NELSON VCE

FOUNDATION MATHEMATICS

UNITS 1 2

SUE THOMSON
JUDY BINNS
SERIES EDITOR: DIRK STRASSER



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NELSON
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Sue Thomson, Judy Binns

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Preface

Introducing ***VCE Foundation Mathematics***, a new book written by successful author team Sue Thomson and Judy Binns. This book covers the revised VCE study design (2015) in 17 short chapters written in plain English, with an emphasis on numeracy and literacy. Themes include health and medicine, creating geometrical designs, scale diagrams, house plans, travelling and measurement. The book also features three optional extension topics for students who wish to advance to the Further Mathematics course in Year 12: Interesting figures, So you've got a right angle and Fitting the data.

This book caters for students heading towards the workforce or further training after school, and teaches useful mathematical skills and applications for life. The author team are experts in teaching and writing for senior practical mathematics courses, and have published for both state and Australian curriculums.

In this book, teachers will find familiar features that strongly contribute to effective mathematics teaching and learning: clear worked examples, graded exercises, investigations, practical activities, language activities, and a combined glossary and index.

We have endeavoured to produce a practical text that captures the spirit of the course, providing relevant, current and interesting examples of mathematics being used in society and industry. This book is accompanied by a ***NelsonNet*** teacher website of worksheets and other teaching resources.

We thank our families for their continued support and patience, publisher **Robert Yen** for his guidance, and editors **Alan Stewart** and **Anna Pang** for swiftly transforming our manuscript into this fine text.

About the authors

Sue Thomson is an experienced teacher and educational leader. She was an examination writer, assessor, marker and curriculum writer for the NSW Board of Studies, and a councillor for the Australian Association of Mathematics Teachers. Sue is a prolific and successful author, well-known for writing and presenting for senior practical mathematics, especially thematic maths. Sue's interests are in language development, financial literacy and making mathematics accessible to all. With her husband, Ian Forster, she has written many successful books for NSW, Queensland and WA, including the ***Hands-On Maths in Practice*** and ***Access to General Mathematics*** series.

Judy Binns is a mathematics coordinator and experienced author who has taught in urban and rural schools. She has an interest in motivating students with learning difficulties, and has taught senior applied mathematics courses at her school. Judy often presents teacher workshops at local and state conferences. More recently, Judy and Sue have written ***New Century Maths 11–12 Mathematics General 1*** for NSW, and ***Nelson Senior Maths 11–12 Essentials*** for WA and the ACT.

Sue dedicates this book to the memory of her husband and co-author, **Ian Forster**.

Other titles in the Nelson VCE Maths series:

- General Mathematics Unit 1, Unit 2
- Further Mathematics Unit 3, Unit 4
- Mathematical Methods Unit 1, Unit 2, Unit 3, Unit 4
- Specialist Mathematics Units 1 and 2, Units 3 and 4

For more information, visit www.nelsonsecondary.com.au/vcemaths

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User's guide



At the beginning of each chapter

- A **Chapter problem** to be solved, a chapter table of contents, a list of chapter outcomes, and a **How are we ever going to use this?** list of applications.

Target heart rate formula:

$H_t = (H_{max} - H_{rest}) \times I + H_{rest}$

where H_{max} = maximum heart rate

H_{rest} = resting heart rate

I = exercise intensity as a decimal

In each chapter

- Important facts and formulas are highlighted in a **Summary** box.

EXERCISE 10.2

Energy in food

1 Calculate the energy content of each meal.

a Jane's breakfast contains a serving of muesli with milk, an apple and a banana.



EXAMPLE 3

10.2

- Graded exercises are linked to worked examples and include problem-solving and realistic applications.

Investigation What's your pulse?

You will need a watch and access to some stairs.

What you have to do

- Search on the Internet to watch a video that shows you how to take your pulse.
- Measure your resting heart rate in beats / minute.
- Exercise for 2 minutes by walking up and down the stairs.
- Measure your exercise pulse in beats / minute.
- Copy and complete the table for members of your group by measuring their pulse at 1, 2, 5 and 10 minutes after the step exercise.

- **Investigations** and **Practical activities** explore the study design in more detail, through group work, discovery and modelling activities.
- **Spreadsheets** promote ICT in the classroom.

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At the end of each chapter

- **Keyword activity** focuses on the mathematical language and terminology learned in the chapter.

Keyword activity

Match the terms in the left column with their meaning on the right.

	Term		Meaning
1	calories	a	A measure of the concentration of alcohol in a person's blood.
2	diastolic pressure	b	A unit for measuring electricity.
3	kilojoule	c	The rate at which our heart beats, usually expressed in beats/minute.
4	kilowatt	d	An old unit for measuring the energy in food.
5	substitute	e	The pressure in our arteries when the heart is beating.
6	systolic pressure	f	The modern unit for measuring energy in food.
7	pulse	g	To replace a pronumeral with a value in an algebraic expression or formula.
8	BAC	h	The pressure in our arteries when the heart is relaxed.

- **Solution to the chapter problem** revisits the problem introduced at the start of the chapter and presents a solution to the problem.

Chapter Problem

Brett is usually a very alert, safe driver. His reaction time is 0.16 seconds. However, when his blood alcohol content (BAC) is 0.05, his reaction time increases to 0.22 seconds.

Brett is driving at 100 km/h when he suddenly needs to brake. How much further does his car travel during his reaction time when his BAC is 0.05, compared to when he hasn't been drinking?

Solution

We can use the formula $D = \frac{5vt}{18}$ (from page 250) to calculate the distance a car travels during the driver's reaction time at different speeds and reaction times.

D = distance in metres

v = the car's speed in km/h

t = the driver's reaction time in seconds.

Brett is driving his car at a speed of 100 km/h.

When $t = 0.16$ Reaction distance $= \frac{5 \times 100 \times 0.16}{18}$

$= 4.44$ m correct to the nearest cm

When $t = 0.22$ Reaction distance $= \frac{5 \times 100 \times 0.22}{18}$

$= 6.11$ m correct to the nearest cm.

The car travels an additional $6.11 - 4.44 = 1.67$ m during Brett's reaction time when his BAC is 0.05.

At the end of the book

- A combined **glossary and index** is a useful reference of course terminology.

NelsonNet teacher website

The NelsonNet teacher website **www.nelsonnet.com.au** contains additional resources that can be downloaded from the ***NelsonNet*** teacher website.

- **Worksheets** that are write-in enabled PDFs
- **Spreadsheets**: ***Excel*** files
- **Chapter PDFs** of the textbook
- **Resource Finder**: search engine for NelsonNet resources

Note: Complimentary access to NelsonNet is only available to teachers who use this book as a core educational resource in their classroom. Contact your sales representative for information about access codes and conditions.



Chapter Problem

Brian bought an $11.25 box of fish and chips for lunch and he paid with a $50 note. The shop assistant gave him a $20 and a $5 note and a few coins for his change. Could the change be wrong?



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CHAPTER 1

WHAT'S THE SCORE?

1.1 Target practice
1.2 Playing darts
1.3 Negative scores
1.4 Order of operations
1.5 How much do I get?
1.6 Estimating costs
1.7 Rounding decimals
Keyword activity
Solution to the chapter problem

By the end of this chapter, you should know how to:

- calculate with numbers mentally, by pen-and-paper and calculator, including with negative numbers and decimals
- use integers and decimals to solve everyday problems, including those involving money
- use 'order of operations' to do mixed calculations
- estimate the total of a bill and how much change you should receive
- round answers to decimal places

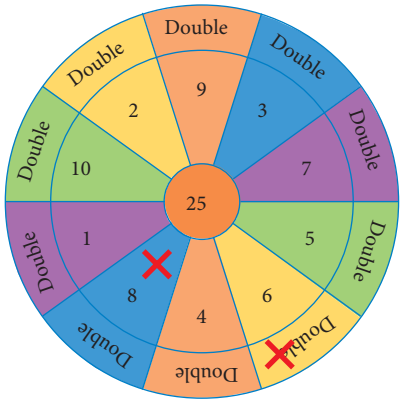
How are we ever going to use this?

- Keeping score in sporting events
- Shopping
- Earning wages
- Budgeting: for yourself or for a party

1.1 Target practice

Alan coaches a junior AFL team. He designed a ball-throwing competition to help his players improve their passes. Each player throws a football at the target two times each turn. The first player to reach 100 points is the winner.

The two red crosses on the diagram show where Marc's two throws landed. One throw landed in **double** 6 and the other in 8.



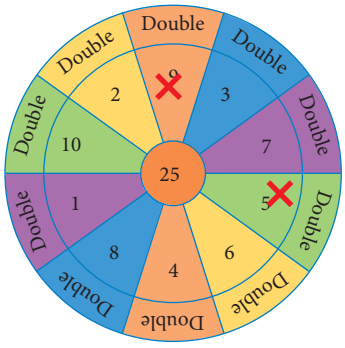
Marc's score $= 2 \times 6 + 8$
$= 12 + 8$
$= 20$

EXERCISE 1.1

Target practice

1 Without using a calculator, work out the scores for each turn.

a



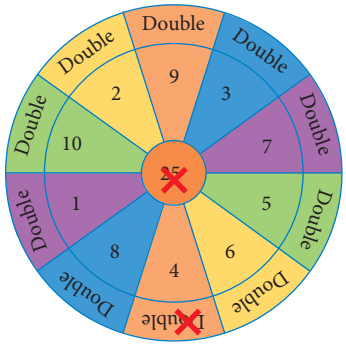
b



c



d



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2 Add each set of scores as quickly as you can.

a A 7 and a 9 **b** Double 5 and 8 **c** Double 6 and double 4

d Double 9 and double 5 **e** Bull's-eye and double 10 **f** A 3 and double 7

3 Jason's score from two throws was 13. Suggest five different combinations of points that can make a score of 13.

4 How many different ways can you get a score of 7 from two throws? List the ways.

5 To win the game, a player's score must add to exactly 100. David's score is 77.

a How many more points does David need to score to win?

b What strategy can David use to win on his next turn?

6 What is the smallest number of throws required to make a total of exactly 100? How is it achieved?

7 Walid's score is 81. He decided to aim for a 9 and a 10. With his first throw, Walid missed the 9 and his ball landed on 3. How can Walid still make 100 on his second throw?

8 When Alan designed the positions of the numbers on the target, he didn't put any big numbers next to each other. He put small numbers on each side of the big numbers. Why do you think Alan arranged the numbers in this way?

Practical activity Design a practice competition

What you have to do

1 Design a practice competition to develop the skills of children learning to play a sporting game that you enjoy.

2 Determine the rules and how you will score the game.

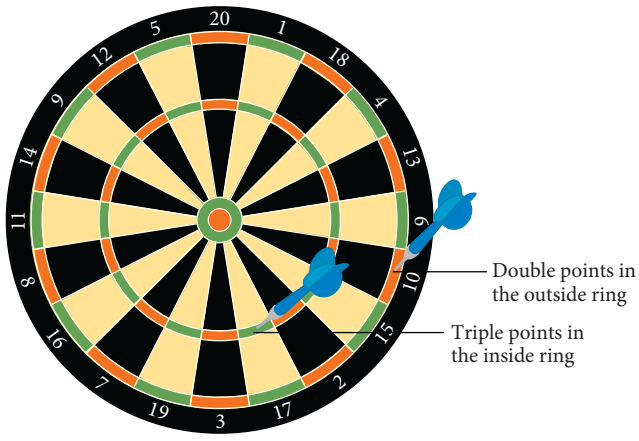
3 Use the practice competition with some younger children in your area.

4 How successful was the competition in developing the children's skills?

5 Did the scoring help to improve the children's number skills?

1.2 Playing darts

There are 20 numbers on a dartboard and each player throws three darts on their turn. If a dart lands in the **outside ring** of a number, the dart scores **double points**. It scores **triple points** if it lands in the inside ring. The **bull's-eye** is worth 50 points. The green ring around the bull's-eye is worth 25 points.

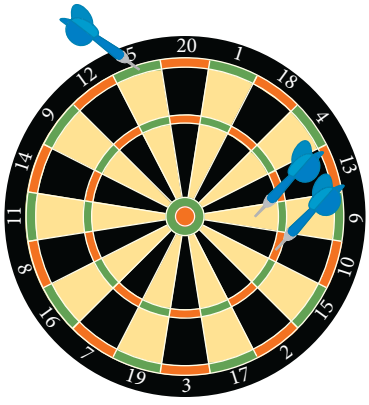


EXERCISE 1.2

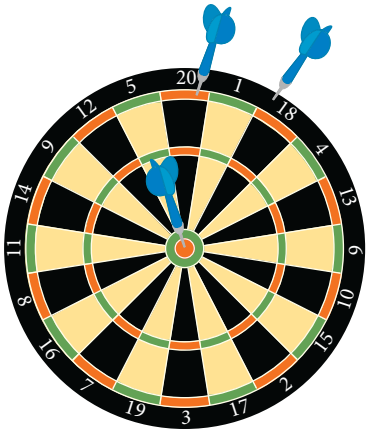
Playing darts

1 Calculate the score for each player.

a



b



2 What is the highest score possible with three darts?

3 How can you score a total of 37 using three darts? Suggest two possible ways.

4 In the dart game '201', players start with 201 points and they keep subtracting the points they score with three darts. The winner is the first to reach zero, but they must finish with a double. If a player's score on one turn is more than their remaining points, *no* points are subtracted for that turn. Use the rules of the game 201 to answer the rest of the questions in this exercise.

With her first set of three darts, Madeleine scored a 7, a double 5 and a triple 11. How many points did Madeleine have left after her first set of three darts?

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5 After the second round, Tim had 152 points left and Rebecca had 147. Who is winning the game: Tim or Rebecca?

6 Fiona had 126 points remaining before her turn. After her turn she had 103 points. One of her darts landed on double 1 and another on 5. Where could her third dart have landed?

7 Divya's score was 86. She threw two double 20s and a triple 19. What was Divya's score after her turn?

8 Before he had his turn Santo had 136 points remaining. His first dart landed on 20, his second landed on triple 4 and his third missed the board. How many points did Santo have left after his turn?

9 Andre needs to score 34 points, finishing with a double, to win a game of 201. How could he win using:

a one dart? **b** two darts? **c** three darts?

10 What is the largest number of points a player can have and still win on their next turn? Explain your answer.

11 Renata and Samantha are playing 201, writing their progress scores on a board.

Turn	Renata	Samantha
	~~201~~	~~201~~
1	~~162~~	~~175~~
2	~~102~~	~~143~~
3	~~63~~	~~85~~
4	~~39~~	~~55~~
5	~~24~~	29
6	0	

a In what number turn did Samantha score a total of 30 points?

b Who shot three darts in the single number 20 in the same turn? In which turn did she do it?

c What was Samantha's biggest score in one turn?

d In which turn did Samantha throw a double 3, triple 2 and single 20?

e Renata finished when she threw a double 7. Explain how it was possible for her to finish with a double 7.

f Samantha can tie the game if she can score a total of 29, finishing with a double. Explain why Samantha will need more than one throw to score 29.

g What strategy do you suggest Samantha should adopt to try to tie the game?

1.3 Negative scores

Integers are positive and negative whole numbers and zero.



Shutterstock.com/Martin Podzorny

Scores in golf and indoor cricket both make use of **negative numbers**. In golf a negative number is a good score, but in indoor cricket it's a bad score.

In golf, one stroke is equivalent to hitting the ball once. The course rating is called 'par', meaning average score. If a golfer's score is –2, it means that she took 2 fewer strokes than the course rating to complete her game. If an indoor cricketer's score is –2, it means that he lost 2 more runs than he scored.

Golf has a language of its own.

- A 'birdie' is 1 stroke under par, or –1.
- An 'eagle' is 2 strokes under par, or –2.
- An 'albatross' is 3 strokes under par, or –3.
- A 'bogie' is 1 stroke over par, or +1.
- A 'double-bogie' is 2 strokes over par, or +2.

Example 1

Jake and Megan played very well in a golf championship. Jake finished with a score of –2 and Megan's score was –3.

a What do scores of –2 and –3 mean?

b Did Jake or Megan play the better game? Explain your answer.

Solution

a In golf, negative scores are good scores. Every 1 negative number means 1 better than the course rating.

–2 means 2 strokes better than the course rating, and –3 means 3 strokes better.

b In golf, –3 is a better score than –2.

Megan played the better game.

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EXERCISE 1.3

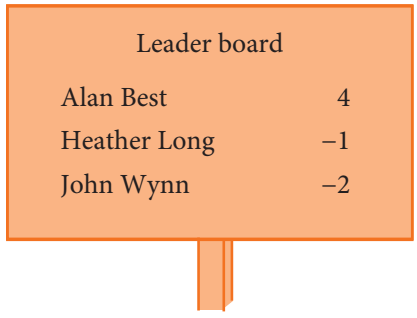
Negative scores

EXAMPLE 1

1 Angelo and Brett are playing golf. Angelo's score is +2 and Brett's score is −1. Who is winning the game?

2 Last week, Darren's golf score was +3. Today, his score was 4 strokes better. What was Darren's score today?

3 This sketch shows the leaders' board at the end of a golf tournament. Par, or the course rating, is 72 strokes.



Remember that a negative golf score tells you how many strokes below the course rating a player took to complete the course.

a How many strokes did John take to complete the course?

b How many more strokes did Alan take than Heather?

4 Shari is going snow skiing.

a What does a temperature of −10°C mean?

b Which temperature is the coldest: 5°C, −4°C or 0°C?

c The temperature on the ski fields today is −3°C. Tomorrow's forecast temperature will be 4°C colder. What temperature is forecast for tomorrow?

d At lunchtime the temperature was −7°C. The temperature went up 2°C between lunch and afternoon tea. What was the temperature at afternoon tea time?

5 In indoor cricket, batters score runs by hitting the ball into the side nets and by running between the batting and running creases. Each batter has a 4-over innings. Every time a batter is given out, 5 runs are deducted from the score, but the batter continues with his or her allocated 4-over innings.

a Imran scored 8 runs and was given out twice. What was his final score?

b Shane scored 46 runs but his final score was only 31. How many times was he given out?

c Grant and Mark are batting together. Here are their statistics.

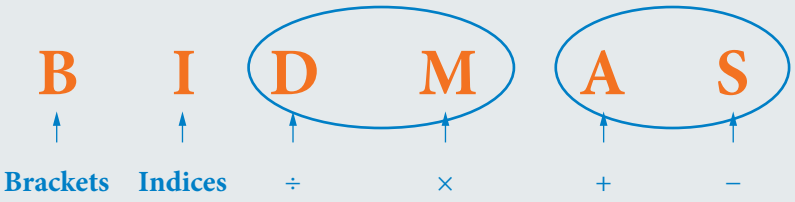
Batter	Zone A 1 run hits	Zone B 2 run hits	Zone C 4 run hits	Zone D 6 run hits	Runs scored by running	Number of outs
Mark	4	8	2	1	12	3
Grant	2	4	6	3	21	2

Together, Mark and Grant, scored $\frac{1}{4}$ of their team's runs. How many runs did the team score?

d The batting team is 8 runs in front and there are 4 balls remaining. How is it possible for the batting team to lose?

1.4 Order of operations

BIDMAS is an easy way to remember the order in which to do a **mixed calculation**.



Always do what's inside brackets first, then any indices (powers), followed by division and multiplication from left to right, and then any addition and subtraction from left to right.

Example 2

Evaluate each mixed expression.

a $60 - 2 \times 5^2$

b $(12 + 5 \times 3) \div 9 + 4$

c $11 + (5 + 11) \div (12 \div 3)$

Solution

a There are no brackets. I is for indices, and the 5 is squared, so do this. Next comes DM for divide and multiply. So, after the square, I have to multiply. Subtraction comes last.

$$\begin{aligned} 60 - 2 \times 5^2 &= 60 - 2 \times 25 \\ &= 60 - 50 \\ &= 10 \end{aligned}$$

b Do what is inside the brackets first, where the × must be done before the +. Then do the ÷. The + 4 comes last.

$$\begin{aligned} (12 + 5 \times 3) \div 9 + 4 &= (12 + 15) \div 9 + 4 \\ &= 27 \div 9 + 4 \\ &= 3 + 4 \\ &= 7 \end{aligned}$$

c Do both sets of brackets first. Then do the division. The addition outside the brackets comes last.

$$\begin{aligned} 11 + (5 + 11) \div (12 \div 3) &= 11 + 16 \div 4 \\ &= 11 + 4 \\ &= 15 \end{aligned}$$

Most calculators 'know' the order of operations. If you press the = key only at the end of the calculation, your calculator's answer will be right.

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EXERCISE 1.4

Order of operations

EXAMPLE 2

1 Is each statement true or false? Write the correct value for any false statements.

a $4 + 2 \times 3 = 18$ **b** $12 - 3 \times 4 = 0$ **c** $20 \div 2 + 2 = 12$

d $6 + 18 \div 3 = 12$ **e** $2 \times 4^2 = 64$ **f** $48 \div 4 \times 3 = 4$

g $20 - 5 + 8 = 7$ **h** $5 \times (20 - 3 \times 4) = 40$

2 Find the value of each expression.

a $18 - 3 \times 5$ **b** $24 \div (5 + 3)$ **c** $8 \times 3 - 10 \div 5$

d $(2 + 10) \times (12 - 9)$ **e** $3 \times (1 + 4)^2$ **f** $36 \div 12 \div 3$

g $30 \div 5 \times 2$ **h** $40 \times 2 \div 8$ **i** $4 \times (7 - 2) \div (3^2 + 1)$

j $4^2 + 5^2 - 3 \times 9$ **k** $10 + 5 \times 6$ **l** $300 - 20 \times 8$

m $120 \div 4 \times 5$ **n** $10 + 4^2$ **o** $5 \times (12 - 3 \times 2)$

3 Copy each statement and insert brackets to make the statement true.

a $4 + 7 \times 5 = 55$ **b** $60 \div 5 + 7 = 5$ **c** $3 \times 2^2 = 36$

d $6 + 8 \times 9 - 5 = 56$ **e** $3 \times 4 + 5 \times 2 = 34$ **f** $28 - 4 \times 5 \times 2 = 16$

4 When Siobhan used her calculator to evaluate $3 + 6 \times 5$, she got the wrong answer. She pressed these keys: 3 [+] 6 [=] [×] 5 [=]

Explain why the answer on Siobhan's calculator was wrong.

5 In the game of **snooker** the points scored are determined by the colours of the balls sunk.

Colour	Red	Yellow	Green	Brown	Blue	Pink	Black
Points	1	2	3	4	5	6	7

Brad sank four red balls, two brown balls, one pink and two blacks, but he lost 15 points for foul shots.

a What does the expression $4 \times 1 + 2 \times 4 + 6 + 2 \times 7$ represent?

b Determine Brad's score for the game.

6 Mr Healy, the school Principal, has a parent complaining about the marking of his daughter's maths exam. He claims his daughter's answer is marked wrong when it is right.

This is the question: $48 - 8 \times 3$. His daughter's answer is 120.

a Why is the daughter's answer wrong?

b How could Mr Healy explain why the daughter's answer is wrong?

c Put brackets in $48 - 8 \times 3$ to make the daughter's answer correct.

1.5 How much do I get?

We put numbers to work in every aspect of our lives, from earning and spending money, through organising schedules, preparing food, assisting with our leisure activities to scoring sporting events.

Example 3

Muspha's annual pay is $42 768. How much does he earn per month?

Solution

Annually means per year. There are 12 months in a year. Divide the annual amount by 12 to calculate the monthly amount.	Monthly pay = 42 768 ÷ 12 = 3564
Write your answer.	Each month Muspha earns $3564.

Example 4

Gillian bought 3 books online and she paid with her debit card. She paid $6.40 for postage and $7.95 for each book. Before Gillian bought the book the balance of her debit card was $160. Calculate the balance after her online purchase.

Solution

Calculate the cost of the postage and the books. Remember that we do multiplication before addition.	Cost = 6.40 + 3 × 7.95 = 30.25 The books and postage cost $30.25.
Subtract the amount Gillian spent from the balance of her debit card.	160 − 30.25 = 129.75
Write your answer.	After Gillian bought the books the balance of her debit card was $129.75.

EXERCISE 1.5

How much do I get?

EXAMPLE 3

1 Finn's annual salary is $82 440. Calculate his monthly pay.

2 Gazi earns $3250 per month. Calculate Gazi's annual pay.

3 Jessie earns $17.10 per hour. How much does she earn when she works a 7-hour day?

4 Martin earns $1840 per fortnight.

a How much does Martin earn per week?

b Calculate the amount Martin earns annually.

Remember! A fortnight is 2 weeks long and there are 26 fortnights in a year.

EXAMPLE 4

1.5

5 Rob ordered 2 burgers and some fries for his lunch. The burgers cost $4.80 each and the fries cost $3.20. Rob paid with a $20 note. How much change should he get?

6 Samantha spends 45 minutes at the gym 6 days per week. How many hours does Samantha spend at the gym each week?

7 Bruce earns $21 per hour for the first 8 hour he works per day. He receives time-and-a-half for any work over 8 hours per day.

Time-and-a-half means 1.5 times the normal rate of pay.

a How much does Bruce earn for working a 7-hour day?

b How much does Bruce earn per hour when he is being paid at time-and-a-half?

c Calculate the amount Bruce earned for working 13 hours last Monday.

8 Reah hates TV advertisements. During a one-hour TV show she counted there were twelve 30-second ads and twenty-four 15-second ads. How many minutes of ads were included in the one-hour TV show?



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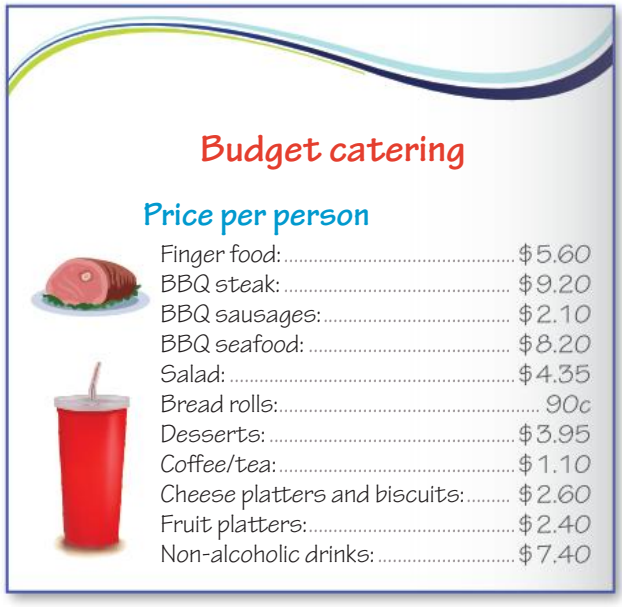
9 The Rockets and the Rascals are opposing teams in a Rugby League match. In league 'tries' are worth 4 points and 'goals' 2 points. During the game the Rockets scored 5 tries and 3 goals and the Rascals scored 6 tries. Which team won and by how many points?

10 The Boomers and the Snakes are two AFL teams in the local competition finals. In AFL, teams score 6 points for a goal and 1 point for a behind. At three-quarter time the Boomers had scored 18 goals and 2 behinds while the Snakes had 17 goals and 8 behinds on the scoreboard. Who is winning?

11 The BBQ Specialists are catering for Annie's garden party. They are supplying catering and bar service for 42 people. How much will Annie have to pay the BBQ Specialists?



Investigation The Year 11 party



Chantelle is responsible for organising the catering for the Year 11 end-of-year party. The organising committee has allocated $4000 to spend on the food and non-alcoholic drinks for the 180 people expected to attend.

Your group's task is to help Chantelle decide on an interesting menu for food and drink. Remember to stay within your budget.

Estimating costs

1.6

No one can keep an accurate record of the total cost of the items in their supermarket trolley in their head but careful shoppers can keep track of the cost by using approximation techniques.

Example 5

Patrick is purchasing the supermarket items shown in the photo: 5 cans of dog food, caesar salad, blueberries, teabags, strawberries and macadamia nuts. The price of each item is shown.



Scout Kozakiewicz

Patrick has $54 in his wallet. Estimate whether he has enough to pay for his shopping.

Solution

Mentally round each price *up* to the nearest dollar.	Estimated price = 15 + 6 + 5 + 7 + 5 + 14 = 52
Dog food 5 × $3 = $15 Caesar salad $6 Blueberries $5 Teabags $7 Strawberries $5 Macadamia nuts $14	The items cost approximately $52.
When we estimated the costs each estimate was more than the true cost.	The true cost of the items is less than $52.
Patrick has $54. Write the answer to the question.	Patrick has enough money to pay for his shopping.

EXERCISE 1.6

Estimating costs

Answer all questions *without* using a calculator.

For Questions **1** to **5**, select the correct answer **A**, **B**, **C** or **D**.

1 Which is the best estimate of the value of $\$9.99 \times 8$?

A \$72 **B** \$80 **C** \$90 **D** \$100

2 What is the best approximate value of \$25.99 + \$4.10 + \$15.05 + \$3.90?

A \$45 **B** \$49 **C** \$55 **D** \$60

3 Gavin ordered 19.5 m of material at a cost of \$29.90 per metre. Estimate the cost of the material.

A \$60 **B** \$90 **C** \$600 **D** \$900

4 Sam is buying small chocolates that cost 49c each. Approximately how much will 50 cost?

A \$9 **B** \$25 **C** \$99 **D** \$2500



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5 Davinia buys bags of nuts and bolts to use in her furniture business. The bags contain between 48 and 51 nuts and bolts. She has 11 bags in her supply cupboard. Approximately how many nuts and bolts does she have?

A 50 **B** 550 **C** 1100 **D** 2500

6 Christina earns \$18.90 per hour in her weekend job.

a Approximately how much will she earn for working 11.9 hours?

b Her pay slip showed that she earned \$105.91 for working 11.9 hours. Is Christina's pay slip right?

EXAMPLE 5

7 When Bryce bought 2 hamburgers @ \$8.95 each and a large fries for \$5.80 he paid with a \$50 note. Approximately how much change should he get?

8 Chloe only has $60 remaining on her EFTPOS card until payday. She has these items in her supermarket basket:

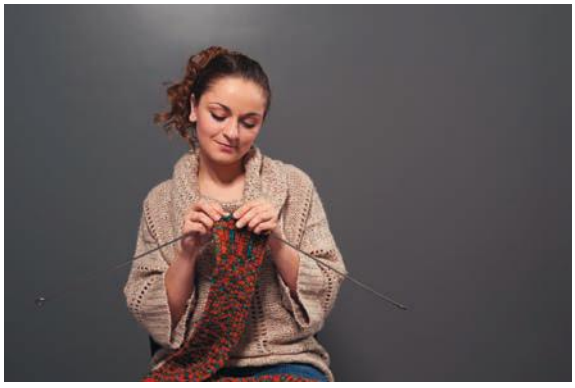
- 2 packets of sausages @ $5.99 per packet
- 1 packet of potatoes $4.99
- 1 carton of milk $3.95
- 1 carton of yoghurt $6.95
- 1 packet of breakfast cereal $5.95

a Estimate the cost of the items in Chloe's supermarket basket.

b Will she have sufficient funds in her account to pay for her shopping?

c Does she have enough money to add a $9.80 carton of soft drink to her shopping?

9 Paige is knitting a scarf. It takes her 29 seconds to complete 1 row.



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a Explain why we can use the calculation $80 \div 2 = 40$ to calculate the number of minutes Paige takes to complete 80 rows of knitting.

b Paige needs to knit 50 more rows to finish the scarf. Approximately how long will it take her?

Investigation My shopping docket

To complete this investigation you will need samples of supermarket shopping dockets.

What you have to do

- Estimate the total cost of the items on the docket.
- Compare your estimate to the total on the docket. Was your estimate more or less than the actual total?
- What is the difference between your estimate and the true cost?
- Use the formula below to calculate the percentage your estimate is different to the true cost.

$$\text{Percentage error} = \frac{\text{difference}}{\text{true value}} \times 100\,\%$$

- Repeat the process for a different shopping docket. Does your percentage error get smaller with practice?

1.7 Rounding decimals

When Imran used his calculator to divide 5.7 by 1.3, his calculator displayed the answer 4.384 615 385. Calculators often show a lot of **decimal places**. Imran wants the answer with only 1 decimal place, so he has to **round off** his answer.

Imran has to decide whether 4.384 615 385 is closer to 4.3 or 4.4. He will check using just the first 3 decimal places: 4.384.

$$\begin{array}{r} 4.384 \\ -\ 4.300 \\ \hline 0.084 \end{array} \qquad \begin{array}{r} 4.400 \\ -\ 4.384 \\ \hline 0.016 \end{array}$$

4.300 is the same as 4.3 and 4.400 is the same as 4.4.

Imran can see that 4.384 is 0.084 from 4.3 and 0.016 from 4.4. As 0.016 is smaller than 0.084, 4.384 is closer to 4.4. Imran will round to 4.4.

When the figure following the decimal place you want to round to is:

- **4 or smaller** – just leave all the extra figures off.
- **5 or bigger** – add 1 to the figure in the decimal place you want to round to, and leave all the extra figures off.

This table shows the results when some decimals are correctly rounded to 1 decimal place.

Original decimal	Rounded to one decimal place
6.802 956	6.8
6.830 5	6.8
6.849 99	6.8
6.853	6.9
6.893	6.9

Example 6

Round 18.3721 to 1 decimal place.

Solution

I have to decide whether 18.3721 is closer to 18.3 or to 18.4. The figure following the 1st decimal place is 7. I need to round the 3 up to 4.

$18.3721 \approx 18.4$.

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EXERCISE 1.7

Rounding decimals

EXAMPLE 6

1 Round each number correct to 1 decimal place.

a 16.7256 **b** 29.7681 **c** 14.6472

d 13.2836 **e** 104.565 **f** 115.649

2 Write 124.727 56 correct to 2 decimal places.

3 Darryn correctly rounded a number to 16.3. Suggest two possible values the original number could have been.



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4 In surfboard-riding competitions, scores are rounded to 1 decimal place. This is how scores are calculated.

Step 1: Each of five judges awards a result from 0.0 to 10.0 for the wave.

Step 2: The lowest and highest scores are removed.

Step 3: The **average** of the three remaining scores is determined by adding the three scores together, then dividing by 3. The average is rounded to 1 decimal place.

Step 4: Each competitor's best two scores are added to obtain their final score for the competition.

Here are the scores that the judges gave Pam for her first wave. Use calculations to show that Pam's score for her first wave is 7.7.

7.5	7.3	8.1	7.9	7.6

5 Mia and Elissa are competing against each other in a surfboard-riding competition. This table shows the points judges awarded them on four waves.

Mia's scores			
Wave 1	Wave 2	Wave 3	Wave 4
4.7	7.2	8.5	7.4
5.1	7.1	8.5	7.8
4.9	7.5	8.4	7.6
5.0	7.3	8.2	7.9
4.9	7.5	8.6	7.5

Elissa's scores			
Wave 1	Wave 2	Wave 3	Wave 4
6.9	7.8	5.2	8.2
7.0	7.9	5.1	8.1
7.2	7.8	5.3	8.4
6.9	8.0	5.2	8.0
7.1	7.8	5.0	8.9

a Calculate the score awarded to each surfer for each wave.

b Calculate each surfer's final score for the competition.

c Who won the event?

6 Express the total of 2 m, 148 cm and 384 mm in metres correct to 1 decimal place.

7 Ben's car travels 8.9 km on 1 L of petrol. The car's fuel tank holds 60.5 L of petrol. How far can the car travel on one full tank of petrol? Express your answer in kilometres correct to 1 decimal place.

8 Pauline buys knitting wool in hanks from a wool mill. She bought five hanks. Two of the hanks contained 1560 m each, and the other three hanks contained 1875 m each.

a Calculate the total length of wool in Pauline's five hanks.

b How many kilometres of wool are in the five hanks? Express your answer correct to 1 decimal place.

9 Thanh bought 79.3 L of petrol at 155.8c/L.

a Explain why the value of 80×1.5 will give the approximate cost of the petrol in dollars.

b Approximately how much will the petrol cost?

c Use a calculator to determine the cost of the petrol correct to the nearest 5c.

d How much different is your approximation to the actual cost?

10 Jaye earns $15.17 per hour. She worked a 7 hours 45 minutes day.

a Jaye wanted to estimate the amount she earned. Will the calculation 15×8 give her a reasonable estimate? Give reasons your answer.

b Explain how you know that 7 h 45 min = 7.75 hours.

c Calculate the value of 15.17×7.75 and express the answer correct to 2 decimal places.

d How much did Jaye earn for working the 7 h 45 min day?

11 Simone earned $48.62 for working for 3 hours. Calculate the amount she earns per hour correct to the nearest cent.

Practical activity Accuracy in measuring lengths

In this activity you are going to investigate how accurately each member of your group can measure lengths. Each group will need a tape measure that measures in metres, centimetres and millimetres.

What you have to do

1. Choose three suitable distances in your school environment; for example, the length of the school verandah, or the distance from your classroom doorway to the nearest tree.
2. Each member of the group measures the lengths as accurately as possible.
3. Compare your group's measurements. At what level of accuracy (for example, answers in metres correct to 1 decimal place) are the measurements the same?

Keyword activity

Copy the find-a-word puzzle below. Solve each of the following clues to find 11 keywords. Then find the same 11 keywords in the puzzle.

1. Multiply by two.
2. Multiple by three.
3. The centre of a dartboard (two words)
4. A positive or negative whole number
5. The initials we use to remember the 'order of operations' rules
6. Above zero
7. Another word for powers, beginning with I
8. For order of operations, we calculate what's inside these first.
9. Below zero
10. A game played on a circular board
11. Electronic Funds Transfer at Point Of Sale (initials).

X	P	O	S	I	T	I	V	E	Y	T
D	M	I	C	T	N	P	D	L	S	G
I	R	A	U	B	I	D	M	A	S	W
N	B	N	E	G	A	T	I	V	E	J
T	U	R	W	Z	C	F	R	C	J	A
E	L	O	A	K	T	S	X	N	E	I
G	L	D	M	C	L	R	H	C	Q	S
E	S	H	Y	W	K	G	I	V	O	B
R	E	S	A	J	R	E	F	P	O	Y
B	Y	E	H	D	A	R	T	S	L	R
K	E	L	P	O	X	F	N	S	R	E
D	O	U	B	L	E	M	F	O	Z	U

Chapter Problem

Brian bought fish and chips for $11.25 and paid with a $50 note. The cashier gave him a $20 and a $5 note and a few coins as change. Could the change be wrong?

Solution

The box of fish and chips cost approximately $10. Brian's change should have been approximately $50 – $10 = $40. Brian's change included a $20 and a $5 note plus a few coins, so the change was probably between $25 and $30.

Yes, he has probably been short-changed (by about $10).



Chapter Problem

Tanika sells cosmetics. She earns commission at the following rates.

Commission on Tanika's monthly sales	
First $500 of sales	5%
On the next $1000	4%
Remainder of sales	3.5%

Calculate Tanika's commission in a month when her sales were $4200.



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CHAPTER 2

BUYING AND SELLING

2.1 Fractions
2.2 Percentages
2.3 Percentage of a quantity
2.4 What percentage of …?
2.5 Percentage increase and decrease
2.6 Profit and loss
2.7 Percentage problems
Keyword activity
Solution to the chapter problem

By the end of this chapter, you should know how to:

- solve problems involving fractions and percentages, especially involving sales and discounts
- calculate a percentage of an amount
- write one amount as a fraction or percentage of another amount
- increase or decrease an amount by a percentage

How are we ever going to use this?

- To calculate discounts on items we want to buy
- To calculate statistics in sport
- Whenever we come across a percentage – which happens often!

2.1 Fractions

To **simplify a fraction**, keep dividing the numerator and denominator by the same number (preferably a large number such as their highest common factor [HCF]).

Example 1

Simplify the fraction $\frac{36}{60}$.

Solution

Divide 36 and 60 by 6 first.

Then divide 6 and 10 by 2.

Stop when there are no more common factors.

$$\frac{36}{60} = \frac{36 \div 6}{60 \div 6} = \frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$$

A faster way is to divide 36 and 60 by their highest common factor, 12.

$$\text{or } \frac{36}{60} = \frac{36 \div 12}{60 \div 12} = \frac{3}{5}$$

Or using the calculator: 36 $a^{b/c}$ 60 =

$$\text{or } \frac{36}{60} = \frac{3}{5}$$

Fraction of a quantity

Example 2

Find:

a $\frac{1}{5} \times 45$ **b** $\frac{7}{8} \times \$32$ **c** $\frac{1}{3}$ of 1 year **d** $\frac{4}{5}$ of 2 kg (in grams)

Solution

a Finding $\frac{1}{5}$ of an amount is the same as dividing the amount by 5.

$$\frac{1}{5} \times 45 = 45 \div 5$$
$$= 9$$

b Finding $\frac{7}{8}$ of an amount is the same as finding $\frac{1}{8}$ of it first, then multiplying by 7.

$$\frac{7}{8} \times \$32 = \left(\frac{1}{8} \times \$32\right) \times 7$$
$$= \$4 \times 7$$
$$= \$28$$

c First convert 1 year to 12 months.

$$\frac{1}{3} \text{ of 1 year} = \frac{1}{3} \times 12 \text{ months}$$
$$= 4 \text{ months}$$

d First convert 2 kg to 2000 g

$$\frac{4}{5} \text{ of } 2 \text{ kg} = \frac{4}{5} \times 2000 \text{ g}$$
$$= \left(\frac{1}{5} \times 2000 \text{ g}\right) \times 4$$
$$= 400 \text{ g} \times 4$$
$$= 1600 \text{ g}$$

EXERCISE 2.1

Fractions

A calculator can also be used.

EXAMPLE 1

1 Simplify each fraction.

a $\frac{5}{10}$ **b** $\frac{4}{12}$ **c** $\frac{12}{26}$ **d** $\frac{18}{24}$

e $\frac{15}{25}$ **f** $\frac{16}{28}$ **g** $\frac{32}{48}$ **h** $\frac{60}{100}$

i $\frac{44}{77}$ **j** $\frac{20}{35}$ **k** $\frac{21}{35}$ **l** $\frac{72}{80}$

EXAMPLE 2

2 Find:

a $\frac{3}{5} \times 40$ **b** $\frac{1}{4} \times 28$ **c** $\frac{1}{6} \times 24$

d $\frac{2}{3} \times 15$ **e** $\frac{7}{10} \times 60$ **f** $\frac{5}{8} \times 16$

g $\frac{3}{4}$ of 1 km (in metres) **h** $\frac{1}{3}$ of 1 day (in hours) **i** $\frac{2}{5}$ of 1 L (in mL)

j $\frac{1}{8}$ of 1 t (in kg) **k** $\frac{5}{6}$ of 1 year (in months) **l** $\frac{7}{12}$ of 1 hour (in minutes)

3 Taffy the cat had a mass of 2.7 kg, then it increased by $\frac{1}{4}$. How much did she gain?

4 An Airbus can seat 351 passengers. If the plane flew with $\frac{2}{3}$ of the seats occupied, how many passengers were on the flight?

5 One-twentieth of the cars produced in a factory had steering defects. There were 340 cars produced. How many of them had steering defects?

6 Glenda planted 128 seedlings in her garden but $\frac{1}{8}$ did not grow. How many plants grew in her garden?

7 In Binnsville's population of 40 530, $\frac{3}{5}$ of the people are adults. How many people in Binnsville are adults?

2.2 Percentages

A **percentage** is a fraction whose denominator is always 100, for example, $65\% = \frac{65}{100}$. The symbol % is an abbreviation of '/100'. Percentages, fractions and decimals are different versions of the same thing: $50\% = \frac{1}{2} = 0.5$. We can change between them and compare them.

Example 3

Convert each percentage to a simplified fraction.

a 55% **b** 130%

Solution

a Write the percentage as a fraction 'over 100' and simplify.

$$55\% = \frac{55}{100}$$
$$= \frac{11}{20}$$

b Write the percentage as a fraction 'over 100' and simplify.

$$130\% = \frac{130}{100}$$
$$= \frac{13}{10}$$
$$= 1\frac{3}{10}$$

Example 4

Convert each percentage to a decimal.

a 8% **b** 43.6%

Solution

a Divide the percentage by 100 by moving the decimal point 2 places to the left.

$$8\% = \frac{8}{100}$$
$$= 8 \div 100$$
$$= 0.08$$

b Divide the percentage by 100.

$$43.6\% = \frac{43.6}{100}$$
$$= 43.6 \div 100$$
$$= 0.436$$

To convert a fraction or a decimal to a percentage, multiply it by 100%.

Example 5

Convert each fraction to a percentage.

a $\frac{11}{20}$ **b** $\frac{7}{8}$

Solution

a Since 100% = 1 whole, multiply the fraction by 100%.

$$\frac{11}{20} = \frac{11}{20} \times 100\%$$
$$= 55\%$$

b Multiply the fraction by 100%.

$$\frac{7}{8} = \frac{7}{8} \times 100\%$$
$$= 87\frac{1}{2}\%$$

Example 6

Convert each decimal to a percentage.

a 0.65 **b** 0.267 **c** 0.8

Solution

a Since 100% = 1 whole, multiply the decimal by 100% by moving the decimal point 2 places to the right.

$$0.65 = 0.65 \times 100\%$$
$$= 65\%$$

b Multiply the decimal by 100%.

$$0.267 = 0.267 \times 100\%$$
$$= 26.7\%$$

c Multiply the decimal by 100%.

$$0.8 = 0.8 \times 100\%$$
$$= 80\%$$

Example 7

Which is larger: 35% or $\frac{3}{8}$?

To compare fractions, percentages and decimals, convert each to a percentage first.

Solution

Convert $\frac{3}{8}$ to a percentage.

$$\frac{3}{8} \times 100\% = 37\frac{1}{2}\%$$

$37\frac{1}{2}\%$ is larger than 35%.

$\frac{3}{8}$ is larger than 35%.

EXERCISE 2.2

Percentages

EXAMPLE 3

1 Convert each percentage to a simplified fraction.

a 60%	**b** 75%	**c** 31%	**d** 8%
e 30%	**f** 85%	**g** 99%	**h** 3%
i 160%	**j** 135%	**k** 25%	**l** 250%

EXAMPLE 4

2 Which decimal is equal to $62\frac{1}{2}\%$? Select the correct answer **A**, **B**, **C** or **D**.

A 62.12	**B** 62.5	**C** 0.625	**D** 0.0625

3 Convert each percentage to a decimal.

a 18%	**b** 82%	**c** 2%	**d** 50%
e 120%	**f** 51.1%	**g** 79%	**h** $12\frac{1}{2}\%$
i 16.3%	**j** 4%	**k** 18.7%	**l** $5\frac{1}{4}\%$

EXAMPLE 5

4 Convert each fraction to a percentage.

a $\frac{17}{100}$	**b** $\frac{7}{10}$	**c** $\frac{13}{50}$	**d** $\frac{11}{20}$
e $\frac{5}{8}$	**f** $\frac{24}{25}$	**g** $\frac{8}{5}$	**h** $\frac{2}{3}$
i $\frac{5}{4}$	**j** $1\frac{2}{5}$	**k** $\frac{27}{40}$	**l** $\frac{1}{16}$

EXAMPLE 6

5 Convert each decimal to a percentage.

a 0.38	**b** 0.55	**c** 0.96	**d** 0.625
e 0.08	**f** 0.054	**g** 0.6	**h** 0.003
i 1.9	**j** 0.405	**k** 1.26	**l** 0.114

6 Copy and complete this table.

	a	**b**	**c**	**d**	**e**	**f**
Fraction				$\frac{1}{2}$	$\frac{1}{8}$	$\frac{5}{8}$
Decimal	0.35	0.6				
Percentage			20%			

EXAMPLE 7

7 For each pair of numbers, determine which is larger.

a 75% and 0.73	**b** $\frac{4}{25}$ and 14%	**c** $\frac{2}{3}$ and 0.64
d 18% and $\frac{1}{7}$	**e** 0.22 and $\frac{1}{4}$	**f** 55% and $\frac{6}{11}$
g 0.93 and $\frac{19}{20}$	**h** 60% and $\frac{13}{25}$	**i** $\frac{1}{6}$ and 0.08

Percentage of a quantity

2.3

Example 8

Find:

a 23% of $1650 **b** 75% of 620 kg

Solution

a Write the percentage as a fraction or decimal and then multiply by the quantity.

Remember: 'of' means multiply.

Remember: we need two decimal places when the answer includes money in cents.

$$23\% \text{ of } \$1650 = \frac{23}{100} \times \$1650$$
$$\text{or } 0.23 \times \$1650$$
$$= \$379.50$$

b Write the percentage as a fraction or decimal and multiply by the quantity.

$$75\% \text{ of } 620 \text{ kg} = \frac{75}{100} \times 620 \text{ kg}$$
$$\text{or } 0.75 \times 620 \text{ kg}$$
$$= 465 \text{ kg}$$

Example 9

Find 32% of 4 m. Give your answer in cm.

Sometimes it is necessary to change the units of the quantity before we calculate the percentage.

Solution

First convert 4 m to centimetres.

$$4 \text{ m} = 4 \times 100 \text{ cm}$$
$$= 400 \text{ cm}$$

Write the percentage as a fraction or decimal and multiply by the quantity.

$$32\% \text{ of } 400 = \frac{32}{100} \times 400 \text{ cm}$$
$$\text{or } 0.32 \times 400 \text{ cm}$$
$$= 128 \text{ cm}$$

EXERCISE 2.3

Percentage of a quantity

EXAMPLE 8

1 Find:

a 11% of $300 **b** 60% of 140 kg **c** 42% of 600 cm

d 25% of $1230 **e** 90% of 240 marks **f** 85% of 700 000 people

g 19.4% of 785 kg **h** 2.5% of 200 m **i** 35% of 840 students

j 5.9% of $5300 **k** 12.5% of 96 L **l** 17.75% of 640 hectares

EXAMPLE 9

2 Calculate each quantity:

- **a** 12% of 8 m, giving your answer in cm
- **b** 35% of 10 days, giving your answer in hours
- **c** 60% of 1 year, giving your answer in days
- **d** 15% of 26 L, giving your answer in mL
- **e** 10% of $2.30, giving your answer in cents
- **f** 87.5% of 16 weeks, giving your answer in days
- **g** 22.5% of 6 km, giving your answer in metres
- **h** 40% of 1.5 tonnes, giving your answer in kg

3 20% of students at Nelson Secondary College study Chinese. There are 1290 students at the school. How many study Chinese?

4 Antony earns 5.5% interest per year on an investment account. He has $7500 in the account. How much interest will Antony earn in one year?

5 Employees at the GMC Steel Company are given a 2.5% pay increase.

- **a** Dean earns $32.40 per hour. How much extra will he be paid per hour after the increase?
- **b** Minnie is paid $82 500 per year. How much extra will she be paid per year after the increase?

6 Jane's laptop can be used for 6 hours without recharging. Her laptop shows she has 62% of her time left. How much time does she have left before her laptop needs recharging?

7 Michael planted 850 seedlings in his market garden. 92% of the seedlings survived to full maturity. How many seedlings survived?

8 When Frankie orders stationery for her business, she must pay 10% GST. Calculate the GST payable on stationery purchases of $524.

9 In a poll of 1500 people, 85% of people agreed 'that Mathematics is the most useful subject to study at school'. How many people is this?

10 At the Strolling Bones concert, 65% of seats need to be sold for the promoter to make a profit. The venue seats 18 000 people. How many seats need to be sold to make a profit?

11 Jason decided to go on a weight loss program. His starting weight was 187 kg. In the first six weeks he lost 13% of his body weight. How many kilograms did Jason lose in the first six weeks?

12 Tayou is downloading the latest album of his favourite band. The file is 750 megabytes. So far he has downloaded 45% of the album. How many megabytes has Tayou downloaded so far?

What percentage of …?

2.4

We often change quantities into percentages so that we can compare them. For example, we calculate our marks in tests as percentages so we can compare our performance in each subject. To be able to do this, we must first write one quantity as a fraction of another. Then we can change it to a percentage.

Example 10

There were 250 people at the school fete, and 160 of them were children. What fraction of the people at the fete were children?

Solution

Write as a fraction $\frac{\text{Number of children}}{\text{Total number of people}}$.

$\text{Fraction} = \frac{160}{250} = \frac{16}{25}$

Write your answer.

$\frac{16}{25}$ of the people at the fete were children.

Example 11

Express 36 minutes as a fraction of an hour.

Solution

Quantities need to be expressed in the same units, so change 1 hour to 60 minutes.

$\text{Fraction} = \frac{36}{60} = \frac{3}{5}$

Write your answer.

36 minutes is $\frac{3}{5}$ of one hour.

$$\text{Percentage} = \frac{\text{amount}}{\text{whole amount}} \times 100\%$$

or amount ÷ whole amount × 100%

Example 12

18 out of the 25 students in the class had part-time jobs. What percentage of the class was this?

Solution

Write as a fraction and multiply by 100%.

$\frac{18}{25} \times 100\% = 72\%$

Write your answer.

72% of the class had part-time jobs.

Example 13

Express 9 months as a percentage of 2 years.

Solution

Quantities need to be expressed in the same units, so change 2 years to 24 months.

$$\frac{9}{24} \times 100\% = 37\frac{1}{2}\%$$

Write your answer.

9 months is $37\frac{1}{2}\%$ of 2 years.

EXERCISE 2.4

What percentage of …?

EXAMPLE 10

1 Convert each test mark to a simplified fraction.

a 50 out of 100 **b** 38 out of 50 **c** 8 out of 12 **d** 45 out of 120

2 The World Cup cricket final attracted 86 000 people to the game, but 94 500 tickets were sold before the match. What fraction of the sold tickets were used?

EXAMPLE 11

3 Express each measurement as a simplified fraction.

a 40 min of 4 h **b** 250 mL of 1 L **c** 700 m of 1 km **d** 230 kg of 1 t

e 300 g of 2 kg **f** $3.80 of $14 **g** 12 h of 4 days **h** 800 m of 1.5 km

EXAMPLE 12

4 Convert each test mark to a percentage.

a 32 out of 40 **b** 11 out of 20 **c** 60 out of 75 **d** 71 out of 80

5 In her driving test, Melanie answered 21 out of 24 questions correctly. The pass mark on the driving test is 95%. Did Melanie pass the test?

6 Joe earns $670 per week and pays $210 in tax. What percentage of his earnings does Joe pay in tax? Answer correct to one decimal place.

EXAMPLE 13

7 Express each measurement as a percentage. Remember to have both quantities in the same units. Answer correct to one decimal place.

a 5 minutes of 1 hour **b** 18 hours of 4 days **c** 75c of $6

d 75 mm of 20 cm **e** 520 kg of 1 tonne **f** 400 mL of 3.5 L

8 In a survey of 1500 people, 1125 of the respondents agreed that breakfast is the most important meal of the day. What percentage is this?

9 Joanne sells jeans in her boutique for $85. She increases the price by $15. What percentage increase is this? Answer correct to one decimal place.

10 Australia has an area of 7.7 million square kilometres. In Australia the area of desert is 3.8 million square kilometres. What percentage of Australia is desert? Answer correct to one decimal place.



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11 The cost of Junior's new glasses is $295. His health fund pays him a benefit of $165. What percentage is his benefit of the cost of his glasses? Answer correct to one decimal place.

12 Vanessa bought 6 m of rope to tie up her plants. She used 315 cm in her backyard. What percentage of the rope has she used?

13 George and Sue are travelling to see their parents. The trip takes 7 hours. They take a break after 2 hours 40 minutes. What percentage of the trip have they completed when they take the break? Answer correct to one decimal place.

14 In a class of 23 students, 8 walk to school and 11 travel by bus. Calculate correct to one decimal place the percentage of students who:

a walk to school

b travel by bus

15 Kyle scored the following results in his half-yearly exams.

English: 45 out of 75

Mathematics: 38 out of 70

Science: 80 out of 125

a Calculate Kyle's percentage in each subject. Answer correct to one decimal place.

b Which subject was Kyle's highest result?

Investigation How much sugar is in the food you eat?

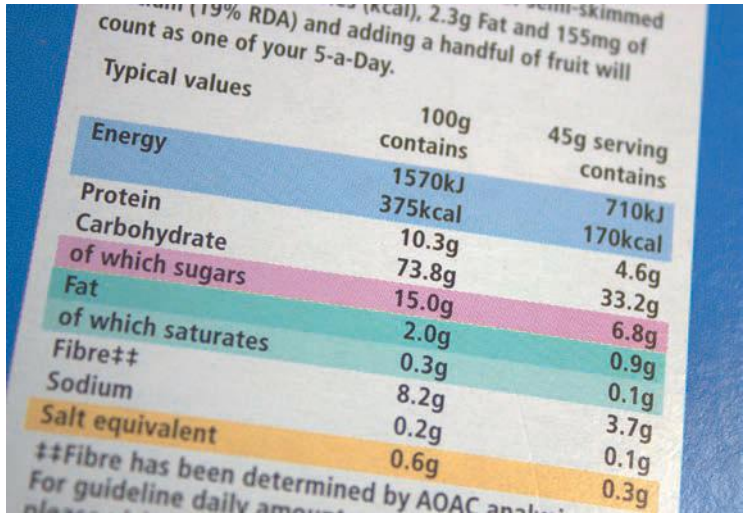
Sugar is one of the contributing factors to weight gain. In this investigation you will examine the percentage of sugar in what you eat and drink.

Did you know that sugar has many different names? Some of them include:

- Sucrose – this is normal table sugar
- Corn sugar – made from cornstarch
- Fructose – sugar in honey, plants and some fruit
- Lactose – found in milk
- Mannitol – a sugar alcohol in many plants

What you need to do

1 Choose 10 items that you regularly eat or drink. You will need to choose items that have product information on the packaging. You could include a soft drink, cereal, pasta or rice, some canned food and a variety of other foods.



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2 For each item:

   a write down the serving size

   b write down the amount of sugar per serving

   c calculate the percentage of sugar in the product

3 Which product had the highest percentage of sugar? Which had the lowest? Write a list of the items from most sugar to least sugar.

4 Compare your findings with other students in your class.

5 Write a report on sugar levels in food. Include recommendations about how a person could reduce the amount of sugar in their diet.

Percentage increase and decrease 2.5

Percentage increase means to make a quantity bigger by a given percentage, for example, in pay increases or adding GST to prices.

Percentage decrease means to make a quantity smaller by a given percentage, for example, in discounts and sale prices.

Example 14

a Increase $500 by 12%.

b Decrease $500 by 7%.

There are two ways to do this. Choose the method you like!

Solution

a Find 12% of $500 and add it to $500.

$12\% \times \$500 = \$60$

$\$500 + \$60 = \$560$

OR New amount is 100% plus 12%.

$100\% + 12\% = 112\%$

$112\% \times \$500 = \$560$

b Find 7% of $500 and subtract it from $500.

$7\% \times \$500 = \$35$

$\$500 - \$35 = \$465$

OR New amount is 100% minus 7%.

$100\% - 7\% = 93\%$

$93\% \times \$500 = \$465$

Example 15

a All Government employees receive a 2.5% pay increase. Georgina is on a salary of $82 000 per year. What is her new salary?

b During an end-of-financial-year sale, computer games are discounted by 25%. Find the cost of a game regularly priced at $115.

Solution

a Find 2.5% of $82 000.

$\frac{2.5}{100} \times 82\,000 = \$2050$

Add this amount to $82 000.

$\$82\,000 + \$2050 = \$84\,050$

Answer the question.

Georgina's new salary is $84 050.

	OR New amount is 100% plus 2.5%.	100% + 2.5% = 102.5%
		102.5% × 82 000 = $84 050
b	Find 25% of $115.	$\frac{25}{100} \times 115 = \$28.75$
	Subtract this amount from $115.	$115 – $28.75 = $86.25
	Answer the question.	The computer game will cost $86.25.
	OR New amount is 100% minus 25%.	100% – 25% = 75%
		75% × 115 = $86.25

EXERCISE 2.5

Percentage increase and decrease

EXAMPLE 14

1 **a** Increase 95 kg by 60%
b Increase $2500 by 6%
c Increase 150 m by 5%
d Increase 10 L by 33%
e Decrease $150 by 57%
f Decrease 2000 L by 28%
g Decrease 110 kg by 4.5%
h Decrease 840 students by 15%

2 Increase a fare of $7.50 by 20%.

3 Increase a price of $620 by 10%.

4 Decrease a cost of $330 by 25%.

5 Decrease a population of 32 780 by 5%.

EXAMPLE 15

6 Jenna earns $32.75 per hour. Her manager gives her a pay increase of 8%. What is her new wage rate?

7 In the January sales all whitegoods are discounted by 15%. Liong buys a new washing machine usually priced at $799. Calculate the amount he pays for the washing machine during the sale.

8 Saria runs a fashion business. She calculates the selling price of each item of clothing by increasing the cost price by 95%. Find the selling price of a jacket that costs Saria $120.

9 Non-essential items in Australia attract a GST (goods and services tax) of 10%. Find the GST-inclusive price of a car costing $18 900.

10 Last year the Australian road toll was 1603 deaths. This year the road toll decreased by 10.3%. Calculate the road toll for this year, rounded to the nearest whole number.

11 Renee and her sister Elise have singing lessons. Renee pays full fees of $450 per term, but Elise gets a 5% family discount.

- **a** Calculate the discount for Elise.
- **b** Calculate the total amount of fees paid for the two sisters.

12 Ken and Amanda buy a house for $64 000 and sell it 10 years later at a profit of 147%. What is the selling price of the house?

13 La Plage restaurant charges a 15% surcharge on public holidays. Find the total charge for a meal costing $160 on Australia Day.

14 The Luxury Hotel charges an additional 1.5% when the customer pays with a credit card. Find the total cost of 3 nights' accommodation at $185 per night which is charged to a credit card. Round your answer to the nearest cent.

15 An electricity company offers a 2% discount when customers pay the account on time.

- **a** Calculate the discount if a bill of $1978.70 is paid on time. Round your answer to the nearest cent.
- **b** How much does the customer pay?

16 Katie works at the local supermarket. She is given a staff discount of 3.5% on all purchases. Find how much she pays for purchases totalling $178.25.

17 At the 'End of model' sale, Honest John's car yard offers a discount of 30% on all cars on the lot. Find the cost of a family car originally priced at $39 990.



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18 For tax purposes the value of a computer is depreciated (decreased) at a rate of 20% per year. Sunil buys a computer priced at $1290. What will be its depreciated value after one year?

19 Filomena buys TVs at a cost price of $295. She adds 75% of the cost price to get her selling price then she adds 10% GST to calculate the final selling price.

- **a** Find the price of the TV after Filomena has added her 75% markup.
- **b** Find the price of the TV after she adds GST. Round your answer to the nearest cent.
- **c** It is unlikely that Filomena will charge her customers exactly that price. Why?

20 Harry's Hardware is having an end-of-financial-year sale. All items in the store are discounted by 18%.

- **a** Lauren is a painter. She buys ten 4 L tins of paint regularly priced at $67 per tin.
  - **i** Calculate the cost of the paint at its regular price.
  - **ii** Calculate the cost of the paint after the 18% discount.
- **b** Lauren is also entitled to a trade discount of 10%. Calculate the cost of the paint to Lauren.

2.6 Profit and loss

All successful businesses make a **profit**. Businesses that make a **loss** quickly close down. Most businesses express the profit or loss they make as a percentage of their costs.

$$\text{Percentage profit} = \frac{\text{profit}}{\text{cost price}} \times 100\%$$

$$\text{Percentage loss} = \frac{\text{loss}}{\text{cost price}} \times 100\%$$

The **cost price** is how much a business buys the item for, from the supplier.

The **selling price** is how much the business sells the item for, to the customer.

Example 16

Jane buys a necklace for her shop for $36. She sells it for $47.

a How much profit does Jane make when she sells the necklace?

b Calculate the percentage profit. Answer correct to 1 decimal place.



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Solution

a Profit = selling price – cost price

$\$47 - \$36 = \$11$

b Write the profit over the cost price and multiply by 100%.

$\frac{11}{36} \times 100\% = 30.6\%$

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Example 17

During a sale, a bed is sold for $599. The bed cost the store $750.

a Calculate the loss for the store on the bed.

b Calculate the percentage loss, correct to 1 decimal place.



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Solution

a Loss = cost price – selling price

$\$750 - \$599 = \$151$

b Write the loss over the cost price and multiply by 100%.

$\frac{151}{750} \times 100\% = 20.1\%$

EXERCISE 2.6

Profit and loss

1 Calculate correct to one decimal place the percentage profit for each sale.

EXAMPLE 16

a cost price $35, selling price $45

b cost price $625, selling price $810

c cost price $149.75, selling price $274.25

d cost price $2.60, selling price $4.05

2 Calculate correct to one decimal place the percentage loss for each sale.

EXAMPLE 17

a cost price $516, selling price $420

b cost price $2175, selling price $1300

c cost price $12 500, selling price $11 000

d cost price $5.20, selling price $4.40

3 Caron pays $40 for a small cupboard at a garage sale and paints it. She sells it for $75 to a furniture dealer. Calculate her percentage profit.

4 Jason pays $165 for a ticket to the football grand final. When his team doesn't make the grand final he sells the ticket for $90. Calculate his percentage loss.

5 Voula buys an old bicycle for $20. She spends $115 replacing some parts and painting it. She sells it for $150.

a Has she made a profit or a loss? How much is the profit or loss?

b Calculate the percentage profit/loss.



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6 Keira buys a second-hand car for $2500 and spends $1750 replacing some parts. At the end of the year she sells it for $3750.

a Does she make a profit or a loss? How much is the profit or loss?

b Calculate the percentage profit or loss.

c Comment on whether you think Keira would be happy with this outcome.

7 Matt needs to make 65% profit on the items in his shop to cover costs. What is the minimum price he can charge for hiking boots that cost him $165?

8 Mardi buys 2000 shares valued at $23 400. She pays a stockbroker's fee of 2.5% of the value of the shares. She later sells the shares for $10.55 per share.

a Calculate the total amount Mardi paid for the shares.

b How much money does she get for her shares when she sells them?

c Has she made a profit or a loss on the shares?

d Calculate correct to the nearest whole number the percentage profit or loss on the shares.

9 Gino buys 7 boxes of bananas for $45 each. Each box contains 10 kg of bananas. He sells 4 of the boxes at $5.10 per kg. He reduces the price to $4 per kg to clear the remaining boxes before the fruit goes off.

a How much do the bananas cost Gino?

b How much does Gino earn from the bananas?

c Does Gino make a profit or a loss? Calculate Gino's percentage profit or loss. Answer correct to one decimal place.



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10 A shoe shop bought pairs of shoes for $48 each and priced them at $90 per pair. During the end-of-season sale the manager reduced the price by $20. Calculate the percentage profit the shop makes when it sells the shoes in the sale. Answer correct to 1 decimal place.

Percentage problems

2.7

In this exercise you will apply all the skills you have learned in this chapter to solve problems involving percentages.

EXERCISE 2.7

Percentage problems

1 Nelson Secondary College's hockey team won 12 games out of 15. What percentage is this?

2 55% of the students at one university are female. The university population is 85 300. How many *males* attend the university?

3 A muesli bar weighs 35 g.
   a It contains 6.9 g of fat. What percentage is this? Answer correct to one decimal place.
   b The muesli bar is 64% carbohydrate. How many grams is this?

4 Luka, a real estate agent, earns 2.5% commission on a sale of $345 000. How much was his commission?



Newspix/Ellen Smith

5 Natasha buys a second-hand car for $14 000 and sells it at a loss of 20%. Find its selling price.

6 Lachlan's pay rate of $18.45/hour increased to $19.85/hour. Find the increase as a percentage of the original pay rate. Answer correct to one decimal place.

7 Sarah earns $2346 per fortnight. Her rent is 30% of her income. How much rent does Sarah pay?

8 Alfredo's Spaghetti Sauce contains 11.9% sugar according to the label. How many grams of sugar are in a 580 g jar?

9 Zoe sells kitchenware by visiting homes and is paid a retainer of $320 per week to cover her expenses, plus a commission of 15% of all sales. Zoe's sales for the first week in April totalled $2896. Calculate her pay for the first week in April.

10 Andrea writes books with a recommended retail price of $54. She is paid 10% royalty on the first 4000 copies sold and 12.5% royalty on the remaining sales. Calculate the royalty Andrea will receive on the sales of 7000 copies of her book.

11 Jane is buying a house that had a price of $345 000 last year. House prices have risen by 1.5% since then. How much will the house cost now?

12 When a new model is coming, car dealers offer discounts on the current model. Tony is interested in buying a car that has a price of $36 990. The car dealer offered him this car for $31 990. What percentage discount is this?

13 Kaitlyn runs Computers R Us. She purchased a batch of laptop computers that cost her $395 each. She calculates their selling price by adding on 75% of the cost price. What is the selling price of one laptop computer?

14 Daniel's watch loses 3 minutes every 6 hours.

a What percentage time loss is this?

b Daniel set his watch correctly at 8 a.m. He finished work at 5 p.m. that afternoon. What time will his watch show when he finishes work?

15 So far this season, the Collingwood AFL team has scored 1150 points and had 1012 points scored against them. Calculate their points percentage using the formula below.

$$\text{Points percentage} = \frac{\text{points for}}{\text{points against}} \times 100\%.$$

16 The Costello family donates 2% of their annual income to medical research into dementia. Calculate how much they donated in a year when the family income is $225 000.

17 A 500 g can of chicken and corn soup has 110 g of corn in it. What percentage of the weight of the can of soup is corn?

18 Shanice bought her wedding dress for $1850. Several months after her wedding, she sold the dress on the Internet for $1280. Calculate her percentage loss on the wedding dress.

Investigation Uses of percentages

1 Look on the Internet, newspapers, magazines and brochures for examples of different ways we use percentages in real life. Find photographs or draw situations where percentages are used. Present the information you have found as a 1-page display or as a PowerPoint presentation.

2 Select an example from what you have found in part 1. Write a set of 10 questions for the example, using what you have learnt about where and how we use percentages in real life. Calculate the answers.

3 Swap the questions you have written with others in the class. Each person should answer at least 3 sets of questions. Ask the student who prepared the questions to mark your answers. If there are any disagreements, check with your teacher.

Keyword activity

Each of the words below has been used in this chapter.

commission	cost price	decrease	discount
GST	increase	interest	loss
profit	selling price		

1 Some of these words are opposites. Write down two pairs of words that are opposites. Write a short paragraph that shows the meanings of each of these words.

2 Which word is a way of earning money? Write a sentence explaining its meaning.

3 Describe the difference between **cost price** and **selling price**.

4 There are three remaining words: discount, GST and interest. Write a sentence explaining the meaning of these words.

Chapter Problem

Tanika sells cosmetics. She earns commission at the following rates.

Commission on Tanika's monthly sales	
First $500 of sales	5%
On the next $1000	4%
Remainder of sales	3.5%

Calculate Tanika's commission in a month when her sales were $4200.

Solution

Commission on first $500 = 5% of $500
= $25

Commission on next $1000 = 4% of $1000
= $40

Remainder of sales = $4200 – $500 – $1000
= $2700

Commission on remainder = 3.5% of $2700
= $94.50

Total commission = $25 + $40 + $94.50
= $159.50



Chapter Problem

Jake is going to Scotland to bag (climb) as many Munros as he can. Munros are Scottish mountains that are more than 3000 feet high. The Scottish mountain Ben Alder is 1148 metres high. Is Ben Alder high enough to be one of Jake's Munros?



CHAPTER 3

METRES, LITRES AND GRAMS

3.1 Estimating lengths
3.2 Measuring lengths
3.3 Measuring capacity, mass and time
3.4 Reading medical scales
3.5 Measuring medicine
3.6 Error in measurement
3.7 Perimeter
3.8 Pythagoras' theorem
Keyword activity
Solution to the chapter problem

By the end of this chapter, you should know how to:

- estimate and measure lengths
- convert between metric units for length, capacity, mass and time
- use and interpret non-metric units
- apply measurement to medical scales and medicine dosages
- calculate perimeters
- apply Pythagoras' theorem for right-angled triangles

How are we ever going to use this?

- Estimating lengths when we don't have measuring equipment
- Converting different metric units
- Calculating the length of fencing we require
- Building and renovating

3.1 Estimating length

There are a few tricks we can use to estimate the length of an object. Our body parts can act as rulers for us; they're convenient and we always have them with us! If you record the length of your little finger, the width of your palm, the length of your shoe and the length of your stride on your phone, you can use them to estimate lengths. Your phone probably has a calculator that will help too!

The photos below show some useful lengths. (Your own measurements may be slightly different from these.)



iStockphoto/Hanis



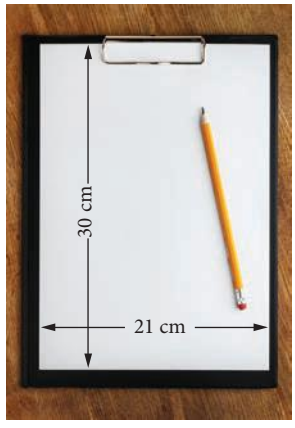
Scout Kozakiewicz



iStockphoto/Marek Mnich



iStockphoto/Sandymaya



Shutterstock.com/Boltenkoff

Example 1

Paula used her shoe to estimate the length of a table. She counted that it was 5 shoes long. Approximately how long is the table?

Solution

According to the photo, a shoe is approximately 30 cm long, so multiply the number of shoes by 30.

$5 \times 30 = 150$ cm

Write your answer.

The table is approximately 150 cm long.

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EXERCISE 3.1

Estimating length

Use the photos on the previous page to help you answer questions 1, 2 and 3.

EXAMPLE 1

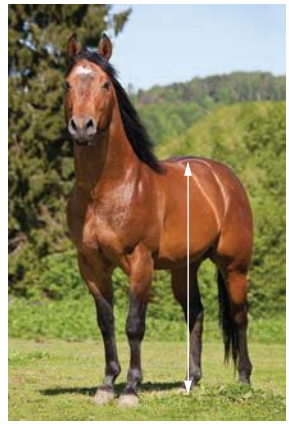
1 Emma's verandah is 8 shoe lengths wide. Approximately how many centimetres wide is Emma's verandah?

2 Copy and complete this table.

	Item	Length in body parts or other useful measures	Approximate length in cm
a	Children's story book	4 little fingers	
b	Length of a work bench	4 lengths of an A4 sheet of paper	
c	Height of a cake	2 thumbs	
d	Height of a skate board jump	3 widths of an A4 sheet of paper	
e	Child's height	$3\frac{1}{2}$ hand spans	
f	The distance across the room	11 paces	

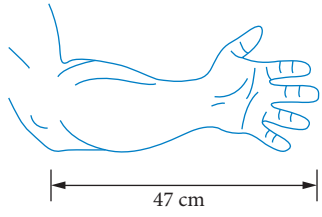
3 **a** This horse is 14 hand widths high. Calculate the height of the horse in cm.

b Why do you think we measure a horse's height as it is shown in the photo and not as the distance of the top of its ears to the ground?

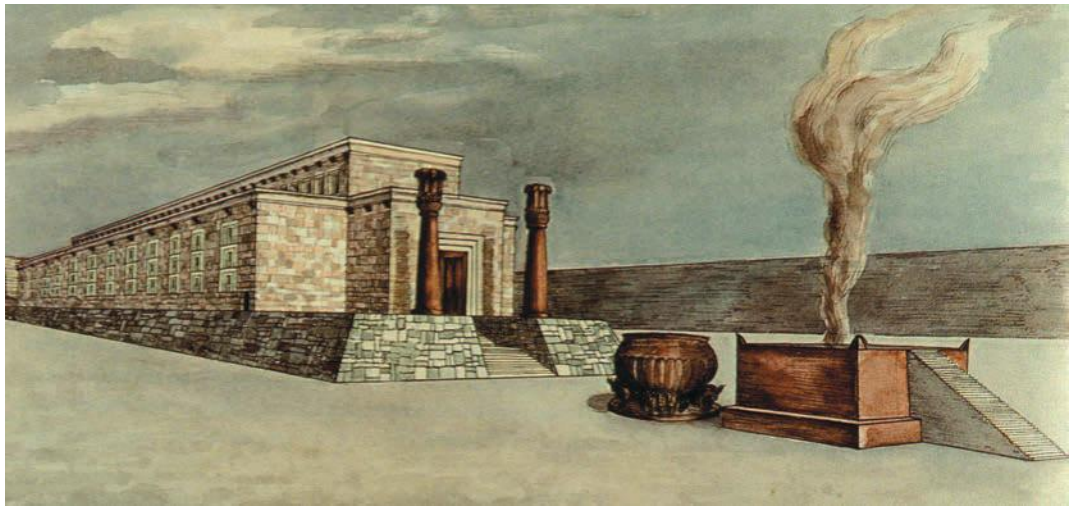


Shutterstock.com/Lenkadan

4 A **cubit** is the distance from the top of our fingers to our elbow. Alan measured a piece of rope as 15 cubits long. Calculate the length of the rope in cm.



5 The Bible includes the measurements of King Solomon's Temple, which is thought to have been built around 1000 BCE. The temple's length was 60 cubits, its width 20 cubits, and its height 30 cubits.

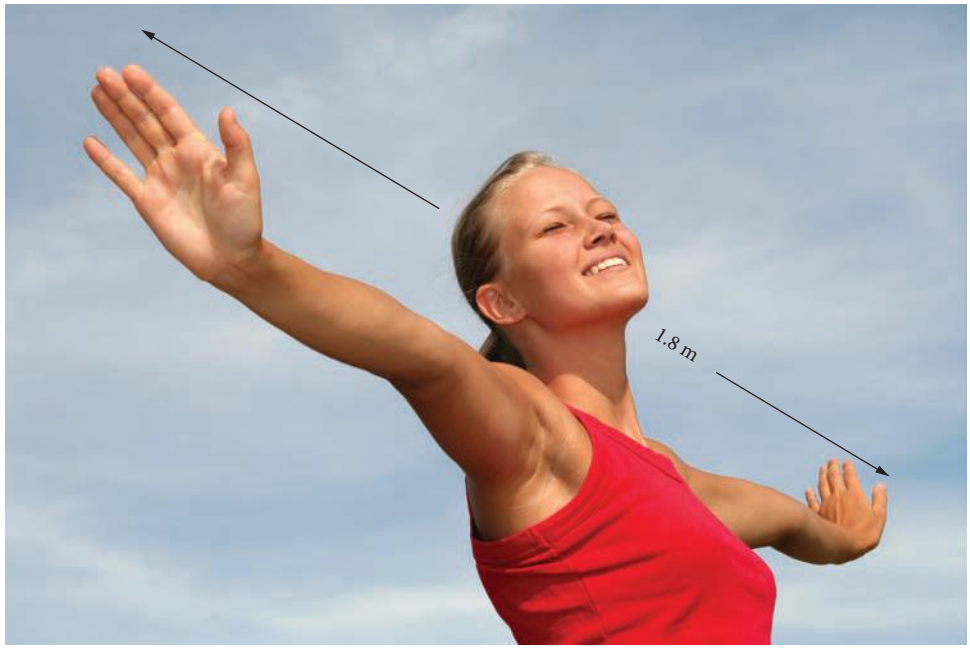


Alamy/www.BibleLandPictures.com

a Calculate the length, width and height of Solomon's Temple in centimetres.

b Divide each measurement by 100 to convert them to metres.

6 A **fathom** is an old English measurement that we can use to help us approximate the length of fabric or fencing wire. A fathom is the distance from the finger tips on one hand to the finger tips on the other when the arms are stretched wide. A fathom is 1.8 metres long. Alexis measured that she has 8 fathoms of fabric. How many metres of fabric does she have?



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Investigation Your body as a ruler

To complete this investigation, each member of your group needs a ruler and a copy of the table below, which can be downloaded from the NelsonNet website.

Part 1

Measure each body part and record the length in the table.

Name:		
Body part	**Approximation use**	**Length**
Top section of thumb	Small lengths	
Little finger	Small lengths	
Hand span	Short lengths	
Foot (shoe)	Short distances on the ground	
Pace	Walking distances	
Fathom	Lengths of fabric or rope	

Part 2

Use your body measurements and a calculation to approximate each length.

1. The length of your classroom
2. The width of a page from this book
3. The length of your calculator
4. The length of a pen
5. The height of your desk above the floor
6. The width of a window in your classroom
7. The distance from your classroom door to the school canteen
8. The height of the classroom door above the floor

Part 3

Use a ruler or tape measure to check the accuracy of your approximations. How good is your body as a ruler?

3.2 Measuring lengths

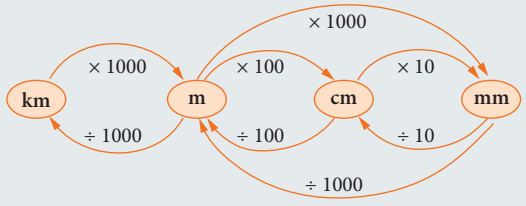
Using body parts to estimate lengths can be convenient, but it's not very accurate because measurements vary from person to person. In the 1790s, the French Academy of Sciences developed a standardised measurement system that was based on powers of 10 and it's what we use today, called the **metric system**.

In the metric system, the **metre** is the basic unit for measuring length. Originally one metre was defined to be one ten-millionth of the distance from the Equator to the North Pole at sea level.

Unit	Relationships	How we use it
kilometre (km)	1 km = 1000 m	Used to measure long distances For example, the distance between cities or countries
metre (m)	1 m = 100 cm 1 m = 1000 mm	Used to measure medium lengths For example, the length of a sports field or the width of a room
centimetre (cm)	1 cm = 10 mm	Used to measure small lengths For example, the dimensions of a brick
millimetre (mm)		Used to measure very small lengths For example, the length of an insect or the size of your toenail

To change from a small unit to a bigger unit, **divide** by the conversion factor.

To change from a big unit to a smaller unit, **multiply** by the conversion factor.



Example 2

Convert:

a 30 m to cm **b** 780 m to km **c** 65 cm to mm.

Solution

a There are 100 cm in 1 metre. The conversion factor is 100. Changing from m to cm is changing to a smaller unit. We need to multiply by the conversion factor.

$30 \text{ m} = 30 \times 100 \text{ cm}$
$= 3000 \text{ cm}$

b There are 1000 m in 1 km. Changing from m into km is changing into a bigger unit. We need to divide by the conversion factor.

$780 \text{ m} = 780 \div 1000 \text{ km}$
$= 0.78 \text{ km}$

c There are 10 mm in 1 cm. Changing from cm into mm is changing into a smaller unit. We need to multiply by the conversion factor.

$65 \text{ cm} = 65 \times 10 \text{ mm}$
$= 650 \text{ mm}$

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Non-metric units

Before Australia adopted the metric system in 1974, we used the Imperial system of measurement based on feet (for length), pounds (for mass) and gallons (for capacity). The Imperial system is still used in the USA today.

Sometimes we use non-metric lengths. Distances at sea and in the air are often given in **nautical miles**. The depth of water can be stated in **fathoms**. Many people still use **miles**, **yards**, **feet** and **inches** to measure distances. The conversion factors are shown in the table below for interest. Note that they are *not* based on powers of 10.

Unit	Metric equivalent	How the unit is used
inch	2.54 cm	To measure small lengths.
foot	30.48 cm	To measure small lengths. 1 foot = 12 inches
yard	0.9144 m	To measure lengths where we would use metres today. 1 yard = 3 feet
fathom	1.8288 m	To measure the depth of water. 1 fathom = 6 feet
furlong	201.168 m	A length used in horse racing.
mile	1.609 km	To measure large distances where we would use kilometres today. 1 mile = 1760 yards
nautical mile	1.852 km	To measure long distances across water or in the air.

Example 3

The distance from Adelaide to Perth by air is 1144 nautical miles. What is this distance in kilometres?

Solution

1 nautical mile = 1.852 km. Changing to km is changing to a smaller unit. Multiply by the conversion factor.	1144 nautical miles = 1144 × 1.852 = 2118.688 km
Write the answer.	The air distance from Adelaide to Perth is 2119 km correct to the nearest km.

Example 4

The pattern for the shawl Rhea plans to knit requires 856 yards of wool. She has 820 m of wool. Is this enough?

Solution

1 yard = 0.9144 m. Changing 820 m into yards is changing to the bigger unit. We have to divide the number of metres by 0.9144.

820 m = 820 ÷ 0.9144
= 896.7629 yards

Answer the question.

Rhea needs 856 yards of wool and she has about 896 yards. She has enough wool for the shawl.

EXERCISE 3.2

Measuring lengths

EXAMPLE 2

1 Copy and complete each conversion.

a 3 cm = ___ mm
b 5 m = ___ cm
c 400 m = ___ km
d 2 km = ___ m
e 30 mm = ___ cm
f 200 cm = ___ m
g 500 mm = ___ m
h 250 m = ___ km
i 60 cm = ___ m
j 60 cm = ___ mm
k 4500 m = ___ km
l 0.8 km = ___ m
m 8 mm = ___ cm
n 90 m = ___ cm
o 90 m = ___ km
p 6.5 m = ___ mm

2 Follow each step to convert 0.64 km into cm.

a Change 0.64 km into m.
b Change your answer to part **a** into cm.

3 Follow each step to convert 85 000 cm into km.

a Convert 85 000 cm into m.
b Change your answer to part **a** into km.

4 Copy and complete each conversion.

a 0.45 km = ___ cm
b 4 800 000 mm = ___ km
c 1.3 km = ___ mm
d 41 750 cm = ___ km

5 Every morning Scott goes swimming training. This morning Scott completed 78 laps at training. Each lap was 50 m long.

a How many metres did Scott swim this morning?
b How many kilometres did Scott swim at training this morning?
c Scott's coach wants him to swim 4.5 km tomorrow. How many laps of the 50 m pool will Scott have to complete to swim 4.5 km?

6 Brent is making 75 cm-long elastic bandages from a bulk roll of bandage 120 m long. How many bandages will he be able to make?

7 When Yasmin went canoeing she paddled down three 540-metre rapids and two 860 metre rapids.

a Calculate the total length of the rapids in metres.

b Express the distance in kilometres.

8 Copy and complete each conversion, correct to 1 decimal place. EXAMPLE 3

a 80 nautical miles = ___ km **b** 15 inches = ___ cm **c** 5 furlongs = ____ m

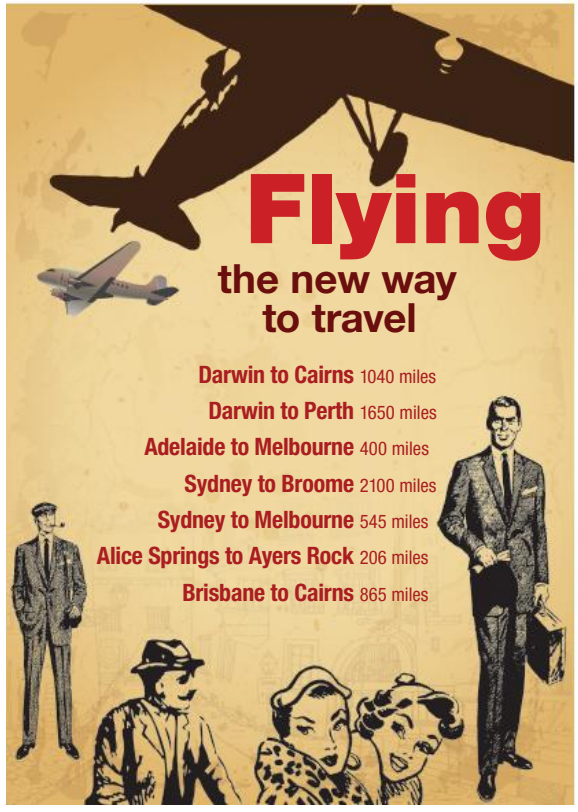
d 2000 feet = ___ m **e** 4 fathoms = ___ m **f** 7 miles = ___ km

9 Convert 120 yards to metres correct to 1 decimal place. EXAMPLE 4

10 Which is the longer horse race: over 8 furlongs or over 1600 m?

11 An old navigation chart shows that the water is 9 fathoms deep. How deep is the water in metres? Answer correct to 1 decimal place.

12 This travel poster is from the 1950s. Design a new poster that shows the distances in km.



13 Rolls of carpet are traditionally 12 feet wide. What is the width of a roll of carpet in metres correct to 2 decimal places?

14 Before we used metric lengths in Olympic events, Olympic swimming pools were 55 yards long. Now Olympic swimming pools are 50 m long. Are the Olympic swimming pools we use now longer or shorter than the old pools, and by how much?

15 How many furlongs are there in a mile?

3.3 Measuring capacity, mass and time

Unit	Relationships
Capacity	
millilitre (mL)	
litre (L)	1 L = 1000 mL
kilolitre (kL)	1 kL = 1000 L
megalitre (ML)	1 ML = 1 000 000 L
Mass	
milligram (mg)	
gram (g)	1 g = 1000 mg
kilogram (kg)	1 kg = 1000 g
tonne (t)	1 t = 1000 kg
megatonne (Mt)	1 Mt = 1 000 000 t
Time	
second (s)	
minute (min)	1 min = 60 s
hour (h)	1 h = 60 min = 3600 s
day (day)	1 day = 24 h

Example 5

Convert:

a 2500 g to kg **b** 1.2 kL to mL

Solution

a 1 kg = 1000 g. Because we are converting to a larger unit (kg), we divide by 1000.

$2500 \text{ g} = 2500 \div 1000$

$= 2.5 \text{ kg}$

b First convert kL to L, and then L to mL.

1 kL = 1000 L and 1 L = 1000 mL.

Because we are converting to a smaller unit (mL), we multiply.

$1.2 \text{ kL} = 1.2 \times 1000 \text{ L}$

$= 1200 \text{ L}$

$= 1200 \times 1000 \text{ mL}$

$= 1\,200\,000 \text{ mL}$

After the decimal point

While 1.5 always means one and a half, 1.5 hours doesn't mean 1 hour and 5 minutes and 1.5 years doesn't mean 1 year and 5 months. The degrees-minutes-seconds key DMS or °′″ on a scientific calculator can be used to convert decimal hours to hours and minutes, or decimal minutes to minutes and seconds.

Example 6

What does the .45 represent in each measurement?

a $16.45 **b** 16.45 metres

Solution

a When we're writing money the first two numbers to the right of the decimal point represent cents.

In the amount $16.45, the .45 represents 45 cents.

b When we write a measurement in metres the first two numbers to the right of the decimal point represent centimetres.

In the amount 16.45 m, the .45 represents 45 cm.

Example 7

Write 2.7 hours in hours and minutes.

Solution

Use the DMS or °′″ key to change 2.7 hours to hours and minutes:

2.7 DMS gives 2° 42′ 0″

2.7 hours = 2 hours 42 minutes

Or can change 0.7 hours into minutes first.

0.7 hours = 0.7 × 60
= 42 minutes

Write the answer.

2.7 hours = 2 h 42 min

EXERCISE 3.3

Measuring capacity, mass and time

1 What unit would be the most suitable to use if you were measuring:

a the time it takes to eat lunch?
b your best friend's mass?
c how long it takes for a plant seed to shoot?
d the amount of medicine on a teaspoon?
e the amount of water in a bucket?
f the time it takes to drive from Sydney to Melbourne?
g the mass of one egg?
h the amount of water in an Olympic-sized swimming pool?
i the mass of a truck?
j how long it takes to tie your shoe lace?
k the mass of a feather?

EXAMPLE 5

2 Convert each mass to grams.

a 1800 mg **b** 9.5 kg **c** 365 mg **d** 0.2 t

3 Convert each capacity to litres.

a 12.2 kL **b** 6340 mL **c** 550 mL **d** 0.98 kL

4 Copy and complete each conversion.

a 32 min = _____ s **b** 550 g = _____ kg

c 45 g = _____ mg **d** 15.2 L = _____ mL

e 0.8 h = _____ min **f** 96 h = _____ days

g 65 700 L = _____ kL **h** 16 h = _____ min

i 12 300 kg = _____ t **j** 3.2 t = _____ g

k 3 days = _____ min **l** 56 800 mg = _____ kg

5 Hamish delivers 6750 kg of grapes to the market in Sydney. How many tonnes is this?

6 A large egg weighs 60 g on average. The eggs are sold in cartons containing 12 eggs per carton.

a What is the mass of eggs in one carton?

b For delivery to the supermarket, the eggs are packaged in containers holding 24 cartons each. How many kilograms of eggs are there in a package?

c The vehicle transporting the eggs can carry 20 of these packages. What is the mass of eggs transported in kilograms and in tonnes?

7 Elizabeth has some pet rabbits which she feeds with a special milk supplement. Each rabbit is given 125 mL of the supplement each day.

a How many litres of the supplement will she need per day to feed 12 rabbits?

b Elizabeth buys the supplement in 4.5 L containers. How many 125 mL feeds will it make?



iStockphoto/Graffizone

8 It takes Yestin $3\frac{1}{2}$ minutes to sow each row of corn. Today he intends to plant out 48 rows. How many hours will it take him?

EXAMPLE 6

9 What does the .24 represent in each measurement?

a $7.24 **b** 7.24 m

10 What does the .9 represent in each measurement?

a $18.9 **b** 18.9 m

EXAMPLE 7

11 Convert each time to hours and minutes.

a 2.5 h **b** 3.8 h **c** 1.4 h **d** 2.9 h

12 Follow these steps to convert 4.5 years into years and months.

- **a** How many months are there in 1 year?
- **b** Multiply the number of months in a year by 0.5.
- **c** 4.5 years is 4 years and the number of months you calculated in part **b**.

13 Convert 2.25 years into years and months.

14 **a** In the time 5.75 years, how many months are equivalent to 0.75 years?

- **b** Write 5.75 years in years and months.

Investigation Estimating mass

To complete this investigation your group will need a set of scales that measures small masses, for example a set of kitchen scales, and scales that measure weights in kg, for example a set of bathroom scales. You will also need a range of items to estimate their mass.

This table shows some common items and their approximate mass.

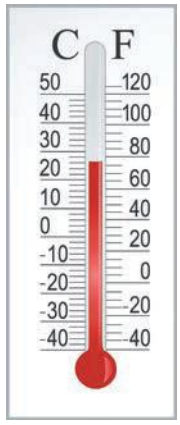
Mass	Items with this approximate mass
1 mg	grain of sand tiny insect like a sandfly
2 g	5c coin
100 g	iPhone without a case
1 kg	1 litre of water 5 medium-sized oranges
50 kg	A thin woman or fashion model
100 kg	A tall, solid or muscular man

What you have to do

1. Choose an item, for example a debit card.
2. Select an item in the right column of the table that has a similar mass. For example, a debit card is similar in mass to a 5c coin.
3. Compare the two items and estimate the mass of the unknown item. A debit card is about twice as heavy as a 5c piece. It will be about twice as heavy as 2 g. It will be about 4 g.
4. Use the scales to check the accuracy of your estimate.

Investigation Celsius vs Fahrenheit

There are two different units for measuring temperature: degrees Celsius (°C) and degrees Fahrenheit (°F).



Wolfgang Grossmann Wolfgang/123RF Stock Photo

1 Investigate the Celsius and Fahrenheit temperature scales. Which one belongs to the metric system?

2 Copy and complete this table to compare common temperatures.

Temperature	Celsius °C	Fahrenheit °F
Freezing point (temperature) of water		
Comfortable air-conditioning temperature		
Normal human body temperature		
The maximum temperature on a hot day		
Boiling point (temperature) of water		

3 Find a formula or graph to convert between the two temperature scales and use it to copy and complete each conversion.

50°C = _____°F

50°F = _____°C

4 American ovens use °F and Australian ovens use °C. Design a conversion table that shows common cooking temperatures in both scales.

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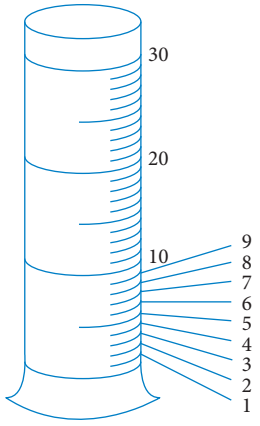
Reading medical scales

3.4

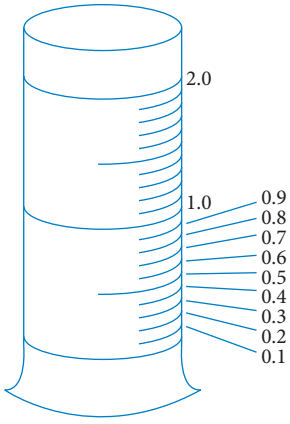
Medications are an important part of modern medicine. When we are measuring medications it is important to measure and interpret the quantities accurately. Making a mistake can affect someone's health and well-being.

Reading scales

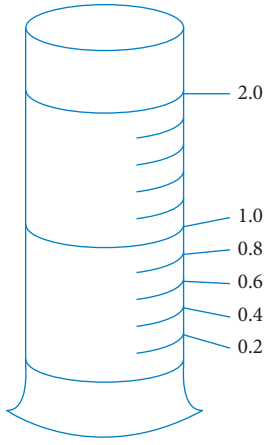
Scales always go up by a constant amount.



In this measuring cylinder the scale goes up by ones.



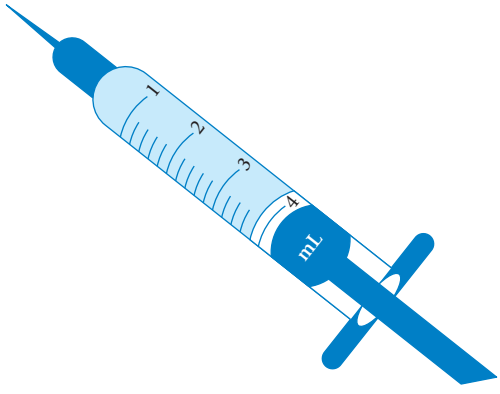
The scale on this cylinder goes up by 0.1.



The scale on this cylinder goes up by 0.2.

Example 8

How much penicillin is in this syringe?



Solution

The scale is going up by 0.2 mL.

The amount is between 3 and 4 so it will be 3 point something.

It's on the 4th line, $4 \times 0.2 = 0.8$.

There is 3.8 mL of penicillin in the syringe.

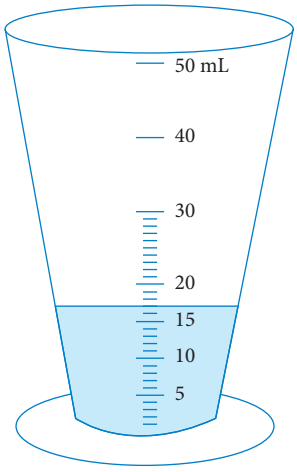
EXERCISE 3.4

Reading medical scales

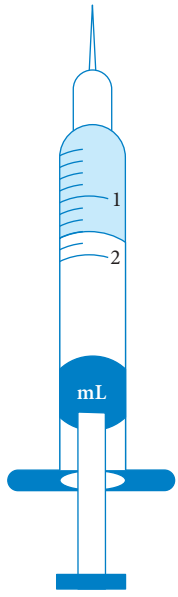
EXAMPLE 8

1 Read each scale carefully and write down the volume shown in each container.

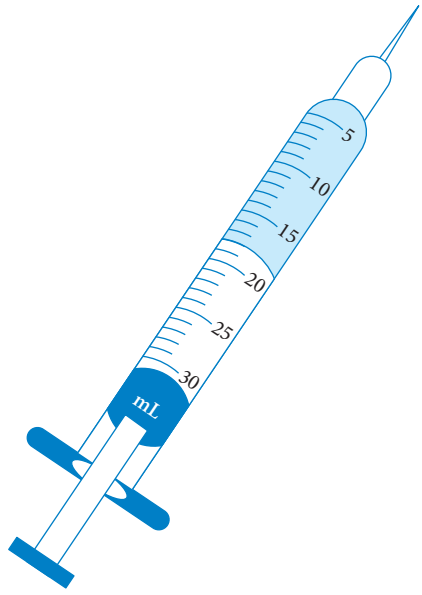
a



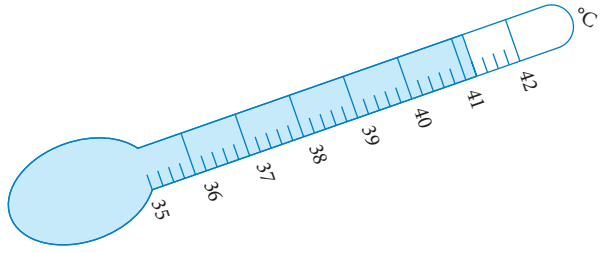
b



c



2 Joanne feels very hot. Her mother took her temperature with this thermometer. Read the thermometer to find out her temperature.



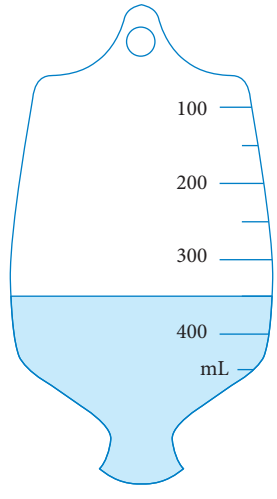
3 The scale on a 'drip bag' tells you how much of the drip has already gone into the patient. When it was full this bag held 500 mL.

a How much of the drip has the patient received already?

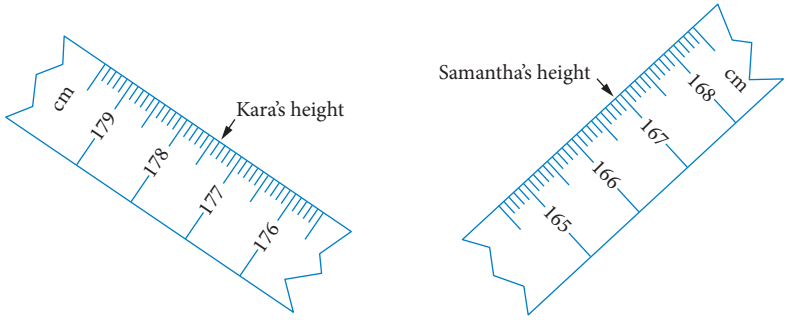
b How much is left in the drip bag?



iStockphoto/araraadt

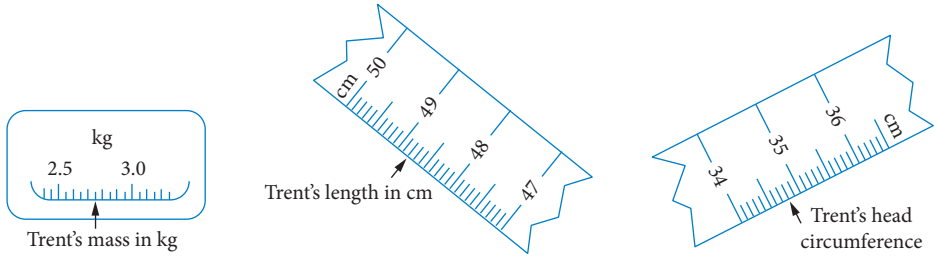


4 The doctor in the sports clinic measured Kara and Samantha's height.



How tall is each girl?

5 Trent is a newborn baby. The midwife measured Trent just after he was born.



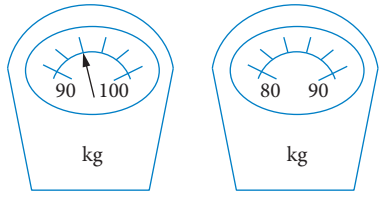
What is Trent's mass, length and head circumference?

For the rest of this exercise, copy each scale to write upon or print the 'Scales sheet' from NelsonNet.

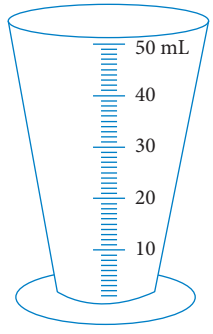
6 Paul's family doctor asked him to stand on the scales.

a How much does Paul weigh?

b Paul needs to lose 8 kg. His doctor told him to follow a special diet and to do regular exercise. Mark Paul's target mass on the set of scales.



7 Jody has a fever. Her doctor wants her to take 35 mL of paracetamol to help bring down her temperature. Show the volume of paracetamol Jody should take on the measuring container.



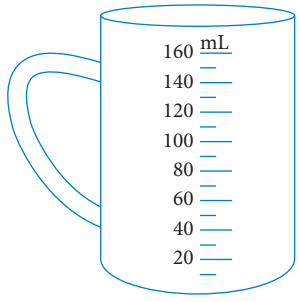
8 Anh is on a 'restricted fluids diet'. He is only allowed 1.2 L of liquid per day. The nurse gives Anh a drink every 2 hours. His first drink is at 0600 (6 a.m.) each day and the last drink at 2000.

a How many drinks is Anh allowed per day?

b Express 1.2 L in mL.

c How many mL is in each of Anh's drinks?

d Mark the level of Anh's drink on his cup.



3.5 Measuring medicine

A dose of medicine (liquid or tablet) is often quite small, usually measured in millilitres or milligrams.

Example 9

A 500 g bottle in the hospital pharmacy contains 12 500 pills. Calculate the mass of a single pill in mg.

Solution

Divide 500 g by 12 500 to calculate the mass of 1 pill. Then convert the answer to milligrams by multiplying by 1000.

Mass of 1 pill = 500 ÷ 12 500 g
= 0.04 g
= 0.04 × 1000 mg
= 40 mg

EXERCISE 3.5

Measuring medicine

EXAMPLE 9

1 A pharmacist ordered a 120 g bottle that contains 2000 tablets.
- **a** Calculate the mass of 1 tablet in g.
- **b** What is the mass of each tablet in mg?

2 Copy and complete each conversion.
- **a** 0.5 g = ____ mg
- **b** 700 mL = _____ L
- **c** 300 mg = ____ g
- **d** 0.4 L = ____ mL

3 After Anthony had a heart attack, his doctor told him to take one 100 mg aspirin tablet every morning to help stop his blood from forming clots.
- **a** How many grams of aspirin does Anthony take in a 30-day month?
- **b** The packet contains 168 aspirin tablets. Calculate the mass of the packet in grams.

4 The nurse on the mobile blood bank van gives everyone who donates blood a 150 mL glass of orange juice.
- **a** How much orange juice will the nurse use to fill 12 glasses?
- **b** How many glasses can the nurse fill from a 5 litre cask of orange juice?

5 When Natasha had a severe infection her doctor prescribed a penicillin injection. The injection contained 0.0022 L of liquid. How many millilitres is this?

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6 Vitamin C powder contains 90% ascorbic acid and 10% calcium.

- **a** How many milligrams of calcium are in 40 mg of Vitamin C powder?
- **b** How many milligrams of ascorbic acid are in 60 mg of Vitamin C powder?
- **c** Calculate the number of milligrams of calcium in 2.4 g of Vitamin C powder.

7 The total mass of a bottle of 500 tablets is 155 g. The bottle weighs 20 g.

- **a** Calculate the mass of the tablets in grams.
- **b** What is the mass of the tablets in milligrams?
- **c** Calculate the mass of one tablet in milligrams.

8 Ordinary table salt contains 40% sodium.

- **a** How much sodium is in 100 g of salt?
- **b** Kim is allowed 2 g of salt in her diet per week. How many milligrams of sodium is she allowed per week?

9 How many 50 mg injections can a doctor make from a 1 gram container of streptomycin?

10 Kate is 35 weeks pregnant and has a urinary tract infection. Her doctor prescribed her an antibiotic to clear up the infection before the baby is born. The appropriate dose is 20 mg per kg of body weight per day. Kate weighs 74 kg.

- **a** How many grams of antibiotic does she require per day?
- **b** Kate will take the medication every 3 hours. How many mg of antibiotics does she need in each dose?

11 Ceftriaxone is an antibiotic that treats pneumonia. The daily intravenous dose is 100 mg per kg of body weight, with a maximum dose of 4 g. How much antibiotic should a 65 kg pneumonia patient receive daily?



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3.6 Error in measurement

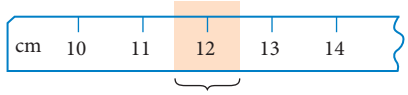
We make many types of errors when we take measurements.

Parallax error results from looking at the measuring scale from an angle instead of from directly in front.

Zero error happens if the measuring device is not properly set to zero beforehand or if the zero mark on a ruler is not placed exactly at the start of the object we are measuring.

Reaction error occurs when we measure time, with a stopwatch for example, when we take a short time to react at the start of the measurement.

We can avoid most of these errors with a bit of care. Another way we can reduce their effect is to take a number of measurements and average them.



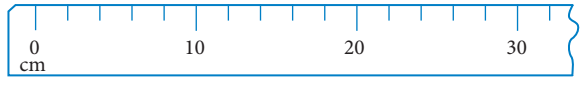
Even if we reduce these human errors, our measurements are only as good as the precision allowed by the measuring device we are using. The ruler's scale below has a **precision** of 1 cm (the size of one unit), so a measurement anywhere in the shaded zone will be recorded as 12 cm but can involve an error of up to half a centimetre either side of the 12 cm mark.

This type of error due to the precision of the measuring device is called the **tolerance** or **margin for error**.

The **tolerance** is half of the **precision** of the measuring device being used. We record the tolerance as ±0.5 of one unit.

Example 10

While repairing her boundary fence, Shannon measured the distance between the strands of wire to be 24 cm using the ruler shown.



a What is the precision shown on the ruler scale?

b What is the tolerance of Shannon's measurement?

c Between what range of values does the actual distance between the strands lie?

Solution

a There are 5 small divisions between 0 and 10 cm on the ruler so each small division is 10 ÷ 5 = 2 cm.

2 cm

b The tolerance is half the smallest division, that is ±0.5 × 2 = ± 1 cm

±1 cm

c So the shortest length the measurement could be is 24 – 1 = 23 cm, and the longest length is 24 + 1 = 25 cm.

The distance between the strands is between 23 cm and 25 cm.

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Example 11

Gina measured her weight on a set of bathroom scales to be 58 kg. What is the tolerance involved in this measurement?

Solution

Because the measurement is given to the nearest kilogram, we assume that the precision of the scales is 1 kg.

Tolerance = $\pm 0.5 \times 1$ kg = ± 0.5 kg ± 0.5 kg

EXERCISE 3.6

Error in measurement

1 For each measuring instrument shown, state:

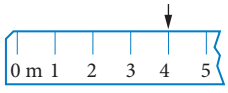
EXAMPLE 10

i the size of one unit

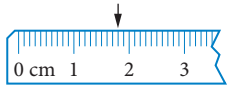
ii the reading to the nearest unit

iii the tolerance of the measurement

a



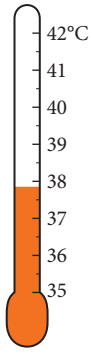
b



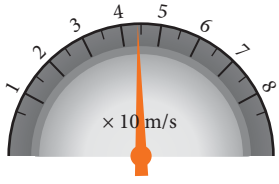
c



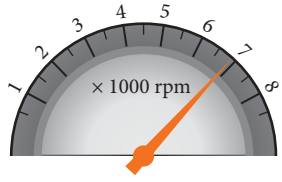
d



e



f



2 The time it takes Saxon to drive his tractor from one end of his property to the other was measured as 7 minutes.

EXAMPLE 11

a What is the tolerance for this measurement?

b What is this margin for error expressed in seconds?

3 Find the missing values in this table.

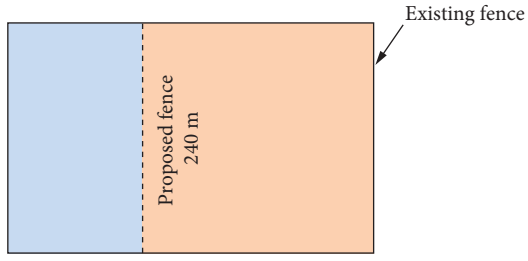
	Measurement	Tolerance	Smallest it could be	Largest it could be
a	18 L correct to the nearest litre	**i**	**ii**	**iii**
b	78 cm correct to the nearest cm	**i**	**ii**	**iii**
c	35 kg correct to the nearest 5 kg	**i**	**ii**	**iii**
d	9.4 seconds correct to 1 decimal place	**i**	**ii**	**iii**
e	730 km correct to the nearest 10 km	**i**	**ii**	**iii**

4 Lucy has decided to divide one of her existing paddocks into two by erecting a new fence. She has measured the width of the field to be 240 metres (to the nearest 10 metres).

a What is the tolerance of the measured width?

b Between what values will the true length of the fence lie?

c What length of fencing wire should Lucy buy to ensure she has enough to do the job?



5 James has listed the stock of a number of the materials he has in his farm shed. Copy and complete the table.

	Item	Quantity	Recorded to the nearest	Tolerance	Possible range of values
a	Fuel	280 L	5 L		
b	Fence posts	350	50		
c	Fencing wire	540 m	20 m		
d	Fertiliser	870 kg	10 kg		

6 We can obtain a more reliable measurement by taking repeated measurements and averaging them. Find the average value of each set of repeated measurements.

a Length of a shed: 12.3 m, 11.9 m, 12.2 m, 12.3 m, 12.1 m

b Mass of a carton of fruit: 15.9 kg, 15.7 kg, 16.1 kg, 15.5 kg

c Capacity of a water trough: 75 L, 77 L, 75 L, 72 L, 79 L, 77 L, 76 L

We find the average by adding all the measurements and dividing this result by the number of measurements.

Practical activity Taking accurate measurements

For this group activity you will need a 30 cm ruler, tape measure, trundle wheel, stopwatch, kitchen scales that measures in grams and kilograms and a 1 litre measuring cylinder.

Each group will measure:

a the longest width of the classroom

b the width of a classroom desk

c the thickness of a $2 coin

d the length of the longest building in the school

e the mass of 4 pencils

f the mass of 5 textbooks

g the volume of water in a glass tumbler

h the time it takes for a ball to hit the ground when dropped from the second floor.

1 Copy this table.

Measurement	Instrument	Units	Tolerance	Results of measurement					Average
				Group 1	Group 2	Group 3	Group 4	Group 5	
a									
b									
c									
d									
e									
f									
g									
h									

2 In your groups, make the measurements. Take care to avoid parallax and zero errors.

3 What problems were encountered in making the measurements?

4 Did you feel averaging all the groups' results gave a more reliable outcome?

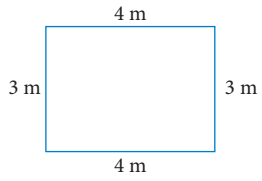
5 What errors other than calibration errors may have occurred in any of the measurements?

3.7 Perimeter

When we measure the distance around the outside of a shape, we are measuring its **perimeter**. Home renovators use perimeter calculations to work out the quantities of a variety of materials they need to order.

Example 12

Emma is putting a wallpaper **frieze** around the top of the walls to decorate the nursery for her new baby. The diagram shows the dimensions of the room.



The frieze is available in 5 m and 10 m rolls. How many rolls of frieze will Emma need?

Solution

Calculate the perimeter of the room by adding up the lengths of all sides.

Perimeter = 3 + 4 + 3 + 4 m
= 14 m

Then compare the length Emma needs to the lengths of the rolls.

Emma needs to buy one 10 m roll and one 5 m roll. She will have 1 m left over.



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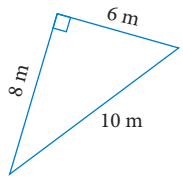
EXERCISE 3.7

Perimeter

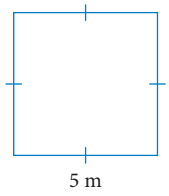
1 Calculate the perimeter of each shape.

EXAMPLE 12

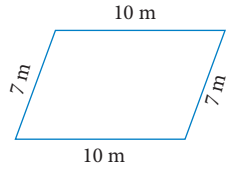
a



b



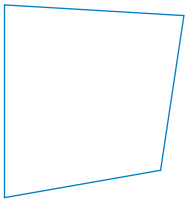
c



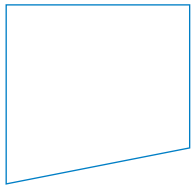
2 The sides of a **rectangle** are 4 m and 11 m long. Calculate the perimeter of the rectangle.

3 Measure the perimeter of each shape as accurately as you can.

a



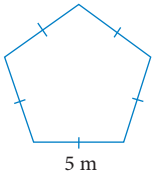
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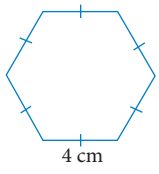
4 The perimeter of a rectangle is 36 m. What could the lengths of the sides of the rectangle be? Suggest two possible sets of sides.

5 All the sides in a **regular polygon** are the same size. The diagrams show three regular polygons. Write the name of each polygon and its perimeter.

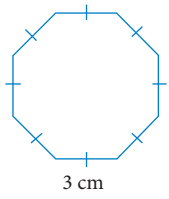
a



b



c

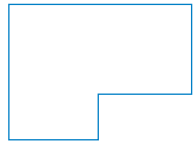


6 The perimeter of a 10-sided regular polygon is 60 m. How long are the sides of the polygon?

7 Katrina needs to replace the fence around her pool and BBQ area. In her **scale drawing**, 1 cm represents 1 m.

a What is the perimeter of Katrina's scale drawing?

