CambridgeMATHS NSW

Stage 5 – Core & Advanced/Extension Paths

THIRD EDITION

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INCLUDES INTERACTIVE TEXTBOOK POWERED BY CAMBRIDGE HOTMATHS

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About the authors

Stuart Palmer was born and educated in NSW. He is a fully qualified high school mathematics teacher with more than 25 years' experience teaching students from all walks of life in a variety of schools. He has been Head of Mathematics in two schools. He is very well known by teachers throughout the state for the professional learning workshops he delivers. Stuart also assists thousands of Year 12 students every year as they prepare for their HSC Examinations. At the University of Sydney, Stuart spent more than a decade running tutorials for pre-service mathematics teachers.

David Greenwood is the Head of Mathematics at Trinity Grammar School in Melbourne and has 30+ years' teaching mathematics from Years 7 to 12. He is the lead author for the Cambridge Essential series and has authored more than 80+ titles for the Australian Curriculum and for the syllabuses of other states. He specialises in analysing curriculum and the sequencing of course content for school mathematics courses. He also has an interest in the use of technology for the teaching of mathematics.

Sara Woolley was born and educated in Tasmania. She completed an Honours degree in Mathematics at the University of Tasmania before completing her education training at the University of Melbourne. She has taught mathematics from Years 7 to 12 since 2006 and is currently a Head of Mathematics. She specialises in lesson design and creating resources that develop and build understanding of mathematics for all students.

Jenny Goodman has taught in schools for over 28 years and is currently teaching at a selective high school in Sydney. Jenny has an interest in the importance of literacy in mathematics education, and in teaching students of differing ability levels. She was awarded the Jones Medal for education at Sydney University and the Bourke Prize for Mathematics. She has written for *CambridgeMATHS NSW* and was involved in the *Spectrum* and *Spectrum Gold* series.

Jennifer Vaughan has taught secondary mathematics for over 30 years in New South Wales, Western Australia, Queensland and New Zealand, and has tutored and lectured in mathematics at Queensland University of Technology. She is passionate about providing students of all ability levels with opportunities to understand and to have success in using mathematics. She has had extensive experience in developing resources that make mathematical concepts more accessible; hence, facilitating student confidence, achievement and an enjoyment of maths.

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Introduction

The third edition of CambridgeMATHS NSW Stages 4 and 5 has been carefully prepared for the NSW Syllabus implemented from 2024 onwards. The series and is packed with new features. It is intended for students across a wide range of ability levels and has been designed to provide the best possible preparation for students as they make their way from the end of Stage 3 towards Stage 6 mathematics.

Coverage of Core and optional Path topics in Stage 5

As with the previous editions, Stage 5 is covered with two differentiated pairs of books:

- The *Core & Advanced/Extension Paths* books replace the Stage 5.1/5.2/5.3 books
- The *Core & Standard Paths* books replace the old Stage 5.1/5.2 books

Care has been taken to choose the content in each book carefully and to present it in a way that reflects the spirit of the new syllabus as well as the practicalities of the classroom. 'Paths', in the plural, refers to the fact that the Path topics are optional, and that a student's journey through Stage 5 may incorporate the core content in combination with none, some or all the Path topics in the books, depending upon the student's needs. Detailed teaching programs and Scope and Sequence documents are available via the Online Teaching Suite.

Beyond Stage 5

Students who have used these *Core & Advanced/Extension Paths* books in Years 9 and 10 will be thoroughly prepared for Year 11 Mathematics Advanced, and possibly Extension if the appropriate sections are covered. Our treatment of all the algebra topics empowers students to make a flying start in the calculus-based Stage 6 courses. We believe this to be one of the main strengths of our books.

Learning Intentions, Success Criteria checklist, and 'Past, present and future learning'

Every lesson now includes a set of Learning Intentions that describe what the student can expect to learn in the lesson. At the end of the chapter, these appear again in the form of a checklist of Success Criteria; students can use this to check their progress through the chapter. Every criterion is listed with an example question to remind students of what the mathematics mentioned looks like. These checklists can also be downloaded and printed so that students can physically check them off as they accomplish their goals.

Also included at the beginning of each section, under the Learning Intentions, is a short set of dot points that locate the topic at hand in the context of **past, present and future learning**.

Now you try

Every worked example now contains additional questions, without solutions, called 'Now you try', which give students immediate practice at the same type of question as the example. We also anticipate these questions will be useful for the teacher to do in front of the class, given that students will not have seen the solution beforehand.

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Building Understanding and changes to the exercise structure

To improve the flow of ideas from the beginning of each lesson through to the end of the exercise, a few structural changes have been made in each lesson for the *Core & Advanced/Extension Paths* books. First, the Understanding questions have been taken out of the exercise, simplified into discussion-style questions, and placed immediately after the Key ideas. These questions are now called 'Building Understanding' and are intended to consolidate the skills and concepts covered by the Key ideas, which students will then encounter in the worked examples. Each exercise now starts at Fluency, and the first question in each exercise has been revised to ensure that it links directly to the first worked example in the lesson. The exercise then continues as before through Problem-solving, Reasoning and Enrichment.

Working Mathematically tasks

A Working Mathematically task now accompanies an Investigation in each chapter, with the goal of familiarising students with using the Working Mathematically process to define, solve, verify and then communicate their solutions when solving non-routine problems.

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Guide to the working programs in exercises

The suggested working programs in the exercises in this book provide three pathways to allow differentiation for Foundation, Standard and Advanced students (schools will likely have their own names for these levels).

Each exercise is structured in subsections that match the mathematical proficiencies of Fluency, Problem-solving and Reasoning, as well as Enrichment (Challenge). (Note that Understanding is covered by 'Building understanding' in each lesson.) In the exercises, the questions suggested for each pathway are listed in three columns at the top of each subsection:

- The left column (lightest shaded colour) is the Foundation pathway
- The middle column (medium shaded colour) is the Standard pathway
- The right column (darkest shaded colour) is the Advanced pathway.

Foundation	Standard	Advanced
FLUENCY		
1, 2–4(½)	2–5(½)	2–5(½)
PROBLEM-SOLVING		
6, 7	6–8	7–9
REASONING		
10	10–12	12–14
ENRICHMENT		
–	–	15

The working program for Exercise 3A in Year 7. The questions recommended for a Foundation student are: 1, 2, 3(½), 4, 6 and 9.

Gradients within exercises and proficiency strands

The working programs make use of the two difficulty gradients contained within exercises. A gradient runs through the overall structure of each exercise – where there is an increasing level of mathematical sophistication required from Fluency to Problem-solving to Reasoning and Enrichment – but also within each proficiency; the first few questions in Fluency, for example, are easier than the last Fluency question.

The right mix of questions

Questions in the working programs are selected to give the most appropriate mix of *types* of questions for each learning pathway. Students going through the Foundation pathway should use the left tab, which includes all but the hardest Fluency questions as well as the easiest Problem-solving and Reasoning questions. An Advanced student can use the right tab, proceed through the Fluency questions (often half of each question), and have their main focus be on the Problem-solving and Reasoning questions, as well as the Enrichment questions. A Standard student would do a mix of everything using the middle tab.

Choosing a pathway

There are a variety of ways to determine the appropriate pathway for students through the course. Schools and individual teachers should follow the method that works for them. If required, the prior-knowledge pre-tests (now found online) can be used as a tool for helping students select a pathway. The following are recommended guidelines:

- A student who gets 40% or lower should complete the Foundation questions
- A student who gets between 40% and below 85% should complete the Standard questions
- A student who gets 85% or higher should complete the Advanced questions.

* The nomenclature used to list questions is as follows:

- 3, 4: complete all parts of questions 3 and 4
- 1-4: complete all parts of questions 1, 2, 3 and 4
- 10(½): complete half of the parts from question 10 (a, c, e, ... or b, d, f, ...)
- 2-4(½): complete half of the parts of questions 2, 3 and 4
- 4(½), 5: complete half of the parts of question 4 and all parts of question 5
- —: do not complete any of the questions in this section.

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Guide to this resource

PRINT TEXTBOOK FEATURES

1 **NSW Syllabus:** content strands, outcome groups and outcomes are listed at the beginning of the chapter to assist with course planning (see teaching programs and Syllabus mapping grids for more detailed guidance)

2 **In this chapter:** an overview of the chapter contents

3 **Chapter introduction:** sets context for students about how the topic connects with the real world and the history of mathematics

4 NEW **Learning intentions:** sets out what a student will be expected to learn in the lesson

5 NEW **Past, present and future learning** connects the lesson to related topics that students have covered or will cover in the NSW Syllabus

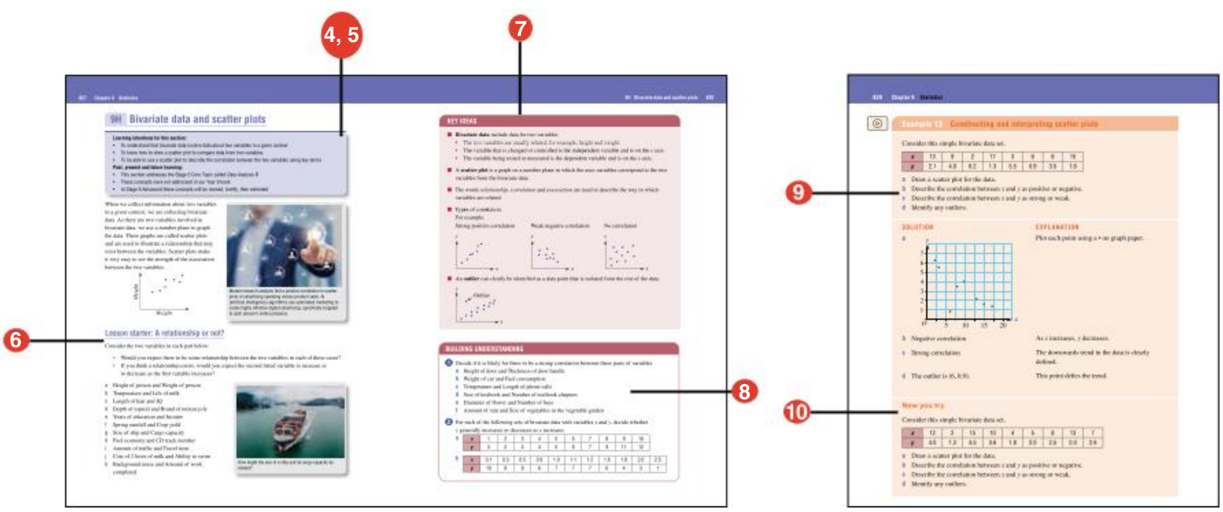
6 **Lesson starter:** an activity, which can often be done in groups, to start the lesson

7 **Key ideas:** summarises the knowledge and skills for the lesson

8 NEW **Building understanding:** a small set of discussion questions to consolidate understanding of the Key ideas (replaces Understanding questions formerly inside the exercises)

9 **Worked examples:** solutions and explanations of each line of working, along with a description that clearly describes the mathematics covered by the example

10 NEW **Now you try:** try-it-yourself questions provided after every worked example in exactly the same style as the worked example to give immediate practice



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11 **Revised exercise structure:** the exercise now begins at Fluency, with the first question always linked to the first worked example in the lesson

12 **Working programs:** differentiated question sets for three ability levels in exercises

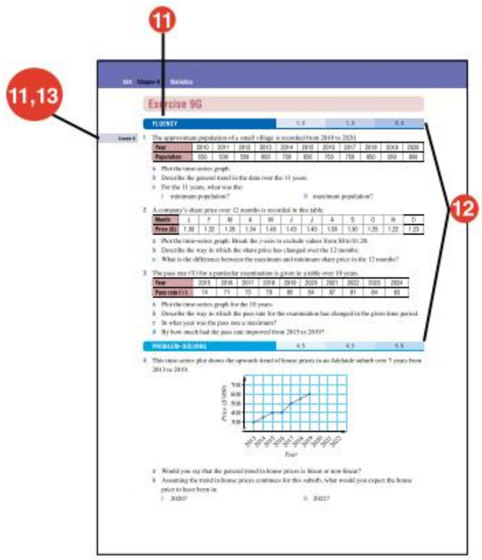
13 **Example references:** show where a question links to a relevant worked example – the first question is always linked to the first worked example in a lesson

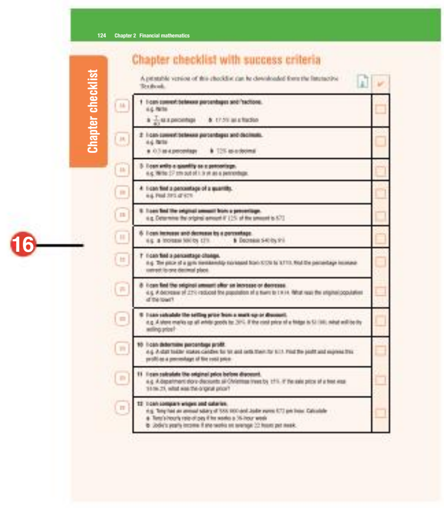
14 **Problems and challenges:** in each chapter provides practice with solving problems connected with the topic

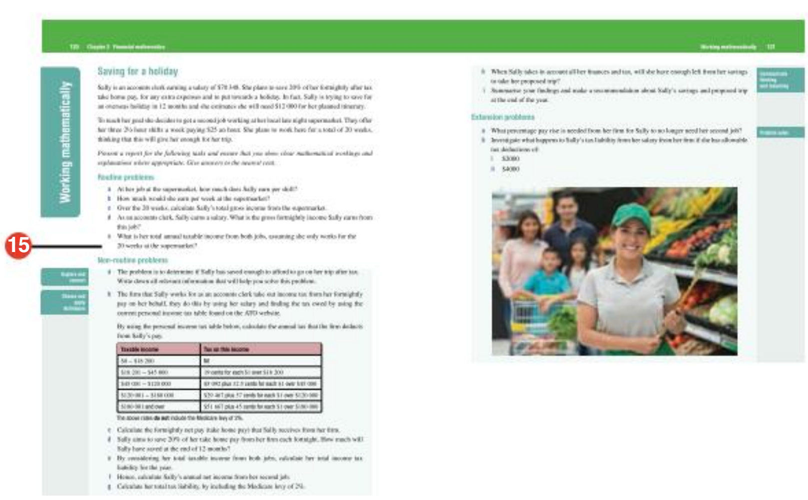
15 NEW **Working mathematically** investigative tasks in each chapter give students the opportunity to apply their learning to solve non-routine problems and practise for Stage 6 investigative-style assignments

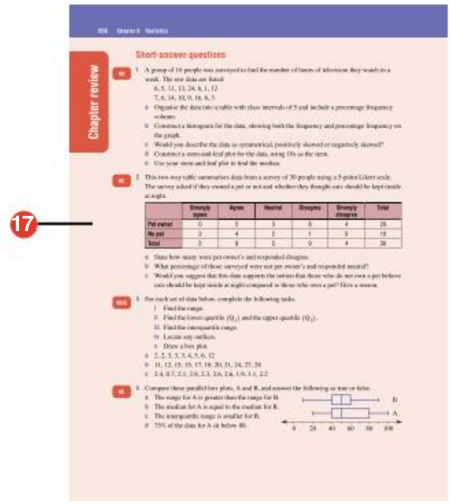
16 NEW **Success criteria checklist:** a checklist of the learning intentions for the chapter, with example questions

17 **Chapter reviews:** with short-answer, multiple-choice and extended-response questions; questions that are extension are clearly signposted









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INTERACTIVE TEXTBOOK FEATURES

18 NEW **Targeted Skillsheets,** one for each lesson, focus on a small set of related Fluency-style skills for students who need extra support, with questions linked to worked examples

19 **Workspaces:** almost every textbook question – including all working-out – can be completed inside the Interactive Textbook by using either a stylus, a keyboard and symbol palette, or uploading an image of the work

20 **Self-assessment:** students can then self-assess their own work and send alerts to the teacher. See the Introduction on page xii for more information.

21 **Interactive question tabs** can be clicked on so that only questions included in that working program are shown on the screen

22 **HOTmaths resources:** a huge catered library of widgets, HOTsheets and walkthroughs seamlessly blended with the digital textbook

23 A revised set of **differentiated auto-marked practice quizzes** per lesson with saved scores

24 **Scorcher:** the popular competitive game

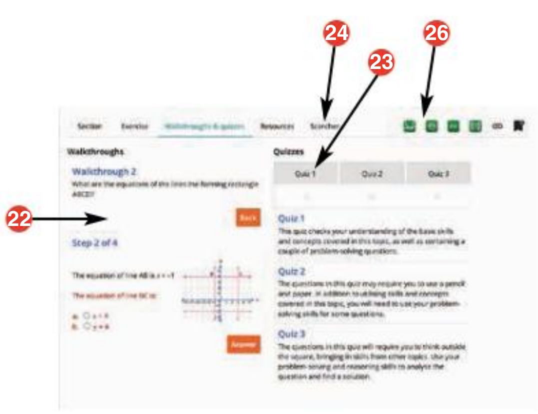
25 **Worked example videos:** every worked example is linked to a high-quality video demonstration, supporting both in-class learning and the flipped classroom

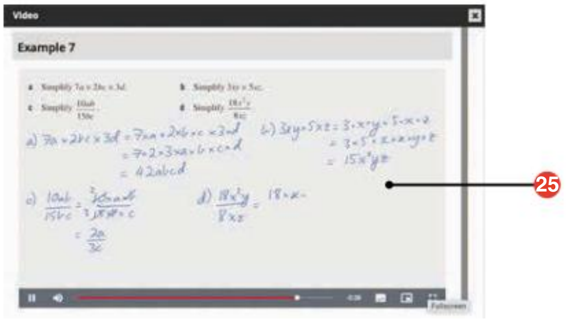
26 **Desmos graphing calculator**, scientific calculator and geometry tool are always available to open within every lesson

27 **Desmos interactives:** a set of Desmos activities written by the authors allow students to explore a key mathematical concept by using the Desmos graphing calculator or geometry tool

28 **Auto-marked prior knowledge pre-test** for testing the knowledge that students will need before starting the chapter

29 **Auto-marked progress quizzes and chapter review multiple-choice questions** in the chapter reviews can now be completed online





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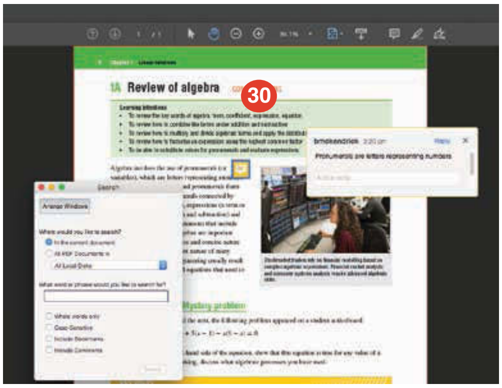
DOWNLOADABLE PDF TEXTBOOK



30 In addition to the Interactive Textbook, a **PDF version of the textbook** has been retained for times when users cannot go online. PDF search and commenting tools are enabled.

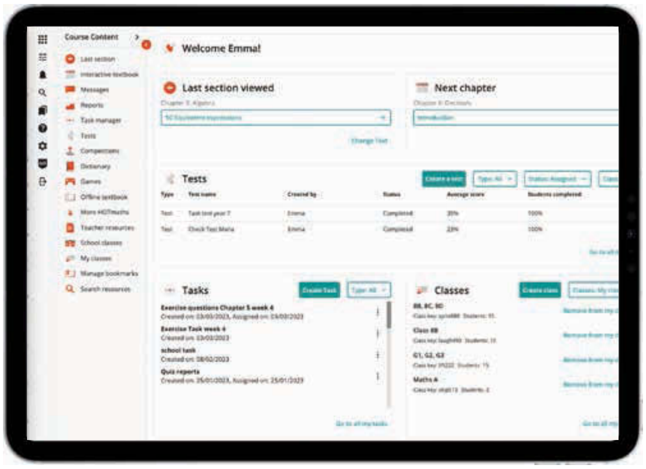
ONLINE TEACHING SUITE

31 NEW **Diagnostic Assessment Tool** included with the Online Teaching Suite allows for flexible diagnostic testing, reporting and recommendations for follow-up work to assist you to help your students to improve

32 NEW **PowerPoint lesson** summaries contain the main elements of each lesson in a form that can be annotated and projected in front of class

33 **Learning Management System** with class and student analytics, including reports and communication tools

34 **Teacher view of student's work and self-assessment** allows the teacher to see their class's workout, how students in the class assessed their own work, and any 'red flags' that the class has submitted to the teacher



35 **Powerful test generator** with a huge bank of levelled questions as well as ready-made tests

36 **Revamped task manager** allows teachers to incorporate many of the activities and tools listed above into teacher-controlled learning pathways that can be built for individual students, groups of students and whole classes

37 **Worksheets and four differentiated chapter tests in every chapter,** provided in editable Word documents

38 **More printable resources:** all Pre-tests, Progress quizzes and Applications and problem-solving tasks are provided in printable worksheet versions

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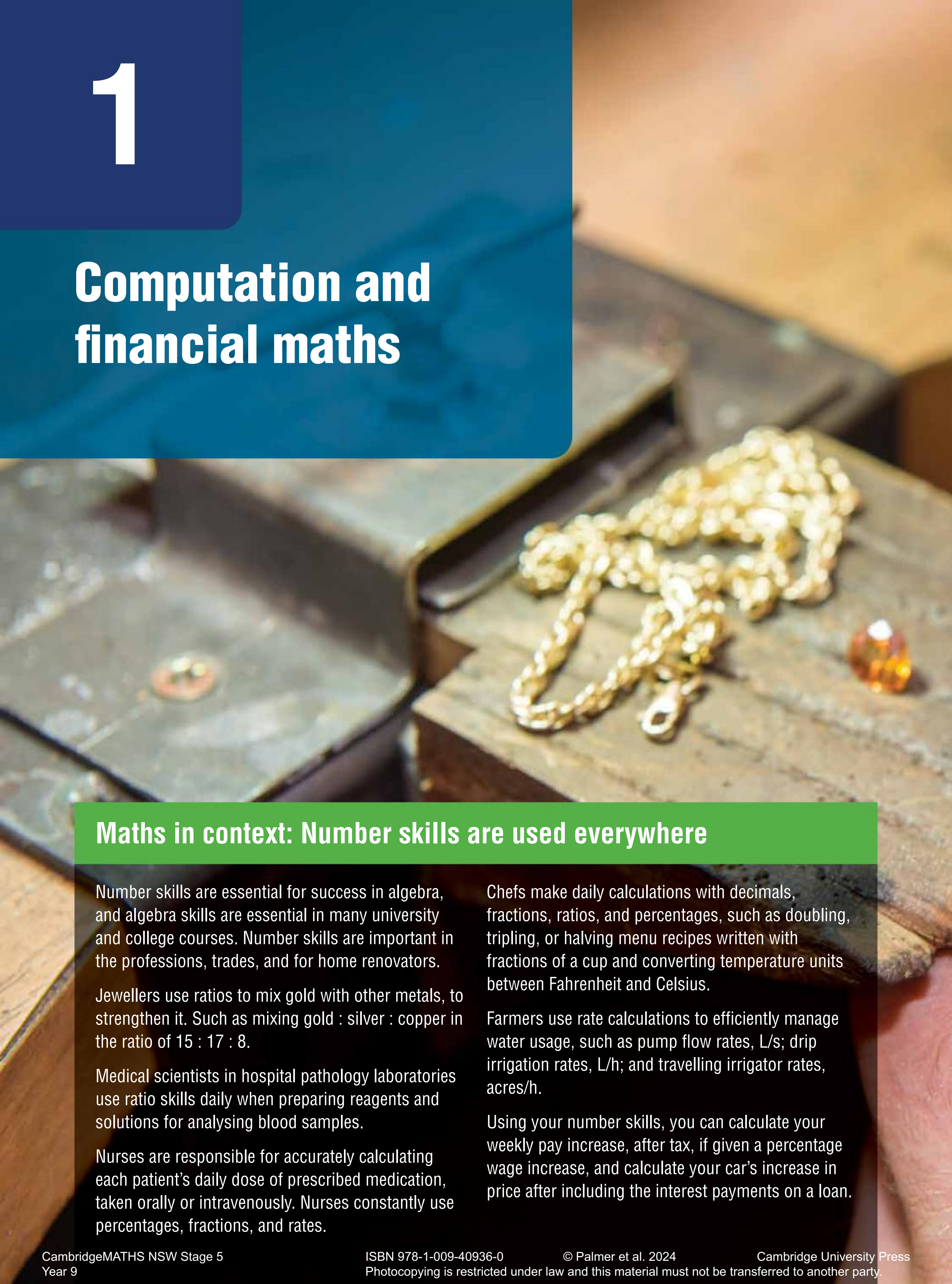

1

Computation and financial maths

Maths in context: Number skills are used everywhere

Number skills are essential for success in algebra, and algebra skills are essential in many university and college courses. Number skills are important in the professions, trades, and for home renovators.

Jewellers use ratios to mix gold with other metals, to strengthen it. Such as mixing gold : silver : copper in the ratio of 15 : 17 : 8.

Medical scientists in hospital pathology laboratories use ratio skills daily when preparing reagents and solutions for analysing blood samples.

Nurses are responsible for accurately calculating each patient's daily dose of prescribed medication, taken orally or intravenously. Nurses constantly use percentages, fractions, and rates.

Chefs make daily calculations with decimals, fractions, ratios, and percentages, such as doubling, tripling, or halving menu recipes written with fractions of a cup and converting temperature units between Fahrenheit and Celsius.

Farmers use rate calculations to efficiently manage water usage, such as pump flow rates, L/s; drip irrigation rates, L/h; and travelling irrigator rates, acres/h.

Using your number skills, you can calculate your weekly pay increase, after tax, if given a percentage wage increase, and calculate your car's increase in price after including the interest payments on a loan.

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Chapter contents

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- solves financial problems involving simple interest, earning money and spending money (MA5-FIN-C-01)
- solves financial problems involving compound interest and depreciation (MA5-FIN-C-02).

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

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1A Computations with integers CONSOLIDATING

Learning intentions
- To know the rules for the mathematical operations applied to negative numbers
- To understand the notation for powers and roots
- To be able to evaluate operations on integers including negative numbers, powers and roots and apply order of operations

Past, present and future learning
- This section consolidates Stage 4 concepts which are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- In Chapter 2 of this book, we apply this learning to algebraic expressions
- Expertise with integers is assumed knowledge for Stage 6 Advanced and may also be required in non-calculator examinations such as NAPLAN and industry aptitude tests

Throughout history, mathematicians have developed number systems to investigate and explain the world in which they live. The Egyptians used hieroglyphics to record whole numbers as well as fractions, the Babylonians used a place-value system based on the number 60, and the ancient Chinese and Indians developed systems using negative numbers. Our current base 10 decimal system (the Hindu–Arabic system) has expanded to include positive and negative numbers, fractions (rational numbers), as well as numbers that cannot be written as a fraction (irrational numbers), for example, π and $\sqrt{2}$. All the numbers in our number system, not including imaginary numbers, are called real numbers.

The abacus is an efficient calculating tool and was widely used for thousands of years across China and Europe.

Lesson starter: Special sets of numbers

Here are some special groups of numbers. Can you describe what special property each group has? Try to use the correct vocabulary, for example, factors of 12.

- 7, 14, 21, 28, …
- 1, 4, 9, 16, 25, …
- 1, 2, 3, 4, 6, 9, 12, 18, 36.
- 1, 8, 27, 64, 125, …
- 0, 1, 1, 2, 3, 5, 8, 13, …
- 2, 3, 5, 7, 11, 13, 17, 19, …

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KEY IDEAS

- The **integers** include … , −3, −2, −1, 0, 1, 2, 3, …
- **The rules of negative numbers**
  - $a + (-b) = a - b$ For example: $5 + (-2) = 5 - 2 = 3$
  - $a - (-b) = a + b$ For example: $5 - (-2) = 5 + 2 = 7$
  - $a \times (-b) = -ab$ For example: $3 \times (-2) = -6$
  - $-a \times (-b) = ab$ For example: $-4 \times (-3) = 12$
  - $a \div (-b) = -\frac{a}{b}$ For example: $8 \div (-4) = -2$
  - $-a \div (-b) = \frac{a}{b}$ For example: $-8 \div (-4) = 2$
- **Squares and cubes**
  - $a^2 = a \times a$ and $\sqrt{a^2} = a$ (if $a \geqslant 0$) For example: $6^2 = 36$ and $\sqrt{36} = 6$
  - $a^3 = a \times a \times a$ and $\sqrt[3]{a^3} = a$ For example: $4^3 = 64$ and $\sqrt[3]{64} = 4$
- **LCM, HCF and primes**
  - The **lowest common multiple** (LCM) of two numbers is the smallest multiple shared by both numbers. For example: the LCM of 6 and 9 is 18.
  - The **highest common factor** (HCF) of two numbers is the largest factor shared by both numbers. For example: the HCF of 24 and 30 is 6.
  - **Prime numbers** have only two factors: 1 and the number itself. The number 1 is not considered a prime number.
  - **Composite numbers** have more than two factors.
- **Order of operations**
  - Deal with brackets first.
  - Deal with indices next.
  - Do multiplication and division next from left to right.
  - Do addition and subtraction last from left to right.

$$\begin{aligned} &5 \times (3 - (-2)^2) + 3^3 \div (-9) \\ &= 5 \times (3 - 4) + 27 \div (-9) \\ &= 5 \times (-1) + (-3) \\ &= -5 + (-3) \\ &= -8 \end{aligned}$$

BUILDING UNDERSTANDING

1 State these sets of numbers.

a The factors of 56
b The HCF (highest common factor) of 16 and 56
c The first 7 multiples of 3
d The LCM (lowest common multiple) of 3 and 5
e The first ten prime numbers starting from 2

2 Evaluate the following.

a 11^2 **b** $\sqrt{144}$ **c** 3^3 **d** $\sqrt[3]{8}$

3 Evaluate the following.

a $-3 + 2$ **b** $2 + (-3)$ **c** $11 - (-4)$
d $2 \times (-3)$ **e** $-11 \times (-2)$ **f** $18 \div (-2)$

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Example 1 Operating with integers

Evaluate the following.

a $-2-(-3\times 13)+(-10)$ **b** $(-20\div(-4)+(-3))\times 2$ **c** $\sqrt[3]{8}-(-1)^2+3^3$

SOLUTION

a $$\begin{aligned}-2-(-3\times 13)+(-10) &= -2-(-39)+(-10)\\ &= -2+39+(-10)\\ &= 37-10\\ &= 27\end{aligned}$$

b $$\begin{aligned}(-20\div(-4)+(-3))\times 2 &= (5+(-3))\times 2\\ &= 2\times 2\\ &= 4\end{aligned}$$

c $$\begin{aligned}\sqrt[3]{8}-(-1)^2+3^3 &= 2-1+27\\ &= 28\end{aligned}$$

EXPLANATION

Deal with the operations in brackets first.

$-a-(-b)=-a+b$

$a+(-b)=a-b$

$-a\div(-b)=\frac{a}{b}$

Deal with the operations inside brackets before doing the multiplication.

$5+(-3)=5-3$

Evaluate powers first.

$\sqrt[3]{8}=2$ since $2^3=8$

$(-1)^2=-1\times(-1)=1$

$3^3=3\times 3\times 3=27$

Now you try

Evaluate the following.

a $-4-(-1\times 12)+(-3)$ **b** $(-24\div(-6)+(-2))\times 2$ **c** $\sqrt[3]{27}-(-2)^2-4^3$

Exercise 1A

FLUENCY	1–4(½)	1–5(½)	1(¼), 2–5(½)

Example 1a,b

1 Evaluate the following showing your steps.

a $-4-3\times(-2)$
b $-3\times(-2)+(-4)$
c $-2\times(3-8)$
d $2-7\times(-2)$
e $2-3\times 2+(-5)$
f $4+8\div(-2)-3$
g $(-24\div(-8)+(-5))\times 2$
h $-7-(-4\times 8)-15$
i $-3-12\div(-6)\times(-4)$
j $4\times(-3)\div(-2\times 3)$
k $(-6-9\times(-2))\div(-4)$
l $10\times(-2)\div(-7-(-2))$
m $6\times(-5)-14\div(-2)$
n $(-3+7)-2\times(-3)$
o $-2+(-4)\div(-3+1)$
p $-18\div((-2-(-4))\times(-3))$
q $-2\times 6\div(-4)\times(-3)$
r $(7-14\div(-2))\div 2$
s $2-(1-2\times(-1))$
t $20\div(6\times(-4\times 2)\div(-12)-(-1))$

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2 Find the LCM (lowest common multiple) of these pairs of numbers.

a 4, 7 **b** 8, 12 **c** 11, 17 **d** 15, 10

3 Find the HCF (highest common factor) of these pairs of numbers.

a 20, 8 **b** 100, 65 **c** 37, 17 **d** 23, 46

Example 1c

4 Evaluate the following.

a $2^3 - \sqrt{16}$ **b** $5^2 - \sqrt[3]{8}$ **c** $(-1)^2 \times (-3)$

d $(-2)^3 \div (-4)$ **e** $\sqrt{9} - \sqrt[3]{125}$ **f** $1^3 + 2^3 - 3^3$

g $\sqrt[3]{27} - \sqrt{81}$ **h** $\sqrt[3]{27} - \sqrt{9} - \sqrt[3]{1}$ **i** $(-1)^{101} \times (-1)^{1000} \times \sqrt[3]{-1}$

5 Evaluate these expressions by substituting $a = -2$, $b = 6$ and $c = -3$.

a $a^2 - b$ **b** $a - b^2$ **c** $2c + a$

d $b^2 - c^2$ **e** $a^3 + c^2$ **f** $3b + ac$

g $c - 2ab$ **h** $abc - (ac)^2$ **i** $\sqrt{abc} - ac^2$

PROBLEM–SOLVING	6, 7	6–8	7–10

6 Insert brackets into these statements to make them true.

a $-2 \times 11 + (-2) = -18$ **b** $-6 + (-4) \div 2 = -5$

c $2 - 5 \times (-2) = 6$ **d** $-10 \div 3 + (-5) = 5$

e $3 - (-2) + 4 \times 3 = -3$ **f** $(-2)^2 + 4 \div (-2) = -2^2$

7 Margaret and Mildred meet on a Eurostar train travelling from London to Paris. Margaret visits her daughter in Paris every 28 days. Mildred visits her son in Paris every 36 days. When will Margaret and Mildred have a chance to meet again on the train?

8 How many different answers are possible if any number of pairs of brackets is allowed to be inserted into this expression?

$-6 \times 4 - (-7) + (-1)$

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9 **a** The sum of two numbers is 5 and their difference is 9. What are the two numbers?
b The sum of two numbers is -3 and their product is -10. What are the two numbers?

10 Two opposing football teams have squad sizes of 24 and 32. For a training exercise, each squad is to divide into smaller groups of equal size. What is the largest number of players in a group if the group size for both squads is the same?

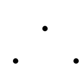
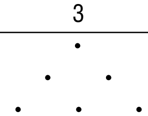
REASONING	11	11	11, 12

11 **a** Evaluate:
i 4^2 **ii** $(-4)^2$
b If $a^2 = 16$, write down the possible values of a.
c If $a^3 = 27$, write down the value of a.
d Explain why there are two values of a for which $a^2 = 16$ but only one value of a for which $a^3 = 27$.
e Find $\sqrt[3]{-27}$.
f Explain why $\sqrt{-16}$ cannot exist (using real numbers).
g -2^2 is the same as -1×2^2. Now evaluate:
i -2^2 **ii** -5^3 **iii** $-(-3)^2$ **iv** $-(-4)^2$
h Decide if $(-2)^2$ and -2^2 are equal.
i Decide if $(-2)^3$ and -2^3 are equal.
j Explain why the HCF of two distinct prime numbers is 1.
k Explain why the LCM of two distinct prime numbers a and b is $a \times b$.

12 If a and b are both positive numbers and $a > b$, decide if the following are true or false.
a $a - b < 0$ **b** $-a \times b > 0$ **c** $-a \div (-b) > 0$
d $(-a)^2 - a^2 = 0$ **e** $-b + a < 0$ **f** $2a - 2b > 0$

ENRICHMENT: Special numbers	–	–	13

13 **a** Perfect numbers are positive integers that are equal to the sum of all their factors, excluding the number itself.
i Show that 6 is a perfect number.
ii There is one perfect number between 20 and 30. Find the number.
iii The next perfect number is 496. Show that 496 is a perfect number.
b Triangular numbers are the number of dots required to form triangles as shown in this table.
i Complete this table.

Number of rows	1	2	3	4	5	6
Diagram						
Number of dots (triangular number)	1	3				

ii Find the 7th and 8th triangular numbers.
c Fibonacci numbers are a sequence of numbers in which each number is the sum of the two preceding numbers. The first two numbers in the sequence are 0 and 1.
i Write down the first ten Fibonacci numbers.
ii If the Fibonacci numbers were to be extended in the negative direction, what would the first four negative Fibonacci numbers be?

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1B Decimal places and significant figures

Learning intentions
- To consolidate the process of rounding off decimals to any number of decimal places
- To introduce and understand the concept of significant figures
- To be able to round a decimal or whole number any number of significant figures

Past, present and future learning
- This section consolidates Stage 4 decimals and introduces Stage 5 significant figures
- Section 6G of this book extends significant figures to scientific notation
- From here onwards, you may be required to round off numbers in two different ways
- This concept is assumed knowledge for Stage 6 Advanced and may also be required in non-calculator examinations such as NAPLAN and industry aptitude tests

Numbers with and without decimal places can be rounded depending on the level of accuracy required. When using numbers with decimal places, it is common to round off the number to leave only a certain number of decimal places. The time for a 100 m sprint race, for example, might be 9.94 seconds.


When civil engineers design a new road cutting through a hill, the volume of earth to be removed only needs to be calculated to two or three significant figures.

Due to the experimental nature of science and engineering, not all the digits in all numbers are considered important or 'significant'. In such cases we are able to round numbers to within a certain number of significant figures (sometimes abbreviated to sig. fig. or simply s.f.). The number of cubic metres of gravel required for a road, for example, might be calculated as 3485 but rounded to 3500. This number is written using two significant figures.

Lesson starter: Plausible but incorrect

Johny says that the number 2.748 when rounded to one decimal place is 2.8 because:

- the 8 rounds the 4 to a 5
- then the new 5 rounds the 7 to an 8.

What is wrong with Johny's theory?

KEY IDEAS

- To round a number to a required number of **decimal places**:
  - Locate the digit in the required decimal place.
  - Round down (leave as is) if the next digit (**critical digit**) is 4 or less.
  - Round up (increase by 1) if the next digit is 5 or more.

  For example:
  - To two decimal places, 1.543 rounds to 1.54 and 32.9283 rounds to 32.93.
  - To one decimal place, 0.248 rounds to 0.2 and 0.253 rounds to 0.3.

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- To round a number to a required number of **significant figures**:
  - Locate the first non-zero digit counting from left to right.
  - From this first significant digit, count the number of significant digits including zeros.
  - Stop at the required number of significant digits and round this last digit.
  - Replace any non-significant digit to the left of a decimal point with a zero.

  For example, these numbers are all rounded to three significant figures:
  $2.5391 \approx 2.54$, $0.002713 \approx 0.00271$, $568\,810 \approx 569\,000$.

BUILDING UNDERSTANDING

1 Choose the number to answer each question.

a Is 44 closer to 40 or 50?
b Is 266 closer to 260 or 270?
c Is 7.89 closer to 7.8 or 7.9?
d Is 0.043 closer to 0.04 or 0.05?

2 Choose the correct answer if the first given number is rounded to three significant figures.

a 32 124 is rounded to 321, 3210 or 32 100
b 431.92 is rounded to 431, 432 or 430
c 5.8871 is rounded to 5.887, 5.88 or 5.89
d 0.44322 is rounded to 0.44, 0.443 or 0.44302

3 Using one significant figure rounding, 324 rounds to 300, 1.7 rounds to 2 and 9.6 rounds to 10.

a Calculate $300 \times 2 \div 10$.
b Use a calculator to calculate $324 \times 1.7 \div 9.6$.
c What is the difference between the answer in part **a** and the exact answer in part **b**?

Example 2 Rounding to a number of decimal places

Round each of these numbers to two decimal places.

a 256.1793 **b** 0.04459 **c** 4.8972

SOLUTION	EXPLANATION
a $256.1793 \approx 256.18$	The number after the second decimal place is 9, so round up (increase the 7 by 1).
b $0.04459 \approx 0.04$	The number after the second decimal place is 4, so round down. 4459 is closer to 4000 than 5000.
c $4.8972 \approx 4.90$	The number after the second decimal place is 7, so round up. Increasing by 1 means 0.89 becomes 0.90.

Now you try

Round each of these numbers to two decimal places.

a 138.4681 **b** 0.03268 **c** 2.7961

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Example 3 Rounding to a number of significant figures

Round each of these numbers to two significant figures.

a 2567 **b** 23 067.453 **c** 0.04059

SOLUTION

a $2567 \approx 2600$

b $23\,067.453 \approx 23\,000$

c $0.04059 \approx 0.041$

EXPLANATION

a The first two digits are the first two significant figures. The third digit is 6, so round up. Replace the last two non-significant digits with zeros.

b The first two digits are the first two significant figures. The third digit is 0, so round down.

c Locate the first non-zero digit, i.e. 4. So 4 and 0 are the first two significant figures. The next digit is 5, so round up.

Now you try

Round each of these numbers to two significant figures.

a 3582 **b** 49 018.47 **c** 0.02864

Example 4 Estimating using significant figures

Estimate the answer to the following by rounding each number in the problem to one significant figure. Use your calculator to check how reasonable your answer is.

$27 + 1329.5 \times 0.0064$

SOLUTION

$27 + 1329.5 \times 0.0064$
$\approx 30 + 1000 \times 0.006$
$= 30 + 6$
$= 36$

The estimated answer is reasonable.

EXPLANATION

Round each number to one significant figure and evaluate. Recall that multiplication occurs before the addition.

By calculator (to one decimal place):
$27 + 1329.5 \times 0.0064 = 35.5$

Now you try

Estimate the answer to the following by rounding each number in the problem to one significant figure. Use your calculator to check how reasonable your answer is.

$19 + 2143.1 \times 0.0032$

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Exercise 1B

FLUENCY	1, 2–7(½)	1, 2(½), 4–7(½)	2(¼), 4–6(½), 7(⅓)

1 Round each of these to two decimal places.

Example 2a a i 124.2694 ii 830.4428

Example 2b b i 0.02163 ii 0.06714

Example 2c c i 7.6941 ii 13.4953

2 Round each of the following numbers to two decimal places.

a 17.962 b 11.082 c 72.986 d 47.859
e 63.925 f 23.807 g 804.5272 h 500.5749
i 821.2749 j 5810.2539 k 1004.9981 l 2649.9974

3 Round these numbers to the nearest integer.

a 6.814 b 73.148 c 129.94 d 36 200.49

4 Use division to write these fractions as decimals rounded to three decimal places.

a $\frac{1}{3}$ b $\frac{2}{7}$ c $\frac{13}{11}$ d $\frac{400}{29}$

Example 3

5 Round each of these numbers to two significant figures.

a 2436 b 35 057.4 c 0.06049 d 34.024
e 107 892 f 0.00245 g 2.0745 h 0.7070

6 Round these numbers to one significant figure.

a 32 000 b 194.2 c 0.0492 d 0.0006413

Example 4

7 Estimate the answers to the following by rounding each number in the problem to one significant figure. Check how reasonable your answer is with a calculator.

a $567 + 3126$ b $795 - 35.6$ c 97.8×42.2
d $965.98 + 5321 - 2763.2$ e $4.23 - 1.92 \times 1.827$ f $17.43 - 2.047 \times 8.165$
g $0.0704 + 0.0482$ h 0.023×0.98 i $0.027 \div 0.0032$
j 41.034^2 k 0.078×0.9803^2 l $1.8494^2 + 0.972 \times 7.032$

PROBLEM-SOLVING	8, 9	8–10	9–11

8 An electronic timer records the time for a running relay. Team A's time is 54.283 seconds and team B's time is 53.791 seconds. What would be the difference in the times for teams A and B if the times were written down using:

a 1 decimal place
b 4 significant figures
c 2 significant figures
d 1 significant figure

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9 $28.4 \times 2.94 \times 11.31$ is calculated by first rounding each of the three numbers. Describe the type of rounding that has taken place if the answer is:
a 900
b 893.2
c 924

10 Eighteen posts and 150 m of fencing are used to create an area in the shape of an equilateral triangle. Posts are used in the corners and are evenly spaced along the sides. Find the distance between each post. Write your answer in metres rounded to the nearest centimetre.

11 A tonne (1000 kg) of soil is to be equally divided between 7 garden beds. How much soil is placed in each garden bed? Write your answer in tonnes rounded to the nearest kilogram.

REASONING	12	12, 13	13, 14

12 Should 2.14999 be rounded down or up if it is to be rounded to one decimal place? Give reasons.

13 A scientific experiment uses very small amounts of magnesium (0.0025 g) and potassium (0.0062 g).
Why does it make sense to use two significant figures instead of two decimal places when recording numbers in a situation like this?

14 Consider the two numbers 24 and 26.
a Calculate:
i $24 + 26$
ii 24×26
b Find the sum (+) of the numbers after rounding each number to one significant figure.
c Find the product (×) of the numbers after rounding each number to one significant figure.
d What do you notice about the answers for parts **b** and **c** as compared to part **a**? Give an explanation.

ENRICHMENT: *n*th decimal place	–	–	15

15 a Use division to express $\frac{2}{11}$ as a decimal correct to 8 decimal places.
b Using the decimal pattern from part **a**, find the digit in the:
i 20th decimal place
ii 45th decimal place
iii 1000th decimal place.
c Express $\frac{1}{7}$ as a decimal correct to 13 decimal places.
d Using the decimal pattern from part **c** find the digit in the:
i 20th decimal place
ii 45th decimal place
iii 1000th decimal place.
e Can you find any fraction whose decimal representation is non-terminating and has no pattern? Use a calculator to help.

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1C Rational numbers CONSOLIDATING

Learning intentions
- To know and understand the difference between rational numbers and irrational numbers
- To be able to convert between fractions and decimals (terminating and recurring)
- To be able to use both notations for recurring decimals
- To be able to compare fractions with different denominators

Past, present and future learning
- This section is mostly revision of Stage 4 but the concept of irrational numbers may be new
- Some of these questions are more challenging than those in our Stage 4 books
- Irrational numbers will appear in various topics in the future, such as trigonometry
- Expertise with fractions is assumed knowledge for Stage 6 Advanced and may also be required in non-calculator examinations such as NAPLAN and industry aptitude tests

Under the guidance of Pythagoras in about 500 BCE, it was discovered that some numbers could not be expressed as a fraction. These special numbers, called irrational numbers, when written as a decimal continue forever and do not show any pattern. So to write these numbers exactly, you need to use special symbols such as $\sqrt{\ }$ and π. If, however, the decimal places in a number terminate or if a pattern exists, the number can be expressed as a fraction. These numbers are called rational numbers.

This is $\sqrt{2}$ to 100 decimal places:

1.4142135623730950488016887242096980785696718753769480731766797379907324784621070388503875343276415727

Numerous people, including scientists, engineers and trade workers, regularly use formulas that contain rational and irrational numbers. This steel sphere is used for chemical analysis and has volume: $V = \frac{4}{3}\pi r^3$.

Lesson starter: Approximating π

To simplify calculations, the ancient and modern civilisations have used fractions to approximate π. To ten decimal places, $\pi = 3.1415926536$.

- Using single-digit numbers, what fraction best approximates π? $\left(\text{For example: } \frac{5}{2}\right)$.
- Using single and/or double-digit numbers, find a fraction that is a good approximation of π. Compare with other students to see who has the best approximation. $\left(\text{For example: } \frac{35}{11}\right)$.

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KEY IDEAS

- An **infinite** decimal is one in which the decimal places continue indefinitely.

Real numbers

Rational numbers (fractions)
(e.g. $\frac{48}{6}, \frac{1}{3}, \frac{7}{1}$)

Terminating decimals
(e.g. 8, 1.2, −6.123)

Recurring decimals
(e.g. $0.\dot{3}$, $-1.\dot{4}$, $2.\overline{345}$)

Irrational numbers
(e.g. $\sqrt{2}$, pi (π), phi (φ) – the golden ratio)

Infinite non-recurring decimals
(e.g. $\pi = 3.14159...$, $\varphi = 1.618033...$, $\sqrt{2} = 1.414213...$)

- **Equivalent fractions** have the same value.
  For example: $\frac{2}{3} = \frac{6}{9}$
- Recall that $\frac{a}{b}$ means $a \div b$.
- A fraction can be simplified by dividing the **numerator** and the **denominator** by their highest common factor.
  For example: $\frac{\cancel{8}^{2}}{\cancel{20}^{5}} = \frac{2}{5}$
- If $\frac{a}{b}$ is a **proper fraction** then $a < b$.
  For example: $\frac{2}{7}$
- If $\frac{a}{b}$ is an **improper fraction** then $a \geq b$.
  For example: $\frac{10}{3}$ and $\frac{3}{3}$
- A **mixed numeral** is written as a whole number plus a proper fraction.
  For example: $2\frac{3}{5}$
- Fractions can be compared using a **common denominator**. This should be the lowest common multiple of both denominators.
  For example: $\frac{2}{3} < \frac{4}{5}$ because $\frac{10}{15} < \frac{12}{15}$
- A dot or bar is used to show a pattern in a recurring decimal number.
  For example: $\frac{1}{6} = 0.16666\ldots = 0.1\dot{6}$ or $\frac{3}{11} = 0.272727\ldots = 0.\overline{27}$

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BUILDING UNDERSTANDING

1 Give these numbers as mixed numerals.

a $\frac{7}{5}$ **b** $\frac{13}{3}$ **c** $\frac{48}{11}$

2 Give these numbers as improper fractions.

a $1\frac{4}{7}$ **b** $5\frac{1}{3}$ **c** $9\frac{1}{2}$

3 Simplify these fractions by cancelling.

a $\frac{4}{10}$ **b** $\frac{8}{58}$ **c** $4\frac{20}{120}$

4 Give the missing number.

a $\frac{3}{5} = \frac{\square}{15}$ **b** $\frac{5}{6} = \frac{20}{\square}$ **c** $\frac{\square}{4} = \frac{21}{28}$ **d** $\frac{3}{\square} = \frac{15}{50}$

Example 5 Writing fractions as decimals

Write these fractions as decimals.

a $2\frac{3}{8}$ **b** $\frac{5}{13}$

SOLUTION

a
$$\begin{array}{r} 0.\ 3\ 7\ 5 \\ 8\overline{)3.{}^{3}0{}^{6}0{}^{4}0} \end{array}$$

$2\frac{3}{8} = 2.375$

b
$$\begin{array}{r} 0.\ 3\ \ 8\ \ 4\ 6\ 1\ 5\ 3 \\ 13\overline{)5.{}^{5}0{}^{11}0{}^{6}0{}^{8}0{}^{2}0{}^{7}0{}^{5}0} \end{array}$$

$\frac{5}{13} = 0.\overline{384615}$

EXPLANATION

Find a decimal for $\frac{3}{8}$ by dividing 8 into 3 using the short-division algorithm.

Divide 13 into 5 and continue until the pattern repeats.
Add a bar over the repeating pattern.
Writing $0.\dot{3}8461\dot{5}$ is an alternative.

Now you try

Write these fractions as decimals.

a $4\frac{5}{8}$ **b** $\frac{2}{11}$

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Example 6 Writing decimals as fractions

Write these decimals as fractions.

a 0.24

b 2.385

SOLUTION

a $0.24 = \frac{24}{100}$

$= \frac{6}{25}$

b $2.385 = 2\frac{385}{1000}$ OR $\frac{2385}{1000}$

$= 2\frac{77}{200}$ $\quad = \frac{477}{200}$

$= 2\frac{77}{200}$

EXPLANATION

Write as a fraction using the smallest place value (hundredths) then simplify using the HCF of 4.

The smallest place value is thousandths. Simplify to an improper fraction or a mixed numeral. 5 is a common factor.

Now you try

Write these decimals as fractions.

a 0.45

b 1.425

Example 7 Comparing fractions

Decide which is the larger fraction.

$\frac{7}{12}$ or $\frac{8}{15}$

SOLUTION

LCM of 12 and 15 is 60.

$\frac{7}{12} = \frac{35}{60}$ and $\frac{8}{15} = \frac{32}{60}$

$\therefore \frac{7}{12} > \frac{8}{15}$

EXPLANATION

Find the lowest common multiple of the two denominators (lowest common denominator).

Write each fraction as an equivalent fraction using the common denominator. Then compare numerators (i.e. 35 > 32) to determine the larger fraction.

Now you try

Decide which is the larger fraction.

$\frac{3}{5}$ or $\frac{7}{11}$

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Exercise 1C

FLUENCY	1–4(½), 6	1–4(⅓), 5, 6	1–4(¼), 5, 6(½)

Example 5a

1 Write these fractions as decimals.

a $\frac{11}{4}$ b $\frac{7}{20}$ c $\frac{37}{16}$ d $\frac{15}{8}$

e $2\frac{5}{8}$ f $3\frac{4}{5}$ g $3\frac{2}{5}$ h $1\frac{7}{32}$

Example 5b

2 Write these fractions as recurring decimals.

a $\frac{3}{11}$ b $\frac{7}{9}$ c $\frac{9}{7}$ d $\frac{5}{12}$

e $\frac{10}{9}$ f $3\frac{5}{6}$ g $7\frac{4}{15}$ h $\frac{29}{11}$

Example 6

3 Write these decimals as fractions.

a 0.35 b 0.06 c 3.7 d 0.56

e 1.07 f 0.075 g 3.32 h 7.375

i 2.005 j 10.044 k 6.45 l 2.101

Example 7

4 Decide which is the larger fraction in the following pairs.

a $\frac{3}{4}$ or $\frac{5}{6}$ b $\frac{13}{20}$ or $\frac{3}{5}$ c $\frac{7}{10}$ or $\frac{8}{15}$ d $\frac{5}{12}$ or $\frac{7}{18}$

e $\frac{7}{16}$ or $\frac{5}{12}$ f $\frac{26}{35}$ or $\frac{11}{14}$ g $\frac{7}{12}$ or $\frac{19}{30}$ h $\frac{7}{18}$ or $\frac{11}{27}$

5 Place these fractions in descending order.

a $\frac{3}{8}, \frac{5}{12}, \frac{7}{18}$ b $\frac{1}{6}, \frac{5}{24}, \frac{3}{16}$ c $\frac{8}{15}, \frac{23}{40}, \frac{7}{12}$

6 Express the following quantities as simplified fractions.

a \$45 out of \$100

b 12 kg out of 80 kg

c 64 baskets out of 90 shots in basketball

d 115 mL out of 375 mL

PROBLEM–SOLVING	7, 8	7(½), 8, 9	8–10

7 These sets of fractions form a pattern. Find the next two fractions in the pattern.

a $\frac{1}{3}, \frac{5}{6}, \frac{4}{3}, \frac{\square}{\square}, \frac{\square}{\square}$ b $\frac{6}{5}, \frac{14}{15}, \frac{2}{3}, \frac{\square}{\square}, \frac{\square}{\square}$

c $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{\square}{\square}, \frac{\square}{\square}$ d $\frac{1}{2}, \frac{4}{7}, \frac{9}{14}, \frac{\square}{\square}, \frac{\square}{\square}$

8 The 'Weather forecast' website says there is a 0.45 chance that it will rain tomorrow. The 'Climate control' website says that the chance of rain is $\frac{14}{30}$. Which website gives the smaller chance that it will rain?

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9 A jug has 400 mL of half-strength orange juice. The following amounts of full-strength juice are added to the mix. Find a fraction to describe the strength of the orange drink after the full-strength juice is added.

a 100 mL **b** 50 mL **c** 120 mL **d** 375 mL

10 If x is an integer, determine the values that x can take in the following.

a The fraction $\frac{x}{3}$ is a number between (and not including) 10 and 11.

b The fraction $\frac{x}{7}$ is a number between (and not including) 5 and 8.

c The fraction $\frac{34}{x}$ is a number between 6 and 10.

d The fraction $\frac{23}{x}$ is a number between 7 and 12.

e The fraction $\frac{x}{14}$ is a number between (and not including) 3 and 4.

f The fraction $\frac{58}{x}$ is a number between 9 and 15.

REASONING	11	11, 12	11, 12

11 $a\frac{b}{c}$ is a mixed numeral with unknown digits a, b and c. Write it as an improper fraction.

12 Given that $\frac{a}{b}$ is a fraction, answer the following questions with reasons.

a Is it possible to find a fraction that can be simplified by cancelling if one of a or b is prime?

b Is it possible to find a fraction that can be simplified by cancelling if both a and b are prime? Assume $a \neq b$.

c If $\frac{a}{b}$ is a fraction in simplest form, can a and b both be even?

d If $\frac{a}{b}$ is a fraction in simplest form, can a and b both be odd?

ENRICHMENT: Converting recurring decimals to fractions	–	–	13

13 Here are two examples of how to convert recurring decimals to fractions.

$$\begin{aligned} 0.\dot{6} &= 0.6666\ldots \\ \text{Let } x &= 0.6666\ldots\ [1] \\ 10x &= 6.6666\ldots\ [2] \\ [2]-[1]\ 9x &= 6 \\ x &= \frac{6}{9}=\frac{2}{3} \\ \therefore 0.\dot{6} &= \frac{2}{3} \end{aligned}$$

$$\begin{aligned} 1.\overline{27} &= 1.272727\ldots \\ \text{Let } x &= 1.272727\ldots\ [1] \\ 100x &= 127.2727\ldots\ [2] \\ [2]-[1]\ 99x &= 126 \\ x &= \frac{126}{99} \\ \therefore 1.\overline{27} &= \frac{126}{99}=1\frac{27}{99}=1\frac{3}{11} \end{aligned}$$

Convert these recurring decimals to fractions using the method above.

a $0.\dot{8}$ **b** $0.\dot{2}$ **c** $0.\overline{81}$ **d** $3.\overline{43}$

e $9.\overline{75}$ **f** $0.\overline{132}$ **g** $2.\overline{917}$ **h** $13.\overline{8125}$

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1D Computation with fractions CONSOLIDATING

Learning intentions
- To understand that to add or subtract fractions a common denominator is required
- To be able to add and subtract fractions, including those in mixed numeral form, using the lowest common denominator
- To understand that to multiply fractions it is simpler to first cancel any common factors between numerators and denominators
- To know that to divide a number by a fraction we multiply by the reciprocal of the fraction
- To know to express mixed numerals as improper fractions before multiplying or dividing
- To be able to multiply and divide fractions

Past, present and future learning
- This section consolidates Stage 4 concepts which are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- In Chapter 2 of this book, we apply this learning to algebraic fractions
- Expertise with fractions is assumed knowledge for Stage 6 Advanced and may also be required in non-calculator examinations such as NAPLAN and industry aptitude tests

Operations with integers can be extended to include rational numbers, which are numbers that can be expressed as fractions. The operations include addition, subtraction, multiplication and division. Addition and subtraction of fractions is generally more complex than multiplication and division because there is the added step of finding common denominators.

Can you name some everyday applications of fractions illustrated by this photo?

Lesson starter: The common errors

Here are incorrect solutions to four problems involving fractions.

- $\frac{2}{3} \times \frac{5}{3} = \frac{2 \times 5}{3} = \frac{10}{3}$
- $\frac{2}{3} + \frac{1}{2} = \frac{2+1}{3+2} = \frac{3}{5}$
- $\frac{7}{6} \div \frac{7}{3} = \frac{7}{6} \div \frac{14}{6} = \frac{\frac{1}{2}}{6} = \frac{1}{12}$
- $1\frac{1}{2} - \frac{2}{3} = 1\frac{3}{6} - \frac{4}{6} = -1\frac{1}{6}$

In each case describe what is wrong and give the correct solution.

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KEY IDEAS

- To add or subtract fractions, first convert each fraction to **equivalent fractions** that have the same **denominator**.
  - Choose the lowest common denominator (LCD).
  - Add or subtract the numerators and retain the denominator.
- To multiply fractions, multiply the numerators and multiply the denominators after the following steps.
  - Convert mixed numerals to improper fractions before multiplying.
  - Cancel the highest common factor between any numerator and any denominator before multiplying.
  - Note: The word 'of' usually means 'multiply'.
    For example: $\frac{1}{3}$ of $24 = \frac{1}{3} \times 24$
- The **reciprocal** of a number multiplied by the number itself is equal to 1.
  - For example: the reciprocal of 2 is $\frac{1}{2}$ since $2 \times \frac{1}{2} = 1$
    the reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$ since $\frac{3}{5} \times \frac{5}{3} = 1$
- To divide a number by a fraction, multiply by its reciprocal.
  For example: $\frac{2}{3} \div \frac{5}{6}$ becomes $\frac{2}{3} \times \frac{6}{5}$
  - Whole numbers can be written using a denominator of 1.
    For example: $3 = \frac{3}{1}$

BUILDING UNDERSTANDING

1 State the lowest common denominator for these pairs of fractions.

a $\frac{1}{2}, \frac{1}{3}$ **b** $\frac{3}{7}, \frac{5}{9}$ **c** $\frac{1}{2}, \frac{1}{4}$ **d** $\frac{5}{12}, \frac{7}{30}$

2 Convert these mixed numerals to improper fractions.

a $2\frac{1}{3}$ **b** $7\frac{4}{5}$ **c** $10\frac{1}{4}$

3 State the missing numbers to complete the given working.

a $\frac{3}{2} + \frac{4}{3} = \frac{\square}{6} + \frac{\square}{6}$
$= \frac{\square}{6}$

b $1\frac{1}{3} - \frac{2}{5} = \frac{\square}{3} - \frac{2}{5}$
$= \frac{\square}{15} - \frac{\square}{15}$
$= \frac{14}{\square}$

c $\frac{5}{3} \div \frac{2}{7} = \frac{5}{3} \times \frac{\square}{2}$
$= \frac{\square}{6}$

4 State the reciprocal of the following fractions.

a 5 **b** $\frac{1}{4}$ **c** $\frac{2}{3}$ **d** $\frac{5}{4}$

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Example 8 Adding and subtracting fractions

Evaluate the following.

a $\frac{1}{2}+\frac{3}{5}$ **b** $1\frac{2}{3}+4\frac{5}{6}$ **c** $3\frac{2}{5}-2\frac{3}{4}$

SOLUTION

a $\frac{1}{2}+\frac{3}{5}=\frac{5}{10}+\frac{6}{10}$

$=\frac{11}{10}$ or $1\frac{1}{10}$

b $1\frac{2}{3}+4\frac{5}{6}=\frac{5}{3}+\frac{29}{6}$

$=\frac{10}{6}+\frac{29}{6}$

$=\frac{39}{6}$

$=\frac{13}{2}$ or $6\frac{1}{2}$

Alternative method:

$1\frac{2}{3}+4\frac{5}{6}=1\frac{4}{6}+4\frac{5}{6}$

$=5\frac{9}{6}$

$=6\frac{3}{6}$

$=6\frac{1}{2}$

c $3\frac{2}{5}-2\frac{3}{4}=\frac{17}{5}-\frac{11}{4}$

$=\frac{68}{20}-\frac{55}{20}$

$=\frac{13}{20}$

EXPLANATION

The lowest common denominator of 2 and 5 is 10. Rewrite as equivalent fractions using a denominator of 10 and add the numerators.

Change each mixed numeral to an improper fraction.

Remember the lowest common denominator of 3 and 6 is 6. Change $\frac{5}{3}$ to an equivalent fraction with denominator 6, then add the numerators and simplify by cancelling HCF of 3.

Alternatively, add whole numbers and fractions separately, after finding a common denominator for the fractions $\left(\frac{2}{3}=\frac{4}{6}\right)$.

$\frac{9}{6}=1\frac{3}{6}$

Convert to improper fractions then rewrite as equivalent fractions with the same denominator. Subtract the numerators.

Now you try

Evaluate the following.

a $\frac{1}{3}+\frac{2}{5}$ **b** $1\frac{1}{2}+3\frac{5}{8}$ **c** $4\frac{3}{5}-2\frac{3}{4}$

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Example 9 Multiplying fractions

Evaluate the following.

a $\frac{2}{3} \times \frac{5}{7}$

b $1\frac{2}{3} \times 2\frac{1}{10}$

SOLUTION

a $\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7}$

$= \frac{10}{21}$

b $1\frac{2}{3} \times 2\frac{1}{10} = \frac{{}^{1}\cancel{5}}{{}_{1}\cancel{3}} \times \frac{\cancel{21}^{7}}{\cancel{10}_{2}}$

$= \frac{7}{2}$ or $3\frac{1}{2}$

EXPLANATION

No cancelling is possible as there are no common factors between numerators and denominators.

Multiply the numerators and the denominators.

Rewrite as improper fractions.

Cancel common factors between numerators and denominators and then multiply remaining numerators and denominators.

Now you try

Evaluate the following.

a $\frac{4}{5} \times \frac{3}{7}$

b $2\frac{1}{3} \times 3\frac{2}{7}$

Example 10 Dividing fractions

Evaluate the following.

a $\frac{4}{15} \div \frac{12}{25}$

b $1\frac{17}{18} \div 1\frac{1}{27}$

SOLUTION

a $\frac{4}{15} \div \frac{12}{25} = \frac{4}{15} \times \frac{25}{12}$

$= \frac{{}^{1}\cancel{4}}{{}_{3}\cancel{15}} \times \frac{\cancel{25}^{5}}{\cancel{12}_{3}}$

$= \frac{5}{9}$

b $1\frac{17}{18} \div 1\frac{1}{27} = \frac{35}{18} \div \frac{28}{27}$

$= \frac{{}^{5}\cancel{35}}{{}_{2}\cancel{18}} \times \frac{\cancel{27}^{3}}{\cancel{28}_{4}}$

$= \frac{15}{8}$ or $1\frac{7}{8}$

EXPLANATION

To divide by $\frac{12}{25}$ we multiply by its reciprocal $\frac{25}{12}$.

Cancel common factors between numerators and denominators then multiply fractions.

Rewrite mixed numerals as improper fractions.

Multiply by the reciprocal of the second fraction.

Now you try

Evaluate the following.

a $\frac{2}{9} \div \frac{4}{15}$

b $1\frac{11}{12} \div 1\frac{3}{8}$

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Exercise 1D

FLUENCY	1–6(½)	1–4(½), 5–6(⅓)	3–4(½), 5–6(¼)

Example 8a

1 Evaluate the following.

a $\frac{2}{5}+\frac{1}{5}$ b $\frac{3}{9}+\frac{1}{9}$ c $\frac{5}{7}+\frac{4}{7}$ d $\frac{3}{4}+\frac{1}{5}$
e $\frac{1}{3}+\frac{4}{7}$ f $\frac{3}{8}+\frac{4}{5}$ g $\frac{2}{5}+\frac{3}{10}$ h $\frac{4}{9}+\frac{5}{27}$

2 Evaluate the following.

a $\frac{4}{5}-\frac{2}{5}$ b $\frac{4}{5}-\frac{7}{9}$ c $\frac{3}{4}-\frac{1}{5}$ d $\frac{2}{5}-\frac{3}{10}$
e $\frac{8}{9}-\frac{5}{6}$ f $\frac{3}{8}-\frac{1}{4}$ g $\frac{5}{9}-\frac{3}{8}$ h $\frac{5}{12}-\frac{5}{16}$

Example 8b

3 Evaluate the following.

a $3\frac{1}{4}+1\frac{3}{4}$ b $2\frac{3}{5}+\frac{4}{5}$ c $1\frac{3}{7}+3\frac{5}{7}$
d $2\frac{1}{3}+4\frac{2}{5}$ e $2\frac{5}{7}+4\frac{5}{9}$ f $10\frac{5}{8}+7\frac{3}{16}$

Example 8c

4 Evaluate the following.

a $2\frac{3}{4}-1\frac{1}{4}$ b $3\frac{5}{8}-2\frac{7}{8}$ c $3\frac{1}{4}-2\frac{3}{5}$
d $3\frac{5}{8}-2\frac{9}{10}$ e $2\frac{2}{3}-1\frac{5}{6}$ f $3\frac{7}{11}-2\frac{3}{7}$

Example 9

5 Evaluate the following (recall that $6=\frac{6}{1}$).

a $\frac{2}{5}\times\frac{3}{7}$ b $\frac{3}{5}\times\frac{5}{6}$ c $\frac{2}{15}\times\frac{5}{8}$ d $\frac{6}{21}\times1\frac{5}{9}$
e $6\times\frac{3}{4}$ f $8\times\frac{2}{3}$ g $\frac{5}{6}\times9$ h $1\frac{1}{4}\times4$
i $2\frac{1}{2}\times6$ j $1\frac{5}{8}\times16$ k $\frac{10}{21}\times1\frac{2}{5}$ l $\frac{25}{44}\times1\frac{7}{15}$
m $1\frac{1}{2}\times1\frac{1}{2}$ n $1\frac{1}{2}\times2\frac{1}{3}$ o $2\frac{2}{3}\times2\frac{1}{4}$ p $1\frac{1}{5}\times1\frac{1}{9}$

Example 10

6 Evaluate the following (recall that the reciprocal of 8 is $\frac{1}{8}$ and for $\frac{3}{4}$ it is $\frac{4}{3}$).

a $\frac{4}{7}\div\frac{3}{5}$ b $\frac{3}{4}\div\frac{2}{3}$ c $\frac{5}{8}\div\frac{7}{9}$ d $\frac{3}{7}\div\frac{4}{9}$
e $\frac{3}{4}\div\frac{9}{16}$ f $\frac{4}{5}\div\frac{8}{15}$ g $\frac{8}{9}\div\frac{4}{27}$ h $\frac{15}{42}\div\frac{20}{49}$
i $15\div\frac{5}{6}$ j $6\div\frac{2}{3}$ k $12\div\frac{3}{4}$ l $24\div\frac{3}{8}$
m $\frac{4}{5}\div8$ n $\frac{3}{4}\div9$ o $\frac{8}{9}\div6$ p $14\div4\frac{1}{5}$
q $6\div1\frac{1}{2}$ r $1\frac{1}{3}\div8$ s $2\frac{1}{4}\div1\frac{1}{2}$ t $4\frac{2}{3}\div5\frac{1}{3}$

PROBLEM–SOLVING	7–9	8, 9, 10(½), 11	9, 10(½), 11, 12

7 To remove impurities, a mining company filters $3\frac{1}{2}$ tonnes of raw material. If $2\frac{5}{8}$ tonnes of impurities are removed, what quantity of material remains?

8 When a certain raw material is processed it produces $3\frac{1}{7}$ tonnes of mineral and $2\frac{3}{8}$ tonnes of waste. How many tonnes of raw material were processed?

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9 In a $2\frac{1}{2}$ hour maths exam, $\frac{1}{6}$ of that time is allocated as reading time. How long is the reading time?

10 Evaluate these mixed-operation problems.

a $\frac{2}{3} \times \frac{1}{3} \div \frac{7}{9}$
b $\frac{4}{5} \times \frac{3}{5} \div \frac{9}{10}$
c $\frac{4}{9} \times \frac{6}{25} \div \frac{1}{150}$
d $2\frac{1}{5} \times \frac{3}{7} \div 1\frac{3}{14}$
e $5\frac{1}{3} \times \frac{13}{24} \div 1\frac{1}{6}$
f $2\frac{4}{13} \times \frac{3}{8} \div 3\frac{3}{4}$

11 A road is to be constructed with $15\frac{1}{2}\,\text{m}^3$ of crushed rock. If a small truck can carry $2\frac{1}{3}\,\text{m}^3$ of crushed rock, how many truckloads will be needed?

12 Regan worked for $7\frac{1}{2}$ hours in a sandwich shop. Three-fifths of her time was spent cleaning up and the rest serving customers. How much time did she spend serving customers?

REASONING	13	13, 14	14, 15

13 Here is an example involving the subtraction of fractions in which improper fractions are not used.

$$2\frac{1}{2} - 1\frac{1}{3} = 2\frac{3}{6} - 1\frac{2}{6}$$
$$= 1\frac{1}{6}$$

Try this technique on the following problem and explain the difficulty that you encounter.

$2\frac{1}{3} - 1\frac{1}{2}$

14 a A fraction is given by $\frac{a}{b}$. Write down its reciprocal.

b A mixed numeral is given by $a\frac{b}{c}$. Write an expression for its reciprocal.

15 If a, b and c are integers, simplify the following.

a $\frac{b}{a} \times \frac{a}{b}$
b $\frac{a}{b} \div \frac{b}{a}$
c $\frac{a}{b} \div \frac{a}{b}$
d $\frac{a}{b} \times \frac{c}{a} \div \frac{a}{b}$
e $\frac{abc}{a} \div \frac{bc}{a}$
f $\frac{a}{b} \div \frac{b}{c} \times \frac{b}{a}$

ENRICHMENT: Fraction operation challenge	–	–	16(½)

16 Evaluate the following. Express your answers using improper fractions where necessary.

a $2\frac{1}{3} - 1\frac{2}{5} \times 2\frac{1}{7}$
b $1\frac{1}{4} \times 1\frac{1}{5} - 2\frac{1}{2} \div 10$
c $1\frac{4}{5} \times 4\frac{1}{6} + \frac{2}{3} \times 1\frac{1}{5}$
d $\left(1\frac{2}{3} + 1\frac{3}{4}\right) \div 3\frac{5}{12}$
e $4\frac{1}{6} \div \left(1\frac{1}{3} + 1\frac{1}{4}\right)$
f $\left(1\frac{1}{5} - \frac{3}{4}\right) \times \left(1\frac{1}{5} - \frac{3}{4}\right)$
g $\left(2\frac{1}{4} - 1\frac{2}{3}\right) \times \left(2\frac{1}{4} + 1\frac{2}{3}\right)$
h $\left(3\frac{1}{2} + 1\frac{3}{5}\right) \times \left(3\frac{1}{2} - 1\frac{3}{5}\right)$
i $\left(2\frac{2}{3} - 1\frac{3}{4}\right) \times \left(2\frac{2}{3} + 1\frac{3}{4}\right)$
j $\left(4\frac{1}{2} - 3\frac{2}{3}\right) \div \left(1\frac{1}{3} + \frac{1}{2}\right)$

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1E Ratios, rates and best buys CONSOLIDATING

Learning intentions
- To revise the meaning, use and applications of ratios and rates
- To be able to simplify ratios, recognise equivalent ratios and divide a quantity in a given ratio
- To be able to simplify rates and recognise equivalent rates
- To be able to solve problems involving ratios and rates, including the use of the unitary method
- To be able to solve problems involving speed, best buys and other rates

Past, present and future learning
- This section consolidates Stage 4 concepts which are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- The Path Topic *Variation and Rates of Change* can be found in Chapter 7 of our Year 10 book
- Expertise with ratios and rates is assumed knowledge for Stage 6 Advanced and may also be required in non-calculator examinations such as NAPLAN and industry aptitude tests

Fractions, ratios and rates are used to compare quantities. A leaf blower, for example, might require $\frac{1}{6}$ of a litre of oil to make a petrol mix of 2 parts oil to 25 parts petrol, which is an oil to petrol ratio of 2 to 25 or 2 : 25. The leaf blower's fan might then spin at a rate of 1000 revolutions per minute (1000 revs/min).

A vehicle's fuel efficiency is a rate, usually given in litres per 100 km. Fuel consumption rates can vary from 5 L/100 km for a small car, up to 25 L/100 km for a large car towing a caravan.

Lesson starter: The lottery win

\$100 000 is to be divided among three lucky people in a ratio of 2 to 3 to 5 (2 : 3 : 5). How should the money be divided?

- Clearly write down your method and answer. There may be many different ways to solve this problem.
- Write down and discuss the alternative methods suggested by other students in the class.

KEY IDEAS

- **Ratios** are used to compare quantities with the same units.
  - The ratio of a to b is written $a : b$.
  - Ratios in simplest form use whole numbers that have no common factor.
- The **unitary method** involves finding the value of one part of a total.
  - Once the value of one part is found then the value of several parts can easily be determined.
- A **rate** compares related quantities with different units.
  - The rate is usually written with one quantity compared to a single unit of the other quantity. For example: 50 km per 1 hour or 50 km/h.
- Ratios and rates can be used to determine **best buys** when purchasing products.

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BUILDING UNDERSTANDING

1 State the missing number.

a $2:5 = \square : 10$ **b** $3:7 = \square : 28$

c $8 : \square = 640 : 880$ **d** $\square : 4 = 7.5 : 10$

2 Consider the ratio of sheep to chickens of 4 : 5 on a farm.
- **a** What is the total number of parts?
- **b** What fraction of the total are sheep?
- **c** What fraction of the total are chickens?
- **d** If there were 18 in total, how many of them are sheep?
- **e** If there were 18 in total, how many of them are chickens?

3 A car is travelling at a rate (speed) of 80 km/h.
- **a** How far would it travel in:
  - **i** 3 hours? **ii** $\frac{1}{2}$ hour? **iii** $6\frac{1}{2}$ hours?
- **b** How long would it take to travel:
  - **i** 400 km? **ii** 360 km? **iii** 20 km?

4 How much would 1 kg cost if:

a 2 kg costs $8? **b** 5 kg costs $15? **c** 4 kg costs $10?

Example 11 Simplifying ratios

Simplify these ratios.

a 38 : 24 **b** $2\frac{1}{2} : 1\frac{1}{3}$ **c** 0.2 : 0.14

SOLUTION

a $38 : 24 = 19 : 12$

EXPLANATION

The HCF of 38 and 24 is 2, so divide both sides by 2.

b
$$\begin{aligned} 2\tfrac{1}{2} : 1\tfrac{1}{3} &= \tfrac{5}{2} : \tfrac{4}{3} \\ &= \tfrac{15}{6} : \tfrac{8}{6} \\ &= 15 : 8 \end{aligned}$$

Write as improper fractions using the same denominator.
Then multiply both sides by 6 to write as whole numbers.

c
$$\begin{aligned} 0.2 : 0.14 &= 20 : 14 \\ &= 10 : 7 \end{aligned}$$

Multiply by 100 to remove all the decimal places and simplify.

Now you try

Simplify these ratios.

a 12 : 8 **b** $2\frac{2}{3} : 1\frac{1}{2}$ **c** 0.4 : 0.24

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Example 12 Dividing into a given ratio

\$300 is to be divided in the ratio 2 : 3.
Find the value of the larger portion using the unitary method.

SOLUTION

Total number of parts is $2 + 3 = 5$.
5 parts = \$300

$$1 \text{ part} = \frac{1}{5} \text{ of } \$300$$
$$= \$60$$

$$\text{Larger portion} = 3 \times \$60$$
$$= \$180$$

EXPLANATION

Use the ratio 2 : 3 to find the total number of parts.

Calculate the value of each part, $300 \div 5$.

Calculate the value of 3 parts.

Now you try

\$250 is to be divided in the ratio 1 : 4.
Find the value of the larger portion using the unitary method.

Example 13 Simplifying rates

Write these rates in simplest form.

a 120 km every 3 hours

b 5000 revolutions in $2\frac{1}{2}$ minutes

SOLUTION

a $$120 \text{ km per 3 hours} = \frac{120}{3} \text{ km/h}$$
$$= 40 \text{ km/h}$$

b 5000 revolutions per $2\frac{1}{2}$ minutes
= 10 000 revolutions per 5 minutes
$$= \frac{10\,000}{5} \text{ revs/min}$$
= 2000 revs/min

EXPLANATION

Divide by 3 to write the rate compared to 1 hour.

First multiply by 2 to remove the fraction.

Then divide by 5 to write the rate using 1 minute.

Now you try

Write these rates in simplest form.

a 70 km every 2 hours

b 600 revolutions in $1\frac{1}{2}$ minutes.

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Example 14 Finding best buys

Which is better value:

a 5 kg of potatoes for $3.80 or 3 kg for $2.20?

b 400 mL of shampoo A at $3.20 or 320 mL of shampoo B at $2.85? Compare the cost of 100 mL of each product then decide which is the better buy. Assume the products are of similar quality.

SOLUTION	EXPLANATION
a **Method A.** Price per kg	
5 kg bag: 1 kg costs $3.80 ÷ 5 = $0.76 3 kg bag: 1 kg costs $2.20 ÷ 3 = $0.73 ∴ the 3 kg bag is better value.	Divide each price by the number of kilograms to find the price per kilogram. Then compare.
Method B. Amount per $1 5 kg bag: $1 buys 5 ÷ 3.8 = 1.32 kg 3 kg bag: $1 buys 3 ÷ 2.2 = 1.36 kg ∴ the 3 kg bag is better value.	Divide each amount in kilograms by the cost to find the weight per $1 spent. Then compare.
b Shampoo A: 100 mL costs $3.20 ÷ 4 = $0.80	Alternatively, divide by 400 to find the cost of 1 mL then multiply by 100.
Shampoo B: 100 mL costs $2.85 ÷ 3.2 = $0.89 ∴ shampoo A is the better buy.	Alternatively, divide by 320 to find the cost of 1 mL then multiply by 100.

Now you try

Which is better value:

a 7 kg of onions for $5.60 or 5 kg of onions for $4.25?

b 300 mL of juice A at $2.40 or 450 mL of juice B at $3.15?

