hours and minutes are there between:
- **a** 8.00 a.m. and 12.00 p.m.?
- **b** 9.00 a.m. and 2.30 p.m.?
- **c** 7.30 a.m. and 5.15 p.m.?

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3.01 Wages, salaries and overtime

Wages and salaries

Wages

Wages and overtime

A person's **wage** is calculated on the number of hours they worked in a given period, with **overtime** pay being paid at a higher rate for working beyond normal hours. Types of overtime are shown in the table on the right. People who earn a wage include waiters, mechanics and receptionists.

Type of overtime	Meaning
Time-and-a-half	1.5 × normal rate
Double time	2 × normal rate
Double time-and-a-half	2.5 × normal rate
Triple time	3 × normal rate

EXAMPLE 1

This week, Christy worked at a childcare centre for 37 hours at $24.70 per hour plus 6 hours overtime at time-and-a-half. Calculate her earnings for the week.

Normal pay $= 37 \times \$24.70 = \$913.90$

Overtime pay $= 6 \times 1.5 \times \$24.70 = \$222.30$

Total earnings $= \$913.90 + \$222.30 = \$1136.20$

Overtime

EXAMPLE 2

Ian works as a bartender at an RSL club. His timesheet for three days he worked in April is shown below.

Day	Start time	Finish time
Tuesday	3.00 p.m.	10.00 p.m.
Wednesday (Anzac Day holiday)	9.00 a.m.	2.00 p.m.
Thursday	12.00 noon	7.00 p.m.

The rate for bartenders is $18.17 per hour between 8.30 a.m. and 6.00 p.m., time-and-a-half between 6.00 p.m. and 11.30 p.m., and double time on public holidays. Calculate Ian's pay for the three days he worked.

Solution

Day	Normal hours	Time-and-a-half	Double time
Tuesday	3	4	0
Anzac Day	0	0	5
Thursday	6	1	0
Total hours	9	5	5

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Normal pay $= 9 \times \$18.17 = \$163.53$

Time-and-a-half pay $= 5 \times 1.5 \times \$18.17 = \$136.275$

Double-time pay $= 5 \times 2 \times \$18.17 = \$181.70$

Total pay $= \$163.53 + \$136.275 + \$181.70$

$= \$481.505$

$\approx \$481.51$

Salaries

A **salary** is a fixed amount of annual pay (yearly pay), paid weekly, **fortnightly** or monthly. People who earn a salary include teachers, website designers and police officers.

To convert a yearly salary to a weekly amount, divide by 52.18 because one year is not exactly 52 weeks.

Number of weeks in an 'average year' $= 365\frac{1}{4} \div 7$ 1 week = 7 days

$= 52.1785$

≈ 52.18

Every four years, a leap year has 366 days so an 'average year' has $365\frac{1}{4}$ days.

Time conversions

1 year = 12 months

1 fortnight = 2 weeks

1 year = 52.18 weeks for salary earners

1 year = 52 weeks for wage earners (because they are paid 52 times in a year)

Note: 1 month ≠ 4 weeks. To convert between weeks and months, convert to years first.

EXAMPLE 3

Siobhan earns a salary of $98 025 as an IT consultant. To the nearest cent, how much is she paid:

a monthly? **b** fortnightly?

Solution

a Monthly pay $= \$98\,025 \div 12$

$= \$8168.75$

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b Fortnightly pay = $98025 ÷ 52.18 × 2 (1 year = 52.18 weeks)

= $3757.1866

≈ $3757.19

Answers to money problems should be rounded to the nearest cent, not 5 cents. Only cash payments are rounded to the nearest 5 cents.

Exercise 3.01 Wages, salaries and overtime

1 Ken is a gardener who works from 6.30 a.m. to 5.00 p.m., Monday to Friday, and earns $27.45 per hour. Calculate his weekly pay, correct to the nearest cent.

2 Sandra is paid $28.50 per hour as a computer operator. She works 9 hours per day, Monday to Friday, and 5 hours on Saturday, for which she is paid time-and-a-half. Calculate her weekly wage.

Example 1

3 For an annual salary of $120 000, what is the weekly amount of pay? Select **A**, **B**, **C** or **D**.

A $2307.69 **B** $2299.73 **C** $2307.70 **D** $2299.75

4 If a labourer earns a weekly wage of $936.70, how much does he earn in a year? Select **A**, **B**, **C** or **D**.

A $48 708.00 **B** $48 877.01 **C** $48 708.40 **D** $48 877.40

5 Jacinta, an Internet website designer, works a 38-hour week and earns $3115.35 every fortnight. Calculate her earnings:

a per year **b** per week **c** per hour.

6 Liam is a trainee nurse who works 36 hours at an ordinary rate of $27.44 and 10 hours at time-and-a-half. Calculate his total pay.

7 A childcare worker earns $948.48 per week. How much will she earn in:

a a year? **b** a month?

8 On a long weekend, a waitress is paid time-and-a-half on Sunday and double time on Monday. How much does she earn for working 9 hours on each of the days if her normal wage rate is $22.54 per hour?

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Example 2

9 A chef earns $41.20 per hour, but is paid time-and-a-half for working after 8.00 p.m. on weekdays and any time on weekends. The timesheet below shows the times he worked last week. Copy and complete the table, then calculate the chef's total pay for the week.

Day	Start time	Finish time	Normal hours	Overtime hours
Monday	12.00 noon	6.00 p.m.		
Tuesday	12.00 noon	6:00 p.m.		
Wednesday	12:00 noon	7:00 p.m.		
Thursday	12:00 noon	10:00 p.m.		
Friday	12:00 noon	12:00 midnight		
Saturday	4:00 p.m.	12:00 midnight		
Sunday	4:00 p.m.	10:00 p.m.		
		Total hours		

10 Dom is a personal assistant who earns $32.30 per hour for a 35-hour week, time-and-a-half for the first 4 hours of overtime, and double time thereafter. Calculate his total earnings for a week in which he worked from 8.30 a.m. to 5.30 p.m. from Tuesday to Saturday.

Example 3

11 A Member of Parliament earns a salary of $205 130. How much is this if paid:

a fortnightly? **b** monthly? **c** weekly?

12 A law clerk earns a salary of $83 215. How much does she earn:

a a month? **b** a fortnight? **c** a weekday?

13 Rachna is a computer systems engineer who earns a salary of $104 235. If she works a 42-hour week, calculate how much she earns:

a per hour **b** per fortnight **c** per month.

14 The table below shows the hours worked by a hotel porter. If he earns $23.78 per hour from 0600 to 1900 (in 24-hour time) and time-and-a-half otherwise, calculate his weekly pay.

Day	Start time	Finish time
Tuesday	0630	1400
Wednesday	0800	1530
Friday	1500	2330
Sunday	0500	1330

15 A newspaper printer worked for 33 hours at $26.72 per hour during the week before Good Friday, then for 6 hours at double-time-and-a-half on Good Friday. Calculate the printer's total pay.

16 A survey field-hand earns $37.18 per hour, working from 7.00 a.m. to 4.30 p.m. every weekday. Calculate how much he is paid:

a a day **b** a week **c** a month.

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17 Which person earns the most? Select **A**, **B**, **C** or **D**.

- **A** Ali: $28.70 per hour for a 35-hour week
- **B** Boun: $984 per week
- **C** Carlos: $4384 per month
- **D** Dimitri: $51 600 salary

18 Simon is paid $205.72 each day for working at a bakery from 4.30 a.m. to 1.00 p.m. If he works 6 days per week, how much does he earn:

a per hour? **b** per week? **c** per year?

19 Each day, a storeman earns $28.10 per hour for 9 hours and time-and-a-half for any extra hours. Last week, he worked the following number of hours each day: 9, 8, 10, 14 and 12.5. Calculate his total pay for the week.

20 Alex worked 32 hours at the normal rate and 4 hours at time-and-a-half, earning a total of $909.20. Find her normal rate of pay per hour.

TECHNOLOGY

Payroll calculations

Enter the following data into a spreadsheet and use appropriate formulas to calculate the normal and overtime pays.

	A	B	C	D
1		**Number of hours**	**Pay rate per hour**	**Pay**
2	Normal	37	$18.65	
3	Time-and-a-half	3.5		
4	Double time	1		
5	**Total hours**		**Total pay**	

Use your spreadsheet to check the answers for Questions **8**, **9**, **10**, **14**, **19** and **20** in Exercise 3.01.

3.02 Commission, piecework and royalties

Some workers are not paid by the amount of time they work, but by the number of items the make or process.

- **Commission** is **income** for selling items, calculated as a percentage of the value of items sold. A real estate agent earns commission on each house sold.
- **Piecework** is income for making or processing items, calculated as a fixed rate per item. Fruit pickers and garment makers earn money this way.
- **Royalties** are income based on the number of copies sold. Musicians and authors earn royalties from sales of their music and books.

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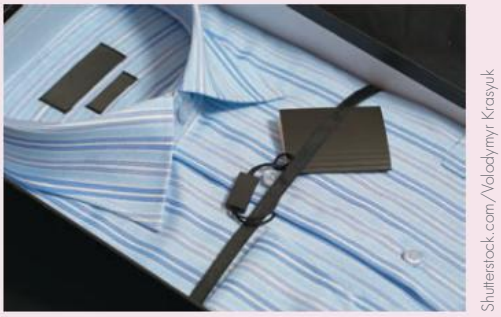
EXAMPLE 4

Josh is paid 73c for each shirt folded and boxed. Last week he processed 1440 shirts. How much did he earn?



Shutterstock.com/Volodymyr Krasyuk

Solution

Earnings = $1440 \times \$0.73 = \$1051.20$

Percentage calculations

Mental percentages

EXAMPLE 5

Ally earns a **retainer** of \$415 per week plus commission of 8.5% on the value of Wonder mops sold.

A **retainer** is a fixed amount earned that does not depend on the number of items sold.

a To the nearest cent, how much does Ally earn for selling \$1595 worth of mops in one week?

b If Ally earned \$700 in a week, what was the value of mops sold that week, to the nearest dollar?

Solution

a Earnings = $\$415 + 8.5\% \times \$1595$

$= \$550.575$

$\approx \$550.58$

b Let $\$V$ = value of mops sold

Earnings = retainer + $8.5\% \times V$

$700 = 415 + 0.085V$

$285 = 0.085V$

$$V = \frac{285}{0.0085}$$

$= 3352.9411 \ldots$

≈ 3352.94

The value of the mops sold is \$3353.94.

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Exercise 3.02 Commission, piecework and royalties

Example 4

1 Brett is paid $4.30 for each bucket of cherries he picks. If he picked 31 buckets of cherries, which of the following is the amount he earned, correct to the nearest 5c? Select **A**, **B**, **C** or **D**.

A $133.30 **B** $103.00 **C** $133.00 **D** $129.00

2 Danny is paid 13.5c for each envelope he folds and addresses. If he can process 14 envelopes in 5 minutes:

a how many envelopes can he process in 1 hour?

b how much does he earn in $2\frac{1}{2}$ hours?

3 Leila is paid $1.05 for each garment she irons. If a basket contains 18 items and she irons five basketsful, which of the following is the amount she earns? Select **A**, **B**, **C** or **D**.

A $18.90 **B** $63 **C** $90 **D** $94.50

4 A stockbroker receives a commission of 2% of the selling price of shares.

a If he sold 350 shares at $4.30 each, calculate his commission.

b What was the value of the shares he sold if he earned $135 commission?

5 A car salesman's commission is 4% for the first $30 000 worth of sales and 2.5% for the rest. If he is also paid a retainer of $420, calculate his total earnings in a week in which he sold $350 000 worth of cars.

6 A real estate agent charges the following commission for selling properties, based on this sliding scale of property prices:

- 2% for the first $500 000
- 1.5% for the next $250 000
- 0.75% for the remaining amount.

What commission will the real estate agent earn for selling:

a an apartment for $342 400?

b a villa for $620 000?

c a house for $1.1 million?

7 A songwriter earns a royalty of 47c every time her song is played on the radio. If, in one week, her song was played 434 times on radio stations, calculate her royalty payment for that week.

8 An inventor is paid 1.8% of the total sales of a can-opener that uses his original design. How much does the inventor earn if 8400 can openers are sold at $8.95 each?

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9 Taylor, a singer-songwriter, earns 8% of each song downloaded and 12.5% of the sales of her latest album. In 3 months, 1 650 000 of her songs were downloaded at $2.59 each and 836 254 copies of her album were sold at $16.99 each. Calculate:

a the value of Taylor's royalties from song downloads

b the total value of Taylor's album sales

c Taylor's total royalties from album sales and song downloads.

Example 5

10 Truong earns 11 cents for every advertising brochure he delivers to letterboxes.

a How much does Truong earn in a day in which he delivers 1329 brochures?

b If Truong earned $275 one day, how many brochures did he deliver?

11 Nick earns $7.24 for every car he vacuums. He vacuums an average of 3.6 cars per hour. How much will he earn in a $6\frac{1}{2}$-hour shift?

12 Libby earns 18% commission for selling cosmetics door-to-door. What value of cosmetics did she sell this week if she earned $648?

13 An author earns 13% in royalties on the sale of her novels. If 34 700 copies of her novels sold for $32.95 each, calculate the author's royalty payment.

14 Pranathi earns $15.40 for every dress she sews. If she can make 7.5 dresses per day, calculate her earnings for a 5-day week.

15 Jules is paid $4.62 for every toy he assembles. In a week, she receives an extra $1.20 for each toy he has assembled after the first 150. Calculate his total earnings if she assembles 216 toys in a week.

16 Debbie earns 15% commission on all the storage containers she sells. If she earned $251.70 this week, what was the total value of the containers she sold?

17 An agent earned, as his commission, $2024 of the actor's fee in a TV commercial. What percentage commission does this represent if the actor's fee was $23 000?

18 Grant is paid 15.5 cents for every newspaper he delivers. How many newspapers did he deliver if he was paid $214.21?

19 At Christmas time, Shane earns $1.65 for every item he gift-wraps.

a Calculate Shane's earnings for wrapping 114 gifts in one day.

b How many gifts did Shane wrap on the next day if his earnings were $224.40?

20 Salma earns $8.40 each time she taste-tests a new brand of food. How many tastings must she perform to earn over $500?

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21 Travis is paid the following commission rates for selling mobile phone plans:

- 4.8% for the first $3000
- 6% for the next $3000
- 7.5% for the remaining amount.

Calculate Travis' commission for selling plans amounting to $8758 in value.

3.03 Bonuses, allowances and annual leave loading

Bonuses, allowances and annual leave loading are examples of extra pay.

A **bonus** is paid to employees who produce work of a high standard or volume, or for meeting an important quota, goal or deadline. The owners of some businesses give their employees a Christmas bonus to reward them for a productive year's work.

An **allowance** is paid to employees for work-related **expenses** (such as travelling, tools or laundry), or who work under difficult or dangerous conditions (such as in wet weather, in isolated areas or great heights).

Annual leave loading (also called **holiday loading**) is extra pay given during annual leave. It is usually paid at a rate of 17.5% of 4 weeks normal pay.

EXAMPLE 6

A construction worker in an isolated area earns a weekly wage of $1280.25 plus a site allowance of $104.80 per week, a meal allowance of $16.20 per day and a height allowance of $5.60 per hour. If she works 9 hours a day for 5 days, calculate her total weekly pay.

Solution

Write all incomes as weekly amounts.

Wage $= \$1280.25$

Site allowance $= \$104.80$

Meal allowance $= 5 \times \$16.20$

$= \$81.00$

Height allowance $= 5 \times 9 \times \$5.60$

$= \$252.00$

Total weekly pay $= \$1280.25 + \$104.80 + \$81.00 + \$252.00$

$= \$1718.05$

Holiday pay

EXAMPLE 7

Hayden earns a yearly salary of $46 598. Calculate his holiday loading, which is 17.5% of the normal pay for 4 weeks, correct to the nearest cent.

Solution

4 weeks' pay $= \$46\,598 \div 52.18 \times 4$

$= \$3572.096\ldots$

Do not round your answer. Leave it on the calculator display.

Holiday loading $= 17.5\% \times \$3572.096\ldots$

$= \$625.116\ldots$

$\approx \$625.12$

Exercise 3.03 Bonuses, allowances and annual leave loading

1 Name an occupation that may require working in each situation.

a wet weather
b isolated areas
c cramped conditions
d hot temperatures
e cold temperatures
f high places
g dangerous conditions
h dirty conditions.

Example 6

2 A painter on the Sydney Harbour Bridge earns $35.70 per hour, as well as a height allowance of $25.10 per day. If he works 10 hours per day for 5 days, calculate his total earnings.

Example 7

3 A police officer earns a salary of $88 275. Calculate her annual leave loading if it is 17.5% of 4 weeks normal pay.

4 At the end of the year, the owners of a small business gave their employees a Christmas bonus of 60% of a normal week's pay. Calculate Lisa's total pay for that week if her usual pay is $720.48.

5 A miner earns a wage of $28.42 per hour, plus an allowance of $9.97 per hour for working underground in confined spaces. If the miner works from 8.30 a.m. to 4.00 p.m. each day from Monday to Friday, calculate:

a his total weekly income
b his yearly income.

6 Scientists working at the South Pole earn a salary of $83 204 plus a weekly allowance of $177.35 for working under extreme and isolated conditions. Calculate their fortnightly pay.

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7 For meeting a target one week ahead of schedule, the owners of a firm gave their staff a bonus equal to 15% of the normal pay for 1 month. What is Amanda's bonus if her salary is $53 045?

8 A window washer earns $27.90 per hour and a multi-storey allowance of $43.40 per day. If he works 10 hours each weekday, calculate his total weekly earnings.

9 Each day, an electrician is paid $298.80 plus site allowance $12.25, meal allowance $8.40 and laundry allowance $3.21.

a If the electrician works 6 days per week, calculate his total weekly earnings.

b Calculate the electrician's annual leave loading if it is 17.5% of the normal pay for 4 weeks (without allowances).

10 An oil rig worker earns a wage of $52.56 per hour, plus a dirt allowance of $9.49 per hour. If she works $9\frac{1}{2}$ hours a day, calculate her total weekly earnings for a 5-day week.

Shutterstock.com/Polina Petrenko

11 The owners of a biscuit factory gave their casual staff a bonus of $472 at Christmas, while the permanent staff received a holiday loading of 17.5% of the pay for 4 weeks. If the normal weekly pay for the permanent staff was $952, what was the difference between their holiday loading and the bonus given to the casual staff?

12 Brock works at a chemical factory and is paid $28.58 per hour, plus an additional allowance of $5.10 per hour for working with toxic substances. What is his total weekly pay if he works from 7.00 a.m. to 3.00 p.m. for 5 days a week?

13 When it rains, road workers are paid an extra $4.60 per hour. Last week, Paul worked 9 hours a day for 5 days at $29.43 per hour. If it rained continuously on two of those days, calculate Paul's total earnings for the week.

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14 Each week, a real estate agent earns a retainer of $510, a commission of 3.5% of the value of properties sold, and a car allowance of $128.70. Calculate her total earnings for a week in which she sold $943 658 worth of properties.

15 A telephone interviewer employed by a market research company earns $27.28 per hour for a 35-hour week. Calculate her holiday loading if it is 17.5% of the normal pay for 4 weeks.

16 One year, Kirsten received a holiday loading of $703.12. What was her normal weekly wage?

17 Anneliese normally works a 40-hour week. She goes on holidays and receives four weeks' wages plus 17.5% holiday loading. If she earns $24.55 per hour, calculate (correct to the nearest cent) Anneliese's:

a holiday loading

b total holiday pay. ← Holiday pay = 4 weeks' normal pay + holiday loading

3.04 Government allowances and pensions

Earning money

Centrelink is an Australian government agency that supports people in need and assists them to become self-sufficient. Various allowances and benefits are available for the aged, the unemployed, students, parents and people with disabilities.

Age pension

The **Age pension** is a fortnightly payment to seniors, aged 65 or over, who are unable to support themselves in their retirement. To be eligible, a person needs to have low income and assets. The payment and income test tables are shown below.

Marital status	Full payment
Single	$797.90
A couple	$601.50 each

Marital status	To get full payment, earnings per fortnight* can be up to:
Single	$164 plus $124.70 per dependent child
A couple (combined income)	$292 plus $124.70 per dependent child

*Income over these amounts reduces the rate of pension payable by 50 cents in the dollar (single), and 50 cents in the dollar each (for couples).

Source: www.humanservices.gov.au, 2017

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EXAMPLE 8

Use the tables to calculate the fortnightly Age pension payment for each person or couple.

a Beryl, aged 72, receives $400 per year dividend from her shares.

b Dulcie (aged 65) and Bert (aged 69), are legal guardians of their granddaughter and have a combined income of $230 per week.

c Stanley, aged 63, has no income and lives alone.

Solution

a Fortnightly income = $400 ÷ 52.18 × 2

= $15.331…

$15.331 < $164

Beryl is single and earns less than $164 per fortnight, so she receives the full payment of $797.90.

b Combined fortnightly income = $230 × 2

= $460

To receive the full payment, the couple must earn less than:

$292 + $124.70 = $416.70 $124.70 for the granddaughter

The payment is reduced by 50c for each dollar earned over $416.70.

Combined pension = (2 × $601.50) — 0.5 × ($460 – $416.70)

= $1181.35

Dulcie and Bert receive a combined pension of $1181.35 per fortnight.

c Stanley is not eligible for the Age pension because he is under 65 years old.

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Youth and study allowances

There are several allowances available for young people or students.

Allowance	Conditions for eligibility
Youth Allowance	Aged 16–21 years, looking for full-time work or doing approved activities Aged 18–24 years, studying full-time Aged 16–24 years, undertaking a full-time apprenticeship Aged 16–17 years, finished Year 12 or equivalent, need to live away from home to study or are independent
Austudy	Aged over 25 years and studying full-time or undertaking an apprenticeship
ABSTUDY	Indigenous students or apprentices

Youth Allowance (for people aged 16 to 24)

Conditions for eligibility	Maximum fortnightly payment
Single with no children, under 18 years and living at home	$239.50
Single with no children, under 18 years and not living at home	$437.50
Single with no children, 18 years or older and living at home	$288.10
Single with no children, 18 years or older and not living at home	$437.50
Single with children	$573.30
Member of a couple, no children	$437.50
Member of a couple, with children	$480.50

Austudy (for people aged 25 or more)

Conditions for eligibility	Maximum fortnightly payment
Single	$437.50
Single, with children	$573.30
Partnered, no children	$437.50
Partnered, with children	$480.50

Source: www.humanservices.gov.au, 2017

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EXAMPLE 9

Calculate the fortnightly youth allowance or Austudy payment for each person.

a Pablo, aged 17, a first-year apprentice plumber living with his parents

b Cassie, aged 27, studying her HSC full-time at TAFE while raising her son alone

c Zina, aged 32, studying part-time and living with her husband and two children

d Greg, aged 16, has left school, is homeless, and is looking for full-time employment

Remember to check the eligibility criteria as well.

Solution

a Pablo is under 18 and is living at home, so he receives a Youth Allowance of $239.50.

b Cassie is over 25 and a single parent, so she receives an Austudy payment of $573.30.

c Zina is not eligible for Austudy, because she is not studying full-time.

d Greg is under 18 and is not living at home, so he receives a Youth Allowance of $437.50.

ABSTUDY

Conditions for eligibility	Allowance per fortnight
Single, no children	
Under 18, at home	$239.50
18–21 years, at home	$288.10
Under 22, not at home	$437.50
22–59, not at home	$528.70
60 or older, not at home	$571.90
Partnered, no dependent children	
Younger than 22 years	$437.50
22 years or older	$477.40
Single with dependent child	
Younger than 22 years	$573.30
22 years or older	$571.90
Partnered with dependent child	
Younger than 22 years	$480.50
22 years or older	$477.40

Source: www.humanservices.gov.au, 2017

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EXAMPLE 10

Calculate the fortnightly ABSTUDY payment for each Indigenous student.

a William, aged 18, who is living with his partner and their two-year-old son

b Annie, aged 14, who is living at home with her parents

c Jackie, aged 27, who is living with her husband and has no children

Solution

a William is younger than 22 and is partnered, with a dependent child, so he receives $480.50.

b Annie is under 18 and is living at home so she receives $239.50.

c Jackie is over 22 and is partnered, with no children, so she receives $477.40.

Exercise 3.04 Government allowances and pensions

Use the tables on the previous pages to answer the questions in this exercise.

Example 8

1 Calculate the fortnightly Age pension payment for each person or couple, assuming they all satisfy the government assets test.

a Georgia, aged 89, receives $1250 per year from her investments.

b Rodney, aged 71, and Bernadette, aged 75, care for their disabled daughter full-time and have a combined income of $470 per week.

c James, aged 62 years, has no savings, no job and lives alone.

2 Why do you think that age pensioners receive less if they are partnered rather than single?

3 The Newstart allowance is paid to unemployed people aged 21 years or over. The maximum fortnightly rates are shown in the table.

Status	Dependants	Newstart allowance
Single	No children	$528.70
Single	Children	$571.90
Partnered		$477.40

a Hani is unemployed, and is single with no children. How much Newstart allowance would she receive if it was paid:

i fortnightly? **ii** yearly? **iii** monthly?

b Jay is married, unemployed and has two children. How much does he receive over 6 months?

c Does a single unemployed person receive more, less or the same as one with a partner? Do you think this is fair? Give two or three reasons for your answer.

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4 Carrie is 18 years old and is studying full-time at a senior high school. She lives with her parents and has a two-year-old daughter. Which of the following is her fortnightly Youth Allowance? Select **A**, **B**, **C** or **D**.

Example 9

A $239.50 **B** $437.50 **C** $573.30 **D** $480.50

5 The parenting payment is paid to parents of children aged 18 years or younger and is not dependent on the number of children they have. The maximum fortnightly rates are shown in the table.

Status	Parenting payment
Partnered	$477.40
Partnered but separated due to illness, respite or prison	$571.90
Single	$738.50

a The Taylor family has three children: Daniel 15, Julianne 12 and Liam 8. How much does the Taylor family receive as a parenting payment per fortnight?

b The Lovejoys have ten children under 18 at home and two children living away from home. Mr Lovejoy is currently in prison. What parenting payment does Mrs Lovejoy receive per fortnight?

c Jayda is separated from her partner and has a boy aged 9 years and a girl aged 7 years. What is her fortnightly parenting payment?

d Kell's wife is in hospital for the next 12 months. He is looking after their twin 11-year-old boys. How much parenting payment will he receive for:

i 1 year? **ii** 1 week?

6 Shaun is a 19-year-old Indigenous student living at home with his parents. Which of the following is his fortnightly ABSTUDY allowance? Select **A**, **B**, **C** or **D**.

Example 10

A $239.50 **B** $288.10 **C** $437.50 **D** $573.30

7 Use the ABSTUDY table from Example **10** on page 106 to determine the fortnightly payment for each Indigenous student.

a Susan is 17 years old, and is living with her partner

b Maxim is 28 years old, and is living at home with his wife and two children under 10 years

c Zelda is 21 years old, and is living away from her parents' home with her boyfriend

d Jasmine is 15 years old, and is living at home with her grandparents

INVESTIGATION

VISIT CENTRELINK

a Investigate the different allowances by visiting a Human Services/Centrelink branch or the **Human Services** government website.

b Research one type of allowance, looking at the eligibility conditions, residential qualifications, and income and assets tests.

c Write a report and present this to the class.

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Pay day

3.05 Gross pay and net pay

Gross pay is the amount a person earns, while **net pay** is the amount remaining after income tax and other **deductions** have been taken from gross pay.

Net pay

Net pay = gross pay – tax – other deductions

Other deductions include:

- **superannuation** contributions (retirement fund)
- private health fund instalments
- union fees
- direct debits for bills
- savings.

EXAMPLE 11

Diane earns $1029.60 per week. Deductions of $303.73 for tax, $42.17 for superannuation, $26.34 for a health fund contribution and $17.50 for gym fees are taken from her gross wage.

Calculate her:

a net pay

b income tax as a percentage of her gross pay, correct to one decimal place.

Solution

a Net pay = $1029.60 – $303.73 – $42.17 – $26.34 – $17.50

= $639.86

Diane's net pay is $639.86

b $$\text{Percentage tax} = \frac{\text{tax}}{\text{gross pay}} \times 100\%$$

$$= \frac{\$303.73}{\$1029.60} \times 100\%$$

$$= 29.4998...$$

$$\approx 29.5\%$$

Diane's income tax is 29.5% of her gross pay.

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EXAMPLE 12

Aidan earns a salary of $67 510, pays 34% tax and has $28.70 per fortnight deducted for union fees. Correct to the nearest cent, calculate:

a how much tax Aidan pays per fortnight

b Aidan's gross fortnightly pay

c Aidan's net fortnightly pay.

Solution

a Yearly tax $= 34\% \times \$67\,510$

$= \$22\,953.40$

Fortnightly tax $= \$22\,953.40 \div 52.18 \times 2$

$= \$879.7776\ldots$

$\approx \$879.78$

Aidan pays $879.78 tax per fortnight.

b Gross fortnightly pay $= \$67\,510 \div 52.18 \times 2$

$= \$2587.5814\ldots$

$= \$2587.58$

Aidan's gross fortnightly pay is $2587.58.

c Net fortnightly pay $= \$2587.58 - \$879.78 - \$28.70$

$= \$1679.10$

Aidan's net fortnightly pay is $1679.10.

Exercise 3.05 Gross pay and net pay

1 Alice earns $827.25 per week and has the following deductions from her gross wage: Tax $244.04, superannuation $64.83, union fees $18.20 and health fund $26.15. Which of the following is Alice's net pay? Select **A**, **B**, **C** or **D**.

A $827.25 **B** $583.21 **C** $474.03 **D** $718.07

2 Melanie earns $945.30 per week and has deductions of $235.90 for tax, $55.17 for superannuation, $24.78 for a health fund contribution and $17.50 for union fees. Calculate her:

Example 11

a net pay

b income tax as a percentage of her gross pay, correct to one decimal place.

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Example 12

3 Zhi earns a salary of $45 674. Each week, he pays 28% tax, $30.76 to his superannuation fund and $86.11 in other deductions. Calculate Zhi's:

a gross weekly pay **b** tax **c** net weekly pay.

4 Fatima earns a gross weekly wage of $1035.25, from which is deducted $34.11 for superannuation and $7.90 for union fees.

a If Fatima's net pay is $692.06, how much tax does she pay?

b What percentage of Fatima's gross wage is paid in tax (correct to two decimal places)?

c What percentage of Fatima's gross wage is paid in superannuation (correct to two decimal places)?

5 Jeff pays 27.5% of his gross pay in tax and 8% in superannuation. Each week, he also has deducted $7.10 for union fees and $14.84 for life insurance premiums. If Jeff's gross weekly pay is $1238.20, calculate his:

a weekly tax

b weekly superannuation payment

c total weekly deductions

d net weekly pay.

6 Lindy pays 22% of her gross pay in tax and 10% in superannuation. She also contributes $4.70 towards a staff social fund every week.

a If Lindy pays $235.52 tax each week, find her gross weekly pay.

b Calculate Lindy's weekly superannuation payment.

c Find Lindy's net weekly pay.

7 **a** Fady earns a salary of $54 251. Calculate his fortnightly pay.

b Each fortnight, Fady's deductions from his pay are $307.26 tax, $145.60 for a car loan repayment and $27.14 to a health fund. What is Fady's net pay?

c What percentage of Fady's gross pay is paid in tax, correct to one decimal place?

8 Melinda earns $29.38 per hour for a 39-hour week and pays 25% tax. Each week, she has $17.40 deducted from her pay for accident insurance. Calculate her:

a gross weekly pay **b** tax **c** net weekly pay.

9 Matthew earns a salary of $108 275. He is paid monthly but, each payday, $4330 is deducted in tax, $1080.50 is deducted for superannuation and $107.54 is deducted for medical insurance.

a Calculate Matthew's monthly gross and net incomes.

b What percentage of Matthew's gross pay is paid in tax?

c What percentage of Matthew's gross pay is his net pay?

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10 Copy and complete the pay statement below, using a calculator or spreadsheet.

Springfield Nuclear Power

Employee: H. Simpson			Employee number: 5521716	
Pay period: 5.11.18–11.11.18			Hourly rate: $21.48	
	Hours	**Pay**	**Deductions**	
Normal	35		Tax (25%)	
Time-and-a-half	4		Superannuation	$60.65
Double time	1		Union	$18.15
	Gross wage		Health fund	$24.80
	Total deductions			
	Net wage			

11 Copy and complete the employee pay slip below.

State Education Board

Employee: E. Krabappel		Serial number: 8805285	
Pay period: 14.3.22–27.3.22		Salary: $87 026	
Other deductions		**Pay for this fortnightly period**	
Superannuation	$71.60	Gross pay	
Union	$16.22	Tax (31%)	
Loan repayment	$43.55	Other deductions	
Car insurance	$20.84	Net pay	
Total deductions			

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TECHNOLOGY

Pay slip spreadsheet

Lachlan is a forklift driver. He is paid an hourly rate of $20.91, for a normal 8-hour day working from 9 a.m. until 6 p.m. on weekdays.

He is paid time-and-a-half for working on Saturdays and double time for any nights working from 6 p.m. until midnight. He is also paid an extra $8.06 per hour for each hour or part of an hour worked after 6 p.m.

a Copy Lachlan's incomplete pay slip into a spreadsheet, as shown below.

	A	B	C	D	E	F	G	H	I
1	Payslip Monday 27th March - Saturday 1st April								
2									
3	Employee	Lachlan O'Connell			Normal Hourly Pay Rate	20.91		Extra Pay/hr	8.06
4									
5	**Date**	**27-Mar**	**28-Mar**	**29-Mar**	**30-Mar**	**31-Mar**	**01-Apr**	**TOTAL**	
6	**Hours**	9 am – 5 pm	10:30 am – 6 pm	6:30 pm – 10 pm	11 am – 6:30 pm	9:45 am – 5:30 pm	9:30 am – 2 pm		
7	**Normal Hours**								
8	**Time-and-half Hours**								
9	**Double Time Hours**								
10	**Extra Pay for Nights**								
11							**TOTAL PAY**		

Complete rows 7, 8 and 9 for the total number of hours for each pay rate.

In cells H7, H8 and H9, enter formulas for the totals. In cell D10 enter '=D9*I3'. In cell H10 enter a formula for the total. In cell H11, enter a formula for the total amount Lachlan was paid for this week.

b The following week, Lachlan worked these hours:

Mon 4 Apr	9.30 a.m. – 5.00 p.m.	Thur 7 Apr	12.00 p.m. – 7.00 p.m.
Tues 5 Apr	6.00 p.m. – 12.00 a.m.	Sat 9 Apr	10.00 a.m. – 4.00 p.m.
Wed 6 Apr	6.30 p.m. – 12.00 a.m.		

Modify your spreadsheet to calculate Lachlan's total pay for this week.

ISBN 9780170413565

3.06 Income tax and Medicare levy

Income tax tables

Income tax

Taxable income and allowable tax deductions

Anyone who earns an income pays **income tax** to the government. The government uses this money to fund public services such as schools, hospitals, roads, trains, the defence services and the police force. Not all of a person's **gross income** is taxed, however. If we use some of our income for work-related expenses or to donate money to charities, these amounts are called **allowable tax deductions** and are not taxed. The remaining part of our income that is taxed is called **taxable income**.

Taxable income

Taxable income = gross income – allowable tax deductions

Here are some examples of allowable tax deductions:

- work-related expenses, such as tools of trade, uniforms, car-related expenses, trade journals, subscriptions to professional organisations and travel to conferences
- donations to charities
- depreciation of home office equipment which is used to earn income
- bank fees on investments for which interest is earned.

Income tax

Income tax is calculated on our taxable income (rounded down to the nearest dollar) for a financial year (1 July of one year to 30 June of the next year) and operates on a sliding scale so that the more we earn, the higher the rate of tax we pay.

Income tax rates for Australian residents	
Taxable income	**Tax on this income**
$0 – $18 200	Nil
$18 201 – $37 000	19c for each $1 over $18 200
$37 001 – $87 000	$3572 plus 32.5c for each $1 over $37 000
$87 001 – $180 000	$19 822 plus 37c for each $1 over $87 000
$180 001 and over	$54 232 plus 45c for each $1 over $180 000

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Income tax

EXAMPLE 13

Cyndi, earned $56 375 and claimed $326.20 in work-related expenses and $125 in donations to charities. Find the amount of income tax Cyndi should pay, using the table above.

Solution

Cyndi's taxable income = $56 375 – $326.20 – $125

= $55 923.80

≈ $55 923 Rounded down to the nearest dollar

Cyndi's income is in the 3rd tax bracket ($37 001–$87 000) for which the tax is '$3572 plus 32.5c for each $1 over $37 000'.

Cyndi's tax payable = $3572 + 0.325 × ($55 923 – $37 000)

= $9721.975

≈ $9721.98

Cyndi should pay $9721.98 in income tax.

Medicare levy

The standard **Medicare levy** for Australian residents is 2% of taxable income, although some low-income earners pay a reduced levy or no levy.

Note: The Medicare levy is due to increase to 2.5% in July 2019.

EXAMPLE 14

Calculate the Medicare levy for Suyen, who has a taxable income of $62 379.

Solution

Medicare levy = 2% × $62 379

= $1247.58

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Exercise 3.06 Income tax and Medicare levy

Example 13

1 Find the amount of income tax payable for each person.

- **a** Kamel who earned $31 425 and claimed $285 in work-related expenses
- **b** Susie who earned $131 412 and claimed $1036 in travel expenses and $643 depreciation of her home office equipment
- **c** Toula who received $14 615 for the year.

Example 14

2 Jenni earns a salary of $91 262 and claims a tax deduction of $1810.15 for work-related expenses.

- **a** Calculate Jenni's taxable income.
- **b** Calculate Jenni's amount of tax payable.
- **c** Calculate the amount of Jenni's Medicare levy.

3 Catriona earns a salary of $87 210 and claims these deductions:

- Travel and entertainment expenses $650.25
- Depreciation of home computer $314.80
- Membership of a professional association $120.50

- **a** Calculate Catriona's taxable income.
- **b** Calculate Catriona's income tax payable.
- **c** What is the amount of Catriona's Medicare levy?

4 Luke earns $1884.62 per week as a teacher. Last financial year, he also earned $286.10 bank interest, $2050.96 share dividends and $892.51 book royalties. He is able to make tax deductions of $241.60 for union fees, $345.80 for car expenses, $175.80 for professional journals and $843.50 for expenses related to the writing of his book. For the financial year, calculate Luke's:

a taxable income **b** income tax payable **c** Medicare levy.

5 The table shows the tax rates for Australian citizens who live in another country.

Income tax rates for Australian foreign residents	
Taxable income	**Tax on this income**
$0–$87 000	32.5c for each $1
$87 001–$180 000	$28 275 + 37c for each $1 over $87 000
$180 001 and over	$62 685 + 45c for each $1 over $180 000
Non-residents are not required to pay the Medicare levy.	

© Australian Taxation Office for the Commonwealth of Australia

Find the income tax payable by each of the following Australian citizens.

- **a** Claude, who lives in France and has a taxable income of $37 850
- **b** Marie, who has a taxable income of $102 670 and lives in Canada
- **c** Benny, who lives in Sweden and has a taxable income of $203 500.

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TECHNOLOGY

Income tax calculator

Create and complete the spreadsheet below for calculating the income tax payable when you enter the taxable income in the correct tax bracket. Two formulas have already been given.

	A	B	C	D
1	**Tax bracket**			
2	**From**	**To**	**Taxable income**	**Income tax**
3	$0	$18 200		$0
4	$18 201	$37 000		=(C4-B3)*0.19
5	$37 001	$87 000		=3572+(C5-B4)*0.325
6	$87 001	$180 000		
7	$180 001	and over		

Create a similar spreadsheet to calculate the Medicare levy as well.

DID YOU KNOW?

Income tax in other countries

Denmark has the highest income tax rates, ranging from 38% to 65%, while the United Arab Emirates (UAE) has no income tax at all. The **Worldwide Tax** website gives the current tax rates around the world. Here are the tax rates for seven countries.

Country	Rate	Country	Rate
Brazil	7.5% to 27.5%	Spain	19% to 45%
Hong Kong	2% to 17%	Sweden	0% to 57%
Russia	13%	USA	0% to 39.6%
Saudi Arabia	20%		

Find the current tax rates in five more countries.

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3.07 PAYG and tax returns

To avoid having to pay income tax as a huge lump sum at the end of a financial year, tax is deducted every payday from a person's gross pay and is called **Pay As You Go (PAYG) tax**. The employer estimates how much tax to take out of each pay and, at the end of the financial year, the actual income tax payable is calculated.

Every income earner must submit a **tax return** to the Australian Tax Office (ATO) at the end of the financial year. A PAYG Payment Summary, showing total gross earnings and PAYG tax paid, is issued by the employer and used by the employee to complete the tax return, including any allowable tax deductions. The ATO then calculates the income tax and Medicare levy payable. Any earner who has paid more PAYG tax than necessary will receive a **tax refund**. However, anyone who has not paid enough PAYG tax will have a **tax debt** and will need to pay the difference owing.

EXAMPLE 15

Mia earns a salary of $115 412.70 and claims tax deductions of $1743. Last year she paid a total of $37 412 PAYG tax. Calculate her:

a taxable income **b** income tax payable

c Medicare levy **d** tax refund or tax debt.

Solution

a Taxable income $= \$115\,412.70 - \$1743$

$= \$113\,669.70$

$\approx \$113\,669$ Rounding down to the nearest dollar

Mia's taxable income is $113 669.

b Tax payable $= \$19\,822 + 0.37 \times (\$113\,669 - \$87\,000)$ From the table on page 113

$= \$29\,689.53$

Mia's income tax is $29 689.53.

c Medicare levy $= 2\% \times \$113\,669$

$= \$2273.38$

Mia's Medicare levy is $2273.38.

d Total taxes $= \$29\,689.53 + \$2273.38 = \$31\,962.91$

Amount of PAYG tax paid = $37 412

Tax refund $= \$37\,412 - \$31\,962.91$ Refund because PAYG tax paid > total taxes

$= \$5449.09$

Mia's tax refund will be $5449.09.

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Exercise 3.07 PAYG and tax returns

1 Jasmine paid $314.68 PAYG tax out of her weekly wage of $1014.65.

a How much PAYG tax did Jasmine pay for the year?

b How much did Jasmine earn for the year, after tax?

2 Hope earned a salary of $73 489 per year. She received her weekly pay, after PAYG tax of $475.15 and $26.80 health insurance were taken out.

a How much PAYG tax did Hope pay in a year?

b How much was Hope's weekly pay?

Use the table of income tax rates on page 113 to answer Questions 3 to 7.

Example 15

3 Gabby is an Australian resident who earned a yearly salary of $204 560. She claimed tax deductions of $10 534 and paid a total of $72 156 PAYG tax. Calculate her:

a taxable income **b** income tax payable

c Medicare levy **d** tax debt or tax refund.

4 Anand had a taxable income of $278 639 and paid $104 568 in PAYG tax over the financial year. Find his:

a income tax payable **b** Medicare levy **c** tax debt or tax refund.

5 Liam earned a wage of $989.60 per week and paid PAYG tax of $229.02. He was able to claim $38.25 per week in allowable tax deductions and $150 per year in donations. Find his:

a taxable income **b** income tax payable

c Medicare levy **d** tax debt or tax refund.

6 Dave paid PAYG tax of $7836 and had a taxable income of $34 589. Which of the following is Dave's tax refund? Select **A**, **B**, **C** or **D**.

A $2672.31 **B** $4788.20 **C** $6622.09 **D** $4030.31

7 Kim had a taxable income of $381 459 and paid $138 639 in PAYG tax. Which of the following is Kim's tax debt? Select **A**, **B**, **C** or **D**.

A $6249.55 **B** $13 878.73 **C** $9642.17 **D** $8281.60

8 Use the tax table for Australian foreign residents on page 115 to find the tax refund or tax debt for each of the following Australian citizens.

a Cecily, who has a taxable income of $93 457 and who paid $734 per fortnight PAYG tax

b Alice, who paid $5279 PAYG tax every quarter and earned a taxable income of $98 534

c Simon, who paid $37 460 tax per month and earned a taxable income of $1.2 million.

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INVESTIGATION

TAX RETURNS

You may use a spreadsheet to complete this task.

1. Choose the profile of a person with a particular occupation and salary or wage. Decide on what PAYG tax that person pays each week, fortnight or quarter.
2. Research the allowable tax deductions for this occupation. These may include travel expenses, special clothing, tools of trade, books and journals, attendance at conferences and seminars, home office expenses and self-education expenses.
3. Add other income you wish, such as bank interest or money from a part-time job. If you do this, there may be other allowable tax deductions, such as bank fees or travel between jobs, which are claimable.
4. Use the form from a Tax Pack to complete a tax return for the person.
5. Calculate the person's Medicare levy.
6. Calculate the person's tax payable and any tax refund or tax debt.

3.08 GST and VAT

While income tax is a tax on earning, the GST and VAT are taxes on spending or consuming.

The **goods and services tax (GST)** in Australia is 10% of the cost of the goods or services. New Zealand, Canada and Singapore also have a GST. In other countries, it is called the **value-added tax (VAT)**.

- Prices in Australia are usually printed with the GST included.
- Prices in other countries are often printed 'before tax', with the tax added when the consumer pays.
- The GST is not charged on essential items such as basic foods, childcare or renting a home.
- The consumer pays GST for goods and services, such as electrical appliances, lease of premises, plumbing work, IT support.

The GST/VAT rate in some countries (correct in 2017) is shown in the table.

GST/VAT rates			
Australia	10%	Singapore	7%
Canada	7%	South Africa	14%
China	17%	Sweden	25%
Mexico	16%	Taiwan	5%
New Zealand	15%	Thailand	7%

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EXAMPLE 16

a A camera costs $568 plus 10% GST. What is its selling price?

b A TV costs $1995, inclusive of GST. What was the original price of the TV, to the nearest 5 cents, before GST was added?

Solution

a Selling price = 110% × $568 — Increasing the price by 10%

= $624.80

The selling price of the camera is $624.80.

b 110% of original price = $1995

1% of original price = $1995 ÷ 110 — Using the **unitary method:** Find 1% first

= $18.136 36 …

Original price = $18.136 36 … × 100 — To find 100%

= $1813.636

≈ $1813.65 — Rounded to the nearest 5 cents

The original price of the television was $1813.65.

EXAMPLE 17

A handbag in Norway costs 425 kroner before VAT and 531.25 kroner after. What is the rate of VAT charged?

Solution

VAT = 531.25 − 425 = 106.25 kroner

$$\text{VAT rate} = \frac{106.25}{425} \times 100\%$$

= 25%

The rate of VAT charged is 25%.

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Exercise 3.08 GST and VAT

Example 16

1 A hamburger in Sydney costs $5.50 plus 10% GST. What is the hamburger's selling price? Select **A**, **B**, **C** or **D**.

A $5.60 **B** $5.00 **C** $6.05 **D** $6.50

2 A computer was purchased in Ireland for €1500 (Euro). What is the computer's selling price when 21% VAT is added?

3 A watch in Thailand costs 3724 baht plus 7% VAT. What is the watch's selling price?

4 A car costs $42 000 including 10% GST. What is the car's price before GST, correct to the nearest dollar? Select **A**, **B**, **C** or **D**.

A $38 182 **B** $37 800 **C** $38 181 **D** $46 200

5 A train ticket from Sydney to Bomaderry costs $25.50 including 10% GST. What is the price of the ticket before tax?

6 A night's accommodation in London is £126.90 (pound) including 17.5% VAT. What was the price before tax?

7 An overseas telephone call from South Africa costs 53.01 rand after a VAT of 14% is imposed. What was its price before VAT, rounded to two decimal places?

8 A box of chocolates from the Netherlands costs €6.50, plus €1.24 VAT. What percentage VAT, to the nearest whole number, has been imposed?

Example 17

9 The cost of a laundry service in Greece was €9.88 after a VAT of €1.91 was added.

a What was the original price of the laundry service?

b What was the rate of VAT charged, to the nearest percentage?

10 In New Zealand, a 15% GST of $4.62 was added to the original price of a set of steak knives. What was:

a the original price of the knives? **b** the final price of the knives?

11 A Korean camera costs 142 900 won before VAT, and 157 190 won after VAT.

a What amount of VAT was imposed?

b What percentage VAT was imposed?

12 An Italian suit costs €220 before VAT and €264 after VAT.

a What VAT is charged?

b What percentage VAT is charged?

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13 A television set costs $1245 plus $224.10 GST. What percentage rate of GST has been imposed?

14 A painting was bought in Luxembourg and a 15% GST of 31.17 euros was added to the original price. Find the selling price of the painting.

15 A roof tiler in New Zealand charges $152.72 for a day of work after a GST of 15% is added. What was the charge before GST?

16 If an Akubra hat has a 10% GST of $17.35 included in its selling price, find its selling price.

iStock.com/AWSeebaran

17 A necklace in Thailand costs 4419 baht after a VAT of 7% has been added. What was its original price, to the nearest baht?

18 If a video store charges $6.50 rental before GST, and $6.93 after GST, what percentage rate of GST is imposed? Answer to two decimal places.

19 At a Canadian cafe, $2.40 GST was added to the cost of a dinner for two. If this represents 5% of the original cost of the meal, calculate the cost of the meal before and after GST.

20 A Belgian key-cutter charges €3, plus €0.57 in VAT, for every key cut. What percentage VAT is being charged?

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INVESTIGATION

SAME DIFFERENCE, MORE-OR-LESS?

1 Why can we find the GST paid on an item by simply dividing its marked price by 11?

2 The Great Gals electrical store adds 10% GST to the prices of its goods. If a washing machine had an original price of $1200, what is its advertised price with GST included?

3 At the Great Gals end-of-year sale, customers get a 10% discount on all goods.

- **a** Will the discounted price be more than, less than or the same as the original price before GST was added?
- **b** What is the discounted price of the washing machine in Question **2** at the end-of-year sale?
- **c** How does the discounted price of the washing machine compare with the original price?
- **d** Would it make a difference to the discounted price of the washing machine if the 10% discount was applied first before the 10% GST was added?

4 What can you say generally about an item that has 10% added then 10% subtracted?

5 Calculate the overall percentage change when 10% is added to an amount then 10% is subtracted.

SAMPLE HSC PROBLEM

Nathan paid PAYG tax of $237.80 per week from his weekly wage of $789.40.

- **a** How much PAYG tax did Nathan pay in the year?
- **b** What was Nathan's gross income for the year?
- **c** If he had $986.50 in allowable tax deductions, what was Nathan's taxable income?
- **d** Use the income tax table on page 113 to calculate Nathan's income tax and Medicare levy.
- **e** Will Nathan receive a tax refund or will he need to pay a tax debt? Calculate the amount.

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Study tip

Showing your working

Why is it important to show your working when answering a mathematics question?

- To demonstrate that you know your mathematics and the steps required to solve the problem
- So that, in an exam, you can receive 'part marks' for correct working, even if you get the wrong answer
- To show that you really do know how to answer the question and aren't simply guessing or bluffing
- To remind yourself how you solved the problem, especially when studying
- So that you can describe how you achieved your answer (explaining something to someone often reinforces your learning and understanding).

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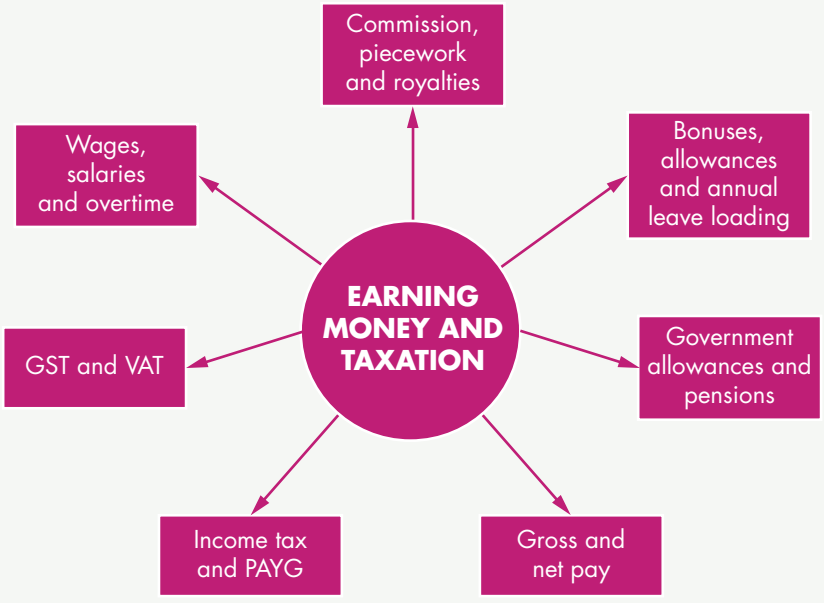
3. CHAPTER SUMMARY

This chapter, Earning money and taxation, looked at the different ways people earn or receive incomes and pay taxes based on earning and spending. You should be able to calculate incomes earned in a variety of ways, including commission, annual leave loading, allowances, pensions and overtime. You have learned to calculate income tax and the Medicare levy, and the tax refund or debt after PAYG tax has been paid. Finally, you looked at GST and VAT.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map is shown below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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3. TEST YOURSELF

Earning and taxation crossword

Exercise 3.01

1 Convert each wage to a yearly amount, correct to the nearest cent.

a weekly wage of $831.15

b fortnightly wage of $1948

c monthly wage of $3786.

Exercise 3.01

2 Calculate to the nearest cent the pay amount for an annual salary of:

a $138 611 paid weekly

b $98 503 paid fortnightly

c $263 764 paid monthly.

Exercise 3.01

3 Marley works three days a week as a part-time carer. She is paid $25.70 per hour up to 5 p.m. and time-and-a-half after that. Use the roster to calculate how much she earned this week.

Day	Start time	Finish time
Monday	1000	1900
Wednesday	0900	1300
Friday	1600	2200

Exercise 3.02

4 Huon picked 28 buckets of grapes and was paid $5.65 per bucket.

a How much did Huon earn?

b How many buckets would Huon need to pick to earn $395.50?

Exercise 3.02

5 Maggie made cupcakes and sold them to a shop at $4.80 per dozen, or $38 for 100. How much more per cupcake did she earn by selling them in lots of 100 rather than by the dozen?

Exercise 3.02

6 Goran earned a 10% royalty on each of his books sold. If his book sells for $45.95, how much will he earn if 2537 books are sold?

Exercise 3.03

7 Byron works in a mine and earns an hourly rate of $39.65. He also gets a below ground allowance of $9.84 per hour, double his hourly rate on weekends and a meal allowance of $27.40 per day if he works six or more hours.

Calculate Byron's total income in a week he worked the following shifts:

Monday: 12 noon–5 p.m. above ground

Wednesday: 2 p.m.–7 p.m. below ground

Friday: 9 a.m.–12 noon above ground, and
2 p.m.–6 p.m. below ground

Saturday: 10 a.m.–2 p.m. below ground, and
3 p.m.–5 p.m. above ground

Exercise 3.03

8 The Great Gals electrical store pays a holiday loading of 17.5% of four weeks' pay to each employee. How much holiday loading is paid to Nirmal, the manager, who earns a salary of $86 430?

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9 Use the table from section 3.04 on page 102 to calculate the fortnightly Age pension payment of Roger (aged 66) and Tatiana (aged 68) who are legal guardians of their two school-aged grandsons and have a combined income of $280 per week doing administrative work in an office.

Exercise 3.04

10 Jack earns a salary of $94 762, pays $594.10 PAYG tax per week and has $28.70 per week deducted for union fees. Correct to the nearest cent, calculate:

Exercise 3.05

a how much tax Jack pays per year
b how much tax Jack pays per fortnight
c Jack's gross fortnightly pay
d Jack's net pay per fortnight.

11 Melissa earned a yearly salary of $118 764. She paid $1424.30 PAYG tax per fortnight and claimed tax deductions of $13 102.20 per year. Calculate her:

Exercise 3.06

Exercise 3.07

a taxable income (rounded down to the nearest dollar)
b income tax payable (using the table on page 113)
c Medicare levy
d tax debt or tax refund.

12 A dishwasher has a selling price in Australia of $1246 (including GST). If GST is 10% of the original price, how much GST is included in this price?

Exercise 3.08

13 Use the table to find the VAT to be added on goods priced at:

Exercise 3.08

a 1262 pesos
b 235 new lei
c 26 950 rand.

Country	Currency	VAT
Hungary	forint	20%
Mexico	peso	15%
Romania	new lei	19%
South Africa	rand	14%

14 Use the table to calculate the price before VAT was added to:

Exercise 3.08

a a car purchased for 200 000 pesos
b a TV bought for 171 400 forint.

Qz
Chapter quiz

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Practice paper 1

Recommended time: One hour

Section 1
10 multiple-choice questions: 1 mark each
Select the correct answer, A, B, C or D.

Question 1

Exercise 2.01

Simplify $4ab + b^2 - 3ab - 4b^2$.

A $-2ab^3$ **B** $ab - 3b^2$
C $7ab - 3b^2$ **D** $ab - 5b^2$

Question 2

Exercise 1.03

Which one of these is an example of discrete data?

A the capacity of Warragamba Dam
B the number of tries scored by a football team over a season
C the height of a person in your class
D the time it took a triathlete to complete her race

Question 3

Exercise 2.01

Simplify $\frac{8t^6}{16t^2}$.

A $2t^4$ **B** $2t^3$ **C** $\frac{t^3}{2}$ **D** $\frac{t^4}{2}$

Question 4

Exercise 1.04

To decide upon the theme of the next school disco, Tasha surveyed a stratified sample of 50 students from the school, based on the following numbers of students in each Year group.

Year	7	8	9	10	11	12
Number of students	114	120	114	128	105	96

How many Year 12 students were in Tasha's sample?

A 4 **B** 7 **C** 8 **D** 14

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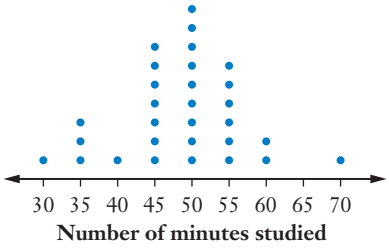
Question 5

Exercise 1.07

The dot plot shows the total number of minutes students studied to prepare for a mathematics assessment task.



Between which two values is the data clustered?

A 35 and 55 **B** 40 and 60 **C** 45 and 55 **D** 45 and 60

Question 6

Exercise 2.04

Solve $5 - 2x = -6$.

A $x = -5\frac{1}{2}$ **B** $x = 5\frac{1}{2}$ **C** $x = -\frac{1}{2}$ **D** $x = \frac{1}{2}$

Question 7

Exercise 2.02

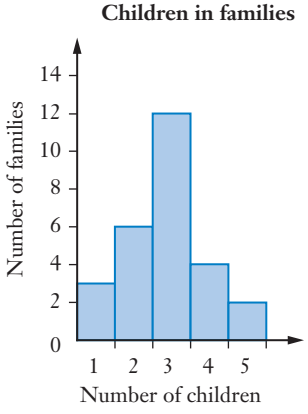
Use the formula $v = u + at$ to find the value of v if $u = 8$, $a = 10$ and $t = 5$.

A $v = 90$ **B** $v = 58$ **C** $v = 42$ **D** $v = 113$

Question 8

Exercise 1.06

This frequency histogram shows the results of a survey about the number of children in each family.



How many families were surveyed?

A 12 **B** 15 **C** 27 **D** 77

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Exercise 3.02

Question 9

Jamie's cookbook sells for $29.00 per copy. He earns a royalty of 12% of the selling price of the book. How much money will Jamie receive if 7200 books are sold in total?

A $25 056 **B** $11 232 **C** $13 824 **D** $38 880

Exercise 3.07

Question 10

A camera has a price of $137.50 after a GST of 10% is added. What was the camera's price before GST?

A $123.75 **B** $125.15 **C** $127.50 **D** $125

Section 2
3 questions: 10 marks each

Exercise 2.04

Question 11 (10 marks)

a Solve $9x - 11 = 4x + 7$. **2 marks**

Exercise 1.07

b The waiting times, in minutes, for patients at the Westvale Health Centre are shown in the stem-and-leaf plot.

Stem	Leaf
0	2 3 4 5
1	0 1 3 6 □ 8
2	1 1 3 4 4
3	1 2
4	1
5	1

i One entry is missing and is shown as □.
What *waiting time* could be represented by this entry? **1 mark**

ii How many patients were surveyed? **1 mark**

iii Calculate, correct to one decimal place, the percentage of patients who waited for longer than half an hour. **1 mark**

iv What was the shortest waiting time? **1 mark**

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c The table below shows the weekly pay of four employees at Wonka's chocolate factory. Overtime is paid at time-and-a-half. Complete the table by calculating the missing values at **i**, **ii**, **iii** and **iv**. **4 marks**

Exercise 3.01

Name	Pay rate (per hour)	Normal hours	Overtime hours	Weekly pay
Ilaria	$25.00	38	2	**i**
Kai	$33.50	12	**ii**	$452.25
Lauren	**iii**	38	4	$1204.72
Harry	$31.00	**iv**	3	$976.50

Question 12 (10 marks)

a Expand and simplify $3(2p + 5) - 4(p + 3)$. **1 mark**

Exercise 2.01

b Zayn needs to give his son some medicine. He uses Clark's rule: $D = \frac{kA}{70}$

where D is the child dosage, k is the mass of the child in kilograms and A is the adult dosage.

Zayn's son is 6 years old and weighs 22 kg. The adult dosage is 10 mL, taken every morning and every night. How many days will a 300 mL bottle of medicine last for Zayn's child? **2 marks**

Exercise 2.03

c Bree earns $925 per week. Calculate her holiday loading if it is 17.5% of 4 weeks pay. **2 marks**

Exercise 3.03

d Jarryd earns 2% commission from each house he sells. How much would he earn in a month in which he sells three houses for $810 000, $675 000 and $408 600? **2 marks**

Exercise 3.02

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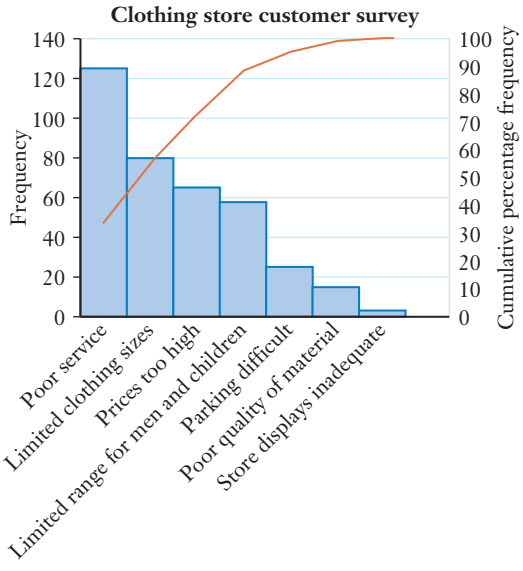
Exercise 1.05

e This Pareto chart shows the result of a customer survey for a clothing store.



i What are the two most common complaints? **1 mark**

ii Is the following statement true or false?

'More customers complained about the limited range of clothing than the poor quality of the material.' **1 mark**

iii What two things could the manager do to improve the store? **1 mark**

Question 13 (10 marks)

Exercise 2.05

a The following formula gives the number of hours it takes for a man's blood alcohol content (BAC) to return to zero after he has been consuming alcohol:

$$\text{Number of hours} = \frac{\text{BAC}}{0.015}$$

If Ben has a BAC of 0.026, how many hours and minutes will it take for his BAC to return to zero? **2 marks**

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b The masses, in kilograms, of 20 Year 11 students were recorded:

58	43	62	52	49	48	52	45	68	72
54	41	65	41	48	51	60	66	57	69

i Are people's masses discrete data or continuous data? **1 mark**

ii Organise the data into a frequency table with classes 40–< 45, 45–< 50, and so on. **1 mark**

iii Draw a frequency histogram to represent the data. **2 marks**

iv What percentage of students were 60 kg or more? **1 mark**

c Phoebe has a gross weekly income of $1874.60. Last financial year, she had allowable tax deductions totalling $2415.00. Calculate Phoebe's taxable income for that year, then use the table below to calculate her income tax. **3 marks**

Income tax rates for Australian residents	
Taxable income	**Tax on this income**
$0–$18 200	Nil
$18 201–$37 000	19c for each $1 over $18 200
$37 001–$87 000	$3572 plus 32.5c for each $1 over $37 000
$87 001–$180 000	$19 822 plus 37c for each $1 over $87 000
$180 001 and over	$54 232 plus 45c for each $1 over $180 000

END OF EXAMINATION

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STATISTICAL ANALYSIS

4.

PROBABILITY

Insurance originated in ancient Babylon and China around 3000 BCE, when voyagers and traders formed financial societies to insure their ships against damage or loss. In 600 CE, the Greeks and Romans organised similar schemes to help families during times of illness and to pay funeral expenses upon death. In 1693, the English astronomer and mathematician Edmund Halley, who discovered Halley's Comet, calculated the chances of people dying at different ages. Modern insurance companies use statisticians called actuaries to study the chances of losses and deaths occurring, such as:

- dying in a plane crash — 1 chance in 720 000
- being struck by lightning — 1 chance in 200 000
- dying in a car crash — 1 chance in 5000
- being in a house fire — 1 chance in 800
- going to prison — Male: 1 chance in 800. Female: 1 chance in 24 000.

CHAPTER OUTLINE

S2 4.01 Probability of simple events
S2 4.02 Tables and tree diagrams
S2 4.03 Complementary events
S2 4.04 Relative frequency
S2 4.05 Comparing relative frequency and theoretical probability
S2 4.06 Probability tree diagrams

IN THIS CHAPTER YOU WILL:

- use the formula $P(\text{event}) = \dfrac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$
- recognise that P(event) has a range of 0 to 1
- use tables and tree diagrams to solve problems involving multi-stage events
- understand complementary events and use the formula P(an event does not occur) = 1 – P(the event does occur).
- perform simple experiments and calculate relative frequency
- compare relative frequencies with theoretical probabilities

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TERMINOLOGY

at least
chance
event
likely
probability
sample space
calculated probability
complementary event
frequency
multi-stage event
probability tree digram
tree diagram
certain
equally likely
impossible
outcome
relative frequency
unlikely

Assignment 4

SkillCheck

1 Simplify each fraction.

a $\frac{28}{100}$ **b** $\frac{7}{77}$ **c** $\frac{13}{52}$ **d** $\frac{24}{36}$

2 Evaluate:

a $1-\frac{13}{40}$ **b** $1-\frac{22}{25}$ **c** $1-\frac{9}{72}$ **d** $1-\frac{16}{36}$

3 Convert each fraction to a decimal.

a $\frac{17}{20}$ **b** $\frac{9}{36}$ **c** $\frac{25}{40}$ **d** $\frac{4}{15}$

4 Convert each fraction to a percentage.

a $\frac{10}{25}$ **b** $\frac{45}{72}$ **c** $\frac{4}{52}$ **d** $\frac{16}{48}$

5 How many:

a cards in a normal deck of playing cards?
b numbers on a die?
c months in a year?
d letters in the alphabet?
e Aces in a deck of playing cards?
f vowels in the alphabet?

6 A sock drawer contains 8 white socks, 2 black socks, 4 grey socks and 6 blue socks.

a What fraction of the socks are grey?
b What percentage of the socks are blue?
c What fraction of the socks are not white?
d What percentage of the socks are black or blue?

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7 A sample of students was surveyed on the number of cars owned by their families and the results are shown in the table.

Number of cars	Frequency
0	4
1	16
2	11
3	0
4	1

a Illustrate this data on a frequency histogram.

b What fraction of the sample owned no cars?

c What percentage of the sample owned three cars?

d What percentage (correct to one decimal place) of the sample owned at least one car?

e What was the most frequent number of cars?

8 Rate each of the following events as being very unlikely (VU), unlikely (U), likely (L) or almost certain (AC).

a It will rain in your area tomorrow.

b A car number plate contains numbers.

c A mother has triplets.

d There is a traffic jam in Sydney on Friday morning.

e You will live to be 100 years old.

f You arrive at school on time on Monday.

g There is a hailstorm in your area this week.

h You will send an email today.

i A double six appears when a pair of dice is rolled.

j An adult is married.

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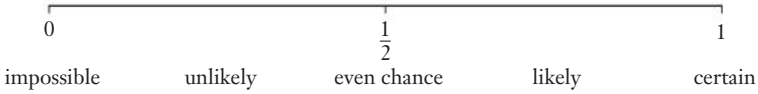
Games of chance

4.01 Probability of simple events

Probability or **chance** can be measured on a 'likelihood scale' between 'impossible' and 'certain', or on a numerical scale between 0 and 1.



The language of chance

Sample space

Probability terminology

An **outcome** is the result of an experiment or game. When a die is rolled, there are six possible outcomes: 1, 2, 3, 4, 5 or 6.

The **sample space** is the set of all possible outcomes, for example {1, 2, 3, 4, 5, 6} when a die is rolled.

An **event** is a group of one or more outcomes, for example the event of rolling an even number on a die is {2, 4, 6}.

EXAMPLE 1

List the sample space and count the number of outcomes for each situation.

a tossing a coin

b the colour shown on a traffic light

c the last digit of a phone number

Solution

a {head, tail}, 2 outcomes

b {red, amber, green}, 3 outcomes

c {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, 10 outcomes

Equally likely outcomes

We assume that all outcomes of an experiment or game are **equally likely** (have an equal chance). However, sometimes this is not the case because:

- a biased coin may show heads more often
- loaded dice show some numbers more often
- in a competition some players or teams may be more skilled.

Can you think of another situation where outcomes are not equally likely?

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EXAMPLE 2

A letter of the alphabet is chosen **at random** from the page of a magazine.

'At random' means that every letter on the page has an equal chance of being chosen.

a How many outcomes are there in the sample space?

b Are all the outcomes equally likely?

Solution

a There are 26 outcomes, because there are 26 letters in the alphabet.

b The outcomes are not equally likely, because some letters appear in words more often (for example, 'e').

Probability of an event

The probability of an event occurring, where all outcomes in the sample space are equally likely, is given by the formula:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

Probability can be expressed as a fraction, a decimal or a percentage.

EXAMPLE 3

A bag of jelly lollies contains 6 red lollies, 7 green lollies, 6 orange lollies, 5 yellow lollies and 4 white lollies. If one lolly is chosen at random, what is the probability that it is:

a red? **b** orange or yellow?

c blue? **d** not white?

Solution

Total number of outcomes = 6 + 7 + 6 + 5 + 4 = 28

There are 28 lollies and, therefore, 28 possible outcomes in the sample space.

a $P(\text{red}) = \frac{6}{28} = \frac{3}{14}$ 6 red lollies

b $P(\text{orange or yellow}) = \frac{6+5}{28} = \frac{11}{28}$

c $P(\text{blue}) = \frac{0}{28} = 0$ No blue lollies

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d $P(\text{not white}) = \dfrac{6+7+6+5}{28}$ 24 non-white lollies

$= \dfrac{24}{28}$

$= \dfrac{6}{7}$

Exercise 4.01 Probability of simple events

Example 1

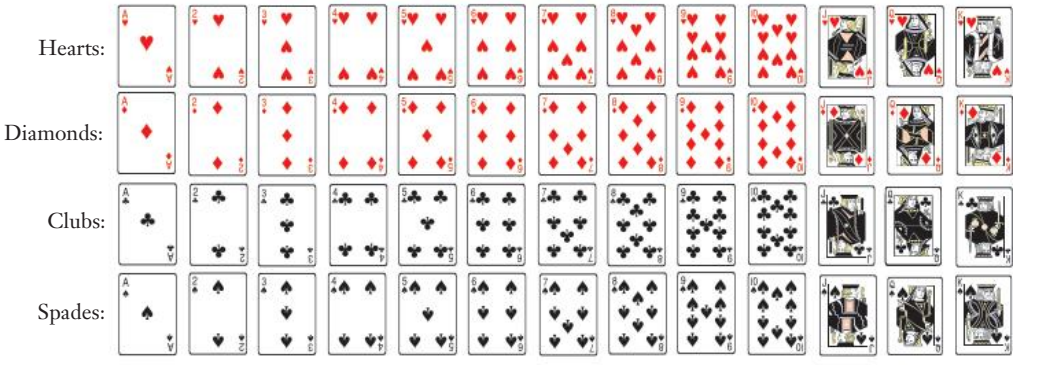
1 List the sample space and write the number of outcomes for each of the following situations.

- **a** selecting a day of the week to go shopping
- **b** choosing a letter from the word 'MATHEMATICS'
- **c** the classification rating of a movie
- **d** the gender of the next baby born at a hospital
- **e** the result of a driving test

2 Count the number of outcomes in the sample space for each of these.

- **a** the consonants from the alphabet
- **b** choosing the month to set a wedding date
- **c** choosing a four-digit PIN for accessing an ATM
- **d** the Year group of a primary school student
- **e** rolling a die in the shape of a triangular pyramid

3 A normal deck of playing cards contains 52 cards. There are four suits of 13 cards each.



One card is drawn at random from a deck. Count the number of possible outcomes for:

- **a** a red card
- **b** a 10
- **c** an even number
- **d** a black Ace
- **e** a picture card
- **f** the Queen of hearts

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4 If England plays Brazil in a soccer match: Example 2

a what are the three possible outcomes in the sample space for the result of the game?

b are all three outcomes equally likely?

5 Explain what is wrong with the following statement: 'Since the weather is either fine or raining each day, the probability of a fine day is $\frac{1}{2}$.'

6 A bag contains 6 blue marbles and 3 white marbles. If one marble is selected without looking, what is the probability that it is: Example 3

a blue? **b** white? **c** red? **d** blue or white?

7 The numbers 1 to 15 are written on separate cards. A card is selected at random. What is the probability that this number:

a is even? **b** is odd?

c is 9? **d** contains the digit 1?

8 A letter is chosen at random from the words 'NEW CENTURY'. Express as a decimal the probability that the chosen letter is:

a E **b** a vowel

c W or Y **d** a letter after N in the alphabet.

9 A multiple-choice question has answers labelled A, B, C and D. If you know the answer is not B, what is the probability of guessing the correct answer?

10 Adriana's purse has four $2 coins, one $1 coin, two 50-cent coins, six 20-cent coins, eight 10-cent coins and three 5-cent coins. If she takes out a coin at random, what is the percentage probability that it is:

a a 10-cent coin? **b** a $1 coin? **c** a silver coin?

d not a 20-cent coin? **e** a $2 coin or a 50-cent coin?

11 Ken, Lydia, Melanie, Nathan, Olivia and Paula wrote their names on separate cards. Which of the following is the probability that a card chosen at random will have a girl's name on it? Select **A**, **B**, **C** or **D**.

A 25% **B** $33\frac{1}{3}\%$ **C** 50% **D** $66\frac{2}{3}\%$

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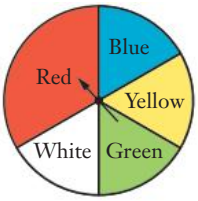
12 This spinner has a red sector that is twice as large as each sector of another colour. If the spinner is spun, find the probability that the arrow points to:

a red **b** white **c** red or green **d** not blue.



13 One card is selected at random from a deck of playing cards. What is the probability it is:

a a red card? **b** a black picture card?

c a King? **d** a number card?

e a red 4? **f** the Ace of spades?

g the Queen of hearts or the Jack of clubs? **h** a diamond?

14 A total of 1200 raffle tickets were sold at the school fete. Ramy bought eight tickets. What is the probability that he wins first prize?

15 What is the probability that a person chosen at random was born in a month beginning with A? Express the answer as a recurring decimal.

16 Ryan says that, because there are 16 teams in the NRL football competition, his favourite team, the Eels, has a chance of $\frac{1}{16}$ of winning the competition this year. Why is he incorrect in saying this?

17 A number from 1 to 20 is chosen at random by a computer. Find the percentage probability that it is:

a greater than 8 **b** a square number **c** divisible by 3.

18 A loaded die is biased so that it lands on 6 twice as often as it lands on any other number. Find the probability that, when it is rolled, it lands on:

a 6 **b** an even number **c** 5

d a number greater than 2 **e** a number other than 6 **f** 7

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TECHNOLOGY

Tossing a coin

A spreadsheet can be used to simulate tossing a coin 20 times, by making the spreadsheet generate, at random, either the number 0 (for tails) or 1 (for heads).

Type in cell A1 of a new spreadsheet the bold heading 'Tossing a coin 20 times', and in cell A2, type '0 = tails, 1 = heads'.

In cell B4, enter the formula **=INT(RAND()*2)** to generate a random number 0 or 1 in that cell.

For an explanation of the INT and RAND functions on a spreadsheet, see 'Technology: Generating random numbers' on page 160.

Fill right to copy the formula across the row to cell F4 and simulate four more tosses of the coin.

Fill down to copy the formulas from rows 4 to 7, for a total of 20 tosses. This should give a random series of 0s and 1s, similar to the spreadsheet shown below.

	A	B	C	D	E	F
1	**Tossing a coin 20 times**					
2	0 = tails, 1 = heads					
3						
4		1	1	0	0	1
5		1	0	0	0	1
6		1	0	0	1	0
7		0	0	0	1	0

Decide how many times you would you expect heads (1) to come up in 20 tosses of a coin. Are there close to this many 1s in your 20 tosses?

To simulate the toss of the coin another 20 times, press the **F9** key to 'recalculate' another set of random numbers. Count how many 'heads' this time. (Is the number close to 10?)

Press the **F9** key each time you want to toss the coin another 20 times. Count how many heads each time.

We can make the spreadsheet count the number of heads by adding all 20 random values together, since heads = 1, tails = 0. In cell B9, type 'Number of heads' and in cell C9, type **=sum(B4:F7)**.

Press the **F9** key a few more times to see how many heads appear in every 20 tosses of the coin.

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Tree diagrams

4.02 Tables and tree diagrams

A **multi-stage experiment** has two or more stages occurring, for example:

- rolling 2 dice (2 stages)
- tossing a coin 3 times (3 stages)
- observing the weather for each day of a long weekend (3 stages)
- drawing out 5 winning raffle tickets from a barrel. (5 stages)

A **multi-stage event** consists of 2 or more events occurring together, such as rolling a sum of 10 on a pair of dice, or getting rain 3 days in a row. The sample space of a multi-stage experiment can be found using a **list**, a **table** or a **tree diagram**.

EXAMPLE 4

A coin is tossed twice.

a Show all the outcomes in the sample space using:

i a list **ii** a table **iii** a tree diagram.

b What is the probability of tossing:

i 2 heads? **ii** at least one tail?

Solution

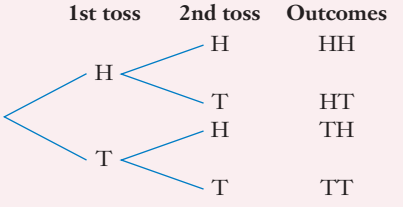
a Let H = heads, and T = tails.

i Using a list:
Sample space = {HH, HT, TH, TT}

ii Using a table:

2nd toss \ 1st toss	H	T
H	HH	TH
T	HT	TT

iii Using a tree diagram:



A tree diagram shows all the alternative pathways at each stage.

For the 1st toss, you could get heads or tails. If heads, then for the 2nd toss you could get heads or tails. If you get tails for the 1st toss, then for the 2nd toss you could get heads or tails.

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b **i** $P(2 \text{ heads}) = P(\text{HH})$

$$= \frac{1}{4}$$

ii $P(\textbf{at least} \text{ one tail}) = P(\text{HT or TH or TT})$

'At least one' means 1 or more (that is, not none).

$$= \frac{3}{4}$$

Tables and tree diagrams are effective because they ensure that all possible arrangements are included.

A **table** can list the possible outcomes of a *two-stage* experiment.

A **tree diagram** lists the possible outcomes of an experiment with *two or more stages*. Branches stretch out to show the possible pathways of outcomes at each stage. An outcomes column at the end of the diagram lists the sample space.

EXAMPLE 5

Tables and tree diagrams

There are 8 people in a dance class: 3 men (Grant, Rick, Stefan) and 5 women (Bethany, Tania, Elise, Chloë, Felicity).

a List all possible male–female dancing couples, using:

i a table **ii** a tree diagram.

b How many dancing couples are possible?

c If one of these dancing couples is selected at random, what is the probability that:

i it is Rick and either Tania or Chloë?

ii it does not include Grant or Felicity?

iii it includes Stefan but not Bethany?

Solution

a Let: G = Grant R = Rick S = Stefan B = Bethany
T = Tania E = Elise C = Chloë F = Felicity

i Using a table:

		Men		
		G	**R**	**S**
Women	**B**	GB	RB	SB
	T	GT	RT	ST
	E	GE	RE	SE
	C	GC	RC	SC
	F	GF	RF	SF

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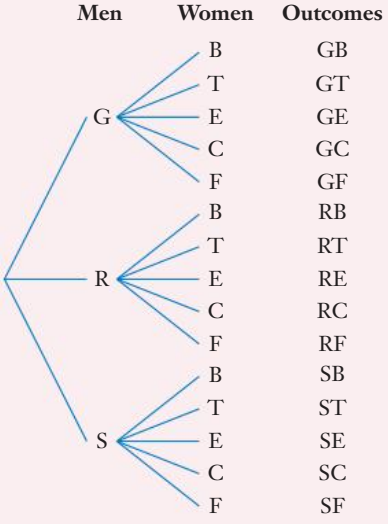
ii Using a tree diagram:



b There are 15 possible dancing couples.

Note that, when there are 3 men and 5 women, there are $3 \times 5 = 15$ possible couples.

c **i** $P(\text{Rick, Tania/Chloë}) = P(\text{RT or RC})$

$$= \frac{2}{15}$$

ii $P(\text{not Grant, not Felicity}) = P(\text{all outcomes without a G or F})$

$$= \frac{8}{15}$$

iii $P(\text{Stefan, not Bethany}) = P(\text{ST or SE or SC or SF})$

$$= \frac{4}{15}$$

Exercise 4.02 Tables and tree diagrams

1 Match each description (**a** to **i**) below with the correct set of *all* counting numbers that meet that description (**A** to **G**).

a	at least 3	**b**	from 1 to 4	**c**	greater than 3	**d**	3 or more
e	between 1 and 4	**f**	at most 3	**g**	3 or less	**h**	less than 3
i	fewer than 5						
A	{2, 3}	**B**	{0, 1, 2, 3}	**C**	{0, 1, 2, 3, 4}	**D**	{3, 4, 5, …}
E	{4, 5, 6, …}	**F**	{0, 1, 2}	**G**	{1, 2, 3, 4}		

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Example 4

2 **a** How many outcomes are possible when:

i tossing a coin? **ii** tossing a die?

b A coin and a die are tossed together. Show all outcomes in the sample space, using:

i a list **ii** a table **iii** a tree diagram.

c How many outcomes are in the sample space?

d Find the probability of tossing:

i heads and an even number

ii tails and a number less than 3

iii heads and not 1.

3 The digits 4, 5, 7 and 8 are written on separate cards and placed in a box. Two cards are drawn out at random to form a two-digit number. Note that the same digit cannot be used twice, so '75' is allowed but '77' is not.

a Use a tree diagram to list all possible two-digit numbers.

b How many two-digit numbers are possible?

c Find, as a percentage, the probability that the number drawn:

i has a first digit of 7 **ii** is odd

iii is less than 60 **iv** is divisible by 5.

4 A coin is tossed three times.

a Show all the possible outcomes of heads (H) and tails (T) using:

i a list

ii a tree diagram with three stages.

b How many outcomes are there in the sample space?

c Find, as a decimal, the probability of tossing:

i exactly one tail

ii at least two heads

iii three heads

iv more tails than heads.

5 How many possible outcomes are there when a coin is tossed:

a once? **b** two times?

c four times? **d** 10 times?

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Example 5

6 Two dice are rolled together and their sum is calculated.

a Copy and complete this table of possible sums.

		1st die					
	+	1	2	3	4	5	6
2nd die	1						
	2						
	3						
	4						
	5						
	6						

b What is the total number of possible outcomes?

c Find the probability of rolling a sum:

i of 10 **ii** of 6 **iii** less than 4

iv of 1 **v** that is odd **vi** of 7 or 11.

d Find the percentage probability of rolling:

i a 'double one' (1 and 1)

ii a 'double' anything (the same number twice).

e Which sum(s) is:

i most likely? **ii** least likely?

7 For breakfast, when staying at a hotel, Simone can choose one item from each course of a set menu.

1st course	2nd course
Cereal (C)	Bacon and eggs (B)
Raisin toast (R)	Pancakes (P)
Watermelon (W)	Sausages and hash browns (S)
Yoghurt (Y)	

a Use a table to list all the different two-course breakfasts available.

b How many two-course breakfasts are possible?

c If one of the combinations is chosen at random, find the probability that it includes:

i cereal or raisin toast

ii watermelon but not pancakes

iii yoghurt or pancakes

iv yoghurt and pancakes.

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8 Harry, Rina, Alison, Kirstie and Thuc are on the formal committee. They need to select a chairperson and a secretary.

a Use a table to determine all the possible pairings of chairperson and secretary. (Remember that a person cannot hold both positions.)

b If each person is equally likely to be selected, find the decimal probability that:

i Kirstie is chairperson and Thuc is secretary

ii Rina is either chairperson or secretary

iii Alison is secretary

iv Harry and Thuc fill the positions (in any order).

9 Consider a family with three children.

a Use a tree diagram to list the possible order of boys and girls for the children in the family.

b Which of the following is the number of possible outcomes? Select **A**, **B**, **C** or **D**.

A 8 **B** 16 **C** 7 **D** 6

c Find the probability of having:

i no boys

ii exactly two boys

iii at least one girl

iv at most one girl

v the eldest and youngest both being boys

vi more than one girl.

10 Suppose that the weather forecast for each day of the week is sunny, cloudy or raining, each being equally likely.

a Use a tree diagram to show all the possible outcomes for the weather for Saturday and Sunday, the two days of the weekend.

b Which of the following is the number of stages in this experiment? Select **A**, **B**, **C** or **D**.

A 2 **B** 3 **C** 4 **D** 6

c How many outcomes are possible?

d Find the probability that:

i it rains on both days

ii the weather is the same on both days

iii it doesn't rain on both days

iv it is sunny on at least one of the days

v it is cloudy on one of the days and sunny on the other.

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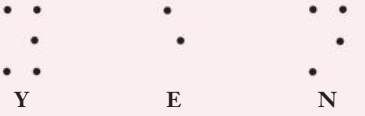
DID YOU KNOW?

The Braille alphabet

The Braille alphabet is a system of raised dots that allows blind people to read by touch. Letters and other symbols are represented by a cell of 1 to 6 dots printed in two columns. For example, the cells for the letters Y, E and N are:



There are also Chinese, Russian, Vietnamese and Korean versions of Braille. Frenchman Louis Braille invented the Braille alphabet in 1821. He became blind at 4 years of age after poking his eye accidentally with a saddler's awl (a tool for making holes in leather).

How many different cells are possible using the Braille system? (*Note*: A completely blank cell is allowed.)

4.03 Complementary events

The range of probability

Because the probability of an event is a fraction, as shown below, its value ranges from 0 to 1 (or, as a percentage, from 0% to 100%):

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

- A **certain** event must occur and has a probability of 1.
- An **impossible** event cannot occur and has a probability of 0.

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EXAMPLE 6

A die is rolled. Find:

a $P(\text{rolling a 3}) + P(\text{not rolling a 3})$

b $P(\text{rolling a number less than 5}) + P(\text{rolling 5 or more})$

c $P(\text{rolling an odd number}) + P(\text{rolling an even number})$.

Solution

a $P(3) + P(\text{not } 3) = P(3) + P(1, 2, 4, 5, 6) = \frac{1}{6} + \frac{5}{6} = 1$

b $P(< 5) + P(\geq 5) = P(1, 2, 3, 4) + P(5, 6) = \frac{2}{3} + \frac{1}{3} = 1$

c $P(\text{odd}) + P(\text{even}) = P(1, 3, 5) + P(2, 4, 6) = \frac{3}{6} + \frac{3}{6} = 1$

The probability of an event not occurring

'Rolling 3 on a die' and 'not rolling 3 on a die' are examples of **complementary events** (or one event and its 'opposite'). 'Complement' means something that completes or match with something to make a whole, and complementary events cover all the possibilities of a experiment. Because of this, the probabilities of complementary events must add to 1. Other examples of complementary events are:

- 'rolling a number less than 5', and 'rolling a number 5 or more'
- 'rolling an odd number', and 'rolling an even number'.

The 'complement of an event E' means all those outcomes that are not E, written as $\overline{\text{E}}$, and $P(\overline{\text{E}})$ is the probability of E *not occurring*.

Complementary events

$$P(\text{event does occur}) + P(\text{event does not occur}) = 1$$

$$P(\text{event does not occur}) = 1 - P(\text{event does occur})$$

$$P(\overline{\text{E}}) = 1 - P(\text{E})$$

If the probability is in percentage form, then $P(\overline{\text{E}}) = 100\% - P(\text{E})$.

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EXAMPLE 7

A ball is selected at random from a bag containing five green, seven red and two yellow balls.

Find the probability that the ball selected is:

a yellow **b** not yellow **c** blue

d not blue **e** not red **f** not red or not green.

Solution

Total number of balls = 5 + 7 + 2 = 14

a $P(\text{yellow}) = \frac{2}{14} = \frac{1}{7}$

b $P(\text{not yellow}) = 1 - P(\text{yellow})$

$$= 1 - \frac{1}{7}$$

$$= \frac{6}{7}$$

c $P(\text{blue}) = \frac{0}{14} = 0$ There are no blue balls. This is an impossible event.

d $P(\text{not blue}) = 1 - P(\text{blue})$

$$= 1 - 0$$

$$= 1$$

All balls are not blue. This is a certain event.

e $P(\text{not red}) = 1 - P(\text{red})$

$$= 1 - \frac{7}{14}$$

$$= \frac{7}{14}$$

$$= \frac{1}{2}$$

f $P(\text{not red or green}) = 1 - P(\text{red or green})$

$$= 1 - \frac{7+5}{14}$$

$$= \frac{1}{7}$$

Why is this answer the same as for part **a**?

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Exercise 4.03 Complementary events

1 Write the complementary event for each of the following events.

a getting a tail when a coin is tossed
b selecting a heart card from a deck of playing cards
c failing a driving test
d having over three children in your family
e selecting a black sock from a drawer of black socks and white socks
f the weather today being rainy
g winning a swimming race
h a traffic light showing green

2 A die is rolled. What is the probability that the result is: Example 6

a not a 6? **b** not less than 3? **c** not a multiple of 3?

3 Two coins are tossed together. List the sample space, then find the probability that the result is:

a two tails **b** not two tails **c** one tail
d not one tail **e** no tails **f** at least one tail.

4 Repeat Question **3** but with three coins tossed together.

5 A computer randomly selects a number from 1 to 20. Find the decimal probability that it is: Example 7

a not less than 10 **b** not divisible by 3 **c** not a two-digit number
d not containing a 7 **e** not divisible by 5 **f** not a factor of 20.

6 Samantha bought a ticket in a raffle in which 1000 tickets were sold. If there are eight prizes, which of the following is the probability that Samantha doesn't win a prize? Select **A**, **B**, **C** or **D**.

A $\frac{124}{125}$ **B** $\frac{7}{8}$ **C** $\frac{1007}{1008}$ **D** $\frac{999}{1000}$

7 Each of the letters in the word 'PROBABILITY' is written on a piece of paper and put into a box. What is the probability that a letter drawn out at random is:

a not 'P'? **b** not 'B'?
c not a vowel? **d** not 'A' or 'B'?

8 What is the probability that a baby chosen at random from a maternity hospital was not born on a Saturday or Sunday?

9 The ratio of green to yellow to white lollies in a jar is 4 : 3: 2. One lolly is selected at random. Find the probability that it is:

a not green **b** yellow **c** not white
d not yellow **e** not red **f** not green or yellow.

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10 In the dice game 'Craps', players lose if they roll a sum of 2 or 12 on a pair of dice. Which of the following is the probability of not losing? Select **A**, **B**, **C** or **D**.

A $\frac{5}{6}$ **B** $\frac{17}{18}$ **C** $\frac{35}{36}$ **D** $\frac{8}{9}$

11 The probability of rain this weekend is 23%. What is the probability that it will not rain?

12 A deck of cards is shuffled and one card is selected at random. Find:

a P(not a 10) **b** P(not a picture card) **c** P(not a club)

d P(not a black card) **e** P(not a red Ace) **f** P(not a black 7, 8, 9 or 10).

13 A traffic light is red for 63 seconds, green for 99 seconds and amber for 2 seconds. Express, correct to three decimal places, the probability that a car faces a traffic light that is:

a red

b not red

c not amber

d not green

e not red or amber

f not red, green or amber.

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14 What is the probability that a student chosen, at random, from your class was not born on 3 June?

15 What is the percentage probability that a mobile phone number selected at random does not end in 0 or 9?

16 In a football match, the Knights have a 42% chance of beating the Dragons, while the Dragons have a 51% chance of beating the Knights.

a What other outcome is possible?

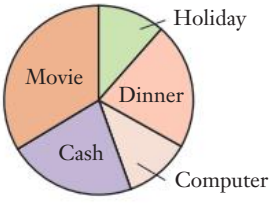
b What is the probability of this outcome?

17 The probability that Westvale High will win the debate is $\frac{13}{20}$. What is the probability that Westvale High will not win?

18 Lisa spins the wheel shown to determine her prize. What is the probability that she:

a does not win the holiday?

b does not win the cash or computer?



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4.04 Relative frequency

Experiments or surveys are used to predict the probability of an event when it is not feasible to calculate the probability of that event, for example, testing the effectiveness of batteries or predicting the outcome of a football match. The values obtained from repeated trials of an event are used to find the **experimental probability**. We call this the **relative frequency** of the event.

Matching probabilities

A page of spinners

Coins probability

Dice probability

Greedy Pig game

Relative frequency

$$\text{Relative frequency of an event} = \frac{\text{frequency of the event}}{\text{total frequency}}$$

EXAMPLE 8

This table shows the times of day that car accidents occurred in Uptown last month.

Time of accident	Frequency
midnight–4.00 a.m.	4
4.00 a.m.–8.00 a.m.	13
8.00 a.m.–noon	17
noon–4.00 p.m.	20
4.00 p.m.–8.00 p.m.	23
8.00 p.m.–midnight	7

Based on these results, what is the relative frequency of a car accident in Uptown occurring:

a between midnight and 4.00 a.m.?

b between 8.00 a.m. and 4.00 p.m.?

c after midday?

d earlier than 8.00 p.m.?

Solution

Total frequency = 4 + 13 + 17 + 20 + 23 + 7 = 84

There were 84 road accidents that month.

a $P(\text{midnight–4.00 a.m.}) = \frac{4}{84} = \frac{1}{21}$

b $P(\text{8.00 a.m.–4.00 p.m.}) = \frac{17+20}{84} = \frac{37}{84}$

c $P(\text{after midday}) = \frac{20+23+7}{84} = \frac{50}{84} = \frac{25}{42}$

d $P(\text{earlier than 8.00 p.m.}) = 1 - P(\text{after 8.00 p.m.})$

$$= 1 - \frac{7}{84}$$

$$= \frac{11}{12}$$

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EXAMPLE 9

A pair of dice was rolled 50 times and their sum calculated each time. The results are shown in this table.

Sum	2	3	4	5	6	7	8	9	10	11	12
Frequency	0	2	4	6	5	5	9	6	8	3	2

a Draw a dot plot to illustrate this data.

b Which sum is:

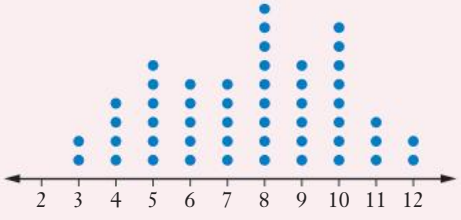
i most likely? **ii** least likely?

c Calculate, as a decimal, the relative frequency of rolling a sum:

i of 7 or 11 **ii** that is an odd number **iii** greater than 9.

Solution

a



b **i** 8 is most likely — It has the highest frequency (9).

ii 2 is least likely — It has the lowest frequency (0).

c **i** $P(7 \text{ or } 11) = \frac{5+3}{40} = \frac{8}{40} = 0.2$

ii $P(\text{odd}) = \frac{2+6+5+6+3}{40} = \frac{22}{40} = 0.55$

iii $P(> 9) = \frac{8+3+2}{40} = \frac{13}{40} = 0.325$

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Exercise 4.04 Relative frequency

1 A sample of matchboxes was taken and the number of matches in each box was counted. The results are shown in this table.

Example 8

Number of matches	Frequency
48	2
49	15
50	70
51	56
52	6
> 52	1

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a How many matchboxes were in the sample?

b What is the relative frequency of a matchbox containing:

i 51 matches?

ii more than 50 matches?

iii 49 or fewer matches?

2 A die was rolled 80 times, with the results shown below.

Outcome	1	2	3	4	5	6
Frequency	11	13	9	13	12	22

a Are the outcomes equally likely?

b Do you think this die is loaded? Give a reason for your answer.

c Express as a percentage the relative frequency of rolling 6 on this die.

d If this die was rolled 100 times, how many 6s would you expect?

3 Than surveyed the number of children in each house in her street, and found the following results:

Number of children	0	1	2	3	4	5
Frequency	5	10	14	7	3	1

a Which number of children is second-most likely?

b If a house is selected at random from Than's street, what is the relative frequency (as a decimal) that it contains:

i two children?

ii no children?

iii at least one child?

iv more than one child?

v three or more children?

vi at most three children?

c How many four-child families would you expect to find in a similar sample of 150 houses?

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4 In all of their home games, the Dragons won 16 matches and lost 12 matches. Which of the following is their experimental probability of losing a home game? Select **A**, **B**, **C** or **D**.

A 25% **B** $42\frac{6}{7}\%$ **C** $57\frac{1}{7}\%$ **D** 75%

5 The daily weather in Springfield during the month of June is shown in the table.

Weather	Number of days
Fine	13
Rain	10
Cloudy	7

a Are the outcomes equally likely?

b Find the relative frequency of the Springfield weather in June being:

i rainy

ii not cloudy.

6 At the 2016 Census, Australia's population was 23 401 892, including 11 855 248 females.

a What is the probability that a person chosen at random from the Australian population in 2016 was female? Express your answer as a percentage, correct to two decimal places.

b Is a person chosen from the 2016 Census more likely to be male or female? Why?

c When Australia's population reaches 25 000 000, how many *males* would you expect there to be, to the nearest thousand?

Example 9

7 Copy the following table and use it to record the results for throwing a drawing pin 30 times and observing whether it lands point up or point down.

Outcome	Tally	Frequency
Point up		
Point down		

a Are the outcomes equally likely?

b Graph the results on a sector graph.

c What is the relative frequency that a drawing pin lands point up?

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8 Copy and complete this table by noting the first letter of the surname of every student in your class.

Surname initial	Tally	Frequency
A–E		
F–J		
K–O		
P–T		
U–Z		

a Which class interval of initials is:

i least likely? **ii** most likely?

b What is the relative frequency of a student selected at random from the class having a surname beginning with a letter:

i from A to E? **ii** after O in the alphabet? **iii** not from K to O?

c If there are 120 students in Year 11, estimate how many students have a surname beginning with a letter from F to J.

9 Toss three coins together 40 times. Copy and complete this table by recording the number of tails that come up each time.

Alternatively, you could simulate tossing three coins together using a calculator or spreadsheet. (See 'Technology: Generating random numbers' next page.)

Number of tails	Tally	Frequency
0		
1		
2		
3		

a Are the outcomes equally likely? If not, which outcome is least likely?

b Express as a decimal the relative frequency of tossing:

i no tails **ii** fewer than three tails **iii** two heads.

c If three coins were tossed 100 times, estimate how many times two tails would result.

10 Draw a card at random from a deck of playing cards and record its suit in a table similar to the one shown. Do this 40 times, returning the card to the deck each time and reshuffling.

Suit	Tally	Frequency
Clubs		
Spades		
Hearts		
Diamonds		

a If a card is selected at random 40 times from a deck, how often should a hearts card be selected?

b Represent the data from your completed table on a sector graph.

c Find the relative frequency of selecting a card that is:

i a spade **ii** not hearts **iii** red.

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11 Copy and complete this table by recording the current ages of the students in your class.

Age	Tally	Frequency
15		
16		
17		
18		
19+		

a Represent this data on a dot plot.

b Which age is most frequent?

c Find the relative frequency of a student from your class:

i being aged 16 **ii** being aged below 18 **iii** not being aged 17.

d If there are 125 students in Year 11, estimate how many of them are 17 years old.

TECHNOLOGY

Generating random numbers

Random numbers can be generated on a calculator or spreadsheet using the RAN# or =RAND() functions. This function outputs a random decimal between 0 and 1 but we can use it to create a random number of any range.

To generate a random whole number from 0 to *n*

Multiply the random decimal generated (between 0 and 1) by $(n + 1)$, then ignore the decimal part of the number (for example read 1.092 as 1).

For example, to create a random number from 0 to 4:

- on a scientific calculator, type RAN × 5 =. Press = repeatedly for more random numbers.
- on a graphics calculator, enter **int(ran#×5)** from the probability functions and press EXE. Press EXE repeatedly for more random numbers.

The INT function outputs the integer (whole) part of a decimal only.

- on a spreadsheet, type into a cell **=INT(RND()*5)** and press the Enter key. Note that, unlike other spreadsheet functions, =RAND() does not require a cell address inside its brackets.

To generate a random whole number from 1 to *n*

Multiply the random decimal generated (between 0 and 1) by *n*, then add 1, and ignore the decimal part of the number.

For example, to create a random number from 1 to 6:

- on a scientific calculator, type RAN × 6 + 1. Press = repeatedly for more random numbers.
- on a graphics calculator, enter int(ran#×6+1) from the probability functions and press EXE. Press EXE repeatedly for more random numbers.
- on a spreadsheet, type into a cell **=INT(RND()*6+1)** and press the Enter key.

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TECHNOLOGY

Rolling a pair of dice

A spreadsheet can be used to simulate the rolling of two dice and for calculating their sum.

Step 1: Enter the labels on the right into a blank spreadsheet.

Step 2: We will simulate the rolling of two dice 10 times so, in the 'Trial' column, we need the numbers 1 to 10. In cell A5, enter **=A4+1** to get 2, then use **Fill down** to copy that formula to cell A13 to generate the rest of the Trial numbers.

	A	B	C	D
1	**Rolling a pair of dice**			
2				
3	Trial	Die 1	Die 2	Sum
4	1			
5				
6				
7				
8				
9				
10				
11				

Step 3: In cell B4, enter **=INT(RAND()*6+1)** to generate a random integer from 1 to 6 and simulate the rolling of the first die. Copy or use **Fill right** to simulate the rolling of the second die and check that you have two numbers from 1 to 6 for each trial.

Step 4: In cell D4, enter an appropriate formula to calculate the sum for the pair of dice.

Step 5: **Fill down** the formulas in columns B, C and D to row 13 so that there are 10 trials for the rolling of two dice.

Step 6: To generate another 10 trials, press the **F9** key to 'recalculate' another set of random dice numbers. Do these numbers seem reasonable for the rolling of dice?

Step 7: Keep pressing the **F9** key to simulate rolling the die another 10 times and observe the sums are generated. Does 7 come up more often than other numbers?

Step 8: Extend this spreadsheet to simulate 100 trials of rolling a pair of dice.

Step 9: Modify this spreadsheet to simulate the tossing of two coins (tails = 0, heads = 1) and change the 'Sum' column so it calculates the 'Number of heads'.

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Rolling a die

Probability of $\frac{1}{2}$

4.05 Comparing relative frequency and theoretical probability

How is the relative frequency (experimental probability) of an event related to its theoretical probability?

EXAMPLE 10

A die is rolled 60 times and the results are as shown.

Outcome	1	2	3	4	5	6
Frequency	6	13	12	8	9	12

a Find the relative frequency (experimental probability) of rolling a 4:

i as a fraction

ii as a decimal, correct to three decimal places.

b Find the theoretical probability of rolling a 4:

i as a fraction

ii as a decimal, correct to three decimal places.

c Are the experimental and theoretical probabilities similar?

d If a die is rolled 60 times, what is the expected number of times a 4 will be rolled?

e The die is rolled 300 times, with the following results.

Outcome	1	2	3	4	5	6
Frequency	46	48	46	52	51	57

Calculate, to three decimal places, the relative frequency of rolling a 4.

f What do you notice about the relative frequency and theoretical probability now?

g If a die is rolled 300 times, what is the expected number of times a 4 will be rolled?

Solution

a Relative frequency:

i $P(4) = \frac{8}{60} = \frac{2}{15}$ **ii** $P(4) = 0.1333\ldots \approx 0.133$

b Theoretical probability:

i $P(4) = \frac{1}{6}$ **ii** $P(4) = 0.1666\ldots \approx 0.167$

c By examining the decimal values, the relative frequency and theoretical probability are close.

d Expected number of 4s $= \frac{1}{6} \times 60 = 10$

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e Experimental $P(4) = \frac{52}{300} = 0.1733... \approx 0.173$

f The relative frequency is now closer to the theoretical probability (0.167).

g Expected number of 4s $= \frac{1}{6} \times 300 = 50$

Relative frequency and theoretical probability

As the number of trials increases, the relative frequency becomes closer to the theoretical probability. This is sometimes called 'the law of averages'.

Exercise 4.05 Comparing relative frequency and theoretical probability

This exercise may be completed as a group activity.

Example 10

1 Ethan tossed three coins together.

a List all the possible outcomes in the sample space.

b Write, as a percentage, the probability of tossing:

i two heads **ii** one or three heads.

c If Ethan tossed three coins 88 times, how many times should he get:

i two heads? **ii** one or three heads?

d Ethan actually had the following results:

Number of heads	0	1	2	3
Frequency	11	35	30	12

Write, as a percentage correct to two decimal places, the relative frequency of getting:

i two heads **ii** one or three heads.

e How do the relative frequencies compare with the theoretical probabilities?

f If three coins are tossed together 200 times, what is the expected number of times that:

i two heads will be tossed? **ii** one or three heads will be tossed?

g Stephanie tossed three coins 200 times with the following results:

Number of heads	0	1	2	3
Frequency	23	72	80	25

Write, as a percentage, the relative frequency of tossing:

i two heads **ii** one or three heads.

h Do the relative frequencies become closer to the theoretical probabilities with more repeated trials?

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2 Roll a die 150 times and record your results in a table similar to the one below.

Alternatively, you could simulate rolling a die using a calculator or spreadsheet. (See 'Technology: Generating random numbers' on page 160.)

Outcome	Tally	Frequency
1		
2		
3		
4		
5		
6		

a Express as a decimal, correct to three decimal places:

i the relative frequency of rolling a 1

ii the theoretical probability of rolling a 1

iii the relative frequency of rolling an even number

iv the theoretical probability of rolling an even number.

b What is the expected number of times a 1 should result when a die is rolled 150 times?

c By pooling your results with those of other students or groups in your class, repeat part a to see whether, with more trials, your relative frequencies become closer to the theoretical probabilities.

3 **a** Copy and complete this table of the possible sums when a pair of dice are rolled together.

		1st die					
	+	1	2	3	4	5	6
2nd die	1						
	2						
	3						
	4						
	5						
	6						

b As a decimal, correct to four decimal places, what is the theoretical probability of rolling a sum of:

i 4?

ii 2 or 12?

iii 7 or 11?

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c Copy this table, then roll a pair of dice 120 times and record your results in the table.

Sum	Tally	Frequency
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

d If a pair of dice is rolled 120 times, what is the expected number of:

i 4s?

ii 2s or 12s?

iii 7s or 11s?

e As a decimal, correct to four decimal places, what is the relative frequency of rolling a sum of:

i 4?

ii 2 or 12?

iii 7 or 11?

4 A card is randomly drawn from a deck of cards to see if it is a picture card (Jack, Queen or King). This is done 78 times, with each card being replaced in the deck before the next draw. The results are shown in the table.

Outcome	Frequency
Picture card	20
Not a picture card	58

a Express as a decimal, correct to three decimal places:

i the calculated probability of selecting a picture card

ii the experimental probability (relative frequency) of selecting a picture card.

b If a card is randomly drawn from a deck of cards 78 times, which of the following is the expected number of times a picture card should be drawn? Select **A**, **B**, **C** or **D**.

A 18 **B** 21 **C** 24 **D** 26

c Use a deck of cards to perform this experiment 78 times, and check whether your experimental probability is close to the calculated probability.

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5 **a** How many outcomes are there in the sample space when a coin and a die are tossed together?

b Tahlia tossed a coin and a die together 48 times and found the frequencies shown.

		Die					
		1	**2**	**3**	**4**	**5**	**6**
Coin	**Heads**	4	5	3	2	4	5
	Tails	3	6	5	4	4	3

Calculate as a decimal, correct to three decimal places:

- **i** the calculated probability of tossing a tail and a 3
- **ii** the relative frequency of tossing a tail and a 3
- **iii** the calculated probability of tossing a head and an even number
- **iv** the relative frequency of tossing a head and an even number
- **v** the calculated probability of tossing a tail and any number but 4
- **vi** the experimental probability of tossing a tail and any number but 4.

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c If a coin and a die are tossed together 48 times, how many times should each of the following events occur?

- **i** a tail and a 3
- **ii** a head and an even number
- **iii** a tail and any number but 4.

6 A coin is tossed repeatedly, and it lands with heads uppermost eight times in a row. On the next toss, is a head more likely to result again, or a tail? Or does each outcome still have an equal chance?

7 Zeinab uses a computer program to simulate tossing a coin but she suspects that the simulated coin is biased when she notices the pattern of results recorded in the table below.

Total number of tosses	50	140	200	540
Number of heads	23	55	84	226

a Calculate as a percentage, correct to one decimal place, the relative frequency of the computer simulation 'tossing heads' when the computer tosses the coin:

i 50 times **ii** 140 times **iii** 200 times **iv** 540 times.

b Is Zeinab justified in believing that the computer program's coin is biased? Give a reason for your answer.

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INVESTIGATION

ARE THE LOTTO NUMBERS EQUALLY LIKELY?

Lotto is a gambling game run by NSW Lotteries, in which 45 numbered balls are placed in a barrel and eight numbers are selected at random from the barrel: six main numbers plus two supplementary numbers. Between the launch of the 45 number game and October 2016, there were a total of 9438 Lotto numbers drawn for the Saturday game. The frequency of drawing each number (from 1 to 45) over this period was as follows:

Number	Frequency	Number	Frequency	Number	Frequency
1	235	16	212	31	215
2	190	17	187	32	211
3	221	18	228	33	221
4	201	19	221	34	200
5	226	20	205	35	184
6	214	21	213	36	221
7	224	22	221	37	211
8	221	23	190	38	207
9	205	24	216	39	196
10	197	25	227	40	227
11	222	26	218	41	225
12	227	27	204	42	222
13	208	28	192	43	205
14	179	29	207	44	181
15	211	30	187	45	203

Source: www.nswlotteries.com.au

1 If, in this period, 9438 numbers were drawn, and each number from 1 to 45 was equally likely, how often would you expect each number to have been drawn?

2 How many of the frequencies from the above table are equal to the expected number you suggested in Question **1**?

3 Which Lotto number came up:

a most often? **b** least often?

4 Do you think the Lotto numbers are equally likely, or is the Lotto draw biased? Give a reason for your answer.

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4.06 Probability tree diagrams

Tree diagrams

A **probability tree diagram** has the probabilities of each stage shown on the branches.

EXAMPLE 11

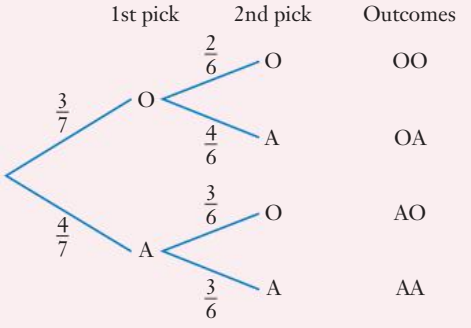
A fruit bowl contains three oranges and four apples. Prue selects two pieces of fruit at random from the bowl. Use a tree diagram to determine the probability that Prue selects:

a two oranges **b** an orange and an apple.

Solution

3 oranges, 4 apples: total 7

3O
4A



See explanation below for this probability tree diagram.

Suppose Prue picks an orange first.

$P(\text{1st pick is an orange}) = \frac{3}{7}$ 3 oranges from 7 fruit

- $P(\text{2nd pick is orange}) = \frac{2}{6}$ 2 oranges from 6 fruit left
- $P(\text{2nd pick is apple}) = \frac{4}{6}$ 4 apples from 6 fruit left

Suppose Prue picks an apple first. 4 apples from 7 fruit

$P(\text{1st pick is an apple}) = \frac{4}{7}$

- $P(\text{2nd pick is orange}) = \frac{3}{6}$ 3 oranges from 6 fruit left

$P(\text{2nd pick is apple}) = \frac{3}{6}$ 3 apples from 6 fruit left

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a $P(\text{OO}) = \frac{3}{7} \times \frac{2}{6}$ Multiply probabilities on OO branches

$= \frac{1}{7}$

b $P(\text{OA or AO}) = P(\text{OA}) + P(\text{AO})$ Two ways of selecting an orange and apple

$= \frac{3}{7} \times \frac{4}{6} + \frac{4}{7} \times \frac{3}{6}$

$= \frac{4}{7}$

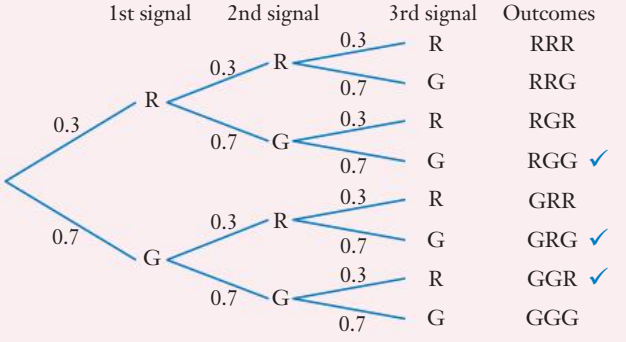
Note: Probabilities on branches from the same point always add to 1, for example, $\frac{3}{7} + \frac{4}{7}$, $\frac{2}{6} + \frac{4}{6}$ on the tree diagram above.

EXAMPLE 12

To drive to work, Ms Gough passes through three sets of traffic lights. The probability of a red signal (including amber) on each light is 0.3. Find the probability that on her way to work Ms Gough gets:

a all green lights **b** two green lights **c** at least one green light.

Solution



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a $P(\text{GGG}) = 0.7 \times 0.7 \times 0.7$

$= 0.343$

b $P(2 \text{ green}) = P(\text{RGG}) + P(\text{GRG}) + P(\text{GGR})$ Ticked on the tree diagram

$= (0.3 \times 0.7 \times 0.7) + (0.7 \times 0.3 \times 0.7) + (0.7 \times 0.7 \times 0.3)$

$= 3 \times (0.3 \times 0.7 \times 0.7)$

$= 0.441$

c $P(\text{at least 1 green}) = 1 - P(\text{no green})$ Using the complementary event rule

$= 1 - P(\text{RRR})$

$= 1 - (0.3 \times 0.3 \times 0.3)$

$= 0.973$

Probability tree diagrams

In a probability tree diagram:

- branches from the same point have probabilities that add to 1
- to calculate the probability of an outcome, multiply the probabilities along that branch
- to calculate the probability of two or more outcomes, add their calculated probabilities
- $P(\text{at least one}) = 1 - P(\text{none})$.

Exercise 4.06 Probability tree diagrams

Example 11

1 A jar contains five red lollies and three green lollies. Maya selects two lollies at random. Calculate the probability that they are of different colours. Select **A**, **B**, **C** or **D**.

A $\frac{13}{28}$ **B** $\frac{15}{32}$ **C** $\frac{15}{28}$ **D** $\frac{15}{56}$

2 Nikolai buys two tickets in a 50-ticket raffle with two prizes. What is the probability that he wins:

a 1st prize? **b** 1st and 2nd prizes?

c no prize? **d** at least one prize?

3 There are ten batteries in a box and two are flat. Danielle takes two batteries out of the box at random. Calculate the probability that:

a both batteries are flat **b** only one of the batteries is flat.

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4 A committee of four women and three men need to select a chairperson and a secretary. If every person is equally likely to be chosen, what is the probability that:

a both positions are filled by women?