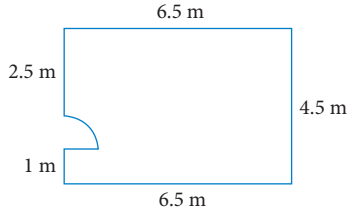
b Katrina is going to put a fence around the perimeter with one gate 90 cm wide in it. How long will the fence be?



8 Alex is replacing the skirting boards in his TV room. These are the strips of wooden boards that surround a room along the base of its walls.

a Calculate the length of skirting boards Alex will need.

b The skirting boards cost $9 per metre. How much will it cost Alex to buy the skirting boards?



3.8 Pythagoras' theorem

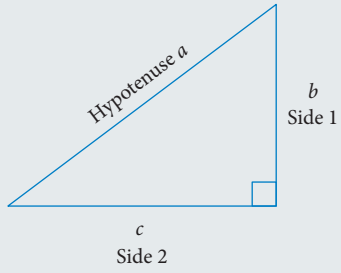
Pythagoras was an ancient Greek mathematician who lived from 580 to 500 BCE. His famous theorem relates the sides of a right-angled triangle.

Pythagoras' theorem states that in a right-angled triangle:

The square of the hypotenuse is equal to the sum of the squares of the other two sides.

$(\text{hypotenuse})^2 = (\text{side 1})^2 + (\text{side 2})^2$

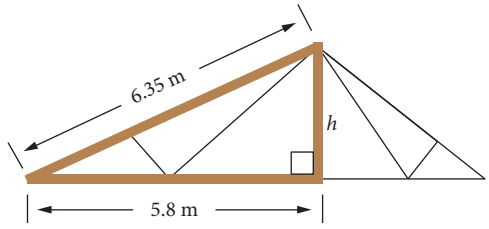
Sometimes we write the formula as $a^2 = b^2 + c^2$



The **hypotenuse** is the longest side of a right-angled triangle.

Example 13

Use Pythagoras' theorem to find the height (h) of this roof truss. Answer correct to 1 decimal place.



Solution

The hypotenuse is 6.35 and one side is 5.8.

Write Pythagoras' theorem.

Solve the equation for h.

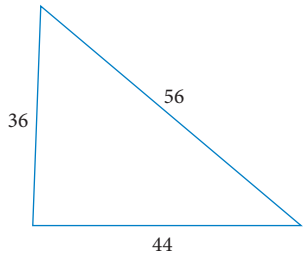
$$a^2 = b^2 + c^2$$
$$6.35^2 = 5.8^2 + h^2$$
$$6.35^2 - 5.8^2 = h^2$$
$$h^2 = 6.6825$$
$$h = \sqrt{6.6825}$$
$$= 2.585$$
$$\approx 2.6 \text{ m to 1 decimal place}$$

Testing for right-angled triangles

We can also use Pythagoras' theorem to test whether a triangle is right-angled. If the theorem works, then it is right-angled. If the theorem doesn't work, then it isn't.

Example 14

Is this triangle right-angled?



Solution

We need to check whether $56^2 = 36^2 + 44^2$.

If both sides of the equation are equal, then the triangle is right-angled.

$56^2 = 36^2 + 44^2$

$3136 = 1296 + 1936$

$3136 = 3232$

This statement is false.

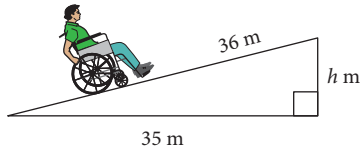
Pythagoras' theorem doesn't work.
The triangle is not right-angled.

EXERCISE 3.8

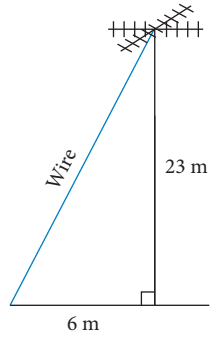
Pythagoras' theorem

EXAMPLE 13

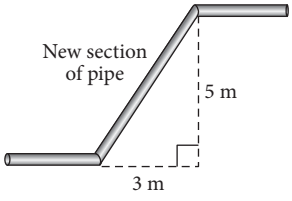
1 The diagram represents a wheelchair ramp. The 36 m long ramp covers a horizontal distance of 35 m. Use Pythagoras' theorem to calculate the rise, h m, of the ramp. Answer to 1 decimal place.



2 What length of wire is required to connect the top of a 23 metre TV antenna to a hook 6 metres from the base of the antenna? Answer correct to 1 decimal place.

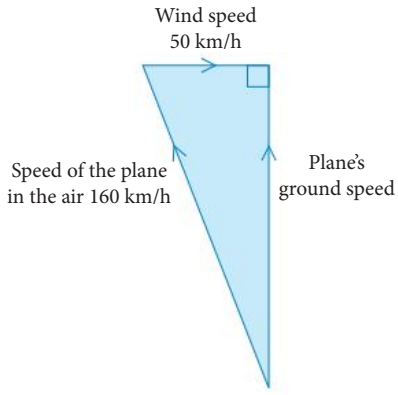


3 Stuart installs a new section of pipe to join two existing pipes. Calculate the length of the new section of pipe. Express your answer in metres correct to the nearest millimetre.



When a measurement is in metres, the nearest millimetre means three decimal places. The nearest centimetre is two decimal places.

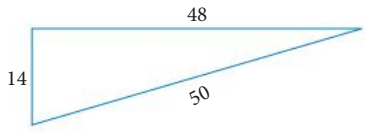
4 Ivana is flying a small plane at 160 km/h against a 50 km/h wind, as shown in the diagram. Calculate the plane's ground speed correct to the nearest km/h.



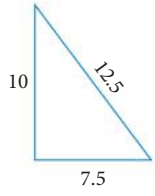
EXAMPLE 14

5 Use Pythagoras' theorem to test whether each triangle is right-angled.

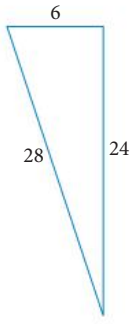
a



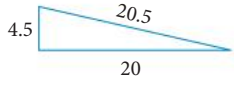
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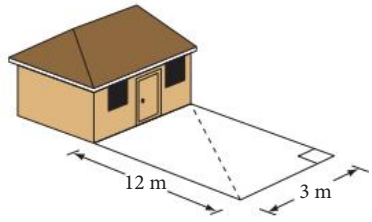
c



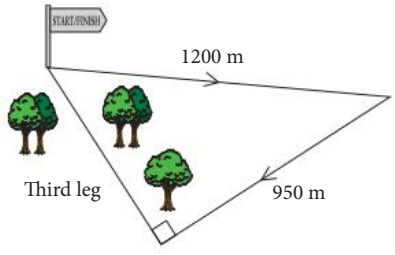
d



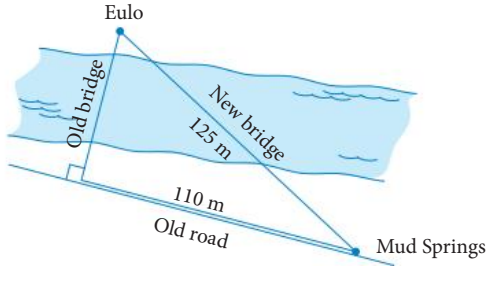
6 John is laying a concrete slab 12 m by 3 m in front of his shed. He uses Pythagoras' theorem to check that the corners of the slab are right angles. How long should the diagonal be? Express your answer in metres correct to the nearest centimetre.



7 The school cross-country course is in the shape of a right-angled triangle. The first leg is 1200 m and the second leg is 950 m. The third leg, through thick scrub, is difficult to measure. Calculate its length correct to the nearest metre.

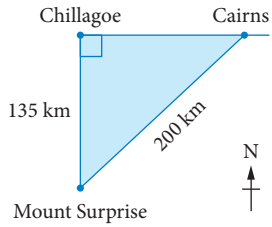


8 Belle had to cross the Paroo River to get from Mud Springs to Eulo. The new bridge across the river was closed for repairs so she had to use the old bridge. How much further did she have to travel using the old road and old bridge compared to the direct route across the new bridge? Answer to one decimal place.



9780170370752

9 David is the pilot of a small plane. He planned to fly 200 km from Mount Surprise to Cairns. Because of poor weather conditions between Mount Surprise and Cairns, David flew 135 km due north to Chillagoe then turned due east and flew to Cairns. Calculate the straight line distance from Chillagoe to Cairns correct to the nearest kilometre.



10 Sketch two different right-angled triangles that have a hypotenuse of length 24 m, writing the lengths of the other two sides, correct to 2 decimal places.

Keyword activity

Use keywords that you have learned in chapter 3 to complete the missing sections in this summary.

Throughout history people used different ways to measure items and it was quite confusing. In the 18th century the French developed a measuring system called the [1]_____ system. In this system the metre is the basic unit for measuring [2]_____. For longer amounts we use [3]_____ which are equivalent to 1000 m. We can use millimetres which are [4]_____ of a metre to measure small lengths.

We use length units when we measure [5]_____, the distance around the outside of a shape. We use units like mL and L when we measure [6]_____.

Chapter Problem

Jake is going to Scotland to bag (climb) as many Munros as he can. Munros are Scottish mountains that are more than 3000 feet high. Ben Alder is 1148 metres high. Is it high enough to be included in the Jake's list of Munros?

Solution

We need to convert 3000 feet to metres.

1 foot = 30.48 cm
= 30.48 ÷ 100 m
= 0.3048 m

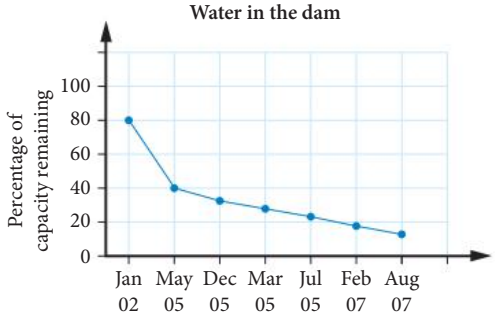
Converting to metres is converting to a larger unit, so multiply by the conversion factor.

3000 feet = 3000 × 0.3048 m
= 914.4 m

So Munros must be more than 914.4 m high. Ben Alder is 1148 metres high, which is higher than 914.4 m. Ben Alder can be included in Jake's list of Munros.

Chapter Problem

During a drought, the water authority produced a graph to show the decreasing supply of water in a dam due to the water usage of local residents. Is the decrease in water supply as bad as the graph seems to indicate? If the graph gives a false impression, identify what has been done to create this impression.





CHAPTER 4

SHOW ME THE GRAPH

4.1 Everyday graphs
4.2 Choosing the best graph
4.3 Graphs and spreadsheets
4.4 Line graphs
4.5 Misleading graphs
4.6 Dot plots and stem-and-leaf plots
Keyword activity
Solution to the chapter problem

By the end of this chapter, you should know how to:

- interpret information presented in graphs
- determine which type of graph is best used to display a set of data
- use spreadsheets to graph data
- draw a variety of graphs to represent data
- identify misleading graphs in the media and advertising

How are we ever going to use this?

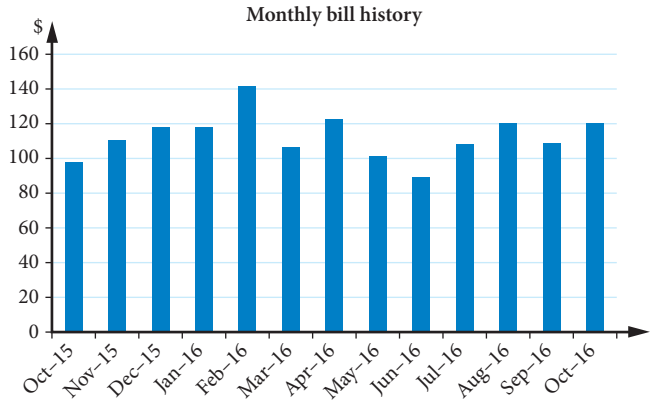
- When reading a graph that is presented in a newspaper, magazine or website
- When drawing a graph to illustrate data to others
- When noticing a graph is being misleading in a report or advertisement

4.1 Everyday graphs

EXERCISE 4.1

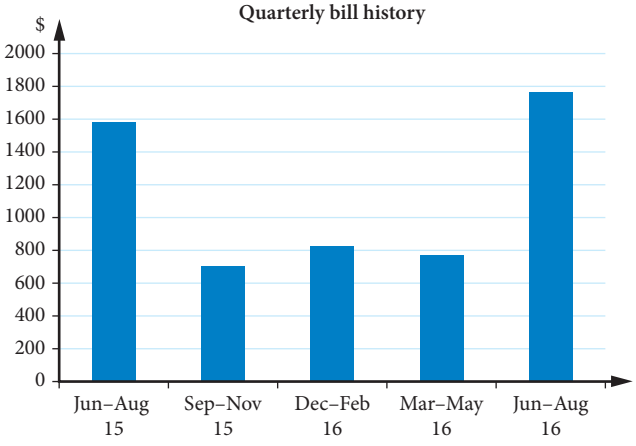
Everyday graphs

1 This column graph was found on a telephone bill.



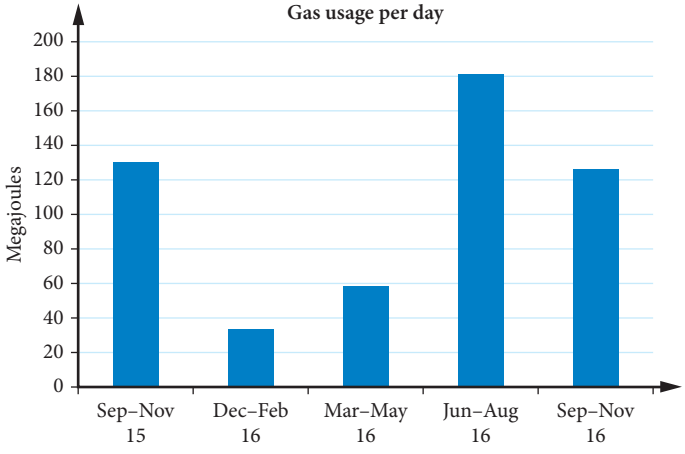
- **a** How much is the most recent bill?
- **b** How does this compare with the same time last year?
- **c** What is the lowest amount paid for one month?
- **d** Why would the telephone company include this in your telephone bill?
- **e** How accurate is it possible to be in reading this graph?

2 This graph comes from an electricity bill.



- **a** How many months does each amount cover?
- **b** What is the most recent amount paid?
- **c** Give a reason why the amounts in June to August are much bigger than for the other months.
- **d** Why is this type of graph useful to customers?

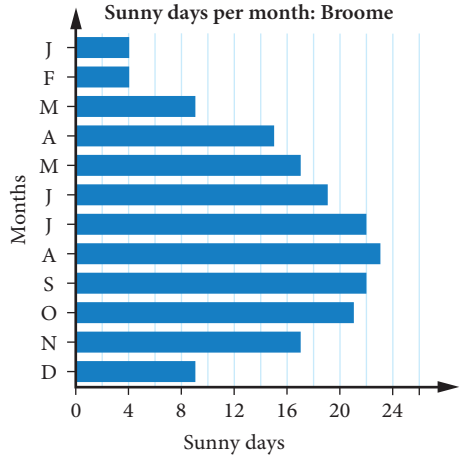
3 This graph comes from a gas bill and shows average daily gas usage for one household, in megajoules (MJ).



- **a** How many months in total does this graph cover?
- **b** What is the highest average daily gas usage?
- **c** At what time of year is the most amount of gas used?
- **d** Why might this occur at the time of year that it does?
- **e** Why is this type of graph useful to customers?

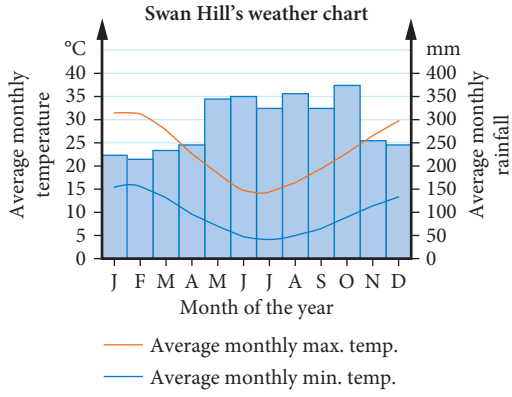
9780170370752

4 There are many graphs which present weather information. This graph shows the number of sunny days per month for 1 year in Broome, Western Australia.



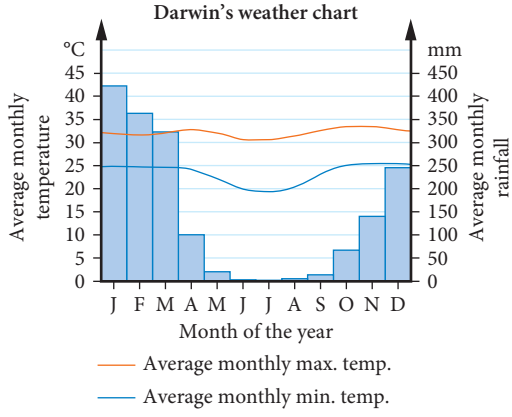
- **a** Which month had the most sunny days?
- **b** Which month had the fewest sunny days?
- **c** Which pairs of months had approximately the same number of sunny days?
- **d** How many months of the year had 16 or more sunny days?
- **e** Andrea is getting married in Broome. She would like a sunny day for her wedding. In which three months should she plan her wedding to be most likely to get a sunny day?
- **f** Give another reason why this information might be useful.

5 Climate graphs combine temperature and rainfall information. Swan Hill's climate graph is shown here.



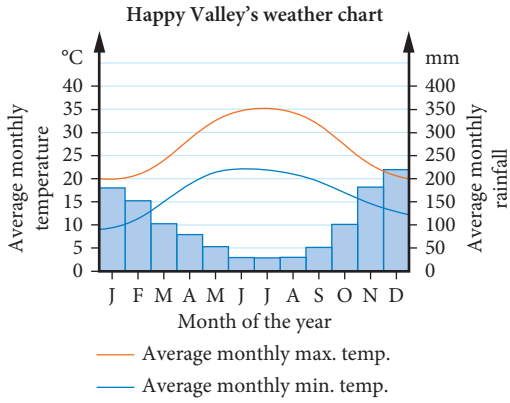
- **a** What is the average maximum temperature in Swan Hill in December?
- **b** Which month has the coldest minimum temperatures?
- **c** Describe Swan Hill's average annual rainfall pattern.

6 This is the climate graph for Darwin.



- **a** Which 3 months have the lowest rainfall?
- **b** What is the difference between the average maximum and minimum temperatures in Darwin in January?
- **c** How much more rain falls during January than during May?
- **d** Explain why it would be useful to have a graph like this for any place you might consider moving to.

7 Alexis found this climate graph for Happy Valley in an atlas. How can you tell that Happy Valley is the northern hemisphere of the world?

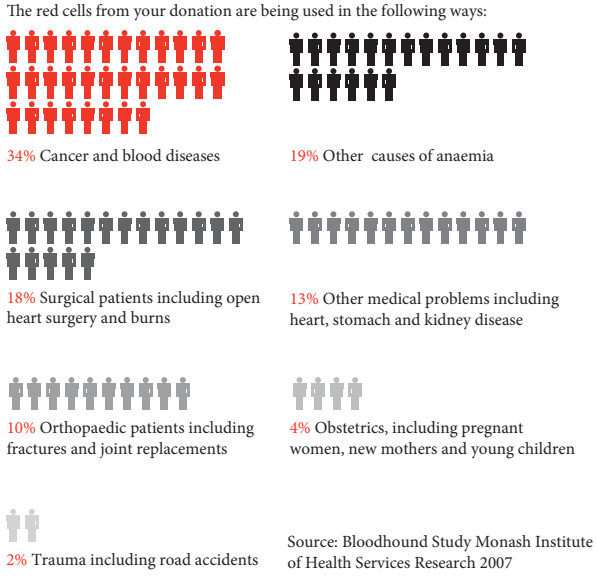




Shutterstock.com/gpointstudio

8 This is a graph from the website of the Australian Red Cross.

- **a** What does each person symbol stand for?
- **b** Explain how this graph helps you understand how blood donations are used.
- **c** Is there any information in this graph which surprises you? Explain your answer.
- **d** Does this graph make you more likely to donate blood?
- **e** If 27 000 litres of blood are collected in one week, how much goes to help patients with cancer and blood diseases?



Investigation Graphs in all subject areas

Graphs are used to record and present data across all subject areas. For each subject you study:

- find examples of graphs used in the subject. (Find as many different types of examples as possible.)
- name the types of graphs used.

Present your findings as a PowerPoint presentation or display them as wall charts.

- Which is the most common type of graph used across different subjects?
- Suggest a reason why this type of graph is used frequently.

Write a short paragraph on the uses of graphs in your areas of study. Explain the similarities and differences. Suggest reasons why different subjects use different types of graphs.

Choosing the best graph

4.2

We use different graphs for different types of data. In this exercise you will learn some hints to help you choose the best graph for each type of data.

EXERCISE 4.2

Choosing the best graph

1 Categorical data is often represented by a **column graph**. Draw a column graph for the following data showing the make of car owned by the families of a group of students in Year 11.

Make of car	Frequency
Holden	12
Ford	10
Mitsubishi	7
Subaru	5
Toyota	8
Other	3

A column graph has columns separated by spaces.

2 When we wish to show each category as a fraction of the whole we use a **pie graph**.

a The pie graph shows the causes of Australian children's deaths in 2011.

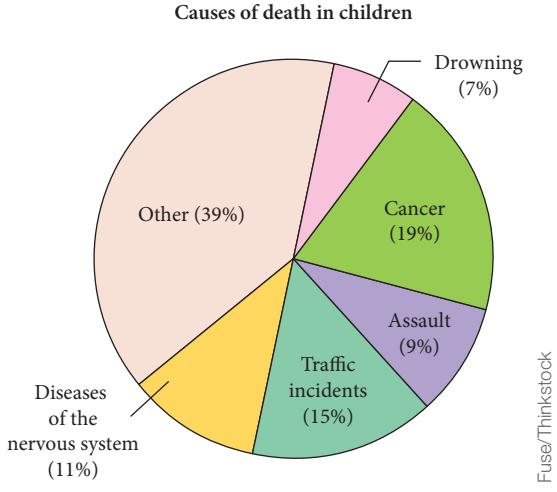
What percentage of the deaths were caused by drowning?

b What was the most common cause of death?

c Suggest what causes of death might be included under 'Other'.

d Drowning, assault and traffic incidents are considered preventable causes of death. What percentage of deaths are from something that could have been prevented?

e If there were 1769 child deaths in 2011, how many were preventable?



Fuse/Thinkstock

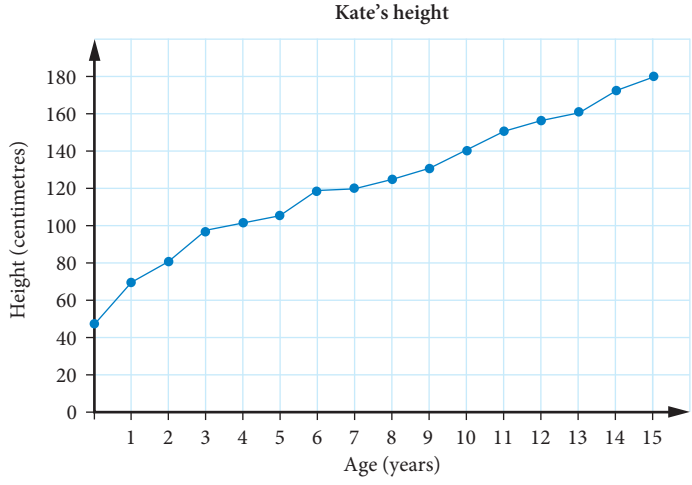
3 A survey of 1080 office workers asked how they travelled to work. This data is a summary of their responses:

Method of transport	Frequency	Angle for sector graph
Train	220	$\frac{220}{1080} \times 360 =$
Tram	280	$\frac{280}{1080} \times 360 =$
Bus	130	$\frac{130}{1080} \times 360 =$
Car	230	$\frac{230}{1080} \times 360 =$
Walk	80	$\frac{80}{1080} \times 360 =$
Cycle	140	$\frac{140}{1080} \times 360 =$

a Copy and complete the table to calculate the angle for each sector in a pie graph. Round each answer to the nearest degree.

b Draw a pie graph for this data.

4 A **line graph** is used to represent data that changes over time.



At birth Kate was 48 cm tall.

a What was Kate's height:

i on her first birthday? **ii** at age 10?

b At what age did Kate reach:

i 1 metre? **ii** 150 cm?

c Between which two birthdays did Kate grow the most?

d How long did it take Kate to double her height from birth?

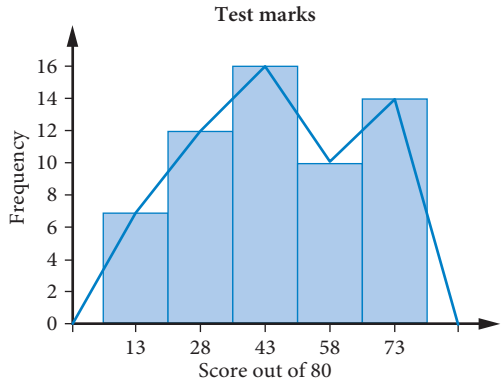
e What do you think the graph would look like from age 15 years to age 20 years?

5 This data shows the temperatures over a 24-hour period in Alice Springs in November.

Time	Noon	3 p.m.	6 p.m.	9 p.m.	Midnight	3 a.m.	6 a.m.	9 a.m.	Noon
Temperature	32°C	35°C	33°C	29°C	26°C	24°C	23°C	29°C	34°C

Draw a line graph for this data, showing Time on the horizontal axis and Temperature on the vertical axis.

6 A group of students was given a Maths test and the test marks out of 80 were grouped into class intervals, then graphed on this frequency histogram (column graph) and polygon (line graph).



a Copy and complete the frequency table using the graph.

Test marks	Frequency
6–20	
21–35	
36–50	
51–65	
66–80	
Total	

b How many students sat the test?

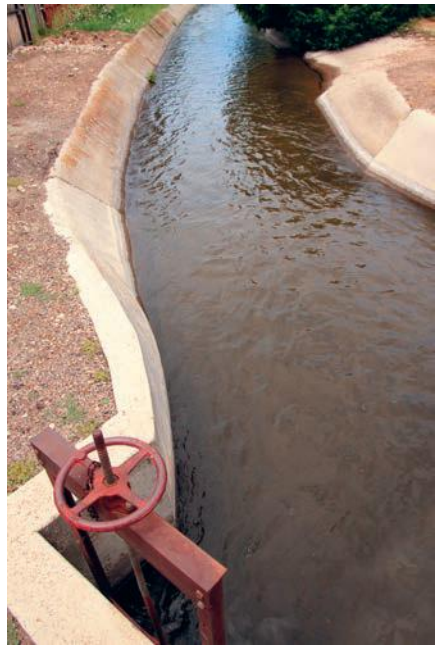
c What was the most common class of test marks?

d How many students scored less than 36 in this test?

7 The water level of a canal was measured over 44 days and recorded below.

Water level (cm)	Frequency
152	5
153	6
154	8
155	12
156	4
157	8
Total	44

Draw a frequency histogram for this data.



iStockphoto/Baxternator

4.3 Graphs and spreadsheets

In this exercise you will use Excel spreadsheet software to produce some of the more common graphs. Follow the instructions to create each graph, save it and show it to your teacher.

You will need to download the 'Statistical graphs' spreadsheet with the data for this exercise from NelsonNet. You should save this spreadsheet to your own computer.

EXERCISE 4.3

Graphs and spreadsheets

1 Column graph

To draw a column graph that shows the numbers of people immigrating to Australia by their region of origin in 1988–89:

- select all of the cells from A3 to B8
- select the **Insert** tab at the top
- under Charts, select **Column** and then the first of the 2D column types
- click on the chart title, highlight the text '1988–9' and change it to 'Immigration by region of origin'
- ensure the chart is selected (**Chart Tools** appears at the top of the sheet), select the **Layout** tab, then axis titles, go to **Primary Vertical Axis Title** and select **Rotated Title**. This will insert vertical script on the vertical axis. Highlight this script and change it to 'Thousands'.
- with the chart selected, choose the **Design** tab and pick **Move Chart**. In the dialogue box select the Object in option and immigration as the sheet and then OK. Move the whole chart to the top left of this sheet (click and hold mouse on border of chart, move to desired location and unclick).

2 Clustered column graph

Several years of information can be provided in the one graph by using clustered columns. For example, data for both years 1998–89 and 2008–09 can be shown.

- Return to the data sheet.
- Select all cells from B3 to C8.
- Select the **Insert** tab, and under **Charts** choose **Column** and select the first one of the 2D column types.
- Go to the **Layout** tab, click **Chart Title** and select **Above Chart**. Change the title text to the same as for the previous chart.
- Give the vertical axis the label 'Thousands' as for the previous chart.
- Move the chart to the immigration sheet as you did for the last one and place it just below the previous column chart.

3 Find the latest data for immigration by region of origin and add a third column to the data provided. Draw a clustered column graph for all three years. Place the graph on the immigration sheet.

4 **Pie graphs**

The same data will be used to draw sector charts of the immigration statistics to show the contribution from the various regions.

- Return to the data sheet and select cells A2 to B8.
- Select the **Insert** tab, and under **Charts** choose **Pie** and select the first 2D pie type.
- Check out the different possible **Chart Layouts** (top of screen). Try them out and choose the chart which you think shows the most information in the way you think looks best.
- Move the chart to the immigration sheet as you have done previously and place it just to the right of the first simple column chart.

  Now create another sector chart for the 2008–09 data. The steps are much the same as before except the initial selections of cells is a little more complicated.
- Return to the data sheet. Select cells A3 to A8. Then press and hold the Ctrl button while you select C3 to C8 with the mouse.
- Insert the pie chart as before.
- This time the title needs to be changed. Click the cursor in front of 2008–9 in the title and add 'Immigration to Australia by region of origin' in front of it.
- When you are happy with its format, place it below the first sector chart on the immigration sheet.

5 **Line graphs**

Use the data for temperatures during a spring day in Launceston in cells A24 to J25 of the data sheet.

- Select cells A24 to J25.
- Choose **Insert**, **Line** and select the first of the 2D chart types.
- Go to the **Layout** tab, go to **Chart Title** and select **Above Chart**. Replace the words Chart Title with 'Temperatures in Launceston'.
- Place the graph in the 'lines' worksheet.

6 Use the vehicle and home theft figures in cells A10 to F13 of the data sheet. Both will be plotted on the same graph. Draw a line graph for this data. Place the graph in the 'lines' worksheet.

7 Use the data on Bazza's gym from the data sheet (A17 to M20). Draw a line graph for this data. Place the graph in the 'lines' worksheet.

Investigation More graphs on spreadsheets

Other types of graphs can be created using a spreadsheet. For this investigation you will continue to use the 'Statistical graphs' spreadsheet you used in Exercise 4.3. You should have this spreadsheet saved to your own computer.

Scatter graphs

Part A

A scatter graph can be used to give some idea of whether two variables are related. In this example, data collected on the heights and foot lengths of 20 individuals will be used to see if there might be some relationship between foot size and height. This data is displayed in the data sheet in cells A27 to U29.

- Select cells A28 to U29 in the data sheet.
- Choose **Insert**, **Scatter** and select the first option.
- In **Chart Layouts**, select the first option (rightmost).
- Change the title to 'Foot length versus height', replace Axis Title on the bottom axis with 'Foot length (cm)' and Axis Title on the vertical axis with 'Height (cm)'.
- Click on the legend and delete it.
- Place the chart in the scatter sheet. Enlarge it in this sheet by pulling down the bottom right-hand corner.

Part B

The data in Part A was obtained from the Census at School website. Go to this website:

- obtain data for two variables that might be linked. For example, arm span and height, height of belly button and foot size, reaction times. Save this data in the spreadsheet to your own computer.
- Draw a scatter graph for your data. Complete appropriate titles and axis labels, and save it.

Radar charts

Radar charts are like a circular version of a line graph and are used for data that is recorded over time. You will be plotting the maximum average monthly temperatures for Darwin and for where you live.

We need to go to the Bureau of Meteorology website to collect the data for each place. The data for Darwin will go in cells B33 to M33. The data for your own town will go in cells B34 to M34 on the spreadsheet with all the other data.

To collect data:

- Search for and go to the Bureau of Meteorology home page
- Select **Climate Information**
- Go down to and click on **Weather and climate data – (Climate Data Online)**.
- Then, for Darwin, select as follows:
  - Data about: Temperature
  - Type of data: 'Monthly' then choose 'Mean maximum temperature'
  - Under 'Select a weather station' write 'Darwin' and click 'Find'
  - Click 'Darwin, NT' in the next window and 'Darwin Airport' in the next
  - Go down and click 'Get Data' (this will open a new window)
  - Scroll down to 'Summary Statistics' for all years and take monthly temperatures shown as 'Mean'. Enter these into cells B29 to M29.
- For your town, close the current window, scroll up and repeat the process for where you live.

Once the data is entered, draw the graph as follows.

- Select cells A32 to M34 in the data sheet.
- Choose **Insert**, **Other Charts** and then the first of the radar charts.
- Choose the first of the **Chart Layouts** and change Chart Title to 'Average Maximum Temperature'.
- Place the chart in the radar sheet and expand it.

Take some time to experiment with all the different types of graphs in the spreadsheet! You can changes colours, fonts, grids and all sorts of things.

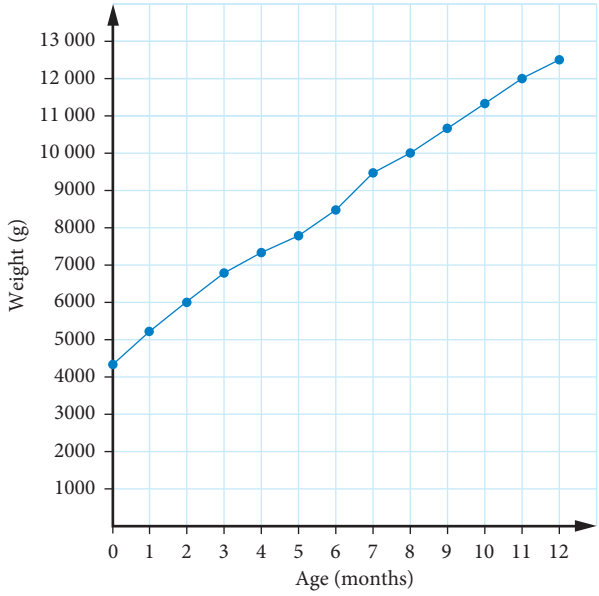
4.4 Line graphs

Line graphs are usually used to graph data that changes over time. They can also be used to compare two or more lots of data by colour-coding them.

EXERCISE 4.4

Line graphs

1 Simon was born weighing 4300 g. This line graph shows his weight over his first 12 months.



a What did Simon weigh at:

i 2 months? **ii** 7 months?

b When did Simon weigh:

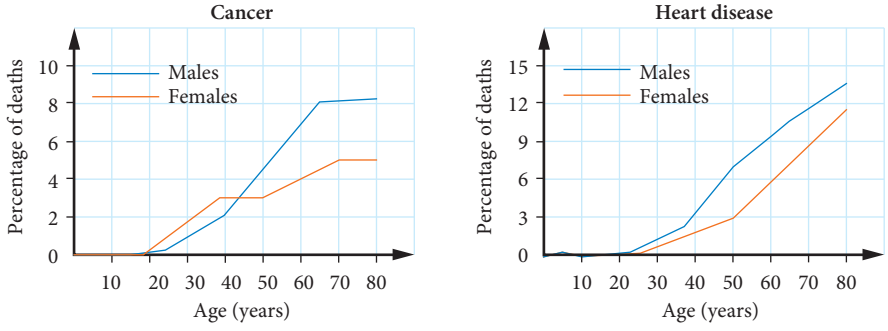
i 10 000 g? **ii** 12 000 g?

c Between which two months did Simon gain the most weight?

d Approximately how long did it take Simon to double his birth weight?

e Suggest what this graph might look like over the next 12 months. Give reasons for your answer.

2 These graphs show the percentage of deaths from cancer and heart disease at different ages.



a For males aged 70, what percentage of deaths are due to cancer?

b Between what two ages do 3% of women die of cancer?

c For males aged 50, what percentage of deaths are caused by heart disease?

d Generally, according to both graphs, what happens as people get older?

e 'More men die of heart disease than women.' True or false? Give one reason why.

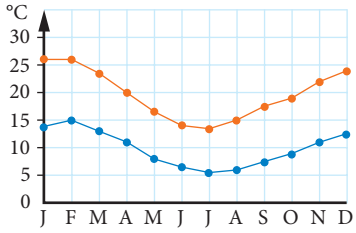
f At what age does the rate of cancer–related death in men increase?

g Between what ages do more women die of cancer than men? Discuss why.

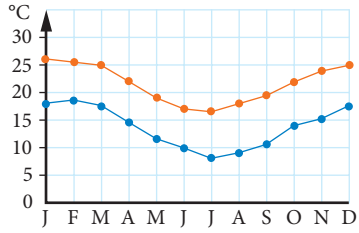
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3 The eight graphs below illustrate the mean (average) maximum (orange) and minimum (blue) monthly temperatures of the Australian capital cities (Adelaide, Brisbane, Canberra, Darwin, Hobart, Melbourne, Perth and Sydney) over a 12-month period, but they are presented in jumbled order.

A



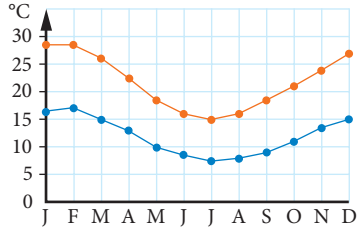
B



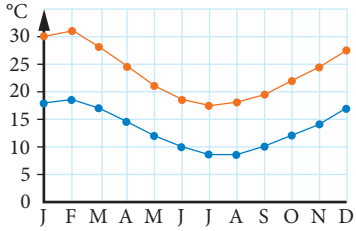
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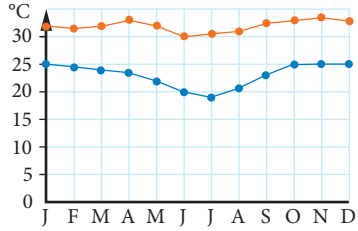
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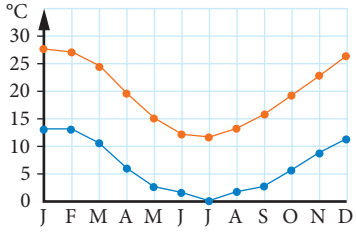
E



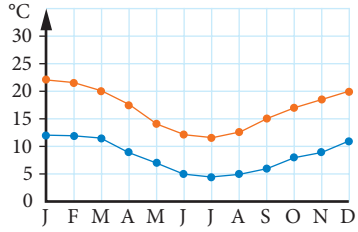
F



G



H



a From your knowledge of geography and climate, decide which graph belongs to which city.

b For graph G, list the maximum temperature of each month.

c Graph E has two months with exactly the same minimum temperature. Which months are they?

d Find which graphs have one month with a minimum of 10°C. For each graph, state which month it is.

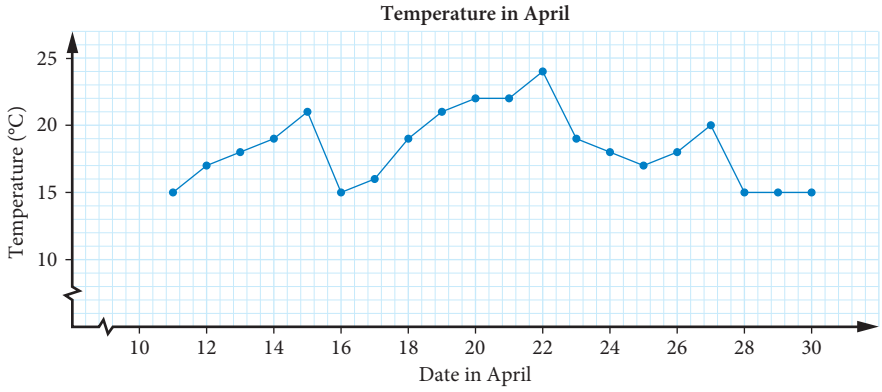
e Which graph has all the maximum temperatures approximately the same?

f Which graph has the greatest difference between the highest maximum temperature and the lowest minimum temperature for one month? What is this difference?

g Find the highest maximum temperature and state which graph shows it and in which month it occurs.

h Find the lowest minimum temperature and state which graph shows it and in which month it occurs.

4 This line graph shows the temperature for the last 20 days of April.



a On what date was the highest temperature, and what was that temperature?

b What was the lowest temperature?

c What was the range (the difference between the highest temperature and the lowest temperature)?

d What is the scale shown on the vertical axis?

e What was the temperature on 17 April?

f On what days was the temperature 22°C?

g Which day experienced the biggest drop in temperature?

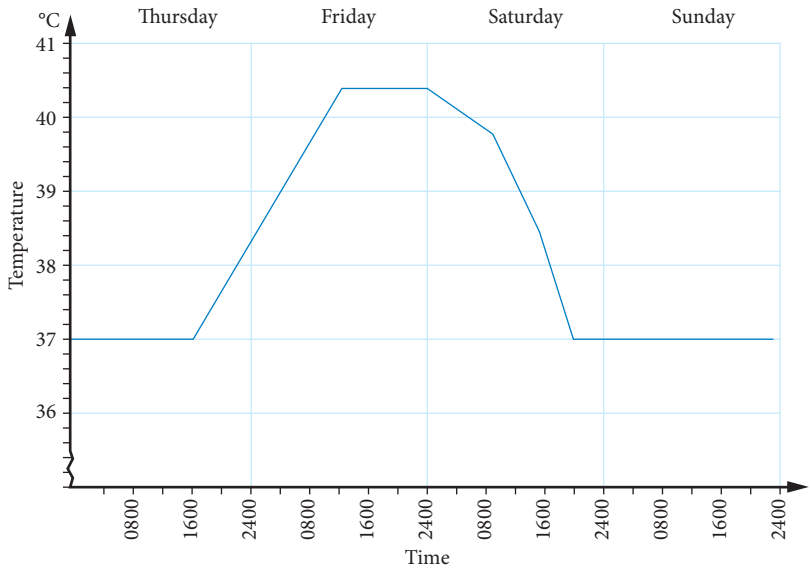
h On how many days was it colder than the day before?



Fairfax Syndication/Dominic O'Brien

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5 Sarah is in hospital recovering from having her tonsils removed.



a What is the size of each unit on the Time axis?

b What was Sarah's temperature at 0800 Friday?

c When did Sarah's temperature first reach 38.3°C?

d What was Sarah's maximum temperature?

e How long was her temperature at this maximum value?

f Increases in temperature can indicate an infection. Sarah's doctor prescribed some antibiotics. When did her temperature return to the normal 37°C?

6 Angelina was admitted to hospital at 10 p.m. with a high temperature. This table shows her temperature each hour for the next 12 hours.

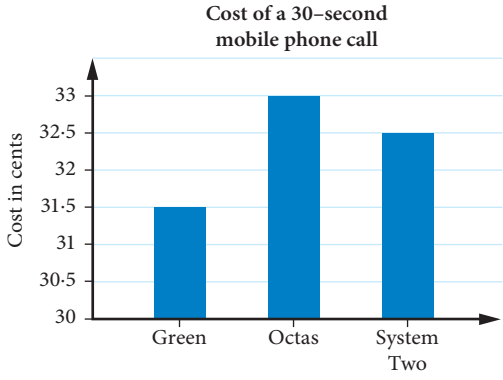
Time	10 p.m.	11	12	1 a.m.	2	3	4	5	6	7	8	9	10 a.m.
Temperature (°C)	42	42	41	40	40	41	39	40	38	39	37	37	37

Draw a line graph showing this information.

4.5 Misleading graphs

Sometimes graphs are deliberately drawn to give a false impression, like the one below. This can be done by:

- not having a scale on the vertical axis
- showing only part of the scale or an irregular scale
- not showing zero on the vertical scale/only showing a small part of the vertical axis
- using pictures or three-dimensional figures on the graph to exaggerate the differences



This graph shows the cost of a 30-second call on three different mobile phone plans. If you view the graph quickly, it appears that the calls are much cheaper on the Green plan but a closer look will show that there really isn't a big difference in cost between the three plans. While the information on the graph is correct, it gives a **misleading** impression.

Only showing a small part of the vertical axis on a graph is the most common way to create a misleading impression. As you work through the questions in the next exercise you will learn about other ways to create misleading impressions.

EXERCISE 4.5

Misleading graphs

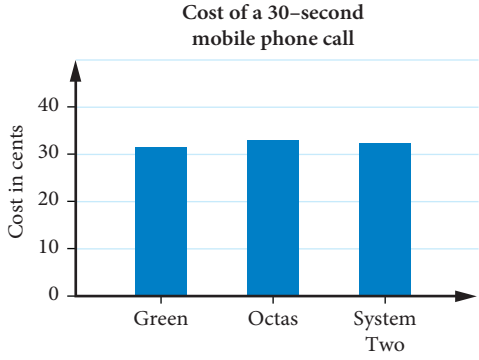
1 Use the misleading graph above to answer the following questions.

a How much does a 30-second mobile phone call cost on each of the three plans?

b How much cheaper is a call on the Green plan than on the System Two plan?

c The graph on the right shows the same information as the above graph.

In which of the two graphs does the cost of a call on the Green plan look much cheaper than the cost on the other plans?

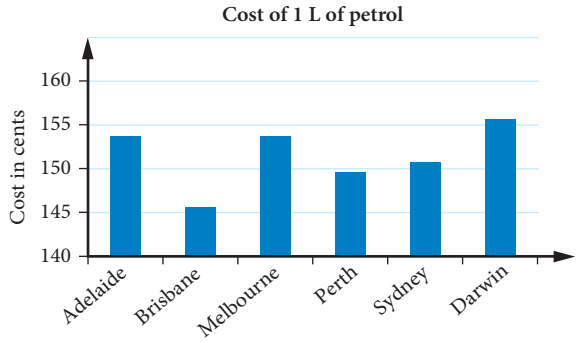


d Write a sentence to describe how the scales on the vertical axes are different in the two graphs.

e Sales people from the Green company and from the System Two company produced the two graphs. Which company do you think produced the graph at the top of the page? Give a reason for your answer.

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2 David drew a column graph to show the average cost of a litre of petrol in six capital cities.



a In which city is the price of petrol the cheapest?

b Which city has the highest petrol price?

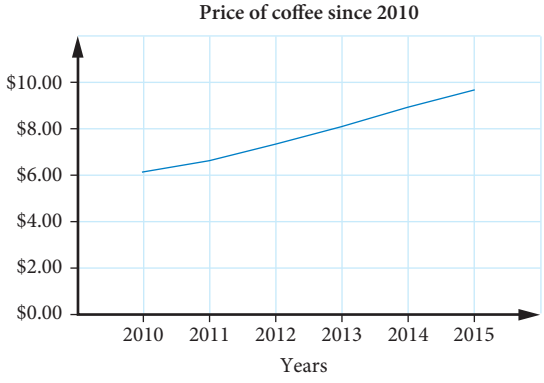
c Use the graph to estimate the price of petrol in Perth.

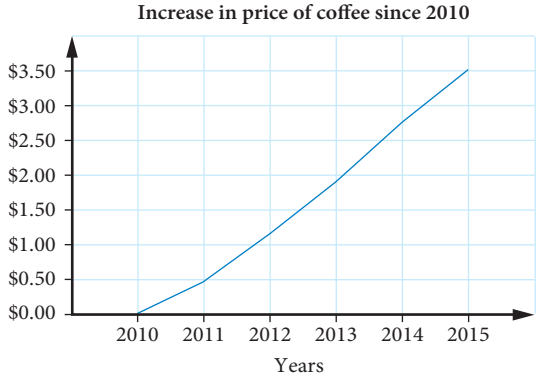
d Approximately how many cents cheaper is the price of petrol in Brisbane than in Perth?

e David included this graph in an advertisement for motoring holidays in Brisbane. He claimed that the graph shows that petrol prices are much lower in Brisbane than they are in other capital cities. Explain what David did to the graph to make the price of petrol look a lot cheaper in Brisbane.

3 This table shows the increase in the price of a brand of coffee since 2010, with the data graphed in two different ways below.

Year	2010	2011	2012	2013	2014	2015
Price of coffee	$6.15	$6.60	$7.30	$8.05	$8.90	$9.65
Increase since 2010	$0	$0.45	$1.15	$1.90	$2.75	$3.50



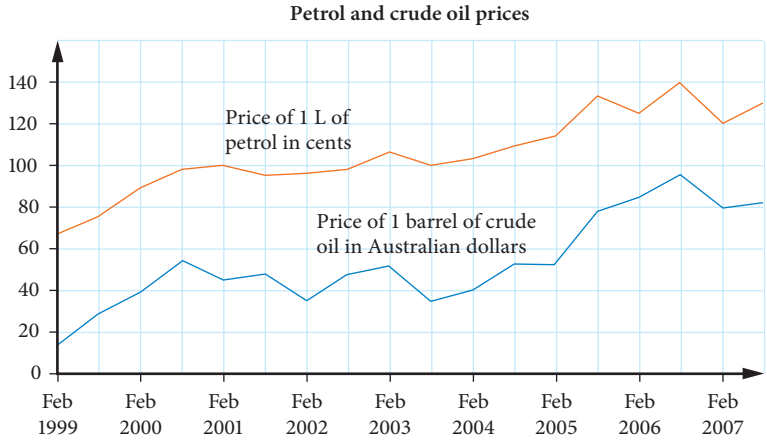


The two graphs illustrate how graphs can make **increases** look small or big. Imagine that you want to draw a graph to show how the population in your area has increased. Describe how you could make the increase look small and how you could make it look big.



iStockphoto/djgunner

4 Scott thinks that oil companies are ripping people off. He decided to investigate and graph the price of petrol and the cost of crude oil.



a What was the price of 1 L of petrol in February 1999?

b How much did a barrel of crude oil cost in February 1999?

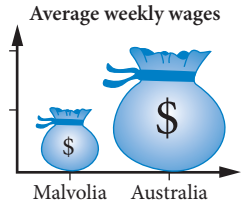
c Most people know the relationship:

Profit = selling price – cost price

The graph shows the oil cost at 53 and the petrol cost at 109 in February 2003, giving the impression that the profit is 109 – 53 = 56. Explain why this impression is completely false.

d What relationship does the graph show between the price of petrol and the price of crude oil?

5 This graph compares the average weekly wages in Malvolia ($700) and Australia ($1400).



a What misleading impression does this graph give?

b Explain two things that are wrong about this graph.

c What should be drawn on the graph instead of pictures?

d Redraw this graph correctly.

6 This question uses data from the spreadsheet from Exercise 4.3 **Drawing statistical graphs using a spreadsheet**. It uses the results of a survey of households about their favourite commercial television network. The data are in cells A37 to B41.

a Select cells A38 to B41 and make a basic column chart from the data.

b Select **Layout**, then **Axes**, **Primary Vertical Axis**, **More Primary Vertical Axis**, **Options**.

c In the dialogue box for **Minimum**, select **Fixed** and enter 3700 next to it. Close the dialogue box.

d Give the graph the obvious title it deserves 'Nine blitzes the other networks' and put it in the misleading sheet.

e Do the same again but in the vertical axis dialogue box make the minimum zero. Close the box.

f This is the same data but the impression is very different. Give this graph a title that explains the situation seen.

g Move it to the misleading sheet directly beneath the first one.

Dot plots and stem-and-leaf plots 4.6

Dot plots are used for small sets of numerical data that are close together.

Stem-and-leaf plots are used for larger sets of numerical data and list the actual data values.

Example 1

The daily maximum temperature (in °C) in Cairns in July was recorded for 15 days.

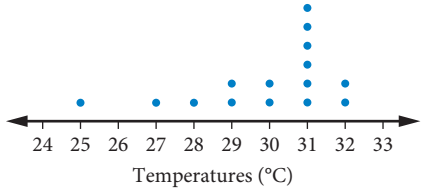
32, 30, 31, 32, 31, 30, 31, 31, 31, 31, 29, 25, 28, 27, 29

Construct a dot plot for this data.

Solution

The temperatures go from 25° to 32°, so the line for the dot plot should go from 24 to 33.

Add a dot above the line for each data value.



Example 2

Sarah works for law enforcement and recorded the number of malicious property damage reports in the inner-city each month over 3 years.

54, 41, 55, 49, 37, 38, 37, 48, 51, 44, 52, 44, 58, 70, 60, 46, 63, 54

45, 43, 46, 55, 55, 67, 49, 66, 90, 45, 66, 62, 51, 51, 53, 53, 38, 52

Draw an ordered stem-and-leaf plot for this data.

Solution

The smallest number is 37 and the largest number is 90, so the stem will go from 3 to 9.

The leaf will be the units digit for each number in the data. This is called an unordered stem-and-leaf plot because the 'leaves' are not in order.

Stem	Leaf
3	7 8 7 8
4	1 9 8 4 4 6 5 3 6 9 5
5	4 5 1 2 8 4 5 5 1 1 3 3 2
6	0 3 7 6 6 2
7	0
8	
9	0

key: 3|7 = 37

Then we write the 'leaves' in order. This is called an ordered stem-and-leaf plot.

Stem	Leaf
3	7 7 7 8
4	1 3 4 4 5 5 6 6 8 9 9
5	1 1 1 2 2 3 3 4 4 5 5 5 8
6	0 2 3 6 6 7
7	0
8	
9	0

key: 3|7 = 37

EXERCISE 4.6

Dot plots and stem-and-leaf plots

EXAMPLE 1

1 Ahmed surveyed his Maths class to find how many hours each student spent on the Internet each week. These are the results:

14, 15, 17, 18, 13, 16, 19, 19, 18, 14, 17, 15, 13, 13, 14, 19, 13, 14, 17, 18

a Draw a dot plot for Ahmed's data.

b How many students were in the class?

c How many students used the Internet for 14 hours per week?

d How many students used the Internet for more than 16 hours per week?

2 The number of motor vehicle thefts in an inner city suburb was recorded each month for one year.

15 9 11 16 13 11 13 12 10 15 13 9

a Draw a dot plot for this data.

b In how many months were 13 motor vehicles stolen?

c What percentage of months had 13 motor vehicles stolen?

d In how many months were fewer than 12 motor vehicles stolen?

3 Lisa surveyed her group of friends about the amount of money they spent on fuel last week, rounded to the nearest dollar.

$20 $28 $25 $26 $22 $26 $28 $28 $24 $22 $29 $28

a Draw a dot plot for this data.

b How many friends did Lisa survey?

c What was the most common amount of money spent on fuel?

d How many of Lisa's friends spent less than $26 on fuel?

EXAMPLE 2

4 Mrs White, the school canteen manager, records the daily number of students visiting the canteen over three weeks.

105 76 97 88 114 86 124 101
112 98 95 105 117 81 112

a Show this information on a stem-and-leaf plot.

b On what percentage of days were more than 100 students served? Answer correct to the nearest whole percentage.

5 A security firm recorded the monthly number of shoplifting incidents over three years.

20 20 23 11 12 33 22 30 16 17 35 48
25 27 25 34 20 23 25 17 12 14 13 13
48 42 55 33 24 39 26 41 33 31 19 55

a Show this information on a stem-and-leaf plot.

b How many months had the number of incidents in the thirties?

c What percentage of the total months was this? Answer correct to the nearest whole percentage.

6 A class of Year 11 students were surveyed on the number of hours of part-time work they did in one month.

42	16	35	27	9	0	33	21	14	11	26
29	31	22	8	24	5	0	15	25	17	

a Complete a stem-and-leaf plot to show this information.

b What was the most hours worked by anyone in this sample?

c How many students worked between 10 and 20 hours in the month?

d How many students worked more than 20 hours in the month?

e How many students were in this class?

f What percentage of students worked more than 20 hours in the month? Answer correct to the nearest whole percentage.

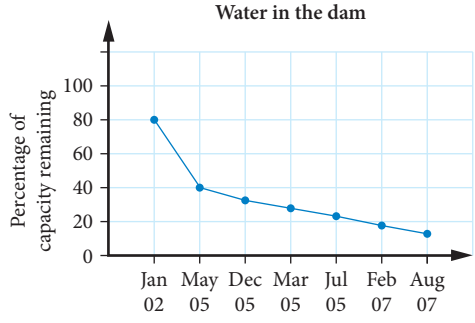
Keyword activity

Summarise in your own words the important features of each type of graph.

1 column graph
2 climate graph
3 clustered column graph
4 dot plot
5 histogram
6 line graph
7 pie graph
8 stem-and-leaf plot

Chapter Problem

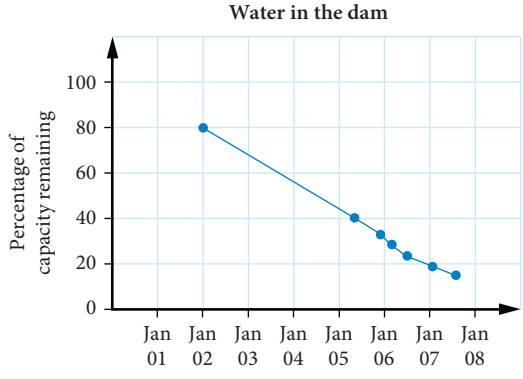
During a drought, the water authority produced a graph to show the decreasing supply of water in a dam due to the water usage of local residents. Is the decrease in water supply as bad as the graph seems to indicate? If the graph gives a false impression, identify what has been done to create this impression.



Solution

The graph shows that the supplies of water are running very low. However, the graph is misleading because the scale on the horizontal axis isn't consistent. The big drop in dam levels between January 2002 and May 2005, compared with the small drop between February and August in 2007, gives the impression that water was being wasted from 2002 to 2005.

The graph to the right shows the same information in a non-misleading way.





Chapter Problem

Lara needs 24 mL of purple paint for a flower scene she is painting. To make the shade of purple she wants she will mix some cyan and magenta paint in the ratio of 1 : 2. How much of each colour should she use?



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CHAPTER 5

WHAT'S MY SHARE?

5.1 Writing and simplifying ratios
5.2 Ratio problems
5.3 Dividing a quantity in a given ratio
5.4 Rate problems
Keyword activity
Solution to the chapter problem

By the end of this chapter, you should know how to:

- write and simplify ratios and rates
- solve practical problems involving ratios and rates
- divide a quantity in a given ratio

How are we ever going to use this?

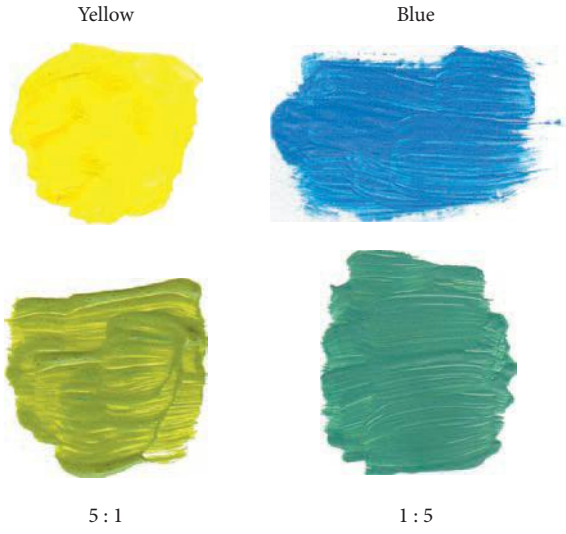
- When mixing paint, hair colour and cake ingredients
- When mixing fertiliser, bleach, pesticide or other concentrates
- When measuring while cooking, renovating and gardening
- When calculating costs of items and speeds of cars

5.1 Writing and simplifying ratios

A **ratio** shows the relative size of two or more quantities of the same type.

Lara uses ratios to make the colours she uses in her paintings. She discovered that the order of a ratio is very important. If she gets the order wrong the resulting colour is wrong and she wastes paint.

Lara mixed yellow and blue paint. When she used the ratio of 5 : 1, she mixed 5 parts yellow with 1 part blue. When she used the ratio of 1 : 5, she mixed 1 part yellow and 5 parts blue. The resulting two colours were quite different. Several mistakes taught her that she has to be careful with the order of ratios.



Example 1

Lara mixed 12 mL of white paint with 8 mL of red paint to make dark pink paint.

a What is the ratio of red to white paint in the mixture?

b What fraction of the mixture was red paint?

Solution

a The order is important. In the ratio of red : white we need to write the quantity of red first.

red : white = 8 : 12

b 12 + 8 = 20 mL of total paint and 8 mL came from red paint. That's 8 mL out of 20 mL.

Fraction of the paint that was red paint = $\frac{8}{20}$

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Simplifying ratios

Just like we can simplify fractions by multiplying or dividing both the numerator (top) and the denominator (bottom) by the same amount, we can simplify ratios by multiplying or dividing both numbers in the ratio by the same number.

Example 2

Lara's red and white paint mixture is in the ratio of 8 : 12. Simplify the ratio 8 : 12.

Solution

Divide 8 and 12 by 4.

$8 \div 4 = 2$ and $12 \div 4 = 3$.

$8 : 12 = 2 : 3$

Write your answer.

When Lara mixes 8 mL red with 12 mL white paint she is mixing the paint in a ratio of 2 : 3.

Example 3

Zack is painting his lounge room. He wanted the colour to be white with a hint of brown. He mixed 10 mL of brown with 8 L of white paint to achieve the colour he wanted. What ratio of brown to white should he write on the lid so that he can make the same colour in the future?

We have to be careful with units. All quantities have to be in the same unit before we can write the quantities in a ratio.



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Solution

Change 8 L into mL so the units are the same and make sure we write the brown amount first.

8 L = 8000 mL

Brown : white = 10 mL : 8000 mL

= 10 : 8000

Simplify the ratio by dividing each number by 10.

10 : 8000 = 1 : 800

Write your answer.

Zack should write brown : white = 1 : 800 on the lid.

EXERCISE 5.1

Writing and simplifying ratios

EXAMPLE 1

1 Lara mixed 5 mL of red with 9 mL of green paint to make brown to create a painting of some trees.

a What is the ratio of green to red paint in the brown mixture?

b What fraction of the brown paint came from the red paint?

EXAMPLE 2

2 Lara mixed 6 mL of green paint with 2 mL of blue to make a blue-green paint for the leaves.

a What is the ratio of green : blue paint in the blue-green mixture?

b Simplify your answer to part **a**.

c What fraction of the blue-green mixture came from the green paint?

3 Simplify each ratio.

a 4 : 10	**b** 6 : 9	**c** 20 : 10	**d** 18 : 12
e 15 : 10	**f** 12 : 6	**g** 100 : 20	**h** 25 : 100
i 36 : 20	**j** 200 : 20	**k** 70 : 7000	**l** 50 : 1000
m 2 : 60	**n** 80 : 8	**o** 24 : 18	**p** 200 : 1000

4 Lara made two containers of orange paint by mixing red and yellow. She mixed 20 mL of red with 40 mL yellow in one mixture and 8 mL of yellow with 4 mL of red in the other.

She is confused. Both colours are the same!

What is the explanation?

Basic colours



Mixed colour



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5 Zack added 5 mL of brown paint to 2 L of white paint to paint the woodwork in his lounge room. EXAMPLE 3

a What is the ratio of brown to white paint in mL?

b Write your answer to part **a** in its simplest form.



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6 Simplify each ratio. Remember to make the units the same first.

a 40 mL : 1 L

b 3 L : 20 mL

c 2 L : 500 mL

d 50 cm : 1 m

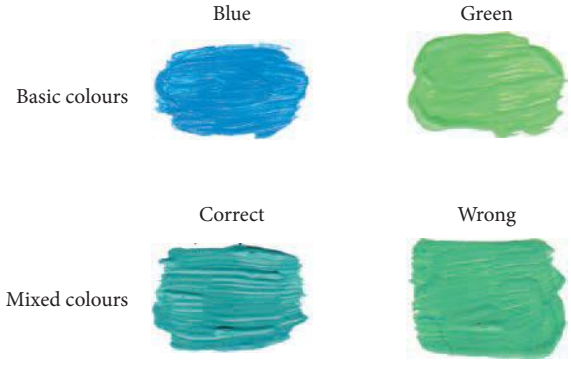
e 3 m : 60 cm

f 5 kg : 500 g

g 2 hour : 40 minutes

h 45 minutes : $\frac{1}{2}$ hour

7 Lara needs to make a particular shade of blue-green by mixing blue and green in the ratio of 4 : 1, but when she mixed the paint she got the wrong colour. What could she have done wrong?



8 Lara makes colours lighter by mixing them with white; the more white the lighter the colour. Here are four shades of blue Lara made by mixing blue and white in the ratios of 3 : 1, 3 : 2, 1 : 1 and 1 : 2 but she forgot to label the diagrams. Match the ratios to the colour samples.

a 

b 

c 

d 

Investigation Making colours

To complete this activity you will need paint in the three **primary colours** (red, yellow and blue), a surface for mixing paint and some paint brushes.

Your challenge is to work out which colours and in what ratio you need to mix them to make the colours in the table below. A sample one has been done for you.

	Colour	Primary colour parts used	Ratio
		1 part red, 1 part blue	1 : 1
1			
2			
3			
4			



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Ratio problems

5.2

When we prepare recipes, whether it is an infant's food formula, a hair dye or a fertiliser for the garden, it is important that we mix the ingredients in the required ratio. Too much or too little of one of the ingredients can cause damage or have unintended effects.

Most ratio problems can be solved using the **unitary method**.

Example 4

Grant makes food for his chickens by mixing grain, protein meal and vegetable matter in the ratio 5 : 2 : 1.

a Grant is going to use 30 kg of grain to make chicken food. How much protein meal and vegetable matter will he need?

b How much chicken food will this mixture make?



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Solution

a The ratio 5 : 2 : 1 means that the food is made up of 5 parts grain, 2 parts protein meal and 1 part vegetable matter.

5 parts (grain) = 30 kg
1 part = 30 ÷ 5
= 6 kg

2 parts (protein meal) = 2 × 6
= 12 kg

1 part (vegetable matter) = 1 × 6
= 6 kg

Write the answer.

Grant will need 12 kg of protein meal and 6 kg of vegetable matter.

b The total mixture will contain 30 kg grain, 12 kg of protein meal and 6 kg of vegetable matter.

Total mass = 30 + 12 + 6
= 48 kg

Mentally check that the answer is reasonable then write the answer.

The total mass will be 48 kg.

EXERCISE 5.2

Ratio problems

EXAMPLE 4

1 To make a fertiliser for his rose garden, Ahmed mixes manure and grass clippings in the ratio of 3 : 2.

a How many spadefuls of grass does Ahmed need to mix with 15 spadefuls of manure?

b When he uses 6 spadefuls of grass, how much manure will he need to mix with it?

2 Nickel brass is a metal alloy containing copper, zinc and nickel mixed in the ratio 14 : 5 : 1. Nickel brass is used in many musical instruments and is the metal used to make the English pound coin.



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a A manufacturer used 70 g of copper to make a batch of nickel brass. How much zinc and nickel did he need?

b Van has 8 g of nickel. How much copper and zinc will he need to mix with it to make nickel brass?

c How much nickel brass can Van make?

3 When Padmina makes scones for the school fete, she mixes self-raising flour and milk in the ratio of 3 : 1 by volume.

a How much milk would she add to 15 cups of flour?

b If she uses 6 cups of milk, how many cups of flour are required?

4 Abi mixes a spray to kill weeds by mixing a concentrated solution of the poison with water. She needs different mixtures for different types of weeds.

Weed	Ratio of concentrate to water
Flat-leaf weeds	1 : 10
General perennial weeds	3 : 20
Woody weeds	1 : 50

a Abi needs to kill some blackberries at the back of her land. Blackberries are classed as woody weeds. How many millilitres of water should she mix with 40 mL of concentrate to make the spray?

b How many litres of water is this?

c She is going to clear a small area to make a new garden. The area is infested with perennial weeds. How much concentrate does she need to add to 1 L of water to make a spray to kill the weeds?

Convert 1 L to mL first.

d Abi has flat-leaved weeds coming up in her lawn. How much concentrate should she mix with 250 mL of water to make the spray for these weeds?

e Abi has made a spray using 45 mL of concentrate mixed with 300 mL of water. What type of weed will she use this for?

5 Eddie makes and sells saddles. Before he sells them, he treats them with a leather dressing made up from lanolin and neatsfoot oil mixed in the ratio 2 : 3.

a How much neatsfoot oil should Eddie mix with 40 mL of lanolin to make his leather dressing?

b How much leather dressing will this make?

c How much leather dressing is Eddie making when he uses 45 mL of neatsfoot oil?

6 Nadim makes garden compost by mixing his vegetable and plant waste with straw and soil in the ratio 12 : 3 : 2.

a How much straw he will need if he has 480 L of vegetable and plant waste?

b How much soil should he add to a mixture containing 12 L of straw?

c How much compost will he make by mixing 60 L of straw with the other ingredients?

7 Gemma has a vegetable garden and a small fruit orchard. To fertilise them she makes mixtures containing nitrogen (N), phosphorus (P) and potassium (K) but the ratios for the vegetable garden and the orchard are different. The table shows the ratios Gemma uses.

Plant	N : P : K	Nitrogen (kg)	Phosphorus (kg)	Potassium (kg)	Total mixture (kg)
Vegetables	6 : 4 : 5	54			
				35	
Fruit	7 : 1 : 2		15		
		140			

Copy and complete the table to show the masses of different ingredients Gemma must use to make her fertilisers.

8 A recipe for making pizza bases mixes plain flour, water and yeast in the ratio of 16 : 10 : 1 by weight.

a Jasmine uses 400 g of flour to make pizzas at home. How many grams of water and yeast will she need to add to make the pizza dough?

b Spiro runs a small pizza shop and makes all the dough for his pizza bases in the morning. He works out that he needs to mix 2.5 kg of water with the other ingredients. How much flour does he use?

c A factory making pizza bases uses 3 kg of yeast each day. How much flour and water does it use?



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9 Connor is renovating his house and makes mortar by mixing cement, sand and lime. As the table shows, he mixes the mortars using different ratios depending on what type of work he is doing.

Job	cement : lime : sand in mortar
New brick construction	1 : 0 : 4
Old brick renovation	2 : 3 : 10
Rendering	1 : 1 : 6

a On what type of job does Connor not use any lime in his mortar?

b Connor constructed a new brick fireplace. To make the mortar he used 8 buckets of sand. How many buckets of cement did he need to add?

c Connor is going to render the new fireplace. The mortar he uses contains 12 buckets of sand. How much lime and cement will he need?

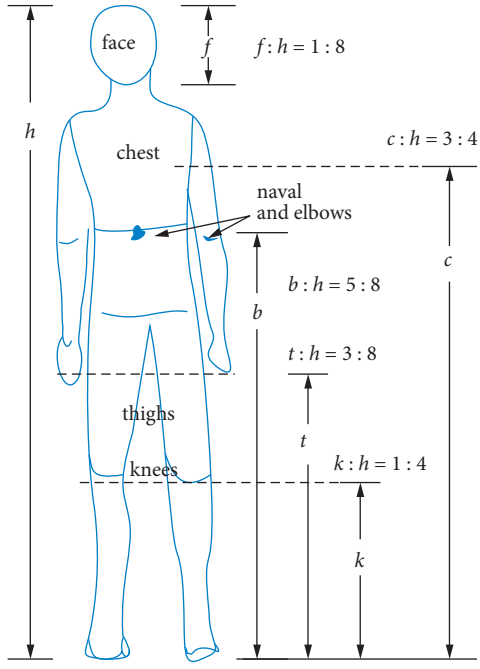
d Connor is going to repair one of the walls of his house using old bricks he has saved. How much sand and lime will be needed if he mixes his mortar using 4 buckets of cement?

Investigation Ratios, body parts and supermodels

Artists use body part ratios to make sketches of people look realistic. The ratios of body parts are slightly different in males and females. In this investigation you are going to compare the ratios that artists use in sketches to the ratios of body parts in everyday people and supermodels.

Men and women's bodies have a different shape. Women's necks and waists are thinner while their hips and thighs are wider. This table shows the different ratios for adult males and females.

	Males	Females
Shoulder width : head length	7 : 3	2 : 1
Shoulder width : head length	15 : 7	1 : 1
Hips width : head length	2 : 1	3 : 2



What you have to do

- Find some full-length photographs of people online or in magazines.
- Use a ruler to measure the lengths f, h, c and t.
- Calculate the ratios $f : h$, $c : h$ and $t : h$.
- Describe how the ratios of body parts in supermodels are similar to and different from the ratios artists use to draw realistic people. What conclusions can you make?

Dividing a quantity in a given ratio 5.3

Sometimes we want to divide or share quantities into unequal amounts. For example, the owners of businesses often share the profits and expenses according to the size of their share in the business.

Example 5

Janet and Darryl own a small market garden, which made a profit this year of $135 000. They have agreed to share the profit in the ratio 2 : 3. How much does each person receive?

Solution

The profits are to be divided into 2 parts for Janet and 3 parts for Darryl.

$$\text{Janet's share} = \frac{2}{5} \times \$135\,000$$
$$= \$54\,000$$

Total number of parts = 2 + 3 = 5.

$$\text{Darryl's share} = \frac{3}{5} \times \$135\,000$$
$$= \$81\,000$$

Check the answer. Do the separate amounts add to the total profit and are the separate amounts a realistic size?

$\$54\,000 + \$81\,000 = \$135\,000$

Write the answer.

Janet's share is $54 000 and Darryl's share is $81 000.

Example 6

Hamish needs to mix 180 kg of concrete. Concrete is made using cement, sand and gravel in the ratio 1 : 2 : 3. How much of each product will Hamish need?

Solution

Total number of parts = 1 + 2 + 3 = 6.

$$\text{Cement} = \frac{1}{6} \times 180$$
$$= 30 \text{ kg}$$
$$\text{Sand} = \frac{2}{6} \times 180$$
$$= 60 \text{ kg}$$
$$\text{Gravel} = \frac{3}{6} \times 180$$
$$= 90 \text{ kg}$$

Check the answer.

$30 + 60 + 90 = 180$

Write the answer.

Hamish will need 30 kg cement, 60 kg of sand and 90 kg of gravel.

EXERCISE 5.3

Dividing a quantity in a given ratio

EXAMPLE 5

1 Divide $2100 in the ratio:

a 2 : 3 **b** 5 : 1 **c** 4 : 2 : 1 **d** 3 : 4 : 7

2 Divide:

a 855 L in the ratio 1 : 4

b 35 lollies in the ratio 4 : 3

c 24 300 m^2 in the ratio 4 : 5

d 819 kg in the ratio 7 : 2

e 52 min in the ratio 3 : 4 : 6

f $2800 in the ratio 8 : 1 : 5

3 Each week Michael produces 3.5 tonnes of fruit to be sent to the markets. He will send 4 parts to the central market and 3 parts to a local market. How much will be sent to the local market?

4 At Jemma's school there are 741 students. The ratio of boys to girls is 7 : 6. How many boys are there in the school?

5 Ava is making a bracelet using silver beads, porcelain balls and crystal eyedrops in the ratio of 4 : 3 : 2. She uses 45 pieces to make the bracelet. How many of each type will she require?

6 A flat white coffee contains black espresso coffee and steamed milk in the ratio 1 : 2. How much milk is there in a 240 mL mug of flat white coffee?

7 Rose gold (or red gold) is used in jewellery and high-quality flutes because of its attractive appearance. It is an alloy of gold, copper and silver blended in the ratio 15 : 4 : 1.

a How much silver is in a 40 g piece of rose gold jewellery?

b A flute made of rose gold weighs 420 g. How much gold and copper does it contain?

EXAMPLE 6

8 Sam makes mocktails using 3 parts grape juice, 2 parts cranberry juice and 4 parts sparkling mineral water.

a How many millilitres of grape juice will he need to make two 225 mL mocktails?

b How much mineral water does Sam use to make 900 mL of the mocktail?

c How many mL of cranberry juice does this mixture require?

9 A insecticide concentrate is mixed with water before it is sprayed on vegetable crops. The mixing ratio depends on the vegetable as the table shows. Jenni needs to make 3 L of insecticide for broccoli, cauliflower and leafy vegetables.

Vegetable	Ratio of concentrate to water	mL of concentrate required	mL of water required
Broccoli	1 : 14		
Cauliflower	1 : 19		
Leafy vegetables	1 : 9		

a Convert 3 L to millilitres.

b Copy and complete the table to show how much concentrate and water Jenni needs to mix for each vegetable.

10 Filomena is a hairdresser who needs to dye a customer's hair. Her colour chart shows that she needs to mix two dyes (dye A and dye B) in the ratio 5 : 3 to make the required colour. If Filomena needs to make 120 mL of the mixture, how much of each dye will she need to use?

11 Boris has two machines on his farm that use 2-stroke fuel, a mixture of petrol and oil. The table shows the petrol to oil ratio required for each.

Machine	petrol : oil
Ride-on mower	24 : 1
Cultivator	39 : 1

a Boris mixes the fuel for the ride-on mower in a 15 L drum. How much petrol must he use?

b How much oil is required (in litres)?

c Express your answer to part **b** in mL.

d For the cultivator, Boris makes the fuel in batches of 20 L. How much oil does he require for each batch? Give your answer in mL.

12 Martin mixes a fertiliser combining nitrogen, phosphorus and potassium in the ratio 5 : 1 : 2. How much of each ingredient will he use to make 120 kg of the fertiliser?

Investigation Ratios and TV screens



123RF.com/Erik Reis

1 Measure the width and height of a variety of TV screens. Alternatively find the measurements of TV screens on the Internet.

2 Calculate the width : height ratio for each TV.

3 Comment on any similarities and differences.

4 The distance you sit from the TV and the length of the TV's diagonal should be in a ratio of 3 : 1 or bigger. What is the length of the diagonal of the largest TV you should have in your TV room?

5.4 Rate problems

While ratios compare two quantities of the same type, **rates** compare two different types of quantities.

Example 7

Write each situation as a rate.

a Andrew runs 200 m in 25 seconds.

b Boris' cultivator fuel cost $35 for 20 L.

Unlike ratios, rates have units. We express these as the units of the first quantity **per** unit of the second quantity.
'Per' means 'for each one'

Solution

a The units for this rate will be metres per second, or m/s.

Divide the number of metres by the number of seconds.

$$\text{Rate} = \frac{200\,\text{metres}}{25\,\text{seconds}}$$
$$= 8\ \text{m/s}$$

b The units will be dollars per litre.

Divide the number of dollars by the number of litres.

$$\text{Rate} = \frac{\$35}{20\,\text{litres}}$$
$$= \$1.75/\text{L}$$

- To solve problems involving rates, we either multiply or divide by the rate.
- Identify the rate and express the units of the rate as a fraction (for example, $\frac{\text{km}}{\text{h}}$).
- To find the top quantity of the fraction, multiply by the rate.
- To find the bottom quantity of the fraction, divide by the rate.

Example 8

Noah hires a small car for $42 per day. How much will the hire cost be if Noah keeps the car for 16 days?

Solution

The rate is $42/day. We can express the units as $\frac{\$}{\text{day}}$.

We are asked to find the number of dollars which is in the top of the fraction, so we multiply by the rate.

$$\text{Hire cost} = \text{days} \times \text{rate}$$
$$= 16\ \text{days} \times \$42/\text{day}$$
$$= \$672$$

Check that the answer is a realistic size then write the answer.

It will cost Noah $672 to hire the car for 16 days.

Example 9

Amy sends the fruit she grows on her farm to market in cartons that hold 12 kg of fruit per carton. How many cartons will she need to pack 180 kg of fruit?

Solution

The rate is 12 kg/carton. The units are $\frac{\text{kg}}{\text{carton}}$.

We are asked to find the number of cartons which is in the bottom of the fraction, so we divide by the rate.

Number of cartons = mass ÷ rate
= 180 kg ÷ 12 kg/carton
= 15 cartons

Check that the answer is a realistic amount then write the answer.

Amy will need 15 cartons.

EXERCISE 5.4

Rate problems

1 Write each situation as a rate.

EXAMPLE 7

a 200 km travelled in 4 hours
b 75 words typed in 3 minutes
c 250 L used in 5 hours
d \$87.50 for 5 kg
e 400 m in 55 s
f \$65 earned in $2\frac{1}{2}$ hours
g 500 g in 2 L
h 1800 revolutions in 3 minutes

2 Marcella uses 90 litres of composting manure on her flower bed which has an area of 5 m^2. What is this as a rate in L/m^2?

3 Goran used his sprinklers to water his garden for $2\frac{1}{2}$ hours. He used 3375 litres. What is his rate of water use in L/h?

4 Marcus runs a sausage sizzle for his sport club's market day. He has purchased the food shown in the table. Calculate the cost rate, including units, for each item.

Item	Cost and amount	Cost rate
Sausages	\$57.80 for 6.8 kg	**a**
Steak	\$93.60 for 7.5 kg	**b**
Onions	\$9.80 for 4 kg	**c**
Rolls	\$132 for 240 rolls	**d**
Tomato sauce	\$19.60 for 8 bottles	**e**

5 Lizzie plants out her tomato seedlings in rows at the rate of 28 seedlings per row. How many seedlings are there in:

EXAMPLE 8

a 4 rows? **b** 7 rows? **c** 12 rows?

6 Jayden earns $12.40/h as a barista in a cafe. How much is he paid for working 20 hours?

EXAMPLE 9

7 Jane proudly claims that she can hike at an average rate of 5 km/h even in difficult conditions. At this rate, how long will it take her to walk:

a 15 km? **b** $7\frac{1}{2}$ km? **c** 24 km?

8 Mandarins cost $3.20/kg. How many kilograms of mandarins can Peter buy for $20?

9 Danielle is growing nectarine trees. She can expect a yield of 80 kg of fruit per tree.

a How many kilograms of nectarines can Danielle expect to produce if she plants 9 trees?

b How many trees would Danielle need to plant if she wanted to produce 1200 kg of nectarines?

10 A plane flies at a speed of 840 km/h.

a How far does the plane travel in $6\frac{1}{2}$ hours?

b Calculate how long it would take the plane to fly from Melbourne to each city, given their distances. Answer in hours correct to one decimal place.

i Brisbane, 1670 km

ii Auckland, New Zealand, 2626 km

iii Perth, 3430 km

iv Los Angeles, USA, 12 773 km

11 The yields Olivia achieves from different vegetable crops are shown below:

Crop	Yield (vegetables/m^2)
Beetroot	40
Lettuce	18
Cabbage	8

a How many beetroots can Olivia expect to get from her 24 m^2 beetroot plot?

b How many lettuces will she grow in a 15 m^2 garden?

c What area does she need to grow 450 lettuces?

d What area does she need to grow 760 cabbages?

12 Sunil is saving to purchase a car. He can save $400 per week.

a How much can Sunil save in 20 weeks?

b The car he wants costs $14 000. How long will it take Sunil to save this amount?

c A newer car is available but it costs $15 200. How much longer would it take Sunil to save for this car?

13 Aaron uses fertiliser on his farm at the rate of 24 kg/hectare. How many kilograms of fertiliser does he require to cover 32 hectares?

14 A dripping tap leaks water at the rate of 25 mL/minute.

a How much water will leak from the tap in 30 minutes?

b How much water will be lost in 1 hour?

c Convert your answer to part **b** to litres.

d How many litres will be lost in 24 hours?

e How many minutes will it take for the tap to leak 1 litre of water?

f How many hours will it take for the tap to leak 60 L of water?

Remember there are 60 minutes in an hour.

Remember there are 1000 mL in 1 L.

15 Chloe earns $14.50/h as a secretary. How many hours will she need to work to earn:

a $261.00? **b** $507.50? **c** $348.00?

Keyword activity

1 What keywords can you find in this chapter? Search through the chapter and list the keywords together with their meaning.

2 Use your own words to describe the process of dividing a quantity in a ratio.

Chapter Problem

Lara needs 24 mL of purple paint for a flower scene she is painting. To make the shade of purple she wants she will mix some cyan and magenta paint in the ratio of 1 : 2. How much of each colour should she use?

Solution

Lara needs cyan : magenta in the ratio of 1 : 2.

Total number of parts = 1 + 2 = 3.

Amount of cyan = $\frac{1}{3}$ x 24 mL = 8 mL

Amount of magenta = $\frac{2}{3}$ x 24 mL = 16 mL

Lara will need 8 mL of cyan and 16 mL of magenta.



Chapter Problem

Daniel has moved out of home to live in a unit. He needs to be at work by 8:30 a.m. and catches a bus to the station and then a train. The bus stop is 10 minutes from home and the trip takes 15 minutes. Buses leave every 15 minutes from 6:00 a.m. The train trip takes 10 minutes and trains leave every 20 minutes from 6:05 a.m. Daniel's workplace is a 5-minute walk from the station. What time does Daniel need to leave home to get to work on time?



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CHAPTER 6

IT'S ABOUT TIME

6.1 0600 hours
6.2 How long will it take?
6.3 Times across Australia
6.4 Times across the world
6.5 When will it arrive?
Keyword activity
Solution to the chapter problem

By the end of this chapter, you should know how to:

- write the time of day in 12- and 24-hour time
- calculate time differences
- solve problems involving Australian and international time zones
- solve problems involving timetables and schedules

How are we ever going to use this?

- Looking up timetables and making plans for travel
- Checking television programs or performance times
- Any time you need to calculate anything to do with time!

6.1 0600 hours



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24-hour time	12-hour time
0000	12 a.m. (midnight)
0100	1 a.m.
0200	2 a.m.
0300	3 a.m.
0400	4 a.m.
0500	5 a.m.
0600	6 a.m.
0700	7 a.m.
0800	8 a.m.
0900	9 a.m.
1000	10 a.m.
1100	11 a.m.

24-hour time	12-hour time
1200	12 p.m. (midday)
1300	1 p.m.
1400	2 p.m.
1500	3 p.m.
1600	4 p.m.
1700	5 p.m.
1800	6 p.m.
1900	7 p.m.
2000	8 p.m.
2100	9 p.m.
2200	10 p.m.
2300	11 p.m.

Example 1

Write 8:25 a.m. as 24-hour time.

Solution

8:25 a.m. is in the morning. Leave out the dots and put a zero at the front.

8:25 a.m. = 0825

Example 2

Write 1615 as 12-hour time.

Solution

1615 is bigger than 1200 so it's in the afternoon (p.m.). For the hour, take 12 away from 16. 16 – 12 = 4.

1615 = 4:15 p.m.

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EXERCISE 6.1

0600 hours

EXAMPLE 1

1 Write as 24-hour time.

a 11:44 a.m. **b** 6:35 p.m. **c** 2:51 a.m. **d** 9:54 p.m.

EXAMPLE 2

2 Write as 12-hour time.

a 0845 **b** 1320 **c** 2331 **d** 1045

3 Write the time shown on each clock in 12-hour time.

a 

b 

c 

4 Write the time shown on each clock in 12-hour time. All times are in the morning.

a 

b 

c 

5 Write the time shown on each clock in 12-hour time. All times are in the afternoon or evening.

a 

b 

c 

6 Sue wants to program her DVR to record a television show when she's out. The show starts at 8:30 p.m. and lasts for an hour and a half. Her DVR uses 24-hour time.

a At what time will Sue need to set her DVR to start recording the program?

b What time should she set for her finish time? Allow an extra 5 minutes for the recording.

7 Jessica was on police duty one Saturday night from 2200 to 0600 on Sunday morning. She received a report of a fight at 2245 and again at 0220.

a How long was Jessica on duty?

b How long was it between the two fight reports?

c A patient was taken from the second fight to hospital and arrived 45 minutes after the incident was reported. What time did the patient arrive at hospital?

6.2 How long will it take?

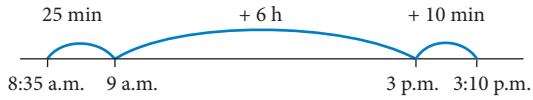
How long do I have before my favourite TV show starts? How long is my stopover in Hong Kong? How long did I work yesterday? We can answer all these questions by doing a time calculation.

Example 3

What is the difference in time between 8:35 a.m. and 3:10 p.m.?

Solution

Use a timeline and count to the next full hour.



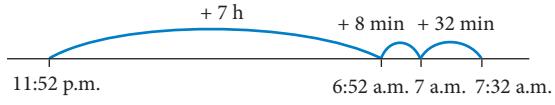
From 8:35 a.m. to 9:00 a.m.	25 minutes
From 9:00 a.m. to 3:00 p.m.	6 hours
From 3:00 p.m. to 3:10 p.m.	10 minutes
Add together.	Total time difference = 6 h + 25 min + 10 min = 6 h 35 min

Example 4

What is the time 7 hours and 40 minutes after 11:52 p.m.?

Solution

Use a timeline and add the hours first:



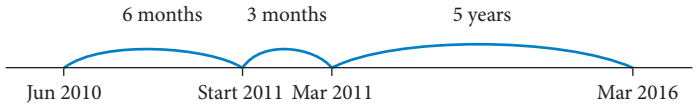
11:52 p.m. + 7 hours	6:52 a.m.
Count to the next full hour and add the number of minutes.	8 minutes to 7 a.m.
To make up 40 minutes, add another 32 minutes: 7 a.m. + 32 min.	7:32 a.m.

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Example 5

In March 2016, Kane's car was 5 years and 9 months old. When was Kane's car made?

Solution



Subtract 5 years from March 2016.	2016 − 5 = 2011 March 2011
Count back to the start of 2011 and subtract the number of months.	3 months to start of 2011
To make up 9 months, subtract another 6 months: Start of 2011 − 6 months	June 2010

EXERCISE 6.2

How long will it take?

1 Calculate the time difference between each pair of times.

EXAMPLE 3

a 7:27 a.m. and 1:12 p.m. **b** 4:09 a.m. and 9:53 a.m.

c 3:42 p.m. and 6:02 p.m. **d** 11:15 p.m. and 3:08 a.m.

2 What is the time difference between each pair of 24-hour times?

a 0800 and 1100 **b** 0500 and 1500

c 0940 and 1455 **d** 1340 and 2150

3 Vamsee's flight lands in Hong Kong at 0950. His next flight leaves Hong Kong at 1920. How long does Vamsee have in Hong Kong airport?

4 At Nelson Secondary College the school day begins at 8:45 a.m. and ends at 3:10 p.m. How long is the school day?

5 What time will it be:

EXAMPLE 4

a 5 hours after 3:00 p.m.?

b 28 minutes after 7:15 p.m.?

c 3 hours 32 minutes after 9:45 a.m.?

d 9 hours 10 minutes after 5:14 p.m.?

6 What time was it:

a 4 hours before 6:15 p.m.?

b 45 minutes before 3:20 a.m.?

c 2 hours 10 minutes before 1:35 p.m.?

d 3 hours 35 minutes before 11:25 a.m.?

7 Eoin has to take his tablets $1\frac{1}{2}$ hours after he finishes lunch. He finished his lunch at 1.45 p.m. What time should Eoin take his medicine?

8 Ramona wants to go for a jog for 50 minutes, but be back in time to watch *Zombie Vampire Athletes* on TV at 8:35 p.m. What is the latest time she can leave for her jog?

EXAMPLE 5

9 Jamie bought a computer in December 2014. How old is it:

a in August 2018? **b** in January 2019? **c** today?

10 A box of chocolates bought in November 2015 has a 'Use by' date of March 2017. How long, in months, will the chocolates last?



11 In February 2015 Stephanie's television was $3\frac{1}{2}$ years old. When did Stephanie buy the television?

12 Ask your teacher when they started teaching at your school. Work out how long they have been at your school in years and months.

13 How old are you today in years, months and days?

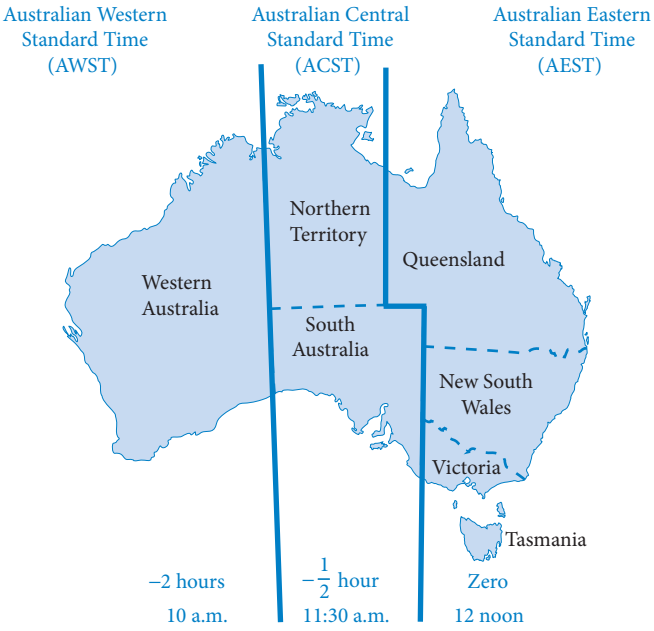
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Times across Australia

6.3

Australia has three time zones:

- Australian Eastern Standard Time (AEST), covering Queensland, New South Wales, the ACT, Victoria and Tasmania
- Australian Central Standard Time (ACST), covering South Australia and the Northern Territory and $\frac{1}{2}$ hour behind AEST
- Australian Western Standard Time (AWST), covering Western Australia and 2 hours behind AEST



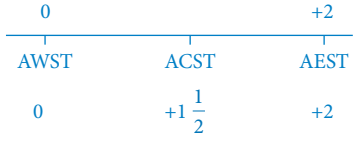
The best way to represent time zones is on a timeline with your location being 0 hours.

Example 6

Show the Australian time zones on a timeline with Western Australia as 0 hours.

Solution

If Western Australia is 0, then Eastern Australia is +2 hours and Central Australia is $+1\frac{1}{2}$ hours



When moving east, add on the time difference.

When moving west, subtract the time difference.

Example 7

a It is 9 a.m. in Perth. What time is it in Adelaide?

b It is 2 p.m. in Melbourne. What time is it in Adelaide?

Solution

a Moving east from Perth to Adelaide: add the time difference.

$9 \text{ a.m.} + 1\frac{1}{2} \text{ hours} = 10{:}30 \text{ a.m.}$

b Moving west from Melbourne to Adelaide: subtract the time difference.

$2 \text{ p.m.} - \frac{1}{2} \text{ hour} = 1{:}30 \text{ p.m.}$

EXERCISE 6.3

Times across Australia

EXAMPLE 6

1 Show the time zones on a timeline with the Northern Territory as zero hours.

2 State whether each location is ahead of, behind or has the same time as Adelaide.

a Sydney **b** Melbourne **c** Darwin **d** Canberra
e Geraldton (WA) **f** Mt Isa (Qld) **g** Ballarat (Vic) **h** Ceduna (SA)
i Hobart **j** Broome (WA)



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EXAMPLE 7

3 It is 10:30 a.m. in Adelaide. What time is it in:

a Cairns (Qld)? **b** Fremantle (WA)? **c** Alice Springs (NT)?
d Launceston (Tas)? **e** Bendigo (Vic)? **f** Broome (WA)?

4 What is the time in each city when it is 11 p.m. in Melbourne?

a Brisbane **b** Adelaide **c** Perth

d Darwin **e** Hobart **f** Canberra

5 The AFL Grand Final started in Melbourne at 2:15 p.m. and was shown live in Adelaide. What time did the game start on Adelaide TV?



Getty Images/Quinn Rooney

6 **a** Joe's flight from Sydney to Perth will take 4 hours. He leaves Sydney at 2 p.m. What time does he land in Perth? Give your answer as Perth local time.

b When Joe flies home, he leaves Perth at 11:30 a.m. What time does he land in Sydney? Give your answer as Sydney local time.

7 When Joanna was holidaying at Margaret River in WA, she phoned her parents in Geelong. She rang at 6:30 p.m. Western Standard Time. What time was it in Geelong?

8 Simone, in Townsville, QLD, uses the Internet to talk with her cousins in Broome, WA. At what time should Simone log on to catch her cousins at 8:00 p.m. Broome time?

9 **a** Use the Internet to find out when daylight savings begins and ends in Victoria.

b Why do some states have daylight saving?

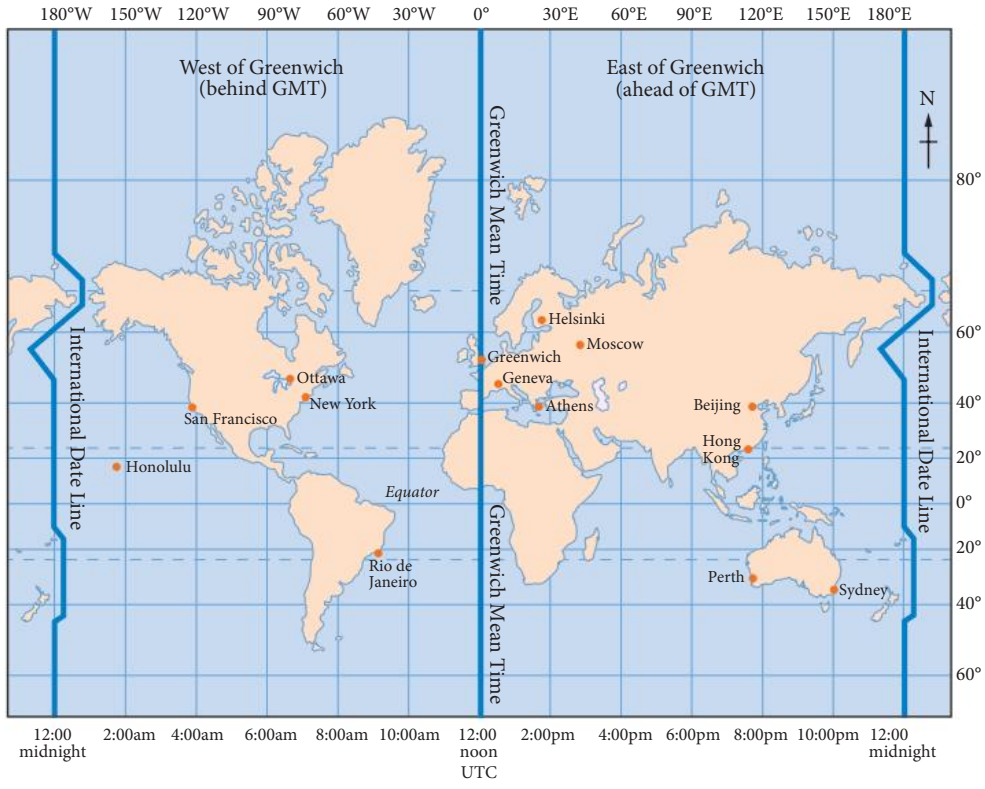
c How does daylight saving affect the different time zones?

d When it is 12:30 p.m. in Western Australia (not on daylight saving), what time is it in Victoria on Eastern Daylight Saving Time?

10 Broken Hill in NSW operates on Central Standard Time rather than Eastern Standard Time like the rest of NSW. Suggest a reason why the people of Broken Hill might have chosen to do this. (Hint: look at a map.)

6.4 Times across the world

This map shows time zones across the world, which are measured relative to the Greenwich Observatory near London in England. The time in London is called **Greenwich Mean Time** (GMT) or Coordinated Universal Time (UTC).



Places west of England are behind GMT and places east of England are ahead of GMT.



Example 8

State whether each country is ahead of or behind Greenwich Mean Time.

a Australia **b** USA **c** Chile **d** China

Solution

a On the map, Australia is east (to the right) of England. Australia is ahead of GMT.

b USA is west (to the left) of England. USA is behind GMT.

c Chile is in South America, west of England. Chile is behind GMT.

d China is in Asia, east of England. China is ahead of GMT.

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This table shows us the time in various cities in relation to GMT.

City	Hours from GMT	Cities	Hours from GMT
Honolulu	−10	Beirut, Helsinki, Cairo	+2
Anchorage	−9	Nairobi, Mecca, Baghdad, Moscow	+3
Las Vegas, Los Angeles, Vancouver	−8	Port Louis, Dubai	+4
Banff, La Paz	−7	Islamabad	+5
Chicago, Mexico City	−6	Astana, Dhaka	+6
Atlanta, New York, Lima	−5	Jakarta, Phnom Penh, Hanoi	+7
Bridgetown, Santiago	−4	Perth, Denpasar, Hong Kong, Singapore	+8
Rio de Janeiro, Buenos Aires	−3	Tokyo, Seoul, Dili	+9
South Sandwich Islands	−2	Brisbane, Melbourne, Port Moresby, Hagatna	+10
Azores	−1	Port Vila, Honiara	+11
London, Dublin, Reykjavik	0	Christchurch, Suva	+12
Florence, Algiers, Paris, Berlin	+1	Nuku'alofa	+13

Example 9

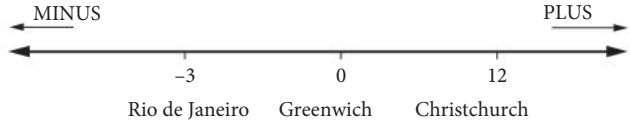
a What is the time difference between Rio de Janeiro (Brazil) and Christchurch (New Zealand)?

b When is 2 a.m. in Rio de Janeiro, what time is it in Christchurch?

c What time is it in Rio de Janeiro when it is 7 a.m. in Christchurch?

Solution

a Rio de Janeiro is −3 from GMT.
Christchurch is +12 from GMT.



Time difference = 3 + 12
= 15 hours

b Christchurch is 15 hours ahead. 2 a.m. + 15 hours = 5 p.m. the same day

c Rio de Janeiro is 15 hours behind. 7 a.m. − 15 hours = 4 p.m. the day before

Remember to include which day it is in your answer.

EXERCISE 6.4

Times across the world

Use the map and table from the previous pages to complete this exercise.

EXAMPLE 8

1 For each country, state whether it is ahead or behind Greenwich Mean Time.

a Alaska **b** Japan **c** India

d Argentina **e** Canada **f** Saudi Arabia

EXAMPLE 9

2 What is the time difference between each pair of cities? Remember: a line diagram can help you work it out!

a Mecca (Saudi Arabia) and Las Vegas (USA)

b Lima (Peru) and Melbourne

c Honolulu (Hawaii) and Buenos Aires (Argentina)

d Helsinki (Finland) and Suva (Fiji)

e Banff (Canada) and Florence (Italy)

3 Use your answers to Question **2** to answer the following:

Remember to include which day it is in your answer.

a When it is 9 a.m. in Las Vegas, what time is it in Mecca?

b When it is 4 p.m. in Mecca, what time is it in Las Vegas?

c What time is it in Melbourne when it is 3 a.m. in Lima?

d When it is 10 a.m. in Melbourne, what time is it in Lima?

e What time is it in Honolulu when it is 2 p.m. in Buenos Aires?

f What time is it in Buenos Aires when it is 10 p.m. in Honolulu?

g When it is 10 a.m. in Suva, what time is it in Helsinki?

h When it is 10 p.m. in Helsinki, what time is it in Suva?

i When it is 8 p.m. in Florence, what time is it in Banff?

j What time is it in Florence when it is 11 a.m. in Banff?

4 **a** When it is 5 a.m. in Mexico City, what time is it in Helsinki?

b When it is 4 p.m. in Hong Kong, what time is it in Vancouver (Canada)?

c What time is it in Algiers (Algeria) when it is 10 a.m. in Honolulu?

d When it is 1 p.m. in La Paz (Bolivia), what time is it in Reykjavik (Iceland)?

e If it is 9 p.m. in Atlanta (USA), what time is it in Dhaka (Bangladesh)?

f When it is 11 a.m. in Chicago (USA), what time is it in Port Vila (Vanuatu)?

5 At 9:00 a.m. in Bridgetown (Barbados), Wesley phones his cousin in Cairo (Egypt). What time is it in Cairo?

6 James is playing an online game at 10 p.m. in Perth. He is multiplaying with Juan in Santiago (Chile). What time is it in Santiago?

7 Michelle caught a flight from Melbourne to Christchurch. This flight left Melbourne at 1030.

a What time was it in Christchurch when Michelle's flight left Melbourne?

b The flight arrived in Christchurch at 1800 (New Zealand time). How long did Michelle's flight take?

8 Carol flew from New York to Honolulu. Her flight left New York at 1520 on Saturday and took 11 hours to get to Honolulu. At what local time did she arrive in Honolulu?

9 Simon flies from Brisbane to Los Angeles. He leaves Brisbane at 1030 on Saturday. He arrives in Los Angeles at 0900 on the same day (Saturday).

a How is this possible?

b How long is the flight from Brisbane to Los Angeles?



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Investigation How did we get time zones?

Use the Internet to research the history of time zones, including:

- how people kept time before clocks were widely used and why this worked
- why Greenwich was established as 'zero time'
- what developments led to the need for common time zones
- what people were involved in the development of time zones
- when time zones became established

Prepare a presentation showing what you have found.

Investigation The International Date Line

Use the Internet to research the following questions.

- What is the International Date Line?
- How is it related to the Prime Meridian through Greenwich?
- What happens when you cross the International Date Line when you are travelling west, for example when flying from the USA to Australia?
- What happens when you cross the International Date Line when you are travelling east, for example when flying from Australia to the USA?
- Name an island on each side of the International Date Line.
- Two countries have recently changed where they are in relation to the International Date Line. What are they and why did they make the change?
- Find and copy a map showing the International Date Line.

6.5 When will it arrive?

Example 10

A section of a train timetable is shown here.

Armadale/Thornlie to Perth

	AM	AM	AM	AM	AM	AM	AM	AM
Armadale	7:16		7:46		8:16		8:31	
Sherwood	7:18		7:48		8:18		8:33	
Challis S	7:20		7:50		8:20		8:35	
Kelmscott	7:22		7:52		8:22		8:37	
Seaforth	7:25		7:55		8:25		8:40	
Gosnells	7:27		7:57		8:27		8:42	
Maddington	7:30		8:00		8:30		8:45	
Kenwick	7:32		8:02		8:32		8:47	
Beckenham	7:34		8:04		8:34		8:49	
Thornlie		7:35		8:05		8:35		8:50
Cannington	7:36	7:39	8:06	8:09	8:36	8:39	8:51	8:54
Queens Park		7:40		8:10		8:40		8:55
Welshpool		7:42		8:12		8:42		8:57
Oats Street	7:39	7:44	8:09	8:14	8:39	8:44	8:54	8:59
Carlisle		7:45		8:15		8:45		9:00
Victoria Park		7:48		8:18		8:48		9:03
Burswood		7:50		8:20		8:50		9:05
Belmont Park				8:22		8:52		9:07
Claisebrook	7:47	7:54	8:17	8:24	8:47	8:54	9:02	9:09
McIver	7:49	7:56	8:19	8:26	8:49	8:56	9:04	9:11
Perth	7:50	7:58	8:20	8:28	8:50	8:58	9:05	9:13



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Perth to Armadale/Thornlie

	PM	PM	PM	PM	PM	PM	PM	PM
Perth	4:37	4:45	4:52	5:00	5:07	5:15	5:22	5:30
McIver	4:38	4:46	4:53	5:01	5:08	5:16	5:23	5:31
Claisebrook	4:40	4:48	4:55	5:03	5:10	5:18	5:25	5:33
Belmont Park	4:42		4:57		5:12		5:27	
Burswood	4:44		4:59		5:14		5:29	
Victoria Park	4:46		5:01		5:16		5:31	
Carlisle	4:48		5:03		5:18		5:33	
Oats Street	4:49	4:54	5:04	5:09	5:19	5:24	5:34	5:39
Welshpool	4:51		5:06		5:21		5:36	
Queens Park	4:53		5:08		5:23		5:38	
Cannington	4:55	4:58	5:10	5:13	5:25	5:28	5:40	5:43
Beckenham		4:59		5:14		5:29		5:44
Kenwick		5:01		5:16		5:31		5:46
Maddington		5:03		5:18		5:33		5:48
Gosnells		5:07		5:22		5:37		5:52
Seaforth		5:09		5:24		5:39		5:54
Kelmscott		5:12		5:27		5:42		5:57
Challis		5:14		5:29		5:44		5:59
Sherwood		5:16		5:31		5:46		6:01
Armadale		5:19		5:34		5:49		6:04
Thornlie	5:00		5:15		5:30		5:45	

Armadale Thornlie Line

Louise catches the 7:52 a.m. train from Kelmscott to get to work at Burswood. What time does she get to Burswood station?

Solution

The 7:52 a.m. train from Kelmscott does not stop at Burswood. She will need to change trains at Oats Street at 8:06 a.m. She can then catch a train at 8:09 a.m. to Burswood.

The train arrives at Burswood station at 8:20 a.m.

EXERCISE 6.5

When will it arrive?

Use the timetable in Example 10 to answer each question.

EXAMPLE 10

1 What time does the 7:16 a.m. train from Armadale arrive at Perth?

2 What time does the 5:03 p.m. train from Carlisle arrive at Thornlie?

3 Ravi lives at Queens Park and works in Perth. He starts work at 8:45 a.m. What is the latest train he can catch at Queens Park to get to work on time?

4 Ravi usually finishes work at 4:30 p.m. It takes him 10 minutes to walk to the station.

a What time can he catch the train home?

b What time does he get to Queens Park?

c Three days a week Ravi catches up with friends after work for half an hour. On these days, what time does he catch the train home?

5 Ruby catches the 5 p.m. train from Perth. What time will she arrive at Gosnells?

6 Katarina is going to catch the train at Maddington to meet her friend Sumana at Belmont Park. They have agreed to meet for shopping and coffee at 9:15 a.m. What time does Katarina need to catch the train?

7 It takes 1 hour and 10 minutes to travel by bus from Thornlie to Perth.

a How much longer does it take by bus than by train?

b Suggest reasons why the bus might take longer.

This is the bus timetable for Adelaide to Alice Springs and return.
Use it to answer Questions **8** to **12**.

ALICE SPRINGS–ADELAIDE

TOWN	CODE		GX850 DAILY EST	PICK-UP AND SET-DOWN POINT
ALICE SPRINGS	ASP	Dep	10:30A	Greyhound Terminal, Todd St
Erldunda	ERL		12:35P	Desert Oaks Resort, Cnr Stuart & Lassester Hwy
Kulgera	KUL	Arr	1:25P	Kulgera Roadhouse Hotel, Stuart Hwy
		Dep	2:00P	
SOUTH AUSTRALIA				
Indulkna Turnoff	IDK		3:20P	Roadside mailbox at turn off
Marla	MBR		4:00P	Traveller's Rest, Stuart Hwy
Cadney Park	CDY		4:55P	Mobil Roadhouse
Coober Pedy	CPD	Arr	6:30P	Terminal, Hutchison St
		Dep	7:25P	
Bilgunnia Turnoff	BGN		8:55P	Turn Off
Glendambo	GBO	Arr	10:10P	Mobil Roadhouse
		Dep	10:30P	
Pimba	PIM	Arr	11:45P	Pimba Roadhouse
		Dep	12:01A	
Port Augusta	PUG		1:50A	Post Office, Commerical Road
Port Augusta Meal Break	PUB	Arr	2:00A	Gull Service Station, Lot 8 Highway
		Dep	2:45A	
Port Pirie	PIR	Arr	3:40A	BP Service Station
Port Wakefield	PWF	Dep	5:00A	BP Service Station
Bolivar	BLV		5:45A	BP Service Station
Cavan	CAV		6:00A	Bus Stop 26, Port Wakefield Road
ADELAIDE	ADL	Arr	6:25A	Greyhound Terminal, Franklin St

ADELAIDE–ALICE SPRINGS

TOWN	CODE		GX580 DAILY EST	PICK-UP AND SET-DOWN POINT
ADELAIDE	ADL	Dep	6:00P	Greyhound Terminal, Franklin St
Cavan	CAV		6:20P	Bus Stop 26, Port Wakefield Rd
Bolivar	BLV		6:30P	Caltex Service Station
Port Wakefield	PWF		7:25P	BP Service Station, Snowtown Rd
Port Pirie	PIR		8:40P	BP Service Station
Port Augusta Meal Break	PUB	Arr	9:45P	Gull Service Station, Lot 8 Highway
		Dep	10:30P	
Port Augusta	PUG		10:50P	Post Office, Commerical Rd
Pimba	PIM	Arr	12:45A	Shell Roadhouse
		Dep	1:00A	
Glendambo	GBO	Arr	2:15A	Shell Roadhouse
		Dep	2:30A	
Bulgunnia Turnoff		BGN	3:35A	Turn Off
Coober Pedy	CPD	Arr	5:15A	Terminal, Hutchison St
		Dep	5:50A	
Cadney Park	CDY	Arr	7:30A	Mobil Roadhouse
Marla	MBR		8:20A	Traveller's Rest, Stuart Hwy
			9:05A	
Indulkna Turn Off	IDK		9:35A	Roadside mailbox at turn off
NORTHERN TERRITORY				
Kulgera	KUL		11:00A	Kulgera Roadhouse Hotel, Stuart Hwy
Erldunda	ERL	Arr	11:45A	Desert Oaks Resort, Cnr Stuart & Lasseter Hwy
		Dep	12:20P	
ALICE SPRINGS	ASP	Arr	2:30P	Greyhound Terminal, Shop 2/76Todd St

Greyhound Australia

8 How long does the trip from Adelaide to Alice Springs take?

9 How many times during the trip does the bus stop for more than 20 minutes? Where are these stops?



Newspix/Steve Pohlner

10 Harry joins the bus at Port Augusta to travel to Coober Pedy.

a How long is his bus trip?

b When Harry returns from Coober Pedy he will travel all the way to Adelaide. How long will it take?

11 Ariana and Axel are two friends. Ariana lives in Adelaide and Axel lives in Alice Springs. They are planning to meet in the middle to spend the weekend together.

a Where is the closest stop to 'the middle'?

b Taking into account the time of day and when each person would arrive, where would be the best place to meet along this route? Justify your answer.

12 The train from Adelaide to Alice Springs leaves Adelaide at 12:20 p.m. Sunday and arrives in Alice Springs at 1:45 p.m. Monday.

a How long does it take to travel by train from Adelaide to Alice Springs?

b How does this time compare to the bus trip?

c Would you prefer to travel by bus or train from Adelaide to Alice Springs? Justify your answer.

This timetable is for CityCat ferries operating on the Brisbane River.
Use it to answer Questions **13** to **15**.

13 How long does the ferry take to travel from Bretts Wharf to South Bank 2?

Departs Terminal:	am	am	am	am	am	am	am	am	pm
Northshore Hamilton	…..	…..	…..	…..	…..	…..	…..	…..	…..
Apollo Wharf	10:25	10:37	10:50	11:02	11:15	11:27	11:40	11:52	12:05
Bretts Wharf	10:28	10:40	10:53	11:05	11:18	11:30	11:43	11:55	12:08
Bulimba	10:34	10:46	10:59	11:11	11:24	11:36	11:49	12:01	12:14
Teneriffe	10:37	10:49	11:02	11:14	11:27	11:39	11:52	12:04	12:17
Hawthorne	10:41	10:53	11:06	11:18	11:31	11:43	11:56	12:08	12:21
New Farm Park	10:46	10:58	11:11	11:23	11:36	11:48	12:01	12:13	12:26
Mowbray Park	10:50	11:02	11:15	11:27	11:40	11:52	12:05	12:17	12:30
Sydney Street	10:53	11:05	11:18	11:30	11:43	11:55	12:08	12:20	12:33
Riverside	11:01	11:13	11:26	11:38	11:51	12:03	12:16	12:28	12:41
QUT Gardens Point	11:09	11:21	11:34	11:46	11:59	12:11	12:24	12:36	12:49
South Bank 2	11:12	11:24	11:37	11:49	12:02	12:14	12:27	12:39	12:52
North Quay	11:16	11:28	11:41	11:53	12:06	12:18	12:31	12:43	12:56
Regatta	11:24	11:36	11:49	12:01	12:14	12:26	12:39	12:51	1:04
Guyatt Park	11:28	11:40	11:53	12:05	12:18	12:30	12:43	12:55	1:08
West End	11:31	11:43	11:56	12:08	12:21	12:33	12:46	12:58	1:11
University of Queensland	11:35	11:47	12:00	12:12	12:25	12:37	12:50	1:02	1:15

CityCat Ferries



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14 a Phillip needs to be at work at a restaurant at Riverside by 11:30 a.m. What is the latest time he can catch the ferry at Teneriffe?

b After work he travels out to the University of Queensland to meet up with friends. How long does the ferry trip take?

c At the end of the evening he returns home. How long will this trip take if he chooses to go by ferry? Assume the reverse journey will take the same length of time as the forward journey.

d To travel from the University of Queensland to Teneriffe by bus takes approximately 55 minutes. Why might people choose to travel by bus rather than ferry?

15 Why is one part of the timetable white and one part green?

16 The City Explorer Bus stops at places of interest in the city and operates in Melbourne, Sydney and Canberra.

a How many buses are needed to meet this Explorer Bus timetable? Explain how you arrived at your answer.

