

MATHEMATICS

OXFORD
MATHS
9
STAGE 5

SERIES CONSULTANT:
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NSW CURRICULUM

obook ^{pro}

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UNIVERSITY PRESS

N S W
CURRICULUM

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Published in Australia by
Oxford University Press
Level 8, 737 Bourke Street, Docklands, Victoria 3008, Australia.

© Karen Bellamy, Alexander Blanksby, Morgan Levick, Elizabeth Busch, Echo Gu,
Jennifer Nolan, Melanie Koetsveld & Joe Marsiglio
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First published 2024

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ISBN 9780190342685

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Edited by Heather Jacobson
Typeset by Newgen KnowledgeWorks Pvt. Ltd., Chennai, India
Proofread by Kim Wilson
Indexed by Max McMaster
Printed in Singapore by Markono Print Media Pte Ltd

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Warning to First Nations Australians

Aboriginal and Torres Strait Islander peoples are advised that this publication may include images or names of people now deceased.

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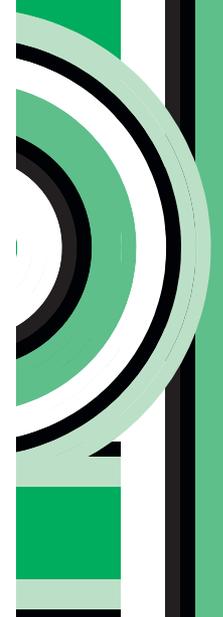


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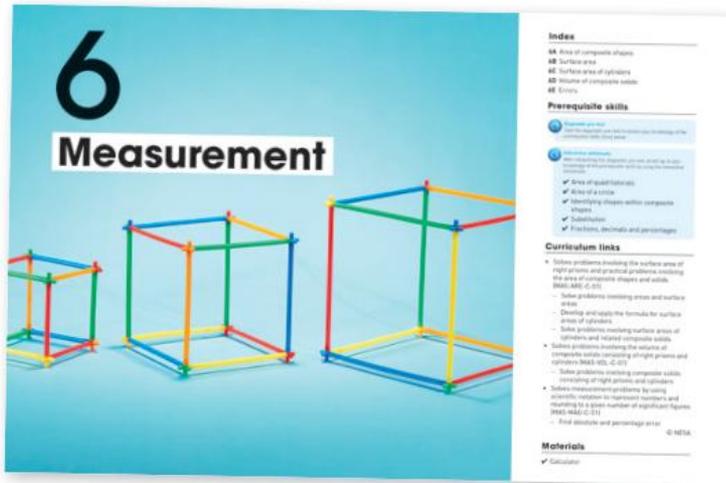
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Oxford Maths 7–10 NSW Curriculum utilises an innovative suite of print and digital resources to guide students on a focused mathematics journey. The series makes maths accessible to students with differing levels of understanding, increasing engagement by giving learners the opportunity to achieve success at their own skill level while also providing comprehensive syllabus coverage.

Key features of Student Books

- > Complete access to all digital resources available on Student obook pro.
- > Australian Maths Trust (AMT) spreads offer unique questions designed to challenge students and build engagement.
- > STEAM projects encourage inter-disciplinary thinking.
- > Semester reviews provide an opportunity to revise key concepts from each semester.



Each chapter opens with:

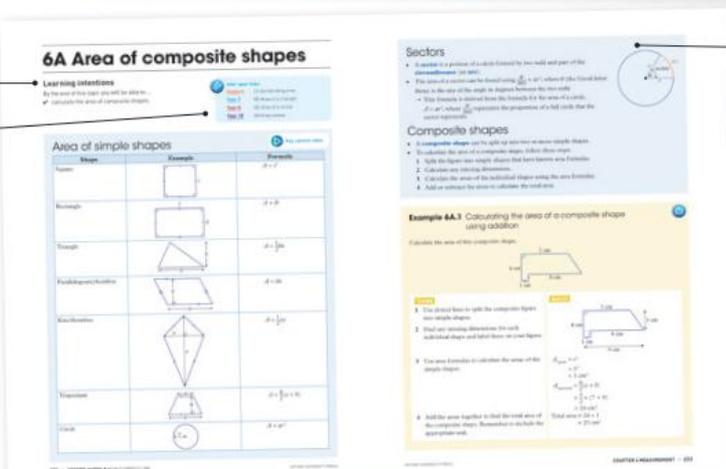
- **Prerequisite skills** with reference to an online diagnostic pre-test and interactive skillsheets.
- **Curriculum links** to all relevant content descriptions in the NSW Curriculum.
- **Materials** used to complete the exercises.

Learning intentions

- Signpost the foundational skills being developed in each section.

Inter-year links

- Provide easy access to support and extension material from each of the 7–10 Student Books as students build knowledge year on year.

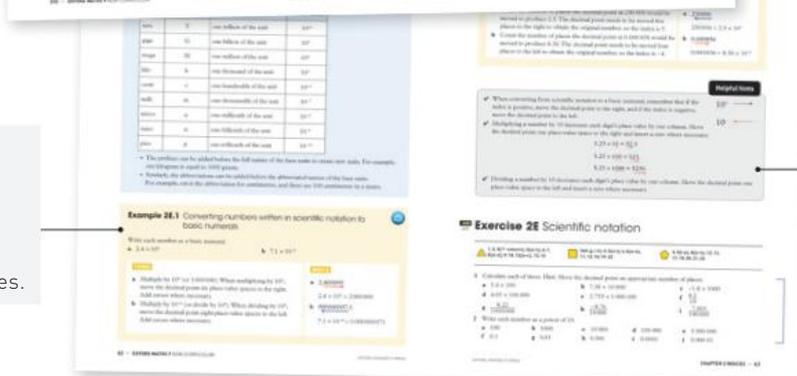


New theory

- Backed by the latest pedagogical research to promote engagement with the material.
- Filled with precise diagrams that bring key concepts to life, and aid understanding.

Worked examples

- Outline a step-by-step thought process for solving essential questions with direct reference to the exercises.



Helpful hints

- Provide additional strategies for tackling problems.
- Highlight important elements of the theory.
- Point out common misconceptions.

Understanding and fluency

- Basic exercises dedicated to practising key concepts.

Problem solving and reasoning

- Comprehensive exercises bring together new ideas and provide engaging contexts from real-world problems.

Challenge

- Advanced exercises designed to build engagement and anticipate future learning outcomes.

Chapter summary

- Condenses all the theory from each section into one accessible revision page.

Differentiated learning pathways

- Each exercise is separated into three pathways, tailoring for students of all skill levels.
- Each pathway can be assigned based on results of the diagnostic pre-tests that are recommended at the beginning of every chapter.

Checkpoint

- A section in the middle of each chapter dedicated to summarising key skills and encouraging memory retention.
- Reference to an online checkpoint quiz to gauge student progress.

Chapter review

- Additional practice questions to further consolidate understanding at the end of each chapter.
- Reference to an online chapter review quiz to track results.
- Reference to Quizlet test to revise new terminology.

Integrated STEAM projects

- Take the hard work out of cross-curricular learning with engaging STEAM projects. Two fully integrated projects are included at the end of each book in the series, and are scaffolded and mapped to the Science, Maths and HSEI curricula.

Problem solving through design thinking

- Each STEAM project investigates a real-world problem that students are encouraged to problem-solve using design thinking.

Full digital support

- Each STEAM project is supported by a wealth of digital resources, including student booklets (to scaffold students through the design-thinking process of each project), videos to support key concepts and skills, and implementation and assessment advice for teachers.

Key features of Student obook pro

- > Student obook pro is a completely digital product delivered via Oxford's online learning platform, **Oxford Digital**.
- > It offers a complete digital version of the Student Book with interactive note-taking, highlighting and bookmarking functionality, allowing students to revisit points of learning.
- > A complete ePDF of the Student Book is also available for download for offline use and read-aloud functionality.

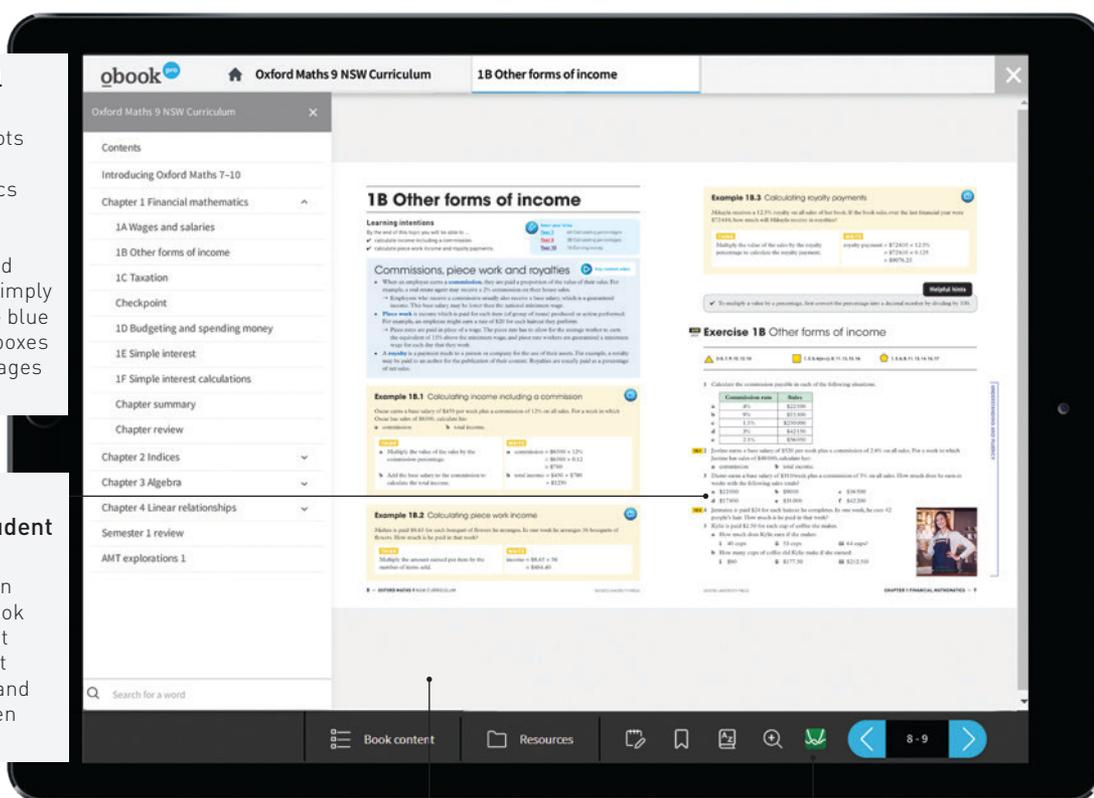
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Integrated digital resources

- Integrated hotspots allow students to access diagnostics tests, quizzes, interactive skill sheets, videos and inter-year links simply by clicking on the blue digital resource boxes throughout the pages of the book.

Complete digital version of the Student Book

- The digital version of the Student Book is true to the print version, making it easy to navigate and transition between print and digital.



Toolbar features

- Notes can be added and saved to the text by simply selecting and highlighting.
- Bookmarks can be saved to any page.
- *Australian Concise Oxford Dictionary* can provide immediate definitions to any word within the text.

Desmos integration

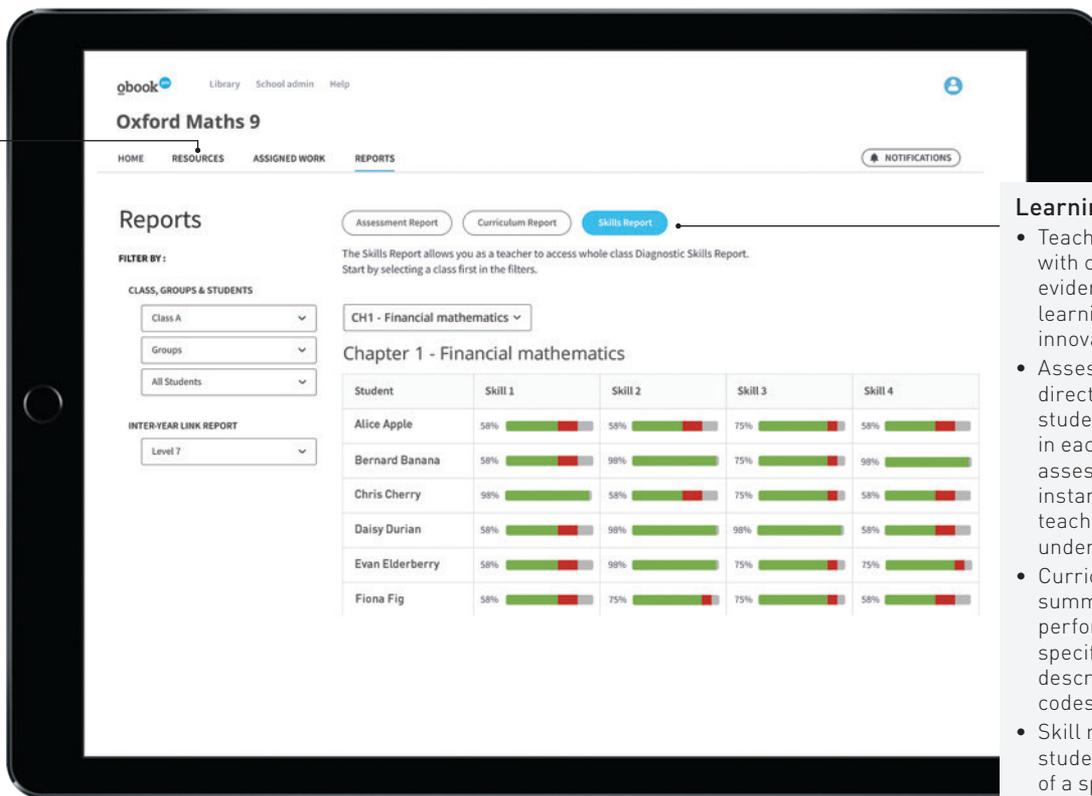
- Our partnership with Desmos allows students to access a suite of calculator tools as they read through the text, providing convenient graphical support as well as the opportunity to investigate plane geometry and Cartesian coordinates.

- > Integrated *Australian Concise Oxford Dictionary* look-up feature
- > Targeted instructional videos for every worked example question
- > Groundwork resources to support assumed knowledge
- > Interactive assessments to consolidate understanding
- > Auto-marked practice exam question sets
- > Integrated Quizlet sets, including real-time online quizzes with live leaderboards
- > Access to online assessment results to track progress

Benefits for students

Key features of Teacher obook pro

- > Teacher obook pro is a completely digital product delivered via Oxford's online learning platform, **Oxford Digital**.
- > Each chapter and topic of the Student Book is accompanied by full teaching support, including assessment reporting, worked solutions, chapter tests, detailed teacher notes and lesson plans.
- > Teachers can use their Teacher obook pro to share notes and easily assign resources or assessments to students, including due dates and email notifications.



Learning pathway reports

- Teachers are provided with clear and tangible evidence of student learning progress through innovative reports.
- Assessment reports directly show how students are performing in each online interactive assessment, providing instant feedback for teachers about areas of understanding.
- Curriculum reports summarise student performance against specific curriculum content descriptors and curriculum codes.
- Skill reports indicate the students' understanding of a specific skill in mathematics.

Additional resources

- Each chapter of the Student Book is accompanied by additional interactive skillsheets, worksheets, investigations and topic quizzes to help students progress.

- > Diagnostic pre-tests and chapter tests that track students' progress against Study Design key knowledge, providing detailed learning pathway reports that differentiate each student's ability in each skill
- > Assign reading and assessments to students either individually, or in groups – administration is taken care of!
- > Ability to set-up classes, monitor student progress and graph results
- > Worked solutions for every Student Book question
- > Detailed teacher notes, teaching programs and lesson plans

Benefits for teachers

1

Financial

mathematics



Index

- 1A Wages and salaries
- 1B Other forms of income
- 1C Taxation
- 1D Budgeting and spending money
- 1E Simple interest
- 1F Simple interest calculations

Prerequisite skills



Diagnostic pre-test

Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.



Interactive skillsheets

After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✓ Equivalent fractions
- ✓ Multiplying and dividing decimals
- ✓ Rounding decimals
- ✓ Calculating percentages

Curriculum links

- Solves financial problems involving simple interest, earning money and spending money (MA5-FIN-C-01)
 - Solve problems involving earning money
 - Solve problems involving simple interest
 - Solve problems involving spending money

© NESAs

Materials

- ✓ Calculator

1A Wages and salaries

Learning intentions

By the end of this topic you will be able to ...

- ✓ understand the difference between a wage and a salary
- ✓ calculate income for working a given period of time
- ✓ apply penalty rates and leave loading.



Inter-year links

Year 8

3C Mark-ups and discounts

Year 10

1A Earning money

Wages, penalty rates and leave loading



Key content video

- **Income** is money received over a period of time in exchange for work or investment.
- A **wage** is a fixed rate of income an employee receives for working a specified time period, usually per hour. For example, an employee may receive a wage of \$25 per hour, or \$25/hour.
 - An hourly wage can be calculated by dividing the amount someone earns by the number of hours they worked.
 - When given a wage, you can work out how much an employee will earn for a given number of hours.
- **Penalty rates** are higher rates of pay that are given to employees for working particular hours or days, such as weekends, public holidays or overtime.
 - Time-and-a-half means an employee is paid 1.5 times their standard wage.
 - Double time means an employee is paid 2 times their standard wage.
- All full-time employees are entitled to a minimum of four weeks of paid annual leave per year.
Leave loading is an extra payment on top of annual leave pay and is usually calculated as 17.5% of normal pay for this period.

Salaries and bonuses

- A **salary** is a fixed income which is paid to an employee on a regular basis. Salaries are usually expressed as an annual figure, with regular payments being made each week, fortnight or month.
 - When calculating weekly or fortnightly pay, it is assumed there are 52 weeks and 26 fortnights in a year.
 - Monthly pay can be calculated by dividing an annual salary by 12.
 - Salaried employees do not typically receive penalty rates or leave loading.
- Some employees receive a **bonus** on top of their salary, as an incentive or to reward good performance. Bonuses are often given as a percentage of an employee's salary.
 - Bonuses may also be given to employees who receive a wage instead of a salary.

Example 1A.1 Calculating wages



In a week, Petra works for 26 hours at her local cafe and earns \$650 without penalty rates. What is her hourly wage?

THINK

Divide the amount earned by the number of hours to get the hourly wage.

WRITE

$$\frac{\$650}{26 \text{ hours}} = \$25/\text{hour}$$

Example 1A.2 Calculating income for a given period of time



Jordan works for 35 hours per week and is paid \$32.50 per hour. How much does he earn in a fortnight?

THINK

- 1 Multiply the hourly wage by the number of hours worked each week to calculate the weekly wage.
- 2 Multiply the weekly wage by 2 to calculate the fortnightly wage.

WRITE

$$\begin{aligned}\text{weekly wage} &= \$32.50 \times 35 \\ &= \$1137.50 \\ \text{fortnightly wage} &= \$1137.50 \times 2 \\ &= \$2275\end{aligned}$$

Example 1A.3 Calculating income with penalty rates



Miriam's standard wage is \$38.30 per hour and on weekends she is paid time-and-a-half.

- a What is Miriam's weekend rate of pay?
- b How much does Miriam earn in a week when she works 6 hours per weekday and 5 hours on Saturday?

THINK

- a Multiply the weekday rate by the penalty rate (1.5) to calculate the weekend rate.
- b
 - 1 Calculate the weekday income by multiplying the weekday rate of pay by the number of hours worked on weekdays.
 - 2 Calculate the weekend income by multiplying the weekend rate of pay by the number of hours worked on the weekend.
 - 3 Add the weekday income to the weekend income to get the total income.

WRITE

$$\begin{aligned}\text{a weekend rate} &= \$38.30 \times 1.5 \\ &= \$57.45 \\ \text{b weekday income} &= \$38.30 \times (6 \times 5) \\ &= \$38.30 \times 30 \\ &= \$1149 \\ \text{weekend income} &= \$57.45 \times 5 \\ &= \$287.25 \\ \text{total income} &= \$1149 + \$287.25 \\ &= \$1436.25\end{aligned}$$

Example 1A.4 Calculating leave loading



Will works 38 hours per week and is paid \$26 per hour. He takes one week of annual leave, for which he receives an additional leave loading of 17.5%. How much does Will get paid for his week of annual leave?

THINK

- 1 Calculate the weekly wage by multiplying the hourly wage by the number of hours worked in a week.
- 2 Calculate the leave loading by multiplying the weekly wage by 17.5%.
- 3 Add the leave loading to the weekly wage to calculate the annual leave weekly wage.

WRITE

$$\begin{aligned}\text{weekly wage} &= \$26 \times 38 \\ &= \$988 \\ \text{leave loading} &= \$988 \times 17.5\% \\ &= \$988 \times 0.175 \\ &= \$172.90 \\ \text{annual leave weekly wage} &= \$988 + \$172.90 \\ &= \$1160.90\end{aligned}$$

- ✓ When calculating a monthly salary, you do not need to consider how many days that month has. All months should be considered as $\frac{1}{12}$ of a year. To calculate a monthly salary, divide the annual salary by 12. It should not be calculated by multiplying the weekly salary by 4.

ANS
p424

Exercise 1A Wages and salaries

1-3, 6-8, 11, 12, 15, 16

2-4, 7-9, 12, 13, 16, 17

2, 5, 8-10, 12, 14, 17-19

UNDERSTANDING AND FLUENCY

- 1A.1** 1 Last week Iona worked for 32 hours and earned \$1318.40. What is her hourly wage?
 2 Calculate the hourly wages for the given amount of time worked and money earned. Assume that no penalty rates were applied.

	Time worked	Amount earned
a	20 hours	\$560
b	35 hours	\$1575
c	12 hours	\$280.80
d	18.5 hours	\$444
e	27.25 hours	\$915.60

- 1A.2** 3 Hiro works for 26 hours per week and is paid \$39.20 per hour. How much does he earn in a fortnight?

- 4 a Rafael works as a courier delivering parcels around the city. He earns \$28.50 per hour. Calculate Rafael's income for a week in which he works 20 hours.
 b In one particular week, Rafael's wage totalled \$684. How many hours did Rafael work in this week?
 5 Copy and complete the following table.

Hourly wage	Time worked	Amount earned
\$32.20	25 hours	
	30 hours	\$735
\$27		\$756
\$30.30	38 hours	
	32 hours	\$1638.40



- 6 For each of these hourly wages, calculate:
 i the time-and-a-half hourly wage
 ii the double time rate.
- a \$18 b \$24 c \$18.80
 d \$25.90 e \$32.60 f \$29.90
- 1A.3** 7 Jo's standard wage is \$25.80 and on weekends she is paid time-and-a-half.
 a What is Jo's weekend rate of pay?
 b How much does Jo earn in a week when she works 5 hours per weekday and 7 hours on Saturday?
- 1A.4** 8 Raj works 34 hours per week and is paid \$31 per hour. He takes one week of annual leave, for which he receives an additional leave loading of 17.5%. How much does Raj get paid for his week of annual leave?

- 9 Calculate the annual salary for the following given values.
- a** \$6450 per month **b** \$4480 per month **c** \$1240 per week
d \$950 per week **e** \$5100 per fortnight **f** \$3330 per fortnight
- 10 Lina works in research and earns a salary of \$66 612.
- a** If Lina is paid monthly, what is her salary each month?
b If Lina is paid fortnightly, what is her salary each fortnight?
- 11 Dina earns a wage of \$34.50 per hour and works for 28 hours each week.
- a** What is Dina's weekly income?
b Use your answer to part **a** to calculate Dina's yearly income.
c Use your answer to part **b** to calculate Dina's monthly income.
- 12 As a bricklayer, Luka receives a standard rate of \$27.40 per hour. He receives a penalty rate of one-and-a-quarter for any overtime (above 30 hours per week).
- a** How much does Luka earn in a week in which he works 35 hours?
b How many hours does Luka work in a week in which he earned \$890.50?
- 13 Rahim earns \$3200 per month and Sal earns \$780 per week. Rahim argues that he earns more money than Sal, because if you multiply Sal's wage by 4 the result is less than his wage, and there are four weeks in a month. Is Rahim correct? Justify your answer by showing appropriate calculations.
- 14 The following table shows the hours a group of workers worked for over the past week, as well as their hourly wage. All workers receive time-and-a-half for working on weekends. Calculate each worker's weekly income.



Worker	Hourly wage	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Nathan	\$31.50	9–5	9–5	–	9–3	9–1	9–3	–
Kristina	\$33.20	10–4	10–4	10–4	10–4	10–6	–	–
Jack	\$26.50	9–3	–	9–3	–	9–3	9–3	9–3
Mina	\$29.40	–	8–5	8–5	9–5	–	10–5	9–3

- 15 For two weeks of annual leave, Lola receives \$3172.50, including leave loading of 17.5%. Lola works a 36-hour week. What is Lola's hourly wage?
- 16 For the 2023–2024 financial year, the minimum wage in Australia is \$23.23.
- a** How much does someone receiving the minimum wage earn if they work a 38-hour week?
b A junior is an employee under 21 years old. Juniors usually get a percentage of the adult pay rate. If a junior is entitled to 80% of the adult pay rate, what is their minimum wage per hour to the nearest cent?
- 17 Marty works in a cafe on weekdays and at a bar on weekends. His cafe job pays \$24.80 an hour and his bar job pays \$37 an hour. One week Marty works for 22 hours at the cafe. How many hours did Marty need to work at the bar for his income for the week to total \$1000 or more?
- 18 Shivani works at an internet security company and receives a fortnightly salary of \$4246. At her end of year performance review, she's informed that she will receive a 6.5% bonus on her annual salary. How much is her bonus worth, correct to the nearest dollar?
- 19 Exactly halfway through the financial year, Henrik received a promotion at work which gave him a 15% increase in his salary. In this financial year Henrik also received a 3% bonus on his new salary. If Henrik's income from work for the year was \$75 446, what was his salary at the end of the year?

Check your Student obook pro for these digital resources and more:

pro



Interactive skillsheet
Wages and salaries



Topic quiz
1A

1B Other forms of income

Learning intentions

By the end of this topic you will be able to ...

- ✓ calculate income including a commission
- ✓ calculate piece work income and royalty payments.



Inter-year links

- [Year 7](#) 4H Calculating percentages
- [Year 8](#) 3B Calculating percentages
- [Year 10](#) 1A Earning money

Commissions, piece work and royalties



Key content video

- When an employee earns a **commission**, they are paid a proportion of the value of their sales. For example, a real estate agent may receive a 2% commission on their house sales.
 - Employees who receive a commission usually also receive a base salary, which is a guaranteed income. This base salary may be lower than the national minimum wage.
- **Piece work** is income which is paid for each item (of group of items) produced or action performed. For example, an employee might earn a rate of \$20 for each haircut they perform.
 - Piece rates are paid in place of a wage. The piece rate has to allow for the average worker to earn the equivalent of 15% above the minimum wage, and piece rate workers are guaranteed a minimum wage for each day that they work.
- A **royalty** is a payment made to a person or company for the use of their assets. For example, a royalty may be paid to an author for the publication of their content. Royalties are usually paid as a percentage of net sales.

Example 1B.1 Calculating income including a commission



Oscar earns a base salary of \$450 per week plus a commission of 12% on all sales. For a week in which Oscar has sales of \$6500, calculate his:

- a** commission **b** total income.

THINK

- a** Multiply the value of the sales by the commission percentage.
- b** Add the base salary to the commission to calculate the total income.

WRITE

- a** commission = $\$6500 \times 12\%$
= $\$6500 \times 0.12$
= \$780
- b** total income = $\$450 + \780
= \$1230

Example 1B.2 Calculating piece work income



Mishra is paid \$8.65 for each bouquet of flowers he arranges. In one week he arranges 56 bouquets of flowers. How much is he paid in that week?

THINK

Multiply the amount earned per item by the number of items sold.

WRITE

$$\begin{aligned} \text{income} &= \$8.65 \times 56 \\ &= \$484.40 \end{aligned}$$



Example 1B.3 Calculating royalty payments

Mikayla receives a 12.5% royalty on all sales of her book. If the book sales over the last financial year were \$72 610, how much will Mikayla receive in royalties?

THINK

Multiply the value of the sales by the royalty percentage to calculate the royalty payment.

WRITE

$$\begin{aligned}
 \text{royalty payment} &= \$72\,610 \times 12.5\% \\
 &= \$72\,610 \times 0.125 \\
 &= \$9076.25
 \end{aligned}$$

Helpful hints

- ✓ To multiply a value by a percentage, first convert the percentage into a decimal number by dividing by 100.

ANS p424 Exercise 1B Other forms of income

▲ 2-5, 7, 9, 10, 12, 14

■ 1, 3, 5, 6(a-c), 8, 11, 13, 15, 16

◆ 1, 3, 6, 8, 11, 13, 14, 16, 17

- 1 Calculate the commission payable in each of the following situations.

	Commission rate	Sales
a	4%	\$22 500
b	9%	\$15 300
c	1.5%	\$250 000
d	3%	\$42 150
e	2.1%	\$56 050

- 1B.1 2 Justine earns a base salary of \$520 per week plus a commission of 2.6% on all sales. For a week in which Justine has sales of \$48 000, calculate her:

a commission b total income.

- 3 Damo earns a base salary of \$310/week plus a commission of 5% on all sales. How much does he earn in weeks with the following sales totals?

a \$22 000 b \$9000 c \$36 500
 d \$17 800 e \$31 000 f \$42 200

- 1B.2 4 Jermaine is paid \$24 for each haircut he completes. In one week, he cuts 42 people's hair. How much is he paid in that week?

- 5 Kylie is paid \$2.50 for each cup of coffee she makes.

a How much does Kylie earn if she makes:

i 40 cups ii 53 cups iii 64 cups?

b How many cups of coffee did Kylie make if she earned:

i \$90 ii \$177.50 iii \$212.50?



- 1B.3 6** Calculate the amount authors will receive in royalties given the following rates and sales figures.

	Royalty rate	Sales
a	11%	\$135 000
b	1.2%	\$240 000
c	4.5%	\$65 000
d	15%	\$72 000
e	6%	\$110 500

- 7 Last year, Juan received a royalty payment of \$557.20 on a 3.5% royalty. What were the sales for that year?
- 8 Hanna receives a royalty payment of \$1108.80 on sales of \$46 200. What is her royalty rate?
- 9 Denis received a base wage of \$400/week and a commission of 3.2% on sales. Denis' income for a week is \$1459.20. How much product did Denis sell that week?
- 10 Luna receives \$4.50 for every candle and \$3.50 for every bath bomb she sells. In one afternoon, Luna earned \$161.50. If Luna sold 18 candles, how many bath bombs did she sell?
- 11 Three friends, Abi, Beth and Craig, all start new jobs at the same time. Abi earns a wage of \$24.50 plus penalty rates, Beth earns a base salary of \$200 plus a commission of 4% on all sales, and Craig is paid \$25 for each item he produces. In one week:
- Abi works 20 hours at her standard rate, plus 6 hours at time-and-a-half
 - Beth sells \$18 000 worth of product
 - Craig produces 58 items.

Put the amount the three friends earn in this week in ascending order.

- 12 Jerome receives \$32 for each phone screen he replaces. If Jerome replaces 25 phone screens in 8 hours, what is his equivalent hourly rate of pay?
- 13 Greta earns \$2.80 for each small cup of coffee she makes and \$3.50 for each large cup of coffee she makes. If Greta makes 35 small cups of coffee and 14 large cups of coffee in 3 hours, what is her equivalent hourly rate of pay?
- 14 Owen receives \$3.50 for the first 20 trees he plants and \$4.40 for each additional tree planted.
- a If Owen plants 34 trees, how much does he earn?
- b If Owen earns \$188.80, how many trees does he plant?
- 15 Gabby earns a base salary of \$250/week. If, in one fortnight, Gabby earns \$1472 on sales of \$27 000, what is Gabby's rate of commission?
- 16 Patrick earns \$3280 on \$80 000 worth of sales and Erica earns \$3010 on \$70 000 worth of sales.
- a Who receives the higher rate of commission?
- b If their rates of commission were swapped, how much would they each earn?
- 17 When starting her new job, Nina is given the following options for her pay:
- Option A: base salary \$600, 2.4% commission on sales
 - Option B: base salary \$400, 2.8% commission on sales
 - Option C: base salary \$250, 3.0% commission on sales
- a Which option would give the highest income for Nina if she expects to sell \$55 000 of product per week?
- b For what amount of expected sales should Nina choose each option?



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Interactive skillsheet

Commission, piece work and royalties



Topic quiz

1B

1C Taxation

Learning intentions

By the end of this topic you will be able to ...

- ✓ calculate an individual's taxable income
- ✓ calculate income tax.



Inter-year links

[Year 7](#)

4D Multiplying decimals

[Year 8](#)

2F Multiplying and dividing decimals

Income tax and deductions



Key content video

- A person's total income includes wages, salaries, bonuses, royalties and interest earned.
- **Tax deductions** are expenses that can be subtracted from an individual's total income to determine their **taxable income**. Tax deductions include work-related expenses, gifts and donations, the cost of managing tax affairs, and self-education expenses.
taxable income = total income – tax deductions
- **Income tax** is tax which is paid on an individual's taxable income.
 - Income tax is a progressive tax, which means that the tax rate increases as the taxable income increases. This is based on the concept of the ability to pay, meaning that those who can afford to pay more, do pay more.
 - Personal income tax is the largest source of government revenue in Australia, allowing the government to fund community services such as education and emergency services.
- The following table details the income tax rates for the 2023–2024 financial year:

Taxable income	Tax on this income
0–\$18 200	Nil
\$18 201–\$45 000	19 cents for each \$1 over \$18 200
\$45 001–\$120 000	\$5092 plus 32.5 cents for each \$1 over \$45 000
\$120 001–\$180 000	\$29 467 plus 37 cents for each \$1 over \$120 000
\$180 001 and over	\$51 667 plus 45 cents for each \$1 over \$180 000

- For example, if someone has an income of \$70 000 for the 2023–2024 financial year, their income tax payable would be:

$$\begin{aligned} \$5092 + 0.325 \times (\$70\,000 - \$45\,000) &= \$5092 + 0.325 \times \$25\,000 \\ &= \$5092 + \$8125 \\ &= \$13\,217 \end{aligned}$$

Pay-As-You-Go (PAYG) tax

- **Pay-As-You-Go (PAYG)** is a system whereby the approximate income tax owed by an employee is withheld from their pay, meaning they are paying their income tax in instalments instead of having to pay a lump sum at the end of the year.
 - The PAYG taxation system helps to ensure people pay their income tax.
 - Any discrepancies between the amount of income tax paid under the PAYG system and the amount of income tax owed in a financial year are resolved by completing a tax return. After reviewing the tax return, the Australian Taxation Office (ATO) will either provide a refund or let you know how much is still owed for each financial year.

Tax tables

- The ATO produces weekly, fortnightly and monthly **tax tables**, which detail the amount of tax that should be withheld given an exact amount of earnings for that specified period.
 - These tax tables can be downloaded from the ATO website. To determine the amount of tax to be withheld, look up the earnings in the first column and read the amount to be withheld from the second or third columns.
 - The first \$18 200 an Australian earns in a financial year is tax-free. This is known as the tax-free threshold.

Weekly earnings	Amount to be withheld	
	With tax-free threshold	No tax-free threshold
	1 2	3
\$	\$	\$
1,026.00	171.00	292.00
1,027.00	171.00	293.00
1,028.00	172.00	293.00
1,029.00	172.00	293.00
1,030.00	172.00	294.00

Medicare levy and net earnings

- The **Medicare levy** is a 2% tax on taxable income which helps to fund the health care system.
 - Some individuals may be either exempt from the Medicare levy or eligible to pay a reduced levy.
- A person's net income can be calculated by subtracting their income tax and Medicare levy from their taxable income.

$$\text{net income} = \text{taxable income} - \text{income tax} - \text{Medicare levy}$$

Example 1C.1 Calculating taxable income



In the last financial year, Reina:

- earned a salary of \$77 800
- received a \$2500 bonus from work
- earned \$232 in interest on her investments
- had \$950 of work-related expenses
- donated \$300 to charity
- paid an accountant \$120 to manage her tax affairs.

Calculate Reina's taxable income.

THINK

- Calculate the total income by adding the salary, bonus and interest together.
- Calculate the total deductions by adding the work-related expenses, charity donations and cost of managing tax affairs together.
- Subtract the tax deductions from the total income to determine the taxable income.

WRITE

$$\begin{aligned} \text{total income} &= \$77\,800 + \$2\,500 + \$232 \\ &= \$80\,532 \\ \text{total deductions} &= \$950 + \$300 + \$120 \\ &= \$1\,370 \\ \text{taxable income} &= \$80\,532 - \$1\,370 \\ &= \$79\,162 \end{aligned}$$

Example 1C.2 Income tax



Use the 2023–2024 financial year tax table to calculate the income tax payable on the following incomes.

a \$65 230

b \$144 900

THINK

- a** **1** Identify the row in the table into which the income falls. \$65 230 lies between \$45 001 and \$120 000.
- 2** Follow the instructions in the second column of the table. To calculate ‘\$5092 plus 32.5 cents for each \$1 over \$45 000’, start by calculating the income over \$45 000. Then, multiply this figure by 0.325. Finally, add the result to \$5092.
- b** **1** Identify the row in the table into which the income falls. \$144 900 lies between \$120 001 and \$180 000.
- 2** Follow the instructions in the second column of the table. To calculate ‘\$29 467 plus 37 cents for each \$1 over \$120 000’, start by calculating the income over \$120 000. Then, multiply this figure by 0.37. Finally, add the result to \$29 467.

WRITE

- a** $\$65\,230 - \$45\,000 = \$20\,230$
income tax = $\$5092 + \$20\,230 \times 0.325$
= $\$5092 + \6574.75
= $\$11\,666.75$
- b** $\$144\,900 - \$120\,000 = \$24\,900$
income tax = $\$29\,467 + \$24\,900 \times 0.37$
= $\$29\,467 + \9213
= $\$38\,680$

Helpful hints

✓ ‘37 cents for each \$1 over \$120 000’ means $\frac{37}{100}$ or 0.37 of the income over \$120 000.

ANS p424 Exercise 1C Taxation

 3, 4(a-c), 5, 6(a-c), 7, 9, 10(a-c), 11, 15

 1, 2, 4(d-f), 6(d-f), 8, 10(d-f), 11, 13(a-c), 16

 1, 4(d-f), 7, 9, 11, 12, 13(d-f), 14, 16, 17

- 1C.1** **1** In the last financial year, Lisa:
- earned a salary of \$57 500
 - received a \$1500 bonus from work
 - earned \$88 in interest on her bank account
 - had \$550 of work-related expenses
 - donated \$100 to charity
 - paid an accountant \$80 to manage her tax affairs.

Calculate Lisa’s taxable income.

- 2 In the last financial year, Kosta's income from his two jobs was \$55 000 and \$34 800, respectively. Kosta also earned \$395 in interest on his savings account and made a monthly donation of \$30 to his favourite charity. Calculate Kosta's taxable income.
- 3 In each of the following situations, calculate the taxable income.
- income \$4981.95; work-related expenses \$56.80; donations \$9.45
 - 36 hours of work at \$25.70 per hour; additional income \$187.50; work-related expenses \$38.90
 - annual salary \$91 200; work-related expenses \$61.25 per month; donations \$27 per month
- 1c.2** 4 Use the 2023–2024 financial year tax table to calculate the income tax payable on the following incomes.
- | | |
|-------------|-------------|
| a \$59 000 | b \$111 000 |
| c \$80 300 | d \$16 000 |
| e \$231 500 | f \$99 900 |
- 5 Last year, Julie had a salary of \$72 000 and received a 5% bonus at work. She earned \$1045 in interest on her investments and had tax deductions that totalled \$2420.
- Calculate Julie's taxable income.
 - How much income tax does Julie need to pay?
- 6 Calculate the Medicare levy payable on the following incomes.
- | | |
|-------------|------------|
| a \$82 000 | b \$46 000 |
| c \$104 000 | d \$66 500 |
| e \$133 000 | f \$59 200 |
- 7 In the 2023–2024 financial year, Mika earned \$122 400 and had tax deductions of \$4200.
- What is Mika's taxable income?
 - How much income tax must Mika pay?
 - How much does Mika have to pay for the Medicare levy?
 - What is Mika's net income?
- 8 Cody has a taxable income of \$91 500 after deductions. What is Cody's net income after deducting income tax and the Medicare levy?
- 9 Last year, Portia had a salary of \$86 520 and each month her employee withheld \$1699 in tax for her. Portia earned an additional \$872 in interest on her investments, had work-related expenses of \$610 and donated \$480 to charity.
- Calculate Portia's income tax for the year.
 - Is Portia due a tax refund, or does she owe more money? By how much?
- 10 The tax brackets are named after the amount of cents paid for each \$ over the given amount in that bracket. Explain why someone whose income falls into the 32.5% tax bracket does not pay 32.5% of their income in income tax.
- 11 Calculate the percentage, correct to one decimal place, of these taxable incomes that needs to be paid in income tax.
- | | |
|------------|-------------|
| a \$94 000 | b \$48 500 |
| c \$70 500 | d \$114 000 |
| e \$37 040 | f \$142 100 |
- 12 Explain how the fixed values in the 2023–2024 tax table (\$5092, \$29 467 and \$51 667) are calculated.



- 13 Use the ATO website to look up the weekly, fortnightly and monthly tax tables and to state the amount of income tax that should withheld for the following earnings. State the amount with the tax-free threshold.
- a** \$774 per week **b** \$1950 per week **c** \$3122 per fortnight
d \$2150 per fortnight **e** \$4797 per month **f** \$7644 per month
- 14 This is a section from the ATO fortnightly tax table for the 2023–2024 financial year:

Fortnightly earnings	Amount to be withheld	
	With tax-free threshold	No tax-free threshold
	1 \$	2 \$
2,622.00	540.00	782.00
2,624.00	540.00	782.00
2,626.00	542.00	782.00
2,628.00	542.00	784.00
2,630.00	542.00	784.00

Fortnightly earnings	Amount to be withheld	
	With tax-free threshold	No tax-free threshold
	1 \$	2 \$
2,792.00	598.00	840.00
2,794.00	600.00	840.00
2,796.00	600.00	842.00
2,798.00	600.00	842.00
2,800.00	602.00	842.00

- a** In 2023–2024, Ben earned \$38.80 per hour and worked a 36-hour week. What was his fortnightly income?
b How much did Ben’s employer withhold in tax for him each fortnight?
- 15 Copy and complete the information in this table.

Income	Deductions	Taxable income	Income tax	Medicare levy	Net income
\$83 100	\$300	\$82 800	\$17 377	\$1656	
\$55 400	\$550	\$54 850		\$1097	
\$102 310	\$1100				
\$68 440	\$810				

- 16 The tax brackets are scheduled to be changed to the following structure for the 2024–2025 financial year:

Taxable income	Tax on this income
0–\$18 200	Nil
\$18 201–\$45 000	19 cents for each \$1 over \$18 200
\$45 001–\$200 000	\$5092 plus 30 cents for each \$1 over \$45 000
\$200 001 and over	\$51 592 plus 45 cents for each \$1 over \$200 000

Compared to the 2023–2024 tax table, how much would people with the following taxable incomes save?

- a** \$55 000 **b** \$97 000 **c** \$165 000
- 17 An alternative tax system is proposed, where all individuals pay 25% of their taxable income in income tax.
- a** If someone pays exactly the same income tax under the 2023–2024 system and this alternative system, what is their taxable income?
b Who would be worse off and who would be better off under this proposed tax system?
c Calculate the difference in income tax payable between the 2023–2024 and this alternative system for people with the following taxable incomes.
i \$38 600 **ii** \$82 100 **iii** \$163 000
d Use your answers to parts **a–c** to comment on the merits of using a progressive income tax system.

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Interactive skillsheet
Income tax



Topic quiz
1C

Checkpoint



Checkpoint quiz

Check your knowledge of the first part of this chapter.

- 1A 1** Copy and complete the following table.

Hourly wage	Time worked	Amount earned
\$32.20	12 hours	
\$23.90		\$478.00
\$40.05	15 hours	
\$36.25		\$1051.25
	33 hours	\$1402.50

- 1A 2** For each of these hourly wages, calculate:

i the time-and-a-half hourly wage

ii the double time rate.

- a** \$23 **b** \$29 **c** \$38 **d** \$26.50 **e** \$34.20 **f** \$44.40

- 1A 3** Calculate the annual salary for the following given values.

- a** \$5740 per month **b** \$4510 per month **c** \$1205 per week
d \$2070 per week **e** \$1840 per fortnight **f** \$3430 per fortnight

- 1A 4** Grant works 30 hours per week and is paid \$26.60 per hour. He takes one week of annual leave, for which he receives an additional leave loading of 17.5%. How much does Grant get paid for his week of annual leave?

- 1B 5** Calculate the piece work rate in each of these situations.

	Items produced	Income
a	59	\$442.50
b	112	\$403.20
c	46	\$791.20
d	2450	\$294.00
e	440	\$171.60

- 1B 6** Last year, Angel received a royalty payment of \$3380 on a 5.2% royalty. What were the sales for that year?

- 1C 7** In the last financial year, Isaac earned a salary of \$74 000 plus a 3.5% bonus. Isaac also earned \$247 in interest on his bank account. He had \$350 of work-related expenses, donated \$240 to charity and paid an accountant \$100 to manage his tax affairs. Calculate Isaac's taxable income.

- 1C 8** This is the income tax table for the 2022–2023 financial year.

Taxable income	Tax on this income
0–\$18 200	Nil
\$18 201–\$45 000	19 cents for each \$1 over \$18 200
\$45 001–\$120 000	\$5092 plus 32.5 cents for each \$1 over \$45 000
\$120 001–\$180 000	\$29 467 plus 37 cents for each \$1 over \$120 000
\$180 001 and over	\$51 667 plus 45 cents for each \$1 over \$180 000

Use the tax table to calculate the income tax payable on the following incomes.

- a** \$75 300 **b** \$94 200 **c** \$47 000 **d** \$53 990 **e** \$115 040 **f** \$66 095

- 1C 9** Calculate the Medicare levy payable on the following incomes.

- a** \$50 500 **b** \$63 420 **c** \$76 100 **d** \$82 090 **e** \$104 820 **f** \$137 950

- 1C 10** In the 2023–2024 financial year, Dina earned \$81 300 and had tax deductions of \$1800.

- a** What is Dina's taxable income? **b** How much income tax must Dina pay?
c How much does Dina have to pay for the Medicare levy? **d** What is Dina's net income?

1D Budgeting and spending money

Learning intentions

By the end of this topic you will be able to ...

- ✓ evaluate payment options
- ✓ create a budget.



Inter-year links

[Year 7](#)

1D Multiplying whole numbers

[Year 8](#)

1C Multiplying and dividing whole numbers

Payment options



Key content video

- When purchasing a product or service there may be a variety of different payment options. By understanding all the different available options, you can make the best decision in each situation.
- To **buy an item on terms** means to make an initial deposit on the item, and then to pay the remaining amount in regular instalments.
 - Buying an item on terms is usually more expensive than buying an item outright.
- **Buy now, pay later** services allow a consumer to buy a product or service and delay the payment. These services are usually interest-free; however, other fees may be charged, including late fees, account-keeping fees, payment processing fees and establishment fees.
- A **lay-by agreement** is a contract where a consumer pays for goods in two or more instalments and does not receive the goods until the full price has been paid.
 - A consumer can cancel a lay-by agreement at any time before receiving the product; however, they may have to pay a termination fee.

Budget

- A **budget** is a financial plan. Budgets can be used to make sure you have enough money for future spending and to help you avoid going into debt.
- To prepare a budget you need to estimate your expected income and expenses. Expenses can be split up into essential items, such as food and housing, and discretionary items, such as entertainment and gifts.
 - To avoid going into debt, the total expenses in a budget should be less than the total income.

Example 1D.1 Evaluating payment options



Lea is shopping for a new television set and wants to purchase a set which is listed at \$799. The sales assistant tells Lea that instead of paying the full amount up front, she can pay a deposit of \$200 today, followed by 12 weekly instalments of \$56.

- If Lea chooses to buy this item on the offered terms, how much will she end up spending?
- What is the percentage increase in the cost of the item if it is purchased on terms as opposed to being paid for up front? Give your answer correct to two decimal places.

THINK

- Multiply the weekly instalment cost by the number of weeks and add the deposit to determine the total cost.

WRITE

- $$\begin{aligned} \text{total cost} &= \$56 \times 12 + \$200 \\ &= \$672 + \$200 \\ &= \$872 \end{aligned}$$

- b** 1 Calculate the increase in cost if purchasing the items on terms by subtracting the listed price from the cost for purchasing on terms.
- 2 Divide the increase in cost by the list price and multiply by 100% to determine the percentage increase in cost.
- 3 Round the answer to two decimal places.

$$\begin{aligned} \text{b increase in cost} &= \$872 - \$799 \\ &= \$73 \end{aligned}$$

$$\begin{aligned} \text{percentage increase in cost} &= \frac{\$73}{\$799} \times 100\% \\ &= 9.136\dots\% \\ &\approx 9.14\% \end{aligned}$$

Example 1D.2 Creating an annual budget



Shai has just started a new job and wants to create a budget to keep track of his finances. He will earn a net income of \$1200 per week, and his estimated weekly expenses are \$300 on rent, \$165 on food, \$100 on entertainment and \$50 on personal items. Each month he also spends \$60 on his mobile phone and \$120 on clothes.

- a** Prepare an annual budget for Shai.
- b** Shai aims to save \$20 000 each year to put towards a house deposit. Does his budget reach that target?

THINK

- a** 1 Calculate the annual income by multiplying the weekly net income by 52.
- 2 Calculate the annual expense for each item by multiplying the weekly expenses by 52 and multiplying the monthly expenses by 12.
- 3 Calculate the sum of the annual expenses.
- 4 Subtract the total expenses from the income to calculate the annual savings.

- b** Compare the annual savings in Shai's budget to his target and comment.

WRITE

a Annual income

$$\$1200 \times 52 = \$62\,400$$

Annual expenses

$$\text{Rent: } \$300 \times 52 = \$15\,600$$

$$\text{Food: } \$165 \times 52 = \$8\,580$$

$$\text{Entertainment: } \$100 \times 52 = \$5\,200$$

$$\text{Personal items: } \$50 \times 52 = \$2\,600$$

$$\text{Mobile phone: } \$60 \times 12 = \$720$$

$$\text{Clothes: } \$120 \times 12 = \$1\,440$$

$$\begin{aligned} \text{Total: } & \$15\,600 + \$8\,580 + \$5\,200 + \$2\,600 + \\ & \$720 + \$1\,440 = \$34\,140 \end{aligned}$$

Annual savings

$$\$62\,400 - \$34\,140 = \$28\,260$$

- b** Yes, Shai's budget reaches his target of saving \$20 000 this year.

Helpful hints

- ✓ A budget is only an estimate of future income and expenses, which is unlikely to be 100% accurate.
- ✓ Make sure to include all your potential expenses when creating a budget. A good way to make sure you include everything is to look at your expenses over the past year.
- ✓ If you need to save more money, it is easier to cut discretionary spending than it is to cut essential spending.

Exercise 1D Budgeting and spending money

▲ 1, 2, 4, 5, 8, 11, 13

■ 2, 3, 6, 7, 10, 11, 13

◆ 3, 5, 6, 9, 10, 12, 14

- 1 What is the total payable for each of the following purchasing options?
- a** \$500 deposit plus 20 payments of \$49 **b** \$150 deposit plus 52 payments of \$29
- c** \$5000 deposit plus 24 payments of \$499 **d** \$399 deposit plus 10 payments of \$399

- 1D.1** 2 Damon is shopping for a new sound system and finds his perfect system for a listed price of \$1599. The sales assistant tells Damon that instead of paying the full amount up front, he can pay a deposit of \$250 today, followed by 12 monthly instalments of \$120.



- a** If Damon chooses to buy this item on the offered terms, how much will he end up spending?
- b** What is the percentage increase in the cost of the item if it is purchased on terms as opposed to being paid for up front? Give your answer correct to two decimal places.
- 3 Cat purchases a new handbag using a buy now, pay later service. This allows her to pay for the \$399 handbag with 10 payments of \$39.90. Cat has to pay \$5.95 to set up the payment system, and each payment has a \$1.50 processing fee.
- a** How much does Cat end up paying in total?
- b** What is the percentage increase in cost with the buy now, pay later service? Give your answer correct to two decimal places.
- 4 Sheldon comes up with the following list of expected expenses for the year. What are his expected total annual expenses?

- Rent: \$350/week
- Food: \$150/week
- Entertainment: \$200/month
- Gifts: \$300/year
- Clothes: \$600/year
- Mobile phone: \$45/month
- Transport: \$240/month
- Other items: \$100/month



- 1D.2** 5 Andrea has just started a new job and wants to create a budget to keep track of her finances. She will earn a net income of \$2600 per fortnight, and her estimated weekly expenses are \$420 on rent, \$140 on food, \$80 on entertainment and \$100 on personal items. Each month she also spends \$75 on her mobile phone, \$150 on clothes and \$100 on transport.
- a** Prepare an annual budget for Andrea.
- b** Andrea wants to purchase a car that will cost \$25 000 at the end of the year. Does her budget allow enough savings for the car?
- 6 Gil is creating an annual budget. His net income is \$61 500 per year, and his rent is \$360 per week. He estimates that he spends \$175 on food and \$50 on public transport each week.
- a** Prepare an annual budget for Gil.
- b** What expenses may Gil not have considered when planning for his budget?

- 7 If someone is conservative when estimating their expenses, they will tend to overestimate them. Explain why being conservative with your expense estimates may be a good idea.
- 8 Explain why weekly expenses can't be multiplied by 4 to get monthly expenses.
- 9 Some companies offer an interest-free period on purchases without any fees or penalties. Give some reasons why companies may make this offer with no obvious benefits for them.
- 10 When purchasing an item using a lay-by agreement, the customer usually pays a deposit, followed by equal weekly payments which cover the full purchasing price. There is often an additional setup fee for a lay-by agreement.
 - a What do you think are the potential benefits of a lay-by agreement? Consider this type of agreement compared to other purchasing options.
 - b What are the potential downsides of a lay-by agreement?
- 11 A mobile phone can be purchased outright for \$1250 or purchased with a 24-month contract for \$55 per month.
 - a How much extra will someone be paying if they decide to purchase the phone on a contract?
 - b Give reasons why some people may prefer to purchase the phone outright, and why other people may prefer to go with the contract.
- 12 Buy now, pay later services make money by charging fees. These fees may include account-keeping fees, establishment fees and late-payment fees.
 - a Comment on how buy now, pay later services may affect purchasing decisions.
 - b Considering your answer to part a, why do some buy now, pay later services decide to only charge late-payment fees?
- 13 Amber wants to start saving money, so she decides to create an annual budget. Her weekly net income is \$935, and she estimates her expenses as:

• Rent: \$325/week	• Food: \$130/week
• Entertainment: \$250/month	• Clothes: \$350/month
• Mobile phone: \$60/month	• Car: \$400/month
• Health insurance: \$110/month	• Other items: \$100/month

 - a How much will Amber be able to save each month?
 - b Make some suggestions as to where Amber could reduce her expenses to increase her savings.
- 14 Sam has prepared a mystery budget for his friends to decipher. He provides the following details:
 - His rent is exactly $\frac{1}{3}$ of his net income.
 - His weekly food expenditure is the same as his weekly transport expenditure.
 - The sum of his monthly mobile phone and entertainment expenses is equal to the amount he spends on transport each week.
 - His other weekly expenditures are half his weekly rent.
 - The amount he spends on food each week is \$200 less than his rent.
 - a If Sam's annual net income is \$54 600, how much will he be able to save each month?
 - b If Sam spends \$210 each week on transport, how much will he be able to save each month?



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Interactive skillsheet
Payment options



Investigation
Short-term loans



Topic quiz
1D

1E Simple interest

Learning intentions

By the end of this topic you will be able to ...

- ✓ calculate interest using the simple interest formula
- ✓ calculate the simple interest on an investment
- ✓ calculate the simple interest on a loan.



Inter-year links

- Year 7** 6C Terms, expressions and equations
- Year 8** 6A Equations
- Year 10** 1B Simple interest

Loans and investments



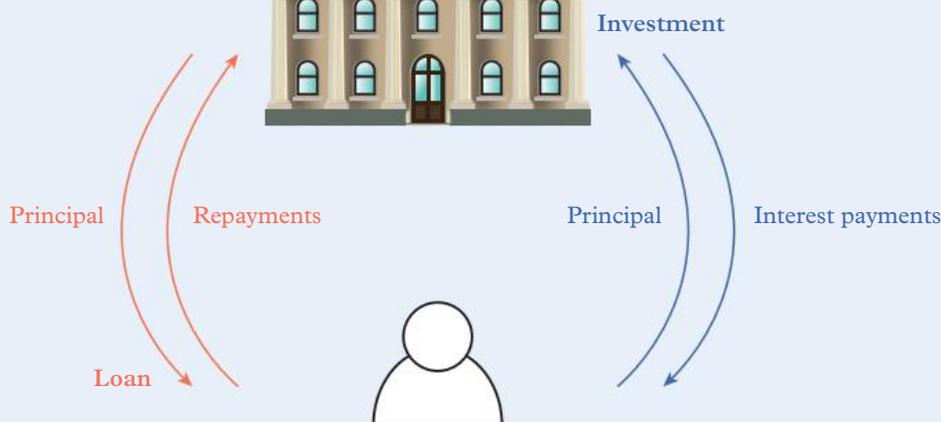
Key content video

- A **loan** is when money is borrowed and **interest** is paid on the amount of money borrowed. If you take out a loan from a bank, the total of your repayments will be more than the amount borrowed. Banks charge interest for providing access to their money.
- An **investment** is when money is deposited and interest is earned on the deposited money. If you invest money with a bank, the bank pays you interest for giving them access to your money.

- 1 Borrow money from bank
- 2 Bank charges interest
- 3 Must pay back loan and interest



- 1 Deposit money in bank
- 2 Bank pays interest
- 3 Money must stay in bank to gain interest



Simple interest

- The amount of interest you pay on a loan (or earn on an investment) depends on the original amount you borrow (or invest), the interest rate charged per time period and the number of time periods.

- **Simple interest** can be calculated using the formula:

→ I = interest

→ P = **principal**, the original amount of money borrowed or invested

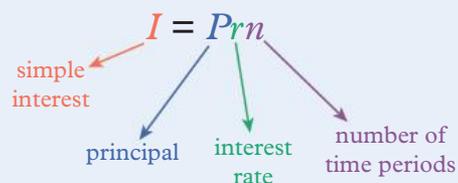
→ r = **interest rate** per time period, usually a percentage converted to a fraction or decimal

For example, 5% would be substituted as $\frac{5}{100}$ or 0.05. The interest rate 5% p.a. means 5% per annum or 5% per year.

→ n = number of time periods

- The total you repay (or receive) includes the original amount plus the interest.

$$\text{Total amount} = P + I$$





Example 1E.1 Calculating interest rates

Write an 8% interest rate as:

a a fraction in simplest form

b a decimal.

THINK

- a** To convert a percentage to a fraction, the percentage becomes the numerator and the denominator is 100. Simplify the fraction if possible.
- b** To convert a percentage to a decimal, divide the percentage by 100.

WRITE

- a** $8\% = \frac{8}{100}$
 $= \frac{2}{25}$
- b** $8\% = 8 \div 100$
 $= 0.08$



Example 1E.2 Calculating simple interest

For an investment of \$5200 at a simple interest rate of 6% p.a. for 4 years, calculate:

a the amount of simple interest earned

b the value of the investment after 4 years.

THINK

- a**
- Write the simple interest formula and identify the key terms: principal, interest rate, and number of time periods. The rate must be written as a fraction or a decimal.
 - Substitute the values into the formula and calculate the result.
 - Write the answer.
- b**
- The value of the investment after 4 years is calculated by adding the interest to the principal.
 - Write your final answer.

WRITE

- a** $I = Prn$
 $P = \$5200$
 $r = 6\% = 0.06$ per year
 $n = 4$ years
- $$I = \$5200 \times 0.06 \times 4$$
- $$= \$1248$$
- The simple interest earned in 4 years is \$1248.
- b** Total amount = $P + I$
 $= \$5200 + \1248
 $= \$6448$
- The value of the investment after 4 years is \$6448.

Helpful hints

- ✓ Be careful when converting your interest rate to a decimal or fraction.
- ✓ When using cash, round your answers to the nearest 5 cents. For all other transactions, round to the nearest 1 cent.
- ✓ If you are finding the simple interest formula difficult, write it out in words to help you.
 Simple interest = principal \times interest rate \times number of time periods
- ✓ Remember that the principal is the initial amount invested or borrowed.
- ✓ Make sure that the interest rate (r) and number of time periods (n) are in the same unit. If the interest rate is per annum, then the number of time periods must also be in years. If the interest rate is per month, then the number of time periods must be in months.

Exercise 1E Simple interest

▲ 1–5, 7, 8(2nd column), 9, 10, 11, 13, 15

■ 1–3, 4(b, d, f), 6, 7, 8(c, f, g, h),
10(a, c, e, g, i), 12, 13, 16, 18, 19(a)

◆ 2, 3, 7, 8(c, e, g, i), 10(f–i), 13, 14, 16–20

You can use your calculator for all questions in this exercise.

1E.1 1 Write these interest rates as:

i a fraction in simplest form

ii a decimal.

a 7%

b 11%

c 24%

d 6%

e 10%

f 12%

1E.2 2 For each investment, calculate:

i the amount of simple interest

ii the value of the investment after the time period listed.

a a simple interest investment of \$5000 at an interest rate of 5% p.a. for 2 years

b a simple interest investment of \$4800 at an interest rate of 4% p.a. for 3 years

c a simple interest investment of \$12 500 at an interest rate of 8% p.a. for 5 years

3 For each loan, calculate:

i the amount of simple interest

ii the total amount to be repaid.

a a loan of \$7500 at a simple interest rate of 5% p.a. over 3 years

b a loan of \$10 800 at a simple interest rate of 12% p.a. over 5 years

c a loan of \$25 000 at a simple interest rate of 7% p.a. over 8 years

4 Calculate the simple interest given each of these. All interest rates are per annum.

a $P = \$4000, r = 6\%, n = 5$ years

b $P = \$8650, r = 7\%, n = 4$ years

c $P = \$15\,000, r = 8\%, n = 10$ years

d $P = \$9200, r = 4\%, n = 3$ years

e $P = \$19\,999, r = 15\%, n = 6$ years

f $P = \$20\,000, r = 20\%, n = 5$ years

5 Christian invests \$3500 in a bank that offers the simple interest rate of 4.8% per annum. He plans to leave the money invested for 2 years.

a Identify the values of P , r and n .

b How much simple interest does Christian earn?

c What is the total value of Christian's investment after 2 years?

6 Jenna plans to start her business in massage therapy and needs to borrow \$44 000 to assist with her set-up costs. She obtains an agreement with her lender to repay the money over 5 years with simple interest charged at 9.5% p.a.

a Identify the values of P , r and n .

b How much simple interest is Jenna charged?

c What is the total amount that Jenna repays?

7 **a** Calculate the amount of simple interest in each of the following situations.

i \$5000 is invested at a simple interest rate of 0.4% per month for 30 months

ii \$5000 is borrowed at a simple interest rate of 0.4% per month for 30 months.

b Compare each of the answers in part **a**. Briefly explain how the simple interest formula is used for investment and loan situations.

c Given that the simple interest calculations involving loans and investments are identical, how are the calculations different when they are interpreted?

Term deposit
4.8%
p.a.
fixed rate



- 8 Convert each time period to years. Where appropriate, write the fraction in simplest form.
- | | | |
|-------------|-------------|-------------|
| a 11 months | b 7 weeks | c 26 weeks |
| d 3 months | e 271 days | f 155 days |
| g 15 months | h 48 months | i 84 weeks |
| j 1241 days | k 30 months | l 286 weeks |
- 9 A simple interest investment is made for 4 years and 3 months. Matthew thinks this is equivalent to 4.3 years but Lizzy is certain that Matthew is wrong. How is 4 years and 3 months written as a decimal in years?
- 10 For the values given in the table below, calculate:
- the amount of simple interest
 - the total amount at the end of the term.

	Principal \$	Simple interest rate p.a.	Time
a	9000	6%	3 years
b	10 500	4.5%	6 months
c	7500	9.8%	130 weeks
d	29 000	3.2%	90 days
e	8600	6.4%	35 days
f	155 570	12.5%	11 months
g	19 999	19.9%	25 weeks
h	45 950	14.05%	2 years and 5 months
i	208 654	8.75%	5 weeks and 4 days

- 11 Sade is investigating which is the best way to calculate her simple interest for a short-term investment. She invests \$2400 for the month of June at a simple interest rate of 4.6% p.a.
- Calculate the simple interest amount after writing the time as a fraction of the total number of months in the year.
 - Now calculate the simple interest amount after writing the number of days in June as a fraction of the total number of days in the year.
 - Which method of calculation would Sade hope would be used? Briefly explain why.
 - If the values given represented a short-term loan instead of an investment, which method of calculation would Sade prefer? Briefly explain why.
- 12 A bank is offering the simple interest rates shown in the table below for its customers to invest in a term deposit. The interest is calculated at the end of the investment. Jasmine has \$20 000 to invest and plans to invest it for 12 months.
- What simple interest rate will Jasmine receive for her investment?
 - How much interest does she earn?
 - Jasmine's brother informed her that she would have earned more interest if she invested the money for one day less than 12 months. Investigate whether this statement is true and show working to support your finding.



Term (months)	Interest on investment amount (% p.a.)		
	\$5000–<\$10 000	\$10 000–<\$50 000	\$50 000–<\$100 000
1–2	2.5	2.5	2.8
3–6	3.25	3.25	3.25
7–11	5.5	5.55	5.5
12–24	5.3	5.25	5.2

- 13** Simple interest assumes the same amount of interest is added every period (year, month, day, etc.). Therefore, if we know the interest amount per period, we can multiply that by the number of periods to determine the total interest using the unitary method. For each of the following calculate:

For example: $P = \$4000$, $r = 4\%$ p.a., $n = 5$ years, $I = \$?$
 4% p.a. \times $\$4000 = \160 per 1 year
 So for 5 years:
 $\$160 \times 5$ per 1 \times 5 years = $\$800$ per 5 years

- i** the amount of interest per period **ii** the total amount of interest
iii the total amount owing for a loan or in an account for an investment.

- a** $P = \$200$, $r = 5\%$ p.a., $n = 10$ years, $I = \$?$ **b** $P = \$1400$, $r = 6.2\%$ p.a., $n = 8$ years, $I = \$?$
c $P = \$500\,000$, $r = 8\%$ p.a., $n = 12.5$ years, $I = \$?$ **d** $P = \$750$, $r = 8\%$ p.a., $n = 20$ months, $I = \$?$
e $P = \$26\,000$, $r = 5\%$ p.a., $n = 72$ weeks, $I = \$?$ **f** $P = \$4380$, $r = 3.5\%$ p.a., $n = 153$ days, $I = \$?$

- 14** The total amount owing for a loan or in an account for an investment can be determined by adding the interest to the principal. Alternatively, we can find the percentage increase for the given time period, then apply the percentage increase to the principal.

For example: $P = \$4000$, $r = 4\%$ p.a., $n = 5$ years, $I = \$?$
 4% per 1 year
 So for 5 years:
 $4\% \times 5$ per 1 \times 5 year = 20% per 5 years
 $(100\% + 20\%) \times \$4000 = \4800

For each part in question **13**:

- i** find the multiplier to determine the total amount using a percentage increase. Give your answers correct to four decimal places where appropriate
ii recalculate the total amount using the percentage increase in part **i**.

- 15** Banks vary in the ways in which they calculate simple interest on savings and transaction accounts.

Some accounts earn no interest while others attract bonus interest rates if certain conditions are met. If an account provides interest, it is most likely to be calculated on the daily account balance. Consider the account balances for the month of February shown.

- a** The opening balance of $\$640.90$ applies for the first 7 days of the month and each new balance applies from the date the transaction is made. How many days does each balance on this account apply for?

Date	Transaction	Credits \$	Debits \$	Balance \$
01/02	Opening balance			640.90
08/02	Withdrawal at Handybank		-100.00	540.90
15/02	Deposit	+240.00		780.90
24/02	EFTPOS purchase		-125.40	655.50
28/02	Interest			

- b** The account attracts simple interest at a rate of 2.1% p.a. For each new balance in the account, calculate the simple interest based on the number of days each balance applies.
c Add all the amounts from part **b** to calculate the total interest for the month.
d What is the account balance at the end of February, if the total interest is added at the end of each month?

- 16** This bank statement shows the transactions made during the month of August. Interest is calculated daily at a rate of 1.8% p.a.

Date	Transaction	Credits \$	Debits \$	Balance \$
01/08	Opening balance			345.50
09/08	ATM Withdrawal		-50.00	295.50
14/08	Deposit - Pay	+370.00		665.50
16/08	ATM Withdrawal		-120.00	545.50
19/08	EFTPOS Purchase		-85.95	459.55
28/08	Deposit - Pay	+370.00		829.55
31/08	Interest			

- a** How much simple interest is earned during the month? **b** What is the final account balance?

17 A bank offers a simple interest rate of 1.5% p.a. on its savings accounts, calculated daily. An extra 3.2% p.a. bonus rate is offered if no more than one withdrawal is made in the month and the account balance has increased by at least \$200 for the month. Consider each of the account statements shown.

A	Date	Transaction	Credits \$	Debits \$	Balance \$
	01/09	Opening balance			1200.85
	15/09	Deposit – Pay	+450.75		
	24/09	Deposit – at branch	+820.00		
	29/09	EFTPOS Purchase		-245.85	
	30/09	Interest			

B	Date	Transaction	Credits \$	Debits \$	Balance \$
	01/01	Opening balance			1548.90
	08/01	EFTPOS Purchase		-246.20	
	15/01	Deposit – Pay	+1920.00		
	29/01	EFTPOS Purchase		-85.94	
	31/01	Interest			

- a Will either of these accounts receive the bonus simple interest rate? Provide a reason to support your answer.
- b Calculate the total interest earned on each account. You will need to determine the account balances following each transaction first.
- c State the final account balance for each statement at the end of the month.

18 Joel plans to buy a second-hand car for \$12 500. He has saved \$2500 and plans to borrow the remaining money from his bank at a simple interest rate of 8.5% p.a. for 3 years.



- a The car-seller asks for a deposit of 15% of the selling price. Are Joel's savings enough to cover the deposit? Note: A deposit is the first part of a payment often used as a promise to pay.
- b How much does Joel borrow to buy the car?
- c Calculate the total amount, including simple interest, that Joel pays for the car.

19 You have \$2000 and wish to double this amount over 3 years. You explore some different options to earn the most amount of simple interest possible.

- a What is the annual simple interest rate that will enable your investment to double in 3 years?
- b Explore how this rate changes if the time of the investment increases to:
 - i 4 years
 - ii 5 years
 - iii 6 years.
- c Explore how this rate changes if the time of the investment decreases to:
 - i 2 years
 - ii 1 year.

20 Provide three different annual interest rates and their corresponding time periods that would result in an investment of \$5000 earning \$1250 in simple interest.

Check your Student obook pro for these digital resources and more:



Interactive skillsheet
Calculating simple interest



Worksheet
Calculating simple interest



Topic quiz
1E

1F Simple interest calculations

Learning intentions

By the end of this topic you will be able to ...

- ✓ calculate the number of time periods for a loan or investment using the simple interest formula
- ✓ calculate the principal using the simple interest formula
- ✓ calculate the interest rate using the simple interest formula.



Inter-year links

Year 7

6H Solving equations using inverse operations

Year 8

6D Changing the subject of a formula

Year 10

1B Simple interest

Calculating the principal, interest rate and number of time periods



Key content video

- The simple interest formula is $I = Prn$.

$$I = Prn \quad \text{Solve for principal}$$

$$I = Prn \quad \text{Solve for interest rate}$$

$$I = Prn \quad \text{Solve for number of time periods}$$

- To solve the simple interest equation for P , r or n :
 - 1 Write the simple interest formula and identify the known variables.
 - 2 Substitute the values into the formula and simplify the calculation.
 - 3 Solve the equation for the unknown value using inverse operations.
 - 4 Write the answer.

Example 1F.1 Calculating the number of time periods for an investment



How long will it take for an investment of \$4000 at an interest rate of 4% p.a. to earn \$800 in simple interest?

THINK

- 1 Write the simple interest formula and identify the variables. Write r as a fraction or a decimal.
- 2 Substitute the values into the formula and simplify the calculation.
- 3 Solve the equation for n .
- 4 Write the answer and include the unit 'years' because r is 'per annum'.

WRITE

$$I = Prn$$

$$P = \$4000$$

$$r = 4\% = 0.04 \text{ p.a.}$$

$$I = \$800$$

$$\$800 = \$4000 \times 0.04 \times n$$

$$\$800 = \$160 \times n$$

$$\frac{\$800}{\$160} = \frac{\$160 \times n}{\$160}$$

$$n = 5$$

It will take 5 years for an investment of \$4000 to earn \$800 in simple interest.



Example 1F.2 Calculating the principal value

How much needs to be invested at an interest rate of 6% p.a. for 3 years to earn \$1440 in simple interest?

THINK

- 1 Write the simple interest formula and identify the variables. r should be written as a fraction or a decimal.
- 2 Substitute the values into the formula and simplify the calculation.
- 3 Solve the equation for P .
- 4 Write the answer.

WRITE

$$I = Prn$$

$$r = 6\% = 0.06 \text{ p.a.}$$

$$n = 3 \text{ years}$$

$$I = \$1440$$

$$\$1440 = P \times 0.06 \times 3$$

$$\$1440 = P \times 0.18$$

$$\frac{\$1440}{0.18} = \frac{P \times 0.18}{0.18}$$

$$P = \$8000$$

\$8000 needs to be invested to earn \$1440 in simple interest over 3 years.



Example 1F.3 Calculating the interest rate

At what rate is \$6000 invested at over 6 years if \$864 is earned in simple interest?

THINK

- 1 Write the simple interest formula and identify the variables. r should be written as a fraction or a decimal.
- 2 Substitute the values into the formula and simplify the calculation.
- 3 Solve the equation for r using inverse operations.
- 4 Convert r to a percentage by multiplying the decimal by 100.
- 5 Write the answer and include the unit p.a. because n is in years.

WRITE

$$I = Prn$$

$$P = \$6000$$

$$n = 6 \text{ years}$$

$$I = \$864$$

$$\$864 = \$6000 \times r \times 6$$

$$\$864 = \$36\,000 \times r$$

$$\frac{\$864}{\$36\,000} = \frac{\$36\,000 \times r}{\$36\,000}$$

$$r = 0.024$$

$$= 2.4\%$$

\$6000 is invested at 2.4% p.a. to earn \$864 in simple interest.

Helpful hints

- ✓ Remember the order of operations (BIDMAS) when solving equations.
- ✓ Remember to round your answers to the nearest cent.
- ✓ To find the solution, the pronumeral does not have to appear on the left-hand side of the equation – if the pronumeral is by itself on either side of the equation, you have found the solution!

Exercise 1F Simple interest calculations

 1-9, 11-14, 15(a-c)

 1(d-f), 2-4, 7, 8, 10, 12-14, 15(d-f)

 4, 7(b, d, f), 8, 13-17

You can use your calculator for all questions in this exercise.

1 Calculate the simple interest in each case. All interest rates are given per annum.

- a $P = \$2000, r = 7\%, n = 3$ years
- b $P = \$250, r = 11\%, n = 1$ year
- c $P = \$8500, r = 5\%, n = 4$ years
- d $P = \$25\,000, r = 4\%, n = 5$ years
- e $P = \$100\,000, r = 9.5\%, n = 4$ years
- f $P = \$16\,000, r = 6\%, n = 2.5$ years

1F.1 2 Find the value for n in each of these situations.

- a How long will it take for an investment of \$8000 at a simple interest rate of 3% p.a. to earn \$1200 in simple interest?
- b How long will it take for an investment of \$1250 at a simple interest rate of 4% p.a. to earn \$350 in simple interest?
- c How long does a loan of \$15000 at a simple interest rate of 9% p.a. take to earn \$5400 in simple interest?
- d How long will it take for an investment of \$5600 at a simple interest rate of 5% p.a. to earn \$1120 in simple interest?

1F.2 3 Find the value for P in each of these situations.

- a How much needs to be invested at a simple interest rate of 8% p.a. for 5 years to earn \$2000 in simple interest?
- b How much is borrowed at a simple interest rate of 10% p.a. over 4 years if \$6000 is paid in simple interest?
- c How much is borrowed at a simple interest rate of 9% p.a. over 5 years if \$1800 is paid in simple interest?
- d How much needs to be invested at a simple interest rate of 6% p.a. for 2 years to earn \$576 in simple interest?

4 Find the unknown value in each of these situations. All interest rates are given per annum.

- a $I = \$600, P = \$3000, r = 4\%, n = ?$
- b $I = \$1200, P = ?, r = 5\%, n = 4$ years
- c $I = \$450, P = ?, r = 9\%, n = 2$ years
- d $I = \$850, P = \$8500, r = 5\%, n = ?$
- e $I = \$1000, P = ?, r = 8\%, n = 4$ years
- f $I = \$5060, P = \$9200, r = 11\%, n = ?$

5 Jessica has invested \$4500 in a bank that offers simple interest of 5.0% p.a. She plans to earn \$675 in interest.

- a From the simple interest formula, which variable do you not know the value of?
- b What variables do each of the given values represent?
- c How long does the money need to be invested to earn \$675 in simple interest?
- d At a higher interest rate of 7.5% p.a., how much sooner can Jessica earn \$675 in simple interest?

6 Throughout the course of a simple interest investment, Stefan's money increased in value from \$8400 to \$8946. The interest was earned at a simple interest rate of 0.25% per month.

- a What is the total amount of interest earned on this investment?
- b How many months was the initial amount of money invested for?

1E.3 7 For the values given in the table, calculate the simple interest rate per annum that applies.

	Simple interest \$	Principal \$	Time
a	1640	8200	4 years
b	420	3500	2 years
c	985	9850	48 months
d	1680	12 000	30 months
e	3936	18 000	3 years and 5 months
f	680	6400	4 years and 3 months

8 Use the simple interest formula to determine the values for the missing amounts in the table.

	Simple interest \$	Principal \$	Rate % p.a.	Time
a		3700	5.6	4.5 years
b	234		4.8	13 months
c	42 532	70 000		6.2 years
d	3549	19 500	5.2	
e	1711		14.5	48 months
f	2631.60	15 480		130 weeks
g	56.88	948		1.2 years
h	1534.40	13 700	6.4	

9 Priyansha borrowed a sum of money from her parents to help her buy her first laptop. They agreed to charge interest at a rate of 4% p.a. over a period of 3 years. The total interest charge for the term of the loan is \$144.

- From the simple interest formula, which variable is unknown?
- Which variable does each of the given values represent?
- How much money does Priyansha borrow from her parents?



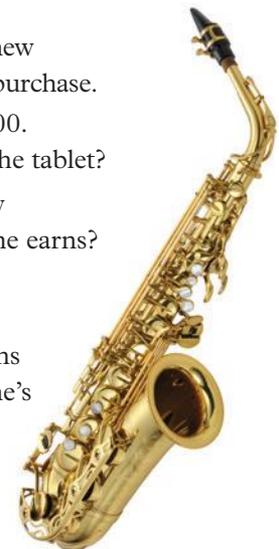
- Priyansha plans to pay her parents \$35 each month for 3 years and believes this will cover the agreed terms of their loan. Determine whether Priyansha's plans are correct and show working to support your finding.
- What are the exact monthly payments Priyansha needs to make to repay her loan?

10 The cost of the latest tablet is \$873. Although Gabriella has the savings to purchase the new tablet, she would rather let the interest earned from her investment cover the cost of the purchase.

- One bank offers her a simple interest rate of 7.2% p.a. for her investment of \$10 000. How long does this money need to be invested to earn enough interest to pay for the tablet?
- Gabriella decides on 12 months to reach her goal. At the same rate of interest, how much does she need to invest in order to fully pay for the tablet with the interest she earns?

11 Daniel has decided to learn the alto saxophone through his school music program. To encourage his development, his parents bought the saxophone shown through a purchase program arranged by his school valued at \$1200. The repayment conditions involve quarterly payments over 3 years. The simple interest charged on the saxophone's cost is \$162.

- What is the quarterly interest rate charged?
- What is the amount of each quarterly payment required?



- 12 The simple interest formula can be used to find the values of the principal, rate or number of time periods if you know the interest amount and two other values, but you can also use your knowledge of rates and percentages. You can use the unitary method or a multiplier with the interest amount per annum to find the number of time periods for the desired interest amount.

$$P = \$4000, r = 4\% \text{ p.a.}, n = ? \text{ years}, I = \$800$$

Unitary method

$$\begin{aligned} 4\% \text{ p.a.} \times \$4000 &= \$160 \text{ per 1 year} \\ &= \$160 \div 160 \text{ per } 1 \div 160 \text{ years} \\ &= \$1 \text{ per } \frac{1}{160} \text{ years} \\ &= \$1 \times 800 \text{ per } \frac{1}{160} \times 800 \text{ years} \\ &= \$800 \text{ per 5 years} \end{aligned}$$

Multiplier method

$$\begin{aligned} 4\% \text{ p.a.} \times \$4000 &= \$160 \text{ per 1 year} \\ &= \$160 \times \frac{800}{160} \text{ per } 1 \times \frac{800}{160} \text{ years} \\ &= \$800 \text{ per 5 years} \end{aligned}$$

When the interest rate per annum or principal amount are not known, you can start by finding the interest per year then either writing it as a percentage of the principal or determining what percentage it is of the interest rate.

For example:

$$P = \$?, r = 6\% \text{ p.a.}, n = 3 \text{ years}, I = \$1440$$

$$\begin{aligned} \$1440 \text{ per 3 years} &= \frac{\$1440}{3} \text{ per } \frac{3}{3} \text{ years} \\ &= \$480 \text{ per year} \end{aligned}$$

$$\begin{aligned} P &= \frac{\$480 \text{ per year}}{6\% \text{ per year}} \\ &= \frac{\$480 \text{ per year}}{0.06 \text{ per year}} \\ &= \$8000 \end{aligned}$$

$$P = \$6000, r = ?\% \text{ p.a.}, n = 6 \text{ years}, I = \$864$$

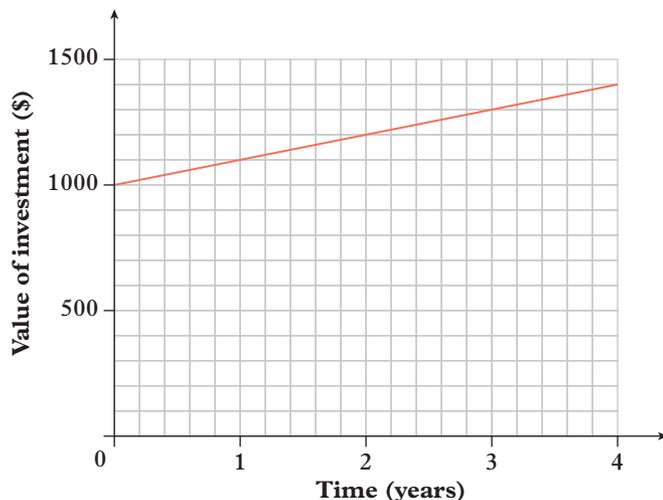
$$\begin{aligned} \$864 \text{ per 6 years} &= \frac{\$864}{6} \text{ per } \frac{6}{6} \text{ years} \\ &= \$144 \text{ per year} \end{aligned}$$

$$\begin{aligned} r &= \frac{\$144 \text{ per year}}{\$6000} \times 100 \\ &= 2.4\% \text{ p.a.} \end{aligned}$$

Determine the values for the missing amounts in the table using rates and percentages.

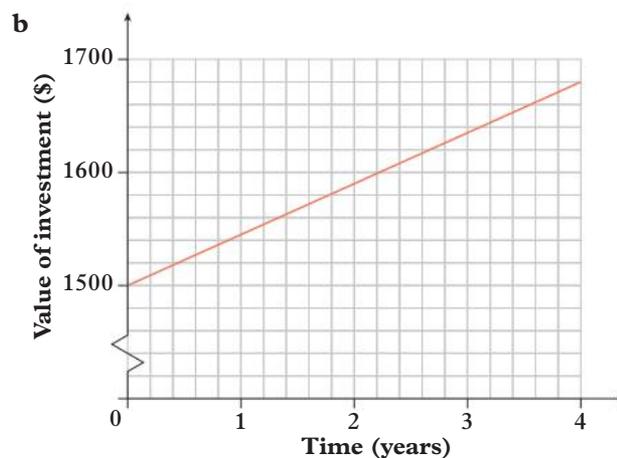
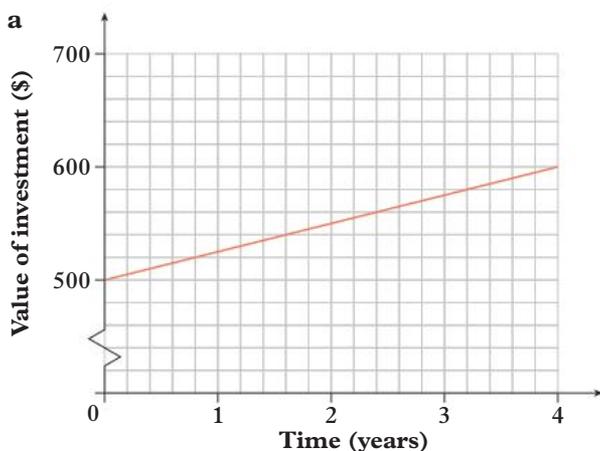
	Simple interest \$	Principal \$	Rate % p.a.	Time
a	297.50	850	5	? years
b	786.24	?	4.2	12 years
c	569.43	999	?	10 years
d	125	?	8	15 months
e	548.80	3200	?	91 weeks
f	2 947 000	7 300 000	5	? days

- 13 The following graph details the value of a simple interest investment.



- What is the value of the principal?
- What is the value of the investment after 4 years?
- What is the interest rate of the investment?

14 Determine the interest rate for each of these simple interest investments.



- 15 **a** After 92 weeks Alex is charged \$63.25 on a simple interest loan with an annual interest rate of 6.5%. How much interest will he be charged after 100 weeks?
- b** Charlotte borrows \$1000 at a simple interest rate of 3.65% p.a. and is charged \$500 interest after a number of days. After how many days will Charlotte be charged \$1000 interest?
- c** Helen has a simple interest investment of 20c. After 1000 months, she earns \$1.05 in interest. Correct to the nearest year, how long will it take Helen to earn \$1000 in interest?
- 16 This bank statement is linked to a savings account and shows the transactions made during the month of April.

Date	Transaction	Amount \$	Balance \$
01/04	Opening balance		2905.60
03/04	Deposit – Pay	1230.75	4136.35
08/04	ATM Withdrawal	250.00	3886.35
15/04	EFTPOS Purchase	499.95	3386.40
17/04	Deposit – Pay	1230.75	4617.15
	Monthly interest	7.54	4624.69

What is the annual interest rate (% p.a.) that applies to this account? Remember that each new balance applies from the day of the transaction. Round your answer to one decimal place.

- 17 The statement shown is linked to a credit card where interest is charged from the day of purchase. To avoid additional charges, the total amount spent, plus interest, is to be paid each month.

Date	Description	Amount \$
06/07	BPAY to electricity provider	290.00
08/07	Gym membership	72.00
11/07	Petrol	45.00
20/07	NRL tickets	85.00
21/07	Clothing store	189.95
24/07	Petrol	52.87
	Interest charge for the month of July	6.31

- a** How much needs to be paid at the end of the month to avoid any additional charges?
- b** What is the annual interest rate (% p.a.) that is charged to this credit account?

Check your Student ebook pro for these digital resources and more:

pro



Interactive skillsheet
Calculating simple interest rate



Interactive skillsheet
Calculating the number of time periods for an investment



Interactive skillsheet
Calculating the principal by simple interest formula



Investigation
What percentage interest are you really paying?



Topic quiz
1F

Chapter summary

Wages and salary

- Income: money received in exchange for work or investment
- Wage: a fixed rate of income for working a given period, e.g., \$25/hour
- Salary: fixed income paid on a regular basis

$$\frac{\$78\,000}{12} = \$6\,500 \text{ per month}$$

$$\frac{\$78\,000}{52} = \$1\,500 \text{ per week}$$

Penalty rates and leave loading

- Time-and-a-half: $1.5 \times$ standard wage
- Double time: $2 \times$ standard wage
- Leave loading: 17.5% additional pay for annual leave

Commissions, royalties and piece work

- Commission: being paid a proportion of sales
- Royalties: payments made for the use of assets
 $2\% \text{ of } \$44\,000 = 0.02 \times \$44\,000$
 $= \$880$
- Piece work: income paid for each item produced or action performed

Income tax

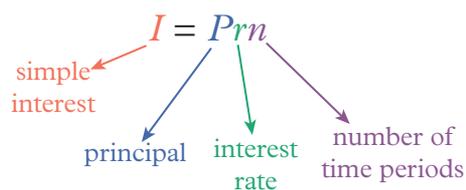
- Total income: wages, salaries, bonuses, royalties and interest earned
- Tax deductions: work-related expenses, gifts and donations and the cost of managing tax affairs
- Taxable income = total income – tax deductions
- PAYG tax is withheld to ensure employees meet their tax obligations
- Discrepancies are resolved by completing a tax return

Taxable income	Tax on this income
0–\$18 200	Nil
\$18 201–\$45 000	19 cents for each \$1 over \$18 200
\$45 001–\$120 000	\$5092 plus 32.5 cents for each \$1 over \$45 000
\$120 001–\$180 000	\$29 467 plus 37 cents for each \$1 over \$120 000
\$180 001 and over	\$51 667 plus 45 cents for each \$1 over \$180 000

Medicare levy and net earnings

- 2% tax on taxable income to fund the health care system
- Net income = taxable income – income tax – Medicare levy

Simple interest



- I = interest
- P = principal, the original amount of money borrowed or invested
- r = interest rate, usually a percentage converted to a fraction or decimal
- n = number of time periods



Chapter review quiz

Assess your knowledge of this chapter.

Quizlet

Test your knowledge of this topic by working individually or in teams.

Mathematical literacy review

The following key terms are used in this chapter:

- bonus
- budget
- buy now, pay later
- buying an item on terms
- commission
- double time
- expenses
- income
- income tax
- interest
- interest rate
- investment
- lay-by agreement
- leave loading
- loan
- Medicare levy
- net income
- PAYG tax
- payment options
- penalty rates
- piece work
- principal
- royalties
- salary
- simple interest
- tax deductions
- tax tables
- taxable income
- time-and-a-half
- wage

- 1 What percentage tax (on taxable income) is the Medicare levy?
- 2 How can you calculate a fortnightly salary if given an annual salary?
- 3 What does the acronym PAYG stand for?
- 4 Identify the key terms being referenced in each of these definitions.
 - a when money is borrowed, and interest is paid on the amount borrowed
 - b a fixed rate of income an employee receives for working a given period
- 5 Using an example, provide a definition in your own words for the following key terms.
 - a leave loading
 - b simple interest
- 6 Complete the following sentences using words from the key term list.
 - a _____ is income which is paid for each item produced or action performed.
 - b _____ are expenses that can be subtracted from a person's total income to determine their _____.

Multiple choice

- 1A** 1 Emma worked for 30 hours and earned \$855. What is her hourly wage?
 A \$28/hour B \$28.50/hour C \$30/hour D \$825/hour E \$25 650/hour
- 1A** 2 Mo receives a standard rate of \$30.20 per hour. He receives a penalty rate of one-and-a-quarter for any overtime (above 25 hours per week). How much does Mo earn in a week in which he works 32 hours?
 A \$264.25 B \$755 C \$966.40 D \$1019.25 E \$1208
- 1B** 3 Vita receives a base income of \$350/wage plus a 3.5% commission on sales. How much does Vita earn in a week in which her sales total \$25 000?
 A \$875 B \$1100 C \$1145 D \$1225 E \$25 350
- 1B** 4 Arthur receives a piece work rate for every haircut he performs. If Arthur cuts 17 people's hair in one day, he will earn \$399.50. What is his piece work rate of pay?
 A \$23 B \$23.50 C \$24 D \$24.50 E \$25.50
- 1C** 5 Last year, Saheed had a salary of \$8420 per month and received a 4% bonus at work. He earned \$2304 in interest on his investments, spent \$1105 on work-related expenses and paid his accountant \$100 to manage his tax affairs. What was Saheed's taxable income?
 A \$9835 B \$101 573 C \$107 386 D \$105 082 E \$106 181

- 1D 6** Scott purchases a new kitchen on terms, with a deposit of \$5000 plus 24 monthly payments of \$599. The retail price of the kitchen is \$18 999. What is the percentage increase in price for purchasing on the given terms, correct to two decimal places?
A 1.98% **B** 1.95% **C** 2.95% **D** 2.69% **E** 7.54%
- 1D 7** Jasmine comes up with the following list of expected expenses for the year. What are her expected total annual expenses?
 • Rent: \$475/week
 • Utility bills: \$450/quarter
 • Food: \$160/week
 • Home insurance: \$50/year
 • Health insurance: \$108/month
 • Entertainment: \$100/month
 • Gifts: \$250/year
 • Clothes: \$500/year
 • Mobile phone: \$55/month
 • Transport: \$200/month
 • Other items: \$100/month
A \$2448 **B** \$29 376 **C** \$40 689 **D** \$42 376 **E** \$127 296
- 1E 8** \$12 000 is invested at 4.2% p.a. simple interest for 18 months. Which values should be substituted into the simple interest formula?
A $P = 12\,000, r = 4.2, n = 18$ **B** $P = 12\,000, r = 0.42, n = 1.5$
C $P = 12\,000, r = 4.2, n = 1.5$ **D** $P = 12\,000, r = 0.042, n = 18$
E $P = 12\,000, r = 0.042, n = 1.5$
- 1E 9** Ginger invests \$6400 in a bank that offers a simple interest rate of 7.2% per annum. What is the value of her investment after 3 years?
A \$921.60 **B** \$8243.20 **C** \$7782.40 **D** \$7321.60 **E** \$6860.80
- 1F 10** A loan of \$4500 with simple interest 8.5% p.a. is charged \$1530 in interest so that \$6030 is now owed. Which simple interest variable do you not know the value of?
A number of time periods **B** principal
C interest rate **D** interest amount
E total amount

Short answer

- 1A 1** Copy and complete the following table.

Hourly wage	Time worked	Amount earned
\$26.40	24 hours	
\$33.20	28.5 hours	
	30 hours	\$1131
\$41.80		\$877.80
\$23.90		\$872.35

- 1A 2** Katie works 36 hours per week and is paid \$32.50 per hour. She takes two weeks of annual leave, for which she receives an additional leave loading of 17.5%. How much does Katie earn for her two weeks of annual leave?
- 1B 3** Calculate the amount authors will receive in royalties given the following rates and sales.

	Royalty rate	Sales
a	5.5%	\$45 000
b	3.2%	\$98 000
c	1.8%	\$30 000
d	9%	\$143 000
e	13%	\$57 200

Analysis

Julia manages a bookstore and earns an annual salary of \$80 000. The normal hourly rate of \$24.50 applies to her casual staff, although the opportunity for overtime is available. The store's rent is \$1800 per week and Julia allows an extra \$200 per week to cover other costs.



- Julia's annual tax deductions total \$1240. What is Julia's monthly net income after subtracting her income tax and the Medicare levy? Use the tax table for the 2023–2024 financial year.
- One week, Julia has three staff working. Simon works 24 hours at the normal hourly rate. Melanie works 15 hours at the normal rate, 3 hours at time-and-a-half and 5 hours at double time. Tahlia works 30 hours at the normal rate and 5 hours at time-and-a-half. Calculate the weekly income for each employee.
- What amount of money must Julia's store make each week to cover the cost of staff pay and store costs? The owners receive a quote for \$48 000 to re-fit the store. They have half of this amount in savings and plan to borrow the remaining amount.
- The bank lends the money at a simple interest rate of 8.2% p.a. over 3 years. What is the total amount of money that must be repaid?
- If the money is repaid in equal monthly instalments, what is the amount?
- In total, how much will the store makeover cost?

Chapter checklist

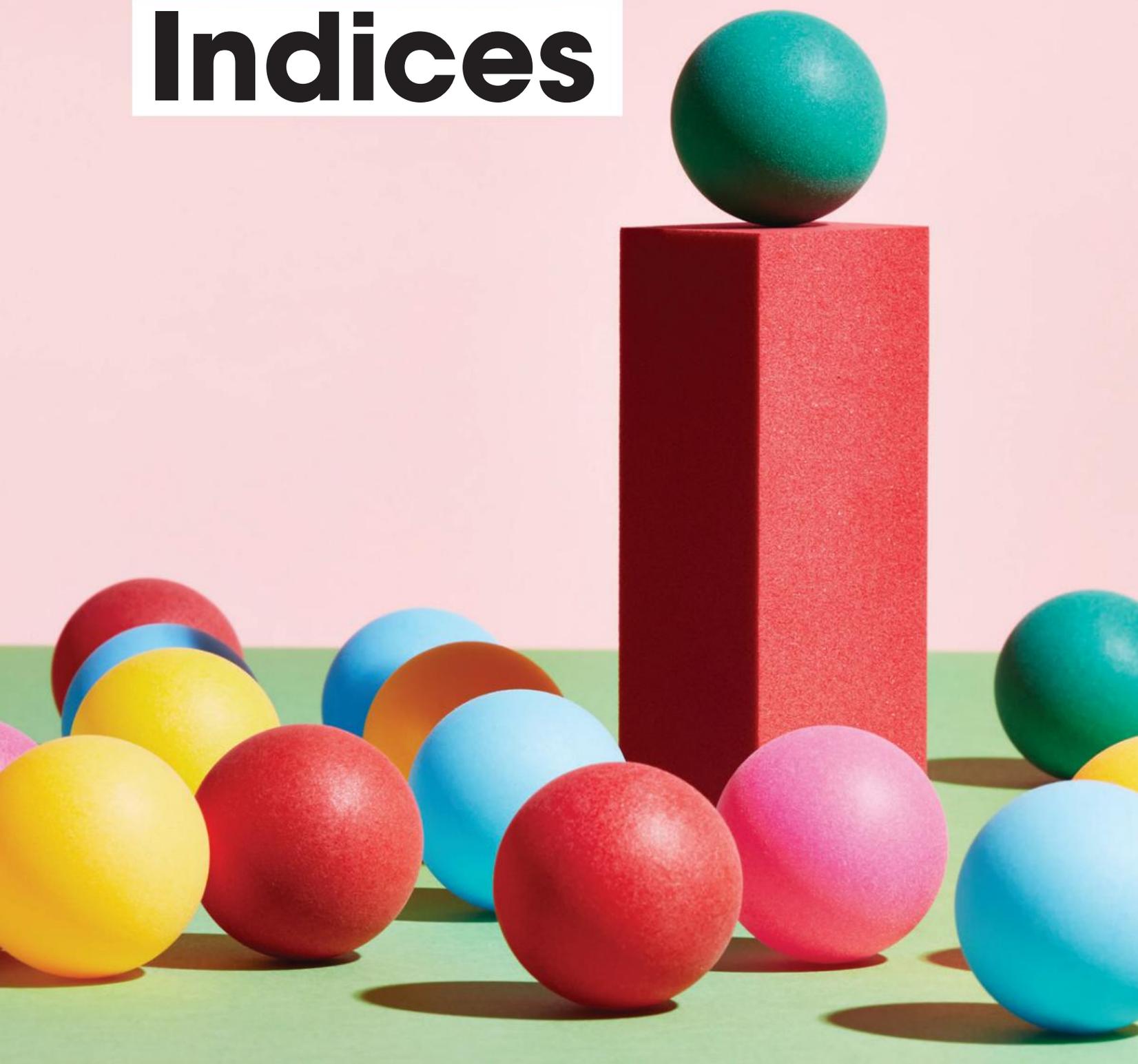


Now that you have completed this chapter, reflect on your ability to do the following.

I can do this	I need to review this
<input type="checkbox"/> Understand the difference between a wage and a salary <input type="checkbox"/> Calculate income for working a given period of time <input type="checkbox"/> Apply penalty rates and leave loading	<input type="checkbox"/> Go back to Topic 1A Wages and salaries
<input type="checkbox"/> Calculate income including a commission <input type="checkbox"/> Calculate piece work income and royalty payments	<input type="checkbox"/> Go back to Topic 1B Other forms of income
<input type="checkbox"/> Calculate an individual's taxable income <input type="checkbox"/> Calculate income tax	<input type="checkbox"/> Go back to Topic 1C Taxation
<input type="checkbox"/> Evaluate payment options <input type="checkbox"/> Create a budget	<input type="checkbox"/> Go back to Topic 1D Budgeting and spending money
<input type="checkbox"/> Calculate interest using the simple interest formula <input type="checkbox"/> Calculate the simple interest on an investment <input type="checkbox"/> Calculate the simple interest on a loan	<input type="checkbox"/> Go back to Topic 1E Simple interest
<input type="checkbox"/> Calculate the number of time periods for a loan or investment using the simple interest formula <input type="checkbox"/> Calculate the principal using the simple interest formula <input type="checkbox"/> Calculate the interest rate using the simple interest formula	<input type="checkbox"/> Go back to Topic 1F Simple interest calculations

2

Indices



Index

- 2A Indices
- 2B Products and quotients of powers
- 2C Raising indices and the zero index
- 2D Negative indices
- 2E Scientific notation
- 2F Rounding and estimating

Prerequisite skills



Diagnostic pre-test

Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.



Interactive skillsheets

After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✓ Prime factorisation
- ✓ Multiplying terms
- ✓ Dividing terms
- ✓ Rounding integers
- ✓ Ordering and comparing large numbers

Curriculum links

- Simplifies algebraic expressions involving positive-integer and zero indices, and establishes the meaning of negative indices for numerical bases (MA5-IND-C-01)
 - Extend and apply the index laws to variables, using positive-integer indices and the zero index
 - Simplify algebraic products and quotients using index laws
 - Apply index laws to numerical expressions with negative-integer indices
- Solves measurement problems by using scientific notation to represent numbers and rounding to a given number of significant figures (MA5-MAG-C-01)
 - Identify and describe very small and very large measurements
 - Estimate and round numbers to a specified degree of accuracy
 - Express numbers in scientific notation

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Materials

- ✓ Calculator

2A Indices

Learning intentions

By the end of this topic you will be able to ...

- ✓ convert between index notation and expanded form
- ✓ calculate the value of numbers in index notation
- ✓ express integers as a product of prime factors.



Inter-year links

- [Year 7](#) 1G Indices and square roots
- [Year 8](#) 4A Indices
- [Year 10](#) 2A Indices

Index notation



Key content video

- **Index notation** (or **index form**) is used to represent repeated multiplication.
 - 3^4 is read as '3 to the power of 4'.
 - a^3 is read as 'a to the power of 3'.
- The **base** is the number or variable that is multiplied repeatedly.
- The **index** (or **exponent**) indicates the number of times the base is multiplied.
- If no index is indicated, then the base has an index of 1.

$$2 = 2^1$$

$$x = x^1$$

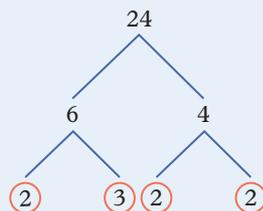
- Index notation can also be used to represent powers of negative numbers.

$$\begin{array}{l} \text{index/exponent} \\ \downarrow \\ \text{base} \longrightarrow (-2)^3 = (-2) \times (-2) \times (-2) = -8 \\ \text{index notation} \quad \text{expanded form} \quad \text{basic numeral} \end{array}$$

- If the base is negative and the index is an even number, the basic numeral will be positive.
- If the base is negative and the index is an odd number, the basic numeral will be negative.

Prime factorisation

- The **prime factorisation** of a positive integer is the product of all prime factors of that integer.
 - Prime factorisation is often expressed in index notation with the bases listed in ascending order.
 - For example, the prime factorisation of 24 is: $24 = 2 \times 2 \times 2 \times 3$
 $= 2^3 \times 3$.
- The prime factorisation of a positive integer can be found using **factor trees**. In factor trees, composite numbers are broken down into pairs of factors until all remaining factors are prime numbers.



Example 2A.1 Calculating the value of a number in index notation



Write the following in expanded form and evaluate.

a 2^5

b $(-4)^3$

c $(\frac{2}{5})^4$

THINK

- a** 1 Identify the base and the index. The base is 2 and the index is 5, so 2 is multiplied by itself 5 times.
2 Perform the multiplications.
- b** 1 Identify the base and the index. The base is -4 and the index is 3, so -4 is multiplied by itself 3 times.
2 Perform the multiplications. Recall that if the base is negative and the index is an odd number, then the basic numeral will be negative.
- c** 1 Identify the base and the index. The base is $\frac{2}{5}$ and the index is 4, so $\frac{2}{5}$ is multiplied by itself 4 times.
2 Perform the multiplications. Recall that to multiply fractions, you multiply the numerators together and the denominators together.

WRITE

a $2^5 = 2 \times 2 \times 2 \times 2 \times 2$

$$\begin{aligned} &= 4 \times 2 \times 2 \times 2 \\ &= 8 \times 2 \times 2 \\ &= 16 \times 2 \\ &= 32 \end{aligned}$$

b $(-4)^3 = -4 \times -4 \times -4$

$$\begin{aligned} &= 16 \times -4 \\ &= -64 \end{aligned}$$

c $(\frac{2}{5})^4 = \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}$

$$\begin{aligned} &= \frac{2 \times 2 \times 2 \times 2}{5 \times 5 \times 5 \times 5} \\ &= \frac{16}{625} \end{aligned}$$

Example 2A.2 Writing variables in expanded form



Write the following in expanded form.

a x^4

b $(-ab)^3$

c $2xy^2z$

THINK

- a** Identify the base and the index. The base is x and the index is 4, so x is multiplied by itself 4 times.
- b** Identify the base and the index. The base is $-ab$ and the index is 3, so $-ab$ is multiplied by itself 3 times.
- c** There are four bases in this term. Identify the bases and the matching index. Recall that if a base doesn't have an indicated index, then the index is 1. Therefore, 2, x and z each have an index of 1, and y has an index of 2.

WRITE

a $x^4 = x \times x \times x \times x$

b $(-ab)^3 = -ab \times -ab \times -ab$

c $2xy^2z = 2 \times x \times y \times y \times z$



Example 2A.3 Prime factorisation using factor trees

Use a factor tree to express each number as the product of its prime factors. Write your answers in index notation.

a 20

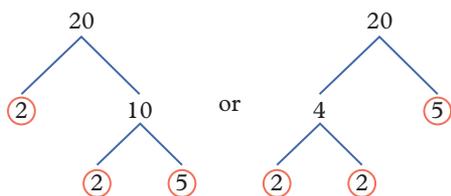
b 315

THINK

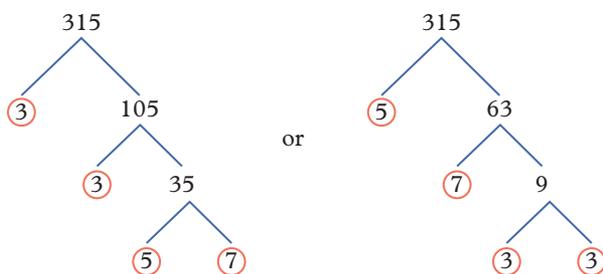
- 1 Identify a factor pair by dividing the composite number by its smallest prime factor. The smallest prime factor of an even number is always 2. Remember that if the sum of all the digits in a number is divisible by 3, then that number is also divisible by 3, and that any number ending in 0 or 5 is divisible by 5.
- 2 Continue to split factors into factor pairs until all remaining factors are prime.
- 3 Write the composite number as a product of its prime factors. Write the answer in index notation and list the bases in ascending order.

WRITE

a $20 = 2 \times 2 \times 5$
 $= 2^2 \times 5$



b $315 = 3 \times 3 \times 5 \times 7$
 $= 3^2 \times 5 \times 7$



Helpful hints

- ✓ Remember that raising a number to an index and multiplying are different operations. For example: $2^4 \neq 2 \times 4$, $2^4 = 2 \times 2 \times 2 \times 2$.
- ✓ Take care when writing indices – they should be a smaller size than the base and sit high up on the shoulder of the base to avoid confusion between 3^4 and 34 .
- ✓ When creating factor trees, remember that if a branch doesn't end on a prime number, then keep dividing the composite number until the branch ends on a prime.
- ✓ Recall that the first 10 prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29.

Exercise 2A Indices

 1-8, 9(a, c, e, g), 10, 11, 13,
14, 17(a, b)

 1-3(b, d, f, h), 4, 6-8, 9(b, d, f, h),
12, 13, 15, 17, 19

 1-3(g, h), 4, 6(e-h), 8(e, f), 12-14,
16, 17(c, d), 18-20

2A.1 1 Write the following in expanded form and evaluate.

a 6^4	b 8^3	c $(-2)^5$	d $(-3)^6$
e $\left(\frac{5}{4}\right)^3$	f $\left(\frac{1}{2}\right)^7$	g $\left(-\frac{2}{3}\right)^4$	h $\left(-\frac{3}{5}\right)^5$

2A.2 2 Write the following in expanded form.

a b^6	b $(-n)^5$	c $(-cd)^2$	d $(2pq)^4$
e $2pq^4$	f $-4a^2b^3c$	g $(3m^2)^5$	h $3(m^2)^5$

3 Write the following in index notation.

a $5 \times 5 \times 5 \times 5$
b $a \times a \times a \times a$
c $v \times k \times k \times v \times k \times v \times v \times 7$
d $qu \times qu \times qu \times qu \times qu$
e $-h \times -h \times -h$
f $-(h \times h \times h)$
g $n^3 \times n^3 \times n^3 \times n^3 \times n^3 \times n^3$
h $5b^3d^4 \times 5b^3d^4$

2A.3 4 Express each number as the product of its prime factors. Write your answers in index notation.

a 50	b 72	c 135	d 378
e 152	f 812	g 550	h 1665

5 Evaluate the following.

a $(0.2)^2$	b $(-0.2)^2$	c $(0.02)^2$
d $(0.2)^3$	e $(-0.2)^3$	f $(0.02)^3$
g $(0.2)^4$	h $(-0.2)^4$	i $(0.02)^4$

6 Write the following in index notation without brackets.

a $(-5)^4$	b $(-5)^3$	c $(ab)^4$	d $(5xy)^8$
e $5(xy)^8$	f $\left(\frac{11}{2}\right)^6$	g $(-3abc)^5$	h $(-3abc)^8$

7 Substitute in the given values and evaluate the expressions.

a x^3 , where $x = 7$
b $6a^4b^2$, where $a = -2$ and $b = \frac{1}{4}$
c $\frac{p^4}{qr^3}$, where $p = 3$, $q = 5$ and $r = -4$
d $2x^3 + 8x^2 + x + 7$, where $x = 10$

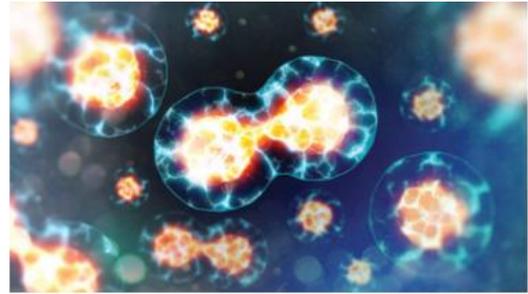
8 Write the following in index notation.

a $2 \times 2 \times 2 \times 3 \times 3$
b $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 6$
c $13 \times 13 \times 13 \times 13 \times 17 \times 17 \times 17 \times 17 \times 17$
d $101 \times 101 \times 103 \times 103 \times 103 \times 103 \times 103$
e $4 \times 4 \times 4 \times x \times x \times x \times x$
f $7 \times 7 \times xy \times xy \times xy \times xy \times xy \times xy \times xy$

- 9 Use the fact that $300 = 2^2 \times 3 \times 5^2$ to help find the prime factors of each of the following numbers and then write the numbers in index notation.
- a** 600 **b** 150 **c** 900 **d** 1500
e 3000 **f** 1800 **g** 2100 **h** 2400
- 10 Express each of the following in index notation.
- a** $xxxxxxx$ **b** $aaabb$ **c** $3rssttt$ **d** $4eeeeeeeff$
- 11 Explain the mistake in each of the following. Then change the right-hand side so that the equation is correct.
- a** $tk^5 = t \times k \times t \times k \times t \times k \times t \times k \times t \times k$
b $(2rw)^4 = 2 \times r \times w \times w \times w \times w$
c $-3 \times (-2)^4 = 6^4$
- 12 Substitute in the given values and evaluate each expression.
- a** $(2x + 3)^8$, where $x = -2$
b $\left(\frac{y}{3}\right)^3 + 4\sqrt{y}$, where $y = 9$
c $ab^3 - ba^2$, where $a = 5$ and $b = -3$
d $2r^3 + 8r^2 - 3r$, where $r = -\frac{3}{2}$
- 13 **a** Evaluate each of the following.
- i** $(-1)^2$ **ii** $(-1)^3$ **iii** $(-1)^4$ **iv** $(-1)^5$ **v** $(-1)^6$
vi $(-1)^7$ **vii** $(-1)^8$ **viii** $(-1)^9$ **ix** $(-1)^{10}$ **x** $(-1)^{11}$
- b** Copy and complete the following sentences.
- i** When the index n is odd, the basic numeral of $(-1)^n$ is _____.
ii When the index n is even, the basic numeral of $(-1)^n$ is _____.
- c** Decide for each of the following whether the basic numeral will be positive or negative. Do not evaluate.
- i** $(-2)^{15}$ **ii** $(-4)^{27}$
iii $(-24)^{30}$ **iv** $(-17)^{198}$
v $(-16)^7 \times (-34)^{11}$ **vi** $(-8)^{14} \times (-5)^{27}$
vii $(-78)^{99} \times (-81)^{45} \times (-21)^{68}$ **viii** $\left(-\frac{77}{101}\right)^{108} \times \left(-\frac{301}{22}\right)^{404}$
- 14 Consider each pair of numbers written in index notation.
- i** Using a calculator, evaluate each pair.
ii Describe how the two numbers are similar and how they are different in their index notation and as a basic numeral.
- a** $(0.7)^3$ and $(0.07)^3$
b $(-0.4)^3$ and $-(0.4)^3$
c $(-1.2)^3$ and $(-1.2)^4$
d $(2.1)^3$ and $(2.01)^3$
- 15 A farmer's herd of cattle grows by approximately 20% each year. In 2023, the farmer had 20 cows.
- a** By what number can the number of cows be multiplied to increase it by 20%?
b Predict the size of the farmer's herd in 2024, 2025 and 2028. Round your answers to the nearest whole number.



16 Three different groups of bacteria, Bacteria A, Bacteria B and Bacteria C, reproduce at different rates.



- a** Bacteria A splits into two bacteria every day. How many times larger will a population of this bacteria be after 3, 8 and 12 days? Write your answers in index notation.
- b** Bacteria B splits into two bacteria twice each day. How many times larger will a population of this bacteria be after 3, 8 and 12 days? Write your answers in index notation.
- c** Bacteria C splits into two bacteria once every two days. How many times larger will a population of this bacteria be after 3, 8 and 12 days? Write your answers in index notation.
- d** Populations of Bacteria A, B and C each have three bacteria initially. Determine the size of each bacteria population after three days.

17 The lowest common multiple is the product of the largest index of each prime factor or pronumeral in each term. The highest common factor is the product of the smallest index of each prime factor or pronumeral in each term. Find the lowest common multiple and highest common factor of each pair of terms. Write your answers in index notation.

- a** $2^8 \times 3^5 \times 5^2 \times 7$ and $2^4 \times 3^{15} \times 5^2 \times 7^4$
- b** $a^8b^5c^2d$ and $a^4b^{15}c^2d^4$
- c** $pq^5r^7s^2$ and $pq^3r^{10}s^4$
- d** $8x^3y^9z^4$ and $12xy^3z^4$

18 For each of the following, state how many different sequences of answers there are. Write your answers in index notation.

- a** a quiz that has 10 true or false questions
- b** a quiz that has 10 multiple-choice questions each with options A, B, C, D, E
- c** a quiz that has 12 true or false questions and 8 multiple-choice questions with options A, B, C, D, E



19 Using positive and negative whole numbers (integers), see how many different index expressions you can find that equal 64.

$$\boxed{}^{\boxed{}} = 64$$

20 Evaluate each of the following.

- a** $\frac{ab^2}{c^3}$, where $a = 6$, $b = \frac{1}{3}$ and $c = -2$
- b** $\frac{p^3}{q^{25}}$, where $p = \frac{3}{2}$ and $q = \frac{2}{3}$
- c** $\frac{r^4}{(mn)^3}$, where $m = -0.5$, $n = 0.2$ and $r = -0.7$

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pro



Interactive skillsheet
Indices



Worksheet
Index notation



Worksheet
Writing terms in expanded form



Interactive skillsheet
Using indices to determine how card tricks work



Topic quiz
2A

2B Products and quotients of powers

Learning intentions

By the end of this topic you will be able to ...

- ✓ simplify products of numbers and variables in index form with the same base
- ✓ simplify quotients of numbers and variables in index form with the same base.



Inter-year link

Support

Adding and subtracting whole numbers

Year 7

1B Adding whole numbers

Year 8

4B Products and quotients of powers

Year 10

2A Indices

Product of powers law



Key content video

- The index laws are rules that apply to all expressions (numeric and algebraic) containing indices.
- When multiplying terms in index notation with the same base, add the indices and write the result with the same base. Writing the terms in expanded form and then simplifying achieves the same result, only at a slower pace.

$$2^3 \times 2^5 = 2^{(3+5)} = 2^8 \quad a^3 \times a^5 = a^{(3+5)} = a^8$$

For example, $2^3 \times 2^5 = 2 \times 2$ is the same as $2^3 \times 2^5 = 2^{3+5} = 2^8$.

- To multiply terms where variables have indices and coefficients:
 - 1 Multiply the coefficients of each term.
 - 2 Apply the product of powers law and add the indices of any common bases.
 - 3 Write the coefficient first, followed by the variables listed in alphabetical order.

Quotient of powers law

- When dividing terms in index notation with the same base, subtract the second index from the first index and write the result with the same base.

$$2^5 \div 2^3 = 2^{(5-3)} = 2^2 \quad a^5 \div a^3 = a^{(5-3)} = a^2$$

- Remember that **quotients** can be written as fractions. When simplifying fractional quotients, subtract the index of the term in the denominator from the index of the term in the numerator.

$$\frac{2^5}{2^3} = 2^{(5-3)} = 2^2 \quad \frac{a^5}{a^3} = a^{(5-3)} = a^2$$

- To divide terms where variables have indices and coefficients:
 - 1 Divide the coefficients by their highest common factor.
 - 2 Apply the quotient of powers law and subtract the indices of any common bases.
 - 3 Write the coefficient first, followed by the variables listed in alphabetical order.

Example 2B.1 Simplifying numerical expressions using an index law



Use an appropriate index law to simplify each expression. Leave each answer in index notation.

a $3^4 \times 3^2$

b $7^8 \div 7^5$

c $\frac{8^5}{8^2}$

- ✓ 'Simplify' and 'evaluate' are different commands:
 - To simplify in this chapter, use index laws to combine the terms and hence reduce the complexity of the calculation or numerical expression.
 - To evaluate or 'find the value' of a calculation or numerical expression, convert the expression from index notation into a basic numeral.
- ✓ Indices only apply to the number or pronumeral immediately to the left of the index. For example, in the term $4gh^3$, the index of 3 only applies to the variable h , so $4gh^3 = 4 \times g \times h \times h \times h$.
- ✓ Recall the rules for multiplying positive and negative numbers. $+ \times - = -$
 $- \times - = +$

ANS
p429

Exercise 2B Products and quotients of powers

 1-5, 6-8(1st, 3rd columns),
9, 11(a, b), 14

 1(f, h, j, l), 3-9(2nd, 4th columns), 10,
11(b, c), 12(a, b), 14, 15(a-d)

 5-9(2nd, 4th columns),
10, 11(c, d), 12(b, c), 13, 15, 16

2B.1 1 Use the appropriate index law to simplify each expression. Leave each answer in index notation.

a $3^5 \times 3^4$	b $7^8 \div 7^2$	c $(-2)^7 \times (-2)^5$	d 6×6^2
e $(-8)^{13} \div (-8)^6$	f $10^2 \times 10^9$	g $3^6 \div 3^5$	h $5^3 \div 5$
i $\frac{4^7}{4^5}$	j $\frac{(-9)^{18}}{(-9)^9}$	k $\frac{13^9}{13^6}$	l $2^5 \times 2^2 \times 2^3$

2 Using a calculator, calculate the basic numeral for question 1 parts **a-d**.

2B.2 3 Using the product of powers law, simplify each expression.

a $3y^3 \times y^6$	b $g^2 \times 7g^5$	c $2b^8 \times 3b^3$	d $-6k^5 \times 2k^8$
e $-2b^8 \times -3b^3$	f $-5g^5 \times -2g \times -8g^5$	g $3c \times 3c^7 \times 3c^6$	h $p^6 \times -3p^2 \times -5p^2$

2B.3 4 Using the quotient of powers law, simplify each expression.

a $a^6 \div a^4$	b $d^7 \div d^6$	c $g^{11} \div g$	d $p^{10} \div p^7$
e $a^8 \div a^3$	f $n^{14} \div n^{11}$	g $r^9 \div r$	h $8x^{17} \div x^6$

5 Use the product and quotient of powers laws to simplify each expression.

a $3x^5 \times 4x^6$	b $5x^4 \times 2x^3$	c $-8x^2 \times 3x^7$	d $-6x^{10} \times -9x$
e $6x^7 \div (2x^3)$	f $-20x^6 \div (-5x^2)$	g $4x^8 \div (10x^7)$	h $15x^{12} \div (9x^4)$
i $\frac{-24t^{18}}{3t^6}$	j $\frac{-20r^8}{-32r^2}$	k $\frac{10c^7}{2c^3}$	l $\frac{15y^{12}}{6y^5}$

6 Use the product and quotient of powers laws to simplify:

a $\frac{ax^{13}}{ax^4}$	b $\frac{a^2b}{a^3}$	c $\frac{m^5n}{m^5}$	d $\frac{b^{20}d}{b^{12}d}$
e $\frac{x^4 \times x^3}{x^2}$	f $\frac{m^7 \times m^6}{m^9}$	g $\frac{6a^2 \times a^8}{a^4}$	h $\frac{n^5 \times n^7}{n^3 \times n^4}$
i $\frac{5d^6 \times d^3}{d^9}$	j $\frac{8t^2 \times t^3}{-2t^3}$	k $\frac{-4k \times 3k^9}{-6k^{10}}$	l $\frac{15e^{13}}{3e^8 \times 5e^5}$

7 Simplify each expression.

a $a^3b^4 \times a^6b^2$	b $6m^5n^2 \times -3m^6n$	c $\frac{c^2d^9}{d^7}$	d $\frac{k^3m^8}{km^5}$
e $x^2 \times y^5 \times x^6 \times y^2$	f $3g^4 \times 5h^3 \times 2g^6$	g $a^5b^4 \times a^3b^2$	h $5x^6y^5 \times 3x^2y^5$
i $9w^4x^8 \times 6x^5y^4$	j $\frac{1}{tu^3} \times \frac{tu^7}{5}$	k $\frac{-6e^5f^{11}}{8e^4f}$	l $-4v^9 \times -9y^3 \times -3v^8y^7$

8 Use the product and quotient of powers laws to simplify each expression.

a $\frac{x^7 \times x^3}{x^4}$

b $\frac{2k^4 \times k^5}{k^6}$

c $\frac{4a^2 \times 3a^6}{2a^7}$

d $\frac{5m^2 \times 2x^4}{10x^6}$

e $\frac{a^5b^7 \times a^3b^6}{a^8b^{10}}$

f $\frac{n^{17}p^{13}}{n^3p^2 \times np^8}$

g $\frac{-6jq^5 \times 5j^7q^2}{15j^3q}$

h $\frac{6zw^9x^6 \times 3zw^4x^5}{9w^5x^4 \times w^6x^3}$

9 Use the product and quotient of powers laws to determine whether each statement is true or false. Explain your reasoning. For each false statement, change the right-hand side to make the statement true.

a $x^3 \times x^4 = x^{12}$

b $k^3 + k^3 = k^6$

c $y^7 \div y = y^6$

d $a^5 \times a \times a^5 = a^{10}$

e $m^3n^5 \times m^2n^4 = m^{14}n^{14}$

f $100^8 \div 100^2 = 100^4$

g $\frac{m^3 \times m^7}{m^{11}} = \frac{1}{m}$

h $\frac{a^5b^6}{a^2b^4} \times \frac{a^3b^5}{a^4b} = a^2$

10 If the index of the denominator is greater than the index of the numerator, we can instead subtract the numerator's index from the denominator's index, leaving the base on the denominator. For example:

$$\frac{2^3}{2^5} = \frac{1}{2^{5-3}} = \frac{1}{2^2}$$

a Simplify the following. Write your answers in index notation.

i $\frac{3^4}{3^{10}}$

ii $\frac{5^2}{5^8}$

iii $\frac{2^5 \times 3^2}{2^9 \times 3^7}$

iv $\frac{2^5 \times 3^7}{2^9 \times 3^2}$

v $\frac{2^9 \times 3^2}{2^5 \times 3^7}$

b Copy and complete the following.

i $\frac{2^3}{2^5} = \frac{2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2} = \frac{1}{\square \times \square} = \frac{1}{2^\square} = \frac{1}{2^{(\square - \square)}}$

ii $\frac{2^4}{2^8} = \frac{\square \times \square \times \square \times \square}{\square \times \square \times \square \times \square \times \square \times \square \times \square \times \square} = \frac{\square}{\square \times \square \times \square \times \square} = \frac{\square}{2^\square} = \frac{\square}{2^{(\square - \square)}}$

iii $\frac{5^6}{5^7} = \frac{\square \times \square \times \square \times \square \times \square \times \square}{\square \times \square \times \square \times \square \times \square \times \square \times \square} = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square^{(\square - \square)}}$

11 Write the following products in index notation with prime number bases.

a $2 \times 4 \times 8 \times 16 \times 32$

b $3 \times 9 \times 27 \times 81 \times 243$

c $6 \times 36 \times 216$

d $4 \times 16 \times 64 \times 256 \times 1024$

12 Determine the values of the unknowns.

a $2^5 \times 3^{4x} \times 5^{12} \times 7^{z+3} = 2^w \times 3^{12} \times 5^{6y} \times 7^{11}$

b $(5^x \times 7^{4y} \times 11^z) \times (3^9 \times 5^6 \times 11) = 3^9 \times 5^{15} \times 7^{24} \times 11^5$

c $\frac{11^a \times 13^b \times 17^{2c} \times 19^8}{11^5 \times 13^6 \times 17^3 \times 19^d} = 11^{11} \times 13^3 \times 17^5 \times 19^2$

13 Do the product and quotient of powers laws work when the terms have different bases? Explain, using $2^4 \times 3^2$ and $y^8 \div x^5$ as examples.

14 Fill in the box to make each statement true.

a $2^\square = 8$

b $3^\square = 27$

c $5^\square = 25$

d $10^\square = 10\,000$

15 Fill in the box to make each statement true. Start by writing the base on the right as a power of the base on the left. For example, $8^4 = (2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3 = 2^{12}$.

a $2^\square = 8^4$

b $3^\square = 27^5$

c $5^\square = 25^9$

d $10^\square = 10\,000^3$

e $4^\square = 16^7$

f $2^\square = 32^6$

g $6^\square = 216^2$

h $3^\square = 243^6$

16 Simplify the following expressions.

a $a^m b^x \times a^n b^y$

b $a^m b^x \div (a^n b^y)$

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Interactive skillsheet

Products and quotients of powers



Worksheet

Multiplying and dividing using index form



Topic quiz

2B

2C Raising indices and the zero index

Learning intentions

By the end of this topic you will be able to ...

- ✓ raise a term in index notation by another index
- ✓ simplify and evaluate expressions and calculations involving the zero index.



Inter-year links

Support

Multiplying and dividing whole numbers

Year 7

1D Multiplying whole numbers

Year 8

4C Raising indices and the zero index

Year 10

2A Indices

Power of a power law



Key content video

- When raising a power to another power, multiply the indices. Writing the term in expanded form and applying the product of powers law achieves the same result, only at a slower pace.

For example, $(2^3)^5 = 2^3 \times 2^3 \times 2^3 \times 2^3 \times 2^3$ is the same as $(2^3)^5 = 2^{3 \times 5}$
 $= 2^{15}$ $= 2^{15}$.

$$(2^3)^5 = 2^{(3 \times 5)} \quad (a^2)^3 = a^{(2 \times 3)}$$
$$= 2^{15} \quad = a^6$$

- To raise an index by another index:
 - 1 Multiply the index of every base inside the brackets by the index outside the brackets. If there is no indicated index for a term, the index is 1 and must still be multiplied.
 - 2 Write the coefficient first, followed by the variables listed in alphabetical order.
- Every term inside brackets should have its index multiplied by the index outside the brackets.

$$(2 \times 3)^5 = 2^5 \times 3^5 \quad (ab)^3 = a^3 \times b^3$$

$$\left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5} \quad \left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$$

The zero index

- Excluding 0, any base with an index of 0 is equal to 1. This is because for every non-zero base, the index indicates the number of times we multiply 1 by the base. If we multiply 1 by the base zero times, we haven't performed any multiplications and are left with 1.

$$2^0 = 1 \quad (-k)^0 = 1$$

- The zero index law can be demonstrated by considering the fact that any non-zero value divided by itself is equal to 1, and then applying the quotient of powers law.

$$\text{For example, } 1 = \frac{a^m}{a^m}$$
$$= a^{(m-m)}$$
$$= a^0.$$

Therefore, $a^0 = 1$.

- The order of operations also applies to simplification. Calculations in grouping symbols should be simplified first. Remember **BIDMAS**: Brackets, Indices, Division and Multiplication, Addition and Subtraction.

Example 2C.1 Raising numerical powers by another index



Use the power of a power law to simplify the following expressions. Give your answers in index notation.

a $(3^4)^5$

b $(4^3 \times 7)^2$

c $\left(\frac{5}{2}\right)^4$

d $3(2^2)^4$

THINK

- a** Multiply the index of the base by the index outside the brackets.
- b** Multiply the index of every base inside the brackets by the index outside the brackets. Remember that base numbers that do not have an indicated index have an index of 1, so $7 = 7^1$.
- c** Multiply the index of every base inside the brackets by the index outside the brackets. Remember that $5 = 5^1$.
- d** Multiply the index of the base by the index outside the brackets. The index outside the brackets only applies to the term inside the brackets.

WRITE

a $(3^4)^5 = 3^{(4 \times 5)}$
 $= 3^{20}$

b $(4^3 \times 7)^2 = 4^{3 \times 2} \times 7^{1 \times 2}$
 $= 4^6 \times 7^2$

c $\left(\frac{5}{2}\right)^4 = \frac{5^{1 \times 4}}{2^{1 \times 4}}$
 $= \frac{5^4}{2^4}$

d $3(2^2)^4 = 3 \times 2^{2 \times 4}$
 $= 3 \times 2^8$

Example 2C.2 Raising algebraic powers by another index



Using the power of a power law, simplify each expression.

a $(2x^4)^3$

b $(3a^2b^3)^2$

c $\left(\frac{-x}{y^2}\right)^3$

d $4(a^2b)^4$

THINK

- a** **1** Multiply the index of every base inside the brackets by the index outside the brackets. Remember that $2 = 2^1$.
2 Write the coefficient first, followed by the variables listed in alphabetical order.
- b** **1** Multiply the index of every base inside the brackets by the index outside the brackets.
2 Write the coefficient first, followed by the variables listed in alphabetical order.
- c** Multiply the index of every base inside the brackets by the index outside the brackets. Recall that if the base is negative and the index is an odd number, then the basic numeral will be negative.
- d** **1** Multiply the index of every base inside the brackets by the index outside the brackets. The index only applies to the terms inside the brackets.
2 Write the coefficient first, followed by the variables listed in alphabetical order.

WRITE

a $(2x^4)^3 = 2^{1 \times 3} x^{4 \times 3}$
 $= 2^3 x^{12}$ or $8x^{12}$

b $(3a^2b^3)^2 = 3^{1 \times 2} a^{2 \times 2} b^{3 \times 2}$
 $= 3^2 a^4 b^6$ or $9a^4 b^6$

c $\left(\frac{-x}{y^2}\right)^3 = \frac{(-x)^{1 \times 3}}{y^{2 \times 3}}$
 $= \frac{-x^3}{y^6}$
 $= -\frac{x^3}{y^6}$

d $4(a^2b)^4 = 4 \times a^{2 \times 4} b^{1 \times 4}$
 $= 4a^8 b^4$

Example 2C.3 Simplifying expressions using the zero index



Use the property $a^0 = 1$ to simplify each expression.

a 23^0

b $3x^0$

c $(x^2)^0$

THINK

- a** Any number, excluding 0, raised to an index of 0 is equal to 1.
- b** Any variable raised to an index of 0 is equal to 1. Recall that an index only applies to the term immediately to its left.
- c** **1** Multiply the index of every base inside the brackets by the index outside the brackets.
2 Any variable raised to the index of 0 is equal to 1.

WRITE

a $23^0 = 1$

b $3x^0 = 3 \times x^0$
 $= 3 \times 1$
 $= 3$

c $(x^2)^0 = x^{2 \times 0}$
 $= x^0$
 $= 1$

Example 2C.4 Simplifying expressions using the index laws



Use the index laws to simplify each expression.

a $(x^3)^5 \times x^2$

b $\frac{4x^8 \times 3x^5}{2x^4 \times (x^3)^3}$

THINK

- a** **1** Use the power of a power law to simplify the first term. Multiply the index of every base inside the brackets by the index outside the brackets.
2 Apply the product of powers law and add the indices of the common base, x .
- b** **1** Simplify the brackets using the power of a power law. Remember BIDMAS.
2 Simplify the numerator and simplify the denominator.
3 Divide the numerator by the denominator. Divide the coefficients. Keep the base and subtract the indices.
4 Use the property $a^0 = 1$ to simplify further.

WRITE

a $(x^3)^5 \times x^2 = x^{3 \times 5} \times x^2$
 $= x^{15} \times x^2$

$= x^{(15+2)}$
 $= x^{17}$

b $\frac{4x^8 \times 3x^5}{2x^4 \times (x^3)^3} = \frac{4x^8 \times 3x^5}{2x^4 \times x^9}$

$= \frac{12x^{13}}{2x^4 \times x^9}$

$= \frac{12x^{13}}{2x^{13}}$
 $= 6x^0$

$= 6 \times 1$
 $= 6$

Helpful hints

- ✓ Take care not to mix up the index laws.
 - across a multiplication sign, add indices
 - across a division sign, subtract indices
 - across brackets, multiply indices
- ✓ Remember that $2^0 = 1$, not 0.

$$a^5 \times a^3 = a^{5+3}$$

$$a^5 \div a^3 = a^{5-3}$$

$$(a^5)^3 = a^{5 \times 3} \quad (ab)^3 = a^3b^3 \quad \left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$$

$$a^0 = 1$$

ANS p430 **Exercise 2C** Raising indices and the zero index

 1–10 (1st, 3rd columns),
12, 13 (a–e), 15 (a, b)

 3–8 (2nd, 4th columns),
9, 10 (c, h, i, l), 11, 13–15, 16 (b), 17

 6–8 (2nd, 4th columns), 9, 10, 13, 15,
16 (c, d), 18–20

2C.1 1 Use the power of a power law to simplify the following expressions. Give your answer in index notation.

a $(6^4)^3$	b $(3^2)^2$	c $(3^3 \times 4)^2$	d $(2^6)^4$
e $(5 \times 2^7)^4$	f $\left(\frac{3}{4}\right)^2$	g $\left(\frac{5^3}{2^2}\right)^4$	h $\left(\frac{1}{8^5}\right)^5$
i $(-3^4)^7$	j $(-3^5)^4$	k $(-7^4 \times -11^3)^7$	l $\left(\frac{13^8}{-17^4}\right)^6$

2C.2 2 Using the power of a power law, simplify each expression.

a $(b^5)^2$	b $(m^4)^2$	c $(j^5)^2$	d $(j^2)^5$	e $(n^{10})^8$	f $(p^{11})^9$
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3 Using the power of a power law, simplify each expression. Give your answer in index notation.

a $(xy)^6$	b $(2d)^3$	c $(-5k)^7$	d $(9p)^{10}$
e $(-3m)^4$	f $\left(\frac{8}{p}\right)^2$	g $\left(\frac{x}{y}\right)^6$	h $(gh)^2$
i $(ab)^5$	j $\left(\frac{k}{m}\right)^3$	k $(-2x)^8$	l $\left(-\frac{d}{3}\right)^5$

4 Using the power of a power law, simplify each expression. Give your answer in index notation.

a $(3x^6)^4$	b $5(a^4b)^7$	c $\left(\frac{2m}{n}\right)^3$	d $\left(\frac{a^2}{b^5}\right)^4$
e $-2(u^3)^4$	f $\frac{4}{3}(v^7w^3)^{10}$	g $\frac{7}{9}\left(\frac{p}{q^6}\right)^7$	h $\frac{1}{2^3}(3^2r^9)^5$
i $8\left(\frac{1}{5^6t^{11}}\right)^4$	j $7(3i^{17})^2$	k $2^3(2^4c^5)^8$	l $\frac{5}{7}\left(\frac{5x^{25}}{7y^{30}}\right)^3$

2C.3 5 Use the property $a^0 = 1$ to simplify each expression.

a 34^0	b $(18)^0$	c y^0	d $(7a)^0$
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6 Use the property $a^0 = 1$ to simplify each expression.

a $2x^0$	b $(2x)^0$	c $-7y^0$	d $(-7y)^0$
e $-(-3c)^0$	f $8^0 + 4^0$	g $2 \times 5^0 - 3^0$	h $m^0 + m^0$
i $n^0 + p^0$	j $a^0 + b^0 + c^0$	k $(x + y)^0$	l $(-a^0)^4$
m $(5^3)^0$	n $(-8)^0$	o -8^0	p $-(-3)^0$

2C.4 7 Use the index laws to simplify each expression.

a $(x^2)^4 \times x^5$	b $(x^5)^3 \times x^7$	c $x^3 \times (x^4)^6$	d $(x^3)^2 \times (x^7)^3$
e $\frac{x^4 \times (x^3)^5}{x^9}$	f $\frac{(zw^2)^4 \times (zw^5)^2}{(zw^4)^3}$	g $\frac{6(b^4)^4 \times (b^3)^2}{18b^{21}}$	h $\frac{e^5 \times e^8}{e^3 \times e^4}$
i $\frac{(x^6)^2 \times x^3}{x^5 \times (x^2)^5}$	j $\frac{4a^6 \times 6(a^3)^4}{2a^4 \times 3a^5}$	k $\frac{t^8}{(t^2)^5} \times \frac{(t^6)^7}{t^{15}}$	l $(f^6)^9 \times \left(\frac{f^7}{f^2}\right)^{11}$

8 Use the index laws to simplify each expression.

a $a^3 \div a^3$	b $-7x^9 \div x^9$	c $(m^2)^3 \div m^6$	d $-18(b^4)^5 \div [-6(b^5)^4]$
e $y^7 \times y \div y^8$	f $(k^6)^0 \times k^2$	g $5g^4 \times 2(-g^7)^0$	h $3(zw^5)^2 \div (zw^2)^5$
i $x^8 \times (x^2)^5 \div x^3$	j $4p^7 \times 3p^2 \div (6p^9)$	k $16(b^3)^3 \div [-2(b^2)^4]$	l $4m^5 \times m \div [10(m^3)^2]$

9 Use the index laws to simplify each expression.

a $\frac{5(n^7)^2 \times -6(n^2)^3}{15n^2 \times (n^3)^6}$	b $\frac{(k^8)^2 \times k \times k^9}{k^3 \times (k^4)^2 \times k^5}$	c $(x^4)^2y^7 \times x^3y^2$
d $\frac{4(m^3)^4n^2 \times (m^2)^3}{8m^5n^6 \times mn}$	e $\frac{-3h^7k^5 \times 2h^6(k^3)^2}{(h^2)^6k^3 \times -6h(k^4)^2}$	f $\frac{(b^2)^2 \times ac}{a^2b \times c^3} \times bc \div a$

10 Use the index laws to simplify each expression.

a $(xy)^3 \times x^6y^4$	b $(2k)^5 \times (7k)^2$	c $(-3x^6)^4$	d $-5(a^4b)^7$
e $\frac{x^4}{y^5} \times \left(\frac{x}{y}\right)^6$	f $\left(\frac{2m}{n}\right)^3$	g $\left(\frac{a^2}{b^5}\right)^4$	h $\left(\frac{w^5x^3}{y^4}\right)^2$
i $\left(\frac{k^3m}{n^2}\right)^5 \times \left(\frac{n^3}{k^2m}\right)^4$	j $\left(\frac{t^4}{r^2p^3}\right)^5 \times \frac{(4r^5)^2}{p^6t^7}$	k $\frac{(a^3b^2)^5 \times (ab^4)^6}{(a^5b)^4}$	l $\frac{(3e^4)^2(2h^6)^3}{(e^2h^3)^4}$

11 **a** Simplify $a^3 \div a^3$ by first writing the expression as a fraction with each term in expanded form.

b Simplify $a^3 \div a^3$ using an index law. Leave your answer in index notation.

c Use your answers to parts **a** and **b** to explain why $a^0 = 1$.

12 Use the index laws to decide whether each statement is true or false. Explain your reasoning. For each false statement, change the right-hand side to make the statement true.

a $(3g)^4 = 3^4 \times g^4$	b $-8^0 = -1$	c $\left(\frac{x}{y}\right)^6 = \frac{x^6}{y}$	d $\frac{(k^3)^2 \times k^4}{k^2} = k^5$
e $6 + k^0 = 7$	f $100^9 \div 100^9 = 0$	g $\frac{m^3 \times m^8}{m^{11}} = 1$	h $a^0 \times b^0 = 0$

13 Find the value of x that will make each statement true.

a $2^x = 2^7$	b $5^x \times 5^2 = 5^6$	c $4^x = 1$	d $7^x \div 7^3 = 7^5$
e $(9^x)^2 = 9^6$	f $\left(\frac{2}{3}\right)^x = \frac{32}{243}$	g $\frac{6^x \times 6^3}{6^5} = 6^5$	h $(3a^x)^4 = 81a^{20}$

14 Eden simplified $3^4 \times (3^5)^3$ as $(3^9)^3 = 3^{27}$. Explain and correct her mistake.

15 The power of a power law can be explained using the product of powers law.

Complete the following.

a $(2^3)^5 = (2^3) \times (2^3) \times \square \times \square \times \square$ $= 2^{3+3+\square+\square+\square}$ $= 2^3 \times \square$	b $(x^7)^4 = (x^7) \times \square \times \square \times \square$ $= x^{7+\square+\square+\square}$ $= x^7 \times \square$
c $(2 \times 3)^4 = (2 \times 3) \times \square \times \square \times \square$ $= 2 \times \square \times \square \times \square \times 3 \times \square \times \square \times \square$ $= 2^\square \times 3^\square$	d $\left(\frac{2}{3}\right)^6 = \frac{2}{3} \times \frac{\square}{\square} \times \frac{\square}{\square} \times \frac{\square}{\square} \times \frac{\square}{\square} \times \frac{\square}{\square}$ $= \frac{2^\square}{3^\square}$

16 We can describe repeated addition in terms of multiplication, $2 + 2 + 2 = 2 \times 3$, and repeated multiplication in terms of raising a base to the power of an index, $2 \times 2 \times 2 = 2^3$. However, repeatedly raising a number to the same index does not require a new operation, as it can be simplified using the index laws.

For example: $((2^5)^5)^5 = 2^{5 \times 5 \times 5} = 2^{125} = 2^{125}$.

Write the following in index notation with a single index.

a $(((((3^2)^2)^2)^2)^2)^2$	b $((((5^3)^3)^3)^3)$	c $(((((7^4)^4)^4)^4)^4)$	d $(((((10^5)^5)^5)^5)^5)$
------------------------------------	------------------------------	----------------------------------	-----------------------------------

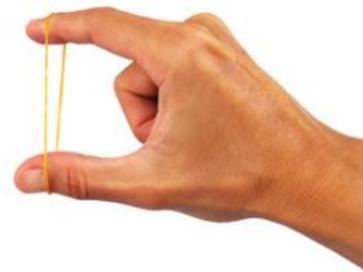
17 A cube has side lengths of 8^5 cm. What is the volume of the cube in cm^3 ? Write your answer in index notation.

18 A rubber band is stretched to $\frac{4}{3}$ of its current length, and this is repeated another four times until it snaps. How many times longer was the rubber band when it snapped than it was originally?

19 Use the power of a power law to show that $(a^m)^n = (a^n)^m$.

20 Solve the following equation for x .

$$\frac{12x^6y^7}{7x^2y^{10}z^4} \times \frac{35y^3z^4}{3x^3} = 3$$



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Interactive skillsheet
Raising indices and the zero index



Topic quiz
2C

Checkpoint



Checkpoint quiz

Check your knowledge of the first part of this chapter.

2A 1 Write the following in expanded form, then evaluate.

a 2^6 **b** $(-3)^4$ **c** $(-4)^3$ **d** $\left(\frac{5}{6}\right)^3$

2A 2 Write the following in expanded form.

a a^6 **b** $(-b)^4$ **c** $(3y)^5$ **d** $3(xy)^5$

2A 3 Write the following in index notation.

a $8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$ **b** $u \times u \times u \times u$
c $4b \times 4b \times 4b \times 4b \times 4b$ **d** $-7 \times k \times k \times k \times k \times h \times h \times h \times h \times h$

2A 4 Write the following numbers as a product of their prime factors. Express your answers in index notation.

a 28 **b** 72 **c** 484 **d** 270

2B 5 Use the index laws to simplify each expression. Express your answers in index notation.

a $8^5 \times 8^6$
b $5^7 \times 7^4 \times 5^3 \times 7^8$
c $\frac{6^8}{6^3}$
d $\frac{3^{14} \times 10^{12}}{3^6 \times 10^5}$

2B 6 Use the index laws to simplify each expression.

a $a^3 \times a^9$
b $4b^{11}c^8 \times -3b^7c^{13}$
c $\frac{u^{14}}{u^9}$
d $\frac{-15p^{17}q^{21}}{-21p^3q^4}$

2B 7 Use the index laws to simplify each expression. Write your answers in index notation.

a $\frac{3^7 \times 3^{12}}{3^9}$
b $\frac{k^{23}}{k^7 \times k^8}$
c $\frac{c^3t^8}{c^{14}t^7}$
d $\frac{9d^7w^4}{10d^{12}w^7} \times \frac{25d^{17}w^{12}}{6dw^2}$

2C 8 Use the index laws to simplify each expression.

a 87^0 **b** $t^5 \div t^5$ **c** $-(4g)^0$ **d** $7a^0 + (8b)^0$

2C 9 Use the index laws to simplify each expression. Write your answers in index notation.

a $(3^4)^6$ **b** $(j^5)^9$ **c** $(-5a^3b^7)^6$ **d** $-\left(\frac{3p^5}{2q^7}\right)^8$

2C 10 Use the index laws to simplify each expression. Write your answers in index notation.

a $\frac{t^4 \times (t^2)^3}{t^{10}}$
b $\frac{3(g^2)^8 \times (3g^5)^3}{(3g)^{11}}$
c $\frac{(5m^{11}n^{10})^8 \times (5mn^6)^6}{(5m^9n^7)^2 \times (5m^2n^3)^5}$
d $\frac{(8j^5p)^0 \times 6(j^0p^4)^3}{(j^7p^2)^6}$

2D Negative indices

Learning intentions

By the end of this topic you will be able to ...

- ✓ write a term with a negative index as a term with a positive index
- ✓ write a term with a positive index as a term with a negative index
- ✓ apply index laws to numerical expressions with negative indices
- ✓ simplify and evaluate numerical expressions with negative indices.



Inter-year links

- Year 7** 3F Multiplying fractions
- Year 8** 2C Multiplying and dividing fractions
- Year 10** 2B Negative indices

Reciprocals and negative indices



Key content video

- The **reciprocal** of a number can be found by dividing 1 by that number.
For example, the reciprocal of 3 is $\frac{1}{3}$.
→ The product of a number and its reciprocal is 1.
For example, $3 \times \frac{1}{3} = 1$.
- The reciprocal of a fraction can be found by swapping the numerator with the denominator.
For example, the reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$.
- A **negative index** is the reciprocal of the base with a positive index.
For example, $2^{-1} = \frac{1}{2^1} = \frac{1}{2}$ and $4^{-2} = \frac{1}{4^2}$.
- Negative indices can be used to write fractions in index notation.
- The index laws also apply to expressions containing terms with negative indices.

Example 2D.1 Determining the reciprocals of numbers



Determine the reciprocal of each of the following.

- a** 3 **b** $\frac{3}{2}$ **c** $\frac{1}{4}$

THINK

- 1 Write the base with a negative index as a fraction if it is not already.
- 2 Find the reciprocal of the fraction. Swap the numerator and denominator.
- 3 Simplify the result.

WRITE

- a** $3^{-1} = \frac{1}{3}$
- b** $\left(\frac{3}{2}\right)^{-1} = \frac{2}{3}$
- c** $\left(\frac{1}{4}\right)^{-1} = \frac{4}{1}$
 $= 4$

Example 2D.2 Writing a term with a positive index



Write each power with a positive index.

a 3^{-3}

b 7^{-4}

THINK

- 1 Write the reciprocal of the base.
- 2 Change the negative index to a positive index.

WRITE

a $3^{-3} = \frac{1}{3^3}$

b $7^{-4} = \frac{1}{7^4}$

Example 2D.3 Writing fractions with positive indices



Write each fraction in index notation with a positive index.

a $\frac{1}{3^{-2}}$

b $\left(\frac{2}{5}\right)^{-3}$

THINK

- 1 Write the reciprocal of the fraction.
- 2 Change the negative index to a positive index.
- 3 Use the power of a power law to remove the brackets. Recall that a number without an indicated index has an index of 1.
- 4 Simplify the result.

WRITE

a $\frac{1}{3^{-2}} = \frac{3^{-2}}{1}$
 $= 3^2$

b $\left(\frac{2}{5}\right)^{-3} = \left(\frac{5}{2}\right)^3$
 $= \frac{5^3}{2^3}$

Example 2D.4 Simplifying expressions with negative indices using index laws



Use an appropriate index law to simplify each expression. Write your answers using positive indices.

a $3^5 \times 3^{-7}$

b $2^4 \div 2^{-3}$

c $(5^{-6})^2 \times 5^3$

THINK

- a**
- 1 Apply the product of powers law to multiply the terms. Write the base and add the indices.
 - 2 Find the reciprocal of the fraction and write the index as a positive number.
- b**
- 1 Apply the quotient of powers law to divide the terms. Write the base and subtract the indices.
- c**
- 1 Apply the power of a power law to simplify the first term. Multiply the index of every base inside the brackets by the index outside the brackets.
 - 2 Apply the product of powers law to multiply the terms. Write the base and add the indices.
 - 3 Find the reciprocal of the fraction and write the index as a positive number.

WRITE

a $3^5 \times 3^{-7} = 3^{(5 + (-7))}$
 $= 3^{-2}$
 $= \frac{1}{3^2}$

b $2^4 \div 2^{-3} = 2^{(4 - (-3))}$
 $= 2^7$

c $(5^{-6})^2 \times 5^3 = 5^{-6 \times 2} \times 5^3$
 $= 5^{-12} \times 5^3$
 $= 5^{(-12 + 3)}$
 $= 5^{-9}$
 $= \frac{1}{5^9}$

✓ If you want to move a number or a variable from the numerator to the denominator, remember that 1 will be left in its place, not zero.

For example: $4^{-2} = \frac{1}{4^2}$.

✓ Don't confuse negative indices with negative numbers. For example: $2^{-3} = \frac{1}{2^3}$ and $2^{-3} \neq -(2^3)$.

ANS
p431

Exercise 2D Negative indices

▲ 1-9(a-d), 11(a-d), 12, 15, 18(a, b), 22(a, b)

■ 6-9(e-h), 11(e-h), 13, 16, 19, 21, 22(e-h)

◆ 1(e-h), 6-7(e-h), 8-10(c-f), 11(e-h), 14, 17, 18, 20, 22(i-p)

2D.1 1 Determine the reciprocal of each of the following.

- | | | | |
|------------------|-----------------|-----------------|------|
| a $\frac{8}{7}$ | b $\frac{1}{2}$ | c -3 | d 9 |
| e $-\frac{7}{3}$ | f -9 | g $\frac{2}{9}$ | h 15 |

2D.2 2 Write each power with a positive index.

- | | | | |
|------------|---------------|---------------|-------------|
| a 5^{-1} | b 8^{-1} | c $(-2)^{-1}$ | d 4^{-2} |
| e 2^{-6} | f $(-9)^{-3}$ | g $(-5)^{-4}$ | h -7^{-8} |

3 Write each of the following in index notation with a negative index.

- | | | | |
|-----------------|-------------------|------------------|--------|
| a $\frac{1}{5}$ | b $-\frac{1}{13}$ | c 5 | d -8 |
| e $\frac{2}{7}$ | f -21 | g $-\frac{4}{3}$ | h 12 |

4 Write each fraction in index notation with a negative index.

- | | | | |
|----------------------|---------------------|--------------------|-----------------------|
| a $\frac{1}{3^4}$ | b $\frac{1}{4^7}$ | c $\frac{1}{6^5}$ | d $\frac{1}{(-5)^3}$ |
| e $\frac{1}{(-9)^2}$ | f $-\frac{1}{11^6}$ | g $-\frac{1}{4^3}$ | h $\frac{-1}{(-6)^3}$ |

5 Write each fraction in index form with a prime base and a negative index.

- | | | | |
|------------------|-------------------|------------------|------------------|
| a $\frac{1}{49}$ | b $\frac{1}{125}$ | c $\frac{1}{16}$ | d $\frac{1}{81}$ |
|------------------|-------------------|------------------|------------------|

2D.3 6 Write each fraction in index notation with a positive index.

- | | | | |
|-------------------------|-----------------------|--------------------------|--------------------------|
| a $\frac{1}{2^{-3}}$ | b $\frac{1}{5^{-6}}$ | c $\frac{1}{(-8)^{-4}}$ | d $\frac{1}{3^{-9}}$ |
| e $\frac{1}{(-7)^{-5}}$ | f $-\frac{1}{4^{-2}}$ | g $-\frac{1}{(-7)^{-3}}$ | h $-\frac{1}{(-8)^{-7}}$ |

7 Write each fraction in index notation with a positive index.

- | | | | |
|-----------------------------------|------------------------------------|---------------------------------------|---------------------------------------|
| a $\left(\frac{4}{5}\right)^{-2}$ | b $\left(\frac{7}{3}\right)^{-1}$ | c $\left(-\frac{3}{4}\right)^{-3}$ | d $\left(-\frac{9}{7}\right)^{-11}$ |
| e $\left(\frac{6}{5}\right)^{-7}$ | f $\left(-\frac{9}{2}\right)^{-9}$ | g $\left(-\frac{13}{17}\right)^{-14}$ | h $\left(\frac{500}{43}\right)^{-11}$ |

2D.4 8 Use an appropriate index law to simplify each expression. Write your answers in index notation with positive indices.

- | | | | |
|-----------------------|---------------------------|------------------------------|--------------------------------|
| a $4^{-5} \times 4^2$ | b $7^3 \times 7^{-4}$ | c $2^{-6} \times 2^8$ | d $(-3)^{-1} \times (-3)^{-5}$ |
| e $5^7 \times 5^{-3}$ | f $(-2)^{-4} \div (-2)^3$ | g $9^5 \div 9^7$ | h $3^6 \div 3^{-2}$ |
| i $4^{-1} \div 4^8$ | j $10^{-7} \div 10^{-4}$ | k $211^{-9} \times 211^{-5}$ | l $13^{-87} \div 13^{13}$ |

9 Use an appropriate index law to simplify each expression. Write your answers in index notation with positive indices.

a $(5^{-3})^2$	b $(3^{-2})^4$	c $(-2^{-4})^{-1}$	d $(3^{-1})^4 \times 3^2$
e $(6^{-5})^3 \times 6^{11}$	f $(4^{-2})^3 \times (4^{-5})^{-1}$	g $9^3 \times 9^{-6} \times 9^2$	h $\frac{5^4 \times 5^{-2}}{5^{-6}}$
i $\frac{7^{-5} \times 7^{-3}}{7^{-4} \times 7^{-7}}$	j $\frac{2^8 \times (2^{-2})^3}{2^5}$	k $\frac{(99^{-12})^{-6} \times 99^{15}}{(99^8)^{-5}}$	l $\frac{(15^{-9})^8 \times (15^7)^6}{(15^{11})^{12}}$

10 Using a calculator, calculate the basic numeral for parts **a–f** in question 7. Write your answers as an integer or fraction.

11 Find the value of x that will make each statement true.

a $2^x = \frac{1}{2^3}$	b $5^x = \frac{1}{5^7}$	c $3^x = \frac{1}{3}$	d $6^x = \frac{1}{6^{-2}}$
e $4^x = \frac{1}{16}$	f $3^x = \frac{1}{27}$	g $5^x = \frac{1}{25}$	h $10^x = \frac{1}{10\,000}$

12 **a** Complete this table.

Index notation	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}
Basic numeral	32	16	8			1		$\frac{1}{4}$			

b Describe the pattern you can see in the table.

c Following the pattern, write 2^{-6} as a fraction.

d Use the fact that $2^{10} = 1024$ to write the value of 2^{-10} as a fraction.

e Use the fact that $2^{-7} = \frac{1}{128}$ to write the value of 2^7 .

13 **a** Complete this table.

Index notation	3^5	3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}	3^{-4}	3^{-5}
Basic numeral		81	27			1		$\frac{1}{9}$			

b Describe the pattern you can see in the table.

c Following the pattern, write 3^{-6} as a fraction.

d Use the fact that $3^8 = 6561$ to write the value of 3^{-8} as a fraction.

e Use the fact that $3^{-7} = \frac{1}{2187}$ to write the value of 3^7 .

14 **a** Complete this table.

Index notation	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
Basic numeral			100			$\frac{1}{10}$	$\frac{1}{100}$		

b Describe the pattern you can see in the table.

c Write the value of each of the following terms as a whole number.

i 10^5	ii 10^6	iii 10^7	iv 10^8	v 10^9
-----------------	------------------	-------------------	------------------	-----------------

d Write the value of each of the following terms as a fraction.

i 10^{-5}	ii 10^{-6}	iii 10^{-7}	iv 10^{-8}	v 10^{-9}
--------------------	---------------------	----------------------	---------------------	--------------------

e Write 10^{-1} as:

i a fraction	ii a decimal.
---------------------	----------------------

f Write each of the following terms as a decimal. Hint: Use the matching fractions from your table.

i 10^{-2}	ii 10^{-3}	iii 10^{-4}
--------------------	---------------------	----------------------

g Write each fraction from your answers to part **d** as a decimal.

h Explain any shortcuts you used to obtain your answers to parts **c–g**.

15 A microscopic worm is 4^{-3} mm in length. Using a calculator, write this length in millimetres:

- a** as a fraction **b** as a decimal.

16 The time for light to travel 3 m is about 10^{-8} s. Using a calculator, write this time in seconds:

- a** as a fraction **b** as a decimal.

17 The diameter of a strand of human hair is about 5^{-6} m. Using a calculator, write this measurement in metres:

- a** as a fraction **b** as a decimal.

18 **a** Without using a calculator, find the whole number value of each of the following. Hint: What shortcut can you use when multiplying by a positive power of 10?

- i** 2×10^4 **ii** 7×10^3 **iii** 3×10^5
iv 4×10^{11} **v** 9×10^7

b Write each expression as a fraction involving positive indices.

- i** 5×10^{-2} **ii** 8×10^{-5} **iii** 2×10^{-3}
iv 7×10^{-4} **v** 6×10^{-9}

c Without using a calculator, find the decimal value of each result in part **b**. Hint: What shortcut can you use when dividing by a positive power of 10?

d Use your results from part **c** to describe a shortcut that can be used when multiplying by a negative power of 10.

19 **a** Complete the following by writing the missing numerals and operations.

i $2 \times 3^{-1} = 2 \times \frac{\square}{\square} = \frac{2}{\square} = 2\square3$ **ii** $2 \div 3^{-1} = \frac{2}{\square} = 2 \times \frac{\square}{\square} = 2\square3$

b Explain the connection between multiplication, division and reciprocals.

20 **a** Evaluate the following.

i $(3^{-1})^{-1}$ **ii** $(5^{-1})^{-1}$ **iii** $\left(\left(\frac{5}{4}\right)^{-1}\right)^{-1}$

b Use index laws to explain why $(a^{-1})^{-1} = a$.

c Explain what $(a^{-1})^{-1} = a$ means in terms of reciprocals.

21 Write the following as products without fractions by using negative indices.

For example: $\frac{9x^3}{y^2} = 9x^3y^{-2}$.

- a** $\frac{2}{a^2b}$ **b** $-\frac{3t^2}{v^3}$ **c** $\frac{x^4}{5y^4}$ **d** $\frac{1}{pq^5r^{-2}}$

22 Use an appropriate index law to simplify each expression. Write your answers using positive indices only.

- a** $x^4 \times x^{-6}$ **b** $x^{-3} \times x^{-1}$ **c** $4x^{-2} \times 2x^5$ **d** $5x^{-8} \times 6x^3$
e $3x^7 \times x^{-7}$ **f** $x^5 \div x^{-4}$ **g** $x^{-10} \div x^{-7}$ **h** $4x^3 \div (2x^{-2})$
i $6x^{-6} \div (18x^4)$ **j** $8x^7 \div (14x^{11})$ **k** $(x^{-2})^3 \times x^4$ **l** $(x^4)^5 \times x^{-9}$
m $(x^{-5})^3 \times 4x^2$ **n** $2x^{-3} \times (x^{-1})^5$ **o** $(x^{-4})^2 \times (x^{-3})^{-1}$ **p** $(xy)^{-7}$



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Interactive skillsheet
Negative indices



Topic quiz
2D

2E Scientific notation

Learning intentions

By the end of this topic you will be able to ...

- ✓ convert numbers written in scientific notation to basic numerals
- ✓ convert numbers written as basic numerals to scientific notation.



Inter-year links

- Support** Place value
- Year 7** 1A Place value
- Year 8** 1A Rounding and estimating

Scientific notation



Key content video

- **Scientific notation** (or **standard form**) is a way of writing very large and very small numbers.
- A number is written in scientific notation if it is the product of a number, a , between 1 (inclusive) and 10 (exclusive) or -1 and -10 , and a power of 10, written in index notation.
 - That is, $a \times 10^m$, where $1 \leq a < 10$ or $-10 < a \leq -1$ and m is an integer.
- When the value of a is positive:
 - If m is a positive integer, the number is larger than or equal to 10.
 - If m is a negative integer, the number is between 0 and 1.
 - If m is zero, the number is between 1 and 10.
- The index laws can be used to perform operations on numbers in scientific notation.
- To convert a number in scientific notation to a basic numeral, the index indicates the number of places the decimal point is moved.
 - If the index is positive, move the decimal point to the right.
 - If the index is negative, move the decimal point to the left.
- To write a number in scientific notation, place the decimal point after the first non-zero digit and multiply by the appropriate power of 10.

$$\begin{array}{l} 1230 = 1.23 \times 10^3 \\ \text{basic numeral} \quad \text{scientific notation} \end{array}$$

$$\begin{array}{l} 0.00123 = 1.23 \times 10^{-3} \\ \text{basic numeral} \quad \text{scientific notation} \end{array}$$

10^+ →

10^- ←

$$\begin{array}{l} 31500 = 3.15 \times 10^4 \leftarrow \text{index of } 4 \\ \text{move four spaces to the left} \end{array}$$

$$\begin{array}{l} 0.042 = 4.2 \times 10^{-2} \leftarrow \text{index of } -2 \\ \text{move two spaces to the right} \end{array}$$

- While we can write the expanded form of a basic numeral using powers of 10, scientific notation uses only the highest power of 10 from the expansion.

Place value	Thousands	Hundreds	Tens	Ones	.	Tenths	Hundredths	Thousandths
Index notation	10^3	10^2	10^1	10^0	.	10^{-1}	10^{-2}	10^{-3}
Basic numeral	1000	100	10	1	.	0.1	0.01	0.001

Metric prefixes

- A prefix is a letter or group of letters added to the beginning of a word to change its meaning. **Metric prefixes** can be used for very small or very large units of measurement, meaning that measurements of weight, distance and time can be expressed to a high degree of accuracy by using only a few digits.
 - Grams (g), metres (m) and seconds (s) are the base units for weight, distance and time, respectively.
- The following table details the most common metric prefixes:

Prefix	Abbreviation	Meaning	Power of 10
tera	T	one trillion of the unit	10^{12}
giga	G	one billion of the unit	10^9
mega	M	one million of the unit	10^6
kilo	k	one thousand of the unit	10^3
centi	c	one-hundredth of the unit	10^{-2}
milli	m	one-thousandth of the unit	10^{-3}
micro	μ	one-millionth of the unit	10^{-6}
nano	n	one-billionth of the unit	10^{-9}
pico	p	one-trillionth of the unit	10^{-12}

- The prefixes can be added before the full names of the base units to create new units. For example, one kilogram is equal to 1000 grams.
- Similarly, the abbreviations can be added before the abbreviated names of the base units. For example, cm is the abbreviation for centimetres, and there are 100 centimetres in a metre.

Example 2E.1 Converting numbers written in scientific notation to basic numerals



Write each number as a basic numeral.

a 2.4×10^6

b 7.1×10^{-8}

THINK

- a** Multiply by 10^6 (or 1 000 000). When multiplying by 10^6 , move the decimal point six place-value spaces to the right. Add zeroes where necessary.
- b** Multiply by 10^{-8} (or divide by 10^8). When dividing by 10^8 , move the decimal point eight place-value spaces to the left. Add zeroes where necessary.

WRITE

a 2.400000

$2.4 \times 10^6 = 2400\ 000$

b 00000007.1

$7.1 \times 10^{-8} = 0.000\ 000\ 071$



Example 2E.2 Converting numbers written as basic numerals to scientific notation

Write each number in scientific notation.

a 230 000

b 0.000 856

THINK

- a** Count the number of places the decimal point in 230 000 would be moved to produce 2.3. The decimal point needs to be moved five places to the right to obtain the original number, so the index is 5.
- b** Count the number of places the decimal point in 0.000 856 would be moved to produce 8.56. The decimal point needs to be moved four places to the left to obtain the original number, so the index is -4 .

WRITE

a 230000
 $230\,000 = 2.3 \times 10^5$

b 0.000856
 $0.000\,856 = 8.56 \times 10^{-4}$

Helpful hints

- ✓ When converting from scientific notation to a basic numeral, remember that if the index is positive, move the decimal point to the right, and if the index is negative, move the decimal point to the left.
- ✓ Multiplying a number by 10 increases each digit's place value by one column. Move the decimal point one place-value space to the right and insert a zero where necessary.

$$5.23 \times 10 = 52.3$$

$$5.23 \times 100 = 523.$$

$$5.23 \times 1000 = 5230.$$

- ✓ Dividing a number by 10 decreases each digit's place value by one column. Move the decimal point one place-value space to the left and insert a zero where necessary.

$10^+ \rightarrow$

$10^- \leftarrow$

ANS p432 Exercise 2E Scientific notation

1, 2, 3(1st column), 5(a-h), 6, 7, 8(a-d), 9, 10, 12(a-c), 15, 16

3(d, g, i, k), 4, 5(e-l), 6, 8(e-h), 11, 12, 14, 19, 22

4, 5(i-p), 8(e-h), 12, 13, 17, 18, 20, 21, 23

1 Calculate each of these. Hint: Move the decimal point an appropriate number of places.

a 5.4×100

b $7.36 \times 10\,000$

c -1.8×1000

d $4.05 \times 100\,000$

e $2.753 \times 1\,000\,000$

f $\frac{6.1}{10}$

g $\frac{8.22}{1\,000\,000}$

h $\frac{-9.76}{10\,000}$

i $\frac{7.003}{100\,000}$

2 Write each number as a power of 10.

a 100

b 1000

c 10 000

d 100 000

e 1 000 000

f 0.1

g 0.01

h 0.001

i 0.0001

j 0.000 01

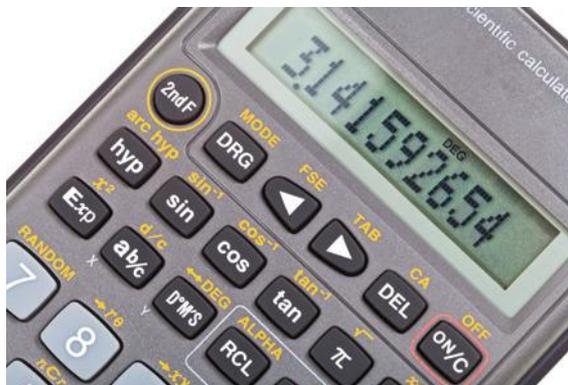
2E.1 3 Write each number as a basic numeral.

- | | | |
|---------------------------------|-------------------------------|----------------------------------|
| a 3.2×10^5 | b 8.14×10^9 | c -5.0×10^2 |
| d -2.345×10^7 | e 1.1×10^4 | f 6.4×10^{-3} |
| g 7.28×10^{-6} | h 9×10^{-7} | i -3.02×10^{-5} |
| j -5.41×10^{-2} | k 4.5×10^{11} | l 6.12×10^{-9} |
| m 5.7×10^{-1} | n 1.3068×10^3 | o 2.7316×10^{-4} |

4 Calculators use different methods for entering in and displaying numbers in scientific notation. Most scientific calculators have a button for entering numbers in scientific notation quickly. It is usually labelled with a bold **E**, **Exp**, $\times 10^x$ or $\times 10^n$.

Check with your teacher if you cannot find this button. To use the button, type the value of a , press the scientific notation button, and then type the index of 10.

- a** Use a calculator to verify each number in question 3.
b Were there any numbers that you could not easily display on your calculator? Explain.



2E.2 5 Write each number in scientific notation.

- | | | | |
|----------------------|--------------------|-----------------------|--------------------|
| a 4500 | b 7 320 000 | c 200 000 | d -190 |
| e 3216 | f 0.0063 | g 0.000 000 18 | h 0.05 |
| i -0.000 0702 | j 0.427 | k 11 220 | l 0.000 004 |
| m -568.2 | n 0.000 249 | o 679 300 | p -0.0102 |

6 Consider the following numbers **A–H**.

- | | | | |
|----------------------------|--------------------------------|------------------------------|------------------------------|
| A 3.4×10^4 | B 2.03×10^{-3} | C -0.58×10^6 | D 60.34×10^2 |
| E 0.009 | F -4.19×10^3 | G 700×10^5 | H 9×10^{-4} |

- a** Which numbers are written in scientific notation?
b Which numbers are larger than 10?
c Which numbers are less than 1?

7 Put the following numbers expressed in scientific notation in ascending order.

3.51×10^3 , 2.814×10^4 , 8.02×10^3 , 7.422×10^2 , 9.10×10^3 , 5.76×10^2

8 Convert the following metric units into the units specified in brackets. Hint: Use the metric prefix table.

- | | |
|---|--|
| a 13 centimetres (metres) | b 2 kilolitres (litres) |
| c 99 kilometres (centimetres) | d 5 milliseconds (microseconds) |
| e 0.4 litres (microlitres) | f 3000 nanoseconds (seconds) |
| g 0.001 gigalitres (centilitres) | h 755 nanometres (millimetres) |

9 Write each approximate measurement in scientific notation.

- a** A medium-sized grain of sand has a length of 0.0005 m.
b Lake Eucumbene has a capacity of approximately 4 800 000 ML.
c The thickness of the epidermal layer of skin on your eyelid is 0.048 mm.
d An estimate for the world's population in 2050 is 9 300 000 000.

10 Write each approximate measurement as a basic numeral.

- a** The number of times the wings of a hummingbird flap in a minute is 6.4×10^3 .
b The diameter of a virus is 8×10^{-5} mm.
c The distance from the Sun to Earth is 1.496×10^8 km.
d The radius of an electron is 2.8×10^{-13} cm.



- 11 Complete the table below by writing the numbers as a product with each of the powers of 10. Underline the answers that are in scientific notation. The first row has been completed for you.

	1234.56	4.0191	0.0492	0.007 40
$\times 10^3$	<u>$1.234\ 56 \times 10^3$</u>	$0.004\ 0191 \times 10^3$	$0.000\ 0492 \times 10^3$	$0.000\ 007\ 40 \times 10^3$
$\times 10^2$				
$\times 10^1$				
$\times 10^0$				
$\times 10^{-1}$				
$\times 10^{-2}$				
$\times 10^{-3}$				

- 12 We can perform arithmetic operations in scientific notation. Multiplication and division can be performed by multiplying or dividing the values of a and then the product and quotient of powers laws to multiply or divide the powers of 10. For example:

$$\begin{aligned}
 (2.1 \times 10^7) \times (8.4 \times 10^3) &= (2.1 \times 8.4) \times (10^7 \times 10^3) \\
 &= 17.64 \times 10^{10} \\
 &= (17.64 \div 10) \times 10^{(10+1)} \\
 &= 1.764 \times 10^{11}
 \end{aligned}
 \qquad
 \begin{aligned}
 (2.1 \times 10^7) \div (8.4 \times 10^3) &= (2.1 \div 8.4) \times (10^7 \div 10^3) \\
 &= 0.25 \times 10^4 \\
 &= (0.25 \times 10) \times 10^{(4-1)} \\
 &= 2.5 \times 10^3
 \end{aligned}$$

Evaluate the following products and quotients. Write your answers in scientific notation.

- a** $(1.7 \times 10^5) \times (4 \times 10^2)$
 b $(8 \times 10^7) \div (4 \times 10^5)$
 c $(-5 \times 10^{-5}) \times (-9 \times 10^8)$
d $(-6 \times 10^9) \div (1.5 \times 10^5)$
 e $(4.1 \times 10^{-6}) \times (-3 \times 10^4)$
 f $(7.2 \times 10^{-2}) \div (2.4 \times 10^{-7})$

- 13 Addition and subtraction require digits with the same place value to be added together. Therefore, in scientific notation, both numbers must be written using the same power of 10 so that the digits in the values of a have the same place value relative to the decimal point. For example:

$$\begin{aligned}
 (2.1 \times 10^5) + (8.4 \times 10^3) &= (2.1 \times 10^5) + (0.084 \times 10^5) \\
 &= (2.1 + 0.084) \times 10^5 \\
 &= 2.184 \times 10^5
 \end{aligned}
 \qquad
 \begin{aligned}
 (2.1 \times 10^{-2}) - (8.4 \times 10^{-3}) &= (2.1 \times 10^{-2}) - (0.84 \times 10^{-2}) \\
 &= (2.1 - 0.84) \times 10^{-2} \\
 &= 1.26 \times 10^{-2}
 \end{aligned}$$

Evaluate the following sums and differences. Write your answers in scientific notation.

- a** $(3.4 \times 10^2) + (7.3 \times 10^5)$
 b $(8.52 \times 10^4) - (1.6 \times 10^3)$
 c $(6.03 \times 10^{-3}) + (2.7 \times 10^{-4})$
d $(8.2 \times 10^{-3}) - (3.5 \times 10^{-2})$
 e $(-9.8 \times 10^3) + (-7.7 \times 10^2)$
 f $(1.01 \times 10^5) - (7.5 \times 10^3)$

- 14 Light travels at a speed of approximately 3.00×10^{10} cm/s.

- a** How many kilometres does it travel in 1 hour? Give your answer in scientific notation.
b How many kilometres does it travel in 1 day? Give your answer in scientific notation.
c As defined by the International Astronomical Union, a light-year is the distance light travels in 365.25 days. How far is 1 light-year in km? Give your answer in scientific notation.

- 15 The following table lists the mass and diameter of all the planets in the solar system.

Planet	Mass (kg)	Diameter (km)
Mercury	3.30×10^{23}	4.88×10^3
Venus	4.87×10^{24}	1.21×10^4
Earth	5.98×10^{24}	1.28×10^4
Mars	6.42×10^{23}	6.79×10^3
Jupiter	1.90×10^{27}	1.43×10^5
Saturn	5.69×10^{26}	1.21×10^5
Uranus	8.68×10^{25}	5.11×10^4
Neptune	1.02×10^{26}	4.86×10^4

- a** Compare the masses of the planets and list them in ascending order.
b Compare the diameters of the planets and list them in ascending order.

- 16** Earth revolves around the Sun at an average speed of 10^5 km/h.
- What distance does Earth travel in 1 day?
 - How many days would it take Earth to travel 9.6×10^8 km?
- 17** The Sun is 1.52×10^8 km from Earth. Light from the Sun travels towards Earth at a speed of 3×10^8 m/s. How long does it take this light to reach Earth? Give your answer to the nearest minute.
- 18** The Australian \$1 coin has a mass of 9 g and a thickness of 3×10^{-1} cm.
- Sarah has a pile of these coins on her desk. She stacks as many of them as she can on top of each other between two shelves in a bookcase. The distance separating the shelves is 26 cm.
 - How many coins are in the stack?
 - What would be the mass of these coins?
 - Ben takes Sarah's stack of coins and places them end-to-end in a line. The line stretches to a length of 2.15 m. What is the diameter of a \$1 coin?



- 19** Write the following in seconds in scientific notation.
- 47 minutes
 - 14 days
 - 40 weeks
 - 1 year (not a leap year)
- 20** A number written in engineering notation is the product of a number, a , between positive or negative 1 (inclusive) and positive or negative 1000 (exclusive) and a power of 1000 written as a power of 10 in index notation. That is, $a \times 10^{3m}$, where $1 \leq a < 1000$ or $-1000 < a \leq -1$ and m is an integer. For example, 3.456×10^9 , 34.56×10^6 , 345.6×10^{-6} are in engineering notation while 0.3456×10^9 , 3456×10^6 , 345.6×10^{-5} are not in engineering notation.

- a** Write the following in seconds in engineering notation.
- | | |
|-----------------------------------|--|
| i 7.3 kiloseconds (7.3 ks) | ii 9.1 microseconds (9.1 μ s) |
| iii 54 nanoseconds (54 ns) | iv 82 teroseconds (82 Ts) |
| v 129 megaseconds (129 Ms) | vi 974 picoseconds (974 ps) |
- b** Write the following times in engineering notation using the appropriate prefix.
- | | |
|--|---|
| i 5.601×10^7 seconds | ii 9.2×10^5 seconds |
| iii 4.31×10^{-5} seconds | iv 7.88×10^{-7} seconds |
| v 8×10^{-2} seconds | vi 1.0356×10^{14} seconds |

- 21** Sound travels at 330 m/s, whereas light travels at 3×10^8 km/s.

- Compare the speed of light and the speed of sound. A timekeeper stands at the end of a 100 m straight running track. The starting gun at the beginning of the track goes off.
- How long does it take:
 - for the sight of the smoke to reach the timekeeper
 - for the sound of the gun to reach the timekeeper?
- What advice should you give the timekeeper in order to have an accurate recording of the time of the race?



- 22** The circumference of a hydrogen atom is 7.98×10^{-9} cm. How far would a line of 1 million hydrogen atoms stretch if placed next to each other?
- 23** Consider the multiplication problem $2^{350} \times 3^2 \times 4^3 \times 5^{355}$. Write the exact answer in scientific notation.

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Investigation
Measuring large units
in our solar system



Topic quiz
2E

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2F Rounding and estimating

Learning intentions

By the end of this topic you will be able to ...

- ✓ round numbers to a specified degree of accuracy
- ✓ estimate the results of calculations
- ✓ determine the effect that rounding during calculations has on the accuracy of results.



Inter-year links

[Year 7](#)

4B Ordering and rounding decimals

[Year 8](#)

1A Rounding and estimating

Rounding to a given number of decimal places



Key content video

- When **rounding** a number, you are replacing the number with an **approximation** that is easier to interpret and use in calculations. The approximately equal (\approx) symbol should be used when rounding occurs.

$$3.5\boxed{7}34 \approx 3.57$$



$$4.0\boxed{1}69 \approx 4.02$$



- To round to a given number of decimal places, put a box around the digit that you are rounding to and look at the value of the digit to the right of the box. If it is 5 or greater, round up; if it is less than 5, round down.

$$3.5\boxed{7}34 \approx 3.57 \quad 4.0\boxed{1}69 \approx 4.02$$

Significant figures

- **Significant figures** are the number of digits required to express a number to a specified degree of accuracy.
- When counting significant figures, start by counting the first non-zero digit from left to right.
 - All non-zero digits are significant. For example, 7.789 has four significant figures, as all digits are non-zero.
 - Zeroes between two non-zero digits are significant. For example, 4056 has four significant figures including the zero between 4 and 5.
 - Leading zeroes are not significant. For example, 0.051 has two significant figures, as both the zeroes are leading zeroes.
 - Trailing zeroes to the right of the decimal point after the last non-zero significant digit are significant. For example, 112.00 has five significant figures.
 - Trailing zeroes in an integer are not significant. For example, 8300 has two significant figures.
- If a number is expressed in scientific notation, all the digits in the value of a are significant. For example, 2.301×10^{-2} has four significant figures.
- Rounding to a given number of significant figures involves the same process as rounding to a given number of decimal places. Put a box around the digit that you are rounding to and look at the value of the digit to the right of the box. If it is 5 or greater, round up; if it is less than 5, round down.

Truncation

- To **truncate** a number, the digits after a specified point are cut off. When truncating a number, you do not need to consider the value of the digit after the point at which you are truncating, and all the digits before that point will stay the same.

$$11.2|93 \approx 11.2$$

Estimations and accuracy

- An **estimate** is an approximate value which is close to the actual value. The results of calculations can be estimated by using rounded values instead of the actual values.
- The **level of accuracy** of an approximate value describes the closeness of that approximation to the exact value. The higher the degree of accuracy, the closer the approximation is to the exact value.
 - Rounding to a greater number of decimal places or significant figures will increase the level of accuracy of an estimate.

Example 2F.1 Identifying significant figures



How many significant figures are shown in each number?

- a** 5.42 **b** 20 803 **c** 6.200 **d** 4000 **e** 0.0082

THINK

- a** All non-zero digits are significant.
- b** Zeroes between non-zero digits are significant.
- c** Zeroes at the end of a decimal number are significant.
- d** Zeroes at the end of an integer are not significant.
- e** Zeroes to the left of the first non-zero digit in a decimal number are not significant.

WRITE

- a** 5.42 has three significant figures.
- b** 20 803 has five significant figures.
- c** 6.200 has four significant figures.
- d** 4000 has one significant figure.
- e** 0.0082 has two significant figures.

Example 2F.2 Writing numbers in scientific notation using significant figures



Write each number in scientific notation with the number of significant figures indicated in brackets.

- a** 53 726 (2) **b** 0.084 03 (3) **c** -13.6007 (5)

THINK

- a** **1** This number has five significant figures. Round to two significant figures (the nearest thousand). Remember that zeroes at the end of an integer are not significant.
2 Write in scientific notation.
- b** **1** This number has four significant figures. Round to three significant figures (the nearest ten-thousandth).
2 Write in scientific notation. Remember that zeroes at the end of a decimal are significant.
- c** **1** This number has six significant figures. The minus sign does not impact how many significant figures a number has. Round to five significant figures (the nearest thousandth).
2 Write in scientific notation.

WRITE

- a** $53\,726 \approx 54\,000$
 $= 5.4 \times 10^4$
- b** $0.084\,03 \approx 0.0840$
 $= 8.40 \times 10^{-2}$
- c** $-13.6007 \approx -13.601$
 $= -1.3601 \times 10^{-1}$

Example 2F.3 Estimating the results of calculations



Estimate the result of 535.6×38.3 by first rounding each value to two significant figures.

THINK

- 1 Place a box around the digit that you are rounding to and look at the value of the digit to the right of the box. If it is 5 or greater, round up; if it is less than 5, round down.
- 2 Complete the calculation using the rounded values.
- 3 Write the answer, using the approximately equal to symbol (\approx).

WRITE

$$\boxed{5}35.6 \approx 540$$

$$\boxed{3}83 \approx 38$$

$$540 \times 38 = 20\,520$$

$$535.6 \times 38.3 \approx 20\,520$$

Helpful hints

- ✓ When truncating a positive number, the truncated value will always be smaller than the original value.
- ✓ A truncation will never be more accurate than rounding to the same number of decimal places or significant figures.

ANS
p433

Exercise 2F Rounding and estimating

 1-2(a-h), 3(a-d), 4-5(a-h),
6(a-d), 7(a, c, e), 10

 2(e-l), 3(e-h), 5(e-l), 6(e-h),
7(b, d, f, h), 8-10, 13

 3(e-h), 5(e-h), 7(c, d, g, h),
10-14

1 Round the following numbers to:

i one decimal place

ii two decimal places.

a 14.851

b 9.549

c 24.020

d 103.999

e -3.2612

f 17.1164

g -99.293

h 5.5454

i 72.8848

j -125.094

k 10.0030

l -356.8261

2F.1 2 How many significant figures are shown in each of the following numbers?

a 345

b 25 000

c 5072

d 400

e -809

f 0.59

g -0.003

h 1.472

i 48.062

j -7.300

k 36 020

l 0.009 04

3 How many significant figures are shown in each of the following numbers?

a 2.4×10^3

b 5.06×10^{-4}

c 1.900×10^7

d 8.0×10^5

e -3.206×10^{-9}

f 7.00×10^5

g -15.120×10^{-2}

h 220.10×10^{10}

4 Round each number to the number of significant figures indicated in brackets.

a 2.58×10^5 (2)

b -5.037×10^4 (3)

c 9.1042×10^6 (4)

d -6.00×10^3 (2)

e 458 (2)

f 73 051 (4)

g 1279 (1)

h 40 008 (1)

i -5.1437 (3)

j 0.0349 (2)

k -42.0607 (4)

l 0.852 (1)

2F.2 5 Write each number in scientific notation with the number of significant figures indicated in brackets.

- a** 327 (2) **b** 48 654 (3) **c** -190 760 (4) **d** 2621 (1)
e 0.4031 (3) **f** -0.0544 (2) **g** 0.000 207 193 (4) **h** -0.008 327 (1)
i 758.4 (2) **j** -20 703.02 (4) **k** 40.155 (3) **l** 54 007.63 (5)

6 Truncate the following numbers after:

- i** one decimal place
ii two decimal places.

- a** 9.024 15 **b** 13.416 02 **c** -110.1415 **d** 80.000 01
e -4.1539 **f** -33.317 33 **g** 255.0542 **h** 1090.0148

2F.3 7 Estimate the result of the following calculations by first rounding each value to two significant figures.

- a** $358.35 + 40.51$ **b** $94.61 - 16.65$ **c** $2450.45 + 432.91$ **d** $540.67 - 249.43$
e 27.98×11.31 **f** $123.45 \div 24.19$ **g** 8.53×16.49 **h** $2984 \div 154.9$

8 **a** Round each of the following to one, two and three significant figures.

- i** 1.901 **ii** 1.994 **iii** 1.997
iv 2.003 **v** 2.006 **vi** 2.098

b Explain how significant trailing zeroes are important in determining to how many significant figures a number is rounded.

9 Consider 0.41, 0.000 000 000 0012 and -0.000 034.

- a** Round each number to one significant figure.
b If leading zeroes were significant, what would each value be when rounded to one significant figure?
c Explain why not including leading zeroes as significant is more useful than including them. Consider how including them is similar to rounding to a place value or number of decimal places.

10 Explain the mistake each student made.

- a** Jane rounded 4.1025 to three significant figures as 4.103.
b Kaleb rounded 0.0432 to three significant figures as 0.04.
c Lisa rounded 102 948.3618 to three significant figures as 102 900.
d Marius rounded 102 948.3618 to three significant figures as 103.

11 Explain why rounding to a given number of decimal places will never give a less accurate approximation of a number when compared to truncating after the same number of decimal places.

12 Does rounding to a greater number of significant figures before performing a calculation always result in a more accurate estimate? Provide an example with your answer.

13 Convert the following measurements into the units in brackets, giving the answers in scientific notation correct to three significant figures.

- a** 10 000 seconds (hours) **b** 1 000 000 seconds (days)
c 1 000 000 000 seconds (years) **d** 1 000 000 000 000 seconds (millennia)

14 Find the value of the following numbers in scientific notation to three significant figures without using a calculator. Hint: $(a \times 10^b)^2 = a^2 \times 10^{2b}$.

- a** 2^4 **b** 2^8 **c** 2^{16} **d** 2^{32}

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Topic quiz
2F

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Chapter summary

<p>Indices</p> <p style="text-align: center;">index/exponent ↓</p> <p>base → $3^4 = 3 \times 3 \times 3 \times 3 = 81$</p> <p style="text-align: center;">index notation expanded form basic numeral</p>	<p>Product of powers law</p> $2^3 \times 2^5 = 2^{(3+5)} \quad a^3 \times a^5 = a^{(3+5)}$ $= 2^8 \quad = a^8$
<p>Prime factorisation</p> <div style="display: flex; align-items: center;"> <div style="margin-right: 20px;"> </div> <div> $315 = 3 \times 3 \times 5 \times 7$ $= 3^2 \times 5 \times 7$ </div> </div>	<p>Quotient of powers law</p> $2^5 \div 2^3 = 2^{(5-3)} \quad a^5 \div a^3 = a^{(5-3)}$ $= 2^2 \quad = a^2$
<p>Reciprocals and negative indices</p> <p>The reciprocal of 3 is $\frac{1}{3}$</p> <p>The reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$</p> $2^{-1} = \frac{1}{2^1} = \frac{1}{2}$ $4^{-2} = \frac{1}{4^2}$	<p>Power of a power law</p> $(2^3)^5 = 2^{(3 \times 5)} \quad (a^2)^3 = a^{(2 \times 3)}$ $= 2^{15} \quad = a^6$ $(2 \times 3)^5 = 2^5 \times 3^5 \quad (ab)^3 = a^3 \times b^3$ $\left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5} \quad \left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$
	<p>The zero index</p> $2^0 = 1 \quad a^0 = 1$
	<p>Truncation</p> $11.2 93 \approx 11.2$
<p>Scientific notation</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> $1230 = 1.23 \times 10^3$ <small>basic numeral scientific notation</small> </div> <div style="text-align: center;"> <p>index of 4 ↓</p> $3.1500 \times 10^4 = 31500$ <small>move four spaces to the right</small> </div> <div style="text-align: center;"> <p>index of 4 ↓</p> $31500 = 3.15 \times 10^4$ <small>move four spaces to the left</small> </div> </div> <div style="display: flex; justify-content: space-around; align-items: flex-start; margin-top: 20px;"> <div style="text-align: center;"> <p>index of -2 ↓</p> $0.00123 = 1.23 \times 10^{-3}$ <small>basic numeral scientific notation</small> </div> <div style="text-align: center;"> <p>index of -2 ↓</p> $004.2 \times 10^{-2} = 0.042$ <small>move two spaces to the left</small> </div> <div style="text-align: center;"> <p>index of -2 ↓</p> $0.042 = 4.2 \times 10^{-2}$ <small>move two spaces to the right</small> </div> </div>	
<p>Rounding</p> $3.5\boxed{7}34 \approx 3.57$ $4.0\boxed{1}69 \approx 4.02$ <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>4 3 2 1 0</p> <p>↓</p> <p>Round down</p> </div> <div style="text-align: center;"> <p>9 8 7 6 5</p> <p>↑</p> <p>Round up</p> </div> </div>	<p>Significant figures</p> <p>7.789 has four significant figures</p> <p>4.056 has four significant figures</p> <p>0.051 has two significant figures</p> <p>112.00 has five significant figures</p> <p>8300 has two significant figures</p> <p>2.301×10^{-2} has four significant figures</p>



Chapter review quiz

Assess your knowledge of this chapter.

Quizlet

Test your knowledge of this topic by working individually or in teams.

Mathematical literacy review

The following key terms are used in this chapter:

- approximation
- base
- basic numeral
- BIDMAS
- decimal points
- estimate
- expanded form
- exponent
- factor
- factor tree
- index
- index form
- index notation
- integer
- leading zeroes
- level of accuracy
- magnitude
- metric prefix
- negative index
- place value
- positive index
- power
- prime factor
- prime factorisation
- prime number
- product
- quotient
- reciprocal
- rounding
- scientific notation
- significant figures
- standard form
- trailing zeroes
- truncate
- zero index

- 1 Which key term can be used to help find the prime factorisation of a number?
- 2 Which description best explains the term prime factorisation of a number?
 - A the sum of two or more integers that equals the given number
 - B the sum of two or more prime factors that equals the given number
 - C the product of two or more integers that equals the given number
 - D the product of two or more prime factors that equals the given number
 - E the product of two or more composite numbers that equals the given number
- 3 Use one of the numbers 7, 12 or 15 to clearly explain the difference between factors and prime factors.
- 4 Show how 24 can be written in expanded form and index notation.
- 5 Complete the following sentences using words from the key terms list.
 - a The _____ of a value written in _____ is the number or variable that is multiplied repeatedly.
 - b Taking the negative index of a fraction is the same as taking the _____ of the _____.
- 6 Determine if the following statements about significant figures are true or false. If they are false, give an example to support your claim.
 - a All non-zero digits are significant.
 - b All zeroes are not significant.
 - c All leading zeroes are not significant.
 - d All trailing zeroes are significant.

Multiple choice

- 2A 1 Which of the following is not equivalent to $9(xy)^4$?

A $9 \times xy \times xy \times xy \times xy$	B $-3^2x^4y^4$	C $9x^4y^4$
D $9xxxxxyyyy$	E $(-3)^2x^4y^4$	
- 2A 2 Which of the following is the prime factorisation of 360?

A $6^2 \times 10$	B $4 \times 9 \times 10$	C $2^3 \times 3^2 \times 5$	D $2^2 \times 3^2 \times 10$	E $3 \times 10 \times 12$
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- 2B 3 Which expression shows $\frac{6ab^2c}{18a^2c}$ in simplified form?

A $\frac{6ab^2}{18a}$	B $\frac{ab^2c}{3a^2c}$	C $\frac{b^2}{3a}$	D $\frac{6ab^2}{18a^2}$	E $3ab^2$
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- 2B** 4 Which statement does not correctly represent one of the index laws?
A $m^5 \times m^2 = m^{5+2}$ **B** $(p \times q)^8 = p^8 \times q^8$ **C** $w^7 \div w^5 = w^{7-5}$
D $a^5 \times a = a^6$ **E** $\left(\frac{x}{y}\right)^4 = \frac{x^4}{y}$
- 2C** 5 Using the index laws, $\frac{5x^{13} \times 2x^4}{4x^8 \times x^0}$ fully simplifies to:
A $\frac{10x^9}{4x^8}$ **B** $\frac{5x^9}{2}$ **C** $\frac{5x^{17}}{2x^8}$ **D** $\frac{5}{2x^9}$ **E** $10x^9$
- 2D** 6 Which of the following is *not* the reciprocal of $\frac{4}{3}$?
A $\frac{3}{4}$ **B** 3×4^{-1} **C** $\left(\frac{4}{3}\right)^{-1}$ **D** $\frac{1}{3 \times 4}$ **E** $1 \div \frac{4}{3}$
- 2D** 7 Which statement is false?
A $\frac{1}{7} = 7^1$ **B** $4^{-2} = \frac{1}{16}$ **C** $\frac{1}{3^6} = 3^{-6}$ **D** $7^3 \times 7^{-5} = \frac{1}{49}$ **E** $\frac{5^{-3}}{5^4} = 5^{-7}$
- 2E** 8 Which number is equivalent to 6.4724×10^2 ?
A 0.64724 **B** 64.724 **C** 0.064724 **D** 64724 **E** 647.24
- 2F** 9 What is the value of 9.1517×10^{-2} when rounded to three significant figures?
A 9.151×10^{-2} **B** 9.15×10^{-2} **C** 9.1517×10^1 **D** 9.1517×10^{-5} **E** 9.15×10^1
- 2F** 10 What is the value of $-38.725\ 04$ when it is truncated after two decimal places?
A -38 **B** -39 **C** -38.72 **D** -38.73 **E** -38.725

Short answer

- 2A** 1 Evaluate the following.
a 3^4 **b** $(-5)^3$ **c** -4^3 **d** $\left(\frac{3}{2}\right)^5$ **e** $(0.6)^3$ **f** $(1.2)^4$
- 2A** 2 Write the following in index notation.
a $17 \times 17 \times 17 \times 17 \times 17 \times 17$ **b** $-5b^2 \times -5b^2 \times -5b^2 \times -5b^2 \times -5b^2$
c $-10 \times f \times f \times f \times v \times v \times v \times v \times v \times v \times v$ **d** $\frac{b^4d^5}{6n^3} \times \frac{b^4d^5}{6n^3} \times \frac{b^4d^5}{6n^3} \times \frac{b^4d^5}{6n^3}$
- 2C** 3 Simplify each expression using the index laws.
a $a^{11} \times a^5$ **b** $b^9 \div b^8$ **c** $(c^8)^2$
d $18d^7 \div (54d^4)$ **e** $(e^5)^5 \times (e^{11})^2$ **f** $5a^0 + 3b^0 + 1c^0$
- 2C** 4 Simplify each expression.
a $\frac{m^3n^4 \times m^9n^{11}}{m^7n^7}$ **b** $\frac{(3k^5l^2)^3 \times (2k^3l^3)^4}{(2k^3l^2)^3}$
- 2D** 5 Write each term with a positive index.
a 5^{-5} **b** $(-11)^{-3}$ **c** $\frac{1}{4^{-4}}$ **d** $\left(-\frac{4}{5}\right)^{-3}$
- 2D** 6 **a** If $4^8 = 65\ 536$, write the value of 4^{-8} as a fraction.
b If $7^{-3} = \frac{1}{343}$, write the value of 7^3 .
- 2E** 7 Write each number as a basic numeral.
a 5.876×10^4 **b** 9.02×10^{-6}
- 2E** 8 Write each number in scientific notation.
a 540 000 **b** 0.000 76
- 2E** 9 A scientist estimates that there are 3.40×10^4 bacteria in one sample and 4.6×10^3 in a second sample. Write the total number of bacteria:
a as a basic numeral **b** in scientific notation.
- 2F** 10 State the number of significant figures in each part of question 7.
- 2F** 11 Round each of the following to the number of significant figures indicated in brackets.
a 879 (2) **b** 2.58×10^5 (1)

Analysis

- 1 Thy and Asha are playing a game. They are using die rolls and a coin flip to generate the product of three numbers in index notation per round. They each roll the dice to determine the value of the bases and indices and flip the coin to determine if each index is positive or negative.

The products generated after each round are multiplied together with the goal to end up with the least number of remaining factors after three rounds.

The table below shows the numbers Thy and Asha got in their three rounds.

	Thy	Asha
Round 1	$2^4 \times 4^3 \times 5^{-3}$	$1^3 \times 3^6 \times 5^{-5}$
Round 2	$3^2 \times 5^6 \times 6^{-3}$	$2^{-3} \times 3^1 \times 6^4$
Round 3	$2^{-6} \times 2^3 \times 3^4$	$3^{-5} \times 4^2 \times 5^3$

- Use the facts that $4 = 2^2$ and $6 = 2 \times 3$ to write Thy and Asha's round 1, 2 and 3 numbers in index notation with positive indices using only the bases 2, 3 and 5.
- Determine Thy and Asha's final number for their game by multiplying their round 1, 2 and 3 numbers together and simplifying the products in index notation with positive indices.
- Who won the game with the least number of factors? Hint: Find the sum of the positive indices.
- Did the winner have the smaller value? Explain.

Thy and Asha decide to play one more round of the game.

- What products do Thy and Asha need to generate to end up with a total product of 1?

Thy and Asha decide to change the rules so that they can choose which base gets which index. Their products from the first two rounds are given in the table below.

	Thy	Asha
Round 1	$4^{-3} \times 5^4 \times 6^5$	$2^{-3} \times 5^{-2} \times 6^4$
Round 2	$1^5 \times 2^3 \times 3^{-4}$	$3^{-3} \times 4^4 \times 5^2$

- Determine the product of round 1 and 2 for Thy and Asha. Write the products in index notation.

For round 3:

- Thy gets the bases 2, 4 and 6 and the indices -6 , -4 and 1.
 - Asha gets the bases 1, 5 and 5 and the indices -4 , 3 and 6.
- Determine which index should go with which base so that Thy and Asha get the minimum number of factors remaining for the game.
 - Who wins this game and by how many factors?

- 2 a Complete the following table.

Base (n)	n^2	n^3	n^4	n^5	n^6	n^7
3						
5						
7						
11						
13						
17						
19						

- Describe the patterns in the final digits for the different powers of each prime number in part a.
- Predict the last digit of n^{100} for each of the prime numbers in part a.
- Use the patterns you observed in part a to predict the last digit of $(a \times b)^{100}$ for the following products.

i 3×5	ii 3×7	iii 3×11	iv 7×19
v 13×17	vi $3^2 \times 7$	vii $13^2 \times 17$	

- 3 In June 2022, the population of each Australian state was recorded. The figure for each state is shown in the table.
- a** Which states and territories have a population listed to:
- four significant figures
 - five significant figures?
- b** Copy the table and add three additional columns.
- c** In the first new column, write the population of each state and territory in full.
- d** In the second new column, round each population to its leading digit.
- e** In the third new column, write each population in scientific notation to one significant figure.
- f** Use your answers from part **d** to determine the following. Write the values in scientific notation.
- Which state or territory has the highest population?
 - Which state or territory has the lowest population?
 - Calculate the difference between the highest and the lowest population.
 - Calculate the total population of SA, Tas, ACT and NT combined.
 - Calculate the total population of Australia.
- g** The actual total population value recorded at the end of June 2022 was 25 978 935. Calculate the difference between your answer to f part v and the actual value. Why is there a difference?

State	Population at 30 June 2022 ('000)
NSW	8153.6
Vic	6613.7
Qld	5322.1
SA	1820.5
WA	2785.3
Tas	571.5
NT	250.6
ACT	456.7

Source: ABS

Chapter checklist

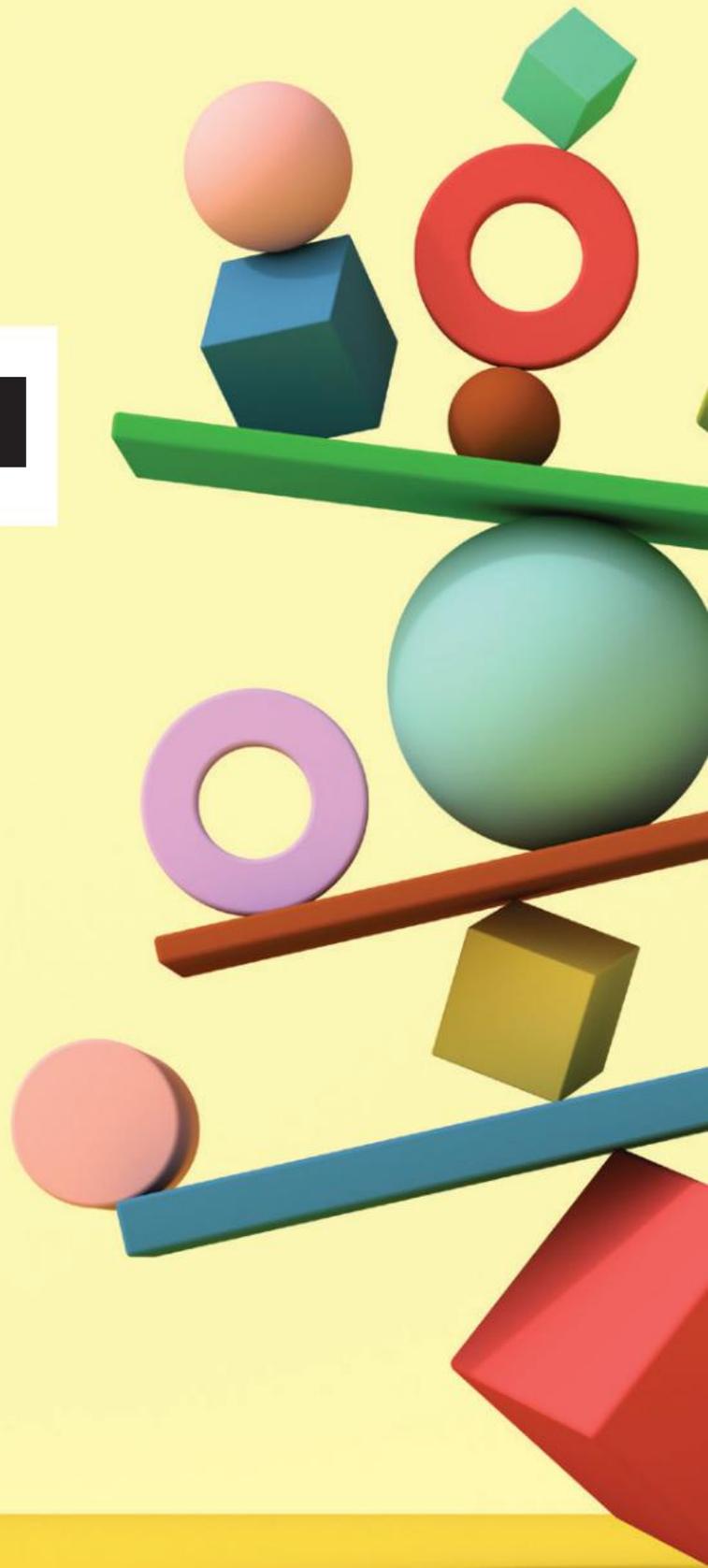


Now that you have completed this chapter, reflect on your ability to do the following.

I can do this	I need to review this
<input type="checkbox"/> Convert between index notation and expanded form <input type="checkbox"/> Calculate the value of numbers in index notation <input type="checkbox"/> Express integers as a product of prime factors	<input type="checkbox"/> Go back to Topic 2A Indices
<input type="checkbox"/> Simplify products of numbers and variables in index form with the same base <input type="checkbox"/> Simplify quotients of numbers and variables in index form with the same base	<input type="checkbox"/> Go back to Topic 2B Products and quotients of powers
<input type="checkbox"/> Raise a term in index notation by another index <input type="checkbox"/> Simplify and evaluate expressions and calculations involving the zero index	<input type="checkbox"/> Go back to Topic 2C Raising indices and the zero index
<input type="checkbox"/> Write a term with a negative index as a term with a positive index <input type="checkbox"/> Write a term with a positive index as a term with a negative index <input type="checkbox"/> Apply index laws to numerical expressions with negative indices <input type="checkbox"/> Simplify and evaluate numerical expressions with negative indices	<input type="checkbox"/> Go back to Topic 2D Negative indices
<input type="checkbox"/> Convert numbers written in scientific notation to basic numerals <input type="checkbox"/> Convert numbers written as basic numerals to scientific notation	<input type="checkbox"/> Go back to Topic 2E Scientific notation
<input type="checkbox"/> Round numbers to a specified degree of accuracy <input type="checkbox"/> Estimate the results of calculations <input type="checkbox"/> Determine the effect that rounding during calculations has on the accuracy of results	<input type="checkbox"/> Go back to Topic 2F Rounding and estimating

3

Algebra



Index

3A Simplifying

3B Algebraic fractions with numerical denominators

ADV 3C Algebraic fractions with algebraic denominators

3D Expanding

ADV 3E Factorising using the HCF

ADV 3F Factorising monic quadratic expressions

Prerequisite skills



Diagnostic pre-test

Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.



Interactive skillsheets

After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✓ Multiplying and dividing with negative numbers
- ✓ Grouping symbols
- ✓ Index laws
- ✓ Highest common factor

Curriculum links

- Simplifies algebraic fractions with numerical denominators and expand algebraic expressions (MA5-ALG-C-01)
 - Apply the 4 operations to simplify algebraic fractions with numerical denominators
 - Apply the distributive law to the expansion of algebraic expressions and collect like terms where appropriate
- ADV** • Simplifies algebraic fractions involving indices, and expands and factorises algebraic expressions (MA5-ALG-P-01)
 - Apply the 4 operations involving algebraic fractions with pronumerals in the denominator
 - Factorise algebraic expressions by taking out a common algebraic factor
 - Expand binomial products and factorise monic quadratic expressions

3A Simplifying

Learning intentions

By the end of this topic you will be able to ...

- ✓ simplify algebraic terms involving addition and subtraction
- ✓ simplify algebraic terms involving multiplication and division.



Inter-year links

Year 8

5C Adding and subtracting algebraic terms

Year 10

2C Simplifying

Terms and expressions

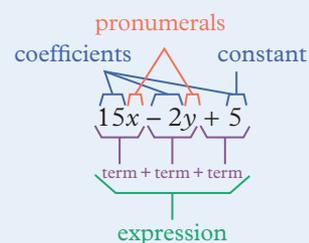
- A **pronumeral** is a letter or symbol that is used in place of a number. Pronumerals can be used to represent an **unknown** or a **variable**.
- An **expression** is a quantity that is represented by a sequence of numbers and/or pronumerals that are connected by mathematical operations.
- A **term** is part of an expression that is separated from the other parts by a plus or minus sign (as long as the plus or minus sign is not inside any brackets).