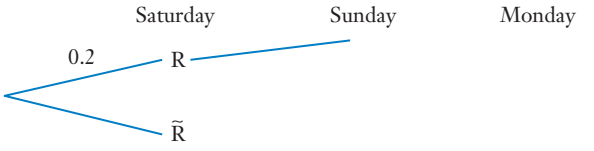
b both positions are filled by men?

c the chairperson is female and the secretary is male?

5 For a long weekend (Saturday to Monday), the probability of rain on any day is 0.2.

Example 12

a Copy and complete this tree diagram.



b Calculate the percentage probability that over the long weekend there is:

i exactly one rainy day **ii** no rainy days **iii** at least one rainy day.

6 A die is rolled three times. What is the probability that 6 does not come up in any roll?

7 A biased coin comes up tails 63% of the time. If the coin is tossed three times, calculate the probability, correct to three decimal places, that:

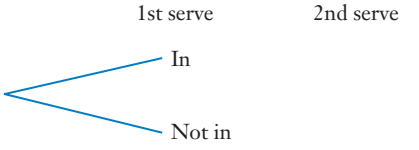
a a tail comes up every time **b** a tail comes up twice.

8 Five cards are numbered 1, 2, 3, 4 and 5. If two cards are randomly selected, what is the probability of selecting:

a two even numbers? **b** two odd numbers?

9 A tennis player gets a second serve only if her first serve is not in. Daria's first serve has a 0.78 probability of going in and her second serve has a 0.94 probability of going in.

a Copy and complete this tree diagram.



b A double fault occurs when both serves do not go in. What is the probability that Daria serves a double fault?

c What is the probability that one of Daria's serves goes in?

10 A student council has eight Year 10 students, six Year 11 students and four Year 12 students. Two students are selected from the council at random to represent the school at the mayor's lunch. Calculate the probability that:

a both students are from Year 10

b there is one student from each of Years 10 and 11

c at least one of the students is from Year 12

d each student is from a different Year.

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11 Three people are selected at random. What is the probability that all were born in March? Select **A**, **B**, **C** or **D**.

A 0.0579% **B** 1.56% **C** 2.78% **D** 3.70%

12 In a town, 8% of people have a virus that can be detected by a medical test which gives a correct reading 90% of the time. If Sam is tested for the virus, what is the percentage probability that he:

a has the virus but it is not detected by the test?

b has the virus and it is detected?

c does not have the virus but it is falsely detected?

INVESTIGATION

MORTALITY RATES

Insurance companies use **mortality rates** to calculate premiums for life **insurance** policies. Mortality rates are the probabilities that a person of a certain age will die that year. This table shows the mortality rates of Australians aged 0 to 100 in 2013–2015.

Age	Males	Females	Age	Males	Females
0	0.00354	0.00249	40	0.00093	0.00076
1	0.00024	0.00021	45	0.00135	0.00116
2	0.00018	0.00018	50	0.00232	0.00179
3	0.00015	0.00015	55	0.00388	0.00258
4	0.00013	0.00012	60	0.00616	0.00371
5	0.00011	0.00011	65	0.01010	0.00559
10	0.00006	0.00006	70	0.01688	0.00895
15	0.00015	0.00012	75	0.02819	0.01581
17	0.00030	0.00014	80	0.04729	0.02892
20	0.00048	0.00015	85	0.08123	0.05925
25	0.00051	0.00016	90	0.15332	0.11670
30	0.00056	0.00026	95	0.24057	0.20842
35	0.00070	0.00036	100	0.32793	0.29473

Source: Australian Bureau of Statistics. Life tables 2013–2015

a How do you think these figures were calculated?

b What patterns do you notice in the mortality rates as people get older? Why?

c Why do you think babies have a higher mortality rate?

d Why do you think there is a difference between male and female mortality rates?

e These mortality rates were lower than those in the year 2010, and this year's mortality rates will be lower than these ones. Why do you think this is so?

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SAMPLE HSC PROBLEM

Two dice are rolled together and the difference between the larger and smaller number is calculated.

a Copy and complete this table of possible differences.

		1st die					
		1	**2**	**3**	**4**	**5**	**6**
2nd die	**1**						
	2				2		
	3						
	4						
	5	4					
	6						0

b Which difference is most likely?

c Find the probability of rolling a difference:

i of 2

ii that is an odd number

iii less than 5.

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Study tip

When and where to study

Are you an early bird or a night owl? do you work best in the morning, in the afternoon or at night? Identify your **peak performance period** and study during this time each day, so that it becomes a habit. There will be times when you can't study due to other commitments but it's important to develop a **regular routine** so that your family and friends know when you should not be interrupted.

Students often ask, 'How many hours should I study each night?', but it is the **quality** of study that is more important than the **quantity**. It is a question of the effort and commitment required, not the number of hours. The amount of time you should study is the amount of time necessary for you to fulfil all of your study tasks and demands. Some students like to do the same amount each day, some do more or less on weekends, or have one day that is study-free. Find out what works best for you, and stick to it.

Students often study by themselves in their rooms or at the library. Some prefer company and use the dining table, because they like the noise and space. Others like to sit outside in the fresh air. You can choose different places for different types of homework.

A good study place has:

- plenty of space to spread out work, such as on a big desk
- minimal noise, few distractions and interruptions
- good lighting and ventilation, and is neither too hot nor too cold
- comfortable and supportive seating.

Study is a serious business. You will be concentrating for a while, so choose a place where you won't be easily distracted. Use a good desk lamp and open the window to prevent sore eyes and drowsiness.

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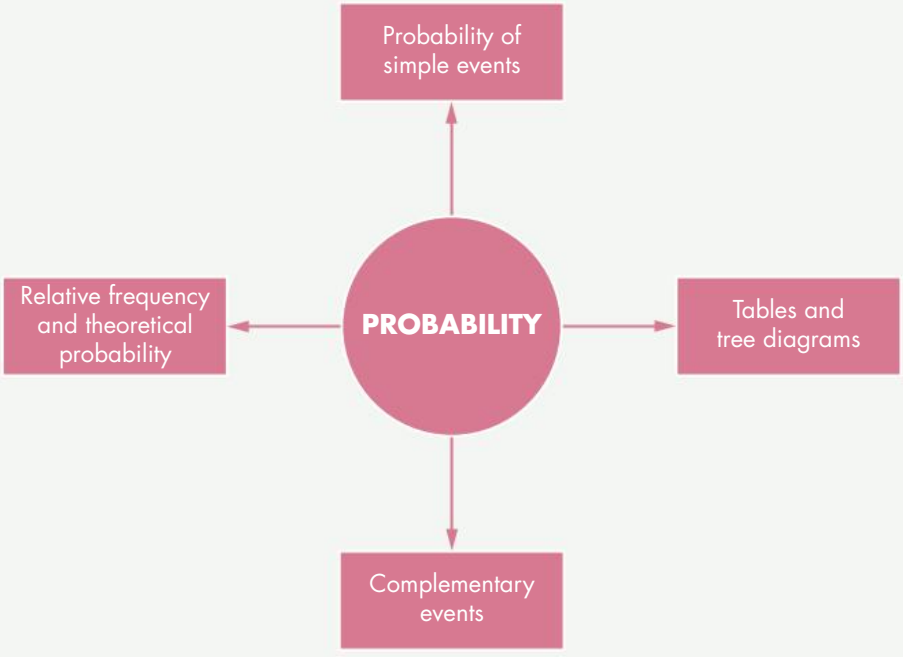
4. CHAPTER SUMMARY

This chapter, Probability, looked at the probabilities of simple and multi-stage events. You should have a good understanding of probability concepts, including complementary events and determining sample spaces using lists, tables and tree diagrams. You should be able to find and compare theoretical probabilities and relative frequencies using given data or through repeated trials of an experiment.

Probability crossword

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map is shown below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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Probability review

4. TEST YOURSELF

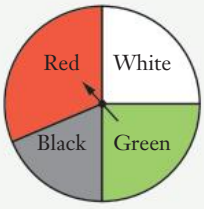
Exercise 4.01

1 A letter is selected at random from the alphabet. Find the probability that it is:

a P **b** a vowel or Y **c** not W, X, Y or Z.

Exercise 4.02

2 A spinner is divided into sectors as shown, with 'Red' taking up one-third of the circle. What is the percentage probability that the arrow points to:



a green?

b black?

c red or green?

d a sector that is not red?

Exercise 4.01

3 Explain what is wrong with the following statement: 'Since a person can be either married or single, the probability of a person being married is 50%'.

Exercise 4.02

4 **a** Use a tree diagram to list the sample space of the different possible combinations of boys (B) and girls (G) in a family of three children.

b If a three-child family is selected at random, what is the probability that it contains:

i all girls? **ii** exactly one boy?

iii more boys than girls? **iv** at least one girl?

v at most one girl? **vi** a boy as the middle child?

Exercise 4.02

5 Two dice are rolled together. What is the probability that the two dice:

a show the same number? **b** show a sum of 8?

Exercise 4.03

6 If Santi has a 68% chance of missing a basketball shot, what is the probability that she scores with the shot?

Exercise 4.03

7 In a batch of 240 instant (scratch) lottery tickets, 54 contain a prize. Which of the following is the decimal probability of not winning a prize from one of these tickets? Select **A**, **B**, **C** or **D**.

A 0.225 **B** 0.443 **C** 0.46 **D** 0.775

Exercise 4.03

8 A sample of students was surveyed about how they travelled to school. The results are shown in the table.

Outcome	Frequency
Walk	43
Car	30
Bicycle	18
Bus	28
Train	21

What is the probability that a student, chosen at random, travels by:

a car?

b public transport?

c a method other than bicycle?

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9 A factory tests a batch of batteries and finds that 77 of them are good while 3 are faulty.

Exercise 4.04

a Express, as a percentage, the probability that a battery selected at random is good.

b If the factory makes 5000 batteries, how many would you expect to be faulty?

10 Roll a die 48 times and record in a table, similar to the one shown, the number of times the die lands with '6' uppermost.

Exercise 4.04

Exercise 4.05

Outcome	Tally	Frequency
6		
Not 6		

a Represent this data on a divided bar graph.

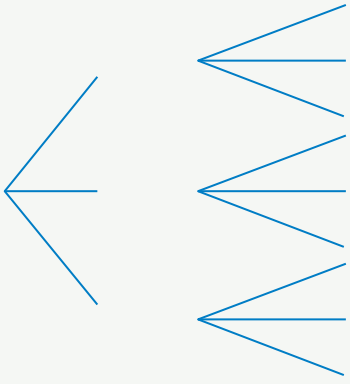
b Find the relative frequency of rolling a 6 on this die.

c What is the theoretical probability of rolling a 6 on a die?

d What is the expected number of 6s when a die is rolled 48 times?

11 Which situation can be illustrated by this tree diagram? Select **A**, **B** or **C**.

Exercise 4.06



A tossing a coin three times

B selecting two balls from a bag of red, blue and green balls

C three students passing or failing an exam.

12 Tegan buys five tickets in a raffle in which 80 tickets are sold. There are three prizes. Use a tree diagram to find the probability that Tegan:

Exercise 4.06

a wins all three prizes

b wins at least one prize

c wins exactly one prize.

Express answers as percentages, correct to two significant figures.

Qz Chapter quiz

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5.

MEASUREMENT

One gigalitre is a billion litres. Sydney Harbour contains approximately 500 gigalitres of water. The entire Sydney urban area uses the equivalent of four Sydney Harbours each year, while the whole of Australia uses the equivalent of 50 Sydney Harbours per year.

CHAPTER OUTLINE

M1.1, 1.3	5.01	Metric units
M1.1	5.02	Error in measurement
M1.1	5.03	Significant figures
M1.1	5.04	Scientific notation
M1.2	5.05	Perimeters of circular and composite shapes
M1.2	5.06	Area
M1.2	5.07	Land surveying
M1.2	5.08	Volume and surface area of a prism
M1.2	5.09	Volume and surface area of a cylinder
M1.2	5.10	Volume and surface area of a sphere
M1.2	5.11	Volume and surface area of composite solids

IN THIS CHAPTER YOU WILL:

- convert between metric units
- investigate error in measurement, including absolute error, limits of accuracy and percentage error
- express numbers in scientific notation and round to significant figures
- calculate perimeters and areas of shapes, including circular and composite shapes
- calculate perimeters and areas of irregular shapes using offset surveys and the trapezoidal rule
- calculate surface areas, volumes and capacities of prisms, cylinders, spheres and composite solids

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TERMINOLOGY

absolute error
accuracy
arc
capacity
circumference
hemisphere
notebook entry
offset survey
percentage error
precision
prism
quadrant
scientific notation
sector
significant figures
sphere
trapezoidal rule
volume

SkillCheck

Assignment 5

1 Evaluate without using a calculator:

a 20.83×1000 **b** $970.2 \div 10$ **c** $6.59 \times 10\,000$

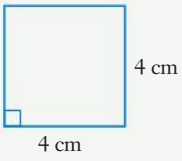
d $72.5 \div 100$ **e** $10.4 \div 1000$ **f** 0.0735×10

2 Evaluate:

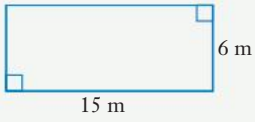
a 10^7 **b** 10^4 **c** 10^{-2}

3 Find the area of each shape. Express the answer to part **c** to two decimal places.

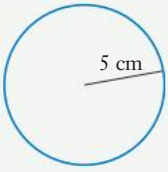
a



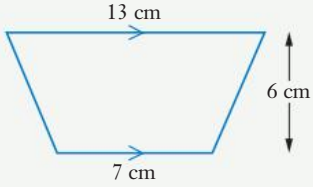
b



c

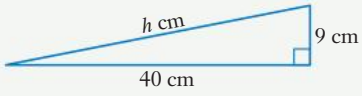


d



4 **a** Find the area of this triangle.

b Find the value of h.



5 **a** If bananas cost \$5.10 per kilogram, how much would we pay for 250 grams?

b If potatoes cost \$4.15 for 5 kg, how much would 3 kg of potatoes cost?

6 Convert 210 cm to:

a metres **b** millimetres **c** kilometres.

7 Convert 8.1 L to:

a kilolitres **b** millilitres.

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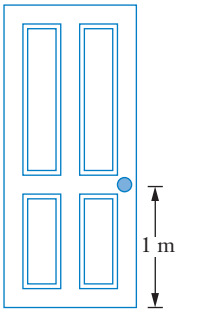
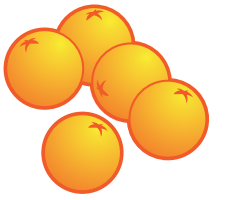
5.01 Metric units

This table shows some common metric units for length, mass, time and capacity.

Unit	Relationships
Length	
micrometre (μm)	
millimetre (mm)	
centimetre (cm)	1 cm = 10 mm
metre (m)	1 m = 100 cm = 1000 mm = 1 000 000 μm
kilometre (km)	1 km = 1000 m
Mass	
milligram (mg)	
gram (g)	1 g = 1000 mg
kilogram (kg)	1 kg = 1000 g
tonne (t)	1 t = 1000 kg
megatonne (Mt)	1 Mt = 1 000 000 t
Time	
second (s)	
minute (min)	1 min = 60 s
hour (h)	1 h = 60 min = 3600 s
day (day)	1 day = 24 h
Capacity	
millilitre (mL)	
litre (L)	1 L = 1000 mL
kilolitre (kL)	1 kL = 1000 L
megalitre (ML)	1 ML = 1 000 000 L



A **metre** is about the height of a door handle.



A **kilogram** is the mass of about five oranges.

A **litre** is the capacity of a tall carton of milk.

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Metric prefixes

- micro- means 'one-millionth' or $\frac{1}{1\,000\,000}$
- milli- means 'one-thousandth' or $\frac{1}{1000}$
- centi- means 'one-tenth' or $\frac{1}{10}$
- kilo- means 'one thousand' or 1000
- mega- means 'one million' or 1 000 000

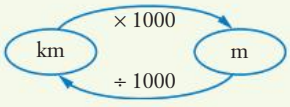
EXAMPLE 1

Convert:

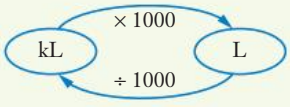
a 23.5 km to metres **b** 450 L to kilolitres **c** 84 000 mg to kilograms

Solution

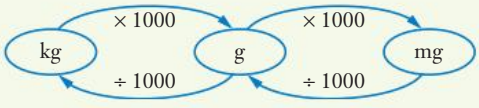
a 23.5 km = 23.5 × 1000 m
= 23 500 m



b 450 L = 450 ÷ 1000 kL
= 0.45 kL



c 84 000 mg = 84 000 ÷ 1000 ÷ 1000 kg
= 0.084 kg



Converting units

- To convert a larger unit to a smaller one, we *multiply* to get more of the smaller units.
- To convert a smaller unit to a larger one, we *divide* to get fewer larger units.

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Exercise 5.01 Metric units

Example 1

1 Convert:

a	6.3 cm to mm	**b**	4.36 m to cm	**c**	7200 mm to m
d	285 g to kg	**e**	6.9 mm to μm	**f**	58 000 mL to L
g	5.32 kg to g	**h**	3400 kg to t	**i**	4720 L to kL
j	6000 mg to kg	**k**	7.5 h to s	**l**	9.4 km to cm

2 Which of the following is the number of grams in 100 kg? Select **A**, **B**, **C** or **D**.

A 100 000 **B** 1000 **C** 1 000 000 **D** 10 000

3 Oliver's height is 169 cm. Convert this to:

a millimetres **b** metres.

4 Mai weighs 57.5 kg. What is her mass in grams?

5 Rhys swam in a 1500 m race. How many kilometres did he swim?

6 Erin left home at 7.41 a.m. and arrived at work at 9.18 a.m. What was her travelling time in:

a minutes? **b** seconds? **c** hours (correct to one decimal place)?

7 Yusuf's water consumption for July was 59 kL. How many litres of water did he use?

8 What metric unit would you use to measure each of the following?

a your mass
b your height
c the length of your bedroom
d the distance from Perth to Sydney
e the capacity of your kitchen sink
f the mass of an ant
g the width of a sheet of paper
h the amount of water in an Olympic-sized pool
i the mass of a train
j the thickness of a strand of hair

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9 Taylor's pet mouse weighs 102 g and is 5.2 cm long. Write its:

a mass in kilograms **b** length in metres.

10 The Great Pyramid of Cheops (or Khufu) at Giza in Egypt contains 2 000 000 blocks, each weighing 2.45 tonnes. What is the total mass of the pyramid in megatonnes?

11 Josh competed in a triathlon. He took 12 min for the 5400 m bike ride, 38 min for the 3.8 km swim and 58 min for the 9.5 km run.

a What was the total distance that Josh covered, in kilometres?

b What was Josh's total time, in hours and minutes?

12 Mia's swimming pool holds 8560 L of water. How many kilolitres is this?

13 Which is the most likely measurement for each of the following? Select **A**, **B**, **C** or **D** each time.

a The height of a 10-storey building

A 400 m **B** 40 m **C** 4 m **D** 4000 m

b The capacity of a backyard swimming pool

A 30 L **B** 300 L **C** 3000 L **D** 30 000 L

c The mass of an orange

A 3 mg **B** 3 g **C** 300 g **D** 3000 g

d The distance from Sydney to Brisbane

A 100 km **B** 500 km **C** 1000 km **D** 5000 km

e The diameter of a DVD

A 12 cm **B** 12 mm **C** 120 m **D** 1.2 cm

f The capacity of a soft drink can

A 380 mL **B** 3800 mL **C** 3.8 mL **D** 38 mL

14 On a 2.5 km walk, John covers 80 cm with each step, while his daughter Anna covers 55 cm with each of her steps. How many more steps does Anna take than her father during the walk?

15 How many seconds are there in one day?

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DID YOU KNOW?

Metric prefixes

Prefix	Abbreviation	Meaning	Example
atto	a	10^{-18}	
femto	f	10^{-15}	
pico	P	10^{-12}	
nano	N	10^{-9}	nanosecond
micro	M	$10^{-6} = \frac{1}{1\,000\,000}$	microgram
milli	M	$10^{-3} = \frac{1}{1000}$	millibar
centi	C	$10^{-2} = \frac{1}{100}$	centilitre
deci	D	$10^{-1} = \frac{1}{10}$	decibel
deca	Da	$10^{1} = 10$	decametre
hecto	H	$10^{2} = 100$	hectopascal
kilo	k	$10^{3} = 1000$	kilojoule
mega	M	$10^{6} = 1000\,000$	megahertz
giga	G	10^{9}	gigawatt
tera	T	10^{12}	terabyte
peta	P	10^{15}	
exa	E	10^{18}	

Choose four of the units from the 'Example' column in this table and find out what they are used to measure.

5.02 Error in measurement

No measurement is ever exact. When we use a device to take a measurement, there is always some error which can be caused by one or more of the following:

- a fault in the measuring device, such as bathroom scales not set to 0 for no mass
- an inappropriate measuring device, such as using a metre stick to measure the length of a calculator
- human error, such as not looking directly at the mark on the scale.

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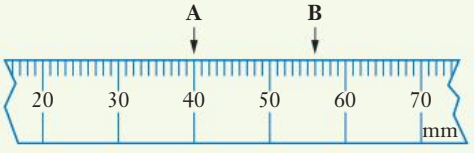
Absolute error

Any measurement should only be recorded correct to the smallest unit on the scale of the measuring device. This unit is called the **precision** of the device. For example, the precision of bathroom scales could be to the nearest kilogram, while the precision of a digital stopwatch is 0.01 seconds.

The **absolute error** of a measurement is the greatest possible error or the 'furthest by which you can be wrong'. It is equal to $\pm\frac{1}{2}$ of the smallest unit marked on the scale.

EXAMPLE 2

This diagram shows a section of a builders' tape.



a What is the precision of the tape?

b How would you record the measurements **A** and **B**?

c What is the absolute error of measurement **A**?

Solution

a The precision of the tape is 1 mm.

The smallest unit

b **A** = 40 mm and **B** = 56 mm. Measurements can only be recorded correct to the nearest mm.

We should not estimate fractions of a millimetre when measuring with this ruler.

c Absolute error $= \pm\frac{1}{2} \times 1 \text{ mm} = \pm 0.5 \text{ mm}$

A measured length recorded as 40 mm means that the true length could be anywhere in the range (40 ± 0.5) mm, which means that the **limits of accuracy** of the measurement are 39.5 mm to (but not including) 40.5 mm.

(40 ± 0.5) mm means '40 millimetres, give or take half a millimetre'.

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EXAMPLE 3

Complete the table below.

Measurement	Smallest unit	Absolute error	Limits of accuracy
40 cm	1 cm	±0.5 cm	39.5 cm to 40.5 cm
56 s			
3.4 mL			
600 kg (to the nearest kg)			
600 kg (to the nearest 10 kg)			

Solution

Measurement	Smallest unit	Absolute error	Limits of accuracy
40 cm	1 cm	±0.5 cm	39.5 cm to 40.5 cm
56 s	1 s	±0.5 s	55.5 s to 56.5 s
3.4 mL	0.1 mL	±0.05 mL	3.35 mL to 3.45 mL
600 kg (to the nearest kg)	1 kg	±0.5 kg	599.5 kg to 600.5 kg
600 kg (to the nearest 10 kg)	10 kg	±5 kg	595 kg to 605 kg

Percentage error

The **percentage error** of a measurement is the size of the absolute error as a percentage of the measurement. For example, an error of 0.5 m in 1 km is relatively small compared to an error of 0.5 m in 10 m.

Percentage error

$$\text{Percentage error} = \frac{\text{absolute error}}{\text{measurement}} \times 100\%$$

EXAMPLE 4

The measured length of the Sydney Harbour Bridge is 503 m, and the measured length of a basketball court is 26 m.

a Find the percentage error of each measurement given above, correct to three decimal places.

b Which of the two measurements is more accurate?

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Solution

Both measurements are measured to the nearest metre, so they have the same absolute error of ±0.5 m.

a Sydney Harbour Bridge: $\text{Percentage error} = \frac{\pm 0.5}{503} \times 100\%$

$= \pm 0.099\,40\ldots\%$

$\approx \pm 0.099\%$

Basketball court: $\text{Percentage error} = \frac{\pm 0.5}{26} \times 100\%$

$= \pm 1.923\,07\ldots\%$

$= \pm 1.923\%$

b The Sydney Harbour Bridge measurement has a smaller percentage error, so it is more accurate.

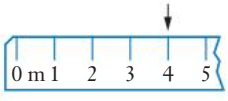
Exercise 5.02 Error in measurement

Example 2,3

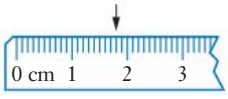
1 For each measuring device shown, write:

- **i** its precision
- **ii** the measurement shown
- **iii** the absolute error and limits of accuracy of the measurement.

a

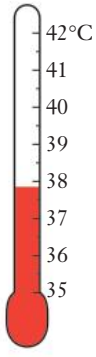


b

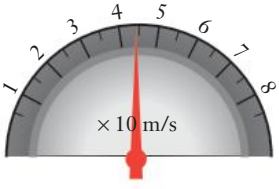


c

d



e



f

2 When using a jug marked in millilitres, what is the absolute error of a measurement? Select **A**, **B**, **C** or **D**.

A ±1 mL **B** ±0.5 mm **C** ±0.5 mL **D** ±1 L

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3 Which of the following lengths could be measured using a measuring tape with a precision of 1 cm? Select all correct answers.

A 37 cm **B** 24.6 cm **C** 125 mm
D 9 cm **E** 8 mm **F** 16.45 cm

4 Which of the following angle sizes are incorrectly recorded if using a protractor marked in degrees? Select all correct answers.

A 32° **B** 145° **C** 103.5° **D** $64\frac{1}{2}°$ **E** 170° **F** 88.4°

5 State the absolute error for each measurement.

a 25 mL **b** 5560 mm
c 6.5 g **d** 78.1 s
e 500 g (to the nearest g) **f** 24 m
g 600 mL (to the nearest mL) **h** 500 g (to the nearest 10 g)
i 8.6 cm **j** 5000 g (to the nearest 100 g)

6 To minimise error, Samantha measured the length of her bedroom four times. She recorded the results 2964 mm, 3021 mm, 2938 mm and 2899 mm and calculated the average correct to the nearest millimetre. What was her answer?

7 The width of an A4 sheet of paper is 21 cm. For this measurement, find:

Example 4

a the absolute error
b the limits of accuracy
c the percentage error, correct to two decimal places.

8 A truck weighs 2.3 t. What is the percentage error (correct to two decimal places) of this measurement?

9 Earth has a diameter of 12 683 km. Find the percentage error of this measurement, to four decimal places.

10 By comparing their percentage errors, determine which of the following measurements is the most accurate.

a 15.2 mg **b** 800 g (to nearest 10 g) **c** 0.15 g

11 A Swans AFL match at the SCG had a crowd of 52 350. Why can't we find a percentage error for this figure?

12 Explain the difference between 38 L of petrol and 38.0 L of petrol.

13 Jessie says the length of the classroom is 45 rulers and Eddy says it is 44 rulers. List as many reasons as you can to explain the different results.

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INVESTIGATION

REPEATING AND AVERAGING MEASUREMENTS

You will need builder's tape, paper, pen and calculator.

1 Choose a netball court, classroom or part of a building to measure. Work with a partner and measure the length correct to the nearest centimetre.

2 Repeat the measurement 4 times.

3 Find the mean (average) of your 5 measurements.

4 What was your range of measurements?

5 Which of your 5 measurements:

a was closest to the mean? **b** was furthest from the mean?

6 Why is repeating and averaging measurements a good way of determining a length?

5.03 Significant figures

Significant figures

Numbers can be rounded to decimal places or **significant figures** to show their level of **accuracy**.

EXAMPLE 5

Round:

a 3812.4, correct to two significant figures

b 1005, correct to three significant figures

c 0.003 572 correct to two significant figures.

Solution

a **38**12.4

The first two significant figures are **38** and the next digit is **1** *(which is less than 5)*, so we round down to **8** and use two 0s as placeholders.

$3812.4 \approx 3800$ (correct to two significant figures)

b **100**5

The first three significant figures are **100** and the next digit is **5**, so we round up to **10** *and use a 0 as a placeholder*.

$1005 \approx 1010$ (correct to three significant figures)

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c 0.00**35**72

For decimals, ignore any zeros at the front of the number. The first two significant figures are **35** and the next digit is **7** *(which is greater than 5)* so we round the **5** up to **6**.

0.003 572 ≈ 0.0036 (correct to two significant figures)

Significant figures

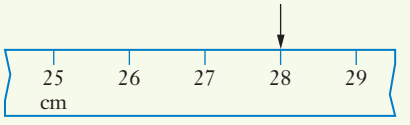
- The first significant figure in a number is the first non-zero digit.
- When rounding to significant figures, start counting from the first digit that is not 0.
- If it is a large number, you may need to insert 0s at the end as placeholders.
- Zeros at the end of a whole number or at the beginning of a decimal are not significant.

Significant figures in measurement

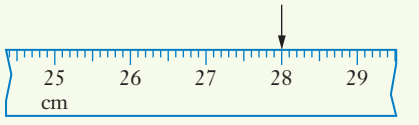
As we can only record a measurement correct to the precision of the measuring device, this means that we must use the appropriate number of significant figures.

EXAMPLE 6

Sections of two tape measures are shown. Tape A has a precision of 1 cm, tape B has a precision 0.1 cm.



Tape A



Tape B

a Write the measurement shown on each tape and state how many significant figures each measurement has.

b Which of the two measurements is more accurate?

c Vincent recorded a measurement of 57.4 cm using tape A. What should he have written instead? Why?

d Rose recorded a measurement of 19 cm using tape B. What should she have written instead? Why?

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Solution

a Tape A: 28 cm, two significant figures

Tape B: 28.0 cm, three significant figures

b 28.0 cm is more accurate than 28 cm because it has more significant figures and is correct to the nearest 0.1 cm.

c 57 cm; we can only record to the nearest centimetre, using two significant figures.

d 19.0 cm; we should record correct to 0.1 cm, using three significant figures.

Exercise 5.03 Significant figures

Example 5

1 What is 305.725 rounded to two significant figures? Select **A**, **B**, **C** or **D**.

A 306 **B** 310 **C** 300 **D** 305.73

2 Write each number correct to two significant figures.

a 3812.4 **b** 2068 **c** 0.006 102

d 245 698 **e** 14 569 235 **f** 0.000 467

3 Write each number correct to three significant figures.

a 129.805 **b** 4982 **c** 0.010 609

d 1 359 048 **e** 25 430 082 **f** 0.000 680 32

4 Write each number correct to one significant figure.

a 2914.23 **b** 3.2548 **c** 11 950

d 0.005 134 **e** 20.46 **f** 0.6517

Example 6

5 Measurements made using a rain gauge marked in millimetres should be recorded correct to two significant figures for rainfall from 10 mm to 99 mm, and to one significant figure for rainfall less than 10 mm. State whether each rainfall measurement has been recorded correctly.

a 9 mm **b** 7.23 mm

c 24 mm **d** 6.5 mm

e 31 mm **f** 10 mm

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6 A micrometer (μm) is one-thousandth of a millimetre and is used for very small measurements. If each of the following measurements should be recorded correct to two significant figures, which measurements have been recorded incorrectly?

a 23 μm **b** 104 μm **c** 19.6 μm
d 832 μm **e** 28.6 μm **f** 3 μm

7 Evaluate each expression, correct to two significant figures:

a $0.2 \div 0.3$ **b** $11 \div 1990$ **c** $16 \div 12$
d $\sqrt{0.0075}$ **e** $9\,300\,000 \times 0.085$ **f** 2.7^2
g $\sqrt{560}$ **h** $\sqrt{5.6}$ **i** 3.4×9.9

8 The population of New South Wales in 2016 was 7 618 259. Round this number to three significant figures.

9 Express each measurement correct to three significant figures.

a mass of an aeroplane, 351 540 kg
b average depth of the Pacific Ocean, 4188 m
c speed of a greyhound, 67.14 km/h
d capacity of a tablespoon, 14.79 mL
e distance from Earth to the Sun, 149 573 881 km

5.04 Scientific notation

Scientific notation puzzle

Scientific notation (or **standard form notation**) is a shorthand way of writing very large or very small numbers, using powers of 10.

Scientific notation

Scientific notation has the form: $m \times 10^n$

where m is a number between 1 and 10
n is an integer.

EXAMPLE 7

Write each number in scientific notation.

a 23 500 000 **b** 0.004 762

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Solution

a Write a decimal between 1 and 10 using the significant figures in the number: 2.35

Count how many places to move the decimal point to make the original number: 7

Or count the number of places after the first significant figure, 2

So $23\ 500\ 000 = 2.35 \times 10^7$

b Write a decimal between 1 and 10 using the significant figures in the number: 4.762

Count how many places to move the decimal point to make the original number: –3

Or count the number of 0s before the first significant figure, 4

So $0.004\ 762 = 4.762 \times 10^{-3}$

EXAMPLE 8

Write 354 780 in scientific notation, correct to two significant figures.

Solution

$354\ 780 = 3.5478 \times 10^5$ Written in scientific notation

$\approx 3.5 \times 10^5$ Rounded to two significant figures

We could also round to two significant figures first, then write in scientific notation.

EXAMPLE 9

Write each number in normal decimal form.

a 6.92×10^4 **b** 3.716×10^{-5}

Solution

a $6.92 \times 10^4 = 69\ 200$ Moving the decimal point 4 places to the right (or making 4 places after the 6)

b $3.716 \times 10^{-5} = 0.000\ 037\ 16$ Moving the decimal point 5 places to the left (or inserting 5 zeros before the 3)

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Scientific notation on a calculator

To enter a number in scientific notation on a calculator, use the $\times 10^x$ or EXP key.

EXAMPLE 10

Evaluate:

a $(6.8 \times 10^5) \times (7.4 \times 10^{-2})$

b $(3.2 \times 10^{-4}) \div (9.1 \times 10^6)$, correct to two significant figures.

Solution

a $(6.8 \times 10^5) \times (7.4 \times 10^{-2}) = 50\,320.$

Calculator steps: 6.8 $\times 10^x$ 5 $\times$ 7.4 $\times 10^x$ (–) 2 =

b $(3.2 \times 10^{-4}) \div (9.1 \times 10^6) = 3.516\,483\,\ldots \times 10^{-11}$

$\approx 3.5 \times 10^{-11}$ Rounded to two significant figures

Calculator steps: 3.2 $\times 10^x$ (–) 4 ÷ 9.1 $\times 10^x$ 6 =

Exercise 5.04 Scientific notation

1 Write each number in scientific notation. Example 7

a	42 130 000	**b**	0.0181	**c**	3400
d	20 000	**e**	0.0035	**f**	0.0002
g	0.33	**h**	0.004	**i**	230
j	0.000 0723	**k**	610 000 000	**l**	0.000 000 08

2 Write each number in scientific notation, correct to two significant figures. Example 8

a	53 467 892	**b**	146 089	**c**	2453
d	0.000 4573	**e**	0.002 652	**f**	0.102 05
g	0.000 033 38	**h**	0.444 44	**i**	6.5207

3 What is 356 587 239 in scientific notation correct to three significant figures? Select **A**, **B**, **C** or **D**.

A 357 000 000 **B** 3.57×10^8 **C** 3.6×10^8 **D** 3.56×10^8

4 Write each number in normal decimal form. Example 9

a	7.4×10^5	**b**	3.12×10^{-1}	**c**	1.85×10^3
d	6.6×10^{-4}	**e**	2.54×10^{-3}	**f**	4.751×10^8
g	9.8×10^{-2}	**h**	3×10^2	**i**	5.497×10^{-2}
j	1.216×10^4	**k**	8.02×10^{-1}	**l**	6.309×10^3

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5 Write each measurement in scientific notation.

a The age of the universe is about 13 700 000 000 years.

b One micrometre (μm) is $\frac{1}{1\,000\,000}$ m.

6 Write each measurement in normal decimal notation.

a The diameter of an atom is 3×10^{-8} mm.

b One light year, the distance light travels in one year, is equal to 9.461×10^{9} m.

c The diameter of the flu virus is 2×10^{-6} mm.

d The distance of Earth from the Sun is 1.526×10^{8} km.

7 The world population in 2016 was 7 463 234 065. Write this number in scientific notation, correct to three significant figures.

Example 10

8 Evaluate each expression using scientific notation.

a $(6.7 \times 10^{4}) \times (3.2 \times 10^{2})$

b $(1.92 \times 10^{10}) \div (6 \times 10^{4})$

c $(3.75 \times 10^{4}) - (2.5 \times 10^{3})$

d $(4.8 \times 10^{-3})^{2}$

e $(8.1 \times 10^{4})^{3}$

f $\frac{9.4 \times 10^{-2}}{2.5 \times 10^{-3}}$

g $(4.2 \times 10^{5}) \times (3 \times 10^{-2})$

h $\sqrt{5.29 \times 10^{10}}$

9 Evaluate each expression in scientific notation, correct to two significant figures.

a $(5.1 \times 10^{3}) + (2.5 \times 10^{4})$

b $(7.8 \times 10^{-5}) - (3.7 \times 10^{-3})$

c $(9.6 \times 10^{8}) \times (4.3 \times 10^{3})$

d $(3.2 \times 10^{-3})^{3}$

e $\frac{3.2 \times 10^{2}}{5.4 \times 10^{-3}}$

f $\sqrt[3]{3.8 \times 10^{-5}}$

g $\sqrt{8.5 \times 10^{5}}$

h $\frac{6 \times 10^{3}}{5.1 \times 10^{7}}$

10 Evaluate each expression, expressing the answer in normal decimal form.

a $96 \div 20\,000$

b $480\,000 \times 91\,000$

c $\frac{0.235}{25}$

d $13\,700^{2}$

e $(4.4 \times 10^{4}) \times (3.1 \times 10^{2})$

f $(1.9 \times 10^{7}) + (6.3 \times 10^{4})$

g $\sqrt{2.704 \times 10^{-9}}$

h $\frac{7.7 \times 10^{9}}{2.2 \times 10^{4}}$

11 What is the largest number you can enter into your calculator in scientific notation? What is the smallest number?

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5.05 Perimeters of circular and composite shapes

A page of circular shapes

Circumference

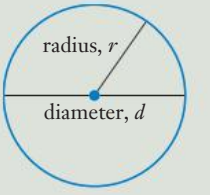
- The **perimeter** of a shape is the distance around the shape.

  It is the sum of the lengths of the sides of the shape.

- The perimeter of a **circle** is called its **circumference**.

  The **radius** is the distance from the centre of a circle to the circumference.

  The radius is half of the **diameter**.



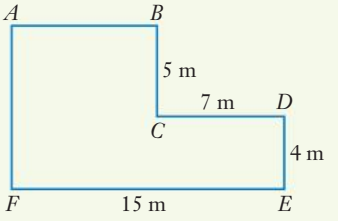
- The **circumference of a circle** with **diameter**, d, or radius, r, is given by the formula:

$$C = \pi d \text{ or } C = 2\pi r$$

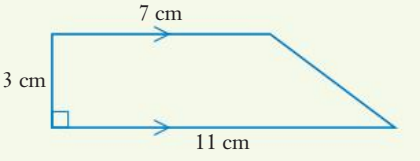
EXAMPLE 11

Find the perimeter of each shape.

a



b



Solution

a Find the lengths of the unknown sides first.

$AB = 15 - 7 = 8$

$AF = 5 + 4 = 9$

Perimeter $= 8 + 5 + 7 + 4 + 15 + 9 = 48$ m

It is possible to make this shape into one big rectangle by moving the 5 m side to the right and the 7 m side upwards. This rectangle has a length of 15 m and a width of 9 m, so its perimeter (15 + 15 + 9 + 9 = 48 m) should be the same as the perimeter of the composite shape.

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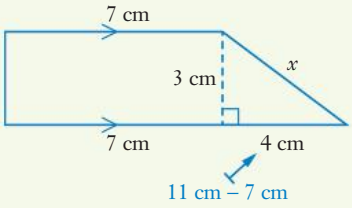
b Let x be the length of the unknown slanted side, which can be found using **Pythagoras'** theorem.



$x^2 = 3^2 + 4^2 = 25$

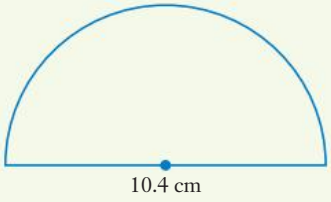
$x = \sqrt{25} = 5$ cm

Perimeter = 7 + 5 + 11 + 3 = 26 cm

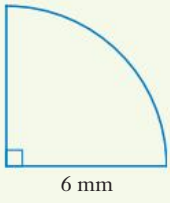
EXAMPLE 12

Find, correct to one decimal place, the perimeter of each shape.

a



b



Solution

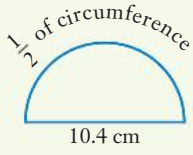
a Perimeter of semicircle $= (\frac{1}{2} \times$ circumference $) + 10.4$

$= (\frac{1}{2} \times \pi \times 10.4) + 10.4$ $\quad C = \pi d$

$= 26.73628...$

≈ 26.7 cm

From the diagram, a perimeter of 26.7 cm looks reasonable.

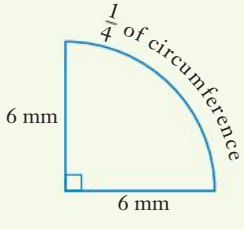


b Perimeter of quadrant $= (\frac{1}{4} \times$ circumference$) + 6 + 6$

$= (\frac{1}{4} \times 2 \times \pi \times 6) + 12$ $\quad C = 2\pi r$

$= 21.424777...$

≈ 21.4 mm



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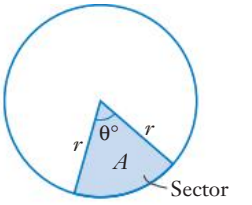
Perimeter of a sector

A **sector** is a fraction of a circle 'cut' along two radii, like a pizza slice.

'radii' is the plural of 'radius': one radius, two radii.



An **arc** is a part of the circumference of a circle. Its length (l) is a fraction of the circumference ($2\pi r$) of the circle. There are 360° in a circle.

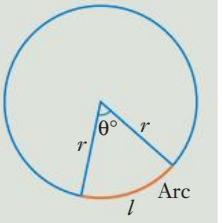
Length of an arc

$$l = \frac{\theta}{360} \times 2\pi r$$

where l = arc length

$\theta°$ = the size of the central angle

r = radius



Perimeter of a sector

$$P = \frac{\theta}{360} \times 2\pi r + 2r \quad \text{(arc length + radius + radius)}$$

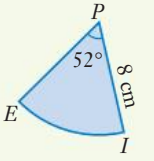
Note: If $\theta = 90°$, the sector is called a **quadrant** (quarter of a circle) and if $\theta = 180°$, the sector is a **semicircle** (half of a circle).

EXAMPLE 13

Find, correct to two significant figures:

a the length of arc *EI*

b the perimeter of sector *PIE*.



Solution

a $l = \frac{\theta}{360} \times 2\pi r$

$= \frac{52}{360} \times 2 \times \pi \times 8$

$= 7.2605...$

≈ 7.3 cm

Length of arc *EI* is 7.3 cm.

b Perimeter = 7.2605… + 8 + 8 Arc length + radius + radius

= 23.2605…

≈ 23 cm

Perimeter of sector *PIE* is 23 cm.

Perimeter and area of a sector

Perimeter and area of a sector

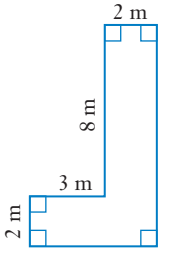
ISBN 9780170413565

Exercise 5.05 Perimeter

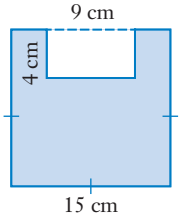
Example 11

1 Find the perimeter of each shape.

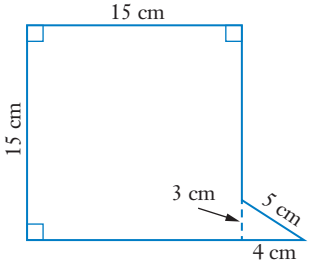
a



b



c



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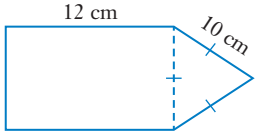
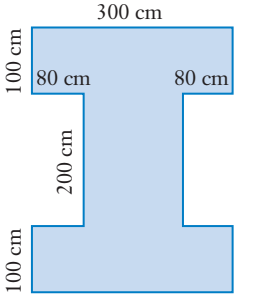


e



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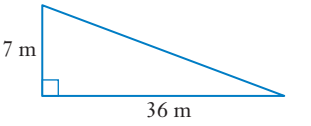
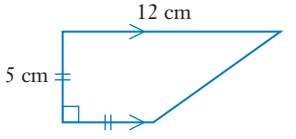
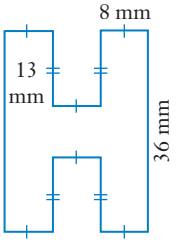


g



h



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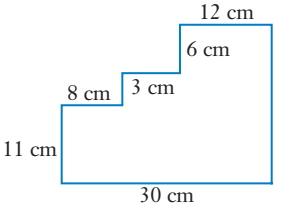
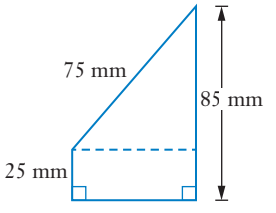


j



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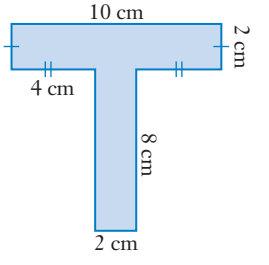

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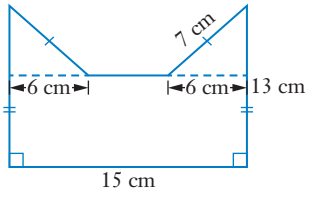

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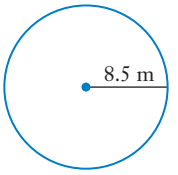
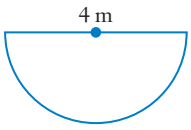
Example 12

2 Find, correct to one decimal place, the perimeter of each shape.

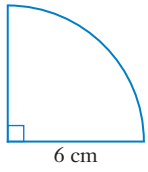
a



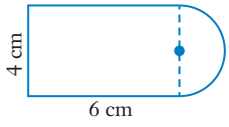
b



c



d



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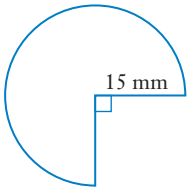

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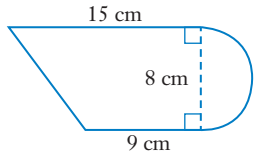

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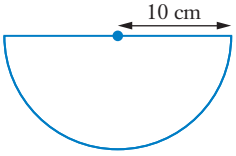
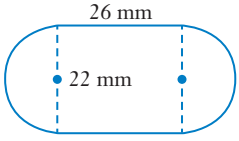
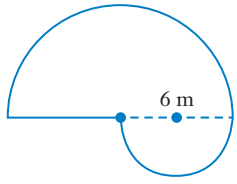


h



i



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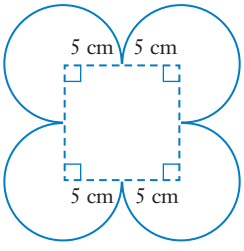

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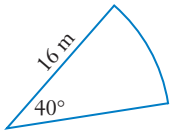

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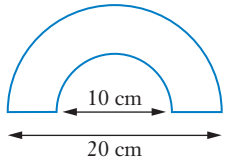
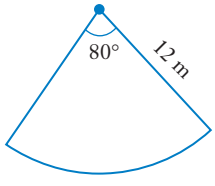


Example 13

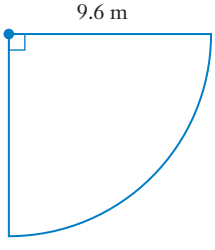
3 For each sector, find correct to one decimal place:

i the length of its arc **ii** its perimeter.

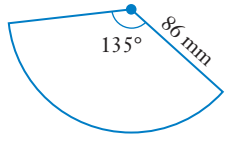
a



b



c



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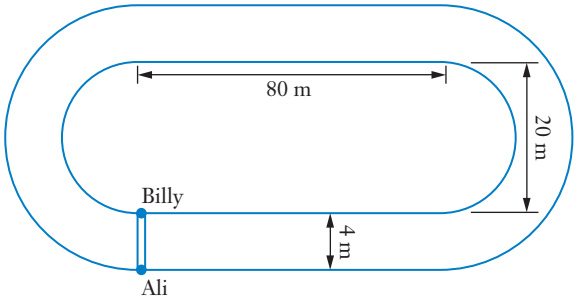
4 Ali and Billy ran around the racetrack shown below along different lanes. Calculate how much more distance Ali ran than Billy in one lap of the track. Answer correct to the nearest 0.1 metre.



5 A pizza has diameter 30 cm and is cut into eight equal pieces. Find:

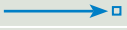
a the central angle θ for one slice

b the perimeter of one slice, correct to one decimal place.

5.06 Area

The **area** of a plane (flat) shape is the amount of surface it occupies, measured in **square units**.

Metric units for area

Unit	Relationship	Size
square millimetre (mm^2)	$1\ mm^2 = 1\ mm \times 1\ mm$	The area of this tiny 1 mm square 
square centimetre (cm^2)	$1\ cm^2 = 1\ cm \times 1\ cm$ $= 10\ mm \times 10\ mm$ $= 100\ mm^2$	The area of this 1 cm square
square metre (m^2)	$1\ m^2 = 1\ m \times 1\ m$ $= 100\ cm \times 100\ cm$ $= 10\ 000\ cm^2$	The area of a 1 m square (about the size of the floor of a large shower recess)
hectare (ha)	$1\ ha = 100\ m \times 100\ m$ $= 10\ 000\ m^2$	The area of a 100 m square (about the size of two football fields)
square kilometre (km^2)	$1\ km^2 = 1\ km \times 1\ km$ $= 1000\ m \times 1000\ m$ $= 1\ 000\ 000\ m^2$	The area of a 1 km square (about the size of a large theme park)

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When converting units of area, we need to multiply or divide by two lots of units.
For example, if 1 m = 100 cm, then 1 m^2 = (100 × 100) cm^2 = 10 000 cm^2.

EXAMPLE 14

Convert:

a 55 m^2 to cm^2 **b** 2350 m^2 to ha **c** 0.8 km^2 to ha

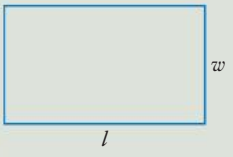
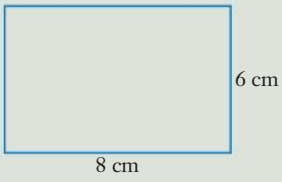
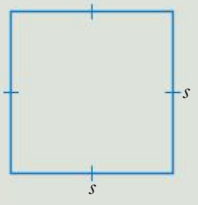
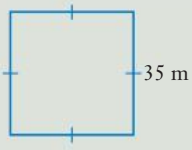
Solution

a 55 m^2 = (55 × 100 × 100) cm^2 Because 1 m^2 = 100 cm × 100 cm
= 550 000 cm^2

b 2350 m^2 = (2350 ÷ 10 000) ha Because 1 ha = 10 000 m^2
= 0.235 ha

c 0.8 km^2 = (0.8 × 1000 × 1000) m^2 Because 1 km^2 = 1000 m × 1000 m
= 800 000 m^2 Converting to m^2 first
= (800 000 ÷ 10 000) ha Because 1 ha = 10 000 m^2
= 80 ha

Area formulas

Shape	Area formula	Example
Rectangle 	A = length × width $A = lw$	 $A = 8 \times 6 = 48$ cm^2
Square 	A = side × side = $(\text{side})^2$ $A = s^2$	 $A = 35^2 = 1225$ m^2

Areas of composite shapes
A page of circular shapes

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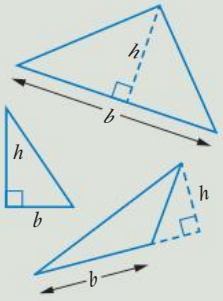
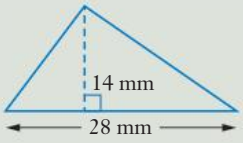
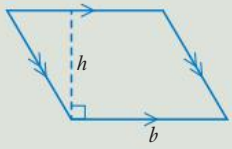
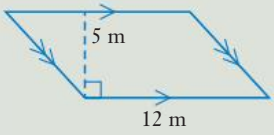
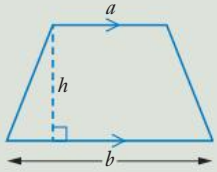
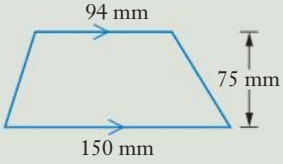
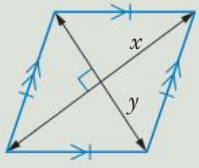
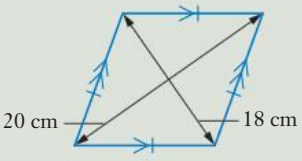
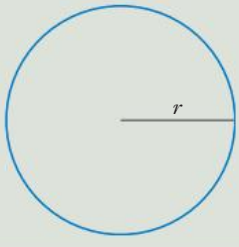
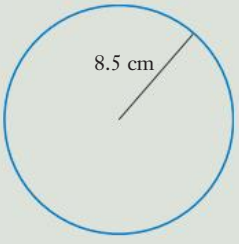
Shape	Area formula	Example
Triangle 	$A = \frac{1}{2} \times$ base $\times$ perpendicular height $A = \frac{1}{2} bh$	 $A = \frac{1}{2} \times 28 \times 14 = 196 \text{ mm}^2$
Parallelogram 	$A =$ base $\times$ perpendicular height $A = bh$	 $A = 12 \times 5 = 60 \text{ m}^2$
Trapezium 	$A = \frac{1}{2} \times$ sum of parallel sides $\times$ perpendicular distance between sides $A = \frac{1}{2}(a + b)h$ $= \frac{h}{2}(a + b)$	 $A = \frac{75}{2} \times (94 + 150)$ $= 9150 \text{ mm}^2$
Rhombus or kite 	$A = \frac{1}{2} \times$ product of diagonals $A = \frac{1}{2} xy$	 $A = \frac{1}{2} \times 20 \times 18 = 180 \text{ cm}^2$
Circle 	$A = \pi \times (\text{radius})^2$ $A = \pi r^2$	 $A = \pi \times 8.5^2 = 227 \text{ cm}^2$

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Area of a sector

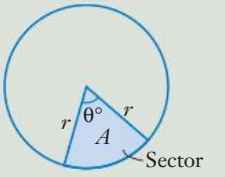
A sector's area (A) is a fraction of the area (πr^2) of the circle.

Area of a sector

$$A = \frac{\theta}{360} \times \pi r^2$$

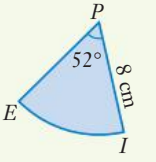
where A = area

$\theta°$ = is the size of the central angle.



EXAMPLE 15

Find, correct to two significant figures, the area of sector *PIE*.



Solution

$$A = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{52}{360} \times \pi \times 8^2$$

$$= 29.0422...$$

$$\approx 29 \text{ cm}^2$$

Area of sector *PIE* is 29 cm^2.

Perimeter and area of a sector

Perimeter and area of a sector

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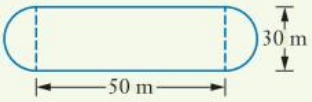
EXAMPLE 16

A local park has the shape of a rectangle with a semicircle on each end as shown.



a What is the area of this park, correct to the nearest square metre?

b What would it cost to cover this park with new grass if the grass costs $10.50 per square metre?

Solution

a Area of park = rectangle + circle

$= 50 \times 30 + \pi \times 15^2$

$= 2206.858\ldots$

$\approx 2207\ \text{m}^2$

2 semicircles = 1 circle

Radius of circle = $\frac{1}{2} \times 30 = 15$ m

b Cost of grass = $2207 \times \$10.50$

$= \$23\ 173.50$

Exercise 5.06 Area

1 How many square centimetres in 300 m^2? Select **A**, **B**, **C** or **D**.

A 3 **B** 3000 **C** 3 000 000 **D** 300

2 How many hectares in 5 km^2? Select **A**, **B**, **C** or **D**.

A 500 **B** 50 **C** 50 000 **D** 0.5

Example 14

3 Convert:

a 5 m^2 to cm^2

b 2500 cm^2 to mm^2

c 72 000 m^2 to ha

d 6800 cm^2 to m^2

e 3.09 km^2 to m^2

f 3.6 km^2 to ha

g 4.73 m^2 to mm^2

h 540 ha to km^2.

4 The area of NSW is 801 600 km^2. How many hectares is this in scientific notation?

5 Builders measure lengths in millimetres, and carpet layers quote per square metre. How many square metres of carpet are needed in a room that measures 2300 mm by 1830 mm? Answer this question using the following two methods:

a calculate the carpet area in mm^2, and then convert to m^2.

b convert the lengths to metres first, and then calculate the carpet area in m^2.

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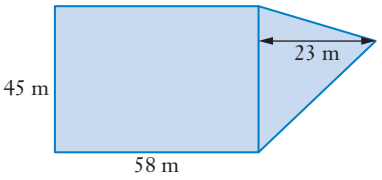
6 Find the area of this park. Select **A**, **B**, **C** or **D**.

A 3127.5 m^2 **B** 3944 m^2

C 3645 m^2 **D** 1972 m^2

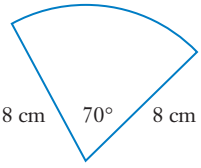


7 Find the area of this sector, correct to one decimal place. Select **A**, **B**, **C** or **D**.

Example 15

A 3.09 cm^2 **B** 9.7 cm^2

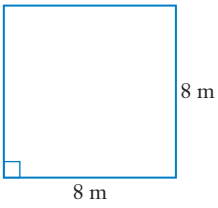
C 39.1 cm^2 **D** 9.8 cm^2



8 Find the area of each shape, correct to one decimal place where needed. (*Note*: Some questions require the use of Pythagoras' theorem first.)

Example 16

a



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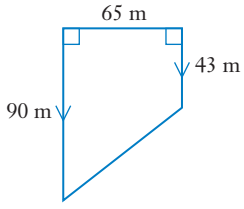

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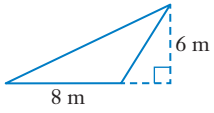

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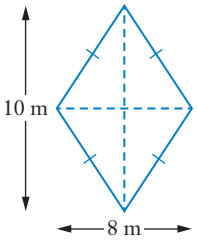

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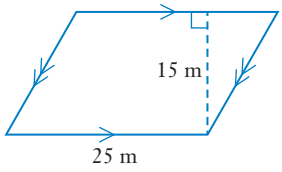

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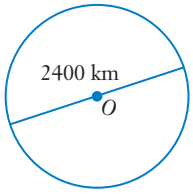

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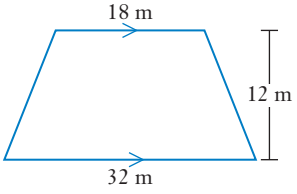

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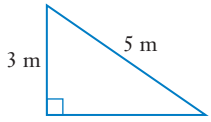

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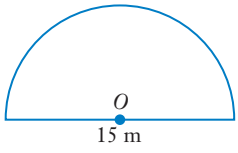
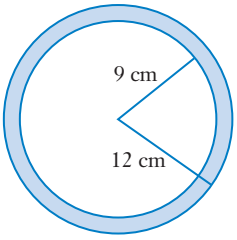
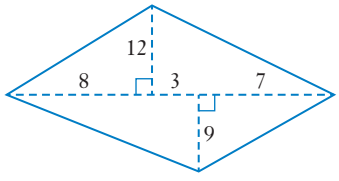


j Find the area of the shaded annulus.



k Measurements are in metres



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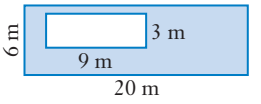
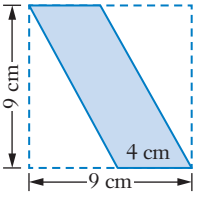
9 Find each shaded area, correct to the nearest square unit where necessary.

a



b



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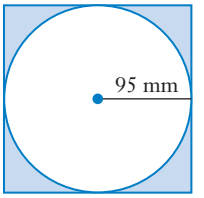

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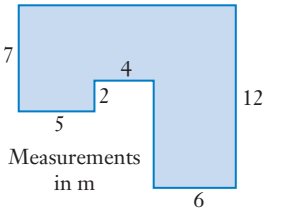

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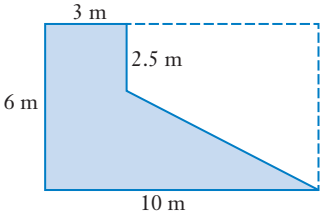

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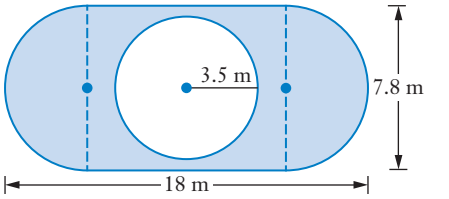
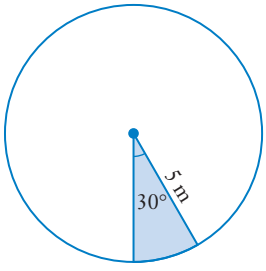


10 Find the area of each shaded sector, correct to the nearest square unit.

a



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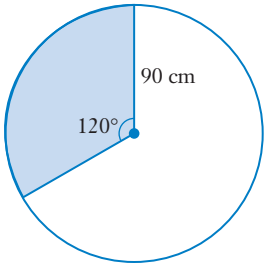

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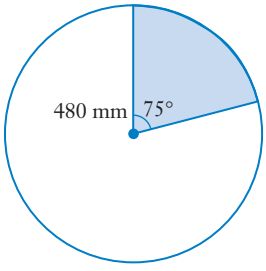

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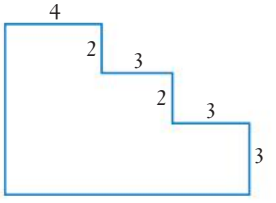
11 Find the area of each shape, correct to the nearest square metre where necessary. All measurements are in metres.

a



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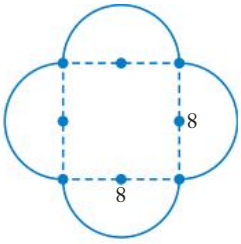

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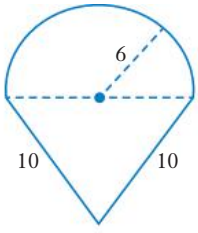

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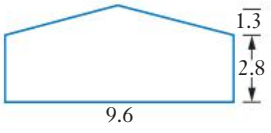

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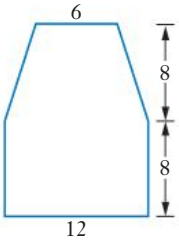

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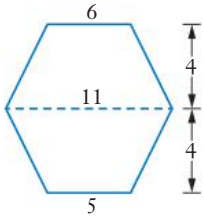
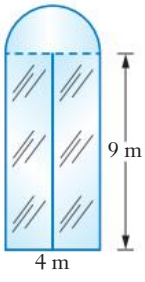


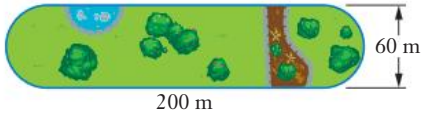
12 Find, correct to the nearest 0.1 m^2, the area of the glass in this window.



13 **a** Find the area of this garden, correct to two significant figures.

b What is the perimeter of this garden, correct to one decimal place?

c What is the cost of fencing the garden if fencing costs $124 per metre?

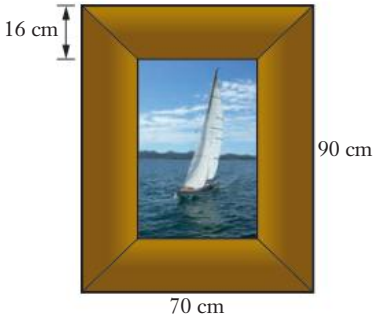


Example 16

14 A wooden picture frame is specifically made for the picture shown. The cost is $135 per square metre. Calculate:

a the area of the frame, in square metres

b the cost of the frame, to the nearest cent.



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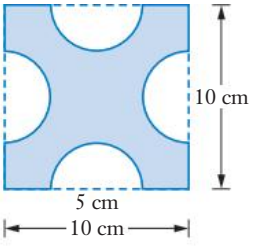
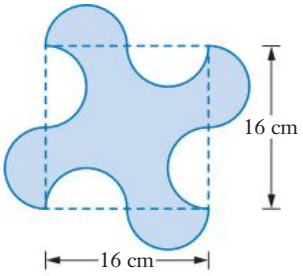
15 a Find the area of each plastic throwing toy, correct to one decimal place.

i



ii



b Can you see an easy way of determining the area of the toy in part **ii** above?

5.07 Land surveying

Offset surveys

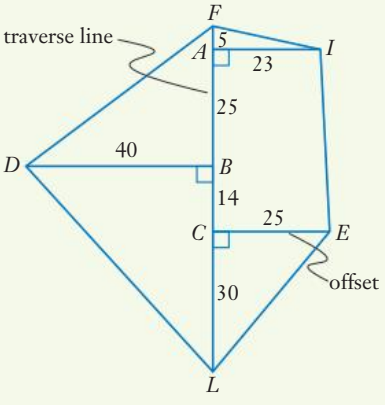
Offset surveys

One method that surveyors use to measure the area of an irregular block of land is an **offset survey**, also called a **traverse survey**. The surveyor chooses a diagonal of the field to be the **traverse line** (traverse means 'to cross'), then measures the offsets (perpendicular distances) to each corner.

EXAMPLE 17

An offset survey was conducted on an irregular block of land, as shown on the **field diagram**, using LF as the traverse line. All measurements are in metres. Find the area of the block.

As most measurements in the diagram are correct to two significant figures, we should write the final answer correct to two significant figures.



Solution

The block is made up of one big triangle on the left, two small triangles and a trapezium on the right.

$$\text{Area of } \Delta FLD = \frac{1}{2} \times 74 \times 40 \qquad LF = 5 + 25 + 14 + 30 = 74$$

$$= 1480 \text{ m}^2$$

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Area of $\Delta FIA = \frac{1}{2} \times 23 \times 5$

$= 57.5 \text{ m}^2$

Area of trapezium $\Delta IECA = \frac{39}{2} \times (23 + 25)$ $\quad AC = 25 + 14 = 39$

$= 936 \text{ m}^2$

Area of $\Delta CEL = \frac{1}{2} \times 25 \times 30$

$= 375 \text{ m}^2$

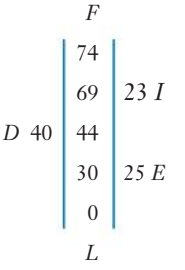
Total area of block $= 1480 + 57.5 + 936 + 375$

$= 2848.5$

$\approx 2800 \text{ m}^2$ Correct to two significant figures

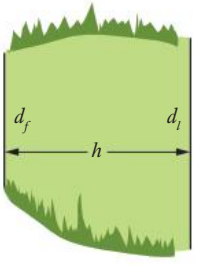
When conducting an offset survey, the surveyor marks the measurements in a notebook first. The **notebook entries** for the field in Example **17** look like those shown on the right.

The surveyor uses a tape measure to measure the increasing distances along the traverse line from *L* to *F*. For example, 74 m is the **entire** distance from *L* to *F*, whereas the distance $CB = 44 - 30 = 14$ m.



The trapezoidal rule

The **trapezoidal rule** is an approximation method for calculating the area of an irregular shape, such as one with a curved side. Approximate the shape as a trapezium with width, *h*, and take two vertical measurements, d_f and d_l, as shown.



The trapezoidal rule

$$A \approx \frac{h}{2}\left(d_f + d_l\right)$$

where A = area

h = distance between successive measurements

d_f = first measurement

d_l = last measurement

or in words:

$$\text{Area} \approx \frac{\text{width of strip}}{2} (\text{first measurement} + \text{last measurement})$$

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EXAMPLE 18

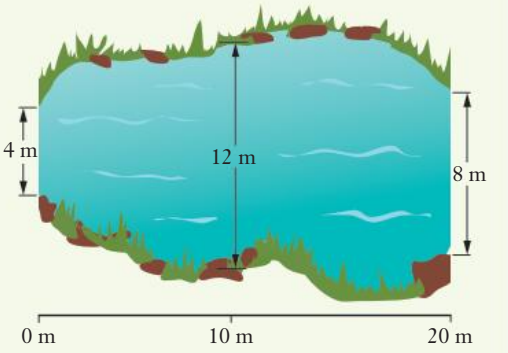
A surveyor took three vertical measurements across a pond at 10 m intervals.

Find the approximate area of the pond using:

a one application of the trapezoidal rule

b two applications of the trapezoidal rule.

Which answer is more accurate?



Solution

a For one application, we treat the pond as a trapezium with parallel sides 4 m and 8 m, so $h = 20$, $d_f = 4$, $d_l = 8$.

Ignore the 12 m in the middle of the pond.

$$A \approx \frac{h}{2}\left(d_f + d_l\right)$$
$$= \frac{20}{2}(4+8)$$
$$= 120 \text{ m}^2$$

The approximate area of the pond is 120 m^2.

b For two applications, treat the pond as two trapeziums side-by-side, joined by the 12 m side.

For the left trapezium, $h = 10$, $d_f = 4$, $d_l = 12$.

$$A \approx \frac{h}{2}\left(d_f + d_l\right)$$
$$= \frac{10}{2}(4+12)$$
$$= 80 \text{ m}^2$$

For the right trapezium, $h = 10$, $d_f = 12$, $d_l = 8$.

$$A \approx \frac{h}{2}\left(d_f + d_l\right)$$
$$= \frac{10}{2}(12+8)$$
$$= 100 \text{ m}^2$$

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Total area = 800 + 100

= 180 m^2

The approximate area of the pond is 180 m^2.

The 180 m^2 is more accurate, because the more applications (trapeziums) we use, the closer the answer is to the exact area of the pond.

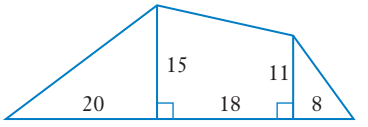
Exercise 5.07 Land surveying

In this exercise, all measurements are in metres.

1 Find the area of each field, correct to two significant figures.

Example 17

a



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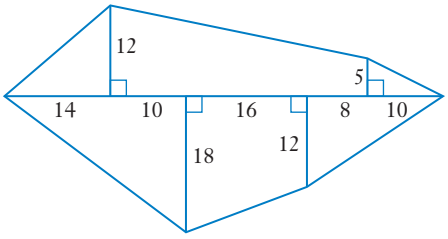

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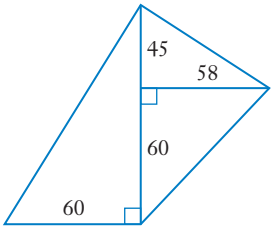
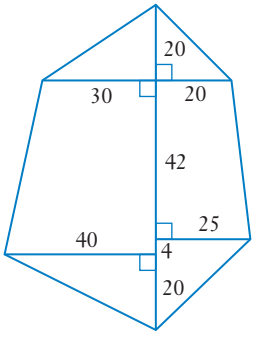
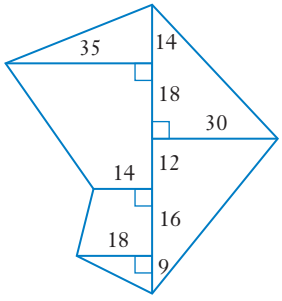


d



e



2 For each surveyor's notebook entry shown, sketch a field diagram and then calculate its area, correct to the nearest square metre.

a

	P	
	0	
	8	16 *Q*
S 7	18	
	23	
	R	

b

	F	
	52	
	34	36 *G*
	20	15 *H*
J 40	12	
	0	
	I	

c

	V	
	38	
	26	28 *W*
	20	36 *X*
Z 26	0	
	Y	

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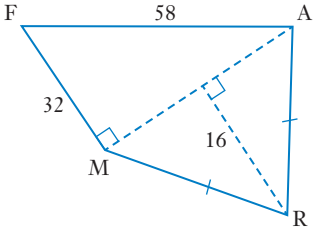
3 **a** Find the area of this farm, correct to the nearest square metre.

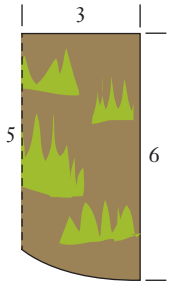
b The farm is to be fenced around its perimeter. What length of fence is needed? Answer to the nearest metre.



Example 18

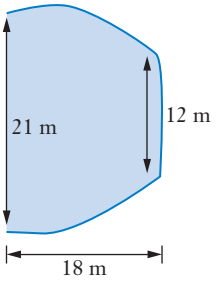
4 A garden has the following measurements in metres. Use the trapezoidal rule to approximate its area to the nearest square metre. Select **A**, **B**, **C** or **D**.

A 7 **B** 17 **C** 18 **D** 45

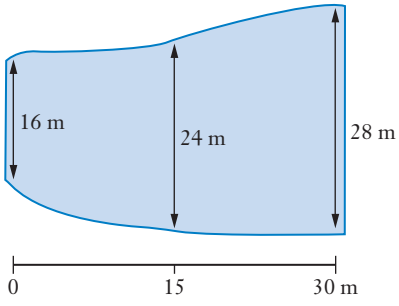


5 Approximate the area of each field using one application of the trapezoidal rule.

a



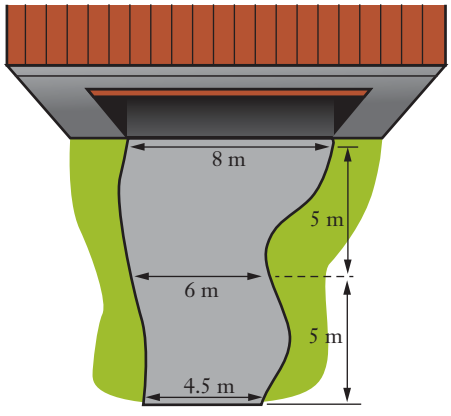
b



6 To calculate an approximate area for her driveway, Olivia took three measurements at 5 m intervals.

a Use two applications of the trapezoidal rule to find the area of the driveway, correct to the nearest m^2.

b What is the cost of sealing the driveway if sealer costs \$45/L and 1 L covers 12 m^2?



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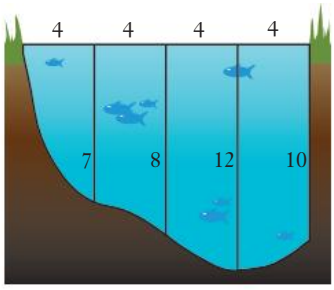
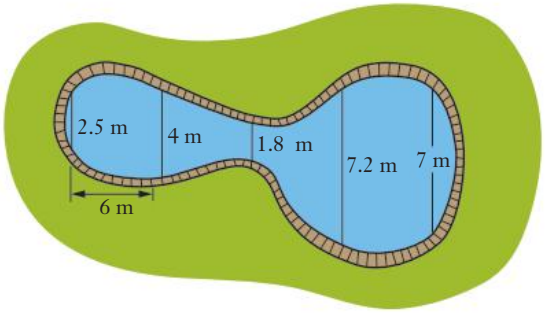
7 The **cross-section** of a river is shown, with measurements in metres. Use four applications of the trapezoidal rule to approximate the area of the cross-section.



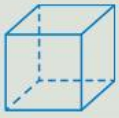
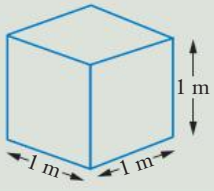
8 The shape of a new swimming pool is marked out on the lawn and five vertical measurements are taken at 6 m intervals. Use four applications of the trapezoidal rule to find the area of the pool.



5.08 Volume and surface area of a prism

Estimating area and volume

A prism is a solid three-dimensional shape with two identical, parallel polygon bases. The **volume** of a solid is the amount of space it occupies, measured in **cubic units**.

Unit	Relationship	Size
cubic millimetre (mm^3)	$1\ mm^3 = 1\ mm \times 1\ mm \times 1\ mm$	The volume of a 1 mm cube (about the size of a grain of raw sugar)
cubic centimetre (cm^3)	$1\ cm^3 = 1\ cm \times 1\ cm \times 1\ cm$ $= 10\ mm \times 10\ mm \times 10\ mm$ $= 1000\ mm^3$	The volume of a 1 cm cube (about the size of a peanut) 
cubic metre (m^3)	$1\ m^3 = 1\ m \times 1\ m \times 1\ m$ $= 100\ cm \times 100\ cm \times 100\ cm$ $= 1\ 000\ 000\ cm^3$	The volume of a 1 m cube (about the size of two washing machines) 

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When converting units of volume, we need to multiply or divide by three lots of units.

For example, if 1 m = 100 cm, then 1 m^3 = (100 × 100 × 100) cm^3 = 1 000 000 cm^3.

EXAMPLE 19

Convert:

a 74 m^3 to cm^3 **b** 4600 mm^3 to cm^3

Solution

a 74 m^3 = (74 × 100 × 100 × 100) cm^3 Because 1 m^3 = (100 × 100 × 100) cm^3

= 74 000 000 cm^3

b 4600 mm^3 = (4600 ÷ 10 ÷ 10 ÷ 10) cm^3 Because 1 cm^3 = (10 × 10 × 10) mm^3

= 4.6 cm^3

Volume and capacity

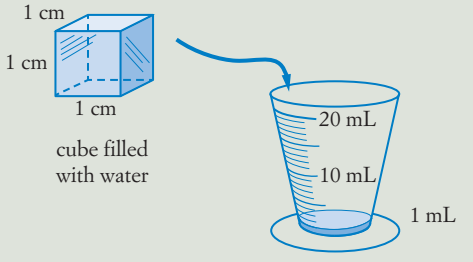
While volume is the amount of space taken up by a solid, **capacity** is the amount of material (usually liquid) that a solid or container can hold.

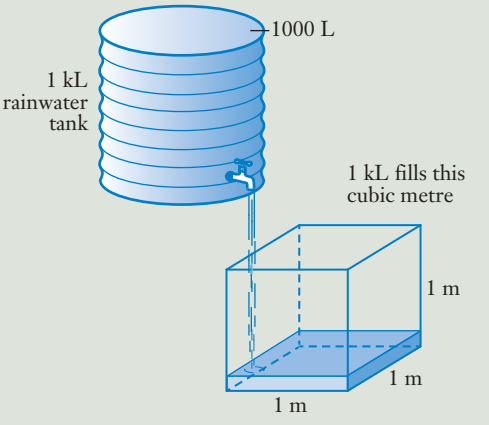
Volume and capacity units

Volume and capacity

1 cm^3 = 1 mL

1 m^3 = 1 kL = 1000 L





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EXAMPLE 20

Convert:

a 46 L to cm^3 **b** 10 400 cm^3 to L **c** 3500 m^3 to ML

Solution

a 46 L = 46 000 mL — 1 L = 1000 mL

= 46 000 cm^3 — 1 cm^3 = 1 mL

b 10 400 cm^3 = 10 400 mL — 1 cm^3 = 1 mL

= 10.4 L

c 3500 m^3 = 3500 kL — 1 m^3 = 1 kL

= 3 500 000 L

= 3.5 ML — 1 ML = 1 million litres

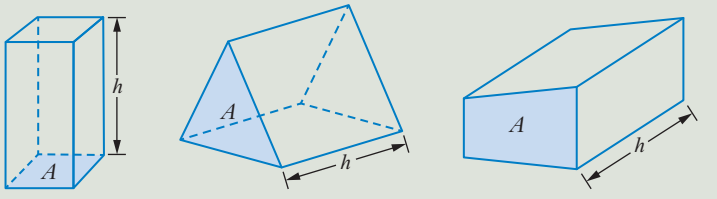
Volume of a prism

$$V = Ah$$

where V = volume

A = area of the base

h = height



Volumes of prisms and cylinders

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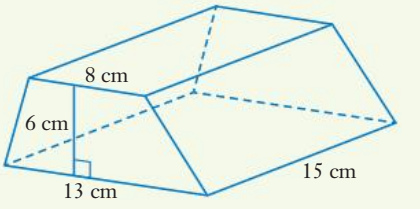
EXAMPLE 21

Find the volume of this trapezoidal prism.



Solution

The base is a trapezium where:

$$A = \frac{6}{2} \times (8 + 13)$$
$$= 63 \text{ cm}^2$$
$$V = Ah$$
$$= 63 \times 15 \text{ cm}$$
$$= 945 \text{ cm}^3$$

Surface area of a prism

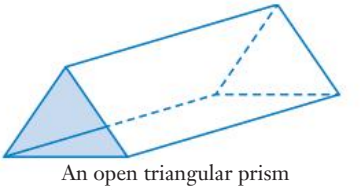
Surface area of a prism

The surface area of a prism is the sum of the areas of its faces, measured in square units.

Nets of solids

The solid can be closed or open.

A closed rectangular prism has 6 faces.

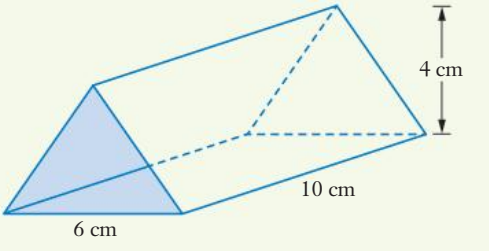


An open triangular prism with no end faces has 3 faces.

EXAMPLE 22

This triangular prism is open at one end. Calculate its surface area.

'Surface area' here means the external (outside) surface area.



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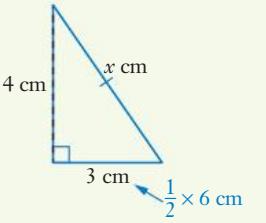
Solution

The open prism has one triangular face and three rectangular faces. To find the areas of the side faces, we must first find the slant height (x cm) of the triangle using Pythagoras' theorem.



$x^2 = 3^2 + 4^2$

$= 25$

$x = \sqrt{25}$

$= 5$

Surface area = area of triangle + area of base rectangle + 2 × area of side rectangles

$= \frac{1}{2} \times 6 \times 4 + (10 \times 6) + 2 \times (10 \times 5)$

$= 172$

The surface area of this triangular prism is 172 cm^2.

Exercise 5.08 Volume and surface area of a prism

1 Convert: Example 19

a 7 m^3 to cm^3 **b** 50 cm^3 to mm^3 **c** 89 000 cm^3 to m^3

d 0.468 m^3 to cm^3 **e** 2400 mm^3 to cm^3 **f** 5 600 000 cm^3 to m^3

g 9 100 000 mm^3 to cm^3 **h** 12 m^3 to cm^3.

2 Convert: Example 20

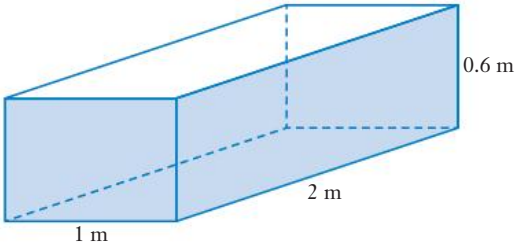
a 680 cm^3 to mL **b** 8500 cm^3 to L **c** 22 m^3 to L

d 8000 L to m^3 **e** 3.5 m^3 to mL **f** 690 L to cm^3

g 55 m^3 to L **h** 4300 m^3 to kL **i** 9500 L to m^3

j 8.5×10^4 cm^3 to L **k** 4.3×10^{-3} kL to cm^3 **l** 10^6 m^3 to ML.

3 Find the volume of this rectangular prism. Select **A**, **B**, **C** or **D**. Example 21



A 1.2 m^3 **B** 7.6 m^3 **C** 7.2 m^3 **D** 2.4 m^3

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4 A fish tank is 1.2 m long, 80 cm wide and 90 cm high.

a What is its volume in cubic metres?

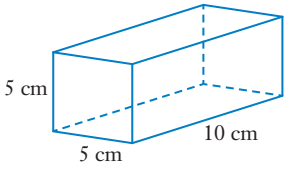
b What is its capacity in litres?

5 What is the volume of a cube of length 2.5 m? Select **A**, **B**, **C** or **D**.

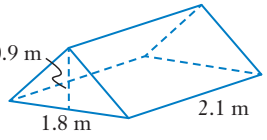
A 6.25 mm^3 **B** 15.625 m^3 **C** 30 m^3 **D** 37.5 m^3

6 Find the volume of each prism.

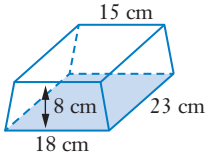
a



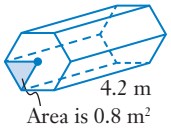
b



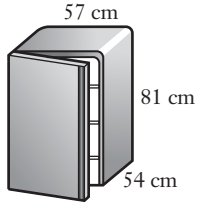
c



d

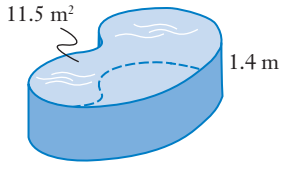


7 What is the capacity of this bar fridge in litres?



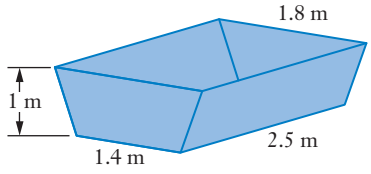
8 The cross-section of a kidney-shaped swimming pool has an area of 11.5 m^2. The pool has a constant depth of 1.4 m. What volume of water does it hold:

a in cubic metres? **b** in litres?



9 **a** This skip has the shape of a trapezoidal prism. What volume of rubbish will it hold?

b If building rubbish costs $16.50 per cubic metre to dump at the local tip, how much will it cost to dump four full skips?



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10 Warragamba Dam supplies Sydney with water and has a capacity of 2.031×10^6 ML. Use scientific notation to express the dam's capacity:

a in litres **b** in cubic metres.

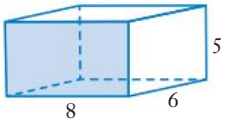
11 Find the surface area of the prism in question **3** if it is open at the top. Select **A**, **B**, **C** or **D**.

A 172 m^2 **B** 3.6 m^2 **C** 5.6 m^2 **D** 6.6 m^2

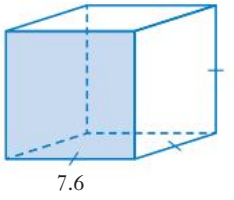
Example 22

12 Find the surface area of each closed prism, correct to the nearest whole number. All measurements are in metres.

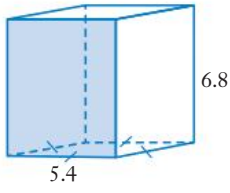
a



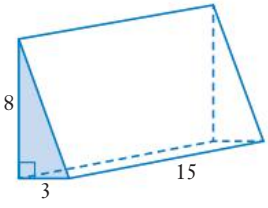
b



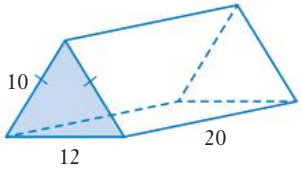
c



d

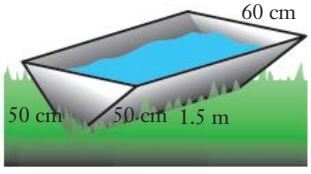


e

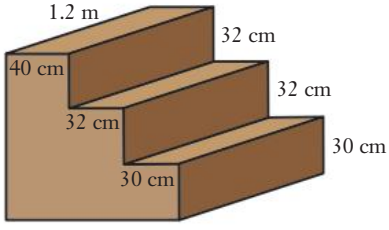


13 **a** Find the external surface area of this open water trough, correct to two significant figures.

b What is the capacity of the trough, to the nearest litre?



14 If the cross-section of a prism has area, A, and perimeter, P, and the length of the prism is h, then its surface area can be calculated using the formula $S = 2A + Ph$. The $2A$ represents the area of the two identical end faces, while Ph is the total area of the rectangular side faces, which combine to make one big rectangle.



a A portable set of stairs has dimensions as shown. Use the formula to calculate its surface area in square metres.

b Calculate its volume in cubic metres.

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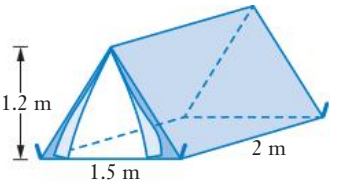
15 **a** Find the volume of this tent.

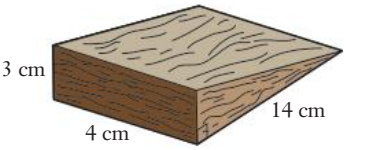
b How many square metres (correct to one decimal place) of canvas were used to make this tent, including the floor?



16 This timber wedge is used to prop open a door.

a Calculate its volume.

b Kobi applies varnish to a box of wedges. One 250 mL pot of varnish covers one square metre of surface area. How many wedges can he varnish with 5 pots?



DID YOU KNOW?

Mass and capacity

One litre of water has a mass of one kilogram. For water, 1 L = 1 kg.

What is the mass of:

a one cubic metre of water?

b one cubic centimetre of water?

Measurement in the home

Sweet areas and volumes

Volumes of prisms and cylinders

5.09 Volume and surface area of a cylinder

Volume of a cylinder

A cylinder has a circular base and its volume can also be found using the formula $V = Ah$ where $A = \pi r^2$.

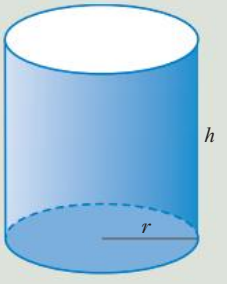
Volume of a cylinder

$$V = \pi r^2 h$$

where V = volume

r = radius of circular base

h = perpendicular height



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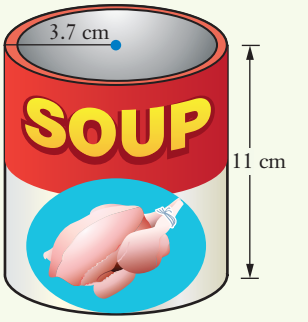
EXAMPLE 23

A can of soup has base radius 3.7 cm and height 11 cm. What is its volume, correct to the nearest millilitre?



Solution

$V = \pi r^2 h$

$= \pi \times 3.7^2 \times 11$

$= 473.0924\ldots \text{ cm}^3$

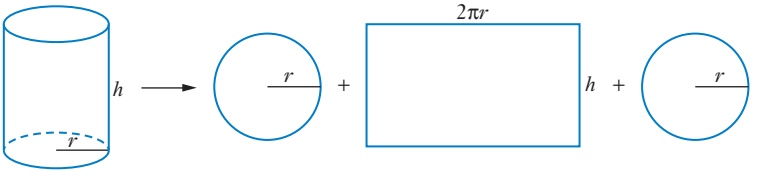
$= 473.0924\ldots \text{ mL}$ $\quad 1 \text{ cm}^3 = \text{mL}$

$\approx 473 \text{ mL}$

Volume of the can is 473 mL.

Surface area of a cylinder

A closed cylinder has two circular ends and a curved surface. The curved surface is a rectangle whose length is equal to the circumference ($2\pi r$) of the circular end.



Surface area = area of 2 circles + area of rectangle

$= 2 \times \pi r^2 + 2\pi r \times h$

$= 2\pi r^2 + 2\pi rh$

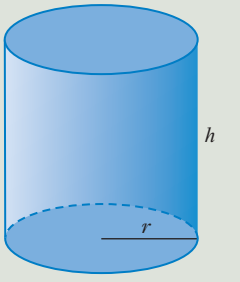
Surface area of a closed cylinder

$$A = 2\pi rh + 2\pi r^2$$

where A = surface area

r = radius of circular base

h = perpendicular height



Surface area of a cylinder

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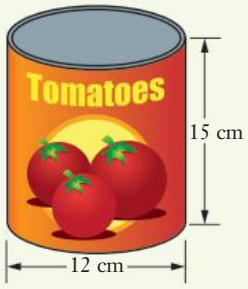
EXAMPLE 24

A can of tomatoes has base diameter 12 cm and height 15 cm. Calculate to the nearest square centimetre:

a the surface area of the can

b the area of the label wrapped around the can if there is a 2 cm overlap.



Solution

a $A = 2\pi rh + 2\pi r^2$

$= 2 \times \pi \times 6 \times 15 + 2 \times \pi \times 6^2$

$= 791.6813\ldots$

$\approx 792 \text{ cm}^2$

The surface area of the can is 792 cm^2.

$r = \frac{1}{2} \times 12 \text{ cm} = 6 \text{ cm}$

b Length of label $= 2\pi r + 2$, height of label $= h$

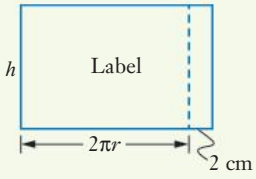
Area of label $= (2\pi r + 2) \times h$

$= (2 \times \pi \times 6 + 2) \times 15$

$= 595.4866\ldots$

$\approx 595 \text{ cm}^2$

The area of the label is 595 cm^2.

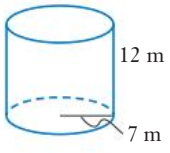


Exercise 5.09 Volume and surface area of a cylinder

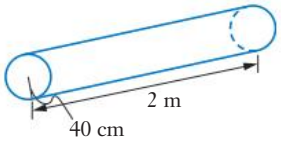
Example 23

1 Find, correct to two significant figures, the volume of each cylinder. Answer parts **b** and **d** in m^2.

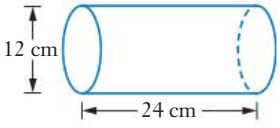
a



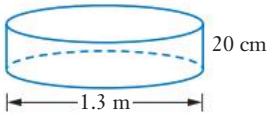
b



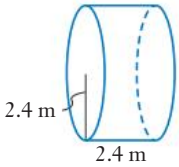
c



d



e



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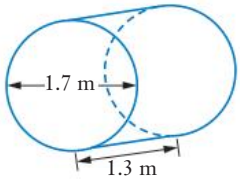

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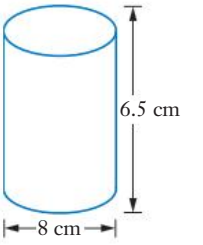
Example 24

2 Find, correct to two significant figures, the surface area of each cylinder in Question **1**.

3 A V6 Holden car has six cylinders. Each cylinder has a bore (diameter) of 8 cm and a stroke (height) of 6.5 cm.

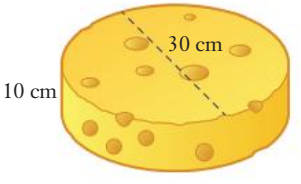
a Find the volume of one cylinder, to the nearest cm^3.

b Find the engine capacity (sum of the capacities of the six cylinders) of the car to the nearest litre.

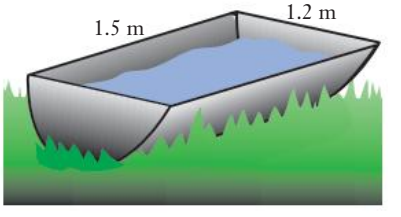


4 Find, correct to two significant figures, the volume of each solid.

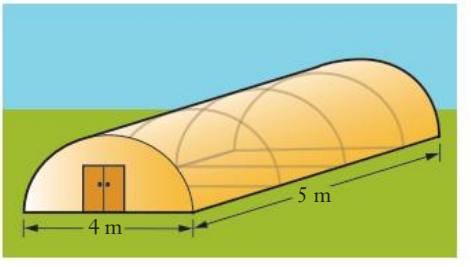
a a wheel of Swiss cheese



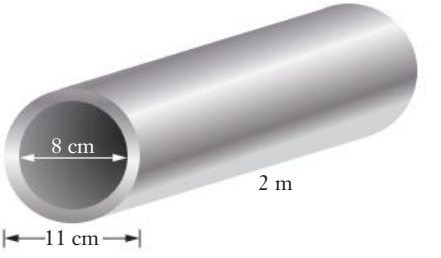
b a water trough in the shape of a half-cylinder



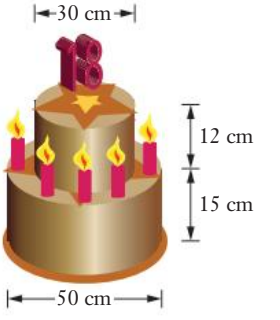
c a greenhouse



d a piece of plumber's pipe



5 All surfaces of this two-tiered cake except the base are iced. Find the area that is iced, correct to the nearest square centimetre.



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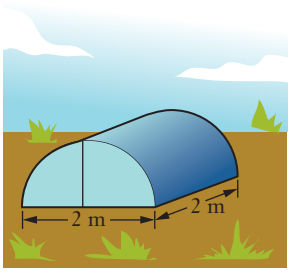
6 A tent has the shape of a half-cylinder. The floor area is 2 m by 2 m. Calculate, correct to one decimal place:

a its volume

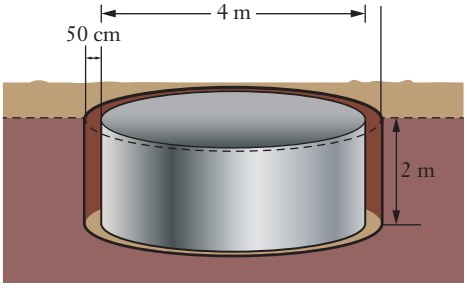
b its surface area, including the floor.



7 A cylindrical tank with diameter 4 m is placed in a 2 m deep circular hole so that there is a gap of 50 cm between the side of the tank and the hole. The top of the tank is level with the ground.

a What volume of dirt was removed to make the hole? Answer to the nearest m^3.

b What is the capacity of the tank, to the nearest litre?



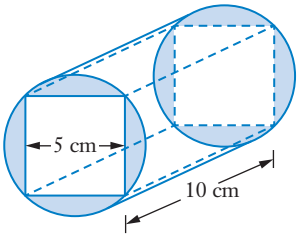
8 A cylinder has a base radius of 5 cm. If the radius is doubled, what happens to the volume? Select **A**, **B**, **C** or **D**.

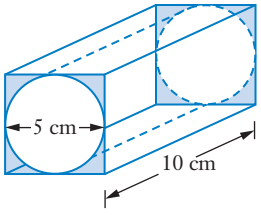
A doubles **B** triples **C** 4 times larger **D** halved

INVESTIGATION

SQUARE OR ROUND PEG?

Have you ever heard the saying that 'you can't put a square peg in a round hole'?





1 Compare the volumes occupied by a square peg of side 5 cm in a round hole of depth 10 cm and a round peg of diameter 5 cm in a square hole of depth 10 cm.

2 Which peg occupies more space?

3 What percentage of the hole does each peg occupy?

4 Is it better to put a square peg in a round hole or a round peg in a square hole? Justify your answer.

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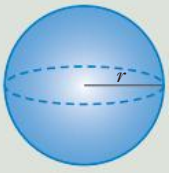
5.10 Volume and surface area of a sphere

Measurement formulas chart
Volumes of solids
Formula matching game

A **sphere** is a perfectly round solid and every point on its surface is the same distance from its centre. This distance is the sphere's **radius**.

Volume of a sphere

$$V = \frac{4}{3}\pi r^3$$



where V = volume

r = radius

Surface area of a sphere

$$A = 4\pi r^2$$

where A = area

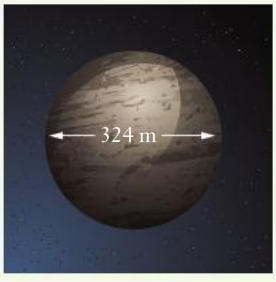
r = radius

EXAMPLE 25

An asteroid is in the shape of a sphere with diameter 324 m.

a What is its volume in scientific notation, correct to three significant figures?

b What is its surface area, correct to the nearest hectare?



Solution

a $V = \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 162^3$

$r = \frac{1}{2} \times 324 = 162$ m

$= 17\,808\,758.84$

$\approx 1.78 \times 10^7$ m^3

The volume of the asteroid is approximately 1.78×10^7 m^3.

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b $A = 4\pi r^2$

$= 4 \times \pi \times 162^2$

$= 329\,791.8304 \text{ m}^2$

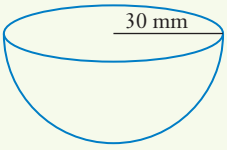
$= 32.9791\ldots \text{ ha}$ $\qquad 1 \text{ ha} = 10\,000 \text{ m}^2$

$\approx 33 \text{ ha}$

The surface area of the asteroid is approximately 33 ha.

EXAMPLE 26

Find the surface area of a closed **hemisphere** with radius 30 mm. Give your answer correct to the nearest 10 mm^2.



Solution

Area of curved surface $= \frac{1}{2} \times 4\pi r^2$

$= 2\pi r^2$

Area of circular top $= \pi r^2$

Surface area of closed hemisphere $= 2\pi r^2 + \pi r^2$

$= 3\pi r^2$

$= 3 \times \pi \times 30^2$

$= 8482.3001\ldots$

$\approx 8480 \text{ mm}^2$

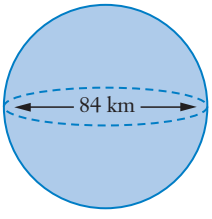
The surface area of the hemisphere is approximately 8480 mm^2.

Exercise 5.10 Volume and surface area of a sphere

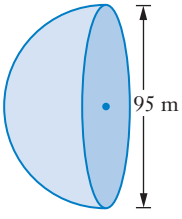
Example 25

1 Find the volume of each sphere or hemisphere, correct to three significant figures.

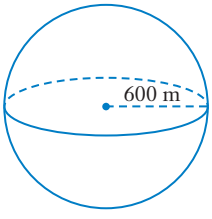
a



b



c



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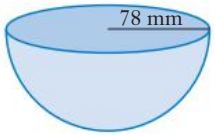
d



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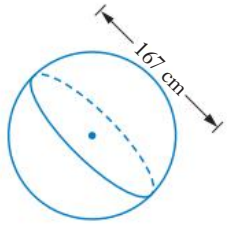

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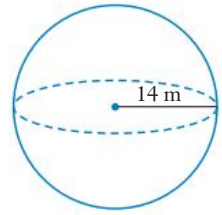


2 Earth is a sphere with a diameter of approximately 12 683 km. Four-fifths of Earth's surface is covered by water.

a Calculate in scientific notation, correct to four significant figures:

i the area of Earth's surface that is covered by land

ii Earth's volume in cubic kilometres.

b If Earth's mass is 5.974×10^{21} tonnes, what is the mass per volume, or density, in t/km^3?

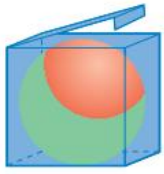
3 A hemispherical metal bird bath has an internal radius of 0.5 m.

Example 26

a What is its capacity, correct to the nearest litre?

b What is the internal surface area, correct two decimal places?

4 Christmas baubles have the shape of a sphere with a diameter of 6 cm. They are packed in individual boxes so they just touch each side of the box.



a What is the volume of each bauble, to the nearest cm^3?

b What volume of air, to the nearest cm^3, is in a box containing one bauble?

c Each bauble is covered in fabric costing \$15 per m^2. How much does it cost to cover 100 baubles?

5 The volume of a sphere is 400 cm^3. What is its radius in cm, correct to one decimal place? Select **A**, **B**, **C** or **D**.

A 4.6 **B** 3.2 **C** 5.6 **D** 7.3

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6 A mirror ball with a radius of 0.28 m hangs over the dance floor at a disco. Its surface is covered with 1 cm^2 mirror tiles.

a What is the volume of the mirror ball in cm^3, correct to three significant figures?

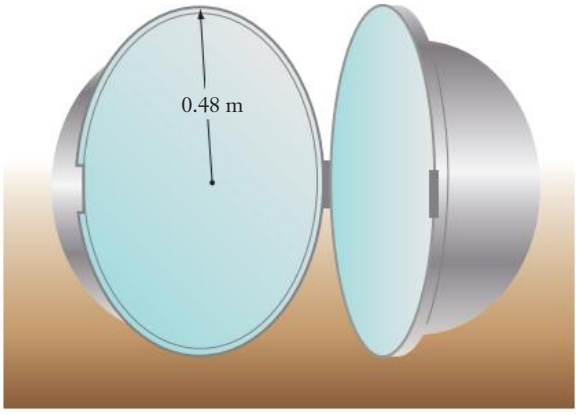
b How many whole mirror tiles cover the surface of the ball?

7 A very powerful computer-operated light is in the shape of two hemispheres hinged together. The curved surface of the light is solid steel and the front of each hemisphere is special glass. If the radius of the light is 0.48 m, find, to two significant figures:

a the area of steel covering the light

b the area of glass

c the volume of the light.



DID YOU KNOW?

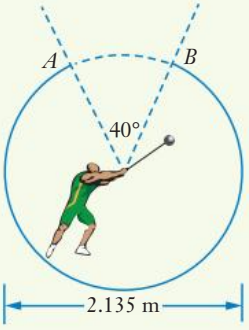
Circles and spheres in sport

The shotput and hammer throw are two athletics field events requiring the participant to start in a throwing circle of diameter 2.135 m and launch into a 40° sector.

The **shot** is a spherical iron ball weighing at least 7 kg for a man and 4 kg for a woman. The shot is 'put' (not thrown) from the throwing circle into the marked sector.

The **hammer** has a spherical head attached to a handle by a length of steel about 1 m long and is launched from the throwing circle using a circular wind-up action.

The shot and hammer can be launched from any position in the circle but must land within the marked sector outside the circle.



The hammer throw

What is the length of arc *AB*?

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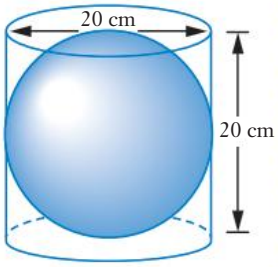
INVESTIGATION

SPHERE INSIDE A CYLINDER

The sphere shown just fits into a cylinder of diameter and height both 20 cm.



a Calculate the volume of the sphere and the cylinder. Do not round your answers.

b What fraction of the cylinder's volume is taken up by the sphere?

c Is this fraction the same for any sphere and cylinder with equal diameters? Experiment with different diameters.

d Is there any way of proving this result?

5.11 Volume and surface area of composite solids

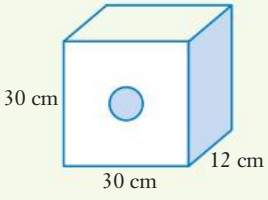
A page of solid shapes

Volumes of water

A composite solid is made up of two or more solids.

EXAMPLE 27

The concrete block shown is a square prism with a cylindrical hole of diameter 10 cm drilled through it.



a What was the volume of the concrete block before drilling?

b What is the volume of the concrete block after drilling, to the nearest cm^3?

c What percentage, correct to one decimal place, of the concrete was removed to make the hole?

Solution

a Volume of concrete block $= 30 \times 30 \times 12$

$= 10\,800\ cm^3$

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b First find the volume of the cylindrical hole.

$V = \pi r^2 h$ where $r = \frac{1}{2} \times 10 = 5$ and $h = 12$

$= \pi \times 5^2 \times 12$

$= 942.47\ldots \text{ cm}^3$

Volume remaining $= 10\,800 - 942.47...$

$= 9857.52\ldots$

$\approx 9858 \text{ cm}^3$

c Percentage removed $= \frac{942.47...}{10\,800} \times 100\%$

$= 8.726\ldots\%$

$\approx 8.7\%$

INVESTIGATION

VOLUME AND PACKAGING

1 a Design a cardboard package to hold six golf balls (each with a diameter of 37 mm) so that the minimum amount of cardboard is used.

b Would the same packaging be suitable to hold:

i 6 billiard balls?

ii 6 ping pong balls?

Why?

2 a Design a cardboard carton to hold 12 cans of soup:

i in one layer

ii in two or more layers.

b Which carton uses the least amount of cardboard?

c Which carton is the most appropriate for this product?

Why?

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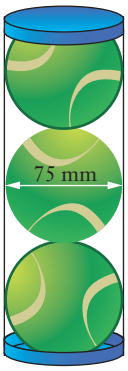
Exercise 5.11 Volume and surface area of composite solids

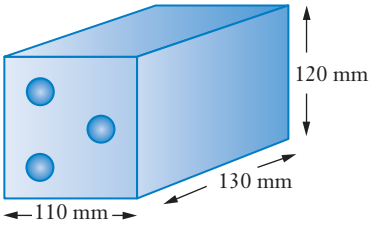
1 A cylindrical can contains three tightly-packed tennis balls, each with a diameter of 75 mm.



- **a** What is the volume of the can (to the nearest cubic millimetre)?
- **b** What percentage (correct to one decimal place) of the can's volume is taken up by the tennis balls?
- **c** If the three balls were packed tightly into a square prism instead of a cylinder, what would be the volume of the prism?
- **d** Why do manufacturers pack tennis balls into cylindrical cans? Justify your answer.

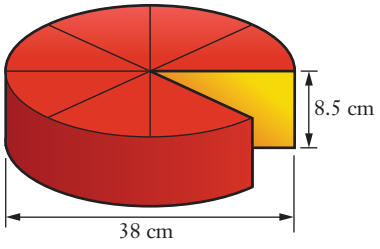
Example 27

2 A metal-fabricating company makes a machine part by drilling three holes of diameter 20 mm through a block of metal.



- **a** What is the volume of metal before drilling?
- **b** What is the volume of metal (to the nearest cm^3) after drilling?
- **c** What percentage (correct to the nearest whole number) of metal was removed when the holes were drilled?
- **d** What is the surface area of the metal before drilling?

3 This cheese wheel is covered with red wax to keep it fresh. One of the sectors has been eaten.



Find, correct to two significant figures:

- **a** the volume of the cheese remaining
- **b** the surface area of the cheese remaining
- **c** the volume of one sector of cheese
- **d** the area of red wax on one sector of cheese.

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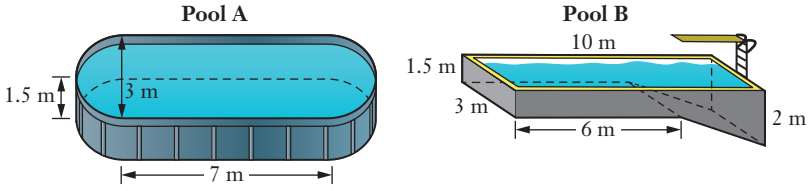
4 Two neighbours are arguing over whose swimming pool holds more water.



a What is the volume of water in each pool, correct to the nearest litre?

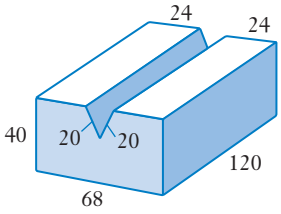
b Which pool holds more water, and by how much?

c If the base and sides of pool B are to be tiled, what area is to be tiled? Answer to the nearest square metre.

5 This is a woodworker's bench, with measurements in centimetres. Find:

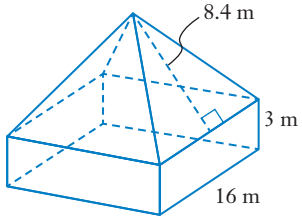
a its volume in cubic metres, correct to two significant figures

b its total surface area, correct to the nearest square centimetre.



6 This marquee with a square floor was hired for a family celebration.

How much canvas (correct to the nearest 0.1 square metre) was used to make this marquee? (Do not include the floor.)



SAMPLE HSC PROBLEM

Police investigating a road crash site measured that a car travelled a braking distance of 20.7 metres before stopping.

a How many significant figures are there in this measurement?

b Write the limits of accuracy of this measurement.

c Calculate, correct to three significant figures, the percentage error of this measurement.

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Study tip

Making lists and managing your time

To demonstrate that you know your mathematics and the steps required to solve the problem:

- You cannot just make up your study routine as you go along. You need to have a plan, even if it is a rough one. Make two lists of the things you want to achieve:
  - An 'A list' of urgent tasks, such as homework and reading
  - A 'B list' of less pressing demands, such as research assignments and revising for an exam.
- Write in your diary what you want to accomplish each day or each week, and include all important dates, deadlines and reminders.
- Make sure your 'A list' is realistic, taking into account the time and effort required for each task. Remember that your time is both precious and limited, so it is not possible to do everything. Make the most of the time that you have and don't be afraid to cut corners. Set realistic goals and spend time only on what is important. Start studying straight away and avoid timewasters, such as cleaning your room, decorating title pages and even devising elaborate study timetables (if you do these, then you are really putting things off).
- From your 'A list', choose the easiest task to begin with, so that you can ease yourself into the routine. Alternatively, some students like to start with a difficult task while their mind is fresh and alert. Once you have completed a significant amount of work, give yourself a break and a small reward.

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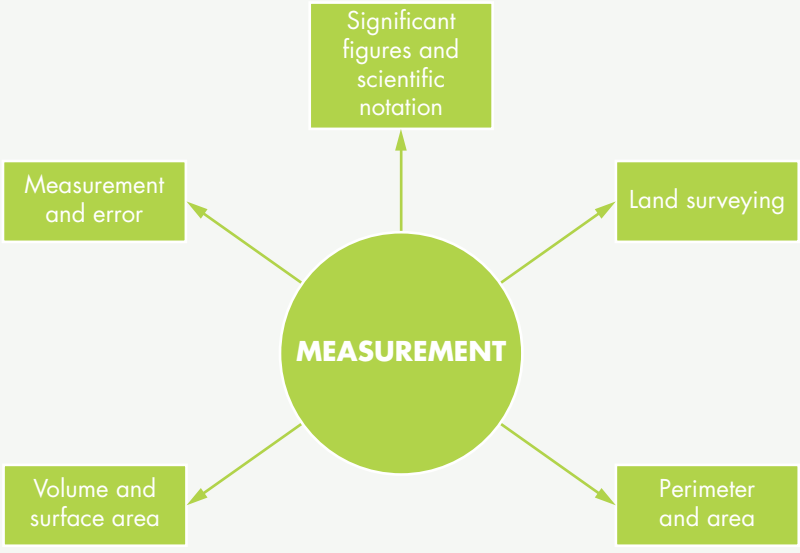
5. CHAPTER SUMMARY

This chapter, Measurement, builds on the measurement work covered in Years 9 and 10. You examined the perimeters, areas, surface areas, volumes and capacities of more complex and composite shapes, including sectors, prisms, cylinders and spheres. Offset surveys and the trapezoidal rule were used to approximate areas of irregular shapes. You should have a good understanding of error in measurement, significant figures and scientific notation. There are many formulas in this chapter, so be sure to include them in your topic summary along with appropriate diagrams and examples.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map is shown below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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5. TEST YOURSELF

1 Convert:

a 28.5 km to m **b** 6.4 t to kg **c** 340 mL to L.

Exercise 5.01

2 Farmer Joe's dam holds 43 ML of water. How many kilolitres does it hold?

Exercise 5.01

3 The width of a room is measured as 3.66 m. Find:

a the absolute error of this measurement

b the limits of accuracy of this measurement

c the percentage error, correct to two decimal places.

Exercise 5.02

4 Write each value correct to two significant figures.

a 38.915 **b** 1036 **c** 0.008 72 **d** 6 587 200

Exercise 5.03

5 A bank has \$8 350 000 000 in its vault. Write this value in scientific notation.

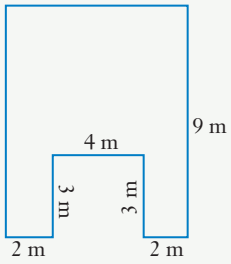
Exercise 5.04

6 A cell is 4.6×10^{-6} mm in diameter. Write this in normal decimal notation.

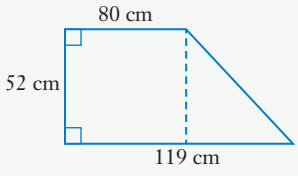
Exercise 5.05

7 Calculate the perimeter of each shape (correct to two decimal places for shape **c**).

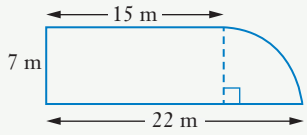
a



b



c



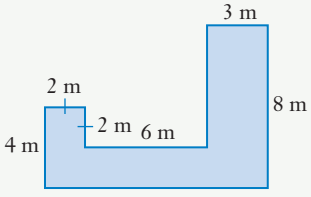
8 Convert:

a 8400 mm^2 to cm^2 **b** 5.6 ha to m^2.