Exercise 1E

FLUENCY	1–3(½), 5(½), 7(½)	1–5(½), 6, 7(½), 8	1–8(⅓)

Example 11

1 Simplify these ratios.

a 6 : 30 **b** 8 : 20 **c** 42 : 28 **d** 52 : 39

e $1\frac{1}{2} : 3\frac{1}{3}$ **f** $2\frac{1}{4} : 1\frac{2}{5}$ **g** $\frac{3}{8} : 1\frac{3}{4}$ **h** $1\frac{5}{6} : 3\frac{1}{4}$

i 0.3 : 0.9 **j** 0.7 : 3.5 **k** 1.6 : 0.56 **l** 0.4 : 0.12

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2 Write each of the following as a ratio in simplest form. (*Hint*: Convert to the same unit first.)

a 80c : \$8 **b** 90c : \$4.50 **c** 80 cm : 1.2 m
d 0.7 kg : 800 g **e** 2.5 kg : 400 g **f** 30 min : 2 hours
g 45 min : 3 hours **h** 4 hours : 50 min **i** 40 cm : 2 m : 50 cm
j 80 cm : 600 mm : 2 m **k** 2.5 hours : 1.5 days **l** 0.09 km : 300 m : 1.2 km

Example 12

3 Divide \$500 in these ratios using the unitary method.

a 2 : 3 **b** 3 : 7 **c** 1 : 1 **d** 7 : 13

4 Divide \$70 in these ratios.

a 1 : 2 : 4 **b** 2 : 7 : 1 **c** 8 : 5 : 1

Example 13

5 Write these rates in simplest form.

a 150 km in 10 hours **b** 3000 revolutions in $1\frac{1}{2}$ minutes
c 15 swimming strokes in $\frac{1}{3}$ of a minute **d** 56 metres in 4 seconds
e 180 mL in 22.5 hours **f** 207 heart beats in $2\frac{1}{4}$ minutes

6 Hamish rides his bike at an average speed of 22 km/h. How far does he ride in:

a $2\frac{1}{2}$ hours? **b** $\frac{3}{4}$ hours? **c** 15 minutes?

Example 14a

7 Determine the best buy (cheaper deal) in each of the following.

a 2 kg of washing powder for \$11.70 or 3 kg for \$16.20
b 1.5 kg of Red Delicious apples for \$4.80 or 2.2 kg of Royal Gala apples for \$7.92
c 2.4 litres of orange juice for \$4.20 or 3 litres of orange juice for \$5.40
d 0.7 GB of internet usage for \$14 or 1.5 GB for \$30.90 with different service providers

Example 14b

8 Find the cost of 100 g of each product below then decide which is the best buy (cheaper deal).

a 300 g of coffee A at \$10.80 or 220 g of coffee B at \$8.58
b 600 g of pasta A for \$7.50 or 250 g of pasta B for \$2.35
c 1.2 kg of cereal A for \$4.44 or 825 g of cereal B for \$3.30

PROBLEM–SOLVING	9, 10	10–12	12–15

9 Kirsty manages a restaurant. Each day she buys watermelons and mangoes in the ratio of 3 : 2. How many watermelons did she buy if, on one day, the total number of watermelons and mangoes was 200?

10 If a prize of \$6000 was divided among Georgia, Leanne and Maya in the ratio of 5 : 2 : 3, how much did each girl get?

11 When a crate of twenty 375 mL soft drink cans is purchased it works out to be \$1.68 per litre. If a crate of 30 of the same cans is advertised as being a saving of 10 cents per can compared with the 20-can crate, calculate how much the 30-can crate costs.

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12 The dilution ratio for a particular chemical with water is 2 : 3 (chemical to water). If you have 72 litres of chemical, how much water is needed to dilute the chemical?

13 Julie is looking through the supermarket catalogue for her favourite cookies-and-cream ice-cream. The catalogue is advertising 2 L of triple-chocolate ice-cream for $6.30. The cookies-and-cream ice-cream is usually $5.40 for 1.2 L. What saving does there need to be on the price of the 1.2 L container of cookies-and-cream ice-cream for it to be of equal value to the 2 L triple-chocolate container?

14 The ratio of the side lengths of one square to another is 1 : 2. Find the ratio of the areas of the two squares.

15 A quadrilateral (with angle sum 360°) has interior angles in the ratio 1 : 2 : 3 : 4. Find the size of each angle.

REASONING	16	16, 17	17, 18

16 2.5 kg of cereal A costs $4.80 and 1.5 kg of cereal B costs $2.95. Write down at least two different methods for finding which cereal is the better buy (cheaper deal).

17 If $a : b$ is in simplest form, state whether the following are true or false.

a a and b must both be odd.

b a and b must both be prime.

c At least one of a or b is odd.

d The HCF of a and b is 1.

18 A ratio is $a : b$ with $a < b$ and a and b are positive integers. Write an expression for:

a the total number of parts

b the fraction of the smaller quantity out of the total

c the fraction of the larger quantity out of the total.

ENRICHMENT: Mixing drinks	–	–	19

19 Four jugs of cordial have a cordial to water ratio as shown in the table. The total volume is also shown.

Jug	Cordial to water ratio	Total volume
1	1 : 5	600 mL
2	2 : 7	900 mL
3	3 : 5	400 mL
4	2 : 9	330 mL

a How much cordial is in:

i jug 1?

ii jug 2?

b How much water is in:

i jug 3?

ii jug 4?

c Jugs 1 and 2 were mixed together to give 1500 mL of drink.

i How much cordial is in the drink?

ii Find the ratio of cordial to water in the drink.

d Find the ratio of cordial to water when the following jugs are mixed.

i jugs 1 and 3

ii jugs 2 and 3

iii jugs 2 and 4

iv jugs 3 and 4

e Which combination of two jugs gives the strongest cordial to water ratio?

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1F Percentages and money CONSOLIDATING

Learning intentions
- To review the conversions between percentages, fractions and decimals
- To be able to express one quantity as a percentage of another quantity
- To be able to find a percentage of a quantity
- To be able to use division or the unitary method to find an original amount from a percentage of the amount

Past, present and future learning
- This section consolidates Stage 4 concepts which are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- These concepts are not revised in our Year 10 book
- Expertise with percentages is assumed knowledge for Stage 6 Advanced and may also be required in non-calculator examinations such as NAPLAN and industry aptitude tests

We use percentages for many different things in our daily lives. Some examples are loan rates, the interest rate on a term deposit and the percentage discount on purchases.

We know from our previous studies that a percentage is a number expressed out of 100. 'Per cent' comes from the Latin term *per centum* and means 'out of 100'.

Businesses convert fractions to percentages to make comparison easier, such as, for comparing the fractions of total profit made by each department or branch of a business.

Lesson starter: Which is the largest piece?

Four people receive the following portions of a cake:

- Milly 25.5%
- Tom $\frac{1}{4}$
- Adam 0.26
- Mai what's left

a Which person gets the most cake and why?
b How much cake does Mai get? What is her portion written as a percentage, a decimal and a fraction?

KEY IDEAS

- To express a number as a **percentage**, *multiply* by 100.
- To express a percentage as a **fraction** or **decimal**, *divide* by 100.

$$\text{Decimal } 0.3,\ \text{Fraction } \tfrac{3}{10} \xrightarrow{\times 100} \text{Percentage } 30\% \qquad \text{Percentage } 30\% \xrightarrow{\div 100} \text{Decimal } 0.3,\ \text{Fraction } \tfrac{3}{10}$$

- A percentage of a number can be found using multiplication.

  For example: $25\% \text{ of } \$26 = 0.25 \times \$26 \text{ or } \frac{25}{100} \times \$26$
  $= \$6.50$

- One quantity can be expressed as a percentage of another quantity by writing as a fraction using the same units and converting to a percentage.

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- To find an original amount, use the **unitary method** or use division.
  For example: 3% of an amount is \$36.
  - Using the unitary method: 1% of the amount is $\$36 \div 3 = \$12$
    $\therefore$ 100% of the amount is $\$12 \times 100 = \$1200$
  - Using division: 3% of the amount is \$36

$$\begin{aligned} 0.03 \times \text{amount} &= \$36 \\ \text{amount} &= \$36 \div 0.03 \\ &= \$1200 \end{aligned}$$

BUILDING UNDERSTANDING

1 Divide these percentages by 100 to express them as fractions. For example, $9\% = \frac{9}{100}$.
Simplify where possible.

a 3% **b** 11% **c** 35% **d** 8%

2 Divide these percentages by 100 to express them as decimals. For example, $9\% = 0.09$.

a 4% **b** 23% **c** 86% **d** 46.3%

3 Express these simple decimals and fractions as percentages.

a 0.5 **b** 0.25 **c** $\frac{3}{4}$ **d** $\frac{1}{5}$

Example 15 Converting between percentages, decimals and fractions

a Express 0.45 as a percentage.
b Express 25% as a decimal.
c Express $3\frac{1}{4}\%$ as a fraction.

SOLUTION

a $$\begin{aligned} 0.45 \times 100 &= 45 \\ \therefore 0.45 &= 45\% \end{aligned}$$

EXPLANATION

Multiply by 100. This moves the decimal point two places to the right.

b $25\% = 25 \div 100 = 0.25$

Divide by 100. This moves the decimal point two places to the left.

c $$\begin{aligned} 3\tfrac{1}{4}\% &= 3\tfrac{1}{4} \div 100 \\ &= \frac{13}{4} \times \frac{1}{100} \\ &= \frac{13}{400} \end{aligned}$$

Divide by 100.
Write the mixed numeral as an improper fraction and multiply by the reciprocal of 100 (i.e. $\frac{1}{100}$).

Now you try

a Express 0.75 as a percentage.
b Express 60% as a decimal.
c Express $2\frac{3}{4}\%$ as a fraction.

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Example 16 Expressing one quantity as a percentage of another quantity

Express 50c as a percentage of \$2.50.

SOLUTION

$$\begin{aligned} \text{50c out of \$2.50} &= \frac{{}^{1}\cancel{50}}{{}_{5}\cancel{250}} \times 100\% \\ &= 20\% \end{aligned}$$

EXPLANATION

Convert to the same units (\$2.50 = 250c) and write as a fraction. Multiply by 100, cancelling first.

Now you try

Express 20c as a percentage of \$1.60.

Example 17 Finding a percentage of a quantity

Find 15% of \$35.

SOLUTION

$$\begin{aligned} 15\% \text{ of } \$35 &= \frac{{}^{3}\cancel{15}}{{}_{20}\cancel{100}} \times \$35 \\ &= \$5.25 \end{aligned}$$

EXPLANATION

Write the percentage as a fraction out of 100 and multiply by \$35. This is the same as $0.15 \times \$35$. (Note: 'of' means to 'multiply'.)

Now you try

Find 35% of \$24.

Example 18 Finding the original amount

Determine the original amount if 5% of the amount is \$45.

SOLUTION

Method 1: Unitary method

$$\begin{aligned} 5\% \text{ of the amount} &= \$45 \\ 1\% \text{ of the amount} &= \$9 \\ 100\% \text{ of the amount} &= \$900 \end{aligned}$$

So the original amount is \$900.

EXPLANATION

To use the unitary method, find the value of 1 part or 1% then multiply by 100 to find 100%.

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Method 2: Division

$$\begin{aligned} 5\% \text{ of the amount} &= \$45 \\ 0.05 \times \text{amount} &= \$45 \\ \text{amount} &= \$45 \div 0.05 \\ &= \$900 \end{aligned}$$

So the original amount is $900.

Write 5% as a decimal and set up an equation using multiplication for 'of'. Then divide both sides by 0.05 to find the original amount.

Now you try

Determine the original amount if 4% of the amount is $12.

Exercise 1F

FLUENCY	1–3(½), 4, 5–7(½)	1–3(½), 4, 5–7(½)	1–3(¼), 5–8(⅓)

Example 15a

1 Express each decimal as a percentage.

a 0.34 **b** 0.4 **c** 0.06 **d** 0.7
e 1 **f** 1.32 **g** 1.09 **h** 3.1

Example 15b

2 Express each percentage as a decimal.

a 67% **b** 30% **c** 250% **d** 8%
e $4\frac{3}{4}\%$ **f** $10\frac{5}{8}\%$ **g** $30\frac{2}{5}\%$ **h** $44\frac{1}{4}\%$

Example 15c

3 Express each part of Question **2** as a simplified fraction.

4 Copy and complete these tables. Use the simplest form for fractions.

Percentage	Fraction	Decimal
10%		
	$\frac{1}{2}$	
5%		
		0.25
		0.2
	$\frac{1}{8}$	
1%		
	$\frac{1}{9}$	
		$0.\dot{2}$

Percentage	Fraction	Decimal
	$\frac{3}{4}$	
15%		
		0.9
37.5%		
$33\frac{1}{3}\%$	$\frac{1}{3}$	
$66\frac{2}{3}\%$		
		0.625
	$\frac{1}{6}$	

Example 16

5 Convert each of the following to a percentage.

a $3 out of $12 **b** $6 out of $18 **c** $0.40 out of $2.50
d $44 out of $22 **e** $140 out of $5 **f** 45c out of $1.80

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Example 17

6 Find:

a 10% of \$360 **b** 50% of \$420 **c** 75% of 64 kg
d 12.5% of 240 km **e** 37.5% of 40 apples **f** 87.5% of 400 m
g $33\frac{1}{3}$% of 750 people **h** $66\frac{2}{3}$% of 300 cars **i** $8\frac{3}{4}$% of \$560

Example 18

7 Determine the original amount when:

a 10% of the amount is \$12 **b** 6% of the amount is \$42
c 3% of the amount is \$9 **d** 40% of the amount is \$2.80
e 90% of the amount is \$0.18 **f** 35% of the amount is \$140

8 Determine the value of x in the following.

a 10% of x is \$54 **b** 15% of x is \$90 **c** 25% of x is \$127
d 18% of x is \$225 **e** 105% of x is \$126 **f** 110% of x is \$44

PROBLEM–SOLVING	9,10	10–12	12–14

9 Bad weather stopped a cricket game for 35 minutes of a scheduled $3\frac{1}{2}$ hour match. What percentage of the scheduled time was lost?

10 Joe lost 4 kg and now weighs 60 kg. What percentage of his original weight did he lose?

11 About 80% of the mass of the human body is water. If Clare weighs 60 kg, how many kilograms of water make up her body weight?

12 In a class of 25 students, 40% have been to England. How many students have not been to England?

13 20% of the cross country runners in a school team weigh between 60 kg and 70 kg. If 4% of the school's 1125 students are in the cross country team, how many students in the team weigh between 60 and 70 kg?

14 One week Grace spent 16% of her weekly wage on a new bookshelf that cost \$184. What is her weekly wage?

REASONING	15	15	15, 16

15 Consider the equation P% of $a = b$ (like 20% of 40 = 8 or 150% of 22 = 33).

a For what value of P is P% of $a = a$?
b For what values of P is P% of $a < a$?
c For what values of P is P% of $a > a$?

16 What can be said about the numbers x and y if:

a 10% of x = 20% of y? **b** 10% of x = 50% of y?
c 5% of x = 3% of y? **d** 14% of x = 5% of y?

ENRICHMENT: More than 100%	–	–	17

17 **a** Find 120% of 60.
b Determine the value of x if 165% of x is 1.5.
c Write 2.80 as a percentage.
d Write 325% as a fraction.
e An investment of \$2000 in a bank account increases to \$5000 over a period of time. By how much has the amount increased as a percentage?

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1G Percentage increase and decrease CONSOLIDATING

Learning intentions
- To be able to increase or decrease an amount by a given percentage
- To be able to calculate the percentage by which an amount has increased or decreased

Past, present and future learning
- This section consolidates Stage 4 concepts which are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- These concepts are not revised in our Year 10 book
- Expertise with percentages is assumed knowledge for Stage 6 Advanced and may also be required in non-calculator examinations such as NAPLAN and industry aptitude tests

Percentages are often used to describe by how much a quantity has increased or decreased. The price of a car in the new year might be increased by 5%. On a $70 000 car, this equates to a $3500 increase. The price of a shirt might be marked down by 30% and if the shirt originally cost $60, this provides an $18 discount. It is important to note that the increase or decrease is calculated on the original amount.

Tourists often change money into another currency. A foreign exchange trader buys and sells money on the International Money Market. Up-to-date currency exchange rates and percentage increases and decreases are available online.

Lesson starter: The quicker method

Nicky and Mila consider this question: When $250 is increased by 15%, what is the final amount?
Nicky puts his solution on the board with two steps.

Step 1.	15% of $250	=	0.15 × $250
		=	$37.50
Step 2.	Final amount	=	$250 + $37.50
		=	$287.50

Mila says that the same problem can be solved with only one step using the number 1.15.

- Can you explain Mila's method? Write it down.
- What if the question was altered so that $250 is decreased by 15%. How would Nicky and Mila's methods work in this case?
- Which of the two methods do you prefer and why?

KEY IDEAS

- To increase an amount by a given percentage, multiply by 100% plus the given percentage.
  For example: to increase by 30%, multiply by 100% + 30% = 130% = 1.3
- To decrease an amount by a given percentage, multiply by 100% minus the given percentage.
  For example: to decrease by 25%, multiply by 100% − 25% = 75% = 0.75

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- To find a **percentage change** use:

  $$\text{Percentage change} = \frac{\text{change}}{\text{original amount}} \times 100\%$$

BUILDING UNDERSTANDING

1 State the missing number.
- **a** To increase a number by 40% multiply by _____.
- **b** Multiplying by 1.21 increases a number by _____.
- **c** To decrease a number by 73% multiply by _____.
- **d** Multiplying by 0.94 decreases a number by _____.

2 The price of a watch increases from \$120 to \$150.
- **a** What is the price increase?
- **b** Express this increase as a percentage of the original price.

3 A person's weight decreases from 108 kg to 96 kg.
- **a** What is the weight decrease?
- **b** Express this decrease as a percentage of the original weight. Round to one decimal place.

Example 19 Increasing by a percentage

Increase \$70 by 15%.

SOLUTION

$100\% + 15\% = 115\%$
$= 1.15$

$\$70 \times 1.15 = \$80.50$

EXPLANATION

First add 15% to 100%.

Multiply by 1.15 to give \$70 plus the increase in one step.

Now you try

Increase \$40 by 25%.

Example 20 Decreasing by a percentage

Decrease \$5.20 by 40%.

SOLUTION

$100\% - 40\% = 60\%$
$= 0.6$

$\$5.20 \times 0.6 = \$3.12$

EXPLANATION

First subtract the 40% from 100% to find the percentage remaining.

Multiply by 60% = 0.6 to get the result.

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Now you try

Decrease \$6.80 by 30%.

Example 21 Finding a percentage change

a The price of a mobile phone increased from \$250 to \$280. Find the percentage increase.
b The population of a town decreases from 3220 to 2985. Find the percentage decrease and round to one decimal place.

SOLUTION	EXPLANATION
a $\text{Increase} = \$280 - \$250$ $= \$30$	First find the actual increase.
$\text{Percentage increase} = \frac{30}{250} \times \frac{100}{1}\%$ $= 12\%$	Divide the increase by the original amount and multiply by 100%.
b $\text{Decrease} = 3220 - 2985$ $= 235$	First find the actual decrease.
$\text{Percentage decrease} = \frac{235}{3220} \times \frac{100}{1}\%$ $= 7.3\%$ (to 1 d.p.)	Divide the decrease by the original population. Round as indicated.

Now you try

a The price of a TV increased from \$300 to \$375. Find the percentage increase.
b The population of a town decreases from 6250 to 5500. Find the percentage decrease.

Example 22 Finding the original amount

After rain, the volume of water in a tank increased by 24% to 2200 L. How much water was in the tank before it rained? Round to the nearest litre.

SOLUTION	EXPLANATION
$100\% + 24\% = 124\%$ $= 1.24$	Write the total percentage as a decimal.
$\text{Original volume} \times 1.24 = 2200$ $\therefore \text{Original volume} = 2200 \div 1.24$ $= 1774$ litres	The original volume is increased by 24% to give 2200 litres. Divide to find the original volume. Round as indicated. Alternatively use the unitary method.

Now you try

The area of a rectangular deck is increased by 32% to 300 m^2. What was the area of the original deck? Round to the nearest square metre.

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Exercise 1G

FLUENCY	1–2(½), 3–5	1–2(½), 3, 5, 6	1–2(¼), 4, 6, 7

Example 19

1 Increase the given amounts by the percentage given in the brackets.

a $50 (5%) b 35 min (8%) c 250 mL (50%) d 1.6 m (15%)
e 24.5 kg (12%) f 25 watts (44%) g $13 000 (4.5%) h $1200 (10.2%)

Example 20

2 Decrease the given amounts by the percentage given in the brackets.

a 24 cm (20%) b 35 cm (30%) c 42 kg (7%) d 55 min (12%)
e $90 (12.8%) f 220 mL (8%) g 25°C (28%) h $420 (4.2%)

Example 21a

3 The length of a bicycle sprint race is increased from 800 m to 1200 m. Find the percentage increase.

4 From the age of 10 to 17, Nick's height increased from 125 cm to 180 cm. Find the percentage increase.

Example 21b

5 The temperature at night decreased from 25°C to 18°C. Find the percentage decrease.

6 Brett, a rising sprint star, lowered his 100 m time from 13 seconds flat to 12.48 seconds. Find the percentage decrease.

7 Find the percentage change in these situations, rounding to one decimal place in each case.

a 22 g increases to 27 g b 86°C increases to 109°C
c 136 km decreases to 94 km d $85.90 decreases to $52.90

PROBLEM–SOLVING	8–10	8–11	9–13

Example 22

8 After a price increase of 20% the cost of entry to a museum rose to $25.80. Find the original entry fee.

9 The average attendance at a sporting match rose by 8% in the past year to 32 473. Find the average attendance in the previous year to the nearest integer.

10 A car when resold had decreased in value by 38% to $9235. What was the original price of the car to the nearest dollar?

11 After the redrawing of boundaries, land for a council electorate had decreased by 14.5% to 165 420 hectares. What was the original size of the electorate to the nearest hectare?

12 The total price of an item including GST (at 10%) is $120. How much GST is paid to the nearest cent?

13 A consultant charges a school a fee of $300 per hour including GST (at 10%). The school hires the consultant for 2 hours but can claim back the GST from the tax office. Find the net cost of the consultant for the school to the nearest cent.

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REASONING	14	14, 15	15–17

14 An investor starts with \$1000.

a After a bad day the initial investment is reduced by 10%. Find the balance at the end of the day.

b The next day is better and the balance is increased by 10%. Find the balance at the end of the second day.

c The initial amount decreased by 10% on the first day and increased by 10% on the second day. Explain why the balance on the second day didn't return to \$1000.

15 During a sale, all travel guides in a bookstore are reduced from \$30 by 20%. What percentage increase is required to take the price back to \$30?

16 The cost of an item is reduced by 50%. What percentage increase is required to return to its original price?

17 The cost of an item is increased by 75%. What percentage decrease is required to return to its original price? Round to two decimal places.

ENRICHMENT: Repeated increase and decrease	–	–	18, 19

18 If the cost of a pair of shoes was increased three times by 10%, 15% and 8% from an original price of \$80, then the final price would be

$\$80 \times 1.10 \times 1.15 \times 1.08 = \$109.30$

Use a similar technique to find the final price of these items. Round the answer to the nearest cent.

a Skis starting at \$450 and increasing by 20%, 10% and 7%

b A computer starting at \$2750 and increasing by 6%, 11% and 4%

c A smart watch starting at \$280 and decreasing by 10%, 25% and 20%

d A circular saw starting at \$119 and increasing by 18%, 37% and 11%

19 If an amount is increased by the same percentage each time, powers can be used. For example, 50 kg increased by 12% three times would increase to

$50\text{ kg} \times 1.12 \times 1.12 \times 1.12$
$= 50\text{ kg} \times (1.12)^3$
$= 70.25\text{ kg (to 2 d.p.)}$

Use a similar technique to find the final value in these situations. Round the answer to two decimal places.

a A rat with an initial mass of 60 grams grows at a rate of 10% every month for 3 months.

b The cost of a new lawnmower with an initial cost of \$80 000 increases by 5% every year for 4 years.

c The value of a house initially at \$380 000 decreases by 4% per year for 3 years.

d As a result of being sharpened, the length of a pencil initially 16 cm long decreases by 15% each week for 5 weeks.

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Progress quiz

1A **1** Evaluate the following.

a $-45 + (-3 \times 6 + 9)$

b $-4 \times 8 \div (-2) - 2^3$

1B **2** Round each of these numbers as indicated in the brackets.

a 3.45678 (2 decimal places)

b 45.89985 (1 decimal place)

c 0.007856473 (2 significant figures)

d 46 789 000 (3 significant figures)

1C **3** Write these fractions as decimals.

a $\frac{3}{4}$

b $\frac{4}{5}$

c $\frac{7}{20}$

d $\frac{1}{3}$

1C **4** Write these decimals as fractions.

a 0.9

b 0.85

c 0.125

1D **5** Evaluate the following.

a $\frac{1}{2} + \frac{5}{6}$

b $\frac{6}{7} - \frac{2}{3}$

c $1\frac{1}{2} \times \frac{4}{5}$

d $\frac{5}{8} \div \frac{3}{10}$

1E **6** Simplify these ratios.

a 45 : 81

b \$24 to 80 cents

c 2.4 : 0.36

d $\frac{3}{4} : 8$

1E **7** Divide:

a \$400 in the ratio of 5 : 3

b 6 kg in the ratio of 3 : 7

c 1000 cm in the ratio of 4 : 5 : 6

1E **8** Write these rates in simplest form.

a \$350 in 5 hours

b 200 km in 2.5 hours

1E **9** Which is the best buy (cheaper deal)?

Product A, which costs \$3.45 for 500 grams, or product B, which costs \$4.38 for 680 grams.

1F **10** Write the following as percentages.

a $\frac{4}{5}$

b 0.96

c $3\frac{3}{4}$

d 40 cents of \$5

1F **11** Find 34% of 6000 cm.

1F **12** Determine the original amount if 8% of the amount is \$32.

1G **13** a Increase \$450 by 12%.

b Decrease 500 kg by 5%.

1G **14** Find the percentage change if 40 g is reduced to 24 g.

1G **15** The volume of a tank is increased by 12% to 952 L. Find the original volume.

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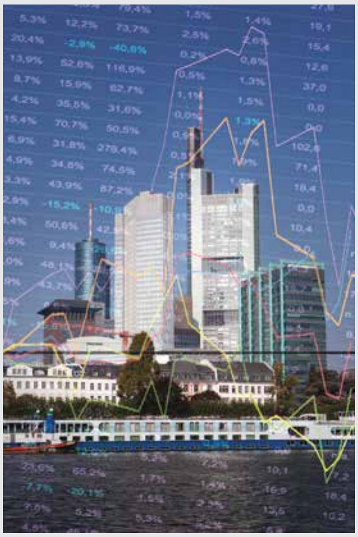
1H Profits and discounts CONSOLIDATING

Learning intentions
- To know and understand the terminology associated with profits, losses and discounts
- To be able to solve problems involving percentage profits, losses and discounts

Past, present and future learning
- This section consolidates Stage 4 concepts which are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- These concepts are not revised in our Year 10 book and are not included in Stage 6 Advanced
- Expertise with percentages is assumed knowledge for Stage 6 Advanced and may also be required in non-calculator examinations such as NAPLAN and industry aptitude tests

Percentages are widely used in the world of finance. Profits, losses, commissions, discounts and taxation are often expressed and calculated using percentages.


Percentages are used by businesses to describe profits and losses.

Lesson starter: The best discount

Two adjacent shops are selling the same jacket at a discounted price. The recommended retail price for the jacket is the same for both shops. Each shop has a sign near the jacket with the given details:

- Shop A: Discounted by 25%
- Shop B: Reduced by 20% then take a further 10% off that discounted price.

Which shop offers the bigger discount?
Is the difference between the discounts equal to 5% of the retail price?

KEY IDEAS

- **Profit** is the amount of money made on a sale. If the profit is negative we say that a loss has been made.
  - Profit = selling price − cost price
- **Mark-up** is the amount added to the cost price to produce the selling price.
  - Selling price = cost price + mark-up
- The **percentage profit** or loss can be found by dividing the profit or loss by the cost price and multiplying by 100.
  - $\%\,\text{Profit or loss} = \dfrac{\text{profit or loss}}{\text{cost price}} \times 100\%$
- **Discount** is the amount by which an item is marked down.
  - New price = original price − discount
  - Discount = % discount × original price

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BUILDING UNDERSTANDING

1 State the missing numbers in the table.

Cost price ($)	7	18		3250
Selling price ($)	10	15.50	11.80	
Profit/Loss ($)			4.50 profit	1180 loss

2 The following percentage discounts are given on the price of various products. State the sale price as a percentage of the original price.

a 10% **b** 15%

Example 23 Determining profit

A manufacturer produces an item for $400 and sells it for $540.

a Determine the profit made.

b Express this profit as a percentage of the cost price.

SOLUTION

a Profit $= \$540 - \$400$
$= \$140$

b $\%\text{ profit} = \dfrac{140}{400} \times 100\%$
$= 35\%$

EXPLANATION

Profit = selling price − cost price

$\%\text{ profit} = \dfrac{\text{profit}}{\text{cost price}} \times 100\%$

Now you try

A manufacturer produces an item for $320 and sells it for $464.

a Determine the profit made.

b Express this profit as a percentage of the cost price.

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Example 24 Calculating selling price from mark-up

An electrical store marks up all entertainment systems by 30%.
If the cost price of an entertainment system is \$8000, what will be its selling price?

SOLUTION

$$\begin{aligned}\text{Selling price} &= 130\% \text{ of cost price}\\ &= 1.3 \times \$8000\\ &= \$10\,400\end{aligned}$$

Alternative method:

$$\begin{aligned}\text{Mark-up} &= 30\% \text{ of } \$8000\\ &= 0.3 \times \$8000\\ &= \$2400\end{aligned}$$

$$\begin{aligned}\therefore \text{selling price} &= \$8000 + \$2400\\ &= \$10\,400\end{aligned}$$

EXPLANATION

As there is a 30% mark-up added to the cost price (100%), it follows that the selling price is 130% of the cost price.

Change percentage to a decimal and evaluate.

Selling price = cost price + mark-up.

Now you try

A nursery marks up all plants by 40%.
If the cost price of a plant is \$12, what will be its selling price?

Example 25 Finding the discount amount

An electronics shop advertises a 15% discount on all audio-visual equipment as a Christmas special. Find the sale price on a projection system that has a marked price of \$18 000.

SOLUTION

$$\begin{aligned}\text{New price} &= 85\% \text{ of } \$18\,000\\ &= 0.85 \times \$18\,000\\ &= \$15\,300\end{aligned}$$

Alternative method:

$$\begin{aligned}\text{Discount} &= 15\% \text{ of } \$18\,000\\ &= 0.15 \times \$18\,000\\ &= \$2700\end{aligned}$$

$$\begin{aligned}\therefore \text{new price} &= \$18\,000 - \$2700\\ &= \$15\,300\end{aligned}$$

EXPLANATION

Discounting by 15% means the new price is 85%, i.e. (100 − 15)%, of the original price.

Change the percentage to a decimal and evaluate.

New price is original price minus discount.

Now you try

A shop has items discounted by 16% for its Easter sale. Find the sale price if an item was marked at \$204.

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Example 26 Calculating the original price after discount

A toy shop discounts a toy by 10% in a sale. If the sale price was \$10.80, what was the original price?

SOLUTION

Let \$$x$ be the original price.

$$\begin{aligned} 0.9 \times x &= 10.8 \\ x &= 10.8 \div 0.9 \\ x &= 12 \end{aligned}$$

The original price was \$12.

EXPLANATION

The percentage discount = 100% − 10% = 90% = 0.9.

Thus \$10.80 is 90% of the original price. Write an equation representing this and solve.

Write the answer in words.

Now you try

A car is discounted by 20%. If the sale price was \$32 000 what was the original price?

Exercise 1H

FLUENCY	1–5, 8, 10	1, 2(½), 3–6, 8–11	3–5, 7, 9, 11

Example 23

1 A shop buys an item for \$500 and sells it for \$575.
  - a Determine the profit made.
  - b Express this profit as a percentage of the cost price.

2 A manufacturer produces and sells items for the prices shown.
  - i Determine the profit made.
  - ii Express this profit as a percentage of the cost price.

  - a Cost price \$10, selling price \$12
  - b Cost price \$20, selling price \$25
  - c Cost price \$120, selling price \$136.80
  - d Cost price \$1400, selling price \$3850

3 Dom runs a pizza business. Last year he took in \$88 000 and it cost him \$33 000 to run the business. What is his percentage profit for the year? Round to two decimal places.

4 It used to take 20 hours to fly to Los Angeles. It now takes 12 hours. What is the percentage decrease in travel time?

Example 24

5 Helen owns a handicrafts store that has a policy of marking up all of its items by 25%. If the cost price of one article is \$30, what will be its selling price?

6 Lenny marks up all computers in his store by 12.5%. If a computer cost him \$890, what will be the selling price of the computer, to the nearest cent?

7 A used-car dealer purchases a vehicle for \$13 000 and sells it for \$18 500. Determine the percentage mark-up on the vehicle to one decimal place.

Example 25

8 A store is offering a 15% discount for customers who pay with cash. Rada wants a microwave oven marked at \$175. How much will she pay if she is paying with cash?

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9 A camera store displays a camera marked at \$595 and a lens marked at \$380. Sam is offered a discount of 22% if he buys both items. How much will he pay for the camera and lens?

Example 26

10 A refrigerator is discounted by 25%. If Paula pays \$460 for it, what was the original price? Round to the nearest cent.

11 A membership for a football team is discounted by 15% during a season. If the new price was \$187, what was the original cost?

PROBLEM–SOLVING	12	12, 13	13, 14

12 A store marks up a \$550 widescreen television by 30%. During a sale it is discounted by 20%. What is the percentage change in the original price of the television?

13 An armchair was purchased for a cost price of \$380 and marked up to a retail price. It was then discounted by 10% to a sale price of \$427.50. What is the percentage mark-up from the cost price to the sale price?

14 Pairs of shoes are manufactured for \$24. They are sold to a warehouse with a mark-up of 15%. The warehouse sells the shoes to a distributor after charging a holding fee of \$10 per pair. The distributor sells them to 'Fine Shoes' for a percentage profit of 12%. The store then marks them up by 30%.

a Determine the price of a pair of shoes if you buy it from one of the 'Fine Shoes' stores (round to the nearest 5 cents).

b What is the overall percentage mark-up of a pair of shoes to the nearest whole per cent?

REASONING	15	15, 16	16, 17

15 The price of an item to be sold includes a percentage mark-up as well as a sale discount. Does it make a difference in which order the mark-up and discount occur? Explain your answer.

16 Depreciation relates to a reduction in value. A computer depreciates in value by 30% in its first year. If its original value is \$3000, find its value after one year.

17 John buys a car for \$75 000. The value of the car depreciates at 15% per year. After 1 year the car is worth 85% of its original value, i.e. 85% of $\$75\,000 = 0.85 \times 75\,000 = \$63\,750$.

a What is the value of the car, to the nearest cent, after:

i 2 years?   ii 5 years?

b After how many years will the car first be worth less than \$15 000?

ENRICHMENT: Deposits and discounts	–	–	18

18 A car company offers a special discount deal. After the cash deposit is paid, the amount that remains to be paid is discounted by a percentage that is one tenth of the deposit percentage.

For example, a deposit of \$8000 on a \$40 000 car represents 20% of the cost. The remaining \$32 000 will be discounted by 2%.

Find the amount paid for each car given the following car price and deposit. Round to two decimal places where necessary.

a Price = \$35 000, deposit = \$7000   b Price = \$45 000, deposit = \$15 000

c Price = \$28 000, deposit = \$3500   d Price = \$33 400, deposit = \$5344

e Price = \$62 500, deposit = \$5000   f Price = \$72 500, deposit = \$10 150

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1I Income

Learning intentions

- To know and understand the different ways that workers can be paid for their labour
- To use rates and percentages to calculate salaries, wages, commissions, royalties etc
- To be able to make conversions between pay per hour, day, week, fortnight, month and year

Past, present and future learning

- This section introduces the Stage 5 Core Topic called *Financial Mathematics A*
- These concepts are not revised in our Year 10 book and are not included in Stage 6 Advanced

Most people's income is made up largely of the money they receive from their paid work – their job. Depending on the job, this payment can be made using a number of different methods. Many professional workers will receive an annual fixed *salary* which may be paid monthly or fortnightly. Casual workers, including those working in the retail area or restaurants, may receive a *wage* where they are paid a rate per hour worked. This rate may be higher out of regular working hours such as weekends or public holidays. Many sales people, including some real estate agents, may receive a weekly fee (a *retainer*) but may also receive a set percentage of the amount of sales they make (a *commission*). From their income, people have to pay living costs such as electricity, rent, groceries and other items. In addition, they have to pay tax to the government, which funds many of the nation's infrastructure projects and welfare. The method in which this tax is paid from their income may also vary.

A laboratory technician might receive a 1% pay rise giving her a $600 p.a. increase in gross pay. This amount is then decreased by 32.5% tax, giving the technician a net pay increase of $405 p.a.

Lesson starter: Which job pays better?

Ben and Nick are both looking for part-time work and they spot the following advertisements.

Kitchen hand
$9.40 per hour, $14.10 per hour on weekends.

Office assistant
Receive $516 per month for 12 hours work per week.

- Nick gets the job as the kitchen hand. In his first week he works 4 hours during the week and 8 hours at the weekend. How much did he earn?
- Ben gets the job as the office assistant. How much does he earn per week if he works 4 weeks in a month? What does his hourly rate turn out to be?
- If Nick continues to work 12 hours in a week, does he earn more than Ben if he only works on week days? How many weekday hours must Nick work to match Ben's pay?
- Out of the 12 hours, what is the minimum number of hours Nick must work at the weekend to earn at least as much as Ben?

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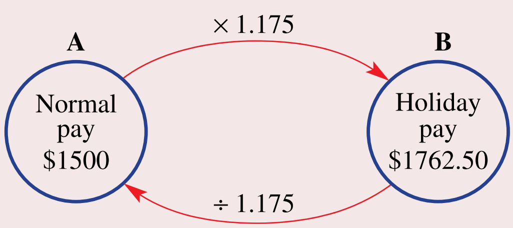
KEY IDEAS

- Workers who earn a **wage** (for example, a casual waiter) are paid a fixed rate per hour. Hours outside the normal working hours (public holidays etc.) are paid at a higher rate called **overtime**. This can occur in a couple of common ways:
  - **Time and a half**: pay is 1.5 times the usual hourly rate
  - **Double time**: pay is twice the usual hourly rate
- Workers who earn a **salary** (for example, an engineer) are paid a fixed amount per year, say \$95 000. This is often paid monthly or fortnightly.
  - 12 months in a year and approximately 52 weeks in a year = 26 fortnights
- Some wage and salary earners are paid **leave loading**. When they are on holidays, they earn their normal pay plus a bonus called leave loading. This is usually 17.5% of their normal pay.

$$\begin{aligned} 100\% + 17.5\% &= 117.5\% \\ &= 1.175 \end{aligned}$$



Leave loading = 17.5% of A = \$262.50

- Some people are paid according to the number of items they produce. This is called **piecework**.
- **Commission** is a proportion of the overall sales amount. Salespeople may receive a commission of their sales as well as a set weekly or monthly fee called a **retainer.**
  - commission = %commission × total sales
- **Royalties:** Some people who write books or music are paid **royalties**, which could be 10% of the sale price of every book or song which is sold.
- A person's **gross income** is the total income that they earn. The **net income** is what is left after deductions, such as tax and union fees, are subtracted.
  - Net income = gross income − deductions
- **Taxation** is paid to the government once a person's taxable income passes a set amount. The amount paid depends on the person's taxable income.

BUILDING UNDERSTANDING

1 A young employee has an annual salary of \$47 424 and works 38-hour weeks. Find the average earnings:

a per month **b** per week **c** per hour.

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2 A shop assistant is paid \$11.40 per hour. Calculate their earnings for the following hours worked.

- **a** 7 hours
- **b** 5.5 hours
- **c** 4 hours at twice the hourly rate (double time)
- **d** 6 hours at 1.5 times the hourly rate (time and a half)

3 Find the commission earned on the given sales if the percentage commission is 20%.

- **a** \$1000
- **b** \$4500

Example 27 Comparing wages and salaries

Ken earns an annual salary of \$88 450 and works a 38-hour week. His wife Brooke works part time in retail and earns \$53.20 per hour.

- **a** Calculate how much Ken earns per week.
- **b** Determine who has the higher hourly rate of pay.
- **c** If Brooke works on average 18 hours per week, what is her yearly income?

SOLUTION	EXPLANATION
a Weekly rate $= \$88\,450 \div 52$ $= \$1700.96$ $\therefore$ Ken earns \$1700.96 per week	\$88 450 pay in a year. There are approximately 52 weeks in a year.
b Brooke: \$53.20 per hour Ken: $\$1700.96 \div 38$ $= \$44.76$ per hour $\therefore$ Brooke is paid more per hour.	Ken works 38 hours in week. Hourly rate = weekly rate ÷ number of hours. Round to the nearest cent. Compare hourly rates.
c Weekly income $= \$53.20 \times 18$ $= \$957.60$ Yearly income $= \$957.60 \times 52$ $= \$49\,795.20$	Weekly income = hourly rate × number of hours. Multiply by 52 weeks to get yearly income.

Now you try

Maria earns an annual salary of \$124 500 and works a 40-hour week. Her husband Gary works part time in catering and earns \$64.50 per hour.

- **a** Calculate how much Maria earns per week.
- **b** Determine who has the higher hourly rate of pay.
- **c** If Gary works an average of 20 hours per week, what is his yearly income?

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Example 28 Calculating overtime

Georgie works some weekends and late nights in addition to normal working hours and has overtime pay arrangements with her employer.

a Calculate how much Georgie earns in one week if she works 16 hours during the week at the normal hourly rate of \$18.50 and 6 hours on the weekend at time and a half.

b Georgie's normal hourly rate is changed. In a week she works 9 hours at the normal rate, 4 hours at time and a half and 5 hours at double time. If she earns \$507.50, what is her new normal hourly rate?

SOLUTION	EXPLANATION
a Earnings at normal rate $= 16 \times \$18.50$ $= \$296$	16 hours at standard rate
Earnings at time and a half $= 6 \times 1.5 \times \$18.50$ $= \$166.50$	Time and half is 1.5 times the normal rate.
Total earnings $= \$296 + \$166.50$ $= \$462.50$	Combine earnings.
Alternative method: 6 hours of time and half $= 6 \times 1.5$ $=$ 9 hours of normal time $9 + 16 = 25$ $25 \times \$18.50 = \$462.50$	
b Equivalent hours worked in the week $= 9 + (4 \times 1.5) + (5 \times 2)$ $= 9 + 6 + 10$ $=$ 25 hours	Calculate the number of equivalent hours worked. 4 hours at time and a half is the same pay as for working 6 hours (4×1.5). 5 hours at double time is the same as working 10 hours (5×2).
Normal hourly rate $= \$507.50 \div 25$ $= \$20.30$	Divide weekly earnings by the 25 equivalent hours worked.

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Now you try

Matt works some weekends and nights in addition to normal working hours and has overtime pay arrangements with his employee.

a Calculate how much Matt earns in one week if he works 20 hours at the normal hourly rate of \$15.20, and 5 hours at time and a half.

b Matt's normal hourly rate is changed. In a week he works 10 hours at the normal rate, 4 hours at time and a half and 2 hours at double time. If he earns \$330 in a week, what is his new normal hourly rate?

Example 29 Calculating commission and piecework

a A saleswoman is paid a retainer of \$1500 per month. She also receives a commission of 1.25% on the value of goods she sells. If she sells goods worth \$5600 during the month, calculate her earnings for that month.

b David is paid for picking fruit. He is paid \$35 for every bin he fills. Last week he worked 8 hours per day for 6 days and filled 20 bins. Calculate his hourly rate of pay.

SOLUTION	EXPLANATION
a $\text{Commission} = 1.25\% \text{ of } \$5600$ $= 0.0125 \times \$5600$ $= \$70$	Calculate the commission on sales. Change the percentage to a decimal and evaluate.
$\text{Earnings} = \$1500 + \$70$ $= \$1570$	Earnings = retainer + commission
b $\$35 \times 20 = \$700$ $8 \times 6 = 48 \text{ hours}$	This is David's pay for the week: 20 bins at \$35 each.
$\$700 \div 48 = \$14.58 \text{ (to 2 d.p.)}$	This is David's hourly rate.

Now you try

a A salesman is paid a retainer of \$1250 per month. He also receives a commission of 1.1% on the value of the goods he sells. If he sells goods worth \$4500 during the month, calculate his earnings for that month.

b Toby gets paid for walking dogs. He is paid \$10 per dog walk. Last week he worked 2 hours walking dogs per day for 5 days and walked 20 dogs. Calculate his hourly rate of pay.

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Exercise 1I

FLUENCY	1–5, 7, 9	1, 2, 3–4(½), 5, 7, 9	4(½), 6, 8, 9, 10

Example 27

1 Georgia earns an annual salary of $95 000 and works a 40-hour week. Her husband Mike works part time and earns $45.20 per hour.

- **a** Calculate how much Georgia earns per week.
- **b** Determine who has the higher hourly rate of pay.
- **c** If Mike works on average 16 hours per week, what is his yearly income?

2 **a** Calculate the hourly rate of pay for a 38-hour working week with an annual salary of:

- **i** $38 532
- **ii** $53 352
- **iii** $83 980

b Calculate the yearly income for someone who earns $24.20 per hour and in a week works, on average:

- **i** 24 hours
- **ii** 35 hours
- **iii** 16 hours

Example 28a

3 A job has a normal working hours pay rate of $18.40 per hour. Calculate the pay, including overtime, for the following hours worked.

- **a** 3 hours and 4 hours at time and a half
- **b** 4 hours and 6 hours at time and a half
- **c** 14 hours and 3 hours at double time
- **d** 20 hours and 5 hours at double time
- **e** 10 hours and 8 hours at time and a half and 3 hours at double time
- **f** 34 hours and 4 hours at time and a half and 2 hours at double time

4 Calculate how many hours at the standard hourly rate the following working hours are equivalent to:

- **a** 3 hours and 2 hours at double time
- **b** 6 hours and 8 hours at time and a half
- **c** 15 hours and 12 hours at time and a half
- **d** 10 hours time and a half and 5 hours at double time
- **e** 20 hours and 6 hours at time and a half and 4 hours at double time
- **f** 32 hours and 4 hours at time and a half and 1 hour at double time

Example 28b

5 Jim, a part-time gardener, earned $261 in a week in which he worked 12 hours during normal working hours and 4 hours overtime at time and a half. What was his hourly rate of pay?

6 Sally earned $329.40 in a week in which she worked 10 hours during the week and 6 hours on Saturday at time and a half and 4 hours on Sunday at double time. What was her hourly rate of pay?

Example 29a

7 Amy works at Best Bookshop. During one week she sells books valued at $800. If she earns $450 per week plus 5% commission, how much does she earn in this week?

8 Jason works for an alarm system company and earns $650 per month plus 4% commission on sales. How much does he earn in a month in which he sells $84000 worth of alarm systems?

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Example 29b

9 Mike delivers the local newspaper. He is paid $30 to deliver 500 newspapers. How much is he paid for each one?

10 Lily is paid for raking leaves. She is paid $10 for every bag she fills. Last week she worked for 6 hours and filled 15 bags. Calculate her hourly rate of pay.

PROBLEM–SOLVING	11, 12	12–14	13–16

11 Arrange the following workers in order of most to least earnt in a week of work.
- Adam has an annual salary of $33 384.
- Bill works for 26 hours at a rate of $22.70 per hour.
- Cate earns $2964 per month. (Assume 52 weeks in a year.)
- Diana does shift work 4 days of the week between 10 p.m. and 4 a.m. She earns $19.90 per hour before midnight and $27.50 per hour after midnight.
- Ed works 18 hours at the normal rate of $18.20 per hour, 6 hours at time and a half and 4 hours at double time in a week.

12 Stephen earns an hourly rate of $17.30 for the first 38 hours he works, time and a half for the next 3 hours and double time for each extra hour above that. Calculate his earnings if he works 44 hours in a week.

13 Jessica works for Woods Real Estate and earns $800 per week plus 0.05% commission. If this week she sold three houses valued at $334 000, $210 000 and $335 500 respectively, how much will she have earned?

14 A door-to-door salesman sells 10 security systems in one week at $1200 each. For the week, he earns a total of $850 including a retainer of $300 and a commission. Find his percentage commission correct to two decimal places.

15 Mel is taking her holidays. She receives $2937.50. This includes her normal pay and 17.5% leave loading. How much is the leave loading?

REASONING	17	17, 18	18, 19

16 Karl is saving and wants to earn $90 from a casual job paying $7.50 per hour.

a How many hours must he work to earn the $90?

b Karl can also work some hours where he is paid time and a half. He decides to work x hours at the normal rate and y hours at time and a half to earn the $90. If x and y are both positive integers, find the possible combinations for x and y.

17 A car salesman earns 2% commission on sales up to $60 000 and 2.5% on sales above that.

a Determine the amount earned on sales worth:

i $46 000 **ii** $72 000

b Write a rule for the amount, A dollars, earned on sales of $\$x$ if:

i $x \leqslant 60\,000$ **ii** $x > 60\,000$

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18 Kim has a job selling jewellery. She has to choose one of two new payment plans below.
Plan A: $220 per week plus 5% of sales
Plan B: 9% of sales and no set weekly retainer
- a What value of sales gives the same return from each plan?
- b Explain how you would choose between Plan A and Plan B.

ENRICHMENT: Payslips and superannuation	–	–	20

19 The following payslip is given at the end of every fortnight to a part-time teacher who is employed for 3 days per week (6 days per fortnight).

PAYSLIP

EMPLOYER: ABC School
EMPLOYEE: J. Bloggs
Fortnight ending: 17/06/2023 AWARD: Teachers

EARNINGS	QUANTITY	RATE	AMOUNT
Ordinary pay	6.00	339.542307	$2037.25

DEDUCTIONS	AMOUNT	DETAILS
Tax	$562.00	

PROVISIONS	EARNED	TAKEN	BALANCE
Superannuation	$183.35	0.00	$2756.02

SUMMARY

PAY DATE	17/06/2012	Year to date
GROSS	$2037.25	$30 766.42
TAX	$562.00	$8312.00
DEDUCTIONS	$0.00	$0.00
NET	$1475.25	$22 454.42

- a Use the internet to research and answer parts i–iv.
  - i Find the meaning of the word ‘superannuation’.
  - ii Why is superannuation compulsory for all workers in Australia?
  - iii What percentage of gross income is compulsory superannuation?
  - iv What happens to the superannuation that is paid to employees?
- b This employee earned $2037.25 (before tax) plus $183.35 superannuation. Was this the correct amount of superannuation?
- c According to the ‘year to date’ figures, how much tax has been paid?
- d Using the ‘year to date’ figures, express the tax paid as a percentage of the gross income.
- e Complete this sentence: For this worker, approximately _______ cents in every dollar goes to the Australian Taxation Office.

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1J The PAYG income tax system

Learning intentions
- To know and understand the terminology gross income, net income and taxable income
- To be able to calculate net income from gross income and deductions
- To be able to use the ATO table to calculate tax payable

Past, present and future learning
- This section continues the Stage 5 Core Topic called *Financial Mathematics A*
- These concepts are not revised in our Year 10 book and are not included in Stage 6 Advanced

The Australian Taxation Office (ATO) collects taxes on behalf of the government to pay for education, hospitals, roads, railways, airports and services such as the police and fire brigades.

In Australia, the financial year runs from 1 July to 30 June the following year. People engaged in paid employment are normally paid weekly or fortnightly. Most of them pay some income tax every time they are paid for their work. This is known as the Pay-As-You-Go (PAYG) system.

At the end of the financial year (30 June), workers complete an income tax return to determine if they have paid the correct amount of income tax during the year. If they have paid too much, they will receive a tax refund. If they have not paid enough, they will be required to pay more tax.

The rates of tax that are paid depend on your income and these are subject to change depending on government policy.

Lesson starter: The ATO website

The ATO (Australian taxation office) website has some income tax calculators. Use one to find out how much income tax you would need to pay if your taxable income is:

- $5200 per annum ($100 per week)
- $26 000 per annum ($500 per week)
- $78 000 per annum ($1500 per week).

Does a person earning $1000 per week pay twice as much tax as a person earning $500 per week?
Does a person earning $2000 per week pay twice as much tax as a person earning $1000 per week?

ISBN 978-1-009-40936-0

KEY IDEAS

- At the end of a financial year (July 1 – June 30) individuals complete a **tax return** form which includes the following:
  - *all* forms of income, including interest from investments.
  - legitimate **tax deductions** shown on receipts and invoices, such as work-related expenses and donations.
- **Taxable income** is calculated using the formula:
  Taxable income = gross income − tax deductions
- The ATO website includes tables and calculators, such as the following table. Each row in the table is called a tax bracket. These are the personal income tax rates from July 2024 and are subject to change.

Taxable income	Tax on income
\$0 – \$18 200	Nil
\$18 201 – \$45 000	19c for each \$1 over \$18 200
\$45 001 – \$200 000	\$5092 plus 30c for each \$1 over \$45 000
\$200 001 and over	\$51 592 plus 45c for each \$1 over \$200 000

  - Individuals may also need to pay the **Medicare levy**. This is a scheme in which all Australian taxpayers share in the cost of running the medical system. In the 2023–2024 financial year, it was 2% of taxable income.
  - It is possible that you may have paid too much tax during the year and will receive a **tax refund**.
  - It is also possible that you may have paid too little tax and will receive a letter from the ATO asking for the **tax liability** to be paid.

BUILDING UNDERSTANDING

1. Complete this statement:
   Taxable income = ____________ income minus ________________.
2. Is the following statement true or false?
   The highest income earners in Australia pay 45 cents tax for every dollar they earn.
3. In the financial year, Greg paid no income tax. What could his taxable income have been?
4. Ann's taxable income was \$87 000, and Ben's taxable income was \$87 001. Ignoring the Medicare levy, how much more tax does Ben pay than Ann?

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Example 30 Calculating tax

Richard earned \$1050 per week (\$54 600 per annum) from his employer and other sources such as interest on investments. He has receipts for \$375 for work-related expenses and donations.

a Calculate Richard's taxable income.

b Use this tax table to calculate Richard's tax payable.

Taxable income	Tax on income
\$0 – \$18 200	Nil
\$18 201 – \$45 000	19c for each \$1 over \$18 200
\$45 001 – \$200 000	\$5092 plus 30c for each \$1 over \$45 000
\$200 001 and over	\$51 592 plus 45c for each \$1 over \$200 000

c Richard also needs to pay the Medicare levy of 2% of his taxable income. How much is the Medicare levy?

d Add the tax payable and the Medicare levy.

e Express the total tax in part **d** as a percentage of Richard's taxable income. Round to one decimal place.

f During the financial year, Richard's employer sent a total of \$7797 in tax to the ATO. Has Richard paid too much tax, or not enough? Calculate his refund or liability.

SOLUTION

a Gross income = \$54 600
Deductions = \$375
Taxable income = \$54 225

b Tax Payable:
$\$5092 + 0.3 \times (\$54\,225 - \$45\,000)$
$= \$7859.50$

c $\frac{2}{100} \times \$54\,225 = \$1084.50$

d $\$7859.50 + \$1084.50 = \$8944$

e $\frac{8944}{54\,225} \times 100\% = 16.5\%$ (to 1 d.p.)

f Richard paid \$7797 in tax during the year.
He should have paid \$8944.
Richard has not paid enough tax.
He needs to pay another \$1147 in tax.

EXPLANATION

a Taxable income = gross income − deductions

b Richard is in the middle tax bracket in the table, in which it says:
\$5092 plus 30c for each \$1 over \$45 000.
Note 30c is \$0.30.

c Medicare levy is 2% of the taxable income

d This is the total amount of tax that Richard should have paid.

e This implies that Richard paid approximately 16.5% tax on every dollar. This is sometimes read as '16.5 cents in the dollar'.

f This is known as a shortfall or a liability. He will receive a letter from the ATO requesting payment.

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Now you try

Bella earned $2500 per week ($130 000 per annum) from her employer and other sources such as interest on investments. She has receipts for $1800 for work-related expenses and donations.

a Calculate Bella's taxable income.

b Use this tax table to calculate Bella's tax payable.

Taxable income	Tax on income
$0 – $18 200	Nil
$18 201 – $45 000	19c for each $1 over $18 200
$45 001 – $200 000	$5092 plus 30c for each $1 over $45 000
$200 001 and over	$51 592 plus 45c for each $1 over $200 000

c Bella also needs to pay the Medicare levy of 2% of her taxable income. How much is the Medicare levy?

d Add the tax payable and the Medicare levy.

e Express the total tax in part **d** as a percentage of Bella's taxable income. Round to one decimal place.

f During the financial year, Bella's employer sent a total of $35 000 in tax to the ATO. Has Bella paid too much tax, or not enough? Calculate her refund or liability.

Exercise 1J

FLUENCY	1–4	1, 2(½), 3, 5	2(½), 4, 5

Example 30

1 James earned $1800 per week ($93 600 per annum) from his employer and other sources such as interest on investments. He has receipts for $2000 for work-related expenses and donations.

a Calculate James' taxable income.

b Use this tax table to calculate James' tax payable.

Taxable income	Tax on income
$0 – $18 200	Nil
$18 201 – $45 000	19c for each $1 over $18 200
$45 001 – $200 000	$5092 plus 30c for each $1 over $45 000
$200 001 and over	$51 592 plus 45c for each $1 over $200 000

c James also needs to pay the Medicare levy of 2% of his taxable income. How much is the Medicare levy?

d Add the tax payable and the Medicare levy.

e Express the total tax in part **d** as a percentage of James' taxable income.

f During the financial year, James' employer sent a total of $20 000 in tax to the ATO. Has James paid too much tax, or not enough? Calculate his refund or liability.

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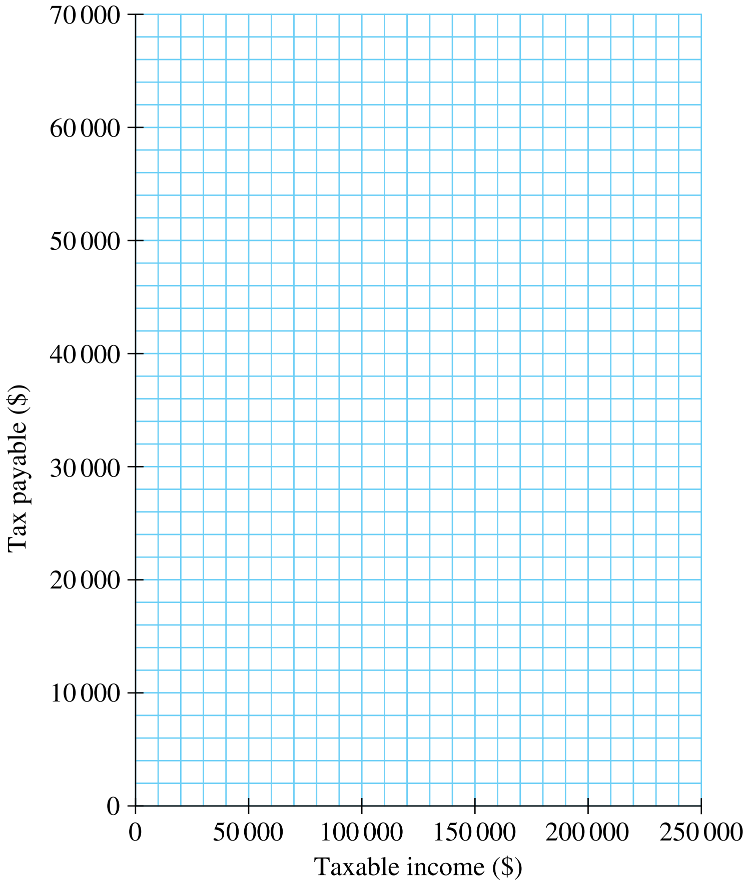
2 Use the tax table given in Question 1 to calculate the income tax payable on these taxable incomes. Ignore the Medicare levy.

a $30 000
b $60 000
c $150 000
d $200 000

3 a Use the tax table to complete this table (ignoring the Medicare levy).

Taxable income	$0	$18 200	$45 000	$200 000	$250 000
Tax payable					

b Copy this set of axes, plot the points, from part a, then join the dots with straight line segments.



4 Jim worked for three different employers. They each paid him $15 000. How much income tax should he have paid?

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5 Lee has come to the end of her first financial year as a teacher. On 30 June she made the notes in the table below about the financial year.

Gross income from employer	\$58 725
Gross income from casual job	\$7500
Interest on investments	\$75
Donations	\$250
Work-related expenses	\$425
Tax paid during the financial year	\$13 070

a Calculate Lee's taxable income.
b Use the tax table to calculate Lee's tax payable.
c Lee also needs to pay the Medicare levy of 2% of her taxable income. How much is the Medicare levy?
d Add the tax payable and the Medicare levy.
e Express the total tax in part d as a percentage of Lee's taxable income.
f Has Lee paid too much tax, or not enough? Calculate her refund or liability.

PROBLEM–SOLVING	6, 7	6–8	7–9

6 Brad's Medicare levy was \$1750. This was 2% of his taxable income. What was his taxable income?

7 Tara is saving for an overseas trip. Her taxable income is usually about \$20 000. She estimates that she will need \$5000 for the trip, so she is going to do some extra work to raise the money. How much extra will she need to earn in order to have an extra \$5000 after tax?

8 When Sue used the tax table to calculate her income tax payable, it turned out to be \$22 722. What was her taxable income?

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9 An example of a progressive taxation system is given in the table.

Income	Tax rate	Tax payable
\$0–\$10 000	0%	\$0
\$10 001–\$30 000	20%	\$0 + 20% of each dollar over \$10 000
\$30 001–\$100 000	30%	\$4000 + 30% of (income – \$30 000)
\$100 001–	40%	\$25 000 + 40% of (income – \$100 000)

a Using the table above, find the tax payable on the following incomes.
i \$20 000 **ii** \$55 000 **iii** \$125 000

b Copy and complete the details in the following progressive tax system.

Income	Tax rate	Tax payable
\$0–\$15 000	0%	\$0
\$15 001–\$40 000	15%	\$0 + 15% of each dollar over \$15 000
\$40 001–\$90 000	25%	
\$90 001–	33%	

c A different system on 'Taxation Island' looks like this.

Income	Tax rate	Tax payable
\$0–\$20 000	10%	10% of total income
\$20 001–\$80 000	30%	30% of total income
\$80 001–	50%	50% of total income

Find the tax payable on an income of:
i \$20 000 **ii** \$21 000
iii \$80 000 **iv** \$80 001

d By referring to your answers in part **c**, describe the problems associated with the taxation system on Taxation Island.

REASONING	10, 11	10, 11	11, 12

10 Explain the difference between gross income and taxable income.

11 Explain the difference between a tax refund and a tax liability.

12 Josh looked at the last row of the tax table and said 'It is unfair that people in that tax bracket pay 45 cents in every dollar in tax'. Explain why Josh is incorrect.

ENRICHMENT: What are legitimate tax deductions?	–	–	13

13 a Choose an occupation or career in which you are interested. Imagine that you are working in that job. During the year you will need to keep receipts for items you have bought that are legitimate work-related expenses. Do some research on the internet and write down some of the things that you will be able to claim as work-related expenses in your chosen occupation.

b i Imagine your taxable income is \$80 000. What is your tax payable?

ii You just found a receipt for a \$100 donation to a registered charity. This decreases your taxable income by \$100. By how much does it decrease your tax payable?

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The following problems will investigate practical situations drawing upon knowledge and skills developed throughout the chapter. In attempting to solve these problems, aim to identify the key information, use diagrams, formulate ideas, apply strategies, make calculations and check and communicate your solutions.

Bedroom makeover

1 Phil and Deb are mixing their own paint to form a special colour for their bedroom makeover. They have tested the following ratios and feel that these two options, A and B, suit their tastes.

Colour	White	Yellow	Red	Blue
Ratio A	20	3	1	3
Ratio B	24	4	2	3

The white paint costs \$8 per litre and the colours all cost \$25 per litre. The bedroom is 6 m long, 5 m wide and 2.5 m high, and the paint covers 10 m^2 per litre.

Phil and Deb are interested in the amount and cost of paint required to cover the walls and ceiling of the bedroom using one of these colour combinations.

a What fraction of:
  i red paint is in ratio A?
  ii blue paint in ratio B?
  iii white paint in ratio B?

b If 45 L of paint is required in total, find the amount of the following correct to the nearest litre.
  i white paint required using ratio A
  ii yellow paint required using ratio B

c If the walls and ceiling require two coats of paint find:
  i the total area of the walls and ceiling
  ii the amount of paint required in total
  iii the amount of red paint needed using ratio A, correct to one decimal place
  iv the amount of blue paint needed using ratio B, correct to one decimal place.

d All paint can only be purchased in a whole number of litres. Find the minimum cost of painting the room with two coats using:
  i ratio A
  ii ratio B.

Deals on wheels

2 During the summer, Casey makes two types of gift hampers and delivers them to customers' homes.

- 5 kg hamper, retail price: \$24.90
- 8 kg hamper, retail price: \$34.90

The cost to Casey to put the hampers together is:

- 5 kg hamper: \$15
- 8 kg hamper: \$23

Casey also charges a \$5 delivery fee, but her cost on average per km of travel is \$0.50.

Casey is interested in the possible profits that can be made in her business and ways in which she can boost sales by offering special combination deals.

Where necessary round any decimal answers to the nearest cent, e.g. \$3.57.

a For the given hamper weights and retail prices:
  i which hamper is better value for money?
  ii if the 5 kg hamper was re-priced based on the cost per kg of the 8 kg hamper, what would be its new retail price?

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b Casey looks at two sales and deliveries scenarios.
- Scenario A: 60 deliveries covering 500 km
- Scenario B: 100 deliveries covering 1000 km

Find the total profit from:
i Scenario A **ii** Scenario B.

Scenario A

Hamper	Sales
5 kg	45
8 kg	25

Scenario B

Hamper	Sales
5 kg	80
8 kg	50

c To encourage sales, Casey offers a combination deal of one 5 kg hamper and one 8 kg hamper for \$50.
i What is the overall profit for a combination deal, assuming a single delivery charge of \$5 and a 10 km journey?
ii Compared to selling one of each of the 5 kg and 8 kg hampers, how much of a discount does the combination deal represent? Express as a percentage correct to one decimal place. Assume that the delivery charges are the same.

d In scenario B in part **b** above, 30 people buy two hampers, including one of each type of hamper, and all the other sales are single hamper purchases. Casey predicts that 50% of the customers who buy just one single hamper can be convinced to buy the combination deal. Will this mean that Casey is better off and by how much? (Note: The 30 people who bought both types of hampers now only pay \$50.)

Taxing commission

3 Dihan works in sales for a large surfboard company and receives a \$600 per week retainer, plus \$50 for each surfboard he sells. Tax is taken out of his yearly income by the company according to the table opposite.

Income bracket	Tax rate	Tax payable
\$0–\$10 000	0%	\$0
\$10 001–\$20 000	10%	\$0 + 10% of each dollar over \$10 000
\$20 001–\$50 000	20%	\$$a$ + 20% of each dollar over \$20 000
\$50 001–	30%	\$$b$ + 30% of each dollar over \$50 000

Dihan is interested in the number of sales required for a particular income goal and the amount of tax payable on that income.

a Assume that Dihan works and is paid for 48 weeks in a year.
i Calculate the total income for the year if Dihan sells on average 6 surfboards each week.
ii Write a rule for Dihan's income per year, \$$A$, if he averages n sales per week.
iii Find the minimum average number of sales per week if Dihan wants to earn \$50 000 for the year.

b The values of a and b in the tax table can be calculated as the maximum amount of tax payable from the previous bracket. Find the values of a and b.

c Find Dihan's tax payable if he earns the following amounts in a year.
i \$36 000 **ii** \$85 000

d Find Dihan's yearly income if he is required to pay the following amounts of tax.
i \$6000 **ii** \$12 000

e Dihan's total living expenses are \$$E$ per year and he wants to save close to \$20 000 in a year after living expenses and taxes are paid. If $E = 25\,000$, find the average number of sales required each week to achieve his goal. Assume he works for 48 weeks and give a whole number answer.

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1K Simple interest

Learning intentions
- To understand the concept of simple interest and how it is calculated
- To be able to use the simple interest formula to perform calculations
- To be able to use the simple interest formula to solve problems about investments and loans

Past, present and future learning
- This section continues the Stage 5 Core Topic called *Financial Mathematics A*
- These concepts are not revised in our Year 10 book
- Simple interest may be revisited briefly in Stage 6 Advanced
- You may be expected to recall the formula from memory in the HSC Examination

When paying back the amount borrowed from a bank or other financial institution, the borrower pays interest to the lender. It is like rent paid on the money borrowed. A financial institution might be the lender, giving you a loan, or the borrower, when you invest your savings with them (effectively when you lend them your money). In either case, interest is calculated as a percentage of the amount borrowed. For simple interest, the percentage is calculated on the amount originally borrowed or invested and is paid at agreed times, such as once a year.

Buying an item with a loan greatly increases its price. A car bought with a $750 deposit and a $10 000 loan at 8% p.a. simple interest repaid over 5 years, plus fees, has had its price increased to around $15 000.

Lesson starter: Developing the rule

$5000 is invested in a bank and 5% simple interest is paid every year. In the table at right, the amount of interest paid is shown for Year 1, the amount of accumulated total interest is shown for Years 1 and 2.

Year	Interest paid that year	Accumulated total interest
1	$\frac{5}{100} \times \$5000 = \$250$	$1 \times \$250 = \$250$
2		$2 \times \$250 = \$500$
3		
4		
t		

- Complete the table, writing an expression in the last cell for the accumulated total interest after n years.
- Now write a rule using $\$P$ for the initial amount, n for the number of years and r for the interest rate to find the total interest earned, $\$I$.

KEY IDEAS

- To calculate **simple interest**, we apply the formula:

  $I = Prn$

  where
  - I is the amount of **simple interest** (in $)
  - P is the **principal** amount; the money invested, borrowed or loaned (in $)
  - r is the interest rate per time period (expressed as a fraction or decimal)
  - n is the number of time periods

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- When using simple interest, the principal amount is constant and remains unchanged from one period to the next.
- The total amount ($\$A$) equals the principal plus interest:
  $A = P + I$

BUILDING UNDERSTANDING

1 **a** How many years is the following number of months?
 i 36 **ii** 18 **iii** 66
 b How many months is the following number of years?
 i 4 **ii** 2.5 **iii** 7.25

2 Find the following percentages.
 a 5% of \$1000 **b** 4% of \$1500 **c** 8% of \$3000

3 \$12 000 is invested at 6% p.a. for 42 months.
 a What is the principal amount?
 b What is the interest rate?
 c What is the time period in years?
 d How much interest is earned each year?
 e How much interest is earned after 2 years?
 f How much interest is earned after 42 months?

Example 31 Using the simple interest formula

Calculate the simple interest earned if the principal is \$1000, the rate is 5% p.a. and the time is 3 years.

SOLUTION	EXPLANATION
$P = 1000$, $r = 0.05$, $n = 3$	List the information given.
$r = 5\% = 5 \div 100 = 0.05$	Convert the interest rate to a decimal.
$I = P \times r \times n$ $= 1000 \times 0.05 \times 3$ $= 150$	Write the formula and substitute the given values.
Interest = \$150	Answer the question.

Now you try

Calculate the simple interest earned if the principal is \$1500, the rate is 7% p.a. and the time is 4 years.

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Example 32 Calculating the final balance

Allan and Rachel plan to invest some money for their child Kaylan. They invest \$4000 for 30 months in an account that pays 4.5% p.a. Calculate the simple interest and the amount available at the end of the 30 months.

SOLUTION

$P = 4000$, $r = 4.5 \div 100 = 0.045$,
$n = \frac{30}{12} = 2.5$

$$\begin{aligned} I &= P \times r \times n \\ &= 4000 \times 0.045 \times 2.5 \\ &= 450 \end{aligned}$$

Interest = \$450

$$\begin{aligned} \text{Total amount} &= \$4000 + \$450 \\ &= \$4450 \end{aligned}$$

EXPLANATION

n is written in years since interest rate is per annum.

Write the formula, substitute the known information and evaluate.

Total amount = principal + interest

Now you try

Rohan plans to invest some money to save for a new tractor. He invests \$20 000 for 42 months at the rate of 6.5%. Calculate the simple interest and the amount available at the end of the 42 months.

Example 33 Determining the investment period

Remy invests \$2500 at 8% p.a. simple interest for a period of time and receives \$50 interest. For how long (in months) did she invest the money?

SOLUTION

$I = 50$, $P = 2500$, $r = 8 \div 100 = 0.08$

$$\begin{aligned} I &= P \times r \times n \\ 50 &= 2500 \times 0.08 \times n \\ 50 &= 200n \\ n &= \frac{50}{200} \\ &= 0.25 \end{aligned}$$

$$\begin{aligned} \text{Time} &= 0.25 \text{ years} \\ &= 0.25 \times 12 \text{ months} \\ &= 3 \text{ months} \end{aligned}$$

EXPLANATION

List the information given.
Convert the interest rate to a decimal

Write the formula, substitute the known information and simplify.

Solve the remaining equation for n.

Convert decimal time to months where appropriate.

Now you try

Mia invests \$4200 at 6% p.a. simple interest for a period of time and receives \$189 interest. For how long (in months) did she invest the money?

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Exercise 1K

FLUENCY	1–6	1, 3, 5–7	2, 3, 5, 6, 8

Example 31

1 Use the rule $I = Prn$ to find the simple interest earned in these financial situations.
   a Principal \$6000, rate 12% p.a., time 5 years
   b Principal \$5200, rate 4% p.a., time 24 months

2 Calculate the simple interest earned if the principal is \$2000, the value is 6% p.a. and the time is 5 years.

Example 32

3 Wally invests \$15 000 at a rate of 6% p.a. for 3 years. Calculate the simple interest and the amount available at the end of 3 years.

4 Annie invests \$22 000 at a rate of 4% p.a. for 27 months. Calculate the simple interest and the amount available at the end of 27 months.

5 A finance company charges 14% p.a. simple interest. If Lyn borrows \$2000 to be repaid over 2 years, calculate her total repayment.

Example 33

6 Zac invests \$3500 at 8% p.a. simple interest, for a period of time, to produce \$210 interest. For how long did he invest the money?

7 If \$4500 earns \$120 simple interest at a flat rate of 2% p.a., calculate the duration of the investment.

8 Calculate the principal amount that earns \$500 simple interest over 3 years at a rate of 8% p.a. Round to the nearest cent.

PROBLEM–SOLVING	9–11	10–12	11, 12

9 Wendy wins \$5000 during a chess tournament. She wishes to invest her winnings, and has the two choices given below. Which one gives her the greater total at the end of the time?

Choice 1: 8.5% p.a. simple interest for 4 years
Choice 2: 8% p.a. simple interest for 54 months

10 Charlotte borrows \$9000 to buy a secondhand car. The loan must be repaid over 5 years at 12% p.a. simple interest. Calculate:
   a the total amount to be repaid
   b the monthly repayment amount if the repayments are spread equally over the 5 years.

11 An investment of \$5000 earns \$6000 simple interest in 12 years. Calculate the interest rate.

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12 Chris invests $\$P$ and wants to double this amount of money.
  a How much interest must be earned to double this initial amount?
  b What simple interest rate is required to double the initial amount in 8 years?
  c The simple interest rate is 5% p.a.
    i How many years will it take to double the investment?
    ii How many years will it take to triple the investment amount?
    iii How do the investment periods in parts i and ii compare?

REASONING	13	13, 14	13, 14

13 To find the total amount $\$T$ including simple interest, the rule is $T = P(1 + rn)$.
  a Use this rule to find the total amount after 10 years when \$30 000 is invested at 7% p.a.
  b Use the rule to find the time that it takes for an investment to grow from \$18 000 to \$22 320 when invested at 6% p.a. simple interest.

14 Rearrange the rule $I = Prn$ to find a rule for:
  a P in terms of I, r and n
  b n in terms of I, P and r
  c r in terms of I, P and n.

ENRICHMENT: Property investing	–	–	15

15 Many investors use interest-only loans to buy shares or property. For such loans the principal stays constant and only the interest is paid back each month.

Sasha buys an investment property for \$300 000 and borrows the full amount at 7% p.a. simple interest. She rents out the property at \$1500 per month and pays \$3000 per year in rates and other costs to keep the property.

  a Find the amount of interest Sasha needs to pay back every month.
  b Find Sasha's yearly income from rent.
  c By considering the other costs in keeping the property, calculate Sasha's overall loss in a year.
  d Sasha hopes that the property's value will increase enough to cover any loss she is making. By what percentage of the original price will the property need to increase in value per year?

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1L Compound interest and depreciation

Learning intentions

- To understand the concept of compound interest and how it is calculated
- To be able to use the compound interest formula to perform calculations
- To be able to use the compound interest formula to solve problems about investments and loans

Past, present and future learning

- This section continues the Stage 5 Core Topic called *Financial Mathematics B*
- These concepts are not revised in our Year 10 book
- Compound interest may be revisited briefly in Stage 6 Advanced

When interest is added to an investment total before the next amount of interest is calculated, we say that the interest is compounded. Interest on a \$1000 investment at 8% p.a. gives \$80 in the first year. If the interest is compounded, the interest calculated in the second year is 8% of \$1080. This is repeated until the end of the investment period. Other forms of growth and decay work in a similar manner.

Assets, such as investment accounts, land and buildings, that grow by a percentage each year increase in total value at a faster rate over time. Their possible future value can be predicted using the compound interest formula.

Lesson starter: Power play

\$10000 is invested at 5% interest compounded annually. Complete this table showing the interest paid and the balance (original investment plus interest) at the end of each year.

Year	Opening balance	Interest paid that year	Closing balance
1	\$10 000	$0.05 \times \$10\,000$ $= \$\_\_\_\_$	$\$10\,000 \times 1.05 = \$\_\_\_\_$
2	\$______	$0.05 \times \$\_\_\_\_$ $= \$\_\_\_\_$	$\$10\,000 \times 1.05 \times 1.05$ $= \$10\,000 \times 1.05^2$ $= \$\_\_\_\_$
3	\$______	$0.05 \times \$\_\_\_\_$ $= \$\_\_\_\_$	$\$10\,000 \times \_\_\_\_$ $= \$10\,000 \times 1.05^{\square}$ $= \$\_\_\_\_$

- What patterns can you see developing in the table?
- How can you use the *power* button on your calculator to help find the balance at the end of each year?
- How would you find the balance at the end of 10 years without creating a large table of values?
- Do you know how to build this table in a computerised spreadsheet?

KEY IDEAS

- A repeated product can be written and calculated using a power.
  - For example: $1.06 \times 1.06 \times 1.06 \times 1.06 = (1.06)^4$
    $0.85 \times 0.85 \times 0.85 = (0.85)^3$
- **Compound interest** is interest which is added to the investment amount before the next amount of interest is calculated.

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- For example: \$5000 invested at 6% compounded annually for 3 years gives $\$5000 \times 1.06 \times 1.06 \times 1.06 = 5000 \times (1.06)^3$.
- 6% compounded annually can be written as 6% p.a.
- p.a. means 'per annum' or 'per year'.
- The initial investment or loan is called the **principal**.
- The total interest earned = final amount − principal.

■ Depreciation is the reverse situation in which the value of an object decreases by a percentage year after year.

BUILDING UNDERSTANDING

1 \$2000 is invested at 10% compounded annually for 3 years.
- **a** Find the interest earned in the first year.
- **b** Find the total balance at the end of the first year.
- **c** Find the interest earned in the second year.
- **d** Find the total balance at the end of the second year.
- **e** Find the interest earned in the third year.
- **f** Find the total balance at the end of the third year.

2 Find the value of the following correct to two decimal places. What do you notice?
- **a** $2500 \times 1.03 \times 1.03 \times 1.03$
- **b** $420 \times 1.22 \times 1.22 \times 1.22 \times 1.22$
- **c** $2500 \times (1.03)^3$
- **d** $420 \times (1.22)^4$

3 State the missing numbers to help describe each situation.
- **a** \$4000 is invested at 20% compounded annually for 3 years.

  $\$4000 \times (\_\_\_\_)^3$
- **b** \$15 000 is invested at 7% compounded annually for 6 years.

  $\$\_\_\_\_ \times (1.07)^{\square}$
- **c** \$825 is invested at 11% compounded annually for 4 years.

  $\$825 \times (\_\_\_\_)^{\square}$

Example 34 Calculating a balance using compound interest

Find the total value of the investment if \$8000 is invested at 5% compounded annually for 4 years.

SOLUTION

$$100\% + 5\% = 105\% = 1.05$$

$$\text{Investment total} = \$8000 \times (1.05)^4 = \$9724.05$$

EXPLANATION

Add 5% to 100% to find the multiplying factor.

Multiplying by $(1.05)^4$ is the same as multiplying by $1.05 \times 1.05 \times 1.05 \times 1.05$.

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Now you try

Find the total value of the investment when \$6000 is invested at 4% compounded annually for 3 years.

Example 35 Finding the initial amount

After 6 years a loan grows to \$62 150. If the interest was compounded annually at a rate of 9%, what is the amount of the initial loan to the nearest dollar?

SOLUTION

$$\begin{aligned} 100\% + 9\% &= 109\% \\ &= 1.09 \end{aligned}$$

$$\begin{aligned} \text{Initial amount} \times (1.09)^6 &= \$62\,150 \\ \text{Initial amount} &= \$62\,150 \div (1.09)^6 \\ &= \$37\,058 \end{aligned}$$

EXPLANATION

Add 9% to 100% to find the multiplying factor.

Write the equation including the final total.
Divide by $(1.09)^6$ to find the initial amount and round as required.

Now you try

After 7 years a loan grows to \$43 200. If the interest was compounded annually at a rate of 8%, what is the amount of the initial loan to the nearest dollar?