Note: If the term is separated on the left by a minus sign, then the term is negative.

- A **coefficient** is the number acting as a multiplier in an algebraic term, usually written before the pronumeral. A pronumeral without a number preceding it has a coefficient of 1.
- A **constant** is a term without any pronumerals. It also counts as a coefficient.



Key content video



Adding and subtracting algebraic terms

- **Like terms** contain the same pronumerals with the same indices
 - The order of pronumerals can be different in two like terms. For example, xyz , $4yxz$, and $7zxy$ are all like terms.
 - You can write terms containing indices in **expanded form** to determine whether they are like terms. For example, $a^2b = a \times a \times b$ and $ab^2 = a \times b \times b$, so a^2b and ab^2 are not like terms.

$$\underbrace{3a^2b}_{\text{index form}} = \underbrace{3 \times a \times a \times b}_{\text{expanded form}}$$

- Like terms can be added or subtracted by adding or subtracting the coefficients of the terms. For example, $a^2b + 2a^2b = 1a^2b + 2a^2b = 3a^2b$

Multiplying algebraic terms

- To multiply algebraic terms:

1 Write the coefficients and pronumerals for each term in expanded form, without expanding index form.

$$3a^3b \times (-2a^2b^2) = 3 \times (-2) \times a^{3+2} \times b^{1+2} = -6a^5b^3$$

2 Multiply the coefficients together.

3 Apply the product of powers law to multiply the numbers in index form. Keep the base and add the indices.

4 Simplify by leaving out the multiplication signs. Write the coefficient first, followed by the pronumerals listed in alphabetical order.

Dividing algebraic terms

- **Algebraic fractions** are fractions that contain at least one pronumeral.
- Remember that quotients can be expressed as fractions.

For example, $x \div 7 = \frac{x}{7}$.

- To divide algebraic terms:
 - 1 Write the coefficients and pronumerals for each term in expanded form.
 - 2 Divide the coefficients by the HCF.
 - 3 Cancel any common pronumeral in both the numerator and the denominator.
 - 4 Write the coefficient first, followed by the pronumerals listed in alphabetical order.

$$\begin{aligned} \frac{12abc}{9b} &= \frac{3 \times 4 \times a \times b \times c}{3 \times 3 \times b} \\ &= \frac{3 \times 4 \times a \times \cancel{b} \times c}{\cancel{3} \times 3 \times \cancel{b}} \\ &= \frac{4ac}{3} \end{aligned}$$

Example 3A.1 Adding and subtracting algebraic terms



Simplify each expression.

a $10a - 6a + a$

b $4xy + 2x - 5xy$

c $-5bc + 2b - 3cb - \frac{b}{2}$

d $6x^2y - 3y^3 - 2y^2x + y^3$

THINK

- a** Identify like terms and simplify by adding and subtracting the coefficients.
- b** **1** Rearrange the expression so that like terms are grouped together.
2 Simplify by adding and subtracting the coefficients.
- c** **1** Rearrange the expression so that like terms are grouped together. Check that the + or - sign in front of each term has moved with that term.
2 Simplify by adding and subtracting the coefficients. Remember that fractional coefficients can be written in two ways, so $\frac{b}{2} = \frac{1}{2}b$.
- d** **1** Rearrange the expression so that like terms are grouped together. Check that the + or - sign in front of each term has moved with that term. Note that xy^2 and x^2y are not like terms.
2 Simplify by adding and subtracting the coefficients. Rearrange pronumerals in alphabetical order.

WRITE

a $10a - 6a + a = (10 - 6 + 1)a$
 $= 5a$

b $4xy + 2x - 5xy = 2x + 4xy - 5xy$
 $= 2x + (4 - 5)xy$
 $= 2x - xy$

c $-5bc + 2b - 3cb - \frac{b}{2} = -5bc - 3cb + 2b - \frac{b}{2}$
 $= (-5 - 3)bc + \left(2 - \frac{1}{2}\right)b$
 $= -8bc + \frac{3}{2}b$

d $6x^2y - 3y^3 - 2y^2x + y^3 = 6x^2y - 3y^3 + y^3 - 2y^2x$
 $= 6x^2y + (-3 + 1)y^3 - 2xy^2$
 $= 6x^2y - 2y^3 - 2xy^2$



Example 3A.2 Multiplying algebraic terms

Simplify the following products.

a $4de \times 7ab$

b $5x^2y^5 \times (-2kwx^3y^3)$

THINK

- a**
- 1 Write both terms in expanded form.
 - 2 Multiply the coefficients together.
 - 3 Simplify by leaving out the multiplication signs. Write the pronumerals in alphabetical order.
- b**
- 1 Write the coefficients and pronumerals for each term in expanded form, without expanding the index form.
 - 2 Multiply the coefficients together.
 - 3 Apply the product of powers law for the multiplication of indices, so $x^2 \times x = x^{(2+1)}$ and $y^5 \times y^3 = y^{(5+3)}$.
 - 4 Simplify by leaving out the multiplication signs.

WRITE

a $4de \times 7ab = 4 \times d \times e \times 7 \times a \times b$
 $= 28 \times d \times e \times a \times b$
 $= 28abde$

b $5x^2y^5 \times (-2kwx^3y^3)$
 $= 5 \times x^2 \times y^5 \times (-2) \times k \times w \times x \times x \times y^3$
 $= -10 \times x^{2+1} \times y^{5+3} \times k \times w$
 $= -10 \times x^3 \times y^8 \times k \times w$
 $= -10kwx^3y^8$



Example 3A.3 Dividing algebraic terms

Simplify the following quotients.

a $\frac{2a}{8}$

b $12ab \div 3b$

c $\frac{-15xy}{10x}$

THINK

- a** Divide the coefficients of the numerator and the denominator by the HCF of 2.
- b**
- 1 Write the coefficients and pronumerals for each term in expanded form.
 - 2 Cancel any common factors from the dividend and divisor.
 - 3 Write the answer by placing the coefficient first and leave out the multiplication sign.
- c**
- 1 Write in expanded form.
 - 2 Cancel the coefficients, and divide -15 and 10 by the HCF of 5 . Cancel any common pronumerals from the numerator and denominator.
 - 3 Simplify the numerator and the denominator.

WRITE

a $\frac{2a}{8} = \frac{2 \times a}{2 \times 4}$
 $= \frac{2 \times a}{2 \times 4}$
 $= \frac{a}{4}$

b $12ab \div 3b = 3 \times 2 \times 2 \times a \times b \div (3 \times b)$
 $= \cancel{3} \times 2 \times 2 \times a \times \cancel{b} \div (\cancel{3} \times b)$
 $= 2 \times 2 \times a$
 $= 4a$

c $\frac{-15xy}{10x} = \frac{-15 \times x \times y}{10 \times x}$
 $= \frac{-15^3 \times x^1 \times y}{10^2 \times x^1}$
 $= -\frac{3y}{2}$

- ✓ Recall the rules for writing algebraic notation.
 - Products are simplified by leaving out the multiplication sign and placing the coefficient first.
For example: $7 \times x = 7x$ and $7 \times (x + 2) = 7(x + 2)$.
 - When a pronumeral is multiplied by 1, the 1 is not shown.
For example: $1 \times x = x$.
 - Quotients are represented by fractions.
For example: $x \div 7 = \frac{x}{7}$.
 - Terms with fractional coefficients can be written in two ways.
For example: $\frac{x}{7} = \frac{1}{7}x$.
 - Write pronumerals in a term in alphabetical order
For example: $ba^2c = a^2bc$.
- ✓ Recall the rules for multiplying and dividing positive and negative numbers.

$$\begin{array}{l}
 + \times - = - \qquad + \times + = + \\
 - \times + = - \qquad - \times - = +
 \end{array}$$

ANS p435 **Exercise 3A Simplifying**

 1, 2, 3-4(1st column), 5,
6-8(1st column), 10-12, 13(a, d, e, f),
14(a, c), 15, 16

 4, 5, 6-8(2nd column), 9, 10,
14, 17-18

 5, 7, 8(2nd column), 9, 10, 15-18

- 1 Consider these terms: $3x, 7xy, -x, 2x^2, xzw, 20x$.
 - a Which are like terms?
 - b Explain how you can tell.
 - c What is the coefficient of each term?
- 2 List each group of like terms from these terms: $2ba^2, 3a, 2b^2a, 6a^3, aaa, 6a, 3aab, 6ab^2, 6a^2a, 6a^2b, 3abb$.

3A.1 3 Simplify each expression where possible.

- | | |
|----------------------------|--------------------------|
| a $6a - 4a + 8a$ | b $4k - 5k - 7k$ |
| c $x^2 + 3x^2 + 2x^2$ | d $3cd + cd - 9cde$ |
| e $3x + 4y + 9x + 2y$ | f $7a + 5b - 3a + b$ |
| g $m - 2p + 4p + 8m$ | h $3 + 5k - 2 - 6k$ |
| i $4xy + 3x^2 - xy + 2x^2$ | j $d + de^2 + d - 5de^2$ |
| k $5m^3 + 7 - m^3 - 5$ | l $abc + ab + ac + 3ab$ |
- 4 Simplify each expression.

a $6x + 3y - x + 2y + 5x - 4y$	b $8ab - 4b - b + b^2 + a - 3ab$
c $2k + 3km - 6k + 4 + 4k - km$	d $4x^2 - 7x^2 - 3x + 5 + 6x - 9$
e $9a - 4a^2 + a^3 + 5a^2 - 3 - 7a$	f $m^2n + 3m^2 + 5nm^2 - 2n^2 + 4mn^2 - 3m^2$
 - 5 Simplify each expression.

a $7a^2 + 5(3a)^2 + (-5a)^2 - 9aa$	b $-5bbb + 40b^2b - (2b)^3 + 2(-3b)^3$
c $9c^2d^2 - (6cd)^2 + 15(cd)^2 + 9d^2c^2$	d $r(qp)^2 + prq^2 + ppqqr + p^2q^2r + pq^2r$
e $2ab + 3cd + 4ac + 5ba + 6dc$	f $4yx^2 + 3xy - 2xy + yx + 3x^2y$

3A.2 6 Simplify each expression.

a $2ab \times 3cd$

b $-5xy \times 4mp$

c $9gh \times g$

d $4km \times (-6kn)$

e $7jp \times 8bpt$

f $-x^2y \times (-ay)$

g $6a^2b \times 3acd$

h $-10hk \times 2hkp$

i $3b \times (-2b) \times b$

j $m^2n \times 4n \times kn$

k $-5xy \times x^2 \times (-3xy)$

l $8abc \times 7a^3c \times b^2$

7 Simplify each expression by first using index laws.

a $(5a^3z^8)^2 \times 3(a^3z^5)^4$

b $(-3by^4)^3 \times (-2yb^7)^4$

c $5(cm^2x)^5 \times c(-2mx)^3 \times mx(-2c)^2$

d $4(d^3n^2w)^5 \times 11(d^2n^5w^3)^4$

e $5e^3p^{12}t^5 \times (e^{10}p^4t^2)^0 \times (-3e^4p^6t^7)^2$

f $-6(g^2q^5u^3)^5 \times 4(-g^4q^4u^5)^3 \times -7(g^2qu^7)^4$

3A.3 8 Simplify each expression.

a $\frac{3a}{18}$

b $12abc \div 12b$

c $\frac{25xy}{15}$

d $36ab \div 24a$

e $12mn \div 36n$

f $\frac{abc}{ac}$

g $\frac{12cd}{3d}$

h $\frac{7ef}{-14ef}$

i $\frac{-5xy}{-20x}$

j $\frac{4mn}{22mn}$

k $\frac{15mn^2}{9m}$

l $\frac{-3a^2bc}{12ab}$

9 Simplify each expression following the correct order of operations.

a $\frac{3a}{2} \times 6b + 8ab \div 2$

b $144b \div 12 \times 8ab + 2ab^2$

c $(6a^2 + 8a^2b \div b) \div 7$

d $(44ab + 5a \times 11b) \div \left(\frac{66a}{22} + 30a\right)$

10 Determine whether each statement is true or false.

- a** Two like terms can be added to form one new term.
- b** Any term can be subtracted from another term to form one new term.
- c** Two terms can be multiplied to form one new term only if they are like terms.
- d** Any term can be divided by another term to form one new term.

11 Students in a class were asked to simplify two algebraic expressions. Three sets of working for each expression are shown below. One set is correct and the other two sets contain errors.

For each expression, choose the correct set and then identify the errors in the other two sets of working.

a Expression 1: $4a - 3b + 2 + 2a + 8b - 7$

Set A	Set B	Set C
$4a - 3b + 2 + 2a + 8b - 7$	$4a - 3b + 2 + 2a + 8b - 7$	$4a - 3b + 2 + 2a + 8b - 7$
$= 4a + 2a + 2 + 7 + 3b + 8b$	$= 4a + 2a + 2 - 7 - 3b + 8b$	$= 4a - 2a + 2 - 7 - 3b + 8b$
$= 6a + 9 + 11b$	$= 6a - 5 + 5b$	$= 2a + 9 + 5b$

b Expression 2: $-3ab \times 4bc$

Set A	Set B	Set C
$-3ab \times 4bc$	$-3ab \times 4bc$	$-3ab \times 4bc$
$= -3 \times 4 \times a \times b^{1+1} \times c$	$= -3 \times 4 \times a \times b^{1+1} \times c$	$= -3 \times 4 \times a \times b^{1+1} \times c$
$= -12ab^2c$	$= 12 \times a \times b^2 \times c$	$= -12 \times a \times b \times 2 \times c$
	$= 12abc$	$= -24abc$

12 If $x = 3$ and $y = -2$, evaluate each expression. To make it easier, simplify each expression first.

a $5x - 6y + 7y + 3x$

b $3xy - 8xy - xy$

c $5x \times 3y$

d $\frac{10xy}{y}$

e $xy \times xy^2$

f $12xy \div 24x$

g $x + y - 2xy + 5y - x$

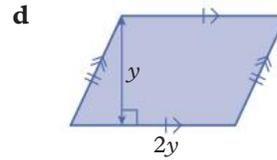
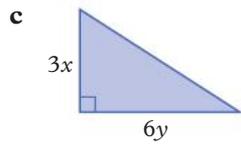
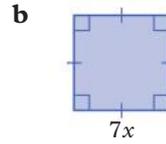
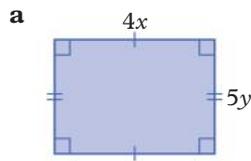
h $x \times 3x - 2x^2 + 4x - y$

i $49x^2y^2 \div 14x^2$

13 Evaluate each expression if $a = 2$, $b = -1$ and $c = 5$. Remember to simplify each expression first.

- | | |
|---|---|
| a $3a + 2b + 7c - a - 5c + b$ | b $7ab + 4a - 5a + ab$ |
| c $a^2b + ab^2 + ac - 3a^2b + 2ac$ | d $2abc \times bc \times 5a$ |
| e $18ab^2c \div (6bc)$ | f $3ac^2 \times 4ab \div (9abc)$ |

14 Write the shaded area in each shape as an algebraic expression in simplest form.



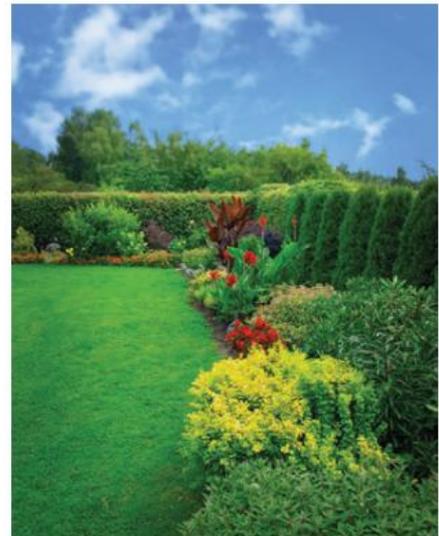
15 Calculate the area shaded in each shape in question 14 for $x = 3$ m and $y = 2$ m.

16 A rectangle has a breadth of k .

- a** If the length of the rectangle is twice the breadth, write an expression for:
- the perimeter of the rectangle
 - the area of the rectangle.
- b** Calculate the perimeter and the area of the rectangle when $k = 5$ cm.

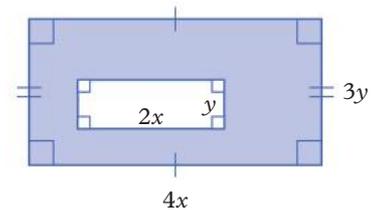
17 Lana plants a 1-metre-wide flowerbed around a square section of lawn.

- a** If the lawn has a length of x metres, write an expression for:
- the perimeter of the lawn
 - the area of the lawn
 - the perimeter around the outer edge of the flowerbed
 - the area of the flowerbed, given that the total area of the lawn and flowerbed is $(x^2 + 4x + 4)$ m².
- b** When $x = 8$, calculate:
- the area of the flowerbed
 - the length of edging needed around the inner edge of the flowerbed
 - the length of edging needed around the outer edge of the flowerbed
 - the area to be mown.



18 Consider the shaded region of this shape.

- a** Write an algebraic expression for the area of the shaded region.
- b** If $x = 4$ cm and $y = 5$ cm, calculate the area of the shaded region.
- c** Write an expression for the total length of the outer and inner edges of the shape.
- d** Use your sets of values from part **b** to calculate the total length of the outer and inner edges of the shape.
- e** If $x = 12$ cm and the area of the shaded region is 210 cm², determine the total length of the outer and inner edges of the shape.



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Dividing terms



Worksheet
Simplify algebraic expressions



Topic quiz
3A

3B Algebraic fractions with numerical denominators

Learning intentions

By the end of this topic you will be able to ...

- ✓ multiply and divide algebraic fractions with numerical denominators
- ✓ add and subtract algebraic fractions with numerical denominators.



Inter-year links

- [Year 7](#) 3B Equivalent fractions
- [Year 8](#) 5E Dividing algebraic terms
- [Year 10](#) 2C Simplifying

Multiplying and dividing algebraic fractions



Key content video

- The product of two algebraic fractions is the product of the numerators over the product of the denominators. For example,

$$\begin{aligned}\frac{3x}{2} \times \frac{2x}{9} &= \frac{3^1x \times 2x}{2 \times 9^3} \\ &= \frac{x^2}{3}.\end{aligned}$$

- The quotient of two algebraic fractions is found by multiplying the fraction before the division sign by the reciprocal of the fraction after the division sign. For example,

$$\begin{aligned}\frac{3x}{2} \div \frac{2x}{9} &= \frac{3x}{2} \times \frac{9}{2x} \\ &= \frac{3x \times 9}{2 \times 2x} \\ &= \frac{27}{4}.\end{aligned}$$

Adding and subtracting algebraic fractions

- Adding and subtracting algebraic fractions is no different from adding and subtracting any other fractions.
 - 1 Identify the lowest common denominator (LCD) by identifying the lowest common multiple (LCM) of the denominators.
 - 2 Write each fraction as an equivalent fraction with the LCD.
 - 3 The sum of the algebraic fractions with a common denominator is then found by adding the numerators. For example, $\frac{x}{4} + \frac{x}{6}$.

The LCD of $\frac{x}{4}$ and $\frac{x}{6}$ is 12, so:

$$\begin{aligned}\frac{x}{4} + \frac{x}{6} &= \frac{x}{4} \times \frac{3}{3} + \frac{x}{6} \times \frac{2}{2} \\ &= \frac{3x}{12} + \frac{2x}{12} \\ &= \frac{3x + 2x}{12} \\ &= \frac{5x}{12}.\end{aligned}$$

- The difference of algebraic fractions with a common denominator is found by subtracting the numerators. So, using the same algebraic fractions from the previous example, we have:

$$\begin{aligned}\frac{x}{4} - \frac{x}{6} &= \frac{x}{4} \times \frac{3}{3} - \frac{x}{6} \times \frac{2}{2} \\ &= \frac{3x}{12} - \frac{2x}{12} \\ &= \frac{3x - 2x}{12} \\ &= \frac{x}{12}.\end{aligned}$$

Example 3B.1 Multiplying and dividing algebraic fractions with numerical denominators



Write each of these expressions as one fraction in simplest form.

a $\frac{x}{2} \times \frac{xy}{5}$

b $\frac{xy}{12} \div \frac{12x}{5}$

c $\left(\frac{xy}{12}\right)^2$

THINK

- a**
- Write the expression as a single fraction by multiplying the numerators together and the denominators together.
 - Look for common factors. Divide both the numerator and the denominator by 2.
 - Simplify.
- b**
- Write the expression as the product of two fractions.
 - Write the expression as a single fraction by multiplying the numerators together and the denominators together.
 - Look for common factors. Divide both the numerator and the denominator by x , then simplify.
- c**
- Write the expression as a product of two identical fractions.
 - Multiply the numerators together and the denominators together.
 - Simplify.

WRITE

a $\frac{x}{2} \times \frac{12xy}{5}$

$$\begin{aligned}&= \frac{x \times 12xy}{2 \times 5} \\ &= \frac{x \times 6xy}{5} \\ &= \frac{6x^2y}{5}\end{aligned}$$

b $\frac{xy}{12} \div \frac{12x}{5}$

$$\begin{aligned}&= \frac{xy}{12} \times \frac{5}{12x} \\ &= \frac{xy \times 5}{12 \times 12x} \\ &= \frac{y \times 5}{12 \times 12} \\ &= \frac{5y}{144}\end{aligned}$$

c $\left(\frac{xy}{12}\right)^2$

$$\begin{aligned}&= \frac{xy}{12} \times \frac{xy}{12} \\ &= \frac{xy \times xy}{12 \times 12} \\ &= \frac{x^2y^2}{144}\end{aligned}$$

Example 3B.2 Adding and subtracting algebraic fractions with numerical denominators



Write each of these expressions as one fraction in simplest form.

a $\frac{2x}{7} + \frac{x}{3}$

b $\frac{7x}{10} - \frac{x}{6}$

c $\frac{x}{2} - \frac{4x}{3} + \frac{3x}{5}$

THINK

- a**
- The LCM of 7 and 3 is 21. So write each fraction as an equivalent fraction with an LCD of 21.
 - Write the expression as a single fraction by adding the numerators.
 - Simplify the numerator by adding and subtracting like terms.
- b**
- The LCM of 10 and 6 is 30. So write each fraction as an equivalent fraction with an LCD of 30.
 - Write the expression as a single fraction by subtracting the numerators.
 - Simplify the numerator by adding and subtracting like terms.
 - Simplify by dividing the numerator and the denominator by their HCF of 2.
- c**
- When more than one fraction are being added or subtracted, find the LCD of all the denominators. The LCM of 2, 3 and 5 is 30.
 - Write each fraction as an equivalent fraction with an LCD of 30.
 - Write the expression as a single fraction by adding and subtracting the numerators.
 - Simplify by adding and subtracting like terms.

WRITE

a
$$\begin{aligned} \frac{2x}{7} + \frac{x}{3} &= \frac{2x}{7} \times \frac{3}{3} + \frac{x}{3} \times \frac{7}{7} \\ &= \frac{3(2x)}{21} + \frac{7x}{21} \\ &= \frac{6x + 7x}{21} \\ &= \frac{13x}{21} \end{aligned}$$

b
$$\begin{aligned} \frac{7x}{10} - \frac{x}{6} &= \frac{7x}{10} \times \frac{3}{3} - \frac{x}{6} \times \frac{5}{5} \\ &= \frac{21x}{30} - \frac{5x}{30} \\ &= \frac{21x - 5x}{30} \\ &= \frac{16x}{30} \\ &= \frac{8x}{15} \end{aligned}$$

c
$$\begin{aligned} \frac{x}{2} - \frac{4x}{3} + \frac{3x}{5} &= \frac{x}{2} \times \frac{15}{15} - \frac{4x}{3} \times \frac{10}{10} + \frac{3x}{5} \times \frac{6}{6} \\ &= \frac{15x}{30} - \frac{40x}{30} + \frac{18x}{30} \\ &= \frac{15x - 40x + 18x}{30} \\ &= -\frac{7x}{30} \end{aligned}$$

Helpful hints

- ✓ You don't need to find the LCD to add and subtract fractions! While the LCD is often the simplest to work with, any common denominator will do. The easiest way to find equivalent fractions with common denominators is to multiply each fraction by the denominator of the other fraction.

$$\begin{aligned} \frac{a}{c} + \frac{b}{d} &= \frac{a}{c} \times \frac{d}{d} + \frac{b}{d} \times \frac{c}{c} \\ &= \frac{ad}{cd} + \frac{bc}{dc} \\ &= \frac{ad + bc}{cd} \end{aligned}$$

Exercise 3B Algebraic fractions with numerical denominators

▲ 1-4, 5(a, c, e, g, i, j),
6(a, c), 7-9(1st, 2nd columns), 12(a-c)

■ 3-4(2nd column), 6(b, d),
7-9(3rd, 4th columns), 10,
12(d, e, f), 14, 15

◆ 3(3rd column), 6(b, d), 7-9(3rd column),
11, 13, 16-18

1 Simplify the following expressions.

a $2 \times \frac{x}{3}$

b $4 \times \frac{xy}{6}$

c $\frac{25m}{6} \times 3$

d $\frac{5}{16} \times \frac{4s}{25}$

e $\frac{x}{3} \div x$

f $\frac{p}{36} \div \frac{1}{6}$

g $\frac{n}{3} \div \frac{1}{3}$

h $\frac{x^2}{24} \div \frac{1}{4}$

2 Simplify the following fractions.

a $\frac{3 \times x \times y}{2 \times 2 \times 3 \times 5}$

b $\frac{m \times n \times n \times 5}{2 \times 2 \times 5 \times 5}$

c $\frac{x \times y \times 3}{2 \times 7 \times 3 \times 5}$

d $\frac{s \times 3 \times s \times 3}{2 \times 3 \times 3 \times 5}$

e $\frac{3 \times x \times 5 \times y \times 7}{2 \times 7 \times 5}$

f $\frac{m \times m \times m \times 8 \times n}{2 \times 2 \times 3}$

g $\frac{m \times n \times 8 \times n}{2 \times m}$

h $\frac{6 \times s \times 8 \times t}{3 \times s \times 2}$

i $\frac{3x \times 5 \times 2y}{6x \times 12}$

j $\frac{m \times m \times m \times 8}{2 \times 3m}$

k $\frac{m \times 8n}{12 \times m}$

l $\frac{6s \times 16}{36 \times 8s}$

3 Write each of the following quotients as a product of two algebraic fractions.

a $\frac{x}{2} \div \frac{x}{5}$

b $\frac{x}{12} \div \frac{3x}{5}$

c $\frac{xy}{2} \div \frac{12y}{7}$

d $\frac{xy}{16} \div \frac{x}{4}$

e $\frac{x^2}{32} \div \frac{x}{25}$

f $\frac{21x}{12} \div \frac{3x}{2}$

g $\frac{5xy}{2} \div \frac{12y}{7}$

h $\frac{x^2y}{2} \div \frac{21y}{15}$

3B.1 4 Write each of these expressions as a single fraction in simplest form.

a $\frac{x}{2} \times \frac{x}{5}$

b $\frac{x}{12} \times \frac{3x}{5}$

c $\frac{xy}{2} \div \frac{y}{7}$

d $\frac{x}{2} \times \frac{12yz}{7}$

e $\frac{x^2}{12} \times \frac{36x}{5}$

f $\frac{21x}{12} \div \frac{3x}{2}$

g $\frac{15xz}{2} \times \frac{2y}{27}$

h $\frac{x^2y}{2} \div \frac{2y}{15}$

i $\frac{3xyz}{23} \times \frac{2y}{15}$

j $\frac{8n^2}{3} \times \frac{mn}{4}$

k $\frac{25mn}{12} \div \frac{2n}{3}$

l $\frac{16st}{36} \div \frac{3t}{2}$

5 Solve each of the products from question 3.

6 Write each of these expressions as a single fraction in simplest form.

a $\frac{x}{4} \times \frac{x}{12} \times \frac{x}{5}$

b $\frac{25x}{12} \times \frac{3x}{5} \times \frac{4x}{5}$

c $\frac{x}{2} \times \frac{y}{7} \times \frac{y}{2}$

d $\frac{z}{2} \times \frac{12yz}{7} \times \frac{x}{6}$

e $\frac{x^2z}{12} \times \frac{36y}{5} \times \frac{z}{2}$

f $\frac{7x}{2} \times \frac{x}{12} \div \frac{3x}{2}$

g $\frac{x}{2} \div \frac{2x}{7} \times \frac{x}{3}$

h $\frac{x^2}{21} \div \frac{2x}{15} \times \frac{5y}{2}$

7 Simplify the following expressions.

a $\left(\frac{x}{4}\right)^2$

b $\left(\frac{3x}{4}\right)^2$

c $\left(\frac{mn}{3}\right)^2$

d $\left(\frac{3xy}{2}\right)^2$

e $\left(\frac{a}{5}\right)^3$

f $\left(\frac{xy}{2}\right)^3$

g $\left(\frac{abc}{4}\right)^3$

h $\left(\frac{2x}{5}\right)^3$

8 Find the LCD of each of the following groups of fractions.

a $\frac{x}{3}$ and $\frac{x}{5}$

b $\frac{x}{10}$ and $\frac{x}{5}$

c $\frac{x}{12}$ and $\frac{x}{8}$

d $\frac{13x}{20}$ and $\frac{x}{5}$

e $\frac{3x}{2}$ and $\frac{7x}{8}$

f $\frac{x}{3}$, $\frac{x}{6}$ and $\frac{x}{12}$

g $\frac{x}{15}$, $\frac{x}{25}$ and $\frac{x}{35}$

h $\frac{5x}{6}$, $\frac{3x}{20}$ and $\frac{7x}{5}$

3B.2 9 Write each of these expressions as a single fraction in simplest form.

a $\frac{x}{4} + \frac{x}{5}$

b $\frac{x}{2} - \frac{x}{7}$

c $\frac{x}{6} + \frac{x}{3}$

d $\frac{x}{8} - \frac{x}{10}$

e $\frac{2x}{3} + \frac{x}{2}$

f $\frac{3x}{4} - \frac{x}{8}$

g $\frac{4x}{5} + \frac{3x}{7}$

h $\frac{5a}{3} + \frac{3a}{5}$

i $-\frac{a}{3} - \frac{a}{5}$

j $-\frac{5n}{13} + \frac{13n}{5}$

k $-\frac{a}{7} + \frac{a}{5}$

l $-\frac{5s}{15} - \frac{3s}{25}$

10 Find the sum of each group of fractions in question 8.

11 Find the product of each group of fractions in question 8.

12 Simplify each of these expressions using the most appropriate method.

a $\frac{x}{3} + \frac{2x}{5} + \frac{x}{2}$

b $\frac{5x}{4} + \frac{x}{8} - \frac{x}{2}$

c $\frac{4x}{3} - \frac{5x}{6} - \frac{3x}{8}$

d $\frac{x^2}{2} - \frac{x^2}{4}$

e $\frac{5p^2}{2} + \frac{5p^2}{4}$

f $\frac{x^2}{2} + \frac{x^2}{4} + \frac{x^2}{8}$

g $\frac{5mn}{3} + \frac{7mn}{5} + \frac{9mn}{7}$

h $\frac{6x}{7} - \frac{5x}{6} - \frac{4x}{5}$

13 Fill in an algebraic expression in simplest form to make each of the following a true statement.

a $\frac{x}{4} \times \underline{\hspace{2cm}} = \frac{xy}{5}$

b $\frac{3x^2}{4} \div \underline{\hspace{2cm}} = \frac{x}{2}$

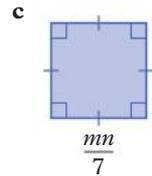
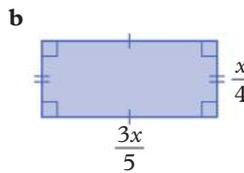
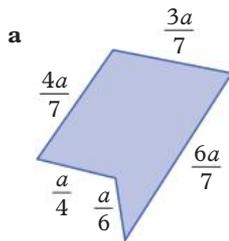
c $\frac{x}{2} + \underline{\hspace{2cm}} = \frac{7x}{4}$

d $\underline{\hspace{2cm}} - \frac{7x}{4} = \frac{17x}{4}$

e $\frac{x}{12} \times \underline{\hspace{2cm}} = \frac{xy}{24}$

f $\frac{3x^2}{4} - \underline{\hspace{2cm}} = \frac{x^2}{2}$

14 Write the perimeter of each shape as an algebraic expression in simplest form.



15 Simplify the following expressions.

a $\left(\frac{x}{12} + \frac{x}{2}\right)^2$

b $\left(\frac{2b}{3} - \frac{b}{2}\right)^3$

c $\left(\frac{2a}{5}\right)^2 + \frac{3a^2}{25}$

d $n^2 \div \left(\frac{2n}{5}\right)^2$

16 Consider the equation $\frac{m}{2} - \frac{n}{4} = \frac{x}{4}$.

- a** Substitute $m = x$ and $n = x$ into the equation to verify if the equation is a true statement.
- b** Substitute $m = 2x$ and $n = 3x$ into the equation to verify if the equation is a true statement.
- c** Find another set of algebraic expressions for m and n that makes the equation a true statement.

17 Consider the equation $\frac{a}{2} + \frac{b}{3} = \frac{10y}{3}$.

- a** Substitute $a = 4y$ and $b = 4y$ into the equation to verify if the equation is a true statement.
- b** Substitute $a = 2y$ and $b = 7y$ into the equation to verify if the equation is a true statement.
- c** Find another set of algebraic expressions for a and b that makes the equation a true statement.

18 Consider the equation $\frac{p}{2} - \frac{z}{3} = \frac{q}{6}$.

- a** Substitute $p = z$ and $q = z$ into the equation to verify if the equation is a true statement.
- b** Substitute $p = 3z$ and $q = 7z$ into the equation to verify if the equation is a true statement.
- c** Find another set of algebraic expressions for p and q that makes the equation a true statement.

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Interactive skillsheet
Algebraic fractions with
numerical denominators



Topic quiz
3B

3C Algebraic fractions with algebraic denominators

Learning intentions

By the end of this topic you will be able to ...

- ✓ multiply and divide algebraic fractions with algebraic denominators
- ✓ add and subtract algebraic fractions with algebraic denominators.



Inter-year links

Year 7	3B Equivalent fractions
Year 8	5E Dividing algebraic terms
Year 10	2E Algebraic fractions

Simplifying algebraic fractions



Key content video

- Algebraic fractions are fractions that contain at least one pronumeral. They can be simplified just like any other fractions, by identifying common factors of the numerator and the denominator.
- The fastest way to simplify a fraction is to divide the numerator and the denominator by their highest common factor (HCF).

For example, the HCF of $2x^2$ and $4x$ is $2x$, so to simplify $\frac{2x^2}{4x}$, we divide the numerator and denominator by $2x$, as shown below.

$$\frac{2x^2}{4x} = \frac{x}{2}$$

$\div 2x$

This gives us $\frac{x}{2}$.

Multiplying and dividing algebraic fractions

- The product of two algebraic fractions is the product of the numerators over the product of the denominators. For example,

$$\begin{aligned} \frac{3ab}{2} \times \frac{2}{6b} &= \frac{3ab \times 2}{2 \times 6b} \\ &= \frac{\cancel{3} \times a \times \cancel{b} \times \cancel{2}}{2 \times \cancel{2} \times \cancel{3} \times b} \\ &= \frac{a}{2} \end{aligned}$$

- The quotient of two algebraic fractions is found by multiplying the fraction before the division sign by the reciprocal of the fraction after the division sign. For example,

$$\begin{aligned} \frac{3}{8y} \div \frac{2x}{9y} &= \frac{3}{8y} \times \frac{9y}{2x} \\ &= \frac{3 \times 9y}{8y \times 2x} \\ &= \frac{27}{16x} \end{aligned}$$

Adding and subtracting algebraic fractions

- Adding and subtracting algebraic fractions is no different from adding and subtracting any other fractions.
 - 1 Identify the lowest common denominator (LCD) by identifying the lowest common multiple (LCM) of the denominators.
 - 2 Write each fraction as an equivalent fraction with the LCD.
 - 3 The sum of the algebraic fractions with a common denominator is then found by adding the numerators.

For example, $\frac{1}{4x} + \frac{6}{x}$

The LCD of $\frac{1}{4x}$ and $\frac{6}{x}$ is $4x$, so:

$$\begin{aligned}\frac{1}{4x} + \frac{6}{x} &= \frac{1}{4x} + \frac{6}{x} \times \frac{4}{4} \\ &= \frac{1}{4x} + \frac{24}{4x} \\ &= \frac{1+24}{4x} \\ &= \frac{25}{4x}\end{aligned}$$

Example 3C.1 Simplifying algebraic fractions with algebraic denominators



Simplify each of these algebraic fractions.

a $\frac{7a}{7ab}$

b $\frac{8a^3b}{2a^2b^2}$

THINK

- a**
- 1 Look for factors common to both the numerator and the denominator to determine the highest common factor (HCF). The HCF is $7a$.
 - 2 Divide both the numerator and the denominator by $7a$.
- b**
- 1 Write the coefficients and pronumerals for each term in expanded form, without expanding index form.
 - 2 Divide each coefficient by the HCF.
 - 3 Apply the quotient of powers law to cancel out repeated pronumerals that appear in both the numerator and denominator. Keep the base and subtract the indices.
 - 4 Simplify the numerator and the denominator.

WRITE

a $\frac{7a}{7ab} = \frac{7a^1}{7a^1b}$

$$= \frac{1}{b}$$

b $\frac{8a^3b}{2a^2b^2} = \frac{8 \times a^3 \times b}{2 \times a^2 \times b^2}$

$$\begin{aligned}&= \frac{8^1}{2^1} \times \frac{a^3}{a^2} \times \frac{b}{b^2} \\ &= 4 \times a^{3-2} \times \frac{1}{b}\end{aligned}$$

$$= \frac{4a}{b}$$



Example 3C.2 Multiplying and dividing algebraic fractions with algebraic denominators

Write each of these expressions as one fraction in simplest form.

a $\frac{3m^2}{2n^2} \times \frac{12}{m^2n^4}$

b $\frac{3a^2}{8b^5} \div \frac{12a}{b}$

THINK

- a**
- 1 Write the expression as a single fraction by multiplying the numerators together and the denominators together.
 - 2 Write the numerator and denominator in expanded form without expanding index form.
 - 3 Rearrange the factors in the numerator and denominator in alphabetical order.
 - 4 Rewrite the fraction as a product of a coefficient and two fractions, where one fraction only contains the pronumeral n , and one fraction only contains the pronumeral m .
 - 5 Simplify each fraction using the HCF.
 - 6 Apply the index law to simplify the answer.
- b**
- 1 Write the expression as a product of two fractions.
 - 2 Write the expression as a single fraction by multiplying the numerators together and the denominators together.
 - 3 Write the numerator and denominator in expanded form without expanding index form.
 - 4 Rearrange the factors in the numerator and denominator in alphabetical order.
 - 5 Rewrite the fraction as a product of a coefficient and two fractions, where one fraction only contains the pronumeral a , and one fraction only contains the pronumeral b .
 - 6 Simplify each fraction using the HCF and the index law.
 - 7 Write the answer as one fraction.

WRITE

a

$$\begin{aligned} \frac{3m^2}{2n^2} \times \frac{12}{m^2n^4} &= \frac{3m^2 \times 12}{2n^2 \times m^2n^4} \\ &= \frac{3 \times m^2 \times 12}{2 \times n^2 \times m^2 \times n^4} \\ &= \frac{3 \times 12 \times m^2}{2 \times n^2 \times n^4 \times m^2} \\ &= \frac{3 \times 12}{2} \times \frac{1}{n^2 \times n^4} \times \frac{m^2}{m^2} \\ &= 18 \times \frac{1}{n^{2+4}} \\ &= \frac{18}{n^6} \end{aligned}$$

b

$$\begin{aligned} \frac{3a^2}{8b^5} \div \frac{12a}{b} &= \frac{3a^2}{8b^5} \times \frac{b}{12a} \\ &= \frac{3a^2 \times b}{8b^5 \times 12a} \\ &= \frac{3 \times a^2 \times b}{8 \times b^5 \times 12 \times a} \\ &= \frac{3 \times a^2 \times b}{8 \times 12 \times a \times b^5} \\ &= \frac{3}{8 \times 12} \times \frac{a^2}{a} \times \frac{b}{b^5} \\ &= \frac{1}{32} \times a \times \frac{1}{b^4} \\ &= \frac{a}{32b^4} \end{aligned}$$



Example 3C.3 Adding and subtracting algebraic fractions with algebraic denominators

Write each of these expressions as one fraction in simplest form.

a $\frac{1}{3x} + \frac{4}{x}$

b $\frac{3}{5x} - \frac{1}{6x}$

THINK

- a** 1 The LCM of $3x$ and x is $3x$. So write each fraction as an equivalent fraction with an LCD of $3x$.
- 2 Write the expression as a single fraction by adding the numerators.
- b** 1 The LCM of $5x$ and $6x$ is $30x$. So write each fraction as an equivalent fraction with an LCD of $30x$.
- 2 Write the expression as a single fraction by subtracting the numerators.

WRITE

a
$$\begin{aligned} \frac{1}{3x} + \frac{4}{x} &= \frac{1}{3x} + \frac{4}{x} \times \frac{3}{3} \\ &= \frac{1}{3x} + \frac{12}{3x} \\ &= \frac{1+12}{3x} \\ &= \frac{13}{3x} \end{aligned}$$

b
$$\begin{aligned} \frac{3}{5x} - \frac{1}{6x} &= \frac{3}{5x} \times \frac{6}{6} - \frac{1}{6x} \times \frac{5}{5} \\ &= \frac{18}{30x} - \frac{5}{30x} \\ &= \frac{18-5}{30x} \\ &= \frac{13}{30x} \end{aligned}$$

Helpful hints

- ✓ Take care not to mix up the index laws and definitions.
 - Across a multiplication sign, add indices.
 - Across a division sign, subtract indices.
 - Across brackets, multiply indices.
 - Zero index: $a^0 = 1$

Index law	Example
Product of powers law	$a^5 \times a^3 = a^{5+3}$
Quotient of powers law	$a^5 \div a^3 = a^{5-3}$
Power of a power law	$(a^5)^3 = a^{5 \times 3}$

ANS
p436

Exercise 3C Algebraic fractions with algebraic denominators

1-4, 5(a, c, e, g), 6(a, c), 7(a-c), 8(a, c), 10(a, b), 11, 12-14(a-c), 15, 17(a)

1-7(a-c), 8(1st column), 9, 10(a-d), 11, 12-14(1st column), 17(b), 18

3-7(3rd column), 8(2nd column), 9(c, d), 10(e-h), 16-19

3C.1 1 Simplify the following expressions.

a $\frac{kmn}{kp}$

b $\frac{-16zw}{8xv}$

c $\frac{18ab}{15ac}$

d $\frac{18a^2}{3a}$

e $\frac{6x^2y}{-2x}$

f $\frac{2a^2bc}{8abd}$

g $\frac{6x^2y}{2xy}$

h $\frac{xy^{12}}{x^4y^8}$

2 Simplify the following expressions.

a $2 \times \frac{y}{6x}$

b $4n \times \frac{n}{6m}$

c $\frac{25}{6p} \times 3p$

d $\frac{5}{16s} \times \frac{4}{25s}$

e $\frac{3}{x} \div 6$

f $\frac{p}{36q} \div \frac{1}{q}$

g $\frac{n}{3m} \div \frac{1}{3m}$

h $\frac{1}{24} \div \frac{1}{4x}$

3c.2 3 Write each of these expressions as a single fraction in simplest form.

a $\frac{1}{2y} \times \frac{1}{5y}$

b $\frac{1}{12m} \times \frac{1}{n}$

c $\frac{q}{2p} \times \frac{12p}{7q}$

d $\frac{7y}{2x^3} \times \frac{12x^2}{7y}$

e $\frac{x^2}{32y} \times \frac{8x}{25y^3}$

f $\frac{21x}{12y^6} \times \frac{3x}{2y^3}$

4 Write each of these expressions as a single fraction in simplest form.

a $\frac{xy}{2} \div \frac{x}{y}$

b $\frac{x}{y} \div \frac{12yz}{5}$

c $\frac{x^2}{y} \div \frac{36x}{y^2}$

d $\frac{21p^2}{12q} \div \frac{3p^3}{2q^2}$

e $\frac{25mn^2}{12} \div \frac{2n}{3m^2}$

f $\frac{12}{5abc} \div \frac{ab}{2c}$

5 Write each of these expressions as a single fraction in simplest form.

a $\frac{1}{4x} \times \frac{1}{12x} \times \frac{1}{x}$

b $\frac{25}{12m} \times \frac{3}{5n} \times \frac{4}{5m}$

c $\frac{1}{2x} \times \frac{y}{7} \times \frac{1}{2y}$

d $\frac{x}{2yz} \times \frac{12y}{7x} \times \frac{1}{6z}$

e $\frac{x^2z}{12} \times \frac{36y}{5z^4} \times \frac{z}{2x^3}$

f $\frac{7x}{2yz} \times \frac{xy}{12z} \div \frac{3}{2x^2}$

g $\frac{z}{2x} \div \frac{2x}{7yz} \times \frac{y}{3z}$

h $\frac{x^2}{21yz} \div \frac{2x}{15yz} \times \frac{5y}{2xz}$

3c.3 6 Write each of these expressions as a single fraction in simplest form.

a $\frac{1}{x} + \frac{1}{x}$

b $\frac{1}{3a} + \frac{1}{a}$

c $\frac{5}{3a} + \frac{3}{5a}$

d $\frac{1}{3a} - \frac{2}{5a}$

e $-\frac{5}{13n} + \frac{13}{5n}$

f $-\frac{5}{2p} + \frac{3}{4p}$

g $-\frac{1}{12ab} + \frac{12}{ab}$

h $-\frac{7}{4xyz} + \frac{4}{7xyz}$

7 Simplify each of these expressions using the most appropriate method.

a $\frac{3}{m^2} + \frac{5}{3m^2} + \frac{1}{5m^2}$

b $\frac{5}{3st} + \frac{7}{5st} + \frac{9}{7st}$

c $\frac{6x}{7y} - \frac{5x}{6y} - \frac{4x}{5y}$

d $\frac{2x^2}{7} \times \frac{3}{10x} \times \frac{5}{x}$

e $\frac{4}{x} \div \frac{12}{x^2} - \frac{5x}{7}$

f $\frac{y^2}{2x^2} + \frac{y^2}{4x^2} + \frac{y^2}{8x^2}$

8 Fill in an algebraic expression in simplest form to make each of the following a true statement.

a $\frac{1}{4xy} \times \text{---} = \frac{xy}{5}$

b $\frac{3x^2}{4} \div \text{---} = \frac{z}{2xy}$

c $\frac{1}{2x} + \text{---} = \frac{7}{4x}$

d $\text{---} - \frac{7}{4x} = \frac{17}{4x}$

e $\frac{x}{4} \times \text{---} = \frac{3x}{5y^2}$

f $\frac{2}{3x^3} \div \text{---} = \frac{x}{2}$

g $\frac{x}{12y} \times \text{---} = \frac{xy}{24}$

h $\text{---} - \frac{5}{6mn} = \frac{1}{2mn}$

9 Simplify the following expressions.

a $\left(\frac{1}{12a} + \frac{1}{2a}\right)^2$

b $\left(\frac{2}{3b} - \frac{1}{2b}\right)^3$

c $\left(\frac{2}{5xy}\right)^2 + \frac{3}{25x^2y^2}$

d $mn^2 \div \left(\frac{2n}{5m}\right)^2$

10 Simplify each expression following the correct order of operations.

a $\frac{ab}{2} - \frac{4a}{3} \times \frac{3b}{5}$

b $\frac{x}{2} + \frac{5}{6} \times \frac{3x}{5}$

c $\frac{x}{2} \times \frac{4}{3} + \frac{x}{2}$

d $\frac{x}{2} + \frac{7}{4} \times \frac{3x}{7}$

e $\frac{x}{2} \div \frac{5x}{6} - \frac{3}{5}$

f $\frac{x}{2} - \frac{4xy}{3} \div \frac{3y}{4}$

g $\frac{mn}{2} + \frac{4m}{3} \times \frac{3n}{5}$

h $\frac{3}{7} + \frac{2x}{5} \div \frac{3x}{5}$

11 Eve completed a worksheet on algebra. Identify the errors in Eve's working.

$$\begin{aligned} \frac{n}{2m} - \frac{4n}{7m} + \frac{2n}{5m} &= \frac{n}{2m} \times \frac{1}{35} - \frac{4n}{7m} \times \frac{1}{10} + \frac{2n}{5m} \times \frac{1}{14} \\ &= \frac{n}{70m} - \frac{4n}{70m} + \frac{2n}{70m} \\ &= \frac{n - 4n + 2n}{70m} \\ &= -\frac{n}{70m} \end{aligned}$$

12 Find the LCD of each of the following groups of fractions.

a $\frac{1}{xy}$ and $\frac{x}{5}$

b $\frac{x}{y}$ and $\frac{y}{x}$

c $\frac{y}{12x}$ and $\frac{x^2}{8}$

d $\frac{x}{3y}$, $\frac{1}{6x}$ and $\frac{y}{12x^2}$

e $\frac{1}{15mn}$, $\frac{1}{25m}$ and $\frac{1}{5n}$

f $\frac{1}{6xy}$, $\frac{1}{20x^2}$ and $\frac{1}{5y^3}$

13 Find the sum of each group of fractions in question 12.

14 Find the product of each group of fractions in question 12.

15 The area of a rectangle is $16xy$.

a If the length is $8x$, write an expression for the breadth of the rectangle.

b If the length of the rectangle is 16 m and y is 5 m, calculate the breadth and the area of the rectangle using the result from part a.

16 The area of a right-angled triangle is $6x^2$.

a If the base length is $4x$, write an expression for the height of the triangle.

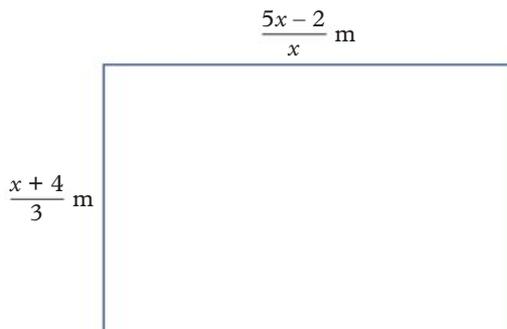
b If the height of the triangle is 12 cm, calculate the area and the base length of the triangle using the result from part a.

17 Write each of these expressions as a single fraction.

a $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$

b $\frac{a}{b} + \frac{b}{c} + \frac{c}{a}$

18 Consider the rectangle below.



a Write an expression for the area of the rectangle.

b Write an expression for the perimeter of the rectangle, expressed as just a single fraction.

19 a Simplify $\frac{1}{(n-1)a} + \frac{1}{na} + \frac{1}{(n+1)a}$ as a single fraction in simplest form by filling in the missing terms.

$$\begin{aligned} \frac{1}{(n-1)a} + \frac{1}{na} + \frac{1}{(n+1)a} &= \frac{n(n+1) + (n-1)(n+1) + \underline{\hspace{2cm}}}{(n-1)n(n+1)a} \\ &= \frac{n^2 + n + n^2 - 1 + n^2 - n}{(n-1)n(n+1)a} \\ &= \frac{\underline{\hspace{2cm}} - 1}{(n-1)n(n+1)a} \end{aligned}$$

b Choose an appropriate value of n and use the conclusion from part a to simplify $\frac{1}{13a} + \frac{1}{14a} + \frac{1}{15a}$.

c Verify your answer in part b by adding the algebraic fractions.

Check your Student obook pro for these digital resources and more:



Interactive skillsheet
Algebraic fractions with algebraic denominators



Topic quiz
3C

Checkpoint



Checkpoint quiz

Check your knowledge of the first part of this chapter.

- ✓ Core
- ✓ Advanced

3A 1 Simplify the expressions.

- a** $-3a + 6b - 5a - 8b + 12a - 5$
b $9t - t^2 + 4t - 6 + 5t^2 + 2 - t$
c $7x^3 - 4x^2 - 3x + 2 + 8x - 3x^2 - 5x^3$
d $7cd^2 - 7d^2 + 2c - 8d^2 - 3d^2c + 5d^2$

3A 2 Simplify the expressions.

- a** $5a \times (-7b)$
b $-8ac \times 3a \times (-2bc)$
c $\frac{45gh}{20g}$
d $\frac{18t^2u}{54tu}$

3A 3 Simplify the expressions.

- a** $(-5a^3b^6)^2 \times (-2a^2b^4)^5$
b $(3a + 2b - 3a) - (2b - 2a + b)$
c $2a^2 \times 3b \times 4ab \times b^2$
d $mn^2 \times 3n \div (4m \times n^2)$

3B 4 Simplify the following.

- a** $\frac{n}{4} \times \frac{2m}{5}$
c $\frac{25xy}{2} \div \frac{2y}{5}$

3B 5 Simplify the following.

- a** $\frac{2a}{3} + \frac{5a}{6}$
c $\frac{x^2}{3} - \frac{x^2}{6}$

3B 6 Simplify the following.

- a** $\left(\frac{2ab}{3}\right)^2$
c $\frac{5z^2}{3} + \frac{7z^2}{5} + z^2$

3C 7 Simplify the following.

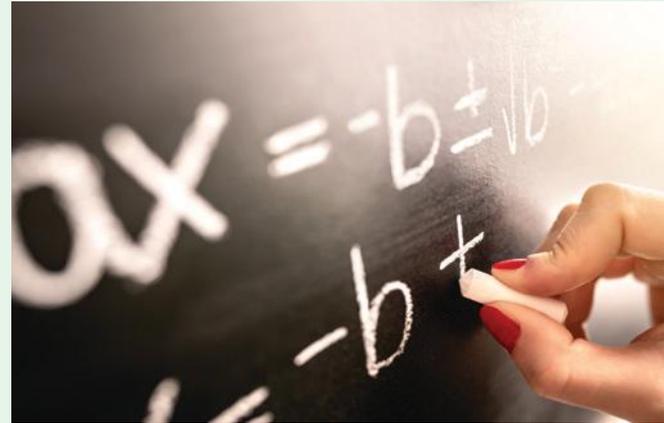
- a** $\frac{36b^2c}{9abd}$
c $\frac{pq^{12}}{p^4q^4}$

ADV 3C 8 Simplify the following.

- a** $\frac{x^2y}{z} \div \frac{36z}{y^2}$
c $\frac{xy^{12}}{32y} \times \frac{8x}{5y^3}$

ADV 3C 9 Simplify the following.

- a** $\frac{c}{ab} \times \frac{25ab^2}{12} \div \frac{2a}{3c}$
c $\frac{q}{2p} \div \frac{-q}{p^2} \times \frac{2pq}{-7}$



b $\frac{ab}{12} \times \frac{3a}{5}$

d $\frac{x}{12} \div \frac{2x}{7}$

b $\frac{5m}{2} - \frac{m}{4}$

d $\frac{5xyz}{2} + \frac{5xyz}{4}$

b $\left(\frac{a}{3} - \frac{a}{2} + \frac{a}{3}\right)^2$

d $\frac{6pq}{5} - \frac{5pq}{6}$

b $\frac{6x^2yz}{12xyz}$

d $\frac{6y^2z}{-2xy}$

b $\frac{21p^4}{2q} \div \frac{p^3}{12q^2}$

d $\frac{11n}{2m^6} \times \frac{3n}{2m^3}$

b $\frac{12}{5mn} \div \left(\frac{mn}{2} \times \frac{1}{n}\right)$

d $\frac{7s}{s^7} \times \frac{7t^2}{4s^3} \times \frac{12s^2}{7t}$

3D Expanding

Learning intentions

By the end of this topic you will be able to ...

- ✓ expand algebraic expressions of the form $a(b + c)$
- ✓ expand algebraic expressions of the form $(a + b)(c + d)$.



Inter-year links

Year 8 5F Expanding

Year 10 2D Expanding

The distributive law



Key content video

- A **binomial** is an expression containing two terms that are either added or subtracted. For example, $x - 7$, $2x + y$ and $x^2 + y^2$ are all binomials.
- A **trinomial** is an expression containing three terms that are either added or subtracted. For example, $x - 7 + y$, $2x^2 + 2y + y$ and $x^2 + 2xy + y^2$ are all trinomials.
- The **distributive law** is used to distribute or **expand** products over addition and subtraction.

$$a(b + c) = ab + ac$$

$$a(b + c + d) = ab + ac + ad$$

Note: The distributive law only applies if the expression inside the brackets is not raised to the power of any index (other than 1). For example, in $a(b + c)^2$, $(b + c)^2$ needs to be expanded first to remove the index before being multiplied to a .

- To expand algebraic expressions with one pair of brackets:
 - 1 Multiply each term inside the brackets by the term outside the brackets.
 - 2 Simplify the results by performing any multiplication, addition and subtraction.
- The distributive law for expanding over a **binomial product** is: $(a + b)(c + d) = ac + ad + bc + bd$

$$(a + b)(c + d) = ac + ad + bc + bd$$

- To expand binomial products:
 - 1 Multiply each term inside the second pair of brackets, c and d , by each term in the first pair of brackets, a and b .
 - 2 Simplify the results by performing any multiplication, addition and subtraction.
- Where possible, expanded expressions should be simplified.
- The distributive law for expanding a binomial product can be derived by treating the first binomial as a single term and then applying the distributive law for expanding over one pair of brackets.

$$\begin{aligned}(a + b)(c + d) &= (a + b)c + (a + b)d \\ &= c(a + b) + d(a + b) \\ &= ca + cb + da + db \\ &= ac + ad + bc + bd\end{aligned}$$

Example 3D.1 Expanding over one pair of brackets



Expand and simplify each expression using the distributive law.

a $-5(3a + 8)$

b $-3a(5a - 4b + 3c)$

THINK

- a** 1 Multiply each term inside the brackets by the term outside the brackets.
- 2 Simplify the results by performing the multiplications. Take care with + and - signs when simplifying.
- b** 1 Multiply each term inside the brackets by the term outside the brackets. Enclose negative terms in brackets to avoid errors.
- 2 Simplify the results by performing the multiplications. Remember the product of two negative numbers gives a positive number.

WRITE

a $-5(3a + 8) = -5 \times 3a + (-5) \times 8$

$$= -15a - 40$$

b $-3a(5a - 4b + 3c) = -3a \times 5a + (-3a) \times (-4b) + (-3a) \times (3c)$

$$= -15a^2 + 12ab - 9ac$$

Example 3D.2 Expanding binomial products



Expand each algebraic expression to remove the brackets.

a $(a + 9)(b + 2)$

b $(x + 4)(x - 6)$

THINK

- a** 1 Multiply each term inside the second pair of brackets, b and 2 , by the first term in the first pair of brackets, a , and then the second term in the first pair of brackets, 9 .
- 2 Simplify each term.
- b** 1 Multiply each term inside the second pair of brackets, x and -6 , by the first term in the first pair of brackets, x , and then the second term in the first pair of brackets, 4 .
- 2 Simplify each term.
- 3 Simplify any like terms.

WRITE

a $(a + 9)(b + 2) = a \times b + a \times 2 + 9 \times b + 9 \times 2$

$$= ab + 2a + 9b + 18$$

b $(x + 4)(x - 6) = x \times x + x \times (-6) + 4 \times x + 4 \times (-6)$

$$= x^2 - 6x + 4x - 24$$
$$= x^2 - 2x - 24$$

- ✓ When multiplying by a negative term, use brackets for clarity.
For example: $-7p(3q - 4) = -7p \times 3q + (-7p) \times (-4)$
- ✓ Show all your working to avoid arithmetical errors and to ensure all signs are correct.
- ✓ After expanding expressions containing brackets, always simplify your results by looking for like terms.
Simplify, simplify, simplify!
- ✓ A good way of remembering which terms to multiply when expanding a binomial product is to memorise 'FOIL' (First terms, Outer terms, Inner terms, Last terms).

$$(a + b)(c + d) = ac + ad + bc + bd$$

First Outer F. O. I. L.
Inner Last

Note: This rule only works for expressions of the form $(a + b)(c + d)$.

ANS
p437

Exercise 3D Expanding

 1 (1st column), 2, 3-4 (1st, 2nd column),
5-8 (1st, 2nd columns), 9-15, 17, 19, 21

 2-8 (2nd column), 11-16,
18, 20, 24

 4(b, d, f), 5-8 (2nd column),
11, 15, 16, 22-25

3D.1 1 Expand each algebraic expression to remove the brackets.

a $4(a + 3)$

b $7(b + 5)$

c $3(c - 2)$

d $5(d - 1)$

e $6(4 + e)$

f $-2(f + 8)$

g $-3(g + 4)$

h $-8(h - 5)$

i $-4(x - 9)$

j $-5(2 - j)$

k $k(p + 6)$

l $a(b - 4)$

m $6(3m + k)$

n $n(2p + 4q)$

o $x(x - 7y)$

2 Expand and simplify each algebraic expression.

a $2(2ab + 4bc + 6ac)$

b $-3(xy + x + y)$

c $-(m^2 - n^2 + 2m^2n^2)$

d $a^2(a^2 - b^2 + 2ab)$

e $-\frac{1}{4}(4pq - 16p + 32q)$

f $\frac{4abc(4b + 2a + 8c)}{8}$

3 Expand and simplify each algebraic expression.

a $3(x + 2) + 8x$

b $11 + 5(p - 1)$

c $a(b + 4) - 2a$

d $-7(1 - y) + 4y + 3$

e $5k + 2 + 4(h - k)$

f $m(m - 6) - m^2$

4 Expand and simplify each algebraic expression.

a $2(x + 5) + 3(x - 6)$

b $8(k - 3) + 5(k + 4)$

c $3(p + 7) - 4(5 - p)$

d $x(x + 1) + 3(x + 4)$

e $m(m + 2) + 3(m + 2)$

f $y(y - 5) - 2(y - 5)$

5 Expand and simplify each expression.

a $3a(4z + 5)$

b $8b(7 + 5y)$

c $2c(7 - 3x)$

d $-5d(4w + 5)$

e $-6e(8v - 9)$

f $-10e(4t + 3u)$

g $4g(2g - 7t)$

h $-6h(5r - 7h)$

i $3ij(4ik + 5jk)$

3D.2 6 Expand each algebraic expression to remove the brackets.

a $(a + 3)(b + 4)$

b $(c + 2)(d + 7)$

c $(m + 5)(n + 1)$

d $(j - 9)(k - 5)$

e $(2a + 7)(b + 3)$

f $(5c + 2)(3d - 4)$

7 Expand each algebraic expression to remove the brackets.

a $(a + 2)(a + 3)$

b $(x + 5)(x + 10)$

c $(d + 4)(d - 6)$

d $(y + 3)(y - 8)$

e $(k - 7)(k + 9)$

f $(m - 6)(m + 3)$

g $(5e - 2)(e - 4)$

h $(7a - 8)(3a - 1)$

i $(3y - 5)(2y - 1)$

8 Expand each algebraic expression to remove the brackets.

a $(5 - a)(b + 4)$

b $(x + 6)(3 - x)$

c $(2c + 3)(5 - 4c)$

d $(11d - 9e)(4d - 6e)$

e $(6p + 5q)(8q - 7p)$

f $(x + 2)(x - 3) + 3x^2 - 8x + 5$

g $(x^3 + x^2)(x^5 + x)$

h $(4x^2 + 7)(12 - 5x^2)$

i $(x + 2)(x + 3) + (x + 1)(x + 5)$

9 Expand each algebraic expression to remove the brackets.

a $(a + 3)(a - 3)$

b $(b + 2)(b - 2)$

c $(c + 5)(c - 5)$

d $(3p - 8)(3p + 8)$

e $(h - 1)(h + 1)$

f $(k + m)(k - m)$

10 Describe the pattern or shortcut you can see in question 9. What is special about the two binomial factors that are multiplied together?

11 The pattern you observed in question 10 is known as the difference of two squares. This rule can be written as:

$$(a + b)(a - b) = a^2 - b^2$$

a Why do you think the rule is called the difference of two squares?

b Does it matter whether the product is $(a + b)(a - b)$ or $(a - b)(a + b)$? Explain.

c Use the rule (or shortcut) to expand each algebraic expression.

i $(x - 2)(x + 2)$

ii $(y + 9)(y - 9)$

iii $(m + 6)(m - 6)$

iv $(3d - 10)(3d + 10)$

v $(m + n)(m - n)$

vi $(3 + x)(3 - x)$

12 **a** Use the difference of two squares expansion rule to expand $(100 + 3)(100 - 3)$.

b Use your answer to work out the result for 103×97 .

c Work out the result for each product without using a calculator.

i 102×98

ii 95×105

iii 1001×999

iv 994×1006

13 Expand each algebraic expression to remove the brackets and simplify.

a $(a + 2)(a + 2)$

b $(b + 7)(b + 7)$

c $(c + 4)(c + 4)$

d $(3p - 5)(3p - 5)$

e $(h - 1)(h - 1)$

f $(m + n)(m + n)$

14 Describe the pattern or shortcut you can see in question 13. What is special about the two binomial factors that are multiplied together?

15 The pattern you observed in question 14 is known as the expansion of a perfect square. This rule can be written as: $(a + b)^2 = a^2 + 2ab + b^2$.

a Why do you think the rule is called the expansion of a perfect square?

b Use the rule (or shortcut) to expand each algebraic expression.

i $(x + 3)^2$

ii $(y + 6)^2$

iii $(m + 2)^2$

iv $(4b + 11)^2$

v $(m + n)^2$

vi $(5 + x)^2$

c Is $(a - b)^2 = a^2 - 2ab + b^2$ also an expansion of a perfect square? Explain.

d Expand each algebraic expression.

i $(a - 2)^2$

ii $(b - 4)^2$

iii $(c - 7)^2$

iv $(5w - 6)^2$

v $(k - p)^2$

vi $(3 - x)^2$

16 Expand each algebraic expression.

a $2x(3w - 5x + 7y - 4z)$

b $(3a + 5b - 6)(2x + 3y)$

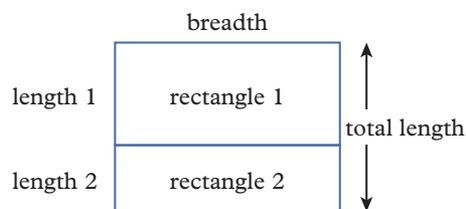
c $(x + 3y + 6)(2x + y + 3)$

d $(x + 2)(x^2 + 6x + 7)$

17 The area of the large rectangle in this diagram can be determined in two ways:

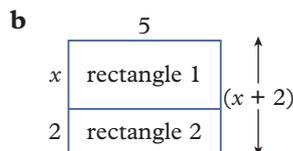
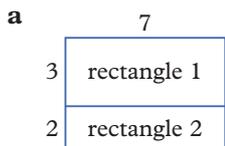
Area of large rectangle = length \times total breadth

Area of large rectangle = area of rectangle 1 + area of rectangle 2
 = length \times breadth 1 + length \times breadth 2

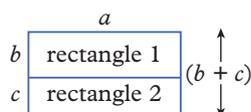


For each diagram below:

- i write an expression for the area of the large rectangle by multiplying the length by the total breadth
- ii write an expression for the area of the large rectangle by adding the area of rectangle 1 and rectangle 2.



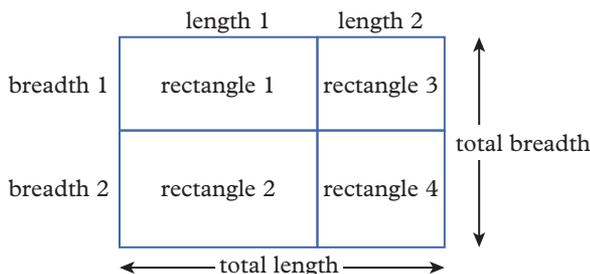
c Use the rectangle below to explain how $a(b + c)$ can be expanded to obtain $ab + ac$.



18 The area of the largest rectangle in this diagram can be determined in two ways:

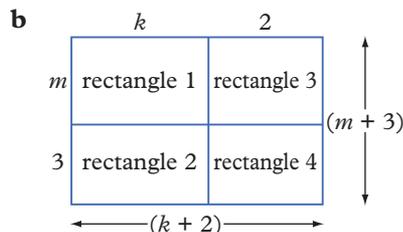
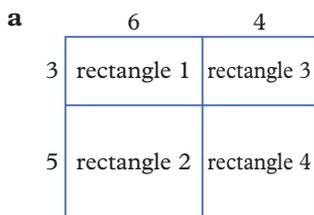
Area of largest rectangle = total length \times total breadth

Area of largest rectangle = area of rectangle 1 + area of rectangle 2 + area of rectangle 3 + area of rectangle 4
 = breadth 1 \times length 1 + breadth 2 \times length 1 + breadth 1 \times length 2
 + breadth 2 \times length 2

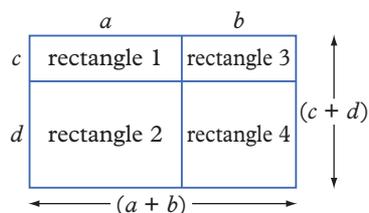


For each diagram below:

- i write an expression for the area of the large rectangle by multiplying the total length by the total breadth
- ii write an expression for the area of the large rectangle by adding the area of rectangle 1, rectangle 2, rectangle 3 and rectangle 4.

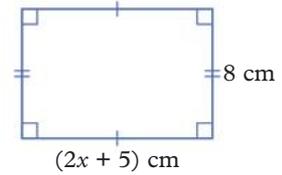


c Use this rectangle to explain how $(a + b)(c + d)$ can be expanded to obtain $ac + ad + bc + bd$.



- 19 Which of the following is not equivalent to $3(x + 2)(x + 8)$?
- A** $(3x + 6)(x + 8)$ **B** $(x + 2)(3x + 24)$ **C** $3(x^2 + 10x + 16)$
- D** $3x^2 + 30x + 48$ **E** $(3x + 6)(3x + 24)$

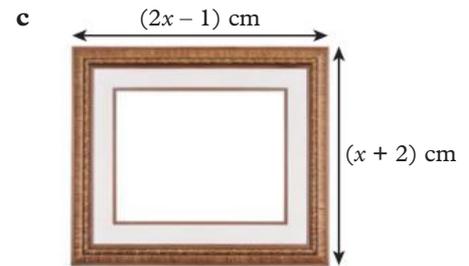
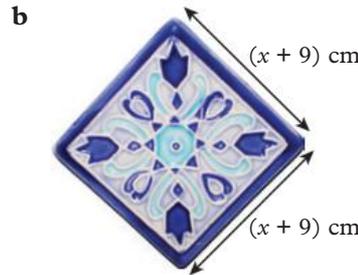
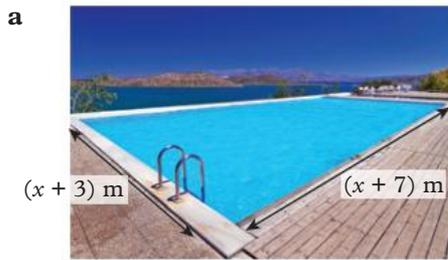
- 20 **a** For the rectangle on the right, write the area as a product of two factors.
b Expand the expression to remove brackets.
c When $x = 3$, calculate the area using your answer to:



- i** part **a** **ii** part **b**.
d Compare your answers in part **c**. How does this show that you have expanded the area expression correctly?

- 21 For each of the following rectangles:

- i** write the area as a binomial product
ii expand the binomial product to remove brackets
iii calculate the area when $x = 5$.



- 22 A rectangular trampoline is 6 m long and 4 m wide. Safety matting that is p metres wide is to be placed around the perimeter of the trampoline.

- a** Draw a labelled diagram of the top of the trampoline and the matting around it.
b Write an expression for:
i the total length of the trampoline and matting
ii the total breadth of the trampoline and matting.
c Write an expression for the total area taken up by the trampoline and matting. Simplify the expression by expanding to remove any brackets.
d Write an expression for the area of the matting only.
e If $p = 2$, calculate the area taken up by:
i the trampoline **ii** the trampoline and matting **iii** the matting.

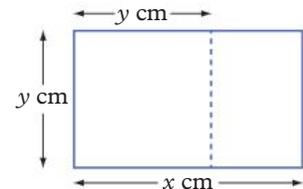
- 23 Expand and simplify each expression.

- a** $(10y - 7)^2$ **b** $x^4(x^3 - x^2)$ **c** $(x^2 - 5)(x^2 + 5)$
- d** $(y^6 + 3y)^2$ **e** $a^3(a^2 + 4) - a^2(a^3 + 9a)$ **f** $x^5y^2(xy^4 + 2w)$

- 24 Show that the expression $(a - b)^2 + (c - d)^2$ is equivalent to $(b - a)^2 + (d - c)^2$.

- 25 A rectangular piece of paper is x cm long and y cm wide. The paper is torn along a line parallel to its breadth, forming a square of side length y cm, and another rectangle.

- a** Write the dimensions of the newly formed square and rectangle.
b Prove that the area of the original rectangle is the same as the total area of the two new shapes.



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pro



Interactive skillsheet

Expanding over one pair of brackets



Interactive skillsheet

Expanding binomial products



Worksheet

Expanding



Investigation

It's a mystery



Topic quiz

3D

3E Factorising using the HCF

Learning intentions

By the end of this topic you will be able to ...

- ✓ find the HCF of two or more algebraic terms
- ✓ factorise algebraic expressions by taking out the HCF
- ✓ factorise algebraic expressions by grouping terms.



Inter-year links

- Year 7** 2D Factors and the highest common factor
- Year 8** 5G Factorising
- Year 10** 2F Factorising

Factorising terms

- **Factorising** an algebraic expression is the opposite to expanding an algebraic expression.
- An expression is in simplest factorised form if all the terms inside the brackets have no common factors.
- An expression is in expanded form if it has no brackets and is simplified.



Key content video

$$\begin{array}{l}
 \text{expanding} \longrightarrow \\
 7(a + 2) = 7a + 14 \\
 \longleftarrow \text{factorising} \\
 \\
 2x^2 - 6x = 2x(x - 3) \\
 \text{expanded form} \quad \text{factorised form}
 \end{array}$$

The highest common factor (HCF) of an expression

- The **highest common factor (HCF)** of two or more algebraic terms is the ‘highest’ or ‘largest’ combination of coefficients, pronumerals and algebraic expressions that are factors of every term.
- The HCF of an expression can have numerical and pronumeral terms.
- The HCF can be a binomial factor.
- To identify the HCF of an expression:
 - 1 Find the HCF of the coefficients.
 - 2 Find the HCF of the pronumerals.
 - 3 Multiply the factors from steps 1 and 2, and then simplify.

Algebraic expressions	HCF
$x + 7x$	x
$6m + 12mn + 8mp$	$2m$
$15a^3bc + 9a^2c$	$3a^2c$
$x(x - 1) + 2(x - 1)$	$(x - 1)$

Factorising algebraic expressions

- To factorise an expression:
 - 1 Identify the HCF of the expression.
 - 2 Write the HCF in front of the brackets. Inside the brackets, divide each term in the original expression by the HCF.
 - 3 Simplify the bracketed expression.

$$\begin{aligned}
 7a + 14 &= 7 \left(\frac{7^1 a}{7^1} + \frac{14^1}{7^1} \right) \\
 &= 7(a + 2)
 \end{aligned}$$

Factorising by grouping terms

- To factorise an expression with four terms by grouping:
 - 1 Identify pairs of terms with common factors. Group the pairs of terms, remembering to move the positive or negative sign with the term.
 - 2 Factorise each pair of terms by dividing out the HCF.
 - 3 Factorise using the binomial factor and simplify the expression.

$$\begin{aligned}
 x^2 + 2x - 3x - 6 &= x(x + 2) - 3(x + 2) \\
 &= (x + 2)(x - 3)
 \end{aligned}$$



Example 3E.1 Identifying the HCF

Find the highest common factor (HCF) of each pair of terms.

a 24 and 84

b a^2b and b^2ac

c $12x^2y^3$ and $-18x^3y$

THINK

- a**
- 1 Write the prime factorisation for each number.
 - 2 Multiply the common factors to find the HCF.
- b**
- 1 Write each term in expanded form.
 - 2 Multiply the common factors to find the HCF.
- c**
- 1 Find the HCF of the coefficients.
The negative sign can be represented using the factor '-1'.
 - 2 Find the HCF of the pronumerals.
 - 3 Multiply the common factors to find the HCF. Simplify the product.

WRITE

- a** $24 = 2 \times 2 \times 2 \times 3$
 $84 = 2 \times 2 \times 3 \times 7$
 HCF = $2 \times 2 \times 3$
 = 12
- b** $a^2b = a \times a \times b$
 $b^2ac = b \times b \times a \times c$
 HCF = $a \times b$
 = ab
- c** The HCF of 12 and -18 is 6.
 The HCF of x^2y^3 and x^3y is x^2y .
 The HCF of $12x^2y^3$ and $-18x^3y$ is $6x^2y$.

Example 3E.2 Factorising algebraic expressions



Factorise each expression.

a $6a + 18$

b $x^2 - 7x$

c $10k^3m^2 + 15k^2m$

THINK

- a**
- 1 Identify the HCF.
 - 2 Write the HCF in front of the brackets.
Inside the brackets, divide each term by the HCF.
 - 3 Simplify the bracketed expression.
- b**
- 1 Identify the HCF.
 - 2 Write the HCF in front of the brackets.
Inside the brackets, divide each term by the HCF.
 - 3 Simplify the bracketed expression.
Use the index laws to help you.
- c**
- 1 Identify the HCF.
 - 2 Write the HCF in front of the brackets.
Inside the brackets, divide each term by the HCF.
 - 3 Simplify the bracketed expression.
Use the index laws to help you.

WRITE

- a** HCF = 6
 $6a + 18 = 6\left(\frac{6^1a}{6^1} + \frac{18^3}{6^1}\right)$
 = $6(a + 3)$
- b** HCF = x
 $x^2 - 7x = x\left(\frac{x^2}{x} - \frac{7x}{x}\right)$
 = $x(x^{2-1} - 7x^{1-1})$
 = $x(x - 7)$
- c** HCF = $5k^2m$
 $10k^3m^2 + 15k^2m$
 = $5k^2m\left(\frac{10^2k^3m^2}{5^1k^2m} + \frac{15^3k^2m}{5^1k^2m}\right)$
 = $5k^2m(2k^{3-2}m^{2-1} + 3k^{2-2}m^{1-1})$
 = $5k^2m(2km + 3)$



Example 3E.3 Factorising using an HCF that is a binomial factor

Factorise each expression.

a $y(x + 3) + 7(x + 3)$

b $4k(2 - m) - 5(2 - m)$

THINK

- a**
- 1 Identify the HCF. It is the binomial factor of $(x + 3)$.
 - 2 Write the HCF in front of the brackets. Inside the brackets, divide each term by the HCF.
 - 3 Simplify the bracketed expression.
- b**
- 1 Identify the HCF. It is the binomial factor of $(2 - m)$.
 - 2 Write the HCF in front of the brackets. Inside the brackets, divide each term by the HCF.
 - 3 Simplify the bracketed expression.

WRITE

a HCF = $(x + 3)$

$$y(x + 3) + 7(x + 3) = (x + 3) \left(\frac{y(x + 3)^1}{(x + 3)^1} + \frac{7(x + 3)^1}{(x + 3)^1} \right)$$

$$= (x + 3)(y + 7)$$

b HCF = $(2 - m)$

$$4k(2 - m) - 5(2 - m) = (2 - m) \left(\frac{4k(2 - m)^1}{(2 - m)^1} - \frac{5(2 - m)^1}{(2 - m)^1} \right)$$

$$= (2 - m)(4k - 5)$$

Example 3E.4 Factorising by grouping terms



Factorise $xy + 2x + 3y + 6$ by grouping terms.

THINK

- 1 Check for an HCF of all four terms. Group the terms in pairs and identify the HCF for each pair.
- 2 Factorise each pair of terms by dividing out the HCF.
- 3 Factorise using the binomial factor of $(y + 2)$. Simplify the expression.

WRITE

$$xy + 2x + 3y + 6 = (xy + 2x) + (3y + 6)$$

HCF of xy and $2x$ is x .
 HCF of $3y$ and 6 is 3 .

$$xy + 2x + 3y + 6 = x(y + 2) + 3(y + 2)$$

$$= (y + 2)(x + 3)$$

Helpful hints

- ✓ When looking for the highest common factor, remember to consider any coefficients and all pronumerals.
- ✓ The divisibility rules can help you to find the HCF. Recall that a number is:
 - divisible by 2, if the number is even
 - divisible by 3, if the number's digits add to a multiple of 3
 - divisible by 5, if the number ends in 0 or 5.
- ✓ The great thing about factorising is that you can always check your answer by expanding the brackets!

$$2x - 16 = 2(x - 8) = 2x - 16$$

Exercise 3E Factorising using the HCF

 1-4(1st, 2nd columns), 5, 6,
7-10(1st, 2nd columns), 11, 12, 15,
16(a, b), 17

 2-4(3rd column), 5-6(e-h),
7-10(2nd, 3rd columns), 13-16,
18, 20, 21, 24

 4(g-i), 7-10(2nd, 3rd columns),
15, 16, 19-25

3E.1 1 Find the highest common factor (HCF) of each pair of terms.

- | | | |
|-------------------------|----------------------------|----------------------------|
| a 4 and 28 | b 6 and 10 | c 15 and 35 |
| d d^2 and $3d$ | e $2e$ and $2k$ | f 3 and -6 |
| g 12 and -8 | h $9h$ and $-15h^2$ | i $24x^2$ and $36x$ |

2 Find the HCF of each pair of terms.

- | | | |
|--------------------------|-----------------------------|----------------------------|
| a bc and cd | b $2xy$ and $2y$ | c $18mn$ and $-9m$ |
| d $abcd$ and bd | e $8xy$ and $28y$ | f $6k^2$ and $-10k$ |
| g p and $11p^2$ | h $45ab$ and $-40cd$ | i $3pq$ and $6p$ |

3 Factorise each expression using the HCFs from question 2.

- | | | |
|----------------------|------------------------|-----------------------|
| a $bc + cd$ | b $2xy + 2y$ | c $18mn - 9m$ |
| d $abcd + bd$ | e $8xy + 28y$ | f $6k^2 - 10k$ |
| g $p + 11p^2$ | h $45ab - 40cd$ | i $3pq + 6p$ |

4 Find the HCF of each pair of terms.

- | | | |
|----------------------------------|-------------------------------------|---|
| a a^4 and a^7 | b $5b^6$ and $3b^2$ | c $16c^5$ and $-2c^3$ |
| d $6d^4z^7$ and $6d^3z^5$ | e $7e^6y^5$ and $14e^7y^8$ | f $-20fx^{12}$ and $6f^3x^9$ |
| g $-2g^2$ and $-2gh^5$ | h $12i^6k^5$ and $-18j^3k^8$ | i $x^9y^{11}z^{13}$ and $-x^{11}y^7z^{16}$ |

3E.2 5 Factorise each expression.

- | | | | |
|----------------------|---------------------|-------------------------|---------------------------|
| a $10x + 5$ | b $3y - 21$ | c $8k + 12$ | d $15 - 6d$ |
| e $28x + 21y$ | f $20n - 50$ | g $x^2y + 3xy^4$ | h $n^3m^2 - 9n^2m$ |

6 Factorise each expression.

- | | | | |
|------------------------|----------------------|------------------------|-----------------------|
| a $20ab - 5b$ | b $8d + 8cde$ | c $15x^2 + 10x$ | d $4k^2 - 22k$ |
| e $30n - 18n^2$ | f $16a^2 + a$ | g $2h^2 - 14h$ | h $6p + 6p^2$ |

7 Factorise each expression. Remember to use the index laws to help you simplify.

- | | | |
|-------------------------------------|---------------------------------|--|
| a $24m^6 + 16m^4$ | b $21q^5r^2 + 35q^3r^6$ | c $15t^4u^2 - 5t^8u^7$ |
| d $12c^9d^5 + 6c^4d^4$ | e $7e^{11}f^3 - 7e^5f^3$ | f $3i^5j^2k^6 + 27ij^7k$ |
| g $a^7b^{12}c^7 + b^3c^9d^4$ | h $5mpq - 3np^{10}q^5$ | i $u^4w^5y^2z^3 + v^4w^5x^2z^3$ |

8 Factorise each expression by factoring out a negative HCF. Remember that $-5mn = -1 \times 5 \times m \times n$.

- | | | |
|-----------------------|-------------------------|--------------------------|
| a $-5mn - 10n$ | b $-14xy - 7x$ | c $-6c + 6cd$ |
| d $-a^2 - 3a$ | e $-4k^2 - 2k$ | f $-8x^2 + 8x$ |
| g $-12 - 3xy$ | h $-16m - 10m^2$ | i $-9x^2y + 18xy$ |

3E.3 9 Factorise each expression.

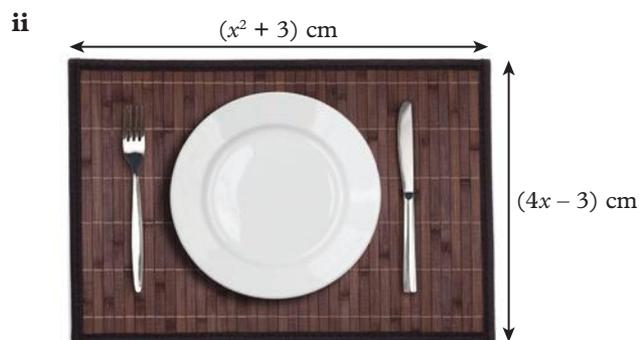
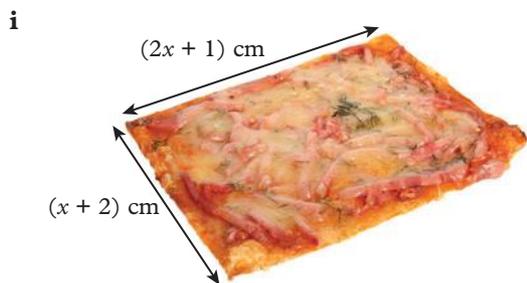
- | | | |
|----------------------------------|----------------------------------|--------------------------------------|
| a $x(zw + 4) + 2(zw + 4)$ | b $y(x - 1) + 7(x - 1)$ | c $a(a + 6) - 3(a + 6)$ |
| d $p(5 - n) + 8(5 - n)$ | e $3k(4 - k) - 5(4 - k)$ | f $2x(3x - 4) + 9(3x - 4)$ |
| g $4g(2g + 1) + (2g + 1)$ | h $2h(8n - d) - (8n - d)$ | i $3a(7x + 6y) + 2b(7x + 6y)$ |

3E.4 10 Factorise each expression by grouping terms.

- | | | |
|-------------------------------|-------------------------------|-------------------------------|
| a $ab + 5b + 4a + 20$ | b $xy - 6x + 7y - 42$ | c $mn + 4m - 2n - 8$ |
| d $y^2 + 3y + 5y + 15$ | e $k^2 - 7k + 2k - 14$ | f $6x + 18 + x^2 + 3x$ |
| g $a^2 - 7a - 2a + 14$ | h $p^2 + 5p - 2p - 10$ | i $6c^2 - 2c + 9c - 3$ |

11 Explain the difference between expanding an expression and factorising an expression.

12 a Write an expression in factorised form for the perimeter of each rectangular shape.



b Calculate the perimeter of each rectangle when $x = 5$.

13 Write an expression for the missing side length of each rectangular object, given the area shown.

a area of rug is $(8x + 20)$ m²
4 m



b area of stained glass panel is $(m^2 + 15m)$ cm²
 m cm



14 Write an expression for the perimeter of each item in question 13 in factorised form.

15 Expressions with more than two terms can be factorised in the same way as those with two terms. For example, to factorise $8a + 12b - 6c$ write the HCF, 2, in front of the brackets and divide each term inside the brackets by the HCF. So, $2\left(\frac{8a}{2} + \frac{12b}{2} - \frac{6c}{2}\right) = 2(4a + 6b - 3c)$. Factorise each expression.

a $27x - 9y + 15z$

b $45p - 50q - 5$

c $4 - 20i + 40j - 60k$

d $a^5 + a^3 + a^2$

e $18b^3c^5 - 36b^4c^4 + 24b^8c^5$

f $84r^{12}t^8 + 7r^5t^7 + 49r^8t^{14} + 14r^9t^8$

16 We can take fractions out as a factor so that all terms in the brackets no longer involve fractions. For example:

$$\begin{aligned} \frac{5}{3}a + \frac{5}{6}b &= \frac{10}{6}a + \frac{5}{6}b \\ &= \frac{1}{6}\left(\frac{10}{6}a \times 6 + \frac{5}{6}b \times 6\right) \\ &= \frac{1}{6}(10a + 5b) \\ &= \frac{5}{6}\left(\frac{10^2a}{5} + \frac{5^1b}{5}\right) \\ &= \frac{5}{6}(2a + b). \end{aligned}$$

Factorise each expression.

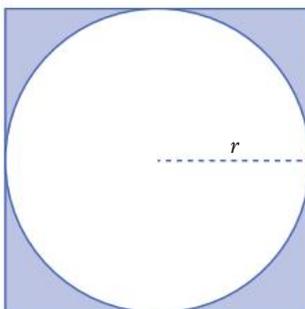
a $\frac{5}{2}x + \frac{3}{2}y$

b $\frac{9}{5}g + \frac{3}{5}h$

c $\frac{5}{2}p + \frac{15}{4}q$

d $\frac{7}{3}m - \frac{5}{4}n$

- 17 A rectangle has an area of $(10x - x^2)$ mm².
- Write the area in factorised form.
 - List possible expressions for the length and breadth of the rectangle.
 - Suggest values for the length and breadth of three different rectangles that have the given area.
- 18 A triangle has an area of $(21x - 3x^2)$ cm².
- List possible expressions for the height and base length of the triangle.
 - Suggest numerical values for the height and base length of three different triangles that have the given area.
- 19 A square-based prism has a volume of $(9x^3 + 45x^2)$ cm³ and a height of $(x + 5)$ cm. Determine the length of the square base of the prism in terms of x .
- 20 Consider the expression $x^2 + 3x - 4x - 12$.
- Factorise the expression by grouping the first two terms together, and the last two terms together.
 - Perform the factorisation again by grouping the first and third terms together, and the second and fourth terms together.
 - Compare your answers to parts **a** and **b**.
- 21 A number is represented by the pronumeral n .
- Write down the next two consecutive numbers in terms of n .
 - Write the sum of these three consecutive numbers.
 - Factorise your expression from part **b**. Explain the answer.
 - Investigate to see whether the same outcome results from the sum of three consecutive even numbers.
- 22 In question 21, an odd number (three) of consecutive numbers were added together. Investigate to see the outcome when an even number of consecutive numbers are added together. Write a factorised expression for the sum of 10 consecutive numbers, where the first number is n .
- 23 Completely factorise each expression.
- $x(y + 5) - (3y + 15)$
 - $p(2q - 3) - 2q + 3$
 - $2(a^2 - 3a) + (9 - 3a)$
- 24 A circle of radius r cm fits within a square as shown in the diagram. Write a factorised expression for the area of the square not covered by the circle. Leave your answer in terms of r and π .



- 25 It is possible to take any term or expression as a factor of another expression by dividing it out. For example, to take 7 out as a factor of $3x + 5$, write the 7 in front of the brackets; inside the brackets, divide each term by 7. So, $3x + 5 = 7\left(\frac{3x}{7} + \frac{5}{7}\right)$.
- Take 5 out as a factor of $9x + 17$.
 - Take 12 out as a factor of $12x^2 + 3x + 6$.
 - Take x out as a factor of $3x^2 + 5xy + y^2$.
 - Take $x + 2$ out as a factor of $7(x + 2)^2 + 5(x + 2) + 9$.

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Interactive skillsheet

Factorising using the HCF



Interactive skillsheet

Factorising by grouping terms



Worksheet

Factorising



Investigation

Surface area of a soft drink can



Topic quiz

3E

3F Factorising monic quadratic expressions

Learning intentions

By the end of this topic you will be able to ...

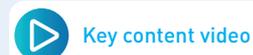
- ✓ factorise monic quadratic trinomials
- ✓ factorise quadratic trinomials by first taking out a common factor.



Inter-year links

- [Year 7](#) 2D Factors and the highest common factor
- [Year 8](#) 5G Factorising
- [Year 10](#) 2G Factorising quadratic expressions

Quadratic trinomials



- A **quadratic expression** is an algebraic expression that contains a squared pronumeral, with no index greater than 2 in the expression.
For example, $6t^2$, $x^2 + 5$, $k^2 + 14k + 30$, and $4b^2 - a^2$ are all quadratic expressions.
- A **quadratic trinomial** is an expression of the form $ax^2 + bx + c$, where a , b and c are constants.

$$ax^2 + bx + c$$

- ax^2 is the **leading term**
- bx is the **linear term**
- c is the **constant term**

- Many quadratic trinomials can be factorised to produce a binomial product.

$$\begin{array}{ccc}
 & \xleftarrow{\text{expanding}} & \\
 & x^2 + 4x + 3 = (x + 1)(x + 3) & \\
 \text{quadratic trinomial} & & \text{binomial product} \\
 \text{expanded form} & & \text{factorised form} \\
 & \xrightarrow{\text{factorising}} &
 \end{array}$$

Factorising monic quadratics of the form $x^2 + bx + c$

- A **monic quadratic trinomial** is a quadratic trinomial in which the leading coefficient is equal to 1.
$$x^2 + bx + c$$
- Expanding a binomial product of the form $(x + m)(x + n)$, where m and n are constants, gives a quadratic trinomial of the form $x^2 + bx + c$.

$$\begin{aligned}
 (x + m)(x + n) &= x(x + m) + n(x + m) \\
 &= x^2 + mx + nx + mn \\
 &= x^2 + (m + n)x + mn \\
 &= x^2 + bx + c
 \end{aligned}$$

$$\text{where } b = m + n \text{ and } c = m \times n$$

- The process can be reversed to factorise quadratic trinomials of the form $x^2 + bx + c$, by finding two numbers (m and n) that add to give b and multiply to give c .

$$\begin{aligned}
 \text{For example, } x^2 + 4x + 3 &= x^2 + (3 + 1)x + (3 \times 1) \\
 &= (x + 3)(x + 1)
 \end{aligned}$$

**Example 3F.1** Finding the numbers for a given product and sum

Identify which two numbers add to give the first number and multiply to give the second number.

a sum: 5, product: 6**b** sum: -2, product: -8**c** sum: 1, product: -6**THINK**

- a** 1 List the factor pairs of 6. Remember that if the positive/negative sign is changed for both factors then the product will be the same, so there are two combinations of factor pairs with the same numerals.
- 2 Add the factor pairs together and determine which pair adds to 5.
- b** 1 List the factor pairs of -8. Remember that a negative multiplied by a positive is a negative, so there are two combinations of factor pairs with the same numerals.
- 2 Add the factor pairs together and determine which pair adds to -2.
- c** 1 List the factor pairs of -6. Remember that a negative multiplied by a positive is a negative, so there are two combinations of factor pairs with the same numerals.
- 2 Add the factor pairs together and determine which pair adds to 1.

WRITE

a $1 \times 6 = 6 \rightarrow 1 + 6 = 7$
 $-1 \times (-6) = 6 \rightarrow -1 + (-6) = -7$
 $2 \times 3 = 6 \rightarrow 2 + 3 = 5$
 $-2 \times (-3) = 6 \rightarrow -2 + (-3) = -5$

The numbers are 2 and 3.

b $-1 \times 8 = -8 \rightarrow -1 + 8 = 7$
 $1 \times (-8) = -8 \rightarrow 1 + (-8) = -7$
 $-2 \times 4 = -8 \rightarrow -2 + 4 = 2$
 $2 \times (-4) = -8 \rightarrow 2 + (-4) = -2$

The numbers are 2 and -4.

c $-1 \times 6 = -6 \rightarrow -1 + 6 = 5$
 $1 \times (-6) = -6 \rightarrow 1 + (-6) = -5$
 $-3 \times 2 = -6 \rightarrow -3 + 2 = -1$
 $3 \times (-2) = -6 \rightarrow 3 + (-2) = 1$

The numbers are 3 and -2.

Example 3F.2 Factorising simple quadratic trinomials

Factorise the quadratic trinomial $x^2 + 7x + 10$.

THINK

- 1 List the factor pairs of the constant term, 10. Remember that if the positive/negative sign is changed for both factors then the product will be the same, so there are two combinations of factor pairs with the same numerals.
- 2 Add the factor pairs together and identify which pair adds to the linear coefficient, 7.
- 3 Write the expression in factorised form.
- 4 Check your result by expanding.

WRITE

$1 \times 10 = 10 \rightarrow 1 + 10 = 11$
 $-1 \times (-10) = 10 \rightarrow -1 + (-10) = -11$
 $2 \times 5 = 10 \rightarrow 2 + 5 = 7$
 $-2 \times (-5) = 10 \rightarrow -2 + (-5) = -7$

The numbers are 2 and 5.

$$x^2 + 10x + 7 = (x + 2)(x + 5)$$

Check: $(x + 2)(x + 5) = x^2 + 2x + 5x + 10$
 $= x^2 + 7x + 10$



Example 3F.3 Factorising more complex quadratic trinomials (+ and -)

Factorise each quadratic trinomial.

a $m^2 + 2m - 3$

THINK

- List the factor pairs of the constant term.
Remember that if the signs for both factors are changed at the same time then the product will be the same, so there are two combinations of factor pairs with the same numerals.
- Add the factors in each pair together and identify which pair adds to the linear term.
- Write the expression in factorised form.
- Check your result by expanding.

b $x^2 - 7x - 8$

WRITE

a $1 \times (-3) = -3 \rightarrow 1 + (-3) = -2$
 $-1 \times 3 = -3 \rightarrow -1 + 3 = 2$
 $m^2 + 2m - 3 = (m + 3)(m - 1)$
 Check: $(m + 3)(m - 1) = m^2 - m + 3m - 3$
 $= m^2 + 2m - 3$

b $1 \times (-8) = -8 \rightarrow 1 + (-8) = -7$
 $-1 \times 8 = -8 \rightarrow -1 + 8 = 7$
 $2 \times (-4) = -8 \rightarrow 2 + (-4) = -2$
 $-2 \times 4 = -8 \rightarrow -2 + (4) = 2$
 $x^2 - 7x - 8 = (x - 8)(x + 1)$
 Check: $(x - 8)(x + 1) = x^2 + x - 8x - 8$
 $= x^2 - 7x - 8$

Example 3F.4 Factorising quadratic trinomials by first taking out a common factor



Factorise by first taking out a common factor.

a $2x^2 - 14x + 12$

THINK

- If the coefficient of x^2 is not 1, check for a common factor of the three terms. Write the HCF in front of the brackets. Inside the brackets, divide each term by the HCF. A common factor can be -1 .
- Factorise the expression inside the brackets. List the factor pairs of the constant term.
- Add the factor pairs together and identify which pair adds to the linear term.
- Write the expression in factorised form.
- Check your result by expanding.

b $-x^2 + 4x - 3$

WRITE

a $2x^2 - 14x + 12 = 2(x^2 - 7x + 6)$
 $1 \times 6 = 6 \rightarrow 1 + 6 = 7$
 $-1 \times -6 = 6 \rightarrow -1 + (-6) = -7$
 $2 \times 3 = 6 \rightarrow 2 + 3 = 5$
 $-2 \times -3 = 6 \rightarrow -2 + (-3) = -5$
 $2(x^2 - 7x + 6) = 2(x - 6)(x - 1)$
 Check: $2(x - 6)(x - 1) = 2(x^2 - x - 6x + 6)$
 $= 2(x^2 - 7x + 6)$
 $= 2x^2 - 14x + 12$

b $-x^2 + 4x - 3 = -(x^2 - 4x + 3)$
 $1 \times 3 = 3 \rightarrow 1 + 3 = 4$
 $-1 \times -3 = 3 \rightarrow -1 + (-3) = -4$
 $-(x^2 - 4x + 3) = -(x - 3)(x - 1)$
 Check: $-(x - 3)(x - 1) = -(x^2 - x - 3x + 3)$
 $= -(x^2 - 4x + 3)$
 $= -x^2 + 4x - 3$

- ✓ The order of the binomial products doesn't matter. This is because of the commutative law for multiplication. For example: $2 \times 3 = 3 \times 2$ and $(x - 2)(x + 3) = (x + 3)(x - 2)$
- ✓ The order of the terms in a quadratic trinomial does not change the value of the expression. This is because of the commutative law for addition. For example: $x^2 + 6x + 8 = 6x + 8 + x^2 = 8 + x^2 + 6x$
- ✓ Always look for the HCF of the three terms before factorising. You might get lucky and find that it makes factorising easier.
- ✓ You can check your factorisation of the binomial products by expanding the brackets!

$$\begin{aligned}
 x^2 + 4x + 3 &= (x + 1)(x + 3) \\
 &= x^2 + 3x + x + 3 \\
 &= x^2 + 4x + 3
 \end{aligned}$$

- ✓ You don't always need to consider all the factor pairs of the constant term, c , to factorise a quadratic of the form $x^2 + bx + c$.
If the constant term, c , is positive, then both factors will have the same sign. So:
 - if b is positive, then both factors will be positive
 - if b is negative, then both factors will be negative.

ANS p439 Exercise 3F Factorising monic quadratic expressions

 1-3(1st, 2nd columns), 4, 5-6(1st, 2nd columns), 7-13, 16(a-d), 18

 4, 6-11, 13, 14, 16(b, e, g, h, i), 18, 19, 21(a, c, d, f)

 4(c, f, i, l), 6(c, f, i, l), 7-11, 14, 15, 16(d, g, h, i), 17, 20, 21(b, c, d, f, i), 22, 23

- 3F.1** 1 Identify which two numbers add to give the first number and multiply to give the second number.
- | | | |
|-------------------------------|-------------------------------|-------------------------------|
| a Sum: 5, product: 4 | b Sum: 6, product: 8 | c Sum: 13, product: 22 |
| d Sum: 9, product: 20 | e Sum: 10, product: 24 | f Sum: 7, product: 12 |
| g Sum: 13, product: 42 | h Sum: 12, product: 35 | i Sum: 8, product: 16 |
- 2 Identify which two numbers add to give the first number and multiply to give the second number.
- | | | |
|-------------------------------|--------------------------------|--------------------------------|
| a Sum: 2, product: -8 | b Sum: -1, product: -6 | c Sum: -8, product: 12 |
| d Sum: 3, product: -10 | e Sum: -8, product: -9 | f Sum: -5, product: 6 |
| g Sum: 5, product: -6 | h Sum: -6, product: -27 | i Sum: -12, product: 11 |
- 3 Use your results from question 1 to factorise each of these quadratic trinomials.
- | | | |
|---------------------------|---------------------------|---------------------------|
| a $x^2 + 5x + 4$ | b $x^2 + 6x + 8$ | c $x^2 + 13x + 22$ |
| d $x^2 + 9x + 20$ | e $x^2 + 10x + 24$ | f $x^2 + 7x + 12$ |
| g $x^2 + 13x + 42$ | h $x^2 + 12x + 35$ | i $x^2 + 8x + 16$ |
- 3F.2** 4 Factorise each quadratic trinomial.
- | | |
|---------------------------|---------------------------|
| a $a^2 + 4a + 3$ | b $b^2 + 9b + 14$ |
| c $c^2 + 7c + 6$ | d $d^2 + 10d + 21$ |
| e $e^2 + 8e + 7$ | f $f^2 + 8f + 15$ |
| g $g^2 + 11g + 28$ | h $h^2 + 13h + 36$ |
| i $x^2 + 9x + 18$ | j $j^2 + 14j + 45$ |
| k $k^2 + 11k + 30$ | l $y^2 + 13y + 40$ |

5 Use your results from question 2 to factorise each of these quadratic trinomials.

a $x^2 + 2x - 8$

b $x^2 - x - 6$

c $x^2 - 8x + 12$

d $x^2 + 3x - 10$

e $x^2 - 8x - 9$

f $x^2 - 5x + 6$

g $x^2 + 5x - 6$

h $x^2 - 6x - 27$

i $x^2 - 12x + 11$

3F.3 6 Factorise each quadratic trinomial.

a $a^2 + 2a - 3$

b $b^2 - 2b - 15$

c $c^2 - 5c + 4$

d $d^2 + 5d - 14$

e $e^2 - 10e + 24$

f $f^2 - 3f - 10$

g $g^2 + g - 12$

h $h^2 - 8h + 15$

i $x^2 - 5x - 24$

j $j^2 - 10j + 16$

k $k^2 + 3k - 18$

l $y^2 - y - 2$

3F.4 7 Factorise each quadratic trinomial by first taking out a common factor.

a $3x^2 + 9x + 6$

b $2x^2 + 16x + 30$

c $5x^2 + 15x - 20$

d $-4x^2 - 20x - 24$

e $-6x^2 + 36x - 30$

f $-x^2 - 2x + 35$

8 Factorise each quadratic trinomial by first taking out a common factor.

a $ax^2 + 8ax + 12a$

b $bx^2 - 11bx + 28b$

c $10cx^2 - 10cx - 300c$

d $-2dx^2 + 34dx - 120d$

e $p^2x^2 + 13p^2x + 30p^2$

f $q^2x^2 + 15qx^2 + 44x^2$

9 Factorise each quadratic expression.

a $x^2 - 6x + 9$

b $x^2 + 0x - 9$

c $x^2 - 6x + 0$

10 Factorise each quadratic expression.

a $x^2 - 25$

b $x^2 - 10x + 25$

c $x^2 - 25x$

d $x^2 + 14x + 49$

e $x^2 - 49x$

f $x^2 - 49$

11 Factorise each quadratic by first reordering and simplifying the terms of each expression.

a $21 + 10a + a^2$

b $2b - 35 + b^2$

c $3c + c^2 - 18$

d $48 + 2d - d^2$

e $-24 - 10e - e^2$

f $10f - f^2 - 16$

g $10g + 70 + g^2 + 7g + 2$

h $-2h + 5 + h^2 - 19 - 3h$

i $50 - 20x - x^2 + 7x - 86$

12 **a** Determine all the non-negative integers (including zero) that can be substituted into the constant term of $x^2 + 6x + \square$ so that it can be factorised. Write both the expanded and factorised forms for each possible value.

b Determine all the positive integers up to 40 that can be substituted into the constant term of $x^2 + 6x - \square$ so that it can be factorised. Write both the expanded and factorised form for each possible value.

c When the coefficient of x is positive, how can you tell if the factors are both positive or if one is positive and one negative?

13 **a** Determine all the positive integers that can be substituted into the constant term of $x^2 - 6x + \square$ so that it can be factorised. Write both the expanded and factorised forms for each possible value.

b Determine all the non-negative (including zero) integers up to 40 that can be substituted into the constant term of $x^2 - 6x - \square$ so that it can be factorised. Write both the expanded and factorised form for each possible value.

c When the coefficient of x is negative, how can you tell if the factors are both negative or if one is positive and one negative?

14 **a** Determine all the positive integers that can be substituted into the x term of $x^2 + \square x + 64$ so that it can be factorised. Write both the expanded and factorised forms for each possible value.

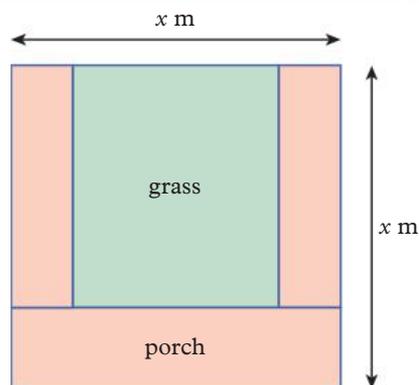
b Determine all the positive integers that can be substituted into the x term of $x^2 - \square x + 64$ so that it can be factorised. Write both the expanded and factorised forms for each possible value.

c When the constant term is positive:

i how can you tell if the factors need to be positive or negative

ii is the coefficient of x the sum or difference of the positive values of the factors?

20 Two friends, Melissa and Lena, want to renovate their square-shaped backyards by adding a full-length rectangular porch and planting a rectangular section of grass with garden beds on either side. Both friends want to have the same breadth for their porches and their garden beds but have different-sized backyards, so they decide to use algebra to determine the area of grass that they need for both. They've already decided on the breadth of their porches, and have narrowed down the area of grass to four options:



- i $(x^2 - 5x + 6) \text{ m}^2$
- ii $(x^2 - 7x + 10) \text{ m}^2$
- iii $(x^2 - 4x + 4) \text{ m}^2$
- iv $(x^2 - 6x + 8) \text{ m}^2$

- a Factorise each quadratic expression.
- b Consider the factors of each equation and relate them to the possible side lengths of the grass. Judging from Melissa and Lena's four options, what is the breadth of Melissa and Lena's porches?
- c Lena also suggested the grass areas $(x^2 - x - 6) \text{ m}^2$ and $(x^2 + x - 6) \text{ m}^2$. Explain why Melissa said these areas wouldn't be possible.
- d Melissa and Lena decide to have 2 m of space either side of the lawn for their garden beds. Which quadratic expression did they decide on?
- e If Melissa's backyard is 6 m by 6 m, determine what area of grass she will plant.
- f If Lena's backyard is 7.5 m by 7.5 m, determine what area of grass she will plant.
- g Determine how much area Melissa and Lena will each have for planting flowers.



21 Factorise each quadratic expression.

- | | | |
|------------------------------|--------------------------------|--------------------------------|
| a $(3x)^2 + 5(3x) - 14$ | b $(11x)^2 + 2(11x) - 80$ | c $(5x)^2 - 12(5x) + 32$ |
| d $(x + 2)^2 + 3(x + 2) - 4$ | e $(x - 4)^2 - 11(x - 4) + 30$ | f $(x - 9)^2 + 14(x - 9) + 45$ |
| g $(3x)^2 - (3x) - 42$ | h $(2x)^2 + 7(2x) - 30$ | i $(8x)^2 + 32(2x) - 48$ |

22 Fully factorise each expression.

- | | | |
|--------------------------|--------------------------|--------------------------|
| a $(x^2)^2 - 13x^2 + 36$ | b $(x^2)^2 - 12x^2 - 64$ | c $(x^2)^2 + 24x^2 - 25$ |
|--------------------------|--------------------------|--------------------------|

23 We can factorise quadratic trinomials by splitting the linear term once we have determined the two factors and then using grouping. For example, two numbers that add to 7 and multiply to 12 are 3 and 4 so:

$$\begin{aligned} x^2 + 7x + 12 &= x^2 + 3x + 4x + 12 \\ &= x(x + 3) + 4(x + 3) \\ &= (x + 3)(x + 4) \end{aligned}$$

Factorise each of the following by splitting the linear term and using grouping.

- | | | |
|-------------------|------------------|--------------------|
| a $x^2 + 5x + 4$ | b $x^2 + 6x + 8$ | c $x^2 + 13x + 22$ |
| d $x^2 + 3x - 10$ | e $x^2 - 8x - 9$ | f $x^2 - 5x + 6$ |

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Interactive skillsheet
Factorising quadratic expressions



Worksheet
Factorising quadratic expressions



Topic quiz
3F

Chapter summary

Simplifying

- Like terms can be added and subtracted by adding and subtracting their coefficients.

$$a^2b + 2a^2b = 1a^2b + 2a^2b \\ = 3a^2b$$

$$3a^3b \times (-2a^2b^2) = 3 \times (-2) \times a^{3+2} \times b^{1+2} \\ = -6a^5b^3$$

- Products and quotients of algebraic terms can be simplified using the HCF and the index laws.

$$\frac{12abc}{9b} = \frac{3 \times 4 \times a \times b \times c}{3 \times 3 \times b} \\ = \frac{3 \times 4 \times a \times b \times c}{3 \times 3 \times b} \\ = \frac{4ac}{3}$$

Multiplying and dividing algebraic fractions

- Product: product of the numerators over the product of the denominators.
- Quotient: multiply the first fraction by the reciprocal of the fraction after the division sign.

$$\frac{3x}{2} \times \frac{2x}{9} = \frac{3x \times 2x}{2 \times 9} = \frac{x^2}{3}$$

$$\frac{3x}{2} \div \frac{2x}{9} = \frac{3x}{2} \times \frac{9}{2x} \\ = \frac{3x \times 9}{2 \times 2x} = \frac{27}{4}$$

Adding and subtracting algebraic fractions

- Write each fraction as an equivalent fraction with the LCD.
- The sum or difference of the algebraic fractions with a common denominator is then found by adding or subtracting the numerators.

$$\frac{x}{4} + \frac{x}{6} = \frac{3x}{12} + \frac{2x}{12} = \frac{3x+2x}{12} = \frac{5x}{12}$$

$$\frac{x}{4} - \frac{x}{6} = \frac{3x}{12} - \frac{2x}{12} = \frac{3x-2x}{12} = \frac{x}{12}$$

Expanding

- Use the distributive law to expand brackets

$$a(b+c) = ab+ac$$

$$(a+b)(c+d) = ac+ad+bc+bd$$

Factorising using the HCF

$$\xrightarrow{\text{factorising}} \\ 2x^2 - 6x = 2x(x-3) \\ \xleftarrow{\text{expanding}} \\ \text{expanded form} \quad \text{factorised form}$$

Factorising by grouping terms

$$x^2 + 2x - 3x - 6 = x(x+2) - 3(x+2) \\ = (x+2)(x-3)$$

Factorising quadratic trinomials

To factorise a quadratic trinomial of the form $x^2 + bx + c$:

- find two numbers that add to give b and multiply to give c

$$(x+m)(x+n) = x(x+m) + n(x+m) \\ = x^2 + mx + nx + mn \\ = x^2 + (m+n)x + mn \\ = x^2 + bx + c$$

where $b = m + n$ and $c = m \times n$

- substitute those two numbers into the binomial product factorised form.

$$\xrightarrow{\text{factorising}} \\ x^2 + 4x + 3 = (x+1)(x+3) \\ \xleftarrow{\text{expanding}} \\ \text{quadratic trinomial} \quad \text{binomial product} \\ \text{expanded form} \quad \text{factorised form}$$

Chapter review



Chapter review quiz

Assess your knowledge of this chapter.

- ✓ Core
- ✓ Advanced

Quizlet

Test your knowledge of this topic by working individually or in teams.

Mathematical literacy review

The following key terms are used in this chapter:

- algebraic fraction
- algebraic expression
- binomial
- binomial product
- coefficient
- constant
- difference of two squares
- distributive law
- expanded form
- expand
- expression
- factorising
- FOIL
- highest common factor (HCF)
- index form
- leading term
- like terms
- linear term
- monic quadratic trinomial
- pronumeral
- quadratic expression
- term
- trinomial
- variable

1 Complete the following sentences using words from the list of key terms.

- a** An algebraic expression contains one or more _____.
A _____ has only two terms and a _____ has three terms.
- b** A _____ has three terms and the leading coefficient of the x^2 term is 1.
- c** _____ is the strategy used to expand the product of two _____ expressions.
- d** _____ is the opposite process to expanding. The first step is to identify if there is a _____.
- e** An _____ is a fraction with a _____ in the numerator or in the denominator or both.

2 For the expression $9x^2 - 6x + 12$, match the following in the left-hand column with a key term in the right-hand column.

- | | |
|--------------------------------|----------------------------------|
| a 12 | I leading coefficient |
| b $9 \times x \times x$ | II constant |
| c -6 | III linear term |
| d x | IV quadratic expression |
| e 9 | V highest common factor |
| f $9x^2 - 6x + 12$ | VI expanded form |
| g $-6x$ | VII variable / pronumeral |
| h 3 | VIII coefficient |

3 State which of the following **I–VI** are used in the equations below. There may be more than one option.

- | | |
|--|------------------------------|
| a $4(x^2 - 2) = 4x^2 - 8$ | I expanding |
| b $\frac{x}{4} + \frac{3x}{8} = \frac{5x}{8}$ | II distributive law |
| c $(x + 1)(x - 2) = x^2 - x - 2$ | III FOIL |
| d $\frac{3x^2y}{6y^3} = \frac{x^2}{2y^2}$ | IV factorising |
| e $x^2 - 10x + 24 = (x - 6)(x - 4)$ | V dividing by the HCF |
| | VI none of the above |

Multiple choice

- 3A 1** Which is not a like term to $4ab^3$?
A $3abbb$ **B** $4ba^3$ **C** $-4b^3a$ **D** $2bab^2$ **E** $\frac{ab^3}{7}$
- 3A 2** If $x = -3$ and $y = 2$, then $-5y^3x^2$ is equal to:
A -360 **B** 360 **C** -540 **D** 540 **E** -9000
- 3B 3** What does $\frac{a^2b^2}{12} \times \frac{ab}{2} \div \frac{ab}{-2}$ equal?
A $-\frac{1}{12}$ **B** $-\frac{a^2b^2}{12}$ **C** $\frac{a^2b^2}{12}$ **D** $\frac{a^4b^4}{48}$ **E** $-\frac{a^2b^2}{3}$
- 3B 4** What does $\frac{x}{12} + \frac{3x}{2}$ equal?
A $\frac{19x}{12}$ **B** $\frac{18x}{12}$ **C** $\frac{3x}{2}$ **D** $\frac{x^2}{8}$ **E** $\frac{7x}{12}$
- ADV 3C 5** Which expression shows $\frac{6ab^2c}{18a^2c}$ in simplified form?
A $\frac{6ab^2}{18a}$ **B** $\frac{ab^2c}{3a^2c}$ **C** $\frac{b^2}{12a}$ **D** $\frac{6ab^2}{18a^2}$ **E** $\frac{b^2}{3a}$
- ADV 3C 6** What does $\frac{x^2}{12x} \times \frac{xy}{y} \div \frac{2x}{y^2}$ equal?
A $\frac{xy^2}{6}$ **B** $\frac{xy^2}{24}$ **C** $\frac{x}{24y^2}$ **D** $\frac{y^2}{24x^2}$ **E** $\frac{x^2y}{24}$
- ADV 3C 7** What does $\frac{2}{mn} + \frac{1}{2mn}$ equal?
A $\frac{5}{2mn}$ **B** $\frac{2}{mn}$ **C** $\frac{3}{3mn}$ **D** $\frac{1}{mn}$ **E** $\frac{1}{m^2n^2}$
- 3D 8** Which expression is equivalent to $8 - 2(3 - 5g)$?
A $18 - 30g$ **B** $10g + 2$ **C** $10g + 10$ **D** $2 - 10g$ **E** $10 - 10g$
- 3D 9** Which expression is *not* equivalent to $(3x + 4)(2y - 5)$?
A $2y(3x + 4) - 5(3x + 4)$ **B** $6xy - 20$
C $6xy - 15x + 8y - 20$ **D** $3x \times 2y + 3x \times (-5) + 4 \times 2y + 4 \times (-5)$
E $3x(2y - 5) + 4(2y - 5)$
- 3D 10** Which expression cannot be expanded using the difference of two squares rule?
A $(x + 6)(x - 6)$ **B** $(7 - p)(7 + p)$ **C** $(2x - 7)(2x + 7)$
D $(q + 9)(-q + 9)$ **E** $(d + 5)(d - 2)$
- 3D 11** Which statement is *incorrect*?
A $(d + 3)(d - 7) = d^2 + 4d - 21$ **B** $-3(b + 5) = -3b - 15$
C $(m - 4)(m + 4) = m^2 - 16$ **D** $(2b + 5)(3b - 2) = 6b^2 + 11b - 10$
E $(e - 6)^2 = e^2 - 12e + 36$
- ADV 3E 12** When fully factorised, the expression $9x^2 - 3x^3$ factorises to:
A $3(3x^2 - x^3)$ **B** $3x^2(3 - x)$ **C** $x^2(9 - 3x)$
D $3x(3x - x^2)$ **E** $-3x(x^2 - 3x)$
- ADV 3E 13** Which of the following is *not* a factor of $12x^2y - 24xy^2$?
A x^2 **B** $12xy$ **C** 12 **D** $6y$ **E** $3x$
- ADV 3E 14** The expression $3x - 2$ is *not* a factor of which expression?
A $24xy + 15x - 18y - 10$ **B** $6(3x - 2) - y(3x - 2)$ **C** $3x(5y + 9) - 2(5y + 9)$
D $21x - 14$ **E** $3x - 2$
- ADV 3F 15** The expression $t^2 + 3t - 18$ factorises to:
A $(t + 9)(t - 2)$ **B** $(t - 6)(t + 3)$ **C** $(t + 6)(t - 3)$
D $(t - 9)(t + 2)$ **E** $(t + 5)(t - 2)$

- ADV 3F 16** When fully factorised, the expression $3y^2 - 33y + 84$ factorises to:
- A** $3(y - 7)(y - 4)$ **B** $(3y - 7)(3y - 4)$ **C** $(3y - 21)(y - 4)$
D $3(y + 7)(y + 4)$ **E** $(y - 7)(3y - 12)$
- ADV 3F 17** Which of the following is not equivalent to $b^2 - 8b + 16$?
- A** $(b - 4)^2$ **B** $(4 - b)^2$ **C** $(b - 4)(4 - b)$
D $(4 - b)(4 - b)$ **E** $(b - 4)(b - 4)$

Short answer

- 3A 1** Simplify each expression.
- a** $15t - 7t + 8t$ **b** $a - 7p - 11p + 12a$
c $3k + 5km - 7k - 15 + 2k - 4km$ **d** $6m^2n - 2m^2 + 7nm^2 + 11n^2 - 4mn^2 - 3m^2$
- 3A 2** Simplify each expression.
- a** $4xy \times 11xyz$ **b** $9mnp \times 2m^3p \times 4n^2$
c $15de \div (18d)$ **d** $11klmn \div (-22klm)$
- 3B 3** Simplify each of these expressions.
- a** $\frac{7a}{12} \times \frac{36b}{7}$ **b** $\frac{3x^2}{2} \times \frac{x}{6}$ **c** $\frac{12mn}{5} \div \frac{3m}{2}$ **d** $\frac{x^2}{12} \div \frac{3x}{2}$
- 3B 4** Simplify each of these expressions.
- a** $\frac{3x}{22} - \frac{2x}{11}$ **b** $\frac{7p^2q}{12} + \frac{3p^2q}{4}$ **c** $\frac{a}{20} + \frac{a}{16} + \frac{a}{8}$ **d** $\frac{5m}{3} - \frac{7m}{5} + \frac{9m}{7}$
- ADV 3C 5** Simplify each of these expressions.
- a** $\frac{5n^2}{32m} \times \frac{8mn}{25n^3}$ **b** $\frac{5ab}{12} \div \frac{5b}{6a}$ **c** $\frac{t}{12t^6} \times \frac{3t}{2s^3} \times \frac{2s}{t}$ **d** $\frac{ab}{5c} \div \frac{b}{-ac} \div \frac{15a}{b}$
- ADV 3C 6** Simplify each of these expressions.
- a** $\frac{5}{12x} + \frac{1}{5x}$ **b** $\frac{1}{3a} - \frac{1}{5a}$ **c** $\frac{13}{15n} - \frac{5}{13n} + \frac{13}{5n}$ **d** $\frac{5}{2p} + \frac{3}{4p} - \frac{1}{6p}$
- 3D 7** Expand each product.
- a** $4(z - 7)$ **b** $-8(5 - 3y)$ **c** $5x(7 - 6w)$
d $(u + 3)(t - 4)$ **e** $(5r + 6)(8 - 3v)$ **f** $(9p + 11q)(7m - 3n)$
- 3D 8** Expand and simplify each expression to remove the brackets.
- a** $5(a + 2) - 3(7 - a)$ **b** $(b - 11)(b + 2)$ **c** $(3c - 2)(4c - 5)$
d $(d + w)(d - w)$ **e** $(6 + e)^2$ **f** $(f - 9)^2$
- 3D 9** Expand and simplify each product.
- a** $x^4(x^3 - x^2)$ **b** $y^3z^4(y^9 - z^2)$ **c** $(x^5 - y^3)(x^6 + y^4)$
- ADV 3E 10** Factorise each expression.
- a** $4a - 24$ **b** $36pq^2 + 144pq$
c $7d(8 - d) - 4(8 - d)$ **d** $5e + 15ef + 2 + 6f$
- ADV 3E 11** Factorise each expression.
- a** $6z^5 - 5z^4$ **b** $9d^2y + 15dx^2$
c $-36r^{10}t^8u^6 - 96r^7t^8u^3$ **d** $a^5b^2c^7d^8 + a^6b^5c^9d^5 + a^4b^2c^8d^7$
- ADV 3E 12** Factorise each expression.
- a** $24x^2(x^2 + 2) - 8x(x^2 + 2)$ **b** $(r + q)(t + 5) + (r + q)(p - 3)$ **c** $4x^2 + 16x - 6x - 24$
- ADV 3F 13** Factorise each quadratic trinomial.
- a** $a^2 + 6a + 5$ **b** $b^2 - 7b + 12$ **c** $c^2 + 4c - 21$ **d** $d^2 - 16d - 36$
- ADV 3F 14** Factorise each quadratic trinomial.
- a** $15 + 8x + x^2$ **b** $60 + 20x - 5x^2$
c $32x - 2x^2 - 120$ **d** $240 - 6x - 3x^2$

a $98 - 2x^2$

b $16y^2 + 64x^2$

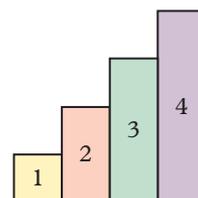
c $x^2 - 10x + 25$

d $9x^2 - 36x$

Analysis

1 Malak and Samara are investigating how to quickly add the positive integers from 1 to n :
 1, 2, 3, 4, 5, ..., n

Malak finds the expression $\frac{1}{2}n(n + 1)$ and Samara finds the expression $\frac{(n + 1)^2 - (n + 1)}{2}$, which will give the sum of the integers from 1 to 4, if $n = 4$. Visually, the first four positive integers can be represented using bars.



a Fully expand both of Malak's and Samara's expressions to show that they are equivalent.

b Calculate the sum of the first five positive integers:

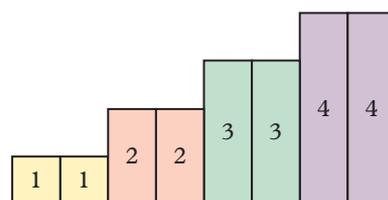
i by manually computing the sum $1 + 2 + 3 + 4 + 5$

ii by using one of the expressions.

c Calculate the sum of the first 100 positive integers.

Malak decides to investigate the sum of the first n positive even numbers:
 2, 4, 6, 8, 10, ..., n

Malak finds the expression $n^2 + n$, which will give the sum of the positive even numbers from 2 to 8, if $n = 4$. Visually, the first four even numbers can be represented using double bars.

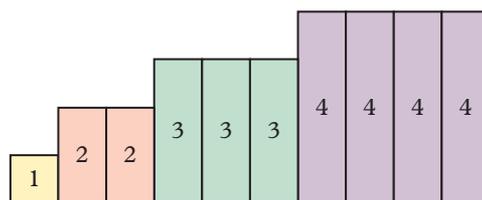


d Calculate the sum of the first 100 positive even numbers.

e Describe the connection between the sum of the first n positive even numbers and the sum of the first n positive integers.

Samara decides to investigate the sum of the first n square numbers: 1, 4, 9, 16, 25, ..., n

Samara finds the expression $\frac{1}{6}(n^2 + n)(2n + 1)$, which will give the sum of the square numbers from 1 to 16, if $n = 4$. Visually, the first four square numbers can be represented using squares of bars.



f Fully expand $\frac{1}{6}(n^2 + n)(2n + 1)$.

g Calculate the sum of the first five square numbers:

i by manually computing the sum $1 + 4 + 9 + 16 + 25$

ii by using one of the expressions.

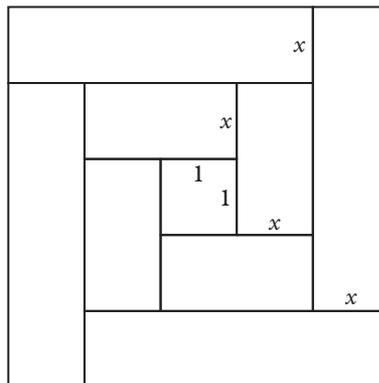
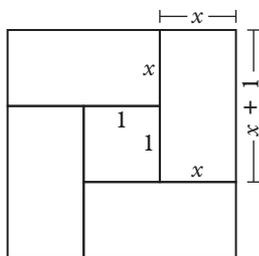
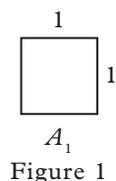
h Calculate the sum of the first 100 square numbers.

2 The following diagrams show different tile patterns made by adding rectangular tiles of breadth x cm around a central tile.

In this pattern, the first tile is a square of side length 1 cm, and the area of Figure 1 is A_1 . In Figure 2, each added rectangle has length $(x + 1)$ cm and a breadth of x cm. The area of Figure 2 is A_2 .

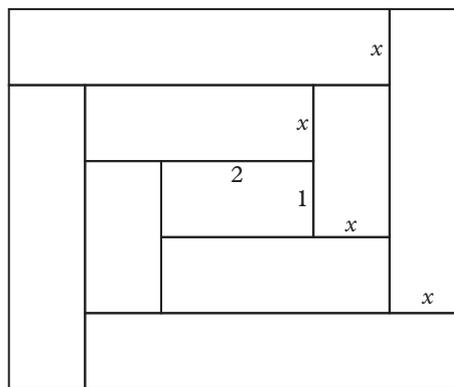
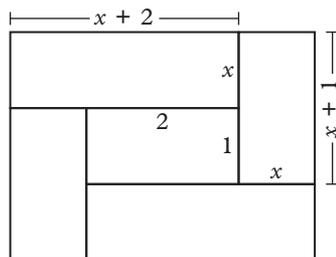
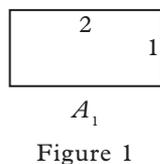
a Complete the following table with algebraic expressions for the overall side length of each new pattern, the area A_n as a square of the side length and A_n expanded into quadratic expression form. Identify the pattern to complete the table for Figure 4.

Figure	Side length (cm)	$A_n = (\text{Side length})^2$	$A_n = ax^2 + bx + c$
1	1	$(1)^2$	1
2			$4x^2 + 4x + 1$
3			
4			



- b** By looking for a pattern in the coefficient of x in the side length, determine the side length for Figure 10.
c Write a general expression for the side length of Figure n and its area, A_n , in factored form.
d For Figure 3, write an expression for the remaining area if the original square is removed.

In a similar pattern, the original shape with area A_1 is a rectangle of length 2 cm and breadth 1 cm. In Figure 2, the added rectangles have lengths $(x + 2)$ cm or $(x + 2)$ cm and each has a breadth of x cm.



- e** Complete the following table with algebraic expressions for the breadth and length of each new pattern, the area A_n as the product of the two side lengths, and A_n expanded into quadratic expression form. Identify the pattern to complete the table for Figure 4.

Figure	Length and breadth	$A_n = \text{length} \times \text{breadth}$	$A_n = ax^2 + bx + c$
1	2 and 1	2×1	2
2			
3			
4			

- f** By looking for a pattern in the coefficient of x in the table, determine the length and breadth for Figure 10.
g Write a general expression for the side length of Figure n and its area, A_n , in factored form.
h What difference do you notice between your answers for part **c** and part **g**? Explain why.
i If the original rectangle was 4 cm \times 5 cm, write down the area in factored form and quadratic expression form for Figure 6 in the pattern.

ADV 3 Sara Lee wants to transport her apple pies in a Styrofoam box. She finds that the internal volume of the large Styrofoam can be calculated using $V = 30x^2 + 600x \text{ cm}^3$ and the apple pie box is a square prism of height 5 cm. Sara wants the apple pie boxes to fit exactly with no wasted space.

- a** The height of the Styrofoam box is a positive integer and the dimensions of the base involve x . Use factorisation to work out all the possible dimensions of the inside of the box.
- b** Sara can stack six apple pie boxes on top of each other in the Styrofoam box. Determine which dimensions from part **a** are correct.

Sara now needs to calculate the full outside volume of the box. The thickness of the Styrofoam is 2 cm and the lid is a piece of Styrofoam which is the same size as the base of the box and is also 2 cm thick.

- c** Write down the external dimensions of the box in terms of x .
- d** Write down an expression for calculating the external volume in factored form.
- e** Expand your answer to part **d** as a quadratic trinomial.

Sara has an order for 72 pies and they fit exactly in one Styrofoam box using three rows of four pies in each layer.

- f** Determine the dimensions of each apple pie box.
- g** What is the value for x in this case?
- h** Use your value for x and your equation in part **d** to find the external volume of the Styrofoam box.

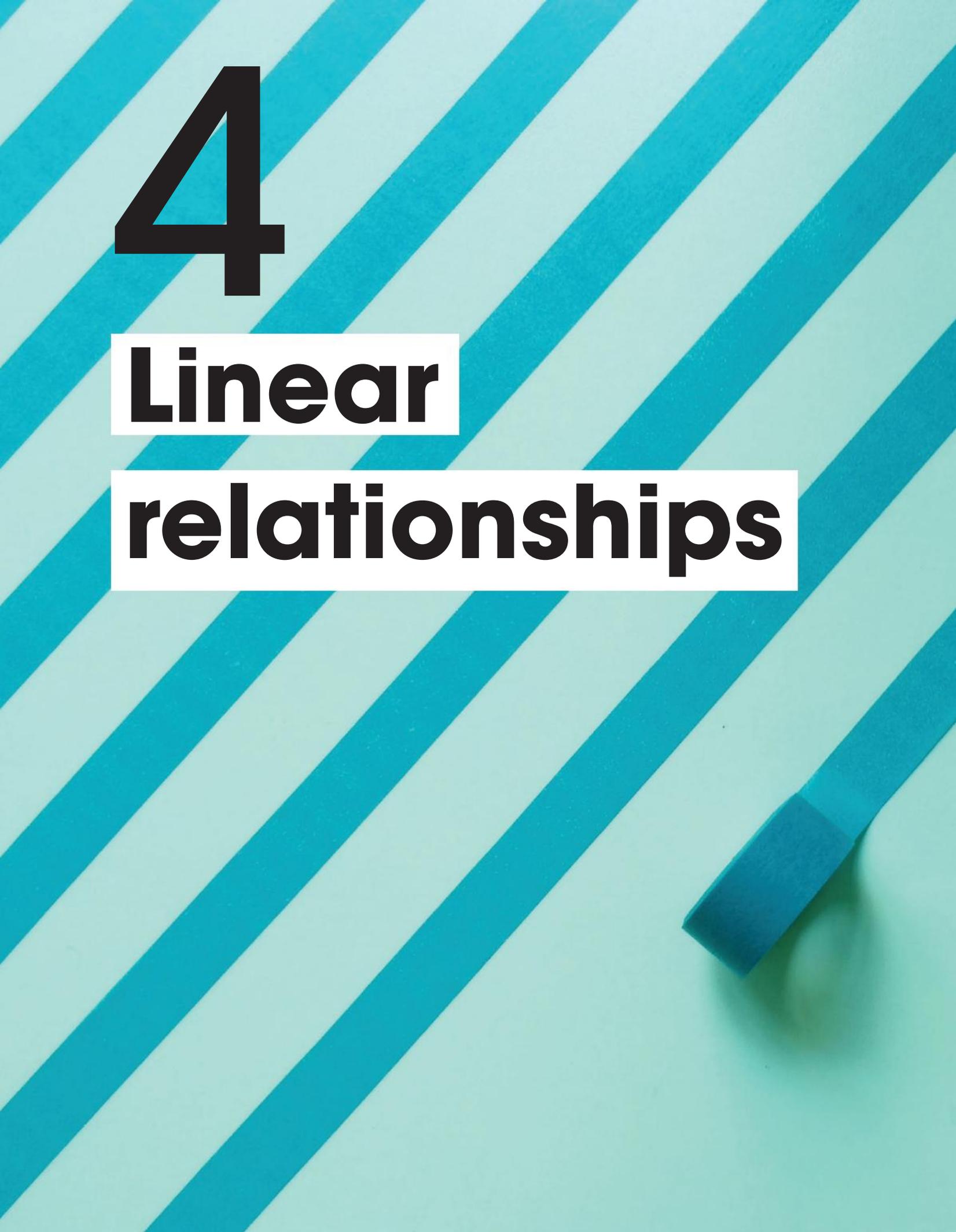


Chapter checklist



Now that you have completed this chapter, reflect on your ability to do the following.

I can do this	I need to review this
<input type="checkbox"/> Simplify algebraic terms involving addition and subtraction <input type="checkbox"/> Simplify algebraic terms involving multiplication and division	<input type="checkbox"/> Go back to Topic 3A Simplifying
<input type="checkbox"/> Multiply and divide algebraic fractions with numerical denominators <input type="checkbox"/> Add and subtract algebraic fractions with numerical denominators	<input type="checkbox"/> Go back to Topic 3B Algebraic fractions with numerical denominators
<input type="checkbox"/> Multiply and divide algebraic fractions with algebraic denominators <input type="checkbox"/> Add and subtract algebraic fractions with algebraic denominators	<input type="checkbox"/> Go back to Topic 3C Algebraic fractions with algebraic denominators
<input type="checkbox"/> Expand algebraic expressions of the form $a(b + c)$ <input type="checkbox"/> Expand algebraic expressions of the form $(a + b)(c + d)$	<input type="checkbox"/> Go back to Topic 3D Expanding
<input type="checkbox"/> Find the HCF of two or more algebraic terms <input type="checkbox"/> Factorise algebraic expressions by taking out the HCF <input type="checkbox"/> Factorise algebraic expressions by grouping terms	<input type="checkbox"/> Go back to Topic 3E Factorising using the HCF
<input type="checkbox"/> Factorise monic quadratic trinomials <input type="checkbox"/> Factorise quadratic trinomials by first taking out a common factor	<input type="checkbox"/> Go back to Topic 3F Factorising monic quadratic expressions

The background features a light teal color with several thick, dark teal diagonal stripes running from the top-left to the bottom-right. In the bottom-right corner, there is a small, curled piece of teal paper or tape.

4

Linear

relationships

Index

4A Solving linear equations

ADV **4B** Solving linear inequalities

4C Plotting linear relationships

4D Gradient and intercepts

4E Sketching linear graphs

4F Determining linear equations

STN ADV **4G** Direct variation

4H Midpoint and length of a line segment

4I Parallel and perpendicular lines

Prerequisite skills



Diagnostic pre-test

Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.



Interactive skillsheets

After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✓ Number lines
- ✓ Order of operations
- ✓ Expanding over one pair of brackets
- ✓ The Cartesian plane
- ✓ Plotting graphs

Curriculum links

- Solves linear equations of up to 3 steps, limited to one algebraic fraction (MA5-EQU-C-01)
- Determines the midpoint, gradient and length of an interval, and graph linear relationships, with and without digital tools (MA5-LIN-C-01)
- Graphs and interprets linear relationships using the gradient/slope-intercept form (MA5-LIN-C-02)
- STN ADV** • Identifies and solves problems involving direct and inverse variation and their graphical representations (MA5-RAT-P-01)
- ADV** • Solves monic quadratic equations, linear inequalities and cubic equations of the form $ax^3 = k$ (MA5-EQU-P-01)
 - Solve linear inequalities and graph their solutions on a number line

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Materials

- ✓ Calculator
- ✓ Graph paper
- ✓ Ruler

4A Solving linear equations

Learning intentions

By the end of this topic you will be able to ...

- ✓ solve linear equations using inverse operations
- ✓ solve linear equations with the unknown on both sides using inverse operations.



Inter-year links

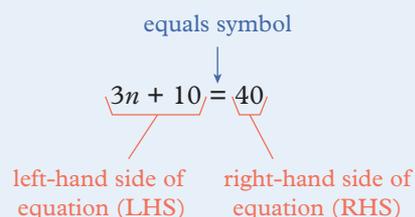
- Year 7** 6H Solving equations using inverse operations
- Year 8** 6C Solving equations with the unknown on both sides
- Year 10** 4A Solving linear equations

Linear equations

- An **equation** is a mathematical statement that shows equivalence between the expression on the left-hand side (LHS) and the right-hand side (RHS) of the equation.
- An equation may contain one or more pronumerals (such as x , y , a or b) that represent values, sometimes referred to as unknowns. If the pronumeral represents an unknown that can have more than one value, then it is called a variable.
- A **linear equation** is an equation containing only pronumerals that are raised to a power of 1 and no two such pronumerals are multiplied together.



Key content video



Linear equations	Non-linear equations
$y = x$	$y = x^2$
$\frac{n}{4} = 2$	$\frac{mn}{4} = 2$
$2a - \frac{3}{5}b = 10$	$a^3 + 6b = 0$

Solving linear equations using inverse operations

- A **solution** is a value for an unknown that makes the equation a true statement.
- To check whether a value is a solution to an equation, substitute that value into the equation to see whether it makes a true statement.
- To solve equations using **inverse operations**, identify and apply the inverse operation(s) required to reverse the operation(s) and isolate the unknown on the LHS of the equation ($x = \dots$).
 - For equations involving more than one operation, inverse operations must be performed in the reverse order to BIDMAS.
 - A useful shorthand is to put the inverse operation in brackets to the right of the equation for each line of working out.
 - To solve an equation in which the unknown appears on both sides of the equation, use inverse operations to eliminate the pronumeral term from one side of the equation, then solve the equation using inverse operations.

Operation	Inverse operation
+ 3	− 3
− 3	+ 3
× 3	÷ 3
÷ 3	× 3

BIDMAS

For example, $4x - 2 = 3x + 1$ (− 3x)

$$x - 2 = 1 \quad (+ 2)$$

$$x = 3.$$

Example 4A.1 Solving two-step equations using inverse operations



Solve the following equations using inverse operations.

a $\frac{x}{3} - 5 = 7$

b $10 = -2(x + 6)$

THINK

- a** Identify and apply the inverse operations to both sides of the equation in the reverse order of BIDMAS, and then write the solution to the equation.
- b** Divide both sides by -2 , remembering that the sign changes when you multiply or divide by a negative number. Subtract 6 from both sides. Note that $-11 = x$ is the same as $x = -11$.

WRITE

a $\frac{x}{3} - 5 = 7$ (+ 5)
 $\frac{x}{3} = 12$ ($\times 3$)
 $x = 36$

b $10 = -2(x + 6)$ ($\div (-2)$)
 $-5 = x + 6$ ($- 6$)
 $x = -11$

Example 4A.2 Solving three-step equations using inverse operations



Solve:

a $12 = \frac{7x}{2} - 9$

b $\frac{-x-4}{3} = 1$

THINK

- a** Identify and apply the inverse operations to both sides of the equation in the reverse order to BIDMAS, and then write the solution to the equation.
- b** Identify and apply the inverse operations to both sides of the equation in the reverse order to BIDMAS, then write the solution to the equation. Note that $-x = 7$ is not the solution, as the $-x$ has a coefficient of -1 .

WRITE

a $12 = \frac{7x}{2} - 9$ (+ 9)
 $21 = \frac{7x}{2}$ ($\times 2$)
 $42 = 7x$ ($\div 7$)
 $x = 6$

b $\frac{-x-4}{3} = 1$ ($\times 3$)
 $-x - 4 = 3$ (+ 4)
 $-x = 7$ ($\div (-1)$)
 $x = -7$

Example 4A.3 Solving equations with the unknown on both sides



Solve each equation for x .

a $4x + 7 = 2x - 3$

b $3(2x + 1) = -17 - 4x$

THINK

- a** **1** Eliminate the pronumeral term from one side of the equation by subtracting the pronumeral with the smaller coefficient, $2x$, from both sides of the equation.
- 2** Solve the equation using inverse operations.

WRITE

a $4x + 7 = 2x - 3$ ($- 2x$)
 $2x + 7 = -3$ ($- 7$)
 $2x = -10$ ($\div 2$)
 $x = -5$

- b 1** Remove the brackets by expanding the expression on the left-hand side of the equation.
- 2** Eliminate the negative pronumeral term by adding $4x$ to both sides of the equation.
- 3** Solve the equation using inverse operations.

$$\mathbf{b} \quad 3(2x + 1) = -17 - 4x$$

$$6x + 3 = -17 - 4x \quad (+ 4x)$$

$$10x + 3 = -17 \quad (- 3)$$

$$10x = -20 \quad (\div 10)$$

$$x = -2$$

Helpful hints

- ✓ Writing the inverse operation beside the appropriate line of working out is a great way to keep track of your calculations! $\frac{x}{3} - 5 = 7 \quad (+ 5)$
- ✓ Remember that you have to apply the inverse operation to both sides of the equation.
- ✓ Remember to define your pronumerals before writing equations to represent the variables in worded problems. For example, let n = number of eggs in a carton or let w = weight of eggs (grams).
Note: ' n = eggs in a carton' would not be correct, as a pronumeral must always represent a quantity. Similarly, ' w = weight of eggs' would not be correct because it doesn't specify a unit of measurement.

ANS
p442

Exercise 4A Solving linear equations

 1-6, 7(a, c, f, g), 8-14

 2-13, 15, 16, 18

 2-5, 7(e-h), 9, 11-13, 16-18

4A.1 1 Solve the following equations using inverse operations.

a $4x + 5 = 29$

b $\frac{x+3}{2} = 4$

c $\frac{x}{4} - 2 = 7$

d $\frac{2x}{5} = 4$

e $-2(x+6) = 28$

f $-17 = -3x - 5$

g $9 = \frac{x}{5} + 6$

h $\frac{x-4}{5} = -1$

i $-20 = 4(x-2)$

j $-5x + 1 = 16$

k $\frac{-3x}{2} = -18$

l $\frac{x}{2} + 7 = 4$

4A.2 2 Solve the following equations using inverse operations.

a $\frac{3x+4}{5} = 2$

b $\frac{-3x}{4} + 1 = 7$

c $\frac{5(x-1)}{2} = 20$

d $4(x+3) - 2 = 30$

e $\frac{x-2}{6} - 3 = 0$

f $1 = \frac{11x}{4} + 23$

g $3 = \frac{-7x+6}{2}$

h $-5(x+2) - 7 = 3$

i $2 = \frac{2(x+8)}{3}$

j $-16 = 4(5-x) + 4$

k $6 = \frac{-x}{2} - 4$

l $\frac{2-3x}{5} = -2$

3 Verify if $x = -2$ is a solution to the following equations using substitution.

a $\frac{x-2}{4} = 1$

b $\frac{x-4}{3} + 11 = 9$

c $\frac{3x}{4} - 1.9 = -3.4$

d $\frac{5x+1}{3} = -3$

4 a Solve $5(x-2) = 20$ by first dividing both sides by 5.

b Another way to solve this equation is first to expand the expression on the left-hand side. Try this method. Do you obtain the same solution?

c Solve $5(x-2) = 18$ by first dividing both sides by 5.

d Solve $5(x-2) = 18$ by first expanding the expression on the left side.

e Which method did you find easier to use when solving $5(x-2) = 18$? Explain.

5 Solve the following equations using the appropriate method from question 4. Where relevant, write the solution as an improper fraction.

a $4(x - 1) = 8$

b $3(x + 7) = -6$

c $2(x - 3) = 5$

d $5(x + 4) = 8$

e $-4(x + 2) = -24$

f $-6(x - 2) = 1$

4A.3 6 Solve each equation for x .

a $6x + 5 = 4x + 9$

b $3x - 11 = x + 3$

c $3x - 8 = 6x - 5$

d $-15 - 2x = 4x + 3$

e $3x + 7 = -3 - 2x$

f $9x - 4 = 10x - 11$

g $10 - x = 7x - 22$

h $-5x - 5 = 11 - x$

7 Solve each equation for x . Use substitution to check that your solution is correct.

a $3(x - 2) = 8x - 1$

b $2x - 1 = 5(x - 2)$

c $2(3x - 4) = 5x - 1$

d $-3(-2x - 1) = -18 - x$

e $4(x + 3) = 5(x + 1)$

f $5(x + 9) = -3(x - 7)$

g $-6(1 - x) = 3(x - 8)$

h $-6(x + 1) = -10(x - 3)$

8 Trent is sharing a bag of jellybeans equally with three of his friends and finds that there are two left over. Consider the number of jellybeans that each person receives, including Trent, if there were 34 jellybeans in the bag.

a Define a pronumeral to represent the unknown quantity in this problem.

b Use this pronumeral to write an equation to represent the problem.

c Solve the equation using inverse operations.

d How many jellybeans did each person receive?

9 Lily is saving to buy a pair of sneakers that cost \$395. She is able to save \$70 per month. If she currently has \$115, consider the number of months it will take for Lily to buy the shoes.

a Define a pronumeral to represent the unknown quantity in the problem.

b Use this pronumeral to write an equation to represent the problem.

c Solve the equation using inverse operations.

d In how many months can Lily buy the shoes?

10 For each problem, set up an equation and solve it using inverse operations.

a Darnell buys three model planes online for a total cost of \$590, which includes the delivery charge of \$35. What is the cost of each model plane?

b Emma and Maggie score a total of 35 goals in a basketball match. Maggie scores seven more goals than Emma. How many goals did Emma score?

c The perimeter of a rectangular playing field is 100 m. If the length is 12 m longer than the breadth, what are the dimensions of the playing field?

d The cost of hiring a party venue is \$500. There is a \$26 per person charge for food. If Kasey has a budget of \$2700 for the party, what is the maximum number of people that can attend?

11 Nylah and Josh have the same amount of money. Nylah buys seven sushi rolls and has \$1.50 left over. Josh buys four sushi rolls and has \$12 left over.

a If x represents the cost of one sushi roll (\$), which equation fits this situation?

A $7x + 150 = 4x + 12$

B $7x + 1.5 = 4x + 12$

C $7x - 1.5 = 4x - 12$

D $7x - 150 = 4x - 12$

E $7x + 12 = 4x + 1.5$

b Solve the equation to find the cost of one sushi roll.



- 12 The length, l , of a rectangle is twice its breadth, b .
- Write an expression for the perimeter, P , of the rectangle in terms of l .
 - If the perimeter of the rectangle is 60 m, find the breadth of the rectangle.

- 13 Violetta cooked sausages for the school sausage sizzle. Each sausage was placed in bread with tomato sauce. Twenty of these were sold with mustard. Half of those left were sold with fried onions. If there were 18 sausages sold with fried onions, how many sausages did Violetta cook?



- 14 One angle in a triangle is 30° . The second angle is twice the size of the third angle in the triangle. Find the size of the largest angle.
- 15 The sum of three consecutive integers is 13 more than the smallest of the three numbers. Identify the three numbers using algebra.

- 16 The linear equations you have dealt with have sometimes been equal. That is, for a particular value of the pronumeral, the left- and right-hand sides are equal. The linear equations $2x + 5 = 2x + 5$, $3x + 5 = 3x + 5$ and $3x + 6 = 3x + 6$ are equal for all values of x . That is, regardless of the value of x , the equations are always equal. The linear equations $2x + 5 = 2x + 6$, $3x + 5 = 3x + 6$ and $3x + 4 = 3x + 6$ are equal for no value of x . That is, regardless of the value of x , the equations are never equal.

- For each of the following, determine whether the equation is always, sometimes or never equal.

i $2x + 5 + 3x = 1 + 5x + 4$	ii $7x - 2 + 4 = 3x + 4x - 6$	iii $2x + 3x - 5 = 6x - 9$
iv $15 - 12x = -3(4x + 5)$	v $4(6x + 10) = 8(3x + 5)$	vi $4(3 - 4x) + x = 4 - 3(5x - 3)$
- When are the two sides of a linear equation with one pronumeral equal?
- In which cases are the two sides of a linear equation with one pronumeral never equal?

- 17 A truck driver travels a total of 650 km at a constant speed for a particular trip.
- Define two pronumerals for the unknown quantities in this problem.
 - Use these pronumerals to write an equation to represent the problem.
 - If the truck driver had increased their speed by 20 km/h, write an expression to represent the distance they would have covered in the same time with the two pronumerals defined in part **a**.
 - If the truck driver travelled 850 km at the new speed in the same time, how long has the driver been on the road?
 - What is the new speed?



- 18 A factory worker made 440 identical parts at a constant speed for a particular shift. If the factory worker had decreased their speed by 10 parts/h, they would have made 40 fewer parts in the same shift. How many parts was the worker making in an hour originally?

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Interactive skillsheet
Solving equations using inverse operations



Interactive skillsheet
Solving equations with the unknown on both sides



Worksheet
Solving linear equations



Topic quiz
4A

4B Solving linear inequalities

Learning intentions

By the end of this topic you will be able to ...

- ✓ represent linear inequalities using a number line
- ✓ solve linear inequalities using inverse operations.



Inter-year links

- Year 7** 6H Solving equations using inverse operations
- Year 8** 6C Solving equations with the unknown on both sides
- Year 10** 4B Solving linear inequalities

Linear inequalities

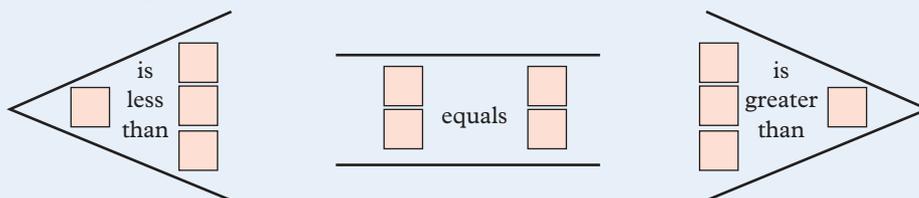


Key content video

- An **inequality** is a mathematical statement that compares the values of two unequal expressions. Inequalities are written using inequality signs, like the ones shown in the table below.

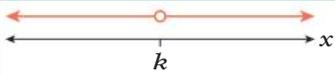
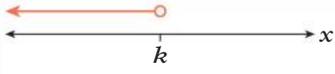
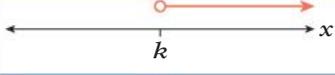
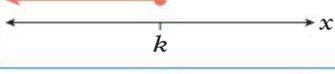
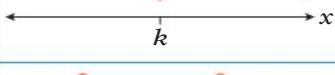
Inequality sign	Meaning	Example	Meaning of example
\neq	'is not equal to '	$x \neq 2$	x is not equal to 2
$<$	'is less than '	$x < 2$	x is less than 2
$>$	'is greater than '	$x > 2$	x is greater than 2
\leq	'is less than or equal to '	$x \leq 2$	x is less than or equal to 2
\geq	'is greater than or equal to '	$x \geq 2$	x is greater than or equal to 2

- A **linear inequality** is an inequality containing pronumerals only raised to the power of 1 and no two pronumerals are multiplied together.



Representing linear inequalities on a number line

- Inequalities involving a single variable can be represented on a number line, using rays and line segments to represent all of the values that satisfy the inequality (see the table on the next page).
On a number line:
 - a line or line segment indicates that all the values the line passes through are included in the possible range of values for the variable
 - a ray (arrowed line) indicates that the possible values for the variable have no limit and continue to positive or negative infinity in the direction indicated
 - a closed circle (solid dot ●) indicates that the endpoint value is included, meaning the value is either 'less than or equal to' (\leq) or 'greater than or equal to' (\geq) the value represented by that point on the number line
 - an open circle (hollow dot ○) indicates that the value is not included, meaning the value is either 'less than' ($<$) or 'greater than' ($>$) the value represented by that point on the number line.

Inequality	Number line representation	Meaning
$x \neq k$		x is not equal to k
$x < k$		x is less than k
$x > k$		x is greater than k
$x \leq k$		x is less than or equal to k
$x \geq k$		x is greater than or equal to k
$k \leq x < m$		x is greater than or equal to k but less than m

Solving linear inequalities

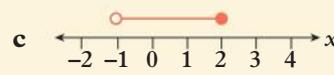
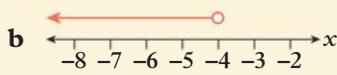
- In most cases, if the same operation is performed on both sides of an inequality sign, the result is an equivalent linear inequality of a different form and the possible values of the unknown remain the same.
- Inverse operations can be used to solve linear inequalities. However, the inequality sign is reversed when both sides of the inequality are multiplied or divided by a negative number.

For example, $5 - 2x \leq 1$ (-5)
 $-2x \leq -4$ $(\div (-2))$
 $x \geq 2.$

Example 4B.1 Representing linear inequalities on a number line



Write the inequality represented by each of these number lines.



THINK

- a** The line has a closed circle at 3, indicating that x can be equal to 3. The ray points to the right, indicating that x can be any value greater than 3. So x is greater than or equal to 3.
- b** The line has an open circle at -4 , indicating that x cannot be equal to -4 . The ray points to the left, indicating that x can be any value less than -4 . So x is less than -4 .
- c** The line has an open circle at -1 , indicating that x cannot be equal to -1 . The line segment extends to a closed circle at 2. So, x is greater than -1 but less than or equal to 2.

WRITE

- a** $x \geq 3$
- b** $x < -4$
- c** $-1 < x \leq 2$



Example 4B.2 Solving linear inequalities

Solve each of these inequalities.

a $\frac{x}{4} - 8 \leq 0$

b $-2 > 10x + 3$

c $3 > -1 - \frac{x}{7}$

THINK

- a**
- 1 Apply the inverse operations to both sides in the correct order.
 - 2 Check your solution. Substitute $x = 32$ into the LHS of the inequality. The result should equal the RHS. Then substitute any value for x that is less than 32 into the original inequality to check the inequality sign. The result should be less than or equal to the RHS.
- b**
- 1 Apply the inverse operations to both sides in the correct order.
 - 2 Write the solution with x on the LHS, reverse the inequality sign if needed.
 - 3 Check your solution. Substitute $x = -\frac{1}{2}$ into the RHS of the inequality. The result should equal the LHS. Then substitute any value for x that is less than $-\frac{1}{2}$ into the original inequality to check the inequality sign. The result should be less than the LHS.
- c**
- 1 Apply the inverse operations to both sides in the correct order.
 - 2 Write the solution with x on the LHS, reverse the inequality sign if needed.
 - 3 Check your solution. Substitute $x = -28$ into the RHS of the inequality. The result should equal the LHS. Then substitute any value for x that is greater than -28 into the original inequality to check the inequality sign. The result should be less than the LHS.

WRITE

a $\frac{x}{4} - 8 \leq 0 \quad (+ 8)$

$$\frac{x}{4} \leq 8 \quad (\times 4)$$

$$x \leq 32$$

Check by substituting $x = 32$ into $\frac{x}{4} - 8 \leq 0$.

$$\text{LHS} = \frac{32}{4} - 8$$

$$= 0$$

$$= \text{RHS} \quad (\text{For } x = 32, \text{ LHS} = \text{RHS as required.})$$

Check by substituting a value less than 32

(for example, $x = 8$).

$$\text{LHS} = \frac{8}{4} - 8$$

$$= -7$$

$$-7 < 0 \quad (\text{For } x < 32, \text{ LHS} < \text{RHS as required.})$$

b $-2 > 10x + 3 \quad (- 3)$

$$-5 > 10x \quad (\div 10)$$

$$-\frac{1}{2} > x$$

$$x < -\frac{1}{2}$$

Check by substituting $x = -\frac{1}{2}$ into $-2 > 10x + 3$.

$$\text{RHS} = 10\left(-\frac{1}{2}\right) + 3$$

$$= -2$$

$$= \text{LHS} \quad (\text{For } x = -\frac{1}{2}, \text{ LHS} = \text{RHS as required.})$$

Check by substituting a value less than $-\frac{1}{2}$

(for example, $x = -2$).

$$\text{RHS} = 10(-2) + 3$$

$$= -17$$

$$-2 > -17 \quad (\text{For } x < -\frac{1}{2}, \text{ LHS} > \text{RHS as required.})$$

c $3 > -1 - \frac{x}{7} \quad (+ 1)$

$$4 > -\frac{x}{7} \quad (\times (-7))$$

$$-28 < x$$

$$x > -28$$

Check by substituting $x = -28$ into $3 > -1 - \frac{x}{7}$.

$$\text{RHS} = -1 - \frac{-28}{7}$$

$$= -1 + 4$$

$$= 3$$

$$= \text{LHS} \quad (\text{For } x = -28, \text{ LHS} = \text{RHS as required.})$$

Check by substituting a value greater than -28

(for example, $x = 0$).

$$\text{RHS} = -1 - \frac{0}{7}$$

$$= -1$$

$$3 > -1 \quad (\text{For } x > -28, \text{ LHS} > \text{RHS as required.})$$



Example 4B.3 Solving inequalities with the unknown on both sides

Solve each of these inequalities.

a $7x - 2 \geq 3x - 10$

b $4 - 5x < 3 - 8x$

THINK

- Use inverse operations to remove the pronumeral on the side where the coefficient is a number less than the coefficient on the other side.
- Identify and apply the correct inverse operations, in order, to both sides of the inequality.
- Check your solution by substituting appropriate values for x into the original inequality to see if true inequalities result.

WRITE

a $7x - 2 \geq 3x - 10$ $(-3x)$
 $4x - 2 \geq -10$ $(+2)$
 $4x \geq -8$ $(\div 4)$
 $x \geq -2$

Check by substituting $x = -2$ into $7x - 2 \geq 3x - 10$.

$$\begin{array}{ll} \text{LHS} = 7(-2) - 2 & \text{RHS} = 3(-2) - 10 \\ = -16 & = -16 \end{array}$$

(For $x = -2$, LHS = RHS as required.)

Check by substituting a value greater than -2 (for example, $x = 0$).

$$\begin{array}{ll} \text{LHS} = 7(0) - 2 & \text{RHS} = 3(0) - 10 \\ = -2 & = -10 \end{array}$$

(For $x > -2$, LHS > RHS as required.)

b $4 - 5x < 3 - 8x$ $(+8x)$
 $4 + 3x < 3$ (-4)
 $3x < -1$ $(\div 3)$
 $x < -\frac{1}{3}$

Check by substituting $x = -\frac{1}{3}$ into $4 - 5x < 3 - 8x$.

$$\begin{array}{ll} \text{LHS} = 4 - 5\left(-\frac{1}{3}\right) & \text{RHS} = 3 - 8\left(-\frac{1}{3}\right) \\ = \frac{17}{3} & = \frac{17}{3} \end{array}$$

(For $x = -\frac{1}{3}$, LHS = RHS as required.)

Check by substituting a value less than $-\frac{1}{3}$ (for example, $x = -1$).

$$\begin{array}{ll} \text{LHS} = 4 - 5(-1) & \text{RHS} = 3 - 8(-1) \\ = 9 & = 11 \end{array}$$

(For $x < -\frac{1}{3}$, LHS < RHS as required.)

Helpful hints

- ✓ Be careful when multiplying and dividing inequalities by negative numbers.
- ✓ If you need to swap the LHS and RHS of an inequality, remember to reverse the inequality sign so that it is still pointing towards the lesser expression. For example, if $5 \leq x$, then $x \geq 5$.
- ✓ When solving linear inequalities with the unknown on both sides, always remove the pronumeral on the side where the coefficient is less. That way you will always have a positive value for the remaining coefficient.
- ✓ Remember, $4 > 2$ but $-4 < -2$.

Exercise 4B Solving linear inequalities

 1-4, 5(1st, 2nd columns),
6, 7, 8 (a-e), 9, 11, 13

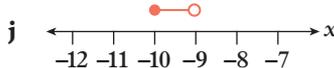
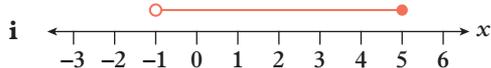
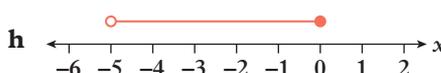
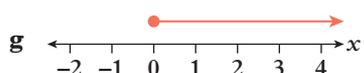
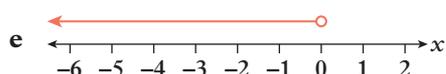
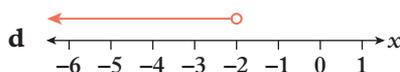
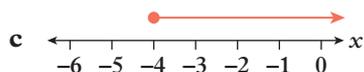
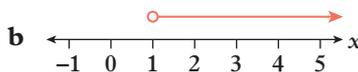
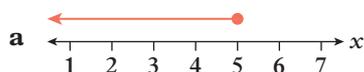
 2-3(2nd column), 5-7(3rd, 4th columns),
8(e-i), 9, 10, 12, 14

 2-3(2nd column), 6-8(e-h), 9, 12, 14-17

1 Write each of the following statements as an inequality.

- | | |
|---|--|
| a Two is less than three. | b Three is greater than two. |
| c Three is greater than or equal to two. | d Negative three is less than or equal to negative two. |
| e Three is not equal to negative three. | f Negative three is not equal to three. |

4B.1 2 Write the inequality represented on each of these number lines.



3 For each of these inequalities, show the possible values of x on a number line.

- | | | |
|-------------------------|-----------------------|-------------------------|
| a $x > 4$ | b $x < 3$ | c $x \geq -2$ |
| d $x < 0$ | e $x \neq 2$ | f $x \neq -2$ |
| g $1 \leq x < 5$ | h $-4 < x < 2$ | i $0 < x \leq 6$ |

4 Apply the following operations to each side of the following inequalities. Ensure the correct inequality symbol is written between the two numbers.

- | | |
|-------------------------------------|---|
| a $-4 < 9$ | b $x \geq 6$ |
| i add five | ii subtract nine |
| iii multiply by three | iv multiply by one-half |
| v multiply by negative three | vi multiply by negative one-half |
| vii divide by three | viii divide by negative three |

4B.2 5 Solve each of these inequalities.

- | | | | |
|------------------------------|-------------------------------------|----------------------------------|--------------------------------|
| a $x + 5 > 19$ | b $\frac{x}{10} \leq 30$ | c $-20 > 4x$ | d $-3x \geq -15$ |
| e $7 < -\frac{x}{10}$ | f $\frac{x}{5} + 10 \leq 11$ | g $2x - 9 \geq -11$ | h $26 > 14 - 4x$ |
| i $-x + 3 \geq 12$ | j $5(x + 3) \leq 50$ | k $\frac{x - 8}{15} > -3$ | l $1 < 10x + 14$ |
| m $12 - 14x > 14$ | n $-17 \leq -6x + 12$ | o $-6(x - 4) < 13$ | p $-11(3x + 5) \geq 40$ |

6 Solve each of these inequalities and show the solution on a number line each time.

- | | | | |
|------------------------------------|--------------------------------|------------------------------------|-------------------------------------|
| a $3x + 2 \geq -13$ | b $1 - 7x < -6$ | c $5 - \frac{x}{6} \leq 2$ | d $\frac{x - 4}{2} < 5$ |
| e $\frac{7 - x}{3} \geq -1$ | f $\frac{2 + x}{5} > 3$ | g $\frac{3x}{2} - 5 \leq 4$ | h $\frac{8 - x}{7} - 3 < -2$ |

4B.3 7 Solve each of these inequalities. Use substitution to check your solutions.

- a** $4x - 3 > 2x + 5$ **b** $7x + 1 \leq 3x - 7$ **c** $5x - 9 \geq 9 - x$ **d** $2x + 4 < 5x - 8$
e $3 - x > 4x - 2$ **f** $x + 11 \geq 7 - 3x$ **g** $8 - 3x \leq 18 - 5x$ **h** $1 - 6x > -3x - 5$

8 Solve each of these inequalities by first expanding to remove brackets.

- a** $2(x - 3) > x + 5$ **b** $4x - 7 \leq 3(x + 2)$ **c** $5(x + 1) \geq -3x - 11$
d $3(2x - 5) < 7x + 4$ **e** $9(x + 5) < 10(x + 6)$ **f** $3(3x - 1) \leq -2(x - 4)$
g $4(x + 3) \geq 2(5x - 3)$ **h** $2(2 - x) > 3(1 - 2x)$ **i** $5(3 - 2x) \leq -4(2x - 7)$

9 Linear inequalities have an infinite number of solutions unless additional conditions are added. For each inequality in question 7, state the number of solutions if x is:

- i** a positive integer
ii a negative integer.

10 Write an inequality statement to represent each of the following situations. Use a pronumeral for the unknown quantity each time.

- a** A boat is sold for at least \$750 000. What could the boat have sold for?
b To fit in an economy class seat on an aeroplane, a person must be less than 200 cm tall. What could be the height of a person in such a seat?
c A Vespa cannot travel faster than 55 km/h. At what speeds could a Vespa travel?
d To fit in a new sleeping bag, a person must be less than 196 cm tall. How tall could a person be to fit in the sleeping bag?



11 Emily and Klaus are selling watermelons at a market. They start with 20 melons and agree to share any that are left at the end of the day.

- a** Write an expression for the number of melons they each will take home if they sell x watermelons.
b Emily and Klaus aim to take home no more than three watermelons each. Use your expression from part **a** to write an inequality for this situation and then solve it.
c How many watermelons can they sell to meet their goal?

12 Ella runs a business selling candles where customers can pay for delivery or pick them up from her store. She sells each candle for \$2.50 and can mail a package containing one candle for \$3, two candles for \$5.50 and four candles for \$10.

A customer has \$26 to spend on candles. They live too far away to visit the store, so they must pay the delivery fee.

- a** How many candles could they buy if each candle is packaged individually?
b If the candles in part **a** are packaged in groups of two, can the customer afford an extra candle and the delivery fee?
c The customer decides to prioritise buying packs of four candles instead. If they have enough money left over, they will buy a pack of two candles or a single candle. How many candles can the customer afford now? Make sure to include the delivery fee in your calculations.



- 13 Todd is deciding how many packs of playing cards he should buy. Each pack costs \$3 and he has \$25 in his wallet.
- Write an inequality to represent this situation and then solve it.
 - List the number of packs of playing cards Todd could possibly buy.
- 14 a Square all parts of the following inequalities. Ensure the correct inequality symbols are written between the numbers.

i $2 < 3$ ii $-2 > -3$ iii $2 > -3$ iv $-2 < 3$
 v $\frac{1}{2} > \frac{1}{3}$ vi $-\frac{1}{2} < \frac{1}{3}$ vii $-2 \neq 2$

- Explain why we need to be careful when squaring both sides of an inequality.
- Apply the reciprocal to all parts of the following inequalities. Ensure the correct inequality symbols are written between the numbers.

i $3 > \frac{1}{2}$ ii $-3 < \frac{1}{2}$ iii $-3 < -\frac{1}{2}$
 iv $3 > -\frac{1}{2}$ v $\frac{1}{3} < \frac{1}{2}$ vi $-\frac{1}{3} < \frac{1}{2}$

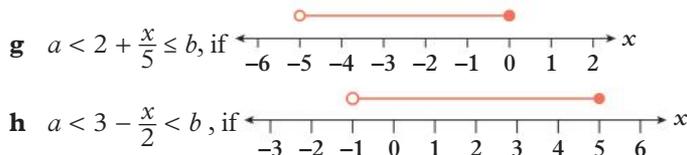
- Explain why we need to be careful when applying the reciprocal to both sides of an inequality.

- 15 A company makes and sells two products: X and Y. It costs the company \$4 for each of product X to be made and \$5 for each of product Y. The company needs the cost of producing the two products to be no more than \$100 per day. Let x be the number of product X the company makes and let y be the number of product Y the company makes.

- Write an inequality describing the cost of producing products X and Y for one day.
- State two other inequalities that must be true given that x and y represent the number of each product.
- Determine the greatest number of product X that could be produced in one day.
- Determine the greatest number of product Y that could be produced in one day.
- Determine the greatest number of product X that could be produced in one day if eight of product Y are produced.

- 16 Determine a and b for the each of the following inequalities.

- $3x + 8 \neq a$ if $x \neq -2$
- $3x + 8 \leq a$ if $x \leq 6$
- $a < 3x - 1 < b$ if $-7 < x < -4$
- $a \leq 3 - x < b$ if $-7 < x \leq 6$
- $ax \leq 2$ if $x \geq -10$
- $-2 < ax - 4 \leq 2$ if $1 < x < 3$



- 17 Solve each of these inequalities.

a $-3 \leq 5 - 2x \leq 7$ b $1 \leq \frac{5}{2} - \frac{5x}{3} < 10$ c $-2 < 4 - \frac{x}{2} < 8$

Check your Student obook pro for these digital resources and more:

pro



Interactive skillsheet

Representing linear inequalities on a number line



Interactive skillsheet

Solving linear inequalities



Worksheet

Solving linear inequalities



Topic quiz

4B

4C Plotting linear relationships

Learning intentions

By the end of this topic you will be able to ...

- ✓ plot linear relationships from tables of values and equations.



Inter-year links

Support

The Cartesian plane

Year 7

5D The Cartesian plane

Year 8

6F Plotting linear relationships

Year 10

5A Linear relationships

Linear relationships

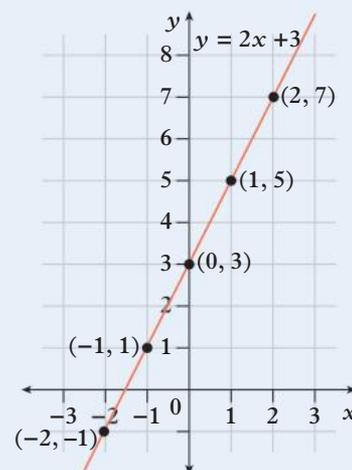
- The relationship between two variables can be represented by an algebraic equation, a table of values, a set of **coordinate points** or a graph.
- A **linear relationship** is a relationship between two variables, the independent variable and the dependent variable, which produces a linear graph. The standard form of a linear relationship is $y = mx + c$.
- A **linear graph** is a straight line on a **Cartesian plane**.
- Substitution can be used to determine whether a point lies on a line.

Plotting linear relationships

- A **plot** is composed of individual coordinate points. A linear graph is a continuous line made up of an infinite number of coordinate points.
- To sketch a graph from an equation follow these steps:
 - 1 Construct a table of values by selecting values for x , then substituting each value of x into the equation to find the corresponding value of y .
 - 2 Write out the coordinate points listed in the table.
 - 3 Plot the coordinate points on the Cartesian plane.
 - 4 Join the points using a straight line.



Key content video



Example 4C.1 Determining whether a point lies on a line



Determine whether the following points lie on the line $y = -4x - 6$.

a $(-1, -2)$

b $(1, 2)$

THINK

Substitute the x -coordinate into the equation and simplify. If the result is equal to the y -coordinate, then the point lies on the line. If not, then the point does not lie on the line.

WRITE

$$\begin{aligned}\mathbf{a} \quad y &= -4x - 6 \\ y &= -4(-1) - 6 \\ &= 4 - 6 \\ &= -2\end{aligned}$$

The point lies on the line.

$$\begin{aligned}
 \text{b } y &= -4x - 6 \\
 y &= -4(1) - 6 \\
 &= 4 - 6 \\
 &= -2 \\
 &\neq 2
 \end{aligned}$$

The point does not lie on the line.

Example 4C.2 Determining whether a relationship is linear



Use the table of values to construct a plot of the relationship between x and y . Is the relationship linear or non-linear?

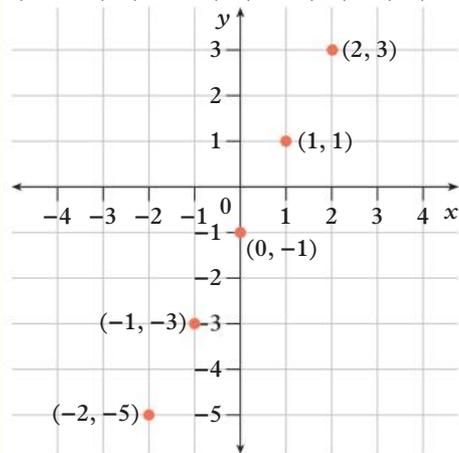
x	-2	-1	0	1	2
y	-5	-3	-1	1	3

THINK

- Write out the coordinate points listed in the table.
- Plot the points on the Cartesian plane.
- Consider whether the points form a straight line. As the points form a straight line, the relationship is linear.

WRITE

$(-2, -5), (-1, -3), (0, -1), (1, 1), (2, 3)$



The relationship is linear.

Example 4C.3 Plotting linear relationships from a table of values



Plot a graph of $y = -x - 3$ by first completing a table of values for x from -3 to 2 .

THINK

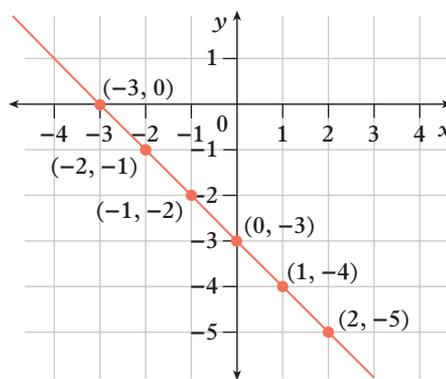
- Construct a table of values for x from -3 to 2 . Substitute each value of x into the equation to find the corresponding value of y .
- Write out the coordinate points listed in the table.

WRITE

x	-3	-2	-1	0	1	2
y	0	-1	-2	-3	-4	-5

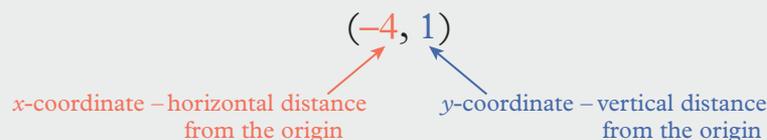
$(-3, 0), (-2, -1), (-1, -2), (0, -3), (1, -4), (2, -5)$

- 3 Plot the points on the Cartesian plane.
- 4 Join the points with a straight line.



Helpful hints

- ✓ Remember that in Cartesian coordinates, the x -coordinate is always listed first, followed by the y -coordinate.



- ✓ When constructing plots and sketches, always label your x - and y -axes and label your graph with the equation of the graph.

ANS
p444

Exercise 4C Plotting linear relationships

▲ 1, 2, 3-4(1st column), 5-9, 12

■ 2, 3-4(2nd column), 6, 8, 10, 11-14

◆ 3(3rd column), 4(iv), 6, 8, 11-15

UNDERSTANDING AND FLUENCY

4C.1 1 Determine whether each of the following points lies on the lines below.

i $(0, 0)$

ii $(1, -5)$

iii $(-4, -24)$

a $y = 6x$

b $y = x - 6$

c $y = 2x - 16$

d $y = 5x - 10$

4C.2 2 a Use the following tables of values to construct a plot of each relationship between x and y .

i

x	-3	-2	-1	0	1	2	3
y	-2	-1	0	1	2	3	4

ii

x	-3	-2	-1	0	1	2	3
y	11	6	3	2	3	6	11

iii

x	-3	-2	-1	0	1	2	3
y	10	9	8	7	6	5	4

iv

x	-3	-2	-1	0	1	2	3
y	-27	-8	-1	0	1	8	27

b Classify each relationship as linear or non-linear.

4C.3 3 For each of the following linear relationships, construct a table of values for x from -3 to 3 and then write out the coordinate points listed in the table.

a $y = x + 2$

b $y = x - 4$

c $y = 3 - x$

d $y = 2 - x$

e $y = -x - 3$

f $y = 4x$

g $y = 2x + 1$

h $y = 3x - 2$

i $y = 4 - 2x$

4 a Complete the following tables of values using the equations provided.

i $x + y = 24$

x	0	3	6	9	12	15	18
y							

ii $xy = 24$

x	-12	-6	-2	2	6	12
y						

iii $y = x(x + 2)$

x	-3	-2	-1	0	1	2	3
y							

iv $x = 4y - 2$

x							
y	-3	-2	-1	0	1	2	3

b Plot the coordinates from the tables of values and classify each relationship as linear or non-linear. Do not attempt to join the points with curves or lines.

5 a Sketch a graph of each linear relationship on the same set of axes by first constructing a table of values.

i $y = x$

ii $y = 2x$

iii $y = 3x$

iv $y = 4x$

b Describe the similarities and differences between each of the four graphs.

c Describe the relationship between the coefficient of x and the steepness of the corresponding graph.

d Without using a table of values, sketch an approximate graph of the following relationships on the same set of axes.

i $y = 5x$

ii $y = 3.5x$

e Use your answers to b and c to describe the graph of $y = 0.5x$.

6 a Sketch a graph of each linear relationship on the same set of axes by first constructing a table of values.

i $y = -x$

ii $y = -2x$

iii $y = -3x$

iv $y = -4x$

b How do these graphs differ from those sketched in question 5?

c Describe the relationship between the coefficient of x and the feature of the graph identified in part b.

d Without using a table of values, sketch the graphs of the following relationships on the same set of axes.

i $y = -5x$

ii $y = -1.5x$

7 a Sketch a graph of each linear relationship on the same set of axes by first constructing a table of values.

i $y = x$

ii $y = x + 1$

iii $y = x + 2$

iv $y = x + 3$

b Describe the similarities and differences between each of the four graphs.

c Describe the relationship between the equations and corresponding graphs.

d Without using a table of values, sketch an approximate graph of the following relationships on the same set of axes.

i $y = x + 4$

ii $y = x + 1.5$

8 a Sketch a graph of each linear relationship on the same set of axes by first constructing a table of values.

i $y = x$

ii $y = x - 1$

iii $y = x - 2$

iv $y = x - 3$

b Describe the similarities and differences between each of the four graphs.

c Describe the relationship between the equations and corresponding graphs.

d Without using a table of values, sketch an approximate graph of the following relationships on the same set of axes.

i $y = x - 4$

ii $y = x - 5$

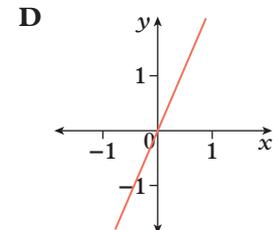
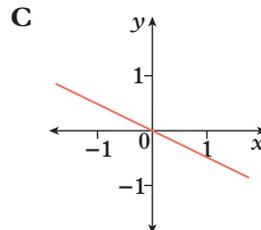
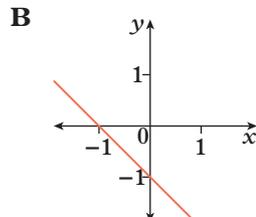
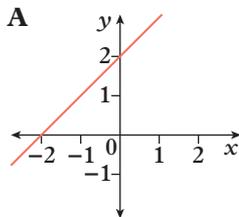
9 Use your answers to questions 5 to 8 to match each equation with its corresponding graph.

a $y = x + 2$

b $y = 2x$

c $y = -x - 1$

d $y = -0.5x$



10 Use your answers to questions 5–8 to sketch the graphs of the following linear relationships. In each part, sketch the two graphs on the same set of axes.

a i $y = 2x$

ii $y = 2x + 1$

b i $y = -x$

ii $y = -x - 1$

c Describe the relationship between the first and second graph in parts **a** and **b**.

11 Javier is training for a cross-country race. He runs laps of the oval every other day and records the number of laps he runs.

Day	1	3	5	7	9	11	13
Number of laps	3	3.5	4	4.5	5	5.5	6

a i What feature(s) of the table of values indicates that the graph is likely to be linear?

ii Hence, what is the minimum number of points you need to plot to sketch the line?

b Sketch the relationship on a Cartesian plane and extend the line to day 25.

c Use the graph to determine:

i the number of laps Javier should run on day 17

ii the day Javier should run 8 laps.

12 A bus is hired for a school trip to the snow. The school pays \$250 towards the bus hire and charges each student an additional \$40.

a If m is the total amount of money collected for bus hire and n students go on the trip, write an equation for the relationship between m and n .

b Calculate how much money would be collected for bus hire if:

i 0 students go on the trip

ii 20 students go on the trip.

c Use your answer to part **b** to sketch a graph of this relationship using a scale from 0 to 30 along the horizontal axis.

d Is the relationship linear? Explain.

e Use the graph to find the total amount of money collected if 30 students go on the trip.

f Use the graph to determine how many students need to go on the trip to collect a total of \$850 for bus hire.

g The hire cost of the bus is \$1300. Use the graph to determine the minimum number of students who need to go on the trip to cover the hire cost.

13 The formula $F = 1.8C + 32$ describes the relationship between temperatures in degrees Celsius, C , and temperatures in degrees Fahrenheit, F .

a Plot the graph of this relationship. Show a scale from -50 to 50 along the horizontal axis.

b Use the graph to find the temperature in $^{\circ}\text{F}$ for 30°C .

c Use the graph to find the temperature in $^{\circ}\text{C}$ for -22°F .

14 Use the graph in question 13 to find where the temperature in $^{\circ}\text{C}$ has the same numerical value as the matching temperature in $^{\circ}\text{F}$.

15 For each of the following pairs of linear relationships, plot two graphs on the same Cartesian plane to determine the coordinates where the two lines intersect.

a $y = x + 1$ and $y = 2x$

b $y = 5x$ and $y = \frac{1}{3}x$

c $y = \frac{1}{2}x - 1$ and $y = -x + 2$

d $y + 2x = 12$ and $3y + x = 16$

Check your Student obook pro for these digital resources and more:



Interactive skillsheet
Plotting linear relationships



Worksheet
Plotting graphs of linear relationships



Investigation
How accurate is a 'rule of thumb' temperature formula?



Topic quiz
4C

4D Gradient and intercepts

Learning intentions

By the end of this topic you will be able to ...

- ✓ identify the x - and y -intercepts of a linear graph
- ✓ determine the gradient of a line segment and a graph.



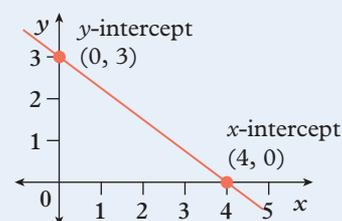
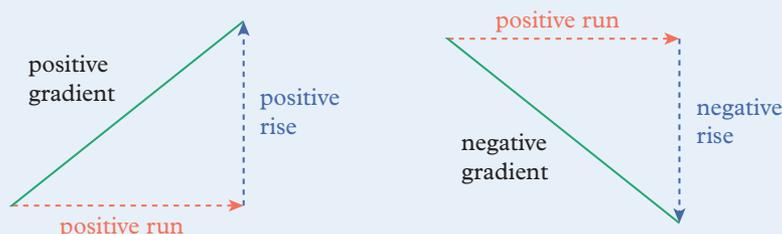
Inter-year links

- Support** The Cartesian plane
- Year 7** 5D The Cartesian plane
- Year 8** 6I Finding linear equations
- Year 10** 5A Linear relationships

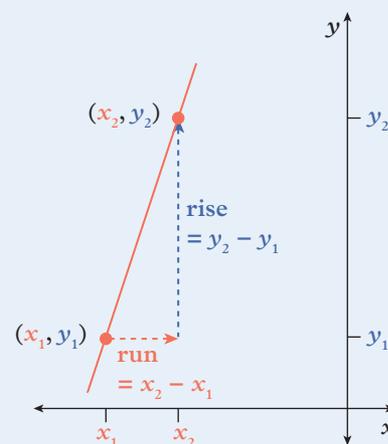
Features of linear graphs



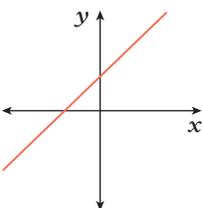
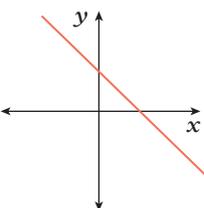
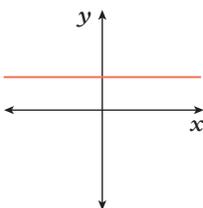
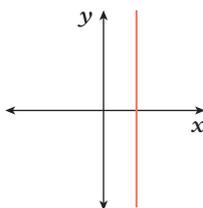
- The features of a linear graph include its x -intercept, y -intercept and gradient.
- The **x -intercept** is the point where the graph crosses the x -axis.
- The **y -intercept** is the point where the graph crosses the y -axis.
- The **gradient** is a numerical measure of the slope of the graph. It is also referred to as the rate of change between the two variables.



- The gradient of a linear graph is a constant because the gradient between any two points on the line is the same. This means that the rate of change between two variables in a linear relationship does not change.
- The value of the gradient is the number of units that the graph increases in the vertical direction for every 1 unit that it increases in the horizontal direction. So, if the gradient is 3, then the linear graph increases 3 units up in the vertical direction for every 1 unit that it increases to the right in the horizontal direction.
- The formula for the gradient, m , between any two points, (x_1, y_1) and (x_2, y_2) , is: $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Gradients can be positive, negative, zero or undefined.



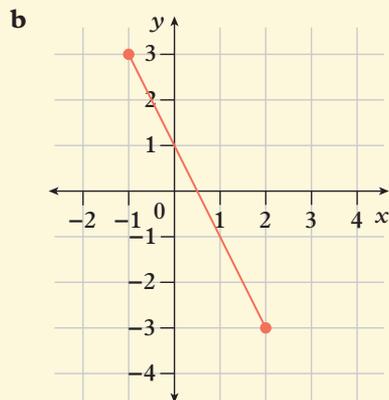
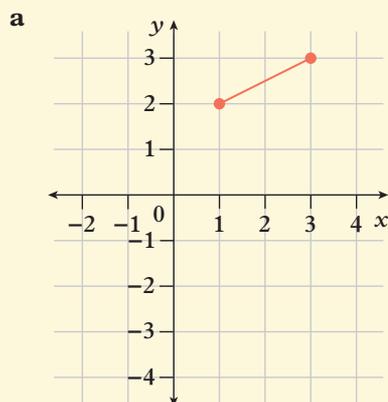
$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Positive	Negative	Zero	Undefined
			



Example 4D.1 Determining the gradient of a line segment by identifying rise and run

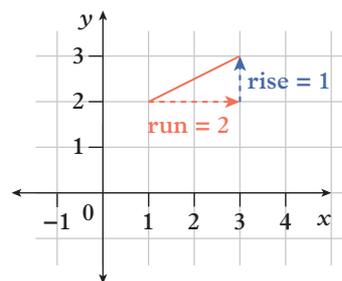
Find the gradient of each line segment.



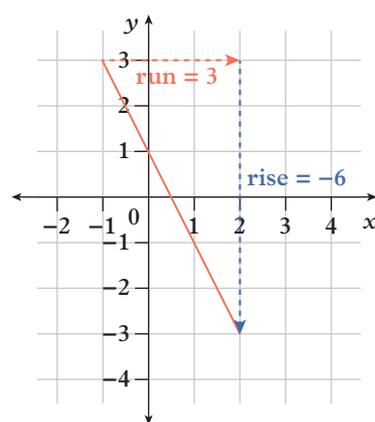
THINK

- a**
- 1 Determine the run, the horizontal distance between the endpoints of the line segment. The run is 2 units.
 - 2 Determine the rise, the vertical distance between the endpoints of the line segment. The rise is 1 unit.
 - 3 Calculate the gradient by dividing the rise by the run. Simplify the gradient where possible.
- b**
- 1 Determine the run, the horizontal distance between the endpoints of the line segment. The run is 3 units.
 - 2 Determine the rise, the vertical distance between the endpoints of the line segment. The rise is -6 units.
 - 3 Calculate the gradient by dividing the rise by the run. Simplify the gradient where possible.

WRITE



$$\begin{aligned}\text{Gradient} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{1}{2}\end{aligned}$$



$$\begin{aligned}\text{Gradient} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{-6}{3} \\ &= -2\end{aligned}$$

Example 4D.2 Determining the gradient, x-intercept and y-intercept

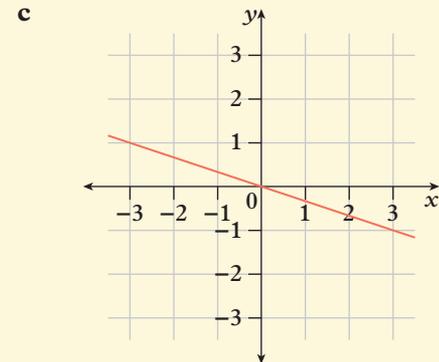
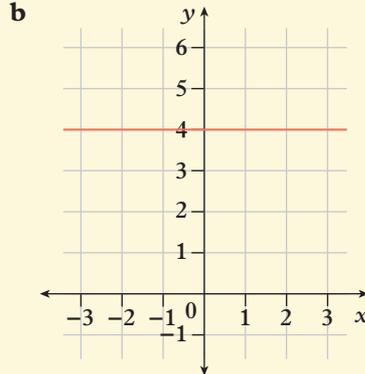
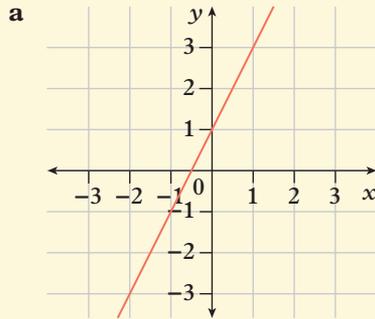


For each of the linear graphs shown, determine the:

i gradient

ii x-intercept

iii y-intercept.

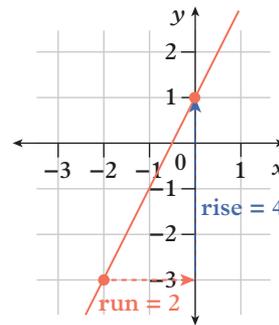


THINK

- a i** Select any two points that have integer coordinates. Determine the rise and run, and then calculate the gradient.
- ii** State the coordinates of the point at which the graph crosses the x-axis.
- iii** State the coordinates of the point at which the graph crosses the y-axis.
- b i** The graph is a horizontal line, so it has a zero gradient.
- ii** The graph does not cross the x-axis, so it does not have an x-intercept.
- iii** State the coordinates of the point at which the graph crosses the y-axis.
- c i** Select any two points that have integer coordinates. Determine the rise and run, and then calculate the gradient. Remember that if the graph is sloping down to the right then the gradient is negative.
- ii** State the coordinates of the point at which the graph crosses the x-axis.
- iii** State the coordinates of the point at which the graph crosses the y-axis.

WRITE

a i



$$\begin{aligned} \text{Gradient} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{4}{2} \\ &= 2 \end{aligned}$$

ii x-intercept: $(-\frac{1}{2}, 0)$

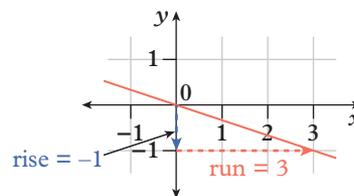
iii y-intercept: $(0, 1)$

b i Gradient = 0

ii no x-intercept

iii y-intercept: $(0, 4)$

c i



$$\begin{aligned} \text{Gradient} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{-1}{3} \\ &= -\frac{1}{3} \end{aligned}$$

ii x-intercept: $(0, 0)$

iii y-intercept: $(0, 0)$

Example 4D.3 Determining gradient using two coordinate points



Use the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to calculate the gradient of the line segment joining the points (3, 2) and (9, 5).

THINK

- 1 Define the points (x_1, y_1) and (x_2, y_2) .
The order of the coordinates does not affect the value of the gradient.
- 2 Substitute the x - and y -coordinates into the gradient formula.
- 3 Calculate and simplify the gradient.

WRITE

Let $(x_1, y_1) = (3, 2)$ and $(x_2, y_2) = (9, 5)$

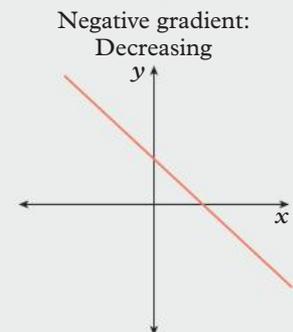
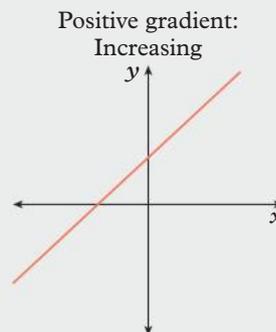
$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{5 - 2}{9 - 3} \\ &= \frac{3}{6} \\ &= \frac{1}{2} \end{aligned}$$

Helpful hints

- ✓ The order in which you substitute points into the formula for the gradient of a line won't affect your final value – you just need to make sure the x - and y -coordinates of a given point match up vertically!
- ✓ Graphs with positive gradients are described as increasing, as the value of y increases from left to right.
- ✓ Graphs with negative gradients are described as decreasing, as the value of y decreases from left to right.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

point 2 (at y_2, x_2) and point 1 (at y_1, x_1)



ANS
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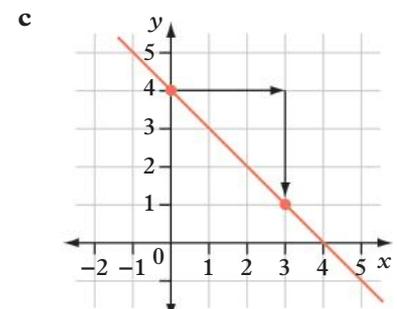
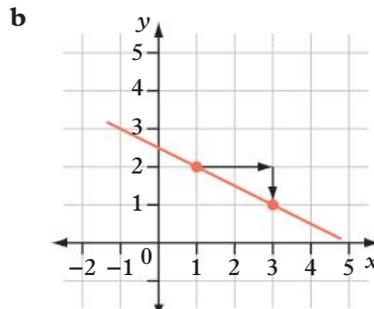
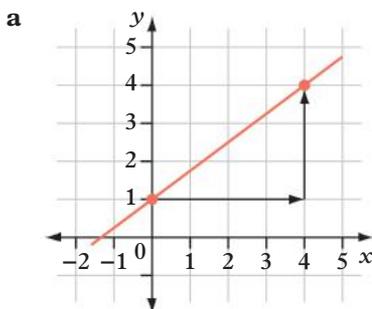
Exercise 4D Gradient and intercepts

▲ 1–5, 6(1st column), 7–10, 12, 16

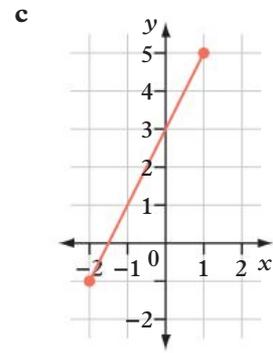
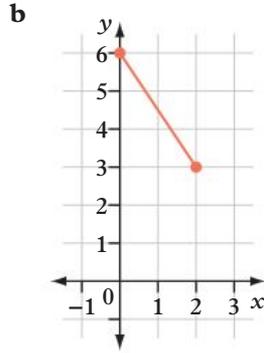
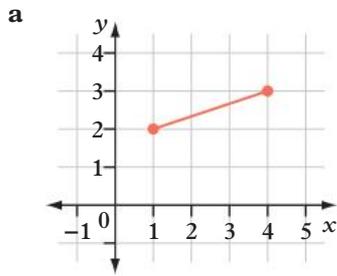
■ 3–5, 6(2nd column), 8–11,
13, 14, 15(a–c)

◆ 3, 4(c, e), 6(2nd column), 7, 9, 13–16

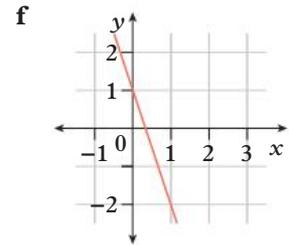
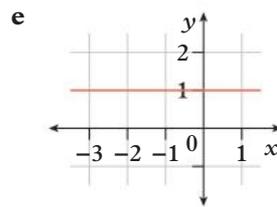
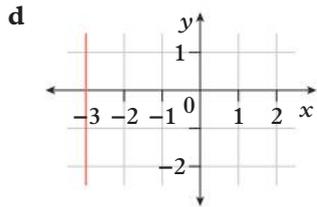
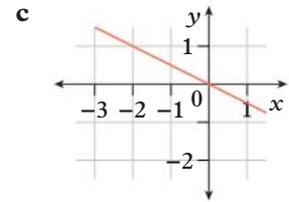
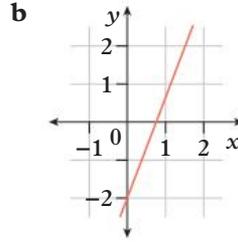
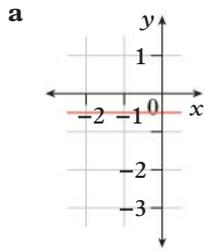
- 1 Use the rise and run marked on the graphs below to determine the gradient of each line.



4D.1 2 Find the gradient of each line segment.

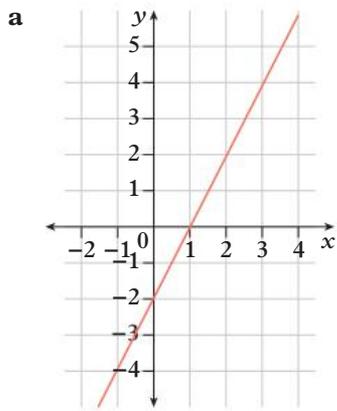


3 Classify the gradients of the following lines as positive, negative, zero or undefined.

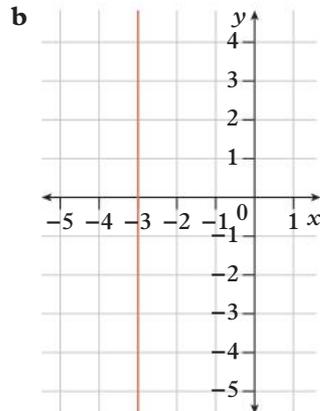


4D.2 4 For the linear graphs shown, determine the:

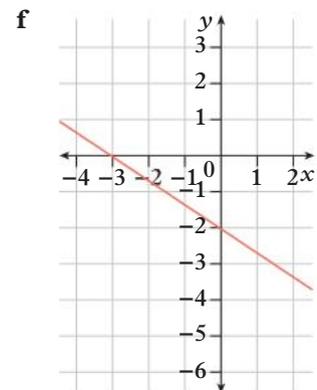
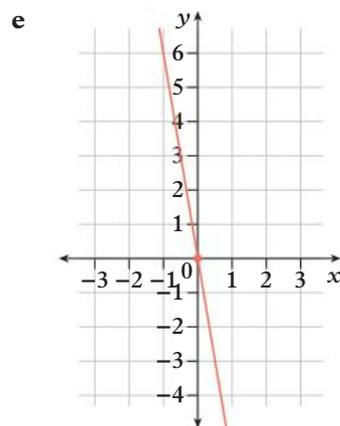
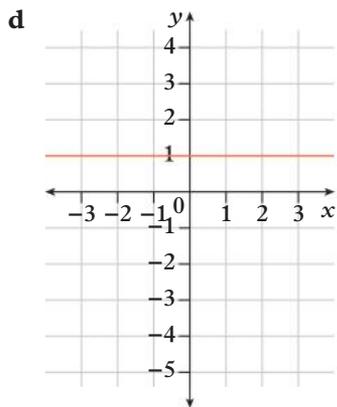
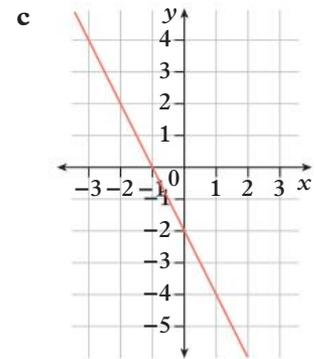
i gradient



ii x-intercept



iii y-intercept.

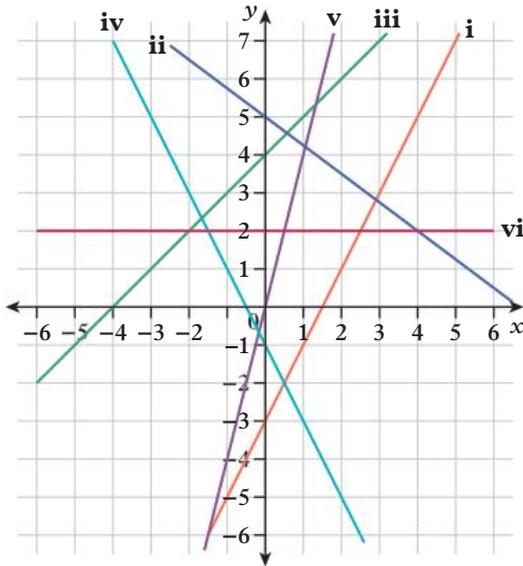


13 Complete the table below.

Run	8	-20			4	
Rise			-4	-16		30
Gradient	$\frac{3}{2}$	$-\frac{4}{5}$	$\frac{1}{7}$	$-\frac{2}{5}$	7	-5

14 Consider these linear graphs.

a Complete the following table by calculating the gradient and identifying the coordinates of the y -intercept of each graph.



	Equation	Gradient	y -intercept
i	$y = 2x - 3$		
ii	$y = -\frac{3}{4}x + 5$		
iii	$y = x + 4$		
iv	$y = -2x - 1$		
v	$y = 4x$		
vi	$y = 2$		

b What pattern can you see in the table that allows you to identify the gradient and the y -intercept from the equation for the graph?

c Determine the gradient and y -intercept of the linear graphs with the following rules:

i $y = 6x + 4$

ii $y = x - 5$

iii $y = -3x$

15 Determine the value of the unknown in each of the following.

a A line with a gradient of 3 passes through the points (1, 4) and (x, 10).

b A line with a gradient of -2 passes through the points (1, 4) and (3, y).

c A line with a gradient of 2 passes through the points (1, 4) and (3, y).

d A line with a gradient of 2 passes through the points (0, 4) and (3, y).

e A line with a gradient of $-\frac{1}{2}$ passes through the points (1, 4) and (3, y).

f A line with a gradient of $\frac{3}{2}$ passes through the points (-1, -4) and (x, 8).

16 The gradient of a line is 2 and it passes through the points (0, 4) and (x, y).

a Write an equation for the gradient of this line.

b Rearrange the equation in part a to solve for y in terms of x .

The gradient of another line is 2 and it passes through the points (1, 4) and (x, y).

c Write an equation for the gradient of this line.

d Rearrange the equation in part c to solve for y in terms of x .

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Interactive skillsheet
Gradients



Interactive skillsheet
Intercepts



Worksheet
Finding the gradient of a linear graph



Worksheet
Identifying features of a linear graph



Topic quiz
4D

4E Sketching linear graphs

Learning intentions

By the end of this topic you will be able to ...

- ✓ determine the x - and y -intercepts of a linear graph from its equation
- ✓ sketch linear graphs with two intercepts using the x - and y -intercepts
- ✓ sketch linear graphs with one intercept.



Inter-year links

- Support** The Cartesian plane
- Year 7** 5D The Cartesian plane
- Year 8** 6F Plotting linear relationships
- Year 10** 5A Linear relationships

Sketching linear graphs with two intercepts



Key content video

- A minimum of two coordinate points are required to **sketch** a linear graph.
- The x -intercept is the point where the linear graph crosses the x -axis and $y = 0$.
- The y -intercept is the point where the linear graph crosses the y -axis and $x = 0$.
- Sketch a linear graph with two intercepts by first finding the x - and y -intercepts:
 - 1 Determine the x -intercept by substituting $y = 0$ into the equation of the line and solving for x .
 - 2 Determine the y -intercept by substituting $x = 0$ into the equation of the line and solving for y .
 - 3 Plot and label the x - and y -intercepts on the Cartesian plane.
 - 4 Draw a straight line through the two points.

For example, to sketch the graph of $y = 2x + 3$:

For x -intercept, let $y = 0$:

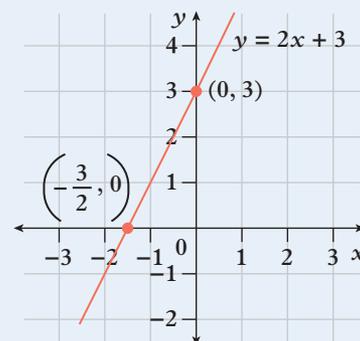
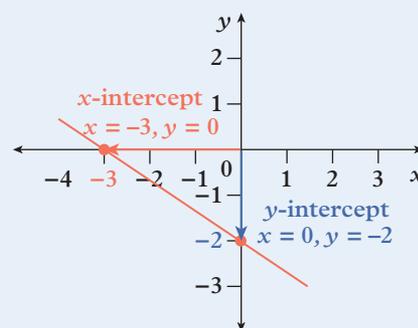
$$\begin{aligned} 0 &= 2x + 3 \\ -3 &= 2x \\ x &= -\frac{3}{2} \end{aligned}$$

x -intercept: $(-\frac{3}{2}, 0)$

For y -intercept, let $x = 0$:

$$\begin{aligned} y &= 2 \times 0 + 3 \\ y &= 0 + 3 \\ y &= 3 \end{aligned}$$

y -intercept: $(0, 3)$



Sketching linear graphs with one intercept

- There are three cases in which a linear graph has only one intercept:

Description	General equation	Intercept	Gradient
Vertical lines	$x = a$	x -intercept: $(a, 0)$	undefined
Horizontal lines	$y = b$	y -intercept: $(0, b)$	0
Lines that pass through the origin	$y = mx$	origin: $(0, 0)$	m

where a , b and m are constants.

- To sketch a linear graph that passes through the origin $(0, 0)$, determine a second point on the graph by substituting any value of x into the equation and then solve for y .

The gradient–intercept method

- The equation for all linear relationships can be expressed in the general **gradient–intercept form** shown below, where:
 - m is the gradient of the line
 - c is the y -coordinate of the y -intercept (the point where the line crosses the y -axis).
- The gradient–intercept form can be used to identify the gradient and y -intercept, enabling us to sketch the graph.

$$y = mx + c$$

↙ ↘

gradient y-intercept

Example 4E.1 Calculating x - and y -intercepts



Determine the coordinates of the x - and y -intercepts of the graphs of the linear relationships:

a $x + 5y = 10$

b $y = 3x - 4$

THINK

- a 1** To determine the x -coordinate of the x -intercept, substitute $y = 0$ into the equation and solve for x . The coordinates of the x -intercept have the form $(x, 0)$.
- 2** To determine the y -coordinate of the y -intercept, substitute $x = 0$ into the equation and solve for y . The coordinates of the y -intercept have the form $(0, y)$.
- b 1** To determine the x -coordinate of the x -intercept, substitute $y = 0$ into the equation and solve for x . The coordinates of the x -intercept have the form $(x, 0)$.
- 2** To determine the y -coordinate of the y -intercept, substitute $x = 0$ into the equation and solve for y . The coordinates of the y -intercept have the form $(0, y)$.