Depart									
Explorer depot	10:00	10:25	10:50	11:15	11:45	12:00	12:25	12:50	1:15
City cathedral	10:08	10:33	10:58	11:23	11:53	12:08	12:33	12:58	1:23
Railway station	10:15	10:40	11:05	11:30	12:00	12:15	12:40	1:05	1:30
Parliament	10:24	10:49	11:14	11:39	12:09	12:24	12:49	1:14	1:39
Museum	10:35	11:00	11:25	11:50	12:20	12:35	1:00	1:25	1:50
City square	10:45	11:10	11:35	12:00	12:30	12:45	1:10	1:35	2:00
Zoo	11:00	11:25	11:50	12:15	12:45	1:00	1:25	1:50	2:15
Dockland shops	11:12	11:37	12:02	12:27	12:57	1:12	1:37	2:02	2:27
Arts centre	11:19	11:44	12:09	12:34	1:04	1:19	1:44	2:09	2:34
Water gardens	11:30	11:55	12:20	12:45	1:15	1:30	1:55	2:20	2:45
Hall of fame	11:38	12:03	12:28	12:53	1:23	1:38	2:03	2:28	2:53
Arrive									
Explorer depot	11:50	12:15	12:40	1:05	1:35	1:50	2:15	2:40	3:05

City Explorer Bus

b Vo, Binh and Vicki came to the city by train, arriving at the station at 11:42 a.m. They caught the Explorer Bus to the zoo. What is the earliest time they could expect to arrive at the zoo? Explain your answer.

c Manuel and Sofia are dropped off by car at the City cathedral at 10:25 a.m. They arrange to meet their hosts at the 'Hall of fame' at 2:45 p.m. They want to spend at least half an hour at the museum, photograph the City square and do some souvenir shopping at the Dockland shops. Plan a list of times for them to catch the Explorer Bus to do these things and meet their hosts on time.

d In summer, extra Explorer tours leave the depot at 11:30 a.m., 1:30 p.m. and 2:30 p.m. Make a list of departure times that appear in the timetable for each of these tours.

Investigation Planning a trip

You are visiting one of the state capitals in Australia. You are going to plan a trip from the airport to the CBD and on to an island off the coast.

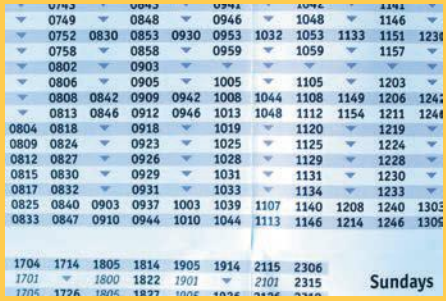
1. Choose which capital you are going to visit.
2. You need to travel from the airport to the CBD in the city you're visiting for your accommodation. Choose a hotel in the capital city to stay at. Find out the best way to travel from the airport to the hotel, how long it will take and what it will cost.
3. You are going to visit one of the islands off the coast near the city you have chosen (Rottnest, Kangaroo or Stradbroke for example). Find out how to get there from your hotel (train, bus and/or ferry), how long it will take and what it will cost.
4. Choose another tourist attraction in the city you are visiting and find out how to get there from your hotel.
5. Find out if there any special public transport deals available in the capital city you have chosen that could save you money when you are visiting the city.

Keyword activity

Describe each phrase in a sentence and match it with one of the photos below:

12-hour time 24-hour time timeline time zone timetable

A



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B



Shutterstock.com/Matteo Festi

C



Alamy/BCS

D



Shutterstock.com/Iraidka

E



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Chapter Problem

Daniel has moved out of home to live in a unit. He needs to be at work by 8:30 a.m. and catches a bus to the station and then a train. The bus stop is 10 minutes from home and the trip takes 15 minutes. Buses leave every 15 minutes from 6:00 a.m. The train trip takes 10 minutes and trains leave every 20 minutes from 6:05 a.m. Daniel's workplace is a 5-minute walk from the station. What time does Daniel need to leave home to get to work on time?

Solution

We work backwards from work to home. Daniel has to arrive at work by 8:30 am so he needs to arrive at the station at least 5 minutes earlier (5 minutes to walk)	8:30 a.m. – 5 min = 8:25 a.m.
Daniel must catch the train at least 10 minutes earlier (10 minute train trip)	8:25 a.m. – 10 min = 8:15 a.m.
Trains leave at 6:05, 6:25, 6:45, 7:05, 7:25, 7:45, 8:05, and 8:25	Daniel must catch the train at 8:05 a.m.
Daniel must catch the bus at least 15 minutes earlier (15 minute bus trip)	8:05 a.m. – 15 min = 7:50 a.m.
Buses leave at 6:00, 6:15, 6:30, 6:45, 7:00, 7:15, 7:30, 7:45, and 8:00	Daniel must catch the bus at 7:45 a.m.
Daniel must leave home at least 10 minutes earlier (5 minutes to walk)	7:45 a.m. – 10 min = 7:35 a.m. Daniel should leave home about 7:35 a.m.

Depending on the reliability of the bus and train, he may choose to leave earlier to be sure to get to work on time.



Chapter Problem

Andrew and Catriona run an interior design business. They have a client who wants a bathroom renovation. They need to present the client with a choice of designs for a border at the top of the wall tiles. What are some possible designs?



CHAPTER 7

CREATING DESIGNS

7.1 2D shapes
7.2 3D shapes
7.3 Designing a logo
7.4 Sketching solids
7.5 3D in 2D
7.6 It's symmetrical
7.7 Slide, flip or spin?
7.8 Tile patterns
Keyword activity
Solution to the chapter problem

By the end of this chapter, you should know how to:

- draw and recognise properties of common 2D and 3D shapes
- create geometric designs
- interpret 2D representations of 3D objects
- recognise line and rotational symmetry
- identify the different types of tessellations and transformations

How are we ever going to use this?

- Whenever we use shapes or solids in design
- To know how to make models of 3D objects
- To make decorative patterns for house renovations, craft activities and paving
- In an occupation where design is an important element, for example graphic design

7.1 2D shapes

2D (two-dimensional) shapes are flat and have two dimensions: length and width.

3D (three-dimensional) shapes are solids and have three dimensions: length, width and depth.

EXERCISE 7.1

2D shapes

Use the word list to complete the following paragraphs. You can use words more than once. You may like to cut out some shapes and use them to help you fill in the blanks.

1 Triangles



Alamy/Mediablitzimages



Alamy/DIZ München GmbH



Shutterstock.com/Arnab Maity

Word list:

angle	angles	equilateral
scalene	sides	two
acute	obtuse	right

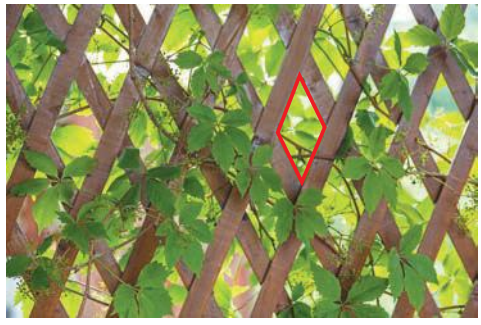
We name triangles according to the lengths of their _____. A triangle with three sides the same length is called an _____ triangle. This triangle also has three _____ equal. Each _____ is equal to 60°. An isosceles triangle has _____ sides and _____ angles equal. A _____ triangle has no sides the same length.

We can also name triangles according to the types of angles in them. A triangle with three angles less than 90° is called an _____-angled triangle. A triangle with one angle between 90° and 180° is called an _____-angled triangle. A right-angled triangle has one _____ angle.

2 The parallelogram and rhombus



Shutterstock.com/Christian Mueller



Shutterstock.com/
Pavel L Photo and Video

9780170370752

Word list:

bisect	equal	four	opposite
parallel	90°	rectangle	square

Each of these shapes has _____ sides. A parallelogram has both pairs of _____ sides equal. Each pair of opposite sides is also _____. The opposite pairs of angles are _____. The diagonals of a parallelogram cut each other in half. We say they _____ each other.

A rhombus has all of the features of a parallelogram. In addition all four sides of the rhombus are _____. This means the diagonals bisect each other at an angle of _____.

A parallelogram looks like a _____ pushed over and the rhombus looks like a _____ pushed over.

3 The rectangle and square

Word list:

bisect	diagonals	equal
parallelogram	rhombus	90°

A rectangle is a special type of _____. However in a rectangle all the angles are _____ and the diagonals are _____ lengths.

A square has all the features of a _____. In addition, the sides are all _____. The angles are all _____. The _____ are the same length.

4 The trapezium and kite



Shutterstock.com/RTimages



Shutterstock.com/Hurst Photo

Word list:

diagonals	equal	parallel
quadrilateral	90°	

Any shape with four sides is called a _____. The trapezium is a quadrilateral with one pair of opposite sides _____. A kite is a bit different. The 'top' pair of sides is _____ and the 'bottom' pair of sides is also _____. The angles at the 'sides' are _____. The _____ intersect at an angle of _____.

5 More polygons



Shutterstock.com/ Tony Magdaraog y



Shutterstock.com/irin-k



Shutterstock.com/Koosen

Word list:

decagon	hexagon	number	octagon
pentagon	quadrilaterals	regular	

A polygon is a shape with straight sides. We use the _____ of sides they have to name each polygon. We have already looked at triangles and _____ in detail. We have the following names for polygons with more than 4 sides:

Name	Number of sides
	5
	6
heptagon	7
	8
nonagon	9
	10
undecagon	11
dodecagon	12

When all the sides of a polygon are equal it is called a _____ polygon. The equilateral triangle and the square are _____ polygons.

3D shapes

7.2

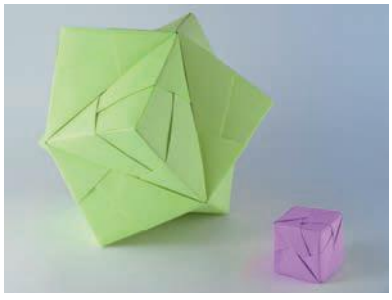
We live in a 3D world. Everywhere around us we see 3D solids – books, buildings, boxes.

EXERCISE 7.2

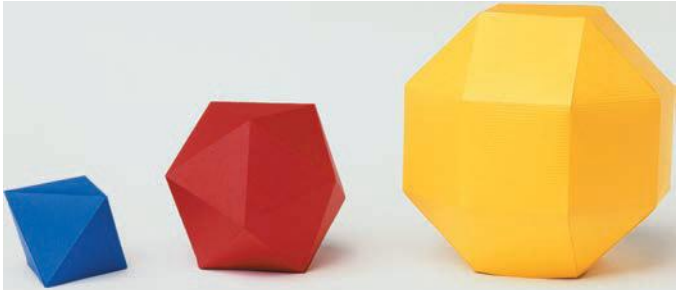
3D shapes

Use the word list to complete each paragraph.

1 Polyhedrons



iStockphoto/vuduchild



Getty Images/Steve Gorton

Word list:

edge	flat	hexahedron	pentahedron
prism	pyramid	rectangular	vertex

Solids can have flat faces and curved faces. Solids that only have _____ faces are called polyhedrons. Some of the common polyhedrons are the _____ prism, the triangular _____ and the square _____. Polyhedrons have similar names to polygons. We call a solid with five faces a _____. A solid with six faces is called a _____. When the faces of a polyhedron meet they form an _____. When three or more edges meet they form a _____.

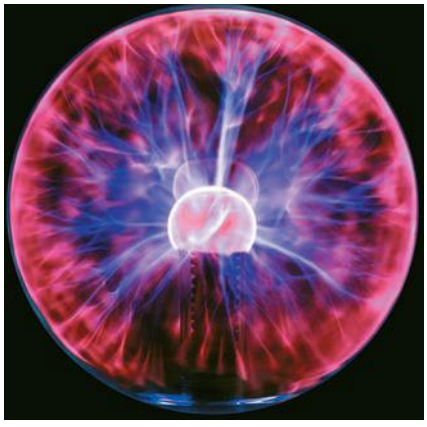
2 Solids with curved surfaces



iStockphoto/enviromantic



Alamy/David Moore/Australia



Shutterstock.com/WHITE RABBIT83

Word list:

circle cone curved flat rectangle sphere

Some solids are not polyhedrons because they have _____ faces. A solid that has only one curved face is the _____. The cylinder has two _____ faces and one _____ face. When we flatten the curved face it is a _____. A _____ has one flat face and one curved face. When we flatten the curved section it is a sector of a _____.

3 Copy and complete this table.

Solid	Number of faces	Shapes of faces	Number of identical faces
cube			
cylinder			
square pyramid			
triangular prism			
rectangular prism			
cone			
triangular pyramid			

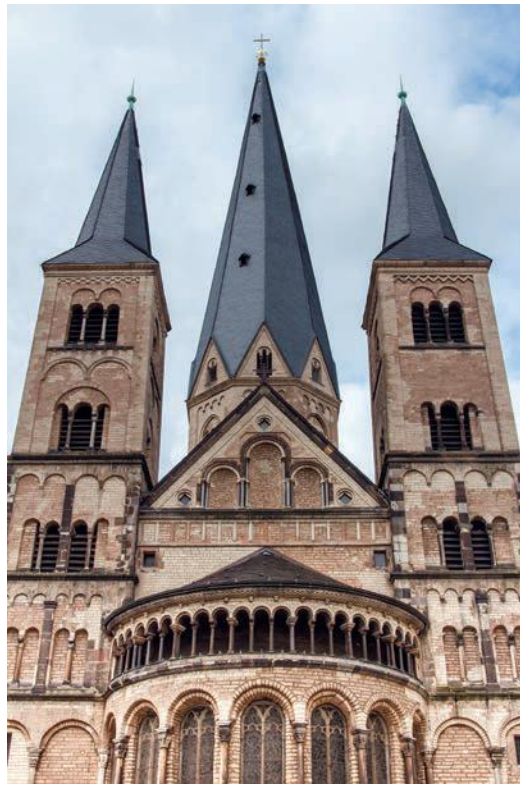
4 Prisms and pyramids



© Pawel Opaska/Dreamstime.com



iStockphoto/falun



Shutterstock.com/Tobias Arhelger

Word list:

bottom	cross-section	end
pointed	rectangle	shape
size	square	triangle

A prism has the same _____ from one end to the other. Both ends are the same _____. Prisms take their names from the shape at each _____. A rectangular prism has a _____ at each end and a triangular prism has a _____ at each end.

A pyramid has a _____ top, called the apex. The shape at the _____ of the pyramid gives the pyramid its name. A square pyramid has a _____ at the bottom. A pyramid's _____ is NOT the same from the bottom to the top. Each cross-section is the same shape but not the same _____.

5 Name each solid shape shown in these pictures.

a



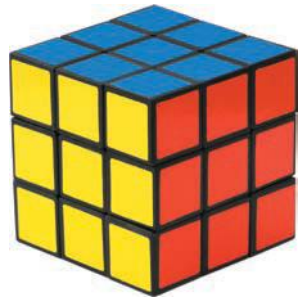
Shutterstock.com/ dean bertoncelj

b



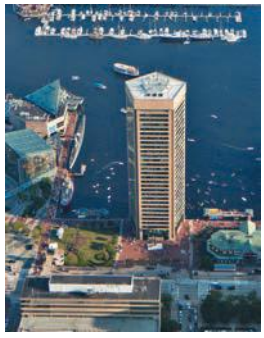
Shutterstock.com/M. Unal Ozmen

c



Shutterstock.com/PeterVrabel

d



Getty Images/Greg Pease

e



Shutterstock.com/ Issarawat Tattong

f



Shutterstock.com/PeterG

Investigation Stacking and packaging

1. Look around your classroom. Why are bricks the shape of rectangular prisms? Why is the room a rectangular prism? What shape are most books? Why do you think this is?
2. Imagine you are in a supermarket. What solids are used for packaging? Why do you think manufacturers use different shapes? How are items stacked on the shelves? What part does packaging play in this?
3. Research the solid shape bees use to make a beehive and why they use it.
4. Find other 3D structures in your environment. What solid shapes have been used? Why might this be so? If you can't think of any, use the Internet to find some examples of different structures.

7.3 Designing a logo

Geometric designs are all around us. They are used in creating logos for businesses and organisations. They are part of the design of products we see in the supermarket and in other graphic design work.



Alamy/Peter Lane



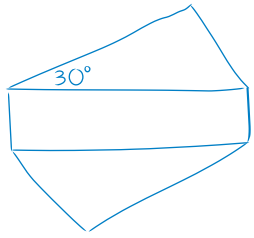
Alamy/Mark Richardson



Alamy/Kevin Foy

Example 1

Chloe works for a design company. When she produces a design for a logo she does a rough sketch first. Draw this design accurately.

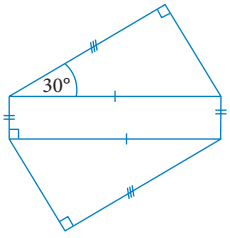


Solution

This logo is made up of a rectangle and two identical right-angled triangles. Draw the rectangle first.



Draw two triangles with a 30° angle and a right angle.



9780170370752

Example 2

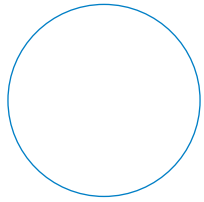
Sometimes, Chloe writes down a description of the design and other people are responsible for drawing it. This can be done by hand or using a computer drawing program.

Chloe described the design for a new logo as follows: 'The logo is a circle overlaid by an equilateral triangle. The vertices of the triangle are outside the circle and the segments of the circle outside the triangle are shaded.

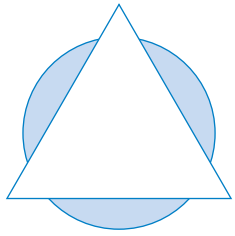
Draw this logo.

Solution

Draw the circle first.



Add the equilateral triangle and shade the segments of the circle outside the triangle.



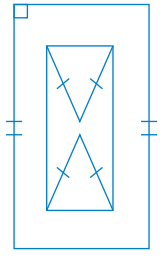
EXERCISE 7.3

Designing a logo

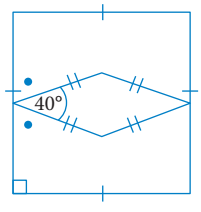
EXAMPLE 1

1 Draw each design accurately, using a ruler and protractor or using technology.

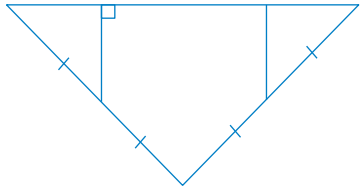
a In the large rectangle the length is twice the width.
In the small rectangle the width is half the width of the large rectangle.



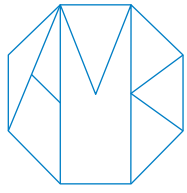
b



c



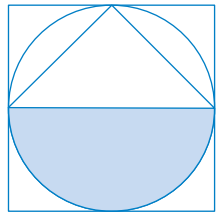
d



All sides of the octagon are equal.

2 Design your own logo. Make a rough sketch first, then an accurate sketch. Use your own initials somewhere in the logo.

3 Use the following words to complete the description of the logo.



Word list: circle, circumference, diameter, four, isosceles, semicircle, square, vertex.

The perimeter of the logo is a square. Inside the _____ there is a large circle whose _____ touches each of the _____ sides of the square. The lower _____ is coloured blue. There is an _____ triangle in the top semicircle. Two vertices of the triangle are on either ends of the _____ and the third _____ is on the circumference of the _____.

EXAMPLE 2

4 Read each description of a logo, then draw the logo.

a The vertices of an equilateral triangle are on the circumference of a circle. The segments between the triangle and the circle are shaded.

b The four vertices of a square are on the circumference of a circle. Both diagonals of the square have been drawn. Two of the four identical triangles have been shaded.

c A regular hexagon has two equilateral triangles inside it. One vertex of each triangle is at the centre of the hexagon and the triangles are opposite each other. One triangle has the letter R in it and the other has the letter M in it.

d A rectangle contains the letters B and B, back-to-back.

5 Choose three logos of well-known companies or products. What shapes are they based on? Write a description of them. Share your descriptions with your friends and see if they can guess what logos you have chosen.

Investigation Road signs

Road signs in Australia are based on a number of geometrical shapes. Use the Internet to help you research the following questions.

- The regular octagon is used for only one sign. What sign is it? Draw or copy an example.
- What signs use the equilateral triangle? Draw or copy an example.
- Many signs are rectangular. Name some of the signs that use rectangles and draw or copy some examples.
- Warning signs are usually black on yellow. What shape are they? Draw or copy an example.
- Some signs have the shape below. What sort of shape is it? Draw or copy an example.



Present your findings as a PowerPoint presentation.

Sketching solids

7.4

Solids or 3D objects can be difficult to draw when you are completing design work. This exercise will give you some ideas to help you sketch common solids more easily.

EXERCISE 7.4

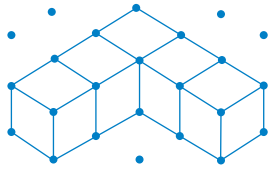
Sketching solids

1 Copy each diagram on isometric dot paper (which can be downloaded from NelsonNet).

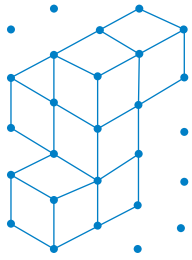
a



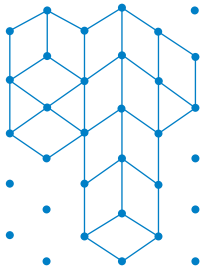
b



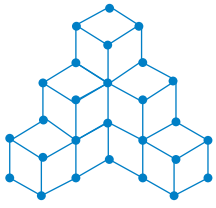
c



d



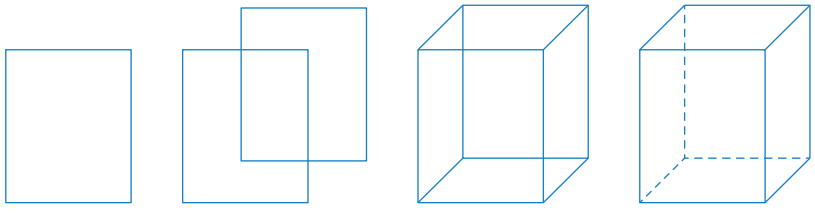
e



f



2 Sketch a rectangular prism using these diagrams and instructions.



a Draw a rectangle that will be the front face of the prism.

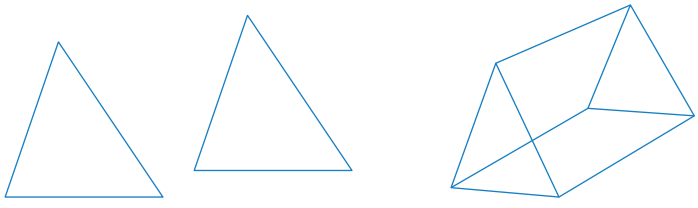
b Draw another rectangle the same size, but position it slightly above and to the right of the first rectangle.

c Join the matching vertices (corners).

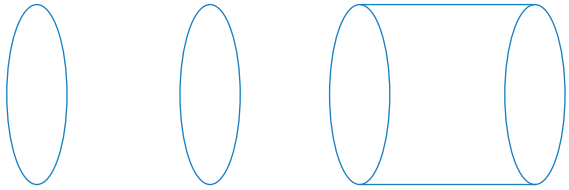
d Make the outside edges of the prism darker and use lighter or broken lines for the inside edges.

3 Draw each solid using the method shown in Question **2**.

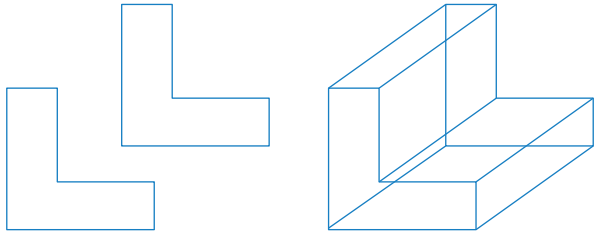
a A short, triangular prism



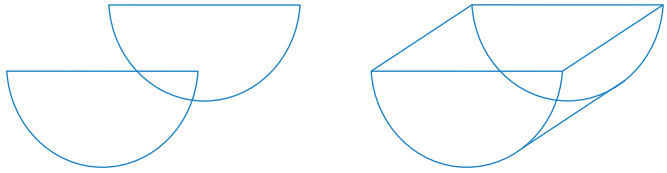
b A cylinder



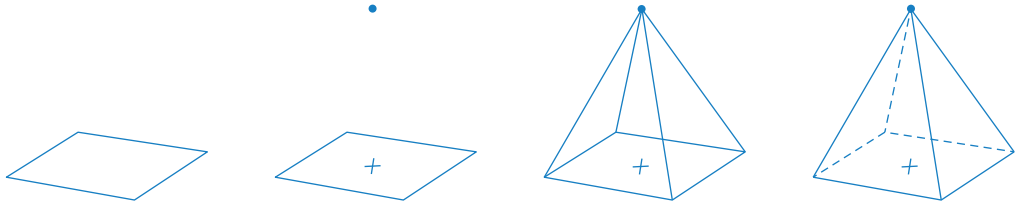
c An L-shaped prism



d A half-cylinder



4 Sketch a rectangular pyramid using these diagrams and instructions.



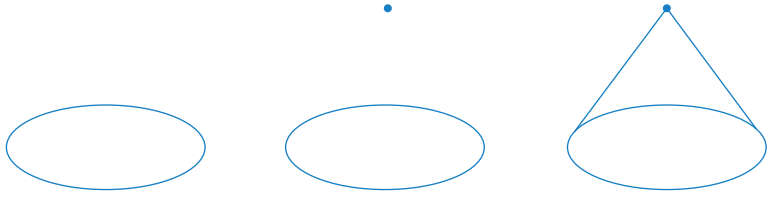
a Draw a parallelogram for the rectangular base.

b Determine the centre of the parallelogram (where the diagonals cross) and place a dot above this position: this dot will be the top of the pyramid.

c Join the four vertices of the parallelogram to the dot.

d Make the outside edges of the pyramid darker and the inside edges lighter or broken.

5 Sketch a cone using these diagrams and instructions.



a Draw an oval for the circular base.

b Draw a dot above the centre of the oval.

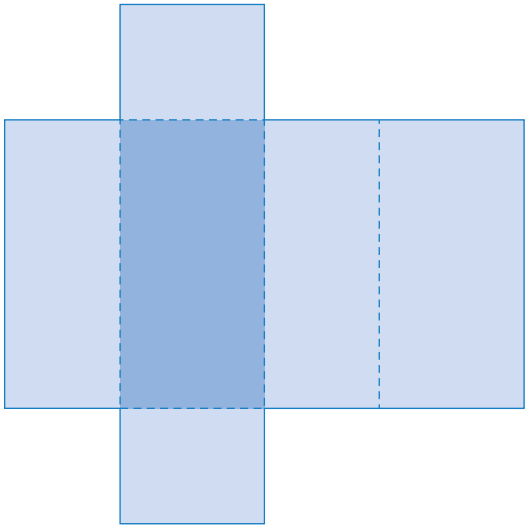
c Join the dot to the oval.

3D in 2D

7.5

Nets of solids

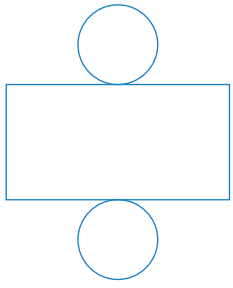
A cardboard box starts out as a flat piece of cardboard. We cut out a particular shape and fold it to make the box. The 2D shape we cut out is called the **net** of a solid. For the box, the net would look like this:



We have not included the tabs we would need to glue it together.

Example 3

What solid would this net make?



Solution

The rectangle would curve around to match the circles.

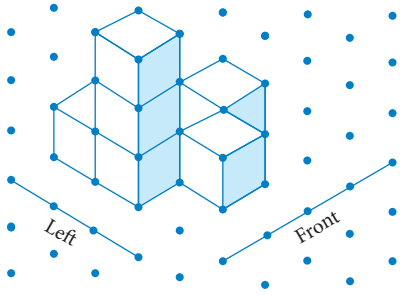
This is the net of a cylinder.

Different views of solids

We can also draw a solid from different perspectives (points of view). We can draw what we would see when looking from the front or the top or one of the sides.

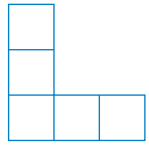
Example 4

For this solid, draw the front, left and top views.



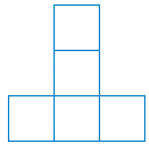
Solution

Imagine you are standing in front of the solid – this is what you would see.



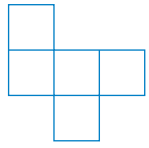
Front view

Imagine you are standing on the left of the solid – this is what you would see.



Left view

Looking down at the solid from above – this is what you would see.



Top view

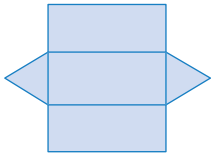
EXERCISE 7.5

3D in 2D

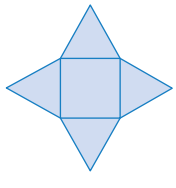
EXAMPLE 3

1 Match each net with the solids they make in parts **i** to **vi**. Write the name of each solid.

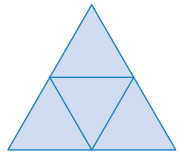
a



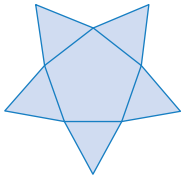
b



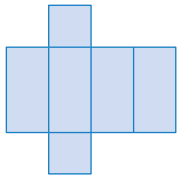
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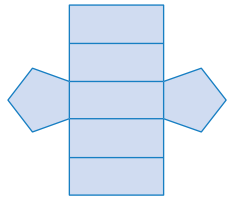
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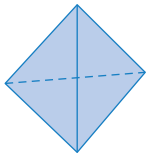
e

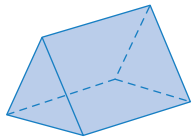


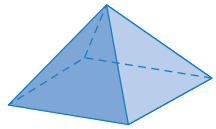
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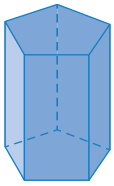


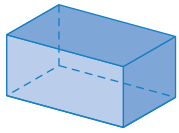
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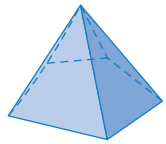
i 

ii 

iii 

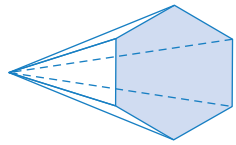
iv 

v 

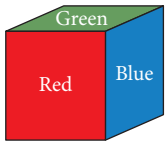
vi 

2 **a** What is the name of this solid?

b Draw the net of this solid.

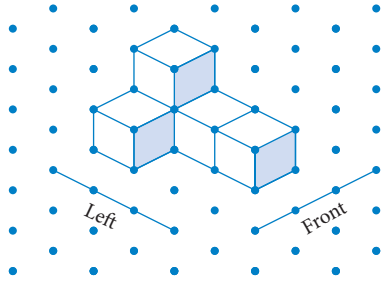


3 Draw the net of this cube, showing the correct positions of the three coloured faces shown.

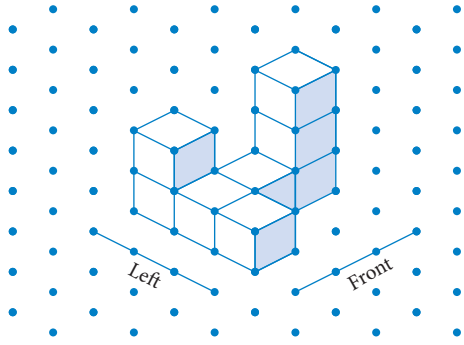


4 For each solid, draw each view.

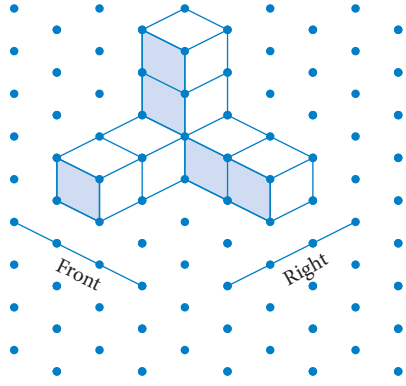
EXAMPLE 4

a 

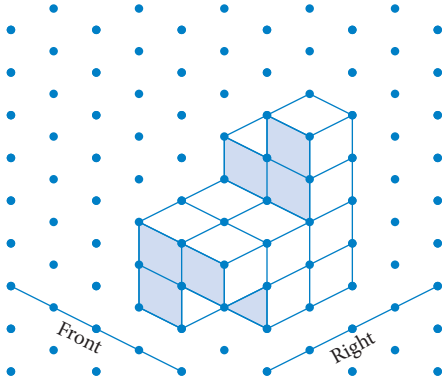
i front view
ii left view
iii top view

b 

i left view
ii back view
iii top view

c 

i front view
ii right view
iii top view

d 

i back view
ii right view
iii top view

5 Draw the top and front views of each object.

a



Shutterstock.com/ Maxx-Studio

b



Shutterstock.com/ Vereshchagin Dmitry

c



Shutterstock.com/ Olga Kovalenko

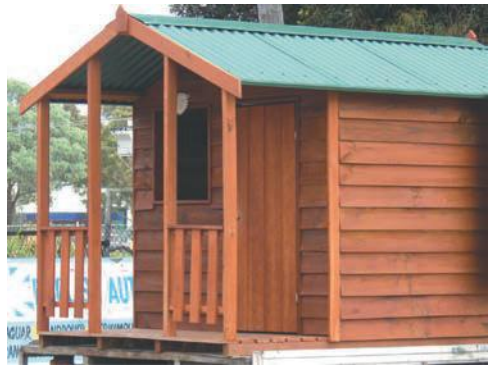
d



Shutterstock.com/ Africa Studio

6 Sketch the front, left and right views of this children's cubby house.





7 Sketch the front, left and 'bird's eye' views of this small country church.



Left back



Left front



Right front

'Bird's eye' view means the top view.

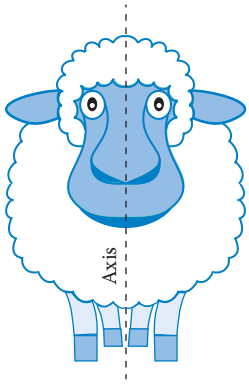
9780170370752

It's symmetrical

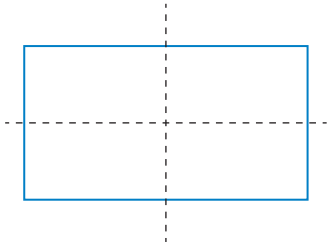
7.6

Symmetry is used in many designs to make them attractive to the eye. There are two types of symmetry.

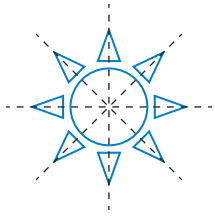
A shape has **line symmetry** if it can be divided by a line into two identical parts which are mirror images of one another. The line is called the **axis** of symmetry.



One axis of symmetry

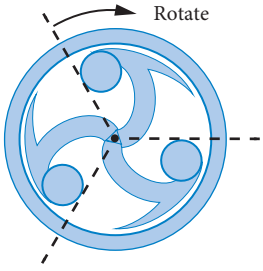


2 axes of symmetry

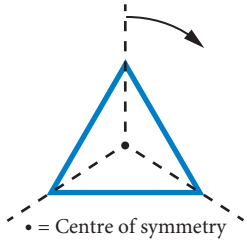


4 axes of symmetry

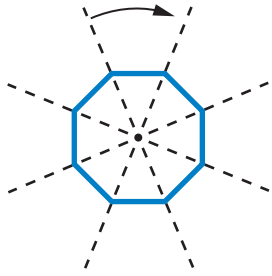
A shape has **rotational symmetry** if it can be spun about a point so that it repeats its shape more than once in a 360° rotation. The point is called the **centre** of symmetry. The number of times the shape repeats itself in 360° is called the **order** of rotational symmetry.



Rotational symmetry of order 3



Rotational symmetry of order 3



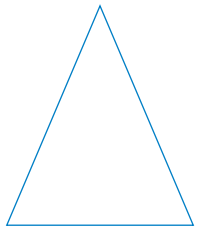
Rotational symmetry of order 8

EXERCISE 7.6

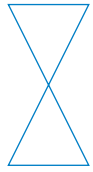
It's symmetrical

1 Copy each shape, draw in any axes of symmetry and count them.

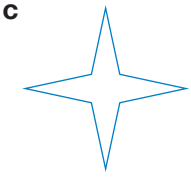
a



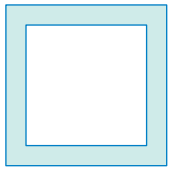
b



c



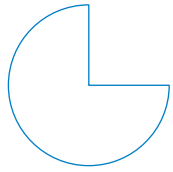
d



e

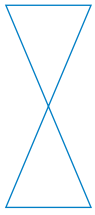


f

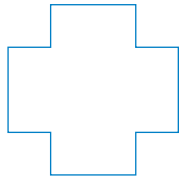


2 Copy each shape, mark the centre of rotational symmetry and state the order of rotational symmetry.

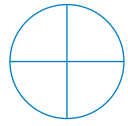
a



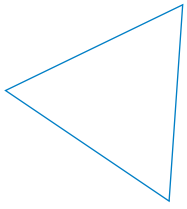
b



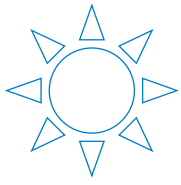
c



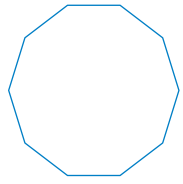
d



e



f



3 For each car tyre, state the order of rotational symmetry (ignore any bolts or holes for bolts).

a



b



c



d



e

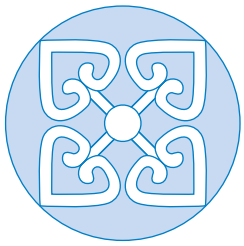


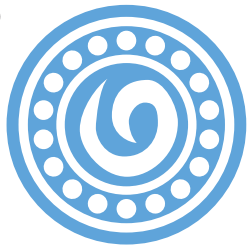
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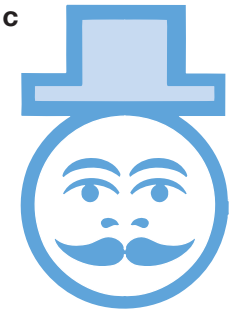


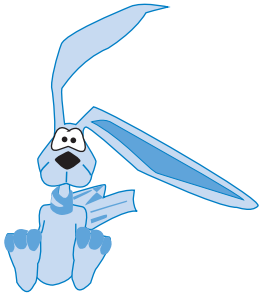
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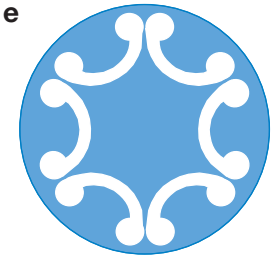
4 Classify each design as having line symmetry, rotational symmetry, both types of symmetry or no symmetry.

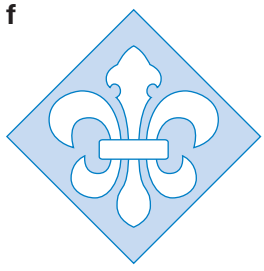
a 

b 

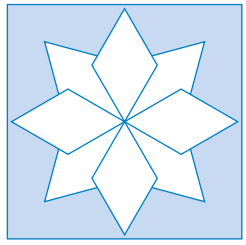
c 

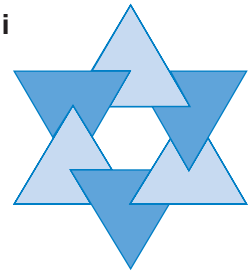
d 

e 

f 

g 

h 

i 

Practical activity Are symmetrical faces more appealing?

In this activity you will create 'symmetrical faces' of students in your class by taking photos of them and using photo-editing software such as Picture Manager from Microsoft Office. You need a digital camera, PowerPoint or Word software and a scanner (if you want to edit photos of 'beautiful people' from magazines).

1 Take photos of students in your class against a plain background. Make sure each person is looking straight at the camera.

2 Download the photos to a computer.

3 Select a photo and save it under a new name. Crop the left half of the person's face.

4 Copy this left-face image and paste it into a PowerPoint or Word file.

5 Make another copy of the left-face image, then 'flip horizontally' to make a symmetrical image.





Left



Left flipped



Left side face

6 Move the two half-faces until they are together to make a whole 'left-side' face.

7 Open the original photo of the person and paste it into the file to compare.

8 Now use the photo-editing software to crop the right half of the person's face.

9 Copy this right-face image, paste it into the file, make another copy, then 'flip horizontally' to make a symmetrical image.

10 Move the two half-faces until they are together to make a whole 'right-side' face.



Right flipped



Right



Right side face

11 Compare the three faces. Are they all the same? Decide whether the original face is symmetrical.



Original whole face



Left side face

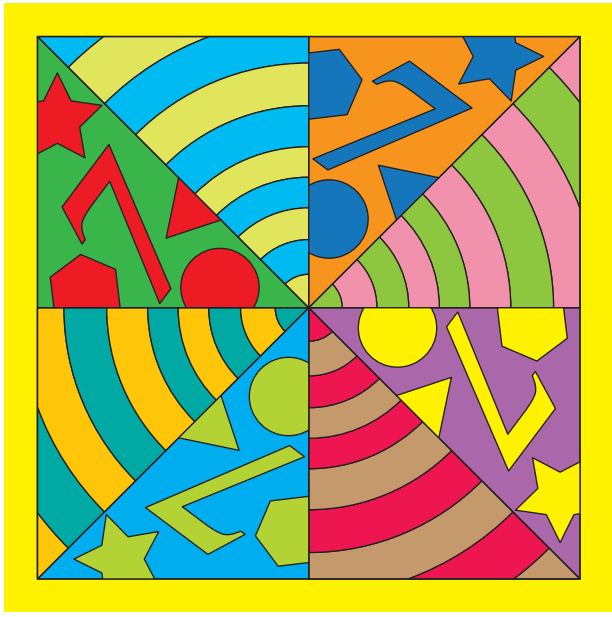


Right side face

12 Repeat for the other photos you have taken. Which students have symmetrical faces? Are they more appealing?

13 Scan some photos of 'beautiful people' from magazines to investigate whether they have symmetrical faces. Write a paragraph summarising your conclusions.

Investigation Symmetry in art and architecture



Oxford Designers & Illustrators



Shutterstock.com/Dmitry Strizhakov

1 Use the Internet to find examples of both line symmetry and rotational symmetry in art and architecture similar to those shown above. The symmetry may be in part of the artwork or in the design or decoration of the building.

2 Print out an image of a work of art AND of a building showing:

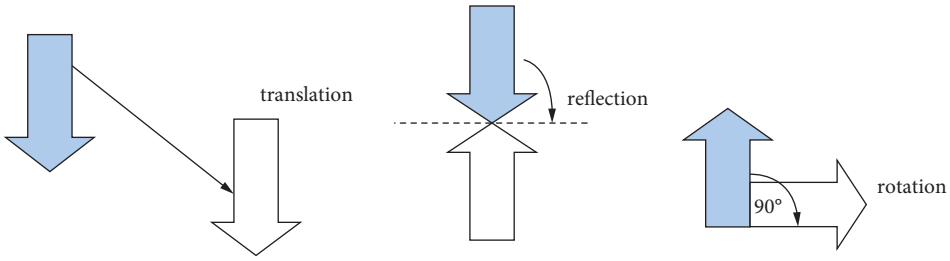
   a line symmetry

   b rotational symmetry

3 On each image, draw the lines of symmetry, mark the centre of rotational symmetry and state the order of rotational symmetry.

7.7 Slide, flip or spin?

Patterns in tiles, wallpaper and paving are usually made by taking a basic shape and repeating it. A pattern can be created by **sliding**, **flipping** or **spinning** the shape. These movements have special names.



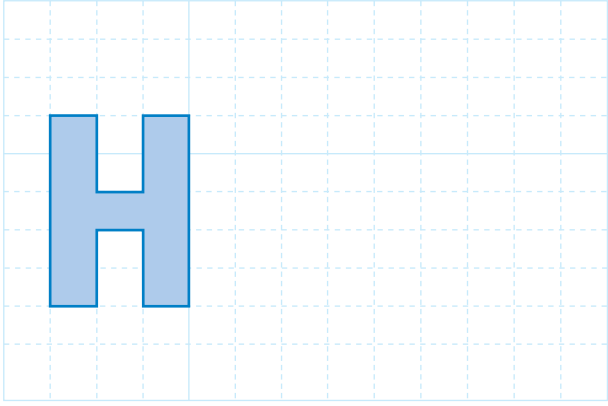
A 'slide' is called a **translation**. A 'flip' is called a **reflection**. A 'spin' is called a **rotation**.

Together, these are called **transformations**, which means change.

- The original shape is called the **original**.
- The transformed shape is called the **image**.

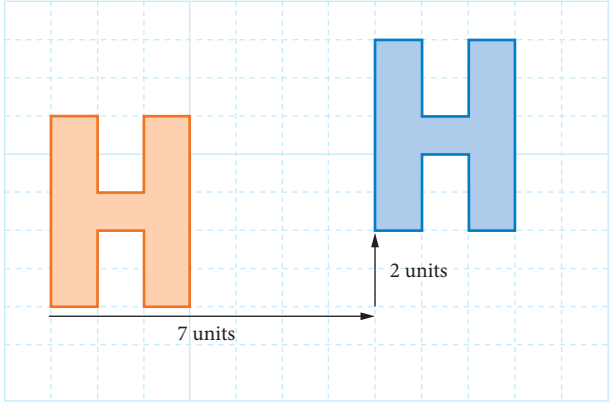
Example 5

Translate the H-shape below 7 units right and 2 units up.



Solution

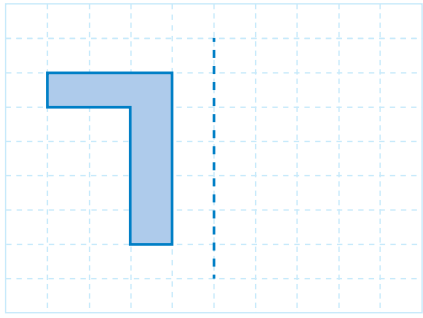
Slide each point 7 squares across and 2 squares up and draw the whole shape.



9780170370752

Example 6

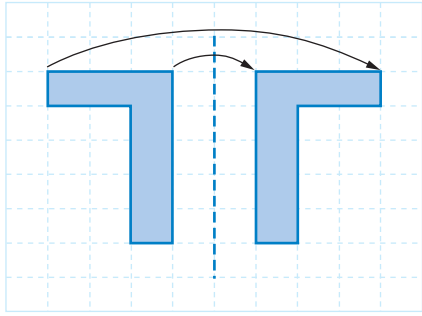
Reflect the L-shape across the dotted line.



The dotted line is called the line of reflection.

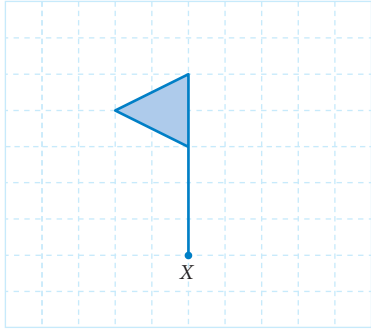
Solution

Flip to make the mirror-image of the original, back-to-front and the same distance from the line of reflection.



Example 7

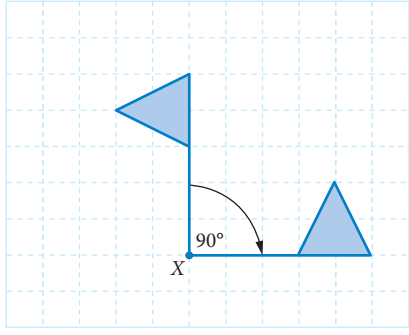
Rotate the flag shape 90° clockwise about the point X.



X is called the centre of rotation.

Solution

Spin clockwise around X to make the image. The original and image are the same distance from the centre of rotation, so the point X does not move.



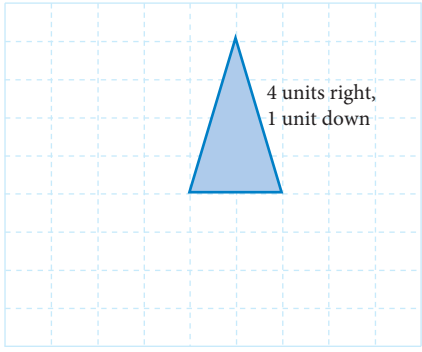
EXERCISE 7.7

Slide, flip or spin?

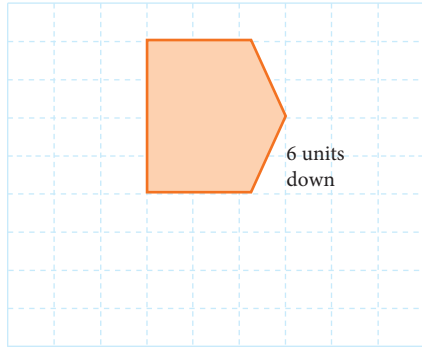
EXAMPLE 5

1 Copy each shape on grid paper and translate it according to the directions given.

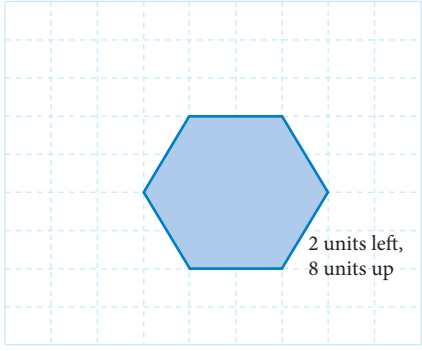
a



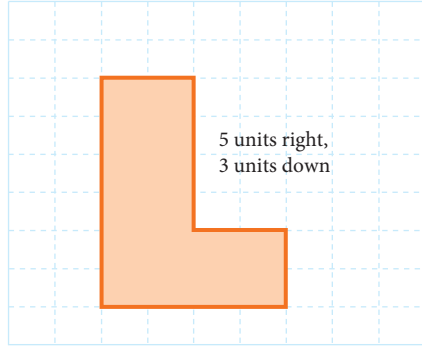
b



c



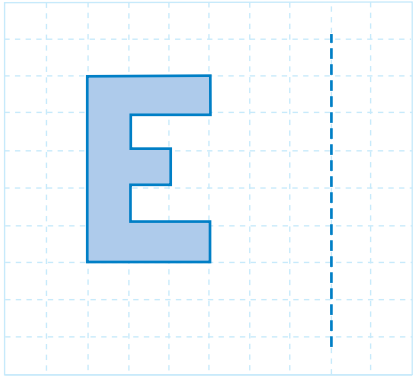
d



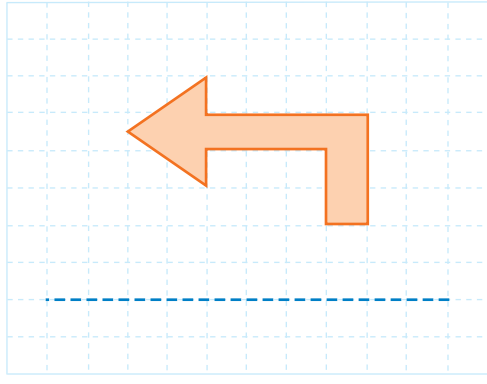
EXAMPLE 6

2 Copy each shape on square dot paper and reflect it across the dotted line.

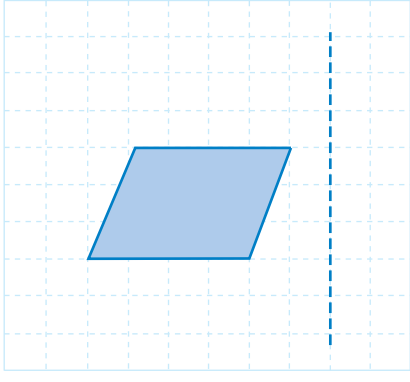
a



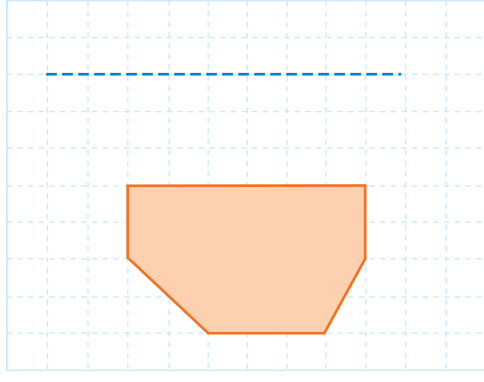
b



c



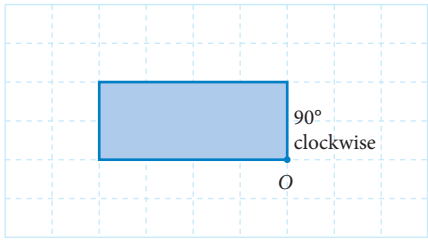
d



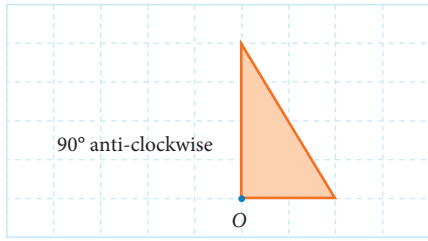
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3 Rotate each shape about the point O by the given angle.

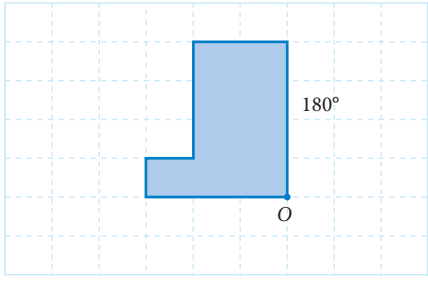
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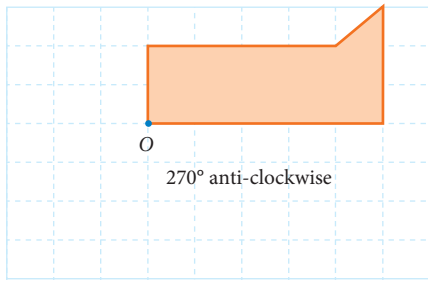
b



c



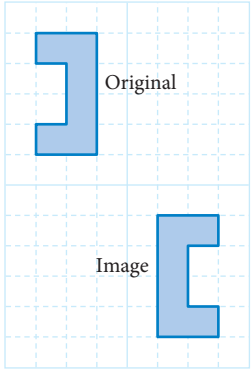
d



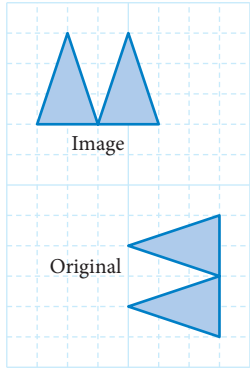
4 Why don't rotations of 180° need the label 'clockwise' or 'anti-clockwise'?

5 Combinations of translations, reflections and rotations can be applied one after the other. For each diagram below, state the combination of two transformations used on the original figure to create the image.

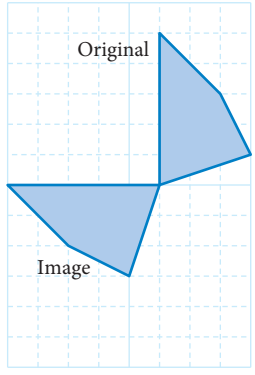
a



b

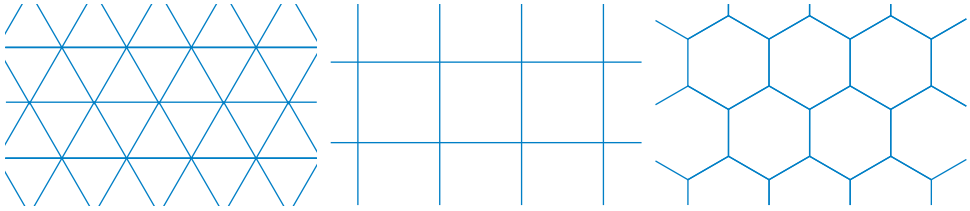


c

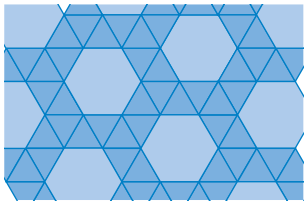


7.8 Tile patterns

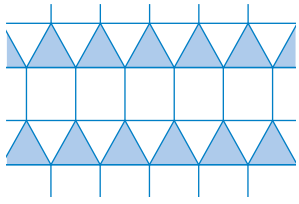
A 'tile pattern' of repeated shapes covering a flat surface so that there is no overlapping or gaps is called a **tessellation**. A **regular tessellation** uses only one shape, a **regular polygon.** There are only 3 regular tessellations, shown below.



A **semi-regular tessellation** uses two or more regular polygons. There are only 8 semi-regular tessellations. Here are two examples.

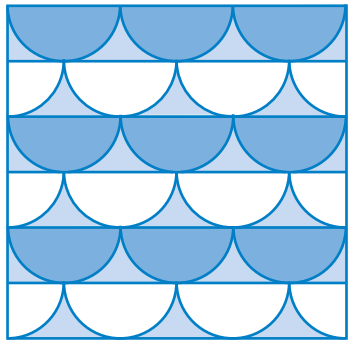


Hexagon and triangle



Square and triangle

A **non-regular tessellation** uses shapes that are not regular polygons.

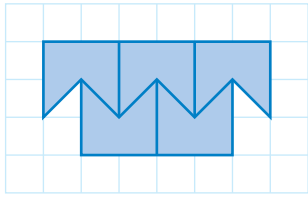


Tessellations are created by using **transformations** of a shape: translating, reflecting, rotating or a combination of them.

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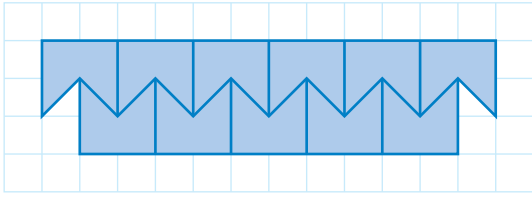
Example 8

Extend this tessellation by repeating the pattern.



Solution

Rotate the shape upside-down to create the next row of patterns.



EXERCISE 7.8

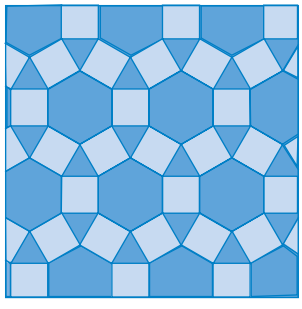
Tile patterns

1 Which three shapes are used in drawing **regular tessellations**?

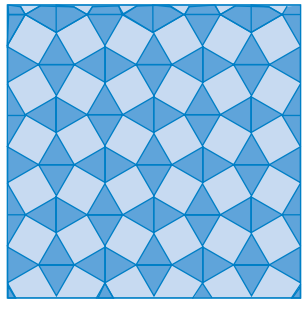
2 Search for **semi-regular tessellations** on the Internet. Each one is made up of two or more shapes. Write down which shapes are used in each of the eight semi-regular tessellations.

3 What regular polygons have been used to make each tessellation shown?

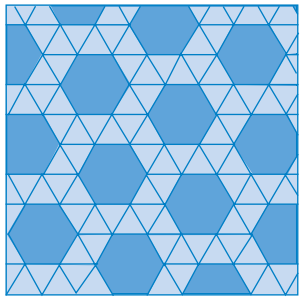
a



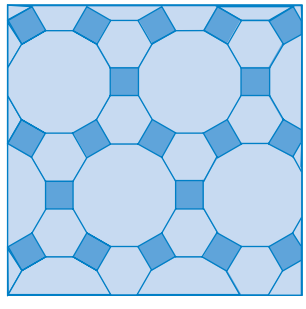
b



c



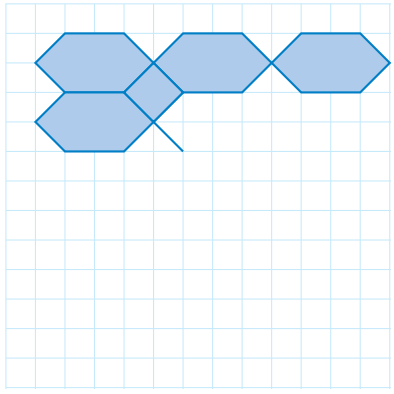
d



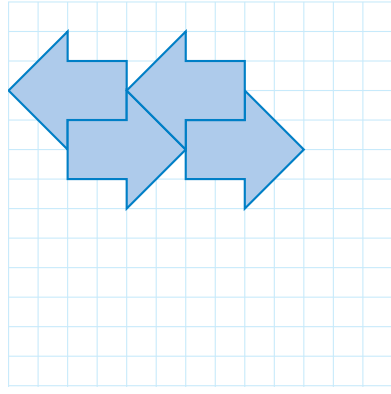
EXAMPLE 8

4 Copy and complete each tessellation on grid paper. Colour them to show the different shapes used. (These tessellations can also be found on the worksheet 'Tessellations' which can be downloaded from NelsonNet.)

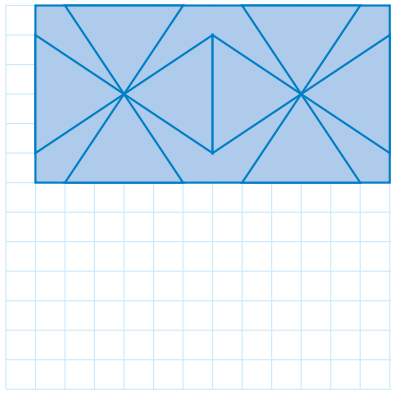
a



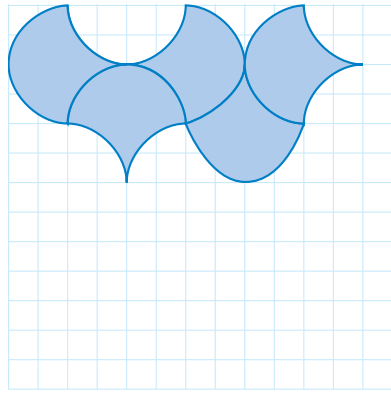
b



c



d



5 Recreate your favourite tessellation or create one of your own, using any combination of the transformations. If your first shape doesn't work out, keep trying different shapes.

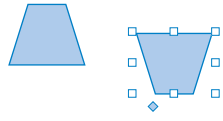
Practical activity Making tessellations using PowerPoint

1 Start a new PowerPoint file on your computer and select **Insert, Shapes** and insert a trapezium into the file.

2 Use copy-and-paste to make 8 copies of the trapezium.



3 To make the trapeziums tessellate, rotate one of the trapeziums upside-down by selecting it and spinning the green dot.



4 Slide the rotated trapezium into position.



5 Repeat the process to complete the tessellation.

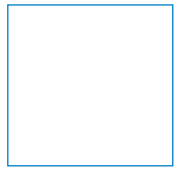
6 Create a tessellations of regular hexagons.

7 Create another tessellation of your choice.

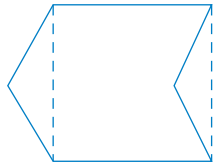
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Practical activity Creating new tessellations

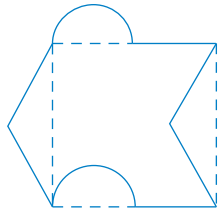
1 Construct a square or any other tessellating shape.



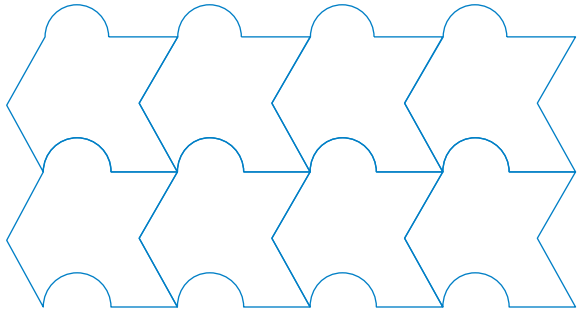
2 Add a piece to one side of the square but take an identical piece out of the opposite side.



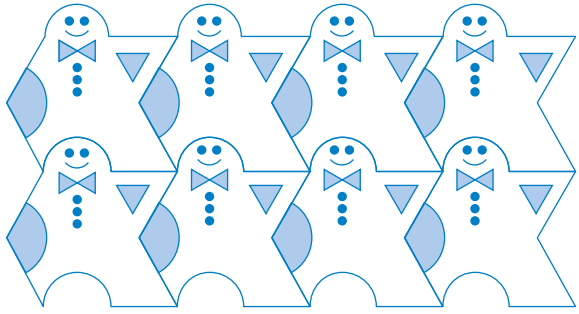
3 Add a piece to the top of the square but take an identical piece out of the bottom side.



4 Make copies of this new shape and join them together.



5 Design a picture to fit in the shape and repeat the design in all the shapes in the pattern.



Investigation — Researching tessellations

Escher

1. Who is Escher? Why is he famous?
2. How did he use tessellations?
3. Print or reproduce an example of his work using tessellations.

Quilting patterns

1. How do quilting patterns use tessellations?
2. Find 4 examples of quilting patterns using tessellations. Print or reproduce them.
3. Name the shapes that are used in each one.

Islamic art

1. Why do we find so many tessellations in Islamic art?
2. Find 4 examples of Islamic art using tessellations. Print or reproduce them.
3. Name the shapes that are used in each one.

Keyword activity

Match each word with its definition.

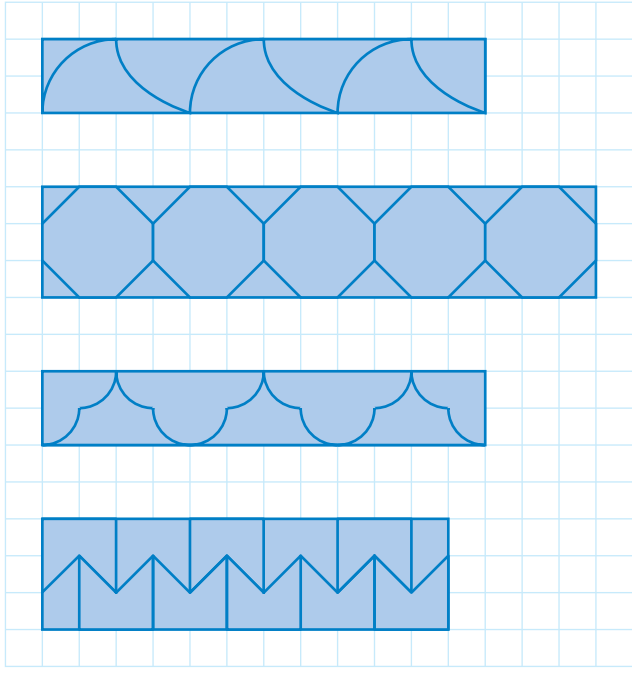
	WORDS		DEFINITIONS
1	line symmetry	A	A solid with all flat faces
2	net	B	Describes an object that has length, width and height
3	polyhedron	C	A pattern of repeated shapes that cover a flat surface with no overlapping or gaps
4	prism	D	A four-sided shape
5	pyramid	E	When one half of a shape is the mirror image of the other half
6	quadrilateral	F	A three-sided shape
7	rotational symmetry	G	Sliding a shape to a new position
8	reflection	H	Describes an object that has length and width only
9	rotation	I	A solid that comes to a point
10	tessellation	J	Spinning a shape to a new position
11	3D	K	A solid whose cross-section is the same from end to end
12	transformation	L	A shape that can be spun about a point so that it repeats its shape
13	translation	M	Flipping a shape to a new position
14	triangle	N	Translation, reflection and rotation are examples of this
15	2D	O	What you get when you 'flatten' a solid into its faces

Chapter Problem

Andrew and Catriona run an interior design business. They have a client who wants a bathroom renovation. They need to present the client with a choice of designs for a border at the top of the wall tiles. What are some possible designs?

Solution

We could use symmetry, tessellations, transformations or any combination of these to create a border. Some possible borders are shown below.



Now make one of your own!



Chapter Problem

Simone is a journalist with a local newspaper. She is writing an article about house prices in the area and has collected some information on sale prices in recent months:

$395 000	$296 000	$415 000
$479 000	$270 000	$269 000
$410 000	$419 000	$289 000
$375 000	$320 000	$440 000
$170 000	$359 000	$369 000
$725 000	$650 000	$495 000

What type of average should Simone use in her article?



CHAPTER 8

IT'S BETTER THAN AVERAGE

8.1 Categorical vs numerical data
8.2 Sample vs census
8.3 That's biased
8.4 What's the average?
8.5 What's the outlier?
8.6 What's the spread?
8.7 Boxplots
Keyword activity
Solution to the chapter problem

By the end of this chapter, you should know how to:

- identify categorical and numerical data
- evaluate the advantages and disadvantages of a census and a sample
- identify bias in data
- calculate measures of central tendency: the mean, the median and the mode
- calculate measures of spread: the range and interquartile range
- complete a five-number summary for a set of data
- construct boxplots for a set of data

How are we ever going to use this?

- When reading media reports or advertising claims involving statistics
- When evaluating the results of surveys

8.1 Categorical vs numerical data

Data means a collection of facts or information. There are two main types of data.

Categorical data is information that is grouped into categories, such as the colour of cars or favourite ice-cream flavours.

Numerical data is information that is counted or measured, such as the number of goals scored or women's heights.

Example 1

Classify each type of data as categorical or numerical.

a makes of cars

b people's salaries

c weights of athletes

d favourite radio stations

Solution

a Makes of cars are words. — Categorical

b Salaries involve numbers. — Numerical

c Weights of athletes involve measurement on a numerical scale. — Numerical

d Radio stations involve words. — Categorical

EXERCISE 8.1

Categorical vs numerical data

EXAMPLE 1

1 Classify each type of data as categorical (C) or numerical (N):

a masses of Year 11 students

b brands of computers

c prices of large pizzas

d number of TV sets owned by families

e amounts of rainfall in Shepparton

f causes of tooth decay

g sets won in a tennis match

h speed of trucks

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2 Classify each type of data as categorical (C) or numerical (N):

- **a** people's hair colours
- **b** customer ratings of a restaurant's level of service (from poor to excellent)
- **c** homes with Internet access
- **d** times taken for athletes to run 100 metres
- **e** countries where cars are manufactured
- **f** school populations
- **g** heights of bridges
- **h** classification rating of films

3 You are asked to do a survey on people's preferences about anything to do with cars. Write a questionnaire with 6 questions: 3 questions whose answers are categorical and 3 questions whose answers are numerical.



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8.2 Sample vs census

To collect information, we usually survey a representative group. This process is called taking a **sample**.

To collect information about a whole population, *all* people or items must be surveyed. This is called taking a **census**. People who are members of small groups can be missed in a sample. We always use a census when we want to make sure that the views or characteristics of small groups are included.

The Australian Bureau of Statistics conducts a national census every five years. The census is in each year ending in a 1 or a 6.

Sample	Census
• Surveys a selected group of people or items	• Surveys all people or items in the population
• Gives approximate information about the population	• Gives exact information about the population
• Simple and inexpensive	• Complex and expensive
• Can be done quickly	• Takes a lot of time to collect and process the information

Example 2

Andrew is evaluating the facilities and services that the United Club provides to its 14 000 members. Should he use a census or a sample to gather information?

Solution

A census would be expensive and it would take a long time to process the information.

A sample should be used in this situation.

Example 3

From what population should you take a sample to investigate the reputation of the United Club in the local community?

Solution

You should only survey people who live close enough to use the club.

The population would be the residents of the town or suburb where the United Club is located.

EXERCISE 8.2

Sample vs census

EXAMPLE 2

1 Write a paragraph in your own words describing the advantages and disadvantages of using a census to collect information.

2 Should a census or a sample be used for each investigation? Give a reason for your answer.

- **a** The most popular car colour in Australia
- **b** The number of retired people living in Mildura
- **c** The number of Australians who watch the AFL Grand Final
- **d** The use of soap versus bodywash
- **e** Testing coffee for taste
- **f** The number of babies born in Moe last year
- **g** The number of people using the emergency department at large city hospital on Saturday night
- **h** Length of time a certain type of car battery lasts
- **i** Which political party people in the Goulburn Valley support
- **j** The favourite TV show of your 10 best friends
- **k** Percentage of successful first serves in tennis by Sarena Williams
- **l** Internet usage by Year 11 students at your school

EXAMPLE 3

3 What is the population for each investigation?

- **a** Are girls better than boys at Maths?
- **b** Voting intentions for the next state election
- **c** The best song of the last decade
- **d** Student attitudes to school uniform at your school
- **e** Favourite make of car in Western Australia
- **f** How much people are prepared to spend on going to the gym each week
- **g** Which venue Year 12 should use for their formal
- **h** Giving to charities by rich people
- **i** What factors influence the choice of supermarket
- **j** The batting performance of the Australian cricket team



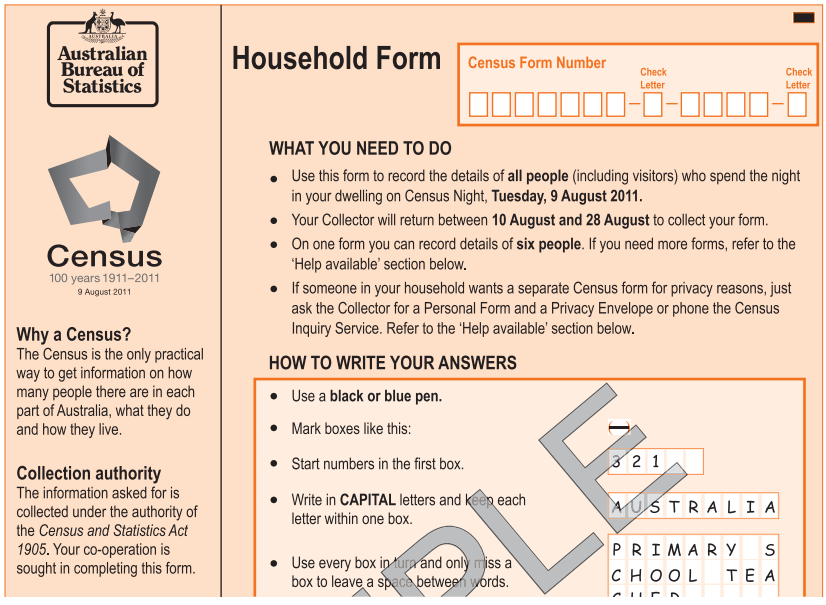
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Alamy/Ed Simons

Investigation The Australian Census

The Australian census is conducted by the Australian Bureau of Statistics. Visit their website www.abs.gov.au and click on 'Census' to find the answers to these questions.



Australian Bureau of Statistics

Census
100 years 1911–2011
9 August 2011

Why a Census?
The Census is the only practical way to get information on how many people there are in each part of Australia, what they do and how they live.

Collection authority
The information asked for is collected under the authority of the *Census and Statistics Act 1905*. Your co-operation is sought in completing this form.

Household Form

Census Form Number
Check Letter
Check Letter

WHAT YOU NEED TO DO

- Use this form to record the details of **all people** (including visitors) who spend the night in your dwelling on Census Night, **Tuesday, 9 August 2011.**
- Your Collector will return between **10 August and 28 August** to collect your form.
- On one form you can record details of **six people**. If you need more forms, refer to the 'Help available' section below.
- If someone in your household wants a separate Census form for privacy reasons, just ask the Collector for a Personal Form and a Privacy Envelope or phone the Census Inquiry Service. Refer to the 'Help available' section below.

HOW TO WRITE YOUR ANSWERS

- Use a **black or blue pen.**
- Mark boxes like this:
- Start numbers in the first box.
- Write in **CAPITAL** letters and keep each letter within one box.
- Use every box in turn and only miss a box to leave a space between words.

3 2 1

A U S T R A L I A

P R I M A R Y S
C H O O L T E A

SAMPLE

Source: Australian Bureau of Statistics. This material has been used under a Creative Commons Attribution 2.5 Australia licence.

1 When was the first national census conducted?

2 When was the last census?

3 Find five examples of census questions that will give categorical answers.

4 Find five examples of census questions that will give numerical answers.

5 Find three questions that have been asked in the past but were not asked in the most recent census.

6 Were there any questions in the most recent census that have not been asked before? If so, what were they?

7 Are all questions in the Census compulsory?

8 Give three examples of how census information is used.

That's biased

8.3

To be fair and reliable, a sample being surveyed should represent the population truly. **Bias** is an unwanted influence that favours some sections of the population more. Bias can happen if a sample was not selected fairly, or if a questionnaire has poorly-written questions.

Bias also occurs in the media when survey results are misrepresented to support a particular point of view. A good understanding of the statistical process will help you interpret results properly.

Example 4

Andrew has written some questions to ask United Club members about the club bistro.

- How often do you eat at our fabulous bistro?
- Rate your last meal: Great Yummy Quite nice

In what way are these questions biased? Rewrite them so they are not biased.

Solution

Using the word 'Fabulous' in the first question encourages the person answering the question to give a positive answer.	It would be better to ask: 'How often do you eat at the bistro?'
All the options in the second question are positive.	The options should include Poor, Average, Good.



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Example 5

Andrew is considering two ways to select a sample of club members to complete his survey:

a Ask 100 people as they come through the door on a Friday night

b Ask 100 people playing Bingo on a Wednesday morning

Why would each of these samples be biased? How could you ensure that Andrew obtains a representative sample?

Solution

a Younger members are more likely to be in club on a Friday night.

All age groups would not be represented fairly.

b People who are not working would be most likely to be in the club on a Wednesday morning.

Working members would be under-represented in the survey.

Andrew could obtain an alphabetical list of the members and choose every 10th member. (Other answers are possible.)

Example 6

Alex stood at the school gate one morning and asked 10 students if they like the school uniform. Five students said something negative about the uniform. Alex submitted a report to the school principal, claiming that half of the students hate the uniform. Explain three things that are wrong with this conclusion.

Solution

Alex only asks 10 students and it is not a random selection.

The sample is unlikely to be representative of the student body.

The question is vague and it's not clear what students don't like.

It may not be the whole uniform that students don't like but possibly some item.

Alex exaggerates the answers when he says 'hate'.

We don't know how strongly students feel about the uniform.

EXERCISE 8.3

That's biased

EXAMPLE 4

1 In what way is each survey question below biased? Rewrite them so they are not biased.

- **a** Do you prefer to holiday in sunny Brisbane or rainy Melbourne?
- **b** Do you agree that we should increase the tax on those disgusting cigarettes?
- **c** Rate the last book you read: OK Good Fantastic
- **d** Do you prefer exciting soccer or boring old cricket?
- **e** Do you think the Club is open long enough or should we be able to party into the early morning?
- **f** Rate your Health fund: useless not much good reasonable
- **g** Are there enough events for young people in this quiet town?
- **h** Isn't this just the greatest movie you've ever seen?
- **i** Are you one of those poor people who walk to school?
- **j** Rate the gym you attend: Best ever Good OK

EXAMPLE 5

2 In what way is each sample biased? How could you ensure that an accurate sample is found?

- **a** asking people their favourite sport in the crowd at an AFL match
- **b** calling every sixth phone number on a page in the phone book to ask about mobile phone usage
- **c** surveying people about their preferred supermarket in a town with only one supermarket
- **d** asking people in Sydney their view on drought support by the Government
- **e** asking the teachers what music should be played at the Year 12 formal
- **f** surveying the Maths staff on what books should be bought for the school library
- **g** finding out what people think of a new design for a football club jersey by asking people who walk past the office door
- **h** asking taxi drivers who they will vote for in the next election
- **i** asking the people in the United Club on a Thursday morning how many hours they spend on the Internet each week
- **j** asking McDonalds customers which is their favourite coffee shop

EXAMPLE 6

3 Jane asks five Year 11 students whether they like the food served at the school canteen. Three students say they don't like it. Jane writes an article for the school newspaper with the headline '60% of students hate canteen food'. Explain three things that are wrong with this headline.

4 The Hightop Hotel e-mails its customers to complete a survey about its service. It receives 25 replies, each of which makes some positive comments about the hotel. An advertisement is published saying 'Customers 100% happy with our company'. Explain what is wrong with this claim.

5 A newspaper headline reads 'Government program for disadvantaged students a disaster'. In the article, it is revealed that 35 students out of 1000 students receiving government support have dropped out of school. Explain why this headline is misleading.

8.4 What's the average?

Sets of data are often too big to understand just by looking at them. We need to analyse them and one way to do this is to find a typical or central value to represent all of the data. There are three types of **average** or **measures of central tendency**.

- The **mode** is the most common or frequent score(s).
- The **median** is the middle score when the scores are placed in order from smallest to largest.
- The **mean** is calculated by adding all the scores and dividing by the number of scores. This is what most people call the 'average'.

Example 7

The following data shows the daily maximum temperature (in °C) in Ceduna, South Australia for 15 days in November.

22, 25, 24, 35, 22, 25, 24, 20, 23, 29, 33, 32, 30, 25, 21

For this data find:

a the mode **b** the median **c** the mean

Solution

a 25 occurs 3 times, which is more than the number of times any other temperature occurs.

Mode is 25°C.

b Put the scores in order.

20, 21, 22, 22, 23, 24, 24, 25, 25, 25, 29, 30, 32, 33, 35

There are 15 scores, so the middle one will be the 8th score.

20, 21, 22, 22, 23, 24, 24, 25, 25, 25, 29, 30, 32, 33, 35

Median is 25°C.

c Add all the scores and divide by 15.

$\frac{390}{15} = 26$

Mean = 26°C.

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Example 8

Find the mode for each set of data.

a 3 3 4 5 5 6 7 9 10

b 1 2 3 5 7 10

Sometimes there is more than one mode and sometimes there is no mode at all!

Solution

a Both 3 and 5 occur twice. The modes are 3 and 5.

b All scores occur only once. There is no mode for this data.

Example 9

Find the median for this set of data:

68 81 64 57 95 62

When you have an even number of scores in the data there are two middle scores. To find the median you average the two middle scores.

Solution

Arrange the scores in order. 57 62 64 68 81 95

There are two middle scores.

Find the average of the two middle scores by adding them and dividing by 2. $\frac{64+68}{2} = 66$

The median is 66.

EXERCISE 8.4

What's the average?

1 The scores in a class test were: EXAMPLE 7

8 9 7 7 10 5 9 10 9 8 9 6 7

a What is the mode for this data?

b What is the median?

c What is the mean?

2 What is the mode for each set of data? EXAMPLE 8

a 6 9 2 1 2 9 2

b 67 43 89 45 54 86 45 76 54

c 12 3 6 5 8 2 7 1

d blue, green, yellow, green, blue, red, green, yellow, red, green, red, blue

e heart, spade, spade, diamond, spade, club, heart, heart, diamond, spade

EXAMPLE 9

3 Find the median for each set of data.

Remember to arrange the scores from smallest to largest before you select the middle one.

a 12 17 14 15 16 16 18

b 36 40 38 37 40 30

c 64.1 100 99.2 71.4 91.5 60.8 72.9

d 74 50 0 27 14 99 51 66

4 Last week a security company recorded the number of minutes it took one of their security guards, Leon, to respond to each alarm call. The results were:

6, 12, 13, 7, 26, 10, 13, 9

a How many alarm calls did Leon respond to last week?

b What is the mode of this data?

c What is Leon's median response time?

d Calculate his mean response time.

e Which of the three measures most accurately describes the typical response time? Justify your answer.



123RF.com/moodboard

5 This table shows the number of people who breached their bail conditions in a city over one year.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
19	17	14	12	13	14	12	14	10	11	14	16

For this set of data find:

a the median

b the mode

c the mean (correct to 1 decimal place)

d In which months was the number of people breaching bail conditions greatest? Suggest a reason for this.

6 This table shows the number of people who breached their bail conditions in a country region over one year.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
7	9	7	6	3	4	5	3	2	1	1	4

For this set of data find:

a the median

b the mode

c the mean (correct to 1 decimal place)

7 **a** What are the differences in the statistics for Questions **5** and **6**? Suggest a reason for these differences.

b What are the similarities in the statistics for Questions **5** and **6**? Suggest a reason for these similarities.

8 Steve and his dad are having an argument about the number of phone calls Steve makes each night. The statistics are shown in this table.

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Number of phone calls	9	3	6	2	13	1	1

a Find the mode, the median and the mean for this data.

b Which of these measures could Dad use to demonstrate that Steve makes too many phone calls? Explain your answer.

c Which of these measures could Steve use to show that he doesn't make many calls? Explain your answer.

d Which of the three measures gives the best indication of the typical number of calls Steve makes each night? Justify your answer.

9 Andrea asked 50 students how many text messages they sent within the last 2 hours.

Number of text messages (Score)	Frequency	Score × frequency
0	8	$0 \times 8 = 0$
1	4	$1 \times 4 = 4$
2	10	
3	10	
4	15	
5	3	
Total	50	

You can use your calculator or a spreadsheet to calculate the mean from a table.

a What is the mode for this data?

b Copy and complete the above table.

c Use the formula Mean = $\frac{\text{Total of (score} \times \text{frequency)}}{\text{Total of frequency}}$ to calculate the mean for this data.

10 Centrelink officers want to encourage jobseekers to keep applying for jobs. They recorded how many jobs each person applied for before they received their first job interview.

Score	Frequency
1	2
2	5
3	10
4	6
5	11
6	13
7	3

a How many people were surveyed?

b What is the mode for this data?

c Copy the table, add a 'score × frequency' column, then complete it.

d Calculate the mean for this data.

e Jon applied for 4 jobs and he hasn't got an interview yet. If you were his Centrelink case manager, what could you say to him to encourage him to keep applying for jobs? Mention the mean, mode or median in your answer.

8.5 What's the outlier?

An **outlier** is a score that is very different from the rest of the data. It is either much bigger than the other scores or much smaller than the other scores. An outlier can have a significant effect on the mean but has no impact on the median. In the following exercise you will see the difference an outlier can make.

EXERCISE 8.5

What's the outlier?

1 This data shows the ages of members of the Binns and Thomson families.

Ages of the Binns family: 19 31 21 3 6 14 19 24 11

Ages of the Thomson family: 19 31 21 3 6 14 19 24 91

a Calculate the mean age of each family. Answer correct to 1 decimal place.

b What is the main difference between the two sets of data?

c What effect does the difference identified in part **b** have on the mean?

2 Eleven houses were sold in Keswick Street over the last two years. The selling prices are listed below:

$620 000, $625 000, $700 500, $738 000, $625 000, $1 800 000,
$598 000, $612 000, $696 500, $720 000, $705 000

a Find the median sale price for the houses.

b Find the mean sale price.

c Which measure best describes the price of the houses in Keswick Street? Justify your answer.

d Which price is the outlier in this data?

e Calculate the mean of the remaining prices when this outlier is removed. Is this mean closer to the median you found in part a?

3 Mark and Steve's batting scores for six innings of cricket are shown below.

Mark: 45 48 53 38 32 40 51

Steve: 23 57 6 125 65 5 37

a Calculate the mean score for each player. Answer correct to 1 decimal place.

b Which player is better if you use the mean?

c Find the median score for each player.

d Which player is better if you use the median?

e Which player would you rather have in your cricket team? Justify your answer.

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4 Twelve people work in a small business which sells electronics equipment. Their annual salaries are:

$61 000	$57 000	$66 000	$54 000	$51 000	$53 000
$62 000	$56 000	$63 000	$60 000	$245 000	$820 000

a Calculate the mean salary for this company. Answer to the nearest dollar.

b Find the median salary for this company.

c Which measure (mean or median) is the best reflection of a typical salary for this company? Justify your answer.

d If you wanted to attract new employees to the company, which would you advertise as the average salary? Why?

e Which scores are outliers in this data?

f Calculate the mean salary of the remaining scores if the two outliers are removed.

5 Darryl is a market gardener. In the first five years of his business his annual profits were $32 000, $67 000, $71 000, $72 000 and $75 000.

a Find the mean and the median for this data.

b Which measure gives the most accurate impression of Darryl's usual annual profit? Explain your answer.

c In what situation might Darryl use the other measure to describe his usual annual profit?

d Which score is the outlier?

e Calculate the mean and the median of the remaining scores if the outlier is removed.

6 An obstetrician specialises in providing care for expectant mothers and delivering their babies. The data below shows the number of caesarean deliveries performed by a sample of 20 obstetricians in a 12-month period:

22	38	15	204	3	16	21	13	24	32
19	24	22	31	8	21	14	37	28	21

a What is the value of the outlier in this set of data?

b Some people would accuse a doctor who performed a lot of caesarean sections of doing unnecessary operations. Why might one obstetrician need to perform a lot more caesarean deliveries than all the other doctors?

c Calculate the mean and the median of the 20 numbers.

d Calculate the mean and the median without the outlier.

e What do your answers to parts **c** and **d** show?

7 A property developer has 40 new apartments for sale. The 20 apartments on the first 5 floors are priced at $330 000 each. The next 8 apartments on floors 6 and 7 are priced at $380 000, and then the next 8 apartments on floors 8 and 9 are priced at $425 000. The three apartments on the tenth floor are $835 000 each and the penthouse apartment on the top floor is priced at $1.7 million.

a Determine the median price of the apartments.

b Calculate the mean price of the apartments.

c When the developer is advertising the apartments for sale, which 'average' would the developer use? Explain your answer.

d The developer will be speaking to potential investors in his company. What 'average' might he use to make his company look profitable? Explain your answer.

e Which prices are outliers?

f Calculate the mean after removing the outliers.

8.6 What's the spread?

Sometimes is it useful to see how the data is spread out. The **range** and the **interquartile range** are both measures of **spread**.

Range = Highest score – lowest score

Quartiles divide the data into four equal groups. 25% of the data is between each quartile.



Interquartile range = Upper quartile (Q_3) – lower quartile (Q_1)

50% of the data lies within the interquartile range

Example 10

The ages of the 23 people at an Internet café are shown here.

33, 23, 28, 36, 27, 15, 32, 18, 13, 13, 38, 38, 27, 7, 34, 27, 12, 26, 33, 21, 24, 39, 20

For this data, find:

a each quartile **b** the range **c** the interquartile range

Solution

a Place the ages in ascending order and find the middle score – it is the median but it is also the second quartile.

7, 12, 13, 13, 15, 18, 20, 21, 23, 24, 26, 27, 27, 27, 28, 32, 33, 33, 34, 36, 38, 38, 39

$Q_2 = 27$

Find the middle of each half of the scores.

7, 12, 13, 13, 15, 18, 20, 21, 23, 24, 26

27, 27, 28, 32, 33, 33, 34, 36, 38, 38, 39

$Q_1 = 18$, $Q_3 = 33$

The quartiles are 18, 27 and 33.

b Range = Highest score – lowest score

Range = 39 – 7
= 32

c Interquartile range
= Upper quartile (Q_3) – lower quartile (Q_1)

Interquartile range = 33 – 18
= 15

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EXERCISE 8.6

What's the spread?

Keep your answers to this exercise because you will need them for the next exercise.

1 The number of hamburgers sold in a burger store each day during August is shown below. EXAMPLE 10

17, 27, 28, 18, 18, 17, 19, 19, 25, 27, 17, 19, 20, 19, 21, 26,
28, 18, 19, 20, 17, 19, 23, 24, 20, 18, 17, 20, 19, 27, 28

For this data, find:

a the range **b** each of the quartiles **c** the interquartile range

2 The following data shows the daily maximum temperatures (in °C) for 15 days in Cairns in July.

32, 30, 31, 32, 31, 30, 31, 31, 31, 31, 29, 25, 28, 27, 29

For this data, find:

a the range **b** each quartile **c** the interquartile range

3 The heights of 25 Year 11 students in centimetres were:

151	167	181	172	179	155	159	162	169	174
178	180	158	166	171	168	157	160	175	172
150	169	163	170	176					

Find:

a the range **b** each quartile **c** the interquartile range

4 A senior maths class was surveyed to find the number of hours each student spent on homework each week. These are the results:

4, 5, 7, 8, 3, 6, 9, 9, 8, 4, 7, 5, 3, 3, 4, 9, 3, 4, 7, 8

If there is an even number of scores then the quartile is the average of the two middle scores.

Find:

a the range **b** each quartile **c** the interquartile range

5 The following data is a record of the monthly number of thefts from retail stores in two regions.

Inner City: 13, 13, 13, 12, 16, 25, 23, 20, 33, 25, 27, 25,
55, 20, 27, 33, 28, 26, 38, 24, 33, 55, 42, 48

Coastal: 62, 60, 52, 62, 52, 63, 60, 65, 74, 61, 36, 66
36, 69, 70, 47, 39, 64, 69, 55, 40, 60, 58, 52

a For each set of data, find:

i the range **ii** each quartile **iii** the interquartile range

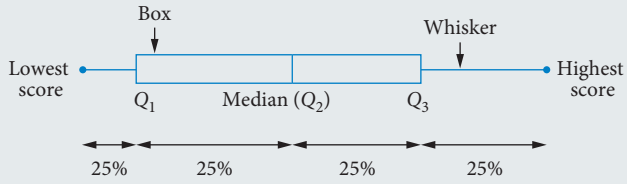
b Comment on the differences between these two sets of data.

8.7 Boxplots

The three quartiles for a set of data and the lowest and highest scores together make a **five-number summary** for the data, which can then be graphed on a **boxplot,** also called a box-and-whisker plot.

The five-number summary for a set of data consists of:

- the lowest score
- the first or lower quartile, Q_1
- the second quartile, the median, Q_2
- the third or upper quartile, Q_3
- the highest score



A boxplot is a graphical representation of a five-number summary.

Example 11

In Example 10 on page 184 we found the quartiles for the ages of users of an Internet cafe.

a Write the five-number summary for this data.

b Draw a boxplot for this data.

Solution

a Listing the required information from Example 10.

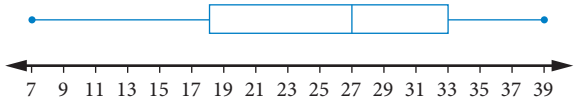
The five-number summary is:

Lowest score = 7

$Q_1 = 18$, $Q_2 = 27$, $Q_3 = 33$

Highest score = 39

b Showing the five-number summary on a boxplot.



EXERCISE 8.7

Boxplots

EXAMPLE 11

In this exercise you will use your answers to Exercise 8.6.

1 Use your answers to Question **1** to:

a write a five-number summary for the data

b draw a boxplot for the data

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2 Use your answers to Question **2** to:

a write a five-number summary for the data

b draw a boxplot for the data

3 Use your answers to Question **3** to draw a boxplot for the data.

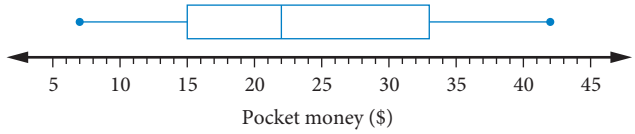
4 The monthly number of home burglaries in a country town was recorded over 27 months.

21, 25, 17, 23, 16, 21, 41, 22, 25, 20, 22, 11, 20, 12, 13,
12, 6, 12, 10, 19, 30, 22, 21, 14, 34, 33, 24

a What is the five-number summary for this data?

b Draw the boxplot for this data.

5 This boxplot represents the amount of pocket money in dollars earned by a sample of 60 children.



a Find the median.

b Find the range.

c How many children earned between:

i $33 and $42? **ii** $15 and $42?

d Find the interquartile range.

6 This stem-and-leaf plot shows the number of students served at the school canteen over a three week period.

Stem	Leaf
7	6
8	1 6 8
9	5 7 8
10	1 5 5
11	2 2 4 7
12	4

Key: 7|6 = 76



Edwina Pickles/Fairfax Syndication

a Find the five-number summary for this data.

b Draw the boxplot for this data.

7 This back-to-back stem-and-leaf plot shows the heights in centimetres of boys and girls in a Year 11 class.

Boys	Stem	Girls
4 4 4 3	16	0 1 3 4 4
9 8	16	5 7
4 2 1 1	17	0 1 2 2 4
9 9 6	17	5 7
2 1 0	18	

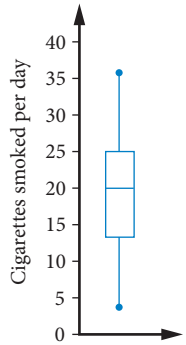
Key: 3|16| = 163
|16|0 = 160

a Find the five-number summary for each set of data.

b Draw the boxplot for each set of data.

c Summarise the differences between the heights of boys and girls.

8 This boxplot illustrates the number of cigarettes smoked per day by a sample of 60 smokers who are trying to quit.





Shutterstock.com/RAYphotographer

a What is the median number of cigarettes smoked per day?

b What is the interquartile range?

c What is the lowest score?

d How many people smoked between 20 and 25 cigarettes per day?

e How many people smoked fewer than 20 cigarettes per day?

9 **a** Use your answers to Question **5** in Exercise 8.6 to write a five-number summary for each set of data.

b Draw a boxplot for each set of data.

c In which region do you think it would be safer to open a shop? Justify your answer.

Keyword activity

Copy and complete each sentence using a word from the list. When you have completed this, you will have a summary of the chapter.

Word list

average	boxplot	categorical	census	interquartile range
median	mode	number	numerical	odd
order	quartiles	range	sample	

1 There are two types of data: _____ and _____.

2 The _____ is the most frequent score.

3 Another word for the mean is the _____.

4 The mean is the sum of the scores divided by the _____ of scores.

5 The _____ is the middle score.

6 There's only one middle score when there are an _____ number of scores.

7 To find the median, you must first put the scores in _____.

8 _____ divide the data into 4 equal parts.

9 The difference between the highest score and the lowest score is called the _____.

10 The difference between the upper quartile and the lower quartile is called the _____.

11 When we take a _____ we ask some people to answer our questions.

12 All people in the population answer the questions in a _____.

13 The graph we draw from a five-number summary is called a _____.

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Chapter Problem

Simone is a journalist with a local newspaper. She is writing an article about house prices in the area and has collected some information on sale prices in recent months:

$395 000	$296 000	$415 000	$479 000	$270 000	$269 000
$410 000	$419 000	$289 000	$375 000	$320 000	$440 000
$170 000	$359 000	$369 000	$725 000	$650 000	$495 000

What type of average should Simone use in her article?

Solution

Mode: there is no mode as all the prices are different. Even if there were two prices the same the mode is unlikely to reflect a typical price.

Mean: Total of prices = $6 818 000

Number of sales = 18

$$\text{Mean} = \frac{\$6\,718\,000}{18}$$
$$= \$373\,222$$

Median: put scores in order

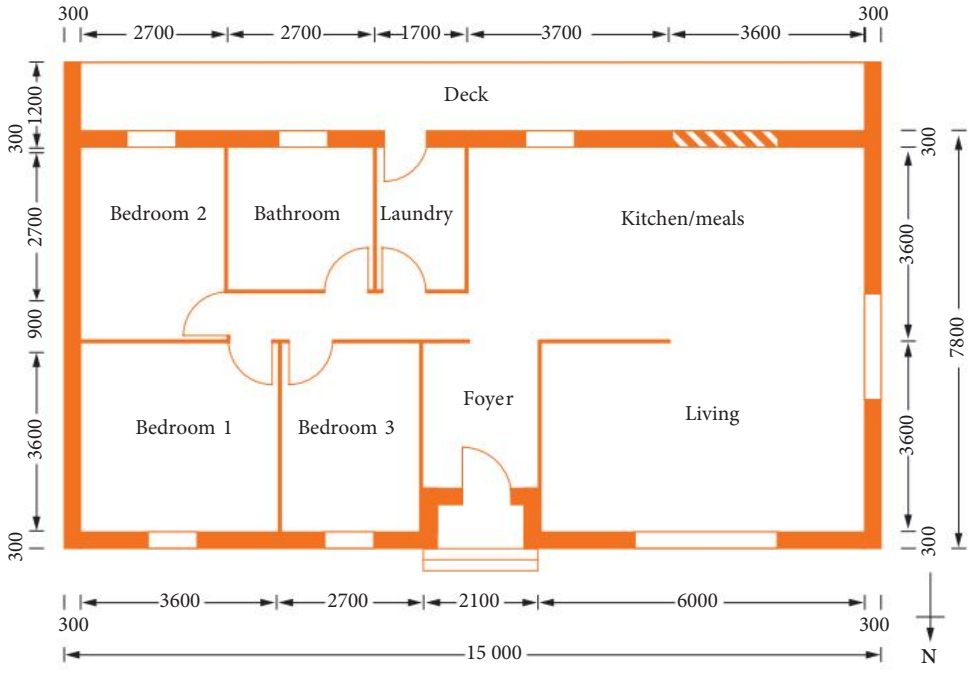
$170 000	$269 000	$270 000	$289 000	$296 000	$320 000
$359 000	$369 000	$375 000	$395 000	$398 000	$410 000
$415 000	$419 000	$440 000	$479 000	$495 000	$550 000

$$\text{Median} = \frac{375\,000 + 395\,000}{2}$$
$$= \$385\,000$$

In this case the median is the best example of the typical price as the mean has been lowered by the outlier of $170 000.

Chapter Problem

Francesca and Michael are building a house according to this plan. They need to order carpet for the living room and the three bedrooms. How many square metres of carpet will they need? Answer to the nearest square metre.




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CHAPTER 9

FROM PAPER TO REALITY

9.1 Scale factors
9.2 Scale diagrams
9.3 House plans and elevations
9.4 Reading house plans
9.5 Renovate my house
9.6 Drawing scale diagrams
Keyword activity
Solution to the chapter problem

By the end of this chapter, you should know how to:

- understand scale factors and their use in enlargements and reductions
- interpret scale drawings including commonly used symbols and abbreviations
- find actual measurements from scale drawings, such as lengths, perimeters and areas
- calculate quantities and costs involving home renovations
- create scale drawings

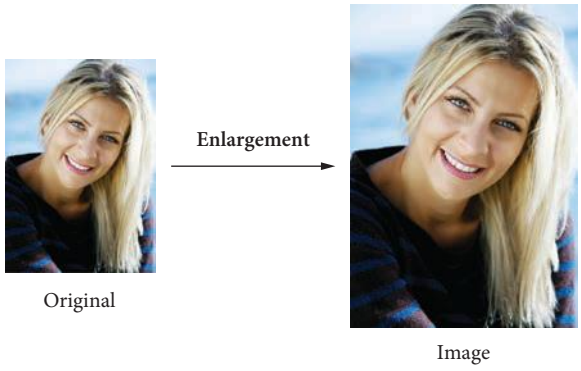
How are we ever going to use this?

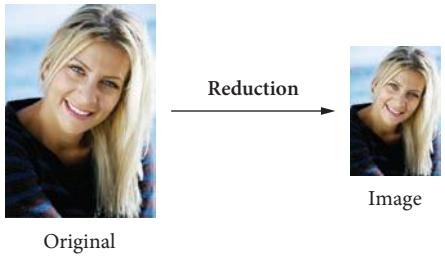
- Any time you need a scale drawing
- If you decide to build a house later in life
- If you are working in a building trade

9.1 Scale factors

If a shape is **enlarged** (made bigger) or **reduced** (made smaller), the shape we start with is called the **original** and the final shape is called the **image**. The **scale factor** describes the change in size between the original and the image.

Printers and graphic designers use enlargements and reductions when printing different sizes of photographs or logos.





Shutterstock.com/Liv friis-larsen

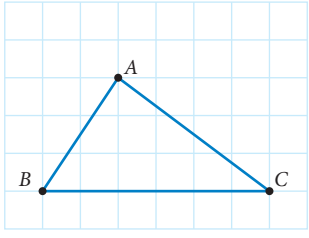
$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$

- A scale factor greater than 1 means that the original has been enlarged.
- A scale factor that is a fraction between 0 and 1 means that the original has been reduced.

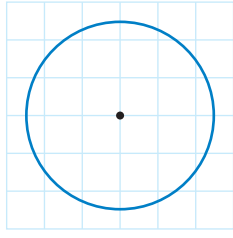
Example 1

Enlarge:

a this triangle by a factor of 3



b this circle by a factor of 2



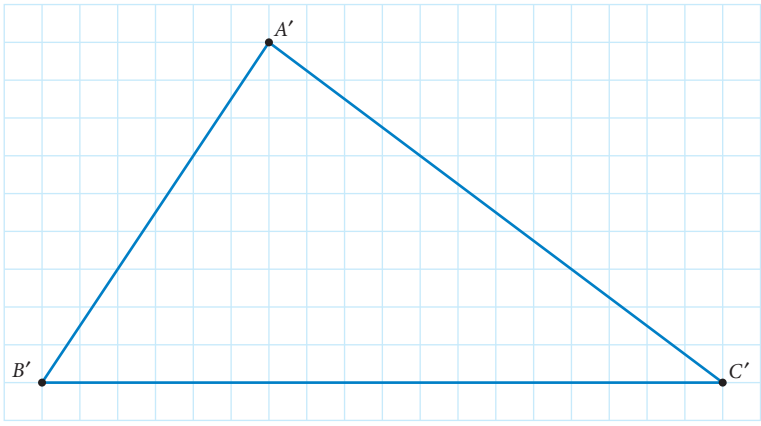
Solution

a Each side needs to be 3 times larger than in the original:

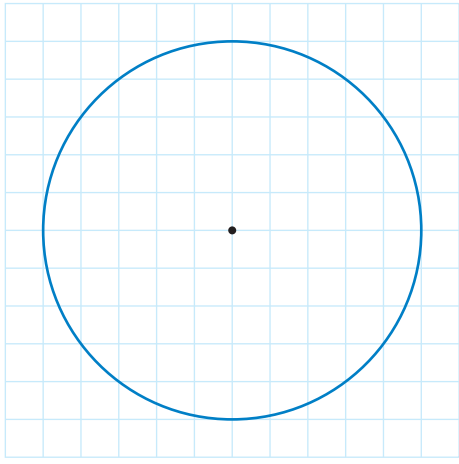
the base BC is 6 units so the new base will be 18 units

the height is 3 units so the new height will be 9 units.

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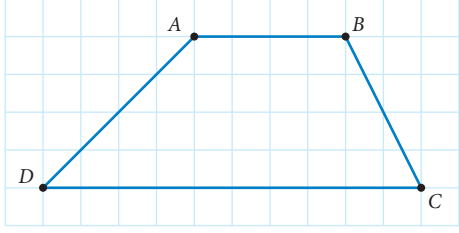


b All measurements need to be twice as big as the original: the radius is 2.5 units so the new radius will be 5 units.



Example 2

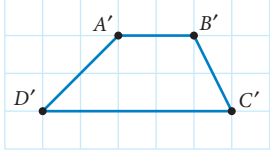
Reduce this shape by a scale factor of $\frac{1}{2}$.



Solution

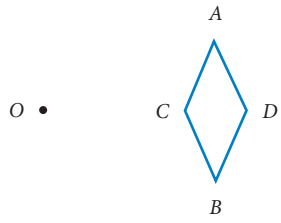
Each side needs to be half the size of the original:

the base DC is 10 units so the new base will be 5 units, the top AB is 4 units so the new top will be 2 units.



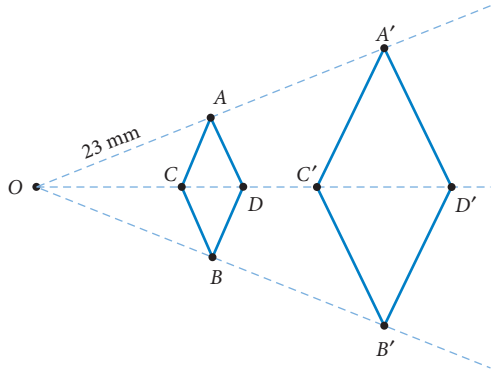
Example 3

Enlarge this shape using the given centre of enlargement, O, and enlargement factor 2.



Solution

Draw a line from O through each of the vertices of the shape.



Measure the distance from O to each vertex.

Multiply this distance by 2 to give the distance to the new vertex.

OA = 23 mm, so OA' = 46 mm

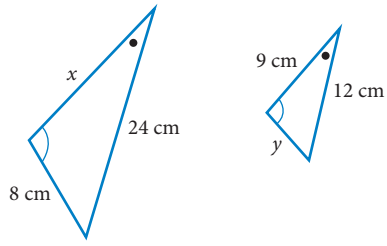
Mark the point A'.

Repeat for each vertex. Draw in the enlarged shape.

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Example 4

These two triangles are similar.



Similar figures have the same shape but not the same size. One is an enlargement or reduction of the other.

a What is the scale factor from the larger triangle to the smaller triangle?

b Calculate the value of y.

c What is the scale factor from the smaller triangle to the larger triangle?

d Calculate the value of x.

Solution

a The 24 cm side in the larger triangle matches with the 12 cm side in the smaller triangle.

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$

$$\text{Scale factor} = \frac{12}{24} = \frac{1}{2}$$

b The side y matches the 8 cm side in the smaller triangle.

Using the scale factor:

$$y = 8 \times \frac{1}{2} = 4 \text{ cm}$$

c From the smaller triangle to the larger triangle

$$\text{Scale factor} = \frac{24}{12} = 2$$

d The side x matches the 9 cm side in the larger triangle.

Using the scale factor:

$$x = 9 \times 2 = 18 \text{ cm}$$

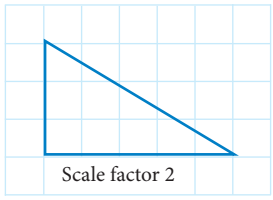
EXERCISE 9.1

Scale factors

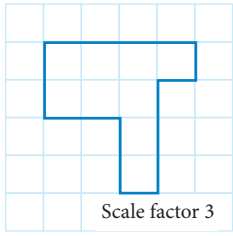
EXAMPLE 1

1 On grid paper, enlarge each shape by the given scale factor.

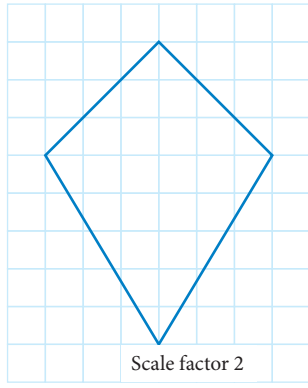
a



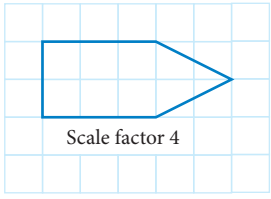
b



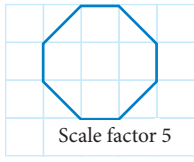
c



d



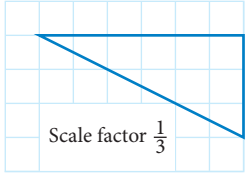
e



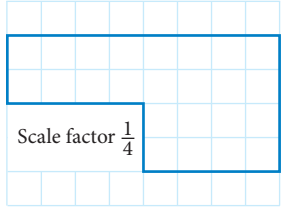
EXAMPLE 2

2 On grid paper, reduce each shape by the given scale factor.

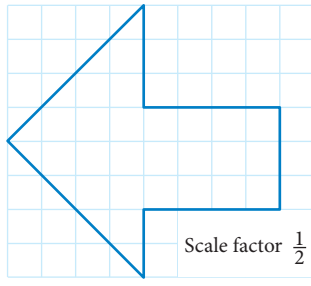
a



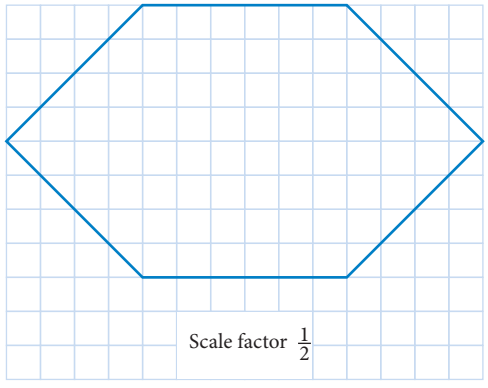
b



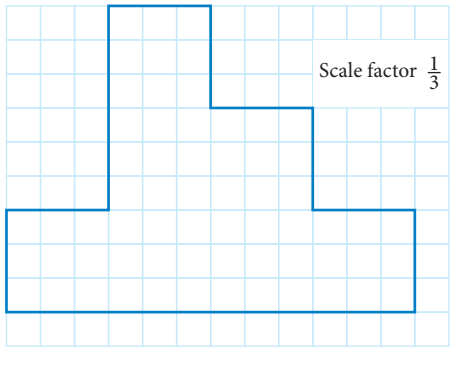
c



d



e



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3 Copy each diagram and enlarge the shape using the given centre of enlargement, O and scale factor.

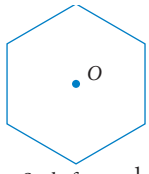
EXAMPLE 3

a



Scale factor 2

b



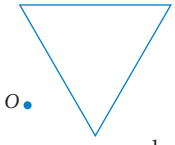
Scale factor $\frac{1}{2}$

c



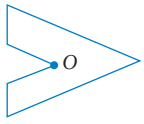
Scale factor 3

d



Scale factor $\frac{1}{3}$

e

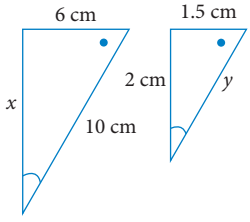


Scale factor 3

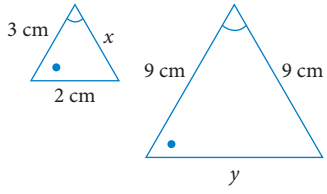
4 Find the value of each pronumeral in each pair of similar figures.

EXAMPLE 4

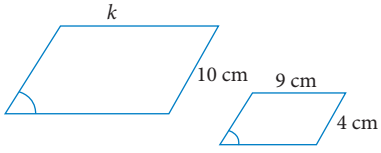
a



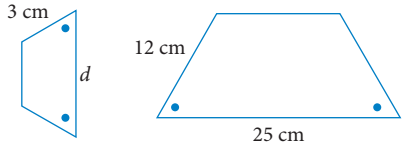
b



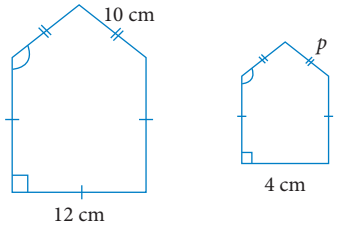
c



d



e



5 Enlarge each of these logos onto an A4 page. Decide what scale factor you will need to use and be as accurate as you can.

a



Shutterstock.com/Dragana Gerasimoski

b



Shutterstock.com/GraphEGO

c



Shutterstock.com/Aquir

6 Make up a simple shape design of your own and then enlarge or reduce it using one of the methods you have learnt in this exercise.

9.2 Scale diagrams

A **scale diagram** is a reduced or enlarged version of a real object. The most common scale diagrams are maps and house plans. By taking measurements on the scale drawing we can calculate the size of objects in real life using the scale given on the diagram.

To calculate a real length on a scale diagram:

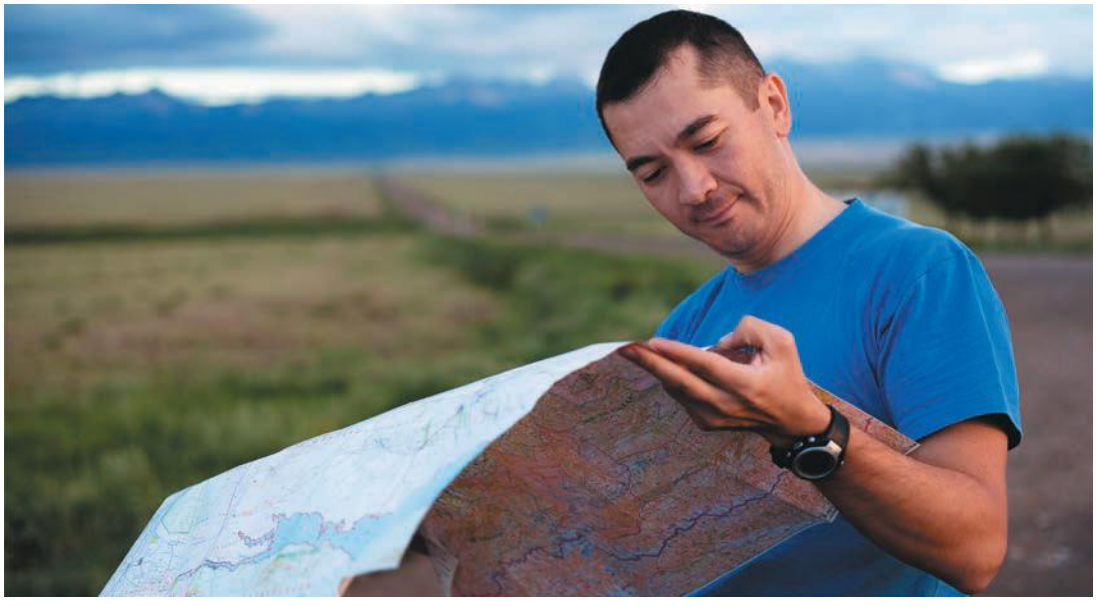
- measure the scaled length on the scale diagram
- multiply by the given scale factor
- convert your answer to the required units if necessary

Scales on a diagram are most commonly given as a statement: 1 cm represents 5 m or a ratio: 1 : 500.