Example 36 Calculating a depreciated value

A car worth \$30 000 will depreciate by 15% p.a. What will the car be worth at the end of the fifth year to the nearest cent?

SOLUTION

$$\begin{aligned} 100\% - 15\% &= 85\% \\ &= 0.85 \end{aligned}$$

$$\begin{aligned} \text{Depreciated value} &= \$30\,000 \times (0.85)^5 \\ &= \$13\,311.16 \text{ (to the nearest cent)} \end{aligned}$$

EXPLANATION

Subtract 15% from 100%.

Multiply by $(0.85)^5$ for 5 years.
Rounding to the nearest cent is rounding correct to two decimal places.

Now you try

A computer worth \$3000 will depreciate by 18% p.a. What will the computer be worth at the end of the third year to the nearest cent?

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Exercise 1L

FLUENCY	1–3, 5	1, 2(½), 3, 4(½), 5, 6(½), 7	2(½), 3, 4(½), 6(½), 7

Example 34

1 Find the total value of the investment when \$3200 is invested at 6% compounded annually for 5 years. Round to the nearest cent.

2 Find the total balance of these investments given the interest rates and time period. Assume interest is compounded annually in each case. Round to the nearest cent.

a \$4000, 5%, 10 years
b \$6500, 8%, 6 years
c \$25 000, 11%, 36 months
d \$4000, 7%, 60 months

3 Barry borrows \$200 000 from a bank for 5 years and does not pay any money back until the end of the period. The compound interest rate is 8% p.a. How much does he need to pay back at the end of the 5 years? Round to the nearest cent.

4 Find the total percentage increase in the value of these amounts, compounded annually at the given rates. Round to one decimal place.

a \$1000, 4% p.a., 5 years
b \$20 000, 6% p.a., 3 years
c \$125 000, 9% p.a., 10 years
d \$500 000, 7.5% p.a., 4 years

Example 35

5 After 5 years a loan grows to \$45 200. If the interest was compounded annually at a rate of 6% p.a., find the size of the initial loan to the nearest dollar.

6 Find the initial investment amount to the nearest dollar given these final balances, annual interest rates and time periods. Assume interest is compounded annually in each case.

a \$26 500, 4%, 3 years
b \$42 000, 6%, 4 years
c \$35 500, 3.5%, 6 years
d \$28 200, 4.7%, 2 years

Example 36

7 Find the depreciated value of these assets to the nearest dollar.

a \$2000 phone at 30% p.a. for 4 years
b \$50 000 car at 15% p.a. for 5 years
c \$50 000 car at 15% p.a. for 10 years

PROBLEM-SOLVING	8, 9	9–11	10–12

8 Average house prices in Hobart are expected to grow by 8% per year for the next 5 years. What is the expected average value of a house in Hobart in 5 years' time, to the nearest dollar, if it is currently valued at \$370 000.

9 The population of a country town is expected to fall by 15% per year for the next 8 years due to the downsizing of the iron ore mine. If the population is currently 22 540 people, what is the expected population in 8 years' time? Round to the nearest whole number.

10 It is proposed that the mass of a piece of limestone lying out in the weather has decreased by 4.5% per year for the last 15 years. Its current mass is 3.28 kg. Find its approximate mass 15 years ago. Round to two decimal places.

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11 Charlene wants to invest \$10 000 long enough for it to grow to at least \$20 000. The compound interest rate is 6% p.a. How many whole number of years does she need to invest the money for so that it grows to her \$20 000 target?

12 A forgetful person lets a personal loan balance grow from \$800 to \$1440 at a compound interest rate of 12.5% p.a. For approximately how many years did the person forget about the loan?

REASONING	13	13	13, 14

13 \$400 is invested for 5 years under the following conditions.

- **i** Simple interest at 7% p.a.
- **ii** Compound interest at 7% p.a.

a Find the percentage increase in the total value of the investment using condition part **i**.

b Find the percentage increase in the total value of the investment using condition part **ii**. Round to two decimal places.

c Explain why the total for condition part **i** is less than the total for condition part **ii**.

14 Find the total percentage increase in these compound interest situations to two decimal places.

a 5% p.a. for 3 years
b 12% p.a. for 2 years
c 4.4% p.a. for 5 years
d 7.2% p.a. for 9 years
e r% for t years

ENRICHMENT: Comparing simple and compound interest	–	–	15–17

15 \$16 000 is invested for 5 years at 8% compounded annually.

a Find the total interest earned over the 5 years to the nearest cent.

b Find the simple interest rate that would deliver the same overall interest at the end of the 5 years. Round to two decimal places.

16 \$100 000 is invested for 10 years at 5.5% compounded annually.

a Find the total percentage increase in the investment to two decimal places.

b Find the simple interest rate that would deliver the same overall interest at the end of the 10 years. Round to two decimal places.

17 Find the simple interest rate that is equivalent to these annual compound interest rates for the given periods. Round to two decimal places.

a 5% p.a. for 4 years

b 10.5% p.a. for 12 years

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1M Using a formula for compound interest and depreciation

Learning intentions
- To know the formulas for compound interest and depreciation
- To be able to use the compound interest formula to find a future value
- To be able to use a formula to find the salvage value after depreciation

Past, present and future learning
- This section concludes the Stage 5 Core Topic called *Financial Mathematics B*
- These concepts are not revised in our Year 10 book
- Compound interest may be revisited briefly in Stage 6 Advanced
- You may be expected to recall the formula from memory in the HSC Examination

We saw earlier that interest can be calculated, not just on the principal investment, but on the actual amount present in the account. Interest can be calculated using updated applications of the simple interest formula, or by developing a new formula known as the compound interest formula.

If the original amount loses value over time, it is said to have depreciated. The compound interest formula can be adapted to cover this scenario as well. Cars, computers, boats and mobile phones are examples of consumer goods that depreciate in value over time.

Lesson starter: Repeated increases and decreases

If \$4000 increased by 5% in the first year, and then this amount was increased by 5% in the second year:
- what was the final amount at the end of the second year?
- what was the overall increase in value?
- what could you do to calculate the value at the end of 100 years, if the interest was added to the amount each year before the new interest was calculated?

If \$4000 decreased in value by 5% each year, how would you find the value at the end of:
- 1 year?
- 2 years?
- 100 years?

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KEY IDEAS

- The **compound interest** formula is:

  $A = P(1 + r)^n$,

  Where
  - A is the total amount (i.e. the principal and interest)
  - P is the principal investment (i.e. the original amount invested)
  - r is the percentage interest rate per compounding period (expressed as a decimal).
  - n is the number of compounding periods

  The formula is sometimes written as:

  $FV = PV(1 + r)^n$,

  Where
  - FV is the future value
  - PV is the present value

  It can also be written as, $FV = PV(1 + r)^n$ where r is the interest rate, per period, expressed as a decimal.
- Compound interest = amount (A) − principal (P)
- **Depreciation** occurs when the principal decreases in value over time. The formula is:

  $A = P\left(1 - \frac{r}{100}\right)^n$
- Common rounding used in monetary problems is two decimal places as this rounds to the nearest cent. This rounding is used throughout this section.

BUILDING UNDERSTANDING

1. Which of the following can be used to calculate the final value of a \$1000 investment at 10% p.a. for 3 years compounded annually?

   A $1000 \times 0.1 \times 0.1 \times 0.1$ **B** $1000 \times 1.1 \times 3$

   C $1000\left(1 + \frac{10}{100}\right)^3$ **D** $1000\left(1 - \frac{10}{100}\right)^3$

2. Complete the table of values showing how a car worth \$20 000 depreciates at a rate of 10% p.a. over 4 years.

Year	Value at start of year	10% depreciation	Value at end of year
1	\$20 000	\$2000	\$18 000
2	\$18 000	\$1800	
3			
4			

3. Which of the following gives the value of a \$40 000 car once it has depreciated for 4 years at a rate of 30% p.a.?

   A $40\,000 \times \left(1 + \frac{30}{100}\right)^4$ **B** $40\,000 \times \frac{70}{100} \times 4$

   C $40\,000 \times \left(1 - \frac{30}{100}\right)^4$ **D** $40\,000 - \left(\frac{70}{100}\right)^4$

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Example 37 Using the compound interest formula

Callum invested \$10000 with a bank. The bank pays interest at the rate of 7% p.a., compounded annually.

a What is the value of Callum's investment at the end of 5 years to the nearest cent?

b How much interest was earned over this time?

SOLUTION

a $A = P(1 + r)^n$

$P = 10\,000$

$r = 0.07$

$n = 5$

$A = 10\,000(1 + 0.07)^5$

$= 14\,025.517...$

Final value is \$14 025.52.

b Interest = \$4025.52

EXPLANATION

Write down the compound interest formula.
Write down the values of the terms you know.

$n = 5$ (once a year for 5 years)
Substitute and evaluate. The round to the nearest cent.

The interest is found by subtracting the principal from the final amount.

Now you try

Moshie invests \$4000 in a fund which pays 6.5% p.a. compounded annually.

a. What is the value of Moshie's investment at the end of 7 years to the nearest cent?

b. How much interest is earned over this time?

Example 38 Finding compounding amounts using months

Victoria's investment of \$6000 is invested at 8.4% p.a. for 4 years. Calculate the final value of her investment if interest is compounded monthly. Round to the nearest cent.

SOLUTION

$A = P(1 + r)^n$

$P = 6000$

$r = 0.084 \div 12 = 0.007$

$n = 4 \times 12 = 48$

$A = 6000(1 + 0.007)^{48}$

$= 8386.2119\ldots$

Final amount is \$8386.21.

EXPLANATION

Write down the compound interest formula.
Write down the values of the terms you know.
The interest rate of 8.4% is *divided by* 12 to find the *monthly* rate.
The number of compounding periods (n) is 12 times a year for 4 years or $n = 48$.
Substitute and evaluate.

Now you try

Millie's investment of \$24 000 is invested at 6.2% p.a. for 3 years. Calculate the final value of her investment if interest is compounded monthly. Round to the nearest cent.

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Example 39 Calculating the depreciated value

Naomi bought a computer three years ago for $5680. She assumed the value of the computer would depreciate at a rate of 25% p.a.
What is the value of the computer at the end of the three years (assuming her assumption was correct)?

SOLUTION	EXPLANATION
$A = P(1 - r)^n$	Use $A = P(1 - r)^n$ since the computer is depreciating in value.
$A = 5680(1 - 0.25)^3$ $= 2396.25$	Substitute $P = 5680$, $r = 0.25$ and $n = 3$.
The value at the end of the three years is $2396.25.	

Now you try

Roger purchased a car five years ago for $50 000. He assumed the value of the car would depreciate at a rate of 10% p.a.
What is the value of the car at the end of the five years (assuming his assumption was correct)?

Exercise 1M

FLUENCY	1, 2–4(½)	1, 2–4(½)	2–4(⅓)

Example 37

1 Brodie invests $7000 in a bank at 5% p.a. compounded annually.
  a What is the value of Brodie's investment at the end of 4 years to the nearest cent?
  b How much interest is earned over this time?

2 Calculate the final investment amount if the interest is compounded monthly. Round to the nearest cent.
  a $4000 at 5% p.a. for 8 years
  b $2000 at 5.4% p.a. for 3 years
  c $20 000 at 8% p.a. for 6 years
  d $10 000 at 16% p.a. for 6 months
  e $500 at 3.8% p.a. for 25 years
  f $350 at 6.78% p.a. for 2 year
  g $68 000 at 2.5% p.a. for 10 years
  h 1 000 000 at 1% p.a. for 100 years

Example 38

3 Calculate the final investment amount if the interest is compounded monthly. Round to the nearest cent.
  a $4000 at 12% p.a. for 4 years
  b $5000 at 6% p.a. for 2 years
  c $15 000 at 9% p.a. for 3 years
  d $950 at 6.6% p.a. for 30 months
  e $90 000 at 7.2% p.a. for $1\frac{1}{2}$ years
  f $34 500 at 4% p.a. for 1 year

Example 39

4 It can be assumed that most cars depreciate at a rate of about 15% p.a. Calculate the projected value of each of the following cars at the end of 3 years, by using the rate of 15% p.a. Answer correct to the nearest dollar.
  a Bentley Continental GT $189 900
  b Audi A3 $30 850
  c Hyundai Accent $16 095
  d Lexus RX $44 000
  e Toyota Camry $29 500
  f Mazda 6 $29 800

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PROBLEM-SOLVING	5–7	5–8	6–10

5 How much interest is earned on an investment of \$400 if it is invested at:
- **a** 6% p.a. simple interest for 2 years?
- **b** 6% p.a. compounded annually over 2 years?
- **c** 6% p.a. compounded monthly over 2 years? Round to the nearest cent.

6 **a** Tom invests \$1500 in an account earning $6\frac{1}{2}\%$ p.a. compounded annually. He plans to keep this invested for 5 years. How much interest will he earn on this investment? Round to the nearest cent.
- **b** Susan plans to invest \$1500 in an account earning a simple interest rate of $6\frac{1}{2}\%$ p.a. How long does Susan need to invest her money to ensure that her return is equal to Tom's? Answer to one decimal place.
- **c** If Susan wishes to invest her \$1500 for 5 years as Tom did, yet still insists on a simple interest rate, what annual rate should she aim to receive so that her interest remains the same as Tom's? Answer to one decimal place.

7 Inflation is another application of the compound interest formula, as the cost of goods and services increases by a percentage over time. Find the cost of these household items, in 3 years, if the annual rate of inflation is 3.5%. Round to the nearest cent.
- **a** Milk \$3.45 for 2 L
- **b** Loaf of bread \$3.70
- **c** Can of cola \$1.80
- **d** Monthly electricity bill of \$120
- **e** Dishwashing powder \$7.99

8 Talia wishes to invest a lump sum today so that she can save \$25 000 in 5 years for a European holiday. She chooses an account earning 3.6% p.a. compounded monthly. How much does Talia need to invest to ensure that she reaches her goal? Round to the nearest cent.

9 **a** Find the compound interest to the nearest cent when \$10 000 is invested at 6% p.a. over 2 years compounding:
- **i** annually
- **ii** every 6 months (twice a year)
- **iii** quarterly
- **iv** monthly
- **v** daily

b What conclusions can be made about the interest earned versus the number of compounding periods?

10 What annual simple interest rate corresponds to an investment of \$100 at 8% p.a. compounded monthly for 3 years? Round to two decimal places.

REASONING	11	11, 12	12, 13

11 Write down a scenario that could explain each of the following lines of working for compound interest and depreciation.
- **a** $300\,(1.07)^{12}$
- **b** $300\,(1.02)^{36}$
- **c** $1000\,(1.005)^{48}$
- **d** $1000\,(0.95)^{5}$
- **e** $9000\,(0.8)^{3}$

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12 **a** Calculate the interest earned on an investment of \$10 000 at 5% p.a. compounded annually for 4 years. Round to the nearest cent.

b Calculate the interest earned on an investment of \$20 000 at 5% p.a. compounded annually for 4 years. Is it double the interest in part **a**?

c Calculate the interest earned on an investment of \$10 000 at 10% p.a. compounded annually for 2 years. Is this the same amount of interest as in part **a**?

d Comment on the effects of doubling the principal invested.

e Comment on the effect of doubling the interest rate and halving the time period.

13 **a** Calculate the interest earned to the nearest cent when \$1000 is invested at 6% p.a. over 1 year compounded:

i annually **ii** six-monthly **iii** quarterly
iv monthly **v** fortnightly **vi** daily

b Explain why increasing the number of compounding periods increases the interest earned on the investment.

ENRICHMENT: Doubling your money	–	–	14

14 How long does it take to double your money?

Tony wishes to find out how long he needs to invest his money at 9% p.a., compounding annually, for it to double in value.

He sets up the following situation, letting P be his initial investment and $2P$ his desired final amount.

He obtains the formula $2P = P(1.09)^n$, which becomes $2 = (1.09)^n$ when he divides both sides by P.

a Complete the table below to find out the number of years needed for Tony to double his money. Round to three decimal places.

n	1	2	3	4	5	6	7	8	9
$(1.09)^n$									

b How long does it take if the interest rate is:

i 8% p.a.? **ii** 12% p.a.? **iii** 18% p.a.?

c What interest rate is needed to double Tony's money in only 3 years?

d Use your findings above to complete the following.

Years to double × interest rate = ____________

e Use your rule to find out the interest rate needed to double your money in:

i 2 years

ii $4\frac{1}{2}$ years

iii 10 years

f Investigate the rule needed to triple your money.

g Investigate a rule for depreciation. Is there a rule for the time and rate of depreciation if your investment is halved over time?

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Investing in art

Matilda is a keen art investor and has the opportunity to purchase a new work from an auction house. The auctioneer is saying that the estimated value of the painting is \$10 000.

Matilda's main investment goal is for each of her investments to at least double in value every 10 years.

Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate. Round all monetary values to the nearest cent.

Routine problems

a If Matilda purchases the painting for \$10 000 and assumes a growth rate of 5% p.a., calculate the value of the investment after:

i 1 year **ii** 2 years **iii** 3 years.

b The rule connecting the value of the \$10 000 investment (\$$A$) growing at 5% p.a. after n years is given by $A = 10\,000(1 + 0.05)^n$.

i Check your answers to part **a** by substituting $n = 1$, $n = 2$ and $n = 3$ into the given rule and evaluating the value of A using technology.

ii Construct a similar rule for an investment value of \$12 000 and a growth rate of 3%.

iii Construct a similar rule for an investment value of \$8000 and a growth rate of 8%.

c Using $A = 10\,000(1 + 0.05)^n$ find the value of a \$10 000 investment at 5% after 10 years. (i.e. Calculate the value of A if $n = 10$.)

Non-routine problems

Explore and connect

a The problem is to determine an investment growth rate that delivers at least a doubling of Matilda's initial investment amount after 10 years. Write down all the relevant information that will help solve this problem.

b Explain what the numbers 10 000 and 5 mean in the rule $A = 10\,000(1 + 0.05)^n$ in relation to Matilda's investment.

Choose and apply techniques

c Use the rule $A = 10\,000(1 + r)^n$ to determine the value of Matilda's \$10 000 investment after 10 years for the following growth rates (r%).

i $r = 4$ **ii** $r = 7$ **iii** $r = 10$

d Choose your own values of r using one decimal place accuracy, and determine the growth rate for which the investment doubles in value after 10 years.

Communicate thinking and reasoning

e By considering values of r either side of your chosen value found in part **d**, demonstrate that your answer is correct to one decimal place.

f Refine your answer so that it is correct to two decimal places.

g Summarise your results and describe any key findings.

Extension problems

Problem solve

a Decide if changing the initial investment value changes the total percentage increase in value after the same number of years. Justify your answer.

b If Matilda only paid \$8000 for the artwork but still wanted it to be valued at \$20 000 after 10 years, determine the required growth rate of the artwork. Round to two decimal places.

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Investigation

Compounding investments

Banks offer many types of investments that pay compound interest. Recall that for compound interest you gain interest on the money you have invested over a given time period. This increases your investment amount and therefore the amount of interest you gain in the next period.

Calculating yearly interest

Mary invests \$1000 at 6% per annum. This means Mary earns 6% of the principal every year in interest. That is, after 1 year the interest earned is 6% of $\$1000 = \frac{6}{100} \times \$1000 = \$60$.

a The interest earned is added to the principal at the end of the year, and the total becomes the principal for the second year.
  i How much interest will she earn at the end of the second and third year?
  ii What total amount will Mary have at the end of the third year?

b Write down a rule that calculates the total value of Mary's investment after t years. Use an initial investment amount of \$1000 and an annual interest rate of 6% p.a.

c Use your rule from part **b** to calculate:
  i the value of Mary's investment after 5 years
  ii the time it takes for Mary's investment to grow to \$2000.

Using a spreadsheet

This spreadsheet will calculate the compound interest for you if you place the principal in cell B3 and the rate in cell D3.

In Mary's case put 1000 in B3 and $\frac{6}{100}$ in D3.

	A	B	C	D
1	COMPOUND	INTEREST	SIMULATOR	
2				
3	PRINCIPAL	. . .	RATE OF PERIOD	. . .
4				
5	PERIOD	OPENING BALANCE	INTEREST EARNED	NEW BALANCE
6	1	=B3	=B6*$D$3	=B6 + C6
7	2	=D6	=B7*$D$3	=B7 + C7
8	3	=D7	=B8*$D$3	=B8 + C8
9	4	=D8	=B9*$D$3	=B9 + C9

a Copy the spreadsheet shown using 'fill down' at cells B7, C6 and D6.

b What will be Mary's balance after 10 years? Extend your spreadsheet to find out.

c Draw a graph of investment value versus time. Plot points using the results from your spreadsheet and join them with a smooth curve. Discuss the shape of the curve.

d Now try altering the interest rate. What would Mary's investment grow to in 10 years if the interest rate was 10%?

e What interest rate makes Mary's investment grow to \$2000 in 8 years? Use trial and error to get an answer correct to two decimal places. Record your investigation results using a table.

f Investigate how changing the principal changes the overall investment amount. Record your investigations in a table, showing the principal amounts and investment balance for a given interest rate and period.

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1 By only using the four operations $+, -, \times$ and $\div$ as well as brackets and square root ($\sqrt{\ }$), the number 4 can be used exactly 4 times to give the answer 9 in the following way: $4 \times \sqrt{4} + 4 \div 4$.
Use the number 4 exactly 4 times (and no other numbers) and any of the operations (as many times as you like) to give the answer 0 or 1 or 2 or 3 or … or 10.

2 What is the value of n if n is the largest of 5 consecutive numbers that multiply to give 95 040?

3 Evaluate.

a $\dfrac{1}{1+\dfrac{1}{1+\frac{1}{3}}}$

b $\dfrac{2}{1+\dfrac{2}{1+\frac{2}{5}}}$

4 A jug has 1 litre of 10% strength cordial. How much pure cordial needs to be added to make the strength 20%? (The answer is not 100 mL.)

5 An old table in a furniture store is marked down by 10% from its previous price on five separate occasions. What is the total percentage reduction in price correct to two decimal places?

6 What simple interest rate is equivalent to a compound interest rate of 6% p.a. over 10 years correct to two decimal places?

7 Brendon has a rectangular paved area in his yard.

a If he increases both the length and breadth by 20%, what is the percentage increase in area?

b If the length and breadth were increased by the same percentage and the area increases by 21%, what is the percentage increase of the length and breadth?

8 A rectangular sandpit is shown on a map that has a scale of 1 : 100. On the map the sandpit has an area of 20 cm^2. What is its actual area?

9 Arrange the numbers 1 to 15 in a row so that each adjacent pair of numbers sums to a square number.

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Chapter summary

Reviewing number and financial mathematics

Review of numbers

$2 + (-3) = -1$
$-4 - (-2) = -2$
$15 \div (-5) = -3$
$-7 \times (-3) = 21$
$(-2)^2 = 4$
$4^3 = 64$

For 8 and 12
HCF is 4
LCM is 24

$\sqrt{36} = 6$
$\sqrt[3]{27} = 3$

Operations with fractions

+/– : need same denominator

e.g. $1\frac{5}{6} - \frac{7}{12} = \frac{11}{6} - \frac{7}{12}$
$= \frac{22}{12} - \frac{7}{12}$
$= \frac{15}{12}$
$= 1\frac{1}{4}$

× : Cancel then multiply
÷ : × by reciprocal

e.g. $1\frac{1}{2} \div \frac{7}{2} = \frac{3}{\cancel{2}_1} \times \frac{\cancel{2}^1}{7}$
$= \frac{3}{7}$

Rounding

1.284 to 1 d.p. is 1.3
to 2 sig. fig. is 1.3
472.543 to 1 d.p. is 472.5
to 1 sig. fig. is 500
0.0624 to 2 d.p. is 0.06
to 2 sig. fig. is 0.062

Rates, ratios and best buy

Rate: 180 km per 3 h = 60 km/h
Ratio: $\frac{3}{7} : \frac{1}{2} = \frac{6}{14} : \frac{7}{14} = 6 : 7$
\$200 divided into 7 : 3
10 parts is \$200
∴ 1 part is \$20
3 parts is \$60
7 parts is \$140
Best buy: 3 kg of carrots for \$6.45
5 kg for \$10.20
3 kg bag: $6.45 \div 3 = \$2.15$/kg
5 kg bag: $10.20 \div 5 = \$2.04$/kg
∴ 5 kg is best buy

Rational numbers (fractions)

$1\frac{3}{8} = \frac{11}{8} = 1.375$
mixed numeral, improper fraction, terminating decimal

$\frac{1}{6} = 0.166... = 0.1\dot{6}$
proper fraction, recurring decimal

$\frac{2}{3}$ and $\frac{4}{6}$ are equivalent fractions
Irrational numbers cannot be expressed as a fraction

Compound interest

$A = P(1+r)^n$
\$3000 at 5% for 3 years
Amount $= \$3000 \times (1.05)^3$
$= \$3472.88$

Percentages

Fraction or decimal $\xrightarrow{\times 100\%}$ Percent
Percent $\xrightarrow{\div 100\%}$ Fraction or decimal

$\frac{3}{4} = \frac{3}{4} \times 100\% = 75\%$
$25\frac{1}{2}\% = 25.5 \div 100\% = 0.255$
20% of $60 = 0.2 \times 60 = 12$
5% of amount = 32
∴ amount $= 32 \div 0.05$
$= 640$

Depreciation

\$50 000 at 10% for 4 years
Amount $= \$50\,000 \times (1 - 0.1)^4$
$= \$32\,805$

Simple interest

\$2000 at 5% for 4 years
$I = Prn = \frac{\$2000 \times 0.05 \times 4}{100}$
$= \$400$
Amount $= P + I$

Income and tax

Employees can be paid:
wage: hourly rate with overtime at time and a half = 1.5 or double time
salary: annual amount
commission: % of sales
net income
= gross income – tax
Calculate PAYG tax using a tax table.

Applications of percentages

Percentage profit or loss
$= \frac{\text{profit or loss}}{\text{cost price}} \times 100\%$
Mark-up and discount
Commission/tax

Percentage increase and decrease

Increase	Decrease
20 by 6%:	20 by 5%:
$20 \times 1.06 = 21.2$	$20 \times 0.95 = 19$

Percentage change $= \frac{\text{change}}{\text{original}} \times 100\%$

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Chapter checklist with success criteria

A printable version of this checklist is available in the Interactive Textbook

		✔
1A	**1. I can apply order of operations with positive and negative integers.** e.g. Evaluate $3 \times (-5) + (-10 - (-2))$.	☐
1A	**2. I can work with roots and powers.** e.g. Evaluate $(-2)^2 - 3^2 \div \sqrt{9}$.	☐
1B	**3. I can round to a required number of decimal places.** e.g. Round 3.4572 to two decimal places.	☐
1B	**4. I can round to a required number of significant figures.** e.g. Round 24 532 and 0.0423 to two significant figures.	☐
1B	**5. I can estimate using significant figures.** e.g. Estimate the answer to $1986.2 + 37 \div 0.0234$ by rounding each number in the problem to one significant figure.	☐
1C	**6. I can express a fraction as a decimal.** e.g. Express $\frac{7}{12}$ as a decimal.	☐
1C	**7. I can express a decimal as a fraction.** e.g. Express 2.35 as a fraction.	☐
1C	**8. I can compare fractions.** e.g. Decide which is smaller: $\frac{11}{15}$ or $\frac{18}{25}$.	☐
1D	**9. I can add and subtract proper fractions.** e.g. Evaluate $\frac{5}{8} + \frac{1}{6}$.	☐
1D	**10. I can add and subtract mixed numerals.** e.g. Evaluate $2\frac{2}{5} - 1\frac{3}{4}$.	☐
1D	**11. I can multiply fractions.** e.g. Evaluate $3\frac{1}{3} \times 1\frac{3}{5}$.	☐
1D	**12. I can divide fractions.** e.g. Evaluate $1\frac{4}{5} \div 1\frac{5}{7}$.	☐
1E	**13. I can simplify ratios.** e.g. Simplify 32 : 20 and 0.3 : 0.12.	☐
1E	**14. I can divide into a given ratio.** e.g. 350 cm is to be divided in the ratio 2 : 5. Find the length of the larger portion.	☐
1E	**15. I can simplify rates.** e.g. Write \$140 every 4 hours in simplest form.	☐
1E	**16. I can determine a best buy.** e.g. What is better value: 2 kg of rice for \$3.60 or 5 kg of rice for \$17?	☐

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Chapter checklist

1F	**17. I can convert between percentages, decimals and fractions.** e.g. Express 24.5% as a decimal and 8% as a fraction.	☐
1F	**18. I can express a quantity as a percentage.** e.g. Express 34 out of 40 as a percentage.	☐
1F	**19. I can find a percentage of a quantity.** e.g. Find 36% of $60.	☐
1F	**20. I can find the original amount from a percentage.** e.g. Determine the original amount if 12% of the amount is $72.	☐
1G	**21. I can increase and decrease by a percentage.** e.g. Increase $30 by 8% and decrease $40 by 15%.	☐
1G	**22. I can find a percentage change.** e.g. Find the percentage change when a plant grows from 120 cm to 210 cm.	☐
1G	**23. I can find the original amount from a word problem.** e.g. The cost of petrol rises by 12% over a week to $1.54 per litre. What was the price at the start of the week? Round to the nearest cent.	☐
1H	**24. I can calculate the percentage profit.** e.g. Mark restores an antique chair he bought for $120. If he sells it for $210, what is his percentage profit?	☐
1H	**25. I can determine the selling price from a mark-up or discount.** e.g. Before school holidays an airline marks up all flights by 30%. If a flight costs $840, what will be its selling price for the holidays?	☐
1H	**26. I can calculate a sale saving.** e.g. A pair of jeans is discounted by 15%. If the sale price was $68, what was the original price?	☐
1I	**27. I can calculate wages and salaries.** e.g. Ryan earns an annual salary of $74 500. He works 48 weeks of the year and works 38-hour weeks. Determine his hourly rate of pay to the nearest cent.	☐
1I	**28. I can calculate overtime.** e.g. Ben's normal hourly rate of pay is $24.30. Calculate his earnings in a week if he works 12 hours at the normal rate, 6 hours at time and a half and 8 hours at double time.	☐
1I	**29. I can calculate commission.** e.g. A jeweller is paid a retainer of $250 per week and she also receives a commission of 6% on the value of goods she sells. If she sells $10 000 worth of jewellery during the week, how much will she earn?	☐
1J	**30. I can calculate tax using the PAYG system.** e.g. Dara has an annual income of $115 000 and $5000 worth of receipts of work-related expenses and donations. Use the tax table to find Dara's tax payable.	☐
1K	**31. I can calculate simple interest using the formula.** e.g. $2000 is invested at 4% p.a. simple interest for 3 years. Calculate the interest earned.	☐
1K	**32. I can find other unknowns using the simple interest formula.** e.g. An investment of $3000 at 4% p.a. simple interest earns $280 interest over a certain period of time. Calculate how long the money was invested for.	☐

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1L	**33. I can calculate the final balance using compound interest.** e.g. Find the total value of an investment if $12 000 is invested at 4% compounded annually for 5 years. Round to the nearest cent.	☐
1L	**34. I can find the initial amount using compound interest.** e.g. After 4 years a loan increases to $120 300. If the interest was compounded annually at a rate of 7%, find the size of the initial loan to the nearest dollar.	☐
1L	**35. I can find the final amount after depreciation.** e.g. A car worth $60 000 will depreciate by 12% p.a. What will the car be worth at the end of the fifth year to the nearest cent?	☐
1M	**36. I can use the compound interest formula to calculate a final balance compounded annually.** e.g. Riley invests $20 000 in a 5% p.a. interest account for 8 years compounded annually. How much interest is earned over this time correct to the nearest cent?	☐
1M	**37. I can use the compound interest formula to calculate a final balance compounded monthly.** e.g. Sophie invests $6000 in a 8% p.a. interest account for 5 years compounded monthly. What is the value of the investment, correct to the nearest cent, after the 5 years?	☐
1M	**38. I can calculate the future value of an item after depreciation has occurred.** e.g. A TV is purchased for $6000 and it depreciates at a rate of 16% p.a. Find the value of the TV after 5 years correct to the nearest cent.	☐

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Chapter review

Short-answer questions

1A
1 Evaluate the following.
a $-4 \times (2 - (-3)) + 4$
b $-3 - 4 \times (-2) + (-3)$
c $(-8 \div 8 - (-1)) \times (-2)$
d $\sqrt{25} \times \sqrt[3]{8}$
e $(-2)^2 - 3^3$
f $\sqrt[3]{1000} - (-3)^2$

1B
2 Round these numbers to three significant figures.
a 21.483
b 29 130
c 0.15271
d 0.002414

1B
3 Estimate the answer by first rounding each number to one significant figure.
a $294 - 112$
b 21.48×2.94
c $1.032 \div 0.493$

1C
4 Write these fractions as decimals.
a $2\frac{1}{8}$
b $\frac{5}{6}$
c $\frac{13}{7}$

1C
5 Write these decimals as fractions.
a 0.75
b 1.6
c 2.55

1D
6 Simplify the following.
a $\frac{5}{6} - \frac{1}{3}$
b $1\frac{1}{2} + \frac{2}{3}$
c $\frac{13}{8} - \frac{4}{3}$
d $3\frac{1}{2} \times \frac{4}{7}$
e $5 \div \frac{4}{3}$
f $3\frac{3}{4} \div 1\frac{2}{5}$

1E
7 Simplify these ratios.
a $30 : 12$
b $1.6 : 0.9$
c $7\frac{1}{2} : 1\frac{2}{5}$

1E
8 Divide 80 into the given ratio.
a $5 : 3$
b $5 : 11$
c $1 : 2 : 5$

1E
9 Dry dog food can be bought from store A for \$18 for 8 kg or from store B for \$14.19 for 5.5 kg.
a Determine the cost per kilogram at each store and state which is the best buy (cheaper deal).
b Determine to the nearest integer how many grams of each brand you get per dollar.

1F
10 Copy and complete the table below.

Decimal	Fraction	Percentage
0.6		
	$\frac{1}{3}$	
		$3\frac{1}{4}\%$
	$\frac{3}{4}$	
1.2		
		200%

1F
11 Find:
a 25% of \$310
b 110% of 1.5

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1G

12 Determine the original amount if:

a 20% of the amount is 30

b 72% of the amount is 18

1G

13 a Increase 45 by 60%.

b Decrease 1.8 by 35%.

c Find the percentage change if $150 is reduced by $30.

1G

14 The mass of a cat increased by 12% to 14 kg over a 12 month period. What was its starting mass?

1H

15 Determine the discount given on a $15 000 car if it is discounted by 12%.

1H

16 The cost price of an article is $150 and its selling price is $175.

a Determine the profit made.

b Express the profit as a percentage of the cost price.

1I

17 Determine the hourly rate of pay for each case.

a A person with an annual salary of $36 062 working a 38-hour week

b A person who earns $429 working 18 hours at the hourly rate and 8 hours at time and a half

1J

18 Jo earns a salary of $124 000 and has work related expenses and donations of $4000. Use the tax table to calculate Jo's tax payable. Ignore the Medicare levy.

1K

19 Find the simple interest earned on $1500 at 7% p.a. for 5 years.

1K

20 Rob invests $10 000 at 8% p.a. simple interest to produce $3600. For how long was the money invested?

1L/M

21 Find the total value of an investment if $50 000 is invested at 4% compounded annually for 6 years. Round to the nearest cent.

1L/M

22 After 8 years a loan grows to $75 210. If the interest was compounded annually at a rate of 8.5%, find the size of the initial loan to the nearest dollar.

1L/M

23 A $4000 computer depreciates at a rate of 12% p.a. over 6 years. Find the value of the computer after the 6 years correct to the nearest cent.

1M

24 Mik's $24 000 investment pays interest of 9% p.a. compounded monthly. What is the value of the investment after 3 years correct to the nearest cent?

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Chapter review

Multiple-choice questions

1B **1** $\frac{2}{7}$ written as a decimal is:

A 0.29 **B** 0.286 **C** $0.28\dot{5}$ **D** $0.\overline{285714}$ **E** $0.28571\dot{4}$

1B **2** 3.0456 written to three significant figures is:

A 3.04 **B** 3.05 **C** 3.045 **D** 3.046 **E** 3.45

1C **3** 2.25 written as a fraction in simplest form is:

A $2\frac{1}{2}$ **B** $\frac{5}{4}$ **C** $\frac{9}{4}$ **D** $9\frac{1}{4}$ **E** $\frac{225}{100}$

1D **4** $1\frac{1}{2} - \frac{5}{6}$ is equal to:

A $\frac{2}{3}$ **B** $\frac{5}{6}$ **C** $-\frac{1}{2}$ **D** $\frac{2}{6}$ **E** $\frac{1}{2}$

1D **5** $\frac{2}{7} \times \frac{3}{4}$ is equivalent to:

A $\frac{8}{11}$ **B** $\frac{3}{7}$ **C** $\frac{5}{11}$ **D** $\frac{8}{12}$ **E** $\frac{3}{14}$

1D **6** $\frac{3}{4} \div \frac{5}{6}$ is equivalent to:

A $\frac{5}{8}$ **B** 1 **C** 21 **D** $\frac{4}{5}$ **E** $\frac{9}{10}$

1E **7** Simplifying the ratio 50 cm : 4 m gives:

A 50 : 4 **B** 8 : 1 **C** 25 : 2 **D** 1 : 8 **E** 5 : 40

1F **8** 28% as a fraction in its simplest form is:

A 0.28 **B** $\frac{28}{100}$ **C** $\frac{0.28}{100}$ **D** $\frac{2.8}{100}$ **E** $\frac{7}{25}$

1F **9** 15% of \$1600 is equal to:

A 24 **B** 150 **C** \$240 **D** \$24 **E** 240

1I **10** Jane is paid a wage of \$7.80 per hour. If she works 12 hours at this rate during a week plus 4 hours on a public holiday for which she gets paid at time and a half, her earnings for the week are:

A \$140.40 **B** \$124.80 **C** \$109.20 **D** \$156 **E** \$62.40

1I **11** Simon earns a weekly retainer of \$370 and 12% commission of any sales he makes. If he makes \$2700 worth of sales in a particular week, he will earn:

A \$595 **B** \$652 **C** \$694 **D** \$738.40 **E** \$649.60

1L/M **12** \$1200 is increased by 10% for two years with compound interest. The total balance at the end of the two years is:

A \$252 **B** \$1452 **C** \$1450 **D** \$240 **E** \$1440

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Extended-response questions

1 Pauline buys a formal dress at cost price from her friend Tila. Pauline paid $420 for the dress, which is normally marked up by 55%.

a How much did she save?

b What is the normal selling price of the dress?

c If Tila gets a commission of 15%:

i how much commission did she get in dollars?

ii how much commission did Tila lose by selling the dress at cost price rather than the normal selling price?

2 Matilda has two bank accounts with the given details.

- Investment: Principal $25 000, interest rate 6.5% compounded annually
- Loan: 11.5% compounded annually

a Find Matilda's investment account balance after:

i 1 year

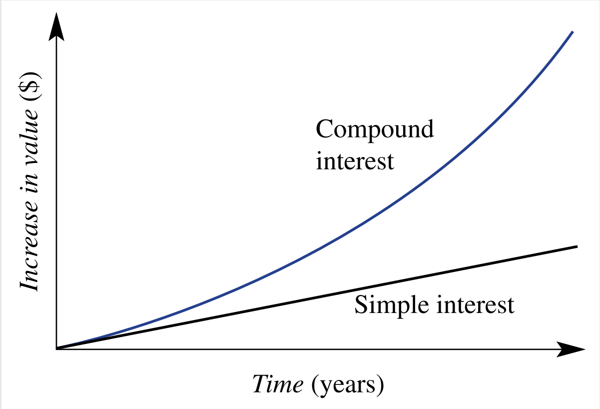
ii 10 years (to the nearest cent)

b Find the total percentage increase in Matilda's investment account after 10 years correct to two decimal places.

c After 3 years Matilda's loan account has increased to $114 250. Find the initial loan amount to the nearest dollar.

d Matilda reduces her $114 250 loan by $30 000. What is this reduction as a percentage to two decimal places?

e For Matilda's investment loan, what simple interest rate is equivalent to 5 years of the compounded interest rate of 6.5%? Round the answer to one decimal place.



This graph shows the effect of the same rate of compound and simple interest on the increase in value of an investment or loan, compared over time. Simple interest is calculated only on the principal, so yields a balance that increases by the same amount every year, forming a straight-line graph. After the first year, compound interest is calculated on the principal and what has been added by the previous years' interest, so the balance increases by a greater amount each year. This forms an exponential graph.

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2

Expressions, equations and inequalities

Maths in context: Finding the price for highest profit

Mathematicians called Operations Research Analysts develop pricing strategies in areas such as the airline industry, computer services, financial engineering, healthcare, manufacturing, mining, transportation, and the military.

Large corporations that trade globally want to price their products for high profits and quick sales. For example, consider an electric car's price as set by the manufacturer. If this price is too low, no profit is made. A high price increases profit enabling more cars to be manufactured. But this assumes all higher priced cars are quickly sold, which doesn't happen in reality. How do we find a selling price suitable for both the manufacturer and its customers?

To determine the best price for a car, analysts solve simultaneous equations for Supply (i.e., production) and Demand (i.e., sales). If $p =$ price of one car, and $n =$ the number sold, then:

- the linear equation for Supply shows that as p increases, n increases.
- the linear equation for Demand shows that as p increases, n decreases.

Simultaneously solving the Supply and Demand equations finds the solution (n, p) that makes both equations true. The best car price for the highest profit is when the number of manufactured cars equals the number of definite sales.

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Chapter contents

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- simplifies algebraic fractions involving indices, and expands and factorises algebraic expressions (MA5-ALG-P-01)
- solves monic quadratic equations, linear inequalities and cubic equations (MA5-EQU-P-01)
- solves linear equations of more than 3 steps, monic and non-monic quadratic equations, and linear simultaneous equations (MA5-EQU-P-02).

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

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2A Algebraic expressions CONSOLIDATING

Learning intentions
- To review the terminology associated with algebraic expressions
- To identify terms, coefficients and constant terms in expressions
- To be able to convert worded scenarios into algebraic expressions
- To be able to substitute numbers into algebraic expressions

Past, present and future learning
- This section consolidates Stage 4 concepts which are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- These concepts will be revised very briefly in Chapter 1 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

Algebra is central to the study of mathematics and is commonly used to solve problems in a vast array of theoretical and practical problems. Algebra involves the representation and manipulation of unknown or varying quantities in a mathematical context. Pronumerals (or variables) are used to represent these unknown quantities.

Dentists use an algebraic formula to calculate the quantity of local anaesthetic required for a procedure. The pronumerals in the formula represent the concentration of the anaesthetic being used and the patient's weight.

Lesson starter: Remembering the vocabulary

State the parts of the expression $5x - 2xy + (4a)^2 - 2$ that match these words.

- pronumeral (or variable)
- term
- coefficient
- constant term
- squared term

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KEY IDEAS

- In algebra, letters are used to represent numbers. These letters are called **pronumerals** or **variables**.
- An **expression** is a combination of numbers and pronumerals connected by any of the four operations $+, -, \times$ and $\div$. Brackets can also be used.
  For example: $5x^2 + 4y - 1$ and $3(x + 2) - \frac{y}{5}$
- A **term** is a combination of numbers and variables connected with only multiplication and division. Terms are separated with the operations $+$ and $-$.
  For example: $5x + 7y$ is a two-term expression.
- **Coefficients** are the numbers being multiplied by pronumerals.
  For example: the 3 in $3x$, the -2 in $5 - 2x$ and the $\frac{1}{2}$ in $\frac{x^2}{2}$ are coefficients.
- **Constant terms** consist of a number only.
  For example: -2 in $x^2 + 4x - 2$ (The sign must be included.)
- Expressions can be **evaluated** by substituting a number for a pronumeral.
  For example: if $a = -2$ then $a + 6 = -2 + 6 = 4$.
- **Order of operations** should be followed when evaluating expressions:
  1. Operations in brackets, then
  2. Powers, then
  3. Multiplication and division (from left to right), then
  4. Addition and subtraction (from left to right).

BUILDING UNDERSTANDING

1 State the number of terms in these expressions.

a $5x + 2y$ **b** $1 + 2a^2$ **c** $b^2 + ca - 1$ **d** $\frac{x^2}{2}$

2 Match each item in the left column with an item in the right column.

a Product	**A** Division
b Sum	**B** Subtraction
c Difference	**C** Multiplication
d Quotient	**D** Addition
e x^2	**E** the reciprocal of a
f $\frac{1}{a}$	**F** the square of x

3 State the coefficient in these terms.

a $5xy$ **b** $-2a^2$ **c** $\frac{x}{3}$ **d** $-\frac{2a}{5}$

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Example 1 Writing algebraic expressions from words

Write an algebraic expression for:

a the number of tickets needed for 3 boys and r girls
b the cost of P pies at $3 each
c the number of grams of peanuts for one child when 300 g of peanuts is shared equally among C children.

SOLUTION	EXPLANATION
a $3 + r$	3 tickets plus the number of girls
b $3P$ dollars	3 multiplied by the number of pies
c $\frac{300}{C}$	300 g divided into C parts

Now you try

Write an algebraic expression for:

a the number of seats needed for a adults and 7 children
b the cost of n apples at $1.20 each
c the number of pieces of chocolate for one person when 60 pieces is equally shared among p people.

Example 2 Converting words to expressions

Write an algebraic expression for:

a five less than x
b three more than twice x
c the sum of a and b is divided by 4
d the square of the sum of x and y.

SOLUTION	EXPLANATION
a $x - 5$	5 subtracted from x
b $2x + 3$	Twice x plus 3
c $\frac{a + b}{4}$	The sum of a and b is done first $(a + b)$ and the result divided by 4.
d $(x + y)^2$	The sum of x and y is done first and then the result is squared.

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Now you try

Write an algebraic expression for:

a six more than x

b one less than three times x

c two less than a is divided by 3

d the square root of the sum of a and b.

Example 3 Substituting values into expressions

Evaluate these expressions when $a = 5$, $b = -2$ and $c = 3$.

a $7a - 2(a - c)$

b $b^2 - ac$

SOLUTION

a
$$\begin{aligned} 7a - 2(a - c) &= 7 \times 5 - 2(5 - 3) \\ &= 35 - 2 \times 2 \\ &= 35 - 4 \\ &= 31 \end{aligned}$$

EXPLANATION

Substitute the values for a and c. When using order of operations, evaluate brackets before moving to multiplication and division then addition and subtraction.

b
$$\begin{aligned} b^2 - ac &= (-2)^2 - 5 \times 3 \\ &= 4 - 15 \\ &= -11 \end{aligned}$$

Evaluate powers before the other operations: $(-2)^2 = -2 \times (-2) = 4$

Now you try

Evaluate these expressions when $a = 3$, $b = -1$ and $c = -2$.

a $3(a - b) + 2c$

b $c^2 - 3ab$

Exercise 2A

FLUENCY	1, 2–3(½)	1, 2–4(½)	2–3(½), 4

Example 1

1 Write an algebraic expression for the following.

a The number of tickets required for:

i 4 boys and r girls

ii t boys and 2 girls

iii b boys and g girls

iv x boys, y girls and z adults.

b The cost, in dollars, of:

i P pies at \$6 each

ii 10 pies at \$$n$ each

iii D drinks at \$2 each

iv P pies at \$5 each and D drinks at \$2 each.

c The number of grams of lollies for one child when 500 g of lollies is shared equally among C children.

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Example 2

2 Write an algebraic expression for each of the following.

- **a** The sum of 2 and x
- **b** The sum of ab and y
- **c** 5 less than x
- **d** The product of x and 3
- **e** The difference between $3x$ and $2y$
- **f** Three times the value of p
- **g** Four more than twice x
- **h** The sum of x and y is divided by 5
- **i** 10 less than the product of 4 and x
- **j** The square of the sum of m and n
- **k** The sum of the squares of m and n
- **l** The square root of the sum of x and y
- **m** The sum of a and its reciprocal
- **n** The cube of the square root of x

Example 3

3 Evaluate these expressions when $a = 4$, $b = -3$ and $c = 7$.

- **a** $b - ac$
- **b** $bc - a$
- **c** $a^2 - c^2$
- **d** $b^2 - ac$
- **e** $\dfrac{a + b}{2}$
- **f** $\dfrac{b^2 + c}{a}$
- **g** $\dfrac{1}{c} \times (a - b)$
- **h** $a^3 - bc$

4 Evaluate these expressions when $x = -2$, $y = -\frac{1}{2}$ and $z = \frac{1}{6}$.

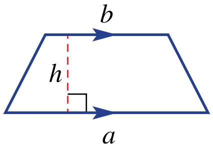
- **a** $xy + z$
- **b** $y^2 + x^2$
- **c** xyz
- **d** $\dfrac{xz + 1}{y}$

PROBLEM–SOLVING	5, 6	5, 6	6, 7

5 A rectangular garden bed is 12 m long and 5 m wide.

- **a** Find the area of the garden bed.
- **b** The length is increased by x m and the breadth is decreased by y m. Find the new length and breadth of the garden.
- **c** Write an expression for the area of the new garden bed.

6 The expression for the area of a trapezium is $\frac{h}{2}(a + b)$ where a and b are the lengths of the two parallel sides and h is the distance between the two parallel sides.



- **a** Find the area of the trapezium with $a = 5$, $b = 7$ and $h = 3$.
- **b** A trapezium has $h = 4$ and area 12. If a and b are positive integers, what possible values can the variable a have?

7 The cost of 10 identical puzzles is $\$P$.

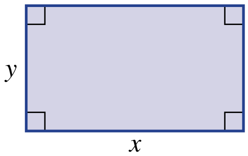
- **a** Write an expression for the cost of one puzzle.
- **b** Write an expression for the cost of n puzzles.

REASONING	8	8, 9	9, 10

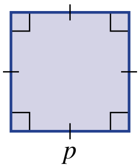
8 For each of these shapes, write an expression for:

- **i** the perimeter
- **ii** the area

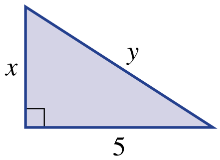
a



b



c



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9 Decide if the following statements refer to the same or different expressions. If they are different, write an expression for each statement.

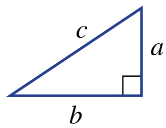
a **A** Twice the sum of x and y

B The sum of $2x$ and y

b **A** The difference between half of x and half of y

B Half of the difference between x and y

10 For a right-angled triangle with hypotenuse c and shorter sides a and b, Pythagoras' theorem states that $c^2 = a^2 + b^2$.



a Which of these two descriptions also describes Pythagoras' theorem?

A The square of the hypotenuse is equal to the square of the sum of the two shorter sides.

B The square of the hypotenuse is equal to the sum of the squares of the two shorter sides.

b For the incorrect description, write an equation to match.

ENRICHMENT: The sum of the first n positive integers	–	–	11

11 The rule for the sum of the first n positive integers is given by:

The product of *n* and one more than *n* all divided by 2.

a Write an expression for the above description.

b Test the expression to find these sums

i $1 + 2 + 3 + 4$ $(n = 4)$

ii $1 + 2 + 3 + \ldots + 10$ $(n = 10)$

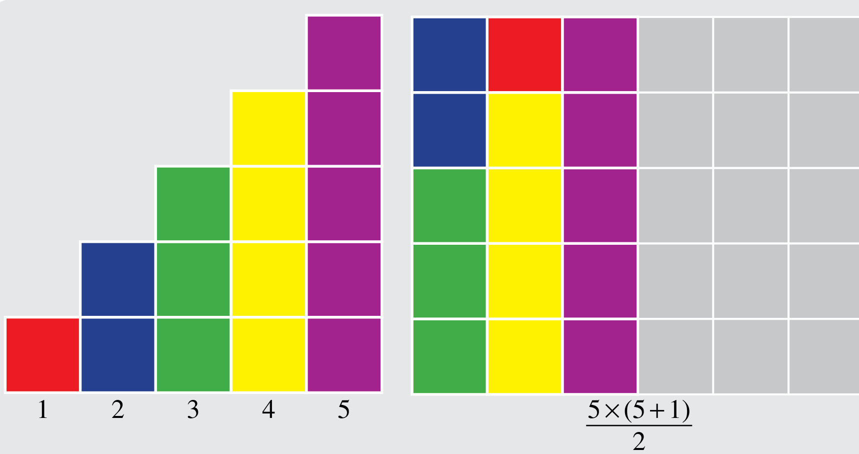
c Another way to describe the same expression is:

The sum of half of the square of *n* and half of *n*.

Write the expression for this description.

d Check that your expressions in parts **a** and **c** are equivalent (the same) by testing $n = 4$ and $n = 10$.

e $\frac{1}{2}(n^2 + n)$ is also equivalent to the above two expressions. Write this expression in words.



A diagram representing the sum of the first five positive integers arranged according to the expression in Question 11

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2B Simplifying algebraic expressions CONSOLIDATING

Learning intentions
- To be able to identify like terms
- To understand that only like terms can be combined under addition or subtraction
- To be able to collect and combine like terms using addition or subtraction
- To review the conventions and processes for multiplication and division of algebraic terms
- To be able to multiply algebraic terms by multiplying coefficients and pronumerals
- To be able to divide algebraic terms by cancelling the highest common factor

Past, present and future learning
- This section consolidates Stage 4 concepts which are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- These concepts will be revised very briefly in Chapter 1 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

Just as $2 + 2 + 2 + 2 = 4 \times 2$, so $x + x + x + x = 4 \times x$ or $4x$. We say that the expression $x + x + x + x$ is simplified to $4x$. Similarly, $3x + 5x = 8x$ and $8x - 3x = 5x$.

All these expressions have like terms and can be simplified to an expression with a smaller number of terms.

A single term such as $2 \times 5 \times x \div 10$ can also be simplified using multiplication and division, so $2 \times 5 \times x \div 10 = \frac{10x}{10} = x$.

Manufacturing companies, such as globe makers, find that an item's selling price, \$$x$, affects the number of sales and the revenue. Algebraic expressions often need to be simplified when developing formulas for a company's expenses, revenue and profit.

Lesson starter: Are they equivalent?

All these expressions can be separated into two groups. Group them so that the expressions in each group are equivalent.

$2x$	$2x - y$	$4x - x - x$	$10x - y - 8x$
$\frac{24x}{12}$	$y + x - y + x$	$2 \times x - 1 \times y$	$-y + 2x$
$0 + \frac{1}{2} \times 4x$	$\frac{x}{\left(\frac{1}{2}\right)} + 0y$	$\frac{6x^2}{3x}$	$-1 \times y + \frac{x^2}{\frac{1}{2}x}$

KEY IDEAS

- The symbols for multiplication ($\times$) and division ($\div$) are usually not shown in simplified algebraic terms.
  For example: $5 \times a \times b = 5ab$ and $-7 \times x \div y^2 = -\frac{7x}{y^2}$

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- When dividing algebraic expressions, common factors can be cancelled.

  For example: $\frac{7x}{14} = \frac{x}{2}$, $\frac{a^2b}{a} = \frac{\cancel{a}^1 \times a \times b}{\cancel{a}^1} = ab$

  $\frac{7xy}{14y} = \frac{x}{2}$ and $\frac{15a^2b}{10a} = \frac{3 \times \cancel{5}^1 \times a \times \cancel{a}^1 \times b}{2 \times \cancel{5}^1 \times \cancel{a}^1} = \frac{3ab}{2}$

- **Like terms** have the same pronumeral factors.
  - For example: $5x$ and $7x$ are like terms and $3a^2b$ and $-2a^2b$ are like terms.
  - Since $a \times b = b \times a$ then ab and ba are also like terms.

- The pronumeral part of a term is often written in alphabetical order.

- **Like terms** can be collected (added and subtracted) to form a single term.

  For example: $5ab + 8ab = 13ab$

  $4x^2y - 2yx^2 = 2x^2y$

- **Unlike terms** do not have the same pronumeral factors.

  For example: $5x$, x^2, xy and $\frac{4xyz}{5}$ are all unlike terms.

BUILDING UNDERSTANDING

1. State the missing word or expression.
   a. Like terms have the same __________ factors.
   b. $3x + 2x$ simplifies to __________.
   c. $3a^2b$ and $2a$ are __________ terms.

2. Simplify these fractions.
   a. $\frac{20}{8}$
   b. $\frac{30}{10}$
   c. $\frac{8}{40}$
   d. $-\frac{5}{100}$

3. Decide if the following pairs of terms are like terms.
   a. $4ab$ and $3ab$
   b. $2x$ and $7xy$
   c. 5 and $4m$
   d. $3t$ and $-6tw$
   e. $2mn$ and $9nm$
   f. $3x^2y$ and $7xy^2$

Example 4 Multiplying algebraic terms

Simplify the following.

a. $3 \times 2b$

b. $-2a \times 3ab$

SOLUTION	EXPLANATION
a. $3 \times 2b = 3 \times 2 \times b$ $= 6b$	Multiply the coefficients.
b. $-2a \times 3ab = -2 \times 3 \times a \times a \times b$ $= -6a^2b$	Multiply the coefficients and simplify.

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Now you try

Simplify the following.

a $5 \times 7n$

b $-3x \times 6xy$

Example 5 Dividing algebraic terms

Simplify the following.

a $\dfrac{6ab}{18b}$

b $12a^2b \div (3ab)$

SOLUTION

a $\dfrac{{}^{1}\cancel{6}a\cancel{b}^{1}}{{}_{3}\cancel{18}\cancel{b}_{1}} = \dfrac{a}{3}$

b $12a^2b \div (3ab) = \dfrac{{}^{4}\cancel{12}\cancel{a^2}^{1}\cancel{b}^{1}}{{}_{1}\cancel{3}\cancel{a}_{1}\cancel{b}_{1}}$
$= 4a$

EXPLANATION

Deal with numerals and pronumerals separately, cancelling where possible.

Write as a fraction first. Cancel where possible, recall $a^2 = a \times a$.

Now you try

Simplify the following.

a $\dfrac{4xy}{8x}$

b $-20ab^2 \div (10ab)$

Example 6 Collecting like terms

Simplify the following by collecting like terms.

a $3x + 4 - 2x$

b $3x + 2y + 4x + 7y$

c $8ab^2 - 9ab - ab^2 + 3ba$

SOLUTION

a $3x + 4 - 2x = 3x - 2x + 4$
$= x + 4$

b $3x + 2y + 4x + 7y = 3x + 4x + 2y + 7y$
$= 7x + 9y$

c $8ab^2 - 9ab - ab^2 + 3ba$
$= 8ab^2 - ab^2 - 9ab + 3ab$
$= 7ab^2 - 6ab$

EXPLANATION

Collect like terms ($3x$ and $-2x$). The sign belongs to the term that follows. Combine their coefficients: $3 - 2 = 1$.

Collect like terms and combine their coefficients.

Collect like terms. Remember $ab = ba$ and $ab^2 = 1ab^2$.
$8 - 1 = 7$ and $-9 + 3 = -6$

Now you try

Simplify the following by collecting like terms.

a $6a + 2 - a$

b $4m + n + 2m + 6n$

c $7a^2b + ab - 4a^2b - 3ba$

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Exercise 2B

FLUENCY	1–4(½), 6–8(½)	1–8(½)	2–8(⅓)

Example 4a

1 Simplify the following.

a $3 \times 6r$
b $2 \times 8b$
c $-2x \times 7$
d $3 \times (-5p)$
e $-4c \times 3d$
f $5m \times (-3n)$
g $-4r \times 3 \times 2s$
h $5j \times (-4) \times 2k$

Example 4b

2 Simplify the following.

a $7a \times 3ab$
b $5mn \times (-3n)$
c $-3gh \times (-6h)$
d $3xy \times 4xy$
e $-4ab \times (-2ab)$
f $-2mn \times 3mn$

Example 5a

3 Simplify the following by cancelling.

a $\frac{8b}{2}$
b $-\frac{2a}{6}$
c $\frac{4ab}{6}$
d $\frac{3mn}{6n}$
e $-\frac{5xy}{20y}$
f $\frac{10st}{6t}$
g $\frac{u^2v}{u}$
h $\frac{5r^2s^2}{8rs}$
i $\frac{5ab^2}{9b}$

Example 5b

4 Simplify the following by first writing in fraction form.

a $2x \div 5$
b $-4 \div (-3a)$
c $11mn \div 3$
d $12ab \div 2$
e $-10 \div (2gh)$
f $8x \div x$
g $-3xy \div (yx)$
h $7mn \div (3m)$
i $-27pq \div (6p)$
j $24ab^2 \div (8ab)$
k $25x^2y \div (-5xy)$
l $9m^2n \div (18mn)$

5 Simplify the following.

a $x \times 4 \div y$
b $5 \times p \div 2$
c $6 \times (-a) \times b$
d $a \times (-3) \div (2b)$
e $-7 \div (5m) \times n$
f $5s \div (2t) \times 4$
g $6 \times 4mn \div (3m)$
h $8x \times 3y \div (8x)$
i $3ab \times 12bc \div (9abc)$
j $4x \times 3xy \div (2x)$
k $10m \times 4mn \div (8mn)$
l $3pq \times pq \div p$

Example 6a

6 Simplify the following by collecting like terms.

a $3a + 7a$
b $4n + 3n$
c $12y - 4y$
d $5x + 2x + 4x$
e $6ab - 2ab - ba$
f $7mn + 2mn - 2mn$
g $4y - 3y + 8$
h $7x + 5 - 4x$
i $6xy + xy + 4y$
j $5ab + 3 + 7ba$
k $2 - 5m - m$
l $4 - 2x + x$

Example 6b

7 Simplify the following by collecting like terms.

a $2a + 4b + 3a + 5b$
b $4x + 3y + 2x + 2y$
c $6t + 5 - 2t + 1$
d $5x + 1 + 6x + 3$
e $xy + 8x + 4xy - 4x$
f $3mn - 4 + 4nm - 5$
g $4ab + 2a + ab - 3a$
h $3st - 8ts + 2st + 3ts$

Example 6c

8 Simplify the following by collecting like terms.

a $5xy^2 - 4xy^2$
b $3a^2b + 4ba^2$
c $8m^2n - 6nm^2 + m^2n$
d $7p^2q^2 - 2p^2q^2 - 4p^2q^2$
e $2x^2y - 4xy^2 + 5yx^2$
f $10rs^2 + 3rs^2 - 6r^2s$
g $x^2 - 7x - 3x^2$
h $a^2b - 4ab^2 + 3a^2b + b^2a$
i $10pq^2 - 2qp - 3pq^2 - 6pq$
j $12m^2n^2 - 2mn^2 - 4m^2n^2 + mn^2$

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PROBLEM–SOLVING	9, 10	9–11	10–12

9 A farmer has x pigs and y chickens.
- **a** Write an expression for the total number of heads.
- **b** Write an expression for the total number of legs.

10 The length of a rectangle is three times its breadth x metres. Write an expression for:
- **a** the rectangle's perimeter
- **b** the rectangle's area.

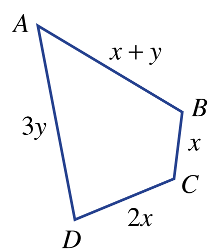
11 A right-angled triangle has side lengths $5x$ cm, $12x$ cm and $13x$ cm. Write an expression for:
- **a** the triangle's perimeter
- **b** the triangle's area.

12 The average (mean) mark on a test for 20 students is x. Another student who scores 75 in the test is added to the list. Write an expression for the new average (mean).

REASONING	13	13	13, 14

13 Decide whether the following are always true for all real numbers.
- **a** $a \times b = b \times a$
- **b** $a \div b = b \div a$
- **c** $a + b = b + a$
- **d** $a - b = b - a$
- **e** $a^2b = b^2a$
- **f** $1 \div \frac{1}{a} = a \ (a \neq 0)$

14 The diagram shows the route taken by a salesperson who travels from A to D via B and C. All distances are in kilometres.



- **a** If the salesperson then returns directly to A, write an expression (in simplest form) for the total distance travelled.
- **b** If $y = x + 1$, write an expression for the total distance the salesperson travels in terms of x only. Simplify your expression.
- **c** When $y = x + 1$, by how much would the distance have been reduced (in terms of x) if the salesperson had travelled directly from A to D and straight back to A?

ENRICHMENT: Higher powers	–	–	15(½)

15 For this question, note this example: $\frac{2a^3}{4a} = \frac{\cancel{2}_1 \times a \times a \times \cancel{a}_1}{{}^2\cancel{4} \times \cancel{a}_1}$

$$= \frac{a^2}{2}$$

Simplify these expressions with higher powers.
- **a** $\frac{a^4}{a}$
- **b** $\frac{3b^3}{9b}$
- **c** $\frac{4ab^3}{12ab}$
- **d** $\frac{6a^4b^2}{16a^3b}$
- **e** $-\frac{2a^5}{3a^2}$
- **f** $-\frac{8a^5}{20a^7}$
- **g** $\frac{4a^3}{10a^8}$
- **h** $\frac{3a^3b}{12ab^4}$
- **i** $\frac{15a^4b^2}{5ab}$
- **j** $\frac{28a^3b^5}{7a^4b^2}$
- **k** $\frac{2a^5b^2}{6a^2b^3}$
- **l** $-\frac{5a^3b^7}{10a^2b^{10}}$

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2C Expanding algebraic expressions

Learning intentions
- To know how the distributive law is used to expand brackets in expressions
- To be able to simplify expressions involving brackets and like terms

Past, present and future learning
- This section addresses the Stage 5 Core Topic called *Algebraic techniques A*
- In Section 2E of this book you will use this skill to solve equations
- These concepts will be revised very briefly in Chapter 1 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

A mental technique to find the product of 5 and 23 might be to find 5×20 and add 5×3 to give 115. This technique uses the distributive law over addition.

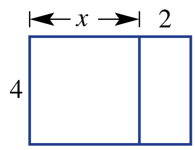
$$\begin{aligned} \text{So } 5 \times 23 &= 5 \times (20 + 3) \\ &= 5 \times 20 + 5 \times 3 \end{aligned}$$

Since pronumerals (or variables) represent numbers, the same law applies for algebraic expressions.

Portable dance floors for hire come in 1 m by 1 m squares of two types, edge squares and inner squares. For a floor x m by y m, the number of edge squares $= 2x + 2(y - 2)$ and the number of inner squares $= (x - 2)(y - 2)$.

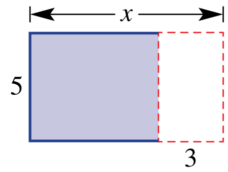
Lesson starter: Rectangular distributions

This diagram shows two joined rectangles with the given dimensions.



- Find two different ways to write expressions for the combined area of the two rectangles.
- Compare your two expressions. Are they equivalent?

This diagram shows a rectangle of length x reduced by a length of 3.



- Find two different ways to write expressions for the remaining area (shaded).
- Compare your two expressions. Are they equivalent?

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KEY IDEAS

- The **distributive law** is used to expand and remove brackets.
  - A term on the outside of the brackets is multiplied by each term inside the brackets.

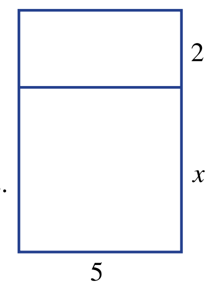
$$a(b + c) = ab + ac \quad \text{or} \quad a(b - c) = ab - ac$$

$$-a(b + c) = -ab - ac \quad \text{or} \quad -a(b - c) = -ab + ac$$

  - If the number in front of the bracket is negative, the sign of each of the terms inside the brackets will change when expanded.
    For example: $-2(x - 3) = -2x + 6$ since $-2 \times x = -2x$ and $-2 \times (-3) = 6$.

BUILDING UNDERSTANDING

1 This diagram shows two joined rectangles with the given dimensions.



- **a** State an expression for the area of:
  - **i** the larger rectangle (x by 5)
  - **ii** the smaller rectangle (2 by 5)
- **b** Use your answers from part **a** to find the combined area of both rectangles.
- **c** State an expression for the total side length of the side involving x.
- **d** Use your answer from part **c** to find the combined area of both rectangles.
- **e** State the missing terms in this statement: $5(x + 2) = \_\_\_\_\_ + \_\_\_\_\_$

2 Complete the working for the following.

a $\begin{aligned} 3(x + 5) &= 3 \times \_\_ + \_\_ \times 5 \\ &= 3x + \_\_ \end{aligned}$

b $\begin{aligned} 4(y - 3) &= 4 \times \_\_ + 4 \times \_\_ \\ &= 4y - \_\_ \end{aligned}$

c $\begin{aligned} -2(x + 3) &= -2 \times x + \_\_ \times 3 \\ &= -2x - \_\_ \end{aligned}$

d $\begin{aligned} -3(x - 1) &= -3 \times x + (-3) \times \_\_ \\ &= -3x + \_\_ \end{aligned}$

Example 7 Expanding simple expressions with brackets

Expand the following.

a $3(x + 4)$ **b** $5(x - 11)$ **c** $-2(x - 5)$

SOLUTION	EXPLANATION
a $3(x + 4) = 3x + 12$	$3 \times x = 3x$ and $3 \times 4 = 12$
b $5(x - 11) = 5x - 55$	$5 \times x = 5x$ and $5 \times (-11) = -55$
c $-2(x - 5) = -2x + 10$	$-2 \times x = -2x$ and $-2 \times (-5) = +10$

Now you try

Expand the following.

a $5(x + 2)$ **b** $3(x - 7)$ **c** $-3(x - 2)$

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Example 8 Expanding brackets and simplifying

Expand the following.

a $4(x + 3y)$

b $-2x(4x - 3)$

SOLUTION

a $4(x + 3y) = 4 \times x + 4 \times 3y$
$= 4x + 12y$

b $-2x(4x - 3) = -2x \times 4x + (-2x) \times (-3)$
$= -8x^2 + 6x$

EXPLANATION

Multiply each term inside the brackets by 4.
$4 \times x = 4x$ and $4 \times 3 \times y = 12y$.

Each term inside the brackets is multiplied by $-2x$.
$-2 \times 4 = -8$, $x \times x = x^2$ and $-2 \times (-3) = 6$

Now you try

Expand the following.

a $3(x + 7y)$

b $-3a(2a - 4)$

Example 9 Simplifying by removing brackets

Expand the following and collect like terms.

a $2 + 3(x - 4)$

b $3(x + 2y) - (3x + y)$

SOLUTION

a $2 + 3(x - 4) = 2 + 3x - 12$
$= 3x - 10$

b $3(x + 2y) - (3x + y)$
$= 3x + 6y - 3x - y$
$= 3x - 3x + 6y - y$
$= 5y$

EXPLANATION

Multiply first by expanding brackets:

$3(x - 4) = 3x - 12$

Then combine any like terms: $2 - 12 = -10$

Expand each set of brackets where

$-(3x + y) = -1(3x + y) = -3x - y$.

Collect like terms and simplify.

Now you try

Expand the following and collect like terms.

a $4 + 2(y - 1)$

b $3(x - 2y) - (2x + 3y)$

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Exercise 2C

FLUENCY	1–4(½)	1–5(½)	2–5(¼)

Example 7a,b

1 Expand the following.

a $2(x + 3)$ **b** $5(x + 12)$ **c** $2(x + 7)$ **d** $7(x - 9)$
e $3(x - 2)$ **f** $2(x - 6)$ **g** $4(7 - x)$ **h** $7(3 - x)$

Example 7c

2 Expand the following.

a $-3(x + 2)$ **b** $-2(x + 11)$ **c** $-5(x - 3)$ **d** $-6(x - 6)$
e $-4(2 - x)$ **f** $-13(5 + x)$ **g** $-20(9 + x)$ **h** $-300(1 - x)$

Example 8

3 Expand the following.

a $2(a + 2b)$ **b** $5(3a - 2)$ **c** $3(4m - 5)$ **d** $-8(2x + 5)$
e $-3(4x + 5)$ **f** $-4x(x - 2y)$ **g** $-9t(2y - 3)$ **h** $a(3a + 4)$
i $d(2d - 5)$ **j** $-2b(3b - 5)$ **k** $2x(4x + 1)$ **l** $5y(1 - 3y)$

Example 9a

4 Expand the following and collect like terms.

a $3 + 2(x + 4)$ **b** $4 + 6(x - 3)$ **c** $2 + 5(3x - 1)$ **d** $5 + (3x - 4)$
e $3 + 4(x - 2)$ **f** $7 + 2(x - 3)$ **g** $2 - 3(x + 2)$ **h** $1 - 5(x + 4)$
i $5 - (x - 6)$ **j** $9 - (x - 3)$ **k** $5 - (3 + 2x)$ **l** $4 - (3x - 2)$

Example 9b

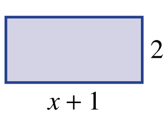
5 Expand the following and collect like terms.

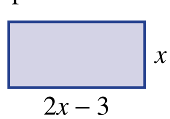
a $2(x + 3) + 3(x + 2)$ **b** $2(x - 3) + 2(x - 1)$ **c** $3(2x + 1) + 5(x - 1)$
d $4(3x + 2) + 5(x - 3)$ **e** $-3(2x + 1) + (2x - 3)$ **f** $-2(x + 2) + 3(x - 1)$
g $2(4x - 3) - 2(3x - 1)$ **h** $-3(4x + 3) - 5(3x - 1)$ **i** $-(x + 3) - 3(x + 5)$
j $-2(2x - 4) - 3(3x + 5)$ **k** $3(3x - 1) - 2(2 - x)$ **l** $-4(5 - x) - (2x - 5)$

PROBLEM-SOLVING	6, 7	6, 7(½), 8	7(½), 8, 9

6 The length of a rectangle is 4 more than its breadth, x. Find an expanded expression for its area.

7 Find the area of these basic shapes in expanded form. All angles at vertices are 90°.

a


b


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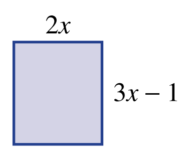

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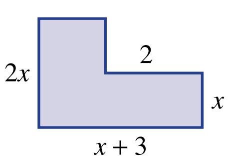

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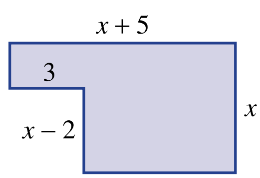

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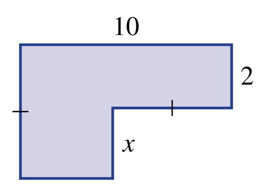

8 Gary gets a \$20 bonus for every computer he sells over and above his quota of 10 per week. If he sells n computers in a week and $n > 10$, write an expression for Gary's bonus in that week. (Give the answer in expanded form.)

9 Jill pays tax at 20c in the dollar for every dollar earned over \$10 000. If Jill earns \$$x$ and $x > 10\,000$, write an expression in expanded form for Jill's tax.

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REASONING	10	10, 11(½)	10, 11(½)

10 Identify the errors in these expressions then write the correct expansion.

a $2(x+6) = 2x+6$
b $x(x-4) = 2x-4x$
c $-3(x+4) = -3x+12$
d $-7(x-7) = -7x-49$
e $5-2(x-7) = 5-2x-14$
$= -9-2x$
f $4(x-2)-3(x+2) = 4x-8-3x+6$
$= x-2$

11 In Years 7 and 8 we explored how to use the distributive law to find products mentally.
For example: $7 \times 104 = 7 \times (100+4)$
$= 7 \times 100 + 7 \times 4$
$= 728$
and $4 \times 298 = 4 \times (300-2)$
$= 4 \times 300 - 4 \times 2$
$= 1192$

Use the distributive law to evaluate these products mentally.

a 6×52
b 9×102
c 5×91
d 4×326
e 3×99
f 7×395
g 9×990
h 6×879

ENRICHMENT: Pronumerals and taxes	–	–	12

12 A progressive income tax system increases the tax rate for higher incomes. Here is an example.

Income	Tax
0–\$20 000	\$0
\$20 001–\$50 000	\$0 + 20% of income above \$20 000
\$50 001–\$100 000	a + 30% of income above \$50 000
\$100 000–	b + 50% of income above \$100 000

a Find the values of a and b in the above table.

b Find the tax payable for the following incomes.
i \$35 000
ii \$72 000
iii \$160 000

c Find an expression for the tax payable for an income of \$$x$ if:
i $0 \leqslant x \leqslant 20000$
ii $20000 < x \leqslant 50000$
iii $50000 < x \leqslant 100000$
iv $x > 100000$

d Check that you have fully expanded and simplified your expressions for part **c**. Add steps if required.

e Use your expressions from parts **c** and **d** to check your answers to part **b** by choosing a particular income amount and checking against the table above.

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2D Linear equations with pronumerals on one side

Learning intentions

- To understand the difference between expressions and equations
- To understand that equivalent equations can be created by applying the same operation to both sides of the equation
- To review the steps for solving simple linear equations
- To be able to solve linear equations with a pronumeral on one side only

Past, present and future learning

- This section addresses the Stage 5 Core Topic called *Equations A*
- These concepts will be revised very briefly in Chapter 1 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

A mathematical statement containing an equals sign, a left-hand side and a right-hand side is called an equation. $5 = 10 \div 2$, $3x = 9$, $x^2 + 1 = 10$ and $\frac{1}{x} = \frac{x}{5}$ are examples of equations. Linear equations can be written in the form $ax + b = c$ where the power of x is 1. $4x - 1 = 6$, $3 = 2(x + 1)$ and $\frac{5x}{3} = \frac{2x + 1}{4}$ are all linear equations. Equations are solved by finding the value of the variable (or pronumeral) that makes the equation true. This can be done by inspection for very simple linear equations (for example, if $3x = 15$ then $x = 5$ since $3 \times 5 = 15$). More complex linear equations can be solved through a series of steps where each step produces an equivalent equation.

Computer programmers solve linear equations when writing software code for various applications, developing websites, troubleshooting network problems, calculating data upload times and adjusting security settings.

Lesson starter: Why are they equivalent?

The following list of equations can be categorised into two groups. The equations in each group should be equivalent.

$5x = 20$	$2x - 1 = -3$	$x = 4$	$1 - x = -3$
$7x = -7$	$3 - 5x = -17$	$\frac{8x}{5} - \frac{3x}{5} = -1$	$x = -1$

- Discuss how you divided the equations into the two groups.
- How can you check to see if the equations in each group are equivalent?

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KEY IDEAS

- **Equivalent equations** are created by:
  - adding or subtracting the same number on both sides of the equation
  - multiplying or dividing both sides of the equation by the same number (not including 0).
- Solve a linear equation by creating equivalent equations using **inverse operations** (sometimes referred to as **backtracking**).
- The solution to an equation can be checked by substituting the solution into the original equation and checking that both sides are equal.

BUILDING UNDERSTANDING

1. State the value of x that is the solution to these equations. No written working is required.
   a $3x = 9$ **b** $\frac{x}{4} = 10$ **c** $x + 7 = 12$ **d** $x - 7 = -1$
2. Use a 'guess and check' (trial and error) method to solve these equations. No written working is required.
   a $2x + 1 = 7$ **b** $4 - x = 2$ **c** $2 + \frac{x}{3} = 6$ **d** $\frac{x+1}{7} = 1$
3. Which of the following equations are equivalent to $3x = 12$?
   A $3x - 1 = 12$ **B** $-3x = -12$ **C** $\frac{3x}{4} = 3$ **D** $\frac{3x}{5} = 10$

Example 10 Solving simple linear equations

Solve each of the following equations.

a $2x + 3 = 4$ **b** $5 - 2x = 12$

SOLUTION	EXPLANATION
a $2x + 3 = 4$	
$2x = 1$	Subtract 3 from both sides.
$x = \frac{1}{2}$	Divide both sides by 2.
Check: $2 \times \left(\frac{1}{2}\right) + 3 = 4$	Check the answer by substituting $x = \frac{1}{2}$ into the original equation.
b $5 - 2x = 12$	
$-2x = 7$	Subtract 5 from both sides.
$x = -\frac{7}{2}$	Divide both sides by -2.
Check: $5 - 2 \times \left(-\frac{7}{2}\right) = 5 + 7 = 12$	Check the answer.

Now you try

Solve each of the following equations.

a $3x + 1 = 10$ **b** $4 - 3x = 7$

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Example 11 Solving linear equations involving fractions

Solve each of the following equations

a $\frac{x}{4} - 3 = 7$ **b** $\frac{2x}{3} + 5 = 7$ **c** $\frac{x+4}{9} = 2$

SOLUTION	EXPLANATION
a $\frac{x}{4} - 3 = 7$	
$\frac{x}{4} = 10$	Add 3 to both sides.
$x = 40$	Multiply both sides by 4.
Check: $\frac{(40)}{4} - 3 = 10 - 3 = 7$	Check the answer by substituting $x = 40$ into $\frac{x}{4} - 3$. Since this equals 7, $x = 40$ is the solution.
b $\frac{2x}{3} + 5 = 7$	
$\frac{2x}{3} = 2$	Subtract 5 from both sides first then multiply both sides by 3.
$2x = 6$	
$x = 3$	Divide both sides by 2.
Check: $\frac{2 \times (3)}{3} + 5 = 2 + 5 = 7$	Check the answer.
c $\frac{x+4}{9} = 2$	
$x + 4 = 18$ $x = 14$	Multiply both sides by 9 first to eliminate the fraction. Solve the remaining equation by subtracting 4 from both sides.
Check: $\frac{(14)+4}{9} = \frac{18}{9} = 2$	Check the answer.