WRITE

- a** For x -intercept, let $y = 0$:

$$x + 5(0) = 10$$

$$x + 0 = 10$$

$$x = 10$$

$$x\text{-intercept: } (10, 0)$$

- For y -intercept, let $x = 0$:

$$0 + 5y = 10$$

$$5y = 10 \quad (\div 5)$$

$$y = 2$$

$$y\text{-intercept: } (0, 2)$$

- b** For x -intercept, let $y = 0$:

$$0 = 3x - 4 \quad (+ 4)$$

$$4 = 3x \quad (\div 3)$$

$$x = \frac{4}{3}$$

$$x\text{-intercept: } \left(\frac{4}{3}, 0\right)$$

- For y -intercept, let $x = 0$:

$$y = 3(0) - 4$$

$$= 0 - 4$$

$$= -4$$

$$y\text{-intercept: } (0, -4)$$

Example 4E.2 Sketching linear graphs with two intercepts



Sketch a graph of $4x - y = 8$ by first finding the x - and y -intercepts.

THINK

- 1 Determine the x -intercept by substituting $y = 0$ into the equation and solving for x . Determine the y -intercept by substituting $x = 0$ into the equation and solving for y .
- 2 Plot and label the x - and y -intercepts on the Cartesian plane.
- 3 Rule a straight line through the points. Label the graph with its equation.

WRITE

For x -intercept, let $y = 0$: For y -intercept, let $x = 0$:

$$4x - 0 = 8$$

$$4x = 8 \quad (\div 4)$$

$$x = 2$$

x -intercept: $(2, 0)$

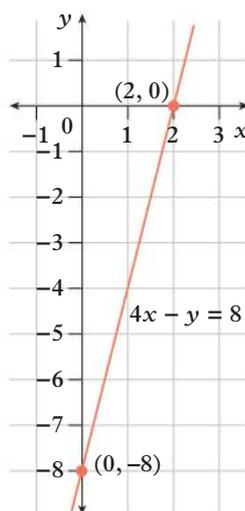
$$4(0) - y = 8$$

$$0 - y = 8$$

$$-y = 8 \quad (\div -1)$$

$$y = -8$$

y -intercept: $(0, -8)$



Example 4E.3 Sketching vertical and horizontal lines



Sketch a graph of the following linear relationships.

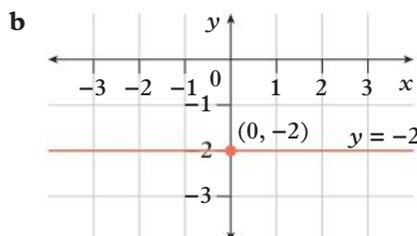
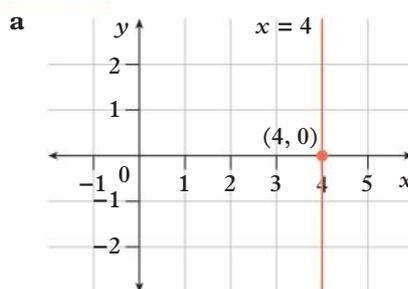
a $x = 4$

b $y = -2$

THINK

- a** The graph of $x = 4$ is a vertical line that passes through the point $(4, 0)$. Rule a vertical line through $(4, 0)$ and label the x -intercept. Label the graph with its equation.
- b** The graph of $y = -2$ is a horizontal line that passes through the point $(0, -2)$. Rule a horizontal line through $(0, -2)$ and label the y -intercept. Label the graph with its equation.

WRITE



Example 4E.4 Sketching linear graphs that pass through the origin



Sketch the graph of $y = -3x$.

THINK

- 1 The x - and y -intercepts are both $(0, 0)$ so the graph passes through the origin $(0, 0)$. Determine a second point on the graph by substituting any value of x into the equation and solving for y .
- 2 Plot and label the two points on the Cartesian plane.
- 3 Rule a straight line through the points and label the graph with its equation.

WRITE

For x -intercept, let $y = 0$:

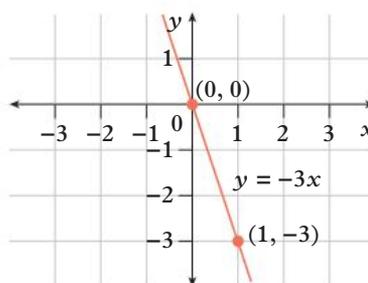
$$0 = -3x \quad (\div -3)$$
$$x = 0$$

x - and y -intercept: $(0, 0)$

For a second point, let $x = 1$:

$$y = -3(1)$$
$$= -3$$

Second point: $(1, -3)$



Example 4E.5 Sketching linear graphs using the gradient-intercept method



Sketch the graph of $y = -\frac{1}{2}x + 3$.

THINK

- 1 Identify the gradient, m , and the y -intercept, c , by comparing the equation to the general gradient-intercept form $y = mx + c$.
- 2 Write the gradient as a fraction and identify appropriate values for the rise and the run.
- 3 Plot a point at the y -intercept and label the coordinates $(0, 3)$.
- 4 Add the rise to the y -coordinate and the run to the x -coordinate of the y -intercept to find a second point on the graph. Move 2 units to the right and 1 unit down to the point $(2, 2)$.
- 5 Rule a straight line through the points. Label the graph with its equation.

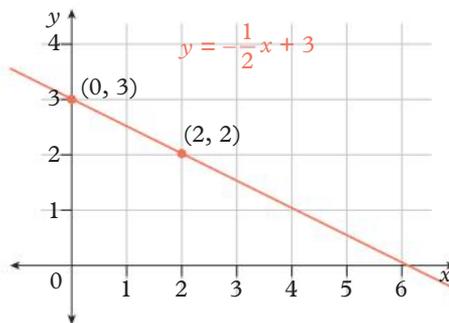
WRITE

$$y = -\frac{1}{2}x + 3$$

$$m = -\frac{1}{2}, c = 3$$

$$\frac{\text{rise}}{\text{run}} = -\frac{1}{2}$$

So let rise = -1 and let run = 2 .



- ✓ To find the x -intercept of a linear relationship, set $y = 0$ and solve for x .
- ✓ To find the y -intercept of a linear relationship, set $x = 0$ and solve for y .
- ✓ Find the x - and y -intercepts before you start sketching so you can plan the scale on the axes of your Cartesian plane.
- ✓ Always label your graph with the following information:
 - the equation of the graph
 - any x - and y -intercepts.

ANS
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Exercise 4E Sketching linear graphs

 1-7, 9, 11, 13, 14

 3-8, 10, 12-14, 16(a, b), 18

 3, 6, 7, 12-19

4E.1 1 Determine the coordinates of the x - and y -intercepts of the graphs of the following linear relationships.

a $x + 4y = 12$

b $y = x + 4$

c $2x + y = 6$

d $y = x - 5$

e $y = -2x + 8$

f $y = 3x - 6$

g $5x + y = -10$

h $y = -x + 7$

i $y = 4 - x$

2 Sketch the linear graphs that have the following x - and y -intercepts.

a x -intercept: $(2, 0)$, y -intercept: $(0, 1)$

b x -intercept: $(-3, 0)$, y -intercept: $(0, 5)$

c x -intercept: $(-1, 0)$, y -intercept: $(0, -1)$

d x -intercept: $(\frac{1}{2}, 0)$, y -intercept: $(0, -\frac{5}{2})$

4E.2 3 Sketch a graph of each of the following linear relationships by first finding the x - and y -intercepts.

a $4x + y = 4$

b $y = x - 2$

c $-2x + 3y = 6$

d $y = -3x + 3$

e $5x + y = -5$

f $y = 2 - x$

g $2y = 2 - 4x$

h $3y - 9x + 12 = 0$

i $4x - 3y - 8 = 0$

4E.3 4 Sketch a graph of the following linear relationships.

a $x = 1$

b $y = 4$

c $x = -3$

d $y = 0$

4E.4 5 Sketch a graph of the following linear relationships.

a $y = 3x$

b $y = -2x$

c $y = 6x$

d $y = -x$

6 Identify the gradient, m , and the coordinates of the y -intercept of the linear graphs with the following equations.

a $y = 2x + 5$

b $y = 4x + 1$

c $y = -3x + 7$

d $y = -5x - 3$

e $y = x - 6$

f $y = 1 - x$

g $y = \frac{4}{3}x + 2$

h $y = \frac{x}{2} - 8$

i $y = -\frac{4x}{3} + \frac{1}{4}$

j $y = 9$

k $y = -7x$

l $y = 5 - \frac{2}{5}x$

4E.5 7 Sketch the graph of each linear relationship using the gradient–intercept method.

a $y = 2x - 4$

b $y = \frac{2}{3}x + 1$

c $y = -3x + 5$

d $y = \frac{5}{4}x - 2$

e $y = -5x - 1$

f $y = -\frac{1}{2}x + 3$

g $y = 3x - 4$

h $y = x + 6$

i $y = -x - 3$

8 Use the most appropriate method to sketch the graph of each linear relationship.

a $2x - 5y = 10$

b $y = 4x + 2$

c $x + y = 6$

d $y = -3x$

e $y = 7$

f $y = 6 - 3x$

g $x = 1$

h $y = 4x - 4$

i $x + 3y - 9 = 0$

j $y = x$

k $y = -3x + 5$

l $y = -7$

9 Without sketching, determine how many axis intercepts the graph of each relationship has.

a $y = 10$

b $y = 2x + 1$

c $x = -12$

d $y = 15x$

e $5x - y = 25$

f $x + y = 1$

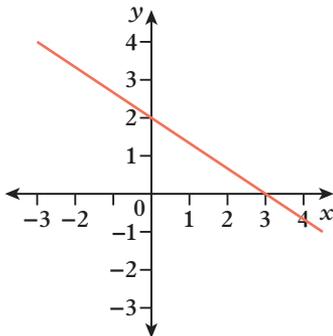
g $y = x$

h $x = 1$

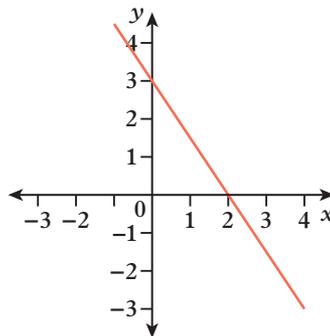
i $-2y = -7x$

10 Which of these three graphs is a correct sketch of $3x + 2y = 6$? Identify the errors in the other two options.

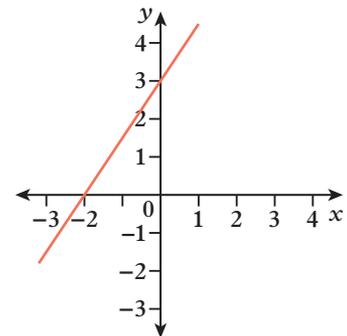
A



B



C



11 Decide whether each statement about the graph of $y = 3x + 6$ is true or false. Correct each false statement.

a The y -intercept is 6.

b The x -intercept is 2.

c The relationship is linear.

d The point $(1, 9)$ lies on the line.

e The line passes through the origin.

f The gradient of the line is 3.

12 Decide whether each statement about the graph of $y = -4x$ is true or false. Correct each false statement.

a The x -intercept is -4 .

b The y -intercept is 0.

c The line passes through the origin.

d The point $(1, 4)$ lies on the line.

e The gradient is negative.

f The gradient of the line is $-\frac{1}{4}$.

13 a Sketch the following lines on the same set of axes:

$2x + 3y = 12$, $2x + 3y = 18$, $2x + 3y = 0$ and $2x + 3y = -12$

b State the similarities between the equations and their graphs.

c Describe the impact of the differences between the equations and their graphs.

14 a Sketch the following lines on the same set of axes:

$2x + 3y = 12$, $2x - 3y = 12$, $-2x + 3y = 12$ and $-2x - 3y = 12$

b State the similarities between the equations and their graphs.

c Describe the impact of the differences between the equations and their graphs.

15 Determine the gradient of the following lines by finding and using the x - and y -intercepts.

a $3x - 5y = 30$

b $7x + 6y + 9 = 0$

c $10y = 3 - 5x$

d $8x = 4y - 3$

16 Tony is buying a skateboard on a purchase plan where he makes a regular payment each week. He creates the formula $y = 300 - 25x$ to describe the relationship between the amount still owed in dollars (y) after a number of weeks (x).

a Explain why Tony has chosen to solve for y (amount still owed in dollars) in terms of x (number of weeks), rather than the other way around. Hint: Think about the relationship between the variables.

b Sketch the graph of this relationship by first determining the coordinates of the x - and y -intercepts.

c What does the y -intercept represent on this graph?

d What does the x -intercept represent on this graph?

e Describe the purchase plan Tony is using. When will he be able to bring his skateboard home?



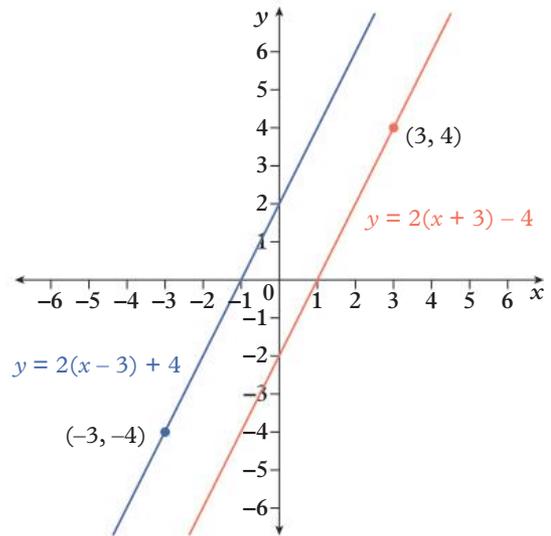
- 17 A rainwater tank has a capacity of 1500 L and feeds a drip system to water the garden. At the beginning of April, the tank is full, but it is empty at the end of the last day of the month. Let x represent the number of days from the start of April and y represent the number of litres of water in the tank. Assume a constant rate of water use and no further rain during April.



- Explain why this relationship can be represented by a linear graph.
- Determine the coordinates of the x -intercept for this relationship.
- Determine the coordinates of the y -intercept for this relationship.
- Use an appropriate method to sketch the graph of the relationship.
- Use the graph to estimate the number of litres of water in the tank at the end of the day on 10 April.
- Use the graph to estimate when there is 600 L of water left in the tank.

- 18 a Explain why the reciprocals of a and b are respectively the x - and y -intercepts of $ax + by = 1$.
 b Explain how you could find the x - and y -intercepts of linear equations in the form $ax + by = d$.
 c For the equation $ax + by = d$, write an expression for d in terms of a and b such that the x -intercept is equal to b and the y -intercept is equal to a .
- 19 The gradient–intercept form $y = mx + c$ is a specific case of the gradient–point form $y = m(x - h) + k$, where (h, k) is a point on the line. For example, $y = 2(x - 3) + 4$ passes through the point $(3, 4)$ and $y = 2(x + 3) - 4$ passes through the point $(-3, -4)$.

- Substitute $(h, k) = (0, c)$ and show that the equation $y = m(x - h) + k$ becomes $y = mx + c$.
- Sketch the following lines by plotting the point (h, k) and then use the gradient to plot another point in the same way you would using the gradient–intercept method.
 - $y = -3(x - 2) + 6$
 - $y = 3(x - 2) + 6$
 - $y = \frac{5}{2}(x + 3) - 4$
 - $y = -\frac{3}{4}(x + 7) + 10$



- Determine the coordinates for the x - and y -intercepts of $y = m(x - h) + k$ in terms of m, h and k .
- Substitute $(h, k) = (b, 0)$ to determine a new general form for writing a linear relationship in terms of the x -intercept.
 - Explain what you can do to the equation $y = mx + c$ to write it in this form.
 - Explain how you can use this form to determine the x -intercept.
- Use your method from part **d** to write the following equations in the new general form and determine their x -intercepts.
 - $y = 3x - 12$
 - $y = 2x - 5$
 - $y = -3x - 12$
 - $y = 2x + 5$

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Interactive skillsheet
Sketching linear relationships using intercepts



Investigation
Making a stained glass window



Topic quiz
4E

Checkpoint

ANS
p453



Checkpoint quiz

Check your knowledge of the first part of this chapter.

- ✓ Core
- ✓ Advanced

4A 1 Solve the following equations for x .

a $\frac{x}{3} + 2 = -7$ **b** $4(x + 2) = 28$ **c** $5(2x - 3) - 8x = -1$

4A 2 Solve the following equations for x .

a $5x + 7 = 2x - 5$ **b** $7 - 4x = 6x - 13$ **c** $7(x - 3) = -61 - x$ **d** $8(2x + 7) = 4(3x + 11)$

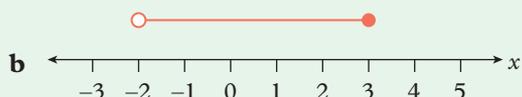
4A 3 Jamie orders five equally priced video games online for a total cost of \$409.70, which includes the delivery charge of \$9.95.

- a Define a pronumeral to represent the unknown quantity in this problem.
- b Use this pronumeral to write an equation to represent the problem.
- c Solve the equation using inverse operations.
- d What is the cost of each game?

ADV 4B 4 Represent the values of x for each inequality on a number line.

a $x \leq 5$ **b** $x > -3$
c $-1 \leq x < 4$ **d** $x \geq 4$

ADV 4B 5 Write the inequality that is represented on each of these number lines.



ADV 4B 6 Solve the following inequalities.

a $6x > 18$ **b** $-4x \geq 20$
c $-13 < \frac{x}{5} - 8$ **d** $7 - 3x < 4$
e $4x + 7 \neq 6x + 13$ **f** $4(3x - 3) \leq 3(3x + 2)$

4C 7 Determine whether the following points lie on the line $y = 4x - 4$.

a $(-1, 0)$ **b** $(2, 4)$ **c** $(0, -4)$ **d** $(-4, -8)$

4C 8 Consider the following tables of coordinate points.

i Plot the points from the following tables.

ii State whether the graph is linear or non-linear.

a

x	-3	-2	-1	0	1	2	3
y	-10	-8	-6	-4	-2	0	2

b

x	-3	-2	-1	0	1	2	3
y	4.5	4	3.5	3	2.5	2	1.5

c

x	-3	-2	-1	0	1	2	3
y	5	2.25	0	-1.75	-3	-3.75	-4

d

x	-3	-2	-1	0	1	2	3
y	-2	0	2	4	6	8	12

4C 9 Sketch a graph of each of the following linear relationships by first completing a table of values for x from -3 to 3 .

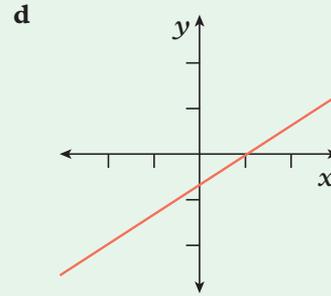
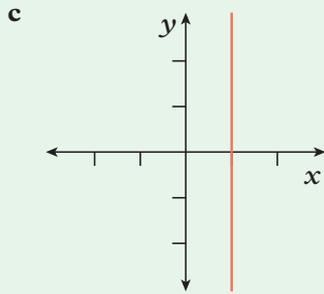
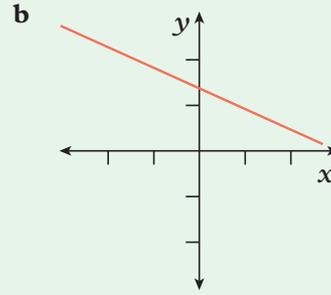
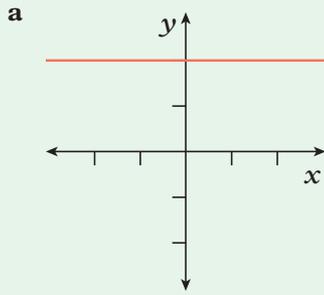
a $y = 2x - 5$

b $y = 5 - x$

c $y = -3x + 1$

d $y = -2x - 3$

4D 10 State whether the following lines have a gradient that is positive, negative, zero or undefined.



4D 11 Determine the gradients of the lines that pass through the following pairs of points.

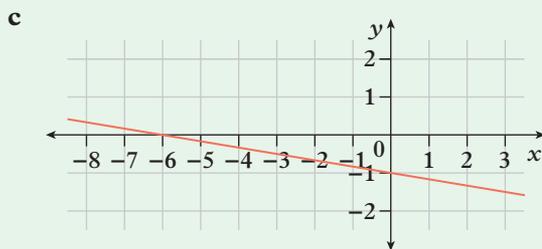
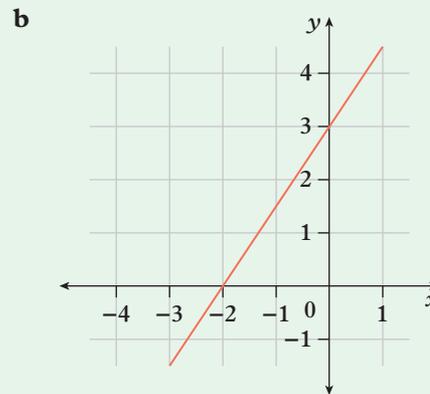
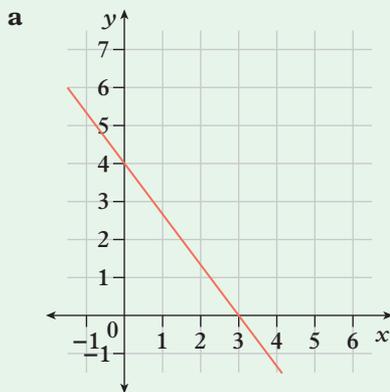
a $(1, 5)$ and $(3, 11)$

b $(-1, 3)$ and $(2, -6)$

c $(0, 5)$ and $(3, 0)$

d $(-6, -2)$ and $(-1, -8)$

4D 12 State the x - and y -intercepts of the following lines as coordinates.



4E 13 Sketch the following linear relationships

a $y = 2x - 4$

b $y = -4x$

c $y = 3$

d $x + 4y = 8$

4F Determining linear equations

Learning intentions

By the end of this topic you will be able to ...

- ✓ determine the equation of a linear graph.



Inter-year links

Year 8

6I Finding linear equations

Year 10

5A Linear relationships

Determining the equation of a linear graph

- To find the equation of a linear graph in the form of $y = mx + c$, determine the value of the:

→ gradient, for any two points (x_1, y_1) and (x_2, y_2) :

$$m = \frac{\text{rise}}{\text{run}}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

→ y -coordinate of the y -intercept: $(0, c)$.

For example, this linear graph has a gradient of 3 and a y -intercept at $(0, 6)$.

$m = 3$ and $c = 6$, so the equation of the graph is $y = 3x + 6$.

Note: The x -intercept at $(-2, 0)$ is not used in this calculation.

- For lines with only one intercept, the equations can be determined using the following general equations where a, b and m are constants.

→ A vertical line with an x -intercept at $(a, 0)$ is given by $x = a$.

For example, the graph of the equation $x = -1$ is a vertical line with an x -intercept at $(-1, 0)$.

→ A horizontal line with an y -intercept at $(0, b)$ is given by $y = b$.

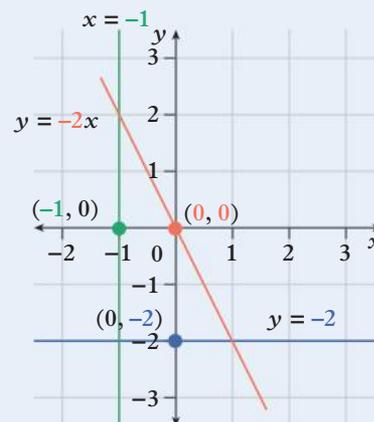
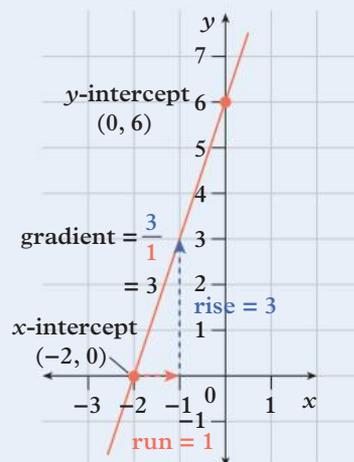
For example, the graph of the equation $y = -2$ is a horizontal line with a y -intercept at $(0, -2)$.

→ A line that passes through the origin $(0, 0)$ with gradient m is given by $y = mx$.

For example, the graph of the equation $y = -2x$ passes through the origin and has a gradient of -2 .



Key content video



Example 4F.1 Determining the equation of a linear graph given the gradient and the y -intercept



Determine the equation of a linear graph with a gradient of -3 and a y -intercept of $(0, 7)$.

THINK

Identify the values of gradient m and the y -coordinate of the y -intercept c , and then substitute into the gradient–intercept form, $y = mx + c$.

WRITE

$$m = -3$$

The y -intercept is $(0, c)$.

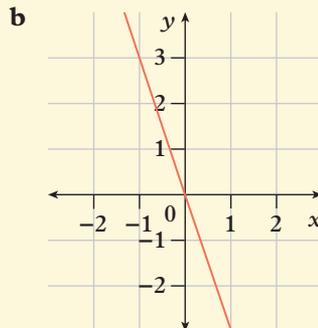
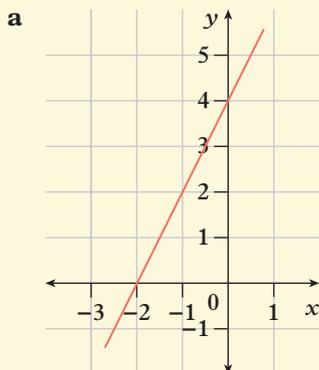
$$c = 7$$

$$y = -3x + 7$$

Example 4F.2 Determining the equation of a linear graph



Determine the equation of each of the following linear graphs.



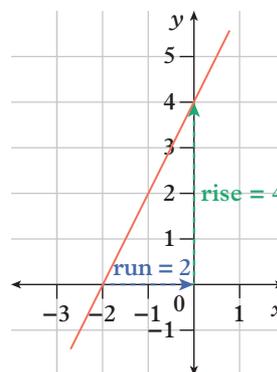
THINK

- a**
- Determine the value of the gradient, m , by identifying the rise and run between any two integer coordinates on the graph.
 - Determine the value of the constant, c , by identifying the y -coordinate of the y -intercept.
 - Substitute the values of m and c into the general equation for a straight line.
- b**
- Determine the value of the gradient, m , by identifying the rise and run between any two integer coordinates on the graph.

- Determine the value of the constant c by identifying the y -coordinate of the y -intercept.
- Substitute the values of m and c into the general equation for a straight line.

WRITE

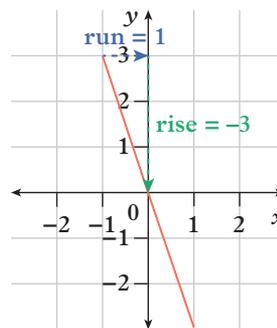
$$\begin{aligned} \mathbf{a} \quad m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{4}{2} \\ &= 2 \end{aligned}$$



y -intercept: $(0, 4)$

$$\begin{aligned} c &= 4 \\ y &= mx + c \\ y &= 2x + 4 \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{-3}{1} \\ &= -3 \end{aligned}$$



y -intercept: $(0, 0)$

$$\begin{aligned} c &= 0 \\ y &= mx + c \\ y &= -3x + 0 \\ y &= -3x \end{aligned}$$



Example 4F.3 Determining the equation of a linear graph given two points

Find the equation of the linear graph that passes through the points (6, 5) and (2, -3).

THINK

- 1 Calculate the value of the gradient, m .
- 2 Substitute the value of the gradient, m , into the general equation for a straight line, $y = mx + c$.
- 3 Select one of the two coordinates, substitute the values of x and y into the partially completed equation, and solve for c .
- 4 State the equation of the line.

WRITE

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5 - (-3)}{6 - 2}$$

$$= \frac{8}{4}$$

$$= 2$$

$$y = 2x + c$$

From the point (6, 5):

Let $x = 6$ and $y = 5$:

$$5 = 2(6) + c$$

$$5 = 12 + c \quad (-12)$$

$$c = -7$$

$$y = 2x - 7$$

Helpful hints

- ✓ The different methods for finding the equation of a line can be overwhelming, so remember that you just need to find the gradient m and the y -intercept $(0, c)$, which you can then substitute into the formula for a linear graph, $y = mx + c$.

ANS p455 Exercise 4F Determining linear equations

 1-2(a-d), 3, 4(1st column), 5, 6, 7(a-e), 9, 11(a, b), 13

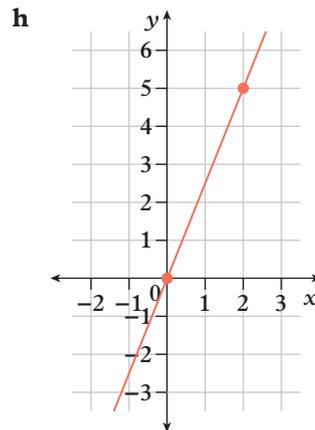
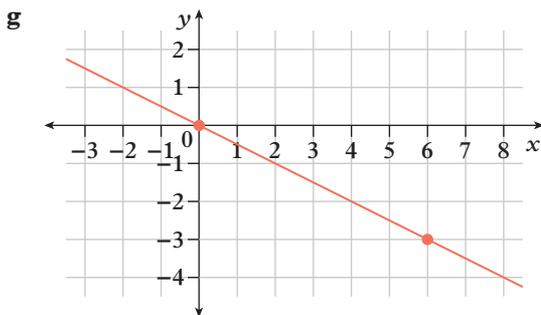
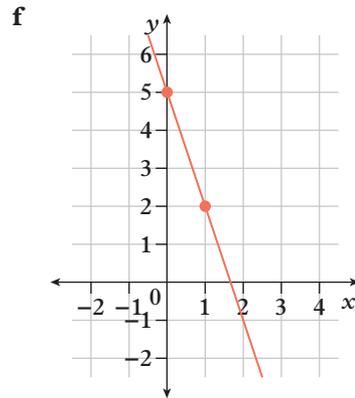
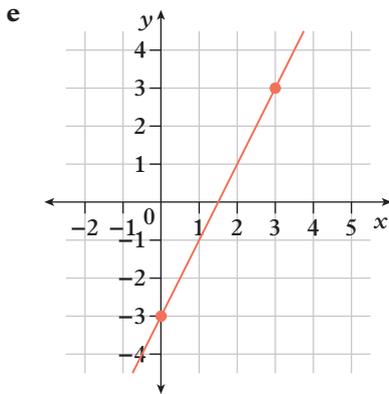
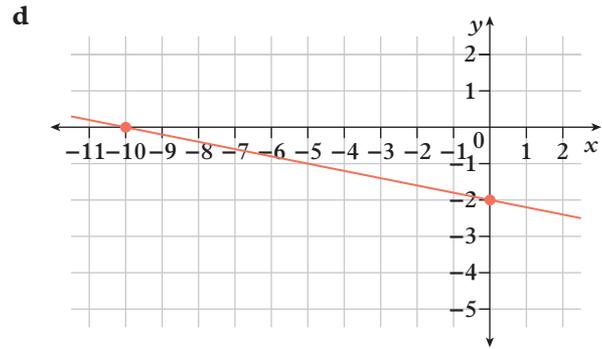
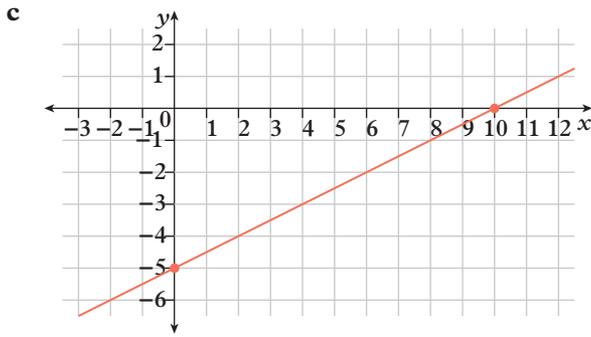
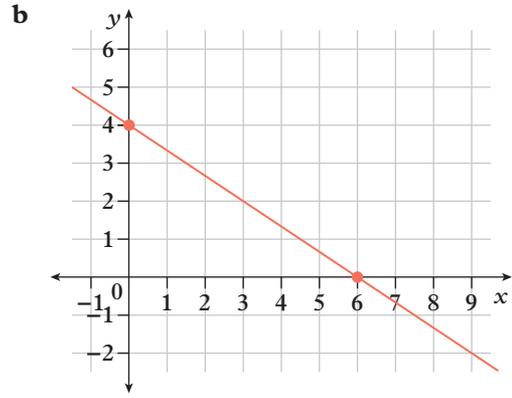
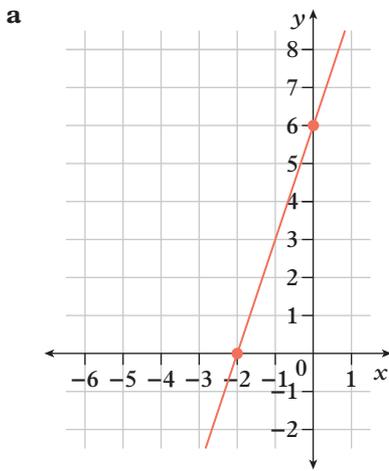
 1-2(e-h), 3(b, e, g, h), 4(b, d, f), 5, 6, 7(e, f), 9, 10, 11(a, b, d), 13, 14(a, b)

 2(2nd column), 3(b, d, f, h), 6(e-h), 7(b, d, f), 10, 11(c, d), 12-15

4F.1 1 Determine the equations of the lines in the following graphs in the form $y = mx + c$.

- a gradient: 3, y -intercept: (0, 4)
- b gradient: -2, y -intercept: (0, 10)
- c gradient: 1, y -intercept: (0, -7)
- d gradient: -12, y -intercept: (0, -1)
- e gradient: -1, y -intercept: (0, 20)
- f gradient: 5, y -intercept: (0, 0)
- g gradient: $\frac{1}{3}$, y -intercept: $(0, \frac{4}{5})$
- h gradient: $-\frac{3}{4}$, y -intercept: (0, 0)

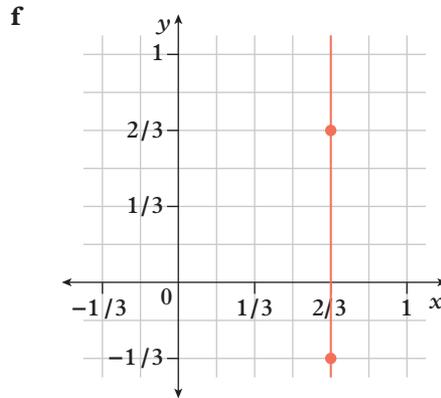
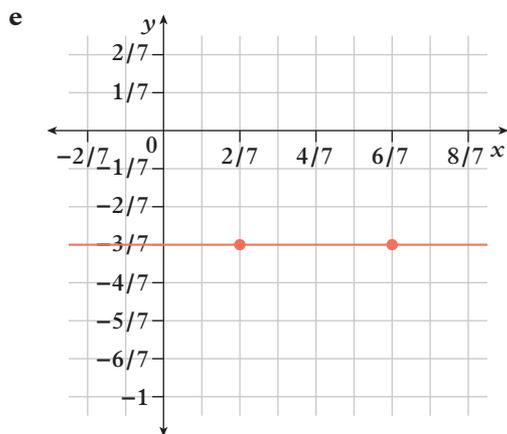
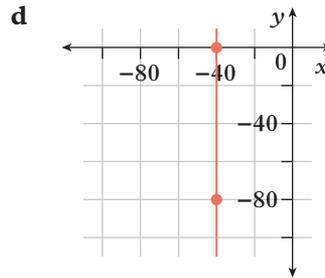
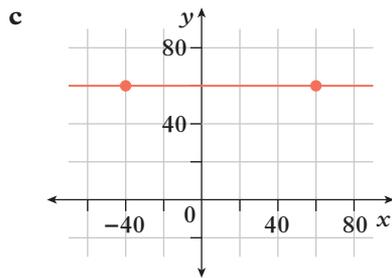
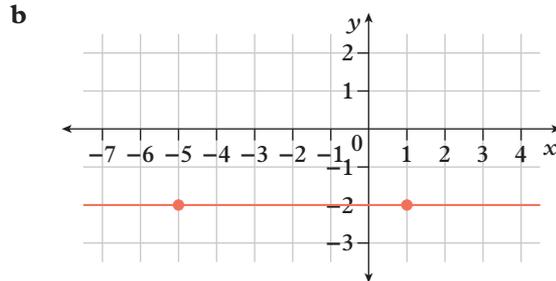
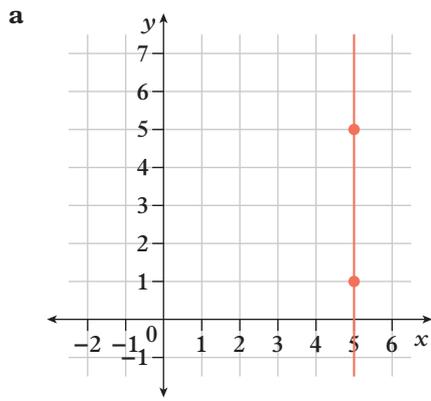
4F.2 2 Determine the equation of each of the following linear graphs.



4F.3 3 Find the equations of the linear graphs that pass through each pair of points.

- | | |
|-------------------------------|---------------------------------|
| a (1, -20) and (11, 0) | b (2, 4) and (3, 2) |
| c (-1, 6) and (3, 2) | d (-4, -12) and (-2, -6) |
| e (2, 5) and (1, 3) | f (9, -4) and (11, 6) |
| g (-1, -2) and (5, -5) | h (6, -3) and (9, 1) |

4 Find the equation of the linear graphs below. Recall that the general equation for a vertical line is $x = a$ and the general equation for a horizontal line is $y = b$, where a and b are constants.

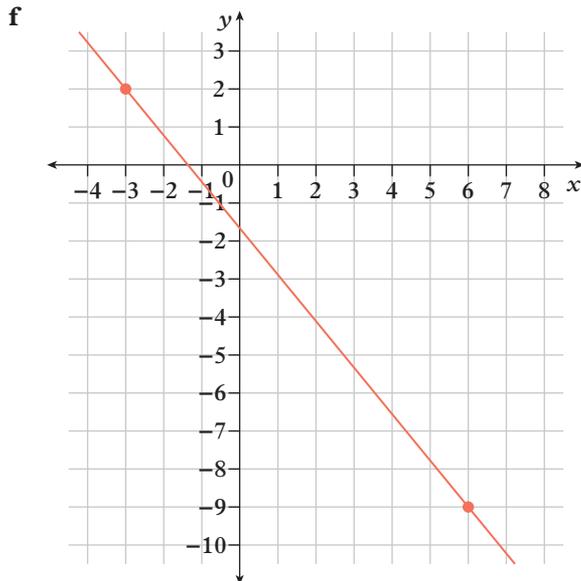
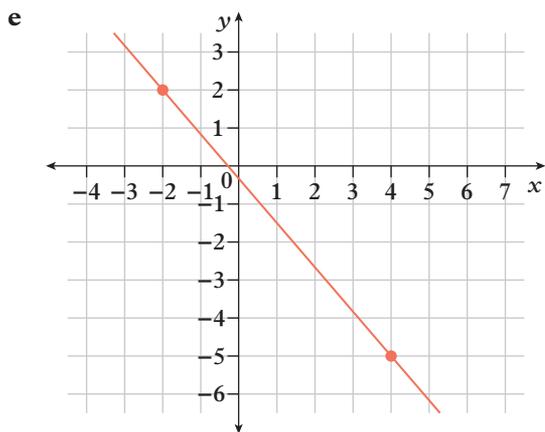
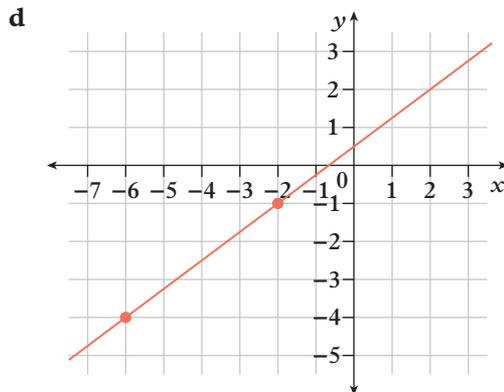
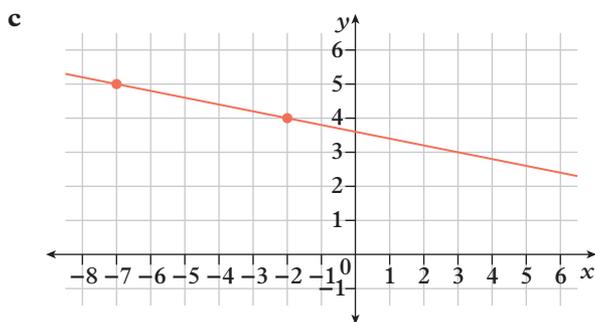
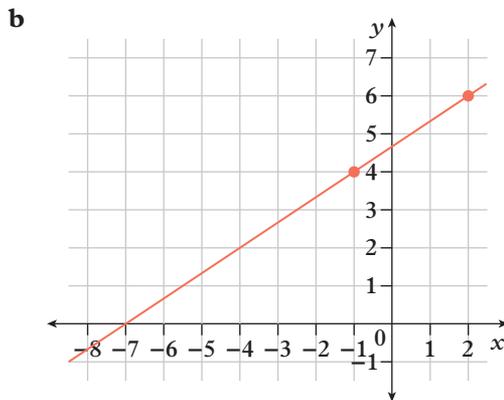
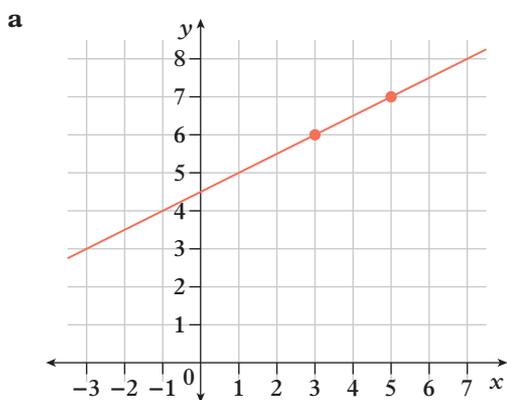


5 Find the equations of the x -axis and y -axis.

6 Determine the equations of the linear graphs that have the following intercepts.

- | | |
|---|--|
| a x -intercept: (4, 0), y -intercept: (0, 12) | b x -intercept: (-5, 0), y -intercept: (0, 10) |
| c x -intercept: (3, 0), y -intercept: (0, -21) | d x -intercept: (-6, 0), y -intercept: (0, -24) |
| e x -intercept: (-1, 0), y -intercept: (0, 10) | f x -intercept: (18, 0), y -intercept: (0, 6) |
| g x -intercept: (5, 0), y -intercept: (0, -2) | h x -intercept: (-6, 0), y -intercept: (0, 9) |

7 Identify two integer coordinates on each linear graph and use these to determine their equations.



- 8 Azami invests some money into a simple interest account. After two years the account has \$2500, and after five years the account has \$3400. Determine the equation that gives the amount in Azami's account, A , after n years.
- 9 Roland recently started a business and realised his profits were increasing at a constant rate. After seven days of being open, he had made a profit of \$450, and after 14 days, he had made \$870 in profit.
- a** Write a linear equation that gives the amount of profit, P , that Roland earns after n days.
Being overly optimistic about his launch, Roland uses his model to predict the profit he will earn one year (365 days) after opening.
- b** What is the predicted profit Roland will have earned exactly one year (365 days) after opening?

10 As water drips from the ceiling of a cave it creates a long crystalline formation that hangs from the ceiling called a stalactite. A specific stalactite is measured to be 85 centimetres long in 2010 and then 10 years later it is measured to be 87 centimetres long.

- a Determine a linear equation for the growth of the stalactite, g , in centimeters n years after 2010.
- b Use your equation to estimate the age of the stalactite in 2010. What year did it begin to form?

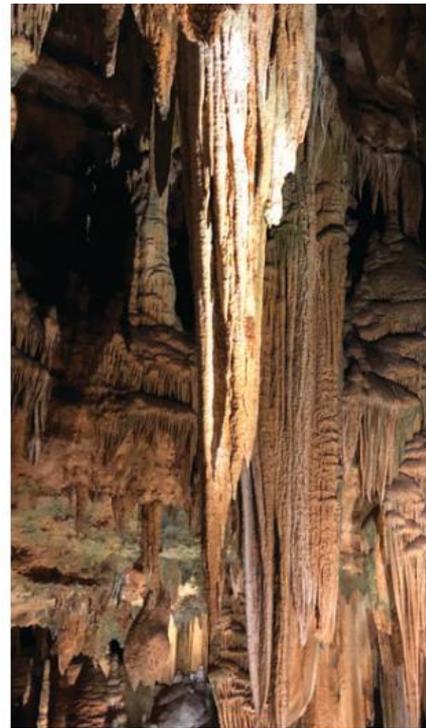
11 Determine the equation of the lines with the following tables of values.

a	x	-3	-2	-1	0	1	2	3
	y	-32	-23	-14	-5	4	13	22

b	x	-3	-1	1	3	5	7	9
	y	-2	1	4	7	10	13	16

c	x	10	20	30	40	50	60	70
	y	48	51	54	57	60	63	66

d	x	1	4	9	16	25	36	49
	y	28	22	12	-2	-20	-42	-68



12 A carpenter sells his wares based on the cost of the materials and the time spent crafting. A particular collection all cost the same in materials but take different amounts of time to craft. A piece that takes 16 hours to craft costs \$845 and a piece that takes 28 hours to craft costs \$1385. The carpenter then marks up the total cost by 120% to sell.

- a Write the equation that gives the total cost, $\$C$, for a piece in this collection that takes n hours to craft.
- b Write the equation that gives the revenue, $\$R$, for a piece in this collection that takes n hours to craft.
- c Write the equation that gives the profit, $\$P$, for a piece in this collection that takes n hours to craft.

13 a Factorise the right-hand side of each of the following equations.

b Find the x -intercept of each of the following equations.

c Describe the connection between the factor form in part a and the x -intercept in part b.

i $y = 3x + 12$

ii $y = -5x - 15$

iii $y = 2x - 10$

iv $y = 6x - 8$

v $y = 21 - 7x$

vi $y = 16x - 40$

14 Find the equations of the linear graphs that pass through each pair of points.

a $(\frac{2}{3}, 0)$ and $(0, \frac{7}{5})$

b $(-\frac{2}{5}, \frac{3}{4})$ and $(\frac{1}{5}, -\frac{1}{4})$

c $(-\frac{9}{2}, -\frac{7}{4})$ and $(-\frac{9}{8}, -\frac{5}{6})$

15 The product of the gradient, m , and y -intercept, c , for a particular line is 99. The sum of the gradient and y -intercept is 20.

- a Write the two possible equations for the line.
- b Find the coordinates of the point where the graphs of the two equations intersect.

Check your Student obook pro for these digital resources and more:

pro



Interactive skillsheet
Determining linear equations



Worksheet
Writing the rule for horizontal and vertical linear graphs



Investigation
The human body



Topic quiz
4F

4G Direct variation

Learning intentions

By the end of this topic you will be able to ...

- ✓ identify when two variables vary directly in an equation or graph
- ✓ solve problems involving direct variation.



Inter-year links

- Support** Multiplying and dividing whole numbers
- Year 7** 5E Line graphs
- Year 8** 3H Rates
- Year 10** 5I Direct and inverse variation

Direct variation

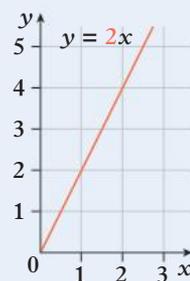
- Two variables x and y are said to **vary directly** when:
 - $x = 0, y = 0$
 - the **rate of change** of y with respect to x is constant.
- It is also said that y is directly proportional, or just proportional, to x .
- **Direct variation** (or **direct proportion**) is denoted using the symbol \propto .
- If $y \propto x$, then the equation for the relationship between x and y is $y = kx$, where k is the rate of change of y with respect to x and is called the **constant of variation**.
- The graph of $y = kx$ is a straight line that passes through the origin $(0, 0)$ and has a gradient of k .
For example, in the relationship represented by the graph to the right, the constant of variation is 2.

$$y = kx$$



Key content video

constant of variation
= rate of change
= gradient



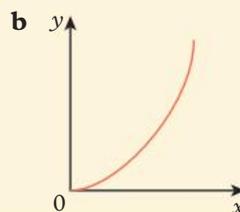
Example 4G.1 Identifying direct variation



Determine whether x and y vary directly in each of the following relationships. If the relationship is not directly proportional, provide a reason for your answer.

a

x	0	1	2	3	4
y	0	-7	-14	-21	-28



THINK

- a**
- 1 Check that $x = 0$ when $y = 0$.
 - 2 Check whether the rate of change is constant. To determine the rate of change from a table of values, calculate $\frac{y}{x}$ for each pair of coordinates excluding $(0, 0)$.
 - 3 State whether or not $y \propto x$.
- b**
- 1 Check that $x = 0$ when $y = 0$.
 - 2 Identify whether the gradient of the graph is constant.
 - 3 State whether or not $y \propto x$.

WRITE

- a** When $x = 0, y = 0$.

$$\frac{y}{x} = \frac{-7}{1} = -7$$

$$\frac{y}{x} = \frac{-14}{2} = -7$$

$$\frac{y}{x} = \frac{-21}{3} = -7$$

$$\frac{y}{x} = \frac{-28}{4} = -7$$

x and y vary directly.

- b** The graph passes through the origin $(0, 0)$.
The graph of y versus x is non-linear and, therefore, does not have a constant gradient.
 x and y do not vary directly.

**Example 4G.2** Finding the constant of variation

Find the constant of variation if $y \propto x$ using the given information, then write the equation for each directly proportional relationship.

a $y = 18$ when $x = 3$

b

x	0	2	4	6	8
y	0	24	48	72	96

**THINK**

- a 1** If $y \propto x$, $y = kx$. Substitute the given values into $y = kx$ and solve for k .
- 2** Substitute into the general equation $y = kx$.
- b 1** Select any pair of values in the table and substitute the corresponding values of x and y into $y = kx$, then solve for k .
- 2** Substitute into the general equation $y = kx$.
- c 1** As the graph passes through the origin, $(0, 0)$, to find the gradient, k , select any point on the graph, substitute the corresponding values of x and y into $y = kx$, then solve for k .
- 2** Substitute into the general equation $y = kx$.

WRITE

a $y = kx$

Let $x = 3$ and $y = 18$:

$$18 = k \times 3$$

$$18 = 3k \quad (\div 3)$$

$$k = 6$$

$$y = 6x$$

b

x	0	2	4	6	8
y	0	24	48	72	96

$$y = kx$$

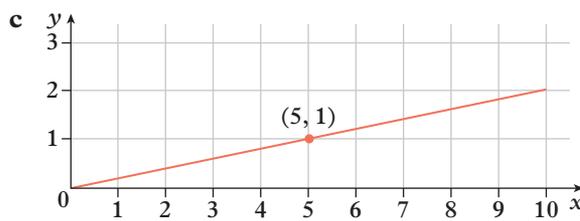
Let $x = 2$ and $y = 24$:

$$24 = k \times 2$$

$$24 = 2k \quad (\div 2)$$

$$k = 12$$

$$y = 12x$$



$$y = kx$$

Let $x = 5$ and $y = 1$:

$$1 = k \times 5$$

$$1 = 5k \quad (\div 5)$$

$$k = \frac{1}{5}$$

$$y = \frac{1}{5}x$$

**Example 4G.3** Solving problems involving direct variation

Jamie completes a 200 metre sprint in 40 seconds.

- a Assuming Jamie runs at a constant speed, sketch a graph of Jamie's distance, d , from the starting line against the time, t , for the duration of the sprint.
- b i Determine the gradient of the graph.
ii Express Jamie's average speed as a rate.
- c Determine the equation for the relationship between d and t .
- d How far had Jamie run after 10 seconds?

THINK

- a Plot the endpoints:
Start $t = 0$ s, Jamie is $d = 0$ m from the starting line: $(0, 0)$.
End $t = 40$ s, Jamie is $d = 200$ m from the starting line: $(40, 200)$.
- b i To find the gradient, select the points $(0, 0)$ and $(40, 200)$, and calculate $m = \frac{\text{rise}}{\text{run}}$.
ii The rate of change of d with respect to t is the same as the gradient of the graph of d vs. t . Jamie runs 5 metres per second.
- c The equation for direct proportion is $y = kx$, where k is the constant of proportionality.
- d Substitute time, $t = 10$ seconds, into the equation and solve for the distance, d .

WRITE

- a
- b i $m = \frac{\text{rise}}{\text{run}}$
 $= \frac{200}{40}$
 $= 5$
ii Speed = 5 m/s
- c $d = 5t$
- d Let $t = 10$
 $d = 5(10)$
 $= 50$ m

Helpful hints

- ✓ Direct variation is an example of linear algebra and linear graphs. Remember that for the relationship $y = kx$, $k = \text{constant of variation} = \text{rate of change} = \text{gradient}$.
- ✓ For a relationship to be directly proportional, the constant of variation k must be a constant, but it can still be fractional, irrational or negative!

Exercise 4G Direct variation

▲ 1-4, 6, 7, 10

■ 1-3, 5, 6, 8, 9, 11

◆ 1, 3, 5, 7, 11, 12

- 4G.1** 1 Determine whether x and y vary directly for each of the following relationships. If the relationship is not directly proportional, provide a reason for your answer.

a

x	0	1	2	3	4
y	0	4	8	12	16

b

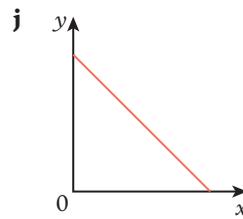
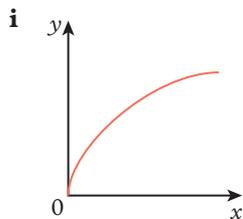
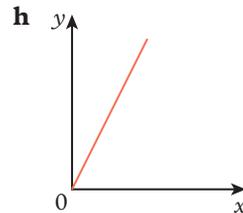
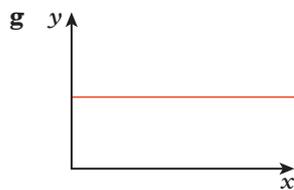
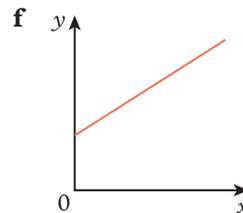
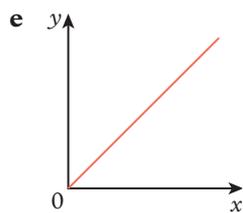
x	0	1	2	3	4
y	0	1	8	27	64

c

x	0	1	2	3	4
y	0	-9	-18	-27	-36

d

x	0	1	2	3	4
y	1	2	4	6	8



- 2 Identify and state the constant of variation for each relationship.

a $y = 4x$

b $m = -10n$

c $h = 3.5t$

d $y = \frac{x}{3}$

- 4G.2** 3 Find the constant of variation using the given information, then write the equation for each directly proportional relationship.

a $y = 50$ for $x = 5$

b $y = 63$ for $x = -7$

c $y = 16$ for $x = 64$

d $y = -12$ for $x = 18$

e

x	0	1	2	3	4
y	0	-6	-12	-18	-24

f

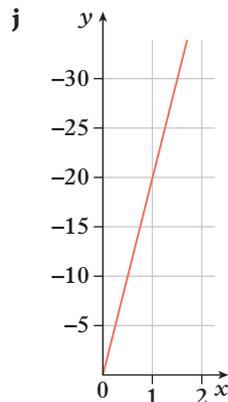
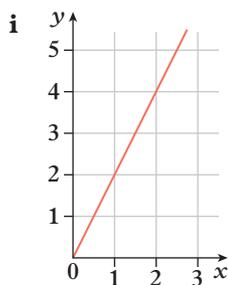
x	0	5	10	15	20
y	0	25	50	75	100

g

x	0	10	20	30	40
y	0	-5	-10	-15	-20

h

x	0	2	4	6	8
y	0	10	20	30	40



k a and b vary directly and $a = 2$ when $b = 7$

l c is directly proportional to d and $c = \sqrt{2}$ and $d = 5$

4G.3 **4** Julian earns \$24 an hour working at the local deli.

a i Sketch a graph of Julian's earnings, c , against t , the number of hours he works during an 8-hour shift.

ii Determine the gradient of the graph.

b Express Julian's hourly wage as a rate.

c Determine the equation for the relationship between c and t .

d If Julian has to leave halfway through the 8-hour shift, how much will he earn in total?

5 a Use the formula $C = 2\pi r$ to explain why the circumference of a circle is directly proportional to the radius of the circle and identify the constant of variation.

b Use the formula $A = \pi r^2$ to explain why the area of a circle is not directly proportional to the radius of the circle.

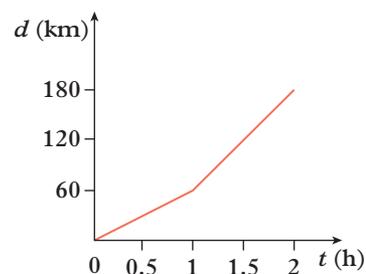
6 The graph on the right shows the distance travelled by a car over time.

a Describe the car's journey, including the speed at which the car is travelling at different times in the journey.

b Write an equation for the relationship between d and t during the first hour of the journey.

c Write an equation for the relationship between d and t during the second hour of the journey.

d Is $d \propto t$? Why or why not?



7 The cost, C dollars, of building a house is proportional to the area, A , of the floor space in square metres. It costs \$90 000 to build a house with a floor space of 150 m^2 .

a Given $C \propto A$, determine the constant of variation.

b Write an equation for the relationship between C and A .

c Use your answer from part **b** to calculate the floor space of a house that you could build with a budget of \$126 000.

8 The Australian dollar (AUD) and the US dollar (USD) are directly proportional to each other where \$1 USD is approximately equal to \$1.50 AUD. When converting between different currencies, the constant of variation is better known as the exchange rate.

a Identify the exchange rate and the equation to use when converting from USD to AUD.

b Draw the graph of the equation from part **a** for values on the horizontal axis from \$0 USD to \$100 USD.

c Use your graph to convert \$50 USD to AUD.

d What is the exchange rate to convert from AUD to USD?



- 9 A physiotherapist sees 160 patients every week.
- If the physiotherapist works 40 hours each week, on average, how many patients does the physiotherapist see per hour?
 - On average, how many minutes does the physiotherapist spend with each patient?
 - If the physiotherapist wishes to earn at least \$10 000 every week, what is the minimum he must charge each patient?
- 10 Jasmine lives 1200 m from school. She walks to school each day at a speed of 1 m/s. Nelson lives 4.5 km from school. He rides his bike to school each day at a speed of 15 m/s.



- If both Jasmine and Nelson arrive at school at 8.30 am, what time did each student leave home?
 - Jasmine and Nelson live 4.8 km apart. They plan to meet each other one morning and both leave their homes at 9.00 am. At what time will they meet?
- 11 Tom is riding in a cycling event. His distance from the start line at given times is recorded. The table shows values for t (number of hours) and d (distance from the start line in km).

t	0	1	4	9	16
d	0	20	40	60	80

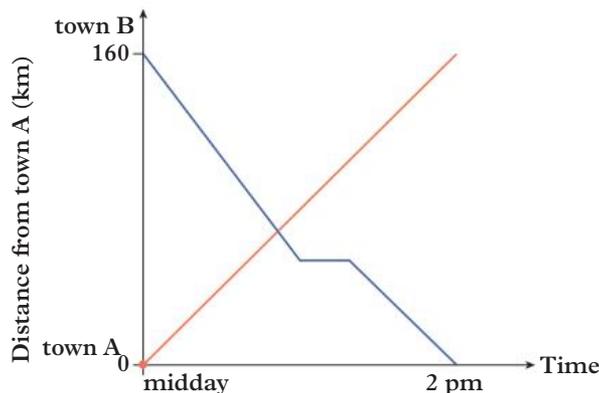
- Plot the points on a Cartesian plane and then join them with a smooth line.
- Is the relationship between t and d an example of direct variation? Explain.
- Complete this table.

\sqrt{t}					
d	0	20	40	60	80

- Plot the points on a Cartesian plane and join them with a smooth line.
- Is the relationship between \sqrt{t} and d an example of direct variation? Explain.
- Determine the constant of proportionality for the relationship between \sqrt{t} and d and hence write an equation for the relationship.
- What distance is Tom from the start line after 36 hours?



- 12 The graph below shows the journeys made by two friends between towns A and B. Both friends leave their homes at midday and arrive at their destinations 2 hours later.
- Helena travels at a constant speed from town A to town B. Julia travels at a constant speed for the first hour, then rests for 15 minutes before continuing on to town A at the same speed travelled as Helena.
- Determine how far each of them was from town A after travelling for half an hour.



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Interactive skillsheet
Direct variation



Investigation
How much gold is actually in a piece of gold jewellery?



Topic quiz
46

4H Midpoint and length of a line segment

Learning intentions

By the end of this topic you will be able to ...

- ✓ determine the midpoint of a line segment on the Cartesian plane
- ✓ calculate the length of a line segment on the Cartesian plane.



Inter-year links

Year 7

1G Indices and square roots

Year 8

7D Pythagoras' theorem

Year 10

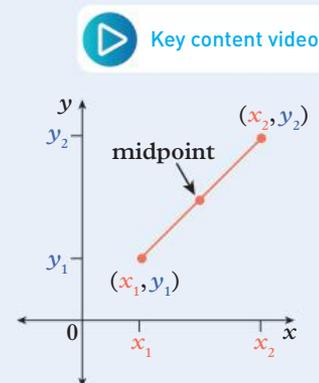
5B Gradient, midpoint and length of a line segment

Midpoint of a line segment

- The **midpoint** of a line segment is the point located exactly halfway between the endpoints of the line segment. If the coordinates of two endpoints of a line segment are (x_1, y_1) and (x_2, y_2) , then:

$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

- The x -coordinate of the midpoint is the average of the x -coordinates of the endpoints.
- The y -coordinate of the midpoint is the average of the y -coordinates of the endpoints.

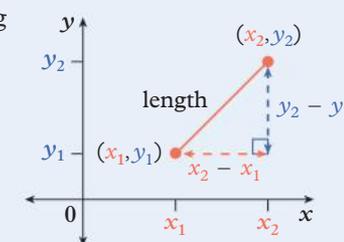


Key content video

Length of a line segment

- The formula for the length of a line segment can be determined by drawing an appropriate right-angled triangle and using Pythagoras' theorem. If the coordinates of two endpoints are (x_1, y_1) and (x_2, y_2) , then the two shorter side lengths of the triangle are $a = x_2 - x_1$ and $b = y_2 - y_1$. The distance, c , between the two points is the length of the hypotenuse, and can be found by taking the positive solution to the equation:

$$c^2 = a^2 + b^2$$



Example 4H.1 Determining the midpoint of a line segment



Find the coordinates of the midpoint of the line segment joining $(-1, 4)$ and $(7, 9)$.

THINK

- 1 Calculate the x -coordinate of the midpoint by averaging the x -coordinates of the endpoints.
- 2 Calculate the y -coordinate of the midpoint by averaging the y -coordinates of the endpoints.
- 3 Write the coordinates of the midpoint.

WRITE

$$x\text{-coordinate of the midpoint} = \frac{-1 + 7}{2} = 3$$

$$y\text{-coordinate of the midpoint} = \frac{4 + 9}{2} = \frac{13}{2}$$

$$\text{Coordinates of the midpoint: } \left(3, \frac{13}{2} \right)$$



Example 4H.2 Calculating the length of a line segment

Calculate the length of the line segment joining $(-1, 4)$ and $(7, 9)$, correct to one decimal place.

THINK

- 1 Define the points (x_1, y_1) and (x_2, y_2) . The order of the points does not affect the result.
- 2 Determine the lengths of the two shorter sides of the corresponding right-angled triangle.
- 3 Substitute the shorter side lengths into Pythagoras' theorem and solve for c .
- 4 Use a calculator to evaluate the root and round the answer to one decimal place as specified by the question.

WRITE

Let $(x_1, y_1) = (-1, 4)$ and $(x_2, y_2) = (7, 9)$

$$a = x_2 - x_1 \qquad b = y_2 - y_1$$

$$= 7 - (-1) \qquad = 9 - 4$$

$$= 8 \qquad = 5$$

$$c^2 = a^2 + b^2$$

$$c^2 = 8^2 + 5^2$$

$$= 64 + 25$$

$$= 89$$

$$c = \sqrt{89}$$

$$c \approx 9.4 \text{ units}$$

Helpful hints

- ✓ Take care when finding the horizontal or vertical distances between two points – watch carefully for any changes in sign!
- ✓ When finding the distance between two points, it may help to plot the two points on the Cartesian plane and identify a suitable right-angled triangle before applying Pythagoras' theorem.

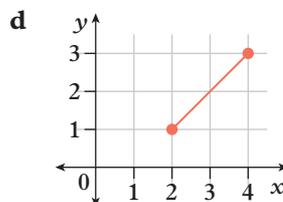
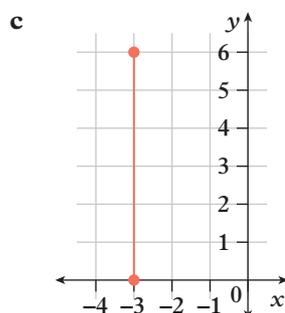
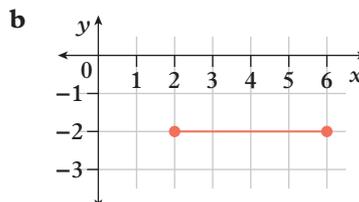
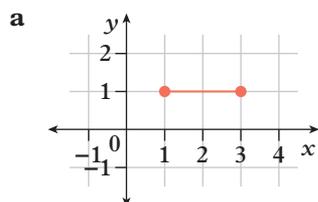
ANS p457 Exercise 4H Midpoint and length of a line segment

▲ 1, 2-3(1st column), 4-7, 9, 11

■ 2-3(2nd column),
4, 8-10, 12, 13(a), 15

⬠ 2-3(g-l), 4, 8, 10, 12, 13(b),
14, 16-18

- 1 State the coordinates of the midpoint of the following line segments.



4H.1 2 Find the coordinates of the midpoint of the line segment joining each pair of points.

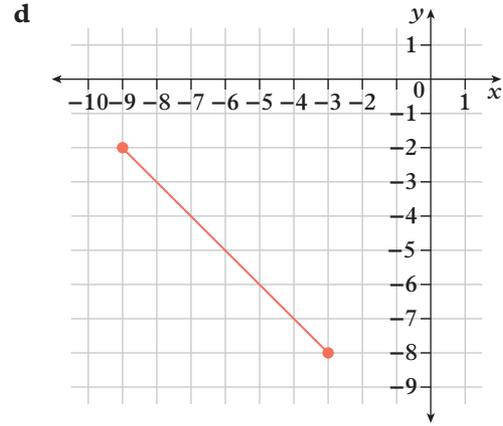
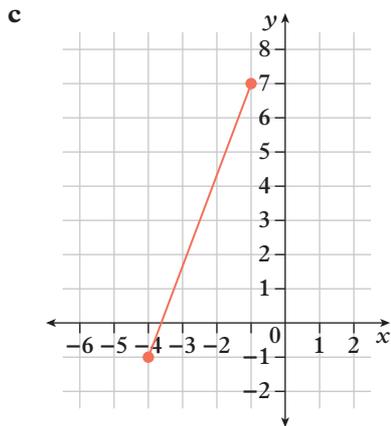
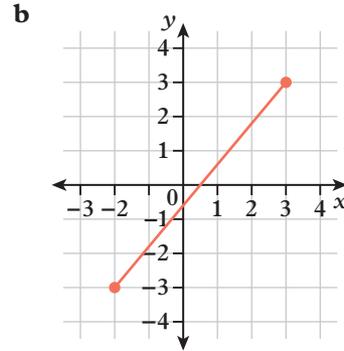
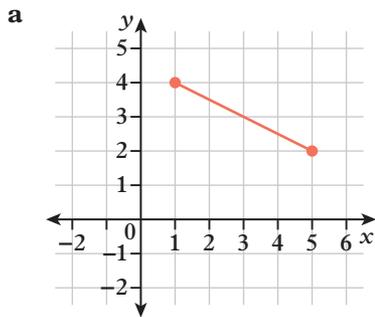
- | | |
|-------------------------------|-------------------------------|
| a (1, 4) and (3, 10) | b (2, 5) and (8, 3) |
| c (1, 0) and (5, 2) | d (2, 6) and (2, 10) |
| e (0, 5) and (8, 9) | f (3, -4) and (7, 6) |
| g (2, -1) and (6, 7) | h (-3, -4) and (5, -4) |
| i (-4, -2) and (-2, 2) | j (3, 9) and (4, 8) |
| k (5, 0) and (8, 11) | l (-5, 7) and (5, -7) |

4H.2 3 Calculate the length of the line segment joining each pair of points correct to one decimal place.

- | | |
|--------------------------------|------------------------------|
| a (2, 5) and (3, 7) | b (3, 4) and (5, 8) |
| c (6, 2) and (9, 3) | d (-4, 5) and (-4, 9) |
| e (2, -4) and (4, 2) | f (5, 0) and (8, -4) |
| g (0, -1) and (1, -2) | h (7, 8) and (-7, 8) |
| i (4, -3) and (6, 0) | j (-3, 6) and (-2, 2) |
| k (-5, -4) and (-1, -2) | l (6, -5) and (-6, 5) |

4 For each line segment, find:

- i** the midpoint **ii** the length (to one decimal place) **iii** the gradient.

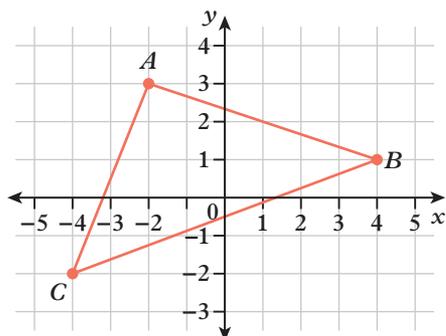


5 Calculate the distance between the midpoint and one of the endpoints of each of the line segments in question 3.

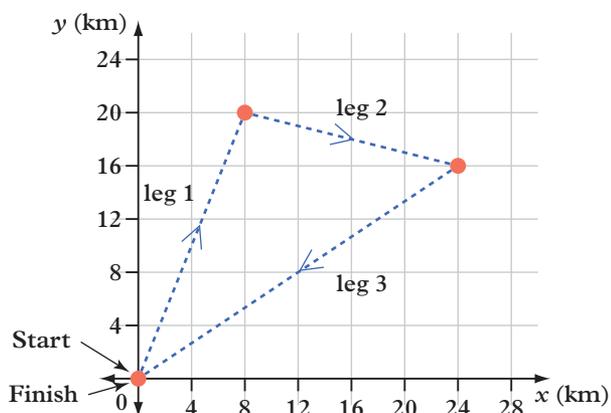
6 The midpoint of a line segment AB has the coordinates (6, 4). If point A has the coordinates (2, 3), find the coordinates of point B .

7 The midpoint of a line segment CD has the coordinates (-5, 1). If point D has the coordinates (4, -7), find the coordinates of point C .

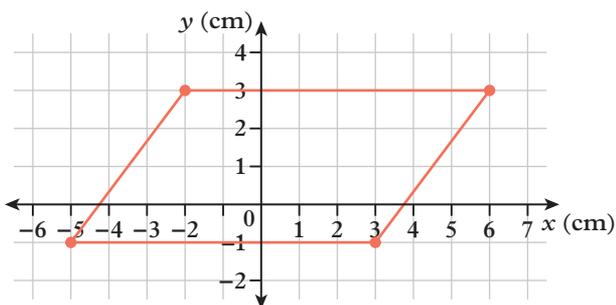
- 8 A point, A , on a circle has the coordinates $(-1, -1)$.
- If the centre of the circle is at $(2, 3)$, calculate the radius of the circle.
 - Identify the coordinates of another point on the circle that forms a diameter with point A .
- 9 Calculate the perimeter of $\triangle ABC$ correct to one decimal place.



- 10 Calculate the perimeter of each shape correct to one decimal place.
- triangle with vertices at $(-3, -2)$, $(-2, 4)$ and $(4, 2)$
 - square with vertices at $(-1, 2)$, $(2, 5)$, $(5, 2)$ and $(2, -1)$
 - rectangle with vertices at $(-4, -2)$, $(2, 4)$, $(4, 2)$ and $(-2, -4)$
 - trapezium with vertices at $(-3, 3)$, $(1, 5)$, $(3, 3)$ and $(2, -2)$
- 11 A yacht race follows a triangular course that has been mapped onto a Cartesian plane. The scales on the axes represent distances in kilometres. The race begins and ends at the origin.



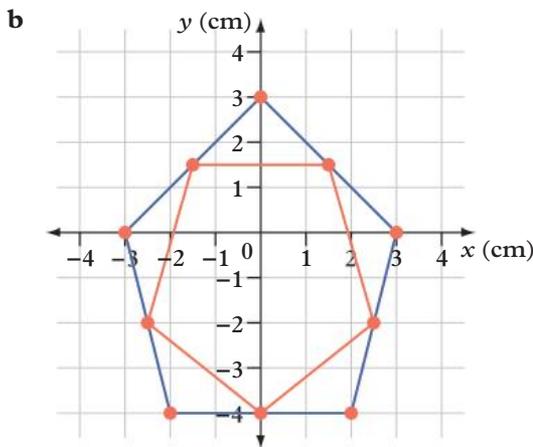
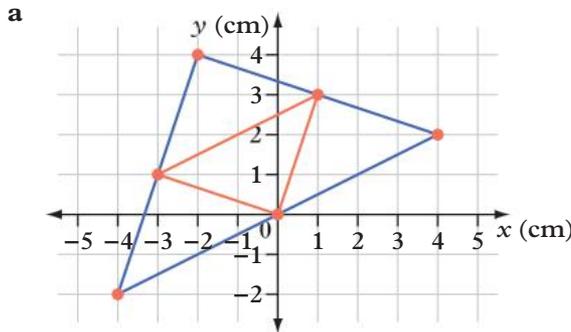
- Calculate the length of each leg of the race correct to one decimal place.
 - Calculate the total distance covered during the race correct to one decimal place.
 - An observer's boat is located close to the midpoint of the second leg of the race. Determine the distance between the observer's boat and the finishing point, correct to one decimal place.
- 12 Consider this parallelogram drawn on a Cartesian plane.
- List the coordinates of the vertices of the parallelogram.
 - Find the coordinates of the midpoint of:
 - the longer diagonal
 - the shorter diagonal.
 - What do you notice about your answers to part **b**?
 - Calculate the perimeter of the parallelogram.
 - Find the difference in length of the two diagonals correct to one decimal place.



- 13** A shape (in blue) is drawn on a Cartesian plane. A smaller shape (in orange) is then formed by joining the midpoints of the vertices of the original shape.

In each case, find the perimeter (correct to one decimal place) of:

- i** the blue shape
- ii** the orange shape.
- iii** Compare the perimeter of the orange shape to the perimeter of the blue shape.



- 14** A quadrilateral $ABCD$ has vertices at $A(-4, 2)$, $B(-1, 4)$, $C(3, -2)$ and $D(0, -4)$. The scales on the axes represent distances in metres.

- a** Determine the length of each side of the quadrilateral correct to one decimal place.
- b** Determine the coordinates of the midpoint of each diagonal.
- c** Determine the distance between each vertex and the midpoint correct to one decimal place.
- d** Use your answers for parts **a–c** to identify the shape of the quadrilateral $ABCD$. Explain your reasoning.
- e** Hence use the correct formula to determine the area of the quadrilateral, correct to one decimal place.

- 15 a** Prove that triangle ABC with vertices at $A(3, 6)$, $B(-1, -2)$ and $C(-5, 2)$ is an isosceles triangle. State any dimensions in simplified surd form.

- b** Calculate the area of the triangle using your knowledge of the midpoint and length of a line segment.

- 16** Prove that quadrilateral $ABCD$ with vertices at $A(-5, 3)$, $B(-1, 5)$, $C(3, -2)$ and $D(-1, -4)$ is a parallelogram. State any dimensions in simplified surd form.

- 17** A line segment has endpoints at $A(-1, -1)$ and $B(2, 5)$. If point C lies between A and B such that AC is $\frac{1}{3}$ the length of AB , determine the exact coordinates of C .

- 18** A line segment has a midpoint at the origin, a length of 10 units and a gradient of 2. Determine the coordinates of the two endpoints. Write your answer in simplified surd form. Hint: Sketch a diagram and then consider only one half of the line segment.

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Interactive skillsheet
Midpoint of a line segment



Interactive skillsheet
Length of a line segment



Investigation
From square to triangle



Topic quiz
4H

4I Parallel and perpendicular lines

Learning intentions

By the end of this topic you will be able to ...

- ✓ determine if two lines are parallel or perpendicular from their equations
- ✓ determine linear equations for parallel lines
- ✓ determine linear equations for perpendicular lines.



Inter-year links

Year 8 6H Intersecting lines

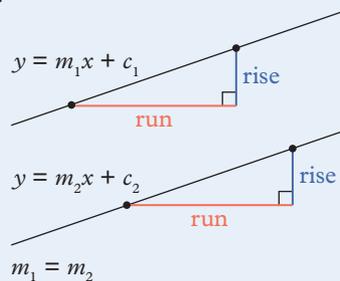
Year 10 5B Gradient, midpoint and length of a line segment

Parallel lines

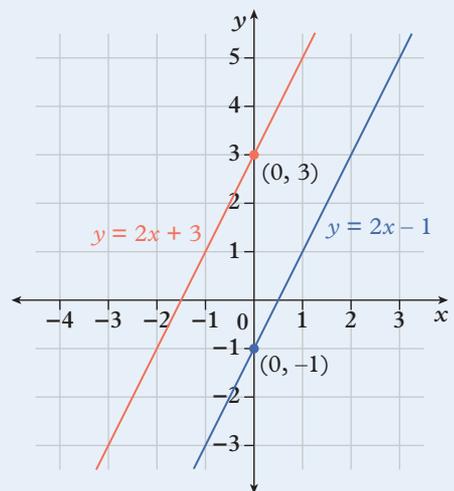
- **Parallel lines** are two or more straight lines that never meet.
- Two lines are parallel if they have the same gradient.

If the gradient–intercept equations for two lines are $y = m_1x + c_1$ and $y = m_2x + c_2$, the lines are parallel if $m_1 = m_2$.

For example, the lines $y = 2x + 3$ and $y = 2x - 1$ shown on the right are parallel.



Key content video

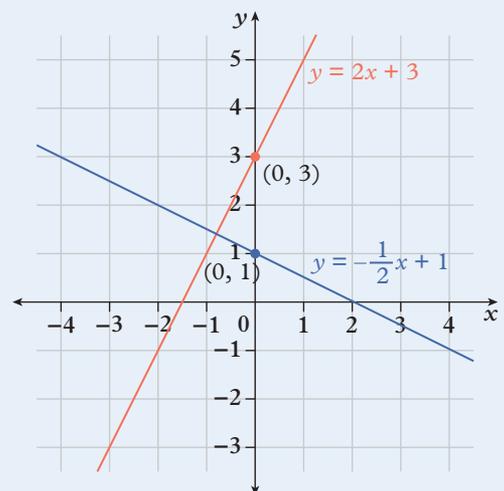
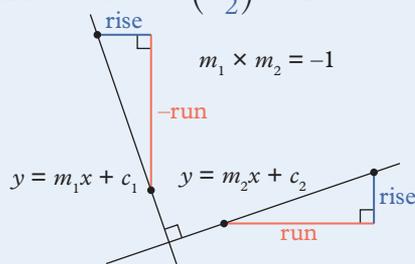


Perpendicular lines

- **Perpendicular lines** are straight lines that meet at right angles.
- Two lines are perpendicular if the product of their gradients is -1 or, expressed another way, if one gradient is the negative reciprocal of the other.

If the equations of two lines written in gradient–intercept form are $y = m_1x + c_1$ and $y = m_2x + c_2$, the lines are perpendicular if $m_1 \times m_2 = -1$ or, expressed another way, if $m_1 = -\frac{1}{m_2}$.

For example, the lines $y = 2x + 3$ and $y = -\frac{1}{2}x + 1$ are perpendicular because $2 \times \left(-\frac{1}{2}\right) = -1$.



Equations of parallel and perpendicular lines

- To determine the equation of a parallel or perpendicular line in the form $y = mx + c$:
 - determine the gradient of the given line, m_2
 - determine the gradient of the new line, m_1
 - If lines are parallel, then their gradients are equal: $m_1 = m_2$.
 - If lines are perpendicular, then their gradients are the negative reciprocal of each other: $m_1 = -\frac{1}{m_2}$.
 - identify the x - and y -coordinates of a point on the new line
 - substitute into the formula $y = mx + c$ to determine c .

Example 4I.1 Determining whether two lines are parallel or perpendicular



For each pair of equations, decide whether the lines are parallel, perpendicular or neither. Give reasons.

a $y - 2 = 5x$
 $y - 5x = 9$

b $1 - 4y = 2x + 3$
 $1 + 4x = 2y$

c $y = 3x - 2$
 $3x + y = -2$

THINK

- a**
- Rearrange $y - 2 = 5x$ to make y the subject and identify the gradient.
 - Rearrange $y - 5x = 9$ to make y the subject and identify the gradient.
 - State whether the lines are parallel, perpendicular or neither, by comparing the two gradients.
- b**
- Rearrange $1 - 4y = 2x + 3$ to make y the subject and identify the gradient.
 - Rearrange $1 + 4x = 2y$ to make y the subject and identify the gradient.
 - State whether the lines are parallel, perpendicular or neither, by comparing the two gradients.
- c**
- Identify the gradient of $y = 3x - 2$ by looking at the coefficient of x .
 - Rearrange $3x + y = -2$ to make y the subject and identify the gradient.
 - State whether the lines are parallel, perpendicular or neither, by comparing the two gradients.

WRITE

- a** $y - 2 = 5x$ (+ 2)
 $y = 5x + 2$
So the gradient of $y - 2 = 5x$ is 5.
 $y - 5x = 9$ (+ 5x)
 $y = 5x + 9$
So the gradient of $y - 5x = 9$ is 5.
The lines are parallel because their gradients are the same.
- b** $1 - 4y = 2x + 3$ (- 1)
 $-4y = 2x + 2$ ($\div -4$)
 $y = -\frac{1}{2}x - \frac{1}{2}$
So the gradient of $1 - 4y = 2x + 3$ is $-\frac{1}{2}$.
 $1 + 4x = 2y$
 $2y = 1 + 4x$ ($\div 2$)
 $y = 2x + \frac{1}{2}$
So the gradient of $1 + 4x = 2y$ is 2.
The lines are perpendicular because their gradients are the negative reciprocals of each other.
- c** The gradient of $y = 3x - 2$ is 3.
 $3x + y = -2$ (- 3x)
 $y = -3x - 2$
So the gradient of $3x + y = -2$ is -3 .
The lines are neither parallel nor perpendicular. Their gradients are not equal, nor are they the negative reciprocals of each other.

Example 41.2 Determining parallel lines



Determine the equation for a line that is parallel to the graph of $y = -7x - 5$ and passes through $(2, 6)$.

THINK

- 1 Identify the gradient of the graph of $y = -7x - 5$ and use this to define the gradient of the parallel line. Parallel lines have equal gradients.
- 2 Substitute the values for the gradient and the coordinates into the gradient–intercept equation: $y = mx + c$.
- 3 Solve the equation for c and write the final equation in gradient–intercept form.

WRITE

The gradient of $y = -7x - 5$ is -7 .

For the parallel line, use $m = -7$ with $x = 2$ and $y = 6$.

$$y = mx + c$$

$$6 = -7 \times 2 + c$$

$$6 = -14 + c \quad (+14)$$

$$c = 20$$

The equation is $y = -7x + 20$.

Example 41.3 Determining perpendicular lines



Determine the equation for a line that is perpendicular to the graph of $y = \frac{2}{3}x + 11$ and passes through $(10, -8)$.

THINK

- 1 Identify the gradient for the graph of $y = \frac{2}{3}x + 11$ and use this to define a gradient for the perpendicular line. Perpendicular lines have gradients that are the negative reciprocal of each other, $m_1 = -\frac{1}{m_2}$.
- 2 Substitute the values for the gradient and the coordinates into the gradient–intercept equation: $y = mx + c$.
- 3 Solve the equation for c and write the final equation in gradient–intercept form.

WRITE

The gradient of $y = \frac{2}{3}x + 11$ is $\frac{2}{3}$.

For the perpendicular line, use

$$m = -\frac{3}{2} \quad \left(\frac{2}{3} \times \left[-\frac{3}{2}\right] = -1\right)$$

Use $x = 10$ and $y = -8$.

$$y = mx + c$$

$$-8 = -\frac{3}{2} \times 10 + c$$

$$-8 = -15 + c \quad (+15)$$

$$c = 7$$

The equation is $y = -\frac{3}{2}x + 7$.

Helpful hints

- ✓ If y is already the subject of the equation, you do not need to rearrange the equation to the form $y = mx + c$ to identify the gradient, m . The coefficient of x will be the gradient.
For example, the gradient of $y = 1 - 4x$ is -4 and the gradient of $y = \frac{1 + 2x}{3}$ is $\frac{2}{3}$.
- ✓ Remember, if you know the y -intercept, $(0, c)$, all you need to do is determine the gradient, m , and substitute both values into the gradient–intercept equation $y = mx + c$.

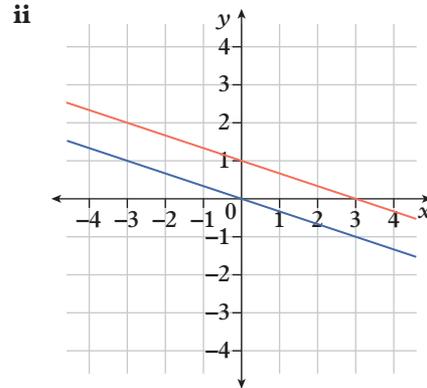
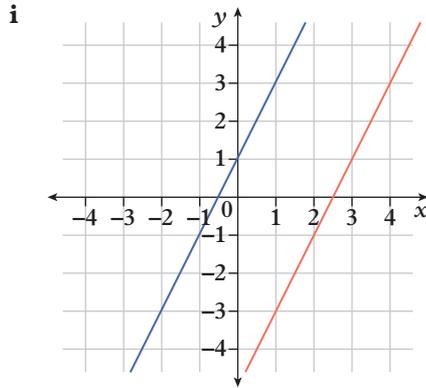
Exercise 4I Parallel and perpendicular lines

△ 1-3, 4(a, c, e, g), 5, 6(a, c, e, g), 7, 8(a, c, e, g), 9-11, 13, 16

■ 4, 5, 6(a, c, e, g), 7, 8(a, c, e, g), 9, 12, 17, 19, 21

⬠ 5, 6(g, h), 7, 8(g, h), 9, 14, 15, 18, 20, 22-26

1 a Calculate the gradients for the linear graphs on each of these Cartesian planes.



b What do you notice about both pairs of lines?

c What can you say about the gradients of parallel lines?

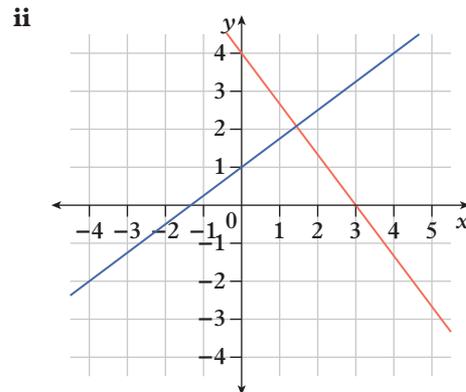
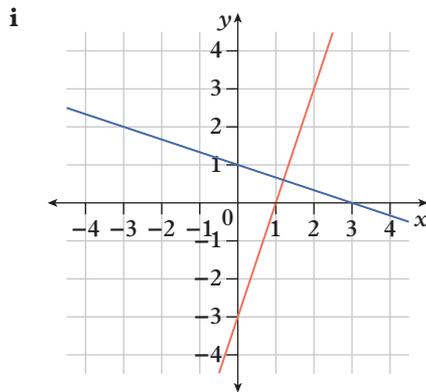
d Decide whether each equation below describes a linear graph parallel to those in part **a i**.

i $y = 2x - 9$

ii $y = 5 + 2x$

iii $y = 3x + 2$

2 a Calculate the gradients for the linear graphs on each of these Cartesian planes.



b Find the product of the gradients for each pair of lines in part **a**.

c What do you notice about both pairs of lines?

d What can you say about the product of the gradients of perpendicular lines?

e Write an equation for determining the gradient m_2 of a line that is perpendicular to another line with gradient m_1 .

f Decide whether each of the following equations describes a linear graph that is perpendicular to those in question 1 part **a ii**.

i $y = 3x - 4$

ii $y = \frac{1}{3}x + 4$

iii $y = 9 - \frac{1}{3}x$

3 If each of the numbers below is the gradient of a line, then find the gradient of the corresponding perpendicular lines.

a 5

b -8

c -1

d $\frac{1}{6}$

e $\frac{4}{5}$

f $-\frac{3}{2}$

4 For each linear equation below:

- i** rearrange the equation so that it is in gradient–intercept form, $y = mx + c$
- ii** write the gradient of a line parallel to the line described by the given equation
- iii** write the gradient of a line perpendicular to the line described by the given equation.

a $3x + y = 11$

b $-4x + y = 7$

c $-\frac{5}{3}x + y = \frac{2}{3}$

d $3x + 2y = 8$

e $5x - 2y = 4$

f $8x + 5y = -11$

g $9y - 3x = 15$

h $x = -5y - 3$

i $\frac{y-2}{x+5} = 1$

4.1.1 5 Decide whether each of these pairs of equations describes lines that are parallel, perpendicular or neither. Give a reason for your answer.

a $y = 4x - 3$ and $4x - y = 5$

b $3y - 2x = 12$ and $3x + 2y = 8$

c $y = \frac{3}{8}x$ and $3y = 4 - 8x$

d $2x + y = 3$ and $y = 2x + 6$

e $5x + y + 4 = 0$ and $x + 5y = 7$

f $2x - 7y = 11$ and $14y = 4x + 1$

g $8x - 12y = 7$ and $18y + 12x = -11$

h $6x + 3y = 7$ and $20y - 10x = 4$

4.1.2 6 Determine the equation for the line that is:

a parallel to the graph of $y = 4x + 3$ and passes through $(2, -5)$

b parallel to the graph of $y = -3x + 7$ and passes through $(-1, -3)$

c parallel to the graph of $y = -x - 6$ and passes through $(-4, 2)$

d parallel to the graph of $y = \frac{1}{2}x + 9$ and passes through $(6, 0)$

e parallel to the graph of $y = -\frac{1}{3}x - 8$ and passes through $(15, 0)$

f parallel to the graph of $y = \frac{2}{7}x + 5$ and passes through $(0, 11)$

g parallel to the graph of $y = \frac{3}{2} - \frac{5}{4}x$ and passes through $(0, -5)$

h parallel to the graph of $y = \frac{5}{6}x - \frac{9}{4}$ and passes through $(-9, 12)$.

7 Rearrange each of these equations to help you determine the equation for a line that is:

a parallel to the graph of $8x + 2y = 10$ and passes through $(5, -2)$

b parallel to the graph of $5x + 10y = 30$ and passes through $(-8, 1)$

c parallel to the graph of $7x - 3y = 5$ and passes through $(0, 6)$

d parallel to the graph of $40y - 24x = 6$ and passes through $(5, 0)$.

4.1.3 8 Determine the equation for the line that is:

a perpendicular to the graph of $y = 5x + 9$ and passes through $(5, -8)$

b perpendicular to the graph of $y = -7x - 3$ and passes through $(0, 3)$

c perpendicular to the graph of $y = x + 4$ and passes through $(-1, -2)$

d perpendicular to the graph of $y = \frac{2}{3}x + 1$ and passes through $(-4, 4)$

e perpendicular to the graph of $y = \frac{4}{9}x - 7$ and passes through $(0, 8)$

f perpendicular to the graph of $y = -\frac{1}{3}x + 2$ and passes through $(-10, 0)$

g perpendicular to the graph of $y = \frac{14}{3} - \frac{5}{6}x$ and passes through $(9, 0)$

h perpendicular to the graph of $y = \frac{4}{7}x - \frac{8}{21}$ and passes through $(-28, 3)$.

9 Rearrange each of these equations to help you determine the equation for a line that is:

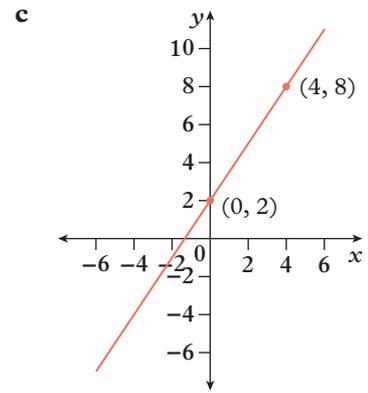
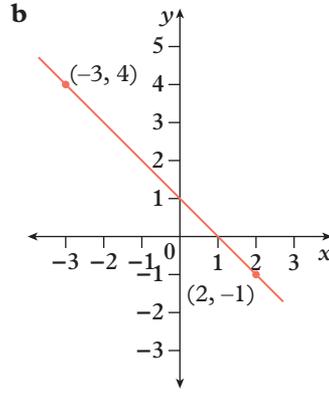
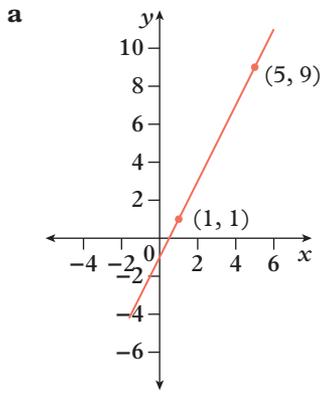
a perpendicular to the graph of $15x + 3y = 12$ and passes through $(-10, 7)$

b perpendicular to the graph of $12x - 36y = 72$ and passes through $(-9, -6)$

c perpendicular to the graph of $9x + 7y = 13$ and passes through $(0, 47)$

d perpendicular to the graph of $45y - 18x = 63$ and passes through $(-40, 0)$.

10 Write the equation for a line that is perpendicular to each line shown and has a y -intercept of 3.



11 For each line shown in question 10, write the equation for a line that has a y -intercept of -4 and is:

- i** parallel to the given line
- ii** perpendicular to the given line.

12 **a** Sketch each pair of graphs on the same Cartesian plane.

- i** $2x + 3y = 12$ and $2x - 3y = 12$
- ii** $2x + 3y = 12$ and $\frac{1}{2}x - \frac{1}{3}y = 12$
- iii** $2x + 3y = 12$ and $2x - 3y = \frac{1}{12}$
- iv** $5x + 2y = 20$ and $2x - 5y = 20$
- v** $5x + 2y = 20$ and $-2x + 5y = 20$
- vi** $5x + 2y = 20$ and $2x + 5y = -20$

b Determine which pairs of graphs are perpendicular.

c Write the equation of a line perpendicular to $ax + by = 1$ in the same form and in terms of a and b .

13 **a** Sketch the graphs of $x = 2$ and $y = 4$ on the same Cartesian plane.

b Explain why each of the following explanations is either correct or incorrect.

i Aamira says the graphs are perpendicular because they meet at a right angle.

ii Jack says the graphs are not perpendicular because $\frac{0}{1} \times \frac{1}{0} = 1 \neq -1$.

c Complete each of the following sentences.

i Vertical lines are perpendicular to horizontal lines because they meet at a _____ degree angle.

ii Vertical lines are perpendicular to horizontal lines, but we cannot use the formula $_ \times _ = _$ because the gradient of a vertical line is _____.

iii Vertical lines are perpendicular to horizontal lines, so the equations _____ and _____ are perpendicular.

14 Show that the line joining $(2, -3)$ and $(4, 5)$ on the Cartesian plane is parallel to the graph of $y = 4x - 7$.

15 Show that the line joining $(-11, -7)$ and $(-1, -2)$ on the Cartesian plane is perpendicular to the graph of $y = -2x + 5$.

16 A line passes through two points, $(-2, 3)$ and $(7, 9)$.

a Find the gradient of the line.

b Write the equation for a linear graph that is:

- i** parallel to the line and passes through $(6, -1)$
- ii** perpendicular to the line and passes through $(-4, 8)$.

17 A line segment joins $(-6, 7)$ and $(0, -11)$. Write the equation for a linear graph that is:

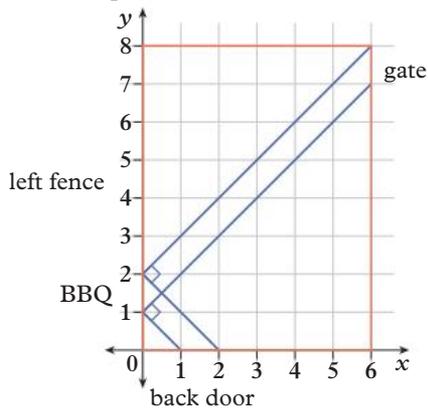
a parallel to this line segment and passes through the origin

b perpendicular to this line segment and passes through $(-6, 7)$

c perpendicular to this line segment and passes through its midpoint.

Hint: Use $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ to find the coordinates of the midpoint.

- 18 In his backyard, Husayn is planning to add a concrete path that goes from the back door to the barbecue on the left fence, then continues to the gate in the right corner. He wants the path to make a right turn at the barbecue. The path is planned to be 1 m wide and the back door is 1 m away from the left fence. Determine the equations of the four lines that mark the edge of the proposed path.



- 19 Four points are plotted on a Cartesian plane: $A(-2, 1)$, $B(5, 4)$, $C(-6, -4)$ and $D(8, 2)$.
- Is line segment AB parallel to line segment CD ?
 - Is line segment AC perpendicular to line segment BD ?
- 20 The following three points are plotted on a Cartesian plane: $A(-2, 3)$, $B(1, 12)$ and $C(7, 10)$.
- Show that line segments drawn between these points form a right-angled triangle with the points A , B and C as vertices.
 - Is the triangle ABC an isosceles right-angled triangle? Show your reasoning.
 - Calculate the perimeter of the triangle, correct to one decimal place.
 - Calculate the area of the triangle, correct to one decimal place.
- 21 Use your knowledge of parallel lines and Pythagoras' theorem to:
- show that quadrilateral $ABCD$, with vertices at $A(1, 4)$, $B(2, 6)$, $C(4, 7)$ and $D(3, 5)$, is a rhombus
 - show that quadrilateral $EFGH$, with vertices at $E(1, 3)$, $F(8, 6)$, $G(4, -2)$ and $H(-3, -5)$, is a parallelogram.
- 22 Prove that quadrilateral $KLMN$, with vertices at $K(0, 0)$, $L(2, 4)$, $M(6, 2)$ and $N(4, -2)$, is a square. Then find its perimeter and area, correct to one decimal place.
- 23 Prove that quadrilateral $PQRS$, with vertices at $P(-7, 5)$, $Q(-3, 1)$, $R(0, 4)$ and $S(-4, 8)$, is a rectangle. Then find its perimeter and area, correct to one decimal place.
- 24 A perpendicular bisector is the line that passes through the midpoint of a line segment between two points at a right angle. Write the equation for the perpendicular bisector of the line segment joining $(-2, 3)$ and $(8, -7)$.
- 25 Prove that quadrilateral $WXYZ$, with vertices at $W(-9, -11)$, $X(-1, 8)$, $Y(6, 9)$ and $Z(7, 2)$, is a kite. Hint: Also consider the diagonals of the quadrilateral.
- 26 Without plotting points on a Cartesian plane, what type of quadrilateral is $ABCD$ if its vertices are $A(2, 1)$, $B(8, 4)$, $C(9, 1)$ and $D(-3, -5)$? Show your reasoning, using your knowledge of parallel and perpendicular lines.

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Interactive skillsheet
Parallel and perpendicular lines



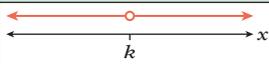
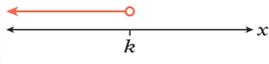
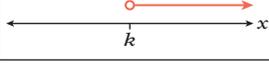
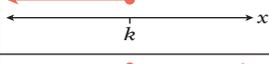
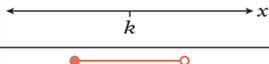
Worksheet
Working with linear relationships



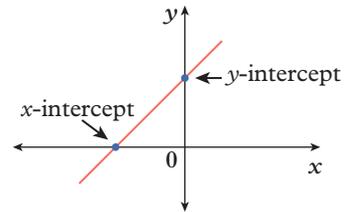
Topic quiz
41

Chapter summary

Linear inequalities

Inequality	Number line representation	Meaning
$x \neq k$		x is not equal to k
$x < k$		x is less than k
$x > k$		x is greater than k
$x \leq k$		x is less than or equal to k
$x \geq k$		x is greater than or equal to k
$k \leq x < m$		x is greater than or equal to k but less than m

Intercepts



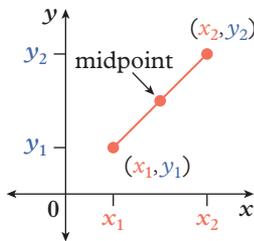
Gradient-intercept form

$$y = mx + c$$

↑ gradient ↑ y-coordinate of y-intercept

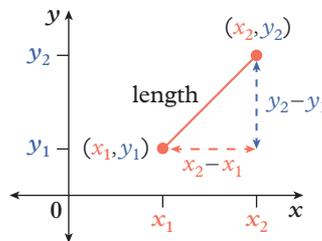
Midpoint

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$



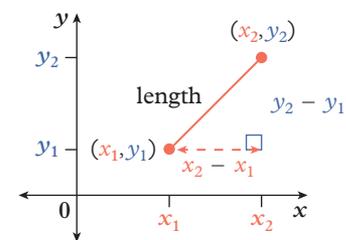
Length

$$\text{Length} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Gradient

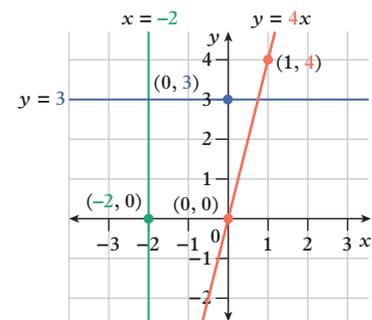
$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$



Linear graphs with one intercept

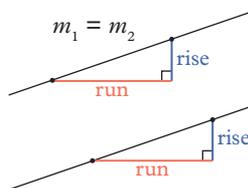
Description	General equation	Intercept	Gradient
Vertical lines	$x = a$	x-intercept: $(a, 0)$	undefined
Horizontal lines	$y = b$	x-intercept: $(0, b)$	0
Lines that pass through the origin	$y = mx$	origin: $(0, 0)$	m

where a , b and m are constants.



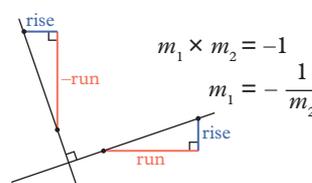
Parallel lines

- Their gradients are equal.



Perpendicular lines

- The product of their gradients is -1 .



Direct variation

- when $x = 0$, $y = 0$
- the rate of change of y with respect to x is constant

$$y \propto x \rightarrow y = kx$$

↑
 = rate of change
 = gradient

Chapter review



Chapter review quiz

Assess your knowledge of this chapter.

- ✓ Core
- ✓ Standard
- ✓ Advanced

Quizlet

Test your knowledge of this topic by working individually or in teams.

Mathematical literacy review

The following key terms are used in this chapter:

- Cartesian plane
- constant
- constant of variation
- coordinate points
- direct variation
- gradient
- gradient–intercept form
- inequality
- inverse operations
- linear equation
- linear inequality
- linear graph
- linear relationship
- midpoint
- number line
- parallel
- perpendicular
- rate of change
- solution
- variable
- vary directly
- x -intercept
- y -intercept

- 1 Complete the following sentences using words from the list of key terms.
 - a A linear _____ is a mathematical statement that compares two unequal expressions. The result can be plotted on a _____.
 - b A _____ relationship is a relationship between two variables that produces a straight line when drawn on the _____.
 - c A linear _____ is a mathematical statement that compares two equal expressions. It contains a _____ that has a power of 1 and is not multiplied by another variable.
- 2 Given two linear equations in gradient–intercept form, $y = mx + b$, what do the two values of m in each equation tell us? State True or False for the following statements.
 - a They indicate if the lines are parallel to each other.
 - b They indicate if the lines are perpendicular.
 - c They indicate what the y -intercepts are.
 - d They indicate what the x -intercepts are.
 - e They indicate the rate of change.
 - f They indicate if there is a direct variation between the variables.
 - g They indicate if the line is horizontal.
- 3 Explain the difference between the following terms using one or both of the following equations as an example.

Equation 1: $y = 3x + 1$ Equation 2 involving cost and time: $C = 6t$

 - a The *constant* in the gradient–intercept form of a linear equation and the *constant of variation*.
 - b A *linear relationship* and a *direct variation*.

Multiple choice

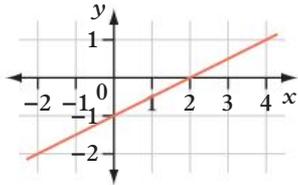
- 4A 1 The solution to the equation $3x - 9 = 12$ is:

A $x = 1$	B $x = 7$	C $x = 12$	D $x = 13$	E $x = 63$
-----------	-----------	------------	------------	------------
- 4A 2 The value of x in the equation $5x - 2 = 3x + 7$ is:

A $\frac{5}{8}$	B $1\frac{1}{8}$	C $2\frac{1}{2}$	D $4\frac{1}{2}$	E $5\frac{1}{2}$
-----------------	------------------	------------------	------------------	------------------
- ADV 4B 3 What is the solution to $2x - 1 < 5$?

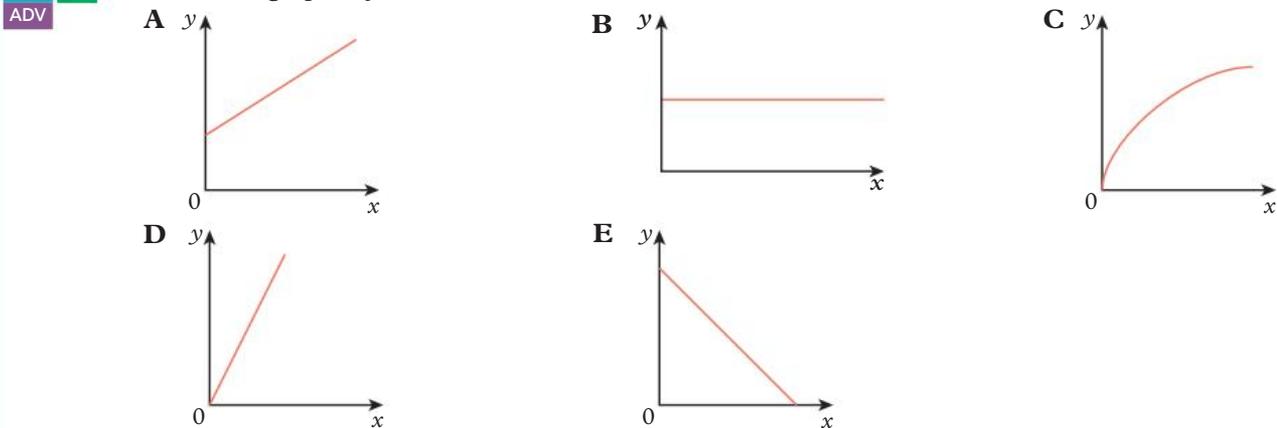
A $x > 3$	B $x < 3$	C $x > 2$	D $x < 2$	E $x < 6$
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- ADV 4B** 4 Which of the following values of x satisfies the inequality $60 < 2x + 12$?
A 36 **B** 24 **C** 12 **D** 6 **E** 0
- 4C** 5 Which point does *not* lie on the graph of $y = 2x - 4$?
A $(0, -4)$ **B** $(1, -2)$ **C** $(4, 4)$ **D** $(-1, -2)$ **E** $(-2, -8)$
- 4D** 6 The gradient of the linear graph shown to the right is:
A -2 **B** $-\frac{1}{2}$ **C** $\frac{1}{2}$ **D** 1 **E** 2



- 4D** 7 The gradient of the line segment joining the points $(-2, 4)$ and $(7, -4)$ is:
A $\frac{8}{9}$ **B** $-\frac{8}{9}$ **C** $\frac{9}{8}$ **D** $-\frac{9}{8}$ **E** 0
- 4D** 8 A line with a gradient of 0:
A is vertical **B** is horizontal **C** is undefined
D increases from left to right **E** decreases from left to right.
- 4E** 9 At the point where a line crosses the x -axis:
A $x = y$ **B** $y = -x$ **C** $y = 1$ **D** $x = 0$ **E** $y = 0$
- 4E** 10 The y -intercept for the graph of $2y = 3x - 4$ is:
A $(0, -4)$ **B** $(0, -2)$ **C** $(0, 0)$ **D** $(0, 2)$ **E** $(0, 4)$
- 4E** 11 In a linear graph, $y = mx + c$, the pronumeral c represents the:
A y -intercept **B** x -intercept **C** rise **D** run **E** gradient
- 4F** 12 A linear graph has a gradient of 3 and a y -intercept of $(0, -2)$. The equation of the graph is:
A $y = -2x + 3$ **B** $y = \frac{1}{3}x - 2$ **C** $y = 3x - 2$ **D** $y = -3x + 2$ **E** $3x - 2y = 1$

- STN 4G** 13 For which graph is $y \propto x$?



- 4H** 14 The coordinates of the midpoint of the line segment joining $(3, 5)$ and $(7, 9)$ are:
A $(5, 7)$ **B** $(7, 5)$ **C** $(2, 2)$ **D** $(4, 8)$ **E** $(1, 2)$
- 4H** 15 The length of the line segment joining $(3, 5)$ and $(7, 9)$ is closest to:
A 4 **B** 5 **C** 6 **D** 7 **E** 8
- 4I** 16 A line parallel to the line $3x + 2y = 6$ would have a gradient of:
A $\frac{2}{3}$ **B** -3 **C** $-\frac{3}{2}$ **D** $\frac{3}{2}$ **E** $-\frac{2}{3}$
- 4I** 17 A line has a gradient of $-\frac{3}{4}$. A line perpendicular to this line would have a gradient of:
A 3 **B** $-\frac{3}{4}$ **C** $\frac{3}{4}$ **D** $-\frac{4}{3}$ **E** $\frac{4}{3}$

Short answer

4A 1 Solve each equation using inverse operations.

a $3x - 4 = 2$

b $2 = 4 - 3x$

c $4(x - 3) = 3$

d $\frac{x}{3} - 2 = -1$

4A 2 Solve each equation for x .

a $7x - 3 = 4x + 9$

b $x + 8 = 1 - 6x$

4A 3 One more than three-fifths of the class is 16 people.

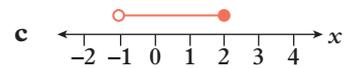
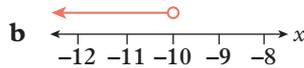
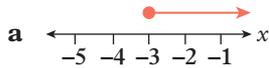
a Define a pronumeral to represent the unknown quantity in this problem.

b Use this pronumeral to write an equation to represent the problem.

c Solve the equation.

d How many people are in the class?

ADV 4B 4 Write the inequality that is represented on each of these number lines.



ADV 4B 5 Solve each of these inequalities.

a $5 - 2x < 7$

b $2 - x > 10 + 2x$

4C 6 Complete a table of values from $x = -2$ to $x = 2$ for each linear relationship, and then plot the relationship between x and y .

a $y = x + 5$

b $y = x - 5$

c $y = -x + 5$

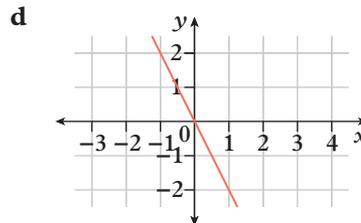
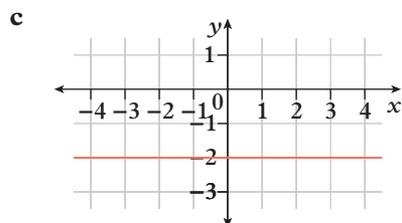
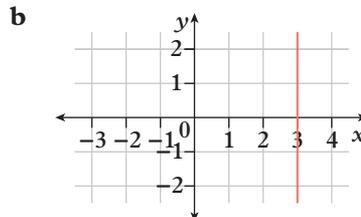
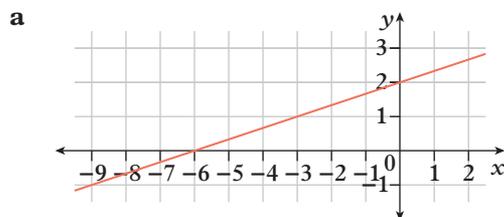
d $y = -x - 5$

4D 7 For each linear graph shown, determine the:

i gradient

ii x -intercept

iii y -intercept.



4D 8 Find the gradient of the line segment joining each pair of points.

a $(2, 3)$ and $(-2, -3)$

b $(-3, -2)$ and $(-2, -3)$

4E 9 For the graph of each linear relationship below, determine the coordinates of the:

i x -intercept

ii y -intercept.

a $2x + 3y = 18$

b $3x - y = 6$

c $y = 4x - 2$

d $2y = 5x - 3$

e $x - 2y = 4$

f $-y = 4 - x$

4E 10 Use your answers from question 9 to sketch each linear graph.

4E 11 Use the most appropriate method to sketch the graph of each linear relationship.

a $3x + 2y = 4$

b $4 - 2x = 3y$

c $y = \frac{1}{3}x$

d $y = 5 - \frac{2}{3}x$

e $y = 5$

f $2x + 3y + 6 = 0$

g $y = \frac{3}{4}x - 2$

h $x = -7$

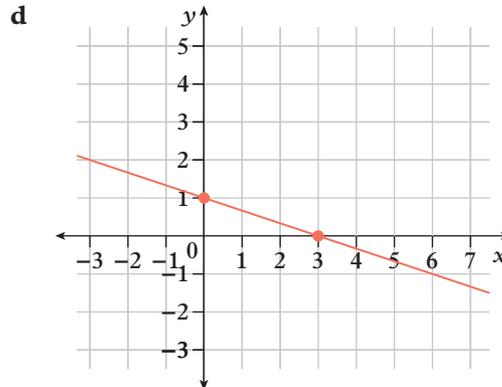
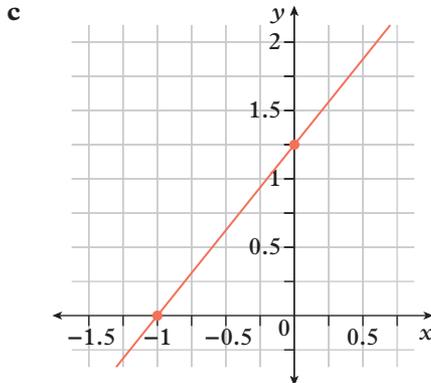
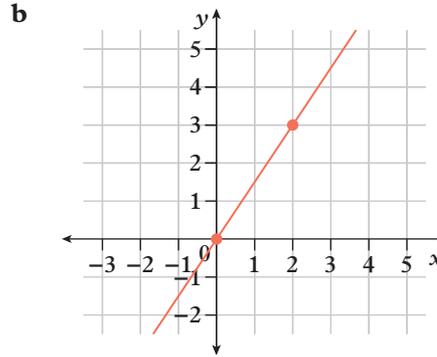
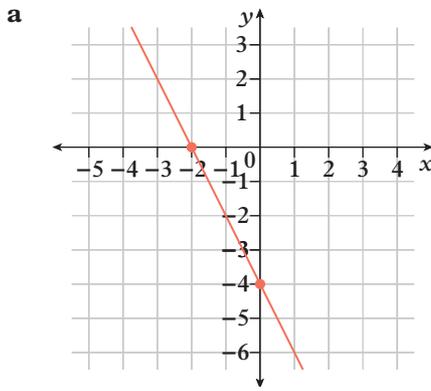
4F 12 Write the equations of the linear graphs with the following properties.

a gradient: 4, y -intercept: $(0, -2)$

b gradient: $\frac{1}{4}$, y -intercept: $(0, 0)$

c gradient: 0, y -intercept: $(0, -\frac{1}{2})$

4F 13 Find the equation of each of the following linear graphs.



STN 4G 14 Determine whether x is directly proportional to y in each of the following relationships. If the relationship is not directly proportional, provide a reason for your answer.

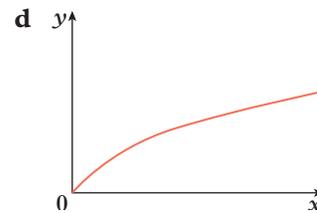
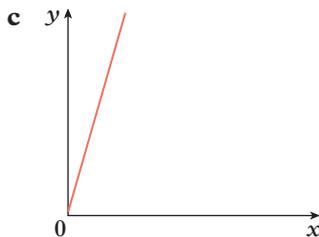
ADV

a

x	0	1	2	3	4
y	0	1	4	9	16

b

x	0	3	6	9	12
y	0	-1.5	-3	-4.5	-6



4H 15 For the line segment that joins each of the following pairs of points, find the:

i midpoint

ii length (correct to one decimal place).

a (2, 3) and (8, 7)

b (-4, 6) and (-3, 5)

c (2, -4) and (-2, 4)

d (-3, -1) and (2, -9)

4H 16 Determine the equations of the linear graphs that pass through each pair of points.

a (-5, 2) and (1, -4)

b (100, 39) and (-47, 39)

c (-3, 4) and (6, -8)

d (7, 12) and (-3, 4)

4I 17 Write an equation for a line that is:

a parallel to the graph of $y = -4x + 5$; with y -intercept of -3

b perpendicular to the graph of $y = 2x - 7$; with y -intercept of -4 .

4I 18 Write an equation for a line that is:

a parallel to the graph of $y = 3x + 2$ and passes through (3, 2)

b perpendicular to the graph of $y = 3x + 2$ and passes through (-3, -2).

Analysis

- 1 For a hot food stall, Sophie buys and then sells sausage rolls and party pies. The sausage rolls cost 71 cents each and the party pies cost 26 cents each. Sophie sells the sausage rolls for \$1.20 each and the party pies for 50 cents each. Sophie has a budget of \$60 per day. Let s be the number of sausage rolls Sophie buys and p be the number of party pies Sophie buys.

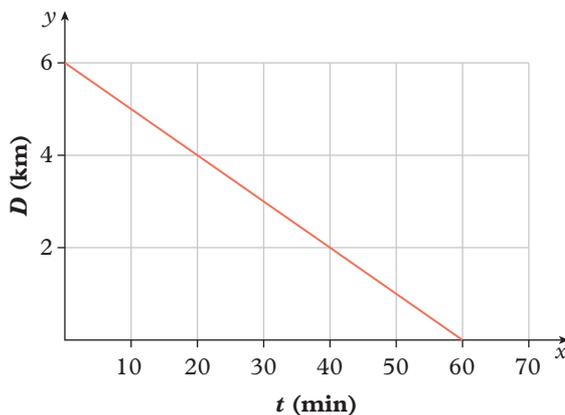
- Write an inequality that describes how Sophie can spend her budget each day.
- If Sophie buys 50 sausage rolls, write an inequality that describes the number of party pies she can buy.
- If Sophie buys 150 party pies, write an inequality that describes the number of sausage rolls she can buy.
- Assume Sophie is able to sell all the sausage rolls and party pies she buys on a particular day. Let P be the profit Sophie will make that day.

Write an equation that describes the profit Sophie will make.

- On another day, Sophie made \$50 profit.
Sketch the graph of the number of sausage rolls against the number of party pies Sophie could have bought and sold. Write the coordinates of the intercepts, correct to two decimal places. Hint: Put sausage rolls on the vertical axis.
- State the maximum number of party pies Sophie could have bought and sold on the day she made \$50 profit.
- On the day Sophie made \$50 profit, the price she paid for the sausage rolls and party pies was \$59.90. Determine the number of sausage rolls and party pies Sophie bought and sold that day.

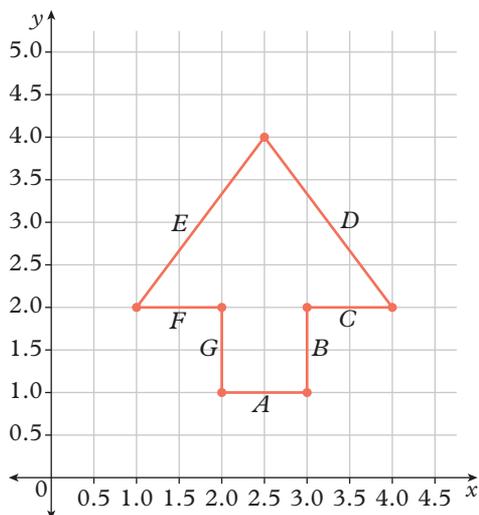


- 2 David decides to visit his friend Steve, who lives 6 km away. It takes him 60 min to get there if he walks quickly. This can be shown in the graph of distance, D , in kilometres from Steve's house and time, t , in minutes.



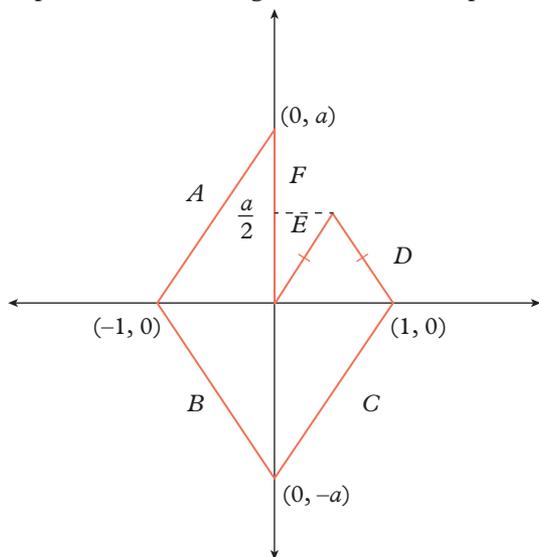
- Determine the gradient of the graph.
- Express David's average speed for walking as a rate in km per minute.
- Determine the equation for David's journey in gradient–intercept form using the variables D and t .
- Steve decides to meet David halfway. What is this distance from his home?
- Steve leaves home on a skateboard at the same time as David leaves and gets to the halfway point after 20 minutes. Plot this point on the graph and draw in line segments to show Steve's journey and him waiting to meet David.
- How long did Steve have to wait to meet David?
- Express Steve's average speed when riding his skateboard as a rate in km per minute.
- Determine the equation for Steve's journey in gradient–intercept form using the variables D and t .

- 3 a The shape below is made from seven line segments. Determine the linear equation and an inequality showing the values of x or y involved to produce each line segment in the following table.



Line segment	Linear equation	x - or y -values involved
A	$y = 1$	$2 \leq x \leq 3$
B		$1 \leq y \leq \underline{\quad}$
C		
D		$2.5 \leq x \leq 4$
E		$1 \leq x \leq 2.5$
F		
G		

- b Complete the table of linear equations and corresponding inequalities showing the values of x or y involved to produce the line segments for this shape.



Line segment	Linear equation	x - or y -values involved
A		
B		
C		
D		
E		
F		

- c Which lines are parallel in part b)?
- d What value of a will make lines A and B perpendicular in part b)?
- 4 A rectangle $DEFG$ has vertices at $D(-2, -1)$, $E(0, 1)$, $F(3, -2)$ and $G(1, -4)$.
- a Draw the rectangle on the Cartesian plane.
- b Calculate the lengths of all the sides of the rectangle, correct to one decimal place.
- c Using your sketch, identify the coordinates of the y -intercepts of the line segments:
- i \overline{DE} ii \overline{EF} iii \overline{DG} iv \overline{FG}
- d Find the gradients of the lines through:
- i \overline{DE} ii \overline{EF} iii \overline{DG} iv \overline{FG}
- e Point P is the midpoint of \overline{DE} , point Q is the midpoint of \overline{EF} , point R is the midpoint of \overline{FG} and point S is the midpoint of \overline{DG} . Find the coordinates of:
- i P ii Q iii R iv S
- f Describe the shape of the figure $PQRS$. Justify the statements you make.
- g If the original figure $DEFG$ had been a square instead of a rectangle, explain how this would affect the shape of $PQRS$. Support your answer with mathematical evidence.

Semester 1 review

Short answer

- Ben works 32 hours per week and is paid \$24 per hour. He takes two weeks of annual leave, for which he receives an additional leave loading of 17.5%. How much does Ben get paid for his two weeks of annual leave?
- What is the total payable for each of the following purchasing options?
 - \$150 deposit plus 15 payments of \$45
 - \$500 deposit plus 24 payments of \$175
 - \$799 deposit plus eight payments of \$399
 - \$1295 deposit plus 12 payments of \$547
- \$4500 is invested at a simple interest rate of 8% p.a. How much interest is earned after nine years?
 - \$150 is borrowed and \$35 interest is charged after two years. At what annual simple interest rate was the \$150 borrowed, correct to two decimal places?
 - \$5 interest is earned on an investment after two weeks with a simple interest rate of 6% p.a. How much was invested, correct to the nearest cent?
 - \$100 000 is borrowed at a simple interest rate of 0.6% per month. After how many months is \$1200 of interest charged?
- Round the following as specified in the brackets.
 - 20.040 194 (three decimal places)
 - 20.040 194 (three significant figures)
 - 0.000 782 718 (four decimal places)
 - 0.000 782 718 (four significant figures)
- Solve the following equations for x .
 - $12x - 7 = 4$
 - $9 - 2x = 15$
 - $4x + 9 = 6x - 7$
 - $\frac{21}{x} = 3$
- Plot the graphs of the following equations by completing each table of values and plotting the coordinates.
 - $y = 3x - 4$

x	0	1	2	3	4
y					
 - $y = -\frac{5}{2}x$

x	-4	-2	0	2	4
y					
- Write the following numbers in index form (as the product of powers of prime factors).
 - 81
 - 500
 - 216
 - 2700
- Write the following as basic numerals.
 - 7^3
 - 3×5^3
 - 8.213×10^4
 - 1.0530×10^{-2}
 - $2^{-3} \times 3^3$
 - $\left(\frac{5}{7}\right)^{-2}$
- Write the following in index form with positive indices.
 - $(2^3 \times 7^2)^4 \times (2^9 \times 7^5)^2$
 - $\frac{11^{12} \times 13^9 \times 17^6}{11^4 \times 13^9 \times 17^{15}}$
 - $101^{-5} \times 97^5 \times 101^{-7} \times 97^{-9}$
 - $\left(\frac{41^{-3}}{57^5}\right)^{-8} \times \left(\frac{57^{-4}}{41^{-7}}\right)^6$
- Simplify the following. Write your answers using positive indices.
 - $(ab^3)^4 \times (ba^6)^2$
 - $\frac{c^2de^8}{c^{12}de^7}$
 - $5g^2 \times 7h^3 \times 2g^5 \times h^4$
 - $(j^2 + k^3)^0$
- State the number of significant figures in the following.
 - 946 025
 - 120 000 000
 - 0.002 520 20
 - 630.00
- Write the following numbers in scientific notation.
 - 751 425 963, correct to four significant figures
 - 25.360 141, correct to five significant figures
 - 0.000 085 945 6, correct to two significant figures
 - 75 000, correct to three significant figures

13 Expand the following products.

a $8(7 - 2x)$

b $-2x^3(3x^2 - 8)$

c $(x + 2)(x - 7)$

d $(5x - 2)(2y + 3)$

e $5 - 4(3x + 7)$

f $2x(x + 9) + 3(x^2 - 4x + 2)$

14 Determine the x - and y -intercepts for the graphs of each of the following equations.

a $y = 3x - 6$

b $5x + 8y = -60$

c $y = 7 - 2x$

d $2x + y = 17$

15 Write each of these expressions as a single fraction in simplest form.

a $\frac{x^3}{5} \times \frac{x^2}{4}$

b $\frac{3x^2}{2} \div \frac{x}{6} \times \frac{y}{3}$

c $-\frac{4p}{5} + \frac{3p}{9}$

d $\frac{4t}{5} - \frac{3t}{2} - \frac{7t}{3}$

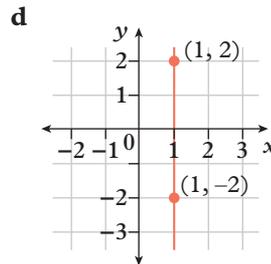
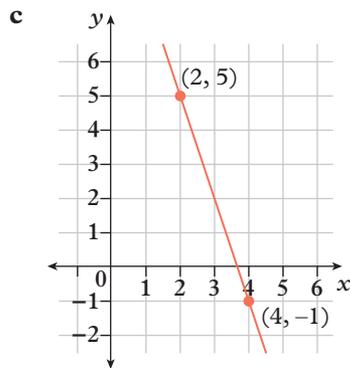
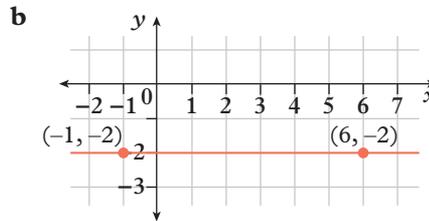
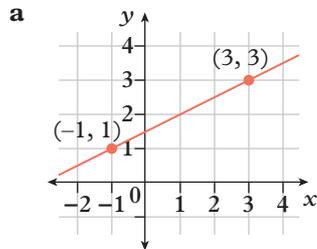
16 Sketch the graphs of the following equations.

a $2x + 3y = 12$

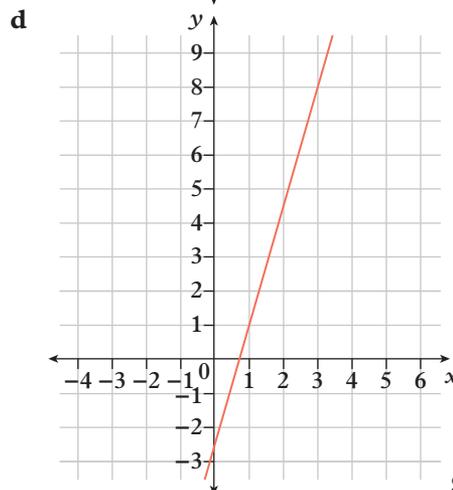
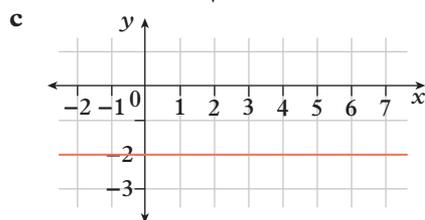
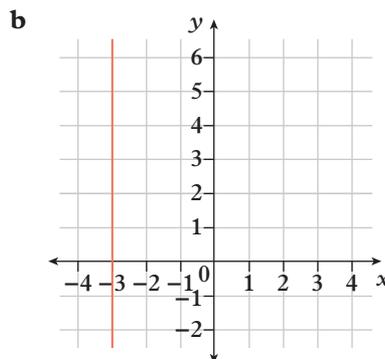
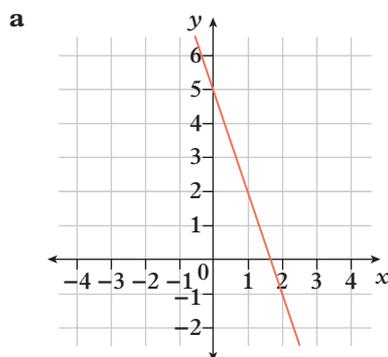
b $y = 2$

c $y = 2x$

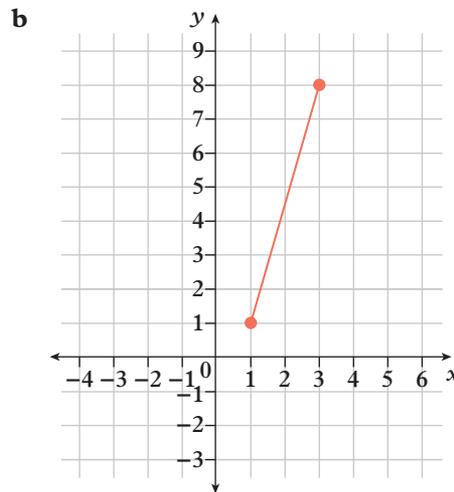
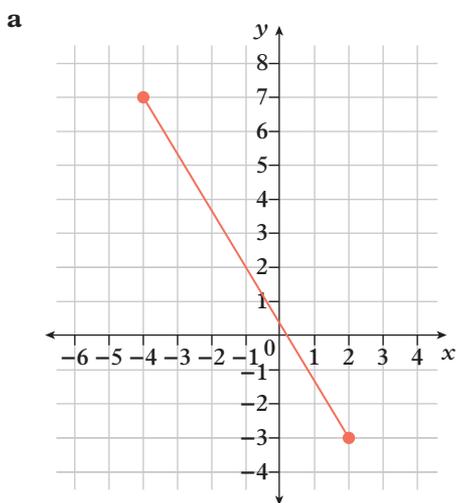
17 Calculate the gradient of the following lines.



18 Determine the equations of the following graphs.



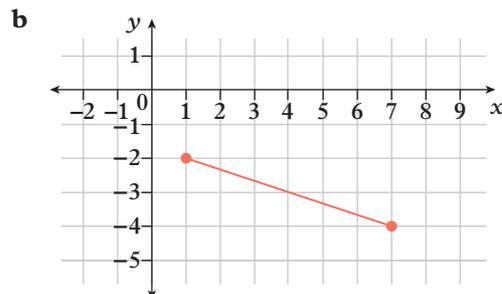
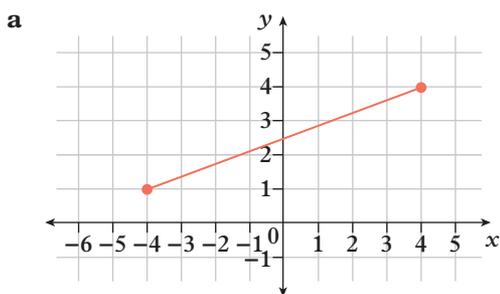
19 Find the midpoint of the following straight-line segments.



20 From the given information, determine the equation of each of the lines in **a–e** below. Write each equation in the form $y = mx + c$.

- a** The gradient of the line is 4 and the line passes through the point $(-2, 5)$.
- b** The line passes through the points $(3, -4)$ and $(8, 12)$.
- c** The line is parallel to $3x - 6y = 12$ and passes through the point $(4, 3)$.
- d** The line is perpendicular to $y = -\frac{4}{3}x + 5$ and passes through the point $(-2, 5)$.
- e** The line is perpendicular to the line segment between the points $(1, 4)$ and $(-2, 5)$ and passes through the midpoint of the line segment.

21 Determine the length of the following line segments, correct to two decimal places.



STN 22 Find the constant of variation using the given information, then write the equation for each directly proportional relationship.

ADV

a $y = 5$ for $x = 20$

b $y = -12$ for $x = 4$

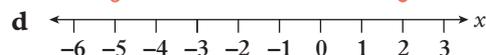
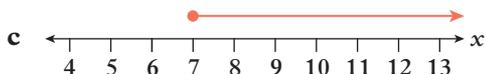
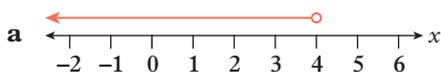
c

x	0	1	2	3	4
y	0	-5	-10	-15	-20

d

x	0	4	8	12	16
y	0	-1	-2	-3	-4

ADV 23 State the inequality represented on each of the following number lines.



ADV 24 Solve each of the following inequalities. Show each solution on a number line.

a $4x + 5 \neq 17$

b $-6x + 4 > 28$

c $8 - 4x \leq 5$

d $5(2x - 3) \leq 4x$

ADV 25 Factorise the following expressions.

a $12a + 18b^2 - 6cd$

b $21g^2 - 7g$

c $3m(n - 5) - 4(n - 5)$

d $vw + 5v + 2w + 10$

e $3p^2 + 3p - 18$

f $r^2 + 5r - 36$

ADV 26 Write each of these expressions as a single fraction in simplest form.

a $\frac{2}{3x} \times \frac{5}{8x} \times \frac{7}{2x}$

b $\frac{3p^2}{4q} \div \frac{2p}{qr} \times \frac{7}{2pq}$

c $-\frac{1}{4abc} + \frac{5}{6abc}$

d $\frac{2c}{5d} + \frac{3c}{6d} + \frac{4c}{7d}$

Analysis

- Jane is driving home from work, which is 30 km from where she lives. It takes her 36 minutes to drive home. Assume Jane drives at a constant speed throughout her journey. Jane's distance from home, d kilometres, at time t minutes after leaving work can be described using a linear equation.
 - Which two features of the graph of d are given in the information above?
 - Determine Jane's speed throughout her journey in kilometres per minute.
 - How does the gradient of d relate to your answer to part **b**?
 - Write the equation of d in gradient–intercept form.
 - Sketch the graph of d and label the axis intercepts.

Jane's daughter Vanessa attends a school located at the exact midpoint of Jane's journey home. Occasionally, Jane picks up Vanessa on her way home. It takes three minutes for Vanessa to get into the car and for Jane to be on her way again at the same constant speed as before.

 - Find the coordinates of the point on your graph that represents Jane stopping to pick up Vanessa.
 - How should you represent the time spent picking up Vanessa from school on your graph? How does this affect the key features of the graph of d ?
 - Sketch the graph representing Jane's journey home on a day when she picks up Vanessa. Label any key coordinates.
 - Your sketch from part **h** can be broken into three linear graph segments. Find the equation of each segment in terms of d and t .
- Lisa works at an aged care centre and receives a wage of \$24 per hour. For hours worked outside of the core working hours (8 am to 6 pm) Lisa receives time-and-a-quarter pay.

Lisa's regular working week is shown in the table below:

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Hours	8–4	8–4	12–8	12–8	8–2	–	–

- How much does Lisa earn in a regular working week?
 - Lisa is provided with 20 days of annual leave each year, for which she receives leave loading of 17.5%. How much is Lisa paid for her four weeks of annual leave?
 - Calculate Lisa's annual income from her job if she works for 48 weeks and takes four weeks of annual leave. Lisa is also a co-author of a book on health care, for which she receives a 4.5% royalty. Over the last financial year, the book recorded sales of \$65 320.
 - Calculate the amount of income Lisa receives from her book royalty.
- Lisa also started the year with \$12 600 of savings, which she put into a simple interest account which paid 7.4% p.a.
- How much interest did Lisa earn on her savings?
 - Given that Lisa had tax deductions which totalled \$484, what is Lisa's taxable income for the year?
 - Use the following tax table to determine Lisa's income tax for the year.

Taxable income	Tax on this income \$35 000
0–\$18 200	Nil
\$18 201–\$45 000	19 cents for each \$1 over \$18 200
\$45 001–\$120 000	\$5092 plus 32.5 cents for each \$1 over \$45 000
\$120 001–\$180 000	\$29 467 plus 37 cents for each \$1 over \$120 000
\$180 001 and over	\$51 667 plus 45 cents for each \$1 over \$180 000

- What is Lisa's net income from the year, after her income tax and the Medicare levy has been deducted from her taxable income?

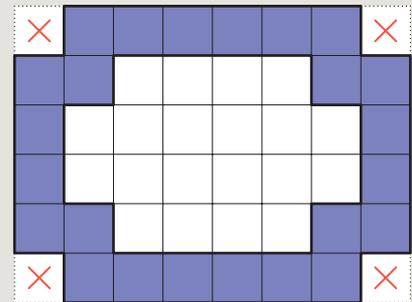
1 Overtake

A car, a van, a truck and a bicycle are all travelling in the same direction on the same road, each at its own constant speed. At 10 am, the car overtakes the van; at noon, it overtakes the truck; at 2 pm it overtakes the bicycle. At 4 pm, the truck overtakes the bicycle. At 6 pm, the van overtakes the truck.

- a** Suppose the speed of the car is 120 km/h and the speed of the truck is 80 km/h.
- Find the speeds of the van and the bicycle. Then find the time at which the van overtakes the bicycle.
 - A motorbike overtakes the van at 10 am and the bicycle at 3 pm. What is its speed?
 - A semi-trailer travelling at 80 km/h in the opposite direction passes the van at 6 pm. At what time does it pass the bicycle?
- b** Let c and t represent the speeds, in km/h, of the car and truck, respectively. Find the speeds of the van and the bicycle in terms of c and t . Then show that the time at which the van overtakes the bicycle is always the same, regardless of the speeds of the car and the truck.

2 Borders

The design shown is formed by removing one square from each corner of a rectangular grid and shading a border of thickness 1 unit. In this 6×8 example, there are 24 shaded squares in the border and 20 unshaded squares in the interior.



- a** Ignoring rotations, we want to find the number of such designs that have the same numbers of border and interior squares. Follow these steps:
- Let the dimensions of the original rectangular grid be $x \times y$. Find expressions, in terms of x and y , for the number of border squares and the number of interior squares.
 - Setting the expressions equal, rearrange the equation to get 0 on the right-hand side.
 - The left-hand side can almost be fully factorised as a binomial product of the form $(x - \underline{\quad})(y - \underline{\quad})$. What number do you need to add to both sides of the equation to make this work?
 - Remember that x and y are positive integers, so the factorised expression on the left-hand side of your equation should match up with a factorisation of the number on the right-hand side. Deduce that there are six possible pairs of positive integer solutions for x and y .
 - Why does this mean that there are only three possible designs?
- b** Adapt the above methods to find, ignoring rotations, the number of such designs in which there are:
- twice as many interior squares as border squares
 - twice as many border squares as interior squares.

3 Absurd surds

What does the infinite surd expression $\sqrt{6 + \sqrt{6 + \sqrt{6 + \dots}}}$ equal? Does it even equal anything? Maybe as you add more and more sixes, the result gets larger and larger without ever settling down.

- a** One way to get an idea of what's going on is to use the 'answer' button on your scientific calculator, as follows. The instructions for your model might be slightly different.
- ▶ Enter $\sqrt{6} =$ – this calculates the first approximation of $\sqrt{6}$ and stores it as the most recent answer.
 - ▶ Enter $\sqrt{6 + \text{Ans}} =$ – this calculates the next approximation $\sqrt{6 + \text{Ans}}$ based on the previous answer.
 - ▶ Now repeatedly enter $=$ – this repeats the previous calculation as often as you like.
 - ▶ What number do the approximations appear to get closer to?
- b** Now let's prove that your observation in part **a** is correct. Let x be the final answer, then notice that $x = \sqrt{6 + x}$. Square both sides and solve the quadratic equation. Which solution can you ignore and why?
- c**
- Adapt the method in part **b** to calculate the exact values of $\sqrt{12 + \sqrt{12 + \sqrt{12 + \dots}}}$ and $\sqrt{2 - \sqrt{2 - \sqrt{2 - \dots}}}$
 - Find two different infinite surd expressions that are equal to 5.

4 Snowflakes

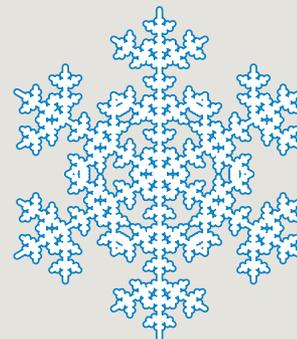
There is a famous conjecture in mathematics about so-called *hailstone* numbers. Starting with any positive integer, we halve the number if it is even, otherwise we triple it and add 1. Now repeat this process to form a sequence of numbers called hailstones, because of the way they rise and fall like real hailstones forming in a cloud. The conjecture states that every number will eventually ‘fall to the ground’ by reaching the number 1. Even though this idea dates back to 1937, when it was first introduced by Lothar Collatz, mathematicians still do not know whether this will always happen. You might like to check what happens when you start with 27; it takes a surprisingly long time for the hailstones to fall!

Let’s look at a gentler version of this problem: *snowflake* numbers. Again, if a number is even we halve it, otherwise we triple it and *subtract* 1. For example, the snowflakes of 11 are 32, 16, 8, 4, 2, 1, 2, 1 ... and so on. The sequence starting with 11 has a cycle of length 2 that repeats indefinitely, which we can represent as follows:

$$11 \rightarrow 32 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1 \rightarrow 2 \rightarrow 1 \rightarrow \dots$$

We call them snowflakes because, being lighter than hailstones, they might never fall to the ground!

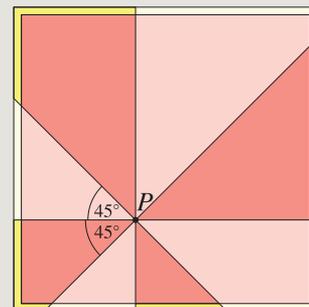
- Show that the snowflakes of 12 also result in a cycle of length 2.
- Show that the snowflakes of 13 result in a cycle of length 5.
- Find a cycle of snowflakes with length greater than 5.
- Explain why no cycle of snowflakes can contain a multiple of 3.
- Find all cycles of length 5.
- Find an odd number whose snowflakes are alternately even and odd, which do not form a cycle for at least 1000 terms.



5 Square pizzas

Angelo likes square pizzas because they fit inside square boxes more efficiently than round pizzas do. However, not having as many symmetries, he worries that it might not be possible to share square pizzas as fairly as round ones. By ‘sharing fairly’, Angelo means that everyone gets the same amount of topping (total area of their slices) and the same amount of crust (total length of boundary shared with the original square).

Angelo makes a cut parallel to one pair of sides, and then makes a second cut parallel to the other pair of sides. Through the point P where these two cuts meet, he makes two more cuts at 45° to the others, as shown.



- A pizza has side length 20 cm and the point P is 8 cm from the left edge and 6 cm from the bottom edge. Find the length of each piece of crust (ignoring its thickness) and the area of each slice (including the crust).
- Verify that the pizza in part **a** will be shared fairly among two people if they take alternate slices, shaded lighter and darker in the diagram.
- Show that, regardless of the size of the pizza and the location of the point P , the method above will always share a pizza fairly among two people. Hint: Use coordinates $(0, 0)$, $(c, 0)$ and (a, b) for the bottom-left corner, bottom-right corner and intersection point P , respectively. What if P is on a diagonal? What if it isn’t?
- Next, Angelo is interested in sharing a square pizza fairly among three people. He wants to do this by making three straight cuts from a common point somewhere inside the square. Is this possible?
- For what values of n is it possible to share a square pizza fairly among n people by making n straight cuts from a common point?

Explorations inspired by the Australian Maths Trust’s competitions and programs: www.amt.edu.au

5

Non-linear

relationships



Index

- ADV 5A** Solving quadratic equations
- 5B** Plotting quadratic relationships
- 5C** Sketching parabolas using intercepts
- 5D** Sketching parabolas using transformations
- 5E** Plotting exponential relationships

Prerequisite skills



Diagnostic pre-test

Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.



Interactive skillsheets

After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✓ Solving equations using inverse operations
- ✓ Factorising quadratic expressions
- ✓ The Cartesian plane
- ✓ Plotting graphs
- ✓ Substitution

Curriculum links

- Identifies connections between algebraic and graphical representations of quadratic and exponential relationships in various contexts (MA5-NLI-C-01)
 - Examine the connection between algebraic and graphical representations of quadratics and exponentials
- Identifies and compares features of parabolas and exponential curves in various contexts (MA5-NLI-C-02)
 - Graph and examine quadratic relationships
 - Graph and examine exponential relationships
 - Distinguish between linear, quadratic and exponential relationships by examining their graphical representations
- ADV** • Solves monic quadratic equations, linear inequalities and cubic equations of the form $ax^3 = k$ (MA5-EQU-P-01)
 - Solve monic quadratic equations

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Materials

- ✓ Calculator
- ✓ Graph paper

5A Solving quadratic equations

Learning intentions

By the end of this topic you will be able to ...

- ✓ solve simple quadratic equations.



Inter-year links

Year 7

6C Terms, expressions and equations

Year 8

6A Equations

Year 10

4D Solving quadratic equations

Quadratic equations



Key content video

- A single variable **quadratic equation** is an equation that, when expanded, has its variable raised to an index of no more than 2.
- The general form of a quadratic equation is: $ax^2 + bx + c = 0$ where a , b and c are constants.
- A **monic quadratic equation** is a quadratic equation in which the coefficient, a , of the leading term, ax^2 , is 1. The general form of a monic quadratic equation is $x^2 + bx + c = 0$.

The Null Factor Law

- The **Null Factor Law** states that if the product of factors is 0, then one or more of the factors must be 0. Given the factors a and b , if $a \times b = 0$, then $a = 0$ or $b = 0$.
- Applying the Null Factor Law to quadratic equations in **factor form**, $(x - p)(x - q) = 0$: if $(x - p)(x - q) = 0$, then $x - p = 0$ or $x - q = 0$.

Solving quadratic equations

- To solve a factorisable quadratic equation in general form:
 - 1 Factorise the quadratic expression.
 - 2 Apply the Null Factor Law by equating each factor to 0.
 - 3 Solve each linear equation.

For example, $x^2 + 2x - 3 = 0$

$$(x + 3)(x - 1) = 0$$

$$x + 3 = 0 \text{ or } x - 1 = 0$$

$$x = -3 \text{ or } x = 1$$

- Quadratic equations can have 0, 1 or 2 real solutions.

Type of quadratic equation	Example	Number of solutions
Factorised form is $(x + a)(x + b) = 0$, where $a \neq b$	$x^2 + 3x + 2 = 0$ $(x + 1)(x + 2) = 0$ $x + 1 = 0 \text{ or } x + 2 = 0$ $x = -1 \text{ or } x = -2$	2
Factorised form is a perfect square: $(x + a)^2 = 0$	$x^2 + 2x + 1 = 0$ $(x + 1)^2 = 0$ $x + 1 = 0$ $x = -1$	1
$ax^2 + bx + c = 0$ cannot be factorised for some values of a , b and c	$x^2 + x + 1 = 0$	0

**Example 5A.1** Solving factorised quadratic equations

Solve each quadratic equation.

a $(x - 6)(x + 2) = 0$

b $x(x - 4) = 0$

c $(2x + 1)(4x - 3) = 0$

THINK

- 1 Check that the quadratic equation is in factor form and is equal to 0.
- 2 Apply the Null Factor Law by equating each factor to 0.
- 3 Solve each linear equation.

WRITE

a $(x - 6)(x + 2) = 0$

$x - 6 = 0$ or $x + 2 = 0$

$x = 6$ or $x = -2$

b $x(x - 4) = 0$

$x = 0$ or $x - 4 = 0$

$x = 0$ or $x = 4$

c $(2x + 1)(4x - 3) = 0$

$2x + 1 = 0$ or $4x - 3 = 0$

$2x = -1$ or $4x = 3$

$x = -\frac{1}{2}$ or $x = \frac{3}{4}$

Example 5A.2 Factorising and solving quadratic equationsSolve the quadratic equation $x^2 - 3x - 10 = 0$.**THINK**

- 1 Factorise the quadratic equation.
- 2 Apply the Null Factor Law by equating each factor to 0.
- 3 Solve each linear equation.

WRITE

$x^2 - 3x - 10 = 0$

$(x + 2)(x - 5) = 0$

$x + 2 = 0$ or $x - 5 = 0$

$x = -2$ or $x = 5$

Example 5A.3 Rearranging quadratic equations before solvingSolve $x^2 + 12 = 8x$.**THINK**

- 1 Rearrange the equation so that one side of the equation is equal to 0, and the x^2 term is positive.
- 2 Factorise the quadratic equation.
- 3 Apply the Null Factor Law by equating each factor to 0.
- 4 Solve each linear equation.

WRITE

$x^2 + 12 = 8x$

$x^2 - 8x + 12 = 0$ (- 8x)

$(x - 2)(x - 6) = 0$

$x - 2 = 0$ or $x - 6 = 0$

$x = 2$ or $x = 6$



Example 5A.4 Solving quadratic equations with common factors

Solve each quadratic equation

a $2x^2 + 22x + 36 = 0$

b $-x^2 + 10x = 25$

THINK

- a**
- 1 Take the highest common factor (HCF) out of the quadratic expression.
 - 2 Divide both sides by the HCF. Note that you can never divide both sides by the variable, x , in case $x = 0$.
 - 3 Factorise the quadratic equation.
 - 4 Apply the Null Factor Law by equating the factors to 0.
 - 5 Solve each linear equation.
- b**
- 1 Rearrange so that the right-hand side is 0.
 - 2 Take out the HCF. Remember, the HCF can be -1 .
 - 3 Divide both sides by the HCF.
 - 4 Factorise and apply the Null Factor Law. Remember, there is only one solution when the factorised form is a perfect square.
 - 5 Solve the linear equation.

WRITE

a

$$2x^2 + 22x + 36 = 0$$

$$2(x^2 + 11x + 18) = 0$$

$$\frac{2(x^2 + 11x + 18)}{2} = \frac{0}{2}$$

$$x^2 + 11x + 18 = 0$$

$$(x + 2)(x + 9) = 0$$

$$x + 2 = 0 \text{ or } x + 9 = 0$$

$$x = -2 \text{ or } x = -9$$

b

$$-x^2 + 10x = 25$$

$$-x^2 + 10x - 25 = 0$$

$$-1(x^2 - 10x + 25) = 0$$

$$\frac{-1(x^2 - 10x + 25)}{-1} = \frac{0}{-1}$$

$$x^2 - 10x + 25 = 0$$

$$(x - 5)^2 = 0$$

$$x - 5 = 0$$

$$x = 5$$

Helpful hints

- ✓ Watch for changes in sign when solving quadratic equations for x .

$$(x + 6)(x - 2) = 0$$

$$x + 6 = 0 \text{ or } x - 2 = 0$$

$$x = -6 \text{ or } x = +2$$

- ✓ Although it is possible to add and subtract multiples of x , you must *not* divide both sides of an equation by x in case $x = 0$, since dividing anything by 0 is *undefined*.

For example,

$$x^2 = 5x \quad (-5x)$$

$$x^2 - 5x = 0$$

$$x(x - 5) = 0$$

$$x = 0 \text{ or } x = 5$$

$$x^2 = 5x \quad (\div x)$$

$$x = 5$$

Solution $x = 0$ is missing

- ✓ Always check the coefficients for a highest common factor (HCF). You can factor out the HCF from any quadratic equation, $ax^2 + bx + c = 0$, by dividing on both sides. This is because $\frac{0}{\text{HCF}} = 0$ for any HCF.

Exercise 5A Solving quadratic equations

 1, 2(a, c, e, g), 3-11, 13, 16

 1, 2(f, g, h), 4-10, 12, 13, 15, 18

 1, 5-7, 9, 10, 12, 15-18

1 Which of the following are quadratic equations?

a $x^2 - 2 = 0$

b $3(x + 1) = 0$

c $x^2 + x = 5$

d $x^2 + 5x + 6$

e $2x^2 + x - 4 = 0$

f $x^3 + 8 = 0$

g $6x + 1 = 2x - 5$

h $x^2 + 7x = x - 3$

2 Use substitution to check whether the value in brackets is a solution to the given quadratic equation.

a $(x - 4)(x - 5) = 0$ ($x = 5$)

b $(x + 2)(x - 8) = 0$ ($x = 2$)

c $x(x - 6) = 0$ ($x = 3$)

d $x^2 + 8x + 7 = 0$ ($x = -1$)

e $x^2 - 4x + 4 = 0$ ($x = -2$)

f $x^2 - 49 = 0$ ($x = 7$)

g $x^2 - 2x - 15 = 0$ ($x = -3$)

h $x^2 - 8x + 12 = 0$ ($x = -4$)

3 Solve each of the following linear equations.

a $x + 3 = 0$

b $x - 6 = 0$

c $4x - 8 = 0$

d $3x + 1 = 0$

5A.1 4 Solve each of the following quadratic equations.

a $(x + 2)(x - 3) = 0$

b $(x - 7)(x - 1) = 0$

c $(x + 4)(x - 4) = 0$

d $x(x - 6) = 0$

e $(x + 5)(x + 1) = 0$

f $x(x + 2) = 0$

g $(x - 8)(x + 8) = 0$

h $(2x + 2)(x - 7) = 0$

i $x(x - 11) = 0$

j $(2x + 3)(x - 5) = 0$

k $(4x - 2)(4x - 2) = 0$

l $(3x + 5)(2x + 3) = 0$

5A.2 5 Factorise and then solve each quadratic equation.

a $x^2 + 7x + 10 = 0$

b $x^2 - 3x + 2 = 0$

c $x^2 + 5x = 0$

d $x^2 - 3x = 0$

e $x^2 - 36 = 0$

f $x^2 + 10x + 21 = 0$

5A.3 6 Rearrange, factorise and then solve each quadratic equation.

a $x^2 - 2x = 8$

b $x^2 = 1$

c $x^2 = -8x$

d $x^2 + 3 = 4x$

e $6x = -x^2 - 9$

f $1 = 2x - x^2$

7 Use substitution to check that your solutions for questions 5 and 6 are correct.

8 Solve each quadratic equation.

a $2(x + 8)(x - 2) = 0$

b $-3(x - 1)(x - 4) = 0$

c $-7(x + 6)(x - 6) = 0$

d $-5x(x + 9) = 0$

5A.4 9 Solve each quadratic equation by first dividing both sides by the highest common factor (HCF).

a $3x^2 + 9x + 6 = 0$

b $2x^2 + 8x = 0$

c $2x^2 - 4x - 16 = 0$

d $3x^2 - 12x + 12 = 0$

e $-5x^2 - 5x + 10 = 0$

f $-4x^2 + 8x = 0$

g $-x^2 - 10x - 21 = 0$

h $-3x^2 - 24x - 48 = 0$

i $-2x^2 + 32 = 0$

10 Solve each equation.

a $x^2 + 14x + 48 = 0$

b $3x^2 - 12x = 0$

c $x - x^2 = 0$

d $x^2 - 64 = 0$

e $-x^2 - 2x + 3 = 0$

f $-x^2 + 8x - 16 = 0$

g $-x^2 + 5x = 6$

h $2x^2 + 50 = -20x$

i $20 = 5x^2$

11 **a** Determine how many solutions each of the following quadratic equations has.

i $(x - 4)(x - 7) = 0$

ii $(x - 4)(x - 4) = 0$

iii $x^2 + 4 = 0$

b Identify and describe the feature(s) of each equation in part **a** that result(s) in the number of solutions that the equation has.

12 State how many real solutions each equation has.

a $x^2 + 3x - 10 = 0$

b $x^2 - 4 = 0$

c $x^2 - 6x + 9 = 0$

d $x^2 + 1 = 0$

e $x^2 + 7x = 0$

f $x^2 + 12x + 32 = 0$

g $x^2 - x - 72 = 0$

h $x^2 + 2x + 5 = 0$

13 The length of a rectangular mouse pad is 8 cm longer than its breadth (b).

a Write an expression for the area of the mouse pad in terms of b .

b Expand the expression.

c The area of the mouse pad is estimated to be 560 cm^2 . Write an equation for the area of the mouse pad.

d Factorise and solve the equation. Which value of b is a feasible solution in this scenario? Explain.

e State the dimensions of the mouse pad.



14 The area of a rectangular sand pit is 35 m^2 . The length is 2 m longer than the breadth (b).

a Write a quadratic equation to represent this scenario.

b Solve the quadratic equation.

c State the dimensions of the sand pit.

15 The breadth of a laptop screen is 12 cm less than its length. If the area of the screen is 640 cm^2 , use algebra to determine the dimensions of the screen.



16 Alec throws a tennis ball back on to the court from the spectator stand. The height of the ball above the surface of the tennis court can be represented by the quadratic relationship $h = -4(t + 1)(t - 2)$, where h is the height in metres after t seconds in the air.

a What is the height of the ball after:

i 1 second

ii 2 seconds?

b What is the height of the ball when Alec releases it from his hand?

c How long does it take for the ball to hit the tennis court after it is thrown?

d Explain why there is only one time value for your answer to part **c** even though you have solved a quadratic equation that has two solutions.

17 The height above the ground, h , of a firework rocket x seconds after it is launched is given by the equation $h = -5x^2 + 40x$.

a When is the rocket at a height of 60 m above the ground?

b Why are there two solutions? Explain the significance of each solution.



18 An equation containing a pronumeral that is taken to the power of three (and no higher) is called a cubic equation. Use the Null Factor Law to solve the following cubic equations.

a $(x - 1)(x - 3)(x - 5) = 0$

b $x(x + 6)(x - 6) = 0$

c $(x + 11)(x - 4)^2 = 0$

d $(2x - 4)(3x + 1)(4x - 6) = 0$

Check your Student obook pro for these digital resources and more:

pro



Interactive skillsheet
Solving quadratic equations



Worksheet
Solving quadratic equations



Topic quiz
5A

5B Plotting quadratic relationships

Learning intentions

By the end of this topic you will be able to ...

- ✓ identify the key features of a parabola
- ✓ plot parabolas from tables of values.



Inter-year links

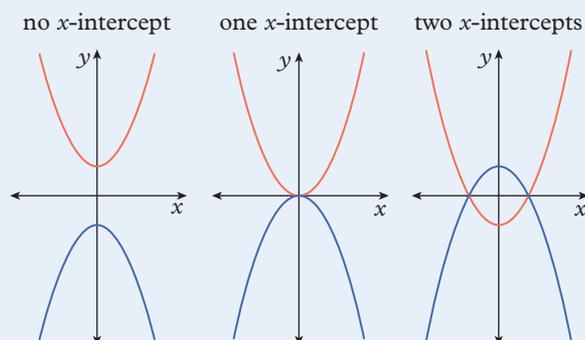
- Support** The Cartesian plane
- Year 7** 5D The Cartesian plane
- Year 8** 6F Plotting linear relationships

Key features of parabolas



Key content video

- The general form of a quadratic relationship is $y = ax^2 + bx + c$, where a , b and c are constants. The graph of a quadratic relationship is called a **parabola**.
- The key features of a parabola include:
 - x - and y -intercepts; while all parabolas have one y -intercept, parabolas can have no, one or two x -intercepts
 - a **turning point** or **vertex**; a quadratic relationship with a positive leading term has a **minimum turning point** and a quadratic relationship with a negative leading term has a **maximum turning point**
 - Concavity; curves with a minimum turning point are concave up, and curves with a maximum turning point are concave down
 - an **axis of symmetry**, $x = -\frac{b}{2a}$, where $-\frac{b}{2a}$ is the x -coordinate of the turning point.
- If there are two x -intercepts, then the axis of symmetry will pass through the midpoint between them.

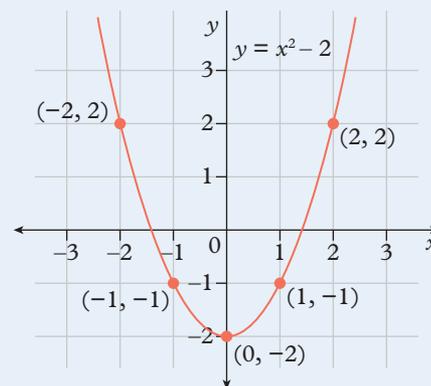


Plotting parabolas

- To plot the graph of a quadratic relationship, create a table of coordinate points and join the points on a Cartesian plane with a smooth curve.

x	-2	-1	0	1	2
$y = x^2 - 2$	2	-1	-2	-1	2

- 1 Construct a table of x - and y -values by selecting values for x , then substituting each value of x into the relationship to find the corresponding value of y .
- 2 Write the coordinate points in the table.
- 3 Plot the coordinate points on a Cartesian plane.
- 4 Join the points using a smooth curve to form a parabola and label the graph with its equation.

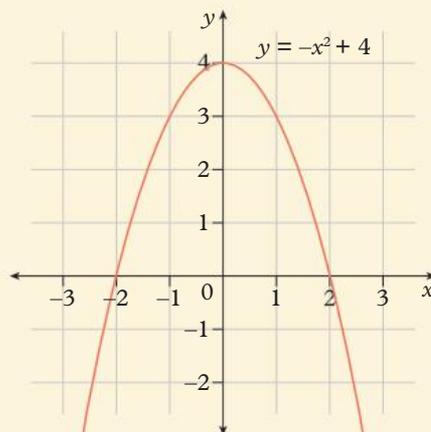


Example 5B.1 Identifying the key features of a parabola



For the parabola shown, identify:

- a the coordinates of the x -intercepts
- b the coordinates of the y -intercept
- c whether the turning point is a maximum or minimum.



THINK

- a State the coordinates of the points where the graph crosses the x -axis.
- b State the coordinates of the point where the graph crosses the y -axis.
- c The turning point is the highest point on the parabola, so it is a maximum turning point.

WRITE

- a x -intercepts: $(-2, 0)$ and $(2, 0)$
- b y -intercept: $(0, 4)$
- c maximum turning point

Example 5B.2 Plotting quadratic relationships



Sketch a graph of $y = 2x^2 - 3$ by first completing a table of values for x from -2 to 2 .

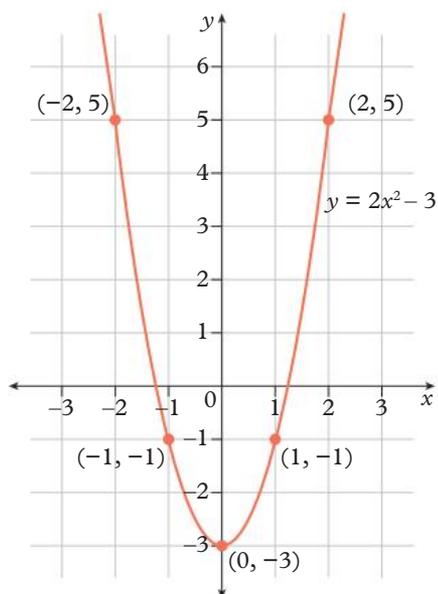
THINK

- 1 Construct a table of values for x from -2 to 2 . Substitute each value of x into the equation to find the corresponding value of y .
- 2 List the coordinate points.
- 3 Plot the points on a Cartesian plane.
- 4 Join the points with a smooth curve. Label the graph with its equation.

WRITE

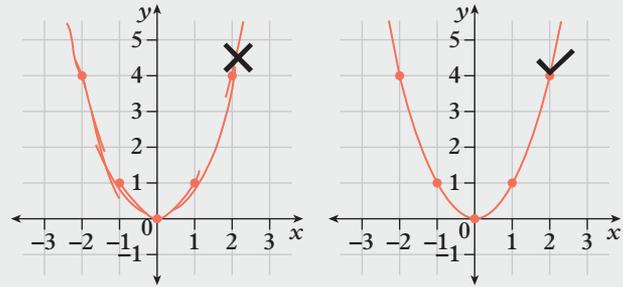
x	-2	-1	0	1	2
$y = 2x^2 - 3$	5	-1	-3	-1	5

$(-2, 5)$, $(-1, -1)$, $(0, -3)$, $(1, -1)$, $(2, 5)$

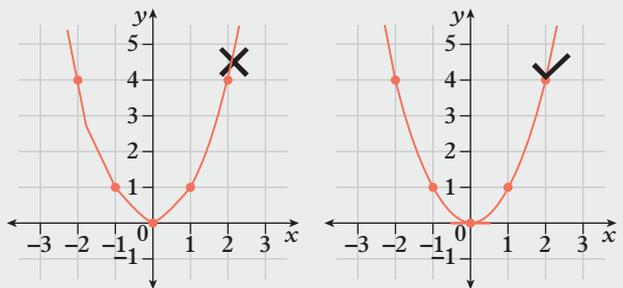


Helpful hints

✓ Rather than drawing 'hairy' parabolas, try starting at the turning point and then drawing two smooth, continuous and symmetrical curves through the other plotted points.



✓ Draw a short horizontal dash where the turning point is to ensure the parabola does not end up with a pointy turning point.



✓ Take care when reading a question. If you are asked to find the coordinates of the x - and y -intercepts, then you must give your answer as coordinates. So, if the coordinates of the intercepts are $(5, 0)$ and $(0, -1)$, do *not* write $x = 5$ and $y = -1$.

ANS
p465

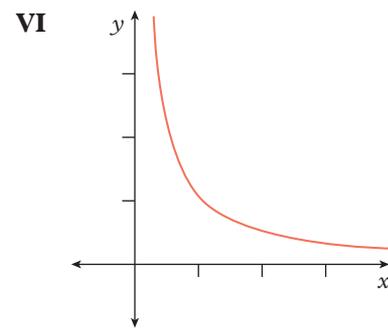
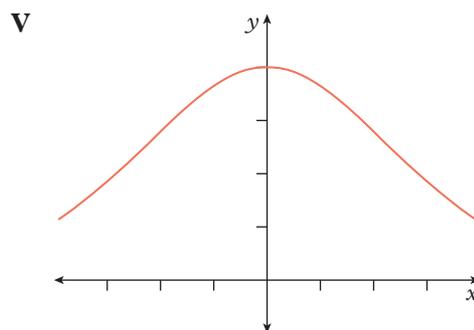
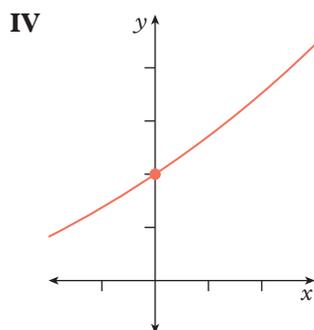
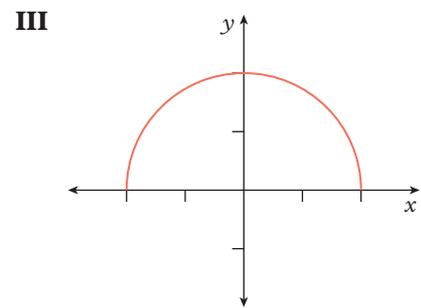
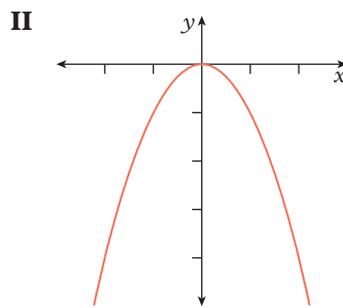
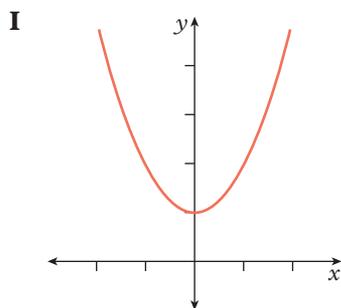
Exercise 5B Plotting quadratic relationships

▲ 1-3, 4(1st, 2nd columns),
5, 6, 8, 9

■ 2, 3, 4(2nd, 3rd columns),
5-7, 9

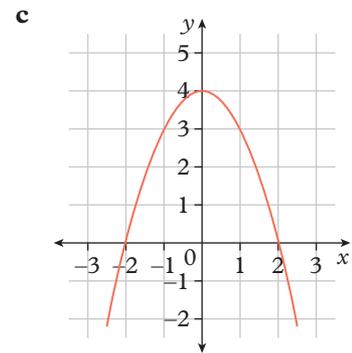
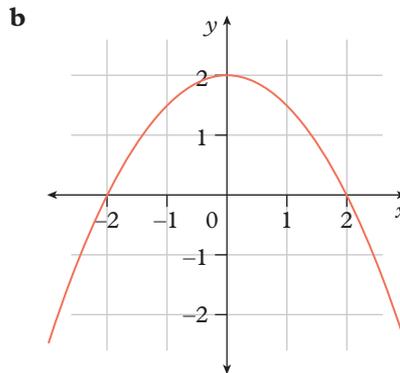
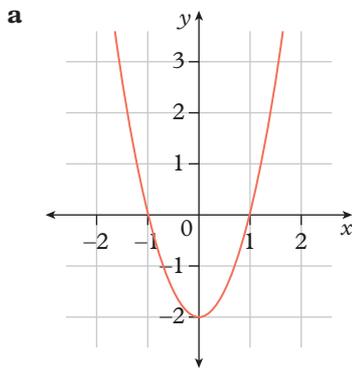
◆ 2(c), 3(b), 4(3rd, 4th columns),
5, 7-10

1 Which of the following graphs are parabolas?



5B.1 2 For each parabola, identify:

- i** the coordinates of the x -intercepts, if any
- ii** the coordinates of the y -intercept
- iii** the coordinates of the turning point
- iv** whether the turning point is a maximum or minimum.



3 For each quadratic relationship:

- i** complete the table of values
- ii** plot the graph
- iii** identify the axis intercepts and the turning point, and then state the equation of the axis of symmetry
- iv** state whether the turning point is a maximum or minimum.

a $y = 2x^2 - 8$

x	-3	-2	-1	0	1	2	3
y							

b $y = 9 - x^2$

x	-4	-3	-2	-1	0	1	2	3	4
y									

5B.2 4 For each quadratic relationship, plot its graph by first completing a table of coordinate points using x -values from -2 to 2 .

a $y = x^2$

b $y = -x^2$

c $y = 2x^2$

d $y = -2x^2$

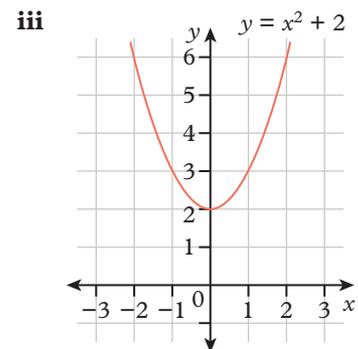
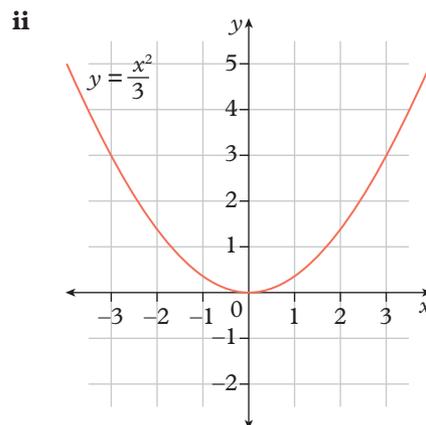
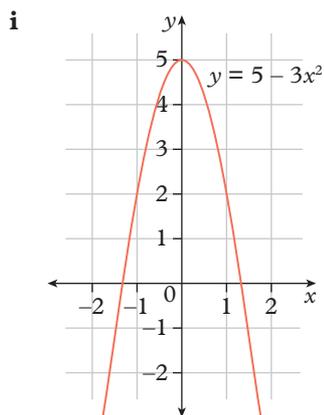
e $y = x^2 + 1$

f $y = 1 - x^2$

g $y = 1 + 2x^2$

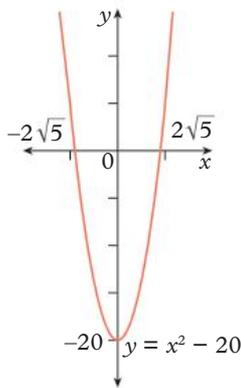
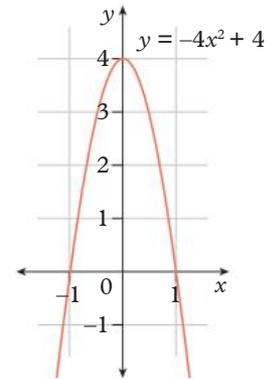
h $y = 1 - 2x^2$

5 a How many x -intercepts do each of the following parabolas have?

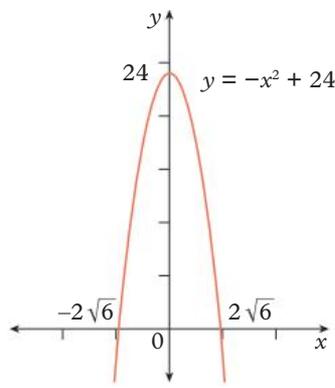


- b** Can a parabola have more than two x -intercepts? Explain.
- c** How many y -intercepts does each parabola in part **a** have?
- d** Describe the link between the y -intercept of a parabola and the general equation, $y = kx^2 + c$.

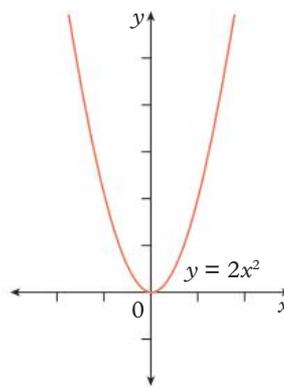
- 6 For each of the following parabolas state the number of:
- i** x -intercepts **ii** y -intercepts.
- a** parabola with a minimum turning point at $(0, 0)$
- b** parabola with a maximum turning point at $(10, 0)$
- 7 Alani's hair clip falls to the ground while she is on a roller coaster. The position of the clip as it falls can be described by the relationship $h = 100 - 4t^2$, where h is the height of the clip above the ground in metres after t seconds.
- a** Plot the graph of the relationship for t -values from 0 to 5.
- b** Why shouldn't you draw the parabola for values less than 0 or greater than 5?
- c** What is the height of the clip above the ground after:
- i** 2 s **ii** 3 s?
- d** From what height above the ground did the hair clip start to fall?
- e** How long did it take for the hair clip to hit the ground?
- 8 Consider the graph of $y = -4x^2 + 4$.
- a** Identify the x -coordinates of the x -intercepts from the graph.
- b** Solve the quadratic equation $-4x^2 + 4 = 0$.
- c** Compare your answers for parts **a** and **b**. What do you notice?
- d** Explain how you can use the graph of $y = x^2 - 10$ to solve $x^2 - 10 = 0$.
- 9 Use your observations from question 8 to solve each quadratic equation below using the following graphs.



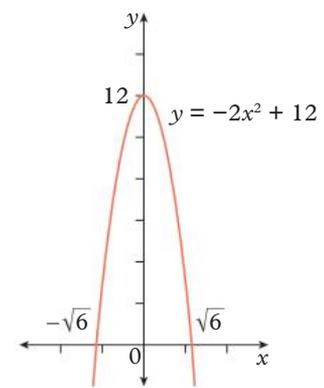
a $x^2 - 20 = 0$



b $-x^2 + 24 = 0$



c $2x^2 = 0$



d $-2x^2 + 12 = 0$

- 10 The equation for a graph with x -intercepts at $(-5, 0)$ and $(5, 0)$ is written as $y = x^2 - a$, where a is a constant.
- a** Determine the x -intercepts of $y = x^2 - a$ in terms of a .
- b** Hence identify an appropriate value for a and write the equation for the graph.
- c** Determine the minimum value of y .
- d** Determine the equation for a graph with x -intercepts at $(-5, 0)$ and $(5, 0)$ and a maximum value of 25.

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Interactive skillsheet
Plotting quadratic relationships



Topic quiz
5B

5C Sketching parabolas using intercepts

Learning intentions

By the end of this topic you will be able to ...

- ✓ find the intercepts of a quadratic relationship in the form $y = kx^2 + c$
- ✓ sketch parabolas using the intercepts.



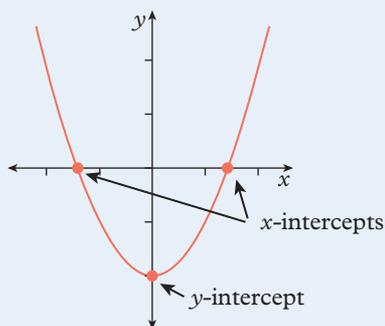
Inter-year links

Support	The Cartesian plane
Year 7	6E Substitution
Year 8	5B Substitution
Year 10	5C Graphing parabolas using intercepts

Sketching parabolas using intercepts



- The intercepts of some quadratic relationships can be used to sketch the parabola.



- Find the coordinates of all the intercepts, plot them on the Cartesian plane and then join the points with a smooth curve.

- To find the x -intercepts of a quadratic relationship in the form $y = kx^2 + c$:

- 1 Substitute $y = 0$ into the quadratic relationship.

$$y = x^2 - 16$$

Let $y = 0$:

$$x^2 - 16 = 0$$

$$x^2 = 16$$

$$x = -4 \text{ or } x = 4$$

x -intercepts: $(-4, 0)$ and $(4, 0)$

- If $kx^2 + c = 0$ does not have a real solution, then the quadratic relationship $y = kx^2 + c$ has no x -intercept.

- To find the y -intercept, substitute $x = 0$ into the quadratic relationship.

$$y = x^2 - 16$$

Let $x = 0$:

$$y = (0)^2 - 16$$

$$y = -16$$

y -intercept: $(0, -16)$

- The axis of symmetry for the graph of a quadratic relationship in the form $y = kx^2 + c$ is the y -axis. This makes the y -intercept of the parabola the turning point.

Example 5C.1 Finding the coordinates of the x - and y -intercepts



Find the coordinates of the x - and y -intercepts of $y = x^2 - 36$.

THINK

- 1 To find the x -coordinate of the x -intercept, substitute $y = 0$ into the quadratic relationship.
- 2 Solve the quadratic equation by adding 36 to both sides of the equation.
- 3 Write the coordinates of the x -intercepts.
- 4 To find the y -coordinate of the y -intercept, substitute $x = 0$ into the quadratic relationship and solve for y .
- 5 Write the coordinates of the y -intercept.

WRITE

$$y = x^2 - 36$$

For x -intercepts, let $y = 0$:

$$x^2 - 36 = 0$$

$$x^2 = 36$$

$$x = -6 \text{ or } x = 6$$

x -intercepts: $(-6, 0)$ and $(6, 0)$

For y -intercept, let $x = 0$:

$$y = (0)^2 - 36$$

$$= -36$$

y -intercept: $(0, -36)$

Example 5C.2 Sketching parabolas using the intercepts



Sketch the graph of $y = -x^2 + 5$ by first finding the x - and y -intercepts.

THINK

- 1 Find the x -intercepts by substituting $y = 0$ into the relationship.
- 2 Solve the equation for x .
- 3 Find the y -intercept by substituting $x = 0$ into the relationship and solving for y .
- 4 Mark the x - and y -intercepts on the axes, then draw the parabola. Label the equation of the graph.

WRITE

$$y = -x^2 + 5$$

For the x -intercepts, let $y = 0$:

$$-x^2 + 5 = 0$$

$$x^2 = 5$$

$$x = \sqrt{5} \text{ or } x = -\sqrt{5}$$

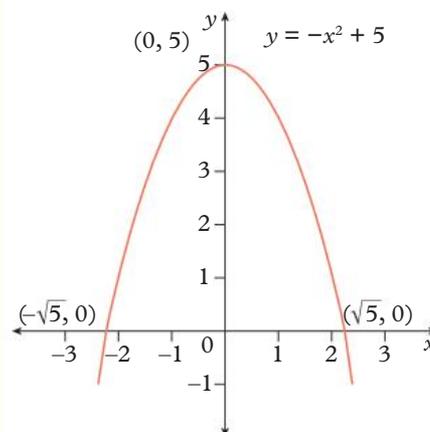
x -intercepts: $(-\sqrt{5}, 0)$ and $(\sqrt{5}, 0)$

For the y -intercept, let $x = 0$:

$$y = -(0)^2 + 5$$

$$= 5$$

y -intercept: $(0, 5)$



- ✓ Label your working out so that it is clear that you are calculating the x -intercepts and the y -intercept. This can make it easier to check your calculations on a test.
- ✓ Always label the key features of a graph such as the x -intercepts and the y -intercept with their values or coordinates when sketching parabolas.
- ✓ A parabola can have 0, 1 or 2 x -intercepts. If your quadratic relationship has no x -intercepts, or the x - and y -intercepts are both at the origin, you will need information about other features of the parabola to sketch the graph.

ANS
p467

Exercise 5C Sketching parabolas using intercepts

▲ 1-2(a-c), 3-5, 9, 10, 11

■ 1-2(d-f), 5-12

◆ 1-2(b, f), 8-14

UNDERSTANDING AND FLUENCY

5C.1 1 For each quadratic relationship, find the coordinates of the:

i x -intercepts

ii y -intercept.

a $y = x^2 - 2$

b $y = 2x^2 - 8$

c $y = -x^2 + 25$

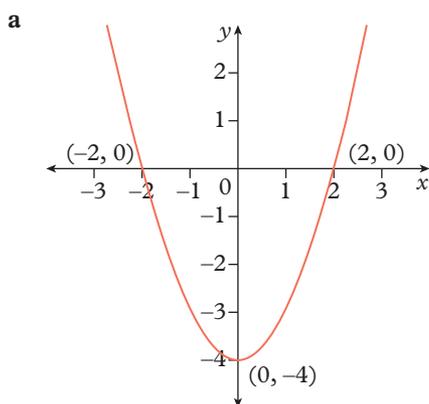
d $y = 2 - \frac{x^2}{2}$

e $y = -4x^2 + 36$

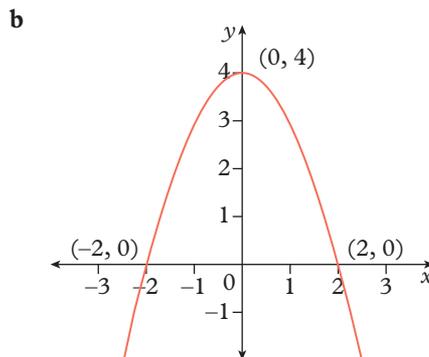
f $y = 25 - \frac{x^2}{4}$

5C.2 2 Sketch the graph of each quadratic relationship in question 1.

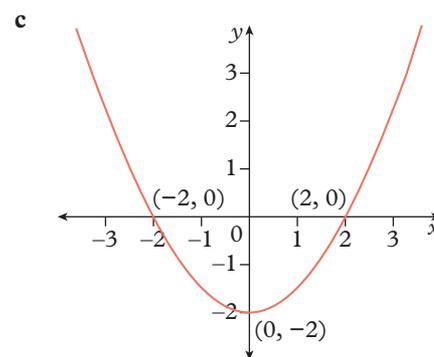
3 Match each graph with its rule from the list below.



I $y = x^2 - 4$



II $y = \frac{x^2}{2} - 2$



III $y = -x^2 + 4$

4 Consider the graphs of $y = \frac{x^2}{2} - 18$ and $y = 18 - \frac{x^2}{2}$.

a Identify the x -intercepts for each graph.

b Identify the y -intercepts for each graph.

c Describe the similarities and differences between the two graphs.

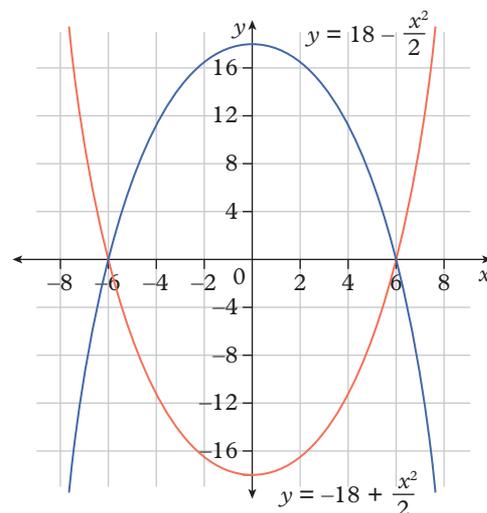
5 Find the y -intercepts of the following relationships.

a $y = 2x^2 - 3$

b $y = x^2 + \frac{1}{4}$

c $y = -2x^2 + \sqrt{5}$

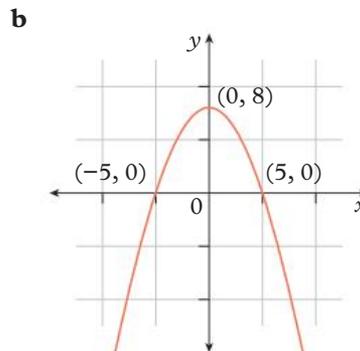
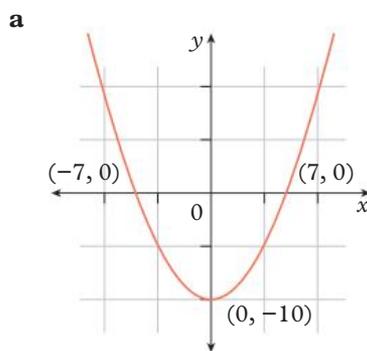
d $y = \frac{x^2}{4} - 6$



- 6** For a quadratic equation in the form $y = kx^2 + c$:
- write the coordinates of the y -intercept in terms k and c
 - write the coordinates of the x -intercept in terms k and c
 - find the value of c if the quadratic relationship only has one x -intercept
 - write a relationship between k and c if there is no x -intercept.
- 7** Use your answer to question **6** to determine how many x -intercepts each of the quadratic relationships in question **5** has.
- 8** For each of the following parts, find:
- the value of c
 - the coordinates of the x -intercept(s)
 - if the relationship has a maximum turning point or a minimum turning point.
- A quadratic relationship in the form $y = x^2 + c$ passes through $(0, -16)$.
 - A quadratic relationship in the form $y = -x^2 + c$ passes through $(0, 64)$.
- 9** Find the equation of the parabola in the form $y = kx^2$ that passes through the following points:
- $(1, 1)$
 - $(1, 2)$
 - $(1, 4)$
 - $(1, \frac{1}{2})$
- 10** Sketch the parabolas in question **9** on one graph; remember your parabola needs to pass the given point. Describe the relationships between the shape of the graph and the coefficient k .
- 11** For each quadratic relationship:
- identify whether its graph will be a concave up or concave down parabola
 - find the coordinates of the x - and y -intercepts
 - write the coordinates of the turning point
 - sketch the graph of the relationship.
- $y = 2x^2 - 18$
 - $y = -\frac{x^2}{2} + 8$
 - $y = \frac{x^2}{3} - 12$
 - $y = 12 - 3x^2$
- 12** The equation for a parabola with at least one x -intercept can be expressed in factorised form:

$$y = a(x - p)(x + p).$$

The values of p are equal to the x -coordinates of the positive x -intercepts. The value of a can be determined by substituting the coordinates of any other points on the graph into the values for x and y . Use the factorised form to determine the equation of each of the following parabolas.



- 13** What is the y -intercept of a quadratic relationship in the form $y = a(x - p)(x + p)$?
- 14** Find the equation of the parabola in the form $y = kx^2 + c$ that has two x -intercepts, one of the x -intercepts at $(3, 0)$, and a y -intercept of $(0, 6)$.

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Interactive skillsheet
Sketching parabolas using intercepts



Topic quiz
5C

Checkpoint



Checkpoint quiz

Check your knowledge of the first part of this chapter.

- ✓ Core
- ✓ Advanced

ADV 5A 1 Solve each quadratic equation using the Null Factor Law.

a $(x - 4)(x - 3) = 0$

b $(x - 2)(x + 2) = 0$

c $(x + 9)(x + 9) = 0$

d $x(x + 4) = 0$

ADV 5A 2 Solve:

a $x^2 - 12x + 35 = 0$

b $x^2 + 3x - 28 = 0$

c $x^2 - 12x + 36 = 0$

d $x^2 - 81 = 0$

ADV 5A 3 Solve the following quadratic equations by first identifying an HCF.

a $2x^2 - 32 = 0$

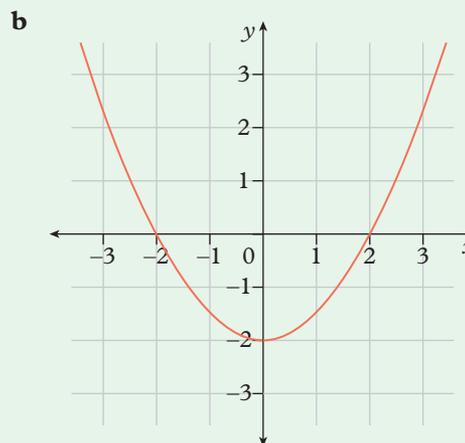
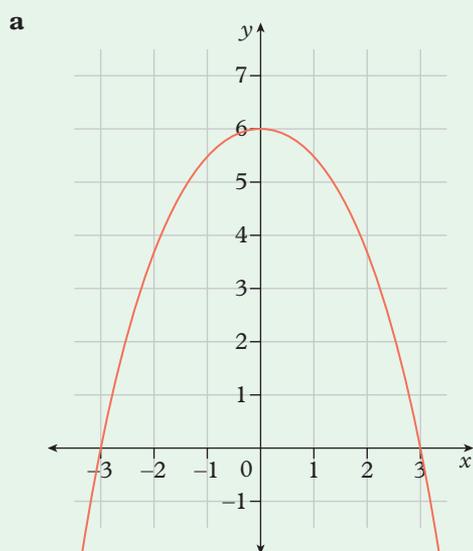
b $8x^2 - 16x - 64 = 0$

c $-2x^2 - 12x - 10 = 0$

d $45x - 3x^2 = 0$

5B 4 For the parabolas shown identify:

- i the coordinates of the x -intercepts
- ii the coordinates of the y -intercept
- iii whether it is a maximum or minimum turning point
- iv the equation of the axis of symmetry.



5B 5 Sketch a graph of $y = 2x^2 - 8$ by first completing a table of values for x from -3 to 3 .

5B 6 A quadratic relationship in the form $y = x^2 + c$ has a minimum turning point at $(0, -2)$.

a What is the value of c ?

b What are the coordinates of the x -intercept(s)?

5C 7 For each quadratic relationship, find the coordinates of the:

i x -intercept(s)

ii y -intercept.

a $y = x^2 - 3$

b $y = 2x^2 - 50$

c $y = x^2 - 8$

d $y = x^2 - 64$

5C 8 Sketch the graph of $y = x^2 - 3$ by first finding the x - and y -intercepts. Label the turning point with its coordinates.

5D Sketching parabolas using transformations

Learning intentions

By the end of this topic you will be able to ...

- ✓ sketch quadratic relationships in the form $y = kx^2 + c$ using reflections, dilations and vertical translations.



Inter-year links

Support

The Cartesian plane

Year 7

5D The Cartesian plane

Year 10

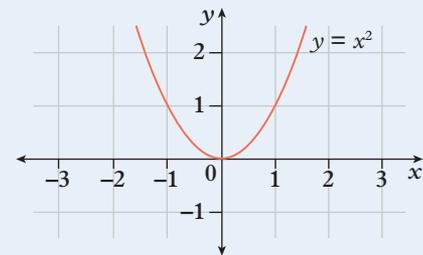
5D Graphing parabolas using transformations

Sketching parabolas using transformations



Key content video

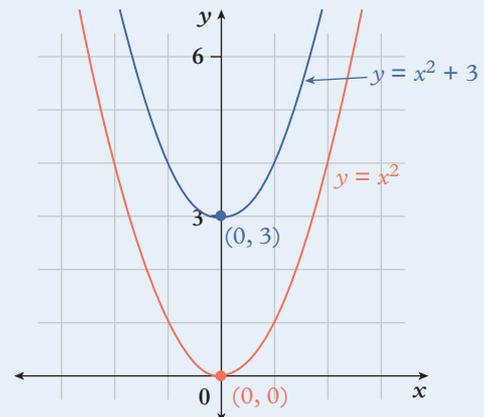
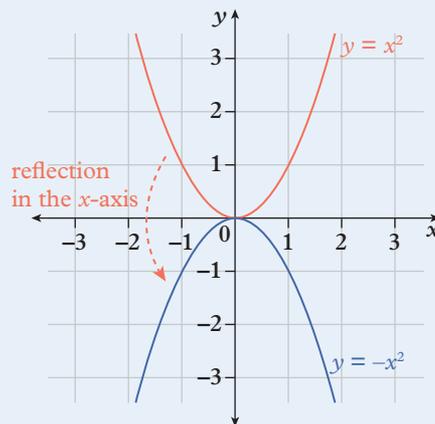
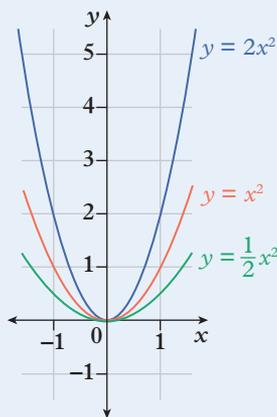
- The graph of $y = x^2$ is an upright parabola with a minimum turning point at the origin, $(0, 0)$.
- **Transformations** can be performed on the graph of $y = x^2$ to sketch a graph of $y = kx^2 + c$, where:
 - the dilation in the y -direction is by a factor of k
 - the coordinates of the turning point are $(0, c)$.
- A **dilation** by a factor of k in the y -direction will:
 - *enlarge* the parabola in the y -direction if $k > 1$
 - *compress* the parabola in the y -direction if $0 < k < 1$.
- A **reflection** in the x -axis will take place if $k = -1$.
- A reflection and a dilation will take place if $k < 0$ and $k \neq -1$.
- A **vertical translation** will move the parabola c units:
 - *up* if $c > 0$
 - *down* if $c < 0$.
- Note: Reflections and dilations must be applied before translations.



$$y = kx^2 + c$$

↗ dilation
 $k > 0$, upright
 $k < 0$, inverted

↖ vertical translation of c units
 $c > 0$, up
 $c < 0$, down



Example 5D.1 Sketching a parabola using translations



Sketch the graph of $y = x^2$ on a Cartesian plane, and then perform a translation to sketch the graph of each quadratic relationship. Label the y -axis at the intercept and label the turning point with its coordinates.

a $y = x^2 - 3$

b $y = 2x^2$

c $y = -x^2$

THINK

- a 1** Identify the transformation. The relationship is in the form $x^2 + k$ where $k = -3$, so the graph of $y = x^2$ undergoes a vertical translation of 3 units down. So the y -intercept is at the turning point.

$$y = x^2 - 3$$

vertical translation of 3 units down

- 2** Sketch the graph of $y = x^2$ and label its turning point, $(0, 0)$. Translate the turning point 3 units down to find the turning point of $y = x^2 - 3$.
- b 1** Identify the transformation. The relationship is in the form $y = kx^2$ where $k = 2$, so the graph of $y = x^2$ is dilated in the y -direction by a factor of 2.

$$y = 2x^2$$

vertical dilation by a factor of 2

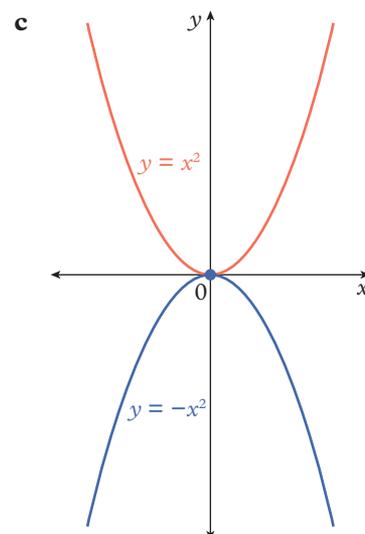
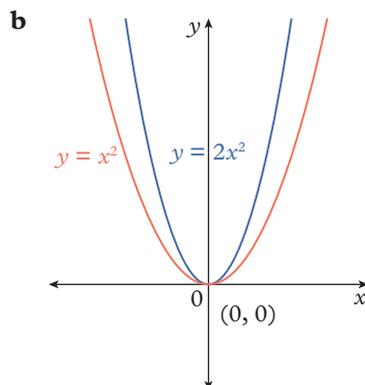
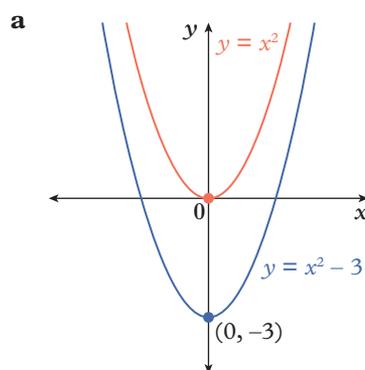
- 2** Sketch the graph of $y = x^2$ and label its turning point $(0, 0)$. Stretch the curve in the y -direction by a factor of 2. That means for any given x -coordinate, every point on $y = 2x^2$ is twice as tall as on $y = x^2$.
- c 1** Identify the transformation. The relationship is in the form $y = kx^2$ where $k = -1$, so the graph of $y = x^2$ is reflected across the x -axis.

$$y = -x^2$$

reflection in the x -axis

- 2** Sketch the graph of $y = x^2$ and label its turning point, $(0, 0)$. Reflect the curve across the x -axis. That means the minimum turning point for the upright parabola at $(0, 0)$ becomes the maximum turning point for the inverted parabola.

WRITE



Example 5D.2 Sketching a parabola using multiple transformations



Sketch the graph of each quadratic relationship by performing transformations on the graph of $y = x^2$. Label the y -axis at the intercept and label the turning point with its coordinates.

a $y = -x^2 + 3$

b $y = 2x^2 - 1$

THINK

a 1 Identify the transformation. The relationship is in the form $y = kx^2 + c$, so the graph of $y = x^2$:

- is inverted, as $k = -1$
- is translated 3 units up, as $c = 3$.

$$y = -x^2 + 3$$

reflected across the x -axis vertical translation of 3 units up

2 Sketch the graph and label the turning point.

b 1 Identify the transformation. The relationship is in the form $y = kx^2 + c$, so the graph of $y = x^2$:

- is stretched in the y -direction by a factor of 2, as $k = 2$
- is translated 1 unit down, as $c = -1$.

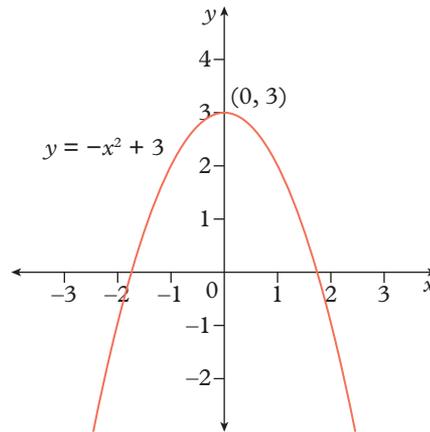
$$y = 2x^2 - 1$$

vertical dilation by a factor of 2 vertical translation of 1 unit down

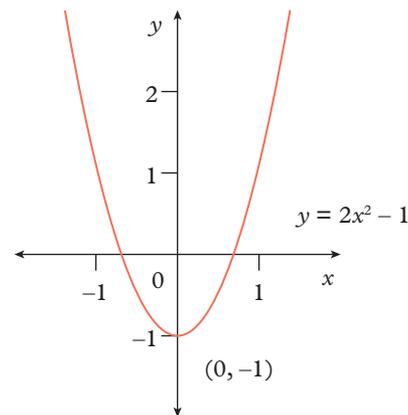
2 Sketch the graph and label the turning point.

WRITE

a

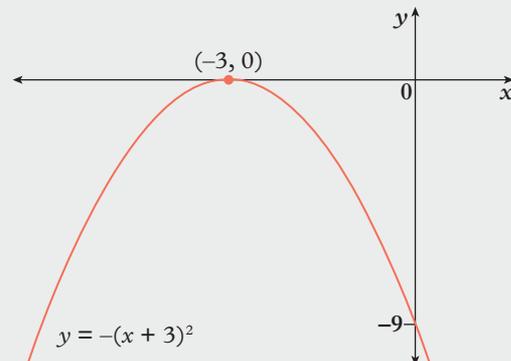


b



Helpful hints

- ✓ Remember that if k is negative, you must reflect the graph in the x -axis before translating.
- ✓ Not all quadratic relationships can be written in the form $y = kx^2 + c$. That means not all parabolas have a turning point at $(0, c)$ and an axis of symmetry at the y -axis. For example, the graph of $y = -(x + 3)^2$ is shown to the right. You will learn more about quadratic relationships in other forms and perform horizontal translations on parabolas in Stage 5 Adv Path.



Exercise 5D Sketching parabolas using transformations

▲ 1, 2(a-c), 3-7, 8(b-d), 9(c), 10-12

■ 1-2(c,d), 3, 6, 7, 8(d-f), 9, 11-15

◆ 2(f), 6, 8(e, f), 12-16

UNDERSTANDING AND FLUENCY

1 Complete the descriptions of transformations below.

a To sketch the graph of $y = x^2 - 10$, the graph of $y = x^2$ is translated _____ units _____.

b To sketch the graph of $y = x^2 + 8$, the graph of $y = x^2$ is translated _____ units _____.

c To sketch the graph of $y = x^2$, the graph of $y = x^2 - 2$ is translated _____ units _____.

d To sketch the graph of $y = x^2 + 2$, the graph of $y = x^2 - 2$ is translated _____ units _____.

5D.1 2 Sketch the graph of $y = x^2$ on a Cartesian plane, then perform a vertical translation to sketch the graph of each quadratic relationship on the same set of axes. Label the turning point on each parabola with its coordinates.

a $y = x^2 + 3$

b $y = x^2 + 1$

c $y = x^2 - 2$

d $y = x^2 + 6$

e $y = x^2 - 4$

f $y = x^2 + 9$

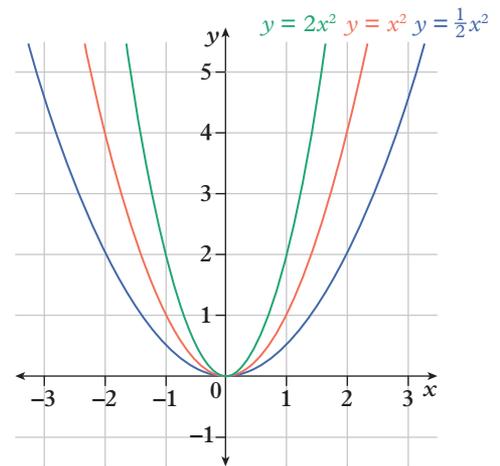
3 Consider the graphs of the form $y = kx^2$.

a What is the value of the dilation factor, k , for $y = x^2$?

b Which graph is narrower than the graph of $y = x^2$? State the value of the graph's dilation factor.

c Which graph is wider than the graph of $y = x^2$? State the value of the graph's dilation factor.

d Describe how the value of the dilation factor affects the width of the corresponding parabola compared to the parabola $y = x^2$.



4 Match each graph on the Cartesian plane on the right with its equation from the list provided below.

I $y = x^2$

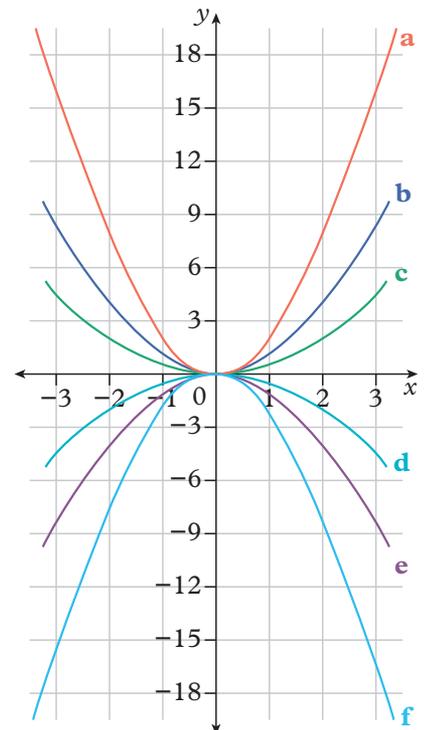
II $y = -2x^2$

III $y = -\frac{1}{2}x^2$

IV $y = \frac{1}{2}x^2$

V $y = 2x^2$

VI $y = -x^2$



5 Sketch the graph of $y = x^2$ on a Cartesian plane, then perform a vertical dilation to sketch the graph of each quadratic relationship on the same set of axes.

a $y = 4x^2$

b $y = \frac{1}{4}x^2$

c $y = -4x^2$

d $y = -\frac{1}{4}x^2$

6 Without plotting the graph, determine if the following quadratic relationships would have a maximum turning point or a minimum turning point.

a $y = x^2 + 2$

b $y = 2x^2 - 12$

c $y = \frac{x^2}{2}$

d $y = -x^2 + 3$

e $y = -3x^2 - 2$

f $y = \frac{x^2}{-3}$

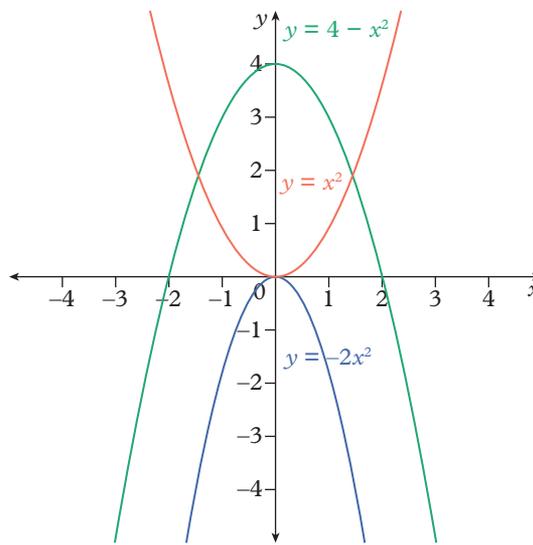
g $y = 10 - 2x^2$

h $y = -2x^2$

7 Complete the following descriptions of transformations.

a To sketch the graph of $y = -2x^2$, the graph of $y = x^2$ is reflected across the _____ and stretched by a factor of 2 in the direction of the _____.

b To sketch the graph of $y = 4 - x^2$, the graph of $y = x^2$ is _____ across the x -axis and translated _____ units _____.