Being able to use whichever type of scale is given is a useful skill.

Example 5

Keiran is using a map with the scale 1 cm represents 2 km. How far would he have to walk if the distance on the map is 6 cm?



iStockphoto/Petrichuk

Solution

Multiply the scaled distance by the scale factor.

Scaled distance = 6 cm.

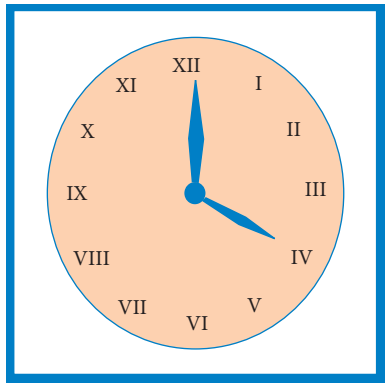
Real distance = 6×2 km = 12 km

Keiran would have to walk 12 km.

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Example 6

This diagram of a clock is drawn to a scale of 1 : 6.
Measure its length and calculate its actual length.



Scale 1 : 6

Solution

Measure the scaled length on the diagram. Scaled length = 5 cm

Multiply by the scale. Actual length = 5 × 6 cm
= 30 cm

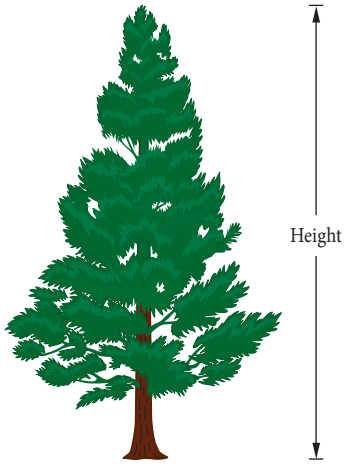
EXERCISE 9.2

Scale diagrams

1 For each scale diagram find the actual length shown by measuring and calculating.

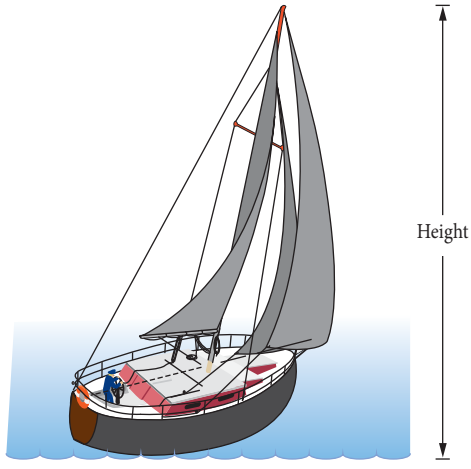
EXAMPLE 5

a



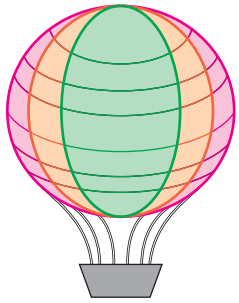
Scale: 1 cm represents 2 m

b



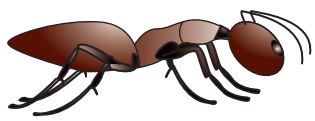
Scale: 1 cm represents 5 m

c



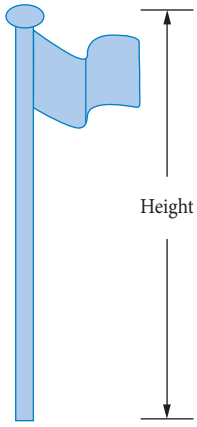
Scale: 1 cm represents 1.5 m

d



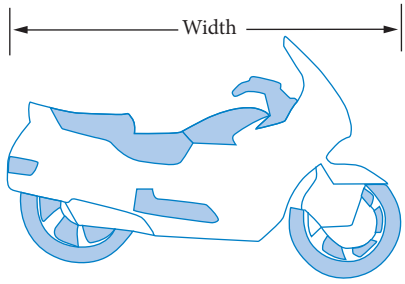
Scale: 1 cm represents 5 mm

e



Scale: 1 cm represents 3 m

f



Scale: 1 cm represents 40 cm

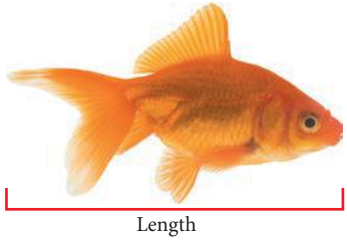
2 A street map uses a scale of 1 cm represents 200 m. Leanne walks from the Town Hall to the shopping centre, a distance that measures 5.5 cm on the map. How far does she walk?

3 Alan has a model of his favourite yacht in his study. The model has been made on the scale of 2 cm represents 1 m. The model is 50 cm long. How long is the yacht?

EXAMPLE 6

4 Measure the length of each scaled-down image below, then use the scale ratio to calculate the actual length of the object.

a Fish 1 : 3



Shutterstock.com/Andrew Burgess

b House 1 : 300



Shutterstock.com/Alexmisu

9780170370752

c Pen 1 : 4



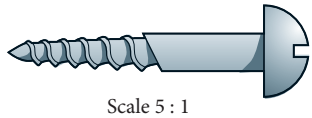
d Tennis racquet 1 : 16



5 Lena collects model cars that have been made in the ratio 1 : 43. Her favourite model is 9.3 cm long. What would be length of the actual car? Give your answer to the nearest metre.

6 A map of south-eastern Victoria has a scale of 1 : 50 000. Dean has to travel a distance measured as 128 mm on the map. How far does Dean have to travel? Express your answer in kilometres.

7 When the ratio for the scale is given in the reverse order this means the object is smaller than the scale drawing. In this case we divide by the scale. This drawing of a screw is drawn to a scale of 5 : 1.



Copy and complete the following calculation to find the length of the screw.

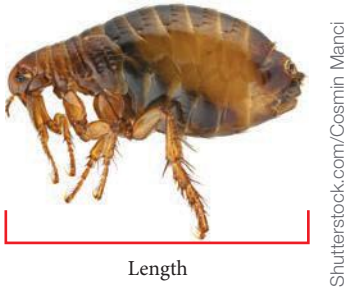
Scaled length of screw = _____

Actual length of screw = _____ ÷ 5

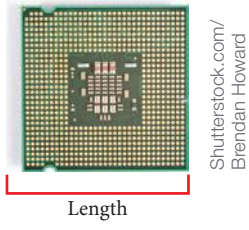
= _____ cm

8 Measure the length of each magnified image below, then use the scale ratio to calculate the actual length of the object.

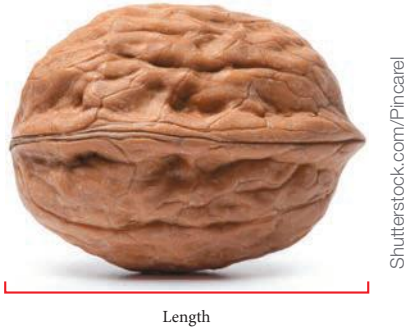
a Flea 100 : 1



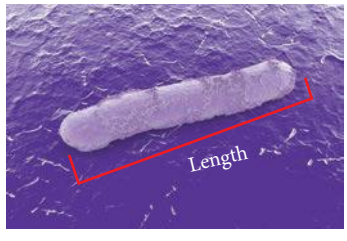
b Microchip 3 : 1



c Nut 2 : 1



d Bacterium 100 : 1



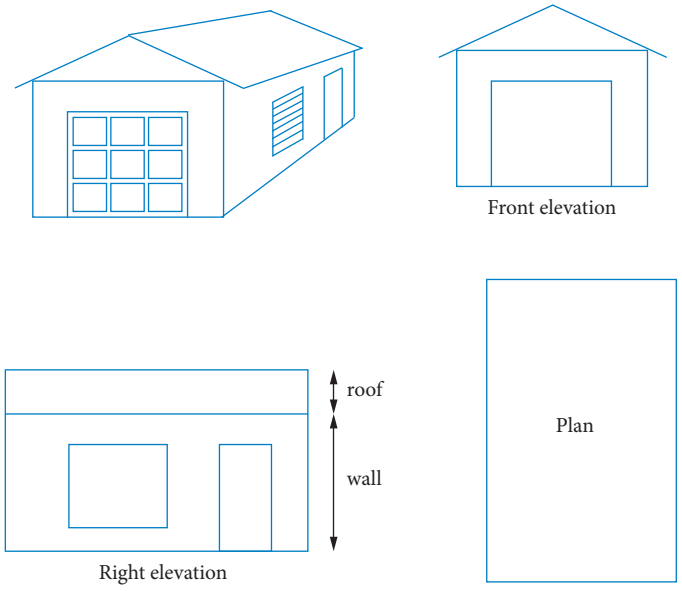
9.3 House plans and elevations



Shutterstock.com/Romakoma

A draftsperson drawing a design for a new building includes different views of the building:

- a **plan**: a diagram of the floor
- the **elevations**: the views of the front, back, left side and right side of the building.

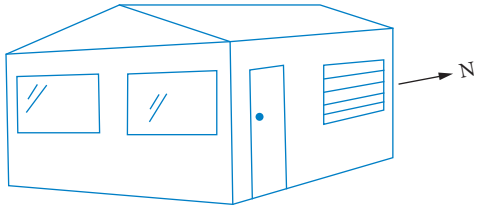


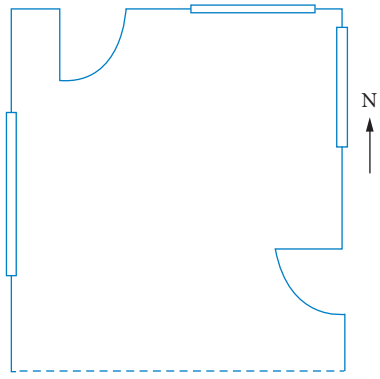
9780170370752

EXERCISE 9.3

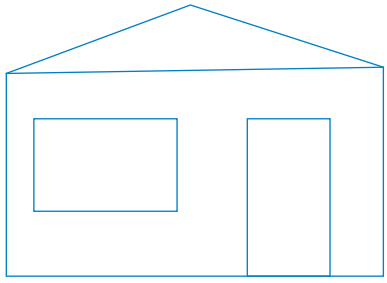
House plans and elevations

1 The plan and two elevations for Erik's work shed are shown below.

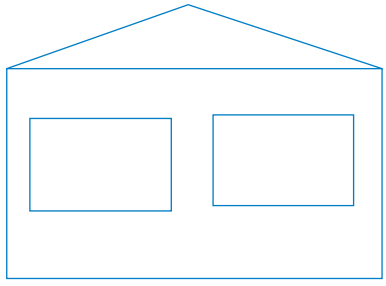




Plan



Northern elevation

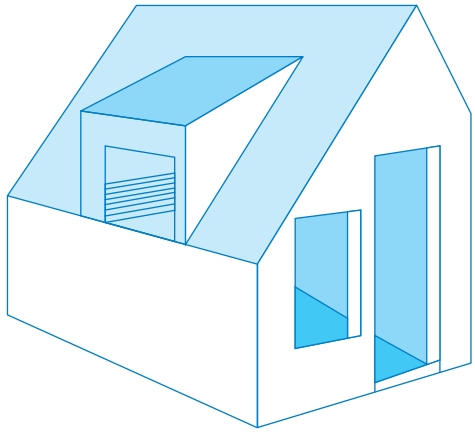


Southern elevation

a The plan is incomplete where the dotted line is shown. Copy and complete the plan.

b Draw the eastern and western elevations.

2 For the house shown, sketch the front and side elevations. You do not need to draw these to scale but use a ruler and be as accurate as you can.

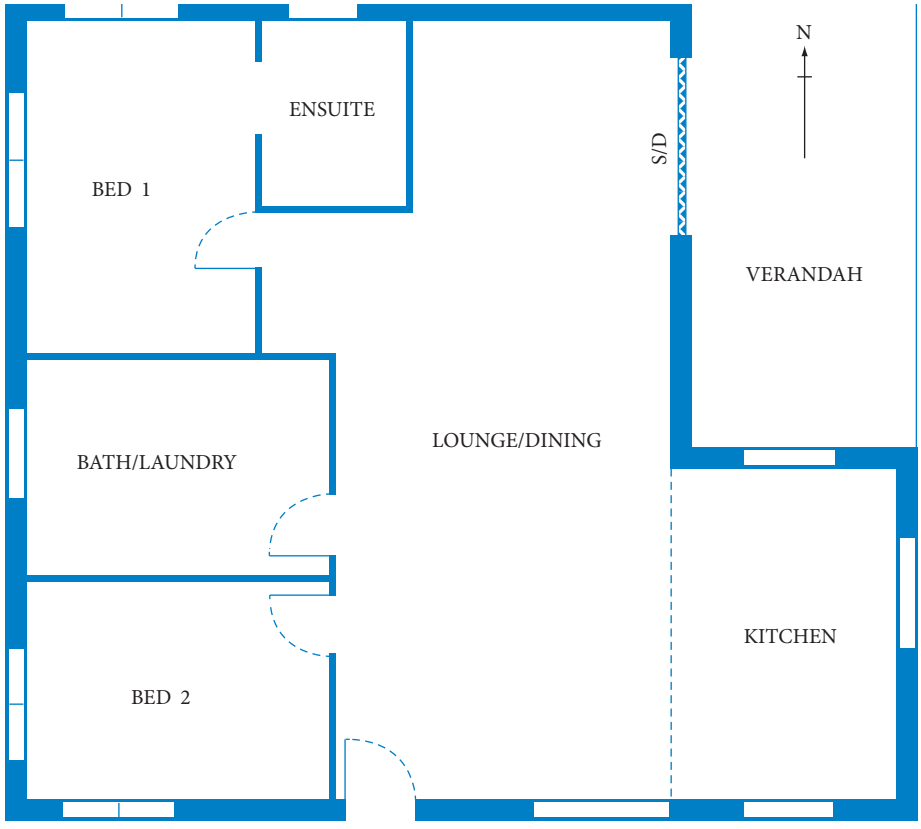


3 For the house shown sketch the front and possible side elevations. You do not need to draw these to scale but use a ruler and be as accurate as you can.



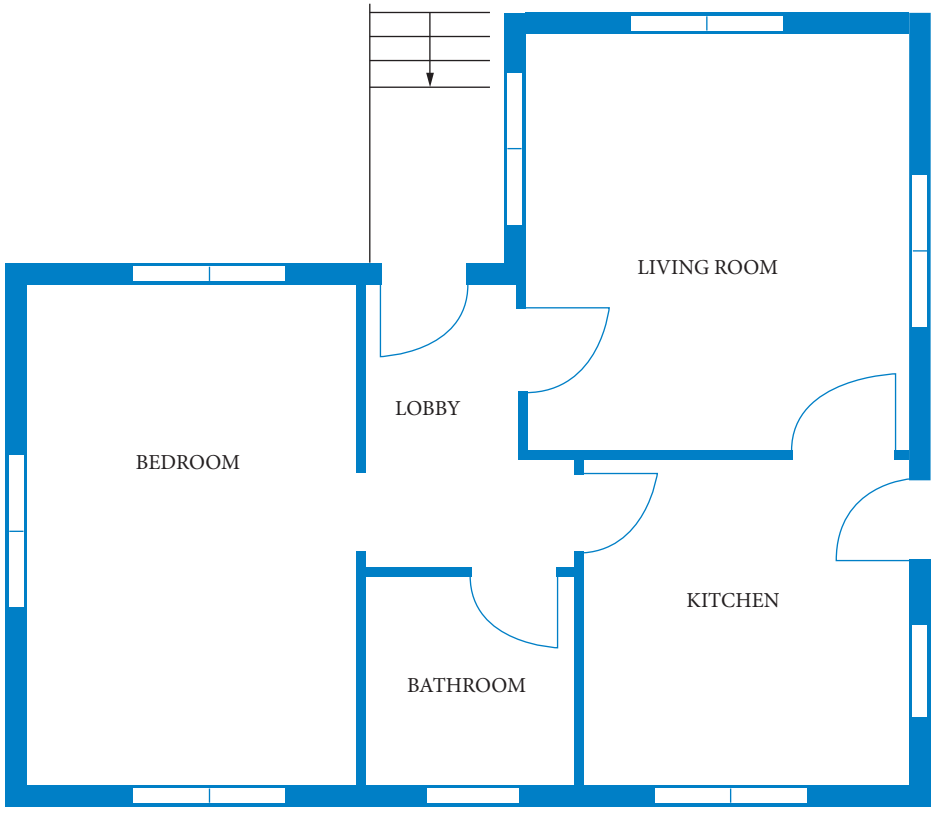
Shutterstock.com/Ewelina Wachala

4 This is the plan of Grigor and Silvana's holiday house.



Sketch the northern and western elevations. You do not need to draw these to scale but use a ruler and be as accurate as you can.

5 This is the plan for a one-bedroom apartment whose front door faces north. Draw elevation views of each side of the house.



Investigation Design your own home

In this investigation, you are going to use the 3D modelling software SketchUp. A free version, SketchUp Make, can be downloaded from www.sketchup.com. You can also find tutorials on the Internet if you don't know how to use SketchUp.

Part A

- Design a simple building. It can be a small house, a cubby or a shed. Design a floor plan, the elevations and the 3D version.
- Place the building on a block of land with a driveway leading to the building. Include an L-shaped swimming pool or a cylindrical water tank on the block.
- Add plants and trees to the block.

Part B (optional)

Design a more complex building such as a house or a set of units.

Save both designs for use in later investigations.

9.4 Reading house plans

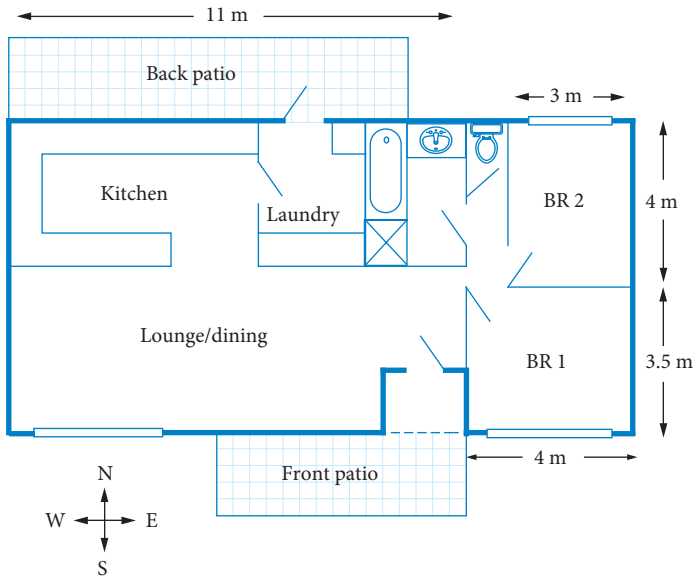
House plans use a lot of different symbols and abbreviations. They are either drawn to scale or have measurements written on them. Measurements are usually shown in **millimetres** to avoid the use of decimal points, which can lead to errors in printing and reading.



Shutterstock.com/Production Perig

Example 7

This is the floor plan for Joe and Michelle's house.



a What are the dimensions of Bedroom 1?

b What is the area of Bedroom 1?

c What is the length of the house?

d What is the width of the house excluding the patios?

Solution

a Find Bedroom 1 on the plan. The dimensions are the length and the width.

The dimensions of Bedroom 1 are 4 m by 3.5 m.

b Area = length × width

$A = 4 \times 3.5$
$= 14 \text{ m}^2$

c Find the full length from the plan.

Length = 11 + 4
= 15 m

d Find the full width from the plan.

Width = 4 + 3.5
= 7.5 m

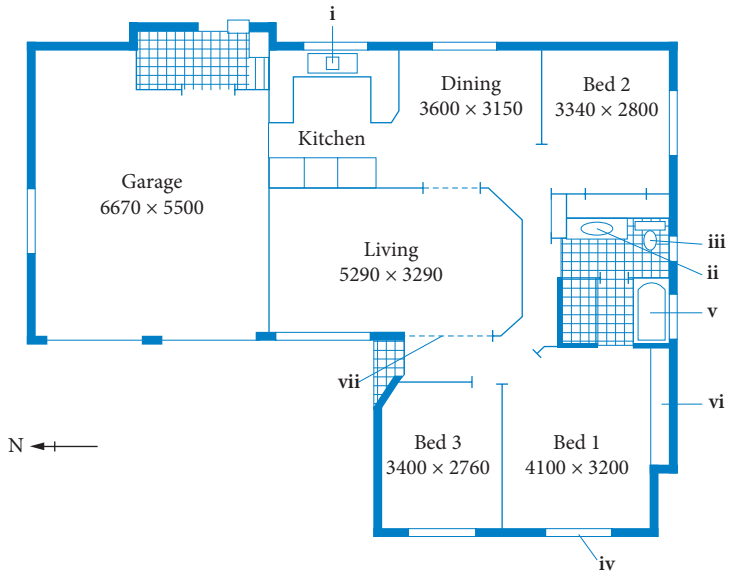
9780170370752

EXERCISE 9.4

Reading house plans

1 This is the plan for Menhal's new house.





a What does each symbol used on the plan represent?

i  **ii**  **iii**  **iv**

iv  **vi**  **vii**

b Three areas on the plan are shaded with this pattern.  What does this mean?

c How many bedrooms has the house?

d Which bedrooms have built-in wardrobes?

e What are the dimensions of bedroom 1?

f Where is the laundry?

g Where is the linen press?

h How many bathrooms has the house?

i How many toilets?

j How many doors lead into the bathroom?

k Which room has dimensions 5.29 m by 3.29 m?

l What are the dimensions of the garage?

m If Menhal looks out of the window of each room mentioned below, in which direction is he facing?

i living room **ii** dining room **iii** bedroom 1

iv bedroom 2 **v** garage

n Which direction do the garage doors face?

o If Menhal stands on the front doorstep, can he see into the bathroom?

p If Menhal stands inside the front doorway, list the four rooms he can see into.

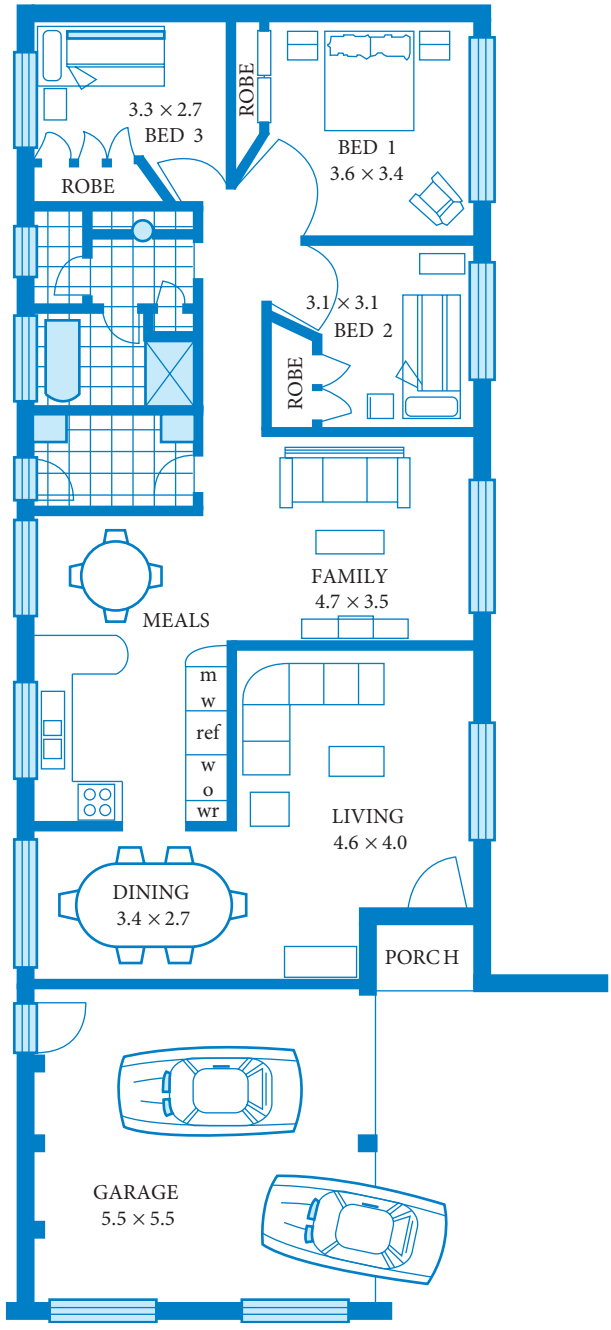
q If Menhal is working in the kitchen, list the three rooms he can see into.

2 **a** How many bedrooms does this house have?

b How many of the bedrooms have built-in wardrobes?

c Is there an ensuite bathroom?

d The floor area of the house is 128 square metres. Calculate the cost of building this house at a rate of $672 per square metre.





iStockphoto/cunfek

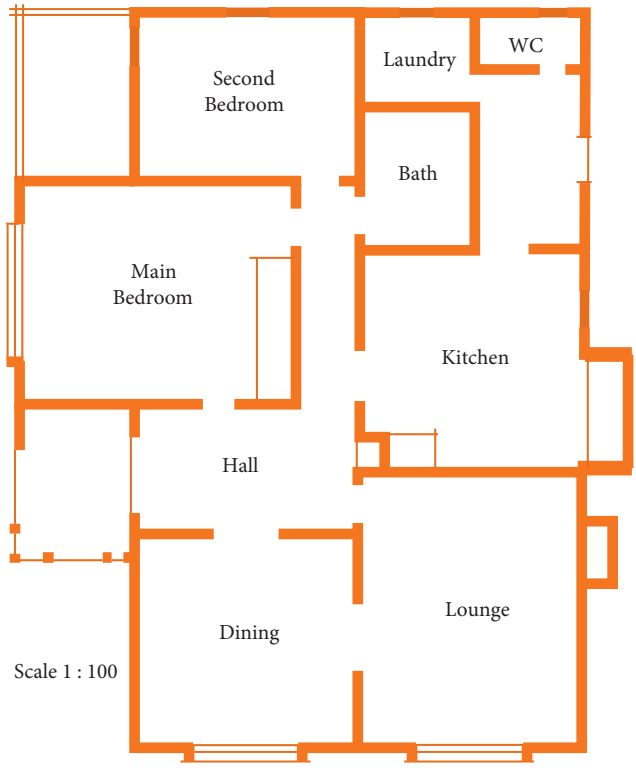
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EXAMPLE 7

9.4

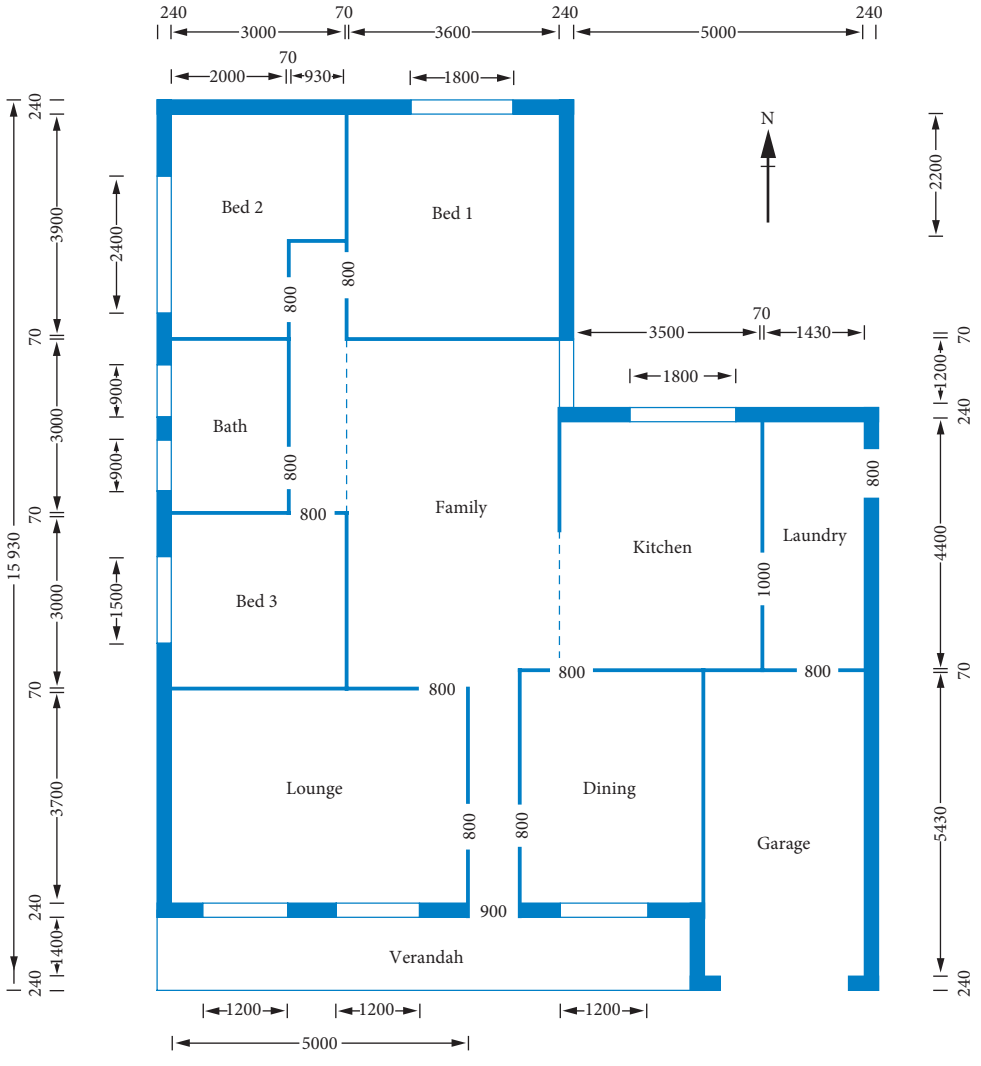
3 This house plan has a scale of 1 : 100.
Measure and calculate:

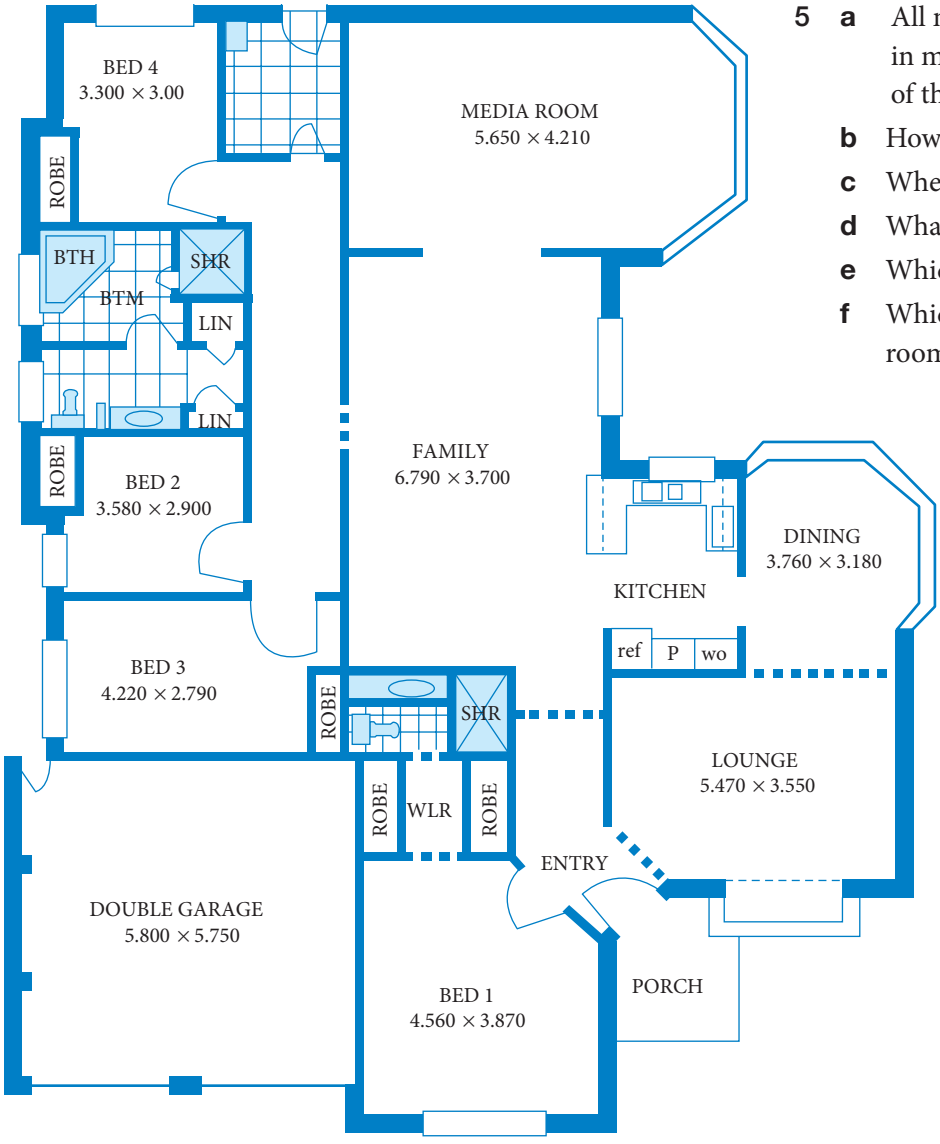
- **a** the length of the main bedroom
- **b** the length of the window in that room
- **c** the length of the laundry
- **d** the area of the bathroom
- **e** the longer side of the lounge room
- **f** the area of the dining room



4 Judy and Keith are buying a house with this floor plan.

- **a** What is the thickness of the external walls of this house in mm?
- **b** What is the thickness of the internal walls?
- **c** What is the front width of the house in metres?
- **d** What is the length of the left side of the house?
- **e** What are the dimensions of the family room?





5 **a** All measurements on this plan are in metres. What are the dimensions of the family room, in millimetres?

b How many toilets are there?

c Where are the two linen presses?

d What does SHR stand for?

e Which is the smallest bedroom?

f Which is bigger in area: the family room or the media room?

Investigation My home

There are lots of websites that contain house floor plans. Search for 'project homes' on the Internet and choose some floor plans for houses that you like.

For each of the plans:

- calculate the floor area of the whole house
- if possible, obtain an estimate for the cost of building the house
- calculate the cost per square metre



iStockphoto/pamspix

Compare and contrast the features of each house.

Decide which house you prefer based on the information you have found above, and present your findings.

Renovate my house

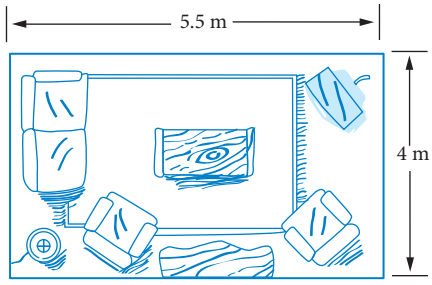
9.5

We can use house plans and sketches to calculate the amount of materials required for a job and to estimate costs.

Example 8

This is a diagram of Jackie's family room.

Jackie is laying cork tiles on the family room floor. The tiles cost $37.50 per square metre. How much will the cork tiles cost?



Solution

We need to find the area of the floor.

$$\text{Area} = 5.5 \times 4$$
$$= 22 \text{ m}^2$$

Multiply the area by the price.

$$\text{Cost} = 22 \times \$37.50$$
$$= \$825$$

EXERCISE 9.5

Renovate my house

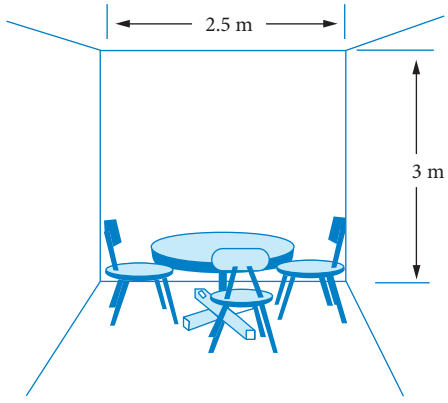
EXAMPLE 8

1 The dining room in Nicole's home unit is very small. To make it look bigger she plans to cover one wall with mirror tiles.

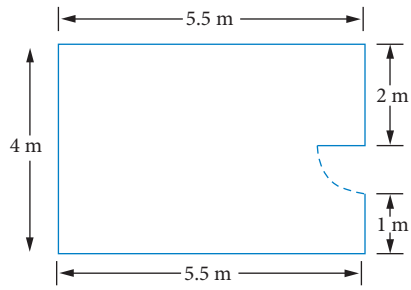
The wall she plans to cover is 3 metres high and 2.5 metres wide.

a What is the area of the wall?

b The mirror tiles Nicole has chosen cost $36.50 per square metre. What will be the total cost of the mirror tiles for the wall in Nicole's dining room?



2 Louis is replacing the skirting boards in his lounge room.

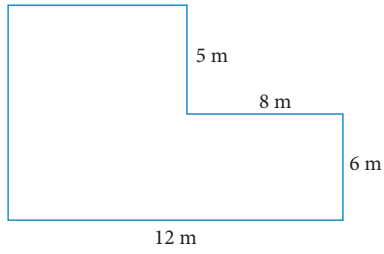


Skirting boards are the strips of wooden boards that run around a room along the base of the walls.

a How many metres of skirting boards will he need?

b The skirting boards cost $11 per metre. Calculate the total cost of the skirting boards for the job.

3 The floor area of Jesse's new house is shown.



a What is the floor area of this house?

b Using a cost of $840 per square metre, calculate the cost of building this house.

Use the floor plan for Joe and Michelle's house from Example **7** on page 206 to answer Questions **4** to **7**.

4 Joe and Michelle are renovating Bedroom 1. The skirting boards need replacing.

a Calculate the perimeter of Bedroom 1.

b The door is 1 m wide. How many metres of skirting board are needed?

c Skirting boards cost approximately $15 per metre. What will be the cost of the skirting boards?

5 Joe and Michelle are also replacing the carpet in Bedroom 1.

a How many square metres of carpet will they need?

b Carpet comes in rolls 3.6 m wide. What length of carpet will Joe and Michelle need?

c The carpet they like costs $79.45 a metre. How much will the carpet cost?

6 Joe and Michelle also need to paint the ceiling.

a What is the area of the ceiling?

b The ceiling requires two coats of paint. One litre of paint covers approximately 12 m^2 of the ceiling.

i Calculate the number of litres of paint they require.

ii The ceiling paint costs $34.90 for 4-litre can. Estimate the cost of paint for the ceiling.

7 **a** In which order should Joe and Michelle do the renovations from Questions **4**, **5** and **6**?

b What is the approximate total cost of these renovations?

Use the floor plan for Judy and Keith's house from Question **4** on page 209 to answer Questions **8** to **12**.

8 Judy and Keith have decided to replace all the floor coverings before they move into the house. They are going to lay wood parquet flooring in the lounge room. The flooring costs $68 per square metre. How much will the flooring cost?

9 Other rooms need new carpet. Judy has chosen a carpet that is 3.66 m wide. How many metres of carpet are required for each of these rooms?

- **a** bedroom 1
- **b** bedroom 2
- **c** bedroom 3

10 The kitchen and laundry are to have new tiled floors. The tiles cost $41.10 per square metre allowing for breakage and wastage. How much will it cost to buy tiles for the floor? Answer correct to the nearest dollar.

11 Judy and Keith are expecting a new baby and they are going to make bedroom 2 the baby's room. Judy wants to decorate the room with a frieze pattern around the walls. It will go above the door and the windows.



iStockphoto/ac_bnphotos

- **a** How many metres long will the frieze be?
- **b** Each roll of frieze is 5 m long. How many rolls of frieze will Judy need to buy?

12 Judy and Keith were surprised by the estimated cost of their renovations. Before signing off on the purchase of this house, they checked out the cost of building a project home of similar style and with the same floor area as this house. The cost of the project home was approximately $594 per square metre. Calculate the estimated cost of building this project home.

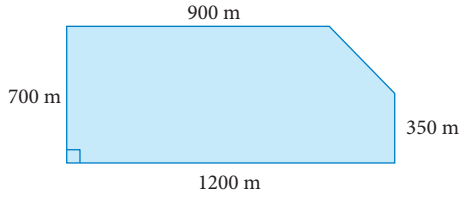
9.6 Drawing scale diagrams

To make a scale diagram, start with a rough sketch with the measurements we need on it. Then choose a scale and draw the diagram accurately.

To calculate a scaled length, divide the real length by the scale.

Example 9

This is a sketch of Freda's farm.
Draw a scale diagram of the farm.



Solution

Choose a suitable scale – we want it to fit easily on the page.

Use the scale 1 cm represents 200 m.

Calculate the scaled length for each measurement by dividing by the scale.

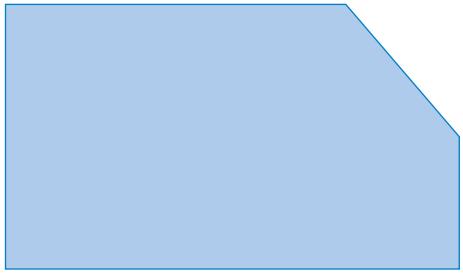
900 ÷ 200 = 4.5 cm

700 ÷ 200 = 3.5 cm

1200 ÷ 200 = 6 cm

350 ÷ 200 = 1.75 cm

Use these measurements to draw your scale diagram.



9780170370752



iStockphoto/compassandcamera

Example 10

Sue is going on a bushwalk from her camping site. She walks 2.5 km due east and then 1.9 km due northwest.

a Draw a scale diagram of Sue's walk.

b By measurement calculate how far Sue is from her campsite.

We can use our scale drawing to find other measurements.

Solution

a Choose a suitable scale.

Use scale 1 cm represents 0.5 km

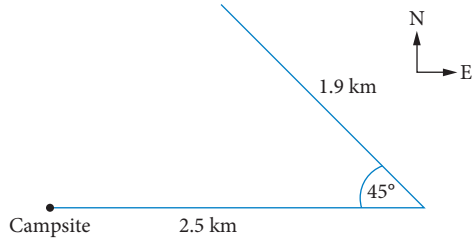
Find the scaled measurements by dividing by the scale.

$2.5 \div 0.5 = 5$ cm

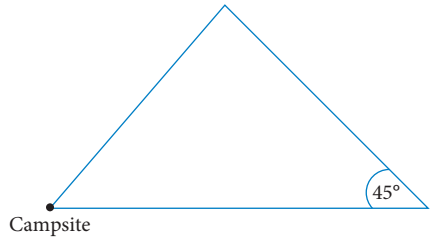
$1.9 \div 0.5 = 3.8$ cm

Use a ruler and a protractor to construct the scale diagram.

Northwest is 45° (halfway) between west and north.



b Join the starting point to the finishing point and measure this line.



The line is 3.5 cm long.

Find the real distance by multiplying by the scale.

$3.5 \times 0.5 = 1.75$ km

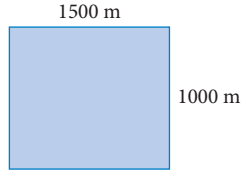
EXERCISE 9.6

Drawing scale diagrams

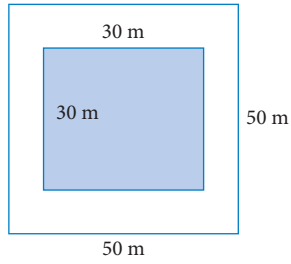
You will need a ruler and a protractor to complete this exercise.

EXAMPLE 9

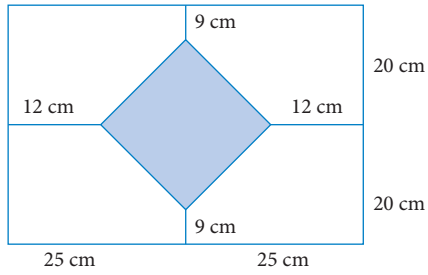
1 Make a scale drawing of this field.



2 Andrew drew a sketch of a courtyard. Make a scale drawing of the courtyard.



3 Samantha designed this cutting plate.
Make a scale drawing of her cutting plate.



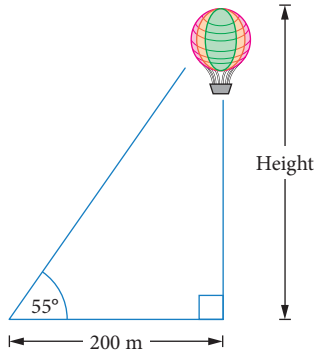
EXAMPLE 10

4 Keith and Rob are setting off on a hike. They walk 5 km due west of their starting point and then turn to walk 7 km south. They stop for lunch and then walk another 6 km in a north-easterly direction before stopping for afternoon tea.

a Draw a scale diagram of Keith and Rob's walk.

b How far are they from their starting point?

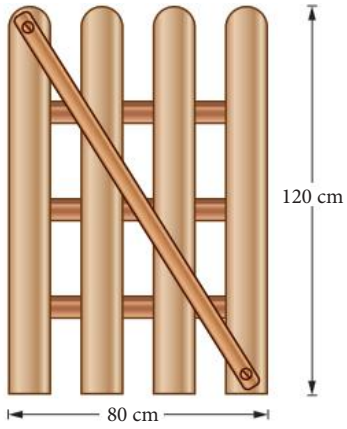
5 Construct a scale drawing to calculate the actual height of the hot air balloon above the ground.



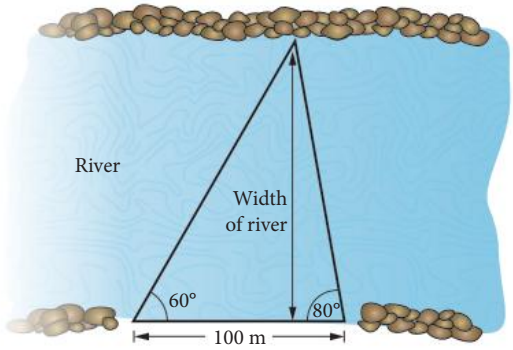
6 An old wooden gate 80 cm wide by 120 cm high needs a diagonal brace for support.

a Construct a scale drawing of the gate and find the actual length of the brace.

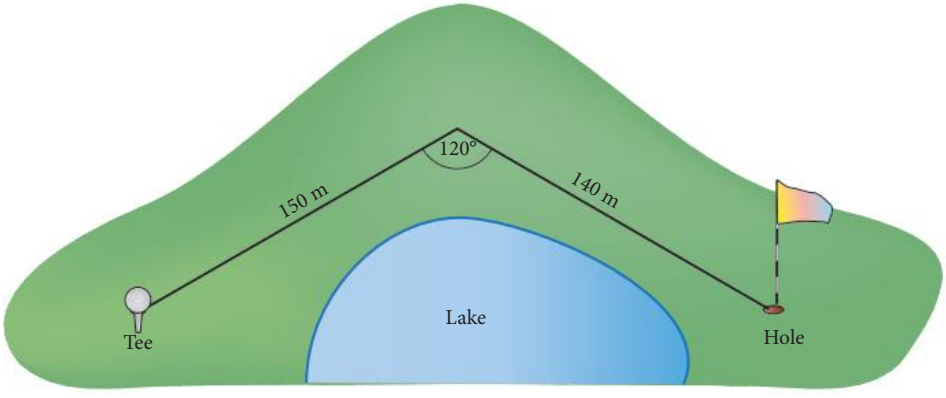
b Use Pythagoras' theorem to check the answer you obtained from the scale drawing. Answer correct to 1 decimal place.



7 Jodie wants to swim across the river. Draw this diagram to scale and calculate the width of the river.



8 A golf course has a large lake as an obstacle. Most golfers follow the dog leg around the lake. How far is it straight across the lake, from the tee to the hole?



Investigation Make my own scale drawing

Part A

- Choose something that is a large rectangle, for example, a paved area, a quadrangle, a brick wall, a whiteboard or the classroom floor.
- Measure the length and width of the rectangle.
- Make a scale drawing of your rectangle. Be sure to include the scale you used.

Part B

- Measure the length and width of the school reception area or foyer.
- There are plans to enlarge the length and width of this area by 50%. What will be the new dimensions of this area?
- Draw a scale diagram of the enlarged area. Add the position of furniture and any other items in the school entrance area. Show the scale you used.

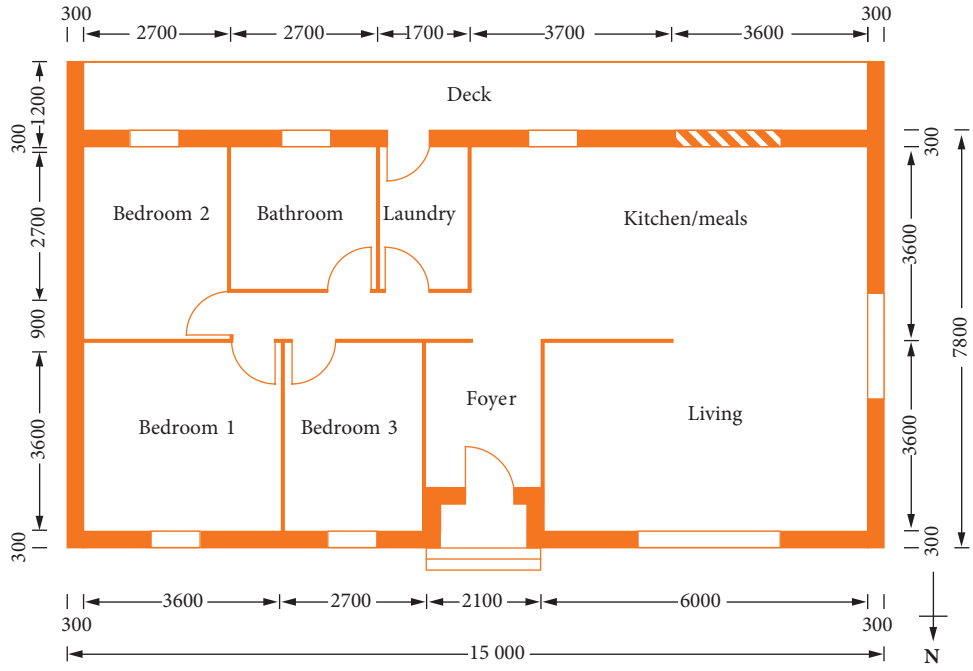
Keyword activity

Match each word in the left column to its correct meaning in the right column.

	Word		Meaning
1	dimensions	A	For an enlargement or reduction, the image length divided by the object length
2	elevation	B	The relationship between a real object and its scale diagram
3	enlarge	C	A unit of length used in house plans
4	house plan	D	To make a drawing or object larger
5	millimetre	E	Icons that illustrate features on a house plan
6	reduce	F	The length and width of a room on a plan
7	scale	G	A diagram showing the arrangement and measurement of rooms in a house
8	scale factor	H	The view of a building from the side
9	symbols	I	To make a drawing or object smaller

Chapter Problem

Francesca and Michael are building a house according to the plan shown below. They need to order carpet for the living room and the three bedrooms. How many square metres of carpet will they need? Answer to the nearest square metre.



Solution

Change the measurements for each room into metres and calculate its area.

Area of living room = 6000 mm × 3600 mm
= 6 m × 3.6 m
= 21.6 m^2

Area of bedroom 1 = 3600 mm × 3600 mm
= 3.6 m × 3.6 m
= 12.96 m^2

Area of bedroom 2 = 2700 mm × (2700 mm + 900 mm)
= 2.7 m × 3.6 m
= 9.72 m^2

Area of bedroom 3 = 2700 mm × 3600 mm
= 2.7 m × 3.6 m
= 9.72 m^2

Total area = 21.6 + 12.96 + 9.72 + 9.72
= 54 m^2



Chapter Problem

Brett is usually a very alert, safe driver. His reaction time is 0.16 seconds. However, when his blood alcohol content (BAC) is 0.05, his reaction time increases to 0.22 seconds.

Brett is driving at 100 km/h when he suddenly needs to brake. How much further does his car travel during his reaction time when his BAC is 0.05, compared to when he hasn't been drinking?



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CHAPTER 10

HEALTHY FIGURES

10.1 Burning energy
10.2 Energy in food
10.3 Gym training
10.4 Healthy formulas
10.5 Heart rate formulas
10.6 Blood pressure
10.7 Equations and formulas
10.8 Staying under 0.05
Keyword activity
Solution to the chapter problem

By the end of this chapter, you should know how to:

- use and apply measurements and rates in health contexts
- use and apply formulas
- solve simple equations to solve problems

How are we ever going to use this?

- Formulas and equations are used in many trades and workplaces
- Formulas are used in health and fitness, for example in BMI (body mass index) and BAC (blood alcohol content) calculations

10.1 Burning energy

Kilojoules (kJ) are metric units of energy. If we use more energy than we eat, we lose body weight. However, if we eat more energy than we use, the extra energy is stored as fat.

This table shows the amount of energy our body needs each day, depending on our age group, sex and level of activity.

Age	Lifestyle	Men kJ/day	Women kJ/day
18–35	inactive active very active	10 500 12 500 14 800	8 000 9 000 10 500
36–70	inactive active very active	10 000 11 800 14 300	8 000 8 800 10 400
Pregnant women Breastfeeding women			10 100 11 800

Example 1

Chloe is 22 years old, works in an office and has an inactive lifestyle. Every day on her way to work she has a cappuccino at the coffee shop. The cappuccino contains 940 kJ.

a How many kilojoules of energy does Chloe need each day?

b What percentage of her daily energy needs does Chloe have in her morning cappuccino?



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Solution

a Chloe is 22, a woman and has an inactive lifestyle. Her energy needs are shown in the top row and right column in the table.

8000

Write your answer.

Chloe needs 8000 kJ per day.

b $\dfrac{\text{kJ in the cappuccino}}{\text{kJ per day}} \times 100\%$

$\dfrac{940}{8000} \times 100 = 11.75\%$

Write your answer.

Chloe's cappuccino contains 11.75% of her daily energy needs.

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Calories (cal) are an older unit of energy. Calories are bigger than kilojoules.

1 calorie = 4.2 kilojoules

Example 2

A 500 mL chai tea latte on full cream milk contains 240 calories. How many kilojoules is this?

Solution

Kilojoules are smaller than calories. To change to a smaller unit, multiply by the conversion factor.

$240 \text{ cal} = 240 \times 4.2 \text{ kJ}$
$= 1008 \text{ kJ}$

EXERCISE 10.1

Burning energy

1 Mark has a very active lifestyle as a 20-year-old bricklayer. EXAMPLE 1

- **a** According to the table on the previous page, how many kilojoules does he require per day?
- **b** Mark eats a big breakfast containing 3250 kJ to give him energy for the day's work. What percentage (to the nearest whole number) of his daily energy requirements does Mark have for breakfast?

2 Shauna is 36 years old. During the week she is inactive and on the weekend she is very active.

- **a** How many kJ does she need each day during the week?
- **b** How many more kJ does she require per day on the weekend than during the week?

3 Courtney, aged 18, is very thin and refuses to eat more than 6000 kJ per day. Every afternoon she works out at the gym for 2 hours. Courtney's mother is worried and knows that the type of gym workout Courtney does burns 2500 kJ/hour.

- **a** How much energy does Courtney use at the gym each day?
- **b** How much energy does she have left for the remaining 22 hours in the day?

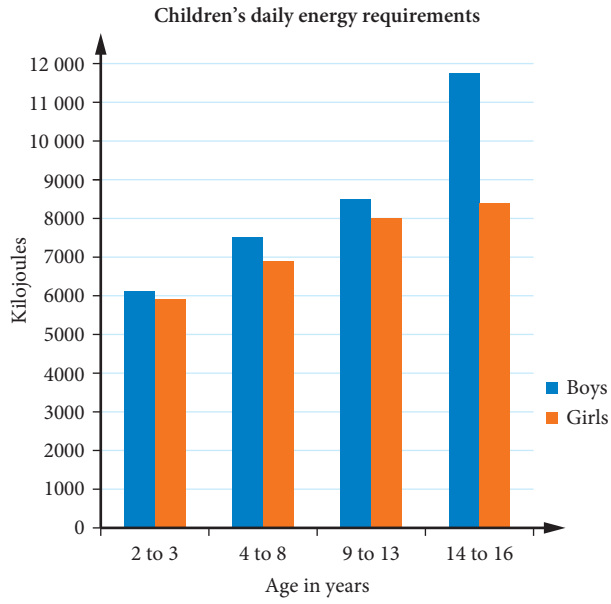
4 Use the conversion, 1 cal = 4.2 kJ to copy and complete each statement. Answer to nearest whole number. EXAMPLE 2

- **a** 500 cal = ___ kJ
- **b** 360 kJ = ____ cal
- **c** 68 cal = ___ kJ
- **d** 25 kJ = ____ cal
- **e** 2460 kJ = ___ cal
- **f** 10 800 kJ = ___ cal

5 **a** The treadmill at the gym shows that Samantha has used 550 cal. How many kJ is this?

- **b** Samantha wants to use 2500 kJ on the machine. How many more calories does she have to burn?

6 This graph shows children's daily energy requirements.



a Do boys or girls need more energy per day?

b At which age do boys and girls need approximately the same amount of energy per day?

c How many more kJ/day do 16-year-old boys need than 16-year-old girls?

d At what age do boys require approximately 7500 kJ/day?

e Calculate the weekly energy requirements for a 12-year-old girl.

f Jon is an 8-year-old boy who usually eats 8600 kJ of food per day. Is he eating an appropriate amount?

g If Jon continues to eat 8600 kJ/day, will his weight increase or decrease?

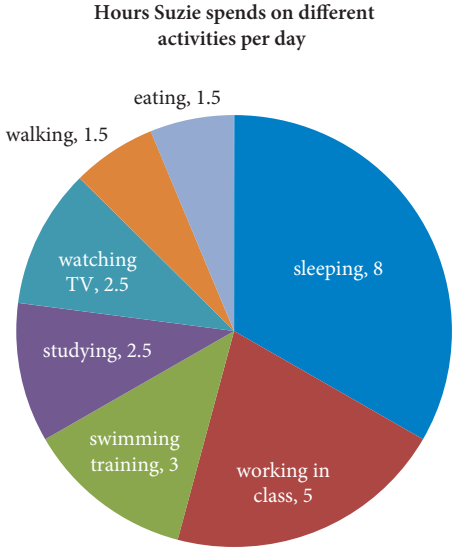
7 This table shows the average length of time it takes an 18-year-old to burn up 1000 kJ for different activities.

Activity	Time required to use 1000 kJ
Sleeping	4 hours
Eating	3 hours
Working in class Studying Watching TV	2 hours 30 minutes
Walking	1 hour
Bike riding	50 minutes
Swimming	30 minutes

a How many kilojoules does Leo use swimming for 1 hour?

b How long will it take Divya to use 500 kJ while she is sleeping?

c Suzie, aged 18, leads a very active life. The pie chart shows the number of hours each day that she spends on different activities.



i According to the table on page 222, how many kilojoules does Suzie need per day?

ii If Suzie stays within this recommendation, will she eat a sufficient amount to meet her energy requirements?

iii What recommendations would you give Suzie concerning her diet?

Energy in food

10.2

The food we eat supplies us with energy. This table shows the amount of energy stored in some normal-sized serves of food.

Food	kJ	Food	kJ	Food	kJ
Roast lamb, gravy	1064	Apple pie	1380	Apple	243
Potato bake	1175	1 slice bread and butter	520	Large tomato Small tomato	120 30
Mixed grill	2600	Yoghurt	315	Chocolate biscuit	493
Steak	3900	Muesli	1470	Can of soft drink	372
Bacon (2 slices)	640	Eggs (2)	735	Orange juice	206
Fish	340	Cheese	115	Milk (glass)	628
Grilled chicken breast	1264	Ham (2 slices)	224	Milk for cereal	400
Chips	1425	Mixed nuts (100 g)	2640	Coffee, milk and sugar	295
Ice-cream	810	Broccoli	98	Banana	546
Sauce for steak or chicken	246	Sauce for fish	265	Beef sausage	176

Example 3

On a plane, Lucas was served a meal of grilled chicken breast with sauce, chips and broccoli. How many kilojoules were in this meal?



Corbis/Ashley Cooper

Solution

Look up the kJ contents of each food in the table.	Chicken breast: 1264 Sauce: 246 Chips: 1425 Broccoli: 98
Add them.	Total kJ = 1264 + 246 + 1425 + 98 = 3033
Write your answer.	The meal contained 3033 kJ.

Example 4

Vivian ate 200 g of mixed nuts. How long will it take her to burn the energy from the nuts if she works out at the gym at a rate of 30 kJ/minute?

Solution

200 g of nuts = 2 serves (from the table)	Total kJ = 2640 × 2 = 5280
Divide the number of kJ by the rate to find the time.	Time = 5280 ÷ 30 = 176 minutes = 2 hours 56 minutes
Write your answer.	Vivian will have to work out for 2 h 56 min to use the energy from the nuts.

Example 5

Athletes running at 15 km/h use 75 kJ/h per kg of their body weight. Claire's body weight is 58 kg. How many kJ does she burn when she runs for 30 minutes at 15 km/h?

Solution

Claire burns 75 kJ per hour per kg of her body weight. She weighs 58 kg so each hour she burns 75 × 58.	kJ burned per hour = 75 × 58 = 4350
30 minutes is half of an hour. Divide by 2.	kJ burned in 30 minutes = 4350 ÷ 2 = 2175
Write your answer.	Claire burns 2175 kJ when she runs for 30 minutes.

EXERCISE 10.2

Energy in food

EXAMPLE 3

1 Calculate the energy content of each meal.

a Danica's breakfast contains a serving of muesli with milk, an apple and a banana.

b Kobi's breakfast comprises 2 slices of ham, 2 beef sausages, 2 eggs and 1 slice of bread and butter.

c Andrew ate a steak with sauce, a large tomato and a serving of chips for his lunch.

d Emma had some grilled fish with sauce and a large sliced tomato for her lunch.



iStockphoto/Serge_Bertasius



iStockphoto/rakratchada

2 George is 16 years old. This is what he eats in a typical day.

Breakfast	Dinner	Lunch
2 slices of bacon	Mixed grill	Ham sandwich with 2 slices of ham
2 eggs	Chips	Apple
Muesli with milk	Broccoli	2 chocolate biscuits
2 slices of toast with butter	Apple pie	Yoghurt
Glass of orange juice	Ice cream	Can of soft drink
	Coffee, milk, sugar	

a Calculate the total number of kilojoules George eats in a typical day.

b An average 16-year-old boy should eat 11 800 kJ per day. What effect will George's typical day's food have on his body weight?

3 Amanda requires a daily diet of 8800 kJ. Use the foods in the table on page 225 to plan a healthy day's menu for Amanda.

EXAMPLE 4

4 A large chocolate cake contains 27 600 kJ. Mai cut the cake into 8 equal slices.

a How many kilojoules are in each slice of cake?

b How many minutes of gym work, at 30 kJ/minute, will it take to use the energy in one slice of the cake?

5 This table shows the energy an average person uses per minute for different activities. Calculate the number of kJ an average person uses when:

Activity	kJ/minute
Sleeping	4
Cleaning	15
Ironing	17
Bricklaying	17
Playing tennis	31
Gardening	23
Circuit training	53
Walking	23

a gardening for 30 minutes

b ironing for 45 minutes

c bricklaying for an hour

d circuit training for $1\frac{1}{4}$ hours

e sleeping for 8 hours

f walking for 50 minutes every day for a week

EXAMPLE 5

6 The more you weigh, the more kilojoules you will burn doing different activities. The table shows the number of kJ per kg of body weight that teens and adults use in 1 hour in various activities.

Activity	kJ used per hour per kg of body weight
Sleeping	3.7
Driving a car	6.4
Class work	6.8
Walking	18
Skiing (15 km/h)	43
Swimming (45-second laps)	29

a Catherine weighs 45 kg. How many kilojoules does she use during a 1-hour maths lesson?

b Fahim's body mass is 75 kg. How many more kilojoules does he burn in 8 hours sleep than Catherine?

c Harry started walking everyday to try to lose weight. He currently weighs 128 kg. How many kJ does he use when he walks for 30 minutes?

d Mili is a keen skier who weighs 63 kg. One Saturday she drove her car for 3 hours to the snowfields. Then she skied at an average speed of 15 km/h for 5 hours. How many kJ of energy did she use in the 8 hours?

e Every morning Scott spends an hour swimming 45-second laps of the pool as part of his training routine. During the hour Scott uses approximately 2200 kJ. What is Scott's approximate body weight?



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Gym training

10.3

Heart rates

The heart is a large muscle that pumps blood around our body. If it stops beating for more than a few minutes, we die. Our hearts beat faster during exercise. After exercise, the time it takes for our hearts to return to its resting rate is a measure of our fitness.

Our **heartbeat rate** or **pulse** is measured in beats per minute. The average rate is 72 beats/min.

We can use a simple formula to convert a pulse rate for any time period into a rate per minute.

$$\text{Pulse in beats per minute} = \frac{\text{number of beats}}{\text{time in seconds}} \times 60$$

Example 6

Darren takes his pulse during an exercise session and counted 52 beats in 20 seconds. Calculate his pulse in beats per minute.

Solution

$$\text{Pulse in beats per minute} = \frac{\text{number of beats}}{\text{time in seconds}} \times 60$$

$$\text{Pulse} = \frac{52}{20} \times 60$$
$$= 156 \text{ beats/min}$$

Exercise machines can generate electricity

Gyms can collect the energy people use on exercise machines and convert it to electricity. One person doesn't generate a lot of energy but when the gym is full of people the electricity generated can be used to power the gym's TV and sound systems, which saves money.



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Example 7

At the gym, each exercise bike rider generates 100 **watts** of electricity per hour. On average, an exercise bike is used for 6 hours per day, 364 days per year (closed Christmas Day) and the gym pays 54c per **kilowatt** for electricity.

1 kilowatt (kW) = 1000 watts (W)

How much in electricity costs does each bike save the gym annually?

Solution

Calculate the electricity a bike generates each day by multiplying the rate by 6 hours.	Electricity generated per day = 100×6 = 600 W
Multiply the daily amount by 364 to get the annual amount.	Electricity generated per year = 600×364 = 218 400 W
Divide by 1000 to change it into kilowatts.	$218\,400 \div 1000 = 218.4$ kW
Calculate the electricity cost by multiplying by the cost rate. Write 54c as \$0.54.	Cost = $218.4 \times \$0.54$ = \$117.94
Write the answer.	On average, each exercise bike saves the gym \$117.94 in electricity costs per year.

EXERCISE 10.3

Gym training

1 Renee's 3-month gym membership costs \$234 and gives her 13 weeks' unlimited use of the gym.

- **a** How much does Renee's membership cost per week?
- **b** Casual users of the gym are charged \$11 per visit. Renee attends the gym 4 times per week. How much will her membership save her compared to the casual rate?
- **c** How many visits to the gym does a member need to make in 3 months to make membership worthwhile?

2 A gym is advertising special membership deals to attract new members.

- **a** Calculate the charge per week for each of the two plans.
- **b** If you were joining the gym, which plan would you choose? Why?



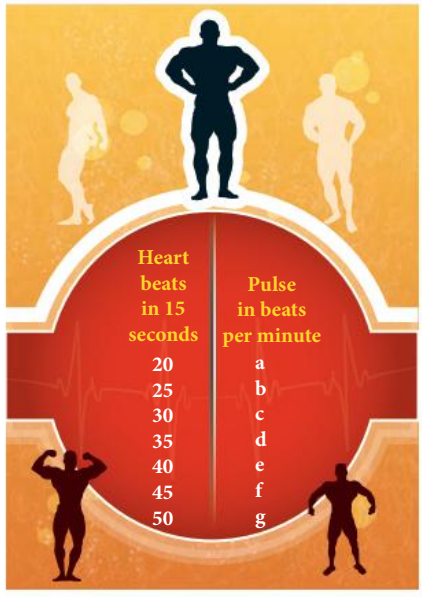
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EXAMPLE 6

3 Convert each pulse rate to beats per minute.

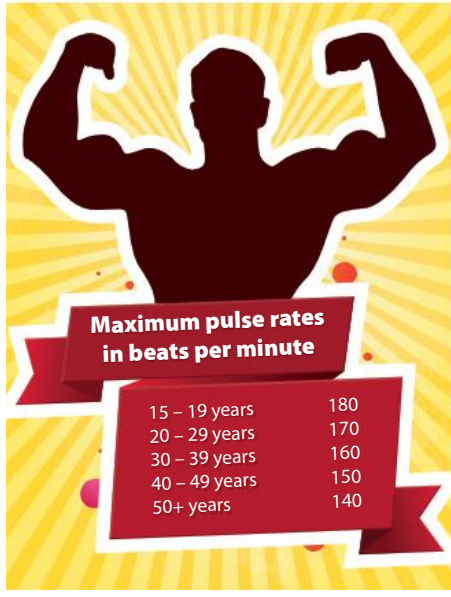
- **a** 25 beats in 20 seconds
- **b** 16 beats in 15 seconds
- **c** 65 beats in 30 seconds

4 Gym supervisors are making a sign to help visitors count their pulse for 15 seconds to convert to beats/minute. Complete the missing values in the sign.



5 During her first aerobics class, Renee counted her pulse as 43 beats in 15 seconds.

- **a** What is Renee's pulse in beats per minute?
- **b** The best pulse rate during exercise is 60% of the maximum pulse rate. Renee is 16 years old. Use the sign to calculate her best exercise pulse rate.
- **c** What advice would you give Renee if she exercises at an intensity which gives her a pulse of 43 beats in 15 seconds?



6 Renee's resting breathing rate (respiration rate) is 16 breaths/minute. When she runs on the mini trampoline she likes to run at a pace that gives her a 75% increase in her respiration rate. What respiration rate does Renee aim for when she uses a mini-trampoline?

7 As a warm-up exercise, Renee rode an exercise bike at a speed of 20 km/h for 5 minutes.

a What fraction of 1 hour is 5 minutes?

b Use your answer to part **a** to calculate how many kilometres (correct to two decimal places) Renee rode on the bike during the 5 minutes.

8 In a circuit class, participants change fitness activities every 30 seconds. There are 28 different activities in the circuit. These are completed at workstations.

a How long will it take Renee to complete 1 circuit using each workstation once?

b In a circuit class the workstations are used for 25 minutes. How many workstations will Renee use per class?

EXAMPLE 7

9 When Renee is using the fitness machines, she generates 85 watts of electricity per hour. She uses the machines for 3 hours per week.

a How many watts of electricity does Renee generate on the machines per week?

b Calculate the amount of electricity Renee will generate during her 13-week membership. Express your answer in kilowatts.

c Calculate the value of the electricity Renee generates in 13 weeks at 54c/kilowatt.

10 The gym has 24 treadmills. On average, each treadmill is in use for 10 hours per day, 364 days per year. Members using a treadmill generate 110 watts of electricity per hour.

a Calculate the total number of kilowatts of electricity generated by members on the treadmills per day.

b How many kilowatts of electricity do members generate on the treadmill per year?

c At 54c/kilowatt, calculate the total annual value of the electricity generated on the treadmills.

Healthy formulas

10.4

Many people use **formulas** at work. Builders use formulas to determine the amount of concrete required for making paths. Nurses use formulas to calculate quantities for injections and medications.

Example 8

Fried's formula is used to calculate the infant dose of medicine.

Fried's formula says $D = \frac{MA}{150}$ where:

D = the infant dose in mL

A = the adult dose in mL

M = the child's age in months

The adult dose of a cough medicine is 30 mL. What dose of cough medicine can a parent give to a 12-month-old child?



Shutterstock.com/Iakov Filimonov

Solution

Substitute the given information into the formula: Adult dose $A = 30$.

The child is 12 months old, so $M = 12$.

$$D = \frac{MA}{150}$$
$$= \frac{12 \times 30}{150}$$
$$= 2.4$$

Write your answer.

The child's dose is 2.4 mL.

EXERCISE 10.4

Healthy formulas

EXAMPLE 8

1 Use Fried's formula $D = \frac{MA}{150}$ to calculate the amount of a cough medicine (correct to one decimal place) that Mia can give to her 18-month-old daughter. The adult dose is 24 mL.

2 We can use the formula $C = \frac{5}{9}(F - 32)$ to change temperatures from degrees Fahrenheit to degrees Celsius. The recipe for Grandma's special cake requires the oven to be set to 350°F. Isabella's oven measures temperature in °C. At what temperature should Isabella set her oven to cook Grandma's special cake? Answer to the nearest degree.

3 Nurses in a hospital nursery use the formula $V = \frac{30DW}{N}$ to calculate the volume of milk required for each feed for babies younger than 7 days old, where:

V = volume of milk in mL

D = the baby's age in days

W = the baby's weight in kg

N = the number of feeds per day.

Rachel is 5 days old and weighs 2.4 kg. The nurse is going to give her 6 bottles (feeds) per day.

a Calculate the volume of milk in each of Rachel's bottles.

b What is the total amount of milk Rachel will receive on the 5th day?

4 We can estimate a baby's adult height in cm, with the formula

Expected adult height = 24 + birth length × 3.

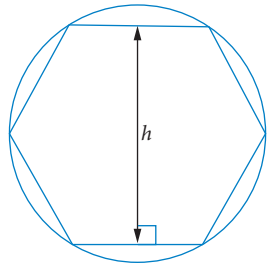
Rachel was 46 cm long when she was born. Use the formula to estimate Rachel's expected adult height.



Getty Images/Roy Hsu

5 Fitters and turners make regular hexagonal nuts from cylindrical pieces of metal. The formula relating the height, h, of the nut that can be made from a cylinder with a radius r is $h = 1.73r$.

What is the height of a nut made from a cylindrical piece of metal with a radius of 12 mm?



6 There are two alternative formulas for approximating the children's dose of adult medicines.

$$\text{Young's rule: Dose} = \frac{\text{age of child in years} \times \text{adult dose}}{\text{age of child in years} + 12}$$

$$\text{Clark's rule: Dose} = \frac{\text{weight of the child in kilograms} \times \text{adult dose}}{70}$$

Emma is 8 years old and weighs only 20 kg. The adult dose of the medicine that her mother wants to give her is 30 mL.

a How much medicine does each of Young's and Clark's rules say Emma can have?

b The amounts are different. Do you think Emma's mother should use the formula based on age or weight? Give a reason for your answer.

7 The formula for calculating blood alcohol content (BAC) in females after they have been drinking alcohol is

$\text{BAC}_{\text{female}} = \dfrac{10N - 7.5H}{5.5M}$ where:

N = the number of standard drinks

H = the number of hours drinking

M = the woman's mass.

Olivia's mass is 65 kg and she had 5 standard drinks in 2 hours. Calculate her blood alcohol content correct to 4 decimal places.

8 The formula for calculating BAC in males is $\text{BAC}_{\text{male}} = \dfrac{10N - 7.5H}{6.8M}$. Oscar (male) and Jenna (female) both have a mass of 70 kg and they each consumed 4 standard drinks in 90 minutes.

a Express 90 minutes in hours.

b Who had the greater BAC and by how much?

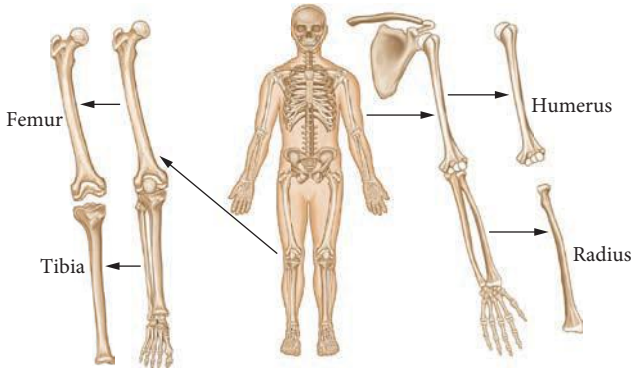
c Why do Oscar and Jenna have different BACs? Refer to the formulas in your explanation.

9 When skid marks are in a straight line, police can use the formula $V = 16\sqrt{FS}$ to estimate a car's speed before the driver applied the brakes. V is the speed of the car in km/h, F is the road surface coefficient of friction and S is the length of the skid marks in metres.

Accident investigators measured the length of a skid mark to be 31 metres. The road's coefficient of friction is 0.85. Calculate the speed of the car that left the skid marks. Answer correct to the nearest km/h.

Investigation Your height can be found in your bones

Forensic experts use science and mathematics to help them determine all kinds of information. For example, the length of one of the four human 'long bones' and the sex of a person is all they require to calculate the height of the person. They use formulas that allow them to determine height from the length of a skeleton's femur, tibia, humerus and radial bones.



	Females	Males
Humerus	$h = \frac{3144H + 64\ 977}{1000}$	$h = \frac{2970H + 73\ 570}{1000}$
Radius	$h = \frac{3876R + 73\ 502}{1000}$	$h = \frac{3650R + 80\ 405}{1000}$
Tibia	$h = \frac{2533T + 72\ 572}{1000}$	$h = \frac{2392T + 81\ 688}{1000}$
Femur	$h = \frac{2317F + 61\ 412}{1000}$	$h = \frac{2238F + 69\ 089}{1000}$

In the formulas above:

h = the person's height

H = the length of the humerus (upper arm bone)

R = the length of the radius (lower arm bone)

T = the length of the tibia (lower leg bone)

F = the length of the femur (upper leg bone)

All measurements are in centimetres.

1 The bones of two people, a male and a female, were found in a shallow grave. The female's humerus was 33.5 cm long and the male's tibia was 41 cm. What was the height of each person when they were alive?

2 Several bones of an unidentified male skeleton were found in a forest. The humerus was 40 cm, the radius was 31 cm and the tibia was 47.5 cm.

a Calculate the height of the unidentified male, based on each bone.

b What height do you think should be recorded?

c Can you suggest a reason why the bones don't give exactly the same height?

3 Measure your height and calculate the length of each of your four long bones.

Heart rate formulas

10.5

Maximum heart rate

If we want to live long and healthy lives, we must look after our hearts.

A person's maximum heart rate depends on their age and sex.

Maximum heart rate formulas:

$M_{\text{malemax}} = 220 - \text{age in years}$

$M_{\text{femalemax}} = 226 - \text{age in years}$

Example 9

Brittany is 16 years old. What is her maximum heart rate?

Solution

Use the $M_{\text{femalemax}}$ formula with age = 16.

$M_{\text{femalemax}} = 226 - 16$
$= 210$

Write your answer.

Brittany's maximum heart rate is 210 beats/min.

Example 10

Ewan exercised at an intensity that made his pulse 140 beats/minute. How many times did his heart beat during his 50 minutes workout?

Solution

Write the units of the rate as a fraction: $\frac{\text{beats}}{\text{min}}$

To find the number of beats, multiply by the rate.

Number of beats $= 50 \times 140$
$= 7000$

Write your answer.

Ewan's heart beat 7000 times.

Target heart rate

We should exercise at different heart rates and intensities to achieve different goals. We exercise at 70% to 80% of our maximum heart rate to improve our cardio system and at 60% to 70% for weight loss.

Target heart rate formula:

$H_t = (H_{\text{max}} - H_{\text{rest}}) \times I + H_{\text{rest}}$

where H_{max} = maximum heart rate

H_{rest} = resting heart rate

I = exercise intensity as a decimal

This table shows the percentage of maximum heart rate required for different exercise intensities.

Recover or weight loss	Aerobic (cardio training)	Anaerobic (strength/muscle training)
60% to 70%	70% to 80%	80% to 90%

Example 11

Ali works out in the gym every day for cardio training. He is 19 years old and his resting pulse is 62 beats/minute. What is his target heart rate for aerobic exercise?

Solution

First find Ali's maximum heart rate using $M_{\text{male max}} = 220 - \text{age in years}$

$H_{\text{max}} = 220 - 19$
$= 201$

Substitute $H_{\text{rest}} = 62$ and for aerobic exercise I has values from 0.7 to 0.8 (70% to 80%) into

$H_t = (H_{\text{max}} - H_{\text{rest}}) \times I + H_{\text{rest}}$

Lower target heart rate $= (201 - 62) \times 0.7 + 62$
$= 159.3$

Upper target heart rate $= (201 - 62) \times 0.8 + 62$
$= 173.2$

Answer the question asked.

Ali's target heart rate for aerobic exercise should be from 159 to 173 beats/min.

EXERCISE 10.5

Heart rate formulas

EXAMPLE 9

1 Calculate the maximum heart rate for each person in the table.

	Name	Sex	Age	Resting pulse
a	Jordan	Male	15	65
b	Olivia	Female	18	55
c	Hunter	Male	20	73
d	Kate	Female	17	50
e	Hassan	Male	21	59
f	Alisha	Female	19	68

EXAMPLE 10

2 While we are asleep, our heart is at its resting pulse.

a Use the above table to calculate how many times Olivia's heart beats in one hour of sleep.

b How many times does Olivia's heart beat while she is asleep for 7 hours?

3 How many times does Hassan's heart beat during a 30-minute nap?

4 When Chris is training in the pool he keeps his heart rate at 135 beats/minute. His training sessions are 45 minutes long. How many times does his heart beat during a training session?



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5 17-year-old Sophie goes to the gym each day and she exercises at a rate that makes her pulse 80% of her maximum heart rate.

a What is Sophie's maximum heart rate?

b Calculate Sophie's exercise pulse rate.

6 Mark's maximum heart rate is 197 beats per minute. How old is he?

7 Megan's maximum heart rate is 215 beats per minute. How old is she?

8 The people in Question **1** go to the gym for different reasons. Copy and complete the table. The first row has been completed as an example.

EXAMPLE 11

	Name	Objective	Lower target heart rate	Upper target heart rate
a	Jordan	Anaerobic	$(205 - 65) \times 0.8 + 65 = 177$	$(205 - 65) \times 0.9 + 65 = 191$
b	Olivia	Aerobic		
c	Hunter	Weight loss		
d	Kate	Anaerobic		
e	Hassan	Aerobic		
f	Alisha	Weight loss		

Recover or weight loss	Aerobic	Anaerobic
60% to 70%	70% to 80%	80% to 90%

9 When we're resting, our heart pumps 70 mL of blood every time it beats. How many litres of blood does Jordan's heart pump every minute when he is resting?

10 When Kate is exercising vigorously, her pulse is 180 beats per minute and her heart pumps 21 L of blood every minute. How much blood does Kate's heart pump with each beat? Answer to the nearest mL.

Investigation What's your pulse?

You will need a watch and access to some stairs.

What you have to do

- Search on the Internet to watch a video that shows you how to take your pulse.
- Measure your resting heart rate in beats/minute.
- Exercise for 2 minutes by walking up and down the stairs.
- Measure your exercise pulse in beats/minute.
- Copy and complete the table for members of your group by measuring their pulse at 1, 2, 5 and 10 minutes after the step exercise.

Name	Resting pulse	Exercise pulse	Pulse after exercise			
			1 minute	2 minutes	5 minutes	10 minutes

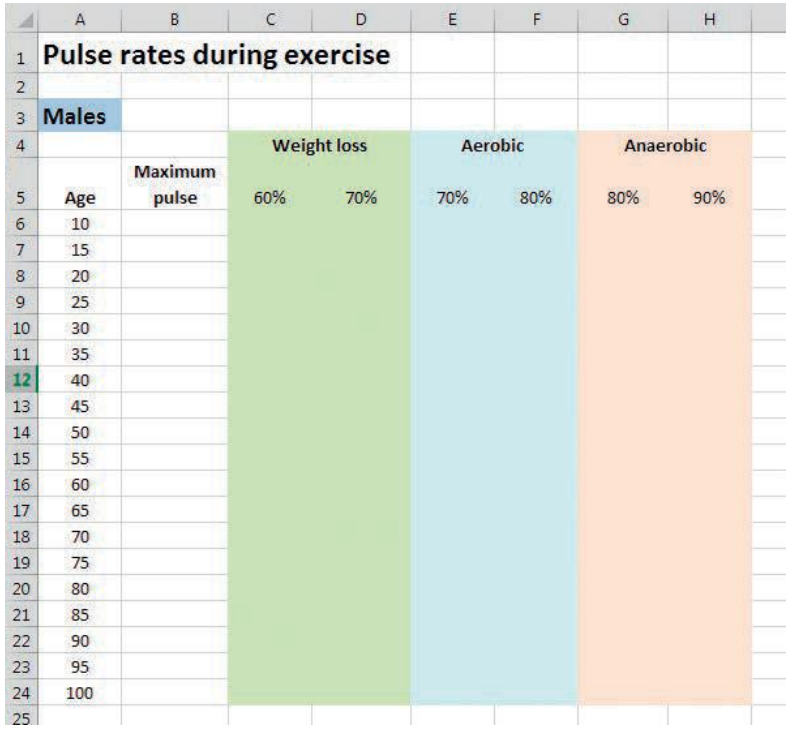
- Construct a scatter plot to show each member of the group's exercise pulse rate as well as their pulse for the first 10 minutes after exercise.

Class discussion questions

1. Who in your class has the lowest resting pulse?
2. Who has the highest resting pulse?
3. How does a resting pulse change during 2 minutes of stair climbing?
4. Whose pulse changed the most during the stair-climbing exercise?
5. Whose pulse was the quickest to return to close to the resting rate after the stair-climbing exercise?
6. Some people claim that the lower a person's resting pulse, the quicker an exercise pulse returns to a resting pulse. Does the data support this claim? Why or why not?

Investigation Heart rate spreadsheet

In this activity you are going to construct a spreadsheet to display target heart rate information, suitable for displaying in a gym.



	A	B	C	D	E	F	G	H
1	**Pulse rates during exercise**							
2								
3	**Males**							
4			**Weight loss**		**Aerobic**		**Anaerobic**	
5	**Age**	**Maximum pulse**	60%	70%	70%	80%	80%	90%
6	10							
7	15							
8	20							
9	25							
10	30							
11	35							
12	40							
13	45							
14	50							
15	55							
16	60							
17	65							
18	70							
19	75							
20	80							
21	85							
22	90							
23	95							
24	100							
25								

What you have to do

- Construct the spreadsheet above, or download a copy from NelsonNet.
- For males, maximum pulse = 220 – age in years. In cell B6 type the formula **=220-A6** then press enter. If you have entered the formula correctly 210 will appear in cell B6.
- Click on cell B6 then **fill down** to cell B24. The maximum pulse column should be complete.
- In cell C6 type the formula **=B6*0.6** then press enter.
- Click on cell C6 and fill down to cell C24.
- Enter the formulas for cells D6 to H6. The formulas required are:

Cell	D6	E6	F6	G6	H6
Formula	=B6*0.7	=D6	=B6*0.8	=F6	=B6*0.9

Pull all the formulas down to row 24.

- When you have completed the spreadsheet for males, construct a similar one for females. Remember that the maximum pulse rate is different for males and females.

Investigation How long does it take your heart to recycle all of your blood?

a Visit the Easy Calculation website and search for the Blood Volume Calculator.



Use the calculator to find the volume of your blood in litres.

b Measure your resting pulse in beats/minute.

c The typical heart pumps 70 mL of blood on each beat. Use the formula below to calculate the volume of blood in litres your heart pumps per minute when you are resting.

$V = H_r \times P_r \times 0.001$ where

H_r = heart rate in beats/minute, and P_r = the heart pump rate in mL/beat

d Calculate the number of minutes it takes for your entire blood to be recycled around your body by dividing the total volume of blood in your body, found in part **a**, by the volume pumped per minute, found in part **c**.

e Repeat the calculations to determine the time required to recycle your blood when you are exercising. Increase the heart pump rate to 90 mL/beat.

Blood pressure

10.6

We need pressure to keep our blood moving around our body delivering oxygen to our cells. Extremely high or low blood pressure can cause damage to our kidneys, heart and brain, but a healthy diet that minimises fats and sugars combined with regular exercise can keep our blood pressure at a suitable level.

Doctors use **two numbers** to describe blood pressure, for example '130 over 85'.

- The first number is **systolic pressure** and is always the larger number, measuring the pressure when our heart is pumping.
- The second number is **diastolic pressure**, measuring the pressure when our heart is relaxed.

Blood pressure is measured in mm of mercury (mm Hg).

'hyper' means high and 'hypo' means low. Remember that there is an 'o' in low and hypo.

Normal blood pressure	Below 120/80
High blood pressure (*Hyper*tension)	140/90 or higher. If one or both of the numbers is higher than 140/90 you have high blood pressure.
Low blood pressure (*Hypo*tension)	Less than 90/60 and causing symptoms of light-headedness. Elite athletes can have blood pressure at 90/60 without it being considered low.



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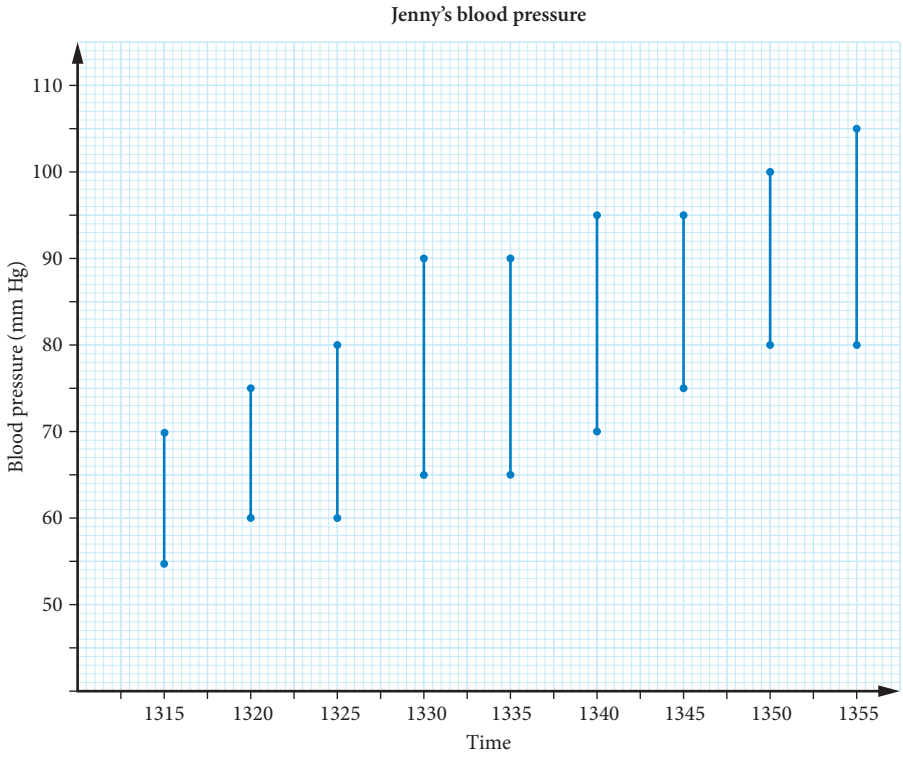
Example 12

When Jenny was 9 weeks pregnant, she had a miscarriage and lost a lot of blood. Nurses treated her low blood volume and monitored her blood pressure every 5 minutes.

Time	1315	1320	1325	1330	1335	1340	1345	1350	1355
Systolic pressure	70	75	80	90	90	95	95	100	105
Diastolic pressure	55	60	60	65	65	70	75	80	80

Graph Jenny's blood pressure readings.

Solution



When we graph blood pressure, we plot the positions for the systolic and diastolic readings and join the readings with a vertical line.

EXERCISE 10.6

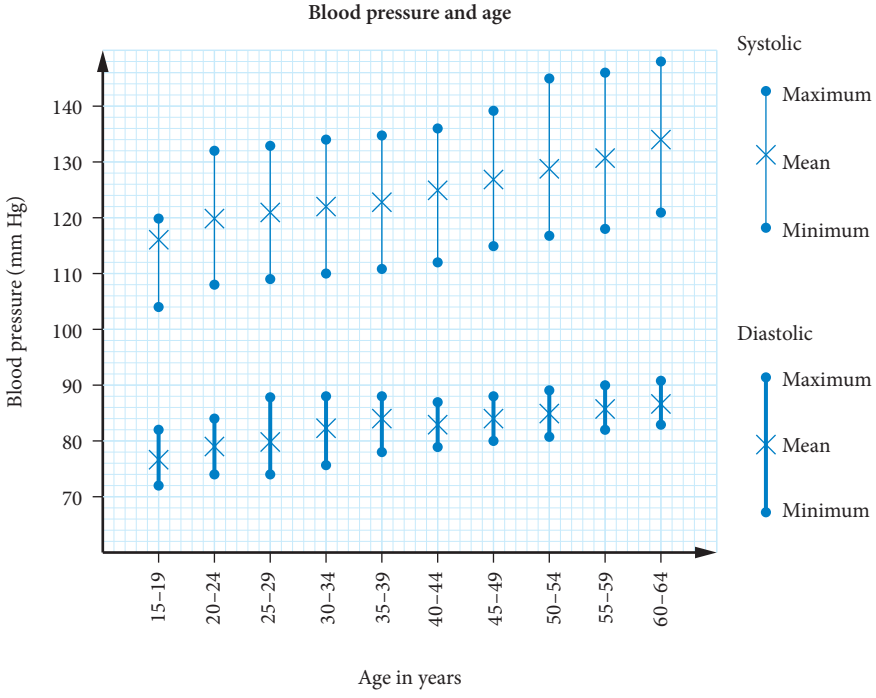
Blood pressure

1 During an operation, medical staff discovered that Ian was allergic to the anaesthetic they were using. Graph this table of values of Ian's blood pressure.

EXAMPLE 12

Time	11 a.m.	1 p.m.	3 p.m.	5 p.m.	7 p.m.	9 p.m.	11 p.m.	1 a.m.	3 a.m.	5 a.m.
Systolic pressure	140	180	180	160	170	180	200	175	150	140
Diastolic pressure	90	130	135	130	130	145	150	130	120	105

2 This graph shows normal blood pressure ranges for different age groups.



a The normal systolic blood pressure range for a 55-to-59-year-old person is 118 to 144 mm Hg. What is the normal diastolic range for a 55-to-59-year-old person?

b What is the mean systolic and diastolic readings for a 55-to-59-year-old person?

c What is the systolic blood pressure range for a 30-to-34-year-old person?

d What is the average systolic and diastolic values for a 20-to-24-year-old person?

e According to the graph, what should your blood pressure be?

f In any age group, how is the range of systolic blood pressure readings different to the range of the diastolic readings?

3 This chart shows blood pressure categories.

Systolic (mm Hg)		Diastolic (mm Hg)	Evaluation
120 or less	and	80 or less	Normal blood pressure
121–139	or	81–89	Pre hypertension (Pre high blood pressure)
140–159	or	90–99	Stage 1 high blood pressure
160 or higher	or	100 or higher	Stage 2 high blood pressure
Higher than 180	or	Higher than 110	Emergency situation

a Darren's blood pressure is 120/85. Does he have normal blood pressure?

b Saul's blood pressure is 123/70. Explain why his doctor said he has pre-high blood pressure.

c Morgan is 25 weeks pregnant and she has developed gestational hypertension (high blood pressure in pregnancy). Her blood pressure is 145/95. If she wasn't pregnant, how would her high blood pressure be classified?

d Mandy doesn't feel very well. She is finding it hard to concentrate and she has a headache. When she took her blood pressure on her home monitor it was 195/108. What do you recommend Mandy do?

High blood pressure in pregnancy is very dangerous. Both the mother's and baby's lives can be in danger.

Practical activity What happens to our blood pressure during exercise?

You will need an automatic blood pressure measuring machine and access to a brisk walking area.

What you have to do

- Visit the Mayo Clinic website or YouTube and search for the video 'How to measure blood pressure using an automatic monitor'.
- Measure each group member's resting blood pressure.
- Go for a brisk group walk for 5 minutes then measure everyone's blood pressure again.
- Construct a scatter plot showing each person's resting blood pressure against their 'after a brisk walk' blood pressure.

Class discussion questions

1. What happens to blood pressure during exercise?
2. Does exercise change everybody's blood pressure in the same way?
3. Describe the resting blood pressure of people whose blood pressure changed the most during exercise.

Equations and formulas

10.7

An equation is like a see-saw, balanced about the equal sign. We can solve an equation if we do the same thing to *both* sides. **Solve** means to find the value of x that makes the equation true. That value is called the **solution** to the equation.

Example 13

Solve the equation $10x + 2 = 63$.

Solution

When we solve an equation, we want to finish with a statement like $x = 2$.

$$10x + 2 = 63$$

Step 1: Subtract 2 from both sides of the equation.

$$10x + 2 - 2 = 63 - 2$$
$$10x = 61$$

Step 2: Divide both sides by 10 to find x.

$$\frac{10x}{10} = \frac{61}{10}$$
$$x = 6.1$$

When the solution is $x = 6.1$ it means that when you substitute 6.1 for x in the original equation, both sides will have the same value.

Example 14

Solve the equation $\frac{5x}{3} = 4$.

Solution

Step 1: Multiply both sides by 3 to cancel out the fraction.

$$\frac{5x}{\cancel{3}} \times \cancel{3} = 4 \times 3$$
$$5x = 12$$

Step 2: Divide both sides by 5 to find x.

$$\frac{5x}{5} = \frac{12}{5}$$
$$x = 2.4$$

Example 15

After age 30, a person's height decreases by 0.06 cm per year. This can be modelled by the formula $h = T - 0.06a + 1.8$ where

h = height in cm

T = height in cm at age 30

a = age in years.

a Mohammed is 48 years old and was 175 cm tall when he was 30. How tall is Mohammed now?

b Grandma was 168 cm tall on her 100th birthday. How tall was she when she was 30 years old?

Solution

a Mohammed is 48: $a = 48$, his height at age 30 was 175 cm: $T = 175$

Substitute into the formula $h = T - 0.06a + 1.8$

$$h = 175 - 0.06 \times 48 + 1.8$$
$$= 173.92$$

Write your answer.

Mohammed is 173.92 cm tall now.

b Grandma is 100, $a = 100$ and her height now is $h = 168$.

Substitute into the formula $h = T - 0.06a + 1.8$ and simplify.

$$168 = T - 0.06 \times 100 + 1.8$$
$$= T - 6 + 1.8$$
$$= T - 4.2$$

Solve the equation by adding 4.2 to both sides.

$$168 + 4.2 = T - 4.2 + 4.2$$
$$172.2 = T$$
$$T = 172.2$$

Write your answer.

Grandma was 172.2 cm tall when she was 30 years old.



Alamy/Frances M. Roberts

EXERCISE 10.7

Equations and formulas

1 Solve each equation. EXAMPLE 13

a $10a + 4 = 42$ **b** $3x + 12 = 27$ **c** $2p + 18 = 48$

d $2d - \frac{1}{2} = 7\frac{1}{2}$ **e** $3m - 5 = 22$ **f** $5y - 1.6 = 8.4$

2 Solve each equation. EXAMPLE 14

a $\frac{2x}{5} = 4$ **b** $\frac{4p}{3} = 20$ **c** $\frac{10k}{7} = 5$

d $8g + 7 = 27$ **e** $2p + 1\frac{1}{2} = 11$ **f** $\frac{4p}{3} = 10$

g $4t - 3 = 27$ **h** $\frac{2y}{7} = 5$ **i** $\frac{2x}{5} = 3\frac{1}{2}$

j $\frac{6p}{11} = 2$ **k** $\frac{10x}{3} = \frac{2}{5}$ **l** $2b - \frac{7}{10} = \frac{3}{5}$

m $\frac{8p}{3} = \frac{5}{4}$ **n** $6 - 2x = 18$ **o** $-\frac{3d}{2} = 7$

You can check your solutions using the equation solver at the WolframAlpha website.

3 The femur bone is found in the top half of a human leg. The formula that relates an adult female's height, h cm, to the length of her femur bone, F cm, is $h = 61.41 + 2.32F$. Dagmar is 174 cm tall. How long is her femur bone correct to one decimal place? EXAMPLE 15

4 The cost, C dollars, of a taxi trip over a distance of d kilometres is given by $C = 2 + 1.2d$. What distance did James travel when his taxi trip cost him $32?

5 The perimeter P of a rectangle is given by the formula $P = 2L + 2W$, where L is the length and W is the width of the rectangle. The perimeter of a rectangle is 71 cm and its width is 17 cm. Calculate the length of the rectangle.

6 Fried's formula for calculating an infant's dose of an adult's medicine is $D = \frac{MA}{150}$ where

D = the infant dose in mL

A = the adult dose in mL

M = the infant's age in months.

16-month old Isabella had a bad cough. During the night her mother gave her a 3.2 mL dose of an adult cough medicine to help ease her cough. How much of the medicine can an adult take?

7 The number of times a cricket chirps in a minute is related to the temperature. The formula $T = \frac{n}{8} + 5$ tells us how the number of cricket chirps, n, and the temperature, T in degrees C, are related.

a On a warm day a cricket chirped 200 times per minute. What was the temperature?

b How many chirps per minute will a cricket make when the temperature is 20°C?

8 In indoor cricket, a batter's score can be calculated using the formula $S = R - 5n$, where S is the batter's score, R is the number of runs made and n is the number of times the batter was given out.

a In one match Rose made 60 runs and her score was 45. How many times was she given out?

b David batted well to score 31. He was given out twice. How many runs did he make?

9 The speed a car travels during a driver's reaction time is given by the formula $D = \frac{5vt}{18}$ where

D = the distance in metres

v = the speed of the car in km/h

t = the driver's reaction time in seconds.

When Pascal was travelling at 90 km/h, his car travelled 45 m during his reaction time. What is Pascal's reaction time?



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Staying under 0.05

10.8



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Alcohol affects people's reaction times and their ability to make good decisions. Every country has regulations prohibiting people from driving when they are affected by alcohol. Most countries use **blood alcohol content (or concentration) (BAC)** to decide whether a person can drive. A BAC of 0.05 means that there is 0.05 g of alcohol in 100 mL of blood. In Australia, it is illegal to drive with a BAC of 0.05 or over.

BAC is affected by the amount of alcohol consumed, the weight of a person, their fitness, sex, liver function and age. There are formulas for estimating BAC.

$$\text{BAC}_{\text{male}} = \frac{10N - 7.5H}{6.8M} \text{ and } \text{BAC}_{\text{female}} = \frac{10N - 7.5H}{5.5M}$$

where N is the number of standard drinks consumed, H is the number of hours of drinking, and M is the person's mass in kilograms.

Example 16

Jane weighs 60 kg. In two hours on Saturday night she drank 5 standard drinks of wine. Calculate Jane's BAC correct to two decimal places.

Solution

$\text{BAC}_{\text{female}} = \dfrac{10N - 7.5H}{5.5M}$.

In this situation $N = 5$, $H = 2$ and $M = 60$.

$$\text{BAC} = \frac{10 \times 5 - 7.5 \times 2}{5.5 \times 60}$$
$$= \frac{35}{330}$$
$$= 0.10606\ldots$$
$$\approx 0.11$$

Write your answer.

Jane's BAC is 0.11

Jane is over the 0.05 limit for driving

Example 17

Dylan's mass is 65 kg. How many standard drinks can he have at most in 3 hours and keep his BAC under 0.05?

Solution

$\text{BAC}_{\text{male}} = \dfrac{10N - 7.5H}{6.8M}$, $H = 3$, $M = 65$ and BAC < 0.05 and we need to find the value of N.

$$\frac{10N - 7.5 \times 3}{6.8 \times 65} < 0.05$$

$$\frac{10N - 22.5}{442} < 0.05$$

< means 'less than'

Multiply both sides by 442.

$$\frac{10N - 22.5}{442} \times 442 < 0.05 \times 442$$

$$10N - 22.5 < 22.1$$

Add 22.5 to both sides.

$$10N - 22.5 + 22.5 < 22.1 + 22.5$$

$$10N < 44.6$$

Divide both sides by 10.

$$\frac{10N}{10} < \frac{44.6}{10}$$

$$N < 4.46$$

N needs to be a whole number because it is the number of drinks.

$N = 4$.

Write your answer.

Dylan can have at most 4 standard drinks in 3 hours.

Example 18

The table below shows the BAC for males at different masses and number of standard drinks.

	Number of standard drinks									
Mass	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
45 kg	.038	.075	.113	.150	.188	.225	.263	.300	.338	.375
50 kg	.036	.071	.107	.142	.178	.214	.249	.285	.320	.356
55 kg	.039	.039	.101	.135	.169	.203	.237	.270	.304	.338
60 kg	.033	.066	.099	.132	.165	.198	.231	.264	.297	.330
65 kg	.031	.062	.093	.124	.155	.186	.217	.248	.279	.310
70 kg	.028	.056	.085	.113	.141	.169	.197	.226	.254	.282
75 kg	.026	.052	.079	.105	.132	.158	.184	.210	.237	.263
80 kg	.025	.049	.074	.098	.123	.147	.172	**.196**	.221	.245
85 kg	.023	.045	.068	.090	.113	.136	.158	.181	.203	.226
90 kg	.021	.041	.062	.083	.104	.124	.145	.166	.186	.207
95 kg	.019	.038	.057	.076	.095	.113	.132	.151	.170	.189
100 kg	.017	.034	.051	.068	.085	.102	.119	.136	.153	.170

Phillip's mass is 79 kg and he consumed 8 standard drinks in 5 hours. Estimate Phillip's BAC after the 5 hours if his BAC decreases by 0.015 per hour of drinking.

Solution

Phillip's mass of 79 kg is not in the table.
Use 80 kg, the closest mass to 79 kg.

A mass of 80 kg and 8 standard drinks gives 0.196.

Phillip's BAC would be 0.196 if he consumed all the drinks at the same time.

Subtract 0.015 for each hour of drinking.

$$\text{BAC} = 0.196 - 0.015 \times 5$$
$$= 0.121$$

Write your answer.

Phillip's BAC was 0.121 when he finished drinking.

Our kidneys remove alcohol from our blood at a constant rate, so BAC should decrease by the same amount every hour.

EXERCISE 10.8

Staying under 0.05

EXAMPLE 16

1 Find correct to 3 decimal places the value of $BAC_{female} = \dfrac{10N - 7.5H}{5.5M}$ with values $N = 4$, $H = 3$ and $M = 55$.

2 Billy's mass is 84 kg. During a 3-hour period he consumed 6 standard drinks. Calculate his BAC correct to 3 decimal places.

3 Holly drank too much at a party. She consumed 8 standard drinks in 2 hours. Her mass is 58 kg. Calculate Holly's BAC correct to 3 decimal places.

4 John and Celine both have a mass of 75 kg. They both consumed 6 standard drinks in 4 hours. How much more was Celine's BAC than John's? Answer to 4 decimal places.

EXAMPLE 17

5 Olivia wants to keep her BAC under 0.05. Her mass is 62 kg. How many standard drinks can she have in 4 hours?

6 Calculate the number of standard drinks each person can have and keep their BAC under 0.05.

	Person	Sex	Mass	Number of hours drinking
a	Zara	female	52 kg	3
b	Jake	male	95 kg	4
c	Jackson	male	78 kg	2
d	Sophia	female	65 kg	5

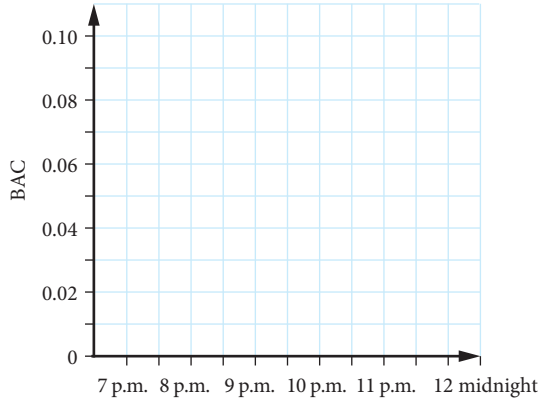
7 Alex went to the club after work on Friday. Before he left the club at 7 p.m. he used the club's BAC machine and found that his BAC was 0.09. Alex's BAC decreases by 0.012 every hour.

a Copy and complete the missing values in the table.

Time	7 p.m.	8 p.m.	9 p.m.	10 p.m.	11 p.m.	12 midnight
Alex's BAC	0.09	0.078				

b Copy the grid and draw a graph of Alex's BAC from 7 p.m. to midnight.

c Alex won't drive unless his BAC is lower than 0.02. Use the graph to determine when he can drive.



8 The formula $N = \frac{BAC}{0.015}$ gives the number of hours a P-plate driver must wait before driving. Swaetha is a P-plate driver and her BAC is 0.03. How long must she wait before she drives a car?

9 Joungwoo is a P-plate driver. At 9 p.m. his BAC was 0.05. What is the earliest time he can drive a car legally?

10 Karin is a P-plate driver who weighs 62 kg. Between 4 p.m. and 7 p.m. she consumed 6 standard drinks. Calculate her BAC correct to two decimal places. Will Karin be able to drive at 11.30 p.m? Justify your answer.

11 The size of a standard drink varies. Different types and brands of drinks contain different amounts of alcohol. By law, the number of standard drinks a bottle contains must be displayed on the label. Determine the number of millilitres that constitute a standard drink for each of the following bottles of alcohol. The first row has been completed as an example.

Type of drink	Size of bottle	Number of standard drinks	Size of a standard drink in mL
Red wine	750 mL	7.7	$750 \div 7.7 = 97$ mL
Dessert wine	375 mL	2.7	
White wine	750 mL	7.1	
Scotch whisky	1 L	32	
Port	750 mL	11	
Beer	330 mL	1.2	

12 Ben had a two-hour dinner in a restaurant with friends. He enjoyed four 150 mL glasses of wine from a 750 mL bottle of wine with his meal. The bottle contained 7.5 standard drinks. Ben's mass is 85 kg and he is a P-plate driver.

a Calculate the number of standard drinks Ben had.

b Use the formula $\text{BAC}_{\text{male}} = \frac{10N - 7.5H}{6.8M}$ to calculate his BAC correct to 3 decimal places.

c How many hours and minutes would Ben have to wait before he could drive home?

13 Blake's mass is 71 kg. During 4 hours he consumed 5 standard drinks. Use the table on page 253 to calculate Blake's BAC.

EXAMPLE 18

14 Use the table on page 253 to calculate each BAC.

	Body mass	Number of standard drinks	Time spent drinking
a	51 kg	5	4 hours
b	62 kg	7	2 h 30 min
c	87 kg	3	90 min

15 In each situation, state whether the BAC will increase, decrease or stay the same.

a Jake had the same number of drinks over a longer time period.

b Sally had more drinks in the same time.

c A person with the same body mass had more standard drinks in the same time.

d Helena had the same number of drinks in the time but they were bigger than standard drinks.

e A person with a smaller body mass had the same number of standard drinks in the same time.

Investigation Comparing BAC estimation techniques

An alternative formula to estimate BAC uses the size of drinks and the percentage alcohol in the drinks. Males and females have different percentages of fluid in their bodies. The average male body is 58% fluid and the average female body is 49% fluid.

1 A 1-litre bottle of red wine contains 14% alcohol. Calculate the amount of alcohol contained in three 160 mL-glasses of the wine.

2 Sienna's mass is 70 kg. Determine the amount of fluid in her body in kg.

3 Sienna consumed three glasses of the red wine in 2 hours. Use the formula $\text{BAC} = \dfrac{0.0637 \times \text{amount of alcohol consumed in mL}}{\text{weight of fluid in the body in kg}}$ to determine Sienna's BAC.

4 Dion's mass is 70 kg. He also consumed three 160 mL glasses of the wine in 2 hours. Use the formula to determine his BAC.

5 Dion and Sienna have the same mass and they drank the same amount of wine. Explain why David's BAC is lower than Sienna's BAC.

6 Determine Sienna's BAC with the formula $\text{BAC}_{\text{female}} = \dfrac{10N - 7.5H}{5.5M}$ if she treated the three glasses of wine as three standard drinks.

7 How much larger was the value of Sienna's BAC when you used the formula involving the amount of alcohol consumed than the formula using standard drinks?

8 Write a sentence explaining how you can get a value much lower than the real BAC when you count drinks rather than measured, standard drinks.

Investigation BAC in other countries

In Victoria, the legal BAC limit for full licence drivers is 0.05. Is this limit the same in other Australian states and in other countries?

1 Visit the website of the International Centre for Alcohol Policies.

2 Search for legal BAC limit in 12 different countries around the world, such as the USA, UK, Sweden, NZ and Singapore.

3 Which country has the highest legal BAC limit?

4 What countries have a zero legal BAC limit?

Keyword activity

Match the terms in the left column with their meaning on the right.

	Term		Meaning
1	calories	a	A measure of the concentration of alcohol in a person's blood.
2	diastolic pressure	b	A unit for measuring electricity.
3	kilojoule	c	The rate at which our heart beats, usually expressed in beats/minute.
4	kilowatt	d	An old unit for measuring the energy in food.
5	substitute	e	The pressure in our arteries when the heart is beating.
6	systolic pressure	f	The modern unit for measuring energy in food.
7	pulse	g	To replace a pronumeral with a value in an algebraic expression or formula.
8	BAC	h	The pressure in our arteries when the heart is relaxed.

Chapter Problem

Brett is usually a very alert, safe driver. His reaction time is 0.16 seconds. However, when his blood alcohol content (BAC) is 0.05, his reaction time increases to 0.22 seconds.

Brett is driving at 100 km/h when he suddenly needs to brake. How much further does his car travel during his reaction time when his BAC is 0.05, compared to when he hasn't been drinking?

Solution

We can use the formula $D = \frac{5vt}{18}$ (from page 250, Question 9) to calculate the distance a car travels during the driver's reaction time at different speeds and reaction times.

D = distance in metres

v = the car's speed in km/h

t = the driver's reaction time in seconds.

Brett is driving his car at a speed of 100 km/h.

When $t = 0.16$ Reaction distance $= \frac{5 \times 100 \times 0.16}{18}$

$= 4.44$ m correct to the nearest cm

When $t = 0.22$ Reaction distance $= \frac{5 \times 100 \times 0.22}{18}$

$= 6.11$ m correct to the nearest cm.

The car travels an additional $6.11 - 4.44 = 1.67$ m during Brett's reaction time when his BAC is 0.05.



Chapter Problem

Francesca is making gift boxes to sell. She plans to make three types of boxes:

Cube: 15 cm long

Cylinder: radius 4.5 cm and height 34 cm

Square pyramid: base 15 cm long, slant height 16 cm

Should she charge the same for each gift box?
Give reasons for your answer.



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CHAPTER 11

ON THE SURFACE

11.1 Measuring area
11.2 Area formulas
11.3 Composite areas
11.4 Surface areas of prisms
11.5 Surface areas of cylinders, spheres and cones
11.6 Surface areas of pyramids
Keyword activity
Solution to the chapter problem

By the end of this chapter, you should know how to:

- use metric units of area
- use formulas to calculate areas of shapes, including composite shapes
- find the surface area of prisms, pyramids, cylinders, cones and spheres

How are we ever going to use this?

- When we are doing craft and need to work out what materials are required
- If we are covering cushions or making quilts
- Many trades require knowledge of area and surface area

11.1 Measuring area

The area of a shape is the amount of surface enclosed by the shape. It is measured in square units, which are based on the length units.

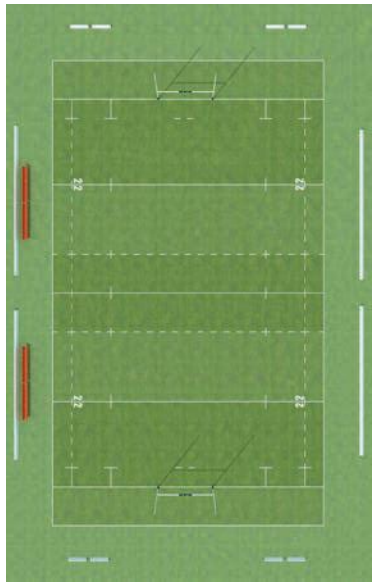
Area unit	The size of a square of length:	Approximately the size of:
square millimetre (mm^2)	1 mm (Actual size: ▪)	
square centimetre (cm^2)	1 cm (Actual size: ■)	a fingernail
square metre (m^2)	1 m	the floor of a large shower recess
hectare (ha)	100 m	the area bounded by an athletics track, or an international rugby pitch



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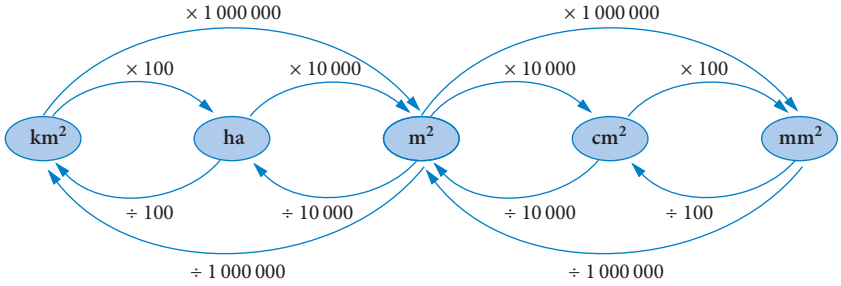


Alamy/Archideaphoto

1 cm = 10 mm $1\ cm^2 = 10 \times 10\ mm^2$ $= 100\ mm^2$	10 mm 1 cm^2 10 mm	1 m = 1000 mm $1\ m^2 = 1000 \times 1000\ mm^2$ $= 1\ 000\ 000\ mm^2$	1000 mm 1 m^2 1000 mm
1 m = 100 cm $1\ m^2 = 100 \times 100\ cm^2$ $= 10\ 000\ cm^2$	100 cm 1 m^2 100 cm	1 ha = $100 \times 100\ m^2$ $= 10\ 000\ m^2$	1 ha 100 m 100 m

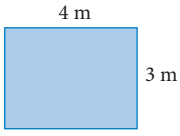
$1\ cm^2 = 100\ mm^2$
$1\ m^2 = 10\ 000\ cm^2 = 1\ 000\ 000\ mm^2$
$1\ ha = 10\ 000\ m^2$

This diagram shows how to convert between units of area.