Now you try

Solve each of the following equations

a $\frac{x}{2} - 5 = 6$ **b** $\frac{5x}{2} + 1 = 4$ **c** $\frac{x+3}{5} = 4$

Exercise 2D

FLUENCY	1–5(½)	1–5(½)	1–5(⅓)

Example 10a

1 Solve each of the following equations. Check your answers using substitution.

a $2x + 5 = 9$ **b** $5a + 6 = 11$ **c** $3m - 4 = 8$
d $2b + 15 = 7$ **e** $3y - 2 = -13$ **f** $3a + 2 = 7$
g $4b + 7 = 25$ **h** $24x - 2 = 10$ **i** $6x - 5 = 3$
j $7y - 3 = -8$ **k** $2a + \frac{1}{2} = \frac{1}{4}$ **l** $5n - \frac{1}{4} = -1$

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Example 10b

2 Solve each of the following equations.

a $12 - 2x = 18$
b $2 - 7x = 9$
c $15 - 5x = 5$
d $3 - 2x = -13$
e $2 - 5x = 9$
f $4 - 7x = 23$
g $5 - 8x = 2$
h $-3 - 4x = -10$

Example 11a

3 Solve each of the following equations.

a $\frac{t}{2} + 5 = 2$
b $\frac{a}{3} + 4 = 2$
c $\frac{y}{5} - 4 = 2$
d $\frac{x}{3} - 7 = -12$
e $\frac{s}{2} - 3 = -7$
f $\frac{x}{4} - 5 = -2$
g $\frac{m}{4} - 2 = 3$
h $1 - \frac{y}{5} = 2$

Example 11b

4 Solve these equations.

a $\frac{2b}{3} = 6$
b $\frac{3x}{2} = 9$
c $\frac{4x}{3} = -9$
d $\frac{2x}{5} = -3$
e $\frac{3x}{4} = \frac{1}{2}$
f $\frac{5n}{4} = -\frac{1}{5}$
g $\frac{2x}{3} - 1 = 7$
h $\frac{3x}{4} - 2 = 7$
i $3 + \frac{2x}{3} = -3$
j $5 + \frac{3d}{2} = -7$
k $11 - \frac{3f}{2} = 2$
l $3 - \frac{4z}{3} = 5$

Example 11c

5 Solve each of the following equations. Check your answers.

a $\frac{x+1}{3} = 4$
b $\frac{x+4}{2} = 5$
c $\frac{4+y}{3} = -2$
d $\frac{6+b}{2} = -3$
e $\frac{1-a}{2} = 3$
f $\frac{5-x}{3} = 2$
g $\frac{3m-1}{5} = 4$
h $\frac{2x+2}{3} = 4$
i $\frac{7x-3}{3} = 9$
j $\frac{3b-6}{2} = 5$
k $\frac{4-2y}{6} = 3$
l $\frac{9-5t}{3} = -2$

PROBLEM–SOLVING	6, 7	6, 7	6(½), 7, 8

6 For each of the following, write an equation and solve it to find the unknown value. Use x as the unknown value.

a If 8 is added to a certain number, the result is 34.
b Seven less than a certain number is 21.
c I think of a number, double it and add 4. The result is 10.
d I think of a number, halve it and subtract 4. The result is 10.
e Four less than three times a number is 20.
f A number is multiplied by 7 and the product is divided by 3. The final result is 8.

7 Five Easter eggs are added to my initial collection of Easter eggs. I share them between myself and two friends and each person gets exactly four. Find how many eggs there were initially by solving an appropriate equation.

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8 My weekly pay is increased by \$200 per week. Half of my pay now goes to pay the rent and \$100 to buy groceries. If this leaves me with \$450, what was my original weekly pay?

REASONING	9	9	9, 10

9 Describe the error made in each of these incorrect solutions.

a $\begin{aligned} 2x - 1 &= 4 \\ x - 1 &= 2 \\ x &= 3 \end{aligned}$

b $\begin{aligned} \frac{5x+2}{3} &= 7 \\ \frac{5x}{3} &= 5 \\ 5x &= 15 \\ x &= 3 \end{aligned}$

c $\begin{aligned} 5 - x &= 12 \\ x &= 7 \end{aligned}$

d $\begin{aligned} \frac{x}{3} - 4 &= 2 \\ x - 4 &= 6 \\ x &= 10 \end{aligned}$

10 An equation like $2(x + 3) = 8$ can be solved without expanding the brackets. The first step is to divide both sides by 2.

a Use this approach to solve these equations.

i $3(x - 1) = 12$

ii $4(x + 2) = -4$

iii $7(5x + 1) = 14$

iv $5(1 - x) = -10$

v $-2(3x + 1) = 3$

vi $-5(1 - 4x) = 1$

b By considering your solutions to the equations in part **a**, when do you think this method is most appropriate?

ENRICHMENT: Changing the subject	–	–	11(½)

11 Make a the subject of each of the following equations; that is, a should be on the left-hand side on its own.

a $a - b = c$ **b** $2a + b = c$ **c** $c - ab = 2d$ **d** $b = \frac{a}{c} - d$

e $\frac{ab}{c} = -d$ **f** $\frac{2a}{b} = \frac{1}{c}$ **g** $\frac{2ab}{c} - 3 = -d$ **h** $b - \frac{ac}{2d} = 3$

i $\frac{a+b}{c} = d$ **j** $\frac{b-a}{c} = -d$ **k** $\frac{ad - 6c}{2b} = e$ **l** $\frac{d - 4ac}{e} = 3f$

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2E Linear equations with brackets and pronumerals on both sides

Learning intentions

- To know that equations involving brackets can be solved by first expanding the brackets
- To know that some equations can be simplified by dividing both sides by a common factor
- To be able to collect pronumerals on one side of the equation
- To be able to solve equations with brackets and pronumerals on both sides of the equation

Past, present and future learning

- This section addresses the Stage 5 Core Topic called *Equations A*
- These concepts will be revised very briefly in Chapter 1 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

More complex linear equations may have pronumerals on both sides of the equation and/or brackets. Examples are $3x = 5x - 1$ or $4(x + 2) = 5x$. Brackets can be removed by expanding and equations with pronumerals on both sides can be solved by collecting like terms using addition and subtraction.

More complex equations require several steps completed in the correct order to maintain the balance and undo all the operations that have been applied around the pronumeral.

Lesson starter: Steps in the wrong order

The steps to solve $3(2 - x) = -2(2x - 1)$ are listed here in the incorrect order.

$$\begin{aligned} 3(2 - x) &= -2(2x - 1) \\ x &= -4 \\ 6 + x &= 2 \\ 6 - 3x &= -4x + 2 \end{aligned}$$

- Arrange them in the correct order working from top to bottom.
- By considering all the steps in the correct order, explain what has happened in each step.

KEY IDEAS

- Equations with brackets can be solved by first expanding the brackets.
  For example: $3(x + 1) = 2$ becomes $3x + 3 = 2$.
- If an equation has pronumerals on both sides, collect to one side by adding or subtracting one of the terms.
  For example: $3x + 4 = 2x - 3$ becomes $x + 4 = -3$ by subtracting $2x$ from both sides.

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BUILDING UNDERSTANDING

1 State the result from expanding and simplifying these expressions.

a $3(x - 4) + x$ **b** $2(1 - x) + 2x$ **c** $3(x - 1) + 2(x - 3)$

2 State the next line of working for the given equations and instructions.

a $2(x + 3) = 5$ (expand the brackets)

b $5 + 2(x - 1) = 7$ (expand the brackets)

c $3x + 1 = x - 6$ (subtract x from both sides)

d $4x - 3 = 2x + 1$ (subtract $2x$ from both sides)

Example 12 Solving equations with brackets

Solve each of the following equations.

a $2(3x - 4) = 11$ **b** $2(x + 3) - 4x = 8$

SOLUTION	EXPLANATION
a $\begin{aligned} 2(3x - 4) &= 11 \\ 6x - 8 &= 11 \\ 6x &= 19 \\ x &= \frac{19}{6} \text{ or } 3\frac{1}{6} \end{aligned}$	Expand the brackets: $2(3x - 4) = 2 \times 3x + 2 \times (-4)$. Add 8 to both sides then divide both sides by 6, leaving your answer in fraction form.
b $\begin{aligned} 2(x + 3) - 4x &= 8 \\ 2x + 6 - 4x &= 8 \\ -2x + 6 &= 8 \\ -2x &= 2 \\ x &= -1 \end{aligned}$	Expand the brackets and collect any like terms, i.e. $2x - 4x = -2x$. Subtract 6 from both sides. Divide by -2.

Now you try

Solve each of the following equations.

a $3(2x - 1) = 4$ **b** $5(x + 2) - 2x = 19$

Example 13 Solving equations with pronumerals on both sides

Solve each of the following equations.

a $5x - 2 = 3x - 4$ **b** $3(2x + 4) = 8(x + 1)$

SOLUTION	EXPLANATION
a $\begin{aligned} 5x - 2 &= 3x - 4 \\ 2x - 2 &= -4 \\ 2x &= -2 \\ x &= -1 \end{aligned}$	Collect x terms on one side by subtracting $3x$ from both sides. Add 2 to both sides and then divide both sides by 2.

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b $$\begin{aligned} 3(2x+4) &= 8(x+1) \\ 6x+12 &= 8x+8 \\ 12 &= 2x+8 \\ 4 &= 2x \\ 2 &= x \\ \therefore x &= 2 \end{aligned}$$

Expand the brackets on each side.
Subtract $6x$ from both sides, alternatively subtract $8x$ to end up with $-2x + 12 = 8$. (Subtracting $6x$ keeps the x-coefficient positive.)
Solve the equation and make x the subject.

Now you try

Solve each of the following equations.

a $7x - 1 = 5x - 7$ **b** $4(2x - 3) = 5(x + 6)$

Exercise 2E

FLUENCY	1–3(½)	1–4(½)	1–4(⅓)

Example 12a
1 Solve each of the following equations by first expanding the brackets.

a $3(m + 4) = 31$ **b** $5(y - 7) = 12$ **c** $4(p + 5) = 35$
d $2(k - 5) = 9$ **e** $4(5 - b) = 18$ **f** $2(1 - m) = 13$
g $5(3 - x) = 19$ **h** $7(2a + 1) = 8$ **i** $4(3x - 2) = 30$
j $3(3n - 2) = 0$ **k** $5(3 - 2x) = 6$ **l** $6(1 - 2y) = -8$

Example 12b
2 Expand and simplify then solve each of the following equations.

a $2(x + 4) + x = 14$ **b** $2(x - 3) - 3x = 4$
c $6(x + 3) + 2x = 26$ **d** $3(x + 2) + 5x = 46$
e $3(2x - 3) + x = 12$ **f** $(3x + 1) + 3x = 19$
g $4(x - 1) + x - 1 = 0$ **h** $3(2x + 3) - 1 - 4x = 4$

Example 13a
3 Solve each of the following equations.

a $5b = 4b + 1$ **b** $8a = 7a - 4$ **c** $4t = 10 - t$
d $3m - 8 = 2m$ **e** $5x - 3 = 4x + 5$ **f** $9a + 3 = 8a + 6$
g $12x - 3 = 10x + 5$ **h** $3y + 6 = 2 - y$ **i** $5m - 4 = 1 - 6m$

Example 13b
4 Solve each of the following equations.

a $5(x - 2) = 2x - 13$ **b** $3(a + 1) = a + 10$ **c** $3(y + 4) = y - 6$
d $2(x + 5) = x - 4$ **e** $5b - 4 = 6(b + 2)$ **f** $2(4m - 5) = 4m + 2$
g $3(2a - 3) = 5(a + 2)$ **h** $4(x - 3) = 3(3x + 1)$ **i** $3(x - 2) = 5(x + 4)$
j $3(n - 2) = 4(n + 5)$ **k** $2(a + 5) = -2(2a + 3)$ **l** $-4(x + 2) = 3(2x + 1)$

PROBLEM–SOLVING	5, 6	5(½), 6	6, 7

5 Using x for the unknown number, write down an equation then solve it to find the number.

a The product of 2 and 3 more than a number is 7.
b The product of 3 and 4 less than a number is -4.
c When 2 less than 3 lots of a number is doubled the result is 5.
d When 5 more than 2 lots of a number is tripled the result is 10.
e 2 more than 3 lots of a number is equivalent to 8 lots of the number.
f 2 more than 3 times the number is equivalent to 1 less than 5 times the number.
g 1 less than a doubled number is equivalent to 5 more than 3 lots of the number.

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6 Since Tara started work her original hourly wage has been tripled, then decreased by \$6. It is now to be doubled so that she gets \$18 an hour. Write an equation and solve it to find Tara's original hourly wage.

7 At the start of lunch, Jimmy and Jake each brought out a new bag of x marbles to play with their friends. By the end of lunch, they were surprised to see they had an equal number of marbles, even though overall Jimmy had gained 5 marbles and Jake had ended up with double the result of 3 less than his original amount. How many marbles were originally in each bag?

REASONING	8, 9	8, 9	9, 10

8 Consider the equation $3(x - 2) = 9$.
- a Solve the equation by first dividing both sides by 3.
- b Solve the equation by first expanding the brackets.
- c Which of the above two methods is preferable and why?

9 Consider the equation $3(x - 2) = 7$.
- a Solve the equation by first dividing both sides by 3.
- b Solve the equation by first expanding the brackets.
- c Which of the above two methods is preferable and why?

10 Consider the equation $3x + 1 = 5x - 7$.
- a Solve the equation by first subtracting $3x$ from both sides.
- b Solve the equation by first subtracting $5x$ from both sides.
- c Which of the above two methods is preferable and why? Describe the differences.

ENRICHMENT: Literal solutions with factorisation	–	–	11(½)

11 Literal equations contain a variable (such as x) and other variables (or pronumerals) such as a, b and c. To solve such an equation for x, factorisation can be used as shown here.

$$\begin{aligned} ax &= bx + c && \\ ax - bx &= c && \text{Subtract } bx \text{ from both sides.} \\ x(a - b) &= c && \text{Factorise by taking out } x. \\ x &= \frac{c}{a - b} && \text{Divide both sides by } (a - b). \\ & && \text{(Note: } a \neq b.\text{)} \end{aligned}$$

Solve each of the following for x in terms of the other pronumerals by using factorisation.

- a $ax = bx + d$
- b $ax + 1 = bx + 3$
- c $5ax = bx + c$
- d $3ax + 1 = 4bx - 5$
- e $ax - bc = xb - ac$
- f $a(x - b) = x - b$
- g $ax - bx - c = d + bd$
- h $a(x + b) = b(x - c) - x$

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2F Solving word problems

Learning intentions

- To be able to define a variable to represent the unknown in a worded problem
- To be able to write an equation to represent a worded problem
- To be able to solve the problem by solving the equation

Past, present and future learning

- This section addresses the Stage 5 Core Topic called *Equations A*
- These concepts will be revised very briefly in Chapter 1 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

Many types of problems can be solved by writing and solving linear equations. Often problems are expressed only in words. Reading and understanding the problem, defining a variable and writing an equation become important steps in solving the problem.

Lawyers can use linear equations when distributing inherited money. For example, if the first recipient receives $\$x$, the second person might receive \$5000 less and the third person could receive twice the second person's amount.

Lesson starter: Too much television?

Three friends, Rick, Kate and Sue, compare how much television they watch in a week at home. Kate watches three times as much television as Rick and Sue watches 4 hours less television than Kate. In total they watch 45 hours of television. Find the number of hours of television watched by Rick.

- Let x hours be the number of hours of television watched by Rick.
- Write expressions for the number of hours of television watched by Kate and by Sue.
- Write an equation to represent the information above.
- Solve the equation.
- Answer the question in the original problem.

KEY IDEAS

- To solve a **word problem** using algebra:
  - Read the problem and find out what the question is asking for.
  - Define a variable and write a statement such as: 'Let x be the number of …'. The variable is often what you have been asked to find.
  - Write an equation using your defined variable to show the relationship between the facts in the question.
  - Solve the equation.
  - Answer the question in words.
  - Check that the solution makes sense.

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BUILDING UNDERSTANDING

1. Give an equation for these situations.
   a. One less than twice a number is 11.
   b. Ben earns $x per week. $100 plus two weeks' earnings gives $2200.
   c. Maggie and Doris score 60 points between them and Doris scores 12 more points than Maggie.
   d. An engineer charges a total of $480 including a $100 up-front fee plus $80 per hour for n hours work.

Example 14 Turning a word problem into an equation

Five less than a certain number is 9 less than three times the number. Write an equation and solve it to find the number.

SOLUTION	EXPLANATION
Let x be the number.	Define the unknown as a pronumeral.
$\begin{aligned} x-5 &= 3x-9 \\ -5 &= 2x-9 \\ 4 &= 2x \\ x &= 2 \end{aligned}$	5 less than x is $x-5$ and this equals 9 less than three times x, i.e. $3x-9$. Subtract x from both sides and solve the equation.
The number is 2.	Write the answer in words.

Now you try

Two more than a certain number is 3 less than double the number. Write an equation and solve it to find the number.

Example 15 Solving word problems

David and Mitch made 254 runs between them in a cricket match. If Mitch made 68 more runs than David, how many runs did each of them make?

SOLUTION	EXPLANATION
Let the number of runs for David be r.	Define the unknown value as a pronumeral.
Number of runs Mitch made is $r+68$. $\begin{aligned} r+(r+68) &= 254 \\ 2r+68 &= 254 \\ 2r &= 186 \\ r &= 93 \end{aligned}$	Write all other unknown values in terms of r. Write an equation: number of runs for David + number of runs for Mitch = 254. Subtract 68 from both sides and then divide both sides by 2.
David made 93 runs and Mitch made $93+68=161$ runs.	Express the answer in words.

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Now you try

Flora and Jo raised \$426 between them in a fundraising effort. If Jo raised \$52 more than Flora, how much did each of them raise?

Exercise 2F

FLUENCY	1–6	1–7	2, 4, 5, 7, 8

Example 14

1 For each of the following examples, make x the unknown number and write an equation.
 a A number when doubled results in a number that is 5 more than the original number.
 b Eight less than a certain number is 2 more than three times the number.
 c Three less than a certain number is 9 less than four times the number.
 d Seven is added to a number and the result is then multiplied by 3. The result is 9.

Example 15

2 Leonie and Emma scored 28 goals between them in a netball match. Leonie scored 8 more goals than Emma.
 a Define a variable for the number of goals scored by Emma.
 b Write the number of goals scored by Leonie in terms of the variable in part **a**.
 c Write an equation in terms of your variable to represent the problem.
 d Solve the equation in part **c** to find the unknown value.
 e How many goals did each of them score?

3 A rectangle is four times as long as it is wide and its perimeter is 560 cm.
 a Define a variable for the unknown breadth.
 b Write an expression for the length in terms of your variable in part **a**.
 c Write an equation involving your variable to represent the problem. Draw and label a rectangle to help you.
 d Solve the equation in part **c**.
 e What is the length and breadth of the rectangle?

4 Toni rented a car for a total cost of \$290. If the rental company charged \$40 per day, plus a hiring fee of \$50, for how many days did Toni rent the car?

5 Anthony walked a certain distance, and then ran twice as far as he walked. He then caught a bus for the last 2 km. If he travelled a total of 32 km, find how far Anthony walked and ran.

6 A prize of \$1000 is divided between Adele and Benita so that Adele receives \$280 more than Benita. How much did they each receive?

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7 Kate is three times as old as her son. If Kate is 30 years older than her son, what are their ages?

8 Andrew, Brenda and Cammi all work part-time at a supermarket. Cammi earns $20 more than Andrew and Brenda earns $30 less than twice Andrew's wage. If their total combined wage is $400, find how much each of these workers earns.

PROBLEM–SOLVING	9–12	9, 11–14	11–15

9 A train station is between the towns Antville and Bugville. The station is four times as far from Bugville as it is from Antville. If the distance from Antville to Bugville is 95 km, how far is it from Antville to the station? (*Hint*: Draw a diagram to help you picture the problem.)

10 If I multiply my age in six years' time by three, the resulting age is my mother's age now. If my mother is currently 48 years old, how old am I?

11 The second leg in a yacht race was half the length of the first leg, the third leg was two-thirds of the length of the second leg, and the last leg was twice the length of the second leg. If the total distance was 153 km, find the length of each leg.

12 Twelve years ago Eric's father was seven times as old as Eric was. If Eric's father is now 54 years old, how old is Eric now?

13 Macy bought a total of 12 fiction and non-fiction books. The fiction books cost $12 each and the non-fiction books cost $25 each. If she paid $248 altogether, how many of each kind of book did she purchase? Define the number of non-fiction books bought in terms of the number of fiction books bought.

14 The Ace Bicycle Shop charges a flat fee of $4, plus $1 per hour, for the hire of a bicycle. The Best Bicycle Shop charges a flat fee of $8, plus 50 cents per hour. Connie and her friends hire three bicycles from Ace, and David and his brother hire two bicycles from Best. After how many hours will their hire costs be the same?

15 Car A left Melbourne for Adelaide at 11:00 a.m. and travelled at an average speed of 70 km/h. Car B left Melbourne for Adelaide at 1:00 p.m. on the same day and travelled at an average speed of 90 km/h. At what time will Car B catch Car A?

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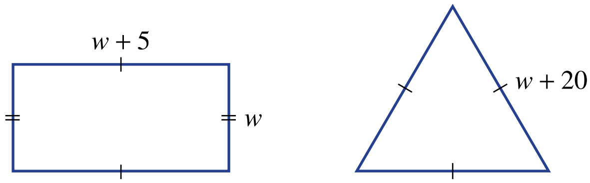
REASONING	16	16, 17	17, 18

16 Two paddocks in the shapes shown below are to be fenced with wire. If the same total amount of wire is used for each paddock, what are the dimensions of each paddock in metres?



17 Consecutive integers can be represented algebraically as x, $x + 1$, $x + 2$ etc.
- a Find three consecutive numbers that add to 84.
- b i Write three consecutive even numbers starting with x.
  - ii Find three consecutive even numbers that add to 18.
- c i Write three consecutive odd numbers starting with x.
  - ii Find three consecutive odd numbers that add to 51.
- d i Write three consecutive multiples of 3 starting with x.
  - ii Find three consecutive multiples of 3 that add to 81.

18 Tedco produces a teddy bear which sells for \$24. Each teddy bear costs the company \$8 to manufacture and there is an initial start-up cost of \$7200.

- a Write a rule for the total cost, \$$T$, of producing x teddy bears.
- b If the cost of a particular production run was \$9600, how many teddy bears were manufactured in that run?
- c Write a rule for the revenue, \$$R$, received by the company when x teddy bears are sold.
- d How many teddy bears were sold if the revenue was \$8400?
- e If they want to make an annual profit of \$54 000, how many teddy bears need to be sold?

ENRICHMENT: Worded challenges	–	–	19–21

19 An art curator was investigating the price trends of two art works that had the same initial value. The first painting, 'Green poles', doubled in value in the first year then lost \$8000 in the second year. In the third year its value was three-quarters of that of the previous year.
The second painting, 'Orchids', added \$10 000 to its value in the first year. In the second year its value was only a third of that of the previous year. In the third year its value improved to double that of the previous year.
If the value of the two paintings was the same in the third year, write an equation and solve it to find the initial value of each painting.

20 Julia drove to her holiday destination over a period of five days. On the first day she travels a certain distance, on the second day she travels half that distance, on the third day a third of that distance, on the fourth day one-quarter of the distance and on the fifth day one-fifth of the distance. If her destination was 1000 km away, write an equation and solve it to find how far she travelled on the first day, to the nearest kilometre.

21 Anna King is x years old. Her brother Henry is two-thirds of her age and her sister Chloe is three times Henry's age. The twins who live next door are 5 years older than Anna. If the sum of the ages of the King children is equal to the sum of the ages of the twins, find the ages of all the children.

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Progress quiz

2A **1** Write an algebraic expression for each of the following.
- **a** The number of tickets required for 5 boys, x girls and y adults
- **b** 15 less than the product of 4 and y
- **c** The sum of the squares of k and p
- **d** The square of the difference between m and n
- **e** The square root of 16 more than x

2A **2** Evaluate these expressions when $x = 4$ and $y = -2$.
- **a** $x^2 - y^2$
- **b** $5y - 3(x - y)$
- **c** $\dfrac{x + y}{x}$
- **d** $\dfrac{3xy}{2}$

2B **3** Simplify the following.
- **a** $2a \times 7b$
- **b** $5pq \times (-8p)$
- **c** $\dfrac{12k}{3km}$
- **d** $30xy^2 \div (-5xy)$

2B **4** Simplify the following by collecting like terms.
- **a** $6x + 7 - 4x$
- **b** $7 - 3mk + 4km + 4$
- **c** $8xy - 3x - 3yx + 2x$
- **d** $a^2b + ba + b^2a - ab + b$

2C **5** Expand the following.
- **a** $4(a + 7)$
- **b** $-3x(2x - 5)$

2C **6** Expand the following and collect like terms.
- **a** $10 + 3(a - 2)$
- **b** $4(2x + y) - 3(x + 2y)$

2D **7** Solve the following equations.
- **a** $3x - 7 = 5$
- **b** $\dfrac{a}{3} - 5 = 2$
- **c** $9 - 2p = 16$
- **d** $\dfrac{3t}{5} - 4 = 8$
- **e** $\dfrac{3 - x}{4} = -2$
- **f** $\dfrac{3k - 6}{3} = 5$

2E **8** Solve the following equations.
- **a** $3(x - 4) = 7$
- **b** $4(3a + 1) - 3a = 31$
- **c** $7m - 4 = 3m - 12$
- **d** $3(2y + 1) = 8(y - 2)$

2F **9** For each of the following examples, make x the unknown number and write an equation.
- **a** Five is added to a number and this is then multiplied by 2. The result is 14.
- **b** Nine less than a certain number is 6 more than four times the number.

2F **10** Andy's soccer team scores 14 more goals for the season than Jed's team. The two teams score a total of 30 goals for the season. Solve an appropriate equation to find the number of goals scored by Andy's team.

2F **11** If I multiply my age in five years' time by four, the resulting age is my grandfather's age now. If my grandfather is currently 88 years old, set up and solve an equation to determine how old I am now.

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2G Linear inequalities

Learning intentions
- To understand that an inequality represents an infinite set of values
- To be able to illustrate an inequality on a number line using known conventions
- To understand when an inequality symbol needs to be reversed
- To be able to solve a linear inequality

Past, present and future learning
- This section addresses the Stage 5 Path Topic called *Equations B*
- These concepts will be revised very briefly in Chapter 1 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

An inequality (or inequation) is a mathematical statement which uses a $<, \leqslant, >$ or $\geqslant$ symbol. Here are some examples of inequalities:

$$2 < 6, \ 5 \geqslant -1, \ 3x + 1 \leqslant 7 \text{ and } 2x + \frac{1}{3} > \frac{x}{4}$$

Inequalities can represent an infinite set of numbers. For example, the inequality $2x < 6$ means that $x < 3$ and this is the infinite set of all real numbers less than 3.

The altitude, h, that passenger planes can successfully and economically fly at can be represented by an inequality: $0 < h \leqslant 40\,000$ feet or $0 < h \leqslant 12$ km.

Lesson starter: Infinite solutions

Greg, Kevin and Greta think that they all have a correct solution to this inequality:

$$4x - 1 \geqslant x + 6$$

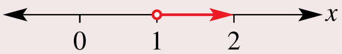
Greg says $x = 4$ is a solution.
Kevin says $x = 10$ is a solution.
Greta says $x = 100$ is a solution.

- Use substitution to show that they are all correct.
- Can you find the smallest whole number that is a solution to the inequality?
- Can you find the smallest number (including fractions) that satisfies the inequality? What method leads you to your answer?

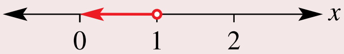
KEY IDEAS

- **Inequalities** can be illustrated using a number line because a line represents an infinite number of points.
  - Use an **open circle** when showing $>$ (greater than) or $<$ (less than).

For example: $x > 1$



$x < 1$



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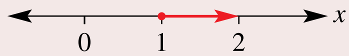
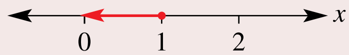
- Use a **closed circle** when showing $\geqslant$ (greater than or equal to) or $\leqslant$ (less than or equal to).

  For example: $x \geqslant 1$



  $x \leqslant 1$



- A **set** of numbers may have both an upper and lower bound.

  For example: $-2 < x \leqslant 3$

- Linear inequalities can be solved in a similar way to linear equations.
  - All the numbers that satisfy an inequality are called a **solution set**.
  - If we multiply or divide both sides of an inequality by a negative number, the inequality symbol is reversed.
    For example: $5 < 8$ but $-5 > -8$, so if $-x > 1$ then $x < -1$.
  - If we swap the sides of an inequality, then the inequality symbol is reversed.
    For example: $3 < 7$ but $7 > 3$ so if $2 > x$ then $x < 2$.

BUILDING UNDERSTANDING

1 State the missing symbol $<$ or $>$ to make each statement true.

a $3 \square 2$ **b** $-1 \square 4$ **c** $-7 \square -3$ **d** $5 \square -50$

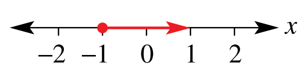
2 Match the inequalities **a**, **b** and **c** with the number lines **A**, **B** and **C**.

a $x < 2$

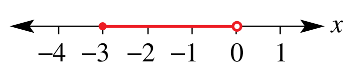
b $x \geqslant -1$

c $-3 \leqslant x < 0$

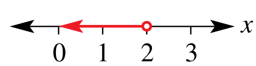
A



B



C



Example 16 Representing inequalities on a number line

Show each of the following examples on a number line.

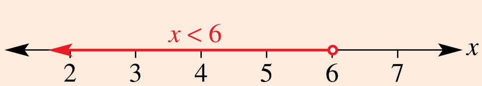
a x is less than 6, ($x < 6$).

b x is greater than or equal to 2, ($x \geqslant 2$).

c x is greater than -2 but less than or equal to 3, ($-2 < x \leqslant 3$).

SOLUTION

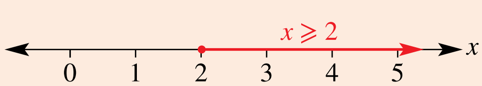
a



EXPLANATION

An open circle is used to indicate that 6 is not included.

b



A closed circle is used to indicate that 2 is included.

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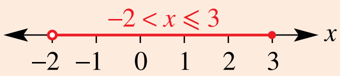
c 

An open circle is used to indicate that -2 is not included and a closed circle is used to indicate that 3 is included.

Now you try

Show each of the following examples on a number line.

a x is less than 4, $(x < 4)$.

b x is greater than or equal to -3, $(x \geqslant -3)$.

c x is greater than -3 but less than or equal to 1, $(-3 < x \leqslant 1)$.

Example 17 Solving linear inequalities

Find the solution set for each of the following inequalities.

a $x - 3 < 7$

b $5 - 2x > 3$

c $\frac{d}{4} - 3 \geqslant -11$

d $2a + 7 \leqslant 6a + 3$

SOLUTION	EXPLANATION
a $x - 3 < 7$ $x < 10$	Add 3 to both sides.
b $5 - 2x > 3$ $-2x > -2$ $x < 1$	Subtract 5 from both sides. Divide both sides by -2, and reverse the inequality symbol.
c $\frac{d}{4} - 3 \geqslant -11$ $\frac{d}{4} \geqslant -8$ $d \geqslant -32$	Add 3 to both sides. Multiply both sides by 4; the inequality symbol does not change.
d $2a + 7 \leqslant 6a + 3$ $7 \leqslant 4a + 3$ $4 \leqslant 4a$ $1 \leqslant a$ $a \geqslant 1$	Gather pronumerals on one side by subtracting $2a$ from both sides. Subtract 3 from both sides and then divide both sides by 4. Place the variable a on the left and reverse the inequality symbol.

Now you try

Find the solution set for each of the following inequalities.

a $x - 2 < 9$

b $4 - 3x > 7$

c $\frac{a}{3} + 1 \geqslant -2$

d $3a - 1 \leqslant 5a + 6$

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Exercise 2G

FLUENCY	1, 2–4(½)	1–6(½)	1–6(⅓)

Example 16

1 Show each of the following inequalities on a number line.

a $x > 2$ **b** $x \leqslant 4$ **c** $x \geqslant -1$
d $-1 < x < 1$ **e** $0 \leqslant x \leqslant 3$ **f** $-3 < x \leqslant 4$

Example 17a

2 Find the solution set for each of the following inequalities.

a $x + 5 < 8$ **b** $b - 2 > 3$ **c** $y - 8 > -2$
d $5x \geqslant 15$ **e** $4t > -20$ **f** $y + 10 \geqslant 0$
g $3m - 7 < 11$ **h** $4a + 6 \geqslant 12$ **i** $7x - 5 < 2$

Example 17b

3 Find the solution set for each of the following inequalities.

a $4 - 3x > -8$ **b** $2 - 4n \geqslant 6$ **c** $4 - 5x \leqslant 1$
d $7 - a \leqslant 3$ **e** $5 - x \leqslant 11$ **f** $7 - x \leqslant -3$
g $-2x - 3 > 9$ **h** $-4t + 2 \geqslant 10$ **i** $-6m - 14 < 15$

Example 17c

4 Find the solution set for each of the following inequalities.

a $\frac{x}{2} - 5 \leqslant 3$ **b** $3 - \frac{x}{9} \geqslant 4$ **c** $\frac{2x}{5} \leqslant 8$
d $\frac{2x + 6}{7} < 4$ **e** $\frac{3x - 4}{2} > -6$ **f** $\frac{1 - 7x}{5} \leqslant 3$

5 Solve these inequalities involving brackets.

a $4(x + 2) < 12$ **b** $-3(a + 5) > 9$ **c** $5(3 - x) \geqslant 25$
d $2(3 - x) > 1$ **e** $5(y + 2) < -6$ **f** $-7(1 - x) < -11$

Example 17d

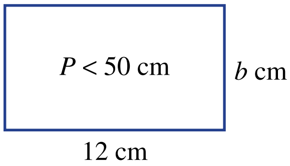
6 Find the solution set for each of the following inequalities.

a $2x + 9 \leqslant 6x - 1$ **b** $6t + 2 > t - 1$ **c** $7y + 4 \leqslant 7 - y$
d $3a - 2 < 4 - 2a$ **e** $1 - 3m \geqslant 7 - 4m$ **f** $7 - 5b > -4 - 3b$

PROBLEM–SOLVING	7–9	8–10	9–11

7 Wendy is x years old and Jay is 6 years younger. The sum of their ages is less than 30. Write an inequality involving x and solve it. What can you say about Wendy's age?

8 The perimeter of a particular rectangle needs to be less than 50 cm. If the length of the rectangle is 12 cm and the breadth is b cm, write an inequality involving b and solve it. What breadth does the rectangle need to be?



9 How many integers satisfy both of the given inequalities?

a $2x + 1 \leqslant 5$ and $5 - 2x \leqslant 5$ **b** $7 - 3x > 10$ and $5x + 13 > -5$
c $\frac{x + 1}{3} \geqslant -2$ and $2 - \frac{x}{3} > 3$ **d** $\frac{5x + 1}{6} < 2$ and $\frac{x}{3} < 2x - 7$

10 The breadth of a rectangular area is 10 m and its height is $(2x - 4)$ m. If the area is less than 80 m^2, what are the possible integer values for x?

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11 Two car rental companies have the following payment plans:
Carz: \$90 per week and 15c per kilometre
Renta: \$110 per week and 10c per kilometre
What is the maximum whole number of kilometres that can be travelled in one week with Carz if it is to cost less than it would with Renta?

REASONING	12	12, 13	13, 14

12 a Consider the inequality $2 > x$.
   i List five values of x between -1 and 2 which make the inequality true.
   ii What must be true about all the values of x if the inequality is true?
b Consider the inequality $-x < 5$.
   i List five values of x which make the inequality true.
   ii What must be true about all the values of x if the inequality is true?
c Complete these statements.
   i If $a > x$ then x __ __.
   ii If $-x < a$ then x __ __.

13 Consider the equation $9 - 2x > 3$.
a Solve the equation by first adding $2x$ to both sides, then solve for x.
b Solve the equation by first subtracting 9 from both sides.
c What did you have to remember to do in part **b** to ensure that the answer is the same as in part **a**?

14 Combine all your knowledge from this chapter so far to solve these inequalities.
a $\frac{2(x+1)}{3} > x + 5$
b $2x + 3 \geqslant \frac{x - 6}{3}$
c $\frac{2 - 3x}{2} < 2x - 1$
d $\frac{4(2x - 1)}{3} \leqslant x + 3$
e $1 - x > \frac{7(2 - 3x)}{4}$
f $2(3 - 2x) \leqslant 4x$

ENRICHMENT: Literal inequalities	–	–	15(½)

15 Given a, b, c and d are positive numbers and that $1 < a < b$, solve each of the following for x.
a $ax - b > -c$
b $b - x \leqslant a$
c $\frac{x}{a} - b \leqslant c$
d $\frac{bx}{c} \leqslant a$
e $\frac{ax + b}{c} < d$
f $\frac{b - 2x}{c} \leqslant d$
g $a(x + b) < c$
h $\frac{ax - b}{c} > -d$
i $a(b - x) > c$
j $ax + b \leqslant x - c$
k $ax + b > bx - 1$
l $b - ax \leqslant c - bx$

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2H Using formulas

Learning intentions

- To know which variable is the subject of a formula
- To be able to work with a formula by substituting known values
- To be able to find the unknown value in a formula by solving an equation
- To be able to change the subject of a formula

Past, present and future learning

- This section addresses the Stage 5 Path Topic called *Equations A*
- These concepts will be revised very briefly in Chapter 1 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

A formula (or rule) is an equation that relates two or more variables. You can find the value of one of the variables if you are given the value of all others. Some common formulas contain squares, square roots, cubes and cube roots. The following are some examples of formulas.

- $A = \pi r^2$ is the formula for finding the area, A, of a circle given its radius, r.
- $F = \frac{9}{5}C + 32$ is the formula for converting degrees Celsius, C, to degrees Fahrenheit, F.
- $d = vt$ is the formula for finding the distance, d, given the velocity, v, and time, t.

Doctors and nurses calculate the amount of medicine, C, for a child using Young's formula: $C = \frac{a}{a + 12} \times D$, where a is the child's age in years, and D is the quantity of the adult dose.

A, F and d are said to be the subjects of the formulas given above.

Lesson starter: Common formulas

As a class group, try to list at least 10 formulas that you know.

- Write down the formulas and describe what each variable represents.
- Which variable is the subject of each formula?

KEY IDEAS

- The **subject** of a **formula** is a variable that usually sits on its own on the left-hand side. For example, the C in $C = 2\pi r$ is the subject of the formula.
- A variable in a formula can be evaluated by substituting numbers for all other variables.

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- A formula can be **transposed** (rearranged) to make another variable the subject.

  $C = 2\pi r$ can be transposed to give $r = \dfrac{C}{2\pi}$.
  - To transpose a formula use similar steps as you would for solving an equation, since variables represent numbers.
- Note that $\sqrt{a^2} = a$ if $a \geqslant 0$ and $\sqrt{a^2 + b^2} \neq a + b$ provided a or b is not zero.

BUILDING UNDERSTANDING

1 State the letter that is the subject of these formulas.

a $A = \frac{1}{2}bh$ **b** $D = b^2 - 4ac$ **c** $M = \dfrac{a+b}{2}$ **d** $A = \pi r^2$

2 Describe the first step when attempting to transpose these formulas to make a the subject.

a $2a + 3 = b$ **b** $\dfrac{a-c}{4} = d$ **c** $\sqrt{4-a} = b$ **d** $b = \sqrt{a} + c$

3 State the steps to solve for x in these equations.

a **i** $3x + 5 = 12$ **ii** $ax + b = c$

b **i** $3x^2 = 75, x > 0$ **ii** $ax^2 = b, x > 0$

Example 18 Substituting values into formulas

Substitute the given values into the formula to evaluate the subject.

a $S = \dfrac{a}{1-r}$, when $a = 3$ and $r = 0.4$

b $E = \frac{1}{2}mv^2$, when $m = 4$ and $v = 5$

SOLUTION	EXPLANATION
a $S = \dfrac{a}{1-r}$	
$S = \dfrac{3}{1-0.4}$ $= \dfrac{3}{0.6}$ $= 5$	Substitute $a = 3$ and $r = 0.4$ and evaluate. $\dfrac{3}{0.6} = \dfrac{30}{6}$ if calculating by hand i.e. multiply numerator and denominator by 10.
b $E = \frac{1}{2}mv^2$	
$E = \frac{1}{2} \times 4 \times 5^2$ $= \frac{1}{2} \times 4 \times 25$ $= 50$	Substitute $m = 4$ and $v = 5$ and evaluate. (Note: Square the value of v before multiplying by the value of m.)

Now you try

Substitute the given values into the formula to evaluate the subject.

a $a = \dfrac{t}{1-b}$, when $t = 5$ and $b = 0.5$

b $s = ut + \frac{1}{2}at^2$, when $u = 25$, $a = 10$ and $t = 4$

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Example 19 Finding the unknown value in a formula

The area of a trapezium is given by $A = \frac{h}{2}(a+b)$. Substitute $A = 12$, $a = 5$ and $h = 4$ and then find the value of b.

SOLUTION

$$\begin{aligned} A &= \frac{h}{2}(a+b) \\ 12 &= \frac{4}{2} \times (5+b) \\ 12 &= 2(5+b) \\ 6 &= 5+b \\ b &= 1 \end{aligned}$$

EXPLANATION

Write the formula and substitute the given values of A, a and h. Then solve for b.

$\frac{1}{2} \times 4 = 2$ and divide both sides by 2 since 2 is a factor of 12. Alternatively, you can expand the brackets.

Now you try

The area of a trapezium is given by $A = \frac{h}{2}(a+b)$. Substitute $A = 24, b = 3$ and $h = 2$ and then find the value of a.

Example 20 Transposing formulas

Transpose each of the following to make b the subject.

a $c = a(x+b)$

b $c = \sqrt{a^2+b^2}\quad (b > 0)$

SOLUTION

a
$$\begin{aligned} c &= a(x+b) \\ \frac{c}{a} &= x+b \\ \frac{c}{a} - x &= b \\ b &= \frac{c}{a} - x \quad \left(\text{or } b = \frac{c-ax}{a}\right) \end{aligned}$$

EXPLANATION

Divide both sides by a.

Subtract x from both sides.

Make b the subject on the left side. An alternative answer has a common denominator, which will also be the answer format if you expand the brackets first.

b
$$\begin{aligned} c &= \sqrt{a^2+b^2} \quad (b > 0) \\ c^2 &= a^2+b^2 \\ c^2-a^2 &= b^2 \\ b^2 &= c^2-a^2 \\ b &= \sqrt{c^2-a^2} \end{aligned}$$

Square both sides to remove the square root.

Subtract a^2 from both sides.

Make b^2 the subject.

Take the square root of both sides, $b = \sqrt{c^2-a^2}$ as b is positive.

Now you try

Transpose each of the following to make a the subject.

a $b = 2(a-c)$

b $a^2 - b = c\quad (a > 0)$

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Exercise 2H

FLUENCY	1–3(½)	1–3(½)	1–3(⅓)

Example 18

1 Substitute the given values into the formula to evaluate the subject. Round to two decimal places where appropriate.

a $A = bh$, when $b = 3$ and $h = 7$

b $F = ma$, when $m = 4$ and $a = 6$

c $m = \frac{a + b}{4}$, when $a = 14$ and $b = -6$

d $t = \frac{d}{v}$, when $d = 18$ and $v = 3$

e $A = \pi r^2$, when $r = 12$

f $I = \frac{MR^2}{2}$, when $M = 12.2$ and $R = 6.4$

g $c = \sqrt{a^2 + b^2}$, when $a = 12$ and $b = 22$

h $Q = \sqrt{2gh}$, when $g = 9.8$ and $h = 11.4$

Example 19

2 Substitute the given values into the formula then solve the equations to determine the value of the unknown pronumeral. Round to two decimal places where appropriate.

a $m = \frac{F}{a}$, when $m = 12$ and $a = 3$

b $A = lw$, when $A = 30$ and $l = 6$

c $A = \frac{h}{2}(a + b)$, when $A = 64$, $b = 12$ and $h = 4$

d $C = 2\pi r$, when $C = 26$

e $S = 2\pi r^2$, when $S = 72$

f $v^2 = u^2 + 2as$, when $v = 22$, $u = 6$ and $a = 12$

g $m = \sqrt{\frac{x}{y}}$, when $m = 8$ and $x = 4$

Example 20

3 Transpose each of the following formulas to make the pronumeral shown in brackets the subject.

a $v = u + at$ $[t]$

b $A = \frac{xy}{2}$ $[x]$

c $p = m(x + n)$ $[n]$

d $d = \frac{a + bx}{c}$ $[x]$

e $A = 2\pi rh$ $[r]$

f $I = \frac{Prt}{100}$ $[r]$

g $V = \pi r^2 h \quad (r > 0)$ $[r]$

h $P = \frac{v^2}{R} \quad (v > 0)$ $[v]$

i $S = 2\pi rh + 2\pi r^2$ $[h]$

j $A = (p + q)^2$ $[p]$

k $\sqrt{A} + B = 4C$ $[A]$

l $T = 2\pi\sqrt{\frac{l}{g}}$ $[g]$

PROBLEM-SOLVING	4, 5	4–6	5–7

4 The formula $s = \frac{d}{t}$ gives the speed s km/h of a car which has travelled a distance of d km in t hours.

a Find the speed of a car which has travelled 400 km in 4.5 hours. Round to two decimal places.

b i Transpose the formula $s = \frac{d}{t}$ to make d the subject.

ii Find the distance covered if a car travels at 75 km/h for 3.8 hours.

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5 The formula $F = \frac{9}{5}C + 32$ converts degrees Celsius, C, to degrees Fahrenheit, F.

a Find what each of the following temperatures is in degrees Fahrenheit.

i 100°C ii 38°C

b Transpose the formula to make C the subject.

c Calculate each of the following temperatures in degrees Celsius. Round to one decimal place where necessary.

i 14°F ii 98°F

6 The velocity, v m/s, of an object is described by the rule $v = u + at$, where u is the initial velocity in m/s, a is the acceleration in m/s^2 and t is the time in seconds.

a Find the velocity after 3 seconds if the initial velocity is 5 m/s and the acceleration is 10 m/s^2.

b Find the time taken for a body to reach a velocity of 20 m/s if its acceleration is 4 m/s^2 and its initial velocity is 12 m/s.

7 The volume of water (V litres) in a tank is given by $V = 4000 - 0.1t$ where t is the time in seconds after a tap is turned on.

a Over time, does the water volume increase or decrease according to the formula?

b Find the volume after 2 minutes.

c Find the time it takes for the volume to reach 1500 litres. Round to the nearest minute.

d How long, to the nearest minute, does it take to completely empty the tank?

REASONING	8	8, 9	8(½), 9

8 Write a formula for the following situations. Make the first listed variable the subject.

a $\$D$ given c cents

b d cm given e metres

c The discounted price $\$D$ that is 30% off the marked price $\$M$

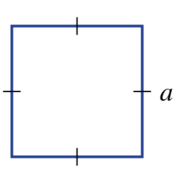
d The value of an investment $\$V$ that is 15% more than the initial amount $\$P$

e The cost $\$C$ of hiring a car at \$50 upfront plus \$18 per hour for t hours

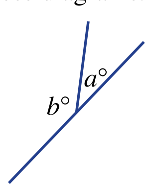
f The distance d km remaining in a 42 km marathon after t hours if the running speed is 14 km/h

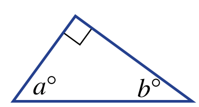
g The cost $\$C$ of a bottle of soft drink if b bottles cost $\$c$

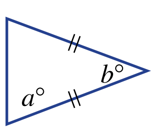
9 Write a formula for the value of a in these diagrams.

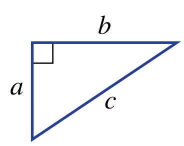
a


Perimeter = P

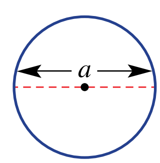
b


c


d


e


(Note: $a^2 + b^2 = c^2$.)

f


Area = A

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ENRICHMENT: Basketball formulas	–	–	10

10 The formula $T = 3x + 2y + f$ can be used to calculate the total number of points made in a basketball game where:

x = number of three-point goals	y = number of two-point goals
f = number of free throws made	T = total number of points

a Find the total number of points for a game in which 12 three-point goals, 15 two-point goals and 7 free throws were made.

b Find the number of three-point goals made if the total number of points was 36 and 5 two-point goals and 5 free throws were made.

c The formula $V = \left(p + \frac{3r}{2} + 2a + \frac{3s}{2} + 2b\right) - \frac{1.5t + 2f + m - o}{g}$ can be used to calculate the value, V, of a basketball player where:

p = points earned	r = number of rebounds
a = number of assists	s = number of steals
b = number of blocks	t = number of turnovers
f = number of personal fouls	m = number of missed shots
o = number of offensive rebounds	g = number of games played

Calculate the value of a player who earns 350 points and makes 2 rebounds, 14 assists, 25 steals, 32 blocks, 28 turnovers, 14 personal fouls, 24 missed shots, 32 offensive rebounds and plays 10 games.

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2I Linear simultaneous equations: substitution

Learning intentions

- To understand that a solution to a pair of simultaneous equations can be found algebraically
- To use the method of substitution to solve problems involving simultaneous equations

Past, present and future learning

- This section shows one way to solve simultaneous equations to find a point of intersection
- It addresses the Stage 5 Path Topic called *Equations C*
- These concepts will be extended in Chapter 1 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

A linear equation with one unknown has one unique solution. For example, $x = 2$ is the only value of x that makes the equation $2x + 3 = 7$ true.

The linear equation $2x + 3y = 12$ has two unknowns and it has an infinite number of solutions. Each solution is a pair of x- and y-values that makes the equation true, for example $x = 0$ and $y = 4$ or $x = 3$ and $y = 2$ or $x = 4\frac{1}{2}$ and $y = 1$.

However, if we are told that $2x + 3y = 12$ and also that $y = 2x - 1$, we can find a single solution that satisfies both equations. Equations like this are called simultaneous linear equations, because we can find a pair of x- and y-values that satisfy both equations at the same time (simultaneously).

Civil engineers solve simultaneous equations to find the required strength of trusses that support bridges and towers. Linear equations are formed by equating the forces acting in opposite directions, horizontally and vertically, on each joint.

Lesson starter: Multiple solutions

There is more than one pair of numbers x and y that satisfy the equation $x - 2y = 5$.

- Write down at least 5 pairs (x, y) that make the equation true.

A second equation is $y = x - 8$.

- Do any of your pairs that make the first equation true, also make the second equation true? If not, can you find the special pair of numbers that satisfies both equations simultaneously?

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KEY IDEAS

- An algebraic method called **substitution** can be used to solve **simultaneous equations**. It is used when at least one of the equations has a single variable as the subject. For example, y is the subject in the equation $y = x + 1$.
- To solve simultaneous equations using substitution:
  1. Substitute one equation into the other, using brackets.
  2. Solve for the remaining variable.
  3. Substitute to find the value of the second variable.

$$\begin{aligned} 2x + 3y &= 8 \text{ and } y = x + 1 \\ 2x + 3(x + 1) &= 8 \\ 2x + 3x + 3 &= 8 \\ 5x + 3 &= 8 \\ 5x &= 5 \\ x &= 1 \\ \therefore y = (1) + 1 &= 2 \end{aligned}$$

BUILDING UNDERSTANDING

1 Find the value of x or y by substituting the known value.

a $y = 2x - 3 \quad (x = 4)$ b $x = 5 - 2y \quad (y = 4)$ c $2x + 4y = 8 \quad (y = -3)$

2 Choose the correct option.

a When substituting $y = 2x - 1$ into $3x + 2y = 5$, the second equation becomes:
A $3x + 2(2x - 1) = 5$ B $3(2x - 1) + 2y = 5$ C $3x + 2y = 2x - 1$

b When substituting $x = 1 - 3y$ into $5x - y = 6$, the second equation becomes:
A $1 - 3x - y = 6$ B $5(1 - 3y) = 6$ C $5(1 - 3y) - y = 6$

3 Check whether $x = -2$ and $y = 2$ is a solution to each pair of simultaneous equations.

a $x - 2y = -6$ and $2x + y = 0$ b $2x + y = -2$ and $x = 4y - 10$

Example 21 Solving simultaneous equations using substitution

Solve each of the following pairs of simultaneous equations by using substitution.

a $x + y = 10$, $y = 4x$ b $3x + 2y = 19$, $y = 2x - 8$ c $4x - y = 6$, $y = 2x - 4$

SOLUTION	EXPLANATION
a $x + y = 10$ [1] $y = 4x$ [2]	Number the equations for reference.
$x + (4x) = 10$	Substitute $y = 4x$ into [1].
$5x = 10$ $x = 2$	Combine like terms and solve for x.
From [2] $y = 4x$	
$= 4 \times (2)$ $= 8$	Substitute $x = 2$ into [2] to find the value of y.
$\therefore x = 2, y = 8$	
Check: $2 + 8 = 10$ and $8 = 4 \times 2$	Check answer by substituting $x = 2$ and $y = 8$ into [1] and [2].

Continued on next page

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b $3x + 2y = 19$ [1]
$y = 2x - 8$ [2]

$$\begin{aligned} 3x + 2(2x - 8) &= 19 \\ 3x + 4x - 16 &= 19 \\ 7x - 16 &= 19 \\ 7x &= 35 \\ x &= 5 \\ \text{From [2]}\quad y &= 2x - 8 \\ &= 2 \times (5) - 8 \\ &= 2 \end{aligned}$$

Substitute $y = 2x - 8$ into [1].
Use the distributive law and solve for x.

Substitute $x = 5$ into [2] to find the value of y.

$\therefore\ x = 5, y = 2$
Check: $3 \times 5 + 2 \times 2 = 19$ and $2 = 2 \times 5 - 8$

Check: substitute $x = 5$ and $y = 2$ into [1] and [2].

c $4x - y = 6$ [1]
$y = 2x - 4$ [2]

$$\begin{aligned} 4x - (2x - 4) &= 6 \\ 4x - 2x + 4 &= 6 \\ 2x + 4 &= 6 \\ 2x &= 2 \\ x &= 1 \\ \text{From [2]}\quad y &= 2x - 4 \\ &= 2 \times (1) - 4 \\ &= -2 \end{aligned}$$

Substitute $y = 2x - 4$ into [1] using brackets.
Use the distributive law and solve for x.
$$\begin{aligned} -(2x - 4) &= -1(2x - 4) \\ &= -1 \times 2x - 1 \times (-4) \\ &= -2x + 4 \end{aligned}$$

Substitute $x = 1$ into [2] to find the value of y.

$\therefore\ x = 1, y = -2$
Check: $4 \times 1 - (-2) = 6$ and $-2 = 2 \times 1 - 4$

Check: substitute $x = 1$ and $y = -2$ into [1] and [2].

Now you try

Solve each of the following pairs of simultaneous equations by using substitution.

a $x + y = 8$
$y = 3x$

b $5x + 3y = 7$
$y = x + 5$

c $3x - y = 9$
$y = 4x - 11$

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Exercise 2I

FLUENCY	1, 2(½), 3	1, 2–3(½)	2–3(½)

Example 2Ia

1 Solve each of the following pairs of simultaneous equations by using substitution.

a $x + y = 3$
$y = 2x$

b $x + y = 6$
$x = 5y$

c $x + 5y = 8$
$y = 3x$

Example 2Ib

2 Solve each of the following pairs of simultaneous equations by using substitution.

a $x + y = 12$
$y = x + 6$

b $2x + y = 1$
$y = x + 4$

c $5x + y = 5$
$y = 1 - x$

d $3x + 2y = 8$
$y = 4x - 7$

e $2x + 3y = 11$
$y = 2x + 1$

f $4x + y = 4$
$x = 2y - 8$

g $2x + 5y = -4$
$y = x - 5$

h $2x - 3y = 5$
$x = 5 - y$

i $3x + 2y = 5$
$y = 3 - x$

Example 2Ic

3 Solve each of the following pairs of simultaneous equations by using substitution.

a $3x - y = 7$
$y = x + 5$

b $3x - y = 9$
$y = x - 1$

c $x + 2y = 6$
$x = 9 - y$

d $y - x = 14$
$x = 4y - 2$

e $3x + y = 4$
$y = 2 - 4x$

f $4x - y = 12$
$y = 8 - 6x$

PROBLEM-SOLVING	4, 5	4, 5	5, 6

4 The sum of two numbers is 48 and the larger number is 14 more than the smaller number. Write two equations and solve them to find the two numbers.

5 The combined mass of two trucks is 29 tonnes. The heavier truck is 1 tonne less than twice the mass of the smaller truck. Write two equations and solve them to find the mass of each truck.

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6 The perimeter of a rectangle is 11 cm and the length is 3 cm more than half the breadth. Find the dimensions of the rectangle.

REASONING	7	7, 8	7, 8

7 One of the common errors when applying the method of substitution is made in this working. Find the error and describe how to avoid it.

Solve $y = 3x - 1$ and $x - y = 7$.

$$\begin{aligned} x - 3x - 1 &= 7 \quad (\text{substituting } y = 3x - 1 \text{ into } x - y = 7) \\ -2x - 1 &= 7 \\ -2x &= 8 \\ x &= -4 \end{aligned}$$

8 If both equations have the same variable as the subject, substitution is still possible.

For example, solve $y = 3x - 1 \ldots [1]$ and $y = 2 - x \ldots [2]$

Substitute [1] into [2].

$$\begin{aligned} 3x - 1 &= 2 - x \\ 4x &= 3 \\ x &= \frac{3}{4} \text{ and } y = \frac{5}{4} \end{aligned}$$

Use this method to solve these simultaneous equations.

a $y = 4x + 1$
$y = 3 - 2x$

b $y = 3 - 4x$
$y = 2x + 8$

c $y = \frac{1}{2}x + 4$
$y = \frac{x + 1}{3}$

ENRICHMENT: Literally challenging	–	–	9

9 Use substitution to solve each of the following pairs of literal simultaneous equations for x and y in terms of a and b.

a $ax + y = b$
$y = bx$

b $ax + by = b$
$x = by$

c $x + y = a$
$x = y - b$

d $ax - by = a$
$y = x - a$

e $ax - y = a$
$y = bx + a$

f $ax - by = 2a$
$x = y - b$

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The following problems will investigate practical situations drawing upon knowledge and skills developed throughout the chapter. In attempting to solve these problems, aim to identify the key information, use diagrams, formulate ideas, apply strategies, make calculations and check and communicate your solutions.

IT consultants

1 Three IT consultants charge fees in different ways:

- Con: \$120 per hour
- Tracy: \$210 upfront plus \$90 per hour
- Rish: Undecided

Rish is investigating different upfront fees and hourly rates to compete with Con and Tracy in his IT consultancy business.

a What are the total fees for 5 hours of work for the following consultants?
  i Con **ii** Tracy

b Write rules for the total fees $\$T$ for t hours work for:
  i Con **ii** Tracy

c Find the number of hours and total cost if Con's cost is equal to Tracy's cost.

d If Rish decides on an upfront fee of \$80 plus a \$100 hourly rate, find the time of hire if Rish's cost is equal to that for:
  i Con **ii** Tracy.

e Rish decides on an upfront fee of \$105 and wants the total cost to be the same as Con and Tracy's for $t = 7$. What should Rish's hourly rate be?

f If Rish wants to be cheaper than both Con and Tracy for $t = 10$, give a possible upfront fee of more than \$100 and an hourly rate of more than \$80 that makes this possible.

Separating fact from fiction

2 Fiona explores the secondhand book stall at her school fair and notices that all fiction books are \$3 each and non-fiction books are \$4 each. She has a total of \$70 with her to spend.

Fiona is interested in the number of books of each type that she can buy for her \$70.

a If Fiona buys 10 of each type of book, does she spend all of her \$70?

b Can Fiona buy the following combinations of books without spending more than \$70?
  i 9 fiction and 11 non-fiction
  ii 12 fiction and 8 non-fiction

c Write an expression for the cost of x fiction and y non-fiction books.

d Fiona decides to buy exactly 21 books, spending exactly \$70 dollars.
  i Let x be the number of fiction books and y be the number of non-fiction books. Write a rule for y in terms of x.
  ii Find the number of each type of book she buys.

e Can Fiona spend exactly \$70 and buy 24 books? Show your working to justify your answers.

f In how many different ways can Fiona buy books if she spends exactly \$70? List the numbers of fiction and non-fiction books in each case.

g Is it possible for Fiona to spend exactly \$50 on books? If so give the combinations.

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Applications and problem-solving

Kit homes

3 A German manufacturer of kit homes offers a basic essential module which is 10 m by 20 m. Additional bedrooms 5 m wide by x m long can be purchased, where x is an integer and $3 \leqslant x \leqslant 7$. All bedrooms purchased must have the same dimensions and when assembled, must share at least one entire wall with the essential module or another bedroom.

The manufacturer wishes to investigate the possible total areas of their kit homes and the perimeter of various arrangements.

- **a** What are the possible lengths of the bedrooms?
- **b** If three bedrooms are purchased:
  - **i** find the total area of the kit home if $x = 5$ is chosen
  - **ii** write an expression for the total area in terms of x
  - **iii** find the value of x if the total area is 290 m^2
  - **iv** find the maximum value of x if the total area is to be less than 250 m^2.
- **c** If n bedrooms are purchased:
  - **i** write an expression for the total area in terms of x and n
  - **ii** find three combinations of x and n that give a total area of 260 m^2.
- **d** If one bedroom is purchased, find:
  - **i** the two possible perimeters of the total kit home if $x = 4$ is chosen
  - **ii** the two possible perimeters of the total kit home in terms of x
  - **iii** the value of x if the perimeter must be 66 m.
- **e** If two bedrooms are purchased, find:
  - **i** the minimum perimeter possible, stating your chosen value of x
  - **ii** the maximum perimeter possible, stating your chosen value of x.

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2J Linear simultaneous equations: elimination

Learning intentions

- To be able to identify and form equations involving a matching pair of terms
- To be able to use the process of elimination to solve simultaneous equations

Past, present and future learning

- This section shows another way to solve simultaneous equations
- It addresses the Stage 5 Path Topic called *Equations C*
- These concepts will be extended in Chapter 1 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

Another method used to solve simultaneous linear equations is called elimination. This involves the addition or subtraction of the two equations to eliminate one of the variables. We can then solve for the remaining variable and substitute to find the value of the second variable.

Traffic engineers solve simultaneous equations to model possible traffic flow in a network of roads. Using known and unknown flow rates, linear equations are formed by equating the number of vehicles entering and leaving intersections.

Lesson starter: To add or subtract?

To use the method of elimination you need to decide if using addition or using subtraction will eliminate one of the variables.

Decide if the terms in these pairs should be added or subtracted to give the result of 0.

- $3x$ and $3x$
- $-x$ and x
- $2y$ and $-2y$
- $-7y$ and $-7y$

Describe under what circumstances addition or subtraction should be used to eliminate a pair of terms.

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KEY IDEAS

- **Elimination** involves the addition or subtraction of two equations to remove one variable.
- Elimination is often used when both equations are of the form $ax + by = d$ or $ax + by + c = 0$.
- Add equations to eliminate terms of opposite sign:

$$\begin{array}{rr} & 3x - y = 4 \\ + & \underline{5x + y = 4} \\ & 8x = 8 \end{array}$$

- Subtract equations to eliminate terms of the same sign:

$$\begin{array}{rr} & 2x + 3y = 6 \\ - & \underline{2x - 5y = 7} \\ & 8y = -1 \end{array}$$

- If terms cannot be eliminated just by using addition or subtraction, first multiply one or both equations to form a matching pair.
  For example:

matching pair

1 $\begin{array}{r} 3x - 2y = 1 \\ 2x + y = 3 \end{array}$ $\begin{array}{c} \longrightarrow \\ \xrightarrow{\times 2} \end{array}$ $\begin{array}{r} 3x - 2y = 1 \\ 4x + 2y = 6 \end{array}$ (Multiply both sides by 2.)

matching pair

2 $\begin{array}{r} 7x - 2y = 3 \\ 4x - 5y = -6 \end{array}$ $\begin{array}{c} \xrightarrow{\times 4} \\ \xrightarrow{\times 7} \end{array}$ $\begin{array}{r} 28x - 8y = 12 \\ 28x - 35y = -42 \end{array}$ (Multiply both sides by 4.) (Multiply both sides by 7.)

BUILDING UNDERSTANDING

1 Choose a '+' or '−' sign for each statement to make them true.

a $3x \square 3x = 0$

b $-2y \square 2y = 0$

c $11y \square (-11y) = 0$

d $-4x \square (-4x) = 0$

2 **a** Decide whether addition or subtraction should be chosen to eliminate the variable x in these simultaneous equations.

i $x + 2y = 3$
$x - 5y = -4$

ii $-2x - y = -9$
$2x + 3y = 11$

iii $y - x = 0$
$3y - x = 8$

b Decide whether addition or subtraction will eliminate the variable y in these simultaneous equations.

i $4x - y = 6$
$x + y = 4$

ii $7x - 2y = 5$
$-3x - 2y = -5$

iii $10y + x = 14$
$-10y - 3x = -24$

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Example 22 Solving simultaneous equations using elimination

Solve the following pairs of simultaneous equations by using elimination.

a $x - 2y = 1$
$-x + 5y = 2$

b $3x - 2y = 5$
$5x - 2y = 11$

SOLUTION

a

$$\begin{aligned} x - 2y &= 1 \quad [1] \\ -x + 5y &= 2 \quad [2] \end{aligned}$$

$[1] + [2]$ $\quad 3y = 3$

$y = 1$

From [1]

$$\begin{aligned} x - 2y &= 1 \\ x - 2 \times (1) &= 1 \\ x - 2 &= 1 \\ x &= 3 \end{aligned}$$

$\therefore x = 3, y = 1$

Check: $3 - 2(1) = 1$

$-3 + 5(1) = 2$

b

$$\begin{aligned} 3x - 2y &= 5 \quad [1] \\ 5x - 2y &= 11 \quad [2] \end{aligned}$$

$[2] - [1]$ $\quad 2x = 6$

$x = 3$

From [1]

$$\begin{aligned} 3x - 2y &= 5 \\ 3 \times (3) - 2y &= 5 \\ 9 - 2y &= 5 \\ -2y &= -4 \\ y &= 2 \end{aligned}$$

$\therefore x = 3, y = 2$

Check: $3(3) - 2(2) = 5$

$5(3) - 2(2) = 11$

EXPLANATION

a Add the two equations to eliminate x since $x + (-x) = 0$. Then solve for y.

Substitute $y = 1$ into equation [1] to find x.

Substitute $x = 3$ and $y = 1$ into the original equations to check.

b Subtract the two equations to eliminate y since they are the same sign, i.e. $-2y - (-2y) = -2y + 2y = 0$.
Alternatively, could do [1] − [2] but [2] − [1] avoids negative coefficients.
Solve for x.
Substitute $x = 3$ into equation [1] to find y.

Substitute $x = 3$ and $y = 2$ into the original equations to check.

Now you try

Solve the following pairs of simultaneous equations by using elimination.

a $-x + y = 1$
$x - 2y = -3$

b $4x - 3y = -16$
$3x - 3y = -15$

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Example 23 Solving simultaneous equations by creating a matching pair

Solve the following pairs of simultaneous equations by using elimination.

a $5x + 2y = -7$
$x + 7y = 25$

b $4x + 3y = 18$
$3x - 2y = 5$

SOLUTION

a

$$\begin{array}{rrl} & 5x + 2y = -7 & [1] \\ & x + 7y = 25 & [2] \\ 5 \times [2] & 5x + 35y = 125 & [3] \\ & \underline{5x + 2y = -7} & [1] \\ [3] - [1] & 33y = 132 & \\ & y = 4 & \end{array}$$

$$\begin{aligned} \text{From [2]} \quad x + 7y &= 25 \\ x + 7 \times (4) &= 25 \\ x + 28 &= 25 \\ x &= -3 \end{aligned}$$

$\therefore x = -3, y = 4$

b

$$\begin{array}{rrl} & 4x + 3y = 18 & [1] \\ & 3x - 2y = 5 & [2] \\ 2 \times [1] & 8x + 6y = 36 & [3] \\ 3 \times [2] & \underline{9x - 6y = 15} & [4] \\ [3] + [4] & 17x \qquad = 51 & \\ & x = 3 & \end{array}$$

$$\begin{aligned} \text{From [1]} \quad 4x + 3y &= 18 \\ 4 \times (3) + 3y &= 18 \\ 12 + 3y &= 18 \\ 3y &= 6 \\ y &= 2 \end{aligned}$$

$\therefore x = 3, y = 2$

EXPLANATION

a There are different numbers of x and y in each equation so multiply equation [2] by 5 to make the coefficient of x equal to that in equation [1].
Subtract the equations to eliminate x.

Substitute $y = 4$ in equation [2] to find x.

Substitute $x = -3$ and $y = 4$ into the original equations to check.

b Multiply equation [1] by 2 and equation [2] by 3 to make the coefficients of y equal in size but opposite in sign. Alternatively we could make the coefficients of x match.
Add the equations to eliminate y.

Substitute $x = 3$ into equation [1] to find y.

Substitute $x = 3$ and $y = 2$ into the original equations to check.

Now you try

Solve the following pairs of simultaneous equations by using elimination.

a $7x + 2y = 12$
$x - 3y = 5$

b $3x + 2y = 5$
$5x - 3y = 21$

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Exercise 2J

FLUENCY	1, 2–4(½)	1, 2–4(½)	2–4(½)

Example 22a

1 Solve these simultaneous equations by first adding the equations.

a $3x - y = 2$
$2x + y = 3$

b $2x - 3y = -2$
$-5x + 3y = -4$

c $4x + 3y = 5$
$-4x - 5y = -3$

Example 22b

2 Solve these simultaneous equations by first subtracting the equations.

a $3x + y = 10$
$x + y = 6$

b $2x + 7y = 9$
$2x + 5y = 11$

c $2x + 3y = 14$
$2x - y = -10$

d $5x - y = -2$
$3x - y = 4$

e $-5x + 3y = -1$
$-5x + 4y = 2$

f $9x - 2y = 3$
$-3x - 2y = -9$

Example 23a

3 Solve the following pairs of simultaneous linear equations by using elimination.

a $4x + y = -8$
$3x - 2y = -17$

b $2x - y = 3$
$5x + 2y = 12$

c $-x + 4y = 2$
$3x - 8y = -2$

d $3x + 2y = 0$
$4x + y = -5$

e $4x + 3y = 13$
$x + 2y = -3$

f $3x - 4y = -1$
$6x - 5y = 10$

g $-4x - 3y = -5$
$7x - y = 40$

h $3x - 4y = -1$
$-5x - 2y = 19$

i $5x - 4y = 7$
$-3x - 2y = 9$

Example 23b

4 Solve the following pairs of simultaneous linear equations by using elimination.

a $3x + 2y = -1$
$4x + 3y = -3$

b $7x + 2y = 8$
$3x - 5y = 21$

c $6x - 5y = -8$
$-5x + 2y = -2$

d $2x - 3y = 3$
$3x - 2y = 7$

e $7x + 2y = 1$
$4x + 3y = 8$

f $5x + 7y = 1$
$3x + 5y = -1$

g $5x + 3y = 16$
$4x + 5y = 5$

h $3x - 7y = 8$
$4x - 3y = -2$

i $2x - 3y = 1$
$3x + 2y = 8$

j $2x - 7y = 11$
$5x + 4y = -37$

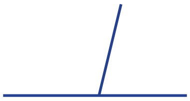
k $3x + 5y = 36$
$7x + 2y = -3$

l $2x - 4y = 6$
$5x + 3y = -11$

PROBLEM-SOLVING	5, 6	5–7	6–8

5 The sum of two numbers is 30 and their difference is 12.
Write two equations and find the numbers.

6 Two supplementary angles differ by 24°. Write two equations and find the two angles.



7 The perimeter of a rectangular city block is 800 metres and the difference between the length and breadth is 123 metres. What are the dimensions of the city block?

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8 A teacher collects a total of 17 mobile phones and iPads before a group of students heads off on a bushwalk. From a second group of students, 40 phones and iPads are collected. The second group had twice the number of phones and three times as many iPads as the first group. How many phones and how many iPads did the first group have?

REASONING	9	9, 10	10, 11

9 Consider the pair of simultaneous equations:

$$2x + y = 5 \ldots \quad [1]$$
$$5x + y = 11 \ldots \quad [2]$$

a Solve the equations by first subtracting equation [2] from equation [1], i.e. [1] – [2].
b Now solve the equations by first subtracting equation [1] from equation [2], i.e. [2] – [1].
c Which method **a** or **b** is preferable and why?

10 To solve any of the pairs of simultaneous equations in this section using the method of substitution, what would need to be done first before the substitution is made?
Try these using substitution.

a $x + y = 5$
$2x - y = 7$

b $3x - y = -2$
$x - 4y = 3$

11 Find the solution to these pairs of simultaneous equations. What do you notice?

a $2x + 3y = 3$
$2x + 3y = 1$

b $7x - 14y = 2$
$y = \frac{1}{2}x + 1$

ENRICHMENT: Literal elimination	–	–	12(½)

12 Use elimination to solve the following pairs of simultaneous equations to find the value of x and y in terms of the other pronumerals.

a $x + y = a$
$x - y = b$

b $ax + y = 0$
$ax - y = b$

c $x - by = a$
$-x - by = 2a$

d $2ax + y = b$
$x + y = b$

e $bx + 5ay = 2b$
$bx + 2ay = b$

f $ax + 3y = 14$
$ax - y = -10$

g $2ax + y = b$
$3ax - 2y = b$

h $2ax - y = b$
$3ax + 2y = b$

i $-x + ay = b$
$3x - ay = -b$

j $ax + 2y = c$
$2ax + y = -c$

k $ax - 4y = 1$
$x - by = 1$

l $ax + by = a$
$x + y = 1$

m $ax + by = c$
$-ax + y = d$

n $ax - by = a$
$-x + y = 2$

o $ax + by = b$
$3x - y = 2$

p $ax - by = b$
$cx - y = 2$

q $ax + by = c$
$dx - by = f$

r $ax + by = c$
$dx + by = f$

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2K Using linear simultaneous equations to solve problems

Learning intentions

- To know the steps involved in solving a word problem with two unknowns
- To be able to choose an appropriate method to solve two equations simultaneously

Past, present and future learning

- This section addresses the Stage 5 Path Topic called *Equations C*
- These concepts will be extended in Chapter 1 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

Many problems can be described mathematically using a pair of simultaneous linear equations from which a solution can be obtained algebraically.

Weather forecasting uses algebraic equations to model changes over time of physical variables, such as air temperature and rainfall. Current observations are entered into supercomputers which simultaneously solve huge systems of equations, rapidly updating predictions.

Lesson starter: The tyre store

In one particular week a total of 83 cars and motorcycles check into a garage to have their tyres changed. All the motorcycles change 2 tyres each and all the cars change 4 tyres each. The total number of tyres sold in the week is 284.

If you had to find the number of motorcycles and the number of cars that have their tyres changed in the week:

- What two variables should you define?
- What two equations can you write?
- Which method (substitution or elimination) would you use to solve the equations?
- What is the solution to the simultaneous equations?
- How would you answer the question in words?

KEY IDEAS

- To solve worded problems with simultaneous equations complete these steps.
  - Define two variables by writing down what they represent.
    For example: Let $\$C$ be the cost of …
    Let x be the number of …
  - Write a pair of simultaneous equations from the given information using your two variables.
  - Solve the equations simultaneously using substitution or elimination.
  - Check the solution by substituting into the original equations.
  - Express the answer in words.

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BUILDING UNDERSTANDING

1 The sum of two numbers is 42 and their difference is 6. Find the two numbers x and y by completing the following steps.

- **a** State a pair of simultaneous equations relating x and y.
- **b** Which method, substitution or elimination, would it be best to use to solve the pair of equations?

2 A rectangular block of land has a perimeter of 120 m and the length l m of the block is three times the breadth b m. Find the dimensions of the block of land by completing the following steps.

- **a** State a pair of simultaneous equations relating l and b.
- **b** Which method, substitution or elimination, would it be best to use to solve the pair of equations?

Example 24 Setting up and solving simultaneous linear equations

The length of a rectangle is 5 cm longer than its breadth. If the perimeter is 84 cm, find the dimensions of the rectangle.

SOLUTION

Let b cm be the breadth of the rectangle and let l cm be the length.

$l = b + 5$ [1]

$2l + 2b = 84$ [2]

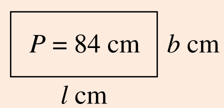
Substitute [1] into [2]

$$\begin{aligned} 2(b+5) + 2b &= 84 \\ 2b + 10 + 2b &= 84 \\ 4b + 10 &= 84 \\ 4b &= 74 \\ b &= 18.5 \\ \text{From [1] } l &= (18.5) + 5 \\ &= 23.5 \end{aligned}$$

$\therefore$ the dimensions of the rectangle are 23.5 cm by 18.5 cm.

EXPLANATION

Define the unknowns. Draw a diagram to help.



The length is 5 cm more than the breadth and the perimeter is 84 cm.

Choose the method of substitution to solve.
Expand and collect like terms to solve for b.

Substitute $b = 18.5$ into [1].
Substitute into original equations to check.
Answer the question in a sentence.

Now you try

The length of a rectangle is 6 cm longer than its breadth. If the perimeter is 52 cm, find the dimensions of the rectangle.

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Example 25 Solving further word problems with simultaneous equations

Andrea bought two containers of ice-cream and three bottles of maple syrup for a total of \$22. At the same shop, Bettina bought one container of the same ice-cream and two bottles of the same maple syrup for \$13. How much does each container of ice-cream and each bottle of maple syrup cost?

SOLUTION	EXPLANATION
Let: $\$x$ be the cost of a container of ice-cream $\$y$ be the cost of a bottle of maple syrup	Define the unknowns. Ask yourself what you are being asked to find.
$2x + 3y = 22$ [1] $x + 2y = 13$ [2]	2 containers of ice-cream and 3 bottles of maple syrup for a total of \$22. 1 container of ice-cream and 2 bottles of maple syrup for \$13.
$2 \times$ [2] $\quad 2x + 4y = 26$ [3] $2x + 3y = 22$ [1] [3] − [1] $\quad y = 4$	Choose the method of elimination to solve. Multiply [2] by 2 to obtain a matching pair. Subtract equation [1] from [3].
Form [2] $\quad x + 2y = 13$ $x + 2 \times (4) = 13$ $x + 8 = 13$ $x = 5$	Substitute $y = 4$ into [2]. Solve for x. Substitute $y = 4$ and $x = 5$ into original equations to check.
The cost of one container of ice-cream is \$5 and the cost of one bottle of maple syrup is \$4.	Answer the question in a sentence.

Now you try

Wally bought three identical hats and four identical scarfs for \$70. From the same store on another day he bought five hats and two scarfs for \$64.40. How much does each hat and each scarf cost?

Exercise 2K

FLUENCY	1–5	1–3, 5, 6	2, 5–7

Example 24

1 The length of a rectangular pool is 10 m longer than its breadth. If the perimeter of the pool is 80 m, find the dimensions of the pool by completing the following steps.

- **a** Define two variables to represent the problem.
- **b** Write a pair of simultaneous equations relating the two variables.
- **c** Solve the pair of equations using substitution or elimination.
- **d** Write your answer in words.

2 Mila plants 820 hectares of potatoes and corn. To maximise his profit he plants 140 hectares more of potatoes than of corn. How many hectares of each does he plant?

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Example 25

3 Mal bought three bottles of milk and four bags of chips for a total of $17. At the same shop, Barbara bought one bottle of milk and five bags of chips for $13. Find how much each bottle of milk and each bag of chips cost by completing the following steps.
   a Define two variables to represent the problem.
   b Write a pair of simultaneous equations relating the two variables.
   c Solve the pair of equations using substitution or elimination.
   d Write your answer in words.

4 Leonie bought seven lip glosses and two eye shadows for a total of $69 and Chrissie bought four lip glosses and three eye shadows for a total of $45. Find how much each lip gloss and each eye shadow costs by completing the following steps.
   a Define two variables to represent the problem.
   b Write a pair of simultaneous equations relating the two variables.
   c Solve the pair of equations using substitution or elimination.
   d Write your answer in words.

5 Steve bought five cricket balls and fourteen tennis balls for $130. Ben bought eight cricket balls and nine tennis balls for $141. Find the cost of a cricket ball and the cost of a tennis ball.

6 At a birthday party for 20 people each person could order a hot dog or a bucket of chips. If there were four times as many hot dogs ordered as buckets of chips, calculate how many hot dogs and how many buckets of chips were ordered.

7 The entry fee for a fun run is $10 for adults and $3 for children. A total of $3360 was collected from the 420 competitors. Find the number of adults and the number of children running.

PROBLEM–SOLVING	8, 9	8–10	9–11

8 Carrie has 27 coins in her purse. All the coins are 5 cent or 20 cent coins. If the total value of the coins is $3.75, how many of each type does she have?

9 Michael is 30 years older than his daughter. In five years' time Michael will be 4 times as old as his daughter. How old is Michael now?

10 Jenny has twice as much money as Kristy. If I give Kristy $250, she will have three times as much as Jenny. How much did they each have originally?

11 At a particular cinema the cost of an adult movie ticket is $15 and the cost of a child's ticket is $10. The seating capacity of the cinema is 240. For one movie session all seats are sold and $3200 is collected from the sale of tickets. How many adult and how many children's tickets were sold?

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REASONING	12	12, 13	13, 14

12 Wilfred and Wendy are in a long distance bike race. Wilfred rides at 20 km/h and has a 2 hour head start. Wendy travels at 28 km/h. How long does it take for Wendy to catch up to Wilfred? Use distance = speed × time.

13 Andrew travelled a distance of 39 km by jogging for 4 hours and cycling for 3 hours. He could have travelled the same distance by jogging for 7 hours and cycling for 2 hours. Find the speed at which he was jogging and the speed at which he was cycling.