5D.2 8 Sketch the graph of each quadratic relationship by performing transformations on the graph of $y = x^2$. Label the turning point on each parabola with its coordinates.

a $y = -x^2 + 4$

b $y = -x^2 - 3$

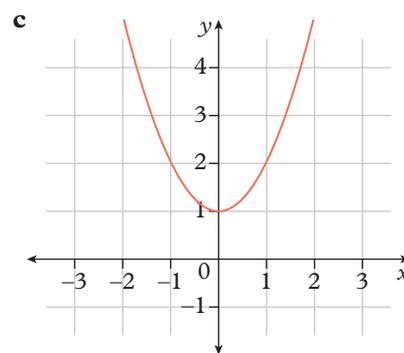
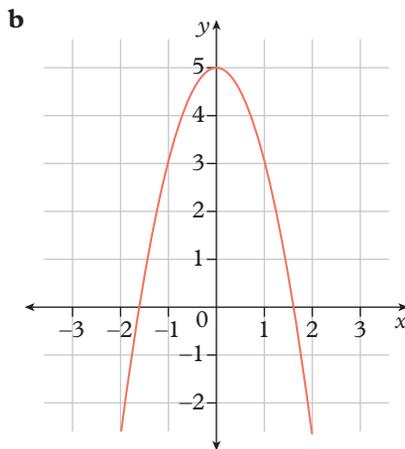
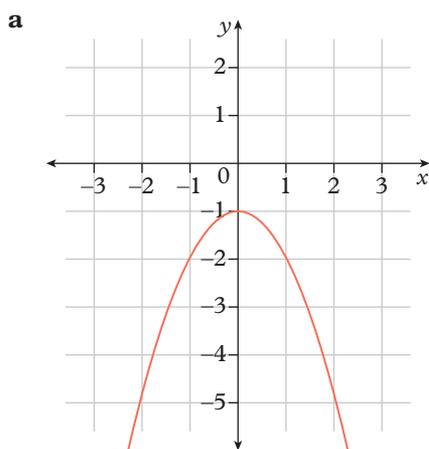
c $y = 2x^2 - 1$

d $y = \frac{x^2}{4} + 1$

e $y = -2x^2 + 4$

f $y = -\frac{x^2}{3} + 2$

9 Consider the parabolas below.



- i** State whether the parabola is concave up or concave down.
 - ii** State whether the graph will have wider, narrower or equal width to the graph of $y = x^2$.
 - iii** Identify the coordinates of the turning point.
- If each graph has an equation of the form $y = kx^2 + c$:
- iv** decide whether k is positive or negative
 - v** determine the value of c .

- 10** Consider the quadratic relationship $y = x^2 + 5$.
- What are the coordinates of the turning point?
 - What is the smallest y -value that this relationship can have?
 - Is there a maximum value for y ? Explain your answer.
- 11** Consider the quadratic relationship $y = -x^2 + 2$. What is the maximum y -value in this relationship? Explain.
- 12** Write the equation for each of the parabolas described below. Assume each parabola has the same shape as $y = x^2$; that is, the dilation factor is $+1$ or -1 .
- | | |
|--|---|
| a concave up, turning point at $(0, 7)$ | b concave up, turning point at $(0, 5)$ |
| c concave down, turning point at $(0, 4)$ | d concave down, turning point at $(0, -1)$ |
- 13** Write an equation for the parabola produced after performing each set of transformations on the graph of $y = x^2$.
- dilation by a factor of 3, then a translation of 2 units up
 - reflection in the x -axis, then a translation of 4 units down
 - dilation by a factor of $\frac{1}{2}$, then a reflection in the x -axis
 - dilation by a factor of 4, reflection in the x -axis, then a translation of 2 units up
- 14** Compare the following pairs of parabolas and determine which parabola would be narrower.
- | | |
|------------------------------------|-----------------------------------|
| a i $y = 2x^2 - 1$ | ii $y = x^2 - 2$ |
| b i $y = -x^2$ | ii $y = -3x^2$ |
| c i $y = 8x^2 - 4$ | ii $y = 4x^2 - 8$ |
| d i $y = \frac{x^2}{4}$ | ii $y = -\frac{x^2}{2}$ |
| e i $y = 12 + 2x^2$ | ii $y = 8 - 4x^2$ |
| f i $y = 2 - \frac{x^2}{4}$ | ii $y = 1 - \frac{x^2}{2}$ |

- 15 a** Perform the following transformations to the graph of $y = x^2$ in the given order.
- Reflect across the x -axis, dilate by a factor of 2, move 3 units up.
 - Dilate by a factor of 2, reflect across the x -axis, move 3 units up.
 - Reflect across the x -axis, move 3 units up, dilate by a factor of 2.
 - Dilate by a factor of 2, move 3 units up, reflect across the x -axis.
 - Move 3 units up, reflect across the x -axis, dilate by a factor of 2.
 - Move 3 units up, dilate by a factor of 2, reflect across the x -axis.
- b** Which of the transformations in part **a** would produce the same graph?
- c** Match the following quadratic relationships to the transformations in part **a**.
- | | | | |
|--------------------------|-----------------------------|------------------------------|-----------------------------|
| I $y = -2x^2 + 3$ | II $y = -(2x^2 + 3)$ | III $y = -2(x^2 + 3)$ | IV $y = 2(-x^2 + 3)$ |
|--------------------------|-----------------------------|------------------------------|-----------------------------|
- 16** A parabola is the set of all points in a plane equidistant from a fixed point and a fixed line.
- Graph $y = \frac{1}{2}x^2 - \frac{1}{2}$ and $y = -1$ on the same Cartesian plane.
 - Verify if the following points are on the parabola.

i $(1, 0)$	ii $(0, -\frac{1}{2})$	iii $(-2, \frac{3}{2})$
-------------------	-------------------------------	--------------------------------
 - What is the vertical distance between each point in part **b** and $y = -1$?
 - Calculate the distance between each point in part **b** and the point $(0, 0)$ and compare it with your solution in part **c**.
 - The fixed line $y = -1$ is called the directrix of the parabola, and the fixed point $(0, 0)$ is called the focus. Choose a point on $y = \frac{1}{2}x^2 - \frac{1}{2}$ and verify if your point of choice is equidistant from the focus and the directrix.

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	Interactive skillsheet Sketching parabolas using transformations		Worksheet Quadratic relationships		Topic quiz 5D
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5E Plotting exponential relationships

Learning intentions

By the end of this section you will be able to ...

- ✓ describe features of exponential curves
- ✓ sketch simple exponential relationships using a table of values.



Inter-year links

Year 10

5G Graphing exponential and logarithmic relationships

Exponential relationships

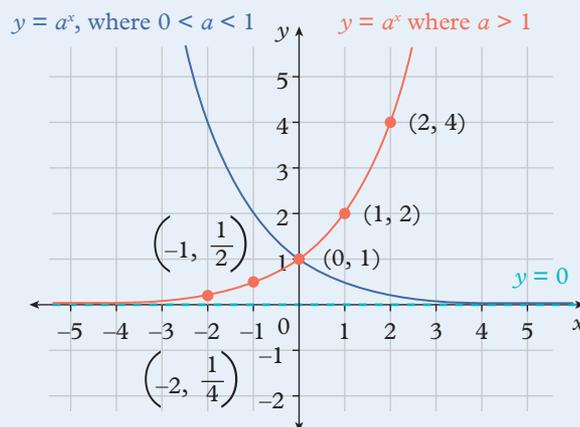


Key content video

- Exponential relationships can be used to model many real situations involving growth and decay; for example, situations involving population growth or investments.
- The basic form of an **exponential relationship** has the following equation:

$$y = a^x$$

index/exponent
base
 where $a > 0$



- The graph has:
 - a y -intercept of 1
 - and a horizontal asymptote along the x -axis.
- An **asymptote** is a boundary line that a curve approaches but never reaches. The asymptote for $y = a^x$ is the horizontal line with equation $y = 0$.
- To plot the graph of an exponential relationship, create a table of coordinate points and join the points on a Cartesian plane with a smooth curve.

- 1 Construct a table of x - and y -values by selecting values for x , then substituting each value of x into the relationship to find the corresponding value of y .

x	-2	-1	0	1	2
$y = 2^x$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4

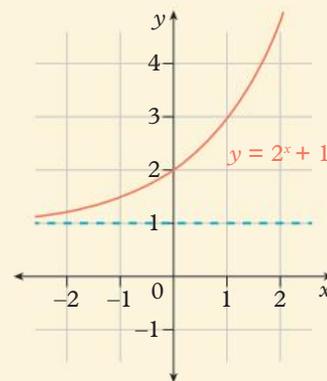
- 2 Write the coordinate points.
 - 3 Plot the coordinate points on a Cartesian plane.
 - 4 Join the points using a smooth curve and label the graph with its asymptote.
- An exponential graph in the basic form can be reflected, dilated and translated to create the graph of other exponential relationships. The new graph would still have a horizontal asymptote and one y -intercept.
 - The new exponential graph can have 0 or 1 x -intercepts. If an x -intercept exists, the x -coordinate can be found by substituting $y = 0$ into the equation.

Example 5E.1 Describing features of an exponential curve



Use the exponential graph of $y = 2^x + 1$ to answer the following questions.

- a Does the curve have an x -intercept?
- b What is the coordinate of the y -intercept of $y = 2^x + 1$?
- c What is the asymptote of $y = 2^x + 1$?
- d As x increases, do the y -values increase, decrease or stay the same?



THINK

- a The red curve does not cross the x -axis, so the curve does not have an x -intercept.
- b The curve crosses the y -axis at 2. Substitute $x = 0$ into the equation to verify:
When $x = 0$, $y = 2^x + 1$
 $\quad = 2^0 + 1$
 $\quad = 1 + 1$
 $\quad = 2$
- c The asymptote of an exponential curve is a straight line that the curve approaches but never reaches. In the graph, the red curve approaches the horizontal line that crossed the y -axis at $y = 1$.
- d The curve trends upwards as we move along the x -axis from left to right, so the y -values increase as x increases.

WRITE

- a The curve does not have an x -intercept.
- b The coordinates of the y -intercept are $(0, 2)$.
- c The asymptote is $y = 1$.
- d The y -values increase as x increases.

Example 5E.2 Plotting exponential equations from a table of values



Sketch the following exponential equations by first completing a table of values for integer values of x from -2 to 2 .

a $y = 3^x$

$$y = \left(\frac{1}{2}\right)^x$$

THINK

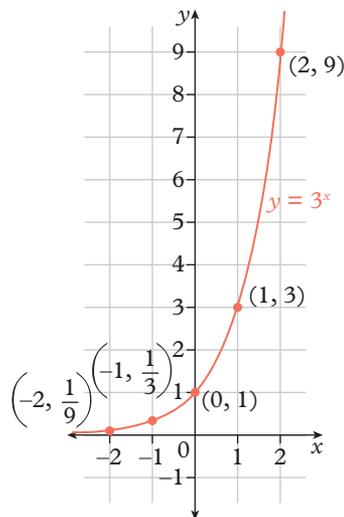
- 1 Construct a table of values for x from -2 to 2 . Substitute each value of x into the equation to find the corresponding values of y .
- 2 Write out the coordinates of each point listed in the table.
- 3 Plot the points on a Cartesian plane, mark the asymptote, and join the points with a smooth curve.

WRITE

a

x	-2	-1	0	1	2
$y = 3^x$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9

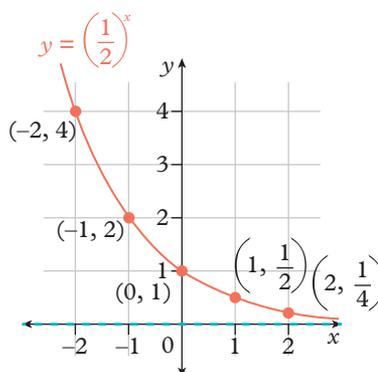
$(-2, \frac{1}{9}), (-1, \frac{1}{3}), (0, 1), (1, 3), (2, 9)$



b

x	-2	-1	0	1	2
$y = (\frac{1}{2})^x$	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$

$(-2, 4), (-1, 2), (0, 1), (1, \frac{1}{2}), (2, \frac{1}{4})$



Helpful hints

- ✓ The graph of an exponential relationship can get increasingly closer to the asymptote but can never cross or touch it.
- ✓ Remember that every exponential graph has an asymptote and you must always include it, even when it is $y = 0$ (which is on the x -axis).

ANS **Exercise 5E** Plotting exponential relationships

1-8, 10

2-11

3-9, 11, 12

1 For each of the following relationships, state whether or not it is exponential. Determine the base for the exponential relationships.

a $y = x^3$
e $y = \frac{1}{x^3}$

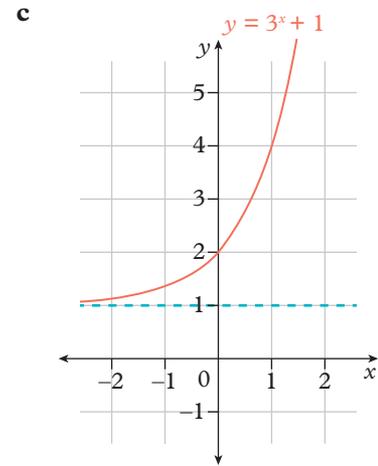
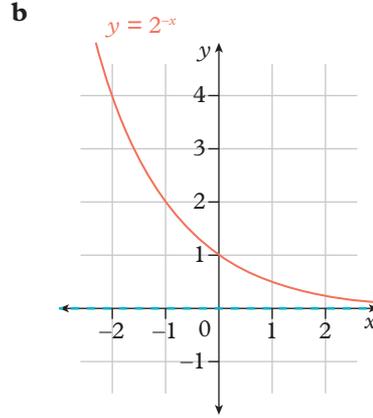
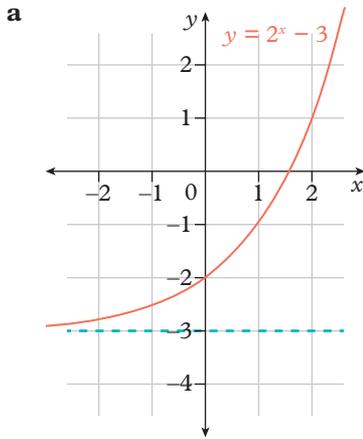
b $y = 3x$
f $y = \frac{1}{3^x}$

c $y = 3^x$
g $y = 3^{-x}$

d $y = \frac{1}{x}$
h $y = (-3)^x$

5E.1 2 Determine the features of the following exponential graphs.

- i** Does the curve have an x -intercept?
- ii** What are the coordinates of the y -intercept?
- iii** What is the asymptote?
- iv** As x increases, do the y -values increase, decrease or stay the same?



3 Complete the following tables of values.

a

x	-2	-1	0	1	2
$y = 4^x$					

b

x	-2	-1	0	1	2
$y = 3^{-x}$					

c

x	-2	-1	0	1	2
$y = \left(\frac{2}{3}\right)^x$					

5E.2 4 Plot the graph of each of the exponential relationships in question 3.

5 Sketch the graph of each exponential relationship using a table of values.

a $y = 2 \times 3^x$

b $y = 3 \times 3^x$

c $y = 0.5 \times 3^x$

d $y = 2 \times \left(\frac{1}{2}\right)^x$

e $y = 3 \times \left(\frac{1}{2}\right)^x$

f $y = 0.5 \times \left(\frac{1}{2}\right)^x$

6 a Draw graphs of $y = 2^x$ and $y = 3^x$ for x -values from -3 to 3 on the same Cartesian plane. You can plot the graphs using a table of values or digital technology.

b Compare the two curves. How are they different?

c Which features are the same?

d Write the y -intercept for each graph.

e Write the rule for the asymptote for each graph.

f Predict how the graph of $y = 5^x$ would compare to the two curves you have drawn.

7 a Draw graphs of $y = \left(\frac{1}{2}\right)^x$ and $y = \left(\frac{1}{3}\right)^x$ for x -values from -3 to 3 on the same Cartesian plane. You can plot the graphs using a table of values or digital technology.

b Compare the two curves. How are they different?

c Which features are the same?

d Write the y -intercept for each graph.

e Write the equation for the asymptote of each graph.

f Predict how the graph of $y = \left(\frac{1}{5}\right)^x$ would compare to the two curves you have drawn.

- 8 Draw the graphs of $y = 2^x$ and $y = 2^{-x}$ for x -values from -3 to 3 on the same Cartesian plane and compare the two curves. You can plot the graphs using a table of values or digital technology.
- How are they different?
 - Which features are the same?
 - Write the y -intercept for each graph.
 - Write the rule for the asymptote for each graph.
 - Describe the transformation that has been performed on the basic graph of $y = 2^x$ to produce $y = 2^{-x}$.
 - What transformations are needed to produce the graph of $y = -2^{-x}$ from the graph of $y = 2^x$?
- 9 Add a row to the table of values you created in question 3 to find the ratio between consecutive values of the dependent variable. What do you notice? Use the index laws to explain your finding.

- 10 Sienna invests \$2000 at 10% p.a. interest compounded annually. The amount, A (in dollars), of the investment after n years can be represented by $A = 2000 \times (1.1)^n$.
- What will the graph of this relationship look like? Sketch the graph showing the key features.
 - Describe what happens to the value of A as n increases.
 - Produce the graph of $A = 2000 \times (1.1)^n$ for $0 \leq n \leq 10$, using a table of values or digital technology to help you. Use the graph to find:
 - the amount of the investment after four years
 - the number of years it takes for the investment amount to be more than \$4000.



- 11 The value of Gabriel's work tools depreciates at the rate of 20% p.a. compounded annually. The asset value, V (in dollars), after n years can be represented by $V = 3500 \times (1.25)^{-n}$.
- What will the graph of this relationship look like? Sketch the graph showing the key features.
 - What was the initial value of Gabriel's tools?
 - Describe what happens to the value of V as n increases.
 - Produce the graph of $V = 3500 \times (1.25)^{-n}$ for $0 \leq n \leq 10$, using a table of values or digital technology to help you. Use the graph to find the number of years it takes for the asset value of the tools to be less than \$1000.

- 12 The equation $x^2 = 2^x$ has three solutions.

- a Complete the following table to find two of the solutions.

x	-2	-1	0	1	2	3	4	5
x^2								
2^x								

- b Use the table to explain why the third solution must be between -1 and 0 .
- c Sketch the graphs of $y = x^2$ and $y = 2^x$ using appropriate technology, and find the value of the third solution to $x^2 = 2^x$, rounded to three decimal places.

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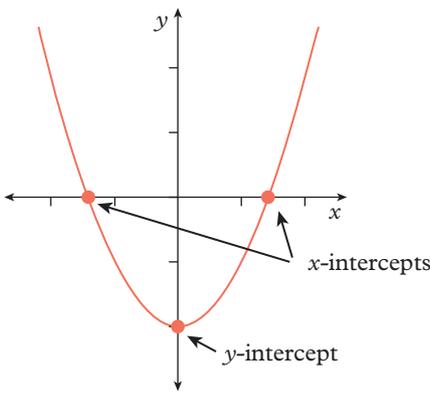
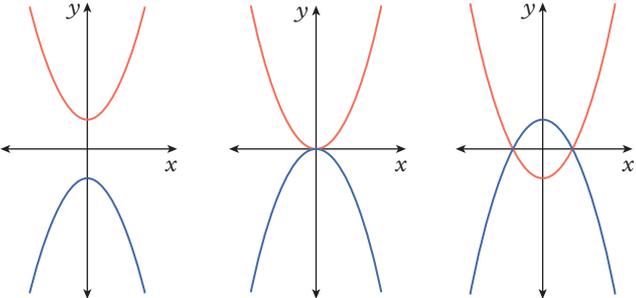
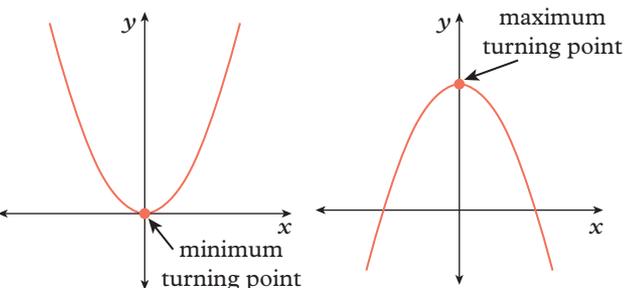
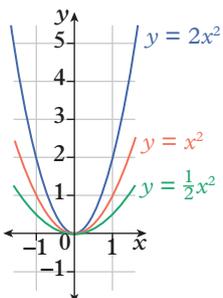
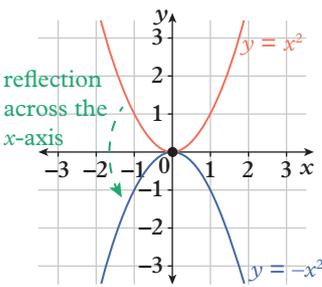
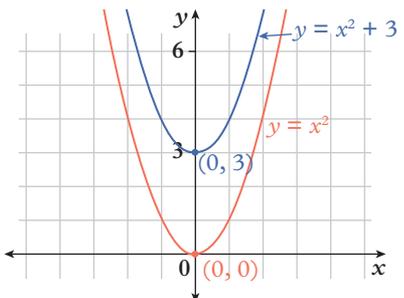
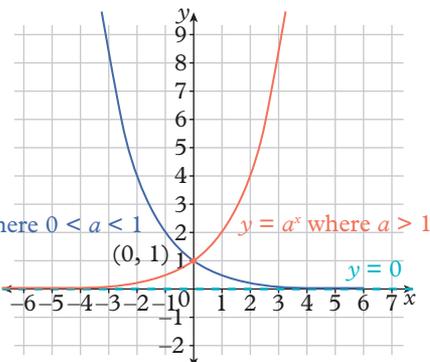


Interactive skillsheet
Exponential relationships



Topic quiz
5E

Chapter summary

<p>The Null Factor Law</p> <ul style="list-style-type: none"> If $a \times b = 0$, then $a = 0$ or $b = 0$. If $(x - p)(x - q) = 0$, then $x - p = 0$ or $x - q = 0$. 	<p>Parabolas</p> 
<p>Intercepts of parabolas</p>  <p>no x-intercept one x-intercept two x-intercepts</p>	<p>Transformation of parabolas</p> $y = kx^2 + c$ <p>dilation $k > 0$, concave up $k < 0$, concave down</p> <p>vertical translation of c units $c > 0$, up $c < 0$, down</p>
<p>Turning points</p> 	<p>Dilations in the y-direction</p>  <p>Reflection</p>  <p>Translation</p> 
<p>Exponential graphs</p> <p>$y = a^x$ where $a > 0$.</p> <p>The basic exponential equation graph has a y-intercept of $(0, 1)$ and an asymptote along the x-axis with equation $y = 0$.</p>	

Chapter review



Chapter review quiz

Assess your knowledge of this chapter.

- ✓ Core
- ✓ Advanced

Quizlet

Test your knowledge of this topic by working individually or in teams.

Mathematical literacy review

The following key terms are used in this chapter:

- asymptote
- base
- concavity
- dilation
- exponent
- exponential
- factor
- factorise
- highest common factor
- leading coefficient
- leading term
- linear equation
- monic
- non-linear
- Null Factor Law
- parabola
- quadratic equation
- reflection
- symmetry
- transformation
- turning point
- vertex

1 Complete the following sentences using words from the list of key terms.

Solving a quadratic equation involves the following steps:

Step 1: Have all the terms on one side so the equation equals zero.

Step 2: Identify and divide by the _____ if present.

Step 3: _____ the quadratic equation into two binomial terms being multiplied.

Step 4: Apply the _____ by equating each _____ to 0.

Step 5: Solve each _____.

2 Match the following descriptions in the left-hand column with a term in the right-hand column for quadratic relationships.

- | | |
|---|------------------------------------|
| a A form of a quadratic relationship | I Parabola |
| b Indicates the y -intercept | II Concavity |
| c The shape of a quadratic graph | III Turning point or vertex |
| d The maximum or minimum point on the graph | IV $y = kx^2 + c$ |
| e A vertical line of reflection for the graph | V 0, 1 or 2 |
| f The number of x -intercepts | VI The x -intercepts |
| g Describes if the curve opens upwards or downwards | VII The value of c |
| h The solutions to the factored form of a quadratic equation | VIII Axis of symmetry |

3 State true or false for the following statements about the transformation of a quadratic graph $y = kx^2$. If it is false, change the statement to make it true.

- a** If $k > 1$, then the graph has a vertical dilation and is enlarged.
 - b** If $k < -1$, then the graph has a vertical dilation and is compressed.
 - c** If $k = -1$, then the graph has a vertical dilation.
 - d** If $0 < k < 1$, then the graph is compressed vertically.
- A constant, c , and $c \neq 0$, is added to the equation to give $y = kx^2 + c$.
- e** The graph will now be translated vertically.
 - f** The graph will pass through the origin.
 - g** The coordinates of the vertex are $(c, 0)$.

Multiple choice

- ADV 5A 1** The solution(s) to the quadratic equation $x(x + 1) = 0$ are:
A $x = -1$ **B** $x = 1$ **C** $x = -1$ or $x = 1$ **D** $x = 0$ or $x = -1$ **E** $x = 0$ or $x = 1$
- ADV 5A 2** In the quadratic equation $x^2 - 13x - 48 = 0$, $x =$
A -3 or 16 **B** -16 or 3 **C** -12 or 4 **D** -6 or 8 **E** -8 or 6
- 5B 3** Which of the following parabolas has two x -intercepts?
A parabola with a maximum point at $(0, 2)$
B parabola with a maximum point at $(0, -2)$
C parabola with a minimum point at $(0, 0)$
D parabola with a maximum point at $(0, 0)$
E parabola with a minimum point at $(0, 2)$
- 5C 4** What are the coordinates of the y -intercept of $y = \frac{x^2}{2} - 2$?
A $(0, 2)$ **B** $(0, -2)$ **C** $(\sqrt{2}, 0)$ **D** $(-2, 0)$ **E** $(0, \sqrt{2})$
- 5C 5** Which of the following quadratic relationships is concave up?
A $y = 3 - 2x^2$ **B** $y = -3 - 2x^2$ **C** $y = -3x^2 + 2x^2$ **D** $y = 3x^2 - 2$ **E** $y = -3x^2 + 2$
- 5D 6** The graph of $y = x^2$ is translated 5 units down. The equation of the translated graph is:
A $y = x^2 + 5$ **B** $y = 5x^2$ **C** $y = x^2 - 5$ **D** $y = (x + 5)^2$ **E** $y = \frac{1}{5}x^2$
- 5D 7** Which of the following transformations is required to sketch the graph of $y = 4x^2 - 1$?
A $y = x^2$ must be translated to the left by 1 unit.
B $y = x^2$ must be reflected across the y -axis.
C $y = x^2$ must be reflected across the x -axis.
D $y = x^2$ must be dilated.
E $y = x^2$ must be translated up by 1 unit.
- 5D 8** Which of the following parabolas is the narrowest when plotted on one Cartesian plane?
A $y = \frac{x^2}{2}$ **B** $y = \frac{x^2}{4}$ **C** $y = 2x^2$ **D** $y = -4x^2$ **E** $y = x^2 - 2$
- 5E 9** Which of the following relationships have one x -intercept?
A $y = 2^x$ **B** $y = x^2 + 2$ **C** $y = x^2 - 2$ **D** $y = 2^x + 2$ **E** $y = 2^x - 2$
- 5E 10** What are the coordinates of the y -intercept of $y = 3^x$?
A $(0, 3)$ **B** $(0, 1)$ **C** $(3, 0)$ **D** $(-3, 0)$ **E** $(1, 0)$
- 5E 11** Which description best describes an exponential graph, $y = a^x$?
A a graph which is constantly increasing if $a > 1$; has a minimum and has an asymptote at $y = 0$
B a graph which is constantly decreasing if $0 < a < 1$; does not have a minimum and has an asymptote at $y = 0$
C a graph which is constantly increasing if $a > 1$; does not have a minimum and has an asymptote at $x = 0$
D a graph which is constantly increasing if $0 < a < 1$; does not have a minimum and has an asymptote at $x = 0$
E a graph which is constantly decreasing if $0 < a < 1$; has a minimum and has an asymptote at $y = 0$

Short answer

- ADV 5A 1** Solve:
a $x^2 - 5x + 6 = 0$ **b** $x^2 + x - 30 = 0$ **c** $x^2 + 9 = 0$ **d** $3x^2 - 36x = 0$
- ADV 5A 2** Verify if $x = 2$ is a solution to the following quadratic equations.
a $x^2 + 2x + 4 = 0$ **b** $2x^2 + 8 = 8x$ **c** $3x^2 - 6x = 0$ **d** $(4 - 2x)(x + 2) = 0$

- 5B** 3 Sketch a graph of the following relationships by first completing a table of values for x from -3 to 3 . Use your graph to identify:
- the coordinates of the x - and y -intercepts
 - the coordinates of the turning point
 - whether there is a maximum or a minimum turning point
 - the equation of the axis of symmetry.
- a** $y = 4x^2 - 4$ **b** $y = -4x^2 + 4$ **c** $y = x^2 + 4$
- 5C** 4 Complete the following for each quadratic relationship below.
- Find the coordinates of the x - and y -intercepts.
 - Sketch the graph.
- a** $y = 2x^2 - 32$ **b** $y = 64 - x^2$ **c** $y = 15 - 3x^2$
- 5D** 5 Sketch a graph of each quadratic relationship by performing transformations on the graph of $y = x^2$. Label the y -axis at the intercept and label the turning point with its coordinates.
- a** $y = x^2 + 3$ **b** $y = 2x^2$ **c** $y = x^2 - 2$ **d** $y = -x^2$
- 5D** 6 Sketch the graph by performing transformations on the graph of $y = x^2$. Label the turning point on each parabola with its coordinates.
- a** $y = -2x^2$ **b** $y = 2 - x^2$ **c** $y = 3x^2 + 1$ **d** $y = 1 - 2x^2$
- 5E** 7 Find the coordinates of the y -intercept of the following exponential relationships.
- a** $y = 2^x$ **b** $y = -2^x$ **c** $y = 3^x$ **d** $y = \left(\frac{1}{3}\right)^x$
- 5E** 8 Plot the following exponential relationships by first completing a table of values for x from -2 to 2 .
- a** $y = 3^x$ **b** $y = \left(\frac{1}{2}\right)^x$

Analysis

- 1 A parabolic through-arch bridge has an upper arch and a lower arch that are closely modelled by the following quadratic relationships:

$$\text{Lower arch: } h = -\frac{1}{120}d^2 + 30$$

$$\text{Upper arch: } h = -\frac{1}{180}d^2 + 40$$

where h is the height of the arch at a horizontal distance of d from the middle of the bridge. Both h and d are in metres.



- a Sketch the two arches on the same Cartesian plane from $d = -60$ m to $d = 60$ m.
- b For each parabola, determine:
 - i the coordinates of the y -intercepts
 - ii the coordinates of the turning points.
- c What is the height of the upper arch above the lower arch at each end of the bridge?
- d What is the span of the bridge?

The distance between the arches does not remain constant over the span of the bridge.

- e Find the height of the highest point of the:
 - i lower arch
 - ii upper arch.
 - f How far apart are the two arches at their highest point?
 - g How far apart are the two arches at a horizontal distance of 30 m from the middle of the bridge?
- 2 Buzz can hit a golf ball that lands 100 metres away after reaching a height of 80 metres. Assume the course is on level ground and ignore any additional bouncing and rolling. The ball's path can be modelled with the equation $h = a(d + 50)(d - 50)$ where h is the height in metres and d is the horizontal distance, in metres, from the highest point the ball reached. The value a is a constant.
- a Simplify the given equation.
 - b Determine the horizontal intercepts and describe what they represent.
 - c Write down the coordinates for the maximum height of the ball.
 - d Sketch a graph showing the path of the ball.
 - e Use the coordinates for the maximum height to determine the value for a in the equation.
 - f Calculate the height of the ball when it has travelled 20 metres horizontally from where it was hit. Round to one decimal place.
 - g What is the other horizontal position from when the ball was hit when the ball will be at the same height as in part f?
 - h Draw lines on your graph to show your answers to parts f and g.

3 Explore quadratic relationships written in different forms.

- a Complete the following table of values for equations A and B.

Equation	x	-2	-1	0	1	2
A	$y = x^2$					
B	$y = 4x^2$					
C						

- b Describe how Equation A has been transformed to make Equation B.
- c Complete the table for Equation C, $y = (2x)^2$. What do you notice?
- d Write two equations for each of the following transformations of $y = x^2$:
 - i dilation of 9
 - ii dilation of 2
 - iii dilation of -4
 - iv dilation of k , for $k > 0$.
- e Complete the following table of values for equations A and B.

Equation	x	-2	-1	0	1	2
A	$y = x^2$					
B	$y = x^2 - 1$					
C						

- f** Describe how Equation A has been transformed to make Equation B.
- g** Complete the table for Equation C, $y = (x + 1)(x - 1)$. What do you notice?
- h** Write two equations for the following transformations of $y = x^2$:
- i** translation of 4 units down
 - ii** translation of 2 units down
 - iii** translation of c units down, for $c > 0$
 - iv** explain why there is not an equivalent equation for $c < 0$.
- 4** Explore exponential relationships written in different forms.
- a** Complete the following table of values for equations A and B.

Equation	x	-2	-1	0	1	2
A	$y = 2^x$					
B	$y = 2 \times 2^x$					
C						

- b** Describe how Equation A has been transformed to make Equation B.
- c** Complete the table for Equation C, $y = 2^{x+1}$. What do you notice?
- d** Write two equations for a dilation of 4 units.
- e** For the equation $y = k \times 2^x$, what values of k will follow a similar pattern?
- f** Choose appropriate values for a given $k > 0$ and write the equation $y = k \times 3^x$ and the corresponding equation $y = 3^{x+a}$ when:
- i** a is a positive integer
 - ii** a is a negative integer.

Chapter checklist



Now that you have completed this chapter, reflect on your ability to do the following.

I can do this	I need to review this
<input type="checkbox"/> Solve simple quadratic equations	<input type="checkbox"/> Go back to Topic 5A Solving quadratic equations
<input type="checkbox"/> Identify the key features of a parabola <input type="checkbox"/> Plot parabolas from tables of values	<input type="checkbox"/> Go back to Topic 5B Plotting quadratic relationships
<input type="checkbox"/> Find the intercepts of a quadratic relationship in the form $y = kx^2 + c$ <input type="checkbox"/> Sketch parabolas using the intercepts	<input type="checkbox"/> Go back to Topic 5C Sketching parabolas using intercepts
<input type="checkbox"/> Sketch quadratic relationships in the form $y = kx^2 + c$ using reflections, dilations and vertical translations	<input type="checkbox"/> Go back to Topic 5D Sketching parabolas using transformations
<input type="checkbox"/> Describe features of exponential curves <input type="checkbox"/> Sketch simple exponential relationships using a table of values	<input type="checkbox"/> Go back to Topic 5E Plotting exponential relationships

6

Measurement





Index

- 6A Area of composite shapes
- 6B Surface area
- 6C Surface area of cylinders
- 6D Volume of composite solids
- 6E Errors

Prerequisite skills



Diagnostic pre-test

Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.



Interactive skillsheets

After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✓ Area of quadrilaterals
- ✓ Area of a circle
- ✓ Identifying shapes within composite shapes
- ✓ Substitution
- ✓ Fractions, decimals and percentages

Curriculum links

- Solves problems involving the surface area of right prisms and practical problems involving the area of composite shapes and solids (MA5-ARE-C-01)
 - Solve problems involving areas and surface areas
 - Develop and apply the formula for surface areas of cylinders
 - Solve problems involving surface areas of cylinders and related composite solids
- Solves problems involving the volume of composite solids consisting of right prisms and cylinders (MA5-VOL-C-01)
 - Solve problems involving composite solids consisting of right prisms and cylinders
- Solves measurement problems by using scientific notation to represent numbers and rounding to a given number of significant figures (MA5-MAG-C-01)
 - Find absolute and percentage error

© NESAs

Materials

- ✓ Calculator

6A Area of composite shapes

Learning intentions

By the end of this topic you will be able to ...

- ✓ calculate the area of composite shapes.



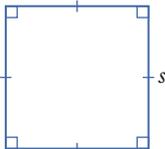
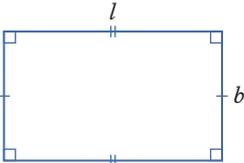
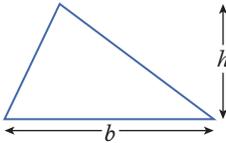
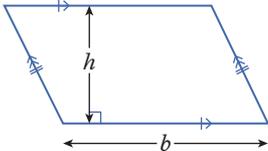
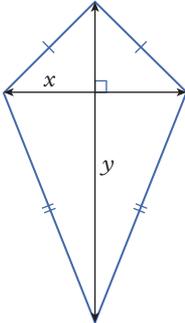
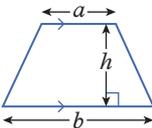
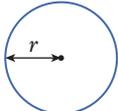
Inter-year links

- Support** Understanding area
- Year 7** 8E Area of a triangle
- Year 8** 8D Area of a circle
- Year 10** 8A Area review

Area of simple shapes

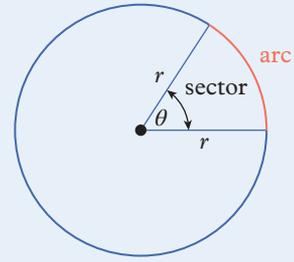


Key content video

Shape	Example	Formula
Square		$A = s^2$
Rectangle		$A = lb$
Triangle		$A = \frac{1}{2}bh$
Parallelogram/rhombus		$A = bh$
Kite/rhombus		$A = \frac{1}{2}xy$
Trapezium		$A = \frac{h}{2}(a + b)$
Circle		$A = \pi r^2$

Sectors

- A **sector** is a portion of a circle formed by two radii and part of the **circumference** (an **arc**).
- The area of a sector can be found using $\frac{\theta}{360} \times \pi r^2$, where θ (the Greek letter theta) is the size of the angle in degrees between the two radii.
 - This formula is derived from the formula for the area of a circle, $A = \pi r^2$, where $\frac{\theta}{360}$ represents the proportion of a full circle that the sector represents.



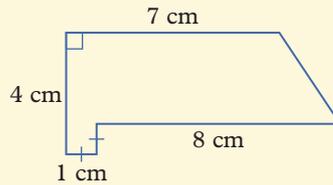
Composite shapes

- A **composite shape** can be split up into two or more simple shapes.
- To calculate the area of a composite shape, follow these steps:
 - 1 Split the figure into simple shapes that have known area formulas.
 - 2 Calculate any missing dimensions.
 - 3 Calculate the areas of the individual shapes using the area formulas.
 - 4 Add or subtract the areas to calculate the total area.

Example 6A.1 Calculating the area of a composite shape using addition



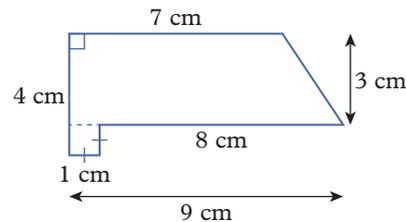
Calculate the area of this composite shape.



THINK

- 1 Use dotted lines to split the composite figure into simple shapes.
- 2 Find any missing dimensions for each individual shape and label these on your figure.
- 3 Use area formulas to calculate the areas of the simple shapes.
- 4 Add the areas together to find the total area of the composite shape. Remember to include the appropriate unit.

WRITE

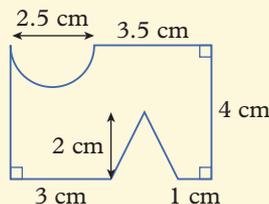


$$\begin{aligned}
 A_{\text{square}} &= s^2 \\
 &= 1^2 \\
 &= 1 \text{ cm}^2 \\
 A_{\text{trapezium}} &= \frac{h}{2}(a + b) \\
 &= \frac{3}{2} \times (7 + 9) \\
 &= 24 \text{ cm}^2 \\
 \text{Total area} &= 24 + 1 \\
 &= 25 \text{ cm}^2
 \end{aligned}$$



Example 6A.2 Calculating the area of a composite shape using subtraction

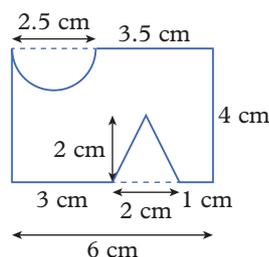
Calculate the area of this composite shape, correct to one decimal place.



THINK

- Use dotted lines to split the composite figure into simple shapes.
- Find any missing dimensions for each individual shape and label these on your figure.
- Use area formulas to calculate the areas of the simple shapes.
- Subtract the smaller areas from the main area to find the total area of the composite shape. Remember to round your answer and include the appropriate unit.

WRITE



$$\begin{aligned} A_{\text{rectangle}} &= lb \\ &= 6 \times 4 \\ &= 24 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} A_{\text{triangle}} &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 2 \times 2 \\ &= 2 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} A_{\text{semicircle}} &= \frac{1}{2}\pi r^2 \\ &= \frac{1}{2} \times \pi \times 1.25^2 \\ &= 2.45\dots \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Total area} &= 24 - 2 - 2.45\dots \\ &= 19.54\dots \\ &\approx 19.5 \text{ cm}^2 \end{aligned}$$

Helpful hints

- ✓ When calculating the areas of composite shapes, a useful technique can be to number the different parts, e.g. A_1 = Area of shape 1. This is useful when there is more than one of a particular type of simple shape in the composite figure.
- ✓ Many composite shapes can be split up in more than one way. There is not a correct way to split up a composite shape, but you should look for the easiest method. However the shape is split up, the total area should be the same!
- ✓ Remember that area is a measure of two-dimensional space so the units of measurement must be squared: mm^2 , cm^2 , m^2 , km^2 .

Exercise 6A Area of composite shapes



1-3, 4(a, b, c, e), 5-7, 8(c, d, f), 9, 12, 13(a, b)

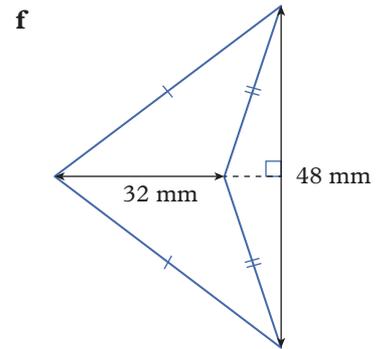
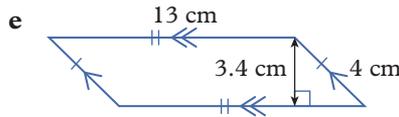
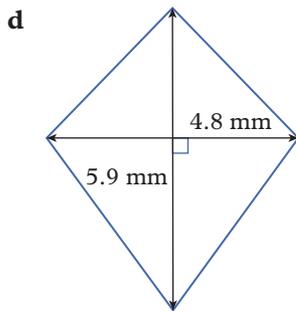
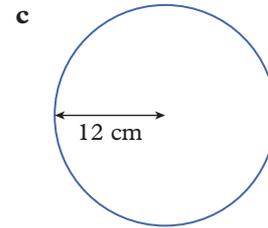
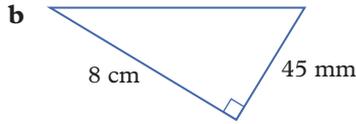
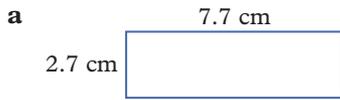


1, 2, 4, 5, 7, 8(b, d, e, f), 10, 12, 13(c, d, f), 15, 18(a, b)

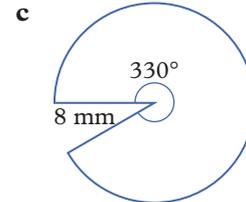
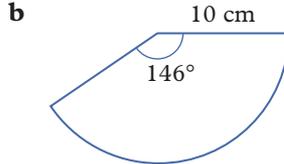
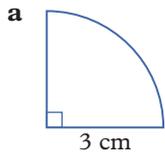


2, 4, 5, 8(b, e, f), 10, 11, 13(d, e, f), 14, 16, 17, 18(b, c)

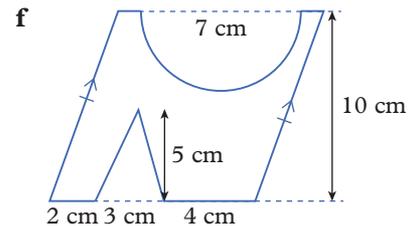
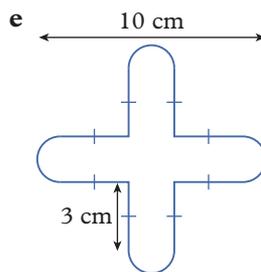
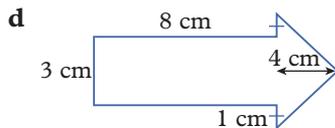
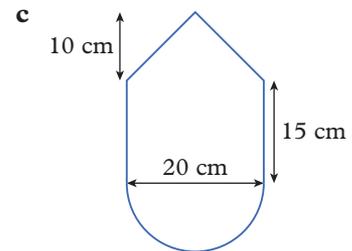
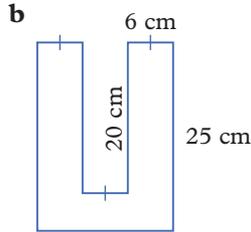
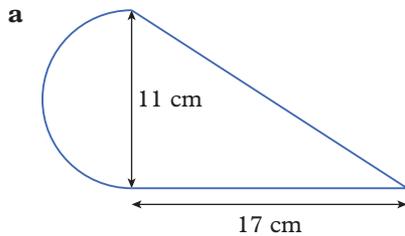
1 Calculate the area of these simple shapes. Give your answers to two decimal places where necessary.



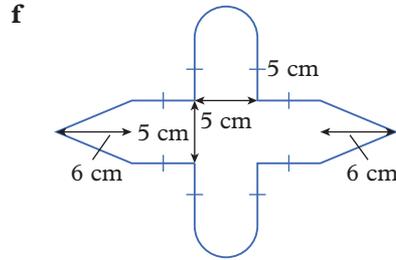
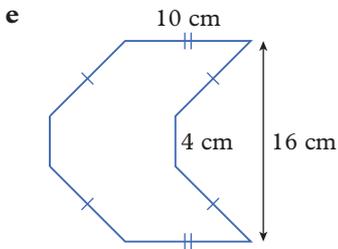
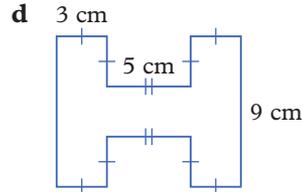
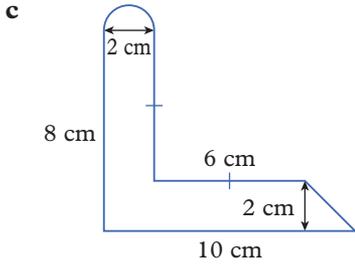
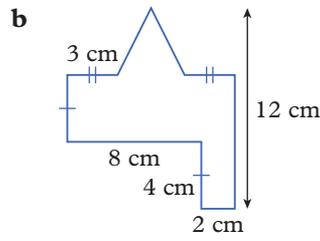
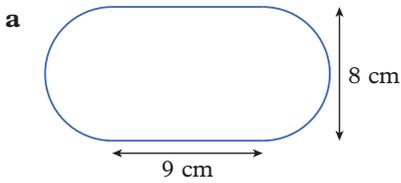
2 Calculate the area of each sector correct to two decimal places.



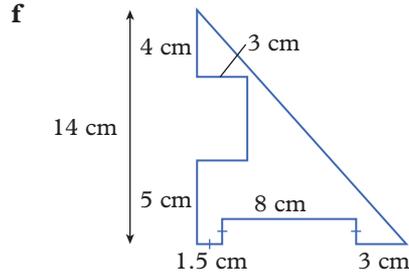
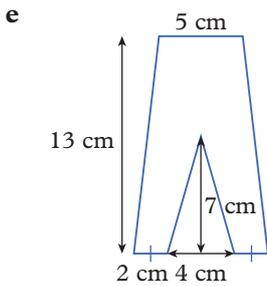
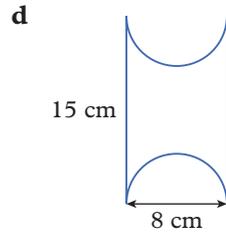
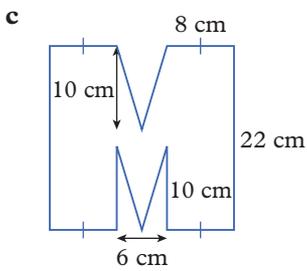
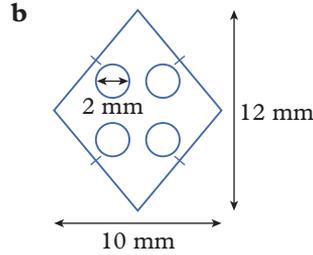
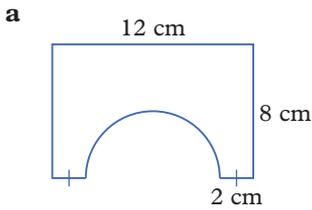
3 Identify the basic shapes within each composite shape.



6A.1 4 Calculate the area of each composite shape. Give your answers to two decimal places where necessary.

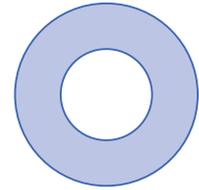


6A.2 5 Calculate the area of each composite shape. Give your answers to two decimal places where necessary.



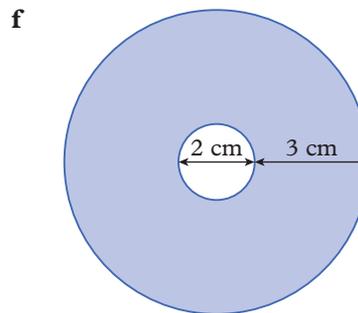
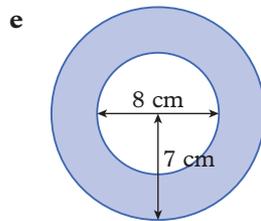
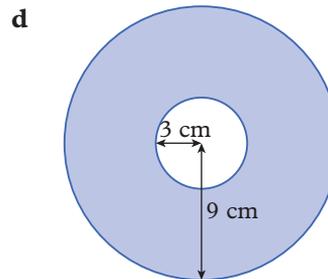
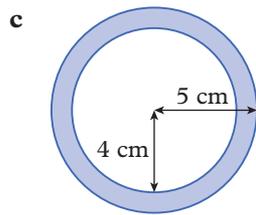
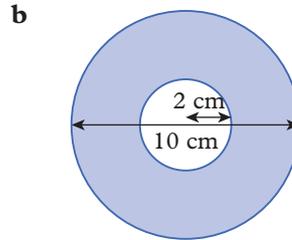
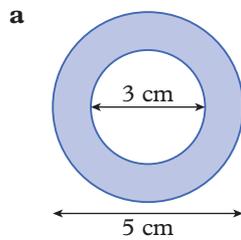
6 Calculate the area of each composite shape in question 3. Give your answers to two decimal places where necessary.

7 This figure is known as an annulus (plural annuli).

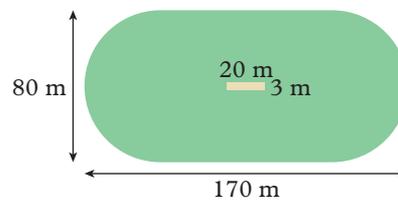


- a What is an annulus?
- b Explain how you can find the area of this annulus using a formula.
- c Find the area of this annulus if it has an outer diameter of 16 cm and an inner diameter of 9 cm. Give your answer to two decimal places.

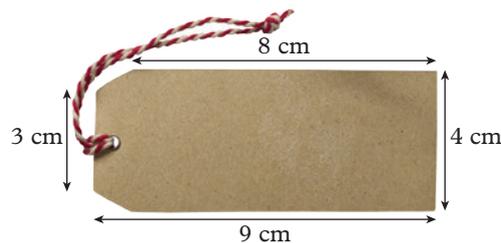
8 Calculate the area of each annulus correct to two decimal places.



9 What area of grass needs to be mowed on this sports oval, assuming that the cricket pitch in the middle is artificial and does not require mowing? Give your answer to two decimal places.



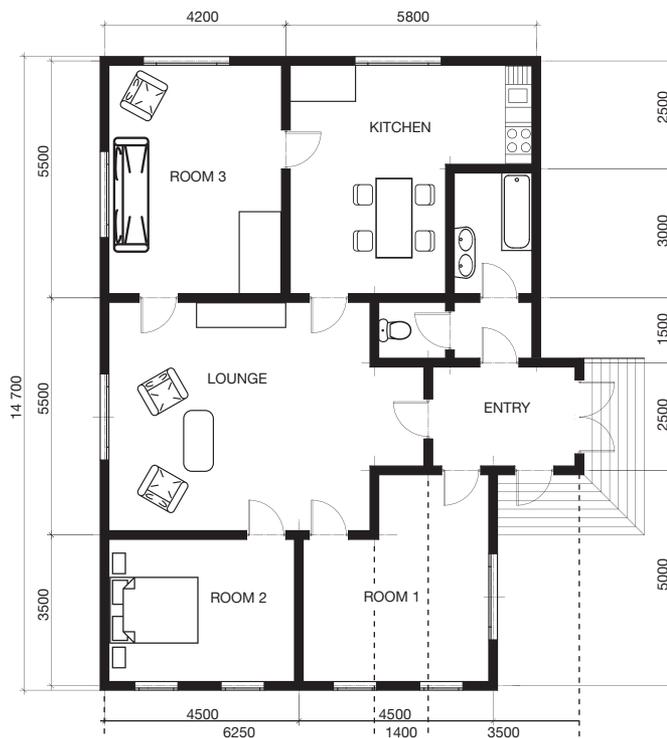
10 Emily makes and sells custom tags through the website *Etsy*.



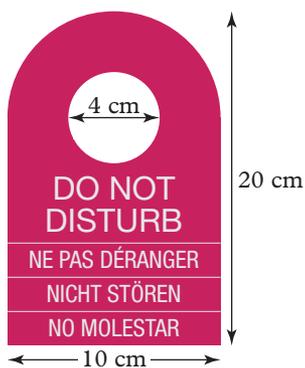
- a What is the area of cardboard in a tag with the dimensions shown?
- b What is the total area of cardboard required to make 550 tags with the dimensions shown? Explain why Emily might need to buy more than this amount of cardboard in order to make the tags.

11 Shannon wants to lay new carpet in his house. Consider the floor plan with measurements in millimetres.

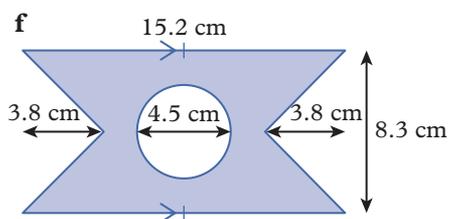
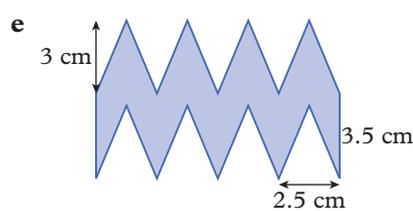
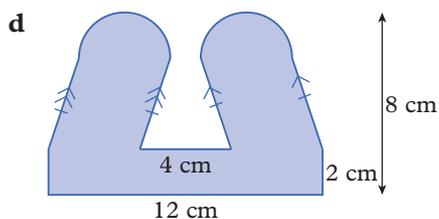
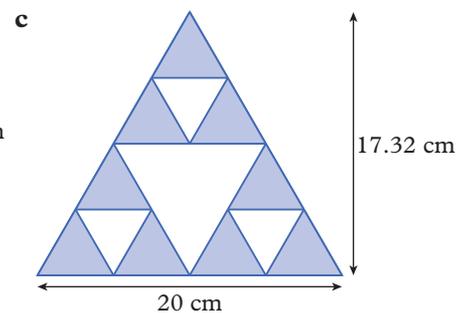
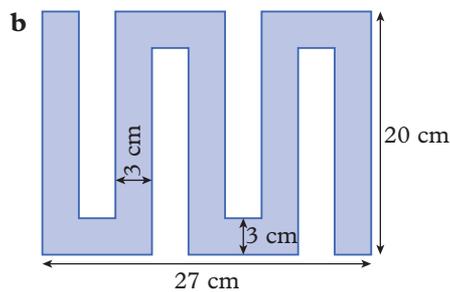
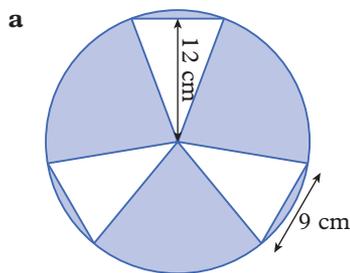
- What area of the house would he need to cover if everything except the bathroom, kitchen and toilet was to be carpeted? Write this in square metres. (Hint: You may find it easier to convert the measurements to metres.)
- How much would it cost if the carpet is priced at \$45/m²?
- How much money would he save if he chose a cheaper carpet at \$30/m²?



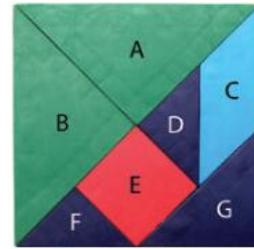
12 Find the amount of cardboard in this DO NOT DISTURB sign correct to two decimal places.



13 Calculate the shaded area of each shape. Give your answers to two decimal places where necessary.

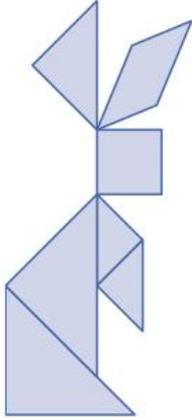


14 The tangram is an ancient Chinese puzzle that consists of seven pieces (usually placed in a square as shown) that can be rearranged to form a variety of shapes.

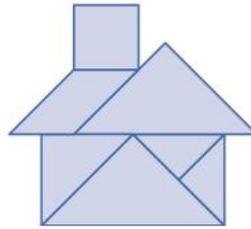


- a** What is the area of each individual piece in this tangram square if the entire square has side lengths of 10 cm?
- b** What is the total area of each of these shapes that can be made from the seven pieces of the tangram?

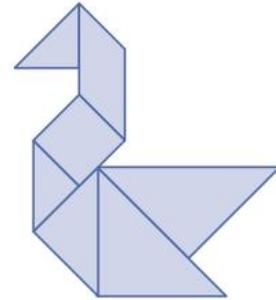
i



ii



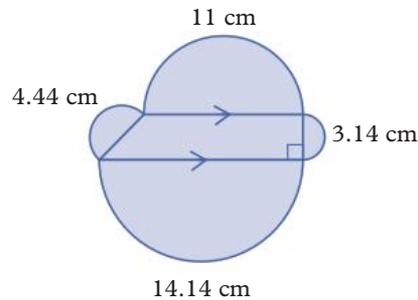
iii



15 Find the area of an annulus with an outer circumference of 35 cm and an inner circumference of 25 cm. Give your answer to two decimal places.

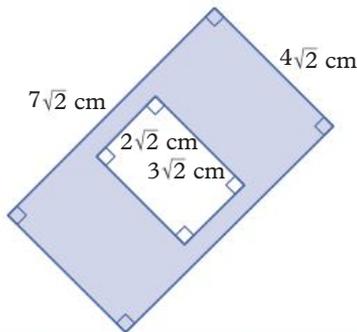
16 A circular skirt, without hems, is cut from a circle of material with a hole for the waist. If a particular circular skirt had an inner circumference of 70 cm and was 60 cm in length when worn, what was the area of material used to create the skirt? Give your answer to two decimal places.

17 Calculate the shaded area correct to two decimal places, given the number shown on each arc is the length of the corresponding semicircle.

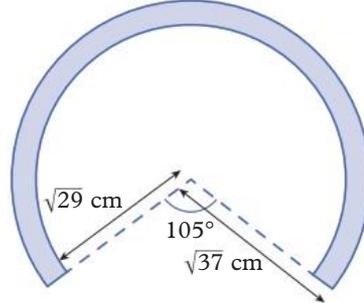


18 Calculate the shaded area of each shape. Give your answers correct to two decimal places where necessary.

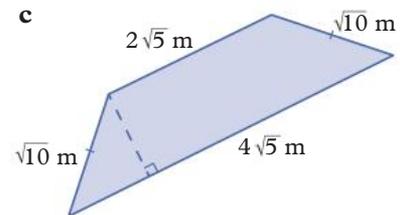
a



b



c



Check your Student obook pro for these digital resources and more:

pro



Interactive skillsheet
Area of composite shapes



Worksheet
Identifying simple shapes within composite shapes



Worksheet
Area of shapes



Topic quiz
6A

6B Surface area

Learning intentions

By the end of this topic you will be able to ...

- ✓ draw the net for right prisms
- ✓ calculate the surface area of right prisms.



Inter-year links

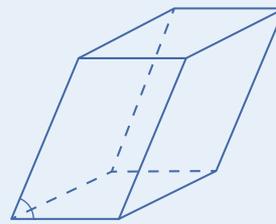
- Year 7** 8C Area of a rectangle
- Year 8** 8C Area of quadrilaterals
- Year 10** 8B Surface area review

Prisms

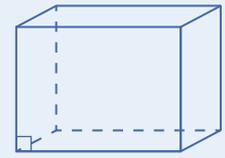


Key content video

- A **prism** is a 3D object with straight edges, two identical bases and a constant cross-section.
 - A **face** is a flat surface on a three-dimensional (3D) object.
 - An **edge** is a line segment that joins two faces.
 - A **cross-section** of a 3D object is the 2D shape obtained when cutting through the object.
- A **right prism** has right angles between the base and the sides.
 - All the sides (non-bases) of a right prism are rectangles.

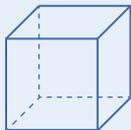


Non-right prism



Right prism

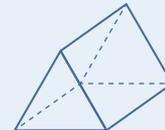
Cube (6 faces)



Rectangular prism (6 faces)



Triangular prism (5 faces)

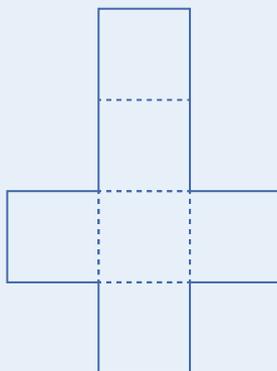


- A **cube** has six faces that are all identical squares.
- A **rectangular prism** has six faces. There are three pairs of identical rectangular faces.
- A **triangular prism** has five faces. The two ends of a triangular prism are identical triangular faces.

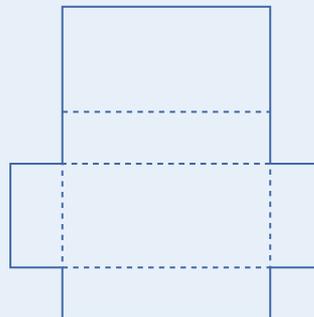
Nets

- A **net** is a 2D plan that can be folded to form a 3D object.
- Nets can be used to show all the faces of 3D objects and hence to help calculate the surface areas of those objects.

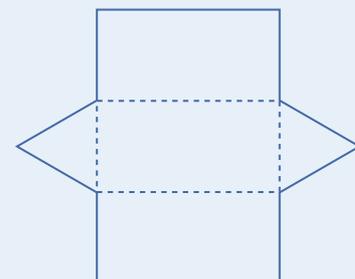
Net of a cube



Net of a rectangular prism

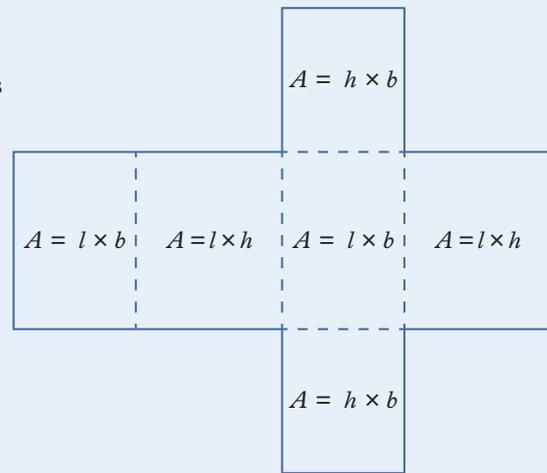


Net of a triangular prism



Surface area

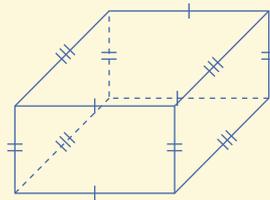
- The **surface area** of a three-dimensional (3D) object is the total area of the outer surface of that object.
- To calculate the surface area of a 3D object:
 - 1 Determine the number of faces.
 - 2 Calculate the area of each face.
 - 3 Add the areas of the faces together.
 → The surface area of a rectangular prism can be calculated using the formula: $SA = 2lb + 2lh + 2hb$, where: l is the length of the base rectangle, b is the breadth of the base rectangle and h is the height of the prism.



Example 6B.1 Drawing the net of a right prism



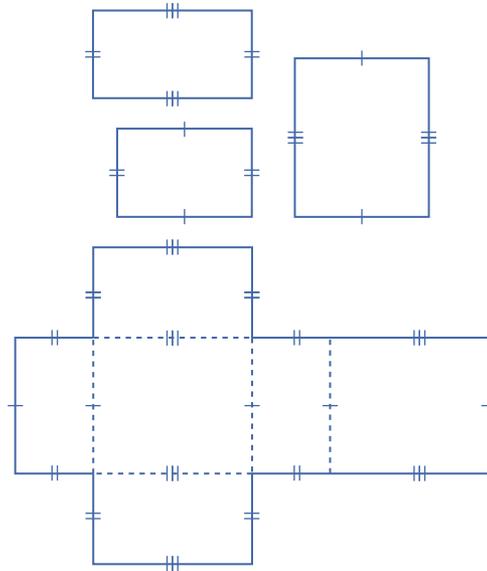
Draw a net for this 3D object.



THINK

- 1 A rectangular prism has six faces, made up of three different kinds. Draw each different one.
- 2 Try to visualise the rectangular prism unfolding – which face is on the bottom? This will form the centre of the net.
- 3 Arrange the remaining faces in a hopscotch style net. Look carefully at which faces are touching the centre face.
- 4 Add tabs if you wish to construct the object.

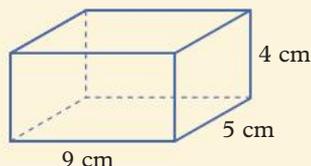
WRITE



Example 6B.2 Calculating the surface area of a rectangular prism



Calculate the surface area of this rectangular prism.



THINK

- 1 Determine the number of faces. A rectangular prism has six rectangular faces; three pairs of identical faces.
- 2 Calculate the area of each face.
- 3 Add the areas of all the faces together.
Check that all six faces have been included in the sum.

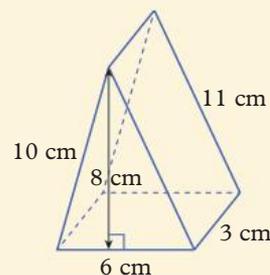
WRITE

$$\begin{aligned}\text{Area of front and back faces} &= 9 \times 4 \\ &= 36 \text{ cm}^2 \\ \text{Area of side faces} &= 5 \times 4 \\ &= 20 \text{ cm}^2 \\ \text{Area of top and bottom faces} &= 9 \times 5 \\ &= 45 \text{ cm}^2 \\ \text{Surface area} &= 2 \times 36 + 2 \times 20 + 2 \times 45 \\ &= 202 \text{ cm}^2\end{aligned}$$

Example 6B.3 Calculating the surface area of a triangular prism



Calculate the surface area of this triangular prism.



THINK

- 1 Determine the number of faces. A right triangular prism has two identical triangular faces and three rectangular faces.
- 2 Calculate the area of each face.
- 3 Add the areas of the faces together.
Include the appropriate unit.

WRITE

$$\begin{aligned}\text{Area of triangular face} &= \frac{1}{2} \times b \times h \\ &= \frac{1}{2} \times 6 \times 8 \\ &= 24 \text{ cm}^2 \\ \text{Area of bottom rectangular face} &= 6 \times 3 \\ &= 18 \text{ cm}^2 \\ \text{Area of left-hand rectangular face} &= 10 \times 3 \\ &= 30 \text{ cm}^2 \\ \text{Area of right-hand rectangular face} &= 11 \times 3 \\ &= 33 \text{ cm}^2 \\ \text{Surface area} &= 2 \times 24 + 18 + 30 + 33 \\ &= 129 \text{ cm}^2\end{aligned}$$

- ✓ All of the non-matching end faces in a right prism are rectangles.
- ✓ A prism can be orientated in any way. Identify the matching ends to determine the shape of the base of a prism.

ANS p477 **Exercise 6B** Surface area

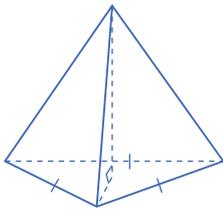
▲ 1, 2, 3(a, b), 4(a-c), 5-8, 10, 12

■ 3(c, d), 4(d-f), 5, 6, 8, 11, 13, 14

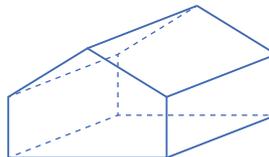
◆ 6, 8-16

1 Which of the following objects are right prisms?

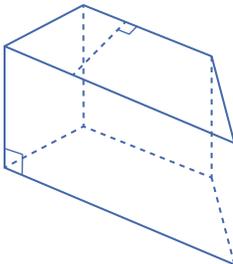
a



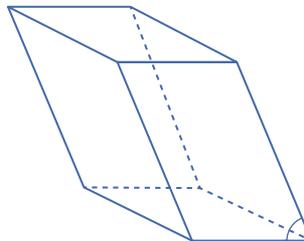
b



c



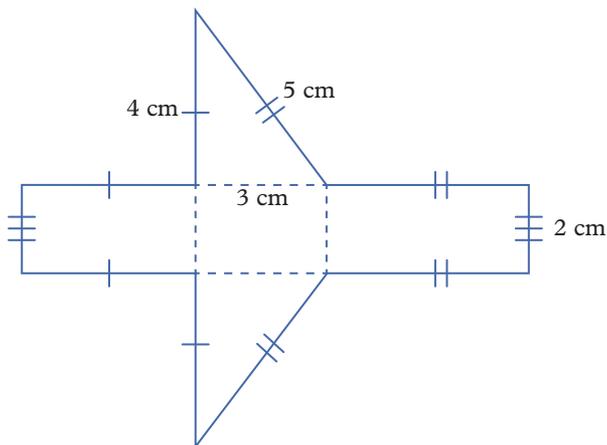
d



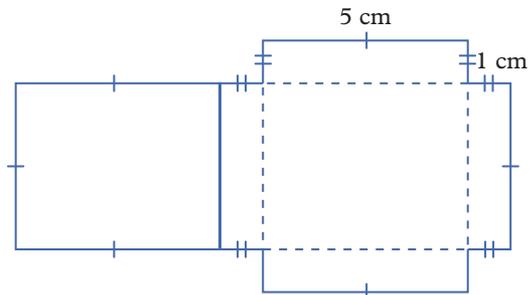
2 For each of the following nets, determine:

- i the name of the 3D objects it folds up to
- ii the surface area of the 3D objects.

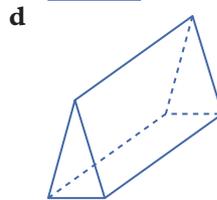
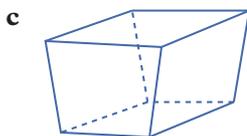
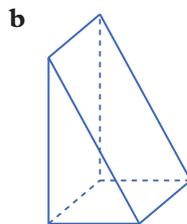
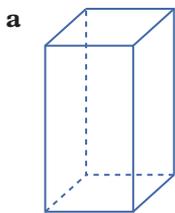
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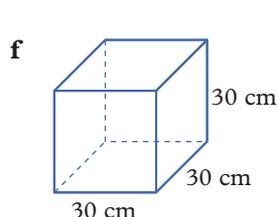
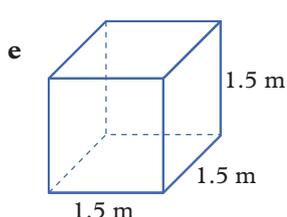
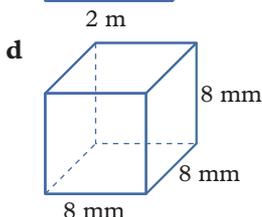
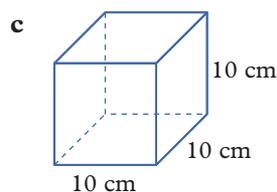
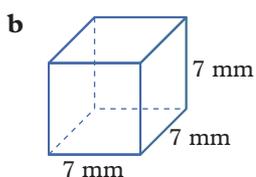
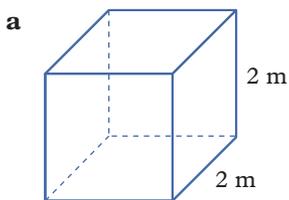
b



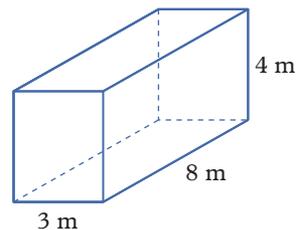
6B.1 3 Draw a net for each 3D object.



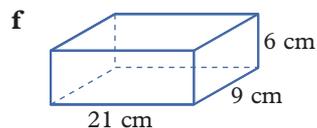
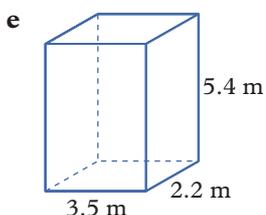
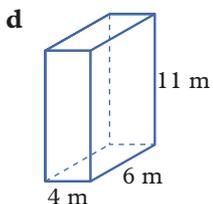
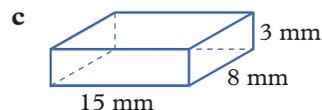
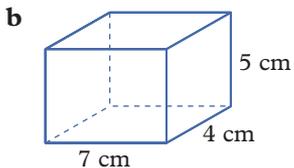
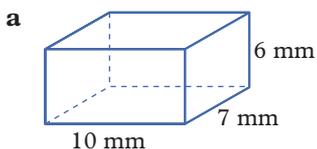
4 Calculate the surface area of each cube.



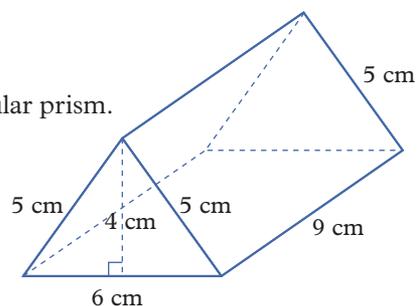
- 5 a** How many faces does the rectangular prism on the right have?
b Calculate the area of each pair of identical faces of the prism.
c Use your answer from part **b** to calculate the surface area of the prism.



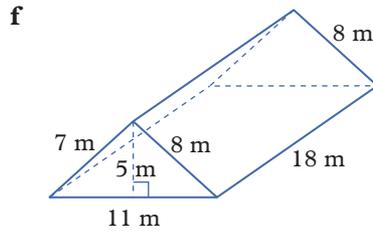
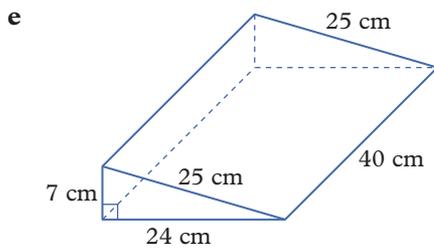
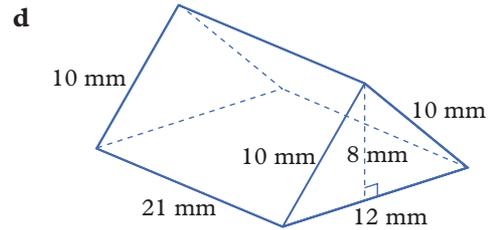
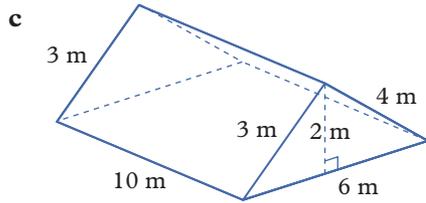
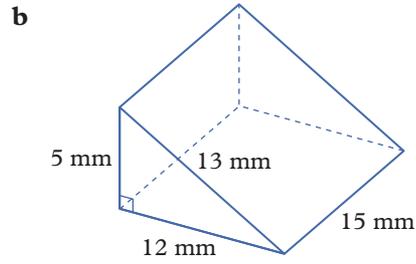
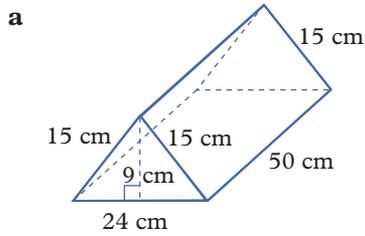
6B.2 6 Calculate the surface area of each of these rectangular prisms.



- 7 a** How many faces does the triangular prism on the right have?
b Calculate the area of each face of the triangular prism.
c Use your answer from part **b** to calculate the surface area of the triangular prism.

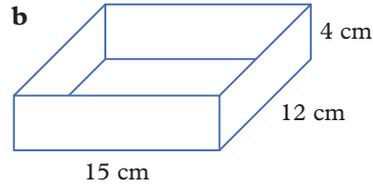
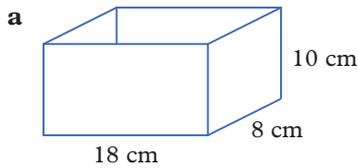


6B.3 8 Calculate the surface area of each triangular prism.

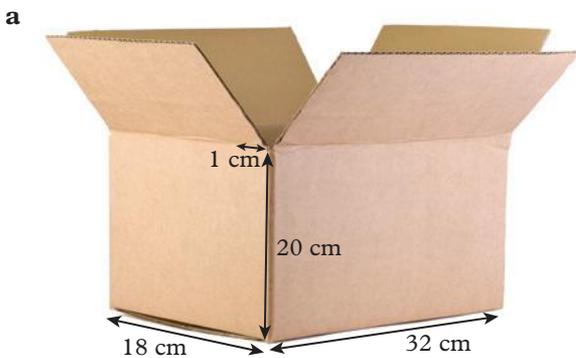
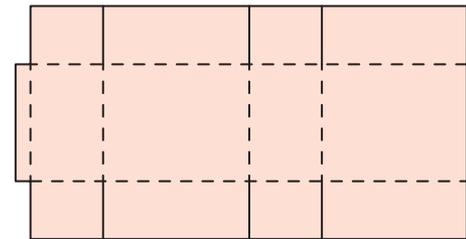


9 Find the surface area of these boxes if you were to consider:

- i** the amount of material required to build the boxes (no top)
- ii** the amount of paint (in cm^2) required to paint all surfaces of the boxes (inside and outside).



10 The net of a basic cardboard box is shown. The dashed lines represent folds, and the solid lines represent cuts. The tab on the left is used to join the central section of the box, and the flaps at the top and bottom are all half of the breadth of the box. Calculate the amount of cardboard required to make these boxes.



- 11** A block of butter is in the shape of a rectangular prism.
- If the block was 7 cm wide, 16 cm long and 6 cm high, calculate its surface area.
 - If the block was cut halfway along the length (that is, to give two pieces 8 cm long), what is the surface area:
 - of each piece
 - in total?
 - How is your answer to part **b ii** different from your answer to part **a**?
 - Imagine that you cut the block of butter into 1-cm cubes. What would be the surface area of the butter now?
 - If somebody wanted to melt butter quickly, what would you recommend to them? Why?

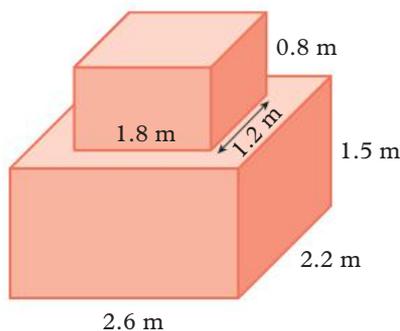


- 12** Billy is having his room painted. His room is 6 m long, 5 m wide and 2.5 m high and has a large window on one wall that measures 150 cm × 95 cm. The walls are to be painted blue and the ceiling is to be painted cream.
- What area is to be painted blue and what area is to be painted cream?
 - How many litres of each colour paint is needed if 1 L of paint covers about 15 m² and the room will take two coats?

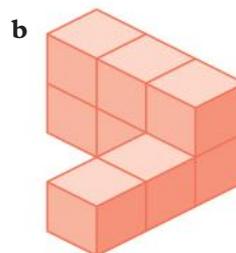
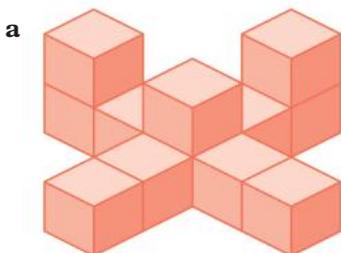
- 13** A cube has a surface area of 150 cm².
- What is the surface area of each face?
 - What is the side length of the cube?
 - Write a formula that will help you determine the side length of any cube if you know its surface area.

- 14** Explain why knowing the surface area of a cube is enough to determine its side lengths but knowing the surface area of a rectangular prism is not.

- 15** A sculpture is constructed in the shape of a rectangular box with a smaller box placed on top. The dimensions of the boxes are shown in the diagram. To finish the sculpture, the artist must work out the surface area to be painted. Calculate the surface area of the sculpture excluding the base, which will not be visible to the public. Note: The overlapping parts of the boxes will also not be painted.



- 16** These objects are made of 1-cm cubic blocks. Work out their surface area (including the base).



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Interactive skillsheet
Surface area of prisms



Worksheet
Surface area of prisms



Investigation
Dissecting cubes



Topic quiz
6B

6C Surface area of cylinders

Learning intentions

By the end of this topic you will be able to ...

- ✓ draw the net of cylinders
- ✓ calculate the surface area of cylinders.

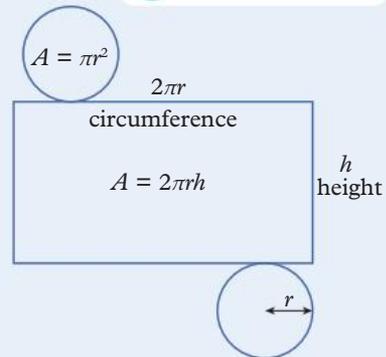
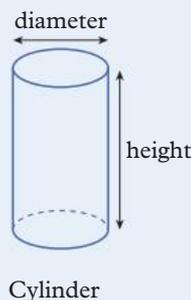


Inter-year links

- Year 7** 8B Circumference of a circle
- Year 8** 8D Area of a circle
- Year 10** 8B Surface area review

Cylinders

- A **cylinder** is a 3D object with a circular base and a constant circular cross-section.
 - The net of a cylinder is formed by a rectangle and two identical circular bases.
 - The surface area of a cylinder can be calculated using the formula $SA = 2\pi rh + 2\pi r^2$, where $2\pi rh$ is the area of the rectangle, $2\pi r^2$ is the area of the two base circles.

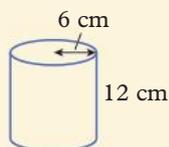


Key content video

Example 6C.1 Drawing the net of a cylinder



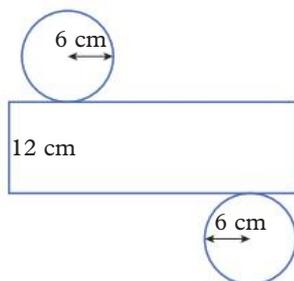
Draw the net of this cylinder, labelling dimensions accurately.



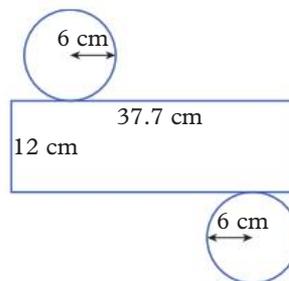
THINK

- 1 Draw a net of a cylinder, remembering that it consists of a rectangle and two circles.
- 2 Label the radius of the circles and the height of the rectangle. Include appropriate units.
- 3 The length of the rectangle is the circumference of the circular ends. Use the formula $C = 2\pi r$ to calculate this length, correct to one decimal place.
- 4 Label the length of the rectangle to complete your net. Include appropriate units.

WRITE



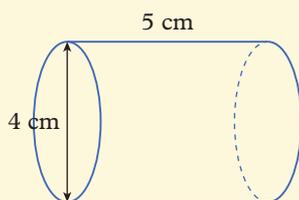
$$\begin{aligned} C &= 2\pi r \\ &= 2\pi \times 6 \\ &\approx 37.7 \text{ cm} \end{aligned}$$





Example 6C.2 Calculating the surface area of a cylinder

Calculate the surface area of this cylinder correct to two decimal places.



THINK

- 1 Identify the measurements for the radius (r) and the height (h) and substitute these into the formula.
- 2 Write the formula for surface area of a cylinder.
- 3 Substitute the values for r and h into the formula and calculate the result correct to two decimal places. Include the appropriate unit.

WRITE

$$\begin{aligned}
 r &= d \div 2 \\
 &= 4 \div 2 \\
 &= 2 \text{ cm} \\
 h &= 5 \text{ cm} \\
 SA &= 2\pi rh + 2\pi r^2 \\
 &= 2 \times \pi \times 2 \times 5 + 2 \times \pi \times 2^2 \\
 &= 20\pi + 8\pi \\
 &= 28\pi \\
 &= 87.964\dots \\
 &\approx 87.96 \text{ cm}^2
 \end{aligned}$$

Helpful hints

- ✓ If you can't remember the formula for the surface area of a cylinder, remember that a cylinder consists of two circles (the ends) and a rectangle (the curved surface). Calculate the area of each individual face and add the areas together.

ANS
p477

Exercise 6C Surface area of cylinders

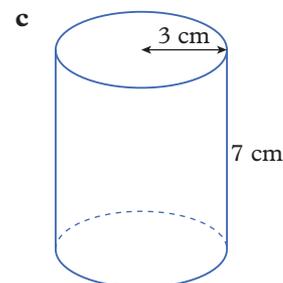
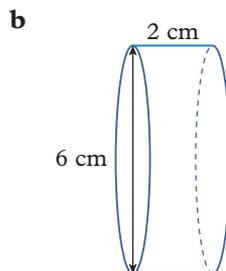
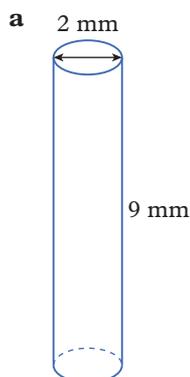
▲ 1-5(a, e, f), 7, 9

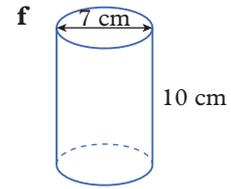
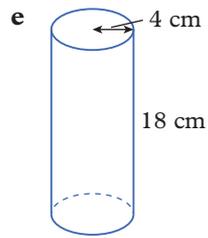
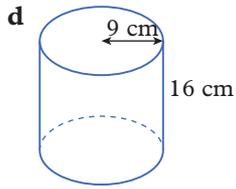
■ 5, 6, 8, 11, 14

◆ 5, 8, 10, 12-16

UNDERSTANDING AND FLUENCY

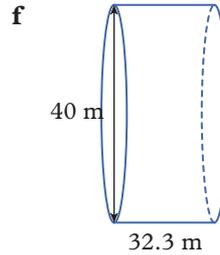
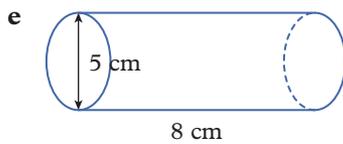
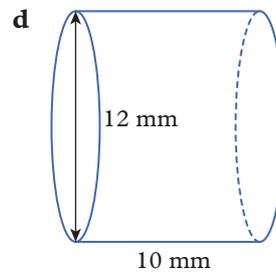
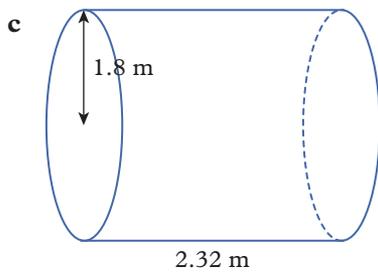
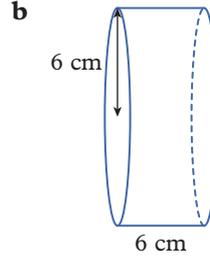
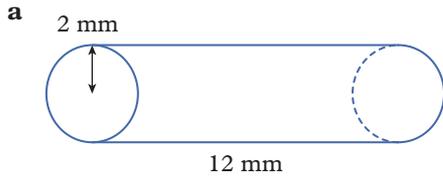
6C.1 1 Draw a net for each cylinder, labelling the dimensions correct to two decimal places where appropriate.





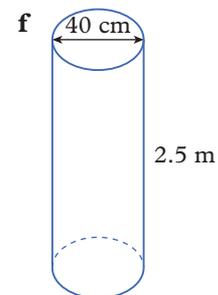
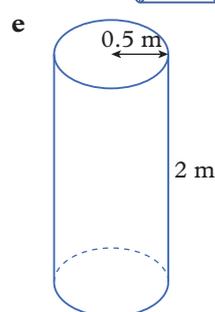
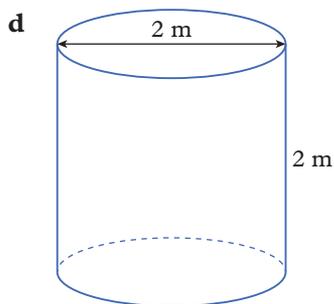
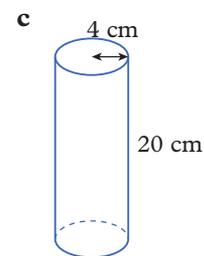
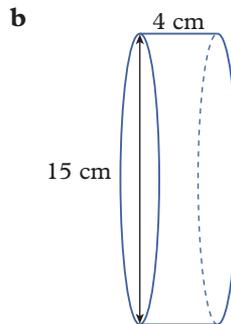
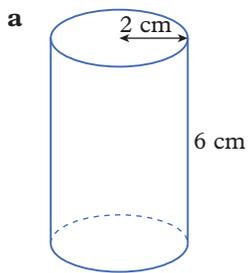
6C.2 2 Calculate the surface area of each cylinder in question 1. Write your answers correct to two decimal places.

3 Draw a net for each cylinder, labelling the dimensions correct to two decimal places where appropriate.



4 Calculate the surface area of each cylinder in question 3. Give your answers to two decimal places.

5 Calculate the surface area for each cylinder correct to two decimal places.

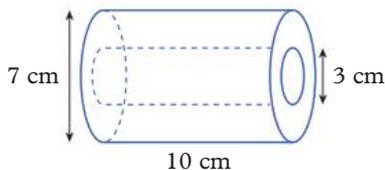


- 6 Calculate the surface area for each cylinder correct to two decimal places.
- | | |
|--|---|
| a $h = 8$ cm, $r = 10$ cm | b $h = 2$ cm, $d = 3.1$ cm |
| c $h = 2.1$ m, $r = 3$ m | d $h = 3.2$ m, $d = 7.3$ m |
| e $h = \frac{1}{2}$ mm, $r = 1\frac{1}{3}$ mm | f $h = \frac{3}{2}$ mm, $d = \frac{4}{3}$ mm |

7 Calculate the outer surface area of this box and its lid, if the box is 22 cm tall and has a radius of 15 cm. The lid is 4 cm tall and has a diameter of 31 cm. Give your answer to two decimal places.



- 8 A cylindrical pool is 2 m deep and has a radius of 6.5 m. How much would it cost, to the nearest dollar, to paint its interior if it needs two coats of paint that costs \$50 per litre? Assume that 1 L covers 15 m².
- 9 Maria has the choice of two paint rollers. One roller is 25 cm long and has a radius of 4 cm. The other roller is 30 cm long and has a diameter of 6 cm. Assuming they have the same absorbency, which roller would need to be re-dipped in paint the least often?
- 10 **a** What is the surface area of the tube below, including the internal exposed surface? Give your answer correct to two decimal places.
- b** Lucian answered part **a** as 296.88 cm² and Curtis said it was 282.74 cm². Explain where they went wrong.



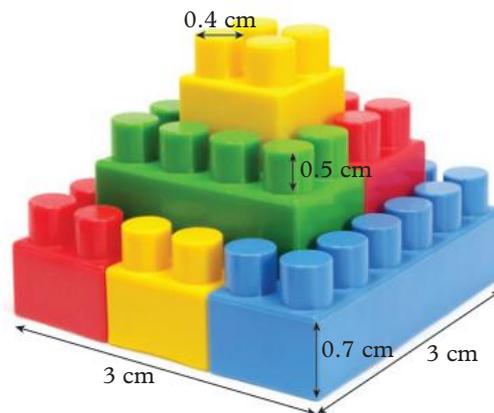
- 11 A cylinder has a radius of 4 cm and a surface area of 300 cm². What is its height to the nearest centimetre?
- 12 The surface area for any prism can be calculated using the formula:
 $SA = 2 \times \text{area of base} + \text{perimeter of base} \times \text{height}$.

Explain why this formula works.

- 13 If you double the height of a cylinder, does its surface area also double? Explain, using an example.
- 14 The surface area of a sphere can be calculated using the formula $SA = 4\pi r^2$. Calculate the surface area of a sphere, correct to two decimal places, that has:

- a** radius 4 cm
- b** radius 7 mm
- c** diameter 10 cm
- d** circumference 20 cm.

- 15 Write a formula that will calculate the height of a cylinder, given its surface area and radius.
- 16 Calculate the exterior surface area, correct to two decimal places, of the structure shown in the image on the right (do not include the base).



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Interactive skillsheet
Surface area of cylinders



Investigation
Making can-coolers



Topic quiz
6C

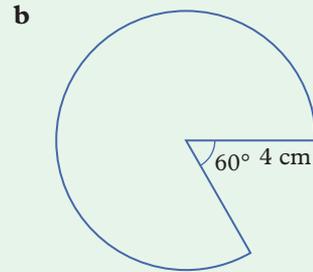
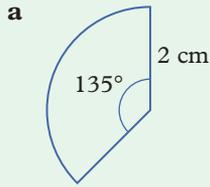
Checkpoint



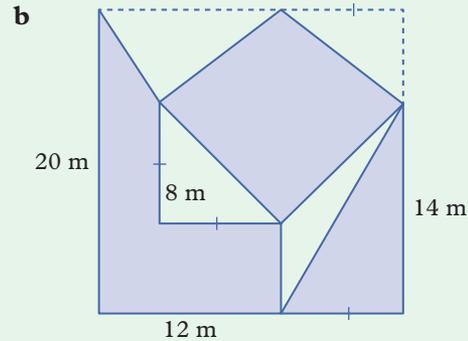
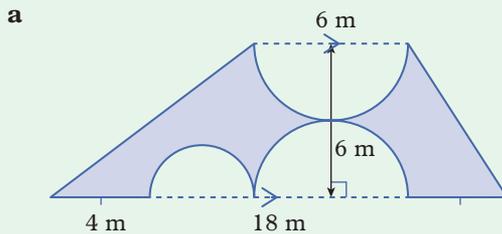
Checkpoint quiz

Check your knowledge of the first part of this chapter.

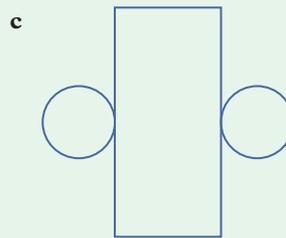
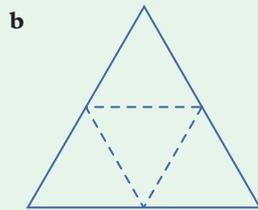
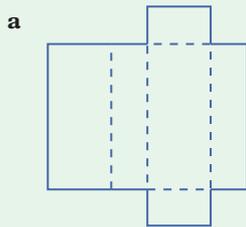
- 6A 1** Calculate the area of these sectors correct to two decimal places.



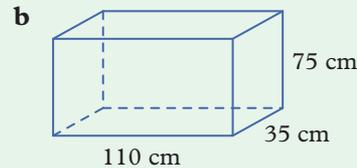
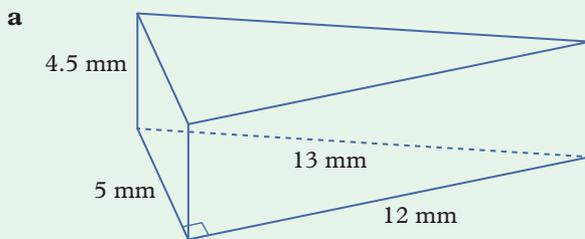
- 6A 2** Calculate the shaded area of these composite shapes correct to two decimal places.



- 6B 3** Determine if the following diagrams represent a net of a right prism.



- 6B 4** Calculate the surface area of these prisms.



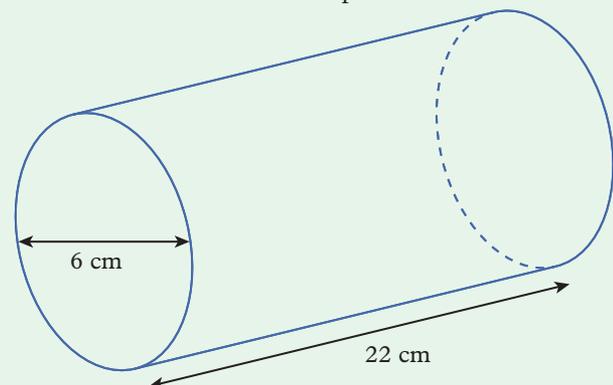
- 6C 5** Calculate the surface area of these cylinders; write your answers correct to two decimal places.

a a cylinder with height 12 cm and diameter 12 cm

b a cylinder with height 6 m and radius 6 m

- 6C 6 a** Draw a net of the cylinder shown on the right and include dimensions accurate to two decimal places where necessary.

b Determine the surface area of the cylinder correct to one decimal place.



6D Volume of composite solids

Learning intentions

By the end of this topic you will be able to ...

- ✓ calculate the volume and capacity of prisms and cylinders
- ✓ calculate the volume of composite solids.



Inter-year links

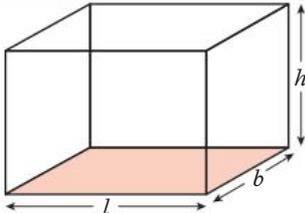
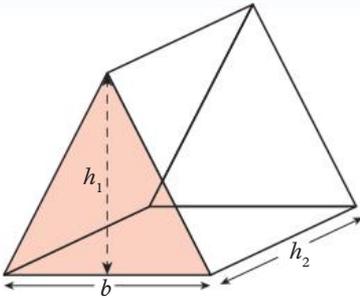
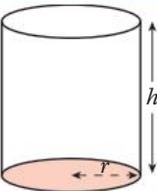
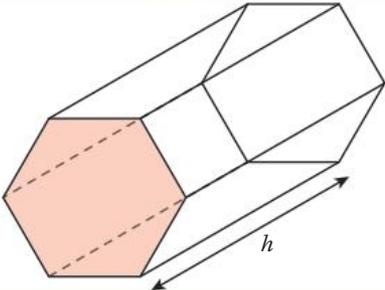
- Support** Volume and capacity
- Year 7** 8F Volume and capacity
- Year 8** 8E Volume and capacity
- Year 10** 8C Volume review

Volume of prisms and cylinders



Key content video

- The **volume** of a 3D object is the amount of three-dimensional space that it occupies.
- The volume of a right prism or a cylinder can be found by multiplying the base area by the height.
 $V = Ah$, where A is the area of the base and h is the height of the prism.

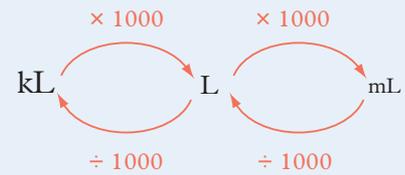
Shape	Diagram	Formula
Rectangular prism		$V = (\text{area of rectangle}) \times h$ $= lbh$
Triangular prism		$V = (\text{area of triangle}) \times h$ $= \frac{1}{2}bh_1h_2$
Cylinder		$V = (\text{area of circle}) \times h$ $= \pi r^2h$
Right prism		$V = (\text{area of base}) \times h$

Capacity

The **capacity** of a 3D object is a measure of how much the object can hold.

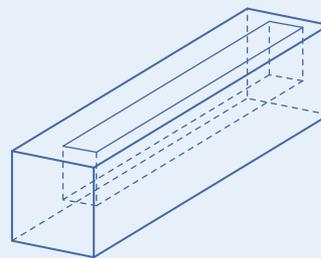
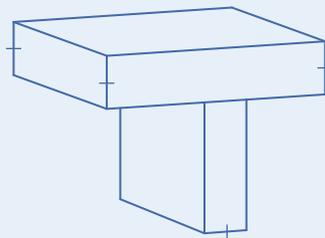
A container with an inside volume of:

- 1 cm³ holds 1 mL of liquid
- 1000 cm³ holds 1 L of liquid
- 1 m³ holds 1 kL of liquid.



Volume of composite solids

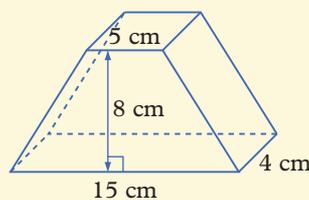
- A **composite solid** consists of two or more simple solids.
- To calculate the volume of a composite solid:
 - 1 Split the object into individual parts.
 - 2 Calculate any missing dimensions.
 - 3 Calculate the volume of each individual part.
 - 4 Add or subtract the part volumes to find the total volume.



Example 6D.1 Calculating the volume of a prism



Calculate the volume of this prism.



THINK

- 1 Identify the shape of the base of the right prism and write the appropriate formula for the area: $A = \frac{h_1}{2}(a + b)$.
- 2 Substitute the values for the base area (A) and the height of the prism (h_2) into the formula for the volume of the prism: $V = Ah_2$.
- 3 Calculate the result. Remember to include the appropriate unit.

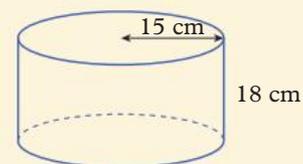
WRITE

$$\begin{aligned}
 A &= \frac{h_1}{2}(a + b) \\
 &= \frac{8}{2} \times (5 + 15) \\
 &= 80 \text{ cm}^2 \\
 V &= Ah_2 \\
 &= 80 \times 4 \\
 &= 320 \text{ cm}^3
 \end{aligned}$$

Example 6D.2 Calculating the volume and capacity of a cylinder



For the cylinder on the right, calculate the volume correct to two decimal places and the capacity correct to the nearest litre.



THINK

- 1 Identify the values for the radius of the base (r) and the height of the cylinder (h).
- 2 Calculate the volume of the cylinder, using the π button on your calculator.
- 3 Convert the volume into capacity by using an appropriate conversion factor.

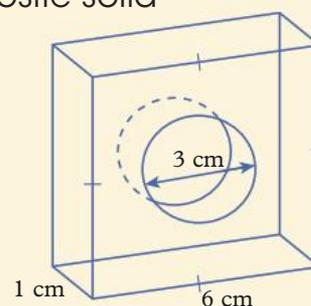
WRITE

$$\begin{aligned}r &= 15 \text{ cm} \\h &= 18 \text{ cm} \\V &= \pi r^2 h \\&= \pi \times 15^2 \times 18 \\&= 4050\pi \\&\approx 12723.45 \text{ cm}^3 \\ \text{Capacity} &= (12723.45 \div 1000) \text{ L} \\&\approx 13 \text{ L}\end{aligned}$$

Example 6D.3 Calculating the volume of a composite solid



Calculate the volume of the following composite solid correct to two decimal places.



THINK

- 1 Split the composite solid into a prism and cylinder. List the dimensions of the prism and the cylinder.
- 2 Use volume formulas to calculate the volume of the prism and cylinder.
- 3 Subtract the volume of the cylinder from the volume of the rectangular prism. Use exact values of π until the last step of your calculation, then round your answer to two decimal places.

WRITE

The length, breadth and height of the rectangular prism are 6 cm, 6 cm and 1 cm.

The diameter and height of the cylinder are 3 cm and 1 cm.

$$\begin{aligned}V_{\text{prism}} &= lbh \\&= 6 \times 6 \times 1 \\&= 36 \text{ cm}^3 \\V_{\text{cylinder}} &= \pi \left(\frac{d}{2}\right)^2 h \\&= \pi \times \frac{3^2}{4} \times 1 \\&= \frac{9}{4}\pi \text{ cm}^3\end{aligned}$$

$$\begin{aligned}\text{Total Volume} &= V_{\text{prism}} - V_{\text{cylinder}} \\&= 36 - \frac{9}{4}\pi \\&\approx 28.93 \text{ cm}^3\end{aligned}$$

- ✓ For rectangular prisms, any of the faces can be designated as the base, but remember that the height must be perpendicular (at right angles) to the base.
- ✓ Remember that volume and capacity are not the same thing! Capacity is a measure of how much an object can hold (mL, L, kL), whereas volume is a measure of how much space an object occupies (mm^3 , cm^3 , m^3).

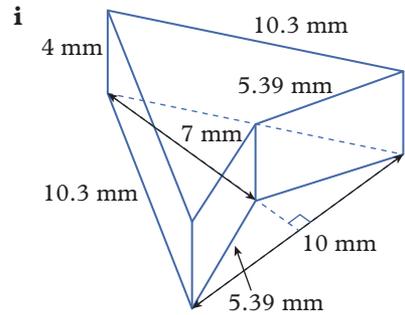
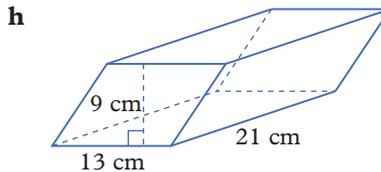
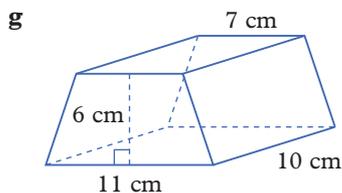
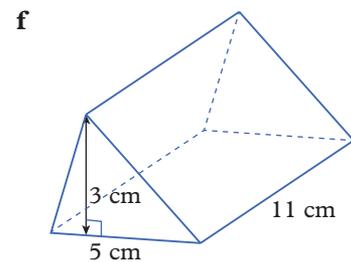
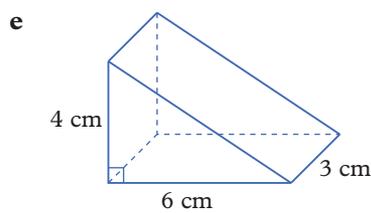
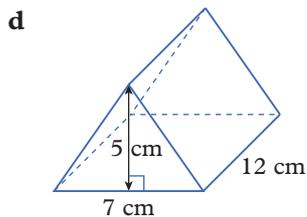
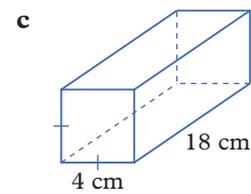
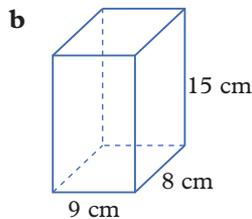
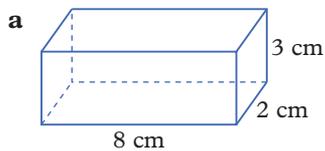
ANS p480 **Exercise 6D** Volume of composite solids

▲ 1, 2, 3(a, b), 4-7, 9, 13

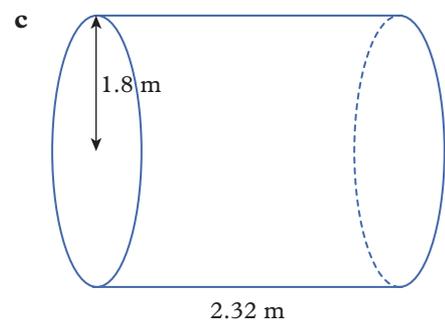
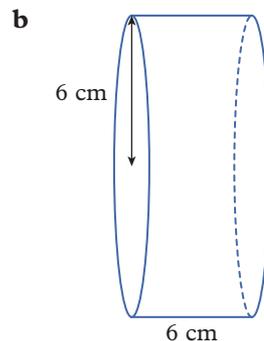
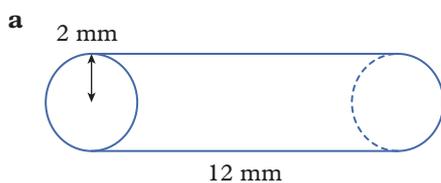
■ 1(d, f, g), 2(d-f), 3(b, c), 4(d-f), 6-9, 11, 14

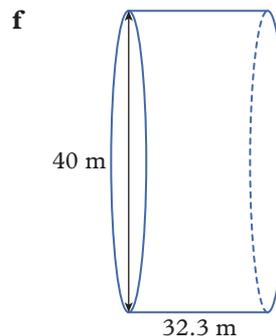
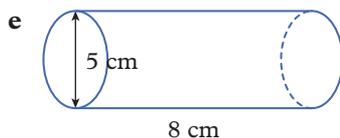
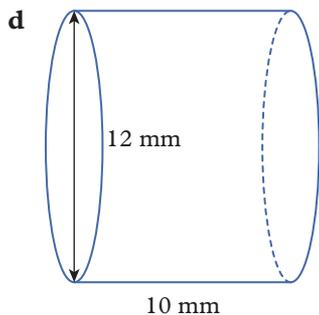
◆ 1(g-i), 2(e, f), 3(c), 4(e, f), 8, 10, 12, 13-15

6D.1 1 Calculate the volume of each prism.

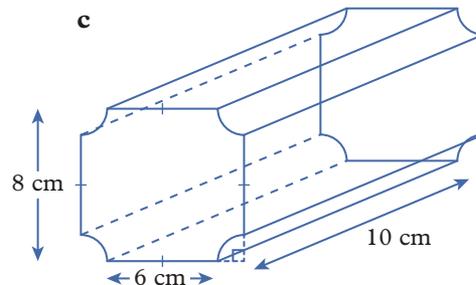
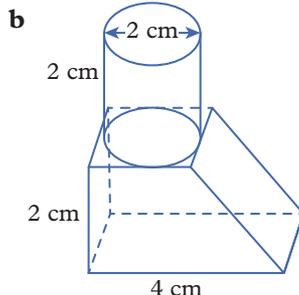
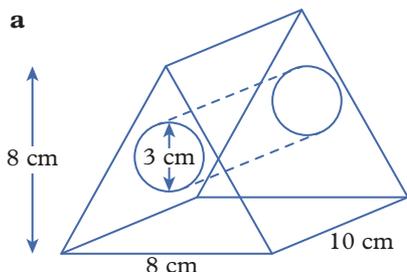


6D.2 2 Calculate the volume of each cylinder. Give your answers to two decimal places.





3 Calculate the volume of the following composite shapes.



4 Determine the capacity of each cylinder in question 2.

5 A prism has a volume of 240 cm^3 .

a If the prism has a height of 8 cm, what is the area of its base?

b If the prism has a base area of 60 cm^2 , find its height.

6 A prism has a volume of 360 cm^3 .

a If the prism has a height of 60 mm, what is the area of its base in cm^2 ?

b If the prism has a base area of 90 cm^2 , find its height in mm.

7 The height of a cube of side length x is increased 4 cm. The surface area of the new rectangular prism formed is 80 cm^2 more than the surface area of the cube.

a Write an expression to represent the surface area of the cube.

b Write an expression to represent the volume of the cube.

c What are the dimensions of the rectangular prism?

d What is the surface area of the rectangular prism?

e What is the value of x ?

f What is the volume of the rectangular prism?

8 The height of a cube is increased 6 cm and the surface area of the new rectangular prism formed is 96 cm^2 more than the surface area of the cube. What is the volume of the rectangular prism?

9 The height of a rectangular prism with a square base is increased 2 cm to form a cube of side length x cm. The surface area of the new cube formed is 48 cm^2 more than the surface area of the rectangular prism.

a Write an expression to represent the surface area of the cube.

b Write an expression to represent the volume of the cube.

c What are the dimensions of the rectangular prism?

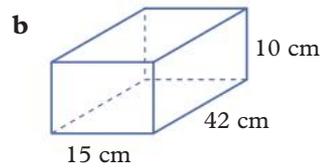
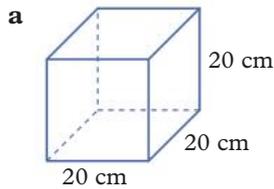
d What is the surface area of the rectangular prism?

e What is the value of x ?

f What is the volume of the rectangular prism?

10 The height of a rectangular prism with a square base is increased 1 cm to form a cube and the surface area of the new cube formed is 12 cm^2 more than the surface area of the rectangular prism. What is the volume of the rectangular prism?

- 11** The length and breadth of a rectangular prism are the same, and the height of the prism is double the length. The sum of the length, the breadth and height is 12 cm. What is the volume of the prism?
- 12** By calculating the surface area and volume of both of these boxes, demonstrate that boxes with the same surface area do not always have the same volume.

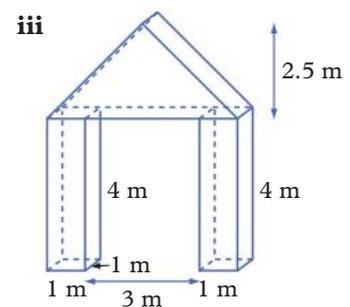
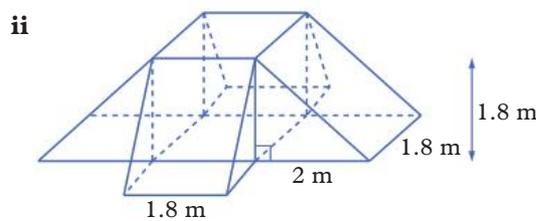
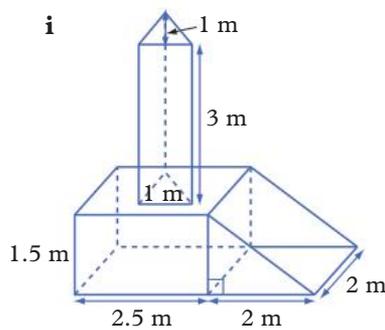


- 13** A new kind of insulation is to be fitted in the roof of a house. It is measured in cubic metres rather than square metres because it fills the entire roof (normally insulation is just a thick layer). The roof is in the form of a triangular prism. If the roof is 2 m high, 9 m wide and 14 m long, calculate:

- a** how much insulation is needed (in cubic metres)
- b** how much it will cost if the insulation costs \$30 per cubic metre.

- 14** Andrew likes to make geometric sculptures.

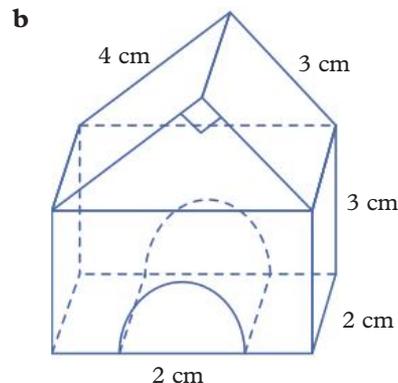
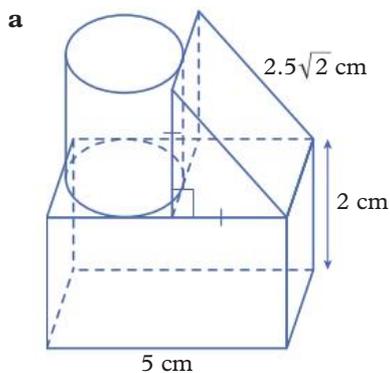
- a** If his materials cost \$2 per cubic metre, calculate how much it costs him to produce each sculpture.



- b** Mel decides she wants to buy one of Andrew's pieces, and she wants to buy the lightest of the three sculptures. Use the following information to recommend which sculpture Mel should buy.

- sculpture **i** weighs 3 kg per cubic metre
- sculpture **ii** weighs 2 kg per cubic metre
- sculpture **iii** weighs 2.5 kg per cubic metre

- 15** Calculate the volume of the following sculptures.



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Interactive skillsheet
Volume of prisms



Interactive skillsheet
Volume of cylinders



Interactive skillsheet
Capacity



Topic quiz
6D

6E Errors

Learning intentions

By the end of this topic you will be able to ...

- ✓ determine the precision of a measuring instrument
- ✓ find the absolute error of measuring instruments
- ✓ calculate the percentage error of a given measurement.



Inter-year links

Year 7

4G Fractions, decimals and percentages

Year 8

1A Rounding and estimating

Errors



Key content video

- All measurements are estimates of exact values.
 - The **precision** of a measuring instrument refers to the level of consistency of a set of measurements taken using the same instrument.
 - For example, suppose a pencil is measured using two different rulers. It is first measured three times using a ruler with smallest division 0.1 cm, and the results are 6.1 cm, 6.2 cm and 6.0 cm. Next, the pencil is measured using a ruler with smallest division 0.5 cm, and the results are 6.5 cm, 6.0 cm and 5.5 cm. The first ruler is more precise than the second ruler because the measurements of 6.1 cm, 6.2 cm and 6.0 cm are more consistent than the measurements of 6.5 cm, 6.0 cm and 5.5 cm.

- The **absolute error** of a measuring instrument is half of the smallest division on the instrument.

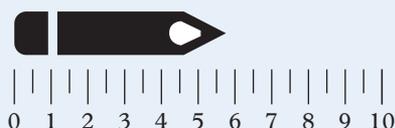
$$\text{absolute error} = \frac{1}{2} \times \text{smallest division}$$

For example, the absolute error of a ruler measuring to the nearest millimetre is 0.5 mm, the absolute error of a ruler measuring to the nearest half centimetre is 0.25 cm.

- Percentage error** of a given measurement is the ratio between the absolute error and the actual measurement expressed as a percentage.

$$\text{percentage error} = \frac{\text{absolute error}}{\text{measurement}} \times 100\%$$

- The **accuracy** of a measurement refers to how close a measured value is to the true value.
- For a given measurement:
 - the **lower bound** is the boundary of the true measurements that can round up to the given measurement. For example, the length of the pencil below is closer to the 6.0 cm mark than the 5.5 cm mark, so the measurement is recorded as 6.0 cm. However, the true measurement lies between 6.0 cm and 5.75 cm.



- the **upper bound** is the boundary of the true measurements that can round down to the given measurement. For example, the length of the pencil below is closer to the 6.0 cm mark than the 6.5 cm mark, so the measurement is recorded as 6.0 cm. However, the true measurement lies between 6.0 cm and 6.25 cm.



- Usually:

$$\text{lower bound} = \text{measurement} - 0.5 \times \text{smallest division}$$

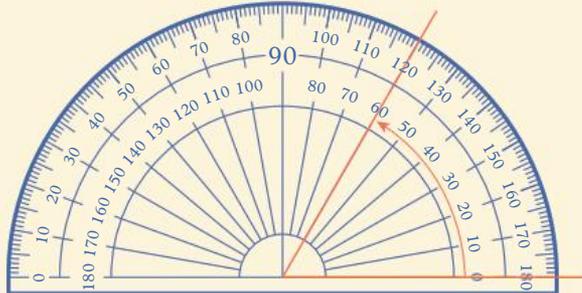
$$\text{upper bound} = \text{measurement} + 0.5 \times \text{smallest division}$$

Example 6E.1 Calculating the absolute error and percentage error



Willa used a protractor to measure an angle in her textbook. Determine:

- a the smallest division of the protractor
- b the absolute error of the protractor
- c the percentage error of her measurement.



THINK

- a There are 10 markings between every two numbers labelled on the protractor. For example, there are 10 markings between 60° and 70° , and each mark represents a change of 1° .
- b The absolute error of an instrument can be calculated using the formula
absolute error = $\frac{1}{2} \times$ smallest division.
Substitute 1° into the equation.
- c The percentage error of an instrument can be calculated using the formula
percentage error = $\frac{\text{absolute error}}{\text{measurement}} \times 100\%$.
The measurement on the protractor is 60° .
Substitute the measurement and the absolute error into the formula.

WRITE

- a The smallest division is 1° .
- b absolute error = $\frac{1}{2} \times$ smallest division
 $= \frac{1}{2} \times 1^\circ$
 $= 0.5^\circ$
- c percentage error = $\frac{0.5^\circ}{60^\circ} \times 100\%$
 $= 0.00833\dots$
 $= 0.83\%$

Example 6E.2 Finding the upper and lower bounds



Mitch measures the length of a pen as 9 cm. Find the lower and upper bounds for this measurement value if the ruler has:

- a 1 mm as the smallest unit of measurement



- b 2 mm as the smallest unit of measurement.



THINK

- a 1** The measurement 9 cm is an estimation of the exact length of the pen. The error of this measurement depends on the accuracy of the ruler being used. Identify the smallest division on the ruler as 1 mm. Convert 1 mm to 0.1 cm.
- 2** Calculate the lower and upper bounds by substituting the measurement value and the smallest division into the lower and upper bound formulas.
- b 1** Identify the smallest division as 2 mm. Convert 2 mm to 0.2 cm.
- 2** Calculate the lower and upper bounds by substituting the measurement value and the division into the lower and upper bound formulas.

WRITE

- a** $1 \text{ mm} = 0.1 \text{ cm}$
 measurement value = 9 cm
- lower bound = measurement $- 0.5 \times$ smallest unit
 $= 9 - 0.5 \times 0.1$
 $= 8.95 \text{ cm}$
- upper bound = measurement $+ 0.5 \times$ smallest unit
 $= 9 + 0.5 \times 0.1$
 $= 9.05 \text{ cm}$
- b** $2 \text{ mm} = 0.2 \text{ cm}$
 measurement value = 9 cm
- lower bound = measurement $- 0.5 \times$ smallest unit
 $= 9 - 0.5 \times 0.2$
 $= 8.9 \text{ cm}$
- upper bound = measurement $+ 0.5 \times$ smallest unit
 $= 9 + 0.5 \times 0.2$
 $= 9.1 \text{ cm}$

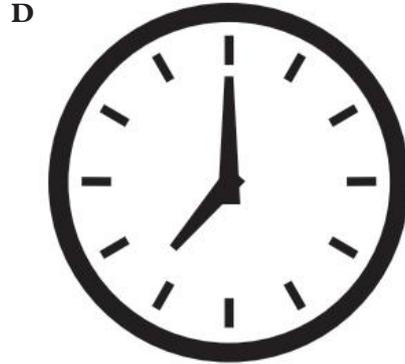
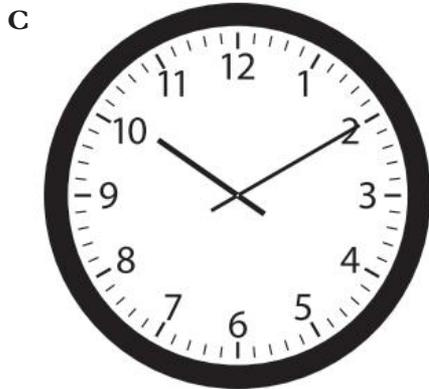
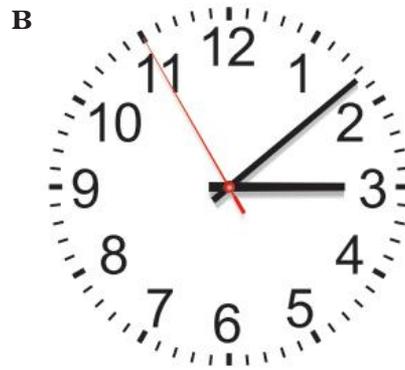
Helpful hints

- ✓ To calculate percentage errors, remember to multiply relative errors by 100% rather than just 100.
- ✓ Convert values to the same unit of measurement before substituting them into a formula.

ANS
p480**Exercise 6E Errors** 1-6, 8, 9 2, 3, 5, 6, 9-12 2, 3, 5, 7-13

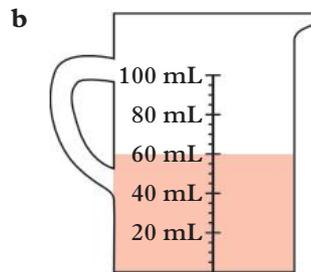
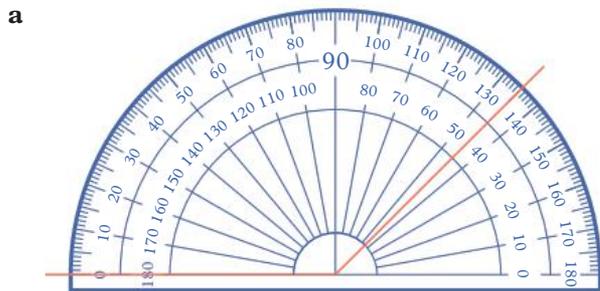
- 1** Find the absolute errors of the following measuring instruments.
- a** A measuring tape has markings every 5 mm. **b** A scale has markings every 0.5 g.
- c** A syringe has markings every 0.1 mL. **d** A thermometer has markings every 0.1°C .
- e** A stopwatch has markings every 0.02 seconds. **f** A water meter has markings every 1 m^3 .
- 2** Len uses a digital scale to measure the following ingredients in his cookie recipe. The smallest unit of measurement of the scale is 0.1 g. What is the percentage error of each of the ingredients?
- a** 100 g granulated sugar **b** 165 g brown sugar
- c** 7.4 g salt **d** 115 g unsalted butter
- e** 4.5 g vanilla extract **f** 155 g all-purpose flour
- g** 2.4 g baking soda **h** 110 g semi-sweet chocolate chunks

3 Which of the following clocks is the most precise?



6E.1 4 For each of the following measurements, determine:

- i the smallest measuring unit of the instrument
- ii the absolute error of the instrument
- iii the percentage error of the measurement.



- 5 a What is the smallest decimal that could result in an answer of 1.7 when rounded to one decimal place?
 b What is the smallest decimal that could result in an answer of 2.33 when rounded to two decimal places?

- 6E.2 6** A scale shows that a block of cheese weighs 430 g. Calculate the lower and upper bounds for this measurement value if the scale has:
- 1 g as the smallest unit of measurement
 - 5 g as the smallest unit of measurement.
- 7 The length of an object is measured as 125 cm using a measuring tape that has markings every 5 cm. Between which two values would the true length of the object lie?
- 8 Jacob measures the side length of a square as 12 cm, correct to the nearest centimetre. Find:
- the lower and upper bounds for the side length of the square
 - the lower and upper bounds for the perimeter of the square
 - the lower and upper bounds for the area of the square.
- 9 The side length of a cube is measured to the nearest centimetre. The measurement is then used to calculate the volume of the cube as 64 cm^3 .
- What is the largest possible volume of the cube, correct to two decimal places?
 - What is the smallest possible volume of the cube, correct to two decimal places?
- 10 Arya wrapped a piece of string around the cylindrical pillar in front of her local art gallery to find the circumference of the circular cross-section of the pillar. The length of the string is then measured with a ruler that has 1 cm as the smallest division. The measurement is recorded as 113 cm.
- What are the lower and upper bounds of the measurement?
 - What are the lower and upper bounds of the radius of the circular cross-section of the pillar, correct to the nearest centimetre?
 - What are the lower and upper bounds of the area of the circular cross-section of the pillar, correct to the nearest square centimetre?
- 11 Deb used a ruler to measure the length of a rope. She wrote down the measurement as 24 cm. Paul suggests that it's better to record it as 24.0 cm. Is Paul's suggestion always valid? Explain.
- 12 Jonny uses a measuring tape to measure his height. The tape's smallest measuring units are 1 cm on one side and 0.1 feet on the other side. Jonny's measurement from the side of the tape using the metric system is 168 cm, and the measurement from the other side, using the imperial system, is 5.5 feet. Which measurement has a bigger percentage error?
- 13 The thickness of a piece of printing paper is 0.08 mm. George has a ruler that can measure to the nearest millimetre. He made a few different stacks of paper and recorded the measurements in the table below.

	Stack A	Stack B	Stack C	Stack D
Number of sheets	20	50	200	500
Thickness (mm)	2	4	17	42

- Stack C has 10 times as many papers as stack A, and stack D has 10 times as many papers as Stack B. Explain why the thicknesses recorded for stacks C and D are not exactly 10 times the thicknesses of stacks A and B.
- Which measurement is likely to give the most accurate data for calculating the thickness of a piece of paper? Calculate the thickness of a piece of paper from this measurement.

Check your Student obook pro for these digital resources and more:

pro



Interactive skillsheet
Errors



Worksheet
Upper and lower bounds



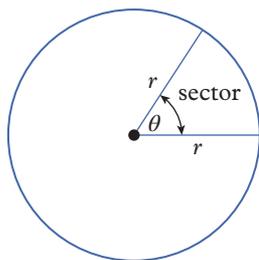
Worksheet
Reading scales



Topic quiz
6E

Chapter summary

Area of a sector

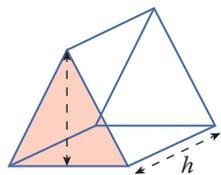


$$A = \frac{\theta}{360} \times \pi r^2$$

Area of composite shapes

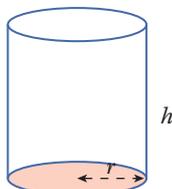
- 1 Split the figure into simple shapes that have known area formulas.
- 2 Calculate any missing dimensions.
- 3 Calculate the areas of the individual shapes using the area formulas.
- 4 Add or subtract the areas to calculate the total area.

Volume of prisms and cylinders



$$V = Ah$$

Where A is the area of the base and h is the height



$$V = \pi r^2 h$$

Volume of composite solids

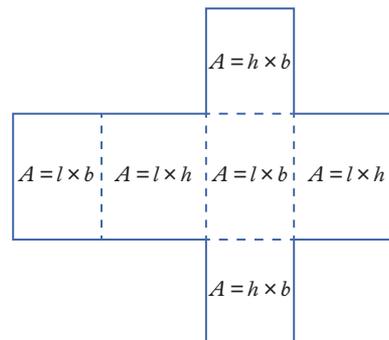
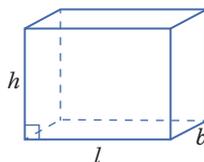
To calculate the volume of a composite solid:

- 1 Split the object into individual parts.
- 2 Calculate any missing dimensions.
- 3 Calculate the volume of each individual part.
- 4 Add or subtract the part volumes to find the total volume.

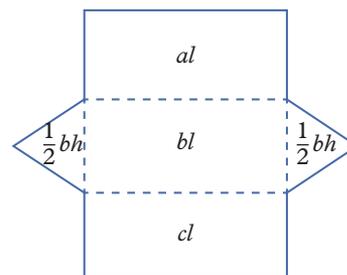
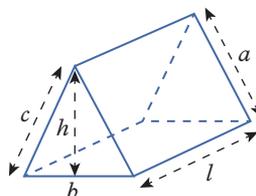
Surface area

- 1 Determine the number of faces.
- 2 Calculate the area of each face.
- 3 Add the areas of the faces together.

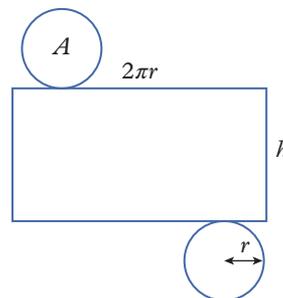
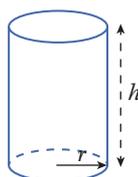
- Rectangular prism $SA = 2lb + 2lh + 2hb$



- Triangular prism $SA = bh + (a + b + c)l$



- Cylinder $SA = 2\pi rh + 2r^2$



Capacity

The capacity of a 3D object is a measure of how much the object can hold. A container with an inside volume of:

- 1 cm^3 holds 1 mL of liquid
- 1000 cm^3 holds 1 L of liquid
- 1 m^3 holds 1 kL of liquid.

Errors

$$\text{absolute error} = \frac{1}{2} \times \text{smallest division}$$

$$\text{lower bound} = \text{measurement} - 0.5 \times \text{smallest division}$$

$$\text{percentage error} = \frac{\text{absolute error}}{\text{measurement}} \times 100\%$$

$$\text{upper bound} = \text{measurement} + 0.5 \times \text{smallest division}$$



Chapter review quiz

Assess your knowledge of this chapter.

Quizlet

Test your knowledge of this topic by working individually or in teams.

Mathematical literacy review

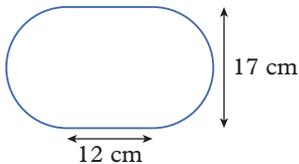
The following key terms are used in this chapter:

- absolute error
- accuracy
- arc
- area
- capacity
- circumference
- composite shape
- cross-section
- cylinder
- face
- lower bound
- net
- percentage error
- precision
- prism
- quadrant
- rectangular prism
- right prism
- sector
- semi-circle
- surface area
- triangular prism
- upper bound
- volume

- 1 **a** The surface area of a _____ can be calculated by adding the area of two circles and a rectangle; the breadth of the rectangle is equal to the _____ of the circle.
 - b** The _____ of a 3D solid is a 2D plan that can be folded to form a 3D solid.
 - c** The _____ of a 3D solid is the amount of space it occupies.
 - d** A _____ is $\frac{1}{4}$ of a circle; a _____ is $\frac{1}{2}$ of a circle.
- 2 Explain the difference between absolute error and percentage error.
 - 3 Write three common units for each of the following and show the conversion between these units:
 - a** area
 - b** volume
 - c** capacity.

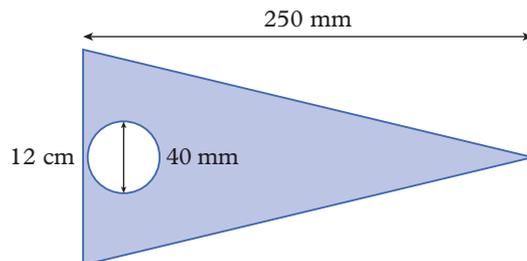
Multiple choice

- 6A** 1 The area of this shape is closest to:



- A** 77 cm^2 **B** 204 cm^2 **C** 431 cm^2 **D** 1112 cm^2 **E** 641 cm^2

- 6A** 2 The shaded area is closest to:

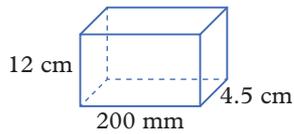


- A** 99.7 cm^2 **B** 137.43 cm^2 **C** 143.72 cm^2 **D** 150 cm^2 **E** 162.57 cm^2

- 6B** 3 Which of the following objects has the largest surface area?

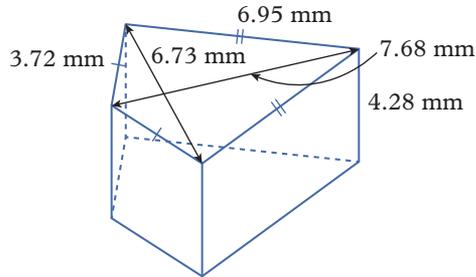
- A** a rectangular prism with length 3 cm, breadth 4 cm and height 5 cm
B a cube with side length 4 cm
C a cylinder with diameter 4 cm and height 4 cm
D a rectangular prism with length 2 cm, breadth 2 cm and height 10 cm
E a cylinder with radius 3 cm and height 5 cm

- 6B** 4 The surface area of this rectangular prism is:



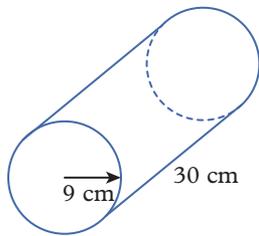
- A** 6708 cm² **B** 1080 cm² **C** 768 cm² **D** 678 cm² **E** 384 cm²

- 6B** 5 The surface area of this prism is:



- A** 150.7444 mm² **B** 143.0216 mm² **C** 110.65512 mm² **D** 221.217792 mm² **E** 194.708 mm²

- 6C** 6 The surface area of the following cylinder is:



- A** 270 cm² **B** 7634.1 cm² **C** 2205.4 cm² **D** 7351.3 cm² **E** 678.6 cm²

- 6C** 7 The surface area of a cylinder increased by 12π cm² after the height of the cylinder is increased by 3 cm. What is the radius of the base?

- A** 4π cm **B** 2π cm **C** 4 cm **D** 2 cm **E** 1 cm

- 6D** 8 Which of the following options is closest to the volume of the cylinder in question 6?

- A** 7700 cm³ **B** 7600 cm³ **C** 700 cm³ **D** 800 cm³ **E** 2000 cm³

- 6D** 9 What is the capacity of a cylinder with height 25 cm and radius 35 cm?

- A** 96 L **B** 67 L **C** 17 L **D** 38 L **E** 24 L

- 6E** 10 Which of the following measurements has the biggest percentage error?

- A** 0.4 g measured to the nearest 0.1 g
B 10 g measured to the nearest 1g
C 2.2 m measured to the nearest 10 cm
D 4 km measured to the nearest 1 m
E 15.5 mL measured to the nearest 0.5 mL

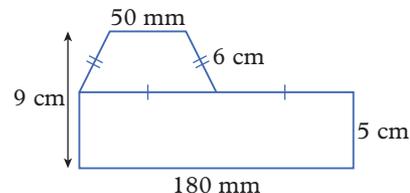
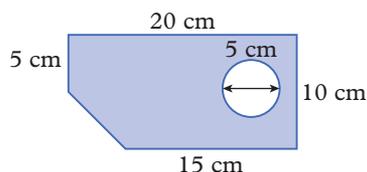
- 6E** 11 The absolute error of a scale is 0.5 g. What is the smallest division on the scale?

- A** 0.25 **B** 0.5 **C** 1 **D** 0.1 **E** 0.05

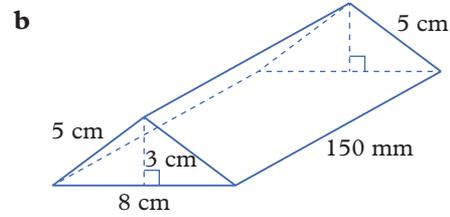
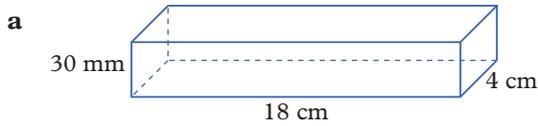
Short answer

- 6A** 1 Calculate the area of the composite shape on the right in square centimetres.

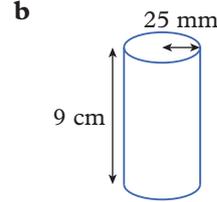
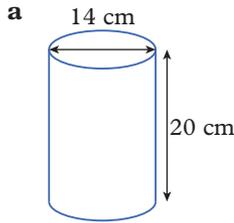
- 6A** 2 Calculate the shaded area correct to two decimal places.



6B 3 Calculate the surface area of each prism.



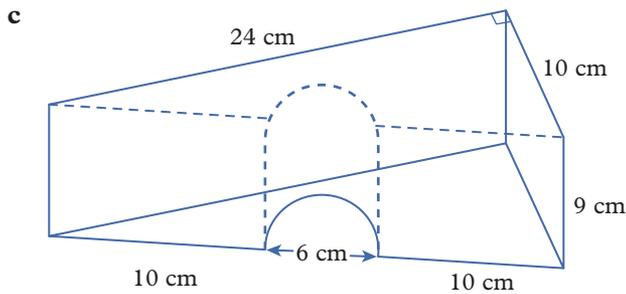
6C 4 Calculate the surface area of each cylinder correct to two decimal places.



6D 5 Calculate the volume of each cylinder in question 4. Write your answers correct to two decimal places.

6D 6 Calculate the volume of these 3D objects; write your answer correct to two decimal places if needed.

- a** a cube with side length 5 cm
b a rectangular prism with breadth 3 mm, length 4 mm and height 5 mm



6D 7 The volume of a cylindrical water tank with a radius of 10 m is 4000 m^3 .

- a** What is the capacity of the water tank?
b Calculate the height of the tank to the nearest metre.

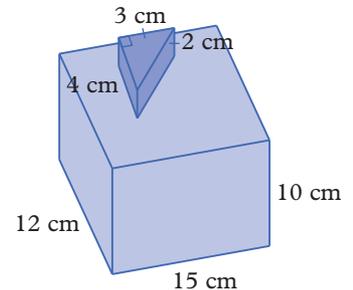
6E 8 Calculate the percentage error for the following measurements.

- a** 55.0 cm using a rule with 1 mm as the smallest division
b 32.5 cm using a tape measure with 5 mm as the smallest division
c 30 g using a scale with 1 g as the smallest unit of measurement
d 108 mL using a measuring cylinder with 1 mL as the smallest division

Analysis

1 Angelique is making a sculpture. The rectangular prism will be painted purple and the triangular prism will be painted pink. The triangular prism has a height of 2 cm and the cross-section is a right-angled triangle with a base length of 3 cm and a perpendicular height of 4 cm.

- a** What is the length of the longest side of the triangle?
b Calculate the area to be painted in each colour (cm^2), assuming all faces, including the bottom, will be painted before assembly.
c Angelique has a tin of purple paint that will cover 1 m^2 . Does she have enough purple paint for two coats?
d After painting two purple coats, what area would the remaining purple paint cover?
e Pink paint comes in smaller tins that contain enough paint to cover 90 cm^2 . What is the maximum number of coats possible from one tin?



2 Kala is constructing a right prism using cardboard with an irregular hexagonal base. She plans to construct the base using right-angled triangles. Her plan is shown on the right.

a Calculate the area of the hexagonal base.

Kala initially wants the prism to be 8 cm tall.

b What is the total amount of cardboard Kala will need?

c How much space will Kala's prism take up?

Kala instead decides to choose the height of the prism such that the value of its surface area and volume are equal. Let h be the height of Kala's prism.

d Write expressions, in terms of h , for the surface area and volume of the prism.

e Determine the height of the prism, correct to two decimal places, if the value of the surface area and volume are equal.

f Kala uses a ruler with smallest division 0.2 cm to mark the height of the prism, where the height is the length determined in part **e**. Assume all other measurements are equal to their true values. Determine, correct to two decimal places where necessary, the:

i absolute error of the ruler

ii percentage error of the height of the ruler

iii minimum and maximum surface areas of the prism.

3 Water is being pumped from a cylindrical water tank with a diameter of 1.2 m and a height of 2.4 m into a rectangular pool 8 m long, 4 m breadth and 1.5 m deep.

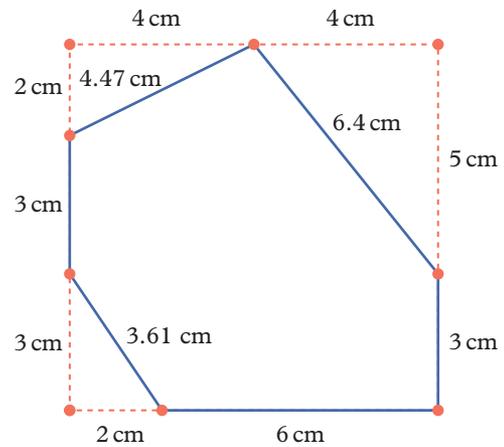
a What is the total area inside the pool that needs to be covered in tiles?

b Calculate the capacity of the water tank, correct to the nearest litre.

c Will a full tank of water be enough to fill the pool? If not, how many more identical tanks are needed to fill the pool?

d A new water tank is installed that is twice as wide as the current tank but only half as high. Will the new tank hold more or less water than the current tank?

e 1 gram of chlorine is needed to maintain the quality of every 1000 L of water. If the dimensions of the pool are measured to the nearest 10 centimeters, what are the maximum and minimum amounts of chlorine needed to maintain the pool? Write your answer correct to two decimal places.

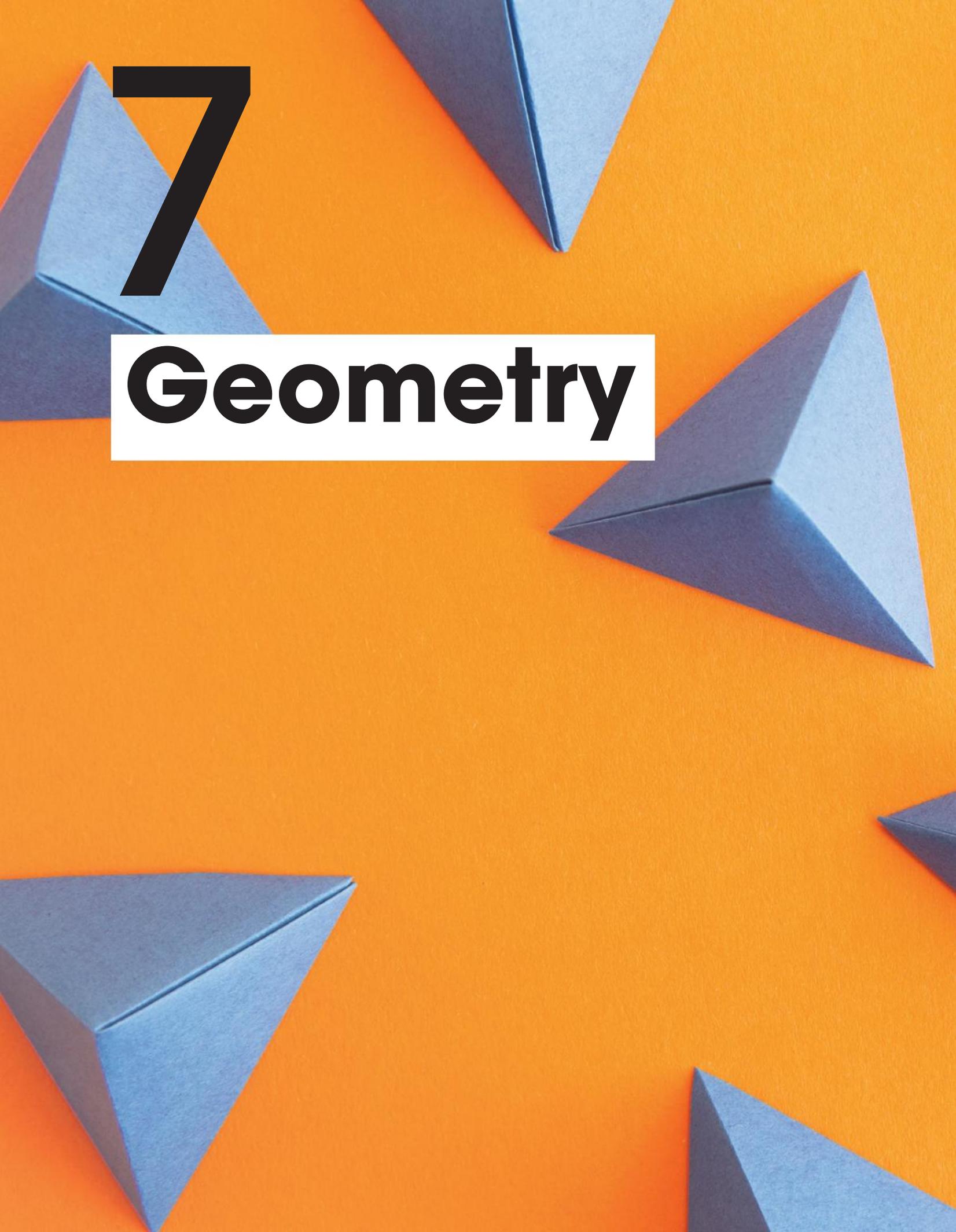


Chapter checklist



Now that you have completed this chapter, reflect on your ability to do the following.

I can do this	I need to review this
<input type="checkbox"/> Calculate the area of composite shapes	<input type="checkbox"/> Go back to Topic 6A Area of composite shapes
<input type="checkbox"/> Draw the net for right prisms	<input type="checkbox"/> Go back to Topic 6B Surface area
<input type="checkbox"/> Calculate the surface area of right prisms	
<input type="checkbox"/> Draw the net of cylinders	<input type="checkbox"/> Go back to Topic 6C Surface area of cylinders
<input type="checkbox"/> Calculate the surface area of cylinders	
<input type="checkbox"/> Calculate the volume and capacity of prisms and cylinders	<input type="checkbox"/> Go back to Topic 6D Volume of composite solids
<input type="checkbox"/> Calculate the volume of composite solids	
<input type="checkbox"/> Determine the precision of a measuring instrument	<input type="checkbox"/> Go back to Topic 6E Errors
<input type="checkbox"/> Find the absolute error of measuring instruments	
<input type="checkbox"/> Calculate the percentage error of a given measurement	



7

Geometry

Index

- ADV 7A** Symmetry and reflections
- ADV 7B** Translations and rotations
- 7C** Dilations and similar figures
- EXT 7D** Area and volume scale factors
- EXT 7E** Congruence
- EXT 7F** Congruent triangles
- EXT 7G** Similar triangles
- 7H** Trigonometric ratios
- 7I** Using trigonometry to find side lengths
- 7J** Using trigonometry to find angles

Prerequisite skills



Diagnostic pre-test

Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.



Interactive skillsheets

After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✓ The Cartesian plane
- ✓ Volume of prisms
- ✓ Triangle properties
- ✓ Substitution
- ✓ Solving equations using inverse operations

Curriculum links

- Applies trigonometric ratios to solve right-angled triangle problems (MA5-TRG-C-01)
 - Demonstrate and explain ...
 - Apply trigonometry to solve ...
- Identifies and applies the properties of similar figures and scale drawings ... (MA5-GEO-C-01)
 - Identify and describe the properties ...
 - Solve problems using ratio and scale ...
- ADV** • Describes and applies transformations, the midpoint, gradient/slope and distance ... (MA5-LIN-P-01)
 - Identify line and rotational symmetries
 - Describe translations, reflections in an axis ...
- EXT** • Establishes conditions for congruent triangles and similar triangles ... (MA5-GEO-P-01)
 - Identify and explain congruence
 - Develop and use the conditions ...
 - Develop and apply the minimum ...
 - Establish and apply properties ...

7A Symmetry and reflections

Learning intentions

By the end of this topic you will be able to ...

- ✓ identify line and rotational symmetry in plane shapes
- ✓ identify line and rotational symmetry in graphs
- ✓ reflect points and objects in axes of symmetry.



Inter-year links

Support Transformations

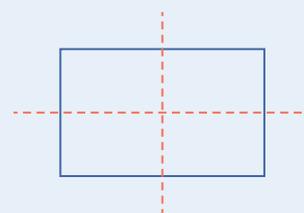
Year 7 5D The Cartesian plane

Line symmetry

- A plane shape has **line symmetry** if it can be divided into two halves that are mirror images of each other.
- The line about which a plane shape is symmetrical is called an axis of symmetry (or line of symmetry).
- The **order of line symmetry** is the number of axes of symmetry for a shape.
For example, the rectangle on the right has two axes of symmetry, so the order of line symmetry is 2.
→ Plane shapes can have none, one or many axes of symmetry.

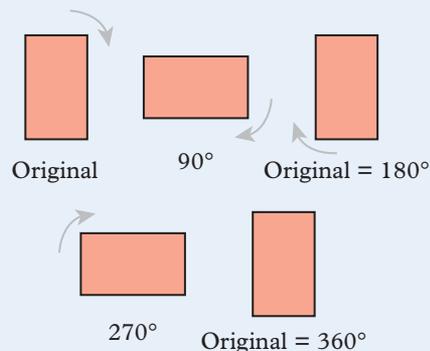


Key content video



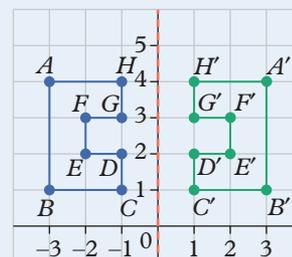
Rotational symmetry

- A plane shape has **rotational symmetry** if it fits exactly onto itself after being rotated less than 360° .
- The **order of rotational symmetry** is the number of times a plane shape fits exactly onto itself when being rotated through 360° .
For example, all non-square rectangles have an order of rotational symmetry of 2.
- Plane shapes with no rotational symmetry have an order of rotational symmetry of 1.



Reflections

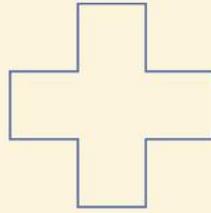
- A transformation is a manipulation of a shape or object. A reflection is an example of a transformation.
→ A transformed shape (also called an **image**) should always be given a name that indicates that it is a transformation – usually by adding a small dash to the top right of the original name. For example, the image of point A is called A' ('A prime').
- To reflect a shape in an axis of symmetry, start by reflecting a single point. This point is called the **point of focus**. Then continue to reflect single points until you can confidently draw the whole image.
→ When performing a reflection, each point in the image must be the same distance from the axis of symmetry as the corresponding original point.



Example 7A.1 Finding the order of line symmetry



What is the order of line symmetry for this shape?

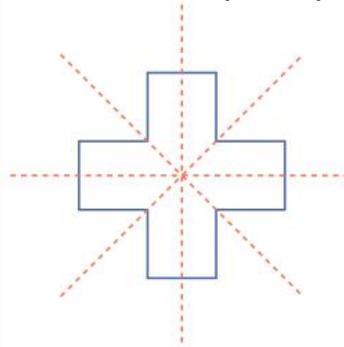


THINK

- 1 Draw all axes of symmetry. Consider horizontal, vertical and diagonal axes of symmetry. Each side of the axis must be a mirror image of the other side.
- 2 Count the number of axes of symmetry.

WRITE

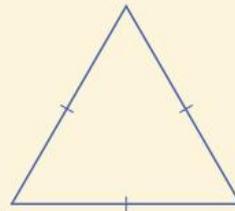
The order of line symmetry for this shape is 4.



Example 7A.2 Finding the order of rotational symmetry



What is the order of rotational symmetry for this shape?

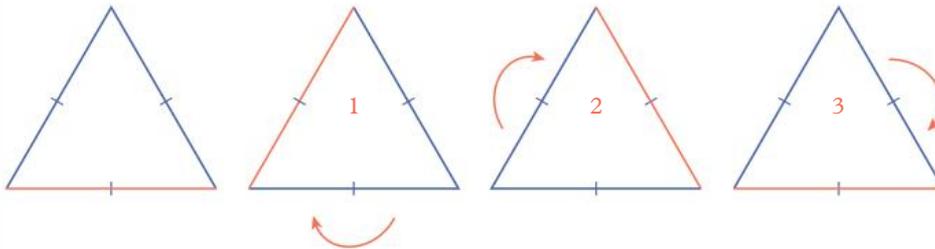


THINK

- 1 Rotate the triangle until it looks exactly the same as when you started.
- 2 Count the number of times the shape looks exactly the same in a 360° rotation.

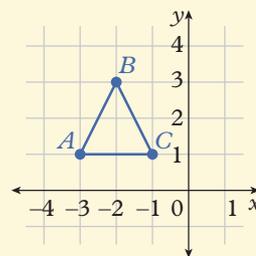
WRITE

The order of rotational symmetry for this shape is 3.



Example 7A.3 Performing a reflection on a Cartesian plane

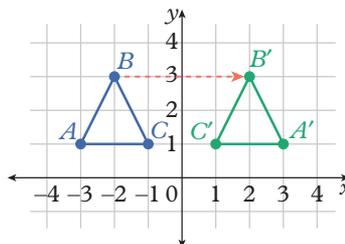
Reflect the object on the right across the y -axis and give the coordinates of the image.



THINK

- 1 Choose a point of focus, B .
- 2 Reflect B across the y -axis. Label the reflected point B' .
- 3 Reflect the other points across the y -axis and draw the reflected shape.
- 4 Give the coordinates of the image.

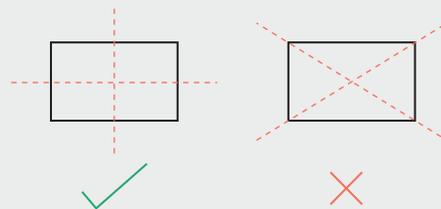
WRITE



The coordinates of the image are $A'(3, 1)$, $B'(2, 3)$ and $C'(1, 1)$.

Helpful hints

- ✓ Be careful with diagonal lines of symmetry! Many people mistakenly think that the diagonals of rectangles and parallelograms are axes of symmetry when they are not.
- ✓ Remember to give your image a name that identifies it as a transformation. For example, for a shape B , the image is named B' .



ANS
p481

Exercise 7A Symmetry and reflections

▲ 1(a-d), 2, 3(a-d), 4-6, 9, 10, 12-14

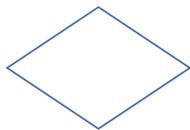
■ 1(c-f), 3(c-f), 5-9, 11, 13-15, 17

◆ 2, 4, 7-9, 13-19

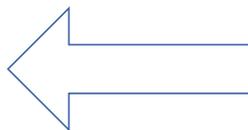
UNDERSTANDING AND FLUENCY

7A.1 1 What is the order of line symmetry for each of the following shapes?

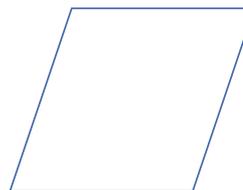
a



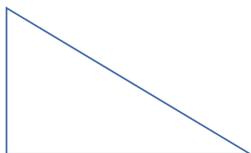
b



c



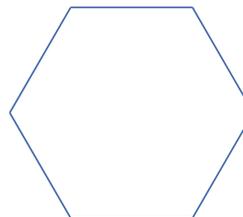
d

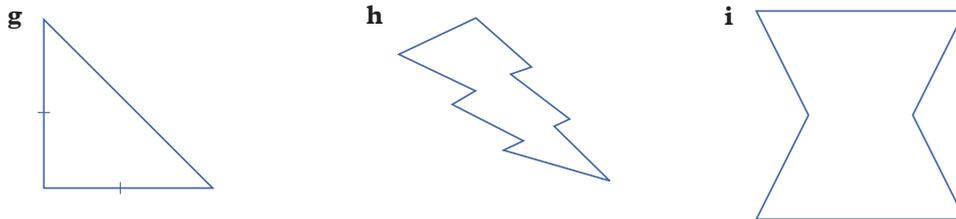


e



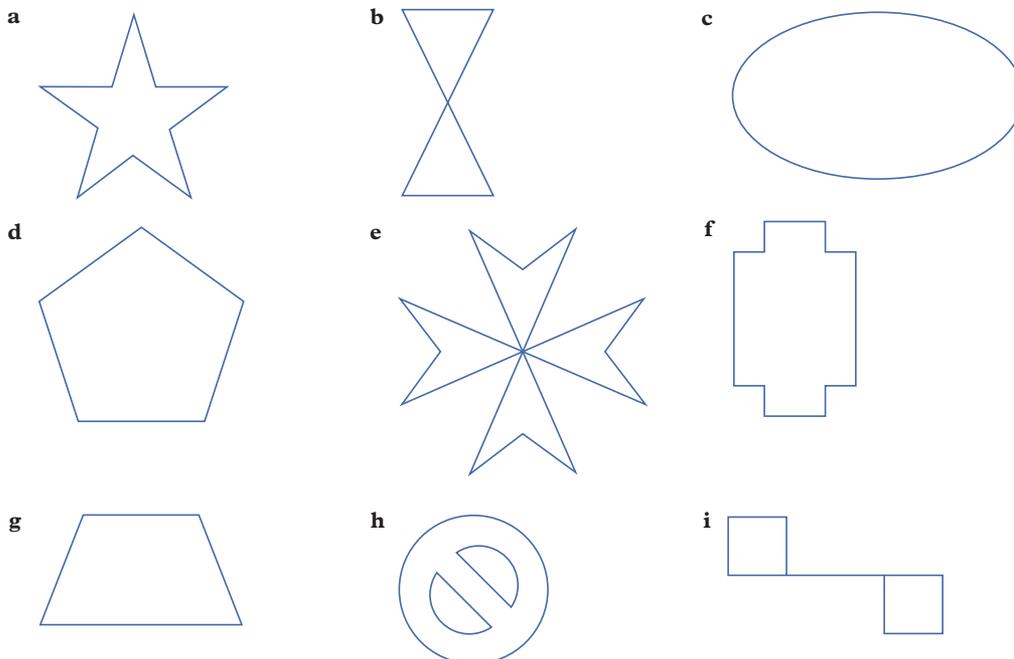
f



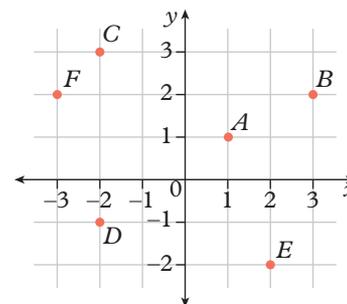


- 2 Consider the 26 letters of the alphabet in upper case.
- Find the axis or axes of symmetry for each letter (if they exist). Consider the letter O to be a circle.
 - Which letters have an order of line symmetry of 0?
 - Which letters have an order of line symmetry of 2 or more?

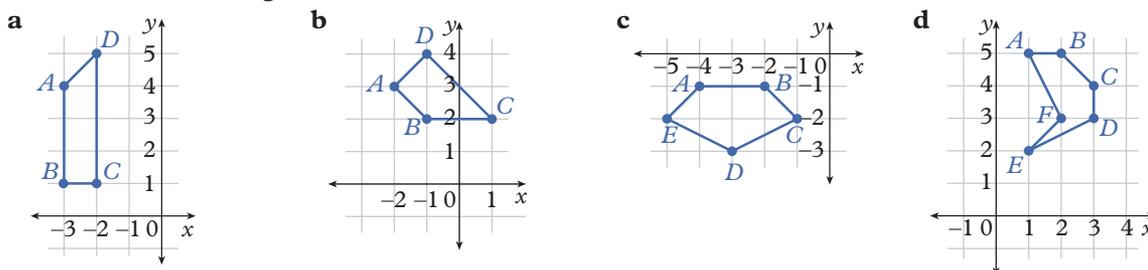
7A.2 3 What is the order of rotational symmetry for each of the following shapes?



- 4 What letters of the alphabet (in upper case) have an order of rotational symmetry that is 2 or more? List these with their orders of rotational symmetry. Consider the letter O to be a circle.
- 5 Reflect each of the labelled points in the diagram on the right across the y -axis and give the coordinates of the images.
- 6 Reflect each of the points in question 5 across the x -axis and give the coordinate of the images.

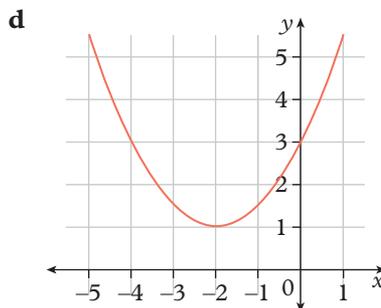
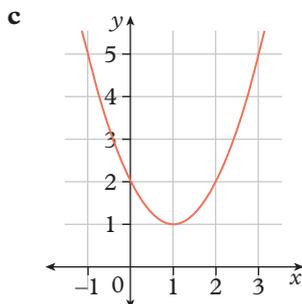
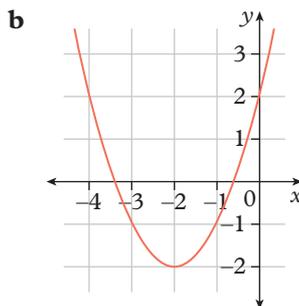
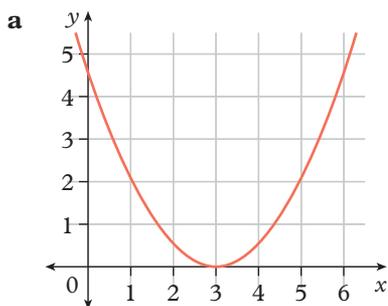


7A.3 7 Reflect each of the following shapes across the y -axis and give the coordinates of the images.

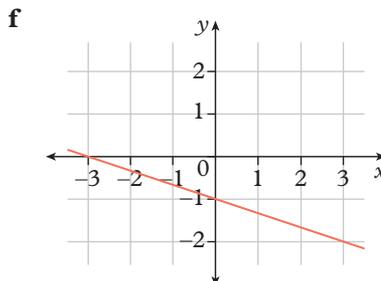
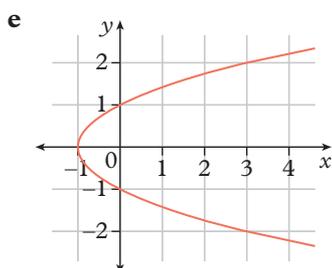
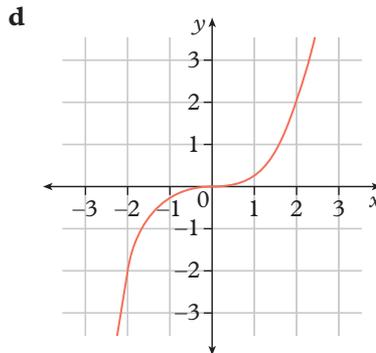
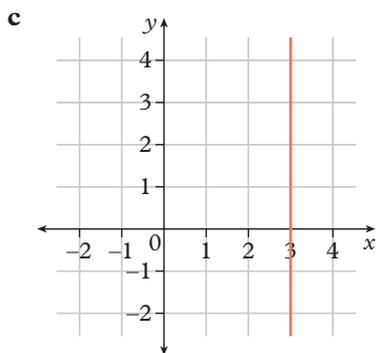
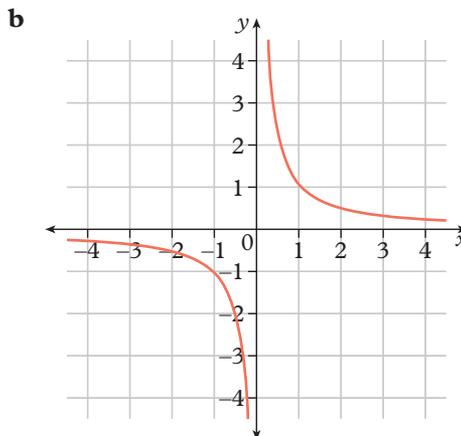
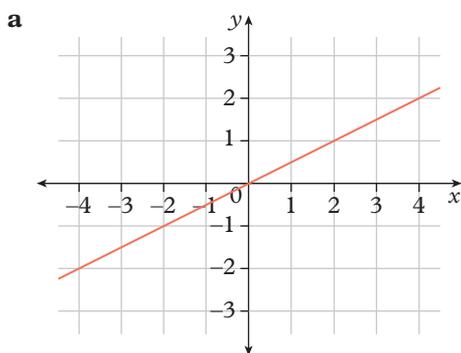


- 8 Reflect each of the shapes in question 7 across the x -axis and give the coordinates of the images.

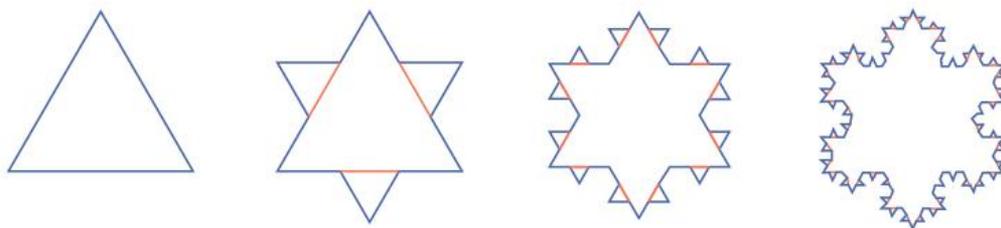
- 9 All graphs of quadratic equations have an axis of symmetry, with the curve being symmetrical on either side of the axis of symmetry. The equation for the axis of symmetry on a quadratic graph will be of the form $x = \underline{\hspace{1cm}}$.
Give the equation of the axis of symmetry for the following quadratic graphs.



- 10 Do these graphs have line symmetry?



- 11 Explain why no quadratic graph has rotational symmetry.
- 12 Which of the graphs from question 10 have rotational symmetry?
- 13 Points A , B and C have the coordinates $(1, 4)$, $(-2, 3)$ and $(5, -1)$, respectively.
- If all three points are reflected in the y -axis, what are the coordinates of A' , B' and C' , respectively?
 - Explain how the coordinates of an image of a point reflected in the y -axis can be determined without looking at a Cartesian plane.
- 14 Points P , Q and R have the coordinates $(0, 3)$, $(2, -1)$ and $(-4, -3)$, respectively.
- If all three points are reflected in the x -axis, what are the coordinates of P' , Q' and R' , respectively?
 - Explain how the coordinates of an image of a point reflected in the x -axis can be determined without looking at a Cartesian plane.
- 15 The Koch snowflake is a fractal curve. It is built up in a sequence of stages, beginning with an equilateral triangle.



- How many lines of symmetry does the first triangle have?
 - How many lines of symmetry does the fourth snowflake have?
 - What can you say about the number of lines of symmetry as the snowflake gets more sides? Explain your answer.
- 16 a Plot the following coordinates on graph paper and join them to make a shape.
 $(2, 2)$, $(6, 2)$, $(4, 5)$
- Reflect the shape across the y -axis. What are the new coordinates?
 - Reflect the original shape across the x -axis. What are the new coordinates?
- 17 Use centimetre graph paper to complete the following.
- Plot the following coordinates on graph paper.
 $(5, 2)$, $(5, 8)$, $(3, 8)$
 - Add one pair of coordinates to make a quadrilateral that has a vertical line of symmetry.
- 18 Use centimetre graph paper to complete the following.
- Plot the following coordinates on graph paper.
 $(2, 2)$, $(5, 4)$, $(2, 8)$
 - Add one pair of coordinates to make a quadrilateral that has a horizontal line of symmetry.
- 19 a A triangle is reflected across the x -axis. The new coordinates of the vertices are $(1, 1)$, $(2, 1)$ and $(1, 2)$. What were the original coordinates?
- A square is reflected across the y -axis. The new coordinates of the vertices are $(1, 1)$, $(2, 1)$, $(2, 2)$ and $(1, 2)$. What were the original coordinates?

Check your Student obook pro for these digital resources and more:

pro



BLM
Exercise 7A



Interactive skillsheet
Line symmetry



Interactive skillsheet
Rotational symmetry



Interactive skillsheet
Reflections



Worksheet
Symmetrical shapes



Investigation
Flags of the world



Topic quiz
7A

7B Translations and rotations

Learning intentions

By the end of this topic you will be able to ...

- ✓ perform translations on a Cartesian plane
- ✓ perform rotations on a Cartesian plane
- ✓ identify coordinates after a point or shape has been transformed.



Inter-year links

Support Transformations

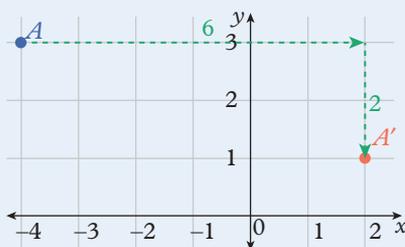
Year 7 5D The Cartesian plane

Translations



Key content video

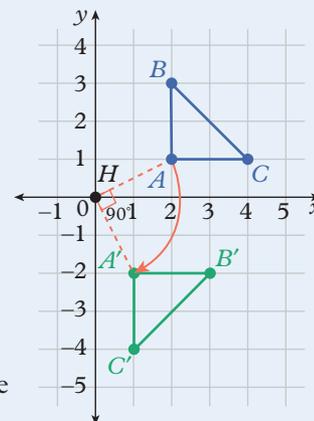
- A **translation** is a transformation which moves a point, line or shape without turning or changing the size. Translations on a Cartesian plane are recorded by counting how many units the object slides up or down and left or right.
 - Translations along the x -axis slide the shape left or right.
 - Translations along the y -axis slide the shape up or down.
 - Point A has been translated 6 units to the right (along the x -axis) and 2 units down (along the y -axis).



- Multiple translations along the same axis can be combined into a single translation.
 - For example, a translation of 2 units down followed by a translation of 3 units down is the same as a translation of 5 units down.

Rotations

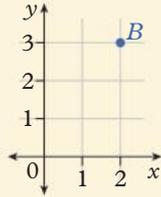
- A **rotation** turns a shape about a fixed point. To perform a rotation, three details are required:
 - 1 the centre of rotation
 - usually the origin or a coordinate
 - 2 the angle of the rotation
 - between 0° and 360°
 - 3 the direction of the rotation
 - clockwise or anticlockwise.
- When performing a rotation, identify a point of focus to concentrate on.
 - After rotating one point you may be able to determine the positions of the other rotated points in relation to this point. If not, proceed by rotating a second point.





Example 7B.1 Performing a translation

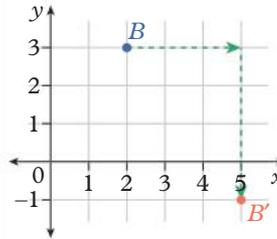
Translate this point 3 units to the right and 4 units down.



THINK

- 1 Count 3 units to the right.
- 2 Count 4 units down.
- 3 Label the translated image B' to show that it is a transformation of the original point.

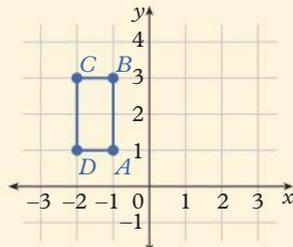
WRITE



Example 7B.2 Performing a rotation on a Cartesian plane



Rotate this shape 90° clockwise about the origin and give the coordinates of the image.

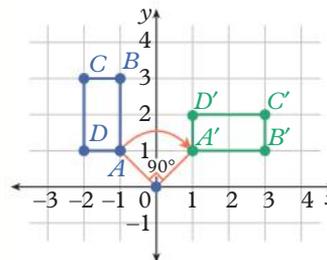


THINK

- 1 Choose a point of focus, A .
- 2 Rotate A by 90° clockwise about the origin. Label the rotated point A' .
- 3 Choose a second point to rotate, B . Rotate B by 90° clockwise about the origin and label the rotated point B' .
- 4 If you can, draw the remaining points of the rotated shape. If needed, rotate a third point from the original shape. The rotated shape will be the same shape and size as the original shape.
- 5 Give the coordinates of the image.

WRITE

Point A is the point of focus.



The coordinates of the image are $A'(1, 1)$, $B'(3, 1)$, $C'(3, 2)$ and $D'(1, 2)$.



Example 7B.3 Finding coordinates after a translation

A point P is located at $(-3, 4)$ on a Cartesian plane. It is translated 6 units down and 10 units to the right. What are the coordinates of the translated point P' ?

THINK

- 1 If translating to the right, add the units to the x -coordinate. If translating to the left, subtract the units from the x -coordinate.
- 2 If translating up, add the units to the y -coordinate. If translating down, subtract the units from the y -coordinate.

WRITE

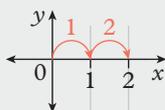
$$-3 + 10 = 7$$

$$4 - 6 = -2$$

The coordinates of P' are $(7, -2)$.

Helpful hints

- ✓ When identifying or performing a translation, do not start counting at the starting position. Start counting at the position of the first square beyond the shape.



ANS
p482

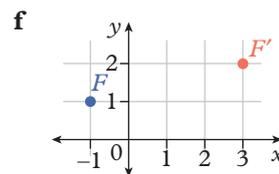
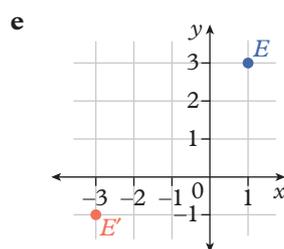
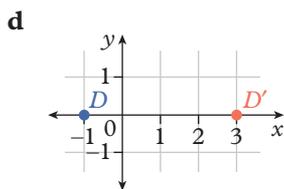
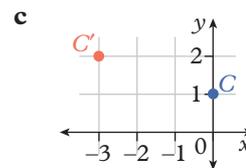
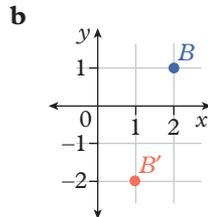
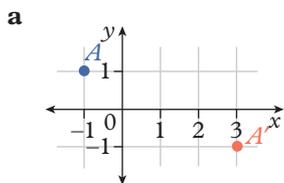
Exercise 7B Translations and rotations

▲ 1-2(a-d), 3(a, b), 5, 6(a-d), 7(a-d), 9(a, b), 11, 14(a, b), 17

■ 2(c-f), 3-4(c-d), 5, 6(c-f), 8, 9(c, d), 12, 13, 14(c, d), 16, 18

◆ 2(c-f), 3(c, d), 5, 9-10(e, f), 12, 14(e, f), 15, 16, 19, 20

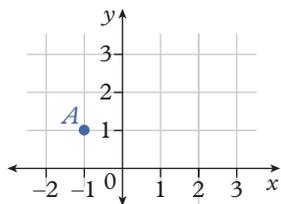
1 Identify the translations shown in the following diagrams.



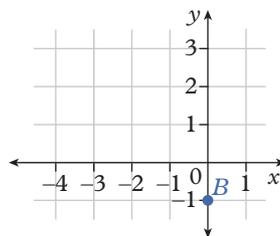
UNDERSTANDING AND FLUENCY

7B.1 2 Translate the following points by the given amount.

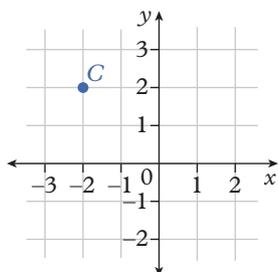
a 2 units right and 1 unit up



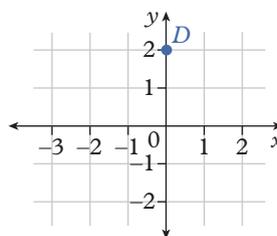
b 4 units left and 3 units up



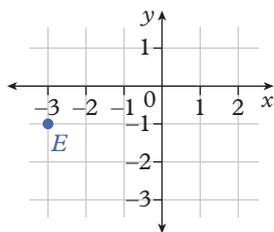
c 4 units down



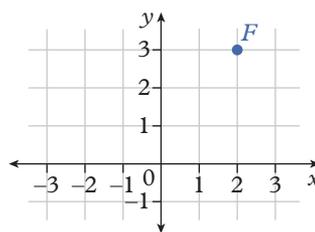
d 3 units left



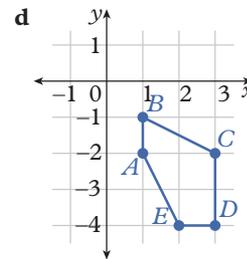
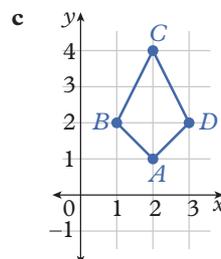
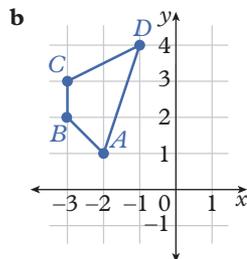
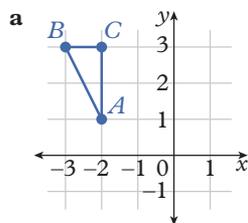
e 4 units right and 2 units down



f 3 units left and 1 unit down



7B.2 3 Rotate each of the shapes 90° clockwise about the origin and give the coordinates of the images.



4 Rotate each of the shapes in question 3 90° anticlockwise about the origin and give the coordinates of the images.

7B.3 5 **a** Point Q is located at $(2, 1)$ on a Cartesian plane. It is translated 7 units to the left and 4 units down. What are the coordinates of the translated point Q' ?

b Point R is located at $(-5, 0)$ on a Cartesian plane. It is translated 2 units to the right and 5 units up. What are the coordinates of the translated point R' ?

c Point S is located at $(2, -4)$ on a Cartesian plane. It is translated 12 units to the left and 4 units up. What are the coordinates of the translated point S' ?

d Point T is located at $(1, 7)$ on a Cartesian plane. It is translated 1 unit to the right and 9 units down. What are the coordinates of the translated point T' ?

e Point U is located at $(-3, -5)$ on a Cartesian plane. It is translated 4 units to the left and 5 units up. What are the coordinates of the translated point U' ?

f Point V is located at $(-10, 2)$ on a Cartesian plane. It is translated 7 units to the right and 9 units down. What are the coordinates of the translated point V' ?

6 Describe the translation required to move the first point to the second point.

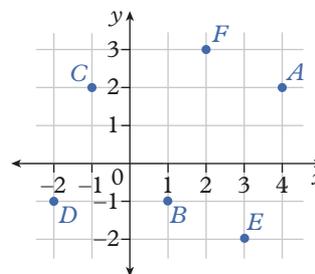
- a $A(2, 1)$ to $A'(5, 5)$
- b $B(-3, 3)$ to $B'(0, 4)$
- c $C(5, 2)$ to $C'(1, -1)$
- d $D(-2, 0)$ to $D'(2, -2)$
- e $E(4, -1)$ to $E'(6, -3)$
- f $F(-2, -3)$ to $F'(3, 3)$

7 Which single translation is equal to these combined translations?

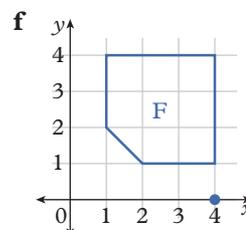
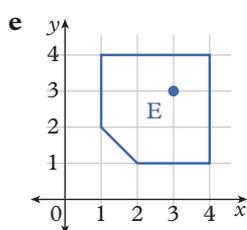
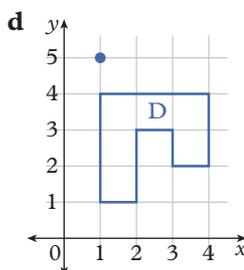
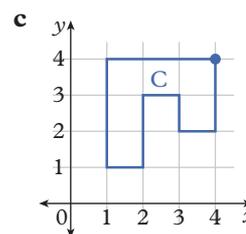
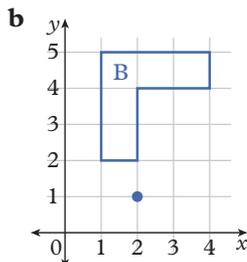
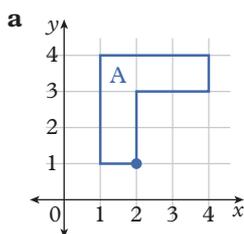
- a 2 units right and 5 units up, followed by 3 units right and 2 units down
- b 5 units right and 1 unit up, followed by 4 units left and 6 units down
- c 1 unit left and 2 units down, followed by 2 units left and 8 units down
- d 4 units right and 3 units up, followed by 9 units right and 3 units up
- e 2 units right and 4 units down, followed by 2 units right and 4 units up
- f 7 units left and 1 unit down, followed by 1 unit left and 10 units up

8 Look at the Cartesian plane on the right.

- a Which point has been translated 1 unit right and 4 units up from point B ?
- b Which point has been translated 1 unit left and 4 units down from point A ?
- c Describe these translations.
 - i point B to point C
 - ii point A to point B
 - iii point D to point A
 - iv point F to point E
 - v point C to point F
 - vi point E to point B

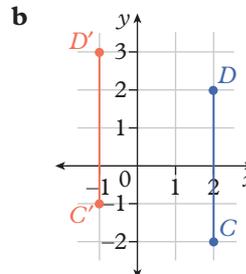
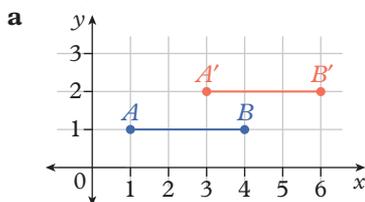


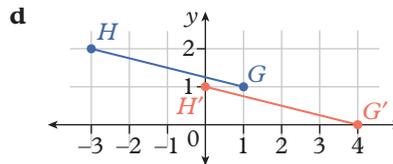
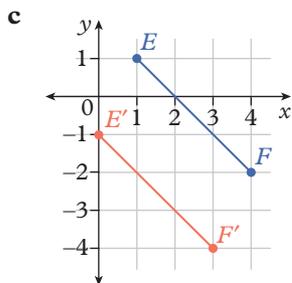
9 Rotate each shape 90° clockwise about the given point.



10 Rotate the shapes in question 9 180° clockwise about the given point.

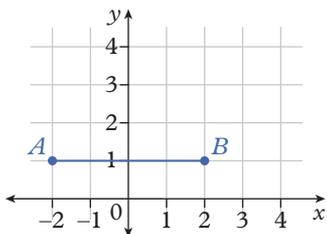
11 Identify the translations shown in the following diagrams.



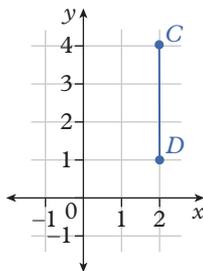


12 Translate the following line segments by the given amounts.

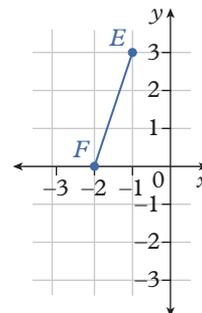
a 2 units right and 3 units up



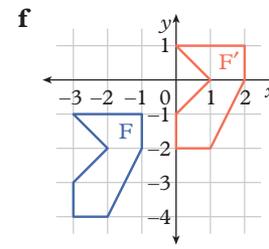
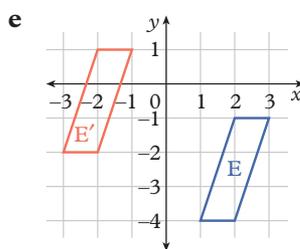
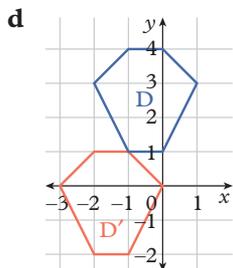
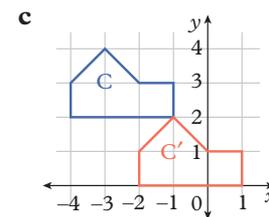
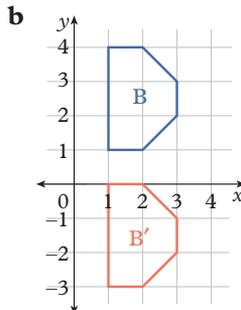
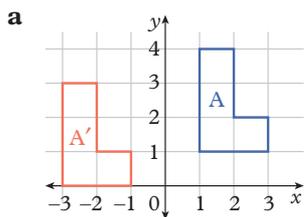
b 3 units left and 2 units down



c 1 unit right and 3 units down

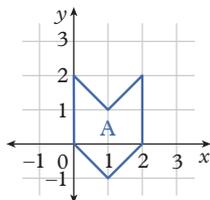


13 Identify the translations shown in the following diagrams.

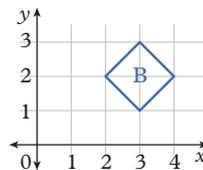


14 Translate the following shapes by the given amount.

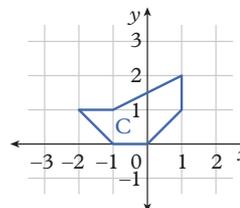
a 4 units up and 3 units left



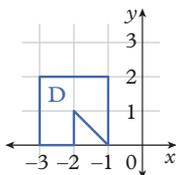
b 4 units down and 5 units left



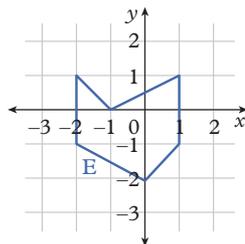
c 1 unit down and 1 unit right



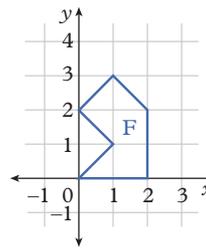
d 5 units up and 5 units right



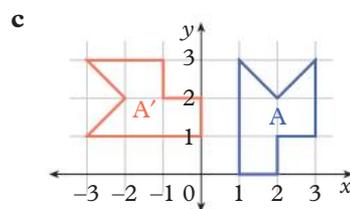
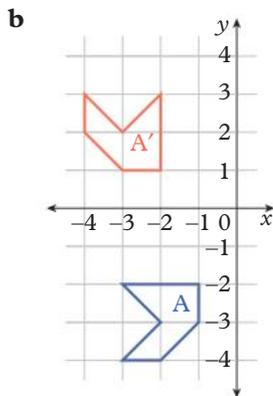
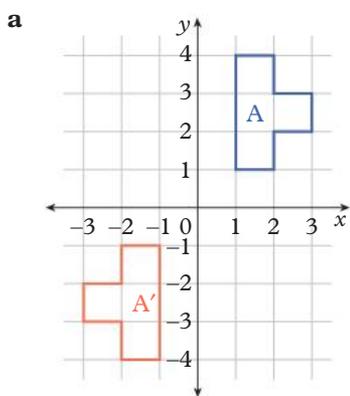
e 3 units down and 2 units right



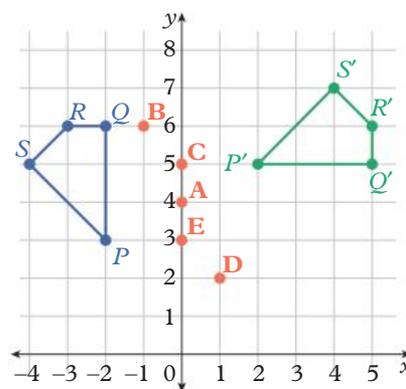
f 2 units up and 2 units right



15 Each of these shapes has been rotated clockwise about the origin. By how many degrees have they each been rotated?



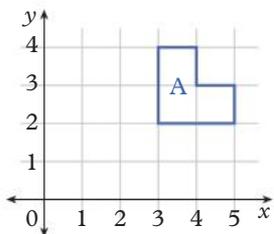
16 About which of the given points has the blue shape in the diagram on the right been rotated by 90° clockwise?



17 Explain why a rotation of 90° clockwise can also be described as rotation of 270° anticlockwise.

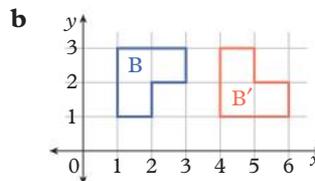
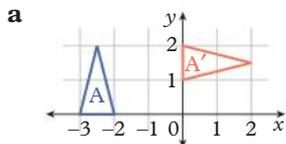
18 A quadrilateral is rotated 90° clockwise about the origin, and the new coordinates of the vertices are $(8, 2)$, $(10, 4)$, $(6, 4)$ and $(8, 6)$. What were the original coordinates?

19 Transform this shape according to the steps listed. Draw each shape.



Translate 3 units to the left and 4 units down, then rotate 90° clockwise about the bottom right corner, then reflect across the y -axis. Label the final image A''' .

20 A rotation clockwise about the origin followed by a translation have been performed in the following diagrams. State the transformations.



Check your Student obook pro for these digital resources and more:

pro



BLM
Exercise 7B



Interactive skillsheet
Translations



Interactive skillsheet
Rotations



Worksheet
Transforming shapes



Investigation
Tetris



Topic quiz
7B

7C Dilations and similar figures

Learning intentions

By the end of this topic you will be able to ...

- ✓ determine scale factors
- ✓ find unknown lengths in similar figures
- ✓ solve problems involving scale drawings.



Inter-year links

Year 7

3B Equivalent fractions

Dilations

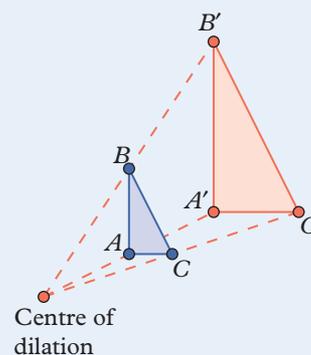


Key content video

- A **dilation** is a transformation that changes the size of an object. For a dilation, the shape and orientation of the image remains the same as the original shape.
 - The angle measurements of a shape do not change after a dilation.
- A dilation can be either an **enlargement** or a **reduction**.
 - An enlargement creates an image that is larger than the original shape.
 - A reduction creates an image that is smaller than the original shape.
- A **centre of dilation** and **scale factor** need to be specified to perform a dilation.
 - If the scale factor is between 0 and 1, the image will be a reduction.
 - If the scale factor is greater than 1, the image will be an enlargement.
- The scale factor of a dilation can be found using the formula:

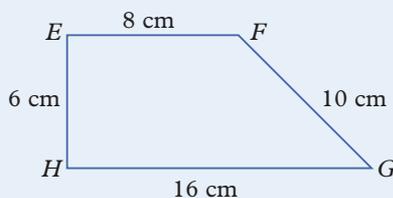
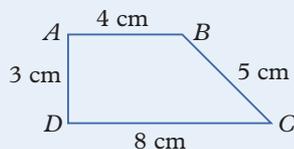
$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$

- The side length of a dilation can be found by multiplying the original side length by the scale factor.
 - For example, when dilating a shape with a side AB :
length of $A'B' = \text{length of } AB \times \text{scale factor}$.



Similar figures

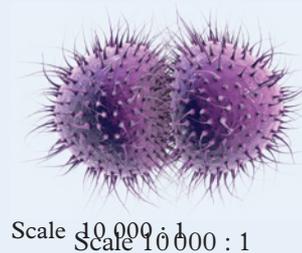
- Two figures are said to be **similar** if they are the same shape. Similar figures do not need to be the same size.
 - An image produced by a dilation will be a figure similar to the original shape. For example, $ABCD$ and $EFGH$ are similar figures.



- If two figures are similar, all corresponding angles are equal in size and all corresponding side lengths are in the same proportion.
- The symbol \sim can be used to indicate similarity. When making a similarity statement, corresponding vertices must be put in the same order.
 - For example, $ABCD \sim EFGH$.
- If two figures are known to be similar, an unknown side length can be found by using the scale factor.

Scale drawings

- A **scale drawing** is a drawing that represents a real-life object. Scale drawings can be used to accurately represent objects that are difficult to draw at their actual size; for example, house plans.
- In a scale drawing, the **scale** represents the ratio 'drawing length : actual length'. For example, if the scale is 1 : 100, each centimetre in the drawing represents 100 cm in the real-life object.
- Scale drawings can also be used to represent objects that are too small to see without using a microscope, for example, bacteria and viruses.
 - If the scale drawing is 10 000 : 1, each centimetre in the drawing represents 0.0001 cm in real life.

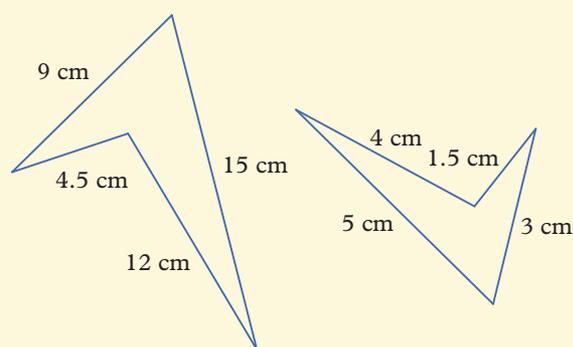


Scale 10 000 : 1
Scale 10 000 : 1

Example 7C.1 Finding the scale factor



Find the scale factor for this pair of similar figures if the figure on the left is the original shape.

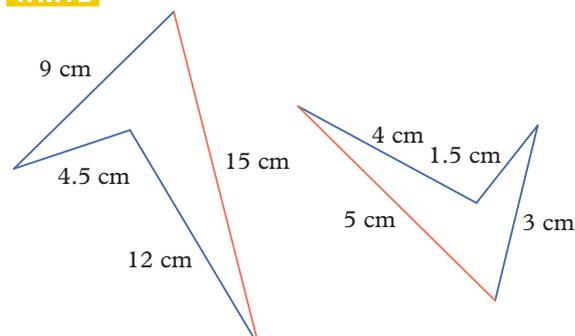


THINK

- 1 Identify two corresponding sides. The longest side of the original figure is 15 cm and the longest side of the image is 5 cm.
- 2 Substitute these lengths into the formula:

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$
- 3 Match the remaining corresponding sides and check that each pair of sides is in the same ratio.
- 4 Check that the scale factor seems reasonable (between 0 and 1 means a reduction).

WRITE



$$\begin{aligned} \text{Scale factor} &= \frac{\text{image length}}{\text{original length}} \\ &= \frac{5 \text{ cm}}{15 \text{ cm}} \\ &= \frac{1}{3} \end{aligned}$$

Check each corresponding pair of sides:

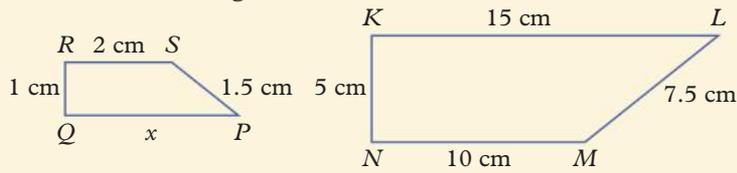
$$\text{Scale factor} = \frac{3}{9} = \frac{1.5}{4.5} = \frac{4}{12} = \frac{1}{3}$$

The scale factor of $\frac{1}{3}$ is a reduction, as required.

Example 7C.2 Finding an unknown side length in similar figures



Find the value of x , given that these two figures are similar.



THINK

- 1 Identify two corresponding sides and calculate the scale factor. Use corresponding vertices to write the side names in the same order. As R corresponds to N and S corresponds to M , then side RS corresponds to side NM (not MN).
- 2 Find a pair of corresponding sides that involve x .
- 3 Substitute in the known values and solve for x .

WRITE

RS and NM are corresponding sides.

$$\begin{aligned} \text{Scale factor} &= \frac{\text{image length}}{\text{original length}} \\ &= \frac{10 \text{ cm}}{2 \text{ cm}} \\ &= 5 \end{aligned}$$

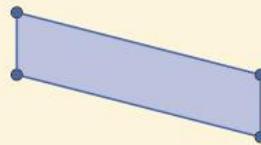
Side QP has length x . QP and KL are corresponding sides.

$$\begin{aligned} \text{Scale factor} &= \frac{\text{image length}}{\text{original length}} \\ 5 &= \frac{15}{x} \\ 5 \times \frac{x}{5} &= \frac{15}{x} \times \frac{x}{5} \\ x &= \frac{15}{5} \\ x &= 3 \text{ cm} \end{aligned}$$

Example 7C.3 Performing a dilation



Dilate this figure by a scale factor of $\frac{1}{2}$ using the centre of dilation O .

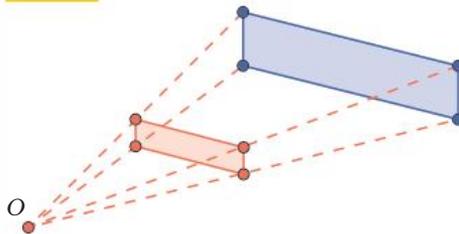


O

THINK

- 1 Use dashed lines to connect the centre of dilation with each of the vertices on the original shape.
- 2 The scale factor is $\frac{1}{2}$ so measure half the distance from the centre of dilation to each of the vertices on the original shape and mark these points. These are the corresponding vertices of the image.
- 3 Join the vertices of the image together to complete the dilation.

WRITE





Example 7C.4 Interpreting scale drawings

The plan for a two-bedroom apartment is drawn to a scale of 1 : 60. If, in the scale drawing, one of the bedrooms measures 8 cm by 5 cm, what is the size of the bedroom in real life? Give your answer in metres.

THINK

- 1 The scale is 1 : 60, so each centimetre in the scale drawing represents 60 cm in real life.
- 2 Multiply the scale drawing dimensions by 60 to determine the real-life dimensions.
- 3 Convert the units to metres by dividing by 100.
- 4 Write the answer.

WRITE

$$8 \times 60 = 480 \text{ cm}$$

$$5 \times 60 = 300 \text{ cm}$$

$$\frac{480}{100} = 4.8 \text{ m}$$

$$\frac{300}{100} = 3 \text{ m}$$

The real-life size of the bedroom is 4.8 m by 3 m.

Helpful hints

- ✓ The image from a dilation will always be in the same orientation as the original shape; however, similar shapes do not need to be in the same orientation.

ANS
p486

Exercise 7C Dilations and similar figures



1-4, 5, 6(a, b), 7, 8, 10(a-c), 11, 14



2-4, 6, 7, 9, 10(d-f), 14, 15, 18, 21



3, 4, 6, 7, 9, 10(e, f), 12, 13, 16-20

UNDERSTANDING AND FLUENCY

- 1 Decide whether each scale factor produces an enlargement or a reduction.

a 4

b $\frac{1}{3}$

c 2

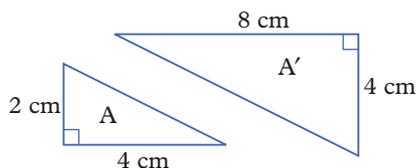
d 6

e $\frac{1}{10}$

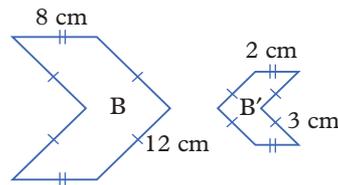
f $\frac{1}{5}$

- 7C.1 2 Find the scale factor for each pair of similar figures.

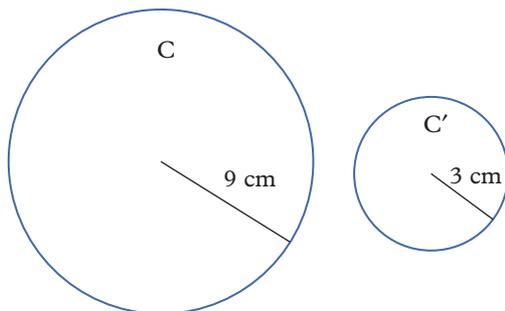
a



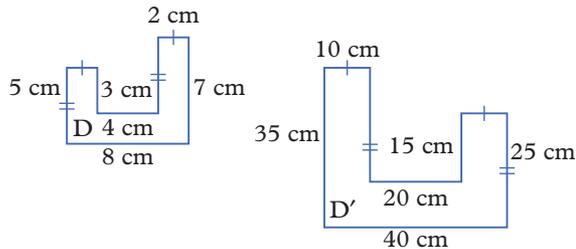
b

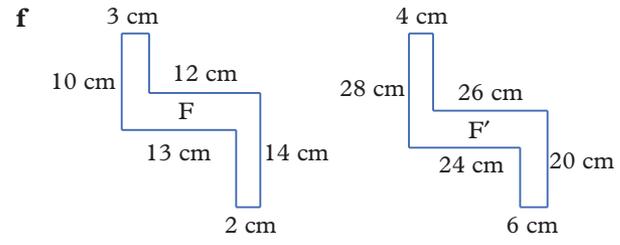
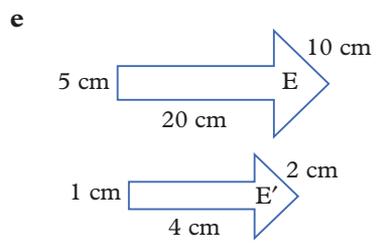


c

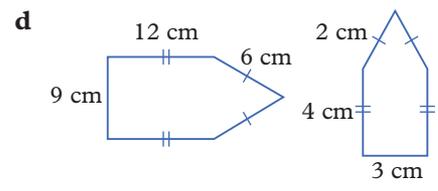
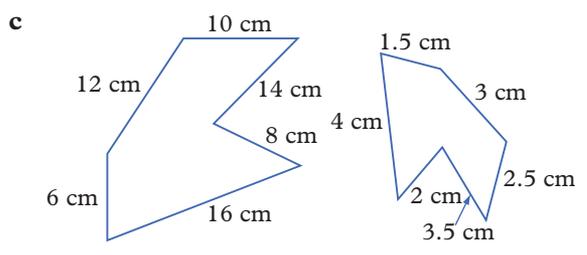
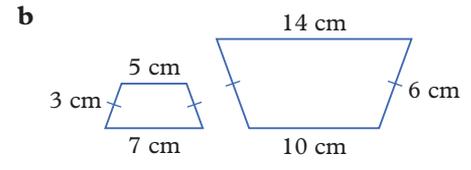
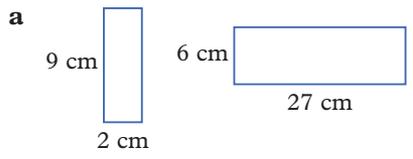


d

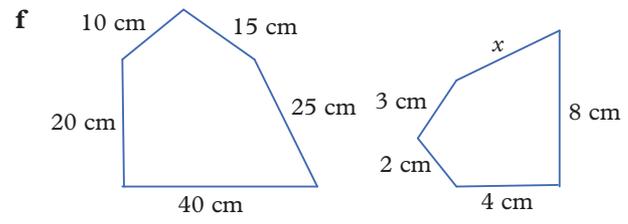
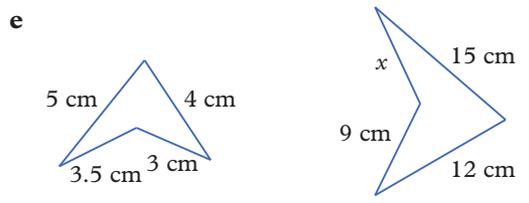
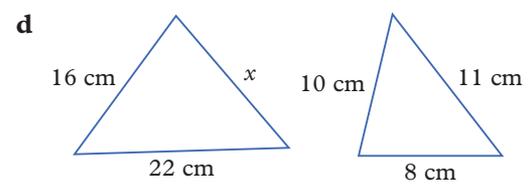
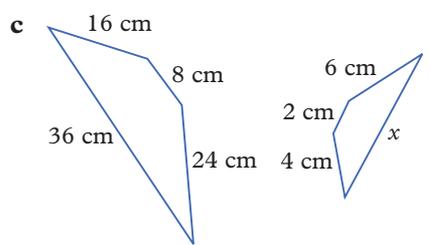
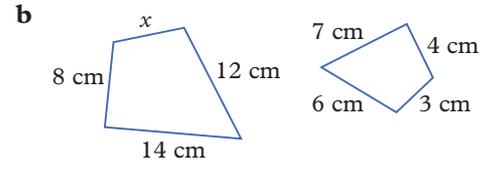
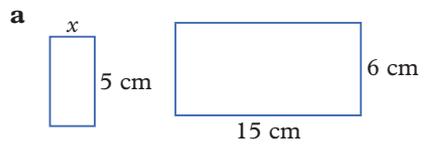




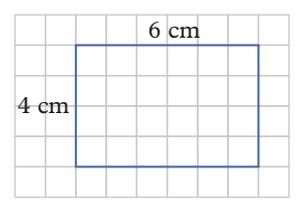
3 Find the scale factor for each pair of similar figures if the figure on the left of each pair is the original shape.



7c.2 4 Find the values of x in the following diagrams, given that each pair of shapes are similar.



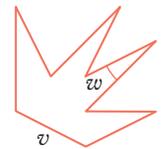
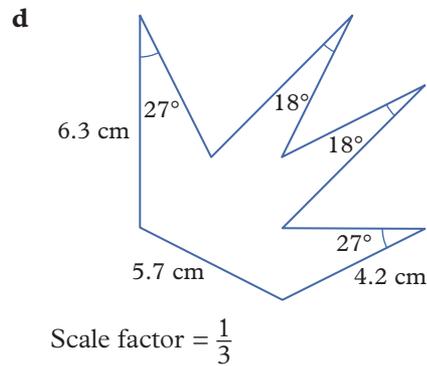
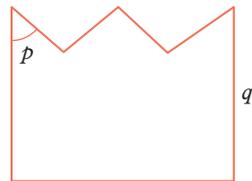
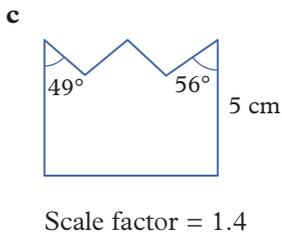
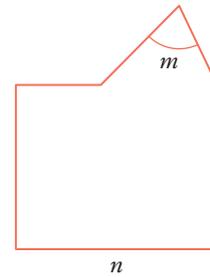
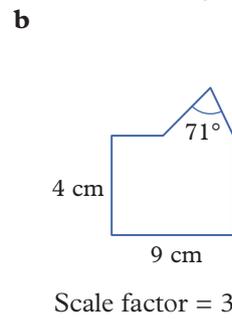
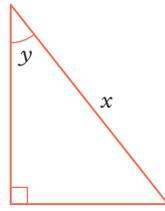
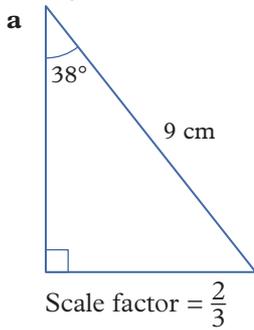
5 The rectangle below has been drawn on grid paper.



Draw a similar rectangle that has dimensions that are:

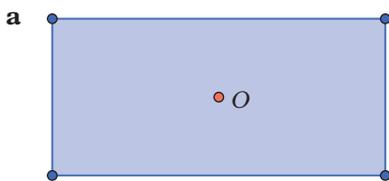
- a** half the size
- b** twice the size.

6 In each pair of shapes, the shape on the left has been dilated by the given scale factor to create the image on the right. Determine the values of the unknown pronumerals in each image.

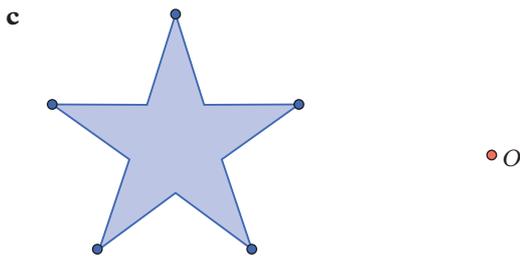
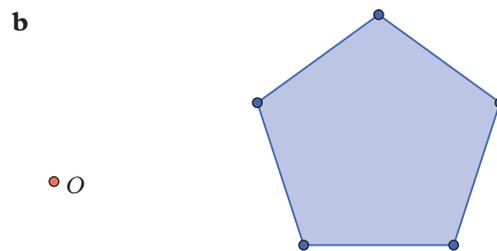


7C.3 7 Use the worksheet available on your gbook pro to dilate each figure using the centre of dilation O and a scale factor of:

i 3



ii $\frac{1}{3}$.



8 The plan for the two-bedroom apartment on the right is drawn to a scale of 1 : 80. If, in the scale drawing, the living room measures 9.8 cm by 4.5 cm, what is the size of the living room in real life? Give your answer in metres.