Example 1

What is the area of this rectangle?



Because the length and width are in metres, the area is in square metres (m^2).

Solution

The area of a rectangle is $A = lw$ where l is the length and w is the width. For this rectangle, $l = 4$ and $w = 3$.

$A = 4 \times 3$
$= 12 \text{ m}^2$

Example 2

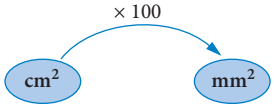
Convert:

a 3 cm^2 to mm^2 **b** 4000 mm^2 to m^2 **c** 81 000 m^2 to ha

Solution

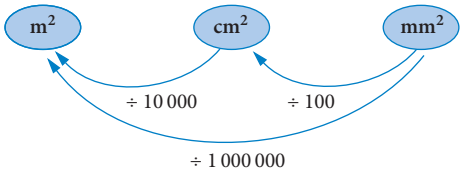
a cm^2 to mm^2, large to small unit: × 100.

$3 \text{ cm}^2 = 3 \times 100 \text{ mm}^2$
$= 300 \text{ mm}^2$



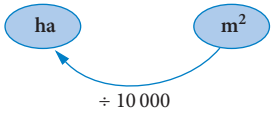
b mm^2 to m^2, small to large unit: ÷ 1 000 000.

$4000 \text{ mm}^2 = 4000 \div 1\,000\,000 \text{ m}^2$
$= 0.004 \text{ m}^2$



c m^2 to ha, small to large unit: ÷ 10 000.

$81\,000 \text{ m}^2 = 81\,000 \div 10\,000 \text{ ha}$
$= 8.1 \text{ ha}$



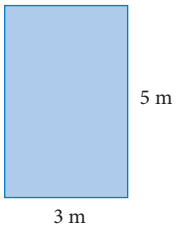
EXERCISE 11.1

Measuring area

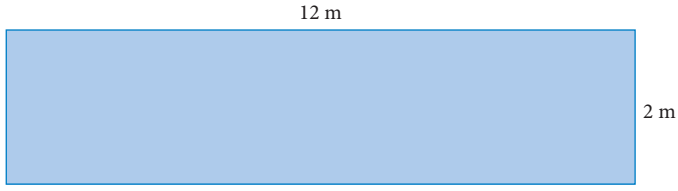
EXAMPLE 1

1 Find the area of each rectangle.

a



b



2 Select the best unit (km^2, ha, m^2, cm^2 or mm^2) for measuring each area.

a	a farm	**b**	the floor of a classroom	**c**	a shirt
d	the school oval	**e**	a sheet of paper	**f**	your eardrum
g	Australia	**h**	a flattened insect	**i**	Mornington Peninsula

EXAMPLE 2

3 Copy and complete each conversion.

a 7.9 cm^2 = _____ mm^2

b 1 500 000 mm^2 = _____ m^2

c 690 mm^2 = _____ cm^2

d 76 000 000 m^2 = _____ km^2

e 865 000 cm^2 = _____ m^2

f 12 ha = _____ m^2

g 0.32 km^2 = _____ m^2

h 4.5 m^2 = _____ cm^2

i 0.75 m^2 = _____ mm^2

j 19 000 m^2 = _____ ha

4 Arrange these measurements in ascending order: 6.5 m^2, 25 050 000 mm^2 and 114 000 cm^2.

To compare sizes, the measurements need to be in the same units.

5 Arrange these amounts in descending order: 990 mm^2, 54 cm^2 and 0.000 032 m^2.

6 The centre of Adelaide is approximately a square bounded by North, South, East and West Terraces. Its area is about 3 521000 m^2. Express the area of this square in:

a hectares

b square kilometres

7 14% of a 4.8 hectare war memorial garden is covered with rose gardens. Calculate the area of the rose gardens in square metres.

9780170370752

Investigation Estimating area

It takes practice to become good at estimating. Complete these practical estimation activities to develop your estimation skills.

To complete these activities, each group will need:

- 4 metre rulers or measuring tapes
- Paper, pencils and scissors

A copy of this table:

Estimating area

Names	Group members' estimates				Real measurement
Foot prints in a square metre					
Hand prints in a square metre					
Area of a car number plate in square centimetres					

Activity 1: Footprints in a square metre



Imagine you are going to cover a square metre with your footprints. How many footprints will fit inside 1 m^2 without any overlaps? Record your group's estimates in your table.

How good was your estimate?

- Make a template of your foot. Put a piece of paper under your foot and trace around the outside. Use scissors to cut out the template. Make several templates.
- Place 4 metre rulers on the floor to outline 1 m^2.
- Systematically place your foot template in the 1 m^2 and count the number required to cover the square.

Activity 2: Handprints in a square metre

Now imagine that you are going to cover a square metre with your handprints without any overlaps. How many handprints will you need?

Use a similar method to that for checking footprints in a square metre to check the accuracy of your group's handprint estimates.

Activity 3: The area of a car number plate

Estimate the area covered by a standard car number plate.

To check your estimate, measure the length and the height of an appropriate number plate, then use the formula Area = $l \times w$ to calculate the area.



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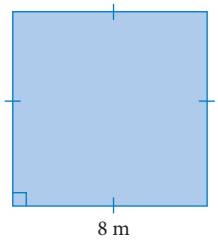
11.2 Area formulas

Shape	Area formula	Meaning of pronumerals
Rectangle	$A = lw$	l = length, w = width
Triangle	$A = \frac{1}{2}bh$	b = base length, h = height
Parallelogram	$A = bh$	b = base length, h = height
Trapezium	$A = \frac{1}{2}h(a + b)$	a, b = lengths of parallel sides, h = height
Circle	$A = \pi r^2$	r = radius
Sector of a circle	$A = \frac{\text{angle in sector}}{360°} \times \pi r^2$	r = radius

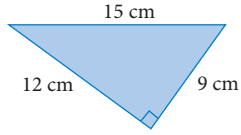
Example 3

Find the area of each shape.

a



b



Solution

a A square is a special rectangle. Use the formula for the area of a rectangle: $A = l \times w$.

$$\text{Area} = 8 \times 8 = 64 \text{ m}^2$$

b For a triangle, $A = \frac{1}{2}bh$. The base, b, and the height, h, are always at right angles to each other. In this triangle $b = 12$ and $h = 9$.

The 9 cm and the 12 cm are at right angles.

$$\text{Area} = \frac{1}{2} \times 9 \times 12 = 54 \text{ cm}^2$$

We don't need to use the hypotenuse of 15 cm.

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EXERCISE 11.2

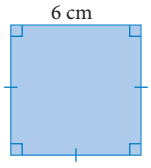
Area formulas

EXAMPLE 3

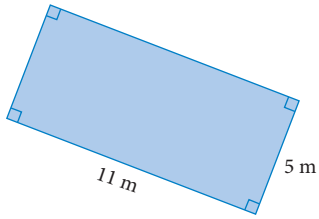
11.2

1 Use the appropriate formula to calculate the area of each quadrilateral.

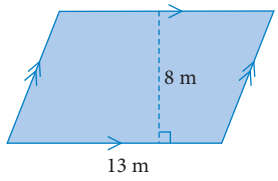
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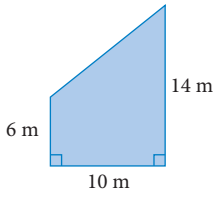
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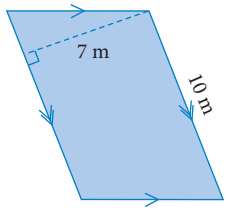
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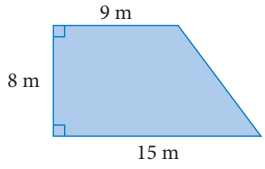
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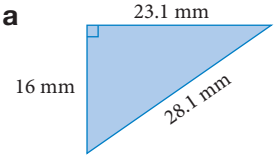
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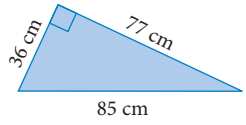
EXAMPLE 3

2 Calculate the area of each triangle.

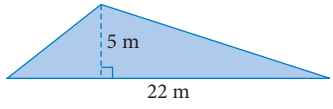
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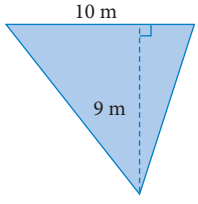
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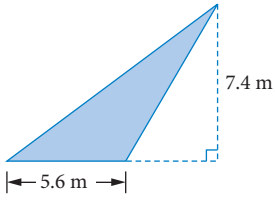
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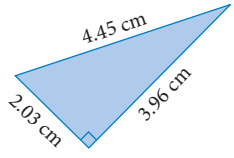
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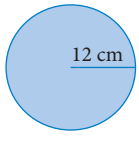


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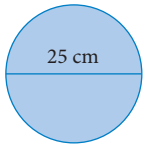


3 Find the area of each circle or sector, correct to two decimal places.

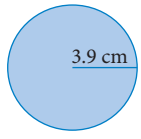
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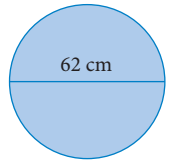
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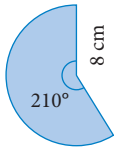
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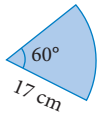
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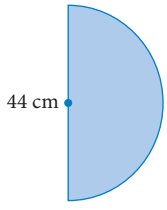
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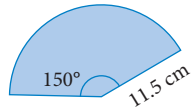
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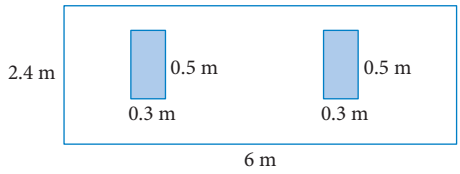
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4 Mark is a bricklayer. He uses 65 bricks to build 1 m^2 of wall.

a How many square metres of wall can Mark build with 715 bricks?

b Mark is going to build this brick fence leaving two holes as requested by the customer. How many square metres of brickwork are in the fence?



c How many bricks will Mark need to build the fence?

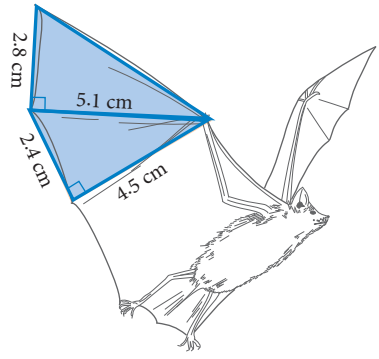
5 Paula is a wheat and sheep farmer. Each of her paddocks is 450 m long by 500 m wide.

a Calculate the area of each paddock in square metres.

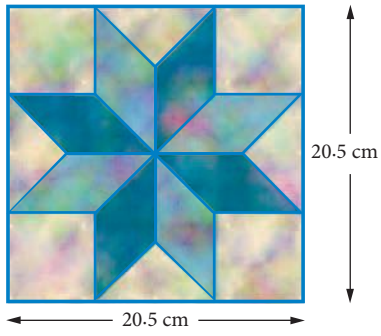
b What is the size of each paddock in **hectares**?

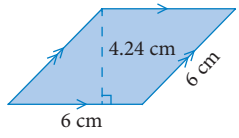
c Paula can have nine sheep per hectare in her paddocks. How many sheep can she have per paddock?

6 Bats are the most common mammal on Earth. The end sections of their wings are triangular. Calculate the area of each triangular section of the bat's wings.



7 Isabella is using parallelograms to make a star pattern on a quilt.



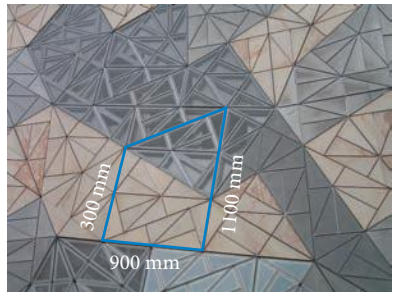


a Determine the area covered by each parallelogram.

b What is the area covered by one star?

c How much of the area of the square is **not** covered by the star?

8 Find the area of the trapezoidal shape in the photo of the exterior of Federation Square, Melbourne.



9 A large pizza is 30 cm in diameter.

a What is the area of the pizza, correct to 2 decimal places?

b The pizza is cut into 8 equal pieces. What is the area of each piece?

c Corey eats 3 slices and Anya eats 5 slices. How many more square centimetres of pizza does Anya eat?



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10 Last century, army barracks were built with a semicircular end with diameter 7 m. Find the area of tin used for one semicircular end of the building, correct to 2 decimal places.



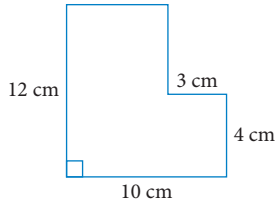
Alamy/Detail Heritage

11.3 Composite areas

Composite shapes are made up of smaller simpler shapes. We can find the areas of composite shapes by adding the areas of simpler shapes together or by subtracting them.

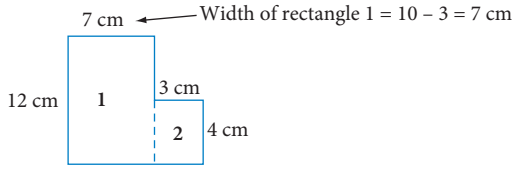
Example 4

Find the area of this shape.



Solution

Divide the shape into 2 rectangles as shown



Calculate the area of each rectangle and add them together.

$$\text{Area of rectangle 1} = 7 \times 12 = 84 \text{ cm}^2$$

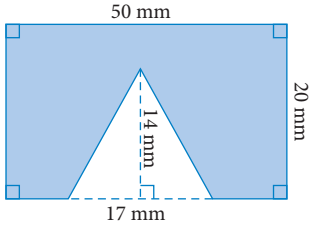
$$\text{Area of rectangle 2} = 3 \times 4 = 12 \text{ cm}^2$$

$$\text{Total area} = 84 + 12 = 96 \text{ cm}^2$$

There is more than one way to divide this shape. All methods result in the same answer.

Example 5

Find the area of this shape.



Sometimes it is easier to find the area by subtraction.

Solution

The easiest way to find the area is to find the area of the rectangle and subtract the area of the triangle.

$$\text{Area} = \text{area of rectangle} - \text{area of triangle}$$

$$= 50 \times 20 - \frac{1}{2} \times 17 \times 14$$

$$= 1000 - 119$$

$$= 881 \text{ mm}^2$$

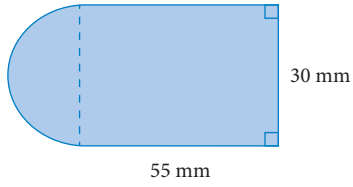
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Example 6

Find the area of this shape correct to two decimal places.



Solution

This shape is a rectangle and a semicircle.
Find the area of the rectangle first.

$$\text{Area of rectangle} = 55 \times 30 = 1650 \text{ mm}^2$$

Area of the semi-circle is $A = \frac{1}{2} \times \pi r^2$

Radius $r = \frac{1}{2} \times 30 = 15$ mm

$$\text{Area of semi-circle} = \frac{1}{2} \times \pi \times 15^2 = 353.4291\ldots$$

Add both areas together.

$$\text{Area} = 1650 + 353.4291\ldots = 2003.4291\ldots \text{ mm}^2 \approx 2003.43 \text{ mm}^2$$

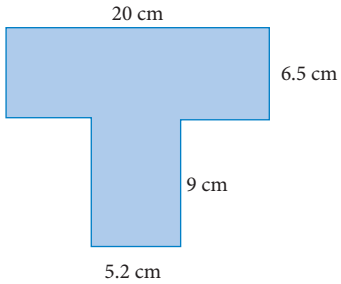
EXERCISE 11.3

Composite areas

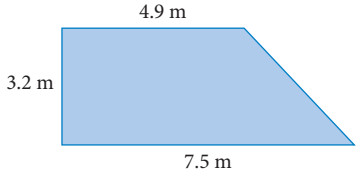
1 Find the area of each shape, correct to two decimal places where necessary.

EXAMPLE 4

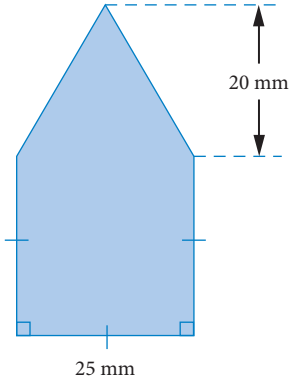
a



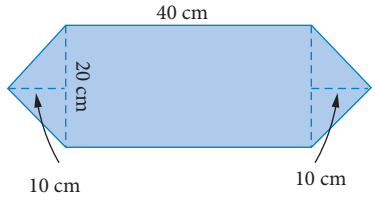
b



c



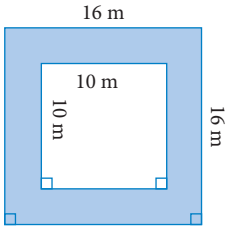
d



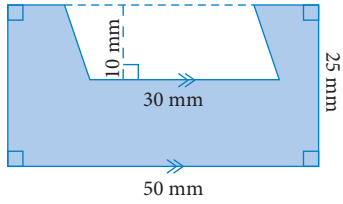
EXAMPLE 5

2 Calculate the area of each shaded region.

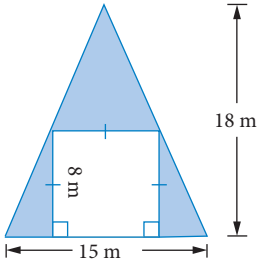
a



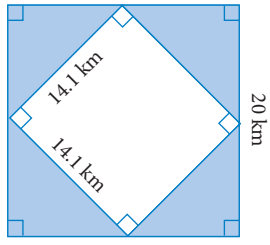
b



c



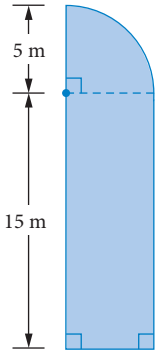
d



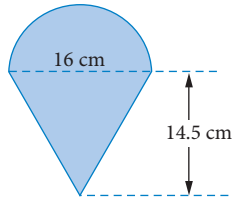
EXAMPLE 6

3 Calculate the area of each shaded region, correct to 2 decimal places.

a



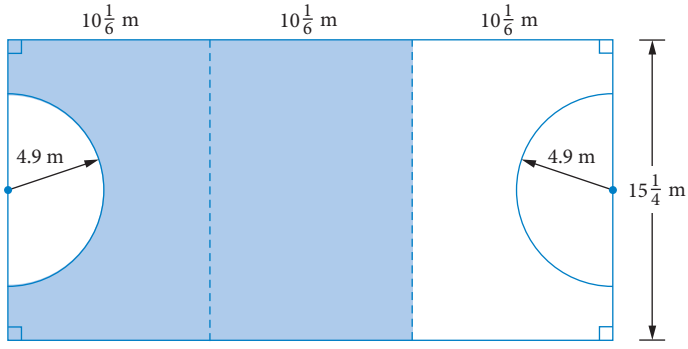
b



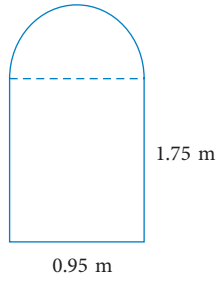
4 The diagram shows a netball court. The shaded area represents the part of the court where the 'wing attack' can play.

a Calculate the area of one of the goal semicircles. Answer correct to two decimal places.

b Calculate correct to one decimal place the size of the area where the wing attack can play.



5 Susan and Ian are putting three arch windows in the front room of their house. The diagram shows one of the windows with its dimensions.

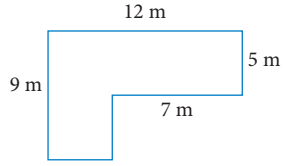


a Find the area of one arch window, correct to two decimal places.

b Find the area of all three arch windows.

c 10% of the area is NOT glass. Find the area of glass used.

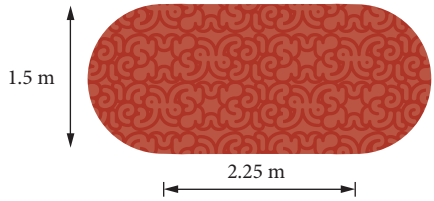
6 A children's playground has an L-shape as shown.



a What is the area of the playground?

b The playground is to be covered with woodchips for safety reasons. How much will the woodchips cost if they are sold for $7.20 per square metre?

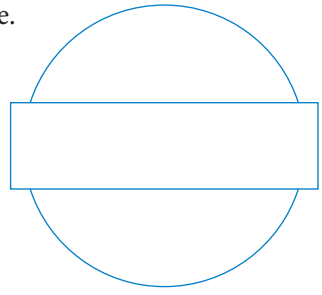
7 Angelina buys a new rug for her living room. The rug is composed of one rectangle and two semi-circles as shown.



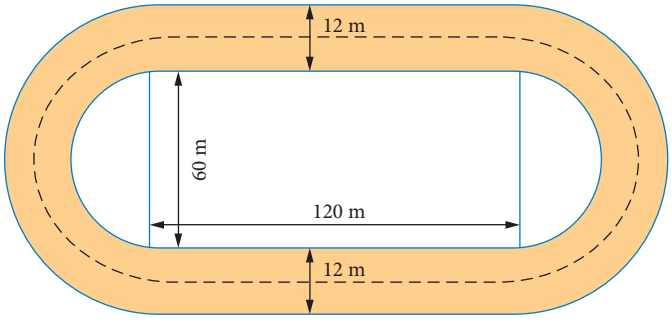
a Calculate the area of the rug, correct to 2 decimal places.

b Angelina's living room floor is a rectangle 3 m by 2.5 m. What area of the floor will NOT be covered by the rug?

8 The badge shown is made up of a metal disc overlaid with a rectangle. The metal disc has a radius of 3.5 cm. The rectangle is 1 cm longer than the diameter of the circle and is 1.5 cm wide. Calculate the total area of the disc and the rectangle correct to 2 decimal places.

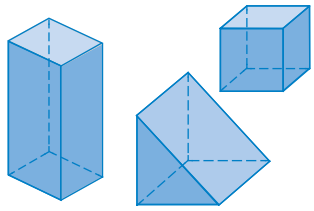


9 Middleton City has a multi-purpose football field surrounded by an athletics track as shown. Calculate the area of the athletics track correct to 2 decimal places.



11.4 Surface areas of prisms

When we want to make packaging we need to know the area of the materials we plan to use. This is called **surface area** because it is the sum of the areas of all the surfaces of the object. The easiest way to calculate the surface area of an object is to start with its net. We learned about the nets of solids in Chapter 7. We calculate the area of each shape individually and then add them together to find the total surface area.

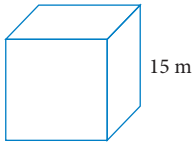


A **prism** is a solid shape that has identical ends. Some examples are shown above: a rectangular prism, triangular prism and cube. All side faces are rectangles.

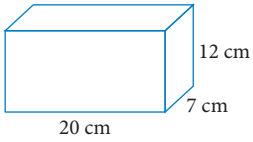
Example 7

Find the surface area of each prism.

a

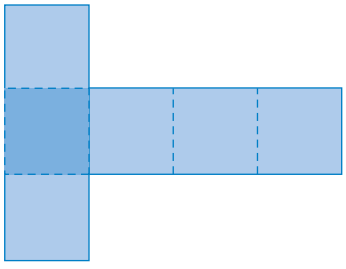


b



Solution

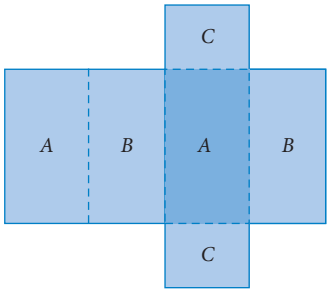
a A cube has 6 identical faces, all squares.
This is the net.



The surface area is 6 times the area of each square.

$$\text{Total surface area} = 6 \times (15 \times 15)$$
$$= 1350 \text{ m}^2$$

b This is the net of a rectangular prism. Notice that the rectangles are in matching pairs.

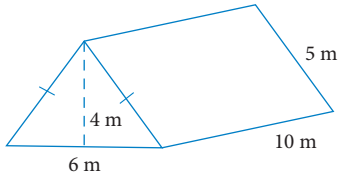


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The ends (C) are the same rectangles.	Area of ends $= 2 \times (7 \times 12)$ $= 168 \text{ cm}^2$
The top and bottom (A) are the same.	Area of top/bottom $= 2 \times (20 \times 7)$ $= 280 \text{ cm}^2$
The front and back (B) are the same.	Area of front/back $= 2 \times (20 \times 12)$ $= 480 \text{ cm}^2$
Add the areas together.	Total surface area $= 168 + 280 + 480$ $= 928 \text{ cm}^2$

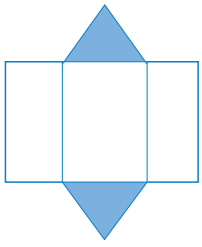
Example 8

Find the surface area of this triangular prism.



Solution

This is the net of a triangular prism. There are three rectangles and two triangles that are the same.



Find the area of the base rectangle.	Area $= 6 \times 10$ $= 60 \text{ m}^2$
Find the area of the two side rectangles.	Area $= 2 \times (5 \times 10)$ $= 100 \text{ m}^2$
Find the area of the two triangles.	Area $= 2 \times (\frac{1}{2} \times 6 \times 4)$ $= 24 \text{ m}^2$
Add the areas together.	Total surface area $= 60 + 100 + 24$ $= 184 \text{ m}^2$

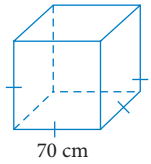
EXERCISE 11.4

Surface areas of prisms

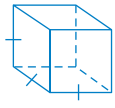
EXAMPLE 7

1 Find the surface area of each cube.

a



b



2 Charlotte has a plastic storage cube that is 35 cm long with no top. Calculate its external surface area.



Shutterstock.com/WathanyuSowong

3 This metal cube sculpture has a side length of 450 cm. The surface is to be covered with a weather-resistant finish. How many square metres need to be covered?

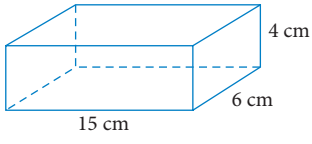


Alamy/Robert K. Chin

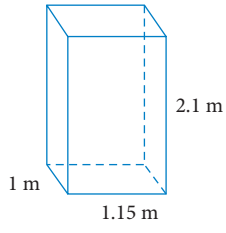
EXAMPLE 7

4 Find the surface area of each prism.

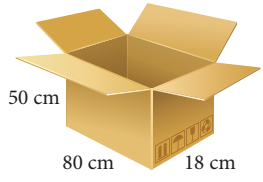
a



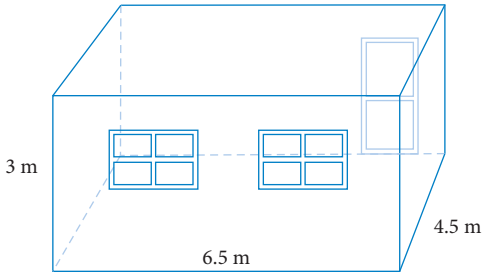
b



5 James' new television was packed in a box with measurements as shown in the diagram. Find the surface area of the box.



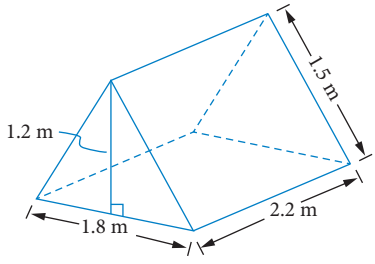
6 Alexa and Jai built a new living room that has dimensions as shown.



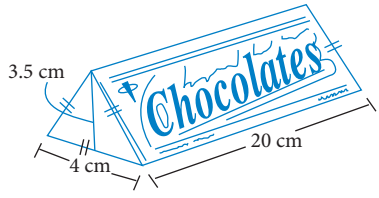
- **a** Calculate the area of the walls and ceiling to be painted. Subtract 6 m^2 for the windows and door.
- **b** If paint costs $23 for a 4 litre can that covers 10 m^2, calculate how much it would cost to paint the room with two coats.
- **c** Calculate the area of the floor to be covered with wood parquet.
- **d** If wood parquet costs $15.70 per square metre, how much would it cost to cover the floor? Answer to the nearest ten dollars.

7 Tom's company makes camping equipment, including the tent as shown. What amount of material is required to make the tent?

EXAMPLE 8

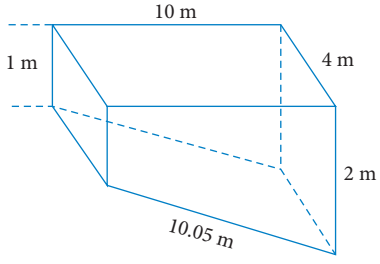


8 The diagram shows a chocolate box in the shape of a triangular prism. The ends of the box are equilateral triangles.



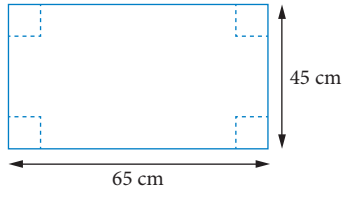
- **a** Calculate the surface area of the box.
- **b** The manufacturer allows an extra 10% of the surface area for tabs and wastage to determine how much cardboard one box requires. How much cardboard is required for one box?

9 Noel and Janet have a backyard pool shaped as shown. It needs to be repainted. There are tiles around the top edge to a depth of 40 cm. Calculate the area to be painted.



Remember to convert 40 cm to m.

10 A metal tray is made by taking a rectangular piece of metal and cutting squares from the corners. The edges are then bent upwards and welded at the corners to form the tray.



- **a** This piece of metal has dimensions 45 cm by 65 cm. Squares of side length 10 cm are cut out of each corner. What are the dimensions of the tray that can be formed?
- **b** The bottom of the tray is to be lined with material. What area of material is required?
- **c** The internal sides and all of the outside are to be enamelled. What area is to be enamelled?

Investigation SketchUp Part 2

In this Investigation you will use the buildings you designed in the Investigation in Chapter 3.

Surface area of one room

- In SketchUp use the dimensions tool to find the dimensions of one of the rooms in your building.
- Calculate the area of each of the four walls and the ceiling of your chosen room. Remember to subtract the area of any doors and windows in your room.
- Calculate how much paint you will need to paint the four walls and ceiling with two coats of paint.
- Use the Internet to find the cost of paint and how much one can of paint covers. Calculate the approximate cost of painting your chosen room with two coats of paint.

Surface area of the inside rooms (optional)

- Repeat the process you have used for one room to find the surface area of all the main rooms in your house.
- Hence find the approximate cost of painting the interior of your house.

The driveway

- In SketchUp use the dimensions tool to find the dimensions of the driveway.
- Calculate the area of the driveway.
- The average cost of building the driveway is $21 per square metre. Calculate the approximate cost of building your driveway.

The block of land

- Find the area of your block of land.
- Find the area of your floor plan.
- What percentage of your block is used by the house?

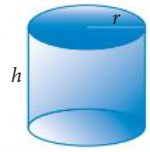
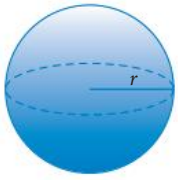
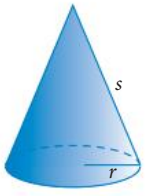


iStockphoto/acilo

Surface areas of cylinders, spheres and cones

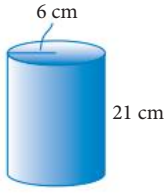
11.5

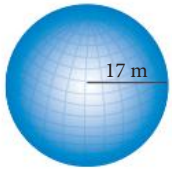
There are special formulas for the surface areas of solids with curved surfaces.

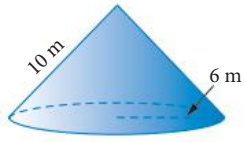
Shape	Surface area formula	Meaning of pronumerals
Cylinder 	SA = area of 2 circles + area of rectangle $SA = 2\pi r^2 + 2\pi rh$	r = radius, h = height
Sphere 	$SA = 4 \times \pi \times (\text{radius})^2$ $SA = 4\pi r^2$	r = radius
Cone 	SA = area of circle + area of curved surface $SA = \pi r^2 + \pi rs$	r = radius, s = slant height

Example 9

Find the surface area of each solid, correct to 2 decimal places.

a 

b 

c 

Solution

a Use the formula for the surface area of a cylinder.
$SA = 2\pi r^2 + 2\pi rh$
In this case, $r = 6$ and $h = 21$.

$SA = 2 \times \pi \times 6^2 + 2 \times \pi \times 6 \times 21$
$= 1017.8760\ldots$
$\approx 1017.88\ \text{cm}^2$

b Use the formula for the surface area of a sphere.
$SA = 4\pi r^2$, where $r = 17$.

$SA = 4 \times \pi \times 17^2$
$= 3631.6811\ldots$
$\approx 3631.68\ \text{m}^2$

c Use the formula for the surface area of a cone.
$SA = \pi r^2 + \pi rs$, where $r = 6$ and $s = 10$.

$SA = \pi \times 6^2 + \pi \times 6 \times 10$
$= 301.5928\ldots$
$\approx 301.59\ \text{m}^2$

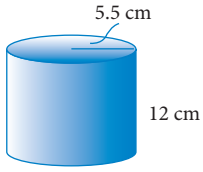
EXERCISE 11.5

Surface areas of cylinders, spheres and cones

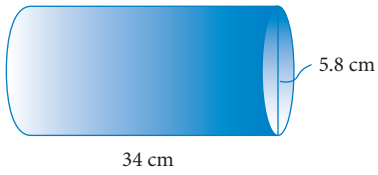
EXAMPLE 9

1 Find the surface area of each cylinder, correct to 2 decimal places.

a



b



2 A can of pineapple pieces has height 11 cm and radius 5 cm. The label is wrapped around the curved surface.

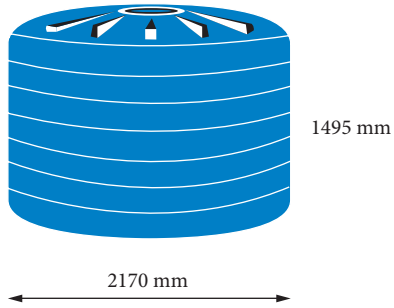
When the label is removed and flattened it is a rectangle.

Calculate the area of the label, correct to 2 decimal places.



Alamy/Mediablitzimages

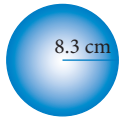
3 How much plastic would be needed to make this water tank? Answer correct to the nearest square metre.



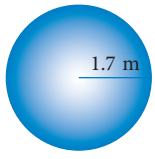
EXAMPLE 9

4 Find the surface area of each sphere, correct to 2 decimal places.

a



b



5 A tennis ball has a diameter of 7cm.

a Calculate the surface area of the tennis ball, correct to the nearest cm^2.

b If the ball is made of two identical pieces, what is the area of each piece?

6 The Earth is approximately a sphere with a radius of 6400 km.

a Calculate the surface area of the Earth, correct to the nearest 10 square kilometres.

b Approximately 70% of the Earth's surface is water. What area of the Earth's surface is covered with water?

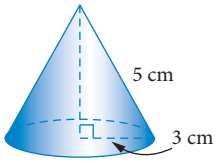
c What area of the Earth's surface is land?

EXAMPLE 9

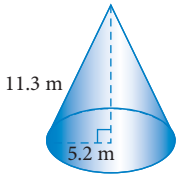
11.5

7 Calculate the surface area of each cone correct to 1 decimal place.

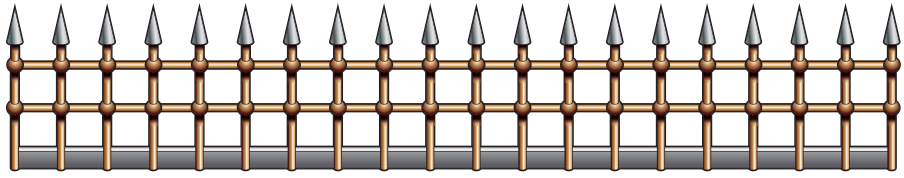
a



b

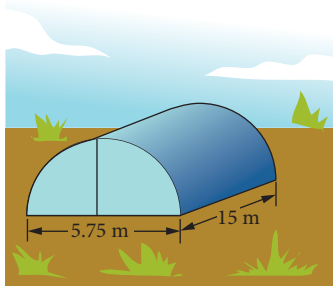


8 Dale is dipping cone-shaped spikes into a silver electroplating solution before he installs the spikes on top of a fence. The curved surface and the circular base of each cone will be electroplated. A total of 20 cone-shaped spikes are needed to complete the fence. The spikes have a slant height of 15 cm and a radius of 2 cm. Calculate, to the nearest square centimetre, the total area to be electroplated by Dale.



9 Natalie has a traffic cone with a radius of 12 cm and a slant height of 40 cm. She must dip the cone into orange paint to cover the sides but not the circular base. What area will Natalie cover with the paint? Give your answer to the nearest cm^2.

10 A hydroponics shed is made in the shape of a half-cylinder. Find the area of sheet metal needed for one shed, correct to the nearest square metre. Do not include the floor.



11 A mirror ball is 33 cm in diameter. It is covered with small mirror tiles which have an area of 1.44 cm^2 each.

a Find the surface area of the sphere, correct to 2 decimal places.

b How many tiles are used to cover the mirror ball?

12 Elise bought a cylinder for sending posters through the mail. It is a cardboard tube with plastic 'plugs' at each end. The tube is 1.1 m long and has a radius of 2.5 cm.

a Calculate the amount of cardboard used in creating the tube, correct to the nearest square centimetre.

b Calculate the total area of the plastic ends, correct to 2 decimal places.

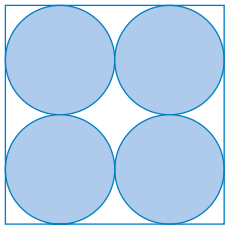


Shutterstock.com/Damon Allen DAVISON

Investigation Christmas bauble boxes

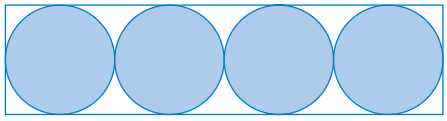
Resource Design has been asked to design a new cardboard box for a set of four spherical Christmas baubles with a diameter of 6.5cm. They have presented two designs:

An open square box



An open box has no top.

An open rectangular box



1. Calculate the area of cardboard required for the open square box. Allow for an extra centimetre on each dimension (length, width, height) to have space for interior packaging. Add 10% extra area for the overlapping sections to glue the box together.
2. Calculate the area of cardboard required for the open rectangular box. Allow an extra centimetre on each dimension for interior packaging space. Add 10% extra area for overlapping sections.
3. Is there another possible design to hold 4 Christmas decorations? If so, provide a sketch and calculate the area of cardboard required for it. Allow an extra centimetre on each dimension for interior packaging space. Add 10% extra area for overlapping sections.
4. Which design would you recommend? Justify your decision.

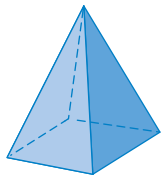


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Surface areas of pyramids

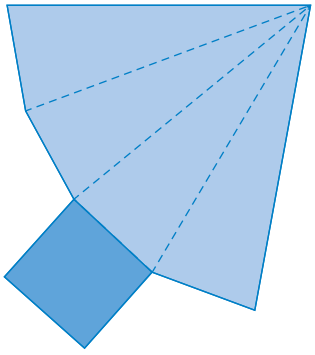
11.6

A **pyramid** has a triangle, quadrilateral or other polygon as its base, then all side faces are triangles that meet at a point called the **apex**. We name pyramids by the shape of their base. For example, this is a square pyramid.

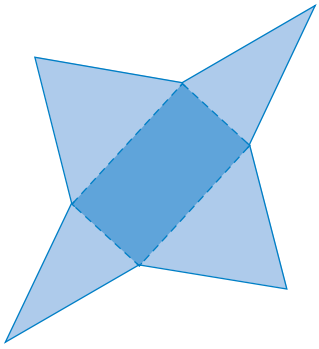


The nets of some pyramids are shown below: these will help us find their surface areas.

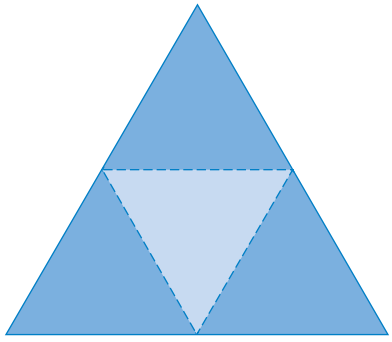
Square pyramid



Rectangular pyramid



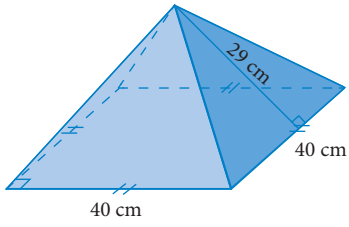
Triangular pyramid

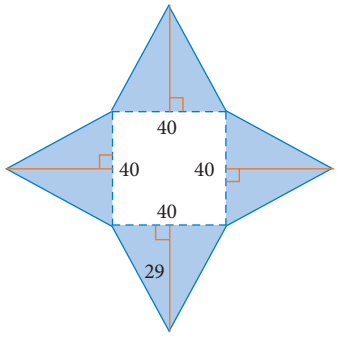


Surface area of a pyramid = area of base + area of all the triangles

Example 10

The diagrams show a square pyramid and its net. Calculate the surface area of the pyramid.





Solution

Calculate the area of the four triangles. They are all the same because the base is a square.

$$\text{Area} = 4 \times (\frac{1}{2} \times 40 \times 29)$$
$$= 2320 \text{ cm}^2$$

Calculate the area of the square base.

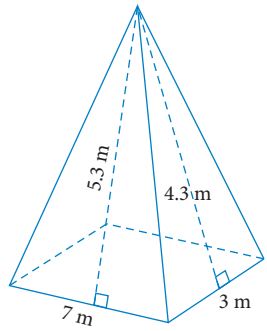
$$\text{Area} = 40 \times 40$$
$$= 1600 \text{ cm}^2$$

Add the areas together.

$$\text{Total surface area} = 2320 + 1600$$
$$= 3920 \text{ cm}^2$$

Example 11

Find the surface area of this rectangular pyramid.



Solution

Opposite triangles are identical.
Find the area of the left/right triangles.

$$\text{Area} = 2 \times (\frac{1}{2} \times 3 \times 4.3) = 12.9 \text{ m}^2$$

Find the area of the front/back triangles.

$$\text{Area} = 2 \times (\frac{1}{2} \times 7 \times 5.3) = 37.1 \text{ m}^2$$

Find the area of the rectangular base.

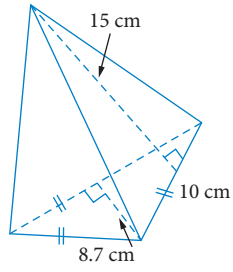
$$\text{Area} = 7 \times 3 = 21 \text{ m}^2$$

Add the areas together.

$$\text{Total surface area} = 12.9 + 37.1 + 21$$
$$= 71 \text{ m}^2$$

Example 12

Find the surface area of this triangular pyramid.

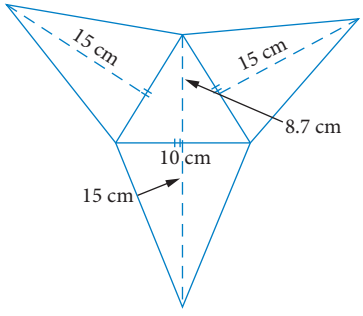


Solution

The net of this pyramid is shown: the base is an equilateral triangle and the three side faces are identical triangles.

Find the area of the three side faces.

$$\text{Area} = 3 \times (\frac{1}{2} \times 10 \times 15)$$
$$= 225 \text{ cm}^2$$



Find the area of the base triangle.

$$\text{Area} = \frac{1}{2} \times 10 \times 8.7$$
$$= 43.5 \text{ cm}^2$$

Add the areas together.

$$\text{Total surface area} = 225 + 43.5$$
$$= 268.5 \text{ cm}^2$$

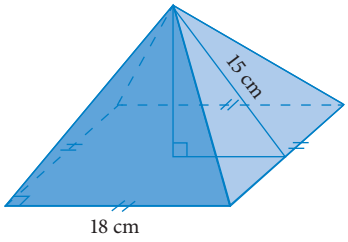
EXERCISE 11.6

Surface areas of pyramids

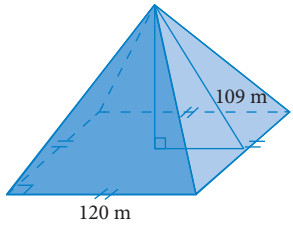
EXAMPLE 10

1 Find the surface area of each square pyramid.

a



b



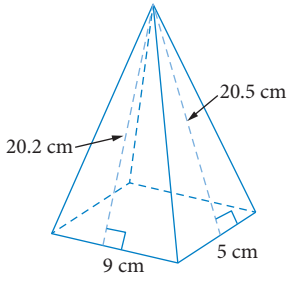
2 Andrea is making a stained glass lampshade in the shape of a square pyramid. The bottom edges are 20 cm long and the slant height of the triangles is 24 cm. Find the total area of glass that Andrea will need to make the lampshade. Remember a lampshade will not have a base!

3 The Great Pyramid in Egypt is a square pyramid with the sides of the base being 230 m and the slant height of the sides 300 m. Calculate the surface area of the Great Pyramid, including the base.

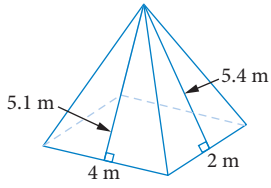
EXAMPLE 11

4 Calculate the surface area of each rectangular pyramid.

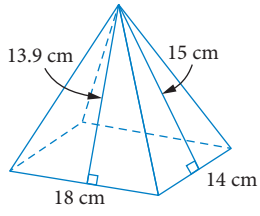
a



b



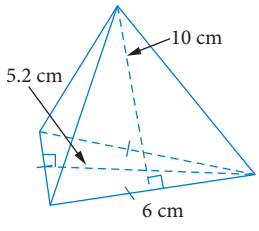
5 Harry believes his seedlings grow best when they are planted in glass pyramids. He uses a glass cover in the shape of a rectangular pyramid which fits over a glass base 18 cm by 14 cm. Calculate the total surface area of the pyramid and the glass base.



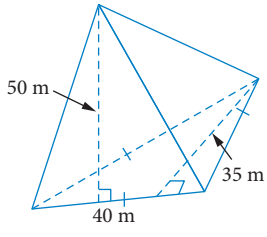
EXAMPLE 12

6 Calculate the surface area of each triangular pyramid.

a



b



7 Edward wins the state public speaking competition and receives a crystal glass trophy in the shape of a triangular pyramid. The base is an equilateral triangle with sides 4 cm and perpendicular height 3.5 cm. The slant height of the sides of the trophy is 22 cm. Calculate the surface area of the glass.

8 The entrance to the Louvre museum in Paris is a large square pyramid made of glass. The base of the pyramid is 35 m long. The slant height of the pyramid is 27.8 m.

a Calculate the area of the four sides of the pyramid.

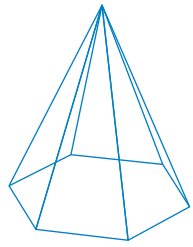
b The sides are made out of glass pieces in the shape of a rhombus. There are approximately 640 of these on the four sides. What is the approximate area of one glass piece?



Shutterstock.com/Nikonaft

9 This is a hexagonal pyramid made to hold chocolates. The base has sides of 8 cm and its area is 83 cm^2. The slant height of the triangles is 15 cm. Calculate the surface area of this chocolate box.

All the sides of a regular hexagon are the same length.



Hexagonal pyramid

10 The roof of one section of Sue's house is a square pyramid. Its base is 10 m long and the slant height 5.3 m. At the top a smaller square pyramid has been sliced off. The removed section has a square base 1.8 m long and a slant height of 1.9 m.



Courtesy of Sue Thomson

a What is the surface area of the roof *before* the top section is removed?

b What is the surface area of the section of roof that has been removed?

c What is the surface area of the roof that Sue has to cover with roof tiles?

Keyword activity

area	composite	net
prism	pyramid	surface area
square millimetres	square centimetre	square metre
hectare	square kilometre	

Write two or three paragraphs that summarise this chapter using ALL of the words listed above. Your paragraphs must make sense and show clearly the meanings of the words.

Chapter Problem

Francesca is making gift boxes to sell. She plans to make three types of boxes:

Cube: 15 cm long

Cylinder: radius 4.5 cm and height 34 cm

Square pyramid: base 15 cm long, slant height 16 cm

Should she charge the same for each gift box? Give reasons for your answer.

Solution

First calculate the surface area for each type of box, to compare the amount of material used.

Cube:

$$\begin{aligned}\text{Surface area} &= 6 \times (15 \times 15) \\ &= 1350 \text{ cm}^2\end{aligned}$$

Cylinder: use the formula $SA = 2\pi r^2 + 2\pi rh$, where $r = 4.5$, $h = 34$

$$\begin{aligned}\text{Surface area} &= 2 \times \pi \times 4.5^2 + 2 \times \pi \times 4.5 \times 34 \\ &= 1088.5618\ldots \\ &\approx 1089 \text{ cm}^2\end{aligned}$$

Square pyramid: base plus 4 triangles

$$\begin{aligned}\text{Surface area} &= 15 \times 15 + 4 \times \frac{1}{2} \times 15 \times 16 \\ &= 705 \text{ cm}^2\end{aligned}$$

No, Francesca should NOT charge the same for each gift box. They will require different amounts of materials and the prices should reflect this. The cube should have the highest cost, then the cylinder, then the square pyramid.



Chapter Problem

Nina is planning a 9 km bush walk. She anticipates that she will only be able to walk at a speed of 2.5 km/h over the steep and bushy land. She plans to leave the car park at 10 a.m. and for safety she will log her walk with the park ranger. What time should she tell the park ranger to expect her back at the car park?



Shutterstock.com/Dudarev Mikhail

CHAPTER 12

GOING PLACES

12.1 Are we there yet?
12.2 Heading north
12.3 Street maps
12.4 Scales on maps
12.5 Regional maps
12.6 Family road trip
Keyword activity
Solution to the chapter problem

By the end of this chapter, you should know how to:

- solve problems involving speed, distance and time
- recognise and use compass directions
- read grid references and information on street maps
- interpret and use scale on maps

How are we ever going to use this?

- Planning holidays and journeys
- Travelling in unfamiliar locations
- Finding an alternate route to avoid traffic problems

12.1 Are we there yet?

The question 'How long will it take to get there?' can easily be answered with a little mathematics. The values for the distance covered, the speed and the time taken are related. When we know two of the values we can calculate the third value.

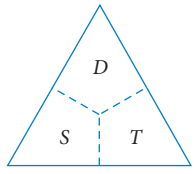
Distance covered = speed × time

or $D = S \times T$

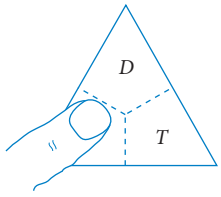
The units for the speed tell us the units for distance and time.

When the speed is in km/h, the distance is in kilometres and the time is in hours.

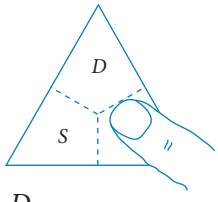
Some students find it easier to use the 'distance, speed and time triangle' to solve problems involving speed. Place the letters D for distance, S for speed and T for time in alphabetical order in the triangle.



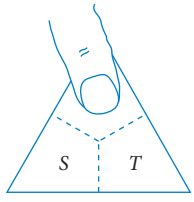
To calculate the speed, cover up S, which leaves $\frac{D}{T}$.
This means that $S = \frac{D}{T}$, or $S = D \div T$.



We can find the two remaining formulas in the same manner.



$T = \frac{D}{S}$ or $T = D \div S$



$D = S \times T$



Shutterstock.com/Bruce Amos

Example 1

A racing greyhound runs at a speed of 18 m/s.

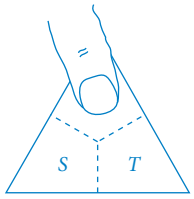
a How far will the greyhound run in 4 seconds?

b How long will it take the greyhound to complete a 1200 m race? Answer correct to the nearest 0.1 s.

Solution

a Use the triangle. To find the distance, cover the D leaving S and T together. $D = S \times T$.

$S = 18$ m/s and $T = 4$ s.



$D = S \times T$

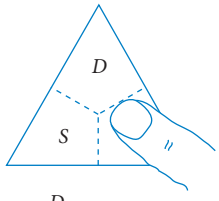
$D = 18 \times 4$
$= 72$ m

Write your answer.

The greyhound will run 72 metres in 4 seconds.

b To find the time, cover the T, leaving D over S. $T = D \div S$.

$D = 1200$ m and $S = 18$ m/s.



$T = \frac{D}{S}$ or $T = D \div S$

$T = 1200 \div 18$
$= 66.666\ldots$ seconds
≈ 66.7 seconds

Write your answer.

The greyhound will complete the race in 66.7 seconds.



Shutterstock.com/James Klotz

Example 2

A kangaroo bounds at a speed of 48 km/h. How far will a kangaroo bound in 20 minutes?

Solution

$D = S \times T$ $S = 48$ km/h

The speed is in km/h so we need the time in hours, not minutes. Divide minutes by 60 to change to hours.

$T = \frac{20}{60} = \frac{1}{3}$ h

$D = 48 \times \frac{1}{3}$
$= 16$ km

Write your answer.

The kangaroo will bound 16 kilometres in 20 minutes.

Example 3

Calvin is driving through a school zone at a speed on 40 km/h. Convert 40 km/h to a speed in m/s correct to one decimal place.

When we want to calculate speed in different units we can change the units before we start the calculation.

Solution

To convert 40 km/h to m/s, change 40 km to m and h to s.

40 km $= 40 \times 1000$ m
$= 40\,000$ m

1 h $= 60 \times 60$ seconds
$= 3600$ s

$S = D \div T$

$S = 40\,000$ m $\div 3600$ s
$= 11.1111\ldots$
≈ 11.1 m/s

Write your answer.

A speed of 40 km/h is equivalent to 11.1 m/s.



Newspix/Brad Hunter

EXERCISE 12.1

Are we there yet?

EXAMPLE 1

1 **a** Use the formula $T = \frac{D}{S}$ to find the value of T when $D = 80$ and $S = 16$.

b In the formula $D = S \times T$, what is the value of D when $S = 80$ and $T = 3$?

2 **a** Karen is driving at a speed of 60 km/h. How far will she drive in 3 hours?

b How long will it take her to drive 240 km?

3 Brian has an appointment 160 km away. He must be there in 2 hours. At what speed must he travel to arrive in time?

4 Go-karts can race at a speed of 110 km/h. At this speed, how many kilometres can a go-kart travel in a $2\frac{1}{2}$-hour race?

5 Wasim is driving through heavy traffic at a speed of 32 km/h. How far will he travel in 15 minutes?



Shutterstock.com/Nicky Rhodes

EXAMPLE 2

6 An ambulance is racing to the scene of a serious freeway accident, at a speed of 100 km/h. The accident is 15 km from the ambulance station.

a How long will the ambulance take to reach the accident? Express your answer as a decimal of an hour.

b Multiply your answer to part **a** by 60 to change the time to minutes.

7 A tactical response team is travelling at 80 km/h reach a hostage situation 10 km away. How long will it take the team to arrive at the scene? Express your answer in minutes.

8 A whitewater rafting team completed three sets of rapids and 1.6 km of calm water in 30 minutes. The lengths of the sets of rapids were 150 m, 80 m and 170 m.

a Calculate the distance that the rafting team covered in 30 minutes. Express your answer in kilometres.

b Explain why you can't use $T = 30$ in the equation $S = D \div T$ to calculate the speed of the raft in km/h.

c Calculate the raft's average speed in km/h.

9 Cheetahs can run at a speed of 31 m/s in short bursts.

a How far can a cheetah run in 9 seconds?

b How long does it take a cheetah to run 140 m? Answer in seconds correct to 1 decimal place.

10 A peregrine falcon's top speed is 90 m/s.

a How far can the falcon fly in 1 minute?

b How long will it take the falcon to fly 1 km?



iStockphoto/Andyworks

11 The table shows distances between several cities. Distances are in kilometres.

Approximate kilometres	Albury	Brisbane	Canberra	Goulburn	Sydney	Tamworth
Albury	–	1610	190	380	600	1040
Brisbane	1610	–	1300	1225	1020	575
Canberra	190	1300	–	95	300	750
Goulburn	380	1225	95	–	205	660
Sydney	600	1020	300	205	–	460
Tamworth	1040	575	750	660	460	–

a How far is it from Canberra to Tamworth?

b How long will it take to drive from Canberra to Tamworth at an average speed of 75 km/h?

c Gina took 5 hours to drive from Goulburn to Albury. What was her average speed?

d Max and Santosh left Albury at 6 a.m. on Monday to drive to Brisbane. They shared the driving and completed the trip at an average speed of 70 km/h. At what time did they arrive in Brisbane?

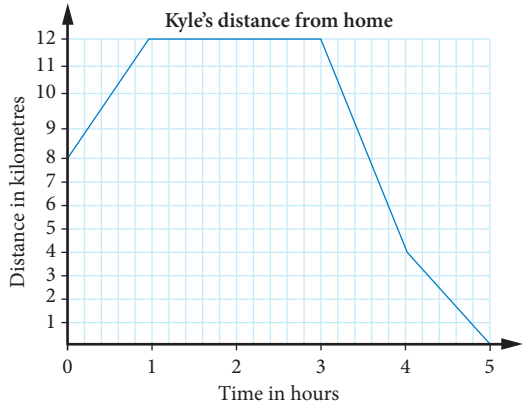
EXAMPLE 3

12 Convert each speed to m/s.

a 60 km/h **b** 90 km/h

13 The minimum speed a rocket needs to escape the Earth's gravity is 11.2 km/s. What is this speed in km/h?

14 Kyle and Anna watched a movie together. The graph shows the distance they were from Kyle's house after the movie finished.



a How far is the cinema from Kyle's house?

b After the movie finished they rode their bikes to Anna's house. How far is Anna's house from the cinema?

c How long did it take them to ride from the cinema to Anna's house?

d What was their average speed riding to Anna's house?

e How long did Kyle stay at Anna's house?

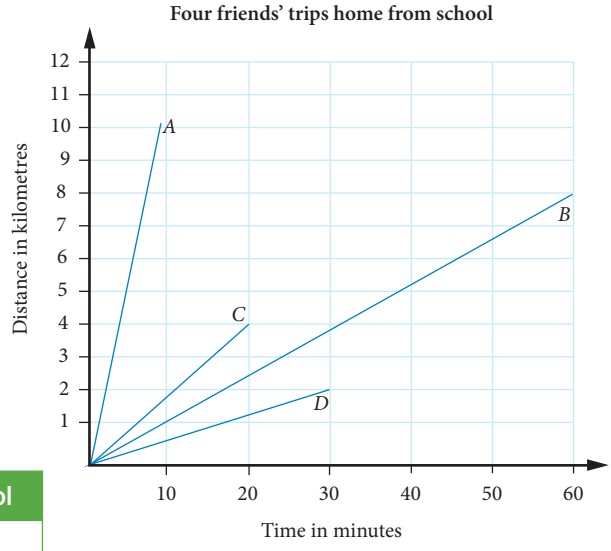
f Calculate Kyle's average speed when he was riding home.

g Part of the trip home is downhill and part is uphill. Did he ride the uphill section during the first or second hour of his ride home? Justify your answer.

15 Copy and complete the table below.
Use the clues to determine which person matches with each graph.

Clues

- Shelby walks home at a speed of 4 km/h.
- Wayne rides his bike at an average speed of 12 km/h.
- Peta travels on the slow bus. With all the stops, the bus only averages 8 km/h.
- Luke drives home at an average speed of 60 km/h.



Graph	Person	Distance from school
A		
B		
C		
D		

16 Construct a graph, similar to the graph in Question **15**, which shows the trip home from school for the members of your group.

Investigation Speed units

Use the Internet to investigate a variety of speeds shown in the table below.

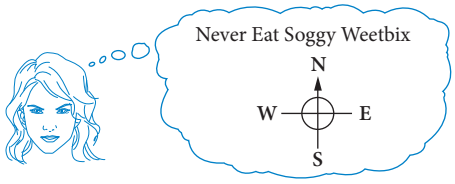
Copy and complete this table, or print a copy of 'Speed units' from NelsonNet.

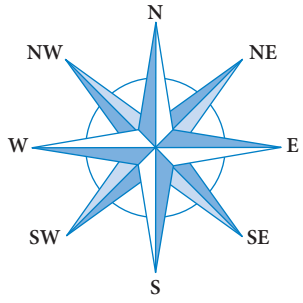
Activity	Typical or record speed
Slow speeds Moving snail Adult walking Athlete running	
Medium speeds Cruise ship Cars Downhill cross-country skiing Olympic speed-skating	
Fast speeds Queensland tilt train Japanese high-speed trains International airline jets	
Very fast speeds Sound Bullet fired from a gun Fighter jet International space station Light	

12.2 Heading north

We can use a compass for directions. The four major directions are **north** (N), **east** (E), **south** (S) and **west** (W).

Remembering compass directions is easy!





Also, we call the direction halfway between north and east, **north-east** (NE). Similarly there are directions called **south-east** (SE), **south-west** (SW) and **north-west** (NW). We use these 8 major directions to describe things like wind direction, the position of the sun and in travel directions.

Example 4

a According to this map, what direction is it from Coober Pedy to Port Augusta?

b In what direction is Coober Pedy from Port Augusta?



Getty Images/pop_jop

Solution

a Position the centre of a compass rose on the place the direction is '*from*'. In this example the bearing is from Coober Pedy.



The direction to Port Augusta is between east and south.

The direction from Coober Pedy to Port Augusta is south-east.

9780170370752

b Now the bearing is from Port Augusta.
Position the compass rose on Port Augusta.



The direction to Coober Pedy is between north and west.

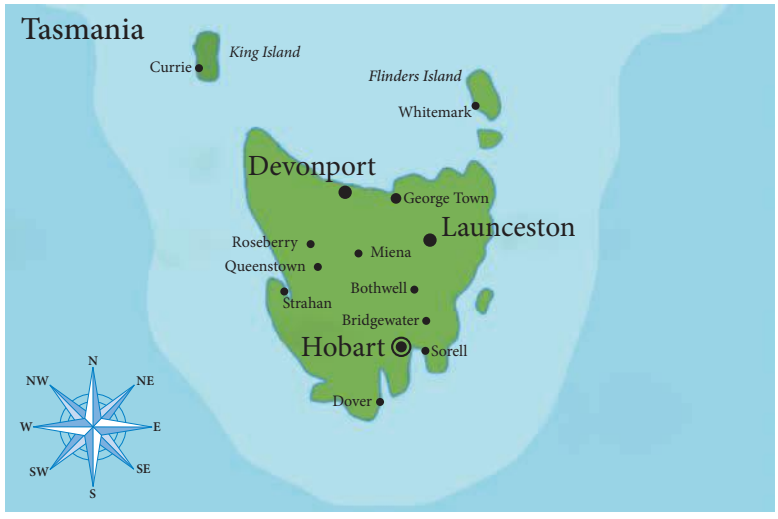
Coober Pedy is north-west of Port Augusta.

EXERCISE 12.2

Heading north

1 Write the compass direction:

EXAMPLE 4



Getty Images/pop_jop

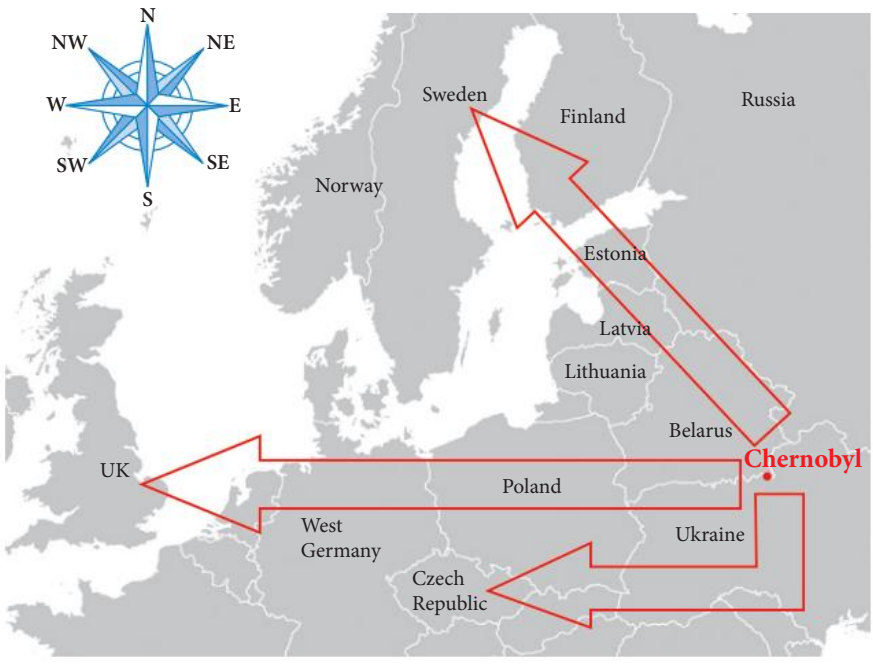
a from Hobart to Roseberry

b from Launceston to Bothwell

c to Roseberry from Queenstown

d to Launceston from George Town

2 After an explosion at the Chernobyl nuclear power station in 1986, winds carried the radiation cloud over a vast area.



Getty Images/Kosmozoo

a What compass direction is it from Chernobyl to the Czech Republic?

b What countries experienced radiation clouds as a result of easterly winds?

3 When Muslim people around the world pray, they always face in the direction of Mecca in Saudi Arabia, Islam's holiest city. What is the approximate compass direction from each place to Mecca?



Getty Images/Kosmozoo

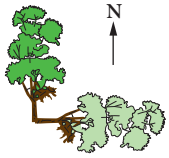
a Australia **b** China **c** Turkey

d Pakistan **e** Mali **f** Argentina

g France **h** Tanzania **i** Mexico

4 Late in the afternoon the sun is in the west. Which of the diagrams **A**, **B**, **C** or **D** shows the correct position of the tree's shadow?

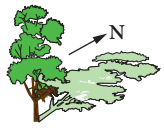
A



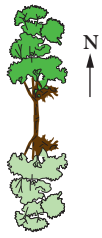
B



C



D



5 The arrow on a wind vane always points to the direction the wind comes *from*. When a weather forecast predicts a 'southerly wind' it means a wind comes from the south.

Which wind in **a** to **d** matches with the windvanes in **i** to **iv**?

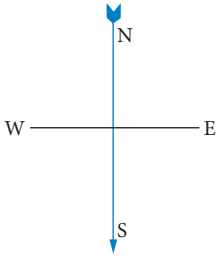
a southerly wind **b** north-easterly wind

c westerly wind **d** south-westerly wind

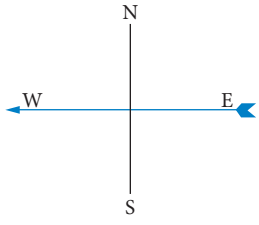


Shutterstock.com/TEVARUK1981

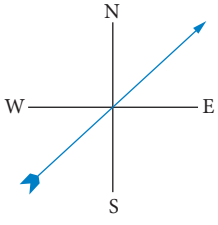
i



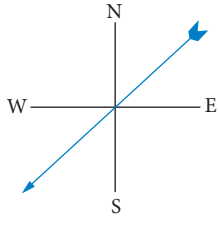
ii



iii



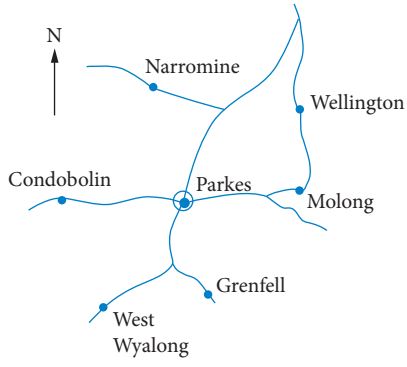
iv



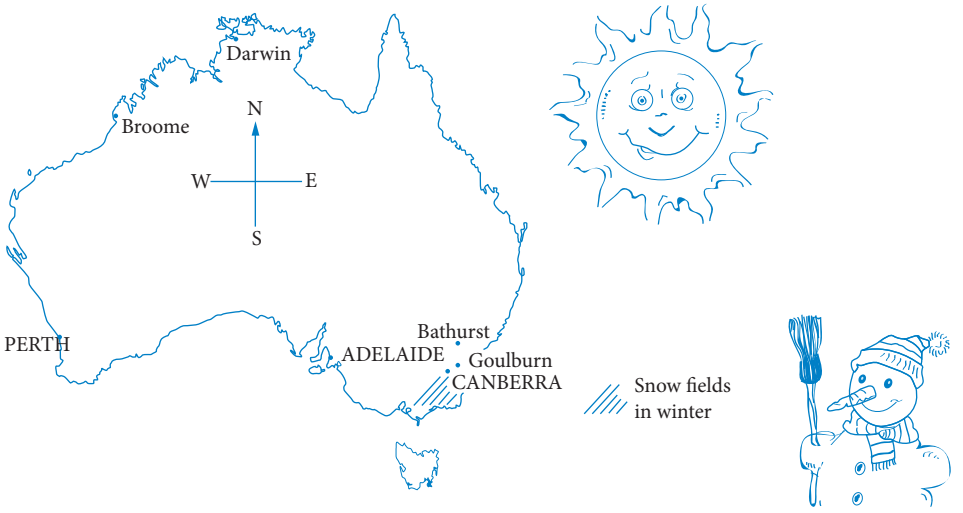
6 A westerly wind brought smoke from a bushfire to Parkes.

a Near which town could the bushfire be?

b If the wind keeps blowing in the same direction, which other town will have smoke blown into it?



7 Winds from over the ocean carry moisture and are likely to bring rain, while winds from over land are likely to be dry. The temperature of the land the winds blow over determines whether the winds will be hot or cold. In winter, winds from over snow fields will be cold, while summer winds from Central Australia are likely to be dry and hot.



Are the following winds likely to be hot, cold, bring rain or fine weather?

a Westerly winds in Perth

b Westerly winds in Bathurst in summer

c North-easterly wind in Goulburn

d Southerly winds in Canberra in winter

e Southerly winds in Darwin in summer

f Southerly winds in Adelaide

g North-westerly winds in Broome

h South-easterly winds in Broome in summer

Street maps

12.3

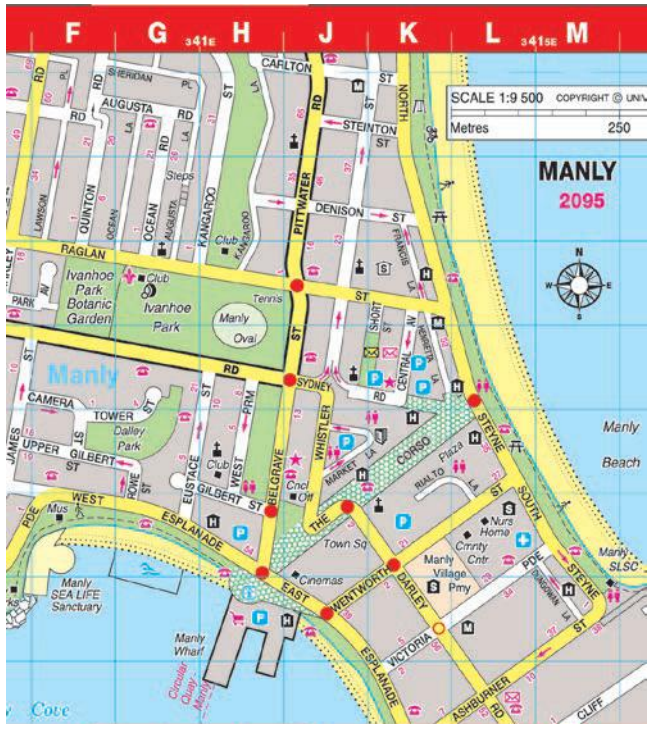
Street maps have a grid coordinate system to help us locate features. Usually we read the coordinates by starting with the horizontal (across) letters, then the vertical (up and down) numbers.

Example 5

a On this map, what is the name of the park at F3?

b What are the position coordinates of Manly Oval?

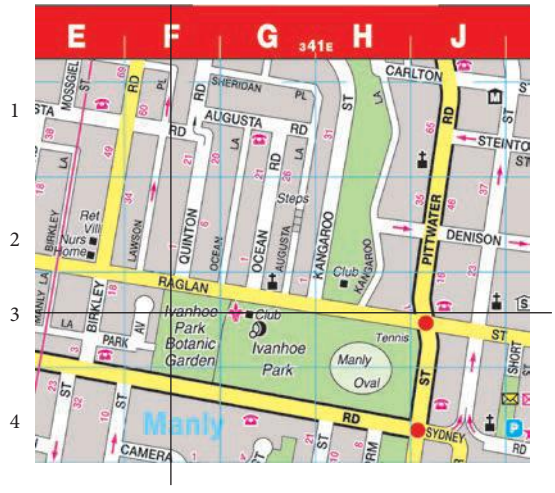
c What direction is it from Manly Oval to Manly Wharf?



UBD Gregory's Sydney Street Directory Map 29

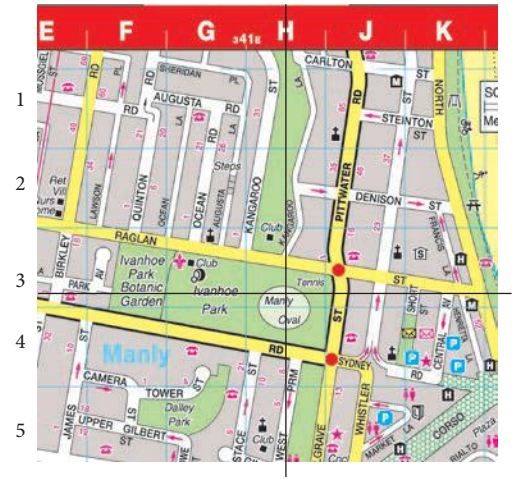
Solution

a Find the position where column F meets row 3.



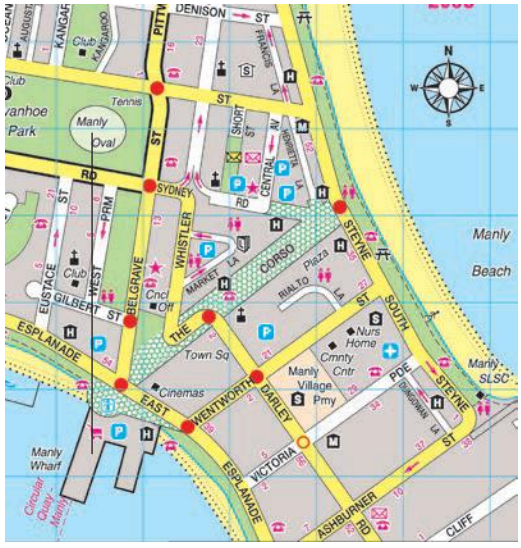
Ivanhoe Park is located at F3.

b Manly Oval is in column H and rows 3 or 4.



Manly Oval is at H3 or H4.

c An imaginary line from Manly Oval to Manly Wharf points south.



Manly Wharf is south of Manly Oval.

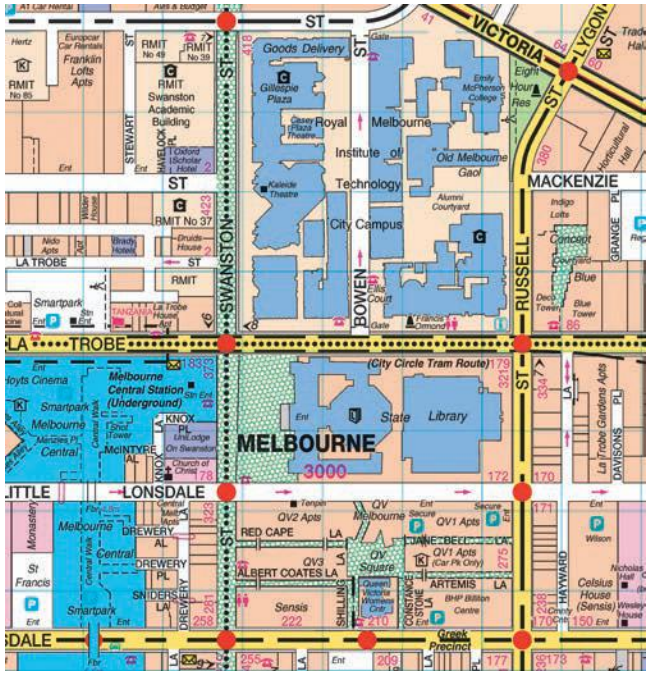
If we need to calculate a distance that isn't along the side of a grid square we can place a chain or piece of string along the distance, then compare the distance to the sides of the grid. In this picture, the length of the chain is roughly 3 grid squares.



9780170370752

Example 6

On this street map of Melbourne CBD, 1 km equals 16 grid squares. Use this scale to calculate the length of the Melbourne Institute of Technology campus along Swanston St.



Map section reproduced with permission of UBD-Gregory's. Copyright Universal Publishers Pty Ltd DD 05/15

Solution

1 km covers 16 grid squares, so calculate the length of one grid square first.

Convert 1 km to 1000 m also.

$$\text{One square} = 1\text{ km} \div 16$$
$$= 1000\text{ m} \div 16$$
$$= 62.5\text{ m}$$

The length of the campus covers 4 grid squares.

Multiply by 62.5 m.

$$\text{Boundary length} = 62.5 \times 4$$
$$= 250\text{ m}$$

Write your answer.

The Swanston St boundary is 250 m long.

EXERCISE 12.3

Street maps

Use map 427 next page to answer the questions in this exercise.

EXAMPLE 5

1 **a** What is the name of the school at E11?
b What does the small black square at D3 represent?
c Which Surf Life Saving Club is located at B1?
d There is a red light traffic camera at the intersection of the Frankston Freeway and Frankston-Dandenong Rd at F12. What symbol does the map use to show a red light camera?
e There are speed humps along Wangarra Rd (L18 to N19) to slow traffic. Name two other roads towards the top of the map that also have speed humps.

2 State the position coordinates of each place.
a Belvedere Park Primary School
b Seaford Golf Driving Range
c Long Island Retirement Village

EXAMPLE 5

3 Name the compass direction:
a from Kananook Railway Station (F9) to the caravan park in C9
b from John Paul Secondary College (G15) to Long Island Country Club Clubhouse (G11)
c from VicRoads (H7) to the Indoor Cricket centre (H7)
d to tennis (B13) from the Arts Centre (B20)

4 Jeeva drove along the Nepean Highway from the tennis courts at B13 to his home in Brodie St (C3). In what direction did Jeeva drive?

5 Every day Madison drives along the Frankston-Dandenong Rd from Ballarto Rd (P5) to Klauer St (K8) to get to her work. In what direct does she drive to work?

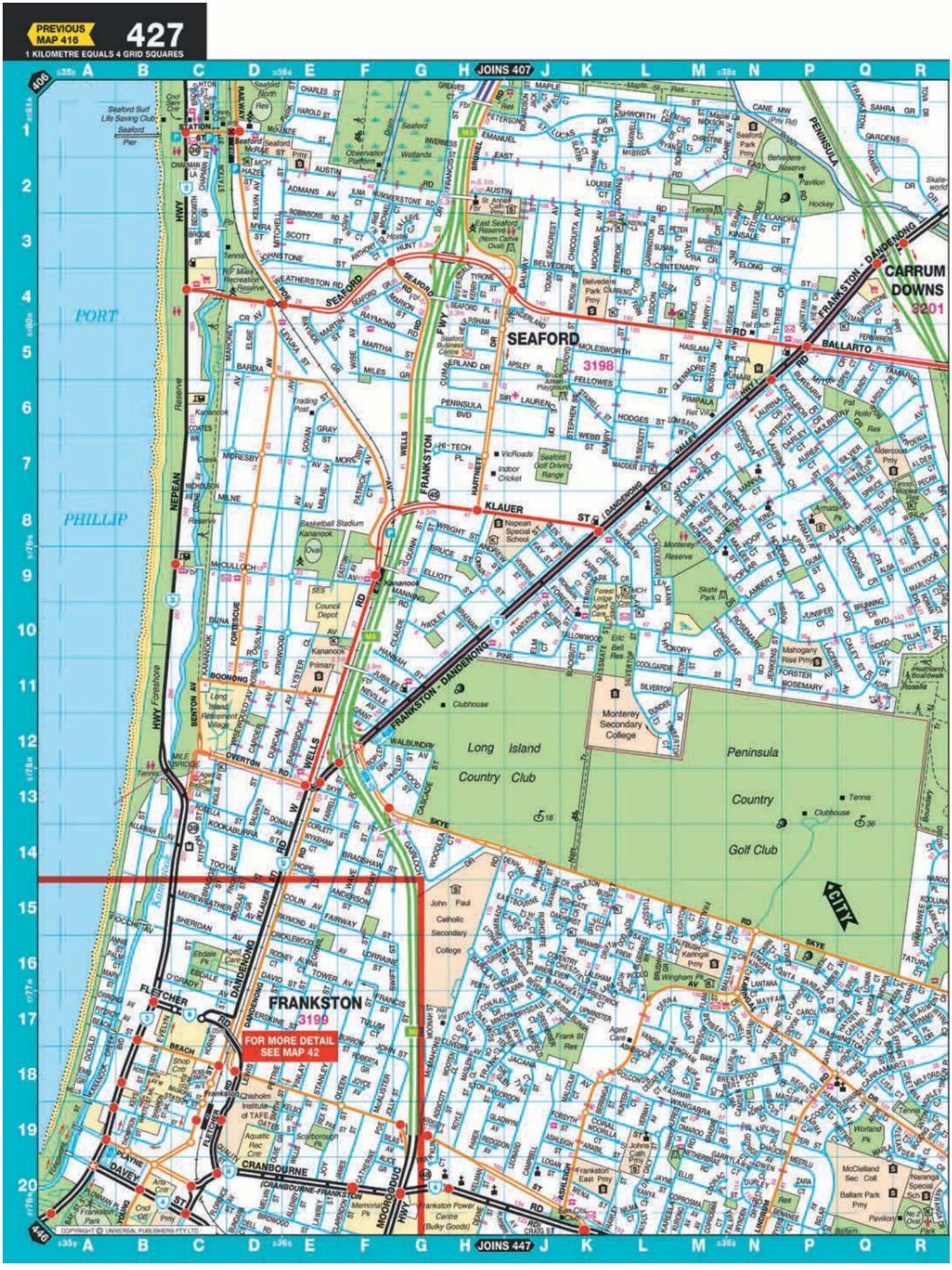
6 On the map 1 kilometre covers 4 grid squares. Calculate the length of 1 grid square in metres.

EXAMPLE 6

7 Approximately how far is it along the Frankston Freeway from the overhead bridge in H2 to the interchange in G8? Answer correct to the nearest 100 metres.

8 Approximately how long is Skye Rd between the traffic lights in F13 to the roundabout in N15?

9 Each afternoon Michael jogs along the beach from his home in Beach St at A17 to the Seaford Pier at B1 and back home.
a How far does Michael jog each afternoon? Answer to the nearest kilometre.
b Michael's afternoon jog takes 2 hours. Calculate Michael's average jogging speed.


PREVIOUS MAP 416
427
1 KILOMETRE EQUALS 4 GRID SQUARES
JOINS 407
JOINS 447
SEAFORD
3198
FRANKSTON
3199
CARRUM DOWNS
3201
FOR MORE DETAIL SEE MAP 42
PORT PHILLIP
Long Island Country Club
Peninsula Country Golf Club
Monterey Secondary College
John Paul Catholic Secondary College
Kanamook Primary
Nepean Special School
Long Island Retirement Village
Seaford Surf Life Saving Club
Seaford Pier
Seaford Wetlands
Seaford Golf Driving Range
Kananook Oval
Basketball Stadium
Council Depot
SES
Skate Park
Monterey Reserve
Eric Bell Res
Frank St Res
Aquatic Rec Cntr
Frankston Chisholm Institute of TAFE
Frankston Power Centre (Bulky Goods)
Frankston Memorial Pk
Frankston Park
Seaford Park Pmy
Belvedere Reserve
East Seaford Reserve (Norm Carlile Oval)
Seaford North Res
Seaford McRae Pmy
Seaford Pmy
Belvedere Park Pmy
Aldercourt Pmy
Mahogany Rise Pmy
Kananook Pmy
Frankston East Pmy
Ballam Park Pmy
Kananook Manning Res
Seaford Business Centre
Trading Post
Hi-Tech Indoor Cricket
VicRoads
Clubhouse
Tennis
Skate-world
Hockey
Pavilion
Aged Care
Shop Cntr
Arts Cntr
Cncl Off
Cncl Serv Cntr
Scarborough
Worland Pk
McClelland Sec Coll
Naranga Special Sch
Ballam Park Pmy
Frankston Park
Res
Oval
Boundary
Tennis
Clubhouse
Aged Care
Shop Cntr
Arts Cntr
Cncl Off
Cncl Serv Cntr
Pmy
Res
Oval
Park

10 Calculate each distance 'as the crow flies'.

'As the crow flies' means 'in a straight line'.

- **a** From Frankston Station (C19) to Kananook Station (F9).
- **b** From Kananook caravan park (C6) to the Post Office in Seaford Business Centre (H5).
- **c** From the swings in Skate Park (M9) to the closest public telephone.

11 How far is it along the roads between each location?

- **a** Corner of Boonong Av and Fortescue Av (D11) to the roundabout on Govan St (E7).
- **b** The traffic lights at the intersection of Nepean Highway and Seaford Rd (C4) to the telephone exchange at N5.

12 Samantha rode her bike from John Paul College to Monterey College. She left from the roundabout in H16 and rode to the south-western corner of Monterey College K12. She rode along McMahons Rd, Syke Rd then along Nats bike track.

- **a** Approximately how far did Samantha ride?
- **b** She rode at an average speed of 3 metres per second. Approximately how long did the trip take?

13 Biljana lives near the roundabout on Buna Ave (C10). She left home and walked north along Kananook Ave. She turned into the first street on the right then left into the third street on the left. She crossed the road and walked to the end of the park to practise her favourite sport. What is her favourite sport?

14 Dieter is visiting his grandmother in the retirement village at G17. Describe the route he could take to walk to his home in Oakhurst Ct (K15)

Investigation Directions to your home

Imagine friends are going to visit you in your home but they have never been there before.

Your task is to:

- use either a street map or Google Maps to obtain a street map of your local area.
- write instructions for your visitors to get from the train or bus station to your home.

Include distance and direction indications such as left and right as well as compass directions to make following your instructions easy.

Scales on maps

12.4

Maps usually have a scale drawn or written as a ratio.

Example 7

The scale on a bushwalking map is 1 : 5000. On the map a walking trail is 4.5 cm long. How long is the trail in real life?

Solution

The scale 1 : 5000 means that 1 unit on the map means 5000 units in real life. Multiply the scaled length on the map by 5000 to find the real length.	Real length = 4.5 cm × 5000 = 22 500 cm
Convert to metres by dividing by 100.	22 500 cm ÷ 100 = 225 m
Write your answer.	The trail is 225 m long.

EXERCISE 12.4

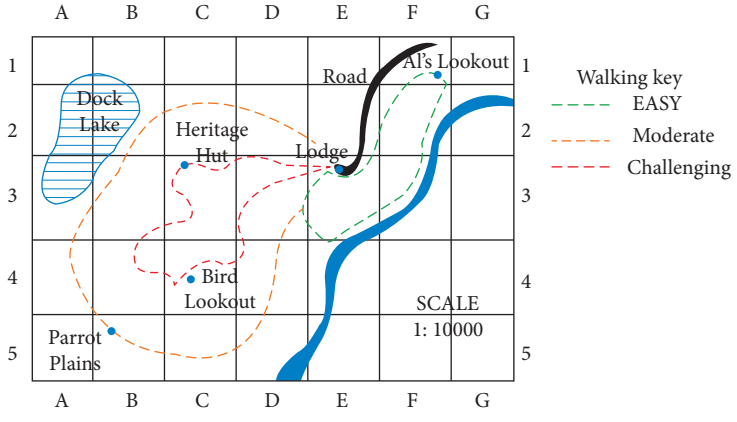
Scales on maps

EXAMPLE 7

1 A map's scale is 1 : 2000. Calculate the real lengths of each feature.

- **a** Lake 1.5 cm wide
- **b** Walking track 5.8 cm long
- **c** Creek 9.5 cm long

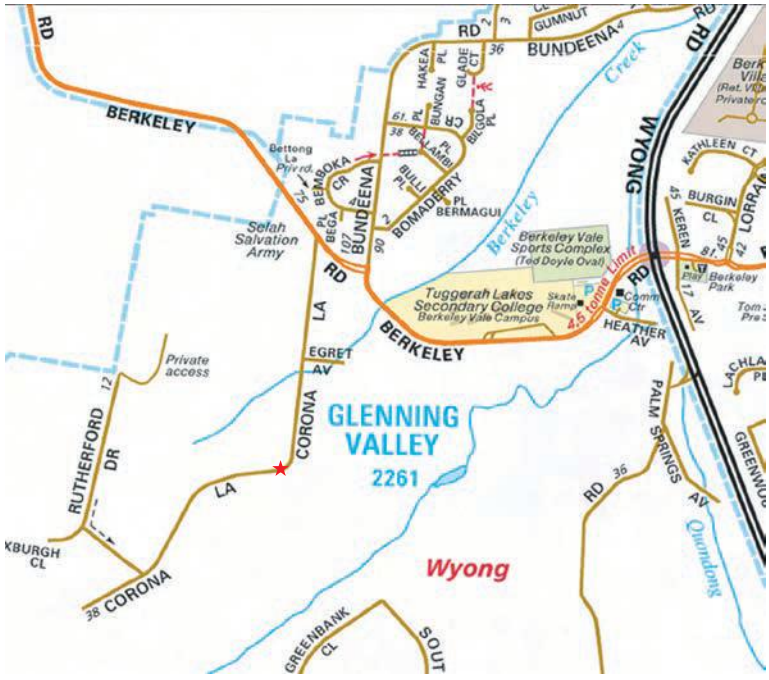
2 This map shows the walking trails that start in the lodge.



- **a** What is the scale?
- **b** What length in real life is represented by 10 cm on the map?
- **c** How long is the creek?
- **d** Calculate the length of the moderate walking track.
- **e** Jude started walking along the moderate walking track at 11 a.m. She is walking at a speed of 3 km/h. When will Jude finish walking the track?

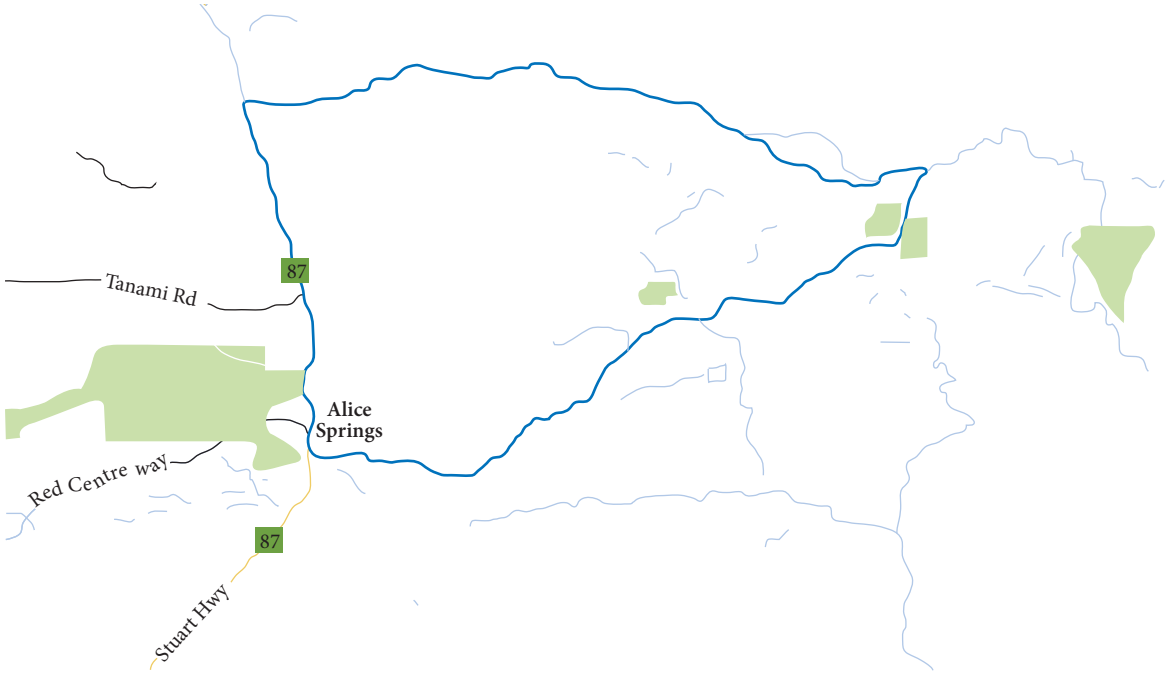
Remember! You can use a chain or a piece of string to help you find lengths on a map.

3 Zack lives in Corona Lane. The star on the map shows the position of his house. Use the scale 1:10 000 to calculate the distance Zack walks to Tuggerah Lakes Secondary College each day.



Map proudly reproduced with permission from Sydway Street Directory

4 This map shows the road around East MacDonnell National Park in Central Australia. The scale is 1 : 1 000 000.

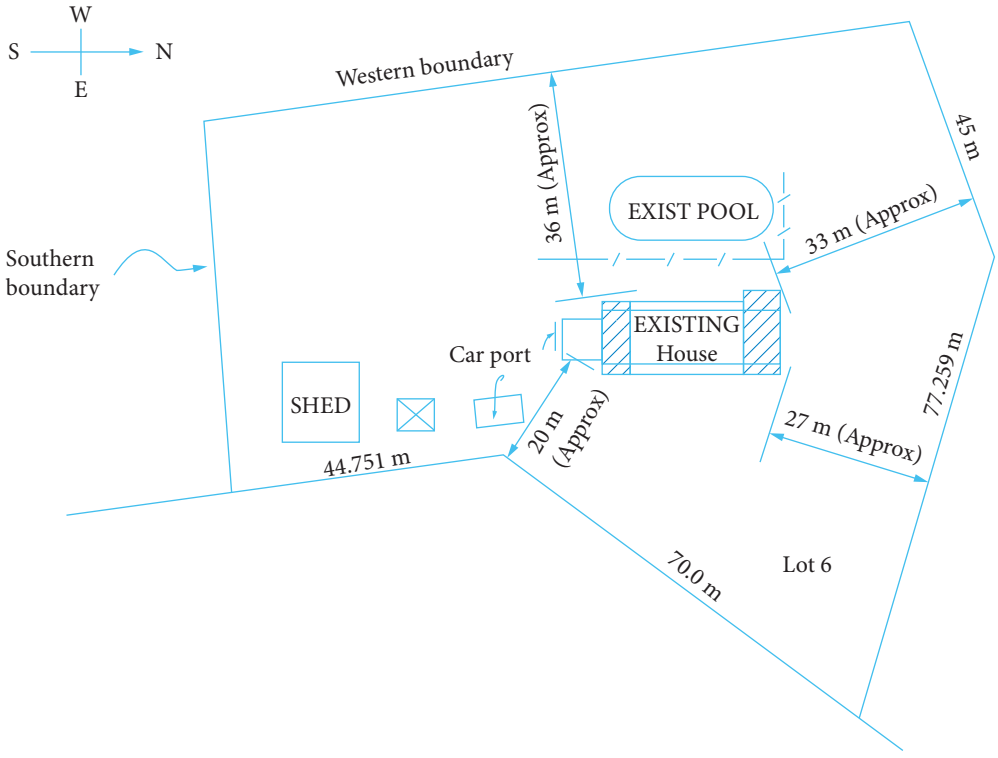


a Use the scale to calculate the length of the road.

b The suggested time to drive one circuit of this road is $5\frac{1}{2}$ hours. At what average speed do drivers need to travel to complete the circuit in $5\frac{1}{2}$ hours? Express your answer correct to the nearest km/h.

c What do you think the surface of the road is like? Give a reason for your answer.

5 Cadastral maps show the boundaries of properties. This part of a cadastral map shows the dimensions of a block of land and the buildings on the land.

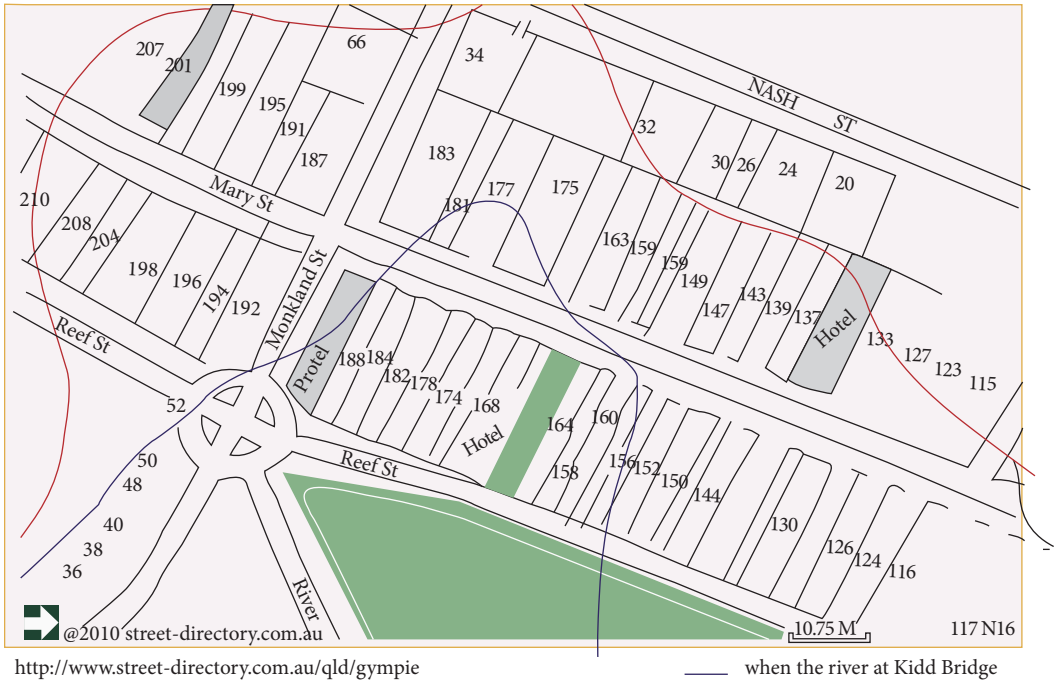


a Approximately how close is the house to the nearest boundary?

b The scale is 1 : 1000. Calculate the length of the southern and western boundaries.

c The owners are going to replace the fence on the southern and western boundaries. The fencing costs $28/m for materials and labour. Calculate the cost of replacing the fencing.

6 Mary St in the township of Gympie in South-East Queensland is prone to flooding. The map shows the height of the flood waters when the height of the Mary River is 18 m and 23.5 m at the Kidd Bridge. The scale is 1 : 1000.



http://www.street-directory.com.au/qld/gympie

when the river at Kidd Bridge reaches 18 m

1-in-50 year flood level 23.5 m at Kidd Bridge

a What are the street numbers of addresses in Mary Street that flood when the Mary River reaches 18 m at the Kidd Bridge?

b Estimate the height of the Mary River at the Kidd Bridge when number 187 Mary St floods.

c Calculate the length of Reef St that is under water when the Mary River is at 18 m at the Kidd Bridge.

d Calculate the length of Mary Street that is under water in 1-in-50 year floods.

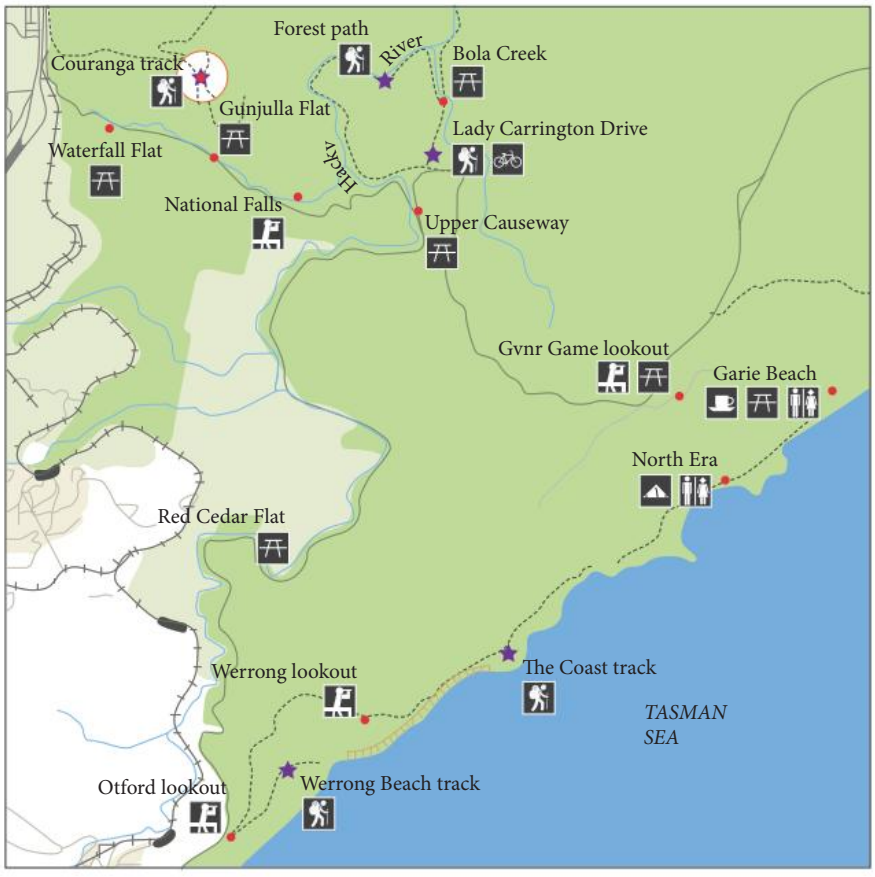
e Rhys is considering buying the hotel at 135 Mary Street. What recommendation could you give him?



Newspix/Nathan Richter

9780170370752

7 Jose, Isaac and Chase were hiking along the Coastal Track in the National Park. They left their bikes at Otford Lookout and set out for Garie Beach.



http://www.environment.nsw.gov.au/NationalParks/parkWalking.aspx?id=N0030#Courangawalkingtrack

a The map scale is 1 : 80 000. How long is the walk from the Otford lookout to Garie Beach?

b The boys hike at an average speed of 4 km/h. How long should the hike take them?

c When they had been hiking for 2 hours Chase lost his footing, fell and broke his leg. Isaac phoned emergency services for help. Approximately where on the track should he tell the rescue helicopter they are?

Investigation A cadastral map of your school

A cadastral map shows information about the boundaries of individual and surrounding blocks of land. In this investigation you are going to make a cadastral map of your school and surrounding properties. Make the map suitable for display in the school's front office foyer.

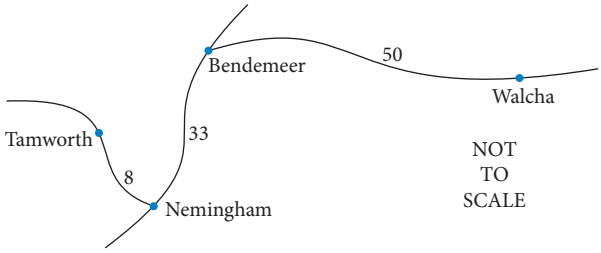
On the map you should include: a compass rose, a scale, the lengths of all boundaries and the position of buildings.

Start your investigation by downloading a map and a picture of the terrain from Google Earth.

12.5 Regional maps

Example 8

Josie is travelling from Tamworth to Walcha. How far is it by road?



Solution

It is 8 km from Tamworth to Nemingham, then 33 km to Bendemeer and 50 km to Walcha. Add the three distances together.

Total distance = 8 + 33 + 50
= 91 km

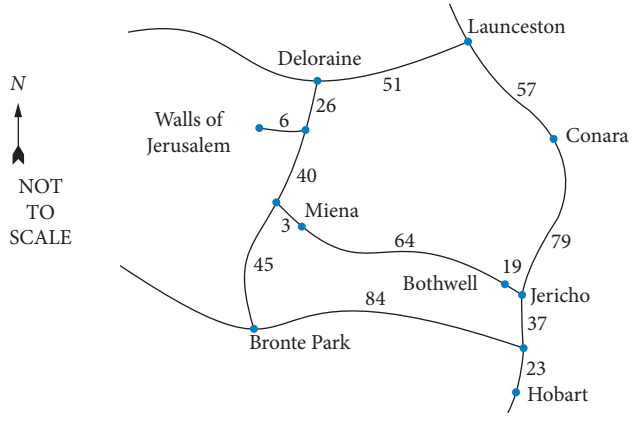
Write your answer.

It is 91 km by road from Tamworth to Walcha.

EXERCISE 12.5

Regional maps

Use this map to answer Questions **1** to **4**.



EXAMPLE 8

1 Tyson drove from Launceston to Bothwell, via Jericho, to stay in the Bothwell Grange Hotel. How far did he drive?

2 How far is it from Launceston, via Deloraine, to the Great Lake Hotel in Miena?

3 Luana drove from her hotel in Miena to see the Walls of Jerusalem and she drove back to her hotel for lunch. What distance did she travel on the round trip?

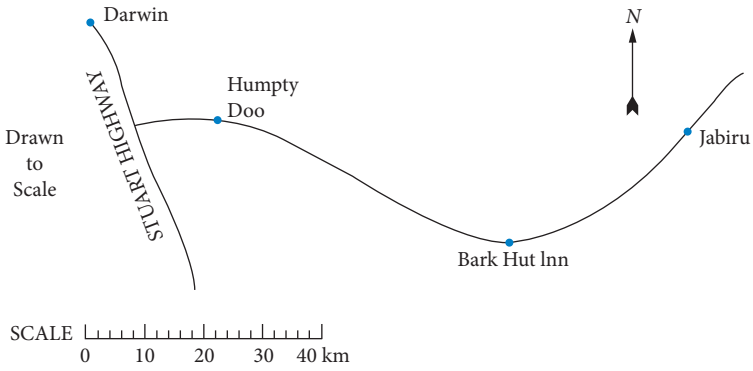
4 When Luana left Miena the next day to travel to Hobart, the odometer on her car showed 3521 She drove south-east and then south.

a What was the reading on her odometer when she reached Hobart?

b The trip took 2 hour 15 minutes. Calculate her average speed from Miena to Hobart correct to the nearest km/h.

9780170370752

5 Mick drove his 4-wheel drive from Darwin to Kakadu Park Headquarters at Jabiru.

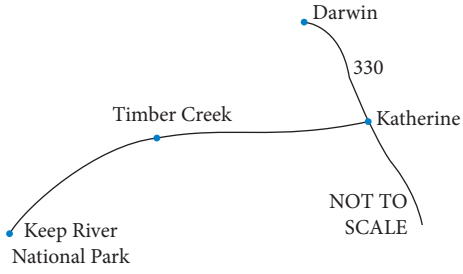


a Use the scale to calculate the distances along each section of the road.

b How far is it from Darwin to Jabiru?

c Mick drove from Darwin to Jabiru at an average speed of 80 km/h. How long did the trip take him?

6 Linda drove from Darwin to Katherine and then on to see the spectacular red sandstone cliffs in Keep River National Park.





Getty Images/Australian Scenics

When she left Darwin the car's odometer reading was 7 3 4 7 5,
and when she reached Keep River National Park it was 7 4 2 6 1.

a How far is it from *Katherine* to the Keep River National Park?

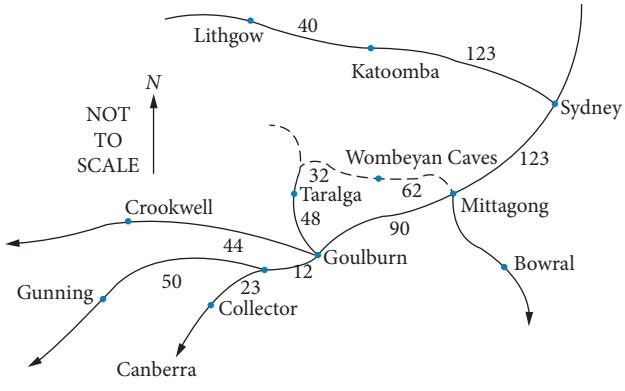
b Linda drove at an average speed of 60 km/h. How long did the drive take her?

7 Clarence and Jimmy drove southwest from Sydney for 123 km and stopped for lunch.

a Where did they stop for lunch?

b Wombeyan Caves was their next stop. They thought the dirt road directly to the caves looked too rough so they went via Goulburn. How far did they travel?

c Their average speed after lunch was 90 km/h. How long did it take them?



8 Their friends Gordon and Gilbert met them at the caves. They drove the 'short', dirt road route from Mittagong to the caves. It took them 1 hour 40 minutes. What was their average speed?

9 At Wombeyan Caves, Clarence and Jimmy camped in their tent and Gordon and Gilbert stayed in an on-site caravan.



Calculate the total accommodation costs for the 4 friends to stay for 2 nights.

10

Cave tours times

Fig Tree cave (Self-guided)	9 am to 5 pm
Wollondilly Junction	1:45 pm 11 am and 2:30 pm
Mulwaree	11:30 am
Kooringa	3:15 pm

Cave inspection fees

	Adult	Child	Family
Self-guided (Fig Tree cave)	$15	$7	$35
Guided tour	$18	$9	$40
Two cave package	$22	$12	$50

a Clarence and Jimmy went on an afternoon tour of the Junction Cave. What time did the tour start?

b How much did the tour cost?

c Gordon and Gilbert toured the Wollondilly and Fig Tree Caves. How much did they save altogether by buying the Two cave package instead of separate tickets?

11 The next day the four friends had breakfast in Goulburn. They left Goulburn at 10:15 a.m. and drove at an average speed of 85 km/h to Gunning. What time did they arrive in Gunning?

Family road trip

12.6

The Winslow family are travelling from Sydney to South Australia for a holiday. They plan to stay in motels, caravan parks and the occasional luxury hotel.

EXERCISE 12.6

Family road trip

1 Buses travel from Sydney to Adelaide every day. Adelaide's local time is 30 minutes behind Sydney and the times in the bus timetable are local times.

Sydney	11:00 a.m.
Wagga Wagga	7:10 p.m.
Cullulleraine	3:05 a.m.
Time change	
Blanchtown	5:13 a.m.
Adelaide	7:10 a.m.
All times are local times	

a The bus doesn't stop at Cullalleraine. However, the time changes after this border crossing. What is the Central Standard Time as the bus passes through Cullulleraine?

b How long does it take to travel from Sydney to Adelaide on the bus?

c The adult fare is $210 each way and children pay 70% of the adult fare. How much would it cost the Winslow family of 2 adults and 3 children to travel from Sydney to Adelaide on the bus?

2 The Indian Pacific train from Sydney to Adelaide takes about the same time as the bus.

SYDNEY TO ADELAIDE TRAIN FARES		
Economy (Sit up all the way)	Adult $375	Child $213
First-class sleeping berth	Adult $850	Child $688

a Which method of travel would be the more expensive for the Winslow family, the bus or the economy train, and by how much?

b How much would it cost the family to travel by first-class rail from Sydney to Adelaide?



Fairfax Syndication/Dallas Kilponen

3 Kanga Air flies from Sydney to Adelaide.

KANGA AIR (All times are local times)			
		Depart	Arrive
From Sydney to Adelaide	Daily	0700	0925
	Daily	1630	1855

a What is the local time in Adelaide when the 7 a.m. flight leaves Sydney?

b How long does the flight take from Sydney to Adelaide?

c The single adult fare is $250 and children pay 40% less than adults. The Winslow family decided to fly to Adelaide. What is the total cost of their flights?

4 After Adelaide, the family visited Kangaroo Island in South Australia. They stayed here for two days and enjoyed watching its friendly wildlife.

Kangaroo Island Tours
2 days/night
$99 per person

or

Organise the trip yourself
Ferry trip $5 each way/person
Kangaroo Island motel (price per night)



$90
1 person

$130
2 persons

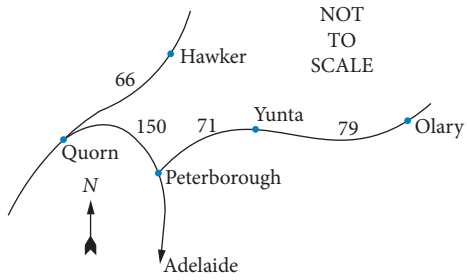
$30 each
Extra person

How much did the family save by using Kangaroo Island Tours instead of organising the trip themselves?



Shutterstock.com/Hypervision Creative

5 The Winslows returned to Adelaide and hired a 4-wheel-drive vehicle to travel north to Peterborough, and then Quorn.



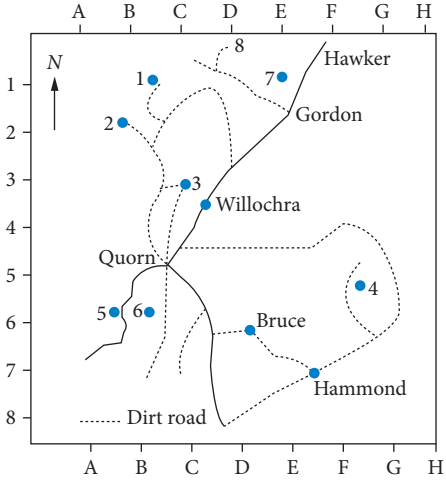
- **a** What compass direction is it from Peterborough to Quorn?
- **b** They left Peterborough at 11 a.m. and arrived at Quorn at 1 p.m. Calculate their average speed.

6 The Winslows stayed at the Quorn Caravan Park, where the charges are shown at right.

	Tariff/night	
Cabin (acc. up to 6)	$75	$10
On-site Van (acc. up to 4)	$55	$5

- **a** How much did it cost the family to stay in a cabin for 2 nights?
- **b** How much more would it have cost them to use the on-site vans for 2 nights?
- **c** Why would it have cost more for the Winslow family to stay in the on-site vans rather than the cabin?
- **d** Will it always be more expensive to stay in vans than in cabins? Explain your answer.

7 The region around Quorn is shown on this map.



Key

1 Middle Gorge
2 Warren Gorge
3 Junction Art Gallery
4 Moockra Tower—Lookout over the Horseshoe Ranges
5 Richi Richi Pass
6 Devils Peaks
7 Kanyaka Ruins
8 Ben's Cave

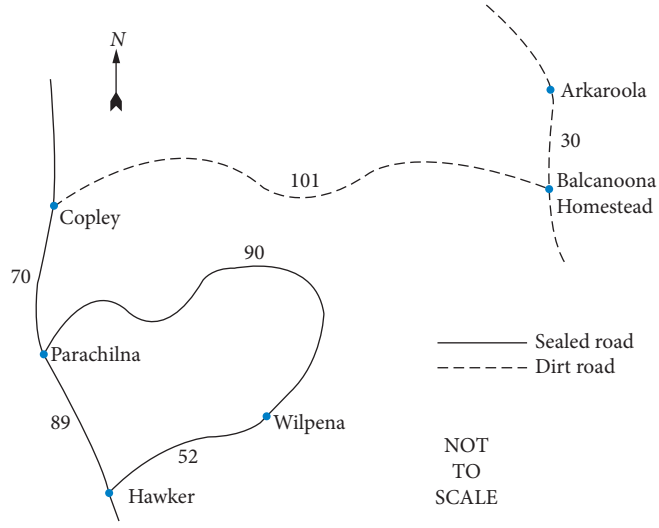
0 10 20 30 km
Scale

- **a** What scenic attraction is at F6?
- **b** What is the name of the town at D7?
- **c** The children wanted to see some kangaroos. The park manager gave them these directions to a place frequented by kangaroos:

  'Travel north-east from Quorn for 24 km and turn left onto a dirt track. Travel another 24 km then take the track to the right.'

  Where did they go to see the kangaroos?
- **d** Mrs Winslow wanted to see the Horsehoe Ranges. She asked for directions to get there from Quorn. What could the directions have been?

8 The next part of the road trip involved travel from Hawker to Arkaroola.



a How far is it from Hawker to Arkaroola via the shortest route?

b The recommended average speed from Hawker to Copley is 80 km/h and from Copley to Arkaroola is 40 km/h. The Winslows followed the recommended average speeds. How long did it take them to drive from Hawker to Arkaroola?