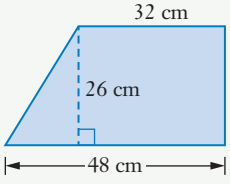
Exercise 5.06

9 Calculate the area of each shape (correct to two significant figures for shape **c**).

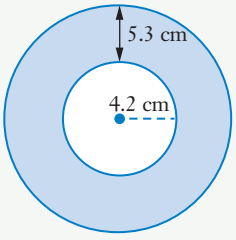
a



b



c



Exercise 5.06

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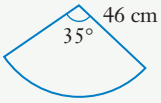
Exercise 5.06

10 For this sector, calculate correct to one decimal place:



a its perimeter

b its area.

Exercise 5.07

11 Felix completed an offset survey on a field, *PQRST*, and recorded the entries on the right in his notebook. Draw a diagram of the field and calculate its area:

			S
R	16	52	
Q	10	32	
		24	15 *T*
		0	
			P

a in m^2

b in ha.

Exercise 5.08

12 Convert:

a 20.7 cm^3 to mm^3

b 1 650 000 cm^3 to m^3.

Exercise 5.08

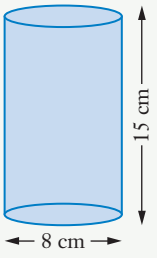
13 For each closed solid find, correct to two significant figures:

i the volume

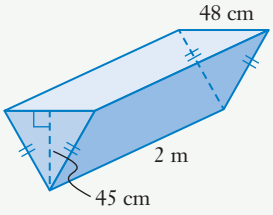
ii the surface area

Exercise 5.09

a



b



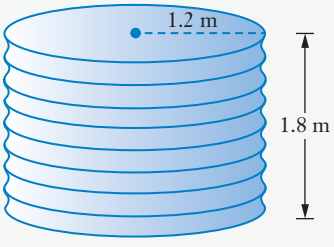
Exercise 5.08

14 **a** How many millilitres will a container of volume 894 cm^3 hold?

b How many litres will a container of volume 6.5 m^3 hold?

Exercise 5.09

15 This water tank has the shape of a cylinder. Find correct to two significant figures:

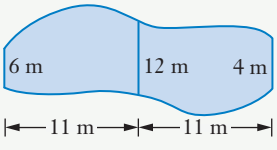


a its volume in cubic metres

b its capacity in litres.

Exercise 5.07

16 Find the area of this field, correct to the nearest square metre, using:



a one application of the trapezoidal rule

b two applications of the trapezoidal rule.

Which answer is more accurate? Why?

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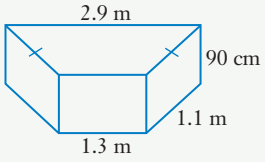
17 Blocks of concrete are used in the construction of a sports stadium. One of the blocks is shown in the diagram. Calculate, correct to one decimal place:



a its surface area **b** its volume.

Exercise 5.08

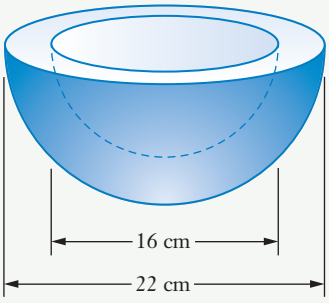
18 A large stainless steel cylinder for storing wine is 7 m high and has a diameter of 2.8 m. How many full 750 mL bottles of wine can be filled from this container?

Exercise 5.09

19 Rosemary is having a party and decides to make an 'ice bowl' to hold punch. She freezes water in the space between two hemispherical bowls of diameter 16 cm and 22 cm. Find, correct to two decimal places:

a the volume of ice used to make the bowl

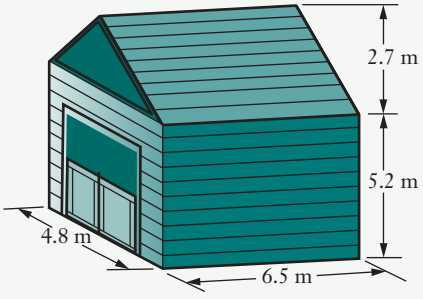
b how many litres of punch the ice bowl will hold.



Exercise 5.10

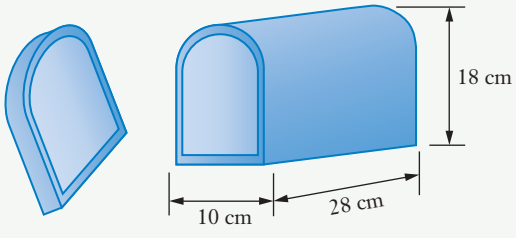
20 **a** Find the volume of the barn shown.

b If the barn is to be painted and one litre of paint covers 14 m^2, how much paint (rounded up to the nearest whole litre) do you need to put two coats on the barn?



Exercise 5.11

21 A 'hi-top' loaf of bread is cut into thick slices in the shape of a rectangle combined with a semicircle. Find, correct to two significant figures:



a the volume of the loaf

b the length of crust on one slice of bread.

Exercise 5.11

Qz Chapter quiz

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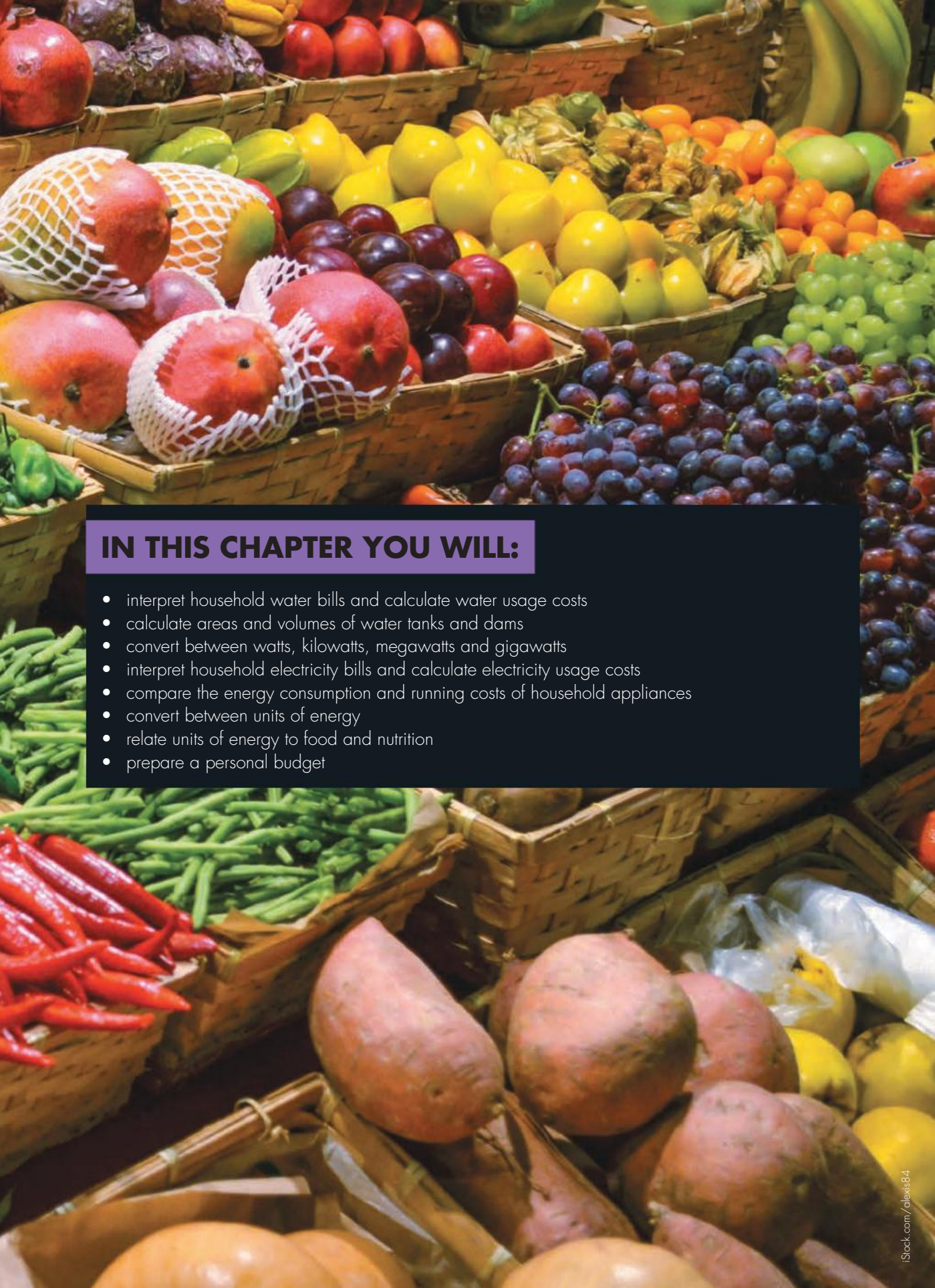
6.

MANAGING A HOME

Many management and financial skills are required to run a household successfully. This chapter ties together the financial, measurement and statistical knowledge necessary to manage the utilities of water and electricity in the 21st century home. The science of nutrition and energy in food is also covered.

CHAPTER OUTLINE

F1.3	6.01	Water usage in the home
M1.2, S1.1	6.02	Water tanks and dams
M1.3, S1.1, F1.3	6.03	Electricity usage in the home
M1.3, F1.3	6.04	Energy consumption and the costs of appliances
M1.3	6.05	Food and energy consumption
F1.3	6.06	Household budgeting



IN THIS CHAPTER YOU WILL:

- interpret household water bills and calculate water usage costs
- calculate areas and volumes of water tanks and dams
- convert between watts, kilowatts, megawatts and gigawatts
- interpret household electricity bills and calculate electricity usage costs
- compare the energy consumption and running costs of household appliances
- convert between units of energy
- relate units of energy to food and nutrition
- prepare a personal budget

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TERMINOLOGY

budget
discretionary spending
fixed spending
joule
kilowatt-hour (kWh)
sewerage
calorie
Energy Rating Label (ERL)
halogen light
kilojoule (kJ)
megajoule (MJ)
watt (W)
capacity
expense
incandescent light globe
kilowatt (kW)
megawatt (MW)

Assignment 6

SkillCheck

1 **a** During exercise, Grace has a heart rate of 135 beats per minute. Convert this rate to beats/second.

b How many times would Grace's heart beat in half an hour?

2 The cost of 55 L of petrol is $78.65. Express this cost as a rate in cents per litre.

3 Convert:

a 8.4 cm to mm **b** 9600 mm to m **c** 36 000 mL to L

d 2610 L to kL **e** 5.2 m^3 to L **f** 660 s to min.

4 The quarterly electricity bill was $423.85 before GST. Calculate the electricity bill after 10% GST is added, correct to the nearest cent.

5 How many days from 05/05/19 to 03/08/19?

6 The electricity account for the period 05/08/20 to 09/11/20 was $468.24. What was the average daily cost for the electricity usage?

7 A phone call costs 74 cents per minute.

a Find the cost of a 7-minute call.

b If Jessie's call cost $9.62, how long was the call?

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6.01 Water usage in the home

Your home has a **water meter** that measures the amount of water used in your home. A water bill is sent to your home every quarter listing the cost of the water service and **sewerage** (which are fixed), and the cost of the water used in your home (which is variable).

Water usage and costs

Water usage in your home

EXAMPLE 1

Yitong's water bill shows the following charges:

Water service	$ 41.39
Wastewater (sewerage)	$162.88
Water usage 33.5 kL @ $2.115 per kL	**A**
Amount due	**B**

Calculate Yitong's total water usage (A) and the total amount due (B).

Solution

Total water usage $= 33.5 \times \$2.115$

$= \$70.8525$

$\approx \$70.85$

Total amount due $= \$41.39 + \$162.88 + \$70.85$

$= \$275.12$

Exercise 6.01 Water usage in the home

Example 1

1 Anil's water bill shows the following charges:

Water service	$ 41.39
Wastewater (sewerage)	$162.88
Water usage 52.1 kL @ $2.115 per kL	**A**
Amount due	**B**

a Calculate Anil's total water usage (**A**).

b Find the total amount due (**B**).

2 For the month of June, a household used 425 L of water per day. How much water did they use for the entire month? Answer in kilolitres. Select **A**, **B**, **C** or **D**.

A 11.9 kL **B** 12.75 kL **C** 13.175 kL **D** 17 kL

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3 Below is a water bill from Riverstone Water.

Riverstone WATER

General Enquiries 13 20 92
Telephone Payments 1300 12 34 58

Last bill	Credits	Balance	This bill	Total amount due
$225.30	$225.30	$0.00	$261.45	**$261.45**

Date of issue 12 May 2018

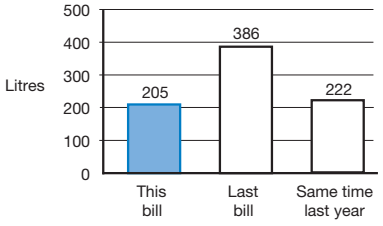
Please pay by **02/06/18**

MR WALTER TANK
3/47 LAKESIDE DRIVE
RIVERSTONE NSW 2765

Account number **4402 4244 418**

Account for strata unit — **3/47 Lakeside Drive, Riverstone**

Charges	**1 April 2018–30 June 2018**	$
Water service		24.00
Sewerage service		118.75
Usage charge	**10 Feb 2018–8 May 2018**	38.07
Water	18 kilolitres at $2.115 per kilolitre	
Your average daily usage	**Total amount due**	**$180.82**



1000 litres = 1 kilolitre

Water Meter Details

	Date read	*Reading*
This reading	8 May 2018	699 kilolitres
Last reading	10 Feb 2018	681 kilolitres

Total water used in 88 days was 18 kilolitres

a How many days are there from the bill's day of issue to the payment's due date?

b Is this bill paid monthly, two-monthly or **quarterly**?

c What are the fixed charges for, and how much are they in total?

d How many kilolitres of water were used in the period covered by this bill?

e Was more water used each day for this bill compared to the last bill? Give a possible reason for this.

f Why is it more useful to compare average daily usage with that of the same time last year?

g Verify the average daily usage of 205 L by dividing the volume of water used by the number of days covered by this bill.

h What is the cost per kilolitre for water usage?

i Verify the water usage charge of $38.07 using this rate.

j A backyard swimming pool contains 40 kL of water. Calculate the cost of filling this pool with water.

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4 For the final quarter of the year (October to December), the Brentnall family used 66.24 kL of water. On average, how many litres of water did they use each day? Select **A**, **B**, **C** or **D**.

A 585 **B** 690 **C** 720 **D** 736

5 Hannah's water bill shows that water usage is charged at $2.115/kL.

a If Hannah used 24 kL of water, how much will she need to pay?

b Hannah paid $32.57 in water usage charges on her latest bill. Correct to one decimal place, how many kilolitres of water did she use?

c Hannah was also charged $41.39 for the water service and $232.88 for the wastewater (sewerage) service. What was the total of Hannah's bill?

6 The Carrozza family has two adults and two teenagers. In one year, their water usage for each quarter was as shown below:

Quarter	Date	Water usage (kL)
1	1 Jan–31 Mar	51.0
2	1 Apr–30 Jun	47.3
3	1 July–30 Sept	46.5
4	1 Oct–31 Dec	54.8

a What was the Carrozza family's total water usage for the year?

b On average, how much did they use per quarter?

c Calculate the average daily water use for 1 January to 31 March.

d The Carrozza family use their dishwasher twice a day, requiring 21 L of water each time it is used. How much water did their dishwasher use over the 3rd quarter?

e Calculate what percentage of the 3rd quarter water usage was for the dishwasher. Answer correct to one decimal place.

7 The Harris family are calculating their water usage for the quarter. This reading is 8764 kL and the previous reading was 8112 kL. What was their water usage for the quarter? Water usage is charged at $2.115/kL. Select **A**, **B**, **C** or **D**.

A $652 **B** $985 **C** $1378.98 **D** $3569.57

8 Jeremy and Annie pay a recycled water charge of $9.37 per quarter. If they also use 12 kL of recycled water at $1.93/kL and 20.2 kL of drinking water at $2.115 kL, calculate their charges for water usage for the quarter.

9 The Green family pay a recycled water charge of $11.58 per bill. On their latest bill they used 39 kL of recycled water at $1.93/kL. How much did they pay in total for recycled water charges? Select **A**, **B**, **C** or **D**.

A $66.30 **B** $77.88 **C** $86.85 **D** $385.32

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10 The average daily water usage for two families, for the quarter 1 January–31 March, is:

- The Baskaran family (4 people), 560 L/day
- The Zhong family (6 people), 710 L/day

For each family, calculate:

a the water usage costs for the quarter if water costs $2.115 per kL and state which family pays more

b the average daily water use per person and state which family uses more.

DID YOU KNOW?

Water usage in the home

• Washing your hands/face	5 L
• Brushing your teeth (tap running)	5 L
• Brushing your teeth (tap not running)	1 L
• Cooking and making coffee/tea	8 L per day
• Flushing the toilet	9 L to 13 L
• Flushing the toilet (half flush)	4.5 L to 6 L
• Household tap	18 L per minute
• Washing the dishes (by hand)	18 L
• Washing the dishes (dishwasher)	25 L per cycle
• Bath use	85 L to 150 L
• Shower (8 minutes)	80 L to 120 L
• Washing machine (front loading)	120 L per cycle
• Washing machine (top loading)	180 L per cycle
• Washing the car (with hose)	100 L to 300 L
• Garden sprinkler	1 kL to 1.5 kL per hour
• Garden hose	1.8 kL per hour
• Swimming pool (backyard)	20 kL to 55 kL

On average, a four-person Sydney house (with garden) uses 936 litres of water per day. Outside taps and toilet flushing account for half of this usage.

How much water does your household use each day? Find out by investigating your water bill.

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INVESTIGATION

WATER USAGE IN YOUR HOME

Visit the Sydney Water and Water rating websites to find out more about water bills and the costs of water usage, including recycled water and wastewater. Also investigate water efficiency in the home.

Over a one-week period, investigate the amount and cost of water used in your home, including showering and bathing, washing clothes, watering the garden, washing a car and using the toilet.

Put a bucket in your shower to measure how much water is used per minute, and time your showers. Find the amount of water that your kitchen sink, washing machine and toilet use each time, and keep a tally of their weekly usage.

1 Draw a graph comparing the amount of water used in different areas of your home.

2 Investigate the Water Efficient Targets on the Sydney Water website and compare your household use with the daily water consumption targets. Give an explanation of your findings.

3 Find a recent water bill for your home and interpret the different costs. Can you recommend any changes to your family's usage to help save water in your home?

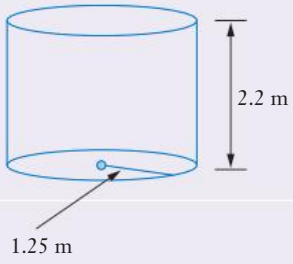
6.02 Water tanks and dams

WS Water tanks

Australia is the world's driest inhabited continent, with water availability affected by drought, making it a valuable resource. Rainwater is collected in water tanks and dams. The amount of rain falling in a location is measured in millimetres. For example, a rainfall of 5 mm means that the amount of water falling would fill a container to a height of 5 mm.

EXAMPLE 2

Calculate the volume of this cylindrical rainwater tank in kilolitres, correct to two decimal places.



Solution

radius $r = 1.25$ m, height $h = 2.2$ m

$V = \pi r^2 h$

$= \pi \times 1.25^2 \times 2.2$

$= 10.7992...\text{ m}^3$

$= 10.7992...\text{ kL}$

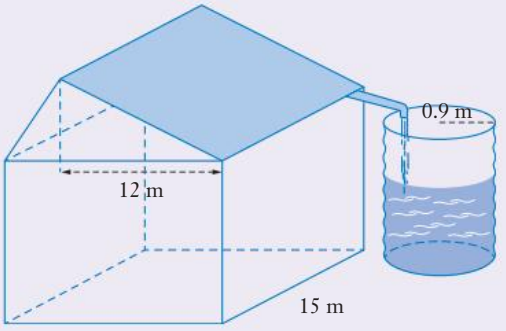
$= 10.80\text{ kL}$

ISBN 9780170413565

EXAMPLE 3

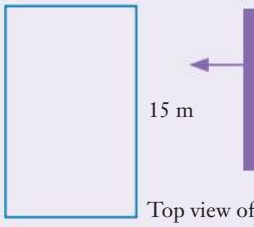
Rainwater falling onto the right face of the roof of a building is collected into an empty cylindrical water tank.

The top view of the right face shows the aerial dimensions of the catchment area.



a After 20 mm of rainfall, how many litres of water are drained into the tank?

b If the radius of the water tank is 0.9 m, what was the water level in the tank after 20 mm of rainfall, correct to the nearest 0.1 m?



Top view of right face

This is the right face's 'horizontal' area as seen from above, not its actual sloped surface area.

Solution

a Area of a rectangle $= 12 \times 15$

$= 180 \text{ m}^2$

Volume of rainfall $= 180 \text{ m}^2 \times 20 \text{ mm}$ Change 20 mm to m

$= 180 \text{ m}^2 \times 0.02 \text{ m}$

$= 3.6 \text{ m}^3$

$= 3.6 \times 1000 \text{ L}$ $1 \text{ m}^3 = 1000 \text{ L}$

$= 3600 \text{ L}$

b Volume of a cylinder $= \pi r^2 h$

$V = 3.6 \text{ m}^3$ from part **a** and $r = 90 \text{ cm} = 0.9 \text{ m}$

$3.6 = \pi \times 0.9^2 \times h$

$3.6 = 2.5446\ldots \times h$

$$h = \frac{3.6}{2.5446\ldots}$$

$= 1.4147\ldots$

$= 1.4 \text{ m}$

The height of the water in the tank was 1.4 m.

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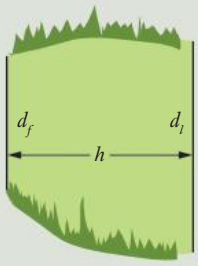
Trapezoidal rule for estimating area

$$A \approx \frac{h}{2}(d_f + d_l)$$



where A = area

h = distance between successive measurements

d_f = first measurement

d_l = last measurement

or in words:

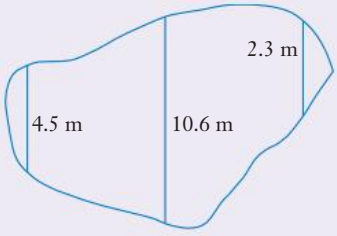
$$\text{Area} \approx \frac{\text{width of strip}}{2}(\text{first measurement} + \text{last measurement})$$

EXAMPLE 4

The width of a dam is measured at 10-metre intervals.

a Use two applications of the trapezoidal rule to estimate the area of the dam.

b Calculate the volume of water in the dam in kilolitres if it is 1.9 metres deep.



Solution

a Using the first two measurements:

$d_f = 4.5, d_l = 10.6$

$$A \approx \frac{10}{2}(4.5 + 10.6)$$
$$= 75.5$$

Using the last two measurements:

$d_f = 10.6, d_l = 2.3$

$$A \approx \frac{10}{2}(10.6 + 2.3)$$
$$= 64.5$$

Total area = 75.5 + 64.5

$= 140 \text{ m}^2$

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b $V = Ah$

$= 140 \times 1.9$

$= 266 \text{ m}^3$

$= 266 \text{ kL}$ $\quad 1 \text{ m}^3 = 1 \text{ kL}$

The volume of water in the dam is 266 kL.

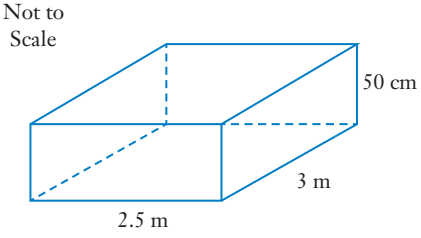
Exercise 6.02 Water tanks and dams

Example 2

1 Calculate the **capacity** (volume) of each tank or pool in kilolitres, correct to two decimal places.

- **a** a cylindrical rainwater tank with diameter 9.36 m and height 2.18 m
- **b** a rectangular swimming pool with dimensions 2.4 by 4.8 m filled to a depth of 1.2 m
- **c** a semicircular pond with a radius of 1.2 m and a depth of 0.4 m

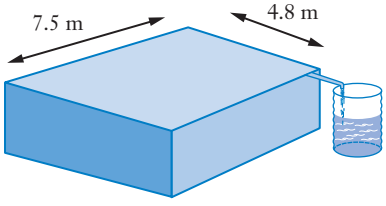
2 Bella has installed two water bladders (water storage bags) under her new house to collect rain. They are both in the shape of the rectangular prism shown.



- **a** Calculate the combined capacity of the water bladders, in litres.
- **b** In April, Bella's family uses an average of 570 L/day. If the water bladders are full, how long will it take to use all the water in the bladders (assuming no rainfall). Answer to the nearest day.

Example 3

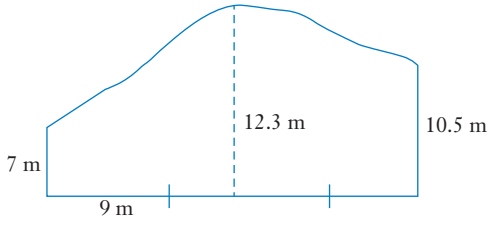
3 Rainwater falls onto the rectangular roof of a shed so that it drains into an empty cylindrical water tank of radius 60 cm and height 2.4 m.



- **a** After 50 mm of rainfall, how many litres of water are drained into the tank?
- **b** What is the water level after 50 mm of rainfall?
- **c** How many millimetres of rain needs to fall onto the roof to completely fill the tank?

Example 4

4 The diagram shows the outline of a lake in a botanical garden.



- **a** Use two applications of the trapezoidal rule to find the approximate surface area of the lake.
- **b** The average water depth in the lake is 1.6 metres. How many whole litres of water can the lake hold when full?

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5 This table shows the monthly storage levels of Warragamba Dam in Sydney in 2016, listing the percentage of dam's capacity and the available storage in megalitres.

	Available storage (%)	Available storage (ML)
Jan 2016	94.4	1 914 444
Feb 2016	93.3	1 891 728
March 2016	90.9	1 842 528
April 2016	89.0	1 803 930
May 2016	87.2	1 768 468
June 2016	99.1	2 008 362
July 2016	100.0	2 027 000
August 2016	96.5	**A**
September 2016	97.2	1 969 816
October 2016	**B**	1 943 221
November 2016	94.6	1 918 134
December 2016	92.4	1 873 640

© State of New South Wales through WaterNSW

a In which month was Warragamba Dam at full available storage?

b What was the level below capacity (in ML) in January 2016?

c What was the available storage (in ML) in August 2016 (**A**)?

d Calculate the available storage percentage (**B**) of Warragamba Dam in October 2016.

e How much did the storage level rise at Warragamba between May and June 2016?

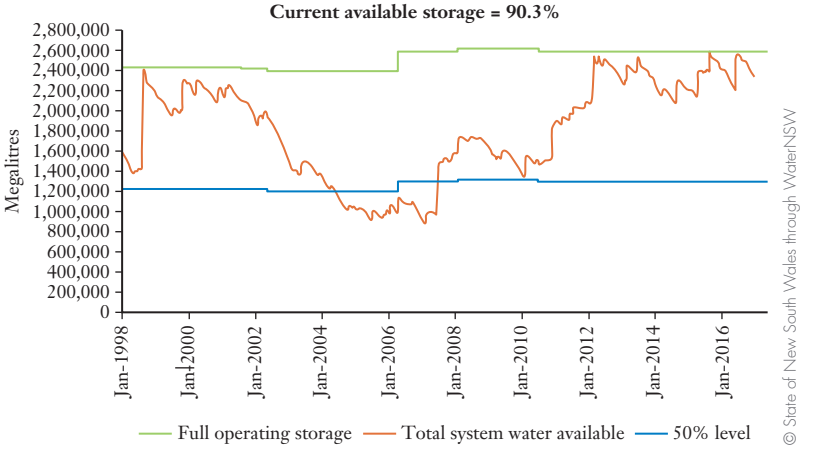
f Calculate the percentage decrease in water available in Warragamba Dam from November to December 2016.

g In which month was the dam at its lowest level?

h Construct a graph of the available storage of the dam over the 12 months.

i Describe the change in the available storage over the 12 months.

6 This graph shows the available water storage for NSW from 1998 to 2016.



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a Estimate the available water storage in January 2016.

b Write another year when the available water storage was at the same level as January 2016.

c When was the available water storage at its lowest and what was this amount?

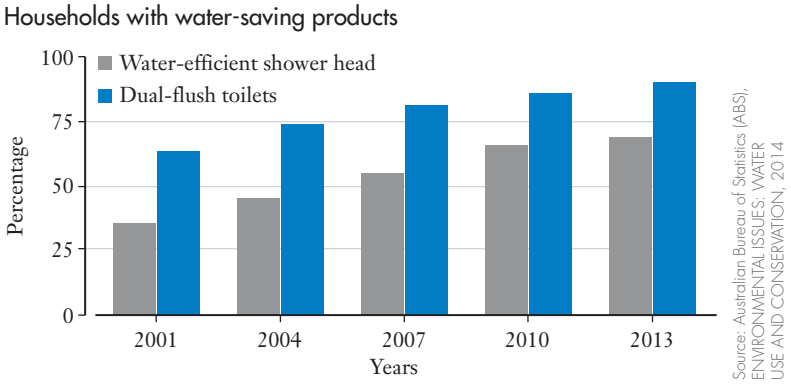
d Why do you think the water storage was so low at this time?

e How many megalitres was 'full operating storage' in January 2016?

f How many megalitres of water was considered to be 'half full' in January 2016?

g In which years was the available water storage at 2 000 000 ML?

7 This graph shows the percentages of Australian bathrooms using water-efficient shower heads and dual-flush toilets from 2001 to 2013.



a Which is the more popular water-saving product?

b What percentage of Australian households had water-efficient shower heads in 2010?

c In which year did 74% of households have dual-flush toilets?

d What was the percentage increase in households with water-efficient shower heads installed between 2007 and 2010?

e In which year did the smallest increase occur for water-efficient shower heads?

f Calculate the overall percentage increase in dual-flush toilets installed from 2001 to 2013.

g Briefly explain why there was a significant increase in households installing water-saving products between 2001 and 2013.

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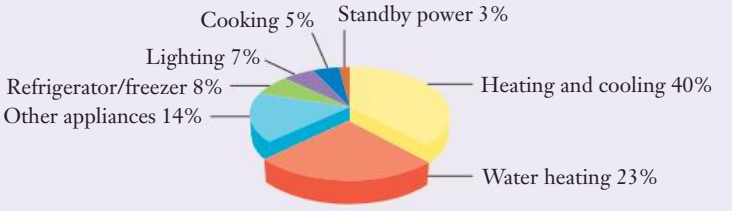
6.03 Electricity usage in the home

Power problems

EXAMPLE 5

This sector graph shows how electricity is used in in an average Australian home.



a Calculate, to the nearest degree, the angle size of the 'Cooking' sector.

b What area consumed the most electricity in the home?

c If the cost of a household electricity bill was $978.58 for the Nov–Feb quarter, how much was spent on heating and cooling?

d What could be included in 'Other appliances'?

e If the cost of 'Other appliances' for one bill was $58.50, what was the total electricity bill?

Solution

a Angle size of 'Cooking' sector $= 5\% \times 360°$

$= 18°$

b Heating and cooling — 40% is the highest percentage

c Cost of heating and cooling $= 40\% \times \$978.58$

$= \$391.432$

$\approx \$391.43$

d 'Other appliances' may include dishwasher, washing machine, iron, kettle, clothes dryer, TV, phones, computers, tablets and other entertainment devices.

e Other appliances (14%) $= \$58.50$ — Using the unitary method

$1\% = \$58.50 \div 14$

$= \$4.17857 \ldots$

Total electricity bill (100%) $= \$4.17857 \ldots \times 100$

$= \$417.857\ldots$

$\approx \$417.86$

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Units of power

The **watt** (abbreviation W) is a unit of **power** and is equal to one joule of energy per second.

Unit	Relationships
microwatt (μW)	$1\ \mu W = \frac{1}{1\,000\,000} W = 10^{-6}\ W$
milliwatt (mW)	$1\ mW = \frac{1}{1000} W = 10^{-3}\ W$
watt (W)	$1\ W = 1000\ mW = 1\,000\,000\ \mu W$
kilowatt (kW)	$1\ kW = 1000\ W$
megawatt (MW)	$1\ MW = 1000\ kW = 1\,000\,000\ W$ or $10^6\ W$
gigawatt (GW)	$1\ GW = 1000\ MW = 1\,000\,000\,000\ W$ or $10^9\ W$

Units of energy usage

Units of energy usage

Electrical energy usage is measured in **watt-hours (Wh)**, which is the amount of electrical energy used by a one-watt load (such as a light globe) drawing power for one hour.

The electricity usage for households is measured in **kilowatt-hours (kWh)**, which is 1000 watt-hours. The **cost** of electricity usage is given in **cents/kilowatt-hour** (c/kWh).

EXAMPLE 6

Convert:

a 8350 W to kW **b** 35 000 MW to W **c** 152 600 GW to kW.

Give answers in scientific notation where necessary.

Solution

a $8350\ W = 8350 \div 1000\ kW$
$= 8.35\ kW$

$1\ kW = 1000\ W$

b $35\,000\ MW = 35\,000 \times 1000\,000\ W$
$= 35\,000\,000\,000\ W$
$= 3.5 \times 10^{10}\ W$

$1\ MW = 1\,000\,000\ W$

c $152\,600\ GW = 152\,600 \times 1\,000\ MW$
$= 152\,600 \times 1000 \times 1000\ kW$
$= 152\,600\,000\,000\ kW$
$= 1.526 \times 10^{11}\ kW$

$1\ GW = 1000\ MW$
$1\ MW = 1000\ kW$

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Domestic and off-peak electricity

Domestic rate electricity refers to electricity that is used during the day.

Off-peak rate electricity refers to electricity that is used during the late evening and early morning, such as between 11 p.m. and 7 a.m., when the demand from households and businesses is much lower. Off-peak rate electricity is charged at a cheaper rate than domestic rate electricity, and is often used to heat water in homes.

EXAMPLE 7

Calculate the cost of electricity usage for a household where domestic usage is 954 kWh at \$0.284500/kWh and off-peak usage is 748 kWh at \$0.120379/kWh.

Solution

$$\begin{aligned}\text{Cost of domestic usage} &= 954 \times \$0.284500 \\ &= \$271.413\end{aligned}$$

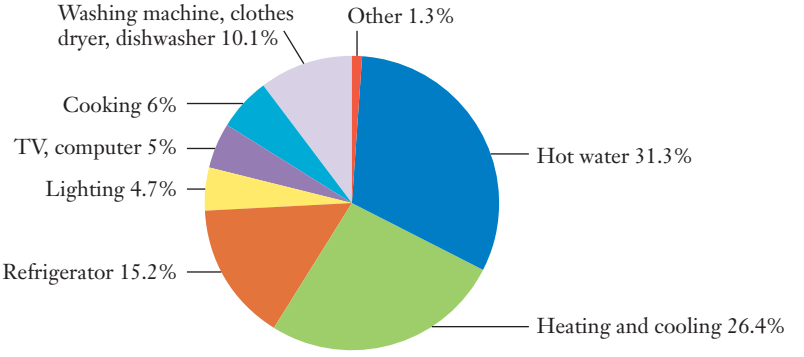
$$\begin{aligned}\text{Cost of off-peak usage} &= 748 \times \$0.120379 \\ &= \$90.043\end{aligned}$$

$$\begin{aligned}\text{Total cost} &= \$271.413 + \$90.043 \\ &= \$361.456 \\ &\approx \$361.46\end{aligned}$$

Exercise 6.03 Electricity usage in the home

Example 5

1 The sector graph shows the energy use for an average household in NSW.



a What fraction of energy use is for cooking?

b What could be included in the 'Other' sector?

c If the household bill for 90 days is \$562.37, how much did 'TV, computers' cost?

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d If lighting costs $0.67 per day, what would be the total electricity bill be for 90 days?

e Calculate, to the nearest degree, the angle size of the 'Hot water' sector.

2 The divided bar graph below shows the electricity use for an average Tasmanian household.

a What fraction of electricity is used for heating?

b What percentage of electricity is used for hot water?

c If the average Tasmanian household uses 10 000 kWh of electricity per year, how much electricity is for lighting?

Example 6

3 Express each measurement of power in scientific notation.

a 5 700 000 MW **b** 3 000 000 kW **c** 4 845 000 000 GW

4 Which of the following is 6 840 000 MW expressed in scientific notation, correct to two significant figures? Select **A**, **B**, **C** or **D**.

A 6 800 000 MW **B** 6.9×10^6 MW

C 6.84×10^6 MW **D** 6.8×10^6 MW

5 Convert:

a 5600 W to kW **b** 27 500 000 kW to MW

c 3 495 000 mW to kW.

6 How many kW in 8 450 000 W? Select **A**, **B**, **C** or **D**.

A 845 kW **B** 8.45 kW **C** 8450 kW **D** 84 500 kW

7 Convert 2.35 GW to:

a megawatts **b** kilowatts **c** watts.

Example 7

8 An electricity bill from Lightning Energy is shown on the next page.

a Is this bill paid monthly, two-monthly or quarterly?

b What was the average daily electricity usage over this period?

c From the graph, find the time of the year when the electricity usage was greatest. Why do you think this is so?

d How much electricity was used in the period covered by this bill?

e How many days of usage were covered by this bill?

f When was the electricity meter read? What was the reading?

g Has the electricity bill increased or decreased since the last bill? By how much?

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Lightning Energy

Account Number: 71433973
Date Due: Thursday 6 December 2018
Amount Due: $385.44

Ms B. SPARKS
13 AMPERE CRESCENT
LIGHTNING RIDGE, NSW 2834

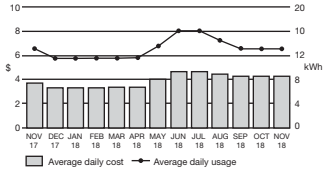
ELECTRICITY ACCOUNT

Account Summary

Description	Amount
Previous Balance	$415.69
Less payment(s) received to 05/09/18	$415.69 CR
Subtotal	$0.00
Bill period: 16 Aug 2018 to Nov 2018 (91 days)	$385.44
Total Amount Due (Incl. GST)	**$385.44**

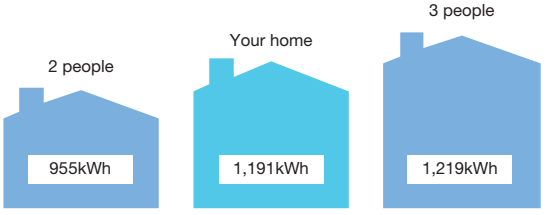
How much energy are you using?

Average daily cost and usage.



Never use more than one adaptor on the one power outlet. Better still use a power board to plug in additional low current applications as they have their own safety overload cut out switch.

Compare with other homes in your area.



Average usage data supplied by Australian Energy Regulator based on homes with gas and no pool during spring.
Visit energymadeeasy.gov.au for more information.

Snapshot.

Average daily cost:
$4.24

Average daily usage:
13.09kWh

Same time last year:
14.38kWh

Your electricity supply details.

Meter No.	Read Date	Days	Start Read	End Read	kWh
0874221	14 Nov 2018	91	57144	58335	1191

Your next meter read is between **13 Feb 2019** and **17 Feb 2019.** Please ensure easy access to your meter on to your meter on these days.

New charges and credits.

Usage and supply charges	Units	Price	Amount
peak	997 kWh	$0.2511	$250.35
Off Peak	194 kWh	$0.2438	$47.30
Supply charge	91 days	$0.7857	$71.50
Other Charges			
Payment processing fee			$2.08
Total charges			$371.23
Credits			
7% Pay On Time Discount			$20.84 CR
TOTAL NEW CHARGES AND CREDITS			$350.39
Total GST			$35.05
TOTAL AMOUNT PAYABLE (incl. GST)			$385.44

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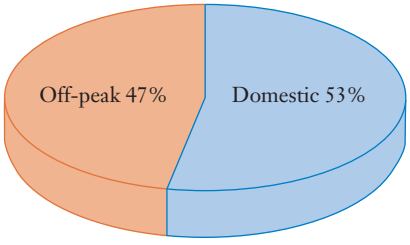
Consider the picture graphs of houses on the bill, under the heading 'Compare with other homes in your area'.

h This bill is for a household of 2 adults and 2 children. Do you think this house is running efficiently? Justify your answer.

i Name two given features of this house which possibly explain the energy efficiency.

j Explain the two different cost rates of electricity usage.

9 This graph shows the domestic and off-peak electricity usage for a Campbelltown household over a 96-day period.



a If the total usage was 3022 kWh, how many kilowatt hours was off-peak usage?

b Calculate the cost of:
- **i** off-peak usage at \$0.092800/kWh
- **ii** domestic usage at \$0.265800/kWh.

c What was the average daily cost for domestic usage?

10 The following questions refer to the electricity bill shown on the next page.

a What was the average daily electricity usage over the period of this bill? Answer separately for peak and off peak.

b From the graph, find the time of the year when electricity usage was the greatest. Why do you think this is so?

c An extra fee is charged in addition to off-peak and peak usage. What is this extra fee for and at what rate is it charged?

d Explain the difference in the two off-peak domestic usage rates in this bill.

e At what time of day are domestic peak rates charged?

f Did this household use more peak or off-peak power?

g By how much has electricity usage increased or decreased when compared to the same period the previous year? Answer separately for peak and off peak.

11 The pricing information for a electricity supplier is given below.

Tariff Type	Cost
Off Peak (cents/kWh)	12.03719
Supply charge (cents/day)	95.6494
Domestic First 1750 kWh/qtr (cents/kWh)	27.027
Remaining Usage kWh/qtr (cents/kWh)	20.46

Calculate the electricity bill for a Penrith household with off-peak usage of 984 kWh and domestic usage of 1985 kWh for a 90-day period.

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NCM Energy

Account Number: 56007899
Date Due: Thursday 23 August 2018
Amount Due: $585.39

MR B POWER
125 FARADAY DR
PICTON 2571

FOR ACCOUNT ENQUIRIES: PHONE 131 002
FOR 24HR EMERGENCY ASSISSTANCE: PHONE 131 002

ELECTRICITY ACCOUNT

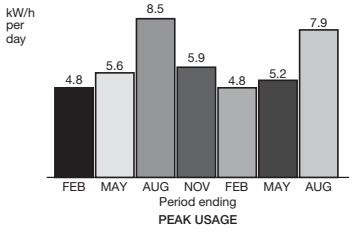
Issue Date: 03/08/2018

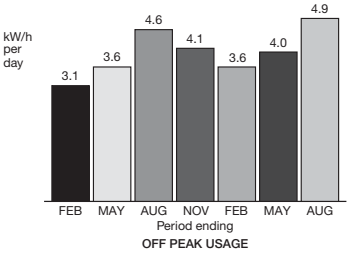
Summary of Your Account

Description		Amount
Balance of last account		$342.15
Less payments and rounding		$342.15 CR
Subtotal		$0.00
Account Summary – 05/05/18 to 03/08/18		
Electricity charges (see next page for details)*	$532.17	
Plus GST payable	$53.22	
Total amount of this account (including GST)		$585.39
Total Amount Due (including GST)		**$585.39**

*GST applies to this item

Your Electricity Usage





Itemised Details

Meter information for Period 05/05/18 to 03/08/18 – 90 days

Meter No.	Const	Days	Usage (kWh)
1234530	1	90	437
OFF PEAK – Total Usage			437
1234532	1	90	718
DOMESTIC – Total Usage			718

Your Account Calculations

Pricing Option	GST	Usage Applies	Rate	Amount
DOMESTIC OFF PEAK(10 pm – 7 am Mon – Fri)				
OFF PEAK Off Peak 1	Yes	369.000	$0.11913	$43.96
DOMESTIC OFF PEAK (SHOULDER USAGE 7 am – 2 pm, 8 pm – 10 pm)				
Off Peak 2	Yes	68.000	$0.21186	$14.41
DOMESTIC PEAK(2 pm – 8 pm Mon – Fri)				
DOMESTIC Domestic	Yes	718.00	$0.54164	$388.90
Daily Supply Chg $0.94336	Yes	90 days		$84.90
Electricity subject to GST				**$532.17**

ISBN 9780170413565

INVESTIGATION

BUILDING AN ENERGY-EFFICIENT HOME

When building a new home, there are many things that can be done to reduce the amount of electricity used. For example, up to 35% of heat gain and loss from a house can be through a ceiling that is not insulated. Insulating the roof, walls and suspended floors will reduce energy use and cut electricity costs.

1 Investigate the following websites to find different ways a house can be made to be more energy efficient:

Environment NSW

Energy Australia and select 'Energy saving and safety'

Your Home (government site)

2 What are 'ideal' settings for heating and for cooling?

3 How can landscaping help reduce your energy bill?

4 What type of lighting should be used in outdoor areas?

DID YOU KNOW?

Watts it all about?

- **Microwatts** are used to measure power from solar cells used in calculators and watches.
- A person's hearing aid uses less than one **milliwatt** while a laser pointer outputs about 5 milliwatts of light power.
- A wind turbine has a power capacity of 1 to 3 **megawatts**, while a lightning strike may contain tens of **gigawatts.**
- The total power of sunlight striking Earth's atmosphere is 170 to 180 **petawatts**.

How much power is generated by a person climbing a flight of stairs? A highly-trained athlete sprinting 100 metres? A car engine while cruising?

ISBN 9780170413565

6.04 Energy consumption and the costs of appliances

Electrical appliances

Heating and cooling make up almost a quarter of a home's energy use. The biggest users of energy in Australian homes are hot water systems. To reduce costs and improve efficiency you need to know how and where energy is used in the home.

EXAMPLE 8

How much energy is used (in kWh) when a 100 W light globe is on for 5 hours?

Solution

Energy used in 5 h = 100 W × 5 h

= 500 Wh

= 500 ÷ 1000 kWh 1 kWh = 1000 Wh

= 0.5 kWh

Running costs of electrical appliances

You can calculate the **running cost** of an electrical appliance by multiplying the power used (in kWh) by the cost of electricity.

Household energy costs

Running costs

Running cost = power used (kWh) × electricity cost ($/kWh)

EXAMPLE 9

Energy consumption

Tamin takes about $1\frac{1}{2}$ hours per week to iron his clothes. If he uses a 2200 W iron and the cost of electricity is $0.267257/kWh, calculate the cost of using the iron in one year.

Solution

Power used in one year = 2200 W × 1.5 × 52

= 171600 Wh

= 171.6 kWh

Annual cost = 171.6 × $0.267257

= $45.86130

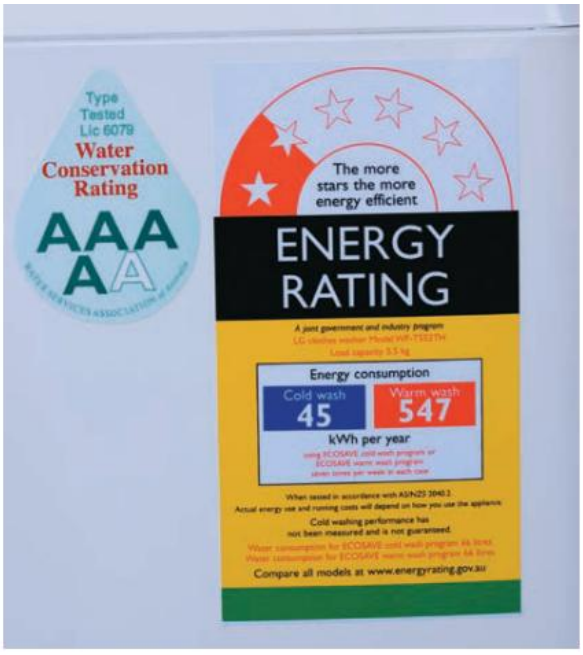
≈ $45.86

ISBN 9780170413565

Energy ratings

Energy ratings

Many household appliances display an **Energy Rating Label (ERL)** to help people choose energy-efficient appliances. These include TVs, washing machines, clothes dryers, refrigerators, freezers, dishwashers, air conditioners and heaters. These labels contain a star rating to show how efficient the appliance is – the more stars, the more efficient the energy use. A standard label has 6 stars but for more efficient appliances there is a label that goes up to 10 stars. The label also lists an estimate of the energy (in kWh) the appliance will use over a year.



Alamy/Mark Boulton

Hot water systems

Water heating can account for up to 30% of the energy used in a home. The types of hot water systems (storage and instantaneous) available are solar, heat pumps, natural gas and off-peak electric. Around half of Australian homes use electric water heaters, but they contribute 80% of hot water greenhouse emissions. By installing an appropriate and efficient water heater for the household size, considering water use patterns and climate, it is possible to save money and reduce greenhouse emissions.

Lighting

About 5% of the electricity used in NSW homes is for lighting. The main types of light globes used in homes are **compact fluorescent lamps** (CFL), **halogen** (used in downlights) and LED lights. In 2009, the traditional **incandescent light globes** started being phased out in favour of more energy-efficient light sources with longer lamp lives. It is estimated that by phasing out incandescent light globes, the average household could save 300 kWh and $75 each year.

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EXAMPLE 10

A traditional (obsolete) incandescent 75 W light globe and the equivalent modern compact fluorescent 10.5 W lamp are used for 5 hours a day. Compare the cost of using both globes over a year if electricity is charged at \$0.258100/kWh.

Solution

Incandescent light globe:

Electricity used in one year $= 75 \times 5 \times 365$
$= 136\ 875$ Wh
$= 136.875$ kWh

Cost of electricity used $= 136.875 \times \$0.258100$
$= \$35.3274375$
$\approx \$35.33$

Compact fluorescent lamp:

Electricity used in one year $= 10.5 \times 5 \times 365$
$= 19\ 162.5$ Wh
$= 19.1625$ kWh

Cost of electricity used $= 19.1625 \times \$0.258100$
$= \$4.94584125$
$= \$4.95$

Over a year, the cost of using the compact fluorescent lamp is approximately one-seventh of the cost of the incandescent light globe.

Appliances

Washing machines, clothes dryers, fridges and freezers, televisions, dishwashers and other appliances can account for 25% of a household's electricity bill. Choosing energy-efficient products can reduce power use and running costs.

EXAMPLE 11

Calculate the lifetime running cost of the following televisions. Use 30.0 cents/kWh as an estimate for the cost of electricity and assume that the average lifetime of major appliances is 10 years.

a A 138 cm, UHD Smart LED TV with a 4.5-star label, energy consumption 380 kWh/year

b A 140 cm, SUHD Smart LED television with a 5-star label, energy consumption 310 kWh/year.

Solution

a Lifetime running cost $= 380 \times \$0.30 \times 10$
$= \$1140$

b Lifetime running cost $= 310 \times \$0.30 \times 10$
$= \$930$

ISBN 9780170413565

Standby power

Many appliances such as sound systems, computers and cordless phones have a **standby mode** so that they can be turned on quickly. **Standby power** is the electricity used by appliances with an electronic clock, when they are turned off or waiting to be turned on by a remote control. Standby power accounts for about 5.9% of total household electricity usage and many household appliances use some standby power.

In Australia, the average daily standby power consumption is 39.4 W per household or 129.4 kWh per year, with an annual cost of $860 million. To save energy costs, you should aim to switch appliances completely off when not in use if possible, either by using the OFF switch or at the power point.

EXAMPLE 12

The power consumption of a cordless phone in standby mode is about 3 W. What is the cost of the standby mode for the phone over a year if electricity is charged at the rate of 25.81 cents/kWh?

Solution

Power consumption $= 3 \times 24 \times 365$ Wh

$= 26\,280$ Wh

$= 26.28$ kWh

Annual cost $= 26.28 \times \$0.2581$

$= \$6.782868$

$= \$6.78$

Exercise 6.04 Energy consumption and the costs of appliances

Example 8

1 Calculate how much energy (in kWh) is used when each light globe is used for 8 hours.

- **a** a 100 W (old style) incandescent light globe
- **b** a 20 W compact fluorescent lamp
- **c** a 12 W LED light globe
- **d** a 50 W halogen light globe

2 How much energy is used if it takes 2 min 36 seconds to boil 1 litre of water in a 2200 W kettle? Select **A**, **B**, **C** or **D**.

A 5.192 kWh **B** 0.5192 kWh **C** 0.09533 kWh **D** 0.9533 kWh

3 How much energy is used over 5 hours by an air conditioner if the power input is 0.78 kW?

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4 The Energy Rating label on page 262 shows the average energy consumption of a washing machine that is used 250 times a year. If it consumes 45 kWh of electricity per year for cold washes and 547 kWh per year for warm washes, calculate how much power is used for:

a a cold wash **b** a warm wash.

5 Simon uses a 1500 W steam mop for 30 minutes a week to clean the tiled floors of his house. Example 9

a How many kWh of energy does the mop use over a year?

b If electricity is charged at the rate of 26.8400 c/kWh, calculate the cost of using the steam mop over a year.

6 The power input of a bar heater is 2400 W and is used on average for 5 hours a day during the winter quarter (92 days). Calculate the cost of using the heater during if power is charged at the rate of $0.288200/kWh.

7 The power input of the wash cycle of a front loader washing machine is 2100 W. The machine is used five times a week and it takes 38 minutes to complete a cold water wash cycle. Electricity is charged at the rate of 29.8210 cents/kWh. What is the total cost of using this washing machine in one year?

8 A reverse cycle air conditioner is on for 260 hours a year for heating and 240 hours a year for cooling. Calculate the cost of running each brand of air conditioner for a year if the cost of electricity is 24.7793 c/kWh.

a an Iamtubizi Electric – 4.5-star energy rating for cooling with power output 2.5 kW, 4.5-star energy rating for heating with power output 3.2 kW.

b a NBG – 5-star energy rating for cooling with power input 2.3 kW, 5-star energy rating for heating with power input 3.0 kW.

c a Tishu – 2.5-star energy rating for cooling with power input 7.1 kW, 3-star energy rating for heating with power input 8.0 kW.

9 A house has 18 lights which are used for an average of 4 hours per day. Electricity is charged at the rate of 24.45 cents/kWh. Calculate the cost of lighting for a year if: Example 10

a 75 W 'vintage style' light globes are used

b 15 W compact fluorescent lamps are used

c 12 W LED lights are used

d 50 W halogen (with a 10 W transformer) lights are used.

10 The energy consumption for a 2-star top-loader washing machine is 123 kWh/year for a cold wash and 636 kWh per year for a warm wash. If electricity is charged at a rate of 29.82 cents/kWh, calculate the difference in cost between cold and warm washing for a year. Example 11

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11 The energy rating label on a small bar refrigerator showed 1.5 stars. The energy consumption was 259 kWh per year. Find the cost of running the bar fridge for one year if electricity costs 29.8210 cents/kWh.

12 **a** A 3-bedroom house has 18 lights. Compact fluorescent lights of 20 W are used and the lights are on for 6 hours per day. The cost of electricity is 30 cents/kWh. What is the cost of lighting for one year?

b If 12 W LED lights are used instead of CFLs, what is the cost of lighting for one year and how much will be saved?

13 **a** For each appliance in the table, calculate the cost of using it over a quarter (91 days), using 30 cents/kWh for the cost of electricity.

Appliance	Average quarterly energy use
Refrigerator	130 kWh
Freezer	70 kWh
Washing machine	10 kWh (based on 4 cold washes/week)
Dishwasher	92 kWh (for hot water wash, 7 times/week)
Clothes dryer	110 kWh (based on 1 cycle/week)
Television	171 kWh (based on television being on 10 h/day)
Computer	216 kWh (based on computer being on 8 h/day)

b What is the total cost?

14 Assuming that the average lifetime of a major appliances is 10 years, calculate the lifetime running cost of each appliance. Use 30 cents/kWh as an estimate for the cost of electricity.

a a dishwasher with a $2\frac{1}{2}$-star energy rating label and energy consumption of 336 kWh per year for a hot water cycle

b a refrigerator with a 1-star energy rating label and energy consumption of 685 kWh per year

c a reverse cycle air conditioner with a 4-star energy rating label for cooling with power input of 3.5 kW and a 4-star energy label for heating with power input 3.7 kW. The air conditioner is cooling for 250 hours per year and heating for 280 hours per year.

Example 12

15 A games console in standby mode uses 5.4 W. Find the cost of standby for the games console for one year if electricity is charged at the rate of 28.0500 cents/kWh.

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16 This table shows the standby power consumption per hour for some common household appliances. If electricity is charged at the rate of 26.675 cents/kWh, calculate the:

a standby power cost per year for each appliance

b total standby power cost for the seven appliances.

Appliance	Standby power consumption (per hour)
Television (LCD)	3.5 W
Computer notebook	8.9 W
Inkjet printer	1.3 W
Microwave oven	3.1 W
Dishwasher	2.7 W
Set-top box (Pay TV)	15.8 W
Mobile phone charger	0.3 W

Source: © Government of South Australia

TECHNOLOGY

Energy calculators

The cost of appliances, heating and cooling a home, and lighting can be calculated using online energy calculators which can be found on the Internet. Search for an energy calculator such as **Energy Use Calculator.**

1 Use the energy calculator to find the yearly cost of running appliances such as washing machines, LCD/LED display or TV screen, laptops and notebooks, refrigerators, freezers, dishwashers, recharging mobile (cell) phones and so on, as well as calculating the cost of lighting, heating and cooling a home.

2 Prepare a report, putting your information in a table so that the total energy use of your home can be calculated over a year.

3 Appliances can be compared in terms of their running costs for different numbers of years. Visit the **Energy Rating** government website and select 'Compare Products', choose an appliance and compare the cost of 2 or 3 different brands, comparing the cost over 1 year and 10 years. Record or print your results.

4 Repeat this process for at least three other appliances that are used in the home.

INVESTIGATION

ENERGY CONSUMPTION IN YOUR HOME

1 Visit the government website **Energy made easy** and select 'Understand and compare your electricity usage' to calculate the average household electricity usage per day, per season and per year for:

a North Ryde **b** Bathurst

c Bondi Beach **d** where you live.

2 Reducing electricity consumption can be achieved by understanding the running costs of your appliances. Fact sheets on saving energy can be found at the **Origin Energy** website by searching for 'State fact sheets'.

a Use this site to obtain fact sheets for NSW, Victoria, Queensland and South Australia.

b Draw a clustered column graph from information on the fact sheets that can be used to compare electricity consumption between the states.

c What advice do the fact sheets give for reducing electricity consumption?

Another useful site is the **Your Energy Savings** government website.

Investigating fast food

6.05 Food and energy consumption

Nutrition in food provides a source of energy to maintain life and also for growth. The right balance of nutrients help the body to function and fight illness and disease. The more active a person is, the more energy they need.

Energy intake is measured in **joules** (J) or **calories** (Cal). Calories are an older unit of energy. Sometimes in food packaging, you will see the word **kilocalories**. When describing energy in food and nutrition, kilocalories (kcal) and calories are the same thing.

Converting units of energy

1 kilojoule (kJ) = 1000 joules (J)

1 megajoule (MJ) = 1 000 000 joules (J) = 1000 kJ

1 calorie (Cal) = 4.184 kJ

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EXAMPLE 13

Convert each unit of energy.

a 300 J to kJ **b** 14 MJ to kJ **c** 82 Cal to kJ
d 1600 kJ to Cal **e** 0.17 MJ to Cal

Solution

a $300\text{ J} = 300 \div 1000$
$= 0.3\text{ kJ}$

b $14\text{ MJ} = 14 \times 1000$
$= 14\,000\text{ kJ}$

c $82\text{ Cal} = 82 \times 4.184$
$= 343.088\text{ kJ}$

d $1600\text{ kJ} = 1600 \div 4.184$
$= 382.4091778$
$\approx 382.4\text{ Cal}$

e $0.17\text{ MJ} = 0.17 \times 1000$
$= 170\text{ kJ}$
$= 170 \div 4.184$
$= 40.6309\ldots$
$\approx 40.6\text{ Cal}$

$1\text{ MJ} = 1000\text{ kJ}$

$1\text{ Cal} = 4.184\text{ kJ}$

Exercise 6.05 Food and energy consumption

Example 13

1 Convert each unit of energy.
a 2350 kJ to J **b** 54 MJ to kJ **c** 760 Cal to kJ
d 1475 kJ to MJ **e** 250 kJ to Cal **f** 0.9 MJ to Cal

2 One glass of orange juice contains 400 kJ of energy. How many calories is this?

3 A box of 6 fruit and nut snack packs contains a total of 14.6 grams of fat.
a If 1 gram of fat = 37 kJ, calculate the total number of kilojoules of fat in the box.
b How many kilojoules of fat are in one of the snack packs?
c How many calories of fat are in one snack pack?

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4 Anthea bought a 1 kg box of rolled oats cereal. The front of the box contained this dietary intake information per 50 g serve.

Calculate the total amount of energy in this box of oats in:

a megajoules (MJ)

b (kilo) calories.

Alamy/Clare Gainey

5 This nutrition information panel shows the percentage of the recommended daily intake (DI) of energy provided by 2 slices of organic bread.

Energy	Protein	Fat	Saturated Fat	Carbs	Sugars	Sodium	Fibre
751 kJ	11.2 g	4.8 g	0.7 g	19.9 g	2.4 g	361 mg	5.6 g
9% DI	22% DI	6% DI	3% DI	6% DI	3% DI	16% DI	20% DI

The recommended daily energy intake for an adult is 8700 kJ. Calculate:

a the number of kilojoules represented by protein

b the number of calories in a loaf of this bread containing 14 slices.

6 Anneliese bought her lunch which consisted of the following items:

Item	Energy (kJ)
One-quarter BBQ chicken	2120
Small fruit salad	744
Iced chocolate milk	1620

a Calculate the total energy intake for Anneliese's lunch.

b What percentage (correct to one decimal place) of the recommended daily intake (of 8700 kJ), does her lunch represent?

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7 Olivia is making a hamburger and milkshake at home. She knows the energy content of her ingredients as shown in the table.

Food	Energy content (kJ)
1 hamburger slice	1280
1 hamburger bun	650
1 egg	300
Potatoes (1 cup)	680
1 teaspoon of oil	185
Milk (1 glass)	645
1 scoop of ice cream	268
1 teaspoon of sugar	80
1 slice of cheese	429
1 serve of tomato sauce	64

a Calculate the total amount of energy, in kilojoules (kJ), contained in Olivia's meal if she consumes the following:

Hamburger	Milkshake
1 hamburger bun	1 glass of milk
1 hamburger slice	2 scoops of ice cream
1 egg	Chocolate syrup (contains 3 teaspoons of sugar)
1 slice of cheese	
1 serve of tomato sauce	

b If Olivia is 25 years old, use this table to determine what percentage of her daily energy intake her meal represents.

Estimated daily energy requirement (kilojoules) for a sedentary person (e.g. office work or study, no strenuous physical activity)		
Age	**Male**	**Female**
12–15 years	8200–9900	7400–8200
16–18 years	10300–10900	8400–8500
19–30 years	10300*	8400*
31–50 years	9900*	8000^
51–70 years	9100*	7600^
Adults over 70 years	8300*	7200^

*estimated energy intake for male 180 cm tall, 71.3 kg
^estimated energy intake for female 170 cm tall, 63.6 kg

Source: National Health and Medical Research Council.

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8 The nutrition information panel for a tub of yoghurt is shown below.

Servings per package: 3 Serving size: 150 g	Quantity per serving	Quantity per 100 g
Energy	608 kJ	405 kJ
Protein	4.2 g	2.8 g
Fat – total	7.4 g	4.9 g
– saturated	4.5 g	3.0 g
Carbohydrate – total	18.6 g	12.4 g
– sugars	18.6 g	12.4 g
Sodium	90 mg	60 mg

a **i** How many suggested servings are there in the tub of yoghurt?

ii Calculate the total weight of the tub of yoghurt.

b Convert the energy content per 100 g into calories.

c **i** Given that 1 g of carbohydrates has 17 kJ, calculate the number of kilojoules of carbohydrates per serving.

ii What percentage of the energy content per serving does this represent?

9 The table below shows the energy used by Michael, an 18-year-old male weighing 75 kg and with a height of 170 cm, in a number of physical activities with a duration of 30 minutes.

Activity	Energy used (kJ)
Weightlifting	465
Lawn bowling	512
Golf	667
Skateboarding	775
Cycling (gentle)	930
Bushwalking	930
Skiing	1085
Soccer	1085
Swimming	1085
Touch football	1240
Running	1240

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a Name at least two activities that would best help Michael use this energy in physical activity and state how long he would need to exercise if he:
- **i** consumes a bacon and egg roll with an energy content of 2194 kJ
- **ii** has a chicken teriyaki salad with an energy content of 1015 kJ.

b Selena is an 18-year-old female, 170 cm tall and weighing 62 kg. Calculate how long it will take Michael and Selena to use the energy given in each actvity below, with the given energy content.
- **i** running/jogging, 1049 kJ
- **ii** swimming, 918 kJ
- **iii** bushwalking, 787 kJ

INVESTIGATION

COMPARING ENERGY CONTENT OF FOODS AND PHYSICAL ACTIVITY

Visit the **Calorie King** website and search for its food database.

Select a food item listed below and compare the energy rates of at least two types of activity from the table below to use the energy contained in the food. Calculate the amount of time needed for you to use the energy in each activity you choose.

a hamburger **b** muffin **c** frozen yoghurt
d noodles **e** soft drink

Activity	Energy rate (kJ/kg/h)
Sitting quietly	1.7
Writing	1.7
Standing relaxed	2.1
Driving a car	3.8
Vacuuming	11.3
Walking rapidly	14.2
Running	29.3
Swimming (4 km/h)	33
Rowing in a race	67

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ISBN 9780170413565

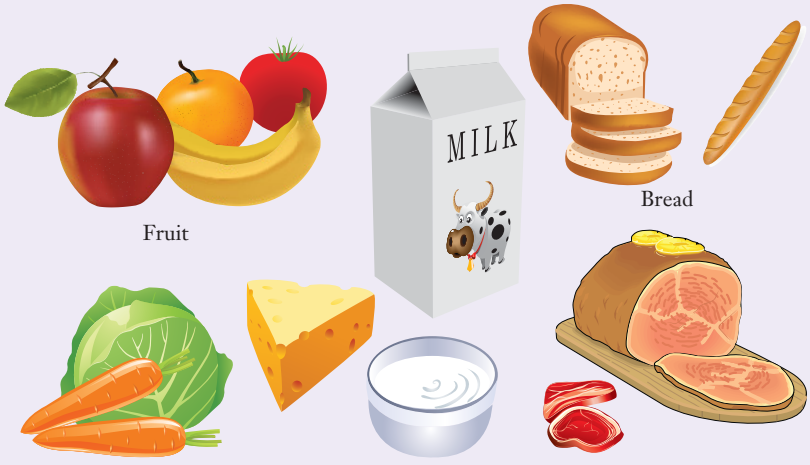
DID YOU KNOW?

The five food groups

To meet the nutritional requirements for good health, a variety of foods from each of the five food groups is recommended. It is not essential to eat from each food group at every single meal because some foods in the food groups are recommended to only be consumed a few times a week.

The five food groups are:

- vegetables and legumes/beans
- fruit
- grain (cereal) foods mostly whole grain and/or high cereal fibre varieties
- lean meats and poultry, fish, eggs, tofu, nuts and seeds
- milk, yoghurt, cheese and/or alternatives, mostly reduced fat.



Fruit Bread

Vegetables Milk Meat

Visit the Eat for health website and select:

- **'How much do we need each day' to find out how many serves of each food group we should aim to eat and how often they should be consumed**
- **'Serve sizes' to find out the mass of serving sizes for different food groups and their average energy content in kilojoules (kJ).**

ISBN 9780170413565

6.06 Household budgeting

Budget grid

Budgeting scenarios

A **budget** is an estimate of income and spending for a certain period of time. It is a plan to help manage your income wisely. Whether you are running a household or a business or just managing your pocket money, it is essential that you do not spend more than you earn. A budget is divided into **income** and **expenses** and a balanced budget is one in which the total expenses equal the total income.

When preparing a budget it is also important to identify items that qualify as **fixed spending** and **discretionary spending**. **Fixed spending** is money spent on necessities such as food, clothes, fuel and household bills. **Discretionary spending** is money spent by consumers on items other than necessities. These purchases could involve technology or entertainment items, movie tickets or upgrading furniture or appliances. Overspending on discretionary items could lead to debt if expenses are more than income.

EXAMPLE 14

Lucy was saving for an overseas trip and accepted some extra casual work. She set up a weekly budget, as shown in the table, and aimed to save $170 per week.

Income		Expenses	
Wages	$1120	Rent	$425
Casual wage	_____	Groceries	$163
		Fares	$103
		Car loan	$188
		Petrol/car maintenance	$93
		Entertainment	$185
		Savings	_____
Total:	$1305	Total:	$_____

a How much did Lucy earn from her casual job?

b What should be Lucy's total expenses?

c To balance her budget, how much can Lucy save per week?

d Name an expense that represents discretionary spending.

e Suggest a way Lucy could increase her savings to $170 per week.

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Solution

a Casual wage = \$1305 – \$1120

= \$185

b Total expenses = \$1305 Equal to total income

c Savings = \$1305 – (\$425 + \$163 + \$103 + \$188 + \$93 + \$185)

= \$148

d Entertainment or possibly the car loan (in addition to fares listed). Perhaps they live near public transport and don't need the car as well.

e Suggestions: cut down on entertainment, rent with a friend, sell the car, or look for weekly grocery specials.

Exercise 6.06 Household budgeting

1 Categorise each item as either income or expense.

- **a** a car insurance payment
- **b** an inheritance from your grandmother
- **c** a birthday present for a friend
- **d** the cost of a plane ticket to London
- **e** a second prize win in the lottery
- **f** the monthly rental of a computer
- **g** a rent payment to a real estate agent
- **h** the admission price to the cinema
- **i** the rent from an investment unit
- **j** pay to an electrician for installing new power points
- **k** your weekly wage
- **l** the cost of a gym class
- **m** pocket money
- **n** a Christmas present from your aunt
- **o** the cost of a Friday night pizza

2 For each expense item from Question **1**, identify it as either fixed spending or discretionary spending.

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Example 14

3 Mariella's budget allowed her to save $60 each week. Her budget is shown below.

Income		Expenses	
Youth allowance	$239.50	Rent	$160
Casual job	_____	Fares	$56
		Groceries	$124
		Clothes	$75
		Entertainment	_____
		Bills	$68
		Savings	$60
Total:	$621.00	**Total:**	$

a How much did Mariella earn for her casual job?

b Is entertainment a fixed or discretionary item? What amount did Mariella spend on entertainment?

c Suggest two ways that Mariella could increase her weekly savings to $80.

4 Jess wants to save $2200 for an end of year trip. She earns $615.40 net a week and has the following weekly expenses: meals $40, rent $138, groceries $52, fares $36, tennis $20, entertainment $60, car loan $85, bills $65 and magazines $8.

a Calculate the amount Jess has left for savings each week.

b Will Jess have enough saved to pay for her trip in one year?

c If the trip price increases by 20%, would Jess have enough saved in one year to pay the new price?

5 The Rogers family has the following net weekly incomes: wages $1901.77, part-time job wages $289.53 and parenting allowance $197.20. The family also has the following weekly expenses: bills $105.30, school fees $85.80, entertainment costs $295, health fund payment $53.40, clothes $132, home maintenance $184, groceries $210.50, petrol $85.40, Friday night takeaway $46, newspapers and magazines $26, home loan repayments $545.60, and car loan repayment $184.22.

a Use a table or spreadsheet to display this data as a budget, adding an entry for savings.

b What amount does the Rogers family save per week?

6 Zac shares a flat and pays $110 a week towards the rent. He spends $64 on food and $46 on public transport. His shares of the quarterly electricity bill and two-monthly phone bill are $45.10 and $12.80 respectively. He earns $551.40 net per week, spends about $45 per week on entertainment and visits a chiropractor once a month at a cost of $42 a visit. Convert Zac's expenses to weekly amounts and draw up a weekly budget, including an amount for savings.

a What is Zac's weekly income?

b What is the total of Zac's weekly expenses?

c How much can Zac save per week?

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INVESTIGATION

PREPARING A BUDGET

Imagine that you are 25 years old with a net monthly income of $3340.

	Amount	Per	Weekly amount
Income			
	$3340.00	month	
Expenses			
Rent	$185.00	week	
Mobile phone		month	
Gym membership	$484.00	year	
Internet	$49.95	month	
Water	$114.80	quarter	
Electricity	$122.50	quarter	
Health fund	$26.40	week	
Groceries		week	
Petrol/car maintenance		week	
Fares		week	
Clothes		week	
Entertainment		week	
Sport		week	
Gifts/donations		week	
Savings		week	
Other		week	

a Use a calculator or spreadsheet to copy this weekly budget and complete it after converting the fixed expenses to weekly amounts.

b Choose the amount you wish to spend per week on each variable expense and enter these amounts.

c Suggest three ways that you could save more per week.

ISBN 9780170413565

SAMPLE HSC PROBLEM

The Luk family has three 1.5 kW solar panels installed on the roof of their house.

a Calculate how much electricity will be generated in one year if the average daily production of each solar panel is 2.09 kWh. Assume it is a normal year.

b The Luk family's average annual household electricity consumption is 8764 kWh. What percentage of the household consumption is generated by the solar panels? Answer correct to two decimal places.

c **i** How much of their annual electricity consumption is *not* generated by the solar panels?

ii If their electricity provider charges \$0.258100/kWh, what is the Luk family's expected annual cost for electricity?

Study tip

Organising your notes

- One feature of effective study is an organised collection of class notes. This allows you to quickly and easily locate your work on any topic. The key to maintaining a neat set of notes is to label everything: the names of the topics, the theory, the class examples, the exercises, your folders.
- Your maths class follows a program of topics, which may correspond roughly to the chapters of this textbook. Ask your teacher for a copy of the program or syllabus and organise your folder according to these topics. Use coloured dividers or title pages to mark the start of each new topic.
- Start each maths lesson with the title of the lesson and the date. Pay attention to your teacher's explanations and copy the class examples into your book. Make sure you write the question as well as each step in the solution. You can add any personal notes, comments or reminders to help you later, when revising the work. If you don't understand something you should ask your teacher to explain again.
- When using a loose-leaf folder, make sure your notes are filed in the correct order. Number the pages of each topic so that you can refile if they get out of order. If your folder becomes too bulky to manage, you should start a new one or remove some of the older topics and file them at home. Realise that you do not need to bring to school all your notes for the entire maths course, just what you are currently working on in class.

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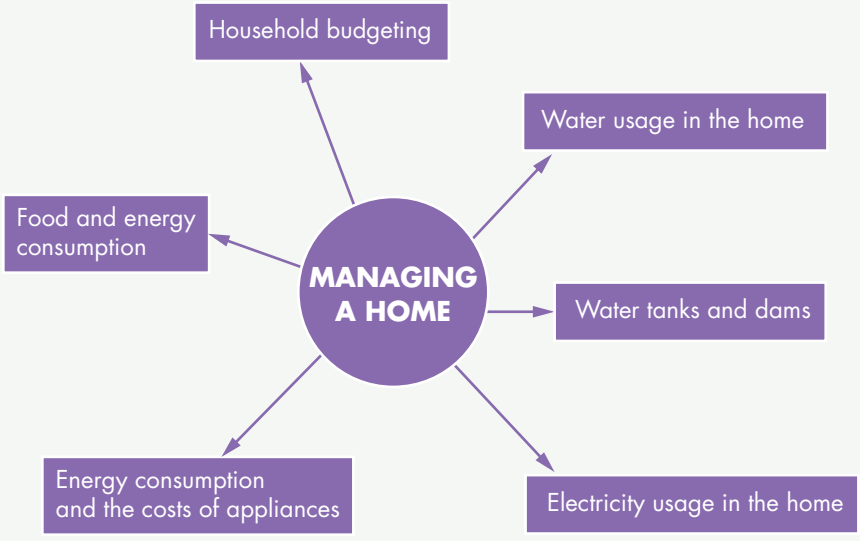
6. CHAPTER SUMMARY

This chapter, Managing a home, examined the financial mathematics, measurement and statistics behind water and electricity use in the home. The science of nutrition and energy in food was also covered. Make sure that you develop a deep understanding of the terminology, calculations and metric units involved in the usage and costs of household water and electricity. Revise volumes and capacities of water tanks and dams, and how to apply the trapezoidal rule to irregular areas and volumes.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map is shown below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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6. TEST YOURSELF

1 Jed's water bill shows that water usage is charged at $2.115/kL. Exercise 6.01

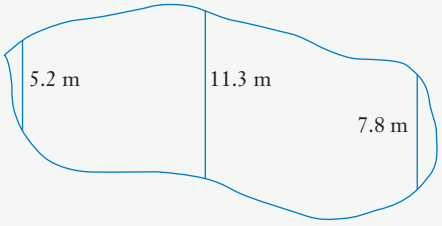
a If Jed used 21.6 kL, how much will he need to pay?

b Jed was also charged $41.39 for the water service and $162.88 for the wastewater (sewerage) service. What was the total of Jed's bill?

2 The Hamper family is calculating their water costs for the quarter. This reading is 8342 kL and the previous reading was 8145 kL. What was their water bill if usage is charged at $2.115/kL? Select **A**, **B**, **C** or **D**. Exercise 6.01

A $197 **B** $416.66 **C** $2374.58 **D** $3487.00

3 The width of a lake is measured at 10-metre intervals. Exercise 6.02



a Use two applications of the trapezoidal rule to estimate the area of the lake.

b Calculate the volume of water in the lake if it is 3.1 metres deep.

4 Convert: Exercise 6.03

a 16 400 kW to MW **b** 7.15 GW to kW **c** 26 000 000 mW to kW.

5 The pricing information for an electricity supplier is given below. Calculate the quarterly (92 days) electricity bill for a household with off-peak usage of 1087 kWh and domestic usage of 2107 kWh. Exercise 6.03

Tariff type	Cost
Off Peak (cents/kWh)	12.03719
Domestic (cents/kWh)	29.5443
Supply charge (cents/day)	95.6494

6 In a household of four people, containing two adults and two teenagers, each person uses on average, 8 minutes of hot water per shower. This house contains a 3.6 kW hot water unit and each person showers twice per day. Exercise 6.03

If electricity is charged at 27.5 c/kWh when the hot water unit is being used, what is the electricity cost for the hot water used by this household in one week? Select **A**, **B**, **C** or **D**.

A $221.52 **B** $61.60 **C** $7.39 **D** $3.70

7 A 2400 W kettle is used on average 15 minutes per day. If the cost of electricity is 24.25 cents/kWh, how much will it cost to use the kettle over one year? Exercise 6.04

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Exercise 6.04

8 The energy consumption for a 4-star 8 kg top-loader washing machine is 85 kWh/year for a cold wash and 360 kWh per year for a warm wash. If electricity costs 28.34 cents/kWh, calculate how much cheaper it is to wash in cold water than in warm.

Exercise 6.05

9 Convert:

a 4.65 kJ to J

b 19 MJ to kJ

c 7235 calories to kJ

d 108.5 kJ to MJ

e 680 kJ to calories

f 0.4 MJ to calories.

Exercise 6.05

10 Bella had a 150 g tub of yoghurt for a snack. The front of the box contained this dietary intake information per 60 g serve.

Calculate the total amount of energy in this tub of yoghurt in:

a kilojoules

b calories (to one decimal place).

Exercise 6.06

11 Chloe earns $1654 per fortnight as a graphic designer as well as collecting $1270 monthly rent from an apartment she owns.

a Calculate Chloe's total weekly income.

b Prepare a balanced weekly budget for Chloe using the items and values below.

- Groceries $142
- Home loan repayment $235
- Health fund $28.20
- Household bills $43
- Newspapers and magazines $18
- Take-away meals $32
- Petrol $70
- Fares $56.70
- Entertainment $?
- Clothes $?
- Savings $?

c Name one way Chloe could reduce her discretionary spending and save $40 more per week.

Qz Chapter quiz

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Practice paper 2

Recommended time: One hour
Marks: 40

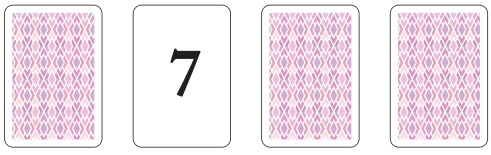
Section 1

10 multiple-choice questions: 1 mark each
Select the correct answer, A, B, C or D.

Question 1

Four cards are marked with the numbers 7, 8, 9 and 10 and one card is turned over as shown.

Exercise 4.01



What is the probability that the next card turned over is an even number?

A $\frac{1}{2}$ **B** $\frac{1}{4}$ **C** $\frac{2}{3}$ **D** $\frac{1}{3}$

Question 2

Exercise 4.02

A coin is tossed three times. What is the probability that 'tails' comes up all three times?

A $\frac{1}{3}$ **B** $\frac{1}{2}$ **C** $\frac{1}{8}$ **D** $\frac{3}{8}$

Question 3

Exercise 5.02

A bridge length was measured to be 35.6 m. What is the percentage error of this measurement?

A ±0.140% **B** ±0.281% **C** ±1.40% **D** ±2.81%

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Question 4

Exercise 6.05

Alex eats a bacon and egg roll with an energy content of 2194 kJ. When he swims, Alex uses 2170 kJ of energy per hour. For how many minutes must he swim to work off the energy from the bacon and egg roll?

A 59 **B** 61 **C** 99 **D** 101

Question 5

Exercise 6.04

How much energy does a 18 W compact fluorescent lamp use if it is on for 6 hours per day for 1 year?

A 6.57 kWh **B** 10.80 kWh **C** 39.42 kWh **D** 584.0 kWh

Question 6

Exercise 6.05

If the cost of electricity is 24.25 cents/kWh, calculate the cost of 780 kWh.

A \$18.92 **B** \$189.15 **C** \$18.72 **D** \$190.00

Question 7

Exercise 6.01

From 25 July to 24 October inclusive, the Gough family used 42.7 kL of water. Calculate their daily water usage, to the nearest litre.

A 516 L **B** 474 L **C** 464 L **D** 388 L

Question 8

Exercise 4.01

If there is a 30% chance of rain today, what is the chance that it does not rain today?

A 60% **B** 70% **C** 15% **D** 20%

Question 9

Exercise 5.04

Evaluate $(4.2 \times 10^5) \times (3.5 \times 10^{-2})$, correct to two significant figures.

A 1.47×10^4 **B** 1.4×10^3 **C** 1.5×10^3 **D** 1.5×10^4

Question 10

Exercise 4.05

A coloured die has two red faces, three blue faces and one green face. If the die is rolled 100 times, how many times can the green face be expected to turn up?

A 10 **B** 17 **C** 33 **D** 60

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Section 2
3 questions: 10 marks each

Question 11 (10 marks)

a Soojung's water bill shows that water usage is charged at \$2.115/kL.

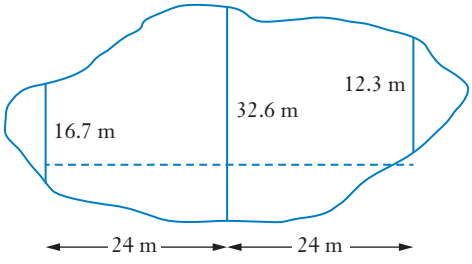
Exercise 6.01

i On her latest bill, she paid \$37.86 in water usage charges. How many kilolitres of water did she use? Answer correct to one decimal place. **1 mark**

ii Soojung was also charged \$41.39 for the water service, \$162.88 for the wastewater service and a recycled water charge of \$11.58. She also used 21 kL of recycled water at \$1.70/kL. What was the total of Soojung's water bill? **2 marks**

b A dam has three vertical measurements taken at 24 m intervals.

Exercise 5.07



Use two applications of the trapezoidal rule to approximate the area of the lake, correct to three significant figures. **3 marks**

$$\left[A \approx \frac{h}{2}\left(d_f + d_1\right)\right]$$

c Tiffany, Rose and Max sit in a row for a photo.

Exercise 4.02

i Use a tree diagram to list all possible seating arrangements. **2 marks**

ii If every seating arrangement is equally likely, what is the probability that Max sits in the middle? **1 mark**

iii What is the probability that Rose sits further left than Tiffany? **1 mark**

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Exercise 4.02

Question 12 (10 marks)

a A biased die has sides 1, 2, 3, 3, 5 and 6. It is rolled twice and the sum of the numbers is calculated.

i The table below can be used to list all possible outcomes.

Copy and complete the table. **1 mark**

		Second roll					
		1	**2**	**3**	**3**	**5**	**6**
First roll	**1**	2	3	4	4	6	7
	2	3	4	5	5	7	8
	3	4	5	6	6	8	9
	3	4	5	6	6	8	9
	5						
	6						

ii How many possible outcomes are there? **1 mark**

iii What is the probability of rolling a sum of 6? **1 mark**

iv What is the probability of rolling a sum that is an even number? **1 mark**

Exercise 6.01

b Rainwater is collected in a water tank from the roof of a block of classrooms at a school. The area of the roof is 300 m^2. During a day of heavy rainfall, 15 mm of rain falls on the roof and is collected in the water tank. How many litres of water were collected? **2 marks**

Exercise 5.10

c The planet Venus has a diameter of 12 104 km. Assuming that it is a sphere, calculate in scientific notation, correct to four significant figures:

i its volume **2 marks**

ii its surface area in hectares. **2 marks**

$$\left[V = \frac{4}{3}\pi r^3,\ A = 4\pi r^2\right]$$

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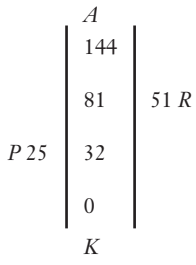
Question 13 (10 marks)

Exercise 5.07

a An offset survey on a field, *PARK*, was conducted and the following notebook entries were made by the surveyor. All measurements are in metres.



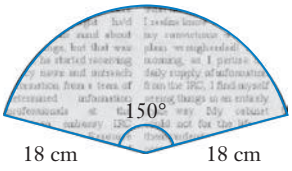
i Draw a diagram of the field, showing all measurements. **1 mark**

ii Calculate the area of the field. **2 marks**

b At the animal park, newspaper is cut in the shape of a sector and rolled to hold animal food. For one piece of newspaper, find:

Exercise 5.05

Exercise 5.06



i its area, correct to the nearest square centimetre **1 mark**

ii its perimeter, correct to the nearest centimetre. **2 marks**

$$\left[\text{Arc length} = \frac{\theta}{360} \times 2\pi r,\ \text{Area of a sector} = \frac{\theta}{360} \times \pi r^2\right]$$

c The digits 7, 2, 3 and 6 are written on separate cards. Two cards are selected at random to form a 2-digit number.

Exercise 4.02

i Use a tree diagram to list all possible outcomes. **2 marks**

ii What is the probability that the number formed is divisible by 3? **2 marks**

END OF EXAMINATION

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ALGEBRA

7.

LINEAR FUNCTIONS

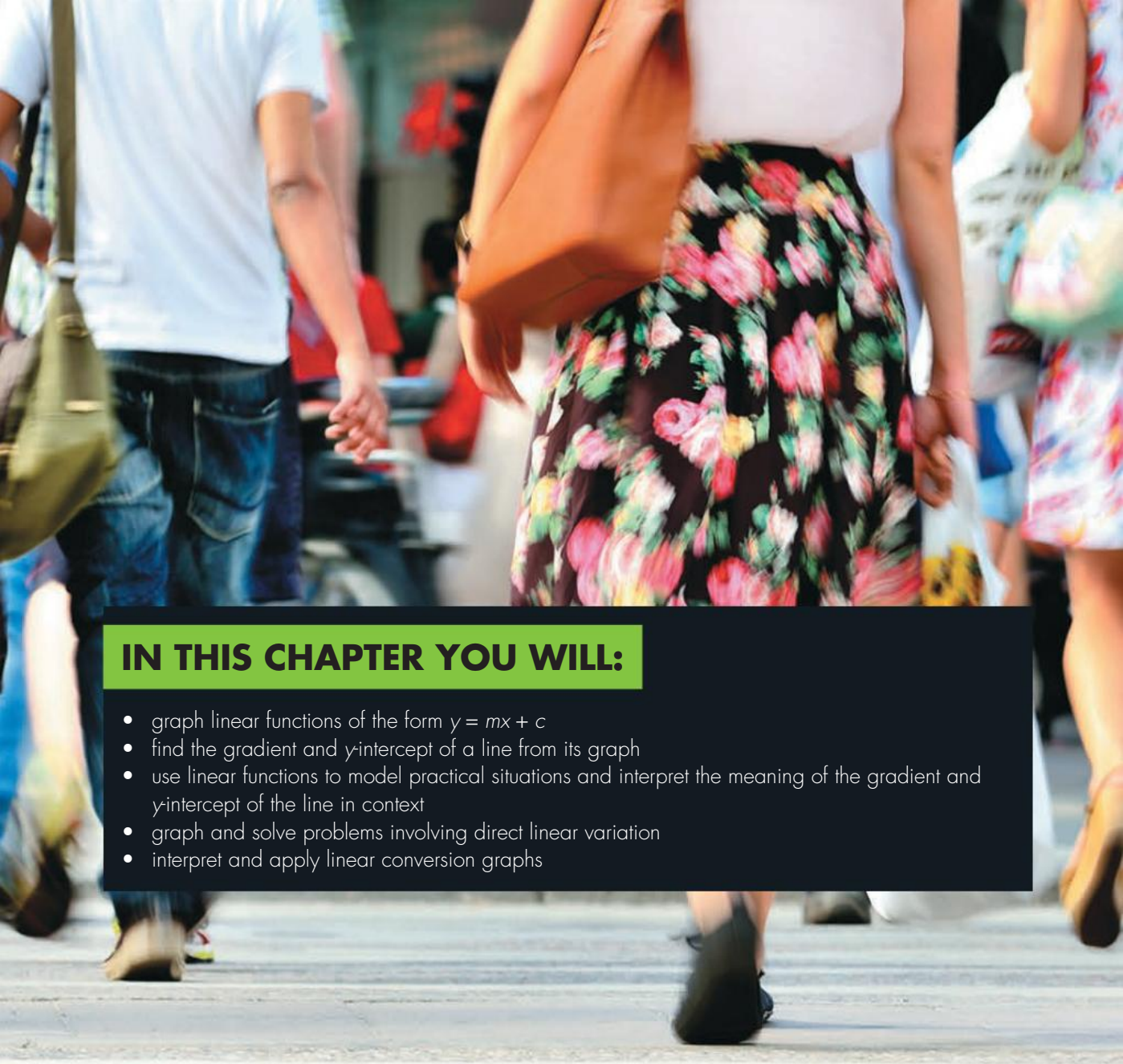
A criminologist discovered that, in a big city, if the number of police on street patrol was increased, the number of crimes committed decreased. She noticed the following pattern:

Number of police	50	150	200	250	300
Number of crimes (per month)	3100	2800	2650	2500	2350

Is it possible for the criminologist to find an algebraic formula to fit this pattern? Algebraic modelling is the study of relationships and the formulation of a mathematical rule or model to describe such relationships.

CHAPTER OUTLINE

A2 7.01 Graphing linear functions
A2 7.02 The gradient formula
A2 7.03 Linear modelling
A2 7.04 Direct linear variation
A2 7.05 Conversion graphs



IN THIS CHAPTER YOU WILL:

- graph linear functions of the form $y = mx + c$
- find the gradient and y-intercept of a line from its graph
- use linear functions to model practical situations and interpret the meaning of the gradient and y-intercept of the line in context
- graph and solve problems involving direct linear variation
- interpret and apply linear conversion graphs

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TERMINOLOGY

constant of variation
direct linear variation
linear function
rise
y-intercept
conversion graph
gradient
linear modelling
run
dependent variable
independent variable
proportional to
vertical intercept

Assignment 7

SkillCheck

1 Graph this table of values on a number plane and rule a line through the points.

x	–1	0	1	2
y	–4	–1	2	5

2 Copy and complete each of these tables of values using the given formula.

a $y = 2x + 7$

x	–1	0	1	2
y				

b $y = -x - 4$

x	–1	0	1	2
y				

3 If $y = mx + c$, find the value of c if $x = 2$, $y = 5$ and $m = 3$.

4 If $y = kx$:

a find the value of y if $k = 0.7$ and $x = 4$

b find the value of k if $x = 2.5$ and $y = 10$.

5 Given the formula $V = -450t + 2575$:

a find the value of V if:

i $t = 3$ **ii** $t = 4.2$.

b find the value of t if:

i $V = 1450$ **ii** $V = 10$.

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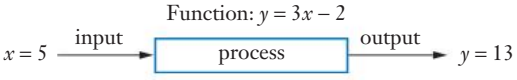
7.01 Graphing linear functions

The formula or equation $y = 3x - 2$ is called a **linear function** because its graph is a straight line. **Linear** means 'of a line', while a **function** is an algebraic rule similar to the 'number machine' shown below, that changes an input value, x, into an output value, y.



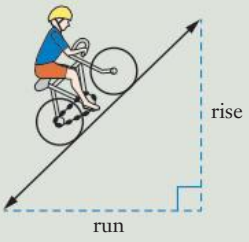
A line has a **gradient** (the measure of its steepness) and a ***y*-intercept** (the value where the line crosses the y-axis).

Gradient of a line

The gradient of a line is given by the formula:

$$m = \frac{\text{rise } (\uparrow)}{\text{run } (\rightarrow)}$$

- The **rise** is the vertical change in position (going up).
- The **run** is the horizontal change in position (going across to the right).



EXAMPLE 1

a Copy and complete this table of values for the linear function $y = 3x - 2$.

x	-1	0	1	2	3
y					

b What pattern do you notice in the y values in the bottom row of the completed table?

c Graph $y = 3x - 2$ on a number plane.

d Find the gradient and y-intercept of the line you drew in part **c**.

Drawing gradients
Graphing linear functions
A page of number planes
Graphing lines: graphics calculator
Graphing linear equations
Gradient and y-intercept
$y = mx + c$

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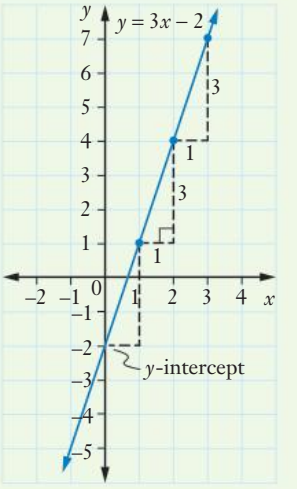
Solution

a

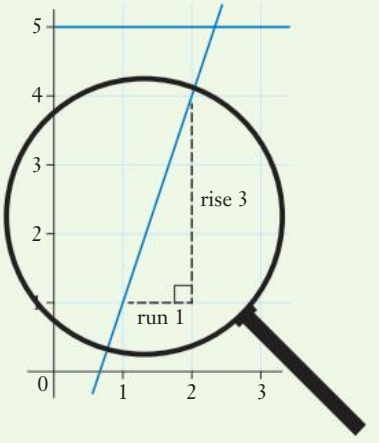
x	–1	0	1	2	3
y	–5	–2	1	4	7

b The y values increase by 3 each time.

c Use the (x, y) ordered pairs from the table to graph the points: (–1, –5), (0, –2), (1, 1), (2, 4), (3, 7). Then rule and label the line through the points.



d Choose two points on the line, say (1, 1) and (2, 4).



$$\text{Gradient } (m) = \frac{\text{rise}}{\text{run}} = \frac{3}{1} = 3$$

The y-intercept is –2.

Gradient and y-intercept of a line

Linear function

A **linear function** has the form $\boldsymbol{y = mx + c}$, where m is the gradient and c is the y-intercept.

$y = mx + c$ — gradient; y-intercept

$y = 3x - 2$ — gradient 3; y-intercept –2

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EXAMPLE 2

a Complete this table of values for the linear function $y = -2x + 4$.

x	-1	0	1	2	3
y					

b What pattern do you notice in the y values in the completed table?

c Graph $y = -2x + 4$ on a number plane.

d Find the gradient and y-intercept of the line you drew in part **c**.

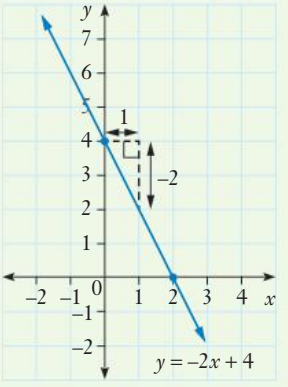
Solution

a

x	-1	0	1	2	3
y	6	4	2	0	-2

b The y values decrease by 2 each time.

c



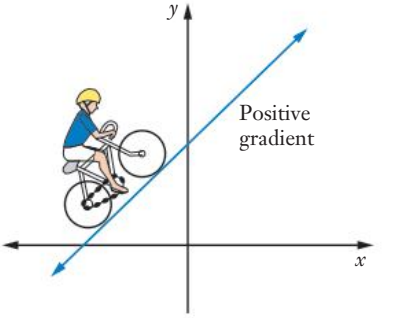
d Using points (0, 4) and (1, 2):

$$\text{Gradient } (m) = \frac{\text{rise}}{\text{run}} = \frac{-2}{1} = -2$$

The y-intercept is 4.

Positive and negative gradients

A line with a **positive gradient** slopes upwards (from left to right) because its y values are *increasing*. A gradient of 3 means that, for each 1 unit we move across, the line goes up 3 units or, as the x values increase by 1, the y values *increase* by 3.



Graph sloping upwards

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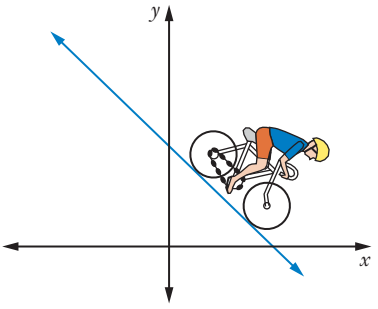
A line with a **negative gradient** slopes downwards (from left to right) because its y values are *decreasing*. A negative gradient has a 'negative rise', or a 'drop'. A gradient of -2 means that, for each unit we move across, the line goes down 2 units, or as the x values increase by 1, the y values *decrease* by 2.



Graph sloping downwards

EXAMPLE 3

Graph each linear function after finding its gradient and y-intercept.

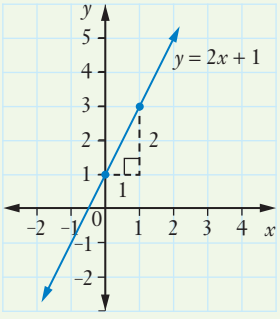
a $y = 2x + 1$ **b** $y = -4x$ **c** $y = \frac{2}{3}x - 4$

Solution

a Gradient = 2, y-intercept = 1

To graph the line:

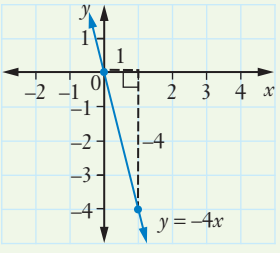
- plot the y-intercept, 1, on the y-axis
- indicate a gradient of 2 from this point, by moving across 1 unit and up 2 units and marking that point
- rule a line through both points you marked
- draw arrows at the ends and label the line with its equation.



b Gradient = -4, y-intercept = 0

To graph the line:

- plot the y-intercept, 0, at the origin
- indicate a gradient of -4 from this point, by moving across 1 unit and down 4 units and marking that point
- rule a line through both points you marked, add arrows and label.



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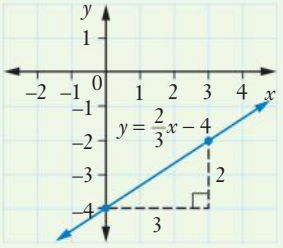
c Gradient $= \frac{2}{3}$, y-intercept $= -4$

To graph the line:

- plot the y-intercept -4 on the y-axis
- indicate a gradient of 3 from this point, by moving across 3 units and up 2 units and marking that point
- rule a line through both points you marked, add arrows and label.



Exercise 7.01 Graphing linear functions

1 Write the equation of the line with:

a gradient 3, y-intercept 7

b gradient -2, y-intercept 1

c gradient 1, y-intercept -1

d gradient $\frac{1}{3}$, y-intercept $\frac{1}{2}$

e gradient $-\frac{5}{4}$, y-intercept 0

f gradient 0, y-intercept 5

2 Graph each linear function.

a $y = 4x - 3$

b $y = \frac{1}{2}x + 2$

c $y = x - 2$

d $y = -2x$

e $y = 2x - 5$

f $y = \frac{3}{5}x - 1$

g $y = -x + 4$

h $y = \frac{4}{3}x + 1$

i $y = \frac{1}{4}x + 3$

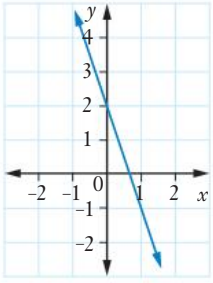
3 What is the gradient of this line? Select **A**, **B**, **C** or **D**.

A $\frac{1}{3}$

B $-\frac{1}{3}$

C 3

D -3



A page of number planes

Number plane grid paper

Example 1

Example 2

Example 3

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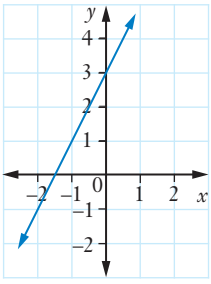
4 Find the gradient and y-intercept of each line below, and write its equation.

a



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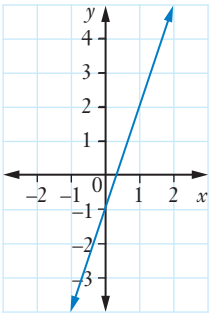

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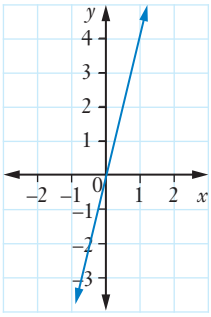

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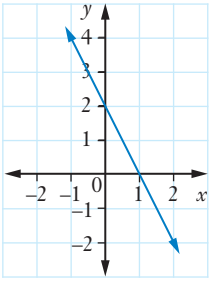

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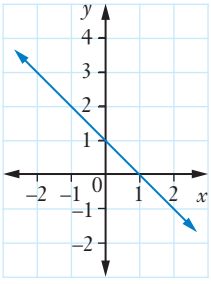

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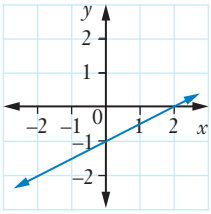

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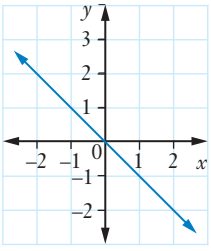

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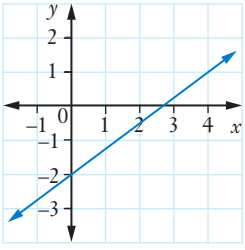

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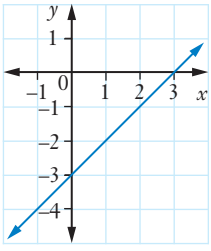

INVESTIGATION

GRADIENTS

a What does a line with a gradient of 0 look like?

b What does a line with a gradient of 1 look like?

c What does a line with a gradient of -1 look like?

d What is the highest gradient a line can have?

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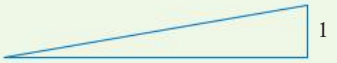
DID YOU KNOW?

Steep roads and hills

Australia is a fairly flat continent but in Europe road signs often display the steepness of a road for drivers (or the steepness of a hill for hikers).

Consider this example:



For this example, the steepness is $\frac{1}{5.8}$, also written as '1 : 5.8' or '1 in 5.8'.

Bulli Pass, north of Wollongong, has a gradient of $\frac{1}{6}$ while Victoria Pass, east of Lithgow, has a gradient of $\frac{1}{8}$. Which pass is steeper?

Shutterstock.com/Bennian

A gradient of 8% (that is $\frac{8}{100}$) means a rise of 8 units for every 100 units of run.

7.02 The gradient formula

The formula for the gradient of a line is useful if:

- the coordinates of two points on the line are given, or
- a table of values is given.

Gradient formula

$$m = \frac{\text{rise } (\uparrow)}{\text{run } (\rightarrow)} = \frac{\text{change in } y}{\text{change in } x}$$

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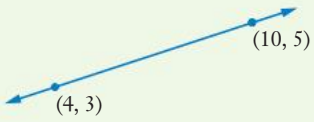
EXAMPLE 4

Find the gradient of each line.

a



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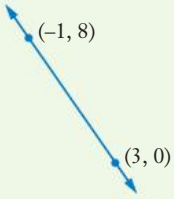

c

Solution

a $m = \dfrac{\text{change in } y}{\text{change in } x}$

$= \dfrac{5-3}{10-4}$

$= \dfrac{2}{6}$

$= \dfrac{1}{3}$ A gradient of $\frac{1}{3}$ is not steep.

b $m = \dfrac{0-8}{3-(-1)}$

$= \dfrac{-8}{4}$

$= -2$ Negative gradient, the line slopes downward

c $m = \dfrac{2-2}{8-3}$

$= \dfrac{0}{5}$

$= 0$ Zero gradient line is flat

EXAMPLE 5

Find the gradient of the line represented by each table of values.

a

x	0	4	8	20
y	5	11	17	35

b

x	2	6	9	12
y	4	–8	–17	–26

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Solution

a Choose any two points from the table, say (0, 5) and (4, 11):

$$m = \frac{\text{change in } y}{\text{change in } x}$$
$$= \frac{11-5}{4-0}$$
$$= \frac{6}{4}$$
$$= \frac{3}{2}$$

The gradient is $\frac{3}{2}$.

b Using (2, 4) and (6, –8) from the table:

$$m = \frac{-8-4}{6-2}$$
$$= \frac{-12}{4}$$
$$= -3$$

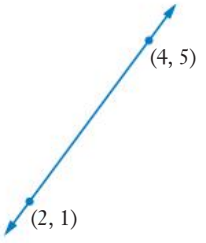
Negative gradient (y decreases as x increases)

Exercise 7.02 The gradient formula

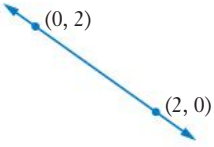
Example 4

1 Find the gradient of each line.

a



b



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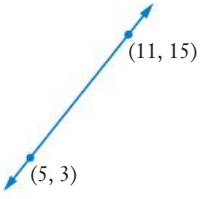

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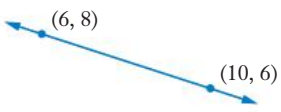

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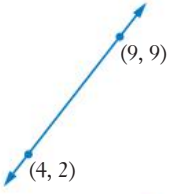

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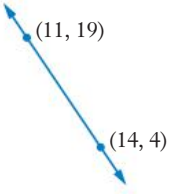

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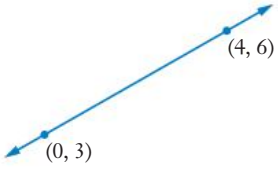

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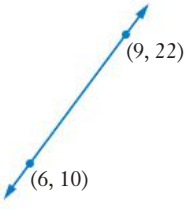

i

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Example 5

2 Find the gradient of the line represented by each table of values.

a

x	0	4	7	10
y	−1	11	20	29

b

x	1	5	7	12
y	1	21	31	56

c

x	0	8	12	20
y	3	5	6	8

d

x	1	3	6	8
y	7	3	−3	−7

e

x	0	10	25	40
y	−4	2	11	20

f

x	2	8	14	16
y	4	1	−2	−3

WS Finding the equation of a line

PS Linear functions code puzzle

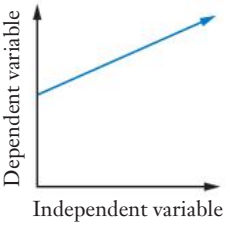
7.03 Linear modelling

Scientists and researchers observing patterns in nature and society often use a mathematical formula to represent a real-life situation. This is called **algebraic modelling** and if the relationship uses the linear function $y = mx + c$, it is called **linear modelling**.

Dependent and independent variables

Because the value of y depends on the value of x, y is called the **dependent variable** and x is called the **independent variable**.

When a function is graphed, the independent variable is shown on the horizontal axis, while the dependent variable is shown on the vertical axis.



Interpreting the gradient and the y-intercept

The **gradient** measures the steepness of a line, but it also shows how quickly the y-values are changing.

Look at this table of values for $y = 5x - 4$:

		+1	+1	+1	+1
x	−1	0	1	2	3
y	−9	−4	1	6	11
		+5	+5	+5	+5

As the x values increase by 1, the y values increase by 5.
If this linear function is graphed, for every 'run' of 1 unit, there is a 'rise' of 5 units, giving a gradient of $\frac{5}{1} = 5$.

The gradient and y-intercept

- The **gradient** of a linear function is the **rate of change** of y.
- The higher the gradient, the steeper the line, and the faster y increases relative to x.
- The ***y*-intercept** is the value of y when $x = 0$.

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EXAMPLE 6

Brett is organising a 21st birthday party. The total cost of the party will include $200 for hiring the hall, $120 for the music and $14 per person for catering. The table below shows the total cost, $C, for different numbers of guests, $n, attending the party.

Number of guests, n	80	100	120	130	150
Party cost, $C	1440	1720	2000	2140	2420

a Which variable, n or C, is the dependent variable?

b Find the gradient of the linear relationship between n and C.

c Find the vertical intercept of this linear relationship.

Vertical intercept is the more general name for the y-intercept, because linear functions can use variables other than x and y.

d Write the linear function for C in terms of n.

e If this function was graphed, which variable would be shown on the horizontal axis?

f What does the vertical intercept of this function represent?

g Find the cost of the party if there were 95 guests.

Solution

a C, because C depends on n.

b Choosing the ordered pairs (80, 1440) and (100, 1720) from the table:

$$m = \frac{1720 - 1440}{100 - 80} = \frac{280}{20} = 14$$

The gradient is 14.

c The linear function is: $C = mn + c$

The gradient is: $m = 14$ From the answer to part **b**

So: $C = 14n + c$

To find the value of c, the vertical intercept, substitute an ordered pair into the function.

Substitute (100, 1720):

$1720 = 14(100) + c$

$1720 = 1400 + c$

$c = 320$

The vertical intercept is 320.

d The function is $C = 14n + 320$.

This formula should work for the other ordered pairs in the table.

e n, because it is the independent variable.

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f The vertical intercept, 320, is the value of C when $n = 0$, which is the cost of the party if there are no guests attending. This makes sense because it is the total fixed cost of hiring the hall and the music ($200 + $120).

g Substitute $n = 95$ into the function:

$$\begin{aligned} C &= 14n + 320 \\ &= 14(95) + 320 \\ &= 1650 \end{aligned}$$

The cost for the party with 95 guests is $1650.

EXAMPLE 7

A criminologist discovered that the number of crimes committed per month, C, in a big city decreased as the number of police officers, P, patrolling the city increased. After graphing her data on a number plane, she found the linear relationship to be $C = -3P + 3250$.

a What is the independent variable in this relationship?

b Copy and complete this table for the equation $C = -3P + 3250$.

Number of police, *P*	50	150	200	250	300
Crimes per month, *C*					

c Graph $C = -3P + 3250$ on the number plane.

d What is the gradient of the graph you drew in part **c** and what does it represent?

e What is the vertical intercept of the graph you drew in part **c** and what does it represent?

f Calculate how many crimes are committed when 100 police officers are on patrol.

g Calculate how many police officers are required to reduce the number of crimes to 1900.

Solution

a P, because C depends on P.

b

Number of police, *P*	50	150	200	250	300
Crimes per month, *C*	3100	2800	2650	2500	2350

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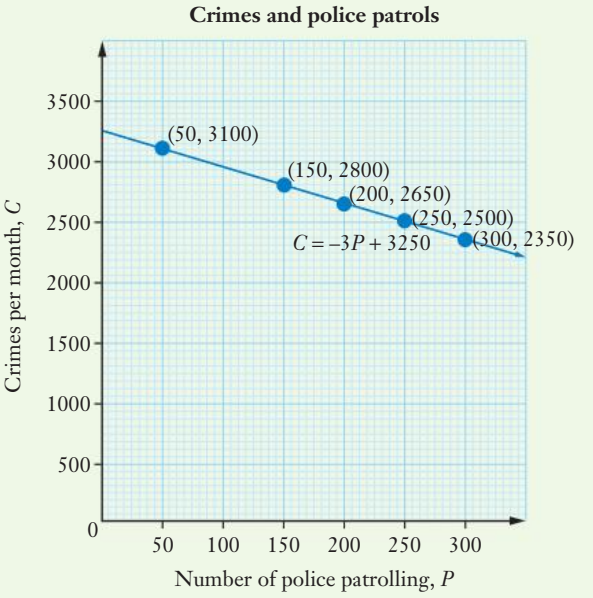
c



d From the equation, the gradient is –3 and it represents the rate of change in crime as the number of police increases. As the number of police increases by 1, the number of crimes decreases by 3.

e The vertical intercept is 3250, and this represents the number of crimes if no police officers were on patrol. It is the value of C when $P = 0$.

f Substitute $P = 100$:

$$C = -3P + 3250$$
$$C = -3(100) + 3250$$
$$= 2950$$

2950 crimes are committed when 100 police are on patrol.

g Substitute $C = 1900$:

$$C = -3P + 3250$$
$$1900 = -3P + 3250$$
$$-3P = -1350$$
$$P = \frac{-1350}{-3}$$
$$= 450$$

450 police officers are required to reduce the number of crimes to 1900.

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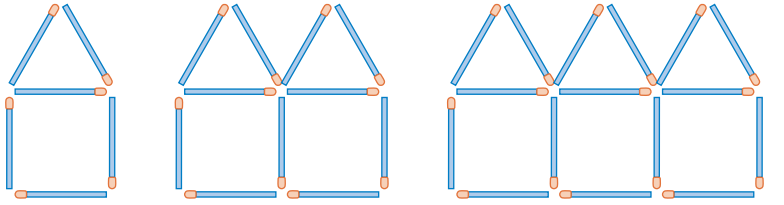
Exercise 7.03 Linear modelling

Example 6

1 This is a matchstick pattern of houses.



One house uses 6 matches.

Two houses use 11 matches.

Three houses use 16 matches.

a Copy and complete this table for the pattern above.

Number of houses, *h*	1	2	3	4	5	6
Number of matches, *N*						

b What is the dependent variable?

c Find the linear relationship in the form $N = mh + c$.

d How many matches are required to make 20 houses?

e How many houses can be made from 81 matches?

f Graph the linear function on a number plane.

g Write the gradient and the vertical intercept of the graphed line.

2 This table shows the cost, C cents, of mobile phone calls under the Oz-Zone Budget Plan, for calls of different lengths, t minutes.

Length of call, *t* (min)	1	2	5	10	15
Cost, *C* (cents)	102	182	422	822	1222

a Find the linear relationship in the form $C = mt + c$.

b If this function was graphed on a number plane, which variable would be shown on the vertical axis?

c Use the relationship you found in part **a** to calculate the cost of an 18-minute call.

d What is the vertical intercept of this function and what does it represent?

e If a phone call is extended by 3 minutes, by how much would its cost increase?

f How long is a phone call under this plan if it cost $5.82?

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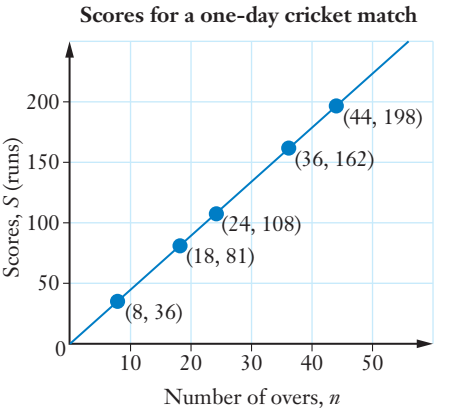
3 A cricket team's progressive score during a one-day cricket match can be approximated by the linear function graphed here.
A one-day match has 50 overs, where an over is a set of 6 balls bowled by the same bowler. The variable, n, represents the number of overs bowled, while S represents the total number of runs scored by the batting team.



a Is S the dependent variable or the independent variable?

b Find the formula for S in terms of n.

c The gradient of this linear function is also the team's run rate. What is the gradient of the function and in what units is this run rate measured?

d What is the vertical intercept and what does it represent?

e What was the score after:

i the 21st over? **ii** the 50th over?

f At the end of which over had the score reached:

i 54 runs? **ii** 180 runs?

g The graph for a real cricket match would not be a straight line but would normally flatten out later in the innings. Why?

Example 7

4 The value of a notebook computer depreciates according to the formula $V = -420t + 1900$, where V is the value in dollars and t is the time in years.

a Copy and complete this table for the formula $V = -420t + 1900$.