14 Malcolm's mother is 27 years older than he is and their ages are both two-digit numbers. If Malcolm swaps the digits in his age he gets his mother's age.

a How old is Malcolm if the sum of the digits in his age is 5?
b What is the relationship between the digits in Malcolm's age if the sum of the digits is unknown.
c If the sum of the digits in Malcolm's two-digit age is unknown, how many possible ages could he be? What are these ages?

ENRICHMENT: Digit swap	–	–	15, 16

15 The digits of a two-digit number sum to 10. If the digits swap places the number is 36 more than the original number. What is the original number? Can you show an algebraic solution?

16 The difference between the two digits of a two-digit number is 2. If the digits swap places the number is 18 less than the original number. What is the original number? Can you show an algebraic solution?

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2L Quadratic equations of the form $ax^2 = c$

Learning intentions
- To know and perform the steps required to solve a simple quadratic equation
- To understand that the number of solutions may be two, one or zero

Past, present and future learning
- This section addresses the Stage 5 Path Topic called *Equations C*
- These concepts will be extended in Chapter 10 of this book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

In a linear equation, if the highest power of the variable is 2, the equation is called a quadratic equation. Since $4 \times 4 = 16$ and $(-4) \times (-4) = 16$, we would say that the equation $x^2 = 16$ has two possible solutions $x = 4$ and $x = -4$. In this section we will solve some simple quadratic equations. Quadratic equations may arise in areas such as the path of a projectile, a falling object or a measurement calculation.

Lesson starter: More than one solution?

Is it possible to have more than one solution to an equation?

Consider the following.

What do you notice when you evaluate:

- 3^2?
- $(-3)^2$?
- $(-5)^2$?
- 5^2?

From the above results, what possible value(s) of x make the following equations true?

- $x^2 = 9$
- $x^2 = 25$

Can you find a value of x to make the equation $x^2 = 29$ true? Why or why not?

KEY IDEAS

- A **quadratic equation** is one in which the highest power of the pronumeral is 2. In this section you will solve quadratic equations of the form $ax^2 = c$, such as these:
  $x^2 = 0$ $\qquad$ $x^2 = 9$ $\qquad$ $2x^2 = 18$ $\qquad$ $2x^2 - 6 = 7$
- To solve equations of the form $ax^2 = c$, make x^2 the subject, then take the square root of both sides of the equation. This process may produce two solutions, one solution or no real solutions.
  - If $2x^2 = 18$, then $x^2 = 9$. This equation has **two solutions**. These are found by taking the square root of both sides of the equation. We know that $3^2 = 9$ and also $(-3)^2 = 9$, so there are two solutions: $x = 3$ and $x = -3$. This can be abbreviated to $x = \pm 3$.

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- If $2x^2 = 0$, then $x^2 = 0$. This equation only has **one solution**, $x = 0$, because the square root of zero is zero.
- If $2x^2 = -18$, then $x^2 = -9$. This equation is said to have '**no real solutions**' because the square root of -9 has no value in the set of real numbers. In future years you may study a topic called *Complex Numbers*, in which equations like $x^2 = -9$ can be solved.

- Some quadratic equations have non-integer solutions. These can be expressed using **surds**. For example, the solution of $x^2 = 10$ is $x = \pm\sqrt{10}$. These are the exact solutions, but they could be expressed as $x = \pm 3.2$ (to 1 d.p.), for example.
- In practical problems such as those involving measurement, the negative solution is usually rejected. For example, if the area of a square is 10 square metres, the exact side length is $\sqrt{10}$ metres, not $-\sqrt{10}$ metres or $\pm\sqrt{10}$ metres.

BUILDING UNDERSTANDING

1 a If $x^2 = 16$, then $x = \pm\square$

b If $x^2 = 0$, then $x = \square$

c If $x^2 = -16$, how many real solutions can be found for x?

2 Copy and complete:

a
$$\begin{aligned} 2x^2 - 16 &= 0 \\ 2x^2 &= \square \\ x^2 &= \square \\ x &= \pm\square \end{aligned}$$

b
$$\begin{aligned} 2x^2 + 16 &= 16 \\ 2x^2 &= \square \\ x^2 &= \square \\ x &= \square \end{aligned}$$

c
$$\begin{aligned} 2x^2 + 16 &= 0 \\ 2x^2 &= \square \\ x^2 &= \square \end{aligned}$$
No solutions

3 How many solutions will each equation have (two, one or none)?

a $x^2 = 81$

b $81x^2 = 0$

c $x^2 + 81 = 0$

d $2x^2 - 32 = 0$

e $2x^2 + 32 = 0$

f $2x^2 + 32 = 32$

Example 26 Solving equations of the form $ax^2 = c$

Find all possible solutions to the following equations.

a $x^2 = 25$

b $2x^2 = 18$

c $\frac{1}{5}x^2 = 20$

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SOLUTION

a
$$\begin{aligned} x^2 &= 25 \\ \therefore x &= \pm\sqrt{25} \\ x &= \pm 5 \end{aligned}$$

EXPLANATION

Take the square root of both sides to solve for x. If $x^2 = c$, $c > 0$, then $x = \pm\sqrt{c}$.
Since 25 is a square number, $\sqrt{25}$ is a whole number.
$\pm$ represents two solutions: $+5$ since $(+5)^2 = 25$ and -5 since $(-5)^2 = 25$.

b
$$\begin{aligned} 2x^2 &= 18 \\ x^2 &= 9 \\ \therefore x &= \pm\sqrt{9} \\ x &= \pm 3 \end{aligned}$$

First make x^2 the subject by dividing both sides by 2.
Take the square root of both sides to solve for x.
This gives two possible values for x, $+3$ and -3.

c
$$\begin{aligned} \frac{1}{5}x^2 &= 20 \\ \frac{x^2}{5} &= 20 \\ x^2 &= 100 \\ \therefore x &= \pm\sqrt{100} \\ x &= \pm 10 \end{aligned}$$

$\frac{1}{5}$ of x^2 is the same as $\frac{x^2}{5}$, so rewrite equation in this form.
Multiply both sides by 5 (or divide both sides by $\frac{1}{5}$ after the first line, but multiplying by 5 is easier.)
Take the square root of both sides to give the two possible solutions for x.

Now you try

Find all possible solutions to the following equations.

a $x^2 = 49$ **b** $3x^2 = 27$ **c** $\frac{1}{4}x^2 = 36$

Example 27 Solving equations with solutions that are surds

Find all possible solutions to the following equations.

a $x^2 = 17$ (answer in exact surd form)
b $3x^2 = 21$ (give answer to one decimal place)

SOLUTION

a
$$\begin{aligned} x^2 &= 17 \\ \therefore x &= \pm\sqrt{17} \end{aligned}$$

EXPLANATION

Take the square root of both sides.
Since 17 is not a square number, leave answer in exact surd form $\pm\sqrt{17}$ as required.

b
$$\begin{aligned} 3x^2 &= 21 \\ x^2 &= 7 \\ \therefore x &= \pm\sqrt{7} \\ x &= \pm 2.6 \text{ (to 1 d.p.)} \end{aligned}$$

Divide both sides by 3.
Take the square root of both sides.
Use a calculator to evaluate $\sqrt{7} = 2.64575\ldots$ and round to 1 decimal place as required.

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Now you try

Find all possible solutions to the following equations.

a $x^2 = 29$ (answer in exact surd form)

b $5x^2 = 55$ (answer correct to one decimal place)

Example 28 Rearranging and solving equations of the form $ax^2 + b = c$

Find exact solution(s), where possible, to the following equations.

a $x^2 - 64 = 0$

b $x^2 + 7 = 0$

c $3x^2 - 5 = 7, x > 0$

d $13 - x^2 = 3, x < 0$

SOLUTION	EXPLANATION
a $\begin{aligned} x^2 - 64 &= 0 \\ x^2 &= 64 \\ \therefore x &= \pm\sqrt{64} \\ x &= \pm 8 \end{aligned}$	Make x^2 the subject by adding 64 to both sides. Solve for x by taking the square root of both sides.
b $\begin{aligned} x^2 + 7 &= 0 \\ x^2 &= -7 \end{aligned}$ $\therefore$ No possible solution	Make x^2 the subject by subtracting 7 from both sides. Since x^2 is positive for all values of x, $x^2 = c$, where c is negative, has no real solution.
c $\begin{aligned} 3x^2 - 5 &= 7 \\ 3x^2 &= 12 \\ x^2 &= 4 \\ \therefore x &= \pm\sqrt{4} \\ x &= \pm 2 \\ x &= 2 \text{ since } x > 0 \end{aligned}$	Make x^2 the subject and then take the square root of both sides to solve for x. Note the restriction that x is a positive value so reject the solution $x = -2$.
d $\begin{aligned} 13 - x^2 &= 3 \\ -x^2 &= -10 \\ x^2 &= 10 \\ \therefore x &= \pm\sqrt{10} \\ x &= -\sqrt{10} \text{ since } x < 0 \end{aligned}$	Subtract 13 from both sides and then divide both sides by -1. Alternatively, add x^2 to both sides to give $13 = x^2 + 3$ and then solve. Leave answer in exact surd form, eliminating $x = \sqrt{10}$ since $x < 0$.

Now you try

Find exact solution(s), where possible, to the following equations.

a $x^2 - 121 = 0$

b $x^2 + 13 = 0$

c $2x^2 - 11 = 39$ $(x > 0)$

d $19 - x^2 = 6$ $(x < 0)$

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Exercise 2L

FLUENCY	1–3	1–3	2–3

Example 22

1 Find the possible solutions for x in the following equations.

a $x^2 = 9$
b $x^2 = 121$
c $x^2 = 1$
d $x^2 = 400$
e $5x^2 = 20$
f $6x^2 = 600$
g $8x^2 = 200$
h $4x^2 = 196$
i $-3x^2 = -108$
j $-4x^2 = -64$
k $\frac{x^2}{2} = 32$
l $\frac{x^2}{3} = 27$
m $\frac{x^2}{10} = 12.1$
n $0.1x^2 = 1.6$
o $\frac{1}{2x^2} = 72$
p $\frac{1}{5x^2} = 20$

Example 23

2 a Solve the following equations, leaving your answer in exact surd form.

i $x^2 = 15$
ii $x^2 = 10$
iii $x^2 = 42$
iv $x^2 = 3$
v $3x^2 = 21$
vi $8x^2 = 88$
vii $\frac{x^2}{2} = 17$
viii $\frac{x^2}{3} = 13$

b Solve these equations, giving your answers to 1 decimal place.

i $x^2 = 5$
ii $x^2 = 17$
iii $3x^2 = 120$
iv $\frac{x^2}{7} = 20$

Example 24

3 Find the possible solutions to the following equations. Leave your answers in exact form.

a $x^2 - 49 = 0$
b $x^2 - 36 = 0$
c $x^2 - 13 = 0$
d $x^2 + 4 = 0$
e $x^2 + 14 = 0$
f $2x^2 - 44 = 0$
g $3x^2 - 75 = 0, x > 0$
h $2x^2 + 32 = 36, x > 0$
i $5x^2 - 32 = 43$
j $4 - x^2 = 0, x < 0$
k $10 - x^2 = -12, x < 0$
l $8 - 4x^2 = 15$

PROBLEM-SOLVING	4–6	4, 6–8	4, 5, 7, 9

4 Solve the following equations by first writing in the form $ax^2 = c$. Answer in exact form.

a $3x^2 = 2x^2 + 9$
b $2x^2 = 4x^2 - 32$
c $x^2 + 3x = 3x + 4$
d $x^2 - 4x = 64 - 4x$
e $5x^2 + 3 = 3x^2 + 15, x > 0$
f $x^2 + 2x + 7 = 2x - 5, x > 0$
g $7 - x^2 = 2x^2 + 4, x < 0$
h $-x^2 - 2x + 4 = 2 - 2x, x < 0$

5 15 is subtracted from the square of a number and the result is 66. What are the possible numbers?

6 A square vegetable garden in a small backyard has side length x metres and area $14\,\text{m}^2$.

a Write an equation to represent this information and find the possible values for x to 1 decimal place.
b Comment on the validity of your result in part **a**, given x represents a side length.

7 Determine the radius of a circle with the following areas, A. Answer to 1 decimal place where necessary. Recall the area of a circle is given by πr^2, where r is the radius.

a $A = 24\,\text{m}^2$
b $A = 45\,\text{cm}^2$
c $A = 4\pi\,\text{m}^2$

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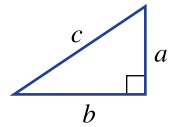
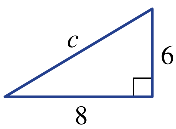


8 Recall Pythagoras' theorem: in a right-angled triangle the sum of the squares of the two shorter sides equals the square of the hypotenuse; $a^2 + b^2 = c^2$

a Solve for the unknown side length in these right-angled triangles. Answer in exact form.

i


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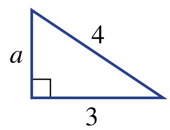

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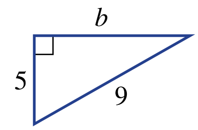
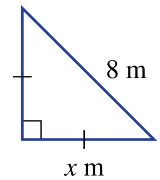
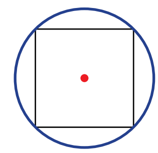


b In the isosceles triangle shown, with hypotenuse 8 m, what is the value of x in exact form?



9 Determine the side length of the largest square that fits inside a circle of diameter 12 m. Round to 1 decimal place.



REASONING	10–12	11–13	12–14

10 A ball is dropped from the top of a tall building. The height of the object above ground after t seconds is given by $h = 60 - 5t^2$.

a What is the height of the ball above ground after 3 seconds?

b After how many seconds will the ball reach ground level? Answer to 2 decimal places.

c After how many seconds was the ball at a height of 40 m?

11 Explain why $x^2 = c$, where $c < 0$, has no solutions.

12 Determine for which values of c the equation $x^2 + c = 0$ has:

a 0 solutions **b** 1 solution **c** 2 solutions

13 Billy says to Maddy: 'When I add 10 to the square of a number, the result is 6.' Maddy replies that this is impossible. Explain why Maddy is correct.

14 a Write down the possible value(s) for x in each of the following.

i $x^2 = 16$ **ii** $x = \sqrt{16}$

b Are the answers in part **a** the same or different? If they are different, explain how.

ENRICHMENT: Kinematics	–	–	15, 16

Kinematics is the study of motion. If the acceleration of an object is constant, there are a number of formulas that can be used to calculate velocity, displacement (distance from starting point), acceleration and time. One such equation is of the form $s = ut + \frac{1}{2}at^2$, where u is the initial velocity of the object, t is time, a is acceleration and s is displacement. Use this formula to answer the following questions.

15 A remote-control car starts from rest and accelerates constantly at $2\,\text{ms}^{-2}$. How long will the car take to travel 100 m?

16 A skydiver is dropped from a plane with an initial velocity of $0\,\text{ms}^{-1}$. If the acceleration (due to gravity) is $9.8\,\text{ms}^{-2}$, how long does the skydiver take to fall 200 m? Answer to 1 decimal place.

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Working mathematically

Investing in art

A small boutique movie theatre is opening soon and will be publishing adult and child ticket prices on its website. After some analysis, the owner decides to make the following assumptions:

- the sum of the cost of a child ticket and an adult ticket will be \$30
- 40 children and 25 adults are expected to attend each evening.

Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate.

Routine problems

a If the cost of a child movie ticket is \$6, find the cost of an adult ticket.

b Find the revenue (total amount earned) for the movie theatre on one particular evening using the assumption that 40 children and 25 adults attend and a child ticket is \$6.

c Solve the following simultaneous equations.

$$\begin{aligned} a + c &= 30 \\ 25a + 40c &= 900 \end{aligned}$$

Non-routine problems

Explore and connect

a The problem is to determine a suitable price for the adult and child tickets to achieve a revenue of at least \$1000. Write down all the relevant information that will help solve this problem.

b Use the following facts to write two equations using \$$a$ for the cost of an adult ticket and \$$c$ for the cost of a child ticket.

- The sum of the cost of a child ticket and an adult ticket will be \$30.
- 40 children and 25 adults attend on one particular evening.
- The total revenue for the evening is \$930.

Choose and apply techniques

c Solve your simultaneous equations from part **b** to find the cost of a child ticket and the cost of an adult ticket.

d Repeat part **c** for the following total revenues.

i \$825 **ii** \$960

e Explore the following special cases for the total revenue earned and decide if they represent reasonable pricing structures.

i \$975 **ii** \$765

Communicate thinking and reasoning

f By considering your results from above, explain why it will be difficult for the theatre to earn more than \$1000 on each evening based on the assumptions made.

g If the sum of the cost of a child ticket and an adult ticket is changed to \$40, do you think it would be reasonable to expect that more than \$1000 revenue can be earned each evening. Show your choices and calculations to justify your answer.

h Summarise your results and describe any key findings.

Extension problems

Problem solve

a By keeping the assumption that the cost of a child ticket and an adult ticket will be \$30, determine the maximum and minimum expected revenue for an evening at the cinema.

b Decide if the cinema owner should increase the price for the total cost of purchasing one adult and one child ticket to achieve a revenue of more than \$1000. Choose a suitable total cost and justify your choice showing all calculations.

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Investigation

Fire danger

In many countries fire indices have been developed to help predict the likelihood of fire occurring. One of the simplest fire-danger rating systems devised is the Swedish Angstrom Index. This index considers the relationship between relative humidity, temperature and the likelihood of fire danger.

The index, I, is given by: $I = \frac{H}{20} + \frac{27 - T}{10}$

where H is the percentage of relative humidity and T is the temperature in degrees Celsius. The table below shows the likelihood of a fire occurring for different index values.

Index	Likelihood of fire occurring
$I > 4.0$	Unlikely
$2.5 < I < 4.0$	Medium
$2.0 < I < 2.5$	High
$I < 2.0$	Very likely

Constant humidity

a If the humidity is 35% ($H = 35$), how hot would it have to be for the occurrence of fire to be:
   i very likely? **ii** unlikely?
   Discuss your findings with regard to the range of summer temperatures for your capital city or nearest town.

b Repeat part **a** for a humidity of 40%.

c Describe how the 5% change in humidity affects the temperature at which fires become:
   i very likely **ii** unlikely.

Constant temperature

a If the temperature was 30°C, investigate what humidity would make fire occurrence:
   i very likely **ii** unlikely.

b Repeat part **a** for a temperature of 40°C.

c Determine how the ten-degree change in temperature affects the relative humidity at which fire occurrence becomes:
   i very likely **ii** unlikely.

Reflection

Is this fire index more sensitive to temperature or to humidity? Explain your answer.

Investigate

Use the internet to investigate fire indices used in Australia. You can type in key words, such as Australia, fire danger and fire index.

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Problems and challenges

1 Transpose each of the following formulas to make the pronumeral shown in brackets the subject.

a $wx - 4 = ax + k$ [x]

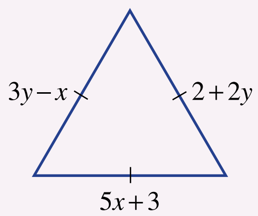
b $\frac{a}{K} - y = \frac{b}{K}$ [K]

c $\sqrt{\frac{a - w}{ak}} = m$ [a]

2 Five consecutive integers add to 195. Find the middle integer.

3 A group of office workers had some prize money to distribute among themselves. When all but one took \$9 each, the last person only received \$5. When they all took \$8 each, there was \$12 left over. How much had they won?

4 The sides of an equilateral triangle have lengths $3y - x$, $5x + 3$ and $2 + 2y$, as shown. Find the length of the sides.



5 a If $a > b > 0$ and $c < 0$, insert an inequality symbol to make a true statement.

i $a + c$ ___ $b + c$

ii ac ___ bc

iii $a - b$ ___ 0

iv $\frac{1}{a}$ ___ $\frac{1}{b}$

b Place a, b, c and d in order from smallest to largest given:

$$a > b$$
$$a + b = c + d$$
$$b - a > c - d$$

6 Find the values of x, y and z when:

$$\begin{aligned} x - 3y + 2z &= 17 \\ x - y + z &= 8 \\ y + z &= 3 \end{aligned}$$

7 Solve these equations for x.

a $\frac{1}{x} + \frac{1}{a} = \frac{1}{b}$

b $\frac{1}{2x} + \frac{1}{3x} = \frac{1}{4}$

c $\frac{x - 1}{3} - \frac{x + 1}{4} = x$

d $\frac{2x - 3}{4} - \frac{1 - x}{5} = \frac{x + 1}{2}$

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Expressions and linear equations

Substitution
The process of replacing a pronumeral with a given value
e.g. if $x = 3$, $2x + 4 = 2 \times 3 + 4$
$= 6 + 4$
$= 10$
e.g. if $x = 2$, $y = -4$, $xy = 2 \times (-4)$
$= -8$

Expanding brackets
e.g. $2(x + 3) = 2 \times x + 2 \times 3$
$= 2x + 6$
$-3(4 - 2x) = -12 + 6x$

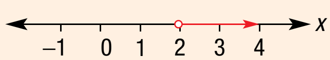
Expressions
e.g. $\boxed{2x} - 3y + 5$
term, coefficient, constant
e.g. 2 more than 3 lots of m is $3m + 2$

Addition/subtraction
Only like terms can be combined under addition or subtraction.
Like terms: e.g. $3a$ and $7a$
$2ab$ and $5ab$
not $4a$ and $7ab$
e.g. $3x + 2y - x + 4y + 7$
$= 3x - x + 2y + 4y + 7$
$= 2x + 6y + 7$

Solving linear equations
Finding the value that makes an equation true
e.g. $2x + 5 = 9$
$2x = 4$ (subtract 5)
$x = 2$ (divide by 2)
e.g. $5(x - 1) = 2x + 7$
$5x - 5 = 2x + 7$ (expand)
$3x - 5 = 7$ (subtract $2x$)
$3x = 12$ (add 5)
$x = 4$ (divide by 3)

Solving word problems
1 Define variable(s)
2 Set up equation(s)
3 Solve equation(s)
4 Check each answer and write in words

Inequalities
These can be represented using $>$, $<$, $\geqslant$, $\leqslant$ rather than $=$.
e.g. $x > 2$ 2 not included (open circle)



Solving inequalities uses the same steps as solving equations except when multiplying or dividing by a negative number. In this case, the inequality symbol must be reversed.
e.g. $4 - 2x < 10$ (-4)
$-2x < 6$ $(\div -2)$
$x > -3$ (reverse symbol)

Simultaneous equations
Use substitution or elimination to find the solution that satisfies two equations.

Substitution
e.g. $2x + y = 12$ [1]
$y = x + 3$ [2]
In [1] replace y with [2]
$2x + (x + 3) = 12$
$3x + 3 = 12$
$x = 3$
Substitute $x = 3$ to find y
In [2] $y = (3) + 3 = 6$

Elimination
Ensure both equations have a matching pair.
Add 2 equations if matching pair has different sign; subtract if same sign.

e.g. $x + 2y = 2$ [1]
$2x + 3y = 5$ [2]
[1] × 2 $2x + 4y = 4$ [3]
[3] – [2] $y = -1$
In [1] $x + 2(-1) = 2$
$x - 2 = 2$
$\therefore x = 4$

Formulas
Some common formulas
e.g. $A = \pi r^2$, $C = 2\pi r$
An unknown value can be found by substituting values for the other variables.
A formula can be transposed to make a different variable the subject.
e.g. make r the subject in $A = \pi r^2$
$A = \pi r^2$
$\frac{A}{\pi} = r^2$
$\sqrt{\frac{A}{\pi}} = r$
$\therefore r = \sqrt{\frac{A}{\pi}}$ where $r > 0$

Simple quadratic equations
$x^2 = 9$
$x = \pm\sqrt{9}$
$x = \pm 3$
$x^2 = -7$ has no solutions

$16x^2 = 9$
$x^2 = \frac{9}{16}$
$x = \pm\sqrt{\frac{9}{16}}$
$x = \pm\frac{3}{4}$

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Chapter checklist

Chapter checklist with success criteria

A printable version of this checklist is available in the Interactive Textbook

		✔
2A	**1. I can convert between algebraic expressions and words.** e.g. Write an algebraic expression for the cost of t tickets at \$20 each.	☐
2A	**2. I can substitute values into expressions and evaluate.** e.g. Evaluate $a^2 + 3(ab - c)$ if $a = -2$, $b = 3$ and $c = -8$.	☐
2B	**3. I can multiply algebraic terms.** e.g. Simplify $4x \times 7xy$.	☐
2B	**4. I can divide algebraic terms.** e.g. Simplify $12xy \div (18x)$.	☐
2B	**5. I can collect like terms under addition and subtraction.** e.g. Simplify $4xy - 5x^2y - xy + 3x^2y$.	☐
2C	**6. I can expand brackets and simplify.** e.g. Expand $4x(3x - 5)$.	☐
2C	**7. I can simplify by removing brackets.** e.g. Expand and simplify $7 - 3(3x - 2)$.	☐
2D	**8. I can solve simple linear equations.** e.g. Solve $10 - 2x = 16$.	☐
2D	**9. I can solve linear equations involving fractions.** e.g. Solve $\frac{4x - 2}{3} = 6$.	☐
2E	**10. I can solve linear equations with brackets.** e.g. Solve $2(4x + 3) = 7$.	☐
2E	**11. I can solve linear equations with pronumerals on both sides.** e.g. Solve $6x + 5 = 3x - 13$.	☐
2F	**12. I can turn a word problem into an equation.** e.g. Write an equation to represent that four more than a certain number is three less than two times the number.	☐
2F	**13. I can solve a word problem using an equation.** e.g. Ava and Elise have 27 soft toys between them. Ava has 5 more than Elise. Use an equation to find how many soft toys they each have.	☐
2G	**14. I can represent an inequality on a number line.** e.g. Show x is less than or equal to 3 ($x \leqslant 3$) on a number line.	☐
2G	**15. I can solve a linear inequality.** e.g. Find the solution set for $8 - 3x < 20$.	☐
2H	**16. I can substitute values into formulas.** e.g. Evaluate the formula $c = \sqrt{a^2 + b^2}$ when $a = 9$ and $b = 12$.	☐
2H	**17. I can find the unknown value in a formula.** e.g. In the formula $s = ut + \frac{1}{2}at^2$, find a if $s = 80$, $u = 10$ and $t = 4$.	☐

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2H	**18. I can transpose formulas.** e.g. Transpose $V = \pi r^2 h$ to make r the subject.	☐
2I	**19. I can solve simultaneous linear equations using substitution.** e.g. Solve the following pair of simultaneous equations using substitution: $x + 2y = 4$ $y = 2x - 3$.	☐
2J	**20. I can solve simultaneous linear equations using elimination.** e.g. Solve the following pair of simultaneous equations using elimination: $3x + 2y = 3$ $2x - y = -5$.	☐
2K	**21. I can solve word problems with simultaneous equations.** e.g. At the football, 2 meat pies and 3 bottles of water cost \$21 while 3 meat pies and 5 bottles of water cost \$33. How much does each meat pie and each bottle of water cost?	☐
2L	**22. I can solve quadratic equations of the form $ax^2 = c$** e.g. Solve the equation $2x^2 - 2 = 2$	☐

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Chapter review

Short-answer questions

2A **1** Write algebraic expressions to represent the following.

a The product of m and 7
b Twice the sum of x and y
c The cost of 3 movie tickets at m dollars each
d n divided by 4 less 3

2A **2** Evaluate the following when $x = 2$, $y = -1$ and $z = 5$.

a $yz - x$
b $\frac{2z - 4y}{x}$
c $\frac{2z^2}{x} + y$
d $x(y + z)$

2B **3** Simplify.

a $2m \times 4n$
b $\frac{4x^2y}{12x}$
c $3ab \times 4b \div (2a)$
d $4 - 5b + 2b$
e $3mn + 2m - 1 - nm$
f $4p + 3q - 2p + q$

2C **4** Expand and simplify the following.

a $2(x + 7)$
b $-3(2x + 5)$
c $2x(3x - 4)$
d $-2a(5 - 4a)$
e $5 - 4(x - 2)$
f $4(3x - 1) - 3(2 - 5x)$

2D **5** Solve the following linear equations for x.

a $5x + 6 = 51$
b $\frac{x + 2}{4} = 7$
c $\frac{2x}{5} - 3 = 3$
d $\frac{2x - 5}{3} = -1$
e $7x - 4 = 10$
f $3 - 2x = 21$
g $1 - \frac{4x}{5} = 9$
h $2 - 7x = -3$

2F **6** Write an equation to represent each of the following and then solve it for the pronumeral.

a A number, n, is doubled and increased by 3 to give 21.
b The number of lollies, l, is decreased by 5 and then shared equally among three friends so that they each get 7 lollies.
c 5 less than the result of Toni's age, x, divided by 4 is 0.

2E **7** Solve the following linear equations.

a $2(x + 4) = 18$
b $3(2x - 3) = 2$
c $8x = 2x + 24$
d $5(2x + 4) = 7x + 5$
e $3 - 4x = 7x - 8$
f $1 - 2(2 - x) = 5(x - 3)$

2F **8** Nick makes an initial bid of \$$x$ in an auction for a signed cricket bat. By the end of the auction he has paid \$550, which is \$30 more than twice his initial bid. Set up and solve an equation to determine Nick's initial bid.

2G **9** Represent each of the following on a number line.

a $x > 1$
b $x \leqslant 7$
c $x \geqslant -4$
d $x < -2$
e $2 < x \leqslant 8$
f $-1 < x < 3$

2G **10** Solve the following inequalities.

a $x + 8 < 20$
b $2m - 4 > -6$
c $3 - 2y \leqslant 15$
d $\frac{7 - 2x}{3} > -9$
e $3a + 9 < 7(a - 1)$
f $-4x + 2 \leqslant 5x - 16$

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2G **11** A car salesman earns \$800 per month plus a 10% commission on the value of sales he makes for the month. If he is aiming to earn a minimum of \$3200 a month, what is the possible value of sales that will enable this?

2H **12** Find the value of the unknown in each of the following formulas.

a $E = \sqrt{PR}$ when $P = 90$ and $R = 40$

b $v = u + at$ when $v = 20$, $u = 10$, $t = 2$

c $V = \frac{1}{3}Ah$ when $V = 20$, $A = 6$

2H **13** Rearrange the following formulas to make the variable in brackets the subject.

a $v^2 = u^2 + 2ax$ $[x]$

b $A = \frac{1}{2}r^2\theta$ $[\theta]$

c $P = RI^2$, where $I > 0$ $[I]$

d $S = \frac{n}{2}(a + l)$ $[a]$

2I/J **14** Solve the following simultaneous equations using the substitution method for parts **a**–**c** and the elimination method for **d**–**f**.

a $x + 2y = 12$
$x = 4y$

b $2x + 3y = -6$
$y = x - 1$

c $7x - 2y = 6$
$y = 2x + 3$

d $x + y = 15$
$x - y = 7$

e $3x + 2y = -19$
$4x - y = -7$

f $3x - 5y = 7$
$5x + 2y = 22$

2K **15** The sum of the ages of Billy and his sister is 32. If Billy is 8 years older than his sister, what are their ages?

16 Solve the following quadratic equations.

a $2x^2 = 8$

b $5x^2 = 35$

c $-3x^2 = -30$

d $6x^2 + 2 = 8, x < 0$

Multiple-choice questions

2A **1** The algebraic expression that represents 2 less than 3 lots of n is:

A $3(n - 2)$ **B** $2 - 3n$ **C** $3n - 2$
D $3 + n - 2$ **E** n

2B **2** The simplified form of $6ab + 14a \div 2 - 2ab$ is:

A $8ab$ **B** $8ab + 7a$ **C** $ab + 7a$
D $4ab + 7a$ **E** $4ab + \frac{7a}{2}$

2D **3** The solution to $\frac{x}{3} - 1 = 4$ is:

A $x = 13$ **B** $x = 7$ **C** $x = 9$
D $x = 15$ **E** $x = \frac{5}{3}$

2F **4** The result when a number is tripled and increased by 21 is 96. The original number is:

A 22 **B** 32 **C** 25
D 30 **E** 27

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2E **5** The solution to the equation $3(x - 1) = 5x + 7$ is:

A $x = -4$ **B** $x = -5$ **C** $x = 5$
D $x = 3$ **E** $x = 1$

2F **6** $\$x$ is raised from a sausage sizzle. Once the \$50 running cost is taken out, the money is shared equally among three charities so that they each get \$120. An equation to represent this is:

A $\frac{x - 50}{3} = 120$
B $\frac{x}{3} - 50 = 120$
C $\frac{x}{50} = 360$
D $\frac{x}{3} = 310$
E $3x + 50 = 120$

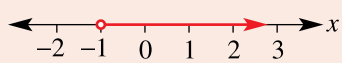
2H **7** If $A = 2\pi rh$ with $A = 310$ and $r = 4$, then the value of h is closest to:

A 12.3 **B** 121.7 **C** 24.7
D 38.8 **E** 10.4

2H **8** The formula $d = \sqrt{\frac{a}{b}}$ transposed to make a the subject is:

A $a = \sqrt{bd}$ **B** $a = d\sqrt{b}$ **C** $a = \frac{d^2}{b^2}$
D $a = \frac{d}{\sqrt{b}}$ **E** $a = bd^2$

2G **9** The inequality representing the x-values on the number line below is:



A $x < -1$ **B** $x > -1$ **C** $x \leqslant -1$
D $x \geqslant -1$ **E** $-1 < x < 3$

2G **10** The solution to the inequality $1 - 2x > 9$ is:

A $x < -4$ **B** $x < 4$ **C** $x < -5$
D $x > -4$ **E** $x > 5$

2I **11** The solution to the simultaneous equations $x + 2y = 16$ and $y = x - 4$ is:

A $x = 4, y = 0$ **B** $x = 8, y = 4$ **C** $x = 6, y = 2$
D $x = 12, y = 8$ **E** $x = 5, y = 1$

2J **12** The solution to the simultaneous equations $2x + y = 2$ and $2x + 3y = 10$ is:

A $x = 0, y = 2$ **B** $x = 2, y = 2$ **C** $x = 2, y = -2$
D $x = -1, y = 4$ **E** $x = -2, y = 6$

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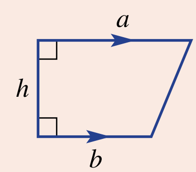
Extended-response questions

1 The area of a trapezium is given by $A = \frac{h}{2}(a + b)$. A new backyard deck in the shape of the trapezium shown is being designed.



Currently the dimensions are set such that $a = 12$ m and $h = 10$ m.

- **a** What range of b values is required for an area of at most $110\,\text{m}^2$?
- **b** Rearrange the area formula to make b the subject.
- **c** Use your answer to part **b** to find the length b m required to give an area of 100 m^2.
- **d** Rearrange the area formula to make h the subject.
- **e** If b is set as 8 m, what does the breadth of the deck (h m) need to be reduced to for an area of 80 m^2?

2 Members of the Hayes and Thompson families attend the local Regatta by the Bay.

- **a** The entry fee for an adult is \$18 and that for a student is \$8. The father and son from the Thompson family notice that after paying the entry fees and for 5 rides for the son and 3 for the adult, they have each spent the same amount. If the cost of a ride is the same for an adult and a student, write an equation and solve it to determine the cost of a ride.
- **b** For lunch each family purchases some buckets of hot chips and some drinks.
  The Hayes family buys 2 drinks and 1 bucket of chips for \$11 and the Thompson family buys 3 drinks and 2 buckets of chips for \$19. To determine how much each bucket of chips and each drink costs, complete the following steps.
  - **i** Define two variables to represent the problem.
  - **ii** Set up two equations relating the variables.
  - **iii** Solve your equations in part **b ii** simultaneously.
  - **iv** What is the cost of a bucket of chips and the cost of a drink?

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3

Right-angled triangles: Pythagoras’ theorem and trigonometry

Maths in context: Aerial and undersea drones

UAVs (unmanned aerial vehicles), or drones, have an increasing number of applications including for aerial photography, mapping, surveying, agricultural monitoring, crop spraying, bird control, firefighting, product deliveries, and emergency responses.

The UAV’s flight dynamics were first represented using mathematics and then coded. Its GPS sensor receives time signals and position data from 4 GPS satellites and the UAV’s algorithms instantly calculate its distance from each satellite. Algorithms apply Pythagoras’ theorem and trigonometry in 3D, computing the UAV’s latitude, longitude, and altitude.

Using data inputs from its GPS, magnetometer and accelerometer, the UAV’s algorithms apply trigonometry in 3D to calculate the direction and distance to fly to a given destination. It can follow detailed instructions such as distances, bearings, or the GPS of where to photograph, turn, hover, orbit, and land.

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Chapter contents

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- applies trigonometric ratios to solve right-angled triangle problems (MA5-TRG-C-01)
- applies trigonometry to solve problems, including bearings and angles of elevation and depression (MA5-TRG-C-02).

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

Underwater drones invented and coded by Australian scientists and engineers are becoming smaller and smarter. They can destroy marine pests like crown-of-thorns starfish and help restore coral reef health by releasing coral larvae. Other applications include surveying, monitoring, and photographing coral reefs and other seafloor structures, such as the foundations of offshore wind turbines. Also, they collect valuable data on ocean temperatures, oxygen, and carbon dioxide levels.

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3A Pythagoras' theorem CONSOLIDATING

Learning intentions
- To know the relationship between the square of the sides of a right-angled triangle
- To be able to use Pythagoras' theorem to find the length of the hypotenuse given two sides
- To be able to find hypotenuse lengths as rounded decimals or exact values

Past, present and future learning
- This section consolidates Stage 4 concepts which are used in Stages 5 and 6
- Some of these questions are more challenging than those in the Year 8 book
- These concepts will be revised and extended in Chapter 4 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced
- You may be expected to recall the formula from memory in the HSC Examination

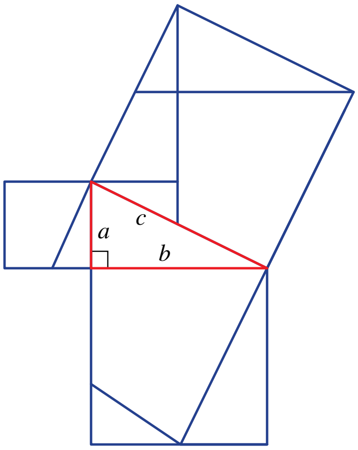
Pythagoras was born on the Greek island of Samos in the 6th century BCE. He received a privileged education and travelled to Egypt and Persia where he developed his ideas in mathematics and philosophy. He settled in Crotone, Italy, where he founded a school. His many students and followers were called the Pythagoreans, and under the guidance of Pythagoras they lived a very structured life with strict rules. They aimed to be pure, self-sufficient and wise, where men and women were treated equally and all property was considered communal. They strove to perfect their physical and mental form, and made many advances in their understanding of the world through mathematics.

Land surveyors measure the horizontal and vertical distance between two locations of different altitudes. The straight line distance of the slope, which forms a hypotenuse, is calculated using Pythagoras' theorem.

The Pythagoreans discovered the famous theorem, which is named after Pythagoras, and the existence of irrational numbers such as $\sqrt{2}$, which cannot be written down as a fraction or terminating decimal. Such numbers cannot be measured exactly with a ruler with fractional parts and were thought to be unnatural. The Pythagoreans called these numbers 'unutterable' numbers and it is believed that any member of the brotherhood who mentioned these numbers in public would be put to death.

Lesson starter: Matching the areas of squares

Look at this right-angled triangle and the squares drawn on each side. Each square is divided into smaller sections.



- Can you see how the parts of the two smaller squares would fit into the larger square?
- What is the area of each square if the side lengths of the right-angled triangle are a, b and c as marked?
- What do the answers to the above two questions suggest about the relationship between a, b and c?

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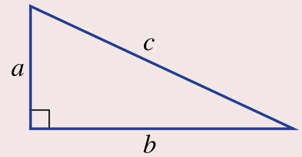
KEY IDEAS

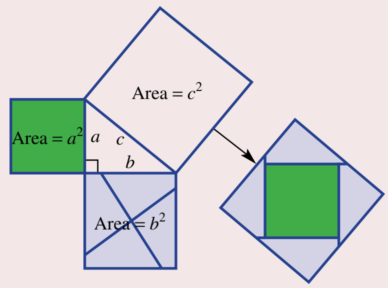
- The longest side of a right-angled triangle is called the **hypotenuse** and is opposite the right angle.
- The **theorem of Pythagoras** says that the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.
  For the triangle shown, it is:

  $c^2 = a^2 + b^2$

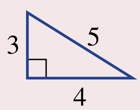
  square of the hypotenuse — squares of the two shorter sides

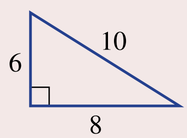


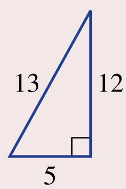
  - The theorem can be illustrated in a diagram like the one on the right. The sum of the areas of the two smaller squares ($a^2 + b^2$) is the same as the area of the largest square (c^2).



- Lengths can be expressed with **exact values** using **surds**.
  $\sqrt{2}$, $\sqrt{28}$ and $2\sqrt{3}$ are examples of surds.
  - When expressed as a decimal, a surd is an infinite non-recurring decimal with no pattern.
    For example: $\sqrt{2} = 1.4142135623\ldots$
- A **Pythagorean** triad is a set of three whole numbers that can be used to form a right-angled triangle. Some of the best known examples are:



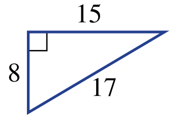




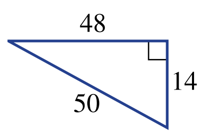
BUILDING UNDERSTANDING

1 State the length of the hypotenuse (c units) in these right-angled triangles.

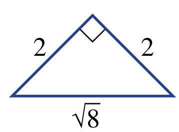
a



b

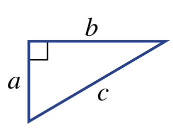


c

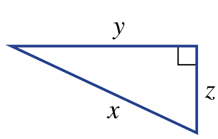


2 State Pythagoras' theorem using the given pronumerals for these right-angled triangles. For example: $z^2 = x^2 + y^2$.

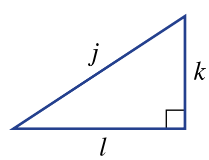
a



b



c



3 Evaluate the following, rounding to two decimal places in part **d**.

a 3.2^2 **b** $3^2 + 2^2$ **c** $\sqrt{64 + 36}$ **d** $\sqrt{24}$

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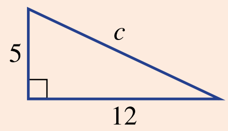
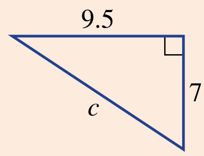
Example 1 Finding the length of the hypotenuse

Find the length of the hypotenuse in these right-angled triangles. Round to two decimal places in part **b**.

a



b



SOLUTION

a
$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 5^2 + 12^2 \\ &= 169 \\ \therefore c &= \sqrt{169} \\ &= 13 \end{aligned}$$

b
$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 7^2 + 9.5^2 \\ &= 139.25 \\ \therefore c &= \sqrt{139.25} \\ &= 11.80 \text{ (to 2 d.p.)} \end{aligned}$$

EXPLANATION

Write the rule and substitute the lengths of the two shorter sides.

If $c^2 = 169$ then $c = \sqrt{169} = 13$.

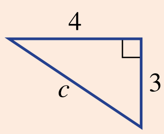
The order for a and b does not matter since $7^2 + 9.5^2 = 9.5^2 + 7^2$.

Round as required.

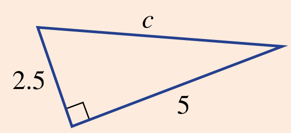
Now you try

Find the length of the hypotenuse in these right-angled triangles. Round to two decimal places in part **b**.

a

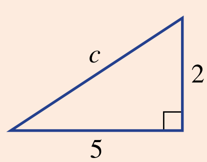


b



Example 2 Finding the length of the hypotenuse using exact values

Find the length of the hypotenuse in this right-angled triangle, leaving your answer as an exact value.



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SOLUTION

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 5^2 + 2^2 \\ &= 29 \\ \therefore c &= \sqrt{29} \end{aligned}$$

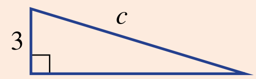
EXPLANATION

Apply Pythagoras' theorem to find the value of c.

Express the answer exactly using a surd.

Now you try

Find the length of the hypotenuse in this right-angled triangle, leaving your answer as an exact value.

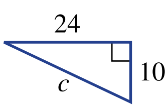


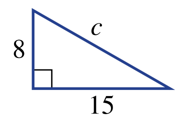
Exercise 3A

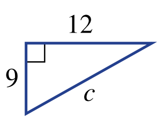
FLUENCY	1–4(½)	1–5(½)	1–5(⅓)

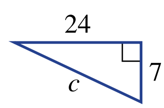
Example 1a

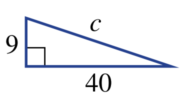
1 Find the length of the hypotenuse in each of the following right-angled triangles.

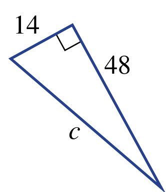
a


b


c


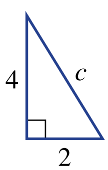
d


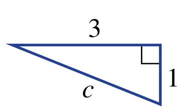
e


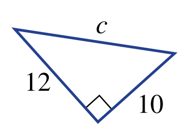
f


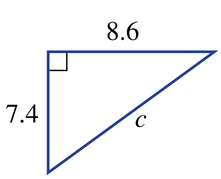
Example 1b

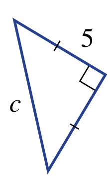
2 Find the length of the hypotenuse in each of these right-angled triangles, correct to two decimal places.

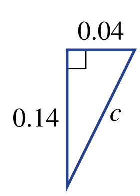
a


b


c


d


e


f


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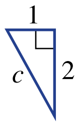
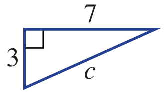
Example 2

3 Find the length of the hypotenuse in these triangles, leaving your answer as an exact value.

a


b


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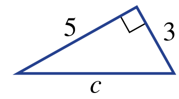
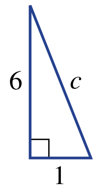
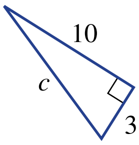


d


e


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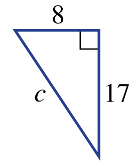
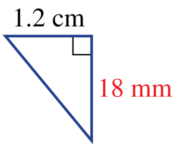


4 Find the length of the hypotenuse in each of these right-angled triangles, rounding to two decimal places where necessary. Convert to the units indicated in red.

a


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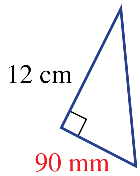

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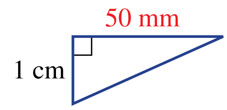
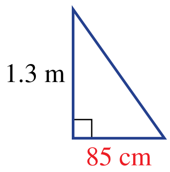


d


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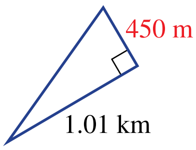

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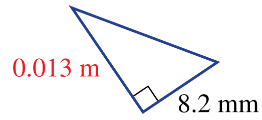
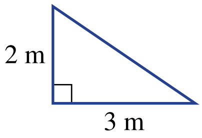


5 For each of these triangles, first calculate the length of the hypotenuse then find the perimeter, correct to two decimal places.

a


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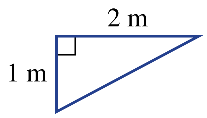

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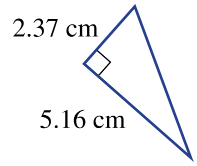

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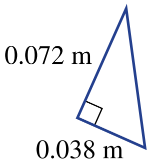

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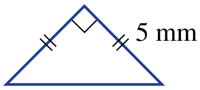

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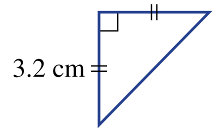
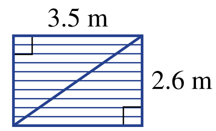


PROBLEM–SOLVING	6–8	6–9	8–11

6 Find the length of the diagonal steel brace required to support a wall of length 3.5 m and height 2.6 m. Give your answer correct to one decimal place.



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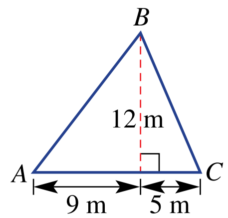
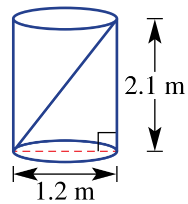
7 Find the perimeter of this triangle. (*Hint*: You will need to find AB and BC first.)

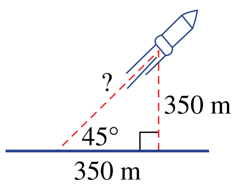


8 A helicopter hovers at a height of 150 m above the ground and is a horizontal distance of 200 m from a beacon on the ground. Find the direct distance of the helicopter from the beacon.

9 Find the length of the longest rod that will fit inside a cylinder of height 2.1 m and circular end surface of diameter 1.2 m. Give your answer correct to one decimal place.



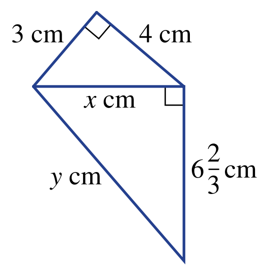
10 A miniature rocket blasts off at an angle of 45° and travels in a straight line. After a few seconds, it reaches a height of 350 m above the ground. At this point it has also covered a horizontal distance of 350 m. How far has the rocket travelled to the nearest metre?



11 For the shape shown, find the value of:

a x

b y (as a fraction).

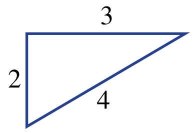


REASONING	12	12, 13(½)	13(½), 14, 15

12 One way to check whether a four-sided figure is a rectangle is to ensure that both its diagonals are the same length. What should the length of the diagonals be if a rectangle has side lengths 3 m and 5 m? Answer to two decimal places.

13 We know that if the triangle has a right angle, then $c^2 = a^2 + b^2$. The converse of this is that if $c^2 = a^2 + b^2$ then the triangle must have a right angle. Test whether $c^2 = a^2 + b^2$ to see if these triangles have a right angle. They may not be drawn to scale.

a



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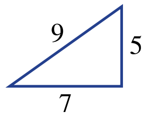

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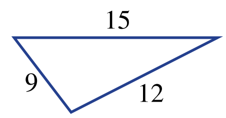

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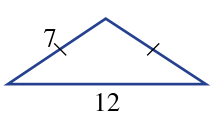

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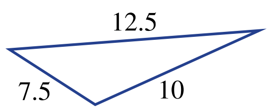

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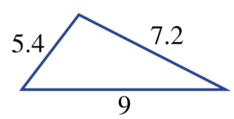

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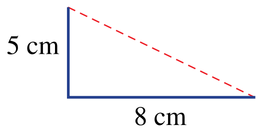
14 We saw in Question **13** the test for a right-angled triangle. We can also use the theorem to determine if a triangle is acute-angled or obtuse-angled.



a If the two shorter sides of a triangle are 5 cm and 8 cm, what is the length of the longest side if the triangle is right-angled? Answer to one decimal place.

b Use your answer to part **a** to determine if a triangle with lengths 5 cm, 8 cm and the length given is acute-angled or obtuse-angled.

i 9 cm

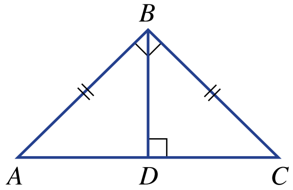
ii 11 cm

c Complete the following.

i If $a^2 + b^2 < c^2$, the triangle is ____________-angled.

ii If $a^2 + b^2 > c^2$, the triangle is ____________-angled.

15 Triangle ABC is a right-angled isosceles triangle, and BD is perpendicular to AC. $DC = 4$ cm and $BD = 4$ cm.



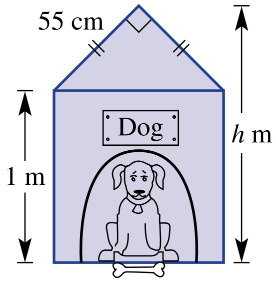
a Find the length of BC correct to two decimal places.

b State the length of AB correct to two decimal places.

c Use Pythagoras' theorem and ΔABC to check that the length of AC is twice the length of DC.

ENRICHMENT: Kennels and kites	–	–	16, 17

16 A dog kennel has the dimensions shown in the diagram on the right. Give your answers to each of the following correct to two decimal places.



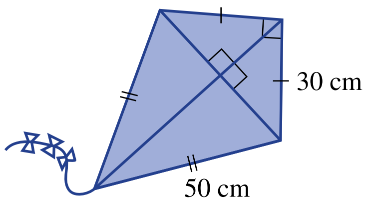
a What is the breadth, in cm, of the kennel?

b What is the total height, h m, of the kennel?

c If the sloping height of the roof was to be reduced from 55 cm to 50 cm, what difference would this make to the total height of the kennel in centimetres? (Assume that the breadth is the same as in part **a**.)

d What is the length, in cm, of the sloping height of the roof of a new kennel if it is to have a total height of 1.2 m? (The height of the kennel without the roof is still 1 m and its breadth is unchanged.)

17 The frame of a kite is constructed with six pieces of timber dowel. The four pieces around the outer edge are two 30 cm pieces and two 50 cm pieces as shown. The top end of the kite is to form a right angle. Find the length of each of the diagonal pieces required to complete the construction. Answer to two decimal places.



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3B Finding the length of the shorter sides CONSOLIDATING

Learning intentions

- To be able to set up Pythagoras' theorem where the unknown is a shorter side
- To be able to solve the resulting equation to find the length of a shorter side

Past, present and future learning

- This section consolidates Stage 4 concepts which are used in Stages 5 and 6
- Some of these questions are more challenging than those in the Year 8 book
- These concepts will be revised and extended in Chapter 4 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced
- You may be expected to recall the formula from memory in the HSC Examination

Throughout history, mathematicians have used known theorems to explore new ideas, discover new theorems and solve a wider range of problems. Similarly, Pythagoras knew that his right-angled triangle theorem could be manipulated so that the length of one of the shorter sides of a triangle can be found if the length of the other two sides are known.

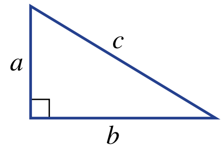
Knowing the vertical height between floors and an escalator's length, engineers can use Pythagoras' theorem to calculate the minimum horizontal breadth of floor space needed by an escalator.

We know that the sum $7 = 3 + 4$ can be written as a difference $3 = 7 - 4$. Likewise, if $c^2 = a^2 + b^2$ then $a^2 = c^2 - b^2$ or $b^2 = c^2 - a^2$.

Applying this to a right-angled triangle means that we can now find the length of one of the shorter sides if the other two sides are known.

Lesson starter: True or false

Some mathematical statements relating to a right-angled triangle with hypotenuse c and the two shorter sides a and b are shown.



Some of these mathematical statements are true and some are false. Can you sort them into true and false groups?

$a^2 + b^2 = c^2$	$a = \sqrt{c^2 - b^2}$	$c^2 - a^2 = b^2$	$a^2 - c^2 = b^2$
$c = \sqrt{a^2 + b^2}$	$b = \sqrt{a^2 - c^2}$	$c = \sqrt{a^2 - b^2}$	$c^2 - b^2 = a^2$

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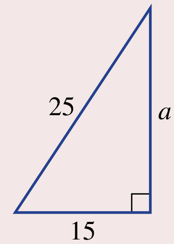
KEY IDEAS

- When finding the length of a side:
  - substitute known values into Pythagoras' theorem
  - solve this equation to find the unknown value.

  For example:
  - If $a^2 + 16 = 30$ then subtract 16 from both sides.
  - If $a^2 = 14$ then take the square root of both sides.
    - $a = \sqrt{14}$ is an **exact** answer (a surd).
    - $a = 3.74$ is a rounded decimal answer.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ 25^2 &= a^2 + 15^2 \\ 625 &= a^2 + 225 \\ 400 &= a^2 \\ a &= \sqrt{400} \\ a &= 20 \end{aligned}$$



BUILDING UNDERSTANDING

1 Find the value of a or b in these equations. (Both a and b are positive numbers.)

a $a = \sqrt{196}$ **b** $a^2 = 144$ **c** $b^2 + 9 = 25$ **d** $36 + b^2 = 100$

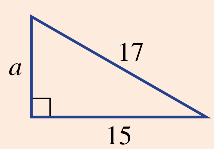
2 Given that $a^2 + 64 = 100$, decide whether the following are true or false.

a $a^2 = 100 - 64$ **b** $64 = 100 + a^2$ **c** $100 = \sqrt{a^2 + 64}$
d $a = \sqrt{100 - 64}$ **e** $a = 6$ **f** $a = 10$

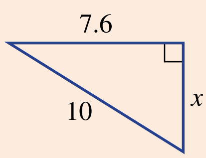
Example 3 Finding the length of a shorter side

In each of the following, find the value of the pronumeral. Round your answer in part **b** to two decimal places and give an exact answer to part **c**.

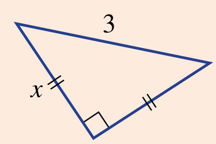
a



b



c



SOLUTION

a
$$\begin{aligned} a^2 + 15^2 &= 17^2 \\ a^2 + 225 &= 289 \\ a^2 &= 64 \\ a &= \sqrt{64} \\ a &= 8 \end{aligned}$$

b
$$\begin{aligned} x^2 + 7.6^2 &= 10^2 \\ x^2 + 57.76 &= 100 \\ x^2 &= 42.24 \\ x &= \sqrt{42.24} \\ x &= 6.50 \text{ (to 2 d.p.)} \end{aligned}$$

EXPLANATION

a Write the rule and substitute the known sides.
Square 15 and 17.
Subtract 225 from both sides.
Take the square root of both sides.

b Write the rule.
Subtract 57.76 from both sides.
Take the square root of both sides.
Round to two decimal places.

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c $x^2 + x^2 = 3^2$ — Two sides are of length x.

$2x^2 = 9$ — Add like terms and then divide both sides by 2.

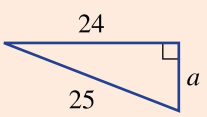
$x^2 = \frac{9}{2}$ — Take the square root of both sides. To express as an exact answer, do not round.

$x = \sqrt{\frac{9}{2}}\left(= \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}\right)$ — Different forms are possible.

Now you try

In each of the following, find the value of the pronumeral. Round your answer in part **b** to two decimal places and give an exact answer to part **c**.

a



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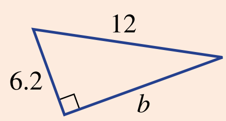

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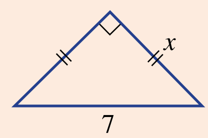


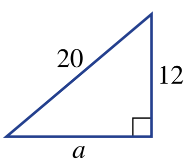
Exercise 3B

FLUENCY	1–3(½), 4	1–3(½), 4	1–3(⅓), 4

Example 3a

1 In each of the following, find the value of the pronumeral.

a



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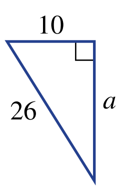

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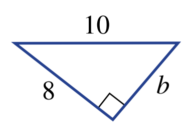

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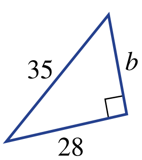

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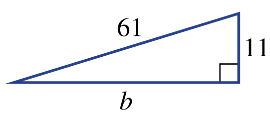

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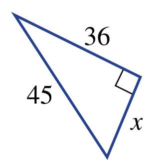
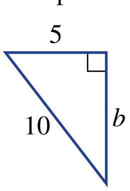


Example 3b

2 In each of the following, find the value of the pronumeral. Express your answers correct to two decimal places.

a



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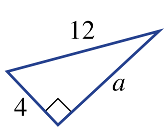

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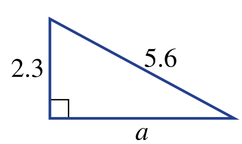

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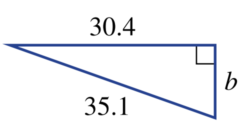

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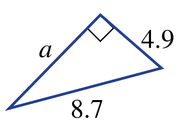

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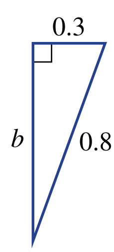

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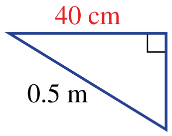
3 Find the length of the unknown side of each of these triangles, correct to two decimal places where necessary. Convert to the units shown in red.

a


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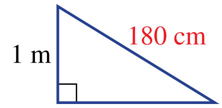

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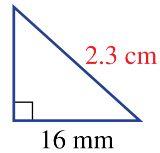

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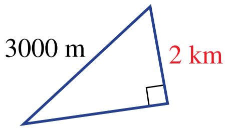

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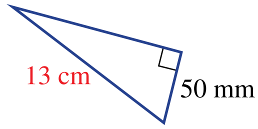

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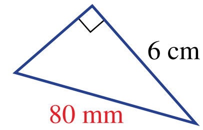
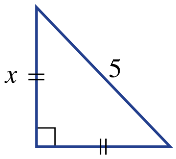
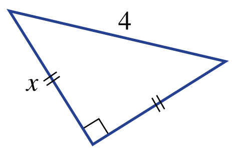


Example 3c

4 In each of the following, find the value of x as an exact answer.

a


b


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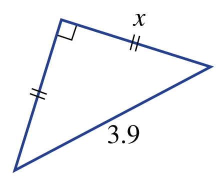
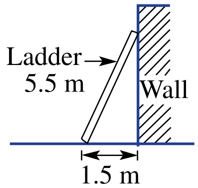


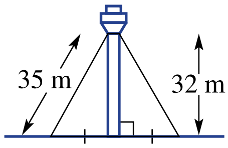
PROBLEM–SOLVING

PROBLEM–SOLVING	5–7	6–8	7–9

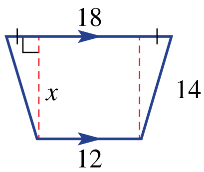
5 The base of a ladder leaning against a vertical wall is 1.5 m from the base of the wall. If the ladder is 5.5 m long, find how high the top of the ladder is above the ground, correct to one decimal place.



6 A 32 m communication tower is supported by 35 m cables stretching from the top of the tower to a position at ground level as shown. Find the distance from the base of the tower to the point where the cable reaches the ground, correct to one decimal place.



7 For each of the following diagrams, find the value of x. Give an exact answer each time.

a


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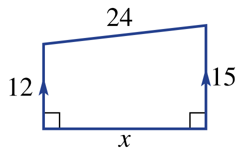
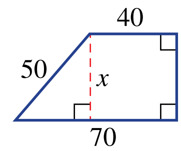
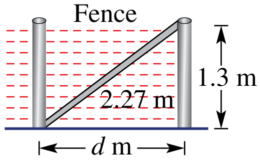


c


8 If a television has a screen size of 63 cm it means that the diagonal length of the screen is 63 cm. If the vertical height of a 63 cm screen is 39 cm, find the breadth of the screen to the nearest centimetre.

9 Two 1.3 m vertical fence posts are supported by a 2.27 m bar, as shown in the diagram on the right. Find the distance (d m) from the base of one post to the base of the second post. Give your answer correct to two decimal places.



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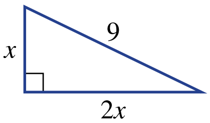
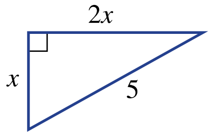
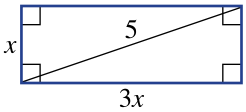
REASONING	10	10	10, 11

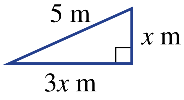
10 For these questions note that $(2x)^2 = 4x^2$ and $(3x)^2 = 9x^2$.
Find the value of x as an exact answer.

a 

b 

c 

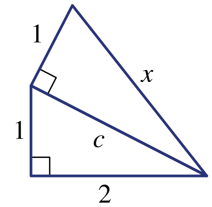
11 A right-angled triangle has a hypotenuse measuring 5 m. Find the lengths of the other sides if their lengths are in the given ratio. Give an exact answer.
(*Hint*: You can draw a triangle like the one shown for part **a**.)



a 1 to 3
b 2 to 3
c 5 to 7

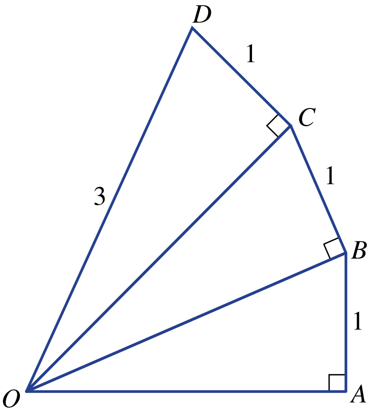
ENRICHMENT: The power of exact values	–	–	12, 13

12 Consider this diagram and the unknown length x.



a Explain what needs to be found first before x can be calculated.
b Now try calculating the value x as an exact value.
c Was it necessary to calculate the value of c or was c^2 enough?
d What problems might be encountered if the value of c is calculated and rounded before the value of x is found?

13 In the diagram at right, $OD = 3$ and $AB = BC = CD = 1$.



a Using exact values find the length of:
i OC ii OB iii OA
b Round your answer in part **a iii** to one decimal place and use that length to recalculate the lengths of OB, OC and OD (correct to two decimal places) starting with ΔOAB.
c Explain the difference between the given length $OD = 3$ and your answer for OD in part **b**.
d Investigate how changing the side length AB affects your answers to parts **a** to **c** above.

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3C Using Pythagoras' theorem to solve two-dimensional problems CONSOLIDATING

Learning intentions
- To be able to identify right-angled triangles in a two-dimensional situation
- To be able to use Pythagoras' theorem to solve a problem

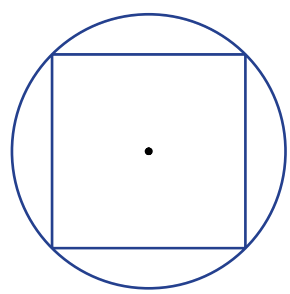
Past, present and future learning
- This section consolidates Stage 4 concepts which are used in Stages 5 and 6
- Some of these questions are more challenging than those in the Year 8 book
- These concepts will be revised and extended in Chapter 4 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced
- You may be expected to recall the formula from memory in the HSC Examination

Initially it may not be obvious that Pythagoras' theorem can be used to help solve a particular problem. With further investigation, however, it may be possible to identify and draw a right-angled triangle that can help solve the problem. As long as two sides of the right-angled triangle are known, the length of the third side can be found.

Engineers and architects regularly use Pythagoras' theorem, for example, to calculate the length of each cable on the Anzac Bridge, Sydney.

Lesson starter: The biggest square

Imagine trying to cut the largest square from a circle of a certain size and calculating the side length of the square. Drawing a simple diagram as shown does not initially reveal a right-angled triangle.



- If the circle has a diameter of 2 cm, can you find a good position to draw the diameter that also helps to form a right-angled triangle?
- Can you determine the side length of the largest square?
- What percentage of the area of a circle does the largest square occupy?

KEY IDEAS

- When applying Pythagoras' theorem, follow these steps.
  - Identify and draw a right-angled triangle that may help to solve the problem.
  - Label the sides with their lengths or with a letter (pronumeral) if the length is unknown.
  - Use Pythagoras' theorem to solve for the unknown.
  - Solve the problem by making any further calculations and answering in words.
  - Check that the answer makes sense.

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BUILDING UNDERSTANDING

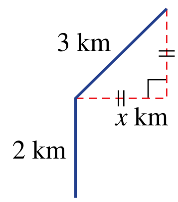
1 Match each problem (**a**, **b** or **c**) with both a diagram (**A**, **B** or **C**) and its solution (**I**, **II**, **III**).

a Two trees stand 20 m apart and they are 32 m and 47 m tall. What is the distance between the tops of the two trees?

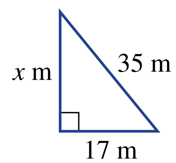
b A man walks due north for 2 km then north-east for 3 km. How far north is he from his starting point?

c A kite is flying with a kite string of length 35 m. Its horizontal distance from its anchor point is 17 m. How high is the kite flying?

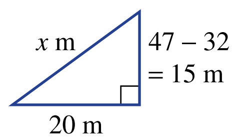
A



B



C



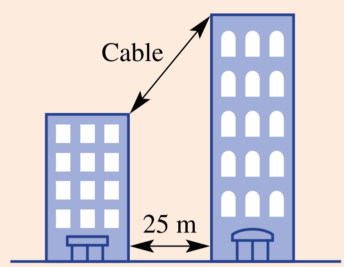
I The kite is flying at a height of 30.59 m.

II The distance between the top of the two trees is 25 m.

III The man has walked a total of $2 + 2.12 = 4.12$ km north from his starting point.

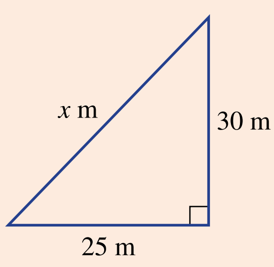
Example 4 Applying Pythagoras' theorem

Two skyscrapers are located 25 m apart and a cable links the tops of the two buildings. Find the length of the cable if the buildings are 50 m and 80 m in height. Give your answer correct to two decimal places.



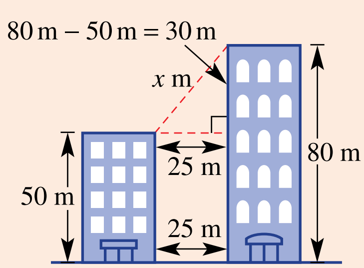
SOLUTION

Let x m be the length of the cable.



EXPLANATION

Draw a right-angled triangle and label the measurements and pronumerals.



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$c^2 = a^2 + b^2$ $x^2 = 25^2 + 30^2$ $x^2 = 625 + 900$ $= 1525$ $x = \sqrt{1525}$ $= 39.05$ (to 2 d.p.)	Set up an equation using Pythagoras' theorem and solve for x.
The cable is 39.05 m long.	Answer the question in words.

Now you try

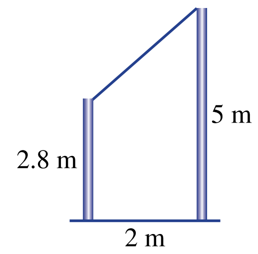
Two poles are located 40 m apart and a rope links the tops of the two poles. Find the length of the rope if the poles are 15 m and 21 m in height. Give your answer correct to two decimal places.

Exercise 3C

FLUENCY	1–4	1–5	2–5

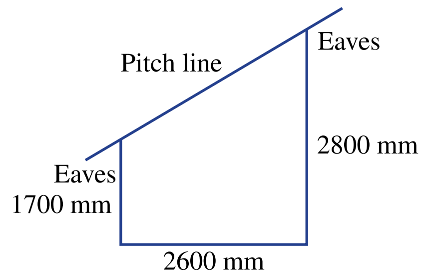
Example 4

1 Two poles are located 2 m apart. A wire links the tops of the two poles. Find the length of the wire if the poles are 2.8 m and 5 m in height. Give your answer correct to one decimal place.



2 Two skyscrapers are located 25 m apart and a cable of length 62.3 m links the tops of the two buildings. If the taller building is 200 metres tall, what is the height of the shorter building? Give your answer correct to one decimal place.

3 A garage is to be built with a skillion roof (a roof with a single slope). The measurements are given in the diagram. Calculate the pitch line length, to the nearest millimetre. Allow 500 mm for each of the eaves.



4 Two bushwalkers are standing on different mountain sides. According to their maps, one of them is at a height of 2120 m and the other is at a height of 1650 m. If the horizontal distance between them is 950 m, find the direct distance between the two bushwalkers. Give your answer correct to the nearest metre.

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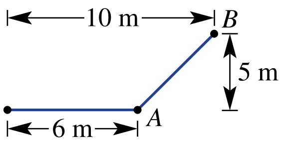
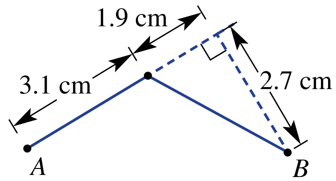
5 Find the direct distance between the points A and B in each of the following, correct to one decimal place.

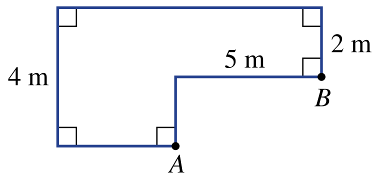
a



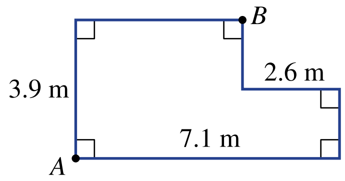
b



c



d



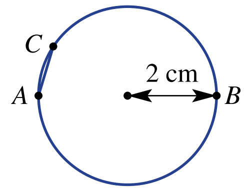
PROBLEM–SOLVING	6, 7	7, 8	7–9

6 A 100 m radio mast is supported by six cables in two sets of three cables. All six cables are anchored to the ground at an equal distance from the mast. The top set of three cables is attached at a point 20 m below the top of the mast. Each of the three lower cables is 60 m long and attached at a height of 30 m above the ground. If all the cables have to be replaced, find the total length of cable required. Give your answer correct to two decimal places.

7 In a particular circle of radius 2 cm, AB is a diameter and C is a point on the circumference. Angle ACB is a right angle. The chord AC is 1 cm in length.

a Draw the triangle ABC as described, and mark in all the important information.

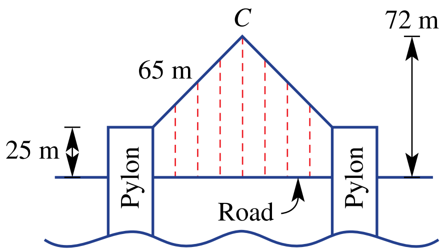
b Find the length of BC correct to one decimal place.



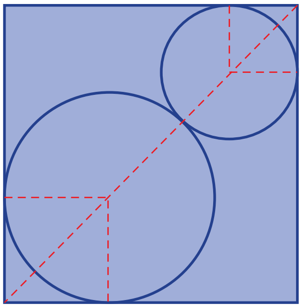
8 A suspension bridge is built with two vertical pylons and two straight beams of equal length that are positioned to extend from the top of the pylons to meet at a point C above the centre of the bridge, as shown in the diagram on the right.

a Calculate the vertical height of the point C above the tops of the pylons.

b Calculate the distance between the pylons, that is, the length of the span of the bridge, correct to one decimal place.



9 Two circles of radii 10 cm and 15 cm respectively are placed inside a square. Find the perimeter of the square to the nearest centimetre. (*Hint*: First find the diagonal length of the square using the diagram shown.)



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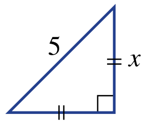
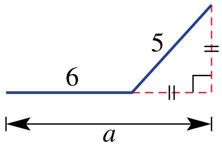
REASONING	10	10	10, 11

10 It is possible to find the length of the shorter sides of a right-angled isosceles triangle if only the hypotenuse length is known.

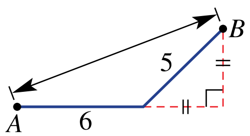
a Find the exact value of x in this right-angled isosceles triangle.



b Now find the exact value of a in this diagram.



c Finally, use your results from above to find the length of AB in this diagram correct to one decimal place.



11 Use the method outlined in Question **10** for this problem.
In an army navigation exercise, a group of soldiers hiked due south from base camp for 2.5 km to a water hole. From there, they turned 45° to the left, to head south-east for 1.6 km to a resting point.
When the soldiers were at the resting point, how far (correct to one decimal place):

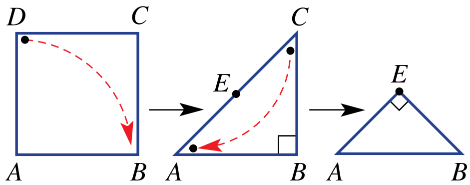
a east were they from the water hole?

b south were they from the water hole?

c were they in a straight line from base camp?

ENRICHMENT: Folding paper	–	–	12

12 A square piece of paper, $ABCD$, of side length 20 cm is folded to form a right-angled triangle ABC. The paper is folded a second time to form a right-angled triangle ABE as shown in the diagram.



a Find the length of AC correct to two decimal places.

b Find the perimeter of each of the following, correct to one decimal place where necessary:

i square $ABCD$ **ii** triangle ABC **iii** triangle ABE

c Use Pythagoras' theorem and your answer for part **a** to confirm that $AE = BE$ in triangle ABE.

d Investigate how changing the initial side length changes the answers to the above.

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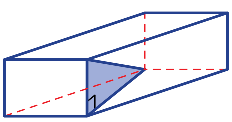
3D Using Pythagoras' theorem to solve three-dimensional problems

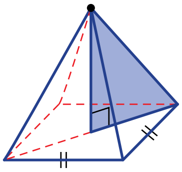
Learning intentions
- To understand how right-angled triangles can arise in three-dimensional situations
- To be able to identify a right-angled triangle in a solid
- To be able to apply Pythagoras' theorem in the identified triangle

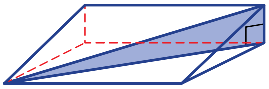
Past, present and future learning
- This section addresses the Stage 5 Path Topic called *Trigonometry C*
- These concepts will be revised and extended in Chapter 4 of our Year 10 book and also in Stage 6 Advanced
- You may be expected to recall the formula from memory in the HSC Examination

If you cut a solid to form a cross-section, a two-dimensional shape is revealed. From that cross-section it may be possible to identify a right-angled triangle that can be used to find unknown lengths. These lengths can then tell us information about the three-dimensional solid.

You can visualise right-angled triangles in all sorts of different solids.



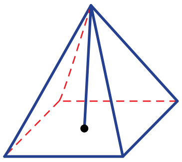




Engineers, architects, builders and carpenters use Pythagoras' theorem to calculate the direct distance between two positions in three dimensions. For example, calculating the length of trusses that support the roof of this Sydney sports stadium.

Lesson starter: How many triangles in a pyramid?

Here is a drawing of a square-based pyramid. By drawing lines from any vertex to the centre of the base and another point, how many different right-angled triangles can you visualise and draw? The triangles could be inside or on the outside surface of the pyramid.



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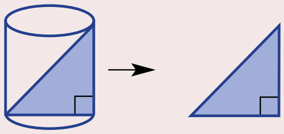
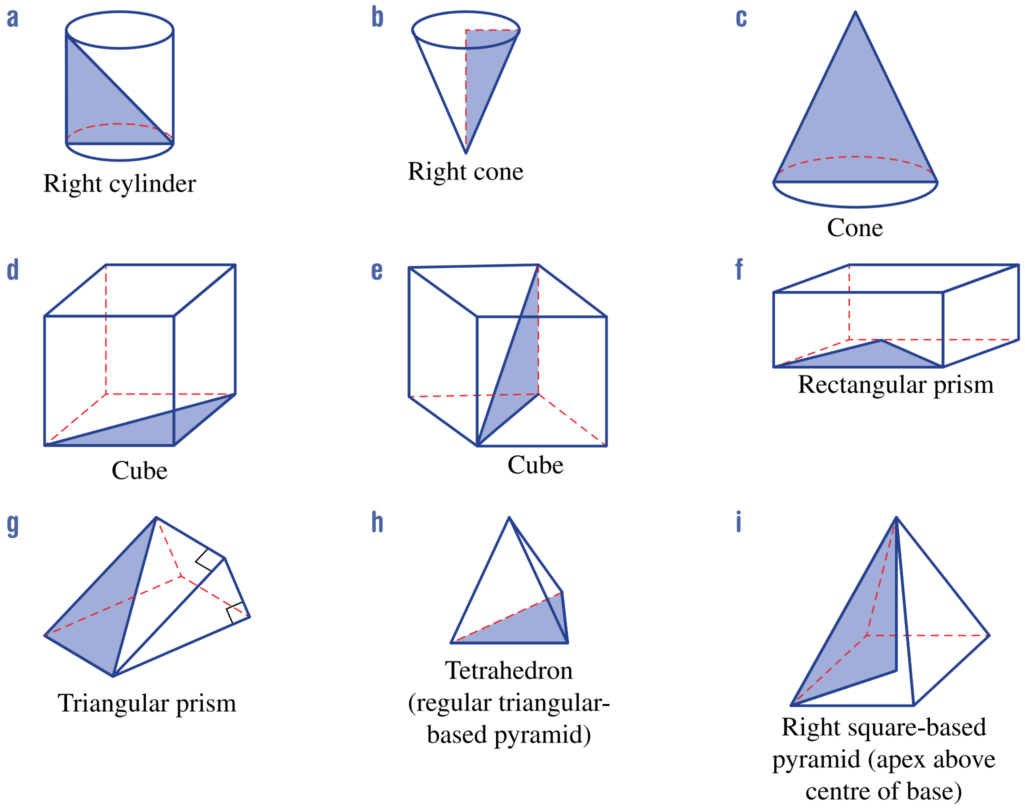
KEY IDEAS

- Right-angled triangles can be identified in many three-dimensional solids.
- It is important to try to draw any identified right-angled triangle using a separate diagram.



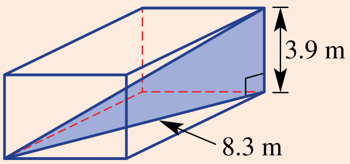
BUILDING UNDERSTANDING

1 Decide if the following shaded regions would form right-angled triangles.



Example 5 Using Pythagoras in 3D

The length of the diagonal on the base of a rectangular prism is 8.3 m and the height of the rectangular prism is 3.9 m. Find the distance from one corner of the rectangular prism to the opposite corner. Give your answer correct to two decimal places.



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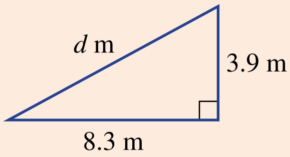
SOLUTION

Let d m be the distance required.