9 At Arkaroola the Winslows had a choice of 2 luxury lodges.

Greenwood Lodge	**Room rates per night**
Basic charge	$199 (up to 2 persons)
Additional adult	$36 (each)
Additional child (aged 2 to 12)	$30 (each)
Room size	1 double bed and 1 single bed

Mawson Lodge	**Room rates per night**
Basic charge	$219 (up to 2 persons)
Additional adult	$36 (each)
Additional child (aged 2 to 12)	$30 (each)
Room size	1 double bed and 3 single beds

The Winslow children are aged 13, 11 and 8.

a Explain why the Mawson Lodge is the more suitable accommodation for the family.

b Calculate the cost of one night's accommodation for the family in the Mawson Lodge.

10 The family decided to fly from Arkaroola to Adelaide. The flight takes 2 hours and their plane flight home to Sydney leaves Adelaide at 1700.

Flights leave Arkaroola at 0600, 0715, 0900, 1345, 1445 and 1600. What flight do you recommend they catch from Arkaroola to Adelaide to be on time for their flight home to Sydney? Give a reason for your answer.

Investigation Where is Arkaroola?

And what is there to see in Arkaroola? Use the Internet to find out some interesting information about Arkaroola and prepare a travel poster to advertise the best features of this location.



AAP Image/South Australian Tourism

12.6

Keyword activity

Copy this crossword, then use the clues to complete it.

Across

5 The letter-number combination that gives the position of a street or object on a street map.

8 What D stands for in the formula $D = S \times T$.

9 A type of direction, for example, north, south, east and west.

Down

1 What 's' stands for in the speed units m/s.

2 Speed is often measured in _____ per hour.

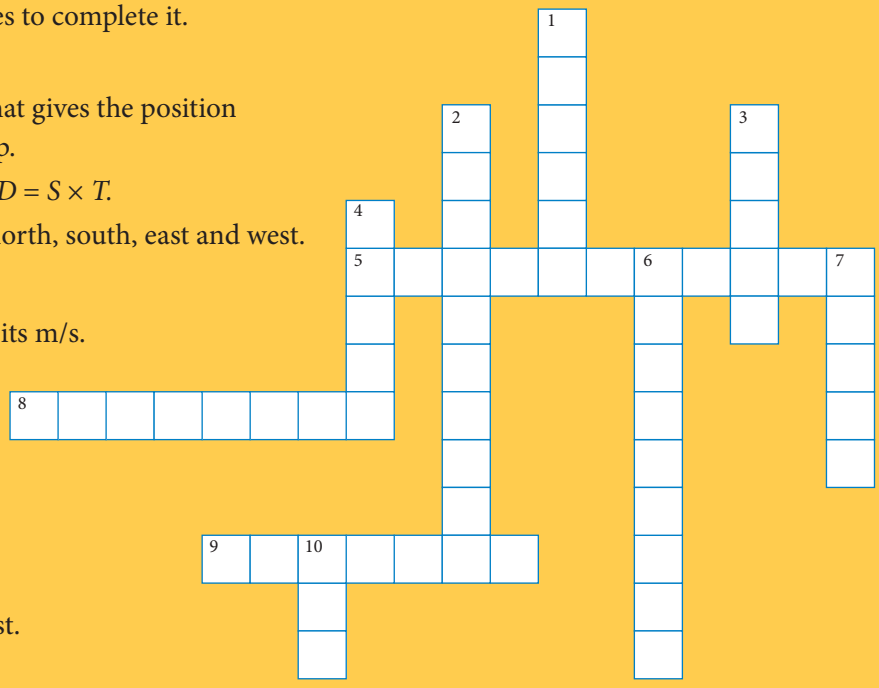
3 On a map, the direction pointing down the page.

4 A ratio that allows us to calculate real distances from a map.

6 The opposite direction to southeast.

7 Distance ÷ time.

10 A scale diagram of a city or region.



Chapter Problem

Nina is planning a 9 km bush walk. She anticipates that she will only be able to walk at a speed of 2.5 km/h over the steep and bushy land. She plans to leave the car park at 10 a.m. and for safety she will log her walk with the park ranger. What time should she tell the park ranger to expect her back at the car park?

Solution

Time taken = distance ÷ speed ($T = D \div S$)
= 9 ÷ 2.5 hours
= 3.6 hours
= 3 hours 36 minutes

Nina will start the walk at 10 a.m. and she will finish 3 h 36 min later. She will finish at 1:36 p.m. She should tell the park ranger to expect her to return before 2 p.m.



Chapter Problem

Ziad and Adrian are arguing about which local rugby league team is the best. The points scored by their favourite teams in each match over the previous season are listed here:

Eagles: 20, 10, 40, 12, 17, 20, 22, 20, 34, 19, 36, 18, 24, 12, 38, 34, 24, 36, 32, 22, 6, 7, 38, 18

Cougars: 14, 18, 24, 39, 14, 4, 4, 14, 10, 13, 28, 22, 16, 18, 18, 12, 18, 28, 21, 6, 10, 18, 36, 12

a What is the best graph to use to show this data? Construct this graph.

b Which is the better team based on this data? Justify your answer.

c What additional data might you need to know in order to decide which is the better team?



CHAPTER 13

COMPARING DATA

13.1 The spread of data
13.2 Standard deviation
13.3 Calculating statistics on a spreadsheet
13.4 Back-to-back stem-and-leaf plots
13.5 Double boxplots
Keyword activity
Solution to the chapter problem

By the end of this chapter, you should know how to:

- use informal ways of describing spread such as spread out/dispersed, tightly packed, clusters, gaps or more/less dense regions
- calculate and interpret statistical measures of spread such as the standard deviation
- compare back-to-back stem-and-leaf plots for different data sets
- compare double boxplots for different data sets

How are we ever going to use this?

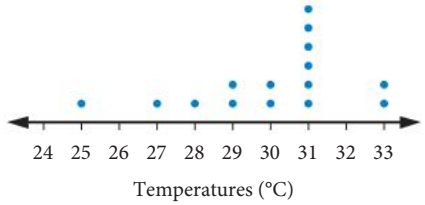
- To calculate summary statistics to allow comparisons of data
- When comparing products for sale
- When examining performances in a variety of sports
- To determine whether data has been analysed correctly in order to support opinions

13.1 The spread of data

When we graph data, such as on a dot plot, we can see the spread of the data scores. They can be spread out or clustered around one place, or there might be gaps or outliers.

Example 1

This dot plot shows the daily maximum temperatures (in °C) in Cairns over 15 days. Comment on the spread of the scores.



Solution

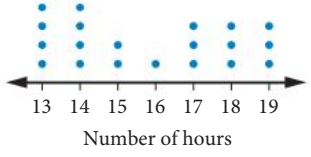
Look for clusters.	These scores are clustered around 31°.
Look for gaps.	There are gaps at the bottom end of the data and at the top end.
The temperatures range from 25 to 33.	These temperatures are quite spread out.
Write a description.	The temperatures are quite spread out but they are clustered around 31°. There are gaps towards the bottom end of the data.

EXERCISE 13.1

The spread of data

EXAMPLE 1

1 Ahmed surveyed his class to find the number of hours each student spent on the Internet each week. Comment on the spread of the scores shown in this dot plot.



2 This stem-and-leaf plot shows the number of malicious property damage reports to police per month during the last few years. Describe the spread of scores shown in this data.

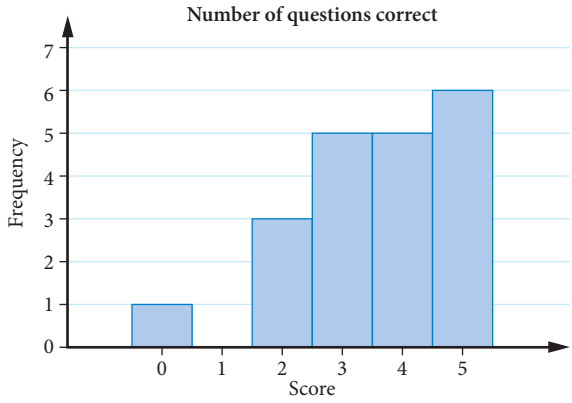
Stem	Leaf
3	7 7 8 8
4	1 3 4 4 5 5 6 6 8 9 9
5	1 1 1 2 2 3 3 4 4 5 5 5 8
6	0 2 3 6 6 7
7	0
8	
9	0

Key: 3|7 = 37

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3 Ms Thomson gave her class a short maths quiz of 5 questions. This frequency histogram shows the results.

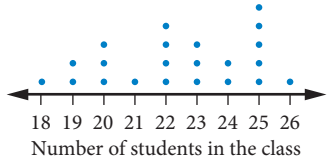
Comment on the spread of the scores shown in this histogram.



4 Copy and complete the descriptions of the following sets of data using words from this list.

spread out	tightly-packed	clustered
gaps	more dense	less dense

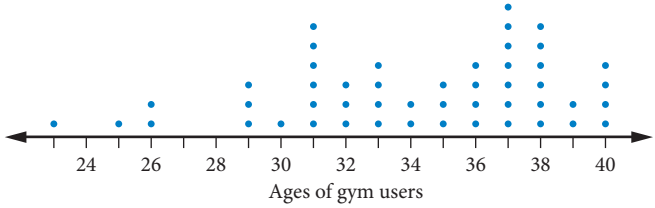
a This dot plot shows the number of students in each class at Cengage Primary School.



Class sizes are _____ between 22 and 25.

The class sizes are _____ from 18 to 26.

b The ages of people exercising at a gym one weekend are shown in the dot plot below.



These ages show _____ between 26 and 29. They are _____ _____ between 36 and 38 but _____ _____ between 23 and 26.

c This stem-and-leaf plot shows the heights of students in a PE class in centimetres.

Stem	Leaf
13	7
14	0 1 3 5
15	3 4 5 7 7 7 8 9
16	2 2 3 6 6 8
17	0 1 5 5 7

Key: 13|7 = 137

These heights are _____ _____ in the 150s and 160s. They are _____ _____ from 137 to 177.

d This stem-and-leaf plot shows the number of people who visited the information desk at Cengage Mall daily over a 3-week period.



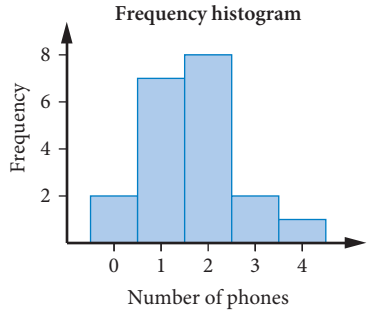
Alamy/travelib europe

Stem	Leaf
7	6
8	1 6 8
9	5 7 8
10	1 5 5
11	2 2 4 7
12	4

Key: 7|6 = 76

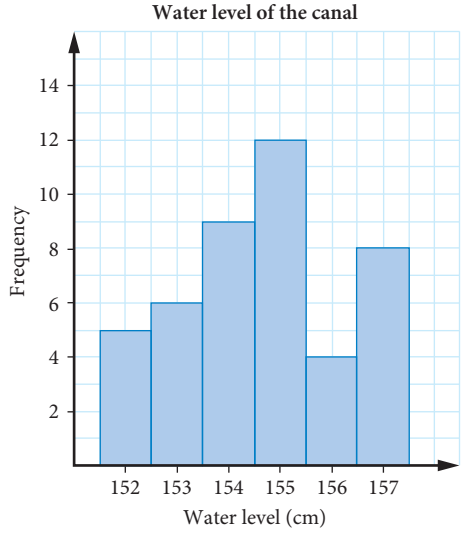
These are evenly _____ _____ across the 80s, 90s, 100s and 110s. The data does not show any _____.

e This frequency histogram shows how many phones were owned in a sample of households.



The scores are _____ at 1 and 2. The responses are not _____ _____ but t_____ _____.

f This histogram shows the water level of a canal recorded over 44 days.



The water level is _____ _____ from 152 to 157. There are no _____.

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Standard deviation

13.2

The **standard deviation** is another **measure of spread**, like the **range** and the **interquartile range**. It describes how far all the scores are from the mean. The bigger the standard deviation, the more spread out the scores are. The formula for standard deviation is quite complicated so you don't need to know it, because you will always use your calculator to find the standard deviation.

Example 2

The net weekly wages of 8 labourers are:

$730	$490	$600	$440
$490	$370	$700	$580

Use your calculator to find:

a the mean wage

b the standard deviation for this data



123RF.com/Dmitry Kalinovsky

Solution

Follow the instructions for statistics mode (SD or STAT) of your calculator. The instructions below may differ from those of your calculator so check with your teacher if you need help.

Operation	Casio scientific	Sharp scientific
Start statistics mode.	MODE STAT 1-VAR	MODE STAT =
Clear the statistical memory.	SHIFT 1 Edit, Del-A	2ndF DEL
Enter data.	SHIFT 1 Data to get table 730 = 490 = , etc. to enter in column AC to leave table	730 M+ 490 M+ , etc.
Calculate the mean. ($\bar{x}$ = 550)	SHIFT 1 Var $\bar{x}$ =	RCL $\bar{x}$
Calculate the standard deviation. (σ_n = 117.26)	SHIFT 1 Var $x\sigma_n$ =	RCL σx
Return to normal (COMP) mode.	MODE COMP	MODE 0

a Look up the mean. Mean = $550

b Look up the standard deviation (σ_n). Standard deviation = $117.26

A large standard deviation means that the wages are quite spread out.

Example 3

Indoor cricket selectors are trying to choose between two pairs of players (Sanjeev/Angus vs Christian/Tyler) for the state team. Their scores (runs) are shown in the table.

Sanjeev/Angus	34	30	36	35	29	34
Christian/Tyler	41	26	37	35	25	34

Which pair is the more consistent?



Newspix/Mark Evans

Solution

Use your calculator to find the standard deviation for each set of data.

Sanjeev/Angus: $\sigma_n = 2.58$
Christian/Tyler: $\sigma_n = 5.74$

The more consistent players are the ones with the lower standard deviation.

Sanjeev/Angus are the more consistent players.

EXERCISE 13.2

Standard deviation

EXAMPLE 2

1 Find the mean and standard deviation of each set of scores. Express your answers correct to 2 decimal places.

a 20, 24, 17, 21, 19, 26, 21, 23, 16

b 60, 51, 82, 65, 27, 38, 32, 64, 18, 20, 74

c 14, 13, 17, 14, 13, 16, 16, 17, 16, 12, 12

d 67, 58, 89, 72, 34, 45, 39, 71, 25, 27, 81

2 Martin is looking for a job in the construction industry. He collected information on the wages of apprentices at a large construction company. The following amounts are the weekly wages of 10 apprentices.

$342, $528, $684, $401, $574, $386, $601, $475, $558, $427

a Find the mean wage.

b Find the standard deviation of this data. Answer correct to 1 decimal place.

c Give a reason for the large standard deviation of this data.

3 Janet is looking for a unit to rent in North Melbourne. She used a website to find the following data about the weekly rent for a number of units in North Melbourne.

$400, $450, $600, $530, $529, $430, $430, $390, $550, $450, $350, $420, $540, $495, $480

a Find the mean rental price.

b Find the standard deviation for this data. Answer correct to 2 decimal places.

c Why can places in the same area have different weekly rents?

d If Janet can afford to pay $420 per week, how difficult will it be for her to find a place to rent?

4 Kate is looking for her ideal place to live. She likes a consistent, warm climate. She is considering two places: Port Paradise and Palm Tree Cove. This table shows the mean monthly temperatures (in °C) of each location.

EXAMPLE 3

	J	F	M	A	M	J	J	A	S	O	N	D
Port Paradise	30	28	26	25	24	24	24	24	26	27	28	30
Palm Tree Cove	32	34	28	26	25	22	20	21	25	29	31	32

a Calculate to 2 decimal places the standard deviations of the temperatures in Port Paradise and Palm Tree Cove.

b Use your answers to part a to advise Kate on which of the two places would suit her better.

5 Brad, Aryn and Kim are competition card players. Each week they play a round of 12 games against other players. The table shows the number of games each of them won per round during the last 6 weeks.

Brad	8	9	7	10	5	11
Aryn	2	4	6	3	7	5
Kim	10	5	8	3	9	4

a Calculate the mean number of games won per round (out of 12) and the standard deviation of games won per round. Answer correct to 1 decimal place.

b Which player is the most consistent? Explain your answer.

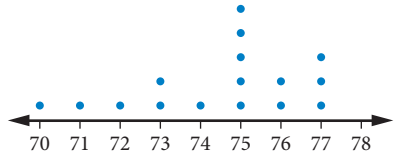
c You have to select one player to represent your club at the regional championships. Who would you choose and why?

6 Paul's golf scores for 16 rounds of golf are shown on this dot plot.

a From the dot plot, write out a list of Paul's scores for the 16 rounds of golf.

b Calculate the mean and the standard deviation for this data.

c Is Paul a consistent player? Explain your answer.



7 This stem-and-leaf plot shows Sofia's pulse rate every 2 minutes while she was exercising on the cross-country machine at the gym.

Stem	Leaf
12	7 9
13	0 3 5
14	3 3 3 6
15	4 5 7

Key: 12|7 = 127

a Would you expect a large or a small standard deviation for this data? Explain your answer.

b Calculate the standard deviation for this data.

c Why do we NOT want a consistent pulse rate in this situation?

13.3 Calculating statistics on a spreadsheet

We can calculate the mode, the mean, the median and other statistics using the statistical functions on a spreadsheet.

EXERCISE 13.3

Calculating statistics on a spreadsheet

1 a Enter into a spreadsheet the following data about the daily maximum temperatures in Alice Springs in one week.

	A	B	C	D	E
1	**Day**	**Temperature (°C)**			
2	Sunday	29		Mean	
3	Monday	31		Mode	
4	Tuesday	30		Median	
5	Wednesday	33		Standard deviation	
6	Thursday	29		Maximum	
7	Friday	28		Minimum	
8	Saturday	35		Range	
9					
10					

b Copy each into the given cells.

Cell E2: =average(B2:B8)
Cell E3: =mode(B2:B8)
Cell E4: =median(B2:B8)
Cell E5: =stdev.p(B2:B8)
Cell E6: =max(B2:B8)
Cell E7: =min(B2:B8)
Cell E8: =E6–E7

Sometimes if you type the first couple of letters, the spreadsheet will suggest the correct word.

c Save your results.

2 a Repeat Question **1** for data for the town or city where you live. Go to the Bureau of Meteorology website www.bom.gov.au to find the data.

b Find data for one month instead of for one week. Repeat Question **1** for this new data. You will need to adjust the formulas you enter for the statistics.

3 a Enter into a spreadsheet the following data showing the monthly number of motor vehicle thefts in a capital city over 2 years.

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	A	B	C	D	E	F
1	**Motor vehicle theft in a capital city**					
2						
3	Jan 2016	20		Jan 2015	15	
4	Feb 2016	19		Feb 2015	19	
5	Mar 2016	26		Mar 2015	17	
6	Apr 2016	17		Apr 2015	21	
7	May 2016	27		May 2015	20	
8	Jun 2016	13		Jun 2015	18	
9	Jul 2016	17		Jul 2015	13	
10	Aug 2016	17		Aug 2015	13	
11	Sep 2016	20		Sep 2015	14	
12	Oct 2016	18		Oct 2015	14	
13	Nov 2016	12		Nov 2015	18	
14	Dec 2016	24		Dec 2015	20	
15						
16	Mean			Mean		
17	Median			Median		
18	Mode			Mode		
19	SD			SD		
20						

b Copy the following formulas into the given cells:

Cell B16: =average(B3:B14)
Cell B17: =median(B3:B14)
Cell B18: =mode(B3:B14)
Cell B19: =stdev.p(B3:B14)
Cell E16: =average(E3:E14)
Cell E17: =median(E3:E14)
Cell E18: =mode(E3:E14)
Cell E19: =stdev.p(E3:E14)

Note: The data for 2015 has many modes, but the spreadsheet only lists one of them!

c Save your results.

d Comment on the differences between the two years.

4 Repeat Question **3** for Melbourne or Sydney. You will need to use the Internet to find the data. Try looking up Crime Statistics and motor vehicle theft.

5 **a** Open a new spreadsheet and create a spreadsheet similar to that in Question **1** for the following data giving the number of hamburgers sold between 11 a.m. and 2 p.m. in Hungry Macs for one week.

Day of week	Number of hamburgers sold
Monday	60
Tuesday	71
Wednesday	63
Thursday	69
Friday	78
Saturday	90
Sunday	86

b Find the mean, the median and the standard deviation for this data and save your spreadsheet.

c Calculate the same statistics for the *weekdays* only.

d Describe the differences between the data for the whole week and the weekday-only data. Why do you think this is so?



Newspix/Craig Greenhill

6 This data shows the heights of 30 students' navels above the ground, in centimetres.

107	101	100	81	97	98	68	96	104	93
90	94	95	86	92	81	85	98	88	90
99	81	100	81	90	104	100	93	98	113

a Create a spreadsheet similar to that in Question **1** for this data.

b Find the mean, the median, the mode and the standard deviation for this data and save your spreadsheet.

c Delete the outlier of 68 cm. Calculate the same statistics for the remaining data and save this new spreadsheet.

d Describe the differences between the two sets of data. Why do you think this is so?

Back-to-back stem-and-leaf plots 13.4

We can use back-to-back stem-and-leaf plots to compare two data sets.

Example 4

Madeline is trying to decide which gym to join. She records the ages of people attending classes at two local gyms in the afternoon.



Alamy/Art Directors & TRIP

Allfit: 17, 22, 51, 12, 27, 43, 39, 20, 21, 15, 15, 43, 15, 20, 32, 21, 23, 16, 34, 22

Superfit: 16, 20, 34, 46, 27, 16, 11, 27, 38, 21, 13, 22, 32, 34, 23, 31, 20, 19, 22, 35

a Draw a back-to-back stem-and-leaf plot for this data.

b Find the median age for each gym.

c Find the range for each gym.

d Are there any outliers in either gym? If so, state the outliers.

e State one similarity between the two sets of data.

f State one difference between the two sets of data.

Solution

a The stem is in the middle.

Allfit		Superfit
7 6 5 5 5 2	1	1 3 6 6 9
7 3 2 2 1 1 0 0	2	0 0 1 2 2 3 7 7
9 4 2	3	1 2 4 4 5 8
3 3	4	6
1	5	

Key: 2\|1\| = 12
\|1\|6 = 16

b	Each gym has 20 scores so the middle scores are the 10th and 11th scores.	Allfit: middle scores are 21 and 22 Median for Allfit $= \frac{21+22}{2}$ $= 21.5$ Superfit: middle scores are 22 and 23 Median for Superfit $= \frac{22+23}{2}$ $= 22.5$
c	Range = highest score − lowest score	Range for Allfit = 51 − 12 = 39 Range for Superfit = 46 − 11 = 35
d	An outlier is an extreme score that is very different from the other scores.	51 is an outlier for Allfit.
e	Look for similar things about the data.	The medians for both sets of data are quite close. Both sets of data are also clustered in the 20s.
f	Look for different things about the data.	Allfit's ages are more spread out than Superfit. Superfit's ages are spread evenly across the 10s, 20s and 30s; the Allfit ages are clustered in the 10s and 20s.

EXERCISE 13.4

Back-to-back stem-and-leaf plots

EXAMPLE 4

1 These are the results of two Year 10 classes in their final mathematics exam.

10A: 84, 71, 79, 82, 78, 89, 71, 95, 93, 81, 85, 65, 70, 95, 91, 89, 89, 75, 62, 71, 69, 88, 94, 81, 85, 76, 80, 67, 60

10B: 88, 60, 66, 74, 42, 59, 61, 68, 54, 65, 56, 46, 65, 71, 57, 72, 40, 65, 41, 38, 35, 39, 51, 47, 39, 42, 39, 42

a Draw a back-to-back stem-and-leaf plot for this data.

b How many students are there in 10A?

c Find the median score for each class.

d Find the range for each class.

e Are there any outliers in either class? If so, state the outliers.

f If one class is the top class and one class is the middle class, which class is which?

2 Mr Farley, the school Principal, is concerned about absences in Year 11 and Year 12. The following shows the number of absentees per day in Year 11 and Year 12 over a 4-week period.

Year 12: 20, 22, 12, 8, 19, 13, 14, 23, 7, 22, 18, 20, 13, 19, 21, 15, 24, 10, 26, 24

Year 11: 30, 15, 30, 23, 39, 20, 31, 42, 22, 41, 30, 25, 23, 30, 22, 30, 29, 15, 15, 44



Alamy/Marmaduke St. John

a Draw a back-to-back stem-and-leaf plot for this data.

b Find the median score for each year group.

c Find the range for each year group.

d Are there any outliers in either year group? If so, state the outliers.

e State one similarity between the two sets of data.

f State one difference between the two sets of data.

g Mr Farley believes there is more absenteeism in Year 11 than in Year 12. What other information would we need to evaluate this statement?

3 For a PE assignment, Kayne compares the heights of males and females in Year 11. The following data he collected are in centimetres.

Males: 178, 183, 167, 184, 181, 170, 190, 181, 181, 200, 183, 160, 165, 172, 178, 178, 168, 191, 181, 190, 180, 184, 180, 175, 170

Females: 178, 166, 166, 150, 168, 166, 163, 162, 167, 159, 157, 185, 176, 164, 165, 164, 160, 185, 176, 177, 171, 152, 173, 173

a Draw a back-to-back stem-and-leaf plot for this data. Use stems of 15, 16, …

b How many males have been measured?

c How many females have been measured?

d Find the five-number summary each for the males and the females.

e Are there any outliers in each group? If so, state the outliers.

f State one difference between the two sets of data.

g Write two or three sentences that Kayne could use as a conclusion for his assignment.

Reminder: Five-number summary is:

- the lowest score
- the lower quartile, Q_1
- the median, Q_2
- the upper quartile, Q_3
- the highest score

4 The following are the daily maximum temperatures for Broome, WA and Kiama, NSW in the month of February.

Broome: 32.1, 32.3, 33.1, 33.4, 31.8, 31.0, 33.7, 34.3, 32.7, 32.7, 31.0, 33.4, 33.6, 34.2, 32.3, 32.5, 31.9, 32.1, 32.8, 31.7, 29.3, 31.9, 31.2, 31.7, 31.2, 32.2, 29.5, 31.0

Kiama: 33.0, 27.9, 32.6, 27.6, 36.3, 29.6, 21.0, 23.0, 22.5, 25.0, 26.4, 22.1, 20.7, 22.3, 24.9, 25.6, 25.5, 24.5, 27.0, 32.5, 21.8, 20.3, 22.5, 24.7, 25.6, 25.9, 26.7, 25.4



Getty Images/Ignacio Palacios

a Draw a back-to-back stem-and-leaf plot for this data. Use stems 20, 21, 22, … up to 36.

b How many days are there in February?

c Find the median score for each town.

d Find the range for each town.

e Are there any outliers in the data for either town? If so, state the outliers.

f State one difference between the two sets of data.

g Write two or three sentences comparing the temperatures in the two towns.

h Which town would you prefer to live in based on this data? Justify your answer.

5 This back-to-back stem-and-leaf plot shows the number of points scored in each match by two basketball teams during last season.

Langley Lynx		Blakely Bears
6 6 5 4 3	4	4 9
8 8 3 0	5	2 3 3 6 8
8 8 6 6 3 1 1	6	5 6 8 9
7 4 3 0	7	0 0 1 3 6
6 6 5	8	2 5 7 7 9 9
2 2	9	0 3 4

Key: 3|4| = 43
|4|9 = 49

a How many games were played in this season?

b Find the range for each team.

c Find the median for each team.

d Find the five-number summary for each team.

e Draw a double boxplot for both teams.

f Comment on the similarities and differences of the points scored by the two teams.

g Which is the better-scoring team?

Double boxplots

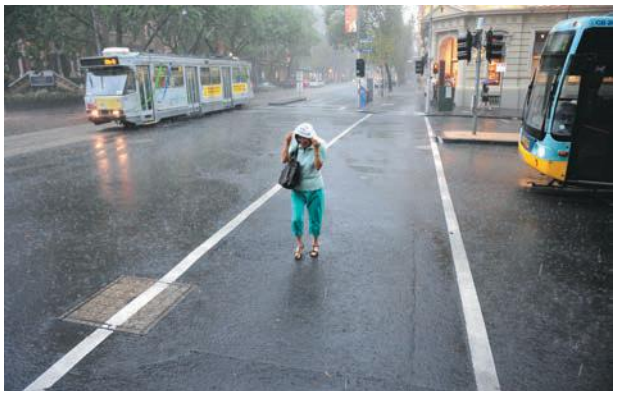
13.5

In statistics, there are many opportunities to compare two sets of data. We can compare sets of data by drawing two or more boxplots using the same scale.

Example 5

The two five-number summaries below describe the number of rainy days per month over two years for Sydney and Melbourne.

Sydney: 9, 11, 13, 14, 15
Melbourne: 7, 10, 14, 16, 19



Fairfax Syndication/Pat Scala

a Draw double boxplots for these summaries.

b Find the median for each city.

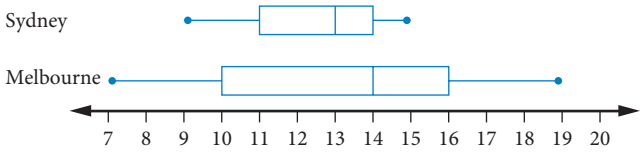
c What is the interquartile range for each city?

d Which city has more rainy days per month?

e If Corrina prefers a more consistent pattern of rainy days, which city would you recommend for her? Justify your answer.

Solution

a Use one scale for both boxplots.



b The median is the middle number in the five-number summary, or use the middle bar in the boxplots.

Median for Sydney = 13.
Median for Melbourne = 14.

c IQR = $Q_3 - Q_1$
In a five-number summary, Q_1 is the 2nd value and Q_3 is the 4th value, or use the ends of the boxes in the boxplots

IQR for Sydney = 14 − 11 = 3.
IQR for Melbourne = 16 − 10 = 6.

d Compare the medians and boxes of the boxplots.

Melbourne has more rainy days per month. Its median is higher and half its scores are above 14 compared to one-quarter of scores for Sydney.

e Consistent means the data is less spread out.

Sydney has the more consistent pattern of rainy days because its range and interquartile range are smaller than Melbourne's. This is shown by its shorter boxplot.

EXERCISE 13.5

Double boxplots

EXAMPLE 5

1 Rigby and Alex are in different Year 11 mathematics classes. The following five-number summaries are for half-yearly exams in each class.

Rigby's class: 48, 64, 75, 87, 96

Alex's class: 47, 57, 69, 80, 97

a Draw double boxplots for these summaries.

b What is the median for :

i Rigby's class? **ii** Alex's class?

c What is the range for :

i Rigby's class? **ii** Alex's class?

d Both Alex and Rigby scored 85% in their half-yearly exams. Who has performed better in relation to their own class? Justify your answer.

e Which class generally performed better in the half-yearly exams? Justify your answer.

f Can we calculate the mean from the given information? Explain.

2 These are the waiting times in minutes for service when calling customer service of two phone companies.

Chatphone: 10, 7, 6, 8, 7, 5, 6, 9, 7, 3, 8, 8, 9, 7, 9, 7, 9, 8

Oztel: 10, 5, 9, 9, 9, 10, 11, 9, 8, 7, 9, 7, 7, 6, 9, 8, 11, 11

a Find the five-number summary for each set of data.

b Construct double boxplots for this data.

c Find the median for each company.

d Find the mode for each company.

e Find the interquartile range for each company.

f Chatphone claims that its waiting times are generally lower than those of Oztel. Is this correct? Justify your answer.

g Is this sufficient information to decide which company you would choose to have your mobile phone with? What other information would you need, if any?

3 The Mayor of Middleton claims that his town is safer for drivers than the nearby town of Blakewell. To test this, James measured the speed (in km/h) of a sample of 20 cars in each town.

Middleton: 60, 65, 70, 68, 62, 75, 80, 83, 82, 69, 73, 75, 85, 72, 67, 88, 90, 85, 72, 63

Blakewell: 76, 64, 58, 82, 72, 70, 68, 75, 63, 67, 74, 70, 79, 80, 73, 75, 71, 68, 72, 73

a Find the five-number summary for each set of data.

b Construct double boxplots for this data.

c Find the median for each town.

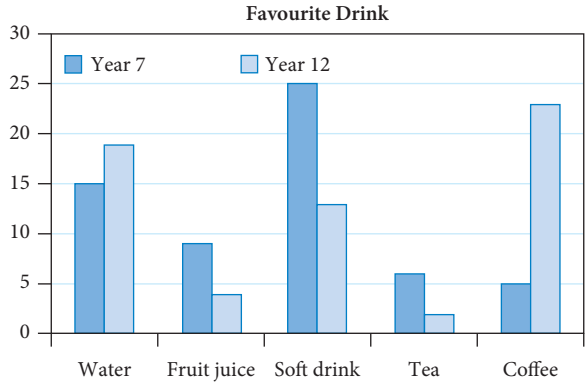
d Find the mode(s) for each town.

e Find the mean for each town.

f Find the interquartile range for each town.

g Is the Mayor of Middleton correct? Justify your answer.

4 Nadine surveyed Year 7 and Year 12 students at her school about their favourite drinks and showed the results on this **clustered column graph**.



a How many Year 7 students did Nadine ask?

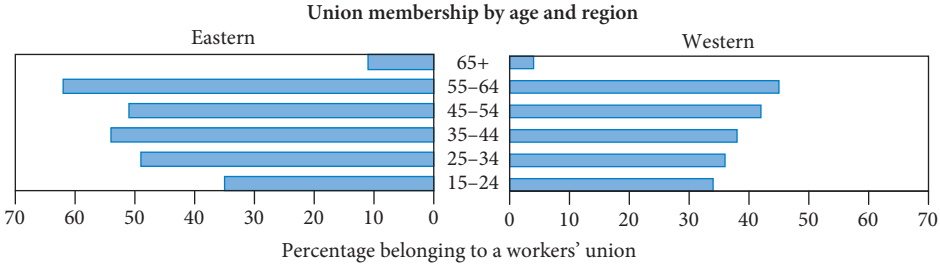
b How many Year 12 students did Nadine survey?

c What is the most popular drink in Year 7?

d What is the most popular drink in Year 12?

e Why do you think the most popular drink is different in Year 7 and Year 12?

5 The workers in two regions were surveyed to find out who belonged to a workers' union, across different age groups. The results are displayed in a back-to-back histogram.



a What percentage of 45- to 54-year-olds in the eastern region belonged to a union?

b Which age group had about the same percentage of union members across both regions?

c Compare the shapes of both distributions.

d Comment on the statement 'People in the eastern region are more likely to join a union'. Justify your answer.



Newspix/Stephen Cooper

6 The monthly average temperatures for Swan Hill and Lakes Entrance are displayed on the radar chart.

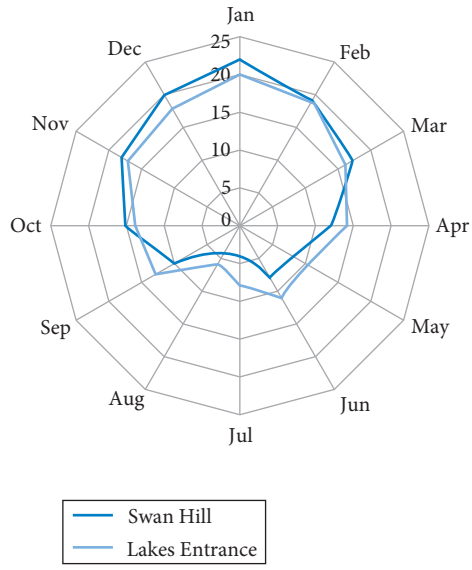
a What was the average temperature in Lakes Entrance in January?

b In which month did both towns have the same average temperature?

c In which months did Swan Hill have the lower average temperature?

d Estimate the range of temperatures in Lakes Entrance.

e Which town would you choose to live in, if you don't like hot summers and cold winters? Justify your answer.



Investigation Comparing sets of data using Census at School

Use data from the CensusAtSchool website www.abs.gov.au/censusatschool to compare two groups.

1. Decide which two groups you will compare, for example, male and female, different states, city vs country, different ages or different years.
2. Decide what numerical data you are going to compare, for example, heights, arm spans, reaction times, foot length.
3. Record 25 values of data for each group you are comparing. Only record sensible measurements and exclude data that is obviously wrong, for example, a height of 60 cm.
4. Draw a back-to-back stem-and-leaf plot for both sets of data.
5. Find the median for each group.
6. Find the range for each group.
7. State one similarity between the two groups.
8. State one difference between the groups.
9. Find the five-number summary for each data set.
10. Construct a double boxplot for both sets of data.
11. Write a short paragraph about the similarities and/or differences of the data based on the box-and-whisker plots.

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Keyword activity

1 Copy and complete: A measure of spread that is based on the mean is called the _____ _____.

2 Describe in your own words a back-to-back stem-and-leaf plot. Draw a rough sketch of what one would look like.

3 What is a five-number summary?

4 Describe in your own words a double boxplot. Draw a rough sketch of what one would look like.

5 Describe in your own words a clustered column graph.

Chapter Problem

Ziad and Adrian are arguing about which local rugby league team is the best. The points scored by their favourite teams in each match over the previous season are listed here:

Eagles: 20, 10, 40, 12, 17, 20, 22, 20, 34, 19, 36, 18, 24, 12, 38, 34, 24, 36, 32, 22, 6, 7, 38, 18

Cougars: 14, 18, 24, 39, 14, 4, 4, 14, 10, 13, 28, 22, 16, 18, 18, 12, 18, 28, 21, 6, 10, 18, 36, 12

a What is the best graph to use to show this data? Construct this graph.

b Which is the better team based on this data? Justify your answer.

c What additional data might you need to know in order to decide which is the better team?

Solution

a This data is best shown on a stem-and-leaf plot.

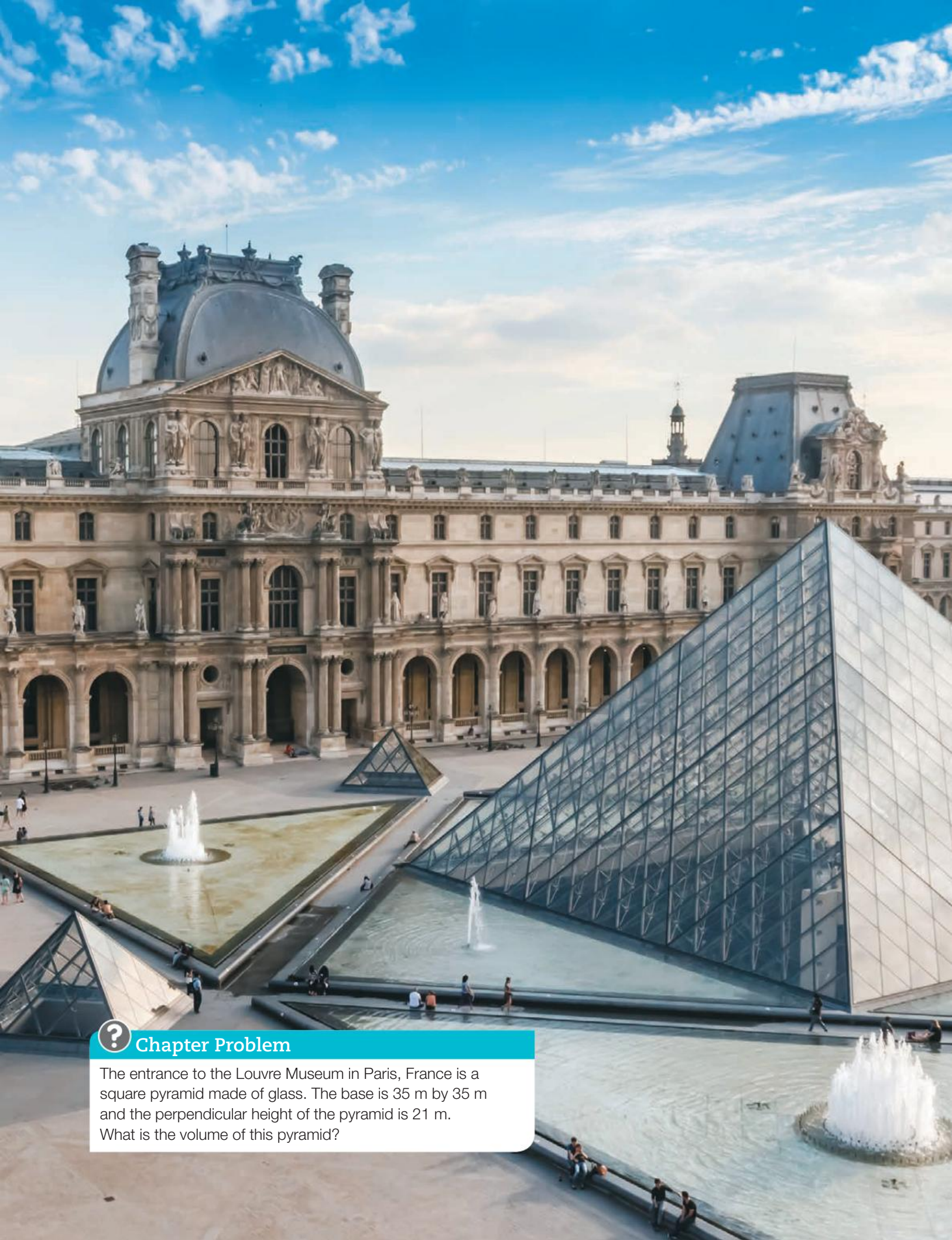
Eagles		Cougars
7 6	0	4 4 6
9 8 8 7 2 2 0	1	0 0 2 2 3 4 4 4 6 8 8 8 8 8
4 4 2 2 0 0 0	2	1 2 4 8 8
8 8 6 6 4 4 2	3	6 9
0	4	

Key: 9|1| = 19
|1|0 = 10

b The median for the Eagles is 21. The median for the Cougars is 17. The scores for the Cougars are clustered in the 10s. The scores for the Eagles are spread evenly across the 10s, 20s and 30s.

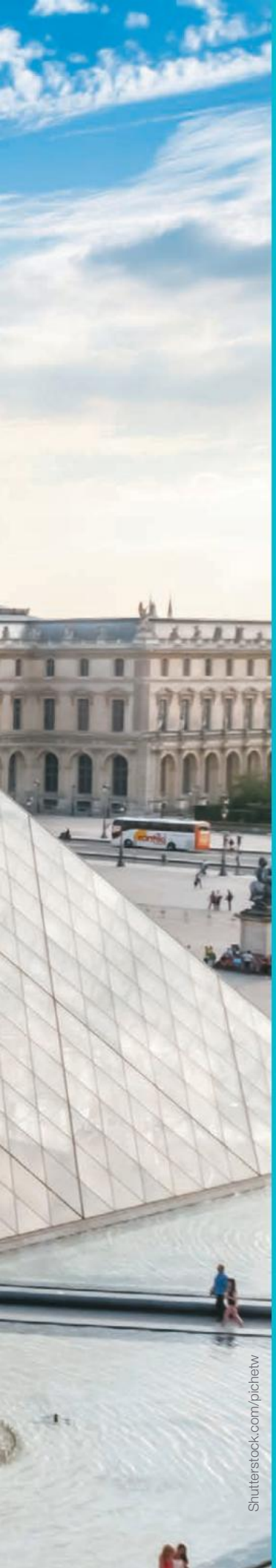
Based on this data the better team is the Eagles.

c You would need to know how many games each team won. It would also help to know how many points were scored against each team.



Chapter Problem

The entrance to the Louvre Museum in Paris, France is a square pyramid made of glass. The base is 35 m by 35 m and the perpendicular height of the pyramid is 21 m. What is the volume of this pyramid?


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CHAPTER 14

TURN UP THE VOLUME

14.1 Measuring volume
14.2 Volumes of rectangular and triangular prisms
14.3 Volume and capacity
14.4 Volumes of prisms and cylinders
14.5 Volumes of pyramids, cones and spheres
Keyword activity
Solution to the chapter problem

By the end of this chapter, you should know how to:

- use metric units of volume and capacity and convert between them
- use formulas to calculate volumes of rectangular prisms, triangular prisms and cylinders
- use formulas to calculate volumes of pyramids, cones and spheres

How are we ever going to use this?

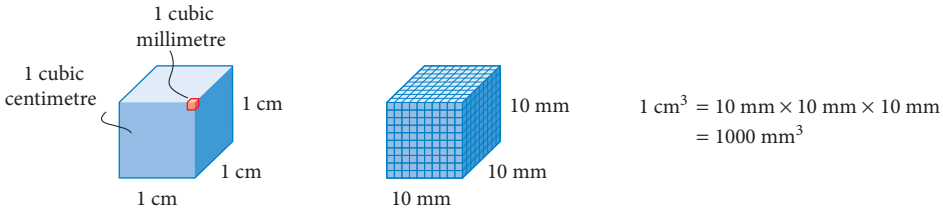
- When calculating the quantity of materials for a job, for example, the amount of soil or mulch we need in the garden
- Volume is a key component in the work of many trades, including plumbers and landscape gardeners
- To determine the capacity of containers

14.1 Measuring volume

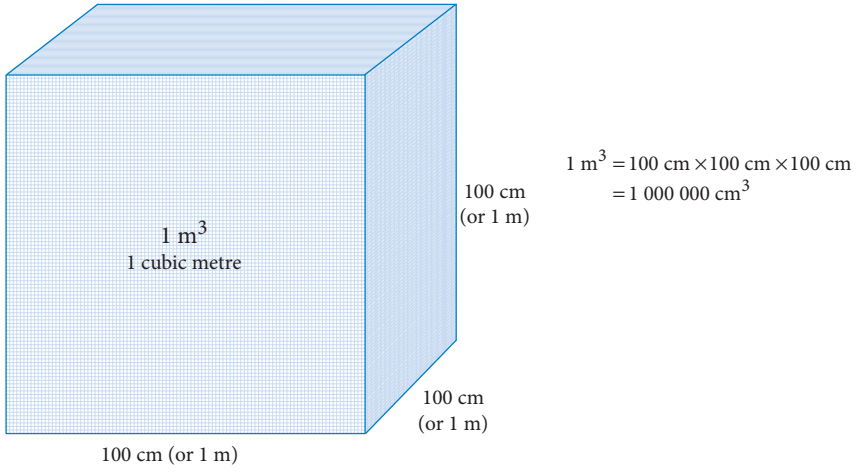
The **volume** of a solid is the amount of space occupied by the solid. Volume is measured in cubic units.

Volume unit	The size of a cube of length:	Approximately the size of:
cubic millimetre (mm^3)	1 mm	a grain of raw sugar or rock salt
cubic centimetre (cm^3)	1 cm	a tooth or a headache tablet
cubic metre (m^3)	1 m	two washing machines

The diagrams below compare a cubic millimetre with a cubic centimetre.



This diagram compares a cubic centimetre with a cubic metre.

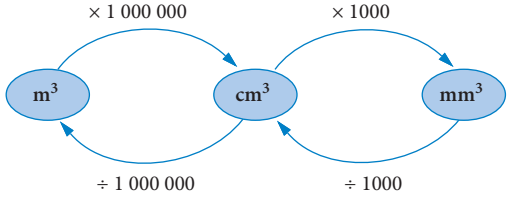


One cubic metre

$1\ cm^3 = 1000\ mm^3$

$1\ m^3 = 1\ 000\ 000\ cm^3$

This diagram shows how to convert between different units of volume.



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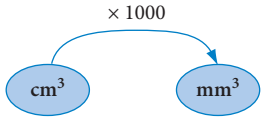
Example 1

Convert:

a 36 cm^3 to mm^3 **b** 84 000 000 cm^3 to m^3

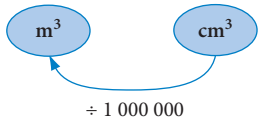
Solution

a cm^3 to mm^3, large to small unit: × 1000.



$36\ cm^3 = 36 \times 1000\ mm^3$
$= 36\,000\ mm^3$

b cm^3 to m^3: small to large unit: ÷ 1 000 000.



$84\,000\,000\ cm^3 = 84\,000\,000 \div 1\,000\,000\ m^3$
$= 84\ m^3$

EXERCISE 14.1

Measuring volume

1 Name which unit of volume you would use to measure the volume of:

a a bedroom
b a backpack
c a mobile phone
d a matchbox
e a concert hall
f a suitcase



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2 Use the information in this table to help you complete each volume conversion.

EXAMPLE 1

Volume unit	Abbreviation	Size description
cubic metre	m^3	1 000 000 cm^3
cubic centimetre	cm^3 or cc	one-millionth of a cubic metre, 1000 mm^3
cubic millimetre	mm^3	one-thousandth of a cm^3

a 5 cm^3 = ____ mm^3 **b** 2 m^3 = _____ cm^3 **c** 500 mm^3 = ___ cm^3
d 0.25 m^3 = ____ cm^3 **e** 24 000 cm^3 = ___ m^3 **f** 36 000 mm^3 = ___ cm^3

3 What units would you use to measure the volume of each object?

a Your calculator
b Your bedroom
c A driver's licence
d A glass of water
e A USB drive
f A box of laundry powder

4 Match the correct volume (**A** to **G**) with each of the items (**a** to **g**) listed.

a bottle of nail polish
b box of tissues
c glass of fruit juice
d bottle of lemonade
e classroom
f school hall
g packet of cereal

A 200 m^3
B 3980 m^3
C 1250 cm^3
D 5000 cm^3
E 20 000 mm^3
F 250 cm^3
G 2200 cm^3

Hint: You may find it helpful to put the items in descending order of volume and then choose the measurements.

5 Copy and complete each conversion.

a 5000 cm^3 = _____ mm^3
b 1.6 m^3 = _____ cm^3
c 6000 cm^3 = _____ m^3
d 4000 mm^3 = _____ cm^3
e 160 000 cm^3 = _____ m^3
f 250 mm^3 = _____ cm^3
g 0.18 m^3 = _____ cm^3
h 0.12 cm^3 = _____ mm^3
i 4 m^3 = _____ mm^3
j 9 600 000 mm^3 = _____ m^3

6 Arrange in ascending order: 42 cm^3, 4210 mm^3, 0.0042 m^3

7 Arrange in descending order: 65 000 cm^3, 0.6 m^3, 7 000 000 mm^3

8 The volume of Marty's chest of drawers is 306 000 cm^3. What is this in cubic metres?

9 Ruchi's lunchbox has a volume of 2520 cm^3. What is the volume of her lunchbox in cubic millimetres?

10 Hans ordered 1 m^3 of pine bark to mulch his vegetable garden. After he finished putting the mulch on his garden he had 20% of the original amount left. How many cubic centimetres of mulch were left over?



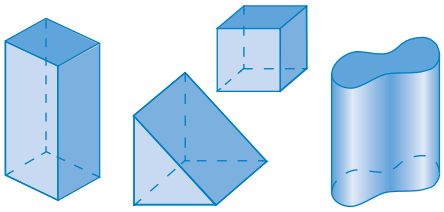
Shutterstock.com/Alison Hancock

Volumes of rectangular and triangular prisms

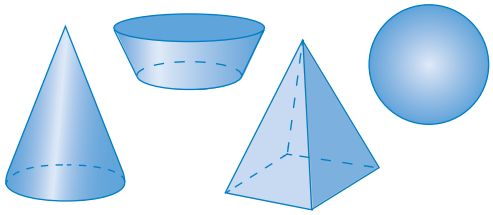
14.2

These diagrams show some solids with identical ends and solids that don't have identical ends.

Examples of solids with identical ends



Examples of solids that don't have identical ends



A cylinder is not a prism because its ends are circles, which are round.

A solid with identical ends and flat sides is called a **prism**. The three prisms shown above on the left are a rectangular prism, triangular prism and cube. The ends of a prism are shapes with straight sides (rectangles, triangles or any other polygon) and its name is part of the name of the prism, for example, rectangular prism, triangular prism.

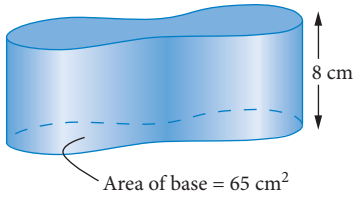
Summary

The volume of a solid with identical ends = $A \times h$

where A is the area of the end or base and h is the height.

Example 2

The area of the base of this solid is 65 cm^2.
What is the volume of the solid?



Solution

Both ends of the solid are identical so we can use the formula $V = A \times h$.

$A = 65$ and $h = 8$.

Volume = 65×8
= 520 cm^3

Formula for the volume of a rectangular prism

For a **rectangular prism**, the base is a rectangle so $A = l \times w$ and the formula becomes $V = l \times w \times h = lwh$.

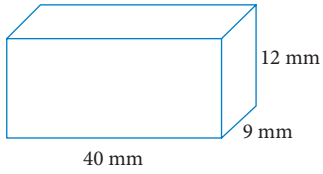
$V = lwh$

where l = length, w = width, h = height

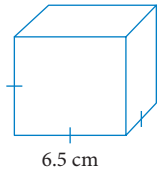
Example 3

Find the volume of each prism.

a



b



Solution

a Use the formula for a rectangular prism.

$V = lwh$

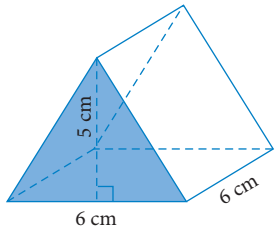
$V = 40 \times 9 \times 12$
$= 4320 \text{ mm}^3$

b $V = lwh$

$V = 6.5 \times 6.5 \times 6.5$ or 6.5^3
$= 274.625 \text{ cm}^3$

Example 4

Find the volume of this triangular prism.



Solution

For a triangular prism, $A = \frac{1}{2}bh$.

$b = 6, h = 5$.

$V = (\frac{1}{2} \times 6 \times 5) \times 6$
$= 90 \text{ cm}^3$

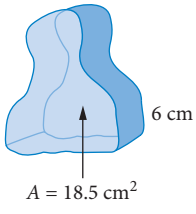
EXERCISE 14.2

Volumes of rectangular and triangular prisms

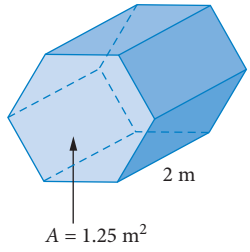
EXAMPLE 2

1 Find the volume of each solid.

a

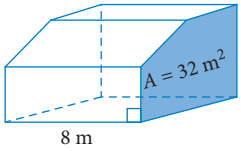


b

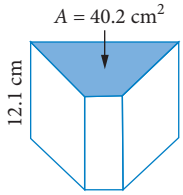


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c



b

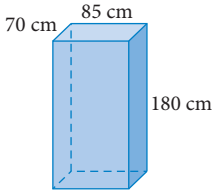


2 Find the volume of each prism.

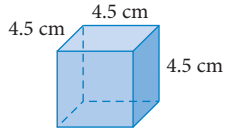
EXAMPLE 3

14.2

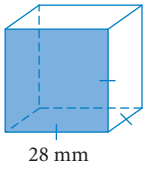
a



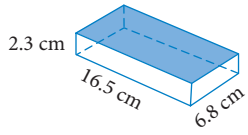
b



c



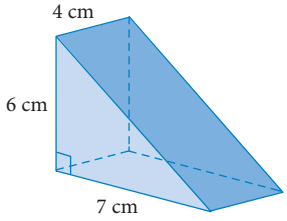
b



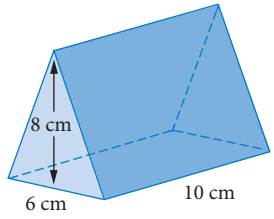
3 Find the volume of each prism.

EXAMPLE 4

a



b



4 Concrete blocks are used to build houses.

a If each concrete block measures 45 cm by 17 cm by 21 cm, calculate the volume of one block.

b A wall is made from 80 concrete blocks. Calculate the volume of the wall.

5 These storage cubes have a side length of 35 cm.

a Calculate the volume of one cube.

b Inga has a stack of 11 cubes in her bedroom to store books. What volume of books can she store?

c Inga's books have an average volume of 1425 cm^3 each. Approximately how many books can she store?



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6 Calculate the volume of each box.

a



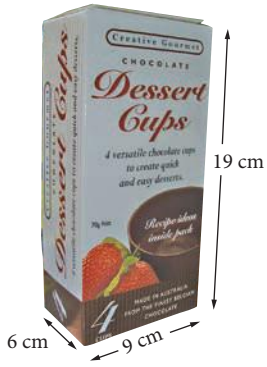
b



c



d



e



Alamy/Mediablitzimages

7 Michael is laying pavers for a new rectangular outdoor area, 4.2 m by 2.7 m. Each paver is 300 mm by 300 mm and 50 mm thick.

a How many pavers will he need?

b Calculate the volume of one paver.

c Calculate the volume for the total number of pavers required for this job.

d Pavers come in boxes of 10. How many boxes will Michael need to purchase?

e Each box costs $56.70. How much will the pavers cost?

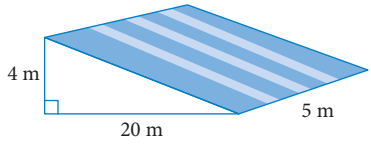
Remember to have all your measurements in the SAME units!



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8 This concrete ramp was built at Hawk's Garden beach to give easy access to the beach. Calculate the volume of concrete required for this ramp.



9 Nazneen plans to install air-conditioning inside her house. The air conditioner is available in 4 sizes.

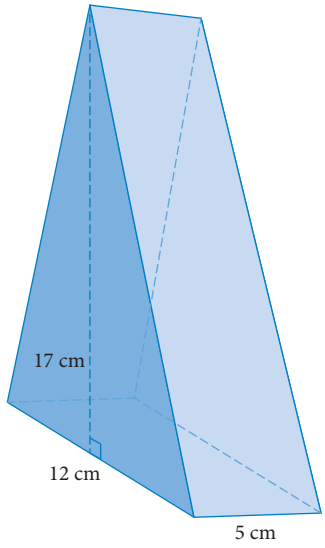
Air conditioner size	Volume of air in house
Small	210 m^3
Medium	350 m^3
Large	500 m^3
Extra large	720 m^3

Nazneen's house is in the shape of a rectangular prism, with the dimensions shown on the photograph.



What size air conditioner will Nazneen need?

10 Rhianna won the Star Performance award at her school concert. Find the volume of glass used to make this trophy.



Investigation Volume of a car

It takes practice to become good at estimating. Complete these practical estimation activities to develop your group's estimation skills.

To complete these activities, each group will need metre rulers or measuring tapes.

Estimating volume					
Names	Group members' estimates				Real measurement
Volume of a car boot					
Volume of a car					

Activity 1: The volume of a car boot

Luggage compartments of cars come in different sizes. Before your group starts this activity, select an appropriate car.

Record the group's estimates for the volume, then measure the width, depth and height of the car boot in metres. Use the formula Volume = Area of the base × height to calculate the volume in m^3.



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Activity 2: The volume of a car

The volume of most cars can be modelled by using 2 or 3 rectangular prisms. Before your group starts this activity, select an appropriate car.



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Record the estimates, then measure the width, depth and height of the sections of the car in metres. Use the formula Volume = Area of the base × height to calculate the volume in m^3 of each section. Then add the sections to determine the total volume.

Volume and capacity

14.3

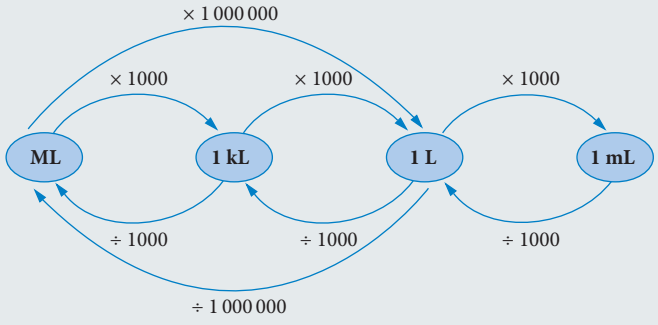
Volume measures the amount of space inside a container while capacity measures the amount of liquid or gas a container will hold. Capacity is measured in millilitres, litres, kilolitres and megalitres.

Capacity unit	Approximately the size of:
millilitre (mL)	a large drop of water
litre (L)	a tall carton of milk
kilolitre (kL)	a small rainwater tank
megalitre (ML)	half an Olympic-sized swimming pool

1 L = 1000 mL

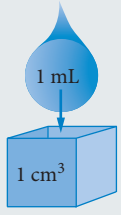
1 kL = 1000 L

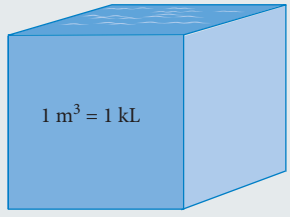
1 ML = 1000 kL = 1 000 000 L



1 cm^3 holds 1 mL

1 m^3 holds 1000 L or 1 kL





Example 5

Convert:

a 5 cm^3 to mL **b** 1850 mL to litres

Solution

a 1 cm^3 holds 1 mL. The number of cm^3 and mL are always the same.

5 cm^3 holds 5 mL

b mL to L: small to large unit: ÷ 1000.

1850 mL = 1850 ÷ 1000 L
= 1.85 L

Example 6

The volume of a large fishpond is 3.4 m^3.
How many litres of water does it hold?



Shutterstock.com/Ron Zmiri

Solution

1 m^3 holds 1000 L. We multiply the number of m^3 by 1000.	3.4 m^3 holds 3.4 × 1000 L = 3400 L
Write your answer.	The fishpond holds 3400 L of water.

EXERCISE 14.3

Volume and capacity

1 State what unit of capacity you would use to measure the size of:

a a glass of milk
b a dam
c a car's petrol tank
d a bottle of medicine
e an office water cooler
f a swimming pool

2 Match the correct capacity (**A** to **J**) with the items (**a** to **j**) listed.

a	car petrol tank	**A**	200 mL
b	a cup of flour	**B**	23 kL
c	bath tub	**C**	5 mL
d	bucket of water	**D**	70 L
e	can of drink	**E**	1250 mL
f	glass of water	**F**	1.875 ML
g	Olympic swimming pool	**G**	250 mL
h	bottle of lemonade	**H**	9 L
i	teaspoon	**I**	375 mL
j	water storage tank	**J**	180 L

Hint: You may find it helpful to put the items in descending order of capacity and then choose the measurements.

EXAMPLE 5

3 Convert each measurement to mL.

a 8 cm^3
b 1500 cm^3
c 425 cm^3

4 Convert each measurement to litres.

a 2000 mL **b** 3500 mL **c** 250 mL

5 **a** The volume of a large container is 5000 cm^3. How many millilitres does the container hold?

b How many litres will a container with a volume of 5000 cm^3 hold?

EXAMPLE 6

14.3

6 How many litres can a 2 m^3 container hold?

7 How many litres of water does a water truck with a volume of 4 m^3 hold?

8 Liam is pouring 1500 cm^3 of liquid chlorine into the swimming pool. Express this quantity in litres.

9 What is the volume of a carton that holds 1 L of milk?

10 What is the volume in cubic centimetres of a 1.25-litre soft drink bottle?

11 Each can in a box of 24 cans of soft drink holds 375 mL. How many litres of soft drink are contained in the box?

12 A tap leaks 10 mL of water every 50 seconds. How much water will the tap lose in:

a 1 second? **b** 1 minute? **c** 3 hours? **d** 1 day?

13 Zina's swimming pool is 5.8 m long and 3.2 m wide. Hot, dry winds from Central Australia caused 11 cm of water in the pool to evaporate.

a What solid shape could be used to represent the volume of water that evaporated from Zina's pool?

b How many cubic metres of water evaporated from the pool?

c How many litres of water are required to top up Zina's pool?

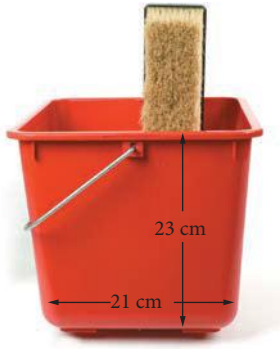
d The pump on Zina's water tank delivers 105 L per minute. For how long will she need to pump water from the tank into her pool to replace the evaporated water? Express your answer correct to the nearest minute.

14 This bucket has a square base with sides of 21 cm.

a The sides of the bucket are 23 cm high. Calculate the volume of the bucket in cm^3.

b How many whole litres of water can the bucket hold?

c One litre of water weighs 1 kg. Approximately how much will the bucket weigh when it is half-full of water?



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14.4 Volumes of prisms and cylinders

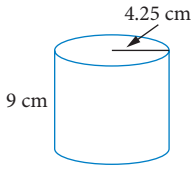
The volume of a solid with identical ends = $A \times h$

where A is the area of the end or base and h is the height.

We can use this formula to find the volume of all prisms and cylinders. For a cylinder, the base is a circle, with area $A = \pi r^2$.

Example 7

Find the volume of this cylinder, correct to 2 decimal places.



Solution

$V = Ah$ where $A = \pi r^2$.

$r = 4.25$ and $h = 9$.

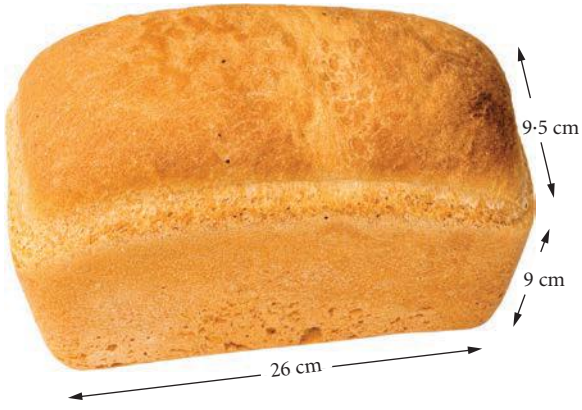
$V = (\pi \times 4.25^2) \times 9$
$= 510.7051\ldots$
$\approx 510.71\ \text{cm}^3$

EXERCISE 14.4

Volumes of prisms and cylinders

1 The end of this loaf of bread is a rectangle 9 cm by 9.5 cm.

- **a** What is the area of the end of the loaf of bread?
- **b** The loaf is 26 cm long. Calculate the volume of the loaf of bread.
- **c** The loaf contains 22 slices of bread. Calculate the average volume of a slice of bread correct to the nearest cm^3.



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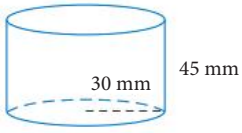
2 The oil in this bottle is 15 cm deep and the area of the bottom of the bottle is 35.5 cm^2. Calculate the volume of oil in the bottle.



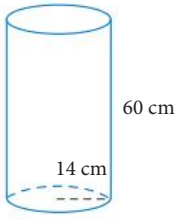
Scout Kozakiewicz

3 Find the volume of each cylinder, correct to 2 decimal places.

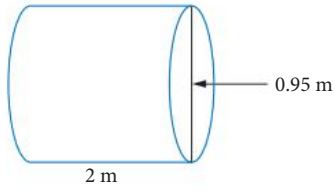
a



b



c



EXAMPLE 7

4 A small cylindrical water tank has a diameter of 720 mm and a height of 970 mm.

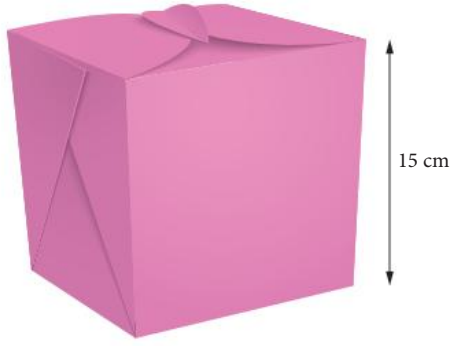
a Calculate the volume of the water tank in m^3, correct to two decimal places.

b What is the capacity of the water tank in litres? Answer correct to the nearest 10 litres.

Hint: Change the measurements to metres before you start.

Remember: 1 m^3 = 1000 L

5 The base of this take-away food box is 300 cm^2. What is the volume of the box?



Shutterstock.com/Natalia Semenchenko

6 Gianni makes soft drink cans. He is experimenting with a new size of can. Which of these options provides the larger volume? Justify your answer with calculations.

A a can of radius 3.95 cm and height 11.8 cm

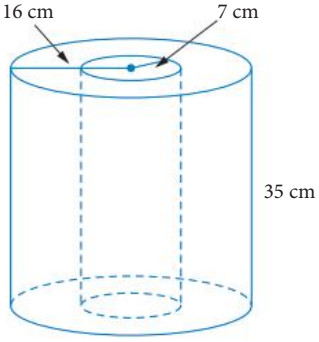
B a can of diameter 6.1 cm and height 14 cm

7 A hollow cylinder is shown.

a Find the volume of a cylinder with radius 16 cm, correct to 2 decimal places.

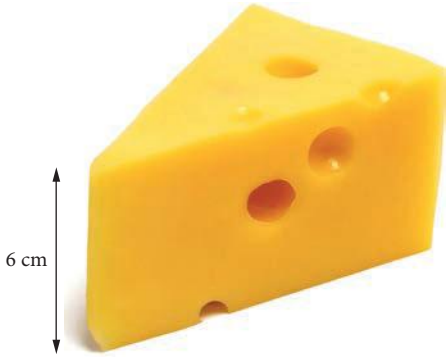
b Find the volume of the 'hole' with radius 7 cm, correct to 2 decimal places.

c Calculate the volume of the hollow cylinder by subtracting your answer to part **b** from your answer to part **a**.



8 Calculate the volume of each food item, given the area of its base.

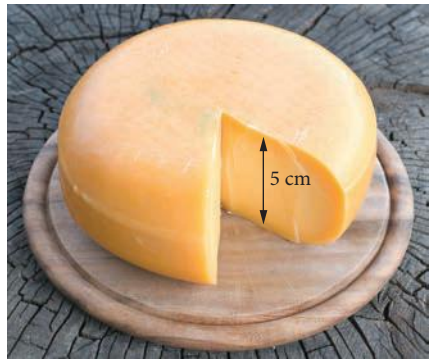
a



iStockphoto/PIKSEL

Area of the base = 32 cm^2

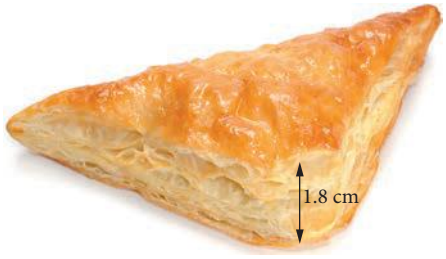
b



© Emprize/Dreamstime.com

Area of the base = 254 cm^2

c



Thinkstock/Vitali Dyatchenko

The area of the base = 24 cm^2

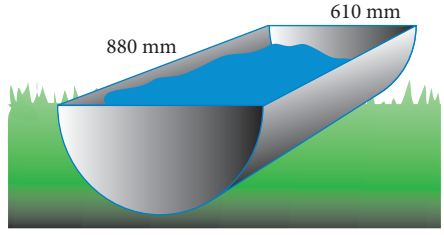
d



Thinkstock/ekinyalgin

The area is 18.5 cm^2 and it is 1.2 cm thick.

9 Silvana runs a farm. She uses half-cylinder water troughs as shown for the animals.



a Calculate the volume of one water trough in cubic metres, correct to 2 decimal places.

b How much water is needed to fill one water trough?

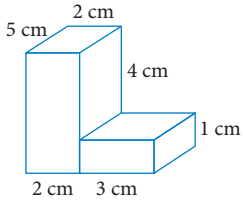
c There 25 water troughs on the farm. How much water is needed to fill all troughs?

d Silvana uses a cylindrical tank on a trailer to fill the water troughs. Calculate the volume of the tank, correct to two decimal places, if its diameter is 1560 mm and its height is 1210 mm.

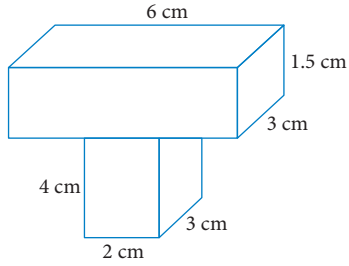
e Can this tank hold enough water to fill all the troughs? If not, how many trips will Silvana have to make?

10 Calculate the volume of each prism.

a



b



9780170370752

11 Stuart installed a large pipe to transport water from a dam to a nearby community. The area of the circular cross-section of the pipe is 0.95 m^2 and the pipe is 8.7 km long.

a What is the length of the pipe in m?

b Calculate the volume of water that the pipe can hold. Answer in cubic metres.

c How many litres of water are in the pipe when it is full?



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Practical activity Designing a smaller can

Changing the dimensions of a package is one method that manufacturers use to disguise price rises. Keeping the price the same but reducing the volume is equivalent to increasing the price.

This can has a base radius of 5 cm and a height of 10 cm. It holds 785 mL. Your task is to design a can that looks almost the same size but contains 25% less volume.



www.dlibrary.com.au/Pedigree

Practical activity SketchUp Part 3

You will use the buildings you designed in the SketchUp practical activity in Chapter **9**.

The swimming pool or water tank

- Find the dimensions of your swimming pool/water tank.
- Calculate the volume of your swimming pool/water tank, firstly in cubic metres and then in litres.

The building

- Calculate the volume of each room in your building.
- Estimate the total volume of your building.

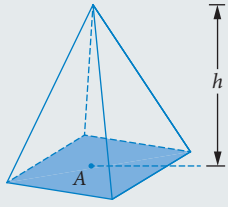
14.5 Volumes of pyramids, cones and spheres

There are special formulas for calculating the volumes of a pyramid, cone and sphere.

Volume of a pyramid

Volume $= \frac{1}{3} \times$ area of the base $\times$ height

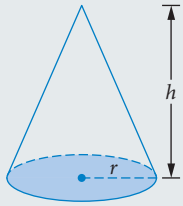
$V = \frac{1}{3}Ah$



Volume of a cone

Volume $= \frac{1}{3} \times$ area of the circular base $\times$ height

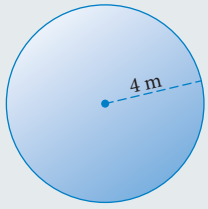
$V = \frac{1}{3}\pi r^2 h$ (because the area of a circle is $A = \pi r^2$)



Volume of a sphere

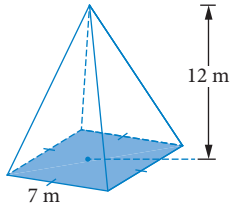
Volume $= \frac{4}{3} \times \pi \times \text{radius}^3$

$V = \frac{4}{3}\pi r^3$



Example 8

Calculate the volume of this pyramid.



Solution

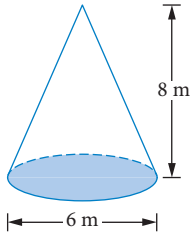
This is a square pyramid. Use the formula for the volume of a pyramid $V = \frac{1}{3}Ah$ where $A = 7 \times 7$.

$V = \frac{1}{3} \times 7 \times 7 \times 12$
$= 196 \text{ m}^3$

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Example 9

Find the volume of this cone. Give your answer correct to the nearest cubic metre.



Solution

Use the formula for the volume of a cone

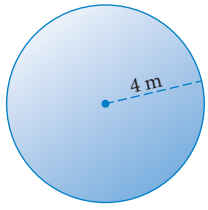
$V = \frac{1}{3}\pi r^2 h$ where $r = \frac{1}{2} \times 6 = 3$ and $h = 8$.

$$V = \frac{1}{3} \times \pi \times 3^2 \times 8$$
$$= 75.398\,22\ldots$$
$$\approx 75 \text{ m}^3$$

Remember to halve the diameter to find the radius!

Example 10

Find the volume of this sphere. Give your answer correct to 2 decimal places.



Solution

Use the formula for the volume of a sphere

$V = \frac{4}{3}\pi r^3$ where $r = 4$.

$$V = \frac{4}{3} \times \pi \times 4^3$$
$$= 268.0825\ldots$$
$$\approx 268.08 \text{ m}^3$$

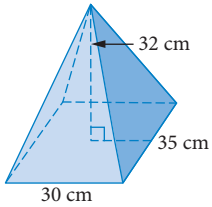
EXERCISE 14.5

Volumes of pyramids, cones and spheres

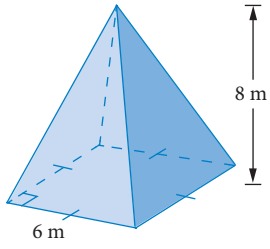
1 Find the volume of each pyramid.

EXAMPLE 8

a



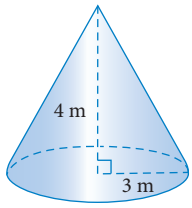
b



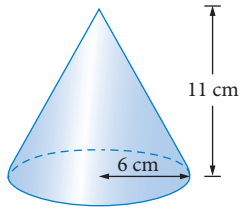
EXAMPLE 9

2 Find correct to one decimal place the volume of each cone.

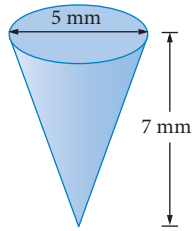
a



b



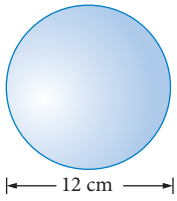
c



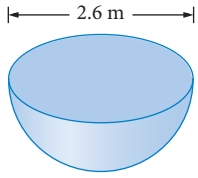
EXAMPLE 10

3 Find correct to one decimal place the volume of the sphere and hemisphere ('half sphere').

a



b



4 The Great Pyramid of Khufu, which was built in 2550 BCE, is in the shape of a square pyramid. The sides of the square are 230 m long, and the original height was 146 m. Calculate the original volume of the pyramid. Express your answer correct to the nearest cubic metre.



iStockphoto/CPaulussen

5 An ice-cream cone is full of ice-cream, and there is a hemisphere of ice-cream on top of the cone.

a What is the radius of the hemisphere?

b What is the radius of the top of the cone?

c Determine the amount of ice-cream that is contained in the cone and the hemisphere. Express your answer correct to the nearest mL.



Shutterstock.com/Stockcreations

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6 A large sphere with a radius of 5.5 m is used to store gas in an oil refinery.

a Calculate the volume of the oil refinery sphere in cubic metres. Give your answer correct to 3 decimal places.

b What is the volume in litres?

7 The top of the roof of this church is a square pyramid on top of a square prism.

a Find the height of the square pyramid.

b Calculate the volumes of the pyramid and the prism.

c How much air is contained inside this prism–pyramid feature?



Alamy/Nick Hawkes

8 Red cones are often used by road workers to indicate that a section of the road is closed to traffic. These cones are 66 cm tall and have a base diameter of 31 cm.

a What is the volume one cone? Give your answer correct to the nearest cubic centimetre.

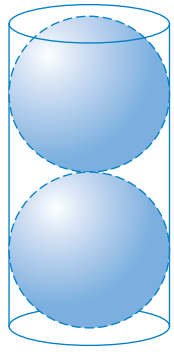
b If these cones were solid plastic they would be very heavy. In fact 85% of their volume is a hollow space inside a thin plastic cone. What is the volume of the plastic used in making one of these cones?

9 The radius of each sphere in the diagram is 6 cm.

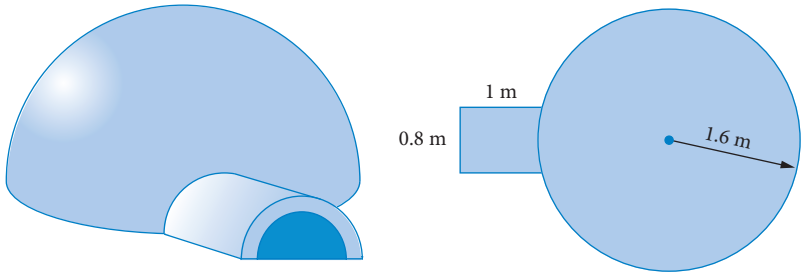
a Calculate the total volume of the two spheres.

b What is the radius and height of the cylinder?

c Calculate the volume of the cylinder.

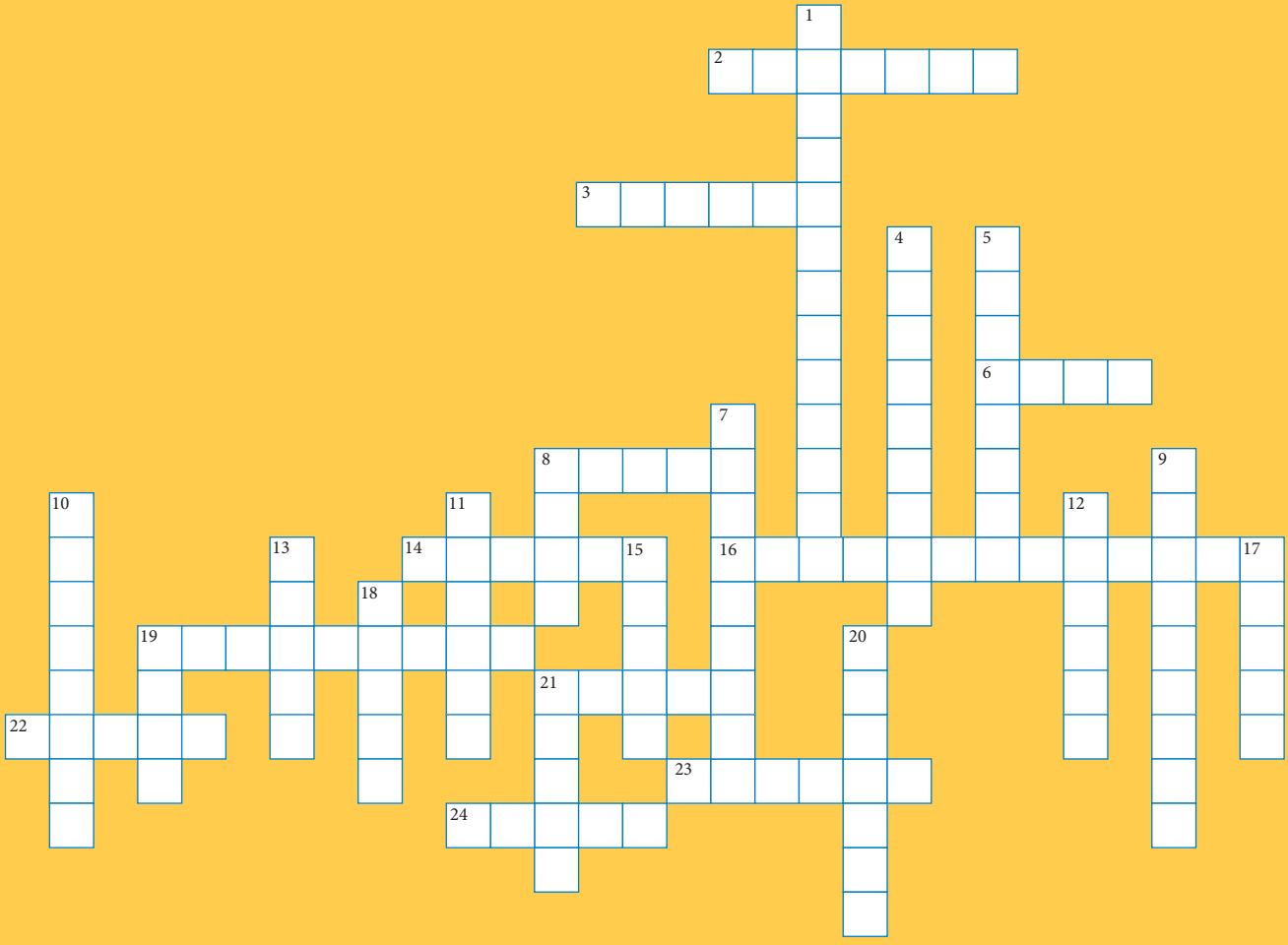


10 Canadian and Alaskan Eskimos used igloos as temporary shelters. The main section was a hemisphere, and the entrance was half a cylinder. The diagram shows the internal dimensions of an igloo. How much air was inside an empty igloo? Express your answer in cubic metres correct to 1 decimal place.



Keyword activity

Make a copy of this crossword and complete it using the clues below.



Across

2 Another word for 'width' beginning with B.

3 A rectangle whose length and width are the same.

6 The amount of surface enclosed by a shape.

8 A millionth of a metre is μm, called a _____-metre.

14 Not length or width.

16 Quadrilateral with opposite sides equal.

19 This unit of capacity has abbreviation kL.

21 A hundredth of a metre is cm, called a _____-metre.

22 A thousandth of a metre is mm, called a _____-metre.

23 How long something is.

24 Solid shape with identical ends and flat sides.

Down

1 At right angles.

4 Its area is length × width.

5 Its area is $\frac{1}{2}$ × base × height.

7 Type of shape made up of two or more shapes.

8 A million litres is ML, called a _____-litre.

9 The distance around the outside of a shape.

10 The amount of material that a container can hold.

11 Our system of measurement.

12 The amount of space occupied by a solid.

13 A 'slice' of a solid shape is called a _____-section.

15 1000 kg

17 1000 mm

18 A unit of capacity.

19 A thouand litres is kL, called a _____-litre.

20 The unit of area that is the size of a 100 m by 100 m square, abbreviation ha.

21 Volume can be measured in _____ metres.

Chapter Problem

The entrance to the Louvre Museum in Paris, France is a square pyramid made of glass. The base is 35 m by 35 m and the perpendicular height of the pyramid is 21 m. What is the volume of this pyramid?

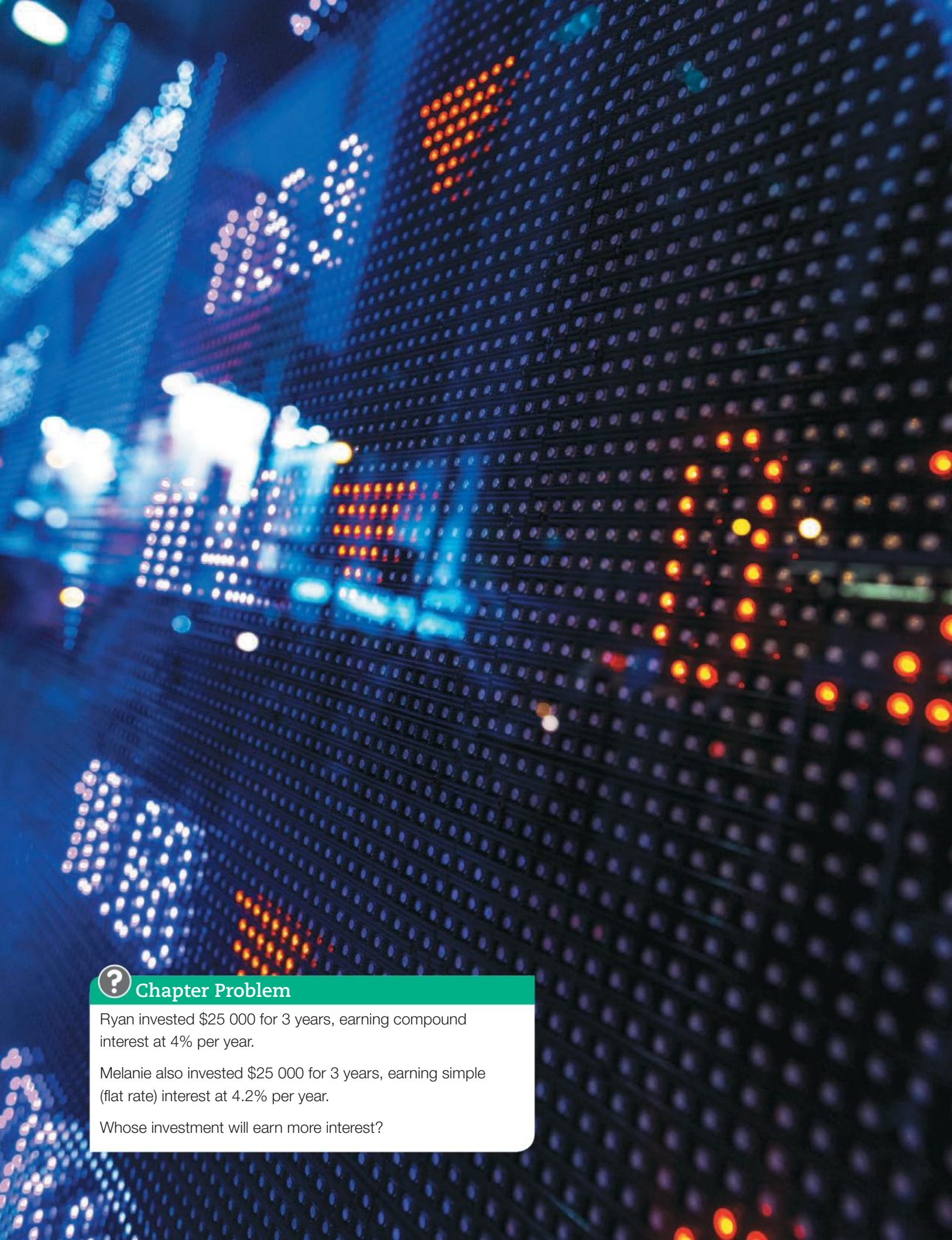
Solution

The volume of a pyramid is given by the formula

$V = \frac{1}{3}Ah.$

$V = \frac{1}{3} \times 35 \times 35 \times 21$

$= 8575 \text{ m}^3$



Chapter Problem

Ryan invested $25 000 for 3 years, earning compound interest at 4% per year.

Melanie also invested $25 000 for 3 years, earning simple (flat rate) interest at 4.2% per year.

Whose investment will earn more interest?



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CHAPTER 15

INTERESTING NUMBERS

15.1 Simple interest
15.2 Compound interest
15.3 The compound interest formula
15.4 Compound interest using technology
15.5 Significant figures
15.6 Scientific notation
Keyword activity
Solution to the chapter problem

By the end of this chapter, you should know how to:

- calculate simple and compound interest
- calculate the future value of an investment
- recognise and avoid potential financial scams

How are we ever going to use this?

- Determine the chance that an investment is safe
- Choose a safe investment that will give us reasonable return
- Make calculations to check that we have received the correct investment returns
- Avoid falling for financial scams

15.1 Simple interest

When we invest money with **simple** (or **flat rate**) **interest**, the amount of interest earned is the same every year. The interest is a **percentage** of the **principal**, the original amount invested.

The simple interest formula is

$I = Prn$

where P = principal (what we invest)

r = the rate of interest per time period, as a decimal

n = number of time periods

Example 1

Selina invested $2000 at 3.25% p.a. simple interest for 3 years.

a How much interest will she earn?

b How much will be in her account at the end of 3 years?

p.a. = per annum = per year

Solution

a In the simple interest formula $I = Prn$, $P = 2000$, $r = 3.25\% = 0.0325$ and $n = 3$.

$\text{SI} = 2000 \times 0.0325 \times 3$
$= 195$

Write your answer.

Selina will earn $195 in simple interest.

b At the end of 3 years, Selina will have her original investment plus the interest.

$\text{Account total} = \$2000 + \$195$
$= \$2195$

Write your answer.

At the end of 3 years there will be $2195 in Selina's account.

Example 2

Raoul invested $5600 at 3.9% p.a. for 18 months. How much interest did he earn?

Solution

The interest rate is per year but the time is in *months*. We much change 18 months to years by dividing by 12.

$\text{Time} = \frac{18}{12}$ years

In the simple interest formula $\text{SI} = Prn$, $P = 5600$, $r = 0.039$, $n = \frac{18}{12}$.

$\text{SI} = 5600 \times 0.039 \times \frac{18}{12}$
$= 327.6$

Write your answer.

Raoul earned $327.60 in interest.

When the interest rate is p.a. the time must be in years. Divide months by 12 to change the time into years.

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Example 3

What percentage interest rate per month is equivalent to 5.16% p.a.?

Solution

We divide an annual interest rate by 12 to change it to a monthly interest rate.	$5.16\% \div 12 = 0.43\%$
Write your answer.	5.16% p.a. is equivalent to 0.43% per month.

EXERCISE 15.1

Simple interest

1 Calculate the simple interest on each principal.

EXAMPLE 1

a \$700 at 6.2% p.a. for 3 years

b \$520 at 4.2% p.a. for 5 years

c \$3400 at 4.85% p.a. for $2\frac{1}{2}$ years

d \$750 at 3.1% p.a. for $1\frac{1}{2}$ years

2 What is the simple interest on each investment?

EXAMPLE 2

a \$940 at 3.25% p.a. for 18 months

b \$12 560 at 7.05% p.a. for 6 months

c \$1560 at $4\frac{1}{2}$% p.a. for 3 months

d \$380 at $5\frac{1}{4}$% p.a. for 1 month

3 What percentage interest rate per month is equivalent to 4.68% p.a.?

EXAMPLE 3

4 The Advantage Bank offers investors 8.4% p.a. simple interest. Express correct to two decimal places this rate of interest as a:

a monthly rate

b weekly rate

c 6-monthly rate

d daily rate

e fortnightly rate

f quarterly rate (3-monthly rate)

Divide the annual rate by 52 for the weekly rate, divide by 2 for the 6-monthly rate and divide by 4 for the quarterly rate.

5 Nicole borrowed \$2400 from a finance company for 2 years at 18% p.a. interest.

a How much interest did she have to pay?

b How much, including interest, did Nicole have to repay the finance company?

6 Matthew owed \$365 on his credit card. The credit card company charged him one month's interest at 22% p.a.

a How much interest was he charged?

b Calculate the total amount he had to repay the credit card company.

7 Roberto won \$5000 as part of his prize for being 'Apprentice of the Year'. He invested it for 1 month at 4.5% p.a. interest. How much interest did he earn?

8 The Great Aussie Credit Union pays different interest rates for large investments.

Balance	Interest rate
$1–$4999	3.00% p.a.
$5000–$9999	3.50% p.a.
$10 000–$19 999	4.00% p.a.
$20 000–$49 999	4.75% p.a.
$50 000 and over	5.50% p.a.

a Kate invested the $38 000 she inherited from her great-aunt with the credit union for 2 years. How much interest will she earn?

b Yuri won $54 000 on the football pools. He invested his win with the credit union for 18 months. How much interest will he earn?

c Felicity saved $15 050 from her part-time job at the hardware store. How much interest will she earn if she invests it with the credit union for 3 months?

9 Sunny borrowed $265 000 from a finance company to set up a plumbing supplies business. He borrowed the money for 8 months and he was charged 16.5% p.a. interest. How much did he have to repay the finance company, including interest?

10 During the global financial crisis, bank interest rates in the UK fell to 0.5% p.a. Keith in London invested $30 000 for 2 years at 0.5% p.a. simple interest.

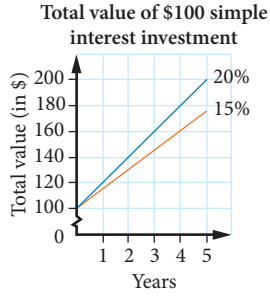
a How much interest did the investment earn?

b At the time he made the investment a man Keith met at a party told him of a non-bank investment that was paying 8% pa.

i How much more interest would Keith earn in the non-bank investment?

ii Why do you think Keith ignored the non-bank investment opportunity?

11 The graph shows the total value of a $100 simple interest investment at 15% p.a. and 20% p.a. for 5 years.



a How much is the investment worth after 4 years at 15% p.a.?

b How much simple interest is earned in 3 years at 20% p.a.?

c How much more interest is earned in 5 years at 20% p.a. than at 15% p.a.?

d Both of the graphs start at 100 on the vertical axis. What is the significance of the 100?

12 Moonsu is creating a spreadsheet to calculate simple interest.

	A	B	C	D	E	F	G
1	**Simple interest calculations**						
2							
3	Enter the principal in F3					3500	
4	Enter the annual interest as a decimal rate in F4					0.05	
5	Enter the length of the investment in years in F5					3	
6							
7							
8		Interest earned =		525			
9		Total in the account =		4025			
10							
11							

What formulas could Moonsu enter in cells D8 and D9?

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Compound interest

15.2

When we invest money with **compound interest**, the amount of interest earned grows as the size of the investment grows. After interest is added to the investment, the next time interest is calculated, it will be based on this larger amount. The interest is a percentage of the **current** size of the investment, so with compound interest, you get 'interest on your interest', or the interest is 'compounded'.

Example 4

Scott invested $2000 at 5% p.a. interest compounded annually.

a Calculate the value of his investment at the end of three years.

b Calculate the total interest earned.

Solution

a We need to calculate the interest for each year separately and add it to the principal before we calculate the following year's interest.

$I = Prn$ where $r = 5\% = 0.05$, $n = 1$ year and P changes each year.

	Interest at the end of the year	Balance at the end of the year
End of the first year	$I = \$2000 \times 0.05 \times 1$ $= \$100$	$\$2000 + \$100 = \$2100$
End of the second year	$I = \$2100 \times 0.05 \times 1$ $= \$105$	$\$2100 + \$105 = \$2205$
End of the third year	$I = \$2205 \times 0.05 \times 1$ $= \$110.25$	$\$2205 + \$110.25 = \$2315.25$

At the end of three years, Scott's investment is worth $2315.25.

b Total interest earned
= Final balance − original principal

$\text{Total interest} = \$2315.25 - \$2000$
$= \$315.25$

or add the three interest amounts in the middle column.

$\text{Total interest} = \$100 + \$105 + \$110.25$
$= \$315.25$

Write your answer.

The total interest earned over the three years was $315.25.

EXERCISE 15.2

Compound interest

EXAMPLE 4

1 Svetlana invested $8000 at 6% p.a. interest compounded annually for 3 years. Copy and complete this table to calculate the value of her investment at the end of three years.

	Interest at the end of the year	Balance at the end of the year
End of the first year	$I = Prn$ $= \$8000 \times 0.06 \times 1$ $= \$_____$	$\$8000 + \$_____ = \$_____$
End of the second year	$I = Prn$ $= \$_____ \times 0._____ \times 1$ $= \$_____$	$\$_____ + \$_____ = \$_____$
End of the third year	$I = Prn$ $= \$_____ \times 0._____ \times 1$ $= \$_____$	$\$_____ + \$_____ = \$_____$

2 Jayden invested $12 000 for 3 years at 4% p.a. compounding yearly.

a How much interest did he earn in the first year?

b How much was in his account at the end of the first year?

c How much was in his account at the end of the second year?

d Calculate the value of his investment at the end of three years.

e How much interest will Jayden earn during his 3-year investment?

3 Letitia invested $4000 at 3.2% p.a. for 2 years compounding annually.

a Calculate the total value of her investment at the end of the 2 years.

b How much interest will she earn during the 2 years?

c How much less interest would Letitia earn if it had been simple interest?



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4 Suresh has saved $14 000 from his after-school job that he plans to spend on a car. He is going to invest the money for 3 months until he has his P-plates. Suresh's investment is going to pay 0.7% per month, interest compounded monthly.



iStockphoto/Jamesbowyer

a Copy and complete the table.

	Interest at the end of the month	Balance at the end of the month
End of the first month	$I = Prn$ $= \$14\,000 \times 0.007 \times 1$ $= \$_____$	$\$14\,000 + \$_____$ $= \$_____$
End of the second month	$I = Prn$ $= \$_____ \times 0._____ \times 1$ $= \$_____$	$\$_____ + \$_____$ $= \$_____$
End of the third month	$I = Prn$ $= \$_____ \times 0.___ \times 1$ $= \$_____$	$\$_____ + \$_____$ $= \$_____$

b How much interest will Suresh make during the 3-month investment?

5 Kirrilly invested some money in monthly compounding interest. The table shows her interest and balance calculations.

	Interest at the end of the month	Balance at the end of the month
End of the first month	$I = Prn$ $= \$9600 \times 0.008 \times 1$ $= \$76.80$	$\$9600 + \$76.80 = \$9676.80$
End of the second month	$I = Prn$ $= \$9676.80 \times 0.008 \times 1$ $= \$77.41$	$\$9676.80 + \$77.41 = \$9754.21$

a How much money did Kirrilly invest?

b What was the monthly rate of compound interest?

c Convert the monthly rate to an annual rate of compound interest.

d How much interest did Kirrilly's investment earn?

6 Last night Zac received a phone call from an investment advisor he doesn't know telling him about an investment opportunity that is virtually risk-free. The offer is likely to be fully subscribed quickly, so he has to agree straight away. Minimum investment is $10 000 and the interest rate is 15% p.a. compounding annually for 3 years.

a Calculate the amount of interest the advisor claims $10 000 will receive in the next 3 years.

b What signs show that this opportunity is most probably a scam?

7 Rachel created a spreadsheet for calculating compound interest that allows her to change the principal and interest rate. The spreadsheet 'Compound interest' is shown below and available on NelsonNet.

	A	B	C	D
1	**Compound interest**			
2	Only enter data in cells shaded blue.			
3				
4	Principal	$1,000.00	Annual rate of interest as a percentage	12%
5				
6				
7		Account balance at the beginning of the year	Interest earned during the year	Account balance at the end of the year
8	Year 1	$1,000.00	$120.00	$1,120.00
9	Year 2	$1,120.00		
10	Year 3			
11	Year 4			
12	Year 5			
13	Year 6			
14	Year 7			
15	Year 8			

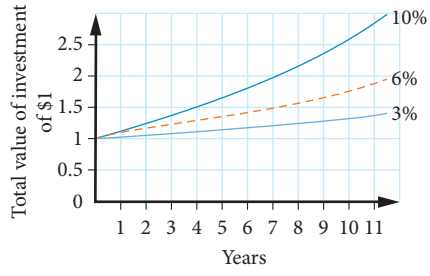
a What formulas could Rachel have used for cells B8, D8 and B9?

b Rachel used the formula =B8*D4. Why did she use the two $ signs?

c 'Fill down' to complete the spreadsheet calculations for 8 years.

d Use the spreadsheet to determine the amount of interest a $4500 investment will earn in 8 years at 4.8% p.a. annually compounding interest.

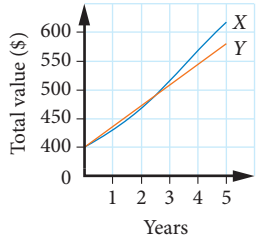
8 This graph shows the total value on $1 compound investment at 3% p.a., 6% p.a. and 10% p.a.



a Describe how the shape of a compound interest graph is different to the shape of a simple interest graph.

b Approximately how long does it take for a $1 investment to double its value at 10% p.a. compound interest?

9 Nick and Adam invested the same amount of money at 9% p.a., but Nick's investment was 9% p.a. compounded annually while Adam's was 9% p.a. simple. The graphs show the value of their investments after 5 years.

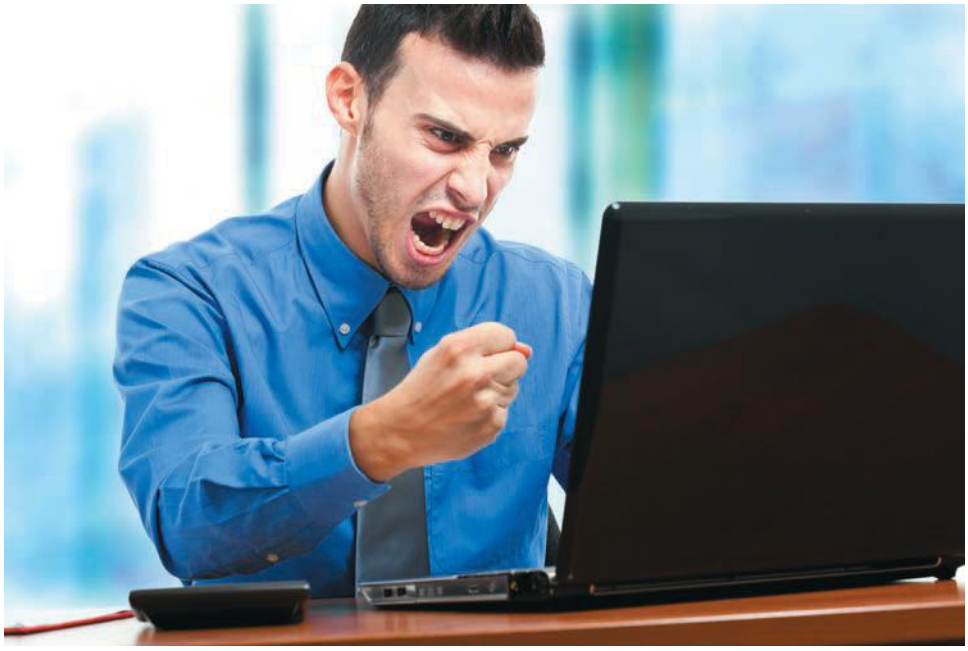


a How much was Nick and Adam's original investment?

b Match the graphs *X* and *Y* to Adam's and Nick's investments.

c After 5 years, state whose investment is higher and calculate by how much (to the nearest dollar).

Investigation Is it a scam?



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Technology and the Internet have changed the way we do things and made life easier for us. But technology has also made life easier for criminals to use scams and financial fraud to rob people of their hard-earned money.

1. Visit the Scamwatch website www.scamwatch.gov.au.
2. Investigate '**How scams work**'**.** List 4 different reasons why people fall for scams.
3. Prepare a list of answers you could give to possible scammers when they are trying to convince you to do something you don't want to do.
4. Investigate '**Have you been scammed?**' Describe what you should do if you have sent money to someone you think is a scammer.

15.3 The compound interest formula

It's quicker and easier to calculate compound interest with a formula.

The formula for compound interest is

$A = P(1 + r)^n$

where A = the final amount or future value

P = principal

r = rate of interest per compounding period as a decimal

n = the number of compounding periods

Example 5

a What will be the final amount of an investment of $850 after three years at 6.5% p.a. compounding annually?

b How much interest will be earned during the three years?

Solution

a Use the formula $A = P(1 + r)^n$
$P = 850$, $r = 0.065$, $n = 3$

$A = 850 \times (1 + 0.065)^3$
$= 850 \times 1.065^3$
$= 1026.7571\ldots$
≈ 1026.76

Write your answer.

$850 will amount to $1026.76 at the end of 3 years.

b Interest = final amount − principal

Interest = $1026.76 − $850
= $176.76

Write your answer.

The interest earned during the three years will be $176.76.

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Example 6

Carl invested $3500 for 2 years at 9% p.a. monthly compounding interest to help him save the deposit for a tradesman's truck. How much interest did his investment earn?

When the interest compounds monthly the time and interest rate must both be in months.

Solution

Interest is compounded monthly so r and n must be in months.

$r = 0.09 \div 12 = 0.0075$ per month
$n = 2 \times 12 = 24$ months

Divide 9% = 0.09 by 12 for a monthly interest rate.

Multiply 2 years by 12 for months.

Use the formula $A = P(1 + r)^n$

$P = 3500$, $r = 0.0075$ per month, $n = 24$ months.

$A = 3500 \times (1 + 0.0075)^{24}$
$= 3500 \times 1.0075^{24}$
$= 4187.4473\ldots$
≈ 4187.45

Interest = final amount (A) − original principal (P)

Interest = $4187.45 − $3500
= $687.45

Write your answer.

The account will earn $687.45 in interest.

EXERCISE 15.3

The compound interest formula

1 Use the compound interest formula $A = P(1 + r)^n$ with $P = 7600$, $r = 0.075$ and $n = 5$ to calculate the value of A.

2 David invested $1870 at 6.8% p.a. compounding annually for 4 years.

EXAMPLE 5

- **a** Write down the value for P, r and n, for the compound interest formula.
- **b** Calculate the value of David's investment at the end of the 4 years.
- **c** How much interest did David earn?

3 Megan won $5260 in lotto and decided to invest it at 7.1% p.a. compounding yearly for 3 years. How much will her investment be worth at the end of the 3 years?

4 Danielle's bank pays 6.36% p.a. compounding monthly on term deposits.

EXAMPLE 6

- **a** Explain how you know that 6.36% p.a. is equivalent to 0.53% monthly.
- **b** Danielle invested $1950 in a term deposit for 18 months. Calculate the value of her investment at the end of 18 months.
- **c** How much interest will Danielle earn from this investment?

5 When Simon retired, he invested $145 600 from his retirement funds at 5.88% p.a. monthly compound interest for 5 years.

a What monthly interest rate is equivalent to 5.88% p.a.?

b For how many months did Simon invest the money?

c How much interest did Simon make from this investment?

6 Calculate the final value of the following investments.

Remember! The interest rate and number of periods must be in the same units as the compounding period.

a $14 000 invested at 6% p.a. monthly compound for 4 years.

b $1350 invested at 5.4% p.a. monthly compound interest for $1\frac{1}{2}$ years.

c $25 000 invested for 5 years at 7.2% p.a. with interest compounding every 6 months.

d $60 000 invested for 7 years at 7.2% p.a. with interest compounding quarterly, that is, every three months.

e $15 000 invested at 6% p.a. for 2 years, interest compounding quarterly.

7 Connor is going to invest $10 000 for 3 years. Which investment is the best option, which will give the biggest return?

A 6% p.a. simple interest

B 5.9% p.a. compounded annually

C 5.85% p.a. compounded half-yearly

8 Ray has $50 000 to invest for 4 years. Which investment option will give him the better return: 7% p.a. simple interest or 6.2% p.a. compounded monthly?

Use calculations to justify your answer.

9 Jill used this spreadsheet to calculate the amount of interest she would earn when she invested $60 000 at 8% p.a. for 4 years with monthly compounding interest.

	A	B	C	D	E	F	G	H
1	**Compound interest**							
2								
3								
4	Enter the principal in G4						$60,000.00	
5	Enter the number of compounding periods in a years in G5						12	
6	Enter the annual interest rate as a decimal in G6						0.08	
7	Enter the term of the investment in years in G7						4	
8								
9		Final value of the investment =			$82,539.97			
10	Interest earned during the investment =				$22,539.97			
11								

a One of the formulas Jill used is =(E9–G4). What does this formula calculate?

b Jill made an error when she entered the formula =G4*(1+G6/G5)^G7*G5 in cell E9. Correct the error in Jill's formula.

c Construct a spreadsheet similar to Jill's spreadsheet and use your spreadsheet to determine the change in the interest Jill will earn if instead of monthly compounding her investment compounds weekly.

Compound interest using technology 15.4

For this section, you will need access to appropriate technology: an online calculator on a website such as www.thecalculatorsite.com, the 'Different compounding periods' spreadsheet available on NelsonNet or a calculator with a TVM mode.

Example 7

Jana invested $2375 for 3 years in monthly compounding interest at 4.5% p.a. Calculate the final value of her investment and the amount of interest she will earn.

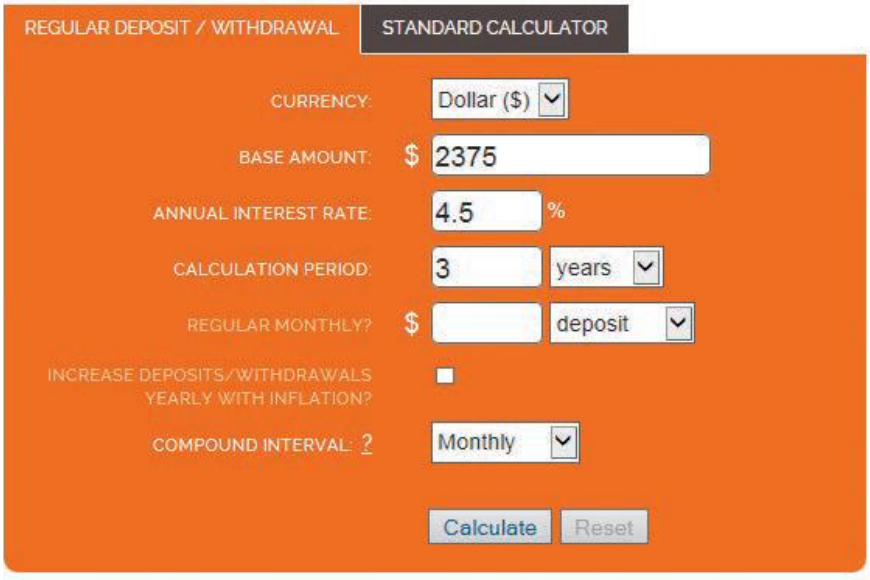
Solution

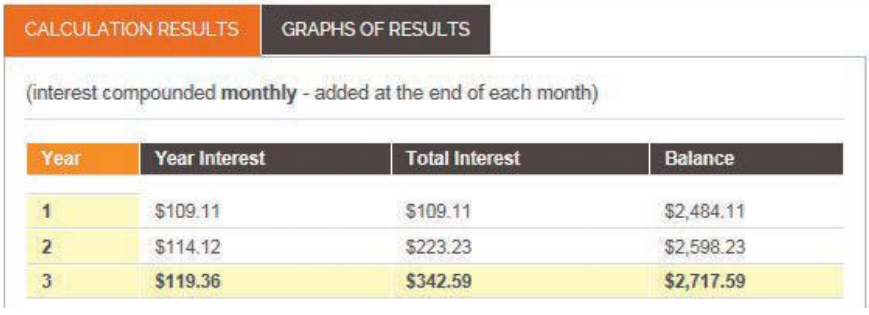
The solution steps will be slightly different depending on the technology you choose to use. The final values can be slightly different depending on the way the technology rounds off.

Using an online compound interest calculator, enter $2375 as the initial (or base) deposit, 4.5% for the interest rate, 3 for the number of years, select monthly for the compounding period then click on calculate.

Final value of the investment = $2717.59

Total interest = $342.59.





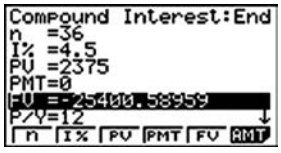
Year	Year Interest	Total Interest	Balance
1	$109.11	$109.11	$2,484.11
2	$114.12	$223.23	$2,598.23
3	$119.36	$342.59	$2,717.59

Using the NelsonNet spreadsheet, enter the 4 values in column B:

	A	B	C	D	E
1	**Different compounding periods**				
2					
3	**How much compound interest will I earn?**				
4	Only enter data in cells shaded blue.				
5	Principal	$2,375.00		Interest rate per compounding period, as a decimal	0.00375
6	Annual rate of interest as a percentage	4.50%		Number of compounding periods	36
7	Number of compounding periods per year	12		Final value of the investment	$2,717.59
8	Length of the investment in years	3		Interest earned during the investment	$342.59

Final value of the investment = $2717.59
Total interest = $342.59.

Using TVM mode on a calculator, enter the values shown. The value for FV is irrelevant.





The final value of the investment is $2717.59.

Interest earned
= final value – present value
= $2717.59 – $2375
= $342.59

EXERCISE 15.4

Compound interest using technology

Use the technology of your choice to answer the questions in this exercise.

EXAMPLE 7

1 Determine the amount of interest each investment will earn.

	Principal	Interest rate	Term	Compounding
a	$10 000	6% p.a.	3 years	monthly
b	$2500	4.2% p.a.	4.5 years	quarterly
c	$1840	3.6% p.a.	5 years	half yearly
d	$3800	4.6% p.a.	2 years	daily

2 Marissa won $5800 and decided to invest it at 7.2% p.a. monthly compounding interest for 3 years.

a How much will Marissa's win grow to in 3 years?

b How much interest will Marissa's investment earn?

3 Billy saved $1660 for a holiday after he finishes Year 12. He invested it at 5% p.a. monthly compounding interest for 1 year. How much will be in his holiday account at the end of the year?



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4 Jack received a $2500 bonus as a result of increased company profits. He is going to invest it for 1 year at 6% p.a. How much more interest will he earn in monthly compounding compared to simple interest?

5 When Kathy was made redundant at work she was given a lump-sum payment of $40 000. She invested it for 5 years. How much more interest will she earn from daily compounding than annually compounding interest at 6% p.a.?

6 On his retirement, Ross received a lump-sum superannuation payment of $450 000 and he is going to invest it for 2 years. He can invest it at 6.4% p.a. monthly compounding or 6.55% p.a. annually compounding interest. Which investment will give him the better return? Justify your answer.

7 Heath has $200 000 to invest to provide for his retirement. He is considering two different investment options.

	Investment term	Fees
Finance company	5% p.a. compounding monthly	$10 per month
Managed funds	6% p.a. daily compounding	6.5% of the interest earned

Which of the two options will give Heath the larger return in 12 months? Explain your answer.

Investigation Different compounding periods

In this investigation you are going to determine a relationship between the amount of interest earned and the frequency of the interest compounding period.

- Use the technology of your choice to complete the missing values in the table.

Invest $10 000 for 4 years at 8% p.a. compounding interest	
Compounding period	Interest earned
Annually	
Every 6 months	
Monthly	
Weekly	
Daily	

- Write a sentence to describe the observations you've made.
- Check the accuracy of your observation by completing a table for another investment.

Invest $50 000 for 7 years at 12% p.a. compounding interest	
Compounding period	Interest earned
Annually	
Every 6 months	
Monthly	
Weekly	
Daily	

- Chris is going to invest $2000 for 3 years at 6% p.a. compounding interest. He can select either monthly compounding or fortnightly compounding interest. Which compounding period will give him the better return? Give a reason for your answer.

15.5 Significant figures

When we're talking about large quantities we often use an approximate amount. In a disaster, for example, the media could report that a bushfire burned out 3 000 000 hectares, or that 20 000 people were stranded because of bad weather. Just as numbers can be rounded to decimal places, they can also be rounded to **significant figures**. With large numbers, we start counting from the first digit. With small decimals, we start counting from the first digit that is not 0.