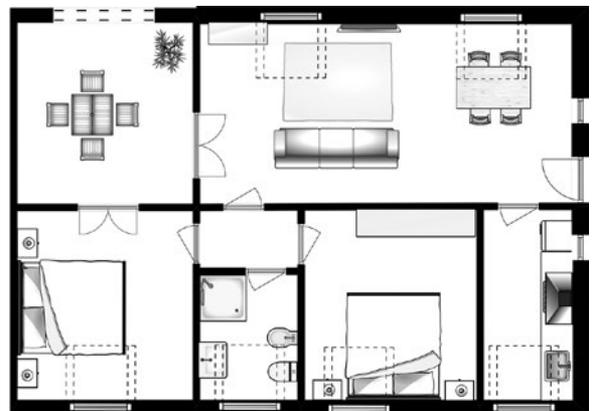
9 Ayesha creates a scale drawing of a house with a scale of 1 : 120. How big are each of the following rooms given their sizes in the scale drawing?

a bathroom (3 cm by 2.3 cm)

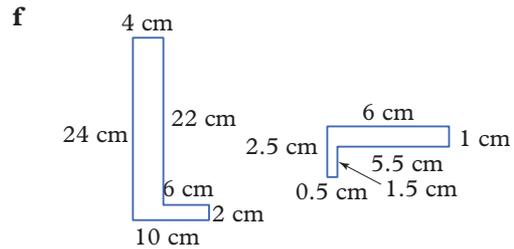
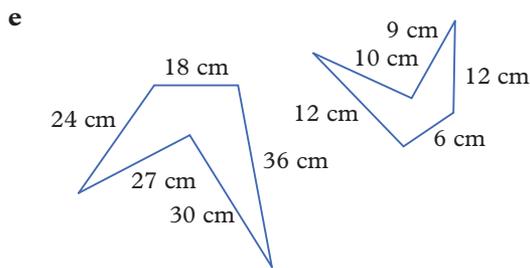
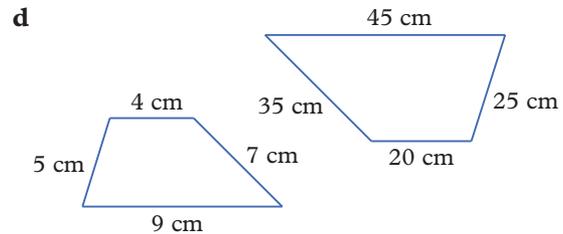
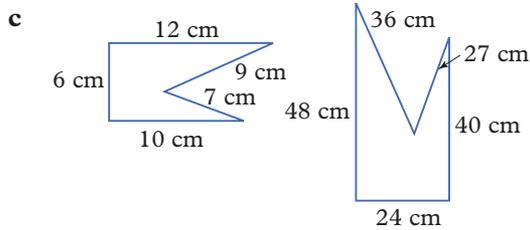
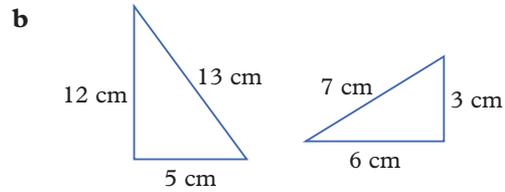
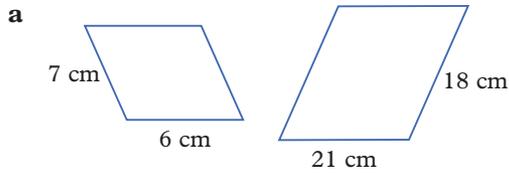
b bedroom (3.5 cm by 2.8 cm)

c kitchen (3.8 cm by 2.2 cm)

d living room (4.2 cm by 4.05 cm)



10 Decide whether each of the following pairs of shapes is similar by calculating and comparing the scale factor for each pair of corresponding sides.



11 A microscope with a magnification factor of 20 000 was used to view a bacterial cell, which appeared 5.2 cm long. What was the actual length of the bacterial cell?

12 Explain why a circle is similar to all other circles and a square is similar to all other squares, but a rectangle is not similar to all other rectangles.

13 Finn was explaining dilations to a classmate and said that a dilation of $\frac{2}{3}$ was the same as dilating by a factor of 2 and then dilating by a factor of $\frac{1}{3}$. Can you explain why he is correct?

14 One standard print size for photographs is 10 cm × 15 cm. Alice wanted to enlarge a photograph to place it onto a canvas measuring 40 cm × 50 cm. Are the photograph and the canvas similar figures?

15 Timothy got a model plane for his birthday, which was constructed using a scale of 1 : 100.

a What does this scale mean?

b If the real-life plane has a length of 34 m, what is the length of the model plane?

c If the model plane has a wingspan of 27 cm, what is the wingspan of the real-life plane?



16 A model car is constructed using a ratio of 1 : 40.

a If the model car has a length of 10 cm, what is the length of the real car?

b If the real car has a height of 1.5 m, what is the height of the model car?

c Do you think that a model car can be a true similar figure to a real car? Explain your answer.



17 Marc has a hiking map with a ratio scale of 1 : 75 000.

a Marc planned to walk the length of a 15 km track. What distance would this be on the map?

b At lunchtime, he had walked a distance shown as 7 cm on the map. How far did he have left to walk on the track?

18 A photo of a red blood cell is printed at a scale of 5000 : 1. In the photo, the diameter of the red blood cell is 3.5 cm. What is the diameter of the red blood cell in micrometres (μm)? Note: 1 cm = 10 000 μm .

- 19 Elliot drew a regular hexagon with side lengths of 3 cm.
- What is the difference between a regular and an irregular hexagon?
 - If Elliot drew another regular hexagon with different side lengths, explain how you know without measuring anything that the two hexagons must be similar.

- 20 Emily bought Timothy a model plane to go into a special glass case that allows for a maximum height of 15 cm.

- If the plane was 30 m tall in real life, what maximum scale factor could be applied to the model plane?

When the plane arrives, Emily finds that the plane actually measures 16 cm in height.

- What scale factor has actually been applied to the model plane?

Assuming that all dimensions must stay in the same ratio, it is important that the plane fits into the glass case.

- What scale factor would the model need to be dilated by to fit into the space? Hint: Which measurement is the original and which is the image?
 - What scale factor would the space need to be dilated by to fit the current model?
 - How are the answers to parts **c** and **d** different and how are they similar?
- 21 This table shows the dimensions of standard paper sizes, rounded to the nearest millimetre.
- Use the table to decide whether any or all of the following paper sizes are similar figures. Compare:
 - sizes within the A group (e.g. A2, A4 and A7)
 - sizes within the B group (e.g. B0, B3 and B6)
 - sizes within the C group (e.g. C1, C5 and C8)
 - sizes between different groups (e.g. A0, B0 and C0).



	A sizes (mm)	B sizes (mm)	C sizes (mm)
0	841 × 1189	1000 × 1414	917 × 1297
1	594 × 841	707 × 1000	648 × 917
2	420 × 594	500 × 707	458 × 648
3	297 × 420	353 × 500	324 × 458
4	210 × 297	250 × 353	229 × 324
5	148 × 210	176 × 250	162 × 229
6	105 × 148	125 × 176	114 × 162
7	74 × 105	88 × 125	81 × 114
8	52 × 74	62 × 88	57 × 81
9	37 × 52	44 × 62	40 × 57
10	26 × 37	31 × 44	28 × 40

- The scale factor to enlarge a paper size to the next larger size (e.g. from A5 to A4) is approximately 1.41. Explain the significance of this value, and what this means in terms of the paper sizes.

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BLM
Exercise 7C



Interactive
skillsheet
Similar figures



Interactive
skillsheet
Scale drawings



Worksheet
Dilations



Topic quiz
7C

7D Area and volume scale factors

Learning intentions

By the end of this section, you will be able to ...

- ✓ calculate area and volume scale factors
- ✓ use scale factors to find measurements of original and image objects.



Inter-year links

Year 7

8E Area of a triangle

Year 8

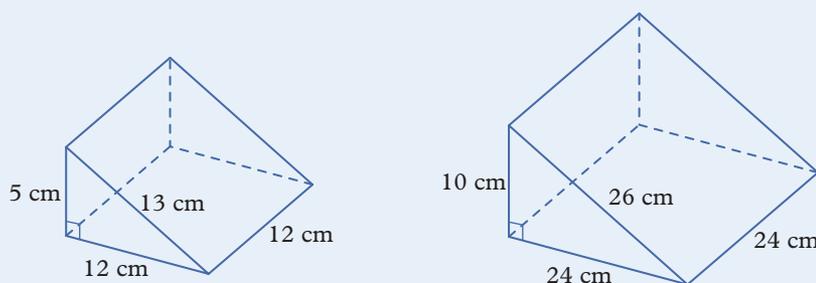
8E Volume and capacity

Similar objects



Key content video

- Two objects are similar if they are exactly the same shape. Similar objects do not need to be the same size.
- If two objects are similar, all side lengths will be in the same proportion. For example, these two prisms are similar, as every side length in the prism on the right is twice the corresponding side length in the prism on the left.



Scale factor

- A scale factor measures the enlargement or reduction factor between similar objects.
- The length scale factor is the ratio between an image side length and the original side length.

$$\text{Length scale factor} = \frac{\text{image length}}{\text{original length}}$$

- The **area scale factor** is the ratio between an image area and the original area, and is equal to the square of the length scale factor between the objects.

$$\text{Area scale factor} = \frac{\text{image area}}{\text{original area}} = \left(\frac{\text{image length}}{\text{original length}} \right)^2$$

- The **volume scale factor** is the ratio between an image volume and the original volume, and is equal to the cube of the length scale factor between the objects.

$$\text{Volume scale factor} = \frac{\text{image volume}}{\text{original volume}} = \left(\frac{\text{image length}}{\text{original length}} \right)^3$$

- For example, if the length scale factor between two similar objects is 4, the area scale factor is $4^2 = 16$ and the volume scale factor is $4^3 = 64$.
- If the scale factor is between 0 and 1, the image is a reduction (smaller than the original object).
- If the scale factor is greater than 1, the image is an enlargement (larger than the original object).



Example 7D.1 Calculating area and volume scale factors

Two dolls from a set of Russian nesting dolls are identical in shape but differ in size. One of the dolls is twice as tall as the other.

- Find the area scale factor for the surface areas of the dolls.
- Find the volume scale factor for the volume of the dolls.
- If the smaller doll has a volume of 200 cm^3 , find the volume of the larger doll.



THINK

- The area scale factor is the square of the length scale factor.
- The volume scale factor is the cube of the length scale factor.
- The volume of the smaller doll multiplied by the volume scale factor gives the volume of the larger doll.

WRITE

- Area scale factor $= 2^2$
 $= 4$
- Volume scale factor $= 2^3$
 $= 8$
- Volume $= 200 \times 8$
 $= 1600 \text{ cm}^3$

Example 7D.2 Using scale factors to find measurements (regular objects)



A rectangular prism has length 5 cm, breadth 4 cm and height 3 cm. The prism is dilated by a scale factor of 2.

- Find the surface area and volume of the original rectangular prism.
- Find the surface area and volume of the new prism by first dilating the length, breadth and height measurements.
- Find the surface area and volume of the new prism using the area and volume scale factors.

THINK

- 1 The surface area of a prism is the sum of the areas of all six faces.
- 2 The volume of a rectangular prism is the product of the length, breadth and height of the prism.

WRITE

- Surface area $= 2 \times (3 \times 4 + 3 \times 5 + 4 \times 5)$
 $= 94 \text{ cm}^2$

Volume $= 3 \times 4 \times 5$
 $= 60 \text{ cm}^3$

- b** 1 Find the dilated dimensions of the prism.
- 2 Find the image surface area using the dilated length, dilated breadth and dilated height measurements.
- 3 Find the image volume by multiplying the dilated length, breadth and height together.
- c** 1 Find the area scale factor as the square of the length scale factor, and then multiply the original surface area by the area scale factor.
- 2 Find the volume scale factor as the cube of the length scale factor, and then multiply the original volume by the volume scale factor.

b Dilated length = 10 cm, dilated breadth = 8 cm, dilated height = 6 cm.

$$\begin{aligned}\text{Surface area of image} &= 2 \times (6 \times 8 + 6 \times 10 \\ &\quad + 8 \times 10) \\ &= 376 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Volume of image} &= 6 \times 8 \times 10 \\ &= 480 \text{ cm}^3\end{aligned}$$

c Area scale factor = 2^2

$$= 4$$

$$\begin{aligned}\text{Surface area of image} &= 4 \times 94 \\ &= 376 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Volume scale factor} &= 2^3 \\ &= 8\end{aligned}$$

$$\begin{aligned}\text{Volume of image} &= 8 \times 60 \\ &= 480 \text{ cm}^3\end{aligned}$$

Example 7D.3 Using scale factors to find measurements (irregular objects)



A gift shop in an airport sells model airplanes as souvenirs with a scale of 1 : 64 compared to the full-sized plane. The full-sized airplane is 38 m long and has a surface area of 500 m².

- a** Find the length of the model airplane in cm.
- b** Find the surface area of the model correct to the nearest cm².

THINK

- a** The ratio means the full-sized plane is 64 times larger than the model. The length of the model is equal to the length of the full-sized plane divided by the scale factor.
- b** The area scale factor is the square of the length scale factor. The area of the model is equal to the surface area of the full-sized plane divided by the area scale factor. Convert m² to cm² by multiplying by 100².

WRITE

- a** Scale factor = 64
- $$\begin{aligned}38 \div 64 &= 0.59375 \text{ m} \\ &= 0.59375 \times 100 \text{ cm} \\ &= 59.375 \text{ cm}\end{aligned}$$
- b** Area scale factor = 64^2
- $$\begin{aligned}500 \div 64^2 &= 0.12207 \text{ m}^2 \\ &= 0.12207 \times 100^2 \text{ cm}^2 \\ &\approx 1221 \text{ cm}^2\end{aligned}$$

Helpful hints

- ✓ For similar objects with length scale factor k , the area scale factor is k^2 and the volume scale factor is k^3 .
- ✓ The side length, area and volume of a dilated object can be found by multiplying the original measurement by the corresponding scale factor.

Exercise 7D Area and volume scale factors

▲ 1-4, 6, 9, 10, 12

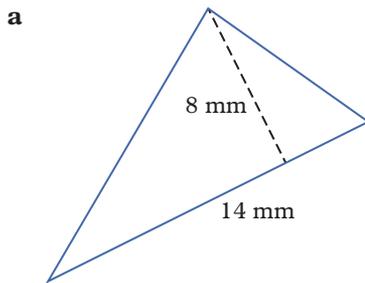
■ 3-6, 8, 10, 13, 15, 19

◆ 4, 7, 8, 11, 14-19

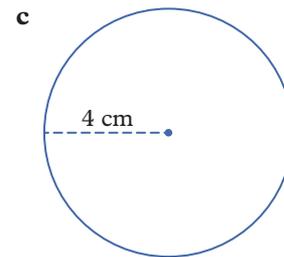
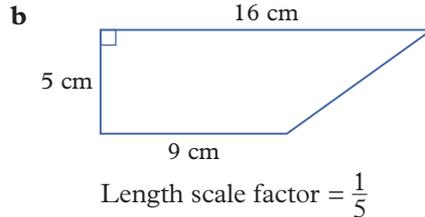
1 Copy and complete this table.

Length scale factor	Area scale factor	Volume scale factor
2		
$\frac{3}{4}$		
	16	
	$\frac{36}{25}$	
		$\frac{1}{27}$

2 For each shape, state the area of the image if the original figure is dilated by the given length scale factor. Give your answers correct to two decimal places where necessary.

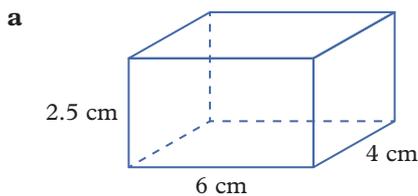


Length scale factor = 2

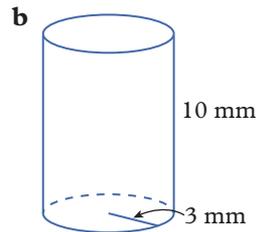


Length scale factor = $\frac{1}{2}$

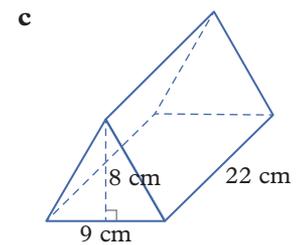
3 For each shape, state the volume of the image if the original figure is dilated by the given length scale factor.



Length scale factor = $\frac{1}{2}$



Length scale factor = 5



Length scale factor = 1.5

7D.1 4 A set of salt and pepper shakers are similar in shape. The salt shaker is 1.5 times taller than the pepper shaker.

- Find the area scale factor for the surface areas of the shakers.
- Find the volume scale factor for the volume of the shakers.
- If the pepper shaker has a volume of 5 cm^3 , find the volume of the salt shaker.

7D.2 5 A cube of side length 5 mm is dilated by a scale factor of 3.

- Find the surface area and volume of the original cube.
- Find the surface area of the new cube.
- Find the volume of the new cube.

6 Calculate the length scale factor for these area scale factors.

- | | | |
|-------------|-------------|--------------|
| a 49 | b 25 | c 100 |
| d 64 | e 4 | f 400 |

- 7 A cylinder with radius 12 cm and height 9 cm is dilated by a scale factor of $\frac{1}{3}$. Leave your answers as exact values for all parts of this question.
- Find the surface area and volume of the original cylinder.
 - Find the surface area of the new cylinder.
 - Find the volume of the new cylinder.

- 7D.3** 8 A handicraftsman created a model of the Louvre Pyramid with a scale of 1 : 300.
- The Louvre Pyramid is 21.6 m high. Find the height of the model in cm.
 - The base surface area of the Louvre Pyramid is 1000 m^2 . Find the base surface area of the model to the nearest cm^2 .

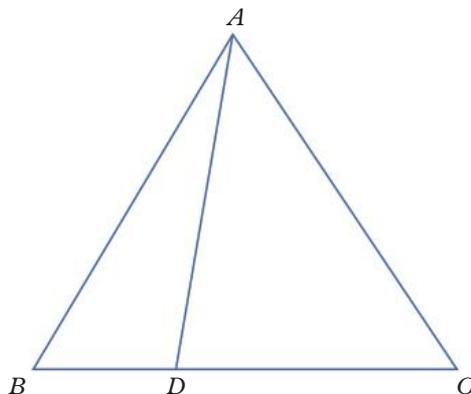


- 9 A floor plan with a scale of 1 : 50 illustrates a rectangular bedroom in a house. The dimension of the room is measured to be 8 cm by 6 cm on the floor plan. Find the area of the bedroom in m^2 .
- 10 Lake Eacham has an area of $640\,000 \text{ m}^2$. On one map, the area of the lake is measured to be 4 cm^2 . Find the scale of this map in the form 1 : n .
- 11 Andrea has a set of similar rectangular containers. She tries to store 400 mL of soup in a container with dimensions $15 \text{ cm} \times 5 \text{ cm} \times 18 \text{ cm}$ and finds that the container is too large. Find the length scale factor between this container and the smallest similar container that is sufficient to store the soup.
- 12 Lillian grows herbs in a pot on her balcony. The herbs thrive under her care, and she needs to find a similar pot that has twice the volume for replanting. What is the scale factor comparing the dimensions of the two pots?
- 13 In low light, the pupils of our eyes dilate to let in more light. In normal light, Chantelle's pupils had a radius of 1.2 mm. When she walked into a dark room, they dilated to have a radius of 3.6 mm.
- What is the length scale factor?
 - What is the area scale factor?
 - Using the formula $A = \pi r^2$ and your answer to part **b**, find correct to two decimal places:
 - the original pupil area
 - the dilated pupil area
 - the difference in these two areas.

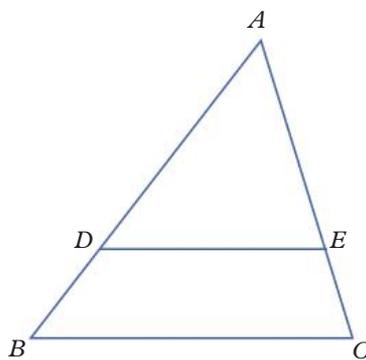


- 14 A radar has a scanning radius of roughly 5 km. If the owner wanted to cover an area of $100\pi \text{ km}^2$, by how much would the radius of the radar need to increase?
- 15 A community recreational centre is reconstructing their rectangular swimming pool of constant depth to accommodate the growing population. The new pool will be twice as long in breadth and length. Due to safety restrictions, the depth of the pool will remain the same as before. Find the scale factor for the volume of the new pool.
- 16 A student uses a 3D printer to create a solid sculpture with 25 g of filament. The student also spent 20 minutes painting its surface. This sculpture is later selected for an exhibition, and the student needs to print the same sculpture at three times the size for display.
- How much filament does the student need to print the larger statue?
 - If this student can paint at a constant rate, approximately how long would it take to paint the surface of the larger sculpture?

- 17 In $\triangle ABC$, point D divides side BC into a ratio of $1 : 2$. A student suggests that the area ratio between $\triangle ABD$ and $\triangle ADC$ is $1 : 4$, calculated as the square of the length ratio. Do you agree with this statement? Explain your reasoning.



- 18 In $\triangle ABC$, points D and E lie on side AB and AC , respectively, and DE is parallel to BC . Point D divides side AB in a ratio of $2 : 1$.



- Find the ratio between the areas of $\triangle ABC$ and $\triangle ADE$ in the form $m : n$.
 - Find the ratio between the areas of $\triangle ADE$ and the quadrilateral $BCED$ in the form $m : n$.
- 19 A spherical balloon has a radius of 10 cm. It loses some air and deflates to half of its original radius over a week. The surface area of a sphere is $SA = 4\pi r^2$, and the volume of a sphere is $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere. Leave your answers as exact values for all parts of this question.
- Find the surface area and the volume of the original balloon.
 - Find the radius of the balloon after a week.
 - Use the new radius to calculate the surface area of the balloon after a week.
 - Calculate the area scale factor. Comment on your result in relation to the radius scale factor.
 - Use the new radius to calculate the volume of the balloon after a week.
 - Calculate the volume factor. Comment on your result in relation to the radius scale factor.



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Interactive skillsheet

Area and volume scale factors



Topic quiz

7D

7E Congruence

Learning intentions

By the end of this topic you will be able to ...

- ✓ determine if shapes are congruent
- ✓ identify congruent shapes
- ✓ match vertices and sides of congruent pairs.



Inter-year links

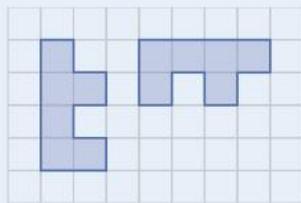
Support 2D shapes

Congruence

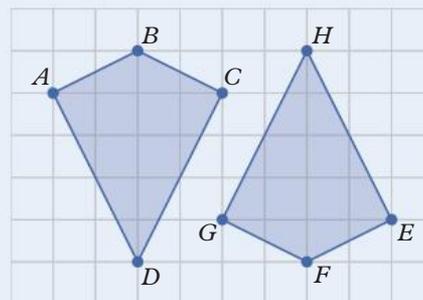


Key content video

- Two figures are **congruent** if they have the same shape and size, or if one shape is the mirror image of the other.
 - Congruent figures can be in any position or orientation.
 - Corresponding side lengths and angles are equal in congruent figures.
 - For example, these shapes are congruent even though they are not in the same orientation.

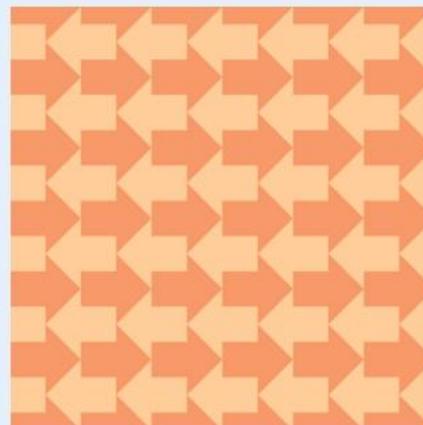


- An **isometric transformation** is a transformation that preserves the original size and shape of the object being transformed. Reflections, translations and rotations are examples of isometric transformations.
 - If isometric transformations can be used to move one figure exactly on top of another figure, the two figures are congruent.
- The symbol \cong is used to indicate congruence. When making a congruence statement, matching vertices must be put in the same order.
 - For example, $ABCD \cong EFGH$.



Tessellations

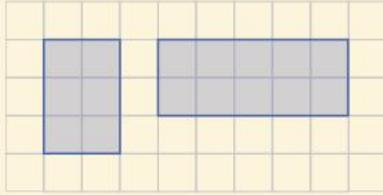
- A **tessellation** is a pattern of repeating shapes that has no overlaps or gaps. Tessellations use congruent figures to create their pattern. For example, the tessellation on the right uses only one shape in the pattern.





Example 7E.1 Determining if two shapes are congruent

Determine if these two shapes are congruent.



THINK

- 1 Determine if the basic shapes are the same.
- 2 Check if the corresponding sides are equal.
- 3 Write your answer.

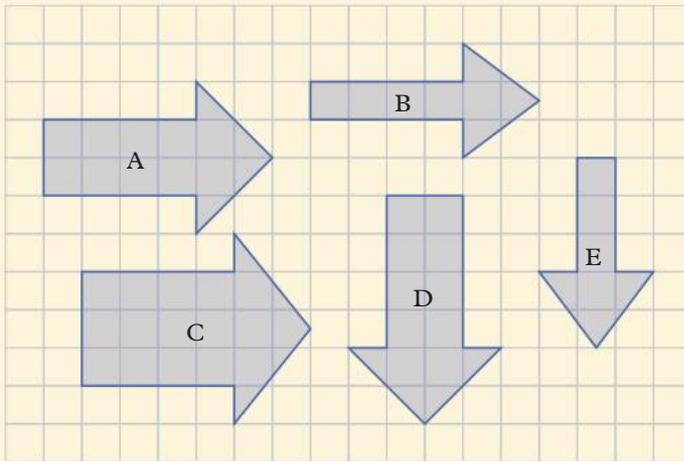
WRITE

The shapes are not congruent because they do not have equal corresponding sides.

Example 7E.2 Identifying congruent pairs



Identify any pairs of congruent shapes in the following diagram.



THINK

- 1 Compare the length and width of all the shapes.
- 2 E is shorter than the other shapes, so it does not have a congruent pair.
- 3 C is wider than the other shapes, so it does not have a congruent pair.
- 4 B is thinner than both A and D, so it does not have a congruent pair.
- 5 Shapes A and D are the same shape and the corresponding sides are equal in length, so they are a congruent pair.

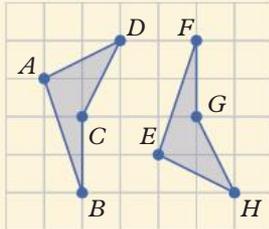
WRITE

Shapes A and D are congruent.



Example 7E.3 Matching sides of congruent pairs

Match the corresponding sides of this pair of congruent quadrilaterals.



THINK

- 1 Pick a side from the first shape that has a different length from all other sides. For example, side AB is the only side that goes down by 3 units and across by 1 unit. Find the corresponding side on the second shape (EF).
- 2 Pick an adjacent side to that chosen in step 1; for example, BC . Find the corresponding side on the second shape (FG).
- 3 Continue picking adjacent sides on the first shape and finding the corresponding sides on the second shape until all sides have been matched.

WRITE

AB and EF , BC and FG ,
 CD and GH , DA and HE

Helpful hints

- ✓ Remember that orientation does not matter when determining congruence, but the size of the shapes does matter!

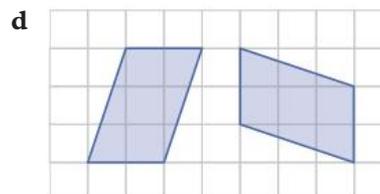
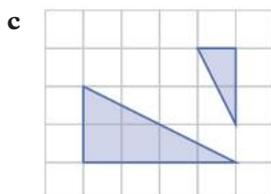
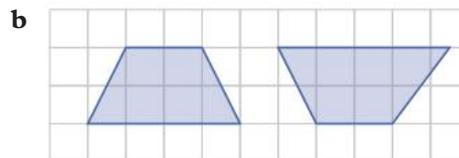
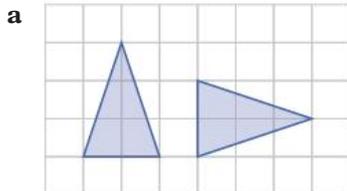
ANS p488 Exercise 7E Congruence

 1-3, 5-11, 14

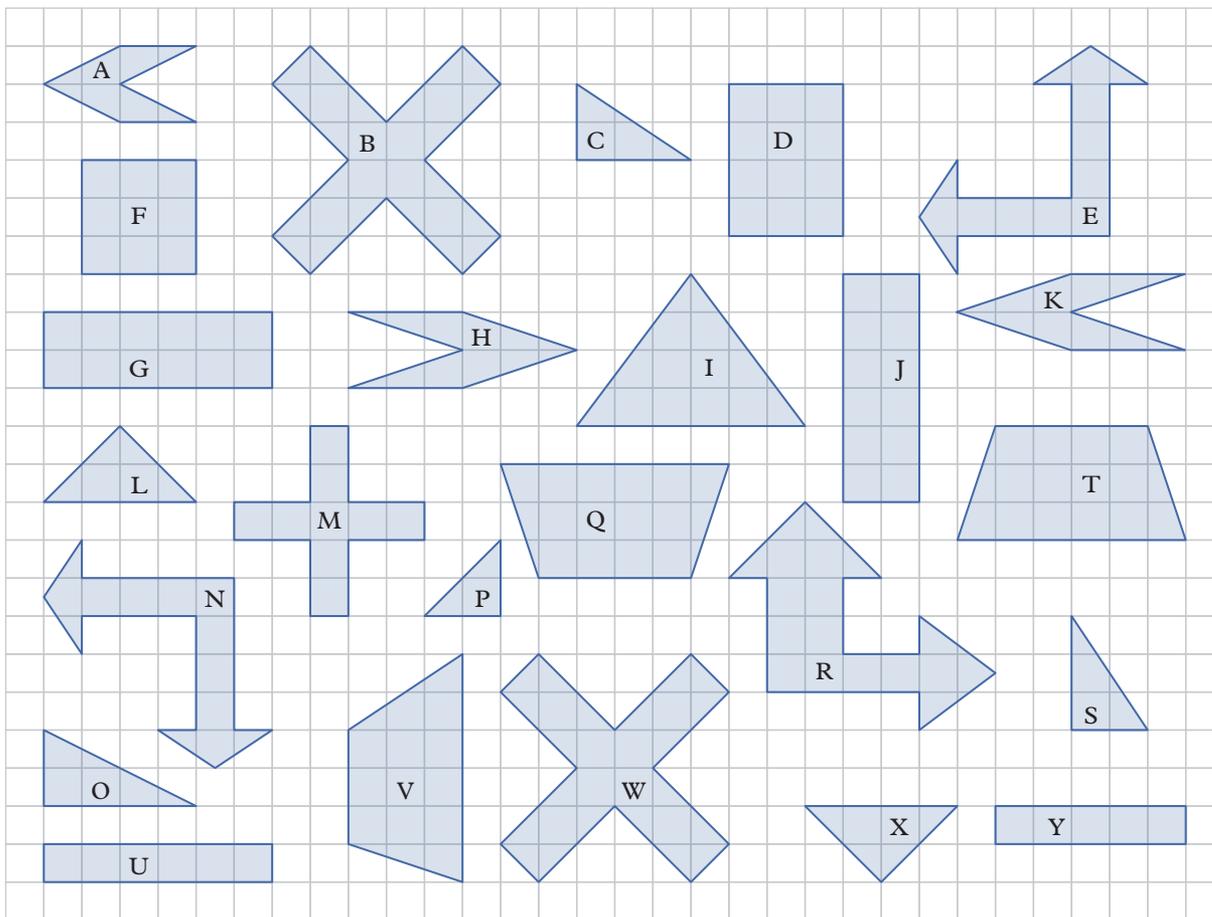
 1-5, 8-10, 12, 14, 16

 2-5, 8, 9, 12, 14, 15, 17

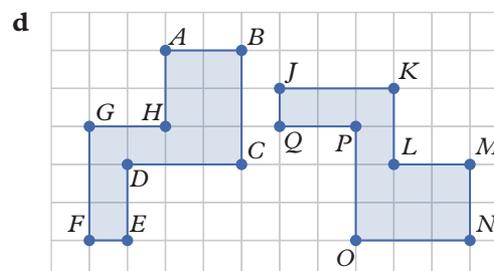
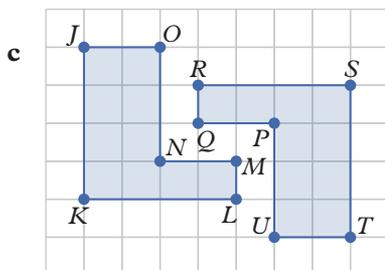
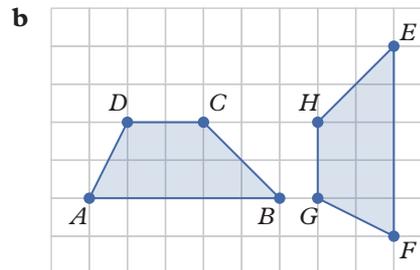
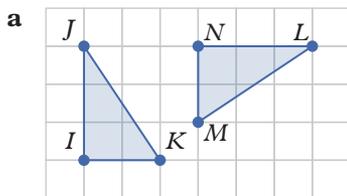
7E.1 1 Decide whether each pair of shapes is congruent.



7E.2 2 Identify any pairs of shapes that are congruent in the following diagram.

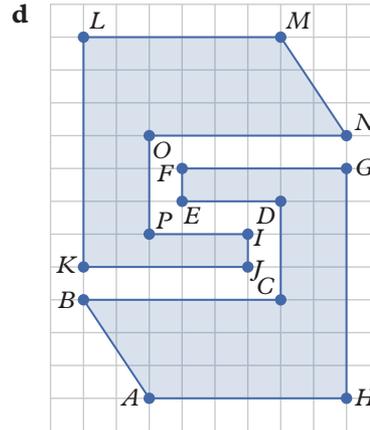
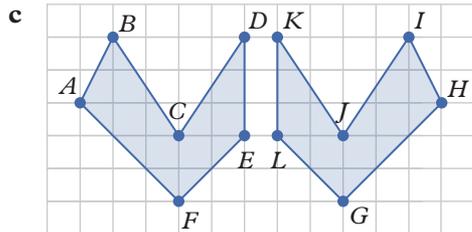
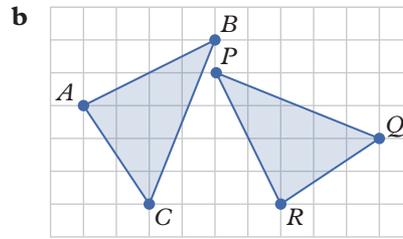
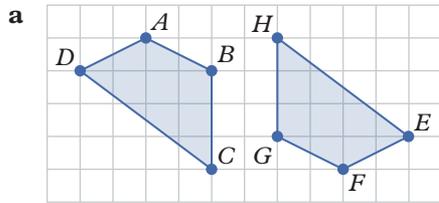


7E.3 3 Match the corresponding sides of each pair of congruent shapes.



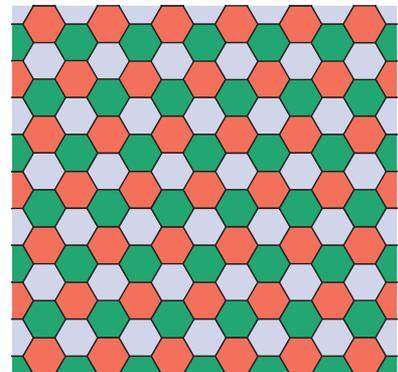
4 Use the symbol for congruence (\cong) to write a congruence statement about each pair of congruent shapes in question 3. Remember that matching vertices must be put in the same order.

5 Match the corresponding vertices of each pair of congruent shapes.

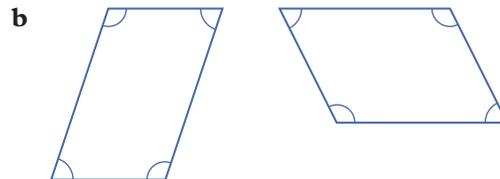
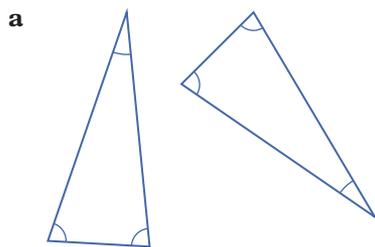


6 The shape that makes up the tessellation in the image on the right is:

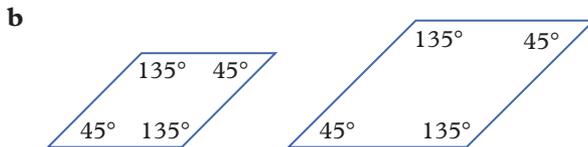
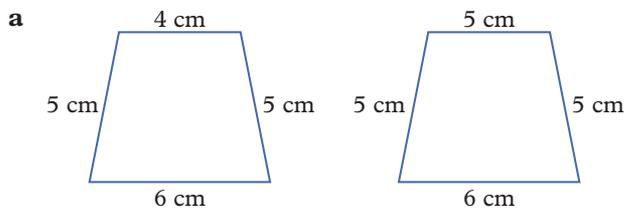
- A a pentagon
- B a hexagon
- C a square
- D a triangle
- E an octagon.



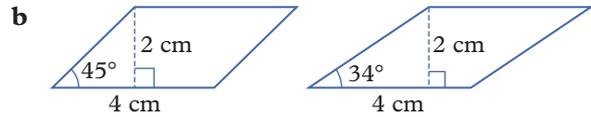
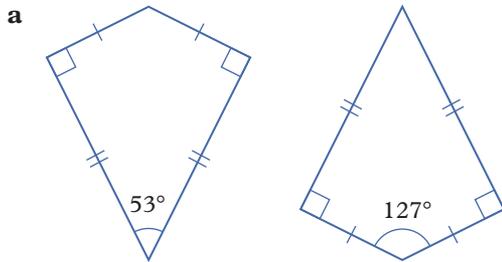
7 For each pair of figures, measure all sides and angles and state if the figures are congruent.



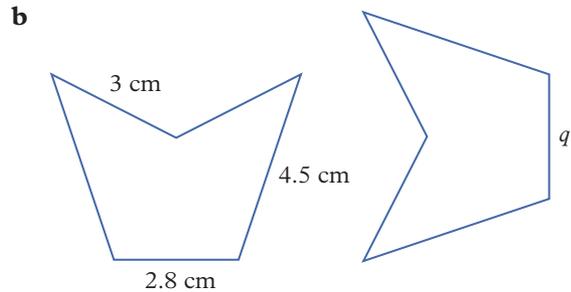
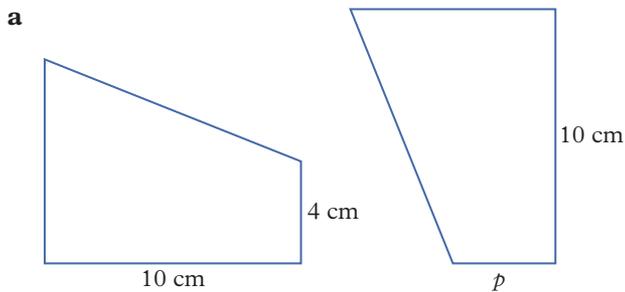
8 Determine if each pair of figures is congruent, justifying your decision.



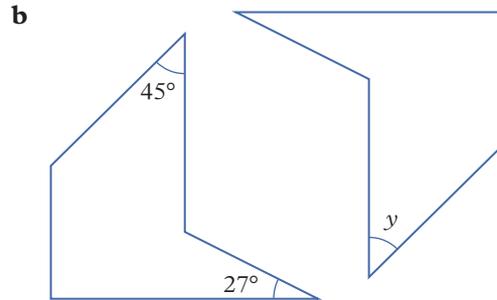
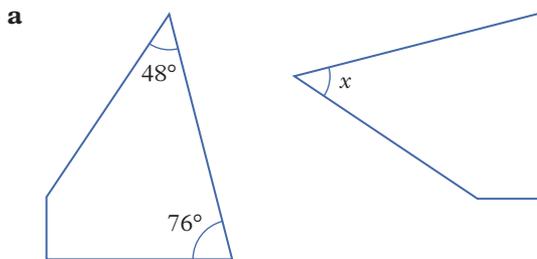
9 Determine if each pair of figures is congruent, justifying your decision.



10 Find the value of the pronumerals, given that each pair of shapes is congruent.



11 Find the value of the pronumerals, given that each pair of shapes is congruent.



12 Answer true or false to the following statements, giving reasons for your answers.

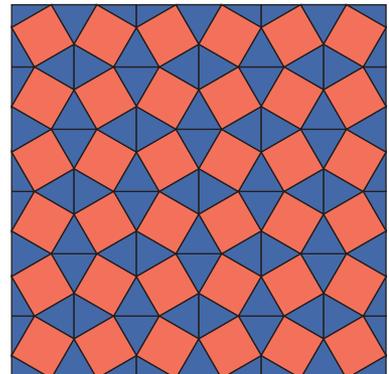
- All squares with the same side length are congruent with each other.
- All rectangles with the same side lengths are congruent with each other.
- All triangles with the same base length and height are congruent with each other.
- All parallelograms with the same base length and height are congruent with each other.
- All circles with the same radii are congruent with each other.

13 Explain why it is not possible to match corresponding sides and vertices on congruent squares, rectangles or parallelograms.

14 A polygon is a closed shape with straight sides. There are three main types of tessellations:

- regular tessellations, which are made from identical, regular polygons
- semi-regular tessellations, which are made from any number of regular polygons
- irregular tessellations, which are made from any number of different shapes (including non-polygons).

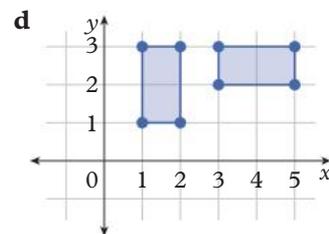
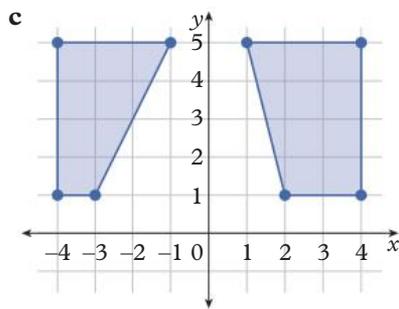
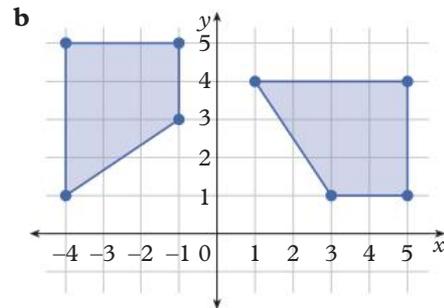
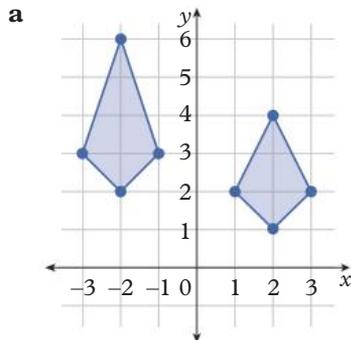
A semi-regular tessellation is shown on the right. What polygons are included in this tessellation?



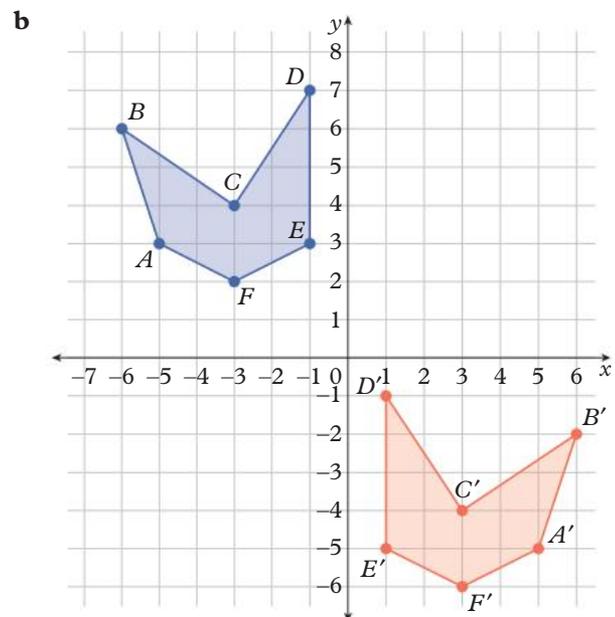
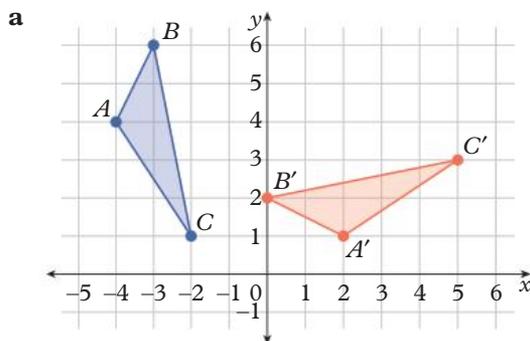
15 Which polygons can be used to create regular tessellations? Justify your answer.

16 For each of the following pairs of shapes:

- identify the single transformation that can be applied to the left shape to show if the two shapes are congruent or not congruent
- state whether the shapes are congruent or not congruent.



17 For each diagram, describe a series of transformations that can be applied to the left shape to show the two shapes are congruent.



Check your Student obook pro for these digital resources and more:

pro



Interactive skillsheet
Congruence



Worksheet
Congruent figures and transformations



Investigation
Tiling patterns in art and design



Topic quiz
7E

Checkpoint



Checkpoint quiz

Check your knowledge of the first part of this chapter.

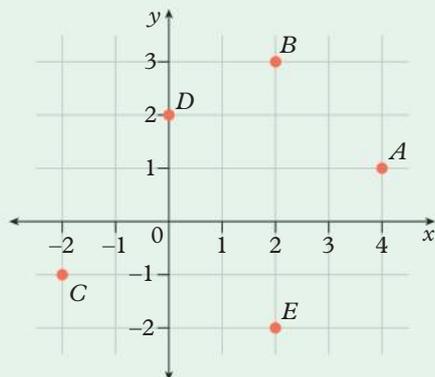
- ✓ Core
- ✓ Advanced
- ✓ Extension



BLM

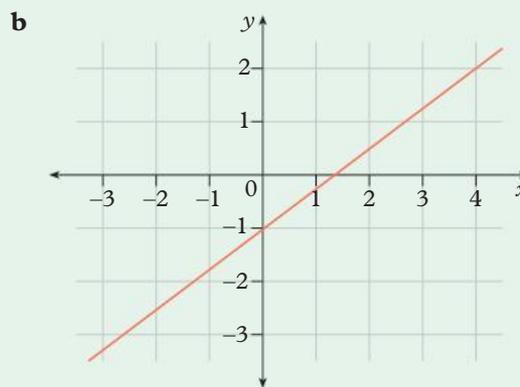
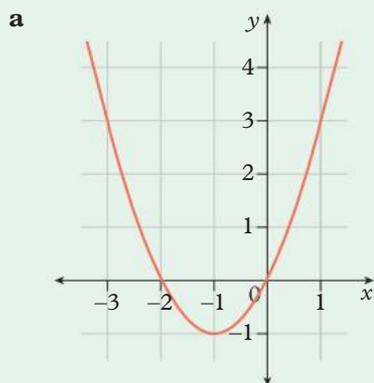
Chapter 7 Checkpoint

- ADV 7A 1** Reflect each of these points across the x -axis and give the coordinates of the images.



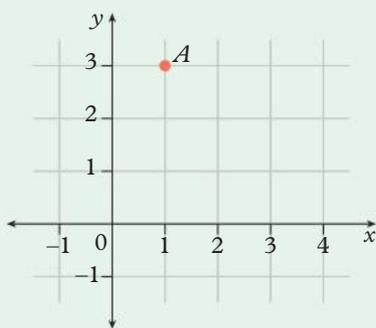
- ADV 7A 2** Determine if the following graphs have:

- i line symmetry
- ii rotational symmetry.

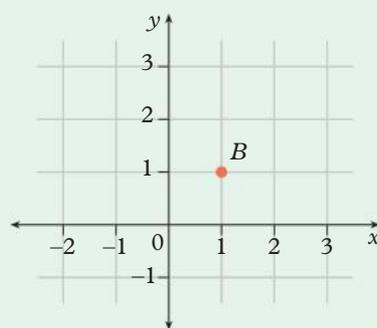


- ADV 7B 3** Translate each point by the given amount.

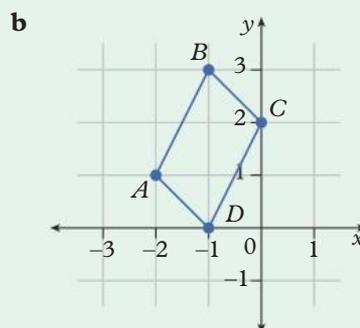
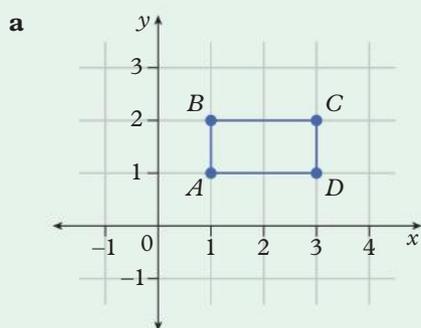
- a** 3 units right and 2 units down



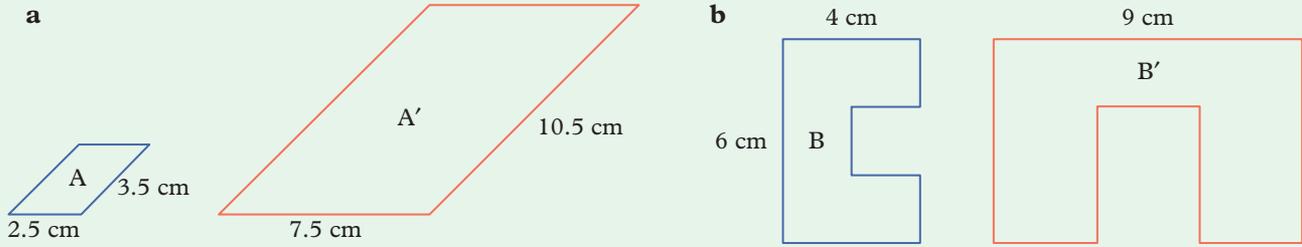
- b** 4 units left and 1 unit up



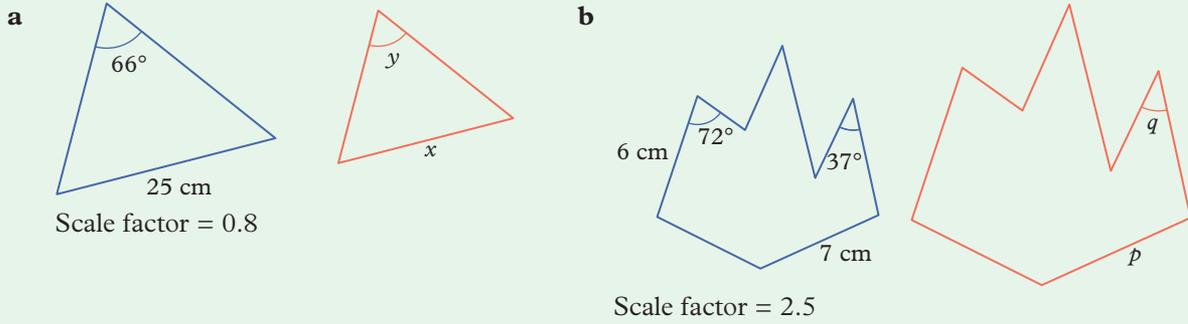
- ADV 7B 4** Rotate each shape by 180° clockwise about the origin.



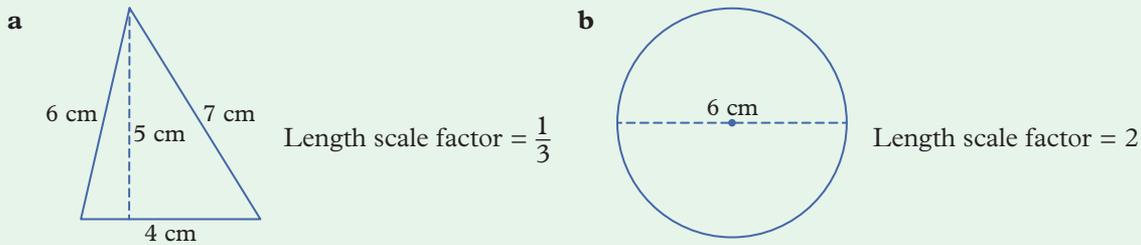
7C 5 Find the scale factor for each pair of similar figures if the figure on the left of each pair is the original shape.



7C 6 The shapes on the left have been dilated by the given scale factor to create the image on the right. Determine the value of the unknown pronumerals in each image.

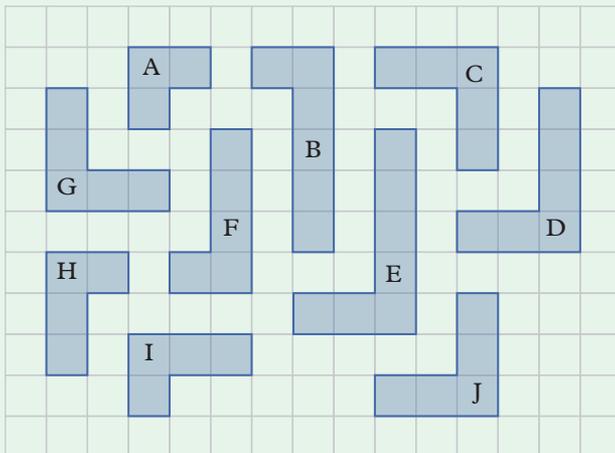


EXT 7D 7 For each shape, state the area of the image if the original figure is dilated by the given length scale factor. Give your answers to two decimal places where necessary.

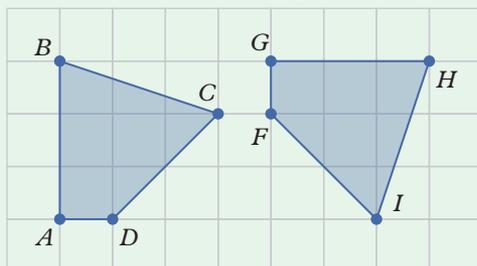


EXT 7D 8 A triangular flag needs its dimensions increased by a factor of 5. If the flag has a 30 cm base and a 45 cm height, find the difference in the area of material needed to make the two flags.

EXT 7E 9 Identify any pairs of shapes that are congruent in the following diagram.



EXT 7E 10 Write a congruence statement for the following pair of congruent figures.



7F Congruent triangles

Learning intentions

By the end of this topic you will be able to ...

- ✓ use congruence conditions to determine if two triangles are congruent.



Inter-year links

Year 7

7D Classifying triangles

Year 8

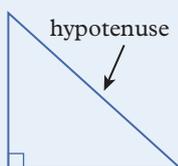
7B Triangles

Congruent triangles



Key content video

- Two triangles are congruent if all their sides and angles are equal.
- The symbol \triangle can be used instead of writing 'triangle'; for example, $\triangle ABC$.
- An **included angle** is the internal angle between two sides.
- The hypotenuse of a right-angled triangle is the side that appears opposite the right angle.



- If a pair of triangles meet any of the following conditions, they are congruent:

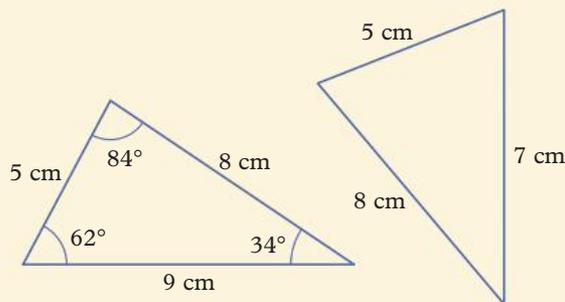
Condition	Name	Meaning	Example
SSS	side–side–side	All three side lengths are equal.	
SAS	side–angle–side	Two side lengths and the included angle (between these sides) are equal.	
AAS (or ASA or SAA)	angle–angle–side	Two angles and a corresponding side length are equal.	
RHS	right angle–hypotenuse–side	The hypotenuse and another side length of two right-angled triangles are equal.	

- In the condition AAS, any corresponding side length is sufficient to prove congruence. This rule may also be known as ASA or SAA.



Example 7F.1 Using the SSS condition

Use the congruence condition SSS to decide if this pair of triangles is congruent.



THINK

- 1 SSS means the triangles are congruent if all three side lengths are equal.
- 2 Go through each of the side lengths on the first shape and check whether there is a matching side length on the second triangle. In the second triangle there is not a matching side length for the 9 cm side in the first triangle.
- 3 If the condition is met, the triangles are congruent; if the condition fails, the triangles are not congruent.

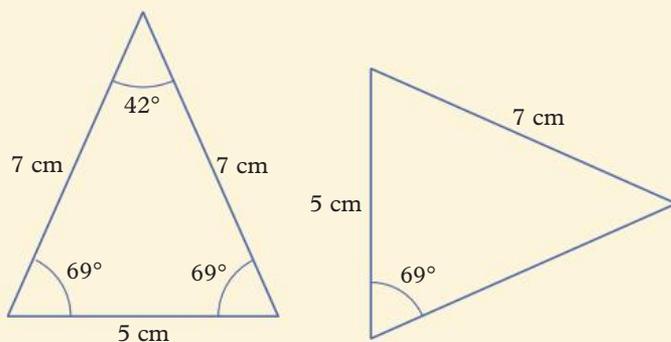
WRITE

The two triangles are not congruent.

Example 7F.2 Using the SAS condition



Use the congruence condition SAS to decide if this pair of triangles is congruent.



THINK

- 1 SAS means the triangles are congruent if two side lengths and the included angle are equal.
- 2 Check that there are two matching side lengths on each triangle.
- 3 Look at the angles between the two matching sides. In the second triangle the angle is not between the sides, so we cannot determine if the SAS condition is met.

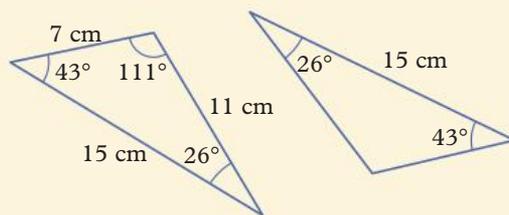
WRITE

The two triangles may or may not be congruent.



Example 7F.3 Using the AAS condition

Use the congruence condition AAS to decide if this pair of triangles is congruent.



THINK

- 1 AAS means the triangles are congruent if two angles and a corresponding side length are equal.
- 2 Check that there are two pairs of equal angles.
- 3 Identify the corresponding sides of each triangle by using the matching angles from step 2.
- 4 If any two corresponding side lengths are equal, the condition is met, and the triangles are congruent.

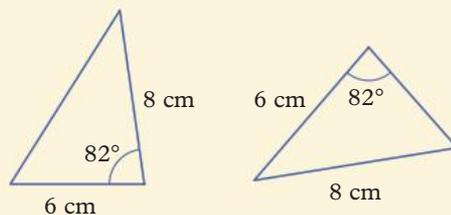
WRITE

The two triangles are congruent.

Example 7F.4 Determining if two triangles are congruent



Decide if this pair of triangles is congruent, giving a reason for your answer.



THINK

- 1 Identify what information is provided in each triangle. Both triangles contain two equal sides and an equal angle.
- 2 Check whether any of the conditions for congruence are met. The first triangle meets the SAS condition; however, the angle is not between the corresponding sides in the second triangle.
- 3 Provide the answer.

WRITE

The given information is insufficient to determine whether the triangles are congruent or not.

Helpful hints

- ✓ When using the SAS condition, make sure the provided angle is between the corresponding sides!
- ✓ When using the AAS condition, make sure that the matching side lengths are corresponding sides!
- ✓ When there is a pair of right-angled triangles, if the information for the RHS condition is not provided remember to check if there is sufficient information for either the SAS or AAS condition.

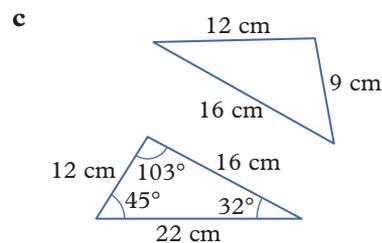
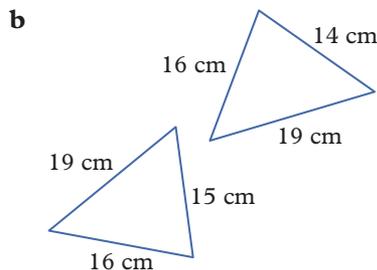
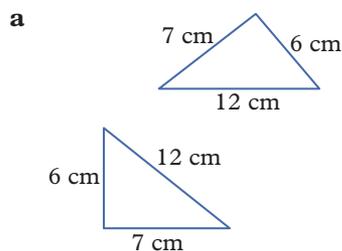
Exercise 7F Congruent triangles

 1-3, 5, 6, 7(a, b), 9, 10

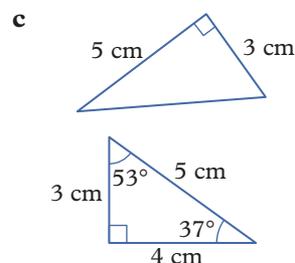
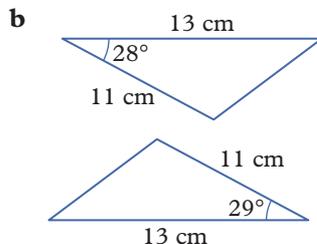
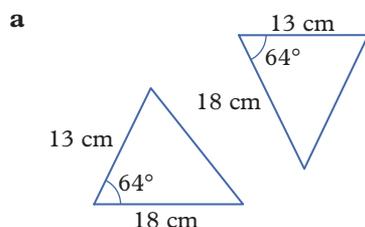
 1-4, 5(c, d), 6, 8, 10, 11, 12(a, b)

 1-3, 5(c, d), 6, 10-13

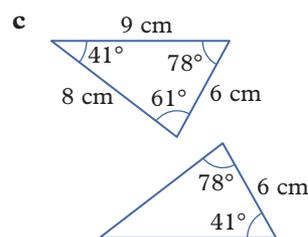
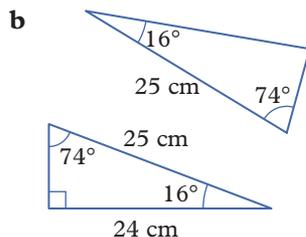
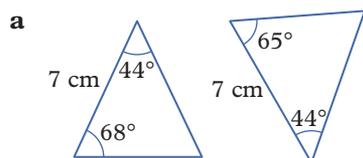
7F.1 1 Use the congruence condition SSS to decide if the following pairs of triangles are congruent.



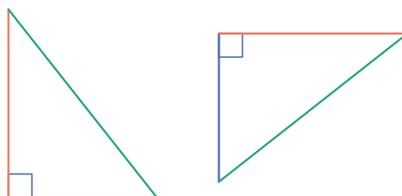
7F.2 2 Use the congruence condition SAS to decide if the following pairs of triangles are congruent.



7F.3 3 Use the congruence condition AAS to decide if the following pairs of triangles are congruent.



4 The congruence condition RHS only applies to right-angled triangles. In every right-angled triangle the longest side (known as the hypotenuse) appears opposite the right angle.



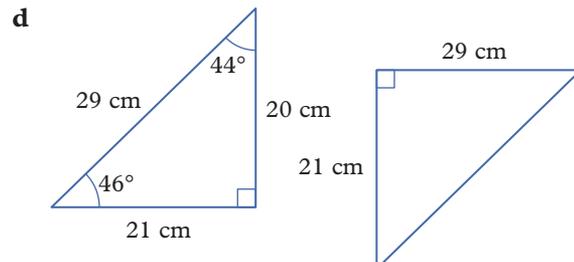
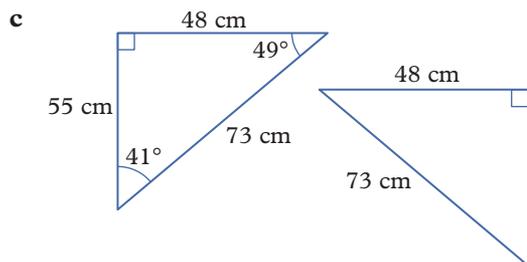
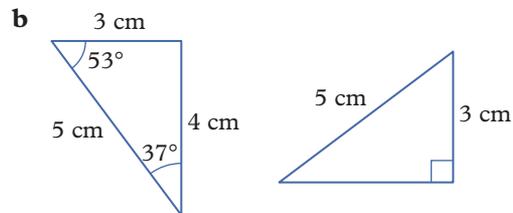
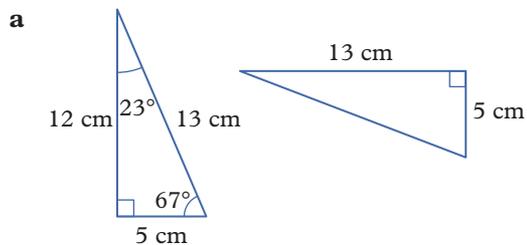
a Why must the hypotenuse be one of the sides for this condition? Hint: If the right angle is between the two sides, what condition does it become?

When the triangle is right-angled and you know the length of two sides, the length of the third side can be calculated using Pythagoras' theorem.

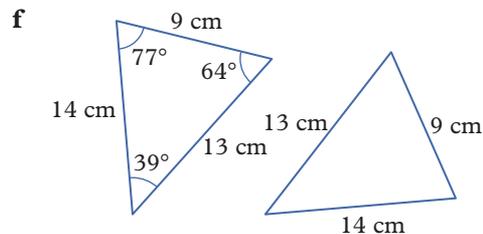
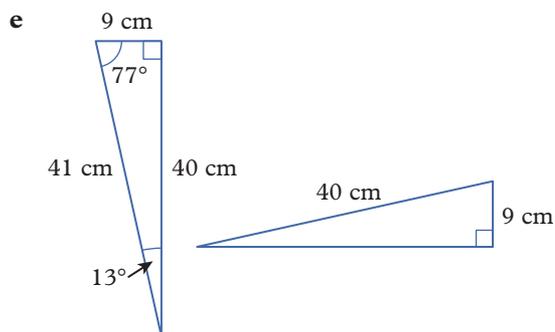
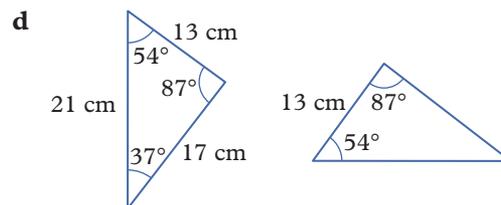
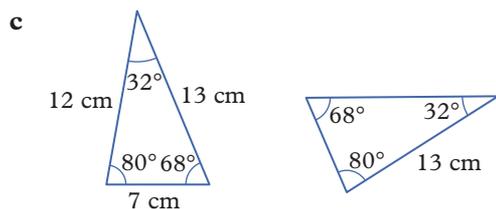
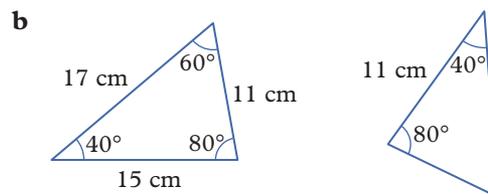
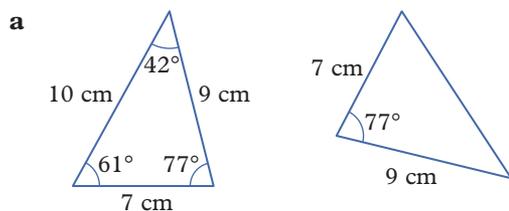
b If you calculated the length of the third side, which condition would this now be? Hint: You would know the lengths of all the sides.

c How does this show that RHS is a condition for congruence?

5 Use the condition RHS to decide if the following pairs of triangles are congruent.

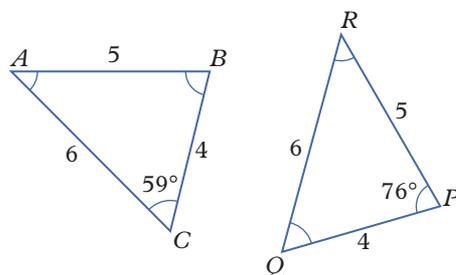


7F.4 6 Decide if the following pairs of triangles are congruent, giving a reason for your answer.



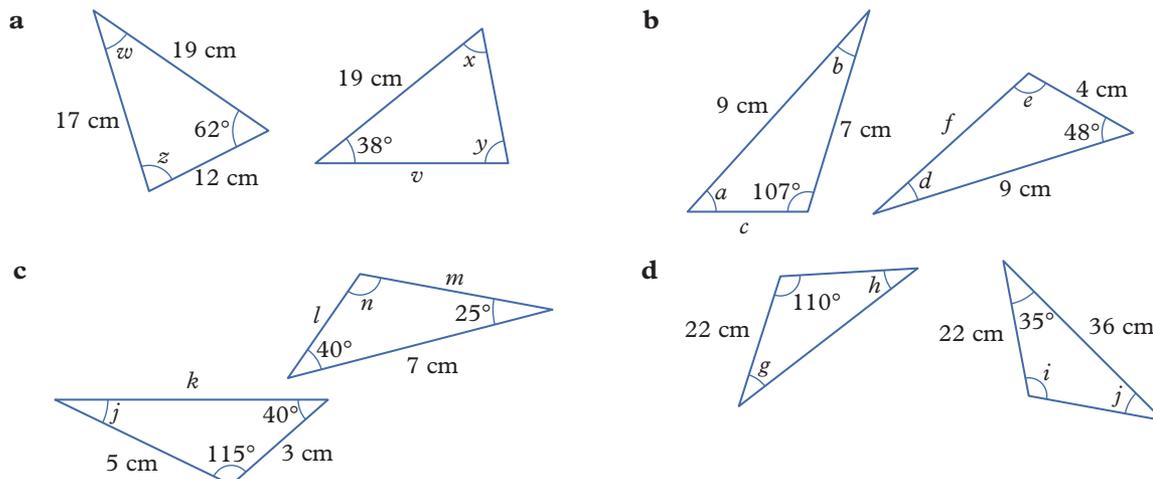
7 **a** State the condition that explains why the following pair of triangles are congruent.

b Determine the size of angles $\angle ABC$ and $\angle BAC$.



8 Each of these pairs of triangles are congruent.

- Find the unknown side lengths and angles in each triangle.
- State the condition(s) that you used to solve each problem.

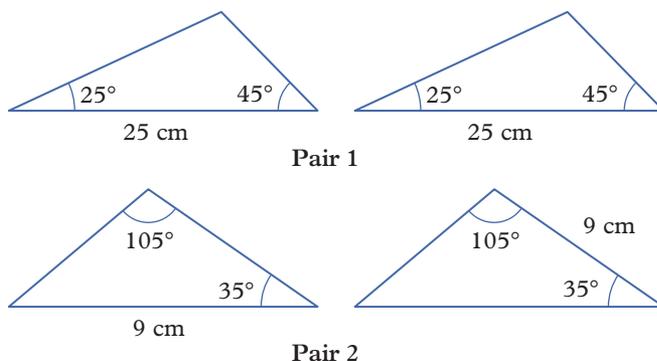


9 Another condition for congruence is ASA.

- If you know two of the angles in a triangle, how do you find the third angle?
- Using your answer to part **a**, explain how you know that two triangles that have two pairs of equal angles must meet the specification AAA.

ASA refers to two matching pairs of angles and the pair of sides in between.

- Explain why pair 1 meets the condition ASA but pair 2 does not.



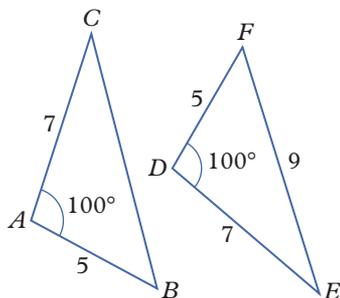
- Can you explain how, if a pair of triangles meet the condition ASA, they also meet the condition AAS?
Hint: You need to find the third angle of the triangles.

10 Demonstrate that SSA and AAA are not valid congruency tests by drawing a pair of triangles that meet these conditions but are not congruent.

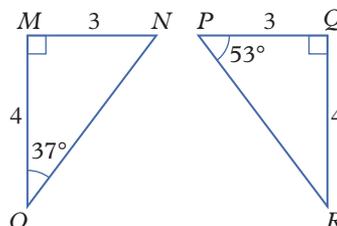
11 For each of the following:

- explain why the given condition is or is not able to show the triangles are congruent
- if the condition cannot show the triangles are congruent, state a condition that can be used.

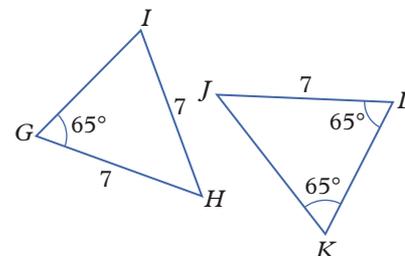
a SSS



b RHS

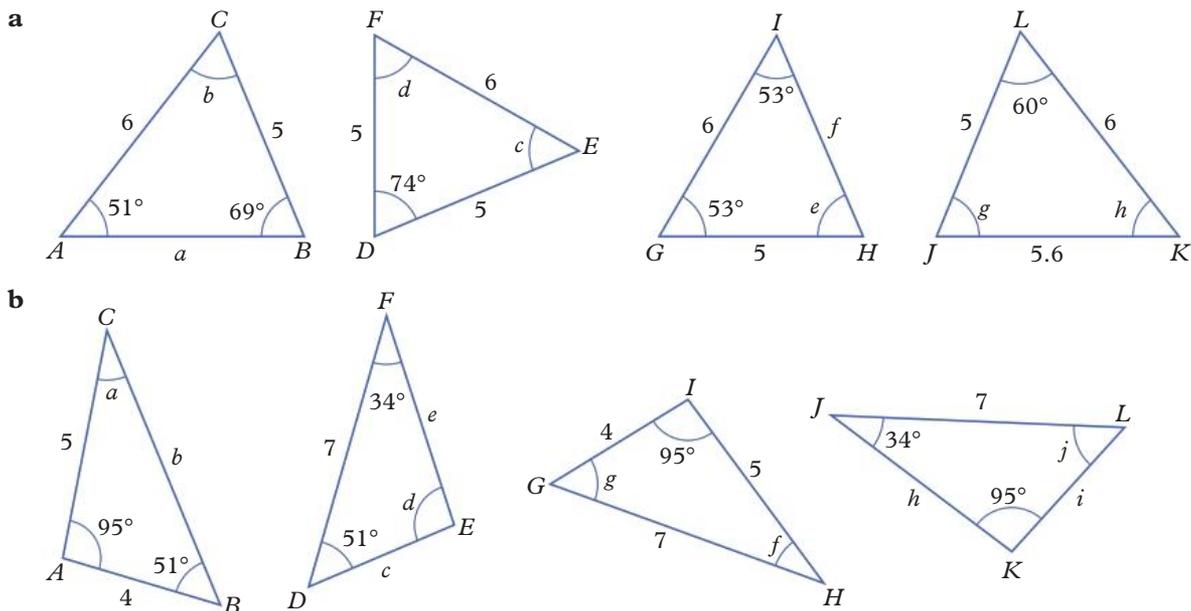


c AAS



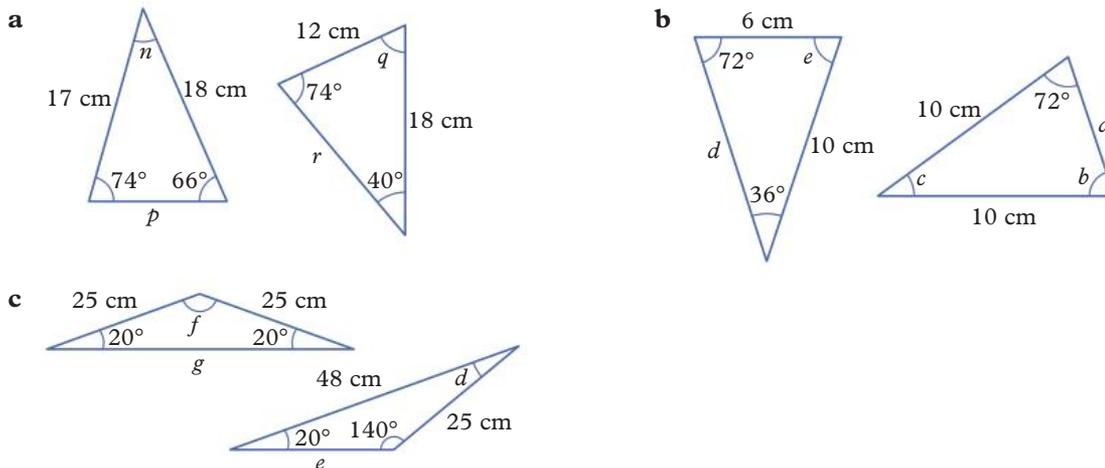
12 For each group of triangles:

- determine which triangles are congruent
- determine the values of the pronumerals.



13 For each pair of triangles below:

- decide if they are congruent, stating which condition you used
- find the value of the pronumerals if the pair is congruent.



14 There are up to 12 values that can be given for a pair of congruent triangles (six angles and six side lengths, three each per triangle). SSS, SAS, AAS and RHS all require at least six values to be given. What is the largest number of values that can be given about two congruent triangles and still not be certain the two are congruent, without determining other lengths or angles? Provide an example pair of triangles using hatch marks and arcs to label equal side lengths and angles.

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Interactive skillsheet
Congruent triangles



Worksheet
Conditions for
congruence



Investigation
Congruence in
construction



Topic quiz
7F

7G Similar triangles

Learning intentions

By the end of this topic you will be able to ...

- ✓ find unknown angles and side lengths in similar triangles
- ✓ determine whether two triangles are similar.



Inter-year links

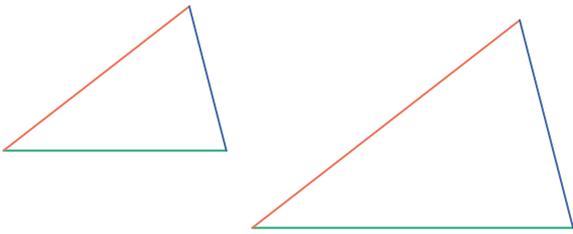
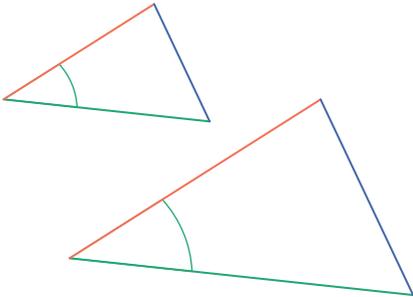
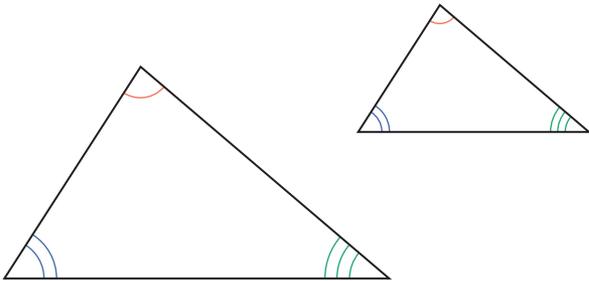
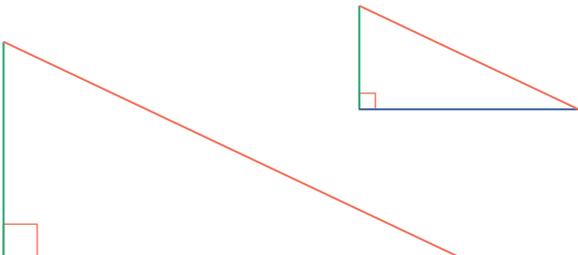
- Year 7** 7D Classifying triangles
- Year 8** 7B Triangles
- Year 10** 7B Congruence and similarity

Similar triangles



Key content video

- Two triangles are similar if:
 - corresponding pairs of angles are equal
 - corresponding pairs of side lengths are in the same ratio.
- If a pair of triangles meets any of the following conditions, they are similar:

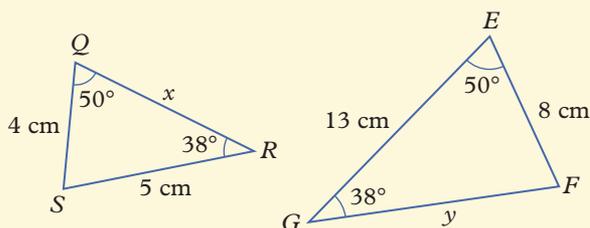
Condition	Meaning	Example
SSS	All three pairs of corresponding side lengths are in the same ratio.	
SAS	Two corresponding pairs of side lengths are in the same ratio and the included angles are equal.	
AAA	All three pairs of corresponding angles are equal.	
RHS	The lengths of the pair of hypotenuses are in the same ratio as another pair of sides in a right-angled triangle.	

- In the condition SAS the matching angles must be between the corresponding pairs of side lengths to demonstrate similarity.
- Other specifications, such as SSA and AAS, are not sufficient to demonstrate similarity.
- The similarity conditions are the minimum set of conditions required to demonstrate similarity. In some questions more information may be provided, however if any of the conditions are met then a pair of triangles will be similar.
- If a pair of triangles are congruent then they will also be similar (with a scale factor of 1).

Example 7G.1 Finding unknown side lengths in similar triangles



Find the values of the unknown side lengths in this pair of similar triangles.



THINK

- 1 Identify the equal corresponding angles and then match the corresponding sides opposite these angles. Sides QS and EF are both opposite the 38° angle. Sides RS and GF are both opposite the 50° angle.
- 2 Using the pair of corresponding sides that have known measurements (QS and EF), find the scale factor between $\triangle QSR$ and $\triangle EFG$. Consider $\triangle QSR$ as the original triangle and $\triangle EFG$ as the image.
- 3 Use the scale factor formula to find y in $\triangle EFG$.
- 4 Use the scale factor formula to find x in $\triangle QSR$.

WRITE

Corresponding pairs: $QS = 4$ cm and $EF = 8$ cm

$$\begin{aligned} \text{Scale factor} &= \frac{\text{image length}}{\text{original length}} \\ &= \frac{8 \text{ cm}}{4 \text{ cm}} \\ &= 2 \end{aligned}$$

Corresponding pairs: $RS = 5$ cm and $GF = y$

$$\begin{aligned} \text{Scale factor} &= \frac{\text{image length}}{\text{original length}} \\ 2 &= \frac{y}{5} \\ y &= 2 \times 5 \text{ cm} \\ y &= 10 \text{ cm} \end{aligned}$$

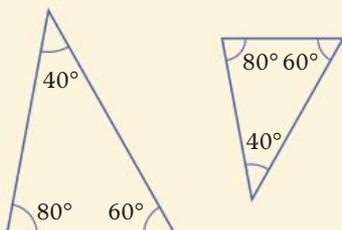
Corresponding pairs: $QR = x$ and $EG = 13$ cm

$$\begin{aligned} \text{Scale factor} &= \frac{\text{image length}}{\text{original length}} \\ 2 &= \frac{13 \text{ cm}}{x} \\ x &= \frac{13 \text{ cm}}{2} \\ x &= 6.5 \text{ cm} \end{aligned}$$



Example 7G.2 Identifying similarity conditions

Which condition would you use to check that these triangles are similar?



THINK

Look at the information provided in each triangle. All angles are given in both triangles.

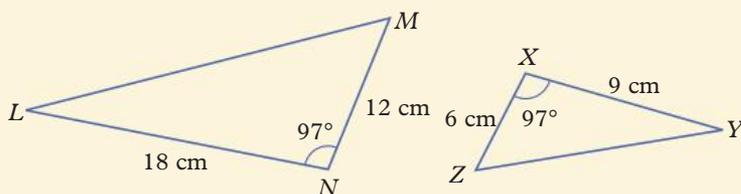
WRITE

The similarity condition AAA can be used to check whether these triangles are similar.

Example 7G.3 Determining if two triangles are similar



Determine whether these two triangles are similar.



THINK

- Both triangles show two sides and an included angle, so we can use the similarity condition SAS to determine whether the two triangles are similar.
- Calculate the scale factor between the two shorter sides (MN and XZ) and the two longer sides (LN and XY).
- Compare the scale factors to determine whether the similarity condition is met.
- Write your final answer.

WRITE

$$\begin{aligned} \text{Scale factor} &= \frac{\text{image length}}{\text{original length}} \\ &= \frac{XZ}{MN} \\ &= \frac{6 \text{ cm}}{12 \text{ cm}} \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{Scale factor} &= \frac{\text{image length}}{\text{original length}} \\ &= \frac{XY}{LN} \\ &= \frac{9 \text{ cm}}{18 \text{ cm}} \\ &= \frac{1}{2} \end{aligned}$$

The two triangles are similar, as they meet the similarity condition SAS.

Helpful hints

- ✓ Make sure that you identify the corresponding sides and angles correctly. This is a key skill. Remember that the given triangles may not be in the same orientation.

Exercise 7G Similar triangles

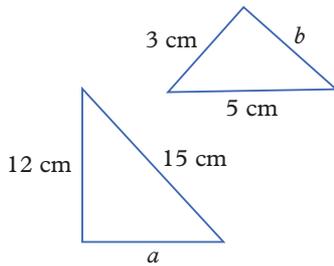
 1, 2, 4-6, 9, 11, 12, 14

 1, 3-5, 7, 8, 10, 13, 15, 16

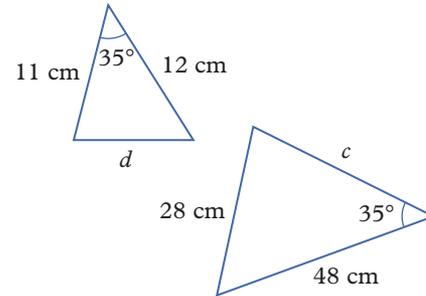
 1, 4, 5, 7, 11, 13, 14, 16-19

7G.1 1 Find the values of the unknown side lengths in each pair of similar triangles.

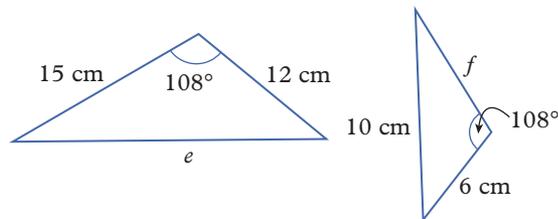
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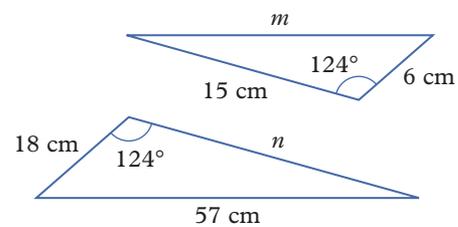
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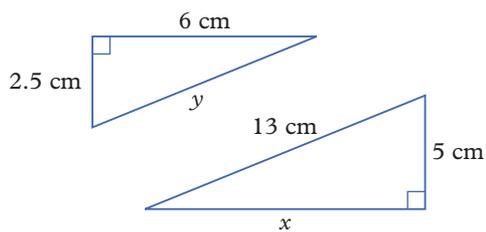
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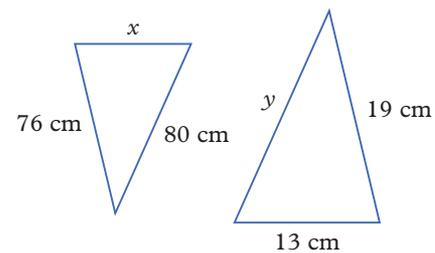
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e

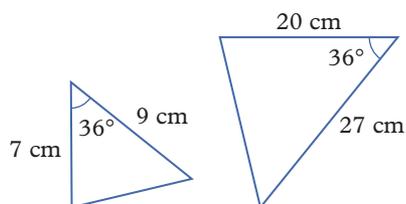


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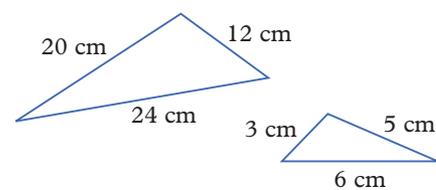


7G.2 2 State the condition you would use to decide whether each pair of triangles is similar.

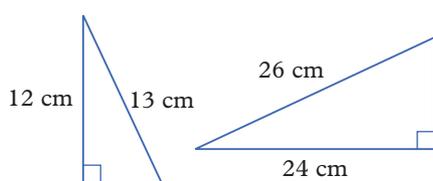
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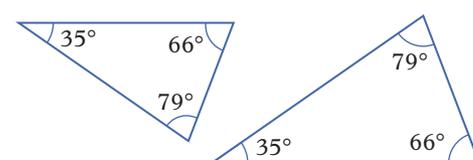
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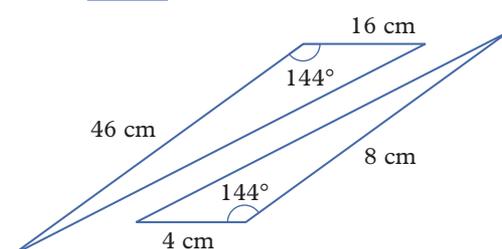
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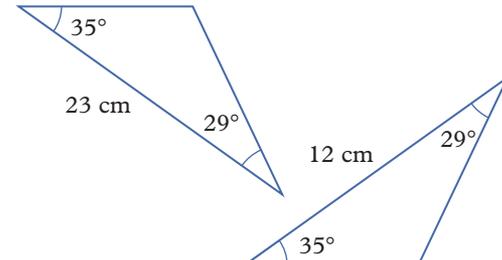
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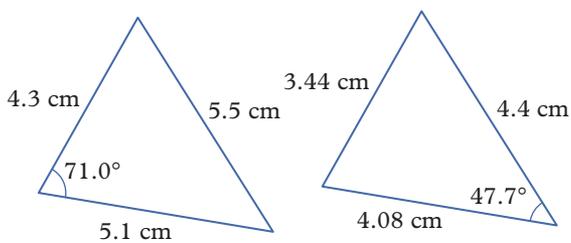


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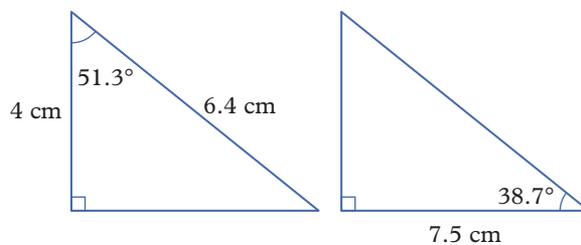


3 Which condition(s) could you use to check that these pairs of triangles are similar?

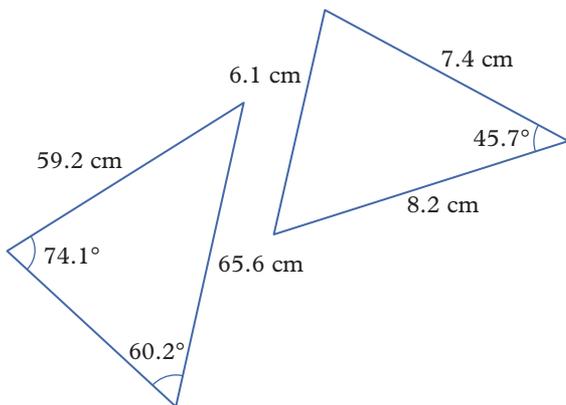
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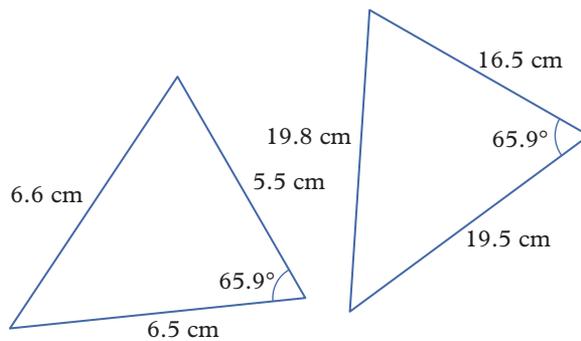
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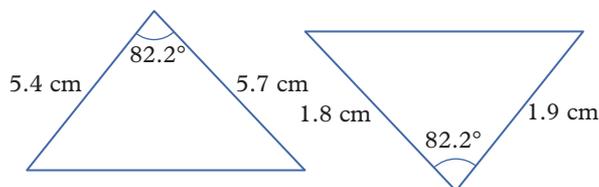


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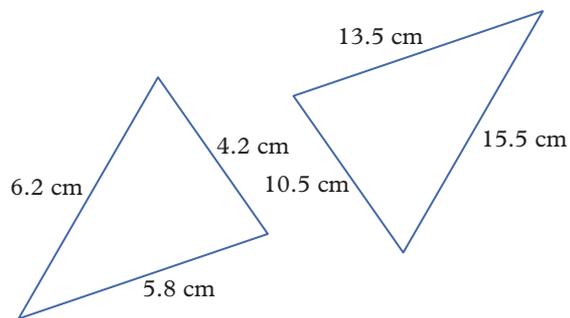


76.3 4 Determine if these pairs of triangles are similar. State the similarity condition used or explain why the triangles are not similar.

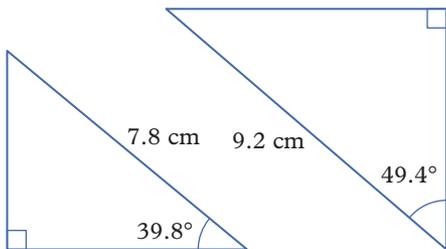
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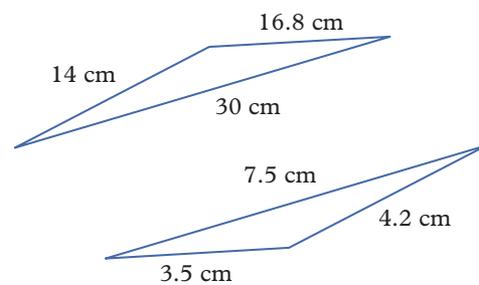
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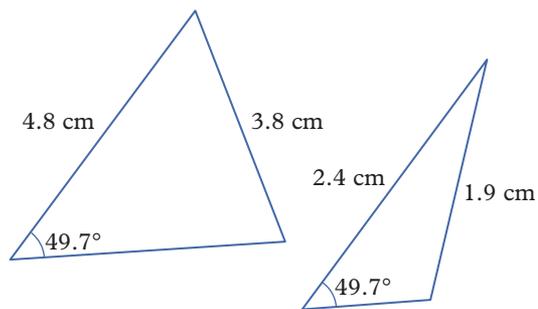
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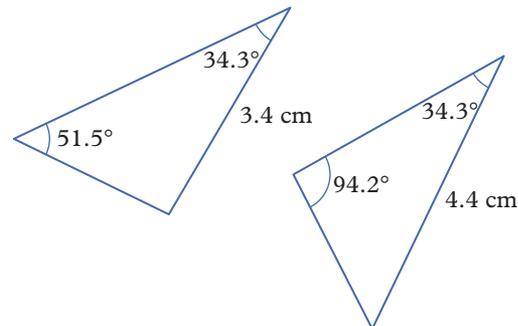
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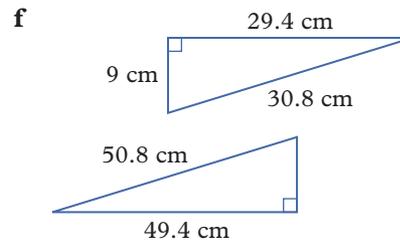
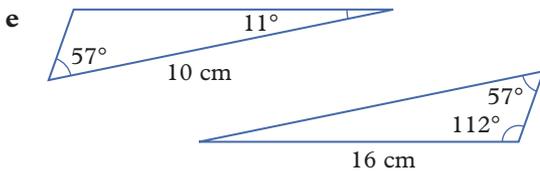
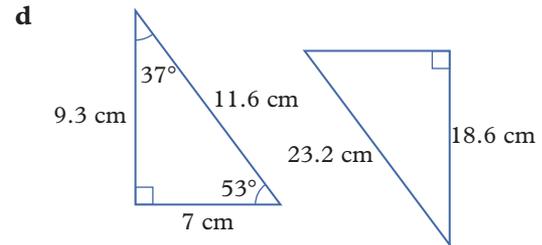
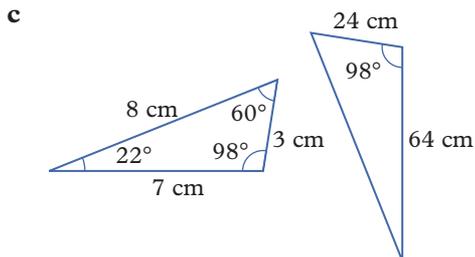
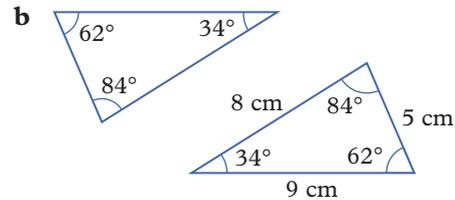
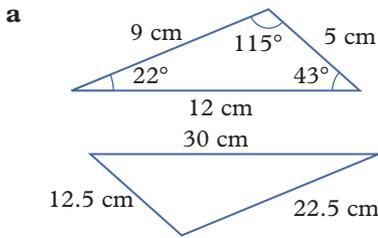
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f



5 Determine whether these pairs of triangles are similar.



6 Determine if the pairs of triangles from question 2 are similar or not.

7 Consider the four similar-triangle conditions. What conditions need to be added to each of the conditions for similar triangles to be congruent?

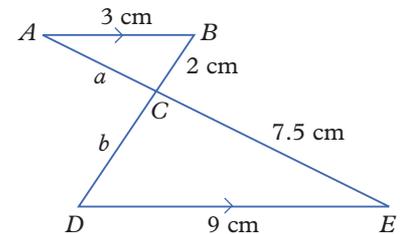
8 Why are all equilateral triangles similar to each other?

9 Explain why only two angles need to be checked when using AAA as a test for similarity.

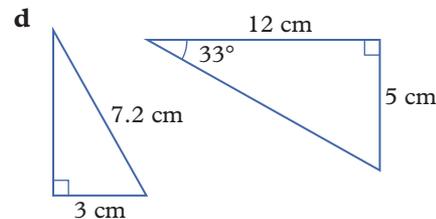
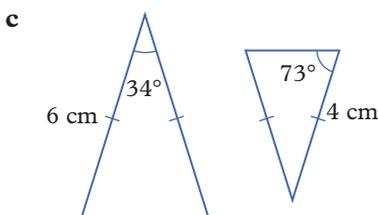
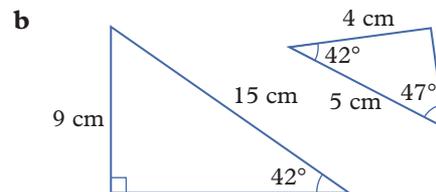
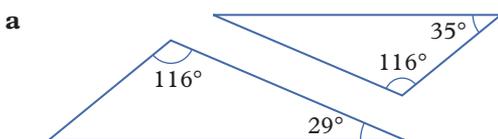
10 **a** Using an understanding of angles and parallel lines, explain why the pair of triangles on the right are similar.

b If triangle ABC is the original figure, find the scale factor between the two triangles.

c Find the lengths of the unknown sides.



11 Use your understanding of triangle properties to determine whether these pairs of triangles are similar.

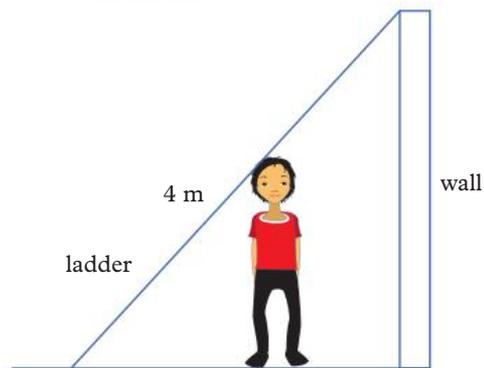


- 12 Similar triangles can be used to find the heights or lengths of objects that are difficult to measure. Consider this diagram showing the shadows cast by a person and a tree. The person is 1.76 m tall and casts a shadow of 2.1 m. If the tree casts a shadow of 7.8 m, how tall is the tree?

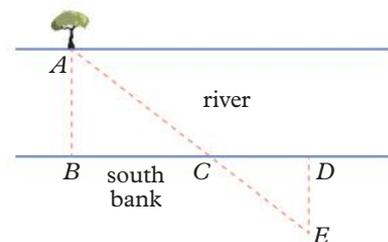


- 13 Luke wants to measure the heights of the walls in his house and then order some wall decorations. He places his 4 m ladder on an angle so that it hits the top of the wall, as shown in the diagram. The ladder has rungs every 50 cm: at 50 cm, 1 m, etc. If Luke, at 170 cm tall, stood directly under the ladder and his head came into contact with the fifth rung, find:

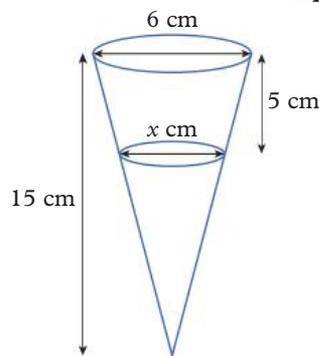
- the length of the ladder where it touched Luke's head
- the height of Luke's walls.



- 14 In order to find the width of a river, Ahmed places three markers along its southern bank. Marker B is directly opposite a tree at position A . Marker C is 50 m east of marker B and 40 m west of marker D . Ahmed then stands at point E , 30 m south of marker D and he can see marker C in a direct line with the tree at point A . Mark in these measurements on the diagram and then use similar triangles to find the width of the river.



- 15 An ice-cream cone is 15 cm high with an opening of diameter 6 cm. If chocolate coats the inside of the cone from the bottom to 5 cm below its top, find the largest diameter of the chocolate coating inside the cone.



- 16 Draw diagrams to help you solve these similar triangle problems.

- Scott has a red triangular flag that is 10 cm high and 15 cm long. He wants to create a larger version of the flag using a piece of red material that is 2 metres long. If he uses its full length, what height would this larger flag have?
- Christy bought a new netball ring but she wasn't sure that it was the official height (3.05 m). She measured her shadow as 117 cm and the netball ring's shadow as 195 cm. If Christy is 1.8 m tall, is the netball ring the official height?
- Oscar wants to know the height of a flagpole. A guide wire positioned 3.5 m from its base reached to exactly half its height. Oscar stood exactly halfway between the flagpole and where the guide wire attached to the ground and where the guide wire touched the top of his head. If Oscar is 1.75 m tall, how tall is the flagpole?

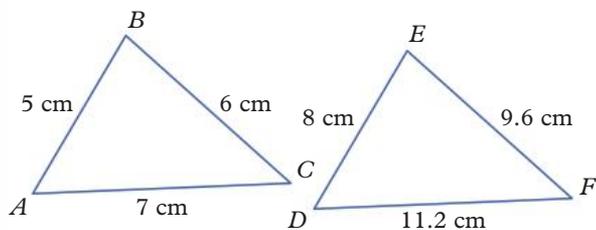
- 17 Sean and Tania want to know whose house is taller. One morning, Sean measures his shadow as 1.9 m and the shadow of his house as 3.42 m. Later that day, Tania measures her shadow as 2.3 m and the shadow of her house as 3.8 m. If Sean is 172 cm tall and Tania is 165 cm tall, whose house is taller?

- 18 Explain why you cannot just compare the shadow lengths of the houses in question 17 to decide which house is taller.

19 The process of showing that two triangles are similar can be formalised and generalised further. We can prove beyond a shadow of a doubt that a given fact is true by writing up bulletproof reasoning behind every step we take. We call these geometric proofs.

Follow the worked examples and then complete the geometric proofs.

Example: Prove that $\triangle ABC \sim \triangle DEF$.



$$\frac{DE}{AB} = \frac{8 \text{ cm}}{5 \text{ cm}} = 1.6$$

$$\frac{EF}{BC} = \frac{9.6 \text{ cm}}{6 \text{ cm}} = 1.6$$

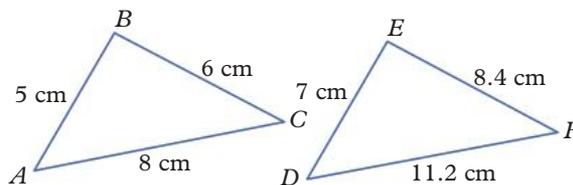
$$\frac{FD}{CA} = \frac{11.2 \text{ cm}}{7 \text{ cm}} = 1.6$$

$$\frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA}$$

Corresponding sides are in the same ratio.

$\therefore \triangle ABC \sim \triangle DEF$ by SSS

a Prove that $\triangle ABC \sim \triangle DEF$.



$$\frac{DE}{AB} = \frac{\square}{\square} =$$

$$\frac{EF}{BC} = \frac{\square}{\square} =$$

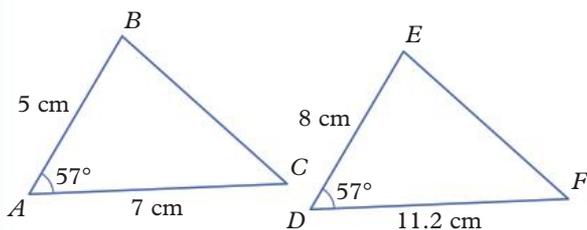
$$\frac{FD}{CA} = \frac{\square}{\square} =$$

$$\frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$$

Corresponding sides are in the same ratio.

$\therefore \triangle \square \sim \triangle \square$ by \square

Example: Prove that $\triangle ABC \sim \triangle DEF$.



$$\frac{DE}{AB} = \frac{8 \text{ cm}}{5 \text{ cm}} = 1.6$$

$$\frac{FD}{CA} = \frac{11.2 \text{ cm}}{7 \text{ cm}} = 1.6$$

$$\angle EDF = 57^\circ$$

$$\angle BAC = 57^\circ$$

$$\frac{DE}{AB} = \frac{FD}{CA}$$

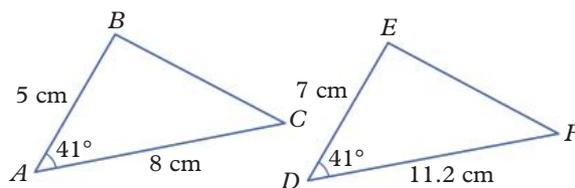
Corresponding sides are in the same ratio.

$$\angle EDF = \angle BAC$$

Corresponding included angles are equal.

$\therefore \triangle ABC \sim \triangle DEF$ by SAS

b Prove that $\triangle ABC \sim \triangle DEF$.



$$\frac{DE}{AB} = \frac{\square}{\square} =$$

$$\frac{FD}{CA} = \frac{\square}{\square} =$$

$$\angle EDF = \square^\circ$$

$$\angle BAC = \square^\circ$$

$$\frac{\square}{\square} = \frac{\square}{\square}$$

Corresponding sides are in the same ratio.

$$\angle \square = \angle \square$$

Corresponding included angles are equal.

$\therefore \triangle \square \sim \triangle \square$ by \square

Check your Student obook pro for these digital resources and more:



Interactive skillsheet
Similar triangles



Investigation
Using mirrors to
calculate length



Topic quiz
76

pro

7H Trigonometric ratios

Learning intentions

By the end of this topic you will be able to ...

- ✓ identify the hypotenuse, opposite and adjacent sides of a right-angled triangle containing a reference angle
- ✓ calculate sine, cosine and tangent ratios of an angle in a right-angled triangle.



Inter-year links

Year 7 3H Ratios

Year 8 3E Ratios

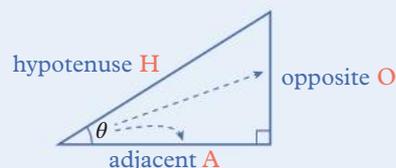
Year 10 9B Trigonometry

Trigonometry



Key content video

- **Trigonometry** is the study of relationships between angles and side lengths in triangles.
- In a right-angled triangle with a reference angle:
 - the **opposite side** is opposite the reference angle
 - the **adjacent side** is next to or adjacent to the reference angle
 - the hypotenuse is always the longest side and is opposite the right angle.
- The symbol θ (*theta*, a Greek letter) is often used to represent the reference angle.



Trigonometric ratios

- For any right-angled triangle with a reference angle of θ :
 - the **sine** of angle θ is the ratio of the lengths of the opposite side to the hypotenuse
 - the **cosine** of angle θ is the ratio of the lengths of the adjacent side to the hypotenuse
 - the **tangent** of angle θ is the ratio of the lengths of the opposite side to the adjacent side.
- The ratios can be written as fractions:

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

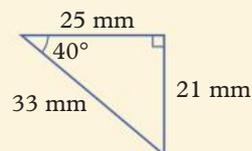
$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

Example 7H.1 Labelling the sides of a right-angled triangle with respect to a reference angle



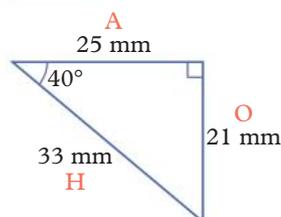
Label the sides of this triangle with O (for opposite side), A (for adjacent side) and H (for hypotenuse) with respect to the angle of 40° .



THINK

- 1 Label the hypotenuse with H. It is the longest side and is opposite the right angle.
- 2 Identify the opposite side and the adjacent side with respect to the reference angle of 40° . Label the side opposite 40° as O and the side next to 40° (adjacent) as A.

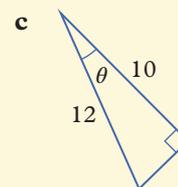
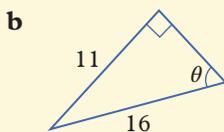
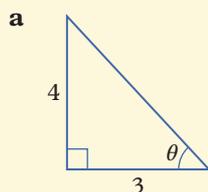
WRITE





Example 7H.2 Determining trigonometric ratios

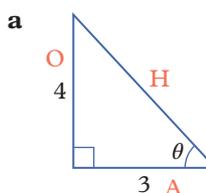
For each triangle, identify which trigonometric ratio can be used and then determine its value in simplest form, based on the side lengths provided.



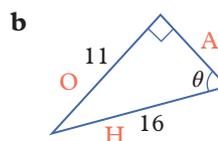
THINK

- a**
- 1 Identify the given sides with respect to the reference angle. The side length opposite the angle is 4. The side length adjacent to the angle is 3. Label the sides accordingly.
 - 2 Decide which trigonometric ratio to use. As O and A are involved, use tangent.
 - 3 Substitute the values into the equation to determine the ratio.
- b**
- 1 Identify the given sides with respect to the angle. The side length opposite the angle is 11. The hypotenuse is 16. Label the sides accordingly.
 - 2 Decide which trigonometric ratio to use. As O and H are involved, use sine.
 - 3 Substitute the values into the equation to determine the ratio.
- c**
- 1 Identify the given sides with respect to the angle. The side length adjacent to the angle is 10. The hypotenuse is 12. Label the sides accordingly.
 - 2 Decide which trigonometric ratio to use. As A and H are involved, use cosine.
 - 3 Substitute the values into the equation to determine the ratio.
 - 4 Simplify the fraction.

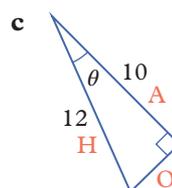
WRITE



$$\begin{aligned} O &= 4, A = 3 \\ \tan(\theta) &= \frac{O}{A} \\ &= \frac{4}{3} \end{aligned}$$



$$\begin{aligned} O &= 11, H = 16 \\ \sin(\theta) &= \frac{O}{H} \\ &= \frac{11}{16} \end{aligned}$$



$$\begin{aligned} A &= 10, H = 12 \\ \cos(\theta) &= \frac{A}{H} \\ &= \frac{10}{12} \\ &= \frac{5}{6} \end{aligned}$$

- ✓ Trigonometry can involve a lot of calculator work. You will need to know how to enter the trigonometric function into your scientific calculator.
- ✓ To remember the ratio of the two different sides according to the reference angle, you can use SOH CAH TOA. Like with Pythagoras' theorem, writing the equation from memory before substituting the values in will help you to memorise it quicker.

$$\sin(\theta) = \frac{O}{H} \quad \cos(\theta) = \frac{A}{H} \quad \tan(\theta) = \frac{O}{A} \quad \longrightarrow \quad \text{SOH CAH TOA}$$

- ✓ The sine, cosine or tangent values do not have units. The units cancel out in the ratio.
- ✓ Do not write $\sin =$, $\cos =$ or $\tan =$ without including the angle. Sine, cosine and tangent do not have a value without an angle.
- ✓ The tangent of the angle is the gradient of the hypotenuse of a triangle in the first quadrant with the reference angle placed at the origin.
- ✓ The prefix 'co-' in cosine is short for complementary, as cosine is the sine of the complementary angle, the other non-right-angle in a right-angled triangle, which is opposite the side adjacent to the reference angle.

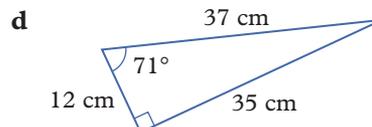
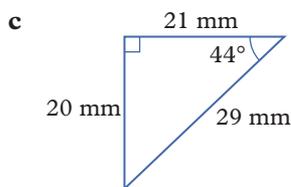
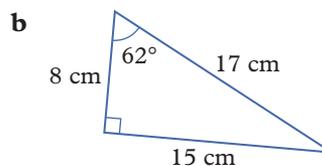
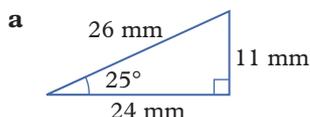
ANS p491 **Exercise 7H** Trigonometric ratios

 1-5, 6(a, c, e), 7(b, d, f), 8-11, 14, 15, 19

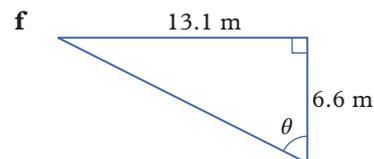
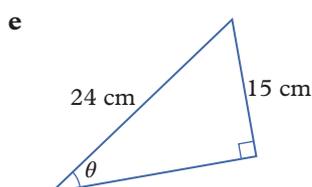
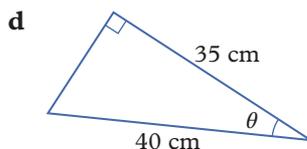
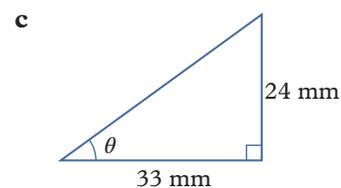
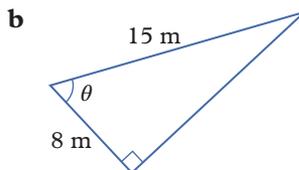
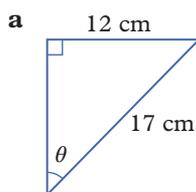
 2-5, 7-10, 12, 16, 17, 19, 20

 3-5, 8-10, 12, 13, 16, 18, 20, 21

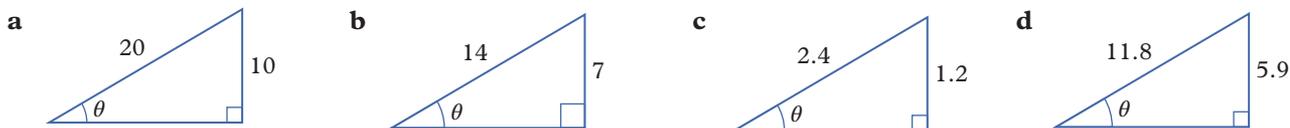
7H.1 1 Label the sides of each triangle with O (for opposite side), A (for adjacent side) and H (for hypotenuse) with respect to the given reference angle.



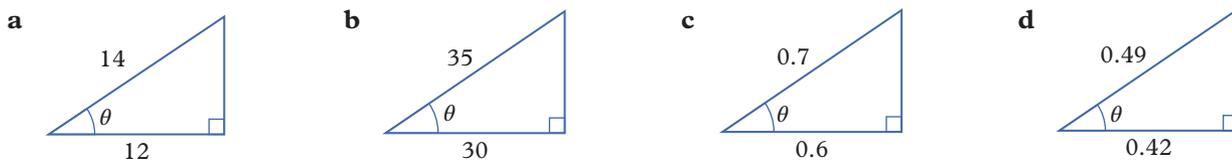
7H.2 2 For each triangle, determine which trigonometric ratio can be used based on the side lengths provided.



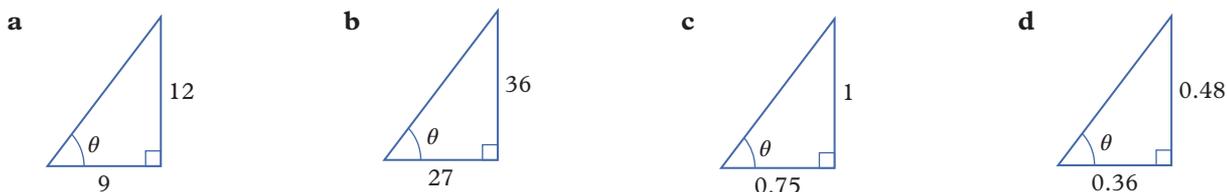
3 For each of the following triangles, determine the ratio equal to the sine of angle θ . Simplify your fractions where possible.



4 For each of the following triangles, determine the ratio equal to the cosine of angle θ . Simplify your fractions where possible.



5 For each of the following triangles, determine the ratio equal to the tangent of angle θ . Simplify your fractions where possible.



6 Evaluate the following using a calculator correct to four decimal places. Ensure your calculator is in degree mode.

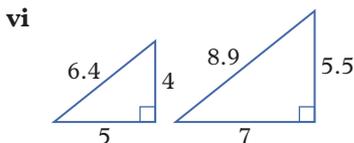
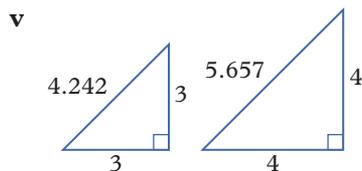
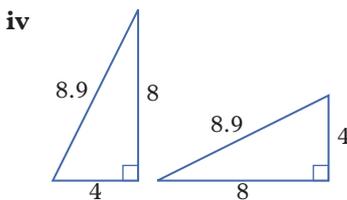
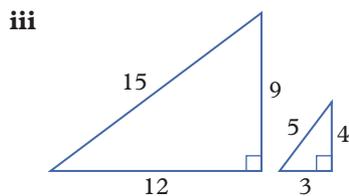
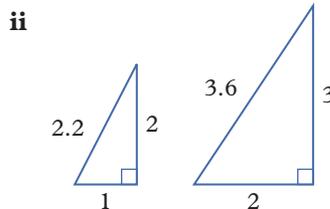
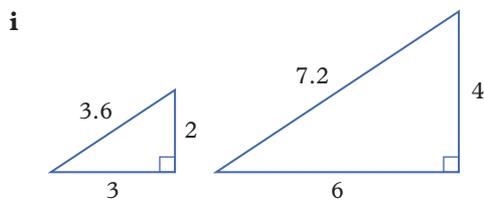
- a $\sin(20^\circ)$ b $\sin(36^\circ)$ c $\cos(36^\circ)$
 d $\tan(36^\circ)$ e $\tan(63^\circ)$ f $\cos(63^\circ)$

7 Evaluate the following using a calculator correct to four significant figures. Ensure your calculator is in degree mode.

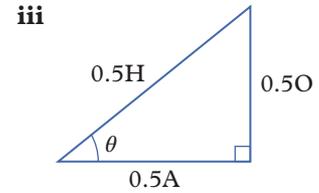
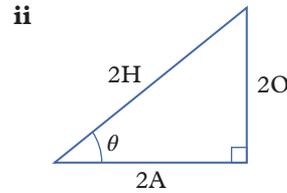
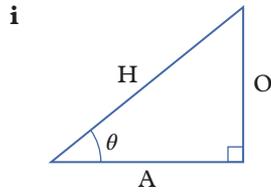
- a $6 \sin(20^\circ)$ b $\frac{6}{\sin(20^\circ)}$ c $\frac{\sin(20^\circ)}{6}$
 d $(\cos(20^\circ))^2 + (\sin(20^\circ))^2$ e $(\cos(20^\circ))^2 - (\sin(20^\circ))^2$ f $\frac{\sin(20^\circ)}{\cos(20^\circ)}$

8 a Describe the relationship between the triangles in questions 3, 4 and 5.

b Determine which pairs of triangles have the same corresponding sine, cosine and tangent values for their non-right angles.



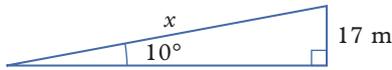
9 a For each of the following similar triangles, determine the ratio equal to the sine, cosine and tangent of angle θ . Simplify your fractions where possible.



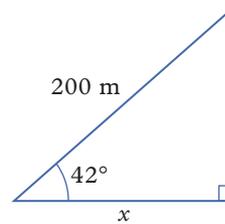
b Write a sentence about the values of the sine, cosine and tangent ratios of angles in similar triangles.

10 For each of the following, use the given approximate trigonometric ratio to determine the value of x .

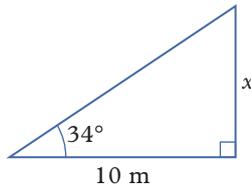
a $\sin(10^\circ) = 0.17$



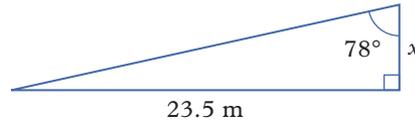
b $\cos(42^\circ) = 0.74$



c $\tan(34^\circ) = 0.67$



d $\tan(78^\circ) = 4.7$



11 a Using your calculator, complete the table below. Write your answers correct to four decimal places.

θ	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$	$\frac{1}{\tan(\theta)}$
1°				
5°				
15°				
30°				
45°				
60°				
75°				
85°				
89°				

b Which column(s) increase(s) as the angle, θ , increases? Which column(s) decrease(s) as the angle, θ , increases?

c Fill in the blanks of the following equations.

i $\sin(27^\circ) = \cos(\underline{\quad}^\circ)$

ii $\sin(\underline{\quad}^\circ) = \cos(56^\circ)$

iii $\tan(12^\circ) = \frac{1}{\tan(\underline{\quad}^\circ)}$

iv $\tan(\underline{\quad}^\circ) = \frac{1}{\tan(47^\circ)}$

d Describe the relationship between sine and cosine. Use the word *complementary* in your description.

e Describe the relationship between tangent and the reciprocal of tangent. Use the word *complementary* in your description.

12 Two special triangles can help us determine exact values of trigonometric ratios for common angles.

Diagram 1 shows a square with side lengths of 1 unit. If we cut the square in half along a diagonal, we obtain a right-angled isosceles triangle, as shown in Diagram 2. This is the first special triangle.

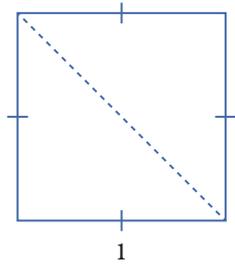


Diagram 1

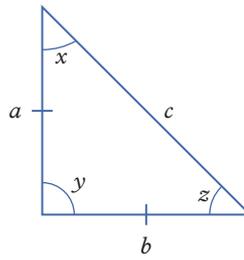


Diagram 2

- a** Determine the values of all side lengths and angles in Diagram 2 using geometry and Pythagoras' theorem. Write your answers exactly, as a square root where required.
- b** Using the values determined in part **a**, write down the exact values of the sine, cosine and tangent ratios of 45° .

The second special triangle is formed by cutting an equilateral triangle of side length 2 units in half down the middle, as shown in Diagrams 3 and 4.

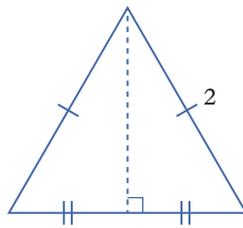


Diagram 3

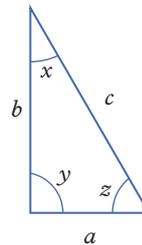
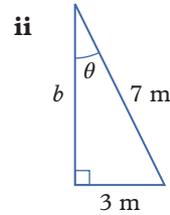
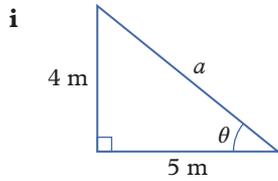


Diagram 4

- c** Determine the values of all side lengths and angles in Diagram 4 using geometry and Pythagoras' theorem. Write your answers exactly, as a fraction or a square root where required.
 - d** Using the values determined in part **c**, write down the exact values of the sine, cosine and tangent ratios of 30° and 60° .
 - e** Evaluate the following on a calculator.

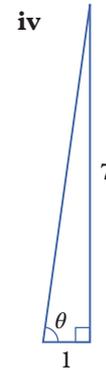
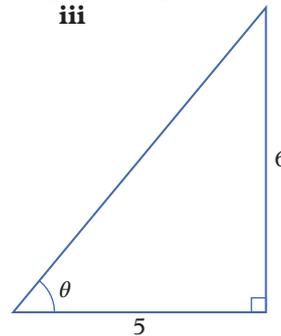
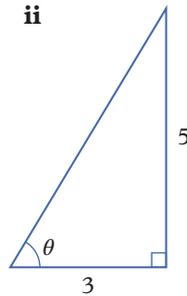
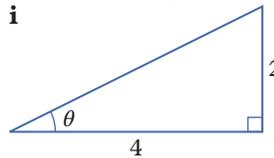
i $(\sin(45^\circ))^2$	ii $(\cos(30^\circ))^2$	iii $(\tan(30^\circ))^2$	iv $(\sin(60^\circ))^2$
v $(\sin(30^\circ))^2$	vi $(\cos(60^\circ))^2$	vii $(\tan(45^\circ))^2$	viii $(\cos(45^\circ))^2$
 - f** Describe how you could use the calculations in part **e** to obtain the exact values of the sine, cosine and tangent ratios of 30° , 45° and 60° without the special triangles.
- 13 a** Explain, by referring to the hypotenuse as the longest side of a right-angled triangle, why the values of sine and cosine of θ are never greater than 1 for $0^\circ < \theta < 90^\circ$.
- b** Explain why the values of tangent, unlike sine and cosine, can be greater than 1 for $0^\circ < \theta < 90^\circ$.
- c** Under what condition(s) is the value of tangent greater than 1?
- 14 a** Add another column to the table in question **11** with the heading $\frac{\sin(\theta)}{\cos(\theta)}$. Complete the column by performing the division $\sin(\theta) \div \cos(\theta)$ for each angle.
- b** Fill in the blanks of the following equations.
- | | |
|--|--|
| i $\frac{\sin(27^\circ)}{\cos(27^\circ)} = \tan(___\circ)$ | ii $\frac{\sin(___\circ)}{\cos(56^\circ)} = \tan(56^\circ)$ |
| iii $\frac{\sin(12^\circ)}{\cos(___\circ)} = \tan(12^\circ)$ | iv $\frac{\sin(___\circ)}{\cos(___\circ)} = \tan(47^\circ)$ |
- c** Describe the relationship between $\frac{\sin(\theta)}{\cos(\theta)}$ and $\tan(\theta)$.

15 a For each of the triangles below, use Pythagoras' theorem to determine the missing side length correct to two decimal places.



b For each triangle in part **a**, write the sine, cosine and tangent ratios of θ as a decimal correct to four decimal places where required.

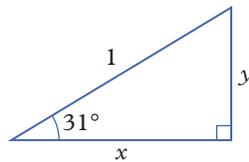
16 a Calculate the gradient of the hypotenuse in each right-angled triangle.



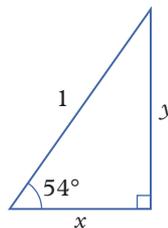
b Write the value of the tangent of θ for each right-angled triangle in part **a**.

c Describe the relationship between the gradient of a line segment and the tangent of the angle from the horizontal.

17 a Consider the following right-angled triangle, with a base of x and an altitude (height) of y . Write two equations using the sine ratio and cosine ratio for this triangle.



b Write two equations using the sine ratio and cosine ratio for this triangle.

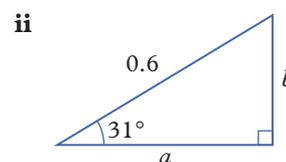
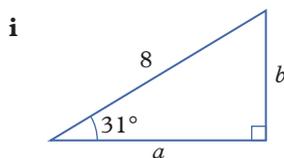


c Complete the following sentences by using the words 'base' and 'altitude'.

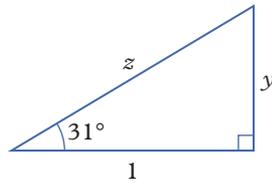
i The sine of an angle is the length of the _____ of a right-angled triangle with a hypotenuse of length 1.

ii The cosine of an angle is the length of the _____ of a right-angled triangle with a hypotenuse of length 1.

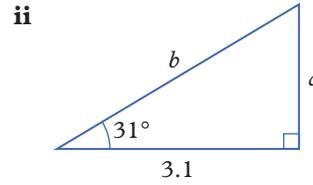
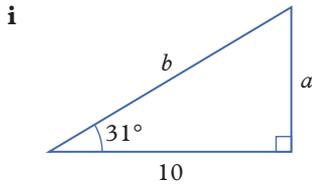
d Use your answer to part **c** and the fact that trigonometric ratios are equal in similar triangles to determine the lengths of a and b in these similar triangles correct to four significant figures.



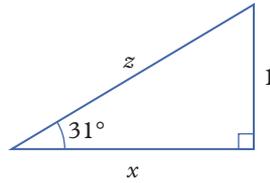
18 a Write two equations using the cosine ratio and tangent ratio for the following triangle.



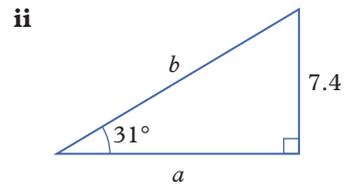
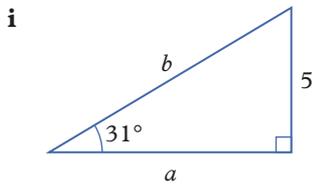
b Determine the lengths of a and b in these similar triangles correct to four significant figures.



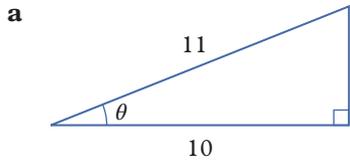
c Write two equations using the sine ratio and tangent ratio for the following triangle.



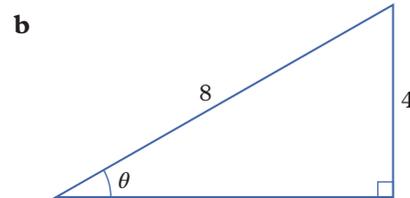
d Determine the lengths of a and b in these similar triangles correct to four significant figures.



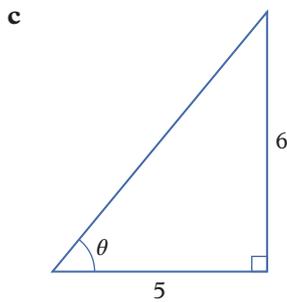
19 Explain and correct the error each person has made when writing the trigonometric ratio.



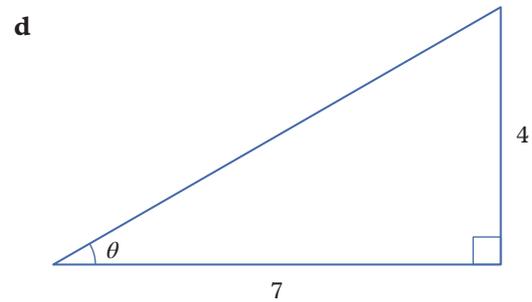
$$\cos = \frac{10}{11}$$



$$\sin(\theta) = \frac{8}{4}$$



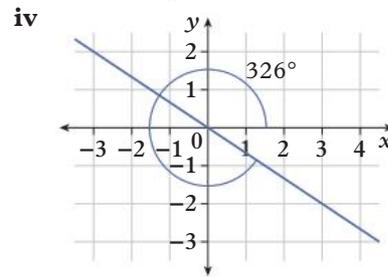
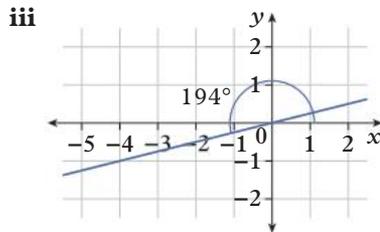
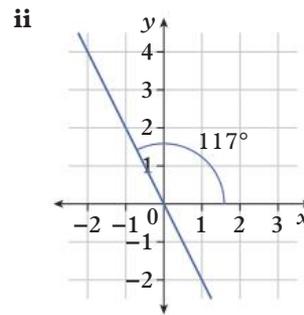
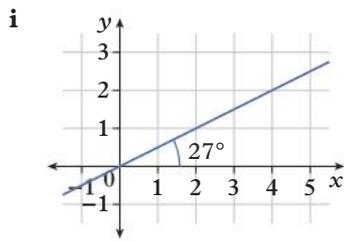
$$\cos(\theta) = \frac{6}{5}$$



$$\tan(\theta) = \frac{7}{4}$$

20 In question 16, we saw that the tangent of the angle from the horizontal is equal to the gradient of the hypotenuse. This is also true for lines inclined at any angle, not just $0^\circ < \theta < 90^\circ$.

a State the tangent of the angle for each by determining the gradient of the line.



b Evaluate the following with a calculator.

i $\tan(45^\circ)$

ii $\tan(135^\circ)$

iii $\tan(225^\circ)$

iv $\tan(315^\circ)$

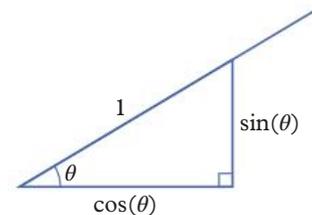
c Explain the similarity of the values in part **b**. Hint: Draw a diagram to help.

d Complete the following sentences.

i The gradient of a horizontal line is _____. Therefore, the tangent of 0° is _____.

ii The gradient of a vertical line is _____. Therefore, the tangent of 90° is _____.

21 We saw in question 14 that $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$. The tangent of the angle from the horizontal is equal to the gradient, so $\frac{\sin(\theta)}{\cos(\theta)} = \frac{\text{rise}}{\text{run}}$. Considering the ray on the right:



- the sine of the angle is the amount of rise when the hypotenuse is 1
- the cosine of the angle is the amount of run when the hypotenuse is 1.

a Evaluate the following with a calculator.

i $\sin(30^\circ)$

ii $\sin(150^\circ)$

iii $\sin(210^\circ)$

iv $\sin(330^\circ)$

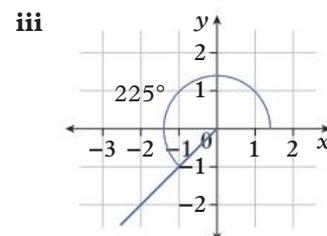
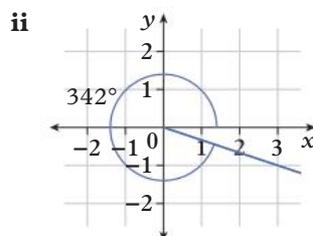
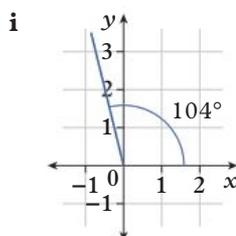
v $\cos(60^\circ)$

vi $\cos(120^\circ)$

vii $\cos(240^\circ)$

viii $\cos(300^\circ)$

b State whether the sine and cosine values are positive or negative for the angles shown.



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Interactive skillsheet
Trigonometry



Investigation
Nested similar triangles



Topic quiz
7H

7I Using trigonometry to find side lengths

Learning intentions

By the end of this topic you will be able to ...

- ✓ use trigonometric ratios to find unknown side lengths in a right-angled triangle.



Inter-year links

Year 7

6H Solving equations using inverse operations

Year 8

6B Solving linear equations

Year 10

9C Applications of trigonometry

Using trigonometry to find a side length

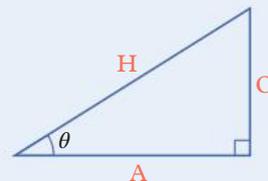


Key content video

- Trigonometric ratios can be used to find an unknown side length in a right-angled triangle if an angle (other than the right angle) and one side length are known.
- To find a side length:
 - 1 Identify the given sides with respect to the angle θ . Label the sides accordingly.
 - 2 Decide which trigonometric ratio to use.
 - 3 Substitute in the known side length and angle.
 - 4 Solve the equation for the unknown.
- Use a calculator to perform the final calculation and, where appropriate, round your answer.

SOH CAH TOA

$$\sin(\theta) = \frac{O}{H} \quad \cos(\theta) = \frac{A}{H} \quad \tan(\theta) = \frac{O}{A}$$



Degrees and minutes

- As well as being measured in decimal degrees, angles can also be measured in degrees and **minutes**. There are 60 minutes in one degree.
 - Minutes are denoted by the symbol $'$. For example, $25^{\circ}35'$ means 25 degrees and 35 minutes.
- Most calculators have a button for inputting an angle measure in degrees and minutes. Alternatively, you can convert minutes into a decimal value by dividing by 60.

Example 7I.1 Solving equations involving trigonometric ratios



Solve the equation $\sin(23^{\circ}) = \frac{m}{4}$. Round your answer to two decimal places.

THINK

- 1 Rearrange the equation to make m the subject of the formula.
- 2 Use a calculator to evaluate the trigonometric ratio $\sin(23^{\circ})$ and perform the multiplication operation ($\times 4$).
- 3 Round the answer to two decimal places.

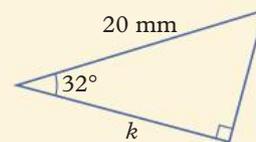
WRITE

$$\begin{aligned} m &= 4 \times \sin(23^{\circ}) \\ &= 1.5629... \\ &\approx 1.56 \end{aligned}$$

Example 71.2 Identifying which trigonometric ratio to use to write an equation



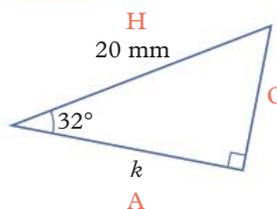
Determine which trigonometric ratio can be used, based on the side lengths provided, and hence write an equation that involves k .



THINK

- 1 Identify the given sides with respect to the angle 32° . The side length adjacent to the angle is k . The hypotenuse is 20 mm. Label the sides accordingly.
- 2 Decide which trigonometric ratio to use. As A and H are involved, use cosine.
- 3 Substitute for θ , A and H to write an equation involving k .

WRITE



$$A = k, H = 20 \text{ mm}$$

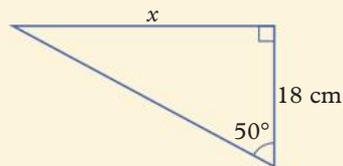
$$\cos(\theta) = \frac{A}{H}$$

$$\cos(32^\circ) = \frac{k}{20 \text{ mm}}$$

Example 71.3 Using trigonometry to find an unknown side length in the numerator



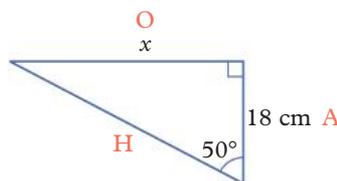
Use trigonometry to find the side length x in this triangle, correct to two decimal places.



THINK

- 1 Identify the given sides with respect to the angle 50° . The side length adjacent to the angle is 18 cm. The side length opposite the angle is x . Label the sides accordingly.
- 2 Decide which trigonometric ratio to use. As O and A are involved, use tangent.
- 3 Substitute for θ , O and A.
- 4 Use a calculator to solve the equation for x .
- 5 Round the value of x to two decimal places.

WRITE



$$O = x, A = 18 \text{ cm}$$

$$\tan(\theta) = \frac{O}{A}$$

$$\tan(50^\circ) = \frac{x}{18 \text{ cm}}$$

$$18 \times \tan(50^\circ) = x$$

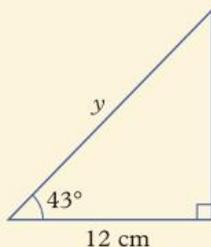
$$21.4515... = x$$

$$x \approx 21.45 \text{ cm}$$



Example 71.4 Using trigonometry to find an unknown side length in the denominator

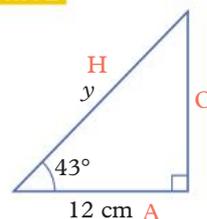
Use trigonometry to find the side length y in this triangle, correct to two decimal places.



THINK

- 1 Identify the given sides with respect to the angle 43° . The side length adjacent to the angle is 12 cm. The hypotenuse is y . Label the sides accordingly.
- 2 Decide which trigonometric ratio to use. As A and H are involved, use cosine.
- 3 Substitute for θ , A and H.
- 4 Solve the equation for y . Multiply both sides of the equation by y , and then divide both sides of the equation by $\cos(43^\circ)$.
- 5 Use a calculator to divide 12 by $\cos(43^\circ)$.
- 6 Round the value of y to two decimal places.

WRITE



$$A = 12 \text{ cm}, H = y$$

$$\cos(\theta) = \frac{A}{H}$$

$$\cos(43^\circ) = \frac{12 \text{ cm}}{y}$$

$$y \times \cos(43^\circ) = 12 \text{ cm}$$

$$y = \frac{12}{\cos(43^\circ)}$$

$$\approx 16.4079\dots$$

$$\approx 16.41 \text{ cm}$$

Helpful hints

- ✓ Make sure your calculator is in degree mode, not radian mode. Otherwise, all your answers will be wrong! Check the DEG or D symbol is on screen, not RAD or R or GRAD or G.
- ✓ Remember when solving an equation for an unknown, you can check your answer by substituting it back into the original equation.
- ✓ Drawing your diagram close to scale (making all angles roughly the correct size) will let you check that your answer is reasonable by referring to the relative length of the other sides. Always ensure that at the end of the question the hypotenuse is the longest side of the triangle.
- ✓ SOH CAHTOA is still a useful mnemonic device for this section. You may find it helpful to write it at the top of your page before starting the exercise.

$$\text{S} \frac{\text{O}}{\text{H}} \text{C} \frac{\text{A}}{\text{H}} \text{T} \frac{\text{O}}{\text{A}}$$

ANS p492 **Exercise 7I** Using trigonometry to find side lengths

UNDERSTANDING AND FLUENCY

 1-6, 8, 9, 13, 14(a-d)

 2, 3, 5-7, 10, 11, 13, 14, 18, 19

 3, 5-7, 12, 13, 15-18, 20

7I.1 1 Solve the following equations for the unknown, correct to two decimal places.

a $0.4226 = \frac{x}{5}$

b $0.4226 = \frac{5}{x}$

c $\sin(25^\circ) = \frac{x}{5}$

d $\sin(25^\circ) = \frac{5}{x}$

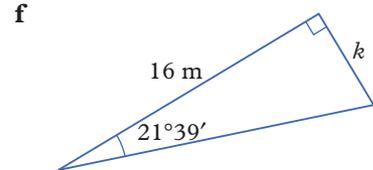
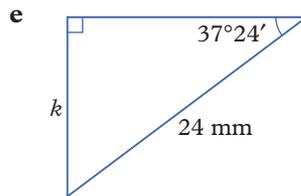
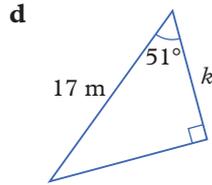
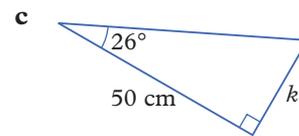
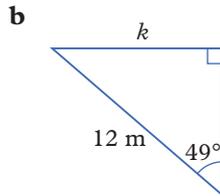
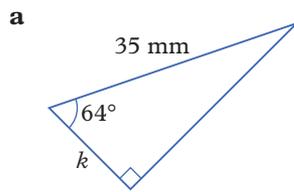
e $\cos(53^\circ) = \frac{x}{11}$

f $\cos(53^\circ) = \frac{11}{x}$

g $\tan(11^\circ) = \frac{x}{20}$

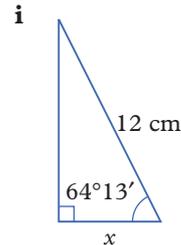
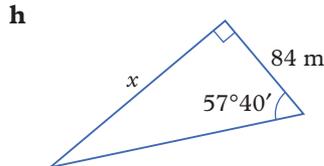
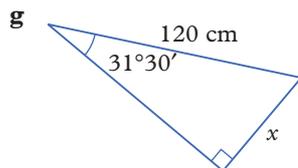
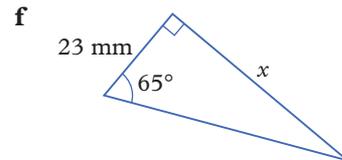
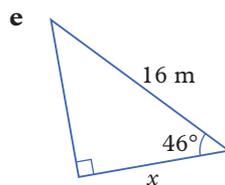
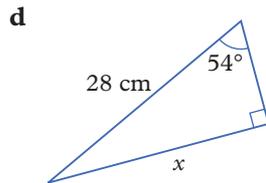
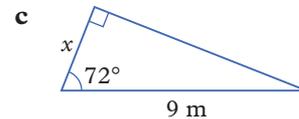
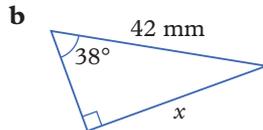
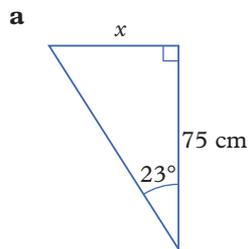
h $\tan(79^\circ) = \frac{20}{x}$

7I.2 2 For each triangle, determine which trigonometric ratio can be used, based on the side lengths provided, and hence write an equation that involves k .

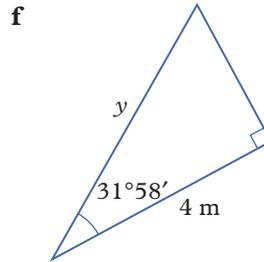
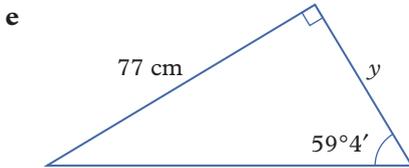
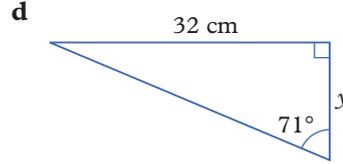
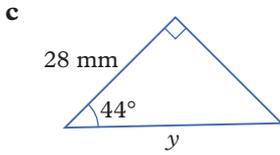
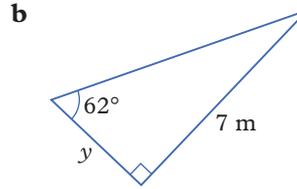
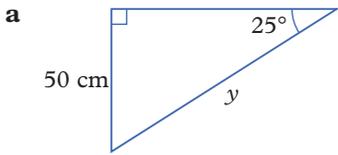


3 Solve each equation formed in question 1 to calculate the value of k , correct to two decimal places.

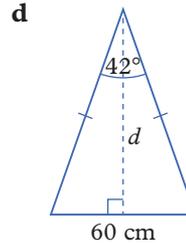
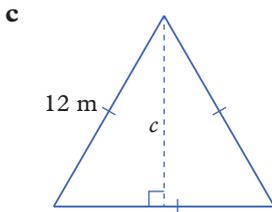
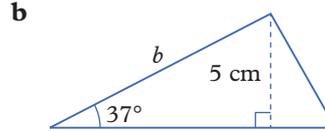
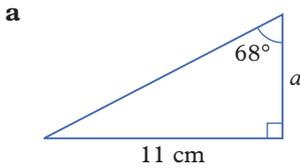
7I.3 4 Use trigonometry to find the side length x in each triangle, correct to two decimal places.



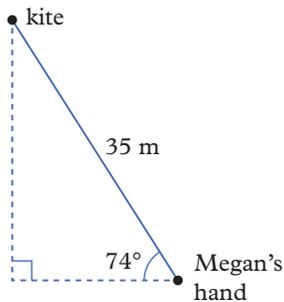
7.4 5 Find the side length y in each triangle, correct to two decimal places.



6 Find the value of the pronumeral in each triangle, correct to two decimal places.

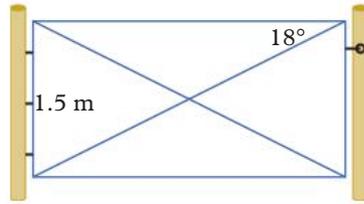


7 Megan holds the string attached to her kite at a height of 1 m above the ground. The 35 m long string makes an angle of 74° with the horizontal.



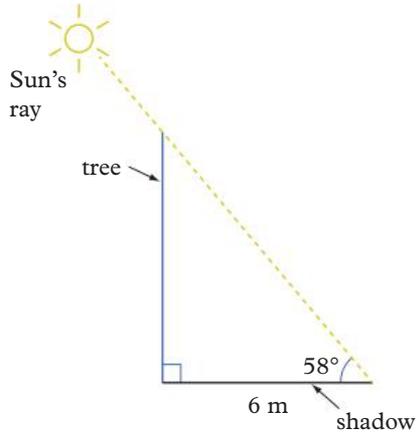
- a** Use trigonometry to calculate, correct to two decimal places, the vertical distance from one end of the string to the other.
- b** What is the height of the kite above the ground, correct to two decimal places?

- 8 A farm gate has diagonal supporting braces that make an angle of 18° with the horizontal. If the height of the braces is 1.5 m, how wide is the gate, correct to two decimal places?



- 9 A shadow is formed when the Sun's rays are blocked by an object. The angle of the Sun's rays determine how long a shadow will be.

At a certain time of day, the Sun's rays make an angle of 58° with the ground and a tree forms a shadow that is 6 m long. Find the following correct to two decimal places.



- a Use the diagram to calculate the height of the tree.
 b At the same time, another tree forms a shadow that is 10 m long. How high is this tree?
- 10 The two identical sides of this ladder meet at an angle of 50° and are 1.8 m apart where they touch the ground. Find the following correct to two decimal places.



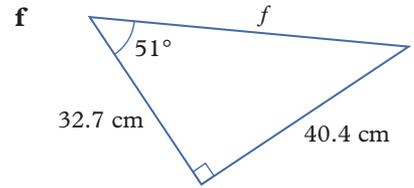
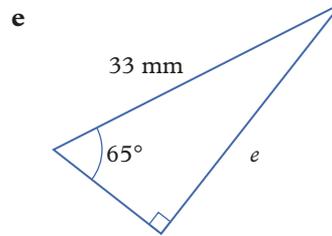
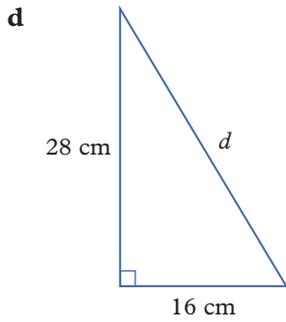
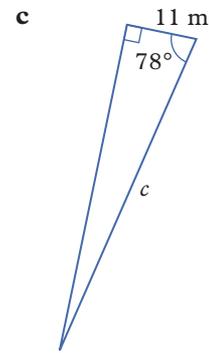
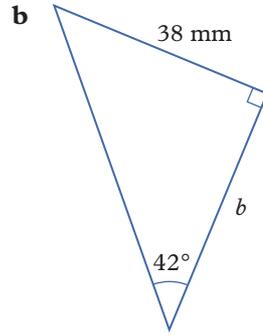
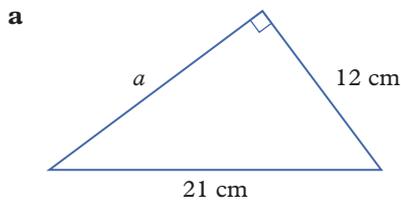
- a How high is the top of the ladder above the ground?
 b How long is each side of the ladder?
- 11 The Great Hall of Parliament House in Canberra displays a tapestry based on a painting by the Australian artist Arthur Boyd. The tapestry measures 20 m wide and is said to be one of the largest tapestries in the world. The tapestry hangs from the ceiling, and reaches all the way to the floor, with the angle between the diagonal and the floor being 24.2° .
- a Draw a diagram to display this information.
 b How high is the ceiling in the Great Hall, correct to the nearest metre?
 c What is the perimeter of the tapestry, correct to the nearest metre?

- 12 A ramp for wheelchair access to a building is to be built at an angle of 3° to the horizontal. The front door of the building is 45 cm above ground level. How long should the ramp be? Give your answer in metres, correct to two decimal places.

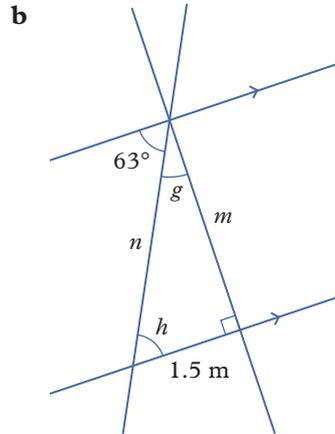
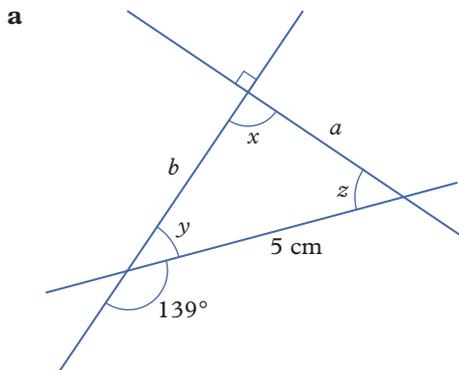


- 13 a What is the minimum amount of information needed to find an unknown side length in a right-angled triangle by using trigonometry?
 b What is the minimum amount of information needed to find an unknown side length in a right-angled triangle by using Pythagoras' theorem?

14 For each triangle, find the length of the unknown side correct to one decimal place.

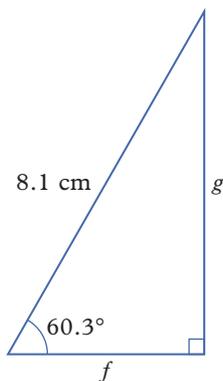


15 For each diagram, determine the values of the pronumerals correct to two decimal places.

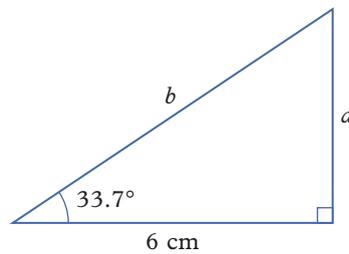


16 Use the specified ratio, geometry and Pythagoras' theorem to determine the value of the pronumerals in these diagrams correct to two decimal places.

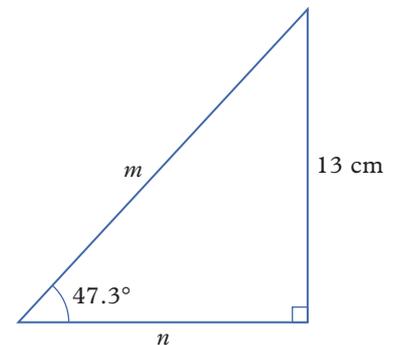
a sine



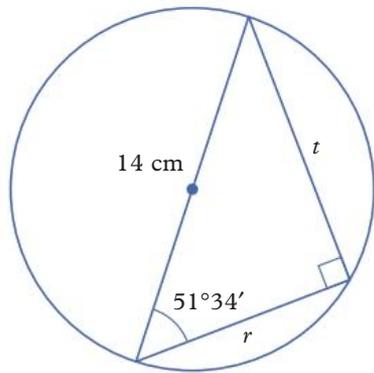
b tangent



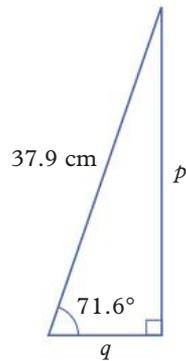
c sine



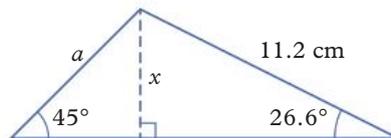
17 In the following diagram, determine the lengths r and t correct to two decimal places.



18 Use only the tangent ratio and Pythagoras' theorem to determine the value of the pronumerals in the following diagram correct to two decimal places. Hint: Start by writing p in terms of q .



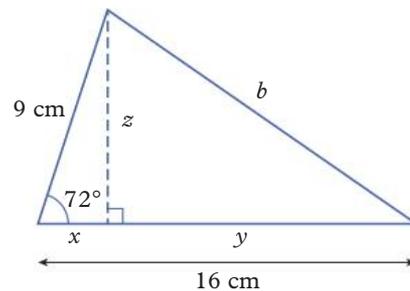
19 We can find lengths in non-right-angled triangles by constructing right-angled triangles made up of known lengths. Consider the non-right-angled triangle shown with a perpendicular height added as a dotted line.



- Calculate the length of x correct to two decimal places.
- Using your answer to part **a**, calculate the length of a correct to two decimal places.

20 Consider the non-right-angled triangle shown on the right.

- Calculate the length of x correct to two decimal places.
- Calculate the length of y correct to two decimal places.
- Calculate the length of z correct to two decimal places.
- Hence, calculate the length of b .



CHALLENGE

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Interactive skillsheet
Finding lengths by trigonometry



Investigation
Using trigonometry to find an approximate value for π



Topic quiz
71

pro

7J Using trigonometry to find angles

Learning intentions

By the end of this topic you will be able to ...

- ✓ find an unknown angle in a right-angled triangle.



Inter-year links

Year 7

6H Solving equations using inverse operations

Year 8

6B Solving linear equations

Year 10

9C Applications of trigonometry

Using trigonometry to find an angle

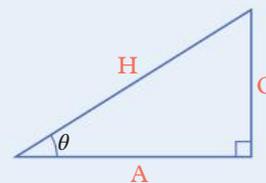


Key content video

- Trigonometric ratios can be used to find an unknown angle in a right-angled triangle if two side lengths are known.
- A calculator can be used to obtain the angle from its sine, cosine or tangent value. This is known as finding the **inverse sine** (\sin^{-1}), **inverse cosine** (\cos^{-1}) or **inverse tangent** (\tan^{-1}) of a value.
- To find an angle:
 - 1 Identify the given sides with respect to the angle. Label the sides accordingly.
 - 2 Decide which trigonometric ratio to use.
 - 3 Substitute in the known side lengths.
 - 4 Rearrange to make θ the subject of the equation using the inverse trigonometric ratio.
 - 5 Use the calculator to find the result.

SOH CAH TOA

$$\sin(\theta) = \frac{O}{H} \quad \cos(\theta) = \frac{A}{H} \quad \tan(\theta) = \frac{O}{A}$$



Example 7J.1 Solving trigonometric equations to find the value of an angle



Solve each equation to find the value of θ , correct to the nearest degree.

a $\tan(\theta) = 2.4$

b $\sin(\theta) = \frac{22}{35}$

THINK

- a**
- 1 Rearrange to make θ the subject of the equation using the inverse tangent.
 - 2 Use the \tan^{-1} key on the calculator to find the inverse tangent of 2.4. Round the value of θ to the nearest degree.
- b**
- 1 Rearrange to make θ the subject of the equation using the inverse sine.
 - 2 Use the \sin^{-1} key on the calculator to find the inverse sine of $\frac{22}{35}$. Round the value of θ to the nearest degree.

WRITE

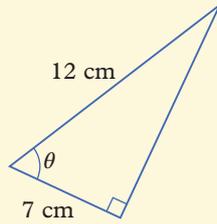
$$\begin{aligned} \tan(\theta) &= 2.4 \\ \theta &= \tan^{-1}(2.4) \\ &= 67.3801\dots \\ &\approx 67^\circ \end{aligned}$$

$$\begin{aligned} \sin(\theta) &= \frac{22}{35} \\ \theta &= \sin^{-1}\left(\frac{22}{35}\right) \\ &= 38.9448\dots \\ &\approx 39^\circ \end{aligned}$$

Example 7J.2 Using trigonometry to find an unknown angle



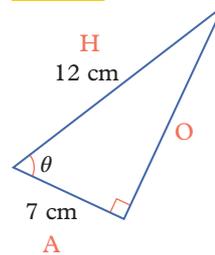
Use trigonometry to find angle θ in this triangle, correct to the nearest degree.



THINK

- 1 Identify the given sides with respect to the angle. The side length adjacent to the angle is 7 cm. The hypotenuse is 12 cm. Label the sides accordingly.
- 2 Decide which trigonometric ratio to use. As A and H are involved, use cosine.
- 3 Substitute for A and H.
- 4 Rearrange to make θ the subject of the equation using the inverse cosine.
- 5 Use the \cos^{-1} key on the calculator to find the inverse cosine of $\frac{7}{12}$. Round the value of θ to the nearest degree.

WRITE



$$A = 7 \text{ cm}, H = 12 \text{ cm}$$

$$\cos(\theta) = \frac{A}{H}$$

$$\cos(\theta) = \frac{7 \text{ cm}}{12 \text{ cm}}$$

$$\theta = \cos^{-1}\left(\frac{7}{12}\right)$$

$$= 54.3146\dots$$
$$\approx 54^\circ$$

Helpful hints

- ✓ While a superscript of -1 , like 2^{-1} , usually indicates 2 to the power of -1 (the reciprocal), \sin^{-1} does *not* mean sine to the power of -1 (the reciprocal of the value of sine); instead it is the inverse of sine.
- ✓ It can be easy to choose the wrong trigonometric function when determining the angle. Make sure you take your time and label the sides of the triangles. This can help to minimise errors.
- ✓ If you get the ratio upside-down for sine and cosine when determining the angle, your calculator will display an error as no real angle has a sine or cosine ratio greater than 1. However, if you get the ratio upside-down for tangent, your calculator will not show an error.
- ✓ To convert a value expressed in decimal degrees to degrees and minutes, multiply the decimal part by 60 and add the whole number part.

Exercise 7J Using trigonometry to find angles

 1-3(a, c, e, f), 4-6, 8, 10, 12

 2-3(b, d, f, h), 4-7, 11-13, 17

 2-3(c, e, f), 4(b, d, f), 5, 7, 9, 13-17

UNDERSTANDING AND FLUENCY

7J.1 1 Calculate each of the following, correct to the nearest degree.

a $\sin^{-1}(0.23)$

b $\cos^{-1}(0.72)$

c $\tan^{-1}(1.46)$

d $\sin^{-1}(0.95)$

e $\sin^{-1}\left(\frac{2}{3}\right)$

f $\cos^{-1}\left(\frac{24}{25}\right)$

g $\tan^{-1}\left(\frac{39}{11}\right)$

h $\cos^{-1}\left(\frac{11}{17}\right)$

2 Find each unknown angle θ , correct to the nearest degree.

a $\sin(\theta) = 0.34$

b $\cos(\theta) = 0.81$

c $\tan(\theta) = 0.65$

d $\tan(\theta) = 0.47$

e $\cos(\theta) = 0.99$

f $\sin(\theta) = 0.01$

g $\tan(\theta) = 3.14$

h $\sin(\theta) = 0.71$

3 Solve each equation to find the value of θ , correct to the nearest degree.

a $\sin(\theta) = \frac{4}{5}$

b $\cos(\theta) = \frac{15}{19}$

c $\tan(\theta) = \frac{8}{21}$

d $\sin(\theta) = \frac{23}{26}$

e $\cos(\theta) = \frac{11}{31}$

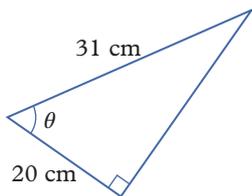
f $\sin(\theta) = \frac{9}{23}$

g $\cos(\theta) = \frac{17}{31}$

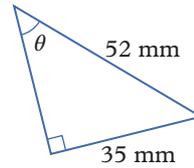
h $\tan(\theta) = \frac{26}{3}$

7J.2 4 Use trigonometry to find angle θ in each triangle, correct to the nearest degree.

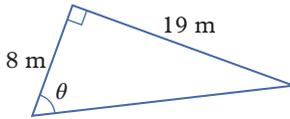
a



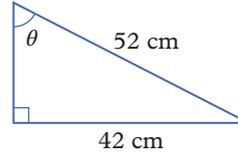
b



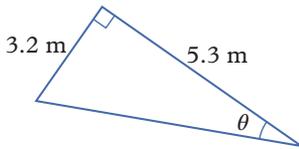
c



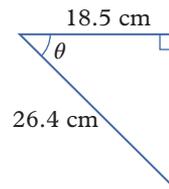
d



e

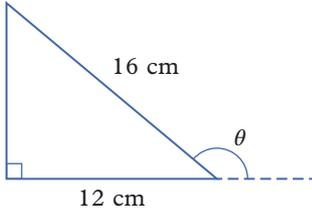


f

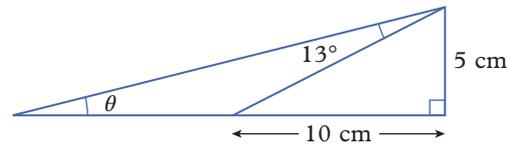


5 Determine the value of θ in each of the following diagrams, correct to one decimal place.

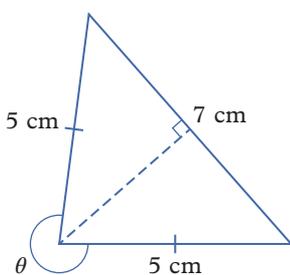
a



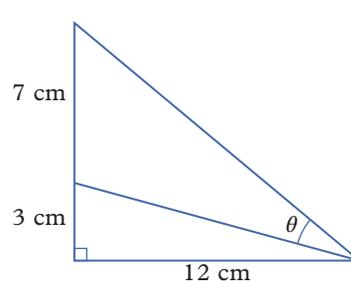
b



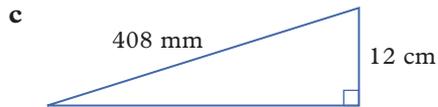
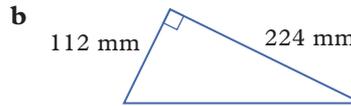
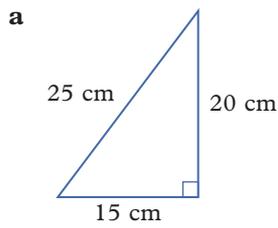
c



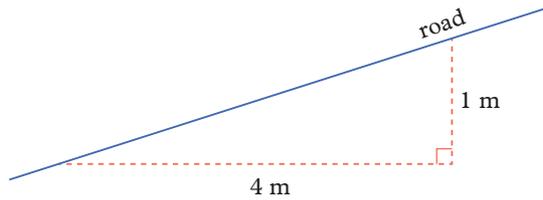
d



- 6 Calculate the size of the non-right angles in the following right-angled triangles. Give your answers correct to the nearest minute. Hint: To convert a value expressed in decimal degrees to degrees and minutes, multiply the decimal part by 60 and add the whole number part.



- 7 A road has a gradient of 1 in 4; that is, it rises 1 m vertically for every 4 m horizontally.



- a** What angle does the road surface make with the horizontal? Give your answer correct to the nearest degree.
b How far have you travelled along the road if you are now 1.7 m higher than when you started? Give your answer correct to two decimal places.
- 8 The roof of a holiday house has the dimensions shown.

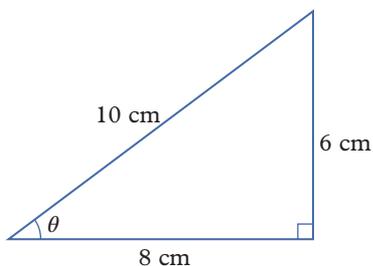


Calculate, correct to the nearest degree:

- a** the angle the roof makes with the horizontal
b the angle formed where the two sections of roof meet.
- 9 An anchor holding a boat in position lies on the seabed at a depth of 7.5 m. It is attached to the boat by a chain that is 8.4 m long.
- a** Draw a diagram of this scenario.
b What angle does the chain make with the vertical? Give your answer correct to the nearest degree.
c If the chain was longer, would this angle be larger or smaller? Explain your answer.



10 Consider the right-angled triangle shown below.



- a** Calculate the value of θ to the nearest degree using a calculator and:
- i** the sine ratio
 - ii** the cosine ratio
 - iii** the tangent ratio.
- b** Draw your own scale diagram of the triangle and measure angle θ with a protractor.
- c** Compare your answers to parts **a** and **b**. Comment on the advantages and disadvantages of using:
- i** trigonometry
 - ii** measurements from a scale diagram.
- 11 Nadia and Alex are set a problem-solving task by their teacher. Their challenge is to work out the height of a tree in their school yard without climbing it or using a ladder. The only allowable equipment is a 1 m ruler, a tape measure and a calculator.

Nadia and Alex discuss some possible ideas, take some measurements simultaneously and draw these diagrams.

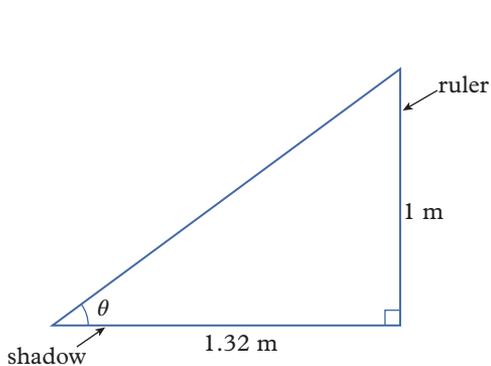


Diagram 1

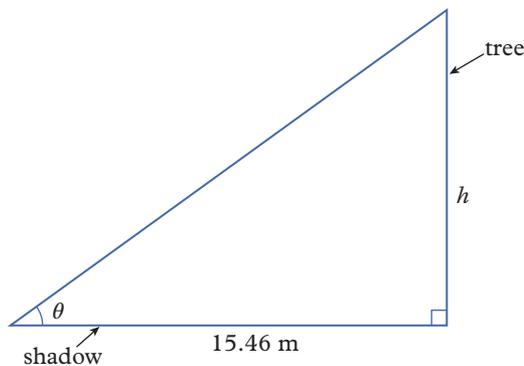
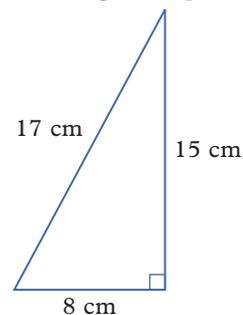


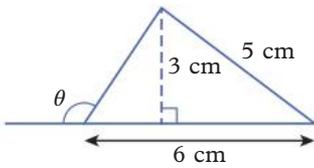
Diagram 2

- a** What do you think their strategy is?
 - b** Use trigonometry to calculate the value of θ to the nearest degree in Diagram 1.
 - c** Explain why this value of θ can be used in Diagram 2.
 - d** Use trigonometry to calculate the height of the tree correct to two decimal places.
 - e** Instead of using trigonometry, Nadia and Alex could have used their knowledge of similar triangles. Explain how this strategy could be used to calculate the height of the tree.
- 12 Consider the right-angled triangle shown on the right.
- a** Correct to the nearest degree, calculate the size of the non-right angles using a calculator and:
 - i** the sine ratio
 - ii** the cosine ratio
 - iii** the tangent ratio.
 - b** After the size of one non-right angle has been determined, how else could the size of the other non-right angle be determined?

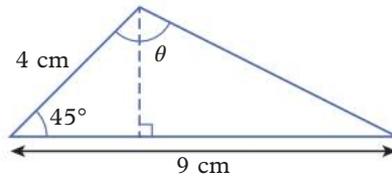


13 Determine the value of θ in each of the following diagrams, correct to the nearest minute.

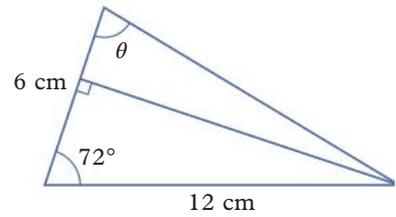
a



b

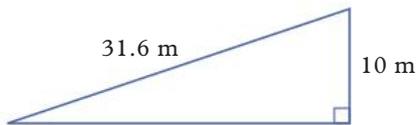


c

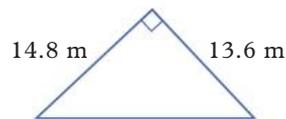


14 Calculate the size of the non-right angles and the missing side length in the following right-angled triangles using the sine ratio and Pythagoras' theorem, correct to one decimal place.

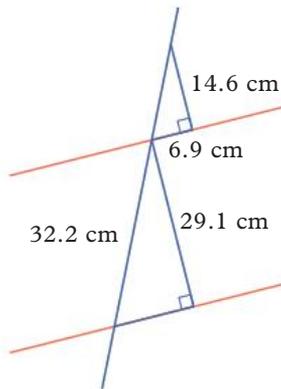
a



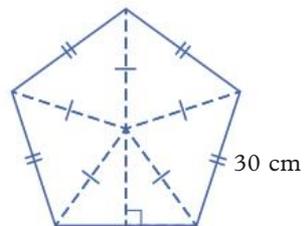
b



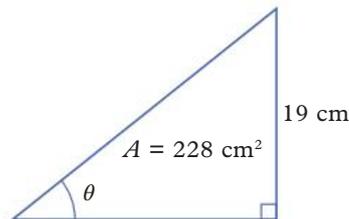
15 Determine if the two red lines below are parallel. Explain why or why not.



16 Calculate the area of the following regular pentagon, correct to one decimal place.



17 The area of the triangle shown below is 228 cm^2 . Determine the size of the angle θ correct to one decimal place.



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Interactive skillsheet

Finding angles by trigonometry



Worksheet

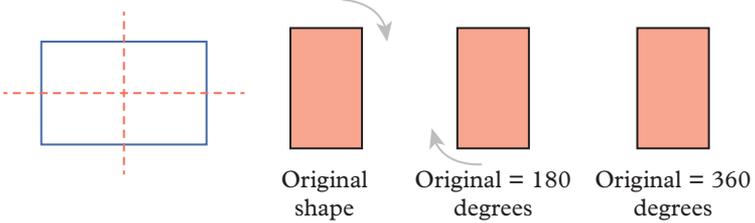
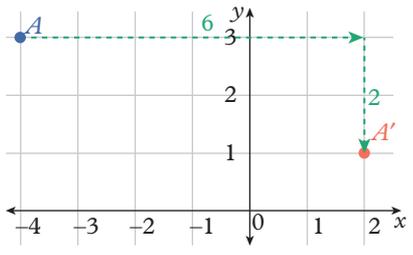
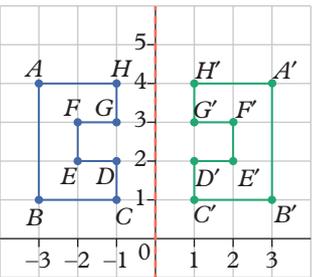
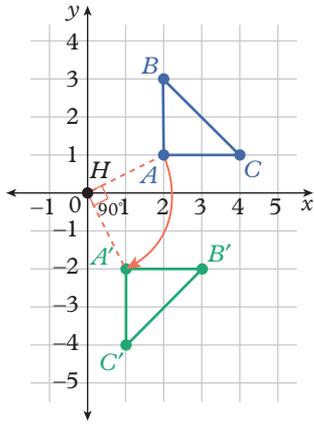
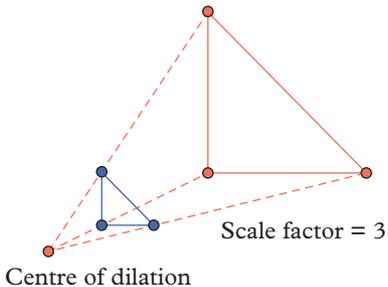
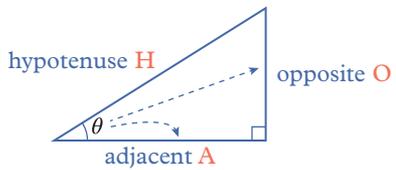
Using trigonometry



Topic quiz

7J

Chapter summary

<p>Line and rotational symmetry</p>  <p>Original shape Original = 180 degrees Original = 360 degrees</p> <ul style="list-style-type: none"> • Order of line symmetry = 2 • Order of rotational symmetry = 2 	<p>Translations</p> 	
<p>Reflections</p> 	<p>Rotations</p> 	<p>Dilations</p>  <p>Scale factor = 3 Centre of dilation</p> <ul style="list-style-type: none"> • Creates similar figures • Scale factor between 0 and 1: reduction • Scale factor greater than 1: enlargement
<p>Similar figures</p> <ul style="list-style-type: none"> • Side lengths are in proportion • Corresponding angles are equal 	<p>Congruent figures</p> <ul style="list-style-type: none"> • Same shape and size. Isometric translations produce congruent figures. • A tessellation is a repeating pattern of shapes with no overlaps or gaps. 	
<p>Scale drawings</p> <ul style="list-style-type: none"> • Scale is 'drawing length : actual length' • 1 : 100 means 1 cm represents 100 cm 	<p>Congruent figures</p> <ul style="list-style-type: none"> • Same shape and size. Isometric translations produce congruent figures. • A tessellation is a repeating pattern of shapes with no overlaps or gaps. 	
<p>Scale factors</p> <p>Length scale factor = $\frac{\text{image length}}{\text{original length}}$ Area scale factor = $\left(\frac{\text{image length}}{\text{original length}}\right)^2$ Volume scale factor = $\left(\frac{\text{image length}}{\text{original length}}\right)^3$</p>		
<p>Congruent triangles</p> <ul style="list-style-type: none"> • SSS (side–side–side): all side lengths equal • SAS (side–angle–side): two side lengths and included angles equal • AAS (angle–angle–side): two angles and one side length equal • RHS (right angle–hypotenuse–side): hypotenuse and another side length of a right-angled triangle equal 	<p>Similar triangles</p> <ul style="list-style-type: none"> • SSS (side–side–side): three side lengths in same ratio • SAS (side–angle–side): two side lengths in same ratio and included angles equal • AAA (angle–angle–angle): all three angles equal • RHS (right angle–hypotenuse–side): hypotenuses in the same ratio as another set of sides in a right-angled triangle 	
<p>Trigonometry</p>  <p>$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$ $\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$</p>	<p>To find a side length:</p> <ol style="list-style-type: none"> 1 Identify given sides with respect to the angle. 2 Decide which trigonometric ratio to use. 3 Substitute in known side length and angle. 4 Solve for the unknown. <p>To find an angle:</p> <ol style="list-style-type: none"> 1 Identify given sides with respect to the angle. 2 Decide which trigonometric ratio to use. 3 Substitute in known side lengths. 4 Rearrange to make the subject of the equation using the inverse trigonometric ratio. 5 Solve for the unknown. 	

Chapter review



Chapter review quiz

Assess your knowledge of this chapter.

- ✓ Core
- ✓ Advanced
- ✓ Extension

Quizlet

Test your knowledge of this topic by working individually or in teams.

Mathematical literacy review

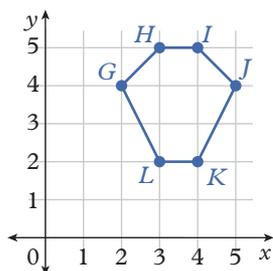
The following key terms are used in this chapter:

- adjacent side
- area scale factor
- axis of symmetry
- centre of dilation
- congruent
- cosine
- degrees
- dilation
- enlargement
- hypotenuse
- image
- included angle
- isometric transformation
- line symmetry
- minutes
- opposite angle
- order of line symmetry
- order of rotational symmetry
- point of focus
- reduction
- reflection
- rotation
- rotational symmetry
- scale drawing
- scale factor
- similar
- sine
- tangent
- tessellation
- transformation
- translation
- trigonometric ratios
- trigonometry
- volume scale factor

- 1 Explain the difference between a pair of shapes that are congruent and a pair of shapes that are similar.
- 2 How can you determine the volume scale factor if you know the length scale factor?
- 3 How many minutes are there in one degree?
- 4 Identify the key terms being referenced in each of these definitions.
 - a the longest side in a right-angled triangle, appearing opposite the right angle
 - b a smaller image produced after a figure has been dilated
 - c a pattern of repeating shapes that has no overlaps or gaps
- 5 Using an example, provide a definition in your own words for the following key terms.
 - a isometric transformation
 - b dilation
 - c order of rotational symmetry
- 6 Complete the following sentences using words from the key term list.
 - a To reflect a shape in an _____, start by reflecting a single point. This point is called the _____.
 - b The cosine of angle θ is the ratio of the lengths of the _____ to the _____.

Multiple choice

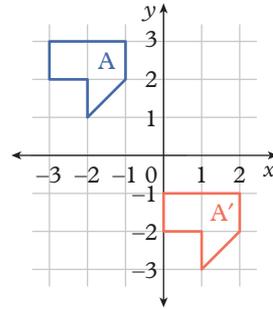
- ADV 7A 1 What will be the coordinates of point J' if this shape is reflected across the y -axis?



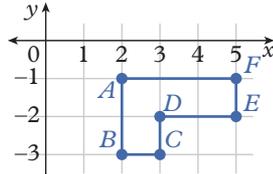
- A $(-5, 4)$ B $(4, -5)$ C $(5, -4)$ D $(-5, -4)$ E $(-4, 5)$

ADV 7B 2 What is the translation shown in this diagram?

- A** 5 units right and 4 units down
- B** 6 units right and 5 units down
- C** 5 units right and 6 units down
- D** 4 units right and 3 units down
- E** 3 units right and 4 units down



ADV 7B 3 What will be the coordinates of point A' if the following shape is rotated 90° clockwise about the origin?

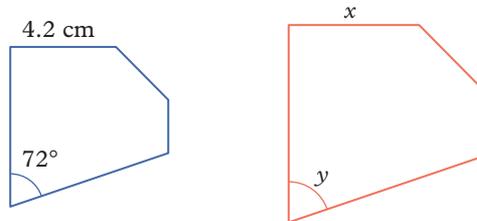


- A** (1, 2) **B** (-1, 2) **C** (-1, -2) **D** (1, -2) **E** (-2, -1)

7C 4 If a scale drawing is made with a scale of 1 : 500, how much does 1.5 cm in the drawing represent in real life in metres?

- A** 5 m **B** 7.5 m **C** 75 m **D** 500 m **E** 750 m

7C 5 The shape below on the left has been dilated by a scale factor of 2 to create the image on the right. What are the values of x and y ?

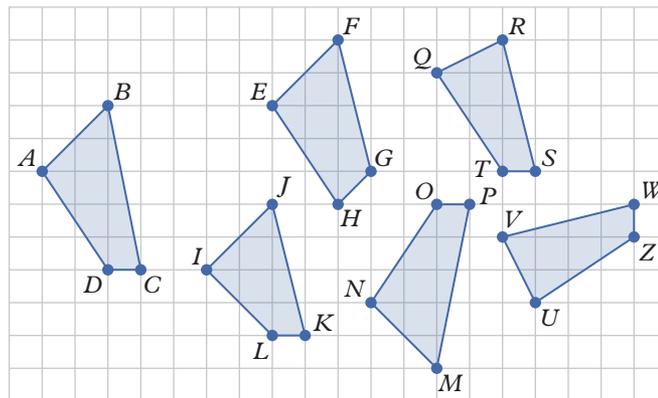


- A** $x = 2.1$ cm, $y = 72^\circ$ **B** $x = 2.1$ cm, $y = 144^\circ$ **C** $x = 4.2$ cm, $y = 144^\circ$
- D** $x = 8.4$ cm, $y = 72^\circ$ **E** $x = 8.4$ cm, $y = 144^\circ$

EXT 7D 6 A figure is dilated by a scale factor of $\frac{1}{2}$. What is its area scale factor?

- A** $\frac{1}{8}$ **B** $\frac{1}{4}$ **C** $\frac{1}{2}$ **D** 2 **E** 4

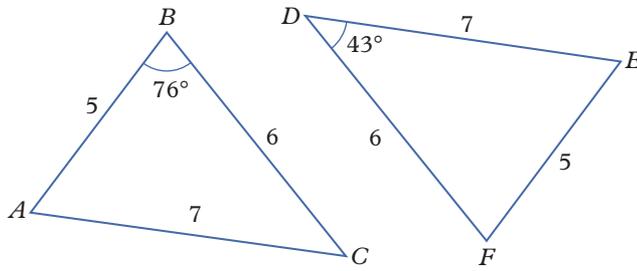
EXT 7E 7 Which of the following shapes is congruent to $ABCD$?



- A** $EFGH$ **B** $IJKL$ **C** $NMPO$ **D** $QRST$ **E** $UVWZ$

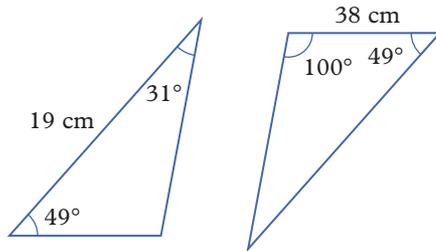
EXT 7F 8 Using the triangles shown below, determine the size of $\angle BAC$.

- A 76°
- B 43°
- C 14°
- D 61°
- E 47°



EXT 7G 9 Which test would you use to decide whether the two triangles below are similar?

- A AAA
- B SSS
- C SAS
- D RHS
- E AAS

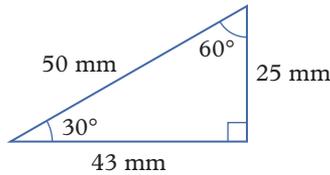


7H 10 Sine is the trigonometric ratio of which side to which side of a right-angled triangle?

- A hypotenuse to opposite side
- B opposite side to adjacent side
- C adjacent side to hypotenuse
- D opposite side to hypotenuse
- E hypotenuse to adjacent side

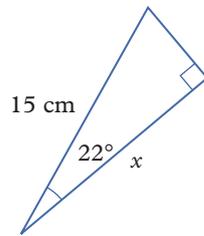
7H 11 Using the lengths on the triangle shown below, the cosine of 30° is:

- A $\frac{43}{50}$
- B $\frac{25}{50}$
- C $\frac{25}{43}$
- D $\frac{50}{43}$
- E $\frac{50}{25}$



7I 12 Which equation can be used to find x in the following triangle?

- A $\cos(22^\circ) = \frac{x}{15}$
- B $\tan(22^\circ) = \frac{x}{15}$
- C $\sin(22^\circ) = \frac{x}{15}$
- D $\cos(22^\circ) = \frac{15}{x}$
- E $\sin(22^\circ) = \frac{15}{x}$

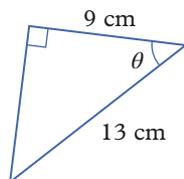


7J 13 If $\sin(\theta) = \frac{12}{15}$, then θ is equal to:

- A $\sin\left(\frac{15}{12}\right)$
- B $\sin^{-1}\left(\frac{12}{15}\right)$
- C $\sin^{-1}\left(\frac{15}{12}\right)$
- D $\sin\left(\frac{12}{15}\right)^{-1}$
- E $\frac{\sin^{-1}(12)}{15}$

7J 14 The angle θ in the following triangle is closest to:

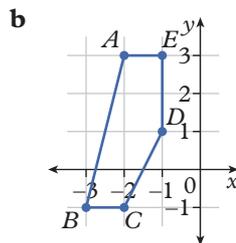
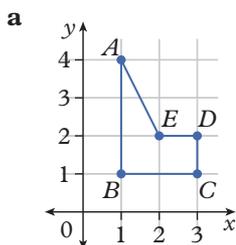
- A 35°
- B 44°
- C 46°
- D 55°
- E 0.81°



Short answer



ADV 7A 1 Reflect the following images in the y -axis.



ADV 7B 2 Describe the translation required to move each first point to the second point.

a $A(7, 3)$ to $A'(6, 1)$

b $B(1, -2)$ to $B'(1, 5)$

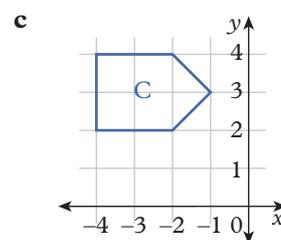
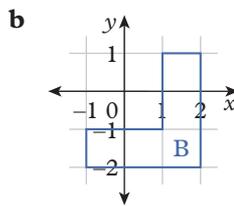
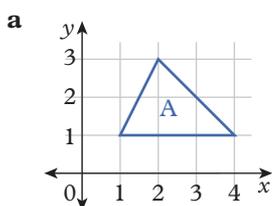
c $C(-3, -2)$ to $C'(2, -1)$

d $D(0, -1)$ to $D'(4, 5)$

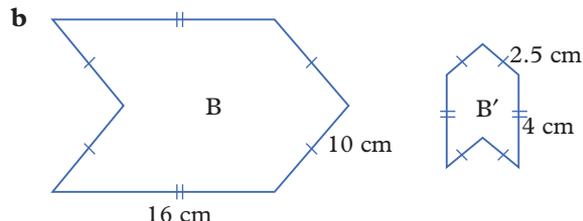
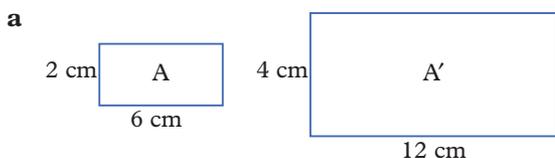
e $E(3, 1)$ to $E'(-2, 4)$

f $F(8, -4)$ to $F'(-1, -5)$

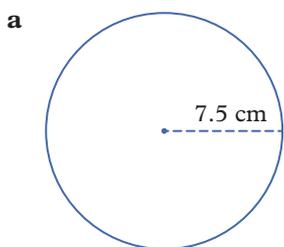
ADV 7B 3 Rotate each of the following shapes 90° clockwise about the origin.



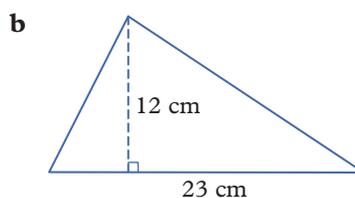
7C 4 Describe each transformation shown below as a reduction or an enlargement and state the scale factor.



EXT 7D 5 For each of the following shapes, state the area of the image if the original figure is dilated by the given scale factor. If necessary, give your answers to two decimals.

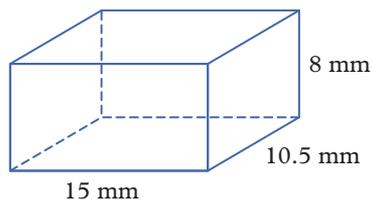


Length scale factor = 3



Length scale factor = $\frac{1}{3}$

EXT 7D 6 Consider the following foam block.

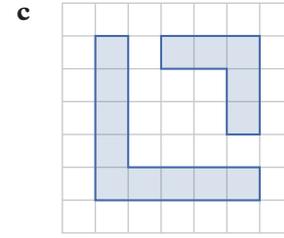
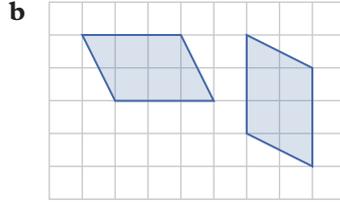
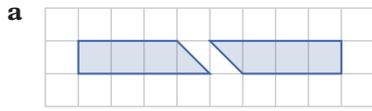


a What is the volume of the foam block?

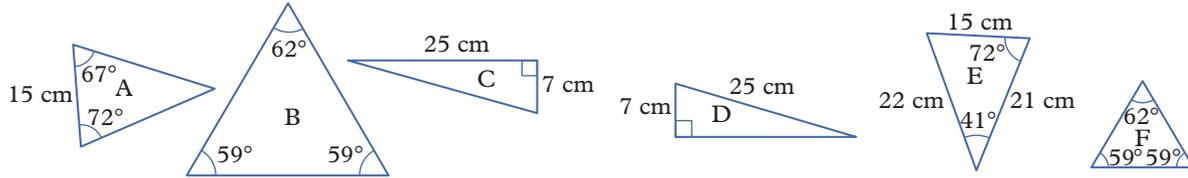
b If the foam block was dilated by a scale factor of 3, what are the dimensions of the dilated foam block?

c What is the volume of the dilated foam block?

EXT 7E 7 Decide whether each of the following pairs of shapes are congruent.

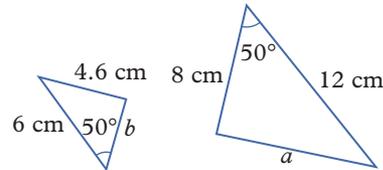


EXT 7F 8 Identify any pairs of congruent triangles from the following selection.

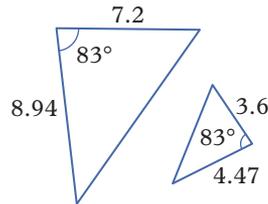


EXT 7F 9 Explain your answers to question 8 using one of the conditions for congruence (SSS, SAS, AAS, RHS).

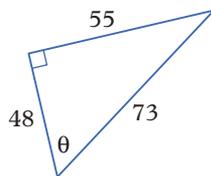
EXT 7G 10 Given that the following two triangles are similar, find the values of the unknown side lengths.



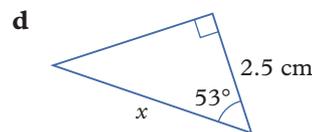
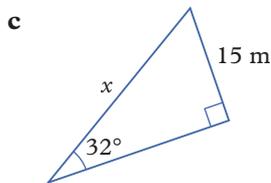
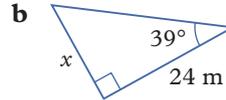
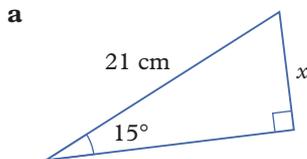
EXT 7G 11 Determine whether the following two triangles are similar. If similar, state the similarity condition you used, or explain why they are not similar.



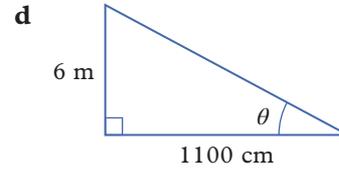
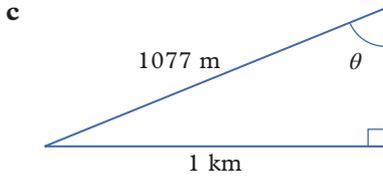
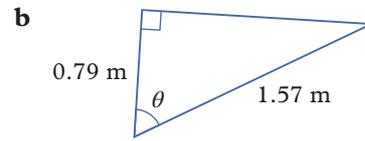
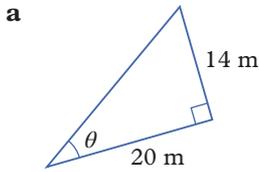
7H 12 Write equations for the sine, cosine and tangent of θ for the following triangle.



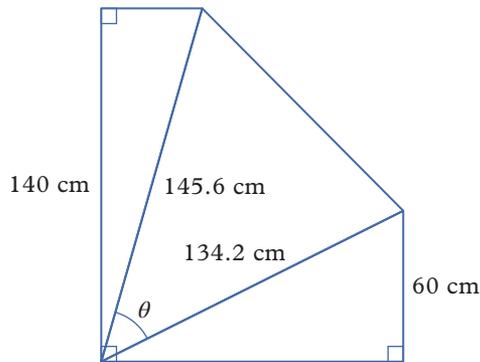
7I 13 Find the side length x in each of the following triangles, correct to two decimal places.



7J 14 Find the angle θ in each of the following triangles, correct to the nearest degree.



7J 15 Calculate the size of the angle θ in the following diagram, correct to one decimal place.



Analysis

ADV 1 a Transform each of the shapes on the right as instructed.

EXT

A: reflect in the y -axis

B: translate 7 units right and 5 units up

C: rotate 90° anticlockwise about the origin

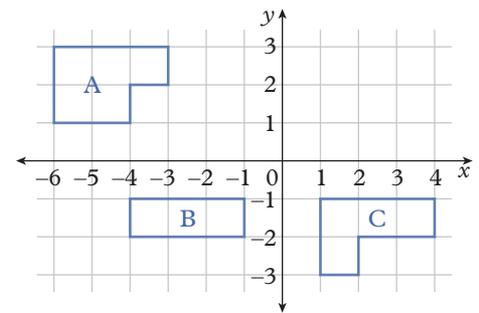
b Together the images from the transformations form a polygon.

State the vertices of the polygon that has been formed.

c The polygon is dilated by a scale factor of 2 using the origin as the centre of dilation. What are the vertices of the image?

d If the length scale factor of the dilation is 2, what is the area scale factor?

e The area of the original polygon is 12 square units. What is the area of the image?

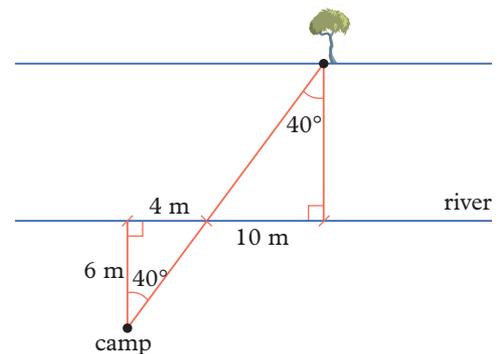


EXT 2 While preparing to study a species of gum tree on a field trip, Stephanie set up a camp 6 m from a river's edge. From the camp, she measures the angle to a gum tree she wishes to get to and records various measurements, as shown in the diagram.

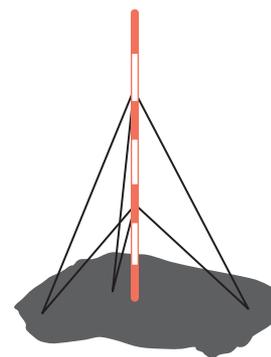
a Are the two triangles congruent or only similar? Provide a reason to support your answer.

b What is the scale factor linking the triangle attached to the camp site to the triangle attached to the gum tree?

c What is the width of the river?



- 3 A 150 m tall AM radio tower is supported by six guy wires equally spaced around the tower connecting to the ground. Three of the wires are connected one-third of the way up the tower, and the other three are connected one-third of the way down from the top of the tower.
- The longer wires are 125 metres in length. Which trigonometric ratio can be used to calculate the angle the longer wires make with the ground?
 - What angle do the longer wires make with the ground, correct to one decimal place?
 - The shorter wires make an angle of 33.7° with the ground. What is the length of the shorter wires correct to the nearest metre?
 - Use your answer to part **b** to calculate the total amount of wire used.



Chapter checklist

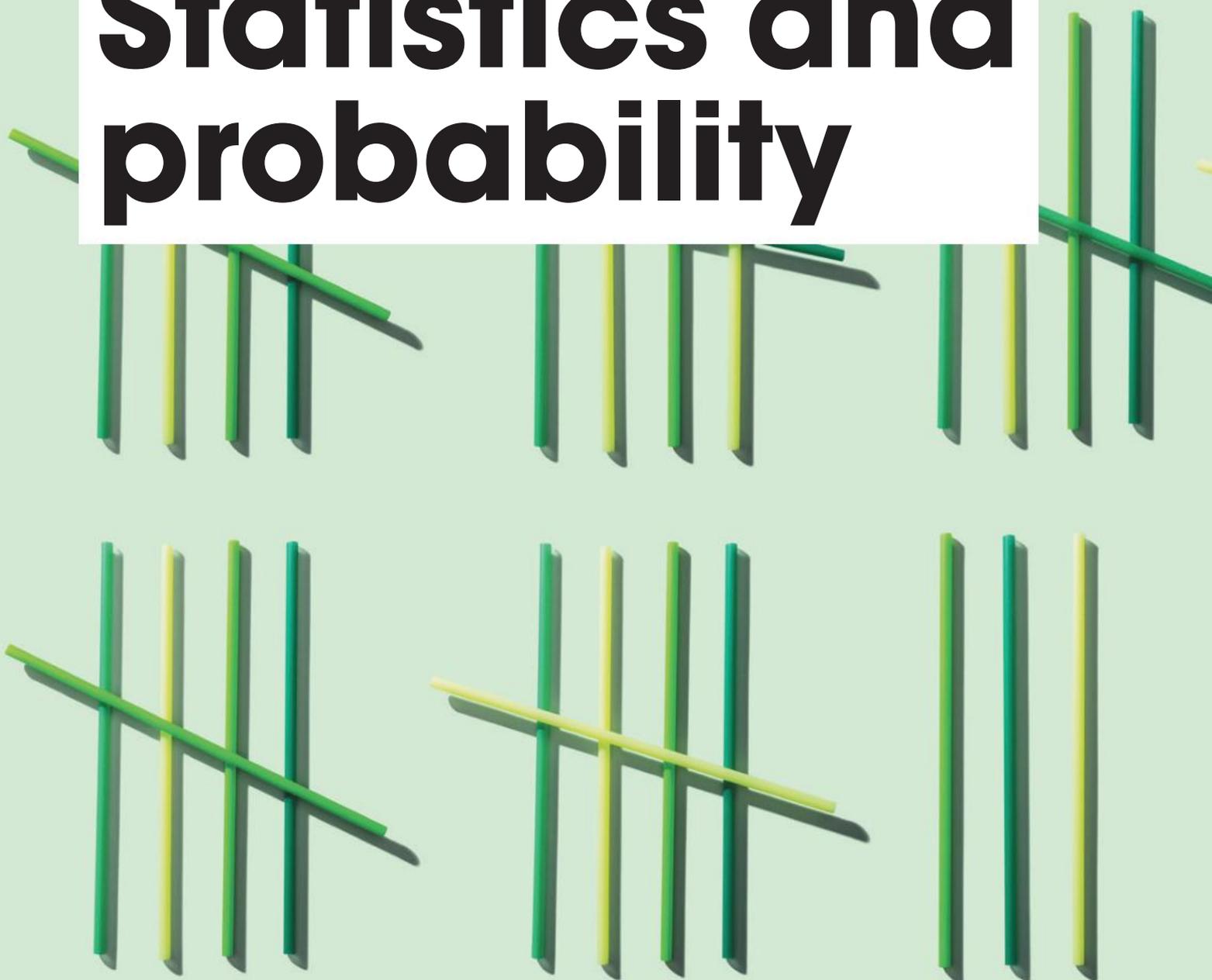


Now that you have completed this chapter, reflect on your ability to do the following.

I can do this	I need to review this
<input type="checkbox"/> Identify line and rotational symmetry in plane shapes <input type="checkbox"/> Identify line and rotational symmetry in graphs <input type="checkbox"/> Reflect points and objects in axes of symmetry	<input type="checkbox"/> Go back to Topic 7A Symmetry and reflections
<input type="checkbox"/> Perform translations on a Cartesian plane <input type="checkbox"/> Perform rotations on a Cartesian plane <input type="checkbox"/> Identify coordinates after a point or shape has been transformed	<input type="checkbox"/> Go back to Topic 7B Translations and rotations
<input type="checkbox"/> Determine scale factors <input type="checkbox"/> Find unknown lengths in similar figures <input type="checkbox"/> Solve problems involving scale drawings	<input type="checkbox"/> Go back to Topic 7C Dilations and similar figures
<input type="checkbox"/> Calculate area and volume scale factors <input type="checkbox"/> Use scale factors to find measurements of original and image objects	<input type="checkbox"/> Go back to Topic 7D Area and volume scale factors
<input type="checkbox"/> Determine if shapes are congruent <input type="checkbox"/> Identify congruent shapes <input type="checkbox"/> Match vertices and sides of congruent pairs	<input type="checkbox"/> Go back to Topic 7E Congruence
<input type="checkbox"/> Use congruence conditions to determine if two triangles are congruent	<input type="checkbox"/> Go back to Topic 7F Congruent triangles
<input type="checkbox"/> Find unknown angles and side lengths in similar triangles <input type="checkbox"/> Determine whether two triangles are similar	<input type="checkbox"/> Go back to Topic 7G Similar triangles
<input type="checkbox"/> Identify the hypotenuse, opposite and adjacent sides of a right-angled triangle containing a reference angle <input type="checkbox"/> Calculate sine, cosine and tangent ratios of an angle in a right-angled triangle	<input type="checkbox"/> Go back to Topic 7H Trigonometric ratios
<input type="checkbox"/> Use trigonometric ratios to find unknown side lengths in a right-angled triangle	<input type="checkbox"/> Go back to Topic 7I Using trigonometry to find side lengths
<input type="checkbox"/> Find an unknown angle in a right-angled triangle	<input type="checkbox"/> Go back to Topic 7J Using trigonometry to find angles

8

Statistics and probability



Index

- 8A** Five-number summary and interquartile range
- 8B** Box plots
- 8C** Distributions of data
- 8D** The mean and standard deviation
- 8E** Two-step chance experiments
- 8F** Experiments with replacement
- 8G** Experiments without replacement
- 8H** Experimental probability and simulations

Prerequisite skills



Diagnostic pre-test

Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.



Interactive skillsheets

After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✓ Stem-and-leaf plots
- ✓ Dot plots
- ✓ The median
- ✓ The mean
- ✓ Calculating theoretical probability

Curriculum links

- Compares and analyses datasets using summary statistics and graphical representations (MA5-DAT-C-01)
 - Examine standard deviation as a measure of spread
 - Determine quartiles and interquartile range
 - Represent datasets using box plots and use them to compare datasets
- Solves problems involving probabilities in multistage chance experiments and simulations (MA5-PRO-C-01)
 - Describe multistage chance experiments involving independent and dependent events
 - Solve problems for multistage chance experiments
 - Design and use simulations to model and examine situations involving probability

8A Five-number summary and interquartile range

Learning intentions

By the end of this topic you will be able to ...

- ✓ find the five-number summary and interquartile range for numerical data sets
- ✓ find the five-number summary and interquartile range from graphical representations.



Inter-year links

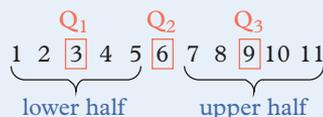
Support	Understanding data
Year 7	9B Summary statistics
Year 8	9A Summary statistics

Measures of data distribution

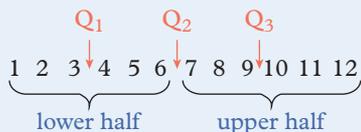


Key content video

- **Quartiles** divide a numerical data set into four quarters.
- The **median** is the middle value of an ordered numerical data set. The median divides a data set into two halves.
→ The median is also known as the second quartile, Q_2 .
- The **lower quartile** (Q_1) is the median of the lower half of an ordered numerical data set.
- The **upper quartile** (Q_3) is the median of the upper half of an ordered numerical data set.
- When calculating the quartiles for a data set with an odd number of data points, the median is not included in either half of the data set.



- If there is an even number of data points, both middle values are included in the quartile calculations.



$$Q_1 = \frac{(3 + 4)}{2}$$

$$= 3.5$$

$$Q_2 = \frac{(6 + 7)}{2}$$

$$= 6.5$$

$$Q_3 = \frac{(9 + 10)}{2}$$

$$= 9.5$$

- The **five-number summary** for a numerical data set consists of the minimum and maximum values, the lower and upper quartiles, and the median.
→ The five values that make up the five-number summary are displayed in ascending order:
minimum, Q_1 , Q_2 (median), Q_3 , maximum
- The **range** of a data set can be found by subtracting the smallest value (minimum) from the largest value (maximum).
- The **interquartile range (IQR)** represents the range of the middle 50% of a data set. It can be found by subtracting the lower quartile from the upper quartile.

$$IQR = Q_3 - Q_1$$

- The range and interquartile range are both **measures of spread**.

Example 8A.1 Finding the five-number summary and interquartile range for a numerical data set



Find the **a** five-number summary and **b** interquartile range for this ordered data set: 2, 2, 3, 4, 5, 6, 6, 6, 8.

THINK

- a**
- The data set is already ordered, so find the median by either circling the middle number or finding the average of the two middle numbers.
 - Divide the data set into two equal halves, remembering not to include the median in either half.
 - Find the lower quartile (Q_1) and the upper quartile (Q_3) by finding the median of each half of the set.
 - Identify the minimum and maximum scores from the data set.
 - Write your answer.
- b** Subtract Q_1 from Q_3 to obtain the IQR.

WRITE

a $\boxed{2 \ 2 \ 3 \ 4 \ 5 \ 6 \ 6 \ 6 \ 8}$
 $Q_2 = 5$

$\boxed{2 \ 2 \ 3 \ 4} \ \boxed{5} \ \boxed{6 \ 6 \ 6 \ 8}$
 $Q_1 = 2.5 \quad Q_3 = 6$

minimum = 2, maximum = 8

The five-number summary is 2, 2.5, 5, 6, 8.

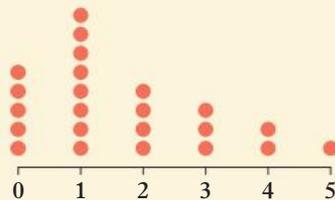
b $IQR = Q_3 - Q_1 = 6 - 2.5 = 3.5$

Example 8A.2 Finding the five-number summary and interquartile range from a graphical representation



Find the **a** five-number summary and **b** interquartile range from the following dot plot.

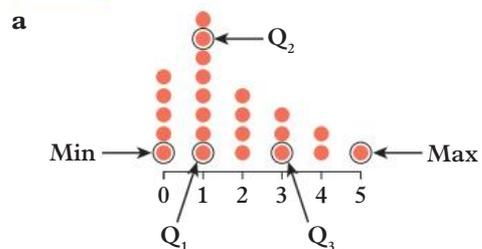
Number of siblings



THINK

- a**
- Examine the dot plot to understand the data distribution. The minimum and maximum values in a dot plot are the dots on both ends of the axis.
 - There are 23 dots in this dot plot. The median divides the data into two halves; so there are 11 dots on both sides of the median. Count from either side of the dot plot: the 12th dot is the median of the data set.
 - Find the upper and the lower quartiles by finding the median of each half of the data set. There are 11 dots in the first half of the data set. Counting from the left side of the dot plot, the sixth dot is the lower quartile. Similarly, counting from the right side of the dot plot, the sixth dot is the upper quartile.

WRITE



minimum = 0, maximum = 5

$Q_2 = 1$

$Q_1 = 1$

$Q_3 = 3$

- 4 Write the five-number summary.
 b Subtract Q_1 from Q_3 to obtain the IQR.