Time, t (years)	1	2	3	4
Value of computer, V (\$)				

b Graph this linear relationship on a number plane.

c What does the gradient of this linear function represent?

d What was the value of the computer after $2\frac{1}{2}$ years?

e What was the original value of the computer?

f This linear model does not work when $t = 5$ and beyond. Why not?

g Find, correct to one decimal place, the time when the computer has zero value.

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5 Yasmin works for a pizza shop and, each day, she earns a base pay of $50, plus $3 for every pizza she delivers.

a If n is the number of pizzas Yasmin delivers in a day and P is her total pay in dollars, write a formula for P in terms of n.

b If this linear function was graphed, which variable would be represented on the horizontal axis?

c What is the vertical intercept of this function and what does it represent?

d How much will Yasmin earn for delivering 28 pizzas in a day?

e If Yasmin earned $98 today, how many pizzas did she deliver?

6 This table shows the linear relationship between distances measured in miles and distances measured in kilometres.

Miles, M	15	25	30	45
Kilometres, K	24	40	48	72

a Graph this linear relationship on a number plane.

b Find the equation of the line.

c What can you say about the value of the vertical intercept? Why?

d What is the gradient and what does it represent?

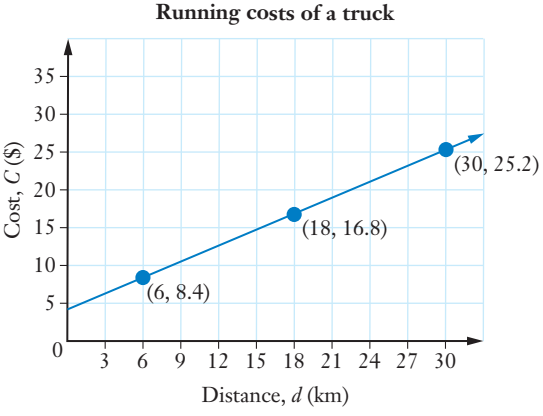
e Use your equation from part **b** to convert:

i 100 miles to kilometres **ii** 100 km to miles.

f Use your graph from part **a** to convert:

i 12 miles to kilometres **ii** 20 kilometres to miles.

7 This graph shows the linear relationship between the distance, d km, travelled by a truck and the running costs, $\$C$, for the trip.



a What is the dependent variable?

b Find the gradient and the vertical intercept of this linear relationship.

c Write the formula for this linear relationship.

d If the length of a trip is extended by 5 km, by how much will the charge increase?

e Calculate the running costs for a trip of length:

i 20 km **ii** 0 km.

f Calculate the distance travelled if the running costs were $37.80.

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8 During summer, crickets chirp faster at night if the temperature is higher. There is a linear relationship between the temperature and a cricket's chirping rate, shown in the table below.

Temperature, T (°C)	12	15	19	22	28
Chirp rate, n (chirps/min)	72	96	128	152	200

a Is T the dependent variable or the independent variable?

b Find the linear function for n in terms of T.

c Graph the linear function you found in part **b**.

d If the temperature increases by 2°C, what happens to the crickets' chirp rate?

e Find the chirp rate of a cricket when the temperature is 26°C.

f At what temperature does a cricket chirp 144 times per minute?

g What is the vertical intercept of this function? Why doesn't the linear model work for this value?

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7.04 Direct linear variation

Ann-Marie noticed that, with her new car, she could drive a distance of 567 km on a full tank of petrol (42 litres). She also discovered the results shown in the table below.

Amount of petrol used, p (L)	10	14	22	32	36	42
Distance travelled, d (km)	135	189	297	432	486	567

When she graphed these values, Anne-Marie found a linear relationship.

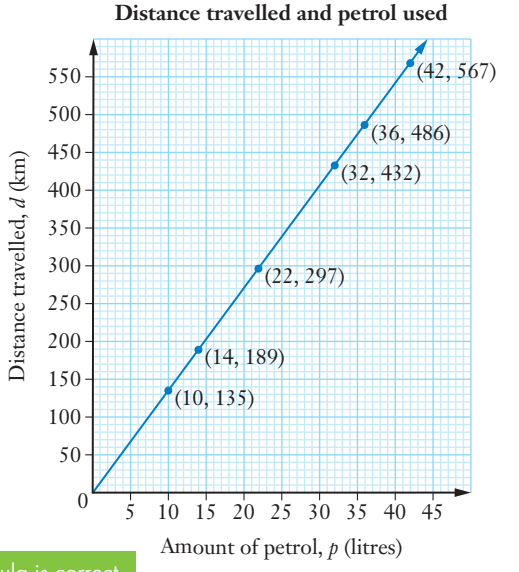
This relationship must be of the form $d = mp + c$, but the vertical intercept is 0 because, when $p = 0$, $d = 0$ (no petrol, no distance). So the linear function is $d = mp$.

To find the value of m, substitute any point from the table into the equation, for example (10, 135).

$$d = mp$$
$$135 = m\,(10)$$
$$m = 10$$
$$= 13.5$$

$\therefore$ The linear function is $d = 13.5p$.

Choose another point from the table to check that this formula is correct.



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Because $d = 13.5p$, the distance, d, is found by multiplying the amount of petrol, p, by a constant amount, 13.5. This is an example of **direct linear variation**, and we can say that 'd varies directly as p', or that 'd is directly **proportional to** p'.

Direct linear variation

If y **varies as** x, or y is **directly proportional to** x, then $y = kx$, where k is a constant. k is called the **constant of variation** or **constant of proportionality**.

A direct linear relationship exists between x and y. If x increases (or decreases), y increases (or decreases). If x is doubled (or halved), y is doubled (or halved).

EXAMPLE 8

The mass, M (in kilograms), of a metal varies directly as its volume, V (in cubic centimetres).

Volume, V (cm^3)	0	48	60	80	116	140
Mass, M (kg)	0	14.4	18	24	34.8	42

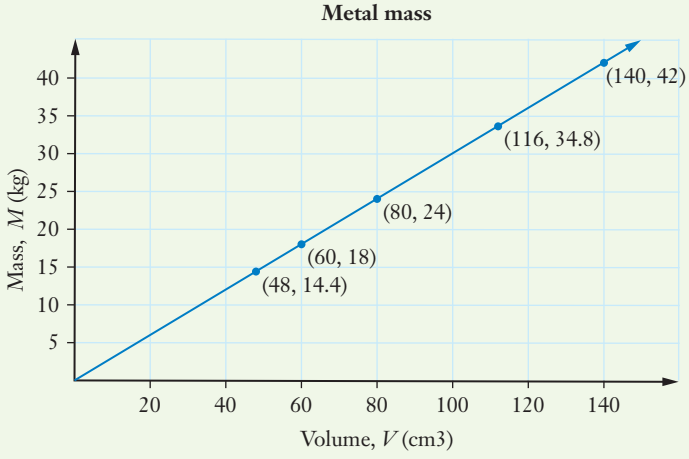
a Graph the relationship between M and V.

b Find the equation for M in terms of V.

c Find the mass of 212 cm^3 of the metal.

Solution

a



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b $M = kV$

Substituting (60, 18) from the table to find k:

$18 = k(60)$

$$k = \frac{18}{60}$$

$$= \frac{3}{10}$$

$$\therefore M = \frac{3V}{10} \quad (\text{or } M = 0.3V)$$

Choose another point from the table to check that this formula is correct.

c Substitute $V = 212$ into the formula.

$$M = \frac{3(212)}{10}$$

$$= 63.6$$

The mass of the metal is 63.6 kg.

EXAMPLE 9

The stretch, S (in centimetres), of a spring varies as the mass, M (in kilograms), of the load pulling it. A load of 24 kg causes the spring to stretch 15 cm.

a Find the variation equation relating S to M.

b What is the stretch caused by a load of 13 kg?

c What is the mass of the load that will cause a stretch of 25 cm?

d What is a limitation of this linear model?

Solution

a $S = kM$

Substitute $M = 24$ and $S = 15$ to find k.

$15 = k(24)$

$$k = \frac{15}{24}$$

$$= \frac{5}{8} \quad (\text{or } 0.625)$$

$$\therefore S = \frac{5M}{8} \quad (\text{or } S = 0.625M)$$

b When $M = 13$:

$$S = \frac{5(13)}{8}$$
$$= 8.125$$

A stretch of 8.125 cm is caused by a load of 13 kg.

c When $S = 25$:

$$25 = \frac{5M}{8}$$
$$5M = 200$$
$$M = 40$$

A 40 kg load will cause a stretch of 25 cm.

d For a very heavy load, the spring will become permanently stretched and will not return to its original length when the load is removed.

Linear variation problems

To solve a linear variation problem:

1 Identify the two variables (say x and y) and form a variation equation, $y = kx$.

2 Substitute values for x and y to find k, the constant of variation.

3 Rewrite $y = kx$ using the value of k.

4 Substitute a value for x or y into $y = kx$ to solve the problem.

Example 8

Exercise 7.04 Direct linear variation

1 Copy and complete this table after finding the value of k in $y = kx$.

x	1	4	6	10	11	15
y		14	21			52.5

Example 9

2 The distance travelled by a bicycle varies directly with the number of revolutions made by the pedals.

a Form a variation equation and find the constant of variation, given that the bicycle travels 55 metres for 20 revolutions of the pedals.

b Calculate the distance travelled for 33 revolutions of the pedals.

c Calculate the number of revolutions of the pedals required for the bike to travel 99 metres.

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3 The increase in pressure experienced by a scuba diver is directly proportional to her depth under the water. The increase in pressure at 25 m is 147 kilopascals (kPa).

a Form a variation equation.

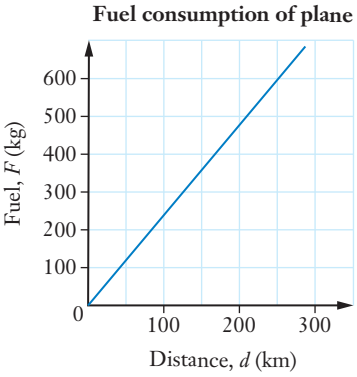
b What increase in pressure is experienced at 40 m?

c At what depth is the increase in pressure 833 kPa?

4 The graph illustrates the fact that the mass of fuel an aeroplane requires varies directly with the distance it flies.

a Form a variation equation, using a point on the graph to find the constant of variation.

b Use the equation to calculate the amount of fuel required to fly 3250 km.



5 A speed expressed in km/h is directly proportional to the same speed expressed in miles/h.

Speed in miles/h, x	30	36	45	50	62.5	74
Speed in km/h, y	48	57.6	72	80	100	118.4

a Find the equation for this table of values.

b Convert 60 miles/h to km/h.

c Convert 120 km/h to miles/h.

6 For an object that is cooling, the drop in temperature varies directly with time. If the temperature drops 8°C in 5 minutes, which of the following is the amount of time it would take for the temperature to drop 10°C? Select **A**, **B**, **C** or **D**.

A 6.25 min **B** 7 min **C** 12.8 min **D** 16 min

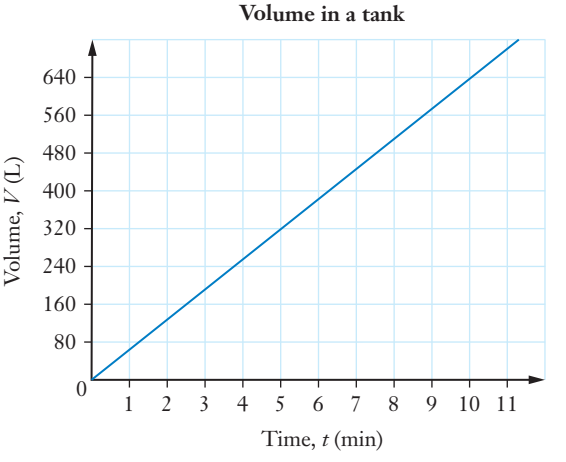
7 The volume, V litres, of water in a tank as it is filled is proportional to the time taken t minutes, as shown on the graph.

a Find the constant of variation.

b What does the constant you found in part **a** represent?

c Calculate the amount of water in the tank after 8 minutes.

d Calculate how long it takes to pump 960 L of water into the tank.



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8 The distance travelled by a marathon runner varies directly with time.

a If he covers 9.75 km in 45 minutes, calculate how long it will take him to run 26 km.

b What is a limitation of this linear model?

9 The weight of an astronaut on Mars is proportional to his weight on Earth. A 72 kg astronaut weighs 27.4 kg on Mars.

a Calculate how much a 60 kg astronaut weighs on Mars, correct to one decimal place.

b If an astronaut weighs 32 kg on Mars, calculate his weight on Earth, correct to one decimal place.

10 Nick noticed that, during his road trip, the amount of petrol used by his car varied with the amount of time he drove. It consumed 45 L of petrol in 4 hours. How long, to the nearest minute, will Nick's car take to consume 100 L of petrol?

11 The download time of a computer file is directly proportional to the size of the file. If a file of 1800 kilobytes requires 36 seconds to download, calculate:

a how long it will take to download a 3000 kilobyte file

b the size of a file that requires 80 seconds to download.

12 The compression of a car spring is proportional to the force applied. A force of 200 N (newtons) causes a compression of 1.6 cm. Which of the following amounts of compression is caused by a 5000 N force? Select **A**, **B**, **C** or **D**.

A 25 cm **B** 40 cm **C** 62.5 cm **D** 200 cm

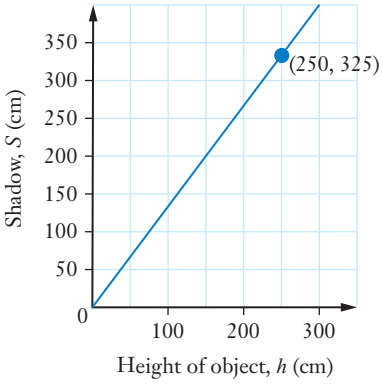
13 At 3:30 p.m. one day, the shadows of objects of different heights were measured, then graphed.

a Find the equation of the line.

b Calculate the length of the shadow of a lamppost of height 244 cm.

c What is the height of a letterbox, correct to the nearest centimetre, if its shadow is 120 cm long?

Shadows of objects of different heights



14 When an object is dropped under gravity, its speed varies with time. Its speed after 5 seconds is 49 m/s.

a Find its speed after 12 seconds.

b Calculate the time taken to reach a speed of 175 m/s (to the nearest second).

c What is a limitation of this linear model?

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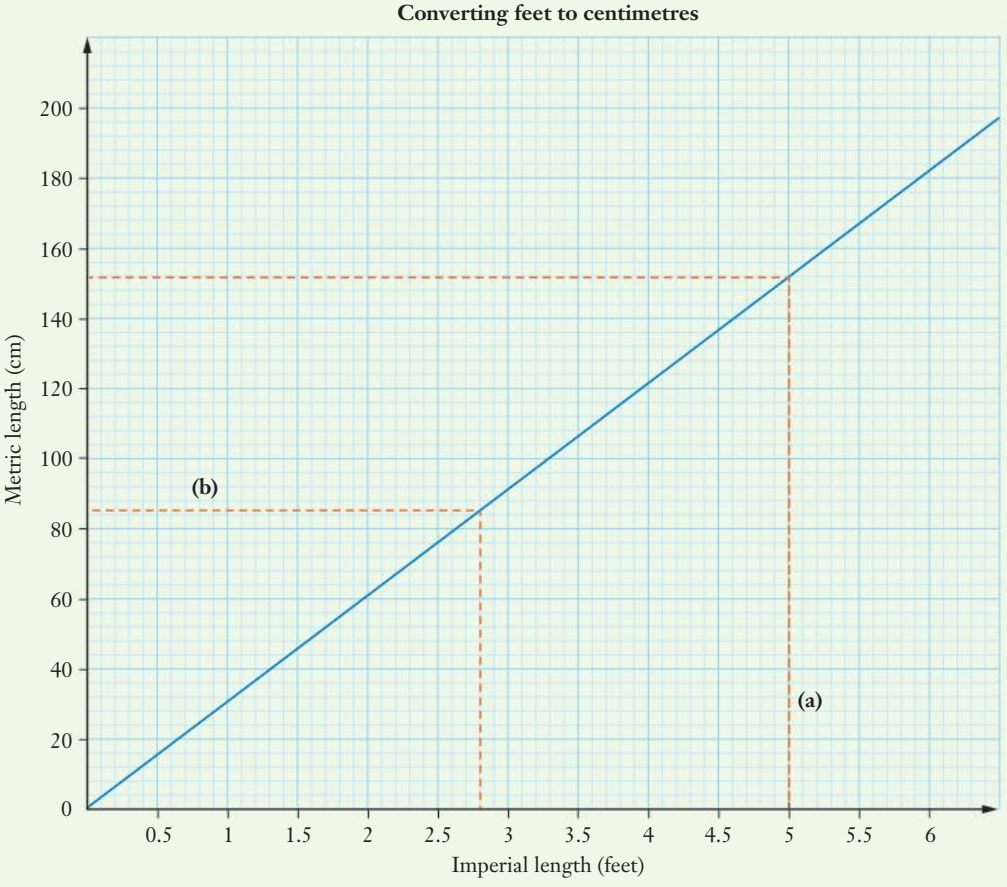
7.05 Conversion graphs

A **conversion graph** is used to convert between different units, such as between metric and imperial units, or between currencies in foreign currency exchange.

EXAMPLE 10

This conversion graph converts between feet (an imperial unit) and centimetres (a metric unit).



a Kylie is 5 feet tall. What is her height in centimetres?

b A doorway is 85 cm wide. Convert this to feet, correct to one decimal place.

c The gradient of the line in the conversion graph is 30.5. What does this value represent?

Solution

a Reading from the graph: 5 feet = 152 cm

b Reading from the graph: 85 cm = 2.8 feet

c The gradient represents the rate of change of centimetres per foot: 30.5 cm = 1 foot

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Exercise 7.05 Conversion graphs

Example 10

1 Use the conversion graph from Example **10** to answer the following questions.

a A bookshelf is 3.5 feet tall. What is its height in centimetres?

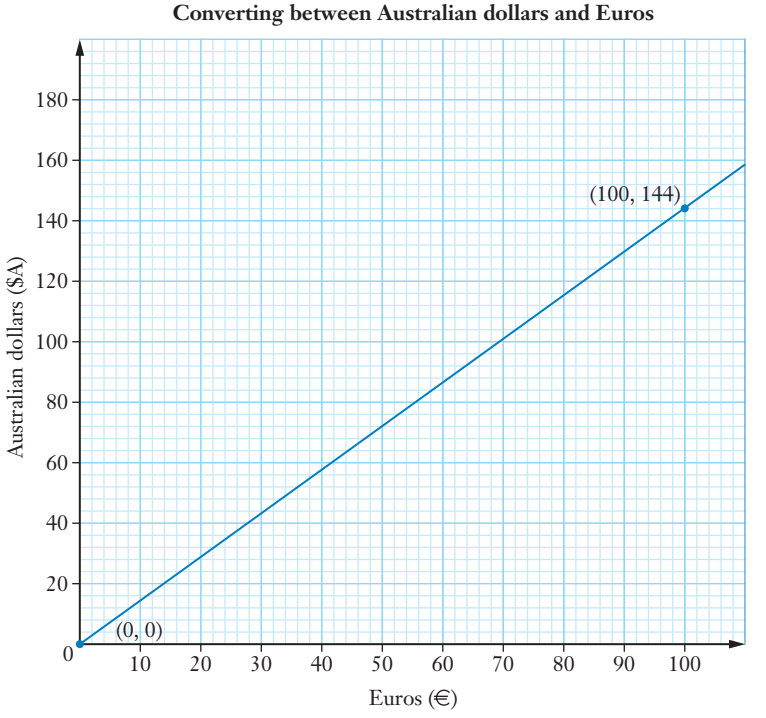
b Michael is 180 cm tall. Convert this height to feet, correct to one decimal place.

c Convert each length to centimetres.

i 2 feet **ii** 2.5 feet **iii** 6 feet

d Convert your own height to feet and inches, given that 1 foot = 12 inches.

2 This currency conversion graph converts between the euro (€) and the Australian dollar (A$).



Convert each European price to Australian dollars.

a a smart watch €88

b theme park entry €36

c a mobile phone €76

d a camera €52

e a city tour €25

f car hire per day €42

3 The conversion graph from Question **2** can also be used to convert small amounts. For example, from the graph: €46 = A$66. So it is also true that €0.46 = A$0.66.

Use the graph to help you convert each of the following prices to Australian dollars.

a a can of drink €1.20

b posting a letter €0.80

c 30 min of Internet use €3.40

d a newspaper €1.25

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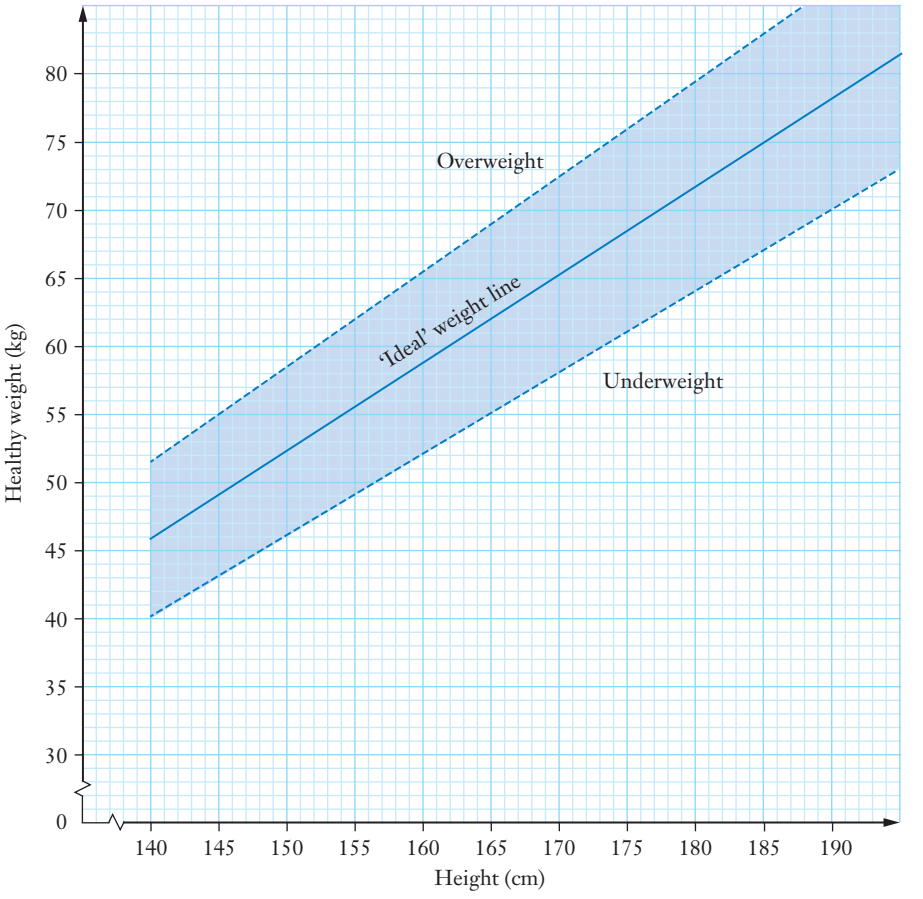
4 Use the conversion graph from Question **2** to convert each of the following Australian prices to euros.

a a movie DVD A$24
b a cordless phone A$76
c a soccer ball A$32
d a computer desk A$130
e an exercise bike A$108
f fish and chips A$7.50

5 Calculate the gradient of the currency conversion graph in Question **2**, correct to two decimal places. What does this value represent?

6 This height–weight graph shows the healthy weight range (shaded) for Australian adults of different heights.

Height–weight graph



a Find the 'ideal' weight of a person whose height is:
i 180 cm **ii** 155 cm **iii** 192 cm.

b Find the height of a person with an 'ideal' weight of:
i 72 kg **ii** 58 kg **iii** 80 kg.

c Calculate the gradient of the 'ideal' weight line. What rate does this gradient value represent?

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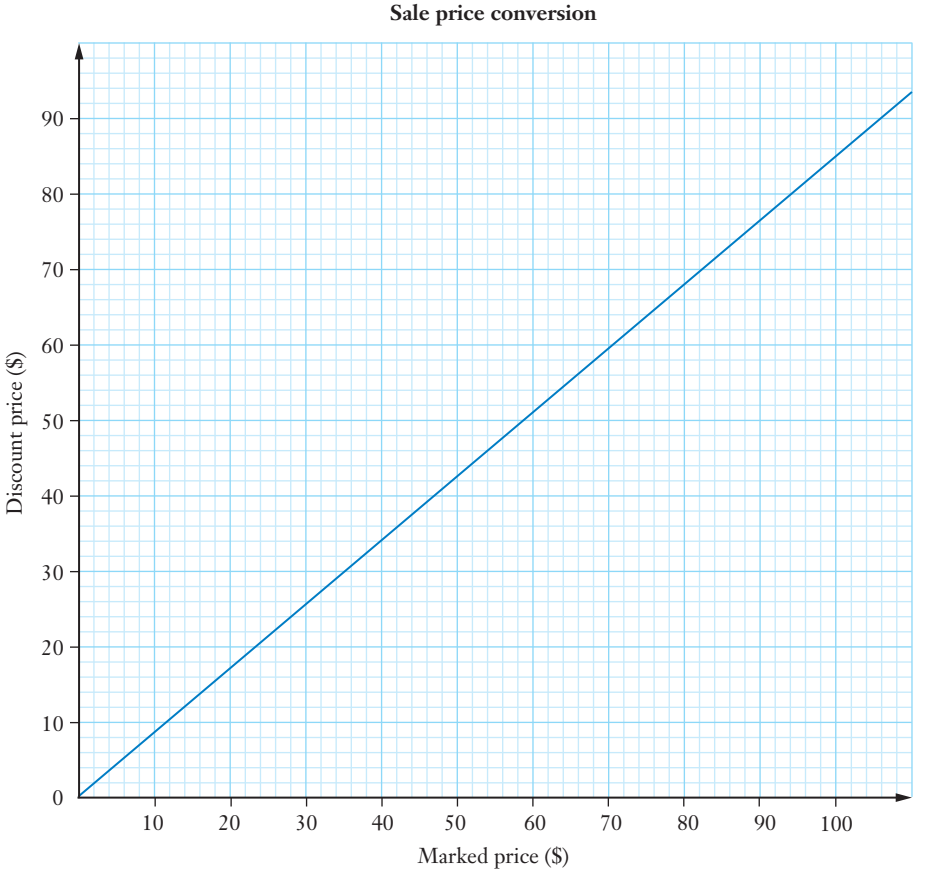
7 This graph converts marked prices to discount prices for a sale at Angry Andy's Bargain Basement.



a Find the discount price of an item with a marked price of:
i \$60 **ii** \$28 **iii** \$82.

b Find the marked price of an item with a discount price of:
i \$25 **ii** \$80 **iii** \$54.

c Calculate the gradient of this graph. What does the gradient value represent?

d Which of the following is the percentage discount given at Angry Andy's Bargain Basement? Select **A**, **B**, **C** or **D**.
A 5% **B** 10% **C** 15% **D** 85%

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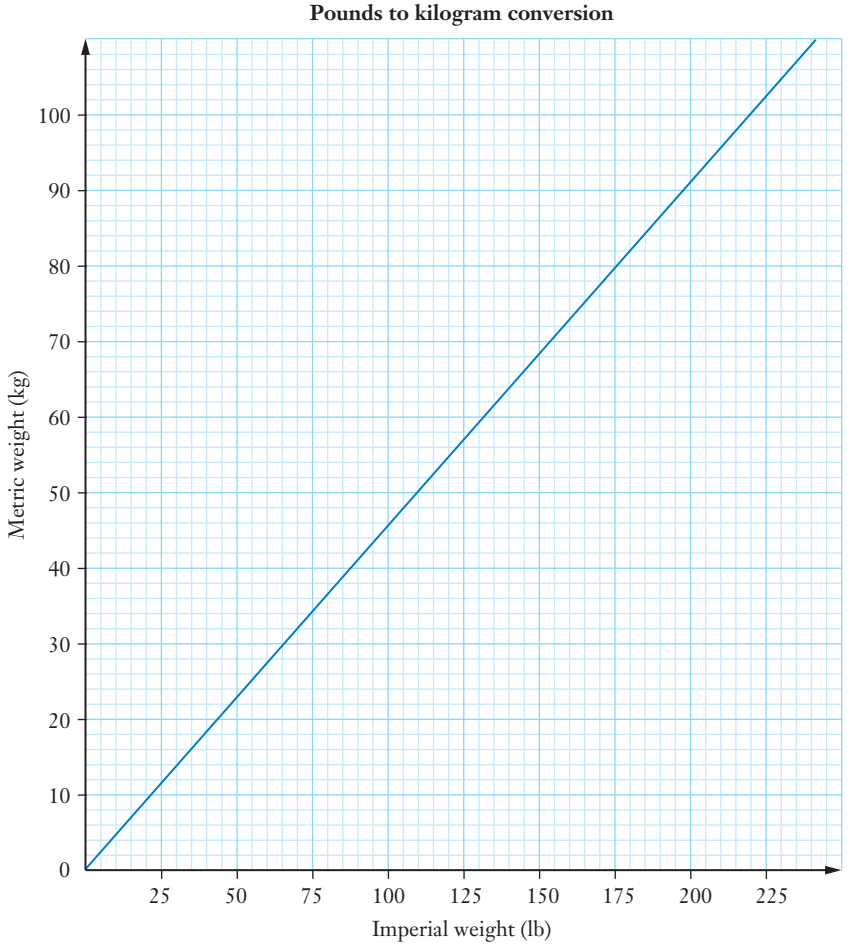
8 The graph below converts between pounds (an imperial unit of mass, with abbreviation 'lb') to kilograms (the metric unit).



a Convert each of these masses to kilograms.

i 90 lb **ii** 135 lb **iii** 160 lb

b Convert each of these masses to pounds.

i 40 kg **ii** 64 kg **iii** 90 kg

c What is the conversion rate in kg/lb?

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SAMPLE HSC PROBLEM

Mr and Mrs Dillon give their children weekly pocket money according to a function based on the age of the child, as shown in the table below.

Age, n years	7	9	10	13	15	18
Pocket money, \$$P$	4	12	16	28	36	48

a Find the formula for P in terms of n.

b What is the gradient for this function and what does it stand for?

c What is the pocket money for a child aged 16?

d What is the age of the child who receives \$20 per week?

e According to Mr and Mrs Dillon, this linear model holds true only for values of n from 7 to 18. Why do you think the formula is not valid for values of n:

i below 7? **ii** above 18?

Study tip

Attacking your weak areas

Most of your study time should be spent on attacking your weak areas to fill in any gaps in your Maths knowledge. Don't spend too much time on work you already know well, unless you need a confidence boost! Ask your teacher, use study guides or other textbooks to improve the understanding of your weak areas and to practise Maths skills. Use your topic summaries for *general revision*, but spend longer study periods on overcoming any difficulties in your mastery of the course.

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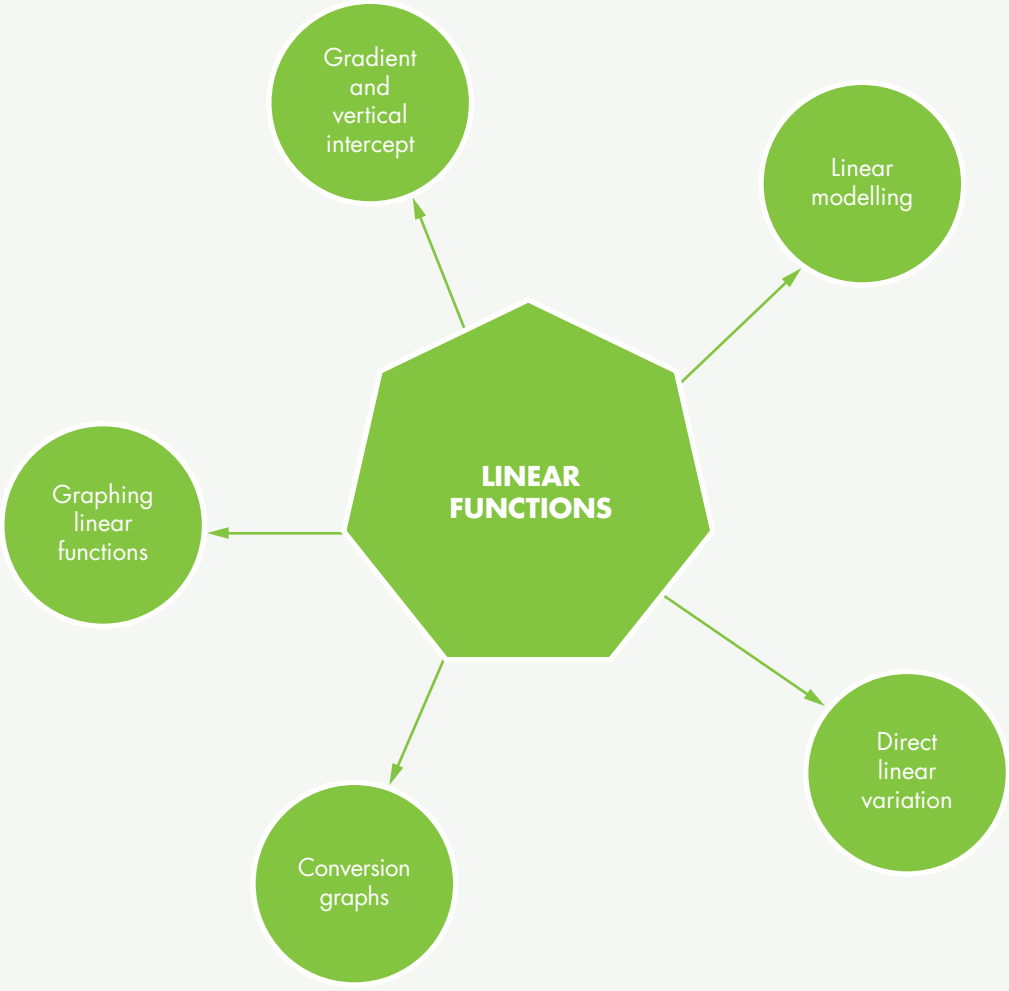
7. CHAPTER SUMMARY

This chapter, Linear functions, introduced the concept of the linear function and its graph and applications. Learn the meanings of the gradient and vertical intercept of a linear function and practise solving problems involving linear modelling, variation and conversion graphs.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map is shown below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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7. TEST YOURSELF

Exercise 7.01

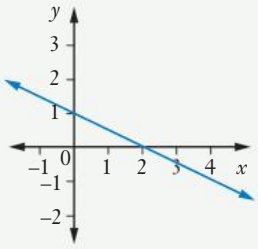
1 Graph each of these linear functions on a number plane.

a $y = -2x + 1$ **b** $y = 4x$

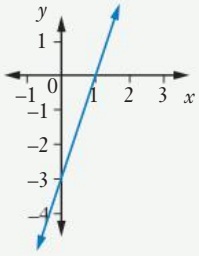
Exercise 7.01

2 Find the equation of each of these lines.

a



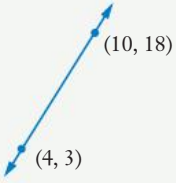
b



Exercise 7.02

3 Find the gradient of each of these lines.

a



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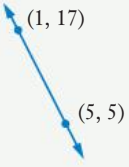

Exercise 7.02

4 Find the gradient of the function for each of these tables of values.

a

x	0	4	6	12
y	12	14	15	18

b

x	2	5	9	11
y	42	30	14	6

Exercise 7.03

5 The length, L inches, of a shoe has a linear relationship with the size, S, of the shoe.

Shoe size, S	2	5	7	8	12
Length, L (inches)	9	10	$10\frac{2}{3}$	11	$12\frac{1}{3}$

a Which is the independent variable?

b Graph this linear relationship.

c What is the gradient, and what does this represent?

d How long is a shoe of size 0? How is this shown on the graph?

e What is the formula for L in terms of S?

f What is the length of a shoe of size $7\frac{1}{2}$?

g What size is a shoe of length 13 inches?

Exercise 7.04

6 The distance travelled by a car is directly proportional to the number of rotations of its tyres. If 950 metres are travelled after 540 rotations, calculate how much distance, to the nearest kilometre, is covered after 10 000 rotations.

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Exercise 7.05

7 This currency conversion graph converts between Thai baht (the currency of Thailand) and Australian dollars (A$).

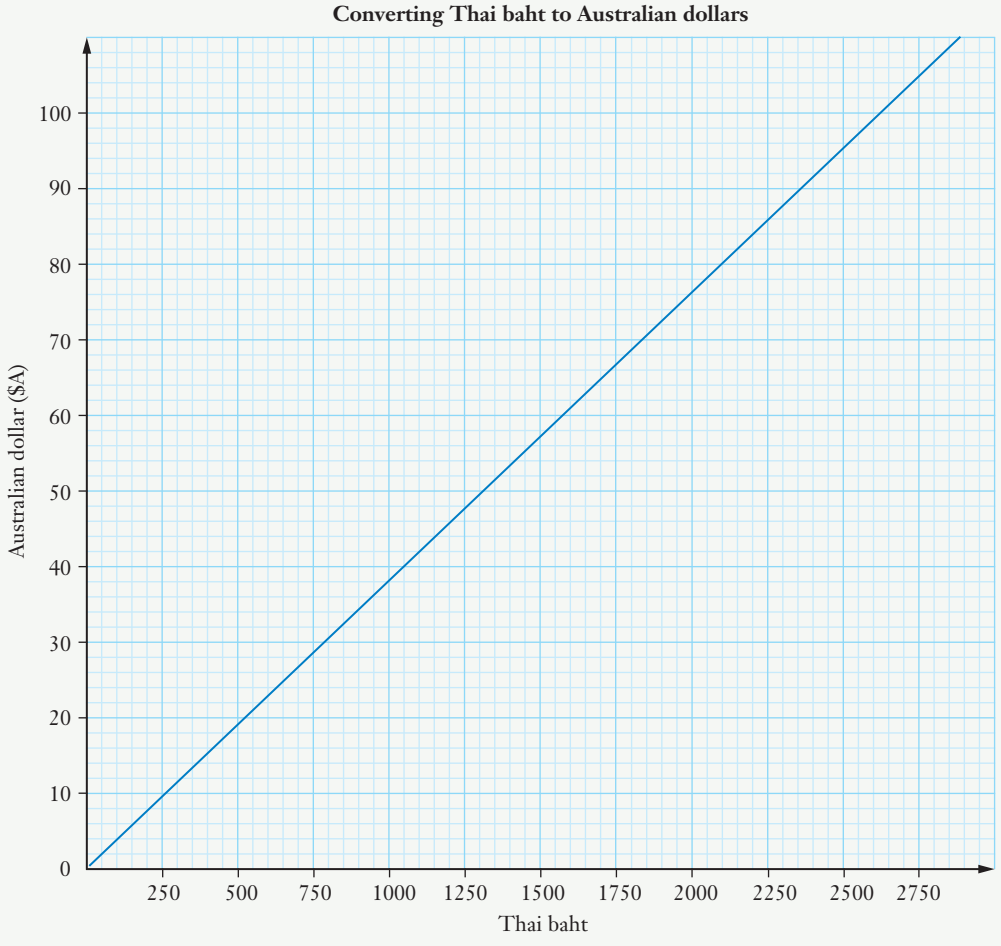
a Convert each of these Thai prices to Australian dollars (to the nearest dollar).

i 800 baht **ii** 2450 baht

b Convert each of these Australian prices to Thai baht (to the nearest 50 baht).

i A$65 **ii** A$82

c Calculate the gradient of this graph, correct to two decimal places. What does this value represent?



Qz Chapter quiz

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8.

INTEREST AND DEPRECIATION

The value of an investment increases over time as a result of interest, simple or compound, being added to it. The value of assets such as office equipment, machinery and cars decrease over time due to usage and age and the price of goods in shops may increase and decrease. These all involve calculating percentage increases and decreases and in some cases lead to formulas to calculate the final value.

CHAPTER OUTLINE

F1.1 8.01 Percentage increase and decrease
F1.1 8.02 Simple interest
F1.1 8.03 Simple interest graphs
F1.1 8.04 Compound interest
F1.1 8.05 Straight-line depreciation

IN THIS CHAPTER YOU WILL:

- calculate repeated percentage changes, for example, an increase of 15% followed by a decrease of 8%, and examine the overall percentage change
- calculate simple interest using $I = Prn$, where P is the principal, r is the interest rate per period and n is the number of periods
- use tables of values for fixed values of P, and hence draw and describe graphs of I against n for different values of r
- calculate and graph compound interest using repeated application of simple interest
- calculate compound interest using $A = P(1 + r)^n$, where A is the final amount, P is the initial principal, n is the number of compounding periods and r is the interest rate per compounding period
- calculate the straight-line depreciation of an asset

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TERMINOLOGY

compounded
compound interest
compounding period
depreciation
final amount
flat rate interest
future value
inflation
present value
principal
quarterly
salvage value
simple interest
straight-line method of depreciation

Assignment 8

SkillCheck

1 Express each percentage as a decimal.

a 2.5% **b** 84% **c** 106% **d** 18.75%

2 Copy and complete these conversions.

a 10% p.a. = _____ % per quarter

b 7.5% p.a. = _____ % per half year

c 5.4% p.a. = _____ % per month

3 Evaluate, correct to two decimal places:

a $2500 \times (1.015)^4$ **b** $11\,640 \times (1.003)^{15}$ **c** $4460 \times (1.0048)^{36}$

4 Find each amount.

a 3% of \$25 780 **b** 2.5% of \$85 400 **c** 11% of \$13 400

d 4% of \$8672 **e** 6.5% of \$56 125 **f** 0.85% of \$15 600

5 **a** Increase \$28.50 by 3%.

b Decrease \$2500 by 5%.

c Increase \$88 by 10%.

6 If $S = V - Dn$, find:

a S if $V = 75\,000$, $D = 4300$, $n = 6$

b V if $S = 0$, $D = 5200$, $n = 8$

c n if $S = 12\,000$, $V = 52\,000$, $D = 4800$.

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8.01 Percentage increase and decrease

Percentage increase and decrease

EXAMPLE 1

a A TV priced at $1200 is discounted by 15%. What is the sale price?

b Saad's annual salary of $92 000 increases by 2.5%. What is his new salary?

Solution

a Decreasing an amount by 15% is the same as finding 85% of that amount, because $100\% - 15\% = 85\%$

$85\% \times \$1200 = 0.85 \times \$1200$

$= \$1020$

The sale price of the TV is $1020.

b Increasing an amount by 2.5% is the same as finding 102.5% of it, because $100\% + 2.5\% = 102.5\%$

$102.5\% \times \$92\,000 = 1.025 \times \$92\,000$

$= \$94\,300$

Saad's new salary is $94 300.

Repeated percentage change

A retail store increases its prices by 10% at the start of summer but has a '10%' clearance sale at the end of summer. Do the increase and decrease cancel each other out, or will there be a combined percentage change?

Suppose the original price was $100.

Increase $100 by 10%: $110\% \times \$100 = 1.1 \times \$100 = \$110$

Decrease $110 by 10%: $90\% \times \$110 = 0.9 \times \$110 = \$99$

The overall change is a decrease of $1, or 1%.

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EXAMPLE 2

Best Goodes are having a '20% off' summer sale for one month. If paying by cash, customers will receive a further 10% discount on the sale price. What percentage discount is equivalent to the combined discounts if paying by cash?

Solution

Method 1: Using numbers

Let the original price be \$100

Decrease \$100 by 20%:

$80\% \times \$100 = 0.8 \times \$100$

$= \$80$

Decease \$80 by 10%:

$90\% \times \$80 = 0.9 \times \$80$

$= \$72$

Combined discount = \$100 – \$72 = \$28

Overall percentage discount = 28%

Method 2: Using percentages only

'Decrease by 20%' means 80%

'Decrease by 10%' means 90%

Overall change $= 0.8 \times 0.9$

$= 0.72$

$= 72\%$

Overall percentage discount = 100% – 72% = 28%

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EXAMPLE 3

The price of petrol decreased by 8% at the beginning of the week but increased by 17% towards the end of the week. What percentage change is equivalent to a decrease of 8% followed by an increase of 17%?

Solution

Method 1: Using numbers

Decrease \$100 by 8%:

$$92\% \times \$100 = 0.92 \times \$100 = \$92$$

Increase \$92 by 17%

$$117\% \times \$92 = 1.17 \times \$92 = \$107.64$$

Combined change = \$107.64 – \$100 = \$7.64 increase

Overall percentage change = 7.64%

Method 2: Using percentages only

'Decrease by 8%' means 92%.

'Increase by 17%' means 117%.

Overall change $= 0.92 \times 1.17$

$= 1.0764$

$= 107.64\%$

Overall percentage change = 107.64% – 100% = 7.64%

Exercise 8.01 Percentage increase and decrease

Example 1

1 A school purchased 200 calculators at \$23 and was given a 12% discount. Find the total paid for the calculators.

2 If Australia's population at the start of the year was 24 287 734, what will be the population at the end of the year if the expected growth for the year is 1.4%. Answer to the nearest whole number.

3 Adash's salary of \$82 450 p.a. increased according to the increase in the Consumer Price Index (CPI) which was 2.3%. What is his new salary?

4 The value of Lisa's computer depreciated (decreased) by 33% this year. If its value was \$2199, calculate its current value.

5 A cafe adds an 11% surcharge on Sundays. Calculate the total to be paid if the bill for lunch for two on a Sunday was \$78.40.

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6 The price of a washing machine is $864 plus 10 % GST. What is the final price of the washing machine?

7 The price of a TV, including 10% GST, is $3799. Calculate the amount of the GST.

Example 2

8 Homegoods Discounts are offering a 15% discount on all items in their store and if payment is by cash, a further discount of 10% will be given. How much will Simon pay for a TV marked at $3299 if he makes a cash payment?

Example 3

9 The Toy Factory stores increased the price of all toys by 15% at the beginning of the financial year in July. The following January they discounted all toys by 8%. For a drone originally priced at $89, calculate:

a the price after the 15%

b the sale price after discount

c the overall change in price and the percentage change on the original price.

10 Which of the following overall percentage increases is equivalent to a percentage increase of 9% followed by another increase of 4%? Select **A**, **B**, **C** or **D**.

A 13% **B** 1.3% **C** 3.36% **D** 13.36%

11 Naveen's investment of $20 000 increased by 5.2%, 4.5% and 4.8% over three years.

a What is the value of Naveen's investment at the end of the three years?

b What was the overall percentage change in Naveen's investment, correct to one decimal place?

12 What percentage change is equivalent to:

a an increase of 20%, followed by a decrease of 20%?

b a decrease of 20%, followed by an increase of 20%?

c an increase of 8%, followed by another increase of 8%?

d an increase of 10%, followed by a decrease of 15%?

e a decrease of 4%, followed by a decrease of 6%?

f a decrease of 5% followed by an increase of 2%.

13 An iceberg with a volume of 254 m^3 has a mass of 234 t. It lost 3% of its mass in the first year, 4% of its mass in the second year and 5% of its mass in the third year.

a What was the mass of the iceberg after 3 years?

b What is the percentage decrease (correct to one decimal place) of the iceberg from its original mass?

WS
Simple interest

8.02 Simple interest

Interest is money earned on an investment with a bank, credit union or other financial institution. **Simple interest** (or **flat rate interest**) is interest calculated as a percentage of the **principal** (the amount of money invested).

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Simple interest formula

The simple interest formula is: $I = Prn$

where I = interest

P = principal or initial amount

r = interest rate per period, expressed as a decimal

n = number of periods

EXAMPLE 4

Julia invested her inheritance of \$33 250 at 4.36% p.a. for 20 months.

p.a. means *per annum* (or 'per year').

Calculate the simple interest Julia earned.

Solution

$P = \$33\ 250$, $r = \dfrac{0.0436}{12}$ per month, $n = 20$ months

r and n are expressed in the same units (months).

$I = Prn$

$= \$33\ 250 \times \dfrac{0.0436}{12} \times 20$

$= \$2416.1666\ldots$

$\approx \$2416.16$

Interest earned is always rounded down to the nearest cent.

EXAMPLE 5

Carmel invested \$3600 in a savings account. After $4\frac{1}{2}$ years, her investment grew to \$4185.12. What simple interest rate, as a percentage to two decimal places, did Carmel receive on her investment?

Solution

Interest, $I = \$4185.12 - \$3600$ Final amount – principal

$= \$585.12$

$P = \$3600$ $n = 4.5$ years

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$$I = Prn$$

$\$585.12 = \$3600 \times r \times 4.5$

$\$585.12 = \$16\,200r$

$$r = \frac{\$585.12}{\$16\,200}$$

$= 0.036\,11\ldots$

$= 3.611\ldots\%$

$\approx 3.61\%$

Simple interest rate is 3.61%.

Exercise 8.02 Simple interest

Example 4

1 Which of the following is the simple interest earned when \$7400 is invested at 5% p.a. for 4 years? Select **A**, **B**, **C** or **D**.

A \$148 **B** \$14.80 **C** \$1480 **D** \$14 800

2 Calculate the simple interest earned from each investment.

a \$2136 invested at 6% p.a. for $2\frac{1}{2}$ years

b \$16 200 invested at 7.65% p.a. for 3 years

c \$12 874 invested at 0.61% per month for 6 months

d \$4510 invested at 0.0301% per day for 31 days

e \$20 016 invested at $7\frac{1}{2}$ % p.a. for 5 months

f \$8250 invested at 10.5% p.a. for 240 days

3 Kalena earned \$262.44 in simple interest from investing an amount for 3 years at 5.4% p.a. What was the amount Kalena invested?

Example 5

4 Alan earned \$5409.25 simple interest from an investment of \$9835 over 5 years. What was the interest rate per annum?

5 For how long must a principal of \$15 750 be invested at 9.8% p.a. simple interest for it to grow to \$18 837?

6 Zoran earned \$675 interest from \$7500 invested for 9 months. What was the interest rate per annum?

7 What principal would earn \$3729.60 in interest if invested for 3 years at 16.8% p.a.?

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8 For how long must a principal of $10 460 be invested at 5.3% p.a. for it to earn $1247.36 simple interest? Select **A**, **B**, **C** or **D**.

A 2 years **B** 2.2 years **C** 2.25 years **D** 2.3 years

9 OzExpress Credit Union has the term deposit accounts shown in the table, in which the principal must be invested for a fixed period. Calculate the simple interest earned on:

a $6300 for 3 years

b $13 750 for 8 months

c $7800 for 5 years

d $14 240 for 2 months.

Term	Interest rate
1–6 months	8.75% p.a.
7–11 months	9.25% p.a.
1–3 years	10.5% p.a.
4–5 years	12% p.a.
Minimum investment: $1000	

10 Kylie earned $80.58 interest from an investment of $2530 over 91 days. What was the interest rate per day, correct to two significant figures?

11 For how long would $4720 need to be invested at 0.67% per month to earn $474.36 in simple interest?

12 An amount of $9020 was invested for 2 years with simple interest and grew to $10 751.84. Calculate the monthly interest rate.

INVESTIGATION

INTEREST RATES

Choose a bank and collect information about the different types of accounts available, and the interest rates and features of each.

Examples: savings account, investment account, fixed term deposit, cheque account, Christmas club account and young saver account.

a Which type of account would you choose for saving your money?

b Compare and contrast the different types of account and suggest the type of person each account would suit.

8.03 Simple interest graphs

Simple interest is represented by the linear function $I = Prn$. If the principal, P, and the interest rate, r, are known, then the interest, I, is the dependent variable and the number of periods, n is the independent variable.

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EXAMPLE 6

Rachel invests $20 000 in an account that earns 6% p.a. simple interest.

a Construct a graph that shows the simple interest, I, earned in dollars for n years, for values of n from 0 to 8.

b What does the gradient of this graph represent?

c Use the graph to estimate:

i the simple interest earned after $4\frac{1}{2}$ years

ii when $8000 in interest will be earned.

Solution

a $I = Prn$ where $P = 20\,000$ and $r = 0.06$

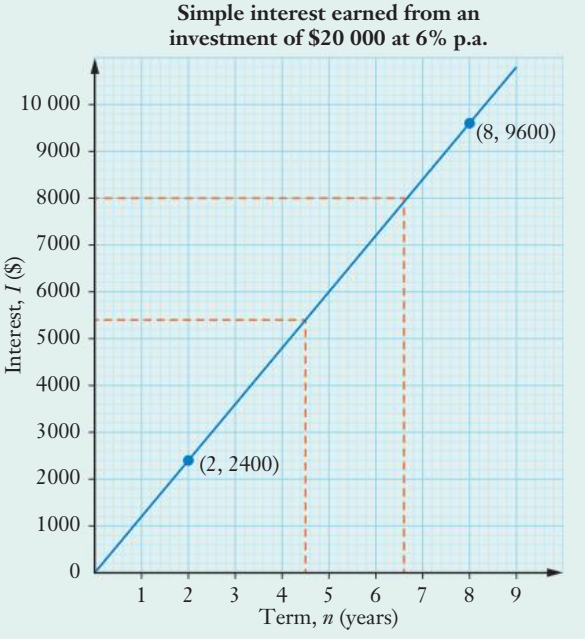
$\therefore I = 20\,000 \times 0.06 \times n$

$I = 1200n$

This is a linear function with gradient 1200 and vertical intercept 0.

Construct a table of values to graph the line:

No. of years, n	0	2	8
Interest, I ($)	0	2400	9600



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b The gradient, 1200, represents the simple interest earned per year.

c **i** \$5400 interest — Reading from the graph at $n = 4\frac{1}{2}$

ii 6.6 years — Reading from the graph at $I = 8000$

Exercise 8.03 Simple interest graphs

Questions **1** to **3** refer to the simple interest graph from Example **6**.

Example 6

1 According to the graph, what is the interest earned on an investment of \$20 000 over 4 years at 6% p.a.? Select **A**, **B**, **C** or **D**.

A \$3150 **B** \$3300 **C** \$4800 **D** \$4900

2 Use the graph to estimate the simple interest earned when \$20 000 is invested at 6% p.a. for:

a 9 years **b** $2\frac{1}{2}$ years **c** 7.6 years.

3 If \$20 000 is invested at 6% p.a., how long will it take to earn simple interest of:

a \$10 000? **b** \$4200? **c** \$8500?

4 Graph on the same axes the simple interest, I, earned for n years, for values of n from 0 to 10, when \$4000 is invested in an account earning:

a 4% p.a. **b** 12% p.a.

5 **a** What is the gradient of the line you drew in Question **4b** and what does it represent?

b Use your graphs from Question **4** to estimate the simple interest earned from an investment of \$4000:

i over 6 years at 4% p.a. **ii** over 8 years at 12% p.a.

c How do the two different interest rates appear on the graphs?

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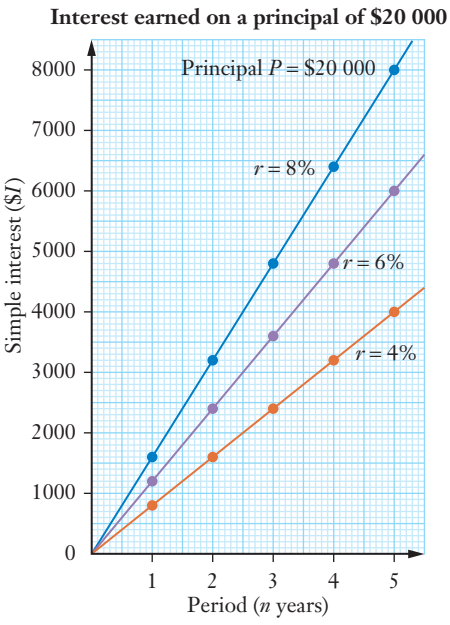
6 This graph shows the simple interest earned on a principal of \$20 000 for up to 5 years at three different interest rates per annum.



According to this graph, how long will it take a principal of \$20 000 to earn \$2400 if the interest rate is 8% p.a.? Select the correct answer **A**, **B**, **C** or **D**.

A $1\frac{1}{2}$ years **B** 2 years

C 2 years 7 months **D** 3 years

7 Use the graph from Question **6** to estimate:

a the interest earned at 6% p.a. after $3\frac{1}{2}$ years

b the interest rate when \$3000 interest is earned after $2\frac{1}{2}$ years

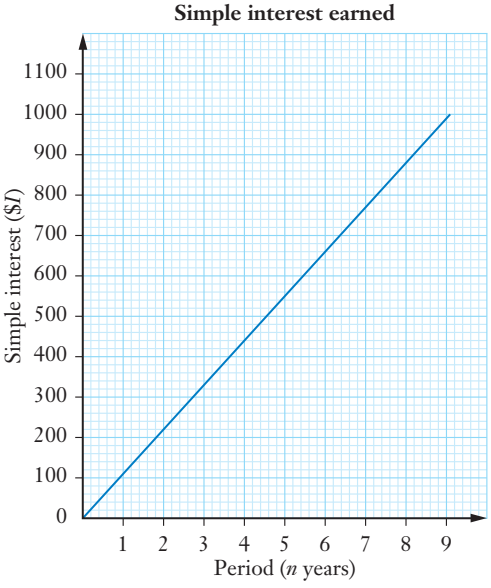
c the number of months it takes to earn \$4500 interest at 6% p.a.

d the time it takes to earn \$7400 in interest at 8% p.a.

e the interest earned after 21 months at 8% p.a.

f the interest rate if \$1800 interest is earned after 27 months.

8 This graph shows the simple interest earned on a principal of \$2000 at $5\frac{1}{2}$% p.a.



a Copy the graph shown and also draw the simple interest graph on:

i a principal of \$2000 at 8% p.a.

ii a principal of \$2000 at 4.6% p.a.

b How much more interest is earned over 8% p.a. than at 4.6% p.a. over 5 years?

c Use your graph to determine how long it takes to earn \$700 at:

i 8% p.a. **ii** 5.5% p.a.

9 The simple interest graph is a linear graph passing through the origin. Explain why the vertical intercept of this graph is 0.

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8.04 Compound interest

Most savings accounts pay **compound interest**, rather than simple interest. Compound interest is interest that is added to the principal and reinvested **(compounded)**. The principal plus interest becomes the new principal on which interest is calculated. In other words, we earn 'interest on our interest' as well as on the original principal.

Compound interest

Simple and compound interest

Compound interest

EXAMPLE 7

A principal of \$3000 is invested at 6% p.a., compounded yearly. Let A_n be the amount of the investment after n years.

a Find A_3, the amount of the investment at the end of 3 years.

b Calculate the compound interest earned after 3 years.

c Find A_5, the amount of the investment after 5 years.

d Find a formula for A_n, the amount of the investment after n years.

Solution

a At the end of the 1st year:

$A_1 = \$3000 + 6\% \times \$3000$ Principal + interest

$= \$3000 \times 1.06$ Increasing \$3000 by 6%

At the end of the 2nd year:

$A_2 = A_1 + 6\% \times A_1$ Increasing A_1 by another 6%

$= A_1 \times 1.06$

$= (\$3000 \times 1.06) \times 1.06$

$= \$3000 \times 1.06^2$

At the end of the 3rd year:

$A_3 = A_2 + 6\% \times A_2$ This is the same as increasing A_2 by another 6%

$= A_2 \times 1.06$

$= (\$3000 \times 1.06^2) \times 1.06$

$= \$3000 \times 1.06^3$

$= \$3573.048$

$\approx \$3573.04$ Rounding down to the nearest cent

b Compound interest earned = final amount – initial principal

$= \$3573.04 - \$3000$

$= \$573.04$

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c $A_5 = \$3000 \times 1.06^5$ Increasing \$3000 by 6% five times

$= \$4014.676\,733$

$\approx \$4014.67$

d Increasing \$3000 by 6% n times:

$A_n = 3000(1.06)^n$

Compound interest

Compound interest formula

The compound interest formula is $A = P(1 + r)^n$.

where A = **final amount**

P = principal or initial amount

r = interest rate per compounding period, expressed as a decimal

n = number of **compounding periods**

The compound interest (I) earned can be found from the formula $I = A - P$.

In the world of finance, the compound interest formula $A = P(1 + r)^n$ is written as

$FV = PV(1 + r)^n$ where FV is called **future value** and PV is called **present value**.

EXAMPLE 8

If \$12 500 is invested at 4.8% p.a. with the interest compounded monthly, calculate the final value and total interest earned over 2 years.

Solution

Because interest is compounded monthly, r and n must be expressed in months:

$P = \$12\,500$ $r = \frac{0.048}{12} = 0.004$ per month $n = 2 \times 12 = 24$ months

Final value $= \$12\,500(1 + 0.004)^{24}$

$= \$12\,500(1.004)^{24}$

$= \$13\,756.8537\ldots$

$\approx \$13\,756.85$

Compound interest $= \$13\,756.85 - \$12\,500$

$= \$1256.85$

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EXAMPLE 9

What present value must be invested at 4.5% p.a. for 8 years so that it grows to $10 000?

Solution

$A = \$10\,000 \qquad r = 0.045 \qquad n = 8$

$A = \mathrm{P}(1 + r)^n$

$\$10\,000 = P(1 + 0.045)^8$

$= P(1.045)^8$

$P = \dfrac{\$10\,000}{(1.045)^8}$

$= \$7031.8512\ldots$ Round the present value up to give the required final amount

$= \$7031.86$

A present value of $7031.86 is needed.

EXAMPLE 10

Use the table below to calculate:

a the final balance when $2000 is invested at 8% p.a. compounded annually for 6 years

b the final balance when $14 000 is invested for 7 months at 2% per month

c the compound interest when $9600 is invested at 10% p.a. for 3 years compounded half-yearly.

Compounded values of $1			
	Interest rate per period		
Periods	2%	5%	8%
1	1.0200	1.0500	1.0800
2	1.0404	1.1025	1.1664
3	1.0612	1.1576	1.2597
4	1.0824	1.2155	1.3605
5	1.1041	1.2763	1.4693
6	1.1262	1.3401	1.5869
7	1.1487	1.4071	1.7138

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Solution

a For a principal of \$1, when $r = 8\%$ and $n = 6$, $A = \$1.5869$ (from the table).

So, for a principal of \$2000, $A = 2000 \times \$1.5869 = \$3173.80$

b For a principal of \$1, when $r = 2\%$ and $n = 7$, $A = \$1.1487$.

So, for a principal of \$14 000, $A = 14\,000 \times \$1.1487 = \$16\,081.80$.

c For a principal of \$1, when $r = \frac{10\%}{2} = 5\%$ per half year and $n = 3 \times 2 = 6$ half years, $A = \$1.3401$.

So, for \$9600, $A = 9600 \times \$1.3401 = \$12\,864.96$.

Compound interest = \$12 864.96 – \$9600 = \$3264.96.

Exercise 8.04 Compound interest

Example 7

1 A principal of \$8000 is invested at 5% p.a., compounded annually.

- **a** Find the amount of the investment at the end of 3 years.
- **b** Calculate the compound interest after 3 years.
- **c** Find the amount of the investment after 5 years.
- **d** Find a formula for the amount after n years.

2 Calculate the final amount of each of the following investments.

- **a** \$7400 invested at 5% p.a. compound interest for 4 years
- **b** \$2840 invested at 6.5% p.a. compound interest for 5 years
- **c** \$4500 invested at 4.9% p.a. compound interest for 2 years
- **d** \$17 000 invested at 0.5% per month compound interest for 10 months
- **e** \$9250 invested at 0.82% per month compound interest for 6 months
- **f** \$9000 invested at 8.4% p.a. for 8 months, compounded monthly

3 An supermarket manager's salary increases with the annual **inflation** rate of 2.7% p.a.

Calculate her salary in three years if it is currently \$76 040 (to the nearest dollar).

Example 8

4 Calculate the final amount of each of the following investments and, hence, the amount of compound interest earned.

- **a** \$12 900 invested at 10.8% p.a. for 1 year, compounded monthly
- **b** \$13 800 invested at 7.5% p.a. for 2 years, compounded half-yearly
- **c** \$13 800 invested at 7.5% p.a. for 2 years, compounded **quarterly**
- **d** \$6920 invested at 9% p.a. for 240 days, compounded daily

1 year = 365 days

One quarter = 3 months

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5 Paul wants to invest some money so that it will grow to $24 000 in 5 years' time, when he will travel through Europe. If the interest rate is 5.5% p.a., what present value should Paul invest, to the nearest cent?

Example 9

6 A sum of $8500 is invested at 7% p.a. for 5 years.

a Calculate the total interest earned if the interest is:

i calculated at a flat rate (simple interest)

ii compounded yearly.

b **i** Why does one type of interest result in a greater amount than the other?

ii By how much is it greater?

7 A principal of $21 000 is invested at 5% p.a. Calculate the future value of the investment after 2 years if the interest is compounded:

a yearly **b** half-yearly **c** quarterly

d monthly **e** daily.

8 Judging from your results for Question **7**, what happens to the amount of interest earned as the frequency of compounding increases? Why?

9 Zara has $4000 in an account earning 4% p.a. interest, compounded yearly. By guessing and checking, determine how long it will take Zara to double her money. Give your answer to the nearest year.

10 How long, to the nearest year, will an investment of $2400 take to grow to $3265 at 8% p.a. interest, compounded yearly?

11 After 8 years, the value of Corrina's investment grew to $6260.14. What was the initial amount of Corrina's investment if the interest rate was 8.75% p.a.?

12 Determine the sum to be deposited if $10 000 is required in 6 years time and terms of 3.75% p.a. (compounded quarterly) are available.

13 A principal of $10 000 is to be invested for 3 years. Which of the following is the best investment option? Select **A**, **B**, **C** or **D**.

A 6% p.a. simple interest **B** 5.9% p.a. compounded annually

C 5.85% p.a. compounded half-yearly **D** 5% p.a. compounded monthly

14 Jack won $1 000 000 in the lottery. If he invests all his winnings in a term deposit earning 7.4% p.a. interest compounded annually, how many years will it take for him to double his money?

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Example 10

15 Use the table to answer these questions.

Compounded values of $1				
	Interest rate per period			
Periods	1%	2%	5%	8%
1	1.0100	1.0200	1.0500	1.0800
2	1.0201	1.0404	1.1025	1.1664
3	1.0303	1.0612	1.1576	1.2597
4	1.0406	1.0824	1.2155	1.3605
5	1.0510	1.1041	1.2763	1.4693
6	1.0615	1.1262	1.3401	1.5869

a When $3000 is invested for 4 years, compounded annually at 5%, which of the following is the final value? Select **A**, **B**, **C** or **D**.

A $12 160 **B** $3646.50

C $3012.16 **D** $3828

b If the final value of an investment is $1469.30 after 5 years at 8% p.a., which of the following is the initial principal? Select **A**, **B**, **C** or **D**.

A $1000 **B** $1 **C** $100 **D** $10

c Calculate the final value for each of these investments.

i $16 000 compounded annually at 2% p.a. for 3 years

ii $3800 at 8% p.a. for 6 years compounded annually

iii $4200 at 12% p.a. compounded monthly for 5 months

d Calculate the compound interest when $32 000 is invested at 4% p.a. for 3 years compounded half-yearly.

WS Simple vs compound interest: Spreadsheet

WS Compounding periods: Spreadsheet

TECHNOLOGY

Compound interest as repeated simple interest

1 Open a new spreadsheet and enter the data as shown below for an investment of $5000 at 8% p.a. compounded annually. Enter **1** in cell A6 to show Year 1 and enter **=B1** in cell B6, which is the investment of $5000.

	A	B	C	D	E
1	Investment	$5,000			
2	Annual interest rate	8%			
3	Compounding periods per year	1			
4					
5	Compounding period	Principal	Interest	Value of investment	Interest earned
6	1	$5,000			
7					
8					

2 In cell C6, enter **=B6*($B$2/$B$3)** to calculate the interest for Year 1 (using $I = P \times r \times 1$).

3 In D6, enter **=C6+B6** to calculate principal + interest.

4 In E6, enter **=C6** to show the total compound interest earned after Year 1.

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5 In A7, enter **=A6+1** to show Year 2.

6 In B7, enter **=D6** to show the new principal for Year 2.

7 Fill down from **A7** and **B7** down to row 30, the fill down from **C6** and **D6** down to row 30.

8 In E7, enter **=E6+C7** to calculate the total compound interest earned after Year 2, then fill down to row 30 to calculate the total compound interest earned after each year.

	A	B	C	D	E
1	Investment	$5,000			
2	Annual interest rate	8%			
3	Compounding periods per year	1			
4					
5	Compounding period	Principal	Interest	Value of investment	Interest earned
6	1	$5,000.00	$400.00	$5,400.00	$400.00
7	2	$5,400.00	$432.00	$5,832.00	$832.00
8	3	$5,832.00	$466.56	$6,298.56	$1,298.56
9	4	$5,298.56	$503.88	$6,802.44	$1,802.44

26	21	$23,304.79	$1,864.38	$25,169.17	$20,169.17
27	22	$25,169.17	$2,013.53	$27,182.70	$22,182.70
28	23	$27,182.70	$2,174.62	$29,357.32	$24,357.32
29	24	$29,357.32	$2,348.59	$31,705.90	$26,705.90
30	25	$31,705.90	$2,536.47	$34,242.38	$29,242.38

9 Select column D and use **Insert Chart** to draw a line graph to show the value of the investment growing.

10 Change the values for the interest rate (B2) and number of compounding periods per year (C2). What effect do the changes you made have on:

a the compound interest earned? **b** the graph?

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INVESTIGATION

COMPOUND INTEREST GRAPH

Blake invests $20 000 in an account that earns 6% p.a. interest, compounded annually.

1 If you graphed the size of his investment growing over 10 years, what do you think the graph would look like? Give reasons for your answer.

2 Copy and complete this table using the formula $A = P(1 + r)^n$, rounding your values to the nearest whole dollar.

3 Use the table to construct a graph that shows the amount of the investment, A, growing over 10 years.

4 What is the vertical intercept of this graph and what does it represent?

5 Comment on what the shape of the graph shows.

6 Use the graph to estimate:

i the value of the investment after $8\frac{1}{2}$ years

ii when the value of the investment reaches $35 200.

No. of years, n	Amount, A ($)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

WS
Straight-line depreciation formula practice

8.05 Straight-line depreciation

A new vehicle loses around 30%–40% of its value in the first year. Loss in value can occur through age, wear and tear, or simply by becoming outdated when a new model is released.

EXAMPLE 11

a A boat was purchased for $37 800 and decreased in value by 11% in its first year. To the nearest dollar, what was its value after 1 year?

b A new Porsche was purchased for $239 850 and was valued at $175 600 twelve months later. What is the percentage decrease in price, to the nearest unit, in the first year?

Solution

a Purchase price = $37 800

11% decrease in value = 89% of price 100% − 11% = 89%

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Value after 1 year $= 89\% \times \$37\,800$

$= \$33\,642$

b Decrease in value $= \$239\,850 - \$175\,600$

$= \$64\,250$

Percentage decrease $= \dfrac{64\,250}{239\,850} \times 100\%$

$= 26.7875\ldots$

$\approx 27\%$

Depreciation is the loss in value of an asset over time. According to the results of a survey by an insurance company, a new car depreciates around 50% in the first 2 or 3 years of its life, and depreciation accounts for about 37% of the costs of running a small car, and about 49% of the costs of running a large four-wheel-drive.

There are two methods for depreciating an item:

- **Straight-line method of depreciation**, where the item's value decreases by the same amount each period
- **Declining balance method of depreciation**, where the item's value decreases by the same percentage each period.

The formula for the straight-line method of depreciation is a **linear function** (of the form $y = mx + c$) and its graph is a straight line.

Straight-line depreciation formula

$$S = V_0 - Dn$$

where S = **salvage** (current) **value** of the asset after n periods

V_0 = initial value of the asset

D = amount of depreciation per period

n = number of periods.

You will learn more about the declining balance method of depreciation in Year 12.

Straight-line depreciation

EXAMPLE 12

A motorcycle is purchased for $16 000 and depreciates by $2400 each year.

a Use the straight-line depreciation formula to find the value of the motorcycle after 3 years.

b When, to the nearest month, will the motorcycle be worth half its purchase price?

c After how many months, to the nearest month, will the motorcycle have a $0 salvage value?

Solution

a $V_0 = 16\,000, D = 2400, n = 3, S = ?$

$S = V_0 - Dn$

$= 16\,000 - 2400 \times 3$

$= 8800$

The value after 3 years is $8800.

b $V_0 = 16\,000, S = 8000, D = 2400, n = ?$

$S = V_0 - Dn$

$8000 = 16\,000 - 2400 \times n$

$2400n = 16\,000 - 8000$

$2400n = 8000$

$n = \dfrac{8000}{2400}$

$= 3.3333\ldots$ years Or $3\frac{1}{3}$ years

$=$ 3 years and 4 months

The motorcycle will be worth half its purchase price after 3 years and 4 months.

c $S = V_0 - Dn$

$0 = 16\,000 - 2400 \times n$ Salvage value is $0

$2400n = 16\,000$

$n = \dfrac{16\,000}{2400}$

$= 6.666\ldots$ years Or $6\frac{2}{3}$ years

$=$ 6 years and 8 months

The motorcycle will have a $0 salvage value after 6 years and 8 months.

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EXAMPLE 13

Office furniture was purchased for $85 000 and depreciated by $5600 per year. The depreciation schedule shows the accumulated depreciation and value for the first 8 years of its life.

Period (years)	Accumulated depreciation ($)	Value at end of year ($)
0	0	85 000
1	5600	79 400
2	11 200	73 800
3	16 800	68 200
4	22 400	62 600
5	28 000	57 000
6	33 600	51 400
7	39 200	45 800
8	44 800	40 200

$V_0 = 85\,000$
$D = 5600$
$n = 8$

1 Use the table to find:

a by how much the office furniture has depreciated after:

i 3 years **ii** 7 years.

b the salvage value at the end of:

i 3 years **ii** 7 years.

c the furniture's value after 10 years by continuing the table for a further 2 years.

2 Construct a straight-line graph with year on the horizontal and value on the vertical axis.

3 Use the graph to find:

a the value, to the nearest $1000, after $4\frac{1}{2}$ years

b after how many months the value is $38 000.

Solution

1 **a** **i** $16 800 **ii** $39 200

b **i** $68 200 **ii** $45 800

c Continue the table for two more years as shown:

9	50 400	34 600
10	56 000	29 000

Value after 10 years is $29 000.

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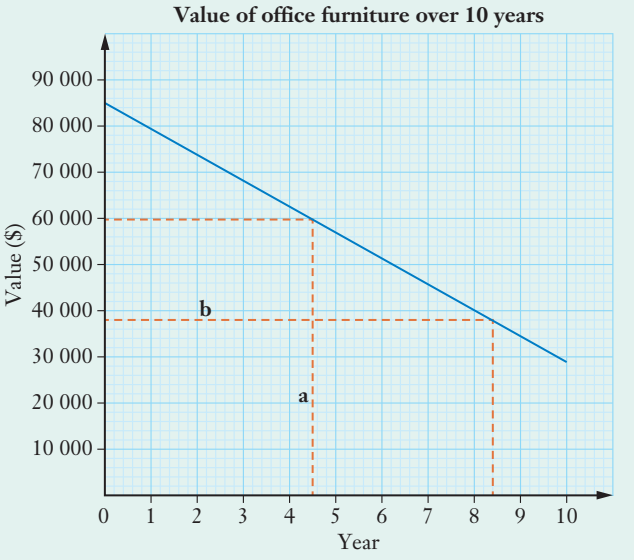
2



3 **a** $60 000 **b** 8.4 years = 101 months

Exercise 8.05 Straight-line depreciation

Example 11

1 A supermarket refrigerator was bought for $64 000 and decreased in value by 35% in the first year. What was its value at the end of the first year? Select **A**, **B**, **C** or **D**.

A $41 600 **B** $21 000 **C** $54 400 **D** $19 200

2 A computer purchased for $5860 depreciated by 12% in the first year. Its value at the end of the first year was:

A $703.20 **B** $6446 **C** $5156.80 **D** $5274

Example 12

3 A photocopier was purchased for $11 700. It was expected to last 6 years and be worthless at the end of this time.

a What is the annual depreciation?

b What was the value after:

i 1 year ? **ii** 3 years?

4 A pizza oven was purchased for $32 200 and depreciated to $5000 at the end of 5 years.

a What was the annual depreciation?

b What was the value after:

i 2 years? **ii** 3 years?

5 A technology classroom was purchased for $46 000. After 3 years it depreciated to half its purchase price. What was the value after 6 years?

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6 New machinery was purchased for $180 000. It had an expected life of 15 years with a final scrap value of $5000.

a What is the yearly depreciation?

b What is the value after 5 years?

7 A baker's van cost $78 000 and is estimated to be worth $35 000 after 5 years.

a What is the annual depreciation?

b Complete the table of values:

No. of years, n	0	1	2	3	4	5
Value, $\$V$	78 000					35 000

c Draw a straight-line graph showing the depreciation over 5 years.

d On your graph, what is the vertical intercept and what does it represent?

e Use the graph to find the value of the van after $2\frac{1}{2}$ years.

f After how many years is the value $40 000?

8 This is the depreciation schedule for an electric scooter.

Example 13

Year	Accumulated depreciation ($)	Value at end of period ($S)
0	0	5600
1	850	4750
2	1700	3900
3	2550	3050
4	3400	2200

Use the schedule to find:

a the purchase price

b the depreciation in the:

i second year **ii** in the first two years

c the salvage value at the end of:

i 2 years **ii** 4 years

d the value after 6 years by continuing the schedule for a further 2 years.

9 For the scooter in Question **8** above:

a draw a straight-line graph to represent the depreciation over 6 years.

b what is the vertical intercept and what does it represent?

c when does the value fall below $1000?

d when is the value of the scooter $3500?

e what is the value after 18 months?

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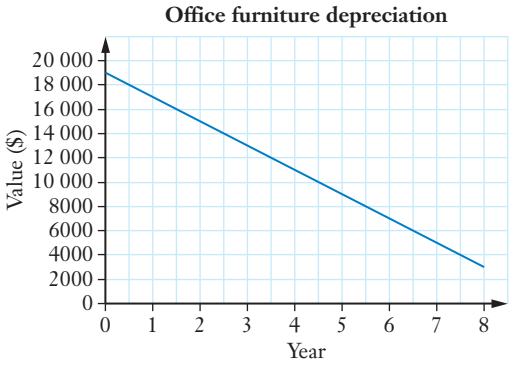
10 The graph on the right shows the depreciation of some office furniture over its useful life.

a What is its purchase price?

b What is the furniture worth after:

i 3 years **ii** 8 years.

c What is the yearly depreciation?



TECHNOLOGY

Straight-line depreciation

1 Complete this depreciation schedule for the first 10 years of the life of the office furniture in Example **13**.

	A	B	C	D	E	F
1	Year	Accumulated depreciation($)	Value at end of year ($)			
2	0	0	=$F$4			
3	=A2+1	=$F$5	=C2–$F$5			
4	=A3+1	=B3+$F$5	=C3–$F$5		V0 =	85000
5					D =	5600
6						
7						

2 Use the chart option to draw the line graph as shown in the Technology section on page 341.

3 Change the purchase price in cell F4 to V0=230000 and the depreciation amount in F5 to D=10050 and the spreadsheet will recalculate cell values. Draw the appropriate line graph.

4 What does the vertical intercept on the graph represent?

5 Compare the table and the graph and give two reasons why:

a the graph is a better mathematical representation of depreciation

b the table is a better mathematical representation of depreciation.

6 Modify your spreadsheet to show that the difference between the simple interest and compound interest earned on a principal of $5000, invested at 4.5% p.a. over 10 years, is $514.85.

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SAMPLE HSC PROBLEM

Brittany invests $15 000 in an account that earns 6.5% p.a. interest.

a Calculate the simple interest that the account will earn over 3 years.

b Calculate the interest the account will earn if interest is compounded annually over 3 years.

c Explain why more interest is earned when compounded annually over the 3 years.

Study tip

Useful exam tips

- Make yourself familiar with the format of the exam: the number of sections and questions, the types of questions (for example, multiple-choice, and short answer), the time allowed, and the number of marks allocated.
- Don't worry if you are nervous. This is normal and will help you perform better. However, being too casual or too anxious can harm your performance.
- Calculate the average amount of time you should spend on each question or section.
- Spend the reading time of the exam browsing through the paper to see what work lies ahead of you.
- Easier questions are usually at the beginning, with harder ones at the end. Do an easy question first to boost your confidence. It will also save time.
- Put a mark next to the harder questions and allow more time for working on them. Leave them if you get stuck and come back to them later.
- Show all working. Even if you get the wrong answer, you may be awarded some marks for correct working.
- Attempt every question. It is better to do most of every question and score some marks than to ignore some questions completely and score nothing for them.
- Don't leave multiple-choice questions unanswered. Even if you guess, there is a chance of being correct. Some students like to leave multiple-choice questions till last so that, if they run out of time, they can make quick guesses. However, some multiple-choice questions can be quite difficult.

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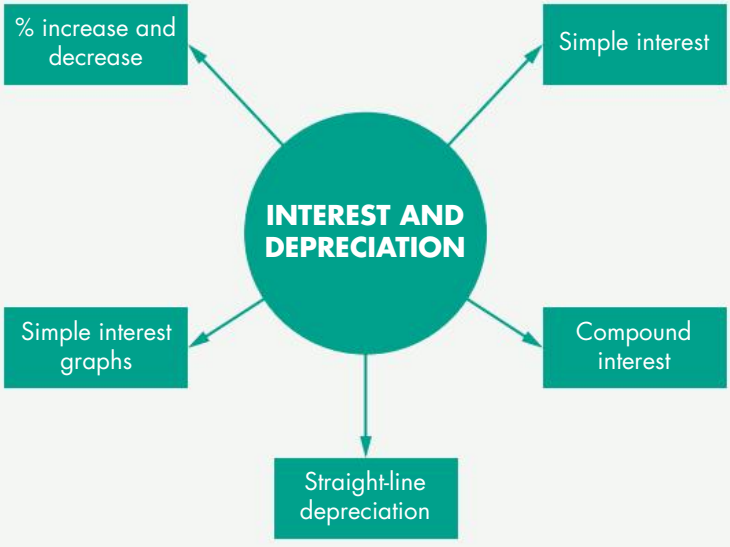
8. CHAPTER SUMMARY

Interest and depreciation crossword

This chapter, Interest and depreciation, examined the mathematics of calculating interest and depreciation. You have applied percentage increase and decrease in various contexts and also calculated simple interest and compound interest in a variety of situations involving investments. Digital technology and graphs have been used to compare simple interest and compound interest graphs for different rates and periods. Depreciation of assets has been calculated using the straight-line method.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map is shown below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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8. TEST YOURSELF

Exercise 8.01

1 Rami, a marketing manager, earns $72 000 per annum. If her salary is increased by 6.4%, calculate her new salary.

2 A shop buys scooters for $145 and then sells them for $260. What is the percentage profit?

3 A mobile phone has a price of $1159 after a 10% GST is added. What was the mobile phone's price before the GST?

4 If a buyer of a $55 000 car received a 10% discount for paying cash and then a further 3% discount in the sale, how much did he pay for the car?

5 Melanie owned $2400 of shares in XYZ Mining. After the stock market crash the shares dropped 18% in value but, in the new year, they gained 5%.

a What was the share price after these two percentage changes?

b What was the overall percentage change?

Exercise 8.02

6 Calculate the simple interest earned from each investment.

a $7200 invested at 6.25% p.a. for 3 years

b $4050 invested at 3% p.a. for 16 months

c $10 300 invested at 0.75% per month for 2 years.

d $12 600 invested at 3.4% p.a. for $3\frac{1}{2}$ years

7 An investment of $8900 earned $2002.50 in simple interest after 5 years. What was the annual interest rate?

8 For how long must a principal of $4000 be invested at 6.6% p.a. for it to earn $2376 in simple interest?

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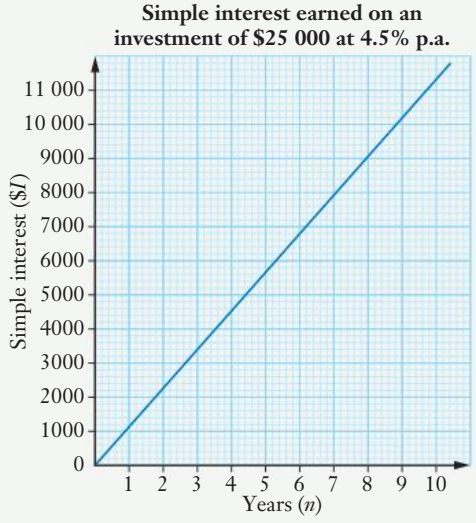
Exercise 8.03

9 The graph below shows the simple interest earned in dollars for n years on an investment of $25 000 at 4.5% p.a.



Use the graph to estimate:

a the simple interest earned after:

i 4 years **ii** $6\frac{1}{2}$ years **iii** 10 years

b when $10 000 in interest will be earned.

10 **a** Construct a graph that shows the simple interest, I, earned in dollars on a principal of $4800 invested at 3% p.a. for n years, for values of n from 0 to 6.

b What is the gradient of this graph and what does it represent?

c Use the graph to estimate when $500 in interest will be earned.

Exercise 8.04

11 For each investment, calculate the final amount and the compound interest earned.

a $7200 at 6.25% p.a. for 3 years

b $4050 at 3% p.a. for 16 months, compounded monthly

c $10 300 at 0.75% per month for 2 years, compounded monthly

12 What principal must be invested at 5.8% p.a., compounded annually, for it to grow to $15 000 in 8 years? (Give your answer to the nearest cent.)

13 A loaf of bread currently costs $3.60. Calculate its cost in 4 years' time if the inflation rate is 2.8% p.a.

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14 Use the table to calculate the final amount and compound interest earned when $9000 is invested at 10% p.a. for 2 years, compounded half-yearly.

Compounded values of $1			
	Interest rate per period		
Periods	1%	5%	10%
1	1.010	1.050	1.100
2	1.020	1.103	1.210
3	1.030	1.158	1.331
4	1.041	1.216	1.461
5	1/051	1.276	1.611

15 Max bought a new tractor for $130 000. He wants to claim its depreciation as a tax deduction. If his tractor depreciated at $13 000 p.a., use the straight-line method of depreciation formula to find:

Exercise 8.05

a the value of the tractor after 8 years

b the depreciation in the 9th year.

16 A new van cost $67 000, including registration and insurance. It depreciates by the same amount each year and after 6 years is expected to be worth $16 000 as a trade-in on another van.

a What is the yearly depreciation of the van?

b Draw up a depreciation schedule showing the value of the van for the first 6 years and the accumulated depreciation.

c What will be the trade-in value of the van after 5 years?

d What is a linear function that can be used to model the depreciation of the van?

Qz Chapter quiz

9.

OWNING A CAR

When buying a car, as well as the initial purchase price (or loan repayments), you need to consider other on-road costs such as insurance, registration and stamp duty. Running costs of a car include fuel consumption, servicing, repairs and tyres. Accidents and speeding fines are also hidden costs of owning a car, with driver distraction causing 44% of road accidents.

CHAPTER OUTLINE

F1.3, S1.1	9.01	Car insurance
F1.3	9.02	Stamp duty
F1.3	9.03	On-road costs
F1.3	9.04	Running costs
F1.3	9.05	Fuel consumption and prices
F1.3	9.06	Choosing the best car

IN THIS CHAPTER YOU WILL:

- identify the different types of car insurance available and the factors that affect insurance premiums
- calculate on-road costs for buying a car, including finance, stamp duty, registration and insurance
- calculate running costs of owning a car, including service and repairs
- identify fuel consumption as a rate and use it to calculate the amount of fuel used on a trip
- consider sustainability when choosing a vehicle to purchase; for example, cost of fuel, recyclable materials and renewable energy sources
- compare the costs of owning different types of car, taking into account economic and sustainability factors

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TERMINOLOGY

comprehensive insurance
Compulsory Third Party (CTP) insurance
excess
fuel consumption
green slip
on-road costs
premium
stamp duty
Third Party Property insurance
vehicle registration

Assignment 9

SkillCheck

1 Copy and complete:

a $148 per month = $..... per year

b $350 per fortnight = $..... per year

c $276 per week = $..... per year

d $2754 per year = $..... per month

e $182 per month = $..... per week [Hint: find amount per year first]

2 A loan repayment is $128 per fortnight. To the nearest cent, how much is this:

a per week? **b** per year? **c** per month?

3 Find these percentages, correct to the nearest cent:

a 3% of $25 680 **b** 2.5% of $64 320 **c** 11% of $12 700

d 4% of $2564 **e** 6.5% of $68 530 **f** 1.9% of $72 350

4 If a car is stolen in Australia every 10 minutes, how many cars are stolen:

a per day? **b** per week? **c** per month? **d** per year?

5 A Range Rover 4WD has a 150 L fuel tank. What is the cost, to the nearest cent, to fill the car with fuel at $1.69 per litre?

6 If the buyer of a $65 000 car received a 10% discount for paying cash then a further 3% discount in the sale, how much did he pay for the car?

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9.01 Car insurance

Investigating car insurance

Car insurance is a financial scheme to protect you in the event of a car accident. In return for paying a regular fee, called the **premium**, the insurance company agrees to pay you a large amount to cover the costs of repair or replacement if you have an accident.

Types of car insurance

There are three main types of car insurance.

- **Compulsory Third Party (CTP) insurance** protects vehicle owners and drivers from legal liability for personal injury or death to any other person ('third party') including drivers, passengers, cyclists and pedestrians. It is also called '**green slip**' insurance because of the colour of the insurance document and it must be obtained before a vehicle can be registered.
- **Third Party Property insurance** is optional and covers damage to other vehicles and property in an accident where you are at fault. It does not cover damage to your vehicle.
- **Comprehensive insurance** is optional and covers damage to all vehicles and property, including your own. It also protects against theft, attempted theft, accidental loss, fire, vandalism or hail damage to your vehicle.

EXAMPLE 1

Mia bought a second-hand utility vehicle ('ute') for her farm. Her comprehensive insurance was $631.08 upfront or $61.75 per month. How much extra would Mia pay per year if she chose to pay her insurance per month?

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Solution

Cost when paid upfront $= \$631.08$

Cost when paid monthly $= \$61.75 \times 12$

$= \$741.00$

Difference $= \$741.00 - \$631.08$

$= \$109.92$

Mia pays an extra $109.92 if she pays per month.

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Insurance premiums and discounts

A motor vehicle insurance **premium** depends on factors such as vehicle type, driver age and driving record, where and how your vehicle is stored (garage, driveway, street parking) and whether the vehicle is financed or fully paid off. Some insurance companies give a **loyalty discount** if you have been with them for several years, a **multi-policy discount** for having more than one type of insurance policy with them, and a **no claim discount** if you have not made a claim for several years.

If you have an accident and it is your fault, an assessor will view the damage and assess the costs involved. You will be asked to complete a **claim form** and pay an **excess**, that is, some money upfront before the company pays the rest. The excess is stated in your policy document and some customers agree to pay a larger excess in return for a cheaper policy.

EXAMPLE 2

This table shows five examples of comprehensive car insurance premiums.

Driver gender	Driver age	Town where vehicle is garaged	Car make	Car year	Basic premium
male	55	Goulburn	Ford	2004	$508
female	33	Wollongong	Mazda	2015	$657
female	24	Dubbo	Mitsubishi	2009	$1339
male	21	Newcastle	Holden	2014	$1161
female	45	Bathurst	Hyundai	2016	$432

Calculate the premium for:

a the female from Bathurst who gets a 10% loyalty discount

b the 55-year-old Ford driver who pays a 5% surcharge for an older car

c the owner of the Mitsubishi who pays a 13% levy as the car is financed

d the Mazda driver from Wollongong who gets a discount of 2.5% for a good driving record as well as a further discount of 1.7% for loyalty.

Solution

a

$$\text{Premium} = \$432$$

$$\text{Discount} = 10\%$$

$$\text{Discounted premium} = 90\% \times \$432$$

$$= \$388.80$$

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b Premium = \$508

Surcharge = 5%

Actual premium = 105% × \$508

= \$533.40

c Basic premium = \$1339

Levy = 13%

Actual premium = 113% × \$1339

= \$1513.07

d Basic premium = \$657

Discount = 2.5% followed by 1.7%

Discounted premium = \$657 × 97.5% × 98.3% [100 – 2.5 = 97.5, 100 – 1.7 = 98.3]

= \$629.6852…

= \$629.69

Theft and accident statistics

Approximately 56 000 motor vehicles are stolen in Australia each year, and the National Motor Vehicle Theft Reduction Council (NMVTRC) says that a car is stolen in Australia almost every 10 minutes. Each year, insurance companies charge \$1 billion more for insurance premiums due to car theft. Older cars are less secure than newer ones and attract more thieves. Insurance costs more in areas that have a higher theft and/or accident rate.

EXAMPLE 3

The percentage probabilities for vehicle theft and recovery in five states and territories each year are shown in the table.

State	Chance of theft (%)	Chance of recovery (%)
ACT	0.26	78
NT	0.63	75
Tas	0.26	92
Vic	0.24	67
Qld	0.19	70

Source: National Motor Vehicle Theft Reduction Council

a Which state or territory has the highest rate of vehicle theft?

b If there are 157 700 cars in Tasmania, how many are expected to be stolen each year?

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c If 7489 vehicles were recovered in Victoria:

i how many were stolen in Victoria?

ii how many vehicles are there in Victoria?

Solution

a Northern Territory, since the chance of theft is greatest at 0.63%.

b Expected no. stolen cars in Tasmania $= 0.26\% \times 157\,700$

$= 0.0026 \times 157\,700$

$= 410.02$

≈ 410

c **i** 67% of stolen vehicles are recovered.

$67\% = 7489$ vehicles

$$1\% = \frac{7489}{67}$$

$$100\% = \frac{7489}{67} \times 100$$

$= 11\,177.611\ldots$

$\approx 11\,178$

11 178 vehicles were stolen in Victoria.

ii 11 178 vehicles is 0.24% of all vehicles in Victoria.

$0.24\% = 11\,178$ From part i

$$1\% = \frac{11\,178}{0.24}$$

$$100\% = \frac{11\,178}{0.24} \times 100$$

$= 4\,657\,500$

There are 4 657 500 vehicles in Victoria.

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DID YOU KNOW?

Car theft

- About 47% of vehicles stolen in Australia are stolen outside the home of the owner or a friend of the owner, so park your car in a garage or in a secure parking area.
- Most vehicles are stolen in the late afternoon or evening, especially on Fridays, Saturdays and Sundays.
- Most cars are stolen for short-term use (such as joyriding, transport) or for resale (either as a whole vehicle or as stripped parts).
- Stolen motorcycles have the lowest rate of recovery with only 43% of thefts recovered.
- About 30% of stolen vehicles are recovered within 10 km from where they were stolen.

Cheaper cars are more likely to be stolen, and account for about 40% of all short-term vehicle thefts. Give a possible reason for this happening.

Exercise 9.01 Car insurance

1 Third Party Property insurance costs $38.17 per month. What is the cost per week? Select **A**, **B**, **C** or **D**.

A $8.80 **B** $9.54 **C** $8.81 **D** $7.63

2 Mia paid her comprehensive car insurance at $150 per month. How much would she save if she paid a lump sum of $1265? Select **A**, **B**, **C** or **D**.

Example 1

A $685 **B** $1115 **C** $0 **D** $535

3 Jack bought a new van for $28 350. His CTP insurance is $626 per year and his comprehensive insurance is $75.78 per month. What is the total cost of insurance per year?

4 Which insurance is the cheapest option? Select **A**, **B** or **C**.

A $985 per year paid in a lump sum

B $83.10 per month

C $38.25 per fortnight

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Example 2

5 The following table gives the Third Party Property insurance premiums for the RIGHT4U insurance company.

Car make	Driver gender	Driver age	Suburb where vehicle is garaged	Place where car is usually kept	Third Party Property premium
Suzuki	male	19	Campbelltown	on street	$345
Honda	female	56	Campbelltown	carport	$198
Mitsubishi	male	42	Parramatta	driveway	$297
Holden	female	23	Parramatta	locked garage	$243
Toyota	male	83	Bondi	on street	$312
Nissan	female	65	Bondi	security compound	$302

What is the Third Party Property insurance premium, to the nearest cent, for:

a the female from Campbelltown who gets a 20% no claim discount?

b the driver who has the car in a locked garage and gets a 3.5% discount?

c the driver under 20 years of age who must pay an 11.6% surcharge as he lives in a high theft area?

d the Toyota driver from Bondi who pays a 24% age levy but also gets a 4.2% loyalty discount?

6 Fahaad paid $864.07 for a new comprehensive insurance policy which included a 15% discount for buying online. What discount did Fahaad receive?

7 The base premium for the Green Slip on Emily's car is $379.67 (GST excluded) and the MCIS Levy (a tax to fund Medical Care and Injury Services) is 24.42% of the base premium. Find the total cost of the Green Slip, including the MCIS levy and the 10% GST on the base premium.

8 Colin bought a motor scooter. His annual CTP Green Slip was $238 and his Third Party Property insurance was $19.35 per month. He also agreed to pay the first 8.5% of a claim if he had an accident that was his fault.

a What was his yearly insurance cost?

b He had an accident that was his fault and the damage bill to the other car was $7320. How much did he contribute to the claim?

c In the same accident, his scooter suffered $1360 worth of damage. How much did Colin pay towards this claim?

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9 The NO FRILLS insurance company advertises a fixed insurance premium of $500 for a car garaged in a metropolitan area and $550 otherwise. The premium is adjusted for age and gender of the driver as shown in the table.

NO FRILLS insurance		
Driver gender	**Driver age**	**% adjustment**
male	<29	+14.6%
female	<29	+13.5%
male	30–69	–4.6%
female	30–69	–6.3%
male	>70	+17.4%
female	>70	+9.8%

Calculate the insurance premium for each driver.

	Driver gender	Driver age	Vehicle garaged
a	male	24	metropolitan area
b	female	76	country town
c	male	32	on a farm
d	female	47	metropolitan area
e	male	28	metropolitan area
f	female	18	country town

10 Refer to the table in Example **3** on page 359 to answer these questions.

Example 3

a If there are 125 000 vehicles in the ACT, how many would you expect to be:

i stolen?

ii recovered?

b In Queensland, 2195 stolen vehicles were never recovered.

i How many were stolen?

ii How many vehicles are there in Queensland?

11 Ellie bought a small second-hand car for $12 350. Her Compulsory Third Party insurance was $286 and she took out comprehensive insurance at $53.95 per month.

a What was her yearly insurance cost?

b She ran into the back of a car which had stopped at the traffic lights and did $6320 damage to the car as well as $4280 to her own car. If Ellie had to pay an excess of 8.5% of the repair costs, how much did the insurance company pay?

c Ellie's comprehensive insurance premium will increase by 13.5% next year. How much will she pay, to the nearest cent?

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12 The six most stolen passenger vehicles in NSW one year are listed.

Car type	No. of thefts	% of thefts recovered
Holden Commodore 1990	317	86.8
Holden Commodore 1989	272	83.5
Holden Commodore 1991	264	83.3
Ford Falcon 1989	214	54.7
Ford Falcon 1990	189	52.9
Toyota Camry 1989	182	83.0

Source: National Motor Vehicle Theft Reduction Council

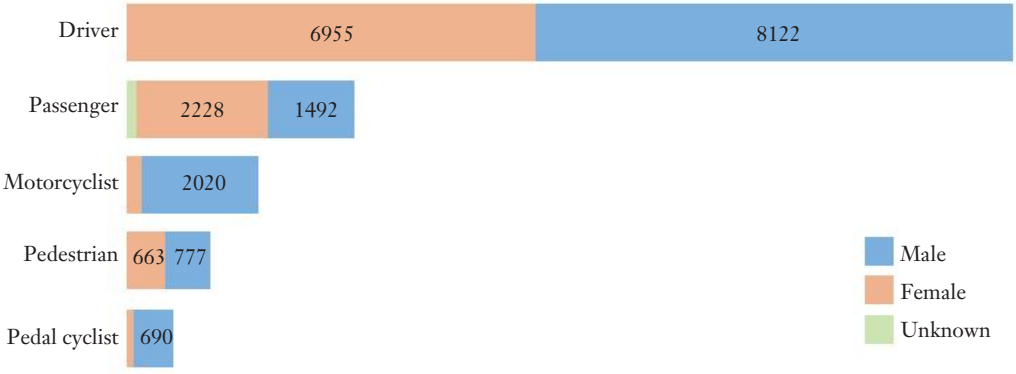
a How many Holden Commodores were stolen?

b How many Ford Falcons were recovered?

c How many Toyota Camrys were not recovered?

d Why do think older cars are more likely to be stolen?

13 The graph below shows crash and casualty statistics for NSW based on gender.



© State of New South Wales (Transport for NSW)

How would this data affect the price of car insurance policies?

Stamp duty

9.02 Stamp duty

Stamp duty is a tax paid to the Office of State Revenue when you buy a new or used vehicle.

It is calculated on the market value of the vehicle or the price actually paid, whichever is the greater. Stamp duty is usually included in the 'on-road' price of a new vehicle.

In NSW, the rates for stamp duty on a motor vehicle are:

Vehicle value	Rate
$0–$44 999	$3 for every $100 (or part of $100)
$45 000 and over	$1350 plus $5 for every $100 (or part of $100) over $45 000

The price of a motor vehicle needs to be rounded up to the nearest $100 when calculating stamp duty.

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EXAMPLE 4

Lexie purchased a vehicle for $56 240. The stamp duty is $3 for every $100 (or part of $100) of the purchase price up to $44 999, and $1350 plus $5 for every $100 (or part of $100) for $45 000 and over.

a How much stamp duty did Lexie pay?

b Complete the table below.

Vehicle value ($)	0	10 000	20 000	30 000	40 000	45 000	50 000	60 000	70 000	80 000
Stamp duty ($)		300			1200			2100		3100

c Use the completed table of values to construct a piecewise linear graph.

d Use the graph to find, to the nearest $100:

i the stamp duty on a $64 000 vehicle

ii the value of a vehicle if the stamp duty is $750.

Solution

a Purchase price = $56 240

≈ $56 300

For stamp duty, round purchase price up to the nearest $100.

Stamp duty = $1350 + 5% × ($56 300 − $45 000)

= $1915

Lexie paid $1915 stamp duty.

b

Vehicle value ($)	0	10 000	20 000	30 000	40 000	45 000	50 000	60 000	70 000	80 000
Stamp duty ($)	0	300	600	900	1200	1350	1600	2100	2600	3100

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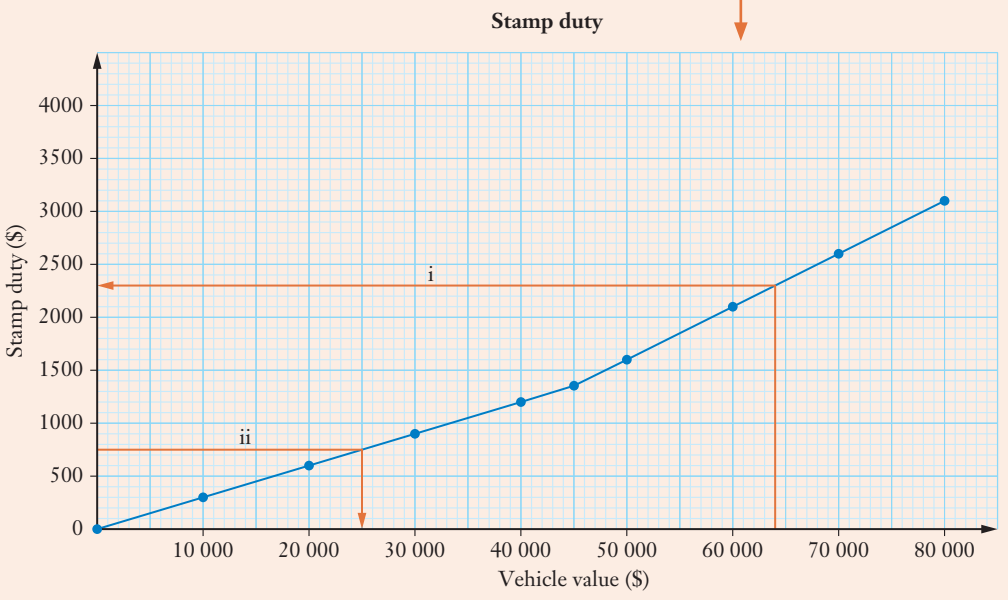
c A piecewise linear graph is made up of line segments with different gradients.



d **i** $2300 **ii** $25 000

Exercise 9.02 Stamp duty

1 Stamp duty is $3 for every $100 (or part of $100) of the purchase price of small car. What is the stamp duty on a small Hyundai with a purchase price of $22 120? Select **A**, **B**, **C** or **D**.

A $660 **B** $664 **C** $662 **D** $666

2 Stamp duty on a vehicle is $3 for every $100 (or part of $100) for the first $45 000 and $5 for every $100 (or part of $100) on the remainder. What is the stamp duty on a $55 000 ute? Select **A**, **B**, **C** or **D**.

A $1350 **B** $1650 **C** $1850 **D** $2750

Example 4

3 Vehicle stamp duty is 3% of the purchase price up to $30 000 plus 11% for every dollar between $30 000 and $40 000 plus 4% for every dollar over $40 000.

a Find the stamp duty on a vehicle costing:
i $30 000 **ii** $45 000 **iii** $60 000

b Construct a piecewise linear graph showing the stamp duty for cars from $0 to $80 000. [Use a scale of 1 cm = $10 000 on the horizontal axis and 1 cm = $500 on the vertical axis.]

c Use your graph to find, to the nearest $100:
i the stamp duty on a $75 000 car
ii the cost of a car if stamp duty is $3000.

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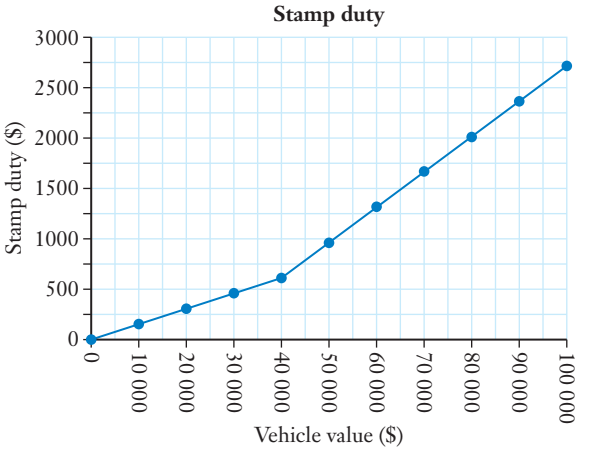
4 If stamp duty on a new vehicle is 2.75% of the purchase price up to $20 000 plus 5.75% for every dollar between $20 000 and $45 000 plus 6.5% for every dollar over $45 000, how much stamp duty will you pay on a car worth:

a $20 000? **b** $45 000? **c** $78 500?

5 Stamp duty charges on vehicles in one state of Australia are shown in the graph.



a To the nearest $100, find the stamp duty on a vehicle worth:
i $33 000 **ii** $66 000 **iii** $85 000.

b To the nearest $1000, find the value of a vehicle if the stamp duty is:
i $2300 **ii** $1700 **iii** $750.

6 In one state of Australia, there is no stamp duty on a car valued at less than $6000. The stamp duty on a car valued at $6000 or more attracts a stamp duty charge of $10 for every $200 or part thereof of its value. To the nearest dollar, what is the stamp duty on a car valued at:

a $6350? **b** $68 980? **c** $146 000?

7 If stamp duty on a vehicle is $60 for the first $3000 of the purchase price plus 4% for every dollar over $3000, calculate the stamp duty payable on a vehicle with purchase price of:

a $8000 **b** $12 500 **c** $69 750.

8 Stamp duty in Victoria is based on the market value of the vehicle, regardless of the price paid. The rates are:

- 2.5% of the market value up to $57 009
- 5% of the market value if it is over $57 009.

Calculate the stamp duty on a vehicle with a market value of:

a $57 100 **b** $22 500 **c** $132 400.

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9 Stamp duty on wrecked cars is calculated using this table.

Car value	Stamp duty
$1000 or less	1% with a minimum of $5
$1001 to $2 000	$10 + 2% on the amount over $1000
$2001 to $3 000	$30 + 3% on the amount over $2000
$3001 or more	$60 + 4% on the amount over $3000

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Find the stamp duty, to the nearest dollar, on a wrecked car worth:

a $980 **b** $2850 **c** $3200.

10 Stamp duty charges based on the value of a car are listed in this table.

Car value	Stamp duty
$600 or less	$18
$600 to $35 000	3%
$35 000 to $40 000	$1050 + 11% for the amount over $35 000
$40 001 or over	4%

Calculate the stamp duty charges, to the nearest cent, for a car valued at:

a $585 **b** $34 800 **c** $105 500

d $890 **e** $38 750.

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TECHNOLOGY

Stamp duty online calculator

Search for 'stamp duty calculator' on the Internet that calculates the duty for different states of Australia.

This calculator shows that the stamp duty in NSW for a vehicle with a purchase price of $40 000 is $1200.

State	NSW
Purchase Price	40000
NSW Stamp Duty: $1,200.00	

1 For a 6-cylinder car with a purchase price of $40 000, find:

- **a** which state has the highest stamp duty
- **b** which state has the lowest fee to transfer the car registration to a new owner
- **c** the total payable in Victoria
- **d** the range of stamp duties
- **e** the average stamp duty for the six states and the two territories.

2 Find the current stamp duty payable on:

- **a** a car worth $22 000 in NSW
- **b** a van with a value of $32 500 in Tasmania
- **c** a hybrid car in Queensland valued at $31 480
- **d** a 4WD in Western Australia worth $102 450.

9.03 On-road costs

You have your licence, you have saved money from your part-time job and now it is time to buy your first car. So what exactly are the costs of purchasing a car?

- Purchase price
- Stamp duty
- **Vehicle registration**: yearly process of paying and obtaining permission for using a vehicle on public roads
- CTP (Compulsory Third Party) insurance – a 'green slip'
- Third Party Property or comprehensive insurance (optional but advisable)
- Loan interest and fees

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The cost of registration and CTP insurance is often included in the purchase price of a car, and stamp duty if the car is new. These are called **on-road costs**.

iStock.com/sturti

EXAMPLE 5

Kayne purchased a van for $40 850 to use in his landscaping business. He set up a table of costs associated with his purchase.

Purchase price	$40 850
GST	10% of purchase price
Stamp duty	$3 for every $100 (or part of $100) of GST price
Registration (per year)	$764
CTP insurance (Green Slip)	$412
Comprehensive insurance (per year)	$963

What was the on-road cost to purchase this vehicle?

Solution

		Cost
GST price	110% × $40 850	$44 935
Stamp duty on GST price	3% × $45 000 (rounding $44 935 up to the nearest $100)	$1350

Total cost = $44 935 + $1350 + $764 + $412 + $963

= $48 424

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Exercise 9.03 On-road costs

1 Angela paid cash for a second-hand motor scooter with these on-road costs.

Purchase price (including GST)	$15 000
Stamp duty	2% of purchase price
Transfer of registration	$200
CTP insurance (Green Slip)	$150
Third party insurance	$240

What was the total cost? Select the correct answer **A**, **B**, **C** or **D**.

A $15 590 **B** $20 090 **C** $15 620 **D** $15 890

2 Naazmi bought a new 4WD with on-road costs as shown. Calculate the total amount paid. Select **A**, **B**, **C** or **D**.

Purchase price	$45 000
GST	10%
Stamp duty	5% of GST price
Registration	$860
Green Slip	$420

A $50 780 **B** $53 255 **C** $48 530 **D** $68 780

3 Libby bought a sports car for $59 600 plus costs as shown.

Example 5

Purchase price	$59 600
GST	10% of purchase price
Stamp duty on GST price	$1350 plus $5 for every $100 (or part of $100) over $45 000
Registration	$615
CTP insurance (Green Slip)	$419
Comprehensive insurance	$1625

What was the total cost of her sports car?

4 Charles bought his second-hand station wagon with the following upfront costs.

Purchase price	$31 280
Stamp duty	1.5% of purchase price
Transfer of registration	$32
CTP Insurance (Green Slip)	$427
Third Party Property insurance	$317

How much did he pay for his vehicle?

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5 A car dealer was offering purchasers a 15% discount on the marked price of all vehicles. The costs of GST, stamp duty, registration and CTP insurance are then added to the discounted price to give the total cost.

Type of car	Registration	CTP Insurance (compulsory)	Third Party Property insurance (optional)	Comprehensive insurance (optional)
Holden	$325	$614	$250	$1268
Ford	$318	$598	$264	$1095
Toyota	$336	$437	$206	$999
Nissan	$384	$503	$295	$1134
Honda	$396	$602	$301	$1003
Subaru	$401	$599	$278	$1236
GST: 10% of discounted price Stamp duty: $3 for every $100 (or part of $100) up to $45 000 or $1350 plus $5 for every $100 (or part of $100) over $45 000 Dealer special: No stamp duty on GST prices of $6500 or less.				

Use the table to find the total cost of a:

a Holden with a marked price of $31 468 and Third Party Property insurance

b Ford with a marked price of $6500 and no optional insurance

c Toyota with a marked price of $65 489 and comprehensive insurance

d Nissan with a marked price of $27 650 and no optional insurance

e Honda with a marked price of $18 639 and comprehensive insurance

f Subaru with a marked price of $58 790 and Third Party Property insurance.

6 Sascha bought a 4WD minivan with the following upfront costs.

Purchase price	$43 860
Stamp duty	1.5% for first $25 000 plus 3% for each $ over $25 000
Registration	$726
CTP Insurance (Green Slip)	$854
Comprehensive insurance	$1352

What was the total cost of his van?

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7 The extra costs involved in a car purchase at Wally's Wheels are listed in the table.

GST	10% of purchase price
Stamp duty (on GST price)	3% of the price inclusive of GST up to $45 000 5% for every dollar over $45 000.
Registration	$743 + transfer fee of $50
CTP Insurance (Green Slip)	$837
Third Party Property insurance	$\$P$ + 5% extra for drivers 25 years and under
Comprehensive insurance	$\$C$ + 10% extra for drivers under 30 years

Note: $\$P$ and $\$C$ are basic amounts depending only on vehicle type. Add age amounts if required.

Find the total costs for purchasing each of the following vehicles.

a $56 780 sports car bought by 29-year-old Josh with comprehensive basic $1034

b $36 204 sedan bought by 40-year-old Natalie with Third Party Property basic $328

c 4WD bought by 26-year-old Heath for $38 759 with Third Party Property basic $407

d Truck bought by 50-year-old 'Highway Harry' for $128 000 with comprehensive basic $6945

e $15 320 car bought by 24-year-old Jasmine with Third Party Property basic $269

f $21 648 van bought by 29-year-old 'Bill the Baker' with comprehensive basic $747

TECHNOLOGY

Purchasing your first car

The Office of Fair Trading provides information on purchasing items, including the booklet *The Car buyer's guide*. Use the booklet or the **Fair Trading** website to answer these questions.

1 What things should be on your vehicle checklist before you buy a car?

2 What are the seven steps to follow when you buy a car privately?

3 What are RMS and PPSR and how can they help you to buy a car?

4 Why should you avoid buying a car advertised on the roadside?

5 What is meant by 'rebirthing'? How does this affect insurance premiums?

Investigate other websites that may assist you in the purchase of a vehicle.

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DID YOU KNOW?

Best-selling car brands

The table shows the ten best-selling car brands in Australia in 2011 and 2016.

	2011	2016
1	Mazda 3	Toyota HiLux
2	Holden Commodore	Toyota Corolla
3	Holden Cruze	Hyundai i30
4	Toyota Corolla	Ford Ranger
5	Toyota Hilux	Mazda 3
6	Hyundai i30	Toyota Camry
7	Mazda 2	Holden Commodore
8	Ford Falcon	Mazda CX-5
9	Ford Territory	Mitsubishi Triton
10	Toyota Camry	Hyundai Tucson

There have been some changes over the 5 years, such as the meteoric rise of the Mazda brand and the Toyota HiLux.

1 **What other notable changes have occurred from 2011 to 2016?**

2 **Find out what the list looks like this year.**

9.04 Running costs

The running costs of a car include fuel, servicing, repairs, tyres, renewal of car registration and insurance premiums, and depreciation. Motorway tolls, parking fees and fines for traffic infringements also need to be considered.

EXAMPLE 6

Rabiba owns a Holden Cruze and drives about 15 000 km per year. Her car was serviced twice during the year, each service capped at $250. She also needed two new tyres at $148 per tyre. Rabiba had to pay $385 for car registration, $585.21 for the green slip and $691.88 for comprehensive insurance. Her weekly fuel bill is $35. Parking fees and tolls amounted to $38 per week.

a How much did Rabiba spend on her car during the year (to the nearest dollar)?

b What was the weekly cost of the car?