When the figure following the significant figure you want to round to is:

4 or smaller – round down.

5 or bigger – add 1 to the figure in the significant figure you want to round to, and round up.

Example 8

Round each quantity correct to 2 significant figures.

a 28 315 008 **b** 32 724 **c** 0.006 952

Solution

a With 28 315 008, the first two significant figures are 28. Decide whether 28 315 008 is closer to 28 000 000 or 29 000 000. The third digit is 3, which is less than 5, so we round down.

$28\,315\,008 \approx 28\,000\,000$

b The first two significant figures are 32. Decide whether 32 724 is closer to 32 000 or 33 000. The third digit is 7, which is bigger than 5, so we need to round up.

$32\,724 \approx 33\,000$

c The first two significant figures are 69 because the 0s are not significant. Decide whether 0.006 952 is closer to 0.0069 or 0.0070. The third digit is 5, so we round up.

$0.006\,952 \approx 0.0070$

When we round an amount to 2 decimal places, we have 2 digits following the decimal point. When we round to 2 significant figures, we only put the two most important digits in the amount.

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EXERCISE 15.5

Significant figures

EXAMPLE 8

15.4

1 The city of Nelson Springs has a population of 372 413. Write this population correct to 2 significant figures.

2 Ian's superannuation account contains $105 496. Round this amount to:

a 2 significant figures **b** 1 significant figure **c** 3 significant figures

3 Express $482 356 correct to:

a 1 significant figure **b** 2 significant figures **c** 3 significant figures

4 Express each decimal correct to 2 significant figures.

a 0.004 623 **b** 0.025 73 **c** 0.019 623 **d** 0.000 466

5 **a** In 2011 the population of Melbourne was 3 999 982. Express this amount correct to 3 significant figures.

b By 2014, Melbourne's population had grown to 4 250 000. Write a possible population that can be rounded to 4 250 000.

c Melbourne's population is predicted to reach 7 739 000 by 2051. By how much is Melbourne's population predicted to increase between 2011 and 2051? Express your answer correct to 3 significant figures.

6 Every 3 months each home owner in Nelson Springs pays an average of $723 in council rates. There are a total of 673 522 home owners in the area.

a Calculate the total amount the council receives in rates every 3 months. Express your answer correct to 3 significant figures.

b How much does the council receive in rates each year? Express your answer correct to 2 significant figures.

7 Write each number correct to one significant figure.

a 2914.23 **b** 3.2548 **c** 11 950

d 0.005 134 **e** 20.46 **f** 0.6517

8 Evaluate each expression, correct to two significant figures.

a $0.2 \div 0.3$ **b** $11 \div 1990$ **c** $16 \div 12$

d $\sqrt{0.0075}$ **e** $9\,300\,000 \times 0.085$ **f** 2.7^2

g $\sqrt{560}$ **h** $\sqrt{5.6}$ **i** 3.4×9.9

9 Express each of the following measurements correct to three significant figures.

a mass of an aeroplane, 351 540 kg

b average depth of the Pacific Ocean, 4188 m

c speed of a greyhound, 67.14 km/h

d capacity of a tablespoon, 14.79 mL

e distance from the Earth to the Sun, 149 573 881 km

15.6 Scientific notation

We often need to use very large numbers, for example when we're describing national debt, the population of large countries or the distances between stars.

Australia's national debt was \$2 300 000 000 000 in 2015. In scientific notation, this amount is $\$2.3 \times 10^{12}$.

Scientific notation is a way of writing very large numbers and very small numbers.

It writes numbers as a decimal between 1 and 10 multiplied by a power of 10.

The diameter of an atom is 0.000 000 03 m. In scientific notation this length is 3×10^{-8} m. Very small numbers have a negative power of 10.

Example 9

Write each number in scientific notation.

a 190 000 **b** 43 000 000 **c** 0.0054 **d** 0.000 087 4

Solution

a Place the decimal point between the 1 and the 9, then count how many places it has moved to the left.

$1.90\,000$ 5 places: 10^5

$190\,000 = 1.9 \times 10^5$

b Place the decimal point between the 4 and the 3, and then count how many places it has moved to the left.

4.3000000 7 places: 10^7

$43\,000\,000 = 4.3 \times 10^7$

c Place the decimal point between the 5 and the 4, and then count how many places it has moved to the right:

$0.005.4$ 3 places to the right: 10^{-3}

$0.0054 = 5.4 \times 10^{-3}$

d Place the decimal point between the 8 and the 7, and then count how many places it has moved to the right:

$0.00008.74$ 5 places to the right: 10^{-5}

$0.000\,087\,4 = 8.74 \times 10^{-5}$

Large numbers have positive powers of 10, small numbers have negative powers of 10.

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Example 10

Express each number in normal decimal form.

a 3.29×10^4 **b** 9.1×10^6 **c** 2.5×10^{-3} **d** 5.8×10^{-5}

Solution

a Move the decimal point 4 places to the right. Add zeros where necessary. $3.29 \times 10^4 = 32\,900$

b Move the decimal point 6 places to the right, adding zeros where necessary. $9.1 \times 10^6 = 9\,100\,000$

c The negative power of 10 means this is a small number. Move the decimal point 3 places to the left, adding zeros where necessary. $2.5 \times 10^{-3} = 0.0025$

d The negative power of 10 means this is a small number. Move the decimal point 5 places to the left, adding zeros where necessary. $5.8 \times 10^{-5} = 0.000\,058$

Scientific notation on a calculator

You can enter numbers in scientific notation into your calculator using the $\times 10^x$ or EXP key.

Example 11

Evaluate $(6.5 \times 10^6) \times (2.2 \times 10^{-2})$.

You don't need to enter the brackets on the calculator.

Solution

6.5 $\times 10^x$ 6 $\times$ 2.2 $\times 10^x$ -2 $=$

$(6.5 \times 10^6) \times (2.2 \times 10^{-2})$
$= 143\,000$ or 1.43×10^5

EXERCISE 15.6

Scientific notation

1 Write each number in scientific notation.

EXAMPLE 9

a 860 000 **b** 9 140 000 000 **c** 2010
d 0.000 36 **e** 0.000 000 0018 **f** 0.000 101

2 Write each number in normal decimal form, without scientific notation.

EXAMPLE 10

a 3.52×10^6 **b** 3.5×10^4 **c** 8.7×10^3 **d** 6.45×10^8
e 6.1×10^{-3} **f** 1.93×10^{-5} **g** 5×10^{-7} **h** 1×10^{-2}

3 A gigametre is a billion (thousand million) metres or 1000 000 000 m. Use scientific notation to express 1 gigametre in metres.

4 The thickness of a soap bubble is 0.000 000 1 metres. Write this thickness in scientific notation.

5 Which has the larger value 7×10^6 or 6×10^7, and by how much?

EXAMPLE 11

6 Evaluate each expression.

a $(5 \times 10^7) \times (2 \times 10^4)$ **b** $(4 \times 10^4) \times (1.6 \times 10^{-2})$ **c** $(3.8 \times 10^6) \times (5.7 \times 10^3)$

d $(7.8 \times 10^7) \times (4.5 \times 10^{-3})$ **e** $(4.4 \times 10^8) \div (4 \times 10^5)$ **f** $(9.6 \times 10^3) \div (1.6 \times 10^{-4})$

g $(7.2 \times 10^6) \div (2.4 \times 10^{-3})$ **h** $(4.7 \times 10^8) \div (1.25 \times 10^{-2})$

7 The diameter of the Earth is 1.275×10^7 metres. What is the radius of the Earth in kilometres? Answer in scientific notation.

8 The Horsehead Nebula, one of the sky's most distinctive features, is 1500 light years from Earth. One light year equals 9.46×10^{12} km. How many kilometres is it from Earth to the Horsehead Nebula? Express your answer in scientific notation.



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9 The diameter of the Universe is 6×10^{22} km and the diameter of the Earth is 1.3×10^4 km. How many times larger is the diameter of the Universe than the diameter of the Earth? Express your answer in scientific notation correct to one decimal place.

10 Geologists say that the Earth's continents are moving. Australia is very slowly moving north towards Asia. Scientists predict that in 50 million years Australia will collide with Asia. How many hours is this? Answer in scientific notation, correct to one decimal place.

11 A bacteria is approximately 3.5×10^{-3} mm wide and an influenza virus is approximately 2×10^{-7} mm wide. Which of them is wider, and by how much?

12 The diameter of a HIV virus particle is approximately 9×10^{-6} mm. Approximately how many particles placed side by side would be required to make a line 1 metre long? Answer to the nearest whole number.

13 Express each number in scientific notation, correct to two significant figures.

a 53 467 892 **b** 146 089 **c** 2453

d 0.000 4573 **e** 0.002 652 **f** 0.102 05

14 Australia's population was 23 483 349 in 2015 and the average debt per person was $16 934. Calculate the total amount of personal debt in Australia in 2015, in scientific notation correct to 3 significant figures.

Keyword activity

Match the terms in the left column with their correct meanings in the right column.

	Terms		Meaning
1	interest	A	Two digits to the right of the decimal point.
2	compound interest	B	The compound interest formula.
3	compounding period	C	The rate of return on an investment, usually annually and expressed as a percentage.
4	two significant figures	D	The interest is only paid on the original investment.
5	scientific notation	E	How often the interest is calculated using compound interest.
6	two decimal places	F	The original amount of an investment.
7	$I = Prn$	G	The first two digits that are not 0 in an amount.
8	interest rate	H	The amount earned on an investment.
9	principal	I	The simple interest formula.
10	$A = P(1 + r)^n$	J	Interest is paid on the current value of the investment, including interest previously added to the principal.
11	simple interest	K	A way of writing very large or small numbers using powers of 10.

Chapter Problem

Ryan invested $25 000 for 3 years, earning compound interest at 4% per year.

Melanie also invested $25 000 for 3 years, earning simple (flat rate) interest at 4.2% per year.

Whose investment will earn more interest?

Solution

Ryan's investment

Final amount = $P(1 + r)^n$

$P = 25\,000$, $r = 0.04$ and $n = 3$

Final amount = $25\,000 \times (1 + 0.04)^3 = 28\,121.60$

Ryan's investment will be worth $28 121.60 at the end of 3 years.

Ryan's interest = $28 121.60 – $25 000 = $3121.60 interest.

Melanie's investment

Simple interest = Prn

$P = 25\,000$, $r = 0.042$ and $n = 3$

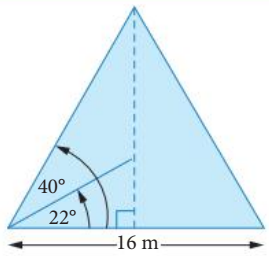
Melanie's interest = $25\,000 \times 0.042 \times 3 = 3150$

Melanie's investment will earn $3150 interest.

Melanie's investment will earn more interest ($28.40 more than Ryan's).

Chapter Problem

Early Australian builders knew that a large roof pitch helped prevent leaks and keep houses cool. The early roof pitches were often as large as 40°. Today most roof pitches are about 22°.



How much higher was the roof of a 16 m-wide early Australian building with a 40° roof pitch than a modern building with the same width and a 22° roof pitch?



CHAPTER 16

SO YOU'VE GOT A RIGHT ANGLE

16.1 The tangent ratio
16.2 Using tan to find an angle
16.3 Elevated or depressed?
16.4 Sine and cosine
16.5 Sin, cos or tan?
16.6 Finding the hypotenuse
16.7 Bearings
Keyword activity
Solution to the chapter problem

By the end of this chapter, you should know how to:

- use trigonometry to calculate the lengths of sides in right-angled triangles
- use trigonometry to calculate the sizes of angles in right-angled triangles
- use bearings in practical situations

How are we ever going to use this?

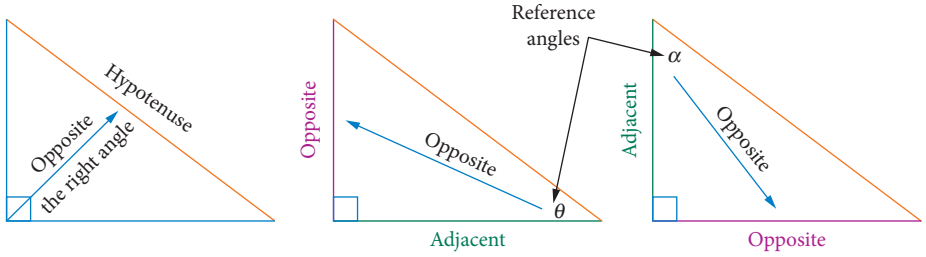
- Determining a length when we can't measure it
- Many people who work in trades, for example, carpenters and builders, use right-angled triangle calculations in their work

16.1 The tangent ratio

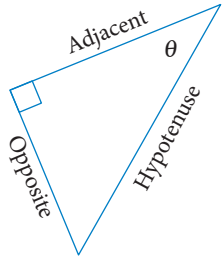
In a right-angled triangle, the **hypotenuse** is the longest side, opposite the right angle. The names of the other two sides are determined by the reference angle.

The **opposite side** is the side facing the reference angle.

The **adjacent side** is next to the reference angle.



The **tangent** ratio, abbreviated **tan**, is the ratio of the length of the **opposite side** to the length of the **adjacent side**.

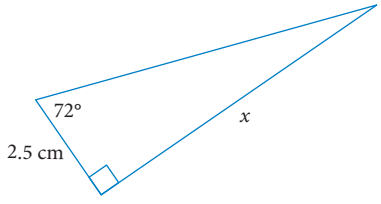


$$\text{Tangent ratio: } \tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

The tangent ratio can be used to calculate the length of a side or an angle in a right-angled triangle.

Example 1

Use the tangent ratio to find the length of side x in the triangle. Express your answer correct to 2 decimal places.



Solution

In the triangle, x is on the side opposite 72° and the adjacent side is 2·5 cm long.
The tan ratio is $\frac{\text{opposite}}{\text{adjacent}}$. We have to make a trig equation, then solve it.

$$\tan(72°) = \frac{x}{2.5} \quad \text{or} \quad \frac{x}{2.5} = \tan(72°)$$

Multiply both sides by 2·5.

$$\cancel{2.5} \times \frac{x}{\cancel{2.5}} = \tan(72°) \times 2.5$$
$$x = 2.5\tan(72°)$$

Enter 2.5 tan 72 = on your calculator.

From the diagram, $x \approx 7{\cdot}69$ cm seems a reasonable answer.

$$x = 7{\cdot}694\,20\ldots$$
$$\approx 7{\cdot}69 \text{ cm}$$

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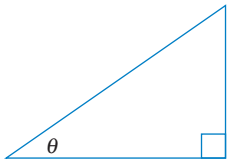
Make sure that your calculator is set in degrees mode DEG or D. If it is set to RAD or GRAD, your calculator will give you the wrong answer.

EXERCISE 16.1

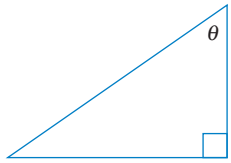
The tangent ratio

1 Copy each triangle and write the names of the sides with respect to the reference angle.

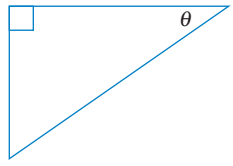
a



b



c

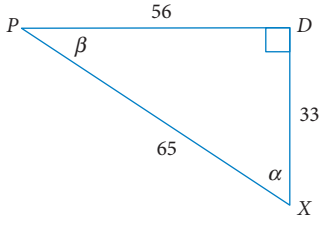


2 For this triangle, how long is:

a the hypotenuse?

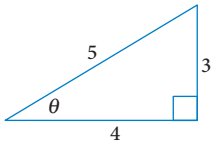
b the side opposite to angle α?

c the side adjacent to angle β?

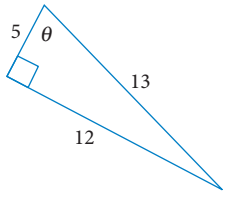


3 For each triangle, write tan (θ) as a fraction.

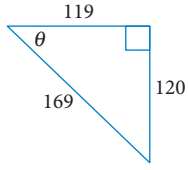
a



b



c



4 Accurately construct a triangle in which $\tan(\theta) = \frac{2}{3}$.

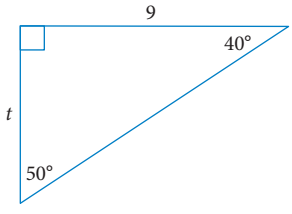
5 Copy and complete the working to calculate the value of t in the diagram. Express your answer correct to 1 decimal place.

EXAMPLE 1

$$\frac{t}{9} = \tan(\square)$$

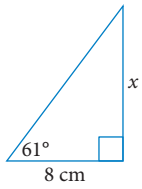
$$\square \times \frac{t}{9} = \tan(\square) \times \square$$

$$t \approx \square$$

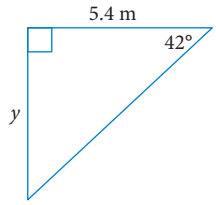


6 Find the value of each pronumeral, correct to 2 decimal places.

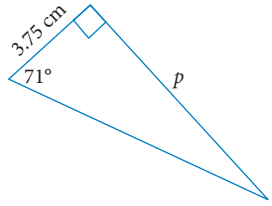
a



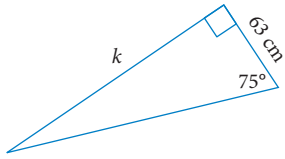
b



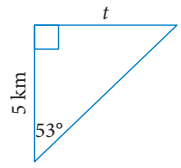
c



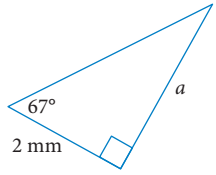
d



e



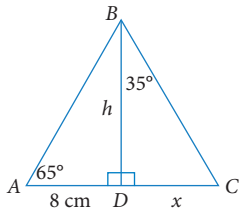
f



7 These two right-angled triangles share a side BD.

a Use $\triangle ABD$ to find h correct to 2 decimal places.

b Use $\triangle BDC$ to find x correct to 1 decimal place.



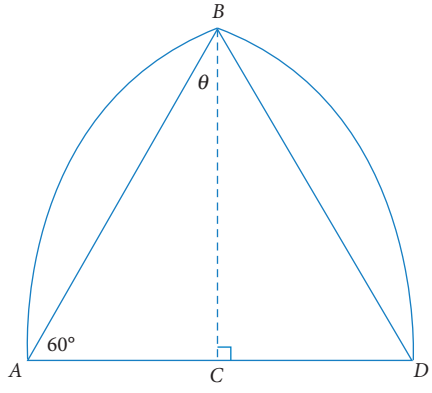
8 Gothic arches are a common feature in the design of Gothic cathedrals. Equilateral triangles are the design basis for most Gothic arches.



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a What must be the size of angle θ in the diagram?

b One Gothic arch is 1.2 m high, so $BC = 1.2$ m. Calculate correct to 1 decimal place the length of AC and so find the width of arch AD.



Remember, the angles in $\triangle ABC$ add to 180°. Also, $\triangle ABD$ is equilateral.

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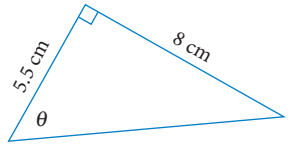
Using tan to find an angle

16.2

We can use the formula $\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$ to calculate angles as well as sides.

Example 2

Calculate the size of angle θ correct to the nearest degree.



Solution

Use $\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$ to find the angle θ.
The opposite side is 8 cm and the adjacent side is 5.5 cm.

$\tan(\theta) = \frac{8}{5.5}$

To 'undo the tan' we need to press
SHIFT tan (8 ÷ 5.5) = .

$\theta = 55.4914\ldots$
$\approx 55°$

From the diagram, the answer $\theta \approx 55°$ looks reasonable.

A common error when substituting into $\frac{\text{opposite}}{\text{adjacent}}$ is to always put the smaller number first and the larger number second. In the tan formula, the smaller number can be first *or* second. Make sure to check which side is the opposite. The opposite side always comes first, and the adjacent side comes second.



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EXERCISE 16.2

Using tan to find an angle

EXAMPLE 2

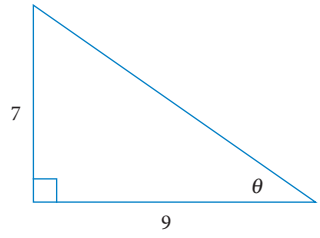
1 Use your calculator to find the value of θ correct to the nearest degree.

a $\tan(\theta) = 0.86$ **b** $\tan(\theta) = 1.07$ **c** $\tan(\theta) = \frac{3}{4}$

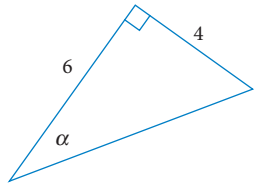
d $\tan(\theta) = \frac{3}{5}$ **e** $\tan(\theta) = \frac{11}{8}$ **f** $\tan(\theta) = 1\frac{1}{3}$

2 Use the tan ratio to find correct to the nearest degree the size of each angle marked with a pronumeral.

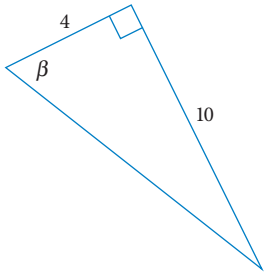
a



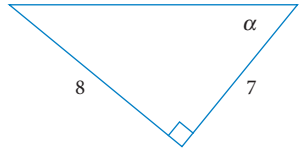
b



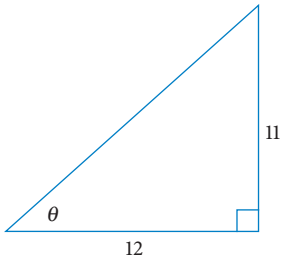
c



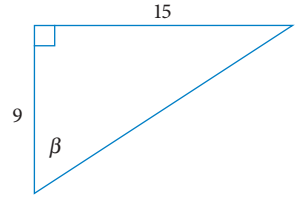
d



e



f



3 The shorter (non-hypotenuse) sides of a right-angled triangle are 4 cm and 6 cm long. Calculate the sizes of the angles in the triangle correct to the nearest degree.

4 The shorter sides in a right-angled triangle are both 5 cm long.

a Find the sizes of the angles in the triangle.

b Explain why $\tan 45° = 1$.

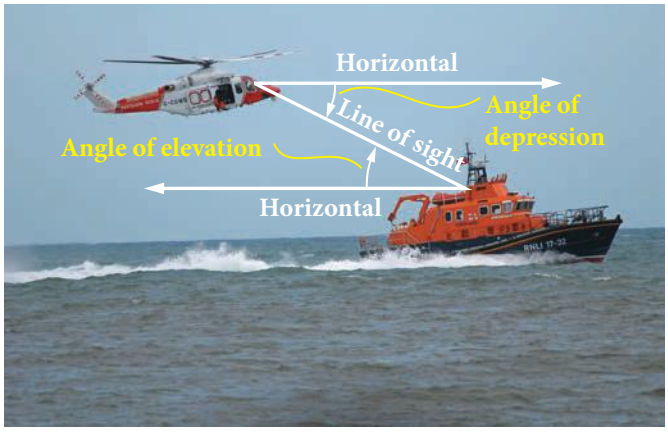
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Elevated or depressed?

16.3

Angles of depression and elevation are used regularly to solve practical measurement problems.

The **angle of depression** is the angle the eye turns down from the horizontal to look at an object in a lower position. The **angle of elevation** is the angle the eye turns up from the horizontal to look at an object in a higher position.



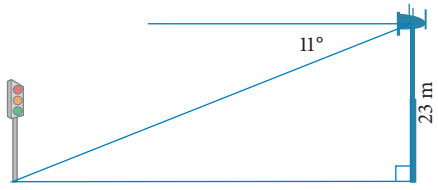
Alamy/stephen How

EXERCISE 16.3

Elevated or depressed?

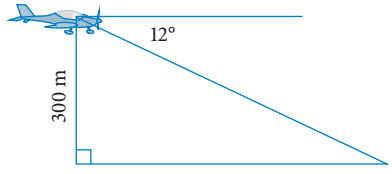
In this exercise, express your answers correct to 1 decimal place (for lengths) or the nearest degree (for angles), unless instructed otherwise.

1 From the top of a 23 m high mobile phone tower, the angle of depression to the bottom of a set of traffic lights is 11°. How far is the set of traffic lights from the base of the mobile phone tower?



The top angle in the triangle is NOT the angle of depression. Subtract the angle of depression from 90° to find the size of the top angle.

2 Pooja is flying an ultra-light aircraft at a height of 300 m above the ground. Her angle of depression to the landing strip is 12°. Calculate the horizontal distance from the ultra-light to the landing strip.

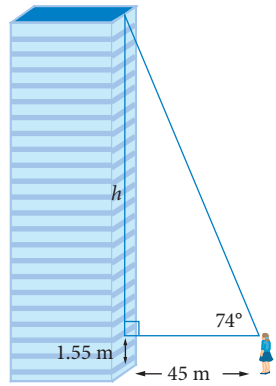


3 A navigation chart shows that the top of a lighthouse is 142 m above sea level. A ship's navigator measured the angle of elevation to the top of the lighthouse as 15°. How far is the ship from the base of the lighthouse?

4 Chantal is 45 m away from a tall office block. She measured the angle of elevation to the top of the office block as 74°.

a Calculate the size of h in metres correct to 3 decimal places.

b Chantal's eyes are 1·55 m above the ground. Calculate the height of the building.



5 When Ali was 810 m away from Uluru he measured the angle of elevation to the top of the rock as 20°.

a Draw a diagram and show Ali's measurements on it.

b Use Ali's measurements to calculate the approximate height of Uluru. Answer correct to the nearest metre.

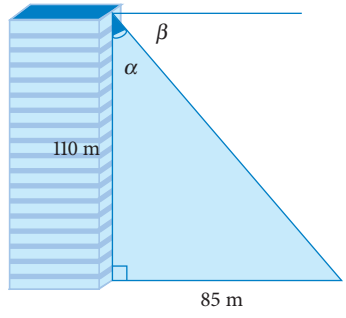
6 The third level of the Eiffel Tower in Paris is 276 m above the ground. When Nhi was standing on the third level of the tower, she measured the angle of depression to a group of her friends on the ground as 19°. How far were Nhi's friends from the base of the Eiffel Tower below Nhi?


Shutterstock.com/Giancarlo Liguori

7 The front of an office block has security lights that come on when they detect movement. The lights are 110 m above the ground, and they light up an 85 m wide strip in front of the building.

a Calculate the size of angle α.

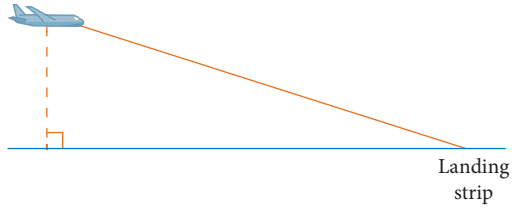
b Find the size of angle β, the light's angle of depression.



8 From a small plane flying at a height of 800 m, the angle of depression to the landing strip is 14°.

a Copy the diagram and show the position of the angle of depression.

b Calculate the horizontal distance from the plane to the landing strip.



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Sine and cosine

16.4

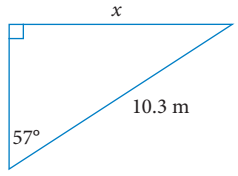
In a right-angled triangle, the **sine** (abbreviated **sin**) and **cosine** (abbreviated **cos**) ratios are used when calculations involve the hypotenuse.

Sine ratio: $\sin(\theta) = \dfrac{\text{opposite}}{\text{hypotenuse}}$

Cosine ratio: $\cos(\theta) = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

Example 3

Find the value of x in the triangle, correct to 1 decimal place.



Solution

For the 57° angle, x is the opposite side, and 10.3 m is the hypotenuse. The sin ratio is $\dfrac{\text{opposite}}{\text{hypotenuse}}$. We have to make a trig equation, then solve it.

$$\sin(57°) = \frac{x}{10.3}$$

$$\text{or } \frac{x}{10.3} = \sin(57°)$$

Multiply both sides by 10.3.

$$\cancel{10.3} \times \frac{x}{\cancel{10.3}} = \sin(57°) \times 10.3$$

$$x = 10.3\sin(57°)$$

Enter 10.3 sin 57 = on your calculator.

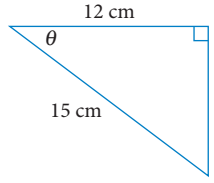
$$x = 8{\cdot}6383\ldots$$

$$\approx 8.6 \text{ m}$$

From the diagram, $x \approx 8.6$ m seems a reasonable answer.

Example 4

Find the size of θ correct to the nearest degree.



Solution

For θ, 12 cm is the adjacent side. 15 cm is the hypotenuse. Use $\cos(\theta) = \dfrac{\text{adjacent}}{\text{hypotenuse}}$.

$$\cos(\theta) = \frac{12}{15}$$

Enter SHIFT cos 12 $a^{b}/_{c}$ 15 =.

$$\theta = 36.8698\ldots$$

$$\approx 37°$$

From the diagram, the answer $\theta \approx 37°$ looks reasonable.

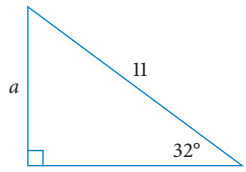
EXERCISE 16.4

Sine and cosine

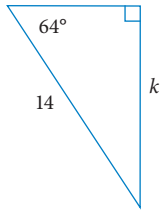
EXAMPLE 3

1 Use the sin ratio to calculate the value of each pronumeral correct to 1 decimal place.

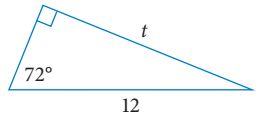
a



b



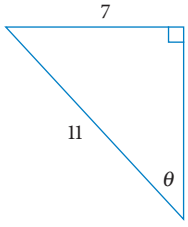
c



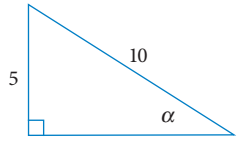
EXAMPLE 4

2 Use the sin ratio to find the size of each angle correct to the nearest degree.

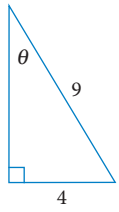
a



b

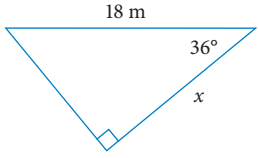


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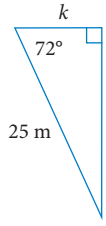


3 Use the cos ratio to calculate the value of each pronumeral correct to the nearest metre.

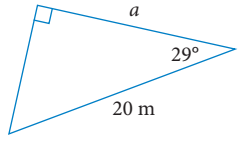
a



b

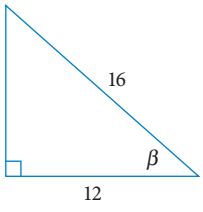


c

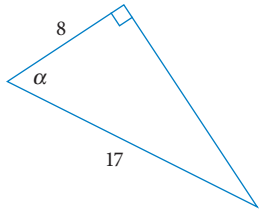


4 Use the cos ratio to find the size of each angle correct to the nearest degree.

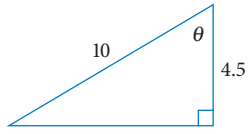
a



b



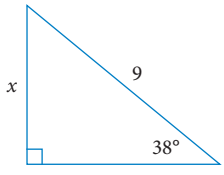
c



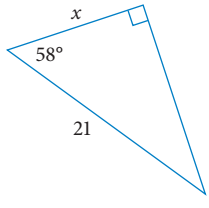
5 For each triangle, choose the appropriate trig ratio to find the value of the pronumeral. Express your answers correct to 1 decimal place.

When the side you know is the hypotenuse, you need to use sin if the side with the pronumeral is opposite and use cos if it is adjacent

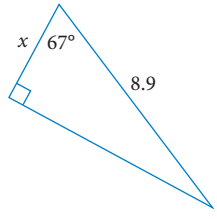
a



b



c



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Sin, cos or tan?

16.5

One way to remember the trig ratio formulas is with the phrase 'Only Half An Hour Of Algebra' and the trig keys on your calculator in order:

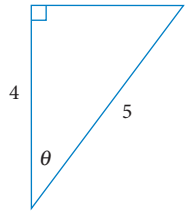
Only **H**alf	**A**n **H**our	**O**f **A**lgebra
$\sin = \frac{\text{opposite}}{\text{hypotenuse}}$	$\cos = \frac{\text{adjacent}}{\text{hypotenuse}}$	$\tan = \frac{\text{opposite}}{\text{adjacent}}$

Some students like to use SOH-CAH-TOA to help them remember the formulas.

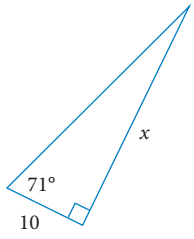
Example 5

Which trig ratio can be used to find the value of the pronumeral in each triangle?

a



b



Solution

a The marked sides are the adjacent, **a**, and the hypotenuse, **h**. 'An hour' is the middle section of the phrase, and the middle trig key on the calculator is cos.

The cos ratio can be used.

b The marked sides are the opposite, **o**, and the adjacent, **a**. 'Of algebra' is the last section of the phrase, and the last trig key on the calculator is tan.

The tan ratio can be used.

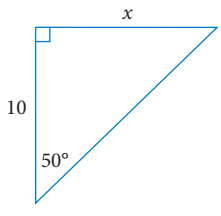
EXERCISE 16.5

Sin, cos or tan?

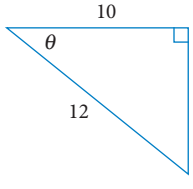
1 In each triangle, which trig ratio could be used to find the value of the pronumeral?

EXAMPLE 5

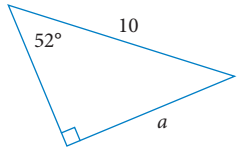
a



b

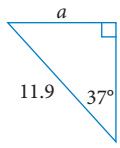


c

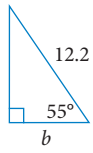


2 Find the value of each pronumeral, correct to 1 decimal place.

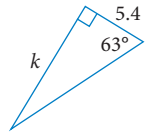
a



b

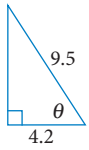


c

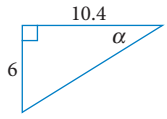


3 Calculate the sizes of the marked angles correct to the nearest degree.

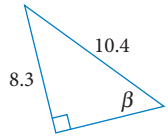
a



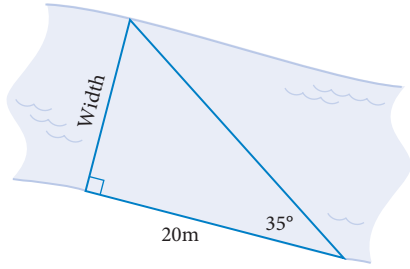
b



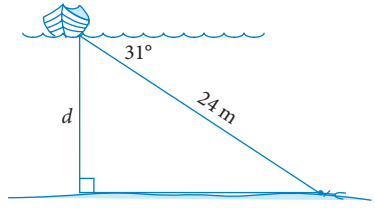
c



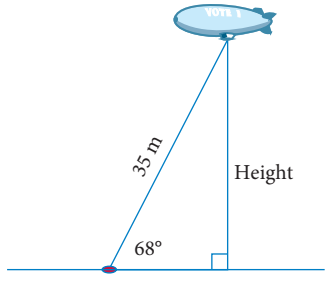
4 During a flood, Grant needed to work out the width of the river. The diagram shows the measurements he took. Use Grant's measurements to determine the width of the flooded river. Answer correct to the nearest metre.



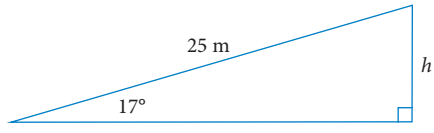
5 A boat's anchor chain is 24 m long, and it is making an angle of 31° with the top of the water. How deep, *d* m, is the water? Express your answer correct to 1 decimal place.



6 An advertising blimp is anchored to the ground by a 35 m long rope. The angle between the rope and the ground is 68°. How high is the advertising blimp above the ground? Answer correct to 1 decimal place.

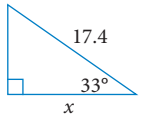


7 A 25 m long skateboard ramp is at 17° to the horizontal. How much higher is one end of the ramp than the other? Answer correct to 1 decimal place.

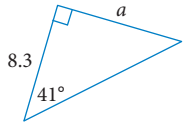


8 Find the value of each pronumeral, correct to one decimal place.

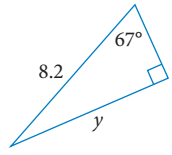
a



b

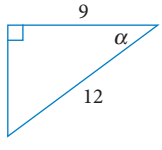


c

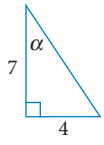


9 Determine the size of α correct to the nearest degree.

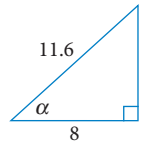
a



b



c

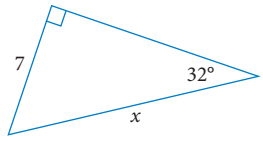


Finding the hypotenuse

16.6

Example 6

Find the value of x in this triangle correct to 1 decimal place.



Solution

In the triangle, x is the hypotenuse and 7 is the side opposite 72°. The sin ratio is $\frac{\text{opposite}}{\text{hypotenuse}}$.

$$\frac{7}{x} = \sin(32°)$$

I can write sin 32° with a denominator of 1.

$$\frac{7}{x} = \frac{\sin(32°)}{1}$$

Take the reciprocal of both sides to get x on top.

$$\frac{x}{7} = \frac{1}{\sin(32°)}$$

Multiply both sides of the equation by 7.

$$\cancel{7} \times \frac{x}{\cancel{7}} = \frac{1}{\sin(32°)} \times 7$$

$$= \frac{7}{\sin(32°)}$$

Enter 7 ÷ sin 32 = on your calculator.

$$x = 13.2095\ldots$$

$$\approx 13{\cdot}2 \text{ cm}$$

From the diagram, $x \approx 13.2$ cm seems a reasonable answer.

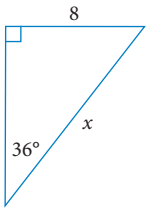
EXERCISE 16.6

Finding the hypotenuse

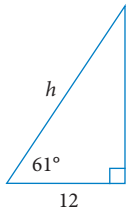
1 Calculate the length of the hypotenuse in each triangle. Express your answers correct to 1 decimal place.

EXAMPLE 6

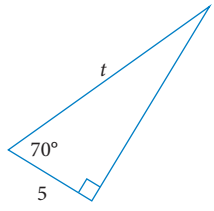
a



b

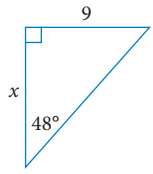


c

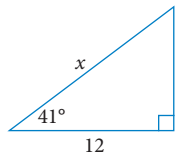


2 Calculate correct to 1 decimal place the length of the side marked with a pronumeral in each triangle.

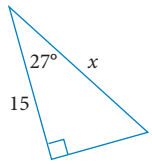
a



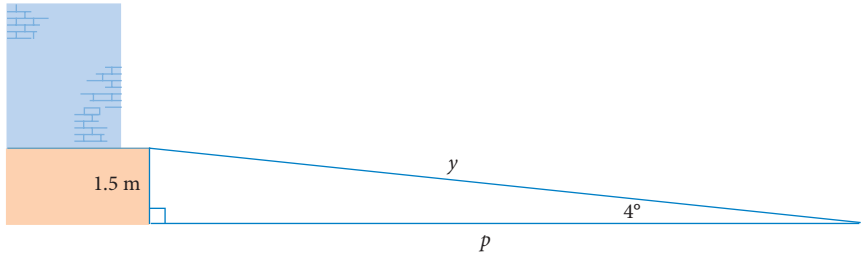
b



c



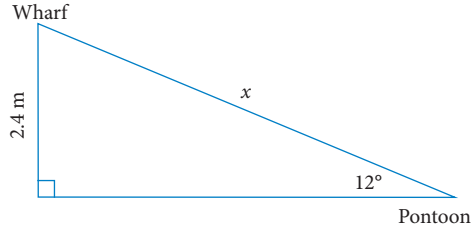
3 Scott is designing a wheelchair access ramp. The ramp will connect the car park with the building. The building is 1.5 m higher than the car park. The angle of inclination must not exceed 4°.



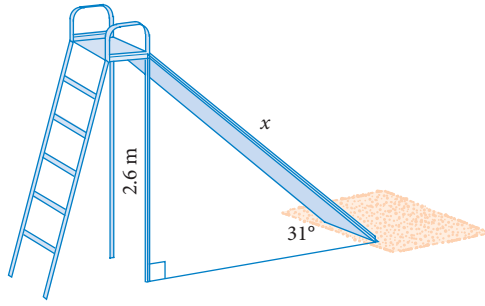
a Calculate the horizontal distance, p, required for the ramp. Express your answer correct to 2 decimal places.

b How long is the ramp, y? Answer correct to 1 decimal place.

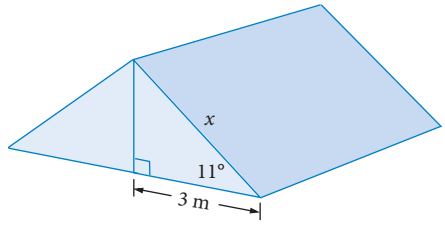
4 At low tide a wharf is 2.4 m above the water. The ramp joining the wharf to a floating pontoon makes an angle of 12° to the horizontal. Calculate the length of the ramp correct to 1 decimal place.



5 The council installs a children's slide in the park. The top of the side is 2.6 m high, and the slide makes an angle of 31° with the ground. Calculate the length of the slide correct to 2 decimal places.

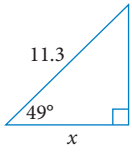


6 The diagram shows a shed roof. Calculate x, the length of each piece of iron sheeting required. Round your answer up to the nearest 10 cm.

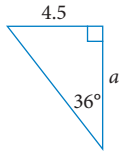


7 Find the value of each pronumeral, correct to 1 decimal place.

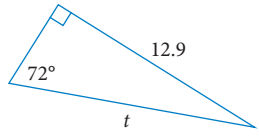
a



b



c



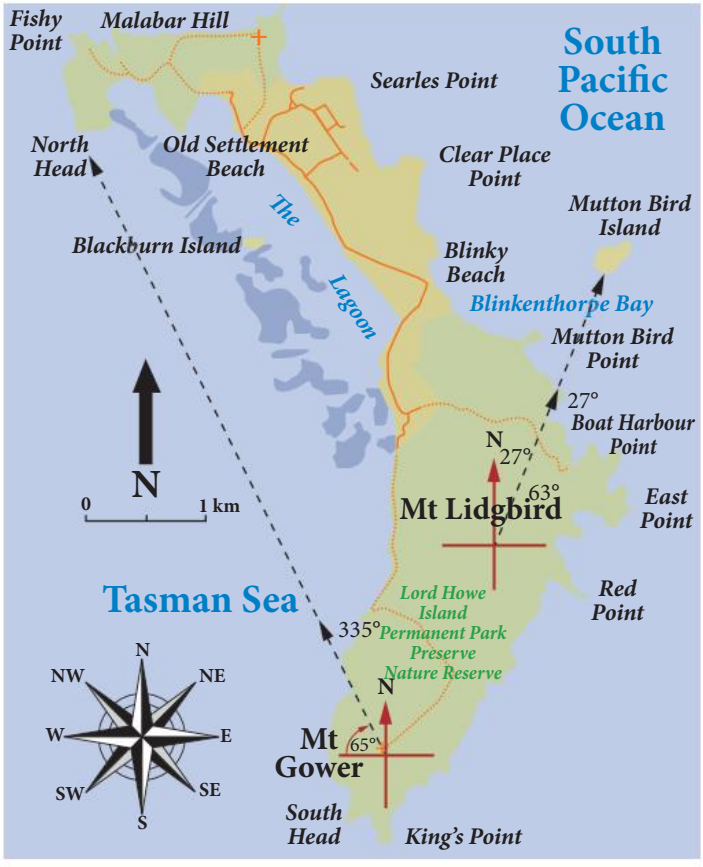
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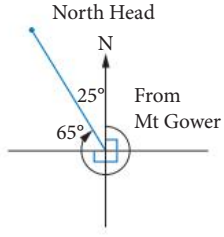
Bearings

16.7

Aircraft and ship navigators are responsible for getting their vessels to their destination. They use **three-figure bearings** (3-digit angles) to specify directions accurately. Three-figure bearings measure angles from true north in a clockwise direction, from 000° to 360°.

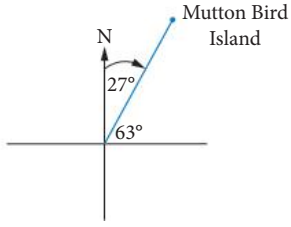
On this map of Lord Howe Island, to find the bearing ***from*** Mt Gower to North Head we draw a compass rose on Mt Gower, draw a line from Mt Gower to North Head, then measure the angles.





We add all the angles in a clockwise direction from north around to the direction line.

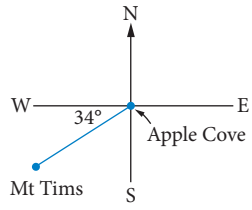
Bearing from Mt Gower to North Head = 90° + 90° + 90° + 65°
= 335°



To find the bearing of Mutton Bird Island ***from*** Mt Lidgbird, position the compass rose on Mt Lidgbird and draw a line from Mt Lidgbird to Mutton Bird Island. The angle between north and the direction line to Mutton Bird Island is 27°, but we write the bearing as 027° because we have to use 3 figures.

Example 7

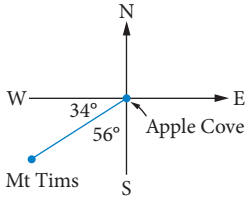
What is the bearing from Apple Cove to Mt Tims on this diagram?



Solution

The angle between Mt Tims and south is
$90° - 34° = 56°$

Add the angles from north in a clockwise direction to the direction of Mt Tims.



Bearing from Apple Cove to Mt Tims
$= 90° + 90° + 56°$
$= 236°$

Example 8

Kevin left his bush camp and walked on a bearing of 117° to the creek.
What bearing should Kevin follow to return to his camp from the creek?

Solution

The bearings of journeys in opposite directions always have a difference of 180°, with one being below 180° and the other being above 180°.

117° is below 180°, so to find the reverse bearing, add 180° to it.

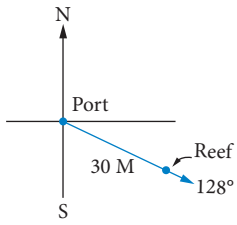
Bearing for the return trip $= 117° + 180°$
$= 297°$

If the bearing is above 180°, then to find the reverse bearing, subtract 180° from it.

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Example 9

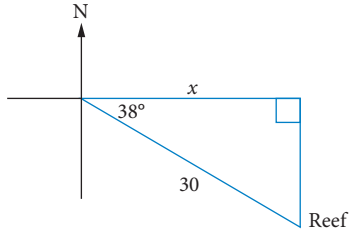
A fishing boat left port and sailed on a bearing of 128° for 30 nautical miles to a good fishing reef. How far is the reef east of port? Answer to the nearest nautical mile.



M is the abbreviation for nautical miles.

Solution

Construct a right-angled triangle by drawing a line from the reef straight up to the east direction line.



The angle in the triangle = 128° – 90°
= 38°

Write a pronumeral such as x for the required distance: how far the reef is east of port.

Then use trigonometry to find x.

$$\cos(38°) = \frac{x}{30}$$
$$x = 30\cos(38°)$$
$$= 23.6403...$$
$$\approx 24 \text{ nautical miles}$$

Write your answer.

The reef is 24 M east of port.

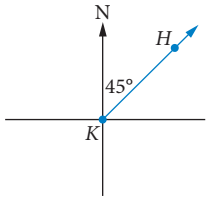
EXERCISE 16.7

Bearings

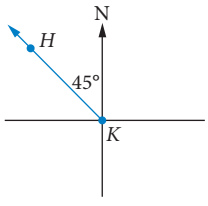
1 What is the three-figure bearing and compass direction from K to H in each diagram?

EXAMPLE 7

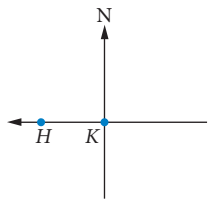
a



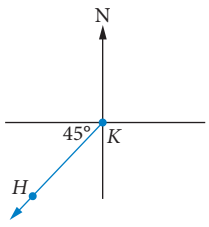
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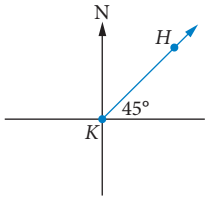
c



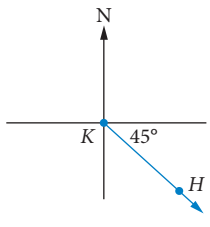
d



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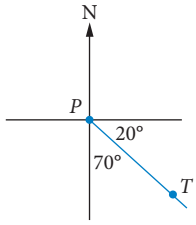


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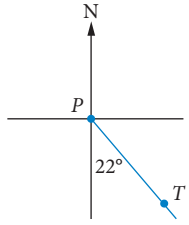


2 What bearing is required to travel from P to T in each diagram?

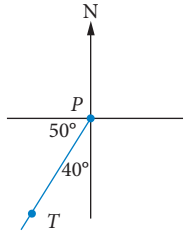
a



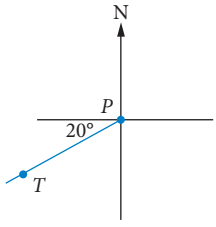
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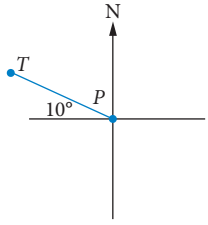
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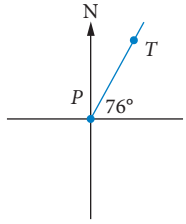
d



e



f



EXAMPLE 8

3 Copy and complete the bearings for each trip.

	Forward trip bearing	Bearing for the return trip
a	134°	
b	092°	
c		316°
d		065°

4 The bearing of Mecca, Saudi Arabia from Brisbane is 280°. What is the bearing from Mecca to Brisbane?



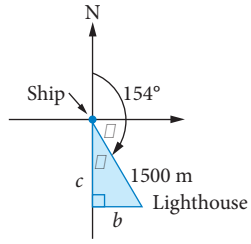
Habib bin Hilal http://www.al-habib.info/qibla-pointer/

5 The bearing from Melbourne to Mecca is 279°. What is the bearing from Mecca to Melbourne?

When we draw straight line compass directions on a map they look curved because a map is a flat representation of the Earth's curved surface.

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6 A lighthouse is 1500 m from a ship on a bearing of 154°.

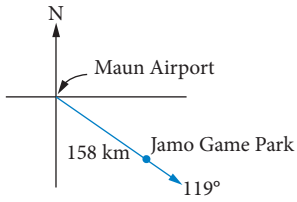


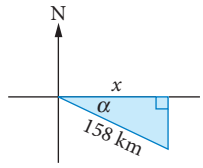
a Explain why $\alpha = 64°$ and $\beta = 26°$ in the diagram.

b How far is the ship west of the lighthouse?

c Calculate the distance that the lighthouse is south of the ship.

7 A light aircraft flew 158 km in a straight line from Maun Airport to Jamo Game Park on a bearing of 119°.

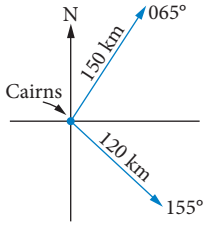


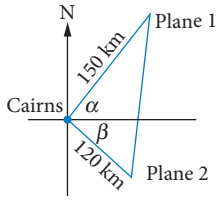


a Calculate the size of the angle α.

b How far is Jamo Game Park east of Maun Airport?

8 Two search-and-rescue planes flew out of Cairns to search for a missing yacht. The first plane flew 150 km on a bearing of 065° and the second plane flew on a bearing of 155° for 120 km.



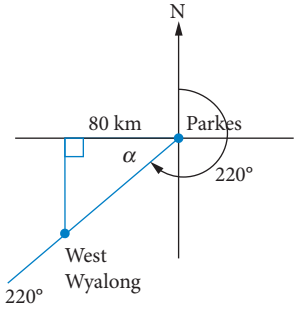


a Calculate the size of angles α and β.

b What is $\alpha + \beta$?

c Use Pythagoras' theorem to calculate how far the planes are apart.

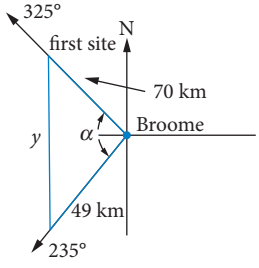
9 Ben drove along the highway from Parkes to West Wyalong, on a bearing of 220°. West Wyalong is 80 km west of Parkes.



a Explain why $\alpha = 50°$.

b How far (correct to the nearest kilometre) is it along the highway from Parkes to West Wyalong?

10 Two diveboats sailed out of Broome heading for popular dive sites. The first sailed on a bearing of 325° for 70 nautical miles. The second boat sailed 49 km on a bearing of 235° until it was due south of the first dive boat.



a Explain why $\alpha = 90°$.

b How far, y, are the two dive sites apart?



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Investigation Where is Mecca?

In this investigation you are going to use a website to find the distance and the compass direction from your home to Mecca.

Search on the Internet for 'Mecca locator' or go to the website www.al-habib.info and select 'Qibla pointer'. Enter the name of your town or city followed by Australia and click 'Find'. Record the distance and the bearing from true north.

For example, from Melbourne, Mecca is 12 741 km and 279°.



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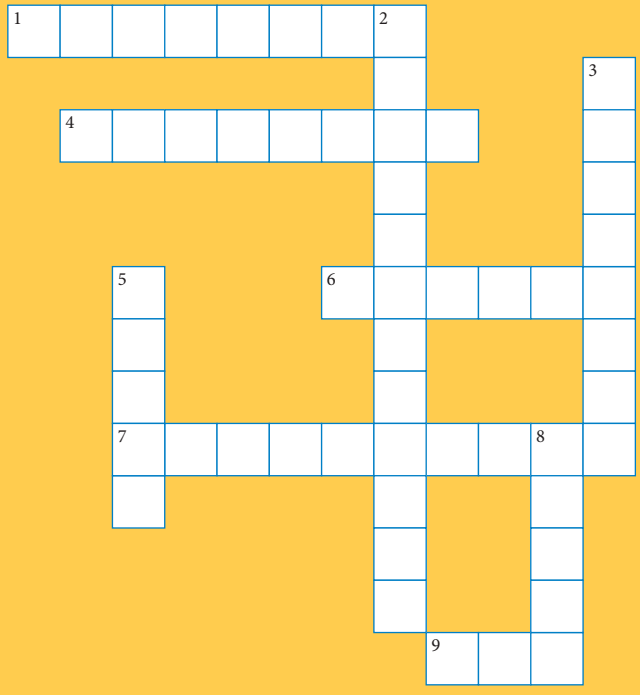
Keyword activity

Use the trigonometry summary below to complete the crossword puzzle. Copy the crossword or print out a copy from NelsonNet.

The word trigonometry comes from two Greek words that combine to mean 'the measurement of triangles'. In modern times [2]_____ has many applications, including determining the size of [6]_____ and [8]_____ in right-angled triangles.

In a [5]_____ angled triangle the side opposite the right angle is called the [7]_____ but the names of the other sides are determined by the reference angle. The [3]_____ side is the side in the triangle that is opposite to the reference angle and the [1]_____ side is next to the reference angle.

The ratio of the opposite side to the adjacent side is called the [4]_____. We use the sin or the [9]_____ ratio when we do calculations involving the hypotenuse.

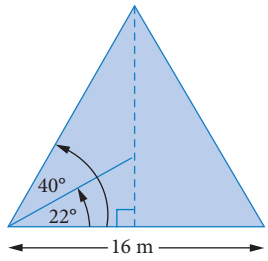


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Chapter Problem

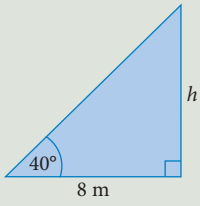
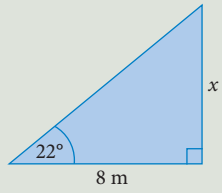
Early Australian builders knew that a large roof pitch helped prevent leaks and keep houses cool. The early roof pitches were often as large as 40°. Today most roof pitches are about 22°.

How much higher was the roof of a 16 m-wide early Australian building with a 40° roof pitch than a modern building with the same width and a 22° roof pitch?



Solution

Use trigonometry to calculate the height of each roof.

Early Australian roof	Modern Australian roof
	
$\tan(40°) = \frac{h}{8}$ $h = 8\tan(40°)$ $= 6.7127\ldots$ ≈ 6.71 m	$\tan(22°) = \frac{x}{8}$ $x = 8\tan(22°)$ $= 3.2322\ldots$ ≈ 3.23 m

Difference in heights = 6.71 – 3.23 = 3.48 m

The roof of the early Australian building was 3.48 m higher than that of the modern building.



Chapter Problem

A consumer association investigated the quality and price of eight pairs of gym shoes, labelled A to H. The investigation team gave each pair of shoes a quality rating out of 10.

This table shows the results.

	A	B	C	D	E	F	G	H
Price	$320	$280	$260	$240	$180	$180	$80	$40
Quality	8	10	9	6	6	4	1	3

Is there a relationship between the price of shoes and the quality rating assigned by the team? If so, what is the relationship?


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CHAPTER 17

FITTING THE DATA

17.1 Scatter plots
17.2 What is the relationship?
17.3 Looking for a pattern
17.4 Drawing a line of fit
17.5 Using a line of fit
17.6 Correlation
17.7 Interpolation and extrapolation
Keyword activity
Solution to the chapter problem

By the end of this chapter, you should know how to:

- describe the patterns and features of bivariate data
- describe the association between two numerical variables in terms of direction (positive/negative), form (linear/non-linear) and strength (strong/moderate/weak)
- draw the line of fit by eye
- calculate the correlation coefficient using technology
- use the line of fit to make predictions, both by interpolation and extrapolation
- recognise the dangers of extrapolation

How are we ever going to use this?

- When we are analysing data to see if variables are related
- To make predictions based on collected data
- Fitness industry professionals use physical data to design personal training plans
- Other jobs use this sort of data to design various items

17.1 Scatter plots

Bivariate data is data with two variables. For example, you might collect data on the height and weight of people – this would be bivariate data.

Bivariate data can be graphed on a **scatter plot**. The first variable is called the **explanatory variable** and is graphed on the **horizontal axis**. The second variable is called the **response variable** and is graphed on the **vertical axis**.

We can look at the scatter plot to see if it has any of the following patterns:

- as one variable increases, the other variable increases
- as one variable increases, the other variable decreases
- there are groups of points
- most of the points are together but a few are out on their own

Example 1

Ten students had their heights and waist heights measured and recorded.

Total height, *H* cm	175	177	178	184	162	172	173	191	163	175
Waist height, *W* cm	101	106	107	115	100	87	106	120	90	108

a Graph this bivariate data in a scatter plot.

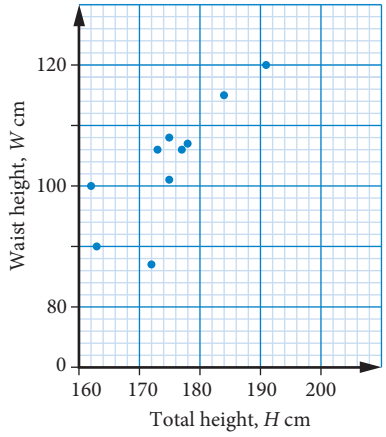
b Which is the explanatory variable and which is the response variable?

c Comment on the features of the scatter plot.

Solution

a The total height will go on the horizontal axis.

The waist height will go on the vertical axis.



b Explanatory variable is on the horizontal axis. The explanatory variable is total height.

Response variable is on the vertical axis. The response variable is waist height.

c Describe any patterns. As total height increases, the waist height increases.

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EXERCISE 17.1

Scatter plots

Graph paper is required for this exercise. Keep your scatter plots to use in later exercises.

1 This table shows the heights of a sample of girls when they were $2\frac{1}{2}$ years old and when they were 18 years old.

EXAMPLE 1

Height at $2\frac{1}{2}$ years (cm)	87	83	88	84	82	81	89	86	85	80
Height at 18 years (cm)	171	168	171	167	161	160	174	170	172	158



iStockphoto/ArtisticCaptures



iStockphoto/Polke

a Graph this bivariate data in a scatter plot.

b Which is the explanatory variable and which is the response variable?

c Comment on the features of the scatter plot.

2 This table shows weekly data for theft reported to two police stations in a central city area.

Police station A	83	78	62	86	67	79	71	91	94	93	98	71
Police station B	45	36	30	28	38	35	48	41	37	34	37	44

a Present this bivariate data in a scatter plot.

b Which is the response variable and which is the explanatory variable?

c Comment on the features of the scatter plot.

3 Simone measured the heights and arm spans of a group of ten senior students.

Height, H (cm)	170	195	181	181	166	200	163	162	183	167
Arm span, A (cm)	171	186	187	178	165	160	147	143	115	169

a Construct a scatter plot to show this bivariate data.

b Which is the explanatory variable and which is the response variable?

c What does the scatter plot show?



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4 The table shows the normal resting pulse of a sample of students and the time it takes each of them to swim 50 m.

Resting pulse (beats/min)	42	70	64	62	55	60	50	72	80
Swimming time (s)	30	48	50	43	40	45	36	49	59

a Construct a scatter plot for this set of data.

b Comment on the features of the scatter plot.

5 This table shows the sales of CD albums and digital albums over 9 years. The data is in thousands.

CD albums	46 174	49 818	44 045	38 659	39 529	33 114	30 223	27 356	14 226
Digital albums	91	418	788	1322	2279	3301	4818	6838	7377

a Present this bivariate data in a scatter plot.

b Describe what the scatter plot shows.



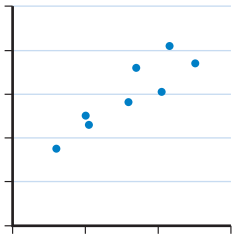
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What is the relationship?

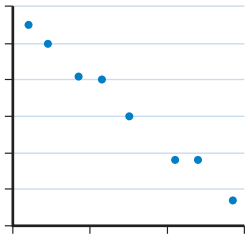
17.2

When data is graphed on a scatterplot we can use it to see if there is a relationship between the two variables. The association between two variables can be described in terms of **direction**, **shape** and **strength**.

The **direction** of the association can be **positive** or **negative**.

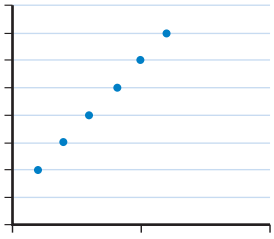


Positive (going up)

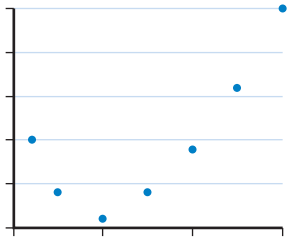


Negative (going down)

The **shape** of the association can be **linear** (in a line) or **non-linear** (in a curve).

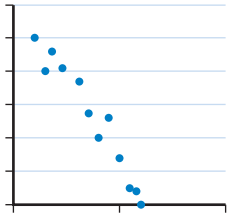


Linear pattern

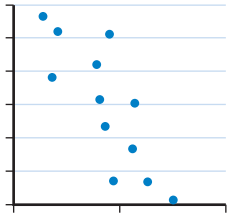


Non-linear pattern

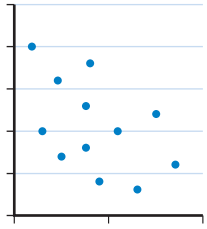
The **strength** of the association can be **strong**, **moderate** or **weak**.



Strong

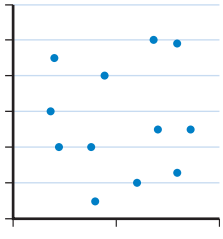


Moderate



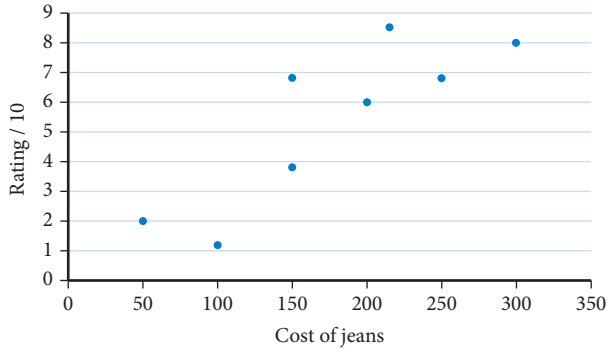
Weak

Sometimes, the points have **no association**.



Example 2

Describe the association between the variables shown in this bivariate scatter plot.



Solution

The dots go up from left to right.	Positive
The dots are close to a straight line.	Linear
The points are spread out.	Moderate
	This association is positive, linear and moderate.

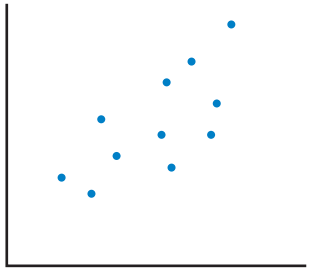
EXERCISE 17.2

What is the relationship?

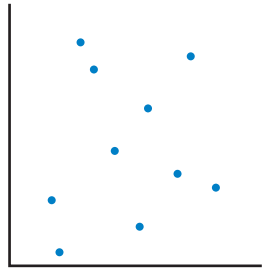
EXAMPLE 2

1 Describe the association (direction, shape and strength) between the two variables shown in each scatter plot.

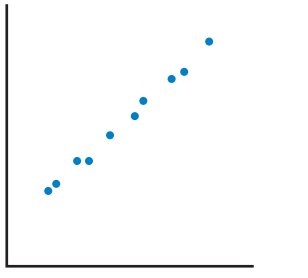
a



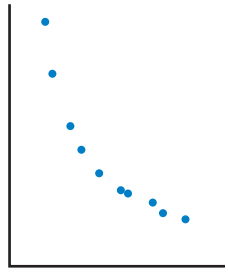
b



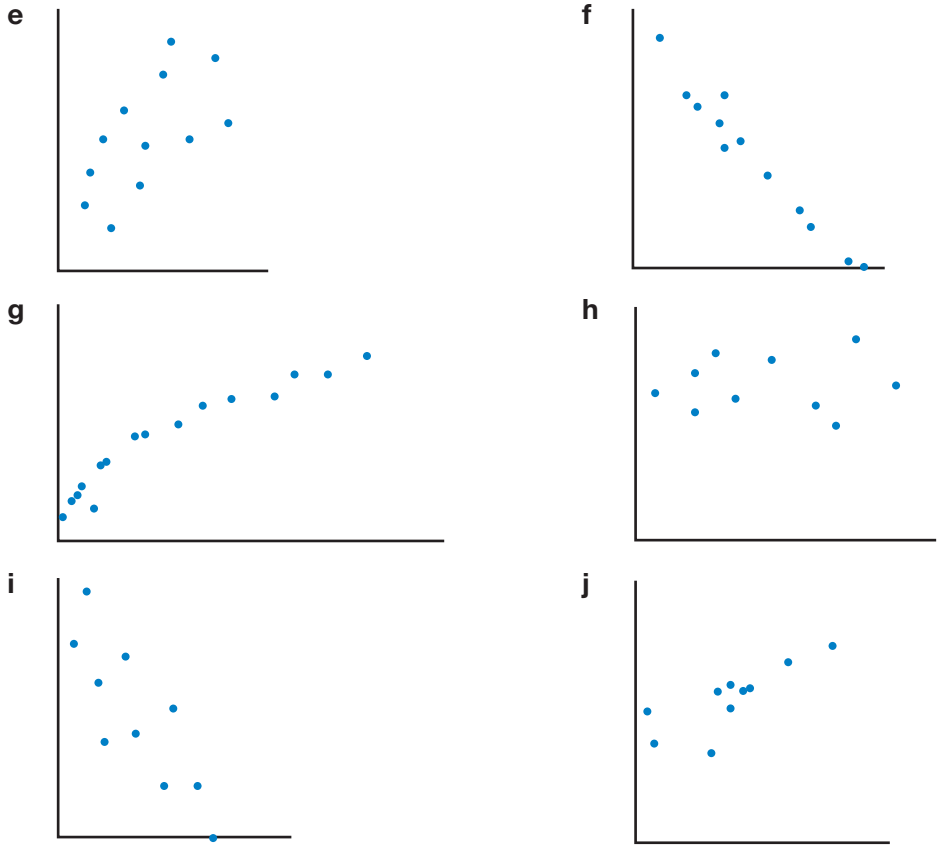
c



d



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2 Look at the scatter plots you drew in Exercise **17.1**. Describe the association of the variables in:

a Question **1** on heights

b Question **4** on pulse and time

c Question **5** on album sales

3 The table shows the ages of 10 boys and time it took them to swim 50 metres.

Age (years)	6	15	7	8	7	9	14	12	10	11
Swim time (s)	76	32	56	49	54	45	36	41	38	36

a Construct a scatter plot to display the information.

b Does there appear to be a relationship between the boy's ages and the time it takes them to swim 50 m? If 'yes', describe the association.



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4 **a** For each student in your class, record the number of letters in their family name and the number of minutes they spend travelling to school (rounded to the nearest 10 minutes).

b Draw a scatter plot of the data with number of letters on the horizontal axis and travelling time on the vertical axis.

c Would you expect these two variables to have any association? Why or why not?

d Is there an association between these two variables? If so, describe it.

5 **a** Collect the following data for each student in your class.

- the circumference of the student's head
- the student's height

b Draw a scatterplot of the data with the circumference of the student's head on the horizontal axis and the student's height on the vertical axis.

c Is there a relationship between these two variables? If so, describe it.

6 Sketch scatter plots to illustrate sets of data that have the following features:

a Weak, positive linear association

b Strong, negative linear association

c Strong, positive, non-linear association

d No association

Investigation My body parts

Measure and record each body part for each student in your class, correct to the nearest cm:

- wrist circumference
- neck circumference
- waist circumference

Construct scatter plots for:

- wrist circumference and neck circumference
- wrist circumference and waist circumference

Describe any relationships you can see in each of these scatter plots.

Looking for a pattern

17.3

In this section we will apply what we have learned to analysing data about a sample of Year 12 students.

EXERCISE 17.3

Looking for a pattern

This table gives information collected from 15 Year 12 students. You can also download this table as a spreadsheet 'Year 12 students' data' from NelsonNet.

Student	Height (cm)	Arm span (cm)	Right foot length (cm)	Time to travel to school (mins)	Hours of homework per week	Resting Pulse (beats per minute)	Hours watching TV per week	Number of siblings
Amy	176	175	26	7	6	64	2	4
Joe	178	175	23	40	6	66	10	2
Annika	151	150	30	10	23	95	4	1
Janine	168	175	25	15	7	76	14	2
Stephen	186	181	37	30	23	60	9	3
Thanh	187	187	28	5	2	74	5	2
Gillian	149	149	22	50	17	70	4	1
Vamsee	174	172	28	4	1	62	3	1
Lalaja	172	178	26	20	16	76	20	4
Darryl	177	177	23	15	10	64	10	3
Lyn	169	160	22	7	15	60	0	3
Jeremy	159	155	24	25	9	83	4	2
Ben	169	184	26	34	0	68	17	1
Abdul	163	159	24	20	3	77	9	2
Miriam	163	165	25	20	4	75	1	3

1 Use the data for arm span and the length of the right foot.

- **a** Draw a scatter plot for this data.
- **b** Name the explanatory and response variables from your graph.
- **c** Describe any features of the scatter plot.
- **d** Is there an association between these two variables? If so, describe it.

2 Use the data in the above table for number of siblings and time to travel to school.

a Would you expect there to be an association between these two variables? Why or why not?

b Construct a scatter plot for this data.

c Describe any features of the scatter plot.

3 Choose two variables from the table that you would expect to have an association.

a Draw a scatter plot for this data.

b Name the explanatory and response variables from your graph.

c What features does the scatter plot have?

d Does the scatter plot show a relationship between these two variables? If so, describe it.

4 Choose two variables from the table that you would NOT expect to have an association.

a Draw a scatter plot for this data.

b Describe any features of the scatter plot.

c Is there an association between these two variables? If so, describe it.



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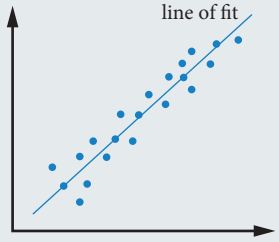
Drawing a line of fit

17.4

If the data on a scatter plot shows a strong linear association, we can approximate the linear relationship by drawing a **line of fit** through the points.

A line of fit:

- represents most or all of the points as closely as possible
- goes through as many points as possible
- has roughly the same number of points above and below it
- is drawn so that the distances of points from the line are as small as possible



Example 3

This table from Question **1** on page 409 shows the heights of a sample of girls when they were 2 $\frac{1}{2}$ years old and when they were 18 years old.

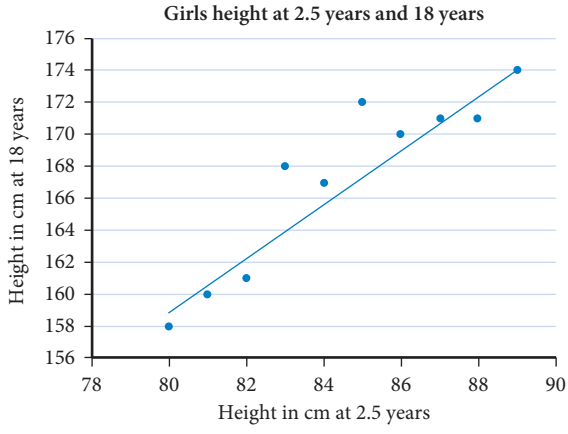
Height at $2\frac{1}{2}$ years (cm)	87	83	88	84	82	81	89	86	85	80
Height at 18 years (cm)	171	168	171	167	161	160	174	170	172	158

Graph this bivariate data on a scatterplot and draw a line of fit for it.

Solution

Graph the data first.

Then draw a line through the middle of the points with roughly the same number of points above and below it.



EXERCISE 17.4

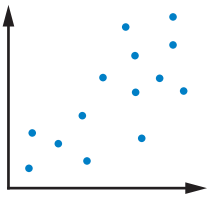
Drawing a line of fit

Keep your answers to this set of exercises. You will be using them again in Exercise **17.6**.

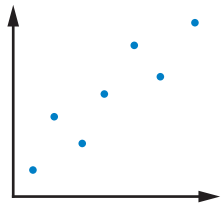
EXAMPLE 3

1 Copy or print each scatter plot and draw in a line of fit. You can print a copy of the graphs as the worksheet 'Scatter plots' from NelsonNet.

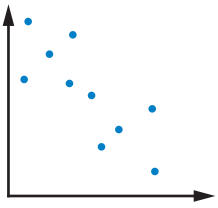
a



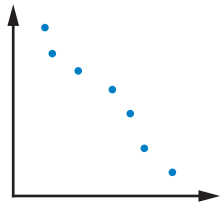
b



c



d



2 This table shows the birth rate and the female life expectancy for a number of countries. The birth rate is births per year per 1000 population. Life expectancy is measured in years.

Country	Birth rate	Female life expectancy
Australia	13.3	85.5
Brazil	14.7	79.8
Canada	11.2	84.6
Fiji	21.4	76
Germany	8.1	83.5
Iraq	31.0	72
Kenya	36.1	61
Laos	28.0	69.5
Nepal	24.3	70
Niger	50.0	57
Rwanda	42.1	61

a Draw a scatter plot of birth rate against life expectancy.

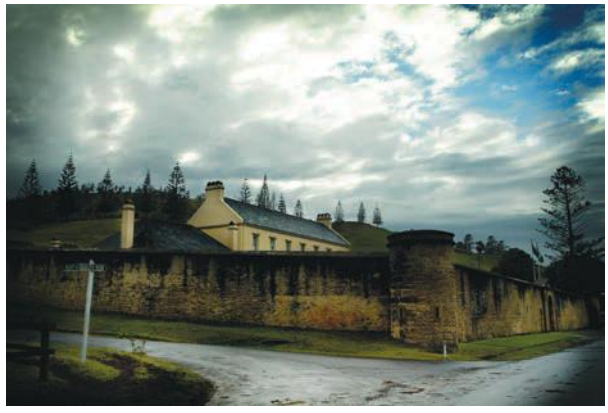
b Draw in a line of fit for this data.

3 This table shows the height above sea level and the average annual rainfall for some places in Australia.

City	Height above sea level (m)	Mean annual rainfall (mm)
Alice Springs	581	282
Ballarat	432	694
Hobart	24	616
Kalgoorlie	387	266
Mount Isa	365	463
Norfolk Island	73	1017
Perth	15	736
Winton	188	382

a Draw a scatter plot of height above sea level against mean annual rainfall.

b Draw in a line of fit for this data.



Fairfax Syndication/FIONA LEE-QUIMBY

4 This table shows information about energy, carbohydrate and fat contained in 100 g of some take-away foods.

Food	Energy (kilojoules)	Carbohydrate (grams)	Fat (grams)
Hamburger	1030	26.6	9.1
Cheeseburger	1070	23.9	11.2
Chicken burger	921	20.2	10.1
Fish burger	988	24.5	10.2
Grilled chicken wrap	771	13.4	10.2
Chicken salad	325	5.5	3.4
Egg and bacon wrap	767	15.3	9.1
Hash browns	1150	26.4	17.2
Chips	1480	39.7	19.0

a Draw a scatter plot of energy against carbohydrate.

b Draw in a line of fit for this data.

c Draw a scatter plot of energy against fat.

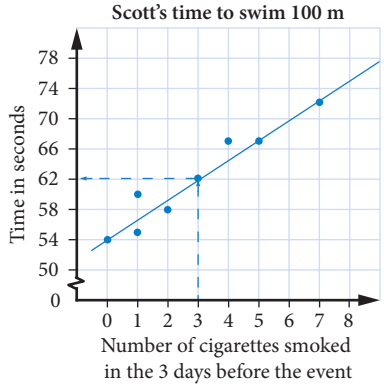
d Draw in a line of fit for this data.

17.5 Using a line of fit

Lines of fit show the association between variables and we can use them to make predictions within the range of the data.

Example 4

Scott is a competitive swimmer. He noticed that his times for the 100 m freestyle event are slower when he smokes cigarettes in the 3 days before the event. He displayed his times for his last six events and the numbers of cigarettes he smoked in the 3 days before on a scatter graph. He drew a line of fit through the data.





Shutterstock.com/BrunoRosa

a What is the explanatory variable?

b What is the response variable?

c What does the line of fit show about the relationship between these two variables?

d Use the line of fit to predict Scott's time for the 100 m freestyle when he has smoked three cigarettes in the 3 days before the event.

Solution

a	Explanatory variable is on the horizontal axis.	The explanatory variable is the number of cigarettes smoked.
b	Response variable is on the vertical axis.	The response variable is the swimming time.
c	Describe the relationship.	As the number of cigarettes smoked increases, the time for the 100 m increases.
d	Find 3 on the horizontal axis, go up to the line and then across to the time.	Scott's time will be approximately 62 seconds.

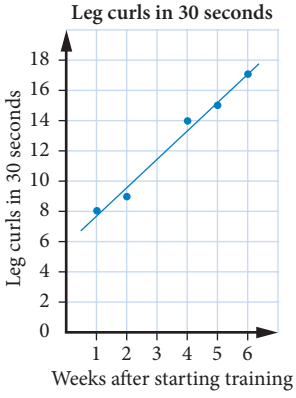
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EXERCISE 17.5

Using a line of fit

1 Anita is trying to strengthen her hamstring muscles. This graph shows the number of leg curls she was able to complete in 30 seconds each week after she started training.

EXAMPLE 4





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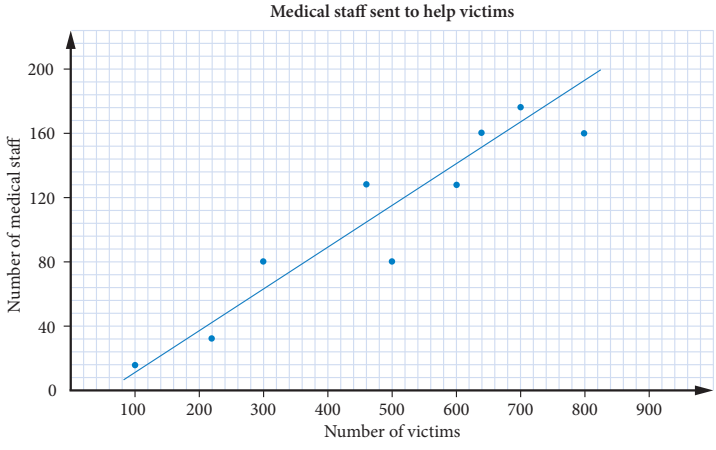
a What is the explanatory variable?

b What is the response variable?

c What does the line of fit show about the relationship between these two variables?

d Anita forgot to record the number of leg curls she could do in week 3. Use the line of fit to predict the number.

2 Humanitarian agencies send people to assist when a disaster occurs. This scatter graph shows the number of medical staff one small agency sent to assist after some disasters and the estimated number of victims in each disaster.



a What is the explanatory variable?

b What is the response variable?

c Use the line of fit to estimate the number of medical staff the agency will send to a disaster with an estimated 200 victims.

d The agency sent 180 medical staff to assist in a disaster. Approximately how many victims were affected by the disaster?

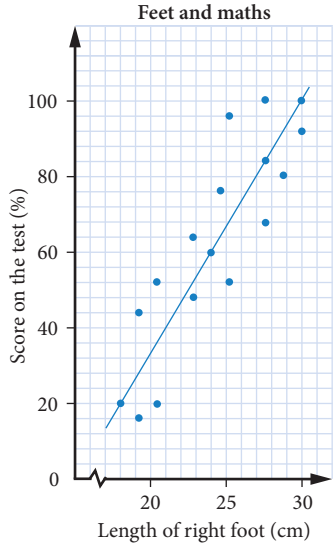
e Describe the relationship between the number of medical staff and the number of victims.

f Kyle noted that the greater the number of medical staff, the more victims there were requiring help. He argued that if the agency decreased the number of medical staff, there would be fewer people needing help. What is wrong with Kyle's argument?



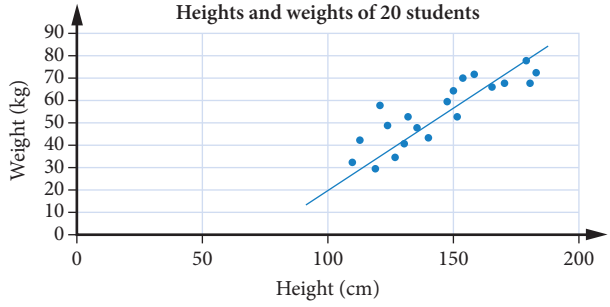
iStockphoto/EdStock

3 Billy announced, 'People with big feet are better at maths than people with small feet'. Billy had given a small maths test to all the people in the school library and he had measured the length of each person's right foot. He displayed the results on a scatter graph.



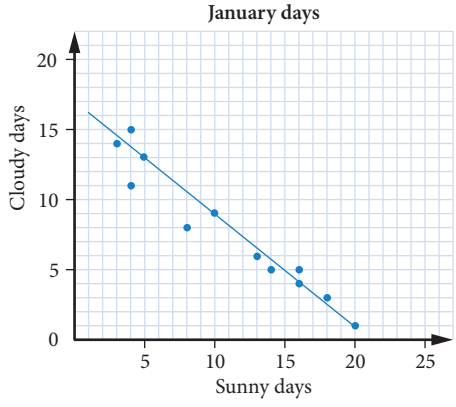
- **a** According to Billy's line of fit, what is the right foot length of a person who scored 58 on the maths test?
- **b** Describe the relationship between the length of the right foot and the score on the test as shown by this line of fit.
- **c** Big feet don't cause high maths scores. How can Billy's results be explained?

4 This graph shows the scatter plot for the heights and weights of 20 students with a line of fit drawn on it.



- **a** Describe the relationship between the height and weight of students as shown by this line of fit.
- **b** Use the line of fit to estimate the weight of a student who is 150 cm tall.
- **c** Does an increase in height **cause** an increase in weight? Why or why not?

5 David graphed the number of sunny days in January against the number of dry cloudy days for 12 Australian locations. He then drew a line of fit on it.

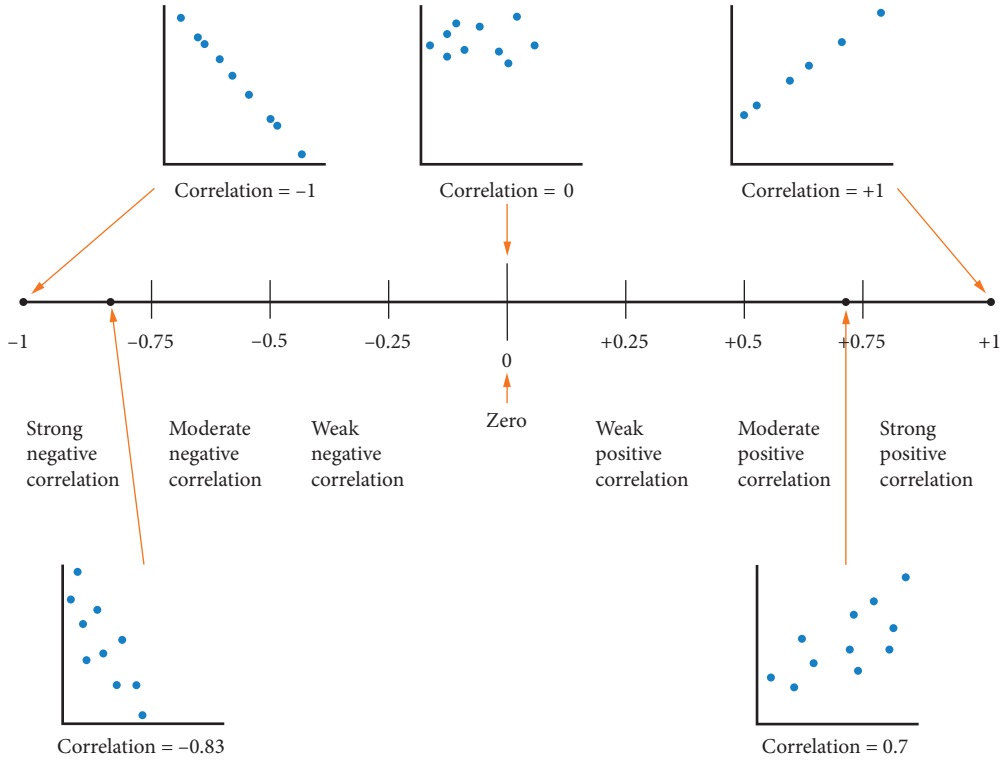


- **a** Use the line of fit to estimate how many dry cloudy days there would be if there were 7 sunny days.
- **b** Describe the relationship between the number of sunny days and the number of cloudy days in January.
- **c** Suggest a reason for this relationship.

Correlation

17.6

Correlation is a measure of how strongly two variables are related to each other. We can measure the strength of the relationship by calculating a statistical number called the **correlation coefficient**. The correlation coefficient can have values from –1 to 1.



The more the data points lie in a straight line, the closer the correlation is to 1 or –1.

The more spread out the data, the closer the correlation is to 0.

Calculating the correlation coefficient using technology

The correlation coefficient can be calculated using an online calculator, scientific calculator or spreadsheet. As with standard deviation, there is a complex formula for calculating the correlation coefficient but you are not required to use it.

Example 5

Calculate the correlation coefficient for the values in the table.

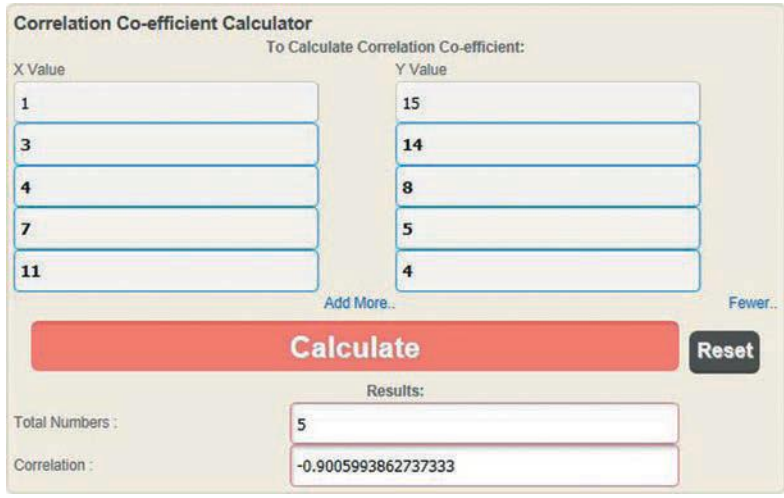
Age	1	3	4	7	11
Number of figurines owned	15	14	8	5	4

Solution

Method 1:
Online calculator

1 Search the Internet for 'correlation coefficient calculator' or use the website www.easycalculation.com.

2 Enter each pair of values, then click Calculate to display the correlation coefficient as –0.9005…



Method 2:
Scientific calculator

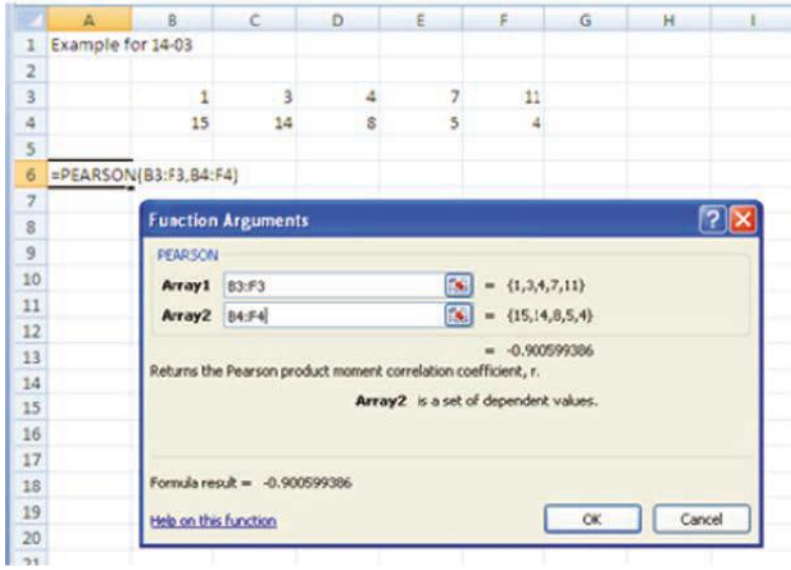
Operation	Casio scientific	Sharp scientific
Start statistics mode.	MODE STAT A+Bx	MODE STAT LINE
Clear the statistical memory.	SHIFT 1 Edit, Del-A	2ndF DEL
Enter data.	SHIFT 1 Data to get table Enter in *X* column 1 = 3 =, etc. Enter in *Y* column 15 = 14 =, etc. AC to leave table	1 2ndF STO 3 M+ 15 2ndF STO 14 M+ etc.
Calculate the correlation coefficient ($r = -0.9005\ldots$)	SHIFT 1 Reg *r* =	ALPHA *r* =

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Method 3: Spreadsheet

1 Type the data into the spreadsheet. The example shows the first row of data in cells B3 to F3 and the second set in B4 to F4.

2 Select the cell where you want to display the correlation coefficient, and select Insert function from the tool bar. In the Search for a function box, type PEARSON then let the spreadsheet help you complete the formula to calculate the correlation coefficient –0.9005…



The correlation coefficient is also called the Pearson Product Moment Correlation Coefficient, which is why the spreadsheet calls the formula PEARSON rather than correlation.

Example 6

A sample of 10 people had their left foot length and left lower arm length (from their elbow to the tip of the middle finger) measured.

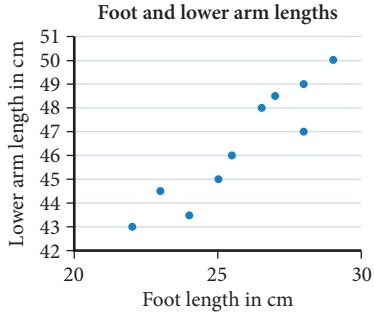
Left foot length (cm)	27	22	23	24	25	25.5	28	26.5	28	29
Left lower arm length (cm)	48.5	43	44.5	43.5	45	46	47	48	49	50

a Construct a scatter plot of this data.

b Calculate correct to 2 decimal places the correlation coefficient and describe the correlation in words.

Solution

a Foot length is on the horizontal axis and arm length is on the vertical axis.



b Calculate the correlation coefficient using one of the three methods shown in Example 5.

The correlation coefficient is 0.93.

The data shows a strong positive linear relationship between the length of the left foot and the length of the lower left arm.

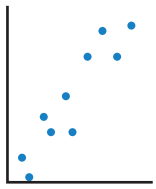
EXERCISE 17.6

Correlation

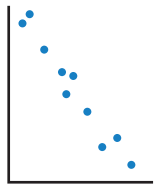
Keep your answers to this exercise for the next exercise, **17.7**.

1 Use the diagram at the beginning of this section to help you complete this question. Match the scatter plots below to their correlation coefficients.

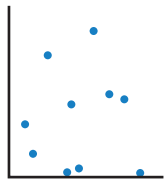
a



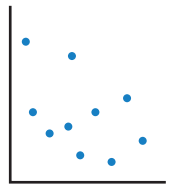
b



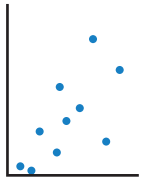
c



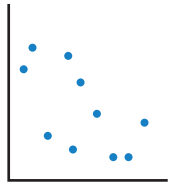
d



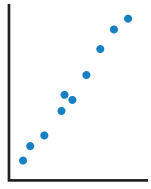
e



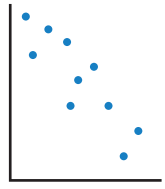
f



g



h



Correlation coefficients

A	−0.623	**B**	0.008	**C**	−0.977	**D**	−0.870
E	0.923	**F**	0.995	**G**	−0.441	**H**	0.681

EXAMPLE 5

2 Kirrily works in a coffee shop. She thinks there is a relationship between the outside temperature and their daily hot chocolate sales.

Temperature (°C)	8	36	12	28	14	22	16	10	24	18	7	31
Hot chocolate drinks sold	42	15	37	24	9	20	35	20	10	30	30	4

a Construct a scatter plot to illustrate Kirrily's information.

b Calculate correct to 2 decimal places the correlation between outside temperature and the number of hot chocolate drinks sold.



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3 Calculate the correlation for the data in Exercise **17.4**:

a Question **2** between birth rate and female life expectancy

b Question **3** between height above sea level and mean annual rainfall

c Question **4** between energy and carbohydrate content

d Question **4** between energy and fat content

e Question **4** between carbohydrate content and fat content

4 Victor wanted to investigate whether the latitude of a city was related to its temperature.

EXAMPLE 6

City	Latitude (°S)	Mean January temperature (°C)
Adelaide	35	30
Alice Springs	23	40
Brisbane	27	31
Darwin	12	33
Hobart	43	24
Melbourne	38	27
Norfolk Island	29	25
Perth	32	32
Townsville	19	33

a Draw a scatter plot showing Victor's data.

b Calculate the correlation coefficient for this data.

c Do you think these two variables are closely related? Why or why not?

d What other factors would influence the mean January temperature in a city?

5 In Exercise **17.1**, Question **5** we looked at the sales of CD and digital albums over 9 years.

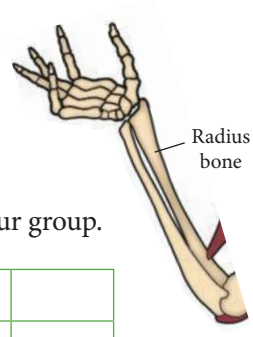
a Calculate the correlation coefficient for this data.

b What does this tell you about the relationship between these two variables?

6 Forensic scientists use the following formulas predict a person's height from the length of the radius bone in the lower arm. All measurements are in centimetres.

Male height = $80.405 + 6.650r$

Female height = $73.502 + 3.876r$



a Copy and complete this table after measuring the radius bone of the members in your group.

Name of person							
Length of radius bone (cm)							
Predicted height using the formula (cm)							
Measured height (cm)							

b Construct separate scatter plots for males and females comparing predicted heights to measured heights.

c Calculate the correlation coefficient.

d Describe the correlation.

e Suggest a reason why the predicted heights may not be very accurate.

17.7 Interpolation and extrapolation

A line of fit helps us to make **predictions** as we did in some of the questions in Exercise 17.5.
If we make a prediction on information that lies *within* the data, this is called **interpolation**.
If we make a prediction on information that lies *outside* the data, this is called **extrapolation**.

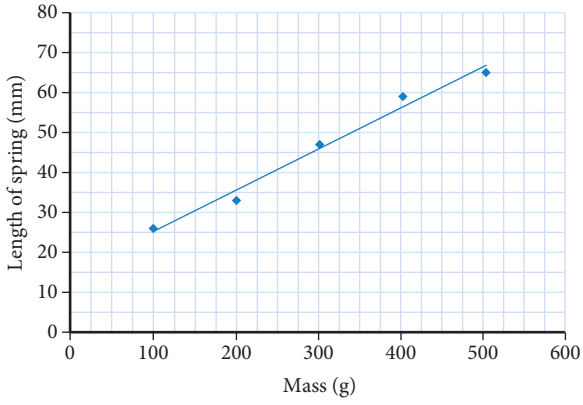
However, we need to be careful when we are making predictions that go outside the data. They can be very unreliable because we can't be sure that the linear relationship continues beyond the data.

Example 7

In a science experiment, Clare and Dilani attached different weights to a spring. They recorded the length of the spring for each weight.

Mass (g)	100	200	300	400	500
Length of spring (mm)	26	33	47	59	65

They graphed the results on the scatter plot and they drew a line of fit.



a Use the line to find:

i the length of the spring for a mass of 250 g

ii the mass when the length of the spring is 30 mm

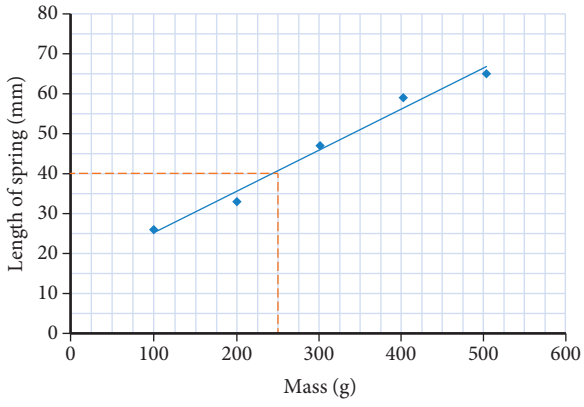
b The equation of the line is $L = 0.104m + 14.8$. Use it to predict the length of the spring when the attached mass is 800 g.

c How reliable is this prediction?

Solution

a **i** Find 250 g on the horizontal axis, go up to the graph and across to length.

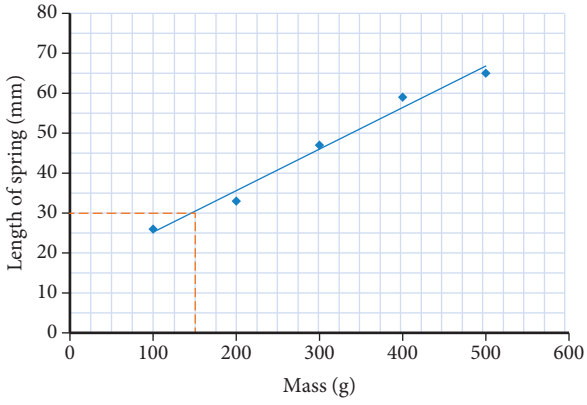
The length of the spring is approximately 40 mm.



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ii Find 30 mm on the vertical axis, go across to the line and down to mass.

The mass is approximately 150 g.



b Substitute $m = 800$ into the formula $L = 0.104m + 14.8$.

$L = 0.104 \times 800 + 14.8 = 98$

The length of the spring is approximately 98 mm.

c A mass of 800 g is outside the data shown on the graph, so it is an example of extrapolation.

The prediction is not necessarily reliable as we don't know if the relationship continues to be linear. It is also possible that at some point the spring might break.

The strength of the correlation also indicates whether predictions on the data will be reliable. The closer the coefficient is to –1 or 1, the more reliable the prediction will be.

Example 8

This student data comes from Example **1** on page 408.

Total height, H cm	175	177	178	184	162	172	173	191	163	175
Waist height, W cm	101	106	107	115	100	87	106	120	90	108

a Calculate the correlation coefficient for this data.

b If we used this data to predict the waist height of a person 170 cm tall, how reliable would it be?

c If we used this data to predict the waist height of a person 130 cm tall, how reliable would it be?

Solution

a Calculate the correlation coefficient using one of the methods shown on pages 424–425.

$r = 0.81$

b The heights in the table range from 162 to 191 cm, so 170 cm is within the data: this is interpolation.

As the correlation coefficient is quite close to 1 and 170 cm is within the data, this prediction would be reasonably reliable.

c 130 cm is outside the data range: this is extrapolation.

The prediction may not be reliable as 130 cm is outside the given data.

EXERCISE 17.7

Interpolation and extrapolation

This exercise requires some of your answers from Exercises **17.4** and **17.6**.

EXAMPLE 7

1 Jason recorded the body mass of a sample of women at the gym and the amount of energy they use per day.

Body mass (kg)	60	70	75	82	89	95
Energy used (kJ)	1040	1140	1190	1260	1300	1340

He graphed the data on a scatter plot and drew a line of fit.

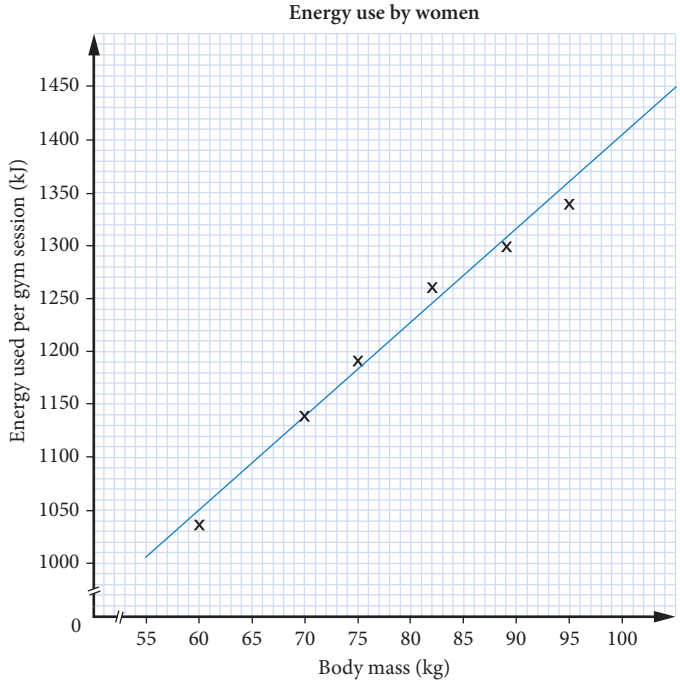
a Use the line of fit to find:

i the amount of energy used by a woman of mass 65 kg

ii the body mass of a woman who uses 1450 kJ of energy per day

b The equation of the line is $E = 16.9B - 143$. Use it to predict the energy used by a woman of mass 120 kg.

c Would you expect this situation to continue to be a linear relationship beyond the data points? Why or why not?



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2 In Exercise **17.4**, Question **2**, you drew a scatter plot for data on the birth rate and female life expectancy in a number of countries. A line of fit has equation $E = -0.725B + 91.2$.

a Predict the female life expectancy in a country where the birth rate is 40.

b Determine the birth rate in a country where the female life expectancy is 65 years.

c What would be the female life expectancy in a country where the birth rate is 60?

d How reliable do you think estimates outside this data would be? Justify your answer.

3 In Exercise **17.4**, Question **3**, you drew a scatter plot for data on the height above sea level and mean annual rainfall for a number of cities. A line of fit has equation $R = -0.769H + 755.5$.

a Predict the mean annual rainfall for a city 300 m above sea level.

b Determine the height above sea level for a city that receives 400 mm of rain per year.

c Calculate the mean annual rainfall for a city 800 m above sea level.

d Predict the mean annual rainfall for a city 1000 m above sea level. Is this possible? What is wrong with this prediction?

e In Exercise **17.6**, Question **4**, you calculated the correlation coefficient for this data. What does it tell you about the reliability of predictions based on this data?

4 Mr Armstrong, the Science teacher, sets up an experiment to measure the pressure of a gas at different temperatures. This table show the results:

EXAMPLE 8

Temperature (°C)	10	20	30	40	50	60	70	80	90
Pressure (g/cm^3)	27.9	30	29.8	32.1	31.9	34.1	33.8	34.8	36.6

a Calculate the correlation coefficient for this data.

b Mr Armstrong used this data to estimate the pressure at 56°C. How reliable is this estimate?

c Mr Armstrong used this data to predict the pressure at –10°C. How reliable is this estimate?

5 In Exercise **17.6**, Question **6**, you calculated the correlation coefficient for data about the latitude and the mean January temperature for a number of places in Australia. Do you think predictions based on this data would be reliable? Justify your answer.

6 In Exercise **17.6**, Question **7**, you calculated the correlation coefficient for data about the sales of CDs and digital albums over a number of years. How reliable would predictions based on this data be? Justify your answer.

7 Ms Cranston has a set of results for her senior class on two Maths tests.

Student	Bill	Ruth	Mary	Ella	Greg	Jim	Meg	Tara	Bob	Clem	Bree	Amy
Algebra test	60	38	65	??	75	48	67	23	82	16	92	80
Data test	35	21	47	31	56	40	54	11	59	20	62	??

Ella and Amy missed a test and Ms Cranston wants to give them an estimate.

a Draw a scatter plot for this data (except Ella and Amy) and add a line of fit.

b Predict the results that Ella and Amy might have received in the test they missed.

c Calculate the correlation coefficient for this data.

d Given this correlation coefficient, how reliable do you think these predictions are? Justify your answer.

8 Ryan investigated the heights (x cm) of a group of his friends and their hand spans (y cm, the maximum distance y between their thumb and little finger).

Height (x cm)	170	178	160	183	168	145	155
Hand span (y cm)	20	21	19	22	20	17	19

a Construct a scatter plot for the data and draw a line of fit on the plot.

b Calculate the correlation coefficient correct to three decimal places.

c A line of fit has equation $y = 0.12x - 0.02$. Use the equation to predict the hand span of Izak, who is 165 cm tall. Answer to the nearest centimetre.

d Robert Wadlow was the tallest man in the world. His height was 272 cm. Use the equation of the line of fit to predict his hand span correct to the nearest cm, and explain why this measurement is unlikely to be correct.

Keyword activity

association	bivariate	correlation
correlation coefficient	explanatory variable	extrapolation
interpolation	linear	line of fit
moderate	negative	non-linear
predictions	response variable	scatterplot
strong	weak	

Use the list of words provided to complete this summary of the chapter.

In this chapter we have studied _____ data, which is data with two variables. We have learnt how to graph this data on a _____. On the graph the variable on the horizontal axis is called the _____ _____ and the variable on the vertical axis is called the _____ _____.

We can use the graph to decide whether there is an _____ between the variables or not. We considered this relationship in terms of its:

Direction: whether it is _____ or _____

Shape: is it _____ or _____?

Strength: is the relationship _____, _____ or _____?

We then added a _____ __ ___ to the scatterplots. We then considered the _____ between the two variables. When we have tables of data, we can calculate the _____ _____.

We can use both the lines of fit and the correlation coefficient to help us make _____ from the data. When we make estimates from within the data range this is called _____ and this is usually reliable if there is a high correlation between the data. When make estimates outside the range of the data, this is called _____ and can often be unreliable or impossible.

Chapter Problem

A consumer association investigated the quality and price of eight pairs of gym shoes, labelled A to H. The investigation team gave each pair of shoes a quality rating out of 10.

This table shows the results.

	A	B	C	D	E	F	G	H
Price	$320	$280	$260	$240	$180	$180	$80	$40
Quality	8	10	9	6	6	4	1	3

Is there a relationship between the price of shoes and the quality rating assigned by the team? If so, what is the relationship?

Solution

Draw a scatter plot to see if there is a relationship.



The scatter plot shows there is relationship between the two variables. Overall the more expensive the gym shoe, the better the quality.

If you wished you could draw a line of fit and find its equation to find the relationship between the two variables in this data.

The relationship is strong, positive and linear. You would expect that the more expensive shoes use better materials and are better constructed than the cheap shoes.

ANSWERS

CHAPTER 1
What's the score?

Exercise 1.1

1 **a** 14 **b** 16 **c** 20 **d** 33

2 **a** 16 **b** 18 **c** 20 **d** 28
e 45 **f** 17

3 Any combination of two scores that add to 13, e.g. double 6 + 1 or 7 + 6

4 6 ways: 4 + 3, 6 + 1, 5 + 2, 5 + double 1, 3 + double 2, 1 + double 3

5 **a** 23
b Double 10 + 3 or double 9 + 5 or double 8 + 7 or double 7 + 9

6 Four throws (4 × 25) **7** 3 + double 8

8 To make it harder to get high scores. If the children want to get high scores they need to place accurate throws.

Exercise 1.2

1 **a** 46 **b** 110

2 180 (3 × triple 20)

3 Many answers are possible, e.g. 20 + 10 + 7 or 10 + 10 + 7

4 151 **5** Rebecca

6 16 or double 8 **7** Still 86

8 104

9 **a** double 17 **b** double 10 + double 7
c 10 + 10 + double 7

10 160

11 **a** 4 **b** Renata on turn 2
c 58 on turn 3 **d** 2
e 2 throws for combined 10, then double 7
f 29 is an odd number (no number on the board doubles to make 29) **g** Teacher to check.

Exercise 1.3

1 Brett **2** −1

3 **a** 70 strokes **b** 5 strokes

4 **a** The temperature is 10° below zero.
b −4°C **c** −7°C **d** −5°C

5 **a** −2 **b** 3 times **c** 376 runs
d Many answers are possible, e.g. batter gets out twice and scores zero on the other 2 balls.

Exercise 1.4

1 **a** False, $4 + 2 \times 3 = 10$
b True **c** True **d** True
e False, $2 \times 4^2 = 2 \times 16 = 32$
f False, $48 \div 4 \times 3 = 12 \times 3 = 36$
g False, $20 - 5 + 8 = 23$ **h** True

2 **a** 3 **b** 3 **c** 22 **d** 36
e 75 **f** 1 **g** 12 **h** 10
i 2 **j** 14 **k** 40 **l** 140
m 150 **n** 26 **o** 30

3 **a** $(4 + 7) \times 5 = 55$ **b** $60 \div (5 + 7) = 5$
c $(3 \times 2)^2 = 36$ **d** $(6 + 8) \times (9 - 5) = 56$
e $(3 \times 4 + 5) \times 2 = 34$ **f** $(28 - 4 \times 5) \times 2 = 16$

4 When she pressed the [=] key in the middle, the calculator added 3 and 6 first, before multiplying by 5.

5 **a** The expression represents the sum of the points for 4 red, 2 brown, 1 pink and 2 black balls.
b 17 points

6 **a** Do the × first
b Explain the order of operations **c** $(48 - 8) \times 3$

Exercise 1.5

1 $6870 **2** $39 000 **3** $119.70

4 **a** $920 **b** $47 840

5 $7.20 **6** 4.5

7 **a** $147 **b** $31.50 **c** $325.50

8 720 s **9** Rockets by 2

10 Neither team is in front. The score is 110 each.

11 $1060.50

Exercise 1.6

1 B **2** B **3** C **4** B **5** B

6 **a** $240 **b** No **7** $26

8 **a** $34 **b** Yes **c** Yes

9 **a** Paige knits approximately 2 rows per minute
b approximately 25 minutes

Exercise 1.7

1 **a** 16.7 **b** 29.8 **c** 14.6
d 13.3 **e** 104.6 **f** 115.6

2 124.73

9780170370752

3 Many answers are possible, e.g. 16.27 or 16.32

4 Teacher to check.

5 **a** Mia: wave 1, 4.9; wave 2, 7.3; wave 3, 8.5; wave 4, 7.6; Elissa: wave 1, 7.0; wave 2, 7.8; wave 3, 5.2; wave 4, 8.2

b Mia: 7.6 + 8.5 = 16.1; Elissa: 7.8 + 8.2 = 16.0

c Mia

6 3.9 m **7** 538.5 km

8 **a** 8745 m **b** 8.7 km

9 **a** 79.3 is close to 80 and 155.8c is close to $1.50

b $120 **c** $123.55 **d** $3.55

10 **a** Yes, $15.17 is approximately $15 and 7 h 45 min is close to 8 hours.

b 45 min = 45 ÷ 60 hours = 0.75

c 117.57 **d** $117.57

11 $16.21

Keyword activity

1 Double **2** Triple **3** Bull's eye
4 Integer **5** BIDMAS **6** Positive
7 Indices **8** Brackets **9** Negative
10 Darts **11** EFTPOS

CHAPTER 2
Buying and selling

Exercise 2.1

1 **a** $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{6}{13}$ **d** $\frac{3}{4}$
e $\frac{3}{5}$ **f** $\frac{4}{7}$ **g** $\frac{2}{3}$ **h** $\frac{3}{5}$
i $\frac{4}{7}$ **j** $\frac{4}{7}$ **k** $\frac{3}{5}$ **l** $\frac{9}{10}$

2 **a** 24 **b** 7 **c** 4
d 10 **e** 42 **f** 10
g 750 m **h** 8 h **i** 400 mL
j 125 kg **k** 10 months **l** 35 min

3 0.675 kg **4** 234 **5** 17
6 112 **7** 24 318

Exercise 2.2

1 **a** $\frac{3}{5}$ **b** $\frac{3}{4}$ **c** $\frac{31}{100}$ **d** $\frac{2}{25}$
e $\frac{3}{10}$ **f** $\frac{17}{20}$ **g** $\frac{99}{100}$ **h** $\frac{3}{100}$
i $1\frac{3}{5}$ **j** $1\frac{7}{20}$ **k** $\frac{1}{4}$ **l** $2\frac{1}{2}$

2 C

3 **a** 0.18 **b** 0.82 **c** 0.02 **d** 0.5
e 1.2 **f** 0.511 **g** 0.79 **h** 0.125
i 0.163 **j** 0.04 **k** 0.187 **l** 0.0525

4 **a** 17% **b** 70% **c** 26% **d** 55%
e 62.5% **f** 96% **g** 160% **h** $66\frac{2}{3}$%
i 125% **j** 140% **k** 67.5% **l** 6.25%

5 **a** 38% **b** 55% **c** 96% **d** 62.5%
e 8% **f** 5.4% **g** 60% **h** 0.3%
i 190% **j** 40.5% **k** 126% **l** 11.4%

6 **a** $\frac{7}{20}$, 35% **b** $\frac{3}{5}$, 60% **c** $\frac{1}{5}$, 0.2
d 0.5, 50% **e** 0.125, 12.5% **f** 0.625, 62.5%

7 **a** 75% **b** $\frac{4}{25}$ **c** $\frac{2}{3}$
d 18% **e** $\frac{1}{4}$ **f** 55%
g $\frac{19}{20}$ **h** 60% **i** $\frac{1}{6}$

Exercise 2.3

1 **a** $33 **b** 84 kg
c 252 cm **d** $307.50
e 216 marks **f** 595 000 people
g 152.29 kg **h** 5 m
i 294 students **j** $312.70
k 12 L **l** 113.6 ha

2 **a** 96 cm **b** 84 hours
c 219 days **d** 3900 mL
e 23c **f** 98 days
g 1350 m **h** 600 kg

3 258 students **4** $412.50
5 **a** $0.81 **b** $2062.50
6 3.72 hours **7** 782 seedlings
8 $52.40 **9** 1275 people
10 11 700 seats **11** 24.31 kg
12 337.5 MB

Exercise 2.4

1 **a** $\frac{1}{2}$ **b** $\frac{19}{25}$ **c** $\frac{2}{3}$ **d** $\frac{3}{8}$

2 $\frac{172}{189}$

3 **a** $\frac{1}{6}$ **b** $\frac{1}{4}$ **c** $\frac{7}{10}$ **d** $\frac{23}{100}$
e $\frac{3}{20}$ **f** $\frac{19}{70}$ **g** $\frac{1}{8}$ **h** $\frac{8}{15}$

4 **a** 80% **b** 55% **c** 80% **d** 88.75%

5 No, because she scored 87.5% **6** 31.3%

7 **a** 8.3% **b** 18.8% **c** 12.5%
d 37.5% **e** 52% **f** 11.4%

8 75% **9** 17.6% **10** 49.4%

11 55.9% **12** 52.5% **13** 38.1%

14 **a** 34.8% **b** 47.8%

15 **a** English 60%, Mathematics 54.3%, Science 64%
b Science

Exercise 2.5

1 **a** 152 kg **b** $2650 **c** 157.5 m
d 13.3 L **e** $64.50 **f** 1440 L
g 105.05 kg **h** 714 students

2 $9 **3** $682 **4** $247.50

5 31 141 **6** $35.37 **7** $679.15

8 $234 **9** $20 790 **10** 1438

11 **a** $22.50 **b** $877.50

12 $158 080 **13** $184 **14** $563.33

15 **a** $39.57 **b** $1939.13

16 $172.01 **17** $27 993 **18** $1032

19 **a** $516.25 **b** $567.88
c Because prices are usually rounded to the nearest dollar or $10.