The five-number summary is
 0, 1, 1, 3, 5.
 b $IQR = Q_3 - Q_1$
 $= 3 - 1$
 $= 2$

Helpful hints

- ✓ When placing a set of data in order, count to make sure the ordered set has the same amount of data values as the unordered set.
- ✓ If there are n values in an ordered data set, the median will be in the $\frac{n+1}{2}$ th position. For example, if there are 11 values, the median will be in the $\frac{11+1}{2} = 6$ th position.

ANS
p495

Exercise 8A Five-number summary and interquartile range

 1-6, 8-10, 13, 15

 2, 3, 5-7, 10-13, 15, 17, 18(a, b)

 3(d-f), 5(d-f), 6, 9-11, 13, 15-18

- 1 Find the range of each of these data sets.

a 5, 9, 6, 12, 16, 3, 8, 19, 11, 8, 12	b 48, 3, 7, 3, 8, 4, 12, 19, 6, 6, 8, 19, 13
c 110, 167, 189, 102, 144, 117, 166	d 54, 46, 78, 11, 19, 22, 26, 57, 67
- 2 Find the five-number summary of each of these ordered data sets.

a 3, 3, 3, 3, 4, 5, 6, 6, 7, 8, 9	b 7, 9, 11, 12, 13, 13, 18, 19, 35, 42, 45, 46, 61, 62, 78
c 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 6, 6, 6, 6, 7, 7, 7, 8, 8, 9, 9	d 2, 4, 4, 6, 7, 7, 8, 8, 9, 11, 16, 16, 18, 19, 19, 19, 21, 27, 32
- 3 Find the five-number summary of each of these data sets.

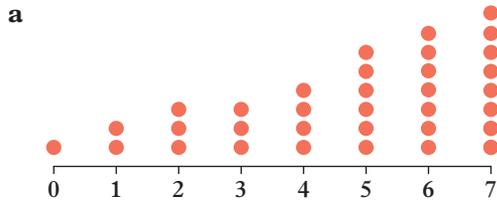
a 4, 8, 9, 12, 7, 6, 6, 6, 7, 4, 3, 2, 8, 9, 11	b 46, 28, 16, 23, 8, 4, 46, 49, 55, 13
c 72, 81, 16, 22, 42, 55, 67, 73, 19, 24, 16, 77, 37	d 724, 169, 843, 120, 564, 786, 341, 186
e 42, 18, 32, 18, 64, 39, 42, 18, 64, 21, 27, 34	f 8, 19, 24, 26, 17, 13, 8, 4, 13, 9, 14, 21, 17, 32, 8, 4, 3, 19
- 4 Given the following five-number summaries, find the interquartile range for each data set.

a 2, 6, 10, 16, 19	b 17, 22, 25, 31, 33	c 7, 9, 13, 19, 24	d 34, 40, 52, 59, 66
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- 8A.1 5 For each of the following data sets, find:

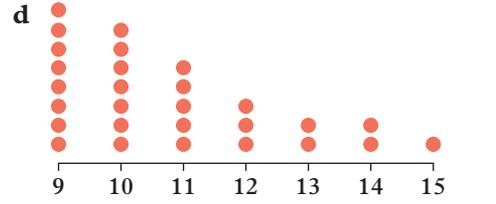
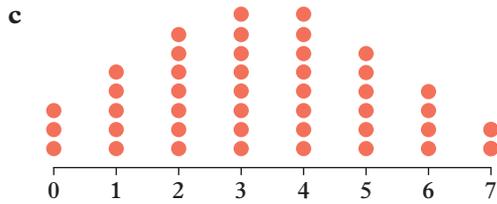
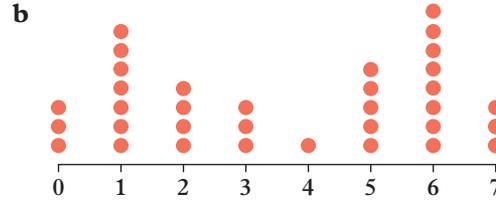
i the five-number summary	ii the interquartile range.
a 46, 48, 32, 12, 62, 27, 19, 34, 86, 16, 16	b 8, 1, 6, 4, 23, 5, 7, 16, 3, 3, 14, 13, 7, 5
c 123, 189, 146, 179, 98, 165, 108, 142, 152	d 19, 62, 53, 14, 8, 43, 62, 9, 46, 13, 19, 20
e 5, 9, 11, 16, 13, 7, 13, 18, 21, 34, 12, 8, 7	f 125, 465, 132, 99, 164, 222, 378

8A.2 6 For each of the following dot plots, find:

i the five-number summary



ii the interquartile range.



7 Write the data in the above dot plots as ordered numerical data sets. Find the five-number summary and interquartile range from each of your numerical data sets to verify your answers to question **6**.

8 The stem-and-leaf plot below shows the number of cupcakes a local bakery sold per day over the course of a month. Find the range and the interquartile range and describe the spread of the data.

Stem	Leaf
4	1 2 2 6 7 8
5	1 6 6 7 9
6	1 2 3 3 6 6 8 9
7	2 4 8 9 9
8	0 1 5 5 7
9	1 2



Key: 4|2 = 42

9 Consider these two data sets.

Set A: 9, 31, 46, 11, 39, 24, 3, 43, 25, 10, 19, 4, 36, 14, 16

Set B: 16, 26, 29, 18, 14, 23, 14, 57, 26, 21, 16, 18, 14, 19, 19

a Find the range for each data set. What do you notice?

b Find the interquartile range for each data set.

c Explain why the interquartile range should be used to describe the difference between the spreads of these two data sets.

10 Consider this data about the number of times a year a sample of Year 9 students have dyed their hair.

Number of times hair has been dyed	0	1	2	3	4	5	6
Frequency	10	2	3	7	11	9	10

a Find the range of the data.

b Find the five-number summary.

c Find the interquartile range.



11 Consider this data set, which is the income (in dollars) per week of some Year 9 students.

10 120 50 60 45 85 250 80 100 65 75 20
 35 45 50 80 25 10 20 50 100 45 80

a Calculate the range and the interquartile range of this data.

b Why is the range so much larger than the interquartile range?

c If the data in a set is spread evenly, the range will be roughly twice the size of the interquartile range. Explain why.

d The presence of an outlier can affect the range. Explain why for data sets with more than five values an outlier would not affect the interquartile range.

- 12 Explain what it means if a data set has:
- a a small interquartile range
 - b a small interquartile range and a large range
 - c a large interquartile range
 - d an interquartile range that is very similar to its range.
- 13 Explain why it is difficult to determine the five-number summary from a histogram that has been compiled using grouped data.
- 14 For each data set below:
- i calculate the range and interquartile range
 - ii describe the spread of the data.
- a 11, 8, 19, 9, 11, 15, 7, 2, 6, 17, 13, 13, 12, 16, 8, 7, 16, 5, 4, 19
- b 32, 68, 43, 31, 77, 44, 67, 42, 37, 55, 59, 79, 81, 83, 11, 56, 57, 61, 82, 63
- c 123, 118, 131, 141, 142, 136, 128, 127, 119, 129, 138, 127, 191, 132, 134

15 Consider the back-to-back stem-and-leaf plot on the right which shows the heights (in cm) of students in two classes.

a For each class, find:

- i the median
- ii the range
- iii the interquartile range.

b Compare the heights of the students in the two classes. Which class would you say is taller? Which class would you say is more diverse?

	Leaf Class A	Stem	Leaf Class B
		14	9
	9	15	6 8
9 8 7 7 7 6 5 4 3 3 2 1 1		16	0 1 3 4 5 5 7 9
9 9 8 7 6 5 5 4 3 2 2 1		17	2 3 4 4 5 5 5 7 9
6 5 4 1		18	1 4 8
		19	7

Key: 15 | 6 = 156

16 Consider the following set of data:

80, 50, 60, 66, 74, 71, 83, 55, 70, 79.

- a Determine the five-number summary for the data.
- b Determine the interquartile range and range for the data.

It turns out the final value, 79, was instead meant to be 9.

- c Determine the correct five-number summary for the data.
- d Determine the correct interquartile range and range for the data.
- e Describe the effect changing 79 to 9 had on the:
 - i median
 - ii range
 - iii interquartile range.
- f Explain why outliers and other extreme values have little to no effect on the interquartile range.

17 Consider this five-number summary created from a list of 10 numbers.

10, 10, 12, 14, 14

- a Determine the number of possible original data sets that would have this five-number summary.
- b If the maximum was increased to 15, determine the number of possible original data sets that would have this five-number summary.

18 Determine the interquartile range for each of the following data sets.

a 21, 4, -11, -2, -24, -18, -3, -1

b $\frac{13}{6}, \frac{17}{3}, \frac{10}{3}, \frac{7}{3}, \frac{1}{4}, 5, \frac{17}{3}, \frac{2}{3}$

c $\frac{18}{3}, \frac{8}{19}, \frac{8}{3}, \frac{13}{10}, \frac{12}{13}, \frac{14}{13}, \frac{9}{11}, \frac{6}{7}$

d $-\frac{17}{18}, -\frac{1}{2}, -\frac{5}{13}, -\frac{11}{8}, -\frac{20}{7}, \frac{9}{10}, \frac{19}{14}, -\frac{13}{17}$

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pro



Interactive skillsheet
Five-number summary
and interquartile range



Worksheet
Measures of centre
and spread



Topic quiz
8A

8B Box plots

Learning intentions

By the end of this topic you will be able to ...

- ✓ create and interpret box plots
- ✓ use parallel box plots to compare data sets.



Inter-year links

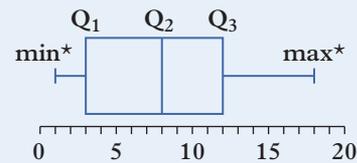
- Support** Understanding data
- Year 7** 9B Summary statistics
- Year 8** 9A Summary statistics

Box plots



Key content video

- A **box plot** (or box-and-whisker plot) can be used to represent the five-number summary visually and to display the centre (median) and spread (range and interquartile range) of a numerical data set.
- A box plot consists of a central box with horizontal lines called 'whiskers' on either side of the box.
 - Box plots should always be accompanied by a clear and even scale.
 - If the minimum or maximum value is equal to the value of the lower or upper quartile, there will be no whisker on that side of the box plot.
 - If the median is equal to either quartile, the median line will not be visible.



*excluding outliers

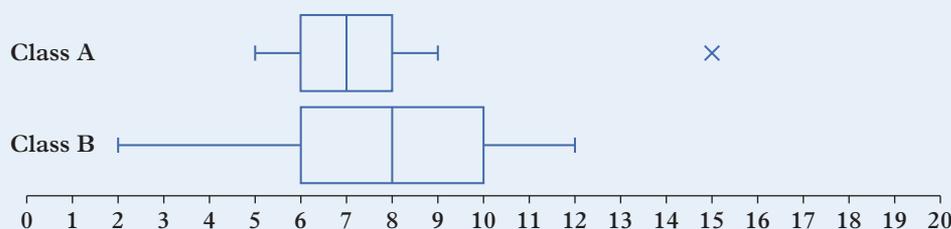
Identifying outliers

- An **outlier** is an extreme value that is much higher or lower than the other values in a data set.
 - Outliers do not have an impact on the median or interquartile range.
- The quartiles and the interquartile range can be used to identify outliers. The **lower fence** lies $1.5 \times \text{IQR}$ below the lower quartile, and the **upper fence** lies $1.5 \times \text{IQR}$ above the upper quartile. Values beyond these fences are classified as outliers.
- Outliers should be identified before box plots are drawn. The values shown by the ends of the whiskers are the minimum and maximum values excluding any outliers. Outliers should be represented on box plots as single points using a cross or dot lying beyond the whiskers.



Using box plots to compare data sets

- **Parallel box plots** can be used to compare data sets on the same scale. Place one box plot directly above the other box plot on the same scale to directly compare key points from each set of data. For example, in the parallel box plots below, the scores of Class A are more consistent than the scores of Class B, except for the outlier in Class A.

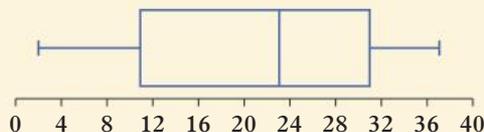




Example 8B.1 Understanding box plots

Use the box plot on the right to state the:

- a five-number summary
- b interquartile range.



THINK

- a The end of the left-hand whisker represents the minimum value, the left-hand side of the box represents the lower quartile, the line inside the box represents the median, the right-hand side of the box represents the upper quartile and the end of the right-hand whisker represents the maximum value.
- b Subtract Q_3 from Q_1 to obtain the IQR.

WRITE

- a $\min = 2,$
 $Q_1 = 11,$
 $Q_2 = 23,$
 $Q_3 = 31,$
 $\max = 37$
 $2, 11, 23, 31, 37$
- b $\text{IQR} = Q_3 - Q_1$
 $= 31 - 11$
 $= 20$



Example 8B.2 Creating a box plot

Draw a box plot to represent this data about the number of books that people own.

45 22 19 4 14 39 152 108 39 19 66 81
 122 42 38 22 18 35 144 33 27 13 31

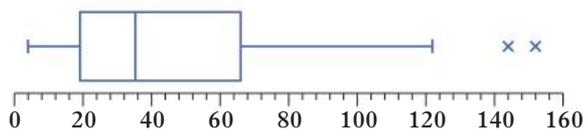
THINK

- 1 Arrange the scores in order and find the value of the median, the lower quartile and the upper quartile.
- 2 Calculate the values of the lower fence (LF) and upper fence (UF).
- 3 Check for outliers by determining whether any values lie below the lower fence or above the upper fence.
- 4 Identify the minimum and maximum values, excluding outliers.
- 5 Draw an even scale that covers all values, including outliers.
- 6 Mark in the position of the minimum and maximum values, the quartiles, the median and any outliers. Complete your box plot using a ruler, providing a title if appropriate.

WRITE

4, 13, 14, 18, 19, 19, 22, 22, 27, 31, 33, 35, 38, 39, 39, 42, 45, 66, 81, 108, 122, 144, 152
 $Q_1 = 19, Q_2 = 35, Q_3 = 66$
 $\text{LF} = 19 - 47 \times 1.5$ $\text{UF} = 66 + 47 \times 1.5$
 $= 19 - 70.5$ $= 66 + 70.5$
 $= -51.5$ $= 136.5$
 144 and 152 are outliers.
 $\min = 4, \max = 122$

Number of books people own



- ✓ The ends of the whiskers do not represent the minimum and maximum values in a data set if there are outliers. If there are outliers, the left whisker will start at the value directly after the closest outlier to the lower fence, and the right whisker will finish at the value directly before the closest outlier to the upper fence.
- ✓ Outliers lie beyond the fences. If a value is exactly equal to the lower fence or upper fence it is not an outlier.
- ✓ The height of a box plot has no particular significance.

ANS p496 **Exercise 8B** Box plots

▲ 1, 2, 3(a-d), 4-6, 8, 9

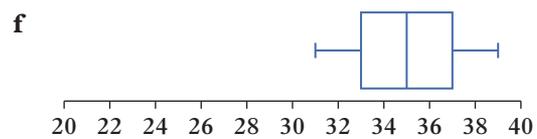
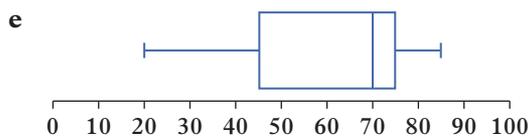
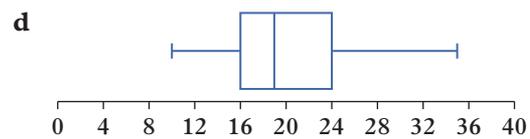
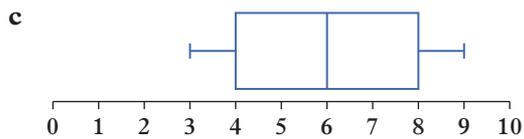
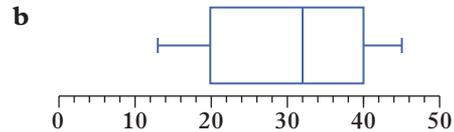
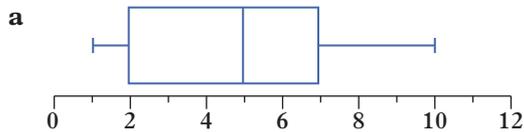
■ 1, 3(e-h), 4-6, 8, 9, 12, 13

◆ 3(e-h), 4-6, 10, 11, 13-15

8B.1 1 For each box plot below, state the:

i five-number summary

ii interquartile range.



2 Consider this data set:

2, 6, 3, 8, 9, 10, 14, 19, 4, 8, 22, 11, 13, 17, 4, 19, 7, 7, 10.

- a Arrange the data in order.
- b Find the lower quartile, median and upper quartile.
- c Calculate the values of the lower and upper fences.
- d Determine whether there are any outliers in this data set.
- e Create a box plot for this data.

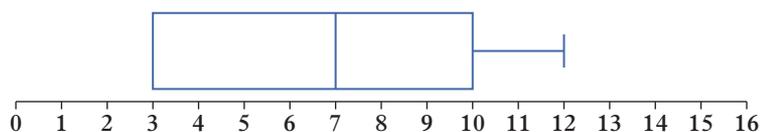
8B.2 3 Draw a box plot to represent each of these data sets.

- | | |
|--|--|
| a 3, 6, 2, 8, 4, 8, 10, 15, 6, 8, 11 | b 11, 15, 14, 19, 20, 26, 17, 13, 6, 22 |
| c 4, 7, 1, 8, 3, 7, 6, 6, 9, 3, 5, 1, 6, 9, 7, 5, 3 | d 12, 36, 45, 92, 42, 85, 49, 66, 51, 19, 27, 78 |
| e 16, 28, 9, 16, 13, 27, 18, 22, 33, 24, 9, 18, 8 | f 5, 7, 9, 19, 6, 9, 8, 5, 5, 4, 3, 9, 9, 1, 6, 5, 4 |
| g 11, 18, 49, 23, 24, 26, 37, 31 | h 8, 11, 19, 21, 6, 7, 12, 18, 19, 31, 4, 6, 24, 7, 9 |

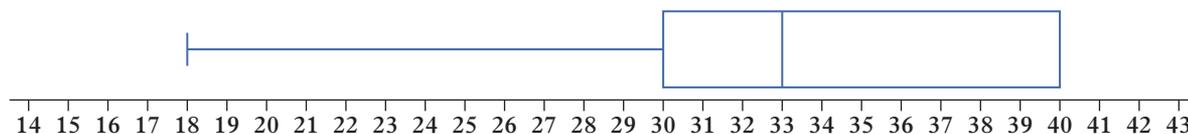
4 For each box plot, state the:

- i five-number summary
- ii interquartile range.

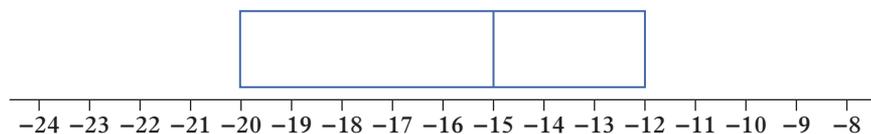
a



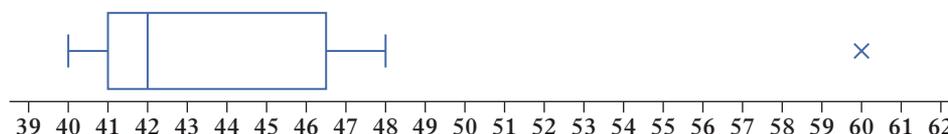
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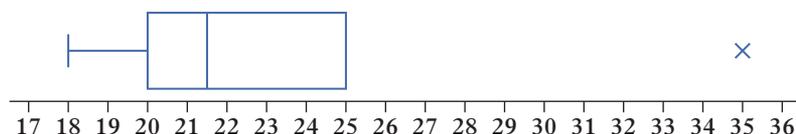
c



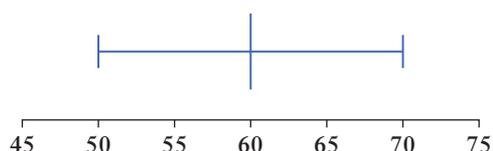
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e

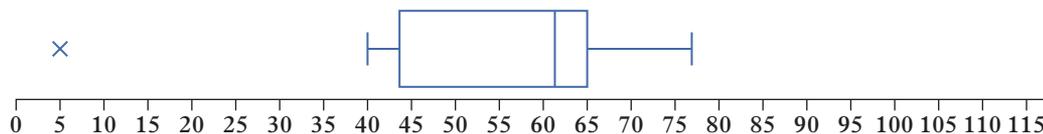


f

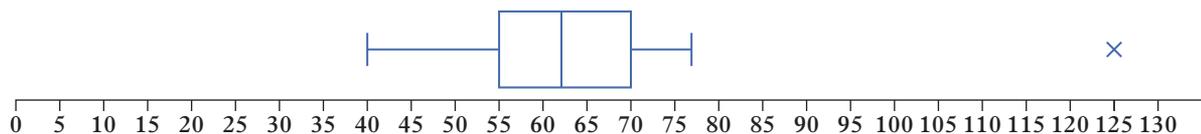


5 For each of the following box plots, add a vertical dashed line where the upper and lower fences lie.

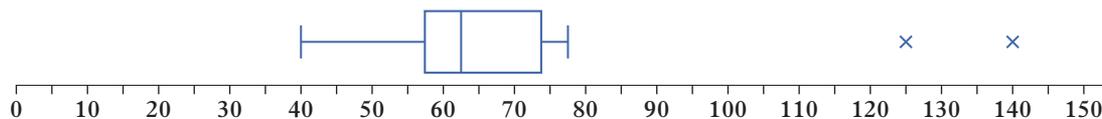
a



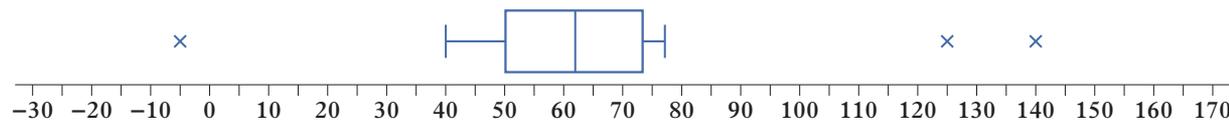
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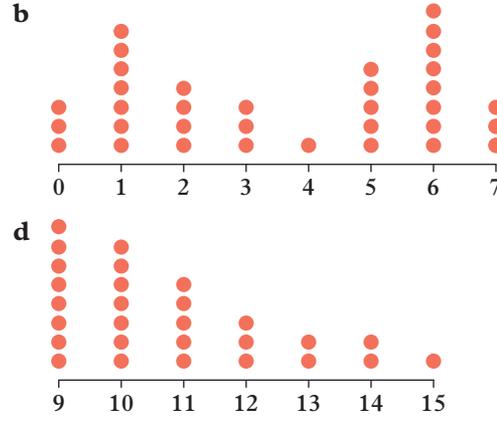
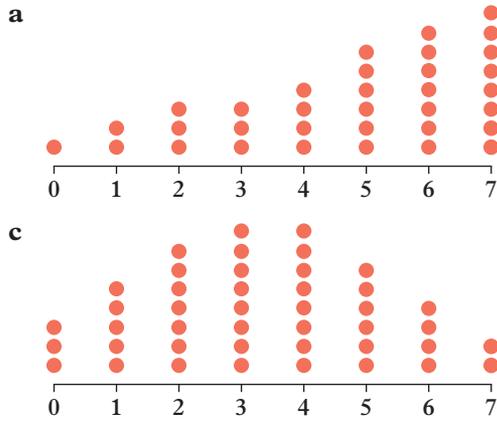
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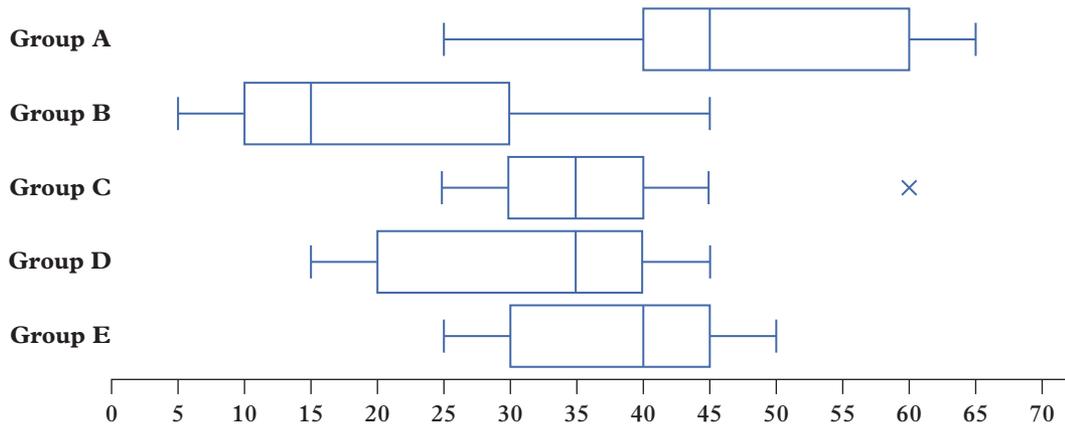
d



6 Display the data in the following dot plots in box plots.



7 Consider the parallel box plots below.



a State the group(s) with the lowest:

i minimum

ii maximum.

b At 45, state the statistic (if any) that group:

i A has

ii B has

iii C has

iv D has

v E has.

c Which group has a range equal to the interquartile range of another group?

8 This data was collected about the number of puppies in different litters:

1	6	4	2	5	4	8	6	4	5	8
3	2	4	6	5	8	7	5	6	5	6
12	4	6	5	7	6	5	6	5		

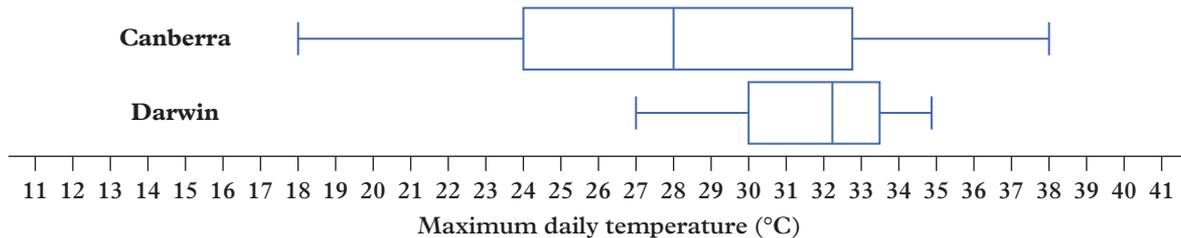


a Find the lower quartile, median and upper quartile of the data set.

b Determine if there are any outliers in this data set.

c Draw a box plot to represent the data.

9 Consider these parallel box plots, showing the distribution of the maximum daily temperatures for a particular month in Darwin and Canberra.



a What is the centre (median) and spread (IQR) of the temperature in Darwin?

b What is the centre (median) and spread (IQR) of the temperature in Canberra?

c Write a paragraph comparing the distributions of the two sets of temperature data. Which would you say is the hotter city?

10 Data was collected on the ages of customers at a laser tag facility on weekdays (set A) and at weekends (set B).

Set A: 19, 18, 21, 22, 23, 18, 24, 25, 22, 21, 20, 19, 18, 18, 19, 20, 19, 20, 24, 22, 21, 23, 27, 19, 21

Set B: 21, 26, 23, 19, 18, 17, 15, 6, 10, 43, 36, 26, 23, 49, 7, 11, 14, 13, 51, 38, 32, 27, 18, 12, 19, 33

- a Draw a set of parallel box plots to represent the data.
- b Describe each data set in terms of its centre (median) and spread (range).
- c Compare the two sets. What differences do you notice?

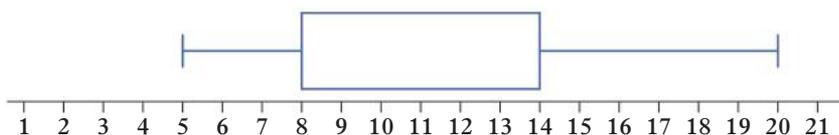
11 The five-number summary for a set of data that shows the number of customers in a shoe shop at 1 pm each day for a month is 4, 6, 7, 8, 10.

- a Draw a possible box plot for this five-number summary.
- b Are other box plots possible? Explain.

The five-number summary for another set of data is 2, 6, 7, 8, 12.

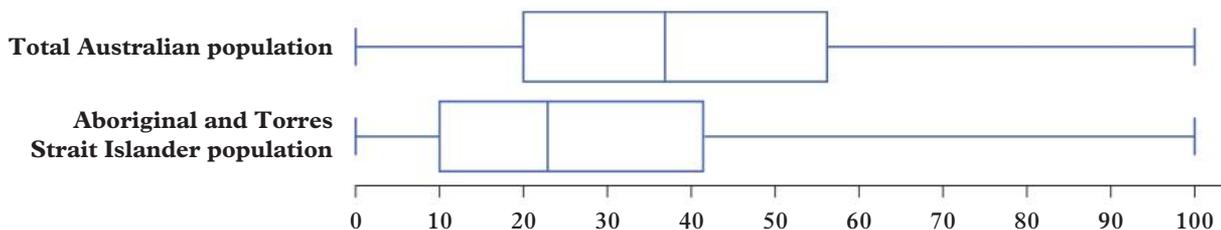
- c Explain why you cannot draw the box plot for this five-number summary with only the information provided.

12 Consider this box plot.



- a Explain why there are two possible five-number summaries when only the box plot (and not the original data) is provided.
- b Provide the two possible five-number summaries for the data the box plot represents.

13 The parallel box plots below show the distribution of the ages of the total Australian population and the distribution of ages of Aboriginal and Torres Strait Islanders in the 2016 census.



Compare the distributions of the ages of all Australians and the ages of Aboriginal and Torres Strait Islanders.

14 Draw a set of three parallel box plots, A, B and C, such that:

- the maximum of A and B are equal
- 75% of the data in B lies within the same values that 25% of the data in A lies within
- 100% of the data in C, excluding an outlier, lies within the same values that the middle 50% of the data of A lies within
- the median of B is equal to the value of the outlier of C
- the medians of A and C are equal to the minimum value of B
- A is perfectly symmetric with each quartile equally spaced, B is negatively skewed, and C is symmetric.

15 For a particular set of data, the lower fence is 15 and the upper fence is 51. Determine the:

- a interquartile range
- b the upper and lower quartiles.

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Interactive skillsheet
Box plots



Worksheet
Box plots



Investigation
How varying data values affect the interquartile range



Topic quiz
8B

8C Distributions of data

Learning intentions

By the end of this topic you will be able to ...

- ✓ describe distributions of data displayed in histograms, dot plots and box plots.



Inter-year links

Year 8

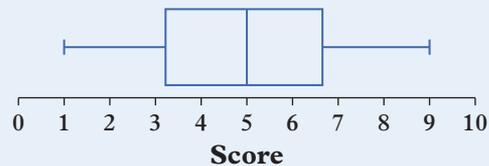
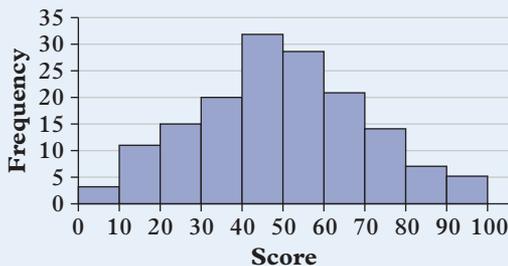
9D Describing data

Describing data distribution

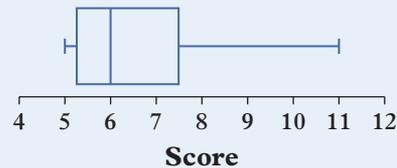
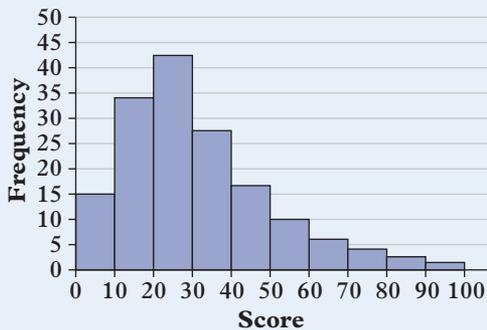


Key content video

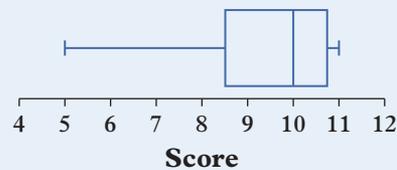
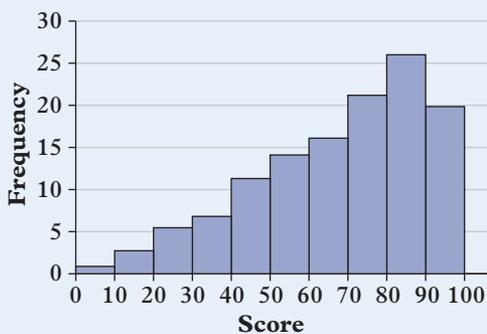
- Symmetrical or **symmetric distributions** are evenly spread on either side of a central peak. The **histogram** and box plot below are examples of symmetric distributions.



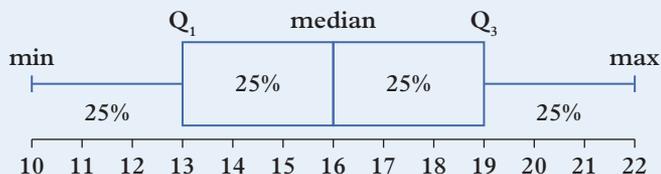
- A **skewed distribution** has a big cluster of data points on one side of the distribution. The other side of the distribution has a tail.
 - A **positively skewed distribution** has a cluster of values around the left-hand side of the distribution and a tail tapering to the right. The histogram and box plot below are examples of positively skewed distributions.



- A **negatively skewed distribution** has a cluster of values around the right-hand side of the distribution and a tail tapering to the left. The histogram and box plot below are examples of negatively skewed distributions.



- To describe the spread of a data set, consider where the scores in the data set lie across the distribution. Remember that 25% of a data set lies between each pair of consecutive terms in a five-number summary, so if any two values in a five-number summary are close together, 25% of scores will be covered in a small range of data (i.e. packed closely together).
- Each section of a box plot represents approximately 25% of a data set.

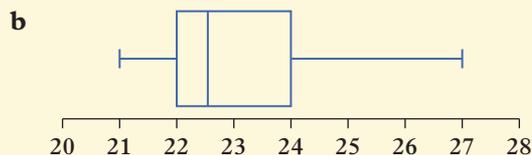
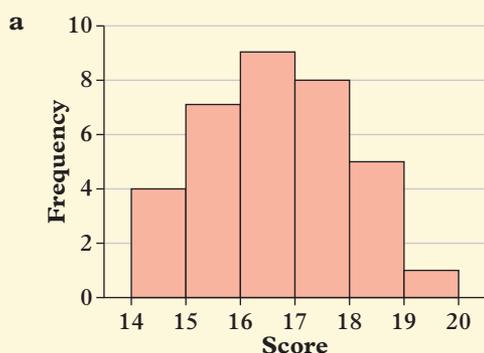


- To describe the centre of a data set, consider where the median score in the data set lies in the distribution.
- If a data set contains an outlier, this should be mentioned when describing the distribution of the data set.

Example 8C.1 Describing a data distribution



Describe the type of distribution of data shown on each of the following.



THINK

- a** Look at the shape of the histogram. The data is approximately symmetrical on either side of the central peak, so this is a symmetric distribution.
- b** Each section of a box plot represents approximately 25% of the data set. There is a cluster of points on the left-hand side of the distribution with a tail tapering to the right, so this is a positively skewed distribution.

WRITE

- a** Symmetric
- b** Positively skewed

Helpful hints

- ✓ Symmetric data does not need to be perfectly symmetric; it only needs to be approximately symmetric.

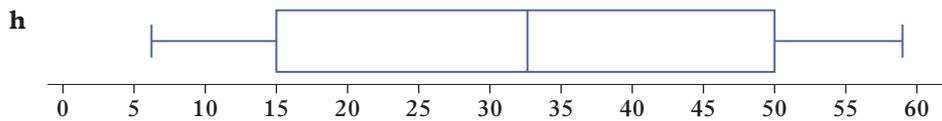
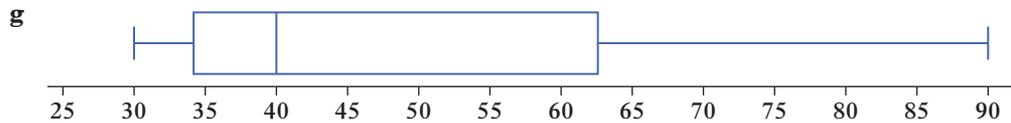
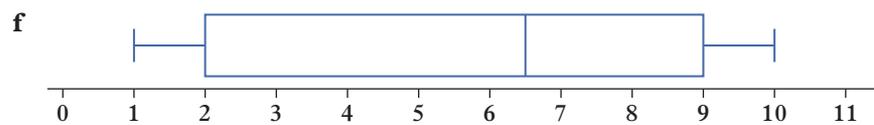
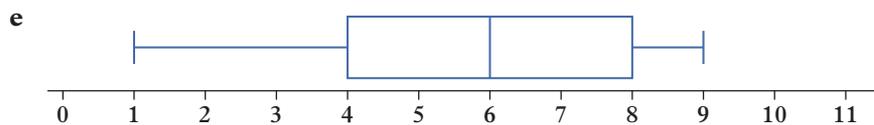
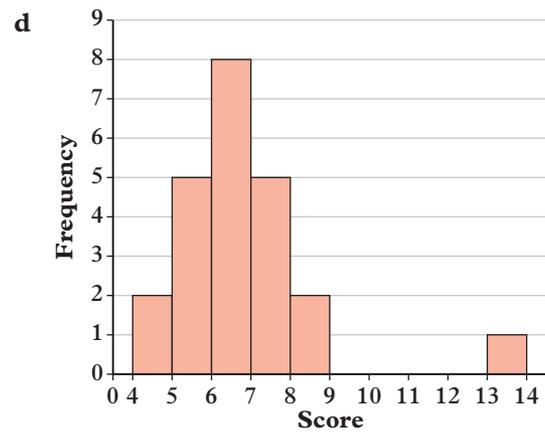
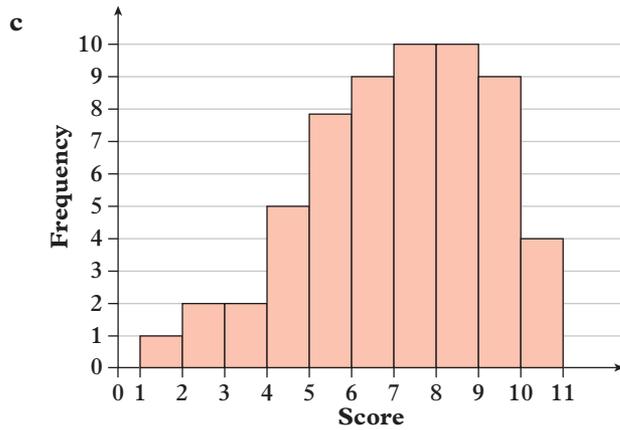
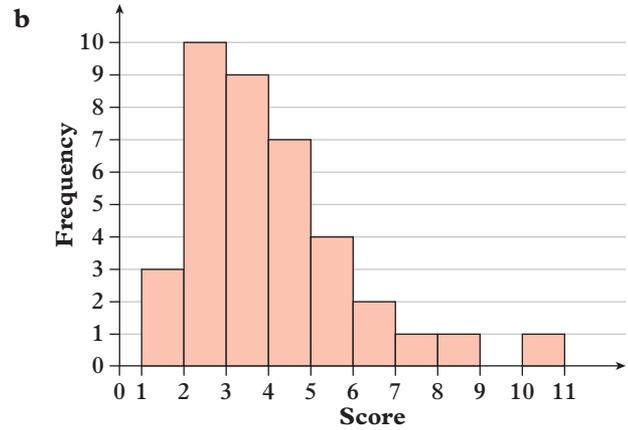
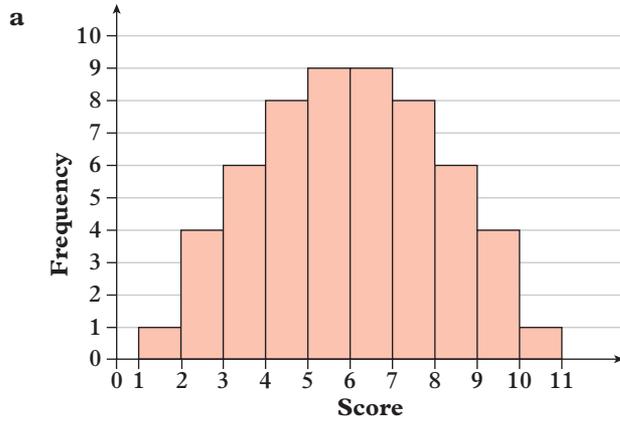
Exercise 8C Distributions of data

 1-4, 6

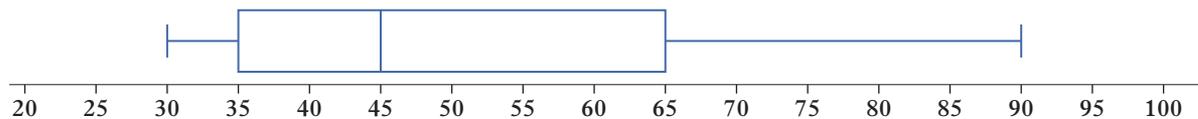
 1, 2, 4-7

 1, 2, 5-8

8C.1 1 Describe each of the following distributions of data.



2 State the percentage of the data that lies in the following ranges of values for this box plot.



a between 35 and 45

b between 35 and 65

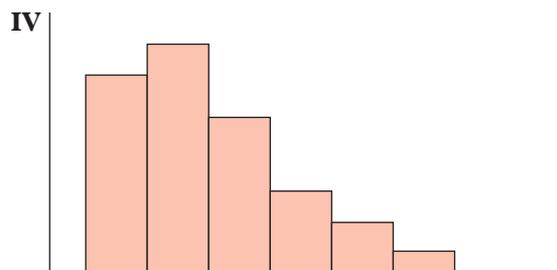
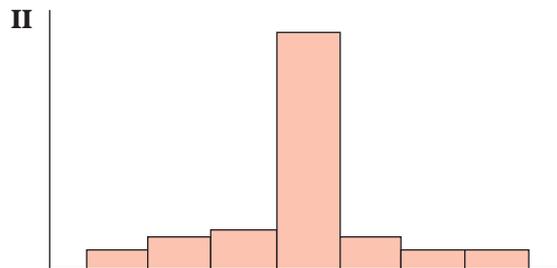
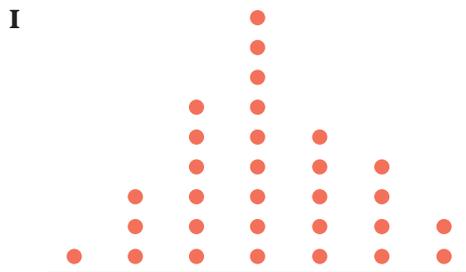
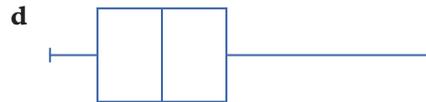
c between 65 and 90

d less than 65

e greater than 35

f less than 35 or greater than 65

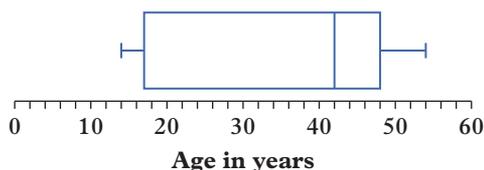
3 Match each of these box plots (a–d) with the correct graph (I–IV).



4 **a** Describe what the box plot on the right tells you about the ages of people attending the Year 9 orientation night, by describing its:

Ages of people at a Year 9 orientation night

- i** shape
- ii** spread
- iii** centre.



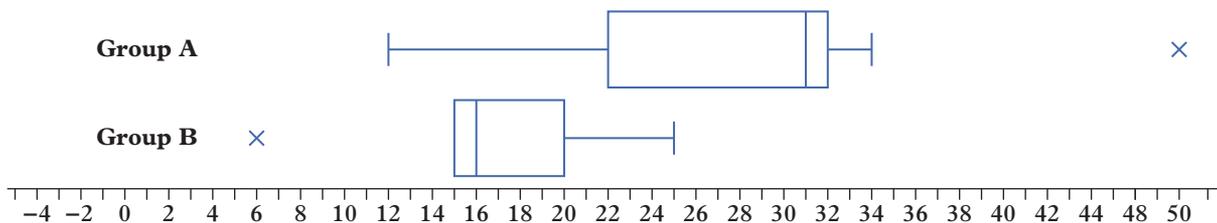
b Can you suggest a reason for the unusual shape of the distribution?

5 A vet collected the following data about the weights (in kilograms) of dogs.

5.1	11.2	35.5	17.9	21.1	84.4	42.2	51.0	9.2	8.6	7.5	18.5	19.2	21.8	22.6
24.8	13.7	13.1	16.9	29.8	58.3	42.7	2.1	3.8	9.6	10.8	46.0	1.9	35.0	39.4

- a** Draw a box plot to represent the data.
- b** Describe the distribution of the data.
- c** Why would a dot plot not be as useful for looking at the distribution of the weights of these dogs?

6 Consider the parallel box plots below.



- a** Describe the distributions of the box plots.
- b** Give three reasons why the median would be a better measure of the centre than the mean to compare these box plots.

8D The mean and standard deviation

Learning intentions

By the end of this topic you will be able to ...

- ✓ calculate the standard deviation of a population and sample
- ✓ use the mean and standard deviation to compare data sets.



Inter-year links

[Year 7](#)

9B Summary statistics

[Year 8](#)

9A Summary statistics

Populations and samples



Key content video

- When working with data, the **population** refers to all potential pieces of data under consideration.
- A **sample** is a selection of data that is part of the population.

Mean and standard deviation

- The **mean** is a numerical average of all values in a data set.
 - The Greek letter μ (mu) is used to represent the population mean.
 - \bar{x} (x -bar) is used to represent a sample mean.
 - The population mean and a sample mean can be calculated in the same way.
- The **standard deviation** is a measure of spread which describes how much the scores in a set differ (or deviate) from the mean.
 - A small standard deviation means that the scores in a data set are close to the mean.
 - A large standard deviation means that the scores in a data set are spread out from the mean.
 - The Greek letter σ (sigma) is used to represent the population standard deviation.
 - The letter s is used to represent a sample standard deviation.
- A population's standard deviation (σ) can be calculated using the formula:

$$\sigma = \sqrt{\frac{(x_1 - \mu)^2 + (x_2 - \mu)^2 + \dots + (x_n - \mu)^2}{n}}$$

- A sample's standard deviation (s) can be calculated using the formula:

$$s = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_n - \bar{x})^2}{n - 1}}$$

- When calculating the standard deviation from a sample mean, the sum is divided by $n - 1$ rather than n because this provides a better estimate of the standard deviation for the population, as the squared deviations from the sample mean underestimate the squared deviations from the population mean.

Using the mean and standard deviation to describe and compare data sets

- The mean describes a central point in a data set. If a data set is positively skewed the mean will be greater than the median, and if the data is negatively skewed the mean will be less than the median.
- When the data is skewed, the median represents the centre of the data better than the mean.
- The standard deviation describes the spread of a data set. When comparing data sets, a larger standard deviation will have scores that are more spread out than a smaller standard deviation.

Example 8D.1 Finding the standard deviation for a list of data



Find the standard deviation for this sample of data, correct to one decimal place:

4, 8, 12, 19, 5

THINK

- 1 Calculate the mean of the sample.
- 2 Substitute the given values into the formula for the standard deviation of a sample and round your answer to one decimal place.

WRITE

$$\bar{x} = \frac{4 + 8 + 12 + 19 + 5}{5}$$

$$= 9.6$$

$$s = \sqrt{\frac{(4 - 9.6)^2 + (8 - 9.6)^2 + (12 - 9.6)^2 + (19 - 9.6)^2 + (5 - 9.6)^2}{5 - 1}}$$

$$= \sqrt{\frac{(-5.6)^2 + (-1.6)^2 + (2.4)^2 + (9.4)^2 + (-4.6)^2}{4}}$$

$$\approx 6.1$$

Example 8D.2 Using a spreadsheet to find the standard deviation



Use Excel (or alternative spreadsheet software) to help you find the standard deviation for the data sample below. Give your answer correct to one decimal place.

10, 15, 11, 23, 18, 9, 22, 17, 14, 10

THINK

- 1 Enter each value from the data sample into a separate cell in column A, in a vertical list.
- 2 Use the formula **=STDEV.S(A1:A10)** to calculate the standard deviation of cells A1 to A10.
- 3 Round the standard deviation to one decimal place.

WRITE

	A
1	10
2	15
3	11
4	23
5	18
6	9
7	22
8	17
9	14
10	10
11	=STDEV.S(A1:A10)

	A
1	10
2	15
3	11
4	23
5	18
6	9
7	22
8	17
9	14
10	10
11	5.043147166

$$s \approx 5.0$$

Note: The formula for finding the standard deviation might vary if you are using a different spreadsheet program.

Helpful hints

- ✓ The formula for the population standard deviation using Excel is =STDEV.P(:), where the range of cells containing the data is placed within the brackets. The cell names of the first and last cells from the listed data are separated by the colon.
- ✓ Most calculators have a function for calculating the standard deviation of a list of data.

Exercise 8D The mean and standard deviation

▲ 1, 2, 3(a, d-f), 4(a, b, d, f), 5, 7, 8, 10, 14, 15

■ 1, 2, 3-4(b, e, f), 5-7, 9, 11, 13, 15, 16

◆ 1, 2, 5, 9, 11, 12, 15-18

- 8D.1 1** Using a calculator, find the standard deviation for each of these sample data sets. Give your answers to one decimal place.
- a** 6, 3, 8, 65, 4 **b** 8, 11, 15, 3, 9, 5 **c** 7, 7, 10, 12, 17, 15
d 13, 29, 21, 22, 22, 11, 19 **e** 140, 156, 120, 99, 187, 147 **f** 159, 166, 122, 171, 136, 129
- 2** Using a calculator, find the standard deviation for each of these population data sets. Give your answers to one decimal place.
- a** 2, 2, 2, 2, 2 **b** 2, 2, 2, 8, 8, 8 **c** 20, 60, 70, 80, 100
d -11, -9, -17, -23, -15 **e** 2.2, 5.0, -0.2, -0.8, 5.0, 3.2 **f** 1, 2, 4, 16, 32, 62
- 8D.2 3** Calculate the standard deviation for these samples of data using technology. Give your answers to one decimal place.
- a** 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 **b** 39, 24, 34, 34, 30, 32, 38, 36, 36, 32
c 10, 100, 1000, 10000, 100000, 1000000 **d** 20, 200, 2000, 20000, 200000, 2000000
e -1000000, -100000, -10000, -1000, -100, -10 **f** $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}, \frac{9}{10}, \frac{10}{11}$
- 4** Calculate the standard deviation for these populations of data using technology. Give your answers to one decimal place.
- a** 89, 104, 97, 114, 97, 90, 107, 82, 109, 103
b 1776.6, 2521.4, 2110.1, 606.5, 926.6, 2395.4, 1572.5, 12.03, 2496.3, 123.7
c 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
d 81, 82, 83, 84, 85, 86, 87, 88, 89, 90
e 5, 6, 7, 8, 9, 10, 11, 12, 13, 14
f 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256
- 5** For each data set below:
- i** decide whether the data set is from a sample or a population
ii calculate the appropriate standard deviation.
- a** This data was collected about the ages of students attending a school formal by surveying everybody who attended.
- | | | | | | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 16 | 17 | 18 | 16 | 17 | 16 | 15 | 14 | 16 | 17 | 18 | 16 | 17 | 17 |
| 16 | 17 | 16 | 17 | 15 | 16 | 17 | 16 | 17 | 16 | 16 | 16 | 15 | 17 |
| 18 | 18 | 17 | 16 | 17 | 16 | 17 | 16 | 17 | 16 | 16 | 16 | 17 | 17 |
- b** This data was collected about the number of hours 20 students in a Year 9 class (of 30 students) spent per week using social networks.
- | | | | | | | |
|----|----|----|----|----|----|----|
| 5 | 10 | 7 | 1 | 19 | 17 | 16 |
| 7 | 10 | 11 | 8 | 9 | 15 | 13 |
| 14 | 6 | 21 | 18 | 14 | 13 | |
- c** This data was collected about the weights of newborn babies in New South Wales:
- | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|
| 3.2 | 4.1 | 2.9 | 3.4 | 3.5 | 3.4 | 3.1 |
| 3.8 | 4.0 | 3.3 | 3.6 | 2.9 | 3.1 | 3.4 |
| 3.5 | 4.2 | 2.6 | 3.0 | 3.6 | 3.4 | 4.8 |
| 2.1 | 3.5 | 3.3 | 3.6 | | | |



- 6 Consider this data set, which shows the ages of every player in two cricket teams.

16	16	15	17	15	16	14	15	16
16	17	16	15	17	16	15	16	16
17	16	17	15	16	16	17	16	



- a Find the standard deviation for the data set. Give your answer to two decimal places.
- b Write a sentence about the spread of ages of players in the cricket teams.

- 7 Consider this data which shows the number of text messages sent by a sample of Year 9 students per day.

1	5	11	17	21	9	8	25	26	19	5	2	8
9	13	34	16	29	31	26	19	18	15	20	10	

- a Find the mean and standard deviation for the data. Give your answers to two decimal places.
- b Write a sentence describing the centre and spread of the data set.

- 8 The following is a list of the number of people at a shoe sale each hour over a 24-hour period:

46	26	35	49	58	13
27	48	46	39	42	48
46	51	35	29	47	41
42	42	49	45	46	51



- a Find the mean for this data. Give your answer to one decimal place.
- b Find the standard deviation. Give your answer to two decimal places.
- c Comment on the centre and spread of the data set.

- 9 Data was collected about the number of minutes a sample of passengers were kept waiting for their trains on two different train lines over the course of a day.

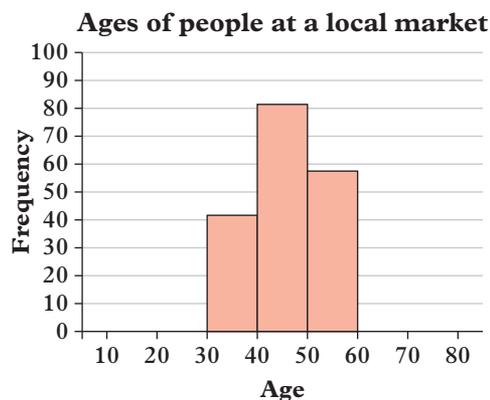
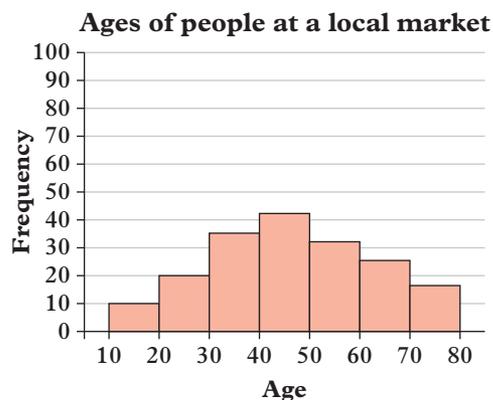
Train line A: 4, 2, 3, 5, 6, 7, 4, 4, 6, 5, 6, 7, 9, 4, 5, 6, 6, 7, 6, 7, 9, 4, 6, 6, 1

Train line B: 2, 9, 15, 3, 4, 3, 11, 1, 1, 6, 2, 3, 4, 2, 3, 19, 2, 3, 2, 2, 2, 3, 1, 4

- a For each train line, find the mean waiting time. Give your answers to one decimal place.
- b Find the standard deviation for the data from each train line. Give your answers to two decimal places.
- c Write a sentence comparing the centre and spread of the two data sets.
- d On which train line would you say that the trains run late more significantly?

- 10 The two histograms below show data collected about the ages of people attending a local market on two different days. The data sets have the same mean and the same number of scores, and have been shown on axes with the same scale for ease of comparison.

Which data set has a small standard deviation and which has a large standard deviation? Explain how you know.



- 11** Consider this data collected about the price (in dollars) of a particular brand of electronic tablet bought from all stores where it is available in a city.

499 519 499 475 469 479 509 549 529 525
 475 499 499 199 489 479 499 475 515 499

- a** Find the standard deviation for this data set.
b You may notice that one of the prices is distinctly different from the rest (an outlier). Provide a possible reason for this.
c Recalculate the standard deviation for the data set excluding the outlier.
d What do you notice about your answers for parts **a** and **c**?
e What measure of spread might you use instead of standard deviation when there is an outlier in the data set?
- 12** The magnitude or absolute value of x , written $|x|$, is its distance from 0 on a number line, ignoring direction. For example, $|5| = |-5| = 5$.

In a data set with n data points, the mean absolute deviation is found using the formula:

$$\text{mean absolute deviation} = \frac{|x_1 - \bar{x}| + \dots + |x_n - \bar{x}|}{n}$$

- a** Calculate the mean absolute deviation and standard deviation for each of the following population data sets.
i 1, 2, 2, 3, 4, 4, 4, 5, 6, 10 **ii** 5, 5, 5, 7, 8, 10, 15, 21
- b** What do you notice when you compare the mean absolute deviations and standard deviations you calculated in part **a**?
- 13** Another measure of spread is the median absolute deviation. In a data set with n data points and median \tilde{x} , the median absolute deviation is the median of the data set

$$|x_1 - \tilde{x}|, \dots, |x_n - \tilde{x}|.$$

For each of the data sets in question **12**:

- a** find the median \tilde{x}
b calculate the median absolute deviation.
- 14 a** When is the median a better measure of the centre of a distribution of data than the mean? Explain.
b When is the interquartile range a better measure of the spread of a distribution of data than the standard deviation? Explain.
- 15** Compare the following pairs of symmetric sample data in terms of their mean and standard deviation. Use Excel (or alternative spreadsheet software) to help you calculate the statistics.
- a** **A:** 20, 24, 28, 28, 30, 30, 31, 32, 35, 40 **b** **A:** 75, 81, 83, 84, 88, 92, 95, 101, 105, 109, 113
B: 24, 25, 25, 25, 26, 27, 29, 31, 34, 36, 36, 37 **B:** 64, 70, 71, 71, 72, 74, 80, 81, 83, 84, 86, 90

- 16** We can compare individual data values by determining the number of standard deviations, s , the value, x , is above or below the mean, \bar{x} . That is, $z = \frac{x - \bar{x}}{s}$. These values are called standard scores (or z -scores). For example, if Julia scored 15 on a test where the mean score was 20 and the standard deviation was 2, then Julia's standard score is $z = \frac{15 - 20}{2} = -2.5$, meaning Julia's score is 2.5 standard deviations below the mean score.

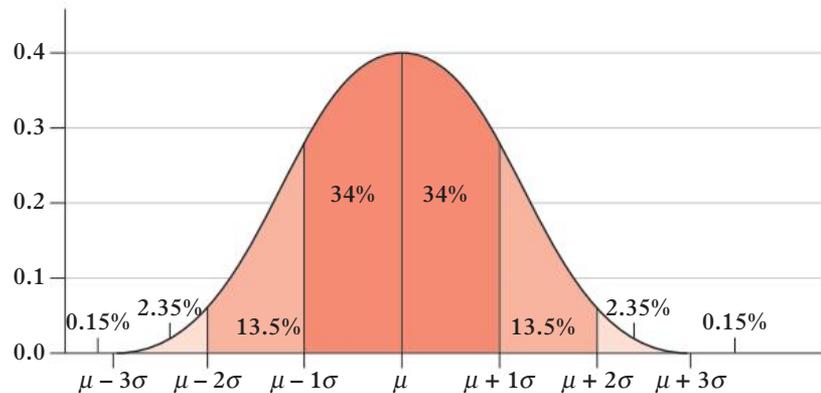
- i** Calculate the standard score for each pair of students' results, correct to two significant figures.
ii Compare the scores of each pair of students from different classes using their standard scores.

	Class A			Class B		
	Mean	SD	Student A	Mean	SD	Student B
a	70	10	85	55	16	85
b	70	20	85	70	15	85
c	50	25	30	75	5	50
d	50	8	26	50	16	2

- 17 A population that has a symmetric distribution where the majority of the data is centred around the mean in a rough bell shape is said to be normally distributed (also known as forming a bell curve). The standard deviation plays a useful role in making predictions about populations that are normally distributed. A normal distribution will have nearly all values (99.7%) within three standard deviations (also known as three sigma) of the mean. Consider the graph below with approximate percentages of the amount of data between different standard deviations using the 68%–95%–99.7% rule.

- a** Show that 99.7% of values lie within three sigma (three standard deviations) of the mean (μ).
- b** What percentage of values lie within:
- one sigma of the mean
 - two sigma of the mean?

A typical example of this kind of data distribution can be seen in the data for IQ (intelligence quotient). IQ scores are a ranking system where the mean (μ) IQ score is set to 100 and the standard deviation is set to 15.



- c** Redraw the graph above so that the scale on the x -axis uses these values (that is, $\mu = 100$, $\mu + 1\sigma = 115$, etc.).
- d** What is the IQ range of 99.7% of the population?
- e** What IQ would you need to have in order to be in the top 0.15% of the population?
- f** What IQ would you need to have in order to be in the bottom 2.5% of the population?
- g** What percentage of the population has an IQ of 130 or greater?
- h** What percentage of the population has an IQ of 85 or less?
- i** A group of highly intelligent people had their IQs tested and the distribution was found to be normal, with a mean in the middle of these scores. If 95% of this group had an IQ between 120 and 152, find the mean and standard deviation of the group.
- 18 If we were to take different samples from a population, the sample mean in each sample would vary in value but would approximate the mean of the population. Regardless of the shape of the distribution of the original data, the distribution of the sample means would be approximately normally distributed with a mean of $\mu_{\bar{x}}$ and a standard deviation of $s_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$, where n is the size of each sample. A particular population has a mean of 40 and a standard deviation of 9.

- a** Find the mean and standard deviation of the distribution of sample means when taking samples of 25.

Use the diagram in question 17 to answer the following.

- b** Determine the probability, as a percentage, that a randomly selected sample mean will be:
- between 38.2 and 41.8
 - greater than 38.2
 - less than 34.6
 - greater than 43.6 or less than 36.4.
- c** Determine the value(s) of the sample mean such that the probability of selecting a sample mean:
- less than this sample mean is 84%
 - greater than this sample mean is 50%
 - between these two sample means is 95%
 - between these two sample means is 15.85%.

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Interactive skillsheet
Standard deviation



Topic quiz
8D

Checkpoint

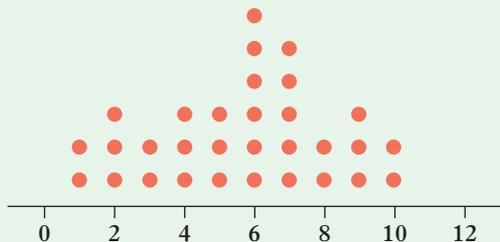


Checkpoint quiz

Check your knowledge of the first part of this chapter.

8A 1 Determine the five-number summary for each of the following.

- a** 14, 16, 50, 23, 17, 24, 32, 19, 22, 32
b 40, 35, 38, 54, 51, 37, 48, 54, 55, 39, 42, 60
c



d

Stem	Leaf
5	4 7 7
6	1 3 5 8 9
7	2 3 5 7 9 9
8	0 1 2

Key: 5 | 4 = 5.4

8A 2 Determine the interquartile range and range for the following five-number summaries:

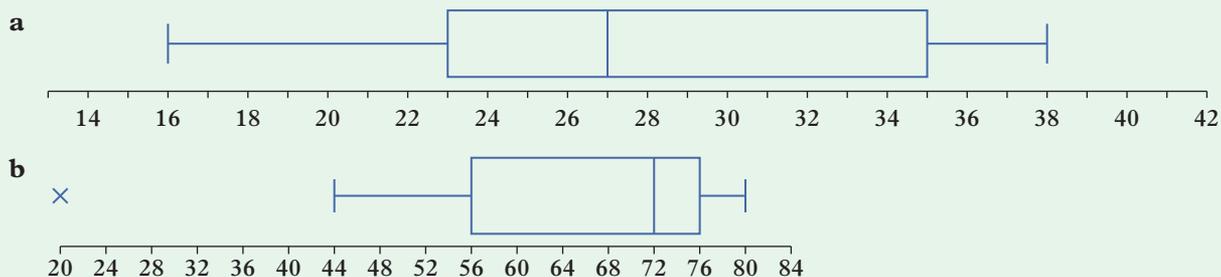
- a** 14, 18, 23, 29, 32
b 58, 64, 85, 91, 93

8B 3 Construct a box plot for each of the following five-number summaries. These box plots will have no outliers.

- a** 120, 130, 145, 150, 175
b 7, 12, 16, 22, 30

8B 4 For each of the following, state the:

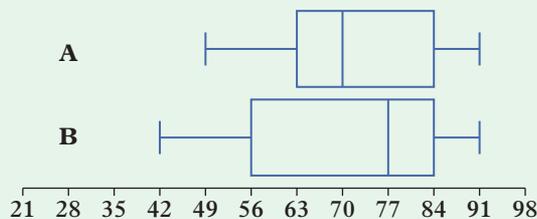
- i** five-number summary
ii interquartile range.



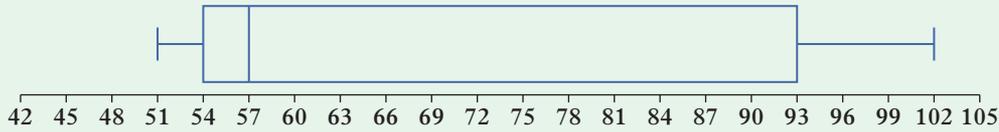
8B 5 Construct a box plot for each set of data. Check each set for potential outliers.

- a** 41, 85, 74, 67, 79, 75, 22, 84, 80, 66, 41, 79, 45, 54, 47, 84
b 56, 68, 68, 71, 56, 60, 79, 68, 76, 74, 64, 75, 62, 77, 80, 72, 76, 55, 62, 71, 72, 35

8B 6 Consider the parallel box plots shown below. Compare the distributions of the data in terms of the centre and spread.

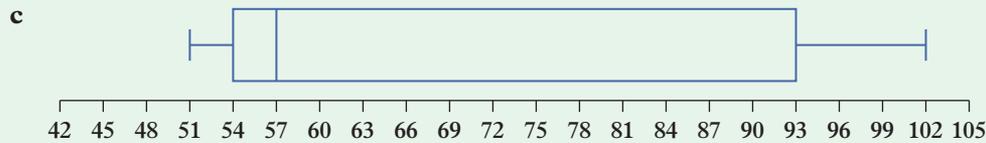
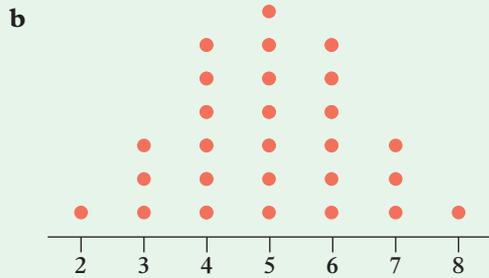
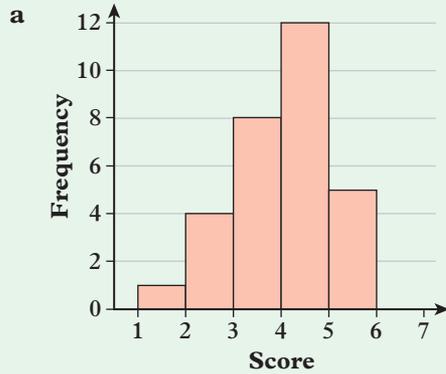


- 8C** 7 State the percentage of the data that lies in the following ranges for the box plot shown below.



- a less than 54
- b between 54 and 93
- c less than 93
- d greater than 57

- 8C** 8 State the shape of the following distributions.



d

Stem	Leaf
1	2 5 8
2	0 1 4 4 5 6 9
3	1 2 3 5 7
4	2 2 9

Key 1 | 2 = 12

- 8D** 9 Hari and Ish are the opening batsmen for a cricket team. In a five-game series against their cross-town rivals they score the following amounts of runs.

Hari: 15, 42, 38, 20, 51

Ish: 9, 18, 94, 21, 1

- a For each batsman, calculate:
 - i the mean score
 - ii the standard deviation (to two decimal places).
- b Which of the two batsmen performed more consistently?

8E Two-step chance experiments

Learning intentions

By the end of this topic you will be able to ...

- ✓ create tree diagrams and arrays to represent sample spaces
- ✓ use tree diagrams and arrays to calculate probabilities.



Inter-year links

Support	Understanding probability
Year 7	9G Theoretical probability
Year 8	9F Theoretical probability
Year 10	11D Theoretical probability

Calculating theoretical probability



Key content video

- The **sample space** of an experiment is a list of all possible outcomes in the **experiment**.
- An **event** is a collection of one or more outcomes from the sample space of an experiment.
- The probability of an event occurring can be written as $\Pr(A)$, where A represents the event.
- If all the outcomes in an experiment are equally likely to occur, the **theoretical probability** of an event occurring is the ratio of the number of outcomes in the event to the number of outcomes in the sample space. It can be expressed as a fraction, decimal or percentage and calculated using the formula:
→ $\Pr(\text{event}) = \frac{\text{number of outcomes in the event}}{\text{number of outcomes in the sample space}}$
- The complement of an event A is the event that A does not occur. Events A and 'not A ' are **complementary events**.
→ Complementary events are denoted using the symbol prime (').
 A and A' are complementary events.
→ All outcomes in the sample space that are not in A must be in A' . $\Pr(A) + \Pr(A') = 1$
- An **independent event** is an event whose outcomes are not affected by the result of any other event.
→ A **dependent event** is an event whose outcomes are affected by the result of another event.
- When calculating the probability of an event involving multiple outcomes, the probabilities of each outcome are added together to give the probability of the event.
→ Make sure you do not include any individual outcome more than once. For example, when a standard six-sided die is rolled, the probability of the event 'rolling an even or a prime number' must only count the outcome '2' once.

Two-step chance experiments

- Two-step chance experiments involve two experiments, which may be completed at the same time, or one after the other. For example, rolling a standard six-sided die and tossing a coin is a two-step chance experiment.
→ If the events in a two-step chance experiment are independent, the probabilities at the second step will not be affected by what happens in the first step.

Arrays

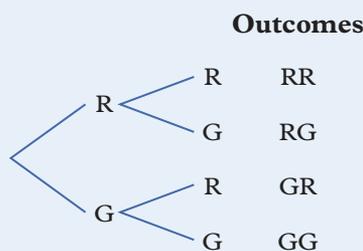
- An **array** is a list of items arranged in rows and columns.
- The sample space of a two-step chance experiment shown in an array has the outcomes of one event in columns and the outcomes of the other event in rows. For example, this array shows the outcomes from rolling a standard six-sided die and tossing a coin.
→ There are two outcomes when a coin is tossed: heads (H) or tails (T).
→ There are six outcomes when a die is rolled: 1, 2, 3, 4, 5, 6.

		Rolling a standard six-sided die					
		1	2	3	4	5	6
Tossing a coin	H	(1, H)	(2, H)	(3, H)	(4, H)	(5, H)	(6, H)
	T	(1, T)	(2, T)	(3, T)	(4, T)	(5, T)	(6, T)

- When calculating probability using an array, all the outcomes are equally likely, and the theoretical probability formula can be used.
- Arrays are best used to display the sample space when there are a large number of outcomes in either step of a two-step experiment.

Tree diagrams

- In a **tree diagram**, the outcomes of each step of a chance experiment are listed vertically, with the events joined with branches. For example, this tree diagram shows the outcomes for selecting two marbles at random from a bag of red and green marbles.

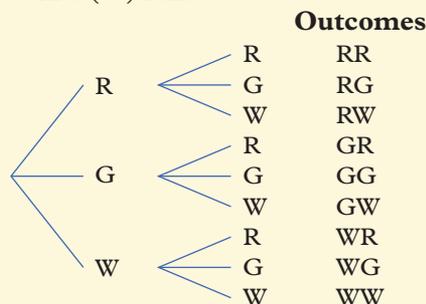


- If the outcomes in a tree diagram are equally likely, the theoretical probability formula can be used. The probability of the final outcomes can be calculated by multiplying together the individual probabilities along the branches. For example, $P(\text{G and R}) = P(\text{G}) \times P(\text{R})$.

Example 8E.1 Understanding tree diagrams



This tree diagram displays the outcomes of selecting two lollies at random from a jar containing red (R), green (G) and white (W) lollies.



Use this tree diagram to find:

- the total number of outcomes
- the number of outcomes containing at least one white lolly.

THINK

- Count the number of final outcomes at the right end of the tree diagram.
- Trace the branches carefully and count the number that contains at least one white lolly.

WRITE

- There are nine possible outcomes: RR, RG, RW, GR, GG, GW, WR, WG, WW.
- Five outcomes contain a white lolly: RW, GW, WR, WG, WW.

Example 8E.2 Calculating probability using an array



Use an array to calculate the probability of a total sum of 10 or above when rolling two standard six-sided dice.

THINK

- 1 Draw an outline for the array, with enough columns to list all the outcomes for the first step, and enough rows to list all the outcomes for the second step.
- 2 Complete the array by showing the result for the first step before the result for the second step in brackets, separated by a comma.
- 3 Count the outcomes that have a total sum 10 or more (these cells have been highlighted in the answer).
- 4 Count the total number of outcomes in the array.
- 5 Use the theoretical probability formula and simplify if possible.

WRITE

		1st dice roll					
		1	2	3	4	5	6
2nd dice roll	1	(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)
	2	(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
	3	(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)
	4	(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)	(6, 4)
	5	(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)	(6, 5)
	6	(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	(6, 6)

$$\begin{aligned} \text{Pr}(10 \text{ or above}) &= \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}} \\ &= \frac{6}{36} \\ &= \frac{1}{6} \end{aligned}$$

Example 8E.3 Calculating probability using a tree diagram

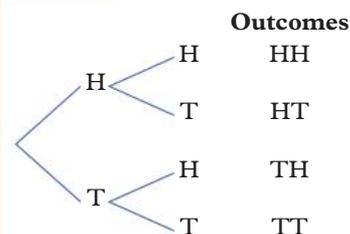


Use a tree diagram to calculate the probability of landing on exactly one tail when tossing a coin twice.

THINK

- 1 Draw the first two branches to represent the first coin toss. Label the ends of these branches with H and T to represent the two different outcomes, heads and tails.
- 2 From each branch, draw another two branches to represent the next coin toss and label them appropriately.
- 3 Write the final outcome on each of the four branches to complete the tree diagram.
- 4 There are four possible outcomes and two outcomes contain exactly one tail (HT, TH). As each of the outcomes are equally likely, the probability formula can be used.

WRITE



$$\begin{aligned} \text{Pr}(\text{exactly one tail}) &= \frac{2}{4} \\ &= \frac{1}{2} \end{aligned}$$

Helpful hints

- ✓ In other contexts, an event means an occurrence or an occasion, but in probability it means a collection of outcomes from the sample space. Don't get confused!
- ✓ When creating a tree diagram, make sure you leave plenty of space to draw in all the branches!

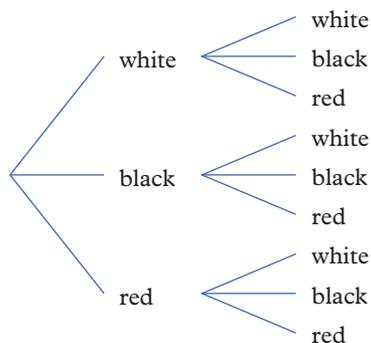
Exercise 8E Two-step chance experiments

1-8, 11, 12

2-4, 6, 8-13

2-4, 6, 9-14

8E.1 1 Consider this tree diagram that shows the results of choosing two coloured balloons at random.



- a How many possible outcomes are there?
- b How many of these outcomes contain at least one red?

8E.2 2 Consider this array, which shows the results of two people randomly picking a number between 1 and 3.

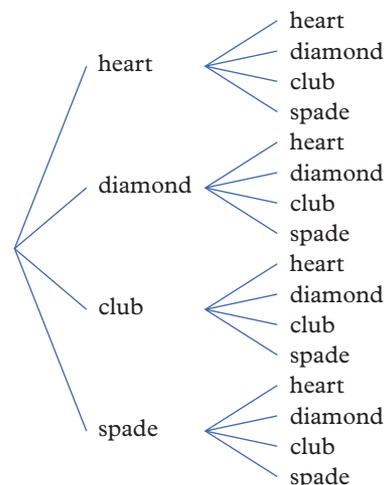
- a How many possible outcomes are there?
- b State the probability of obtaining a sum:
 - i of 3
 - ii that is an odd number
 - iii that is a prime number
 - iv that is a square number
 - v that is a number greater than 6.

		1st number		
		1	2	3
2nd number	1	(1, 1)	(2, 1)	(3, 1)
	2	(1, 2)	(2, 2)	(3, 2)
	3	(1, 3)	(2, 3)	(3, 3)

- 3 Two four-sided dice (numbered 1–4) are rolled and the numbers showing are added together to give the final outcome.
 - a Create an array that lists all the outcomes.
 - b How many equally likely outcomes are detailed in the array?
 - c What is the most likely final outcome? What is the probability of this occurring?
 - d What are the least likely final outcomes? What is the probability of one of these outcomes occurring?
 - e State the probability of rolling two four-sided dice (numbered 1–4) and obtaining a sum:
 - i of 3
 - ii greater than 6
 - iii of an odd number
 - iv less than 5.

4 Two cards are drawn out of a standard deck of cards. Assuming that all outcomes are equally likely, use the tree diagram on the right to find the probability of selecting:

- a at least one diamond
- b a club and a heart
- c two spades
- d not a heart
- e a diamond or a spade
- f a spade then a club.

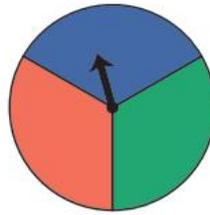


BE.3 5 A coin is tossed twice. Use a tree diagram to find the probability of landing on:

- a** two heads
- b** one head and one tail
- c** no heads.

6 The spinner on the right is spun twice. Use a tree diagram to find the probability of:

- a** two different results
- b** spinning blue at least once
- c** spinning the same colour both times.



7 This array represents a single round of a game of rock (r), paper (p), scissors (s).

		Your choice		
		r	p	s
Opponent's choice	r	(r, r)	(p, r)	(s, r)
	p	(r, p)	(p, p)	(s, p)
	s	(r, s)	(p, s)	(s, s)

rock beats scissors



paper beats rock



scissors beats paper

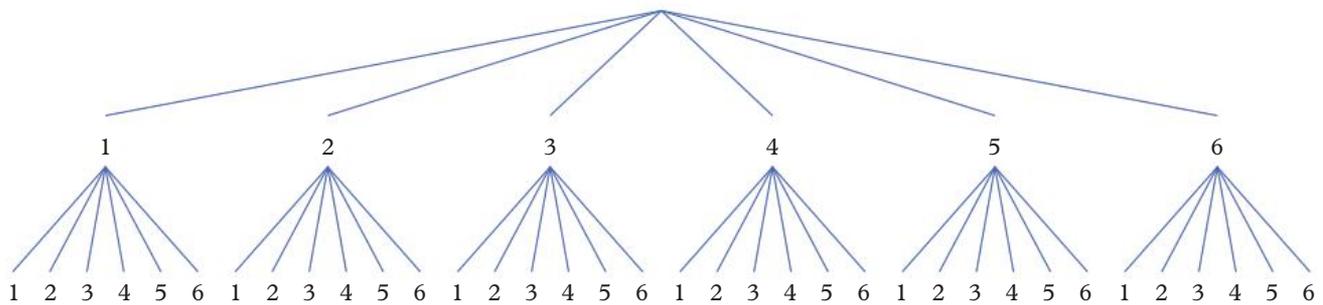


- a** List the outcomes that result in you:
 - i** winning
 - ii** losing
 - iii** drawing the game.
- b** Hence calculate the probability that you win a game of rock, paper, scissors.

8 Consider sitting a quiz consisting of multiple-choice questions, with answers A–D.

- a** Draw a tree diagram to represent the possible different options for the first two questions.
- b** Draw an array to represent the possible different options for the first two questions.
- c** Explain why you could use a tree diagram but not an array to represent guessing the answers to the first three questions.

9 The tree diagram below shows the result of rolling two standard six-sided dice.

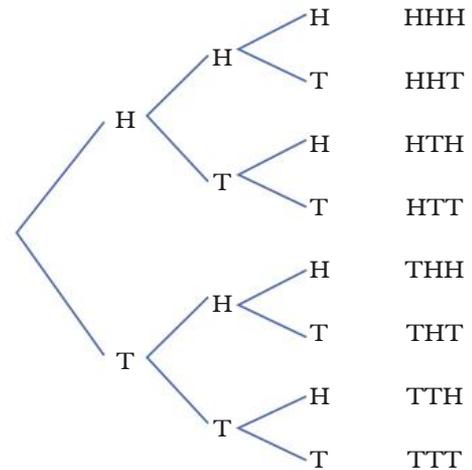


- a** How many possible outcomes are there?
- b** Use the tree diagram to find the probability of rolling:
 - i** two odd numbers
 - ii** one even number
 - iii** a double
 - iv** a total of 6.
- c** Why would it be more appropriate to use an array to display the sample space?

10 Explain why if there are exactly two outcomes at either stage of a two-step chance experiment, the outcomes will be complementary events.

- 11** Tree diagrams aren't limited to repeated trials of the same experiment. They can also be used to display unrelated trials. Imagine that you toss a coin and then roll a standard six-sided die.
- Draw a tree diagram to represent this two-step experiment.
 - How many outcomes are there?
 - What is the probability that you toss a tail:
 - and roll a 6
 - and roll a number less than 4
 - or roll a 6
 - or roll a number less than 4?
 - Explain the mistake made by someone who answers $\frac{8}{12}$ to part **c iii**.
- 12** Similar to tree diagrams, arrays are not limited to repeated trials of the same experiment. They can also be used to display unrelated trials. Imagine that you select a vowel (A, E, I, O, U) at random and roll a standard four-sided die.
- Draw an array to represent this two-step experiment.
 - How many outcomes are there?
 - What is the probability that:
 - you select E and roll a 3
 - you select E or roll a 3
 - you select a vowel with only straight lines and roll a number less than 4
 - you select a vowel with only straight lines or roll a number less than 4?

- 13** Experiments can have more than two steps. When using a tree diagram to represent this, we add more sets of branches to the end of each branch. For example, if a coin is tossed three times (or three coins are tossed), then the tree diagram that represents the sample space would look like this.



- Use the tree diagram shown to determine the probability of tossing a fair coin three times and getting:
 - three heads
 - two heads and one tail in any order
 - two tails and one head in any order
 - an alternating sequence of heads and tails in any order.
 - Explain why you couldn't use an array to show the sample space for an experiment with three or more steps where each set of outcomes at each step is shown. That is, without using the outcomes of a group of steps against the outcomes of another group.
- 14** Anna is investigating the possibilities when rolling four fair six-sided dice. Use a tree diagram to find the probability of rolling:
- four of the same number
 - exactly three of the same number
 - two of the same number and two different numbers
 - two copies of each of two different numbers (e.g. two 2s and two 3s)
 - four different numbers.

Check your Student obook pro for these digital resources and more:

pro



Interactive skillsheet
Arrays



Interactive skillsheet
Tree diagrams



Investigation
Dominoes and tree diagrams



Topic quiz
8E

8F Experiments with replacement

Learning intentions

By the end of this topic you will be able to ...

- ✓ calculate probabilities for two-step chance experiments with replacement.



Inter-year links

- Year 7** 9G Theoretical probability
- Year 8** 9F Theoretical probability
- Year 10** 11E Experiments with and without replacement

Calculating probabilities for two-step experiments



Key content video

- The probabilities for each step of a two-step experiment can be written on the branches of a tree diagram.
- The probabilities for each outcome can be found by multiplying together the probabilities of each branch leading to that outcome. $P(A \text{ and } B) = P(A) \times P(B)$
→ The probabilities of all the outcomes of an experiment always sum to 1.
- If the event you are looking for consists of more than one outcome, the probabilities of each outcome are added together to find the probability of the event.

Experiments with replacement

- Experiments with replacement involve selecting an item at random, recording the result, and then replacing the item before making another selection.
- When replacing the item before making another selection, the probabilities for each step of the experiment remain the same. Therefore, the events at each step are independent events.

Example 8F.1 Representing experiments with equally likely outcomes

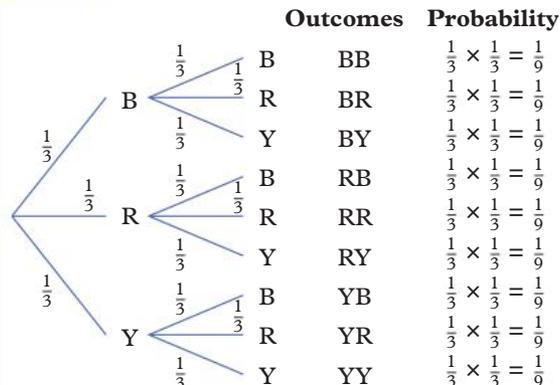


A box contains equal numbers of blue, red and yellow activity cards. A card is selected at random, its colour is recorded and then it is replaced. Another card is then selected at random. Draw a tree diagram to represent this situation, complete with probabilities on each branch and for each final outcome.

THINK

- 1 Draw a tree diagram to represent this two-step experiment, listing all the final outcomes.
- 2 Write the probability of each branch on your tree diagram. Each colour is equally likely so there is a $\frac{1}{3}$ chance of selecting a blue card, a $\frac{1}{3}$ chance of selecting a red card and a $\frac{1}{3}$ chance of selecting a yellow card.
- 3 Multiply the probabilities along each branch to determine the probability of each final outcome.

WRITE



Example 8F.2 Calculating probabilities for experiments with equally likely outcomes



Use the tree diagram from Example 8F.1 to find the probability that:

a two blue cards are selected

b a yellow card is selected first.

THINK

- a** Identify the outcome in which both selections are blue cards.
- b** 1 Identify the outcomes in which a yellow activity card is selected first.
2 Find the sum of the probabilities and simplify.

WRITE

a $\Pr(\text{two blue}) = \Pr(\text{BB})$
 $= \frac{1}{9}$

b $\Pr(\text{yellow first}) = \Pr(\text{YB, YR, YY})$
 $= \frac{1}{9} + \frac{1}{9} + \frac{1}{9}$
 $= \frac{3}{9}$
 $= \frac{1}{3}$

Example 8F.3 Representing experiments with outcomes that are not equally likely



A bag contains 15 red balls and 5 green balls. A ball is selected at random, its colour is recorded and then it is replaced. Draw a tree diagram with probabilities listed on its branches to represent two trials of this experiment and find the probability of each outcome.

THINK

- Draw a tree diagram to represent this experiment, listing all the final outcomes.
- Find the probability of a green ball being selected: $\Pr(\text{G}) = \frac{5}{20} = \frac{1}{4}$
 Find the probability of a red ball being selected: $\Pr(\text{R}) = \frac{15}{20} = \frac{3}{4}$
- Write the probability on each branch.
- Multiply the probabilities along each branch to determine the probability of each final outcome.

WRITE

		Outcomes	Probability
	R	RR	$\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$
	G	RG	$\frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$
	R	GR	$\frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$
	G	GG	$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$



Example 8F.4 Calculating probabilities for experiments with outcomes that are not equally likely

This tree diagram shows the possible outcomes when two customers select a ball at random from a box containing three red balls, two green balls and one blue ball. The ball is returned after the first customer's selection.

Customer 1	Customer 2	Outcomes	Probability
	R	R	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
	G	RG	$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
	B	RB	$\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$
	R	GR	$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$
	G	GG	$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$
	B	GB	$\frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$
	R	BR	$\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$
	G	BG	$\frac{1}{6} \times \frac{1}{3} = \frac{1}{18}$
	B	BB	$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

Calculate the probability of selecting:

- a** exactly one blue ball **b** a blue ball and a green ball.

THINK

- a** Identify the outcomes in which exactly one blue ball is selected. Find the sum of the probabilities of the outcomes and simplify.
- b** Identify the outcomes in which a blue ball and a green ball are selected (in any order). Find the sum of the probabilities of the outcomes and simplify.

WRITE

a $\Pr(\text{exactly one blue}) = \Pr(\text{RB}) + \Pr(\text{GB}) + \Pr(\text{BR}) + \Pr(\text{BG})$

$$= \frac{1}{12} + \frac{1}{18} + \frac{1}{12} + \frac{1}{18}$$

$$= \frac{10}{36}$$

$$= \frac{5}{18}$$

b $\Pr(\text{blue and green}) = \Pr(\text{GB}) + \Pr(\text{BG})$

$$= \frac{1}{18} + \frac{1}{18}$$

$$= \frac{2}{18}$$

$$= \frac{1}{9}$$

Helpful hints

- ✓ The probabilities at each step of a two-step experiment always sum to 1.
- ✓ After calculating the probabilities of the final outcomes, check that the sum of these probabilities is equal to 1.

Exercise 8F Experiments with replacement

 1-9, 13

 2, 4-7, 10, 11, 13-15

 2, 4, 6, 7, 11, 12, 15, 16

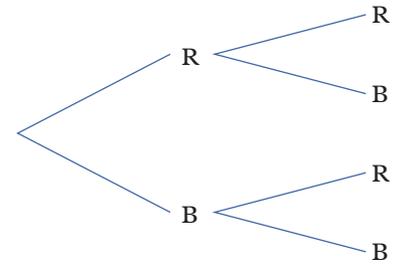
- 1 For each chance experiment, state the theoretical probability of the outcome in brackets in any given trial.
- selecting a marble and recording its colour from a bag with 10 blue, 5 red, 10 yellow and 5 green marbles (selecting a green marble)
 - rolling a standard six-sided die and recording the number on top (rolling a 5)
 - drawing a card from a standard deck of 52 playing cards (drawing a red picture card – jack, queen or king)
 - selecting a gift voucher from a lucky dip containing twelve \$5 vouchers, eight \$10 vouchers and four \$20 vouchers (selecting a \$20 voucher)

8F.1 2 Draw a tree diagram to represent each of the following situations, complete with probabilities on each branch and for each final outcome.

- A pencil case contains an equal number of red and blue pens. A pen is selected, its colour is recorded and then it is replaced. This is repeated one more time.
- A box contains an equal number of \$5, \$20 and \$75 vouchers. A voucher is selected, its value is recorded and then it is replaced. This is repeated one more time.
- A box contains 11 cards, numbered 1 to 11. A card is selected, it is recorded whether the number is even or odd, and then the card is replaced. This is repeated one more time.
- A ball pit contains an equal number of blue, red, green and yellow balls. A ball is selected, its colour is recorded and then it is replaced. This is repeated one more time.

8F.2 3 A bag contains six red counters and six black counters. A counter is drawn twice, being replaced after the first draw. The tree diagram on the right shows the outcomes of the two selections.

- In any given trial, what is the probability of selecting a black counter?
- How many final outcomes are there?
- What is the probability of selecting:
 - two black counters
 - exactly one black counter
 - two red counters?



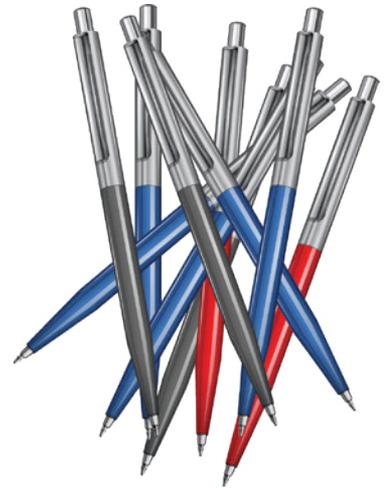
- 4 A box contains milk, dark and white chocolates in equal numbers. A chocolate is selected at random from the box, its flavour recorded and then it is replaced. This is repeated once more. What is the probability that:
- both chocolates are white
 - at least one chocolate is dark
 - the first chocolate is white and the second chocolate is milk
 - one chocolate is white and one chocolate is milk?
- 5 There are 52 cards in a standard pack of playing cards with 13 cards of each suit (clubs, diamonds, hearts and spades). A card is chosen at random from a pack, its suit is recorded and then it is replaced. This is repeated once more. What is the probability of selecting:
- two hearts
 - at least one diamond
 - exactly one club
 - no spades
 - at least one diamond or spade
 - one heart and one club?

- 8F.3 6** Draw a tree diagram with probabilities on the branches for each of these chance experiments.
- A pencil case contains five blue pens and two red pens. A pen is chosen, its ink colour is recorded and then it is replaced. This is repeated one more time.
 - A box contains fifteen \$5 vouchers, ten \$20 vouchers and five \$75 vouchers. A voucher is chosen, its value is recorded and then it is replaced. This is repeated one more time.
 - A box contains 15 cards, numbered 1 to 15. A card is chosen, it is recorded whether the number is even or odd, and then the card is replaced. This is repeated a total of two times.
 - A ball pit contains five blue, four red, three green and two yellow balls. A ball is chosen, its colour recorded, and then it is replaced. This is repeated one more time.

- 8F.4 7** A lucky dip contains 10 pink gift vouchers for \$100 and 40 green gift vouchers for \$10. A voucher is chosen, its value is recorded and then it is replaced. If this is repeated, find the probability that:

- a \$100 voucher is selected twice
- a \$100 voucher is not selected at all
- a \$100 voucher is selected first, and a \$10 voucher selected second.

- 8** A pencil case contains five blue pens, three black pens and two red pens. The owner of the pencil case takes out a pen at random for each lesson.
- Draw a tree diagram to represent the colours of the pens chosen for the first two lessons of the day. Remember to include the probabilities along each branch and for the final outcomes.
 - Find the probability that the owner chooses:
 - a blue pen each time
 - a red pen each time
 - a black pen each time
 - a blue pen, then a black pen
 - a blue pen, then a red pen
 - a red pen, then a black pen.



- 9** The probability of selecting a picture card from a standard 52-card pack is $\frac{3}{13}$. A card is chosen from a standard pack of 52 cards and it is recorded whether it is a picture card or not, before the card is replaced. This is repeated once more.

- Draw a tree diagram to represent this situation. Remember to include probabilities on the branches and calculate the final probability of each outcome as a decimal number rounded to four decimal places.
- Find the probability of selecting:
 - exactly one picture card
 - at least one picture card
 - two picture cards
 - no picture cards.



- 10** In Example 8F.4 part **a** there are four favourable outcomes out of nine possible outcomes. Explain why the theoretical probability of selecting exactly one blue ball is $\frac{5}{18}$ instead of $\frac{4}{9}$.
- 11** A store has a 'lucky dip' sale, where you get a discount based upon the colour of a ball you choose from a box at random. If you take a red ball you get 10% off, if you take a green ball you get 25% off and if you take a blue ball you get 50% off. There are 10 balls of each colour in the box.
- You can have a second chance if you don't take a blue ball out of the box first up, as long as you put the first ball back into the box.
- Use a tree diagram or other means to show that you have a $\frac{5}{9}$ chance of selecting a blue ball from the box.

- 12** An array can be used to list the outcomes of a two-step experiment with replacement.
- Explain why an array may be a better choice to show the outcomes of a two-step experiment with replacement than a tree diagram when there is a large number of equally likely outcomes for each step (even when like outcomes are combined).
 - Explain how you could represent outcomes that are not equally likely in an array.
- 13** A sock drawer contains 10 socks; some are black and some are white. You need to find out how many of each colour are in the drawer, but you can only select one sock at a time and place it back.
- If you selected with replacement 10 times and selected three black socks and seven white socks, does this mean that there are three black and seven white socks in the drawer? Explain.
 - If you selected with replacement 50 times, selecting 21 black socks and 29 white socks, how many socks of each colour would you estimate are in the drawer?
 - If you selected with replacement 80 times, selecting 34 black socks and 46 white socks, does this support your previous estimate?
- 14** A bag contains 12 aqua marbles and some green marbles.
- If the probability of selecting an aqua marble is 0.3, find the number of green marbles.
Three marbles are drawn from the bag and each marble is replaced in the bag before the next is drawn.
 - Draw a tree diagram representing this situation. Include the probability of each step on the branches and calculate the probability of each outcome.
 - Use your tree diagram to find the probability of selecting:
 - three marbles of the same colour
 - exactly two green marbles
 - exactly two marbles of the same colour
 - a green marble first and an aqua marble third.
- 
- 15** A bag contains some white, black and red tiles. Three tiles are drawn from the bag and their colours noted. Each tile is replaced in the bag before the next tile is drawn.
- If you were to represent this situation with a tree diagram, how many different outcomes would there be?
 - Write out all possible outcomes where three different coloured tiles are drawn.
 - If $\Pr(\text{drawing a white tile}) = w$ and $\Pr(\text{drawing a black tile}) = b$, find the probability that three different coloured tiles are drawn.
- 16** A bag contains different numbers of tiles labelled A, B, and C. A tile is taken out of the bag at random, the letter is observed, then the tile is replaced into the bag before a second tile is taken out of the bag at random. Let the probability of selecting a tile labelled A on any given selection be $\Pr(A) = a$, and the probability of selecting a tile labelled B on any given selection be $\Pr(B) = b$. Write an expression for each of the following in terms of a and b .
- the probability of selecting two tiles labelled A
 - the probability of selecting a tile labelled A and a tile labelled B in any order
 - the probability of selecting a tile labelled C on any given selection
 - the probability of selecting a tile labelled A and a tile labelled C in any order in:
 - factorised form
 - expanded form
 - the probability of selecting two tiles labelled C in:
 - factorised form
 - expanded form

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Interactive skillsheet
Experiments with replacement



Investigation
Three-step experiments with replacement



Topic quiz
8F

8G Experiments without replacement

Learning intentions

By the end of this topic you will be able to ...

- ✓ calculate probabilities for two-step chance experiments without replacement.



Inter-year links

- Year 7** 9G Theoretical probability
- Year 8** 9F Theoretical probability
- Year 10** 11E Experiments with and without replacement

Experiments without replacement



Key content video

- Experiments without replacement involve selecting an item at random, recording the result, and then not replacing the item before making another selection.
- The selected items are not replaced, so the probability of the remaining items being selected changes.
- A tree diagram or list of outcomes can help to find the probabilities of individual outcomes or events involving more than one outcome.
- For example, the following tree diagram shows the results of a customer choosing two socks at random from 10 red, 10 green and 10 blue socks. The first sock is not replaced so the probabilities change for subsequent selections.

→ If a pair of red socks is selected, the first selection is from 10 red socks out of 30, and the second selection is from 9 red socks out of 29.

$$\text{Selection 1: Pr(R)} = \frac{10}{30} = \frac{1}{3}$$

$$\text{Selection 2: Pr(R)} = \frac{9}{29}$$

→ If a red sock is selected followed by a green, the first selection is from 10 red socks out of 30 and the second selection is from 10 green socks out of 29.

$$\text{Selection 1: Pr(R)} = \frac{10}{30} = \frac{1}{3}$$

$$\text{Selection 2: Pr(G)} = \frac{10}{29}$$

→ The probabilities of the other outcomes are calculated in a similar manner.

→ The probability of selecting a pair of socks that are the same colour can be calculated:

$$\text{Pr(RR)} + \text{Pr(GG)} + \text{Pr(BB)} = 3 \times \left(\frac{1}{3} \times \frac{9}{29} \right) = \frac{9}{29}$$

→ The probability of selecting a pair of socks that are different colours can be calculated:

$$\text{Pr(RG)} + \text{Pr(RB)} + \text{Pr(GR)} + \text{Pr(GB)} + \text{Pr(BR)} + \text{Pr(BG)} = 6 \times \left(\frac{1}{3} \times \frac{10}{29} \right) = \frac{20}{29}$$

- As the probabilities of the events at the second step of an experiment without replacement depend on the results from the first step, the events at the second step are dependent events.

	Selection 1	Selection 2	Outcomes	Probability
R	$\frac{1}{3}$	R	RR	$\frac{1}{3} \times \frac{9}{29} = \frac{9}{87}$
		G	RG	$\frac{1}{3} \times \frac{10}{29} = \frac{10}{87}$
		B	RB	$\frac{1}{3} \times \frac{10}{29} = \frac{10}{87}$
G	$\frac{1}{3}$	R	GR	$\frac{1}{3} \times \frac{10}{29} = \frac{10}{87}$
		G	GG	$\frac{1}{3} \times \frac{9}{29} = \frac{9}{87}$
		B	GB	$\frac{1}{3} \times \frac{10}{29} = \frac{10}{87}$
B	$\frac{1}{3}$	R	BR	$\frac{1}{3} \times \frac{10}{29} = \frac{10}{87}$
		G	BG	$\frac{1}{3} \times \frac{10}{29} = \frac{10}{87}$
		B	BB	$\frac{1}{3} \times \frac{9}{29} = \frac{9}{87}$

Example 8G.1 Representing experiments without replacement



A lucky dip contains five red gift vouchers for \$50 and five green gift vouchers for \$5. A gift voucher is selected and the customer keeps it. Draw a tree diagram with probabilities listed on its branches to represent two trials of this experiment and find the probability of each possible outcome.

THINK

- 1 Draw a tree diagram to represent this two-step experiment, listing all the final outcomes.
- 2 Write the probability at the first branches of your tree diagram. Initially there is a $\frac{5}{10}$ chance of selecting a red gift voucher, and a $\frac{5}{10}$ chance of selecting a green gift voucher.
- 3 Determine the probabilities at the second branches and write on your tree diagram.
 - If you select a red gift voucher, four red and five green vouchers remain.
 - If you select a green gift voucher, five red and four green gift vouchers remain.
- 4 Multiply the probabilities along each branch to determine the probability of each final outcome.

WRITE

Customer 1	Customer 2	Outcomes	Probability
	R	RR	$\frac{5}{10} \times \frac{4}{9} = \frac{20}{90} = \frac{2}{9}$
	G	RG	$\frac{5}{10} \times \frac{5}{9} = \frac{25}{90} = \frac{5}{18}$
	R	GR	$\frac{5}{10} \times \frac{5}{9} = \frac{25}{90} = \frac{5}{18}$
	G	GG	$\frac{5}{10} \times \frac{4}{9} = \frac{20}{90} = \frac{2}{9}$

Example 8G.2 Calculating probabilities for experiments without replacement



Use the tree diagram from Example 8G.1 to find the probability that:

- a both customers select a \$50 voucher
- b the first customer selects a \$5 voucher and the second customer selects a \$50 voucher.

THINK

- a Identify the outcome where both customers select a \$50 voucher.
- b Identify the outcome where the first customer selects a \$5 voucher and the second customer selects a \$50 voucher.

WRITE

- a $\Pr(\text{both } \$50) = \Pr(\text{RR})$
 $= \frac{20}{90}$
 $= \frac{2}{9}$
- b $\Pr(\text{first } \$5 \text{ and second } \$50) = \Pr(\text{GR})$
 $= \frac{25}{90}$
 $= \frac{5}{18}$

- ✓ If one object is selected in the first step, the number of total outcomes decreases by 1 after the first step of a two-step chance experiment without replacement.
- ✓ The number of favourable outcomes decreases by 1 only for the outcome occurring in the first step.
- ✓ When calculating probabilities involving multiple final outcomes, it is often easier to not simplify fractions until the end of your working, as it is easier to add fractions when they have a common denominator.

ANS
p503

Exercise 8G Experiments without replacement

▲ 1-4, 6, 7, 9, 10, 14

■ 1(b, c), 2, 4-6, 8, 11, 12, 13(a), 15

◆ 1(c), 4-6, 11-13, 15-17

UNDERSTANDING AND FLUENCY

- 8G.1 1** Draw a tree diagram with probabilities on the branches for each of these chance experiments without replacement.
- a** A drawer contains five black socks and five white socks. A sock is selected at random and its colour recorded. This is repeated one more time.
 - b** An esky contains six cans of Coke and six cans of Pepsi. A can is selected at random and its type recorded. This is repeated one more time.
 - c** A box contains five 16 GB SD cards, five 32 GB SD cards and five 64 GB SD cards. A card is selected at random and its capacity recorded. This is repeated once more.

- 8G.2 2** The tree diagram on the right shows how two students can be selected at random from a group of four boys and four girls. Find the probability of selecting:

- a** two boys
- b** no boys
- c** a boy and a girl.

- 3** A lucky dip contains four purple gift vouchers for \$100 and four yellow gift vouchers for \$10. A gift voucher is chosen at random and the customer keeps it. If this was repeated for a second customer, find the probability that:

- a** both customers select a \$100 voucher
- b** the first customer selects a \$100 voucher and the second customer selects a \$10 voucher.

- 4** Each bonbon in a pack of 12 contains one toy, and there are three different kinds of toys: a whistle, a yo-yo and a bouncy ball. In total, the pack contains four of each kind of toy. You and a friend each select a bonbon at random from the pack.

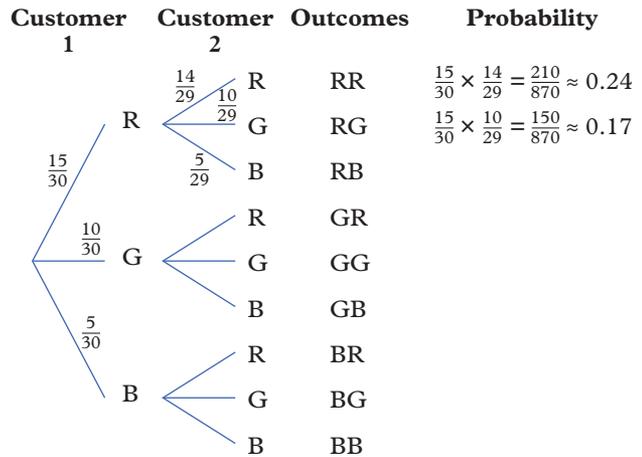
- a** Draw a tree diagram to represent this situation. Be sure to include probabilities on each branch and the final probabilities for each outcome.
- b** Find the probability that:
 - i** you both select a bonbon with a whistle
 - ii** the first bonbon has a yo-yo and the second has a bouncy ball.

Student 1	Student 2	Outcomes	Probability
$\frac{4}{8}$ B	$\frac{3}{7}$ B	BB	$\frac{4}{8} \times \frac{3}{7} = \frac{12}{56} = \frac{3}{14}$
	$\frac{4}{7}$ G	BG	$\frac{4}{8} \times \frac{4}{7} = \frac{16}{56} = \frac{2}{7}$
$\frac{4}{8}$ G	$\frac{4}{7}$ B	GB	$\frac{4}{8} \times \frac{4}{7} = \frac{16}{56} = \frac{2}{7}$
	$\frac{3}{7}$ G	GG	$\frac{4}{8} \times \frac{3}{7} = \frac{12}{56} = \frac{3}{14}$



- 5 Consider the bonbons from question 4. Find the probability that:
- you both select a bonbon with a different toy
 - at least one of you selects a bonbon with a bouncy ball
 - a bonbon with a whistle is not chosen.

6 Experiments without replacement can also start with unequally likely outcomes. A lucky dip contains 15 red, 10 green and 5 blue balls. Customers select one ball at random and they do not replace the ball before the next customer makes a selection.



- Complete the tree diagram on the right.
- Use the tree diagram to calculate the probability that:
 - both customers select a blue ball
 - both customers select a green ball
 - both customers select a red ball
 - the first customer selects a red ball and the second selects a green ball.

- 7 Use the tree diagram from question 6 to calculate the probability of selecting:
- at least one green ball
 - at least one red ball
 - at least one blue ball
 - exactly one red ball
 - exactly one green ball
 - a blue ball and a green ball.

8 There are 52 cards in a standard pack of playing cards, with 13 cards of each suit (clubs, diamonds, hearts and spades). A card is selected and its suit is recorded. The card is not replaced and another card is selected. What is the probability of selecting:

- two hearts
- at least one diamond
- no spades
- one heart and one club?

- 9 A drawer contains two pink socks, two purple socks and two green socks. Use a tree diagram or other means to calculate the probability that you take out a pair when you select two socks from the drawer at random.
- 10 Imagine instead that the drawer from question 9 contains six socks of each colour. How does this change the probability of selecting a pair when you select two socks from the drawer at random?



- 11 If a drawer contained five red socks, four black socks and three white socks, find the probability that the first two socks selected at random from the drawer form a pair.
- 12 A sports team needs to select a captain and a vice-captain. Five people have put their names forward: Adrian, Chantelle, Katie, Guillermo and Sam.
- Draw a tree diagram to represent the selection (start with 'captain' branches).
 - How does this tree diagram differ from the other ones you have created? Hint: Does the second set of branches contain the same number as the first set of branches?
 - How many different combinations of captains and vice-captains are there? Remember that order is important!
 - If each person has an equal chance, then find the probability that:
 - Katie is selected captain
 - Sam is selected either captain or vice-captain
 - Adrian is captain and Chantelle is vice-captain
 - Guillermo does not get a position
 - Katie and Sam both get a position.

13 An array can be used to show the outcomes for a two-step experiment without replacement. It is important to show the outcomes that cannot occur, as they are the repeated outcomes that have been removed after the first step. We show this by placing a cross in that cell. For example, when selecting two marbles from a bag that contains five red marbles and four blue marbles without replacement, the array can be drawn as shown.

a Determine the probability of selecting the following combinations of marbles from the bag with five red marbles and four blue marbles.

- i** two red marbles
- ii** two blue marbles
- iii** a red marble then a blue marble
- iv** a red and a blue marble in either order

b Another bag contains three green marbles and one purple marble. Two marbles are selected from the bag at random without replacement.

- i** Draw an array to show the outcomes for this experiment.
- ii** Determine the probability of selecting one purple marble out of the two marbles.
- iii** Determine the probability of selecting two purple marbles.

14 A box of chocolates contains four milk chocolates, three white chocolates and two dark chocolates. Two chocolates are selected from the box, without looking. Find the probability of selecting:

- a** two white chocolates
- b** no dark chocolates
- c** two different types of chocolate
- d** at least one white chocolate.

15 Johan receives a chocolate box like the one in question 14. He decides to eat three randomly chosen chocolates and share the rest with his friends. Find the probability that he chooses:

- a** three milk chocolates
- b** one of each chocolate
- c** no milk chocolates.

16 A lottery consists of 45 numbered balls, of which six winning balls are chosen at random, with none of the six balls drawn replaced. To win the first division prize, you must pick all six winning numbers. What is the probability of winning the first division prize in this lottery? Express your answer as both a fraction and a decimal correct to three significant figures.

17 An album contains nine songs and the lengths of the songs are shown in the table in minutes and seconds. Find the probability that the lengths of the first two different songs played on shuffle are above the mean length of a track on the album.

4:17	1:08	4:22	1:38	4:16	0:53	4:20	1:44	4:46
------	------	------	------	------	------	------	------	------

		First								
		R	R	R	R	R	B	B	B	B
Second	R	×	RR	RR	RR	RR	BR	BR	BR	BR
	R	RR	×	RR	RR	RR	BR	BR	BR	BR
	R	RR	RR	×	RR	RR	BR	BR	BR	BR
	R	RR	RR	RR	×	RR	BR	BR	BR	BR
	R	RR	RR	RR	RR	×	BR	BR	BR	BR
	B	RB	RB	RB	RB	RB	×	BB	BB	BB
	B	RB	RB	RB	RB	RB	BB	×	BB	BB
	B	RB	RB	RB	RB	RB	BB	BB	×	BB
	B	RB	RB	RB	RB	RB	BB	BB	BB	×



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Interactive skillsheet
Experiments without replacement



Investigation
Draws for tennis tournaments



Topic quiz
8G

8H Experimental probability and simulations

Learning intentions

By the end of this topic you will be able to ...

- ✓ design and conduct a simulation
- ✓ apply reasoning to evaluate a simulation and its related outcomes.



Inter-year links

Year 7

9H Experimental probability

Experimental probability



Key content video

- **Experimental probability** is probability that is based on the results of experiments, instead of mathematical reasoning.
- The **relative frequency** of an event occurring is determined by an experiment involving a number of trials.

$$\text{Relative frequency} = \frac{\text{number of successful trials}}{\text{total number of trials}}$$

- For a small number of trials, the relative frequency may differ significantly from the theoretical probability.
- As the number of trials increases, the relative frequency is likely to approach the theoretical probability. This is known as the **law of large numbers**.

Simulations

- A **simulation** is a recreation of the random element of an experiment, event or situation. A simulation can be used when performing an experiment is impractical, too expensive or time consuming.
- A simulation can be performed using a simple random device (for example, a coin, die or spinner) or technology that generates random outcomes (for example, software or a calculator).
- Before performing a simulation you must know the number of outcomes and the outcomes must be equally likely to occur.

Example 8H.1 Choosing a device for a simulation



A quiz contains 25 questions which are all multiple choice (options A–D). Annabelle wants to simulate guessing the correct answer to each question.

- Choose a device to use for this simulation.
- Assign each device outcome to an outcome of the situation.

THINK

- The situation has four equally likely outcomes (A, B, C and D), so choose a device that has four equally likely outcomes.
- Randomly assign each device outcome (club, diamond, heart, spade) to a situation outcome (A, B, C, D).

WRITE

- A deck of cards could be used for this simulation, as each card is one of four suits: clubs, diamonds, hearts or spades.
- club = A, diamond = B, heart = C and spade = D.

Example 8H.2 Considering the potential limitations of simulations



Discuss the potential limitations of using a spinner with 10 equal sections to simulate collecting a set of 10 cards which are found randomly in packets of chips.

THINK

- 1 Are the outcomes in the simulation equally likely? If not, why not?
- 2 Are there any potential issues with using the chosen device for this simulation?

WRITE

The cards may not be randomly distributed throughout the chip packets.
The spinner must spin smoothly and have an equal chance of landing on each section.
The person spinning the spinner may be able to influence where it points to.

Helpful hints

- ✓ The outcomes of a simulation device should be random and equally likely to occur. Consider whether this is the case when discussing the potential limitations of a simulation.
- ✓ The law of large numbers emphasises the idea that, over a large number of trials, results will tend to even out and become more predictable. However, past outcomes do not influence future results. For example, if you flip a fair coin and get heads five times in a row, you may think you are 'due' for a tails on the next flip, as if the coin is trying to 'balance' the previous outcomes. However, this is incorrect because each flip of the coin is independent, and the probability of getting heads or tails remains the same for each flip.

ANS
p504

Exercise 8H Experimental probability and simulations

▲ 1-5(a-c), 8, 9

■ 1-3(b-d), 4(d-f), 6-9

◆ 4-11

- 8H.1** 1 For each of the following situations:
- i choose a device to use for a simulation
 - ii assign each device outcome to a situation outcome.
- a guessing the answer to a true or false question
 - b being randomly placed into Group A, B, C or D
 - c randomly choosing whether to turn left or right when at a junction in a maze
 - d guessing the day of the week on which someone was born
- 2 Consider each of the situations in question 1. Are the outcomes equally likely? If not, why not?
- 3 For each situation, choose a device for simulation and assign each device outcome to a situation outcome.
- a randomly selecting a Freddo Frog from one of eight available flavours
 - b being randomly assigned to walk in one of the main compass directions
 - c getting one of ten different toys in a Christmas cracker
 - d randomly guessing the month of the year in which someone was born

- 8H.2** 4 Discuss the potential limitations of each of the following simulations.
- using a coin to simulate getting your preferred meal out of two options
 - using a deck of cards to simulate being born in one of the four seasons
 - using a spinner with seven sections to simulate being assigned to one of seven classes
 - using a standard die to simulate being put in one of six sports teams
 - using a 10-sided die to simulate winning the grand final in a competition of 10 teams
 - using a spinner with 25 sections to simulate selecting one person out of a class of 25
- 5 Choose a device to simulate each situation and discuss the assumptions you need to make for each simulation.
- randomly selecting a Chupa Chups from six available flavours
 - a newborn baby being a boy or girl
 - choosing one of four available desserts
 - one person being chosen leader out of a group of five people
 - randomly selecting an M&M and recording its colour (out of six available colours)
 - guessing the answer to a multiple-choice question with options A–E
- 6 There are three main types of Boxer dogs: plain (either fawn or brindle), flashy (with white markings) or white. When two flashy Boxers breed, the puppies can be plain (25%), flashy (50%) or white (25%).
- State these theoretical probabilities as fractions.
 - How could two coins be used to simulate the type of puppy that is born when two flashy Boxers breed?
 - Assign each device outcome to a situation outcome (puppy type).
 - Simulate the birth of 50 puppies and put your results in a table.
 - Determine the relative frequency for each outcome and compare it to the theoretical probability.
 - Discuss the potential limitations of using a simulation to model this situation.
- 7 Consider the simulation from question 6. On average, a litter contains eight puppies.
- Complete 10 experiments with eight trials each and record the number of plain, flashy and white puppies in each litter.
 - Calculate the relative frequency of receiving each type of puppy and compare it to:
 - the theoretical probability
 - your results from question 6 part d.
 - Find the average number of each puppy type born in a typical litter using your simulation results.
 - A breeder can make \$500 from every plain or flashy puppy, but only \$200 from every white puppy.
 - Using your simulation results, how much money could potentially be made from the 10 litters?
 - Find the average amount of money made per litter.
- 8 A promotion is running where one in every six Mars Bar wrappers wins a free bar.
- Explain why even though there are only two outcomes (winning and not winning), you could treat this problem like it has six outcomes.
 - What device could you use to simulate winning a free bar? Assign each device outcome to a situation outcome.
 - How many free Mars Bars would you expect to win if you purchased 30 Mars Bars?
 - Perform 30 trials and record your results. How do they compare to the expected numbers?
 - What would you expect to see in your experiment if you did 3000 trials? Explain your answer.



9 Running 3000 trials of the experiment from question 8 would take a long time. However, you can use a random number generator instead to simulate winning free Mars Bars. One way to generate random numbers is to use a spreadsheet.

- a Open a new file in Excel and enter **=RANDBETWEEN(1, 6)** into a cell. What do you get?
- b Do this 30 times and record your results. Do the results appear to be random?

		B2	fx =1/6	
	A	B	C	D
1	Number	Probability	Count	
2	1	0.166666667		
3	2	0.166666667		
4	3	0.166666667		
5	4	0.166666667		
6	5	0.166666667		
7	6	0.166666667		

Complete the following steps to simulate the 3000 trials of the experiment from question 8.

- 1 Set up the spreadsheet as shown in the screenshot on the right.
 - 2 Enter **=RANDARRAY(3000,1,1,6,TRUE)** in cell E1. This will create 3000 random numbers in column E.
 - 3 Enter **=COUNTIF(E1:E3000,1)** in cell C2. This will count all the 1s in the numbers you have generated.
 - 4 Enter **=COUNTIF(E1:E3000,2)** to count all the 2s in C3. Continue the pattern for C4 to C7.
 - c If the number 6 represents winning a free Mars Bar, how many free Mars Bars were won in your simulation?
 - d Compare the experimental probability for your simulation to the theoretical probability, and comment on the similarities and differences.
- 10 Earth's surface is roughly 70% water.
- a If a meteorite were to crash into Earth, state the theoretical probability of it landing:
 - i in water
 - ii on land.
 - b Out of every 100 meteorites to reach Earth, how many would you expect to land on land?
 - c Choose a device to simulate where a meteorite will land on Earth and assign each device outcome to a situation outcome. Hint: Use a device with 10 equally likely outcomes.
 - d Perform 100 trials of simulation and record your results.
 - e Do the relative frequencies match the theoretical probabilities? Comment on your results.

- 11 A couple want to have children, and would like to have at least one boy and one girl.
- a Assuming that the outcomes are equally likely (50% boy and 50% girl), what device could you use to simulate gender at birth? Assign each device outcome to a situation outcome.
 - b Keep performing trials until you have simulated at least one child of each gender. How many trials did this take?
 - c Repeat part b nine times (to give a total of 10 simulations) and put your results into a table.
 - d Based on your results, what is the mean number of children the couple would need to have in order to have at least one boy and one girl?
 - e What could be done to make the estimate in part d more accurate?
 - f Repeat this experiment another 40 times and tabulate your results. From the 50 simulations, what is the mean number of children the couple would need to have in order to have at least one boy and one girl?
 - g Compare the answer to part f to the answer to part d. What do you find?

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Relative frequency



Worksheet
Simulations



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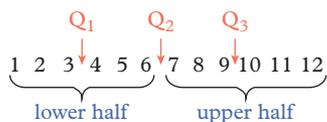


Topic quiz
8H

Chapter summary

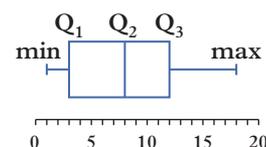
Five-number summary

min, Q_1 , Q_2 (median), Q_3 , max



Box plots

- Lower fence: $1.5 \times \text{IQR}$ below lower quartile
- Upper fence: $1.5 \times \text{IQR}$ above upper quartile
- Outliers: values that lie beyond the fences



Standard deviation

Population's standard deviation:

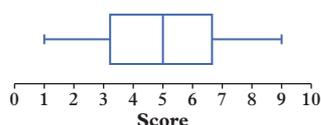
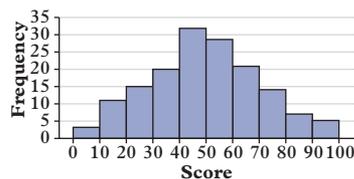
$$\sigma = \sqrt{\frac{(x_1 - \mu)^2 + (x_2 - \mu)^2 + \dots + (x_n - \mu)^2}{n}}$$

Sample's standard deviation:

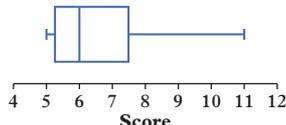
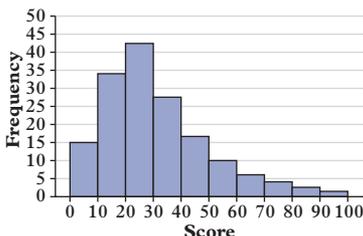
$$s = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_n - \bar{x})^2}{n - 1}}$$

Distributions

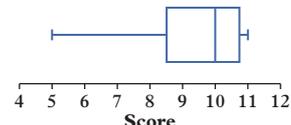
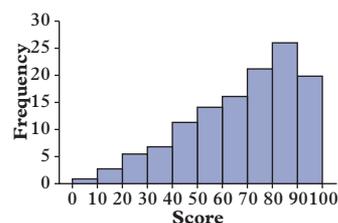
- Symmetric



- Positively skewed



- Negatively skewed



Theoretical probability

- Sample space: all possible outcomes
- Event: collection of one or more outcomes from the sample space
- If outcomes are equally likely:

$$\Pr(\text{event}) = \frac{\text{number of outcomes in the event}}{\text{number of outcomes in the sample space}}$$
- A' means not A . A and A' are complementary events.

$$\Pr(A) + \Pr(A') = 1$$

Arrays

- Use when there is a large number of outcomes in either step of a two-step experiment.

	1	2	3	4	5	6
H	(1, H)	(2, H)	(3, H)	(4, H)	(5, H)	(6, H)
T	(1, T)	(2, T)	(3, T)	(4, T)	(5, T)	(6, T)

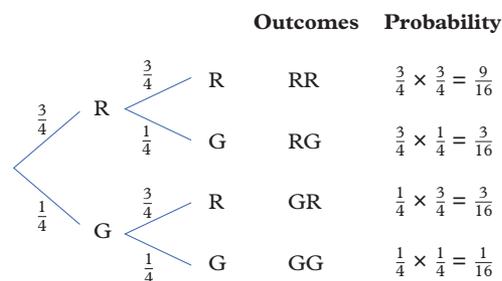
Experiments with/without replacement

- With replacement: probabilities for each step remain the same.
- Without replacement: probabilities for all remaining steps are changed.
- Tree diagrams and lists of outcomes can help find probabilities of multi-step experiments.

Experimental probability and simulations

- Relative frequency = $\frac{\text{number of occurrences}}{\text{total number of trials}}$
- With a small number of trials, results may vary significantly from expected.
- In the long term, relative frequency will approach theoretical probability.
- A simulation can be a simple random device (for example, a coin, die or spinner) or technology that generates random outcomes (for example, software or a calculator).

Tree diagrams





Chapter review quiz

Assess your knowledge of this chapter.

Quizlet

Test your knowledge of this topic by working individually or in teams.

Mathematical literacy review

The following key terms are used in this chapter:

- array
- box plot
- complementary events
- dependent event
- event
- experiment
- experimental probability
- five-number summary
- histogram
- independent event
- interquartile range
- law of large numbers
- lower fence
- lower quartile
- mean
- measure of spread
- median
- negatively skewed distribution
- outcome
- outlier
- parallel box plots
- population
- positively skewed distribution
- probability
- quartiles
- range
- relative frequency
- sample
- sample space
- simulation
- skewness
- standard deviation
- symmetric distribution
- symmetry
- theoretical probability
- tree diagram
- trial
- upper fence
- upper quartile

- 1 Use the benchmark percentages 0%, 25%, 50%, 75% and 100% to explain what the minimum value, lower quartile, median, upper quartile and the maximum value of a data set are.
- 2 Which of the key terms from the above list are measures of spread?
- 3 Explain the difference between a negatively skewed distribution and a positively skewed distribution.
- 4 Identify the key terms being referenced in each of these definitions.
 - a a recreation of the random element of an experiment, event or situation
 - b an event whose outcomes are affected by the result of another event
- 5 Using an example, provide a definition in your own words for the following key terms.

a complementary events	b lower fence
------------------------	---------------
- 6 If a fair coin is flipped twice, list all the outcomes in the following events:

a getting exactly one head	b getting at least one head
c getting no head	d getting two different results
e getting more than one head.	

Multiple choice

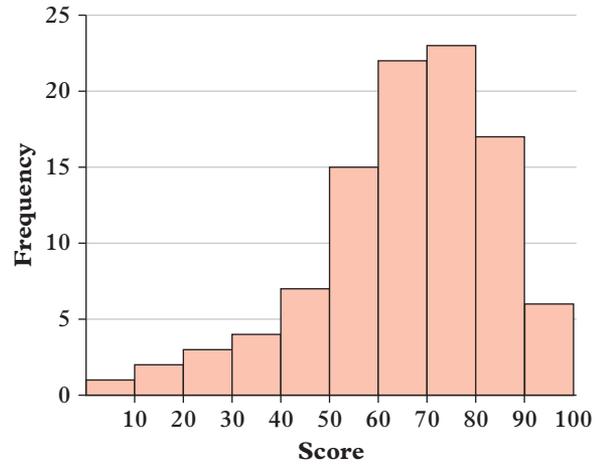
- 8A 1 A data set consists of 14 unique values. Which of the following is correct?
 - A The median and both upper and lower quartiles are values in the set.
 - B The median is a value in the set, but the upper and lower quartiles are not.
 - C The upper and lower quartiles are values in the set, but the median is not.
 - D None of the median, upper or lower quartiles are values in the set.
 - E The median and the upper quartile are values in the set, but the lower quartile is not.
- 8B 2 In a box plot, what does the 'box' part represent?

A mean	B interquartile range	C range	D median	E standard deviation
--------	-----------------------	---------	----------	----------------------
- 8B 3 The length from the lower quartile to the upper quartile in a box plot represents what percentage of the scores of the data set?

A 25%	B 50%	C 75%	D 90%	E 100%
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8C 4 Which option best describes the histogram on the right?

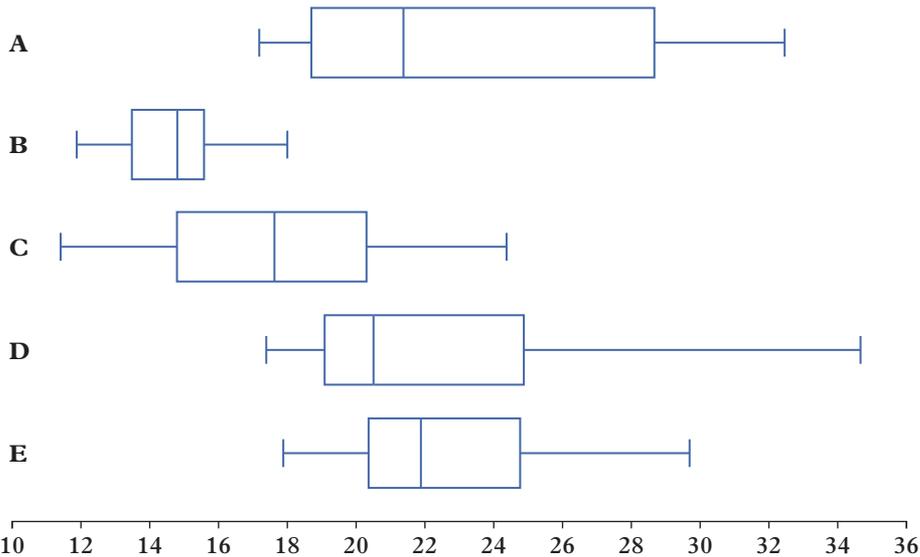
- A positively skewed
- B symmetrical
- C perfectly symmetrical
- D negatively skewed
- E bi-modal



8D 5 What does the standard deviation in a set of data indicate?

- A how large the mean is
- B how the scores are spread out from the mean
- C how many scores there are in the data set
- D where the median lies in the set
- E where the mean lies in the set

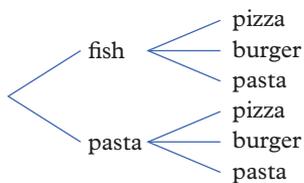
8D 6 Which of the following data sets has the smallest standard deviation?



8E 7 If there is an equal probability of choosing every option on the following menu, the probability that a person orders pasta for both entree and main is:

- A $\frac{1}{3}$
- B $\frac{2}{3}$
- C $\frac{1}{6}$
- D $\frac{1}{2}$
- E $\frac{1}{9}$

Meal choices at a local cafe



8E 8 Two numbers are chosen randomly and their product is recorded. One number is chosen from the set $\{1, 2, 3, 4, 5\}$ and the other is chosen from the set $\{2, 3, 4\}$. The probability of which pair of events is not equal?

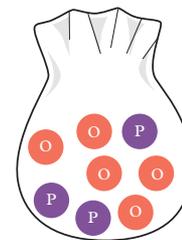
- A $\text{Pr}(4), \text{Pr}(8)$
- B $\text{Pr}(\text{odd}), \text{Pr}(\text{multiple of } 5)$
- C $\text{Pr}(\text{multiple of } 3), \text{Pr}(\text{multiple of } 4)$
- D $\text{Pr}(\text{square number}), \text{Pr}(\text{multiple of } 8)$
- E $\text{Pr}(\text{prime}), \text{Pr}(\text{cube number})$

8F 9 There is a $\frac{1}{13}$ chance of selecting an ace from a standard pack of playing cards. A card is selected at random and an ace is chosen. The card is replaced into the pack. What is the probability of selecting an ace on the second selection?

- A $\frac{1}{13}$
- B $\frac{1}{169}$
- C $\frac{12}{13}$
- D $\frac{144}{169}$
- E $\frac{3}{676}$

Questions 10 and 11 refer to the diagram on the right.

A marble is selected and not replaced, and then a second selection is made.



8G 10 The first marble selected is orange. What is the probability the second marble selected is also orange?

- A** $\frac{4}{7}$ **B** $\frac{3}{7}$ **C** $\frac{1}{7}$ **D** $\frac{5}{7}$ **E** $\frac{2}{7}$

8G 11 The probability of selecting two purple marbles is:

- A** $\frac{3}{8}$ **B** $\frac{2}{7}$ **C** $\frac{5}{14}$ **D** $\frac{3}{28}$ **E** $\frac{3}{4}$

8H 12 Which situation could be simulated using a standard die?

- A** a baby born and its gender recorded
B randomly selecting a marble from a bag containing two blue, four red, five green and three yellow marbles
C randomly guessing the answer to this question
D a winner randomly selected from six finalists
E a red Skittle randomly selected from a bag of Skittles

Short answer

8A 1 For the data in the following stem-and-leaf plot showing the number of ice creams sold per day during the month of January, find:

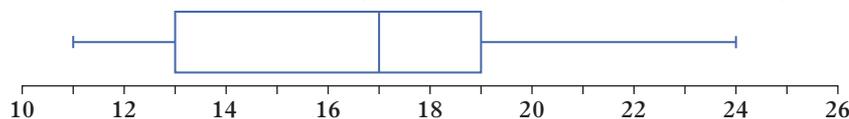
- a** the range **b** the median
c the five-number summary **d** the interquartile range.

Stem	Leaf
4	0 1 3 5 7 9
5	2 5 5 7 8
6	2 3 3 5 7 7 9 9
7	3 5 6 7 8
8	2 4 6 7 8
9	3 6

Key: 4|0 = 40

8B 2 Draw a box plot for the data from question 1 and comment on its shape.

8B 3 State the five-number summary and interquartile range of the data presented in the following box plot.

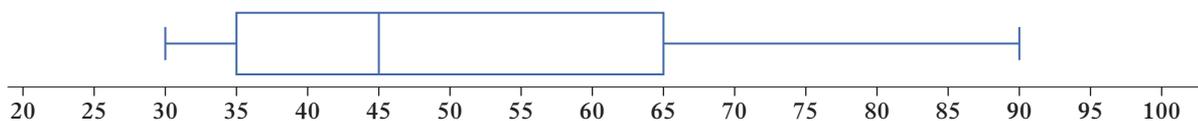


8B 4 State the percentage of the data that lies in the following ranges of values for the box plot above.

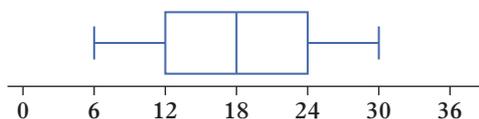
- a** between 17 and 19 **b** less than 11 **c** greater than 13

8C 5 Describe each of the following distributions of data.

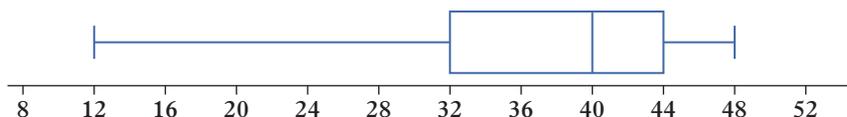
a



b



c



8D 6 Below are the results Lily and Cassy obtained for five tests.

Lily: 72, 77, 67, 69, 75

Cassy: 47, 92, 87, 77, 52

- a** For each student, calculate:
- i** the mean result
 - ii** the standard deviation (to two decimal places).
- b** Which of these two students performed more consistently?

8E 7 A company runs a competition where one out of every four purchases contains the winning bar code for an ebook.

- a** What is the probability of not winning an ebook?
- b** Complete a tree diagram showing all probabilities on the branches for two trials.
- c** Calculate the probability of:
- i** winning twice in a row
 - ii** the complement of winning twice in a row
 - iii** winning on the first try and then not winning on the second try.

8F 8 A coin is tossed and a fair, standard six-sided die is rolled.

- a** Show all possible outcomes on an array.
- b** Calculate the probability that:
- i** the coin lands on heads and the die lands on 2
 - ii** the coin lands on heads or the die lands on 2 or both
 - iii** the coin lands on heads or the die lands on an even number, but not both.
- c** Calculate the probability that the coin lands on heads and the die lands on a multiple of 3.
- d** Calculate the probability that the coin lands on heads or the die lands on a factor of 6 or both.

8F 9 A bag contains six chocolates. Two have orange wrappers, one has a green wrapper and three have pink wrappers. A chocolate is chosen at random and the colour is recorded. The chocolate is replaced and then another is selected.

- a** Show all possible outcomes and probabilities on a tree diagram.
- b** Calculate the probability that:
- i** both wrappers are orange
 - ii** both wrappers are green
 - iii** both wrappers are pink.
- c** Calculate the probability that the first wrapper is green.
- d** Calculate the probability that the second wrapper is pink.

8G 10 There are four suits (clubs, diamonds, hearts and spades) in a standard pack of 52 cards. Two cards are selected from a pack and the suits are noted. Assuming selection without replacement, calculate:

- a** Pr(two spades)
- b** Pr(heart, then spade).

8H 11 Choose a device to simulate each situation.

- a** randomly selecting one piece of sushi from the following options



- b** randomly getting one of six drink options for a meal

Analysis

- 1 The Olympic Games usually take place every four years but were not held during the years of World War I or II, and were delayed one year due to the global pandemic in 2020, so there are some gaps in the record.

Consider this table showing winning times for the 100 m sprint since 1952.

Year	1952	1956	1960	1964	1968	1972	1976	1980	1984
Time (s)	10.79	10.62	10.32	10.06	9.95	10.14	10.06	10.25	9.99

Year	1988	1992	1996	2000	2004	2008	2012	2016	2021
Time (s)	9.92	9.96	9.84	9.87	9.85	9.69	9.63	9.81	9.80



- a From the data in the table, calculate the:
- mean (to two decimal places) and median
 - range and interquartile range
 - five-number summary.
- b Find the standard deviation of the data set correct to two decimal places.
- c
- Draw a box plot to represent the data from the table.
 - Comment on the distribution of the data, and explain what this means in the context of the data.
 - Explain how you think the shape of the box plot will change as future winning times are added to the data.

- 2 Eric is researching the popularity of Ford and Holden cars amongst Year 9 boys and girls. He thinks it will be reasonably evenly distributed.

- a Draw a tree diagram to represent the situation. Hint: Start the first branches with gender.
- b List the sample space (the possible outcomes) and, assuming all are equally likely, provide the theoretical probability for each outcome.

Eric completes a survey and records these results: out of 30 boys surveyed, 23 preferred Holden, and out of 28 girls surveyed, 9 preferred Holden.



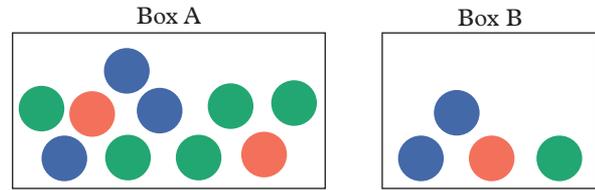
- c How many people were surveyed in total?
- d Find the experimental probability of each outcome.
- e Find the probability that a person chosen randomly from the group:
- prefers Holdens to Fords
 - is a boy.
- f Do you think each outcome is equally likely? Explain.

Eric decides to simulate the situation by rolling a die. He assigns each outcome as in this table.

Outcome	Die roll
Boys who prefer Holden	1 or 2
Boys who prefer Ford	3
Girls who prefer Holden	4
Girls who prefer Ford	5 or 6

- g Perform a simulation and comment on how well it represents the experimental data. What limitations exist?

3 Two boxes each contain balls. Box A contains 5 green, 2 red and 3 blue. Box B contains 1 green, 1 red, and 2 blue.



- a** If one box is chosen at random and then one ball is chosen at random from it, then find the probability that:
- i** box B was chosen
 - ii** the ball is green
 - iii** the ball is green and was chosen from box B
 - iv** the ball is green or box B was chosen.
- b** If two balls are randomly selected, one from each box, then find the probability that:
- i** both balls are red
 - ii** at least one red ball is chosen
 - iii** two red balls are chosen, given that at least one red ball was chosen.
- c** If one box is chosen at random and then two balls are chosen from that box, without replacement, find the probability that:
- i** both balls are blue
 - ii** both balls are blue, given that box B was chosen
 - iii** box B was chosen, given that both balls are blue.

Chapter checklist



Now that you have completed this chapter, reflect on your ability to do the following.

I can do this	I need to review this
<input type="checkbox"/> Find the five-number summary and interquartile range for numerical data sets <input type="checkbox"/> Find the five-number summary and interquartile range from graphical representations	<input type="checkbox"/> Go back to Topic 8A Five-number summary and interquartile range
<input type="checkbox"/> Create and interpret box plots <input type="checkbox"/> Use parallel box plots to compare data sets	<input type="checkbox"/> Go back to Topic 8B Box plots
<input type="checkbox"/> Describe distributions of data displayed in histograms, dot plots and box plots	<input type="checkbox"/> Go back to Topic 8C Distributions of data
<input type="checkbox"/> Calculate the standard deviation of a population and sample <input type="checkbox"/> Use the mean and standard deviation to compare data sets	<input type="checkbox"/> Go back to Topic 8D The mean and standard deviation
<input type="checkbox"/> Create tree diagrams and arrays to represent sample spaces <input type="checkbox"/> Use tree diagrams and arrays to calculate probabilities	<input type="checkbox"/> Go back to Topic 8E Two-step chance experiments
<input type="checkbox"/> Calculate probabilities for two-step chance experiments with replacement	<input type="checkbox"/> Go back to Topic 8F Experiments with replacement
<input type="checkbox"/> Calculate probabilities for two-step chance experiments without replacement	<input type="checkbox"/> Go back to Topic 8G Experiments without replacement
<input type="checkbox"/> Design and conduct a simulation <input type="checkbox"/> Apply reasoning to evaluate a simulation and its related outcomes	<input type="checkbox"/> Go back to Topic 8H Experimental probability and simulations

Semester 2 review

Short answer

- 1 Plot the graphs of the following equations by completing each table of values and plotting the coordinate points.

a $y = \frac{x^2}{2} + 2$

x	-2	-1	0	1	2
y					

b $y = 3 - 2x^2$

x	-2	-1	0	1	2
y					

c $y = 2^x$

x	-2	-1	0	1	2
y					

d $y = 3^x - 1$

x	-2	-1	0	1	2
y					

- 2 Describe the transformation required to produce the following graphs from the graph of $y = x^2$.

a $y = -x^2 + 1$

b $y = 25x^2 - 12$

- 3 Write the equation for each of the quadratic graphs described below. Assume each parabola has the same shape as $y = x^2$; that is, the dilation factor is +1 or -1.

a concave up, turning point at (0, 3)

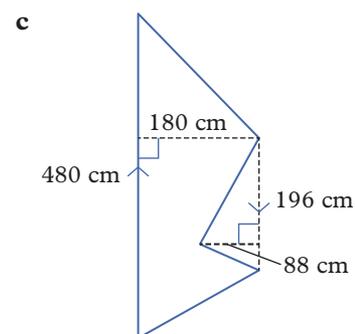
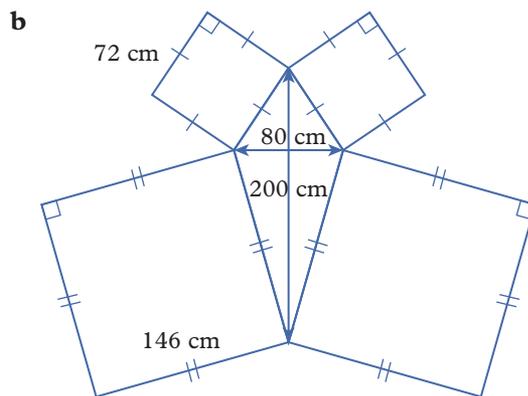
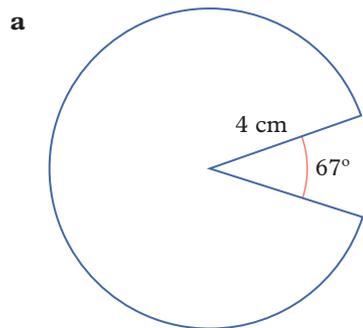
b concave down, turning point at (0, -2)

- 4 Determine the x - and y -intercepts for the graph of each of the following equations.

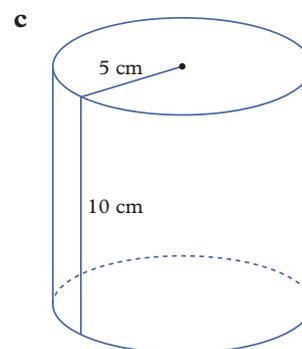
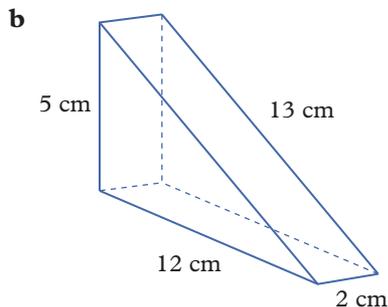
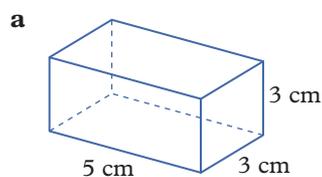
a $y = 5 - \frac{x^2}{5}$

b $y = 4x^2 - 16$

- 5 Calculate the areas of the following composite shapes, correct to two decimal places.

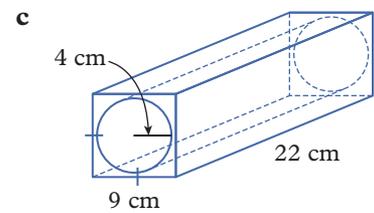
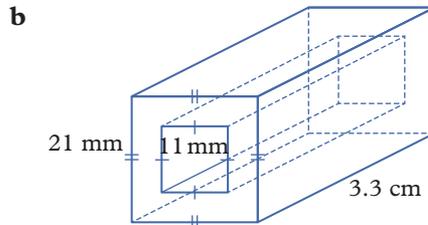
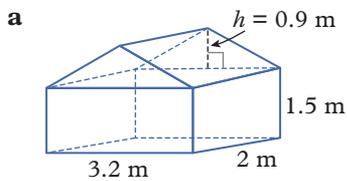


- 6 Calculate the surface area of each of the following objects.

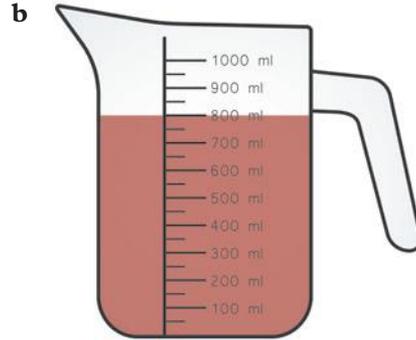
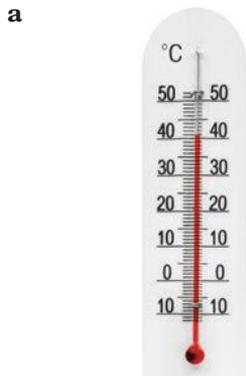


- 7 Calculate the volume of each of the objects in question 6.

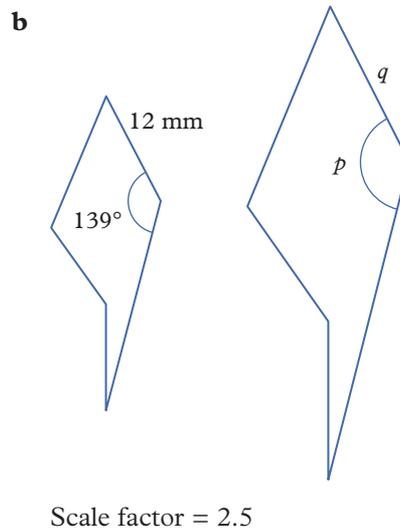
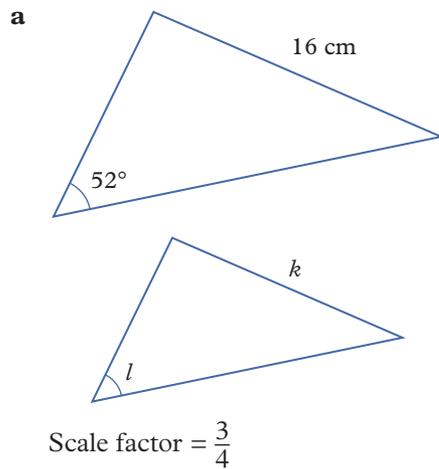
8 Calculate the volume of each of the following composite objects.



9 **i** Determine the absolute error of the following measuring instruments.
ii Determine the percentage error of the following measurements.



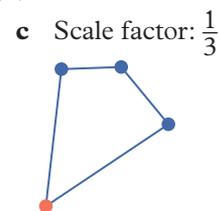
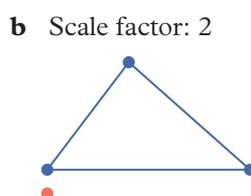
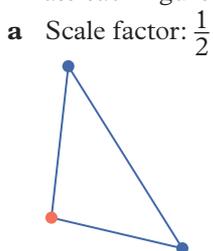
10 The shapes with labelled measurements have been dilated by the given scale factors to create the other shapes. Determine the value of the unknown pronumerals.



11 A scale drawing is drawn to a scale of 1 : 200. What is the size of each of these rooms given the size in the scale drawing?

- a** bathroom (1.6 cm by 1.3 cm)
- b** bedroom (2.4 cm by 1.8 cm)
- c** kitchen (2.8 cm by 1.6 cm)

12 Dilate each figure using the orange centre of dilation and the given scale factor.



13 For each of the following pairs of similar triangles state the following values. Where appropriate, round to four decimal places.

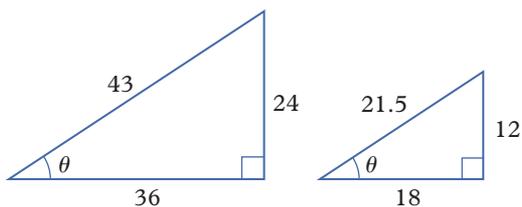
i the scale factor (original on left)

ii $\sin(\theta)$

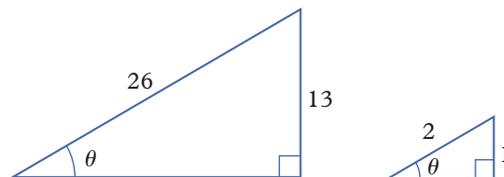
iii $\cos(\theta)$

iv $\tan(\theta)$

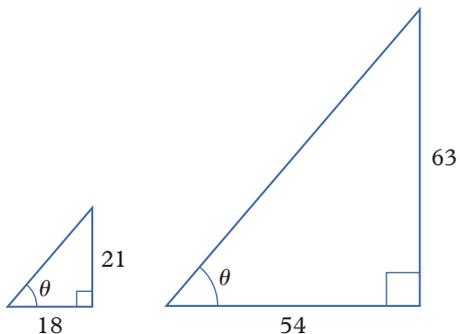
a



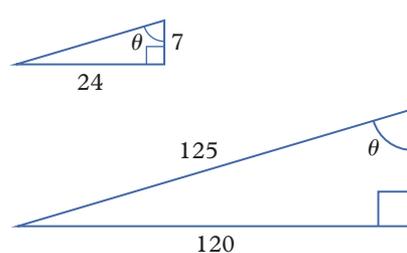
b



c



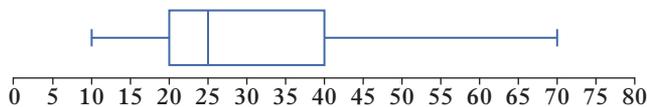
d



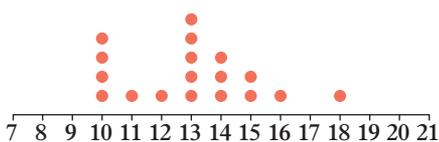
14 Determine the five-number summary and interquartile range for each of the following data sets.

a 68, 32, 62, 56, 59, 75, 38, 76, 50, 64, 48, 48, 55, 41

b



c

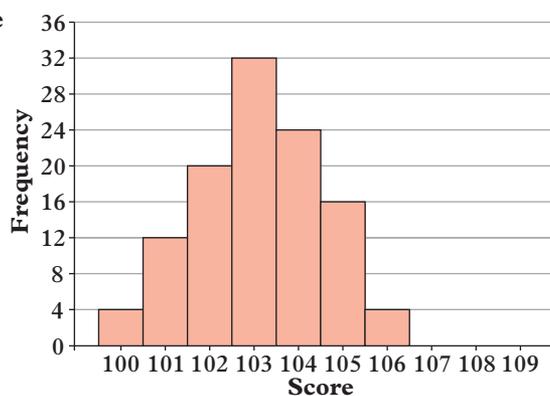


d

Stem	Leaf
1	2 8 9
2	0 1 3 4 7
3	1 2 2 2 4 5 6 7 8 9
4	0 0 0 1 2 3 4

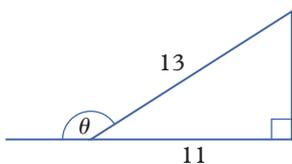
Key: 1 | 2 = 1.2

e

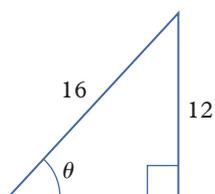


15 Determine the value of θ in each of the following diagrams, correct to the nearest degree.

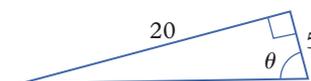
a



b

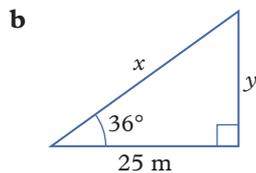
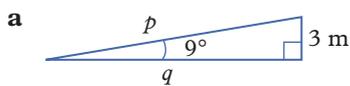


c

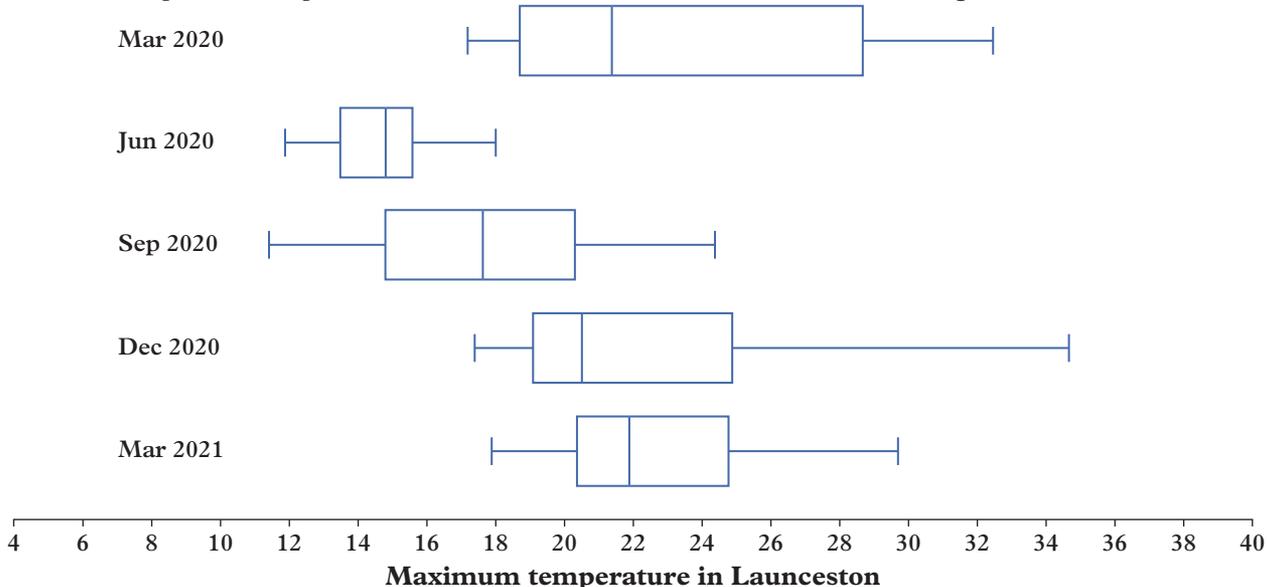


- 16 Construct a box plot for each of the following sets of data. Remember to check for outliers.
- a** 105, 110, 123, 117, 121, 120, 125, 117, 122, 113, 118, 116, 120, 125, 122, 100, 110, 112, 115, 103
- b** 4, 1, 2, 7, 2, 4, 6, 21, 9, 4, 1, 1, 3, 6, 3, 2, 1, 5, 8, 4, 2, 4, 2, 5, 2
- c** 49, 42, 44, 45, 41, 43, 44, 47, 42, 43, 46, 58, 42, 49, 20, 45, 41, 44

17 Determine the lengths of the unknown side lengths, correct to three significant figures.



18 Consider the parallel box plots shown below and state whether each of the following statements is true or false.



- a** The maximum daily temperatures in June 2020 were all less than the lowest of the maximum daily temperatures in March 2021.
- b** The median maximum daily temperatures in March 2020 and March 2021 were both approximately 22°C .
- c** The maximum daily temperatures in December 2020 were positively skewed.
- d** In September 2020, approximately 50% of the days had maximum daily temperatures between 14°C and 20°C .
- e** In March 2020, approximately 25% of the days had maximum daily temperatures between 25°C and 30°C .
- f** Approximately 50% of the days in December 2020 had maximum daily temperatures greater than the median maximum daily temperature in September 2020.

19 A fair six-sided die is rolled and the result is noted. Following the deck and the suit is noted.

- a** Construct an array to show all outcomes in the sample space.
- b** Calculate the probability of:
- rolling a 6 and drawing a spade
 - rolling an even number and drawing a red card (heart or diamond)
 - rolling an even number or drawing a red card or both
 - rolling an even number or drawing a red card but not both.



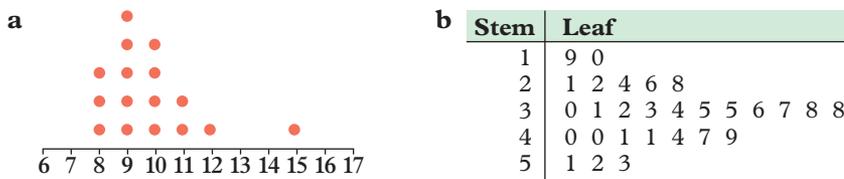
20 Calculate the mean (\bar{x}), sample standard deviation (s) and population standard deviation (σ) for each of the following data sets by hand. Then use a calculator to write your answers correct to two decimal places.

- a** 32, 36, 40, 43 **b** 20, 18, 28, 21, 24 **c** 4, 4, 4, 4, 4, 4, 4, 4, 4

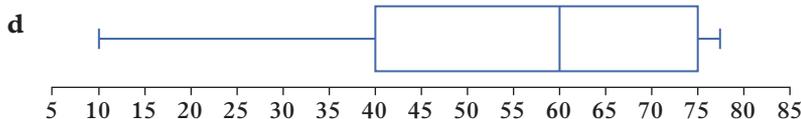
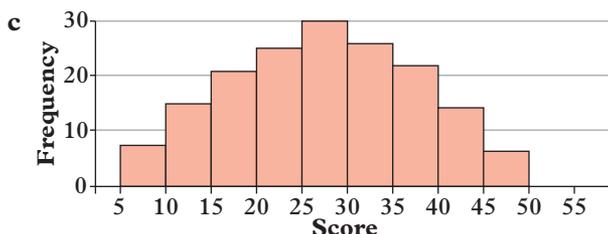
21 Brea plans to play video games on two different consoles this afternoon. She has an Xbox, a PlayStation and a Nintendo Switch to choose from.

- a Construct a tree diagram to show all outcomes in the sample space.
 b Assuming each outcome is equally likely, calculate the probability that Brea plays:
 i Xbox then PlayStation ii Xbox second
 iii Xbox either first or second iv Xbox and PlayStation in either order.

22 Describe the distribution of the data in each of the following data displays.



Key: 1 | 2 = 1200



23 A bag contains 21 coloured buttons: four red, eight blue and nine yellow. Two buttons are selected at random from the bag. Calculate the probability that:

- a a yellow and a blue button are selected, if the first button is replaced
 b a yellow and a blue button are selected, if the first button is not replaced
 c two red buttons are selected, if the first button is not replaced
 d two of the same-coloured buttons are selected, if the first button is not replaced.

24 A chocolate packet is filled with an equal number of milk chocolates, white chocolates, solid chocolates and caramel-filled chocolates. A chocolate is randomly selected from the packet.

- a List the possible outcomes and their theoretical probabilities.
 b Choose a device to simulate the situation.
 c Find how many chocolates, on average, you would have to eat if you wanted to eat at least one of each type.
 d Discuss the limitations of using your device to simulate this situation.

ADV 25 Solve the following equations for x .

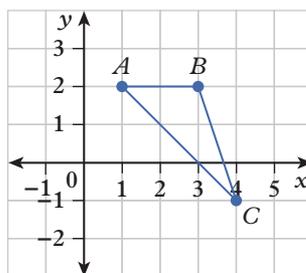
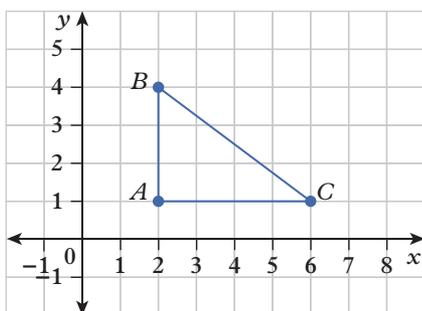
a $0 = (x - 5)(x + 9)$

b $0 = -x^2 - 6x + 40$

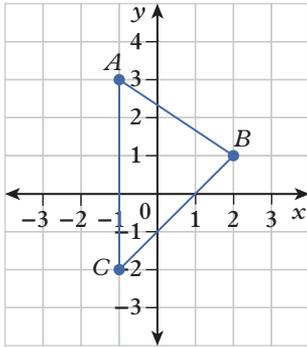
ADV 26 Apply the given transformation(s) to the following shapes.

- a Rotate 90° clockwise about the origin.

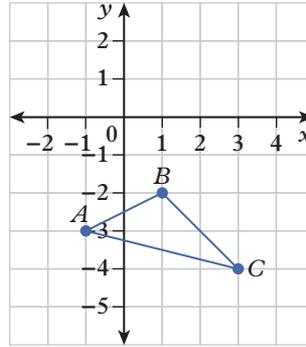
- b Reflect across the x -axis.



c Translate 3 units left and 4 units up.

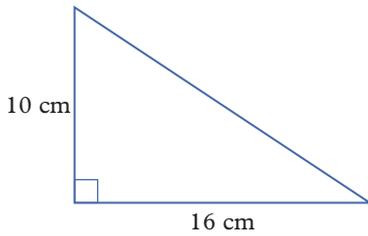


d Rotate 90° anti-clockwise about the origin, then reflect in the y -axis.



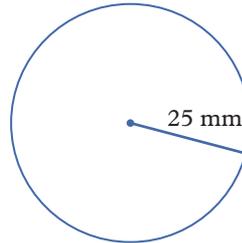
EXT 27 For each shape, state the area of the image if the original figure is dilated by the given length scale factor. Give your answers to two decimal places where necessary.

a



Length scale factor = 2

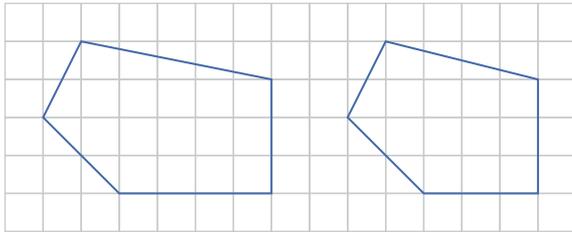
b



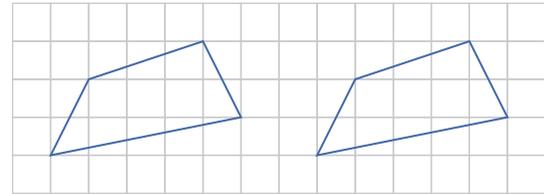
Length scale factor = $\frac{1}{4}$

EXT 28 State if the following pairs of shapes are congruent or not.

a

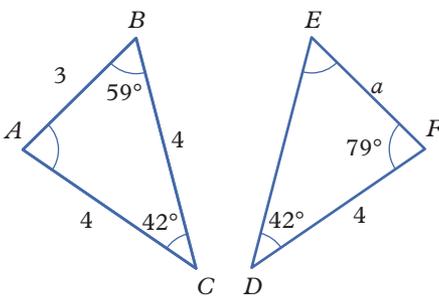


b

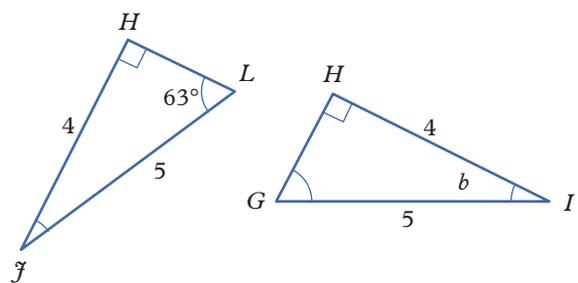


EXT 29 Give the condition that explains why the following pairs of triangles are congruent, then find the values of the pronumerals.

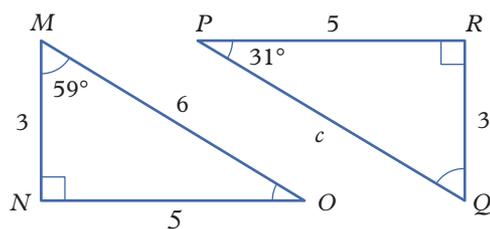
a



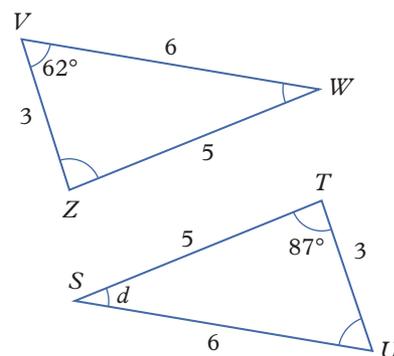
b



c

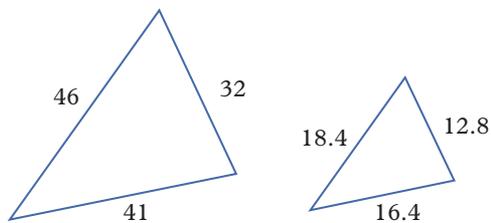


d

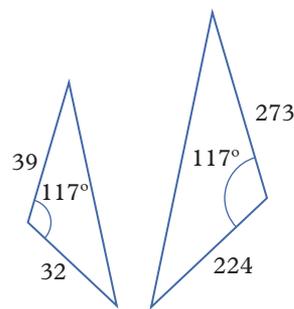


EXT 30 Determine if the following pairs of triangles are similar. If they are similar, state the condition that explains why.

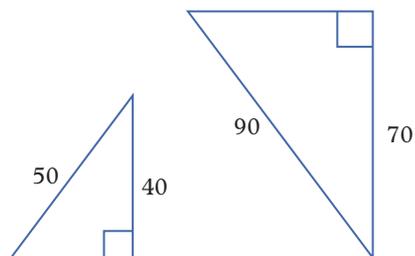
a



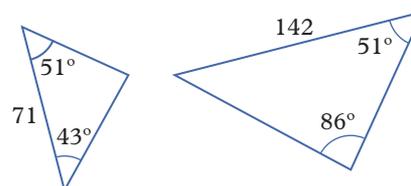
b



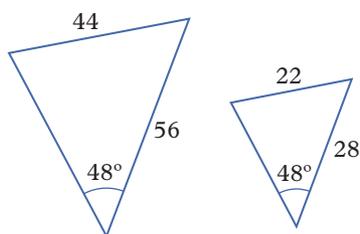
c



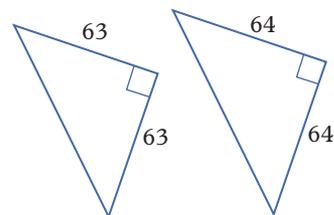
d



e

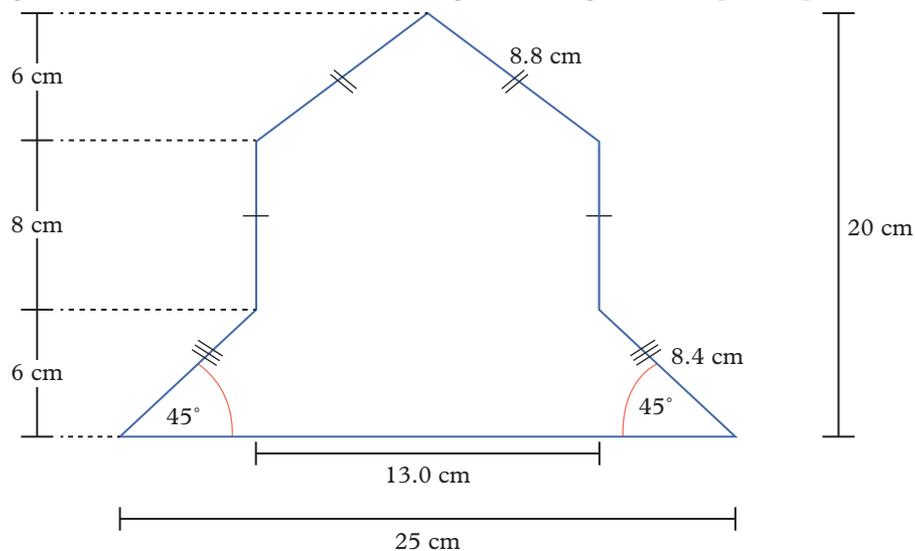


f



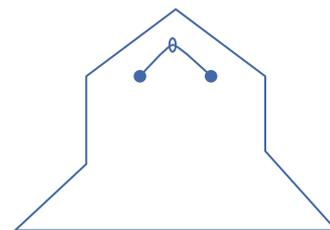
Analysis

1 Xav is planning to create a flat plywood spaceship to mount on his wall. He draws a rough diagram of the spaceship using a scale factor of 1 : 5. The desired lengths and angles in the spaceship are shown.



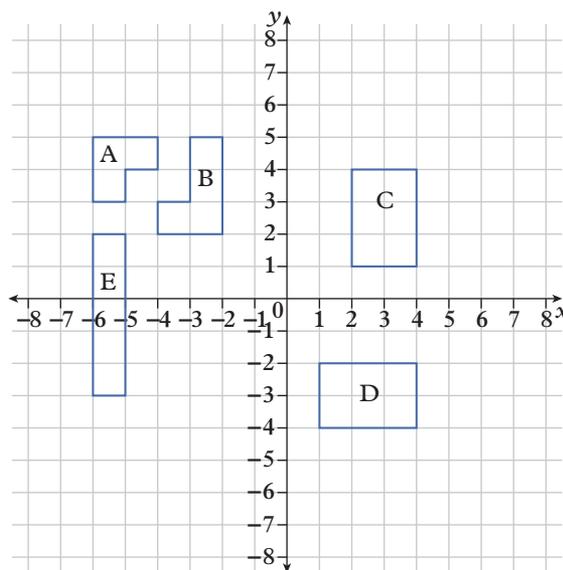
- Most of the measurements on the diagram are accurate, but Xav is unsure about the 8.4 cm and 13.0 cm lengths. Use trigonometry to check whether they are correct, and if they are not provide new lengths correct to one decimal place.
- Draw a labelled diagram of the spaceship showing the correct lengths and angles.
- Calculate the area of:
 - the diagram of the spaceship
 - the actual plywood spaceship.

To mount the plywood spaceship on the wall, Xav wants to attach an 80 cm string to two points on the back to hang on a hook. The hook will pull the string 20 cm up a vertical distance from where the string is connected, as shown on the right.



- d** Determine the angle the string will make with the line between the two attachment points.
 - e** Determine the horizontal distance between the two points where Xav will attach the string, correct to one decimal place.
 - f** Determine whether the string is parallel to the edge of the plywood. Explain why or why not.
- 2** Adriana and Beethal are designing an artwork.

- a** Complete the design by following their instructions:
 - Reflect shape A in the y -axis.
 - Reflect shape B in the x -axis.
 - Rotate shape C 90° anti-clockwise about the point $(2, 0)$.
 - Rotate shape D 90° clockwise about the point $(0, -2)$.
 - Translate shape E 11 units right and 1 unit up.
 - Reflect the image of A (A') in the x -axis.
- b** What single transformation can be applied to shape A to get the image shape A'' ?
- c** For each shape, A, B and C, state the order of:
 - i** rotational symmetry
 - ii** reflection symmetry.
- d** State which of shapes A, B, C, D or E have:
 - i** the greatest area
 - ii** the greatest perimeter.



- 3** An avid puzzle collector has puzzles with 1000, 5000 and 10 000 pieces. Some of the puzzles have irregular-shaped designs, as shown in the table below.

	1000 pieces	5000 pieces	10 000 pieces
Number of puzzles in collection	20	9	1
Number with irregular-shaped designs	5	2	0

- a** Determine the probability of selecting a:
 - i** 1000-piece puzzle out of all puzzles
 - ii** puzzle with an irregular design out of all puzzles
 - iii** 1000-piece puzzle with an irregular design out of all 1000-piece puzzles
 - iv** 1000-piece puzzle with a regular design out of all puzzles with regular designs.

The puzzle collector selects two puzzles at random with replacement.

- b** Determine the probability of selecting:
 - i** an irregular-shaped 1000-piece puzzle and then a regular-shaped puzzle that is not 1000 pieces
 - ii** a 1000-piece puzzle and then an irregular-shaped design.

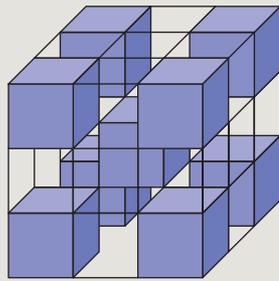


The puzzle collector starts with a rectangular 1000-piece puzzle. The puzzle actually consists of 1026 pieces in a 38-piece by 27-piece grid. The collector separates the corner, edge and centre pieces.

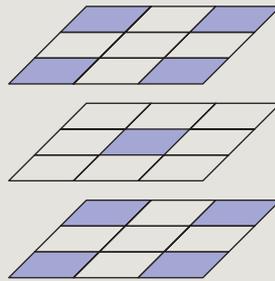
- c** Calculate the probability of selecting at random:
 - i** an edge piece connecting to any corner piece
 - ii** a centre piece connecting to any edge piece.

1 Cube stacks

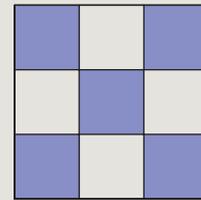
Jali has a large supply of blue and clear cubes of identical size. She arranges 9 blue cubes and 18 clear cubes into a $3 \times 3 \times 3$ stack as shown below. The simpler diagram in the middle shows the position of the blue cubes in each 3×3 horizontal layer. Jali notices that this arrangement results in the same pattern, shown on the right, when she views the stack from all six directions: top, bottom, left, right, front and back. In fact, she realises that if two cubes are removed from the top and bottom layers, it is still possible to see the same final pattern from all directions. Jali wonders what other symmetrical patterns can be seen from all directions and how many blue cubes are needed to make them.



3D view

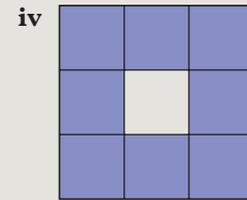
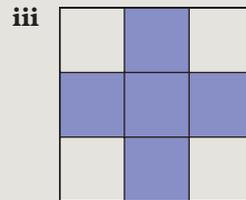
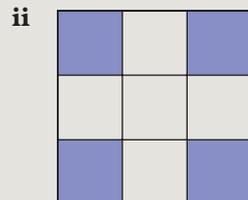
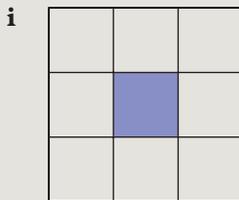


layer view



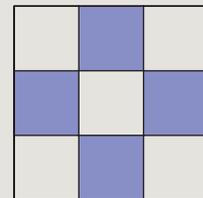
directional view

a For each of the following, determine both the maximum and minimum number of blue cubes that Jali can use in a $3 \times 3 \times 3$ stack so that the resulting pattern can be seen from all six directions. Draw a layer view for each.



b Jali wants to make the symmetrical pattern shown on the right.

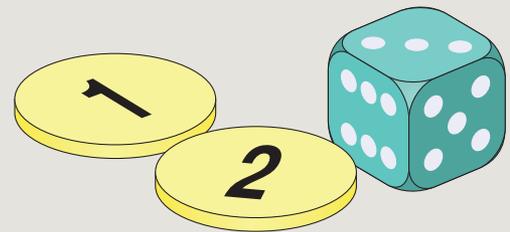
- i** Describe a stack in which this pattern can be seen from exactly two directions.
- ii** Describe a stack in which this pattern can be seen from exactly four directions.
- iii** Explain why it is not possible to see this pattern from all six directions.



2 Counter attack

Connie has a number of counters, each labelled 1 on one side and 2 on the other. She flips the counters and adds up the numbers showing on top. At the same time, Dieter selects a die and rolls it. The game ends in a draw if Connie and Dieter get the same score, otherwise the winner is the player with the higher score.

- a** Connie flips two counters and Dieter rolls a standard six-sided die.
 - i** Calculate the probability that both players get a score of 3.
 - ii** Explain why Dieter is more likely to win than Connie.
- b** Connie flips three counters and Dieter rolls a six-sided die. Is this version of the game fair? That is, do Connie and Dieter have the same chance of winning?
- c** Dieter selects an octahedral die numbered 1 to 8. How many counters must Connie flip for the game to be fair?
- d** Connie flips five counters. Assuming they are numbered 1, 2, 3, ... how many sides must there be on Dieter's die for the game to be fair?
- e** Generalise part **d** as follows: Connie flips n counters and Dieter spins a spinner with k wedges of equal size labelled 1, 2, 3, ..., k . Find the relationship between n and k if the game is fair.
- f** Is there a fair version of the game if Dieter uses a dodecahedral die numbered 1 to 12? What about an icosahedral die numbered 1 to 20?



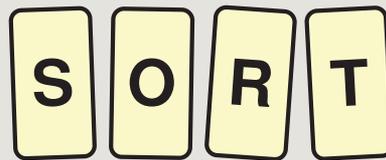
3 Differagons

A differagon is a polygon in which each side length is a different positive integer.

- a** For each of the following types of differagon, find the smallest possible perimeter:
- i** three-sided
 - ii** three-sided with a right angle
 - iii** four-sided
 - iv** four-sided with one right angle
 - v** four-sided with two adjacent right angles
 - vi** four-sided with two opposite right angles
 - vii** five-sided with three consecutive right angles
 - viii** five-sided with three right angles, not all consecutive.
- b** Which types of differagon listed in part **a** can have an odd perimeter? Explain.
- c** Diago claims he has found a 6-sided differagon $ABCDEF$ with the following properties:
- ▶ angles ABC , ACD , ADE and AFE are all right angles
 - ▶ $BC = 2$, $CD = 6$, $DE = 3$ and $EF = 7$.
- Show that Diago must have made a mistake.

4 All sorts

Imagine you have a row of cards in front of you, each with a single letter written on it. The goal is to sort them into alphabetical order. Here are three different approaches to this problem. As you work through them, see what shortcuts you can come up with to answer the questions as efficiently as possible.



- a** The SelectSort algorithm works as follows:
- ▶ Find the letter that appears earliest in the alphabet (in the case of repeats, choose the first occurrence).
 - ▶ Swap it with the letter in the first position.
 - ▶ Repeat the above steps with the remainder of the list.

For example, consider the cards S O R T in that order. The first step swaps O (the earliest letter) with S (the first letter), producing O S R T. Now repeat this procedure with the remaining list S R T, swapping R and S. The final step applied to S T has no effect, since S is earliest and in the first position. So only two steps are required to arrive at the final order, O R S T.

For each of the following, find the number of steps that SelectSort will take to sort the cards into alphabetical order.

- i** ORDERS **ii** SORTED **iii** ARRANGING **iv** DISORGANISED
- b** In the SwapThree algorithm, a move consists of reversing the order of three consecutive cards. For example, S O R T can be converted to R O S T by reversing the first three cards, or to S T R O by reversing the last three. For each of the following, decide whether it is possible to sort the cards into alphabetical order using SwapThree and, if so, determine the minimum number of moves required.
- i** ORDERS **ii** SORTED **iii** ARRANGING **iv** DISORGANISED

- c** Each pass of the iSort algorithm works as follows:
- ▶ Compare the first two letters in the list. Whichever is earlier in the alphabet is moved to a new list. The other stays in the original list.
 - ▶ Repeat the previous step with the remaining letters, moving one letter to the end of the new list at each stage.
 - ▶ When there is one letter remaining, move it to the end of the new list.

For example, comparing the first two letters of S O R T, the letter O starts the new list and S R T remain in the original list. Comparing the first two letters again, R is moved to the new list, leaving S T. Finally, S and then T are moved to the new list, giving O R S T. This is already in alphabetical order, so the original list was sorted in one pass of the iSort algorithm. But in general, more passes may be needed.

For each of the following, find the number of passes of iSort needed to sort the cards into alphabetical order.

- i** ORDERS **ii** SORTED **iii** ARRANGING **iv** DISORGANISED

Explorations inspired by the Australian Maths Trust's competitions and programs: www.amt.edu.au

How can we use sustainable farming practices so that no one goes hungry in the future?

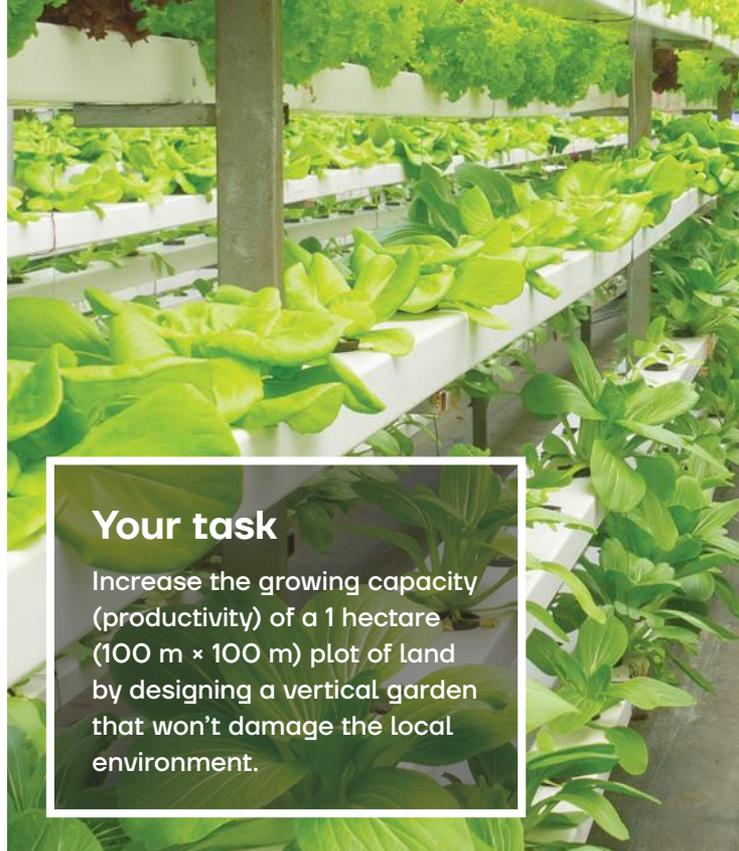
The United Nations ranks food shortages and hunger among the most serious issues affecting humankind. It predicts that more than 840 million people will be hungry by 2030. Even in a high-income country such as Australia, 5 per cent of the population are unable to access enough nutritious food. The experience of having inadequate access to food, or having an inadequate supply of food, is known as food insecurity. Food insecurity is linked to poor general health, higher rates of some cancers and higher mortality.

Rapid climate change is increasing threats to Australia's and the world's food security. Changes in the amount of rainfall, longer droughts and an increase in the number of extreme weather events are expected to disrupt the amount and quality of food that Australia can produce. A hotter climate is expected to cause stress in livestock animals such as chickens, sheep and cattle, and to increase the amount of water needed for crop irrigation.

Sustainable farming

Sustainable farming practices use methods that balance the needs of all members of the community. This means that new and old technologies are used to make sure that food production is:

- economically viable – if farmers cannot make enough money to survive, then the farming practice is not sustainable



Your task

Increase the growing capacity (productivity) of a 1 hectare (100 m × 100 m) plot of land by designing a vertical garden that won't damage the local environment.

Figure 1 Vertical farming allows people to grow more food in a smaller space.



Figure 2 Drought impacts Australia's production of important crops, such as wheat.

- socially supportive – if the lifestyle of the farming community is not supported, then people will not want to live in the area
- ecologically sound – if the local environment is not supported, then the land will be unable to support food production. Sustainable



farming also works to maintain the diversity of the local wildlife. Sustainable farming uses technology to increase the production of fresh, nutritious food while minimising the impact on the local environment.



HSIE

In Geography this year, you will learn about food security around the world and food production in Australia. You will investigate the factors that influence crop yield (such as soil moisture) and how food production can alter a biome. In Commerce, you will study the agricultural resources (such as wheat) that form a large part of Australia's trade economy.

To complete this task successfully, you will need to investigate the environmental constraints on agricultural production in Australia, such as climate and distribution of water resources. You will also need to understand the extent to which agricultural innovations have overcome these constraints.



MATHS

In Maths this year, you will build on your knowledge of measurement and geometry to determine areas and volumes of more complicated shapes. You will study right-angled triangles using Pythagoras' theorem and trigonometry. You will also extend your skills in collecting, representing and investigating data.

To complete this task successfully, you will need to perform calculations involving angles, lengths and areas of two-dimensional and three-dimensional shapes. You will need to apply your understanding of scale factors to build a prototype of your designed product. To consider the situation at local, national and international scales, you will need skills in dealing with ratios and proportions. You may also find it helpful to use scientific notation for very large or very small numbers.

You will find help for applying these maths skills in Chapter 6 'Measurement', Chapter 7 'Geometry', and section 2E 'Scientific notation' of *Oxford Maths 9 NSW Curriculum*.



SCIENCE

In Science this year, you will learn about the carbon cycle and the ways human activity can disrupt it. You will also consider the consequences of disruption, including the enhanced greenhouse effect. You will also learn about asexual reproduction and investigate vegetative propagation.

To complete this task successfully, you will need to understand the factors required to keep a system, such as a vertical garden, alive. You may need to consider how these factors can be monitored and controlled automatically. You will also need to be familiar with the scientific method, and understand how to conduct a fair test.

The design cycle

To successfully complete this task, you will need to complete each of the phases of the design cycle.



Discover

When designing solutions to a problem, you need to know whom you are helping and what they need. The people you are helping, those who will use your design, are called your end-users.

Consider the following questions to help you empathise with your end-users:

- Whom am I designing for?
- What problems are they facing? Why are they facing them?
- What do they need? What do they not need?
- What does it feel like to face these problems?

To answer these questions, you may need to investigate using different resources, or even conduct interviews or surveys.

Define

Before you start to design your vertical garden, you need to define the criteria that you will use to test the success of your vertical garden in achieving your goal.

Define your version of the problem

Rewrite the problem so that you describe the group you are helping, the problem they are

experiencing and why it is important. Use the following phrase as a guide:

‘How can we help (the group) to solve (the problem) so that (the reason)?’

Determine the criteria

- 1 What is the total area of the 100 m × 100 m plot of land? (Remember to use the correct units.)
- 2 If the plants are planted 25 cm apart in a 100 m row, and the rows are placed 1 m apart, how many plants could be planted in the plot of land? HINT: Draw the plot of land to make sure you reach maximum capacity.
- 3 What criteria will you use to measure the success of your solution or design? How will you measure how much the end-users have been helped?

Ideate

Once you know whom you’re designing for, and you know what the criteria are, it’s time to get creative!

As a group, brainstorm ways the problem can be solved. Remember that there are no bad ideas at this stage. One silly thought could lead to a genius innovation!

Once you have many possible solutions, it is time to sort them by possibility. Select three to five ideas that are possible. Research whether these ideas have already been produced by someone else. If they are already on the market, can you make a better version?

Build

Draw your top two vertical garden designs. Label each part of the designs. Include the materials that will be used for their construction.

Include in the designs:

- a the total surface area available for plant growth
- b a description of how food production will be increased

- c a description of how the design (inputs and waste) will impact the local ecosystem
 - d a description of how the workers will access all areas of the design to tend the plants
 - e at least one advantage and disadvantage of each design.
- Select one of the designs to take to the building and testing stage.

Build the prototype

You will need to build at least three versions of your vertical garden design prototype. The first prototype garden will be tested for effectiveness. The second prototype will be used to survey the group you are helping. The third prototype will be used for the presentation. The prototype may be full size, or it may be a scale model (10 cm represents 1 m). Use the following questions as a guideline for your prototype:

- What materials will you use?
- What material will you use to represent the plants?
- How will you represent the height, width and angle of the finished prototype?

Test

Prototype 1

Use the scientific method to design an experiment that will test the effectiveness and strength of your first vertical garden prototype. You will test the prototype more than once, to compare results, so you will need to control your variables between tests. What criteria will you use to determine the success of your prototype? Conduct your tests and record your results.

Prototype 2

If your prototype will be used to help market gardeners, then you will need to generate a survey to test whether the prototype is appropriate for their use. (How would they use it? Would they consider buying it?)

If your prototype will be used to help another group, or native plants and animals, you will need to consider how you could test the impact it will have. (Will the prototype affect normal behaviours? How will the prototype affect the soil or waterways?)

Prototype 3

Use the information you have obtained from testing the first two versions to adapt your last prototype to be more effective and usable for the group you are helping. You may want to use the first two prototypes to demonstrate how the design has been improved.

Communicate

Present your vertical garden design to the class as though you are trying to get your peers to invest in it. Describe the criteria and testing used to measure the effectiveness of your vertical garden design.

In your presentation, you will need to:

- explain why we need to be more efficient with food production
- describe the key features of your design and how they improve or solve the problem of food shortages
- show a labelled, to-scale diagram of your prototype
- describe how the ecosystem will be affected by the installation of the prototype
- explain the relevant scientific principles that support your designed solution (e.g. water cycle, photosynthesis, nitrogen/carbon cycle)
- quantify the increase in food production that your design allows; present calculations to justify your claim
- present a calculation for the estimated cost of producing a full-size model of your design
- explain the implications of your design at a state or national level, by comparing the benefits and costs.

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This video will help you to better understand each phase of the design cycle.



How to manage your project
This 'how-to' video will help you to manage your time throughout the design cycle.



How to pitch your idea
This 'how-to' video will help you with the 'Communicate' phase of your project.

How can we harness technology so that we can live healthier lives?

A disorder or disease is a condition that affects the normal functioning of the body. Different disorders and diseases can affect many parts of the body. They can be caused by infectious agents such as bacteria or viruses that spread from person to person. Some disorders or diseases are inherited. Environmental factors (such as pollution or diet) can also have an impact on the development of disorders or diseases.

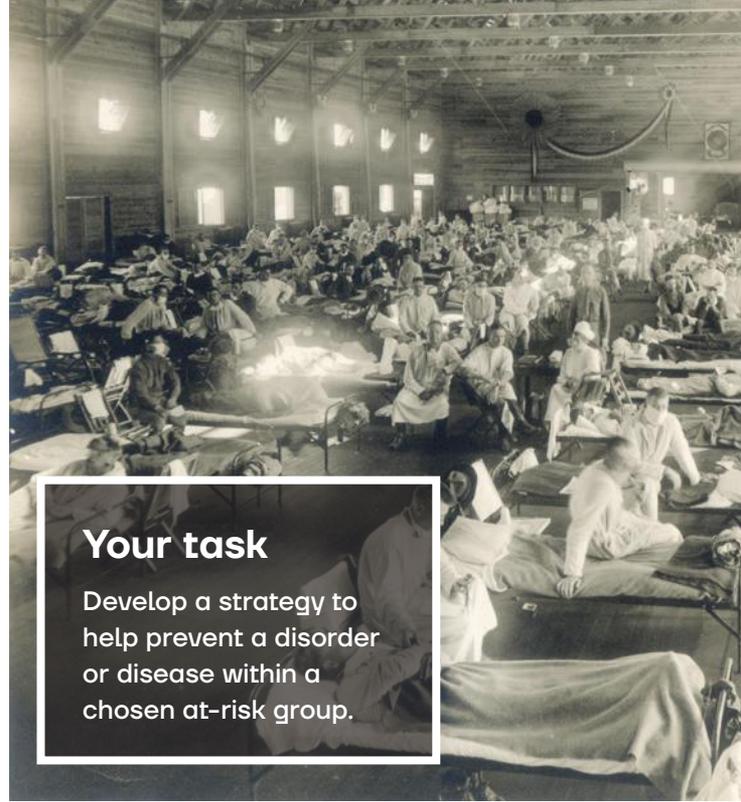
Heart disease, a non-infectious disease, is the leading cause of death globally. Mental health disorders, such as depression, bipolar disorder and dementia, also affect many people around the world.

Disorders and disease affect both high-income and low-income countries, but there are large differences in the ability of different healthcare systems to provide adequate care for people. The need for low-cost health care has led many researchers to investigate how technology can be used to help people live healthier lives.

Prevention of disorders and disease

There are many disorders and diseases that can be prevented through simple, low-cost interventions. Below are a few examples.

- Wearing a helmet or a seat belt has been shown to decrease the risk of brain injury from a road accident. In Vietnam, when wearing a helmet was made mandatory for motorcycle riders, it resulted in a 16 per cent decrease in head injuries.
- The use of mosquito nets can help to prevent malaria, a disease that can lead to life-long neurological impairment, such as epilepsy in children if they have a severe infection.



Your task

Develop a strategy to help prevent a disorder or disease within a chosen at-risk group.

Figure 1 During the 1918 flu pandemic (sometimes called the Spanish flu), an estimated 500 million people, or a third of the world's population, were infected with the virus.



Figure 2 Healthcare workers wear personal protective equipment (PPE) to prevent the spread of infectious disease.

- Providing vaccinations for viruses such as polio and meningitis can also prevent neurological conditions.
- Promoting a healthy lifestyle and educating the population about the importance of diet can reduce the prevalence of stroke. In Japan, campaigns and treatment for high blood pressure have reduced the rate of strokes by 70 per cent.



- Personal protective equipment (PPE) is used to protect people from catching infectious diseases, such as Covid-19.



HSIE

In Commerce this year, you will learn about how health services are provided in Australia. In Geography, you will study how people are interconnected through travel, technology and trade. These connections affect where and how people access the services they need. In History, you will examine the experiences of different groups during the Industrial Revolution, and the reforms made to improve living standards.

To complete this task successfully, you will need to research the demographics of your local area, and the location and accessibility of health services. You should also consider the economic performance of your area to determine what type of preventative strategy would be most successful for your at-risk group.



MATHS

In Maths this year, you will extend your skills in representing and interpreting data. You will consider media reports that use statistics and collect secondary data to investigate social issues. You will relate real-world data to probabilities of events, and compare data sets using summary statistics and different graphical displays. You will evaluate and represent data, both with and without digital technology.

To complete this task successfully, you will need to find data to quantify the problem, work out how much your strategy will cost, and calculate a quantitative, evidence-based estimate of the possible benefits of your strategy. You will need skills in dealing with ratios, proportions and percentages to consider the situation at local, national and international scales.

You will find help for applying these maths skills in Chapter 8 ‘Statistics and probability’ of *Oxford Maths 9 NSW Curriculum*.



SCIENCE

In Science this year, you will learn about how the body coordinates and regulates its internal systems so that it can respond to changes. When things change in the environment (such as the emergence of a disease-causing agent), or a part of the body fails, the normal functioning of the body is interrupted. The body needs to respond and attempt to return to a normal homeostatic state before permanent damage is caused.

To complete this task successfully, you will need to identify how the body’s systems work together to maintain a functioning body. You should consider the type of disorder or disease that you will be fighting, and how it may cause changes in the body’s normal function and response mechanisms.

The design cycle

To successfully complete this task, you will need to complete each of the phases of the design cycle.



Discover

When designing solutions to a problem, you need to know whom you are helping and what they need. The people you are helping, who will use your design, are called your end-users.

Consider the following questions to help you empathise with your end-users:

- Whom am I designing for? Is it the people directly affected by the disorder or disease, or do their families and carers need support too?
- What problems are they facing? Why are they facing them?
- What do they need? What do they not need?
- What does it feel like to face these problems? What words would I use to describe these feelings?

To answer these questions, you may need to investigate using different resources, or even conduct interviews or surveys.

Define

Before you start to design your solution to the problem, you need to define the parameters you are working towards.

Define your version of the problem

Rewrite the problem so that you describe the group you are helping, the problem they are experiencing and why it is important. Use the following phrase as a guide.

‘How can we help (the group) to solve (the problem) so that (the reason)?’

Determine the criteria

- 1 Describe the type of life that the people you are helping lived before their lives were affected by the disorder or disease.
- 2 Describe how the people affected by the disease have needed to change their lives to cope with the effects of the disorder or disease.
- 3 Describe how you will know that you have made their lives better as a result of your prototype strategy.

Ideate

Once you know whom you’re designing for, and you know what the criteria are, it’s time to get creative!

Outline the criteria or requirements your design must fulfil (i.e. usability, accessibility, cost).

Brainstorm at least one idea per person that fulfils the criteria.

Remember that there are no bad ideas at this stage. One silly thought could lead to a genius innovation!

Build

Each group member should draw an individually designed solution. Label each part of the design. Include the material that will be used for its construction.

Include in the designs:

- a a detailed diagram of the design
- b a description of why it is needed by the individual or group

- c a description of any similar designs that are already available to buy
 - d an outline of why your idea or design is better than others that can be purchased.
- Present your design to your group.

Build the prototype

Choose one design and build two or three prototypes.

Use the following questions as a guideline for your prototype.

- What materials or technology will you need to build or represent your prototype design?
- What skills will you need to construct your prototype design?
- How will you make sure your prototype design is able to be used by the people who need it?
- How will you describe the way the prototype design will work?

Test

Prototype 1

Use the scientific method to design an experiment that will test the effectiveness and strength of your first prototype. You will test the prototype more than once, to compare results, so you will need to control your variables between tests.

What criteria will you use to determine the success of your prototype?

Conduct your tests and record your results in an appropriate table.

Prototype 2

If your prototype will be used to help individuals with the disorder or disease, then you will need to generate a

survey to test whether the prototype is appropriate for their use. (How would they use it? Would it make their life easier or harder? Would they consider buying it? How much would they be willing to pay to access the design?)

Prototype 3

Your last prototype should be adapted using the information gathered from testing the first two versions to make it more effective and usable for the group you are helping. You may want to use the first two prototypes as a demonstration of how the design has been improved over time.

Communicate

Present your design to the class as though you are trying to get your peers to invest in your design.

In your presentation, you will need to:

- outline the relevant disorder or disease and how it affects individuals, as well as society as a whole
- create a working model, or a detailed series of diagrams, with a description of how it will be used by an individual and group
- explain how you changed your design as a result of testing or feedback
- describe how the design will improve the life of an individual or group
- describe how many people need or will use the design
- describe how individuals will be able to access the design (will they need to purchase it or will it be publicly funded?)
- describe how the design will improve an individual's ability to contribute to society as a whole.

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How to define a problem
This 'how-to' video will help you to narrow your ideas down and define a specific problem.

Answers

Chapter 1 Financial mathematics

EX 1A Wages and salaries

p6

- 1 \$41.20
- 2 a \$28 b \$45 c \$23.40
d \$24 e \$33.60
- 3 \$2038.40
- 4 a \$570 b 24

5

Hourly wage	Time worked	Amount earned
\$32.20	25 hours	\$805
\$24.50	30 hours	\$735
\$27	28 hours	\$756
\$30.30	38 hours	\$1151.40
\$51.20	32 hours	\$1638.40

- 6 a i \$27 ii \$36
b i \$36 ii \$48
c i \$28.20 ii \$37.60
d i \$38.85 ii \$51.80
e i \$48.90 ii \$65.20
f i \$44.85 ii \$59.80
- 7 a \$38.70 b \$915.90
- 8 \$1238.45
- 9 a \$77 400 b \$53 760 c \$64 480
d \$49 400 e \$132 600 f \$86 580
- 10 a \$5551/month b \$2562/fortnight
- 11 a \$966 b \$50 232 c \$4186
- 12 a \$993.25 b 32 hours
- 13 Rahim is not correct. To calculate Sal's monthly wage, you first need to calculate his annual wage by multiplying by 52, and then you need to divide by 12.
 $\$780 \times 52 = \$40\,560$
 $\$40\,560 \div 12 = \3380
- 14 Nathan: \$1102.50, Kristina: \$1062.40, Jack: \$954, Mina: \$1337.70
- 15 \$37.62
- 16 a \$882.74 b \$18.58
- 17 13 hours
- 18 \$7176
- 19 \$78200

EX 1B Other forms of income

p9

- 1 a \$900 b \$1377 c \$3750
d \$1264.50 e \$1177.05
- 2 a \$1248 b \$1768
- 3 a \$1410 b \$760 c \$2135
d \$1200 e \$1860 f \$2420

- 4 \$1008
- 5 a i \$100 ii \$132.50 iii \$160
b i 36 cups ii 71 cups iii 85 cups
- 6 a \$14850 b \$2880 c \$2925
d \$10800 e \$6630
- 7 \$15 920
- 8 2.4%
- 9 \$33 100
- 10 23
- 11 Abi (\$710.50), Beth (\$920), Craig (\$1450)
- 12 \$100/hour
- 13 \$49/hour
- 14 a \$131.60 b 47
- 15 2.7%
- 16 a Erica b Patrick: \$3440, Erica: \$2870
- 17 a Option B
b Option A: Below \$50 000
Option B: Between \$50 000 and \$75 000
Option C: Above \$75 000

EX 1C Taxation

p13

- 1 \$58 358
- 2 \$89 835
- 3 a \$4915.70 b \$48 259 c \$90 141
- 4 a \$9642 b \$26 542 c \$16 564
d \$0 e \$74 842 f \$22 934
- 5 a \$74 225 b \$14 590
- 6 a \$1640 b \$920 c \$2080
d \$1330 e \$2660 f \$1184
- 7 a \$118 200 b \$28 882 c \$2364
d \$86 954
- 8 \$69 466
- 9 a \$18 515 b Portia is due a tax refund of \$1873.
- 10 People in the 32.5% tax bracket only pay 32.5% of their incomes over \$45 000. The rest of their income is taxed at a lower percentage.
- 11 a 22.4% b 12.8% c 19.0%
d 24.1% e 9.7% f 26.5%
- 12 These are the values of the income tax payable for the amount of income at the top of the previous tax bracket (e.g. \$5092 is payable for an income of \$45 000).
- 13 Answers correct for 2022–2023 financial year.
a \$95 b \$490 c \$712
d \$376 e \$862 f \$1846
- 14 a \$2793.60 b \$600

Income	Deductions	Taxable income	Income tax
\$83 100	\$300	\$82 800	\$17 377
\$55 400	\$550	\$54 850	\$8 293
\$102 310	\$1100	\$101 210	\$23 360
\$68 440	\$810	\$67 630	\$12 466

Income	Medicare levy	Net income
\$83 100	\$1656	\$63 767
\$55 400	\$1097	\$45 460
\$102 310	\$2024	\$75 826
\$68 440	\$1353	\$53 831

- 16 a \$250 b \$1300 c \$5025
- 17 a \$124 442
- b People earning less than \$124 442 would be worse off and people earning more than \$124 442 would be better off.
- c i \$5774 worse off
ii \$3376 worse off
iii \$4627 better off
- d Sample answer: A progressive income tax system reduces the tax burden on the people who can least afford to pay and reduces inequality.

EX p16 **Checkpoint**

Hourly wage	Time worked	Amount earned
\$32.20	12 hours	\$386.40
\$23.90	20 hours	\$478.00
\$40.05	15 hours	\$600.75
\$36.25	29 hours	\$1051.25
\$42.50	33 hours	\$1402.50

- 2 a i \$34.50 ii \$46
b i \$43.50 ii \$58
c i \$57 ii \$76
d i \$39.75 ii \$53
e i \$51.30 ii \$68.40
f i \$66.60 ii \$88.80
- 3 a \$68 880 b \$54 120 c \$62 660
d \$107 640 e \$47 840 f \$89 180
- 4 \$937.65
- 5 a \$7.50 per item b \$3.60 per item
c \$17.20 per item d \$0.12 per item
e \$0.39 per item
- 6 \$65 000
- 7 \$76 147
- 8 a \$14 939 b \$21 082 c \$5742
d \$8013 e \$27 855 f \$11 948
- 9 a \$1010 b \$1268 c \$1522
d \$1642 e \$2096 f \$2759

- 10 a \$79 500 b \$16 304
c \$1590 d \$61 606

EX p19 **1D Budgeting and spending money**

- 1 a \$1480 b \$1658
c \$16 976 d \$4389
- 2 a \$1690 b 5.69%
3 a \$419.95 b 5.25%
- 4 \$33 920
- 5 a **Annual income**
\$2600 × 26 = \$67 600
- Annual expenses**
Rent: \$420 × 52 = \$21 840
Food: \$140 × 52 = \$7280
Entertainment: \$80 × 52 = \$4160
Personal items: \$100 × 52 = \$5200
Mobile phone: \$75 × 12 = \$900
Clothes: \$150 × 12 = \$1800
Transport: \$100 × 12 = \$1200
Total: \$21 840 + \$7280 + \$4160 + \$5200 + \$900 + \$1800 + \$1200 = \$42 380

Annual savings

\$67 600 – \$42 380 = \$25 220

- b Yes, Andrea's budget has enough savings for the \$25 000 car at the end of the year.
- 6 a **Monthly income**
\$5125
- Monthly expenses**
Rent: \$360 × 52 = \$18 720 per year
Food: \$175 × 52 = \$9100 per year
Public transport: \$50 × 52 = \$2600 per year
Total: \$18 720 + \$9100 + \$2600 = \$30 420 per year
30 420 ÷ 12 = \$2535 per month
- Monthly savings**
\$5125 – \$2535 = \$2590
- b Possible answers include: entertainment, personal items, mobile phone, clothes, other items, loan repayments, utility bills, insurance
- 7 Sample answer: By being conservative with your expense estimates you are more likely to spend less than anticipated, resulting in higher savings than expected. Underestimating expenses also leaves more room for unexpected expenses which have not been budgeted for.
- 8 There are more than four weeks in the average month. To calculate monthly expenses from weekly expenses you need to first multiply by 52, and then divide by 12, which is equivalent to multiplying by 4.333...
- 9 Sample answer: Offering an interest-free purchase period may encourage some people to purchase an item at that time instead of waiting for the future, leading to increased sales.

- 10 a** Sample answer: Lay-by agreements help people to budget and ensure that they don't spend beyond their means. They are less risky than buy now, pay later schemes as there are no potential fees or penalties for missing payments, with only a small cancellation fee to consider.
- b** Sample answer: With a lay-by agreement you will usually end up paying more than the labelled price for an item, and you don't receive the item until the final payment has been made.
- 11 a** \$70
- b** Sample answer: Purchasing the phone outright results in a reduced overall outlay; however, some people may not have the available funds to purchase outright and will find it easier to budget for a monthly expense.
- 12 a** Sample answer: Buy now, pay later schemes are likely to encourage more people to purchase items as they do not need to have the available funds at that time to make the purchase.
- b** Sample answer: Some buy now, pay later schemes only charge late payment fees as they are confident that a number of customers will not be able to make their payments on time, which will make the scheme profitable.
- 13 a** \$810
- b** Sample answer: It would be easier for Amber to reduce the expenses of her discretionary spending (entertainment and clothes) than her essential spending (food, accommodation, etc.).
- 14 a** \$825
- b** \$635

EX **1E Simple interest**
p23

- 1 a i** $\frac{7}{100}$ **ii** 0.07
b i $\frac{11}{100}$ **ii** 0.11
c i $\frac{6}{25}$ **ii** 0.24
d i $\frac{3}{50}$ **ii** 0.06
e i $\frac{1}{10}$ **ii** 0.1
f i $\frac{3}{25}$ **ii** 0.12
- 2 a i** \$500 **ii** \$5500
b i \$576 **ii** \$5376
c i \$5000 **ii** \$17 500
- 3 a i** \$1125 **ii** \$8625
b i \$6480 **ii** \$17 280
c i \$14 000 **ii** \$39 000
- 4 a** \$1200 **b** \$2422 **c** \$12 000
d \$1104 **e** \$17 999.10 **f** \$20 000
- 5 a** $P = \$3500, r = 4.8\%, n = 2$ years
b \$336 **c** \$3836
- 6 a** $P = \$44 000, r = 9.5\%, n = 5$ years
b \$20 900 **c** \$64 900

- 7 a i** \$600 **ii** \$600
- b** Answers to parts **a i** and **ii** are identical. Simple interest is calculated in the same manner for investments and loans. In each case, simple interest is found by multiplying principal, rate (as a fraction or decimal) and number of time periods.
- c** Interest on an investment is a payment to the investor and on a loan is a charge.
- 8 a** $\frac{11}{12}$ of a year **b** $\frac{7}{52}$ of a year **c** $\frac{1}{2}$ of a year
d $\frac{1}{4}$ of a year **e** $\frac{271}{365}$ of a year **f** $\frac{31}{73}$ of a year
g $1\frac{1}{4}$ years **h** 4 years **i** $1\frac{8}{13}$ years
j $3\frac{2}{5}$ years **k** $2\frac{1}{2}$ years **l** $5\frac{1}{2}$ years
- 9** 4.25 years
- 10 a i** \$1620 **ii** \$10 620
b i \$236.25 **ii** \$10 736.25
c i \$1837.50 **ii** \$9337.50
d i \$228.82 **ii** \$29 228.82
e i \$52.78 **ii** \$8652.78
f i \$17 825.73 **ii** \$173 395.73
g i \$1913.37 **ii** \$21 912.37
h i \$15 601.94 **ii** \$61 551.94
i i \$1950.77 **ii** \$210 604.77
- 11 a** \$9.20 **b** \$9.07
- c** Sade would hope the method that writes time as a fraction of total number of months in year was used. She earns more interest with this method.
- d** Sade would prefer the method that uses the number of days in June as a fraction of total number of days in year. She would pay less interest on her loan using this method.
- 12 a** 5.25% **b** \$1050
- c** This statement is true. If Jasmine invests her money for 364 days, the calculations are:

$$I = \frac{\$20\,000 \times 0.0555 \times 364}{365} = \$1106.96$$
She receives slightly more interest.
- 13 a i** \$10 **ii** \$100 **iii** \$300
b i \$86.80 **ii** \$694.40 **iii** \$2094.40
c i \$40 000 **ii** \$500 000 **iii** \$1 000 000
d i \$5 per month or \$60 per year
ii \$100 **iii** \$850
e i \$25 per week or \$1300 per year
ii \$1800 **iii** \$27 800
f i \$0.42 per day or \$153.3 per year
ii \$64.26 **iii** \$4444.26
- 14 a i** 1.5 **ii** \$300
b i 1.496 **ii** \$2094.40
c i 2 **ii** \$1 000 000
d i 1.1333 **ii** \$849.98
e i 1.0692 **ii** \$27 799.20
f i 1.0147 **ii** \$4444.39

15 a, b

Balance \$	Days applied for	Interest earned \$
640.90	7 days	0.26
540.90	7 days	0.22
780.90	9 days	0.40
655.50	5 days	0.19

- c \$1.07 d \$656.57
- 16 a \$0.72 b \$830.27
- 17 a Yes, account A will receive the bonus interest rate. They have made only one withdrawal for the month and the account balance has increased by more than \$200. Account B will not receive the bonus because there is more than one withdrawal.
- b Account A: \$6.23, Account B: \$3.06
- c Account A: \$2231.98, Account B: \$3139.82
- 18 a The minimum deposit is \$1875, so Joel's savings are enough to cover it.
- b \$10000 c \$15050
- 19 a $33\frac{1}{3}\%$ p.a.
- b i 25% p.a. ii 20% p.a. iii $16\frac{2}{3}\%$ p.a.
- c i 50% p.a. ii 100% p.a.
- 20 Sample answer: 5% p.a. for 5 years, 2.5% for 10 years, 10% for 2.5 years

Repayments of \$35 per month will not be enough to repay the loan over three years.