EXPLANATION

Draw a right-angled triangle and label all the measurements and pronumerals.

$$\begin{aligned} d^2 &= 3.9^2 + 8.3^2 \\ &= 84.1 \\ \therefore d &= 9.17 \text{ (to 2 d.p.)} \end{aligned}$$

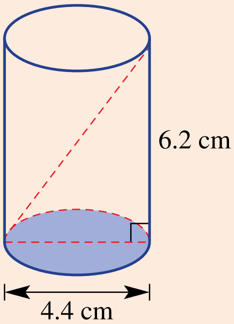
Use Pythagoras' theorem.
Round $\sqrt{84.1}$ to two decimal places.

The distance from one corner of the rectangular prism to the opposite corner is approximately 9.17 m.

Write your answer in words.

Now you try

A cylinder has diameter 4.4 cm and height 6.2 cm. Find the distance from a point on the circumference of the base to a point on the opposite side at the top. Give your answer correct to two decimal places.

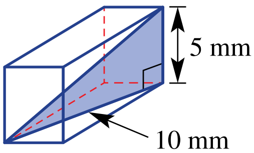


Exercise 3D

FLUENCY	1–5	1–6	2, 3, 5, 6

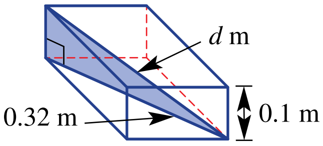
Example 5

1 The length of the diagonal on the base of a rectangular prism is 10 mm and the height of the rectangular prism is 5 mm. Find the distance from one corner of the rectangular prism to the opposite corner. Give your answer correct to two decimal places.

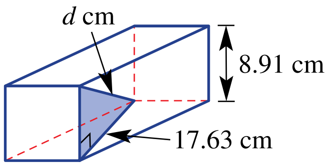


2 Find the distance, d units, from one corner to the opposite corner in each of the following rectangular prisms. Give your answers correct to two decimal places.

a



b



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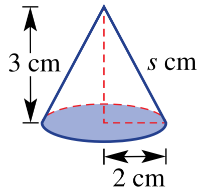
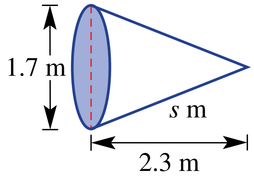
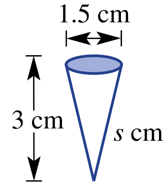
3 Find the slant height, s units, of each of the following cones. Give your answers correct to one decimal place.

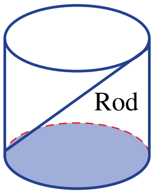
a


b


c


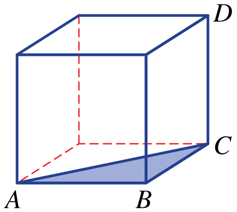
4 Find the length to the nearest millimetre of the longest rod that will fit inside a cylinder of the following dimensions.

a Diameter 10 cm and height 15 cm
b Radius 2.8 mm and height 4.2 mm
c Diameter 0.034 m and height 0.015 m

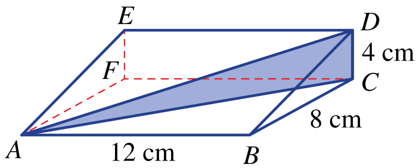


5 The cube in the diagram on the right has 1 cm sides.

a Find the length of AC as an exact value.
b Hence, find the length of AD correct to one decimal place.



6 Consider the shape shown.

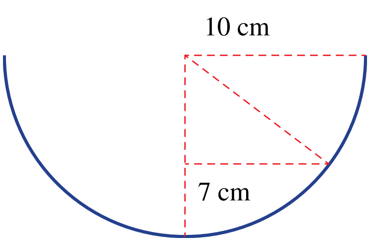


a Find the length of AC as an exact value.
b Hence, find the length of AD correct to one decimal place.

PROBLEM–SOLVING	7	7, 8	8, 9

7 A miner makes claim to a circular piece of land with a radius of 40 m from a given point, and is entitled to dig to a depth of 25 m. If the miner can dig tunnels at any angle, find the length of the longest straight tunnel that he can dig, to the nearest metre.

8 A bowl shown below is in the shape of a hemisphere (half sphere) with radius 10 cm. The surface of the water in the container has a radius of 7 cm. How deep is the water? Give your answer to two decimal places.



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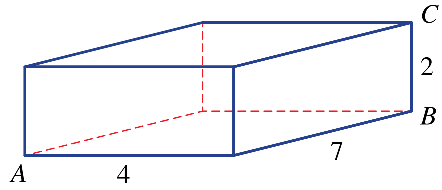
9 A cube of side length l sits inside a sphere of radius r so that the vertices of the cube sit on the sphere. Find the ratio $r : l$.

REASONING	10	10	10, 11

10 For this rectangular prism answer these questions.

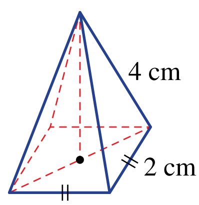


- **a** Find the exact length AB.
- **b** Find AB correct to two decimal places.
- **c** Find the length AC using your result from part **a** and then round to two decimal places.
- **d** Find the length AC using your result from part **b** and then round to two decimal places.
- **e** How can you explain the difference between your results from parts **c** and **d** above?

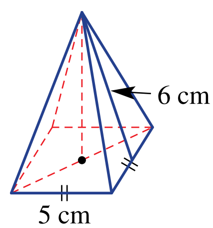
11 There are different ways to approach finding the height of a pyramid depending on what information is given. For each of the following square-based pyramids, find:

- **i** the exact length (using a surd) of the diagonal on the base
- **ii** the height of the pyramid correct to two decimal places.

a

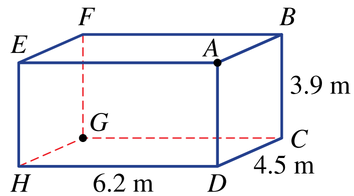


b



ENRICHMENT: Spider crawl	–	–	12

12 A spider crawls from one corner, A, of the ceiling of a room to the opposite corner, G, on the floor. The room is a rectangular prism with dimensions as given in the diagram shown.



- **a** Assuming the spider crawls in a direct line between points, find how far (correct to two decimal places) the spider crawls if it crawls from A to G via:
  - **i** B
  - **ii** C
  - **iii** D
  - **iv** F
- **b** Investigate other paths to determine the shortest distance that the spider could crawl in order to travel from point A to point G. (*Hint*: Consider drawing a net for the solid.)

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3E Introducing the trigonometric ratios

Learning intentions
- To know how to label the sides of a right-angled triangle as hypotenuse, opposite, adjacent
- To understand that the opposite and adjacent sides are relative to the angle involved
- To know the trigonometric ratios for sine, cosine and tangent
- To know that the trigonometric ratios remain constant when a triangle is enlarged
- To be able to use two sides, an angle and a trigonometric ratio to write an equation

Past, present and future learning:
- This section addresses the Stage 5 Core Topic called *Trigonometry A*
- These concepts will be revised, briefly, in Chapter 6 of our Year 10 book and also in Stage 6 Advanced
- You may be expected to recall the three ratios from memory in the HSC Examination

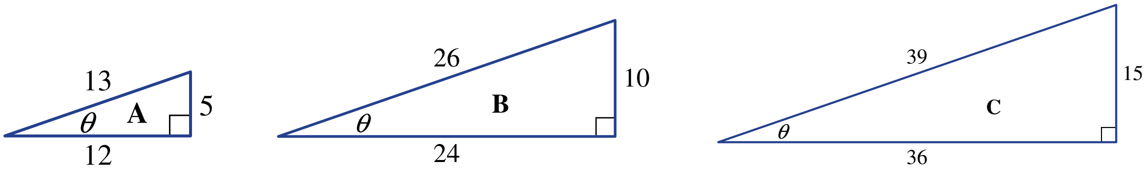
The branch of mathematics called trigonometry deals with the relationship between the side lengths and angles in triangles. Trigonometry dates back to the ancient Egyptian and Babylonian civilisations in which a basic form of trigonometry was used in the building of pyramids and in the study of astronomy. The first table of values including chord and arc lengths on a circle for a given angle was created by Hipparchus in the 2nd century BCE in Greece. These tables of values helped to calculate the position of the planets. About three centuries later, Claudius Ptolemy advanced the study of trigonometry writing by 13 books called the *Almagest*. Ptolemy also developed tables of values linking the sides and angles of a triangle, and produced many theorems which use the sine, cosine and tangent functions.


Everyday users of trigonometry include surveyors, engineers, architects, designers and builders of houses, carports, awnings and holiday cabins.

Lesson starter: What is the sine ratio?

The triangles below are similar:

- The sides of A have been multiplied by 2 to give B.
- The sides of A have been multiplied by 3 to give C.
- The three angles in A are the same as in B and C.
- The Greek letter θ (theta) has been used to mark an angle.



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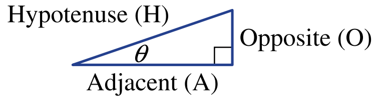
In this topic, the three sides are labelled as follows:

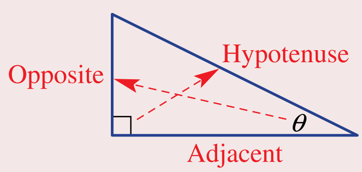


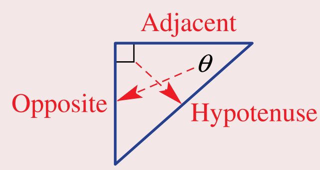
- In triangle A, the ratio $\frac{\text{opposite}}{\text{hypotenuse}} = \frac{\square}{\square}$ (in simplest form)
- In triangle B, the ratio $\frac{\text{opposite}}{\text{hypotenuse}} = \frac{\square}{\square} = \frac{\square}{\square}$ (in simplest form)
- In triangle C, the ratio $\frac{\text{opposite}}{\text{hypotenuse}} = \frac{\square}{\square} = \frac{\square}{\square}$ (in simplest form)

When a right-angled triangle is enlarged, the angles stay the same and the sine ratio remains constant.

KEY IDEAS

- In this topic, the Greek letter θ (theta) is often used to label an unknown angle.
- For a right-angled triangle with another angle θ then:
  - the **hypotenuse** is the longest side, opposite the 90° angle
  - the **opposite** side is opposite θ
  - the **adjacent** side is next to θ but is not the hypotenuse.

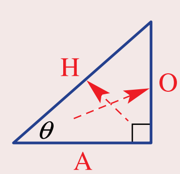




- For a right-angled triangle with a given angle θ, the three ratios **sine (sin)**, **cosine (cos)** and **tangent (tan)** are given by:
  - sine ratio: $\sin\theta = \frac{\text{length of the opposite side}}{\text{length of the hypotenuse}}$
  - cosine ratio: $\cos\theta = \frac{\text{length of the adjacent side}}{\text{length of the hypotenuse}}$
  - tangent ratio: $\tan\theta = \frac{\text{length of the opposite side}}{\text{length of the adjacent side}}$
- For any right-angled triangle with the same angles, these ratios are always the same i.e. the three ratios remain constant when a right-angled triangle is enlarged or reduced using a scale factor.
- The term **SOHCAHTOA** is useful when trying to remember the three ratios.

SOH CAH TOA

$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$ $\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$ $\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$



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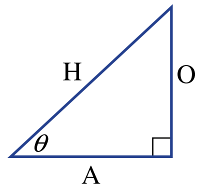
BUILDING UNDERSTANDING

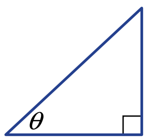
1 State the missing word in these sentences.



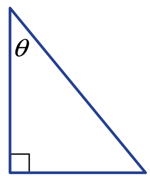
- **a** H stands for the word ___________.
- **b** O stands for the word ___________.
- **c** A stands for the word ___________.
- **d** $\sin\theta =$ ___________ $\div$ hypotenuse.
- **e** $\cos\theta =$ adjacent $\div$ ___________.
- **f** $\tan\theta =$ opposite $\div$ ___________.

2 Decide which side of each triangle should be labelled as opposite to θ (O), adjacent to θ (A) or hypotenuse (H).

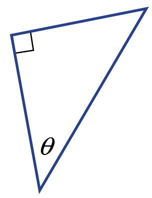
a



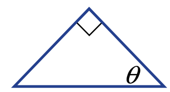
b



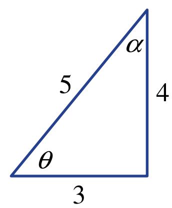
c



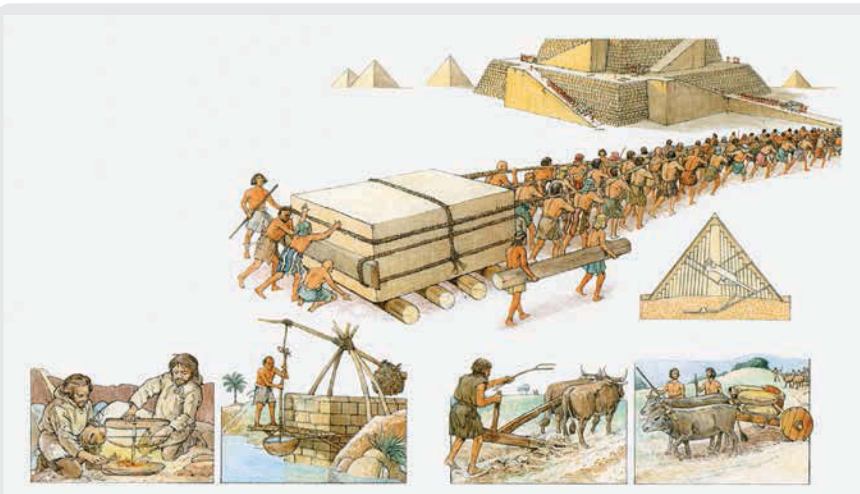
d



3 For the triangle shown, state the length of the side which corresponds to:



- **a** the hypotenuse
- **b** the side opposite angle θ
- **c** the side opposite angle α
- **d** the side adjacent to angle θ
- **e** the side adjacent to angle α.



A basic form of trigonometry was used in the building of pyramids in ancient Egypt.

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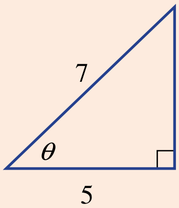
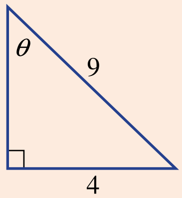
Example 6 Writing trigonometric ratios

Using the given sides, write a trigonometric ratio (in fraction form) for each of the following triangles.

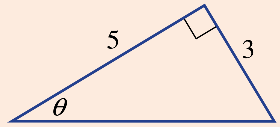
a

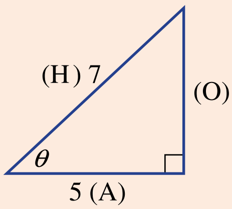
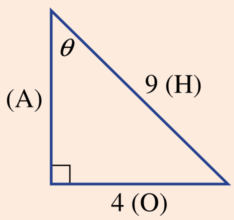
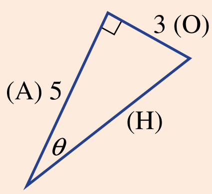


b



c

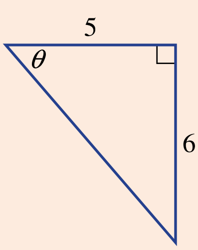


SOLUTION	EXPLANATION	
a $\cos\theta = \frac{A}{H}$ $= \frac{5}{7}$		Side length 7 is opposite the right angle so it is the hypotenuse (H). Side length 5 is adjacent to angle θ so it is the adjacent (A).
b $\sin\theta = \frac{O}{H}$ $= \frac{4}{9}$		Side length 9 is opposite the right angle so it is the hypotenuse (H). Side length 4 is opposite angle θ so it is the opposite (O).
c $\tan\theta = \frac{O}{A}$ $= \frac{3}{5}$		Side length 5 is the adjacent side to angle θ so it is the adjacent (A). Side length 3 is opposite angle θ so it is the opposite (O).

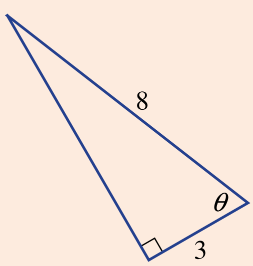
Now you try

Using the given sides, write a trigonometric ratio (in fraction form) for each of the following triangles.

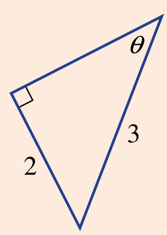
a



b



c



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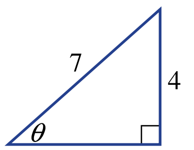
Exercise 3E

FLUENCY | 1(½), 2, 3 | 1(½), 2–4 | 1(⅓), 2–4

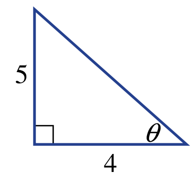
Example 6

1 Using the given sides, write a trigonometric ratio (in fraction form) for each of the following triangles and simplify where possible.

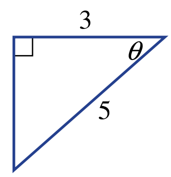
a



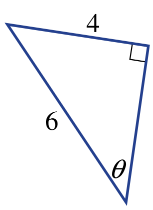
b



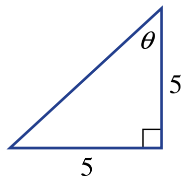
c



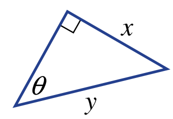
d



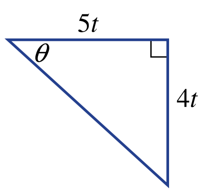
e



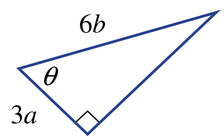
f



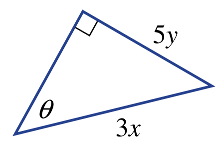
g



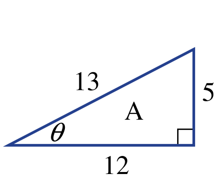
h

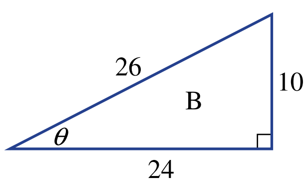


i



2 Here are two similar triangles A and B.





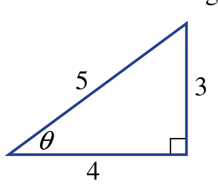
a **i** Write the ratio $\sin\theta$ (as a fraction) for triangle A.
ii Write the ratio $\sin\theta$ (as a fraction) for triangle B.
iii What do you notice about your two answers from parts **a i** and **ii** above?

b **i** Write the ratio $\cos\theta$ (as a fraction) for triangle A.
ii Write the ratio $\cos\theta$ (as a fraction) for triangle B.
iii What do you notice about your two answers from parts **b i** and **ii** above?

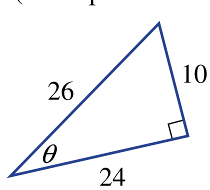
c **i** Write the ratio $\tan\theta$ (as a fraction) for triangle A.
ii Write the ratio $\tan\theta$ (as a fraction) for triangle B.
iii What do you notice about your two answers from parts **c i** and **ii** above?

3 For each of these triangles, write a ratio (in simplified fraction form) for $\sin\theta$, $\cos\theta$ and $\tan\theta$.

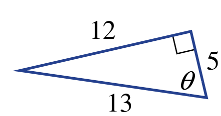
a



b



c



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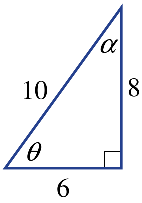
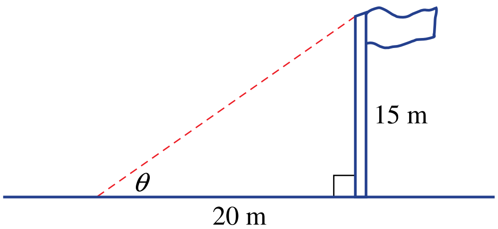
4 For the triangle shown on the right, write a ratio (in fraction form) for:

a $\sin \theta$ **b** $\sin \alpha$ **c** $\cos \theta$
d $\tan \alpha$ **e** $\cos \alpha$ **f** $\tan \theta$



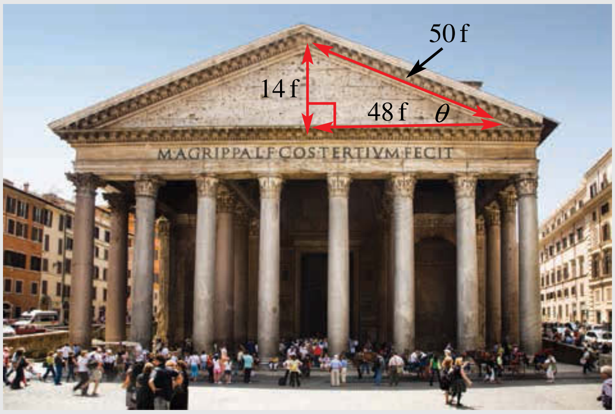
PROBLEM–SOLVING	5, 6	6, 7	7, 8

5 A vertical flag pole casts a shadow 20 m long. If the pole is 15 m high, find the ratio for $\tan \theta$.



6 The facade of a Roman temple has the given measurements in feet (f) below. Write down the simplified ratio for:

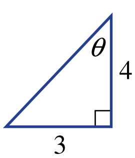
a $\sin \theta$ **b** $\cos \theta$ **c** $\tan \theta$



The Pantheon, a Roman temple that was built in 126 CE

7 For each of the following:

i use Pythagoras' theorem to find the unknown side
ii find the ratios for $\sin \theta$, $\cos \theta$ and $\tan \theta$.

a


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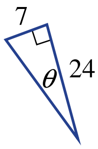

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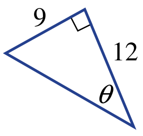

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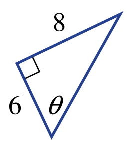

8 **a** Draw a right-angled triangle and mark one of the angles as θ. Mark in the length of the opposite side as 15 units and the length of the hypotenuse as 17 units.
b Using Pythagoras' theorem, find the length of the adjacent side.
c Determine the ratios for $\sin \theta$, $\cos \theta$ and $\tan \theta$.

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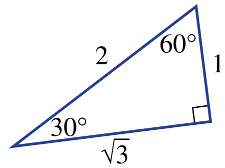
REASONING	9	9, 10	9–11

9 This triangle has angles 90°, 60° and 30° and side lengths 1, 2 and $\sqrt{3}$.



a Write a ratio for:

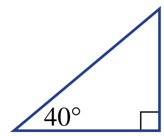
i sin 30° **ii** cos 30° **iii** tan 30°

iv sin 60° **v** cos 60° **vi** tan 60°

b What do you notice about the following pairs of ratios?

i cos 30° and sin 60° **ii** sin 30° and cos 60°

10 **a** Measure all the side lengths of this triangle to the nearest millimetre.



b Use your measurements from part **a** to find an approximate ratio for:

i cos 40° **ii** sin 40° **iii** tan 40°

iv sin 50° **v** tan 50° **vi** cos 50°

c Do you notice anything about the trigonometric ratios for 40° and 50°?

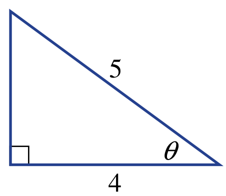
11 Decide if it is possible to draw a right-angled triangle with the given properties. Explain.

a $\tan\theta = 1$ **b** $\sin\theta = 1$

c $\cos\theta = 0$ **d** $\sin\theta > 1$ or $\cos\theta > 1$

ENRICHMENT: Pythagorean extensions	–	–	12

12 **a** Given that θ is acute and $\cos\theta = \frac{4}{5}$, find $\sin\theta$ and $\tan\theta$. (*Hint*: Use Pythagoras' theorem.)



b For each of the following, draw a right-angled triangle then use it to find the other two trigonometric ratios.

i $\sin\theta = \frac{1}{2}$ **ii** $\cos\theta = \frac{1}{2}$ **iii** $\tan\theta = 1$

c Use your results from part **a** to calculate $(\cos\theta)^2 + (\sin\theta)^2$. What do you notice?

d Evaluate $(\cos\theta)^2 + (\sin\theta)^2$ for other combinations of $\cos\theta$ and $\sin\theta$. Research and describe what you have found.

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3F Finding unknown side lengths

Learning intentions

- To know how to use a calculator to evaluate the sine, cosine or tangent of an angle
- To be able to use a trigonometric ratio to set up an equation to find a missing side length
- To know that one angle and one side must be known to find another side length
- To be able to use a trigonometric ratio to find an unknown side length

Past, present and future learning

- This section addresses the Stage 5 Core Topic called *Trigonometry A*
- These concepts will be revised, briefly, in Chapter 6 of our Year 10 book and also in Stage 6 Advanced
- You may be expected to recall the three ratios from memory in the HSC Examination

For similar triangles we know that the ratio of corresponding sides is always the same. This implies that the three trigonometric ratios for similar right-angled triangles are also constant if the internal angles are equal. Since ancient times, mathematicians have attempted to tabulate these ratios for varying angles. Here are the ratios for some angles in a right-angled triangle, correct to up to three decimal places.

Angle (θ)	$\sin\theta$	$\cos\theta$	$\tan\theta$
$0°$	0	1	0
$15°$	0.259	0.966	0.268
$30°$	0.5	0.866	0.577
$45°$	0.707	0.707	1
$60°$	0.866	0.5	1.732
$75°$	0.966	0.259	3.732
$90°$	1	0	undefined

A Segway's gyroscopes and tilt-sensors detect the angle and direction that a rider leans and then the wheels roll to maintain balance. Its microprocessors apply complex motion-control algorithms, including trigonometry calculations using the angle of tilt.

In modern times these values can be evaluated using calculators to a high degree of accuracy and can be used to help solve problems involving triangles with unknown side lengths.

Lesson starter: Calculator start-up

All scientific calculators can produce accurate values of $\sin\theta$, $\cos\theta$ and $\tan\theta$.

- Ensure that your calculator is in degree mode.
- Check the values in the above table to ensure that you are using the calculator correctly.
- Use trial and error to find (to the nearest degree) an angle θ which satisfies these conditions:
  - **a** $\sin\theta = 0.454$
  - **b** $\cos\theta = 0.588$
  - **c** $\tan\theta = 9.514$

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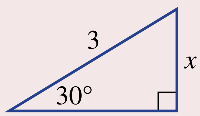
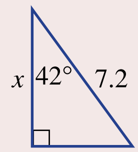
KEY IDEAS

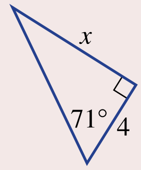
- If θ is in degrees, the ratios for $\sin\theta$, $\cos\theta$ and $\tan\theta$ can be found accurately using a calculator in **degree mode**.
- If the angles and one side length of a right-angled triangle are known, then the other side lengths can be found using the $\sin\theta$, $\cos\theta$ and $\tan\theta$ ratios.



$$\sin 30^\circ = \frac{x}{3}$$
$$\therefore x = 3 \times \sin 30^\circ$$



$$\cos 42^\circ = \frac{x}{7.2}$$
$$\therefore x = 7.2 \times \cos 42^\circ$$



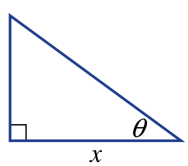
$$\tan 71^\circ = \frac{x}{4}$$
$$\therefore x = 4 \times \tan 71^\circ$$

- Angles that are not whole numbers are sometimes expressed in **degrees**, **minutes** and **seconds**. For example: $22.5 = 22^\circ\ 30'$ and $50.175 = 50^\circ\ 10'\ 30''$
- There are 60 minutes in 1 degree and 60 seconds in 1 minute. Your calculator can be used to make these conversions.

BUILDING UNDERSTANDING

1 For the marked angle θ, decide if x represents the length of the opposite (O), adjacent (A) or hypotenuse (H) side.

a



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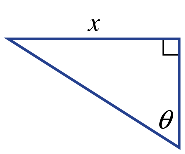
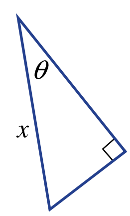
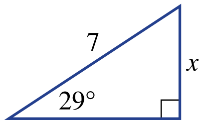


c



2 Decide if you would use $\sin\theta = \frac{\text{O}}{\text{H}}$, $\cos\theta = \frac{\text{A}}{\text{H}}$ or $\tan\theta = \frac{\text{O}}{\text{A}}$ to help find the value of x in these triangles. Do not find the value of x, just state which ratio would be used.

a



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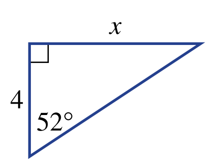
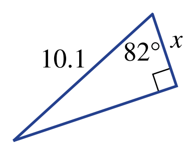


c



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Example 7 Using a calculator

Use a calculator to evaluate the following, correct to two decimal places.

a $\sin 50°$ **b** $\cos 16°$ **c** $\tan 15.6°$ **d** $\sin 22° 30'$

SOLUTION	EXPLANATION
a $\sin 50° = 0.77$ (to 2 d.p.)	$\sin 50° = 0.766044\ldots$ the 3rd decimal place is greater than 4 so round up.
b $\cos 16° = 0.96$ (to 2 d.p.)	$\cos 16° = 0.961261\ldots$ the 3rd decimal place is less than 5 so round down.
c $\tan 15.6° = 0.28$ (to 2 d.p.)	$\tan 15.6° = 0.2792\ldots$ the 3rd decimal place is greater than 4 so round up.
d $\sin 22° 30' = 0.38$ (to 2 d.p.)	$22° 30' - 22.5°$ ($30' = 30$ minutes $= 0.5°$)

Now you try

Use a calculator to evaluate the following, correct to two decimal places.

a $\sin 70°$ **b** $\cos 29°$ **c** $\tan 54.3°$ **d** $\cos 36° 30'$

Example 8 Solving for *x* in the numerator of a trigonometric ratio

Find the value of x in the equation $\cos 20° = \frac{x}{3}$, correct to two decimal places.

SOLUTION

$$\begin{aligned} \cos 20° &= \frac{x}{3} \\ x &= 3 \times \cos 20° \\ &= 2.82 \text{ (to 2 d.p.)} \end{aligned}$$

EXPLANATION

Multiply both sides of the equation by 3 and round as required.

Now you try

Find the value of x in the equation $\sin 40° = \frac{x}{5}$, correct to two decimal places.

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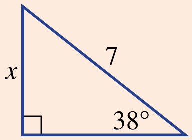
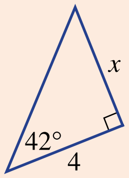
Example 9 Finding unknown side lengths

For each triangle, find the value of x correct to two decimal places.

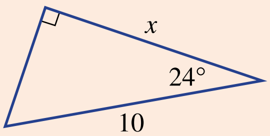
a



b



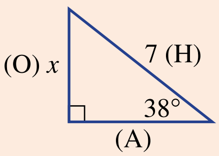
c



SOLUTION / EXPLANATION

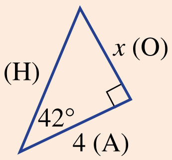
a
$$\begin{aligned}\sin 38^\circ &= \frac{\text{O}}{\text{H}}\\ \sin 38^\circ &= \frac{x}{7}\\ x &= 7\sin 38^\circ\\ &= 4.31 \text{ (to 2 d.p.)}\end{aligned}$$

As the opposite side (O) and the hypotenuse (H) are involved, the $\sin\theta$ ratio must be used. Multiply both sides by 7 and evaluate using a calculator.



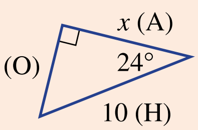
b
$$\begin{aligned}\tan 42^\circ &= \frac{\text{O}}{\text{A}}\\ \tan 42^\circ &= \frac{x}{4}\\ x &= 4\tan 42^\circ\\ &= 3.60 \text{ (to 2 d.p.)}\end{aligned}$$

As the opposite side (O) and the adjacent side (A) are involved, the $\tan\theta$ ratio must be used. Multiply both sides by 4 and evaluate.



c
$$\begin{aligned}\cos 24^\circ &= \frac{\text{A}}{\text{H}}\\ \cos 24^\circ &= \frac{x}{10}\\ x &= 10\cos 24^\circ\\ &= 9.14 \text{ (to 2 d.p.)}\end{aligned}$$

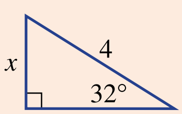
As the adjacent side (A) and the hypotenuse (H) are involved, the $\cos\theta$ ratio must be used. Multiply both sides by 10.



Now you try

For each triangle, find the value of x correct to two decimal places.

a



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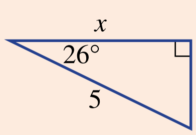

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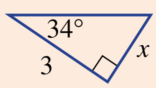

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Exercise 3F

FLUENCY	1, 2–3(½)	1, 2(⅓), 3(½)	2–3(⅓)

Example 7

1 Use a calculator to evaluate the following, correct to two decimal places.

a sin 47° b cos 84° c tan 14.1° d sin 27° 30′

Example 8

2 For each of the following, find the value of x correct to two decimal places.

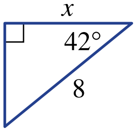
a $\sin 50° = \frac{x}{4}$ b $\tan 81° = \frac{x}{3}$ c $\cos 33° = \frac{x}{6}$

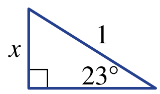
d $\cos 75° = \frac{x}{3.5}$ e $\sin 24° = \frac{x}{4.2}$ f $\tan 42° = \frac{x}{10}$

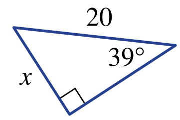
g $\frac{x}{7.1} = \tan 18.4°$ h $\frac{x}{5.3} = \sin 64° 42'$ i $\frac{x}{12.6} = \cos 52° 54'$

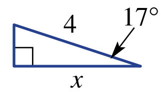
Example 9

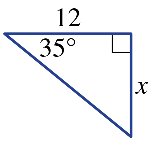
3 For the triangles given below, find the value of x correct to two decimal places.

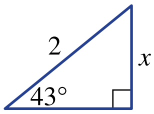
a 

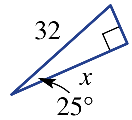
b 

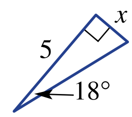
c 

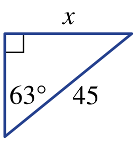
d 

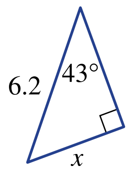
e 

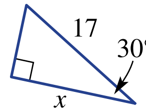
f 

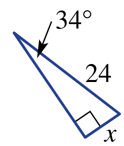
g 

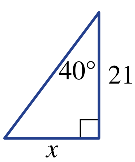
h 

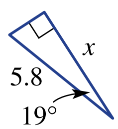
i 

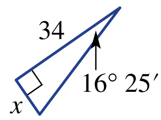
j 

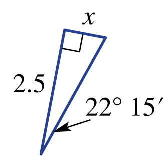
k 

l 

m 

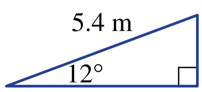
n 

o 

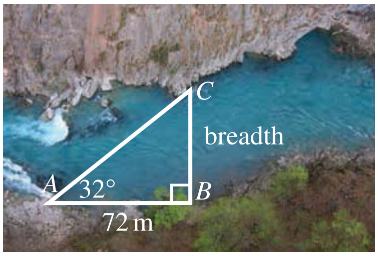
p 

PROBLEM–SOLVING	4, 5	4–6	5–7

4 Amy walks 5.4 m up a ramp that is inclined at 12° to the horizontal. How high (correct to two decimal places) is she above her starting point?



5 Kane wanted to measure the breadth of a river. He placed two markers, A and B, 72 m apart along the bank. C is a point directly opposite marker B. Kane measured angle CAB to be 32°. Find the breadth of the river correct to two decimal places.



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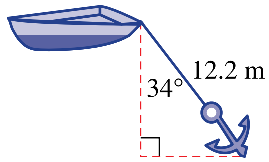
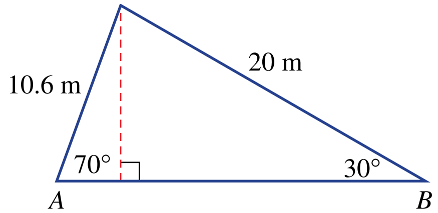
6 One end of a 12.2 m rope is tied to a boat. The other end is tied to an anchor, which is holding the boat steady in the water. If the anchor is making an angle of 34° with the vertical, how deep is the water? Give your answer correct to two decimal places.

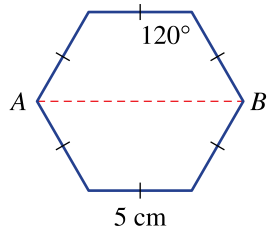


7 Find the length AB in these diagrams. Round to two decimal places where necessary.

a

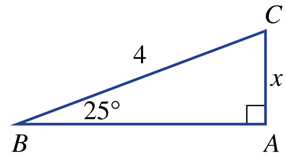


b



REASONING	8	8, 9	8, 9

8 Consider the right-angled triangle shown.

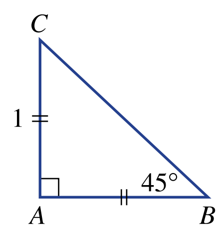


a Find the size of $\angle C$.
b Calculate the value of x correct to three decimal places using the sine ratio.
c Calculate the value of x correct to three decimal places using the cosine ratio.
d Comment on your answers to parts **b** and **c**.

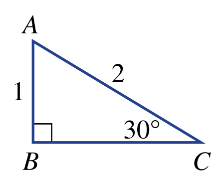
9 Complementary angles sum to 90°.
a Find the complementary angles to these angles.
i 10° **ii** 28° **iii** 54° **iv** 81°
b Evaluate:
i sin 10° and cos 80° **ii** sin 28° and cos 62°
iii cos 54° and sin 36° **iv** cos 81° and sin 9°
c What do you notice in part **b**?
d Complete the following.
i sin 20° = cos _____ **ii** sin 59° = cos _____
iii cos 36° = sin _____ **iv** cos 73° = sin _____

ENRICHMENT: Exact values	–	–	10

10 $\sqrt{2}$, $\sqrt{3}$ and $\frac{1}{\sqrt{2}}$ are examples of exact values.



a For the triangle shown (right), use Pythagoras' theorem to find the exact length BC.
b Use your result from part **a** to write down the exact value of:
i sin 45° **ii** cos 45° **iii** tan 45°
c For the second triangle (below right), use Pythagoras' theorem to find the exact length BC.
d Use your result from part **c** to write down the exact value of:
i sin 30° **ii** cos 30° **iii** tan 30°
iv sin 60° **v** cos 60° **vi** tan 60°



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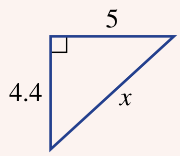
3A/B

1 Find the length of the missing side in these right-angled triangles. Round to two decimal places.

a



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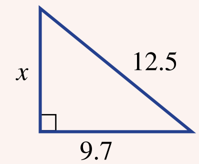
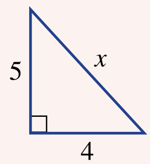


3A/B

2 Find the exact value of x in these right-angled triangles.

a



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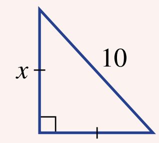


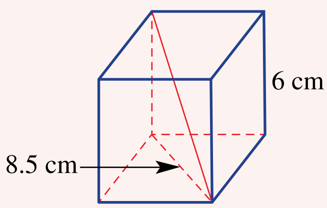
3C

3 A ladder 230 cm long is placed 50 cm from the edge of a building, how far up the side of the building will this ladder reach? Round to one decimal place.

3D

4 Find the length of the diagonals of these prisms, correct to one decimal place.

a



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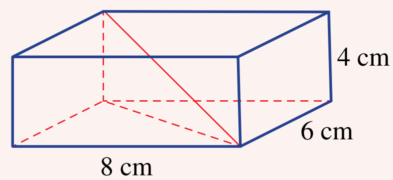


3E

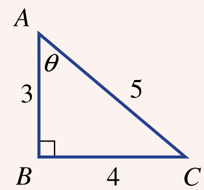
5 Consider the triangle ABC.

a Name the hypotenuse.

b Name the side adjacent to angle ACB.

c Write the ratio for $\cos \theta$.

d Write the ratio for $\tan \theta$.



3F

6 Solve for x, correct to two decimal places.

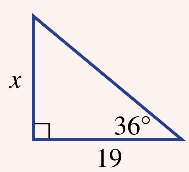
a $x = 12.7 \cos 54°$

b $\tan 30° = \dfrac{x}{12}$

3F

7 Find the value of x, correct to two decimal places.

a



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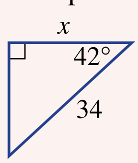
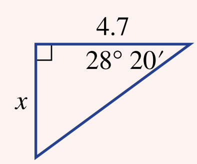


c

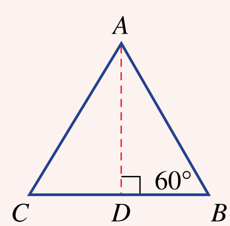


3A–F

8 Triangle ABC is equilateral with a perimeter of 12 cm.

a Find the height AD using any suitable method, correct to three decimal places.

b Calculate the area of the triangle ABC, correct to one decimal place.



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3G Solving for the denominator

Learning intentions
- To be able to use a trigonometric ratio, then find an unknown in the denominator

Past, present and future learning
- This section addresses the Stage 5 Core Topic called *Trigonometry A*
- These concepts will be revised, briefly, in Chapter 6 of our Year 10 book and also in Stage 6 Advanced
- You may be expected to recall the three ratios from memory in the HSC Examination

So far we have constructed trigonometric ratios using a pronumeral that has always appeared in the numerator.

For example: $\frac{x}{5} = \sin 40°$

This makes it easy to solve for x as both sides of the equation can be multiplied by 5.

If, however, the pronumeral appears in the denominator, there are a number of algebraic steps that can be taken to find the solution.

Trigonometry is an essential tool for builders and carpenters. Given the roof pitch (i.e. angle) and its horizontal span, the lengths of the rafters and trusses are calculated using trigonometry.

Lesson starter: Solution steps

Three students attempt to solve $\sin 40° = \frac{5}{x}$ for x.

Nick says $x = 5 \times \sin 40°$.

Sharee says $x = \frac{5}{\sin 40°}$.

Dori says $x = \frac{1}{5} \times \sin 40°$.

- Which student has the correct solution?
- Can you show the algebraic steps that support the correct answer?

KEY IDEAS

- If the unknown value of a trigonometric ratio is in the **denominator**, you need to rearrange the equation to make the pronumeral the subject.

For example: For the triangle shown,

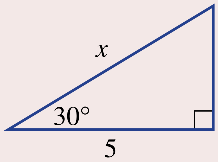
$$\cos 30° = \frac{5}{x}$$

Multiply both sides by x.

$$x \times \cos 30° = 5$$

Divide both sides by $\cos 30°$.

$$x = \frac{5}{\cos 30°}$$



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BUILDING UNDERSTANDING

1 Solve these simple equations for x.

a $\frac{4}{x} = 2$

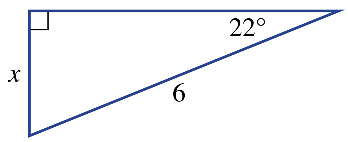
b $\frac{20}{x} = 4$

c $5 = \frac{35}{x}$

d $\frac{10}{x} = 2.5$

2 Decide whether the pronumeral x will sit in the numerator (N) or denominator (D) for the trigonometric ratios for these diagrams.

a



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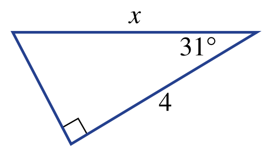

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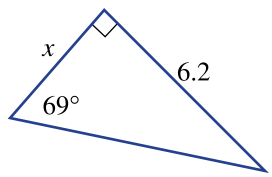

Example 10 Solving for a pronumeral in the denominator

Solve for x in the equation $\cos 35° = \frac{2}{x}$, correct to two decimal places.

SOLUTION	EXPLANATION
$\cos 35° = \frac{2}{x}$	
$x \cos 35° = 2$	Multiply both sides of the equation by x.
$x = \frac{2}{\cos 35°}$	Divide both sides of the equation by $\cos 35°$.
$= 2.44$ (to 2 d.p.)	Evaluate and round to two decimal places.

Now you try

Solve for x in the equation $\sin 62° = \frac{3}{x}$, correct to two decimal places.

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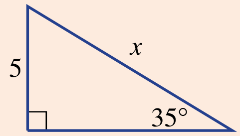
Example 11 Finding unknown side lengths using trigonometry

Find the values of the pronumerals, correct to two decimal places.

a



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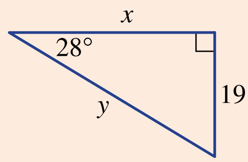


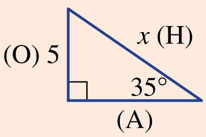
SOLUTION

a

$$\sin 35° = \frac{O}{H}$$
$$\sin 35° = \frac{5}{x}$$
$$x \sin 35° = 5$$
$$x = \frac{5}{\sin 35°}$$
$$= 8.72 \text{ (to 2 d.p.)}$$

EXPLANATION

As the opposite side (O) is given and we require the hypotenuse (H), use sin θ.



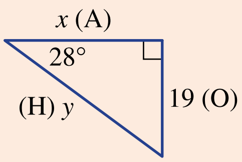
Multiply both sides of the equation by x then divide both sides of the equation by sin 35°.

Evaluate on a calculator and round to two decimal places.

b

$$\tan 28° = \frac{O}{A}$$
$$\tan 28° = \frac{19}{x}$$
$$x \tan 28° = 19$$
$$x = \frac{19}{\tan 28°}$$
$$= 35.73 \text{ (to 2 d.p.)}$$

As the opposite side (O) is given and the adjacent (A) is required, use tan θ.

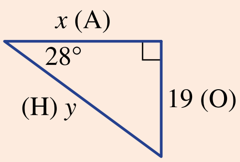


Multiply both sides of the equation by x.

Divide both sides of the equation by tan 28° and round the answer to two decimal places.

$$\sin 28° = \frac{19}{y}$$
$$y \sin 28° = 19$$
$$y = \frac{19}{\sin 28°}$$
$$= 40.47 \text{ (to 2 d.p.)}$$

y can be found by using sin θ.

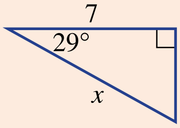


Alternatively, Pythagoras' theorem could be used: $x^2 + 19^2 = y^2$, where $x = \frac{19}{\tan 28°}$

Now you try

Find the values of the pronumerals, correct to two decimal places.

a



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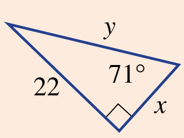

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Exercise 3G

FLUENCY	1–3(½)	1(⅓), 2–3(½)	1–3(⅓)

Example 10

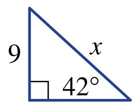
1 For each of the following equations, find the value of x correct to two decimal places.

a $\tan 64° = \frac{2}{x}$ b $\cos 67° = \frac{5}{x}$ c $\sin 12° = \frac{3}{x}$

d $\sin 38.3° = \frac{5.9}{x}$ e $\frac{45}{x} = \tan 21.4°$ f $\frac{18.7}{x} = \cos 32°$

Example 11a

2 Find the value of x correct to two decimal places using the sine, cosine or tangent ratios.

a


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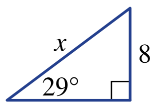

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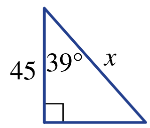
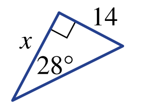


d


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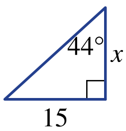

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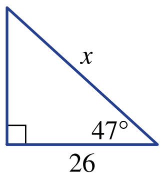

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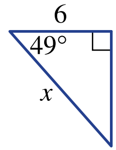

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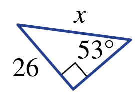

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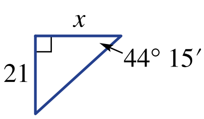

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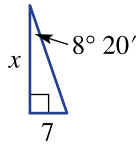

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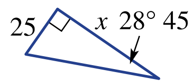

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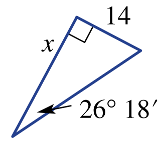
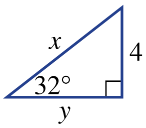


Example 11b

3 Find the value of each pronumeral correct to one decimal place.

a


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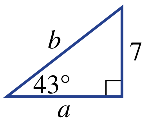

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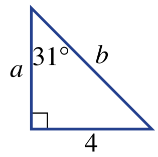

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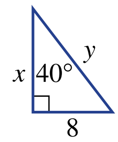

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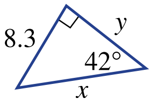

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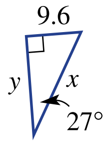

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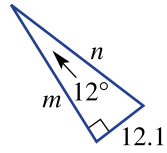

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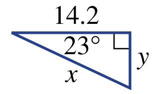
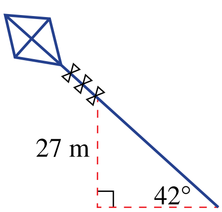


PROBLEM-SOLVING	4, 5	4–6	5, 6

4 A kite is flying at a height of 27 m above the anchor point. If the string is inclined at an angle of 42° to the horizontal, find the length of the string, correct to the nearest metre.



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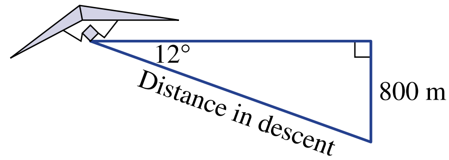
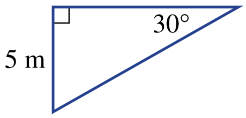
5 A paraglider flying at a height of 800 m descends at an angle of 12° to the horizontal. How far (to the nearest metre) does it travel as it descends to the ground?



6 Find the perimeter of these triangles, correct to one decimal place.

a



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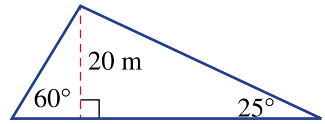
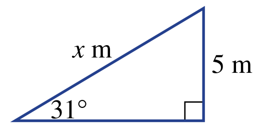


REASONING	7	7	7

7 In calculating the value of x for this triangle, two students come up with these answers, correct to two decimal places.



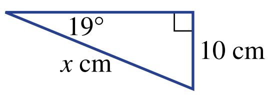
A $x = \dfrac{5}{\sin 31^\circ} = \dfrac{5}{0.52} = 9.62$ **B** $x = \dfrac{5}{\sin 31^\circ} = 9.71$

a Which of the two answers is more accurate and why?

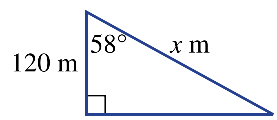
b What advice would you give to the student whose answer is not accurate?

c Find the difference in the answers for these triangles when methods **A** and **B** are used to calculate the value of x correct to two decimal places.

i



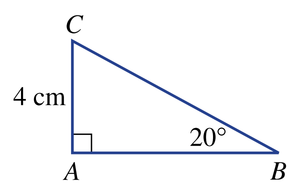
ii



ENRICHMENT: Linking tan θ to sin θ and cos θ	–	–	8

8 **a** For this triangle, find the length of these sides correct to three decimal places.

i AB **ii** BC



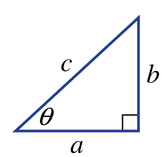
b Calculate these ratios to two decimal places.

i sin 20° **ii** cos 20° **iii** tan 20°

c Evaluate $\dfrac{\sin 20^\circ}{\cos 20^\circ}$ using your results from part **b**. What do you notice?

d For this triangle with side lengths a, b and c, find an expression for:

i $\sin \theta$ **ii** $\cos \theta$ **iii** $\tan \theta$ **iv** $\dfrac{\sin \theta}{\cos \theta}$



e Simplify your expression for part **d iv**. What do you notice?

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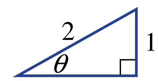
3H Finding unknown angles

Learning intentions
- To know that two sides must be known to find an unknown angle in a right-angled triangle
- To be able to evaluate the unknown angle using a calculator

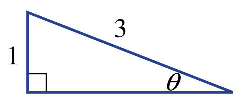
Past, present and future learning
- This section addresses the Stage 5 Core Topic called *Trigonometry A*
- These concepts will be revised, briefly, in Chapter 6 of our Year 10 book and also in Stage 6 Advanced
- You may be expected to recall the three ratios from memory in the HSC Examination



Logically, if you can use trigonometry to find a side length of a right-angled triangle given one angle and one side, you should be able to find an angle if you are given two sides. We know that $\sin 30° = \frac{1}{2}$, so if we were to determine θ when $\sin \theta = \frac{1}{2}$, the answer would be $\theta = 30°$.

Lesson starter: Trial and error can be slow

We know that for this triangle, $\sin \theta = \frac{1}{3}$.

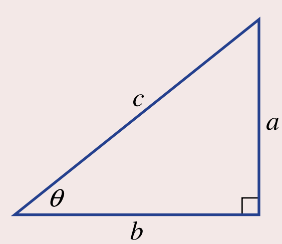


- Guess the angle θ.
- For your guess use a calculator in degree mode to see if $\sin \theta = \frac{1}{3} = 0.333\ldots$
- Update your guess and use your calculator to check once again.
- Repeat this trial-and-error process until you think you have the angle θ correct to three decimal places.
- Now evaluate $\sin^{-1}\left(\frac{1}{3}\right)$ using the $\boxed{\sin^{-1}}$ button on your calculator and check your guess.

The International Space Station has a 16.94 m robotic arm with four segments that's used on spacewalks, and capturing and releasing visiting spacecraft. Trigonometry is used to calculate the required angle between segments for grabbing specific spacecraft.

KEY IDEAS

- Buttons on your calculator labelled **inverse sine** ($\sin^{-1}$), **inverse cosine** ($\cos^{-1}$) and **inverse tangent** ($\tan^{-1}$) can be used to find unknown angles in right-angled triangles. Ensure your calculator is in degree mode.
  - If $\sin \theta = \frac{a}{c}$, press $\sin^{-1}\left(\frac{a}{c}\right)$.
  - If $\cos \theta = \frac{b}{c}$, press $\cos^{-1}\left(\frac{b}{c}\right)$.
  - If $\tan \theta = \frac{a}{b}$, press $\tan^{-1}\left(\frac{a}{b}\right)$.



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- Note that $\sin^{-1} x$ does *not* mean $\dfrac{1}{\sin x}$.
- You can use your calculator to convert angles into degrees, minutes and seconds.
  For example: $28.75° = 28° 45'$

BUILDING UNDERSTANDING

1 Use a calculator to evaluate the following, rounding the answer to two decimal places.

a $\sin^{-1}(0.2)$ **b** $\cos^{-1}(0.75)$ **c** $\tan^{-1}(0.5)$

2 State the missing number.

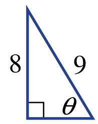
a If $\sin 30° = \frac{1}{2}$ then $30° = \sin^{-1}$ (_____).

b If $\cos 50° = 0.64$ then _____ $= \cos^{-1}(0.64)$.

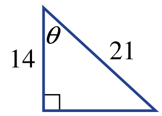
c If $\tan 45° = 1$ then _____ $= \tan^{-1}$ (_____).

3 Which trigonometric ratio should be used to solve for θ?

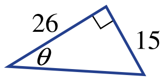
a



b



c



Example 12 Using inverse trigonometric ratios

Find the value of θ to the level of accuracy indicated.

a $\sin \theta = 0.3907$ (nearest degree)

b $\tan \theta = \frac{1}{2}$ (one decimal place)

SOLUTION	EXPLANATION
a $\sin \theta = 0.3907$ $\theta = 22.9988\ldots°$ $= 23°$ (to nearest degree)	Use the $\sin^{-1}$ key on your calculator. Round to the nearest whole number.
b $\tan \theta = \frac{1}{2}$ $\theta = 26.565\ldots°$ $= 26.6°$ (to 1 d.p.)	Use the $\tan^{-1}$ key on your calculator and round the answer to one decimal place.

Now you try

Find the value of θ to the level of accuracy indicated.

a $\cos \theta = 0.416$ (nearest degree)

b $\tan \theta = \frac{5}{3}$ (one decimal place)

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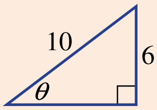
Example 13 Finding an unknown angle

Find the value of θ to the nearest minute.



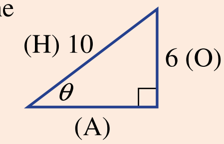
SOLUTION

$$\sin\theta = \frac{O}{H}$$
$$= \frac{6}{10}$$

$$\theta = 36.869...^\circ$$
$$= 36^\circ\, 52'11.63''$$
$$= 36^\circ\, 52 \text{ (to the nearest minute)}$$

EXPLANATION

As the opposite side (O) and the hypotenuse (H) are given, use $\sin\theta$.

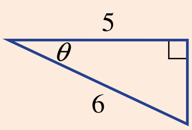


Use the $\sin^{-1}$ key on your calculator and Use you calculator to convert to degrees, minutes and seconds.

Round off to the nearest minute. Since there are 60 seconds in a minute round down since 11.63 is less that 30 seconds.

Now you try

Find the value of θ to the nearest minute.



Exercise 3H

FLUENCY	$1–3(\frac{1}{2})$	$1–2(\frac{1}{3}), 3(\frac{1}{2})$	$1–3(\frac{1}{3})$

Example 12a

1 Find the value of θ to the nearest degree.

a $\cos\theta = 0.8660$ b $\sin\theta = 0.7071$ c $\tan\theta = 0.5774$

d $\sin\theta = 1$ e $\tan\theta = 1.192$ f $\cos\theta = 0$

Example 12b

2 Find the angle θ correct to two decimal places.

a $\sin\theta = \frac{4}{7}$ b $\sin\theta = \frac{1}{3}$ c $\sin\theta = \frac{9}{10}$

d $\cos\theta = \frac{1}{4}$ e $\cos\theta = \frac{4}{5}$ f $\cos\theta = \frac{7}{9}$

g $\tan\theta = \frac{3}{5}$ h $\tan\theta = \frac{8}{5}$ i $\tan\theta = 12$

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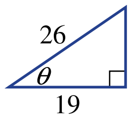
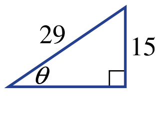
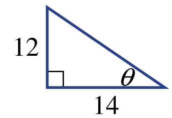
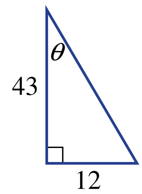
Example 13

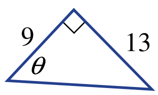
3 Find the value of θ to the nearest minute.

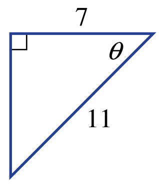
a


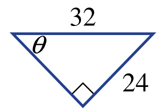
b


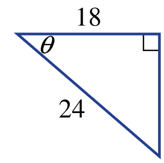
c


d


e


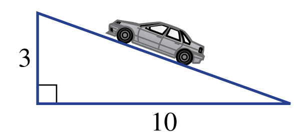
f


g


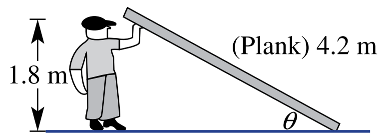
h


PROBLEM–SOLVING | 4–6 | 5–7 | 6–8

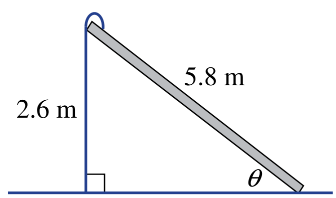
4 A road rises at a grade of 3 in 10. Find the angle (to the nearest degree) the road makes with the horizontal.



5 Adam, who is 1.8 m tall, holds up a plank of wood that is 4.2 m long. Find the angle that the plank makes with the ground, correct to one decimal place.

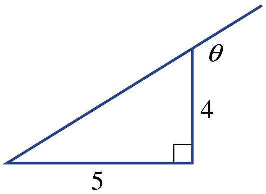


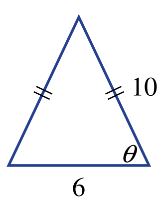
6 A children's slide has a length of 5.8 m. The vertical ladder is 2.6 m high. Find the angle the slide makes with the ground, correct to one decimal place.

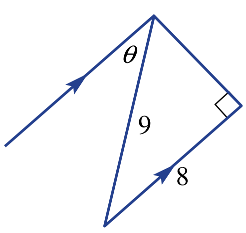


7 When a 2.8 m long seesaw is at its maximum height it is 1.1 m off the ground. What angle (correct to two decimal places) does the seesaw make with the ground?

8 Find the value of θ, correct to one decimal place.

a


b


c


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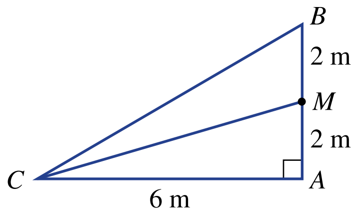
REASONING	9	9, 10	10, 11

9 Find all the angles to the nearest degree in right-angled triangles with these side lengths.
 a 3, 4, 5
 b 5, 12, 13
 c 7, 24, 25

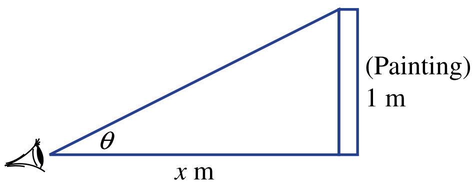
10 For what value of θ is $\sin\theta = \cos\theta$?

11 If M is the midpoint of AB, decide whether $\angle ACM$ is exactly half of angle $\angle ACB$. Investigate and explain.

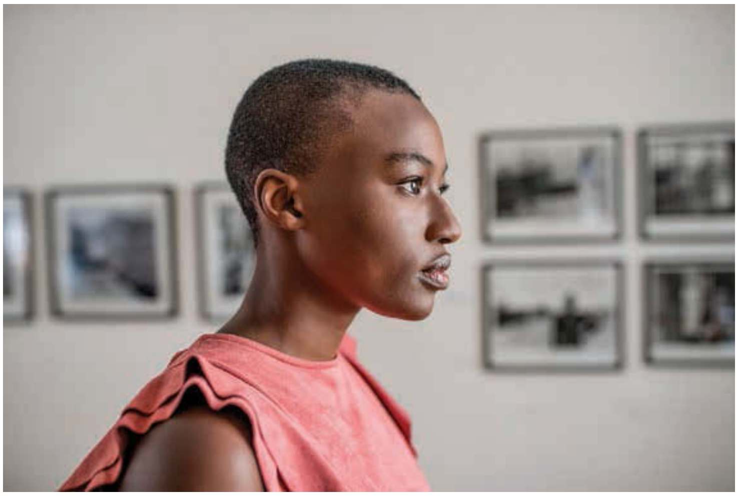


ENRICHMENT: Viewing angle	–	–	12

12 Jo has forgotten her glasses and is trying to view a painting in a gallery. Her eye level is at the same level as the base of the painting and the painting is 1 metre tall. Answer the following to the nearest degree for angles and to two decimal places for lengths.



 a If $x = 3$, find the viewing angle θ.
 b If $x = 2$, find the viewing angle θ.
 c If Jo can stand no closer than 1 metre from the painting, what is Jo's largest viewing angle?
 d When the viewing angle is 10°, Jo has trouble seeing the painting. How far is she from the painting at this viewing angle?
 e Theoretically, what would be the largest viewing angle if Jo could go as close as she would like to the painting?



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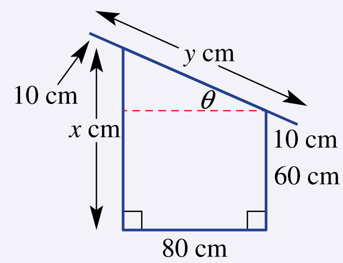
Applications and problem-solving

The following problems will investigate practical situations drawing upon knowledge and skills developed throughout the chapter. In attempting to solve these problems, aim to identify the key information, use diagrams, formulate ideas, apply strategies, make calculations and check and communicate your solutions.

Rabbit roof

1 Sophie is building a hutch for her pet rabbit and wants to include a sloping roof to keep the rain off. A cross-section of the hutch is shown here.



The minimum height at one side is 60 cm and a 10 cm roof overhang is required at each side. The breadth of the hutch is 80 cm, but the taller side, x cm, can vary depending on the slope of the roof (θ). *Sophie is investigating the relationship between the height of the hutch (x cm) and the roof span (y cm). The angle or slope of the roof is also of interest as these measurements affect lengths and the costs of materials.*

a If Sophie chooses the angle for the roof to be 30°, find these lengths correct to one decimal place.
   i Wall height, x cm
   ii Roof span, y cm

b If Sophie chooses the wall height, x cm, to be 90 cm, find these measurements correct to one decimal place.
   i Roof span, y cm
   ii Roof angle θ

c If Sophie chooses the roof span, y cm, to be 120 cm, find these measurements correct to one decimal place where necessary.
   i Wall height, x cm
   ii Roof angle θ

d Write a rule for each of the following:
   i y in terms of θ
   ii y in terms of x
   iii x in terms of θ
   iv x in terms of y.

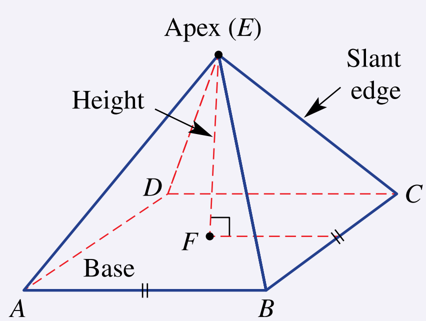
e Given the size of the available materials, Sophie will save money if the roof span, y cm, is less than 110 cm, but she feels that the angle θ needs to be larger than 25° for sufficient water run-off. Investigate to see if this is achievable and give reasons.

Solving the pyramid

2 Richard is studying ancient pyramids and notices that many of them are in fact right square-based pyramids. Such structures have the following properties:

- The base is a square.
- The apex is directly above the centre of the base.

Richard is interested in the properties of such a solid and how Pythagoras' theorem can be used to find various lengths. Given two lengths, Richard is looking to 'solve the pyramid' by finding the lengths of all the other edges, heights or diagonals.



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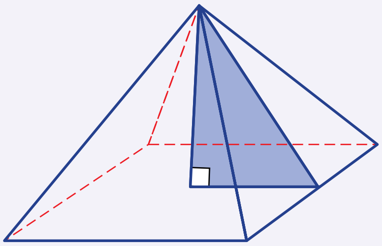


a There are many right-angled triangles that can be drawn inside or on the surface of a right, square-based pyramid. Make three copies of the pyramid and shade in a different right-angled triangle for each one. An example is shown at right.

b Richard is interested in finding the length of a slant edge. A square-based pyramid has base length 100 m and height 50 m. Use various right-angled triangles to find the following lengths. Round each answer correct to one decimal place, but try not to accumulate errors by using rounded answers for subsequent calculations. (Use stored answers in a calculator.) Refer to the diagram at the top of the page.

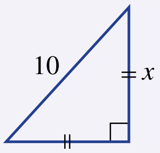
i AC **ii** CF **iii** CE (slant edge)

c A pyramid has base length 100 m and slant edge 80 m. Calculate the height of the pyramid correct to one decimal place.

d A pyramid has slant edge 80 m and height 50 m.

i First check that you can solve for an unknown in the triangle shown here. Round to one decimal place.

ii Now find the base length of the pyramid described above. Round to one decimal place.

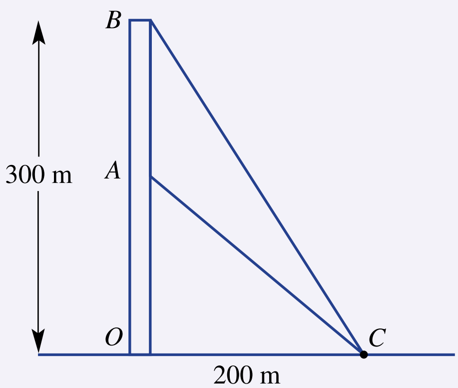


Communications cabling

3 A 300 m high communications tower is to be supported by six cables, including three reaching to the very top and three reaching to a point somewhere up the tower. Each of the three anchor points on the ground is shared by two cables as shown. The anchor points are 200 m from the base of the tower.

The construction designers are interested in the total length of cable required to support the tower, as well as the position of point A for the shorter cable. They are exploring the distance between points A and B and the angles the cables make with the ground.

Round all answers in this word problem to two decimal places where necessary.



a For each long cable that joins to the tower at the very top, find:

i its length **ii** its angle of elevation from ground level.

b If point A is positioned halfway up the tower, find:

i the angle of elevation of the shorter cables

ii the total length of cable required for all six cables.

c If 2000 m of cable is used in total for the six cables, find the position at which the shorter cables connect to the tower (point A).

d The designers decide that the distance AB on a second 300 m high tower should be 100 m. They are hoping to use less than 1900 m of cabling. Will this be possible? Give reasons.

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3I Using trigonometry to solve problems

Learning intentions
- To be able to solve word problems by drawing a diagram then using a trigonometric ratio
- To know that angles of elevation and depression are measured from the horizontal
- To be able to work with angles of elevation and depression in diagrams and word problems

Past, present and future learning
- This section addresses the Stage 5 Core Topic called *Trigonometry B*
- These concepts will be revised, briefly, in Chapter 6 of our Year 10 book and also in Stage 6 Advanced
- You may be expected to recall the three ratios from memory in the HSC Examination

In many situations, angles are measured up or down from the horizontal. These are called angles of elevation and depression. Combined with the mathematics of trigonometry, these angles can be used to solve problems, provided right-angled triangles can be identified. The line of sight to a helicopter 100 m above the ground, for example, creates an angle of elevation inside a right-angled triangle.

Lesson starter: Illustrate the situation

For the situation described below, draw a detailed diagram showing these features:

- an angle of elevation
- an angle of depression
- any given lengths
- a right-angled triangle that will help to solve the problem.

A cat and a bird eye each other from their respective positions. The bird is 20 m up a tree and the cat is on the ground 30 m from the base of the tree. Find the angle their line of sight makes with the horizontal.

Compare your diagram with those of others in your class. Is there more than one triangle that could be drawn and used to solve the problem?

By using trigonometry and higher-level mathematics, it can be shown that the angle of elevation from an observer to a rainbow's highest point, red, is always 42°. In a fainter secondary rainbow, the highest point, violet, always has a 51° angle of elevation.

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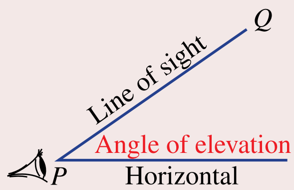
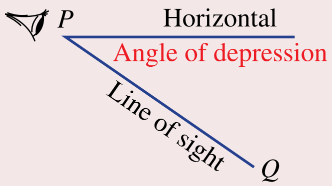
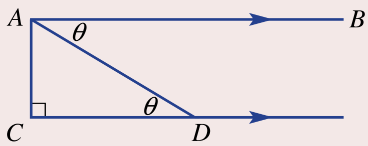
KEY IDEAS

- To solve application problems involving trigonometry follow these steps.
  1. Draw a diagram and label the key information.
  2. Identify and draw the appropriate right-angled triangles separately.
  3. Solve using trigonometry to find the missing measurements.
  4. Express your answer in words.
- The **angle of elevation** or **depression** of a point, Q, from another point, P, is given by the angle the line PQ makes with the horizontal.



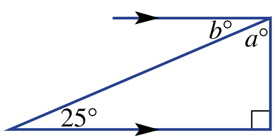


- Angles of elevation or depression are always measured from the horizontal.
- In this diagram the angle of elevation of B from A is equal to the angle of depression of A from B. They are equal alternate angles in parallel lines.



BUILDING UNDERSTANDING

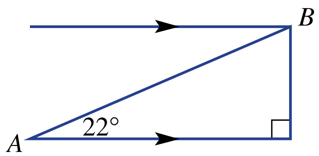
1 State the values of the pronumerals in this diagram.



2 For this diagram:

a what is the angle of elevation of B from A?

b what is the angle of depression of A from B?



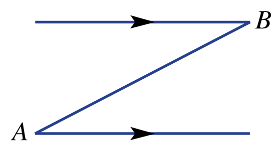
3 Complete these tasks for the diagram shown.

a State where you would mark in the following:

i the angle of elevation (θ) of B from A

ii the angle of depression (α) of A from B.

b Is $\theta = \alpha$ in your diagram? Why?



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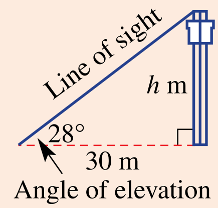
Example 14 Using angles of elevation

The angle of elevation of the top of a tower from a point on the ground 30 m away from the base of the tower is 28°. Find the height of the tower to the nearest metre.



SOLUTION

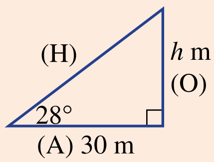
Let the height of the tower be h m.

$$\begin{aligned} \tan 28^\circ &= \frac{\text{O}}{\text{A}} \\ &= \frac{h}{30} \\ h &= 30 \tan 28^\circ \\ &= 15.951\ldots \end{aligned}$$

The height is 16 m, to the nearest metre.

EXPLANATION

As the opposite side (O) is required and the adjacent (A) is given, use $\tan \theta$.



Multiply both sides by 30 and evaluate. Round to the nearest metre and write the answer in words.

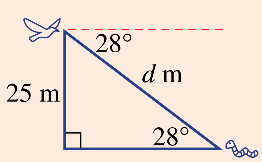
Now you try

The angle of elevation to the top of a tree from a point 50 m away from the base of the tree is 32°. Find the height of the tree to the nearest metre.

Example 15 Using angles of depression

The angle of depression from a bird 25 m up a tree to a worm on the ground is 28°. Find the direct distance between the bird and the worm to the nearest metre.

SOLUTION



Let d m be the distance between the bird and worm.

$$\begin{aligned} \sin 28^\circ &= \frac{25}{d} \\ d \sin 28^\circ &= 25 \\ d &= \frac{25}{\sin 28^\circ} \\ &= 53.2513\ldots \end{aligned}$$

The distance is 53 m to the nearest metre.

EXPLANATION

Draw a diagram, marking in the angle of depression below the horizontal. Use alternate angles to mark the angle inside the triangle.

Use sine given the opposite and hypotenuse are involved.

Multiply both sides by d and divide both sides by $\sin 28^\circ$.

Round to the nearest metre and answer in words.

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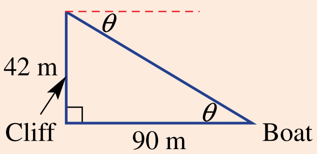
Now you try

The angle of depression from a tennis umpire in a chair 4 m high to a moth on the court is 22°. Find the direct distance between the umpire and the moth to the nearest metre.

Example 16 Finding an angle of depression

From the top of a vertical cliff Andrea spots a boat out at sea. If the top of the cliff is 42 m above sea level and the boat is 90 m away from the base of the cliff, find the angle of depression from Andrea to the boat to the nearest degree.

SOLUTION



$$\tan\theta = \frac{O}{A} = \frac{42}{90}$$

$$\theta = 25.0168\ldots^\circ$$

The angle of depression is 25°, to the nearest degree.

EXPLANATION

Draw a diagram and label all the given measurements. The angle of depression is the angle below the horizontal (red dashed line). Use alternate angles in parallel lines to mark θ inside the triangle.

As the opposite (O) and adjacent sides (A) are given, use $\tan\theta$.

Use the $\tan^{-1}$ key on your calculator.

Round to the nearest degree and express the answer in words.

Now you try

A drone films a concert on a stage from the air. If the drone is 20 m above the level of the stage and a horizontal distance of 50 m from the stage, find the angle of depression from the drone to the stage to the nearest degree.

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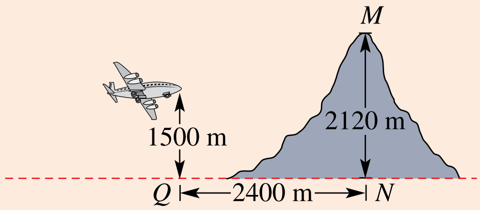
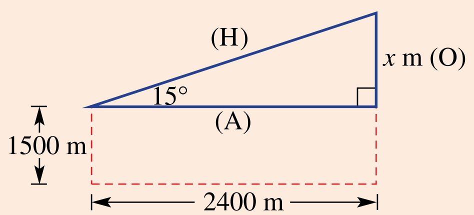
Example 17 Applying trigonometry

An aeroplane flying at an altitude of 1500 m starts to climb at an angle of 15° to the horizontal when the pilot sees a mountain peak 2120 m high, 2400 m away from him horizontally. Will the aeroplane clear the mountain?



SOLUTION



$$\tan 15° = \frac{x}{2400}$$
$$x = 2400 \tan 15°$$
$$= 643.078\ldots$$

x needs to be greater than $2120 - 1500 = 620$

Since $x > 620$ m the aeroplane will clear the mountain peak.

EXPLANATION

Draw a diagram, identifying and labelling the right-angled triangle to help solve the problem. The aeroplane will clear the mountain if the opposite (O) is greater than $(2120 - 1500)$ m $= 620$ m

Set up the trigonometric ratio using tan. Multiply by 2400 and evaluate.

Answer the question in words.

Now you try

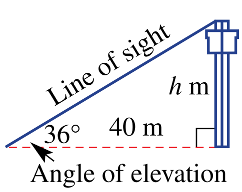
A submarine initially at sea level dives at an angle of 11° to the horizontal. At a horizontal distance of 1200 m there is an uncharted reef at a depth of 240 m. Will the submarine pass over the reef at that point?

Exercise 3I

FLUENCY	1–6	1–7	3, 5, 7, 8

Example 14

1 The angle of elevation of the top of a tower from a point on the ground 40 m from the base of the tower is 36°. Find the height of the tower to the nearest metre.



2 The angle of elevation of the top of a castle wall from a point on the ground 25 m from the base of the wall is 32°. Find the height of the castle wall to the nearest metre.

3 From a point on the ground, Emma measures the angle of elevation of an 80 m tower to be 27°. Find how far Emma is from the base of the tower, correct to the nearest metre.

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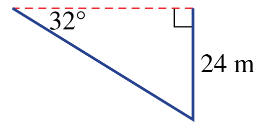
Example 15

4 From a pedestrian overpass, Chris spots a landmark at an angle of depression of 32°. How far away (to the nearest metre) is the landmark from the base of the 24 m high overpass?



5 From a lookout tower, David spots a bushfire at an angle of depression of 25°. If the lookout tower is 42 m high, how far away (to the nearest metre) is the bushfire from the base of the tower?

Example 16

6 From the top of a vertical cliff, Josh spots a swimmer out at sea. If the top of the cliff is 38 m above sea level and the swimmer is 50 m from the base of the cliff, find the angle of depression from Josh to the swimmer, to the nearest degree.

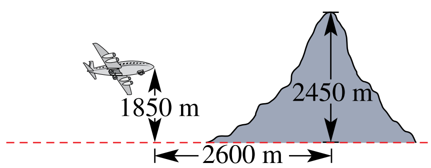
7 From a ship, a person is spotted floating in the sea 200 m away. If the viewing position on the ship is 20 m above sea level, find the angle of depression from the ship to the person in the sea. Give your answer to the nearest degree.

8 A power line is stretched from a pole to the top of a house. The house is 4.1 m high and the power pole is 6.2 m high. The horizontal distance between the house and the power pole is 12 m. Find the angle of elevation of the top of the power pole from the top of the house, to the nearest degree.

PROBLEM–SOLVING	9, 10	9–11	9, 12, 13

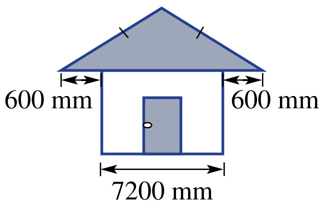
Example 17

9 An aeroplane flying at 1850 m starts to climb at an angle of 18° to the horizontal to clear a mountain peak 2450 m high and a horizontal distance of 2600 m away. Will the aeroplane clear the mountain?

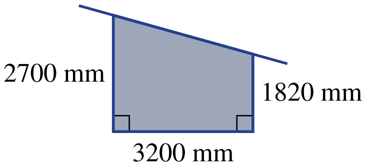


10 A road has a steady gradient of 1 in 10.
 a What angle does the road make with the horizontal? Give your answer to the nearest degree.
 b A car starts from the bottom of the inclined road and drives 2 km along the road. How high vertically has the car climbed? Use your rounded answer from part **a** and give your answer correct to the nearest metre.

11 A house is to be built using the design shown on the right. The eaves are 600 mm and the house is 7200 mm wide, excluding the eaves. Calculate the length (to the nearest mm) of a sloping edge of the roof, which is pitched at 25° to the horizontal.



12 A garage is to be built with measurements as shown in the diagram on the right. Calculate the sloping length and pitch (angle) of the roof if the eaves extend 500 mm on each side. Give your answers correct to the nearest unit.



13 The chains on a swing are 3.2 m long and the seat is 0.5 m off the ground when it is in the vertical position. When the swing is pulled as far back as possible, the chains make an angle of 40° with the vertical. How high off the ground, to the nearest cm, is the seat when it is at this extreme position?

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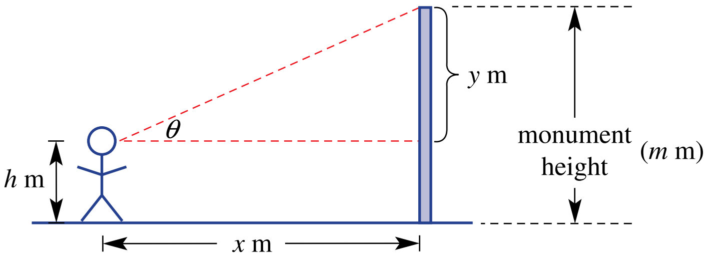
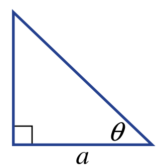
REASONING	14	14	14, 15

14 A person views a vertical monument x metres away as shown.



a If $h = 1.5$, $x = 20$ and $\theta = 15°$, find the height of the monument to two decimal places.
b If $h = 1.5$, $x = 20$ and $y = 10$, find θ correct to one decimal place.
c Let the height of the monument be m metres. Write expressions for:
  i m using (in terms of) y and h
  ii y using (in terms of) x and θ
  iii m using (in terms of) x, θ and h.