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Solution

a Total cost $= \$250 \times 2 + \$148 \times 2 + \$385 + \$585.21 + \$691.88 + \$35 \times 52 + \$38 \times 52$

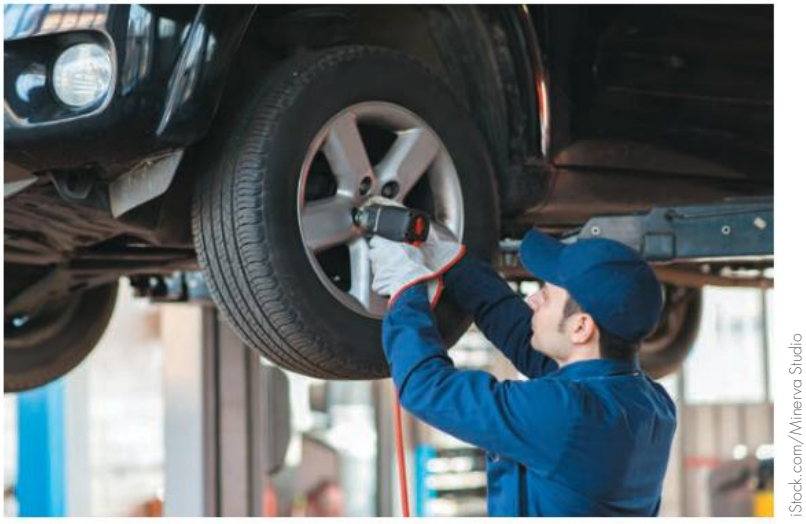
$= \$6254.09$

$\approx \$6254$

b Weekly cost $= \dfrac{\$6254}{52}$

$= \$120.269\ldots$

$\approx \$120.27$



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Exercise 9.04 Running costs

Example 6

1 The average weekly cost of running a Mazda 3 5-door hatchback has been estimated to be \$202.67. What is the yearly cost of running this car?

2 The weekly fuel bill of a car is \$37.69 based on the car travelling 15 000 km per year. If the average fuel price is \$1.28, find:

a the amount of fuel (to the nearest litre) used per week

b the annual fuel cost.

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3 Jackson estimates that he will travel approximately 500 km per week. He estimates the fuel consumption of 11 km per litre and a fuel price of $1.38 per litre. Car registration and Compulsory Third Party insurance will cost $785 and the comprehensive insurance on his car will be $725. Jackson has calculated that $1100 should cover servicing and maintenance of the car and he has paid $180 for membership to the NRMA road service. He pays $50 per week on motorway tolls and parking fees amount to $500 for the year.

a Calculate Jackson's fuel bill for the year.

b How much will Jackson pay for the annual running costs of his car?

c How much will Jackson need to budget per fortnight to cover the cost of running his car?

4 The following table shows the average vehicle running costs for NSW, Queensland and Victoria in 2014.

State	NSW	VIC	QLD
Servicing and maintenance	$953	$1010	$1096
Registration and insurance	$1474	$1232	$1096
Parking and tolls	$298	$311	$181
Fines	$53	$106	$70
Petrol	$2834	$2979	$3494

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a Calculate the average weekly running costs for vehicles for each state.

b What is the difference between the annual cost in NSW and Queensland? Give a possible reason for this difference.

5 The Toyota Hilux was the highest-selling passenger vehicle in 2016. The average total weekly running cost is $235.87. Find the annual running cost of a Toyota Hilux.

6 Sarita bought a new car for $21 000. She is told that the running cost of the car will be 49.29 cents per km. Sarita travels 20 000 km per year. What is her average weekly running cost for the car?

7 Ben's annual fuel cost is $3744. His registration, CTP insurance and comprehensive car insurance amount to $1432. His car has had two services costing $267 and $321. At the last service, Ben was informed he needed four new tyres, which cost $212 each. The NRMA fee for gold membership was $180, and tolls and parking fees amounted to $890 for the year. Parking fines added $320 to his costs.

a What is the weekly running cost of Ben's car?

b If the depreciation on Ben's car amounted to $6000 and is also considered as a running cost, calculate the annual and weekly running cost of the car.

8 The list price for a car is $32 990 and the on-road price is $36 528. The car depreciates by $74.86 per week. What is the value of the car after one year?

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9.05 Fuel consumption and prices

How much petrol?

Graph of petrol prices

The types of fuel used to power vehicles include petrol, diesel, liquefied petroleum gas (LPG), ethanol and electricity. Petrol is the most common fuel and most cars use unleaded petrol (ULP).

Fuel consumption is the rate at which a car uses fuel and is usually measured in litres per 100 km (L/100 km). For example, a large car may use 12.3 L of fuel to travel 100 km whereas a small car may use only 5.8 L. The small car has a better fuel consumption and is more fuel-efficient. The table shows the fuel consumption for some of Australia's best-selling cars.

Vehicle	Fuel consumption (L/100 km)		
	Average	City	Highway
Ford Focus	6.2	8.7	4.8
Ford Ranger (diesel)	6.6	7.8	5.8
Holden Commodore	9	12.5	7
Holden Cruze	7	9.5	5.5
Hyundai i30	7.3	10.3	5.5
Mazda 2	5.4	7.2	4.4
Mazda 3	5.8	7.6	4.8
Mitsubishi Lancer	10.1	13.9	8
Nissan Navara	9.7	13	7.7
Nissan X-trail	8.1	11	6.4
Toyota Camry Hybrid	5.2	5.7	4.9
Toyota Corolla	4.1	3.9	4.1
Toyota Hilux	11.1	15.5	9.4
Toyota Hilux (diesel)	9	10.9	7.8
Toyota Prado	11.6	15.5	9.4
Toyota Yaris	5.8	7.1	5.1
Volkswagen Golf	4.9	5.8	4.5

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The fuel consumption of a vehicle depends on several factors. You use more fuel for:

- stop–start driving in traffic (urban cycle)
- driving on unsealed roads
- using a badly tuned engine
- using a driving style involving hard acceleration and/or braking
- using under-inflated tyres
- towing a trailer or caravan
- using the air conditioner.

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Fuel consumption

EXAMPLE 7

A Mini Cooper has an advertised fuel consumption of 5.8 L/100 km.

a How much fuel, to two decimal places, does it use for a trip of 1530 km?

b If fuel is \$2.04 per litre, what is the fuel cost, to the nearest dollar, for this trip?

c How far, to the nearest kilometre, can the Mini travel on a full tank of 35 L?

d If the Mini used 25 L to go 400 km on an outback trip, how far could it go on 42 L at the same rate?

Solution

a Fuel for 100 km = 5.8 L

Fuel for 1 km = $\frac{5.8}{100}$ L Using the unitary method

Fuel for 1530 km = $1530 \times \frac{5.8}{100}$ L

= 88.74 L

b Fuel cost = $88.74 \times \$2.04$

= \$181.0296

≈ \$181

c 5.8 L is used for 100 km

35 L is used for $\frac{100}{5.8} \times 35$ km = 603.44 827… km

≈ 603 km

The Mini can travel 603 km (to the nearest km) on a full tank of fuel.

d 25 L takes the car 400 km

1 L takes the car $\frac{400}{25}$ km

42 L takes the car $\frac{400}{25} \times 42$ km = 672 km

The Mini can travel 672 km on 42 L.

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EXAMPLE 8

This table shows the average fuel prices (in cents/litre) in state capitals over a 12-month period.

	Regular unleaded petrol	Diesel	LPG
Brisbane	149.2	131.3	72.3
Sydney	132.6	127.8	75.9
Melbourne	144.5	131.0	70.1
Adelaide	127.9	128.9	76.4
Perth	141.2	134.1	73.2
Hobart	151.9	135.6	87.0

a Which capital has the highest fuel price for:

i petrol? **ii** diesel? **iii** LPG?

b What is the average price, in cents, for petrol in the capitals?

c What is the range of LPG prices?

d If Elise filled up her 60 L tank with petrol in Sydney at the price shown:

i how much did she pay?

ii how far could she travel on this tank if her car used 8 L/100 km?

Solution

a **i** Hobart **ii** Hobart **iii** Hobart

b $\text{Average} = \dfrac{149.2+132.6+144.5+127.9+141.2+151.9}{6} = 141.2 \text{ cents}$

c Range = 87.0 – 70.1 = 16.9

d **i** Cost of fuel = 60×132.6

$= 7956$ cents

$= \$79.56$

ii 8 L takes her 100 km

60 L takes her $\dfrac{100}{8} \times 60$ km = 750 km

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Exercise 9.05 Fuel consumption and prices

1 A motorcycle has a fuel consumption of 3.2 L/100 km. How far can it travel on 1 L of petrol? Select **A**, **B**, **C** or **D**.

A 32 km **B** 16.1 km **C** 31.25 km **D** 3.2 km

2 A car travels 160 km on 14 L of petrol. What is its fuel consumption in L/100 km? Select **A**, **B**, **C** or **D**.

A 11.4 **B** 8.7 **C** 22.4 **D** 8.8

Example 7

3 A Mazda has a fuel consumption of 11.5 L/100 km.

a What is the fuel consumption in L/km?

b How much fuel, to the nearest litre, does it use to travel 980 km?

c How far could it travel on a 55 L tank of fuel? Answer to the nearest kilometre.

d If the Mazda used 48 L to go 695 km, how far (to the nearest km) could it go on 102 L at the same rate?

4 A Ford Mondeo has a fuel consumption of 5.5 L/100 km. How much fuel, in litres correct to two decimal places, does it use to travel:

a 10 km? **b** 2347 km?

5 If a car uses 8 L of fuel to travel 100 km, how much will it use to travel:

a 1 km? **b** 150 km?

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6 Refer to the table of fuel consumption of some of Australia's best-selling cars on page 377.

a Which car has the best fuel consumption?

b How far (to the nearest kilometre) can this car travel on a 45 L tank of fuel?

c How much petrol does the Mazda 2 use for a 2500 km trip at the rate shown?

d What is the cost of a 1348 km trip in the Toyota Hilux if:

i the price of petrol is 174.6 c/L?

ii the price of diesel is 142.3 c/L?

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Use the table in Example 8 on page 379 to answer Questions 7 and 8.

Example 8

7 **a** What is the average price, in dollars and cents, for diesel?

b What is the range of petrol prices?

c Tyson filled up his 160 L Hummer fuel tank with petrol in Melbourne at the price shown.

i How much did he pay?

ii If his fuel consumption is 13.2 L/100 km, how many whole kilometres would he go before running out of petrol?

8 **a** What is the average price paid for diesel in the east coast state capitals (Sydney, Melbourne and Brisbane)?

b What is the range of diesel prices in the east coast state capitals?

c What does it cost Angela to fill her 50 L diesel Golf fuel tank at the price shown in Hobart?

d A Volkswagen 'beetle' car travelled 440 km on a 40 L tank of fuel. What was the fuel consumption in L/100 km?

9 The table shows the price of petrol (in c/L) in one week.

Petrol	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Brisbane	143.4	140.4	139.2	150.7	150.6	148.3	146.2
Sydney	149.7	147.6	146.6	159.5	158.0	154.7	151.8
Melbourne	151.1	149.0	148.0	158.9	157.4	155.2	153.0
Adelaide	150.0	147.8	148.0	160.9	158.5	155.9	152.7
Perth	146.7	146.6	146.3	145.9	146.3	146.5	146.6
Hobart	154.1	154.1	154.0	154.0	154.0	154.0	153.9

a What was the average petrol price in Brisbane this week, correct to two decimal places?

b What was the average petrol price on Monday for the cities?

c What was the range of prices on Saturday for the cities?

d What was the range of prices for the week in Perth?

e Which city had the lowest average weekly petrol price?

f On which day was the average petrol price was greatest?

g Which city offered the greatest range of prices over the week?

h Which day showed the smallest range in prices for the cities?

10 Lachlan, Brodie and Mitchell each did a 3-day car trip and recorded their fuel usage.

Brodie	Distance travelled (km)	Fuel used (L)
Friday	497	47
Saturday	364	43
Sunday	579	56

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Lachlan	Distance travelled (km)	Fuel used (L)
Friday	543	68
Saturday	297	43
Sunday	410	56

Mitchell	Distance travelled (km)	Fuel used (L)
Friday	624	56
Saturday	208	39
Sunday	524	45

Whose car had the best average fuel consumption?

11 Copy and complete the table.

Distance travelled (km)	Fuel used (L)	Fuel consumption (L/100 km)
863	62	
	43	11.2
1304		8.4
	86	9.7
2356		12.9
1730	167	

TECHNOLOGY

Calculating fuel consumption and costs

1 Search for an online fuel consumption calculator similar to this one.

Fuel spent litres (L): 60

Kilometres (km): 725

Select units type: Metric units (L, km)

Rounding options: 2 digits after decimal point

SUBMIT

Calculate the fuel consumption in L/100 km for:

a a Jeep that travels 3200 km and uses 380 L fuel

b a Volkswagen that travels 1850 km and uses 160 L fuel

c a Smart car that travels 790 km and uses 60 L fuel.

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2 Search for an online calculator that finds annual fuel costs, such as this one:

Calculate your annual fuel costs					
	Fuel price ($/litre)	Fuel consumption (L/100 km)	Annual distance travelled (km)	Annual fuel cost ($)	
	CAR 1				
	1.95	8.28	15000	CALCULATE	

a Calculate the annual fuel costs for Brad who travels 550 km each week when fuel costs $1.65 per litre. His car can travel 600 km on a 50-litre tank of fuel.

b Survey five friends or family members to see how far each travels per year and the fuel consumption of their cars in L/100 km. Calculate their annual fuel costs at the current petrol price.

DID YOU KNOW?

Ten ways of saving fuel

1. Drive at a moderate speed: not too slow, not too fast. Driving at 110 km/h and beyond uses much more fuel than driving at 80–100 km/h.
2. Service your car regularly and check the tyre pressure.
3. Avoid driving in stop-start traffic during peak times and on busy roads.
4. Avoid excessive braking. Allow the car to come to a natural stop.
5. If waiting in traffic for a long time, switch off the engine.
6. Close the windows when driving to reduce drag.
7. Switch off the air conditioner.
8. Reduce the load on your car by removing heavy objects from the boot and removing roof racks if they are not being used.
9. Plan to do all your small jobs in one trip rather than lots of short trips.
10. Use public transport, walk or ride a bike.

Choose two of the ways above and explain how or why they save fuel.

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9.06 Choosing the best car

Choosing a car to buy will depend on your personal tastes and needs, and most importantly, on your budget. Initial costs that need to be considered are registration, CTP insurance, car insurance, maintenance, repairs, and cost of fuel. If a loan is required to buy the car, then the fortnightly/monthly payments also need to be included in your budget.

Other essential factors that may influence your choice of car are:

- the size of the motor (for example, 1.8 L, 2.0 L, 3.6 L)
- the type of fuel the car uses (petrol, diesel, LPG, hybrids)
- fuel consumption
- warranty periods for a car purchased from a dealership
- security and safety features
- whether to buy a new car or a used car.

EXAMPLE 9

Jasmine decides to buy a new car and apart from the purchase price and on-road costs, she is allowing $6500 per year for running costs. Jasmine borrows $25 000 over 3 years to pay for the car. Her fortnightly repayments are $355.77.

How much will Jasmine have to budget for the car each week (to the nearest dollar)?

Solution

$$\begin{aligned}
\text{Running costs/year} &= \$6500 \\
\text{Loan repayments/year} &= \$355.77 \times 26 = \$9250.02 \\
\text{Total} &= \$6500 + \$9250.02 \\
&= \$15\ 750.02 \\
\text{Weekly amount} &= \frac{\$15\ 750.02}{52} \\
&= \$302.89 \\
&\approx \$303
\end{aligned}$$

Jasmine will have to budget $303 per week.

EXAMPLE 10

Samin bought a car for $39 044, which included on-road costs. Samin paid 40% of the total of his car in cash and borrowed the rest at 8.5% p.a. flat interest over 4 years.

a How much did Samin borrow, rounded up to the nearest dollar?

b How much will he have to budget per week to cover the cost of repayments?

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Solution

a Amount borrowed = 60% × $39 044 — 100% – 40% = 60%

= $23 426.40

≈ $23 427

b Interest = $23 427 × 0.085 × 4

= $7965.18

Total to repay = $23 427 + $7965.18

= $31 392.18

Weekly amount = $\frac{\$31\,392.18}{208}$ — 52 × 4 = 208

= $150.923…

≈ $150.93

Exercise 9.06 Choosing the best car

Example 9

1 The annual running costs for Nicole's car include $780 for registration and Green Slip, $589 for comprehensive insurance, $1800 for fuel, $1100 for maintenance, servicing and tyres. She also has to pay $610 per month for the car loan. Calculate Nicole's weekly budget for the car.

2 Dylan is purchasing a new car. He has selected two cars, which are similar, but one uses diesel and the other uses petrol.

Car 1: Diesel – cost $44 430, average weekly running cost $235.87

Car 2: Unleaded petrol – cost $42 700, average weekly running cost $241.58

Calculate the cost of each car over 5 years to determine which car is the better option.

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3 At a petrol station, the price of regular unleaded petrol is 134.9 c/L and the price of LPG is 71.5 c/L. Two cars, one petrol and the other LPG, both fill their tanks with 65 L of fuel. What is the difference between their fuel bills?

Example 10

4 Mariah paid $32 700 for a second-hand station wagon. She borrowed 75% of the total cost at 9.4% p.a. flat interest over 3 years.

a How much did she borrow, rounded up to the nearest dollar?

b How much will she need to budget per week to cover the cost of loan repayments?

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5 Daniel wants to purchase a new car costing $29 950. He needs finance to pay for the car and has two options:

A: reducible loan at 8% p.a. interest, monthly payments of $731.17 for 4 years

B: reducible loan at 9.5% p.a. interest, fortnightly payments of $441.98 for 3 years.

Which is the better option? Justify your answer.

6 Many car dealerships offer a 5-year warranty or 100 000 km. What does this mean?

7 Copy this table to compare the cost of buying two different cars.

	Car 1	Car 2
Make		
Model		
Year		
Cost of car	$	$
Cash price of car	$	$
Less trade-in or deposit	$	$
Stamp duty and registration transfer	$	$
Comprehensive and third party insurance	$	$
Fees and extras	$	$
	$	$
	$	$
Other costs	$	$
Extended warranty	$	$
Total amount needed for car	$	$

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Complete this table for two cars, using the Internet to obtain information such as price, stamp duty, insurance, cost of registration and so on, to calculate the total amount needed to buy a car.

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SAMPLE HSC PROBLEM

Nitara is buying a used car that has a sale price of $18 700. She will have to pay a transfer of registration fee of $32 and stamp duty which is calculated at $3 for every $100 or part thereof, of the sale price.

a Calculate the stamp duty Nitara has to pay.

b Nitara borrows the total amount of the cost of the car including the transfer of registration and stamp duty. She pays $540 per month over 4 years to repay the loan. Calculate:

i the total amount Nitara borrows

ii the total amount Nitara pays for the car

iii how much Nitara needs to allow for the loan repayments if she works on a weekly budget.

Study tip

Your study routine

- Your study routine will be more effective if you maintain a healthy lifestyle: eat, sleep, rest, exercise, socialise and generally be happy!
- Try to study at the same time and place each day.
- When studying, alternate between easy tasks and hard tasks.
- Stop and reflect on what you have achieved before moving on.
- Use your time wisely and take regular breaks.
- Ask yourself: 'Am I really studying effectively or am I wasting time? Do I feel a sense of achievement?'
- Take a 1-minute break after completing each study or homework task.
- Take a longer break after working for more than 45 minutes continuously.
- Reward yourself when you feel you have completed a significant amount of work: watch a bit of TV, have a snack, play some sport or a game, or go for a walk.

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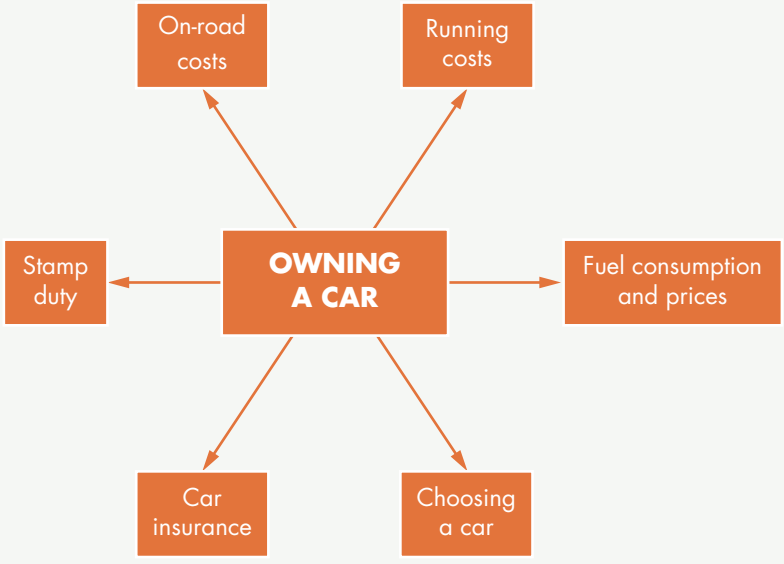
9. CHAPTER SUMMARY

This chapter, Owning a car, looked at the costs of owning a car. These costs include on-road costs such as the purchase price, insurance, registration and stamp duty. It also looked at the running costs of owning a vehicle such as fuel consumption, registration and insurance of the car, fuel consumption, service, maintenance and repairs.

Make a summary of this topic. Use the outline at the start of this chapter as a guide.
An incomplete mind map is shown below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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9. TEST YOURSELF

1 This table gives two examples of Third Party Property insurance premiums.

Exercise 9.01

Car make	Driver gender	Driver age	Town where vehicle is garaged	Place where car is usually kept	Third Party Property premium
Mitsubishi	male	42	Parramatta	driveway	$297
Nissan	female	65	Bondi	security compound	$302

What is the insurance premium for:

a the Nissan driver who gets a 5% discount for secure garaging?

b the male driver who pays an 8% levy for non-secure garaging in a high-theft area?

2 Ben had Third Party Property insurance, paid by a direct debit of $15.80 per fortnight.

Exercise 9.02

a How much would he save if he paid a yearly amount of $402.50 upfront?

b He hit a tree and wrote off his car, worth $15 600. What is his insurance cover?

3 Use the table below to find the stamp duty on cars valued at:

Exercise 9.03

a $23 600 **b** $57 000 **c** $4800 **d** $84 600.

Car value	Stamp duty rate
up to $5000	1.5%
plus each $ between $5001 and $35 000	2.5%
plus each $ over $35 000	5.0%

4 In one state of Australia, stamp duty is based on whichever is higher – the market value or the purchase price.

Exercise 9.03

Value	Stamp duty
$54 500 or less	$5 for every $200 or part thereof
over $54 500	$10 for every $200 or part thereof

Use this table to find the stamp duty on vehicles with:

a purchase price $23 600 and market value $24 500

b purchase price $36 750 and market value $32 400

c market value $69 300 and purchase price $71 260

d market value $59 800 and purchase price $58 000.

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5 The rates for stamp duty in NSW are shown in this table.

Vehicle value	Rate
$0–$44 999	$3 for every $100 (or part of $100)
$45 000 and over	$1350 plus $5 for every $100 (or part of $100) over $45 000

Calculate the stamp duty on a car valued at:

a $21 370 **b** $44 020 **c** $58 700 **d** $89 220.

Exercise 9.03

6 Raphael bought a new car with the following upfront costs..

Purchase price	$24 999
Stamp duty	1.5% up to $5000 plus 2.5% for each dollar between $5000 and $20 000 plus 4.5% for each dollar over $20 000
Registration	$328
CTP Green Slip	$503
Comprehensive insurance	$1204

What was the total upfront cost of his car?

Exercise 9.04

7 Nadia spends $42 on fuel per week and her comprehensive car insurance is $70.81 per month. She also pays $812.68 for car registration and CTP insurance, and services cost her $590 annually. Other repairs and replacement of tyres cost $450. Nadia uses the motorways 5 days a week at an average cost of $62 per week.

Find the total running cost of Nadia's car for:

a the year **b** a week.

Exercise 9.05

8 The fuel consumption of a four-wheel drive vehicle was measured under various conditions.

Conditions	Fuel consumption (L/100 km)	Trip distance (km)	Fuel used (L)
City driving non-peak hour	14.5	35	
City driving in peak hour	12.7	26	
Highways	10.8	460	
Outback dirt roads	8.4	2635	
Towing a caravan	7.3	9870	

a What is the average fuel consumption (in L/100 km) for these five conditions?

b Complete the table by calculating, correct to one decimal place, the fuel used for the trip distances for each condition.

c What are three ways to improve fuel consumption?

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9 The table shows the fuel consumption for some vehicles.

Exercise 9.05

Car type	Fuel consumption (L/100 km)
Ford Falcon	11.7
Holden Commodore	11.3
Honda Jazz	5.7
Hyundai Sonata SLX	8.0
Lexus GS300	9.8
Mini Cooper	5.8
Mitsubishi Colt	5.6
Nissan Patrol	17.2
Peugeot 308	6.7
Toyota Aurion	9.9

a What is the range of fuel consumptions for these ten cars?

b What is the average fuel consumption for these ten cars?

c What is the rate in L/km for the:

i Peugeot 308? **ii** Honda Jazz? **iii** Holden Commodore?

d How many kilometres can the Ford Falcon travel on a 70 L tank of fuel?

e How much fuel does the Lexus use for a trip of 2500 km?

10 Unleaded petrol at a service station is 158.9 cents per litre. Diesel fuel is priced at 128.6 cents per litre.

What is the difference in cost of filling up tanks that hold 70 L?

Exercise 9.06

Chapter quiz

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Practice paper 3

Recommended time: One hour

Section 1

10 multiple-choice questions: 1 mark each
Select the correct answer, A, B, C or D.

Exercise 8.02

Question 1

A principal of $25 000 earns 3.5% p.a. simple interest. What is the interest earned in 7 months?

A $875 **B** $510.42 **C** $150.94 **D** $125

Exercise 9.05

Question 2

A car can travel 465 km on a 50 L tank of petrol. What is its fuel consumption in L/100 km?

A 0.9 **B** 1.1 **C** 9.3 **D** 10.8

Exercise 9.01

Question 3

Linda pays for Third Party Property motor insurance by a direct debit of $41.40 per month. How much would she save by paying a yearly amount of $399 instead?

A nothing **B** $97.80 **C** $169.80 **D** $185.20

Exercise 7.04

Question 4

T is directly proportional to M. When $M = 4$, $T = 10$. What is the value of T when $M = 5$?

A 20 **B** 12 **C** 12.5 **D** 2.5

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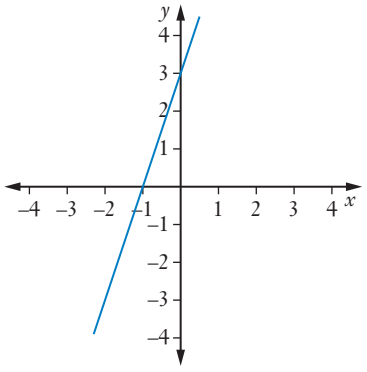
Question 5

Exercise 7.01

Find the equation of the graphed line.



A $y = 3 + x$ **B** $y = x - 3$

C $y = 3 - 3x$ **D** $y = 3x + 3$

Question 6

Exercise 9.02

The stamp duty for purchasing a car is \$3 per \$100 or part thereof up to \$45 000 and \$1350 plus \$5 for every \$100 or part thereof for purchases over \$45 000. How much stamp duty did Taylor pay when buying a car for \$39 450?

A \$1185 **B** \$1183.50 **C** \$1182 **D** \$1975

Question 7

Exercise 8.04

What principal must be invested at 5.7% p.a. compounded yearly for 6 years so that it grows to \$24 000?

A \$3784.30 **B** \$7017.55 **C** \$3784.30 **D** \$17 209.23

Question 8

Exercise 8.02

Cassie borrowed \$12 000 and after two years repaid the loan with a lump sum of \$13 800. What simple interest rate was Cassie charged?

A 7.5% **B** 6.5% **C** 7.25% **D** 0.75%

Question 9

Exercise 7.02

What is the gradient of the line joining the points (3, 5) and (8, 11)?

A $\frac{2}{3}$ **B** $-\frac{5}{6}$ **C** $\frac{6}{5}$ **D** $-\frac{6}{5}$

Question 10

Exercise 8.01

A car was bought for \$87 600 and decreased in value by 35% in the first year. What was its value at the end of the first year?

A \$56 940 **B** \$30 400 **C** \$30 660 **D** \$52 600

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Section 2
3 questions: 10 marks each

Question 11

Exercise 8.05

a Office furniture is purchased for $32 000 and depreciates by $3500 each year.

i Calculate its value after $3\frac{1}{2}$ years. **1 mark**

ii Find when the furniture's value will be $4000. **1 mark**

Exercise 7.01

b **i** What is the gradient and y-intercept of the line $y = -\frac{1}{2}x + 4$? **2 marks**

ii Graph the line on a number plane. **2 marks**

Exercise 9.02

c The stamp duty for purchasing a car is 3% of the purchase price up to $45 000 plus 5% for every dollar over $45 000.

i Copy and complete the table below. **2 marks**

Price $	0	10 000	20 000	30 000	40 000	45 000	50 000	60 000
Stamp duty $	0	300						

ii Use the completed table of values to construct a piecewise graph for stamp duty. **1 mark**

iii Use the graph to estimate the stamp duty on a vehicle purchased for $35 000. **1 mark**

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Question 12

a Briefly explain the main differences between Compulsory Third Party (CTP) insurance and Third Party Property insurance. **2 marks**

Exercise 9.01

b The table below shows the volume of water in kilolitres in a pool as it is drained.

Exercise 7.03

Time, t hours	2	3	6	8	9
Volume, V kL	73	67.5	51	40	34.5

i Find the linear function for V in terms of t. **1 mark**

ii What does the gradient of this function represent? **1 mark**

iii What does the vertical intercept of this function represent? **1 mark**

iv After how many hours, to one decimal place, is the pool drained completely? **1 mark**

c A principal of \$15 000 is invested at 4% p.a. compounded annually.

Exercise 8.04

i Find the interest earned in the first year. **1 mark**

ii What is the principal at the beginning of the second year? **1 mark**

iii Calculate the interest for the second year. **1 mark**

iv How much interest has been earned over the two years? **1 mark**

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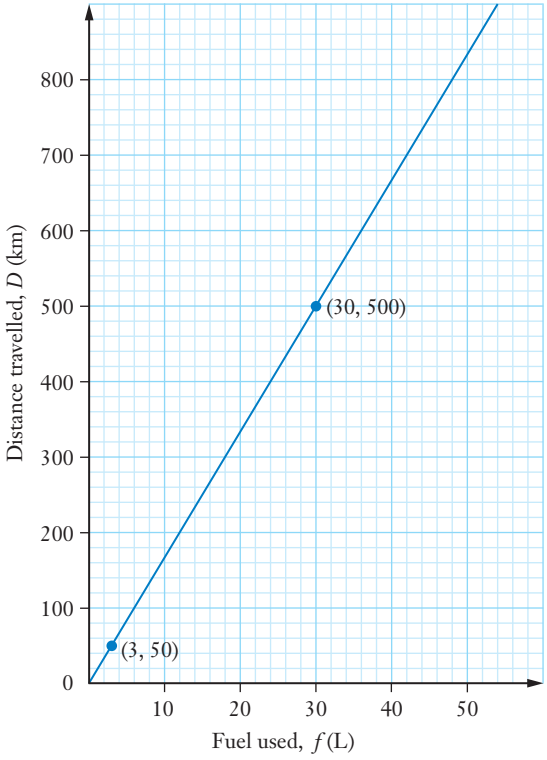
Exercise 7.04

Question 13

a The graph shows that the distance covered varies directly with the amount of fuel used.



i What is the gradient of the line? **1 mark**

ii What is the equation of the line? **1 mark**

iii Use the equation to find amount of fuel used to cover a distance of 900 km. **1 mark**

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b Tamim purchased a new ute for $48 250 to use in his lawn mowing business. His costs associated with his purchase are shown in the table.

Exercise 9.03

Purchase price	$48 250
GST	10% of purchase price
Stamp duty on GST price	$1350 plus $5 for every $100 (or part of $100) over $45 000
Registration per year	$736
CTP insurance	$387
Comprehensive insurance	$682

Calculate the on-road costs that Tamim has to pay for the ute. **3 marks**

c The fuel consumption of Robyn's car is 8.4 L/100 km.

Exercise 9.05

i Robyn drives 47 km each way to work 5 days per week.

Calculate how much petrol Robyn would use going to work each week. **2 marks**

ii She also uses the car on weekends, covering a distance of about 150 km. If Robyn pays 145.9 c/L for petrol, calculate her weekly fuel bill. **2 marks**

END OF EXAMINATION

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10.

ANALYSING DATA

The meteorologists at the Bureau of Meteorology measure and record weather data from over 1000 sites in Australia and Antarctica, and calculate statistics about temperature, rainfall and humidity. Climate averages, such as the median monthly rainfall or the mean number of rainy days per month, are calculated from weather data gathered over many years and can assist farmers to decide on the best times to plant crops.

CHAPTER OUTLINE

S1.2	10.01	The mean, median and mode
S1.2	10.02	Quartiles, deciles and percentiles
S1.2	10.03	The range and interquartile range
S1.2	10.04	The effect of outliers
S1.1	10.05	Cumulative frequency graphs
S1.2	10.06	Box plots
S1.2	10.07	Standard deviation
S1.2	10.08	The shape of a distribution

IN THIS CHAPTER YOU WILL:

- calculate and interpret the mean, median and mode of sets of data, including ungrouped data
- calculate and interpret the quartiles, deciles and percentiles of a set of data
- calculate and interpret the range, interquartile range and standard deviation of sets of data, including ungrouped data
- identify outliers in a set of data and examine their effects on statistical measures
- calculate cumulative frequency and construct cumulative frequency histograms and polygons
- use a cumulative frequency polygon to find the median, quartiles and interquartile range of a data set
- use a five-number summary to construct box plots
- describe the shape of a distribution using its graph or display

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TERMINOLOGY

box plot
class centre
class interval
cumulative frequency
cumulative frequency histogram
cumulative frequency polygon
decile
distribution
extremes
five-number summary
interquartile range
mean
measure of central tendency
measure of spread
median
median class
modal class
mode
ogive
outlier
peak
percentile
population
quartile
range
sample
standard deviation
summary statistics
symmetrical

SkillCheck

Assignment 10

1 This stem-and-leaf plot shows the ages of visitors entering the Royal Easter Show in a five-minute period.

Stem	Leaf
0	3 8 9
1	0 2 2 2 5 6 7 9
2	0 2 3 4 6 7
3	1 3 3 4 9
4	3 4 7 8
5	5 5 8

a How many visitors entered the show during the five-minute period?

b What was the age of the oldest visitor?

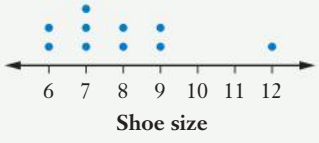
c What was the most common age?

d How many visitors were under 16 years old?

e What was the middle age?

2 Is a frequency histogram a line graph or a column graph?

3 The dot plot shows the shoe sizes of a sample of Year 11 students.



a How many students in the sample?

b What is the most common shoe size for these students?

c Find the outlier and describe the student that has this outlier.

d How many students had a shoe size of 10?

e What percentage of students had a shoe size over 8?

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4 A sample of students was surveyed about the number of cars owned by each of their families. The results are shown in the table.

Number of cars	Frequency
0	4
1	16
2	11
3	0
4	1

a How many families did not own a car?

b What was the most common number of cars owned?

c What was the highest number of cars owned?

d How many students were surveyed?

e What was the total number of cars owned?

5 The masses (in kilograms) of 40 skydivers were recorded. The results are shown below.

58	63	77	82	53	69	65	80	96	105
79	63	52	90	104	85	65	87	68	105
65	87	109	84	62	75	102	78	93	84
68	105	74	59	68	74	88	66	70	62

Copy and complete the frequency table below using the data about the mass of the skydivers.

Mass (kg)	Class centre	Frequency
$50 - < 60$		
$60 - < 70$		
$70 - < 80$		
$80 - < 90$		
$90 - < 100$		
$100 - < 110$		

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Mean, median and mode

Statistical measures

10.01 The mean, median and mode

The mean, **median** and **mode** are three **summary statistics** that represent the centre or average of a set of data. They are called the **measures of central tendency** (or **measures of location**).

The **mean** (or average) has the symbol $\bar{x}$, and is the sum of all scores divided by the number of scores.

The mean

$$\text{Mean, } \bar{x} = \frac{\text{sum of scores}}{\text{number of scores}}$$

$$= \frac{\Sigma x}{n}$$

Σ means 'the sum of'.
x represents a score.
n is the total number of scores.

If the scores in a data set are presented in a **frequency distribution table** then, by adding an 'fx' column, the mean can be calculated using the formula shown below.

Calculating the mean from a frequency table

$$\text{Mean, } \bar{x} = \frac{\text{sum of } fx}{\text{sum of } f}$$

$$= \frac{\Sigma fx}{\Sigma f}$$

The median and mode

When the scores are ordered from lowest to highest, the **median** is:

- the middle score, for an odd number of scores
- the average of the two middle scores, for an even number of scores.

The **mode** is the most common score or category. A set of data can have more than one mode, or no mode at all.

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EXAMPLE 1

For each data set below, find:

- **i** the mean (correct to one decimal place)
- **ii** the median
- **iii** the mode.

a The maximum daily temperature (in °C) in Mudgee for the first two weeks in January:

30 28 26 31 34 35 32 33 21 25 28 32 32 35

b A stem-and-leaf plot of the marks (out of 100) in a maths test for a class of students:

Stem	Leaf
4	4 7 7 8
5	2 6 8 9 9
6	1 3 5 5 7 8
7	0 2 3 4 5
8	3 7 8
9	2 8

Solution

a **i** Sum of scores (Σx) $= 30 + 28 + 26 + 31 + 34 + 35 + 32 + 33 + 21 + 25 + 28 + 32 + 32 + 35$

$= 422$

$$\text{Mean, } \bar{x} = \frac{\text{sum of scores}}{\text{number of scores}}$$

$$= \frac{422}{14}$$

$= 30.142\,85\ldots$

≈ 30.1

Note that the mean temperature of 30.1°C is at the centre of all 14 temperatures.

ii Placing the scores in order:

21 25 26 28 28 30 31 32 32 32 33 34 35 35

For 14 scores, the middle scores are the 7th and 8th scores.

$$\text{Median} = \frac{31+32}{2}$$

$= 31.5$

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iii Mode = 32 — The most common score (it occurred three times)

Note that the mean (30.1), median (31.5) and mode (32) are all around the same central value.

b i Mean, $\bar{x} = \frac{1671}{25}$ ← The sum of the 25 marks is 1671

$= 66.84$

≈ 66.8

ii

Stem	Leaf
4	4 7 7 8
5	2 6 8 9 9
6	1 3 5 **5** 7 8
7	0 2 3 4 5
8	3 7 8
9	2 8

For 25 scores, the middle score is the 13th score

Median = 65

iii Mode = 47, 59 and 65

The statistics mode on a calculator

Statistics mode: graphics calculator

Scientific and graphics calculators have a statistics mode (SD or STAT). Follow the instructions in the table below to calculate the mean of the temperatures from Example **1a** using your calculator's statistics mode.

Operation	Casio Scientific	Sharp Scientific
Start statistics mode.	MODE STAT 1-VAR	MODE STAT =
Clear the statistical memory.	SHIFT 1 Edit, Del-A	2ndF DEL
Enter data.	SHIFT 1 Data to get table 30 = 28 =, etc. to enter in column AC to leave table	30 M+ 28 M+, etc.
Calculate the mean. ($\bar{x} = 30.142\,85\ldots$)	SHIFT 1 Var $\bar{x}$ =	RCL $\bar{x}$
Check the number of scores. ($n = 14$)	SHIFT 1 Var n =	RCL n
Return to normal (COMP) mode.	MODE COMP	MODE 0

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Operation	Casio Graphics	Texas Instruments Graphics
Start statistics mode.	MENU STAT for Lists table	Y= and delete any function by highlighting it and pressing CLEAR STAT EDIT
Clear the statistical memory.	With cursor in List 1 column EXIT F6 DEL-A Yes	With cursor on L1 CLEAR ENTER
Enter data.	30 EXE 28 EXE, etc. to enter in List 1 column	30 ENTER 28 ENTER, etc. to enter in List 1 column
Calculate the mean. ($\bar{x}$ = 30.142 85…) Calculate the sum of scores. (Σx = 422) Check the number of scores. (n = 14)	F6 CALC SET to make these settings (if different): 1Var XList: List 1 1Var Freq: 1 EXE 1VAR to calculate many statistics (scroll down for more)	STAT CALC 1-Var Stats ENTER to calculate many statistics (scroll down for more)

The mean, median and mode from a frequency table

Statistics from a frequency table

EXAMPLE 2

The scores for the players in a nine-hole golf competition were sorted into the frequency table.

Score (x)	Frequency (f)
37	2
38	4
39	7
40	4
41	1

a How many players were there?

b For this data, find:

- **i** the mean (correct to one decimal place)
- **ii** the mode
- **iii** the median.

Solution

a 18 players

Sum of f = 18

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b **i**

Score (x)	Frequency (f)	fx
37	2	74
38	4	152
39	7	273
40	4	160
41	1	41
Totals	$\Sigma f = 18$	$\Sigma fx = 700$

fx means '$f \times x$'
$2 \times 37 = 74$
$4 \times 38 = 152$, etc.

This means that there were two scores of 37, four scores of 38, etc. The 'fx' column groups equal scores and adds them together.

$$\text{Mean, } \bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{700}{18}$$
$$= 38.888\,8\ldots$$
$$\approx 38.9$$

The sum of all 18 scores = 700

ii Mode = 39

39 has the highest frequency, 7

iii To the table, add a cumulative frequency column which keeps a running total of the frequencies.

Score (x)	Frequency (f)	Cumulative frequency	
37	2	2	
38	4	6	$2 + 4 = 6$
39	7	13	$6 + 7 = 13$, etc.
40	4	17	
41	1	18	

Because there are 18 scores, the two middle scores are the 9th and 10th scores. Reading from the cumulative frequency column, the 6th score is 38, and the 13th score is a 39, so the 9th and 10th scores must both be 39.

$$\text{Median} = \frac{39 + 39}{2} = 39$$

Note that the mean (38.9), median (39) and mode (39) are all at the centre of the data set.

Alternatively for part **i**, follow the instructions on the next page to use a calculator's statistics mode to calculate the mean of the golf scores.

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Operation	Casio Scientific	Sharp Scientific
Start statistics mode.	MODE STAT 1-VAR SHIFT MODE scroll down to STAT Frequency? ON	MODE STAT =
Clear the statistical memory.	SHIFT 1 Edit, Del-A	2ndF DEL
Enter data.	SHIFT 1 Data to get table 37 = 38 =, etc. to enter in x column 2 = 4 =, etc. to enter in FREQ column AC to leave table	37 2ndF STO 2 M+ 38 2ndF STO 4 M+ etc.
Calculate the mean. ($\bar{x}$ = 38.888 8…)	SHIFT 1 Var $\bar{x}$ =	RCL $\bar{x}$
Check the number of scores. (n = 18)	SHIFT 1 Var n =	RCL n
Return to normal (COMP) mode.	MODE COMP	MODE 0

Operation	Casio Graphics	Texas Instruments Graphics
Start statistics mode.	MENU STAT for Lists table	Y= and delete any function by highlighting it and pressing CLEAR STAT EDIT
Clear the statistical memory.	With cursor in List 1 column EXIT F6 DEL-A Yes Repeat for List 2	With cursor on L1 CLEAR ENTER Repeat for List 2
Enter data.	37 EXE 38 EXE, etc. to enter in List 1 column 2 EXE 4 EXE, etc. to enter in List 2 column	37 ENTER 38 ENTER, etc. to enter in List 1 column 2 ENTER 4 ENTER, etc. to enter in List 2 column
Calculate the mean. ($\bar{x}$ = 38.888 8…) Calculate the sum of scores. (Σx = 700) Check the number of scores. (n = 18)	F6 CALC SET to make these settings (if different): 1Var XList: List 1 1Var Freq: 1 EXE 1VAR to calculate many statistics (scroll down for more)	STAT CALC 1-Var Stats andtype 'L1, L2' by pressing 2nd 1 , 2nd 2 ENTER to calculate many statistics (scroll down for more)

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The mean of grouped data

For data grouped into **class intervals**, an **estimate** of the mean can be calculated using the **class centres**. It is only an estimate because, with class intervals, we do not know the exact value of every score.

EXAMPLE 3

The ages of the patients at a medical centre in one afternoon were recorded and grouped into this frequency table.

Age	Frequency
0–9	8
10–19	7
20–29	6
30–39	8
40–49	5
50–59	4
60–69	3
70–79	1

a Calculate, correct to one decimal place, the estimated mean age of the patients.

b How many patients went to the medical centre?

Solution

a

Age	Class centre, x	Frequency, f	fx
0–9	4.5	8	36
10–19	14.5	7	101.5
20–29	24.5	6	147
30–39	34.5	8	276
40–49	44.5	5	222.5
50–59	54.5	4	218
60–69	64.5	3	193.5
70–79	74.5	1	74.5
	Totals	$\Sigma f = 42$	$\Sigma f = 1269$

Estimate of the mean, $\bar{x} = \dfrac{\Sigma fx}{\Sigma f}$

$= \dfrac{1269}{42}$

$= 30.214\,2 \ldots$

≈ 30.2

Note that the estimated mean age of 30.2 is a central value of the data set.

b 42 patients $\Sigma f = 42$

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The median class and modal class of grouped data

Median class and modal class

The **median class** is the class interval that contains the median score.

The **modal class** is the most common class interval(s).

EXAMPLE 4

The monthly call costs of a sample of mobile phone users were grouped as shown in the cumulative frequency table on the right.

For this data, find:

a the median class

b the modal class.

Call cost ($)	Frequency	Cumulative frequency
0–< 20	6	6
20–< 40	8	14
40–< 60	13	27
60–< 80	17	44
80–< 100	23	67
100–< 120	20	87
120–< 140	16	103
140–< 160	10	113
160–< 180	4	117
180–< 200	3	120

Solution

a There are 120 scores. The two middle scores are the 60th and 61st scores.

From the cumulative frequency column, the 60th and 61st scores are in the 80–< 100 class.

The median class is 80–< 100.

b The modal class is 80–< 100. This class has the highest frequency, 23.

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Comparing measures of central tendency

A **measure of central tendency**, such as the mean, median or mode, describes the centre or average of a set of data. The following table summarises the three measures of central tendency.

Measure of central tendency	Features	When it is most appropriate
Mean $\bar{x} = \frac{\text{sum of scores}}{\text{number of scores}}$ $\bar{x} = \frac{\Sigma x}{n}$ $\bar{x} = \frac{\Sigma fx}{\Sigma x}$	Depends on all scores in the data set Is affected by outliers	When the data set does not have many outliers
Median Middle score or average of two middle scores	Not affected by outliers	When the data set has many outliers, for example house prices, salaries
Mode Most popular score(s)	Not affected by outliers	When the most common score or category is needed (for example dress size); also useful for categorical data

EXAMPLE 5

Which measure of central tendency is most appropriate for describing each of the following averages?

a the average price of a new car

b the most common number of bedrooms in a house

c a cricket player's batting average

d average weekly income

Solution

a Median, because there would be many outliers (the prices of expensive cars).

b Mode, because the most frequent score is needed.

c Mean, because all scores are required in the calculation.

d Median, because there would be many outliers (the incomes of very rich people).

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EXAMPLE 6

Ten houses were sold this week at Nelson Lakes for the following prices.

$376 000	$1 200 000	$270 000	$308 000	$372 000
$409 000	$387 000	$582 000	$460 000	$238 000

a Calculate the mean house price.

b Calculate the median house price.

c Which measure of central tendency is higher, the mean or the median?

d Which measure is more appropriate to describe the average house price?

Solution

a $$\text{Mean, } \bar{x} = \frac{4\ 602\ 000}{10}$$
$$= \$460\ 200$$

b Prices in order:

$238 000	$270 000	$308 000	$372 000	$376 000
$387 000	$409 000	$460 000	$582 000	$1 200 000

$$\text{Median} = \frac{\$376\ 000 + \$387\ 000}{2}$$
$$= \$381\ 500$$

Note that eight of the ten house prices are below the mean ($460 200).

c The mean is higher.

d The median, because it is not distorted by the outlier of $1 200 000.

Exercise 10.01 The mean, median and mode

Example 1

1 For each set of data below, find:

i the mean **ii** the median **iii** the mode.

a	1	1	2	5	5	7	9	10			
b	37	31	35	39	31	32	34	32	35	38	
c	28	40	38	42	45	29	31	41	30		
d	5	8	14	9	10	7	11	15	8	7	5

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2 The stem-and-leaf plot on the right represents the number of points scored by the Sharks in every round of the football season.

Stem	Leaf
0	6 6
1	2 3 4 4 4 8 8 9
2	0 0 0 5 6
3	0 0 2 4 4 6 7
4	0
5	
6	2

a How many rounds were played in the season?

b Calculate the mean score (correct to the nearest whole number).

c Find the median number of points scored.

d What is the mode?

3 Ngaire is training for a triathlon. She swam the following times, in minutes, in her last 10 races.

28 34 22 24 25 24 26 26 24 27

a Which of the following is Ngaire's mean swim time? Select **A**, **B**, **C** or **D**.

A 24 **B** 25 **C** 25.5 **D** 26

b Which of the following was her median swim time in minutes? Select **A**, **B**, **C** or **D**.

A 24 **B** 25 **C** 25.5 **D** 26

c Which of the following was Ngaire's modal swim time for the 10 races? Select **A**, **B**, **C** or **D**.

A 24 **B** 25 **C** 25.5 **D** 26

Example 2

4 'Average contents 50' is printed on each box of Meg's Matches. A quality controller counted the contents of a sample of 160 matchboxes from the production line and tabulated the results, as shown on the right.

Number of matches (x)	Frequency (f)
48	10
49	45
50	52
51	39
52	9
53	5

a Use an 'fx' column, or your calculator's statistics mode, to calculate the mean number of matches per box, correct to one decimal place.

b Is the claim 'Average contents 50' justified? Give a reason for your answer.

c Find the mode.

d Find the median.

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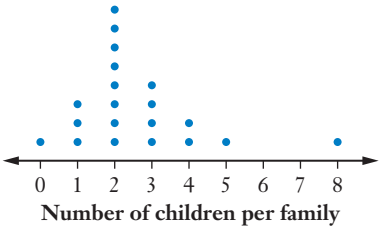
5 This dot plot shows the number of children in each family living on Willard Crescent.



a How many families live on Willard Crescent?

b Use a frequency table, or your calculator's statistics mode, to calculate the mean number of children per family.

c What is the median?

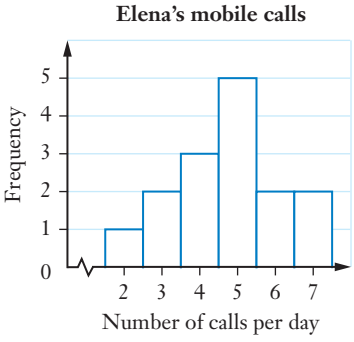
d What is the mode?

e What is the outlier?

f If the outlier is removed from the data set, how does this affect:

i the mean? **ii** the median? **iii** the mode?

6 This frequency histogram shows the number of mobile phone calls made by Elena each day over a number of days.



a Draw a frequency table for thus data, including an 'fx' column.

b Over how many days was the number of calls Elena made recorded?

c Find the mode of this data.

d Find the median of this data.

e Calculate the mean number of phone calls made by Elena per day, correct to one decimal place.

7 The police used radar to check the speeds of motor vehicles driving in a 40 km/h zone outside a local primary school one morning. They recorded the results in the table on the right.

Speed (km/h)	Number of cars, f
36–40	64
41–45	36
46–50	18
51–55	15
56–60	11
61–65	5

Example 3

a Add a column of class centres to the table and calculate an estimate for the mean speed of the vehicles, correct to two decimal places.

b How many motor vehicles had their speeds checked?

8 The heights of young trees in a section of nursery were measured before planting. The results are shown in the table on the right.

Height (cm)	Number of trees
20–29	28
30–39	45
40–49	74
50–59	63
60–69	24

Example 4

For this data, find:

a the median class

b the modal class.

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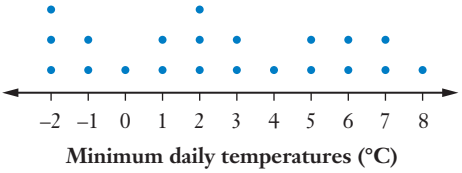
9 This dot plot shows the minimum daily temperatures (in °C) in Camden over a 3-week period.



a What is the mode?

b What is the median?

c Calculate the mean, correct to one decimal place.

10 The weekly wages of the staff at Yen's restaurant are shown in the frequency table.

Wage ($)	Number of employees
100–< 200	5
200–< 300	11
300–< 400	20
400–< 500	4
500–< 600	3
600–< 700	1

a What is the modal class for the wages?

b What is the median class?

11 Decide which *M* (mean, median or mode) is correct for each of the following.

a This *M* takes all scores in the data set into account.

b This *M* is one of the scores if there is an odd number of scores.

c Half of the scores are above this *M*, the other half are below.

d There can be more than one *M* in a set of data.

e This *M* often needs to be rounded to decimal places.

f This *M* can also be used for categorical data.

g This *M* can be distorted by many outliers.

h This *M* must be one of the scores in the data set.

Example 5

12 Which measure of central tendency is most appropriate for describing each average?

a the average exam mark for the class

b the average shirt size for teenage girls

c the average rent paid for a house in Sydney

d the average screen size of a notebook computer

e the average mass of football players in a team

f the average brand of mobile phone

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Example 6

13 A small business employs staff with the following salaries:

- general manager $158 300
- three factory hands $64 300 each
- supervisor $85 600
- two clerical officers $68 500 each

a How many people are there on staff?

b Calculate the mean salary of the staff, correct to the nearest $100.

c Calculate the median salary of the staff.

d Which measure of central tendency is higher, mean or median? Why?

e Which measure of central tendency best describes the average salary at this business?

14 The ages of the maths teachers at Westvale Christian College are:

49 32 37 32 25 41 39 50

a For this data, find:

i the mean **ii** the median **iii** the mode.

b The 39-year-old teacher is replaced by a new teacher, aged 22. Describe how this will affect:

i the mean **ii** the median **iii** the mode.

15 The colours of the new cars sold last week at Huxley Motors were recorded. The results are shown in the table below.

Colour	Black	Blue	Red	Silver	White
Frequency	4	7	7	9	12

a How many new cars were sold?

b What is the mode for this data?

c Why is the mode the only valid measure of central tendency here?

16 The weekly mortgage repayments (in dollars) of 11 home owners are:

370 628 299 417 354 1027 585 435 509 652 481

a For this data, find:

i the mean, correct to the nearest dollar

ii the median

iii the mode.

b Why isn't the mean or mode an appropriate measure of central tendency for this set of data?

c If the outlier is removed from the data, check whether the new mean will be closer to the new median than the mean was to the median for the original set of data.

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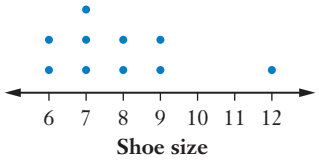
17 The dot plot on the right shows the shoe sizes of a sample of Year 11 students.



a For this data, find:
i the mean **ii** the median **iii** the mode.

b If the outlier is removed, state what will happen to:
i the mean? **ii** the mode?

c A shoe store needs to buy more shoes for a back-to-school sale. Which measure of central tendency is most appropriate for the store to use in this situation?

18 The stem-and-leaf plot on the right shows the maximum daily temperatures (in °C) in Port Macquarie for the last two weeks in December.

Stem	Leaf
2	2 4 4 5 6 6 7 7 7 8 8 9
3	1 4

Source: © Copyright Commonwealth of Australia 2017, Bureau of Meteorology

a For this data, find:
i the mean **ii** the median **iii** the mode.

b Which measure of central tendency is the most appropriate for describing the average maximum daily temperature?

TECHNOLOGY

Calculating measures of central tendency

Step 1: Open a blank spreadsheet to enter the following temperature data about Mudgee from Example **1** on page 403.

	A	B	C	D	E	F	G
1	**Maximum daily temperatures in Mudgee**						
2	30	28	26	31	34	35	32
3	33	21	25	28	32	32	35
4							
5				Mean			
6				Median			
7				Mode			
8							

Step 2: In cell E5, enter the formula =**AVERAGE(A2:G3)** to calculate the mean (30.142 85…).

Step 3: In cell E6, enter the formula =**MEDIAN(A2:G3)** to calculate the median (31.5).

Step 4: In cell E7, enter the formula =**MODE(A2:G3)** to calculate the mode (32).

If there is more than one mode in a data set, the spreadsheet displays *only one* of the modes.

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10.02 Quartiles, deciles and percentiles

Quantiles are points of a distribution or data set that separate the data into equal groups after the data has been sorted into order. Commonly used quantiles are **quartiles**, **deciles** and **percentiles**.

The median and quartiles

Quartiles

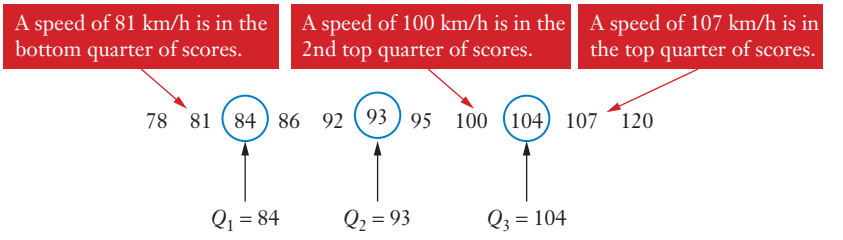
The three **quartiles** of a data set are those values that separate the data into quarters.

- The **lower quartile**, Q_1 or Q_L, separates the bottom quarter (25%) of scores from the rest of the scores.
- The **upper quartile**, Q_3 or Q_U, separates the top quarter (25%) of scores from the rest of the scores.
- The **middle quartile**, Q_2, is the median, and separates the two middle quarters.

These speeds (in km/h) were recorded for 11 cars driving along a major country road:

104 86 95 100 81 120 84 78 93 92 107

When we sort the scores, in ascending order, we can find the quartiles:



Quartiles of a data set

To find the quartiles of a data set:

Step 1: Sort the scores in order, find the median and call it Q_2.

Step 2: Find the median of the bottom half of scores and call it Q_1.

Step 3: Find the median of the top half of scores and call it Q_3.

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EXAMPLE 7

Find the quartiles for each data set below.

a The marks obtained by a class of students for an art project are:

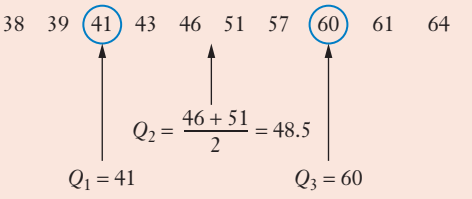
51 41 60 38 46 57 39 61 43 64

b The scores obtained by a golfer for the first nine holes of a golf course are:

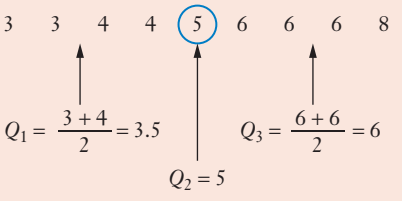
4 3 5 6 4 3 8 6 6

Solution

a First, sort the marks and place them in order:



b



Deciles

Quartiles (Q_1, Q_2 and Q_3) separate data into quarters.

Deciles (D_1, D_2, D_3, D_4, D_5, D_6, D_7, D_8 and D_9) separate data into tenths. *Deci-* means 'one tenth'.

For example:

- D_1 cuts off the lowest 10% of scores.
- D_4 cuts off the lowest 40% of scores.
- D_9 cuts off the lowest 90% of scores (or the top 10% of scores).

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EXAMPLE 8

The lengths (in centimetres) of 20 newborn infants at a hospital were recorded:

51	49	52	49	47	56	48	48	52	50
55	49	48	51	44	52	50	50	53	45

a What is the 3rd decile for this data?

b What is the 5th decile for this data?

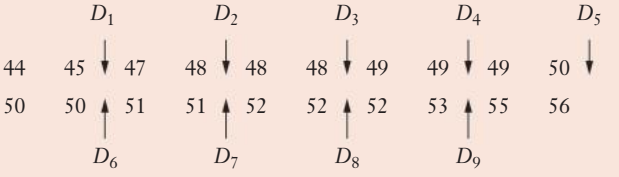
c What is another name for the 5th decile?

d Find the value that separates the bottom 70% of lengths from the top 30%.

e If the length of newborn baby James is in the top 10% of infant lengths, what value must it be greater than?

Solution

Place the values in order first:



a $D_3 = \frac{48+49}{2} = 48.5$

b $D_5 = \frac{50+50}{2} = 50$

c The median, because it cuts off the lowest 50% of scores.

d $D_7 = \frac{51+52}{2} = 51.5$

e James' length must be greater than $D_9 = \frac{53+55}{2} = 54$.

Percentiles

Percentiles ($P_1, P_2, P_3, \ldots P_{99}$) separate data into hundredths.

For example:

- P_{24} cuts off the lowest 24% of scores
- P_{60} cuts off the lowest 60% of scores
- P_{87} cuts off the lowest 87% of scores (or the top 13% of scores).

Deciles and percentiles are only meaningful when analysing large sets of data.

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EXAMPLE 9

The following information is based on population data for the heights of girls aged 16 years.

- The median is 163 cm.
- The 3rd quartile $Q_3 = 167$ cm.
- The 9th decile $D_9 = 171$ cm.
- The 5th percentile $P_5 = 152$ cm.
- The 97th percentile $P_{97} = 175$ cm.

In the following questions, all of the girls mentioned are aged 16.

a Holly's height is 175 cm. Is she tall for her age and what percentage of 16-year-old girls are taller than her?

b Olga is taller than 90% of girls her age. What is her height?

c If $\frac{1}{4}$ of girls her age are taller than Verity, how tall is she?

d What height separates the bottom 5% of heights from the top 95%?

e What percentile is a height of 163 cm?

Solution

a Yes, $P_{97} = 175$ cm, which means Holly is taller than 97% of girls her age. So only 3% of girls aged 16 are taller than her.

b Olga's height $= P_{90} = D_9 = 171$ cm.

c Verity is taller than $\frac{3}{4}$ of girls her age, so her height is $P_{75} = Q_3 = 167$ cm.

d $P_5 = 152$ cm

e 163 cm is the median, so it is also the 50th percentile P_{50} (the height that cuts off the lowest 50% of scores).

The median is the 2nd quartile Q_2, the 5th decile D_5 and the 50th percentile P_{50}.

Exercise 10.02 Quartiles, deciles and percentiles

Example 7

1 Find the quartiles Q_1, Q_2 and Q_3 for each data set below.

a The times, in seconds, to run 100 metres:

8.7 9.1 11.0 13.5 10.6 8.9 10.1 9.6
12.3 9.9 9.0 10.8 9.2 13.1 10.6

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b The number of matches in a box:

49 50 52 48 50 51 49 50 52 51 50 50

c The prices, in dollars, of a bag of potatoes:

3.50 3.20 3.50 4.10 3.00 3.50 3.90 2.80 3.40 3.00

d The weekly rainfall, in millimetres, over three months:

16 24 18 26 21 27 5
7 17 21 9 0 22

2 The stem-and-leaf plot on the right shows the game scores of a group of ten-pin bowlers.

For this data, find:

a the median

b the lower quartile

c the upper quartile.

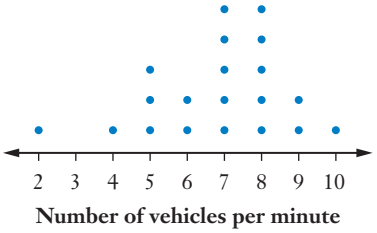
Stem	Leaf
8	2 7 8
9	0 3 4 6 9
10	4 4 5 8 8 8
11	2 3 4 6 7 9 9
12	0 0 5 6 6 8
13	1 1 4 7 9

3 The dot plot on the right shows the number of vehicles driving past Westvale High School per minute in a 20-minute period.

Which of the following is the upper quartile Q_3? Select **A**, **B**, **C** or **D**.

A 7.5 **B** 7

C 8.5 **D** 8



4 The percentage scores of a class of 30 students in a science test are shown below.

Example 8

61	75	46	78	81	95	67	61	50	74
100	57	83	64	69	95	85	89	66	45
71	87	84	80	63	92	64	75	97	60

a What is the 8th decile?

b What is the 3rd decile?

c What is the 40th percentile?

d Find the value that cuts off the lower 20% of scores from the upper 80%?

e What percentage of students scored higher than 79?

5 For the data shown in the dot plot in Question **3**, find:

a the 1st decile

b the 5th decile

c the value that cuts off the lower 70% of scores

d the value that cuts off the top 60% of scores

e the 90th percentile.

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Example 9

6 The following information is based on population data, for the body mass indices (BMI kg/m^2) of boys aged 16 years.

- The 1st quartile $Q_1 = 18.8$.
- The 1st decile $D_1 = 17.6$.
- The 9th decile $D_9 = 25.4$.
- The 50th percentile $P_{50} = 20.6$.
- The 97th percentile $P_{97} = 29.4$.

In the questions below, all of the boys mentioned are aged 16.

a Sanjay has the median BMI for boys his age. What is his BMI?

b Michael has a BMI of 18.8. Is this high for his age? What percentage of boys aged 16 have a BMI lower than him?

c 10% of boys aged 16 have a higher BMI than Harley. What is his BMI?

d Adrian has a BMI of 29.4. Is this high for his age? What percentage of boys aged 16 have a BMI higher than him?

e What percentile is a BMI of 17.6?

7 The information below is based on weather records kept by the Bureau of Meteorology for the maximum daily temperatures in November for Newcastle.

- The mean is 23.5°C.
- The highest temperature on record was 41.0°C (on 19 November 1968).
- The lowest temperature on record was 15.6°C (on 19 November 1986).
- The 1st decile $D_1 = 18.9$°C.
- The 9th decile $D_9 = 28.6$°C.

© Copyright Commonwealth of Australia 2017, Bureau of Meteorology

a What is the range of temperatures?

b What percentage of temperatures were higher than 28.6°C?

c To what value would you expect the median to be close (but not necessarily equal)?

d What value is higher than 10% of all temperatures recorded?

e What is the size of the 9th decile band (the difference between the highest temperature and the 9th decile)?

8 True or false?

a $P_{75} = Q_1$ **b** $P_{60} = D_6$ **c** $P_{50} = \text{Median}$

d $Q_3 = P_{75}$ **e** $D_8 = P_{20}$ **f** $Q_2 = D_5$

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9 This table was published by the University Admissions Centre (UAC) giving the percentiles of different Australian Tertiary Admission Rank (ATAR) for the 2015 HSC.

Percentile	40	50	60	70	80	85	90	95	99	100
ATAR	61.65	68.65	75.25	81.60	87.85	90.90	93.95	96.95	99.40	99.95

© 2016 Universities Admissions Centre (NSW & ACT)

a What percentage of HSC students scored an ATAR:

i below 75.25?

ii above 61.65?

iii between 93.95 and 99.40?

b What percentage of students scored an ATAR above the 90th percentile?

c Only 9.1% of students scored an ATAR of above what value?

d What is the median ATAR for the 2016 HSC?

e What is the percentile of an ATAR of 81.6?

10 This table shows the percentiles for the heights (in cm) of girls aged 2 to 5 years, according to the child growth standards of the World Health Organization (WHO).

Age (years)	P_5	P_{25}	P_{50}	P_{75}	P_{85}	P_{99}
2	80.4	83.5	85.7	87.9	89.1	93.2
2.5	84.9	88.3	90.7	93.1	94.3	98.9
3	88.8	92.5	95.1	97.6	99.0	103.9
3.5	92.4	96.3	99.0	101.8	103.3	108.5
4	95.6	99.8	102.7	105.6	107.2	112.8
4.5	98.7	103.1	106.2	109.2	110.9	116.7
5	101.6	106.2	109.4	112.6	114.4	120.5

© WHO 2017

a What is the median height of a 4-year-old girl?

b Libby is aged 2.5 and is 88.3 cm tall. Is she tall for her age? What percentage of girls her age are shorter than her?

c What is Libby's expected height when she turns 5 years old?

d Only 15% of girls Renee's age are taller than her. How tall is she if she is 3.5 years old?

e Mikayla is 2 years old and 93.2 cm tall. Is she short for her age? What percentage of girls her age are taller than her?

f Mia is aged 3 and her height is at the 3rd quartile. What is her height now and in 18 months time?

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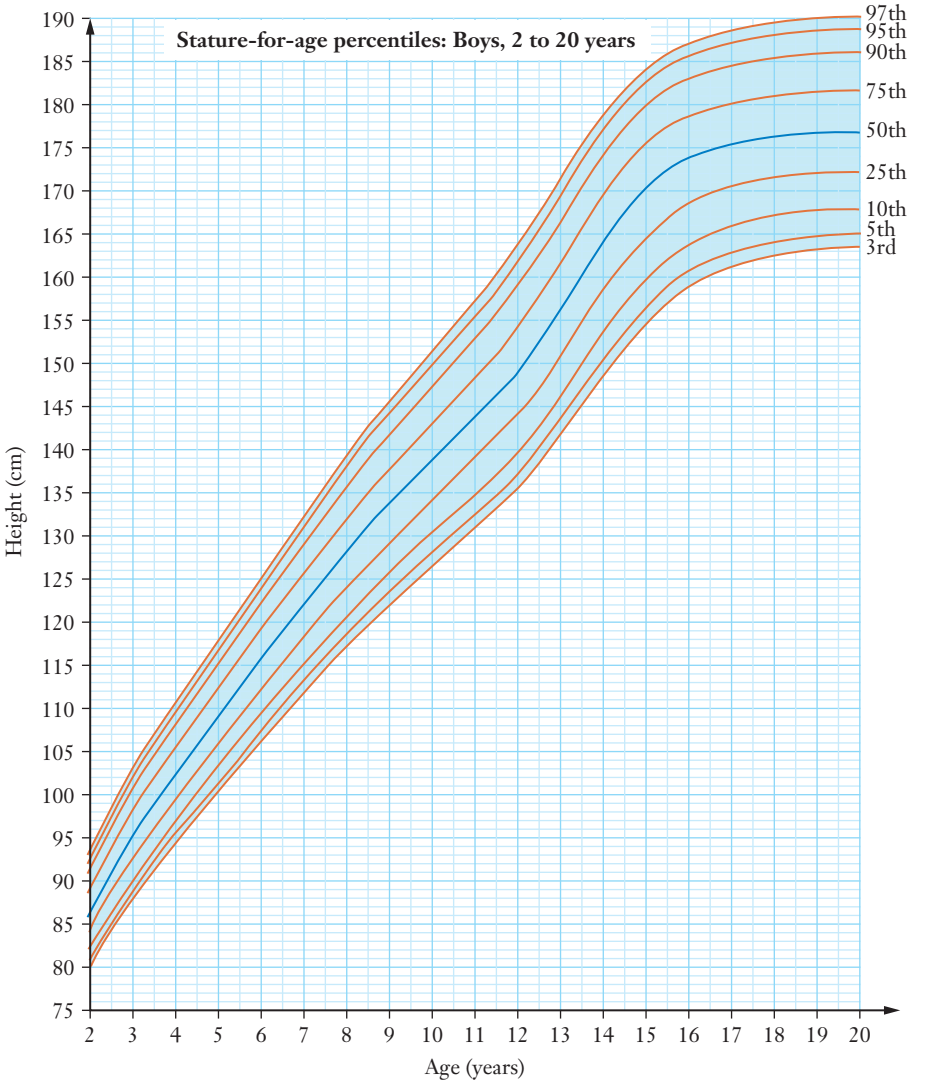
11 This stature-for-age percentiles chart shows the range of heights for boys aged 2 to 20 years.



Source: Developed by the National Center for Health Statistics in collaboration with the National Center for Chronic Disease Prevention and Health Promotion (2000) http://www.cdc.gov/growthcharts

a Adam is aged 9 and 129 cm tall. What percentage of boys his age are shorter than him?

b Justin is 11 years old and 155 cm tall. What percentage of boys his age are shorter than him?

c How tall should Justin be when he turns 18?

d Liong is 103 cm tall, which is at the 1st decile for boys his age. How old is Liong?

e Asam is 16 and his height is at the 3rd quartile.

i What is Asam's height now?

ii What will Asam's height be when he turns 20 years old?

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DID YOU KNOW?

Healthy growth charts for children

In 2006, the World Health Organization (WHO) started publishing growth charts based on good health standards rather than the general population. They selected 8440 children who grew up in optimal healthy environments, from six countries: Brazil, Ghana, India, Norway, Oman and USA. These children were chosen because they were well-fed, breastfed as infants, not obese, their mothers did not smoke, and they had access to good health care where infections were controlled and prevented.

Selecting children from six different countries to represent the world's children is an example of stratified sampling. Why do you think the WHO chose those particular countries?

TECHNOLOGY

Calculating quartiles and percentiles

A spreadsheet can be used to calculate the quartiles, deciles and percentiles of a set of data.

Step 1: Open a blank spreadsheet to enter data in rows 2 and 3 as shown using the infant lengths from Example **8** on page 419.

	A	B	C	D	E	F	G	H	I	J	K
1	LENGTHS OF INFANTS (CM)										
2		44	45	47	48	48	48	49	49	49	50
3		50	50	51	51	52	52	52	53	55	56
4											
5				1st quartile Q1		48					
6				3rd quartile Q3		52					
7				2nd decile D2		48					
8				7th decile D2		51.3					
9				32nd percentile P32		49					
10				95th percentile P95		55.05					

Step 2: In cell F5, enter the formula = **QUARTILE(B2:K3,1)** to calculate $Q_1 = 48$.

Step 3: In cell F6, enter = **QUARTILE(B2:K3,3)** to calculate $Q_3 = 52$.

Step 4: In cell F7, enter = **PERCENTILE(B2:K3,0.2)** to calculate $D_2 = 48$.

Step 5: In cell F8, enter = **PERCENTILE(B2:K3,0.7)** to calculate $D_7 = 51.3$.

Step 6: In cell F9, enter = **PERCENTILE(B2:K3,0.32)** to calculate $P_{32} = 49$.

Step 7: In cell F10, enter = **PERCENTILE(B2:K3,0.95)** to calculate $P_{95} = 55.05$.

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Interquartile range

10.03 The range and interquartile range

While the mean, median and mode describe the centre of a data set, there are three summary statistics that describe the **spread** of data: the **range**, the **interquartile range** and the **standard deviation**. These are called **measures of spread**.

Interquartile range

Interquartile range

Range and interquartile range

Range = highest score – lowest score

Interquartile range (IQR) = upper quartile – lower quartile = $Q_3 - Q_1$

Standard deviation will be explained later in this chapter.

Statistical match-up

EXAMPLE 10

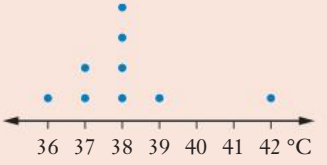
For each data set below, find:

i the range **ii** the interquartile range.

a The maximum daily temperature (in °C) in Mudgee for the first two weeks in January:

30 28 26 31 34 35 32 33 21 25 28 32 32 35

b The body temperatures (in °C) of a sample of hospital patients, as shown in the dot plot on the right.



Patients' temperatures

Solution

a **i** Range = 35 – 21 = 14

ii Placing the scores in order:

21 25 26 (28) 28 30 31 32 32 32 (33) 34 35 35

$Q_1 = 28$ Q_2 $Q_3 = 33$

$$\begin{aligned} \text{IQR} &= Q_3 - Q_1 \\ &= 33 - 28 \\ &= 5 \end{aligned}$$

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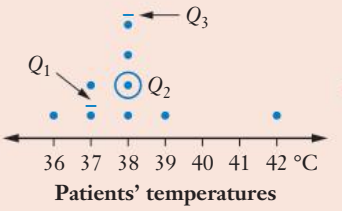
b **i** Range = 42 − 36 = 6



Patients' temperatures

Of 9 scores, the median, Q_2, is the 5th score, counting upwards from the left.

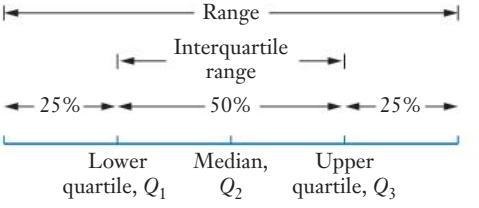
ii $Q_1 = \frac{37+37}{2} = 37$, $Q_3 = \frac{38+39}{2} = 38.5$

$\text{IQR} = Q_3 - Q_1$

$= 38.5 - 37$

$= 1.5$

The **range** represents the total spread of scores but it is not a good measure if there are outliers. The **interquartile range** is not affected by outliers, because it measures the range of the middle two quarters only.



Exercise 10.03 The range and interquartile range

Example 10

1 Calculate the range of each data set.

a Number of accidents per month in a factory:

3 0 0 1 2 1 6 0 0 2 1 0

b A golfer's scores for the first nine holes of a golf course:

4 3 5 6 4 3 8 6 6

c Weekly mortgage repayments, in dollars:

370 628 299 417 354 1027 585 435 509 652 481

d Times, in minutes, for the swim-leg of a triathlon:

28 34 22 24 25 24 26 26 24 27

2 Calculate the interquartile range of each data set in Question **1**.

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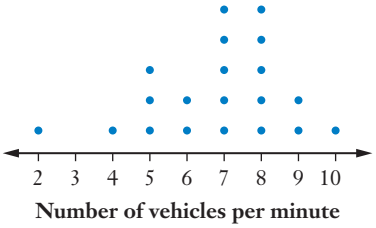
3 The dot plot on the right shows the number of vehicles driving past Westvale High School per minute in a 20-minute period.



Which of the following is the interquartile range of this data set? Select **A**, **B**, **C** or **D**.

A 2.5 **B** 3 **C** 5 **D** 8

4 This stem-and-leaf plot on the right shows the marks out of 100 for a class of students in a maths test.
For this data, find:

a the range

b the interquartile range.

Stem	Leaf
3	0 7
4	2 3 4 6 8
5	0 1 4 5 7 7
6	2 3 5 7 8 8
7	4 5 6 9
8	2 2 7
9	3

5 Fifteen job applicants took a short general knowledge multiple-choice quiz. Their times (in seconds) to complete this test were:

45 37 46 34 26 15 35 61
43 48 52 38 30 44 37

a What was the range of times?

b What was the interquartile range?

c Give a possible reason for the outlier.

6 This stem-and-leaf plot below represents the number of points per match scored by the GWS Giants in a football season.

Stem	Leaf
7	8 9
8	3 5 8 9
9	1 2 3 5 8
10	0 5
11	1 7
12	6 7 9
13	
14	6 9
15	1 8

Getty Images/Matt King

Which of the following is the interquartile range of this data set? Select **A**, **B**, **C** or **D**.

A 80 **B** 38 **C** 44 **D** 42

7 Calculate the range of the data set from Question **6**.

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10.04 The effect of outliers

An **outlier** is a very high or very low score in a data set that is clearly apart from the other scores. It can occur for a variety of reasons and should be investigated. If it was obtained through incorrect measurement, it should be excluded.

Outliers

An **outlier** is a score that is either:

This is only one of many ways of determining whether a score is an outlier.

- less than $Q_1 - 1.5 \times \text{IQR}$ or
- greater than $Q_3 + 1.5 \times \text{IQR}$

where Q_1 (or Q_L) is the lower quartile, Q_3 (or Q_U) is the upper quartile, and IQR is the interquartile range.

EXAMPLE 11

The following scores are marks achieved by students in a test.

11	8	12	12	15	13	10	25	12	11	7
10	13	16	10	12	16	11	12	16	17	20

Test which scores are outliers.

Solution

The scores arranged in order are:

7 8 10 10 10 11 11 11 12 12 12 12 12 13 13 15 16 16 16 17 20 25

Q_1 Q_2 Q_3

$\text{IQR} = Q_3 - Q_1$

$= 16 - 11$

$= 5$

$\therefore 1.5 \times \text{IQR} = 1.5 \times 5$

$= 7.5$

$\therefore Q_1 - 1.5 \times \text{IQR} = 11 - 7.5$

$= 3.5$

$Q_3 + 1.5 \times \text{IQR} = 16 + 7.5$

$= 23.5$

$\therefore$ A score is an outlier if it is less than 3.5 or greater than 23.5.

$\therefore$ 25 is an outlier.

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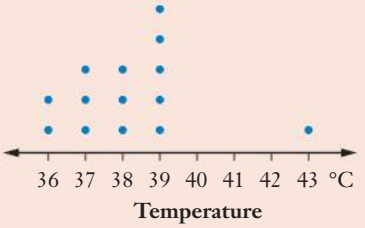
Outliers and measures of central tendency

Outliers can affect the measures of central tendency of a data set.

- The mean is most affected by outliers (because its value depends on every score).
- The median can be affected, but not by much.
- The mode is not affected at all.

EXAMPLE 12

The dot plot shows the temperatures of patients in a hospital ward.



a Calculate the mean, mode and median of this data set.

b What is the outlier temperature?

c Calculate the mean, mode and median of this data set if the outlier is excluded.

d Describe the effect the outlier has on the measures of central tendency of the distribution.

Solution

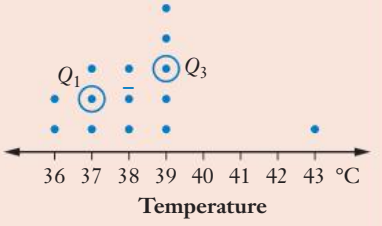
a $\text{Mean} = \dfrac{535}{14} \approx 38.2$

$\text{Mode} = 39$

$\text{Median} = \dfrac{38+38}{2} = 38$ The average of the 7th and 8th scores

b From the dot plot, $Q_1 = 37$, $Q_3 = 39$

$$\begin{aligned}\text{IQR} &= Q_3 - Q_1 \\ &= 39 - 37 \\ &= 2\end{aligned}$$



$$\begin{aligned}1.5 \times \text{IQR} &= 1.5 \times 2 \\ &= 3\end{aligned}$$

If 43 is an outlier, it must be greater than $Q_3 + 1.5 \times \text{IQR}$.

$$\begin{aligned}\therefore\ Q_3 + 1.5 \times \text{IQR} &= 39 + 3 \\ &= 42\end{aligned}$$

$\therefore$ the outlier is 43.

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c $\text{Mean} = \frac{492}{13} \approx 37.8$

Mode = 39

Median = 38

d The high outlier does not affect the mode and median but it increases the mean.

Exercise 10.04 The effect of outliers

Example 11

1 The following scores are the number of goals scored by a hockey team during a season.

3 2 0 0 1 2 3 2 4 8
2 3 5 2 1 3 4 4 2 3

a Find the interquartile range.

b Find the value of:

i $Q_1 - 1.5 \times \text{IQR}$ **ii** $Q_3 + 1.5 \times \text{IQR}$

c Is the score of 8 goals an outlier? Give reasons.

2 Determine whether each data set has outliers.

a 2 5 6 6 7 8 10 10 15

b 9 13 13 14 14 15 15 15 15
16 16 16 16 16 17 17 18

c

Stem	Leaf
1	2 9
2	0 3 4 4 8
3	4 5 6 7
4	1 4 9
5	0 2
6	8

d

Score (x)	Frequency (f)
4	3
5	12
6	4
7	3
8	0
9	1

Example 12

3 The employees at the Bread and Butter Cafe earned the following wages in a week.

\$450 \$520 \$610 \$230 \$900 \$420 \$590

a What is the mean wage?

b What is the median wage?

c Find the interquartile range.

d The manager's wage is an outlier. What is this wage and how do we verify that it is an outlier?

e If the manager's wage is not included, how does this affect the mean and median wage?

f If each employee receives a 10% pay rise, what will be the new mean and median wage? Is it 10% more than the old mean and median?

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4 The cups of coffee drunk by a sample of HSC exam markers in one night is shown in the table.

Cups of coffee	No. of markers
2	1
3	4
4	5
5	9
6	0
7	0
8	1

a How many markers were surveyed?

b What is the outlier?

c What is the mean if the outlier:
i is included? **ii** is not included?

d If the outlier is included, what effect does this have on the mean number of cups of coffee that were drunk?

5 A group of friends goes to the cinema. The ages of the group are

13 12 11 14 12 15 14 13.

If Kait brings her 5-year-old sister as well, what will happen? Select **A**, **B**, **C** or **D**.

A The median age increases.

B The median age decreases.

C The mean age increases.

D The mean age decreases.

6 In a netball tournament of five matches, the points scored by three teams are:

The Wombats	24	18	14	6	22
The Possums	16	16	15	18	15
The Koalas	36	8	14	16	12

a What are the mean and median scores for each team?

b Which team is the most consistent? Why?

c An error was made in the scoring for the Wombats – the score of 6 should have been 16. What are the new mean and median?

d Which team is most consistent now? Why?

7 Sam and Terri sell copiers. The numbers of copiers that they sell each week are sorted in ascending order.

Sam	1	2	3	3	5	6	7	8	12	25
Terri	3	3	3	14	16	18	18	24	32	35

a What is the modal number of copiers sold by each person?

b What could you say about each person if you only knew the mode?

c What is the median number of copiers sold by each person?

d What is the mean number of copiers sold by each person?

e Which measure of central tendency, mean, median or mode, is best for comparing their sales performances?

f Who is the better salesperson? Justify your answer.

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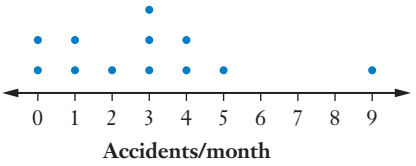
8 This dot plot represents the number of accidents at a factory each month over a year.



a Calculate the mean, mode and median of this data set.

b What is the outlier number of accidents? Explain why.

c Calculate the mean, mode and median of this data set if the outlier is excluded.

d Describe the effect the outlier has on the three measures of central tendency.

9 Rupert's bookstore employs the following people with annual wages as shown.

1 store manager	$73 800
2 cashiers	$34 200 each
2 part-time clerical staff	$28 500 each
3 salespeople	$46 500 each
2 part-time cleaners	$13 500 each

a Find the mean, median and modal annual salary for the 10 employees.

b Which measure of central tendency would Rupert use to make the salaries appear higher? Why?

c Which measure best represents the average wage for an employee at Rupert's bookstore? Why?

DID YOU KNOW?

The *Challenger* space shuttle disaster

In January 1986, an engineer working on the space shuttle program at NASA predicted that at low air temperatures, the potential for damage to the shuttle would be extremely high. For a temperature of 12°C, he calculated a damage index of 11. He compared this to data from previous flights (as shown in the table below) and recommended that the *Challenger* flight be delayed due to the low air temperature on the day.

Year	Data from previous flights				1986
Air temperature (°C)	26	14	19	23	12
Damage index	0	4	0	0	11

However, his advice was ignored and the outlier was not considered important enough to delay the flight. The *Challenger* exploded just after takeoff, killing all seven astronauts. Later it was found that two rubber O-rings had failed to seal a joint at low temperatures, causing the shuttle to disintegrate.

Give another example of when an outlier should not be ignored.

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Cumulative frequency graphs

10.05 Cumulative frequency graphs

A **cumulative frequency histogram** is a column graph of **cumulative frequency**.

A **cumulative frequency polygon**, also called an **ogive** (pronounced 'oh-jive') is drawn by joining the **top right-hand corner** of each column of a cumulative frequency histogram.

EXAMPLE 13

The maximum daily temperatures (in °C) in Campbelltown in June were recorded and grouped into the frequency table.

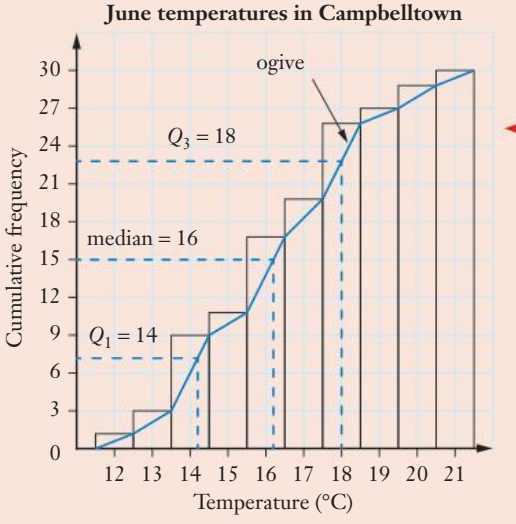
Temperature (°C)	Frequency	Cumulative frequency
12	1	1
13	2	3
14	6	9
15	2	11
16	6	17
17	3	20
18	6	26
19	1	27
20	2	29
21	1	30

a Draw a cumulative frequency histogram and polygon for the data.

b Use the frequency polygon to find the median and calculate the interquartile range.

Solution

a



The ogive (polygon) is contained inside the columns.

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b Draw a horizontal line from the halfway mark (15) on the cumulative frequency axis to where it meets the ogive. The median is the corresponding value on the 'Temperature' axis.

Median = 16

To find Q_1, draw a horizontal line from the quarter mark $\left(\frac{1}{4} \times 30 = 7.5\right)$ on the cumulative frequency axis to where it meets the ogive, then read the temperature value.

$Q_1 = 14$

To find Q_3, draw a horizontal line from the three-quarter mark $\left(\frac{3}{4} \times 30 = 22.5\right)$ on the cumulative frequency axis.

$Q_3 = 18$

Interquartile range $= Q_3 - Q_1$

$= 18 - 14$

$= 4$

EXAMPLE 14

a Use the cumulative frequency graph from Example **13** to find:

i the 4th decile, D_4 **ii** the 7th decile, D_7.

b What value cuts off the top 20% of temperatures?

c Between which two deciles would you find a temperature of 14°C?

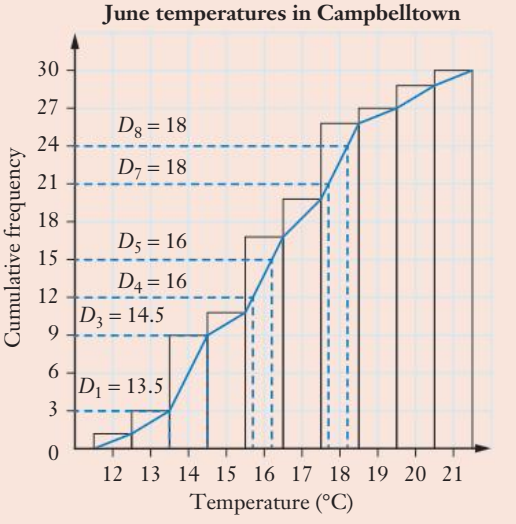
Solution

a The deciles are marked at intervals of 3 units on the cumulative frequency axis.

i $D_4 = 16$ **ii** $D_7 = 18$

b D_8 cuts off the top 20% of temperatures, so the value is 18.

c Between D_1 and D_3.



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EXAMPLE 15

The number of cases of ovarian cancer in women from various age groups is shown below.

Age (years)	Class centre	Frequency	Cumulative frequency
35 –< 45	40	28	28
45 –< 55	50	61	89
55 –< 65	60	65	154
65 –< 75	70	92	246
75 –< 85	80	74	320

Draw an ogive for this data and use it to find an estimate for:

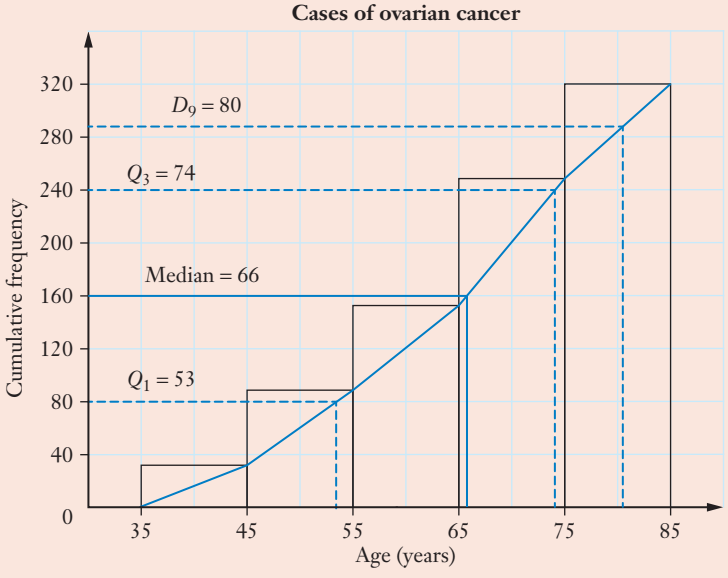
a the median

b the 3rd quartile

c the 9th decile

d the interquartile range.

Solution



a Halfway point on the 'Cumulative frequency' axis = 160

Median ≈ 66 Estimating from the 'Age' axis

All these values are estimates because the data has been grouped into class intervals.

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b The three-quarter point on the 'Cumulative frequency' axis $= \frac{3}{4} \times 320 = 240$

$Q_3 = 74$

c 90% point on the 'Cumulative frequency' axis $= 0.9 \times 320$

$= 288$

$D_9 \approx 80$

d Quarter point on the 'Cumulative frequency' axis $= \frac{1}{4} \times 320$

$= 80$

$Q_1 = 53$

Interquartile range $= Q_3 - Q_1 = 74 - 53$

$= 21$

Exercise 10.05 Cumulative frequency graphs

Example 13

1 A sample of households was surveyed on the number of TVs owned.

TVs owned	Frequency	Cumulative frequency
1	1	
2	7	
3	9	
4	6	
5	0	
6	1	

a Copy the table and complete the cumulative frequency column to find the median.

b Construct a cumulative frequency histogram and polygon.

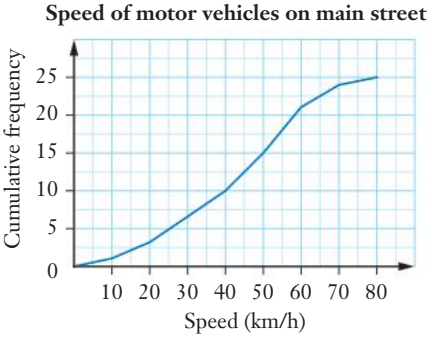
c Use the graphs you drew in part **b** to find:

i the median

ii the interquartile range.

Example 14

2 This ogive shows the speeds of motor vehicles travelling along the main street of a town.



a How many vehicles were in the survey?

b Estimate the median speed of the vehicles

c Estimate the interquartile range.

d Estimate the 9th decile.

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3 A packet of jelly beans is labelled 'Contents 30' but a quality control check found the results shown in the table.

Number of jelly beans	Frequency	Cumulative frequency
28	6	
29	34	
30	56	
31	28	
32	5	
33	1	

a Copy the table and complete the cumulative frequency column.

b Construct an ogive and use it to find an estimate of:

i the median

ii the interquartile range

iii the 4th decile.

Example 15

4 The heights of 50 students were measured and grouped into class intervals.

Height (cm)	Class centre	Frequency	Cumulative frequency
134 –< 141		2	
141 –< 148		3	
148 –< 155		4	
155 –< 162		13	
162 –< 169		15	
169 –< 176		11	
176 –< 183		2	

a Copy and complete the table.

b What is the modal class?

c What is the median class?

d Construct an ogive and use it to estimate:

i the median **ii** the interquartile range **iii** the 7th decile.

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10.06 Box plots

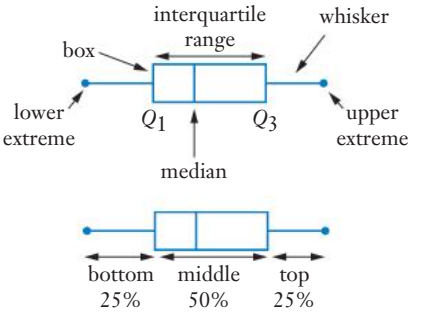
Box-and-whisker plots

Box plots: graphics calculator

A **box plot** (or **box-and-whisker plot**) displays the quartiles of a set of data and the lowest and highest scores. The 'box' represents the middle 50% of scores and the interquartile range, while the 'whiskers' represent the lowest and highest 25% of scores.

A box plot gives a **five-number summary** of a data set:

- the **lower extreme** (lowest score)
- the lower quartile, Q_1
- the median, Q_2
- the upper quartile, Q_3
- the **upper extreme** (highest score).



EXAMPLE 16

The ages of 10 people at a park were:

21 13 64 75 35 83 7 71 18 29

a Find a five-number summary for this data.

b Represent this data on a box plot.

Solution

a In order:

7 13 (18) 21 29 35 64 (71) 75 83

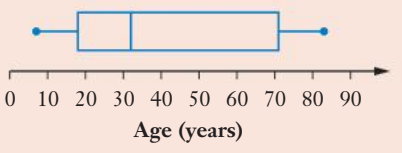
Q_1 Q_2 Q_3

Lower extreme = 7 Lower quartile = 18 Median $= \frac{29+35}{2} = 32$

Upper quartile = 71 Upper extreme = 83

The five-number summary for the ages is 7, 18, 32, 71, 83.

b



This box plot shows that, roughly:
- the bottom 25% of scores lie from 7 to 18
- the next 25% of scores lie from 18 to 32
- the median is 32
- the top 25% of scores lie from 71 to 83.

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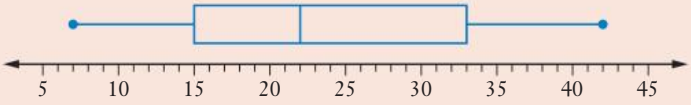
EXAMPLE 17

This box plot represents the amount of pocket money in dollars earned by a sample of 48 children.



Pocket money ($)

a Find the median.

b Find the range.

c How many children earned between:

i $33 and $42? **ii** $15 and $42?

d Find the interquartile range.

Solution

a Median = $22

b Range = $42 − $7 = $35

c **i** $\frac{1}{4} \times 48$ children = 12 children Top 25%

ii $\frac{3}{4} \times 48$ children = 36 children Top 75%

d Interquartile range = $33 — $15 = $18

Double box plots

Parallel box plots

Parallel box plots can be used to represent two or more sets of data. They are drawn on the same scale above each other.

EXAMPLE 18

The mean maximum monthly temperatures for Sydney and Melbourne are shown in this table.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Sydney	25.9	25.8	24.8	22.5	19.5	17.0	16.4	17.9	20.1	22.2	23.7	25.2
Melbourne	26.0	25.8	23.9	20.3	16.7	14.1	13.5	15.0	17.3	19.7	22.0	24.2

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a Find the five-number summary for each city.

b Draw a parallel box plot to display the data.

c For each city, find:

i the range **ii** the interquartile range.

d Compare the temperatures for both cities. Are there significant differences between the spread of the temperatures for Sydney and Melbourne?

Solution

a In order:

Sydney

16.4 17.0 17.9 19.5 20.1 22.2 22.5 23.7 24.8 25.2 25.8 25.9

Q_1 Q_2 Q_3

Lower extreme = 16.4 Lower quartile $= \dfrac{17.9+19.5}{2}$

$= 18.7$

Median $= \dfrac{22.2+22.5}{2}$

$= 22.35$

Upper quartile $= \dfrac{24.8+25.2}{2}$ Upper extreme = 25.9

$= 25.0$

Melbourne

13.5 14.1 15.0 16.7 17.3 19.7 20.3 22.0 23.9 24.2 25.8 26.0

Q_1 Q_2 Q_3

Lower extreme = 13.5 Lower quartile $= \dfrac{15.0+16.7}{2}$

$= 15.85$

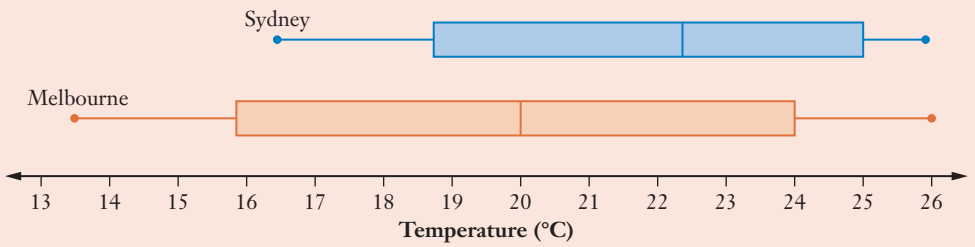
Median $= \dfrac{19.7+20.3}{2}$

$= 20.0$

Upper quartile $= \dfrac{23.9+20.3}{2}$ Upper extreme = 26.0

$= 24.05$

b



c **i** Sydney: Range = 25.9 – 16.4 = 9.5

Melbourne: Range = 26.0 – 13.5 = 12.5

ii Sydney: Interquartile Range = 25.0 – 18.7 = 6.3

Melbourne: Interquartile Range = 24.05– 15.85= 8.2

d The range of temperatures in Melbourne is 3° more than that of Sydney and the IQR is 1.9° more so there is a significant difference. Sydney's mean maximum monthly temperatures are more consistent than Melbourne's.

Example 16

Exercise 10.06 Box plots

1 Tom's scores for the 18 holes of a golf course were:

3 4 6 8 7 9 5 9 11

5 7 4 5 8 6 9 10 5

a Find a five-number summary for this data.

b Represent this data on a box plot.

2 Fifteen job applicants took a short general knowledge multiple-choice quiz. Their times, in seconds, to complete this test were as shown below. Show this data on a box plot.

45 37 46 34 26 15 35 61

43 48 52 38 30 44 37

3 Find a five-number summary for the data in this stem-and-leaf plot of ages of people at the cinema, then draw a box plot for them.

Stem	Leaf
1	4 7 7 8
2	6 8 9 9
3	1 3 5 5 7 8
4	0 2 2 4 5
5	3 7 8
6	2 9

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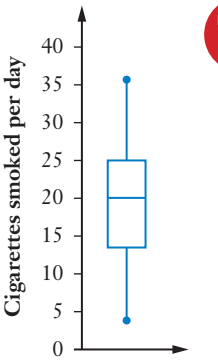
Example 17

4 This box plot illustrates the number of cigarettes smoked per day by a sample of 60 smokers who are trying to quit.



a What is the median number of cigarettes smoked per day?

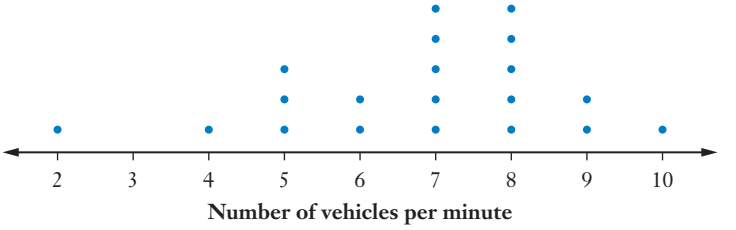
b What is the interquartile range?

c What is the lower extreme?

d How many people smoked between 20 and 25 cigarettes per day?

e How many people smoked fewer than 20 cigarettes per day?

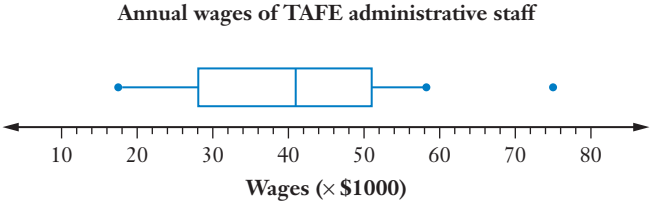
5 This dot plot shows the number of vehicles driving past Westvale High School per minute in a 20-minute period.



a Find the five-number summary for this data and draw a box plot.

b Compare the box plot you drew in part a with the original dot plot. Which one do you prefer? Why?

6 This box plot represents the annual wages (× $1000) of the administration staff at a TAFE college.



a One of the wages is an outlier which was not included in the box plot. What is the outlier?

b What is the median wage?

c Excluding the outlier, what is the range of wages?

d Including the outlier, what is the range of wages?

e Between what two amounts are the middle 50% of staff wages?

f What percentage of the staff earn less than $28 000?

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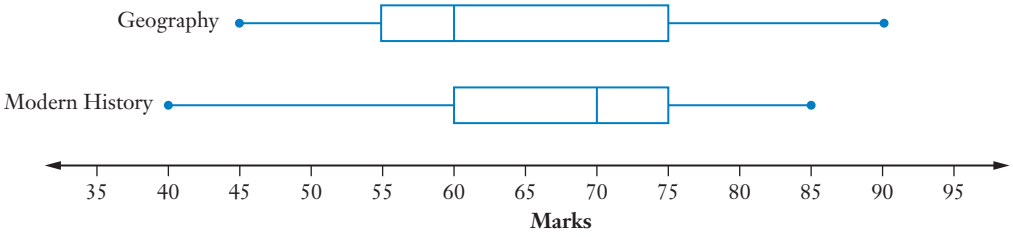
Example 18

7 In Year 11, the results of the first assessment task of 40 students who do both Modern History and Geography, are displayed on the parallel box plot below.



a Find the five-number summary for each subject.

b For each subject, find:

i the range **ii** the interquartile range.

c What is the median for each subject?

d Which subject has the least spread? Give reasons.

e How many students scored between 60 and 75 in:

i Geography? **ii** Modern History?

f In which subject did the Year 11 students perform better? Give reasons.

8 Year 12 students at Baramvale High had their pulse taken. The results are as follows.

Male	106	70	69	58	60	68	64	63	75	70	84	88	59	60	66
Female	68	74	59	75	74	82	82	71	120	55	77	91	73	60	79

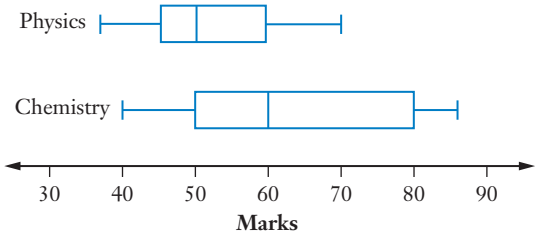
a Find the five-number summary for each group and draw a parallel boxplot to show the information.

b Find the range and interquartile range for each group.

c Compare the spread between the two groups. Are there significant differences between the pulse rates for males and females?

d Which group had the lower pulse rates. Give reasons.

9 The box plot shows the results of tests in Physics and Chemistry.



In Chemistry, 48 students completed the yearly exam and the number of students who scored above 50 or more was the same for both subjects.

How many students completed the Physics exam? Select **A**, **B**, **C** or **D**.

A 24 **B** 12 **C** 54 **D** 72

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10 Fifteen people at a health centre had their reaction times (in seconds) tested first using their dominant hand and then their non-dominant hand. The results are shown in the table on the right.

a Find the five-number summary for both sets of results and draw a parallel box plot to display the data.

b Find the range and interquartile range for the dominant hand and the non-dominant hand.

c Are there significant difference between the two sets of results.

Dominant hand	Non-dominant hand
0.41	0.48
0.31	0.34
0.38	0.38
0.50	0.45
0.38	0.38
0.33	0.35
0.36	0.30
0.46	0.45
0.29	0.9
0.44	0.41
0.52	0.50
0.43	0.41
0.37	0.40
0.31	0.34
0.32	0.35

10.07 Standard deviation

Statistical calculations

Standard deviation is a better **measure of spread** than the range and interquartile range because, like the mean, its value depends on every score in the data set. Standard deviation measures how different each score in a data set is from the mean.

The formula for calculating standard deviation is quite complicated, and does not need to be learnt. Instead, you can use your calculator's statistics mode.

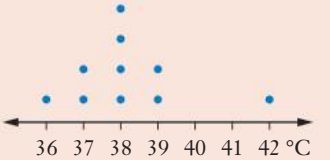
EXAMPLE 19

Calculate, correct to one decimal place, the standard deviation of each data set below.

a The maximum daily temperature (in °C) in Mudgee for the first two weeks in January:

30 28 26 31 34 35 32
33 21 25 28 32 32 35

b The body temperatures (in °C) of a group of hospital patients:



Patients' temperatures

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Solution

a $\sigma = 3.9434\ldots \approx 3.9$

Operation	Casio Scientific	Sharp Scientific
Refer to page 404 to enter the data.		
Calculate the population standard deviation ($\sigma_x = 3.9434\ldots$)	SHIFT 1 Var σ_x =	RCL σx

b $\sigma = 1.5362\ldots$
≈ 1.5

To calculate the standard deviation of data presented in a frequency table, refer to the table of calculator instructions on page 407, then follow the instructions from part **a** above.

EXAMPLE 20

Thirty-six people were given a concentration task and the time taken (in seconds) to complete the exercise are shown below.

Males	32	44	44	29	40	26	64	21	65	32	42	30	66	51	53	30	55	42
Females	35	35	41	41	49	38	33	44	36	53	28	42	37	35	28	54	60	61

a Find the mean and standard deviation of each group.

b Is there a significant difference between the times it took to complete the exercise for males and females? Give reasons.

Solution

a Using the calculator's statistics mode:

Males: $\bar{x} = 42.56, \sigma = 13.58$

Females: $\bar{x} = 41.67, \sigma = 9.77$

b The mean time to complete the task for females was only 0.89 seconds lower than for males. However the standard deviation for females was 3.88 seconds lower than the times for males, showing that the times for females were more consistent than for males.

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Samples and populations

Governments and businesses do not make important decisions based on just one **sample**. Researchers generally take a number of samples from a **population** and calculate the statistics of each sample. The sample means and standard deviations are then used to estimate the population mean and standard deviation respectively.

The **sample mean**, $\bar{x}$, and the **sample standard deviation**, s, or s_x, are called **statistics**.

The **population mean**, μ (the Greek letter 'mu'), and the **population standard deviation**, σ or σ_x (the Greek letter 'sigma') are called **parameters**.

The sample statistics are estimates of the population parameters.

When calculating the standard deviation of a set of data, we will usually use the **population standard deviation**, σ. If the set of data is a sample, however, then use the **sample standard deviation**, s, to estimate the results for a population.

EXAMPLE 21

The ages of 60 people working at Burger Haven this year are:

18	19	18	17	20	20	24	15	24	19
15	35	15	24	22	19	15	17	23	29
15	40	21	17	20	22	23	21	24	23
22	16	36	15	16	24	16	15	19	15
34	19	45	20	15	21	24	27	19	33
18	27	15	30	15	34	17	29	25	17

a Find, correct to one decimal place, the population mean (μ) and population standard deviation (σ) of the Burger Haven employees.

b Randomly select three samples of ten ages from this population of employees and for each sample, calculate (correct to one decimal place) the mean ($\bar{x}$) and the standard deviation (s).

c Estimate the mean and standard deviation of the population from the statistics of the three samples.

d How do the estimates of population mean and standard deviation compare with the answers in part **a**?

Solution

a $\mu = 21.8666\ldots \approx 21.9$ years

$\sigma = 6.7908\ldots \approx 6.8$ years

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b Randomly select three samples of ten ages from the list above. For example:

Sample 1: 21 16 15 15 16 30 19 30 35 24

Sample 2: 19 23 21 16 21 15 20 36 40 15

Sample 3: 18 15 25 27 20 15 24 16 17 21

The mean and standard deviation for the samples are:

Sample 1: $\bar{x} = 22.1$ $s = 7.309\ldots \approx 7.3$

Sample 2: $\bar{x} = 22.6$ $s = 8.604\ldots \approx 8.6$

Sample 3: $\bar{x} = 19.8$ $s = 4.341\ldots \approx 4.3$

c Estimate of the population mean $= \dfrac{22.1+22.6+19.8}{3} = 21.5$

Estimate of the population standard deviation $= \dfrac{7.3+8.6+4.3}{3} = 6.7$

d The estimates to the population mean and standard deviation (21.5 and 6.7) compare favourably with the population mean and standard deviation (21.9 and 6.8).

Exercise 10.07 Standard deviation

Example 19

1 The number of monthly accidents at a construction site over 8 months was:

3 0 4 2 3 0 2 2

a Calculate the mean number of accidents per month.

b Find the standard deviation for the data, correct to one decimal place.

2 An express train from Central Station was late in arriving at Homebush by the following times (in minutes):

6 0 3 –2 5 –1 0 3 –1 6 7 1

a Find the mean, $\bar{x}$.

b Calculate the standard deviation, σ, correct to two decimal places.

c Evaluate $\bar{x}+\sigma$ and $\bar{x}-\sigma$, the values that are, respectively, one standard deviation below and one standard deviation above the mean.

d How many of the given scores lie within one standard deviation of the mean, that is, between the two values you calculated in part **c**?

e What percentage, correct to one decimal place, of scores were within one standard deviation from the mean?

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3 A sample of mobile phone batteries was tested for charge life (in hours).

60 73 65 84 77 64 66 73 88 90 79 81

Find, correct to two decimal places:

a the mean

b the sample standard deviation, s.

4 Blake's weekly commissions, in dollars, for selling Internet plans were:

540 510 1100 1350 780 650 920 590 1080

Calculate for this data, correct to the nearest dollar:

a the mean **b** the standard deviation.

5 Students were surveyed on the number of movies they had downloaded in the last six months, with the results shown in the frequency table.

Score (x)	Frequency (f)
0	6
1	7
2	8
3	10
4	9
5	5
6	5

a For this data, find the mean, $\bar{x}$.

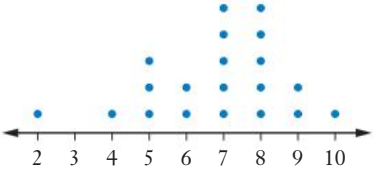
b Calculate, correct to one decimal place, the standard deviation.

c How many scores were within one standard deviation of the mean?

d What percentage of scores were within one standard deviation of the mean?

For many large sets of data, approximately 68% (slightly more than $\frac{2}{3}$) of the scores lie within one standard deviation of the mean.

6 This dot plot shows the number of vehicles driving past Westvale High School every minute for a 20-minute period.



Number of vehicles per minute

a Find the mean.

b Calculate, correct to two decimal places, the standard deviation.

c How many scores were within one standard deviation of the mean?

d What percentage of scores were within one standard deviation of the mean?

7 This table shows the weekly wages of employees at Great Gals electrical store, grouped in classes of $100.

Weekly wage ($)	Class centre	Frequency
$500 –< $600		7
$600 –< $700		20
$700 –< $800		36
$800 –< $900		17
$900 –< $1000		11
$1000 –< $1100		3

a Copy and complete the table.

b Find, to the nearest cent, an estimate for:

i the mean

ii the standard deviation.

ISBN 9780170413565

Example 20

8 The heights (in cm) of males and female students in a Year 11 PDHPE class are shown.

Males	183	160	178	179	171	175	184	172	173	187	179	165
Females	172	160	162	160	173	165	165	163	168	150	160	177

a Find the mean height and sample standard deviation for males and for females.

b Is there a significant difference between the heights of males and females? Give reasons.

9 The results of the first two Maths tests given to a Year 11 class are displayed in the back-to-back stem-and-leaf plot.

Test 1		Test 2
4	3	2
4 3	4	9
9 8 0	5	2 7 9
9 8 7 4 0	6	
9 7 5 5 5 3 1	7	0 1 1 2 4 4 8
9 9	8	0 1 2 4 5 5 7 8

a Find the mean mark and standard deviation for each test.

b Are there significant differences between the means and standard deviations of the two tests?

c In which test did the students perform better? Justify your answer.

10 A group of men and women were timed on the length of time (in seconds) of the last call they made on their mobile phone.

Men	292	360	840	60	60	900	60	328	217	16
	1565	58	22	98	73	537	51	49	1210	15
Women	653	73	202	58	74	75	58	168	354	600
	1560	2220	56	900	481	60	139	80	72	110

a Find the mean and standard deviation for each group.

b Calculate the mean and standard deviation of the times for men and women if the outliers (1565 s and 1210 s for men, 1560 s and 2220 s for women) are excluded.

c Do men or women make longer calls? Justify your answer.

Example 21

11 **a** As in Example **21**, randomly select three samples of ten ages from the population of Burger Haven employees and, for each sample, calculate the mean ($\bar{x}$) and the sample standard deviation (s).

b Estimate the mean and standard deviation of the population from the statistics of the three samples.

c How do the estimates of population mean and standard deviation compare with the answers in part **a**?

12 **a** Randomly select three samples of five ages from the Burger Haven employees and, for each sample, calculate the mean ($\bar{x}$) and the sample standard deviation (s).

b Estimate the mean and standard deviation of the population from the statistics of the three samples.

c How do the estimates of population mean and standard deviation compare with the answers in part **a**?