- e \$37.22 (will be 8 cents short at the end, but can make this adjustment later)
- 10 a 1.2125 years = a year and 78 days or about a year and two and a half months
- b \$12 125
- 11 a 1.125% per quarter b \$113.50
- 12 a 7 years b \$1560 c 5.7%
- d \$1250 e 0.89% f 2947 days
- 13 a \$1000 b \$1400 c 10% p.a.
- 14 a 5% p.a. b 3% p.a.
- 15 a \$68.75 b 10 000 days c 79 365 years
- 16 2.2%
- 17 a \$741.13
- b 16.8% (rounded to one decimal place)

EX
p34

Chapter 1 review

Mathematical literacy review

- 2%
- divide by 26
- Pay-As-You-Go
- a loan b wage
- a Leave loading is an extra payment on top of annual leave that is usually calculated as 17.5% of normal pay for this period. For example, if someone usually earned \$3000 per week and they received leave loading, they would earn $\$3000 \times 1.175 = \3525 for a week of annual leave.
- b Simple interest is interest which is calculated on the principal which has been borrowed or invested. For example, the simple interest on a \$3000 investment at 5% p.a. for two years would be $3000 \times 0.05 \times 2 = \300 .
- a Piece work is income which is paid for each item produced or action performed.
- b Tax deductions are expenses that can be subtracted from a person's total income to determine their taxable income.

EX p29 **1F Simple interest calculations**

- a \$420 b \$27.50 c \$1700
d \$5000 e \$38000 f \$2400
- a $n = 5$ years b $n = 7$ years c $n = 4$ years
d $n = 4$ years
- a $P = \$5000$ b $P = \$15\ 000$ c $P = \$4000$
d $P = \$4800$
- a $n = 5$ years b $P = \$6000$ c $P = \$2500$
d $n = 2$ years e $P = \$3125$ f $n = 5$ years
- a n , number of time periods
b $P = \$4500$, $r = 5.0\%$ p.a., $I = \$675$
c 3 years
d One year sooner ($n = 2$ years compared to $n = 3$ years)
- a \$546 b 26 months
- a 5.0% p.a. b 6.0% p.a. c 2.5% p.a.
d 5.6% p.a. e 6.4% p.a. f 2.5% p.a.
- a \$932.40 b \$4500 c 9.8%
d 3.5 years or 3 years and 6 months
e \$2950 f 6.8% g 5.0%
h 1.75 years or 1 year and 9 months
- a P , amount borrowed
b $r = 4.0\%$ p.a., $n = 3$ years, $I = \$144$
c \$1200
d Total amount to be repaid = $\$1200 + \$144 = \$1344$
Repayments = $\$35 \times 36 = \1260
(3 years = 36 months)

Multiple choice

- 1 B 2 D 3 D 4 B 5 E
6 A 7 D 8 E 9 C 10 A

Short answer

1

Hourly wage	Time worked	Amount earned
\$26.40	24 hours	\$633.60
\$33.20	28.5 hours	\$946.20
\$37.70	30 hours	\$1131
\$41.80	21 hours	\$877.80
\$23.90	36.5 hours	\$872.35

- 2 \$2749.50
- 3 a \$2475 b \$3136 c \$540
d \$12870 e \$7436

4 a \$27 192 b \$16 792 c \$9804
 d \$15 492 e \$3192 f \$56 122

5 \$47 589

6 a \$1125.95 b 3.11%

7 a **Annual income**
 $\$2010 \times 26 = \$52\,260$

Annual expenses

Rent: $\$390 \times 52 = \$20\,280$

Food: $\$170 \times 52 = \8840

Entertainment: $\$100 \times 52 = \5200

Personal items: $\$100 \times 52 = \5200

Mobile phone: $\$35 \times 12 = \420

Clothes: $\$200 \times 12 = \2400

Transport: $\$240 \times 12 = \2880

Utility bills: $\$420 \times 4 = \1680

Total: $\$20\,280 + \$8840 + \$5200 + \$5200 + \$420 +$
 $\$2400 + \$2880 + \$1680 = \$46\,900$

Annual savings

$\$52\,260 - \$46\,900 = \$5360$

b No, Ellie's budget does not allow her to reach her savings target.

8 a i 5.2% p.a.

ii 6.6% p.a.

iii 7.2% p.a.

b i 0.4% per month

ii 0.12% per week

iii 0.95% per quarter

9 a \$600 b \$480 c \$700

10 a $n = 3$ years b $P = \$4800$ c $n = 2.5$ years

d $r = 9\%$ p.a. e $r = 3.4\%$ p.a.

Analysis

a \$5093.40

b Simon: \$588; Melanie: \$722.75; Tahlia: \$918.75

c \$5404.90 per week

d \$29904

e \$830.67

f \$53904

CHAPTER 2 Indices

EX 2A Indices

p43

1 a $6 \times 6 \times 6 \times 6 = 1296$

b $8 \times 8 \times 8 = 512$

c $-2 \times -2 \times -2 \times -2 \times -2 = -32$

d $-3 \times -3 \times -3 \times -3 \times -3 \times -3 = 729$

e $\frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} = \frac{125}{64}$

f $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{128}$

g $-\frac{2}{3} \times -\frac{2}{3} \times -\frac{2}{3} \times -\frac{2}{3} = \frac{16}{81}$

h $-\frac{3}{5} \times -\frac{3}{5} \times -\frac{3}{5} \times -\frac{3}{5} \times -\frac{3}{5} = -\frac{243}{3125}$

2 a $b \times b \times b \times b \times b \times b$

b $-n \times -n \times -n \times -n \times -n$

c $-cd \times -cd$

d $2pq \times 2pq \times 2pq \times 2pq$

e $2 \times p \times q \times q \times q \times q$

f $-4 \times a \times a \times b \times b \times b \times c$

g $3m^2 \times 3m^2 \times 3m^2 \times 3m^2 \times 3m^2$

h $3 \times m^2 \times m^2 \times m^2 \times m^2 \times m^2$

3 a 5^4 b a^4 c $7k^3v^4$

d $(qu)^5$ or q^5u^5 e $(-h)^3$ f $-h^3$

g $(n^3)^6$ h $(5b^3d^4)^2$

4 a 2×5^2 b $2^3 \times 3^2$ c $3^3 \times 5$

d $2 \times 3^3 \times 7$ e $2^3 \times 19$ f $2^2 \times 7 \times 29$

g $2 \times 5^2 \times 11$ h $3^2 \times 5 \times 37$

5 a 0.04 b 0.04 c 0.0004

d 0.008 e -0.008 f 0.000008

g 0.0016 h 0.0016 i 0.00000016

6 a 5^4 b -5^3 c a^4b^4

d $5^8x^8y^8$ e $5x^8y^8$ f $\frac{11^6}{2^6}$

g $-3^5a^5b^5c^5$ h $3^8a^8b^8c^8$

7 a 343 b 6 c $-\frac{81}{320}$ d 2817

8 a $2^3 \times 3^2$ b $5^6 \times 6$ c $13^4 \times 17^5$

d $101^2 \times 103^5$ e $4^3 \times x^4$ f $7^2(xy)^7$ or $7^2x^7y^7$

9 a $2^3 \times 3 \times 5^2$ b $2 \times 3 \times 5^2$

c $2^2 \times 3^2 \times 5^2$ d $2^2 \times 3 \times 5^3$

e $2^3 \times 3 \times 5^3$ f $2^3 \times 3^2 \times 5^2$

g $2^2 \times 3 \times 5^2 \times 7$ h $2^5 \times 3 \times 5^2$

10 a x^7 b a^3b^2 c $3rs^2t^3$ d $4e^7f^2$

11 a Only k has the index 5, so the t should not be repeatedly multiplied. $tk^5 = t \times k \times k \times k \times k \times k$

b All of $2rv$ is raised to the power of 4, so $2rv$ should be repeatedly multiplied.

$(2rv)^4 = 2rv \times 2rv \times 2rv \times 2rv$

c The product was evaluated before the power.
 $-3 \times (-2)^4 = -3 \times 16 = -48$

12 a 1 b 39 c -60

d $\frac{63}{4} = 15\frac{3}{4}$

13 a i 1 ii -1 iii 1 iv -1 v 1
 vi -1 vii 1 viii -1 ix 1 x -1

b i When the index, n , is odd, the basic numeral of $(-1)^n$ is -1 .

ii When the index, n , is even, the basic numeral of $(-1)^n$ is 1.

c i negative ii negative

iii positive iv positive

v positive vi negative

vii positive viii positive

14 a i 0.343 and 0.000343

ii In index form, both are the cube of a decimal with the digit 7 in different place values: one is

7 tenths, the other is 7 hundredths. The basic numerals use the same digits 343 also in different place values; $0.000\ 343$ is $10^3 = 1000$ times smaller than 0.343 .

- b i** -0.064 and -0.064
ii In index form, both are the cube of 0.4 but one has a negative inside the brackets, the other outside. The basic numerals for both are exactly the same.
c i -1.728 and 2.0736
ii In index form, both raise -1.2 to a power: one is to the power of 3 ; the other to the power of 4 . The basic numeral for the power of 3 is negative while the power of 4 is positive. The two numerals have different values.
d i 9.261 and $8.120\ 601$
ii In index form, both are cubes of a number that uses the digits 2 and 1 . The basic numerals share similar digits where $8.120\ 601$ has its digits spaced out one place more than 9.261 does.

- 15 a** 1.2
b $2024: 24, 2025: 29, 2028: 50$
16 a 3 days: 2^3 , 8 days: 2^8 , 12 days: 2^{12}
b 3 days: 2^6 , 8 days: 2^{16} , 12 days: 2^{24}
c 3 days: 2 , 8 days: 2^4 , 12 days: 2^6
d A: 24 , B: 192 , C: 6
17 a LCM: $2^8 \times 3^{15} \times 5^2 \times 7^4$ HCF: $2^4 \times 3^5 \times 5^2 \times 7$
b LCM: $a^8b^{15}c^2d^4$ HCF: $a^4b^5c^2d$
c LCM: $pq^5r^{10}s^4$ HCF: $pq^3r^7s^2$
d LCM: $24x^3y^9z^4$ HCF: $4xy^3z^4$
18 a 2^{10} **b** 5^{10} **c** $2^{12} \times 5^8$
19 $64^1 = 64, 8^2 = 64, (-8)^2 = 64, 4^3 = 64, 2^6 = 64,$
 $(-2)^6 = 64$
20 a $-\frac{1}{12}$ **b** $\frac{243}{32}$ **c** $-\frac{2401}{10} = -240.1$

EX 2B Products and quotients of powers

p48

- 1 a** 3^9 **b** 7^6 **c** $(-2)^{12}$ **d** 6^3
e $(-8)^7$ **f** 10^{11} **g** 3^1 or 3 **h** 5^2
i 4^2 **j** $(-9)^9$ **k** 13^3 **l** 2^{10}
2 a 19683 **b** $117\ 649$ **c** 4096 **d** 216
3 a $3y^9$ **b** $7g^7$ **c** $6b^{11}$ **d** $-12k^{13}$
e $6b^{11}$ **f** $-80g^{11}$ **g** $27c^{14}$ **h** $15p^{10}$
4 a a^2 **b** d **c** g^{10} **d** p^3
e a^5 **f** n^3 **g** r^8 **h** $8x^{11}$
5 a $12x^{11}$ **b** $10x^7$ **c** $-24x^9$ **d** $54x^{11}$
e $3x^4$ **f** $4x^4$ **g** $\frac{2x}{5}$ **h** $\frac{5x^8}{3}$ or $\frac{5}{3}x^8$
i $-8t^{12}$ **j** $\frac{5r^6}{8}$ or $\frac{5}{8}r^6$ **k** $5c^4$ **l** $\frac{5y^7}{2}$ or $\frac{5}{2}y^7$
6 a x^9 **b** $\frac{b}{a}$ **c** n **d** b^8
e x^5 **f** m^4 **g** $6a^6$ **h** n^5
i 5 **j** $-4t^2$ **k** 2 **l** 1

- 7 a** a^9b^6 **b** $-18m^{11}n^3$ **c** c^2d^2
d k^2m^3 **e** x^8y^7 **f** $30g^{10}h^3$
g a^8b^6 **h** $15x^8y^{10}$ **i** $54w^4x^{13}y^4$
j $\frac{u^4}{5}$ or $\frac{1}{5}u^4$ **k** $-\frac{3}{4}ef^{10}$ **l** $-108v^{17}y^{10}$
8 a x^6 **b** $2k^3$ **c** $6a$
d $\frac{m^2}{x^2}$ **e** b^3 **f** $n^{13}p^3$
g $-2j^5q^6$ **h** $2w^2x^4$
9 a False: When multiplying you add, not multiply indices. $x^3 \times x^4 = x^7$
b False: There is no index law for adding terms, but you can add these as they are like terms. $k^3 + k^3 = 2k^3$
c True
d False: There is a power of 1 in the middle term, which must be added. $a^5 \times a \times a^5 = a^{11}$
e False: Add the powers of m and n separately. $m^3n^5 \times m^2n^4 = m^5n^9$
f False: When dividing you subtract, not divide indices. $100^8 \div 100^2 = 100^6$
g True
h False: The first b^6 has been missed. $\frac{a^5b^6}{a^2b^4} \times \frac{a^3b^5}{a^4b} = a^2b^6$
10 a i $\frac{1}{3^6}$ **ii** $\frac{1}{5^6}$ **iii** $\frac{1}{2^4 \times 3^5}$
iv $\frac{3^5}{2^4}$ **v** $\frac{2^4}{3^5}$
b i $\frac{2^3}{2^5} = \frac{2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2} = \frac{1}{2 \times 2} = \frac{1}{2} = \frac{1}{2^{(5-3)}}$
ii $\frac{2^4}{2^8} = \frac{2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2} = \frac{1}{2 \times 2 \times 2 \times 2} = \frac{1}{2^4} = \frac{1}{2^{(8-4)}}$
iii $\frac{5^6}{5^7} = \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5} = \frac{1}{5} = \frac{1}{5^1} = \frac{1}{5^{(7-6)}}$
11 a 2^{15} **b** 3^{15} **c** $2^6 \times 3^6$ **d** 2^{30}
12 a $w = 5, x = 3, y = 2, z = 8$
b $x = 9, y = 6, z = 4$
c $a = 16, b = 9, c = 4, d = 6$
13 No. An index represents the number of times the base is repeatedly multiplied. If you write $2^4 \times 3^2$ as $(2 \times 3)^{(4+2)}$, you are saying that $2 \times 2 \times 2 \times 2 \times 3 \times 3$ is equal to $(2 \times 3) \times (2 \times 3)$, which is not true. Similarly, if you write $y^8 \div x^5$ as $(\frac{y}{x})^{(8-5)}$, you are saying that $\frac{y \times y \times y \times y \times y \times y \times y \times y}{x \times x \times x \times x \times x}$ is equal to $(\frac{y}{x}) \times (\frac{y}{x}) \times (\frac{y}{x})$, which is not true.
14 a $2^3 = 8$ **b** $3^3 = 27$
c $5^2 = 25$ **d** $10^4 = 10\ 000$
15 a $2^{12} = 8^4$ **b** $3^{15} = 27^5$ **c** $5^{18} = 25^9$
d $10^{12} = 10\ 000^3$ **e** $4^{14} = 16^7$ **f** $2^{30} = 32^6$
g $6^6 = 216^2$ **h** $3^{30} = 243^6$
16 a $a^{m+n}b^{x+y}$ **b** $a^{m-n}b^{x-y}$

EX 2C Raising indices and the zero index

p53

- 1 **a** 6^{12} **b** 3^4 **c** $3^6 \times 4^2$ **d** 2^{24}
e $5^4 \times 2^{28}$ **f** $\frac{3^2}{4^2}$ **g** $\frac{5^{12}}{2^8}$ **h** $\frac{1}{8^{25}}$
i -3^{28} **j** 3^{20} **k** $7^{28} \times 11^{21}$ **l** $\frac{13^{48}}{17^{24}}$
- 2 **a** b^{10} **b** m^8 **c** j^{10} **d** j^{10} **e** n^{80} **f** p^{99}
- 3 **a** x^6y^6 **b** 2^3d^3 **c** $(-5)^7k^7 = -5^7k^7$
d $9^{10}p^{10}$ **e** 3^4m^4 **f** $\frac{8^2}{p^2}$
g $\frac{x^6}{y^6}$ **h** g^2h^2 **i** a^5b^5
j $\frac{k^3}{m^3}$ **k** 2^8x^8 **l** $-\frac{d^5}{3^5}$
- 4 **a** 3^4x^{24} **b** $5a^{28}b^7$
c $\frac{2^3m^3}{n^3}$ **d** $\frac{a^8}{b^{20}}$
e $-2u^{12}$ **f** $\frac{4v^{70}w^{30}}{3} = \frac{4v^{70}w^{30}}{3}$
g $\frac{7p^7}{9q^{42}}$ **h** $\frac{3^{10}r^{45}}{2^3} = \frac{3^{10}r^{45}}{2^3}$
i $\frac{8}{5^{24}r^{44}}$ **j** $3^2 \times 7 \times i^{34} = 63i^{34}$
k $2^{35}c^{40}$ **l** $\frac{5^4x^{75}}{7^4y^{90}}$
- 5 **a** 1 **b** 1 **c** 1 **d** 1
- 6 **a** 2 **b** 1 **c** -7 **d** 1
e -1 **f** 2 **g** 1 **h** 2
i 2 **j** 3 **k** 1 **l** 1
m 1 **n** 1 **o** -1 **p** -1
- 7 **a** x^{13} **b** x^{22} **c** x^{27} **d** x^{27}
e x^{10} **f** w^6 **g** $\frac{b}{3}$ **h** e^6
i 1 **j** $4a^9$ **k** t^{25} **l** f^{109}
- 8 **a** 1 **b** -7 **c** 1 **d** 3
e 1 **f** k^2 **g** $10g^4$ **h** 3
i x^{15} **j** 2 **k** $-8b$ **l** $\frac{2}{5}$
- 9 **a** -2 **b** k^{10} **c** $x^{11}y^9$ **d** $\frac{m^{12}}{2n^5}$ **e** 1 **f** $\frac{b^4}{a^2c}$
- 10 **a** x^9y^7 **b** $1568k^7$ **c** $81x^{24}$ **d** $-5a^{28}b^7$
e $\frac{x^{10}}{y^{11}}$ **f** $\frac{8m^3}{n^3}$ **g** $\frac{a^8}{b^{20}}$ **h** $\frac{w^{10}x^6}{y^8}$
i k^7mn^2 **j** $\frac{16t^{13}}{p^{21}}$ **k** ab^{30} **l** $72h^6$
- 11 **a** $a^3 \div a^3 = \frac{a^3}{a^3} = \frac{a \times a \times a}{a \times a \times a} = 1$
b $a^3 \div a^3 = a^{3-3} = a^0$
c The answers to parts **a** and **b** should be the same so $a^0 = 1$.
- 12 **a** True **b** True
c False: Not every power in the bracket has been raised to the sixth power. $(\frac{x}{y})^6 = \frac{x^6}{y^6}$

d False: The bracket has been expanded incorrectly.

$$\frac{(k^3)^2 \times k^4}{k^2} = \frac{k^6 \times k^4}{k^2} = k^8$$

e True **f** False, $100^9 \div 100^9 = 1$

g True

h False, any base with an index of 0 is equal to 1. $a^0 \times b^0 = 1$

13 **a** 7 **b** 4 **c** 0 **d** 8

e 3 **f** 5 **g** 7 **h** 5

14 Eden added the 4 and 5 indices before multiplying the 5 by the 3 index. $3^4 \times (3^5)^3 = 3^4 \times 3^{15} = 3^{19}$

15 **a** $(2^3)^5 = (2^3) \times (2^3) \times (2^3) \times (2^3) \times (2^3)$
 $= 2^{3+3+3+3+3}$
 $= 2^{3 \times 5}$

b $(x^7)^4 = (x^7) \times (x^7) \times (x^7) \times (x^7)$
 $= x^{7+7+7+7}$
 $= x^{7 \times 4}$

c $(2 \times 3)^4 = (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3)$
 $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$
 $= 2^4 \times 3^4$

d $(\frac{2}{3})^6 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$
 $= \frac{2^6}{3^6}$

16 **a** 3^{64} **b** 5^{81} **c** 7^{1024} **d** 10^{3125}

17 8^{15} cm^3

18 $\frac{4^5}{3^5}$

19 $(a^m)^n = a^{m \times n} = a^{n \times m} = (a^n)^m$

20 $x = \frac{3}{20}$

EX Checkpoint

p55

1 **a** $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

b $-3 \times -3 \times -3 \times -3 = 81$

c $-4 \times -4 \times -4 = -64$

d $\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{125}{216}$

2 **a** $a \times a \times a \times a \times a \times a$

b $-b \times -b \times -b \times -b$

c $3y \times 3y \times 3y \times 3y \times 3y$

d $3 \times xy \times xy \times xy \times xy \times xy$

3 **a** 8^7 **b** u^4 **c** $(4b)^5$ **d** $-7k^3h^5$

4 **a** $2^2 \times 7$ **b** $2^3 \times 3^2$

c $2^2 \times 11^2$ **d** $2 \times 3^3 \times 5$

5 **a** 8^{11} **b** $5^{10} \times 7^{12}$ **c** 6^5 **d** $3^8 \times 10^7$

6 **a** a^{12} **b** $-12b^{18}c^{21}$ **c** u^5 **d** $\frac{5}{7}p^{14}q^{17}$

7 **a** 3^{10} **b** k^8

c $\frac{t}{c^{11}}$ **d** $-\frac{15}{4}d^{11}w^7$

8 **a** 1 **b** 1 **c** -1 **d** 8

9 **a** 3^{24} **b** j^{45}

c $(-5)^6a^{18}b^{42} = 5^6a^{18}b^{42}$ **d** $\frac{3^8p^{40}}{2^8q^{56}}$

10 **a** 1 **b** $\frac{g^{20}}{3^7}$ **c** $5^7m^{66}n^{87}$ **d** $\frac{6}{j^{42}}$

EX 2D Negative indices

p58

1 a $\frac{7}{8}$ b 2 c $-\frac{1}{3}$ d $\frac{1}{9}$
 e $-\frac{3}{7}$ f $-\frac{1}{9}$ g $\frac{9}{2}$ h $\frac{1}{15}$

2 a $\frac{1}{5}$ b $\frac{1}{8}$ c $-\frac{1}{2}$
 d $\frac{1}{4^2}$ e $\frac{1}{2^6}$ f $\frac{1}{(-9)^3} = -\frac{1}{9^3}$
 g $\frac{1}{(-5)^4} = \frac{1}{5^4}$ h $-\frac{1}{7^8}$

3 a 5^{-1} b $(-13)^{-1}$ c $\frac{1}{5^{-1}}$
 d $-\frac{1}{8^{-1}}$ e $(\frac{7}{2})^{-1}$ f $-\frac{1}{21^{-1}}$
 g $-(\frac{3}{4})^{-1}$ h $\frac{1}{12^{-1}}$

4 a 3^{-4} b 4^{-7} c 6^{-5}
 d $(-5)^{-3} = -5^{-3}$ e $(-9)^{-2} = 9^{-2}$ f -11^{-6}
 g -4^{-3} h $-(-6)^{-3} = 6^{-3}$

5 a 7^{-2} b 5^{-3} c 2^{-4} d 3^{-4}

6 a 2^3 b 5^6 c $(-8)^4 = 8^4$
 d 3^9 e $(-7)^5 = -7^5$ f -4^2
 g $-(-7)^3 = 7^3$ h $-(-8)^7 = 8^7$

7 a $\frac{5^2}{4^2}$ b $\frac{3}{7}$ c $\frac{4^3}{3^3}$
 d $\frac{7^{11}}{9^{11}}$ e $\frac{5^7}{6^7}$ f $\frac{2^9}{9^9}$
 g $\frac{17^{14}}{13^{14}}$ h $\frac{43^{11}}{500^{11}}$

8 a $\frac{1}{4^3}$ b $\frac{1}{7}$ c 2^2
 d $\frac{1}{(-3)^6} = \frac{1}{3^6}$ e 5^4 f $\frac{1}{(-2)^7} = -\frac{1}{2^7}$

g $\frac{1}{9^2}$ h 3^8 i $\frac{1}{4^9}$
 j $\frac{1}{10^3}$ k $\frac{1}{211^{14}}$ l $\frac{1}{13^{100}}$

9 a $\frac{1}{5^6}$ b $\frac{1}{3^8}$ c -2^4
 d $\frac{1}{3^2}$ e $\frac{1}{6^4}$ f $\frac{1}{4}$
 g $\frac{1}{9}$ h 5^8 i 7^3
 j $\frac{1}{2^3}$ k 99^{127} l $\frac{1}{15^{162}}$

10 a $\frac{1}{64}$ b $\frac{1}{7}$ c 4
 d $\frac{1}{729}$ e 625 f $-\frac{1}{128}$

11 a -3 b -7 c -1 d 2
 e -2 f -3 g -2 h -4

12 a

Index notation	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}
Basic numeral	32	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$

b Each term is half of the term before it. The basic numeral of a term with base 2 and a negative index is the reciprocal of the value of the term with the matching positive index.

c $\frac{1}{64}$ d $\frac{1}{1024}$ e 128

13 a

Index notation	3^5	3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}	3^{-4}	3^{-5}
Basic numeral	243	81	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$	$\frac{1}{81}$	$\frac{1}{243}$

b Each term is one third of the term before it. The basic numeral of a term with base 3 and a negative index is the reciprocal of the value of the term with the matching positive index.

c $\frac{1}{729}$ d $\frac{1}{6561}$ e 2187

14 a

Index notation	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
Basic numeral	10000	1000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10000}$

b Each term is one tenth of the term before it. The basic numeral of a term with base 10 and a negative index is the reciprocal of the value of the term with the matching positive index.

c i 100 000 ii 1 000 000
 iii 10 000 000 iv 100 000 000
 v 1 000 000 000

d i $\frac{1}{100\,000}$ ii $\frac{1}{1\,000\,000}$
 iii $\frac{1}{10\,000\,000}$ iv $\frac{1}{100\,000\,000}$

v $\frac{1}{1\,000\,000\,000}$

e i $\frac{1}{10}$ ii 0.1
 f i $\frac{1}{100} = 0.01$ ii $\frac{1}{1000} = 0.001$

iii $\frac{1}{10\,000} = 0.0001$

g i $\frac{1}{100\,000} = 0.00001$

ii $\frac{1}{1\,000\,000} = 0.000001$

iii $\frac{1}{10\,000\,000} = 0.0000001$

iv $\frac{1}{100\,000\,000} = 0.00000001$

v $\frac{1}{1\,000\,000\,000} = 0.000000001$

h The number of decimal places in the decimal matches the number of zeroes in the denominator of the fraction, which matches the power of 10.

15 a $\frac{1}{64}$ mm b 0.015 625 mm

16 a $\frac{1}{100\,000\,000}$ b 0.000 000 01

17 a $\frac{1}{15\,625}$ b 0.000 064

18 a i 20 000 ii 7000
 iii 300 000 iv 400 000 000 000
 v 90 000 000

b i $\frac{5}{100}$ ii $\frac{8}{100\,000}$ iii $\frac{2}{1000}$

iv $\frac{7}{10\,000}$ v $\frac{6}{1\,000\,000\,000}$

c i 0.05 ii 0.000 08 iii 0.002
 iv 0.0007 v 0.000 000 006

d When multiplying a whole number by a negative power of ten, move the decimal point one place to the left for each zero you see.

19 a i $2 \times 3^{-1} = 2 \times \frac{1}{3} = \frac{2}{3} = 2 \div 3$

ii $2 \div 3^{-1} = \frac{2}{3^{-1}} = 2 \times \frac{1}{3^{-1}} = 2 \times 3$

b Multiplication by the reciprocal of a number is equivalent to division by that number. Division by the reciprocal of a number is equivalent to multiplication by that number.

20 a i 3 ii 5 iii $\frac{5}{4}$

b Using the power of a power law,
 $(a^{-1})^{-1} = a^{(-1) \times (-1)} = a^1 = a$.

c The reciprocal of the reciprocal of a number is the original number.

21 a $2a^{-2}b^{-1}$ b $-3t^2v^{-3}$ c $5^{-1}x^4y^{-4}$ d $p^{-1}q^{-5}r^2$

22 a $\frac{1}{x^2}$ b $\frac{1}{x^4}$ c $8x^3$ d $\frac{30}{x^5}$

e 3 f x^9 g $\frac{1}{x^3}$ h $2x^5$

i $\frac{1}{3x^{10}}$ j $\frac{4}{7x^4}$ k $\frac{1}{x^2}$ l x^{11}

m $\frac{4}{x^{13}}$ n $\frac{2}{x^8}$ o $\frac{1}{x^5}$ p $\frac{1}{x^7y^7}$

EX 2E Scientific notation

p63

1 a 540 b 73 600

c -1800 d 405 000

e 2 753 000 f 0.61

g 0.000 008 22 h -0.000 976

i 0.000 070 03

2 a 10^2 b 10^3 c 10^4 d 10^5 e 10^6

f 10^{-1} g 10^{-2} h 10^{-3} i 10^{-4} j 10^{-5}

3 a 320 000 b 8 140 000 000

c -500 d -23 450 000

e 11 000 f 0.0064

g 0.000 007 28 h 0.000 000 9

i -0.000 030 2 j -0.0541

k 450 000 000 000 l 0.000 000 006 12

m 0.57 n 1306.8

o 0.000 273 16

4 a Check the answers on your calculator with the answers in question 3.

b Some of the very large and very small numbers may be difficult to see on calculators due to limited screen size.

5 a 4.5×10^3 b 7.32×10^6

c 2×10^5 d -1.9×10^2

e 3.216×10^3 f 6.3×10^{-3}

g 1.8×10^{-7} h 5×10^{-2}

i -7.02×10^{-5} j 4.27×10^{-1}

k 1.122×10^4 l 4×10^{-6}

m -5.682×10^2 n 2.49×10^{-4}

o 6.793×10^5 p -1.02×10^{-2}

6 a A, B, F, H b A, D, G c B, C, E, F, H

7 $5.76 \times 10^2, 7.422 \times 10^2, 3.51 \times 10^3, 8.02 \times 10^3,$
 $9.10 \times 10^3, 2.814 \times 10^4$

8 a 0.13 metres b 2000 litres

c 9 900 000 centimetres d 5000 microseconds

e 400 000 microlitres f 0.000 003 seconds

g 100 000 000 centilitres h 0.000 755 millimetres

9 a 5×10^{-4} m b 4.8×10^6 ML

c 4.8×10^{-2} mm d 9.3×10^9 people

10 a 6400 times in a minute

b 0.000 08 mm

c 149 600 000 km

d 0.000 000 000 000 28 cm

11

	1234.56	4.0191
$\times 10^3$	1.23456×10^3	0.0040191×10^3
$\times 10^2$	12.3456×10^2	0.040191×10^2
$\times 10^1$	123.456×10^1	0.40191×10^1
$\times 10^0$	1234.56×10^0	4.0191×10^0
$\times 10^{-1}$	12345.6×10^{-1}	40.191×10^{-1}
$\times 10^{-2}$	123456×10^{-2}	401.91×10^{-2}
$\times 10^{-3}$	1234560×10^{-3}	4019.1×10^{-3}

	0.0492	0.007 40
$\times 10^3$	0.000492×10^3	0.00000740×10^3
$\times 10^2$	0.000492×10^2	0.0000740×10^2
$\times 10^1$	0.00492×10^1	0.000740×10^1
$\times 10^0$	0.0492×10^0	0.00740×10^0
$\times 10^{-1}$	0.492×10^{-1}	0.0740×10^{-1}
$\times 10^{-2}$	4.92×10^{-2}	0.740×10^{-2}
$\times 10^{-3}$	49.2×10^{-3}	7.40×10^{-3}

- 12 **a** 6.8×10^7 **b** 2.0×10^2 **c** 4.5×10^4
d -4.0×10^4 **e** -1.23×10^{-1} **f** 3.0×10^5
- 13 **a** 7.3034×10^5 **b** 8.36×10^4 **c** 6.3×10^{-3}
d -2.68×10^{-2} **e** -1.057×10^4 **f** 9.35×10^4
- 14 **a** 1.08×10^9 km **b** 2.592×10^{10} km
c 9.46728×10^{12} km
- 15 **a** Mercury, Mars, Venus, Earth, Uranus, Neptune, Saturn, Jupiter
b Mercury, Mars, Venus, Earth, Neptune, Uranus, Saturn, Jupiter
- 16 **a** 2.4×10^6 km **b** 400 days
- 17 Approximately 507 seconds or about 8 minutes
- 18 **a i** 86 coins **ii** 774 g
b 0.025 m = 2.5 cm
- 19 **a** 2.82×10^3 seconds
b 1.2096×10^6 seconds
c 2.4192×10^7 seconds
d 3.1536×10^7 seconds
- 20 **a i** 7.3×10^3 seconds
ii 9.1×10^{-6} seconds
iii 54×10^{-9} seconds
iv 82×10^{12} seconds
v 129×10^6 seconds
vi 974×10^{-12} seconds
- b i** 56.01×10^6 seconds, 56.01 megaseconds (56.01 Ms)
ii 920×10^3 seconds, 920 kiloseconds (920 ks)
iii 43.1×10^{-6} seconds, 43.1 microseconds (43.1 μ s)
iv 788×10^{-9} seconds, 788 nanoseconds (788 ns)
v 80×10^{-3} seconds, 80 milliseconds (80 ms)
vi 103.56×10^{12} seconds, 103.56 terraseconds (103.56 Ts)
- 21 **a** Light is much faster than sound.
b i 3.3×10^{-7} seconds
ii about 0.30 seconds
c Watch for the smoke rather than listen for the shot (or listen for the shot and add 0.3 s onto the time).
- 22 Diameter of one atom = 2.54×10^{-9} cm
The line will be 2.54×10^{-3} cm long.
- 23 1.8×10^{356}

- k i** 10.0 **ii** 10.00
l i -356.8 **ii** -356.83
- 2 **a** 3 **b** 2 **c** 4
d 1 **e** 3 **f** 2
g 1 **h** 4 **i** 5
j 4 **k** 4 **l** 3
- 3 **a** 2 **b** 3 **c** 4
d 2 **e** 4 **f** 3
g 5 **h** 5
- 4 **a** 2.6×10^5 **b** -5.04×10^4 **c** 9.104×10^6
d -6.0×10^3 **e** 460 **f** 73 050
g 1000 **h** 40 000 **i** -5.14
j 0.035 **k** -42.06 **l** 0.9
- 5 **a** 3.3×10^2 **b** 4.87×10^4 **c** -1.908×10^5
d 3×10^3 **e** 4.03×10^{-1} **f** -5.4×10^{-2}
g 2.072×10^{-4} **h** -8×10^{-3} **i** 7.6×10^2
j -2.070×10^4 **k** 4.02×10^1 **l** 5.4008×10^4
- 6 **a i** 9.0 **ii** 9.02
b i 13.4 **ii** 13.41
c i -110.1 **ii** -110.14
d i 80.0 **ii** 80.00
e i -4.1 **ii** -4.15
f i -33.3 **ii** -33.31
g i 255.0 **ii** 255.05
h i 1090.0 **ii** 1090.01
- 7 **a** 401 **b** 78 **c** 2930
d 290 **e** 308 **f** 5
g 136 **h** 20
- 8 **a i** 2 (1 s.f.), 1.9 (2 s.f.), 1.90 (3 s.f.)
ii 2 (1 s.f.), 2.0 (2 s.f.), 1.99 (3 s.f.)
iii 2 (1 s.f.), 2.0 (2 s.f.), 2.00 (3 s.f.)
iv 2 (1 s.f.), 2.0 (2 s.f.), 2.00 (3 s.f.)
v 2 (1 s.f.), 2.0 (2 s.f.), 2.01 (3 s.f.)
vi 2 (1 s.f.), 2.1 (2 s.f.), 2.10 (3 s.f.)
- b** If the trailing zeroes were not written as significant, then many of these numbers would round to the same number for different significant figures, which means we would be unable to determine the number of significant figures the number was rounded to consistently.
- 9 **a** 0.4 (1 s.f.), 0.000 000 000 001 (1 s.f.), -0.000 03 (1 s.f.)
b 0, 0, and 0
c If leading zeroes are significant, then all values just more than -0.5 and just less than 0.5 would round to 0 for 1 significant figure, which does not provide useful information about very small numbers and is no different to rounding to the nearest integer (or decimal place for more significant figures).
- 10 **a** Jane rounded to three decimal places, not three significant figures.
b Kaleb included the leading zeroes as significant.

EX 2F Rounding and estimating

p69

- 1 **a i** 14.9 **ii** 14.85
b i 9.5 **ii** 9.55
c i 24.0 **ii** 24.02
d i 104.0 **ii** 104.00
e i -3.3 **ii** -3.26
f i 17.1 **ii** 17.12
g i -99.3 **ii** -99.29
h i 5.5 **ii** 5.55
i i 72.9 **ii** 72.88
j i -125.1 **ii** -125.09

- c Lisa did not include the 0 between the 1 and 2 as significant.
- d Marius did not keep the place value of the significant figures.
- 11 When rounding to a given number of decimal places involves rounding down, truncation will give the same value. When rounding to a given number of decimal places involves rounding up, the rounded value will be an approximation that is closest to the exact value, whereas truncation will give an approximation that is slightly further away from the exact value.
- 12 Rounding to a smaller number of significant figures can result in a more accurate estimate. For example, when the values in $1254 + 1043$ are first rounded to two significant figures the absolute error is 3, whereas when the values are first rounded to three significant figures the absolute error is 7
- 13 a 2.78 hours b 11.6 days c 31.7 years
d 31.7 millennia
- 14 a 1.60×10^1 b 2.56×10^2 c 6.54×10^4
d 4.28×10^9
- 4 a m^5n^8 b $54k^{18}l^2$
5 a $\frac{1}{5^5}$ b $-\frac{1}{11^3}$ c 4^4
d $-\frac{5^3}{4^3}$
6 a $\frac{1}{65\,536}$ b 343
7 a 58760 b 0.00000902
8 a 5.4×10^5 b 7.6×10^{-4}
9 a 38600
b 3.86×10^4 or 3.9×10^4 (rounded)
10 a 4 b 3
11 a 880 b 300000

Analysis

1 a

	Thy	Asha
Round 1	$\frac{2^{10}}{5^3}$	$\frac{3^6}{5^5}$
Round 2	$\frac{5^6}{2^3 \times 3}$	2×3^5
Round 3	$\frac{3^4}{2^3}$	$\frac{2^4 \times 5^3}{3^5}$

- b Thy: $2^4 \times 3^3 \times 5^3$, Asha: $\frac{2^5 \times 3^6}{5^2}$
- c Thy
- d No. Asha had more factors, but the factors of 5 are divided (not multiplied) by the other factors, reducing the value. Thy's value was 54000, whereas Asha's value was 933.12.
- e Thy: $2^{-4} \times 3^{-3} \times 5^{-3} = \frac{1}{2^4 \times 3^3 \times 5^3}$
Asha: $2^{-5} \times 3^{-6} \times 5^2 = \frac{5^2}{2^5 \times 3^6}$
- f Thy: $2^2 \times 3 \times 5^4$, Asha: $2^9 \times 3$
- g Thy: $2^{-6} \times 4^1 \times 6^{-4}$, Asha: $1^6 \times 5^3 \times 5^{-4}$
- h Asha by two factors

2 a

Base (n)	n^2	n^3	n^4	n^5	n^6	n^7
3	9	27	81	243	729	2187
5	25	125	625	3125	15625	78125
7	49	343	2401	16807	117649	823543
11	121	1331	14641	161051	1771561	19487171
13	169	2197	28561	371293	4826809	62748517
17	289	4913	83521	1419857	24137569	410338673
19	361	6859	130321	2476099	47045881	893871739

- b For base numbers ending in 3, the final digit follows the pattern: 3, 9, 7, 1, 3, ...
For base numbers ending in 5, the final digit is always 5.
For base numbers ending in 7, the final digit follows the pattern: 7, 9, 3, 1, 7, ...
For base numbers ending in 9, the final digit follows the pattern: 9, 1, 9, ...
For base numbers ending in 1, the final digit is always 1.

EX Chapter 2 review

p72

Mathematical literacy review

- 1 factor tree
- 2 D
- 3 Sample answer using 12: Factors are numbers that multiply to make 12, such as 2 and 6 ($2 \times 6 = 12$), or 3 and 4 ($3 \times 4 = 12$). These factors are not necessarily prime numbers. Prime factors are factors that are prime, such as 2 and 3 ($2 \times 2 \times 3 = 12$).
- 4 Expanded form: $24 = 2 \times 2 \times 2 \times 3$
Index form: $24 = 2^3 \times 3$
- 5 a The base of a value written in index notation is the number or variable that is multiplied repeatedly.
b Taking the negative index of a fraction is the same as taking the positive index of the reciprocal of the base.
- 6 a True
b False. In 101, the zero is significant (for example).
c True
d False. In 100, the trailing zeroes are not significant (for example).

Multiple choice

- 1 B 2 C 3 C 4 E 5 B
6 D 7 A 8 E 9 B 10 C

Short answer

- 1 a 81 b -125 c -64
d $\frac{243}{32}$ e 0.216 f 2.0736
- 2 a 17^6 b $(-5b^2)^5$ c $-10f^3v^7$ d $\left(\frac{b^4d^5}{6n^3}\right)^4$
- 3 a a^{16} b b c c^{16}
d $\frac{d^3}{3}$ e e^{47} f 9

Base (n)	Last digit of n^{100}
3	1
5	5
7	1
11	1
13	1
17	1
19	1

- d** i 5 ii 1 iii 1
 iv 1 v 1 vi 1
 vii 1

- 3 a** i Tas, NT, ACT
 ii NSW, Vic, QLD, SA, WA

State	Population	Population (rounded to leading digit)	Population (scientific notation)
NSW	8 153 600	8 000 000	8×10^6
Vic	6 613 700	7 000 000	7×10^6
QLD	5 322 100	5 000 000	5×10^6
SA	1 820 500	2 000 000	2×10^6
WA	2 785 300	3 000 000	3×10^6
Tas	571 500	600 000	6×10^5
NT	250 600	300 000	3×10^5
ACT	456 700	500 000	5×10^5

- f** i NSW ii NT iii 7.7×10^6
 iv 3.4×10^6 v 2.64×10^7
g 421 065; the difference is due to rounding.

CHAPTER 3 Algebra

EX 3A Simplifying

p81

- 1 a** $3x, -x, 20x$
b Like terms have exactly the same pronumerals raised to exactly the same powers. The coefficient can vary.
c 3, 7, -1, 2, 1, 20
2 $\{2ba^2, 3aab, 6a^2b\}, \{3a, 6a\}, \{2b^2a, 6ab^2, 3abb\}, \{6a^3, aaa, 6a^2a\}$
3 a $10a$ **b** $-8k$ **c** $6x^2$
d $4cd - 9cde$ **e** $12x + 6y$ **f** $4a + 6b$
g $9m + 2p$ **h** $1 - k$ **i** $3xy + 5x^2$
j $2d - 4de^2$ **k** $4m^3 + 2$ **l** $abc + 4ab + ac$
4 a $10x + y$ **b** $5ab - 5b + b^2 + a$
c $2km + 4$ **d** $-3x^2 + 3x - 4$
e $2a + a^2 + a^3 - 3$ **f** $6m^2n + 4mn^2 - 2n^2$
5 a $68a^2$ **b** $-27b^3$
c $-3c^2d^2$ **d** $3p^2q^2r + 2pq^2r$
e $7ab + 9cd + 4ac$ **f** $7x^2y + 2xy$
6 a $6abcd$ **b** $-20mpxy$ **c** $9g^2h$
d $-24k^2mn$ **e** $56bjp^2t$ **f** ax^2y^2

- g** $18a^3bcd$ **h** $-20h^2k^2p$ **i** $-6b^3$
j $4km^2n^3$ **k** $15x^4y^2$ **l** $56a^4b^3c^2$
7 a $75a^{18}z^{36}$ **b** $-432b^{31}y^{16}$
c $-160c^8m^9x^9$ **d** $44d^{23}n^{30}w^{17}$
e $45e^{11}p^{24}t^{19}$ **f** $-168g^{30}q^{41}u^{58}$
8 a $\frac{a}{6}$ **b** ac **c** $\frac{5xy}{3}$
d $\frac{3b}{2}$ **e** $\frac{m}{3}$ **f** b
g $4c$ **h** $-\frac{1}{2}$ **i** $\frac{y}{4}$
j $\frac{2}{11}$ **k** $\frac{5n^2}{3}$ **l** $-\frac{ac}{4}$

- 9 a** $13ab$ **b** $98ab^2$ **c** $2a^2$
d $3b$
10 a True **b** False **c** False **d** True

- 11 a** Set B is correct. Set A: All signs changed to + when expression rearranged. Set C: The sign in front of the $2a$ term has changed from + to - and $2 - 7$ should be -5 , not 9.
b Set A is correct. Set B: -3×4 should be -12 , not 12, and b should be squared in the last step. Set C: $b \times b$ (or b^2) has been written as $b \times 2$.

- 12 a** $8x + y = 22$ **b** $-6xy = 36$ **c** $15xy = -90$

d $10x = 30$ **e** $x^2y^3 = -72$ **f** $\frac{y}{2} = -1$

g $6y - 2xy = 0$ **h** $x^2 + 4x - y = 23$

i $\frac{7y^2}{2} = 14$

- 13 a** $2a + 3b + 2c = 11$ **b** $8ab - a = -18$
c $-2a^2b + ab^2 + 3ac = 40$ **d** $10a^2b^2c^2 = 1000$
e $3ab = -6$ **f** $\frac{4ac}{3} = \frac{40}{3} = 13\frac{1}{3}$

- 14 a** $20xy$ **b** $49x^2$ **c** $9xy$ **d** $2y^2$
15 a 120 m^2 **b** 441 m^2 **c** 54 m^2 **d** 8 m^2

- 16 a** i $6k$ ii $2k^2$
b 30 cm and 50 cm^2
17 a i $4x \text{ m}$ ii $x^2 \text{ m}^2$
 iii $(4x + 8) \text{ m}$ iv $(4x + 4) \text{ m}^2$
b i 36 m^2 ii 32 m
 iii 40 m iv 64 m^2

- 18 a** $10xy$ **b** 200 cm^2 **c** $12x + 8y$
d 88 cm **e** 158 cm

EX 3B Algebraic fractions with numerical denominators

p87

- 1 a** $\frac{2x}{3}$ **b** $\frac{2xy}{3}$ **c** $\frac{25m}{2}$
d $\frac{s}{20}$ **e** $\frac{1}{3}$ **f** $\frac{p}{6}$
g n **h** $\frac{x^2}{6}$
2 a $\frac{xy}{20}$ **b** $\frac{mn^2}{20}$ **c** $\frac{xy}{70}$
d $\frac{s^2}{10}$ **e** $\frac{3xy}{2}$ **f** $\frac{2m^3n}{3}$

- g $4n^2$ h $8t$ i $\frac{5y}{12}$
 j $\frac{4m^2}{3}$ k $\frac{2n}{3}$ l $\frac{1}{3}$
 3 a $\frac{x}{2} \times \frac{5}{x}$ b $\frac{x}{12} \times \frac{5}{3x}$ c $\frac{xy}{2} \times \frac{7}{12y}$
 d $\frac{xy}{16} \times \frac{4}{x}$ e $\frac{x^2}{32} \times \frac{25}{x}$ f $\frac{21x}{12} \times \frac{2}{3x}$
 g $\frac{5xy}{2} \times \frac{7}{12y}$ h $\frac{x^2y}{2} \times \frac{15}{21y}$
 4 a $\frac{x^2}{10}$ b $\frac{x^2}{20}$ c $\frac{7x}{2}$
 d $\frac{6xyz}{7}$ e $\frac{3x^3}{5}$ f $\frac{7}{6}$
 g $\frac{5xyz}{9}$ h $\frac{15x^2}{4}$ i $\frac{2xy^2z}{115}$
 j $\frac{2mn^3}{3}$ k $\frac{25m}{8}$ l $\frac{8s}{27}$
 5 a $\frac{5}{2}$ b $\frac{5}{36}$ c $\frac{7x}{24}$
 d $\frac{y}{4}$ e $\frac{25x}{32}$ f $\frac{7}{6}$
 g $\frac{35x}{24}$ h $\frac{5x^2}{14}$
 6 a $\frac{x^3}{240}$ b x^3 c $\frac{xy^2}{28}$
 d $\frac{xyz^2}{7}$ e $\frac{3x^2yz^2}{10}$ f $\frac{7x}{36}$
 g $\frac{7x}{12}$ h $\frac{25xy}{28}$
 7 a $\frac{x^2}{16}$ b $\frac{9x^2}{16}$ c $\frac{m^2n^2}{9}$ d $\frac{9x^2y^2}{4}$
 e $\frac{a^3}{125}$ f $\frac{x^2y^3}{8}$ g $\frac{a^3b^3c^3}{64}$ h $\frac{8x^3}{125}$
 8 a 15 b 10 c 24 d 20
 e 8 f 12 g 525 h 60
 9 a $\frac{9x}{20}$ b $\frac{5x}{14}$ c $\frac{x}{2}$
 d $\frac{x}{40}$ e $\frac{7x}{6}$ f $\frac{5x}{8}$
 g $\frac{43x}{35}$ h $\frac{34a}{15}$ i $\frac{8a}{15}$
 j $\frac{144n}{65}$ k $\frac{2a}{35}$ l $\frac{34s}{75}$
 10 a $\frac{8x}{15}$ b $\frac{3x}{10}$ c $\frac{5x}{24}$ d $\frac{17x}{20}$
 e $\frac{19x}{8}$ f $\frac{7x}{12}$ g $\frac{71x}{525}$ h $\frac{143x}{60}$
 11 a $\frac{x^2}{15}$ b $\frac{x^2}{50}$ c $\frac{x^2}{96}$ d $\frac{13x^2}{100}$
 e $\frac{21x^2}{16}$ f $\frac{x^3}{216}$ g $\frac{x^3}{13125}$ h $\frac{7x^3}{40}$
 12 a $\frac{37x}{30}$ b $\frac{7x}{8}$ c $\frac{x}{8}$
 d $\frac{x^2}{4}$ e $\frac{15p^2}{4}$ f $\frac{7x^2}{8}$
 g $\frac{457mn}{105}$ h $\frac{163x}{210}$
 13 a $\frac{4y}{5}$ b $\frac{3x}{2}$ c $\frac{5x}{4}$
 d $6x$ e $2y$ f $\frac{3x^2}{2}$
 14 a $\frac{191a}{84}$ b $\frac{17x}{10}$ c $\frac{4mn}{7}$
 15 a $\frac{49x^2}{144}$ b $\frac{b^3}{216}$ c $\frac{7a^2}{25}$ d $\frac{25}{4}$
 16 a True. $\frac{x}{2} - \frac{x}{4} = \frac{2x}{4} - \frac{x}{4} = \frac{x}{4}$
 b True. $\frac{2x}{2} - \frac{3x}{4} = 1 - \frac{3x}{4} = \frac{x}{4}$

- c Sample answer: $m = 3x, n = 5x$
 17 a True. $\frac{4y}{2} + \frac{4y}{3} = \frac{12y}{6} + \frac{8y}{6} = \frac{20y}{6} = \frac{10y}{3}$
 b True. $\frac{2y}{2} + \frac{7y}{3} = \frac{6y}{6} + \frac{14y}{6} = \frac{20y}{6} = \frac{10y}{3}$
 c Sample answer: $a = 6y, b = y$
 18 a True. $\frac{z}{2} - \frac{z}{3} = \frac{3z}{6} - \frac{2z}{6} = \frac{z}{6}$
 b True. $\frac{3z}{2} - \frac{z}{3} = \frac{9z}{6} - \frac{2z}{6} = \frac{7z}{6}$
 c Sample answer: $p = 4z, q = 10z$

EX p92 **3C Algebraic fractions with algebraic denominators**

- 1 a $\frac{mn}{p}$ b $-\frac{2}{x}$ c $\frac{6b}{3c}$
 d $6a$ e $-3xy$ f $\frac{ac}{4d}$
 g $3x$ h $\frac{y^4}{x^3}$
 2 a $\frac{y}{3x}$ b $\frac{2n^2}{3m}$ c $\frac{25}{2}$
 d $\frac{1}{20s^2}$ e $\frac{1}{2x}$ f $\frac{p}{36}$
 g n h $\frac{x}{6}$
 3 a $\frac{1}{10y^2}$ b $\frac{1}{12mn}$ c $\frac{6}{7}$
 d $\frac{6}{x}$ e $\frac{x^3}{100y^4}$ f $\frac{21x^2}{8y^9}$
 4 a $\frac{y^2}{2}$ b $\frac{5x}{12y^2z}$ c $\frac{xy}{36}$
 d $\frac{7q}{6p}$ e $\frac{25m^3n}{4}$ f $\frac{24}{5a^2b^2}$
 5 a $\frac{1}{48x^3}$ b $\frac{1}{m^2n}$ c $\frac{1}{28x}$
 d $\frac{1}{7z^2}$ e $\frac{3y}{10xz^2}$ f $\frac{7x^4}{36z^2}$
 g $\frac{7y^2z}{12x^2}$ h $\frac{25y}{28z}$
 6 a $\frac{2}{x}$ b $\frac{4}{3a}$ c $\frac{34}{15a}$
 d $\frac{1}{5a}$ e $\frac{144}{65n}$ f $\frac{7}{4p}$
 g $\frac{143}{12ab}$ h $\frac{33}{28xyz}$
 7 a $\frac{73}{15m^2}$ b $\frac{457}{105st}$ c $\frac{163x}{210y}$
 d $\frac{3}{7}$ e $\frac{8x}{21}$ f $\frac{7y^2}{8x^2}$
 8 a $\frac{4x^2y^2}{5}$ b $\frac{3x^3y}{2z}$ c $\frac{5}{4x}$
 d $\frac{6}{x}$ e $\frac{12}{5y^2}$ f $\frac{4}{3x^4}$
 g $2y^2$ h $\frac{16}{3}$
 9 a $\frac{49}{144a^2}$ b $\frac{1}{216b^3}$ c $\frac{7}{25x^2y^2}$
 d $\frac{25m^3}{4}$
 10 a $-\frac{3ab}{10}$ b x c $\frac{7x}{6}$ d $\frac{5x}{4}$
 e 0 f $-\frac{23x}{18}$ g $\frac{13mn}{10}$ h $\frac{23}{21}$

11 When converting terms to equivalent fractions with the same denominators, Eve multiplied the fractions by $\frac{1}{35}$, $\frac{1}{10}$, and $\frac{1}{14}$ which changed the values of the terms.

12 a $5xy$ b xy c $24x$
 d $12x^2y$ e $75mn$ f $60x^2y^3$
 13 a $\frac{5+x^2y}{5xy}$ b $\frac{x^2+y^2}{xy}$ c $\frac{2y+3x^3}{24x}$
 d $\frac{4x^3+2xy+y^2}{12x^2y}$ e $\frac{5+3n+15m}{15mn}$
 f $\frac{10xy^2+3y^3+12x^2}{60x^2y^3}$

14 a $\frac{1}{5y}$ b 1 c $\frac{xy}{96}$
 d $\frac{1}{216x^2}$ e $\frac{1}{1875m^2n^2}$ f $\frac{1}{600x^3y^4}$

15 a $2y$ b $10\text{ m}, 160\text{ m}^2$

16 a $3x$ b $96\text{ cm}^2, 16\text{ cm}$

17 a $\frac{yz+xz+xy}{xyz}$ b $\frac{a^2c+ab^2+bc^2}{abc}$

18 a $\frac{(x+4)(5x-2)}{3x}\text{ m}^2$ b $\frac{2x^2+38x-12}{3x}\text{ m}$

19 a $\frac{1}{(n-1)a} + \frac{1}{na} + \frac{1}{(n+1)a}$

$$= \frac{n(n+1) + (n-1)(n+1) + n(n-1)}{(n-1)n(n+1)a}$$

$$= \frac{n^2+n+n^2-1+n^2-n}{(n-1)n(n+1)a}$$

$$= \frac{3n^2-1}{(n-1)n(n+1)a}$$

b $n = 14, \frac{587}{2730a}$

c $\frac{587}{2730a}$

EX 3D Expanding

p98

1 a $4a+12$ b $7b+35$ c $3c-6$
 d $5d-5$ e $24+6e$ f $-2f-16$
 g $-3g-12$ h $-8h+40$ i $-4x+36$
 j $-10+5j$ k $kp+6k$ l $ab-4a$
 m $18m+6k$ n $2np+4nq$ o x^2-7xy

2 a $4ab+8bc+12ac$ b $-3xy-3x-3y$
 c $-m^2+n^2-2m^2n^2$ d $a^4-a^2b^2+2a^3b$
 e $-pq+4p-8q$ f $2ab^2c+a^2bc+4abc^2$

3 a $11x+6$ b $5p+6$ c $ab+2a$
 d $11y-4$ e $4h+k+2$ f $-6m$

4 a $5x-8$ b $13k-4$ c $7p+1$
 d $x^2+4x+12$ e m^2+5m+6 f $y^2-7y+10$

5 a $12az+15a$ b $56b+40by$ c $14c-6cx$
 d $-20dw-25d$ e $-48ev+54e$ f $-40et-30eu$
 g $8g^2-28gt$ h $-30hr+42h^2$ i $12i^2jk+15ij^2k$

6 a $ab+4a+3b+12$ b $cd+7c+2d+14$
 c $mn+m+5n+5$ d $jk-5j-9k+45$
 e $2ab+6a+7b+21$ f $15cd-20c+6d-8$

7 a a^2+5a+6 b $x^2+15x+50$
 c $d^2-2d-24$ d $y^2-5y-24$
 e $k^2+2k-63$ f $m^2-3m-18$
 g $5e^2-22e+8$ h $21a^2-31a+8$

i $6y^2-13y+5$
 8 a $5b+20-ab-4a$ b $-x^2-3x+18$
 c $-8c^2-2c+15$ d $44d^2-102de+54e^2$
 e $-42p^2+13pq+40p^2$ f $4x^2-9x-1$
 g $x^8+x^7+x^4+x^3$ h $-20x^4+13x^2+84$
 i $2x^2+11x+11$

9 a a^2-9 b b^2-4 c c^2-25
 d $9p^2-64$ e h^2-1 f k^2-m^2

10 The middle two terms always combine to give zero, so you are left with the first term squared minus the second term squared. The two factors are almost the same with one set of brackets containing a sum of two terms and the other containing a difference of the same two terms.

11 a The result is the first term squared minus the second term squared.

b No, as you can multiply two factors in any order.

c i x^2-4 ii y^2-81 iii m^2-36
 iv $9d^2-100$ v m^2-n^2 vi $9-x^2$

12 a $(100+3)(100-3) = 100^2-3^2$

b $103 \times 97 = (100+3)(100-3) = 10000-9 = 9991$

c i 9996 ii 9975 iii 999999
 iv 999964

13 a a^2+4a+4 b $b^2+14b+49$
 c $c^2+8c+16$ d $9p^2-30p+25$
 e h^2-2h+1 f $m^2+2mn+n^2$

14 The result is always the first term squared + 2 times the product of both terms + second term squared. The two factors are exactly the same.

EX Checkpoint

p95

1 a $4a-2b-5$ b $4t^2+12t-4$
 c $2x^3-7x^2+5x+2$ d $4cd^2-10d^2+2c$

2 a $-35ab$ b $48a^2bc^2$

c $\frac{9h}{4}$ d $\frac{t}{3}$

3 a $-800a^{16}b^{32}$ b $2a-b$ c $24a^3b^4$

d $\frac{3n}{4}$

4 a $\frac{mn}{10}$ b $\frac{a^2b}{20}$ c $\frac{125x}{4}$ d $\frac{7}{24}$

5 a $\frac{3a}{2}$ b $\frac{9m}{4}$ c $\frac{x^2}{6}$ d $\frac{15xyz}{4}$

6 a $\frac{4a^2b^2}{9}$ b $\frac{a^2}{36}$ c $\frac{61z^2}{15}$ d $\frac{11pq}{30}$

7 a $\frac{4bc}{ad}$ b $\frac{x}{2}$ c $\frac{q^8}{p^3}$ d $\frac{3yz}{x}$

8 a $\frac{x^2y^3}{36z^2}$ b $126pq$ c $\frac{x^2y^8}{20}$ d $\frac{33n^2}{4m^9}$

9 a $\frac{25bc^2}{8a}$ b $\frac{24}{5m^2n}$ c $\frac{p^2q}{7}$ d $\frac{21t}{s^7}$

- 15 a $(a + b)^2$ is a perfect square.
- b i $x^2 + 6x + 9$ ii $y^2 + 12y + 36$
 iii $m^2 + 4m + 4$ iv $16b^2 + 88b + 121$
 v $m^2 + 2mn + n^2$ vi $25 + 10x + x^2$
- c $(a - b)^2$ is also a perfect square. The rule or pattern holds true. The negative sign in the factor $(a - b)$ means that the middle term is $-2ab$ rather than $2ab$.
- d i $a^2 - 4a + 4$ ii $b^2 - 8b + 16$
 iii $c^2 - 14c + 49$ iv $25z^2 - 60z + 36$
 v $k^2 - 2kp + p^2$ vi $9 - 6x + x^2$

- 16 a $6xz - 10x^2 + 14xy - 8xz$
 b $6ax + 10bx - 12x + 9ay + 15by - 18y$
 c $2x^2 + 7xy + 15x + 3y^2 + 15y + 18$
 d $x^3 + 8x^2 + 19x + 14$

- 17 a i Area of large rectangle = $7(3 + 2)$
 $= 7 \times 5$
 $= 35$
 ii Area of large rectangle = $7 \times 3 + 7 \times 2$
 $= 21 + 14$
 $= 35$
- b i Area of large rectangle = $5(x + 2)$
 ii Area of large rectangle = $5x + 5 \times 2$
 $= 5x + 10$
- c The area of the large rectangle is length \times breadth or $a \times (b + c)$.
 The total area of the two smaller rectangles is $a \times b + a \times c$.

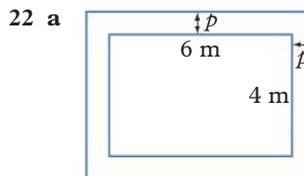
As the two areas are equal, $a(b + c) = ab + ac$.

- 18 a i Area of large rectangle = $(6 + 4)(3 + 5)$
 $= 10 \times 8$
 $= 80$
 ii Area of large rectangle = $6 \times 3 + 6 \times 5 + 4 \times 3 + 4 \times 5$
 $= 18 + 30 + 12 + 20$
 $= 80$
- b i Area of large rectangle = $(k + 2)(m + 3)$
 ii Area of large rectangle = $km + 3k + 2m + 2 \times 3$
 $= km + 3k + 2m + 6$
- c The area of the large rectangle is length \times breadth or $(a + b) \times (c + d)$.
 The total area of the four smaller rectangles is $a \times c + a \times d + b \times c + b \times d$.
 As the two areas are equal,
 $(a + b)(c + d) = ac + ad + bc + bd$.

19 E

- 20 a $8(2x + 5)$ b $16x + 40$
 c i 88 cm^2 ii 88 cm^2
 d The answers should be the same if you have expanded correctly.
- 21 a i $(x + 7)(x + 3)$ ii $x^2 + 10x + 21$
 iii 96 m^2

- b i $(x + 9)(x + 9)$ ii $x^2 + 18x + 81$
 iii 196 cm^2
 c i $(2x - 1)(x + 2)$ ii $2x^2 + 3x - 2$
 iii 63 cm^2



- b i $(2p + 6) \text{ m}$ ii $(2p + 4) \text{ m}$
 c $(2p + 6)(2p + 4) = 4p^2 + 20p + 24$
 d $(4p^2 + 20p + 24) - (6 \times 4) = 4p^2 + 20p$
 e i 24 m^2 ii 80 m^2 iii 56 m^2
- 23 a $100y^2 - 140y + 49$ b $x^7 - x^6$
 c $x^4 - 25$ d $y^{12} + 6y^7 + 9y^2$
 e $-5a^3$ f $x^6y^6 + 2wx^5y^2$
- 24 $(a - b)^2 + (c - d)^2$
 $= (a^2 - 2ab + b^2) + (c^2 - 2cd + d^2)$
 $= (b^2 - 2ab + a^2) + (d^2 - 2cd + c^2)$
 $= (b - a)^2 + (d - c)^2$

- 25 a Square: length = y , breadth = y
 Rectangle: length = y , breadth = $x - y$
 b Area of the original rectangle = xy
 Total area of the two shapes = $(y)(x - y) + y^2$
 $= xy - y^2 + y^2$
 $= xy$

EX
p105

3E Factorising using the HCF

- 1 a 4 b 2 c 5
 d d e 2 f 3
 g 4 h $3h$ i $12x$
- 2 a c b $2y$ c $9m$
 d bd e $4y$ f $2k$
 g p h 5 i $3p$
- 3 a $c(b + d)$ b $2y(x + 1)$ c $9m(2n - 1)$
 d $bd(ac + f)$ e $4y(2x + 7)$ f $2k(3k - 5)$
 g $p(1 + 11p)$ h $5(9ab - 8cd)$ i $3p(q + 2)$
- 4 a a^4 b b^2 c $2c^3$
 d $6d^3z^5$ e $7e^6y^5$ f $2fx^9$
 g $-2g$ h $6k^5$ i $x^9y^7z^{13}$
- 5 a $5(2x + 1)$ b $3(y - 7)$ c $4(2k + 3)$
 d $3(5 - 2d)$ e $7(4x + 3y)$ f $10(2n - 5)$
 g $xy(x + 3y^3)$ h $mn^2(mn - 9)$
- 6 a $5b(4a - 1)$ b $8d(1 + ce)$ c $5x(3x + 2)$
 d $2k(2k - 11)$ e $6n(5 - 3n)$ f $a(16a + 1)$
 g $2h(h - 7)$ h $6p(1 + p)$
- 7 a $8m^4(3m^2 + 2)$ b $7q^3r^2(3q^2 + 5r^4)$
 c $5t^4u^2(3 - t^4u^5)$ d $6c^4d^4(2c^5d + 1)$
 e $7e^5f^3(e^6 - 1)$ f $3ij^2k(i^4k^5 + 9j^5)$
 g $b^3c^7(a^7b^9 + c^2d^4)$ h $pq(5m - 3np^9q^4)$
 i $w^5z^3(u^4y^2 + v^4x^2)$

- 8 **a** $-5n(m+2)$ **b** $-7x(2y+1)$ **c** $-6c(1-d)$
d $-a(a+3)$ **e** $-2k(2k+1)$ **f** $-8x(x-1)$
g $-3(4+xy)$ **h** $-2m(8+5m)$ **i** $-9xy(x-2)$
- 9 **a** $(w+4)(x+2)$ **b** $(x-1)(y+7)$
c $(a+6)(a-3)$ **d** $(5-n)(p+8)$
e $(4-k)(3k-5)$ **f** $(3x-4)(2x+9)$
g $(2g+1)(4g+1)$ **h** $(8n-d)(2h-1)$
i $(7x+6y)(3a+2b)$
- 10 **a** $(a+5)(b+4)$ **b** $(y-6)(x+7)$
c $(n+4)(m-2)$ **d** $(y+3)(y+5)$
e $(k-7)(k+2)$ **f** $(x+3)(6+x)$
g $(a-7)(a-2)$ **h** $(p+5)(p-2)$
i $(3c-1)(2c+3)$

11 Expanding means to write an expression without brackets. Factorising means to write an expression in factor form and generally involves using brackets. Factorising and expanding are opposite processes.

- 12 **a i** $6(x+1)$ cm **ii** $2x(x+4)$ cm
b i 36 cm **ii** 90 cm
- 13 **a** $(2x+5)$ m **b** $(m+15)$ cm
- 14 **a** $2(2x+9)$ m **b** $2(2m+15)$ cm
- 15 **a** $3(9x-3y+5z)$ **b** $5(9p-10q-1)$
c $4(1-5i+10j-15k)$ **d** $a^2(a^3+a+1)$
e $6b^3c^4(3c-6b+4b^5c)$
f $7r^5t^7(12r^7t+1+7r^3t^7+2r^4t)$
- 16 **a** $\frac{1}{2}(5x+3y)$ **b** $\frac{3}{5}(3g+h)$
c $\frac{5}{4}(2p+3q)$ **d** $\frac{1}{12}(28m-15n)$
- 17 **a** $x(10-x)$ mm²
b Length and breadth are x mm and $(10-x)$ mm.
c Sample answers: 6 mm by 4 mm, 7 mm by 3 mm, 9 mm by 1 mm.
- 18 **a** Some possible answers are:
Height = $(7-x)$ cm, base length = $6x$ cm
Height = $2(7-x)$ cm, base length = $3x$ cm
Height = 6 cm, base length = $x(7-x)$ cm
- b** Some possible answers, using the first sample answer in **a**:
Height = 3 cm, base length = 24 cm ($x=4$)
Height = 6 cm, base length = 6 cm ($x=1$)
Height = 2 cm, base length = 30 cm ($x=5$)
- 19 $3x$ cm
- 20 **a** $(x+3)(x-4)$ **b** $(x-4)(x+3)$
c They are equivalent.
- 21 **a** $n+1, n+2$ **b** $3n+3$
c $3(n+1)$; three times middle number
d $n+(n+2)+(n+4)=3n+6=3(n+2)$; also three times middle number
- 22 For four consecutive numbers, the sum is four times the average of the two middle numbers.

For six consecutive numbers, the sum is six times the average of the two middle numbers.

For 10 consecutive numbers, the sum is 10 times the average of the two middle numbers.

$$10 \times \left[\frac{(n+4) + (n+5)}{2} \right] = 10 \times \left(\frac{2n+9}{2} \right) = 5(2n+9)$$

- 23 **a** $(y+5)(x-3)$ **b** $(2q-3)(p-1)$
c $(a-3)(2a-3)$
- 24 Area of square = $l \times b = 2r \times 2r = 4r^2$
Area of circle = πr^2
Area of shaded section = $4r^2 - \pi r^2 = r^2(4 - \pi)$
- 25 **a** $5\left(\frac{9x}{5} + \frac{17}{5}\right)$
b $12\left(x^2 + \frac{x}{4} + \frac{1}{2}\right)$
c $x\left(3x + 5y + \frac{y^2}{x}\right)$
d $(x+2)\left(7(x+2) + 5 + \frac{9}{x+2}\right)$

EX 3F Factorising monic quadratic expressions

- 1 **a** 1 and 4 **b** 2 and 4 **c** 2 and 11
d 4 and 5 **e** 4 and 6 **f** 3 and 4
g 6 and 7 **h** 5 and 7 **i** 4 and 4
- 2 **a** -2 and 4 **b** -3 and 2 **c** -6 and -2
d -2 and 5 **e** -9 and 1 **f** -3 and -2
g -1 and 6 **h** -9 and 3 **i** -11 and -1
- 3 **a** $(x+1)(x+4)$ **b** $(x+2)(x+4)$
c $(x+2)(x+11)$ **d** $(x+4)(x+5)$
e $(x+4)(x+6)$ **f** $(x+3)(x+4)$
g $(x+6)(x+7)$ **h** $(x+5)(x+7)$
i $(x+4)^2$
- 4 **a** $(a+3)(a+1)$ **b** $(b+2)(b+7)$
c $(c+1)(c+6)$ **d** $(d+3)(d+7)$
e $(e+1)(e+7)$ **f** $(f+3)(f+5)$
g $(g+4)(g+7)$ **h** $(h+4)(h+9)$
i $(x+3)(x+6)$ **j** $(j+5)(j+9)$
k $(k+5)(k+6)$ **l** $(y+5)(y+8)$
- 5 **a** $(x-2)(x+4)$ **b** $(x-3)(x+2)$
c $(x-6)(x-2)$ **d** $(x-2)(x+5)$
e $(x-9)(x+1)$ **f** $(x-3)(x-2)$
g $(x-1)(x+6)$ **h** $(x-9)(x+3)$
i $(x-11)(x-1)$
- 6 **a** $(a-1)(a+3)$ **b** $(b-5)(b+3)$
c $(c-4)(c-1)$ **d** $(d-2)(d+7)$
e $(e-6)(e-4)$ **f** $(f-5)(f+2)$
g $(g-3)(g+4)$ **h** $(h-5)(h-3)$
i $(x-8)(x+3)$ **j** $(j-8)(j-2)$
k $(k-3)(k+6)$ **l** $(y-2)(y+1)$
- 7 **a** $3(x+1)(x+2)$ **b** $2(x+3)(x+5)$
c $5(x-1)(x+4)$ **d** $-4(x+2)(x+3)$
e $-6(x-5)(x-1)$ **f** $-(x-5)(x+7)$

- 8 a** $a(x+2)(x+6)$ **b** $b(x-7)(x-4)$
c $10c(x-6)(x+5)$ **d** $-2d(x-5)(x-12)$
e $p^2(x+3)(x+10)$ **f** $x^2(q+11)(q+4)$
- 9 a** $(x-3)^2$ **b** $(x-3)(x+3)$
c $(x+0)(x-6) = x(x-6)$
- 10 a** $(x-5)(x+5)$ **b** $(x-5)^2$
c $x(x-25)$ **d** $(x+7)^2$
e $x(x-49)$ **f** $(x-7)(x+7)$
- 11 a** $(a+3)(a+7)$ **b** $(b-5)(b+7)$
c $(c-6)(c+3)$ **d** $-(d+6)(d-8)$
e $-(e+4)(e+6)$ **f** $-(f-2)(f-8)$
g $(g+8)(g+9)$ **h** $(h-7)(h+2)$
i $-(x+4)(x+9)$
- 12 a** $x^2 + 6x + 0 = (x+0)(x+6)$
 $x^2 + 6x + 5 = (x+1)(x+5)$
 $x^2 + 6x + 8 = (x+2)(x+4)$
 $x^2 + 6x + 9 = (x+3)(x+3)$
b $x^2 + 6x - 7 = (x-1)(x+7)$
 $x^2 + 6x - 16 = (x-2)(x+8)$
 $x^2 + 6x - 27 = (x-3)(x+9)$
 $x^2 + 6x - 40 = (x-4)(x+10)$
c When the coefficient of x is positive, both factors are positive when the constant is positive. When the coefficient of x is positive, one factor is positive and one is negative when the constant is negative.
- 13 a** $x^2 - 6x + 0 = (x-0)(x-6)$
 $x^2 - 6x + 5 = (x-1)(x-5)$
 $x^2 - 6x + 8 = (x-2)(x-4)$
 $x^2 - 6x + 9 = (x-3)(x-3)$
b $x^2 - 6x - 7 = (x+1)(x-7)$
 $x^2 - 6x - 16 = (x+2)(x-8)$
 $x^2 - 6x - 27 = (x+3)(x-9)$
 $x^2 - 6x - 40 = (x+4)(x-10)$
c When the coefficient of x is negative, both factors are negative when the constant is positive. When the coefficient of x is negative, one factor is positive and one is negative when the constant is negative.
- 14 a** $x^2 + 65x + 64 = (x+1)(x+64)$
 $x^2 + 34x + 64 = (x+2)(x+32)$
 $x^2 + 20x + 64 = (x+4)(x+16)$
 $x^2 + 16x + 64 = (x+8)(x+8)$
b $x^2 - 65x + 64 = (x-1)(x-64)$
 $x^2 - 34x + 64 = (x-2)(x-32)$
 $x^2 - 20x + 64 = (x-4)(x-16)$
 $x^2 - 16x + 64 = (x-8)(x-8)$
c i If the coefficient of x is positive, use positive factors when the constant is positive. If the coefficient of x is negative, use negative factors when the constant is positive.
ii Sum
- 15 a** $x^2 - 63x - 64 = (x+1)(x-64)$
 $x^2 - 30x - 64 = (x+2)(x-32)$
 $x^2 - 12x - 64 = (x+4)(x-16)$
 $x^2 - 0x - 64 = (x+8)(x-8)$
b $x^2 + 63x - 64 = (x-1)(x+64)$
 $x^2 + 30x - 64 = (x-2)(x+32)$
 $x^2 + 12x - 64 = (x-4)(x+16)$
 $x^2 + 0x - 64 = (x-8)(x+8)$
c i If the coefficient of x is positive, the larger factor is positive and the smaller factor is negative when the constant is negative. If the coefficient of x is negative, the larger factor is negative and the smaller factor is positive when the constant is negative.
ii Difference
- 16 a** $(x+4)(x+4) = (x+4)^2$
b $(y-5)(y-5) = (y-5)^2$
c $(v+1)(v+1) = (v+1)^2$
d $(2z-3)(2z-3) = (2z-3)^2$
e $(3w+7)(3w+7) = (3w+7)^2$
f $3(h-2)(h-2) = 3(h-2)^2$
g $-(k+9)(k+9) = -(k+9)^2$
h $(f+g)(f+g) = (f+g)^2$
i $(5p-4q)(5p-4q) = (5p-4q)^2$
- 17 a** Check for and factorise out any common factors. Halve the coefficient of the linear term and check if it is equal to the product of the square root of the squared terms (or squared term and constant). Both squared terms must be positive.
b i Both squared terms are not positive.
ii Both squared terms are not positive.
iii Half of the linear term is not the product of the square root of the squared terms.
iv Half of the linear term is not the product of the square root of the squared terms.
- 18 a** $(x+6)$ **b** $(x+3)(x+6)$
c i Length = 8 m, breadth = 5 m,
area = $8 \times 5 = 40 \text{ m}^2$
ii Area = $2^2 + 9 \times 2 + 18 = 40 \text{ m}^2$
- 19 a i** $(x+9) \text{ cm}$ **ii** $(3y+4) \text{ m}$
b 10 cm **c** 7 m
d $x = 2, y = \frac{4}{3}$
e While one of the lengths is positive, the other length and area are both negative, which does not make sense for a physical rectangle. That is, the values for the length, breadth and area for both items must all be positive.
- 20 a i** $(x-2)(x-3)$ **ii** $(x-2)(x-5)$
iii $(x-2)(x-2)$ **iv** $(x-2)(x-4)$
b 2 m

- c $x^2 - x - 6 = (x - 3)(x + 2)$ and
 $x^2 + x - 6 = (x - 2)(x + 3)$
 Both are wider than their backyards' breadth, x m.
- d $x^2 - 6x + 8$
 e 8 m^2
 f 19.25 m^2
 g Melissa: 16 m^2 , Lena: 22 m^2

- 21 a $(3x - 2)(3x + 7)$ b $(11x + 10)(11x - 8)$
 c $(5x - 4)(5x - 8)$ d $(x + 6)(x + 1)$
 e $(x - 9)(x - 10)$ f $x(x - 4)$
 g $3(x + 2)(3x - 7)$ h $2(x + 5)(2x - 3)$
 i $16(2x - 1)(2x + 3)$
- 22 a $(x - 2)(x + 2)(x - 3)(x + 3)$
 b $(x^2 + 4)(x - 4)(x + 4)$
 c $(x^2 + 25)(x - 1)(x + 1)$
- 23 a $(x + 4)(x + 1)$ b $(x + 4)(x + 2)$
 c $(x + 11)(x + 2)$ d $(x + 5)(x - 2)$
 e $(x - 9)(x + 1)$ f $(x - 3)(x - 2)$

- 8 a $8a - 11$ b $b^2 - 9b - 22$
 c $12c^2 - 23c + 10$ d $d^2 - w^2$
 e $36 + 12e + e^2$ f $f^2 - 18f + 81$
- 9 a $x^7 - x^6$ b $y^{12}z^4 - y^3z^6$
 c $x^{11} + x^5y^4 - x^6y^3 - y^7$
- 10 a $4(a - 6)$ b $36pq(q + 4)$
 c $(8 - d)(7d - 4)$ d $(1 + 3f)(5e + 2)$
- 11 a $z^4(6z - 5)$
 b $3d(3dy + 5x^2)$
 c $-12r^7t^8u^3(3r^3u^3 + 8)$
 d $a^4b^2c^7d^5(a^3 + a^2b^3c^2 + cd^2)$
- 12 a $8x(x^2 + 2)(3x - 1)$ b $(r + q)(t + p + 2)$
 c $2(x + 4)(2x - 3)$
- 13 a $(a + 5)(a + 1)$ b $(b - 4)(b - 3)$
 c $(c - 3)(c + 7)$ d $(d - 18)(d + 2)$
- 14 a $(x + 3)(x + 5)$ b $-5(x - 6)(x + 2)$
 c $-2(x - 10)(x - 6)$ d $-3(x + 10)(x - 8)$
- 15 a $2(7 - x)(7 + x)$ b $16(y^2 + 4x^2)$
 c $(x - 5)^2$ d $9x(x - 4)$

EX Chapter 3 review

p116

Mathematical literacy review

- 1 a terms, binomial, trinomial
 b monic quadratic trinomial
 c FOIL, binomial
 d Factorising, highest common factor
 e algebraic fraction, pronumeral or variable
- 2 a II b VI c VIII
 d VII e I f IV
 g III h V
- 3 a I, II b VI c I, II, III
 d V e IV

Multiple choice

- 1 B 2 A 3 B 4 A 5 E
 6 B 7 A 8 B 9 B 10 E
 11 A 12 B 13 A 14 A 15 C
 16 A 17 C

Short answer

- 1 a $16t$ b $13a - 18p$ c $-2k + km - 15$
 d $13m^2n - 5m^2 + 11n^2 - 4mn^2$
- 2 a $44x^2y^2z$ b $72m^4n^3p^2$ c $\frac{5e}{6}$ d $-\frac{n}{2}$
- 3 a $3ab$ b $\frac{x^3}{4}$ c $\frac{8n}{5}$ d $\frac{x}{18}$
- 4 a $-\frac{x}{22}$ b $\frac{4p^2q}{3}$ c $\frac{19a}{80}$ d $\frac{163m}{105}$
- 5 a $\frac{1}{20}$ b $\frac{a^2}{2}$ c $\frac{1}{4t^5s^2}$ d $-\frac{ab}{75}$
- 6 a $\frac{37}{60x}$ b $\frac{2}{15a}$ c $\frac{601}{195n}$ d $\frac{37}{12p}$
- 7 a $4z - 28$ b $-40 + 24y$
 c $35x - 30wx$ d $ut - 4u + 3t - 12$
 e $40r - 15rv + 48 - 18v$
 f $63mp - 27np + 77mq - 33nq$

Analysis

- 1 a $\frac{1}{2}n(n + 1) = \frac{1}{2}n^2 + \frac{1}{2}n, \frac{(n + 1)^2 - (n + 1)}{2} = \frac{1}{2}n^2 + \frac{1}{2}n$
 b i 15 ii 15
 c 5050 d 10 100
 e The sum of the first n even numbers is double the sum of the first n counting numbers.
 f $\frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$
 g i 55 ii 55
 h 338 350

2 a

Figure	Side length	$A = (\text{side length})^2$	$A = ax^2 + bx + c$
1	1	$(1)^2$	1
2	$2x + 1$	$(2x + 1)^2$	$4x^2 + 4x + 1$
3	$4x + 1$	$(4x + 1)^2$	$16x^2 + 8x + 1$
4	$6x + 1$	$(6x + 1)^2$	$36x^2 + 12x + 1$

- b $18x + 1 \text{ cm}$
 c $2(n - 1)x + 1 \text{ cm}, A_n = (2(n - 1)x + 1)^2 \text{ cm}^2$
 d $16x^2 + 8x \text{ cm}^2$
 e

Figure	length and breadth	$A = \text{length} \times \text{breadth}$	$A = ax^2 + bx + c$
1	2 and 1	2	2
2	$(2x + 2)$ and $(2x + 1)$	$(2x + 2)(2x + 1)$	$4x^2 + 6x + 2$
3	$(4x + 2)$ and $(4x + 1)$	$(4x + 2)(4x + 1)$	$16x^2 + 12x + 2$
4	$(6x + 2)$ and $(6x + 1)$	$(6x + 2)(6x + 1)$	$36x^2 + 18x + 2$

- f** $18x + 2$ cm and $18x + 1$ cm
g $2(n - 1)x + 2$ cm and $2(n - 1)x + 1$ cm,
 $A_n = (2(n - 1)x + 2)(2(n - 1)x + 1)$ cm²
h The only difference is in the constant in the binomial terms. The constant comes from the original side lengths. In part **c** they are both 1 since it is a square, and in part **g** they are 1 and 2 as they are side lengths of the rectangle.
i $A_6 = (10x + 4)(10x + 5) = 100x^2 + 90x + 20$
- 3 a** 30 cm \times x cm \times $(x + 20)$ cm
 15 cm \times $2x$ cm \times $(x + 20)$ cm
 15 cm \times x cm \times $2(x + 20)$ cm
 10 cm \times $3x$ cm \times $(x + 20)$ cm
 10 cm \times x cm \times $3(x + 20)$ cm
 5 cm \times $6x$ cm \times $(x + 20)$ cm
 5 cm \times x cm \times $6(x + 20)$ cm
b 30 cm \times x cm \times $(x + 20)$ cm
c 34 cm \times $(x + 4)$ cm \times $(x + 24)$ cm
d $34(x + 4)(x + 24)$
e $34x^2 + 952x + 3264$
f 20 cm \times 20 cm \times 5 cm
g $x = 60$
h $182\,784$ cm³

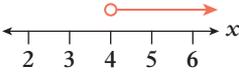
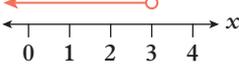
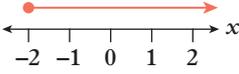
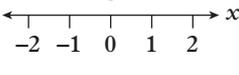
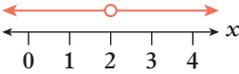
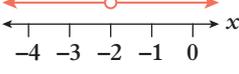
CHAPTER 4 Linear relationships

EX p126 4A Solving linear equations

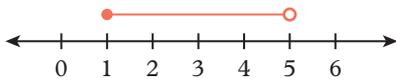
- 1 a** $x = 6$ **b** $x = 5$ **c** $x = 36$ **d** $x = 10$
e $x = -20$ **f** $x = 4$ **g** $x = 15$ **h** $x = -1$
i $x = -3$ **j** $x = -3$ **k** $x = 12$ **l** $x = -6$
- 2 a** $x = 2$ **b** $x = -8$ **c** $x = 9$ **d** $x = 5$
e $x = 20$ **f** $x = -8$ **g** $x = 0$ **h** $x = -4$
i $x = -5$ **j** $x = 10$ **k** $x = -20$ **l** $x = 4$
- 3 a** no **b** yes **c** yes **d** yes
- 4 a** $x = 6$ **b** Yes, $x = 6$ **c** $x = 5.6$ **d** $x = 5.6$
e Expanding brackets first means you do not have to work with fractions or decimals until the last step.
- 5 a** $x = 3$ **b** $x = -9$ **c** $x = \frac{11}{2}$
d $x = -\frac{12}{5}$ **e** $x = 4$ **f** $x = \frac{11}{6}$
- 6 a** $x = 2$ **b** $x = 7$ **c** $x = -1$ **d** $x = -3$
e $x = -2$ **f** $x = 7$ **g** $x = 4$ **h** $x = -4$
- 7 a** $x = -1$ **b** $x = 3$ **c** $x = 7$ **d** $x = -3$
e $x = 7$ **f** $x = -3$ **g** $x = -6$ **h** $x = 9$
- 8 a** Let n = number of jellybeans
b $4n + 2 = 34$
c $n = 8$
d Each person received eight jellybeans.
- 9 a** Let x = number of months
b $70x + 115 = 395$
c $x = 4$ **d** four months

- 10 a** \$185 **b** 14 goals
c 31 m \times 19 m **d** 84 people
11 a B **b** \$3.50
12 a $P = 3l$ **b** $b = 10$ m
13 56 sausages
14 100°
15 Numbers are 5, 6 and 7.
16 a i always **ii** never **iii** sometimes
iv never **v** always **vi** never
b After expanding and simplifying, if the coefficient of the pronumeral is the same on both sides and the constants on both sides are equal. Alternatively, the equation reduces to $0 = 0$.
c After expanding and simplifying, if the coefficient of the pronumeral is the same on both sides and the constants on both sides are not equal. Alternatively, the equation reduces to an equation that is not true, such as $0 = 2$.
17 a s for the speed of the truck, t for travel time
b $st = 650$
c $(s + 20)t$
d 10 hours
e 85 km/h
18 110

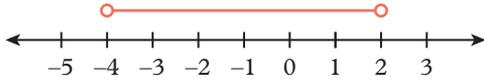
EX p133 4B Solving linear inequalities

- 1 a** $2 < 3$ **b** $3 > 2$ **c** $3 \geq 2$
d $-3 \leq -2$ **e** $3 \neq -3$ **f** $-3 \neq 3$
- 2 a** $x \leq 5$ **b** $x > 1$ **c** $x \geq -4$
d $x < -2$ **e** $x < 0$ **f** $x \neq 3$
g $x \geq 0$ **h** $-5 < x \leq 0$
i $-1 < x < 5$ **j** $-10 \leq x < -9$
- 3 a** $x > 4$
- 
- b** $x < 3$
- 
- c** $x \geq -2$
- 
- d** $x < 0$
- 
- e** $x \neq 2$
- 
- f** $x \neq -2$
- 

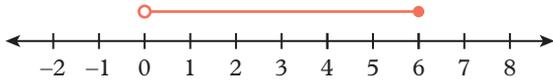
g $1 \leq x < 5$



h $-4 < x < 2$



i $0 < x \leq 6$



4 a i $1 < 14$

ii $-13 < 0$

iii $-12 < 27$

iv $-2 < \frac{9}{2}$

v $12 > -27$

vi $2 > -\frac{9}{2}$

vii $-\frac{4}{3} < 3$

viii $\frac{4}{3} > -3$

b i $x + 5 \geq 11$

ii $x - 9 \geq -3$

iii $3x \geq 18$

iv $\frac{x}{2} \geq 3$

v $-3x \leq -18$

vi $-\frac{x}{2} \leq -3$

vii $\frac{x}{3} \geq 2$

viii $-\frac{x}{3} \leq -2$

5 a $x > 14$

b $x \leq 300$

c $x < -5$

d $x \leq 5$

e $x < -70$

f $x \leq 5$

g $x \geq -1$

h $x > -3$

i $x \leq -9$

j $x \leq 7$

k $x > -37$

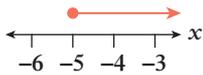
l $x > -\frac{13}{10}$

m $x < -\frac{1}{7}$

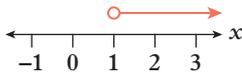
n $x \leq \frac{29}{6}$

o $x > \frac{11}{6}$

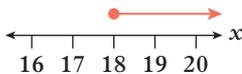
6 a $x \geq -5$



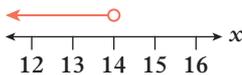
b $x > 1$



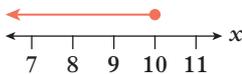
c $x \geq 18$



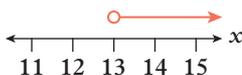
d $x < 14$



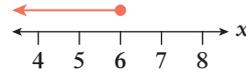
e $x \leq 10$



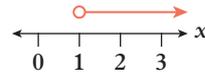
f $x > 13$



g $x \leq 6$



h $x > 1$



7 a $x > 4$

b $x \leq -2$

c $x \geq 3$

d $x > 4$

e $x < 1$

f $x \geq -1$

g $x \leq 5$

h $x < 2$

8 a $x > 11$

b $x \leq 13$

c $x \geq -2$

d $x > -19$

e $x > -15$

f $x \leq 1$

g $x \leq 3$

h $x > -\frac{1}{4}$

i $x \geq -\frac{13}{2}$

9 a i infinite number of solutions

ii no solution

b i no solution

ii infinite number of solutions

c i infinite number of solutions

ii no solution

d i infinite number of solutions

ii no solution

e i no solution

ii infinite number of solutions

f i infinite number of solutions

ii one solution

g i five solutions

ii infinite number of solutions

h i one solution

ii infinite number of solutions

10 a $p \geq 750\,000$, where p is the selling price in dollars

b $h < 200$, where h is the height of a person in centimetres

c $s \leq 55$, where s is the speed in km/h

d $h < 196$, where h is the height of a person in centimetres

11 a $m = \frac{1}{2}(20 - x)$, where m is the number of watermelons they will each take home

b $\frac{1}{2}(20 - x) \leq 3; x \geq 14$

c 14 or more watermelons

12 a 4

b No

c 5

13 a $3n \leq 25$, where n is the number of packs of playing cards Todd could buy; $n \leq \frac{25}{3}$

b Todd could buy 1, 2, 3, 4, 5, 6, 7 or 8 packs of playing cards.

14 a i $4 < 9$

ii $4 < 9$

iii $4 < 9$

iv $4 < 9$

v $\frac{1}{4} > \frac{1}{9}$

vi $\frac{1}{4} > \frac{1}{9}$

vii $4 = 4$

h $y = 3x - 2$

x	-3	-2	-1	0	1	2	3
y	-11	-8	-5	-2	1	4	7

$(-3, -11), (-2, -8), (-1, -5), (0, -2), (1, 1), (2, 4), (3, 7)$

i $y = 4 - 2x$

x	-3	-2	-1	0	1	2	3
y	10	8	6	4	2	0	-2

$(-3, 10), (-2, 8), (-1, 6), (0, 4), (1, 2), (2, 0), (3, -2)$

4 a i

x	0	3	6	9	12	15	18
y	24	21	18	15	12	9	6

ii

x	-12	-6	-2	2	6	12
y	-2	-4	-12	12	4	2

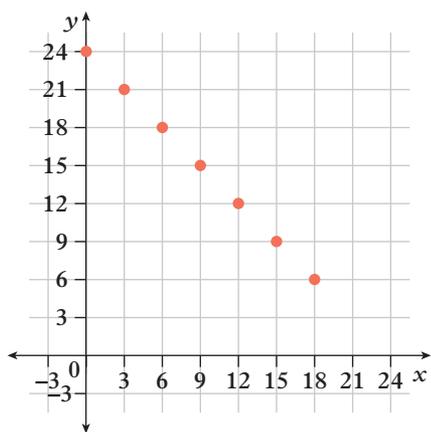
iii

x	-3	-2	-1	0	1	2	3
y	3	0	-1	0	3	8	15

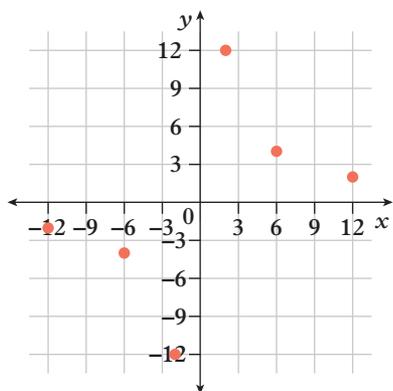
iv

x	-14	-10	-6	-2	2	6	10
y	-3	-2	-1	0	1	2	3

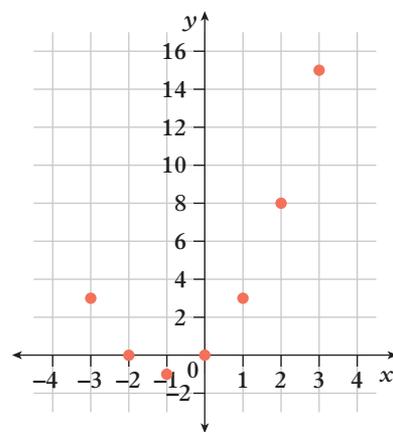
b i linear



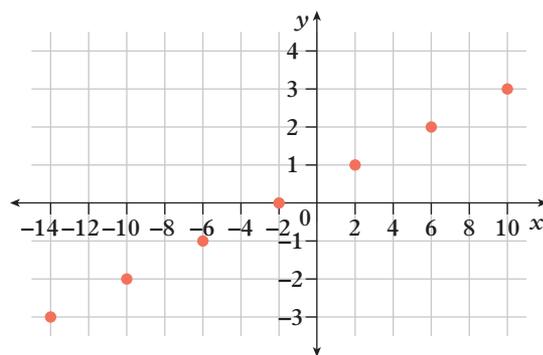
ii non-linear



iii non-linear



iv linear



5 a i $y = x$

x	-3	-2	-1	0	1	2	3
y	-3	-2	-1	0	1	2	3

ii $y = 2x$

x	-3	-2	-1	0	1	2	3
y	-6	-4	-2	0	2	4	6

iii $y = 3x$

x	-3	-2	-1	0	1	2	3
y	-9	-6	-3	0	3	6	9

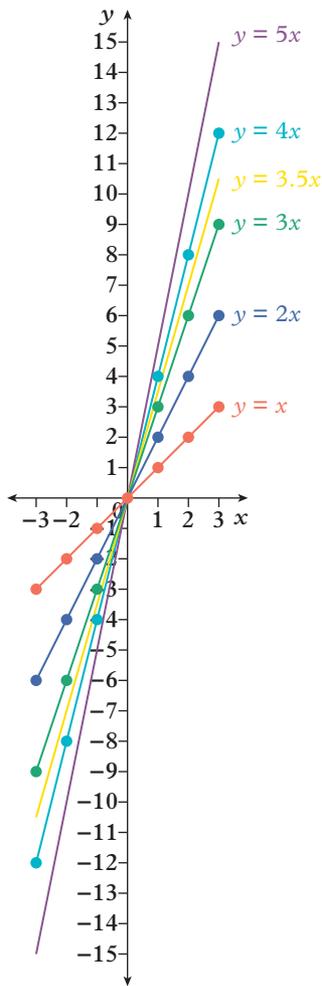
iv $y = 4x$

x	-3	-2	-1	0	1	2	3
y	-12	-8	-4	0	4	8	12

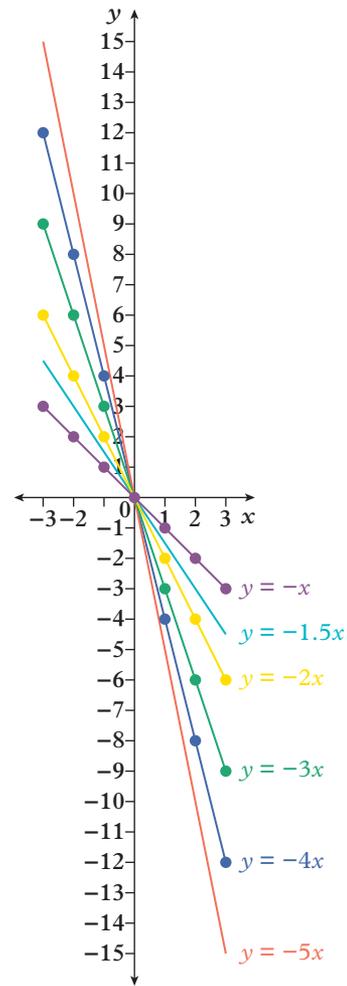
b All four graphs pass through the origin. However, the steepness of each line is different.

c As the coefficient of x increases, the steepness of the corresponding graph increases.

a, d



a, d



e The graph of $y = 0.5x$ is less steep than $y = x$; the graph passes through the origin.

6 a i $y = -x$

x	-3	-2	-1	0	1	2	3
y	3	2	1	0	-1	-2	-3

ii $y = -2x$

x	-3	-2	-1	0	1	2	3
y	6	4	2	0	-2	-4	-6

iii $y = -3x$

x	-3	-2	-1	0	1	2	3
y	9	6	3	0	-3	-6	-9

iv $y = -4x$

x	-3	-2	-1	0	1	2	3
y	12	8	4	0	-4	-8	-12

b The values of y decrease from left to right.

c As the coefficient of x decreases, the steepness of the corresponding graph increases.

7 a i $y = x$

x	-3	-2	-1	0	1	2	3
y	-3	-2	-1	0	1	2	3

ii $y = x + 1$

x	-3	-2	-1	0	1	2	3
y	-2	-1	0	1	2	3	4

iii $y = x + 2$

x	-3	-2	-1	0	1	2	3
y	-1	0	1	2	3	4	5

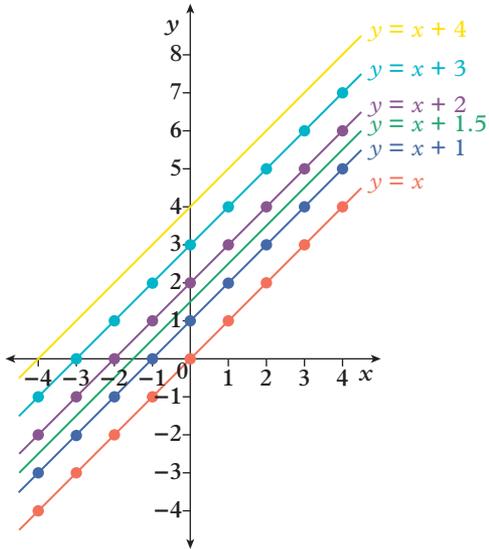
iv $y = x + 3$

x	-3	-2	-1	0	1	2	3
y	0	1	2	3	4	5	6

b All four graphs have the same steepness, but pass through different points on the x - and y -axes.

c As the value added to x increases, the graph moves upwards.

a, d



8 a i

x	-3	-2	-1	0	1	2	3
y	-3	-2	-1	0	1	2	3

ii

x	-3	-2	-1	0	1	2	3
y	-4	-3	-2	-1	0	1	2

iii

x	-3	-2	-1	0	1	2	3
y	-5	-4	-3	-2	-1	0	1

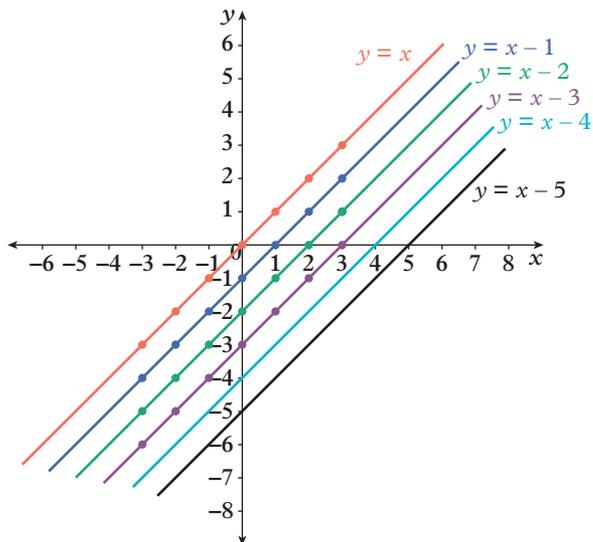
iv

x	-3	-2	-1	0	1	2	3
y	-6	-5	-4	-3	-2	-1	0

b All four graphs have the same steepness, but pass through different points on the x - and y -axes.

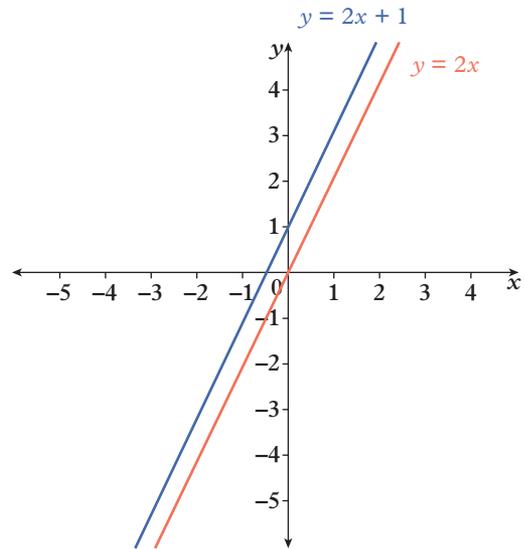
c As the value added to x decreases, the graph moves downwards.

a, d

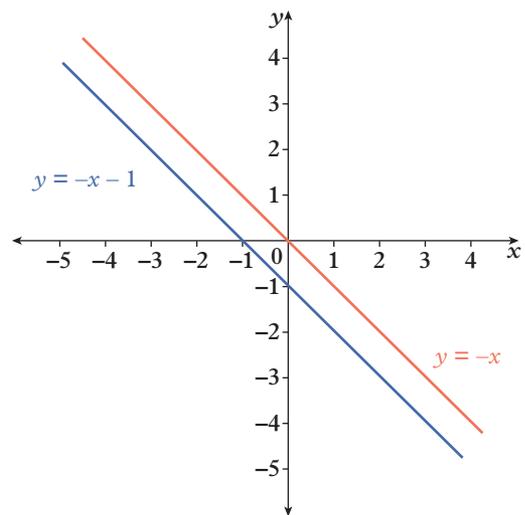


- 9 a A
b D
c B
d C

10 a



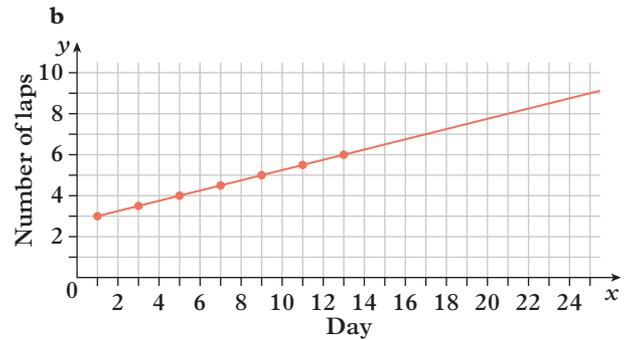
b



c Both graphs have the same steepness in each case, but the second graph is moved up in a and down in b in comparison to the first graph.

11 a i The number of laps increases by the same amount every two days.

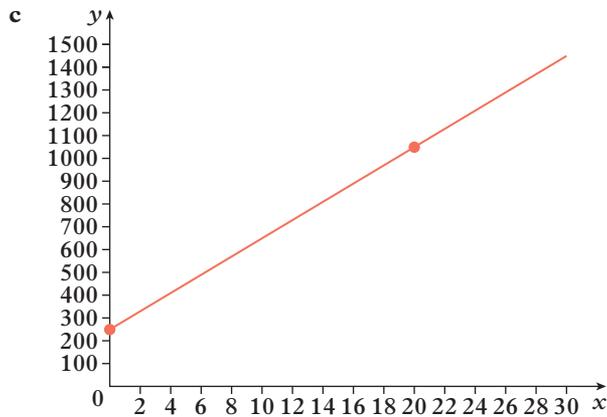
ii 2 points



c i 7 laps ii day 21

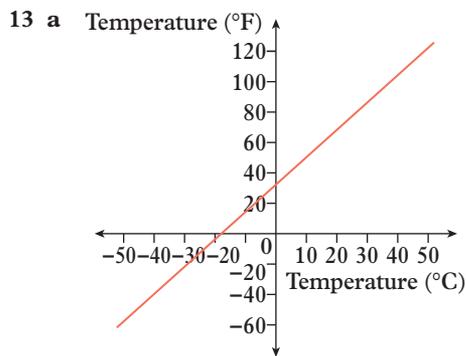
12 a $m = 40n + 250$

b i \$250 ii \$1050



d Relationship is linear because it forms a straight line.

e \$1450 f 15 students g 27 students



b 86°F c -30°C

14 $-40^{\circ}\text{F} = -40^{\circ}\text{C}$

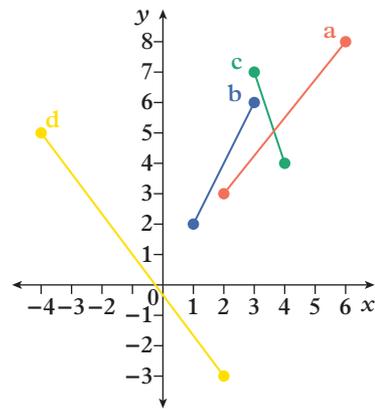
15 a (1, 2) b (0, 0) c (2, 0) d (4, 4)

EX 4D Gradient and intercepts

p144

- | | | |
|--------------------|------------------|------------|
| 1 a $\frac{3}{4}$ | b $-\frac{1}{2}$ | c -1 |
| 2 a $\frac{1}{3}$ | b $-\frac{3}{2}$ | c 2 |
| 3 a zero | b positive | c negative |
| d undefined | e zero | f negative |
| 4 a i 2 | ii 1 | iii -2 |
| b i undefined | ii -3 | iii none |
| c i -2 | ii -1 | iii -2 |
| d i 0 | ii none | iii 1 |
| e i -6 | ii 0 | iii 0 |
| f i $-\frac{2}{3}$ | ii -3 | iii -2 |

5 i



a ii $\frac{5}{4}$

b ii 2

c ii -3

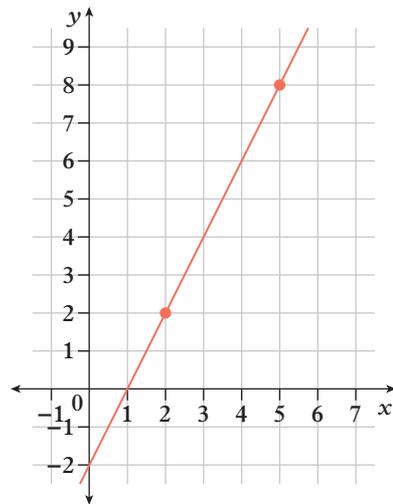
d ii $-\frac{4}{3}$

6 a $\frac{2}{3}$ b $\frac{3}{4}$ c 2 d $-\frac{1}{3}$

e $\frac{2}{5}$ f -3 g undefined h -5

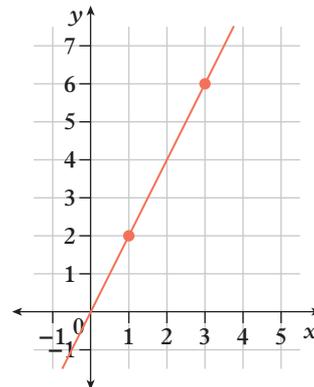
i $\frac{7}{3}$ j undefined k -2 l 0

7 a i

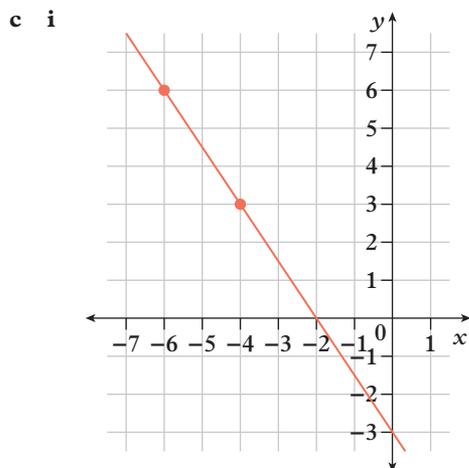


ii (1, 0), (0, -2)

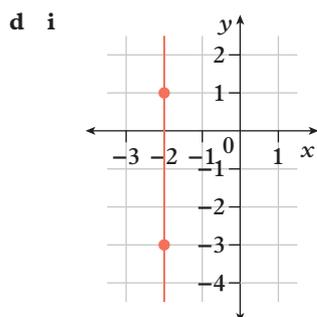
b i



ii (0, 0)



ii $(-2, 0), (0, -3)$



ii $(-2, 0)$

8 a i $\frac{3}{2}$ **ii** $\frac{3}{2}$

b The answers are the same. The gradient of a line is the same as the gradient of any line segment on that line.

9 a i rise 5, run 0

ii the denominator used in the calculation of gradient is the run, which cannot be zero

b i rise 0, run 3

ii the numerator used in the calculation of gradient is the rise; if the rise is zero, the gradient is zero

10 a i 0 **ii** 1 **iii** $\frac{1}{2}$ **iv** -1

b i $\frac{1}{2}$ **ii** 0 **iii** 1 **iv** -1

11 Kane subtracted the second y -coordinate from the first, and the first x -coordinate from the second; $m = -\frac{3}{4}$.

12 The order in which the coordinates are substituted into the gradient formula does not affect the value of the gradient.

13

Run	8	-20	-28	40	4	-6
Rise	12	16	-4	-16	28	30
Gradient	$\frac{3}{2}$	$-\frac{4}{5}$	$\frac{1}{7}$	$-\frac{2}{5}$	7	-5

14 a

	Equation	Gradient	y -intercept
i	$y = 2x - 3$	2	$(0, -3)$
ii	$y = -\frac{3}{4}x + 5$	$-\frac{3}{4}$	$(0, 5)$
iii	$y = x + 4$	1	$(0, 4)$
iv	$y = -2x - 1$	-2	$(0, -1)$
v	$y = 4x$	4	$(0, 0)$
vi	$y = 2$	0	$(0, 2)$

b gradient is coefficient of x ; y -intercept is the constant

c i gradient = 6, y -intercept: $(0, 4)$

ii gradient = 1, y -intercept: $(0, -5)$

iii gradient = -3, y -intercept: $(0, 0)$

15 a $x = 3$ **b** $y = 0$ **c** $y = 8$

d $y = 10$ **e** $y = 3$ **f** $x = 7$

16 a $\frac{y-4}{x} = 2$ **b** $y = 2x + 4$

c $\frac{y-4}{x-1} = 2$ **d** $y = 2x + 2$

EX **4E** Sketching linear graphs

p152

1 a $(12, 0); (0, 3)$ **b** $(-4, 0); (0, 4)$

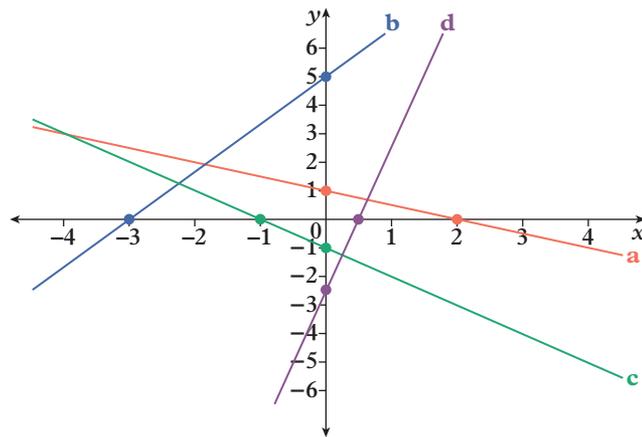
c $(3, 0); (0, 6)$ **d** $(5, 0); (0, -5)$

e $(4, 0); (0, 8)$ **f** $(2, 0); (0, -6)$

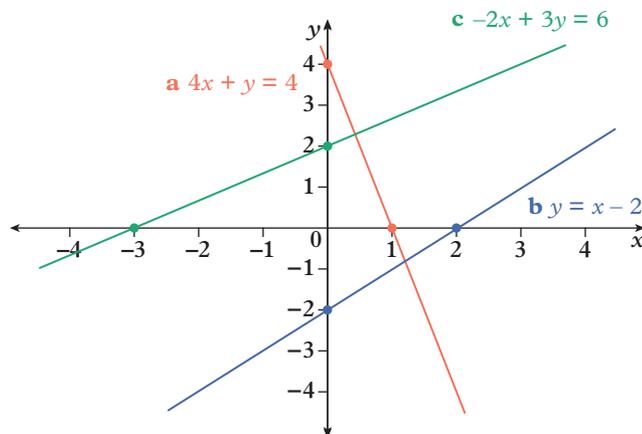
g $(-2, 0); (0, -10)$ **h** $(7, 0); (0, 7)$

i $(4, 0); (0, 4)$

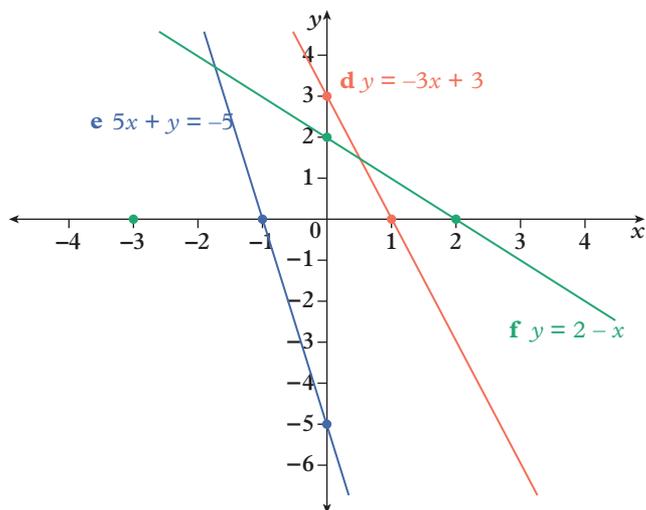
2 a-d



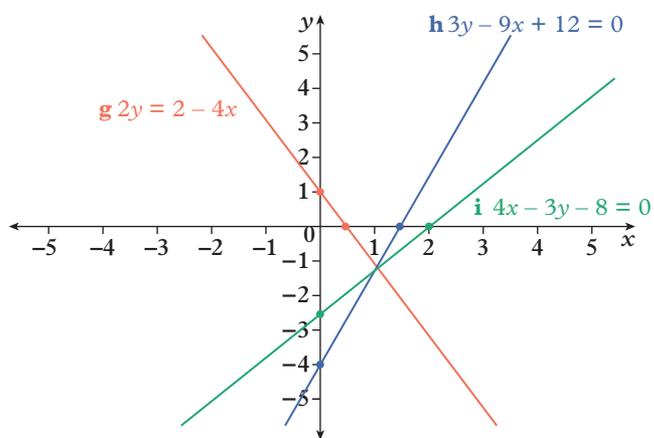
3 a-c



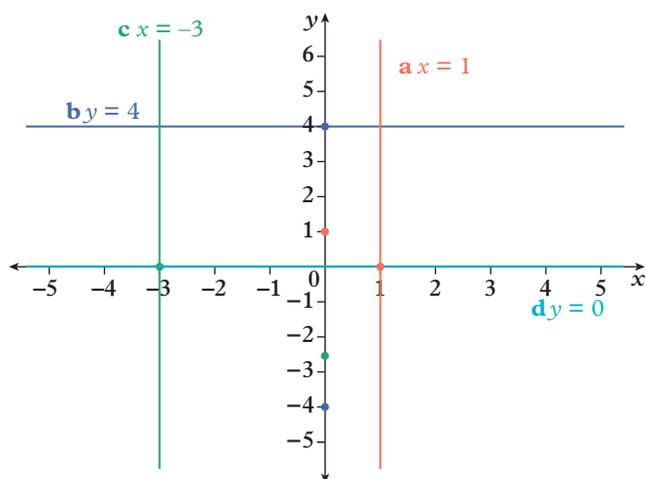
d-f



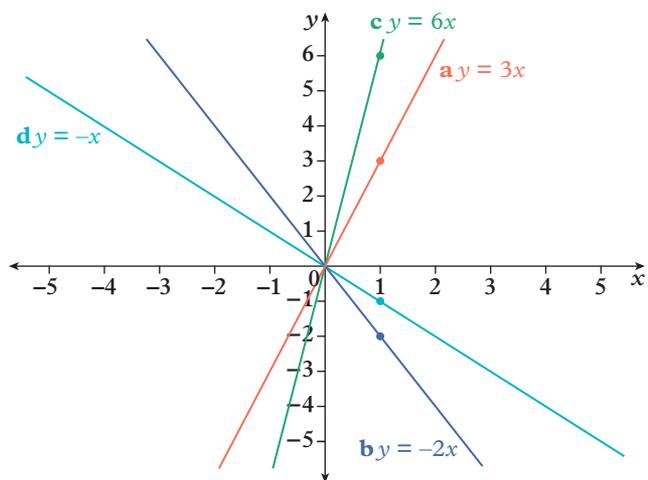
g-i



4



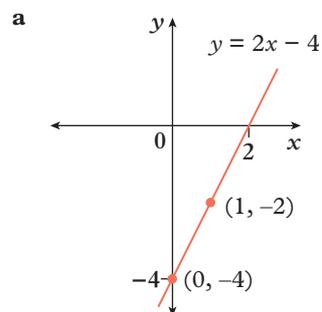
5



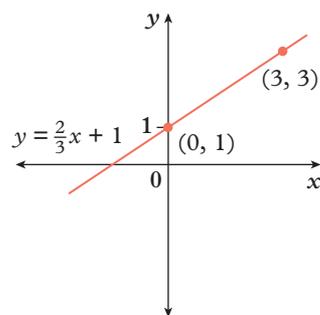
6

- | | | | |
|---|-------------------------------------|---|---------------------------|
| a | $m = 2, c = 5$ | b | $m = 4, c = 1$ |
| c | $m = -3, c = 7$ | d | $m = -5, c = -3$ |
| e | $m = 1, c = -6$ | f | $m = -1, c = 1$ |
| g | $m = \frac{4}{3}, c = 2$ | h | $m = \frac{1}{2}, c = -8$ |
| i | $m = -\frac{4}{3}, c = \frac{1}{4}$ | j | $m = 0, c = 9$ |
| k | $m = -7, c = 0$ | l | $m = -\frac{2}{5}, c = 5$ |

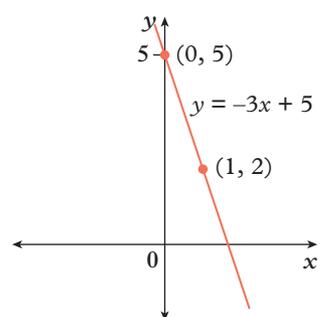
7

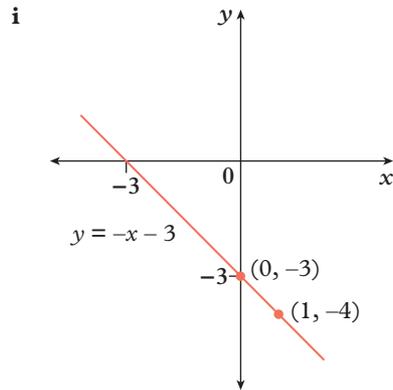
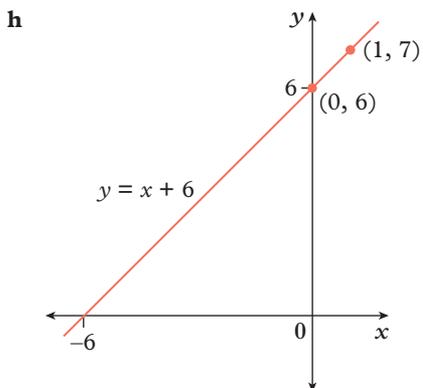
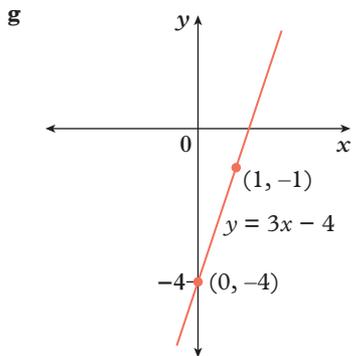
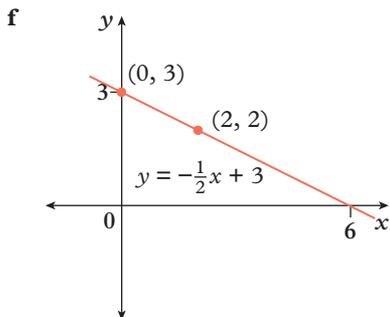
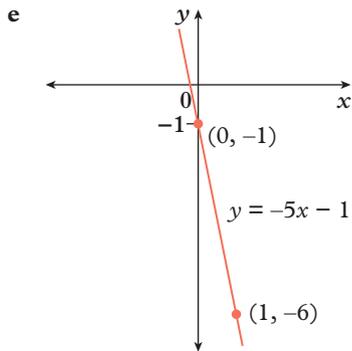
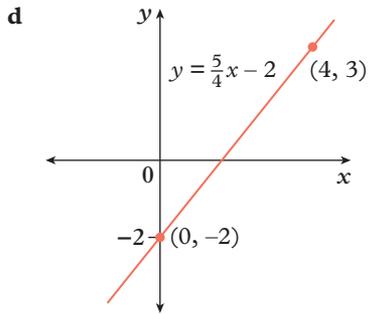


b

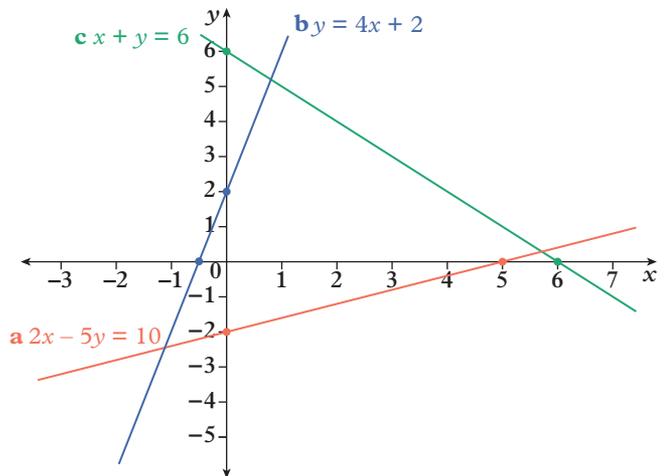


c

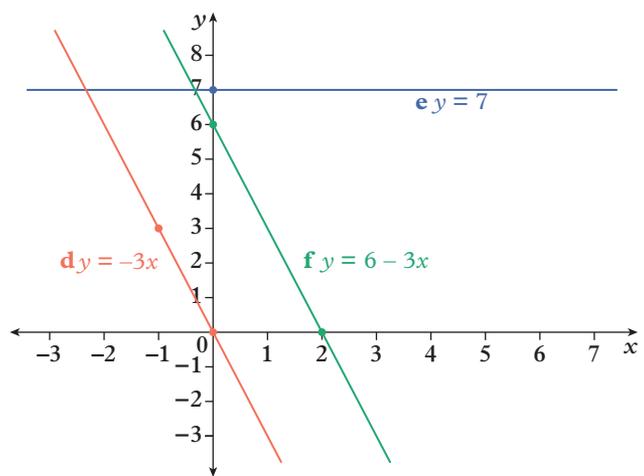




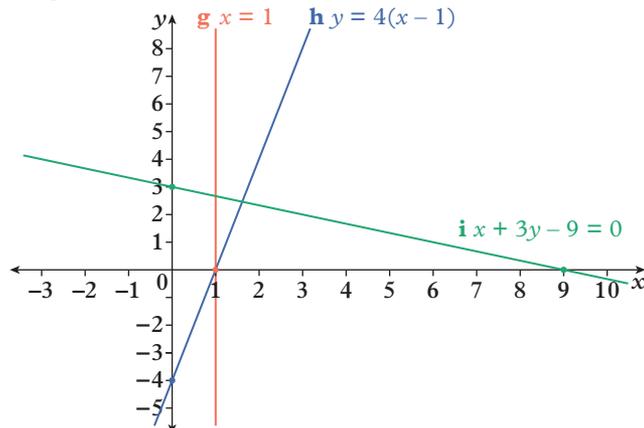
8 a-c



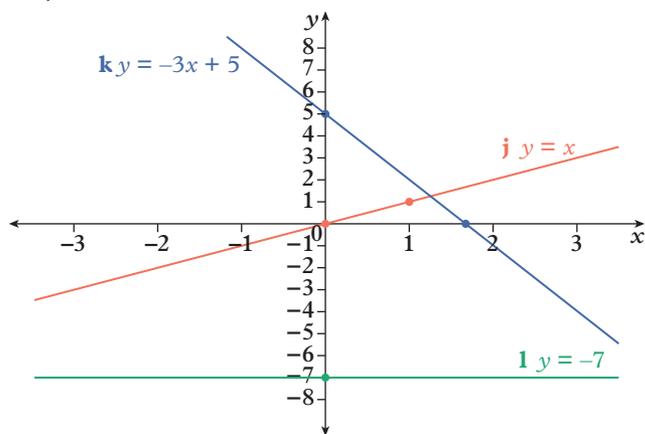
d-f



g-i



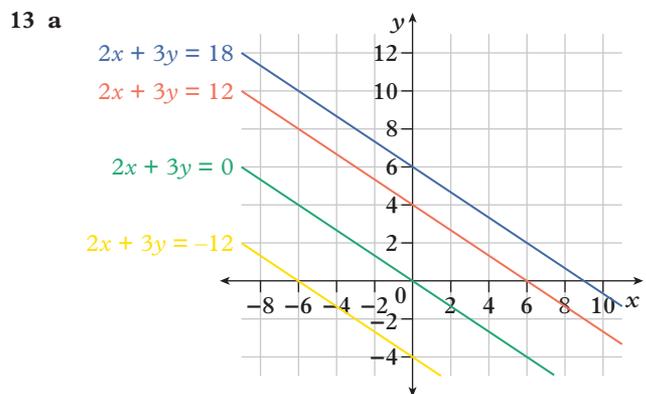
j-1



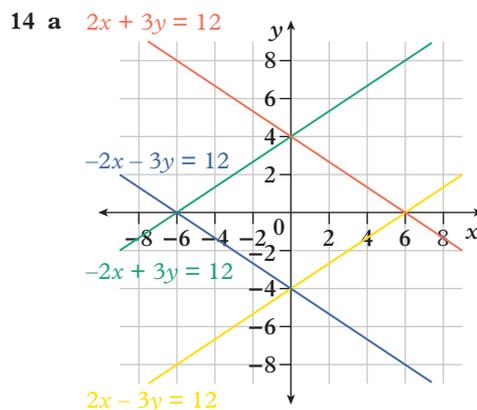
- 9 a 1 b 2 c 1
 d 1 e 2 f 2
 g 1 h 1 i 1

10 Graph B is correct. In graph A, the coefficients of variables have been used as intercepts. Graph B is a correct sketch of $3x + 2y = 6$. Graph C has an x -intercept at -2 instead of $+2$.

- 11 a true
 b false; the x -intercept is -2
 c true
 d true
 e false; the line passes through $(-2, 0)$ and $(0, 6)$
 f true
- 12 a false; the graph passes through the origin
 b true
 c true
 d false; the point $(1, -4)$ lies on the line
 e true
 f false; the gradient of the line is -4



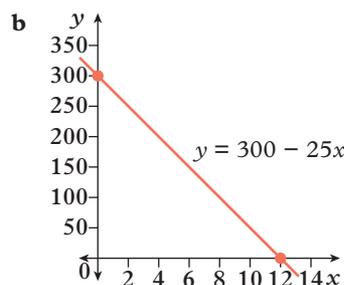
- b The equations have the same variable sum, $2x + 3y$, and are parallel (have the same gradient).
 c Increasing the constant translates the line up and to the right. Decreasing the constant translates the line down and to the left. If the constant is 0, the line passes through the origin. If the constant is positive, the line has positive intercepts. If the constant is negative, the line has negative intercepts.



- b All equations are equal to the same positive constant and have coefficients with the same unsigned values (but not necessarily the same sign).
 c The term(s) with a positive coefficient have positive intercepts. The term(s) with a negative coefficient have negative intercepts. Compared to $2x + 3y = 12$, the line is reflected in the x -axis when the y -term is negative and reflected in the y -axis when the x -term is negative.

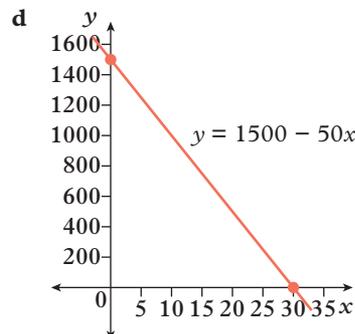
- 15 a $\frac{3}{5}$ b $-\frac{7}{6}$ c $-\frac{1}{2}$ d 2

16 a The independent variable is x (number of weeks) so Tony should calculate y (amount still owed in dollars), which depends on the number of payments over x weeks.



- c original cost of skateboard
 d number of weeks needed to pay off skateboard
 e Tony is paying \$25 a week towards the skateboard. He will pay it off in 12 weeks.
- 17 a Yes, the constant rate of water use and no additions to the tank suggest a constant gradient and hence a linear relationship between time, x , and amount of water, y .

- b $(30, 0)$
 c $(0, 1500)$



e 1000 L

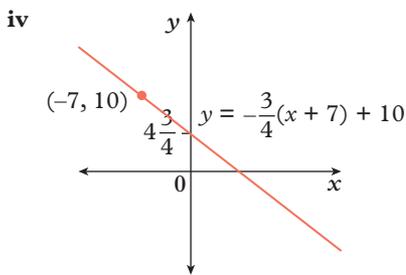
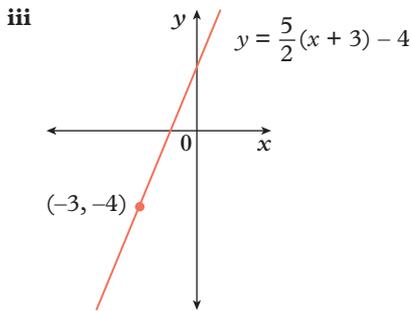
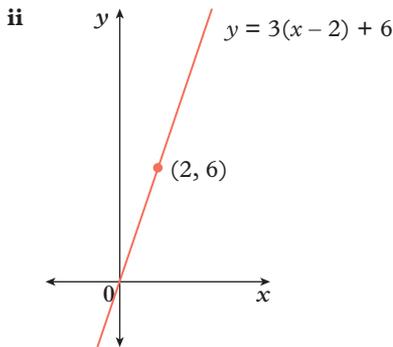
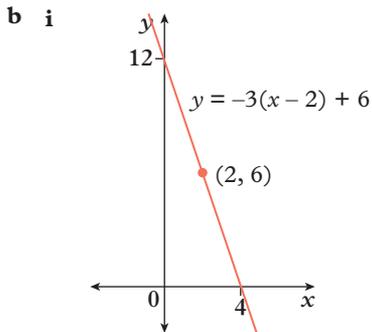
f end of day 18

18 a When $x = 0$, the equation becomes $by = 1$, which has the solution $y = \frac{1}{b}$ (the y -intercept), the reciprocal of b . Likewise, when $y = 0$, the equation becomes $ax = 1$, which has the solution $x = \frac{1}{a}$ (the x -intercept), the reciprocal of a .

b Divide both sides of the equation by d , then the reciprocals of the coefficients of x and y will be the x - and y -intercepts, respectively.

c $d = ab$

19 a $y = m(x - h) + k$
 $y = m(x - 0) + c$
 $y = mx + c$



c $(0, k - mh)$ and $(h - \frac{k}{m}, 0)$

d i $y = m(x - b); x = \frac{y}{m} + b$

This is the x -intercept version of $y = mx + c$.

ii Take the gradient, m , out as if it were a factor of the expression $mx + c$ by dividing each term by m to write the equation as $y = m(x + \frac{c}{m})$.

iii The x -intercept is the negative of the value added to x , $\frac{c}{m}$.

e i $y = 3(x - 4), (4, 0)$

ii $y = 2(x - \frac{5}{2}), (\frac{5}{2}, 0)$

iii $y = -3(x + 4), (-4, 0)$

iv $y = 2(x + \frac{5}{2}), (-\frac{5}{2}, 0)$

EX p155 **Checkpoint**

1 a $x = -27$ b $x = 5$ c $x = 7$

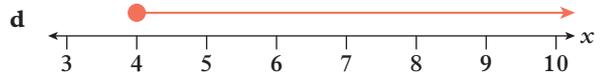
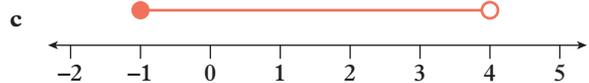
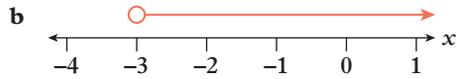
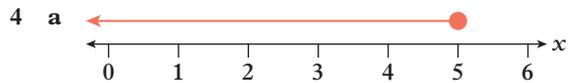
2 a $x = -4$ b $x = 2$ c $x = -5$ d $x = -3$

3 a Let n be the cost of one video game.

b $5n + 9.95 = 409.7$

c $n = 79.95$

d \$79.95



5 a $x < 5$

b $-2 < x \leq 3$

c $x \geq -5$

d $x \leq -2$

6 a $x > 3$

b $x \leq -5$

c $x > -25$

d $x > 1$

e $x \neq -3$

f $x \leq 6$

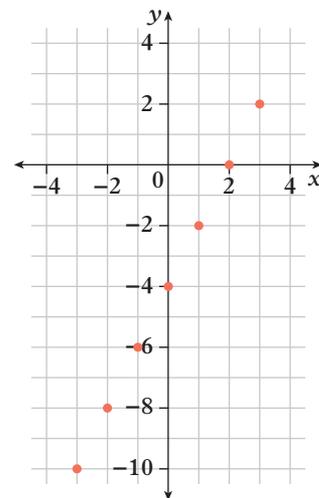
7 a No

b Yes

c Yes

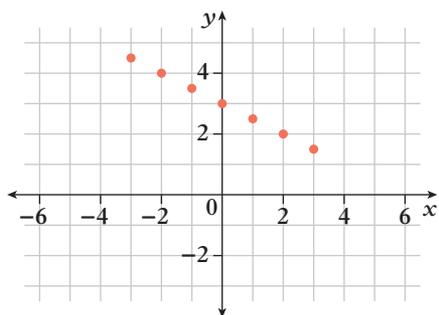
d No

8 a i



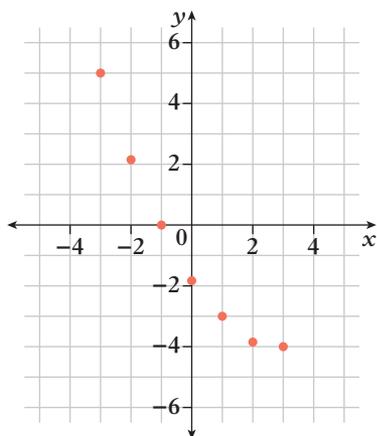
ii linear

b i



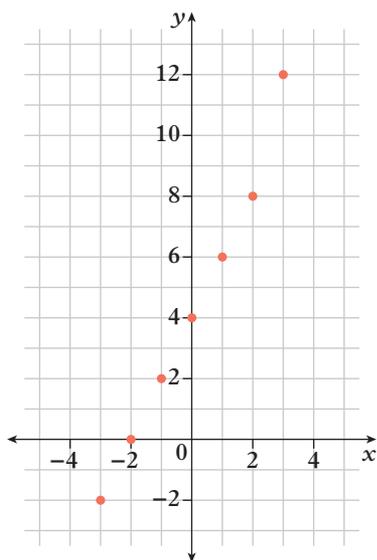
ii linear

c i



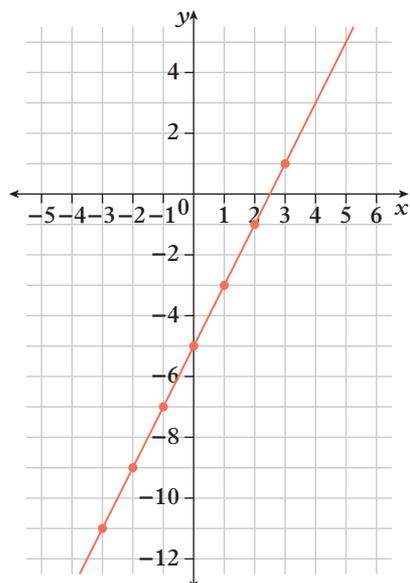
ii non-linear

d i

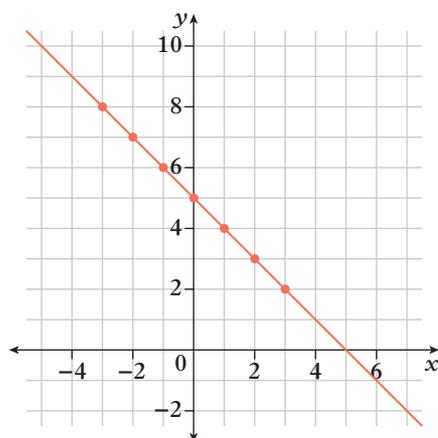


ii non-linear

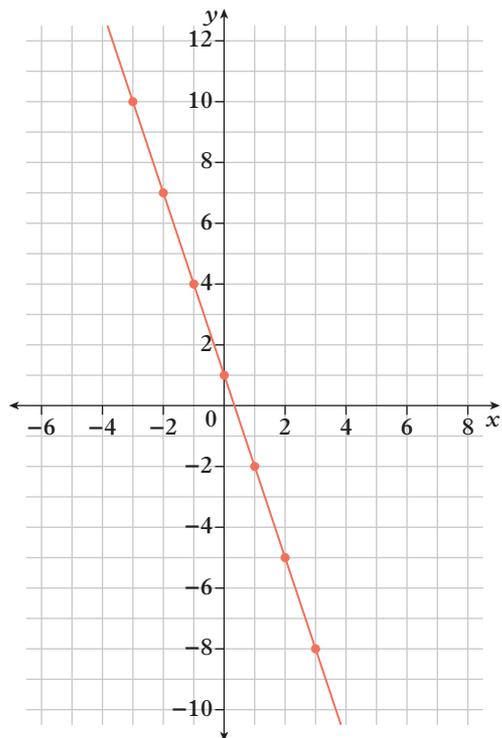
9 a

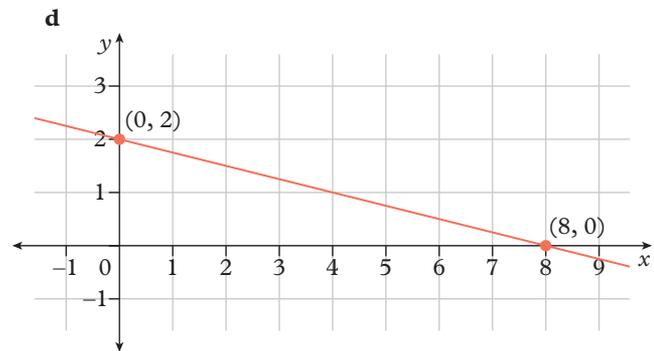
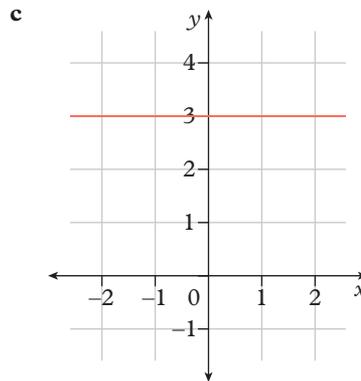
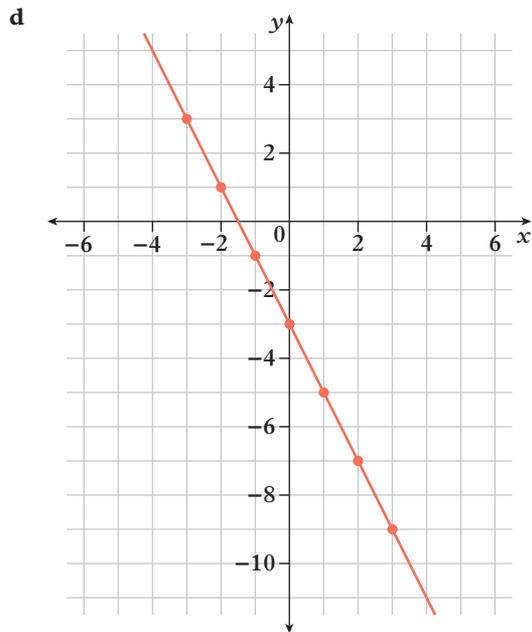


b



c



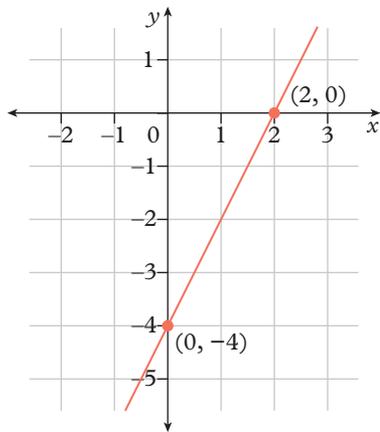


10 **a** zero **b** negative **c** undefined **d** positive

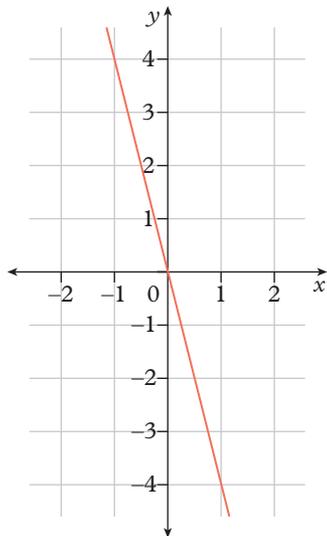
11 **a** 3 **b** -3 **c** $-\frac{5}{3}$ **d** $-\frac{6}{5}$

12 **a** (3, 0), (0, 4) **b** (-2, 0), (0, 3) **c** (-6, 0), (0, -1)

13 **a**



b



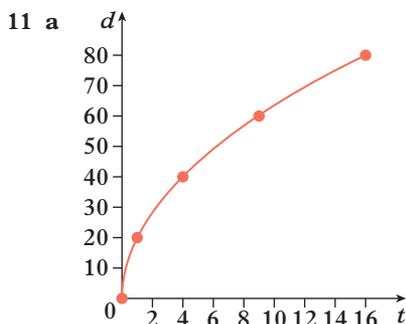
EX 4F Determining linear equations

p159

- | | |
|---|---|
| 1 a $y = 3x + 4$ | b $y = -2x + 10$ |
| c $y = x - 7$ | d $y = -12x - 1$ |
| e $y = -x + 20$ | f $y = 5x$ |
| g $y = \frac{1}{3}x + \frac{4}{5}$ | h $y = -\frac{3}{4}x$ |
| 2 a $y = 3x + 6$ | b $y = -\frac{2}{3}x + 4$ |
| c $y = \frac{1}{2}x - 5$ | d $y = -\frac{1}{5}x - 2$ |
| e $y = 2x - 3$ | f $y = -3x + 5$ |
| g $y = -\frac{1}{2}x$ | h $y = \frac{5}{2}x$ |
| 3 a $y = 2x - 22$ | b $y = -2x + 8$ |
| c $y = -x + 5$ | d $y = 3x$ |
| e $y = 2x + 1$ | f $y = 5x - 49$ |
| g $y = -\frac{1}{2}x - \frac{5}{2}$ | h $y = \frac{4}{3}x - 11$ |
| 4 a $x = 5$ | b $y = -2$ |
| c $y = 60$ | d $x = -40$ |
| e $y = -\frac{3}{7}$ | f $x = \frac{2}{3}$ |
| 5 x-axis: $y = 0$ | |
| y-axis: $x = 0$ | |
| 6 a $y = -3x + 12$ | b $y = 2x + 10$ |
| c $y = 7x - 21$ | d $y = -4x - 24$ |
| e $y = 10x + 10$ | f $y = -\frac{x}{3} + 6$ |
| g $y = \frac{2}{5}x - 2$ | h $y = \frac{3}{2}x + 9$ |
| 7 a $y = \frac{1}{2}x + \frac{9}{2}$ | b $y = \frac{2}{3}x + \frac{14}{3}$ |
| c $y = -\frac{1}{5}x + \frac{18}{5}$ | d $y = \frac{3}{4}x + \frac{1}{2}$ |
| e $y = -\frac{7}{6}x - \frac{1}{3}$ | f $y = -\frac{11}{9}x - \frac{5}{3}$ |

10 a Jasmine 8:10 am, Nelson 8:25 am

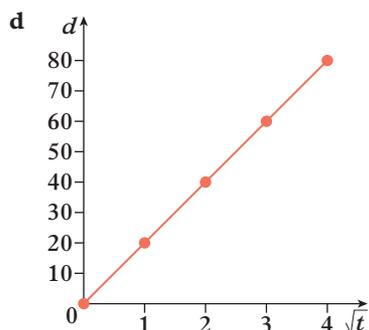
b 9:05 am



b No; as the graph is not a straight line passing through (0, 0)

c

\sqrt{t}	0	1	2	3	4
d	0	20	40	60	80



e Yes; it forms a straight-line graph that passes through (0, 0).

f $k = 20, d = 20\sqrt{t}$ g 120 km

12 Helena is 40 km from town A and Julia is 110 km from town A.

EX 4H Midpoint and length of a line segment

p171

1 a (2, 1) b (4, -2) c (-3, 3) d (3, 2)

2 a (2, 7) b (5, 4) c (3, 1) d (2, 8)

e (4, 7) f (5, 1) g (4, 3) h (1, -4)

i (-3, 0) j (3.5, 8.5) k (6.5, 5.5) l (0, 0)

3 a 2.2 units b 4.5 units c 3.2 units

d 4.0 units e 6.3 units f 5.0 units

g 1.4 units h 14.0 units i 3.6 units

j 4.1 units k 4.5 units l 15.6 units

4 a i (3, 3) ii 4.5 units iii $-\frac{1}{2}$

b i (0.5, 0) ii 7.8 units iii $\frac{6}{5}$

c i (-2.5, 3) ii 8.5 units iii $\frac{8}{3}$

d i (-6, -5) ii 8.5 units iii -1

5 a 1.1 units b 2.2 units c 1.6 units d 2.0 units

e 3.2 units f 2.5 units g 0.7 units h 7.0 units

i 1.8 units j 2.1 units k 2.2 units l 7.8 units

6 (10, 5)

7 (-14, 9)

8 a 5 units b (5, 7)

9 20.3 units

10 a 20.5 units b 17.0 units c 22.6 units d 19.5 units

11 a leg 1: 21.5 km; leg 2: 16.5 km; leg 3: 28.8 km

b 66.9 km c 24.1 km

12 a (-5, -1), (-2, 3), (6, 3) and (3, -1)

b i (0.5, 1) ii (0.5, 1)

c Midpoints of both diagonals have same coordinates

d 26 cm e 5.3 cm

13 a i 21.6 cm ii 10.8 cm

iii Perimeter of orange shape = $\frac{1}{2}$ perimeter of blue shape

b i 20.7 cm ii 16.7 cm

iii Perimeter of orange shape $\approx \frac{4}{5}$ perimeter of blue shape

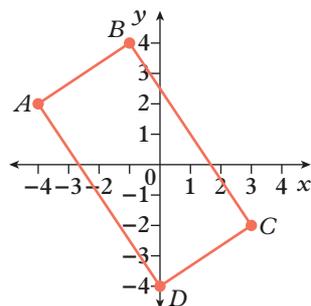
14 a $AB: 3.6$ m, $BC: 7.2$ m, $CD: 3.6$ m and $AD: 7.2$ m

b Midpoint for both diagonals (-0.5, 0)

c $A: 4.0$ m, $B: 4.0$ m, $C: 4.0$ m, $D: 4.0$ m

d The parallelogram is a rectangle because it has two opposite pairs of sides of equal length, diagonals equal in length and bisect each other, with each vertex the same distance from the midpoint.

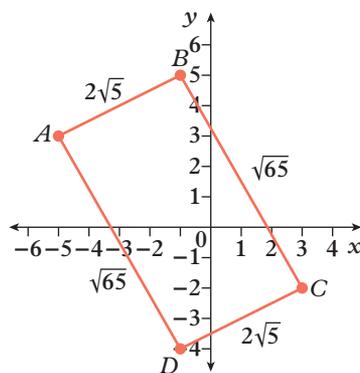
e 26 m^2



15 a Two sides are equal in length, $AB = AC = 4\sqrt{5}$ units and $BC = 4\sqrt{2}$ units.

b 24 units^2

16 Two pairs of equal-length sides, $AB = CD = 2\sqrt{5}$ units and $BC = AD = \sqrt{65}$ units; two pairs of parallel sides with gradients $\frac{1}{2}$ and $-\frac{7}{4}$.



17 (0, 1)

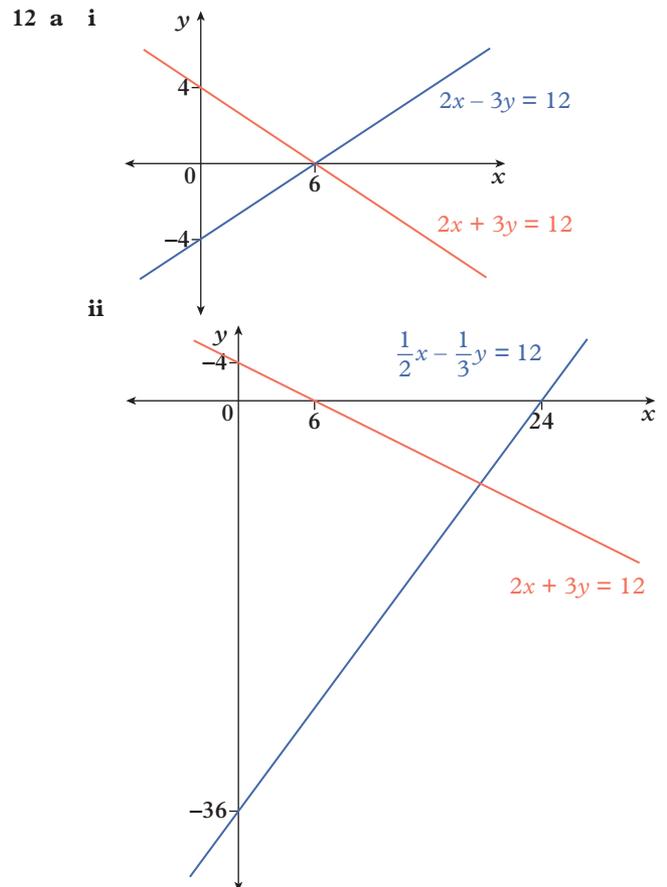
18 $(\sqrt{5}, 2\sqrt{5})$ and $(-\sqrt{5}, -2\sqrt{5})$

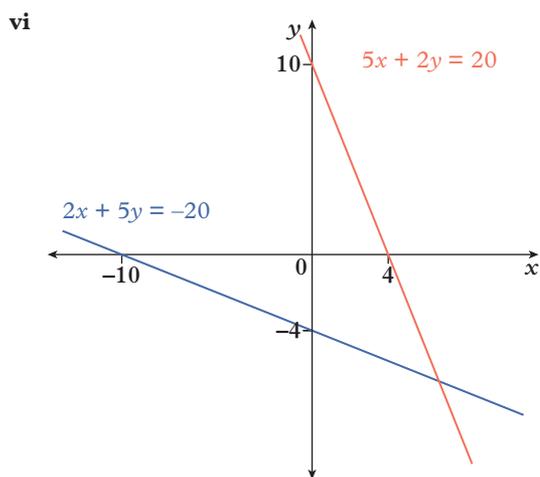
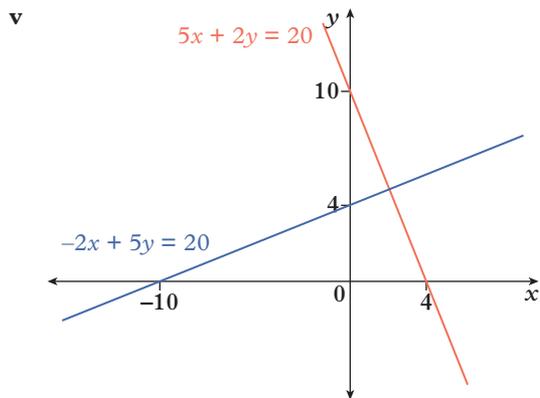
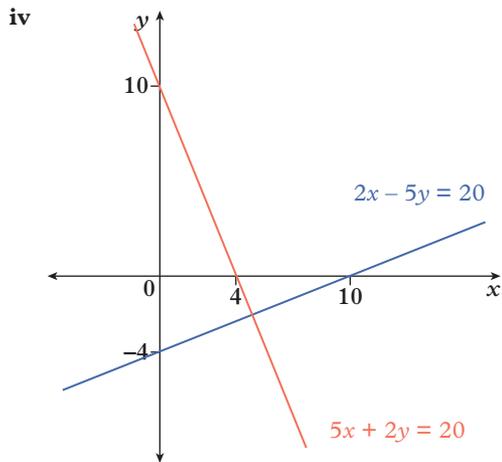
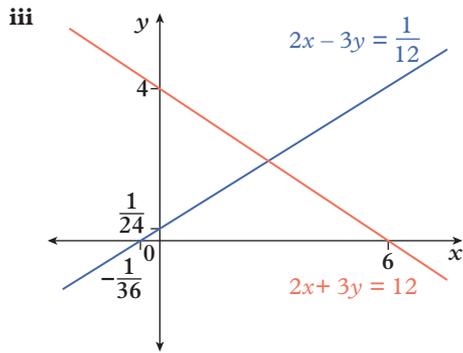
EX 41 Parallel and perpendicular lines

p178

- 1 a i gradients: 2 and 2
 ii gradients: $-\frac{1}{3}$ and $-\frac{1}{3}$
 b Both pairs of lines are parallel.
 c The gradients of parallel lines are always equal.
 d i parallel ii parallel
 iii not parallel
- 2 a i gradients: 3 and $-\frac{1}{3}$
 ii gradients: $\frac{3}{4}$ and $-\frac{4}{3}$
 b Both products are -1 .
 c Both pairs of lines are perpendicular.
 d The product of the gradients of two perpendicular lines is always -1 .
 e $m_2 = -\frac{1}{m_1}$
 f i perpendicular ii not perpendicular
 iii not perpendicular
- 3 a $-\frac{1}{5}$ b $\frac{1}{8}$ c 1
 d -6 e $-\frac{5}{4}$ f $\frac{2}{3}$
- 4 a i $y = -3x + 11$ ii -3 iii $\frac{1}{3}$
 b i $y = 4x + 7$ ii 4 iii $-\frac{1}{4}$
 c i $y = \frac{5}{3}x + \frac{2}{3}$ ii $\frac{5}{3}$ iii $-\frac{3}{5}$
 d i $y = -\frac{3}{2}x + 4$ ii $-\frac{3}{2}$ iii $\frac{2}{3}$
 e i $y = \frac{5}{2}x - 2$ ii $\frac{5}{2}$ iii $-\frac{2}{5}$
 f i $y = -\frac{8}{5}x - \frac{11}{5}$ ii $-\frac{8}{5}$ iii $\frac{5}{8}$
 g i $y = \frac{1}{3}x - \frac{5}{3}$ ii $\frac{1}{3}$ iii -3
 h i $y = -\frac{1}{5}x - \frac{3}{5}$ ii $-\frac{1}{5}$ iii 5
 i i $y = x + 7$ ii 1 iii -1
- 5 a parallel; same gradient of 4
 b perpendicular; negative reciprocal gradients of $\frac{2}{3}$ and $-\frac{3}{2}$
 c perpendicular; negative reciprocal gradients of $\frac{3}{8}$ and $-\frac{8}{3}$
 d neither; gradients of -2 and 2
 e neither; gradients of -5 and $-\frac{1}{5}$
 f parallel; same gradient of $\frac{2}{7}$
 g neither; gradients of $\frac{2}{3}$ and $-\frac{2}{3}$
 h perpendicular; negative reciprocal gradients of -2 and $\frac{1}{2}$
- 6 a $y = 4x - 13$ b $y = -3x - 6$
 c $y = -x - 2$ d $y = \frac{1}{2}x - 3$
 e $y = -\frac{1}{3}x + 5$ f $y = \frac{2}{7}x + 11$

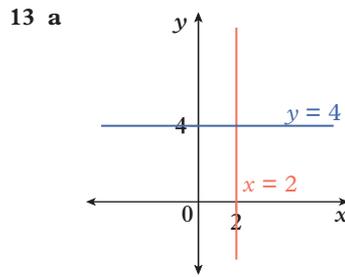
- g $y = -\frac{5}{4}x - 5$ h $y = \frac{5}{6}x + \frac{39}{2}$
- 7 a $y = -4x + 5$; $y = -4x + 18$
 b $y = -\frac{1}{2}x + 3$; $y = -\frac{1}{2}x - 3$
 c $y = \frac{7}{3}x - \frac{5}{3}$; $y = \frac{7}{3}x + 6$
 d $y = \frac{3}{5}x + \frac{3}{20}$; $y = \frac{3}{5}x - 3$
- 8 a $y = -\frac{1}{5}x - 7$ b $y = \frac{1}{7}x + 3$
 c $y = -x - 3$ d $y = \frac{3}{2}x + 10$
 e $y = -\frac{9}{4}x + 8$ f $y = 3x + 30$
 g $y = \frac{6}{5}x - \frac{54}{5}$ h $y = -\frac{7}{4}x - 46$
- 9 a $y = -5x + 4$; $y = \frac{1}{5}x + 9$
 b $y = \frac{1}{3}x - 2$; $y = -3x - 33$
 c $y = -\frac{9}{7}x + \frac{13}{7}$; $y = \frac{7}{9}x + 47$
 d $y = \frac{2}{5}x + \frac{7}{5}$; $y = -\frac{5}{2}x - 100$
- 10 a $y = -\frac{1}{2}x + 3$
 b $y = x + 3$
 c $y = -\frac{2}{3}x + 3$
- 11 a i $y = 2x - 4$ ii $y = -\frac{1}{2}x - 4$
 b i $y = -x - 4$ ii $y = x - 4$
 c i $y = \frac{3}{2}x - 4$ ii $y = -\frac{2}{3}x - 4$





b ii, iv, v

c $\frac{1}{a}x - \frac{1}{b}y = 1$ or $-\frac{1}{a}x + \frac{1}{b}y = 1$ or $bx - ay = 1$
or $-bx + ay = 1$



- b i** Aamira is correct because lines that meet at right angles are, by definition, perpendicular.
ii Jack is incorrect as you cannot divide by zero, even if you divide out that factor.
c i Vertical lines are perpendicular to horizontal lines because they meet at a 90° angle.
ii Vertical lines are perpendicular to horizontal lines but we cannot use the formula $m_1 \times m_2 = -1$ because the gradient of a vertical line is undefined.
iii Vertical lines are perpendicular to horizontal lines, so the equations $x = a$ and $y = c$ are perpendicular.

14 The gradient of the line joining $(2, -3)$ and $(4, 5)$

$$\text{is } \frac{5 - (-3)}{4 - 2} = \frac{8}{2} = 4.$$

The gradient of $y = 4x - 7$ is 4.

So the lines are parallel.

15 The gradient of the line joining $(-11, -7)$ and $(-1, -2)$

$$\text{is } \frac{-2 - (-7)}{-1 - (-11)} = \frac{5}{10} = \frac{1}{2}.$$

The gradient of $y = -2x + 5$ is -2 .

The product of the gradients is -1 , so the lines are perpendicular.

16 a $\frac{2}{3}$

b i $y = \frac{2}{3}x - 5$

ii $y = -\frac{3}{2}x + 2$

17 a $y = -3x$

b $y = \frac{1}{3}x + 9$

c $y = \frac{1}{3}x - 1$

18 $y = -x + 1, y = -x + 2, y = x + 1, y = x + 2$

19 a Yes, both have a gradient of $\frac{3}{7}$.

b No; AC has a gradient of $\frac{5}{4}$, while BD has a gradient of $-\frac{2}{3}$.

20 a The gradient of AB is 3; the gradient of BC is $-\frac{1}{3}$, so line segments AB and BC are perpendicular. This makes triangle ABC right-angled.

b The length of AB is $\sqrt{90}$ units; the length of BC is $\sqrt{40}$ units. So the triangle is not isosceles right-angled.

c 27.2 units

d 30 square units

21 a $AD \parallel BC$, each line segment has a gradient of $\frac{1}{2}$.

$AB \parallel DC$, each line segment has a gradient of 2.

length of AD = length of BC = length of AB = length of DC = $\sqrt{5}$ units

Note: It is only necessary to prove all sides equal in length, but it can also be shown by proving both pairs of opposite sides parallel and two adjacent sides equal in length.

- b** $EF \parallel HG$; both line segments have a gradient of $\frac{3}{7}$.
 $HE \parallel GF$; both line segments have a gradient of 2.

22 gradient of $KN =$ gradient of $LM = -\frac{1}{2}$

gradient of $MN =$ gradient of $LK = 2$

So opposite sides are parallel, and adjacent sides are at right angles.

$KL = LM = MN = NK = 2\sqrt{5} \approx 4.5$ units,

perimeter $= 8\sqrt{5} \approx 17.9$ units,

area $= (2\sqrt{5})^2 = 20$ square units

23 gradient of $PS =$ gradient of $QR = 1$

gradient of $PQ =$ gradient of $SR = -1$

So opposite sides are parallel, and adjacent sides are at right angles.

$PS = QR = 3\sqrt{2}$ units, $PQ = SR = 4\sqrt{2}$ units,

perimeter $= 14\sqrt{2} \approx 19.8$ units

area $= 3\sqrt{2} \times 4\sqrt{2} = 24$ square units

24 $y = x - 5$

25 length of $YX =$ length of $YZ = 7.1$ units

$WX = WZ = \sqrt{425} = 5\sqrt{17} \approx 20.6$ units

This is all that is required. Extra information is as follows:

The gradient of $XZ = -\frac{3}{4}$ and the gradient of $WY = \frac{4}{3}$, so

the diagonals are at right angles. The midpoint of XZ is

$(3, 5)$. This point also lies on the diagonal WY , which has

the equation $y = \frac{4}{3}x + 1$.

26 Gradients of the line segments are: $AB = \frac{1}{2}$, $BC = -3$,

$CD = \frac{1}{2}$, $DA = \frac{6}{5}$, $AC = 0$, $BD = \frac{9}{11}$

This indicates that the quadrilateral is a trapezium (one pair of parallel sides).

EX Chapter 4 review

p183

Mathematical literacy review

- a** inequality, number line

b linear, Cartesian plane

c equation, variable
- a** True **b** True **c** False

d False **e** True **f** False

g True
- a** The *constant* in the gradient–intercept form of a linear equation is the y -value of the y -intercept; for example, the $+ 1$ in Equation 1. The *constant* of proportionality is the coefficient of the dependent variable and is the same as the gradient or rate of change, for example, the 6 in Equation 2.

b A *linear relationship*, $y = mx + c$, is any relationship between two variables that produces a straight-line graph when plotted and a y -intercept at $(0, c)$.

For example, $y = 3x + 1$ has a y -intercept at $(0, 1)$.

A *direct variation* is a specific case of a linear relationship. The straight line passes through the origin and the y -intercept is at $(0, 0)$, so the constant has a value of 0 in the equation, $C = 6t$.

Multiple choice

- 1** B **2** D **3** B **4** A **5** D **6** C
7 B **8** B **9** E **10** B **11** A **12** C
13 D **14** A **15** C **16** C **17** E

Short answer

- 1** **a** $x = 2$ **b** $x = \frac{2}{3}$ **c** $x = 3\frac{3}{4}$ **d** $x = 3$
2 **a** $x = 4$ **b** $x = -1$
3 **a** Let n be the number of people in the class.
b $\frac{3}{5}n + 1 = 16$ **c** $n = 25$ **d** 25 people
4 **a** $x \geq -3$ **b** $x < -10$ **c** $-1 < x \leq 2$
5 **a** $x > -1$ **b** $x < -2\frac{2}{3}$

6 **a**

x	-2	-1	0	1	2
y	3	4	5	6	7

b

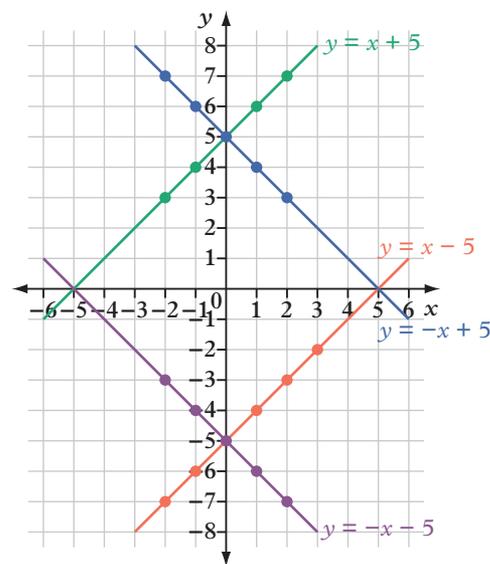
x	-2	-1	0	1	2
y	-7	-6	-5	-4	-3

c

x	-2	-1	0	1	2
y	7	6	5	4	3

d

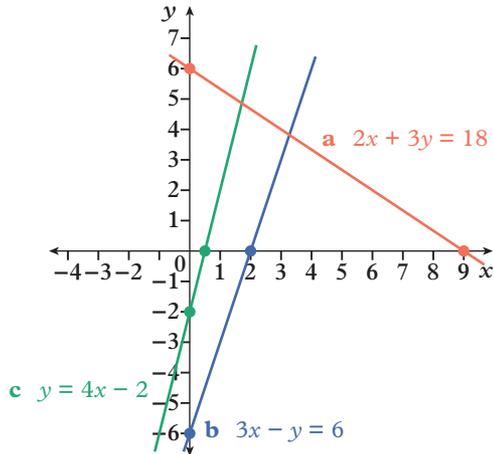
x	-2	-1	0	1	2
y	-3	-4	-5	-6	-7



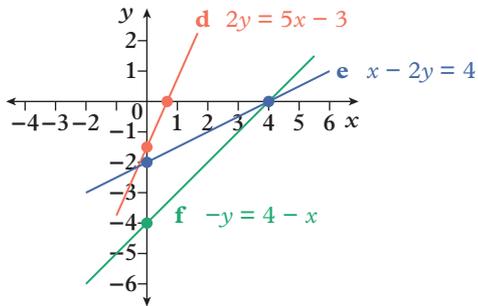
- 7** **a** **i** $\frac{1}{3}$ **ii** $(-6, 0)$ **iii** $(0, 2)$
b **i** undefined **ii** $(3, 0)$
iii no y -intercept
c **i** 0 **ii** no x -intercept **iii** $(0, -2)$
d **i** -2 **ii** $(0, 0)$ **iii** $(0, 0)$
- 8** **a** $\frac{3}{2}$ **b** -1

- 9 a i (9, 0) ii (0, 6)
 b i (2, 0) ii (0, -6)
 c i $(\frac{1}{2}, 0)$ ii (0, -2)
 d i $(\frac{3}{5}, 0)$ ii $(0, -\frac{3}{2})$
 e i (4, 0) ii (0, -2)
 f i (4, 0) ii (0, -4)

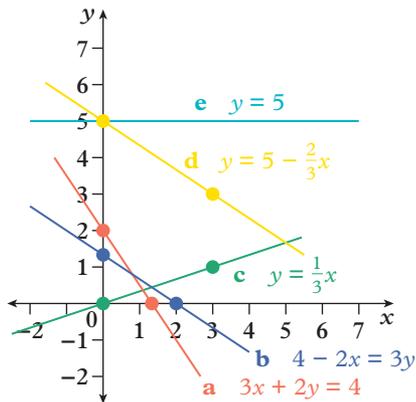
10 a-c



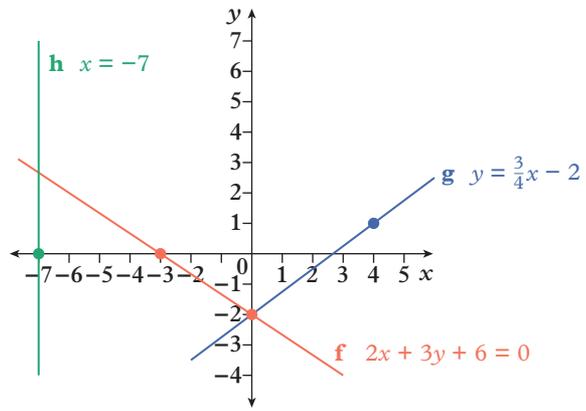
d-f



11 a-e



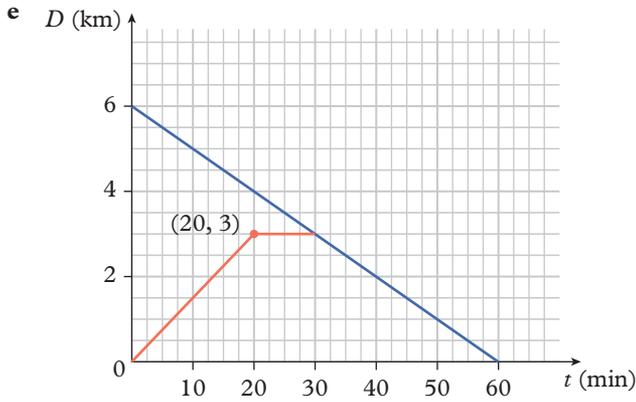
f-h



- 12 a $y = 4x - 2$ b $y = \frac{1}{4}x$ c $y = -\frac{1}{2}$
 13 a $y = -2x - 4$ b $y = \frac{3}{2}x$
 c $y = 1.25x + 1.25$ d $y = -\frac{1}{3}x + 1$
 14 a y is not directly proportional to x ; the rate of change is not constant.
 b $y \propto x$ c $y \propto x$
 d y is not directly proportional to x as the gradient is not constant.
 15 a i (5, 5) ii 7.2 units
 b i (-3.5, 5.5) ii 1.4 units
 c i (0, 0) ii 8.9 units
 d i (-0.5, -5) ii 9.4 units
 16 a $y = -x - 3$ b $y = 39$
 c $y = -\frac{4}{3}x$ d $y = \frac{4}{5}x + \frac{32}{5}$
 17 a $y = -4x - 3$ b $y = -\frac{1}{2}x - 4$
 18 a $y = 3x - 7$ b $y = -\frac{x}{3} - 3$

Analysis

- 1 a $0.71s + 0.26p \leq 60$
 b $p \leq 94$ or $p \leq 94.23$ (two decimal places)
 c $s \leq 29$ or $s \leq 29.58$ (two decimal places)
 d $P = 0.49s + 0.24p$
 e
-
- f 208 party pies
 g 32 sausage rolls and 143 party pies
 2 a $-\frac{1}{10}$ b 0.1 km/min
 c $D = -\frac{1}{10}t + 6$ d 3 km



- f 10 minutes g 0.15 km/min
 h $D = \frac{3}{20}t$

3 a

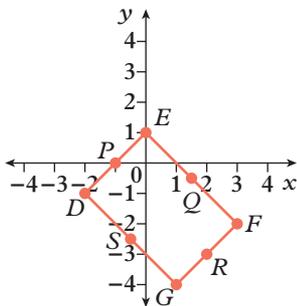
Line segment	Linear equation	x- or y-values involved
A	$y = 1$	$2 \leq x \leq 3$
B	$x = 3$	$1 \leq y \leq 2$
C	$y = 2$	$3 \leq x \leq 4$
D	$y = -\frac{4}{3}x + 7\frac{1}{3}$	$2.5 \leq x \leq 4$
E	$y = \frac{4}{3}x + \frac{2}{3}$	$1 \leq x \leq 2.5$
F	$y = 2$	$1 \leq x \leq 2$
G	$x = 2$	$1 \leq y \leq 2$

b

Line segment	Linear equation	x- or y-values involved
A	$y = ax + a$	$-1 \leq x \leq 0$
B	$y = -ax - a$	$-1 \leq x \leq 0$
C	$y = ax - a$	$0 \leq x \leq 1$
D	$y = -ax + a$	$0.5 \leq x \leq 1$
E	$y = ax$	$0 \leq x \leq 0.5$
F	$x = 0$	$0 \leq y \leq a$

- c Lines A, C and E are parallel
 d $a = 1$

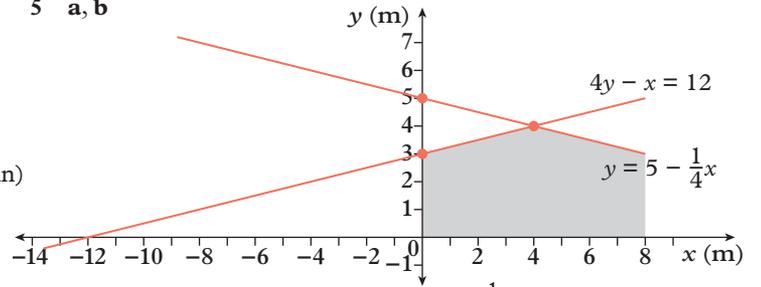
4 a



- b DE and FG are 2.8 units long, EF and DG 4.2 units long.
 c i (0, 1) ii (0, 1) iii (0, -3) iv none
 d i 1 ii -1 iii -1 iv 1
 e i (-1, 0) ii (1.5, -0.5)
 iii (2, -3) iv (-0.5, -2.5)

- f PQRS is a rhombus with all sides equal in length (PQ, RS, SP and QR).
 g If DEFG had been a square, then PQRS would also have been a square because the midpoints of each side would have been the same distance from the midpoints of the diagonals, which cross at 90° .

5 a, b



- c 4 m d 4.1 m e $\frac{1}{4}$
 f Chimney will be at (2, 3.5).

EX p190

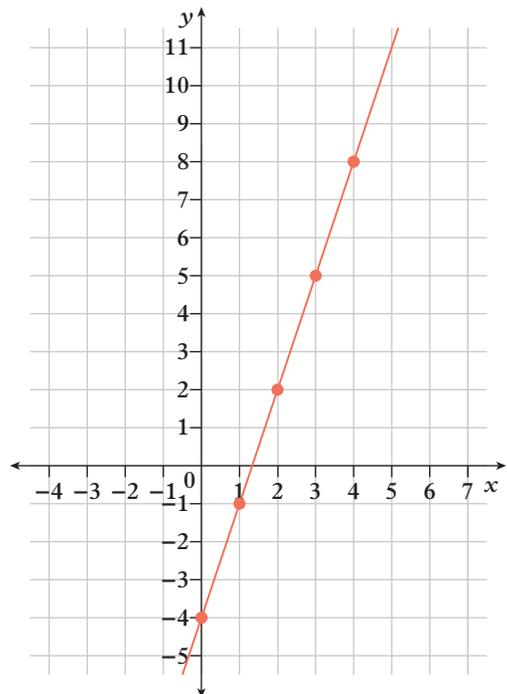
SEMESTER 1 review

Short answer

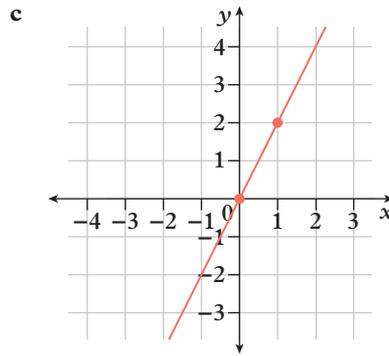
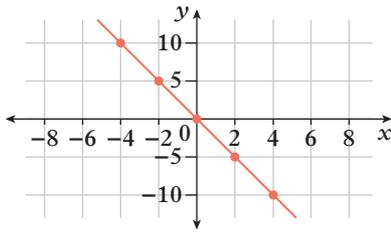
- 1 \$1804.80
 2 a \$825 b \$4700 c \$3991 d \$7859
 3 a \$3240 b 11.67% p.a.
 c 2166.67 d 2 months
 4 a 20.040 b 20.0
 c 0.0008 d 7.827×10^{-4}
 5 a $x = \frac{11}{12}$ b $x = -3$
 c $x = 8$ d $x = 7$

6 a

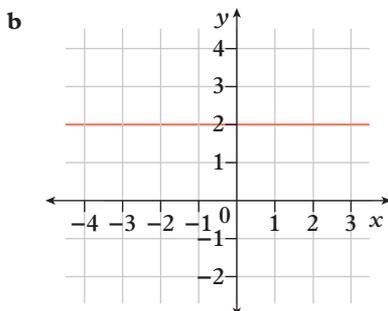
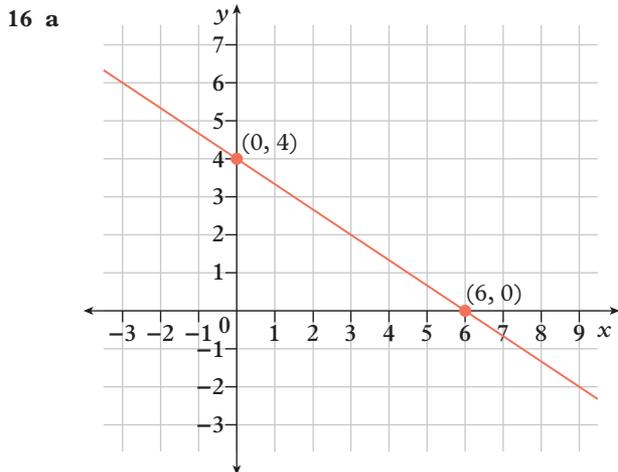
x	0	1	2	3	4
y	-4	-1	2	5	8



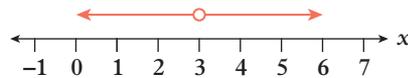
b	x	-4	-2	0	2	4
	y	10	5	0	-5	-10



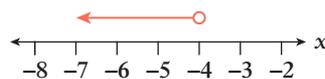
- 7 **a** 3^4 **b** $2^2 \times 5^3$
c $2^3 \times 3^3$ **d** $2^2 \times 3^3 \times 5^2$
- 8 **a** 343 **b** 375
c 82130 **d** 0.010530
e $\frac{27}{8} = 3.375$ **f** $\frac{49}{25} = 1.96$
- 9 **a** $2^{30} \times 7^{18}$ **b** $\frac{11^8}{17^9}$
c $\frac{1}{97^4 \times 101^{12}}$ **d** $41^{66} \times 57^{16}$
- 10 **a** $a^{16}b^{14}$ **b** $\frac{e}{c^{10}}$ **c** $70g^7h^7$ **d** 1
- 11 **a** 6 **b** 2 **c** 6 **d** 5
- 12 **a** 7.514×10^8 **b** 2.5360×10 or 2.5360×10^1
c 8.6×10^{-5} **d** 7.50×10^4
- 13 **a** $56 - 16x$ **b** $-6x^5 + 16x^3$
c $x^2 - 5x - 14$ **d** $10xy + 15x - 4y - 6$
e $-12x - 23$ **f** $5x^2 + 6x + 6$
- 14 **a** (2, 0), (0, -6) **b** (-12, 0), (0, $-\frac{15}{2}$)
c (3.5, 0), (0, 7) **d** (8.5, 0), (0, 17)
- 15 **a** $\frac{x^5}{20}$ **b** $3xy$ **c** $-\frac{21p}{45}$ **d** $-\frac{91t}{30}$



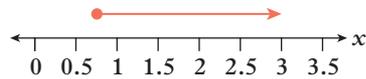
- 17 **a** $\frac{1}{2}$ **b** 0
c -3 **d** undefined
- 18 **a** $y = -3x + 5$ **b** $x = -3$
c $y = -2$ **d** $y = \frac{7}{2}x - \frac{5}{2}$
- 19 **a** (-1, 2) **b** (2, 4.5)
- 20 **a** $y = 4x + 13$ **b** $y = \frac{16}{5}x - \frac{68}{5}$
c $y = \frac{1}{2}x + 1$ **d** $y = \frac{3}{4}x + \frac{13}{2}$
e $y = 3x + 6$
- 21 **a** 8.54 units **b** 6.32 units
- 22 **a** $k = \frac{1}{4}, y = \frac{1}{4}x$ **b** $k = -3, y = -3x$
c $k = -5, y = -5x$ **d** $k = -\frac{1}{4}, y = -\frac{1}{4}x$
- 23 **a** $x < 4$ **b** $x \neq 2$
c $x \geq 7$ **d** $-5 < x \leq 2$
- 24 **a** $x \neq 3$



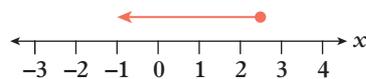
b $x < -4$



c $x \geq \frac{3}{4}$



d $x \leq \frac{5}{2}$

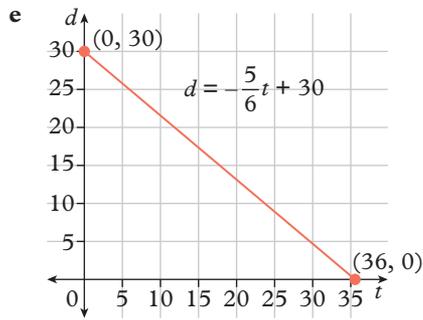


- 25 **a** $6(2a + 3b^2 - cd)$ **b** $7g(3g - 1)$
c $(3m - 4)(n - 5)$ **d** $(v + 2)(w + 5)$
e $3(p + 3)(p - 2)$ **f** $(r - 4)(r + 9)$
- 26 **a** $\frac{35}{24x}$ **b** $\frac{21r}{16q}$ **c** $\frac{7}{12abc}$ **d** $\frac{103c}{70d}$

Analysis

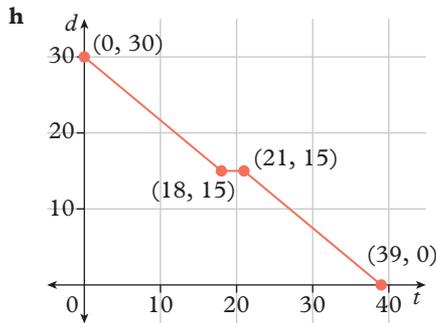
- 1 **a** d -intercept: (0, 30), t -intercept: (36, 0)
b $\frac{5}{6}$ km/min
c From the axis intercepts, the graph must have a negative gradient, so the gradient, m , is the negative of the speed, $m = -\frac{5}{6}$.

d $d = -\frac{5}{6}t + 30$



f (18, 15)

g Add a horizontal line segment at height $d = 15$ for $18 \leq t < 21$ since Jane's car isn't moving for those three minutes. The second half of the original graph is translated horizontally right by three units. The t -intercept moves to (39, 0).



i $d = -\frac{5}{6}t + 30$ ($0 \leq t < 18$)
 $d = 15$ ($18 \leq t < 21$)
 $d = -\frac{5}{6}t + \frac{65}{2}$ ($21 \leq t \leq 39$)

- 2 a \$936 b \$4399.20 c \$49327.20
d \$2939.40 e \$932.40 f \$52715
g \$7599 h \$44062

- 3 a To four decimal places: 2.4495, 2.9068, 2.9844, 2.9974, 2.9996, ..., which appears to approach 3.
b Solve $x^2 - x - 6 = 0$ to give $x = -2$ or 3 but ignore $x = -2$ as x must be positive.
c i 4 and 1
ii $\sqrt{20 + \sqrt{20 + \dots}}$ and $\sqrt{30 - \sqrt{30 - \dots}}$

- 4 a $12 \rightarrow 6 \rightarrow 3 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$
b $13 \rightarrow \dots \rightarrow 14 \rightarrow 7 \rightarrow 20 \rightarrow 10 \rightarrow 5$
c This cycle has length 18. There are other solutions.
 $17 \rightarrow 50 \rightarrow 25 \rightarrow \dots \rightarrow 136 \rightarrow 68 \rightarrow 34$

- d Use the fact that a multiple of 3 can only be the result of halving.
e There is only one: the cycle in part b.
f One solution is $2^{500} + 1$. There are many others.
5 a Clockwise from the top left, the lengths and areas are as follows:

Length (cm)	14	14	12	12	6	6	8	8
Area (cm ²)	80	96	72	54	18	18	30	32

- b Dark length = $14 + 12 + 6 + 8 = 40$ cm
 $= \frac{1}{2}$ of total
Dark area = $80 + 72 + 18 + 30 = 200$ cm²
 $= \frac{1}{2}$ of total
c Show that, for any a and b , the total dark length is always $2c$ and the total dark area is always $\frac{1}{2}c^2$.
d Let P be at the centre and choose any three points on the boundary that divide the perimeter into three equal lengths. Show that cutting from P to the three boundary points creates three fair slices.
e It is possible for all positive integers by adapting the strategy in part d.

EX EXPLORATIONS 1

p194

- 1 a i The van's speed is 90 km/h and the bicycle's is 40 km/h. The van overtakes the bicycle at 4:24 pm.
ii 104 km/h iii 6:40 pm
b Speed of van = $\frac{1}{4}(c + 3t)$ km/h
Speed of bicycle = $(2t - c)$ km/h
The van overtakes the bicycle at 4:24 pm, which is independent of c and t .
2 a i The number of border squares is $2x + 2y - 4$ and the number of interior squares is $xy - 2x - 2y$.
ii $xy - 4x - 4y + 4 = 0$
iii 12, so the equation becomes $(x - 4)(y - 4) = 12$
iv $(x, y) = (5, 16), (6, 10), (7, 8), (8, 7), (10, 6)$ or $(16, 5)$
v Switching x and y rotates the design, so we can ignore the last three solutions.
b i Three: $(x, y) = (7, 34), (8, 20)$ or $(10, 13)$
ii One: $(x, y) = (4, 10)$

CHAPTER 5 Non-linear relationships

EX 5A Solving quadratic equations

p201

- 1 a, c, d, e and h
2 a yes b no c no d yes
e no f yes g yes h no
3 a $x = -3$ b $x = 6$ c $x = 2$ d $x = -\frac{1}{3}$
4 a $x = -2$ or $x = 3$ b $x = 1$ or $x = 7$
c $x = -4$ or $x = 4$ d $x = 0$ or $x = 6$
e $x = -5$ or $x = -1$ f $x = -2$ or $x = 0$
g $x = -8$ or $x = 8$ h $x = -1$ or $x = 7$
i $x = 0$ or $x = 11$ j $x = -\frac{3}{2}$ or $x = 5$
k $x = \frac{1}{2}$ l $x = -\frac{5}{3}$ or $-\frac{3}{2}$
5 a $x = -2$ or $x = -5$ b $x = 1$ or $x = 2$
c $x = 0$ or $x = -5$ d $x = 0$ or $x = 3$
e $x = -6$ or $x = 6$ f $x = -7$ or $x = -3$

- 6 a $x = -2$ or $x = 4$ b $x = -1$ or $x = 1$
 c $x = 0$ or $x = -8$ d $x = 1$ or $x = 3$
 e $x = -3$ f $x = 1$
- 7 See answers in questions 5 and 6.
- 8 a $x = -8$ or $x = 2$ b $x = 1$ or $x = 4$
 c $x = -6$ or $x = 6$ d $x = -9$ or $x = 0$
- 9 a $x = -2$ or $x = -1$ b $x = 0$ or $x = -4$
 c $x = -2$ or $x = 4$ d $x = 2$
 e $x = -2$ or $x = 1$ f $x = 0$ or $x = 2$
 g $x = -7$ or $x = -3$ h $x = -4$
 i $x = -4$ or $x = 4$
- 10 a $x = -8$ or $x = -6$ b $x = 0$ or $x = 4$
 c $x = 0$ or $x = 1$ d $x = -8$ or $x = 8$
 e $x = -3$ or $x = 1$ f $x = 4$
 g $x = 2$ or $x = 3$ h $x = -5$
 i $x = -2$ or $x = 2$
- 11 a i 2 ii 1 iii 0
 b i The equation is factorised into two different linear expressions.
 ii The equation is factorised into a perfect square.
 iii The equation cannot be factorised.
- 12 a 2 c 1 e 2 g 2
 b 2 d 0 f 2 h 0
- 13 a $b(b + 8) \text{ cm}^2$ b $b^2 + 8b \text{ cm}^2$
 c $b^2 + 8b = 560$
 d $b = 20$ or $b = -28$. $b = 20$ is the feasible solution and $b = -28$ is not, as it's not possible to have a negative breadth.
 e breadth = 20 cm and length = 28 cm
- 14 a $b^2 + 2b = 35$ b $b = -7$ or $b = 5$
 c length = 7 m, breadth = 5 m
- 15 $x(x - 12) = 640$, length = 32 cm, breadth = 20 cm
- 16 a i 8 m ii 0 m
 b 8 m c 2 s
 d Alex is above ground level when he throws the ball so it is only at ground level when the ball falls down and time cannot be -1 .
- 17 a 2 seconds and 6 seconds
 b The first time is when the rocket is ascending, the second time is when the rocket is descending.
- 18 a $x = 1, x = 3$ or $x = 5$ b $x = 0, x = -6$ or $x = 6$
 c $x = -11$ or $x = 4$ d $x = 2, x = -\frac{1}{3}$ or $x = \frac{3}{2}$

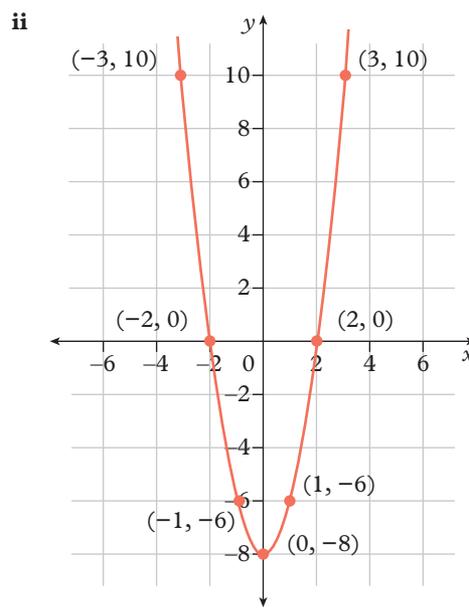
EX 5B Plotting quadratic relationships

p205

- 1 I, II
- 2 a i $(-1, 0), (1, 0)$ ii $(0, -2)$
 iii $(0, -2)$ iv minimum
 b i $(-2, 0), (2, 0)$ ii $(0, 2)$
 iii $(0, 2)$ iv maximum
 c i $(-2, 0), (2, 0)$ ii $(0, 4)$
 iii $(0, 4)$ iv maximum

3 a i

x	-3	-2	-1	0	1	2	3
y	10	0	-6	-8	-6	0	10

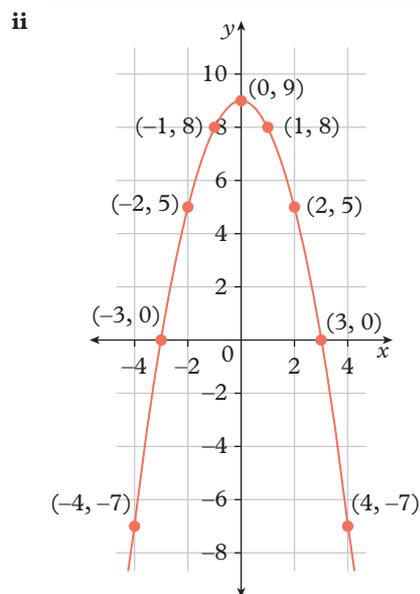


- iii x-intercept: $(2, 0), (-2, 0)$
 y-intercept: $(0, -8)$
 turning point: $(0, -8)$
 axis of symmetry: $x = 0$

iv minimum

b i

x	-4	-3	-2	-1	0	1	2	3	4
y	-7	0	5	8	9	8	5	0	-7

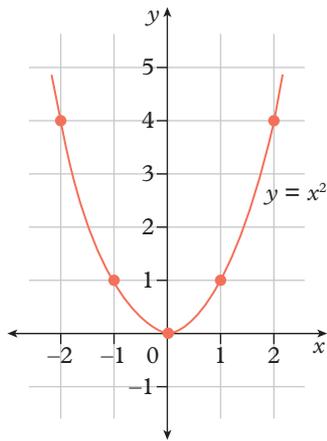


- iii x-intercept: $(3, 0), (-3, 0)$
 y-intercept: $(0, 9)$
 turning point: $(0, 9)$
 axis of symmetry: $x = 0$

iv maximum

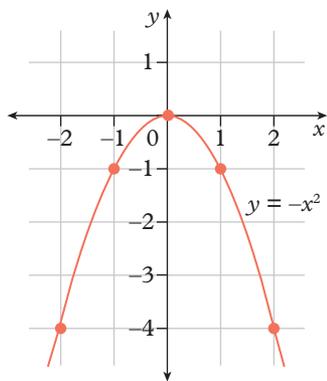
4 a

x	-2	-1	0	1	2
$y = x^2$	4	1	0	1	4



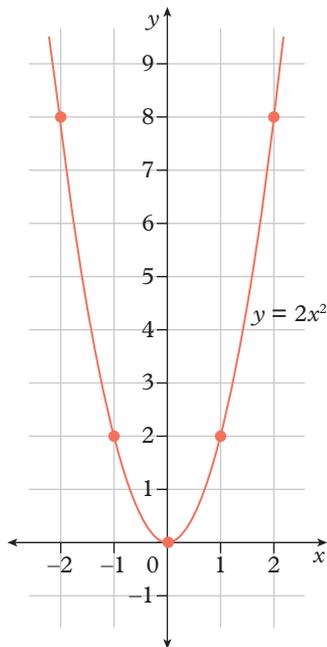
b

x	-2	-1	0	1	2
$y = -x^2$	-4	-1	0	-1	-4



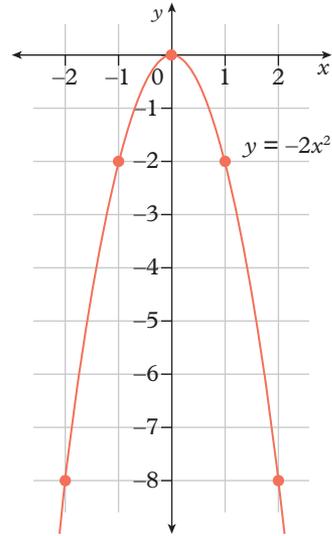
c

x	-2	-1	0	1	2
$y = 2x^2$	8	2	0	2	8



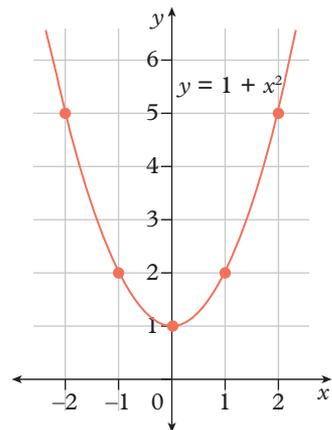
d

x	-2	-1	0	1	2
$y = -2x^2$	-8	-2	0	-2	-8



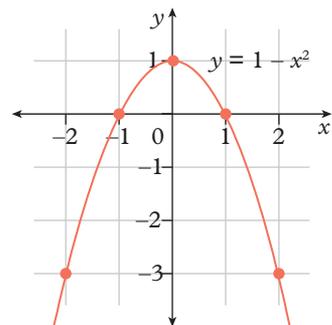
e

x	-2	-1	0	1	2
$y = x^2 + 1$	5	2	1	2	5



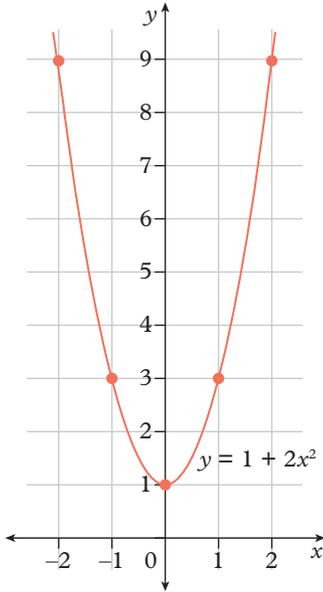
f

x	-2	-1	0	1	2
$y = 1 - x^2$	-3	0	1	0	-3



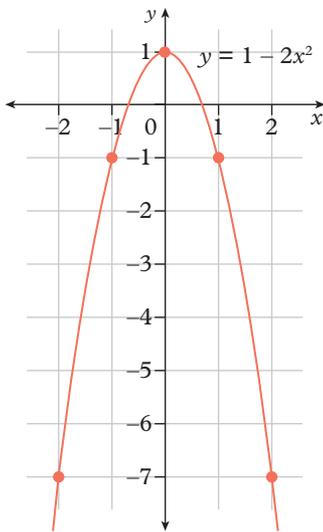
g

x	-2	-1	0	1	2
$y = 1 + 2x^2$	9	3	1	3	9

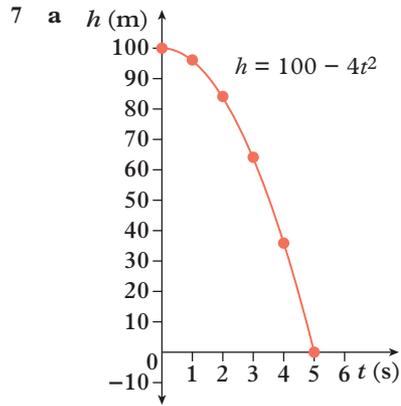


h

x	-2	-1	0	1	2
$y = 1 - 2x^2$	-7	-1	1	-1	-7



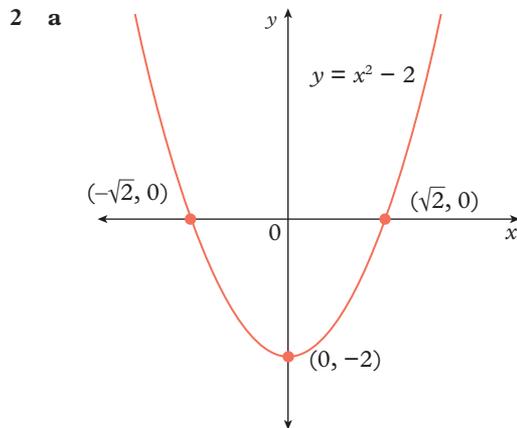
- 5 a** **i** 2 **ii** 1 **iii** 0
b A parabola changes direction only once, so there can only be a maximum of two x -intercepts.
c One
d The y -coordinate of the y -intercept is given by the constant term, c , in the equation.
- 6 a** **i** 1 x -intercept **ii** 1 y -intercept
b **i** 1 x -intercept **ii** 1 y -intercept

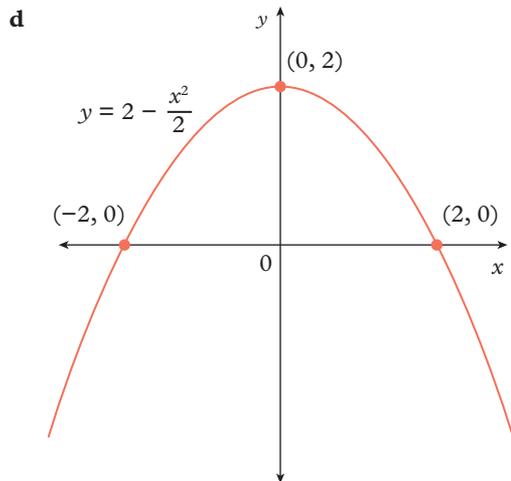
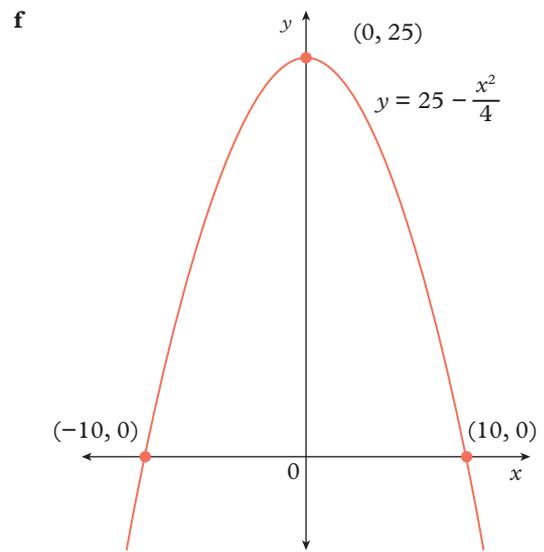
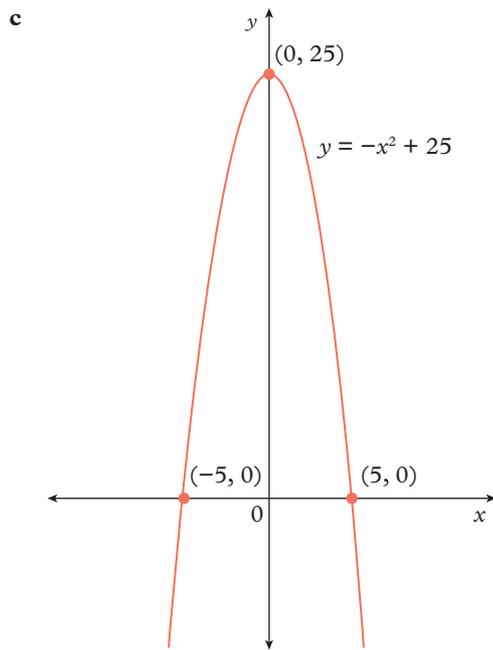
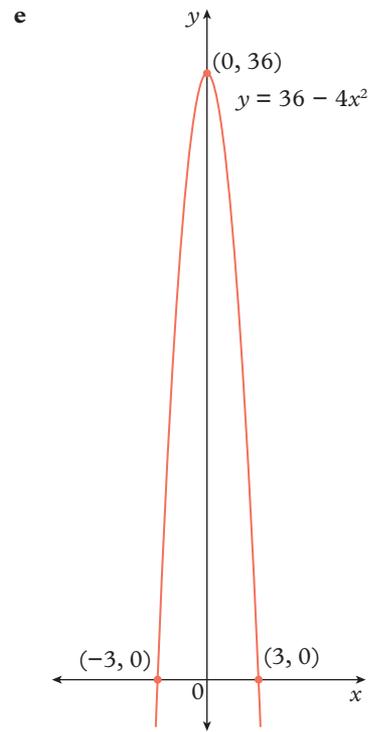
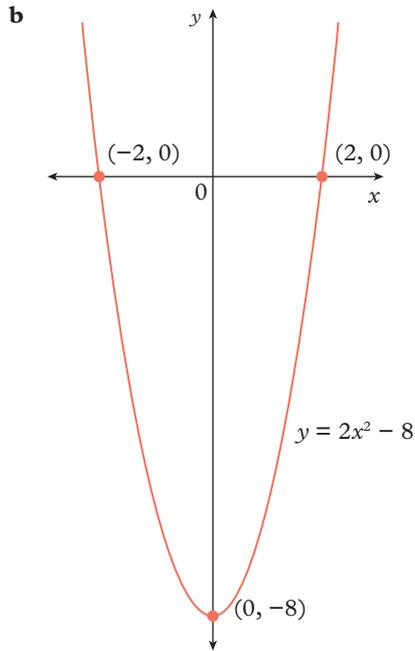


- b** It's not possible to have negative time values and it is not possible for the hairclip to fall below ground level.
- c** **i** 84 m **ii** 64 m
d 100 m **e** 5 s
- 8 a** $(-1, 0)$ and $(1, 0)$ **b** $x = -1$ or $x = 1$
c The solutions are the x -coordinates of the x -intercepts.
d The x -coordinates of the x -intercepts are the solutions of the equation of the parabola.
- 9 a** $-2\sqrt{5}$, $2\sqrt{5}$ **b** $-2\sqrt{6}$, $2\sqrt{6}$
c 0 **d** $-\sqrt{6}$, $\sqrt{6}$
- 10 a** $-\sqrt{a}$, \sqrt{a} **b** $a = 25$, $y = x^2 - 25$
c -25 **d** $y = -x^2 + 25$

EX 5C Sketching parabolas using intercepts
p210

- 1 a** **i** $(\sqrt{2}, 0)$, $(-\sqrt{2}, 0)$ **ii** $(0, -2)$
b **i** $(2, 0)$, $(-2, 0)$ **ii** $(0, -8)$
c **i** $(5, 0)$, $(-5, 0)$ **ii** $(0, 25)$
d **i** $(2, 0)$, $(-2, 0)$ **ii** $(0, 2)$
e **i** $(3, 0)$, $(-3, 0)$ **ii** $(0, 36)$
f **i** $(10, 0)$, $(-10, 0)$ **ii** $(0, 25)$

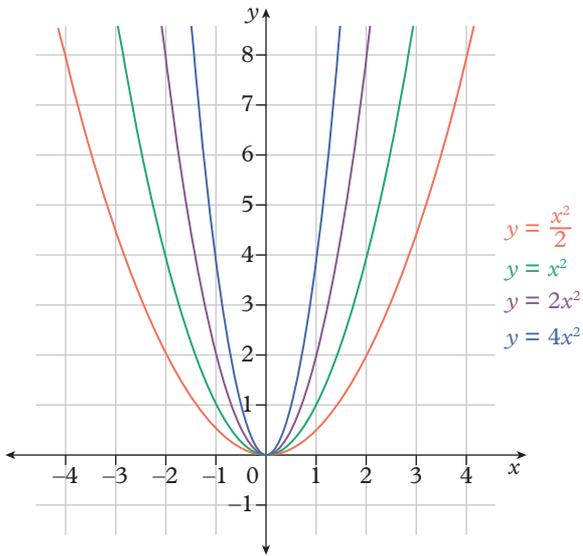




- 3 a I b III c II**
- 4 a** Both graphs have x -intercepts at $(-6, 0)$ and $(6, 0)$.
- b** $y = 18 - \frac{x^2}{2}$ has a y -intercept at $(0, 18)$ and $y = \frac{x^2}{2} - 18$ has a y -intercept at $(0, -18)$.
- c** Both graphs have the same x -intercepts and the same shape, but they are reflections of each other in the x -axis. So $y = \frac{x^2}{2} - 18$ has a minimum turning point and $y = 18 - \frac{x^2}{2}$ has a maximum turning point.
- 5 a** $(0, -3)$ **b** $(0, \frac{1}{4})$
- c** $(0, \sqrt{5})$ **d** $(0, -6)$
- 6 a** $(0, c)$ **b** $\pm\sqrt{\frac{-c}{k}}$
- c** $c = 0$ **d** $\frac{c}{k} > 0$

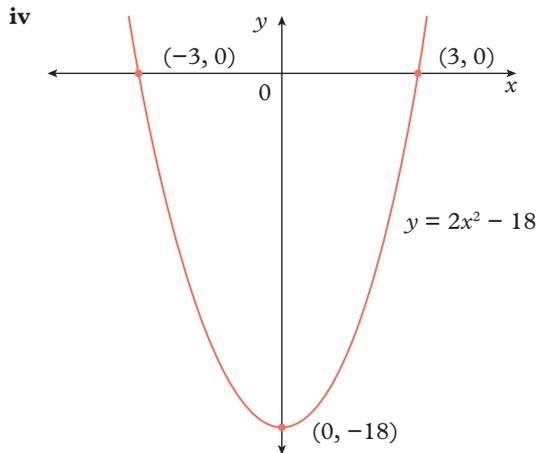
- 7 **a** two **b** zero **c** two **d** two
 8 **a** **i** -16 **ii** (-4, 0) and (4, 0) **iii** minimum point
b **i** 64 **ii** (-8, 0) and (8, 0) **iii** maximum point
 9 **a** $y = x^2$ **b** $y = 2x^2$
c $y = 4x^2$ **d** $y = \frac{x^2}{2}$

10



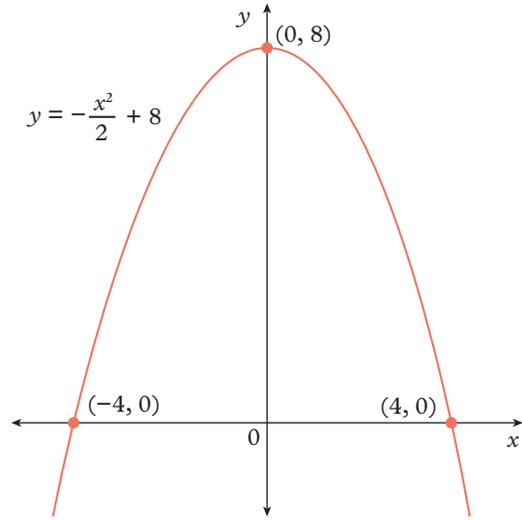
For $k > 0$, the bigger k is, the narrower the parabola becomes.