12 **a** **i** $670 **ii** $549.40
b $494.46

Exercise 2.6

1 **a** 28.6% **b** 29.6% **c** 83.1% **d** 55.8%

2 **a** 18.6% **b** 40.2% **c** 12% **d** 15.4%

3 87.5% **4** 45.5%

5 **a** Profit, $15 **b** 11.1%

6 **a** Loss, $500 **b** 11.8%
c Keira might be happy as she used the car for a year.

7 $272.25

8 **a** $23 985 **b** $21 100 **c** Loss **d** 12%

9 **a** $315 **b** $324 **c** Profit **d** 2.9%

10 45.8%

Exercise 2.7

1 80% **2** 38 385

3 **a** 19.7% **b** 22.4 g

4 $8625 **5** $11 200 **6** 7.6%

7 $703.80 **8** 69.02 g **9** $754.40

10 $41 850 **11** $350 175 **12** 13.5%

13 $691.25

14 **a** 0.83% **b** 4 : 55 : 30

15 113.6% **16** $4500 **17** 22%

18 30.8%

CHAPTER 3
Metres, litres and grams

Exercise 3.1

1 240 cm

2 **a** 20 cm **b** 120 cm **c** 6 cm
d 63 cm **e** 73.5 cm **f** 550 cm

3 **a** 140 cm **b** Teacher to discuss.

4 705 cm

5 **a** length = 2820 cm, width = 940 cm, height = 1410 cm
b length = 28.2 m, width = 9.4 m, height = 14.1 m

6 14.4 m

Exercise 3.2

1 **a** 30 **b** 500 **c** 0.4 **d** 2000
e 3 **f** 2 **g** 0.5 **h** 0.25
i 0.6 **j** 600 **k** 4.5 **l** 800
m 0.8 **n** 9000 **o** 0.09 **p** 6500

2 **a** 640 **b** 64 000

3 **a** 850 **b** 0.85

4 **a** 45 000 **b** 4.8 **c** 1 300 000 **d** 0.4175

5 **a** 3900 **b** 3.9 **c** 90

6 160

7 **a** 3340 m **b** 3.34 km

8 **a** 148.2 **b** 38.1 **c** 1005.8 **d** 609.6
e 7.3 **f** 11.3

9 109.7 **10** 8 furlongs **11** 16.5

12 Teacher to check, distances are 1673 km, 2655 km, 644 km, 3379 km, 877 km, 331 km, 1392 km.

13 3.66 **14** New pools are 29.2 cm shorter. **15** 8

Exercise 3.3

1 **a** minutes **b** kilograms **c** days
d millilitres **e** litres **f** hours
g grams **h** kilolitres **i** tonnes
j seconds **k** milligrams

2 **a** 1.8 g **b** 9500 g **c** 0.365 g **d** 200 000 g

3 **a** 12 200 L **b** 6.34 L **c** 0.55 L **d** 980 L

4 **a** 1920 **b** 0.55 **c** 45 000 **d** 15 200
e 48 **f** 4 **g** 65.7 **h** 960
i 12.3 **j** 3 200 000 **k** 4320 **l** 0.0568

5 6.75

6 **a** 720 g **b** 17.28 kg **c** 345.6 kg, 0.3456 t

7 **a** 1.5 L **b** 36

8 2.8 h or 2 h 48 min

9 **a** 24 cents **b** 24 cm

10 **a** 90c **b** 90 cm

11 **a** 2 h 30 min **b** 3 h 48 min
c 1 h 24 min **d** 2 h 54 min

12 **a** 12 **b** 6 **c** 4 years 6 months

13 2 years 3 months

14 **a** 9 **b** 5 years 9 months

Exercise 3.4

1 **a** 17 mL **b** 1.6 mL **c** 18 mL

2 41.2°C **3** **a** 350 mL **b** 150 mL

4 Kara 177.4 cm, Samantha 166.9 cm

5 **a** 2.75 kg **b** 48.8 cm **c** 34.8 cm

6 **a** 94 kg **b** Teacher to check.

7 Teacher to check.

8 **a** 8 **b** 1200 **c** 150 mL
d Teacher to check

Exercise 3.5

1 **a** 0.06 g **b** 60 mg

2 **a** 500 **b** 0.7 **c** 0.3 **d** 400

3 **a** 3 **b** 16.8 g

4 **a** 1.8 L **b** 33 **5** 2.2 mL

6 **a** 4 **b** 54 **c** 240

7 **a** 135 g **b** 135 000 mg **c** 270 mg

8 **a** 40 g **b** 800 mg **9** 20

10 **a** 1.48 g **b** 185 mg

11 The rate gives 6500 g per day, which is greater than the maximum dose of 4 g daily. The patient should receive 4 g per day.

Exercise 3.6

1 **a** **i** 1 m **ii** 4 m **iii** ±0.5 m
b **i** 1 mm **ii** 28 mm **iii** ±0.5 mm
c **i** 5 km/h **ii** 30 m/h **iii** ±2.5 km/h
d **i** 0.5°C **ii** 38°C **iii** ±0.25°C
e **i** 5 m/s **ii** 45 m/s **iii** ±2.5 m/s
f **i** 500 rpm **ii** 7000 rpm **iii** ±250 rpm

2 **a** ±0.5 min **b** ±30 s

3

	Measurement	Tolerance	Smallest it could be	Largest it could be
a	18 L correct to the nearest litre	±0.5 L	17.5 L	18.5 L
b	78 cm correct to the nearest cm	±0.5 cm	77.5 cm	78.5 cm
c	35 kg correct to the nearest 5 kg	±2.5 kg	32.5 kg	37.5 kg
d	9.4 seconds correct to 1 decimal place	±0.05 s	9.35 s	9.45 s
e	730 km correct to the nearest 10 km	±5 km	725 km	735 km

4 **a** 5 m **b** 235 m and 245 m **c** 245 m

5

	Item	Quantity	Recorded to the nearest	Tolerance	Possible range of values
a	Fuel	280 L	5 L	±2.5 L	277.5 L to 282.5 L
b	Fence posts	350	50	±25 posts	325 to 375 posts
c	Fencing wire	540 m	20 m	±10 m	530 m to 550 m
d	Fertiliser	870 kg	10	±5 kg	865 kg to 875 kg

6 **a** 12.16 m **b** 15.8 kg **c** 77.86 L

Exercise 3.7

1 **a** 24 m **b** 20 m **c** 34 m

2 30 m **3** **a** 14·9 cm **b** 17 cm

4 Any two lengths that add to 18 m, e.g. 12 m by 6 m, or 9 cm by 9 cm

5 **a** Regular pentagon, 25 m

b Regular hexagon, 24 cm

c Regular octagon, 24 mm

6 6 m **7** **a** 14 cm **b** 13·1 m

8 **a** 21 m **b** $189

Exercise 3.8

1 8.4 m **2** 23.8 m **3** 5.831 m **4** 152 km/h

5 **a** yes **b** yes **c** no **d** yes

6 12.37 m **7** 733 m **8** 44.4 m **9** 148 km

10 Teacher to check.

Keyword activity

1 metric **2** length

3 kilometres **4** one-thousandth

5 perimeter **6** capacity or volume

CHAPTER 4
Show me the graph

Exercise 4.1

1 **a** $120 **b** about 22 more **c** about $90

d To allow comparisons between months.

e Only to the nearest $5

2 **a** 3 **b** about $1750

c These are for winter when extra electricity is used for heating and cooking.

d Allows comparisons between quarters/seasons

3 **a** 15 months **b** approximately 180 MJ

c June to August

d heating in winter, others answers possible

e Allows comparisons between quarters/seasons

4 **a** August **b** January and February

c January and February, March and December, May and November

d 7 **e** July, August, September

f Teacher to check.

5 **a** 30°C **b** July

c All months are similar with small amounts of rain, less than 50 mm, falling.

6 **a** June, July, August **b** about 7°C

c 400 mm **d** Teacher to check.

7 Because the hottest (summer) temperatures are in the middle of the year.

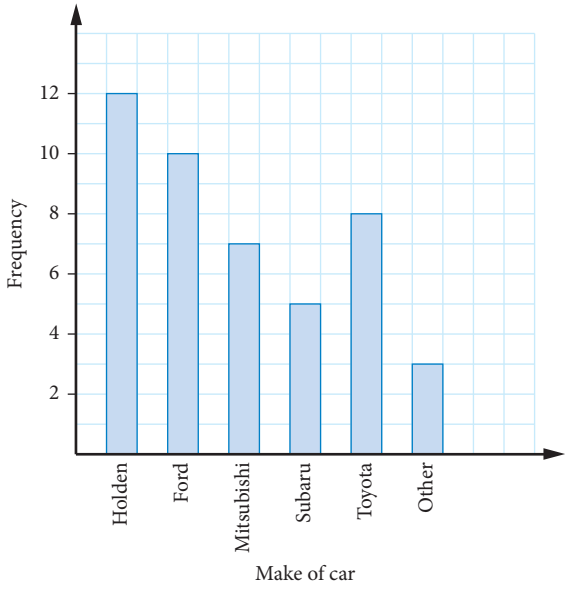
8 **a** 1% of donations **b** Teacher to check.

c Teacher to check. **d** Teacher to check.

e 9180 L

Exercise 4.2

1



2 **a** 7% **b** other

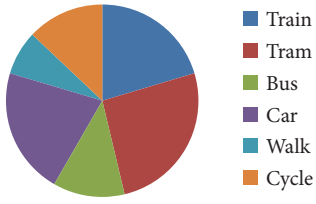
c infections, SIDS, poisons (other answers possible)

d 31% **e** 548

3 **a**

Method of transport	Frequency	Angle for sector graph
Train	220	73°
Tram	280	93°
Bus	130	43°
Car	230	77°
Walk	80	27°
Cycle	140	47°

b



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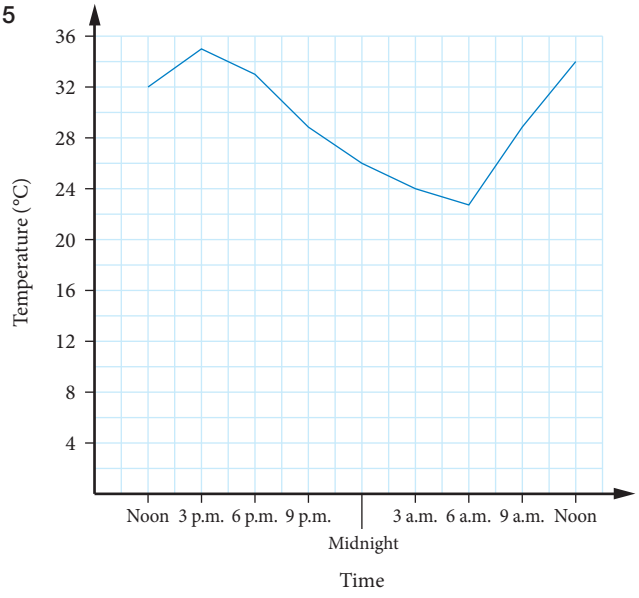
4 **a** **i** 70 cm **ii** 140 cm

b **i** 3 years **ii** 11 years

c 0 and 1 year **d** about 2 years 10 months

e fairly flat

5

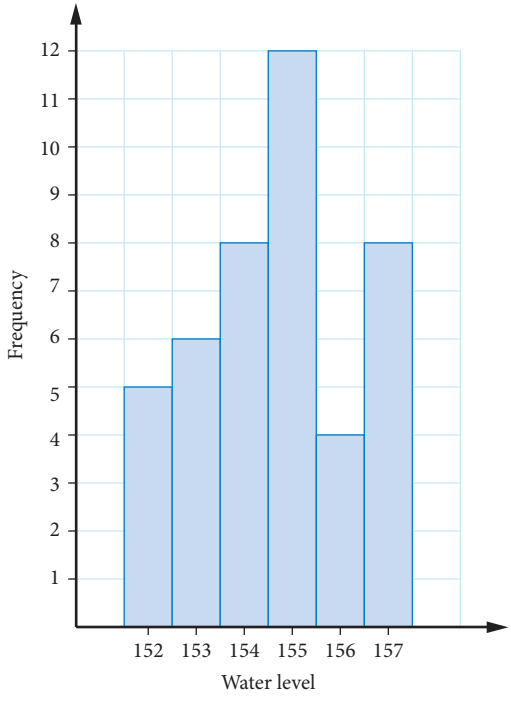


6 **a**

Test marks	Frequency
6–20	7
21–35	12
36–50	16
51–65	10
66–80	14
Total	59

b 59 **c** 36–50 **d** 19

7



Exercise 4.3

Teacher to check all answers.

Exercise 4.4

1 **a** **i** 6000 g **ii** 9500 g

b **i** 7 months **ii** 11 months

c 1 to 2 months **d** 6 months

e Teacher to check.

2 **a** 8% **b** 40 to 50 years of age.

c 7%

d More likely to die from cancer and heart disease.

e True **f** Between 25 and 60.

g Between 18 and 42.

3 **a** A Melbourne, B Sydney, C Brisbane, D Adelaide, E Perth, F Darwin, G Canberra, H Hobart

b Jan 28°, Feb 27°, March 24.5°, April 19.5°, May 15°, June 12°, July 11°, Aug 13°, Sept 16°, Oct 19°, Nov 23°, Dec 26°

c March and December

d B June, C July, D May, E June, September

e F **f** G, 15° in December

g 33°, F, November **h** 0°, G, July

4 **a** April 22, 24°C **b** 15°C

c 9°C **d** 1 unit = 1°C

e 16°C **f** April 20, April 21

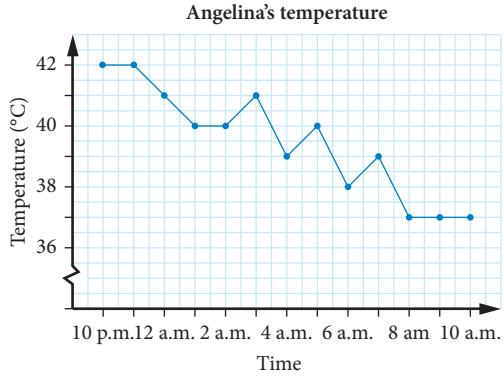
g 16th **h** 5

5 **a** 4 hours **b** 39.7°C

c Midnight Thursday **d** 40.4°C

e 12 hours **f** 2000 (8 p.m.) Saturday

6



Exercise 4.5

1 **a** Green 31.5c, Octas 33c, System Two 32.5c

b 1c **c** the previous graph

d The scale on the second graph starts at zero; the scale on the first graph only goes from 30 to 33.

e Green company, to make their calls seem cheap.

2 **a** Brisbane **b** Darwin **c** 145c **d** about 3c

e David only used from 135c to 155c and this makes the gaps look bigger.

3 Small: graph the actual population; Big: graph the increase in the population.

4 **a** 68c **b** $18

c The cost of petrol is in cents, the cost per barrel is in dollars: vertical scale is different for each graph.

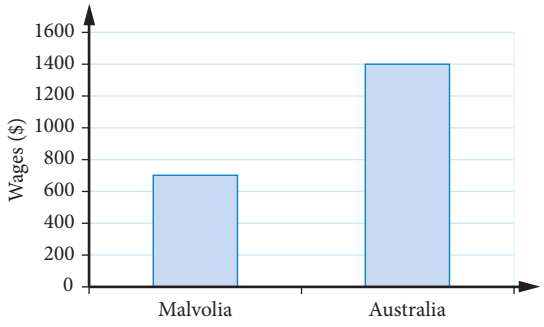
d The two graphs move up and down in a similar fashion: this suggests the two things are related.

5 **a** Australia's average weekly wages are much greater than in Malvolia.

b No vertical scale, using a 3D symbol instead of a column.

c Columns

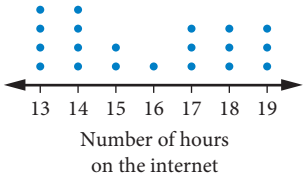
d



6 Teacher to check.

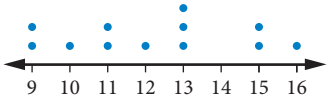
Exercise 4.6

1 **a**



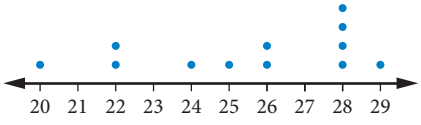
b 20 **c** 4 **d** 9

2 **a**



b 3 **c** 25% **d** 5

3 **a**



b 12 **c** $28 **d** 5

4 **a**

Stem	Leaf
7	6
8	1 6 8
9	5 7 8
10	1 5 5
11	2 2 4 7
12	4

Key: 7|6 = 76

b 53%

5 **a**

Stem	Leaf
1	1 2 2 3 3 4 6 7 7 9
2	0 0 0 2 3 3 4 5 5 5 6 7
3	0 1 3 3 3 4 5 9
4	1 2 8 8
5	5 5

Key: 1|1 = 11

b 8 **c** 22%

6 **a**

Stem	Leaf
0	0 0 5 8 9
1	1 4 5 6 7
2	1 2 4 5 6 7 9
3	1 3 5
4	2

Key: 1|1 = 11

b 42 hours **c** 5 **d** 11

e 21 **f** 52%

CHAPTER 5
What's my share?

Exercise 5.1

1 **a** 9 : 5 **b** $\frac{5}{14}$

2 **a** 6 : 2 **b** 3 : 1 **c** $\frac{3}{4}$

3 **a** 2 : 5 **b** 2 : 3 **c** 2 : 1 **d** 3 : 2

e 3 : 2 **f** 2 : 1 **g** 5 : 1 **h** 1 : 4

i 9 : 5 **j** 10 : 1 **k** 1 : 100 **l** 1 : 20

m 1 : 30 **n** 10 : 1 **o** 4 : 3 **p** 1 : 5

4 The ratios of red : yellow in the mixtures are 20 : 40 and 4 : 8. The ratios are equivalent. They both simplify to 1 : 2.

5 **a** 5 : 2000 **b** 1 : 400

6 **a** 40 : 1000 = 1 : 25 **b** 3000 : 20 = 150 : 1

c 2000 : 500 = 4 : 1 **d** 50 : 100 = 1 : 2

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e 300 : 60 = 5 : 1 **f** 5000 : 500 = 10 : 1

g 120 : 40 = 3 : 1 **h** 45 : 30 = 3 : 2

7 It looks more green than blue. Possibly Lara got the ratio around the wrong way. Maybe she made a 1 : 4 ratio instead of 4 : 1.

8 **a** 3 : 2 **b** 1 : 2 **c** 3 : 1 **d** 1 : 1

Exercise 5.2

1 **a** 10 **b** 9

2 **a** zinc 25 g, nickel 5 g **b** copper 112 g, zinc 40 g

c 160 g

3 **a** 5 cups **b** 18 cups

4 **a** 2000 mL **b** 2 L **c** 150 mL

d 25 mL **e** general perennial weeds

5 **a** 60 mL **b** 100 mL **c** 30 mL

6 **a** 120 L **b** 8 L **c** 340 mL

7

Plant	N : P : K	Nitrogen (kg)	Phosphorus (kg)	Potassium (kg)	Total mixture (kg)
Vegetables	6 : 4 : 5	54	36	45	135
		42	28	35	105
Fruit trees	7 : 1 : 2	105	15	30	150
		140	20	40	200

8 **a** water 250 g, yeast 25 g

b flour 4 kg, yeast 0.25 kg

c flour 48 kg, water 30 kg

9 **a** new brick construction **b** 2

c lime 2, cement 2 **d** sand 20, lime 6

Exercise 5.3

1 **a** \$840, \$1260 **b** \$1750, \$350

c \$1200, \$600, \$300 **d** \$450, \$600, \$1050

2 **a** 171 L, 684 L **b** 20, 15

c 10 800 m^2, 13 500 m^2 **d** 637 kg, 182 kg

e 12 min, 16 min, 24 min **f** \$1600, \$200, \$1000

3 1.5 t **4** 399 boys

5 silver beads 20, porcelain balls 15, crystal eyedrops 10

6 160 mL

7 **a** 2 g **b** gold 315 g, copper 84 g

8 **a** 150 mL **b** 400 mL **c** 200 mL

9 **a** 3000 mL

b

Type of crop	Ratio of concentrate to water	mL of concentrate required	mL of water required
Broccoli	1 : 14	200	2800
Cauliflower	1 : 19	150	2850
Leafy vegetables	1 : 9	300	2700

10 dye A 75 mL, dye B 45 mL

11 **a** 14.4 L **b** 0.6 L **c** 600 mL

d 0.5 L = 500 mL

12 nitrogen 75 kg, phosphorus 15 kg, potassium 30 kg

Exercise 5.4

1 **a** 50 km/h **b** 25 words/min **c** 50 L/h

d \$17.50/kg **e** 7.27 m/s **f** \$26/kg

g 250 g/L **h** 600 revs/min

2 18 L/m^2 **3** 1350 L/h

4 **a** \$8.50/kg **b** \$12.48/kg **c** \$2.45/kg

d 55c/roll **e** \$2.45/bottle

5 **a** 112 **b** 196 **c** 336

6 \$248

7 **a** 3 h **b** 1.5 h **c** 4.8 h or 4 h 48 min

8 6.25 kg **9** **a** 720 kg **b** 15 trees

10 **a** 5460 km

b **i** 2.0 h **ii** 3.1 h **iii** 4.1 h **iv** 15.2 h

11 **a** 960 **b** 270 **c** 25 m^2 **d** 95 m^2

12 **a** \$8000 **b** 35 weeks **c** 3 weeks

13 768 kg

14 **a** 750 mL **b** 1500 mL **c** 1.5 L **d** 36 L

e 40 min **f** 40 h

15 **a** 18 h **b** 35 h **c** 24 h

CHAPTER 6
It's about time

Exercise 6.1

1 **a** 1144 **b** 1835 **c** 0251

d 2154

2 **a** 8:45 a.m. **b** 1:20 p.m. **c** 11:31 p.m.

d 10:45 a.m.

3 **a** 11:23 a.m. **b** 5:54 p.m. **c** 1:16 p.m.

4 **a** 5:20 a.m. **b** 10:05 a.m. **c** 2:45 a.m.

5 **a** 3:15 p.m. **b** 8:35 p.m. **c** 11:55 p.m.

6 **a** 2030 **b** 2205

7 **a** 8 hours **b** 3 hours 35 minutes

c 0305

Exercise 6.2

1 **a** 5 h 45 min **b** 5 h 44 min

c 2 h 20 min **d** 3 h 53 min

2 **a** 3 h **b** 10 h

c 5 h 15 min **d** 8 h 10 min

3 9 h 30 min

4 6 h 25 min

5 **a** 8:00 p.m. **b** 7:53 p.m.

c 1:17 p.m. **d** 2:24 a.m.

6 **a** 2:15 p.m. **b** 2:35 a.m.

c 11:25 a.m. **d** 7:50 a.m.

7 3:15 p.m. **8** 7:45 p.m.

9 **a** 3 years 8 months **b** 4 years 1 month

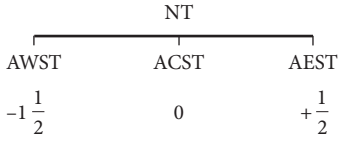
c Teacher to check.

10 16 months **11** August 2011

12 Teacher to check. **13** Teacher to check.

Exercise 6.3

1



2 **a** ahead **b** ahead **c** same **d** ahead

e behind **f** ahead **g** ahead **h** same

i ahead **j** behind

3 **a** 11 a.m. **b** 9 a.m. **c** 10:30 a.m.

d 11 a.m. **e** 11 a.m. **f** 9 a.m.

4 **a** 11 p.m. **b** 10:30 p.m. **c** 9 p.m.

d 10:30 p.m. **e** 11 p.m. **f** 11 p.m.

5 1:45 p.m.

6 **a** 4 p.m. **b** 5:30 p.m.

7 8:30 p.m. **8** 10 p.m.

9 **a** Teacher to check **b** Teacher to check

c Teacher to check **d** 3:30 p.m.

10 Teacher to check.

Exercise 6.4

1 **a** behind **b** ahead **c** ahead **d** behind

e behind **f** ahead

2 **a** 11 hours **b** 15 hours **c** 7 hours **d** 10 hours

e 8 hours

3 **a** 8 p.m. same day **b** 5 a.m. same day

c 6 p.m. same day **d** 7 p.m. day before

e 7 a.m. same day **f** 5 a.m. next day

g Midnight **h** 8 a.m. next day

i Midday same day **j** 7 p.m. same day

4 **a** 1 p.m. same day **b** Midnight

c 9 p.m. same day **d** 8 p.m. same day

e 8 a.m. next day **f** 4 a.m. next day

5 3 p.m. same day **6** 10 a.m. same day

7 **a** 1230 **b** $5\frac{1}{2}$ hours

8 2120

9 **a** Los Angeles is 18 hours behind Brisbane.

b 16 hours 30 minutes

Exercise 6.5

1 7:50 a.m. **2** 5:15 p.m. **3** 8:10 a.m.

4 **a** 4:52 p.m. **b** 5:08 p.m. **c** 5:22 p.m.

5 5:22 p.m. **6** 8:45 a.m.

7 **a** 47 min longer **b** Teacher to check

8 20 h 30 min

9 4, 9:45 p.m., 5:15 a.m. (next day), 8:20 a.m., 11:45 a.m.

10 **a** 6 h 25 min **b** 11 h

11 **a** approximately Bulgunnia Turnoff

b Teacher to check.

12 **a** 25 h 25 min **b** Longer

c Teacher to check.

13 44 min

14 **a** 11:02 a.m. **b** 34 min

c 58 min **d** Teacher to check.

15 white is a.m. times, grey is p.m. times

16 **a** 5 buses, Bus 1 can do the 10 a.m. and noon trips

b 12:45 p.m., first bus after 11:42 a.m. is 12 noon

c Teacher to check.

d Teacher to check.

Keyword activity

12-hour time – D 24-hour time – B

timeline – C time zone – E timetable – A

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CHAPTER 7
Creating designs

Exercise 7.1

1 sides, equilateral, angles, two, two, scalene, acute, obtuse, right

2 four, opposite, parallel, equal, bisect, equal, 90°, rectangle, square

3 parallelogram, 90°, equal, rhombus, equal, 90°, diagonals

4 quadrilateral, parallel, equal, equal, equal, diagonals, 90°

5 number, triangles, quadrilaterals, pentagon, hexagon, octagon, decagon, regular, regular

Exercise 7.2

1 flat, rectangular, prism, pyramid, pentahedron, hexadron, edge, vertex

2 curved, sphere, flat, curved, rectangle, cone, circle

3

Solid	Number of faces	Shapes of faces	Number of identical faces
cube	6	squares	6
cylinder	3	two circles and a cylinder	2
square pyramid	5	square and 4 triangles	4
triangular prism	5	2 triangles and 3 rectangles	2
rectangular prism	6	rectangles	3 pairs of 2
cone	2	circle and a sector	0
triangular pyramid	4	triangles	usually 3

4 cross-section, shape, end, rectangle, triangle, pointed, bottom, square, cross-section, size

5 **a** triangular prism **b** cone and sphere
c cube **d** pentagonal prism
e sphere **f** square or rectangular pyramid

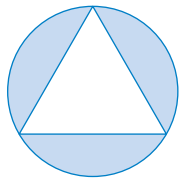
Exercise 7.3

1, 2 Teacher to check.

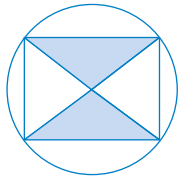
3 square, circumference, four, semicircle, isosceles, diameter, vertex, circle.

4 Other answers possible.

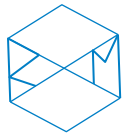
a



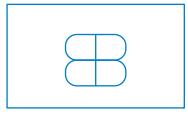
b



c



d



5 Teacher to check.

Exercise 7.4

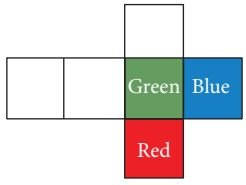
Teacher to check.

Exercise 7.5

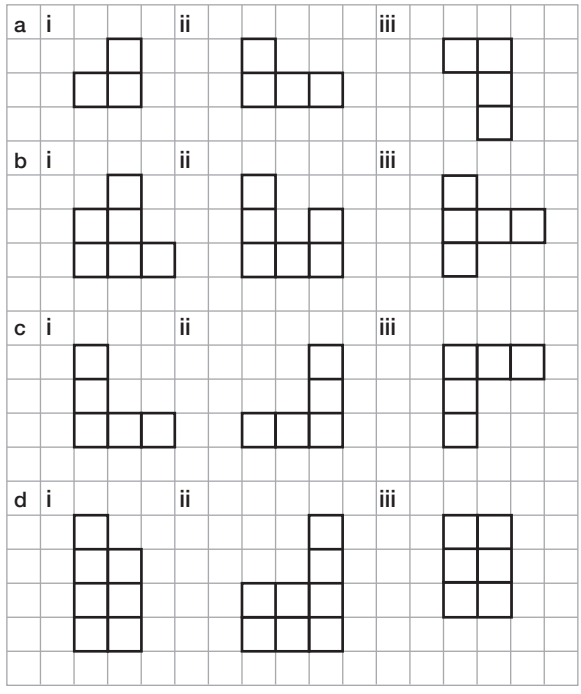
1 **a**-ii, **b**-iii, **c**-i, **d**-vi, **e**-v, **f**-iv

2 **a** hexagonal pyramid **b** Teacher to check.

3 Other answers possible.

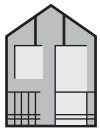


4

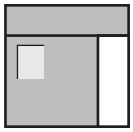


5 Teacher to check.

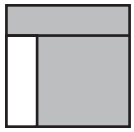
6



Front



Left side

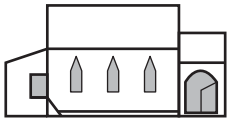


Right side

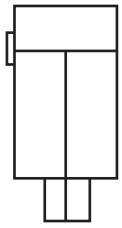
7



Front



Left side



Bird's eye

Exercise 7.6

1 **a** 1 **b** 2 **c** 4
d 4 **e** 2 **f** 1

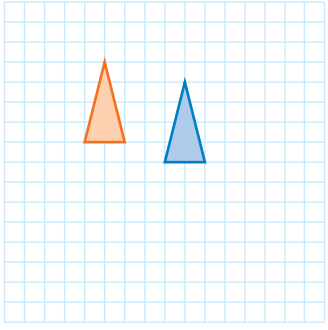
2 **a** 2 **b** 4 **c** 4
d 3 **e** 8 **f** 10

3 **a** 5 **b** 4 **c** 5
d 14 **e** 9 **f** 12

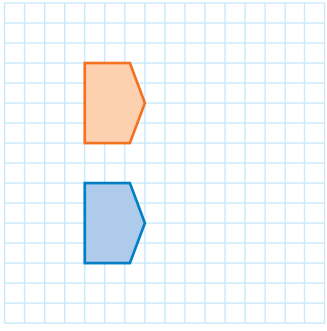
4 **a** both **b** no symmetry **c** line
d no symmetry **e** both **f** line
g line **h** both **i** rotational

Exercise 7.7

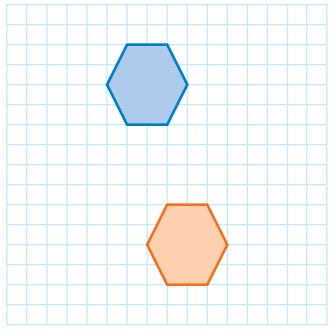
1 **a**



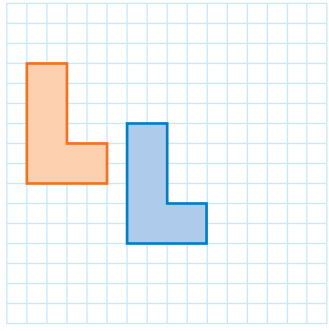
b



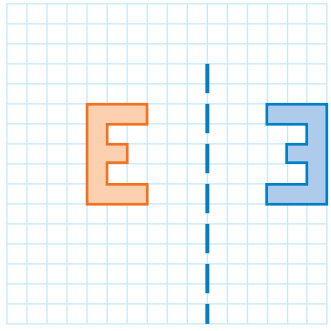
c



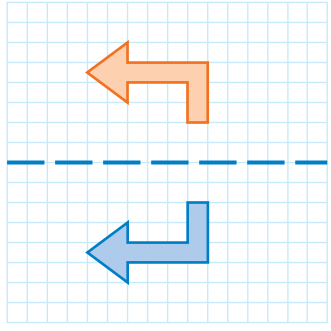
d



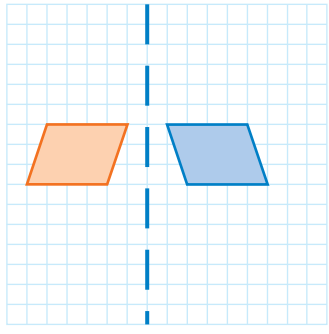
2 **a**



b

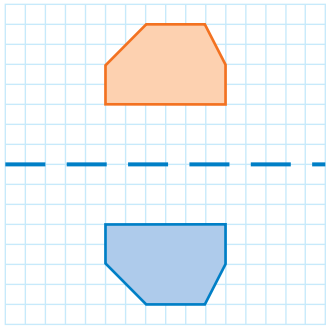


c

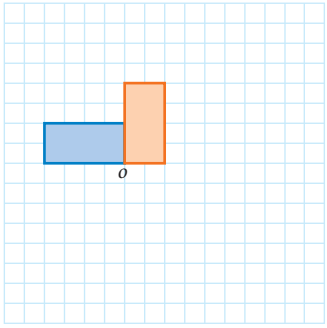


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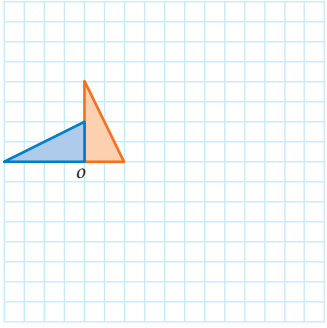
d



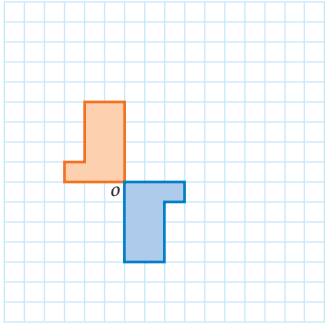
3 a



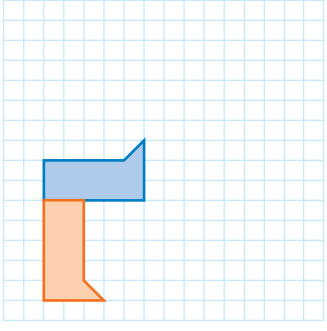
b



c



d



4 The answer is the same.

5 a rotation and translation, or two reflections, or reflection and translation

b rotation and translation

c rotation and reflection

Exercise 7.8

1 equilateral triangle, square, hexagon

2 equilateral triangle and square (2 types); hexagon and equilateral triangle (2 types); octagon and square; dodecagon, square and hexagon; hexagon, square and equilateral triangle; dodecagon and equilateral triangle

3 a equilateral triangle, square, hexagon

b equilateral triangle, square

c hexagon, equilateral triangle

d dodecagon, hexagon, square

4, 5 Teacher to check

Keyword activity

1 E	2 O	3 A	4 K
5 I	6 D	7 L	8 M
9 J	10 C	11 B	12 N
13 G	14 F	15 H	

CHAPTER 8
It's better than average

Exercise 8.1

1 a N	b C	c N	d N
e N	f C	g N	h N
2 a C	b C	c C	d N
e C	f N	g N	h C

3 Teacher to check.

Exercise 8.2

1 Teacher to check.

2 a sample	b census	c sample
d sample	e sample	f census
g census	h sample	i sample
j census	k census	l sample

3 a students

b people of voting age in Victoria

c listeners of a radio station, or readers of a magazine, or musicians

d students at my school

e people of Western Australia

f the general population

g Year 12 students

h rich people

i people who shop at supermarkets

j the Australian cricket team

Exercise 8.3

Teacher to check.

Exercise 8.4

1 a 9 b 8 c 8

2 a 2 b 45, 54 c no mode
d green e spade

3 a 16 b 37.5 c 72.9 d 50.5

4 a 8 b 13 c 11
d 12 e Teacher to check.

5 a 14 b 14 c 13.8
d January, February, December, Teacher to check

6 a 4 b 1, 3, 4, 7 c 4.3

7 Teacher to check.

8 a mode = 1, median = 3, mean = 5
b mean, it is the highest
c mode, it is the smallest
d Teacher to check.

9 a 4 text messages

b

Number of text messages (score)	Frequency	Score × frequency
0	8	0
1	4	4
2	10	20
3	10	30
4	15	60
5	3	15
Total	50	**129**

c 2.58

10 a 50 b 6

c

Number of jobs applied for before getting an interview	Number of people	Jobs applied for × number of people
1	2	2
2	5	10
3	10	30
4	6	24
5	11	55
6	13	78
7	3	21
Total	50	**220**

d 4.4 e Teacher to check

Exercise 8.5

1 a Binns: 16.4, Thomson: 25.3
b the last age
c the outlier 91 makes the mean higher

2 a $696 0500 b $767 273
c Median, it is more typical of most prices
d $1 800 000
e $664 000, much closer to the median

3 a Mark: 43.9, Steve: 45.4 b Steve
c Mark: 45, Steve: 37 d Mark
e Teacher to check.

4 a $137 333 b $60 500
c Median, 10 out of 12 scores are close to the median.
d Mean, its higher e $245 000, $820 000
f $58 300

5 a Mean = $63 400, Median = $71 000
b Median, 4 out of 5 years are close to this value.
c Wanting to minimise tax paid.
d $32 000
e Mean = $71 250, Median = $71 500

6 a 204 deliveries
b This doctor could be a specialist in difficult pregnancies and deliveries.
c Mean = 30.65, median = 21.5
d Mean = 21.5, median = 21
e The outlier has a big effect on the mean but only a very small effect on the median.

7 a $355 000 b $431 125
c Median, it is lower d Mean, it is higher
e $835 000, $1 700 000 f $397 568

Exercise 8.6

1 **a** range = 11 **b** $Q_1 = 18$, $Q_2 = 19$, $Q_3 = 25$
c 7

2 **a** range = 7 **b** $Q_1 = 29$, $Q_2 = 31$, $Q_3 = 31$
c 2

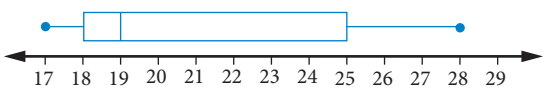
3 **a** range = 31 **b** $Q_1 = 159.5$, $Q_2 = 169$, $Q_3 = 174.5$
c 15

4 **a** range = 6 **b** $Q_1 = 4$, $Q_2 = 5.5$, $Q_3 = 8$
c 4

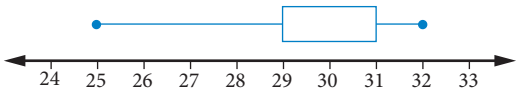
5 **a** Inner City **i** 43
ii $Q_1 = 20$, $Q_2 = 25.5$, $Q_3 = 33$ **iii** 13
Coastal **i** 38
ii $Q_1 = 52$, $Q_2 = 60$, $Q_3 = 64.5$ **iii** 12.5
b Coastal is much higher but the interquartile ranges are much the same.

Exercise 8.7

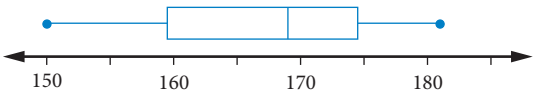
1 **a** 17, 18, 19, 25, 28
b



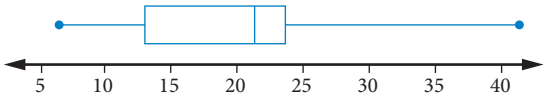
2 **a** 25, 29, 31, 31, 32
b



3

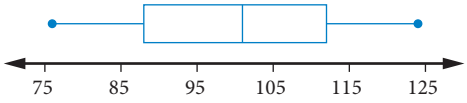


4 **a** 6, 13, 21, 24, 41
b

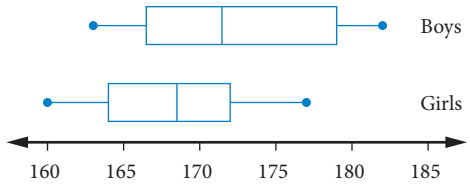


5 **a** 22 **b** 35
c **i** 15 **ii** 45
d 18

6 **a** 76, 88, 101, 112, 124
b

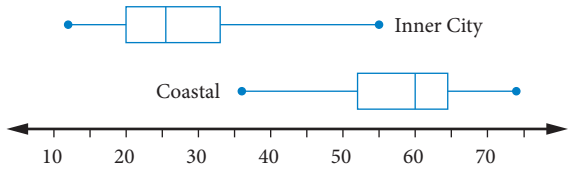


7 **a** Boys: 163, 166.5, 171.5, 179, 182
Girls: 160, 164, 168.5, 172, 177
b



8 **a** 20 **b** 11 **c** 4
d 15 **e** 30

9 **a** Inner City: 12, 20, 25.5, 33, 55
Coastal: 36, 52, 60, 64.5, 74
b



c Inner City, lower risk of theft.

Keyword activity

1 categorical, numerical
2 mode **3** average **4** number
5 median **6** odd **7** order
8 quartiles **9** range **10** interquartile range
11 sample **12** census **13** boxplot

CHAPTER 9
From paper to reality

Exercise 9.1

1–3 Teacher to check.

4 **a** $x = 8$ cm, $y = 2.5$ cm **b** $x = 3$ cm, $y = 6$ cm
c $k = 22.5$ cm **d** $d = 6.25$ cm
e $p = 3\frac{1}{3}$ cm

5–6 Teacher to check.

Exercise 9.2

1 **a** 12 m **b** 30 m **c** 4.5 m **d** 2 cm
e 16.2 m **f** 208 cm = 2.08 km

2 1100 m = 1.1 km **3** 25 m

4 **a** 13.5 cm **b** 7.2 m **c** 16 cm **d** 64 cm

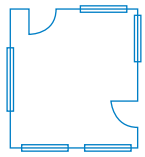
5 4 m **6** 6.4 km

7 Scaled length of screw = 4 cm;
Actual length of screw = 4 ÷ 5 = 0.8 cm

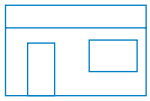
8 **a** 0.4 mm **b** 8 mm **c** 2.4 cm **d** 0.33 mm

Exercise 9.3

1 **a**



b



East elevation



West elevation

2–5 Teacher to check.

Exercise 9.4

1 **a** **i** kitchen sink **ii** vanity (or wash basin)
iii toilet **iv** window
v bath **vi** built-in wardrobe
vii sliding door
b They are all tiled. **c** 3
d 1 and 2 **e** 4100 mm × 3200 mm
f in the back of the garage **g** near the bathroom
h 1 **i** 1 **j** 2
k living room **l** 6670 mm × 5500 mm
m **i** west **ii** east **iii** west
iv south **v** north
n west **o** no
p Living, Dining, Bedroom 1, Bedroom 3
q Dining, Bedroom 2, Living

2 **a** 3 **b** 3 **c** no **d** $86 016

3 **a** 5 m **b** 2.5 m
c 2 m **d** 2 m × 2.5 m = $5m^2$
e 5 m **f** 4 m × 3.8 m = 15.2 m^2

4 **a** 240 mm **b** 70 mm **c** 12.39 m **d** 15.69 m
e 3.6 m by 6.07 m

5 **a** 6790 mm × 3700 mm **b** 2
c outside the bathroom **d** shower
e bedroom 4 **f** family room

Exercise 9.5

1 **a** 7.5 m^2 **b** $273.75

2 **a** 18 m **b** $198

3 **a** 92 m^2 **b** $77 280

4 **a** 15 m **b** 14 m **c** $210

5 **a** 14 m^2 **b** 4 m **c** $317.80

6 **a** 14 m^2 **b** **i** $2\frac{1}{3}$ L ≈ 3 L **ii** $34.90

7 **a** painting, skirting boards, carpet **b** $562.70

8 $1258

9 **a** 3.9 m **b** 3.9 m **c** 3 m

10 $892

11 **a** 11.53 m **b** 3

12 Approximately $87 912

Exercise 9.6

1–3 Teacher to check.

4 **a** Teacher to check. **b** 2.9 km

5 285 m

6 **a** Teacher to check. **b** 144.2 cm

7 133 m **8** 251 m

Keyword activity

1 F **2** H **3** D
4 G **5** C **6** I
7 B **8** A **9** E

CHAPTER 10
Healthy figures

Exercise 10.1

1 **a** 14 800 kJ **b** 22%

2 **a** 8000 kJ **b** 2400 kJ

3 **a** 5000 kJ **b** 1000kJ

4 **a** 2100 **b** 86 **c** 286
d 6 **e** 586 **f** 2571

5 **a** 2310 cal **b** 190 kJ

6 **a** boys **b** 2 to 3 years **c** 3350 kJ
d 4 to 8 years **e** 56 000 kJ
f **i** No, 1100 kJ too many **ii** increase

7 **a** 2000 kJ **b** 2 hours
c **i** 10 500 kJ **ii** no **iii** Eat more or exercise less

Exercise 10.2

1 **a** 2659 kJ **b** 1831 kJ **c** 5691 kJ **d** 725 kJ

2 **a** Breakfast 4491 kJ, lunch 3180 kJ, dinner 6608 kJ, daily total = 14 279 kJ
b Unless he is very, very active, he will put on weight.

3 Teacher to check

4 **a** 3450 kJ **b** 115 minutes

5 **a** 690 kJ **b** 765 kJ **c** 1020 kJ
d 3975 kJ **e** 1920 kJ **f** 8050 kJ

6 **a** 306 kJ **b** 888 kJ **c** 1152 kJ
d 14 754.6 kJ **e** 76 kg

Exercise 10.3

1 **a** $18 **b** $338 **c** 22

2 **a** $18, $16 **b** Teacher to discuss

3 **a** 75 beats/min **b** 64 beats/min
c 130 beats/min

4 **a** 80 **b** 100 **c** 120 **d** 140
e 160 **f** 180 **g** 200

5 **a** 172 beats/min **b** 108 beats/min
c She's working too hard and should slow down.

6 28 breaths/min

7 **a** 5 ÷ 60 = 0.08333 **b** 1.67 km

8 **a** 14 min **b** 50

9 **a** 255 watts **b** 3.315 kilowatts **c** $1.79

10 **a** 26.4 kilowatts **b** 9609.6 kilowatts **c** $5189.18

Exercise 10.4

1 2.9 mL **2** 177°C

3 **a** 60 mL **b** 360 mL

4 162 cm **5** 20.76 mm

6 **a** Young's rule: 12 mL, Clark's rule: 8.6 mL
b Teacher to discuss.

7 0.0979

8 **a** 1.5 hours
b Oscar's BAC = 0.06, Jenna's BAC = 0.75, Jenna's BAC is higher by 0.015
c Teacher to discuss.

9 82 km/h

Exercise 10.5

1 **a** 205 beats/min **b** 208 beats/min
c 200 beats/min **d** 209 beats/min
e 199 beats/min **f** 207 beats/min

2 **a** 3300 **b** 23 100

3 1770 **4** 6075

5 **a** 209 beats/min **b** 167 beats/min

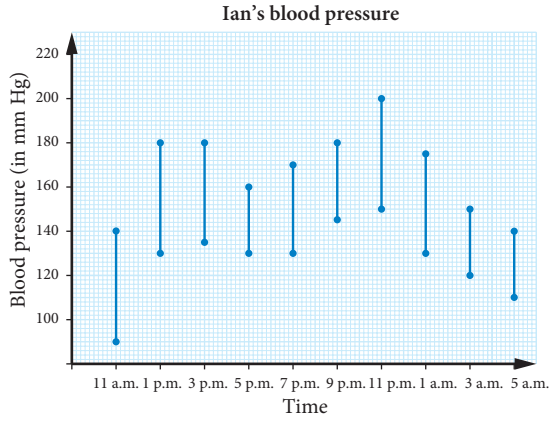
6 23 years **7** 11 years

8 **b** 162, 177 **c** 149, 162
d 177, 193 **e** 157, 171
f 151, 165

9 4.55 L **10** 117 mL

Exercise 10.6

1



2 **a** 82–90 **b** 131 and 86
c 110–134 **d** 120/79
e Teacher to check, probably 116/77.
f Systolic range is much bigger than diastolic range.

3 **a** No, his diastolic pressure is in the pre-hypertension category.
b Her systolic pressure is greater than 120.
c Stage 1 high blood pressure.
d Seek urgent medical advice.

Exercise 10.7

1 **a** $a = 3.8$ **b** $x = 5$ **c** $p = 15$
d $d = 4$ **e** $m = 9$ **f** $y = 2$

2 **a** $x = 10$ **b** $p = 15$ **c** $k = 3.5$ **d** $g = 2.5$
e $p = 4.75$ **f** $p = 7.5$ **g** $t = 7.5$ **h** $y = 17.5$
i $x = 8.75$ **j** $p = 3\frac{2}{3}$ **k** $x = 0.12$ **l** $b = 0.65$
m $p = \frac{15}{32}$ **n** $x = -6$ **o** $d = -4\frac{2}{3}$

3 48.5 cm **4** 25 km **5** 18.5 cm **6** 30 mL

7 **a** 30°C **b** 120 **8** **a** 3 **b** 41

9 1.8 s

Exercise 10.8

1 0.058 **2** 0.066 **3** 0.204

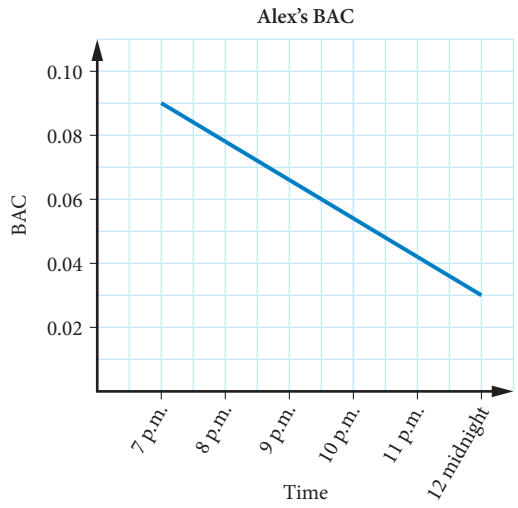
4 0.0139 **5** 4 drinks

6 **a** 3 **b** 6 **c** 4 **d** 5

7 **a**

Time	7 p.m.	8 p.m.	9 p.m.	10 p.m.	11 p.m.	12 midnight
Alex's BAC	0.09	0.078	0.066	0.054	0.042	0.030

b



c approx 1 a.m.

8 2 h **9** 12:20 a.m.

10 0.11; No, she will still be under the influence of alcohol.

11

Type of drink	Size of bottle	Number of standard drinks	Size of a Standard drink in mL
Red wine	750 mL	7.7	750 ÷ 7.7 = 97 mL
Dessert wine	375 mL	2.7	139 mL
White wine	750 mL	7.1	106 mL
Scotch whisky	1 L	32	31 mL
Port	750 mL	11	68 mL
Beer	330 mL	1.2	275 mL

12 **a** 6 **b** 0.078 **c** 5 h 12 min

13 0.081

14 **a** 0.118 **b** 0.194 **c** 0.046

15 **a** decrease **b** increase **c** increase
d increase **e** increase

Keyword activity

1 d **2** h **3** f **4** b
5 g **6** e **7** c **8** a

CHAPTER 11
On the surface

Exercise 11.1

1 **a** 15 m^2 **b** 24 m^2

2 **a** ha **b** m^2 **c** cm^2 **d** m^2
e cm^2 **f** mm^2 **g** km^2 **h** mm^2
i km^2

3 **a** 790 **b** 1.5 **c** 6.9 **d** 76
e 86.5 **f** 120 000 **g** 320 000 **h** 45 000
i 750 000 **j** 1.9

4 6.5 m^2, 114 000 cm^2, 25 050 000 mm^2

5 54 cm^2, 990 mm^2, 0.000 032 m^2

6 **a** 352.1 **b** 3.521 **7** 6720 m^2

Exercise 11.2

1 **a** 36 cm^2 **b** 55 m^2 **c** 104 m^2
d 100 m^2 **e** 70 m^2 **f** 96 m^2

2 **a** 184.8 mm^2 **b** 1386 cm^2 **c** 55 m^2
d 45 m^2 **e** 20.72 m^2 **f** 4.0194 cm^2

3 **a** 452.39 cm^2 **b** 49.87 cm^2 **c** 47.78 cm^2
d 3019.07 cm^2 **e** 117.29 cm^2 **f** 151.32 cm^2
g 760.27 cm^2 **h** 173.11 cm^2

4 **a** 11 m^2 **b** 14.1 m^2 **c** 917 bricks

5 **a** 225 000 m^2 **b** 22.5 ha **c** 202 sheep

6 7.14 cm^2, 5.4 cm^2

7 **a** 25.44 cm^2 **b** 203.52 cm^2 **c** 216.73 cm^2

8 630 000 mm^2

9 **a** 706.89 cm^2 **b** 88.36 cm^2 **c** 176.72 cm^2

10 19.24 m^2

Exercise 11.3

1 **a** 176.8 cm^2 **b** 19.84 m^2 **c** 875 mm^2
d 1000 cm^2

2 **a** 156 m^2 **b** 500 mm^2 **c** 71 m^2
d 201.19 km^2

3 **a** 94.63 m^2 **b** 216.53 cm^2

4 **a** 37.71 m^2 **b** 272.4 m^2

5 **a** 2.02 m^2 **b** 6.06 m^2 **c** 5.45 m^2

6 **a** 80 m^2 **b** $576

7 **a** 5.14 m^2 **b** 2.36 m^2

8 50 48 cm^2

9 **a** 15 621.77 m^2 **b** 10 027.43 m^2 **c** 5594.34 m^2

Exercise 11.4

1 **a** 29 400 cm^2 **b** 600 mm^2

2 6 125 cm^2 **3** 121.5 m^2

4 **a** 348 cm^2 **b** 11.33 cm^2

5 12 680 cm^2

6 **a** 89.25 m^2 **b** $414
c 29.25 m^2 **d** $460

7 12.72 m^2

8 **a** 254 cm^2 **b** 279.4 cm^2

9 71 m^2

10 **a** 45 cm by 25 cm **b** 1125 cm^2 **c** 3925 cm^2

Exercise 11.5

1 **a** 604.76 cm^2 **b** 672.36 cm^2
2 345.58 cm^2 **3** 18 m^2
4 **a** 865.70 cm^2 **b** 36.32 m^2
5 **a** 154 cm^2 **b** 77 cm^2
6 **a** 514 718 540 km^2 **b** 360 302 980 km^2
c 154 415 560 km^2
7 **a** 75.4 cm^2 **b** 269.7 m^2
8 2136 cm^2 **9** 1508 cm^2 **10** 161 m^2
11 **a** 3421.19 cm^2 **b** 2376 tiles
12 **a** 1728 cm^2 **b** 39.27 cm^2

Exercise 11.6

1 **a** 864 cm^2 **b** 40 560 m^2
2 960 cm^2 **3** 190 000 m^2
4 **a** 329.3 cm^2 **b** 39.2 m^2 **5** 712.2 cm^2
6 **a** 105.6 cm^2 **b** 3700 m^2 **7** 139 cm^2
8 **a** 1946 m^2 **b** about 3 m^2 **9** 443 cm^2
10 **a** 106 m^2 **b** 6.84 m^2 **c** 99.16 m^2

Keyword activity

Teacher to check.

CHAPTER 12
Going places

Exercise 12.1

1 **a** $T = 5$ **b** $D = 240$
2 **a** 180 km **b** 4 h
3 80 km/h **4** 275 km **5** 8 km
6 **a** 0.15 **b** 9 min **7** 7.5 min
8 **a** 2 km **b** 30 is minutes not hours
c 4 km/h
9 **a** 279 m **b** 4.5 s
10 **a** 5400 m **b** $11\frac{1}{9}$ s
11 **a** 750 km **b** 10 h
c 76 km/h **d** 5 a.m. Tuesday
12 **a** 16.7 m/s, 25 m/s
13 40 320 km/h
14 **a** 8 km **b** 4 km **c** 1 hour
d 4 km/h **e** 2 hours **f** 6 km/h
g The second hour is uphill. In the first hour his speed was 8 km/h and in the second hour his speed was only 4 km/h.

15

Graph	Who it is	How far their home is from school
A	Luke	10 km
B	Peta	8 km
C	Wayne	4 km
D	Shelby	2 km

16 Teacher to check.

Exercise 12.2

1 **a** NW **b** S **c** N **d** SE
2 **a** SW **b** Poland, West Germany, UK
3 **a** NW **b** SW **c** S **d** W
e E **f** NE **g** SE **h** N
i E
4 A
5 **a** i **b** iii **c** ii **d** iv
6 **a** Condobolin **b** Molong
7 **a** rain **b** hot, dry **c** rain **d** cold
e hot, dry **f** wet **g** wet **h** hot, dry

Exercise 12.3

1 **a** Kananook Primary School
b Tennis court **c** Seaford
d White star inside a red circle.
e Several answers possible, e.g. East Rd, Austin Rd, Mitchell St, Barry St.
2 **a** K4 **b** J7 **c** C11, C12, D11 or D12
3 **a** west **b** north **c** south **d** north
4 north **5** south-west **6** 250 m
7 1400 m **8** 1750 m
9 **a** 8 km **b** 4 km/h
10 **a** 2500 m **b** 2125 m **c** 625 m
11 **a** 1250 m **b** 2750 m
12 **a** 1375 m **b** Between 7 and 8 minutes
13 Basketball **14** Teacher to check

Exercise 12.4

1 **a** 30 m **b** 116 m **c** 190 m
2 **a** 1 : 10 000 **b** 1 km **c** 680 m
d 1.4 km **e** 11:28 a.m.
3 800 m

4 **a** 200 km **b** 36 km/h

c The average speed is low. The road is probably not sealed and rough in places.

5 **a** 20 m **b** 60 m and 115 m **c** $4900

6 **a** Even numbers from 156 to 190 and 175, 177, 181.

b around 21 m **c** 95 m **d** 230 m

e It will flood during 1-in-50 floods.

7 **a** 11.6 km **b** 2 h 54 min

c south-west of North Era

Exercise 12.5

1 155 km **2** 120 km **3** 098

4 **a** 3664 **b** 64 km/h

5 **a** Darwin to turn off 16 km

Turn off to Humpty Doo 14 km

Humpty Doo to Bark Hut Inn 54 km

Bark Hut Inn to Jabiru 36 km

b 120 km **c** $1\frac{1}{2}$ hours

6 **a** 456 km **b** 7 h 36 min

7 **a** Mittagong **b** 170 km **c** 1 h 53 min

8 37 km/h **9** $182

10 **a** 2:30 p.m. **b** $18 each **c** $11 each

11 10:59 a.m.

Exercise 12.6

1 **a** 2:35 a.m. **b** 20 h 40 min **c** $861

2 **a** Bus is cheaper by $1389 – $861 = $528

b $3764

3 **a** 6:30 a.m. **b** 2 h 55 min **c** $950

4 $170

5 **a** north-west **b** 75 km/h

6 **a** $210 **b** $40

c They need 2 caravans but only 1 cabin.

d No, teacher to discuss.

7 **a** Moockra Tower Lookout **b** Bruce

c Middle Gorge **d** Teacher to check.

8 **a** 290 km **b** approx 5 h 17 min

9 **a** They only need 1 room but they would need 2 rooms at the Greenwood Lodge

b $315

10 0900. The 1345 flight lands at 1545 which only leave 75 minutes for any delays and checking in for their flight. It may not be sufficient time and they could miss their flight home.

Keyword activity

Across

5 coordinates **8** distance **9** compass

Down

1 second **2** kilometres **3** south

4 scale **6** north-west **7** speed **10** map

CHAPTER 13
Comparing data

Exercise 13.1

1 The number of hours is evenly spread over a small range.

2 The scores are clustered in the 40s and 50s and tightly packed. There is a gap at the top.

3 The test results are tightly packed with a gap at the bottom.

4 **a** clustered, spread out

b gaps, more dense, less dense

c more dense, spread out

d spread out, gaps

e clustered, spread out, tightly packed

f tightly packed, gaps

Other answers are possible.

Exercise 13.2

1 **a** Mean = 20.78, $\sigma_n = 3.05$

b Mean = 48.27, $\sigma_n = 21.34$

c Mean = 14.55, $\sigma_n = 1.83$

d Mean = 55.27, $\sigma_n = 21.34$

2 **a** Mean = $497.60 **b** $\sigma_n = 103.6$

c Teacher to check.

3 **a** $469.60 **b** $\sigma_n = 67.43$

c Teacher to check.

d Quite difficult as the mean is higher than $420. Other answers possible.

4 **a** Port Paradise: $\sigma_n = 2.17$; Palm Tree Cove: $\sigma_n = 4.46$

b Port Paradise as the temperatures are more consistent (smaller standard deviation)

5 **a** Brad: Mean = 8.3, $\sigma_n = 2.0$; Aryn: Mean = 4.5, $\sigma_n = 1.7$; Kim: Mean = 6.5, $\sigma_n = 2.6$

b Aryn as he has the lowest standard deviation.

c Brad as he has the highest mean and a fairly low standard deviation.

6 a 70, 71, 73, 73, 74, 75, 75, 75, 75, 75, 7, 76, 77, 77, 77

b Mean = 74.6, $\sigma_n = 2.03$

c Paul is a consistent player as his scores show a small standard deviation.

7 a You would expect a large standard deviation as the data is spread from 127 to 157.

b $\sigma_n = 10.08$ c Teacher to check.

Exercise 13.3

1 Mean = 30.71, mode = 29, median = 30, standard deviation = 2.31, maximum = 35, minimum = 28, range = 7

2 Teacher to check.

3 b

	Mean	Median	Mode	SD
2011	19.17	18.5	17	4.45
2010	16.83	17.5	20	2.79

d Theft rose in 2011 and the number of thefts was spread over a wider range in 2011. Other answers possible.

4 Teacher to check.

5 a Teacher to check.

b Mean = 73.9, median = 71, $\sigma_n = 10.5$

c Mean = 68.2, median = 69, $\sigma_n = 6.3$

d Teacher to check.

6 b Mean = 93.4, median = 94.5, mode = 81, $\sigma_n = 9.22$

c Mean = 94.28, median = 95, mode = 81, $\sigma_n = 8.06$

d Without the outlier, the spread is reduced and the mean increases. Other answers possible.

Exercise 13.4

1 a

Class A		Class B
	3	5 8 9 9
	4	0 1 2 2 2 6 7
	5	1 4 6 7 9
9 7 5 2 0	6	0 1 5 5 5 6 8
9 8 6 5 1 1 1 0	7	1 2 4
9 9 9 8 5 5 4 2 1 1 0	8	8
5 5 4 3 1	9	

Key: 0|6| = 60

|3|5 = 35

b 29 c A: 81, B: 56.5

d A: 35, B: 53 e an outlier in B (88)

f A – top, B – middle

2 a

Year 11		Year 12
	0	7 8
5 5 5	1	0 2 3 3 4 5 8 9 9
9 5 3 3 2 2 0	2	0 0 1 2 2 3 4 4 6
9 1 0 0 0 0 0	3	
4 2 1	4	

Key: 5|1| = 15

|1|0 = 10

b 11: 29.5, 12: 19 c 11: 29, 12: 19

d No outliers e Clustered in the 20s

f medians very different

g number of students in each year

3 a

Male		Female
	15	0 2 7 9
8 7 5 0	16	0 2 3 4 4 5 6 6 6 7 8
8 8 8 5 2 0 0	17	1 3 3 6 7 8
4 4 3 3 1 1 1 1 0 0	18	5 5
1 0 0	19	
0	20	

Key: 0|16| = 160

|15|0 = 150

b 25 c 23

d Males: 160, 171, 180, 183.5, 200, Females: 150, 162, 166, 173, 185

e 200 is an outlier in the male group

f Male median is higher than the female median

g Teacher to check.

4 a

Broome		Kiama
	20	3 7
	21	0 8
	22	1 3 5 5
	23	0
	24	5 7 9
	25	0 4 5 6 6 9
	26	4 7
	27	0 6 9
	28	
5 3	29	6
	30	
9 9 8 7 7 2 2 0 0 0	31	
8 7 7 5 3 3 2 1 1	32	5 6
7 6 4 4 1	33	0
3 2	34	
	35	
	36	3

Key: |29| = 293

|20|3 = 203

b 28

c Broome: 32.15°, Kiama: 25.45°

d Broome: 5°, Kiama: 16°

e Broome: 29.3°, 29.5°
Kiama: 36.3°, 33.0°, 32.6°, 32.5°

f Broome temperatures are close together, Kiama temperatures are widely spread out.

g, h Teacher to check.

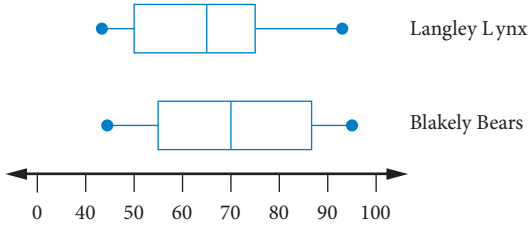
5 a 25

b Langley Lynx: 49, Blakely Bears: 50

c Langley Lynx: 66, Blakely Bears: 70

d Langley Lynx: 43, 51.5, 66, 75.5, 92
Blakely Bears: 44, 57, 70, 87, 94

e

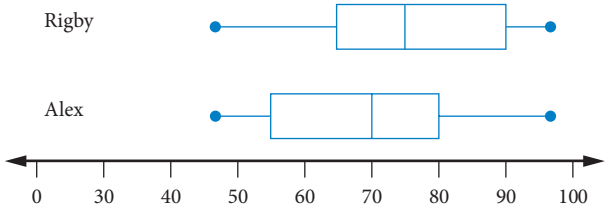


f Teacher to check.

g Blakely Bears

Exercise 13.5

1 a



b i 75 **ii** 69 **c i** 48 **ii** 50

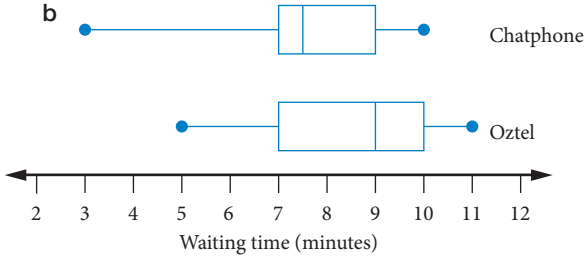
d Alex – his mark is 16% above the median while Rigby's is only 10% above the median

e Rigby's class – more than 50% of the scores are above the median for Alex's class

f No, we don't have any scores or the numbers of students in each class

2 a Chatphone: 3, 7, 7.5, 9 10, Oztel: 5, 7, 9, 10, 11

b



c i 7.5 **ii** 9

d i 7 **ii** 9

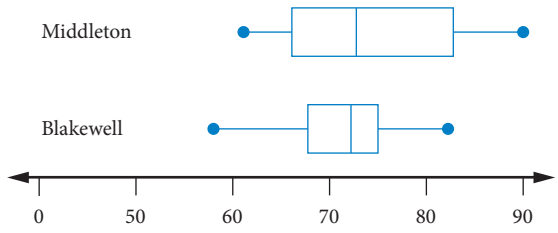
e i 2 **ii** 3

f Yes, Chatphone's median is 1.5 minutes less than Oztel and 75% of Chatphone's calls wait less than 50% of Oztel calls.

g Teacher to check.

3 a Middleton: 60, 67.5, 72.5, 82.5, 90
Blakewell: 58, 68, 72, 75, 82

b



c i 72.5 km/h **ii** 72 km/h

d i 72, 75, 85 **ii** 68, 70, 72, 73, 75

e i 74.2 km/h **ii** 71.5 km/h

f i 15 km/h **ii** 7 km/h

g No, the mean in Blakewell is lower and 25% of the sample in Middleton were faster than the entire Blakewell sample.

4 a 60 **b** 61 **c** soft drink

d coffee **e** Teacher to check.

5 a 52% **b** 15–24 **c** Teacher to check.

d Yes, higher frequency in each age group.

6 a 20° **b** February

c April through to September.

d 14 **e** Teacher to check.

Keyword activity

1 standard deviation **2** Teacher to check.

3 a five figure summary is:

– the lowest score

– the first quartile (Q_1)

– the median (Q_2)

– the third quartile (Q_3)

– the highest score

4, 5 Teacher to check.

CHAPTER 14
Turn up the volume

Exercise 14.1

1 **a** m^3 **b** cm^3 **c** cm^3
d mm^3 **e** m^3 **f** m^3

2 **a** 5000 **b** 2 000 000 **c** 0.5
d 250 000 **e** 0.024 **f** 36

3 **a** cm^3 **b** m^3 **c** mm^3
d cm^3 **e** mm^3 **f** cm^3

4 **a** E **b** G **c** F **d** C
e A **f** B **g** D

5 **a** 5 000 000 **b** 1 600 000 **c** 0.006
d 4 **e** 0.16 **f** 0.25
g 180 000 **h** 120
i 4 000 000 000 **j** 0.0096

6 4210 mm^3, 42 cm^3, 0.0042 m^3

7 0.6 m^3, 65 000 cm^3, 7 000 000 mm^3

8 0.306 m^3 **9** 2 520 000 mm^3

10 200 000 cm^3

Exercise 14.2

1 **a** 111 cm^3 **b** 2.5 m^3
c 256 m^3 **d** 486.42 cm^3

2 **a** 1 071 000 cm^3 **b** 91.125 cm^3
c 21 952 cm^3 **d** 258.06 cm^3

3 **a** 84 cm^3 **b** 240 cm^3

4 **a** 16 065 cm^3 **b** 1 285 299 cm^3

5 **a** 42 875 cm^3 **b** 471 625 cm^3
c approximately 330 books

6 **a** 648 cm^3 **b** 3420 cm^3
c 1068.375 cm^3 **d** 1026 cm^3
e 794.75 cm^3

7 **a** 126 pavers **b** 4 500 000 mm^3
c 567 000 000 mm^3 **d** 13
e $737.10

8 200 m^3

9 Volume = 420 m^3 so Nazneen requires a large air-conditioner

10 510 cm^3

Exercise 14.3

1 **a** mL **b** kL **c** L
d mL **e** L **f** kL

2 **a** D **b** G **c** J **d** H **e** I
f A **g** F **h** E **i** C **j** B

3 **a** 8 mL **b** 1500 mL **c** 425 mL

4 **a** 2 L **b** 3.5 L **c** 0.25 L

5 **a** 5000 mL **b** 5

6 2000 **7** 4000 **8** 1.5 L

9 1000 cm^3 **10** 1250 cm^3 **11** 9 L

12 **a** 0.2 mL **b** 12 mL **c** 2160 mL
d 17.28 L

13 **a** rectangular prism
b 2.042 m^3 **c** 2042 L **d** 19 minutes

14 **a** 10 143 cm^3 **b** 10 L **c** 5 kg

Exercise 14.4

1 **a** 85.5 cm^2 **b** 2223 cm^3 **c** 101 cm^3

2 532.5 cm^3

3 **a** 127 234.50 mm^3 **b** 36 945.13 cm^3
c 1.42 m^3

4 **a** 0.39 m^3 **b** 390 L

5 4500 cm^3

6 A is 454 cm^3, B is 409 cm^3, A has larger volume

7 **a** 28 148.67 cm^3 **b** 5 387.83 cm^3
c 22 760.84 cm^3

8 **a** 192 cm^3 **b** 1270 cm^3 **c** 43.2 cm^3
d 22.2 cm^3

9 **a** 0.13 m^3 **b** 130 L **c** 3250 L
d 2.31 m^3
e No, only holds 2310 L, 2 trips required

10 **a** 65 cm^3 **b** 51 m^3

11 **a** 8700 **b** 8265 **c** 8 265 000

Exercise 14.5

1 **a** 11 200 cm^3 **b** 96 m^3

2 **a** 37.7 m^3 **b** 414.7 cm^3 **c** 45.8 mm^3

3 **a** 904.8 cm^3 **b** 4.6 m^3

4 2 574 467 m^3

5 **a** 2.25 cm **b** 2.25 cm **c** 66 mL

6 **a** 696.910 m^3 **b** 696 910 L

7 **a** 1.1 m **b** Pyramid 1.2 m^3, Prism 2.9 m^3
c 4.1 m^3

8 **a** 16 605 cm^3 **b** 2490.75 cm^3

9 **a** 1810 cm^3 **b** $r = 6$ cm, $h = 24$ cm

c 2714 cm^3 **10** 8.8 m^3

Keyword activity

Across

2 breadth **3** square **6** area

8 micro **14** height **16** parallelogram

19 kilolitre **21** centi **22** milli

23 length **24** prism

Down

1 perpendicular **4** rectangle **5** triangle

7 composite **8** mega **9** perimeter

10 capacity **11** metric **12** volume

13 cross **15** tonne **17** metre

18 litre **19** kilo **20** hectare

21 cubic

CHAPTER 15
Interesting numbers

Exercise 15.1

1 **a** $130.20 **b** $109.20 **c** $412.25 **d** $34.88

2 **a** $45.83 **b** $442.74 **c** $17.55 **d** $1.66

3 0.39%

4 **a** 0.7% **b** 0.162% **c** 4.2% **d** 0.023%

e 0.323% **f** 2.1%

5 **a** $864 **b** $3264

6 **a** $6.69 **b** $371.69

7 $18.75

8 **a** $3610 **b** $4455 **c** $150.50

9 $294 150

10 **a** $300

b **i** $4770

ii It was probably a scam and he would lose his $30 000.

11 **a** $160 **b** $60 **c** $25

d It's the principal.

12 D8 = F3*F4*F5, D9 = F3+D8

Exercise 15.2

1

	Interest at the end of the year	Balance at the end of the year
End of the first year	$I = Prn$ $= \$8000 \times 0.06 \times 1$ $= \$480$	$\$8000 + \$480$ $= \$8480$
End of the second year	$I = Prn$ $= \$8480 \times 0.06 \times 1$ $= \$508.80$	$\$8480 + \$508.80$ $= \$8988.80$
End of the third year	$I = Prn$ $= \$8988.80 \times 0.06 \times 1$ $= \$539.33$	$\$8988.80 + \$539.33$ $= \$9528.13$

2 **a** $480 **b** $12 480 **c** $12 979.20

d $13 498.37 **e** $1498.37

3 **a** $4260.10 **b** $260.10 **c** $4.10

4 **a**

	Interest at the end of the month	Balance at the end of the month
End of the first month	$I = Prn$ $= \$14\,000 \times 0.007 \times 1$ $= \$98$	$\$14\,000 + \$98$ $= \$14\,098$
End of the second month	$I = Prn$ $= \$14\,098 \times 0.007 \times 1$ $= \$98.69$	$\$14\,098 + \$98.69$ $= \$14\,196.69$
End of the third month	$I = Prn$ $= \$14\,196.69 \times 0.007 \times 1$ $= \$99.38$	$\$14\,196.69 + \$99.38$ $= \$14\,296.07$

b $296.07

5 **a** $9600 **b** 0.8% **c** 9.6% **d** $154.21

6 **a** $5208.75

b Scam indicators include:

- cold phone call from someone she doesn't know
- push from the 'advisor' to act quickly
- very high interest rate
- the claim that the opportunity is 'risk-free'
- the minimum investment of $10 000

7 **a** Cell B8 =B4, Cell D8 =B8+C8, Cell B9 =D8

b We use $ signs for an absolute cell reference. This means that the cell reference stays the same when we use the formula in, for example, a 'fill down'.

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c

	A	B	C	D
1	Compound interest spreadsheet			
2	Only enter values in the blue shaded cells			
3				
4	Principal	$1,000.00	Annual rate of interest as a percentage	12%
5				
6				
7		Account balance at the beginning of the year	Interest earned during the year	Account balance at the end of the year
8	Year 1	$1,000.00	$120.00	$1,120.00
9	Year 2	$1,120.00	$134.40	$1,254.40
10	Year 3	$1,254.40	$150.53	$1,404.93
11	Year 4	$1,404.93	$168.59	$1,573.52
12	Year 5	$1,573.52	$188.82	$1,762.34
13	Year 6	$1,762.34	$211.48	$1,973.82
14	Year 7	$1,973.82	$236.86	$2,210.68
15	Year 8	$2,210.68	$265.28	$2,475.96

d $2047.91

8 a The simple interest graph is a straight line. The compound graphs are curves.

b 7 years

9 a $400

b Simple interest is the straight line, *Y*. This is Adam's investment. Compound interest (Nick's) is the curve *X*.

c Nick's investment is greater by approximately $35.

Exercise 15.3

1 10 910.78

2 a $P = 1870$, $r = 0.068$, $n = 4$

b $2432.91 **c** $562.91

3 $6461.81

4 a $6.36 \div 12 = 0.53$ **b** $2144.65

c $194.65

5 a 0.49% **b** 60 **c** $49 623.52

6 a $17 786.85 **b** $1463.63 **c** $35 607.18

d $98 875.58 **e** $16 897.39

7 A $11 800, B $11 876.48, C $11 888.45: option C is the best

8 Simple interest = $64 000, compound = $64 032.10 The compound interest has the better return.

10 a interest

b Needs brackets =G4*(1+G6/G5)^(G7*G5)

c Teacher to check

Exercise 15.4

1 a $1966.81 **b** $517.13 **c** $359.36 **d** $366.16

2 a $7193.75 **b** $1393.75

3 $1744.93 **4** $36.06 **5** $464

6 6.4% p.a. monthly compounding gives the better return with $61 274.92 interest. The annually compounding investment only gives $60 880.61 in interest.

7 The managed funds investment gives the better return; interest = $12 366.26 with $803.81 fees, giving a return of $11 562.45. The finance company returns $10 232.38 less $120 in fees giving $10 112.38.

Exercise 15.5

1 370 000

2 a $110 000 **b** $100 000 **c** $105 000

3 a $500 000 **b** $480 000 **c** $482 000

4 a 0.0046 **b** 0.026 **c** 0.020 **d** 0.000 47

5 a 4 000 000

b Any value from 4 245 000 to 4 254 999

c 3 740 000

6 a $487 000 000 **b** $1 900 000 000

7 a 3000 **b** 3 **c** 10 000

d 0.005 **e** 20 **f** 0.7

8 a 0.67 **b** 0.0055 **c** 1.3

d 0.087 **e** 790 000 **f** 7.3

g 24 **h** 2.4 **i** 34

9 a 352 000 kg **b** 4190 m

c 67.1 km/h **d** 14.8 mL

e 150 000 000 km

Exercise 15.6

1 a 8.6×10^5 **b** 9.14×10^9 **c** 2.01×10^3

d 3.6×10^{-4} **e** 1.8×10^{-9} **f** 1.01×10^{-4}

2 a 3 520 000 **b** 35 000 **c** 8700

d 645 000 000 **e** 0.0061 **f** 0.000 019 3

g 0.000 000 5 **h** 0.01

3 1×10^9 **4** 1×10^{-7} **5** 6×10^7, 53 000 000

6 a 1×10^{12} **b** 640 **c** 2.166×10^{10}

d 3.51×10^5 **e** 1.1×10^3 **f** 6×10^7

g 3×10^9 **h** 3.76×10^{10}

7 6.375×10^4 **8** 1.419×10^{16} **9** 4.6×10^{18}

10 4.4×10^{11} **11** Bacteria, 3.4998×10^{-3}

12 111 111 111

13 a 5.3×10^7 **b** 1.5×10^5 **c** 2.5×10^3

d 4.6×10^{-4} **e** 2.7×10^{-3} **f** 1.0×10^{-1}

14 3.98×10^{11}

Keyword activity

1 H **2** J **3** E **4** G

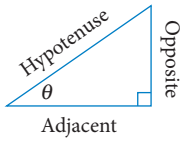
5 K **6** A **7** I **8** C

9 F **10** B **11** D

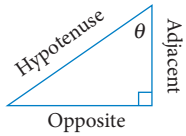
CHAPTER 16
So you've got a right angle

Exercise 16.1

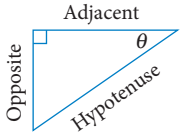
1 a



b



c



2 a 65 **b** 56 **c** 56

3 a $\frac{3}{4}$ **b** $\frac{12}{5}$ **c** $\frac{120}{119}$

4 Teacher to check. **5** 40°; 9, 40°, 9; 7.6

6 a 14·43 cm **b** 4·86 m **c** 10·89 cm
d 235·12 cm **e** 6·64 km **f** 4·71 mm

7 a 17·16 cm **b** 12·0 cm

8 a 30° **b** $AC \approx 0{\cdot}7$ m, $AD \approx 1{\cdot}4$ m

Exercise 16.2

1 a 41° **b** 47° **c** 37°
d 31° **e** 54° **f** 53°

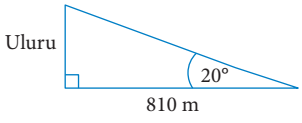
2 a 38° **b** 34° **c** 68°
d 49° **e** 43° **f** 59°

3 90°, 34°, 56°

4 a 90°, 45°, 45°

b The opposite and adjacent sides have the same length.

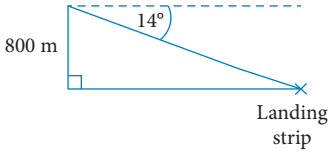
5 a



b 295 m

6 801.6 m **7 a** 38° **b** 52°

8 a



b 3209 m

Exercise 16.3

1 118.3 m **2** 1411.4 m **3** 530.0 m

4 a 156·934 m **b** 158.5 m

Exercise 16.4

1 a 5.8 **b** 12.6 **c** 11.4

2 a 40° **b** 30° **c** 26°

3 a 15 m **b** 8 m **c** 17 m

4 a 41° **b** 62° **c** 63°

5 a 5.5 **b** 11.1 **c** 3.5

Exercise 16.5

1 a tan **b** cos **c** sin

2 a 7.2 **b** 7.0 **c** 10.6

3 a 64° **b** 30° **c** 53°

4 14 m **5** 12.4 m **6** 32.5 m **7** 7.3 m

8 a 14.6 **b** 7.2 **c** 7.5

9 a 41° **b** 30° **c** 46°

Exercise 16.6

1 a 13.6 **b** 24.8 **c** 14.6

2 a 8.1 **b** 15.9 **c** 16.8

3 a 21.45 m **b** 21.5 m

4 11.5 m **5** 5.05 m

6 305.6 cm, which is 310 cm to the nearest 10 cm.

7 a 7.4 **b** 6.2 **c** 13.6

Exercise 16.7

1 a 045°, NE **b** 315°, NW **c** 270°, W
d 225°, SW **e** 045°, NE **f** 135°, SE

2 a 110° **b** 158° **c** 220°
d 250° **e** 280° **f** 014°

3 a 314° **b** 272° **c** 136° **d** 245°

4 100° **5** 099°

6 a Teacher to check. **b** 658 m **c** 1348 m

7 a 29° **b** 138 km

8 a $\alpha = 25°, \beta = 65°$ **b** 90° **c** 192 km

9 a Teacher to check. **b** 124 km

10 a Teacher to check. **b** 85 km

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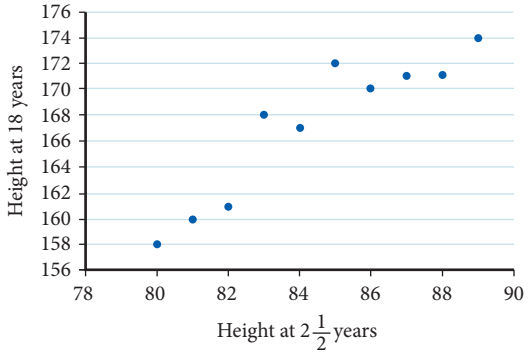
Keyword activity

[1]A	D	J	A	C	E	N	[2]T		
							R		[3]O
	[4]T	A	N	R	A	T	I	O	P
							G		P
							O		O
	[5]R				[6]A	N	G	L	E	S
	I					O				I
	G					M				T
	[7]H	Y	P	O	T	E	N	U	[8]S	E
	T					T			I	
						R			D	
						Y			E	
							[9]C	O	S	

CHAPTER 17
Fitting the data

Exercise 17.1

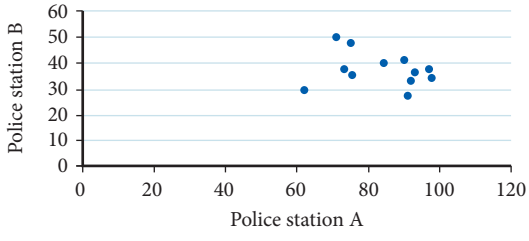
1 a



b Height at $2\frac{1}{2}$ years is the explanatory variable, Height at 18 years is the response variable.

c As height at $2\frac{1}{2}$ years increases, the height at 18 years increases.

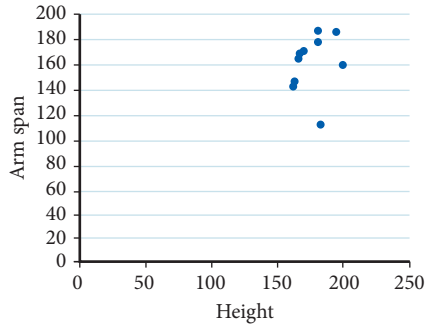
2 a



b Police station A is the explanatory variable and Police station B is the response variable.

c There is no pattern to this data, it is grouped together.

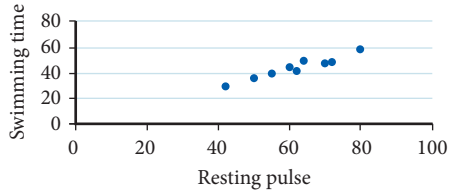
3 a



b Height is the explanatory variable and Arm span is the response variable.

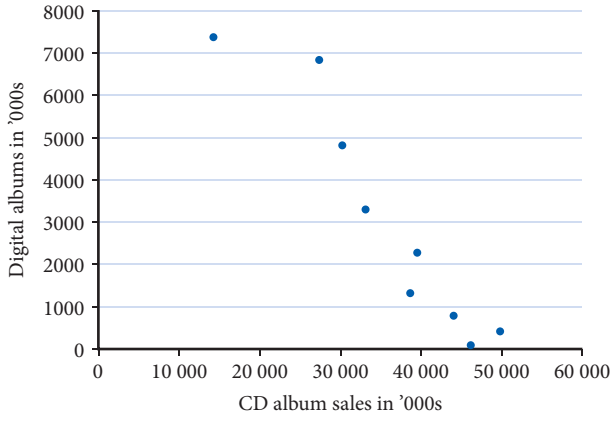
c There is one point out on its own. The other points show that as height increases, the arm span increases.

4 a



b As one variable increases, the other increases.

5 a



b As one variable increases, the other decreases.

Exercise 17.2

1 a positive, linear, moderate

b no association

c positive, linear, strong

d negative, non-linear, strong

e positive, linear, weak

f negative, linear, strong

g positive, non-linear, strong

h no association

i negative, linear, moderate

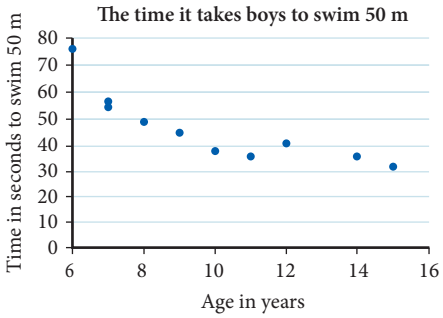
j positive, linear, moderate

2 a positive, linear, strong

b positive, linear, strong

c negative non-linear, moderate

3 a

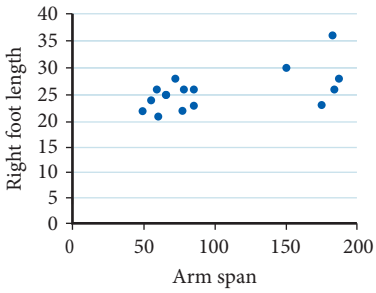


b There is a strong negative linear correlation.

4, 5, 6 Teacher to check.

Exercise 17.3

1 a



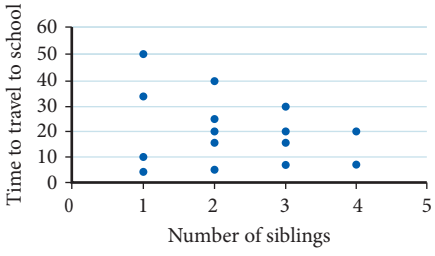
b Explanatory variable – arm span,
response variable – foot length

c As arm span increases, the right foot length increases

d positive, linear, moderate

2 a No, teacher to check reasons.

b



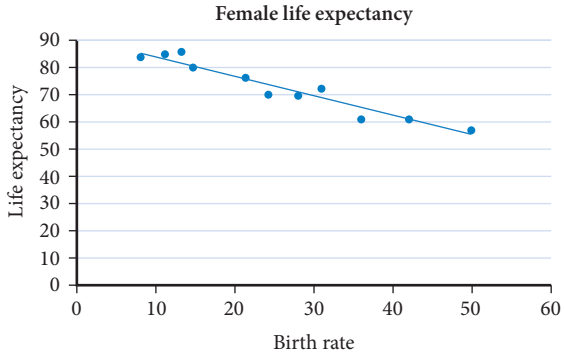
c The scatter plot shows no relationship between the two variables.

3, 4 Teacher to check.

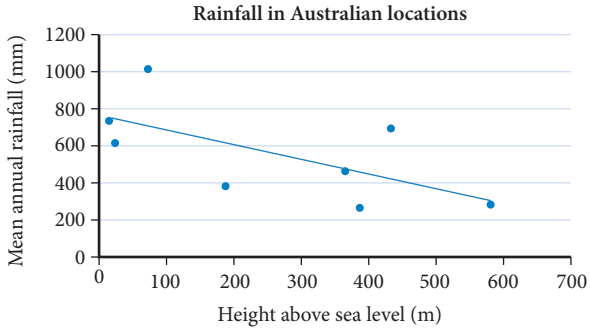
Exercise 17.4

1 Teacher to check.

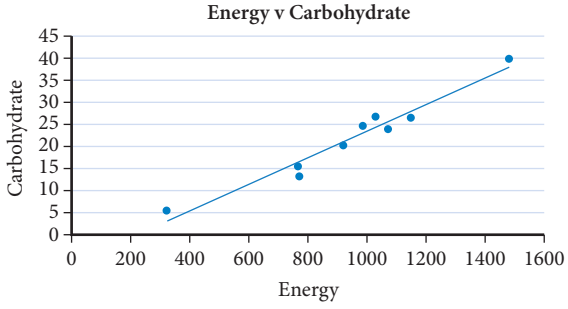
2



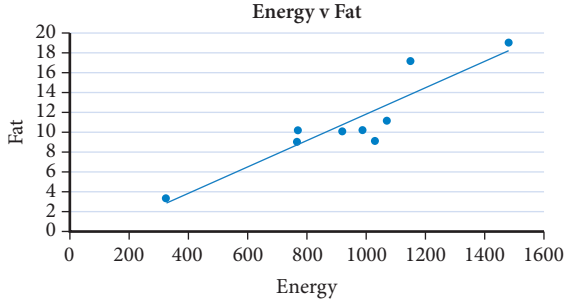
3



4 a, b



c, d



Exercise 17.5

1 a weeks after starting training

b leg curls in 30 seconds

c The longer Anita has trained, the more leg curls she can do in 30 seconds.

d 11 or 12

2 a number of victims b number of medical staff

c about 40 staff d about 750 victims

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e The greater the number of victims, the greater the number of medical staff sent.

f The agency calculated the number of medical staff required based on the number of victims, not the reverse, as Kyle's statement implies.

3 a about 23.5 cm

b the longer the right foot, the higher the test score.

c The age of the students is relevant. Younger students who haven't learned as much maths generally have smaller feet and would score lower on the test.

4 a As the height of students increases, so does the weight of students.

b about 55 kg c No, teacher to check reasoning.

5 a about 10 or 11

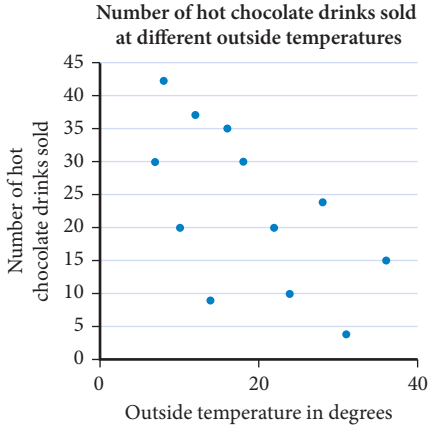
b As the number of sunny days increases, the number of dry cloudy days decreases.

c The total number of sunny, dry cloudy and wet days must be the same as the number of days in the month. As the number of sunny days increases, the number of dry cloudy or wet days must decrease to keep the total the same as the number of days in the month.

Exercise 17.6

1 a E b C c B d G
e H f A g F h D

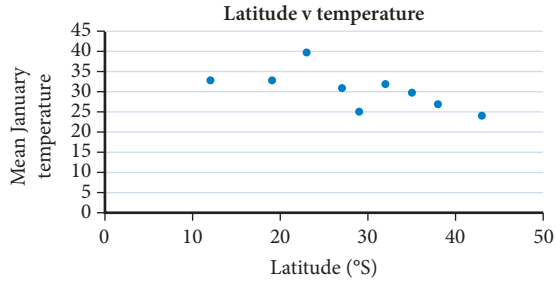
2 a



b −0.611

3 a −0.9662 b −0.633 16 c 0.977 921
d 0.909 538 e 0.852 039

4 a



b −0.65217

c No, the correlation coefficient is only in the moderate range.

d Teacher to check.

5 a −0.9484

b They are closely related. As digital album sales increase, CD sales decrease.

6 Teacher to check.

Exercise 17.7

1 a i 1090 kJ ii 105 kg

b 1885 kJ c Teacher to check.

2 a about 62 years b about 36 births

c about 48 years d Teacher to check.

3 a about 525 mm b about 460 m

c about 140 mm

d −14 mm, not possible as you can't have negative rainfall.

e Not very reliable as the correlation coefficient is in the moderate range.

4 a 0.975 989

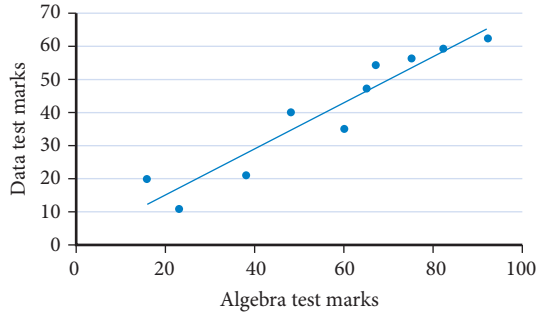
b Very reliable, as the correlation coefficient is close to 1.

c May not be reliable as −10°C is outside the data range.

5 Not very reliable as the correlation coefficient is in the moderate range.

6 Very reliable as the correlation coefficient is close to −1.

7 a

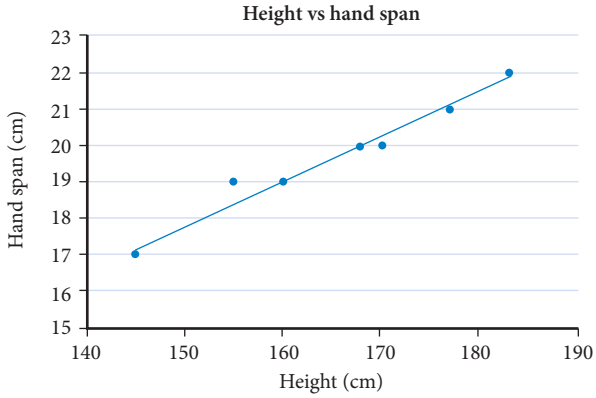


b Ella 43 in Algebra test, Amy 57 in Data test.

c 0.949 98

d Very reliable as the correlation coefficient is close to 1.

8 a



b 0.983

c 20 cm

d 33 cm. The value is very likely to be wrong because Robert Wadlow's height is much greater than the heights used in the scatter plot.

Keyword activity

bivariate, scatter plot, explanatory variable, response variable

association, positive, negative

linear, non-linear

strong, moderate, weak

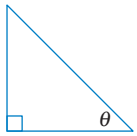
line of fit, correlation, correlation coefficient

predictions, interpolation, extrapolation

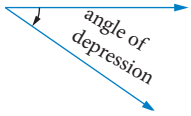
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GLOSSARY AND INDEX

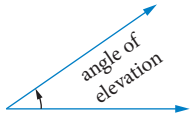
adjacent side In a right-angled triangle, the side that joins the angle to the right angle. (p. 386)



angle of depression The angle through which you turn your eye down from looking straight ahead to look at an object. (p. 391)



angle of elevation The angle through which you turn your eye up from looking straight ahead to look at an object. (p. 391)



area The amount of surface occupied by a flat shape, measured in square units. (p. 260)

association A relationship between two variables. (p. 411)

Australian Central Standard Time (ACST) The time zone for central Australia: the Northern Territory and South Australia. (p. 121)

Australian Eastern Standard Time (AEST) The time zone for eastern Australia: Queensland, NSW, the ACT, Victoria and Tasmania. (p. 121)

Australian Western Standard Time (AWST) The time zone for Western Australia. (p. 121)

back-to-back stem-and-leaf plot Two stem-and-leaf plots in the same diagram. (p. 329)

Allfit		Superfit
7 6 5 5 5 2	1	1 3 6 6 9
7 3 2 2 1 1 0 0	2	0 0 1 2 2 3 7 7
9 4 2	3	1 2 4 4 5 8
3 3	4	6
1	5	

bearing See three-figure bearing. (p. 399)

bias In statistics, something that does not reflect the truth fairly, or when a sample does not represent a population fairly. (p. 175)

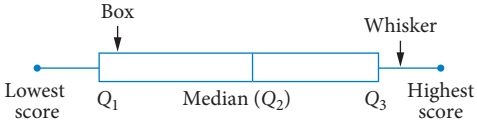
BIDMAS See order of operations. (p. 10)

bivariate data Data involving two variables, such as a person's height and weight, which can be graphed on a scatterplot. (p. 408)

blood alcohol content (BAC) The concentration of alcohol in a person's blood, measured in g/100 mL. (p. 251)

blood pressure A two-part measurement that describes the pressure when the heart is pumping (systolic pressure) and the pressure when the heart is relaxed (diastolic pressure), for example 120/80. (p. 243)

boxplot A diagram that displays the quartiles of a set of data as a box and the extremes as whiskers. (p. 186)



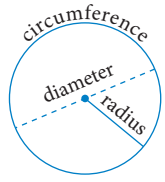
calories (cal) An old measure of the energy contained in food. (p. 223)

capacity Amount of liquid or gas that can be held by a container, usually measured in millilitres (mL) or litres (L). (p. 349)

categorical data Information represented as a category rather than a number, for example, the colours of eyes. Differs from numerical data. (p. 170)

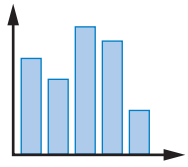
census Collection of information about every member of a population. (p. 172)

circumference The perimeter of a circle. $C = \pi d$ or $C = 2\pi r$, where C is the circumference, π is pi, d is the diameter and r is the radius. (p. 61)

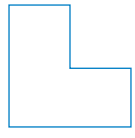


compass direction A direction shown by points on a compass, such as north (N), south (S), east (E), west (W), north-east (NE), north-west (NW), south-east (SE), south-west (SW). (p. 294)

column graph A graph consisting of columns of equal width. (p. 79)



composite shape A shape made up of a combination of shapes, for example, a square and a triangle, a rectangle and a semicircle, and so on. (p. 268)



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compound interest Interest paid on the principal invested as well as on any accumulated interest. Differs from simple interest. (p. 367)

cone A solid shape with a circular base and a curved surface that has a point. (pp.142, 277, 356)



coordinates A pair of values and/or letters for describing a location on a map, for example, D4. (p. 299)

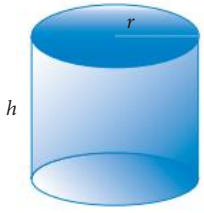
correlation The strength of a relationship between two variables: can be positive or negative, strong or weak. (p. 423)

correlation coefficient A variable that measures the correlation of two variables, and whose value is between −1 and 1. (p. 423)

cosine ratio (cos) In a right-angled triangle, it is $\frac{\text{adjacent side}}{\text{hypotenuse}}$. (p. 393)

cost price The price it costs a shop to buy a product from the factory or wholesaler. (p. 38)

cylinder A solid shape with identical circular ends and a curved surface. (pp.142, 277, 341)

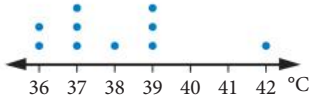


data Statistical information. (p. 170)

diameter The 'width' of a circle though its centre. The diameter is double the radius. See circumference for diagram. (p. 267)

diastolic pressure See blood pressure.

dot plot A graph that uses dots to show frequencies of data scores. (p. 93)



elevation On a plan, the front, back or side view of a building. (p. 202)

equation A mathematical statement that two quantities are equal. An equation contains an equals sign (for example $4x - 5 = 11$). (p. 247)

explanatory variable The first variable in bivariate data, which is graphed on the horizontal axis of a scatterplot. See also response variable. (p. 408)

extrapolation Making a prediction beyond the given data. (p. 428)

error See tolerance.

five-number summary These five values for a set of data: the lowest score, the lower quartile, Q_1, the median, Q_2, the upper quartile, Q_3, and the highest score. (p. 186)

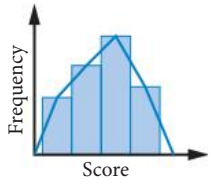
formula A rule written as an algebraic equation, using variables. For example the formula for the area of a triangle is $A = \frac{1}{2}bh$. (p. 233)

frequency The number of times a score or group of scores occurs in a data set. (p. 181)

frequency histogram A column graph in which the height of each column represents the frequency of a score or group of scores. (p. 81)



frequency polygon A line graph formed by joining the midpoints of the tops of the columns of a frequency histogram. (p. 81)



Greenwich mean time (GMT) The reference for the time of day around the world. GMT is the local time in London, England. (p. 124)

heart rate See pulse.

hectare A large unit for measuring area, equal to 10 000 square metres. (p. 260)

histogram See frequency histogram.

hypotenuse The longest side in a right-angled triangle, which is also the side opposite the right angle. (pp. 70, 386)

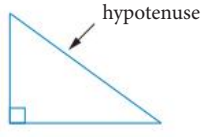


image The name given to a shape after it has been translated, reflected, rotated, enlarged or reduced. (pp. 158, 192)

integer A positive or negative whole number or zero, for example, –3, 7, 0. (p. 8)

interest Money earned on an investment, or money paid for borrowing. (p. 364)

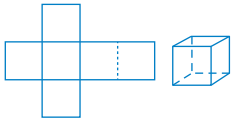
interest rate The percentage of the investment or loan on which interest is calculated. (p. 364)
interpolation Making a prediction within the given data. (p. 428)
interquartile range (IQR) The difference between the upper quartile and lower quartile of a data set ($Q_3 - Q_1$). It is a measure of the spread of the data. (p. 184)

kilojoules (kJ) An measure of the energy contained in food. (p. 222)

linear To do with a line. (p. 411)
line graph A graph made of lines, often used to represent data that changes over time. (pp. 80, 86)
line of fit A straight line that represents a set of points on a scatterplot, that goes through the centre of the points. (p. 417)
line symmetry Symmetry where one half of a shape is the mirror-image of the other half. See also rotational symmetry. (p. 153)
litre A unit for measuring capacity, equal to 1000 mL. (pp. 54, 349)
loss Lost money when selling price is less than cost price. (p. 38)

mean The average of a set of scores, calculated by adding all the scores and dividing by the number of scores. (p. 178)
measure of central tendency A value, such as the mean, median or mode, which describes the centre of a set of data. (p. 178)
measure of spread A value, such as the range, interquartile range or standard deviation, which describes the spread of a set of data. (p. 184)
median The middle score, or the average of the two middle scores, when scores are arranged in order. (p. 178)
mode The most common score(s) in a set of data. (p. 178)

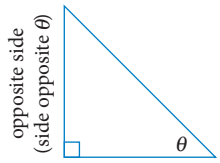
net (of a solid) A flat shape that can be folded to make a solid. The net of a cube is shown. (p. 149)



non-linear Not a straight line. (p. 411)

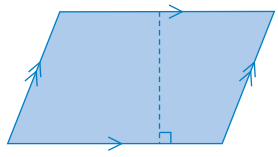
numerical data Data that involves numbers, such as heights. (p. 170)

opposite side In a right-angled triangle, the side opposite the reference angle. (p. 386)

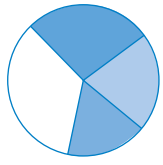


order of operations The order in which you calculate an expression with two or more operations: brackets, indices (powers), multiply and divide from left to right, add and subtract from left to right. This can be remembered using the initials BIDMAS. (p. 10)
outlier An extreme (high or low) score in a data set that is very different from the other scores. (p. 182)

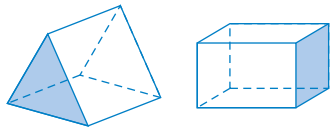
parallelogram A quadrilateral with opposite sides parallel. (p. 138)



per annum (p.a.) Per year. (p. 364)
perimeter The distance around the outside of a shape. (p. 68)
period Amount of time (for example month, week, year). (p. 364)
pie graph A graph using a circle divided into parts called sectors. Also called a sector graph. (p. 79)



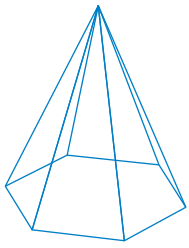
precision The size of one unit on a measuring scale, for example, 1 mm on a ruler, indicates how exact a measurement on this scale can be. (p. 64)
principal The original amount of money invested or borrowed. (p. 364)
prism A solid shape with flat sides that has the same shape at both ends. A triangular prism and square prism are shown below. (pp. 142, 272, 341)



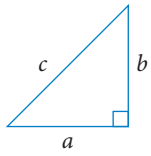
profit Gain or reward when selling price is greater than cost price. (p. 38)

pulse A regular heart beat felt in your wrist or neck. Also called heart rate. (p. 229)

pyramid A solid shape with flat triangular sides that comes to a point. A hexagonal pyramid is shown. (pp. 142, 281, 356)



Pythagoras' theorem In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the two shorter sides. $c^2 = a^2 + b^2$. (p. 70)



quarterly Four times a year, every three months. (p. 365)

quartiles Values that divide a set of data into four equal parts when the scores are arranged in order. The 1st quartile (Q_1) is the lower quartile, the 2nd quartile (Q_2) is the median and the 3rd quartile (Q_3) is the upper quartile. (p. 184)

radius The length from the centre of a circle to its circumference, half of the diameter. See circumference for diagram. (p. 264)

range For a set of data, the highest score minus the lowest score. (p. 184)

rate A comparison of two quantities with different units, for example, km/h. (p. 110)

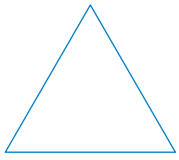
ratio A comparison of two quantities with the same units, for example, 2 : 3. (p. 98)

reflection The process of 'flipping' a shape across a line to give a mirror image. (p. 158)

response variable The second variable in bivariate data, which is graphed on the vertical axis of a scatterplot. See also explanatory variable. (p. 408)

rotation The process of 'spinning' a shape around a fixed point. (p. 158)

rotational symmetry Where a shape can be spun around so that it fits onto itself again before making a complete revolution. This equilateral triangle has rotational symmetry of order 3. See also line symmetry. (p. 153)



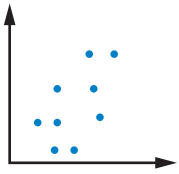
sample A group of items selected from a population for study. (p. 172)

scale A diagram or ratio that compares lengths on a map or diagram with real life, for example 1 : 10 000 means that 1 unit on the diagram represents 10 000 units in real life. (p. 305)

scale diagram A drawing of an object, usually smaller, whose lengths are in the same ratio as the actual lengths of the object. (p. 198)

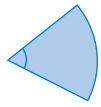
scale factor The ratio $\frac{\text{image length}}{\text{object length}}$ for a pair of similar figures. For example, a scale factor of 3 means every length of the image is 3 times the corresponding length of the object. (p. 192)

scatterplot A graph of points on a number plane showing a relationship between two variables. (p. 408)



scientific notation A way of writing very large and very small numbers, using a decimal multiplied by a power of 10. For example 9.8×10^7 means 98 000 000. (p. 380)

sector A fraction of a circle. (p. 264)



selling price The price a shop sells a product for. (p. 38)

significant figures Meaningful digits in a numeral that tell 'how many'. For example 98 000 000 has two significant figures: 9 and 8, while 0.003 15 has three significant figures: 3, 1 and 5. (p. 378)

similar figures Figures that have the same shape but not necessarily the same size. Matching sides are in the same ratio. (p. 195)



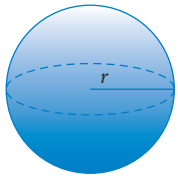
simple interest Interest earned or charged only on the original amount of money (principal) invested or borrowed. Differs from compound interest. (p. 364)

sine ratio (sin) In a right-angled triangle, it is $\frac{\text{opposite side}}{\text{hypotenuse}}$. (p. 393)

solution The answer to a problem, or the value of the pronumeral in an equation that makes the equation true. (p. 247)

speed A rate that compares distance travelled with time taken. Speed is often measured in kilometres per hour (km/h) or metres per second (m/s). Speed = $\frac{\text{distance}}{\text{time}}$. (p. 288)

sphere A solid shape like a ball. (pp. 277, 356)



standard deviation (σ_n) A statistical measure of the spread of a set of scores. Can be found using a calculator's statistical functions. (p. 323)

stem-and-leaf plot A 'number graph' that lists all the data scores, in groups. (p. 93)

Stem	Leaf
4	2 5
5	0 2 8
6	6 7
7	3 5 7 7
8	2

substitute To replace a pronumeral in an expression or formula with a number. (p. 233)

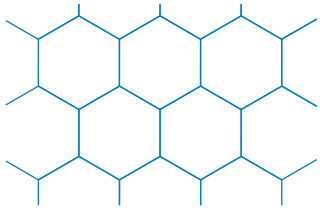
surface area The total area of all the surfaces of a solid shape. (p. 272)

survey To gather information for statistical purposes. (p. 172)

symmetry See line symmetry and rotational symmetry.

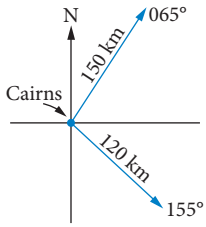
tangent ratio (tan) In a right-angled triangle, it is $\frac{\text{opposite side}}{\text{adjacent side}}$. (p. 386)

tessellation A 'tiling pattern' of repeated shapes with no overlapping or gaps. (p. 162)



three-dimensional (3D) Having length, width and height. A solid shape. (p. 138)

three-figure bearing A direction measured as an angle clockwise from due north, from 000° to 360°. (p. 399)



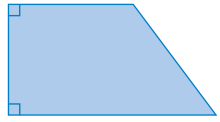
time zone A section of the world where all locations have the same time of day. (p. 121)

tolerance The margin of error of a measurement, equal to half the precision of the measuring device, for example, a measured value of 135 cm on a tape measure may have a tolerance of ±0.5 cm. (p. 64)

transformation In geometry, the process of moving a shape, such as reflection, rotation and translation. (p. 158)

translation The process of 'sliding' a shape a certain distance and direction. (p. 158)

trapezium A quadrilateral with a pair of parallel sides. (p. 139)



12-hour time a.m. or p.m. time, for example, 2:35 p.m. (p. 116)

24-hour time Time expressed in as a 4-digit number with the first two digits representing the hour (from 00 to 23) and the last two digits representing the minutes, for example, 1435 representing 2:35 p.m. (p. 116)

two-dimensional (2D) Having length and width but no depth. A flat shape. (p. 138)

unitary method Dividing to find the value of one item, then multiplying to obtain the value of the required number. (p. 103)

volume The amount of space occupied by a solid, measured in cubic units. (p. 340)

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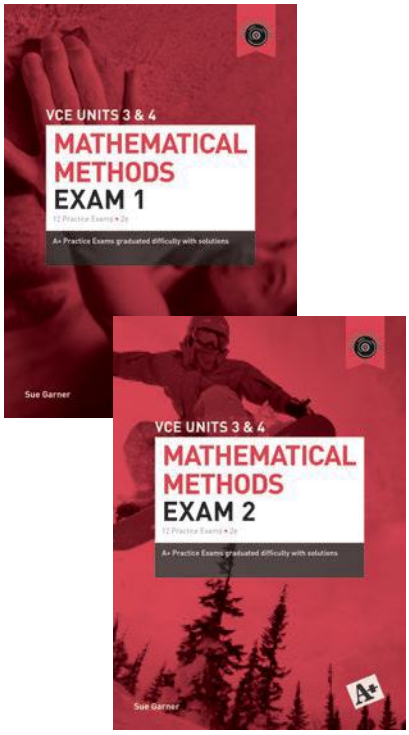
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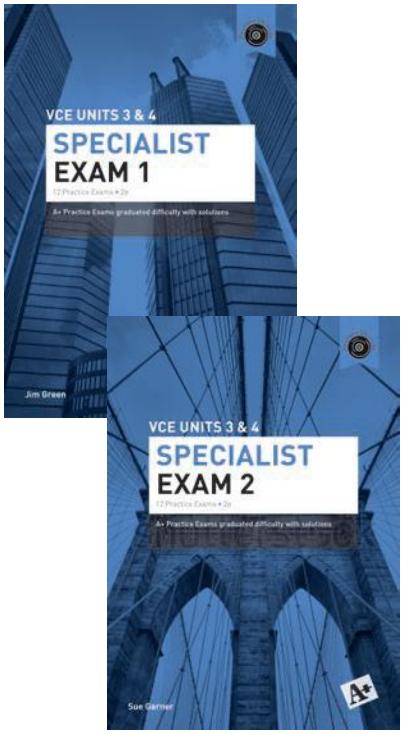
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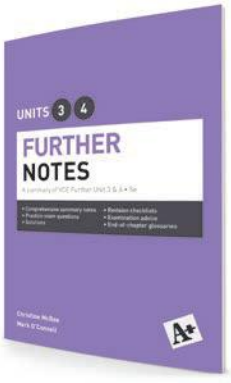
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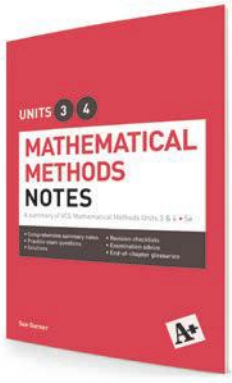


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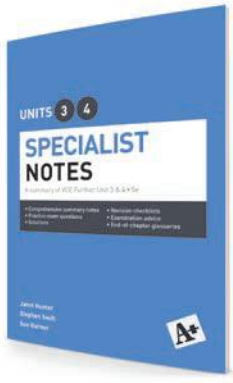
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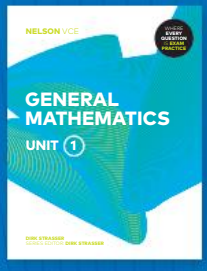
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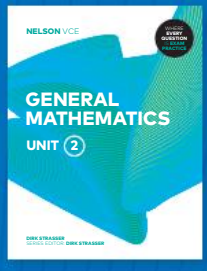
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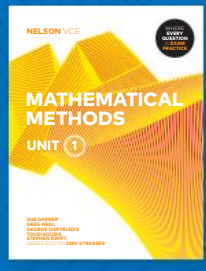
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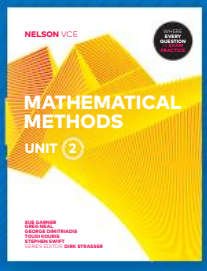
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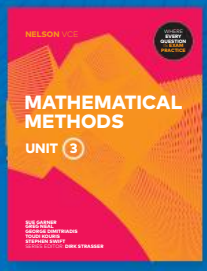
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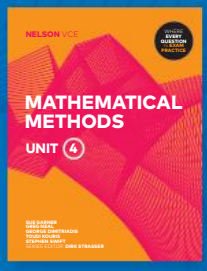
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