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13 Using your results from Questions **11** and **12**, do the sample statistics become more accurate and closer to the values of the population mean and standard deviation with a larger sample size?

TECHNOLOGY

Calculating measures of spread

Step 1: Open a blank spreadsheet and enter the temperature data about Mudgee from Example **19** on page 445.

	A	B	C	D	E	F	G
1	**Maximum daily temperatures in Mudgee**						
2	30	28	26	31	34	35	32
3	33	21	25	28	32	32	35
4							
5			Highest score				
6			Lowest score				
7			Upper quartile, Q_3				
8			Lower quartile, Q_1				
9							
10			Range				
11			Interquartile range				
12			Standard deviation				
13							

Step 2: In cell E5, enter the formula =**MAX(A2:G3)** to calculate the highest score (35).

Step 3: In cell E6, enter =**MIN(A2:G3)** to calculate the lowest score (21).

Step 4: In cell E7, enter =**QUARTILE(A2:G3,3)** to calculate the upper quartile, Q_3 (32.75).

Step 5: In cell E8, enter =**QUARTILE(A2:G3,1)** to calculate the lower quartile, Q_1 (28).

Note: A spreadsheet calculates quartiles using a slightly different method to the method we have described, so its answers for the interquartile range may not be exactly the same as ours, but they should be close.

Step 6: In cell E10, enter =**E5-E6** to calculate the range (14).

Step 7: In cell E11, enter =**E7-E8** to calculate the interquartile range (4.75).

Step 8: In cell E12, enter =**STDEV.P(A2:G3)** to calculate the population standard deviation.

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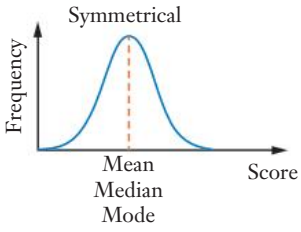
The shape of a frequency distribution

Shapes of distributions

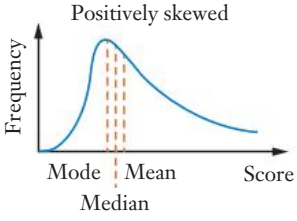
10.08 The shape of a distribution

The **shape** of a statistical **distribution** (data set) shows how the data is spread, and can be seen by drawing a curve around its graph or display.



A distribution is **symmetrical** if the data are balanced or evenly spread about the centre of the distribution, with the mean, median and mode being equal. One example of a symmetrical distribution are students' marks in an HSC examination.

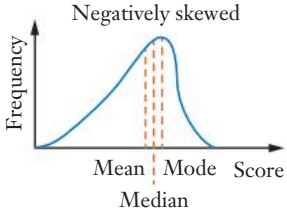
A distribution is **positively skewed** if its tail points to the right (the positive direction), because the mean is above the mode and median.



The word 'skewed' means twisted.

One example of a positively skewed distribution are house prices in a large country town.

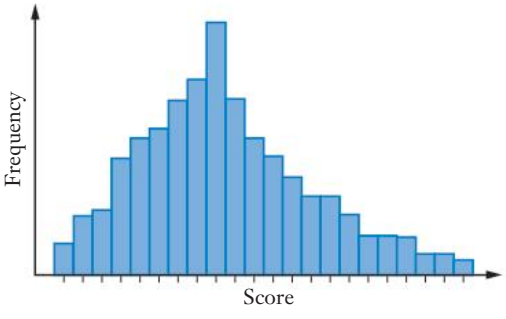
If a distribution is **negatively skewed**, then its tail points to the left (the negative direction) because the mean is below the mode and median.



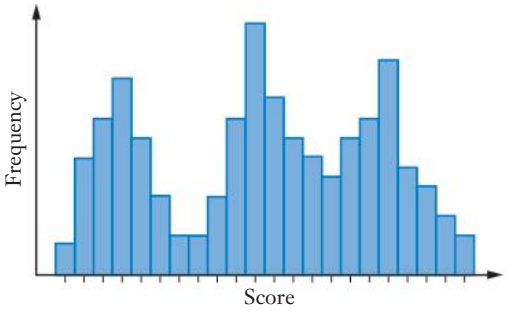
One example of a negatively skewed distribution is the heights of the players in a basketball team.

Peaks are the high points of the distribution and represent the more frequent scores. The highest peak is the mode.

The **modality** is the number of peaks occurring in a distribution. A distribution can have one peak only (**unimodal**) or have more than one peak (**multimodal**).



Unimodal distribution



Multimodal distribution

If a distribution is **bimodal**, it has two peaks. For example, this frequency histogram is bimodal, having two peaks at 2 and 7. The mode, however, is 7.

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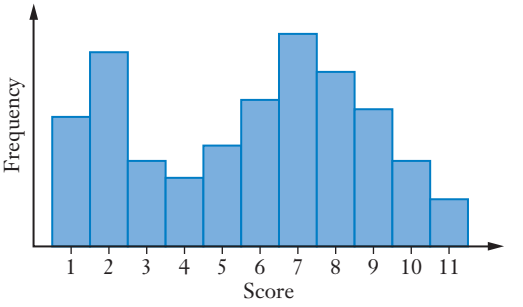


Clusters are groups of scores that are bunched or close together.

EXAMPLE 22

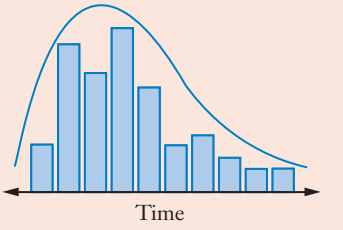
For each distribution shown below:

i describe its shape **ii** state the modality **iii** identify any clusters.

a Marks in a Japanese test

Stem	Leaf
3	1 2
4	3 5 9
5	0 2 6
6	4 5
7	3 5 6 7 7 8 9
8	0 2 4 6 6 8 8 9
9	1 2 4 8 9

b Amount of traffic on Sydney's roads



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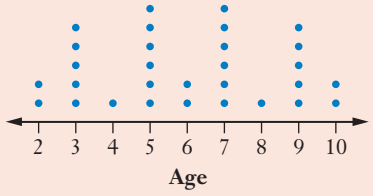
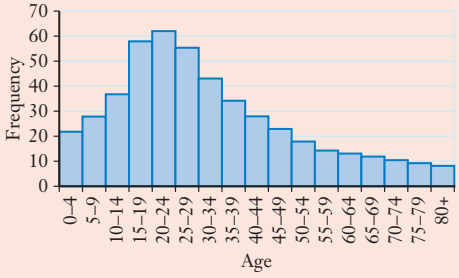
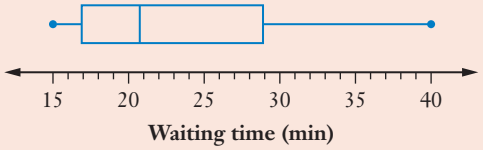
c Ages of children at a cinema



d Ages of people in a small coastal town



e Waiting time in a doctor's surgery



Solution

a **i** negatively skewed (tail points towards the left)
ii multimodal, peaks at 77, 86, 88
iii clusters in the 70-90s

b **i** positively skewed (tail points towards the right)
ii bimodal, 2 peaks
iii clusters at earlier hours

c **i** symmetrical
ii multimodal, peaks at 3,5,7 and 9
iii no clusters

d **i** positively skewed (tail points towards the higher ages)
ii unimodal class, 1 peak
iii cluster from 15 to 29

e **i** positively skewed (tail points towards the right)
ii Unable to determine since individual scores are not known.
iii cluster from 15-17 min (25% of patients)

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Comparing sets of data

Distributions or numerical data sets can be described and compared in terms of modality, shape, measures of central tendency and spread and outliers.

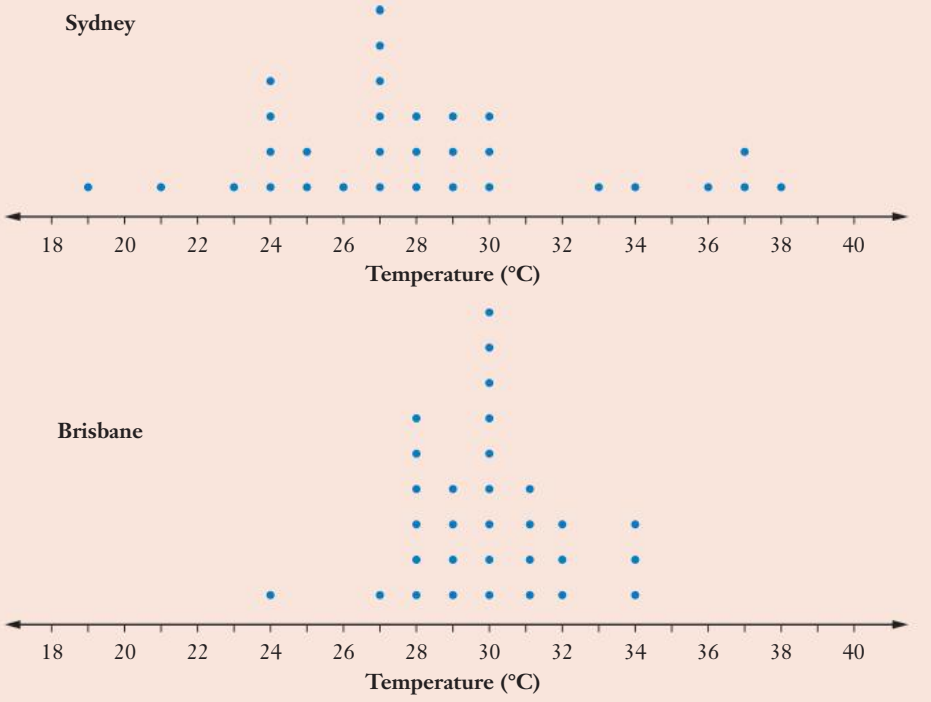
Comparing city temperatures

Comparing word lengths

Comparing sports scores

EXAMPLE 23

The daily maximum temperatures for Sydney and Brisbane for December are shown below.



a Find the mean, the median and modal temperatures for each city.

b Find the range, interquartile range and standard deviation for each city.

c Describe the shape of the distribution of temperatures for each city and identify any outliers and clusters.

d Compare the temperatures in Sydney and Brisbane. Comment on measures of central tendency and measures of spread.

Solution

a Sydney: Mean = 28.2°C Brisbane: Mean = 29.9°C

Median = 27°C Median = 30°C

Mode = 27°C Mode = 30°C

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b Sydney: Range = 38° − 19° = 19°

IQR = $Q_3 - Q_1$

= 30 − 25

= 5

Standard deviation = 4.6

Brisbane: Range = 34° − 24° = 10°

IQR = $Q_3 - Q_1$

= 31 − 28

= 3

Standard deviation = 2.1

c Sydney's distribution of temperatures is positively skewed and 38°C is just an outlier: ($Q_3 + 1.5 \times \text{IQR} = 30 + 1.5 \times 5 = 37.5$).

Brisbane's temperatures have a slight positive skew and has no outliers.

Sydney's temperature are bimodal, with peaks at 24°C and at 27°C, and are clustered at 27–30°C. Brisbane's temperatures are also bimodal, with peaks at 28°C and at 30°C and are clustered at 30°C.

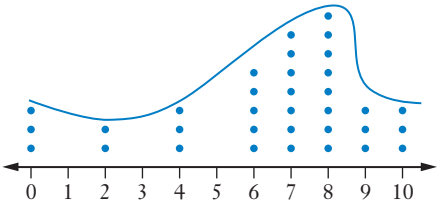
d Brisbane is the warmer city as shown by the mean, median and mode which are 2–3° above those of Sydney.

The spread of Sydney's temperatures is significantly greater than Brisbane's as shown by larger values of the range, interquartile range and standard deviation. Sydney also had the lowest and the highest temperatures in December.

Example 22

Exercise 10.08 The shape of a distribution

1 This dot plot shows the judges' scores in a diving competition. Which of the following statements is true about the distribution? Select **A**, **B**, **C** or **D**.



A The data is positively skewed with a cluster around 6 to 8.

B The data is symmetrical with no modes.

C The data is negatively skewed with one mode.

D The data is positively skewed with a cluster around 0 to 4.

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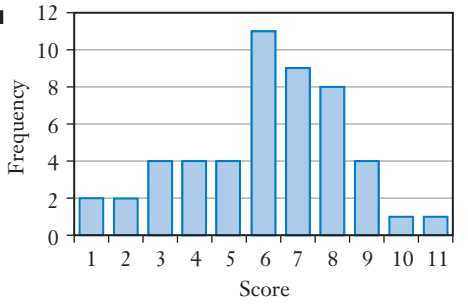
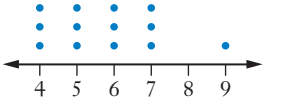
2 For each distribution:

i describe its shape **ii** state the modality **iii** identify any clusters

a



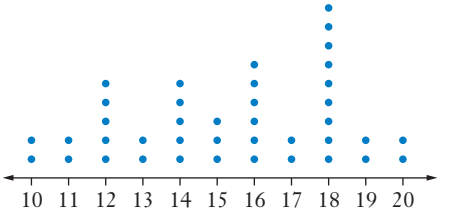
b



c

Stem	Leaf
1	3 4 6 6 6 7 8 9 9
2	0 7
3	1 2 2 5 7 8 8 9
4	0 2 3
5	2 9

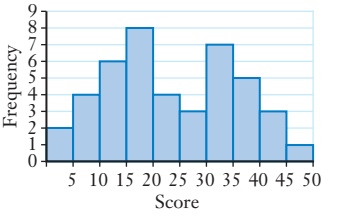
d



e

Stem	Leaf
4	1 3
5	5 5 6
6	0 3 5 5 6 8
7	2 6
8	5 5 8

f



3 This stem-and-leaf plot shows the number of mobile phones sold in January across various OzTel stores in Australia.

a How many OzTel stores were surveyed?

b Describe the shape of the data.

c Where does the clustering occur?

d What is the mode?

Stem	Leaf
2	2 6
3	0 1
4	4 8
5	2 6 9
6	1 3 4 5
7	0 2 3 4 4 5 5 7 7 7 8 9
8	3 5 7 7 8 8 8 8 9
9	2 8

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4 The number of visits to the MyFace website was recorded between 1200 (noon) and 2100 (9 p.m.) one day.

Hour	1201–1300	1301–1400	1401–1500	1501–1600	1601–1700	1701–1800	1801–1900	1901–2000	2001–2100
Hits	1300	800	400	2100	2500	4500	3900	5300	2300

a Draw a histogram to represent this data.

b Comment on the shape of your histogram, also referring to modality and clusters.

c Suggest a possible reason for the skewness of this data.

5 Which statement is true about the data sets below? Select **A**, **B**, **C** or **D**.

A Y is positively skewed.

B X does not have a mode.

C The mean of Y is 5.

D X and Y are both symmetrical.

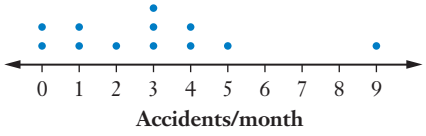
6 These are the ages of employees at the Berry Good Biscuit factory.

16 36 15 16 15 19 55 59 18 20 50 22 21 35 22 19 15 17 43 49

a Draw a stem-and-leaf plot for this data.

b Comment on the shape of the distribution, mentioning skewness, peaks and clusters.

7 This dot plot represents the number of accidents per month at a factory over a year.



a Comment on the shape of the dot plot.

b What is the mode?

c Calculate the mean (correct to one decimal place) and compare it to the mode.

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Example 23

8 This back-to-back stem-and-leaf plot compares the half-yearly exam of two Year 11 Business Studies classes.

11BS1		11BS2
8	3	4
9 7 7	4	5 8
9 7 7 6 6 3	5	2 7 9
9 8 7 4	6	3 4 4 6 7
7 3 2	7	1 1 2 3 4 6 8
5 3	8	2 4

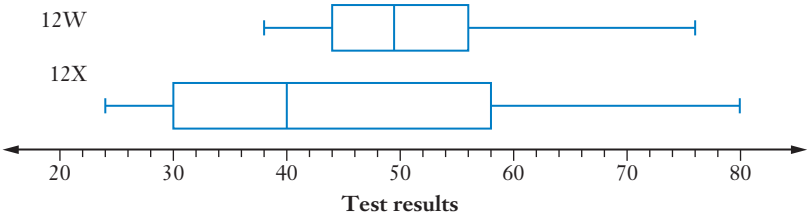
a Find the mean, median and mode for each class.

b Find the range, interquartile range and standard deviation of the marks for each class.

c Describe the shape of the distribution of marks for each class.

d Compare the marks for both classes and determine which class achieved better results, commenting on shape, measures of central tendency and measures of spread.

9 The results of a Year 12 Maths exam are shown on the parallel box plot below.



a What is the median result for each class?

b Find the range and interquartile range for each class.

c Describe the shape of the results for each class.

d Which class had the better test results? Give reasons.

10 A Year 11 Biology class was asked to estimate their test results before completing the test. The estimates and actual test results are shown below.

Estimates	87	80	83	65	82	82	92	73	82	89
	93	77	70	65	85	33	87	77	78	75
	88	89	86	58						
Test results	80	73	86	52	91	91	72	64	91	87
	79	46	78	85	82	32	87	73	79	86
	95	79	49	73						

a Display the data in a back-to-back stem-and-leaf plot.

b Comment on the shape of each set of data, mentioning skewness, modality and clusters.

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c For each group of results, find:
i the mean **ii** the median **iii** the mode.

d For each data set, find:
i the range **ii** the interquartile range
iii the standard deviation.

e Compare the two sets of results. Did the students overestimate their results? Justify your answer.

SAMPLE HSC PROBLEM

The ages, in years, of a sample of patients at a hospital are shown in the stem-and-leaf plot.

Stem	Leaf
1	2 2 3 4 6
2	1 2
3	0 0 0 3
4	4 7 8
5	1 1
7	5 7 8
8	1

a Find the mean age of the patients.

b Find the median age of the patients.

c Is the mean or median more appropriate for describing the average age of the patients? Give a reason for your answer.

d Find the interquartile range of the patients' ages.

e Represent this data set on a box plot.

Study tip

Looking after yourself

- While studying, don't forget to keep it all in perspective.
- Remember to have your own life outside school.
- Look after your physical and mental health.
- Eat properly and have enough sleep.
- Exercise regularly, play sport and go out.
- Plan to do nothing occasionally.
- Relax and rest regularly.
- Talk to your family, visit your friends.
- Be positive and sensible.
- Have confidence in yourself and don't stress.
- Don't worry, be happy.

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10. CHAPTER SUMMARY

This chapter, Analysing data, examined the statistical measures of central tendency (mean, median, mode) and spread (range, interquartile range, standard deviation). You should be competent at making statistical calculations on sets of numerical data, including those represented in frequency tables, class intervals (grouped data), dot plots and stem-and-leaf plots. Make sure you know how to use the statistical functions of your calculator. You should understand the new concepts of quantiles (quartiles, deciles and percentiles), be able to interpret cumulative frequency graphs and construct box plots using a five-number summary. You must also be able to describe, compare and interpret data sets in terms of modality, shape (symmetrical and skewness), measures of central tendency and spread and also look at the effect of outliers.

Statistics review

Statistics crossword

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map is shown below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.

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10. TEST YOURSELF

Exercise 10.01

1 The heights (in centimetres) of a group of ballet dancers are:

165	183	170	168	175	179	168	170
181	168	172	177	171	170	175	179

a Calculate the mean, correct to one decimal place.

b Find the median height.

c What is the mode?

Exercise 10.01

2 Motor vehicles were clocked, by police radar, travelling at the following speeds (in km/h):

78	95	64	77	81	84	77	89	90	78
79	80	82	84	80	79	95	86	84	70
78	65	82	91	89	60	85	81	78	68
90	84	69	70	80	91	85	84	80	76
68	65	85	76	79	83	82	91	84	80

a Sort the data in a frequency table using classes of 60–< 70, 70–< 80, and so on, and include a column of class centres.

b Calculate an estimate for the mean speed.

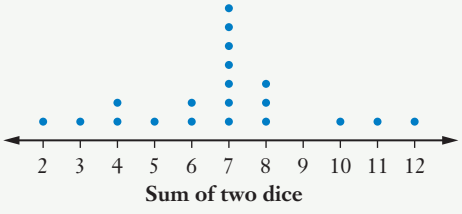
c Find the median class of speeds.

d What is the modal class?

Exercise 10.01

3 The dot plot represents the sum of two dice rolled 20 times.

Find the mean, median and mode of this data.



Exercise 10.01

4 The house prices realised at auction one Saturday in Vincentia were:

$642 000	$585 000	$352 000	$1 480 000
$705 000	$415 000	$680 000	$740 000

a Calculate the mean price.

b Calculate the median price.

c Is the mean or the median the better measure to use as the average price of the houses? Why?

Exercise 10.01

5 Which measure of central tendency is most appropriate for describing each average below? Give a reason for each answer.

a The average men's shoe size

b The average height of Year 11 students

c The average starting salary of an Australian worker

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6 A grouped data frequency table is shown.

Class interval	Frequency
11–15	4
16–20	7
21–25	12
26–30	24
31–35	15

What is the mean? Select **A**, **B**, **C** or **D**.

A 24.1 **B** 25.3

C 26.1 **D** 28.1

7 In a national mathematics test, Simone scored 84.

Exercise 10.02

a This score was above the 7th decile, D_7. Approximately what percentage of students taking the test scored lower than her?

b More specifically, Simone's score was at the 78th percentile, P_{78}. What percentage of students scored higher than her?

8 **a** What is the meaning of 'interquartile range'?

Exercise 10.03

b A random sample of 15 packets of corn chips had the following masses in grams. Find the range and interquartile range of these masses.

52 51 50 49 50 50 48 51

51 50 49 53 50 49 51

9 This stem-and-leaf plot on the right represents the number of points per match scored by the Sharks in a football season. For this data, find:

Exercise 10.03

Stem	Leaf
0	6 6
1	2 3 4 4 4 8 8 9
2	0 0 0 5 6
3	0 0 2 4 4 6 7
4	0
5	
6	2

a the range

b the interquartile range.

10 In a small business, eight employees earn the following wages per week.

Exercise 10.04

$1026 $874 $950 $950 $980 $1140 $1216 $1710

Is the wage of $1710 an outlier for this set of data? Justify your answer with calculation.

11 Consider the set of scores:

Exercise 10.04

4 7 8 8 12 15 19 20

a What is the effect on the mean and median if an outlier of 40 is added to this data set.

b Is the mean or median a better measure of central tendency when there is an outlier in the data set?

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Exercise 10.05

12 Students were surveyed about the number of pairs of shoes they owned, and the results are shown in the table on the right.

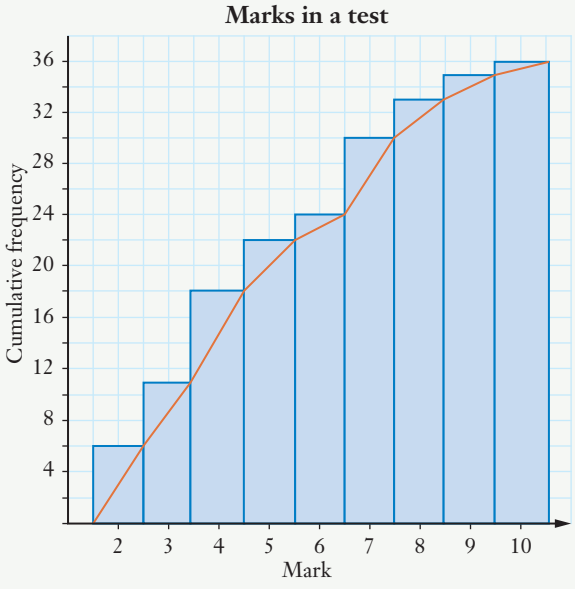
Pairs of shoes	Frequency
5	8
6	11
7	10
8	6
9	5

a Copy the table, adding a cumulative frequency column. Then draw a cumulative frequency histogram and polygon.

b Use your polygon to calculate:
- **i** the median
- **ii** the interquartile range
- **iii** the 3rd decile.

Exercise 10.05

13 The cumulative frequency graph shows the results of an assignment marked out of 10.

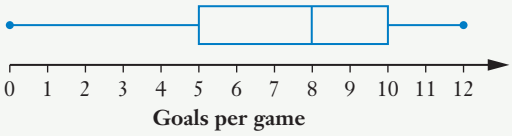


a How many students completed the assignment?

b Use the graph to estimate:
- **i** the median
- **ii** the interquartile range
- **iii** the 6th decile
- **iv** the 45th percentile.

Exercise 10.06

14 This box plot represents the number of goals scored per game by a hockey team over a season.



a What was the lowest score?

b Find the interquartile range.

c In what fraction of games were more than 8 goals scored?

d In what percentage of games were fewer than 5 goals scored?

Exercise 10.06

15 **a** Create a five-number summary for the corn chip packet masses in Question **8b**.

b Represent the mass data on a box plot.

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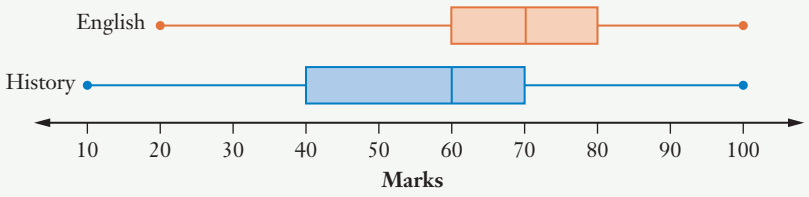
16 The parallel box plots show the distribution of marks for exams in English and History.



a Which subject has the smaller spread of marks? Give reasons.

b The number of students who scored 70 or less is the same for both subjects. If 144 students did the English exam, how many students did the History exam?

17 For quality testing, a manufacturer takes a random sample of 10 screws, each designed to have a length of 2 cm. The actual lengths of the screws, in centimetres, are:

2.00 1.99 1.98 2.01 2.01 1.97 2.03 1.98 2.01 2.00

Exercise 10.07

a Find the mean screw length.

b Find the standard deviation, correct to two decimal places.

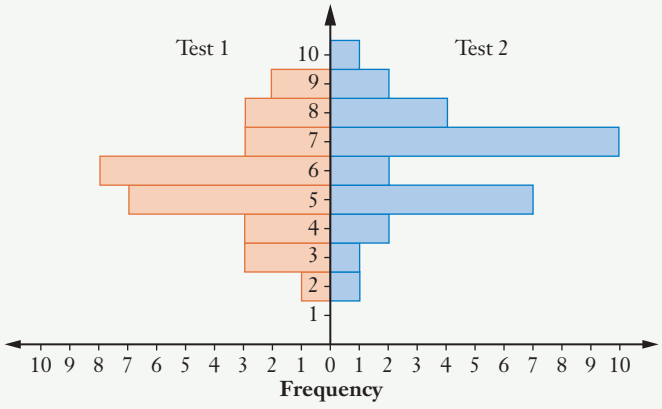
18 For the shoe data from Question **12**, calculate (correct to one decimal place):

Exercise 10.07

a the mean **b** the standard deviation.

19 The results for the multiple-choice section in two tests taken by a Year 11 Mathematics class are shown below.

Exercise 10.08



a Find the mean, median and mode for each test.

b Describe the shape of the data set for each test.

c For each test, find:

i the range **ii** the interquartile range **iii** the standard deviation.

d Are there any significant differences in the results of the two tests? Justify your answer by referring to the measures of central tendency and spread of the tests.

Chapter quiz

ISBN 9780170413565

11.

WORLD LOCATIONS AND TIMES

The Samoan Islands lie in the south Pacific Ocean and are made up of Samoa and American Samoa, 64 km apart. New Zealand governed Samoa until it became an independent country in 1964, and it was called Western Samoa until 1997. American Samoa is a small territory of the USA. Despite being next to each other, when it is Monday in Samoa, it is still Sunday in American Samoa. How can this be so?

CHAPTER OUTLINE

M2 11.01 Latitude and longitude
M2 11.02 Time
M2 11.03 Timetables
M2 11.04 International time zones

IN THIS CHAPTER YOU WILL:

- locate positions on Earth using latitude and longitude
- convert between 12-hour and 24-hour time
- calculate differences between times of day
- interpret transport timetables and use them to solve problems involving travel
- calculate times in different parts of the world and within Australia using international time zones, taking daylight saving into account
- solve problems involving travelling and communicating across the world using international time zones

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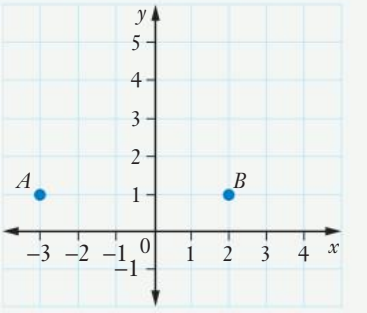
TERMINOLOGY

12-hour time
Coordinated Universal Time (UTC)
Equator
latitude
prime meridian
timetable
24-hour time
coordinates
great circle
longitude
small circle
Australian Eastern Standard Time (AEST)
daylight saving time
International Date Line (IDL)
parallel of latitude
time zone

Assignment 11

SkillCheck



1 **a** Write the coordinates of points A and B.
b What is the length of the interval AB?
c Write the coordinates of point C if it is 3 units up and 4 units left from B.

2 **a** Plot the points $D(3, 5)$ and $E(3, 2)$ on a number plane.
b What is the length of the interval DE?

3 How many hours between:
a 4 a.m. and 1 p.m.?
b 9 a.m. and 2 p.m.?
c 12:30 a.m. and 5:30 p.m.?
d 2:30 p.m. and 10:30 p.m.?

4 How many minutes between:
a 3:00 p.m. and 3:17 p.m.?
b 6:00 a.m. and 6:44 a.m.?
c 10:35 a.m. and 11:00 a.m.?
d 7:17 p.m. and 8:00 p.m.?

5 What is the time:
a 7 hours after 7:00 p.m.?
b 8 hours before 2:00 p.m.?
c 11 h 18 min after 1:15 p.m.?
d 7 h 36 min before 2:20 a.m.?

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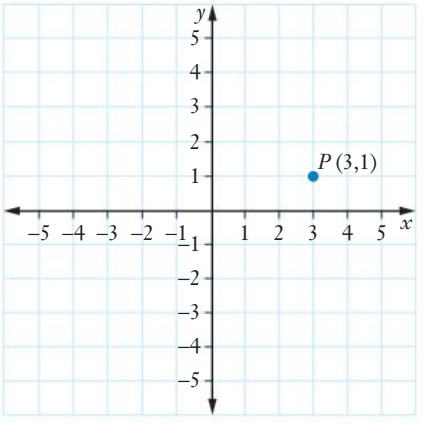
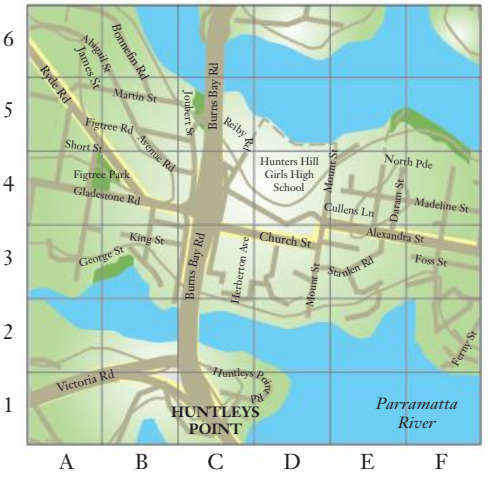
11.01 Latitude and longitude

When describing the location of a point on a number plane or map, we use a **coordinate** system involving ordered pairs (x, y).

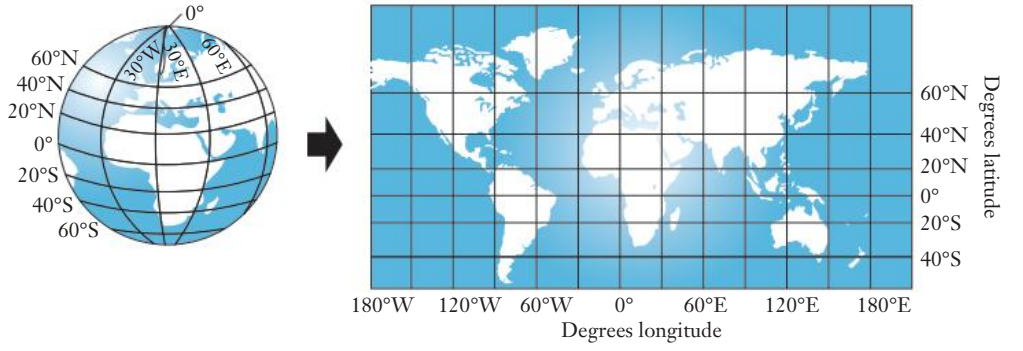


P has coordinates (3, 1) on this number plane.



Hunters Hill Girls High School has coordinates D4 here.

Positions on Earth's surface are described by a coordinate system involving **latitude** and **longitude**. However, because Earth is a sphere, we use a special grid of lines that run across and down a sphere. The diagrams below show this grid on a world globe and a flat world map.



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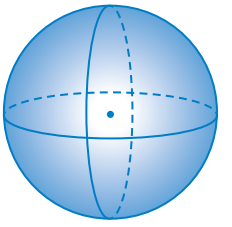
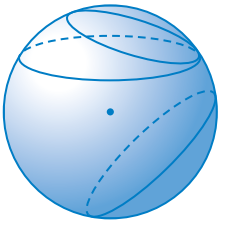
Great and small circles

If you cut a 'slice' through a sphere, its shape is a circle. A slice through the **centre** of a sphere is called a **great circle**, and its radius is the same as that of the sphere. Any other slice is called a **small circle**, because its radius is smaller than that of a great circle.



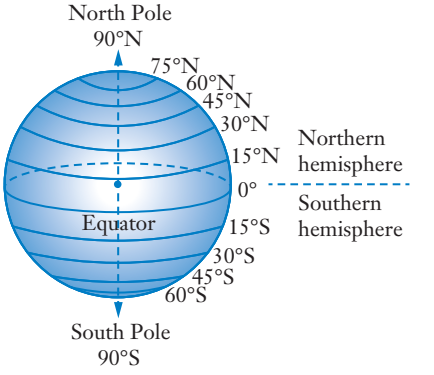
Great circles



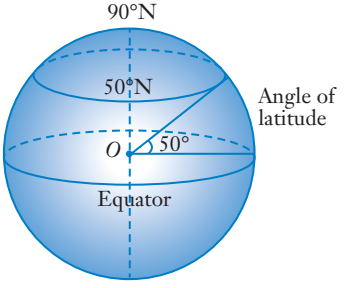
Small circles

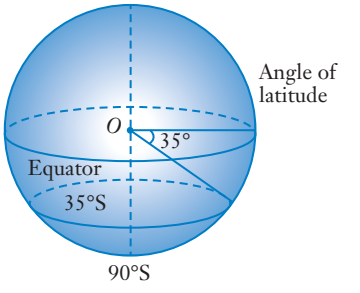
Parallels of latitude

Parallels of latitude are imaginary parallel circles that run around the Earth. The largest parallel of latitude is the **Equator**, 0°, which is a great circle. The other parallels of latitude are all small circles. The North Pole has a latitude 90° north (90°N). The South Pole has a latitude of 90°S.



The **angle of latitude** is the angle the parallel makes with the Equator at the centre, *O*, of the Earth. The diagrams below show the 50°N and 35°S parallels of latitude.





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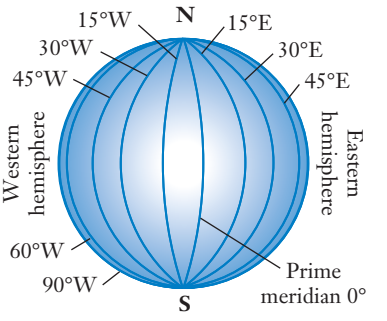
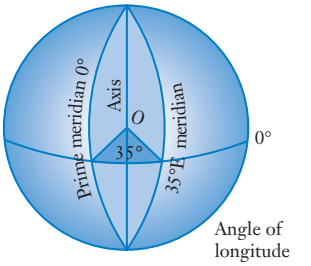
Meridians of longitude

Meridians of longitude are imaginary semicircles that run down the Earth. They are 'half' great circles that meet at the North and South poles. The main meridian of longitude is the **prime meridian** (or **Greenwich meridian**), 0°.

Greenwich is pronounced 'gren-itch'



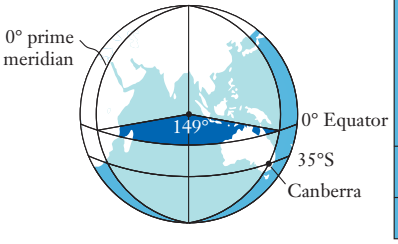
The **angle of longitude** is the angle the meridian makes with the prime meridian at the centre, *O*, of the Earth. The diagram shows the 35°E meridian of longitude.

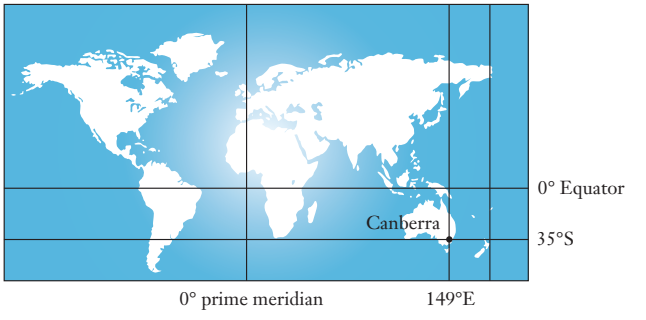


Meridians of longitude range from 180°E to 180°W. 180°E and 180°W are actually the same meridian, on the opposite side of the Earth to the prime meridian. It runs through the Pacific Ocean, east of Fiji and west of Hawaii.

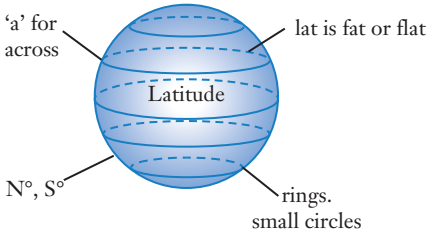
Position coordinates

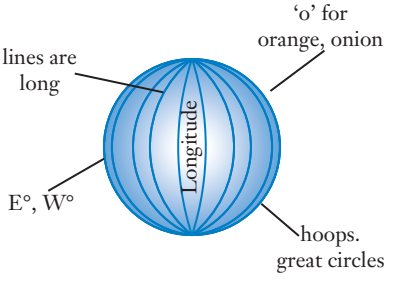
Locations on the Earth are described using latitude (°N or °S) and longitude (°E or °W). For example, Canberra has coordinates (35°S, 149°E), meaning it is 35° south of the Equator and 149° east of the prime meridian.





These diagrams show some mnemonics for understanding latitude and longitude.





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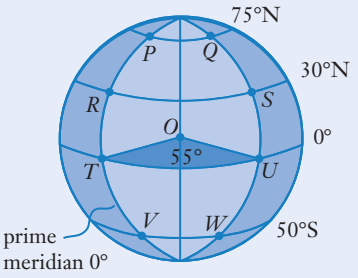
EXAMPLE 1

Match the following coordinates to the points illustrated.

a (50°S, 55°E)
b (30°N, 55°E)
c (75°N, 0°E)
d (0°, 0°)
e (0°, 55°E)
f (75°N, 55°E)



Solution

Since $\angle TOU = 55°$, $QSUW$ is the 55°E meridian of longitude.

a (50°S, 55°E) is W
b (30°N, 55°E) is S
c (75°N, 0°E) is P
d (0°, 0°) is T
e (0°, 55°E) is U
f (75°N, 55°E) is Q

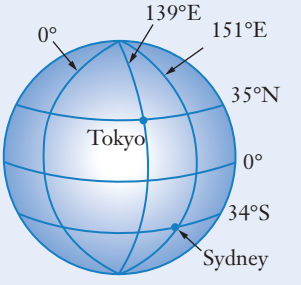
EXAMPLE 2

Sydney's coordinates are (34°S, 151°E) while Tokyo's are (35°N, 139°E).

a Find their difference in latitude.
b Find their difference in longitude.
c Which city is further west?

Solution

Draw a rough sketch to position the cities.



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a Difference in latitude = 35° + 34° = 69° Difference between 35°N and 34°S.

b Difference in longitude = 151° − 139° = 12° Difference between 151°E and 139°E.

c Sydney is further east, so Tokyo is further west.

EXAMPLE 3

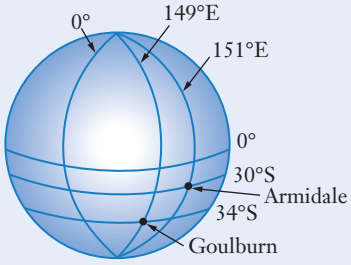
Goulburn has coordinates (34°S, 149°E). If Armidale is 4° north and 2° east of Goulburn, what are its coordinates?

Solution

Latitude = 34°S − 4° = 30°S

Longitude = 149°E + 2° = 151°E

The coordinates of Armidale are (30°S, 151°E).



Exercise 11.01 Latitude and longitude

Note: An atlas and/or a world globe may be helpful for this exercise.

Example 1

1 Match each set of coordinates to a point on Earth.

a (0°, 100°E)

b (40°S, 60°E)

c (60°N, 60°E)

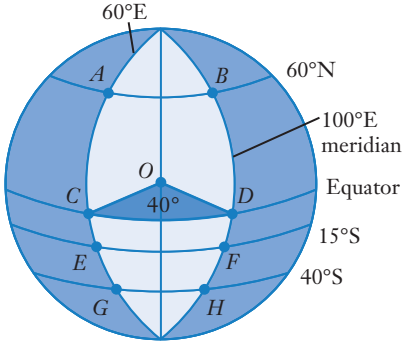
d (0°, 60°E)

e (15°S, 100°E)

f (60°N, 100°E)

g (15°S, 60°E)

h (40°S, 100°E)



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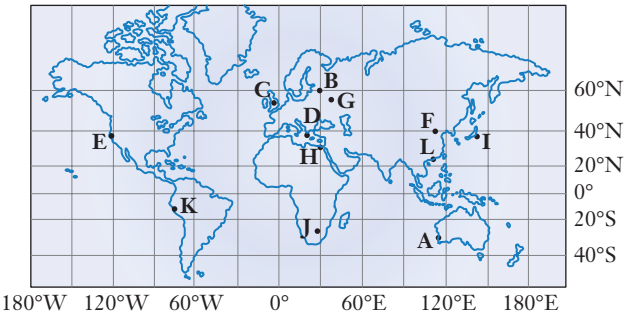
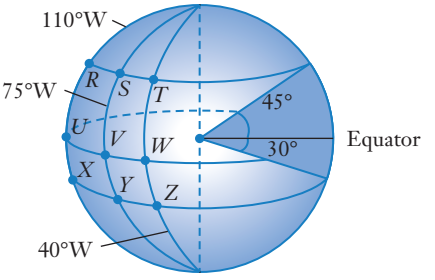
2 Match each city to a letter on the map below.

a Moscow (55°N, 37°E)
b Edinburgh (56°N, 3°W)
c Athens (38°N, 23°E)
d San Francisco (38°N, 122°W)
e Perth (32°S, 116°E)
f Beijing (40°N, 116°E)
g Alexandria (31°N, 30°E)
h Tokyo (35°N, 139°E)
i St Petersburg (60°N, 30°E)
j Johannesburg (26°S, 28°E)
k Lima (12°S, 77°W)
l Hong Kong (22°N, 114°E)



3 Write the coordinates of each of the points on Earth.



a V
b T
c U
d Z
e R
f W
g X
h S

4 What can be found at the 90°S parallel of latitude?

5 Are these small circles or great circles?

a the 30°N parallel
b the prime meridian
c the 145°W meridian

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6 Calculate the difference in latitude between each pair of cities, and state whether the second city listed is north or south of the first.

- **a** Shanghai, China (31°N, 121°E) and New York, USA (40°N, 64°W)
- **b** Nairobi, Kenya (1°S, 37°E) and Bangkok, Thailand (13°N, 100°E)
- **c** Moscow, Russia (55°N, 37°E) and London, UK (51°N, 0°)
- **d** Auckland, New Zealand (37°S, 174°E) and Canberra (35°S, 149°E)
- **e** Melbourne (37°S, 145°E) and Cairo, Egypt (30°N, 31°E)
- **f** Newcastle (33°S, 151°E) and West Wyalong (34°S, 147°E)

7 **a** Deniliquin is 1° due south of Hay. If Hay's coordinates are (34°S, 145°E), what are Deniliquin's coordinates? Select **A, B, C** or **D**.

A (34°S, 144°E) **B** (35°S, 145°E) **C** (33°S, 145°E) **D** (34°S, 146°E)

b Lord Howe Island is 6° due east of Forster. If Forster's coordinates are (32°S, 152°E) what are Lord Howe Island's coordinates? Select **A, B, C** or **D**.

A (32°S, 146°E) **B** (38°S, 152°E) **C** (32°S, 158°E) **D** (26°S, 158°E)

8 Calculate the difference in longitude between each pair of cities, and state whether the second city listed is east or west of the first.

Example 2

- **a** Budapest, Hungary (47°N, 19°E) and Miami, USA (25°N, 80°W)
- **b** Athens, Greece (38°N, 23°E) and Paris, France (49°N, 2°E)
- **c** Havana, Cuba (23°N, 82°W) and Mexico City, Mexico (19°N, 99°W)
- **d** Buenos Aires, Argentina (34°S, 58°W) and Johannesburg, South Africa (26°S, 28°E)
- **e** Manila, Philippines (14°N, 121°E) and Port Moresby, Papua New Guinea (9°S, 147°E)
- **f** Finley (35°S, 145°E) and Bourke (30°S, 146°E)

9 Broken Hill is 7° due west of Dubbo. If Dubbo's position is (32°S, 148°E), what is Broken Hill's position?

10 Ipswich is 2° north and 2° east of Moree (29°S, 150°E). What are Ipswich's coordinates?

Example 3

11 Ballarat is 2° south and 6° west of Batemans Bay (35°S, 150°E). What are the coordinates of Ballarat? Select **A, B, C** or **D**.

A (33°S, 144°E) **B** (37°S, 156°E) **C** (33°S, 156°E) **D** (37°S, 144°E)

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12 **a** In Australia, do Sydney and Tamworth lie on the same line of latitude or longitude?

b Do Athens, Greece and Sicily, Italy, lie on the same line of latitude or longitude?

13 Where on Earth is it possible to stand on every meridian of longitude?

INVESTIGATION

ATLAS ACTIVITIES

Use an atlas or the Internet to locate the positions of different places around the world. Some websites give the coordinates of any city you type in, for example, Google Earth lists the coordinates at the bottom of the page.

1 Use a world map or globe to estimate the coordinates of each city.

a Washington DC, USA **b** Singapore **c** Adelaide, Australia

d Dublin, Ireland **e** Lisbon, Portugal **f** Moscow, Russia

2 Find the actual coordinates of the cities from Question **1**, and then list them in order from:

a west to east **b** north to south.

3 Which city is located at:

a (19°N, 155°W)? **b** (14°N, 121°E)? **c** (34°S, 58°W)?

d (42°N, 12°E)? **e** (12°S, 131°E)? **f** (42°N, 87°W)?

4 How does a flat map distort the positions and sizes of places with high latitudes?

5 Investigate where these regions are in the world and how they got their names.

a the Middle East **b** the Orient **c** Western world

d the Antipodes **e** Ecuador **f** the West Indies

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11.02 Time

Time notation

- **12-hour time:** using a.m. and p.m. and the hours 1 to 12
- **24-hour time:** using 4 digits from 0000 to 2359 with no a.m./p.m.

With **24-hour time,** the hours are numbered from 0 (for midnight) to 12 (for midday) to 23 (for 11 p.m.).

24-hour time	12-hour time
0000	12 a.m. (midnight)
0100	1 a.m.
0200	2 a.m.
0300	3 a.m.
0400	4 a.m.
0500	5 a.m.
0600	6 a.m.
0700	7 a.m.
0800	8 a.m.
0900	9 a.m.
1000	10 a.m.
1100	11 a.m.

24-hour time	12-hour time
1200	12 p.m. (midday)
1300	1 p.m.
1400	2 p.m.
1500	3 p.m.
1600	4 p.m.
1700	5 p.m.
1800	6 p.m.
1900	7 p.m.
2000	8 p.m.
2100	9 p.m.
2200	10 p.m.
2300	11 p.m.

12-hour and 24-hour time

24-hour time on an analog clock

TV times

EXAMPLE 4

Convert each 24-hour time to 12-hour time.

a 1020 **b** 2335 **c** 0048

Solution

We know that 1200 is midday, when a.m. becomes p.m.

a 10 is less than 12, so it is a.m. time.

1020 is 10:20 a.m.

b 23 is greater than 12, so it is p.m. time.

23 – 12 = 11

2335 is 11:35 p.m.

c 00 is the midnight hour

0048 is 12:48 a.m.

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EXAMPLE 5

Convert each 12-hour time to 24-hour time.

a 7:18 a.m. **b** 2:50 p.m. **c** 12:33 a.m. **d** 12:04 p.m.

Solution

a For a.m. times from 1 a.m. onwards, write the time with 4 digits, starting with 0 if needed.

7:18 a.m. = 0718

b For p.m. times from 1 p.m. onwards, add 12 to the hour.

2 + 12 = 14.

2:50 p.m. = 1450

c 12 midnight = 0000

12:33 a.m. = 0033

d 12 midday = 1200

12:04 p.m. = 1204

Time differences

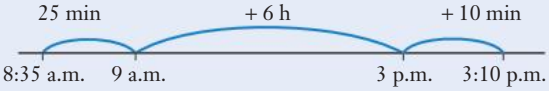
WS Time differences

EXAMPLE 6

What is the difference in time between 8:35 a.m. and 3:10 p.m.?

Solution

Draw a timeline showing the whole hours in between, and calculate the time differences in steps.



Time difference = 6 h + 25 min + 10 min
= 6 h 35 min

Alternative method

Convert to 24-hour time first, then use the calculator's o ' " or DMS key to subtract the times:

Time difference = 3:10 p.m. – 8:35 a.m.
= 1510 – 0835
= 6 h 35 min

15 o ' " 10 o ' " – 8 o ' " 35 o ' " =

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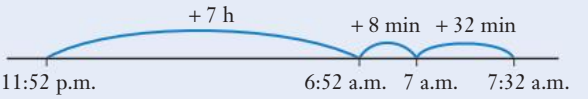
EXAMPLE 7

a What is the time 7 hours 40 minutes after 11:52 p.m.?

b What is the time 9 hours and 30 minutes before 7:38 a.m.?

Solution

a Draw a timeline starting at 11:52 p.m., then add the hours and minutes in steps.



11:52 p.m. + 7 h = 6:52 a.m.

Add the minutes to the next whole hour:
6:52 a.m. + 8 min = 7 a.m.

Add the extra minutes: 7 a.m. + 32 min = 7:32 a.m. 8 min + 32 min = 40 min

So 7 h 40 min after 11:52 p.m. is 7:32 a.m.

b Draw a timeline ending at 7:38 a.m., and subtract the hours and minutes in steps.

7:38 a.m. – 9 h = 10:38 p.m.

Subtract 30 minutes: 10:38 p.m. – 30 min = 10:08 p.m.

So 9 h 30 min before 7:38 a.m. is 10:08 p.m.

Exercise 11.02 Time

1 Convert each 24-hour time to 12-hour time.

a 0845	**b** 1750	**c** 0019	**d** 1105	**e** 1628
f 0915	**g** 0240	**h** 2000	**i** 0625	**j** 1853

2 Convert each 12-hour time to 24-hour time.

a 7:35 p.m.	**b** 11:42 a.m.	**c** 11:59 p.m.	**d** 12:17 a.m.
e 12:30 p.m.	**f** 3:40 a.m.	**g** 7:10 a.m.	**h** 9:54 p.m.
i 10:18 p.m.	**j** 1:59 a.m.		

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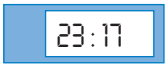
3 Write the time shown on this 24-hour clock in 12-hour time.



4 How would a 24-hour clock show these times?

a 7:30 a.m. **b** 1:15 a.m. **c** 5:10 p.m.

5 Write the time shown on this 12-hour clock in 24-hour time.

6 How would a 12-hour clock show these times?

a 2205 **b** 0545 **c** 1421

Example 6

7 What is the difference in time between 11:42 a.m. and 2:13 p.m.? Select **A, B, C** or **D.**

A 2 h 31 min **B** 3 h 55 min **C** 7 h 29 min **D** 11 h 55 min

8 Calculate the time difference between each pair of times.

a 6:15 p.m. and 8:10 p.m. **b** 1116 and 1206

c 4:10 a.m. and 8:55 a.m. **d** 11:25 p.m. and 3:20 a.m.

e 0725 and 1310 **f** 2120 and 0815

9 A film starts at 3:17 p.m. and ends at 5:09 p.m. How long is the film?

Example 7

10 Find the time:

a 6 hours after 2 p.m. **b** 3 hours before 10 a.m.

c 20 minutes before 7:15 p.m. **d** 2 h 32 min after 10:45 a.m.

e 3 h 29 min after 10:35 p.m. **f** 5 h 40 min before 9:32 a.m.

11 A car rally began at 8:20 a.m. Here are some of the competitors and the times they took. List the competitors in their order of finishing and the time each crossed the finishing line.

Tom 5:24 (5 h 24 min)	Manal 5:23	Eddie 5:44
Robert 6:01	Sarah 5:59	Gianni 5:42

12 Minnie's flight lands in Singapore at 0950 for a stopover. Her next flight leaves for Florida at 1920. How long does Minnie have in Singapore airport?

13 At St Margaret's College the school day begins at 8:25 a.m. and ends at 3:10 p.m. How long is the school day?

14 Klaas wants to go for a run for 1 h 20 min but be back in time to watch Monday Night Football on TV starting at 7:35 p.m. What is the latest time he can leave for his run?

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11.03 Timetables

Public transport trips

A **timetable** is a schedule showing the times at which railway trains, buses, aeroplanes, etc., arrive and depart. It can also be a plan that lists the times things will happen such as the times of your classes at school.

EXAMPLE 8

This is part of a timetable for a bus travelling between Sydney and Wagga Wagga.

Sydney to Wagga Wagga		Wagga Wagga to Sydney	
Sydney	2:30 p.m.	Wagga Wagga	7:15 a.m.
Strathfield	3:00 p.m.	Gundagai	8:25 a.m.
Yagoona	3:20 p.m.	Jugiong	8:54 a.m.
Liverpool	3:45 p.m.	Yass	9:41 a.m.
Mittagong	4:40 p.m.	Goulburn*	10:41 a.m.
Goulburn*	5:40 p.m.	Mittagong	12:10 p.m.
Yass	7:10 p.m.	Liverpool	1:05 p.m.
Jugiong	7:55 p.m.	Yagoona	1:20 p.m.
Gundagai	8:20 p.m.	Strathfield	1:35 p.m.
Wagga Wagga	9:30 p.m.	Sydney	2:05 p.m.

*30 minute meal stop at Goulburn

a What time does the bus from Sydney arrive in Liverpool?

b What time does this bus leave Goulburn?

c How long is the trip from Goulburn to Wagga Wagga?

d After travelling from Wagga Wagga back towards Sydney for 4 hours 55 minutes, where is the bus?

e Addison is waiting at Yass at 9:15 a.m. for the bus to Sydney. How long will he have to wait before the bus arrives?

Solution

a The bus from Sydney arrives in Liverpool at 3:45 p.m.

b This bus stops in Goulburn at 5:40 p.m. for dinner for 30 minutes, so it leaves Goulburn at 6:10 p.m.

c The bus leaves Goulburn at 6:10 p.m. and arrives in Wagga Wagga at 9:30 p.m.

The length of the trip is 3 h 20 min.

d The bus leaves Wagga Wagga at 7:15 a.m. and after 4 h 55 min the time is 12:10 p.m.

The bus is in Mittagong.

e Addison needs to wait from 9:15 a.m. to 9:41 a.m., a period of 26 minutes.

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Exercise 11.03 Timetables

Example 8

1 Refer to the bus timetable from Example **8.**

a How long does the trip from Sydney to Wagga Wagga take?

b How long would the trip take without a meal break?

c What time does the bus from Sydney arrive in Gundagai?

d Ali joins the return bus to Sydney at Jugiong and travels for 4 hours 11 minutes. Where does he stop?

e How long does Cherie have to wait for the bus to Sydney if she arrives in Mittagong at 11:45 a.m.?

f Find the time taken from Sydney to Liverpool and from Liverpool to Sydney. Suggest a reason for the difference.

2 This is part of a bus timetable showing trips between Apollo Bay and Lorne along the Great Ocean Road in Victoria.

Apollo Bay arr	06:05	09:30		14.30
Apollo Bay dep	06:07	09:32		14:32
Skenes Creek	06:10	09:35		14:35
Kennett River	06:30	09:55		14:55
Wye River	06:35	10:00		15:00
Lorne Hotel	06:55	10:20	13:32	15:20
Lorne arr		10:25		15:25
Lorne dep	07:10	10:35	13:47	15:35

a How long is the trip from Wye River to Lorne Hotel?

b You live in Apollo Bay and have an appointment in Lorne at 11 a.m. At what time do you need to catch the bus in Apollo Bay?

c Sia boards the bus at Skenes Creek and her trip takes 50 minutes. Where does she stop?

d At what time does the 2:30 p.m. bus from Apollo Bay get to Wye River?

e Where is this bus at 2:55 p.m.?

3 These tables show the daily flights between Sydney and Merimbula on the NSW south coast.

Sydney to Merimbula									
Flight No	**Depart**	**Arrive**	**Mon**	**Tues**	**Wed**	**Thu**	**Fri**	**Sat**	**Sun**
NC130	09:05	10:45	✈	✈	✈	✈	✈	✈	
NC133	14:20	15:30	✈	✈	✈	✈	✈		✈
NC139	18:50	20:25	✈	✈	✈	✈	✈		✈

ISBN 9780170413565

Merimbula to Sydney									
Flight No	**Depart**	**Arrive**	**Mon**	**Tue**	**Wed**	**Thu**	**Fri**	**Sat**	**Sun**
NC114	06:30	08:15	✈	✈	✈	✈	✈	✈	
NC118	11:05	12:20	✈	✈	✈	✈	✈	✈	
NC134	15:50	17:40	✈	✈	✈	✈	✈	✈	✈

a How many flights are there from Sydney to Merimbula on Saturday?

b What times do the flights leave Merimbula for Sydney on Tuesday? Name these flights.

c You want to be in Sydney by 12 noon on Sunday. What is the latest time you can leave Merimbula?

d What is the latest flight arrival time at Merimbula on Friday?

e How long is the NC134 flight from Merimbula to Sydney?

4 Alexis' netball team flies from Sydney to Brisbane for a tournament. Alexis visited the Nelson Air website and found the following daily flight schedule.

Flight no.	Sydney	Brisbane
NA503	0905	1030
NA511	0935	1100
NA038	1005	1130
NA114	1040	1210
NA514	1105	1230
NA051	1135	1300

a When does flight NA038 leave Sydney and how long is the journey?

b What and when is the latest flight before 11 a.m.?

c The team needs to be at Sydney airport 60 minutes before a flight takes off. When should they be at the airport to catch the NA514 flight?

d The team needs to be at the hotel in Brisbane by 12:30 p.m. and it takes 35 minutes to drive from Brisbane airport to the hotel. What is the latest flight the team can catch from Sydney?

e Which flight takes longer to reach Brisbane than the others? Give two reasons why it might take longer.

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5 Here is part of the train timetable from Goulburn to Sydney.

Goulburn to Sydney — Monday to Friday								
	am	**am**	**am**	**pm**	**pm**	**pm**	**pm**	**pm**
GOULBURN	5:08	7:27	8:17	1:47	2:45	4:26	6:47	7:45
MARULAN	5:26	7:45			3:03			8:03
TALLONG	5:32	7:51	Bookings	Bookings	3:09	Bookings	Bookings	8:09
WINGELLO	5:39	7:58	essential	essential	3:16	essential	essential	8:16
PENROSE	5:44	8:03			3:21			8:21
BUNDANOON	5:50	8:09	8:52	2:22	3:27		7:22	8:27
EXETER	5:55	8:14			3:32			8:32
MOSS VALE	6:05	8:24	9:05	2:35	3:42	5:13	7:35	8:42
BURRADOO	6:10				3:47			8:47
BOWRAL	6:13	8:30	9:11	2:41	3:50		7:41	8:50
MITTAGONG	6:17	8:34	9:16	2:46	3:54		7:46	8:54
YERRINBOOL	6:30				4:07			9:07
BARGO	6:41				4:18			9:18
TAHMOOR	6:48				4:25			9:25
PICTON	6:56	9:08			4:33			9:33
CAMPBELLTOWN	7:23	9:30	10:11	3:41	5:00	6:13	8:42	10:00
STRATHFIELD			10:42	4:17		7:00	9:12	
SYDNEY	8:12	10:12	10:54	4:29	6:20	7:13	9:24	11:04

a Wilder lives in Tallong and has an interview in Sydney at 9:10 a.m.

i At what time must he catch the train in Tallong?

ii How long is the journey?

b What is the difference in the time taken to travel from Goulburn to Sydney on the 7:27 a.m. train and the 8:17 a.m. train?

c Yuki travels from Penrose to Picton, arriving at 9:08 a.m. How long did her trip take?

d A train leaves Goulburn at 1:47 p.m. Liam wants to board this train at Bundanoon. What must he do first and when will he board the train?

e Sonya has been visiting friends in Moss Vale and is returning to Sydney. Which train should she catch and why?

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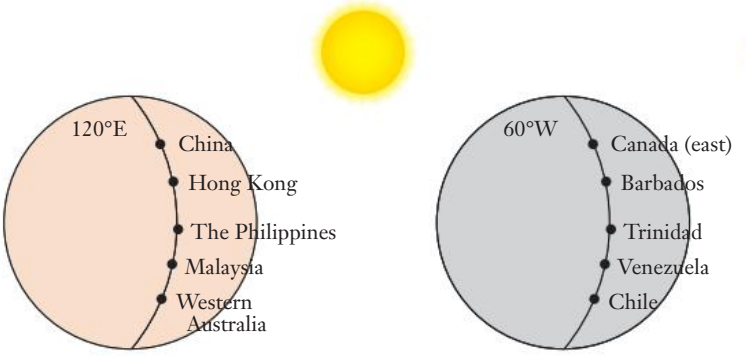
11.04 International time zones

When the Sun shines directly on a meridian of longitude, it is 12 midday at all places along that meridian (meridian means 'midday' in Latin). Directly on the opposite side of the world, it is 12 midnight. For example, when it is 12 midday in Western Australia on the 120°E meridian, it is 12 midnight in Chile on the 60°W meridian.

Table of time zones

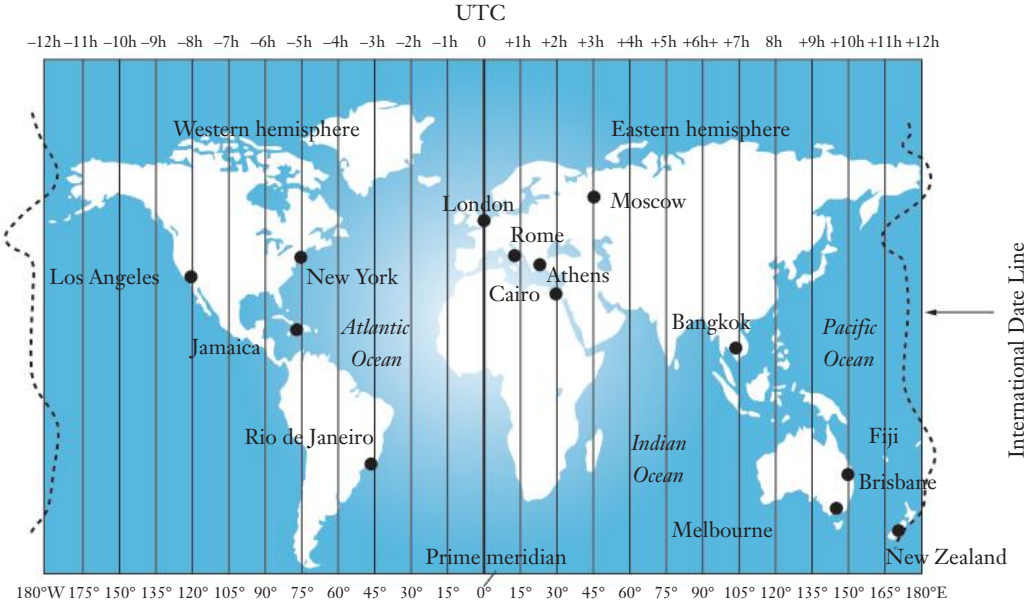
Map of the world



Midday on the 120°E meridian

Midnight on the 60°W meridian

Since the Earth turns 360° in 24 hours, it turns 15° in 1 hour. For every 15° of longitude, the time difference is 1 hour. Local times around the world are measured relative to the time along the prime meridian 0°, called **Coordinated Universal Time (UTC)**, previously called **Greenwich Mean Time (GMT)**. Places east of the prime meridian are *ahead* of UTC, while places west are *behind* UTC.



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Since 1884, the world has been divided into standard **time zones**, with roughly one hour difference for every 15° of longitude. Some countries make variations for geographical and/or administrative reasons. For example, **Australian Central Standard Time (ACST)** is 9.5 hours (not 9 hours) ahead of UTC and China has only one time zone (UTC+8) even though it stretches across four. On the map above, places along the 105°E meridian are 7 hours ahead of UTC, while places along the 150°W meridian are 10 hours behind UTC.

This table lists the international time zones relative to UTC and **Australian Eastern Standard Time (AEST)**, the time zone for eastern Australia, covering Queensland, NSW, ACT, Victoria and Tasmania.

Hours from UTC	Hours from AEST	Places in this zone
–11	–21	American Samoa, Niue Island, Midway Island
–10	–20	Hawaii, Cook Islands, Tahiti
–9	–19	Alaska
–8	–18	USA/Canada Pacific time (west coast), Los Angeles, Vancouver
–7	–17	USA/Canada Mountain time, Mexico (west)
–6	–16	USA/Canada Central time (mid-west), Mexico (east), El Salvador, Nicaragua, Guatemala, Honduras, Belize
–5	–15	USA/Canada Eastern time (east coast), New York, Cuba, Peru
–4	–14	Canada Atlantic time, Chile, Barbados, Brazil (west), Bolivia
–3	–13	Argentina, Brazil (east), Uruguay, Greenland
–2	–12	South Sandwich Islands
–1	–11	Azores
0	–10	UK, Ireland, Iceland, Portugal, Ghana, Liberia, Mali, Morocco
+1	–9	Most of Europe, Algeria, Angola, Chad, Bosnia, Croatia
+2	–8	Finland, Greece, Lebanon, Egypt, South Africa, Bulgaria, Cyprus
+3	–7	Russia (west), Saudi Arabia, Kenya, Madagascar, Iraq
+3.5	–6.5	Iran
+4	–6	Mauritius, United Arab Emirates, Armenia
+4.5	–5.5	Afghanistan
+5	–5	Pakistan, Maldives, Kazakhstan
+5.5	–4.5	India, Sri Lanka
+6	–4	Bangladesh, Bhutan, Kazakhstan, Kyrgystan
+6.5	–3.5	Myanmar (Burma), Cocos Islands

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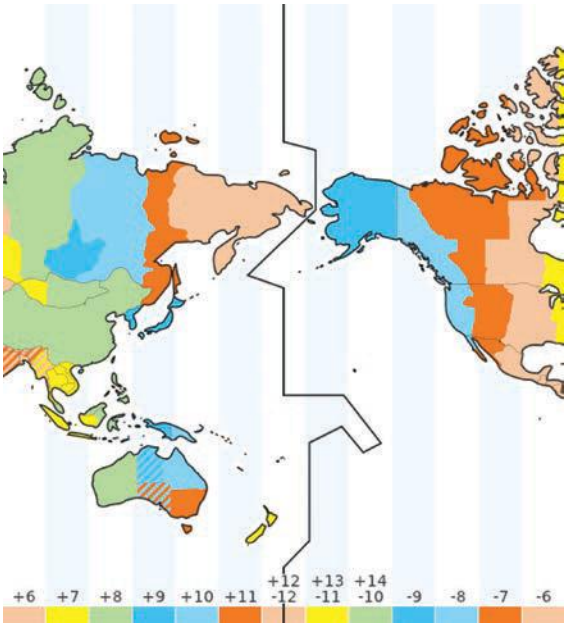
Hours from UTC	Hours from AEST	Places in this zone
+7	–3	Vietnam, Cambodia, Thailand, Indonesia (west), Laos
+8	–2	Australian Western Standard Time (WA), Malaysia, China, the Philippines, Indonesia (central), Mongolia, Taiwan
+9	–1	Japan, North Korea, South Korea, Indonesia (east), East Timor
+9.5	–0.5	Australian Central Standard Time (NT, SA, Broken Hill)
+10	0	Australian Eastern Standard Time, Papua New Guinea, Guam
+10.5	+0.5	Lord Howe Island
+11	+1	Vanuatu, New Caledonia, Solomon Islands, Micronesia
+11.5	+1.5	Norfolk Island
+12	+2	New Zealand, Fiji, Nauru
+13	+3	Samoa, Tonga

The International Date Line

The **International Date Line (IDL)**, on the opposite side of the world to the prime meridian, runs from the North Pole to the South Pole through the Pacific Ocean between Fiji and Hawaii. It is shown by the black line on the map below, and is 'bent' to ensure that it does not run through any countries.



Aman Bhagat/Quora

The IDL also runs between the Samoan islands mentioned at the start of this chapter. Suppose it is 1 am Monday in London, or UTC. Samoa's time zone is 13 hours *east* of UTC, or UTC+13, so its local time is 2 pm Monday. American Samoa's time zone is 11 hours *west* of UTC, or UTC-11, so its local time is 2 pm *Sunday*. So even though the two countries are only 64 km apart, their local times are 24 hours or a day apart.

If you cross the IDL travelling east (such going from Australia to the USA), today becomes yesterday (for example, Monday becomes Sunday) and you 'gain' a day. The reverse occurs when you cross the IDL travelling west (the opposite direction): today becomes tomorrow (for example, Sunday becomes Monday) and you 'lose' a day.

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International time zones

EXAMPLE 9

Habib is flying from New York, USA to Istanbul, Turkey to visit his grandfather. New York is 5 hours behind UTC while Turkey is 2 hours ahead of UTC.

a Habib wants to call his grandfather on the phone before he leaves New York. At what time should he call so that it is 6:00 p.m. in Turkey?

b Habib boards the plane at 1:45 p.m. New York time and the trip lasts 8.5 hours. What is the local time in Istanbul when he arrives?

Solution

a Time difference = 5 + 2 = 7 hours

New York is 7 hours behind Istanbul.

New York time = 6:00 p.m. − 7 hours

= 11:00 a.m.

b Arrival time = 1:45 p.m. + 8.5 hours (New York time)

= 10:15 p.m.

= 10:15 p.m. + 7 hours (Istanbul time)

= 5:15 a.m. (next day)

Australian standard time zones

There are three time zones in Australia (ignoring daylight saving), shown on the following map.

Australian Western Standard time (AWST) UTC + 8	Australian Central Standard time (ACST) UTC + 9.5	Australian Eastern Standard time (AEST) UTC + 10

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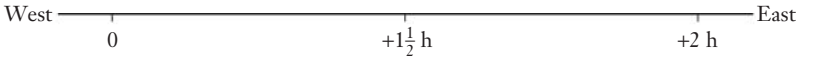
These time zones can also be shown on a timeline.



Western Standard Time	Central Standard Time	Eastern Standard Time
Perth	Adelaide	Sydney
	Alice Springs	Brisbane
	Darwin	Canberra
	Broken Hill (NSW)	Hobart

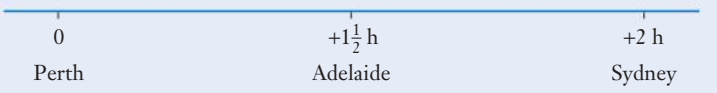
EXAMPLE 10

a Pete is in Perth and it is 11:30 a.m. He wants to ring his mother in Sydney. What time is it in Sydney?

b Nic has just finished work at 5 p.m. in Sydney. What time is it in Adelaide?

Solution

a



Moving from west (Perth) to east (Sydney) **ADD** 2 hours.

Time in Sydney = 11:30 a.m. + 2 hours

= 1:30 p.m.

It is 1:30 p.m. in Sydney.

b Moving from east (Sydney) to central (Adelaide) **SUBTRACT** $\frac{1}{2}$ hour or 30 minutes.

Time in Adelaide = 5 p.m. – 30 min

= 4:30 p.m.

It is 4:30 p.m. in Adelaide.

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Daylight saving time

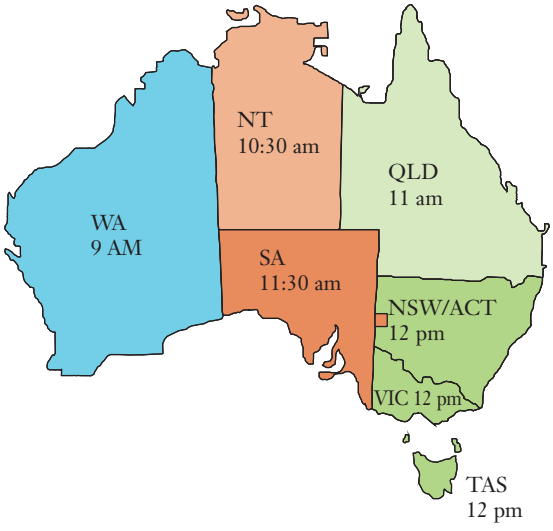
In summer, many countries adopt **daylight saving time**, or **summer time**, and turn their clocks forward one hour to take advantage of the increased hours of sunlight. By 'losing' that hour at the start of daylight saving, people are actually working and sleeping 1 hour earlier and gaining maximum use of the daylight.

Daylight saving has been operating in Australia since 1971, although Queensland, Western Australia and the Northern Territory do not participate in the scheme. Each year, it runs from the first Sunday in October (mid-spring) to the last Sunday in April (mid-autumn).

Daylight saving time

For daylight saving time, add one hour.

This map shows the local times around Australia during daylight saving when it is 12:00 midday in NSW.



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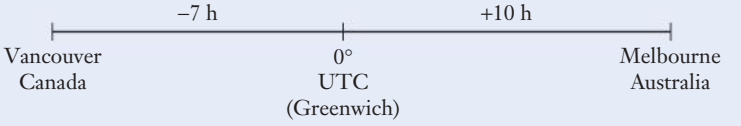
EXAMPLE 11

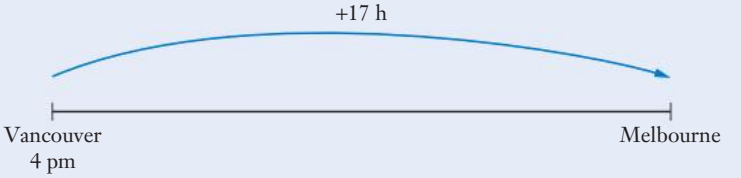
Simone, in Melbourne, wants to chat online with her cousin, Zac, in Vancouver, Canada. Canadian Pacific Standard Time is 8 hours behind UTC while Melbourne is 10 hours ahead of UTC. Daylight saving is operating in Canada. When should Simone log on to reach Zac at 4 p.m. Friday in Vancouver?

Solution

Vancouver time is UTC–8 but adding one hour for daylight saving gives UTC–7.



Time difference = 7 + 10 = 17 hours.



Melbourne is 17 hours ahead of Vancouver.

Log-on time = 4:00 p.m. Friday + 17 hours

= 9:00 a.m. Saturday.

Exercise 11.04 International time zones

Refer to the table on pages 486–487 or the worksheet 'Table of time zones' for this exercise.

Table of time zones

Example 9

1 When it is 4:00 p.m. Saturday in Sydney, what is the time in:

a India? **b** Russia (west)? **c** Greece? **d** Tonga?

2 When it is 10:00 a.m. Tuesday in London, UK, what is the time in:

a Greenland? **b** Bosnia-Herzegovina?

c Myanmar? **d** New Caledonia?

3 What is the time difference (in hours) between:

a Malaysia and New Zealand? **b** Brazil (east) and Chile?

c Iceland and the Philippines? **d** Algeria and Salt Lake City, USA (Mountain time)?

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4 The Australian Open Tennis competition is broadcast live from Melbourne at 8:30 p.m. local time. At what time is it seen in:

a Dallas, USA (Central time)? **b** Japan? **c** Sweden (Europe)?

5 It takes $8\frac{1}{2}$ hours for a plane to fly from Sydney to Hawaii. If Heath leaves Sydney at 1:30 p.m. Friday, what is the local time when he arrives in Hawaii? Select **A, B, C** or **D**.

A 2:00 p.m. Thursday **B** 5:30 p.m. Thursday

C 2:00 a.m. Friday **D** 10:00 p.m. Friday

Example 10

6 If the time in Sydney is 6:00 p.m., what is the time in:

a Melbourne? **b** Hobart? **c** Alice Springs?

d Perth? **e** Adelaide? **f** Surfers Paradise?

7 Queensland, Northern Territory and Western Australia do not participate in daylight saving. If the daylight saving time in Sydney is 10:30 a.m., what is the time in:

a Melbourne? **b** Hobart? **c** Alice Springs?

d Perth? **e** Adelaide? **f** Surfers Paradise?

8 **a** What is the time in Santiago, Chile when it is 12:00 noon in Perth?

b What does this say about the positions of Perth and Santiago on the world globe?

9 A plane takes 12 hours to fly from Adelaide (UTC+9.5) to Cape Town, South Africa (UTC+2). If Shaunna leaves Adelaide at 10:00 a.m., what is the local time when she arrives in Cape Town?

Example 11

10 Robert in Tasmania wants to call his brother in Scotland, UK when the time there is 4:30 p.m. daylight saving time. At what time should Robert call? Select **A, B, C** or **D**.

A 1:30 a.m. **B** 2:30 a.m. **C** 3:30 a.m. **D** 5:30 a.m.

11 Indira flew from New Delhi, India to Sydney, leaving at 6:30 p.m. and arriving the next day at 12:00 p.m. local time. How long did her plane trip take?

12 The Superbowl is broadcast live from New York, USA (Eastern time) at 6:30 p.m. Monday. At what time and day is it seen in:

a Sydney? **b** Berlin, Germany (Europe)? **c** Dublin, Ireland?

13 Wallaby Airlines flies from Adelaide to Mt Isa, Queensland, via Sydney. The flight left Adelaide at 2:30 p.m. and arrived in Sydney 2 h 40 min later. The plane stayed in Sydney for 35 minutes for refuelling and to take extra passengers on board.

The flight from Sydney to Mt Isa took 2 h 45 min. Find the local time when the plane arrived in:

a Sydney **b** Mt Isa.

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14 Laire boards a flight in Coffs Harbour at 9:50 a.m. Tuesday morning. The flight to Sydney takes 40 minutes. Laire then waits in Sydney for one hour and boards a flight to Perth, which takes 4 hours 25 minutes. What time is it in Perth when the plane lands?

15 A plane takes 2 hours and 45 minutes to fly from Wellington, New Zealand (UTC+12) to Sydney (UTC+10). If Paula leaves Wellington at 4:20 p.m. (daylight saving time) in September, what is the local standard time when she arrives in Sydney (standard time)?

16 A plane leaves Sydney at 1:00 p.m. Thursday and flies 20 hours to Paris, France. At what time does the plane arrive in Paris if daylight saving is operating in France but not in Sydney?

17 A plane leaves Hawaii on Monday at 12 noon and flies 6 hours west to Tokyo, Japan.

- **a** When the plane crosses the International Date Line, what day does it become?
- **b** What is the local time in Tokyo when the plane arrives?

18 Liz flew west from Buenos Aires, Argentina (UTC–3) to Melbourne (UTC+10). To her surprise, she discovered that when she arrived in Melbourne it was the same time, 7:30 p.m., as when she left Buenos Aires.

- **a** How long did her plane journey take?
- **b** Was it still the same day? If not, what day was it?

INVESTIGATION

RESEARCHING TIME ZONES

- **a** Why is UTC also known as GMT, Z time, Zulu time or Zebra time?
- **b** How is time measured at the North and South poles, where all the meridians of longitude meet?
- **c** How is time measured in Australia's external territories such as Norfolk Island, Lord Howe Island, Cocos Islands and Christmas Island (Indian Ocean)?
- **d** Which country stretches across 11 time zones?
- **e** Australia, USA and Canada are roughly the same length, but Australia has three time zones while USA and Canada have four and five time zones respectively. Why do you think this is so? (*Hint*: Look at the countries on a world globe or map.)

Longitude and time zones (Syllabus update, 2019): This extra resource is online: download it from NelsonNet.

Longitude and time zones

ISBN 9780170413565

DID YOU KNOW?

Samoa lost an entire day in 2011

The Sun rises in Australia before most of the rest of the world, and compared to us it is still yesterday in the USA. The only places that are 'ahead' of us in time are the Pacific islands such as New Zealand, Fiji, Vanuatu, Samoa and Tonga.

Samoa has undergone much change in the 21st century. In 2009, it switched from driving on the right side of the road to the left side. At the end of 2011, it changed time zones from being UTC-11 like American Samoa to UTC+13. This meant that the date jumped from 29 December to 31 December, skipping the day Friday, 30 December 2011.

Find out the main reasons Samoa changed time zones (and therefore day zones) and driving orientation.

INVESTIGATION

PLAN AN OVERSEAS HOLIDAY

Use an atlas or a world globe to plan a trip to visit up to five different places in the world. Design an itinerary that lists the places you'll be visiting. You will need to take into account:

- distances between places
- mode of transport and travelling speeds: by air, land or sea
- arrival and departure times
- international time zones
- climate/seasonal conditions (these also affect the popularity of destinations).

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SAMPLE HSC PROBLEM

Crystal is travelling by plane from Seattle, USA (48°N, 122°W) to Sydney (34°S, 151°E) for a holiday.

a What is the difference in latitude between Seattle and Sydney?

b Crystal leaves Seattle at 9:45 a.m. on Tuesday and arrives in Sydney at 4:10 a.m. Wednesday *Seattle time*. How long was the journey in hours and minutes?

c Calculate the local time and day in Sydney when Crystal arrives if Seattle is UTC-7 and Sydney is UTC+10.

Study tip

Before an exam

- Make a study plan early – don't leave it to the last minute.
- Read and memorise your topic summaries.
- Work on your weak areas – learn from your mistakes.
- Don't spend too much time studying work you know already.
- Use revision exercises and past exams/assignments for practice.
- Vary the way you study so that you don't become bored: Have someone quiz you, voice-record your summary, design a poster or mind map, explain the work to someone.
- Anticipate the exam:
  - How many questions?
  - What type of questions: multiple-choice, short answer, long answer, problem-solving?
  - Which topics will be tested?
  - How many marks for each section?
  - How long is the exam?
  - How much time should be spent on each question/section?

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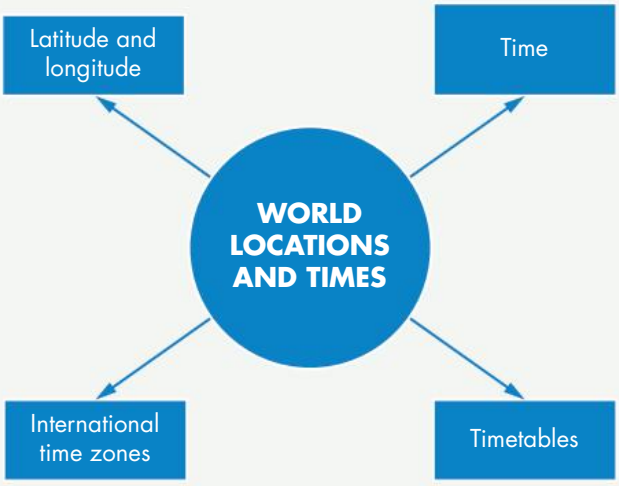
11. CHAPTER SUMMARY

World crossword

This chapter, World locations and times, focused on position and time measurement of locations around the world. Make sure that all theory, rules, terminology and worked examples are included in your summary. Students often experience difficulty in calculating differences in latitude, longitude and time, but such confusion can be overcome by developing a deep and structured understanding of the topic.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map is shown below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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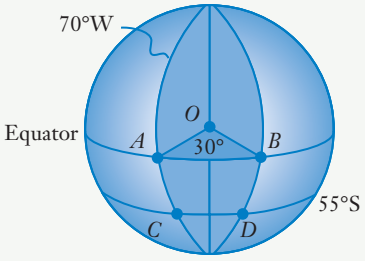
11. TEST YOURSELF

1 What are the position coordinates of A, B, C and D on the Earth?



Exercise 11.01

2 Albury, NSW has coordinates (36°S, 147°E).

Exercise 11.01

a What is the longitude of Albury?

b How many degrees is Albury north of the South Pole?

c Broome has coordinates (18°S, 122°E). What is the difference in longitude between Albury and Broome?

3 Amsterdam, The Netherlands is 15° north and 122° west of Seoul, South Korea (37°N, 127°E). What are the coordinates of Amsterdam?

Exercise 11.01

4 Convert each 24-hour time to 12-hour time.

Exercise 11.02

a 1436 **b** 0505 **c** 2218 **d** 0027

5 Convert each 12-hour time to 24-hour time.

Exercise 11.02

a 5:23 a.m. **b** 4:32 p.m. **c** 9:05 p.m. **d** 12:55 p.m.

6 Calculate the time of day:

Exercise 11.02

a 2 h 34 min after 6:21 a.m. **b** 5 h 8 min before midday

c 7 h 12 min after 0315 **d** 4 h 27 min before 1532.

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Exercise 11.03

7 The City Explorer Bus stops at places of interest in the city. This is the timetable for the Explorer bus in Brisbane.

GPO terminal	9:00	9:45	10:30	11:15	12:00	12:45	1:30	2:15	3:00	3:45	4:30	5:15
City Hall	9:05	9:50	10:35	11:20	12:05	12:50	1:35	2:20	3:05	3:50	4:35	5:20
Treasury Casino Hotel	9:10	9:55	10:40	11:25	12:10	12:55	1:40	2:25	3:10	3:55	4:40	5:25
Riverside Centre	9:15	10:00	10:45	11:30	12:15	1:00	1:45	2:30	3:15	4:00	4:45	5:30
Old Windmill	9:20	10:05	10:50	11:35	12:20	1:05	1:50	2:35	3:20	4:05	4:50	5:35
Transit Centre	9:28	10:13	10:58	11:43	12:28	1:13	1:58	2:43	3:28	4:13	4:58	5:43
Suncorp Stadium	9:32	10:17	11:02	1147	12:32	1:17	2:02	2:47	3:32	4:17	5:02	5:47
Regatra Hotel	9:45	10:30	11:15	12:00	12:45	1:30	2:15	3:00	3:45	4:30	5:15	6:00
Park Rd	9:48	10:33	11:18	12:03	12:48	1:33	2:18	3:03	3:48	4:33	5:18	6:03
Cultural Centre	9:54	10:39	11:24	12:09	12:54	1:39	2:24	3:09	3:54	4:39	5:24	6:09
Southbank	10:00	10:45	11:30	12:15	1:00	1:45	2:30	3:15	4:00	4:45	5:30	6:15
Maritime Museum	10:44	10:49	11:34	12:19	1:04	1:49	2:34	3:19	4:04	4:49	5:34	6:19
City lookout	10:10	10:55	11:40	12:25	1:10	1:55	2:40	3:25	4:10	4:55	5:40	6:25
Chinatown	10:20	11:05	11:50	12:35	1:20	2:05	2:50	3:35	4:20	5:05	5:50	6:35
ANZAC Square	10:24	11:09	11:54	12:39	1:24	2:09	2:54	3:39	4:24	5:09	5:54	6:39
GPO terminal	10:30	11:15	12:00	12:45	1:30	2:15	3:00	3:45	4:30	5:15	6:00	6:45

a How long is a complete round trip on an Explorer bus?

b How many buses are needed to meet the Explorer bus timetable? Explain how you arrived at your answer.

c Vo, Ben and Fatima arrived at Old Windmill at 11:45 a.m. They caught the Explorer bus to Southbank. What is the earliest time they could expect to arrive at Southbank? Explain your answer.

d If you miss a bus, what is the longest you would have to wait before the next one arrives?

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e Manuel and Sofia are dropped off by car at the Riverside Centre at 10:25 a.m. They arrange to meet their friends at the City Lookout at 4:30 p.m. They want to spend at least an hour at Suncorp Stadium, ride on the Ferris wheel at Southbank and do some souvenir shopping at the Maritime Museum. Plan a list of times for them to catch the Explorer bus to do these things and meet their hosts on time.

8 The time in Anchorage, Alaska (USA) is 9 hours behind UTC while the time in Lusaka, Zambia is 2 hours ahead of UTC.

Exercise 11.04

a When the time in Anchorage is 8:00 a.m., what is the time in London?

b When the time in Lusaka is 6:30 p.m., what is the time in Anchorage?

9 What is the time in Brisbane (AEST) when it is:

Exercise 11.04

a 4 p.m. in Perth? **b** 6 a.m. in Adelaide? **c** 2:30 a.m. in Hobart?

10 A plane leaves Darwin on Thursday at 6:00 p.m. and travels east for 8 hours to reach Niue Island.

Exercise 11.04

a When the plane crosses the International Date Line, what day does it become?

b If Darwin is UTC+9.5 and Niue Island is UTC−11, what is the time on Niue Island when the plane arrives there?

11 Kerrie, living in Perth, wants to call her sister in Los Angeles, USA when the time there is 12 noon Sunday. At what time should she call if Perth is UTC+8 and Los Angeles is usually UTC−8 but the USA is on daylight saving time?

Exercise 11.04

Qz

Chapter quiz

ALGEBRA, STATISTICAL ANALYSIS

12.

DRIVING SAFELY

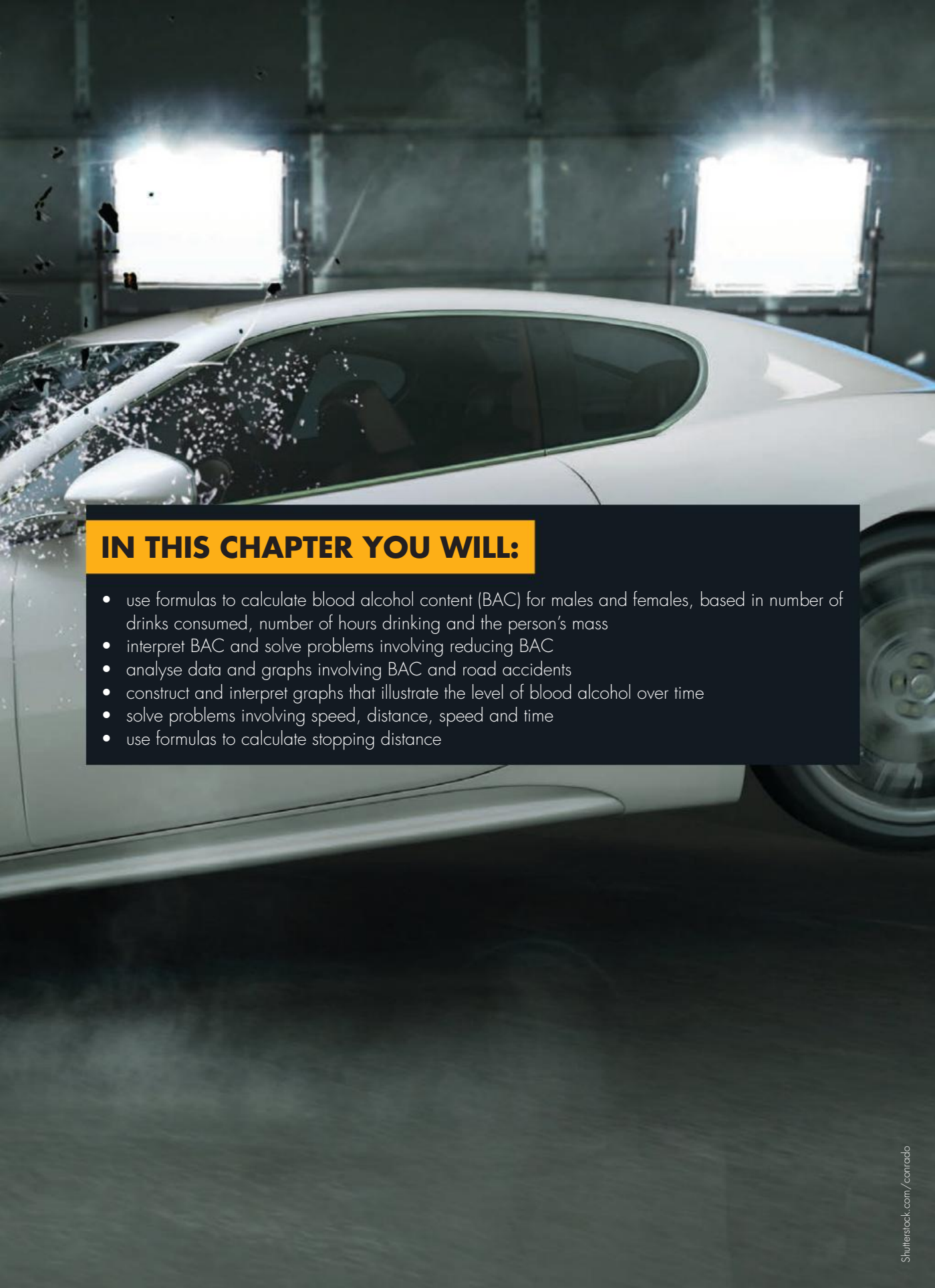
ANCAP, the Australian New Car Assessment Program, crash-tests cars in five ways:

- front-on test where the car hits a barrier at 64 km/h
- side impact test where the car is hit on the driver's side by another car travelling at 50 km/h
- pedestrian test to assess head and leg injuries to pedestrians at 40 km/h
- pole test where the car moves sideways at 29 km/h and hits a pole lined up with driver's head
- whiplash test where the stationary car is 'rear-ended' by another car travelling at 32 km/h.

The 2016 Toyota Prius received a 5-star safety rating, scoring 36.22 out of 37.

CHAPTER OUTLINE

A1, S1.1	12.01	Blood alcohol content (BAC)
S1.1	12.02	Accident statistics
A1	12.03	Speed, distance and time
A1	12.04	Stopping distance



IN THIS CHAPTER YOU WILL:

- use formulas to calculate blood alcohol content (BAC) for males and females, based in number of drinks consumed, number of hours drinking and the person's mass
- interpret BAC and solve problems involving reducing BAC
- analyse data and graphs involving BAC and road accidents
- construct and interpret graphs that illustrate the level of blood alcohol over time
- solve problems involving speed, distance, speed and time
- use formulas to calculate stopping distance

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TERMINOLOGY

blood alcohol content (BAC)
braking distance
fatality
reaction distance
reaction time
standard drink
stopping distance

SkillCheck

Assignment 12

1 Evaluate each expression, correct to two decimal places.

a $\dfrac{0.03}{0.015}$ **b** $\dfrac{0.04}{0.023}$ **c** $\dfrac{0.1}{0.034}$

d $\dfrac{26-15}{5.5\times45}$ **e** $\dfrac{10\times7-5\times7.5}{6.8\times64}$ **f** $\dfrac{2\times10-18}{5.5\times83}$

2 **a** Copy and complete this table for the linear function $y = 4 - 0.3x$.

x	0	1	2	3	4	5
y						

b If this function was graphed, what would be the gradient of the line?

3 For each set of data, find the:

i mean **ii** median **iii** mode
iv range **v** interquartile range.

Answer correct to one decimal place where necessary.

a 5 7 8 9 10 4 5 2 4 1
b 10 12 14 9 0 2 8 12 1

4 Evaluate the subject of each formula.

a $v = u + at$ when $u = 2.3, a = 8.7, t = 3$
b $s = kv(kv + b)$ when $k = 4.1, v = 42.3, b = 7.2$
c $S = \dfrac{D}{T}$ when $D = 128, T = 3.4$

5 Convert:

a 25 km to m **b** 2463 m to km
c 3 h 45 min to hours **d** 5.2 h to hours and minutes
e $7.\dot{3}$ h to hours and minutes **f** 20 km/h to m/s
g 110 km/h to m/s **h** 3.5 m/s to km/h.

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6 A plane moves at an average speed of 840 km/h for 1 day. How many km does it travel?

7 A car travels 245 km in 3 h 45 min. What is its average speed in km/h?

8 Find, correct to the nearest cent:

a 14 % of $35.80 **b** 9.5% of $26 580 **c** 12.5% of $298.60.

12.01 Blood alcohol content (BAC)

Your **blood alcohol content (BAC)** is the concentration of alcohol in your blood. A person with a BAC of 0.05 has 0.05 grams (or 50 mg) of alcohol per 100 mL of blood.

BAC formula practice

Graphing BAC

When alcohol is consumed, skills that safe driving requires become impaired and a driver will have difficulty concentrating on multiple tasks at a time. For example, a driver may be able to monitor their speed but be unable to stay within their lane.

Alcohol affects a driver's:

- **judgement** – ability to reason and respond appropriately
- **concentration** – ability to focus
- **comprehension** – ability to understand a situation quickly
- **reaction time** – ability to respond to a situation quickly.

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Calculating BAC

A **standard drink**, for example a middy of beer or a small glass of wine, contains 10 grams of alcohol.

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A person's BAC depends on four factors:

- the sex of the person
- number of alcoholic drinks consumed
- how quickly the drinks are consumed
- the mass of the person.

Calculating blood alcohol content

$$BAC_{male} = \frac{10N - 7.5H}{6.8M} \text{ and } BAC_{female} = \frac{10N - 7.5H}{5.5M}$$

where N = number of standard drinks consumed
H = number of hours drinking
M = mass in kg

Blood accounts for about 7% of a person's mass so if we assume 1 kg ≈ 1 L then an 80 kg **male driver** will have 5.6 L of blood whereas a 60 kg **female driver** has only 4.2 L of blood. The male has a greater mass to 'soak up' the alcohol and will not get drunk as quickly.

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EXAMPLE 1

Justin and Britney were at a party for 5 hours. Justin, who weighs 75 kg, had 6 standard drinks and Britney, who weighs 58 kg, had 5 standard drinks.

a Who had the lower blood alcohol content (BAC)?

b If 7% of Justin's mass is blood, what is his blood volume, correct to one decimal place?

Solution

a For Justin: $N = 6,\ H = 5,\ M = 75$

$$BAC_{male} = \frac{10N - 7.5H}{6.8M}$$
$$= \frac{10 \times 6 - 7.5 \times 5}{6.8 \times 75}$$
$$= 0.0441...$$
$$\approx 0.044$$

For Britney: $N = 5,\ H = 5,\ M = 58$

$$BAC_{female} = \frac{10N - 7.5H}{5.5M}$$
$$= \frac{10 \times 5 - 7.5 \times 5}{5.5 \times 58}$$
$$= 0.0391...$$
$$\approx 0.039$$

Britney had the lower BAC.

b Justin's blood volume = $7\% \times 75 = 5.25$ L.

Limitations of the formulas

The BAC formulas only give an *approximate* BAC because many other factors affect BAC, such as:

- your health
- the time taken for your liver to process alcohol
- the size and type of drink
- how fast you drink (the faster you drink, the higher your BAC)
- alcohol is absorbed more slowly into the bloodstream if food is in your stomach
- the type of mixer used (water and juice slow absorption while carbonated mixers speed it up)
- the temperature of the drink (warm alcohol is absorbed quicker).

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Getting back to a BAC of zero

A healthy liver can only break down about one standard drink (10 g alcohol) per hour. Not even black coffee or cold showers will speed up the rate at which your body gets rid of alcohol. Once a person stops consuming alcohol and is at a certain BAC level, a normal body can only reduce the BAC by between 0.015 and 0.02 per hour.

Returning to zero BAC

For a body that reduces BAC by 0.015 per hour, the number of hours before zero BAC is reached is:

$$\text{Number of hours} = \frac{BAC}{0.015}$$

For a body that reduces BAC by 0.02 per hour, the number of hours before zero BAC is reached is:

$$\text{Number of hours} = \frac{BAC}{0.02}$$

EXAMPLE 2

Justin and Britney from Example **1** are both on provisional licences (P plates). They stopped drinking at 11 p.m. and waited until one of them had a BAC of zero and could legally drive home. Who drove home and at what time? Assume BAC is reduced by 0.015 per hour.

Solution

For Justin: $BAC = 0.044$

$$\text{No. hours} = \frac{BAC}{0.015} = \frac{0.044}{0.015} = 2.9333\ldots \text{ h} = 2 \text{ h } 56 \text{ min}$$

For Britney: $BAC = 0.039$

$$\text{No. hours} = \frac{BAC}{0.015} = \frac{0.039}{0.015} = 2.6 \text{ h} = 2 \text{ h } 36 \text{ min}$$

Press SHIFT $0'''$ to change decimal hours to hours and minutes.

Britney drove home 2 h 36 min after 11 p.m., that is, at 1.36 a.m.

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Graphing BAC

EXAMPLE 3

After drinking 5 standard drinks, Ben's BAC was 0.09. His BAC decreases over time according to the linear function $B = 0.09 - 0.014H$.

a Use the formula to complete this table of values, correct to three decimal places.

Time after drinking, H hours	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Blood alcohol content, B	0.09		0.08					0.04			

b Graph the linear function on a number plane.

c What is the gradient of the line and what does it represent?

d Use your graph to find:

i Ben's BAC after 3 h 15 min **ii** when his BAC reaches 0.05.

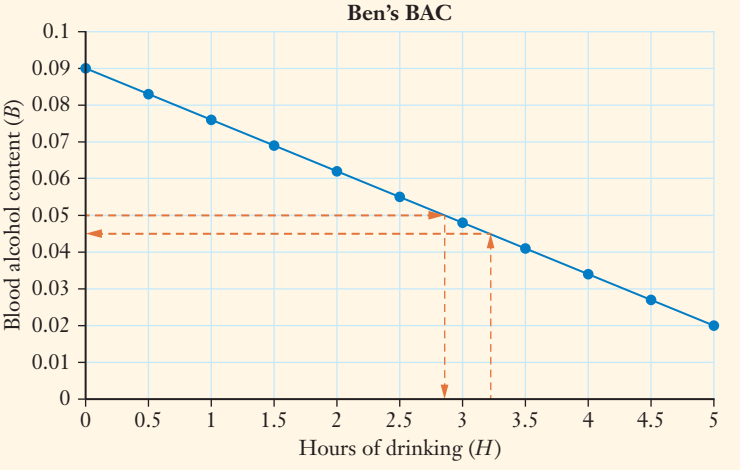
e Use the formula to find when Ben's BAC reaches zero.

Solution

a

Time after drinking, H hours	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Blood alcohol content, B	0.09	0.083	0.076	0.069	0.062	0.055	0.048	0.041	0.034	0.027	0.02

b



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c Choosing two points on the line or from the table, (0, 0.09) and (5, 0.02):

$$\text{Gradient} = \frac{0.02 - 0.09}{5 - 0} = -0.014.$$

The gradient is the rate at which Ben's BAC is decreasing per hour.

d **i** BAC after 3.25 h ≈ 0.045 From graph

ii BAC reaches 0.05 after about 2.9 h (or 2 h 50 min)

e When $B = 0$:

$$0 = 0.09 - 0.014H$$

$$0.014H = 0.09$$

$$H = \frac{0.09}{0.014}$$

$$= 6.428...$$

$$\approx 6\text{ h } 26\text{ min}$$

Ben's BAC reaches zero after 6 h 26 min.

DID YOU KNOW?

Princess Diana's tragedy

In 1997, Princess Diana, the mother of Princes William and Harry, died in a car accident in a road tunnel in Paris, France. Hotel security manager Henri Paul was the driver of the Mercedes Benz that crashed into a pillar, killing himself, the princess and her partner Dodi Fayed. He had a BAC of 0.187, was driving at 105 km/h, and none of them were wearing seat belts. His BAC was over 3 times the legal limit of 0.05, equivalent to drinking nine shots of whisky together quickly.

If Henri Paul's BAC had dropped by 0.017 per hour, how long would it have taken his BAC to return to zero?

Exercise 12.01 Blood alcohol content (BAC)

1 What is the main factor affecting your blood alcohol content? Select **A**, **B**, **C** or **D**.

A age

B number of drinks consumed

C time of day

D heart function

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2 Christine weighs 61 kg, Carla weighs 52 kg and Connie weighs 55 kg. If each drink 8 standard drinks over 6 hours, who has the highest BAC? Select **A**, **B**, **C** or **D**.

A Christine

B Carla

C Connie

D BAC is the same as they are all female

3 Mabel and Madge drank wine at an 80th birthday party over 4 hours. Mabel, who weighs 94 kg, had 4 standard drinks and Madge, who weighed 87 kg, had 3 standard drinks.

Example 1

a Who had the higher blood alcohol content, correct to two decimal places?

b If 7% of Mabel's weight was blood, how much blood did she have?

4 Who has the lowest blood alcohol content (BAC), correct to two decimal places? Select **A**, **B**, **C** or **D**.

A a 72 kg male who drank 8 drinks over 4 hours

B a 65 kg female who drank 9 drinks over 5 hours

C an 82 kg female who drank 7 drinks over 6 hours

D a 93 kg male who drank 10 drinks over 7 hours

5 This table shows the BAC (to two decimal places) for various body masses and number of drinks.

	Number of drinks									
Body mass (kg)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
50	0.03	0.07	0.10	0.14	0.17	0.20	0.24	0.27	0.31	0.34
55	0.03	0.06	0.09	0.13	0.16	0.19	0.22	0.25	0.28	0.31
73	0.02	0.05	0.07	0.09	0.12	0.14	0.16	0.19	0.21	0.23
82	0.02	0.04	0.06	0.08	0.10	0.13	0.15	0.17	0.19	0.21
95	0.02	0.04	0.05	0.07	0.09	0.11	0.13	0.14	0.16	0.18
100	0.02	0.03	0.05	0.07	0.09	0.10	0.12	0.14	0.15	0.17

a How many drinks does a 73 kg person need to drink to reach the legal limit of 0.05?

b How many drinks does a 100 kg person need to drink to reach the legal limit of 0.05?

c What is the mass of a person who has a BAC of 0.16 after 7 drinks?

d What is the BAC of a 55 kg person after 8 drinks?

e Draw a line graph showing the BAC of a 50 kg person for 1 to 10 drinks.

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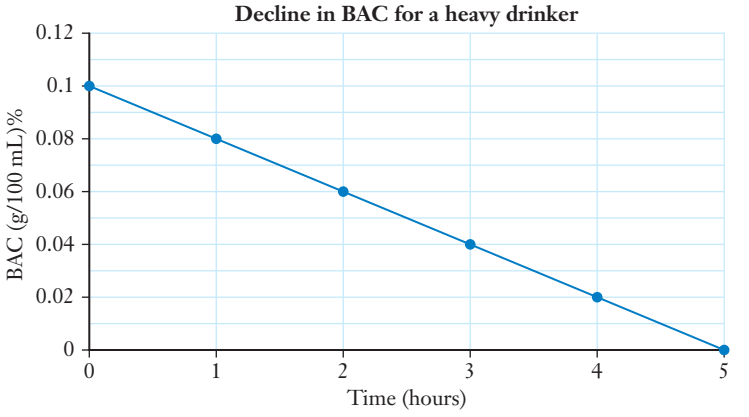
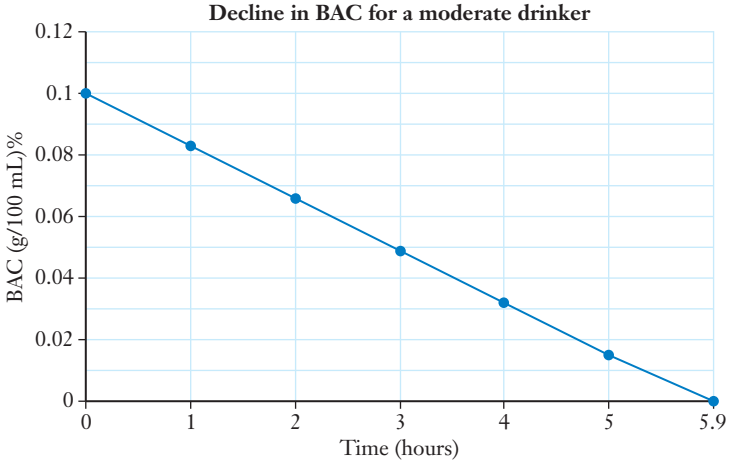
6 A heavy drinker consumes a large quantity of alcohol most days of the week whereas a moderate drinker consumes less alcohol less frequently. The two graphs show the difference in the decline of the BAC for a heavy drinker and a moderate drinker over time.





a Which drinker's BAC returns to zero quicker?

b What is the rate of decline of BAC for a:

i heavy drinker? **ii** moderate drinker?

c What is the BAC of a heavy drinker after 1 h 30 min?

d What is the BAC of a moderate drinker after 4 h 15min?

e What is the difference in the BAC for a heavy and a moderate drinker after 3 hours?

f Write a sentence or two describing the difference in the rate of BAC decrease for each type of drinker and possible reasons for the difference.

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7 Mark is 19, weighs 72 kg and holds a provisional licence with a zero alcohol limit. He started drinking at 6 p.m. and had 10 schooners of full strength beer (15 standard drinks) over 6 hours. He stopped drinking at midnight.

a What was his BAC, to two decimal places, at midnight?

b How long did it take before his BAC was back to zero if his BAC reduced by 0.02 per hour?

c At what time could he legally drive his car?

8 Maria is 17, weighs 61 kg and holds a provisional licence with a zero alcohol limit. She started drinking at 10 p.m. and had 6 mixer (9 standard) drinks over 4 hours.

Example 2

a What was her BAC, to two decimal places, at 2 a.m. when she stopped drinking?

b How long did it take before her BAC was back to zero if her BAC reduced by 0.015 per hour?

c At what time could she legally drive her car?

9 Hugh had a BAC of 0.06 when he stopped drinking and it took 5 hours for his BAC to return to zero. At what percentage rate per hour was his BAC reducing?

10 Chloe has a BAC of 0.08 and burns off alcohol at the rate of 0.016 per hour.

Example 3

a Copy and complete this table of values showing the decline in her BAC over 5 hours.

Time (hours)	BAC
0	0.08
0.5	0.072
1	
1.5	
2	
2.5	
3	
3.5	
4	
4.5	
5	

b Use the table of values to draw a line graph representing this information.

c If Chloe stopped drinking at midnight, then estimate to the nearest 15 minutes, when Chloe's BAC is equal to:

i 0.05 **ii** 0.

d What is the gradient of the line and what does it represent?

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INVESTIGATION

BAC feelings and effects

BAC	Person's feelings	Actual effects
0.03–0.05	relaxed talkative	worse concentration and coordination less able to make a judgement
0.05–0.1	blunted feelings lack of self-control	bad reflexes unable to perceive depth decreased reasoning skills
0.1–0.2	need to be over-expressive or act in a loud and disruptive manner	decrease in reaction time loss of physical control slurred speech
0.2–0.3	can't remember simple things in a daze	loss of understanding loss of sensations possible loss of consciousness
Over 0.3	depressed think they can do anything	can't read signs can't respond to traffic signals impaired breathing loss of bladder control unconsciousness or death

For example, a person in a nightclub with a BAC of 0.15 might talk loudly, disrupt other patrons and express themselves in a way that is not typical of their personality. Their speech may be slurred and they may not be able to walk or dance properly.

a How does a person feel with a BAC of 0.24?

b State three effects of drinking to a BAC of 0.06.

c What is the legal BAC for:

i a learner driver? **ii** a P-plate driver?

d Investigate the penalties for driving with a BAC of:

i 0.05 **ii** 0.1 **iii** 0.4

e Does a BAC below the legal limit mean that it is safe to drive? Explain.

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DID YOU KNOW?

BAC limits for drivers

The BAC limit in NSW (2017) for most drivers is 0.05, but lower if you are a less experienced driver or a driver of a heavy vehicle:

0 for all learner (L) and provisional (P) drivers

0.02 for drivers of buses, taxis, trucks over 13.9 tonnes or carrying dangerous goods

0.05 for all other drivers

Compared to a driver with a zero BAC, the risk of an accident with a BAC of 0.05 is double, 0.08 is 7 times and 0.15 is 20 times.

How many drinks can a provisional driver have if they plan to drive? Justify your answer.

12.02 Accident statistics

Alcohol, excessive speed, driver fatigue and distraction are the main causes of road accidents.

The risk of an accident resulting in death or serious injury is doubled for every 5 km/h you travel over 60 km/h. For example, a car travelling at 65 km/h is twice as likely to crash and a car travelling at 70 km/h is four times as likely to crash!

In a **fatal** accident, one or more persons die but there may be others who suffer serious injuries or permanent disability.

Fatal means 'resulting in death'.

Here are some statistics regarding fatal accidents in NSW.

- 65% occur in country areas.
- 13% involve alcohol.
- 30% affect motorcyclists.
- 88% of drink drivers involved in fatal accidents are male.

iStock.com/vasiliki

WS Investigating drivers in road fatalities

WS Investigating young drivers

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EXAMPLE 4

This table shows the involvement of alcohol in road accidents in 2015 in NSW.

Alcohol involved	Fatal crash	Serious injury	Non-serious injury	Total
Yes	43	386	382	811
No	283	5205	11 976	17 464
Total	**326**	**5591**	**12 358**	**18 275**

Source: Centre for Road Safety, Transport for NSW, State of NSW

a What percentage (correct to one decimal place) of fatal crashes involved alcohol?

b What percentage of the total road accidents had no alcohol involved?

c What percentage of the alcohol-involved road accidents involved a serious injury or fatality?

d If 14% of accidents involving alcohol were head-on crashes, how many was this?

Solution

a Percentage of fatal crashes involving alcohol $= \frac{43}{326} \times 100\%$

$= 13.1901 \ldots \%$

$\approx 13.2\%$

b Percentage of road accidents not involving alcohol $= \frac{17\,464}{18\,275} \times 100\%$

$= 95.5622 \ldots \%$

$\approx 95.6\%$

c Percentage of alcohol-related accidents involving serious injury or fatality $= \frac{43 + 386}{811} \times 100\%$

$= 52.8976 \ldots \%$

$\approx 52.9\%$

d Number of head-on crashes involving alcohol $= 14\% \times 811$

$= 113.54$

≈ 114

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EXAMPLE 5

This table gives the number of road fatalities in NSW from 2006 to 2015.

Year	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Road fatalities	496	435	374	453	405	364	369	333	307	350

Source: Centre for Road Safety, Transport for NSW, NSW Government 'Road traffic casualty crashes in New South Wales: Statistical Statement for the year ended 31 December 2015' roadsafety.transport.nsw.gov.au

a For the 10-year period, find:

- **i** the mean number of fatalities per year
- **ii** the range
- **iii** the median
- **iv** the interquartile range.

b Draw a column graph representing the data.

c Write a sentence discussing the trend in fatalities from 2006 to 2015. Suggest reasons for these trends.

Solution

a **i**
$$\text{Mean} = \frac{\text{sum of fatalities}}{\text{number of years}}$$
$$= \frac{3886}{10}$$
$$= 388.6$$

ii Put the values in ascending order:

307 333 350 369 364 374 405 435 453 496

lowest score; lower quartile; median; upper quartile; highest score

$$\text{Range} = \text{highest score} - \text{lowest score}$$
$$= 496 - 307$$
$$= 189$$

iii $\text{Median} = \frac{364 + 374}{2} = 369$

iv $\text{Interquartile range} = \text{upper quartile} - \text{lower quartile}$
$$= 435 - 350$$
$$= 85$$

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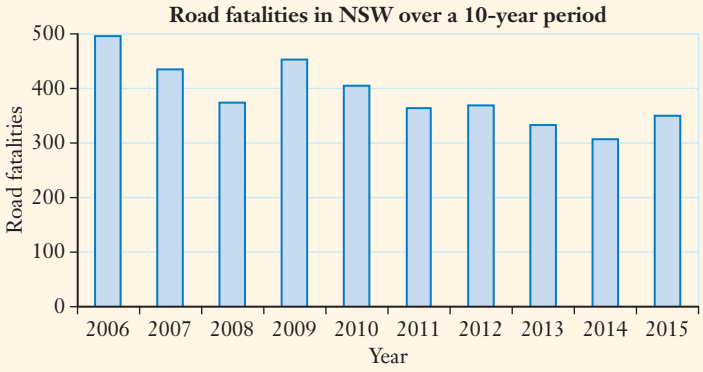
b



c The main trend is the decline in the number of road fatalities. This decline is possibly due to better road safety awareness programs, more police patrols and lower speed limits in school and roadworks zones.