- 11 **a** **i** concave up
ii x-intercepts: (-3, 0) and (3, 0),
 y-intercept: (0, -18)
iii (0, -18)



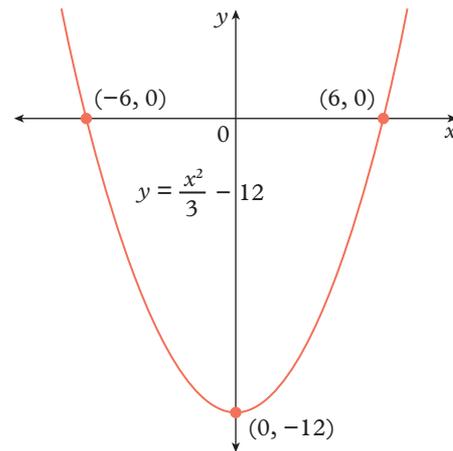
- b** **i** concave down
ii x-intercepts: (-4, 0) and (4, 0),
 y-intercept: (0, 8)
iii (0, 8)

iv



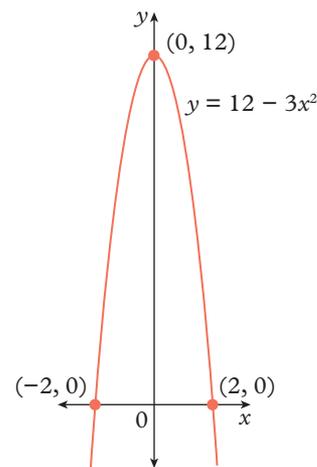
- c** **i** concave up
ii x-intercepts: (-6, 0) and (6, 0),
 y-intercept: (0, -12)
iii (0, -12)

iv



- d** **i** concave down
ii x-intercepts: (-2, 0) and (2, 0),
 y-intercept: (0, 12)
iii (0, 12)

iv



12 **a** $y = \frac{10}{49}(x-7)(x+7)$

b $y = -\frac{8}{25}(x-5)(x+5)$

13 $(0, -ap^2)$

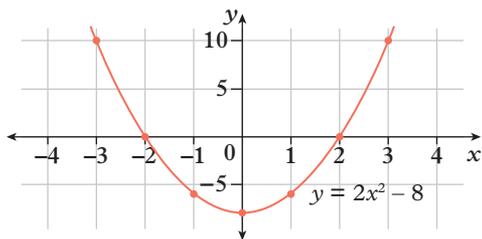
14 $y = -\frac{2}{3}x^2 + 6$

EX Checkpoint

p212

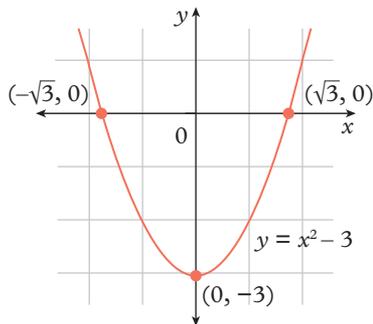
- | | |
|------------------------------|-----------------------|
| 1 a $x = 3$ or $x = 4$ | b $x = -2$ or $x = 2$ |
| c $x = -9$ | d $x = 0$ or $x = -4$ |
| 2 a $x = 5$ or $x = 7$ | b $x = -7$ or $x = 4$ |
| c $x = 6$ | d $x = -9$ or $x = 9$ |
| 3 a $x = -4$ or $x = 4$ | b $x = -2$ or $x = 4$ |
| c $x = -5$ or $x = -1$ | d $x = 0$ or $x = 15$ |
| 4 a i $(-3, 0)$ and $(3, 0)$ | ii $(0, 6)$ |
| iii maximum | iv $x = 0$ |
| b i $(-2, 0)$ and $(2, 0)$ | ii $(0, -2)$ |
| iii minimum | iv $x = 0$ |

5



- | | |
|---------------------------------------|-----------------------------------|
| 6 a -2 | b $(-\sqrt{2}, 0), (\sqrt{2}, 0)$ |
| 7 a i $(-\sqrt{3}, 0), (\sqrt{3}, 0)$ | ii $(0, -3)$ |
| b i $(-5, 0), (5, 0)$ | ii $(0, -50)$ |
| c i $(-2\sqrt{2}, 0), (2\sqrt{2}, 0)$ | ii $(0, -8)$ |
| d i $(-8, 0), (8, 0)$ | ii $(0, -64)$ |

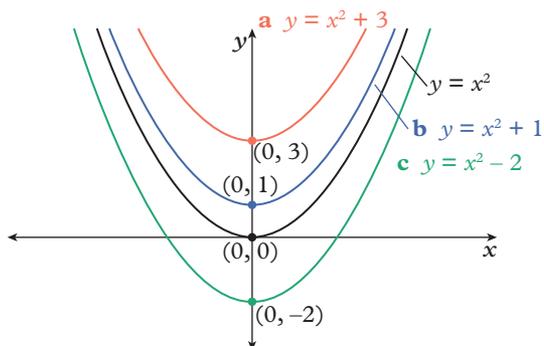
8



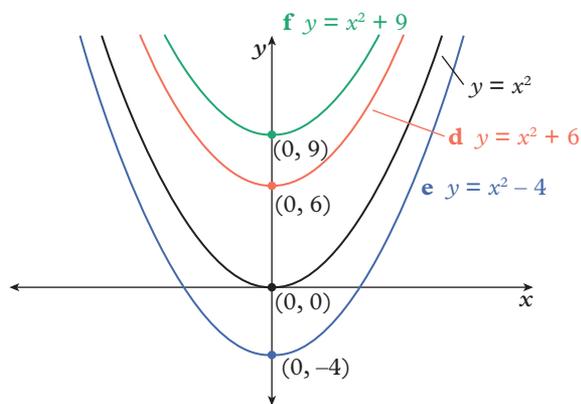
EX 5D Sketching parabolas using transformations

p216

- | | |
|--------------|---------|
| 1 a 10, down | c 2, up |
| b 8, up | d 4, up |
- 2 a-c

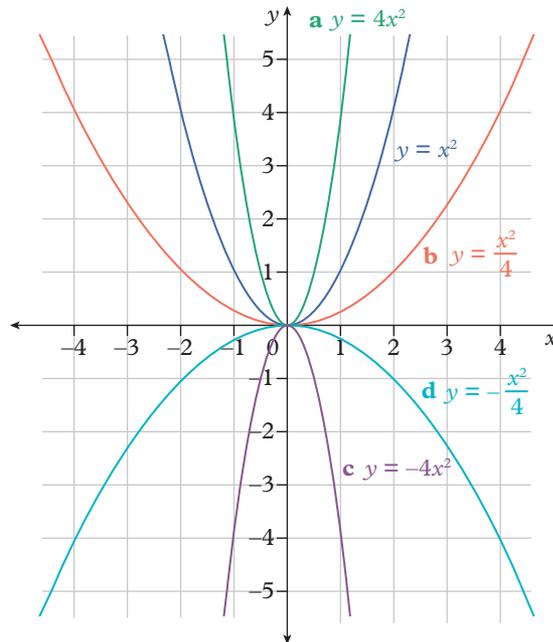


d-f



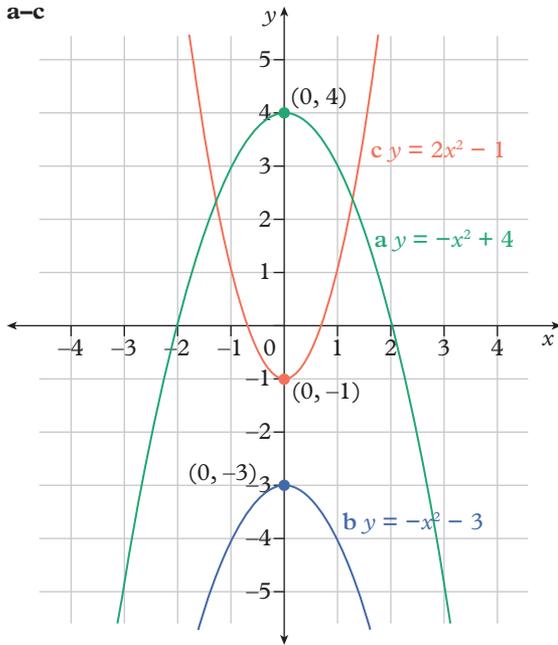
- 3 a $k = 1$
 b $y = 2x^2, k = 2$
 c $y = \frac{1}{2}x^2, k = \frac{1}{2}$
 d If $k > 1$, the graph will be vertically stretched.
 If $0 < k < 1$, the graph will be vertically compressed.

- 4 a V b I c IV d III e VI f II
 5

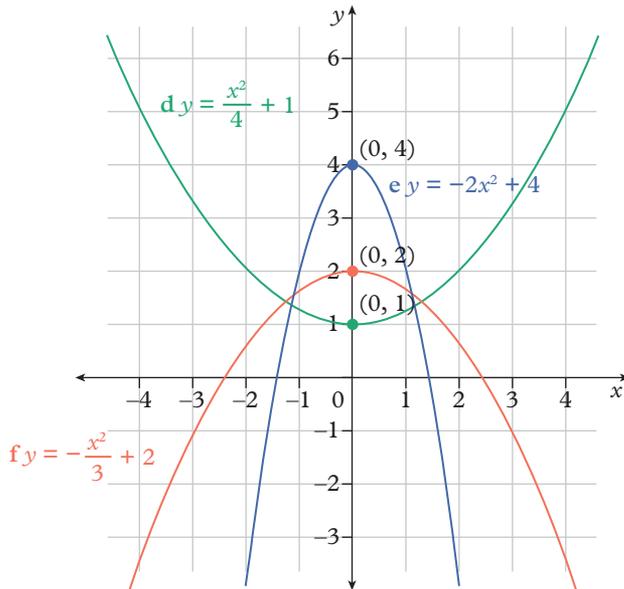


- | | | |
|-------------|-----------|-----------|
| 6 a minimum | b minimum | c minimum |
| d maximum | e maximum | f maximum |
| g maximum | h maximum | |
- 7 a x -axis, y -axis b reflected, 4, up

8 a-c



d-f



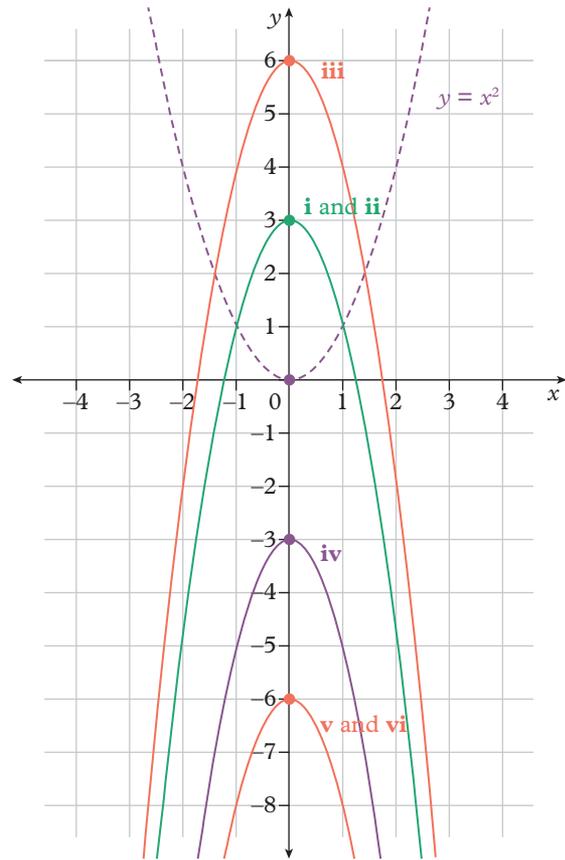
- 9 a i concave down ii same width
 iii (0, -1) iv negative
 v -1
- b i concave down ii narrower
 iii (0, 5) iv negative
 v 5
- c i concave up ii same width
 iii (0, 1) iv positive
 v 1
- 10 a (0, 5) b 5
 c There is no maximum value for y because the parabola is concave up (it has a minimum turning point).

11 Graph is concave down, so it has a maximum turning point. The y -coordinate of the turning point is 2, so the maximum y -value is 2.

- 12 a $y = x^2 + 7$ b $y = x^2 + 5$
 c $y = -x^2 + 4$ d $y = -x^2 - 1$
- 13 a $y = 3x^2 + 2$ b $y = -x^2 - 4$
 c $y = -\frac{x^2}{2}$ d $y = -4x^2 + 2$

14 a i b ii c i d ii e ii f ii

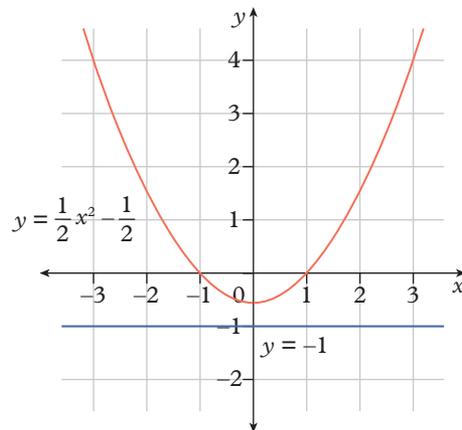
15 a



b i and ii, v and vi

c i I ii I iii IV iv II v III vi III

16 a



- b i yes ii yes iii yes
 c i 1 ii $\frac{1}{2}$ iii $\frac{5}{2}$
 d i 1 ii $\frac{1}{2}$ iii $\frac{5}{2}$
 e Point of choice may vary; distances are the same.

EX 5E Plotting exponential relationships

p221

- 1 **a** No **b** No **c** Yes, 3 **d** No
e No **f** Yes, $\frac{1}{3}$ **g** Yes, $\frac{1}{3}$ **h** No
- 2 **a** **i** yes **ii** $(0, -2)$ **iii** $y = -3$
iv increases
- b** **i** no **ii** $(0, 1)$ **iii** $y = 0$
iv decreases
- c** **i** no **ii** $(0, 2)$ **iii** $y = 1$
iv increases

3 **a**

x	-2	-1	0	1	2
$y = 4^x$	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16

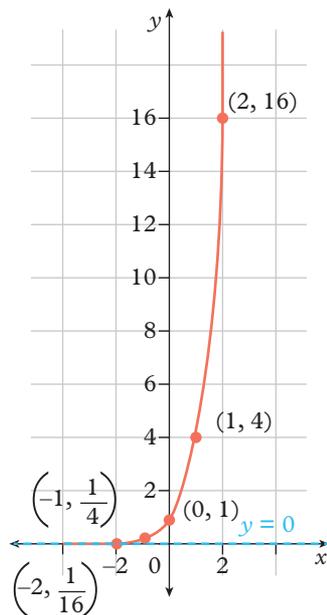
b

x	-2	-1	0	1	2
$y = 3^{-x}$	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$

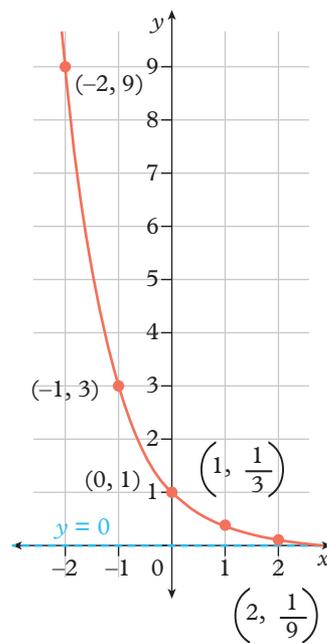
c

x	-2	-1	0	1	2
$y = \left(\frac{2}{3}\right)^x$	$\frac{9}{4}$	$\frac{3}{2}$	1	$\frac{2}{3}$	$\frac{4}{9}$

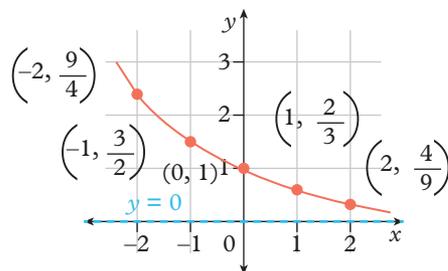
4 **a**



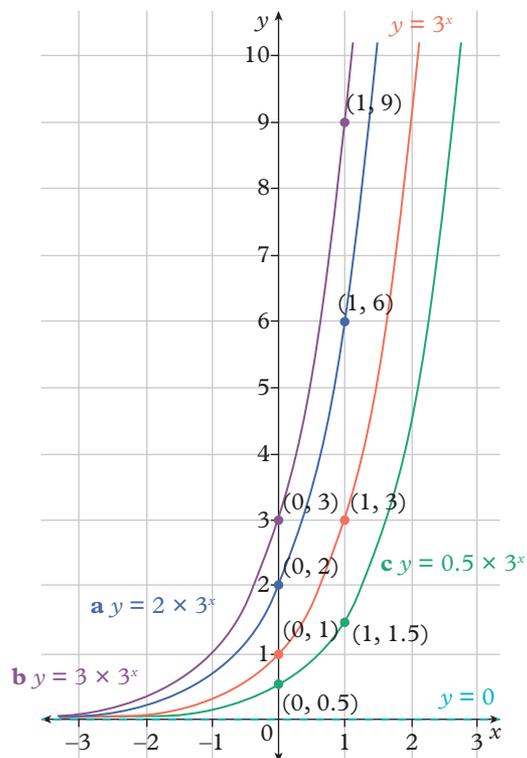
b



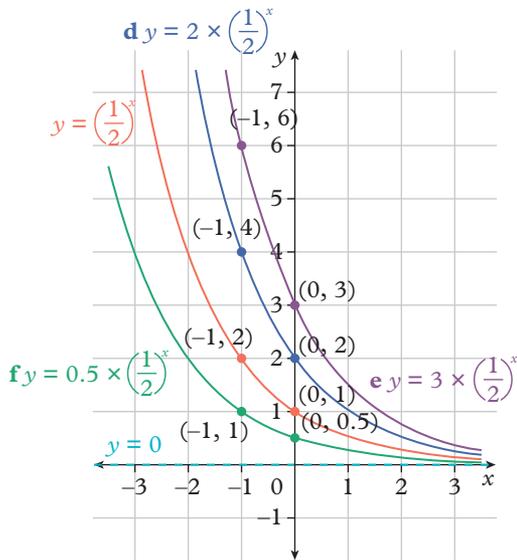
c



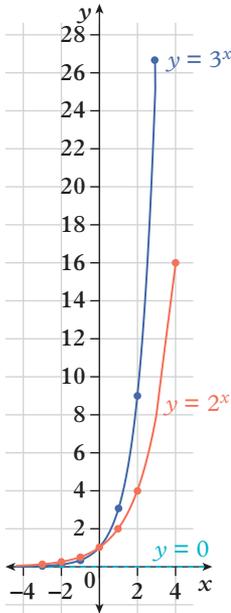
5 **a-c**



d-f

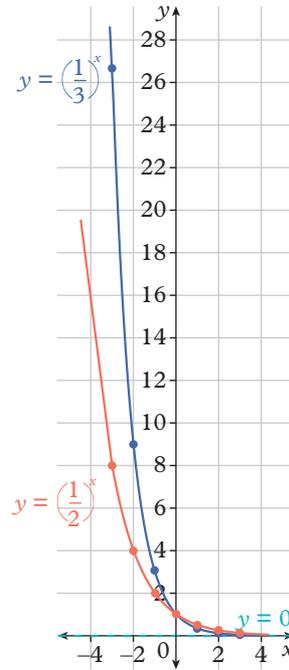


6 a



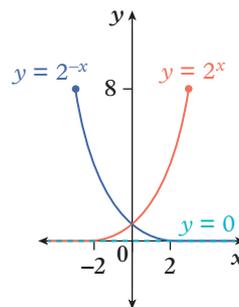
- b** The curve for $y = 3^x$ becomes steeper as the positive x -values increase. That is, the y -values for $y = 3^x$ are bigger than those for $y = 2^x$ when $x > 0$. When $x < 0$, the y -values for $y = 3^x$ are smaller than those for $y = 2^x$.
- c** They have the same asymptote and y -intercept. They are the same general shape.
- d** $(0, 1)$ for both graphs
- e** $y = 0$ for both graphs
- f** The curve for $y = 5^x$ becomes steeper than both as the positive x -values increase. That is, the y -values for $y = 5^x$ are bigger than those for both $y = 2^x$ and $y = 3^x$ when $x > 0$. For $x < 0$, the y -values for $y = 5^x$ are smaller than those for $y = 2^x$ and $y = 3^x$. However, the asymptote and y -intercept would be the same, and the general shape would be the same.

7 a



- b** The curve for $y = \left(\frac{1}{3}\right)^x$ becomes steeper as the x -values decrease. That is, the y -values for $y = \left(\frac{1}{3}\right)^x$ are bigger than those for $y = \left(\frac{1}{2}\right)^x$ when $x < 0$. When $x > 0$, the y -values for $y = \left(\frac{1}{3}\right)^x$ are smaller than those for $y = \left(\frac{1}{2}\right)^x$.
- c** They have the same asymptote and y -intercept. They are the same general shape.
- d** $(0, 1)$ for both graphs
- e** $y = 0$ for both graphs
- f** The curve for $y = \left(\frac{1}{5}\right)^x$ becomes steeper than both as the x -values decrease. That is, the y -values for $y = \left(\frac{1}{5}\right)^x$ are bigger than those for both $y = \left(\frac{1}{2}\right)^x$ and $y = \left(\frac{1}{3}\right)^x$ when $x < 0$. For $x > 0$, the y -values for $y = \left(\frac{1}{5}\right)^x$ are smaller than those for $y = \left(\frac{1}{2}\right)^x$ and $y = \left(\frac{1}{3}\right)^x$. However, the asymptote and y -intercept would be the same, and the general shape would be the same.

8



- a** $y = 2^x$ has positive y -values that increase as x increases. $y = 2^{-x}$ has positive y -values that decrease as x increases.
- b** They have the same asymptote and y -intercept, and the same general shape (reflected in the y -axis).
- c** $(0, 1)$ for both
- d** $y = 0$ for both
- e** reflection in y -axis
- f** a reflection in the x -axis and then a reflection in the y -axis

x	-2	-1	0	1	2
y = 4^x	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16
Ratio	$\frac{1}{4} \div \frac{1}{16} = 4$	$1 \div \frac{1}{4} = 4$	$4 \div 1 = 4$	$16 \div 4 = 4$	

b

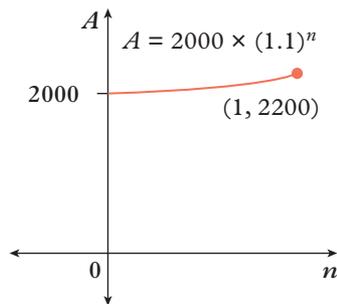
x	-2	-1	0	1	2
y = 3^{-x}	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$
Ratio	$3 \div 9 = \frac{1}{3}$	$1 \div 3 = \frac{1}{3}$	$\frac{1}{3} \div 1 = \frac{1}{3}$	$\frac{1}{9} \div \frac{1}{3} = \frac{1}{3}$	

c

x	-2	-1	0	1	2
y = $(\frac{2}{3})^x$	$\frac{9}{4}$	$\frac{3}{2}$	1	$\frac{2}{3}$	$\frac{4}{9}$
Ratio	$\frac{3}{2} \div \frac{9}{4} = \frac{2}{3}$	$1 \div \frac{3}{2} = \frac{2}{3}$	$\frac{2}{3} \div 1 = \frac{2}{3}$	$\frac{4}{9} \div \frac{2}{3} = \frac{2}{3}$	

The ratio is a constant. For an exponential equation in the form $y = a^x$, the ratio between consecutive values is a constant because $\frac{a^{n+1}}{a^n} = a$.

10 a

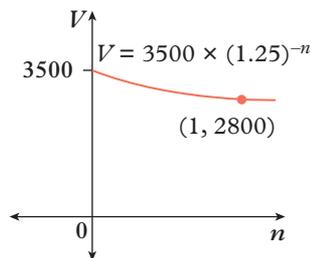


b A increases

c i approximately \$2900 read from the graph (the exact value is \$2928.20)

ii 8 years

11 a The graph will decrease as time increases.



b \$3500

c V decreases

d 6 years

12 a

x	-2	-1	0	1	2	3	4	5
x²	4	1	0	1	4	9	16	25
2^x	0.25	0.5	1	2	4	8	16	32

Therefore, $x = 2$ and $x = 4$.

b As at $x = -1$, $x^2 > 2^x$ but at $x = 0$, $x^2 < 2^x$. This means the expressions must be equal at some value in between -1 and 0 .

c $x = -0.767$

Mathematical literacy review

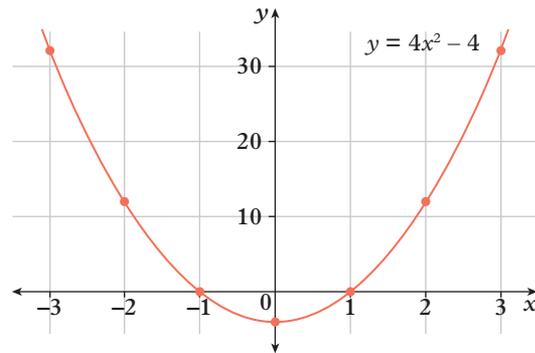
- highest common factor, factorise, Null Factor Law, factor, linear equation
- a** IV **b** VII **c** I **d** III
e VIII **f** V **g** II **h** VI
- a** True
b False. If $k < -1$, then the graph has a vertical dilation and is *enlarged*.
c False. If $k = -1$, then the graph has a vertical *reflection* around the x -axis.
d True
e True
f False. The graph will *not* pass through the origin.
g False. The coordinates of the vertex are $(0, c)$.

Multiple choice

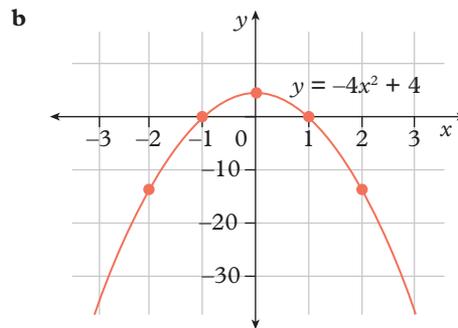
- D 2 A 3 A 4 B 5 D 6 C
- D 8 D 9 E 10 B 11 B

Short answer

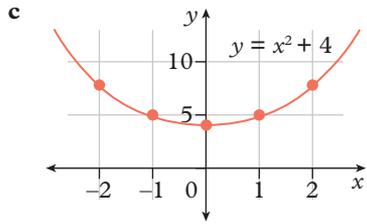
- a** $x = 2$ or $x = 3$ **b** $x = -6$ or $x = 5$
c no solutions **d** $x = 0$ or $x = 12$
- a** no **b** yes **c** yes **d** yes
- a**



- x-intercepts: $(-1, 0)$ and $(1, 0)$, y-intercept: $(0, -4)$
- $(0, -4)$ **iii** minimum **iv** $x = 0$



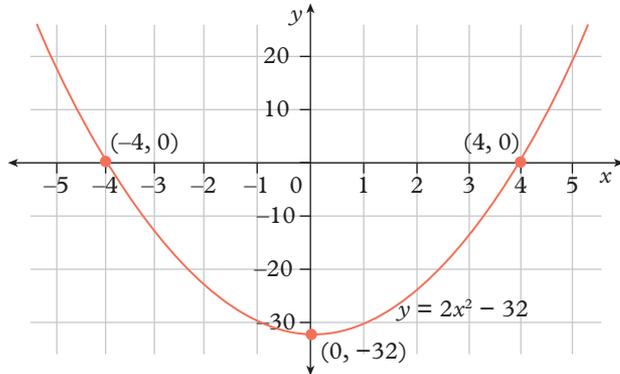
- x-intercepts: $(-1, 0)$ and $(1, 0)$, y-intercept: $(0, 4)$
- $(0, 4)$ **iii** maximum **iv** $x = 0$



- i** no x -intercepts, y -intercept: $(0, 4)$
ii $(0, 4)$ **iii** minimum **iv** $x = 0$

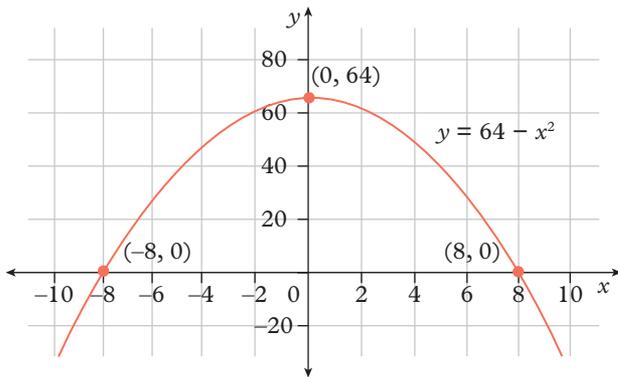
- 4 a i** x -intercepts: $(-4, 0)$ and $(4, 0)$,
 y -intercept: $(0, -32)$

ii



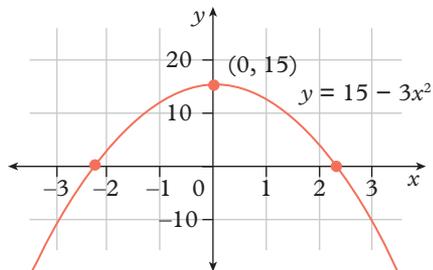
- b i** x -intercepts: $(-8, 0)$ and $(8, 0)$,
 y -intercept: $(0, 64)$

ii

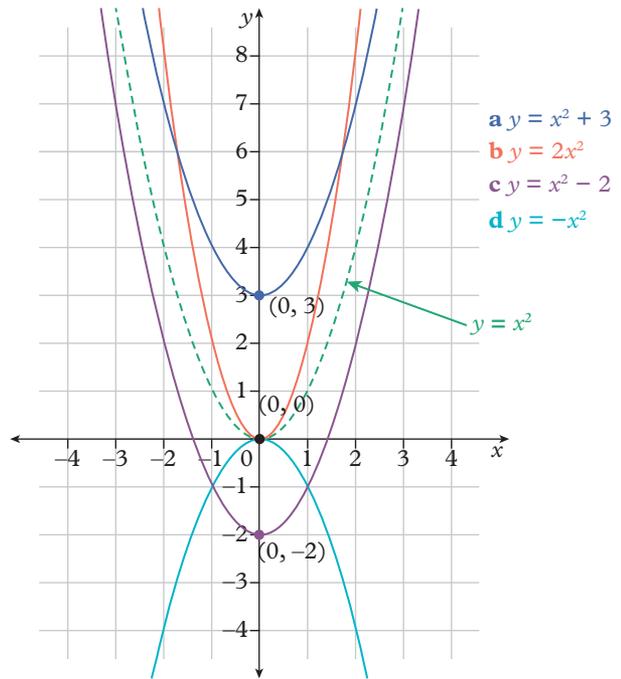


- c i** x -intercepts: $(-\sqrt{5}, 0)$ and $(\sqrt{5}, 0)$,
 y -intercept: $(0, 15)$

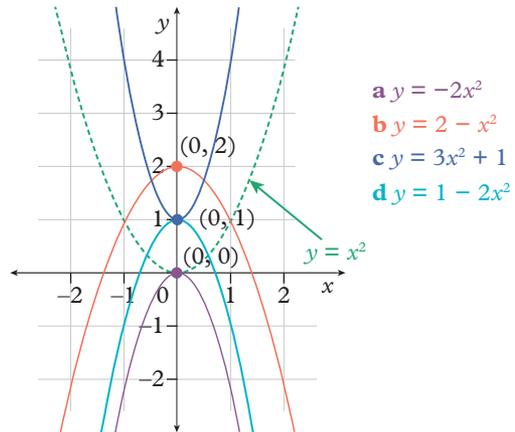
ii



5

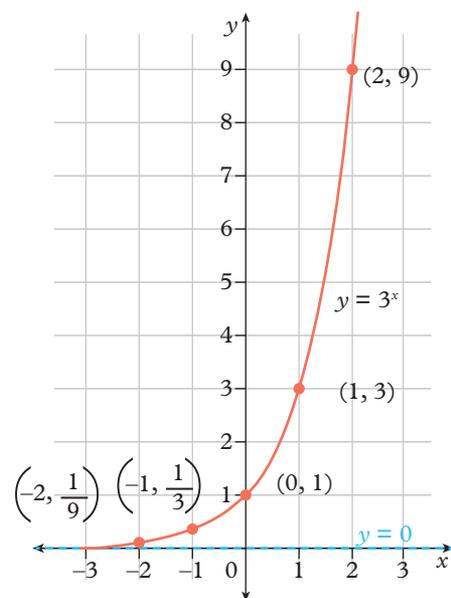


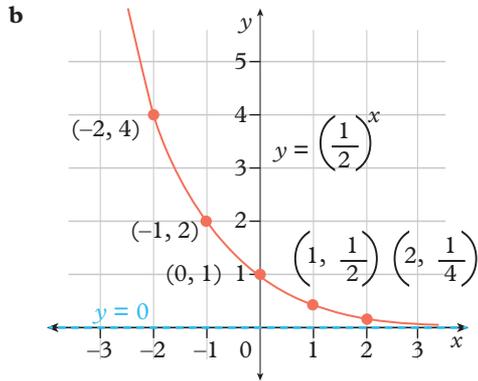
6



- 7 a** $(0, 1)$ **b** $(0, -1)$ **c** $(0, 1)$ **d** $(0, 1)$

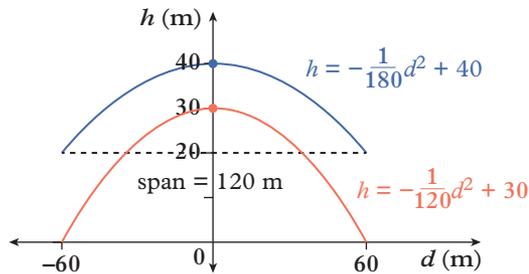
8 a





Analysis

1 a, d



- b i** upper arch: (0, 40), lower arch: (0, 30)
ii upper arch: (0, 40), lower arch: (0, 30)

c 20 m

d 120 m

e i 30 m **ii** 40 m

f 10 m

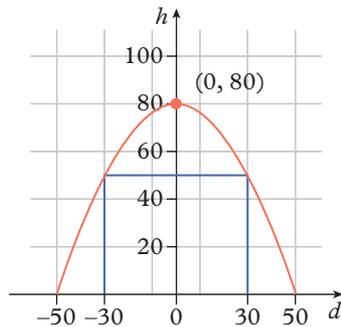
g 12.5 m

2 a $h = ad^2 - 2500a$

b Horizontal intercepts: $d = -50$ is where the ball starts, $d = 50$ is where the ball lands.

c (0, 80)

d, h



e $a = -0.032$

f $h = 51.2$ m

g 80 m

3 a

Equation	x	-2	-1	0	1	2
A	$y = x^2$	4	1	0	1	4
B	$y = 4x^2$	16	4	0	4	16
C	$y = (2x)^2$	16	4	0	4	16

b Dilation by a factor of 4

c Equations B and C will make the same graph.

d i $y = 9x^2$ and $y = (3x)^2$

ii $y = 2x^2$ and $y = (\sqrt{2}x)^2$

iii $y = -4x^2$ and $y = -(2x)^2$

iv $y = kx^2$ and $y = (\sqrt{k}x)^2$

e

Equation	x	-2	-1	0	1	2
A	$y = x^2$	4	1	0	1	4
B	$y = x^2 - 1$	3	0	-1	0	3
C	$y = (x + 1)(x - 1)$	3	0	-1	0	3

f Translated down 1 unit

g Equations B and C will make the same graph.

h i $y = x^2 - 4$ and $y = (x + 2)(x - 2)$

ii $y = x^2 - 2$ and $y = (x + \sqrt{2})(x - \sqrt{2})$

iii $y = x^2 - c$ and $y = (x + \sqrt{c})(x - \sqrt{c})$

iv If $c < 0$, the graph is translated up and so no x -intercepts. It cannot be factored.

4 a

Equation	x	-2	-1	0	1	2
A	$y = 2^x$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
B	$y = 2 \times 2^x$	$\frac{1}{2}$	1	2	4	8
C	$y = 2^{x+1}$	$\frac{1}{2}$	1	2	4	8

b Dilation of 2 units

c Equations B and C have the same graph.

d $y = 4 \times 2^x$ and $y = 2^{x+2}$

e If k is a power of 2 such as 4, 8, 16, etc.

f i $y = 9 \times 3^x$ and $y = 3^{x+2}$

ii $y = \frac{1}{9} \times 3^x$ and $y = 3^{x-2}$

CHAPTER 6 Measurement

EX 6A Area of composite shapes

p235

1 a 20.79 cm² **b** 1800 mm² **c** 452.39 cm²

d 14.16 mm² **e** 44.2 cm² **f** 768 mm²

2 a 7.07 cm² **b** 127.41 cm² **c** 184.31 mm²

3 a triangle, semicircle

b three rectangles

c triangle, rectangle, semicircle

d triangle, rectangle

e rectangles, semicircles

f parallelogram, triangle, semicircle

4 a 122.27 cm² **b** 56 cm² **c** 31.57 cm²

d 69 cm² **e** 160 cm² **f** 174.63 cm²

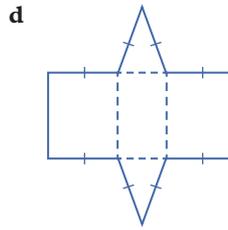
5 a 70.87 cm² **b** 47.43 mm² **c** 424 cm²

d 69.73 cm² **e** 70.5 cm² **f** 60.5 cm²

6 a 141.02 cm² **b** 330 cm² **c** 557.08 cm²

d 34 cm² **e** 34.28 cm² **f** 63.26 cm²

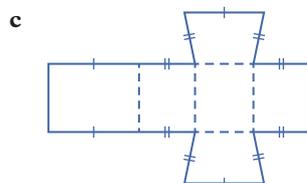
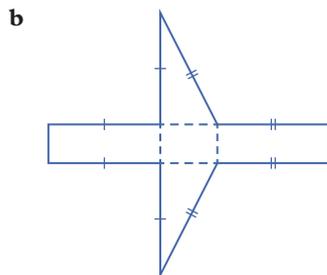
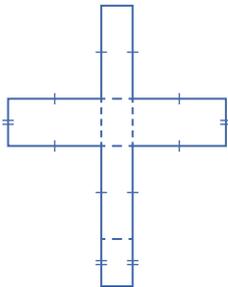
- 7 **a** An annulus is the shape formed by two different-sized circles with a common centre. It is the area between the two circles.
b area of outer circle – area of inner circle
c 137.44 cm^2
- 8 **a** 12.57 cm^2 **b** 65.97 cm^2 **c** 28.27 m^2
d 226.19 cm^2 **e** 103.67 cm^2 **f** 47.12 cm^2
- 9 $12\,166.55 \text{ m}^2$
- 10 **a** 35.5 cm^2
b $19\,525 \text{ cm}^2$
Sample answers: To cater for wastage between tags when they are cut out; in case any mistakes are made in production of tags.
- 11 **a** 105.35 m^2 **b** $\$4740.75$ **c** $\$1580.25$
- 12 176.70 cm^2
- 13 **a** 354.01 cm^2 **b** 336 cm^2 **c** 97.425 cm^2
d 68.57 cm^2 **e** 35 cm^2 **f** 78.72 cm^2
- 14 **a** A, B: 25 cm^2 , C, E, G: 12.5 cm^2 , D, F: 6.25 cm^2
b i 100 cm^2 **ii** 100 cm^2 **iii** 100 cm^2
- 15 47.75 cm^2
- 16 $15\,509.73 \text{ cm}^2$
- 17 71.78 cm^2
- 18 **a** 44 cm^2 **b** 17.80 cm^2 **c** 15 m^2



- 4 **a** 24 m^2 **b** 294 mm^2 **c** 600 cm^2
d 384 mm^2 **e** 13.5 m^2 **f** 5400 cm^2
- 5 **a** Six
b 32 m^2 , 24 m^2 and 12 m^2
c 136 m^2
- 6 **a** 344 mm^2 **b** 166 cm^2 **c** 378 mm^2
d 268 m^2 **e** 76.96 m^2 **f** 738 cm^2
- 7 **a** Five
b 12 cm^2 , 12 cm^2 , 45 cm^2 , 45 cm^2 , 54 cm^2
c 168 cm^2
- 8 **a** 2916 cm^2 **b** 510 mm^2 **c** 142 m^2
d 768 mm^2 **e** 2408 cm^2 **f** 523 m^2
- 9 **a i** 664 cm^2 **ii** 1328 cm^2
b i 396 cm^2 **ii** 792 cm^2
- 10 **a** 3820 cm^2 **b** 2961.2844 cm^2
- 11 **a** 500 cm^2
b i 292 cm^2 **ii** 584 cm^2
c Answer to **b ii** is 84 cm^2 greater because two more faces (7 cm by 6 cm) are created when the block of butter is cut in half.
- d** 4032 cm^2
e Sample answer: Cutting butter into smaller pieces increases the amount of surface exposed to the heat, so butter will melt more quickly.
- 12 **a** cream: 30 m^2 , blue: 53.575 m^2
b cream: 4 L , blue: 7.14 L
- 13 **a** 25 cm^2 **b** 5 cm **c** $l = \sqrt{\frac{\text{TSA}}{6}}$
- 14 Sample answer: A cube has six identical square faces; hence all sides are equal in length. Therefore, the length of one face will determine the length of all sides. A rectangular prism could have a different length, breadth and height.
- 15 24.92 m^2
- 16 **a** 50 cm^2 **b** 30 cm^2

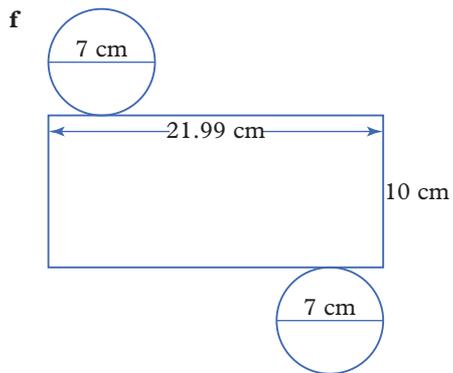
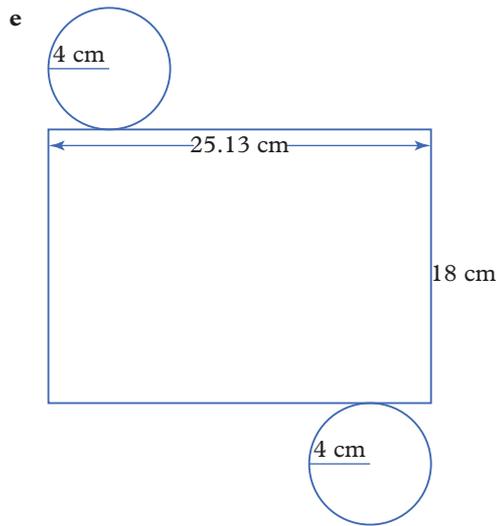
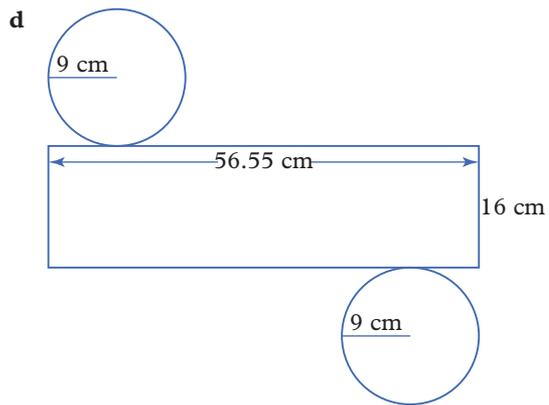
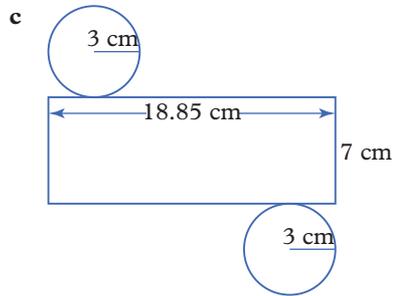
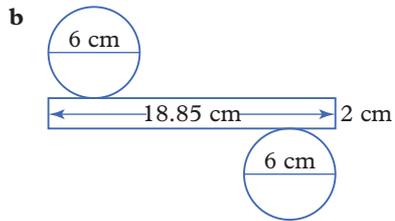
EX p243 **6B Surface area**

- 1 **b, c**
- 2 **a i** triangular prism **ii** 44 cm^2
b i rectangular prism **ii** 70 cm^2
- 3 **a**



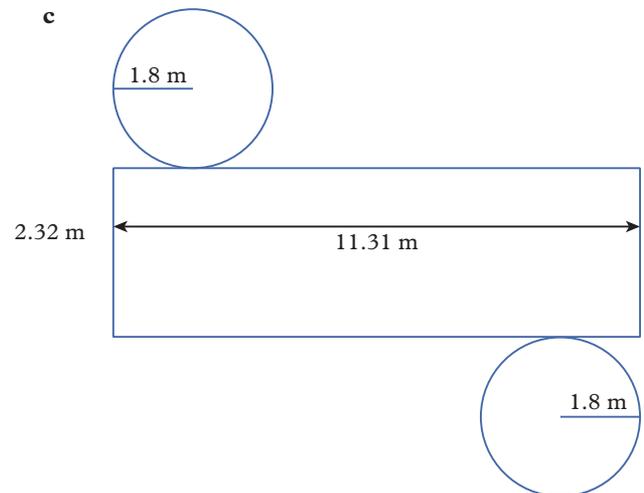
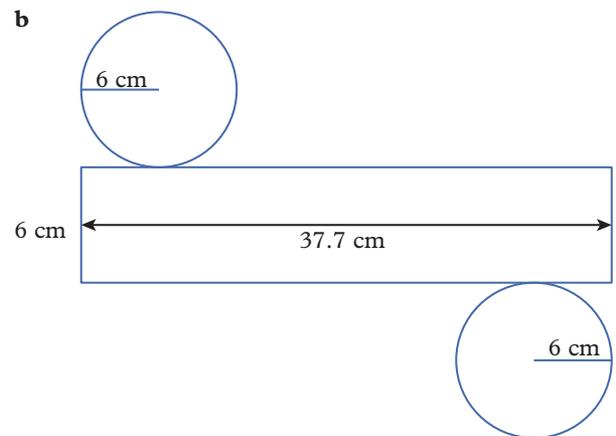
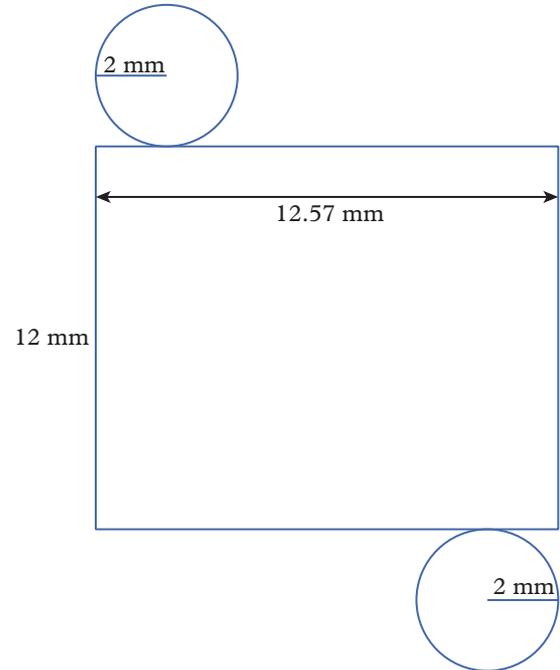
EX p248 **6C Surface area of cylinders**

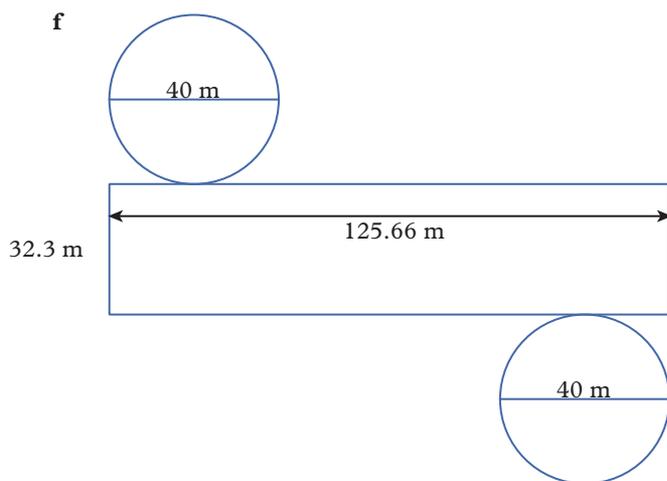
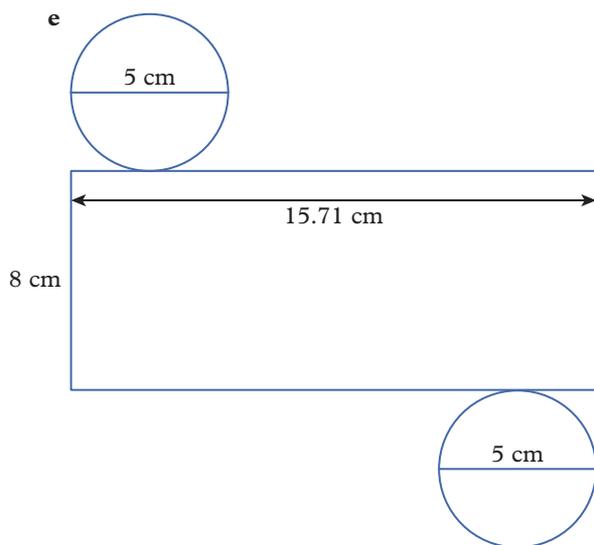
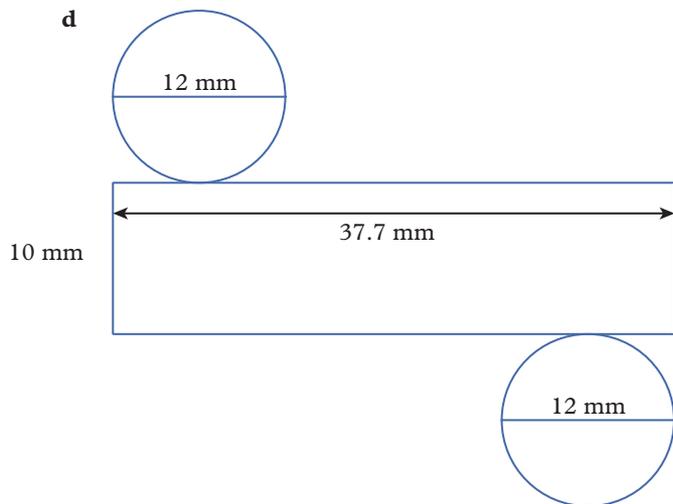
- 1 **a**
-



2 **a** 62.83 mm^2 **b** 94.25 cm^2 **c** 188.50 cm^2
d 1413.72 cm^2 **e** 552.92 cm^2 **f** 296.88 cm^2

3 **a**

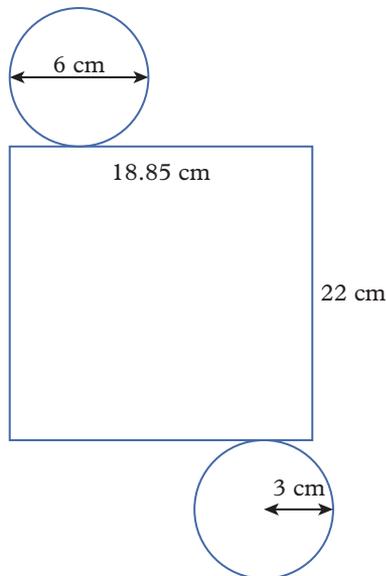




- 7 3924.63 cm²
 8 \$1429 (\$1450 if rounded to needing 29 L of paint)
 9 First roller with radius 4 cm and length 25 cm
 10 **a** 376.99 cm²
b Lucian has calculated the surface area of the whole cylinder (ignoring the hollowed-out section) and Curtis forgot to include the area of the inside surface.
 11 8 cm
 12 Sample answer: 2 × area of base calculates the surface area of the matching ends of a prism. The surface area of each of the rectangular faces joining the matching ends can be calculated by multiplying the section of the perimeter of the base that the face occupies by the height, so together the area of all of the rectangular faces joining the matching ends is perimeter of base × height.
 13 No, surface area will not double if you double the height of the cylinder. The surface area of the curved surface doubles, but the area of the bases does not change.
 14 **a** 201.06 cm² **b** 615.75 mm²
c 314.16 cm² **d** 127.32 cm²
 15 $h = \frac{SA - 2\pi r^2}{2\pi r}$ or $h = \frac{SA}{2\pi r} - r$
 16 48.42

EX Checkpoint
 p251

- 1 **a** 4.71 cm² **b** 41.89 cm²
 2 **a** 37.44 m² **b** 284 m²
 3 **a** Yes **b** No **c** No
 4 **a** 195 mm² **b** 29450 cm²
 5 **a** 678.58 cm² **b** 452.39 m²
 6 **a**



- 4 **a** 175.93 mm² **b** 452.39 cm² **c** 46.60 m²
d 603.19 mm² **e** 164.93 cm² **f** 6572.21 m²
 5 **a** 100.53 cm² **b** 541.92 cm² **c** 603.19 cm²
d 18.85 m² **e** 7.85 m² **f** 3.39 m²
 6 **a** 1130.97 cm² **b** 34.57 cm² **c** 96.13 m²
d 157.10 m² **e** 15.36 mm² **f** 9.08 mm²

- b** 471.2 cm²

EX 6D Volume of composite solids

p255

- 1 **a** 48 cm^3 **b** 1080 cm^3 **c** 288 cm^3
d 210 cm^3 **e** 36 cm^3 **f** 82.5 cm^3
g 540 cm^3 **h** 2457 cm^3 **i** 140 mm^3
- 2 **a** 150.80 mm^3 **b** 678.58 cm^3 **c** 23.61 m^3
d 1.13097 mm^3 **e** 157.08 cm^3 **f** $40\,589.38 \text{ m}^3$
- 3 **a** 249.31 cm^3 **b** 18.28 cm^3 **c** 608.58 cm^3
- 4 **a** 0.1508 mL **b** 678.58 mL **c** $23\,610 \text{ L}$
d 1.13097 mL **e** 157.08 mL **f** $40\,589\,380 \text{ L}$
- 5 **a** 30 cm^2 **b** 4 cm
- 6 **a** 60 cm^2 **b** 40 mm
- 7 **a** $6x^2$ **b** x^3 **c** $x, x, x + 4$
d $6x^2 + 16x$ **e** 5 **f** 225 cm^3
- 8 160 cm^3
- 9 **a** $6x^2$ **b** x^3 **c** $x, x, x - 2$
d $6x^2 - 8x$ **e** 6 **f** 144 cm^3
- 10 18 cm^3
- 11 54 cm^3
- 12 **a** Surface area = 2400 cm^2 , volume = 8000 cm^3
b Surface area = 2400 cm^2 , volume = 6300 cm^3
- 13 **a** 126 m^3 **b** $\$3780$
- 14 **a** **i** $\$24$ **ii** $\$37.58$ **iii** $\$28.50$
b Sculpture **i** weighs 36 kg , sculpture **ii** weighs 37.584 kg , and sculpture **iii** weighs 35.625 kg . She should buy sculpture **iii**.
- 15 **a** 43.52 cm^3 **b** 38.86 cm^3

EX 6E Errors

p260

- 1 **a** 2.5 mm **b** 0.25 g **c** 0.05 mL
d 0.05°C **e** 0.01 seconds **f** 0.5 m^3
- 2 **a** 0.05% **b** 0.03% **c** 0.68% **d** 0.04%
e $1.1\% \text{ g}$ **f** 0.03% **g** 2.08% **h** $0.05\% \text{ g}$
- 3 **B**
- 4 **a** **i** 1° **ii** 0.5° **iii** 0.37%
b **i** 5 mL **ii** 2.5 mL **iii** 4.17%
c **i** 0.1°C **ii** 0.05°C **iii** 0.14%
d **i** 1 kg **ii** 0.5 kg **iii** 1.25%
- 5 **a** 1.65 **b** 2.325
- 6 **a** lower bound: 429.5 g , upper bound: 430.5 g
b lower bound: 427.5 g , upper bound: 432.5 g
- 7 Between 122.5 cm and 127.5 cm
- 8 **a** lower bound: 11.5 cm , upper bound: 12.5 cm
b lower bound: 46 cm , upper bound: 50 cm
c lower bound 132.25 cm^2 , upper bound: 156.25 cm^2
- 9 **a** 614.13 cm^3 **b** 421.88 cm^3
- 10 **a** lower bound: 112.5 cm , upper bound: 113.5 cm
b lower bound: 18 cm , upper bound: 18 cm
c lower bound: 1007 cm^2 , upper bound: 1025 cm^2
- 11 Paul's suggestion is correct if the smallest measuring unit on the ruler is 0.1 cm . Adding a decimal place may suggest that the measurement is more accurate than it really is.

- 12 The measurement in the imperial system has a greater percentage error.
- 13 **a** Because all measurements are estimates. Errors exist in all measurements, especially when the instrument used is not accurate enough.
b Stack D. 0.084 mm

EX Chapter 6 review

p264

Mathematical literacy review

- 1 **a** cylinder, circumference
b net
c volume
d quadrant, semi-circle
- 2 The absolute error of a measuring instrument depends on the smallest measuring unit of the instrument; the absolute error of a given measuring instrument is fixed. The percentage error of a measurement is the ratio between the absolute error of the measuring instrument and the measurement expressed as a percentage. If two measurements are taken using the same measuring instrument, the percentage error may be different.
- 3 Sample answer:
a $1 \text{ hectare} = 10\,000 \text{ square metres} = 100\,000\,000 \text{ square centimetres}$
b $1 \text{ cubic metre} = 1\,000\,000 \text{ cubic centimetres} = 1\,000\,000\,000 \text{ cubic millimetres}$
c $1 \text{ kL} = 1000 \text{ L} = 1\,000\,000 \text{ mL}$

Multiple choice

- 1 **C** 2 **B** 3 **E** 4 **C** 5 **B** 6 **C**
7 **D** 8 **B** 9 **A** 10 **A** 11 **C**

Short answer

- 1 118 cm^2
2 167.87 cm^2
3 **a** 276 cm^2 **b** 294 cm^2
4 **a** 1187.52 cm^2 **b** 180.64 cm^2
5 **a** 3078.76 cm^3 **b** 176.71 cm^3
6 **a** 125 cm^3 **b** 60 mm^3 **c** 952.77 cm^3
7 **a** 4000 kL or 4 ML **b** 13 m
8 **a** 0.09% **b** 0.77% **c** 1.67% **d** 0.46%

Analysis

- 1 **a** 5 cm
b pink: 36 cm^2 , purple: 900 cm^2
c yes
d 0.82 m^2
e 2
- 2 **a** 47 cm^2 **b** 305.85 cm^2 **c** 376 cm^3
d surface area: $26.48h + 94$, volume: $47h$
e $h = 4.58 \text{ cm}$
f **i** 0.1 cm **ii** 2.18%
iii Min SA = 212.63 cm^2 , Max SA = 217.93 cm^2

- 3 a 68 m^2 b 2714 L
 c No, 17 more tanks d More water
 e 45.53 g, 50.53 g

CHAPTER 7 Geometry

EX 7A Symmetry and reflections

p272

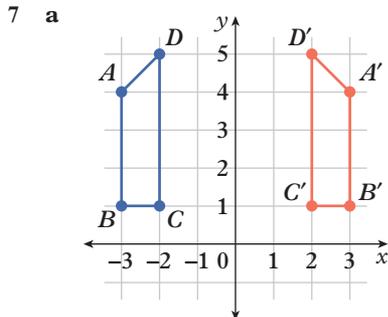
- 1 a Two b One c Zero
 d Zero e Four f Six
 g One h Zero i Two

2 a **A B C D E F G H I J K L M**
N O P Q R S T U V W X Y Z

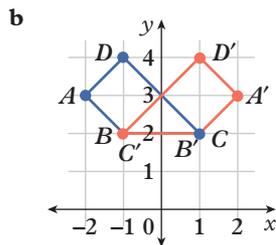
Note: a circular O has infinitely many lines of symmetry.

b E, G, J, L, N, P, Q, R, S, Z c H, I, O, X

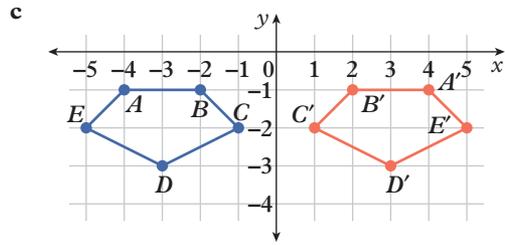
- 3 a Five b Two c Two
 d Five e Four f Two
 g One h Two i Two
- 4 H: 2, I: 2, N: 2, O: infinite, S: 2, X: 2, Z: 2
- 5 $A'(-1, 1)$, $B'(-3, 2)$, $C'(2, 3)$, $D'(2, -1)$, $E'(-2, -2)$, $F'(3, 2)$
- 6 $A'(1, -1)$, $B'(3, -2)$, $C'(-2, -3)$, $D'(-2, 1)$, $E'(2, 2)$, $F'(-3, -2)$



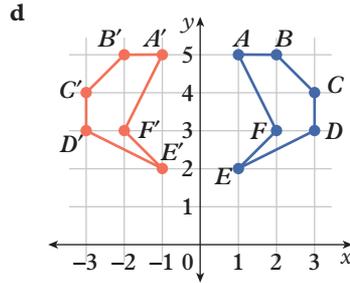
The coordinates of the image are $A'(3, 4)$, $B'(3, 1)$, $C'(2, 1)$, $D'(2, 5)$ 68 m^2 .



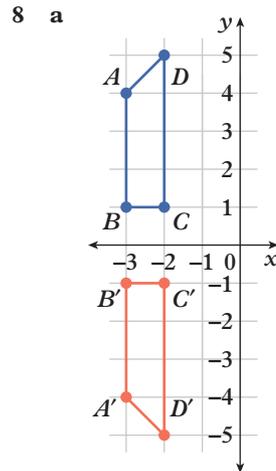
The coordinates of the image are $A'(2, 3)$, $B'(1, 2)$, $C'(-1, 2)$, $D'(1, 4)$.



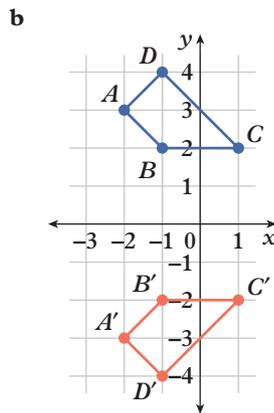
The coordinates of the image are $A'(4, -1)$, $B'(2, -1)$, $C'(1, -2)$, $D'(3, -3)$, $E'(5, -2)$.



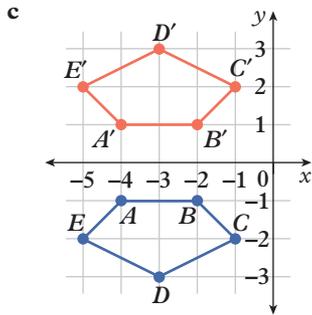
The coordinates of the image are $A'(-1, 5)$, $B'(-2, 5)$, $C'(-3, 4)$, $D'(-3, 3)$, $E'(-1, 2)$, $F'(-2, 3)$.



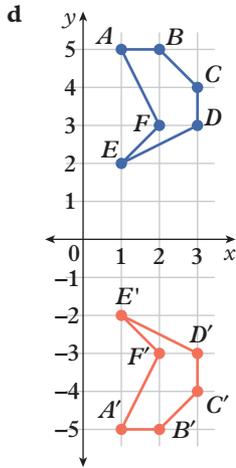
The coordinates of the image are $A'(-3, -4)$, $B'(-3, -1)$, $C'(-2, -1)$, $D'(-2, -5)$.



The coordinates of the image are $A'(-2, -3)$, $B'(-1, -2)$, $C'(1, -2)$, $D'(-1, -4)$.



The coordinates of the image are $A'(-4, 1)$, $B'(-2, 1)$, $C'(-1, 2)$, $D'(-3, 3)$, $E'(-5, 2)$.



The coordinates of the image are $A'(1, -5)$, $B'(2, -5)$, $C'(3, -4)$, $D'(3, -3)$, $E'(1, -2)$, $F'(2, -3)$.

9 a $x = 3$ **b** $x = -2$ **c** $x = 1$ **d** $x = -2$

10 a Yes **b** Yes **c** Yes **d** No

e Yes **f** Yes

11 Every quadratic graph has a U-shape, which does not have rotational symmetry.

12 a Yes **b** Yes **c** Yes

d Yes **e** No **f** Yes

13 a $A'(-1, 4)$, $B'(2, 3)$ and $C'(-5, 1)$

b The x -coordinates have the same magnitude as the object, but the sign is different, whereas the y -coordinates are the same as for the object.

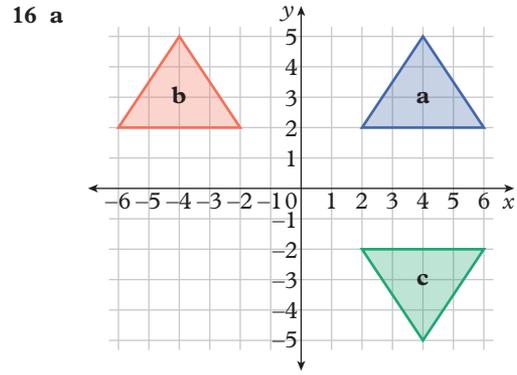
14 a $P'(0, -3)$, $Q'(2, 1)$ and $C'(-4, 3)$

b The x -coordinates are the same as for the object, whereas the y -coordinates have the same magnitude as the object's, but the sign is different.

15 a 3

b 6

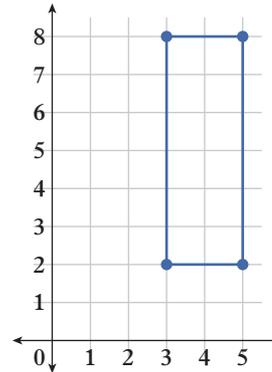
c The number of lines of symmetry remains at 6 as the snowflake gets more sides; this is because the changes happen within each one of the six sections of the snowflake.



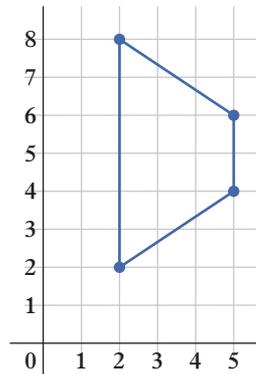
b $(-2, 2)$, $(-6, 2)$, $(-4, 5)$

c $(2, -2)$, $(6, -2)$, $(4, -5)$

17 a, b The new coordinates are $(3, 2)$.



18 a, b The new coordinates are $(5, 6)$.



19 a $(1, -1)$, $(2, -1)$, $(1, -2)$

b $(-1, 1)$, $(-2, 1)$, $(-2, 2)$, $(-1, 2)$

EX
p278

7B Translations and rotations

1 a 4 units right and 2 units down

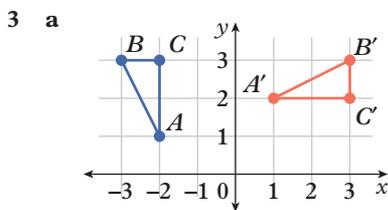
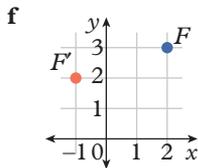
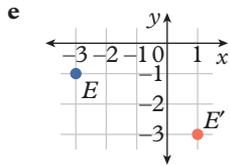
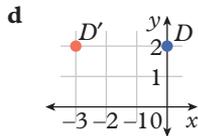
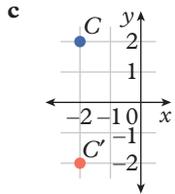
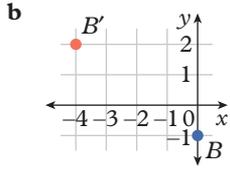
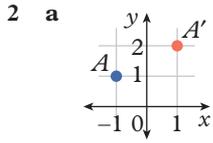
b 1 unit left and 3 units down

c 3 units left and 1 unit up

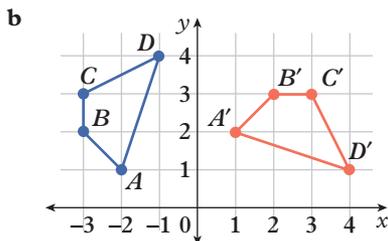
d 4 units right

e 4 units left and 4 units down

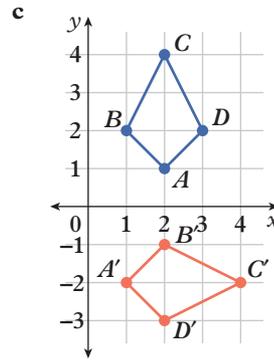
f 4 units right and 1 unit up



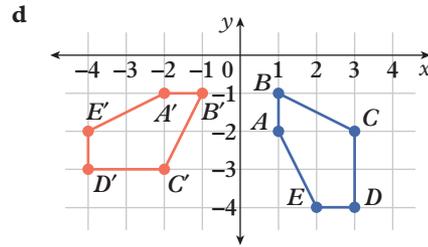
The coordinates of the image are $A'(1, 2)$, $B'(3, 3)$, $C'(3, 2)$.



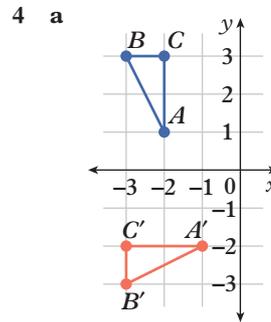
The coordinates of the image are $A'(1, 2)$, $B'(2, 3)$, $C'(3, 3)$, $D'(4, 1)$.



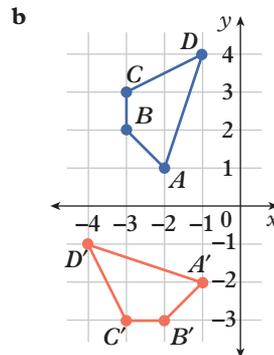
The coordinates of the image are $A'(1, -2)$, $B'(2, -1)$, $C'(4, -2)$, $D'(2, -3)$.



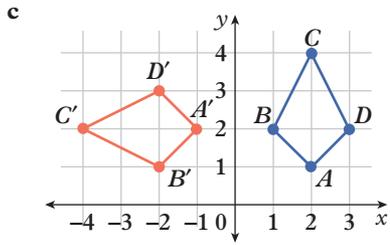
The coordinates of the image are $A'(-2, -1)$, $B'(-1, -1)$, $C'(-2, -3)$, $D'(-4, -3)$, $E'(-4, -2)$.



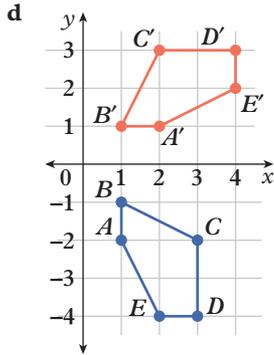
The coordinates of the image are $A'(-1, -2)$, $B'(-3, -3)$, $C'(-3, -2)$.



The coordinates of the image are $A'(-1, -2)$, $B'(-2, -3)$, $C'(-3, -3)$, $D'(-4, -1)$.

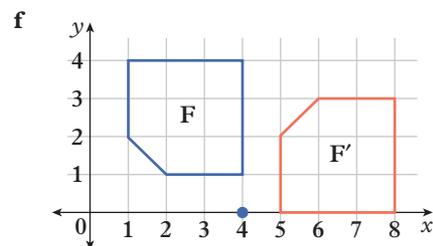
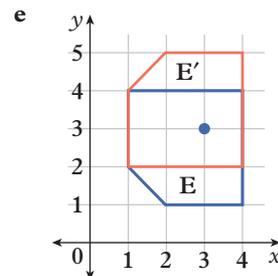
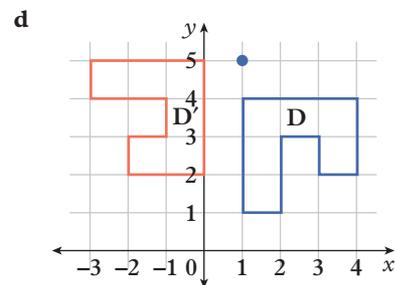
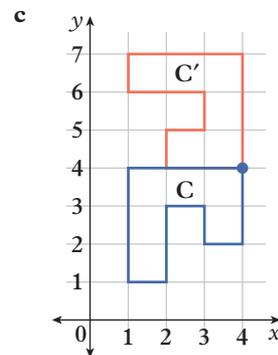
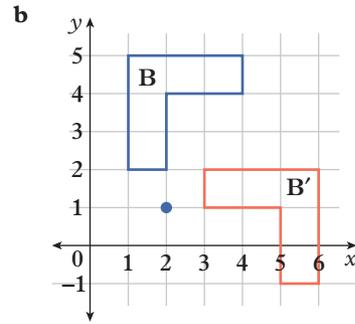
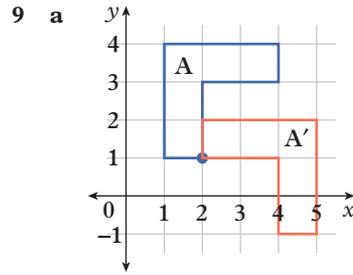


The coordinates of the image are $A'(-1, 2)$, $B'(-2, 1)$, $C'(-4, 2)$, $D'(-2, 3)$.

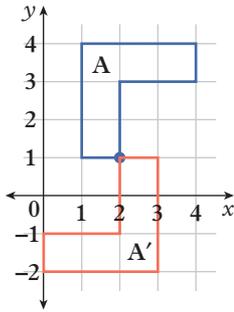


The coordinates of the image are $A'(2, 1)$, $B'(1, 1)$, $C'(2, 3)$, $D'(4, 3)$, $E'(4, 2)$.

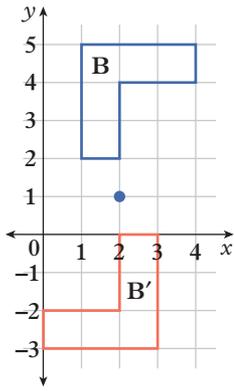
- 5**
- a** $(-5, -3)$
 - b** $(-3, 5)$
 - c** $(-10, 0)$
 - d** $(2, -2)$
 - e** $(-7, -1)$
 - f** $(-3, -7)$
- 6**
- a** 3 units to the right and 4 units up
 - b** 3 units to the right and 1 unit up
 - c** 4 units to the left and 3 units down
 - d** 4 units to the right and 2 units down
 - e** 2 units to the right and 2 units down
 - f** 5 units right and 6 units up
- 7**
- a** 5 units right and 3 units up
 - b** 1 unit right and 5 units down
 - c** 3 units left and 10 units down
 - d** 13 units right and 6 units up
 - e** 4 units right
 - f** 8 units left and 9 units up
- 8**
- a** F
 - b** E
 - c**
 - i** 2 units left and 3 units up
 - ii** 3 units left and 3 units down
 - iii** 6 units right and 3 units up
 - iv** 1 unit right and 5 units down
 - v** 3 units right and 1 unit up
 - vi** 2 units left and 1 unit up



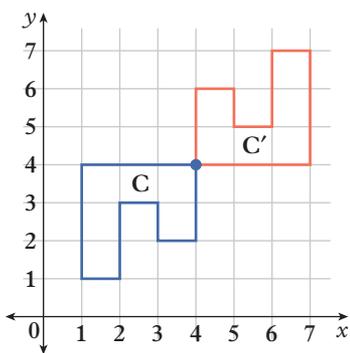
10 a



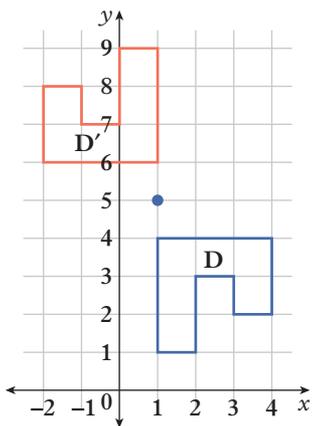
b



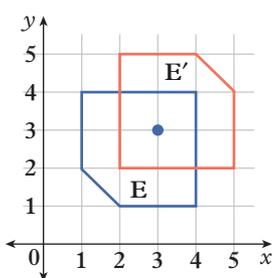
c



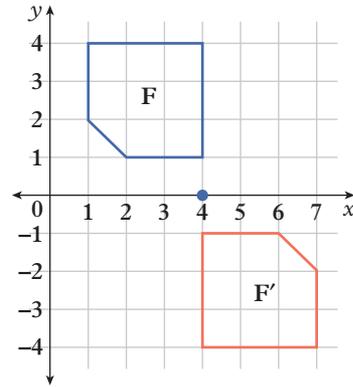
d



e



f



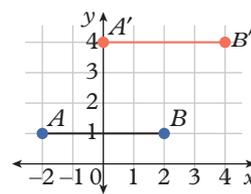
11 a 2 units right and 1 unit up

b 3 units left and 1 unit up

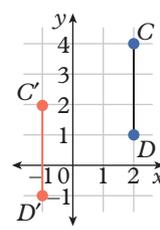
c 1 unit left and 2 units down

d 3 units right and 1 unit down

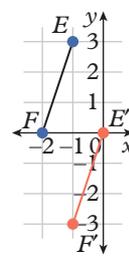
12 a



b



c



13 a 4 units left and 1 unit down

b 4 units down

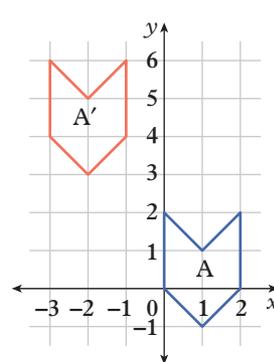
c 2 units right and 2 units down

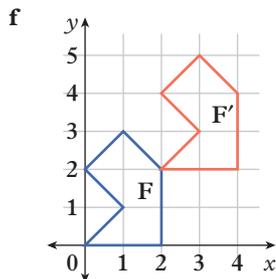
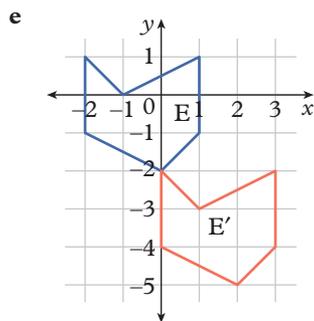
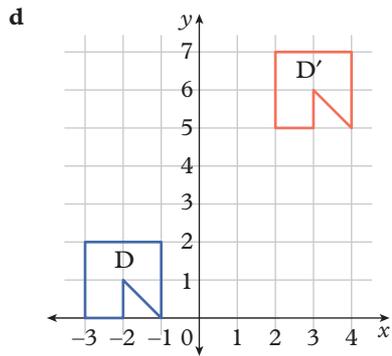
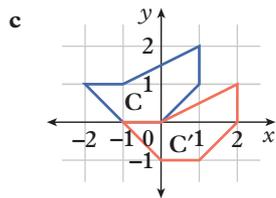
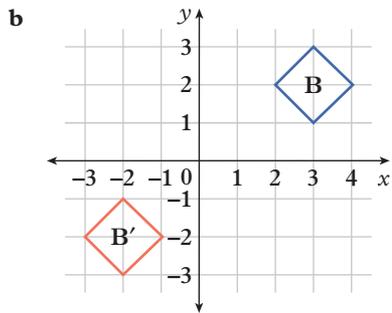
d 1 unit left and 3 units down

e 4 units left and 2 units up

f 3 units right and 2 units up

14 a



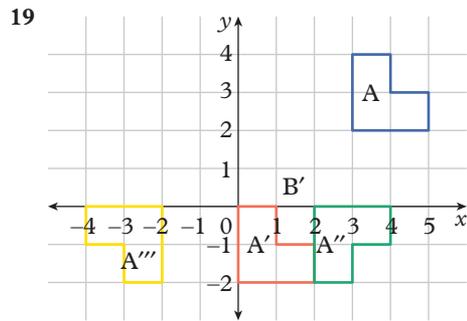


- 15 **a** 180°
b 90°
c 270°

16 **D**

17 Sample answer: Turning 90° clockwise results in the same position as turning 270° anticlockwise because there are 360° in a full circle and $270^\circ = 360^\circ - 90^\circ$.

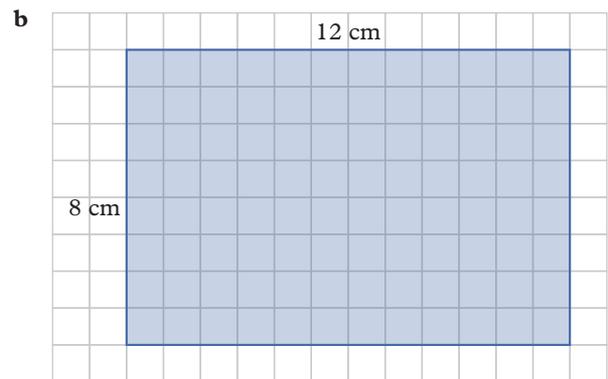
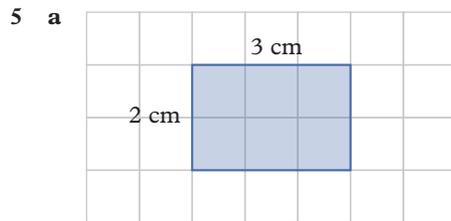
18 $(-2, 8), (-4, 10), (-4, 6), (-6, 8)$



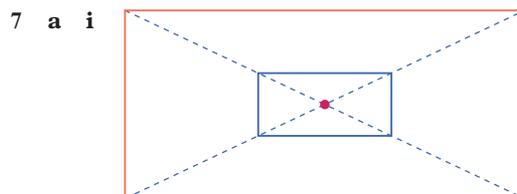
- 20 **a** rotation 90° clockwise around the origin followed by a translation 1 unit down
b rotation 270° clockwise around the origin followed by a translation 7 units right

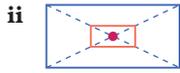
EX p286 **7C Dilations and similar figures**

- 1 **a** enlargement **b** reduction **c** enlargement
d enlargement **e** reduction **f** reduction
- 2 **a** 2 **b** $\frac{1}{4}$ **c** $\frac{1}{3}$
d 5 **e** $\frac{1}{5}$ **f** 2
- 3 **a** 3 **b** 2 **c** $\frac{1}{4}$ **d** $\frac{1}{3}$
- 4 **a** $x = 2$ cm **b** $x = 6$ cm **c** $x = 9$ cm
d $x = 20$ cm **e** $x = 10.5$ cm **f** $x = 5$ cm

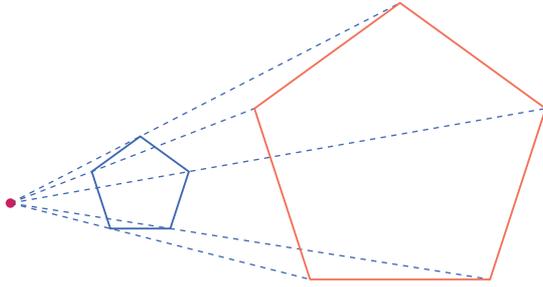


- 6 **a** $x = 6$ cm, $y = 38^\circ$
b $m = 71^\circ, n = 27$ cm
c $p = 49^\circ, q = 7$ cm
d $v = 1.9$ cm, $w = 18^\circ$

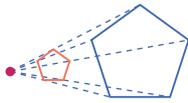




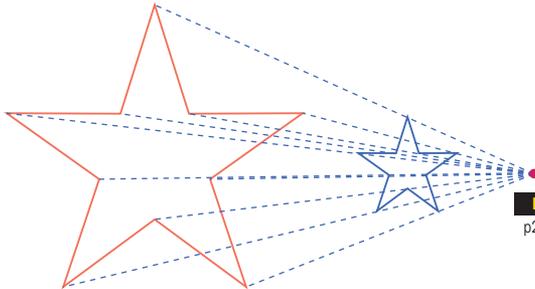
b i



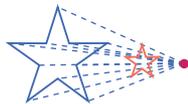
ii



c i



ii



EX
p294

7D Area and volume scale factors

1

Length scale factor	Area scale factor	Volume scale factor
2	4	8
$\frac{3}{4}$	$\frac{9}{16}$	$\frac{27}{64}$
4	16	64
$\frac{6}{5}$	$\frac{36}{25}$	$\frac{216}{125}$
$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$

- 2 a 224 mm² b 2.5 cm² c 12.57 cm²
 3 a 7.5 cm³ b 35342 mm³ c 2673 cm³
 4 a 2.25 b 3.375 c 16.875 cm³
 5 a SA = 150 mm², V = 125 mm³
 b 1350 mm²
 c 3375 mm³
 6 a 7 b 5 c 10
 d 8 e 2 f 20
 7 a SA = 504π cm², V = 1296π cm³
 b 56π cm²
 c 48π cm³
 8 a 7.2 cm b 111 cm²
 9 12 m²
 10 1 : 40 000
 11 1.5
 12 $\sqrt[3]{2}$
 13 a 3
 b 9
 c i 4.52 mm²
 ii 40.72 mm²
 iii 36.19 mm²

- 8 7.84 m by 3.6 m
 9 a 3.6 m by 2.76 m
 b 4.2 m by 3.36 m
 c 4.56 m by 2.64 m
 d 5.04 m by 4.86 m
 10 a similar b not similar c not similar
 d similar e not similar f similar
 11 0.0026 mm
 12 All circles and squares are exactly the same shape, whereas rectangles can vary in shape.
 13 Finn is correct because $\frac{2}{3}$ is the same as $2 \times \frac{1}{3}$.
 14 No
 15 a Size of plane is reduced by a scale factor of 100.
 b 34 cm c 27 m
 16 a 4 m b 3.75 cm
 c If all proportions are in same ratio, yes; however, it would be very difficult to manufacture a model car with every aspect in the same ratio.
 17 a 20 cm b 9750 m or 9.75 km
 18 7 μm
 19 a A regular hexagon has all side lengths equal and all angles equal (120°); an irregular hexagon does not.

- b Because all angles will be the same (they are for every regular hexagon) and all side lengths must be equal, any regular hexagon is similar to any other regular hexagon.
 20 a $\frac{1}{200}$ b $\frac{2}{375}$ c $\frac{15}{16}$ d $\frac{16}{15}$
 e The numerators and denominators are swapped because in part c the plane is the original, but in part d the case is the original.
 21 a i Looking at all paper sizes (from A0 to A8), the sizes are almost all similar (scale factor around 0.71), though rounding limits exact dilations to even numbers and odd numbers.
 ii similar relationship in B-sized paper
 iii similar relationship in C-sized paper
 iv B series covers sizes that are not found in the A series, so they are not similar. The C series sizes are meant to be similar to A sizes, but there are some slight differences due to rounding errors.
 b 1.41 is approximately equal to $\sqrt{2}$. Each larger paper size is double the area of the previous paper size.

- 14 It would need to double to 10 km.
 15 4
 16 a 675 g b 180 min
 17 The area scale factor is the square of the length scale factor only for similar objects. $\triangle ABD$ and $\triangle ADC$ are not similar. The bases have ratio 1 : 2, but the heights of the triangles are the same. So their area ratio is 1 : 2.
 18 a 9 : 4 b 4 : 5
 19 a $SA = 400\pi \text{ cm}^2$, Volume = $\frac{4000}{3}\pi \text{ cm}^3$
 b 5 cm
 c $100\pi \text{ cm}^2$
 d $\frac{1}{4}$. The area scale factor is the square of the radius scale factor.
 e $\frac{500}{3}\pi \text{ cm}^3$
 f $\frac{1}{8}$. The area scale factor is the cube of the radius scale factor.

EX 7E Congruence

p299

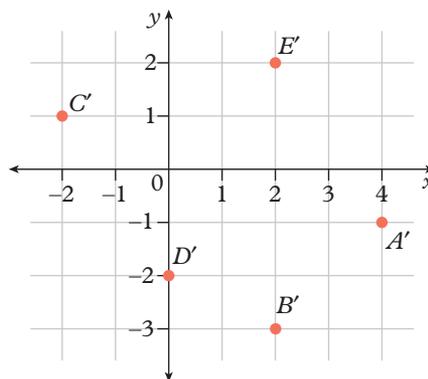
- 1 a yes b no c no d yes
 2 B and W , C and S , E and N , G and J , H and K , L and X , Q and T
 3 a $I\check{J}$ and NL , $\check{J}K$ and LM , KI and MN
 b AD and FG , DC and GH , CB and HE , BA and EF
 c $\check{J}K$ and TS , KL and SR , LM and RQ , MN and QP , NO and PU , $O\check{J}$ and UT
 d AB and MN , BC and NO , CD and OP , DE and PQ , EF and $Q\check{J}$, FG and $\check{J}K$, GH and KL , HA and LM
 4 a $I\check{J}K \cong NLM$
 b $ABCD \cong FEHG$
 c $\check{J}KLMNO \cong TSRQPU$
 d $ABCDEFGH \cong MNOPQ\check{J}KL$
 5 a A and F ; B and G ; C and H ; D and E
 b A and R ; B and P ; C and Q
 c A and H ; B and I ; C and \check{J} ; D and K ; E and L ; F and G
 d A and M ; B and N ; C and O ; D and P ; E and I ; F and \check{J} ; G and K ; H and L
 6 B
 7 a congruent b not congruent
 8 a not congruent; corresponding sides not equal length
 b not congruent; corresponding angles equal but corresponding sides not equal
 9 a congruent; corresponding angles and corresponding side lengths are equal
 b not congruent; corresponding angles are not equal
 10 a $p = 4 \text{ cm}$ b $q = 2.8 \text{ cm}$
 11 a $x = 48^\circ$ b $y = 45^\circ$
 12 a True; all angles in squares are 90° so the corresponding sides and angles are equal.
 b True; all angles in rectangles are 90° so the corresponding sides and angles are equal.

- c False; the angles in triangles with the same base length and height are not always equal.
 d False; the angles in parallelograms with the same base length and height are not always equal.
 e True; all circles with the same radii are the same shape and size.
 13 Rotating a square, rectangle and parallelogram by 180° will give an identical shape in the same orientation; therefore, there is more than one possible set of matching sides and vertices for these shapes.
 14 Squares and triangles
 15 Triangles, squares and hexagons. The internal angles of these polygons can be multiplied to give 360° .
 16 a i Translate 4 units right and 1 unit down.
 ii not congruent
 b i Rotate 90° clockwise about the origin.
 ii congruent
 c i Reflect in the y -axis.
 ii not congruent
 d i Rotate 90° clockwise about (3, 1).
 ii congruent
 17 a Rotate 90° anticlockwise about the origin, then translate 6 units right and 5 units up.
 b Reflect in the y -axis, then translate 8 units down.

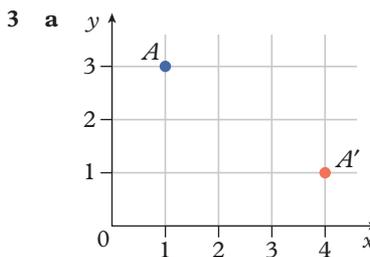
EX Checkpoint

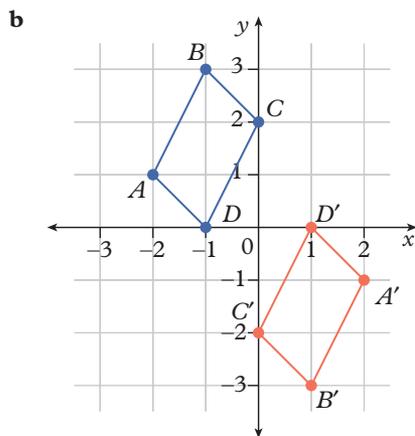
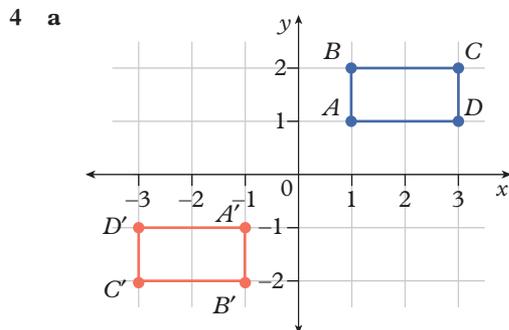
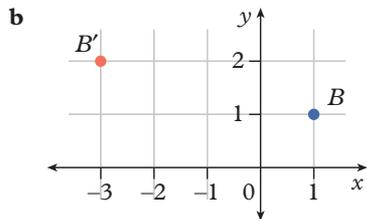
p304

- 1 $A'(4, -1)$, $B'(2, -3)$, $C'(-2, 1)$, $D'(0, -2)$, $E'(2, 2)$



- 2 a i Yes
 ii No
 b i Yes
 ii Yes





- 5 **a** 3 **b** 1.5
 6 **a** $x = 20$ cm, $y = 66^\circ$ **b** $p = 17.5$ cm, $q = 37^\circ$
 7 **a** 1.11 cm² **b** 113.10 cm²
 8 $16\,200$ cm²
 9 C, G and J; H and I
 10 $ABCD \cong GHIF$

EX p309 **7F Congruent triangles**

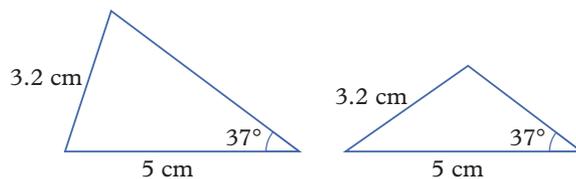
- 1 **a** congruent **b** not congruent
c not congruent
 2 **a** congruent **b** not congruent
c not congruent
 3 **a** not congruent **b** congruent
c not congruent
 4 **a** If the right angle is between the two shorter sides, it becomes the condition SAS.
b SSS
c SSS is a condition for congruence.
 5 **a** not congruent **b** congruent
c congruent **d** not congruent
 6 **a** congruent, SAS (7 cm, 77° , 9 cm)
b not congruent, ASA **c** not congruent, ASA

- d** congruent, ASA (54° , 13 cm, 87°)
e not congruent, RHS
f congruent, SSS (9 cm, 13 cm, 14 cm)

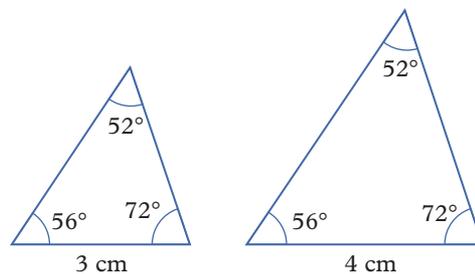
- 7 **a** SSS
b $\angle ABC = 76^\circ$ and $\angle BAC = 45^\circ$
 8 **a i** $v = 17$ cm, $w = 38^\circ$, $x = 62^\circ$, $y = 80^\circ$, $z = 80^\circ$
ii AAS and angle sum of a triangle
b i $a = 48^\circ$, $b = 25^\circ$, $c = 4$ cm, $d = 25^\circ$, $e = 107^\circ$, $f = 7$ cm
ii AAS and angle sum of a triangle
c i $j = 25^\circ$, $k = 7$ cm, $l = 3$ cm, $m = 5$ cm, $n = 115^\circ$
ii angle sum of a triangle and SAS
d i $g = 35^\circ$, $h = 35^\circ$, $i = 110^\circ$, $j = 35^\circ$
ii isosceles triangle facts and SAS
 9 **a** angle sum of a triangle (three angles sum to 180°)
b Both angle pairs will require the same value to make the angle sum equal 180° .
c Pair 1 supports ASA as the corresponding sides between the angles are equal, whereas the equal sides in Pair 2 are not in corresponding positions.
d Once you find the third angle, you have the requirements for AAS (two angles and one corresponding side).

10 Sample answer:

This pair of triangles meets the condition SSA, but the triangles are not congruent.

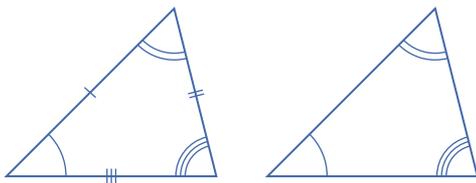


This pair of triangles meets the condition AAA, but the triangles are not congruent.



- 11 **a i** No, because not all three sides are known on both triangles.
ii SAS
b i No, because the length of the hypotenuse is not known.
ii SAS or AAS
c i Yes, because both are isosceles triangles, so all angles can be determined and there is a pair of congruent side lengths.

- 12 a i $\triangle ABC \equiv \triangle JKL$ and $\triangle DEF \equiv \triangle GHI$
 ii $a = 5.6, b = 60^\circ, c = 53^\circ, d = 53^\circ, e = 74^\circ, f = 5, g = 69^\circ, h = 51^\circ$
- b i $\triangle ABC \equiv \triangle DEF \equiv \triangle GHI \equiv \triangle JKL$
 ii $a = 34^\circ, b = 7, c = 4, d = 95^\circ, e = 5, f = 34^\circ, g = 51^\circ, h = 5, i = 4, j = 51^\circ$
- 13 a i congruent; angle sum of a triangle is 180° , ASA
 ii $n = 40^\circ, p = 12 \text{ cm}, q = 66^\circ, r = 17 \text{ cm}$
- b i congruent; isosceles triangles, ASA
 ii $a = 6 \text{ cm}, b = 72^\circ, c = 36^\circ, d = 10 \text{ cm}, e = 72^\circ$
- c i congruent; isosceles triangles, ASA
 ii $d = 20^\circ, e = 25 \text{ cm}, f = 140^\circ, g = 48 \text{ cm}$
- 14 9 values: all 6 angles and 3 different side lengths placed on either triangle

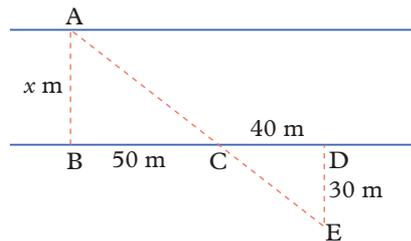


EX 7G Similar triangles

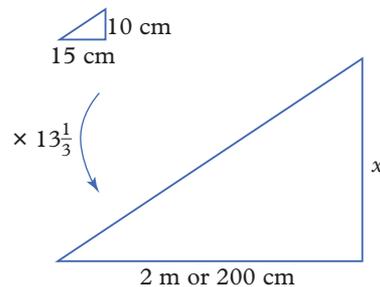
p316

- 1 a $a = 9 \text{ cm}, b = 4 \text{ cm}$
 b $c = 44 \text{ cm}, d = 7 \text{ cm}$
 c $e = 20 \text{ cm}, f = 7.5 \text{ cm}$
 d $m = 19 \text{ cm}, n = 45 \text{ cm}$
 e $x = 12 \text{ cm}, y = 6.5 \text{ cm}$
 f $x = 52 \text{ cm}, y = 20 \text{ cm}$
- 2 a SAS b SSS c RHS
 d AAA e SAS f AAA
- 3 a SSS b AAA c SAS
 d SSS or SAS
- 4 a Yes; SAS
 b No; all corresponding side pairs are not in the same ratio.
 c No; corresponding angles are not equal.
 d Yes; SSS
 e No; SSA is not a condition for similarity.
 f Yes; AAA
- 5 a similar b similar c not similar
 d similar e similar f not similar
- 6 a not similar b similar c similar
 d similar e not similar f similar
- 7 With all the similarity conditions, if one pair of corresponding sides is equal then the triangles are congruent.
- 8 All equilateral triangles have three equal angles (60°), meeting similarity condition AAA.

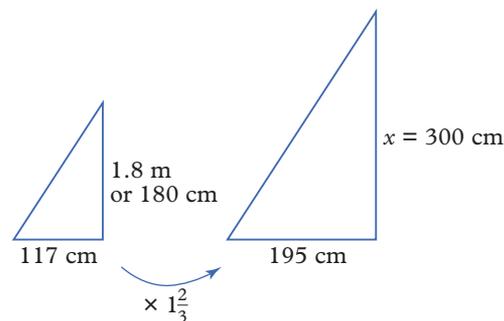
- 9 The third angle is equal to 180° minus the sum of the other two angles. Therefore, since the two triangles have the same two angles, their third angles are also equal.
- 10 a Using knowledge of alternate angles within parallel lines ($\angle ABC = \angle EDC$ and $\angle BAC = \angle DEC$) and vertically opposite angles ($\angle ACB = \angle ECD$), these triangles have three corresponding equal pairs of angles and hence are similar.
 b scale factor = 3
 c $a = 2.5 \text{ cm}, b = 6 \text{ cm}$
- 11 a similar b not similar c similar
 d not similar
- 12 $x = 6.54 \text{ m}$
- 13 a 2.5 m b 2.72 m tall
- 14 $x = 37.5 \text{ m}$



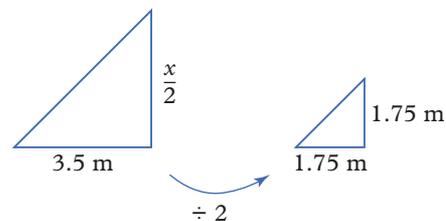
- 15 4 cm
 16 a 133.33 cm



- b Ring is 5 cm too short, at 300 cm tall.



- c 7 m



- 17 Sean's house is 3.1 m tall; Tania's house is 2.7 m tall. Sean's house is taller.

18 You need to have the second similar triangle to compare house heights and shadows because the measurements were taken at different times of day so the lengths of the shadows would change.

19 a $\frac{DE}{AB} = \frac{7}{5} = 1.4$
 $\frac{EF}{BC} = \frac{8.4}{6} = 1.4$
 $\frac{FD}{CA} = \frac{11.2}{8} = 1.4$
 $\frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA}$

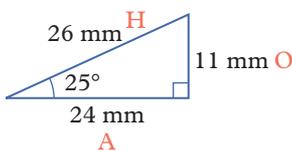
Corresponding sides are in the same ratio
 $\therefore \triangle ABC \sim \triangle DEF$ by SSS

b $\frac{DE}{AB} = \frac{7}{5} = 1.4$
 $\frac{FD}{CA} = \frac{11.2}{8} = 1.4$
 $\angle EDF = 41^\circ$
 $\angle BAC = 41^\circ$
 $\frac{DE}{AB} = \frac{FD}{CA}$

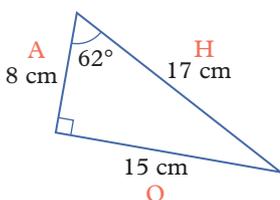
Corresponding sides are in the same ratio.
 $\angle EDF = \angle BAC$
 Corresponding angles are equal
 $\therefore \triangle ABC \sim \triangle DEF$ by SAS

EX p323 **7H Trigonometric ratios**

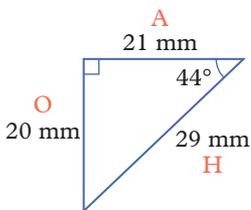
1 a



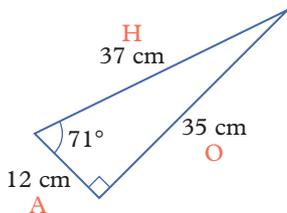
b



c



d



2 a $\sin(\theta) = \frac{12}{17}$

c $\tan(\theta) = \frac{24}{33} = \frac{8}{11}$

e $\sin(\theta) = \frac{15}{24} = \frac{5}{8}$

b $\cos(\theta) = \frac{8}{15}$

d $\cos(\theta) = \frac{35}{40} = \frac{7}{8}$

f $\tan(\theta) = \frac{13.1}{6.6} = \frac{131}{66}$

3 a $\frac{1}{2}$ b $\frac{1}{2}$ c $\frac{1}{2}$ d $\frac{1}{2}$

4 a $\frac{6}{7}$ b $\frac{6}{7}$ c $\frac{6}{7}$ d $\frac{6}{7}$

5 a $\frac{4}{3}$ b $\frac{4}{3}$ c $\frac{4}{3}$ d $\frac{4}{3}$

6 a 0.3420 b 0.5878 c 0.8090
 d 0.7265 e 1.9626 f 0.4540

7 a 2.052 b 17.54 c 0.05700
 d 1 e 0.7660 f 0.3640

8 a They are similar triangles.

b i, iii, iv, v

9 a i $\sin(\theta) = \frac{O}{H}$, $\cos(\theta) = \frac{A}{H}$, $\tan(\theta) = \frac{O}{A}$

ii $\sin(\theta) = \frac{O}{H}$, $\cos(\theta) = \frac{A}{H}$, $\tan(\theta) = \frac{O}{A}$

iii $\sin(\theta) = \frac{O}{H}$, $\cos(\theta) = \frac{A}{H}$, $\tan(\theta) = \frac{O}{A}$

b The values of the sine, cosine and tangent ratios in similar triangles are equal.

10 a 100 m b 148 m c 6.7 m d 5 m

11 a

θ	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$	$\frac{1}{\tan(\theta)}$
1	0.0175	0.9998	0.0175	57.2900
5	0.0872	0.9962	0.0875	11.4301
15	0.2588	0.9659	0.2679	3.7321
30	0.5000	0.8660	0.5774	1.7321
45	0.7071	0.7071	1.0000	1.0000
60	0.8660	0.5000	1.7321	0.5774
75	0.9659	0.2588	3.7321	0.2679
85	0.9962	0.0872	11.4301	0.0875
89	0.9998	0.0175	57.2900	0.0175

b Sine and tangent increase, and cosine and reciprocal tangent decrease.

c i $\sin(27^\circ) = \cos(63^\circ)$

ii $\sin(34^\circ) = \cos(56^\circ)$

iii $\tan(12^\circ) = \frac{1}{\tan(78^\circ)}$

iv $\tan(43^\circ) = \frac{1}{\tan(47^\circ)}$

d The cosine of an angle is the sine of the complementary angle. The sine of an angle is the cosine of the complementary angle.

e The reciprocal of the tangent of an angle is the tangent of the complementary angle.

12 a $a = 1$, $b = 1$, $c = \sqrt{2}$, $x = 45^\circ$, $y = 90^\circ$, $z = 45^\circ$

b $\sin(45^\circ) = \frac{1}{\sqrt{2}}$, $\cos(45^\circ) = \frac{1}{\sqrt{2}}$, $\tan(45^\circ) = 1$

c $a = 1$, $b = \sqrt{3}$, $c = 2$, $x = 30^\circ$, $y = 90^\circ$, $z = 60^\circ$

d $\sin(30^\circ) = \frac{1}{2}$, $\cos(30^\circ) = \frac{\sqrt{3}}{2}$, $\tan(30^\circ) = \frac{1}{\sqrt{3}}$

$\sin(60^\circ) = \frac{\sqrt{3}}{2}$, $\cos(60^\circ) = \frac{1}{2}$, $\tan(60^\circ) = \sqrt{3}$

- e i $\frac{1}{2}$ ii $\frac{3}{4}$ iii $\frac{1}{3}$ iv $\frac{3}{4}$
 v $\frac{1}{4}$ vi $\frac{1}{4}$ vii 1 viii $\frac{1}{2}$

f Determine the square of the value of sine/cosine/tangent of $30^\circ/45^\circ/60^\circ$, then write the square root of the squared value.

- 13 a The values of sine and cosine of θ for $0^\circ < \theta < 90^\circ$ are the ratio of a shorter side to the longer hypotenuse. This means all values of sine and cosine can be considered as a smaller number divided by a larger number, which results in a value less than one.
 b Tangent is the ratio of the two shorter sides. The two shorter sides have no restriction on which is longer or shorter.
 c The tangent ratio is greater than one when the opposite side is longer than the adjacent side, which occurs when $45^\circ < \theta < 90^\circ$.

14 a

θ	$\tan(\theta)$	$\frac{\sin(\theta)}{\cos(\theta)}$
1	0.0175	0.0175
5	0.0875	0.0875
15	0.2679	0.2679
30	0.5774	0.5774
45	1	1
60	1.7321	1.7321
75	3.7321	3.7321
85	11.4301	11.4301
89	57.2900	57.2900

- b i $\frac{\sin(27^\circ)}{\cos(27^\circ)} = \tan(27^\circ)$
 ii $\frac{\sin(56^\circ)}{\cos(56^\circ)} = \tan(56^\circ)$
 iii $\frac{\sin(12^\circ)}{\cos(12^\circ)} = \tan(12^\circ)$
 iv $\frac{\sin(47^\circ)}{\cos(47^\circ)} = \tan(47^\circ)$
 c $\frac{\sin(\theta)}{\cos(\theta)} = \tan(\theta)$

- 15 a i $a = 6.40$ m ii $b = 6.32$ m
 b i $\sin(\theta) = 0.6247$, $\cos(\theta) = 0.7809$, $\tan(\theta) = 0.8$
 ii $\sin(\theta) = 0.4286$, $\cos(\theta) = 0.9035$, $\tan(\theta) = 0.4743$

- 16 a i $\frac{1}{2}$ ii $\frac{5}{3}$ iii $\frac{6}{5}$ iv 7
 b i $\frac{1}{2}$ ii $\frac{5}{3}$ iii $\frac{6}{5}$ iv 7

c The tangent of the angle from the horizontal is equal to the gradient of the hypotenuse.

- 17 a $x = \cos(31^\circ)$, $y = \sin(31^\circ)$
 b $x = \cos(54^\circ)$, $y = \sin(54^\circ)$
 c i The sine of an angle is the length of the altitude of a right-angled triangle with a hypotenuse of length 1.
 ii The cosine of an angle is the length of the base of a right-angled triangle with a hypotenuse of length 1.
 d i $a = 6.857$, $b = 4.120$
 ii $a = 0.5143$, $b = 0.3090$

18 a $y = \tan(31^\circ)$, $\frac{1}{z} = \cos(31^\circ)$ or $z = \frac{1}{\cos(31^\circ)}$

b i $a = 6.009$, $b = 11.67$

ii $a = 1.863$, $b = 3.617$

c $\frac{1}{x} = \tan(31^\circ)$ or $x = \frac{1}{\tan(31^\circ)}$, $\frac{1}{z} = \sin(31^\circ)$ or $z = \frac{1}{\sin(31^\circ)}$

d i $a = 8.321$, $b = 9.708$ ii $a = 12.32$, $b = 14.37$

19 a The person has not written the angle, θ . $\cos(\theta) = \frac{10}{11}$

b The person has written the reciprocal of the sine ratio. So, the fraction is upside down.

$$\sin(\theta) = \frac{4}{8} \quad \frac{1}{\sin(\theta)} = \frac{8}{4}$$

$$\sin(\theta) = \frac{1}{2} \quad \frac{1}{\sin(\theta)} = 2$$

c The ratio between the two given sides is the tangent ratio; however, the person has written a ratio using 'cos' instead of 'tan'. $\tan(\theta) = \frac{6}{5}$

d The person has written the reciprocal of the tangent ratio. So, the fraction is upside down. $\tan(\theta) = \frac{4}{7}$

20 a i $\tan(27^\circ) = \frac{1}{2}$ ii $\tan(117^\circ) = -2$

iii $\tan(194^\circ) = \frac{1}{4}$ iv $\tan(326^\circ) = -\frac{2}{3}$

b i 1 ii -1 iii 1 iv -1

c All four angles (45° , 135° , 225° , and 315°) are 45° rotations from the horizontal, so the gradient of the corresponding line is either 1 or -1 for a line inclined at a 45° angle depending on which way it is rotated.

d i The gradient of a horizontal line is zero. Therefore, the tangent of 0° is zero.

ii The gradient of a vertical line is undefined. Therefore, the tangent of 90° is undefined.

21 a i $\frac{1}{2}$ ii $\frac{1}{2}$ iii $-\frac{1}{2}$ iv $-\frac{1}{2}$

v $\frac{1}{2}$ vi $-\frac{1}{2}$ vii $-\frac{1}{2}$ viii $\frac{1}{2}$

b i sine: positive, cosine: negative

ii sine: negative, cosine: positive

iii sine: negative, cosine: negative

EX 71 Using trigonometry to find side lengths

p333

1 a $x = 2.11$ b $x = 11.83$

c $x = 2.11$ d $x = 11.83$

e $x = 6.62$ f $x = 18.28$

g $x = 3.89$ h $x = 3.89$

2 a $\cos(\theta) = \frac{A}{H}$, $\cos(64^\circ) = \frac{k}{35}$

b $\sin(\theta) = \frac{O}{H}$, $\sin(49^\circ) = \frac{k}{12}$

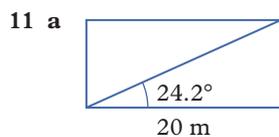
c $\tan(\theta) = \frac{O}{A}$, $\tan(26^\circ) = \frac{k}{50}$

d $\cos(\theta) = \frac{A}{H}$, $\cos(51^\circ) = \frac{k}{17}$

e $\sin(\theta) = \frac{O}{H}$, $\sin(37^\circ 24') = \frac{k}{24}$

f $\tan(\theta) = \frac{O}{A}$, $\tan(21^\circ 39') = \frac{k}{16}$

- 3 a 15.34 mm b 9.06 m c 24.39 cm
 d 10.70 m e 14.58 mm f 6.35 m
- 4 a 31.84 cm b 25.86 mm c 2.78 m
 d 22.65 cm e 11.11 m f 49.32 mm
 g 62.70 cm h 132.70 m i 5.22 cm
- 5 a 118.31 cm b 3.72 m c 38.92 mm
 d 11.02 cm e 46.14 cm f 4.72 m
- 6 a 4.44 cm b 8.31 cm c 10.39 m
 d 78.15 cm
- 7 a 33.64 m b 34.64 m
- 8 4.62 m
- 9 a 9.60 m b 16.00 m
- 10 a 1.93 m b 2.13 m



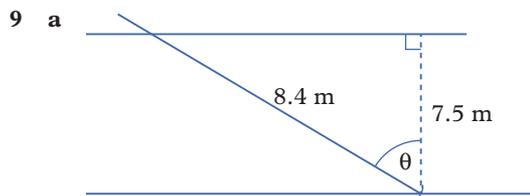
- b 9 m
 c 58 m
- 12 8.60 m
- 13 a One angle size (other than the right angle) and one side length
 b Two side lengths
- 14 a $a = 17.2$ cm b $b = 42.2$ mm c $c = 52.9$ m
 d $d = 32.2$ cm e $e = 29.9$ mm f $f = 52.0$ cm
- 15 a $a = 3.28$ cm, $b = 3.77$ cm, $x = 90^\circ$, $y = 41^\circ$, $z = 49^\circ$
 b $g = 27^\circ$, $h = 63^\circ$, $m = 2.94$ m, $n = 3.30$ m
- 16 a $f = 4.01$ cm, $g = 7.04$ cm
 b $a = 4.00$ cm, $b = 7.21$ cm
 c $m = 17.69$ cm, $n = 12.00$ cm
- 17 $r = 8.70$ cm and $t = 10.97$ cm
- 18 $p = 35.96$ cm, $q = 11.96$ cm
- 19 a $x = 5.01$ cm b $a = 7.09$ cm
- 20 a $x = 2.78$ cm b $y = 13.22$ cm
 c $z = 8.56$ cm d $b = 15.75$ cm

EX 7J Using trigonometry to find angles

p340

- 1 a 13° b 44° c 56° d 72°
 e 42° f 16° g 74° h 50°
- 2 a 20° b 36° c 33° d 25°
 e 8° f 1° g 72° h 45°
- 3 a 53° b 38° c 21° d 62°
 e 69° f 23° g 57° h 83°
- 4 a 50° b 42° c 67° d 54° e 31° f 46°
- 5 a $\theta = 138.6^\circ$ b $\theta = 13.6^\circ$
 c $\theta = 271.1^\circ$ d $\theta = 25.8^\circ$
- 6 a $36^\circ 52'$, $53^\circ 8'$ b $26^\circ 34'$, $63^\circ 26'$
 c $72^\circ 54'$, $17^\circ 6'$ d $25^\circ 40'$, $64^\circ 20'$
- 7 a 14° b 7.01 m
- 8 a 47° b 86°

EX
 p345



- b 27°
 c Larger. If the chain is longer, the angle with the vertical would be greater. If the chain is shorter, the angle is closer to the vertical, hence making a smaller angle size with the vertical.
- 10 a i 37° ii 37° iii 37°
 b $\theta = 37^\circ$
 c i Using trigonometry can be accurate to a required degree of accuracy but is difficult to perform without a calculator.
 ii It is an advantage to use a scale diagram to check the results from trigonometric calculations, but it is difficult to record measurements with sufficient accuracy.
- 11 a First, calculate the angle of the Sun's rays at the time of measurement by using the length of a metre ruler's shadow. Then use the length of the tree's shadow as a measurement to calculate the height of the tree.
 b 37°
 c It refers to the angle of the Sun's rays at the time of measurement, both measurements being taken at the same time.
 d 11.71 m
 e By forming an equivalent ratio statement using the corresponding sides.

$$x : 1 \text{ m} = 15.46 \text{ m} : 1.32 \text{ m}$$

$$x = \frac{1 \text{ m} \times 15.46 \text{ m}}{1.32 \text{ m}}$$

$$= 11.71 \text{ m}$$

- 12 a i $28^\circ, 62^\circ$ ii $28^\circ, 62^\circ$ iii $28^\circ, 62^\circ$
 b The two non-right angles are complementary, so $90^\circ - \theta$.
- 13 a $\theta = 123^\circ 41'$ b $\theta = 110^\circ 23'$ c $\theta = 78^\circ 39'$
- 14 a 30.0 m, 18.4° , 71.6°
 b 20.1 m, 42.6° , 47.4°
- 15 The corresponding angles in the bottom left of each triangle are approximately 64.707° and 64.652° , so the lines are very nearly parallel.
- 16 1548.4 cm^2
 17 $\theta = 38.4^\circ$

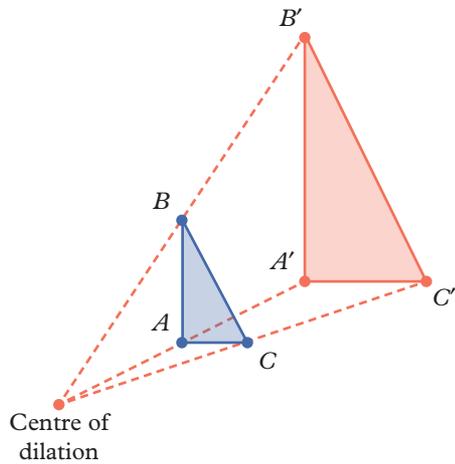
Chapter 7 review

Mathematical literacy review

- 1 Congruent shapes are exactly the same size and shape, whereas similar shapes are exactly the same shape, but do not have to be the same size.
- 2 The volume scale factor is the cube of the length scale factor.

3 60

- 4 **a** hypotenuse **b** reduction **c** tessellation
 5 **a** a transformation that preserves the original size and shape of the object being transformed, for example, a translation
b a transformation that produces an enlargement or a reduction of the original figure, for example:



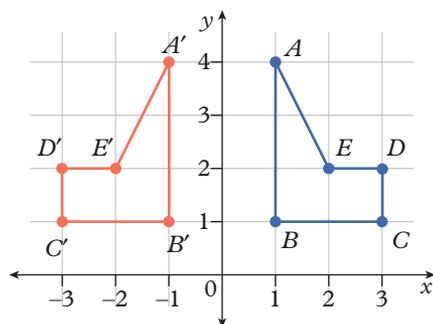
- c** the number of times a plane shape fits exactly onto itself when being rotated through 360° , for example, the order of rotational symmetry of a rectangle is 2
 6 **a** To reflect a shape in an axis of symmetry, start by reflecting a single point. This point is called the point of focus.
b The cosine of angle θ is the ratio of the lengths of the adjacent side to the hypotenuse.

Multiple choice

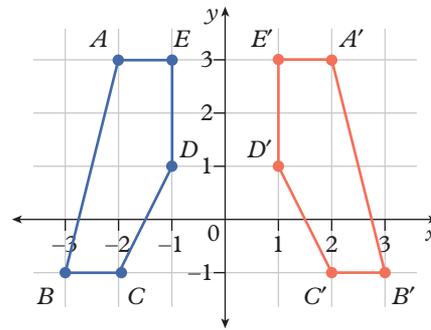
- 1 A 2 E 3 C 4 B 5 D
 6 B 7 C 8 D 9 A 10 D
 11 A 12 A 13 B 14 C

Short answer

1 **a**

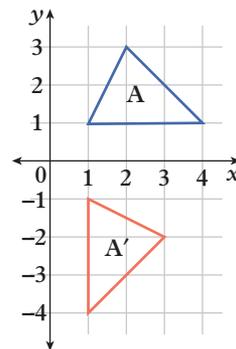


b

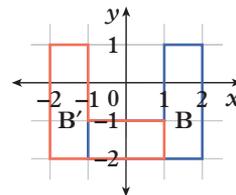


- 2 **a** 1 unit left, 2 units down
b 7 units up
c 5 units right, 1 unit up
d 4 units right, 6 units up
e 5 units left, 3 units up
f 9 units left, 1 unit down

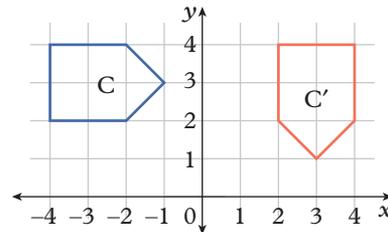
3 **a**



b



c



- 4 **a** enlargement; scale factor = 2
b reduction; scale factor = $\frac{1}{4}$
 5 **a** 1590.43 cm^2
b 15.33 cm^2
 6 **a** 1260 mm^3
b 45 mm by 31.5 mm by 24 mm
c 34020 mm^3
 7 **a** congruent
b congruent
c not congruent
 8 A and E
 9 A and E are congruent due to the condition ASA (AAS).
 10 $a = 9.2 \text{ cm}$, $b = 4 \text{ cm}$

11 Yes, by SAS

12 $\sin(\theta) = \frac{55}{73}$, $\cos(\theta) = \frac{48}{73}$, $\tan(\theta) = \frac{55}{48}$

13 a 5.44 cm b 19.43 m

c 28.31 m d 4.15 cm

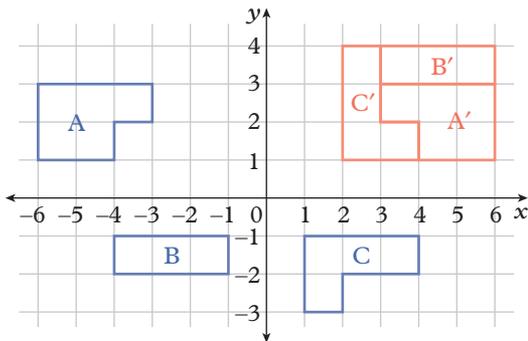
14 a 35° b 60°

c $\theta = 68^\circ$ d $\theta = 29^\circ$

15 $\theta = 47.5^\circ$

Analysis

1 a



b (2, 1), (2, 4), (6, 4), (6, 1)

c (4, 2), (4, 8), (12, 8), (12, 2)

d 4

e 48 square units

2 a The triangles are similar but not congruent. The corresponding angles are the same, but the side lengths are different. The triangles are similar by AAA.

b 2.5

c 15 m

3 a sine b 53.1° c 90 m

d 645 m

CHAPTER 8 Statistics and probability

EX

p356

8A Five-number summary and interquartile range

1 a 16 b 45 c 87 d 67

2 a 3, 3, 5, 7, 9 b 7, 12, 19, 46, 78

c 1, 3, 5, 7, 9 d 2, 7, 11, 19, 32

3 a 2, 4, 7, 9, 12

b 4, 13, 25.5, 46, 55

c 16, 20.5, 42, 72.5, 81

d 120, 177.5, 452.5, 755, 843

e 18, 19.5, 33, 42, 64

f 3, 8, 13.5, 19, 32

4 a 10 b 9 c 10 d 19

5 a i 12, 16, 32, 48, 86 ii 32

b i 1, 4, 6.5, 13, 23 ii 9

c i 98, 115.5, 146, 172, 189 ii 56.5

d i 8, 13.5, 19.5, 49.5, 62 ii 36

e i 5, 7.5, 12, 17, 34 ii 9.5

f i 99, 125, 164, 378, 465 ii 253

6 a i 0, 3, 5, 6, 7 ii 3

b i 0, 1, 3.5, 6, 7 ii 5

c i 0, 2, 3, 5, 7 ii 3

d i 9, 9, 10, 12, 15 ii 3

7 See the answers in question 6.

8 Range = 51; IQR = 23. The difference between the least number of cupcakes sold in the month and the most number of cupcakes sold in the month is 51. On half the days, the number of cupcakes sold falls within a range of 23 (from 56 to 79 cupcakes sold).

9 a Both sets have a range of 43.

b IQR of set A = 26, IQR of set B = 10

c because they have the same range

10 a 6

b minimum = 0, $Q_1 = 2$, Q_2 (median) = 4, $Q_3 = 5$, maximum = 6

c 3

11 a range = \$240, IQR = \$45

b because there is an outlier

c The IQR represents the middle 50% of scores; those scores between Q_1 and Q_3 . If the spread is even, the remaining 50% of scores will have a spread of the same size as the IQR.

d The IQR indicates a region within which the middle 50% of scores lie and the outlier will fall outside this range.

12 a If the IQR is small, 50% of scores are grouped closely around the median.

b If the IQR is small and the range is large, 50% of the scores are grouped closely around the median, while the 25% of scores lying below the lower quartile and the 25% of scores above the upper quartile are spread out.

c If the IQR is large, 50% of the scores are spread widely around the median.

d If the IQR is similar to the range, the range of the 25% of scores lying below the lower quartile plus the range of the 25% of scores above the upper quartile is the same as the range of the middle 50% of scores.

13 To calculate the five-number summary the raw data is required, which is not provided in a histogram that has been compiled using grouped data.

14 a i range = 17, IQR = 8.5

ii The data is spread fairly uniformly throughout the range.

b i range = 72, IQR = 30

ii The scores are packed more closely in the IQR. There appears to be an outlier of 11 in the lower quarter of the scores.

c i range = 73, IQR = 11

ii Scores are packed more closely in the IQR. There appears to be an outlier of 191 in the upper quarter of the scores.

- 15 a class A i 171.5 ii 27 iii 12
 class B i 172.5 ii 48 iii 12.5
- b The median and IQR of heights in the two classes is quite similar, with class B having a slightly larger median; however, the range of heights in class B is greater. Class B has the tallest and shortest members of the group, indicating it is a more diverse group.
- 16 a 50, 60, 70.5, 79, 83 b IQR = 19, range = 33
 c 9, 55, 68, 74, 83 d IQR = 19, range = 74
- e i The median decreased slightly from 70.5 to 68.
 ii The range increased significantly from 33 to 74.
 iii The interquartile range remained at 19.
- f Since outliers and extreme values can only be at the ends of a list of data, they are not close enough to the position of the quartiles to impact the interquartile range.

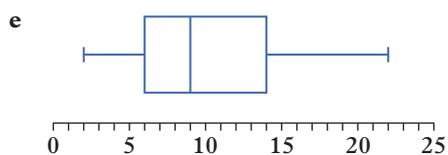
17 a 14 b 28

18 a 16 b $\frac{47}{12}$ c $\frac{5293}{4620}$ d $\frac{16889}{13104}$

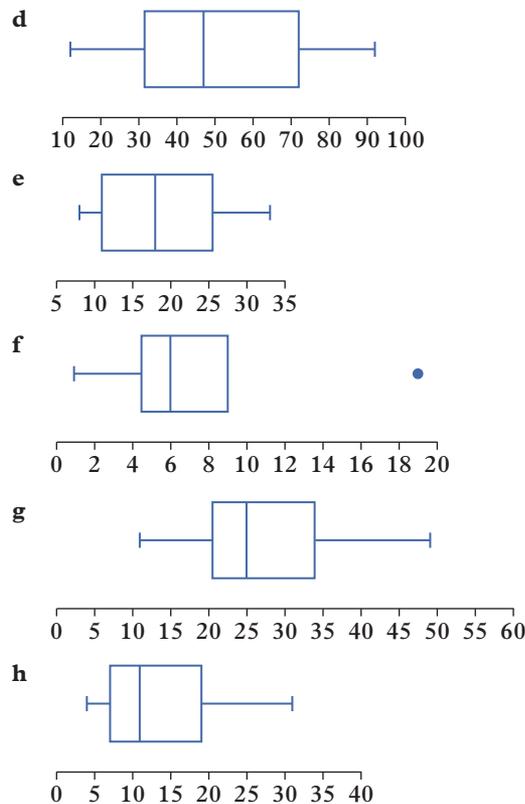
EX 8B Box plots

p361

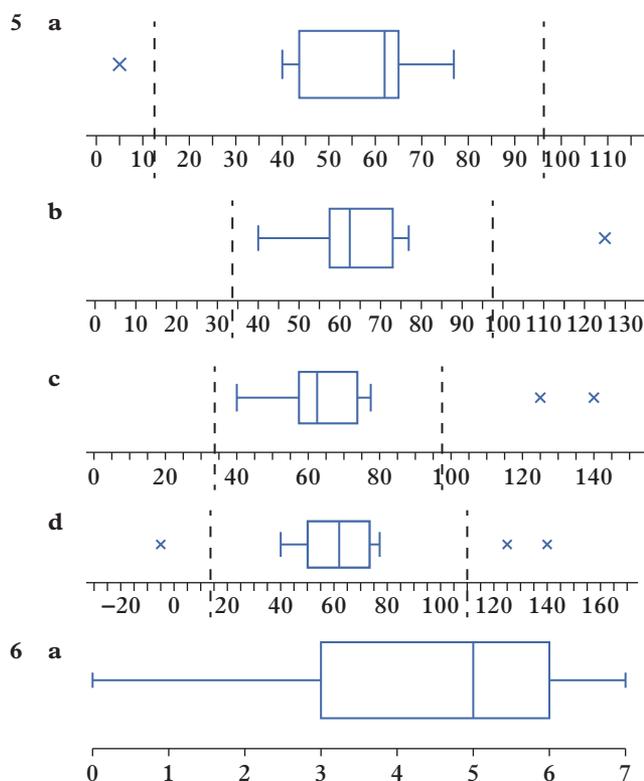
- 1 a i 1, 2, 5, 7, 10 ii IQR = 5
 b i 13, 20, 32, 40, 45 ii IQR = 20
 c i 3, 4, 6, 8, 9 ii IQR = 4
 d i 10, 16, 19, 24, 35 ii IQR = 8
 e i 20, 45, 70, 75, 85 ii IQR = 30
 f i 31, 33, 35, 37, 39 ii IQR = 4
- 2 a 2, 3, 4, 4, 6, 7, 7, 8, 8, 9, 10, 10, 11, 13, 14, 17, 19, 19, 22
 b $Q_1 = 6$, median = 9, $Q_3 = 14$
 c lower fence = -6, upper fence = 26
 d no outliers

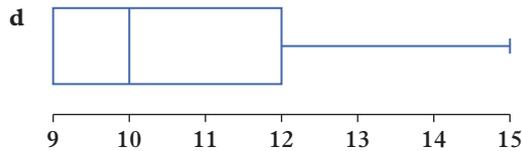
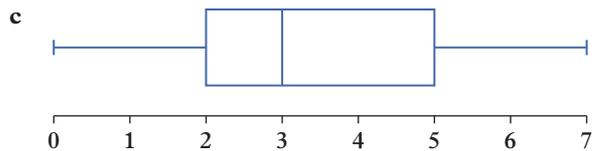
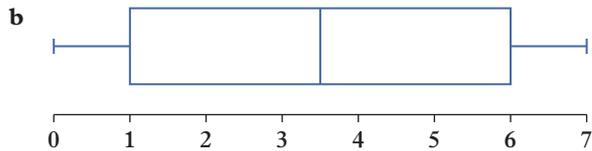


- 3 a
- b
- c



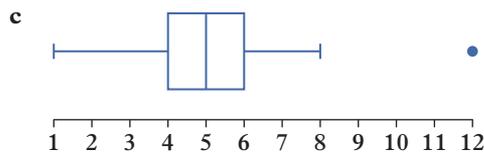
- 4 a i 3, 3, 7, 10, 12 ii 7
 b i 18, 30, 33, 40, 40 ii 10
 c i -20, -20, -15, -12, -12 ii 8
 d i 40, 41, 42, 46.5, 60 ii 5.5
 e i 18, 20, 21.5, 25, 35 ii 5
 f i 50, 60, 60, 60, 70 ii 0





- 7 **a** **i** B **ii** B, D
b **i** median **ii** maximum **iii** none
iv maximum **v** upper quartile
c Group C

- 8 **a** $Q_1 = 4$, median = 5, $Q_3 = 6$
b 12 is an outlier.



Puppies in litter

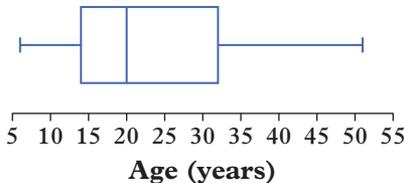
- 9 **a** median $\approx 32.2^\circ\text{C}$, IQR $\approx 3.5^\circ\text{C}$
b median $\approx 28^\circ\text{C}$, IQR $\approx 8.7^\circ\text{C}$
c Darwin's median maximum daily temperature during this month was approximately 32.2°C , which was hotter than Canberra's median maximum daily temperature, 28°C . Darwin's maximum daily temperature was less variable with an interquartile range of approximately 3.5°C compared to Canberra's larger interquartile range of approximately 8.7°C . While Canberra had the largest maximum daily temperature for the month, Darwin is, on average, hotter than Canberra.

10 **a**

Set A:



Set B:

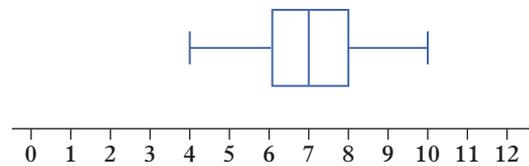


- b** **Set A:** The centre (median) = 21, the spread is 18–27 years, positively skewed. The range of ages is small, indicating that, during the week, the centre is used mainly by young people.
Set B: The centre (median) = 20, the spread is 6–51 years old, positively skewed. The range

of ages is much greater during the weekend, indicating that families most probably visit then.

- c** The range of ages is much greater in set B than in set A, although the median ages are quite similar. More people in a younger and older age range visit during the weekend than attend on weekdays.

11 **a**



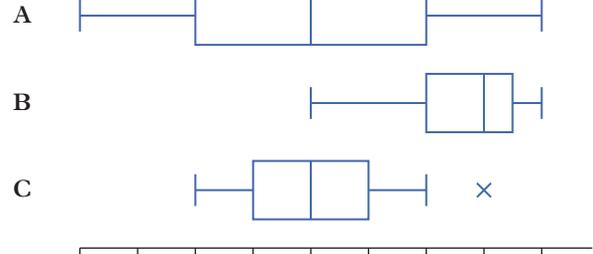
- b** No, as the minimum value (4) is greater than the lower fence (3) and the maximum value (10) is less than the upper fence (11), so the data contains no outliers.
c The minimum value (2) is less than the lower fence (3) and the maximum value (12) is greater than the upper fence (11) so they are both outliers. Therefore, we would need to know at least the next value above the minimum and less than the maximum value (provided they are not also outliers) to know the location of the ends of the whiskers of the box plot.

- 12 **a** Since the median line is not visible, it is equal to either the upper quartile or lower quartile so that it overlaps the line for either quartile.

b 5, 8, 8, 14, 20 or 5, 8, 14, 14, 20

- 13 The median age of all Australians is approximately 37 years, which is greater than the median age of Aboriginal and Torres Strait Islanders (approximately 23 years). The interquartile range of the ages of all Australians is approximately 37 years, which is greater than the interquartile range of the ages of Aboriginal and Torres Strait Islander peoples (approximately 31 years).

14



- 15 **a** IQR = 9 **b** $Q_1 = 28.5$, $Q_3 = 37.5$

EX
p367

8C Distributions of data

- 1 **a** symmetric
b positively skewed
c negatively skewed
d symmetric with an outlier
e negatively skewed
f negatively skewed
g positively skewed
h symmetric
- 2 **a** 25% **b** 50% **c** 25%
d 75% **e** 75% **f** 50%

3 a II b III c I d IV

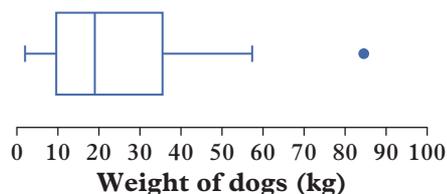
4 a i negatively skewed

ii Scores in the first quartile are very closely packed, making the lower whisker very short. In the second quartile, scores are spread far apart, making the lower half of the box long. Scores in the third and fourth quartiles appear spread out to about the same degree: less closely packed than those in the bottom whisker, but more densely packed than those in the lower half of the box.

iii The median score is close to the values of data at the top end of the distribution.

b There may be multiple scores of the same value in the lower whisker.

5 a



b positively skewed with an outlier

c To one decimal place, the weights are not repeated and so the dot plot would be a long row of scattered dots, which would be difficult to interpret.

6 a Group A: negatively skewed with an outlier; Group B: positively skewed with an outlier

b The mean is affected by both the skew and the outliers the data contains, whereas the median is not affected by these. The mean is also not able to be calculated directly from a box plot, whereas the median can be read off easily.

7 a 11, 14, 25, 32, 34

b 18

c The cumulative frequency always adds to the previous amount, therefore the cumulative frequency cannot decrease. This means the gradient cannot be negative.

d positively skewed

8 a i 25th ii 50th iii 75th

b The 100th percentile would be the value in the data set below which 100% of the data values fall. There is no such value as it would be included in the data set.

c i 4 and 8 ii 40 and 50 iii 301 and 701

d i 3 and 7 ii 30 and 50 iii 300 and 700

e In i and iii, the values differ by 1. In ii, the 30th percentile values are different by one value in the ordered data set and the 70th percentiles are the same.

f The data values would either be the same or very close as we would expect each number from 1 to 100 to appear in the data set many times.

2 a 0.0 b 3.0 c 26.5

d 4.9 e 2.3 f 21.8

3 a 3.0 b 4.4 c 401 083.6

d 802 167.1 e 401 083.6 f 0.1

4 a 9.5 b 922.3 c 2.9

d 2.9 e 2.9 f 73.3

5 a i population ii 0.85

b i sample ii 5.20

c i sample ii 0.54

6 a 0.78

b The spread of ages of players in the cricket teams are tightly packed around the mean value, with all but one player being between 15 and 17 years of age.

7 a mean = 15.88, standard deviation = 9.18

b The data is grouped pretty evenly with a large standard deviation and the mean being very close to halfway between the minimum and maximum.

8 a 41.7 b 9.75

c The spread of data is 13 to 58, with the mean at about 42. The mean is towards the higher end of the data, indicating that the data has a negative skew. The standard deviation of scores from the mean is about 10.

9 a mean for train line A = 5.4

mean for train line B = 4.5

b standard deviation for train line A = 1.89

standard deviation for train line B = 4.57

c The mean for train line A is higher than that for train line B, indicating trains are generally later on line A. Standard deviation for train line A is lower than that for train line B, indicating scores are bunched closely around the mean, so that line A is consistently late. Line B has four outliers of 9, 11, 15 and 19 minutes, which have increased the mean and the standard deviation. Without these outliers, the standard deviation decreases to 1.23 and mean for train line B decreases to 2.65.

d The outliers for train line B might have been caused by unusual circumstances. If these outliers are disregarded, it could be said that the train on line A runs late more often. If the outliers are not disregarded, the train on Line B, on average, runs earlier but is very unpredictable.

10 The first histogram has a larger standard deviation than the second, because the scores are more spread out.

11 a 68.51

b The shop may be a discount shop or might be having a sale; the tablet might be a discontinued old model.

c 20.98

d Standard deviation is much lower with the outlier excluded.

e interquartile range

12 a i $\sigma \approx 2.43$, mean absolute deviation = 1.74

ii $\sigma \approx 5.38$, mean absolute deviation = 4.375

b mean absolute deviation $\leq \sigma$

EX 8D The mean and standard deviation

p372

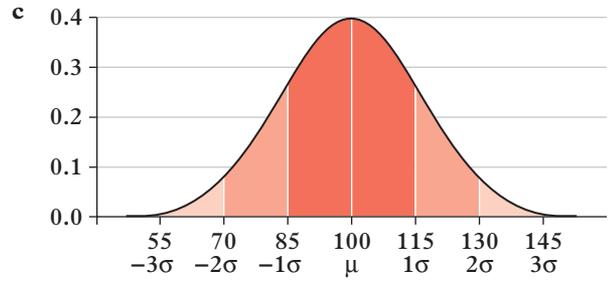
1 a 26.8 b 4.3 c 4.1

d 6.1 e 30.3 f 20.7

- 13 a i $\bar{x} = 4$ ii $\bar{x} = 7.5$
 b i 1.7 ii 4
- 14 a The median is a better measure of the centre than the mean when the data has outliers or is skewed as both of these affect the value of the mean.
 b The interquartile range is a better measure of the spread than the standard deviation when the data has outliers or is skewed as both of these affect the value of the mean and hence the standard deviation.
- 15 a Group A has a mean of 29.8, which is less than group B's mean of 30.1. Group A has a standard deviation of 5.5, which is greater than group B's standard deviation of 5.1. Therefore, on average, group A is less than and more variable than group B.
 b Group A has a mean of 93.3, which is greater than group B's mean of 77.2. Group A has a standard deviation of 12.4, which is greater than group B's standard deviation of 7.9. Therefore, on average, group A is greater than and more variable than group B.
- 16 a i $z_A = 1.5, z_B = 1.875$
 ii Student B scored 1.875 standard deviations above their class mean, which is greater than the 1.5 standard deviations Student A scored above their class mean. This means both students achieved better results than most of their class, with Student B performing better than Student A.
- b i $z_A = 0.75, z_B = 1$
 ii Student B scored 1 standard deviation above their class mean, which is greater than the 0.75 standard deviations Student A scored above their class mean. This means both students achieved better results than their class average, with Student B performing better than Student A.
- c i $z_A = -0.8, z_B = -5$
 ii Student B scored 5 standard deviations below their class mean, which is less than the 0.8 standard deviations Student A scored below their class mean. This means both students achieved worse results than their class average, with Student B performing worse than Student A.
- d i $z_A = -3, z_B = -3$
 ii Student B scored 3 standard deviations below their class mean, which is the same as the 3 standard deviations Student A scored below their class mean. This means both students achieved significantly worse results than their class average.
- 17 a Adding the percentages shown in the given graph:
 $2.35\% + 13.5\% + 34\% + 34\% + 13.5\% + 2.35\% = 99.7\%$

This covers the region of three sigma either side of the mean.

- b i 68% ii 95%

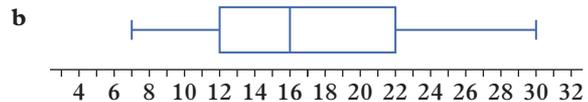
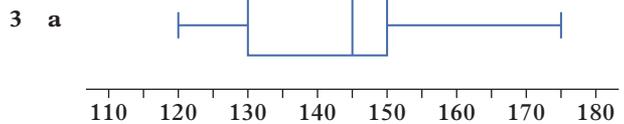


- d 55 to 145
 e higher than 145
 f less than 70
 g 2.5%
 h 16%
 i mean = 136, standard deviation = 8
- 18 a $\mu_x = 40, s_x = \frac{9}{5} = 1.8$
 b i 68% ii 84% iii 0.15%
 iv 5%
 c i 41.8 ii 40 iii 36.4 and 43.6
 iv 34.6 and 38.2 or 41.8 and 45.4

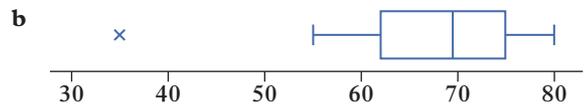
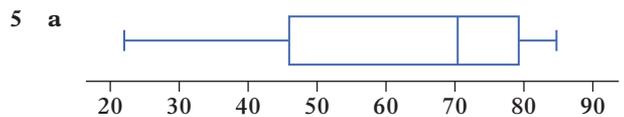
EX Checkpoint p376

- 1 a 14, 17, 22.5, 32, 50 b 35, 38.5, 45, 54, 60
 c 1, 4, 6, 7, 10 d 5.4, 6.2, 7.2, 7.9, 8.2

- 2 a IQR = 11, range = 18
 b IQR = 27, range = 35



- 4 a i 16, 23, 27, 35, 38 ii 12
 b i 20, 56, 72, 76, 80 ii 20



- 6 The median of group A is 70, which is less than the median of group B, 77. The interquartile range of group of A is 21, which is less than the interquartile range of group B, 28. Therefore, group A is, on average, less than group B and less variable.

- 7 a 25% b 50% c 75% d 50%
 8 a negatively skewed b symmetric
 c positively skewed d positively skewed

- 9 a i Hari: 33.2, Ish: 28.6
 ii Hari: 13.59, Ish: 33.45
 b Hari

EX 8E Two-step chance experiments

p381

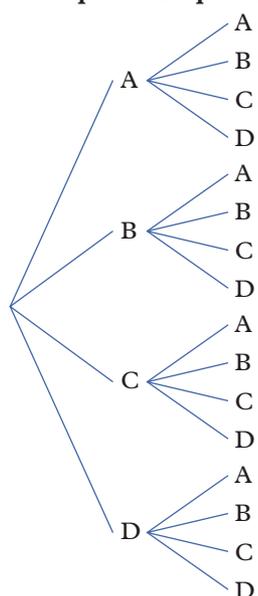
- 1 a 9 b 5
 2 a 9
 b i $\frac{2}{9}$ ii $\frac{4}{9}$ iii $\frac{5}{9}$
 iv $\frac{4}{9}$ v 0

		1st dice roll			
		1	2	3	4
2nd dice roll	1	2	3	4	5
	2	3	4	5	6
	3	4	5	6	7
	4	5	6	7	8

- 3 a
 b 16 c $5, \frac{1}{4}$ d 2 or 8, $\frac{1}{16}$
 e i $\frac{1}{8}$ ii $\frac{3}{16}$ iii $\frac{1}{2}$ iv $\frac{3}{8}$
 4 a $\frac{7}{16}$ b $\frac{1}{8}$ c $\frac{1}{16}$
 d $\frac{9}{16}$ e $\frac{3}{4}$ f $\frac{1}{16}$
 5 a $\frac{1}{4}$ b $\frac{1}{2}$ c $\frac{1}{4}$
 6 a $\frac{2}{3}$ b $\frac{5}{9}$ c $\frac{1}{3}$
 7 a i (r, s), (p, r), (s, p)
 ii (r, p), (p, s), (s, r)
 iii (r, r), (p, p), (s, s)

- b $\frac{1}{3}$

- 8 a
- | | First question | Second question |
|--|----------------|-----------------|
|--|----------------|-----------------|



		Second question			
		A	B	C	D
First question	A	A, A	A, B	A, C	A, D
	B	B, A	B, B	B, C	B, D
	C	C, A	C, B	C, C	C, D
	D	D, A	D, B	D, C	D, D

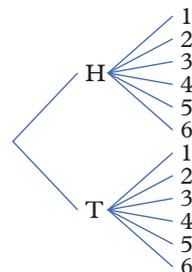
- c With a tree diagram you can add a third step to the right of the second step, whereas it is not possible to add another dimension to an array to represent the third step.

- 9 a 36
 b i $\frac{1}{4}$ ii $\frac{1}{2}$ iii $\frac{1}{6}$ iv $\frac{5}{36}$

- c When there are a large number of outcomes in each step of a two-step chance experiment, the information is easier to interpret in an array than a tree diagram, and it can be easier to locate the favourable outcomes in an array.

- 10 If there are exactly two outcomes at either stage of a two-step chance experiment, then the sample space at this stage will consist of these two outcomes, and one outcome will be the event that the other outcome does not occur, meaning that the two outcomes are complementary events.

- 11 a b 12



- c i $\frac{1}{12}$ ii $\frac{1}{4}$ iii $\frac{7}{12}$ iv $\frac{3}{4}$

- d Tails on the coin toss and a 6 on the die is counted as one outcome, not two.

12 a

		Vowel				
		A	E	I	O	U
Die roll	1	(A, 1)	(E, 1)	(I, 1)	(O, 1)	(U, 1)
	2	(A, 2)	(E, 2)	(I, 2)	(O, 2)	(U, 2)
	3	(A, 3)	(E, 3)	(I, 3)	(O, 3)	(U, 3)
	4	(A, 4)	(E, 4)	(I, 4)	(O, 4)	(U, 4)

- b 20
 c i $\frac{1}{20}$ ii $\frac{2}{5}$ iii $\frac{9}{20}$ iv $\frac{9}{10}$
 13 a i $\frac{1}{8}$ ii $\frac{3}{8}$ iii $\frac{3}{8}$ iv $\frac{1}{4}$

- b It is not possible to add another dimension to an array to represent the third or more steps.

- 14 a $\frac{1}{216}$ b $\frac{5}{54}$ c $\frac{5}{9}$
 d $\frac{5}{72}$ e $\frac{5}{18}$

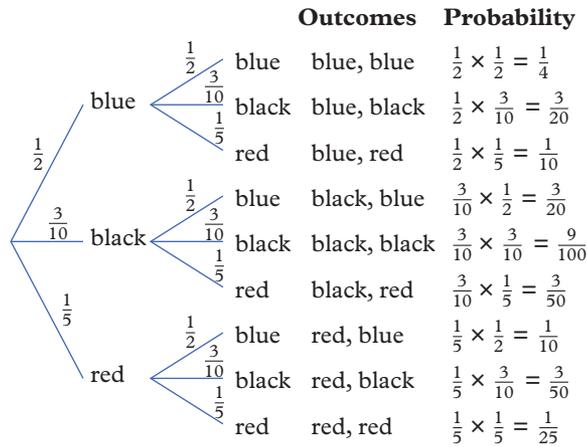
EX 8F Experiments with replacement

p387

- 1 a $\frac{1}{6}$ b $\frac{1}{6}$ c $\frac{3}{26}$ d $\frac{1}{6}$

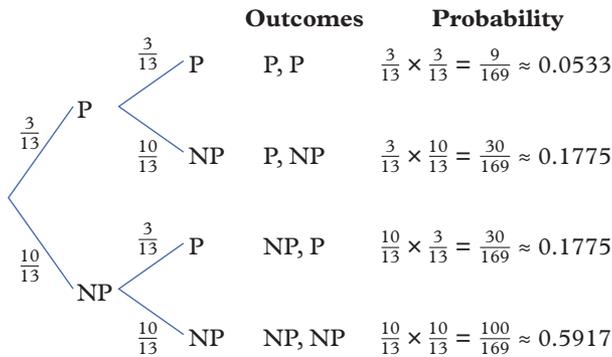
7 a $\frac{1}{25}$ b $\frac{16}{25}$ c $\frac{4}{25}$

8 a



b i $\frac{1}{4}$ ii $\frac{1}{25}$ iii $\frac{9}{100}$
 iv $\frac{3}{20}$ v $\frac{1}{10}$ vi $\frac{3}{50}$

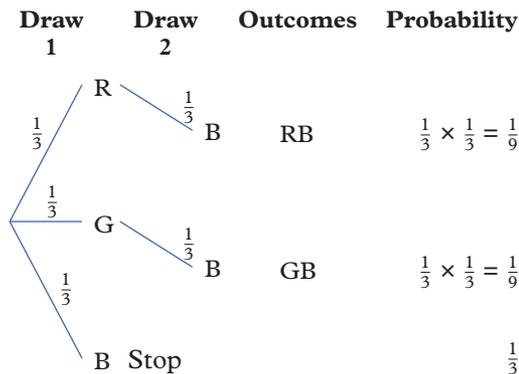
9 a



b i $\frac{60}{169} \approx 0.3550$ ii $\frac{69}{169} \approx 0.4083$
 iii $\frac{9}{169} \approx 0.0533$ iv $\frac{100}{169} \approx 0.5917$

10 The grouped outcomes of this chance experiment are not equally likely, so the theoretical probability is calculated by adding the probabilities of the successful outcomes instead of using the theoretical probability formula.

11 $\Pr(\text{blue ball}) = \Pr(\text{blue ball on first draw})$ or
 $\Pr(\text{not blue, then blue})$
 $= \frac{1}{3} + \left(2 \times \frac{1}{3} \times \frac{1}{3}\right)$
 $= \frac{5}{9}$



12 a If the two-step chance experiment with replacement has lots of outcomes, they may be easier to see in an array than a tree diagram, and it can be easier to locate the favourable outcomes.

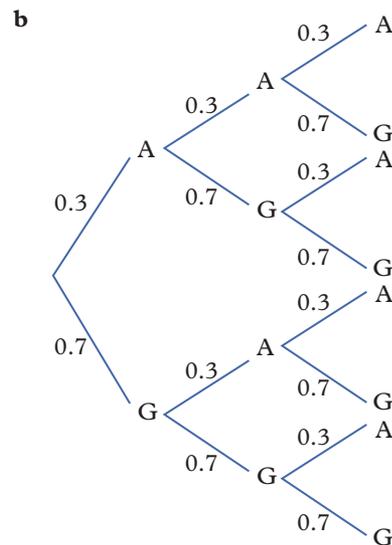
b If outcomes are not equally likely, then include the outcomes in the array in the same proportion as the likelihood. For example, if the first event is a ball is drawn at random from 1 blue and 2 red, then the array would have three columns: blue, red, red.

13 a Sample answer: No, not necessarily. The socks are replaced each time, so you cannot say that this is the exact number of white and black socks. Some of the same socks may have been chosen multiple times.

b four black socks and six white socks

c Yes, this supports the previous estimate.

14 a 28 green marbles



c i 0.37
 ii 0.441
 iii 0.63
 iv 0.21

15 a $3^3 = 27$

b BRW, BWR, RBW, RWB, WBR, WRB

c $6bw(1 - b - w)$

16 a a^2 b $2ab$ c $1 - a - b$

d i $2a(1 - a - b)$ ii $2a - 2a^2 - 2ab$

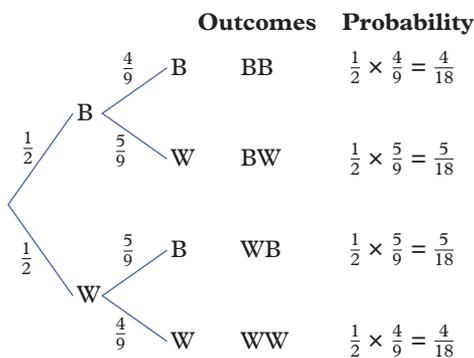
e i $(1 - a - b)^2$

ii $1 - 2a - 2b + 2ab + a^2 + b^2$

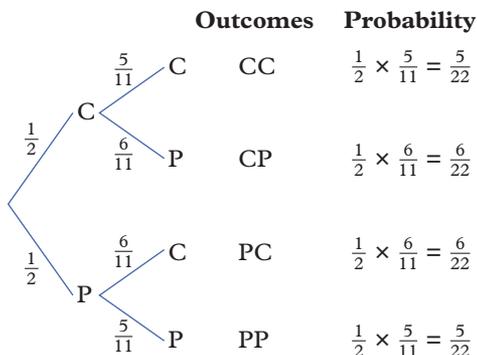
EX 8G Experiments without replacement

p392

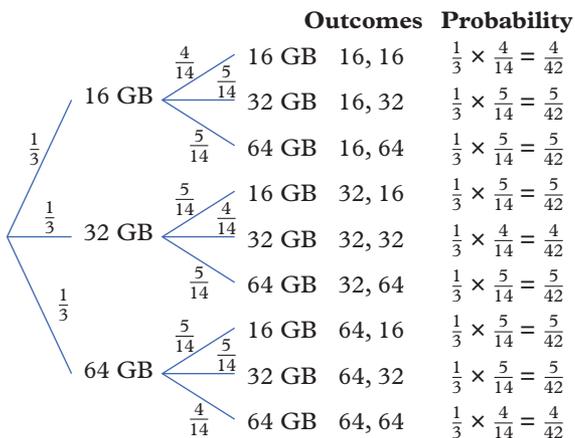
1 a



b



c



2 a 0.21

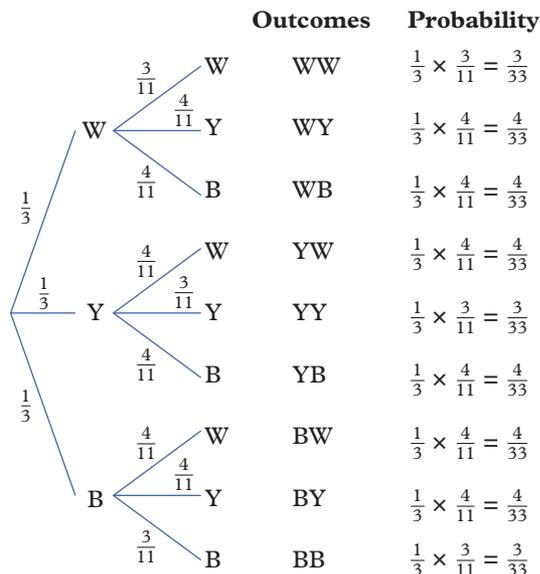
b 0.21

c 0.58

3 a $\frac{3}{14}$

b $\frac{2}{7}$

4 a



b i $\frac{1}{11}$

ii $\frac{4}{33}$

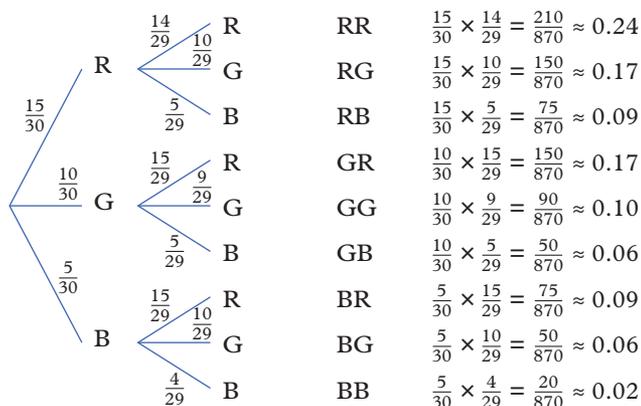
5 a $\frac{8}{11}$

b $\frac{19}{33}$

c $\frac{14}{33}$

6 a

Customer 1 Customer 2 Outcomes Probability



b i $\frac{20}{870} = \frac{2}{87} \approx 0.0230$

ii $\frac{90}{870} = \frac{3}{29} \approx 0.1034$

iii $\frac{210}{870} = \frac{7}{29} \approx 0.2414$

iv $\frac{150}{870} = \frac{5}{29} \approx 0.1724$

7 a $\frac{490}{870} = \frac{49}{87} \approx 0.5632$

b $\frac{660}{870} = \frac{22}{29} \approx 0.7586$

c $\frac{270}{870} = \frac{9}{29} \approx 0.3103$

d $\frac{450}{870} = \frac{15}{29} \approx 0.5172$

e $\frac{400}{870} = \frac{40}{87} \approx 0.4598$

f $\frac{100}{870} = \frac{10}{87} \approx 0.1149$

8 a $\frac{1}{17}$

b $\frac{15}{34}$

c $\frac{19}{34}$

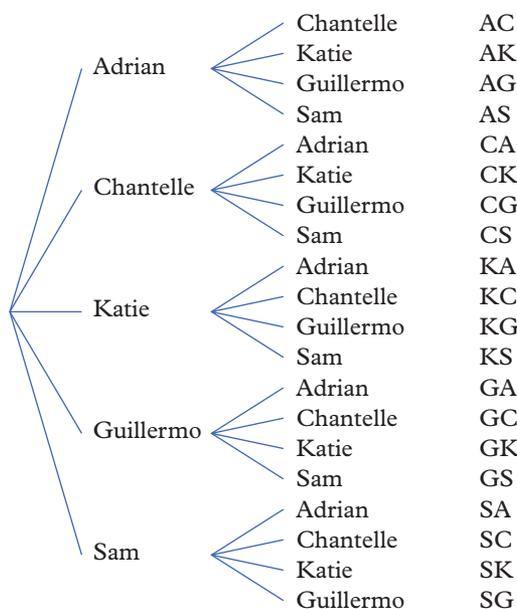
d $\frac{13}{102}$

9 $\frac{1}{5} = 0.2$

10 Pr(pair) = $\frac{5}{17} \approx 0.2941$; probability of a pair increases slightly.

11 $\frac{38}{132} = \frac{19}{66} \approx 0.2879$

12 a **Captain Vice-captain Outcomes**



b The second set of branches has one fewer branch. Once the captain has been selected there is one fewer person to choose from for vice-captain's position.

c 20

d i $\frac{1}{5}$ ii $\frac{2}{5}$ iii $\frac{1}{20}$

iv $\frac{3}{5}$ v $\frac{1}{10}$

13 a i $\frac{5}{18}$ ii $\frac{1}{6}$ iii $\frac{5}{18}$ iv $\frac{5}{9}$

b i

	G	G	G	P
G	X	GG	GG	GP
G	GG	X	GG	GP
G	GG	GG	X	GP
P	PG	PG	PG	X

ii $\frac{1}{2}$ iii 0

14 a $\frac{1}{12}$ b $\frac{7}{12}$ c $\frac{13}{18}$ d $\frac{7}{12}$

15 a $\frac{1}{21}$ b $\frac{2}{7}$ c $\frac{5}{42}$

16 $\frac{1}{8145060} \approx 0.000000123$

17 $\frac{5}{18}$

EX 8H Experimental probability and simulations

p396

Devices listed are suggestions only.

1 a i flipping a coin

ii heads: true, tails: false

b i spinner with four equal sections, labelled A, B, C and D

ii The section the spinner lands on determines the group.

c i flipping a coin

ii tail: left; head: right

d i spinner with seven sections

ii blue: Mon; yellow: Tue; green: Wed; red: Thu; purple: Fri; pink: Sat; orange: Sun.

2 a Potentially not; research would need to be done on a large number of true/false questions to determine whether either outcome is more likely, or whether they are equally likely.

b Yes

c Yes

d Potentially not; research would need to be done to determine if babies are more or less likely to be born on a particular day of the week.

3 a Spinning a spinner with eight equally sized sections; the section the spinner lands on determines the flavour.

b Drawing a card from a deck and recording the suit; clubs: N; diamonds: E; hearts: S; spades: W

c Rolling a 10-sided die, assign each prize to each number on the die

d Spinning a spinner with twelve equally sized sections; the section the spinner lands on determines the month

4 a The person flipping the coin could influence the result; meals may not be evenly distributed

b The person drawing the card could influence the result; babies may be more likely to be born in one or more seasons

c The person spinning the spinner could influence the result; the people selecting the classes may introduce bias; some classes may be bigger and therefore not equally likely to occur; elective choice could influence class placement

d The person rolling the die could influence the result; the people selecting teams may introduce bias; the selection may not be equally likely

e The person rolling the die could influence the result; winning is not equally likely and depends on the skill of the teams.

f The person spinning spinner could influence the result

5 a Rolling a die to represent Chupa Chups flavours; assumes there are equal numbers of each flavour

b Flipping a coin (heads: boy, tails: girl); doesn't allow for genetics or environment affecting gender result

c Rolling a four-sided die; assumes equal availability of desserts and that they are all equally well liked

d Spinning a spinner with five equally sized sections; assumes that each person has same leadership ability

e Rolling a die to represent M&M colours; assumes colours are evenly distributed in each packet

f Spinning a spinner with five sections; assumes correct answers are evenly distributed between options A–E

6 a $25\% = \frac{1}{4}$ and $50\% = \frac{1}{2}$

b flipping two coins produces four possible results: (HH, HT, TH, TT)

c two heads: plain, two tails: white, one head and one tail: flashy

d–e Answers will vary, depending on simulation result

- f The breeding conditions may not be ideal; the breeding pattern may be more complex than stated; the person flipping coins may introduce some bias by flipping them a certain way
- 7 Answers will vary, depending on simulation result
- 8 a Winning and not winning are not equally likely outcomes: there is one in six chances of winning and there are five in six chances of not winning.
- b You may roll a die and let the number 1 represent the chance of winning. The numbers 2, 3, 4, 5 and 6 all represent the chance of losing.
- c 5
- d Answers will vary, depending on simulation result
- e Expect to win about $\frac{1}{6} \times 3000 = 500$ Mars Bars
- 9 a An integer randomly generated from 1 to 6 (inclusive).
- b Yes
- c-d Answers will vary
- 10 a i $\frac{7}{10}$ ii $\frac{3}{10}$
- b 30
- c A 10-sided die may be used. Assign three numbers to represent meteorite crashing on land and remaining seven numbers to represent meteorite crashing on water.
- d-e Answers will vary
- 11 a A coin may be used to simulate genders at birth. Flip twice to represent two children, three times for three children, and so on. Assign a result to a gender; for example, heads is a boy, tails is a girl.

b-g Answers may vary

- b the lower end point in a data set, beyond which any smaller data points are considered outliers. The lower fence lies $1.5 \times \text{IQR}$ below the lower quartile; for example, if the lower quartile is 20 and the IQR is 6, the lower fence would be 11 and any values smaller than 11 would be considered outliers.

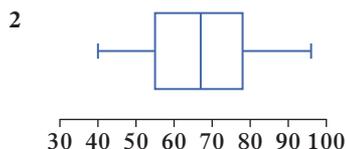
- 6 a HT, TH b HT, TH, HH
c TT d HT, TH e HH

Multiple choice

- 1 C 2 B 3 B 4 D 5 B 6 B
7 C 8 C 9 A 10 A 11 D 12 D

Short answer

- 1 a 56 b 67
c 40, 55, 67, 78, 96 d 23

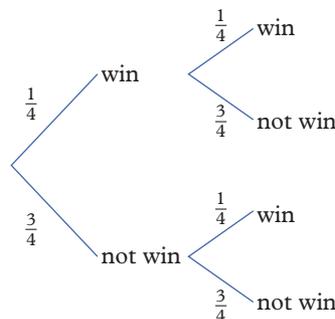


Ice creams sold per day

The box plot shows that 50% of the time between 55 and 78 ice creams are sold each day and 67 ice creams are sold half of the time. The data is pretty evenly spread.

- 3 five-number summary: 11, 13, 17, 19, 24; interquartile range: 6
- 4 a 25% b 0% c 75%
- 5 a positively skewed
b symmetric
c negatively skewed
- 6 a i Lily 72, Cassy 71 ii Lily 3.69, Cassy 18.28
b Lily
- 7 a $\frac{3}{4}$

b Chance of winning an ebook



- c i $\frac{1}{16}$ ii $\frac{15}{16}$ iii $\frac{3}{16}$

8 a

	1	2	3	4	5	6
H	(H, 1)	(H, 2)	(H, 3)	(H, 4)	(H, 5)	(H, 6)
T	(T, 1)	(T, 2)	(T, 3)	(T, 4)	(T, 5)	(T, 6)

- b i $\frac{1}{12}$ ii $\frac{7}{12}$ iii $\frac{1}{2}$
c $\frac{1}{6}$ d $\frac{5}{6}$

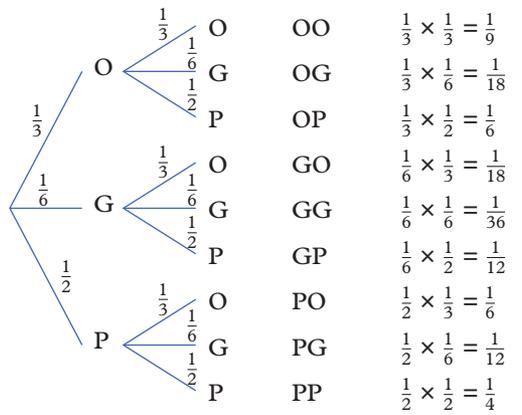
EX Chapter 8 review

p400

Mathematical literacy review

- 1 Sample answer: In an ordered data set, consider 0% as where the data starts: this point is the minimum value. Consider 100% as where the data ends: this is the maximum value in the data set. The quartiles divide the data into four equal parts, with the lower quartile representing the point below which 25% of the data falls. The median is the middle value that splits the data set into two 50% halves. The upper quartile represents the point below which 75% of the data falls.
- 2 interquartile range, standard deviation, range
- 3 A negatively skewed distribution has a cluster of values around the right-hand side of the distribution and a tail tapering to the left, while a positively skewed distribution has a cluster of values around the left-hand side of the distribution and a tail tapering to the right.
- 4 a simulation b dependent event
- 5 a two events which together consist of the entire sample space of an experiment, and where one event is the outcome that the other event does not occur; for example, 'rolling a 3' and 'not rolling a 3' when rolling a standard six-sided die

9 a



b i $\frac{1}{9}$ ii $\frac{1}{36}$ iii $\frac{1}{4}$

c $\frac{1}{6}$ d $\frac{1}{2}$

10 a $\frac{1}{17}$ b $\frac{13}{204}$

11 a Sample answer: Drawing a card from a deck and recording the suit.

b Sample answer: Rolling a standard six-sided die and recording the number.

Analysis

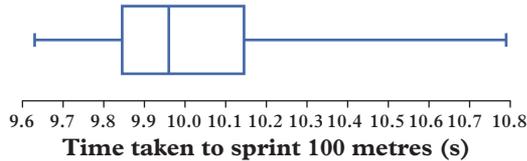
1 a i mean = 10.03, median = 9.955

ii range = 1.16, IQR = 0.3

iii 9.63, 9.84, 9.955, 10.14, 10.79

b 0.295

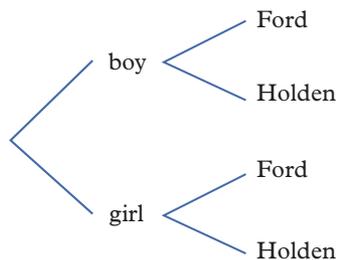
c i



ii Sample answer: The times are positively skewed, being packed more tightly below the median than above the median. This indicates it is becoming increasingly difficult to run faster times, with winning times now appearing within a limited range.

iii The lower whisker will move to the left, and the lower half of the box plot will move left and shrink in size as the times become more densely packed in that region of the plot.

2 a



b {BF, BH, GF, GH};

B = boy, G = girl,

F = Ford, H = Holden

Each outcome has a theoretical probability of 0.25.

c 58 people were surveyed.

d $\Pr(\text{BF}) = \frac{7}{58}$, $\Pr(\text{BH}) = \frac{23}{58}$, $\Pr(\text{GF}) = \frac{19}{58}$,

$\Pr(\text{GH}) = \frac{9}{58}$

e i $\frac{32}{58}$ ii $\frac{30}{58}$ or $\frac{15}{29}$

f No; each result would need to have been around 14 or 15.

g Possible limitations include age and location of people surveyed.

3 a i $\frac{1}{2}$ ii $\frac{3}{8}$ iii $\frac{1}{8}$ iv $\frac{3}{4}$

b i $\frac{1}{20}$ ii $\frac{2}{5}$ iii $\frac{1}{8}$

c i $\frac{7}{60}$ ii $\frac{1}{6}$ iii $\frac{5}{7}$

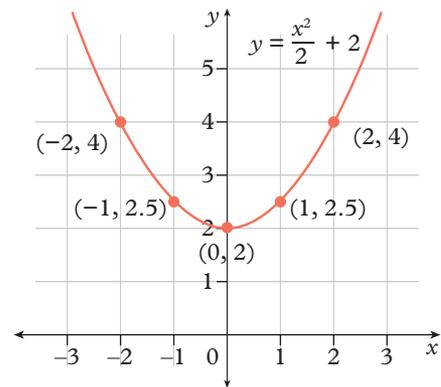
EX SEMESTER 2 review

p406

Short answer

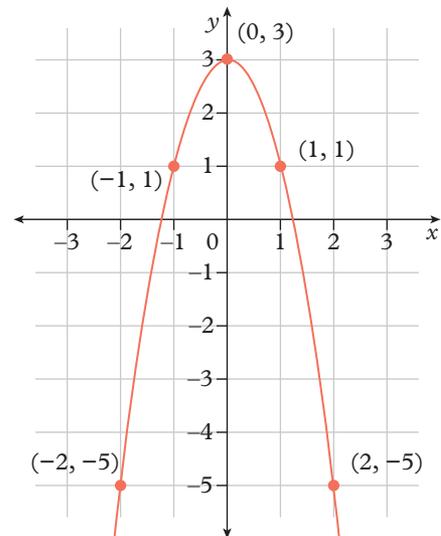
1 a

x	-2	-1	0	1	2
y	4	2.5	2	2.5	4



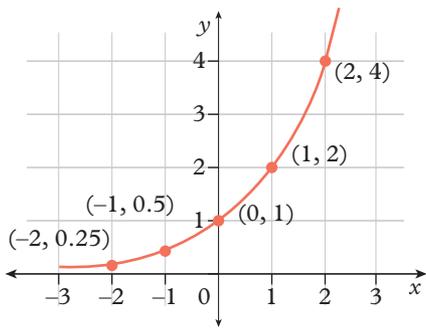
b

x	-2	-1	0	1	2
y	-5	1	3	1	-5



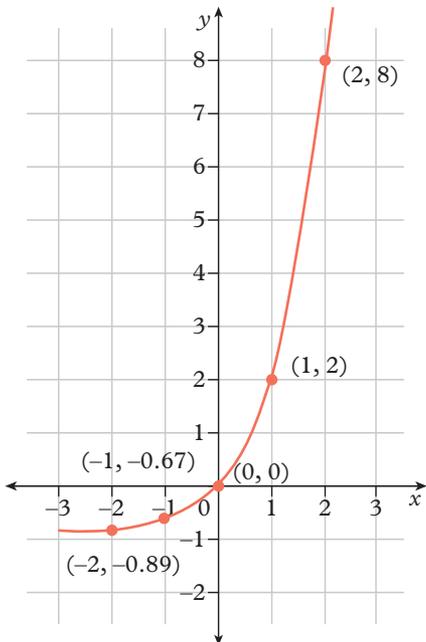
c

x	-2	-1	0	1	2
y	0.25	0.5	1	2	4

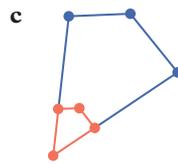
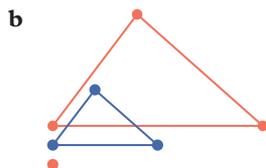
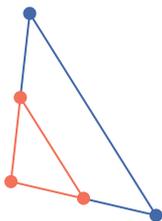


d

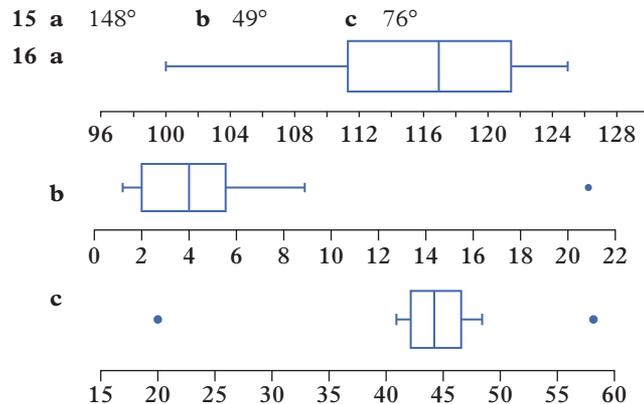
x	-2	-1	0	1	2
y	-0.89	-0.67	0	2	8



- 2 **a** reflected across the x -axis, then translated 1 unit up
b dilated in the y -direction by a factor of 25, then translated 12 units down
- 3 **a** $y = x^2 + 3$ **b** $y = -x^2 - 2$
- 4 **a** (5, 0) and (0, 5)
b (-2, 0), (0, 16) and (2, 0)
- 5 **a** 40.91 cm² **b** 61 000.00 cm²
c 52 216.00 cm²
- 6 **a** 78 cm² **b** 120 cm² **c** 471.24 cm²
- 7 **a** 45 cm³ **b** 60 cm³ **c** 785.40 cm³
- 8 **a** 12.48 m³ **b** 10 560 mm³ **c** 676.2 cm³
- 9 **a** **i** 0.5°C **ii** 1.22%
b **i** 25 mL **ii** 3.125%
- 10 **a** $k = 12$ cm, $l = 52^\circ$ **b** $p = 139^\circ$, $q = 30$ mm
- 11 **a** 3.2 m by 2.6 m **b** 4.8 m by 3.6 m
c 5.6 m by 3.2 m
- 12 **a**



- 13 **a** **i** $\frac{1}{2}$ **ii** 0.5581
iii 0.8372 **iv** 0.6667
- b** **i** $\frac{1}{13}$ **ii** 0.5
iii 0.8660 **iv** 0.5774
- c** **i** 3 **ii** 0.7593
iii 0.6508 **iv** 1.1667
- d** **i** 5 **ii** 0.96
iii 0.28 **iv** 3.4286
- 14 **a** minimum = 32, $Q_1 = 48$, median = 55.5, $Q_3 = 64$, maximum = 76; IQR = 16
b minimum = 10, $Q_1 = 20$, median = 25, $Q_3 = 40$, maximum = 70; IQR = 20
c minimum = 10, $Q_1 = 11$, median = 13, $Q_3 = 14$, maximum = 18; IQR = 3
d minimum = 1.2, $Q_1 = 2.35$, median = 3.4, $Q_3 = 4$, maximum = 4.4; IQR = 1.65
e minimum = 100, $Q_1 = 102$, median = 103, $Q_3 = 104$, maximum = 106; IQR = 2



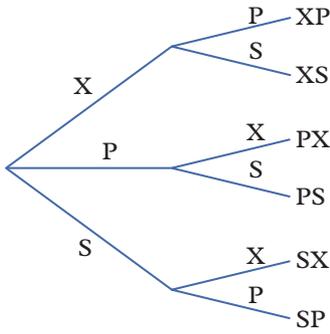
- 17 **a** $p = 19.2$ m, $q = 18.9$ m
b $x = 30.9$ m, $y = 18.2$ m
- 18 **a** false **b** true **c** true
d true **e** false **f** false

19 **a**

	1	2	3	4	5	6
Spade	(1, ♠)	(2, ♠)	(3, ♠)	(4, ♠)	(5, ♠)	(6, ♠)
Club	(1, ♣)	(2, ♣)	(3, ♣)	(4, ♣)	(5, ♣)	(6, ♣)
Diamond	(1, ♦)	(2, ♦)	(3, ♦)	(4, ♦)	(5, ♦)	(6, ♦)
Heart	(1, ♥)	(2, ♥)	(3, ♥)	(4, ♥)	(5, ♥)	(6, ♥)

- b** **i** $\frac{1}{24}$ **ii** $\frac{1}{4}$ **iii** $\frac{3}{4}$ **iv** $\frac{1}{2}$
- 20 **a** $\bar{x} = 37.75$, $s = 4.79$, $\sigma = 4.15$
b $\bar{x} = 22.2$, $s = 3.90$, $\sigma = 3.49$
c $\bar{x} = 4$, $s = 0$, $\sigma = 0$

21 a



b i $\frac{1}{6}$ ii $\frac{1}{3}$ iii $\frac{2}{3}$ iv $\frac{1}{3}$

22 a positively skewed b negatively skewed

c symmetric d negatively skewed

23 a $\frac{16}{49}$ b $\frac{12}{35}$ c $\frac{1}{35}$ d $\frac{1}{3}$

24 a $\Pr(\text{solid milk chocolate}) = \frac{1}{4}$,

$\Pr(\text{solid white chocolate}) = \frac{1}{4}$,

$\Pr(\text{caramel filled milk chocolate}) = \frac{1}{4}$,

$\Pr(\text{caramel filled white chocolate}) = \frac{1}{4}$

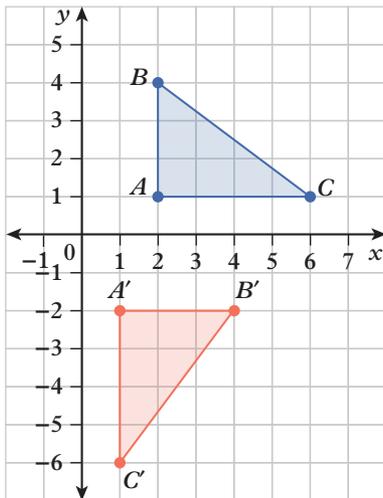
b One possible device is a spinner with four equal sections numbered one to four. Assign a number to each outcome.

c Eight

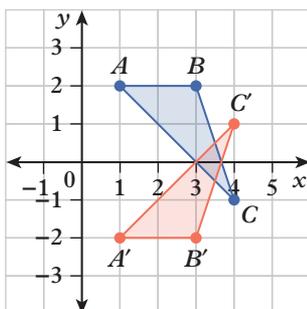
d A spinner may not always produce truly random results.

25 a $x = 5$ or $x = -9$ b $x = 4$ or $x = -10$

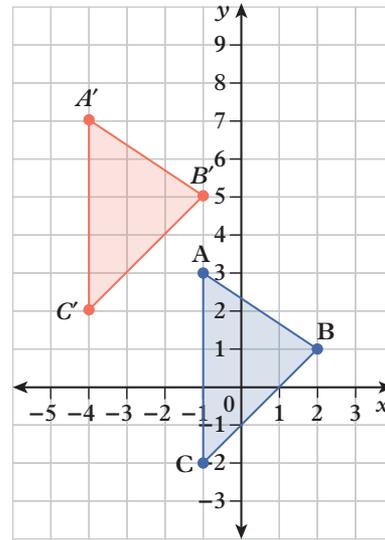
26 a



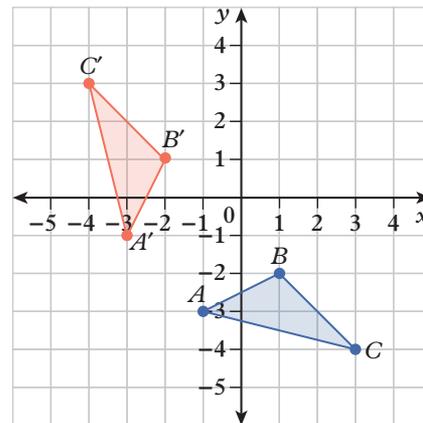
b



c



d



27 a 320 cm^2 b 122.72 mm^2

28 a not congruent b congruent

29 a AAS or ASA, $a = 3$ b RHS, $b = 27^\circ$

c SAS, $c = 6$ d SSS, $d = 31^\circ$

30 a yes, SSS b yes, SAS

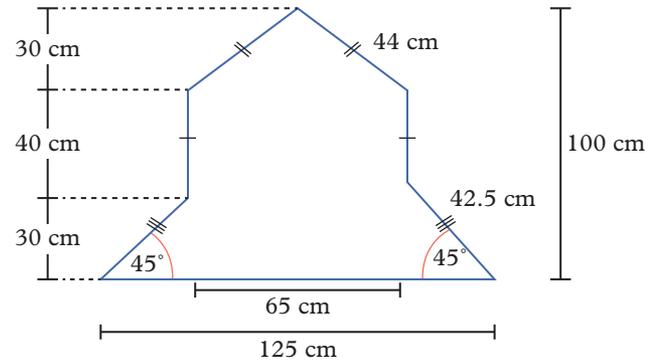
c no d yes, AAA

e yes, SAS f yes, SAS

Analysis

1 a 8.4 cm should be 8.5 cm, correct to one decimal place. 13.0 cm is correct.

b



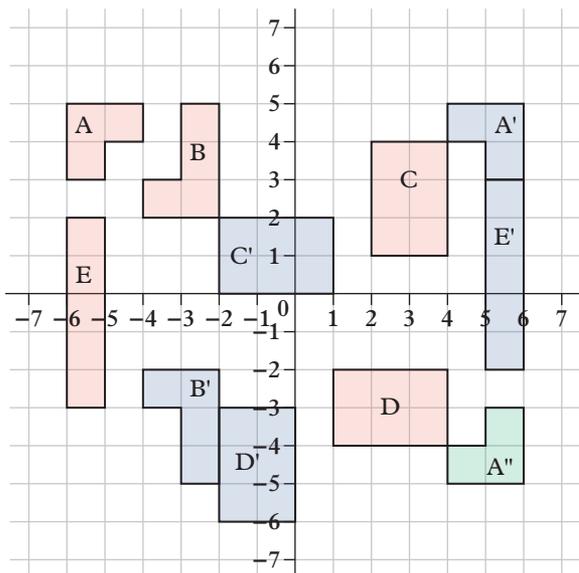
c i 257 cm^2 ii 6425 cm^2

d 30°

e 69.3 cm

f They are not parallel as the angle the edge of the plywood makes with the horizontal ($\approx 42.7^\circ$) is not the same as the angle the string makes with the horizontal (30°), and these are corresponding angles.

2 a



b Rotation of 180° about the origin

c i A: 1, B: 1, C: 2

ii A: 1, B: 0, C: 2

d i C and D

ii E

3 a

i $\frac{2}{3}$

ii $\frac{7}{30}$

iii $\frac{1}{4}$

iv $\frac{15}{23}$

b i $\frac{2}{45}$

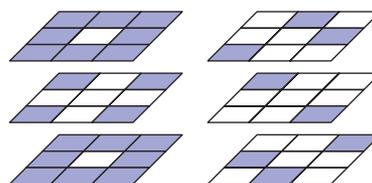
ii $\frac{7}{45}$

c i $\frac{4}{513}$

ii $\frac{59}{513}$

iv maximum = 20

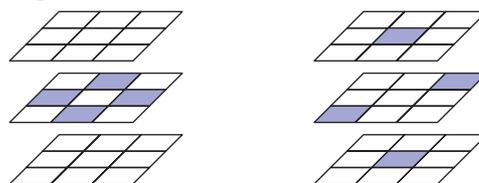
minimum = 8



b One possible arrangement is given for i and ii. There are others.

i Top/bottom views:

ii Four side views:



iii If the pattern can be viewed from all four sides, deduce that the top and bottom layers must have a cube in the centre, as in ii above, so the pattern cannot be seen from above or below.

2 a i $\frac{1}{12}$

ii Dieter has probability $\frac{1}{2}$ of winning but draws are also possible.

b No, Connie has probability $\frac{7}{12}$ of winning.

c 3 coins

d 14 sides

e $k = 3n - 1$

f No for dodecahedral (cannot have $n = 13$ counters). Yes for icosahedral (with $n = 7$ counters).

3 a i 9 ii 12 iii 10 iv 10

v 16 vi 20 vii 18 viii 22

b Types i, iii and iv only.

c $AB = AF$, so not all side lengths are different.

4 a i 2 steps

ii 5 steps

iii 6 steps

iv 9 steps

b i 2 moves

ii impossible

iii 9 moves

iv impossible

c i 2 passes

ii 5 passes

iii 5 passes

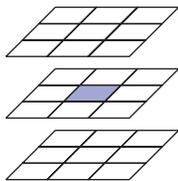
iv 9 passes

EX EXPLORATIONS 2

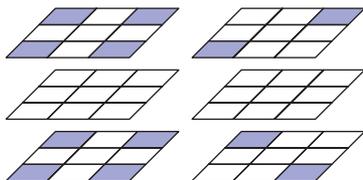
p414

1 a Rotations and reflections of the minimum diagrams in ii and iv are also possible.

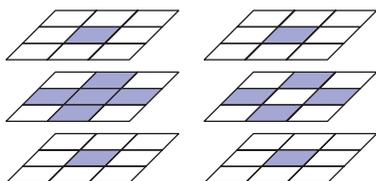
i maximum = minimum = 1



ii maximum = 8 minimum = 4



iii maximum = 7 minimum = 6



Glossary

AAA

abbreviation for angle–angle–angle.
Triangles are similar if the three internal angles are equal

AAS

abbreviation for angle–angle–side.
Triangles are congruent if two angles and a corresponding side length are equal

absolute error

half of the smallest division on a measuring device

$$\text{absolute error} = \frac{1}{2} \times \begin{array}{l} \text{smallest} \\ \text{division} \end{array}$$

accuracy

the closeness of a measurement to its true value

adjacent side

(trigonometry) the side, in a right-angled triangle, next to the reference angle that is not the hypotenuse

algebraic fraction

fraction that contains at least one pronumeral

approximation

a value that is almost the same as the exact value

arc

a portion of the circumference of a circle. Angles are indicated using an arc

area scale factor

the scale factor between the areas of two similar shapes

array

items arranged in rows and columns. Used in probability to display the sample space of a two-step chance experiment

axis of symmetry

imaginary line that divides a symmetrical shape or graph so that one side is a reflection of the other

asymptote

an imaginary line that a graph line approaches but never touches

base

in a value expressed in index notation, the base is the number that is repeatedly multiplied; for example, 2^4 has a base of 2

BIDMAS

order of operations: brackets, indices (exponents), division and multiplication, addition and subtraction

binomial

an expression containing two terms

binomial product

a factorised expression that is the product of two binomial expressions such as $(a + b)(c + d)$

bonus

an incentive or reward for good performance, which is given on top of a salary or wage

box plot

a visual representation of the five-number summary of a numerical data set along a number line. The ‘box’ represents the middle 50% of ordered data, with vertical lines indicating the first quartile, median and third quartile

budget

a financial plan, which details expected income and expenses for a given period of time

buy an item on terms

to make an initial deposit on an item, and to pay the remaining amount in regular instalments

buy now, pay later

services that allow a consumer to buy a product or service and delay the payment

capacity

amount of a substance that a 3D object (container) can hold (mL, L)

Cartesian plane

number plane or region formed by a pair of horizontal and vertical axes that allows any point to be described by a pair of coordinates

centre of dilation

the point from which a dilation is made

circumference

perimeter of a circle

coefficient

the number multiplied by a pronumeral

commission

a payment which is a proportion of the total value of sales

complementary events

a pair of events A and A' (not A) where only one of the two events can take place and their probabilities sum to 1

composite shape

a 2D figure made up of more than one simple shape

composite solid

a 3D object made up of more than one simple solid

congruent

when two shapes are identical in shape and size, but can be in any position or orientation

constant

a term without any pronumeral part

constant of variation

if two quantities x and y are related so that $y = kx$, k is the constant of variation. k is equal to the rate of change of y with respect to x and the gradient of the graph of y vs. x

constant term

the term in an expression that has no variable or the variable is to power 0; for example, -1 is the constant term in $3x^2 + 4x - 1$

coordinate points (coordinates)

pair of numbers that describe the position of a point on a Cartesian plane

cosine (cos)

the cosine of an angle in a right-angled triangle equals the ratio of the length of the adjacent side to the length of the hypotenuse

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

cross-section

the 2D shape obtained when cutting through a 3D object

cube

3D object with six faces that are all identical squares

cylinder

3D object with a uniform circular cross-section

dependent event

an event whose outcomes are affected by the result of another event

dilation

- 1 transformation that can enlarge or compress the graph of a function in the x - or y -direction
- 2 transformation that produces an enlargement or a reduction of the original figure

direct variation (direct proportion)

two quantities are in direct proportion if, as they increase or decrease, the ratio between the quantities stays the same

distributive law

states that the same result is obtained if you add the numbers inside a pair of brackets first before multiplying by the common factor or multiply each number inside the brackets by the number outside the brackets before adding: $a(b + c) = ab + ac$

edge

a line segment which joins two faces in a 3D object

enlargement

larger image produced after a figure has been dilated

equation

two expressions shown to be equivalent by an equals sign

estimate

an approximate value which is close to the actual value

event

outcome or group of outcomes in a sample space; for example, a single outcome is rolling a 2; a group of outcomes is rolling a 2 or a 4

expand

to convert an expression from factor form to expanded form

expanded form

algebraic expression written without brackets, as opposed to factor form; for example, $3(2x + 1)$ expands to $6x + 3$

experiment

repeatable procedure that has a clearly defined set of possible results

experimental probability

probability determined from the results of experiments, not mathematical reasoning

exponent

see index

exponential relationship

a relationship between two variables, one of which is an index (power or exponent); for example, $y = 2^x$

expression

either a single term or the sum or difference of two or more terms

face

a flat surface on a 3D object

factor form

algebraic expression written as the product of its factors, as opposed to expanded form

factor tree

a tree diagram where composite numbers are broken down into factor pairs until prime numbers are found

factorise

to write an algebraic expression as the product of its factors; for example, $6x + 3$ factorises to $3(2x + 1)$ where the two factors are 3 and $2x + 1$

five-number summary

five pieces of information related to a set of numerical data: minimum value, lower quartile, median, upper quartile and maximum value

gradient (m)

numerical measure of the slope of a graph

$$\text{gradient} = m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

gradient-intercept form

general form of a linear equation, $y = mx + c$, where m is the gradient and c is the y -coordinate of the y -intercept of the corresponding linear graph

highest common factor (HCF)

greatest factor that is common to two or more given numbers

histogram

type of bar chart that can be used to display grouped continuous or discrete numerical data where the bounds of each class interval are located at the edges of the columns

image

the result of transforming a shape or an object

included angle

the angle between two given sides

income

money received over a set period of time in exchange for work or investment

income tax

the tax which is paid on an individual's taxable income, calculated by using a tax table or online tax calculator

independent event

an event whose outcomes are not affected by the result of any other event

index

for a value expressed in index notation, the index indicates the number of times the base is written as a repeated multiplication

index notation (index form)

shorter form of writing a repeated multiplication, where a number is written with a base and an index; for example, 2^4 is written in index notation

inequality

a mathematical statement that compares the values of two expressions

interest

money which is paid for lending money, or money which is charged for borrowing money

interest rate

the proportion of a loan that is charged as interest to the borrower, or the proportion of a deposit that is given as interest to the investor

interquartile range (IQR)

a measure of spread for a set of data, calculated by subtracting the lower quartile (Q_1) from the upper quartile (Q_3); that is, $IQR = Q_3 - Q_1$. The interquartile range indicates the spread or range of the middle 50% of the data

inverse cosine (\cos^{-1})

used to obtain an angle from its cosine value

inverse operation

operation that reverses the effect of a previous operation; for example, addition and subtraction are inverse operations

inverse sine (\sin^{-1})

used to obtain an angle from its sine value

inverse tangent (\tan^{-1})

used to obtain an angle from its tangent value

investment

a deposit of money, where interest is earned on the amount of money deposited

isometric transformation

a transformation that preserves the original size and shape of the object being transformed

law of large numbers

a law that states as the number of trials of an experiment increases, the relative frequency is likely to approach the theoretical probability

lay-by agreement

a contract where a consumer pays for goods in two or more instalments and does not receive the goods until the full price has been paid

leading term

the term with the highest power of a variable in an expression

leave loading

an extra payment on top of annual leave that is usually calculated as 17.5% of normal pay for this period

level of accuracy

the closeness of an approximation to the exact value

like terms

terms that contain the same pronumerals

line symmetry

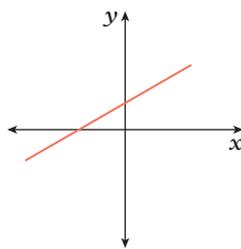
a plane shape has line symmetry if it can be divided into two halves that are mirror images of each other

linear equation

equation that contains a pronumeral term or terms where the highest power is 1 and no two pronumerals are multiplied together; for example, $2x + 3 = 6$

linear graph

straight line graph of a linear relationship

**linear inequality**

an inequality containing pronumerals raised only to the power of 1 and where no two pronumerals are multiplied together; for example, $2x + 3 > 1$

linear relationship

relationship between two variables that produces a linear graph

linear term

the term with a variable of power 1 in an expression; for example, $4x$ is the linear term in $3x^2 + 4x - 1$

loan

a borrowing of money, with interest paid on the amount of money borrowed

lower bound

- 1 abbreviation for lower boundary; the lowest value in a class interval
- 2 the boundary of values that round up to the given measurement

lower fence

the lower end point in a data set beyond which any smaller data points are considered outliers. The lower fence lies $1.5 \times IQR$ below the lower quartile.
see also interquartile range (IQR)

lower quartile

the median of the first half (or lower half) of an ordered set of numerical data, denoted by Q_1

maximum turning point

a point at which a parabola changes direction and has its maximum y -value

mean

a measure of the centre of a data set (numerical average) calculated by adding all the data values and then dividing by the number of data values

measure of spread

number used to indicate how spread out a data set is. The range, interquartile range and standard deviation are measures of spread

median

a measure of the centre of a data set. For an ordered set of data, it is the middle number if there is an odd number of values, and the average of the two middle numbers if there is an even number of values

Medicare levy

a 2% tax on an individual's taxable income that helps to fund the health care system

metric prefix

a selection of letters which can be added to the start of base units of measurement to change their meaning, creating new smaller or larger units of measurement

midpoint

halfway point or middle point of an interval or line segment

minimum turning point

a point at which a parabola changes direction and has its minimum y -value

minutes

a unit to measure angles. There are 60 minutes in one degree

monic quadratic equation

a quadratic equation in which the leading coefficient is equal to 1

monic quadratic trinomial

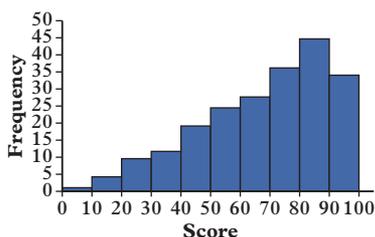
a quadratic trinomial in which the leading coefficient is equal to 1

negative index

index with a negative value indicating repeated division

negatively skewed distribution

describes a distribution that is skewed away from the vertical axis with a tail to the left



net

2D plan that can be folded to form a three-dimensional object

Null Factor Law

states that if the product of two factors is 0, then one or both of the factors must equal 0

opposite side

(trigonometry) the side opposite the reference angle in a right-angled triangle

order of line symmetry

the number of axes of symmetry a plane shape has

order of rotational symmetry

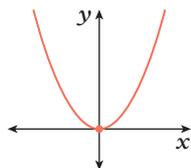
the number of times a plane shape fits exactly onto itself when being rotated through 360°

outlier

extreme piece of data that is much higher or lower than the rest of the data set

parabola

the graph of a quadratic relationship



parallel lines

lines (or rays or segments) that are always the same distance apart so that they never meet. Parallel lines are labelled with matching arrowheads

parallel box plot

two or more box plots placed directly above each other on the same number line for comparison

Pay-As-You-Go (PAYG)

a tax system whereby the approximate income tax owed by an individual is withheld from their pay, to help ensure they fulfil their tax obligations

penalty rates

higher rates of pay that are given to employees for working particular hours or days, such as weekends, public holidays or overtime

percentage error

the ratio between the absolute error of a measuring instrument and the actual measurement expressed as a percentage

$$\text{percentage error} = \frac{\text{absolute error}}{\text{measurement}} \times 100\%$$

perpendicular lines

a description of two lines (or rays, or segments) that meet to form a right angle (90°). Perpendicular lines are labelled using a small box at the right angle

piece work

income which is paid for each item (or group of items) produced or action performed

point of focus

a point to concentrate on when reflecting or rotating a plane shape

plot

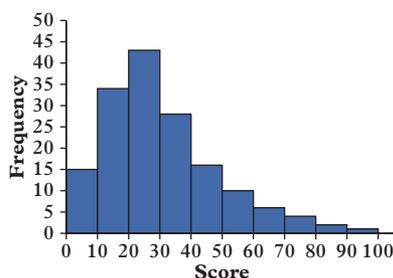
graphical representation of a relationship consisting of individual coordinate points

population

all potential pieces of data under consideration

positively skewed distribution

describes a distribution that is skewed towards the vertical axis with a tail to the right



precision

level of consistency of a set of measurements taken using the same instrument

prime factorisation

the product of all the prime factors of a positive integer

principal

the original amount of money borrowed (for a loan) or deposited (for an investment)

prism

3D object with two ends that are identical polygons and joined by straight edges

pronumeral

letter or symbol that takes the place of a number

quadratic equation

an equation that contains a pronumeral term or terms with a highest power of 2; for example, $x^2 + 3x - 2 = 10$. The general form of a quadratic equation is $ax^2 + bx + c = 0$, where a , b and c are constants

quadratic expression

an algebraic expression that contains a squared pronumeral, with no exponents greater than 2 in the expression

quadratic trinomial

expression of the form $ax^2 + bx + c$, where a , b and c are constants

quartile

the values that divide a numerical data set into four quarters

quotient

the result of dividing one number or algebraic expression by another

range

a measure of spread determined by finding the difference between the highest and lowest values in a data set

rate of change

a measure of how a quantity changes for each unit of change in another quantity

reciprocal

the reciprocal of a number is 1 divided by that number. The product of a number and its reciprocal is 1; for example, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$

rectangular prism

prism with three pairs of identical faces (total of six faces)

reduction

smaller image produced after a figure has been dilated

reflection

transformation where a shape or object is reflected (flipped) across a mirror line to produce its exact mirror image

relative frequency

the number of times a favourable outcome occurs divided by the total number of outcomes

$$\text{relative frequency} = \frac{\text{number of occurrences}}{\text{total number of outcomes}}$$

right prism

prism with sides that make right angles with its base

RHS

abbreviation for right angle–hypotenuse–side. Right-angled triangles are congruent if their hypotenuses are the same length and another pair of side lengths are equal, and are similar if the lengths of their hypotenuses are in the same ratio as the lengths of another pair of sides

rotation

transformation where a shape is turned about a fixed point by a given number of degrees in either a clockwise or anticlockwise direction

rotational symmetry

a plane shape has rotational symmetry if it fits exactly onto itself after being rotated less than 360°

rounding

the process of replacing a number with an approximation that is easier to interpret and use in calculations

royalty

a payment made to a person or company for the use of their assets

salary

a fixed income which is paid to an employee on a regular basis

sample

a selection of data that is part of a population

sample space

list of all the different outcomes possible for a probability experiment; for example, the sample space when rolling a six-sided die is $\{1, 2, 3, 4, 5, 6\}$

SAS

abbreviation for side–angle–side. Triangles are congruent if two corresponding side lengths are equal and the angle between them is equal, and are similar if two corresponding side lengths are in the same ratio and the angle between them is equal

scale

the ratio between the drawing length and the actual length in a scale drawing

scale drawing

a drawing that accurately represents a real-life object, but which is drawn to a different scale

scale factor

indicates how many times larger or smaller an image is after a figure has been dilated

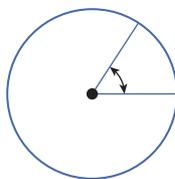
$$\text{scale factor} = \frac{\text{image length}}{\text{original length}}$$

scientific notation

a value written in the form $a \times 10^m$, where $1 \leq a < 10$ or $-10 < a \leq -1$ and m is an integer

sector

portion of a circle formed by two radii and part of the circumference



significant figures

the number of digits required to express a number to a specified degree of accuracy

similar

figures with the same shape but not the same size. Similar figures are produced by a dilation

simple interest

interest which is calculated on the principal which has been borrowed or invested

simulation

using technology or simple devices like coins or dice to simulate real-life events, making performing a large number of trials in a probability experiment easier

sine (sin)

the sine of an angle in a right-angled triangle equals the ratio of the length of the opposite side to the length of the hypotenuse

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

sketch

a simple drawing of a graph in which main features such as intercepts and turning points are labelled

skewed distribution

describes a data set where data points are clustered to the left or right of the distribution

solution

value of the pronumeral that makes an equation a true statement

SSS

abbreviation for side–side–side. Triangles are congruent if all three side lengths are equal, and are similar if all pairs of corresponding side lengths are in the same ratio

standard deviation

a measure of spread representing the difference between individual data values and the mean of a numerical data set

standard form

see scientific notation

surface area

area of the outer surface of an object. The surface area of a prism is the sum of the areas of each face of the prism

symmetrical (symmetric) distribution

describes a data set with a peak in the middle and with data points roughly evenly spread on either side of the peak

tangent (tan)

the tangent of an angle in a right-angled triangle equals the ratio of the length of the opposite side to the length of the adjacent side

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

tax deductions

expenses that can be subtracted from a person's total income, such as work-related expenses, or gifts and donations

tax tables

tables that can be used to calculate the exact amount of tax that should be withheld from an individual given an exact amount of earnings for a specified period of time

taxable income

an individual's total income minus any tax deductions, which is used to calculate the amount of income tax payable

term

a single pronumeral or number, or the product of a number and one or more pronumerals

tessellation

a pattern of repeating shapes that has no overlaps or gaps

theoretical probability

probability determined by mathematical reasoning, not experiments

transformation

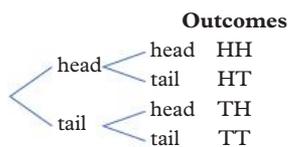
a manipulation of a shape or object

translation

a transformation which moves a point, line or shape without turning or changing the size

tree diagram

a branched display of outcomes for a multi-step experiment; for example, the outcomes when two coins are tossed

**triangular prism**

3D object with five faces consisting of two identical triangular faces whose respective vertices are joined by parallel lines

trigonometry

the study of relationships between angles and side lengths of a triangle

trinomial

an algebraic expression that has three non-zero terms and has more than one variable in the expression

truncate

to cut a number short; for example, to truncate a number after 2 decimal places all digits after that place are removed, with no other digits changing

turning point

the point where a parabola changes direction

unknown

a quantity or measurement required in a problem that has one possible value only and is usually represented by a pronumeral

upper bound

- 1 abbreviation for upper boundary; the largest value in a class interval
- 2 the boundary of values that will round down to the given measurement

upper fence

the upper end point in a data set beyond which any greater data points are considered outliers. The upper fence lies $1.5 \times \text{IQR}$ above the upper quartile
see also interquartile range (IQR)

upper quartile

the median of the upper half of an ordered set of numerical data, denoted by Q_3

variable

quantity that can have different values and can be represented with a word, symbol or pronumeral

vary directly

two variables are said to vary directly if $y = kx$, where k is the constant of variation, shown using the symbol \propto

vertex

see turning point

vertical translation

transformation in which a shape or object is translated (moved) up or down in a straight line without turning or changing size

volume

amount of space that a three-dimensional (3D) object occupies (cm^3 , m^3)

volume scale factor

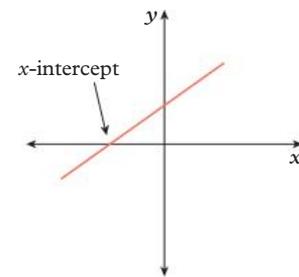
the scale factor between the volumes of two similar objects

wage

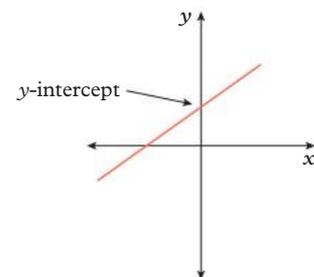
a fixed rate of income an employee receives for working a given time period, for example \$25/hour

x-intercept

point where a line crosses the x -axis of a Cartesian plane

**y-intercept**

point where a line crosses the y -axis of a Cartesian plane



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Pilots need to apply trigonometry when they are making course corrections due to wind. There is a lot of mental maths to be done in the air!