15 Find an expression for the area of this triangle using a and θ.



ENRICHMENT: Plane trigonometry	–	–	16–18

16 An aeroplane takes off and climbs at an angle of 20° to the horizontal, at a speed of 190 km/h along its flight path for 15 minutes.
a **i** Find the distance the aeroplane travels in 15 minutes.
  ii Calculate the height of the aeroplane after 15 minutes correct to two decimal places.
b If the angle at which the aeroplane climbs is twice the original angle but its speed is halved, will it reach a greater height after 15 minutes? Explain.
c If the aeroplane's speed is doubled and its climbing angle is halved, will the aeroplane reach a greater height after 15 minutes? Explain.

17 The residents of Skeville live 12 km from an airport. They maintain that any aeroplane flying at a height lower than 4 km disturbs their peace. Each Sunday they have an outdoor concert from 12:00 noon till 2:00 p.m.
a Will an aeroplane taking off from the airport at an angle of 15° over Skeville disturb the residents?
b When the aeroplane in part **a** is directly above Skeville, how far (to the nearest metre) has it flown?
c If the aeroplane leaves the airport at 11:50 a.m. on Sunday and travels at an average speed of 180 km/h, will it disturb the start of the concert?
d Investigate what average speed (correct to the nearest km/h) the aeroplane can travel at so that it does not disturb the concert. Assume it leaves at 11:50 a.m.

18 Peter observes an aeroplane flying directly overhead at a height of 820 m. Twenty seconds later, the angle of elevation of the aeroplane from Peter is 32°. Assume the aeroplane flies horizontally.
a How far (to the nearest metre) did the aeroplane fly in 20 seconds?
b What is the aeroplane's speed in km/h, correct to the nearest km/h?

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3J Bearings

Learning intentions

- To identify and interpret true bearings and compass bearings
- To be able to solve practical problems involving bearings

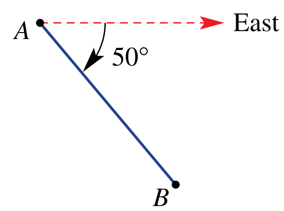
Past, present and future learning

- This section addresses the Stage 5 Core Topic called *Trigonometry B*
- These concepts will be revised, briefly, in Chapter 6 of our Year 10 book and also in Stage 6 Advanced
- You may be expected to recall the three ratios from memory in the HSC Examination

Bearings are used to indicate direction and therefore are commonly used to navigate the sea or air in ships or planes. Bushwalkers use bearings with a compass to help follow a map and navigate bushland. The most common type of bearing is the true bearing measured clockwise from north.

Lesson starter: Opposite directions

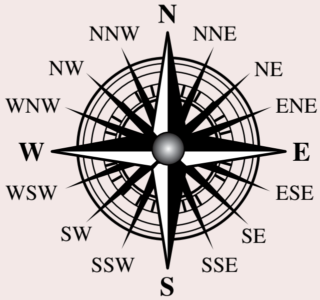
Marg at point A and Jim at point B start walking toward each other. Marg knows that she has to face 50° south of due east.



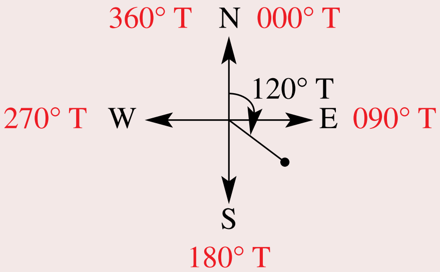
- Measured clockwise from north, can you help Marg determine the true compass bearing that she should walk on?
- Can you find the bearing Jim should walk on?
- Draw a detailed diagram that supports your answers above.

KEY IDEAS

- This mariner's compass shows 16 different directions.



- A **true bearing** is an angle measured clockwise from north.
  - True bearings are written using three digits. For example: 008° T, 032° T or 120° T.
- To describe the true bearing of an object positioned at A from an object positioned at O, we need to start at O, face north then turn clockwise through the required angle to face the object at A.



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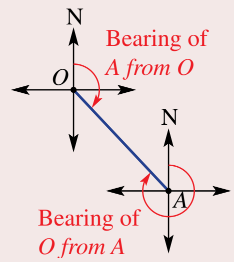
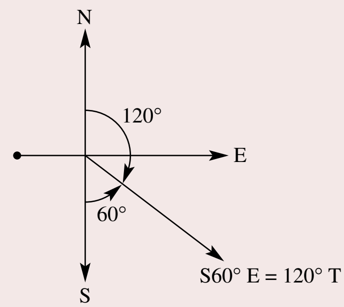
- Opposite directions differ by 180°.
  - $\theta + 180°$ is the opposite bearing if $0° \leqslant \theta < 180°$.
  - $\theta - 180°$ is the opposite bearing if $180° \leqslant \theta < 360°$.



- When solving problems with bearings, draw a diagram including four point compass directions (N, E, S, W) at each point.
- There is another system used, called compass directions.
  E.g. The bearing 120° T is shown in the diagram. It is between South and East.

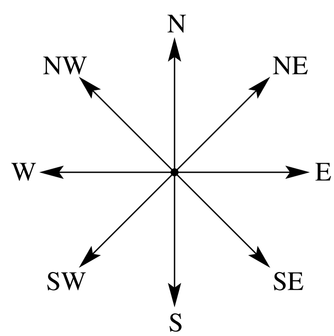
  Starting from South, it is 60 degrees East It is written as S60°E.
  In this system:
  - The first letter is always N or S
  - The last letter is always E or W
  - The angle is always between 0° and 90°



BUILDING UNDERSTANDING

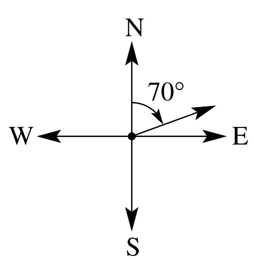
1 Give the true bearings for these common directions.

a North (N)
b North-east (NE)
c East (E)
d South-east (SE)
e South (S)
f South-west (SW)
g West (W)
h North-west (NW)

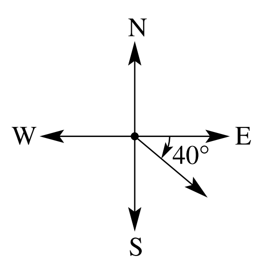


2 State the true bearings and compass bearing shown in these diagrams.

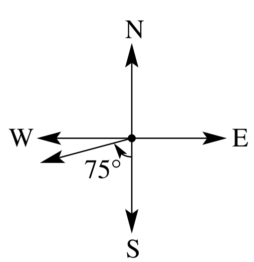
a



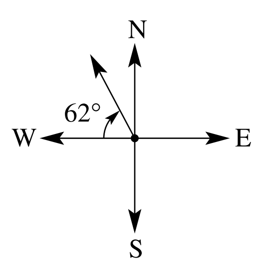
b



c



d



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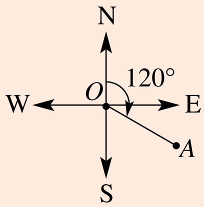
Example 18 Stating true bearings

For the diagram shown give:

a the true bearing of A from O

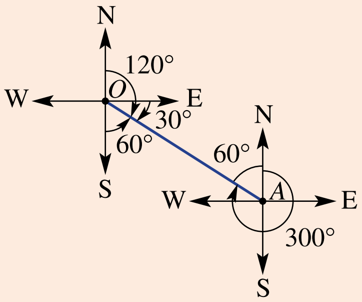
b the true bearing of O from A.



SOLUTION

a The bearing of A from O is 120° T.

b



The bearing of O from A is:
$(360 - 60)^\circ \text{ T} = 300^\circ \text{ T}$

EXPLANATION

Start at O, face north and turn clockwise until you are facing A.

Start at A, face north and turn clockwise until you are facing O. Mark in a compass at A and use alternate angles in parallel lines to mark a 60° angle.

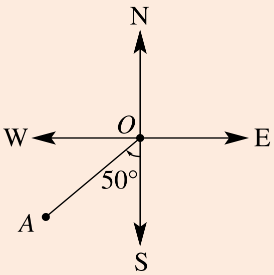
The true bearing is then 60° short of 360°. Alternatively, add 180° to 120° to get the opposite direction.

Now you try

For the diagram shown give:

a the true bearing of A from O

b the true bearing of O from A.



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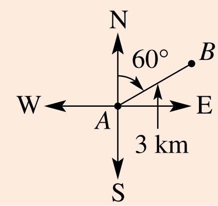
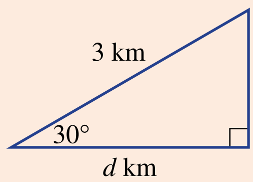
Example 19 Using bearings with trigonometry

A bushwalker walks 3 km on a true bearing of 060° from point A to point B. Find how far east (correct to one decimal place) point B is from point A.



SOLUTION

Let the distance travelled towards the east be d km.



$$\cos 30° = \frac{d}{3}$$
$$d = 3\cos 30°$$
$$= 2.6 \text{ (to 1 d.p.)}$$

$\therefore$ The distance east is 2.6 km.

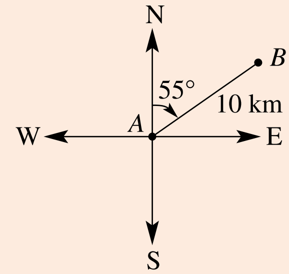
EXPLANATION

Define the distance required and draw and label the right-angled triangle.
As the adjacent (A) is required and the hypotenuse (H) is given, use $\cos\theta$.

Multiply both sides of the equation by 3 and evaluate, rounding to one decimal place.
Express the answer in words.

Now you try

A ship sails 10 km on a true bearing of 055° from point A to point B. Find how far north (correct to one decimal place) point B is from point A.

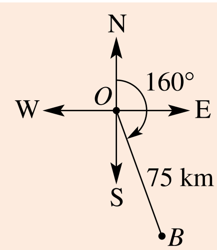


Example 20 Calculating a bearing

A fishing boat starts from point O and sails 75 km on a bearing of 160° T to point B.

a How far east (to the nearest kilometre) of its starting point is the boat?

b What is the true bearing of O from B?



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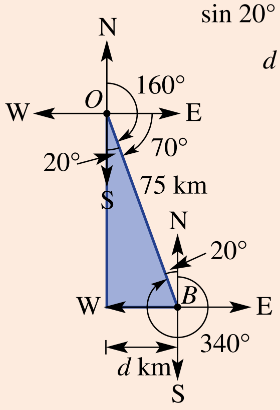
SOLUTION

EXPLANATION

a Let the distance travelled towards the east be d km.

$$\sin 20^\circ = \frac{d}{75}$$
$$d = 75 \sin 20^\circ$$
$$= 25.651\ldots$$



Draw a diagram and label all the given measurements. Mark in a compass at B and use alternate angles to label extra angles.
Set up a trigonometric ratio using sine and solve for d.

The boat has travelled 26 km to the east of its starting point, to the nearest kilometre.

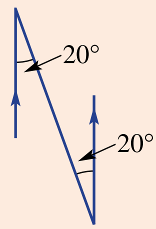
Round to the nearest kilometre and write the answer in words.

b The bearing of O from B is $(360 - 20)^\circ\text{T} = 340^\circ\text{T}$

Start at B, face north then turn clockwise to face O.

Alternatively, add 180° to 160° to get the opposite direction.

Alternate angle = 20°

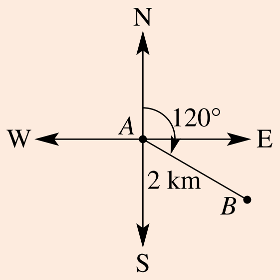


Now you try

A hiker walks 2 km on a true bearing of 120° from point A to point B.

a Find how far east (correct to one decimal place) point B is from point A.

b What is the bearing of A from B?



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Exercise 3J

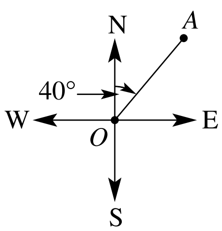
FLUENCY	1–4	1(½), 2, 4, 5	1(½), 3–5

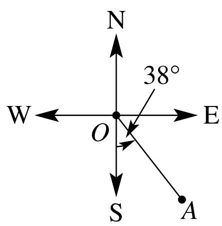
Example 18

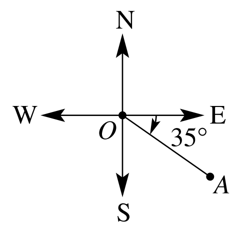
1 For each diagram shown, write:

i the true bearing of A from O

ii the true bearing of O from A.

a


b


c


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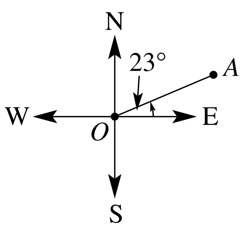

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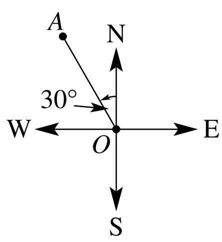

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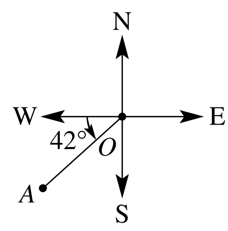
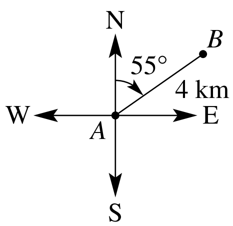
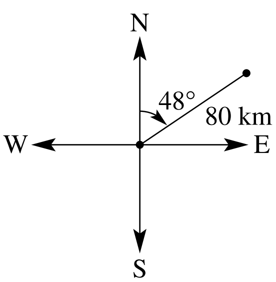


Example 19

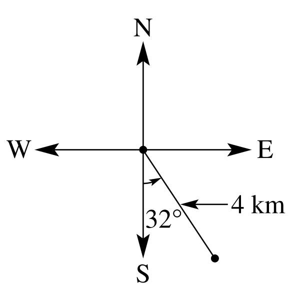
2 A bushwalker walks 4 km on a true bearing of 055° from point A to point B. Find how far east point B is from point A, correct to two decimal places.



3 A speed boat travels 80 km on a true bearing of 048°. Find how far east of its starting point the speed boat is, correct to two decimal places.



4 After walking due east, then turning and walking due south, a hiker is 4 km, 148° T from her starting point. Find how far she walked in a southerly direction, correct to one decimal place.



5 A four-wheel drive vehicle travels for 32 km on a true bearing of 200°. How far west (to the nearest kilometre) of its starting point is it?

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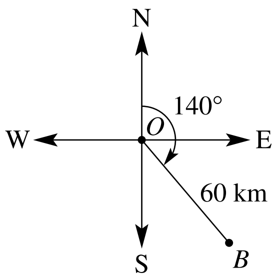
PROBLEM–SOLVING	6–8	6, 8, 9	7–10

Example 20

6 A fishing boat starts from point O and sails 60 km on a true bearing of 140° to point B.

a How far east of its starting point is the boat, to the nearest kilometre?

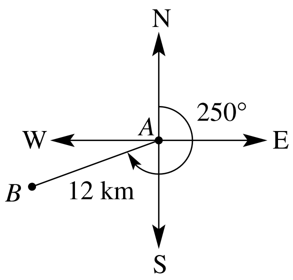
b What is the bearing of O from B?



7 Two towns, A and B, are 12 km apart. The true bearing of B from A is 250°.

a How far west of A is B, correct to one decimal place?

b Find the bearing of A from B.



8 A helicopter flies on a true bearing of 140° for 210 km then flies due east for 175 km. How far east (to the nearest kilometre) has the helicopter travelled from its starting point?

9 Christopher walks 5 km south then walks on a true bearing of 036° until he is due east of his starting point. How far is he from his starting point, to one decimal place?

10 Two cyclists leave from the same starting point. One cyclist travels due west while the other travels on a true bearing of 202°. After travelling for 18 km, the second cyclist is due south of the first cyclist. How far (to the nearest metre) has the first cyclist travelled?

REASONING	11	11	11, 12

11 A true bearing is a°. Write an expression for the true bearing of the opposite direction of a° if:

a a is between 0 and 180

b a is between 180 and 360.

12 A hiker walks on a triangular pathway starting at point A, walking to point B then C, then A again as shown.

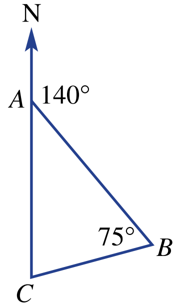
a Find the bearing from B to A.

b Find the bearing from B to C.

c Find the bearing from C to B.

d If the initial bearing was instead 133° and $\angle ABC$ is still 75°, find the bearing from B to C.

e If $\angle ABC$ was 42°, with the initial bearing of 140°, find the bearing from B to C.



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ENRICHMENT: Speed trigonometry	–	–	13, 14

13 An aeroplane flies on a true bearing of 168° for two hours at an average speed of 310 km/h. How far (to the nearest kilometre):

a has the aeroplane travelled?
b south of its starting point is the aeroplane?
c east of its starting point is the aeroplane?

14 A pilot intends to fly directly to Anderly, which is 240 km due north of his starting point. The trip usually takes 50 minutes. Due to a storm, the pilot changes course and flies to Boxleigh on a true bearing of 320° for 150 km, at an average speed of 180 km/h.

a Find (to the nearest kilometre) how far:
 i north the plane has travelled from its starting point
 ii west the plane has travelled from its starting point.
b How many kilometres is the plane from Anderly?
c From Boxleigh the pilot flies directly to Anderly at 240 km/h.
 i Compared to the usual route, how many extra kilometres (to the nearest kilometre) has the pilot travelled in reaching Anderly?
 ii Compared to the usual trip, how many extra minutes (correct to one decimal place) did the trip to Anderly take?

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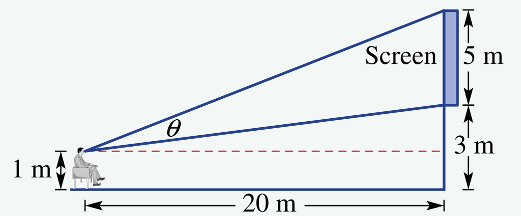
Viewing angle

Lucas is keen to maximise his experience at the outdoor cinema and usually sits at a distance of 20 metres horizontally from the screen. The base of the screen is 3 metres above the ground and the screen is 5 metres in height. Lucas' eye level is 1 metre above ground level as shown.



Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate. Use two decimal places for all rounding.

Routine problems

a What is the vertical distance between Lucas' eye level and the base of the screen?
b Find the angle of elevation from Lucas' eye level to the base of the screen.
c Find the angle of elevation from Lucas' eye level to the top of the screen.
d Find Lucas' viewing angle (θ).

Non-routine problems

Explore and connect

a The problem is to determine how close Lucas should sit to maximise his viewing angle. Write down all the relevant information that will help solve this problem with the aid of a diagram.

Choose and apply techniques

b Determine Lucas' viewing angle if the 20 metre horizontal distance is changed to:
i 14 metres **ii** 8 metres **iii** 2 metres.

Communicate thinking and reasoning

c Choose integer values for the horizontal distance between Lucas and the screen and determine which integer horizontal distance delivers the largest viewing angle and state the corresponding viewing angle.
d Find the viewing angle for the following horizontal distances:
i your answer from part **c** plus 0.1 metres
ii your answer from part **c** minus 0.1 metres.
e Do your results from part **d** confirm that you have found a distance which maximises the viewing angle? Explain why.
f Refine your choices for the horizontal distance and determine a distance correct to one decimal place that maximises Lucas' viewing angle.
g Draw a scale diagram illustrating where Lucas should sit to maximise his viewing angle. Decide if this position for Lucas is reasonable.
h Summarise your results and describe any key findings.

Extension problems

Problem solve

a Investigate how varying the height of the base of the screen above the ground (previously 3 metres) changes the position that Lucas should sit to maximise the viewing angle. Show your calculations and make drawings to suit.
b Investigate how varying the height of the screen (previously 5 metres) changes the position that Lucas should sit to maximise the viewing angle. Show your calculations and make drawings to suit.

Working mathematically

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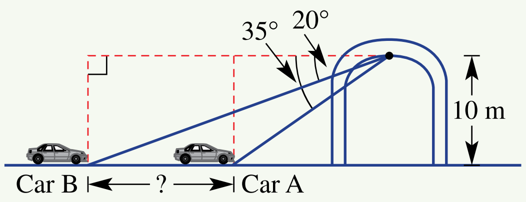
Investigation

Constructing triangles to solve problems

Illustrations for some problems may not initially look as if they include right-angled triangles. A common mathematical problem-solving technique is to construct right-angled triangles so that trigonometry can be used.

Car gap

Two cars are observed in the same lane from an overpass bridge 10 m above the road. The angles of depression to the cars are 20° and 35°.

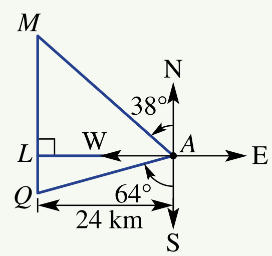


- **a** Find the horizontal distance from car A to the overpass. Show your diagrams and working.
- **b** Find the horizontal distance from car B to the overpass.
- **c** Find the distance between the fronts of the two cars.

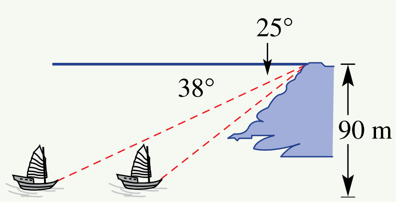
Now try these

Solve these similar types of problems. You will need to draw detailed diagrams and split the problem into parts. Refer to the problem above if you need help.

- **a** An observer is 50 m horizontally from a hot air balloon. The angle of elevation to the top of the balloon is 60° and to the bottom of the balloon's basket is 40°. Find the total height of the balloon (to the nearest metre) from the base of the basket to the top of the balloon.
- **b** A ship (at A) is 24 km due east of a lighthouse (L). The captain takes bearings from two landmarks, M and Q, which are due north and due south of the lighthouse respectively. The true bearings of M and Q from the ship are 322° and 244° respectively. How far apart are the two landmarks?



- **c** From the top of a 90 m cliff the angles of depression of two boats in the water, both directly east of the lighthouse, are 25° and 38° respectively. What is the distance between the two boats to the nearest metre?



- **d** A person on a boat 200 m out to sea views a 40 m high castle wall on top of a 32 m high cliff. Find the viewing angle between the base and top of the castle wall from the person on the boat.

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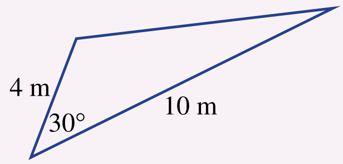
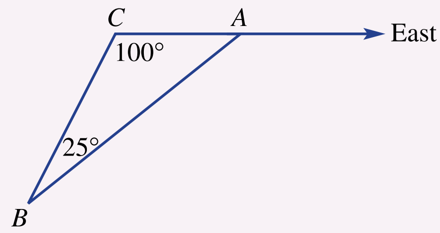
1 A right-angled isosceles triangle has area of 4 square units. Determine the exact perimeter of the triangle.

2 Find the area of the triangle shown using trigonometry. (*Hint*: Insert a line showing the height of the triangle.)



3 A rectangle $ABCD$ has sides $AB = CD = 34$ cm. E is a point on CD such that $CE = 9$ cm and $ED = 25$ cm. AE is perpendicular to EB. What is the length of BC?

4 Find the bearing from B to C in this diagram.



5 Which is a better fit? A square peg in a round hole or a round peg in a square hole. Use area calculations and percentages to investigate.

6 Boat A is 20 km from port on a true bearing of 025° and boat B is 25 km from port on a true bearing of 070°. Boat B is in distress. What bearing (to the nearest degree) should boat A travel on to reach boat B?

7 For positive integers m and n such that $n < m$, the Pythagorean triples (e.g. 3, 4, 5) can be generated using $a = m^2 - n^2$ and $b = 2mn$, where a and b are the two shorter sides of the right-angled triangle.

a Using these formulas and Pythagoras' theorem to calculate the third side, generate the Pythagorean triples for these values of m and n.

i $m = 2, n = 1$

ii $m = 3, n = 2$

b Using the expressions for a and b and Pythagoras' theorem, find a rule for c (the hypotenuse) in terms of n and m.

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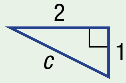
Chapter summary

Pythagoras' theorem and trigonometry

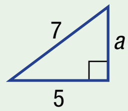
Finding the hypotenuse

$c^2 = 2^2 + 1^2$
$\quad = 5$
$c = \sqrt{5}$
$\quad = 2.24$ (to 2 d.p.)

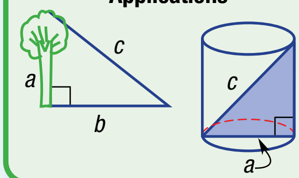


Shorter sides

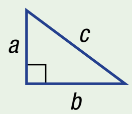
$a^2 + 5^2 = 7^2$
$a^2 = 7^2 - 5^2$
$\quad = 24$
$\therefore a = \sqrt{24}$



Applications



Pythagoras' theorem



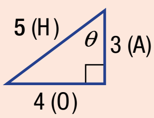
$c^2 = a^2 + b^2$

SOHCAHTOA

$\sin\theta = \frac{\text{O}}{\text{H}} = \frac{4}{5}$

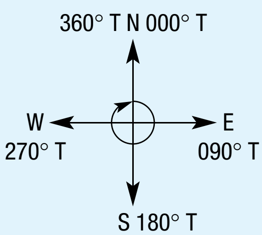
$\cos\theta = \frac{\text{A}}{\text{H}} = \frac{3}{5}$

$\tan\theta = \frac{\text{O}}{\text{A}} = \frac{4}{3}$



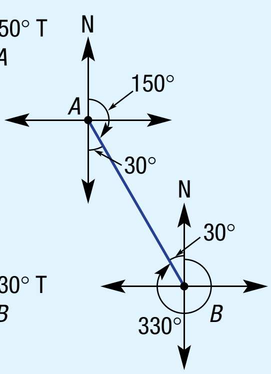
Bearings

True bearings are measured clockwise from north.

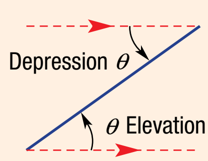


B is 150° T from *A*

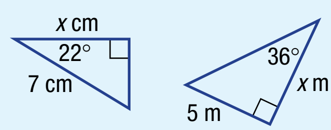
A is 330° T from *B*



Elevation and depression



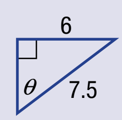
Finding side lengths



$\cos 22° = \frac{x}{7}$
$x = 7\cos 22°$
$\quad = 6.49$
(to 2 d.p.)

$\tan 36° = \frac{5}{x}$
$x \times \tan 36° = 5$
$x = \frac{5}{\tan 36°}$
$\quad = 6.88$ (to 2 d.p.)

Finding angles



$\sin\theta = \frac{6}{7.5}$
$\quad = 53.13°$ (to 2 d.p.)

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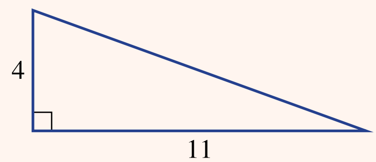
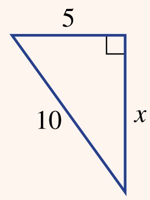
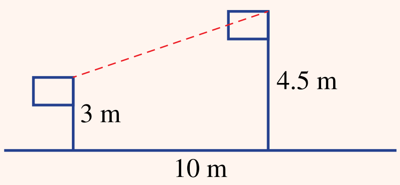
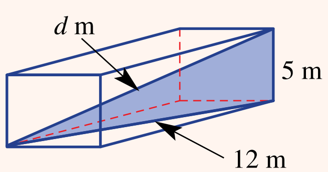
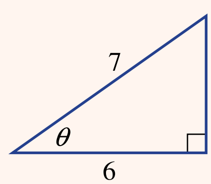
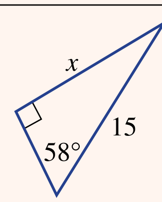
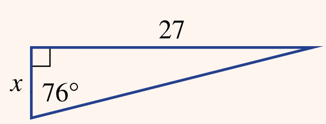
Chapter checklist with success criteria

A printable version of this checklist is available in the Interactive Textbook

Section	Success criteria	✔
3A	**1. I can find the length of the hypotenuse using Pythagoras' theorem.** e.g. Find the length of the hypotenuse as an exact value and rounded to one decimal place. 	☐
3B	**2. I can find the length of a shorter side using Pythagoras' theorem.** e.g. Find the value of the pronumeral correct to one decimal place. 	☐
3C	**3. I can apply Pythagoras' theorem.** e.g. Two flag poles are located 10 m apart. The flag poles are 4.5 m and 3 m high. A cable links the top of the two flag poles. Find the length of the cable correct to one decimal place. 	☐
3D	**4. I can use Pythagoras in 3D.** e.g. Find the distance, d m, from one corner to the opposite corner in the rectangular prism shown. 	☐
3E	**5. I can write a trigonometric ratio.** e.g. Using the given sides, write a trigonometric ratio for the triangle shown. 	☐
3F	**6. I can evaluate a trigonometric ratio on a calculator.** e.g. Evaluate $\cos 35°$ correct to two decimal places.	☐
3F	**7. I can find a side length using a trigonometric ratio.** e.g. Find the value of x, correct to one decimal place. 	☐
3G	**8. I can find a side length using a trigonometric ratio with the unknown in the denominator.** e.g. Find the value of x, correct to one decimal place. 	☐

Chapter checklist

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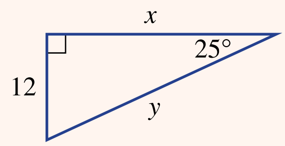
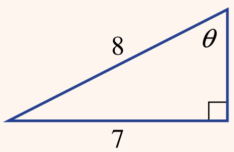
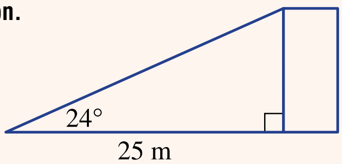
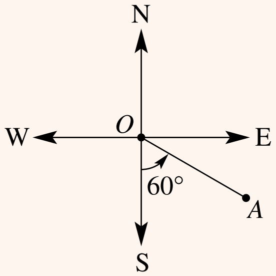
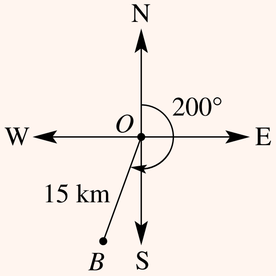
Chapter checklist

3G	**9. I can find multiple side lengths using trigonometry.** e.g. Find the values of the pronumerals, correct to one decimal place, in the right-angled triangle shown.  	☐
3H	**10. I can use an inverse trigonometric ratio.** e.g. Find the value of θ in $\cos\theta = 0.674$ correct to one decimal place.	☐
3H	**11. I can find an angle using trigonometric ratios.** e.g. Find the value of θ, correct to the nearest minute.  	☐
3I	**12. I can apply trigonometry using an angle of elevation.** e.g. The angle of elevation of the top of a building from a point on the ground 25 m from the base of the building is 24°. Find the height of the building to the nearest metre.  	☐
3I	**13. I can apply trigonometry using an angle of depression.** e.g. A duck on a lake spots a fish below the surface. The fish is 8 m below water level and a horizontal direct distance of 6 m from the duck. Find the angle of depression of the fish from the duck to the nearest degree.	☐
3J	**14. I can state true bearings.** e.g. For the diagram shown, give the true bearing of A from O and the true bearing of O from A.  	☐
3J	**15. I can use bearings with trigonometry.** e.g. A yacht sails from point O for 15 km on a bearing of 200° T to point B. How far west is the yacht from its starting point (to the nearest km) and what is the bearing of O from B?  	☐

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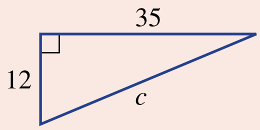
Short-answer questions

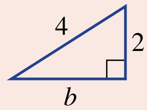
3A/B

1 Find the unknown length in these triangles. Give an exact answer.

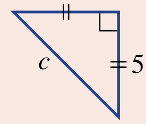
a



b

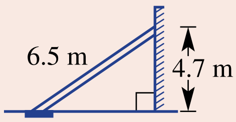


c



3B

2 A steel support beam of length 6.5 m is connected to a wall at a height of 4.7 m above the ground. Find the distance (to the nearest centimetre) between the base of the building and the point where the beam is joined to the ground.

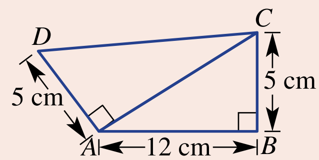


3A

3 For this double triangle, find:

a AC

b CD (correct to two decimal places).



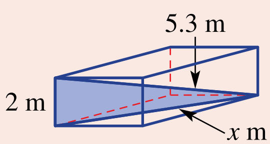
3C

4 Two cafés on opposite sides of an atrium in a shopping centre are respectively 10 m and 15 m above the ground floor. If the cafés are linked by a 20 m escalator, find the horizontal distance (to the nearest metre) across the atrium, between the two cafés.

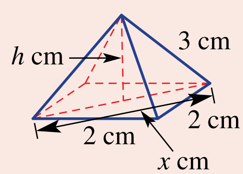
3D

5 Find the values of the pronumerals in the three-dimensional objects shown below. Give the answer correct to two decimal places.

a



b



3F

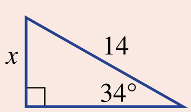
6 Find the following values correct to two decimal places.

a $\sin 40°$ **b** $\tan 66°$ **c** $\cos 44.3°$ **d** $\sin 33°40'$

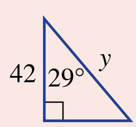
3F/G/H

7 Find the value of each pronumeral correct to two decimal places.

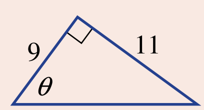
a



b

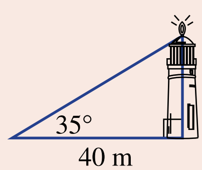


c



3I

8 The angle of elevation of the top of a lighthouse from a point on the ground 40 m from its base is 35°. Find the height of the lighthouse to two decimal places.



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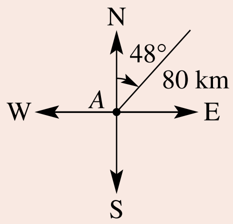
Chapter review

3I

9 From a point on the ground, Geoff measures the angle of elevation of a 120 m tall tower to be 34°. How far is Geoff from the base of the tower, correct to two decimal places?

3J

10 A yacht sails 80 km on a true bearing of 048°.



a How far east of its starting point is the yacht, correct to two decimal places?

b How far north of its starting point is the yacht, correct to two decimal places?

3J

11 A ship leaves Coffs Harbour and sails 320 km east. It then changes direction and sails 240 km due north to its destination. What will the ship's true bearing be from Coffs Harbour when it reaches its destination, correct to two decimal places?

3I

12 From the roof of a skyscraper, Aisha spots a car at an angle of depression of 51°. If the skyscraper is 78 m high, how far away is the car from the base of the skyscraper, correct to one decimal place?

3F

13 Penny wants to measure the breadth of a river. She places two markers, A and B, 10 m apart along one bank. C is a point directly opposite marker B. Penny measures angle BAC to be 28°. Find the breadth of the river to one decimal place.

3I

14 An aeroplane takes off and climbs at an angle of 15° to the horizontal, at a speed of 210 km/h along its flight path for 15 minutes.

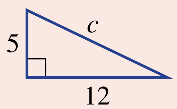
a Find the distance the aeroplane travels.

b Find the height the aeroplane reaches, correct to two decimal places.

Multiple-choice questions

3A

1 For the right-angled triangle shown, the length of the hypotenuse can be found from the equation:



A $c^2 = 5^2 + 12^2$

B $c^2 = 5^2 - 12^2$

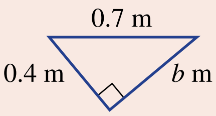
C $c^2 = 12^2 - 5^2$

D $c^2 = 5^2 \times 12^2$

E $(5 + 12)^2$

3B

2 For the right-angled triangle shown, the value of b is given by:



A $\sqrt{0.7^2 + 0.4^2}$

B $\sqrt{0.7^2 - 0.4^2}$

C $\sqrt{0.4^2 - 0.7^2}$

D $\sqrt{0.7^2 \times 0.4^2}$

E $\sqrt{(0.7 - 0.4^2)^2}$

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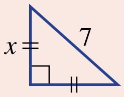
3B **3** For the right-angled triangle shown:



A $x^2 = \frac{49}{2}$

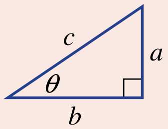
B $7x^2 = 2$

C $x^2 = \frac{7}{2}$

D $x^2 + 7^2 = x^2$

E $x^2 = \frac{2}{7}$

3E **4** For the triangle shown:



A $\sin\theta = \frac{a}{b}$

B $\sin\theta = \frac{c}{a}$

C $\sin\theta = \frac{a}{c}$

D $\sin\theta = \frac{b}{c}$

E $\sin\theta = \frac{c}{b}$

3F **5** The value of $\cos 46°$ correct to four decimal places is:

A 0.7193

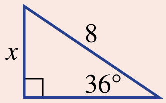
B 0.6947

C 0.594

D 0.6532

E 1.0355

3F **6** In the diagram the value of x, correct to two decimal places, is:



A 40

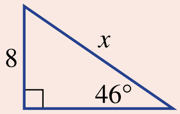
B 13.61

C 4.70

D 9.89

E 6.47

3G **7** The length of x in the triangle is given by:



A $8 \sin 46°$

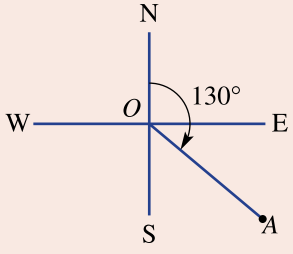
B $8 \cos 46°$

C $\dfrac{8}{\cos 46°}$

D $\dfrac{8}{\sin 46°}$

E $\dfrac{\cos 46°}{8}$

3J **8** The true bearing of A from O is 130°. The true bearing of O from A is:



A 050°

B 220°

C 310°

D 280°

E 170°

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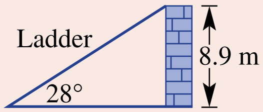
3G

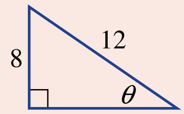
9 A ladder is inclined at an angle of 28° to the horizontal. If the ladder reaches 8.9 m up the wall, the length of the ladder correct to the nearest metre is:



A 19 m
B 4 m
C 2 m
D 10 m
E 24 m

3H

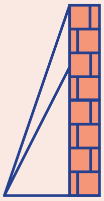
10 The value of θ in the diagram, correct to two decimal places, is:



A 0.73°
B 41.81°
C 48.19°
D 33.69°
E 4.181°

Extended-response questions

1 An extension ladder is initially placed so that it reaches 2 m up a wall. The foot of the ladder is 80 cm from the base of the wall.



a Find the length of the ladder, to the nearest centimetre, in its original position.
b Without moving the foot, the ladder is extended so that it reaches 1 m further up the wall. How far (to the nearest centimetre) has the ladder been extended?
c The ladder is placed so that its foot is now 20 cm closer to the base of the wall.
   i How far up the wall can the extended ladder length found in part **b** reach? Round to two decimal places.
   ii Is this further than the distance in part **b**?

2 From the top of a 100 m cliff, Skevi sees a boat out at sea at an angle of depression of 12°.
a Draw a diagram for this situation.
b Find how far out to sea the boat is to the nearest metre.
c A swimmer is 2 km away from the base of the cliff and in line with the boat. What is the angle of depression to the swimmer, to the nearest degree?
d How far away is the boat from the swimmer, to the nearest metre?

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3 A pilot takes off from Amber Island and flies for 150 km at 040° T to Barter Island where she unloads her first cargo. She intends to fly to Dream Island, but a bad thunderstorm between Barter and Dream islands forces her to fly off-course for 60 km to Crater Atoll on a bearing of 060° T. She then turns on a bearing of 140° T and flies for 100 km until she reaches Dream Island, where she unloads her second cargo. She then takes off and flies 180 km on a bearing of 055° T to Emerald Island.

a How many extra kilometres did she fly trying to avoid the storm? Round to the nearest kilometre.

b From Emerald Island she flies directly back to Amber Island. How many kilometres did she travel on her return trip? Round to the nearest kilometre.

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4

Linear relationships

Maths in context: Animation, cartoon, and game developers

The virtual world of animation has made computer gaming and cartoon videos into global multimillion dollar industries. Animation algorithms apply skills from algebra, coordinate geometry, linear relations, trigonometry, and more advanced mathematics.

When designing a cartoon character, the animator codes a subdivision process. Using the midpoint formula, a 2D polygon with straight sides is morphed into a curve. By repeatedly subdividing lengths and slicing off corners, a virtual curve illusion is created.

A similar subdivision process is coded to build 3D shapes. The initial polyhedral object has flat polygon faces that are then repeatedly subdivided and reduced. The finished shape has countless flat surfaces appearing on a computer screen as a smoothly curved virtual 3D object.

Linear algebra is used to control the position and movement when animating a virtual object. It can be transformed using linear equations that alter the coordinates of its vertices. Up or down movement is achieved by adding or subtracting, enlargement or shrinking uses multiplication or division, and rotation uses trigonometry.

The cartoon Bluey (pictured) is a successful pre-school TV animated cartoon developed in Brisbane and distributed internationally on Disney+.

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Chapter contents

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- determines the midpoint, gradient and length of an interval, and graphs linear relationships, with and without digital tools (MA5-LIN-C-01)
- graphs and interprets linear relationships using the gradient/slope-intercept form (MA5-LIN-C-02)
- describes and applies transformations, the midpoint, gradient/slope and distance formulas, and equations of lines to solve problems (MA5-LIN-P-01).

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

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4A Introducing linear relationships CONSOLIDATING

Learning intentions
- To review the features of the Cartesian plane and plot points using coordinates
- To know that a linear relationship gives a set of points that form a straight line
- To be able to complete a table of values and plot points to form a linear graph
- To be able to arrange the equation of a line in various ways, including general form
- To be able to decide if a point is on a line by substituting into the equation of the line
- To know which points are represented by the x-intercept and y-intercept
- To be able to identify the x-intercept and y-intercept from a table or graph

Past, present and future learning
- This section consolidates Stage 4 concepts which are used in Stages 5 and 6
- Some of these questions are more challenging than those in the Year 8 book
- These concepts will be revised and extended in Chapter 1 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

If two variables are related in some way, we can use mathematical rules to precisely describe this relationship. The most simple kind of mathematical relationship is one that can be illustrated with a straight line graph. These are called linear relationships. The volume of petrol in your car at a service bowser, for example, might initially be 10 L then be increasing by 1.2 L per second after that. This is an example of a linear relationship between *volume* and *time* because the volume is increasing at a constant rate of 1.2 L/s.

A car's annual running costs can be \$4000 p.a., including fuel, repairs, registration and insurance. If a car cost \$17 000, the total cost, C, for n years could be expressed as the linear relationship: $C = 4000n + 17\,000$.

Lesson starter: Is it linear?

Here are three rules linking x and y.

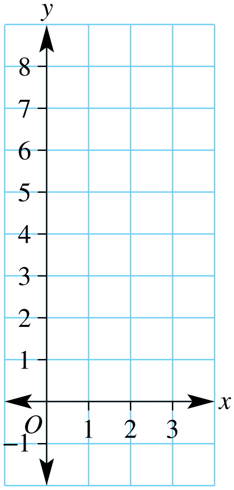
1 $y_1 = \frac{2}{x} + 1$

2 $y_2 = x^2 - 1$

3 $y_3 = 3x - 4$

First complete this simple table and graph.

x	1	2	3
y_1			
y_2			
y_3			



- Which of the three rules do you think is linear?
- How do the table and graph help you decide it's linear?

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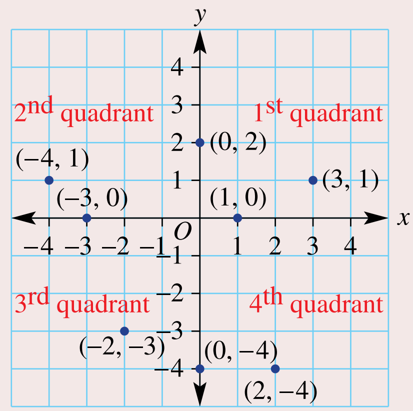
KEY IDEAS

- **Coordinate geometry** provides a link between geometry and algebra.
- The **Cartesian plane** (or number plane) consists of two axes which divide the number plane into four **quadrants**.
  - The horizontal x-axis and vertical y-axis intersect at right angles at the **origin** $O(0, 0)$.
  - A point is precisely positioned on a Cartesian plane using the **coordinate pair** (x, y) where x describes the horizontal position and y describes the vertical position of the point from the origin.

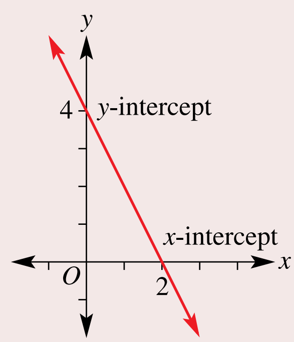


- A **linear relationship** is a set of ordered pairs (x, y) that when graphed give a straight line.
- Linear relationships have rules that may be of the form:
  - $y = mx + c$. For example, $y = 2x + 1$.
  - $ax + by = d$ or $ax + by + c = 0$. For example, $2x - 3y = 4$ or $2x - 3y - 4 = 0$.
- The x-intercept is the point where the graph meets the x-axis.
- The y-intercept is the point where the graph meets the y-axis.

x-intercept

x	−2	−1	0	1	2	3
y	8	6	4	2	0	−2

y-intercept



BUILDING UNDERSTANDING

1 Refer to the Cartesian plane shown.

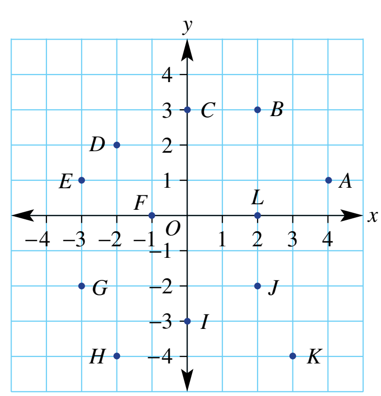
a Give the coordinates of all the points $A - L$.

b State which points lie on:

i the x-axis **ii** the y-axis.

c State which points lie inside the:

i 2nd quadrant **ii** 4th quadrant.



2 For the rule $y = 3x - 4$, find the value of y for these x-values.

a $x = 2$ **b** $x = -1$

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3 For the rule $y = -2x + 1$, find the value of y for these x-values.

a $x = 0$ **b** $x = -10$

4 Give the coordinates of three pairs of points on the line $y = x + 3$.

Example 1 Plotting points to graph straight lines

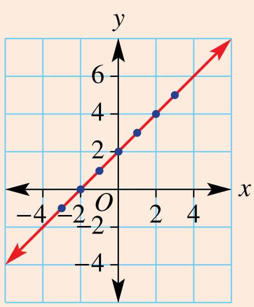
Using $-3 \leqslant x \leqslant 3$, construct a table of values and plot a graph for these linear relationships.

a $y = x + 2$ **b** $y = -2x + 2$

SOLUTION

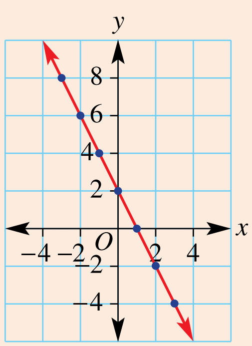
a

x	−3	−2	−1	0	1	2	3
y	−1	0	1	2	3	4	5



b

x	−3	−2	−1	0	1	2	3
y	8	6	4	2	0	−2	−4



EXPLANATION

Use $-3 \leqslant x \leqslant 3$ as instructed and substitute each value of x into the rule $y = x + 2$.

The coordinates of the points are read from the table, i.e. $(-3, -1)$, $(-2, 0)$, etc.

Plot each point and join to form a straight line. Extend the line to show it continues in either direction.

Use $-3 \leqslant x \leqslant 3$ as instructed and substitute each value of x into the rule $y = -2x + 2$.

For example:

$$\begin{aligned} x = -3, y &= -2 \times (-3) + 2 \\ &= 6 + 2 \\ &= 8 \end{aligned}$$

Plot each point and join to form a straight line. Extend the line beyond the plotted points.

Now you try

Using $-3 \leqslant x \leqslant 3$, construct a table of values and plot a graph for these linear relationships.

a $y = x - 1$ **b** $y = -3x + 1$

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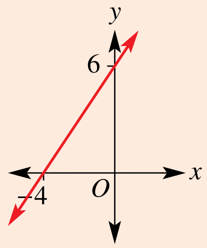
Example 2 Reading off the *x*-intercept and *y*-intercept

Write down the coordinates of the *x*-intercept and *y*-intercept from this table and graph.

a

x	−2	−1	0	1	2	3
y	6	4	2	0	−2	−4

b



SOLUTION

a The x-intercept is at $(1, 0)$.

The y-intercept is at $(0, 2)$.

b The x-intercept is at $(-4, 0)$.

The y-intercept is at $(0, 6)$.

EXPLANATION

a The x-intercept is at the point where $y = 0$ (on the x-axis).

The y-intercept is at the point where $x = 0$ (on the y-axis).

b The x-intercept is at the point where $y = 0$ (on the x-axis).

The y-intercept is at the point where $x = 0$ (on the y-axis).

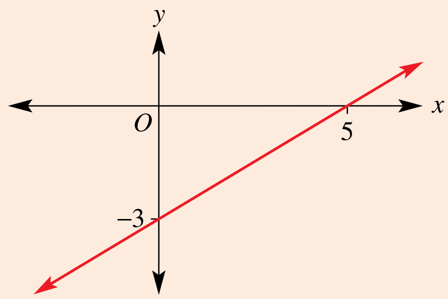
Now you try

Write down the coordinates of the *x*-intercept and *y*-intercept from this table and graph.

a

x	−3	−2	−1	0	1	2
y	4	3	2	1	0	−1

b



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Example 3 Deciding if a point is on a line

Decide if the point $(-2, 4)$ is on the line with the given rules.

a $y = 2x + 10$ **b** $y = -x + 2$

SOLUTION

a $y = 2x + 10$
Substitute $x = -2$
$y = 2(-2) + 10$
$= 6$
$\therefore$ the point $(-2, 4)$ is not on the line.

EXPLANATION

Find the value of y by substituting $x = -2$ into the rule for y.

The y-value is not 4, so $(-2, 4)$ is not on the line.

b $y = -x + 2$
Substitute $x = -2$
$y = -(-2) + 2$
$= 4$
$\therefore$ the point $(-2, 4)$ is on the line.

By substituting $x = -2$ into the rule for the line, y is 4.

So $(-2, 4)$ is on the line.

Now you try

Decide if the point $(-1, 6)$ is on the line with the given rules.

a $y = -2x + 4$ **b** $y = 3x + 5$

Exercise 4A

FLUENCY 1–3(½) 1–4(½) 1–4(½)

Example 1

1 Using $-3 \leqslant x \leqslant 3$, construct a table of values and plot a graph for these linear relations.

a $y = x - 1$ **b** $y = 2x - 3$ **c** $y = -x + 4$ **d** $y = -3x$

Example 2

2 Write down the coordinates of the x- and y-intercepts for the following tables and graphs.

a

x	-3	-2	-1	0	1	2	3
y	4	3	2	1	0	-1	-2

b

x	-3	-2	-1	0	1	2	3
y	-1	0	1	2	3	4	5

c

x	-1	0	1	2	3	4
y	10	8	6	4	2	0

d

x	-5	-4	-3	-2	-1	0
y	0	2	4	6	8	10

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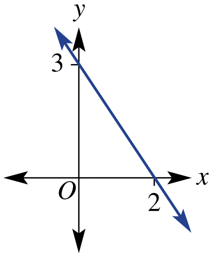
e


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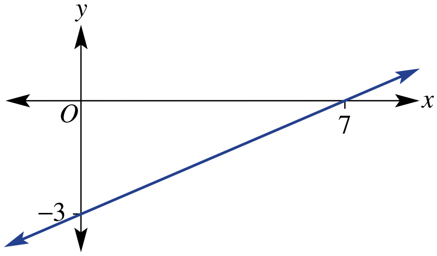

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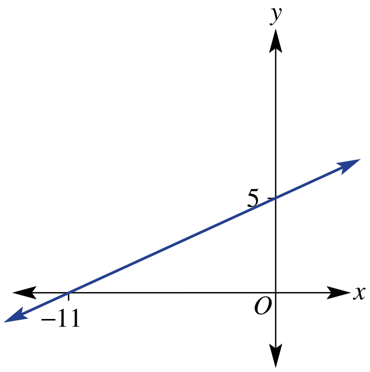

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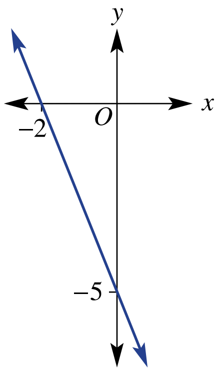

Example 3

3 Decide if the point (1, 2) is on the line with the given rule.

a $y = x + 1$ **b** $y = 2x - 1$ **c** $y = -x + 3$

d $y = -2x + 4$ **e** $y = -x + 5$ **f** $y = -\frac{1}{2}x + \frac{1}{2}$

4 Decide if the point (−3, 4) is on the line with the given rule.

a $y = 2x + 8$ **b** $y = x + 7$ **c** $y = -x - 1$

d $y = \frac{1}{3}x + 6$ **e** $y = -\frac{1}{3}x + 3$ **f** $y = \frac{5}{6}x + \frac{11}{2}$

PROBLEM–SOLVING	5	5, 6	5(½), 6

5 Find a rule in the form $y = mx + c$ (e.g. $y = 2x - 1$) that matches these tables of values.

a

x	0	1	2	3	4
y	2	3	4	5	6

b

x	−1	0	1	2	3
y	−2	0	2	4	6

c

x	−2	−1	0	1	2	3
y	−3	−1	1	3	5	7

d

x	−3	−2	−1	0	1	2
y	5	4	3	2	1	0

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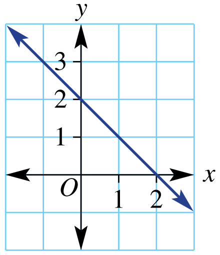
6 Match rules **a**, **b**, **c** and **d** to the graphs **A**, **B**, **C** and **D**.

a $y = 2x + 1$ **b** $y = -x - 1$ **c** $y = -2x + 3$ **d** $x + y = 2$

A



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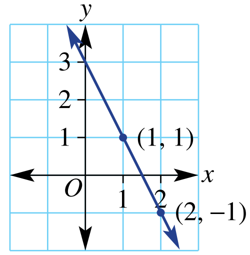
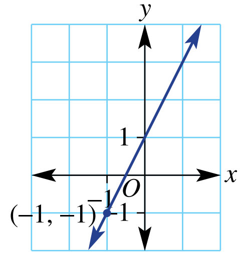
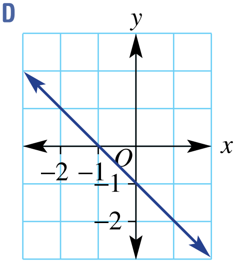


C



D



REASONING

7	7, 8	8, 9

7 Decide if the following rules are equivalent.

a $y = 1 - x$ and $y = -x + 1$

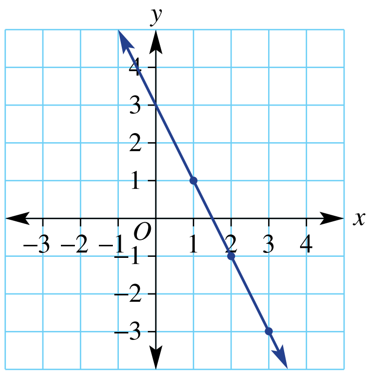
b $y = 1 - 3x$ and $y = 3x - 1$

c $y = -2x + 1$ and $y = -1 - 2x$

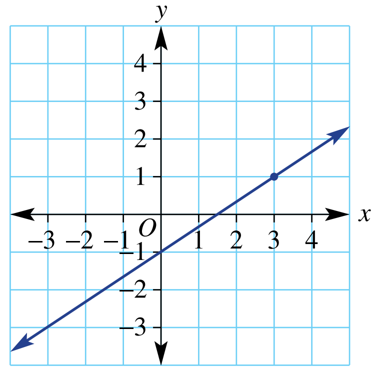
d $y = -3x + 1$ and $y = 1 - 3x$

8 Give reasons why the x-intercept on these graphs is at $\left(\frac{3}{2}, 0\right)$.

a



b



9 Decide if the following equations are true or false.

a $\frac{2x + 4}{2} = x + 4$

b $\frac{3x - 6}{3} = x - 2$

c $\frac{1}{2}(x - 1) = \frac{1}{2}x - \frac{1}{2}$

d $\frac{2}{3}(x - 6) = \frac{2}{3}x - 12$

ENRICHMENT: Tough rule finding

–	–	10

10 Find the linear rule linking x and y in these tables.

a

x	−1	0	1	2	3
y	5	7	9	11	13

b

x	−2	−1	0	1	2
y	22	21	20	19	18

c

x	0	2	4	6	8
y	−10	−16	−22	−28	−34

d

x	−5	−4	−3	−2	−1
y	29	24	19	14	9

e

x	1	3	5	7	9
y	1	2	3	4	5

f

x	−14	−13	−12	−11	−10
y	$5\frac{1}{2}$	5	$4\frac{1}{2}$	4	$3\frac{1}{2}$

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4B Graphing straight lines using intercepts

Learning intentions
- To understand that only two points are required to graph a straight line
- To be able to graph a line by finding the x-intercept and y-intercept

Past, present and future learning
- This section addresses the Stage 5 Path Topic called *Linear relationships C*
- These concepts will be revised, briefly, in Chapter 1 of our Year 10 book and also in Stage 6 Advanced

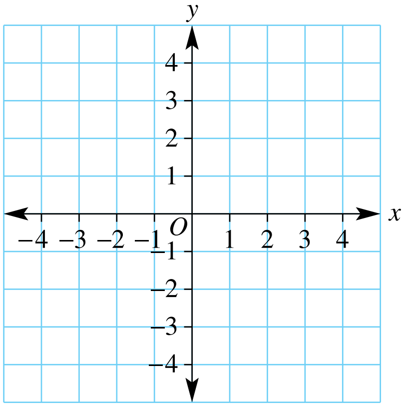
When linear rules are graphed, all the points lie in a straight line. It is therefore possible to graph a straight line using only two points. Two critical points that help draw these graphs are the x-intercept and y-intercept introduced in the previous section.

Businesses claim a tax deduction for the annual decrease in the value, V, of equipment over n years. A straight line segment joining the intercepts $(0, V)$ and $(n, 0)$ shows that the rate of decrease per year is $\frac{V}{n}$.

Lesson starter: Two key points

Consider the relation $y = \frac{1}{2}x + 1$ and complete this table and graph.

x	−4	−3	−2	−1	0	1	2
y							



- What are the coordinates of the point where the line crosses the y-axis? That is, state the coordinates of the y-intercept.
- What are the coordinates of the point where the line crosses the x-axis? That is, state the coordinates of the x-intercept.
- Discuss how you might find the coordinates of the x- and y-intercepts without drawing a table and plotting points. Explain your method.

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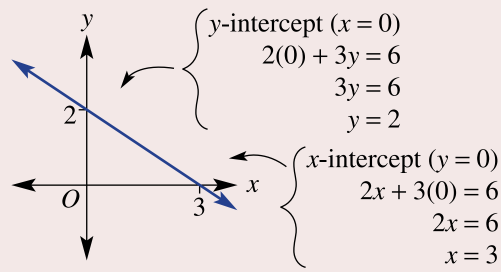
KEY IDEAS

- Two points are required to sketch a straight line graph. Often these points are the axes intercepts.
- The y-**intercept** is the point where the line intersects the y-axis (where $x = 0$).
  - Substitute $x = 0$ to find the y-intercept.
- The x-**intercept** is the point where the line intersects the x-axis (where $y = 0$).
  - Substitute $y = 0$ to find the x-intercept.

Example: $2x + 3y = 6$



BUILDING UNDERSTANDING

1 Mark the following x- and y-intercepts on a set of axes and join in a straight line.

a x-intercept: $(-2, 0)$, y-intercept: $(0, 3)$
b x-intercept: $(4, 0)$, y-intercept: $(0, 6)$

2 **a** Find the value of y in these equations.

i $2y = 6$ **ii** $y = 3 \times 0 + 4$ **iii** $-2y = 12$

b Find the value of x in these equations.

i $-4x = -40$ **ii** $0 = 2x - 2$ **iii** $\frac{1}{2}x = 3$

3 For these equations find the coordinates of the y-intercept by letting $x = 0$.

a $x + y = 4$ **b** $x - y = 5$ **c** $2x + 3y = 9$

4 For these equations find the coordinates of the x-intercept by letting $y = 0$.

a $2x - y = -4$ **b** $4x - 3y = 12$ **c** $y = 3x - 6$

Example 4 Sketching with intercepts

Sketch the graph of the following, showing the x- and y-intercepts.

a $2x + 3y = 6$ **b** $y = 2x - 6$

SOLUTION	EXPLANATION
a $2x + 3y = 6$ y-intercept (let $x = 0$): $2(0) + 3y = 6$ $3y = 6$ $y = 2$	Only two points are required to generate a straight line. For the y-intercept, substitute $x = 0$ into the rule and solve for y by dividing each side by 3.
$\therefore$ the y-intercept is at $(0, 2)$.	State the y-intercept.

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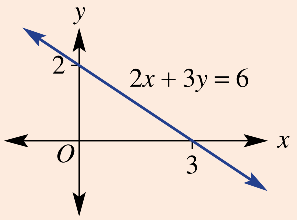
x-intercept (let $y = 0$):
$2x + 3(0) = 6$
$2x = 6$
$x = 3$
$\therefore$ the x-intercept is at (3, 0).

Similarly to find the x-intercept, substitute $y = 0$ into the rule and solve for x.

State the x-intercept.



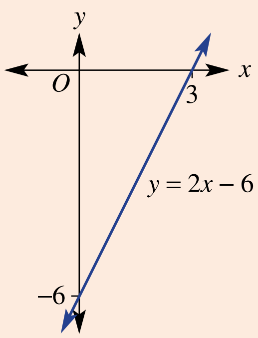
Mark and label the intercepts on the axes and sketch the graph by joining the two intercepts.

b $y = 2x - 6$
y-intercept (let $x = 0$):
$y = 2(0) - 6$
$y = -6$
$\therefore$ the y-intercept is at $(0, -6)$.

Substitute $x = 0$ for the y-intercept.
Simplify to find the y-coordinate.

x-intercept (let $y = 0$):
$0 = 2x - 6$
$6 = 2x$
$x = 3$
$\therefore$ the x-intercept is at (3, 0).

Substitute $y = 0$ for the x-intercept. Solve the remaining equation for x by adding 6 to both sides and then divide both sides by 2.



Mark in the two intercepts and join to sketch the graph.

Now you try

Sketch the graph of the following, showing the x- and y-intercepts.

a $3x + 5y = 15$ **b** $y = 3x - 6$

Exercise 4B

FLUENCY	1–2(½)	1–3(½)	1–3(⅓)

Example 4a

1 Sketch the graph of the following, showing the x- and y-intercepts.

a $x + y = 2$ **b** $x - y = -2$ **c** $2x + y = 4$
d $3x - y = 9$ **e** $4x - 2y = 8$ **f** $3x + 2y = 6$
g $y - 3x = 12$ **h** $-5y + 2x = -10$ **i** $-x + 7y = 21$

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Example 4b

2 Sketch the graph of the following, showing the x- and y-intercepts.

a $y = 3x + 3$
b $y = 2x + 2$
c $y = x - 5$
d $y = -x - 6$
e $y = -2x - 2$
f $y = -2x + 4$

3 Sketch the graph of each of the following mixed linear relationships.

a $x + 2y = 8$
b $3x - 5y = 15$
c $y = 3x - 6$
d $3y - 4x - 12 = 0$
e $2x - y - 4 = 0$
f $2x - y + 5 = 0$

PROBLEM–SOLVING	4, 5	4, 5	5, 6

4 The distance d metres of a vehicle from an observation point after t seconds is given by the rule $d = 8 - 2t$.

a Find the distance from the observation point initially (at $t = 0$).
b Find after what time t the distance d is equal to 0 (substitute $d = 0$).
c Sketch a graph of d versus t between the d and t intercepts.

5 The height, h metres, of a lift above the ground after t seconds is given by $h = 100 - 8t$.

a How high is the lift initially (at $t = 0$)?
b How long does it take for the lift to reach the ground ($h = 0$)?

6 Find the x- and y-axis intercept coordinates of the graphs with the given rules. Write answers using fractions.

a $3x - 2y = 5$
b $x + 5y = -7$
c $y - 2x = -13$
d $y = -2x - 1$
e $2y = x - 3$
f $-7y = 1 - 3x$

REASONING	7	7, 8	8, 9

7 Use your algebra and fraction skills to help sketch graphs for these relations by finding x- and y-intercepts.

a $\frac{x}{2} + \frac{y}{3} = 1$
b $y = \frac{8 - x}{4}$
c $\frac{y}{2} = \frac{2 - 4x}{8}$

8 Explain why the graph of the equation $ax + by = 0$ must pass through the origin for any values of the constants a and b.

9 Write down the rule for the graph with these axes intercepts. Write the rule in the form $ax + by = d$.

a (0, 4) and (4, 0)
b (0, 2) and (2, 0)
c (0, -3) and (3, 0)
d (0, 1) and (-1, 0)
e (0, k) and (k, 0)
f (0, $-k$) and ($-k$, 0)

ENRICHMENT: Intercept families	–	–	10

10 Find the coordinates of the x- and y-intercepts in terms of the constants a, b and c for these relationships.

a $ax + by = c$
b $y = \frac{a}{b}x + c$
c $\frac{ax - by}{c} = 1$
d $ay - bx = c$
e $ay = bx + c$
f $a(x + y) = bc$

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4C Lines with one intercept

Learning intentions
- To be able to use equations to graph vertical lines and horizontal lines
- To know that every line of the form $y = mx$ passes through the origin
- To be able to graph lines of the form $y = mx$ using the origin and one other point

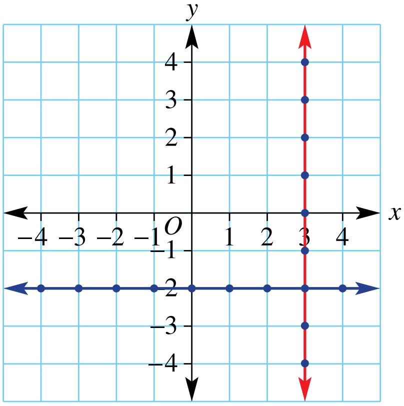
Past, present and future learning
- This section addresses the Stage 5 Core Topic called *Linear relationships A*
- These concepts will be revised, briefly, in Chapter 1 of our Year 10 book and also in Stage 6 Advanced

Lines with one intercept include vertical lines, horizontal lines and lines that pass through the origin.

Linear relationships with graphs through (0, 0) include:
- Fertiliser in kg = usage rate in kg/acre × number of acres;
- annual interest = interest rate p.a. × amount invested;
- weekly pay = pay rate in $/h × number of hours.

Lesson starter: What rule satisfies all points?

Here is one vertical and one horizontal line.
- For the vertical line shown, write down the coordinates of all the points shown as dots.
- What is always true for each coordinate pair?
- What simple equation describes every point on the line?
- For the horizontal line shown, write down the coordinates of all the points shown as dots.
- What is always true for each coordinate pair?
- What simple equation describes every point on the line?
- Where do the two lines intersect?



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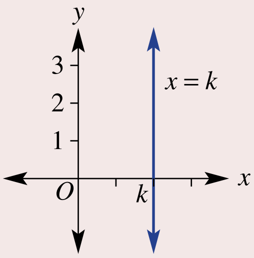
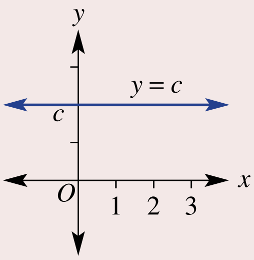
KEY IDEAS

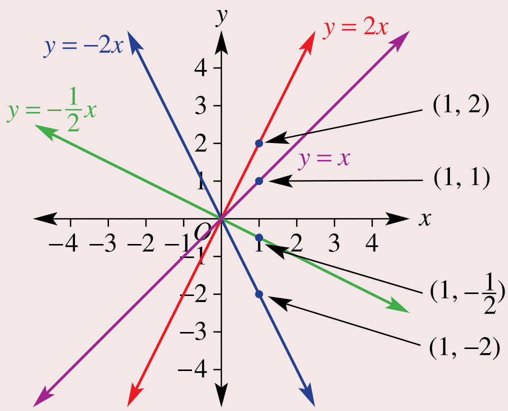
- Vertical line: $x = k$
  - Parallel to the y-axis
  - Equation of the form $x = k$, where k is a constant
  - x-intercept is at $(k, 0)$



- Horizontal line: $y = c$
  - Parallel to the x-axis
  - Equation of the form $y = c$, where c is a constant
  - y-intercept is at $(0, c)$



- Lines through the origin $(0, 0)$: $y = mx$
  - y-intercept is at $(0, 0)$
  - x-intercept is at $(0, 0)$
  - Substitute $x = 1$ or any other value of x to find a second point



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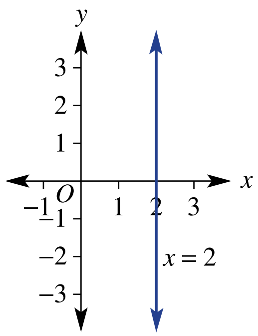
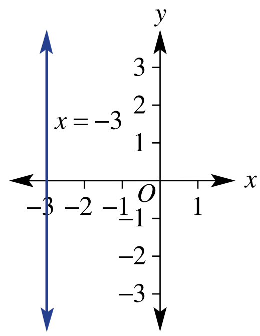
BUILDING UNDERSTANDING

1 State the coordinates of the x-intercept for these graphs.

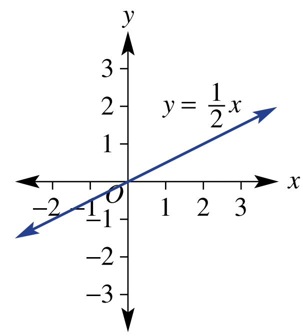
a



b

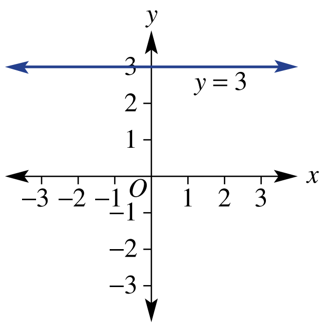


c

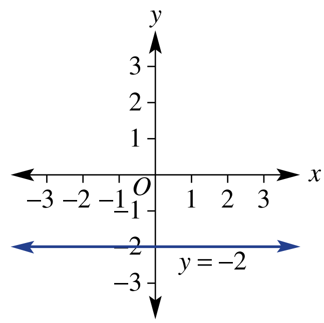


2 State the coordinates of the y-intercept for these graphs.

a



b



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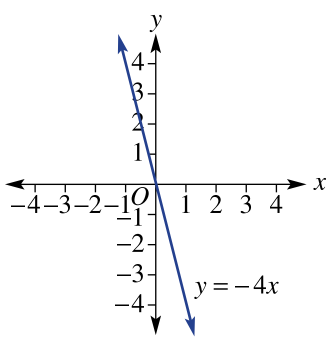

3 Find the value of y if $x = 1$ using these rules.

a $y = 5x$ **b** $y = \frac{1}{3}x$ **c** $y = -4x$ **d** $y = -0.1x$

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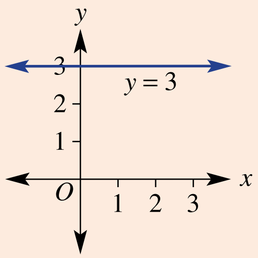
Example 5 Graphing vertical and horizontal lines

Sketch the graph of the following horizontal and vertical lines.

a $y = 3$ **b** $x = -4$

SOLUTION

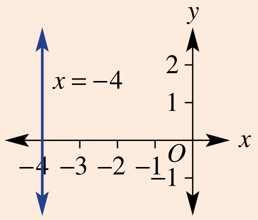
a



EXPLANATION

y-intercept is at (0, 3).
Sketch a horizontal line through all points where $y = 3$.

b



x-intercept is at $(-4, 0)$.
Sketch a vertical line through all points where $x = -4$.

Now you try

Sketch the graph of the following horizontal and vertical lines.

a $y = -1$ **b** $x = 3$

Example 6 Sketching lines that pass through the origin

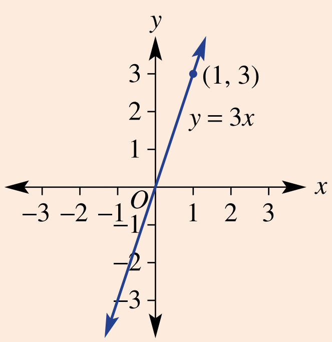
Sketch the graph of $y = 3x$.

SOLUTION

The x- and the y-intercept are both at (0, 0).
Another point (let $x = 1$):

$$y = 3 \times (1)$$
$$y = 3$$

Another point is at (1, 3).



EXPLANATION

The equation is of the form $y = mx$.

As two points are required to generate the straight line, find another point by substituting $x = 1$.
Other x-values could also be used.

Plot and label both points and sketch the graph by joining the points in a straight line.

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Now you try

Sketch the graph of $y = -2x$.

Exercise 4C

FLUENCY | 1–3(½) | 1–4(½) | 1–4(½)

Example 5

1 Sketch the graph of the following horizontal and vertical lines.

a $x = 2$ **b** $x = 5$ **c** $y = 4$ **d** $y = 1$
e $x = -3$ **f** $x = -2$ **g** $y = -1$ **h** $y = -3$

Example 6

2 Sketch the graph of the following linear relationships that pass through the origin.

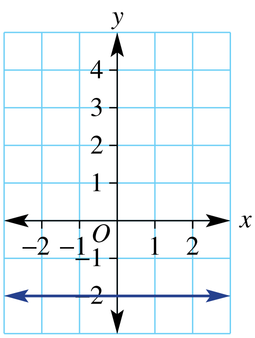
a $y = 2x$ **b** $y = 5x$ **c** $y = 4x$ **d** $y = x$
e $y = -4x$ **f** $y = -3x$ **g** $y = -2x$ **h** $y = -x$

3 Sketch the graphs of these special lines all on the same set of axes and label with their equations.

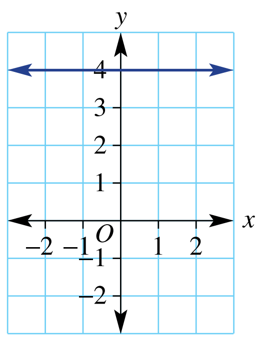
a $x = -2$ **b** $y = -3$ **c** $y = 2$ **d** $x = 4$
e $y = 3x$ **f** $y = -\frac{1}{2}x$ **g** $y = -1.5x$ **h** $x = 0.5$
i $x = 0$ **j** $y = 0$ **k** $y = 2x$ **l** $y = 1.5x$

4 Give the equation of each of the following graphs.

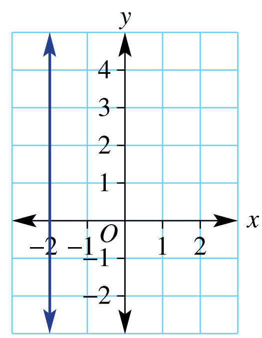
a



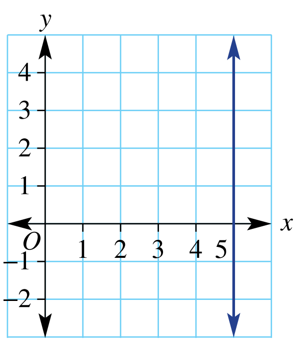
b



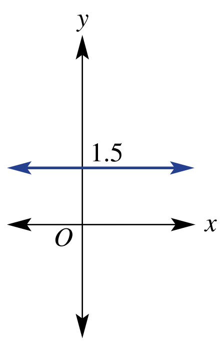
c



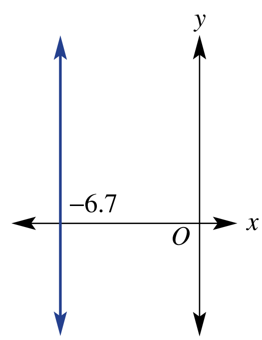
d



e



f



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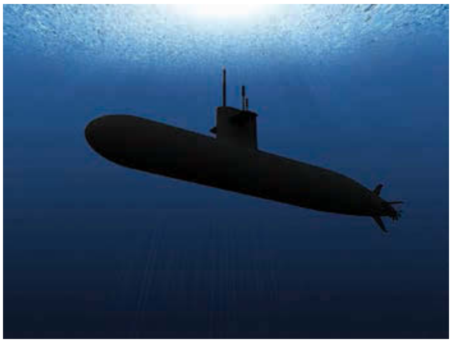
PROBLEM-SOLVING	5, 6	5, 6, 7(½), 8	5(½), 7(½), 8, 9

5 Find the equation of the straight line that is:
a parallel to the x-axis and passes through the point (1, 3)
b parallel to the y-axis and passes through the point (5, 4)
c parallel to the y-axis and passes through the point (−2, 4)
d parallel to the x-axis and passes through the point (0, 0).

6 If the surface of the sea is represented by the x-axis, state the equation of the following paths.
a A plane flies horizontally at 250 m above sea level. One unit is 1 metre.
b A submarine travels horizontally 45 m below sea level. One unit is 1 metre.



7 The graphs of these pairs of equations intersect at a point. Find the coordinates of the point.
a $x = 1, y = 2$
b $x = -3, y = 5$
c $x = 0, y = -4$
d $x = 4, y = 0$
e $y = -6x, x = 0$
f $y = 3x, x = 1$
g $y = -9x, x = 3$
h $y = 8x, y = 40$

8 Find the area of the rectangle contained within the following four lines.
a $x = 1, x = -2, y = -3, y = 2$
b $x = 0, x = 17, y = -5, y = -1$

9 The lines $x = -1$, $x = 3$ and $y = -2$ form three sides of a rectangle. Find the possible equation of the fourth line if:
a the area of the rectangle is:
i 12 square units
ii 8 square units
iii 22 square units
b the perimeter of the rectangle is:
i 14 units
ii 26 units
iii 31 units

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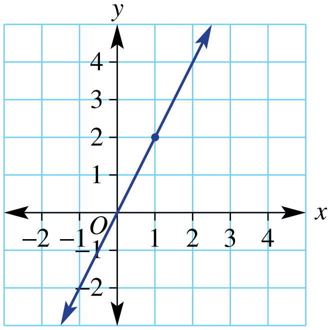
REASONING	10	10–11(½)	10–12(½)

10 The rules of the following graphs are of the form $y = mx$. Use the points marked with a dot to find m and hence state the equation.

a


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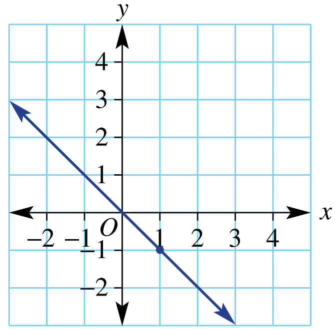

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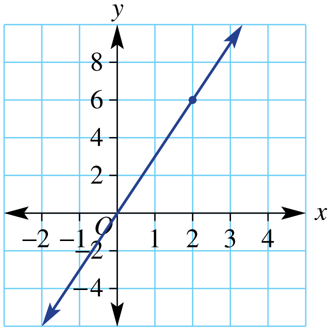
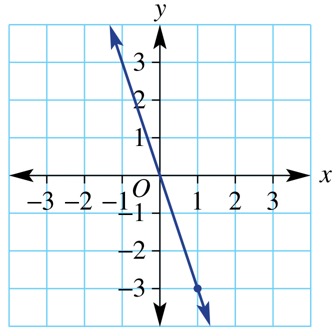


d


11 Find the equation of the line that passes through the origin and the given point.

a $(1, 3)$ b $(1, 4)$ c $(1, -5)$ d $(1, -2)$

12 Sketch the graph of each of the following by first making y or x the subject.

a $y - 8 = 0$
b $x + 5 = 0$
c $x + \frac{1}{2} = 0$
d $y - 0.6 = 0$
e $y + 3x = 0$
f $y - 5x = 0$
g $2y - 8x = 0$
h $5y + 7x = 0$

ENRICHMENT: Trisection	–	–	13–15

13 A vertical line, horizontal line and another line that passes through the origin all intersect at $(-1, -5)$. What are the equations of the three lines?

14 The lines $y = c$, $x = k$ and $y = mx$ all intersect at one point.

a State the coordinates of the intersection point.
b Find m in terms of c and k.

15 The area of a triangle formed by $x = 4$, $y = -2$ and $y = mx$ is 16 square units. Find the value of m given $m > 0$.