Exercise 12.02 Accident statistics

1 In one year, 440 driver deaths on Australian roads were related to alcohol. Of these driver deaths, 224 had a BAC of over 0.05 and 273 were aged under 34.

a What fraction of drivers who died had a BAC over 0.05?

b What percentage (to the nearest whole number) of the drivers were aged under 34?

c What percentage of the drivers had a BAC of 0.05 or less?

Example 4

2 This table shows the number of Australian driver deaths by state and territory over two years.

Year	NSW	Vic	Qld	SA	WA	Tas	NT	ACT
2014	307	248	223	107	183	33	39	10
2015	350	252	243	102	160	34	49	15

Source: Bureau of Infrastructure, Transport and Regional Economics (BITRE), 2016, Road trauma Australia, 2015 statistical summary BITRE, Canberra ACT.

a How many driver deaths were there in Australia in:

i 2014? **ii** 2015?

b Find the average annual number of driver deaths in Australia over the two-year period.

c Which state or territory has the fewest driver deaths? Why do you think?

d Which state or territory had the smallest change between 2014 and 2015?

e Which states and territories had fewer driver deaths in 2015 than they did in 2014?

f Write a sentence describing the similarities and/or differences in the data for the years 2014 and 2015.

g Draw a divided bar graph representing the data for the year 2015.

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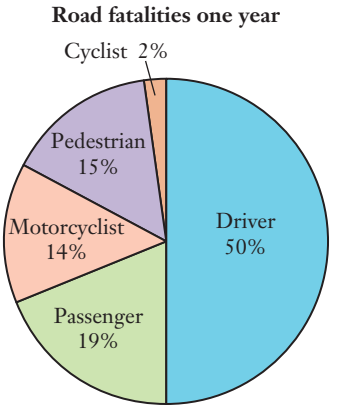
3 An accident survey reported that for fatigued drivers in 18 764 single-vehicle crashes, 70.2% were driving cars, 75.5% were male, 35% were aged under 24 years and 9.6% were aged over 60 years.

a How many drivers were aged 24 to 60?

b How many crashes did not involve cars?

c How many drivers were female?

4 This sector graph shows the category of person killed in road accidents.



a To the nearest degree, what is the sector angle in the graph for:

i Driver? **ii** Passenger? **iii** Cyclist?

b If there were 443 road fatalities last year, how many persons killed were:

i passengers? **ii** motorcyclists?

c If there were 68 pedestrians killed this year, how many drivers were killed?

5 This table shows the drivers under 40 years who died in road accidents over 4 years.

	Year			
Age of driver	**2012**	**2013**	**2014**	**2015**
under 17	70	66	65	65
17–25	284	230	234	226
26–39	300	243	252	272
All ages	1300	1187	1150	1205

Source: Bureau of Infrastructure, Transport and Regional Economics (BITRE), 2016, Road trauma Australia, 2015 statistical summary BITRE, Canberra ACT.

a Find the number of drivers aged 25 or less who were killed over the 4 years.

b Calculate (correct to one decimal place) the percentage of drivers killed in 2012 who were 17–25 years old.

c What was the percentage decrease (correct to one decimal place) of drivers killed aged 26–39 between 2014 and 2015?

d Which year had the fewest road fatalities overall?

e Which year had the fewest road fatalities for people aged 17–25?

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Example 5

6 This table shows the number of Australian road injuries involving different categories of people over six years.

Year	Drivers	Passengers	Pedestrians	Motorcyclists	Pedal cyclists	All road users
2008	3360	943	651	2341	1419	9315
2009	3420	1010	635	2576	1723	9854
2010	3421	988	726	2491	1780	9884
2011	3824	1047	632	2647	2017	10 667
2012	3713	995	643	2680	2157	10 671
2013	3773	1060	658	2881	2517	11 351

Source: Bureau of Infrastructure, Transport and Regional Economics (BITRE), 2016, Road trauma Australia, 2015 statistical summary BITRE, Canberra ACT.

a Which year and category had the highest number of road injuries?

b How many road injuries were there in Australia over the six years?

c What is the mean annual number of injuries involving motorcyclists (correct to one decimal place) over the 6 years?

d What is the range of passenger injuries over the 6 years?

e What is the median number of driver injuries over the 6 years?

f What is the interquartile range of pedal cyclist injuries over the 6 years?

g Represent the data for passengers and pedestrians on a line graph.

7 This table shows the number of Australian fatal crashes over 7 years categorised by number of vehicles and pedestrians involved.

Year	Single-vehicle crashes	Multiple-vehicle crashes	Pedestrian crashes	Total crashes
2009	649	509	189	1347
2010	544	520	169	1233
2011	504	463	184	1151
2012	520	503	167	1190
2013	515	430	156	1101
2014	463	438	149	1050
2015	491	447	163	1101

Source: Bureau of Infrastructure, Transport and Regional Economics (BITRE), 2016, Road trauma Australia, 2015 statistical summary BITRE, Canberra ACT.

a Calculate for single-vehicle crashes:

- **i** the mean number of crashes over the 7 years
- **ii** the median
- **iii** the range
- **iv** the interquartile range.

b Construct a column graph to represent the data for pedestrian crashes.

c Looking at the data for total crashes, describe any general trends over the 7 years and possible reasons for these.

d Why do you think there are more single-vehicle crashes than multiple-vehicle crashes?

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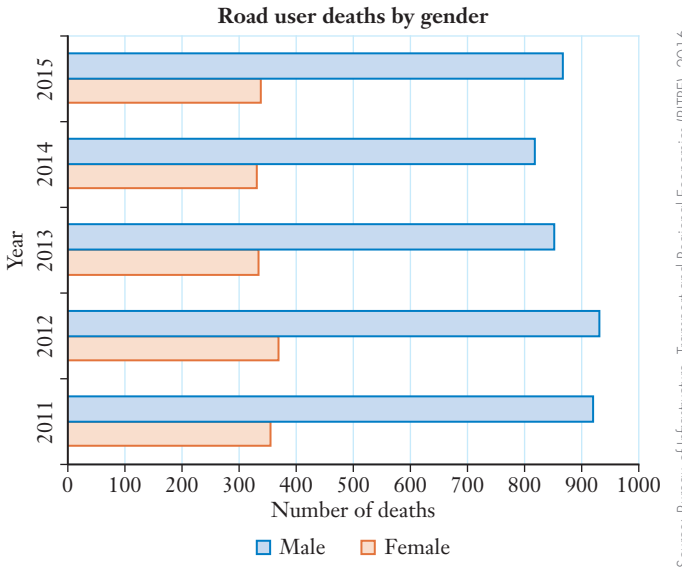
8 This clustered column graph compares male and female road fatalities over five years.



Source: Bureau of Infrastructure, Transport and Regional Economics (BITRE), 2016, Road trauma Australia, 2015 statistical summary BITRE, Canberra ACT.

a How many male deaths, to the nearest 50, occurred in 2013?

b How many female deaths, to the nearest 50, occurred in 2014?

c How many road user deaths, to the nearest 50, occurred in 2015?

d Describe any patterns in the data over the years and any differences between males and females, stating reasons for your comments.

9 This table shows the times of day NSW road casualties occurred in 2015.

Time of day	Road casualties
12 midnight–1:59 a.m.	447
2 a.m.–3:59 a.m.	270
4 a.m.– 5:59 a.m.	429
6 a.m.–7:59 a.m.	1404
8 a.m.–9:59 a.m.	2135
10 a.m.–11:59 a.m.	2080
12 midday–1:59 p.m.	2125
2 p.m.–3:59 p.m.	2608
4 p.m.–5:59 p.m.	2918
6 p.m.–7:59 p.m.	1985
8 p.m.–9:59 p.m.	1119
10 p.m.–11:59 p.m.	752

Source: Centre for Road Safety, Transport for NSW, State of NSW

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a Which time interval had the highest number of casualties? Give a reason why this may be so.

b Which time interval had the lowest number of casualties? Give a reason why this may be so.

c For this data set, find:

i the range **ii** the mean (correct to one decimal place)

iii the median.

d **i** Describe the pattern in the number of road casualties over the 24-hour period.

ii Do you think the pattern would still be the same for today? Give reasons.

10 This table shows the age and sex of speeding drivers involved in road accidents in NSW in 2015.

Age	Male drivers	Female drivers
17–20	357	150
21–25	319	125
26–29	195	71
30–39	374	145
40–49	286	104
All ages	2069	788

Source: Centre for Road Safety, Transport for NSW, State of NSW

a How many speeding drivers were aged 40–49?

b How many speeding female drivers were aged under 30?

c What percentage of speeding drivers aged 17–20 were male? Answer correct to three significant figures.

d What percentage (correct to two decimal places) of female speeding drivers were aged 17–20?

e What patterns do you notice between the numbers of male and female speeding drivers? Why do you think this is so?

f What patterns do you notice in the number of speeding drivers as age increases? Why do you think this is so?

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DID YOU KNOW?

Driver fatigue

Fatigue is mental or physical tiredness due to lack of sleep. In 2015, 16% of road fatalities in NSW involved driver fatigue. Most fatigue-related road fatalities occur between midday and 6 p.m., not at night. Fatigue is a factor in about 20% of head-on collisions where a vehicle crosses to the wrong side of the road due to a driver's slower reaction time or 'microsleeping'.

Here are some statistics on drivers in fatigue-related road accidents.

- 77% were driving cars.
- 72% were male.
- 26% were aged 17 to 25.
- 51% died or were seriously injured.

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12.03 Speed, distance and time

Speed limits are imposed to ensure drivers travel at a safe speed for the road and surrounding environment. Some common speed limits in NSW are:

School zone	40 km/h
Residential area	50 km/h
City street	60 km/h
Highway	100 km/h
Motorway	110 km/h

WS Speed formula practice

WS Speed stories

WS Distance, speed and time

Average speed

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

$$S = \frac{D}{T}$$

This formula can also be written as $D = ST$ or $T = \frac{D}{S}$.

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EXAMPLE 6

Dean left home at 6 a.m. to travel from Sydney to Brisbane, a distance of 1027 km. He stopped for meal breaks and fuel and arrived in Brisbane at 8.30 p.m.

a What was his average speed for the trip, correct to the nearest km/h?

b How far, to nearest kilometre, could he travel at this speed in 50 minutes?

c How long, in hours and minutes, would it take him to drive 864 km at this speed?

Solution

a $D = 1027\,\text{km};\ T = 14.5\,\text{h}$ ← 6 a.m to 8.30 p.m.

$$S = \frac{D}{T}$$
$$= \frac{1027}{14.5}$$
$$= 70.8275...$$
$$\approx 71\ \text{km/h}$$

Average speed was 71km/h.

b $S = 71\,\text{km/h};\ D = ?;\ T = 50\,\text{min} = \frac{50}{60}\,\text{h} = \frac{5}{6}\,\text{h}$

$$D = S \times T$$
$$= 71 \times \frac{5}{6}$$
$$= 59.166...$$
$$\approx 59\,\text{km}$$

He could travel 59 km.

c
$$T = \frac{D}{S}$$
$$= \frac{864}{71}$$
$$= 12.169...$$
$$\approx 12\,\text{h}\ 10\,\text{min}\ 8.45\,\text{s}$$ ← Press °′″ to convert decimal hours to hours, minutes and seconds.

$\approx$ 12 h 10 min

It would take him 12 h 10 min.

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Exercise 12.03 Speed, distance and time

1 Eva drove 93 km from Wollongong to Nowra in 1 hour 30 min. What was her average speed in km/h? Select **A**, **B**, **C** or **D**.

A 46.5 **B** 31 **C** 62 **D** 60

2 Stefan travelled 160 km at an average speed of 76 km/h. How long was his journey in hours and minutes? Select **A**, **B**, **C** or **D**.

A 2 h 10 min **B** 2 h 6 min **C** 2.2 h **D** 2.1 h

Example 6

3 Larni drove 742 km from home to her farm in 9 h 30 min.

a What was her average speed, correct to the nearest km/h?

b How long, in hours and minutes, would it take her to drive 1026 km at this rate?

c How far, to the nearest kilometre, could she travel at this rate in 3 h 10 min?

4 Cathy drove at an average speed of 57 km/h. At this rate, how long, in hours and minutes, will it take her to drive:

a 507 km? **b** 160 km? **c** 440 km?

5 Tan cycled 24 km to Scott's place at an average speed of 16 km/h. He cycled back at 12 km/h. Find:

a the total time for Tan's complete trip

b Tan's average speed, to the nearest km/h, for the trip.

6 Justine's car journey took 9 h 45 min at an average speed of 9.5 m/s. What is her:

a average speed in km/h?

b distance travelled, correct to the nearest metre?

7 How far, in metres to one decimal place, will a vehicle travel in 0.1 seconds when travelling at:

a 40 km/h? **b** 60 km/h? **c** 80 km/h? **d** 100 km/h?

8 Courtney drove a racecar for 402 m at 530 km/h.

a How long, to the nearest second, did it take?

b How far (correct to the nearest metre) could she travel in 10 seconds at this speed?

9 Luis rode his motorbike 460 m in 6.011 seconds.

a What was his speed in m/s, correct to one decimal place?

b How long, to the nearest 0.1 minute, would it take to go 58 km at this rate?

10 **a** Casey's average speed was calculated at 208.33 km/h for a 60.72 km motorbike race. How long, to the nearest 0.1 minute, did he take?

b If he took the same time to complete a 72.4 km race, what was his average speed in km/h for this race, correct to the nearest km/h?

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11 Shellie and Roxy set off for the beach 11.5 km away. Shellie rides her motor scooter at 32 km/h and Roxy cycles at 11 km/h. How many minutes head start should Shellie give Roxy so they arrive at the beach at the same time?

12 A car is travelling at 75 km/h. How far, to the nearest 0.1 m, will it travel in the 5 seconds it takes you to cross the road?

13 Police investigating a car crash determined that the car covered 20.7 m in 1.2 s. At what speed was the car travelling in km/h?

12.04 Stopping distance

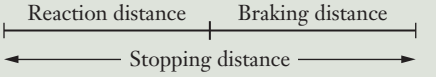
See, think, brake

Imagine you are driving and you see a child run out onto the road in front of you. You hit the brakes to avoid an accident. What is the time and distance from when you sense the danger to when you come to a full stop?

Shutterstock.com/Amy Johansson

Stopping distance

- **Reaction time** is the time between sensing a situation and applying the brakes.
- **Reaction distance** is the distance travelled during your reaction time.
- **Braking distance** is the distance travelled from when you apply the brakes until you stop.
- **Stopping distance** = Reaction distance + braking distance



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Braking distance

In dry conditions, a car travelling at 60 km/h can stop 20 m before one travelling at 80 km/h.

Drivers tend to underestimate stopping distance because the time taken to stop depends on many factors:

- your speed
- your reaction time
- the condition of your brakes and tyres
- the rate at which you decelerate
- the road surface and slope
- the wind speed and weather
- the weight of your vehicle
- driver visibility, whether it is night or day.

Braking distance formula

The approximate braking distance (d m) can be found using the formula:

$$d = kv^2$$

where v km/h is the initial speed of the car and
k is a **constant**.

EXAMPLE 7

Ben has a reaction time of 1.5 s. He was driving at 60 km/h when he saw a tree had fallen across the road. He applied his brakes and stopped 17.4 m later.

a What was his reaction distance, to the nearest metre?

b What was his stopping distance?

c Use the braking distance formula $d = kv^2$ and the given values to find:

- **i** the value of k (correct to two significant figures)
- **ii** the braking distance, to the nearest metre, when travelling at 84 km/h
- **iii** the stopping distance, to the nearest metre, when travelling at 84 km/h
- **iv** the speed, to the nearest km/h, of a vehicle with a braking distance of 100 m.

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Solution

a Reaction distance is the distance car travels in 1.5 s at 60 km/h.

Convert 60 km/h to m/s first.

$60 \text{ km/h} = 60\,000 \text{ m/h}$ Converting km to m

$= \frac{60000}{3600} \text{ m/s}$ Converting h to s

$= 16.666 \text{ m/s}$

$d = s \times t$

$= 25 \text{ m}$

Reaction distance is 25 m.

b Stopping distance = reaction distance + braking distance

$= 25 + 17.4$

$= 42.4 \text{ m}$

c **i** Given $d = 17.4$ m and $v = 60$ km/h,

$d = kv^2$

$17.4 = k \times 60^2$

$= 3600k$

$k = \frac{17.4}{3600}$

$= 0.004\,8333\ldots$

≈ 0.0048

ii Given $v = 84$ km/h and $k = 0.0048$,

$d = kv^2$

$= 0.0048 \times 84^2$

$= 33.8688$

≈ 34

Braking distance is 34 m.

iii Stopping distance = reaction time + braking distance

$= 25 + 34$

$= 59 \text{ m}$

Stopping distance is 59 m.

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iv Given $d = 100$ m and $k = 0.0048$,

$$d = kv^2$$

$$100 = 0.0048v^2$$

$$v^2 = \frac{100}{0.0048}$$

$$= 20833.333...$$

$$v = \sqrt{20833.333...}$$

$$= 144.3375...$$

$$\approx 144$$

Speed is 144 km/h.

Speed and road conditions

It will take longer to stop on a wet road and even longer on an icy road. The braking distance changes with the condition of the road.

EXAMPLE 8

This table shows the different stopping distances at four different speeds and two different road conditions for a driver whose reaction time is 2 s.

Stopping distances for various speeds and road conditions						
Speed (km/h)	**Road**	**Reaction time (s)**	**Reaction distance (m)**	**Braking distance (m)**	**Stopping distance (m)**	**The equivalent of**
40	dry	2	22.2	7.9	30.1	
40	wet	2	22.2	12.6	34.8	
50	dry	2	27.8	12.3	40.1	8 car lengths
50	wet	2	27.8	19.7	47.5	4 bus lengths
60	dry	2	33.3	17.7	51.0	Olympic swimming pool
60	wet	2	33.3	28.3	61.6	3 cricket pitches
100	dry	2	55.6	49.2	104.8	length of a football field
100	wet	2	55.6	78.7	134.3	width of 10 houses

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a What do you notice about the reaction distances?

b Discuss the difference in braking distance for a car travelling at:

- **i** 40 km/h on a wet road and 50 km/h on a dry road
- **ii** 50 km/h on a wet road and 100 km/h on a wet road.

c Discuss the difference in stopping distance for a car travelling at:

- **i** 40 km/h on a dry road and 40 km/h on a wet road
- **ii** 60 km/h on a dry road and 100 km/h on a dry road.

d Why should you reduce speed when travelling on a wet road?

e Draw a clustered column graph to represent the data and say why this is a good representation of the data.

Solution

a The reaction distances increase with speed but do not depend on the road condition.

b **i** There is a 10 km/h difference in speed but the braking distances are about the same.

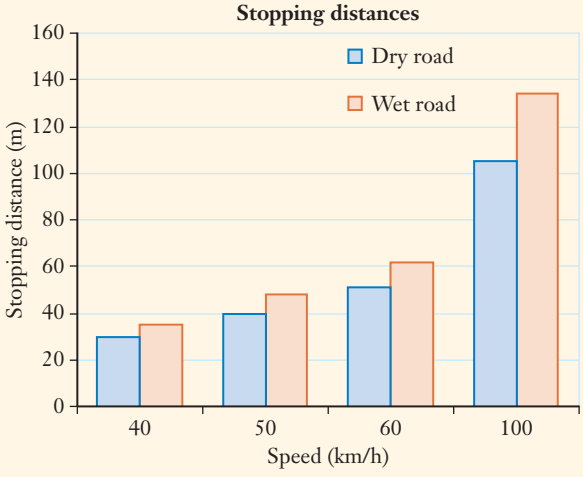
ii The speed is doubled but the braking distance is roughly 4 times greater.

c **i** At 40 km/h there is a 5 m difference in the stopping distance between a dry and wet road and this difference is significant.

ii There is a 40 km/h difference in speed but the stopping distance is doubled: 51.0 m and 104.8 m.

d You take longer to stop on a wet road so you should reduce your speed to avoid any incidents.

e The height of the columns shows clearly that as the speed increases, the difference between the stopping distances on dry and wet roads is greater.



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Exercise 12.04 Stopping distance

1 Darley saw a car stopped 80 m in front of him. He just stopped in time after travelling 51 m while braking. What was his reaction distance? Select **A**, **B**, **C** or **D**.

A 29 m **B** 80 m **C** 51 m **D** 30 m

2 A van moving at 110 km/h travelled a distance of 118.4 m on a wet road before its driver applied the brakes and finally stopped another 79.3 m later. What was the van's stopping distance?

Example 7

3 A motorcyclist with a quick reaction time stopped in 15.6 m after braking for a distance of 9.8 m. How far did he travel during the time it took to react to the danger?

4 This table shows the reaction, braking and stopping distances for a driver on a wet road with a reaction time of one second.

Speed (km/h)	Reaction time (s)	Reaction distance (m)	Braking distance (m)	Stopping distance (m)
40	1	11.1	B	23.7
50	1	A	19.7	33.6
60	1	16.7	C	45.0
100	1	27.8	78.7	D

a Find the values of A, B, C and D.

b When the speed is doubled from 50 km/h to 100 km/h, find to the nearest whole number the percentage increase in:

i braking distance **ii** stopping distance.

c If a driver with a reaction time of 1 s is travelling on a wet road at 100 km/h and sees an obstacle 110 m ahead, will he stop in time? Give reasons for your answer.

5 Two cars were moving side-by-side along a dry road in a 40 km/h school zone when their drivers each saw children crossing 50 m ahead. Fran applied her brakes within 1 second but Hanna took 2 seconds. Use the table of stopping distances from Example **8** on page 527 to find:

Example 8

a how far Hanna travelled under brakes

b how far Fran travelled before she applied her brakes

c if either would stop in time. Why?

6 Two truck drivers were travelling through a town on a dry road. Mark was doing 60 km/h and took 2 s to react to an accident up ahead while Sanjay was travelling at 40 km/h but took 4 s to react. Use the table of stopping distances from Example **8** on page 527 to find:

a how far Sanjay travelled before the brakes were applied

b how far Mark travelled under brakes

c which driver had the greater stopping distance.

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7 Fatima has a reaction time of 1.6 s. When travelling at 80 km/h, she applied his brakes and travelled 62 m before stopping. Use the braking distance formula $d = kv^2$ to find:

a the constant k, correct to four decimal places

b the braking distance, to the nearest 0.1 m, when travelling at 105 km/h

c the speed (to the nearest km/h) of Fatima if she travels 75 m under brakes before stopping

d the stopping distance (to the nearest 0.1 m) from a speed of:

i 80 km/h **ii** 105 km/h.

8 A car travelling at 115 km/h covered 89.6 m under brakes. Use the formula $d = kv^2$ to find:

a the constant k, correct to three significant figures

b the braking distance, correct to one decimal place, if the initial speed is 95 km/h

c the stopping distance for a driver with a reaction time of 2 s who is travelling at 115 km/h.

9 Use the formula $d = 0.02754v^2$ where v km/h is the speed of a car and d m the distance the car travels under brakes, to find (correct to three significant figures) the braking distance of a vehicle travelling at:

a 35 km/h **b** 61 km/h **c** 112 km/h **d** 93 km/h.

10 A car cruising at 80 km/h travelled 41.8 m on an icy road during the time it took for the driver to see a stop sign and start braking. It then travelled 57.2 m under brakes before it stopped.

a What was the driver's reaction time in seconds, to two decimal places?

b What was the stopping distance?

11 Use the formula $d = 0.00435v^2$, where d m is the distance travelled under brakes and v km/h the speed as brakes are applied, to find the speed of a racecar, correct to the nearest km/h, with a braking distance of:

a 142 m **b** 91.2 m **c** 101.4 m **d** 68.9 m.

12 A car travelled 84.3 m under brakes when initially travelling at 106.3 km/h. Use the braking formula $d = kv^2$ to find (correct to three significant figures):

a the constant k

b the braking distance for a car applying brakes from a speed of 110 km/h

c the stopping distance of a car travelling at 110 km/h if the driver's reaction time is 2 s.

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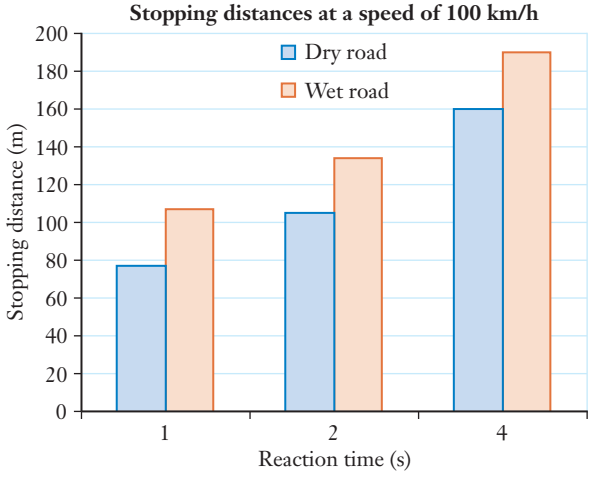
13 The formula $d = \frac{1}{2}x^2 + 5x$, where x km/h is the speed of a vehicle and d m the distance travelled, is only used for very fast cars. How many metres can a racing car travel at a speed of:

a 100 km/h? **b** 250 km/h? **c** 300 km/h?

14 The graph shows the stopping distances on wet and dry roads with various reaction times.



a A driver on a dry road has a reaction time of 4 s. How far will he travel before he stops?

b How many metres does it take for a car to stop when travelling at 100 km/h on a wet road if the driver has a reaction time of:

i 1 s? **ii** 2 s? **iii** 4 s?

c Use the graph to copy and complete this table.

Road condition	Reaction time	Stopping distance

d Compare the benefits of showing data in a table with data represented on a graph.

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INVESTIGATION

STOPPING DISTANCES

1 a Use this table to describe the relationship between road conditions, speed and stopping distance, for a driver with a reaction time of 1.5 s. All values in the table are rounded to one decimal place.

Distances for a reaction time of 1.5 seconds				
Road condition	**Speed (km/h)**	**Reaction distance (m)**	**Braking distance (m)**	**Stopping distance (m)**
Dry	60	25.0	17.4	42.4
	100	41.7	48.2	89.9
Wet	60	25.0	27.8	52.8
	100	41.7	77.2	118.8
Icy	60	25.0	138.9	163.9
	100	41.7	358.8	427.5

This table of values, to one decimal place, was found using an online calculator.

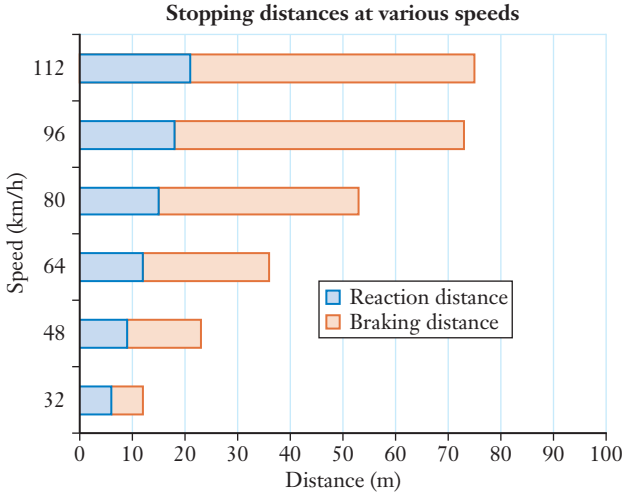
b Find an online calculator and use it to determine the stopping distance for:

i a car travelling at 60 km/h on an icy road if the driver has a reaction time of 1.6 s

ii the difference in stopping distances at 80 km/h for reaction times of 0.9 s and 1.4 s

iii a car travelling on a dry or wet road at 110 km/h, if reaction time is 1.5 s.

2 This graph shows the reaction and braking distances for cars travelling at various speeds. Use the graph to investigate the effect of speed on stopping distances.



Give one trend that you have discovered by looking at stopping distances with road conditions and speed.

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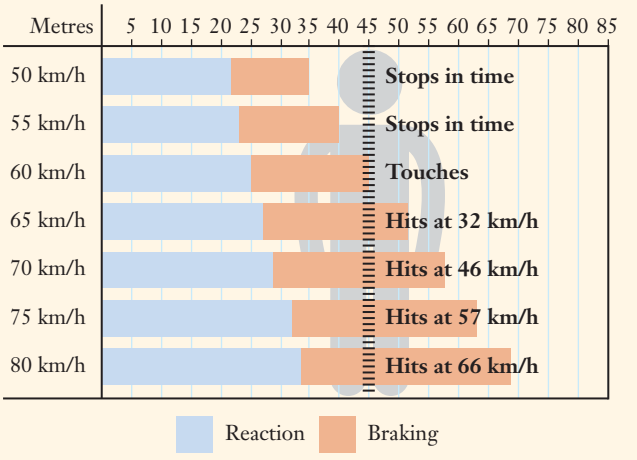
DID YOU KNOW?

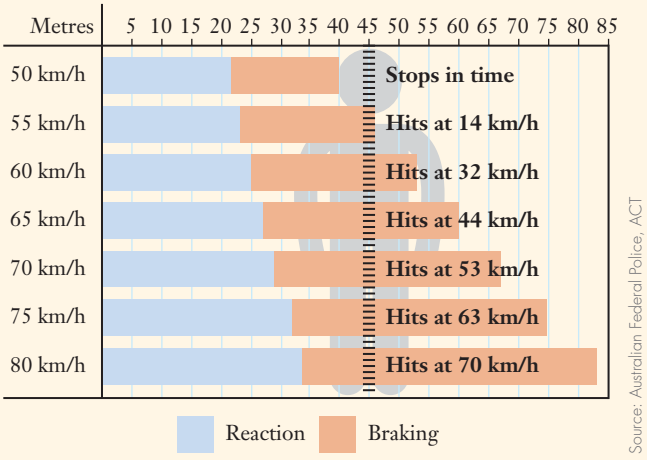
Will you stop in time?

These graphs were published by the Australian Federal Police to display stopping distances for different speeds and road conditions. If you are driving in a 60 km/h zone and a child runs onto the road 45 metres ahead and you brake hard, will you stop in time?

Dry conditions



Wet conditions



Source: Australian Federal Police, ACT

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INVESTIGATION

REACTION TIME TESTS

1 Use an Internet search engine to find a **Reaction Time Test** where the times you take to click the button when the light comes on are recorded and the average reaction time calculated. Work in groups of 4 or 5 and decide who has the best average reaction time.

2 Work in pairs to measure your 'reaction distance'.

- Your partner holds the top of a 30 cm ruler, at the 30 cm end.
- You sit on a chair with your thumb and forefinger around the bottom of the ruler, at the 0 cm end.
- Your partner drops the ruler at a random time.
- You catch the ruler and record the reaction distance.
- Repeat 5 times and find your average reaction distance.
- Swap places with your partner and repeat.

Which student in the class has the fastest reaction time (shortest reaction distance)? What is the class average?

3 Name three activities where a fast reaction time is an advantage.

Study tip

More exam tips

- Read all instructions carefully. Don't rush.
- Calculate the average time you should spend on each question.
- If one answer is taking too long, stop and ask yourself: 'Am I on the wrong track?' Don't get bogged down. You may need to retrace your steps, start again or come back later.
- When you have finished a question, make sure you have actually answered it. Do you need to write the answer in a sentence? Put a circle or box around the answer to highlight it.
- Make sure that your answer is reasonable, especially if it involves money or measurement. Did you round the answer correctly and include the correct units?
- If you have some time left at the end of the exam, double-check your answers, especially for the more difficult or uncertain questions.

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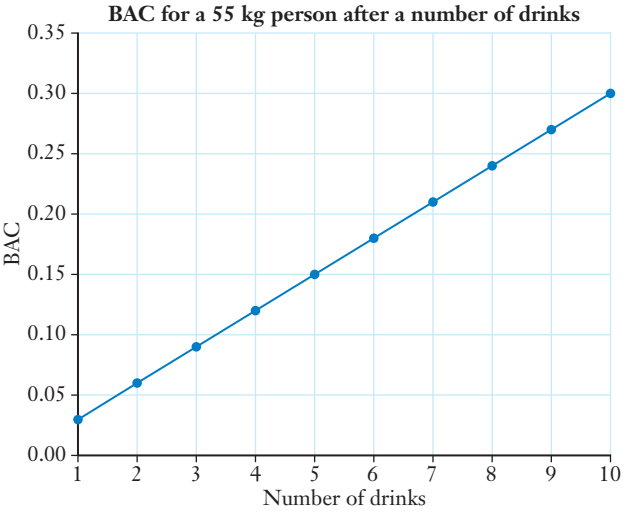
SAMPLE HSC PROBLEM

This graph shows the blood alcohol content (BAC) of a 55 kg woman according to the number of drinks she has had.



a Bianca is 55 kg. Estimate her BAC after 5 drinks.

b How many drinks will cause her to reach a BAC beyond 0.1?

c When BAC reaches 0.3, a person usually passes out. After how many drinks will this occur?

d What is the gradient of the line in the graph and what does it represent?

e If Bianca consumes 8 drinks, how many hours will it take her to return to a BAC of zero? (Use the formula: $h = \dfrac{BAC}{0.015}$)

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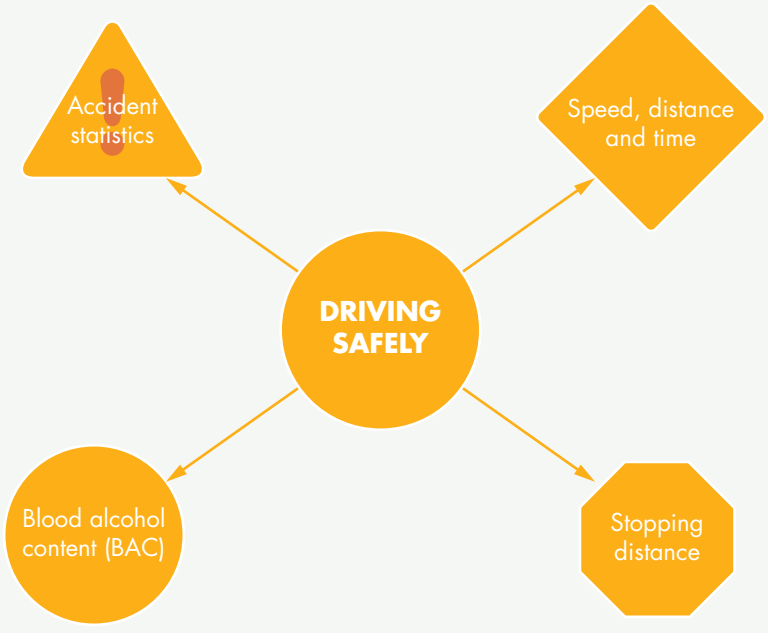
12. CHAPTER SUMMARY

This chapter, Driving safely, looked at the mathematics behind safe driving, covering the areas of measurement (blood alcohol, speed and stopping distance), algebra (formulas) and data analysis (road accident statistics). You should be familiar with the terminology, formulas, graphs and calculations involved with BAC, road accidents, speed and stopping distance.

Make a summary of this topic. Use the outline at the start of this chapter as a guide. An incomplete mind map is shown below. Use your own words, symbols, diagrams, boxes and reminders. Gain a 'whole picture' view of the topic and identify any weak areas.



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12. TEST YOURSELF

1 Jennifer weighs 56 kg and drank 3 glasses of wine at a party over 3 hours. She thought she was safe to drive but when pulled over and tested, she was over the legal limit of 0.05. She later discovered that each wine glass contained $1\frac{1}{2}$ standard drinks. Use the formulas $BAC_{\text{female}} = \frac{10N - 7.5H}{5.5M}$; Number of hours $= \frac{BAC}{0.015}$ to answer the following questions.

Exercise 12.01

a What was her BAC, to three decimal places, when she was tested?

b How long, in hours and minutes, did it take before her BAC was back to zero?

c What would her BAC be, to three decimal places, after 3 standard drinks in 3 hours?

2 This table shows the BAC for women of different weights and number of drinks consumed in an hour.

Exercise 12.01

	Body weight (kg)						
Drinks per hour	**45**	**55**	**64**	**73**	**82**	**91**	**100**
1	0.05	0.04	0.03	0.03	0.03	0.02	0.02
2	0.09	0.08	0.07	0.06	0.05	0.05	0.04
3	0.14	0.11	0.10	0.09	0.08	0.07	0.06

Sheridan is 64 kg. At a party, she consumed 2 drinks in the first hour and 3 drinks in the second hour.

a Calculate Sheridan's BAC:

i at the start of the party

ii after the first hour

iii after the second hour.

b If Sheridan had no more drinks, how many hours until her BAC returns to zero? Assume that her body reduces her BAC at the rate of 0.018 per hour.

3 **a** Why is a heavier person less affected by alcohol?

Exercise 12.01

b Why are females more affected by alcohol?

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Exercise 12.02

4 This table shows the number of people killed in road crashes in an 8-year period in Australia.

Road fatalities in Australia for 2008–2015				
	Drivers		Passengers	
Year	Male	Female	Male	Female
2008	495	175	177	125
2009	521	186	182	148
2010	470	166	154	129
2011	422	146	158	126
2012	459	151	124	136
2013	393	164	101	102
2014	386	146	109	118
2015	424	131	117	132

Source: Bureau of Infrastructure, Transport and Regional Economics (BITRE), 2016, Road trauma Australia, 2015 statistical summary BITRE, Canberra ACT.

a What is the mean number of male drivers killed per year?

b What is the range of female passengers killed over the 8 years?

c In 2015, what percentage (correct to one decimal place) of drivers killed were male?

d In 2015, what percentage (correct to one decimal place) of females killed were drivers?

e What do you notice when you compare the numbers of drivers with the number passengers killed each year? Give a reason why this may be so.

f What is the median number of female drivers killed per year?

g Is the number of drivers killed each year generally increasing or decreasing? Give a reason why this may be so.

Exercise 12.02

5 This table shows the number of Australian fatal crashes over 7 years categorised by number of vehicles and pedestrians involved.

Year	Single-vehicle crashes	Multiple-vehicle crashes	Pedestrian crashes	Total crashes
2009	649	509	189	1347
2010	544	520	169	1233
2011	504	463	184	1151
2012	520	503	167	1190
2013	515	430	156	1101
2014	463	438	149	1050
2015	491	447	163	1101

Source: Bureau of Infrastructure, Transport and Regional Economics (BITRE), 2016, Road trauma Australia, 2015 statistical summary BITRE, Canberra ACT.

a Which category of crash shows a steady decline over the years until 2015?

b In 2015, what percentage (correct to one decimal place) of crashes involved pedestrians?

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c True or false? There are always more single-vehicle crashes than multiple-vehicle crashes.

d Which year had the highest number of:

i multiple-vehicle crashes? **ii** crashes overall?

e Calculate the percentage increase (to one decimal place) in single-vehicle crashes between 2014 and 2015.

f Draw a sector graph to display the data for 2015.

6 Scott took $2\frac{3}{4}$ h to drive from Canberra to Bodalla, 200 km away.

Exercise 12.03

a What was his average speed, correct to the nearest km/h?

b How long, in hours and minutes, would it take him to drive 430 km to Newcastle at this speed?

c How far, to the nearest km, could he travel at this speed in 5 hours?

7 A truck driver was travelling in the Northern Territory at 150 km/h on a road with no speed limits. He began braking 3 seconds after he saw a sign to a truck stop. If he travelled 810 m under brakes before coming to a stop, how far did he travel after seeing the sign?

Exercise 12.04

8 George is travelling on a dry road at 65 km/h and sees a cow in the middle of the road about 70 m ahead. He takes 1.2 s to apply the brakes, then travels 24 m under brakes before coming to a stop. Did he hit the cow? Give reasons for your answer.

Exercise 12.04

9 The following measurements were taken in a school zone.

Exercise 12.04

Stopping distances on a wet road in a school zone			
Reaction time (s)	**Reaction distance (m)**	**Braking distance (m)**	**Stopping distance (m)**
1	11.1	12.6	**A**
2	22.2	12.6	**B**
4	44.4	12.6	**C**

The speed limit in a school zone during school hours is 40 km/h.

a What do you notice about the braking distances?

b What do you notice about the reaction distances?

c Find the values of **A**, **B** and **C**.

d If an 85-year-old driver in a school zone has a slow reaction time of 4 s, would he stop in time if he sees a child crossing the road 50 m ahead?

e Suggest two strategies for this driver that would help him avoid any accidents when driving.

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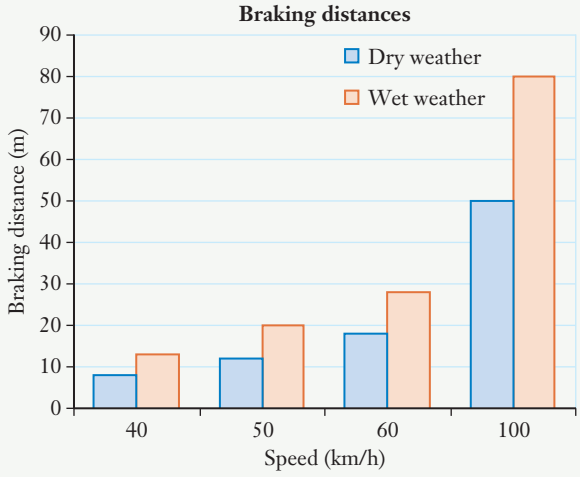
Exercise 12.04

10 This graph gives the braking distance for a car travelling on a dry or wet road at various speeds.



a What is the difference in braking distance on a dry road for two cars travelling at 50 km/h and 100 km/h respectively?

b How much further will a car travel under brakes on a wet road at 60 km/h than on a dry road at the same speed?

c If a car travels 50 m under brakes on a dry road, what is its approximate speed?

d What is the approximate speed of a car on a dry road that travels 12 m after the brakes are applied?

e Write a sentence noting any similarities or differences between braking distances on wet and dry roads, giving reasons for your answer.

Qz Chapter quiz

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Practice paper 4

Recommended time: One hour

Section 1

10 multiple-choice questions: 1 mark each
Select the correct answer, A, B, C or D.

Question 1

Exercise 11.02

What is 12:15 a.m. in 24-hour time?

A 0015 **B** 1215
C 1415 **D** 2415

Question 2

Exercise 10.01

Which statement about the set of data {0, 1, 2, 3, 4, 4, 7} is true?

A The mean and mode are equal.
B The mean and median are equal.
C The median and mode are equal.
D The mean, median and mode are not equal.

Question 3

Exercise 11.01

Port Moresby, Papua New Guinea has coordinates (9°S, 147°E). If Nyngan, NSW is due south of Port Moresby, which one of the following could be the coordinates of Nyngan?

A (4°S, 147°E) **B** (31°S, 147°E) **C** (9°S, 140°E) **D** (9°S, 151°E)

Question 4

Exercise 12.01

Sarah had a blood alcohol content (BAC) reading of 0.062 at 10:30 p.m., at the end of a party. Determine when her BAC will be back to zero by using the formula:

$$\text{Number of hours} = \frac{\text{BAC}}{0.015}$$

A 2:38 a.m. **B** 2:43 a.m. **C** 4:08 a.m **D** 4:13 a.m.

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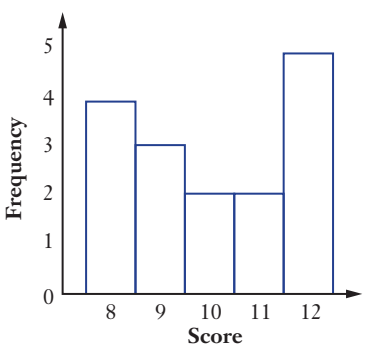
Exercise 10.03

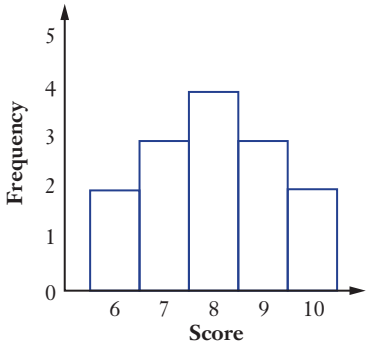
Question 5

The data sets A and B are displayed in these histograms.

A



B



Which one of these statements is true about the mode and range of sets A and B?

A A has a higher mode and B has a higher range.

B A has a higher mode and both ranges are the same.

C Both modes are the same and B has a higher range.

D Both modes are the same and both ranges are the same.

Exercise 11.02

Question 6

Robert needs to board a plane at Sydney airport that departs at 6:35 a.m. Passengers must check-in and board the plane 45 minutes before departure. It takes 50 minutes for Robert to drive to the airport from home. The shuttle bus from the car park to the airport takes up to 20 minutes. What is the latest time that Robert can leave home to board the plane on time?

A 4:40 a.m. **B** 4:50 a.m. **C** 5:00 a.m. **D** 5:20 a.m.

Exercise 12.03

Question 7

Joe left Sydney at 8 a.m. to travel a distance of 1000 km to Melbourne. He arrived in Melbourne at 9 p.m. What was his average speed for the journey, correct to the nearest km/h?

A 77 km/h **B** 83 km/h **C** 91 km/h **D** 111 km/h

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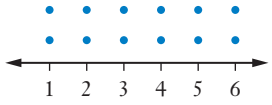
Question 8

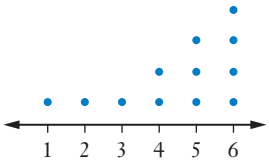
Exercise 10.08

Which dot plot illustrates a distribution that is positively skewed?

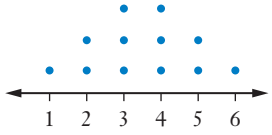
A



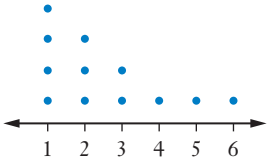
B



C



D



Question 9

Exercise 11.05

Australian Eastern Standard Time AEST is 10 hours ahead of UTC. Calculate the local time in the United Kingdom when it is 6 p.m. in NSW and daylight saving is operating in the United Kingdom.

A 10 a.m. **B** 3 a.m. **C** 4 a.m. **D** 9 a.m.

Question 10

Exercise 10.04

The six employees at the Bug and Beef Cafe earn the following weekly wages:

$350 $420 $510 $130 $320 $460

The mean and standard deviation of this data set were calculated. If a wage of $720 is added to this data set, how will its mean and standard deviation be affected?

A The mean will increase but the standard deviation will decrease.

B The mean will increase but the standard deviation will stay the same.

C The mean and standard deviation will both increase.

D The mean and standard deviation will both decrease.

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Section 2
3 questions: 10 marks each

Question 11 (10 marks)

Exercise 11.04

a This timetable shows flights from Sydney and Melbourne to Christchurch in New Zealand.

	Flight	Depart	Arrive
Sydney - Christchurch	NC482	0855	1355
	NC490	1710	2210
	NC511	1945	0045
Melbourne - Christchurch	NC479	0845	1405
	NC488	0905	1425
	NC491	1755	2315

Exercise 10.05

i When does the first flight leave Sydney and when does it arrive in Christchurch? Write your answers in 12-hour time. **1 mark**

ii The arrival times are shown in New Zealand time, so you need to subtract 2 hours for Australian Eastern Standard Time. Calculate how long it takes to fly between Melbourne and Christchurch. Answer in hours and minutes. **2 marks**

iii Miriam lives in Sydney and needs to be in Christchurch by 10 a.m. on Tuesday for a conference. What is the latest flight she can take? **1 mark**

b A sample of households was surveyed on the number of motor vehicles owned.

Exercise 12.01

No. of vehicles	Frequency	Cumulative frequency
0	1	
1	14	
2	26	
3	7	
4	2	

i Copy and complete the cumulative frequency column of the table. **1 mark**

ii Find the mean for the data. **1 mark**

iii Find the range for the data. **1 mark**

iv Construct a cumulative frequency histogram and polygon for the data. **2 marks**

v Use the cumulative frequency polygon to find the interquartile range. **1 mark**

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Question 12 (10 marks)

a Tegan weighs 56 kg and has had 5 standard alcoholic drinks over 4 hours.

Exercise 12.01

i Calculate, correct to three decimal places, her blood alcohol content using the formula: **1 mark**

$$BAC_{female} = \frac{10N - 7.5H}{5.5M}$$

where N = number of standard drinks consumed

H = number of hours drinking

M = mass in kg

ii If Tegan drank the same number of standard drinks over a longer time, how would her blood alcohol content have been affected? **1 mark**

b Jayden was travelling at 70 km/h when he saw a car stopped ahead of him. He took 1.2 s to react and apply his brakes, then stopped 14 m later.

Exercise 12.04

i What was his reaction distance, to the nearest metre? **1 mark**

ii Use the braking distance formula $d = kv^2$ to find the value of k, correct to two significant figures. **1 mark**

iii Find Jayden's braking distance, correct to one decimal place, if he was travelling at 90 km/h. **1 mark**

c The number of accidents per month on a building site were:

Exercise 9.08

9 4 3 0 1 7 5 6 5 3

For this data, find:

i the mode **1 mark**

ii the interquartile range **1 mark**

iii the standard deviation, correct to two decimal places. **1 mark**

d The following information is based on population data for the body mass indices (BMI kg/m^2) of boys aged 16 years.

Exercise 12.02

- The 1st quartile $Q_1 = 18.8$
- The 5th decile $D_5 = 20.6$
- The 9th decile $D_9 = 25.4$
- The 10th percentile $P_{10} = 17.6$
- The 97th percentile $P_{97} = 29.4$

i Dylan is 16 and has a BMI of 25.4. What percentage of boys aged 16 have a BMI higher than his? **1 mark**

ii Santosh has the median BMI for 16-year-old boys. What is his BMI? **1 mark**

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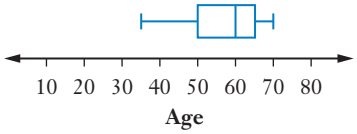
Question 13 (10 marks)

Exercise 10.06

a This box plot shows the ages of members at a lawn bowls club.



i What is the age of the oldest member? **1 mark**

ii Calculate the interquartile range of ages. **1 mark**

iii If there are 124 members in the club, how many of them are aged between 50 and 60? **1 mark**

Exercise 12.02

b This table shows the age and sex of speeding drivers involved in road accidents in NSW in 2015.

Age	Male drivers	Female drivers
17–20	357	150
21–25	319	125
26–29	195	71
30–39	374	145
40–49	286	104
All ages	2069	788

Source: Centre for Road Safety, Transport for NSW, NSW Government 'Road traffic casualty crashes in New South Wales: Statistical Statement for the year ended 31 December 2015' roadsafety.transport.nsw.gov.au

i How many speeding drivers were female? **1 mark**

ii What percentage of speeding male drivers were aged 17–20? Answer correct to 3 significant figures. **1 mark**

iii What patterns do you notice between the numbers of male and female speeding drivers? Why do you think this is so? **1 mark**

Exercise 11.04

c A plane flies east from Manila, the Philippines (14°N, 121°E) to Mexico City (19°N, 99°W). It flies for 19 hours, leaving Manila at 6:00 a.m. Tuesday.

i Which city is further south: Manila or Mexico City? **1 mark**

ii After the plane crosses the International Date Line, what day will it be? **1 mark**

iii If the Philippines is 8 hours ahead of UTC and Mexico City is 8 hours behind UTC, what is the time and day in Mexico City when the plane arrives? **2 marks**

END OF EXAMINATION

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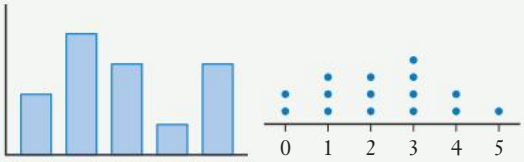
ANSWERS

Chapter 1

SkillCheck

1 **a** sector graph **b** stem-and-leaf plot
c line graph **d** divided bar graph

2



bar chart dot plot

3 **a** 20 **b** 5 **c** 500

4 **a** $\frac{3}{8}$ **b** $37\frac{1}{2}\%$

5 **a** $\frac{7}{20}$ **b** 35%

6 24.5

Exercise 1.01

1 **a** 16% **b** 3% **c** 4648
d 36 389
e It remained stable at 13%.
f 1995: 8%, 2013: 16% which is double, so the statement is true.
g The percentage of people drinking at a risky level will be higher.

2 **a** $\frac{1}{4}$ **b** labour **c** $2.63
d $6500 **e** 54° **f** $1060

3 **a** $\frac{1}{6}$ **b** 35% **c** Suspicious
d 16

4 **a** male **b** 25–34 **c** 14%
d 238 700 **e** 65–74 **f** decreases

5 **a** in-store, online via PC
b **i** in-store **ii** in-store **iii** online via PC
c **i** 17% **ii** 8% **iii** 11%

6 **a** Yes **b** underweight
c 18 **d** 26; lose weight
e 16 to 25

7 **a** 1 mm **b** number of raindays
c 23 mm; 4 **d** October; 24.5 mm
e July; 6 **f** April (lowest rainfall)
g 9

8 A

9 **a** Share prices over 6 days
b 10 c **c** $4.55
d Tuesday **e** It fell, then rose.
f Thursday; lowest price **g** NCM Bank

10 **a** 17 **b** East; 25 on Thursday
c Monday; first working day
d Wednesday; 6
e Saturday; no industry

Exercise 1.02

1 **a** The vertical axis starts at $1.50 so increases in share price appear steeper than they actually are.

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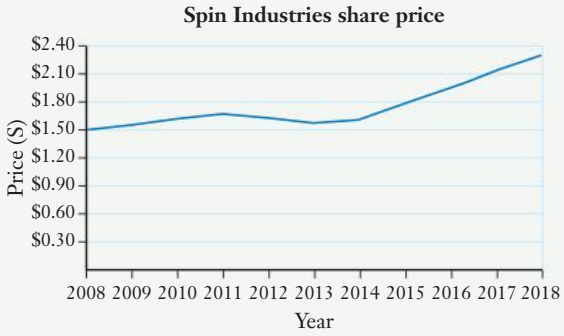
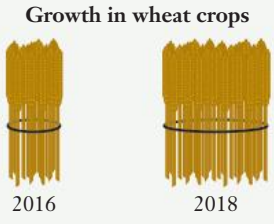
b

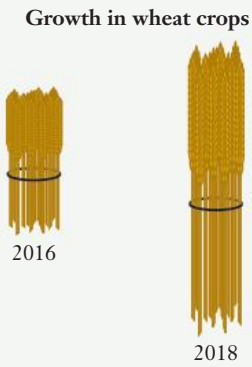


2 **a** Because the 2018 picture is double the length and double the width, it is actually four times the size of the 2016 picture instead of two times the size.

b



or



3 **a** a company's sales

b The new director has made sales go down.

c sales figures and months

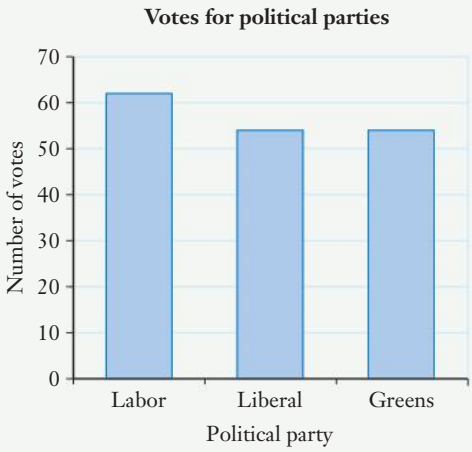
4 **a** The $800 note is double the length and double the width of the $400 note and this makes it four times the size of the $400 note instead of two times the size.

b

5 years ago Now or 5 years ago Now

5 **a** By starting the scale on the vertical axis at 53.

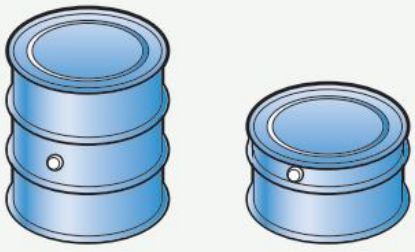
b



c The Labor votes are not overwhelmingly higher. They are just 8 more.

6 **a** The picture for '$100 of oil today' is actually about $\frac{1}{4}$ the size of the picture for '10 years ago'.

b



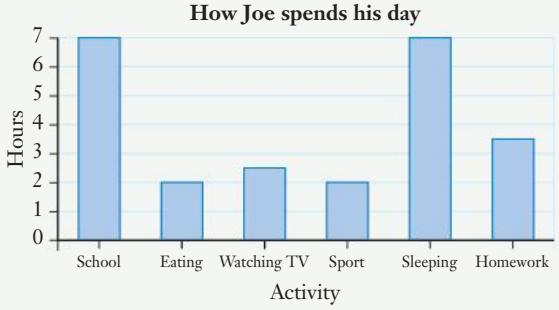
$100 of oil 10 years ago **$100 of oil today**

7 **a** 7

b The hours scale does not begin at 0.

c Big difference between school or sleeping and the other activities.

d



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Exercise 1.03

1 **a** C **b** N **c** C **d** N
e N **f** C **g** C **h** C
i N **j** N **k** N **l** C

2 Categorical data uses words/symbols, for example, type of pet. Numerical data uses quantities/numbers, for example, person's height.

3 **a** N **b** O **c** O **d** N
e N **f** O **g** O **h** N

4 **a** C **b** C **c** C **d** C
e D **f** C **g** D **h** C

5 **a** HSC subjects
b size of hot chips at a fast food store
c number of children in a family
d heights of boys in Year 11

6 **a** numerical **b** categorical
c continuous **d** nominal

7 B

Exercise 1.04

1 **a** C **b** S **c** S **d** C
e S **f** S **g** C **h** S

2 2021, 2026

3 Teacher to check reasons.
a systematic **b** random **c** self-selected
d stratified **e** random **f** random
g random **h** random
i random **j** self-selected

4 138 **5** 188 **6** B

7 **a** Not all teens read magazines.
b Only those diners interested/volunteering to complete the survey are sampled.
c Need to select more students and not only those who arrive early.
d Need to sample workers as well, not only management who may have a biased view.
e People at a premiere are not the usual moviegoers.
f It is restricted to one radio station.
g Relies on passengers to reply to the survey, a self-selected sample. May be biased towards passengers who had a bad experience or a very good experience.

8 b, f and g

9 **a** systematic sample
b shoppers in one store only; only females or males

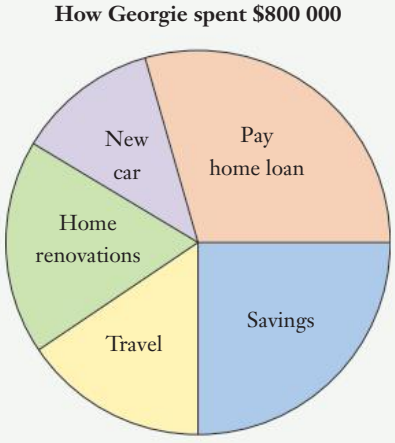
Exercise 1.05

1 **a**

Score	Frequency
0	3
1	6
2	9
3	12
4	7
5	2
6	1
	40

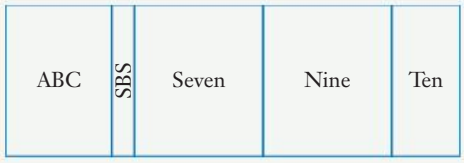
b data is discrete (not continuous)
c 3 **d** 6; 1 student **e** 7.5%

2 **a** shows parts of the whole amount
b



c Saved it **d** $\frac{9}{50}$ **e** 12%

3 **a** 14 970 **b** sector; divided bar; bar chart
c



TV evening news watched in Sydney

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d 23% **e** yes

f No, since the survey only included Sydney households.

4 a 72 **b** bar chart; discrete data

c 9.7% **d** $\frac{11}{18}$

5 D

6 a Delivery delayed, pizza not hot

b 1100 **c** 90% **d** True

e On-time delivery, pizza would be hotter. Discuss with drivers/employees ways to coordinate cooking pizza with prompt delivery to improve efficiency. If delayed delivery is traffic related then perhaps other means of transport needed in certain areas – scooter, bicycle, walking.

7 a Insects

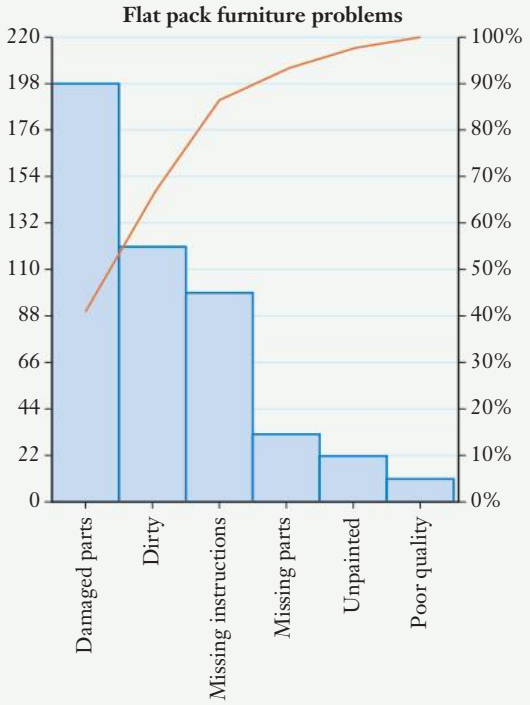
b Insufficient storage, loud music after 11 p.m.

c 100 **d** 25%

e **i** True **ii** False **iii** True

f Install fly screens in rooms and reverse cycle air conditioning

8 a



b Damaged parts, dirty

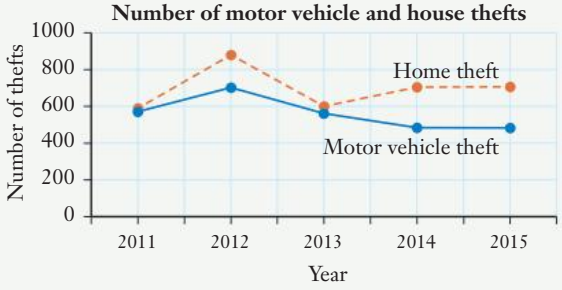
c Checking products are unbroken and clean before packing, providing safer packing and transportation.

9 a

Country of origin of immigrants

b T **c** $\frac{4}{31}$ **d** 8% **e** India

10 a



b 2012 **c** 2015 **d** home theft

e Home thefts rose sharply from 2011 to 2012, then reduced significantly in 2013 to then increase again only slightly until 2015.

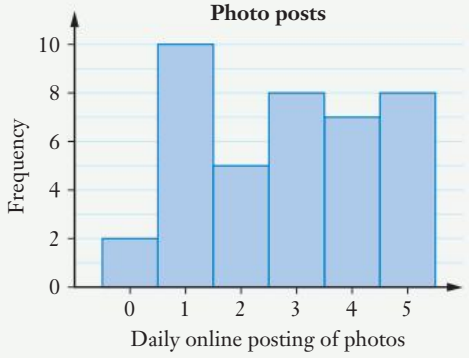
f Decreased, only slightly

Exercise 1.06

1 a

Photo posts/day	Frequency
0	2
1	10
2	5
3	8
4	7
5	8
	40

b



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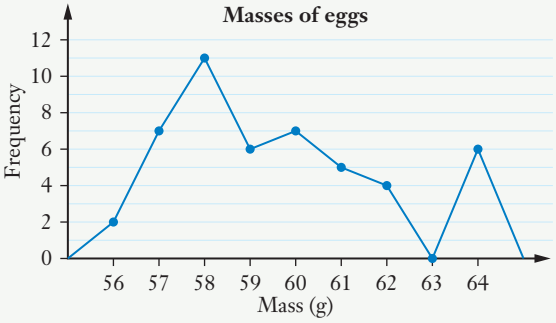
2 a

Mass (g)	Number of eggs
56	2
57	7
58	11
59	6
60	7
61	5
62	4
63	0
64	6
	48

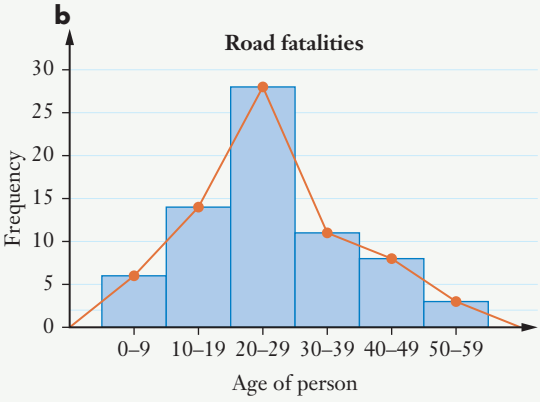
b



c Yes, since 26 out of 48 (about 54%) eggs are lighter than 60 g.

3 a 70

b



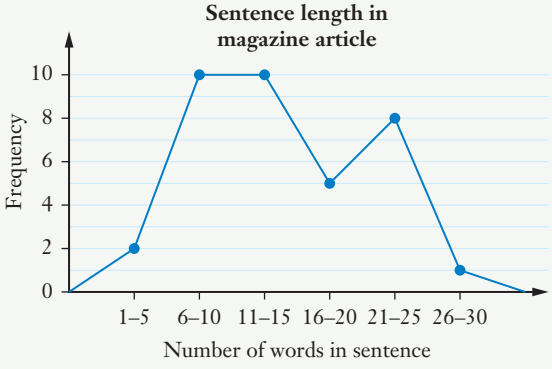
c Most road deaths occur in the (10–39) age group.

4 a discrete

b

Number of words in sentence	Class centre	Tally	Number of sentences
1–5	3	||	2
6–10	8	𝍸 𝍸	10
11–15	13	𝍸 𝍸	10
16–20	18	𝍸	5
21–25	23	𝍸 |||	8
26–30	28	|	1
			36

c



d 6–10 and 11–15 are the two modal classes.

5 a

Class	Class centre	Tally	Frequency
15–19	17	𝍸 𝍸 𝍸 |	16
20–24	22	𝍸 𝍸 𝍸 𝍸	20
25–29	27	||||	4
30–34	32	𝍸	5
35–39	37	|	1
40–44	42	||	2
45–49	47	||	2
			50

b 50

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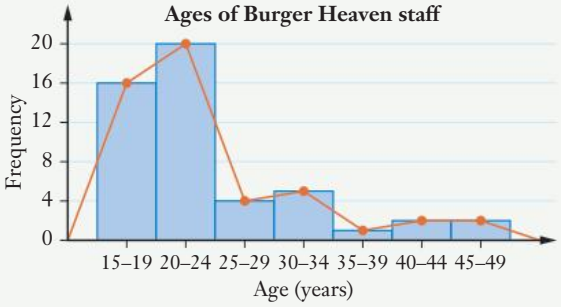
c



d The data is skewed towards younger workers; most of the workers are aged under 25 years.

6 **a** none **b** 43 **c** 16.3%

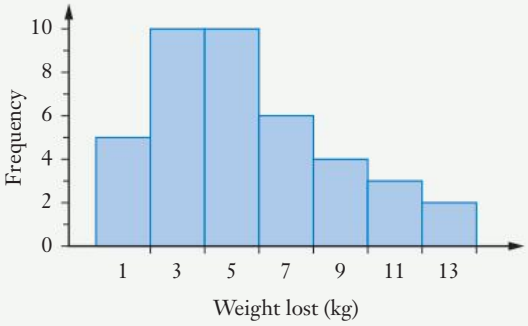
d No, since the last column only tells those who made more than 30 calls.

7 **a** continuous

b

Class	Class centre	Frequency
0 – < 2	1	5
2 – < 4	3	10
4 – < 6	5	10
6 – < 8	7	6
8 – < 10	9	4
10 – < 12	11	3
12 – < 14	13	2
		40

c



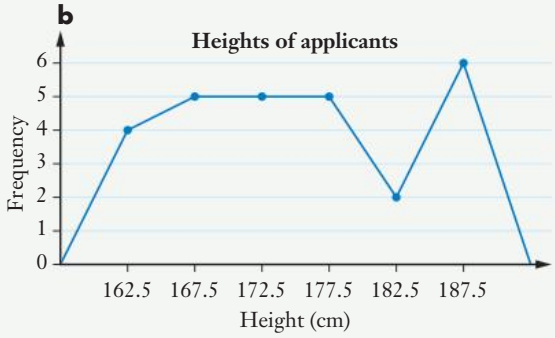
d Modal classes are 2–< 4 and 4–< 6.

e The program seems effective, because most people lost weight.

8 **a**

Height (cm)	Class centre	Tally	Number of applicants
160 – <165	162.2	IIII	4
165 – <170	167.5	𝍸	5
170 – <175	172.5	𝍸	5
175 – <180	177.5	𝍸	5
180 – <185	182.5	II	2
185 – <190	187.5	𝍸 I	6
			27

b

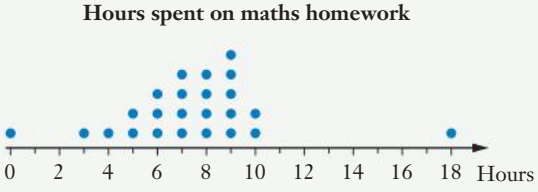


c Modal class interval is 185–< 190

d 160–< 165 **e** 180–< 185

Exercise 1.07

1 **a**



b 24 **c** 0 and 18

d cluster 6–9; gaps at 1–2, 11–17 hours

2 **a** 10 **b** 7

c 12; a student with big feet **d** $\frac{2}{5}$

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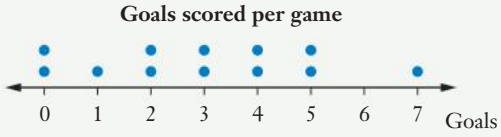
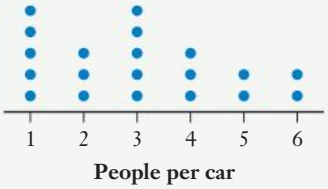
3 a



b 0

c Goals per game ranged from 0 to 7, so the team is not consistent.

4 a



b 20 **c** 6

d no real clusters or outliers

5 a 30 **b** 50% **c** around 6–10

d Mostly good results but 3 quite low scores

6 a

Stem	Leaf
4	3 5 9
5	0 2 7 8
6	1 2 4 5 7 8
7	0 2 3 9
8	2 4 9

b 20 **c** 89 **d** 25%

7 a

Stem	Leaf
2	8
3	
4	2 3 7
5	1 3 7 9
6	1 3 4 6 8 8
7	2 8 9 9
8	1 2 3 4
9	3 4
10	0 3 4
11	0 2 7

b 87% **c** 28; rainy day and few customers

8 a 36 **b** 91 seconds

c about 28%

9 a

Stem	Leaf
5	2 5 6 8
6	0 0 3 3 4 4 4 5 8 8 9 9
7	1 2 2 4 4 6 6 8 9
8	0 1 1 3 3 4 4 5 9
9	0 0 3 4 7
10	
11	0

b 10 **c** 110, heart is beating quickly

d 4

10 a \$975 000 **b** 50 **c** \$385 000

11 a

Stem	Leaf
11	6 8
12	0 1 4
13	6 9
14	3 5 6 6 6
15	4 6 6 7
16	3 5 8
17	2 4 7
18	1 7

b 24 **c** **i** 11.6 s **ii** 18.7 s

d 50.0% **e** $\frac{5}{24}$

Sample HSC problem

a A

b Select some fans from each team's home ground.

c 83°

d players' salaries

e Doesn't show exact values; it is difficult to compare similar values.

Test yourself 1

1 a 4150 **b** pancreatic **c** 6

d false

e These cancers do not represent parts of a whole.

2 a The vertical scale should start at zero.

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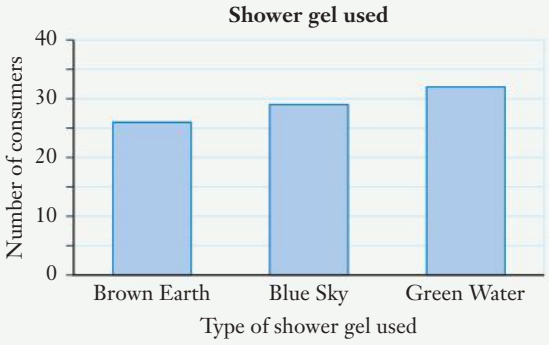


b sector graph, divided bar graph

3 **a** N **b** C **c** C
d N **e** N **f** C

4 **b** N **c** O **f** N

5 **a** D **d** C **e** D

6 systematic, for example, take every 10th bottle and test

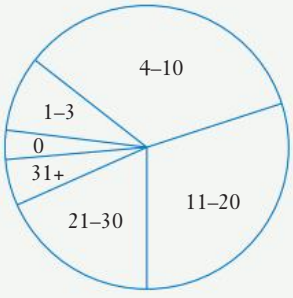
7 **a** The sample may not necessarily take students from each Year into account.

b 9

8 Teacher to check

9 **a** 93

b Number of countries visited



c 12° **d** 34.4% **e** True

f The data is not continuous; the class intervals are not equal in size.

10 **a** traffic, child care

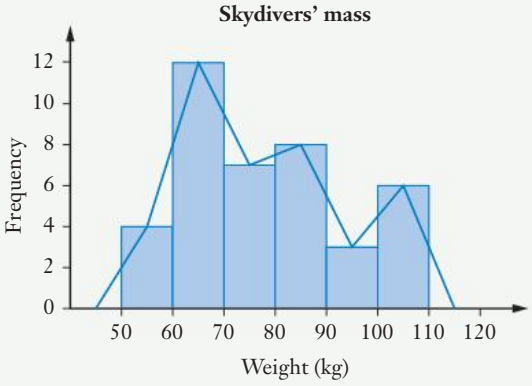
b emergency

c 20 **d** 35%

e provide child care at workplace, change location of company to be near public transport so traffic not a major issue

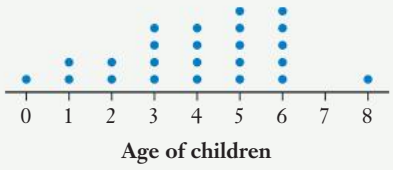
11 **a** continuous

Mass (kg)	Frequency
50–< 60	4
60–< 70	12
70–< 80	7
80–< 90	8
90–< 100	3
100–< 110	6
	40



d 60–< 70 **e** $\frac{1}{5}$

12



b 24 **c** $\frac{1}{4}$

d Data are clustered around 3–6 years.

13 **a** 3 years **b** 29 **c** 12 **d** 41%

Chapter 2

SkillCheck

1 **a** –27 **b** 6 **c** –8
d 25 **e** –4 **f** 2

2 **a** 15 **b** 3 **c** 16
d 6 **e** 243 **f** 18

3 **a** 0 **b** –3 **c** 10
d –128 **e** 9 **f** 9

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4 **a** $x-3$ **b** $20-a$ **c** -3

5 **a** $6x-3$ **b** $28a+35$ **c** $-12p+4$

6 **a** $b=-5$ **b** $w=17$ **c** $r=9$

7 37.5

Exercise 2.01

1 **a** $3x^2+5x$ **b** $7ab+a+b$

c $2m^2+5$ **d** $-4r+2ar$

e y^2+2y-5 **f** a^2+2

g k^2-4k-1 **h** $-3de+2d$

i $-6p$ **j** $4u-15$

k $2t^2+3at$ **l** $18x$

2 **a** $16a^2km$ **b** $-24cd^2$ **c** $\frac{5}{2}t^3w^2$ **d** $25x^6$

e $6n^4$ **f** $14k^5p^2$ **g** $36mn$ **h** $\frac{3}{2}e^3$

i $4x^2y^4$ **j** $-20a^2b^2$ **k** $-n^6$ **l** $9k^2$

3 **a** 2 **b** $7x$ **c** -5

d $2ut$ **e** $\frac{3}{x^2}$ or $3x^{-2}$ **f** $\frac{-3r}{q}$ or $3rq^{-1}$

g $\frac{3g}{5}$ or $\frac{3}{5}g$ **h** $\frac{1}{a}$ or a^{-1} **i** $\frac{5m^2}{3n}$ or $\frac{5}{3}m^2n^{-1}$

j $\frac{-d}{5}$ **k** $\frac{1}{4}$ **l** $\frac{x^2}{4w}$ or $\frac{1}{4}x^2w^{-1}$

4 A

5 **a** $\frac{45y^2}{4}$ **b** $\frac{8ah}{35}$ **c** 2

d $\frac{5}{6}$ **e** $2n^2$ **f** $\frac{5n}{4}$

g $\frac{14a}{5h}$ **h** $\frac{16}{15}$ **i** d

6 **a** $-k^2-2k+8$ **b** $\frac{1}{2}ay^4$ **c** $\frac{1}{4}$

d $-8fg$ **e** $-10c^2+13c$ **f** $-4z^2+5z+8$

g $6a^3$ **h** $\frac{3c}{e}$ or $3ce^{-1}$ **i** $-10m^2$

j $\frac{2n^2}{x}$ or $2n^2x^{-1}$ **k** $4x^2+2xy+3x$ **l** $-3a^2$

Exercise 2.02

1 **a** $3a+6$ **b** $15-10b$

c $-4a-2$ **d** $-6b+12$

e $3x^2-6x$ **f** $3p^2-3ap$

g $-8k-16$ **h** $6t-8t^2$

i $-d^2+5d$ **j** $7k-5k^2$

k $-9b^2+9b$ **l** $14xy+8y^2$

2 **a** $-24n+6n^2$ **b** $5rx^2+10rx$

c $-2a^2+4$ **d** $5a^2b+15b^2-35b$

e $-x^2+4x-10$ **f** $3h^2-21eh-12eh^2$

g $2y^2+3y-y^3$ **h** $d^3e-2de+de^3$

i $9av^2-3v^2+6av$

3 **a** $3x+14$ **b** $d-22$

c $2r+80$ **d** $7f+9$

e $-9x-32$ **f** $3x^2+27x$

g $2b^2+23b$ **h** $3w^2-29w$

i $9k+12p$ **j** $4a$

k $vx+3x$ **l** $-7t-w$

m $4e^2+3e$ **n** $2a-18$

o p^2-q^2

4 **a** 2 and -2; opposites **b** -6 and 6

c yes; $-(b-a)=-b+a$
$=a-b$

Exercise 2.03

1 **a** 91 **b** $-\frac{8}{9}$ **c** 6 **d** 0

e 36 **f** 4 **g** 60 **h** 3

i 42 **j** -12 **k** 7 **l** 169

2 602.63 cm^3

3 **a** 100°C **b** 55°C **c** 28°C

4 **a** \$14.44 **b** \$64.62 **c** \$3465.60

5 **a** v **b** 16 m/s

6 **a** 17 **b** 81 **c** 301

7 12 770.1 cm^3 **8** 3.05 s **9** \$8775.56

10 13.5 m

11 **a** B **b** 20.8

c Increase her weight to around 65 kg.

12 **a** 25°C **b** 38°C **c** 15°C **d** 37°C

13 A

14 **a** 144 mg **b** 129 mg

15 A

16 **a** 12.5 mg **b** 29.2 mg **c** 47.9 mg

17 **a** 111 mg **b** 120 mg

18 62 km

19 \$605.10

20 21 m **21** 207.35 cm^2 **22** B

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Exercise 2.04

1 **a** $d = 6$ **b** $p = 2\frac{1}{2}$ **c** $u = 3\frac{1}{2}$
d $a = -3$ **e** $b = -\frac{1}{3}$ **f** $a = 4\frac{1}{2}$
g $m = -5$ **h** $h = 12$ **i** $r = 13$
j $x = -20$ **k** $y = -5$ **l** $n = -1$
m $y = 5$ **n** $c = 7\frac{1}{2}$ **o** $z = 60$

2 D

3 **a** $k = 11$ **b** $e = -2$ **c** $f = 6\frac{1}{2}$
d $x = 17$ **e** $w = 70$ **f** $d = 3$
g $n = \frac{11}{4}$ or $2\frac{3}{4}$ **h** $w = \frac{-17}{5}$ or $-3\frac{2}{5}$
i $p = 4$ **j** $z = \frac{-26}{5}$ or $-5\frac{1}{5}$
k $b = -3$ **l** $q = 2$

4 A

Exercise 2.05

1 **a** **i** 13 **ii** 31
b **i** 7 **ii** 18

2 12 cm

3 **a** 300 m **b** 60 s

4 2 m **5** 21

6 8 cm **7** \$4387.15

8 **a** 2 min **b** 3.6 min

9 **a** 3.11 miles **b** 0.93 miles

10 **a** **i** 12 **ii** 26
b **i** 11 **ii** 19

11 \$4600 **12** A

13 19.2 km

14 **a** 18 **b** 27

15 3 h

16 **a** **i** 16th **ii** 4th
b **i** 7 s **ii** 12 s

17 D **18** 3 m/s^2

Exercise 2.06

1 **a** $x = \frac{y-4}{2}$ **b** $x = \frac{T+7}{3}$ **c** $x = 3d - 1$
d $x = 18 - p$ **e** $x = \frac{k-r}{4}$ **f** $x = 2(C-n)$
g $x = \frac{s}{10b}$ **h** $x = 6V + 5$ **i** $x \approx \frac{12-z}{a}$

2 **a** $d = St$ **b** $t = \frac{d}{S}$

3 **a** 1800° **b** $n = \frac{A}{180} + 2$ or $n = \frac{A+360}{180}$
c 9

4 D

5 **a** $h = \frac{2A}{a+b}$ **b** $b = \frac{2A}{h} - a$ or $b = \frac{2A-ah}{h}$

6 **a** $r = \sqrt{\frac{A}{\pi}}$ **b** $m = \frac{y-c}{x}$ **c** $s = \frac{v^2-u^2}{2a}$
d $x = sz + m$ **e** $c = \sqrt{\frac{E}{m}}$ **f** $v = \sqrt{\frac{2K}{m}}$
g $P = \frac{A}{(1+r)^n}$ **h** $y = \frac{x^2-3}{6}$ **i** $h = \frac{2A}{b}$
j $l = \frac{T^2 g}{4\pi^2}$ **k** $R = \frac{V+E}{I}$
l $Q = C(e - ir)$ **m** $a = \frac{2(s-ut)}{t^2}$
n $d = Kw - m$

7 $h = \sqrt{\frac{m}{B}}$, 1.55 m **8** 20 cm

9 $F = \frac{9C}{5} + 32$ or $F = \frac{9C+160}{5}$
a 104 **b** 212

10 $P = \frac{10}{\pi hM}$, 1.2

Sample HSC problem

a \$108 **b** $h = \frac{C-60}{32} + 1$ or $h = \frac{C-28}{32}$
c $4\frac{1}{2}$ hours

Test yourself 2

1 **a** $t^2 + 9ut$ **b** $15k^3$ **c** $-6d^2$ **d** $32p^5$
e $-6h + 7$ **f** $8r^2$ **g** $-27d^6$ **h** $\frac{3}{5}$
i $\frac{-vw}{7}$ **j** $8x^2 + 8x$
k $\frac{3c}{b}$ or $3cb^{-1}$ **l** $\frac{-1}{a^2}$ or $-a^{-2}$
m 2 **n** $\frac{2vy}{3}$ **o** $\frac{x}{12}$ **p** $\frac{5}{3a}$

2 **a** $10x - 20$ **b** $-3a - 21$ **c** $48t - 4y$
d $-9r^2 - 18w$ **e** $8m^2n - 8mn^2$ **f** $-8d^2 + 2d^3$

3 **a** $14x - 1$ **b** $2n^2 - n - 1$ **c** $24 - 10d$
d $-4p$ **e** $11u + 8$ **f** $8h^2 + 26h$

4 **a** 128 **b** −28 **c** −400 **d** 10

5 204.20 cm^2 **6** \$700

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7 **a** $p = 5$ **b** $a = -1$ **c** $b = -9$
d $r = \frac{1}{2}$ **e** $n = \frac{55}{4}$ **f** $r = -18$
g $n = -2$ **h** $t = -12\frac{1}{2}$ **i** $g = -1$
8 110.5 m **9** 80
10 $y = 2(P - 110)$ or $y = 2P - 220$, 28 years

Chapter 3

SkillCheck

1 **a** $88 **b** $4.65 **c** $1610
2 **a** $14 473.12 **b** $2932.16 **c** $1284.44
3 6%
4 **a** 1035 **b** 89 500
5 **a** 4 h **b** 5 h 30 min **c** 9 h 45 min

Exercise 3.01

1 $1441.13 **2** $1496.25
3 B **4** C
5 **a** $80 999.10 **b** $1557.68 **c** $40.99
6 $1399.44
7 **a** $49 320.96 **b** $4110.08
8 $710.01 **9** $2678.00 **10** $1711.90
11 **a** $7862.40 **b** $17 094.17 **c** $3931.20
12 **a** $6934.58 **b** $3189.54 **c** $318.95
13 **a** $47.56 **b** $3995.21 **c** $8686.25
14 $826.36 **15** $1282.56
16 **a** $353.21 **b** $1766.05 **c** $7652.88
17 C
18 **a** $24.20 **b** $1234.32 **c** $64 184.64
19 $1636.83 **20** $23.93

Exercise 3.02

1 A
2 **a** 168 **b** $56.70
3 D
4 **a** $30.10 **b** $6750
5 $9620
6 **a** $4848 **b** $11 800 **c** $16 375
7 $203.98 **8** $1353.24
9 **a** $341 880 **b** $14 207 955.46
c $2 117 874.43
10 **a** $146.19 **b** 2500
11 $166.52 **12** $3600
13 $148 637.45 **14** $577.50
15 $1077.12 **16** $1678
17 8.8% **18** 1382
19 **a** $188.10 **b** 136
20 60 **21** $530.85

Exercise 3.03

1 **a** roof tiler **b** nurse
c airline steward **d** steelworker
e scientist in Antarctica **f** scaffolder
g army officer **h** miner
2 $1910.50 **3** $1184.22 **4** $1152.77
5 **a** $1439.63 **b** $74 860.50
6 $3543.81 **7** $663.06 **8** $1612
9 **a** $1935.96 **b** $1254.96
10 $2947.38 **11** $194.40
12 $1347.20 **13** $1407.15
14 $33 666.73 **15** $668.36
16 $1004.46
17 **a** $687.40 **b** $4615.40

Exercise 3.04

1 **a** $797.90 **b** $941.35
c $0 (<65 years)
2 Because a couple live in the same house and the cost of living is shared
3 **a** **i** $528.70 **ii** $13 746.20
iii $1145.52
b $6206.20
c More. This is fair because they have no partner to share costs.
4 C
5 **a** $477.40 **b** $571.90 **c** $738.50
d **i** $14 869.40 **ii** $285.95
6 B
7 **a** $437.50 **b** $477.40
c $477.40 **d** $239.50

Exercise 3.05

1 C
2 **a** $611.95 **b** 25.0%

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3 **a** $875.32 **b** $245.09 **c** $513.36
4 **a** $301.18 **b** 29.09% **c** 3.29%
5 **a** $340.51 **b** $99.06
c $461.51 **d** $776.69
6 **a** $1070.55 **b** $107.06 **c** $723.27
7 **a** $2079.38 **b** $1599.38 **c** 14.8%
8 **a** $1145.82 **b** $286.46 **c** $841.96
9 **a** $9022.92; $3504.88 **b** 48%
c 38.8%

10

	Pay	Deductions	
	$751.80	**Tax (25%)**	**$230.91**
	$128.88	**Superannuation**	$ 60.65
	$ 42.96	**Union**	$ 18.15
Gross wages	**$923.64**	**Health fund**	$ 24.60
Total deductions	**$334.51**		
Net wage	**$589.13**		

11

Gross pay	$3335.61
Tax (31%)	$1034.04
Other deductions	$152.21
Net pay	$2149.36

Total deductions = $152.21

Exercise 3.06

1 **a** $2458.60 **b** $35 633.21 **c** $0
2 **a** $89 451 **b** $20 729.18 **c** $1789.02
3 **a** $86 124 **b** $19 537.30 **c** $1722.48
4 **a** $99 623 **b** $24 492.51 **c** $1992.46
5 **a** $12 301.25 **b** $34 072.90 **c** $73 260

Exercise 3.07

1 **a** $16 363.36 **b** $36 398.44
2 **a** $24 793.33 **b** $906.42
3 **a** $194 026 **b** $60 543.70
c $3880.52 **d** refund of $7731.78
4 **a** $98 619.55 **b** $5572.78
c refund of $375.67
5 **a** $49 320 **b** $7576 **c** $986.40
d refund of $3346.64
6 D
7 B
8 **a** debt of $11 580.09 **b** debt of $11 426.58
c debt of $72 165

Exercise 3.08

1 C **2** €1815
3 3984.68 baht **4** A
5 $23.18 **6** £108
7 46.5 rand **8** 19%
9 **a** €8.30 **b** 19%
10 **a** $30.80 **b** $35.42
11 **a** 14 290 won **b** 10%
12 **a** €44 **b** 20%
13 18% **14** €238.97
15 $132.80 **16** $190.85
17 4129.91 baht **18** 6.62%
19 $48; $50.40 **20** 19%

Sample HSC problem

a $12 365.60 **b** $41 048.80 **c** $40 062
d income tax $4567.15, Medicare levy $801.24
e tax refund of $6997.21

Test yourself 3

1 **a** $43 219.80 **b** $50 648 **c** $45 432
2 **a** $2656.40 **b** $3775.51
c $21 980.33
3 $578.25
4 **a** $158.20 **b** 70
5 Maggie earns 2c less, not 2c more
6 $11 657.52
7 $1332.57
8 $1159.47
9 $1193.70
10 **a** $30 893.20 **b** $1188.20
c $3632.12 **d** $2386.52
11 **a** $105 661 **b** $26 726.57
c $2113.22 **d** refund of $8192.01
12 $113.27
13 **a** 189.3 pesos **b** 44.65 new lei
c 3773 rand
14 **a** 173 913.04 pesos
b 142 833.33 forint

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Practice paper 1

1 B **2** B **3** D **4** B **5** C
6 B **7** B **8** C **9** A **10** D

11 **a** $x = 3\frac{3}{5}$

b **i** It could be 16 minutes, 17 minutes or 18 minutes.

ii 19 patients

iii 21.1%

iv 2 minutes.

c **i** \$1025 **ii** 1 hour **iii** \$27.38 **iv** 27

12 **a** $2p + 3$ **b** 47 days. **c** \$647.50 **d** \$37 872

e **i** Poor service and limited clothing sizes available

ii True

iii Train staff to provide good customer service, order more clothing sizes, reduce prices to compete with other stores, sell more men's and women's clothing.

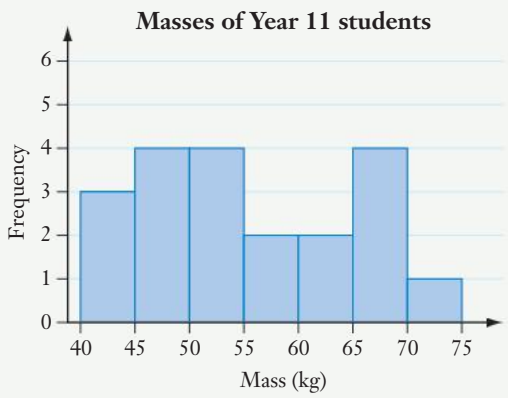
13 **a** 1 h 44 min

b **i** Continuous

ii

Masses (kg)	Frequency
40–<45	3
45–<50	4
50–<55	4
55–<60	2
60–<65	2
65–<70	4
70–<75	1

iii



iv 35%

c Taxable income = \$95 063
Tax payable = \$22 805.68

Chapter 4

SkillCheck

1 **a** $\frac{7}{25}$ **b** $\frac{1}{11}$ **c** $\frac{1}{4}$ **d** $\frac{2}{3}$

2 **a** $\frac{27}{40}$ **b** $\frac{3}{25}$ **c** $\frac{7}{8}$ **d** $\frac{5}{9}$

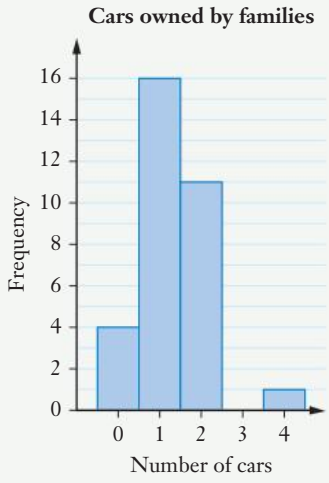
3 **a** 0.85 **b** 0.25 **c** 0.625 **d** $0.2\dot{6}$

4 **a** 40% **b** 62.5% **c** $7\frac{9}{13}\%$ **d** $33\frac{1}{3}\%$

5 **a** 52 **b** 6 **c** 12 **d** 26
e 4 **f** 5

6 **a** $\frac{1}{5}$ **b** 30% **c** $\frac{3}{5}$ **d** 40%

7 **a**



b $\frac{1}{8}$ **c** 0% **d** 87.5% **e** 1

8 **a** Teacher to check. **b** AC
c VU **d** AC
e VU **f** Teacher to check.
g VU **h** Teacher to check.
i U **j** L

Exercise 4.01

1 **a** {Mon, Tues, Wed, Thurs, Fri, Sat, Sun}; 7
b {M, A, T, H, E, I, C, S}; 8
c {G, PG, M, MA, R}; 5
d {boy, girl}; 2 **e** {pass, fail}; 2

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2 **a** 21 **b** 12 **c** 10 000
d 7 **e** 4

3 **a** 26 **b** 4 **c** 20
d 2 **e** 12 **f** 1

4 **a** England wins, Brazil wins, draw **b** No

5 Each event (fine, rain) is not equally likely.

6 **a** $\frac{2}{3}$ **b** $\frac{1}{3}$ **c** 0 **d** 1

7 **a** $\frac{7}{15}$ **b** $\frac{8}{15}$ **c** $\frac{1}{15}$ **d** $\frac{7}{15}$

8 **a** $\frac{1}{5}$ **b** $\frac{3}{10}$ **c** $\frac{1}{5}$ **d** $\frac{1}{2}$

9 $\frac{1}{3}$

10 **a** $33\frac{1}{3}\%$ **b** $4\frac{1}{6}\%$ **c** $79\frac{1}{6}\%$
d 75% **e** 25%

11 D

12 **a** $\frac{1}{3}$ **b** $\frac{1}{6}$ **c** $\frac{1}{2}$ **d** $\frac{5}{6}$

13 **a** $\frac{1}{2}$ **b** $\frac{3}{26}$ **c** $\frac{1}{13}$ **d** $\frac{9}{13}$
e $\frac{1}{26}$ **f** $\frac{1}{52}$ **g** $\frac{1}{26}$ **h** $\frac{1}{4}$

14 $\frac{1}{150}$ **15** $0.1\dot{6}$

16 Each of the 16 teams is not equally likely to win the competition, the levels of ability/success are different.

17 **a** 60% **b** 20% **c** 30%

18 **a** $\frac{2}{7}$ **b** $\frac{4}{7}$ **c** $\frac{1}{7}$
d $\frac{5}{7}$ **e** $\frac{5}{7}$ **f** 0

Exercise 4.02

1 **a** D **b** G **c** E
d D **e** A **f** B
g B **h** F **i** C

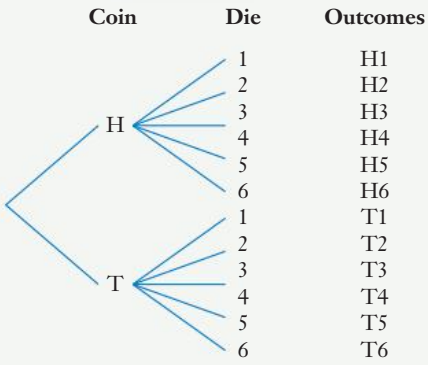
2 **a** **i** 2 **ii** 6

b **i** {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6} where H = head, T = tail

ii

Die	Coin: H	Coin: T
1	H1	T1
2	H2	T2
3	H3	T3
4	H4	T4
5	H5	T5
6	H6	T6

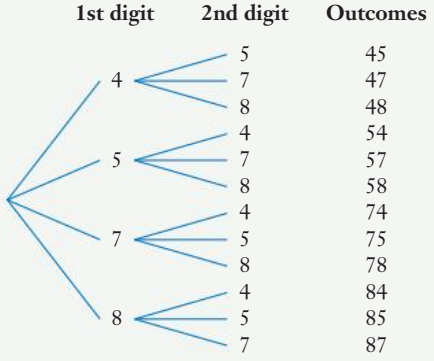
iii



c 12

d **i** $\frac{1}{4}$ **ii** $\frac{1}{6}$ **iii** $\frac{5}{12}$

3 **a**



b 12

c **i** 25% **ii** 50%
iii 50% **iv** 25%

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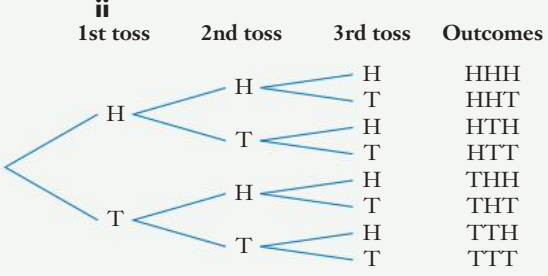
4 a i {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

ii



b 8

c i 0.375 **ii** 0.5 **iii** 0.125 **iv** 0.5

5 a 2 **b** 4 **c** 16 **d** 1024

6 a

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b 36

c i $\frac{1}{12}$ **ii** $\frac{5}{36}$ **iii** $\frac{1}{12}$ **iv** 0 **v** $\frac{1}{2}$ **vi** $\frac{2}{9}$

d i $2\frac{7}{9}\%$ **ii** $16\frac{2}{3}\%$

e i 7 **ii** 2, 12

7 a

2nd course \ 1st course	C	R	W	Y
B	CB	RB	WB	YB
P	CP	RP	WP	YP
S	CS	RS	WS	YS

b 12

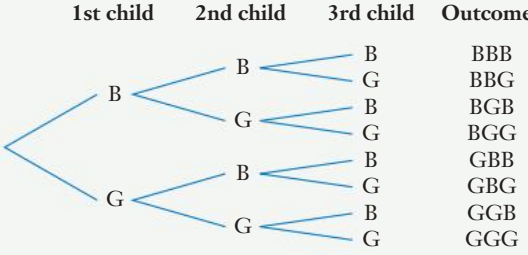
c i $\frac{1}{2}$ **ii** $\frac{1}{6}$ **iii** $\frac{1}{2}$ **iv** $\frac{1}{12}$

8 a

Secretary \ Chairperson	H	R	A	K	T	
H	–	RH	AH	KH	TH	H = Harry
R	HR	–	AR	KR	TR	R = Rita
A	HA	RA	–	KA	TA	A = Alison
K	HK	RK	AK	–	TK	K = Kirstie
T	HT	RT	AT	KT	–	T = Thuc

b i $\frac{1}{20}$ **ii** $\frac{2}{5}$ **iii** $\frac{1}{5}$ **iv** $\frac{1}{10}$

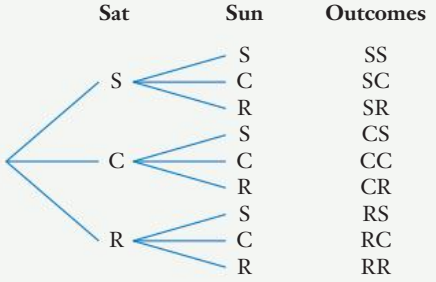
9 a



b A

c i $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{7}{8}$ **iv** $\frac{1}{2}$ **v** $\frac{3}{8}$ **vi** $\frac{1}{2}$

10 a



b A **c** 9

d i $\frac{1}{9}$ **ii** $\frac{1}{3}$ **iii** $\frac{8}{9}$ **iv** $\frac{5}{9}$ **v** $\frac{2}{9}$

Exercise 4.03

1 a getting a head

b selecting a diamonds, clubs or spades card

c passing a driving test

d having 3 or fewer children

e selecting a white sock

f the weather today not being rainy

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g not winning the race

h a traffic light showing red or amber

2 **a** $\frac{5}{6}$ **b** $\frac{2}{3}$ **c** $\frac{2}{3}$

3 {HH, HT, TH, TT}

a $\frac{1}{4}$ **b** $\frac{3}{4}$ **c** $\frac{1}{2}$ **d** $\frac{1}{2}$

e $\frac{1}{4}$ **f** $\frac{3}{4}$

4 {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

a $\frac{3}{8}$ **b** $\frac{5}{8}$ **c** $\frac{3}{8}$ **d** $\frac{5}{8}$

e $\frac{1}{8}$ **f** $\frac{7}{8}$

5 **a** 0.55 **b** 0.7 **c** 0.45

d 0.9 **e** 0.8 **f** 0.7

6 A

7 **a** $\frac{10}{11}$ **b** $\frac{9}{11}$ **c** $\frac{7}{11}$ **d** $\frac{8}{11}$

8 $\frac{5}{7}$

9 **a** $\frac{5}{9}$ **b** $\frac{1}{3}$ **c** $\frac{7}{9}$

d $\frac{2}{3}$ **e** 1 **f** $\frac{2}{9}$

10 B

11 77%

12 **a** $\frac{12}{13}$ **b** $\frac{10}{13}$ **c** $\frac{3}{4}$

d $\frac{1}{2}$ **e** $\frac{25}{26}$ **f** $\frac{11}{13}$

13 **a** 0.384 **b** 0.616 **c** 0.988

d 0.396 **e** 0.604 **f** 0

14 $\frac{364}{365}$

15 80%

16 **a** draw **b** 7%

17 $\frac{7}{20}$

18 **a** $\frac{8}{9}$ **b** $\frac{2}{3}$

Exercise 4.04

1 **a** 150

b **i** $\frac{28}{75}$ **ii** $\frac{63}{150}$ **iii** $\frac{17}{150}$

2 **a** no

b yes, biased towards 6

c $27\frac{1}{2}\%$ **d** 28

3 **a** 1

b **i** 0.35 **ii** 0.125 **iii** 0.875

iv 0.625 **v** 0.275 **vi** 0.9

c 11

4 B

5 **a** no

b **i** $\frac{1}{3}$ **ii** $\frac{23}{30}$

6 **a** 50.66%

b female, probability slightly over 50%

c 12 335 000

7–11 Teacher to check.

Exercise 4.05

1 **a** {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

b **i** $37\frac{1}{2}\%$ **ii** 50%

c **i** 33 **ii** 44

d **i** 34.09% **ii** 53.41%

e They are close.

f **i** 75 **ii** 100

g **i** 40% **ii** 48.5%

h yes

2 **a** **i** Teacher to check. **ii** 0.167 **iv** 0.5

b 25

3 **a**

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b **i** 0.0833 **ii** 0.0556 **iii** 0.2222

d **i** 10 **ii** 7 **iii** 27

4 **a** **i** 0.231 **ii** 0.256

b A

5 **a** 12

b **i** 0.083 **ii** 0.104 **iii** 0.25

iv 0.25 **v** 0.417 **vi** 0.438

c **i** 4 **ii** 12 **iii** 20

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6 Each outcome has an equal chance.

7 **a** **i** 46% **ii** 39.3%
iii 42% **iv** 41.9%

b Yes, the coin seems to be coming up heads around 42% of the time instead of 50%.

Exercise 4.06

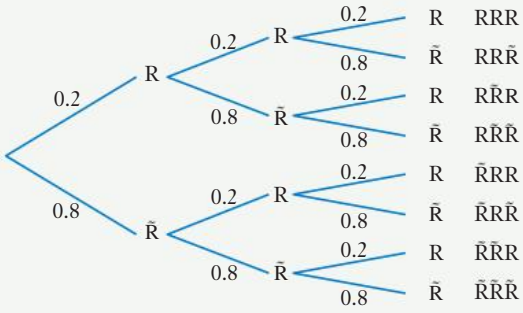
1 C

2 **a** $\frac{1}{25}$ **b** $\frac{1}{1225}$ **c** $\frac{1128}{1225}$ **d** $\frac{97}{1225}$

3 **a** $\frac{1}{45}$ **b** $\frac{16}{45}$

4 **a** $\frac{2}{7}$ **b** $\frac{1}{7}$ **c** $\frac{2}{7}$

5 **a**



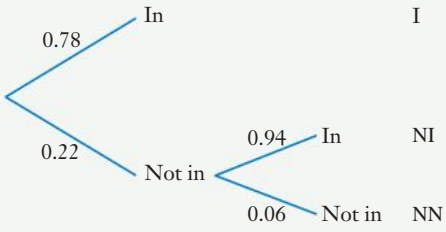
b **i** 38.4% **ii** 51.2% **iii** 48.8%

6 $\frac{125}{216}$

7 **a** 0.250 **b** 0.441

8 **a** $\frac{1}{10}$ **b** $\frac{3}{10}$

9 **a**



b 0.0132 **c** 0.9868

10 **a** $\frac{28}{153}$ **b** $\frac{16}{51}$ **c** $\frac{62}{153}$ **d** $\frac{104}{153}$

11 A

12 **a** 0.8% **b** 7.2% **c** 9.2%

Sample HSC problem

a

2nd die \ 1st die	–	1	2	3	4	5	6
	1	0	1	2	3	4	5
	2	1	0	1	2	3	4
	3	2	1	0	1	2	3
	4	3	2	1	0	1	2
	5	4	3	2	1	0	1
	6	5	4	3	2	1	0

b 1

c **i** $\frac{2}{9}$ **ii** $\frac{1}{2}$ **iii** $\frac{17}{18}$

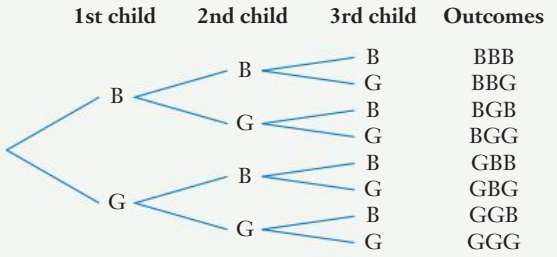
Test yourself 4

1 **a** $\frac{1}{26}$ **b** $\frac{3}{13}$ **c** $\frac{11}{13}$

2 **a** 25% **b** $16\frac{2}{3}\%$ **c** $58\frac{1}{3}\%$ **d** $66\frac{2}{3}\%$

3 Being married and being single are not equally likely outcomes.

4 **a**



b **i** $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{1}{2}$
iv $\frac{7}{8}$ **v** $\frac{1}{2}$ **vi** $\frac{1}{2}$

5 **a** $\frac{1}{6}$ **b** $\frac{5}{36}$

6 32%

7 D

8 **a** $\frac{3}{14}$ **b** $\frac{7}{20}$ **c** $\frac{61}{70}$

9 **a** 92.65% **b** 188

10 **c** $\frac{1}{6}$ **d** 8

11 B

12 **a** 0.012% **b** 18% **c** 17%

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Chapter 5

SkillCheck

1 **a** 20 830 **b** 97.02 **c** 65 900
d 0.725 **e** 0.0104 **f** 0.735

2 **a** 10 000 000 **b** 10 000 **c** 0.01

3 **a** 16 cm^2 **b** 90 m^2
c 78.54 m^2 **d** 60 cm^2

4 **a** 180 cm^2 **b** 41

5 **a** \$1.28 **b** \$2.49

6 **a** 2.1 m **b** 2100 mm **c** 0.0021 km

7 **a** 0.0081 kL **b** 8100 mL

Exercise 5.01

1 **a** 63 mm **b** 436 cm **c** 7.2 m
d 0.285 kg **e** 6900 µm **f** 58 L
g 5320 g **h** 3.4 t **i** 4.72 kL
j 0.006 kg **k** 27 000 s **l** 940 000 cm

2 A

3 **a** 1690 mm **b** 1.69 m

4 57 500 g **5** 1.5 km

6 **a** 97 min **b** 5820 s **c** 1.6 h

7 59 000 L

8 **a** kg **b** cm **c** m **d** km
e L **f** mg **g** cm **h** kL
i t **j** mm

9 **a** 0.102 kg **b** 0.052 m

10 4.9 Mt

11 **a** 18.7 km **b** 1 h 48 min

12 8.56 kL

13 **a** B **b** D **c** C **d** C
e A **f** A

14 about 1420

15 86 400

Exercise 5.02

1 **a** **i** 1 m **ii** 4 m
iii ±0.5 m, 3.5 m to 4.5 m
b **i** 0.1 cm or 1 mm
ii 1.8 cm or 18 mm
iii ±0.5 mm, 1.75 cm to 1.85 cm (or 17.5 mm to 18.5 mm)
c **i** 5 km/h **ii** 50 km/h
iii ±2.5 km/h, 47.5 km/h to 52.5 km/h
d **i** 0.5°C **ii** 38°C
iii ±0.25°C, 37.75°C to 38.25°C
e **i** 5 m/s **ii** 45 m/s
iii ±2.5 m/s, 42.5 m/s to 47.5 m/s
f **i** 500 rpm **ii** 7000 rpm
iii ±250 rpm, 6750 rpm to 7250 rpm

2 C

3 A, D **4** C, D, F

5 **a** ±0.5 mL **b** ±0.5 mm
c ±0.05 g **d** ±0.05 s **e** ±0.5 g
f ±0.5 m **g** ±0.5 mL **h** ±5 g
i ±0.05 cm **j** ±50 g

6 2956 mm

7 **a** ±0.5 cm **b** 20.5 cm and 21.5 cm
c ±2.38%

8 ±2.17%

9 ±0.0039%

10 **a** ±0.33% **b** ±0.63% **c** ±3.33%
So 15.2 mg is the most accurate because it has the smallest percentage error.

11 We don't know the absolute error; the figure could be to the nearest whole number, ten or fifty.

12 38 L is written correct to the nearest 1 L, 38.0 L is correct to 0.1 L. So 38.0 L is more accurate.

13 Teacher to check.

Exercise 5.03

1 B

2 **a** 3800 **b** 2100 **c** 0.0061
d 250 000 **e** 15 000 000 **f** 0.000 47

3 **a** 130 **b** 4980 **c** 0.0106
d 1 360 000 **e** 25 400 000 **f** 0.000 680

4 **a** 3000 **b** 3 **c** 10 000
d 0.005 **e** 20 **f** 0.7

5 **a** Y **b** N **c** Y
d N **e** Y **f** Y

6 b, c, d, e, f

7 **a** 0.67 **b** 0.0055 **c** 1.3
d 0.087 **e** 790 000 **f** 7.3
g 24 **h** 2.4 **i** 34

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8 7 620 000

9 **a** 352 000 kg **b** 4190 m **c** 67.1
d 14.8 mL **e** 150 000 000 km

Exercise 5.04

1 **a** 4.213×10^{7} **b** 1.81×10^{-2} **c** 3.4×10^{3}
d 2.0×10^{4} **e** 3.5×10^{-3} **f** 2.0×10^{-4}
g 3.3×10^{-1} **h** 4.0×10^{-3} **i** 2.3×10^{2}
j 7.23×10^{-5} **k** 6.1×10^{8} **l** 8.0×10^{-8}

2 **a** 5.3×10^{7} **b** 1.5×10^{5} **c** 2.5×10^{3}
d 4.6×10^{-4} **e** 2.7×10^{-3} **f** 1.0×10^{-1}
g 3.3×10^{-5} **h** 4.4×10^{-1} **i** 6.5×10^{0}

3 B

4 **a** 740 000 **b** 0.312
c 1850 **d** 0.000 66
e 0.002 54 **f** 475 100 000
g 0.098 **h** 300
i 0.054 97 **j** 12 160
k 0.802 **l** 6309

5 **a** 1.37×10^{10} **b** 1.0×10^{-6}

6 **a** 0.000 000 03 **b** 9 461 000 000
c 0.000 002 **d** 152 600 000

7 7.46×10^{9}

8 **a** 2.144×10^{7} **b** 3.2×10^{5}
c 3.5×10^{4} **d** 2.304×10^{-5}
e $5.314\,41 \times 10^{14}$ **f** 3.76×10^{1}
g 1.26×10^{4} **h** 2.3×10^{5}

9 **a** 3.0×10^{4} **b** -3.6×10^{-3}
c 4.1×10^{12} **d** 3.3×10^{-8}
e 5.9×10^{4} **f** 3.4×10^{-2}
g 9.2×10^{2} **h** 1.2×10^{-4}

10 **a** 0.0048 **b** 43 680 000 000
c 0.0094 **d** 187 690 000
e 13 640 000 **f** 19 063 000
g 0.000 052 **h** 350 000

11 Answers will vary. Teacher to check.

Exercise 5.05

1 **a** 30 m **b** 68 cm **c** 66 cm
d 54 cm **e** 1720 cm or 17.2 m
f 79.67 m **g** 30.6 cm **h** 172 mm
i 100 cm **j** 230 mm = 23 cm
k 40 cm **l** 58 cm

2 **a** 53.4 m **b** 10.3 m **c** 21.4 cm
d 22.3 cm **e** 100.7 mm **f** 46.6 cm
g 51.4 cm **h** 121.1 mm **i** 34.3 m
j 94.2 cm **k** 43.2 m **l** 57.1 cm

3 **a** **i** 16.8 m **ii** 40.8 m
b **i** 15.1 m **ii** 34.3 m
c **i** 202.6 **ii** 374.6 mm

4 247.96 – 222.83 = 25.1 m

5 **a** 45° **b** 41.8 cm

Exercise 5.06

1 C **2** A

3 **a** 50 000 **b** 25 **c** 7.2
d 0.68 **e** 3 090 000 **f** 360
g 4 730 000 **h** 5.4

4 8.016×10^{7}

5 **a** 4 209 000 mm^2, 4.209 m^2
b $2.3 \times 1.83 = 4.209\ m^2$

6 A **7** C

8 **a** 64 m^2 **b** 4322.5 m^2
c 24 m^2 **d** 40 m^2
e 375 m^2 **f** 4 523 893.4 km^2
g 300m^2 **h** 6 m^2
i 88.4 m^2 **j** 197.9 m^2
k 189 m^2

9 **a** 93 m^2 **b** 36 m^2 **c** 7747 mm^2
d 127 m^2 **e** 30 m^2 **f** 89 m^2

10 **a** 7 m^2 **b** 8482 cm^2
c 150 796 mm^2

11 **a** 52 m^2 **b** 165 m^2 **c** 105 m^2
d 33 m^2 **e** 168 m^2 **f** 66 m^2

12 42.3 m^2

13 **a** 15 000 m^2 **b** 588.5 m **c** $72 974

14 **a** 0.4096 **b** $55.30

15 **a** **i** 60.7 cm^2 **ii** 256 cm^2
b After inserting semicircular gaps, it is equivalent to the area of the dotted square.

Exercise 5.07

1 **a** 430 m^2 **b** 960 m^2 **c** 6200 m^2
d 3800 m^2 **e** 2400 m^2

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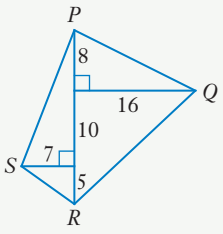
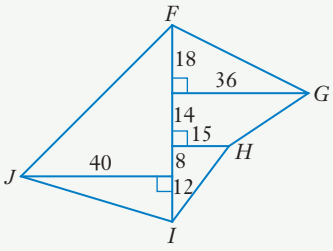
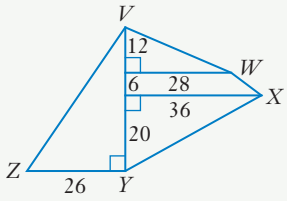
2 **a** 265 m^2



b 1871 m^2



c 1214 m^2



3 **a** 1161 m^2 **b** 148 m

4 B

5 **a** 297 m^2 **b** 660 m^2

6 **a** 61 m^2 **b** $270

7 128 m^2

8 106.5 m^2

Exercise 5.08

1 **a** 7 000 000 cm^3 **b** 50 000 mm^3
c 0.089 m^3 **d** 468 000 cm^3
e 2.4 cm^3 **f** 5.6 m^3
g 9100 cm^3 **h** 12 000 000 cm^3

2 **a** 680 mL **b** 8.5 L
c 22 000 L **d** 8 m^3
e 3.5×10^6 mL **f** 690 000 cm^3
g 55 000 L **h** 4300 kL
i 9.5 m^3 **j** 85 L
k 4300 cm^3 **l** 1000 ML

3 A

4 **a** 0.864 m^3 **b** 864 L

5 B

6 **a** 250 cm^3 **b** 1.701 m^3
c 3036 cm^3 **d** 20.16 m^3

7 249 L

8 **a** 16.1 m^3 **b** 16 100 L

9 **a** 4 m^3 **b** $264

10 **a** 2.031×10^{12} L **b** 2.031×10^9 m^3

11 C

12 **a** 236 m^2 **b** 347 m^2
c 205 m^2 **d** 317 m^2
e 736 m^2

13 **a** 1.7 m^2 **b** 180 L

14 **a** 6 m^2 **b** 0.8 m^3

15 **a** 1.8 m^3 **b** 10.5 m^2

16 **a** 84 cm^3 **b** 298

Exercise 5.09

1 **a** 1800 m^3 **b** 1.0 m^3 **c** 2700 m^3
d 0.27 m^3 **e** 43 m^3 **f** 3.0 m^3

2 **a** 840 m^2 **b** 6.0 m^2 **c** 1100 cm^2
d 3.5 m^2 **e** 72 m^2 **f** 11 m^2

3 **a** 327 cm^3 **b** 2 L

4 **a** 7100 cm^3 **b** 0.85 m^3
c 31 m^3 **d** 9000 cm^3

5 5451 cm^2

6 **a** 3.1 m^3 **b** 13.4 m^2

7 **a** 39 m^3 **b** 25 133 L

8 C

Exercise 5.10

1 **a** 310 000 km^3 **b** 224 000 m^3
c 9.05×10^8 m^3 **d** 994 cm^3
e 2.44 m^3 **f** 11 500 m^3

2 **a** **i** 1.011×10^8 km^2 **ii** 1.068×10^{12} km^3
b 5.592×10^9 t/km^3

3 **a** 262 L **b** 1.57 m^2

4 **a** 113 cm^3 **b** 103 cm^3 **c** $16.96

5 A

6 **a** 92 000 cm^3 **b** 9852

7 **a** 2.9 m^2 **b** 1.4 m^2 **c** 0.46 m^3

Note: The light is a whole sphere but split in half.

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Exercise 5.11

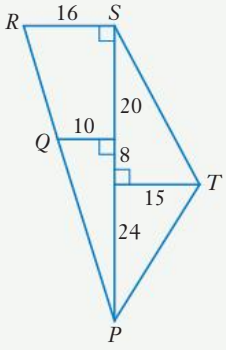
1 **a** 994 020 mm^3 **b** 66.7%
c 1 265 625 mm^3
d It is the most efficient packaging as it has the least amount of air.
2 **a** 1 716 000 mm^3 **b** 1593 cm^3
c 7% **d** 86 200 mm^2
3 **a** 8400 cm^3 **b** 3200 cm^2
c 1200 cm^3 **d** 410 cm^2
4 **a** 42 103 L; 4800 L **b** 2nd pool by 5897 L
c 73 m^2
5 **a** 0.31 m^3 **b** 33 414 cm^2
6 460.8 m^2

Sample HSC problem

a 3 **b** 20.65 m to 20.75 m
c 0.242%

Test yourself 5

1 **a** 28 500 **b** 6400 **c** 0.34
2 43 000
3 **a** ±0.005 **b** 3.655 m to 3.665 m
c 0.14%
4 **a** 39 **b** 1000 **c** 0.0087
d 6 600 000
5 8.35×10^9 **6** 0.000 004 6
7 **a** 40 m **b** 316 cm **c** 55.00 cm
8 **a** 84 cm^2 **b** 56 000 m
9 **a** 44 m^2 **b** 1040 cm^2 **c** 230 cm^2
10 **a** 120.1 cm **b** 646.3 cm^2
11 **a** 810 m^2 **b** 0.081 ha



12 **a** 20 700 mm^3 **b** 1.65 m^3
13 **a** **i** 750 cm^3 **ii** 480 cm^2
b **i** 0.22 m^3 or 220 000 cm^3
ii 3.2 m^2 or 32 000 cm^2
14 **a** 894 mL **b** 6500 L
15 **a** 8.1 m^3 **b** 8100 L
16 **a** 110 m^2 **b** 187 m^2
c 187 m^2 is more accurate, because by using two applications we use more trapeziums to approximate the area more accurately.
17 **a** 8.9 m^2 **b** 1.4 m^3
18 57 470
19 **a** 1715.31 cm^3 **b** 1.07 L
20 **a** 204 m^3 **b** 26 L
21 **a** 4700 cm^3 **b** 52 cm

Chapter 6

SkillCheck

1 **a** 2.25 **b** 4050
2 143 c/L
3 **a** 84 mm **b** 9.6 m **c** 36 L
d 2.61 kL **e** 5200 L **f** 11 min
4 $466.24
5 90 days
6 $4.88
7 **a** $5.18 **b** 13 min

Exercise 6.01

1 **a** $110.19 **b** $314.46
2 B
3 **a** 21 days **b** quarterly
c water and sewerage service, $142.75
d 18 kL
e No, in cooler months fewer showers, water garden less
f similar usage so can compare savings made for conditions
g $\frac{18\,000}{19+30+8+31} = 204.545... = 205\,\text{L}$
h $2.115/ kL **i** $18 \times 2.115 = \$38.07$
j $84.60
4 C
5 **a** $50.76 **b** 15.4 kL **c** $306.84

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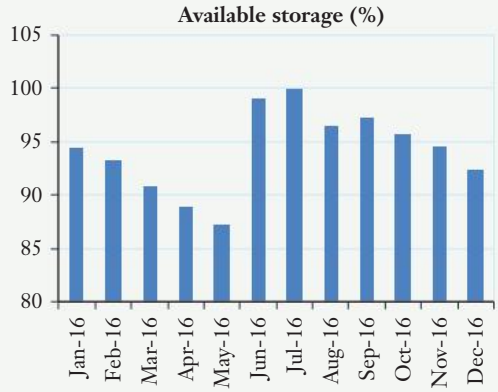
6 **a** 199.6 kL **b** 49.9 kL **c** 567 L/day
d 3.864 kL **e** 8.3%
7 C **8** $75.25 **9** B
10 **a** Baskaran: $106.60 Zhong: $135.15, Zhong's pay more
b Baskaran: 140 L/day Zhong: 118.3 L/day, Baskaran's use more per person per day

Exercise 6.02

1 **a** 150.00 kL **b** 13.82 kL **c** 0.90 kL
2 **a** 7500 L **b** 14 days
3 **a** 1800 L **b** 1.6 m **c** 75.4 mm
4 **a** 189.45 **b** 303 120 L
5 **a** July **b** 112 556 ML
c A = 1 956 055
d 95.9% **e** 239 894 ML
f 2.2% **g** May
h



i Overall the available storage remained fairly constant over the 12 months almost to capacity (despite slight declines in available storage January to May 2016 and September to December 2016).

6 **a** 2 400 000 ML
b 1998, 2012 or 2013
c Jan 2007, 900 000 ML
d Drought conditions
e 2 600 000 ML
f 1 300 000 ML
g 1998, 1999, 2001, 2002, 2011
7 **a** dual flush toilets **b** 65% **c** 2004
d 10% **e** 2013
f 25%
g drought conditions, education about saving water

Exercise 6.03

1 **a** $\frac{3}{50}$
b Charging devices, security system, fish tank, vacuum cleaner, hair dryer
c $28.12 **d** $1282.98 **e** 113°
2 **a** $\frac{2}{5}$ **b** 43% **c** 300 kWh
3 **a** 5.7×10^6 MW
b 3×10^6 kW
c 4.845×10^9 GW
4 D
5 **a** 5.6 kW **b** 27 500 MW
c 3.495 kW
6 C
7 **a** 2350 MW
b 2 350 000 kW or 2.35×10^6 kW
c 2 350 000 000 W or 2.35×10^9 W
8 **a** quarterly **b** 13.09 kWh
c June/July – winter, more electricity used for heating.
d 1191 kWh
e 91 days **f** 14 November, 58 335 kWh
g decreased $30.25
h Yes because the comparison graphs show home usage between 2 and 3 people households
i No pool, also graphs compare homes in the area
j 'Peak' electricity used when most people use it 2 p.m.–8 p.m.; 'Off Peak' low usage times, 7 a.m.–2 p.m., 10 p.m.–7 a.m., 8 p.m.–10 p.m.
9 **a** 1420.34 kWh
b **i** $131.81 **ii** $425.72
c $4.43
10 **a** 7.9 kWh/day for Peak, 4.9 kWh/day for Off Peak
b June to August – winter, when electricity is used for heating.
c Daily supply charge of $0.94336/day
d Off Peak: Mon-Fri 10 p.m.–7 a.m. other off-peak is shoulder usage 7 a.m.–2 p.m. and 8 p.m.–10 p.m.

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e 2 p.m.–8 p.m. Mon–Fri

f Peak

g Peak: decreased by 0.6 kWh/day; Off Peak: increased by 0.3 kWh/day.

11 $725.58 (GST inclusive)

Exercise 6.04

1 **a** 0.8 kWh **b** 0.16 kWh
c 0.096 kWh **d** 0.4 kWh

2 C

3 3.9 kWh

4 **a** 0.18 kWh **b** 2.188 kWh

5 **a** 39 kWh **b** $10.47

6 $318.17 **7** $103.12

8 **a** $354.84 **b** $330.06 **c** $937.65

9 **a** $481.91 **b** $96.38 **c** $77.11
d $385.53

10 $152.98 **11** $77.24

12 **a** $236.52 **b** Cost: $141.91; Saving: $94.61

13 **a** Refrigerator: $39 Freezer: $21
Washing machine: $3 Dishwasher: $27.60
Clothes dryer: $33 Television: $51.30
Computer: $64.80
b Total cost = $239.70

14 **a** $1008 **b** $2055 **c** $5733

15 $13.27

16 **a** Television: (LCD): $8.18
Computer notebook: $20.80
Inkjet printer: $3.04
Microwave oven: $7.24
Dishwasher: $6.31
Set-top box (Pay TV): $36.92
Mobile phone charger: $0.70
b Total cost = $83.19

Exercise 6.05

1 **a** 2 350 000 J **b** 54 000 kJ
c 3179.84 kJ **d** 1.475 MJ
e 59.75 Cal **f** 215.11 Cal

2 95.6 calories

3 **a** 540.2 kJ **b** 90.0 kJ **c** 22 Cal

4 **a** 15.34 MJ **b** 3640 Cal

5 **a** 1914 kJ **b** 1256 cal

6 **a** 4484 kJ **b** 51.5%

7 **a** 4144 kJ **b** 49.3%

8 **a** **i** 3 **ii** 450 g
b 97 Cal
c **i** 316.2 kJ **ii** 52%

9 **a** running/jogging, touch football
i 53 min **ii** 25 min
b **i** 63 min, 29 min **ii** 72 min, 33 min
iii 84 min, 39 min

Exercise 6.06

1 **a** expense **b** income **c** expense
d expense **e** income **f** expense
g expense **h** expense **i** income
j expense **k** income **l** expense
m income **n** income **o** expense

2 fixed: a, f, g discretionary: c, d, h, j, l, o

3 **a** $381.50 **b** Discretionary, $78.
c Spend less on clothes, look for grocery specials.

4 **a** $111.40 **b** yes **c** yes

5 **a** Teacher to check. **b** $435.28

6 **a** $551.40 **b** $279.64 **c** $271.76

Sample HSC problem

a 2288.55 kW **b** 26.11%
c **i** 6475.45 kW **ii** $1671.31

Test yourself 6

1 **a** $45.68 **b** $249.95

2 B

3 **a** 178 m^2 **b** 551.8 m^3

4 **a** 16.4 MW **b** 7 150 000 kW **c** 26 kW

5 $841.34

6 C

7 $53.11

8 $77.94

9 **a** 4650 J **b** 19 000 kJ **c** 30 271.24 kJ
d 0.1085 MJ **e** 162.5 Cal **f** 95.6 Cal

10 **a** 1305 kJ **b** 311.9 Cal

11 **a** $1120.08 **b** Teacher to check.
c Cut down on take-away meals, newspapers and magazines

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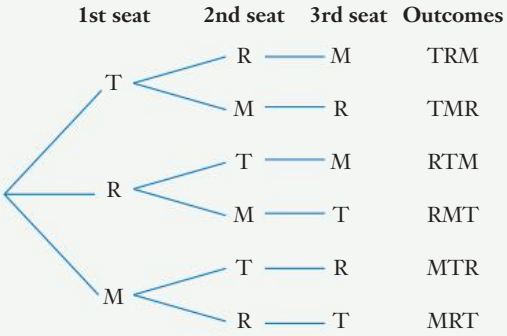
Practice paper 2

1 C **2** C **3** A **4** B
5 C **6** B **7** C **8** B
9 D **10** B

11 a i 17.9 kL **ii** \$289.41

b 1130 m^2

c i T = Tiffany, R = Rose, M = Max



ii $\frac{1}{3}$ **iii** $\frac{1}{2}$

12 a i

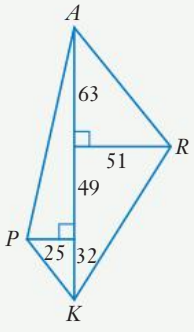
6	7	8	8	10	11
7	8	9	9	11	12

ii 36 **iii** $\frac{1}{6}$ **iv** $\frac{5}{9}$

b 4500 L

c i 9.285×10^{11} km^3 **ii** 4.603×10^{10} ha

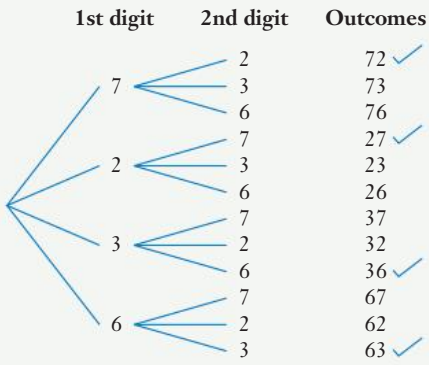
13 a i



ii 5472 m^2

b i 424 cm^2 **ii** 83 cm

c i

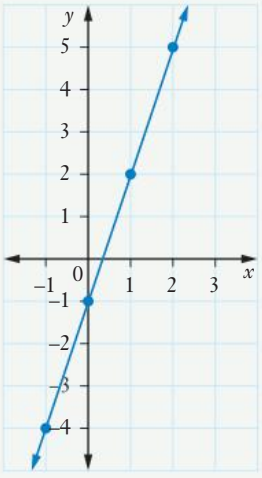


ii $\frac{1}{3}$

Chapter 7

SkillCheck

1



2 a y: 5, 7, 9, 11 **b** y: −3, −4, −5, −6

3 $c = -1$

4 a 2.8 **b** 4

5 a i 1225 **ii** 685

b i 2.5 **ii** 5.7

Exercise 7.01

1 a $y = 3x + 7$ **b** $y = -2x + 1$

c $y = x - 1$ **d** $y = \frac{1}{3}x - \frac{1}{2}$

e $y = -\frac{5}{4}x$ **f** $y = 5$

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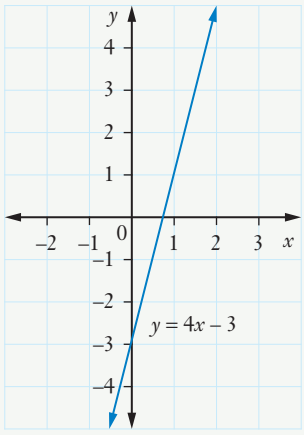
2 a



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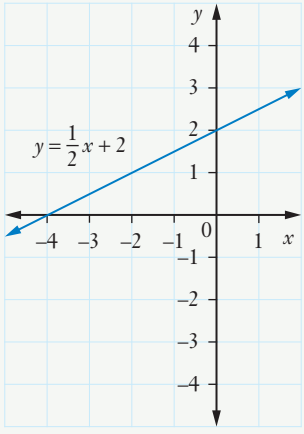

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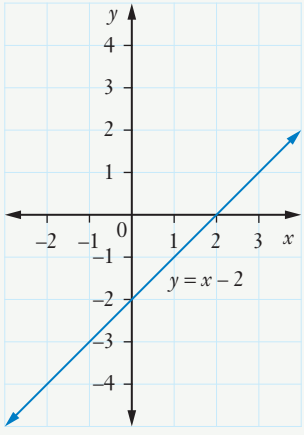

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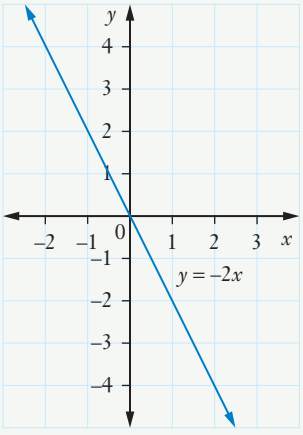

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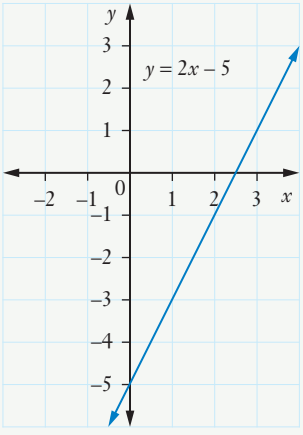

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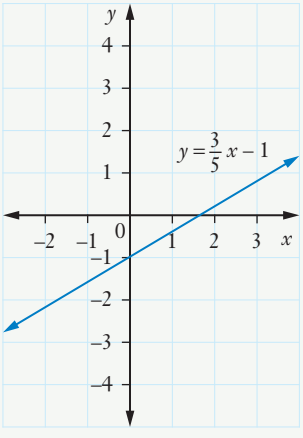

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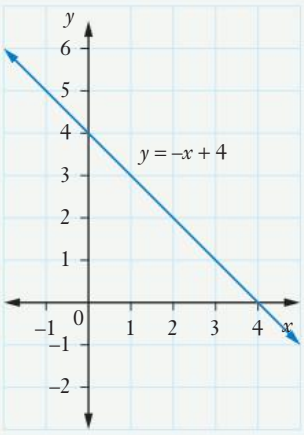
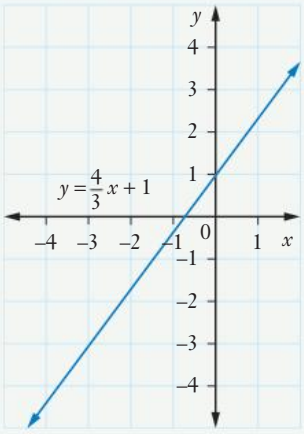
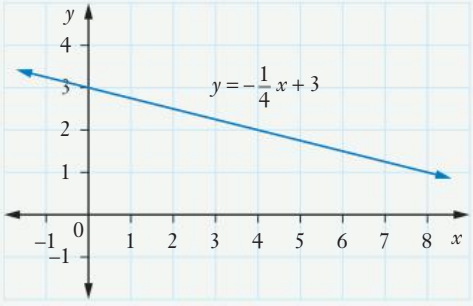
g



h



i



3 D

4 **a** $y = 2x + 3$ **b** $y = 3x - 1$ **c** $y = 4x$

d $y = -2x + 2$ **e** $y = -x + 1$ **f** $y = \frac{1}{2}x - 1$

g $y = -x$ **h** $y = \frac{3}{4}x - 2$ **i** $y = x - 3$

Exercise 7.02

1 **a** 2 **b** -1 **c** 2

d $-\frac{1}{2}$ **e** $\frac{7}{5}$ **f** -5

g $\frac{3}{4}$ **h** 4 **i** $\frac{2}{9}$

2 **a** 3 **b** 5 **c** $\frac{1}{4}$

d -2 **e** $\frac{3}{5}$ **f** $-\frac{1}{2}$

Exercise 7.03

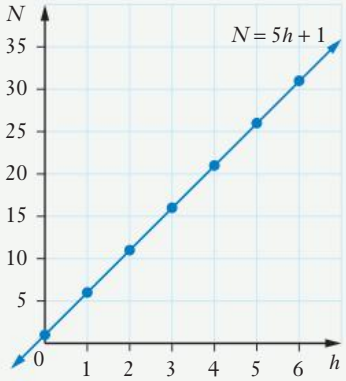
1 **a**

h	1	2	3	4	5	6
N	6	11	16	21	26	31

b N **c** $N = 5h + 1$

d 101 **e** 16

f



g gradient 5, vertical intercept 1

2 **a** $C = 80t + 22$ **b** C

c \$14.62

d 22, starting cost of call (at 0 min)

e \$2.40 **f** 7 min

3 **a** dependent **b** $S = 4.5n$

c 4.5 runs/over

d 0, the number of runs scored after 0 overs

e **i** 95 **ii** 225

f **i** 12 **ii** 40

g Weaker batters bat later, with lower run rate.

4 **a**

t	1	2	3	4
V	1480	1060	640	220

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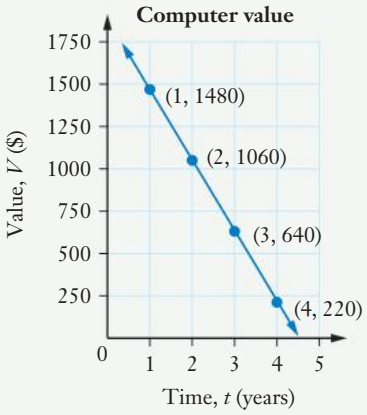
b



c rate of depreciation in dollars per year

d $850 **e** $1900

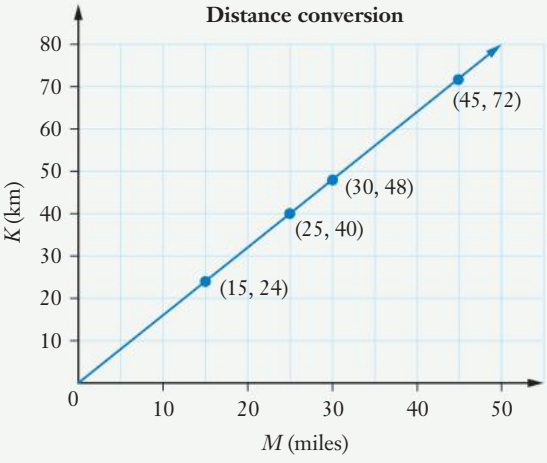
f V becomes negative. **g** 4.5 years

5 **a** $P = 50 + 3n$ **b** n

c 50, the base pay **d** $134

e 16

6 **a**



b $K = 1.6\,M$ **c** 0, 0 miles = 0 km

d 1.6, the number of kilometres in 1 mile

e **i** 160 km **ii** 62.5 miles

f **i** 19 km **ii** 13 miles

7 **a** C

b gradient 0.7, vertical intercept 4.2

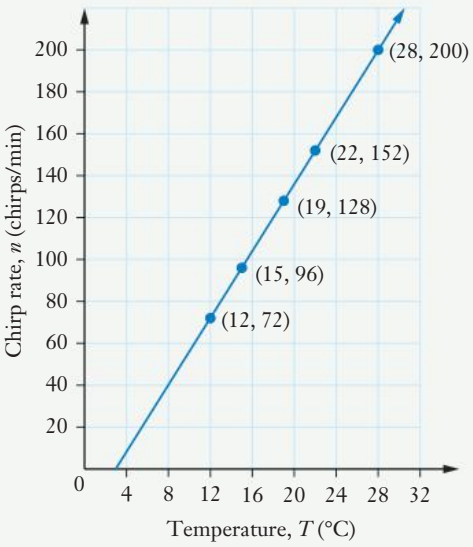
c $C = 0.7d + 4.2$ **d** $3.50

e **i** $18.20 **ii** $4.20

f 48 km

8 **a** independent **b** $n = 8T - 24$

c



d increases by 16 chirps/min

e 184 chirps/min **f** 21°C

g –24, the number of chirps per minute cannot be below zero.

Exercise 7.04

1 $k = 3.5, y = 3.5, 35, 38.5$

2 **a** $d = 2.75r$ **b** 90.75 m **c** 36

3 **a** $p = 5.88d$ **b** 235.2 kPa **c** $141\frac{2}{3}$ m

4 **a** $F = 2.4d$ **b** 7800 kg

5 **a** $y = 1.6x$ **b** 96 km/h **c** 75 miles/h

6 A

7 **a** 64

b increase in water volume per minute

c 512 L **d** 15 min

8 **a** 120 min or 2 hours

b The marathon runner cannot continue to run forever and keep travelling more distance; he needs to rest.

9 **a** 22.8 kg **b** 84.1 kg

10 8 h 53 min

11 **a** 60 seconds or 1 min

b 4000 kilobytes

12 B

13 **a** $S = 1.3h$ **b** 317.2 cm **c** 92 cm

14 **a** 117.6 m/s **b** 18 s

c The object cannot keep falling forever and will stop when it hits the ground.

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Exercise 7.05

1 **a** 107 cm **b** 5.9 ft
c **i** 61 cm **ii** 76 cm **iii** 182 cm

2 **a** \$127 **b** \$52 **c** \$110
d \$75 **e** \$36 **f** \$60

3 **a** \$1.73 **b** \$1.15 **c** \$4.90 **d** \$1.80

4 **a** €17 **b** €53 **c** €22 **d** €90
e €75 **f** €5.20

5 1.44, the no. of A\$ per €, what €1 equals in A\$

6 **a** **i** 72 kg **ii** 56 kg **iii** 80 kg
b **i** 185 cm **ii** 159 cm **iii** 193 cm
c 0.65, rate of change of weight per height in kg/cm

7 **a** **i** \$51 **ii** \$24 **iii** \$70
b **i** \$29 **ii** \$94 **iii** \$63
c 0.85, the discount price as a proportion of the marked price
d C

8 **a** **i** 41 kg **ii** 62 kg **iii** 73 kg
b **i** 88 lb **ii** 140 lb **iii** 198 lb
c 0.46 kg/lb

Sample HSC problem

a $P = 4n - 24$

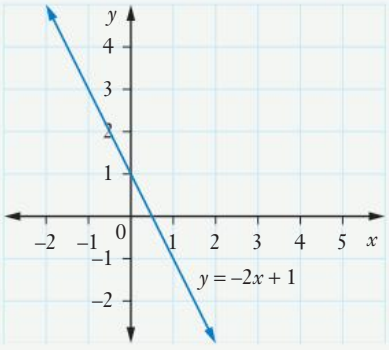
b 4, increase in pocket money in dollars per year of age

c \$40 **d** 11

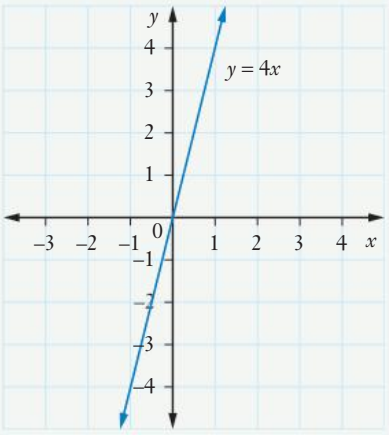
e **i** Values of P will be 0 or less.
ii After 18 years, children become adults and either receive no pocket money or an amount based on a different formula.

Test yourself 7

1 **a**



b



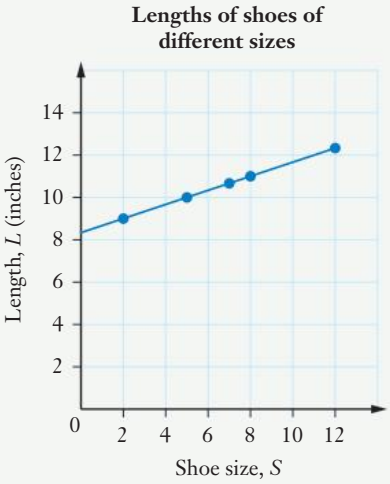
2 **a** $y = -\frac{1}{2}x + 1$ **b** $y = 3x - 3$

3 **a** $2\frac{1}{2}$ **b** -3

4 **a** $\frac{1}{2}$ **b** -4

5 **a** S

b



c $\frac{1}{3}$, the increase in length per shoe size, in inches/size

d $8\frac{1}{3}$ inches, the vertical intercept

e $L = \frac{1}{3}S + 8\frac{1}{3}$

f $10\frac{5}{6}$ **g** 14

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6 18 km

7 **a** **i** \$31 **ii** \$93

b **i** 1700 baht **ii** 2150 baht

c 0.04, number of A\$ per baht

Chapter 8

Skillcheck

1 **a** 0.025 **b** 0.84

c 1.06 **d** 0.1875

2 **a** 2.5 **b** 3.75 **c** 0.45

3 **a** 2653.41 **b** 12 174.94

c 5299.09

4 **a** \$773.40 **b** \$2135 **c** \$1474

d \$346.88 **e** \$3648.13 **f** \$132.60

5 **a** \$29.36 **b** \$2375 **c** \$96.80

6 **a** 49 200 **b** 41 600 **c** 8.33

Exercise 8.01

1 \$4048 **2** 24 627 762

3 \$84 346.35 **4** \$1473.33

5 \$87.02 **6** \$950.40

7 \$345.36 **8** \$2523.74

9 **a** \$102.35 **b** \$94.16

c \$5.16 increase, 5.8% increase

10 D

11 **a** \$23 042.17 **b** 15.2% increase

12 **a** 4% decrease **b** 4% decrease

c 16.64% increase **d** 6.5% decrease

e 9.76% decrease **f** 3.1% decrease

13 **a** 207 t **b** 11.5%

Exercise 8.02

1 C

2 **a** \$320.40 **b** \$3717.90 **c** \$471.18

d \$42.08 **e** \$625.50 **f** \$569.58

3 \$1620 **4** 11%

5 2 years **6** 12%

7 \$7400 **8** C

9 **a** \$1984.50 **b** \$847.91

c \$4680 **d** \$207.66

10 0.035% **11** 15 months

12 0.8%

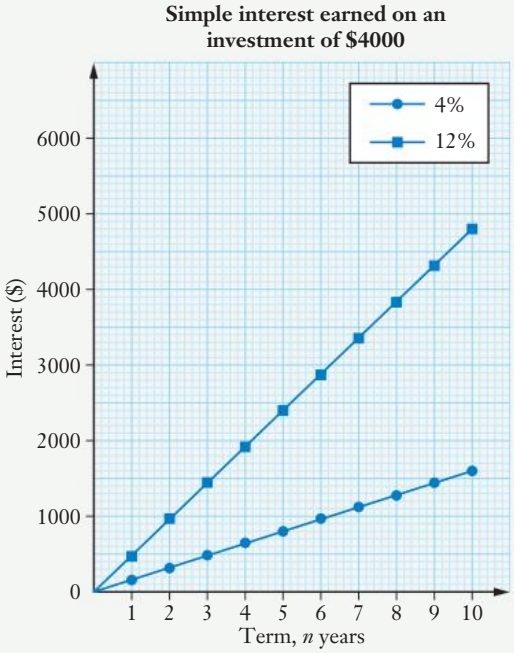
Exercise 8.03

1 C

2 **a** \$10 800 **b** \$3000 **c** \$9120

3 **a** 8.3 years **b** $3\frac{1}{2}$ years **c** 7.1 years

4



5 **a** 480, the interest (\$) earned per year

b **i** \$960 **ii** \$3840

c The second graph is three times as steep as the first graph.

6 A

7 **a** \$4200 **b** 6% **c** 45 months

d 4.6 years **e** \$2800 **f** 4%

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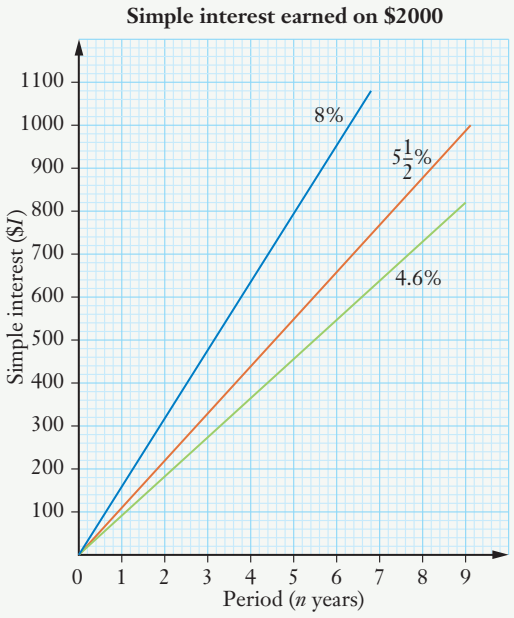
8 a



b $340

c **i** 4.4 years **ii** 6.4 years

9 You earn zero interest if money is invested for zero years.

Exercise 8.04

1 **a** $9261 **b** $1261
c $10 210.25 **d** $8000 \times (1.05)^n$

2 **a** $8994.74 **b** $3891.04 **c** $4951.80
d 17 869.38 **e** $9714.53 **f** $9516.52

3 $82 367

4 **a** $14 364.27, $1464.27
b $15 989.37, $2189.37
c $16 011.05, $2211.05
d $7341.81, $421.81

5 $18 363.23

6 **a** **i** $2975 **ii** $3421.68
b **i** Compound interest is greater because interest is calculated on accumulated interest as well.
ii $446.68

7 **a** $23 152.50 **b** $23.180.07
c $23 194.20 **d** $23 203.76
e $23 208.43

8 The amount of interest increases because you get interest on the interest more often.

9 18 years **10** 4 years
11 $3200 **12** $7993.54
13 C **14** 10 years
15 **a** B **b** A
c **i** $16 979.20 **ii** $6030.22
iii $4414.20
d $4038.40

Exercise 8.05

1 A **2** C

3 **a** $1950
b **i** $9750 **ii** $5850

4 **a** $5440
b **i** $21 320 **ii** $15 880

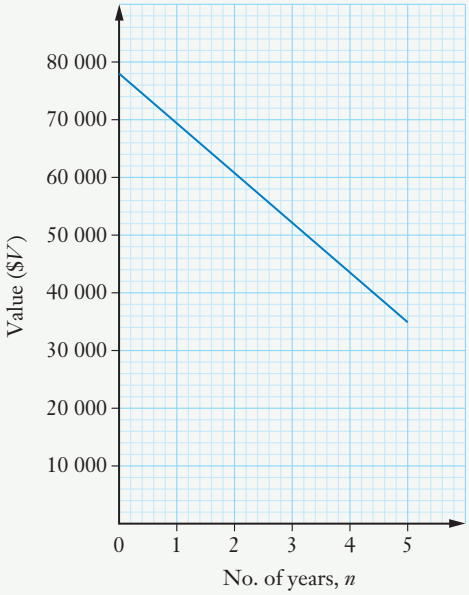
5 $0

6 **a** $11 666.67 (to the nearest cent)
b $121 666.67

7 **a** $8600
b

No. of years, n	0	1	2	3	4	5
Value, $V	78 000	69 400	60 800	52 200	43 600	35 000

c



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d $78 000, the purchase price

e $56 000 **f** 4.4 years

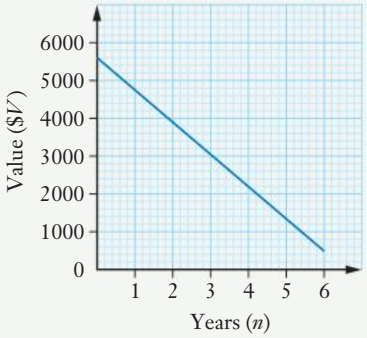
8 a $5600, the purchase price

b i $850 **ii** $1700

c i $3900 **ii** $2200

d $500

9 a



b $5600, the purchase price

c 5.4 years **d** $2\frac{1}{2}$ years

e $4300

10 a $19 000

b i $13 000 **ii** $3000

c $2000

Sample HSC problem

a $2925 **b** $3119.24

c Because interest is calculated on accumulated interest as well.

Test yourself 8

1 $76 608 **2** 79.3%

3 $1053.64 **4** $48 015

5 a $2066.40 **b** 13.9% decrease

6 a $1350 **b** $162 **c** $1854

d $1499.40

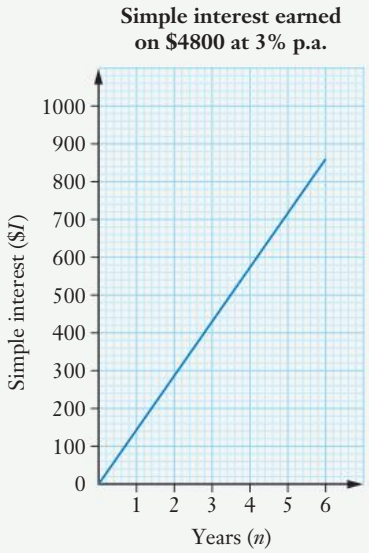
7 4.5% **8** 9 years

9 a i $4500 **ii** $7300

iii $11 300

b 8.9 years

10 a



b 144, the interest earned per year

c $3\frac{1}{2}$ years

11 a $8636.13, $1436.13 **b** $4215.07, $165.07

c $12323.05, $2023.05

12 $9554.46 **13** $4.02

14 $10 944, $1944

15 a $26 000 **b** $13 000

16 a $8500

b

Year	Accumulated depreciation ($)	Value at end of period ($S)
0	0	67 000
1	8500	58 500
2	17 000	50 000
3	25 500	41 500
4	34 000	33 000
5	42 500	24 500
6	51 000	16 000

c $24 500 **d** $S = 67\,000 - 8500n$

Chapter 9

SkillCheck

1 a $1776 **b** $9100 **c** $14 352

d $229.50 **e** $42

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2 **a** $64 **b** $3328 **c** $277.33
3 **a** $770.40 **b** $1608.00 **c** $1397.00
d $102.56 **e** $4454.45 **f** $1374.65
4 **a** 144 **b** 1008
c 4368 **d** 52 416
5 $253.50
6 $56 745

Exercise 9.01

1 C **2** D
3 $1535.36 **4** A
5 **a** $158.40 **b** $234.50
c $385.02 **d** $370.63
6 $129.61 **7** $510.35
8 **a** $470.20 **b** $622.20 **c** $1360
9 **a** $573 **b** $603.90 **c** 524.70
d $468.50 **e** $573 **f** $624.25
10 **a** **i** 325 **ii** 254
b **i** 7317 **ii** 3 850 877
11 **a** $933.40 **b** $9699 **c** $734.80
12 **a** 853 **b** 217
c 31 **d** easier to break into
13 The number of men injured when driving (for driver, motorcyclist, pedal cyclist) is far greater than the number of women driving. This means insurance premiums are more costly for males.

Exercise 9.02

1 D **2** C
3 **a** **i** $900 **ii** $2200 **iii** $2800
b piecewise graph going through (0, 0), (30 000, 900), (40 000, 2000), (80 000, 3600)
c **i** $3400 **ii** $65 000
4 **a** $550 **b** $1987.50 **c** $4165
5 **a** **i** $500 **ii** $1500 **iii** $2200
b **i** $90 000 **ii** $70 000 **iii** $45 000
6 **a** $320 **b** $3450 **c** $7300
7 **a** $260 **b** $440 **c** $2730
8 **a** $2855 **b** $562.50 **c** $6620
9 **a** $9.80 **b** $55.50 **c** $68
10 **a** $18 **b** $1044 **c** $4220
d $26.70 **e** $1462.50

Exercise 9.03

1 D **2** B
3 $70 599 **4** $32 525.20
5 **a** $31 496.58 **b** $6993.50 **c** $65 169.22
d $27 516.75 **e** $19 953.47 **f** $58 096.55
6 $47 732.80
7 **a** $67 448.35 **b** $42 977.13 **c** $45 950.95
d $155 515 **e** $19 270.01 **f** $26 978.89

Exercise 9.04

1 $10 538.84
2 **a** 29 L **b** $1959.88
3 **a** $3261.82 **b** $9151.82
c $352/fortnight
4 **a** NSW – $107.92, VIC – $108.42, Qld – $114.17
b $325, fuel is more expensive in Qld.
5 $12 265.24 **6** $189.58
7 **a** $153.88 **b** $14 002, $269.27
8 $29 097.28

Exercise 9.05

1 C **2** D
3 **a** 0.115 L/km **b** 113 L
c 478 km **d** 1477 km
4 **a** 0.55 L **b** 129.09 L
5 **a** 0.08 L **b** 12 L
6 **a** Toyota Corolla **b** 1097 (average)
c 135 L (average), 180 L (city), 110 L (highway)
d **i** $261.25 (average) **ii** $172.64
7 **a** $1.31
b 24 c/L
c **i** $231.20 **ii** 1212 km
8 **a** 130.0 (to one decimal place)
b 3.5 c/L
c $67.80 **d** 9.09 L/100 km
9 **a** 145.54 c/L **b** 149.17 c/L
c 9.4 c/L **d** 0.8 c/L
e Brisbane (145.54) **f** Thursday (154.98)
g Adelaide (13.1) **h** Sunday (7.7)
10 Brodie's (10.4 L/100 km)
11 7.2, 383.9, 109, 5, 886.6, 303.9, 9.7

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Exercise 9.06

1 $222.87

2 Car 1: $105 756

Car 2: $105 510

Car 2 is slightly cheaper.

3 65 L of petrol costs $41.21 more.

4 **a** $24 525 **b** $201.55

5 **A** $35 096.16, **B** $34 474.44

B is the better option

6 The warranty is for 5 years or 100 000 km, whichever comes first.

So if a car with this warranty covers a distance of 100 000 km in 2 years, the warranty has then lasted only 2 years.

7 Teacher to check.

Sample HSC problem

a $561

b **i** $19 293 **ii** $25 920 **iii** $124.62

Test yourself 9

1 **a** $286.90 **b** $320.76

2 **a** $8.30

b Nothing. He was only insured for damage to other people's property. He needs comprehensive insurance to insure his own property.

3 **a** $540 **b** $1925 **c** $72 **d** $3305

4 **a** $615 **b** $920 **c** $3570 **d** $2990

5 **a** $642 **b** $1323 **c** $2035 **d** $3565

6 $27 708.96

7 **a** $8110.40 **b** $155.97

8 **a** 10.74 L/100 km

b 5.1, 3.3, 49.7, 221.3, 720.5

c Don't speed; accelerate slowly; walk or cycle short distances

9 **a** 11.6 L/100 km **b** 9.17 L/100 km

c **i** 0.067 **ii** 0.057 **iii** 0.113

d 598 km **e** 245 L

10 $21.21

Practice paper 3

1 B **2** D **3** B **4** C

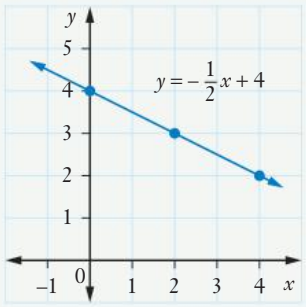
5 D **6** A **7** D **8** A

9 C **10** A

11 **a** **i** $19 750 **ii** 8 years

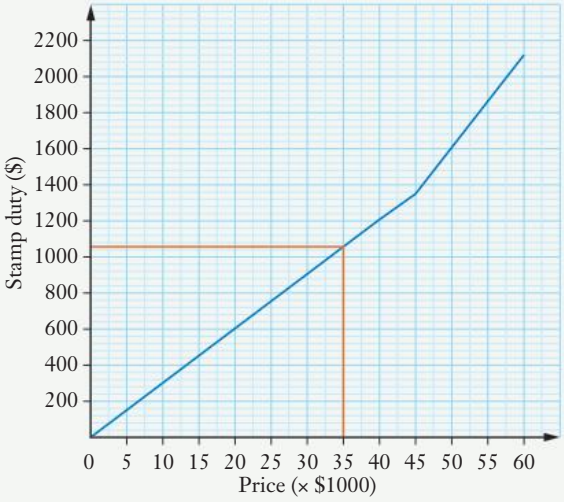
b **i** $m = -\frac{1}{2}$, y-intercept, $c = 4$

ii



c **i** 600, 900, 1200, 1350, 1600, 2100

ii



iii $1050

12 **a** CTP insurance protects owners/drivers from legal liability for personal injury or death to any other person. Third Party Property insurance is optional and covers damage to other vehicles and property where you are at fault. It does not cover damage to your vehicle.

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b **i** $V = -5.5t + 84$

ii The decrease in water volume in kilolitres per hour

iii The amount of water in the pool before you start draining it

iv 15.3 h

c **i** \$600 **ii** \$15 600

iii \$624 **iv** \$1224

13 **a** **i** gradient $= \frac{50}{3}$ **ii** $D = \frac{50}{3}f$

iii 54 L

b \$56 635

c **i** 39.48 L

ii \$75.98

Chapter 10

SkillCheck

1 **a** 29 **b** 58 **c** 12

d 8 **e** 24

2 column graph

3 **a** 10 **b** 7

c 12, wears large shoes, has big feet

d 0 **e** 30%

4 **a** 4 **b** 1 **c** 4

d 32 **e** 42

5

50 – < 60	55	4
60 – < 70	65	12
70 – < 80	75	7
80 – < 90	85	8
90 – < 100	95	3
100 – < 110	105	6

Exercise 10.01

1 **a** **i** 5 **ii** 5 **iii** 1, 5

b **i** 34.4 **ii** 34.5 **iii** 31, 32, 35

c **i** 36 **ii** 38 **iii** no mode

d **i** 9 **ii** 8 **iii** 5, 7, 8

2 **a** 24 **b** 24 **c** 20 **d** 14, 20

3 **a** D **b** C **c** A

4 **a** 50.0 **b** Yes, the mean is 50.0

c 50 **d** 50

5 **a** 20 **b** 2.6 **c** 2

d 2 **e** 8

f **i** decreases **ii** no effect **iii** no effect

6 **a**

Number of calls, x	Frequency f	fx
2	1	2
3	2	6
4	3	12
5	5	25
6	2	12
7	2	14
	15	71

b 15 **c** 5 **d** 5 **e** 4.7

7 **a** 38, 43, 48, 53, 58, 63, $\bar{x} = 44.24$ **b** 149

8 **a** 40 – 49 **b** 40 – 49

9 **a** – 2, 2 **b** 2 **c** 2.6

10 **a** 300 – < 400 **b** 300– < 400

11 **a** mean **b** median **c** median **d** mode

e mean **f** mode **g** mean **h** mode

12 **a** mean **b** mode **c** median

d mode or mean **e** mean **f** mode

13 **a** 7 **b** \$82 000 **c** \$68 500

d Mean, the high salary of the general manager pushes it up.

e median

14 **a** **i** 38.125 **ii** 38 **iii** 32

b **i** decrease **ii** decrease **iii** no effect

15 **a** 39 **b** white

c Data is categorical, not numerical

16 **a** **i** \$523 **ii** \$481 **iii** no mode

b Mean distorted by outlier of \$1027, there is no mode.

c mean \$473, median \$458, yes

17 **a** **i** 7.9 **ii** 7.5 **iii** 7

b **i** decrease **ii** no effect

c mode

18 **a** **i** 27°C **ii** 27°C **iii** 27°C

b mean

Exercise 10.02

1 **a** 9.1, 10.1, 11.0 **b** 49.5, 50, 51

c 3.00, 3.45, 3.50 **d** 8, 18, 23

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2 **a** 113.5 **b** 101.5 **c** 125.5

3 D

4 **a** 88 **b** 64 **c** 68
d 61 **e** 40%

5 **a** 4.5 **b** 7 **c** 8
d 7 **e** 9

6 **a** 20.6 **b** No, 25% **c** 25.4
d Yes, 3% **e** 10th

7 **a** 25.4° **b** 10% **c** 23.5°
d 18.9° **e** 12.4°

8 **a** F **b** T **c** T
d T **e** F **f** T

9 **a** **i** 60% **ii** 60% **iii** 9%
b 6.05% **c** 85 **d** 68.65% **e** 70th

10 **a** 102.7 cm **b** No. 25%
c 106.2 cm **d** 103.3 cm
e No. 1% **f** 97.6 cm, 109.2 cm

11 **a** 25% **b** 95% **c** 188 cm
d 5 years
e **i** 178 or 179 cm **ii** 182 cm

Exercise 10.03

1 **a** 6 **b** 5 **c** 728 **d** 12

2 **a** 2 **b** 2.5 **c** 258 **d** 3

3 A

4 **a** 63 **b** IQR = 75 – 48 = 27

5 **a** 46 **b** 12
c The person who took 15 s to complete the test may know all the work very well or may have guessed the answers.

6 B **7** 80

Exercise 10.04

1 **a** IQR = 1.5
b **i** –0.25 **ii** 5.75
c Yes, since 8 is greater than 5.75 ($= Q_3 + 1.5 \times \text{IQR}$).

2 **a** No, since outliers must be less than –1.25 or greater than 16.75.
b Yes, 9 is an outlier since it is less than 11 ($= Q_1 - 1.5 \times \text{IQR}$).
c No, since outliers must be less than –11 and greater than 81.
d 9 is the outlier since it is greater than 7.5.

3 **a** \$531.43 **b** \$520
c IQR = 610 – 420 = 190
d \$900, $Q_3 + 1.5 \times \text{IQR} = 610 + 1.5 \times 190 =$ \$895, so \$900 is an outlier.
e Mean and median would be lower (\$470, \$485).
f \$584.57, \$572, Yes

4 **a** 20 **b** 8 since $8 > 5 + 1.5 \times 1.5$
c **i** 4.35 **ii** 4.16
d The mean increases.

5 D

6 **a** Wombats: $\bar{x}$ = 16.8 points; median = 18 points
Possums: $\bar{x}$ = 16 points; median = 16 points
Koalas: $\bar{x}$ = 17.2 points; median = 14 points
b Possums, as mean and median are the same.
c $\bar{x}$ = 18.8 points; median = 18 points
d Still the Possums with the Wombats a close second.

7 **a** Sam 3 copiers; Terri 3 copiers
b They are equally good salespeople.
c Sam 5.5 copiers; Terri 17 copiers
d Sam 7.2 copiers; Terri 16.6 copiers
e Median as Sam has an outlier of 25.
f Terri as his median sales are much higher than Sam's.

8 **a** 2.92, 3, 3
b 9 since $9 > 4 + 1.5 \times 3 = 8.5$ ($Q_3 + 1.5 \times \text{IQR}$)
c 2.36, 3, 3
d Increases the mean, no effect on the mode, median

9 **a** mean = \$36 570; median = \$34 200; mode = \$46 500
b mode
c median, since it is not affected by the outlier of \$73 800

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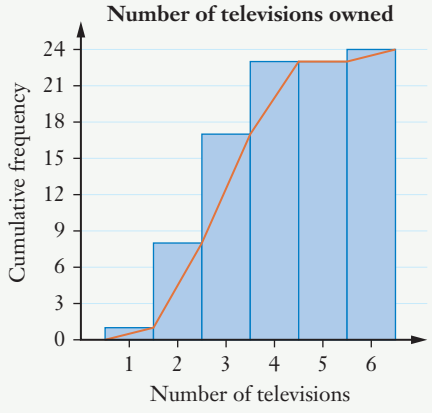
Exercise 10.05

1 **a** 1, 8, 17, 23, 23, 24, median = 3

b



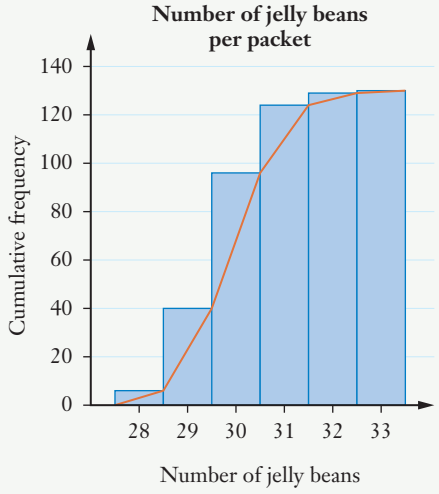
c **i** 3 **ii** 2

2 **a** 25 **b** 45

c 56 – 27 = 29 **d** 65

3 **a** 6, 40, 96, 124, 129, 130

b



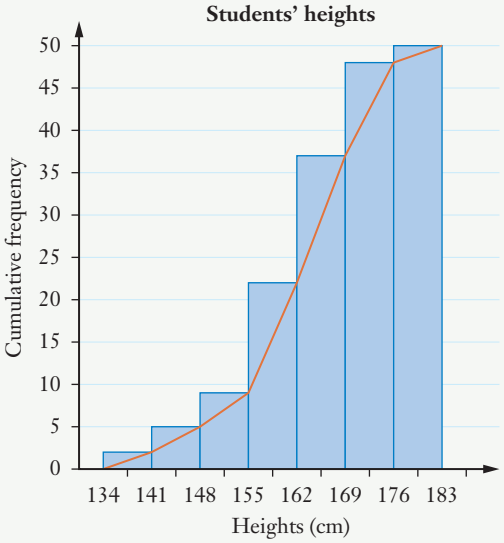
i 30 **ii** 31 – 29 = 2 **iii** 30

4 **a** Class centres: 137.5, 144.5, 151.5, 158.5, 165.5, 172.5,179.5;

Cumulative frequencies: 2, 5, 9, 22, 37, 48, 50

b 162 – < 169 **c** 162 – < 169

d

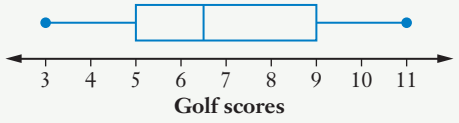


i 164 **ii** 171 – 158 = 13 **iii** 168

Exercise 10.06

1 **a** 3, 5, 6.5, 9, 11

b



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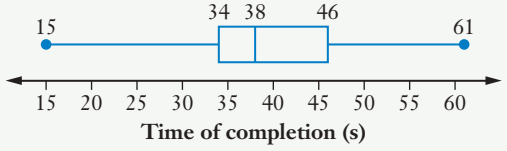

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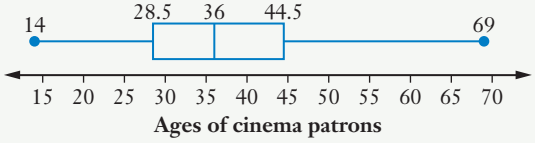
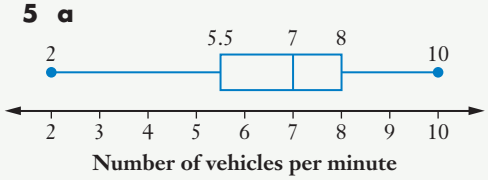


4 **a** 20 **b** 12 **c** 3

d 15 **e** 30

5 **a**



b Different answers possible: The dot plot shows individual scores and any clusters and outliers, while the box-and-whisker plot identifies the quartiles, including the median.

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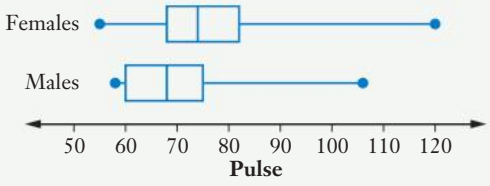
6 **a** $75 000 **b** $41 000 **c** $41 000
d $58 000 **e** $28 000, $51 000
f 25%

7 **a** Geography: 45, 55, 60, 75, 90
Modern History: 40, 60, 70, 75, 80
b Geography: **i** 45 **ii** 20
Modern History: **i** 45 **ii** 15
c Geography 60, Modern History 70
d Modern History as range for both subjects is the same but IQR for History is lower.
e **i** 10 **ii** 20
f Modern History as 30 students scored above 60 compared with 20 students in Geography.

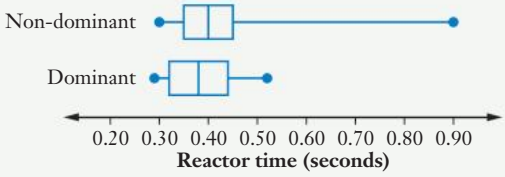
8 **a** Males: 58, 60, 68, 75, 106
Females: 55, 68, 74, 82, 120



b Males: range = 48, IQR = 15
Females: range = 65, IQR = 14
c Range of females is higher due to the outlier of 120. The IQRs are similar, so no significant difference in the spread of both groups.
d Males: 25% of males had pulses above 75 while 50% of female pulses were above 74.

9 D

10 **a** Dominant: 0.29, 0.32, 0.38, 0.44, 0.52
Non-dominant: 0.30, 0.35, 0.40, 0.45, 0.9



b Dominant: range = 0.23, IQR = 0.12
Non-dominant: range = 0.60, IQR = 0.10
c If outlier of 0.9 is omitted, the spread of both groups is similar with dominant hand having slightly better (shorter) reaction times.

Exercise 10.07

1 **a** 2 **b** 1.3

2 **a** 2.25 **b** 3.03 **c** 5.28, –0.78
d 6 **e** 50%

3 **a** 75 **b** $s = 9.84$

4 **a** $836 **b** $277

5 **a** 2.88 **b** 1.8 **c** 27 **d** 54%

6 **a** 6.8 **b** 1.86 **c** 15 **d** 75%

7 **a** 550, 650, 750, 850, 950, 1050
b **i** $764.89 **ii** $119.36

8 **a** Males: $\bar{x} = 175.5, s = 7.87$
Females: $\bar{x} = 164.6, s = 7.24$
b Males are significantly taller than females (difference in means is 10.9).
However, the spread of heights of females is slightly less than the heights of males.

9 **a** Test 1: $\bar{x} = 65.95$ $\sigma = 14.36$
Test 2: $\bar{x} = 71.55$ $\sigma = 14.45$
b The mean for Test 2 is 5.6 marks higher than the mean for Test 1 which is significant, but the standard deviations nearly equal, indicating the spread for both tests is the same.
c Because the mean for Test 2 is significantly higher, the students performed better in Test 2.

10 **a** Men: $\bar{x} = 340.55$ $\sigma = 436.03$
Women $\bar{x} = 399.65$ $\sigma = 559.58$
b Men: $\bar{x} = 224.22$ $\sigma = 269.11$
Women $\bar{x} = 234.06$ $\sigma = 248.21$
c Women make longer calls than men (by 60 s) and the spread of the time for their calls is much more than the time for calls made by men. However, if the outliers are excluded, there is no significant difference, as the mean time of calls for women is only 10 s longer than that for men. Also, the spread of times for calls is now more for men (their standard deviation is 20 more than that for women) but again it is not significant.

11–13 Teacher to check.

Exercise 10.08

1 C

2 **a** **i** negatively skewed
ii unimodal, peak at 6
iii cluster from 6 – 8

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b **i** positively skewed or symmetrical
ii multimodal
iii no clusters

c **i** positively skewed
ii unimodal, peak at 16
iii clustering in the 10s and 30s

d **i** negatively skewed
ii multimodal, peaks at 12, 14, 16 and 18
iii no clusters

e **i** symmetrical
ii multimodal, modes at 55, 65 and 85
iii clustering in the 60s

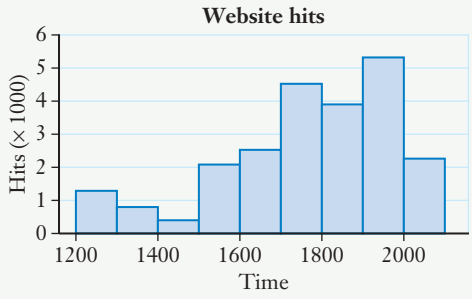
f **i** not symmetrical, not skewed
ii bimodal, peaks at 15–20 and 30–35
iii clustering at 10–20, 30–40

3 **a** 36

b negatively skewed

c 70s and 80s **d** 88

4 **a**



b negatively skewed, cluster from 1700 to 2000, peak 1900–2000

c Most visits to Internet sites occur after work hours.

5 B

6 **a**

Stem	Leaf
1	5 5 5 6 6 7 8 9 9
2	0 1 2 2
3	5 6
4	3 9
5	0 5 9

b positively skewed; cluster 15–19 years; peak mode of 15

7 **a** positively skewed

b 3

c 2.9 (slightly less than mode)

8 **a** 11BS1: mean = 61.9, median = 59, mode = 47, 56, 57

11BS2: mean = 65, median = 66.5, mode = 64, 71

b 11BS1: range = 47, IQR = 19, $\sigma = 12.4$

11BS2: range = 50, IQR = 15.5, $\sigma = 12.5$

c 11BS1: positively skewed

11BS2: negatively skewed

d 11BS2 achieved better results since the mean and median are both greater than the mean and median for 11BS1 (by 3.1, 7.5 respectively). Although 11BS2 had the larger range, the IQR was less and the standard deviation was similar so the spreads were approximately the same.

9 **a** 12W: 50 12X: 40

b 12W: range = 38, IQR = 12

12X: range = 56, IQR = 28

c Both sets of data are positively skewed.

d 12W had the better results as the median score was 50 compared to 40 for 12X. More than 75% of students from 12W scored more than 50% of students from 12X.

10 **a**

Estimates		Test results
3	3	2
	4	6 9
8	5	2
5 5	6	4
8 7 7 5 3 0	7	2 3 3 3 8 9 9 9
9 9 8 7 7 6 5 3 2 2 2 0	8	0 2 5 6 6 7 7
3 2	9	1 1 1 5

b The estimates are negatively skewed, has one mode (82) and are clustered in the 80s. The test results are slightly positively skewed, have 3 modes (73, 79 and 91) and are clustered in the 70s and 80s.

c Estimates: **i** 78.2 **ii** 82 **iii** 82

Test results: **i** 75.4 **ii** 79

iii 73, 79, 91

d Estimates: **i** 60 **ii** 13 **iii** 12.9

Test results: **i** 63 **ii** 14 **iii** 15.8

e The students did overestimate their results but only by 3 when comparing means and medians. Also, the spread of the two sets of scores are similar with the ranges (90 and 93) and IQRs (13 and 14) being very close.

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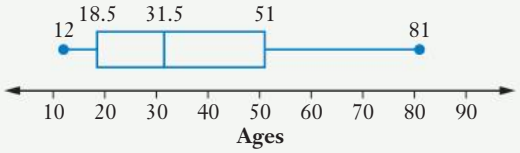
Sample HSC problem

a 39.25 **b** 31.5

c Mean, involves every score and there are no outliers.

d $51 - 18.5 = 32.5$

e



Test yourself 10

1 **a** 173.2 **b** 171.5 **c** 168, 170

2 **a**

Speed (km/h)	Class centre	Frequency
60 – < 70	65	7
70 – < 80	75	13
80 – < 90	85	23
90 – < 100	95	7
		50

b 81 km/h **c** 80–90 **d** 80–90

3 mean = 6.8, median = 7, mode = 7

4 **a** mean \$699 875 **b** median \$661 000

c Median, the outlier of 1 480 000 does not affect its value as it does the mean.

5 **a** mode **b** mean **c** median

6 C

7 **a** 70% **b** 22%

8 **a** Range of middle 50% of scores

b Range = 53 – 48 = 5, IQR = 51 – 49 = 2

9 **a** 56 **b** 19

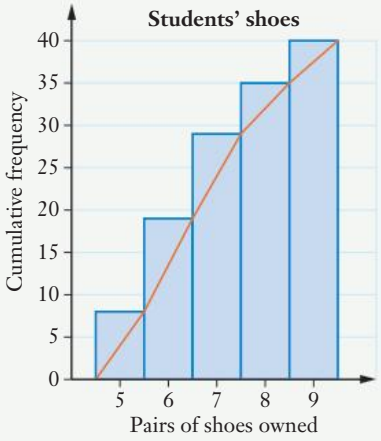
10 $Q_3 + 1.5 \times \text{IQR} = 1178 + 1.5 \times 228 = 1520$

So, 1710 is an outlier.

11 **a** Increases the mean from 11.625 to 14.778 and also increases the median from 10 to 12.

b The median – less affected by the outlier.

12 **a** Cumulative frequency 8, 19, 29, 35, 40



b **i** 7 **ii** 8 – 6 = 2 **iii** 6

13 **a** 36

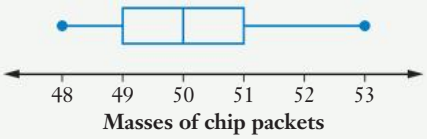
b **i** 4.5 **ii** 7 – 3 = 4

iii 5 **iv** 4

14 **a** 0 **b** 5 **c** $\frac{1}{2}$ **d** 25%

15 **a** 48, 49, 50, 51, 53

b



16 **a** English – smaller range and IQR.

b 96

17 **a** 1.998 cm **b** 0.02 cm

18 **a** 6.7 **b** 1.3

19 **a** Test 1: mean = 5.6, median = 6, mode = 6

Test 2: mean = 6.3, median = 7, mode = 7

b Test 1: Results are symmetrical.

Test 2: Results are negatively skewed.

c Test 1: **i** 7 **ii** 2 **iii** 1.74

Test 2: **i** 8 **ii** 2 **iii** 1.80

d Results of Test 2 are just better than Test 1 as mean, mode and median of Test 2 are higher than for Test 1. The spread for both tests are similar as there is only a difference of 1 between ranges, the IQRs are equal and the standard deviations are approximately equal.

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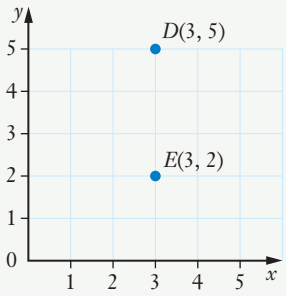
Chapter 11

SkillCheck

1 **a** $A(-3, 1)$, $B(2, 1)$
b 5 units
c $C(-2, 4)$

2 **a**



b 3 units

3 **a** 9 **b** 5 **c** 17 **d** 8

4 **a** 17 **b** 44 **c** 25 **d** 43

5 **a** 2:00 a.m. **b** 6:00 a.m.
c 12:33 a.m. **d** 6:44 p.m.

Exercise 11.01

1 **a** D **b** G **c** A **d** C
e F **f** B **g** E **h** H

2 **a** G **b** C **c** D **d** E
e A **f** F **g** H **h** I
i B **j** J **k** K **l** L

3 **a** (0°, 75°W) **b** (45°N, 40°W)
c (0°, 110°W) **d** (30°S, 40°W)
e (45°N, 110°W) **f** (0°, 40°W)
g (30°S, 110°W) **h** (45°N, 75°W)

4 The South Pole

5 **a** small **b** great **c** great

6 **a** 9°, north **b** 14°, north **c** 4°, south
d 2°, north **e** 67°, north **f** 1°, south

7 **a** B **b** C

8 **a** 99°, west **b** 21°, west **c** 17°, west
d 86°, east **e** 26°, east **f** 1°, east

9 (32°S, 141°E) **10** (27°S, 152°E)

11 D

12 **a** longitude **b** latitude

13 North and South poles

Exercise 11.02

1 **a** 8:45 a.m. **b** 5:50 p.m. **c** 12:19 a.m.
d 11:05 a.m. **e** 4:28 p.m. **f** 9:15 a.m.
g 2:40 a.m. **h** 8 p.m.
i 6:25 a.m. **j** 6:53 p.m.

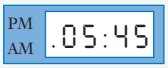
2 **a** 1935 **b** 1142 **c** 2359
d 0017 **e** 1230 **f** 0340
g 0710 **h** 2154
i 2218 **j** 0159

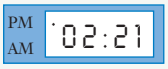
3 11:17 p.m.

4 **a** 07:30 **b** 01:15 **c** 17:10

5 1621

6 **a**

b 

c 

7 A

8 **a** 1 h 55 min **b** 50 min **c** 4 h 45 min
d 3 h 55 min **e** 5 h 45 min **f** 10 h 55 min

9 1 h 52 min

10 **a** 8 p.m. **b** 7 a.m. **c** 6:55 p.m.
d 1:17 p.m. **e** 2:04 a.m. **f** 3:52 a.m.

11 Manal 1:43 p.m.; Tom 1:44 p.m.; Gianni 2:02 p.m.; Eddie 2:04 p.m.; Sarah 2:19 p.m.; Robert 2:21 p.m.

12 9 h 30 min

13 6 h 45 min

14 6:15 p.m.

Exercise 11.03

1 **a** 7 hours **b** 6 hours 30 min
c 8:20 p.m. **d** Liverpool
e 25 min
f Sydney to Liverpool takes 1 hour 15 min, Liverpool to Sydney takes 1 hour. The shorter time for the return trip may be due to it being earlier in the afternoon, when there is less traffic on the road.

2 **a** 20 min **b** 9:32 a.m. **c** Lorne
d 3:00 p.m. **e** Kennett River

3 **a** 1 (at 9:05 a.m.)
b NC114 at 06:30; NC118 at 11:05; NC134 at 15:50.
c 15:50 on Saturday (NC134)
d 20:25 **e** 1 h 50 min

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4 **a** 1005, 1 h 25 min **b** NA114 at 1040
c 1005 **d** NA038 at 1005
e NA114 takes 5 minutes longer. Smaller plane, different flight path

5 **a** **i** 5:32 a.m. **ii** 2 h 40 min
b 8 min **c** 1 h 5 min
d book a ticket, 2:22 p.m.
e 5:13 p.m. or 7:35 p.m., because it's a 'fast train' that doesn't stop at every station.

Exercise 11.04

1 **a** 11:30 a.m. **b** 9:00 a.m.
c 8:00 a.m. **d** 7:00 p.m.

2 **a** 7:00 a.m. **b** 11:00 a.m.
c 4:30 p.m. **d** 9:00 p.m.

3 **a** 4 h **b** 1 h
c 8 h **d** 8 h

4 **a** 4:30 a.m. **b** 7:30 p.m. **c** 11:30 a.m.

5 C

6 **a** 6:00 p.m. **b** 6:00 p.m. **c** 5:30 p.m.
d 4:00 p.m. **e** 5:30 p.m. **f** 6:00 p.m.

7 **a** 10:30 a.m. **b** 10:30 a.m. **c** 9:00 a.m.
d 7:30 a.m. **e** 10:00 a.m. **f** 9:30 a.m.

8 **a** 12 midnight **b** directly opposite sides

9 2:30 p.m.

10 A **11** 13 hours

12 **a** 9:30 a.m. Tuesday **b** 12:30 a.m. Tuesday
c 11:30 p.m. Monday

13 **a** 5:40 p.m. **b** 9:00 p.m.

14 1:55 p.m.

15 4:05 p.m. **16** 1:00 a.m. Friday

17 **a** Tuesday **b** 1:00 p.m. Wednesday

18 **a** 11 h **b** No, the next day

Sample HSC problem

a 82° **b** 18 h 25 min
c 9:10 p.m. Wednesday

Test yourself 11

1 **a** *A* (0°, 70°W), *B* (0°, 40°W), *C* (55°S, 70°W), *D* (55°S, 40°W)

2 **a** 147°E **b** 54° **c** 25°

3 (52°N, 5°E)

4 **a** 2:36 p.m. **b** 5:05 a.m.
c 10:18 p.m. **d** 12:27 a.m.

5 **a** 0523 **b** 1632
c 2105 **d** 1255

6 **a** 8:55 a.m. **b** 6:52 a.m.
c 1027 **d** 1105

7 **a** 1 h 30 min
b 2 (each bus does every second trip)
c 1 p.m. (on the 1220 bus)
d 45 minutes
e Riverside bus at 10:45 a.m.; Suncorp bus at 12:32 p.m.; Southbank bus at 2:30 p.m.; Maritime Museum bus at 4:04 p.m. reaching City Lookout at 4:10 p.m.

8 **a** 5:00 p.m. **b** 7:30 a.m.

9 **a** 6:00 p.m. **b** 6:30 a.m.
c 2:30 a.m.

10 **a** Wednesday
b 5:30 a.m. Wednesday

11 3:00 a.m. Monday

Chapter 12

SkillCheck

1 **a** 2 **b** 1.74 **c** 2.94
d 0.04 **e** 0.07 **f** 0

2 **a**

x	0	1	2	3	4	5
y	4	3.7	3.4	3.1	2.8	2.5

b – 0.3

3

	i	ii	iii	iv	v
a	5.5	5	4, 5	9	4
b	7.6	9	12	14	10.5

4 **a** 28.4 **b** 31 326.7 **c** 37.6

5 **a** 25 000 m **b** 2.463 km **c** 3.75 h
d 5h 12 min **e** 7 h 20 min **f** $5.\dot{5}$ m/s
g $30.\dot{5}$ m/s **h** 12.6 km/h

6 20 160 km **7** 65.3 km/h

8 **a** $5.01 **b** $2525.10 **c** $37.33

Exercise 12.01

1 B **2** B

3 **a** Mabel **b** 6.58 L

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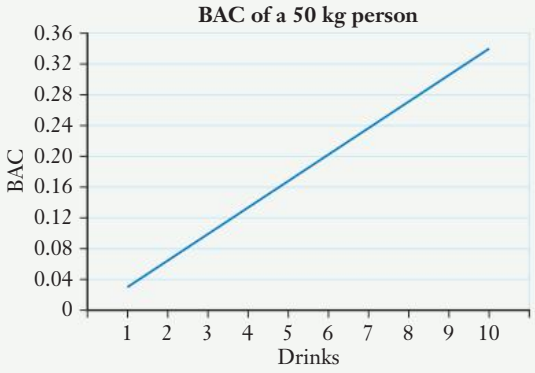
4 C

5 **a** 2 **b** 3 **c** 73 kg
d 0.25 **e** graph



6 **a** heavy drinker
b **i** 0.02 **ii** 0.017
c 0.07 **d** about 0.028
e about 0.01
f A heavy person gets rid of the alcohol in their blood system more quickly.

7 **a** 0.21 **b** 10 h 30 min
c 10:30 a.m. the next day

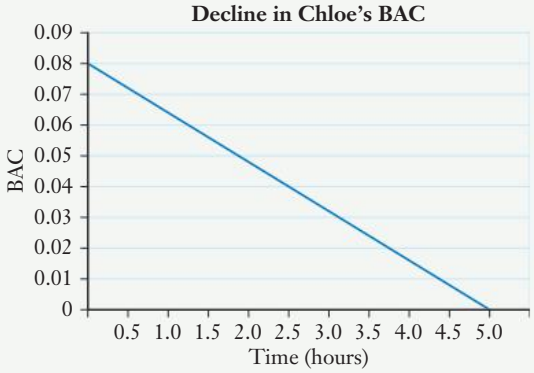
8 **a** 0.18 **b** 12 h
c 2 p.m.

9 0.012 per hour

10 **a**

0	0.08
0.5	0.072
1	0.064
1.5	0.056
2	0.048
2.5	0.04
3	0.032
3.5	0.024
4	0.016
4.5	0.008
5	0

b



c **i** about 1:45 a.m. **ii** 5 a.m.

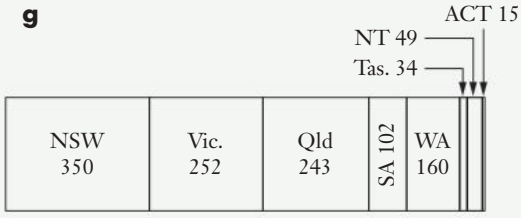
d 0.016, rate at which BAC is decreasing per hour.

Exercise 12.02

1 **a** $\frac{28}{55}$ **b** 62% **c** 49%

2 **a** **i** 1150 **ii** 1205
b 1177.5
c ACT, has the smallest population
d Tasmania
e SA, WA
f Numbers did not change much between 2014 and 2015, except for NSW which showed a large increase.

g



Driver deaths, 2015

3 **a** 55.4% × 18 764 ≈ 10 395
b 5592 **c** 4597

4 **a** **i** 180° **ii** 68° **iii** 7°
b **i** 84 **ii** 62 **c** 227

5 **a** 1240 **b** 21.8% **c** 0.7%
d 2014 **e** 2015

6 **a** Drivers in 2011 **b** 61 742 **c** 2602.7
d 117 **e** 3567 **f** 434

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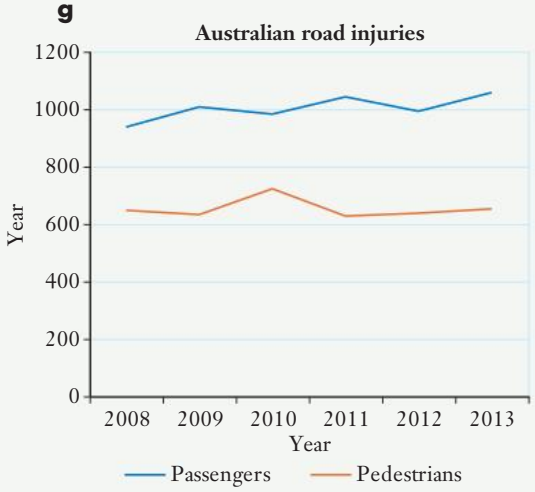
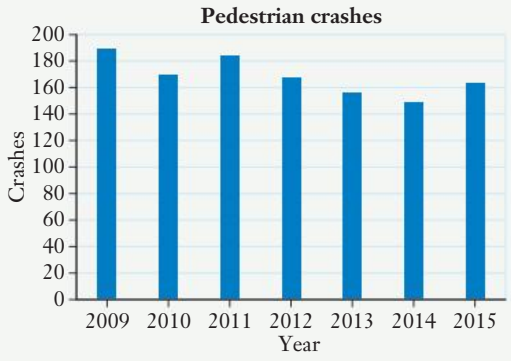
g



7 a i 526.6 **ii** 515 **iii** 186 **iv** 53

b



c Decline in number of crashes perhaps due to road safety campaigns, more police presence, large fines for speeding, more cameras.

d Cars/vehicles are more likely to hit stationary objects such as trees, poles, walls than other vehicles.

8 a 850 **b** 350 **c** 1200

d Number of deaths generally decreasing over the years, male deaths are more than double female deaths (about 2.5 times).

9 a 4 p.m.–5:59 p.m.; peak time when people leaving work or picking up children from school.

b 2 a.m.–3:59 a.m.; quietest time of day when fewest vehicles on roads (people still sleeping).

c i 2648 **ii** 1522.7 **ii** 1694.5

d i The day starts off with low casualties for 6 hours, at its minimum at 2 to 3:59 a.m. After 6 a.m., the number of casualties begins to rise, reaching a peak at 8 to 9:59 a.m. when people are going to work and school. It levels off between 10 a.m. and 2 p.m., then starts to increase as the afternoon peak time begins, at its maximum at 4 to 5:59 p.m. Then in the final 6 hours of the day until midnight, the number of casualties decreases.

ii Yes, because working hours and school hours have not changed. The numbers would be different on a weekend though.

10 a 390 **b** 346

c 70.4% **d** 19.04%

e Male drivers are more than 2.5 the number of female drivers; males drive more aggressively and faster than females.

f Younger drivers (especially aged 17–20) are more likely to have accidents involving speed, the figures dip for ages 26–29, then they increase again for ages 30–39, then start to decrease again.

Exercise 12.03

1 C **2** B

3 a 78 km/h **b** 13 h 8 min **c** 247 km

4 a 8 h 54 min **b** 2 h 48 min **c** 7 h 43 min

5 a $3\frac{1}{2}$ h **b** 14 km/h

6 a 34.2 km/h **b** 333 450 m

7 a 1.1 m **b** 1.7 m **c** 2.2 m **d** 2.8 m

8 a 3 s **b** 1472 m

9 a 76.5 m/s **b** 12.6 min

10 a 17.5 min **b** 248 km/h

11 41 min

12 104.2 m

13 62.1 km/h

Exercise 12.04

1 A **2** 197.7 m

3 5.8 m

4 a A = 13.9 B = 12.6 C = 28.3 D = 106.5

b i 299% **ii** 217%

c Yes. It takes 106.5 m to stop.

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5 **a** 7.9 m **b** 11.1 m
c Both would stop in time. Hanna in 30.1 m and Fran in 19 m.

6 **a** 44.4 m **b** 17.7 m
c Sanjay (52.3 m)

7 **a** $k = 0.0097$ **b** 106.9 m **c** 88 km/h
d **i** 97.6 m **ii** 153.6 m

8 **a** $k = 0.00678$ **b** 61.1 m
c 153.5 m

9 **a** 33.7 m **b** 102 m **c** 345 m **d** 238 m

10 **a** 1.88 s **b** 99 m

11 **a** 181 **b** 145 **c** 153 **d** 126

12 **a** 0.007 46 **b** 90.3 m **c** 151 m

13 **a** 5500 m **b** 32 500 m **c** 46 500 m

14 **a** 160 m
b **i** 108 m **ii** 133 m **iii** 190 m

c

road	reaction time	stopping distance
dry	1	78
dry	2	105
dry	4	160
wet	1	105
wet	2	135
wet	4	190

d You can see exact values in a table but a graph gives a good visual overall picture of the situation.

Sample HSC problem

a 0.15 **b** 4 **c** 10

d 0.03, rate at which BAC is increasing with each drink consumed.

e 16 h

Test yourself 12

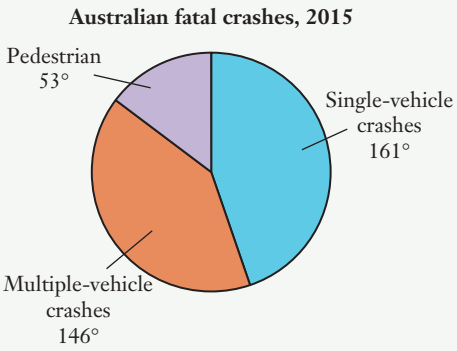
1 **a** 0.073 **b** 4 h 52 min **c** 0.024

2 **a** **i** 0 **ii** 0.07 **iii** 0.17
b 9 h 27 min

3 **a** Heavier people have more blood and water in their bodies to dilute alcohol.
b Females tend to be lighter than males.

4 **a** 446.25 **b** 46
c 76.4% **d** 49.8%
e More drivers are killed than passengers. There are more drivers than passengers in cars overall (many cars have no passengers), drivers sit in the car where they are more likely to be injured
f 157.5
g Generally decreasing (despite increasing populations). Better road safety campaigns, more policing, safer cars and roads.

5 **a** Single-vehicle **b** 14.8% **c** True
d **i** 2010 **ii** 2009
e 6.0%
f



6 **a** 73 km/h **b** 5h 55 min
c 364 km

7 935 m

8 No, he travels 21.7 m before he applies the brakes so his stopping distance is 21.7 m + 24 m = 45.7 m.

9 **a** same
b increases as reaction time increases
c A = 23.7 B = 34.8 C = 57
d No, he needs 57m to stop.
e Drive slower than the speed limit, get someone else to drive you.

10 **a** about 38 m **b** 10 m
c 100 km/h **d** 50 km/h
e It takes much longer to stop on a wet road. If the speed is double, the distance is not. For example, at 100 km/h on a wet road it takes 4 times as long to stop as it does at 50 km/h.

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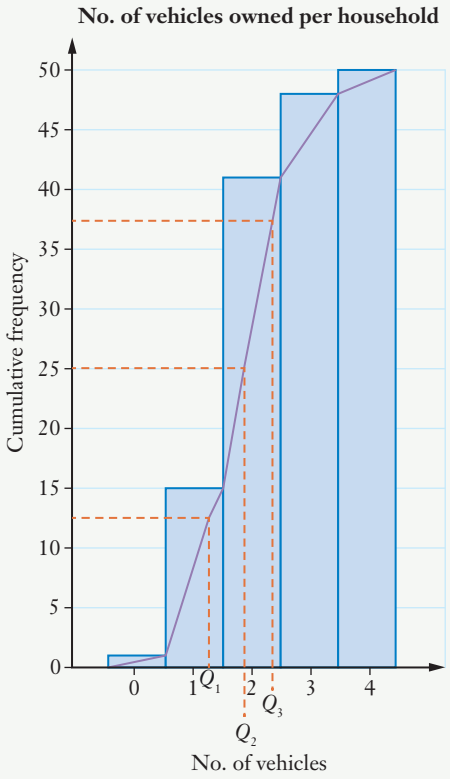
Practice paper 4

1 A **2** B **3** B
4 A **5** B **6** A
7 A **8** D **9** D
10 C

11 **a** **i** 8:55 a.m., 1:55 p.m. **ii** 3 h 20 min
iii NC511 on Monday night
b **i** 1, 15, 41, 48, 50 **ii** 1.9 **iii** 4
iv



v 2 – 1 = 1

12 **a** **i** 0.065 **ii** BAC would have been lower
b **i** 23 m **ii** 0.0029 **iii** 23.1 m
c **i** 3 and 5 **ii** 6 – 3 = 3 **iii** 2.57
d **i** 10% **ii** 20.6

13 **a** **i** 70 **ii** 15 **iii** 31
b **i** 788 **ii** 17.3%
iii More than double the number of male speeding drivers than female, due to more male drivers on the road, more males speeding (risky behaviour).
c **i** Manila **ii** Monday
iii 9 a.m. Tuesday

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GLOSSARY AND INDEX

12-hour time: Time of day written in the usual way using a.m. or p.m. and the hours 1 to 12, for example, 9:27 p.m. *See also* **24-hour time**. (p. 477)

24-hour time: Time of day written using 4 digits (instead of a.m. or p.m.) and the hours 0 to 23. For example, 1745 is the 24-hour time for 5:45 p.m. *See also* **12-hour time**. (p. 477)

absolute error: The maximum possible error for a measurement, equal to $\pm\frac{1}{2}$ of the smallest unit marked on the measurement scale. (p. 186). *See also* **limits of accuracy**.

allowable tax deduction: A part of a person's yearly income that is not taxed, such as work-related expenses or donations to charities. All deductions are subtracted from yearly income to determine **taxable income**. (p. 113)

allowance, worker's: Money paid to a worker for expenses incurred as part of his or her job (for example, for travel, for special clothing, or for working in isolated or dangerous areas). (p. 99)

annual leave loading: Extra payment to a worker based on a percentage (usually 17.5%) of 4 weeks annual leave. (p. 99)

arc: Part of the circumference of a circle. (p. 199)

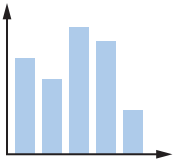
at least: A number equal to or greater than the given value, for example, 'at least 2' means 2 or more. (p. 145)

Australian Central Standard Time (ACST): Standard time zone (UTC+9.5) for central Australia: the Northern Territory and South Australia. (p. 485)

Australian Eastern Standard Time (AEST): Standard time zone (UTC+10) for eastern Australia: NSW, Queensland, Victoria, ACT and Tasmania. (p. 486)

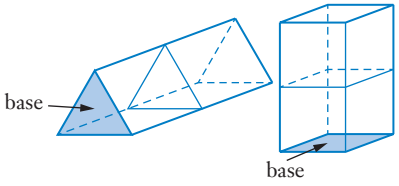
Australian Western Standard Time (AWST): Standard time zone (UTC+8) for Western Australia. (p. 487)

bar chart (or **column graph**): A graph consisting of vertical bars of equal width. (p. 5)



base (in index notation): A number being raised to a power. For example, in 2^5, the base is 2. (p. 61)

base (of a prism): One of the parallel end faces of a prism. (p. 217)



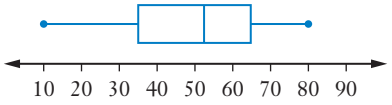
bias: In statistics, an unwanted influence that stops a sample from being representative of a population. (p. 21)

biased sample: A sample that is not truly representative of the population and affects the quality or reliability of a statistical study. (p. 21)

blood alcohol content (BAC): The concentration of alcohol in a person's blood, measured in g/100 mL. (p. 503)

bonus: Extra pay for achieving a high quality or volume of work, such as meeting an important quota, goal or deadline. (p. 99)

box plot: A diagram that displays the quartiles of a set of data as a box and the extremes as whiskers. (p. 439)



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braking distance: The distance travelled between when a driver applies the brakes and when the vehicle stops completely. (p. 524). *See also* **reaction distance** and **stopping distance**.

budget: A plan for managing money. (p. 275)

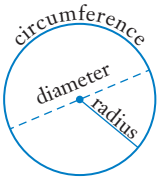
calorie: An older, non-metric unit of energy, equal to 4.184 kilojoules (kJ). (p. 268)

capacity: Maximum volume of liquid that can be held by a container, usually measured in millilitres (mL), litres (L) or kilolitres (kL). (pp. 216, 250)

categorical data: Information or data represented as a category rather than as a number (for example, the makes of cars, or the colour of eyes). Differs from **numerical data**. (p. 15)

census: Collection of information about every member of a population. (p. 18)

circumference: The perimeter of a circle. $C = \pi d$ or $C = 2\pi r$, where C is the circumference, π is pi (3.141 59…), d is the **diameter** and r is the **radius**. (p. 197)

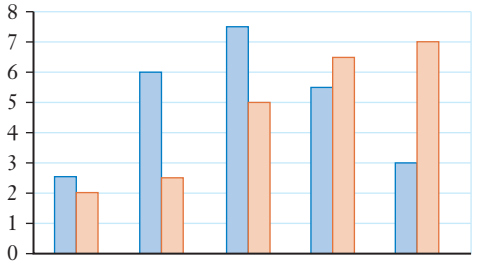


class centre: The centre of a class interval. For example, the class centre of the class interval 10–19 is 14.5. (p. 36)

class interval: In statistics, when there are many data scores, they may be grouped into class intervals. For example, ages of people may be grouped into class intervals of 1–10, 11–20, 21–30, and so on. (pp. 36)

cluster: A group of data scores that are bunched or close together. (p. 45, 453)

clustered bar chart: A bar chart that compares the data of two or more categories. (p. 8)



commission: The earnings of a sales person or agent; usually a percentage of the value of items sold. (p. 95)

complementary event: All the outcomes that are not the event. For example, the complementary event to rolling a 1 on a die is rolling a number that is not 1. (p. 151)

compounded: The addition of interest to the principal sum of a loan or deposit, or in other words, interest on interest. (p. 335)

compound interest: Interest paid on the principal invested as well as on any accumulated interest. Differs from **simple interest**. (p. 335)

compounding period: How often interest is calculated (for example, monthly, quarterly or yearly). (p. 336)

comprehensive insurance: Motor insurance that covers all damage to vehicles and property, including your own, in an accident in which you are at fault. (p. 357). *See also* **CTP insurance** or **Third Party Property insurance**.

Compulsory Third Party (CTP) insurance: Motor insurance that covers personal injury or death to another person ('third party') in an accident in which you are at fault. Also called '**green slip**' because of the insurance certificate is green. (p. 357). *See also* **comprehensive insurance** or **Third Party Property insurance**.

constant: A value that does not change. (p. 525). *See also* **variable**.

constant of variation (or **constant of proportionality**)**:** The constant in a variation equation. For example, if y varies as x, the equation is $y = kx$ and the constant of variation is k. (p. 308)

continuous data: Numerical (quantitative) data that can be measured on a smooth scale of values (without 'gaps'), such as the heights of people. (p. 16)

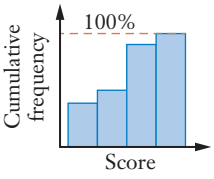
conversion graph: A graph that is used to convert between different units, such as between metric and imperial units of measurement, or between currencies in foreign currency exchange. (p. 313)

Coordinated Universal Time (UTC): *See* **UTC**. (p. 485)

coordinates: A pair of points (e.g. in x and y planes) that describe the location of a point on a number plane, map or involving **latitude** and **longitude**. (p. 468)

cumulative frequency: A running total of frequencies. (p. 434)

cumulative frequency histogram: A histogram in which the height of each column represents the cumulative frequency at each score. (p. 434)



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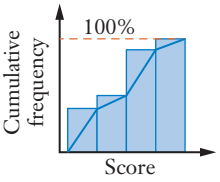
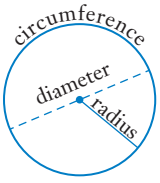
cumulative frequency polygon: A line graph formed by joining the ends of the tops of the columns of the cumulative frequency histogram. Also called an **ogive**. (p. 434)



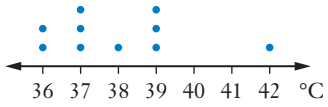
data: Observations or facts which, when collected, organised and evaluated, become information. (p. 15)
data allowance: *See* **included data**.
data set: Collection or group of data scores. (p. 50)
daylight saving time: Scheme where clocks are turned forward an hour to take advantage of increased hours of daylight during summer. (p. 490)
deciles: Values that divide a data set into 10 equal parts when the scores are arranged in order. (p. 418). *See also* **percentiles** and **quartiles**.
deduction (tax): *See* **allowable tax deduction**. (p. 113).
deductions (from pay): Amounts taken out of a person's gross pay (for example, union fees, superannuation, and income tax). (p. 108)
denominator: The number on the bottom of a fraction. For example, in $\frac{3}{5}$ the denominator is 5. (p. 63)
dependent variable: The variable in a function that depends on another variable for its value. For example, if $y = 3x + 5$, y is the dependent variable because its value depends on the value of x. Differs from an **independent variable**. (p. 300)
depreciation: Loss in value of an item or asset over time. (p. 343)
diameter: The length of the interval passing through the centre of a circle and joining two points on the **circumference** of the circle. The diameter is double the **radius**. (p. 197)



direct linear variation (or **direct proportion**): The relationship between two variables (say x and y) is represented by an equation of the form $y = kx$, where k is the constant of variation. (p. 308)
discrete data: Quantitative (numerical) data that can be counted and whose values are separate and distinct, such as the numbers of pets owned, or the numbers of people in families. (p. 16)
discretionary spending: Money spent on items other than necessities. (p. 275)
distribution (or **frequency distribution**): The way the scores of a data set are arranged, especially when graphed. (p. 452). *See also* **shape of a distribution**.

divided bar graph: A graph representing the parts of a whole by a rectangle divided into proportionately sized sections. Displays the same kind of data as a **sector graph**. (p. 5)

dosage: A prescribed amount of medicine to be taken. (p. 69)
dot plot: A graph that uses dots to show frequencies of data scores (for example, the temperatures, in °C, of 10 hospital patients). (p. 45)



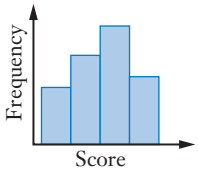
Different from a **stem-and-leaf plot**. (p. 45)
double time: Wages paid at twice the normal rate (for example, for working on a Sunday or a public holiday). (p. 91)

energy rating label (ERL): A label on an electrical appliance showing the star rating of how energy-efficient it is (the more stars, the better), including the estimated number of kilowatt hours used per year (the lower, the better). (p. 262)
equally likely: Having an equal chance of occurring. (p. 138)
equation: A mathematical statement that two quantities are equal. An equation contains an equals sign (for example, $4x - 5 = 11$). (p. 74)
Equator: The 0° parallel of latitude, the great circle running around the middle of the Earth. The latitude of any point on Earth's surface is measured north or south of the Equator. (p. 470)
event: In probability, a result involving one or more outcomes. For example, when rolling a die, the event 'rolling an even number' contains the three outcomes {2, 4, 6}. (p. 138)
excess: Amount the person taking out the insurance agrees to pay upfront in the event of an accident before the company pays the rest. (p. 358)
expense: The cost of spending. (pp. 99, 275)
experimental probability: An estimate of the theoretical or calculated probability; the relative frequency of an event in repeated trials of an experiment. (p. 155)
extremes (upper and lower): The highest and lowest scores in a set of data. (p. 439)

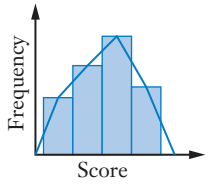
fatality: An accident resulting in death, or a person who dies in an accident. (p. 514)

ISBN 9780170413565

field diagram: A diagram produced from a land survey. (p. 210)
final amount: The value of an investment at the end of the investment term. (p. 336)
five-number summary: Lower extreme, lower quartile, median, upper quartile and upper extreme of a data set. These five numbers are used to draw a **box plot**. (p. 439)
fixed spending: Money spent on necessities such as food, clothes, fuel and household bills. (p. 275)
flat rate interest: Simple interest is charged on the principal borrowed for the term of the loan. (p. 328) *See also* **reducing balance loan**.
formula: A rule written as an algebraic equation, using variables. For example, the formula for the area of a triangle is $A = \frac{1}{2}bh$. (p. 68)
fortnightly: Every two weeks. (p. 92)
frequency: The number of times a score or group of scores occurs in a data set. (p. 37)
frequency distribution: *See* **distribution**.
frequency histogram: A histogram in which the height of each column represents the frequency of a single score or group of scores. (p. 36)



frequency polygon: A line graph formed by joining the midpoints of the tops of the columns of a frequency histogram. (p. 36)



fuel consumption: The rate at which fuel is used by a vehicle, usually measured in L/100 km. (p. 377)
function: The relationship between two variables (for example, $y = 3x - 5$). (p. 291)
future value: The value of an asset or money at a specific date. (p. 336)

giga-: (symbol G) A prefix meaning 'one billion'. (p. 185)
gigalitre: (GL) 1000 ML or 1 000 000 000 L. (p. 178)
gigawatt: (GW) 1000 MW or 1 000 000 000 W. (p. 185)
government allowance: Money paid by the government to support individuals for specific purposes (for example, to support the aged, unemployed, disabled, students and parents). (p. 102)
goods and services tax (GST): The tax a consumer pays on any purchased item or service (for example buying a car, or hiring a painter). (p. 119)
gradient: (symbol m) Slope of a line.
$\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$. (p. 291)
great circle: A slice through the centre of a sphere; its radius is the same as that of the sphere. (p. 470)
green slip: *See* Compulsory Third Party (CTP) **insurance**.
Greenwich Mean Time (GMT): *See* **UTC**.
Greenwich meridian: *See* **prime meridian**.
gross income: Earnings before deductions have been made. (p. 113)
gross pay (or **gross wage):** A person's pay before tax is deducted. Different from **net pay**. (p. 108)

halogen light: Light globe used in downlights. (p. 262)
hectare: A land measure equal to the area of a square 100 m by 100 m. 1 ha = 10 000 m^2. (p. 202)
hemisphere: Half a sphere, or half of the Earth, for example, the southern hemisphere. (p. 228)
histogram: *See* **frequency histogram**.

incandescent light globe: An electric light with a wire filament heated to such a high temperature that it glows with visible light (incandescence). (p. 262)
income: Money that is earned or gained (usually regularly). (p. 95, 275)
income tax: A tax on a person's income, paid to the government. (p. 113)
independent variable: A variable in a function whose values do not depend on any other variable. For example, if $A = \pi r^2$, r is the independent variable. This differs from the **dependent variable**. (p. 300)
inflation: A general increase in prices for goods and services. (p. 338)
insurance: A scheme in which you pay an insurance company regularly in return for being paid a larger amount if an accident occurs. (p. 172)
interest: Money earned on an investment, or money paid to a financial institution for borrowing. (p. 328)
International Date Line (IDL): The imaginary line that runs through the Pacific Ocean and is approximately the 180° meridian of longitude. A day is either gained or lost when this line is crossed. (p. 487)
interquartile range (IQR): The difference between the upper quartile and lower quartile of a data set ($Q_3 - Q_1$). It is a measure of the spread of the data. (p. 426)

ISBN 9780170413565

joule: (J) A unit of energy. (p. 268)

kilo-: (symbol k) A prefix meaning 'one thousand'. For example, a kilolitre is 1000 litres. (p. 182)
kilojoule: (kJ) A unit of energy equal to 1000 joules. (p. 185)
kilowatt: (kW) A unit of power equal to 1000 watts. (p. 254)
kilowatt-hour: (kWh) A unit of electrical energy equivalent to that used by one kilowatt of power in one hour. (p. 254)

land survey: *See* **offset survey**.
latitude: The angular distance north or south of the Equator of a point on Earth's surface; the size of the angle made between the point and the Equator at Earth's centre. (p. 469)
LHS: Left-hand side (of an equation). (p. 75)
likely: Probably will happen; having a probability above 0.5. (p. 138)
like terms: Terms that have exactly the same variable part, even though the numeral part may differ. (p. 61)
limits of accuracy: The limits of a measured value, for example, the limits of accuracy of a measured height of 171 cm are 170.5 cm to 171.5, meaning that the actual height lies within this range. The limits are found by adding to and subtracting from the measured value $\frac{1}{2}$ of the smallest unit marked on the measurement scale. (p. 186). *See also* **absolute error**.
linear: A word used to describe something to do with a line. (p. 291)
linear function: A function of the form $y = mx + c$, whose graph is a straight line. (p. 291)
linear modelling: Using a linear function to approximate a real-life situation. (p. 300)
longitude: The angular distance east or west of the prime meridian of a point on Earth's surface; the size of the angle made between the point and the prime meridian at Earth's centre. (p. 469)

mean: The average of a set of scores.

$$\text{mean (or } \bar{x}) = \frac{\text{sum of scores}}{\text{number of scores}} = \frac{\Sigma x}{n} = \frac{\Sigma fx}{\Sigma f}. \text{ (p. 402)}$$

measure of central tendency (or **measure of location):** A statistical value, such as the mean, median or mode, which describes the centre or average of a set of data. (pp. 402, 410)
measure of spread: A statistical value, such as the range, interquartile range or standard deviation, which describes the spread of a set of data. (p. 426)
median: The middle score of a data set when scores are arranged in ascending order. If there are two middle scores, the median is the average of the two. (p. 402)
median class: The class interval that contains the median score. (p. 409)
Medicare levy: A tax to cover the costs of the public health system, calculated as a percentage (usually 2%) of a person's income. (p. 114)
mega-: (symbol M) A prefix meaning 'one million'. For example, a megatonne is 1 000 000 tonnes. (p. 182)
megajoule: (MJ) 1000 kJ or 1 000 000 J. (p. 268)
megalitre: (ML) 1000 kL or 1 000 000 L. (p. 181)
megatonne: (Mt) 1 000 000 tonnes. (p. 181)
megawatt: (MW) 1000 kW or 1 000 000 W. (p. 260)
meridian of longitude: Great semicircle running down Earth's surface from the North pole to the South pole, measured east or west of the prime meridian (0° longitude). (p. 470)
micro-: (symbol μ) A prefix meaning 'one millionth'. For example, a micrometre is one-millionth of a metre. (p. 182)
microgram: (μg) One-millionth of a gram. 1 g = 1 000 000 μg. (p. 185)
middle quartile: The median, and separates the two middle quarters. (p. 417)
milligram: (mg) One-thousandth of a gram. 1 g = 1000 mg. (p. 181)
milliwatt: (mW) One thousandth of a watt. (p. 260)
modal class: The class interval with the highest frequency. (p. 409)
mode: The most common or frequent score(s) in a set of data. (p. 402)
modelling: Using mathematics to describe a real-life pattern or relationship. (p. 300)
more than: A number greater than the given value. For example, 'more than 2' means any number above 2 (not including 2). (p. 6)
multi-stage event: An event that involves two or more outcomes occurring together. For example, a two-stage event that involves tossing a coin and rolling a die together could be tossing tails and rolling an odd number. (p. 144)

net pay (or **net wage**)**:** A person's pay after tax has been deducted. Differs from **gross pay**. (p. 108)
nominal data: Categorical data that cannot be ordered, for example, colour of eyes. (p. 16)

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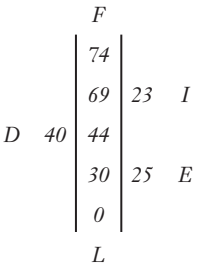
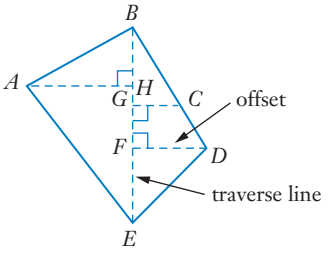
notebook entry: A diagram of measurements drawn by a surveyor in an **offset survey**. (p. 211)



numerical data (or **quantitative data):** Data that involve numbers, such as heights, or the number of DVDs owned. (p. 15)

offset survey (or **traverse survey):** A method of measuring lengths in an irregularly-shaped field, using offsets to each irregularly-shaped field, using offsets to each corner of the field from a traverse line. (p. 210)

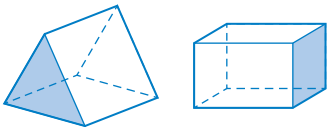


ogive: *See* **cumulative frequency polygon**. (p. 434)
on-road costs: After purchasing a motor vehicle, the additional costs required before it can be used on the road, such as registration, stamp duty and CTP insurance. (p. 370)
ordinal data: Categorical data that can be ordered, for example, level of swimming class. (p. 16)
outcome: In probability, the result of a situation or experiment. For example, when rolling a die, one possible outcome is 4. (p. 138)
outlier: An extreme (high or low) score in a data set that is much different than the other scores, either less than $Q_1 - 1.5 \times \text{IQR}$ or greater than $Q_3 + 1.5 \times \text{IQR}$. (p. 429)
overtime: Time worked beyond usual working hours, usually paid at a higher rate. (p. 91)

parallel of latitude: Small circle running around the Earth's surface, parallel to and measured north or south of the Equator (0° latitude). (p. 470)
Pareto chart: A combined column and line graph, where the columns are displayed in descending order and the line shows the cumulative frequency of the same data. Pareto charts are often used in quality control for businesses. (p. 28)
Pay As You Go (PAYG) tax: Income tax deducted from pay in instalments each payday by employer. (p. 117)
peak: A high point of a distribution, and the highest peak shows the mode. (p. 452)
per annum (p.a.): Per year. (p. 68)
percentage error: The maximum possible error for a measurement, expressed as a percentage.

$$\text{percentage error} = \frac{\text{absolute error}}{\text{measurement}} \times 100\%.$$ (p. 187)

percentiles: Values that divide a data set into 100 equal parts when the scores are arranged in order. (p. 419). *See also* **deciles** and **quartiles**.
pie chart: *See* **sector graph**.
piecework: Earnings based on the number of items processed, made or delivered, paid at a rate per item (rather than per hour). (p. 95)
population: All of the items under investigation. (pp. 18, 447). *See also* **sample**.
power (index): The number of times a base is multiplied by itself. In 2^5, the power is 5. (p. 61)
precision: The smallest unit on the scale of a measuring device. For example, the precision of a ruler may be one millimetre. (p. 186)
premium: The regular cost of an insurance policy. (p. 357)
present value: The current value of a sum of money, in contrast to some future value it will have when it has been invested at compound interest. (p. 336)
prime meridian (or **Greenwich meridian):** The 0° meridian of longitude that passes through Greenwich in London. The longitude of any point on Earth's surface is measured east or west of this meridian. (p. 471)
principal: The original amount of money invested or borrowed. (p. 328)
prism: A solid with flat faces and a uniform cross-section. (p. 217)



probability: A measure of how likely an event E is to occur, written $P(\text{E})$. Its value ranges from 0 to 1. (p. 138)
probability tree diagram: A probability tree diagram has the probabilities of each stage shown on the branches. (p. 168) *See also* **tree diagram**.
proportional to: In the same ratio as. If y is proportional to x, we say $y = kx$ and x and y have a direct linear variation. (p. 308). *See also* **direct linear variation**.

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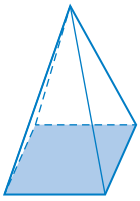
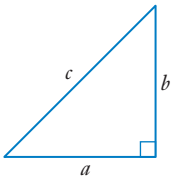
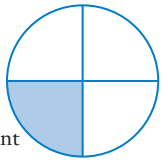
pyramid: A solid with a polygon as a base (for example, a rectangle, or a triangle) and triangular side faces that meet at a point (called the apex). (p. 83)



Pythagoras' theorem: In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the two shorter sides. $c^2 = a^2 + b^2$ (p. 198)



quadrant: Quarter of a circle. (p. 199)

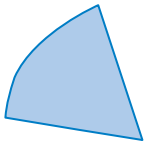


quantitative data: *See* **numerical data**.
quarterly: Every 3 months. (p. 244)
quartiles (upper, middle and lower): The upper quartile is the 3rd **quartile** (Q_3) that cuts off the top 25% of scores in a data set; , the middle quartile (Q_2) is the median, and separates the two middle quarters; the lower quartile is the 1st quartile (Q_1) that cuts off the bottom 25% of scores. (p. 417) *See also* **deciles** and **percentiles**.

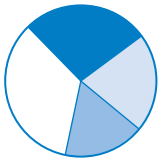
radius (plural: radii): The length of the interval joining the centre of a circle to the circumference. The radius is half of the **diameter**. (p. 197)
random sample: A sample for which every member of a population has an equal chance of selection. (p. 19)
range: The difference between the highest score and the lowest score in a set of data. (p. 426)
reaction distance: The distance travelled during a driver's **reaction time**. (p. 524). *See also* **braking distance** and **stopping distance**.
reaction time: The time between when a driver senses the need to stop and when the brakes are applied. (p. 524)
registration: *See* **vehicle registration**.
relative frequency: The number of times an event or score occurs, written as a fraction of the total number of events or scores. (p. 155)
retainer: A fixed amount paid to a salesperson before commission is added. (p. 96)
RHS: Right-hand side (of an equation). (p. 75)

rise: The vertical change in position between two points on a line; the number of units going up. Used with the **run** to calculate **gradient**. (p. 291)
royalties: Income earned by recording artists and authors, based on the number of copies of their work that are sold. (p. 95)
run: The horizontal change in position between two points on a line; the number of units going to the right. Used with the **rise** to calculate **gradient**. (p. 291)

salary: Fixed earnings quoted as a yearly amount, but paid weekly, fortnightly or monthly. (p. 92). *See also* **wage**.
salvage value: The estimated value that the owner is paid when the item is sold at the end of its useful life. (p.)
sample: A group of items selected from a population. (pp. 18, 447)
sample size: The number of items in a **sample**. (p. 19)
sample space: In probability, the set of all the possible outcomes of a situation or experiment. (p. 138)
scientific notation: A way of writing very large or very small numbers. For example, $98\,000\,000 = 9.8 \times 10^7$. (p. 193)
sector: Part of a circle bounded by an arc and two radii. (p. 199)



sector graph (or pie chart): A graph representing the parts of a whole population using a circle divided into proportionately sized sectors. Displays the same kind of data as a **divided bar graph**. (p. 5)



self-selected sample: A sample in which people volunteer to be part of the sample, such as an SMS poll or website survey, so it is not really random. (p. 19)
sewerage: The system of removing waste water from a home or building. (p. 243)
shape of a distribution: The way the data in a frequency distribution is spread, can be symmetrical, positively skewed or negatively skewed. (p. 452)
significant figures: Meaningful digits in a numeral that tell 'how many'. For example, 98 000 000 has two significant figures: 9 and 8. (p. 190)
simple interest (or flat rate interest): Interest earned or charged only on the original amount of money (principal) invested or borrowed. Differs from **compound interest**. (p. 328)

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simplify: In algebra, simplifying makes an algebraic expression less complex or complicated, which mainly involves collecting like terms. (p. 61)

skewed: The shape of a statistical distribution when most of the data scores are either low (positively skewed) or high (negatively skewed). The tail indicates the direction of the skew. (p. 452)

Negatively skewed Positively skewed

small circle: A slice through a sphere that does not pass through the centre. It has a smaller radius than a **great circle**. (p. 470)

speed: A rate that compares distance travelled with time taken. Speed is often measured in kilometres per hour (km/h) or metres per second (m/s).

$$\text{average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$ (p. 521)

stamp duty: A tax paid to the Office of State Revenue when buying a vehicle, calculated on the market value of the vehicle. (p. 364)

standard deviation: (symbol σ) A statistical measure of the spread of a set of scores. (pp. 426, 445)

standard drink: A drink that contain 10 grams of alcohol, such as a small glass of beer or wine. (p. 504)

standard form notation: *See* **scientific notation**.

statistics: Collection, tabulation and displaying of data for the purpose of analysis. (p. 18)

stem-and-leaf plot: A 'number graph' that lists all the data scores, in groups. This stem-and-leaf plot shows 12 test scores, from 42 to 82. Different from a **dot plot**. (p. 45)

Stem	Leaf
4	2 5
5	0 2 8
6	6 7
7	3 5 7 7
8	2

stopping distance: The distance travelled between when a driver senses the need to stop and when the vehicle stops completely, equal to the sum of the **reaction distance** and **braking distance**. (p. 524)

straight-line method of depreciation: Method of depreciation in which an item's value decreases by the same amount each period. (p. 343)

stratified sample: A sample consisting of a percentage of items from each 'strata' or 'layer' of a population. For example, a stratified sample from a population of 35% children and 65% adults should contain 35% children and 65% adults. (p. 19)

subject (of a formula)**:** The variable on its own on the left side of the '=' sign of a formula, what the formula describes, for example, A (for area) in the formula $A = \frac{1}{2}bh$. (p. 82)

substitute: To replace a pronumeral with a number. (p. 78)

summary statistic: A calculated value that represents or summarises a set of data (for example, the mean or the standard deviation). (p. 402)

summer time: *See* **daylight saving time**.

superannuation: A retirement fund for employees. Every payday, an employee and his/her employer invest a part of the employee's pay into a fund in order to provide income for the employee during retirement. (p. 108)

survey (land): To measure lengths and angles on a field and perform calculations on these measurements. (p. 210)

survey (statistical): To gather information for statistical purposes. (p. 6)

symmetrical: A distribution is symmetrical if the data are balanced or evenly spread about the centre of the distribution, with the mean, median and mode being equal. (p. 452)

systematic sample: A sample chosen by using a set pattern (for example, choosing every 10th number in a phone book). (p. 19)

tax debt: The amount by which the amount of **PAYG tax** already paid is below the amount of tax due. This is owed by the taxpayer to the Australian Office of Taxation. (p. 117)

tax deduction: *See* **allowable tax deduction**.

tax refund: The amount by which the amount of **PAYG tax** already paid is above the amount of tax due; given back to the taxpayer. (p. 117)

tax return: A form completed at the end of a financial year, to account for income earned, allowable deductions and tax already paid. Used to calculate a **tax refund** or **tax debt**. (p. 117)

taxable income: The part of a person's income that is taxed, equal to yearly income minus allowable deductions. (p. 113)

Third Party Property insurance: Motor insurance that covers damage to another person's ('third party') vehicle or property in an accident in which you are at fault. It does not cover damage to your vehicle. (p. 357). *See also* **comprehensive insurance** or **CTP insurance**.

time-and-a-half: Wages paid at 1.5 times the normal rate (for example, for working on a Saturday). (p. 91)

time zone: A zone of the world in which the time is the same for all places. (p. 486)

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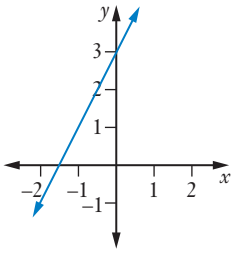
tonne: (t) A unit of mass equal to 1000 kg. (p. 181)
trapezoidal rule: Formula for finding the approximate area of an irregular-shaped block using the area of a trapezium. $A \approx \frac{h}{2}\left(d_f + d_l\right)$. (p. 211)
traverse survey: *See* **offset survey**.
tree diagram: A diagram for listing all the possible outcomes of a multi-stage experiment such as tossing three coins together. (p. 145)

unitary method: A way of calculating an amount by first finding one unit then multiplying. (p. 120)
UTC (Coordinated Universal Time) or **Greenwich Mean Time (GMT):** Local time at the prime meridian (0º longitude) time zone, from which other times in the world are measured. (p. 485)

value-added tax (VAT): A tax on goods or services used in some countries; similar to the GST in Australia. (p. 119)
variable: A pronumeral that can take a range of values (that is, it is not a **constant**). (p. 60)
vehicle registration: The yearly process of paying for and obtaining permission for using a vehicle on public roads. (p. 369)
vertical intercept (or *y*-intercept): The value at which a straight line graph cuts the vertical axis. For example, the vertical intercept of this graph is 3. (p. 301)



volume: The amount of space occupied by a solid, measured in cubic units. (p. 215)

wage: The amount earned by an employee for a set number of working hours, usually paid weekly. (p. 91). *See also* **overtime** and **salary**.
watt: A unit of power equal to one joule of energy per second. (p. 254)

***y*-intercept:** *See* **vertical intercept**.

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OVERVIEW

New Century Maths 11 & 12 has been rewritten and updated to cover the new *Mathematics Standard* HSC courses. ***Mathematics Standard 2*** caters to students heading towards an HSC exam, an ATAR and university studies. The syllabus describes two pathways for Mathematics Standard that start in Year 11. Even though both pathways share a common Year 11 course, students taking each pathway have specific learning needs, so we have published two levels of text for both Years 11 and 12.

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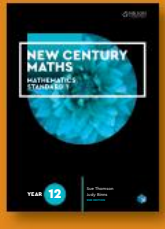

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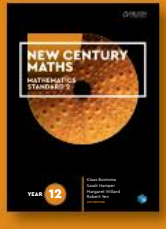

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FEATURES

- Each chapter begins with a chapter table of contents, chapter outcomes, a *Terminology* word list and a *SkillCheck* exercise to review prerequisite skills
- Graded exercises are linked to worked examples and include multiple-choice questions, HSC-style questions and realistic applications
- *Investigations* explore the syllabus in more detail through group work, discovery and modelling activities
- *Technology* promotes ICT in the classroom through the use of spreadsheets, graphics calculators and the Internet
- *Did you know?* sections contain interesting facts and applications of the mathematics learned in a chapter
- Each chapter ends with a *Sample HSC problem*, *Study Tip*, *Chapter Summary* with a mind map exercise and a *Test Yourself* revision set of exercises
- Practice papers after every three chapters revise students' skills and knowledge
- Syllabus grid and codes, answers and glossary/index included

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