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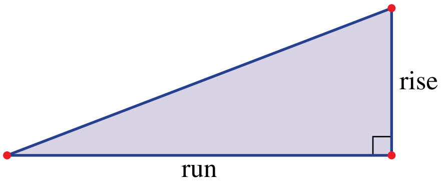
4D Gradient

Learning intentions
- To understand what is meant by the gradient of a line
- To know that the gradient of a line can be positive, negative, zero or undefined
- To be able to find the gradient of a line from a graph or using two given points

Past, present and future learning
- This section addresses the Stage 5 Core Topic called Linear relationships A
- These concepts will be revised, briefly, in Chapter 1 of our Year 10 book and also in Stage 6 Advanced

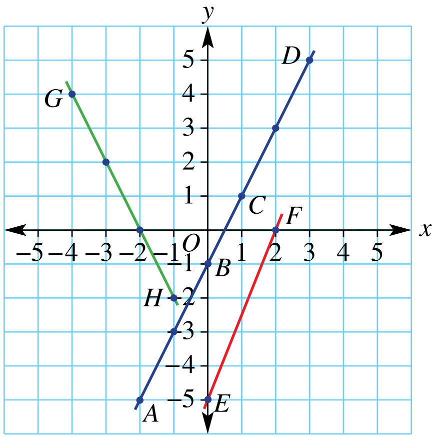
The gradient of a line is a measure of its slope. It is a number that describes the steepness of a line and is calculated by considering how far a line rises or falls between two points within a given horizontal distance. The horizontal distance between two points is called the *run* and the vertical distance is called the *rise*.



Lesson starter: Which line is the steepest?

The three lines shown below right connect the points $A-H$.

- Calculate the rise and run (working from left to right) and also the fraction $\frac{\text{rise}}{\text{run}}$ for these segments.
  a AB **b** BC **c** BD **d** EF **e** GH
- What do you notice about the fractions $\frac{\text{rise}}{\text{run}}$ for parts **a**, **b** and **c**?
- How does the $\frac{\text{rise}}{\text{run}}$ for EF compare with the $\frac{\text{rise}}{\text{run}}$ for parts **a**, **b** and **c**? Which of the two lines is the steeper?
- Your $\frac{\text{rise}}{\text{run}}$ for GH should be negative. Why is this the case?
- Discuss whether or not GH is steeper than AD.



Use computer software (interactive geometry) to produce a set of axes and grid.
- Construct a line segment with endpoints on the grid. Show the coordinates of the endpoints.
- Calculate the rise (vertical distance between the endpoints) and the run (horizontal distance between the endpoints).
- Calculate the gradient as the *rise* divided by the *run*.
- Now drag the endpoints and explore the effect on the gradient.
- Can you drag the endpoints but retain the same gradient value? Explain why this is possible.
- Can you drag the endpoints so that the gradient is zero or undefined? Describe how this can be achieved.

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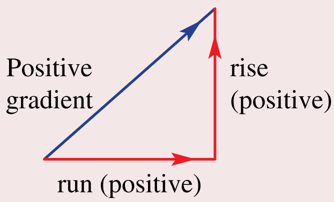
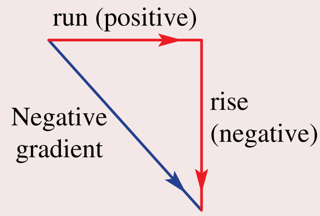
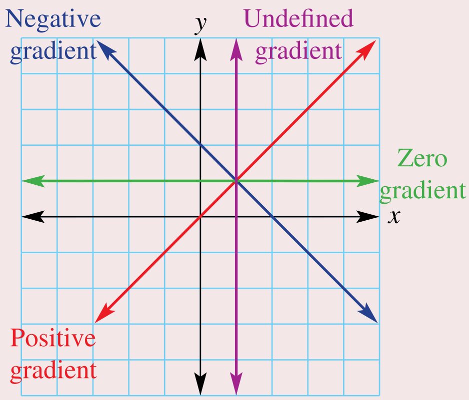
KEY IDEAS

- **Gradient** $(m) = \frac{\text{rise}}{\text{run}}$





- When we work from left to right, the run is always positive.
- The gradient can be positive, negative, zero or undefined.
- A vertical line has an undefined gradient.

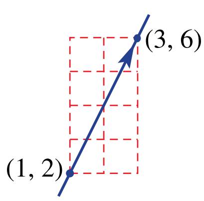


- Gradient is usually expressed as a simplified proper or improper fraction (not a mixed number).

BUILDING UNDERSTANDING

1 Calculate the gradient using $\frac{\text{rise}}{\text{run}}$ for these lines. Remember to give a negative answer if the line is sloping downwards from left to right.

a



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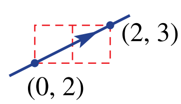
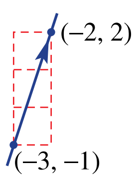
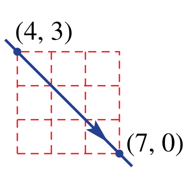


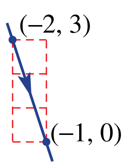
c



d



e



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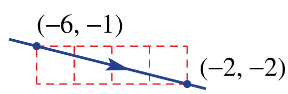

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2 Use the words: positive, negative, zero or undefined to complete each sentence.

a The gradient of a horizontal line is __________.

b The gradient of the line joining (0, 3) with (5, 0) is __________.

c The gradient of the line joining (−6, 0) with (1, 1) is __________.

d The gradient of a vertical line is __________.

3 Use the gradient $m \left(= \frac{\text{rise}}{\text{run}}\right)$ to find the missing value.

a $m = 4$, rise $= 8$, run $= ?$

b $m = 6$, rise $= 3$, run $= ?$

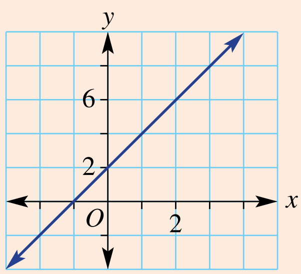
c $m = \frac{3}{2}$, run $= 4$, rise $= ?$

d $m = -\frac{2}{5}$, run $= 15$, rise $= ?$

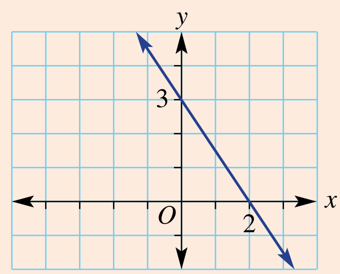
Example 7 Finding the gradient of a line

For each graph, state whether the gradient is positive, negative, zero or undefined, then find the gradient where possible.

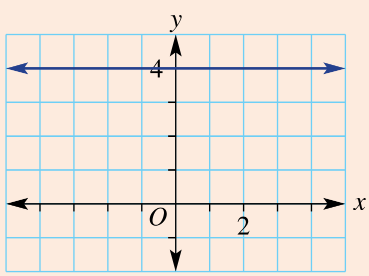
a



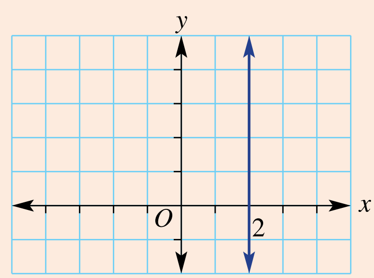
b



c

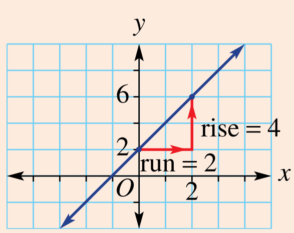


d



SOLUTION

a The gradient is positive.



$$\begin{aligned} \text{Gradient} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{4}{2} \\ &= 2 \end{aligned}$$

EXPLANATION

By inspection, the gradient will be positive since the graph rises from left to right. Select any two points and create a right-angled triangle to determine the rise and run.

Substitute rise = 4 and run = 2.

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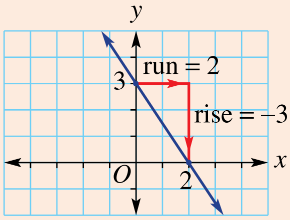
b The gradient is negative.



$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{-3}{2} = -\frac{3}{2}$$

By inspection, the gradient will be negative since y-values decrease from left to right.
Rise $= -3$ and run $= 2$.

c The gradient is 0.

The line is horizontal.

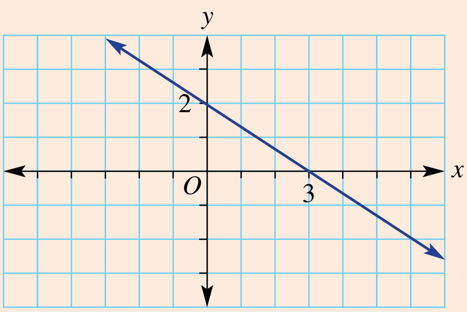
d The gradient is undefined.

The line is vertical.

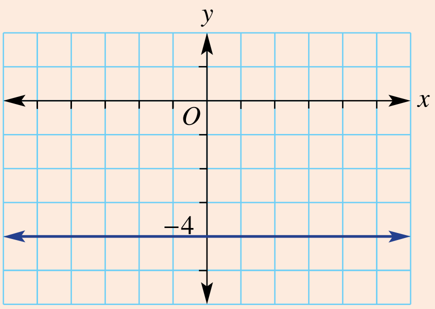
Now you try

For each graph, state whether the gradient is positive, negative, zero or undefined, then find the gradient where possible.

a



b



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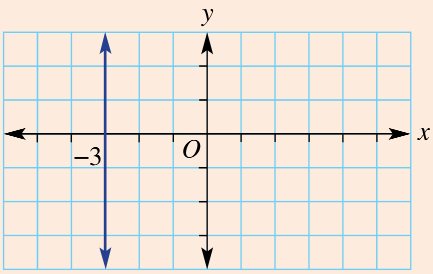

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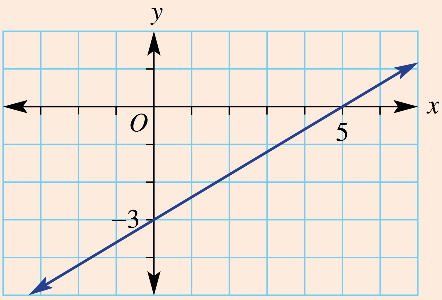

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Example 8 Finding the gradient between two points

Find the gradient (m) of the line joining the given points.

a $A(3, 4)$ and $B(5, 6)$ **b** $A(-3, 6)$ and $B(1, -3)$

SOLUTION

a $m = \frac{\text{rise}}{\text{run}}$

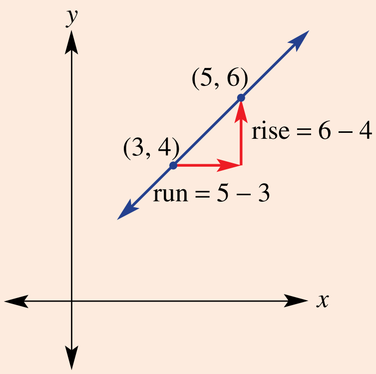
$= \frac{6-4}{5-3}$

$= \frac{2}{2}$

$= 1$

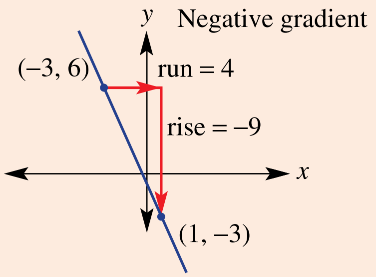
EXPLANATION

Plot points to see a positive gradient and calculate rise and run.



b $m = \frac{\text{rise}}{\text{run}}$

$= \frac{-9}{4}$ or $-\frac{9}{4}$ or -2.25



Now you try

Find the gradient (m) of the line joining the given points.

a $A(2, 6)$ and $B(3, 8)$ **b** $A(-5, 1)$ and $B(2, -3)$

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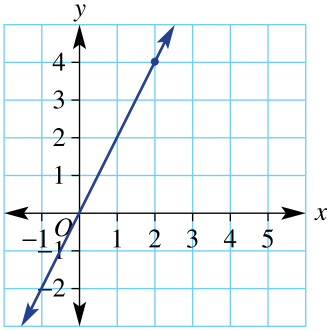
Exercise 4D

FLUENCY	1, 2(½)	1, 2(½)	1–2(½), 3

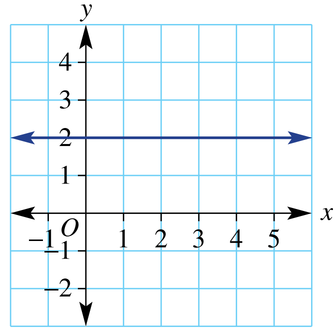
Example 7

1 For each graph state whether the gradient is positive, negative, zero or undefined, then find the gradient where possible.

a



b



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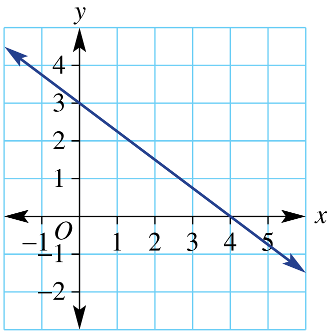

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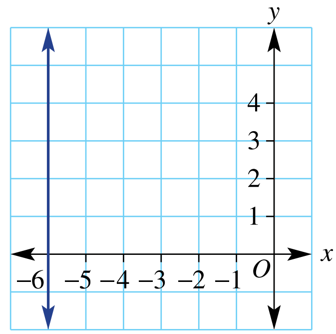

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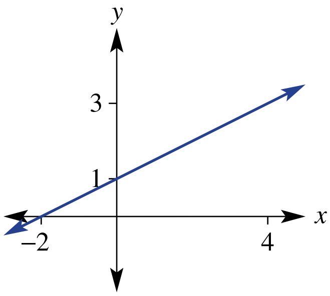

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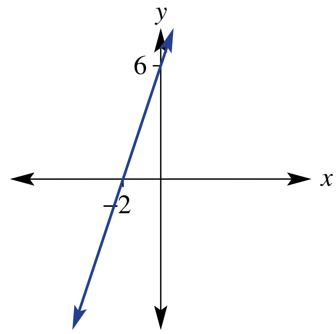

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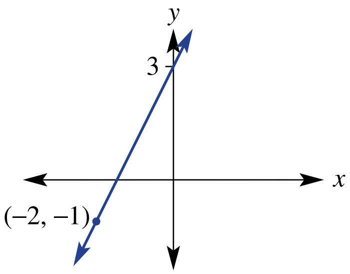

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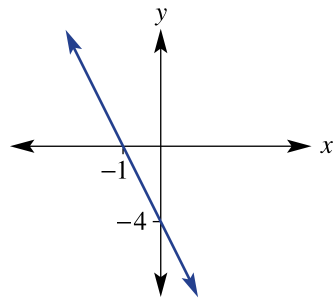

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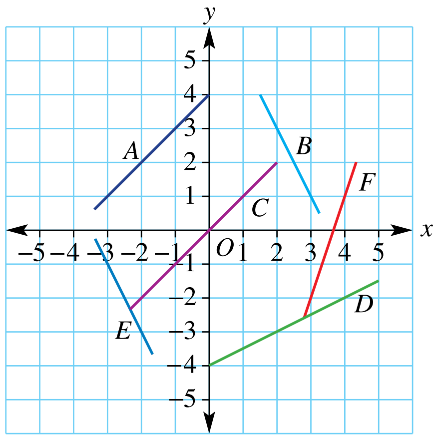
Example 8

2 Find the gradient of the lines joining the following pairs of points.

a $A(2, 3)$ and $B(3, 5)$
b $C(2, 1)$ and $D(5, 5)$
c $E(1, 5)$ and $F(2, 7)$
d $G(-2, 6)$ and $H(0, 8)$
e $A(-4, 1)$ and $B(4, -1)$
f $C(-2, 4)$ and $D(1, -2)$
g $E(-3, 4)$ and $F(2, -1)$
h $G(-1, 5)$ and $H(1, 6)$
i $A(-2, 1)$ and $B(-4, -2)$
j $C(3, -4)$ and $D(1, 1)$
k $E(3, 2)$ and $F(0, 1)$
l $G(-1, 1)$ and $H(-3, -4)$

3 Find the gradient of the lines A–F on this graph and grid.

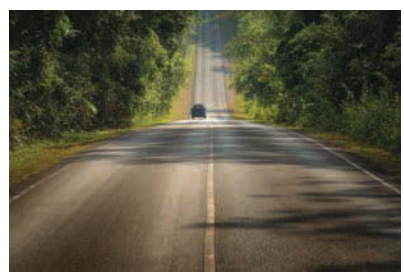


PROBLEM–SOLVING

PROBLEM–SOLVING	4, 5	4–6	5–7

4 Find the gradient corresponding to the following slopes.

a A road falls 10 m for every 200 horizontal metres.
b A cliff rises 35 metres for every 2 metres horizontally.
c A plane descends 2 km for every 10 horizontal kilometres.
d A submarine ascends 150 m for every 20 horizontal metres.



5 Find the missing number.

a The gradient joining the points (0, 2) and (1, ?) is 4.
b The gradient joining the points (?, 5) and (1, 9) is 2.
c The gradient joining the points $(-3, ?)$ and (0, 1) is -1.
d The gradient joining the points $(-4, -2)$ and $(?, -12)$ is -4.

6 A train climbs a slope with gradient 0.05. How far horizontally has the train travelled after rising 15 metres?

7 Complete this table showing the gradient, x-intercept and y-intercept for straight lines.

	A	B	C	D	E	F
Gradient	3	−1	$\frac{1}{2}$	$-\frac{2}{3}$	0.4	−1.25
***x*-intercept**	−3			6	1	
***y*-intercept**		−4	$\frac{1}{2}$			3

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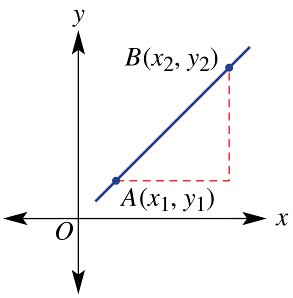
REASONING	8	8, 9	8, 9

8 Give a reason why a line with gradient $\frac{7}{11}$ is steeper than a line with gradient $\frac{3}{5}$.

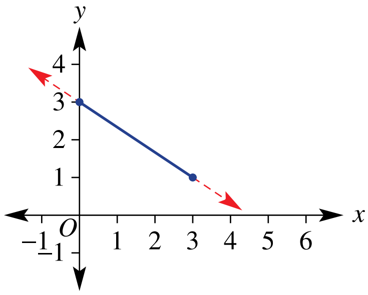
9 The two points A and B shown here have coordinates (x_1, y_1) and (x_2, y_2).



- **a** Write a rule for the run using x_1 and x_2.
- **b** Write a rule for the rise using y_1 and y_2.
- **c** Write a rule for the gradient m using x_1, x_2, y_1 and y_2.
- **d** Use your rule to find the gradient between these pairs of points.
  - **i** (1, 1) and (3, 4)
  - **ii** (0, 2) and (4, 7)
  - **iii** (−1, 2) and (2, −3)
  - **iv** (−4, −6) and (−1, −2)
- **e** Does your rule work for points that include negative coordinates? Explain why.

ENRICHMENT: Where does it hit?	–	–	10

10 The line here has gradient $-\frac{2}{3}$ which means that it falls 2 units for every 3 across. The y-intercept is at (0, 3).



- **a** Use the gradient to find the y-coordinate on the line where:
  - **i** $x = 6$
  - **ii** $x = 9$
- **b** What will be the x-intercept coordinates?
- **c** What would be the x-intercept coordinates if the gradient was changed to the following?
  - **i** $m = -\frac{1}{2}$
  - **ii** $m = -\frac{5}{4}$
  - **iii** $m = -\frac{7}{3}$
  - **iv** $m = -\frac{2}{5}$

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4E Gradient and direct proportion

Learning intentions
- To understand that gradient is the rate of change of one variable with respect to another
- To understand what it means for two variables to be directly proportional
- To be able to form and work with equations involving direct variation

Past, present and future learning
- This section addresses the Stage 5 Core Topics called Linear relationships A and Variation A
- These concepts will be revised, briefly, in Chapter 1 of our Year 10 book and also in Stage 6 Advanced

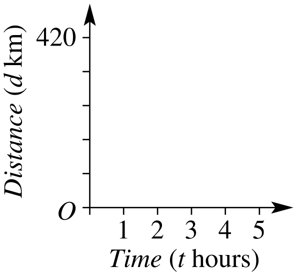
The connection between gradient, rate problems and direct proportion can be illustrated by the use of linear rules and graphs. If two variables are directly related, then the rate of change of one variable with respect to the other is constant. This implies that the rule linking the two variables is linear and can be represented as a straight line graph passing through the origin. The amount of water spraying from a sprinkler, for example, is directly proportional to the time since the sprinkler was turned on. The gradient of the graph of *water volume* versus *time* will equal the rate at which water is spraying from the sprinkler.

A bicycle's speed, v m/min, is in direct proportion to the number of pedal revolutions, P. For example, $v = 6P$ m/min for a wheel of circumference 2 m and gear ratio 3 : 1. If $P = 90$, $v = 540$ m/min, which is a speed of 32.4 km/h.

Lesson starter: Average speed

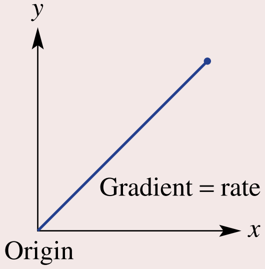
Over 5 hours, Sandy travels 420 km.

- What is Sandy's average speed for the trip?
- Is speed a rate? Discuss.
- Draw a graph of distance versus time, assuming a constant speed.
- Where does your graph intersect the axes and why?
- Find the gradient of your graph. What do you notice?
- Find a rule linking distance (d) and time (t).



KEY IDEAS

- **Direct variation** occurs when two variables are directly proportional. In this situation:
  - the rate of change of one variable with respect to the other is constant
  - the graph is a straight line passing through the origin
  - the rule is of the form $y = mx$ (or $y = kx$)
  - the gradient (m) of the graph equals the rate of change of y with respect to x
  - the letter m (or k) is called the constant of proportionality.



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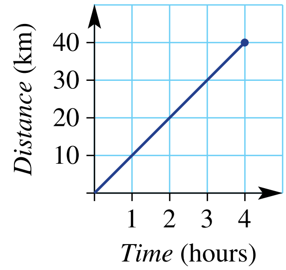
BUILDING UNDERSTANDING

1 This graph shows how far a cyclist travels over 4 hours.



a State how far the cyclist has travelled after:
  i 1 hour
  ii 2 hours
  iii 3 hours.

b State the speed of the cyclist (rate of change of distance over time).

c Find the gradient of the graph.

d What do you notice about your answers from parts **b** and **c**?

2 The rule linking the height of a plant and time is given by $h = 5t$ where h is in millimetres and t is in days.

a Find the height of the plant after 3 days.

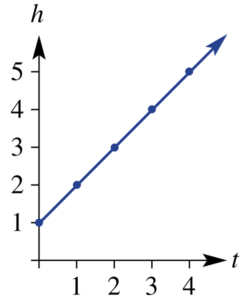
b Find the time for the plant to reach:
  i 30 mm
  ii 10 cm.

c State the missing values for h in this table.

t	0	1	2	3	4
h					

d Choose the correct graph that matches the information above.

A



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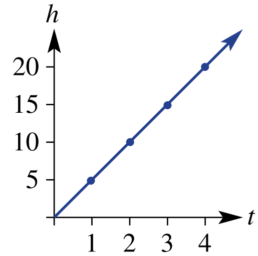

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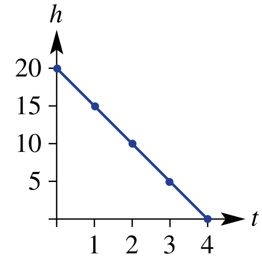

e Find the gradient of the graph.

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Example 9 Exploring direct proportion

Water is poured into an empty tank at a constant rate. It takes 3 hours to fill the tank with 6000 litres.

a What is the rate at which water is poured into the tank?

b Draw a graph of volume (V litres) vs time (t hours) using $0 \leqslant t \leqslant 3$.

c Find:

i the gradient of your graph

ii the rule for V.

d Use your rule to find:

i the volume after 1.5 hours

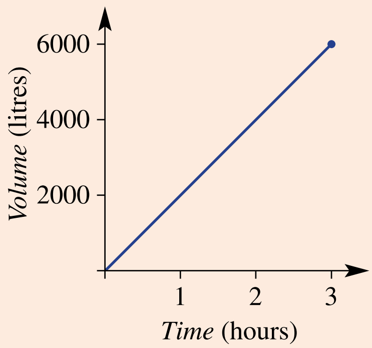
ii the time to fill 5000 litres.

SOLUTION

a 6000 L in 3 hours = 2000 L/h

Explanation: 6000 L per 3 hours = 2000 L per 1 hour

b



Explanation: Plot the two endpoints (0, 0) and (3, 6000) then join with a straight line.

c **i** gradient $= \dfrac{6000}{3} = 2000$

Explanation: The gradient is the same as the rate.

ii $V = 2000t$

Explanation: 2000 L are filled for each hour.

d **i**
$$\begin{aligned} V &= 2000t \\ &= 2000 \times (1.5) \\ &= 3000 \text{ litres} \end{aligned}$$

Explanation: Substitute $t = 1.5$ into your rule.

ii
$$\begin{aligned} V &= 2000t \\ 5000 &= 2000t \\ 2.5 &= t \end{aligned}$$

$\therefore$ it takes $2\frac{1}{2}$ hours.

Explanation: Substitute $V = 5000$ into the rule and solve for t.

Now you try

On a long-distance journey a car travels 450 km in 5 hours.

a What is the rate of change of distance over time (i.e. speed)?

b Draw a graph of distance (d km) versus time (t hours) using $0 \leqslant t \leqslant 5$.

c Find:

i the gradient of your graph

ii the rule for d.

d Use your rule to find:

i the distance after 3.5 hours

ii the time to travel 135 km.

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Exercise 4E

FLUENCY	1–3	1–4	2–4

Example 9

1 A 300 litre fish tank takes 3 hours to fill from a hose.

- **a** What is the rate at which water is poured into the tank?
- **b** Draw a graph of volume (V litres) vs time (t hours) using $0 \leqslant t \leqslant 3$.
- **c** Find:
  - **i** the gradient of your graph
  - **ii** the rule for V.
- **d** Use your rule to find:
  - **i** the volume after 1.5 hours
  - **ii** the time to fill 200 litres.

2 A solar-powered car travels 100 km in 4 hours.

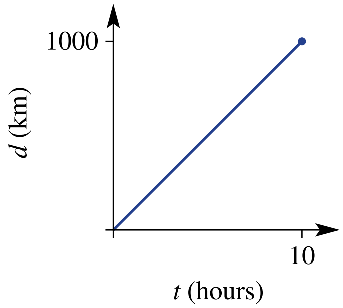
- **a** What is the rate of change of distance over time (i.e. speed)?
- **b** Draw a graph of distance (d km) versus time (t hours) using $0 \leqslant t \leqslant 4$.
- **c** Find:
  - **i** the gradient of your graph
  - **ii** the rule for d.
- **d** Use your rule to find:
  - **i** the distance after 2.5 hours
  - **ii** the time to travel 40 km.

3 Write down a rule linking the given variables.

- **a** I travel 600 km in 12 hours. Use d for distance in km and t for time in hours.
- **b** A calf grows 12 cm in 6 months. Use g for growth height in cm and t for time in months.
- **c** The cost of petrol is \$100 for 80 litres. Use \$$C$ for cost and n for the number of litres.
- **d** The profit is \$10 000 for 500 tonnes. Use \$$P$ for profit and t for the number of tonnes.

4 Use the gradient to find the rate of change of distance over time (speed) for these graphs. Use the units given on each graph.

a



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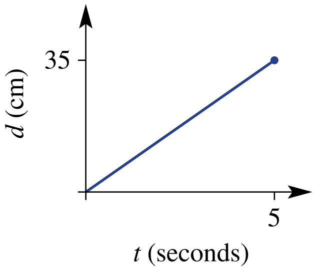
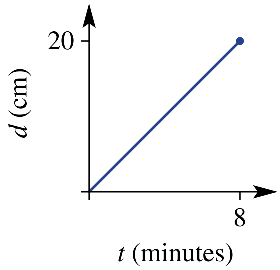
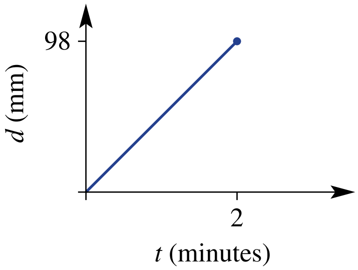


c



d



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PROBLEM–SOLVING	5, 6	5–7	7, 8

5 The trip computer for a car shows that the fuel economy for a trip is 8.5 L per 100 km.

a How many litres would be used for 120 km?

b How many litres would be used for 850 km?

c How many kilometres could be travelled if the capacity of the car's petrol tank was 68 L?

6 Who is travelling the fastest?

- Mick runs 120 m in 20 seconds.
- Sally rides 700 m in 1 minute.
- Udhav jogs 2000 m in 5 minutes.

7 Which animal is travelling the slowest?

- A leopard runs 200 m in 15 seconds.
- A jaguar runs 2.5 km in 3 minutes.
- A panther runs 60 km in 1.2 hours.

8 An investment fund starts at \$0 and grows at a rate of \$100 per month. Another fund starts at \$4000 and reduces by \$720 per year. After how long will the funds have the same amount of money?

REASONING	9	9, 10	10–12

9 The circumference of a circle (given by $C = 2\pi r$) is directly proportional to its radius.

a Find the circumference of a circle with the given radius. Give an exact answer, e.g. 6π.

i $r = 0$ **ii** $r = 2$ **iii** $r = 6$

b Draw a graph of C against r for $0 \leqslant r \leqslant 6$. Use exact values for C.

c Find the gradient of your graph. What do you notice?

10 Is the area of a circle directly proportional to its radius? Give a reason.

11 The base length of a triangle is 4 cm but its height h cm is variable.

a Write a rule for the area of this triangle.

b What is the rate at which the area changes with respect to height h?

12 Over a given time interval, is the speed of an object directly proportional to the distance travelled? Give a rule for speed (s) in terms of distance (d) if the time taken is 5 hours.

ENRICHMENT: Rate challenge	–	–	13, 14

13 Hose A can fill a bucket in 2 minutes and hose B can fill the same bucket in 3 minutes. How long would it take to fill the bucket if both hoses were used at the same time?

14 A river is flowing downstream at a rate of 2 km/h. Murray can swim at a rate of 3 km/h. Murray jumps in and swims downstream for a certain distance then turns around and swims upstream back to the start. In total it takes 30 minutes. How far did Murray swim downstream?

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4F Gradient–intercept form

Learning intentions

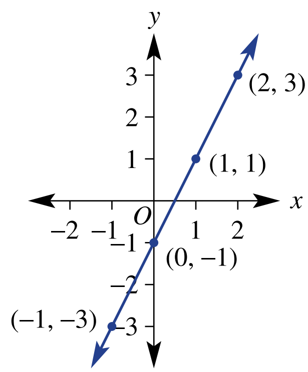
- To know that $y = mx + c$ is the gradient–intercept form of a straight line
- To be able to rearrange a linear equation into gradient–intercept form
- To be able to determine the gradient and y-intercept of a line from the equation
- To be able to sketch a graph using the y-intercept and the gradient

Past, present and future learning

- This section addresses the Stage 5 Core Topics called Linear relationships A and B
- These concepts will be revised, briefly, in Chapter 1 of our Year 10 book and also in Stage 6 Advanced

Shown here is the graph of the rule $y = 2x - 1$. It shows a gradient of 2 and a y-intercept at $(0, -1)$. The fact that these two numbers use numbers in the rule is no coincidence. This is why rules written in this form are called gradient–intercept form. Here are other examples of rules in this form:

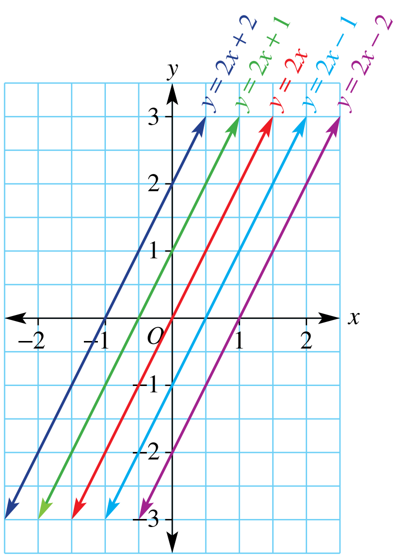
$$y = -5x + 2,\ y = \frac{1}{2}x - 0.5 \text{ and } y = \frac{x}{5} + 20$$



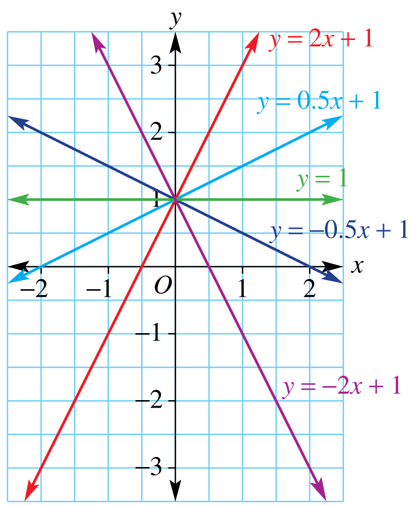
Lesson starter: Family traits

The graph of a linear relation can be sketched easily if you know the gradient and the y-intercept. If one of these is kept constant, we create a family of graphs.

Different y-intercepts and same gradient



Different gradients and same y-intercept



- For the first family, discuss the relationship between the y-intercept and the given rule for each graph.
- For the second family, discuss the relationship between the gradient and the given rule for each graph.

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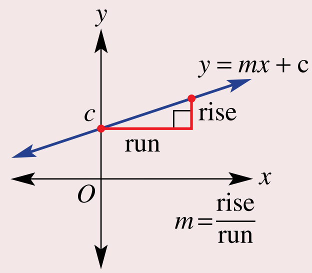
KEY IDEAS

- The **gradient–intercept form** of a straight line equation:

  $y = mx + c$ (or $y = mx + b$ depending on preference)

  m = gradient y-intercept = $(0, c)$



- If the y-intercept is $(0, 0)$, the equation becomes $y = mx$ and these graphs will therefore pass through the origin.
- To sketch a graph using the **gradient–intercept method**, locate the y-intercept and use the gradient to find a second point.
  For example, if $m = \frac{2}{5}$, move 5 across (right) and 2 up.
- The equation of a line can be expressed in several different forms,
  For example, the line with equation $y = 3x - 1$ can also be written as: $3x - y = 1$ or $3x - y - 1 = 0$
  When the RHS is 0, it is said to be in **general form**.

BUILDING UNDERSTANDING

1. State a rule (in gradient–intercept form) for a straight line with the given properties.
   a gradient = 2, y-intercept = $(0, 5)$
   b gradient = -2, y-intercept = $(0, 3)$
   c gradient = -1, y-intercept = $(0, -2)$
   d gradient = $-\frac{1}{2}$, y-intercept = $(0, -10)$
2. Substitute $x = -3$ to find the value of y for these rules.
   a $y = x + 4$
   b $y = 2x + 1$
   c $y = -2x + 3$
3. Rearrange to make y the subject.
   a $y - x = 7$
   b $x + y = 3$
   c $2y - 4x = 10$

Example 10 Stating the gradient and y-intercept

State the gradient and the y-intercept coordinates for the graphs of the following.

a $y = 2x + 1$

b $y = -3x$

SOLUTION	EXPLANATION
a $y = 2x + 1$	The rule is given in gradient-intercept form.
gradient = 2	The gradient is the coefficient of x.
y-intercept = $(0, 1)$	The constant term is the y-coordinate of the y-intercept.
b $y = -3x$	
gradient = -3	The gradient is the coefficient of x including the negative sign.
y-intercept = $(0, 0)$	The constant term is not present, so $c = 0$.

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Now you try

State the gradient and the y-intercept coordinates for the graphs of the following.

a $y = 5x - 2$ **b** $y = 6x$

Example 11 Rearranging linear equations

Rearrange these linear equations into the form shown in the brackets.

a $4x + 2y = 10$ ($y = mx + c$) **b** $y = 4x - 7$ ($ax + by = d$)

SOLUTION

a
$$\begin{aligned} 4x + 2y &= 10 \\ 2y &= -4x + 10 \\ y &= -2x + 5 \end{aligned}$$

b
$y = 4x - 7$
$y - 4x = -7$
i.e. $-4x + y = -7$
or $4x - y = 7$

EXPLANATION

Subtract $4x$ from both sides. Here $10 - 4x$ is better written as $-4x + 10$.
Divide both sides by 2. For the right-hand side divide each term by 2.

Subtract $4x$ from both sides

Multiply both sides by -1 to convert between equivalent forms.

Now you try

Rearrange these linear equations into the form shown in the brackets.

a $10x + 2y = 20$ ($y = mx + c$) **b** $y = -2x + 3$ ($ax + by = d$)

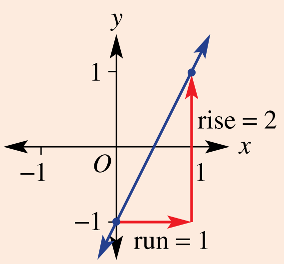
Example 12 Sketching linear graphs using the gradient and y-intercept

Find the value of the gradient and y-intercept for the following and sketch their graphs.

a $y = 2x - 1$ **b** $x + 2y = 6$

SOLUTION

a $y = 2x - 1$
y-intercept $= (0, -1)$
gradient $= 2 = \frac{2}{1}$



EXPLANATION

The rule is in gradient–intercept form so we can read off the gradient and the y-intercept.

Label the y-intercept at $(0, -1)$. From the gradient $\frac{2}{1}$, for every 1 across, move 2 up. From $(0, -1)$ this gives a second point at $(1, 1)$.
Mark and join the points to form a line.

Continued on next page

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b $x + 2y = 6$

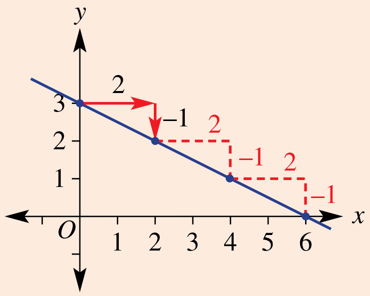
$2y = -x + 6$

$y = \dfrac{-x+6}{2}$

$= -\dfrac{1}{2}x + 3$

so the y-intercept is $(0, 3)$

$m = -\dfrac{1}{2} = \dfrac{-1}{2}$



Rewrite in the form $y = mx + c$ to read off the gradient and y-intercept.

Make y the subject by subtracting x from both sides and then dividing both sides by 2.

$\dfrac{-x}{2}$ is the same as $-\dfrac{1}{2}x$ and $6 \div 2 = 3$.

Link the negative sign to the rise (-1) so the run is positive $(+2)$.

Mark the y-intercept at $(0, 3)$ then from this point move 2 right and 1 down to give a second point at $(2, 2)$.

Note that the x-intercept will be at $(6, 0)$. If the gradient is $\dfrac{-1}{2}$ then a run of 6 gives a fall of 3.

Now you try

Find the value of the gradient and y-intercept for the following and sketch their graphs.

a $y = 3x - 2$ **b** $2x - 5y = 10$

Exercise 4F

FLUENCY	1–3(½)	1–4(½)	1–4(⅓)

Example 10

1 State the gradient and y-intercept coordinates for the graphs of the following.

a $y = 3x + 4$ **b** $y = -5x - 2$ **c** $y = -2x + 3$ **d** $y = \frac{1}{3}x - 4$

e $y = -4x$ **f** $y = 2x$ **g** $y = x$ **h** $y = -0.7x$

Example 11

2 Rearrange these linear equations into the form shown in the brackets.

a $2x + y = 3$ $(y = mx + c)$ **b** $-3x + y = -1$ $(y = mx + c)$

c $6x + 2y = 4$ $(y = mx + c)$ **d** $-3x + 3y = 6$ $(y = mx + c)$

e $y = 2x - 1$ $(ax + by = d)$ **f** $y = -3x + 4$ $(ax + by = d)$

g $3y = x - 1$ (general form) **h** $7y - 2 = 2x$ (general form)

Example 12a

3 Find the gradient and y-intercept for the following and sketch their graphs.

a $y = x - 2$ **b** $y = 2x - 1$ **c** $y = \frac{1}{2}x + 1$ **d** $y = -\frac{1}{2}x + 2$

e $y = -3x + 3$ **f** $y = \frac{3}{2}x + 1$ **g** $y = -\frac{4}{3}x$ **h** $y = \frac{5}{3}x - \frac{1}{3}$

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Example 12b

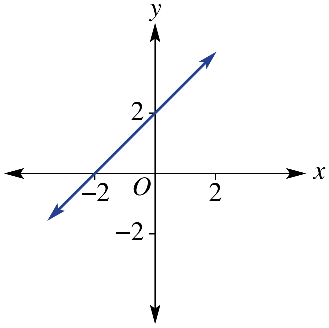
4 Find the gradient and y-intercept for the following and sketch their graphs. Rearrange each equation first.

a $x + y = 4$ b $x - y = 6$ c $x + 2y = 6$
d $x - 2y = 8$ e $2x - 3y = 6$ f $4x + 3y = 12$
g $x - 3y = -4$ h $2x + 3y = 6$ i $3x - 4y = 12$
j $x + 4y = 0$ k $x - 5y = 0$ l $x - 2y = 0$

PROBLEM–SOLVING	5	5, 6(½), 7	6(½), 7

5 Match the following equations to the straight lines shown.

a $y = 3$ b $y = -2x - 1$ c $y = -2x - 4$
d $y = x + 3$ e $x = 2$ f $y = x + 2$

A


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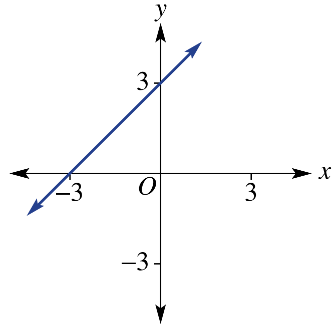

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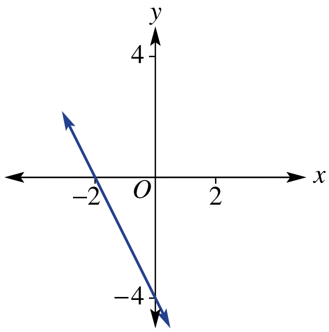

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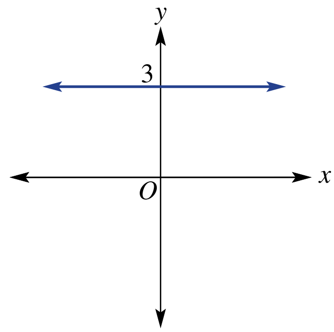

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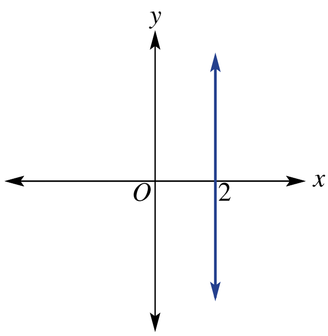

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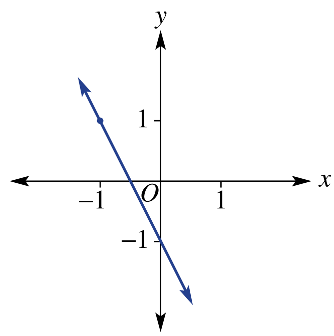

6 Sketch the graph of each of the following linear relationships, by first finding the gradient and y-intercept.

a $5x - 2y = 10$ b $y = 6$ c $x + y = 0$ d $y = 5 - x$
e $y = \frac{x}{2} - 1$ f $4y - 3x = 0$ g $4x + y - 8 = 0$ h $2x + 3y - 6 = 0$

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7 Which of these linear relationships have a gradient of 2 and y-intercept at $(0, -3)$?

A $y = 2(x - 3)$ **B** $y = 3 - 2x$ **C** $y = \dfrac{3 - 2x}{-1}$ **D** $y = 2(x - 1.5)$

E $y = \dfrac{2x - 6}{2}$ **F** $y = \dfrac{4x - 6}{2}$ **G** $2y = 4x - 3$ **H** $-2y = 6 - 4x$

REASONING	8	8, 9	9, 10

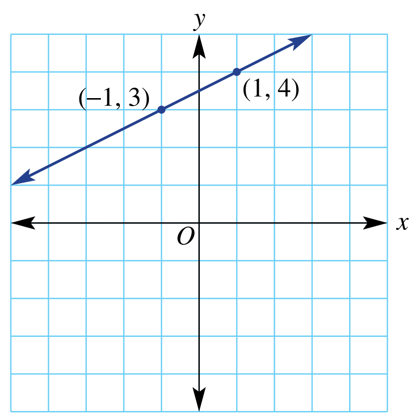
8 Jeremy says that the graph of the rule $y = 2(x + 1)$ has gradient 2 and y-intercept $(0, 1)$.
- **a** Explain his error.
- **b** What can be done to the rule to help show the y-intercept?

9 A horizontal line has gradient 0 and y-intercept at $(0, k)$. Using gradient–intercept form, write the rule for the line.

10 Write the rule $ax + by = d$ in gradient–intercept form. Then state the gradient m and the y-intercept.

ENRICHMENT: The missing y-intercept	–	–	11

11 This graph shows two points $(-1, 3)$ and $(1, 4)$ with a gradient of $\frac{1}{2}$. By considering the gradient, the y-intercept can be calculated to be at $(0, 3.5)$ or $\left(0, \frac{7}{2}\right)$ so $y = \frac{1}{2}x + \frac{7}{2}$.



Use this approach to find the rule of the line passing through these points.
- **a** $(-1, 1)$ and $(1, 5)$
- **b** $(-2, 4)$ and $(2, 0)$
- **c** $(-1, -1)$ and $(2, 4)$
- **d** $(-3, 1)$ and $(2, -1)$

The equation of this straight section of mountain railway could be expressed in the form $y = mx + c$.

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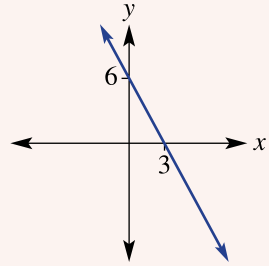
Progress quiz

4A **1** Read off the coordinates of the x- and y-intercepts from this table and graph.

a

x	−2	−1	0	1	2	3
y	−8	−6	−4	−2	0	2

b



4A **2** Decide if the point $(-1, 3)$ is on the line with the given rules.

a $y = 2x - 1$ **b** $y = 2x + 5$

4B **3** Sketch the graph of the following relations, by first finding the x- and y-intercepts.

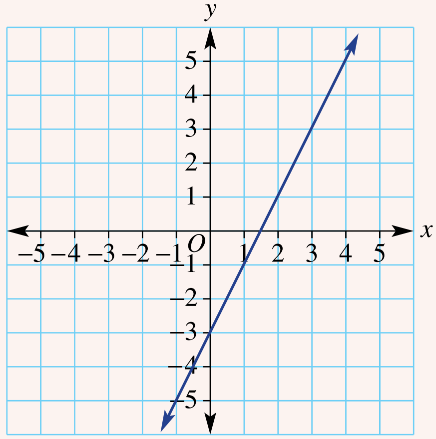
a $3x + 2y = 6$ **b** $y = \frac{3x}{4} + 3$

4C **4** Sketch the graphs of these special lines all on the same set of axes and label with their equations.

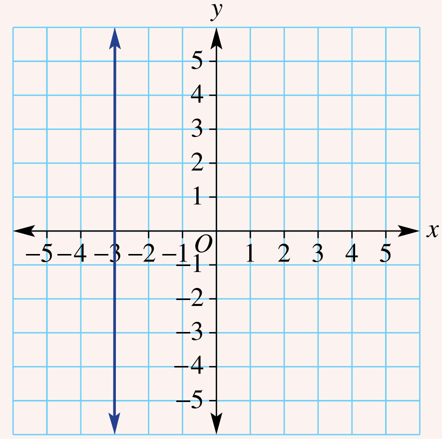
a $y = 2$ **b** $x = 4$ **c** $y = 2x$

4D **5** For each graph state whether the gradient is positive, negative, zero or undefined, then find the gradient where possible.

a



b



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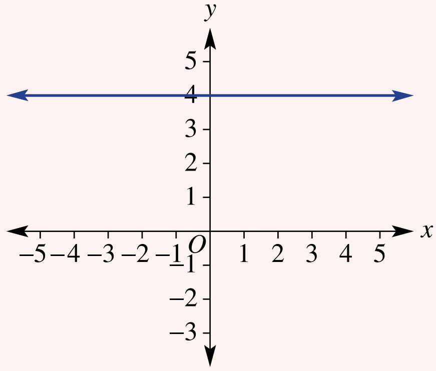

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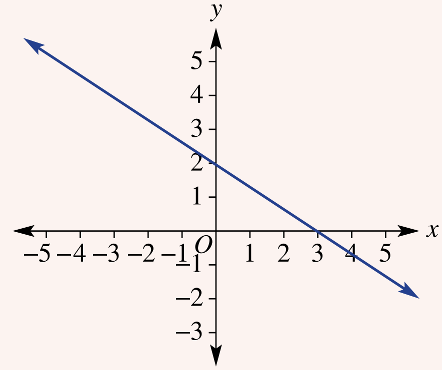

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Progress quiz

4D **6** Find the gradient (m) of the line joining the given points.

a $P(2, 5)$ and $Q(4, 9)$

b $A(-3, 2)$ and $B(1, -10)$

4E **7** An empty tank on a water truck is being filled. It takes 4 hours to fill the tank with 6000 litres.

a What is the rate at which water is poured into the tank?

b Draw a graph of volume (V litres) vs time (t hours) using $0 \leqslant t \leqslant 4$.

c Find:

i the gradient of your graph **ii** the rule for V.

d Use your rule to find:

i the volume after 3 hours **ii** the time to fill 5250 litres.

4F **8** Rearrange these linear equations into the form shown in the brackets.

a $5x + y = 8$ ($y = mx + c$)

b $y = 3x - 5$ ($ax + by = d$)

4F **9** Find the value of the gradient and y-intercept for the following and use these to sketch their graphs.

a $y = 3x - 2$

b $2x + 3y = 9$

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4G Finding the equation of a line using $y = mx + c$

Learning intentions

- To understand that all straight lines can be expressed in the form $y = mx + c$
- To be able to find the equation of a line given the y-intercept and the gradient
- To be able to find the equation of a line given the y-intercept and another point
- To be able to find the equation of a line given the gradient and a point

Past, present and future learning

- This section addresses the Stage 5 Core Topics called Linear relationships A and B
- These concepts will be revised, briefly, in Chapter 1 of our Year 10 book and also in Stage 6 Advanced

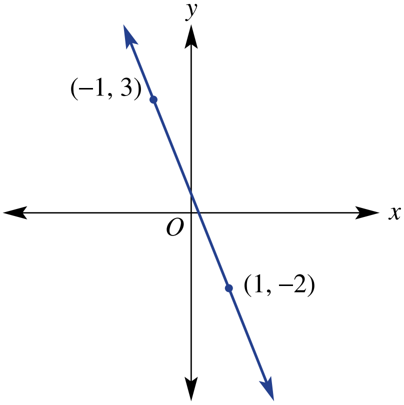
Using the gradient–intercept form, the rule (or equation) of a line can be found by calculating the value of the gradient and the y-intercept. Given a graph, the gradient can be calculated using two points. If the y-intercept is known, then the value of the constant in the rule is obvious. If it is not known, another point can be used to help find its value.

Engineers use linear equations to algebraically model straight supports, such as those in the Madrid airport. Using the slope of the support structure and a location in three dimensions, its linear equation can be found.

Lesson starter: But we don't know the y-intercept!

A line with the rule $y = mx + c$ passes through two points $(-1, 3)$ and $(1, -2)$.

- Using the information given, is it possible to find the value of m? If so, calculate its value.
- The y-intercept is not given on the graph. Discuss what information could be used to find the value of the constant c in the rule. Is there more than one way you can find the y-intercept?
- Write the rule for the line.



KEY IDEAS

- To find the equation of a line in gradient–intercept form $y = mx + c$, you need to find:
  - the value of the gradient (m) using $m = \frac{\text{rise}}{\text{run}}$
  - the value of the constant (c), by observing the y-intercept or by substituting another point.

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BUILDING UNDERSTANDING

1 Substitute the given values of m and c into $y = mx + c$ to find the rule.

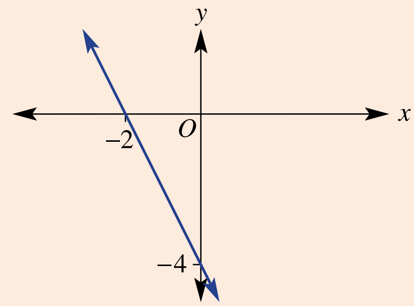
a $m = 2, c = 5$ **b** $m = 4, c = -1$ **c** $m = -2, c = 5$

2 Substitute the point into the given rule and solve to find the value of c. For example, using (3, 4), substitute $x = 3$ and $y = 4$ into the rule.

a $(3, 4), y = x + c$ **b** $(-2, 3), y = 3x + c$ **c** $(3, -1), y = -2x + c$

Example 13 Finding the equation of a line given the *y*-intercept and another point

Determine the equation of the straight line shown here.



SOLUTION

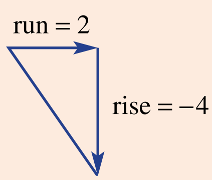
$$y = mx + c$$
$$m = \frac{\text{rise}}{\text{run}} = \frac{-4}{2} = -2$$

y-intercept $= (0, -4)$

$\therefore y = -2x - 4$

EXPLANATION

Write down the general straight line equation.

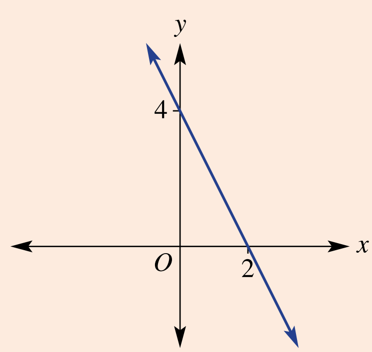


Read from the graph, so $c = -4$.

Substitute $m = -2$ and the value of c into the general equation.

Now you try

Determine the equation of the straight line shown here.



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Example 14 Finding the equation of a line given the gradient and a point

Find the equation of the line which has a gradient m of $\frac{1}{3}$ and passes through the point (9, 2).

SOLUTION

$$\begin{aligned} y &= mx + c \\ y &= \tfrac{1}{3}x + c \\ 2 &= \tfrac{1}{3}(9) + c \\ 2 &= 3 + c \\ -1 &= c \\ \therefore y &= \tfrac{1}{3}x - 1 \end{aligned}$$

EXPLANATION

Substitute $m = \frac{1}{3}$ into $y = mx + c$.

Since (9, 2) is on the line, it must satisfy the equation $y = \frac{1}{3}x + c$; hence, substitute the point (9, 2) where $x = 9$ and $y = 2$ to find c.

Simplify and solve for c.

Write the equation in the form $y = mx + c$.

Now you try

Find the equation of the line which has a gradient m of $\frac{2}{3}$ and passes through the point $(-3, 4)$.

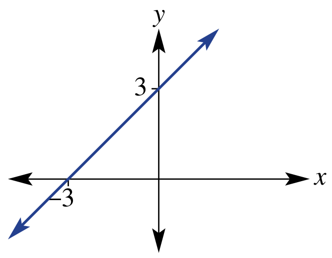
Exercise 4G

FLUENCY	1, 2–3(½)	1–3(½)	1(½), 2–3(⅓)

Example 13

1 Determine the equation of the following straight lines.

a



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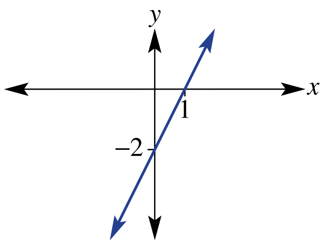
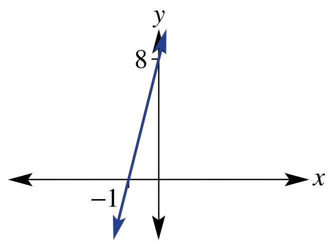


c



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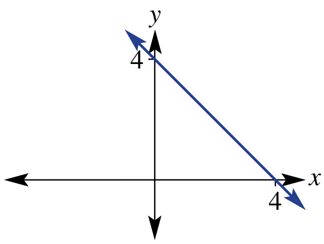

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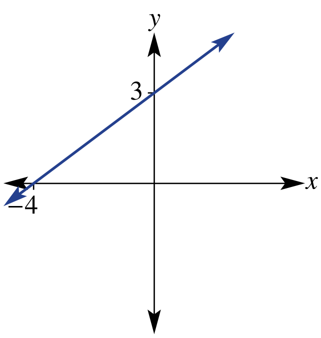
Example 14

2 Find the equation of the line for which the gradient and one point are given.

a $m = 3$, point $(1, 8)$
b $m = -2$, point $(2, -5)$
c $m = -3$, point $(2, 2)$
d $m = 1$, point $(1, -2)$
e $m = -3$, point $(-1, 6)$
f $m = 5$, point $(2, 9)$
g $m = -1$, point $(4, 4)$
h $m = -3$, point $(3, -3)$
i $m = -2$, point $(-1, 4)$
j $m = -4$, point $(-2, -1)$

3 Find the equation of these straight lines, which have fractional gradients.

a



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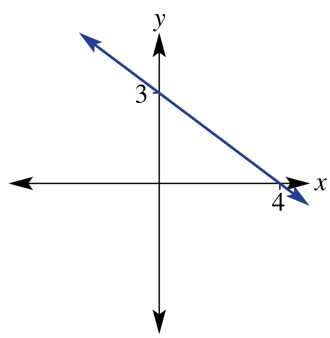

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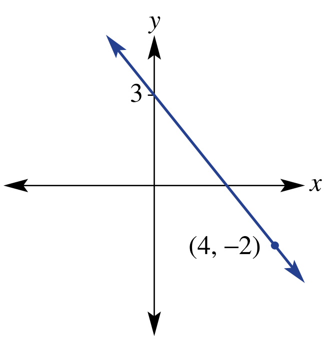

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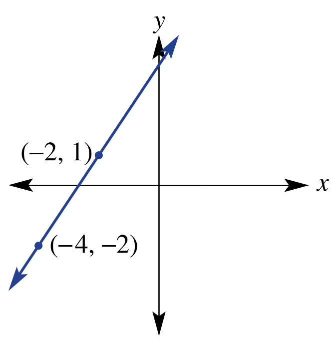
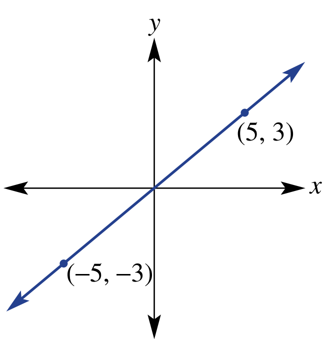
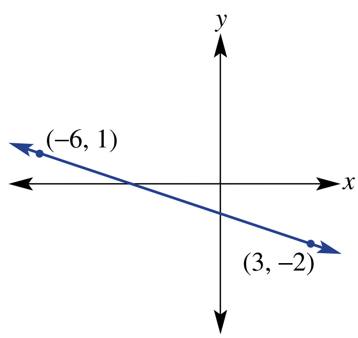


e



f



PROBLEM–SOLVING	4(½), 5	4(½), 5–7	4(½), 6–8

4 For the line connecting the following pairs of points:

i find the gradient
ii find the equation.

a $(2, 6)$ and $(4, 10)$
b $(-3, 6)$ and $(5, -2)$
c $(1, 7)$ and $(3, -1)$
d $(-4, -8)$ and $(1, -3)$

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5 A line has gradient -2 and y-intercept $(0, 5)$. Find its x-intercept coordinates.

6 A line passes through the points $(-1, -2)$ and $(3, 3)$. Find its x- and y-intercept coordinates.

7 The tap on a tank has been left on and water is running out. The volume of water in the tank after 1 hour is 100 L and after 5 hours the volume is 20 L. Assuming the relationship is linear, find a rule and then state the initial volume of water in the tank.

8 The coordinates $(0, 0)$ mark the take-off point for a rocket constructed as part of a science class. The positive x direction from $(0, 0)$ is considered to be east.

a Find the equation of the rocket's path if the rocket rises at a rate of 5 m vertically for every 1 m in an easterly direction.

b A second rocket is fired from 2 m vertically above the launch site of the first rocket. It rises at a rate of 13 m for every 2 m in an easterly direction. Find the equation describing its path.

REASONING	9	9(½), 10	10, 11

9 A line has equation $y = mx - 2$. Find the value of m if the line passes through:

a $(2, 0)$ b $(1, 6)$ c $(-1, 4)$ d $(-2, -7)$

10 A line with rule $y = 2x + c$ passes through $(1, 5)$ and $(2, 7)$.

a Find the value of c using the point $(1, 5)$.

b Find the value of c using the point $(2, 7)$.

c Does it matter which point you use? Explain.

11 A line passes through the origin and the point (a, b). Write its equation in terms of a and b.

ENRICHMENT: The general rule	–	–	12

12 To find the equation of a line between two points (x_1, y_1) and (x_2, y_2) some people use the rule:

$$y - y_1 = m(x - x_1) \text{ where } m = \frac{y_2 - y_1}{x_2 - x_1}$$

Use this rule to find the equation of the line passing through these pairs of points. Write your answer in the form $y = mx + c$.

a $(1, 2)$ and $(3, 6)$ b $(0, 4)$ and $(2, 0)$ c $(-1, 3)$ and $(1, 7)$

d $(-4, 8)$ and $(2, -1)$ e $(-3, -2)$ and $(4, 3)$ f $(-2, 5)$ and $(1, -8)$

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4H Midpoint and length of a line segment from diagrams

Learning intentions

- To be able to find the midpoint of a line segment given the endpoints
- To be able to use Pythagoras' theorem to find the length of a line segment

Past, present and future learning

- This section addresses the Stage 5 Core Topic called Linear relationships A
- These concepts will be extended, in Chapter 1 of our Year 10 book and also in Stage 6 Advanced

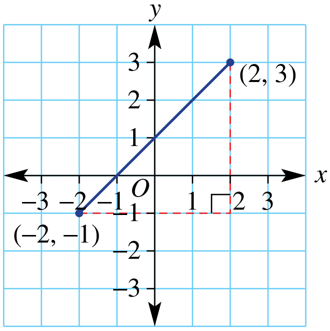
A line segment (or line interval) has a defined length and therefore must have a midpoint. Both the midpoint and length can be found by using the coordinates of the endpoints.

Industrial robots are programmed with the length algorithm to calculate the shortest straight line distance between two points in three dimensions, each located by coordinates (x, y, z).

Lesson starter: Choosing a method

This graph shows a line segment between the points at $(-2, -1)$ and $(2, 3)$.

- What is the horizontal distance between the two points?
- What is the vertical distance between the two points?
- What is the x-coordinate of the point halfway along the line segment?
- What is the y-coordinate of the point halfway along the line segment?
- Discuss and explain a method for finding the midpoint of a line segment.
- Discuss and explain a method for finding the length of a line segment.



Using graphing software or interactive geometry software, produce a line segment like the one shown above. Label the coordinates of the endpoints and the midpoint. Also find the length of the line segment. Now drag one or both of the endpoints to a new position.

- Describe how the coordinates of the midpoint relate to the coordinates of the endpoints. Is this true for all positions of the endpoints that you choose?
- Now use your software to calculate the vertical distance and the horizontal distance between the two endpoints. Then square these lengths. Describe these squared lengths compared to the square of the length of the line segment. Is this true for all positions of the endpoints that you choose?

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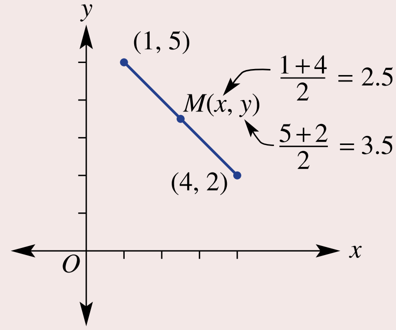
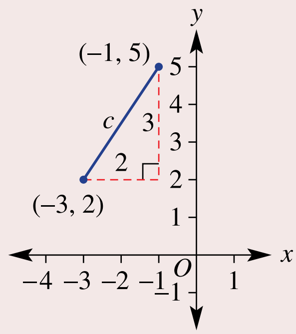
KEY IDEAS

- **The midpoint** (M) of a line segment is the halfway point between the two endpoints.
  - The x-coordinate of the midpoint is the average (mean) of the x-coordinates of the two endpoints.
  - The y-coordinate of the midpoint is the average (mean) of the y-coordinates of the two endpoints.
  - $M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$



- The **length of a line segment** (or line interval) is found using Pythagoras' theorem. This gives the distance between any two points.
  - The line segment is the hypotenuse (longest side) of a right-angled triangle.
  - Find the horizontal distance by subtracting the lower x-coordinate from the upper x-coordinate.



$$\begin{aligned}\text{horizontal distance} &= -1 - (-3)\\ &= 2\\ \text{vertical distance} &= 5 - 2\\ &= 3\end{aligned}$$

$$\begin{aligned}c^2 &= 2^2 + 3^2\\ &= 13\\ \therefore c &= \sqrt{13}\end{aligned}$$

  - Find the vertical distance by subtracting the lower y-coordinate from the upper y-coordinate.

BUILDING UNDERSTANDING

1 Find the number that is 'halfway' between these pairs of numbers.

a 1, 7 **b** 5, 11 **c** −2, 4 **d** −6, 0

2 Find the average (mean) of these pairs of numbers.

a 4, 7 **b** 0, 5 **c** −3, 0 **d** −4, −1

3 Evaluate c correct to two decimal places where $c > 0$.

a $c^2 = 1^2 + 2^2$ **b** $c^2 = 5^2 + 7^2$ **c** $c^2 = 10^2 + 2^2$

Example 15 Finding a midpoint of a line segment

Find the midpoint $M(x, y)$ of the line segment joining these pairs of points.

a (1, 0) and (4, 4) **b** (−3, −2) and (5, 3)

SOLUTION

a
$$x = \frac{1+4}{2} = 2.5$$
$$y = \frac{0+4}{2} = 2$$
$$\therefore M = (2.5, 2)$$

EXPLANATION

Find the average (mean) of the x-coordinates and the y-coordinates of the two points.

Continued on next page

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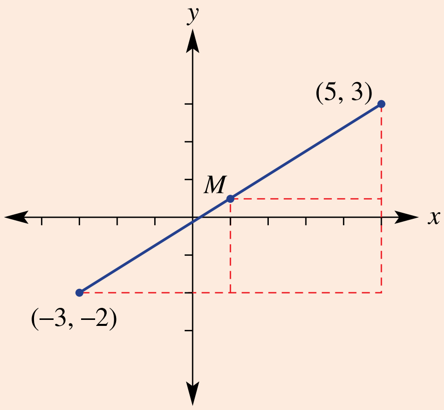
b $x = \dfrac{-3+5}{2} = 1$

$y = \dfrac{-2+3}{2} = 0.5$

$\therefore M = (1, 0.5)$



Now you try

Find the midpoint $M(x, y)$ of the line segment joining these pairs of points.

a (0, 2) and (4, 5) **b** (−4, 1) and (2, −4)

Example 16 Finding the length of a line segment

Find the length of the segment joining (−2, 2) and (4, −1), correct to two decimal places.

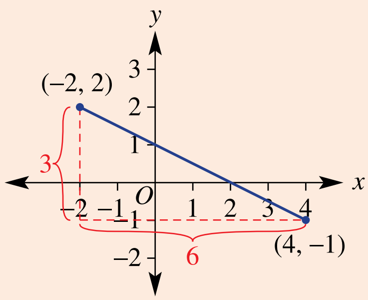
SOLUTION

Horizontal length $= 4 - (-2)$
$= 6$

Vertical length $= 2 - (-1)$
$= 3$

$c^2 = 6^2 + 3^2$
$= 45$
$\therefore c = \sqrt{45}$
$\therefore$ length $= 6.71$ units (to 2 d.p.)

EXPLANATION



Apply Pythagoras' theorem $c^2 = a^2 + b^2$.
Round as required.

Now you try

Find the length of the segment joining (−1, 5) and (3, −2), correct to two decimal places.

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Exercise 4H

FLUENCY	1–2(½)	1–2(½)	1–2(⅓)

Example 15

1 Find the midpoint $M(x, y)$ of the line segment joining these pairs of points.

a (0, 0) and (6, 6)
b (0, 0) and (4, 4)
c (3, 0) and (5, 2)
d (−2, 0) and (0, 6)
e (−4, −2) and (2, 0)
f (1, 3) and (2, 0)
g (−3, 7) and (4, −1)
h (−2, −4) and (−1, −1)
i (−7, −16) and (1, −1)
j (−4, −3) and (5, −2)

Example 16

2 Find the length of the segment joining these pairs of points correct to two decimal places.

a (1, 1) and (2, 6)
b (1, 2) and (3, 4)
c (0, 2) and (5, 0)
d (−2, 0) and (0, −4)
e (−1, 3) and (2, 1)
f (−2, −2) and (0, 0)
g (−1, 7) and (3, −1)
h (−4, −1) and (2, 3)
i (−3, −4) and (3, −1)

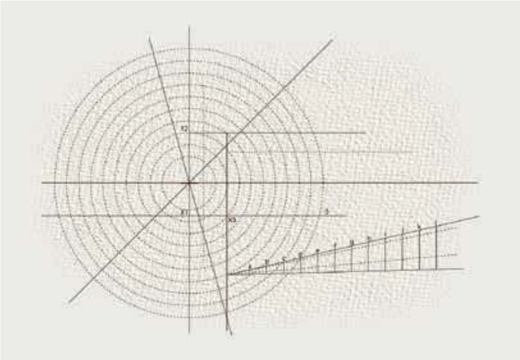
PROBLEM-SOLVING	3, 4	3–5	4–6

3 Find the missing coordinates in this table. M is the midpoint of points A and B.

A	*B*	*M*
(4, 2)		(6, 1)
	(0, −1)	(−3, 2)
	(4, 4)	(−1, 6.5)

4 A circle has centre (2, 1). Find the coordinates of the endpoint of a diameter if the other endpoint has these coordinates.

a (7, 1)
b (3, 6)
c (−4, −0.5)



5 Find the perimeter of these shapes correct to one decimal place.

a A triangle with vertices (−2, 0), (−2, 5) and (1, 3)
b A trapezium with vertices (−6, −2), (1, −2), (0, 4) and (−5, 4)

6 Find the coordinates of the four points which have integer coordinates and are a distance of $\sqrt{5}$ from the point (1, 2). (*Hint*: $5 = 1^2 + 2^2$.)

REASONING	7	7, 8	7, 8

7 A line segment has endpoints (x_1, y_1) and (x_2, y_2) and midpoint $M(x, y)$.

a Write a rule for x, the x-coordinate of the midpoint.
b Write a rule for y, the y-coordinate of the midpoint.
c Test your rule to find the coordinates of M if $x_1 = -3$, $y_1 = 2$, $x_2 = 5$ and $y_2 = -3$.

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8 A line segment has endpoints (x_1, y_1) and (x_2, y_2). Assume $x_2 > x_1$ and $y_2 > y_1$.

a Write a rule for:

i the horizontal distance between the endpoints

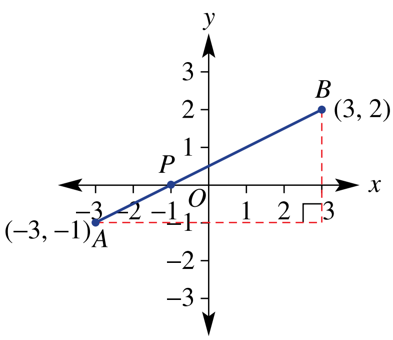
ii the vertical distance between the endpoints

iii the length of the segment.

b Use your rule to show that the length of the segment joining $(-2, 3)$ with $(1, -3)$ is $\sqrt{45}$.

ENRICHMENT: Division by ratio	–	–	9

9 Looking from left to right, this line segment shows the point $P(-1, 0)$, which divides the segment in the ratio 1 : 2.



a What fraction of the horizontal distance between the endpoints is P from A?

b What fraction of the vertical distance between the endpoints is P from A?

c Find the coordinates of point P on the segment AB if it divides the segment in these ratios.

i 2 : 1

ii 1 : 5

iii 5 : 1

d Find the coordinates of point P, which divides the segments with the given endpoints in the ratio 2 : 3.

i $A(-3, -1)$ and $B(2, 4)$

ii $A(-4, 9)$ and $B(1, -1)$

iii $A(-2, -3)$ and $B(4, 0)$

iv $A(-6, -1)$ and $B(3, 8)$

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4I Perpendicular lines and parallel lines

Learning intentions

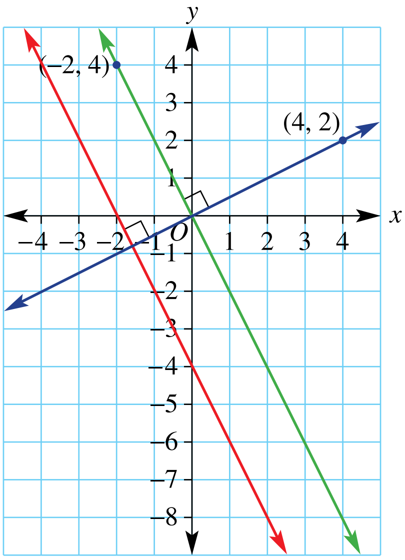
- To know the relationship between the gradients of lines that are either parallel or perpendicular
- To be able to find the equation of a line given a point and a line to which it is parallel or perpendicular
- Past, present and future learning
- This section addresses the Stage 5 Path Topic called Linear relationships C
- These concepts will be revised, briefly, in Chapter 1 of our Year 10 book and also in Stage 6 Advanced

Parallel and perpendicular lines are commonplace in mathematics and in the world around us. Using parallel lines in buildings, for example, ensures that beams or posts point in the same direction. Perpendicular beams help to construct rectangular shapes, which are central in the building of modern structures.

The architect who designed this building has used linear equations to model the sets of parallel lines on its exterior. Shapes with parallel sides are arranged to emphasise the many parallel lines.

Lesson starter: How are they related?

This graph shows a pair of parallel lines and a line perpendicular to the other two. Find the equation of all three lines.



- What do you notice about the equations for the pair of parallel lines?
- What do you notice about the gradient of the line that is perpendicular to the other two lines?
- Write down the equations of three other lines that are parallel to $y = -2x$.
- Write down the equations of three other lines that are perpendicular to $y = -2x$.

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KEY IDEAS

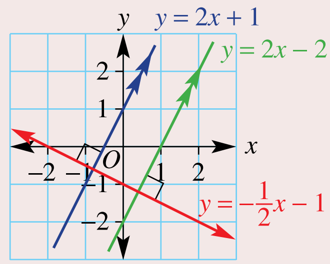
- If two lines are **parallel** then they have the same gradient.
- If two **perpendicular** lines (at right angles) have gradients m_1 and m_2 then:

$$m_1 \times m_2 = -1 \text{ or } m_2 = -\frac{1}{m_1}$$

  - The **negative reciprocal** of m_1 gives m_2.

    For example: If $m_1 = 4$ then $m_2 = -\frac{1}{4}$.

    If $m_1 = -\frac{2}{3}$ then $m_2 = \frac{-1}{\left(-\frac{2}{3}\right)} = -1 \times \left(-\frac{3}{2}\right) = \frac{3}{2}$.



BUILDING UNDERSTANDING

1 Decide if the pairs of lines with these equations are parallel (have the same gradient).

a $y = 3x - 1$ and $y = 3x + 4$
b $y = 7x - 2$ and $y = 2x - 7$
c $y = x + 4$ and $y = x - 3$
d $y = \frac{1}{2}x - 1$ and $y = -\frac{1}{2}x + 2$

2 Two perpendicular lines with gradients m_1 and m_2 are such that $m_2 = -\frac{1}{m_1}$. Find m_2 for the given values of m_1.

a $m_1 = 5$
b $m_1 = -3$

3 Decide if the pairs of lines with these equations are perpendicular (i.e. whether $m_1 \times m_2 = -1$).

a $y = 4x - 2$ and $y = -\frac{1}{4}x + 3$
b $y = 2x - 3$ and $y = \frac{1}{2}x + 4$
c $y = -\frac{1}{2}x + 6$ and $y = -\frac{1}{2}x - 2$
d $y = -\frac{1}{5}x + 1$ and $y = 5x + 2$

Example 17 Finding the equation of a parallel line

Find the equation of a line which is parallel to $y = 3x - 1$ and passes through (0, 4).

SOLUTION

$y = mx + c$
$m = 3$

$c = 4$
$\therefore y = 3x + 4$

EXPLANATION

Since it's parallel to $y = 3x - 1$, the gradient is the same so $m = 3$.

The y-intercept is given in the question, so $c = 4$.

Now you try

Find the equation of a line which is parallel to $y = 4x + 3$ and passes through $(0, -2)$.

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Example 18 Finding the equation of a perpendicular line

Find the equation of a line which is perpendicular to the line $y = 2x - 3$ and passes through $(0, -1)$.

SOLUTION	EXPLANATION
$y = mx + c$	Since it is perpendicular to $y = 2x - 3$,
$m = -\frac{1}{2}$	$m_2 = -\frac{1}{m_1} = -\frac{1}{2}$.
$c = -1$	The y-intercept is given.
$y = -\frac{1}{2}x - 1$	

Now you try

Find the equation of a line which is perpendicular to the line $y = -3x + 1$ and passes through $(0, 4)$.

Exercise 4I

FLUENCY	1, 2(½)	1, 2(½), 3	1, 2(½), 3

Example 17

1 Find the equation of the line that is parallel to the given line and passes through the given point.

a $y = 4x + 2$, $(0, 8)$

b $y = -2x - 3$, $(0, -7)$

c $y = \frac{2}{3}x + 6$, $(0, -5)$

d $y = -\frac{4}{5}x - 3$, $\left(0, \frac{1}{2}\right)$

Example 18

2 Find the equation of the line that is perpendicular to the given line and passes through the given point.

a $y = 3x - 2$, $(0, 3)$

b $y = 5x - 4$, $(0, 7)$

c $y = -2x + 3$, $(0, -4)$

d $y = -x + 7$, $(0, 4)$

e $y = -7x + 2$, $\left(0, -\frac{1}{2}\right)$

f $y = x - \frac{3}{2}$, $\left(0, \frac{5}{4}\right)$

3 a Write the equation of the line parallel to $y = 4$ which passes through these points.

i $(0, 1)$ ii $(-3, -2)$

b Write the equation of the line parallel to $x = -2$ which passes through these points.

i $(3, 0)$ ii $(-3, -3)$

c Write the equation of the line perpendicular to $y = -3$ which passes through these points.

i $(2, 0)$ ii $(0, 0)$

d Write the equation of the line perpendicular to $x = 6$ which passes through these points.

i $(0, 7)$ ii $\left(0, -\frac{1}{2}\right)$

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PROBLEM–SOLVING	4	4, 5	4(½), 5, 6

4 Find the equation of the line which is:

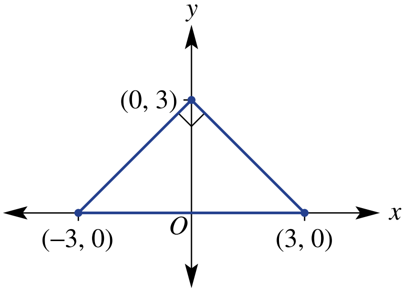
a parallel to the line with equation $y = -3x - 7$ and passes through $(3, 0)$; remember to substitute the point $(3, 0)$ to find the value of the y-intercept

b parallel to the line with equation $y = \frac{1}{2}x + 2$ and passes through $(1, 3)$

c perpendicular to the line with equation $y = 5x - 4$ and passes through $(1, 6)$

d perpendicular to the line with equation $y = -x - \frac{1}{2}$ and passes through $(-2, 3)$.

5 A right-angled isosceles triangle has vertices at $(0, 3)$, $(3, 0)$ and $(-3, 0)$. Find the equation of each side.



6 A parallelogram has two side lengths of 5 units. Three of its sides have equations $y = 0$, $y = 2$, $y = 2x$. Find the equation of the fourth side.

REASONING	7	7, 8	7, 8

7 a Using $m_2 = -\frac{1}{m_1}$, find the gradient of a line perpendicular to the line with the given gradient.

i $m_1 = \frac{2}{3}$　　ii $m_1 = \frac{1}{5}$　　iii $m_1 = -\frac{1}{7}$　　iv $m_1 = -\frac{3}{11}$

b If $m_1 = \frac{a}{b}$, find m_2 given $m_1 \times m_2 = -1$.

8 a Find the gradient of a line that is parallel to these lines.

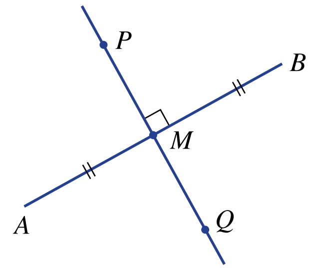
i $2x + 4y = 9$　　ii $3x - y = 8$

b Find the gradient of a line that is perpendicular to these lines.

i $5x + 5y = 2$　　ii $7x - y = -1$

ENRICHMENT: Perpendicular bisectors	–	–	9(½)

9 If a line segment AB is cut by another line PQ at right angles through the midpoint (M) of AB, then PQ is called the perpendicular bisector. By first finding the midpoint of AB, find the equation of the perpendicular bisector of the segment connecting these points.



a $A(1, 1)$, $B(3, 5)$

b $A(0, 6)$, $B(4, 0)$

c $A(-2, 3)$, $B(6, -1)$

d $A(-6, -1)$, $B(0, 2)$

e $A(-1, 3)$, $B(2, -4)$

f $A(-6, -5)$, $B(4, 7)$

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The following problems will investigate practical situations drawing upon knowledge and skills developed throughout the chapter. In attempting to solve these problems, aim to identify the key information, use diagrams, formulate ideas, apply strategies, make calculations and check and communicate your solutions.

Solar car battery

1 The battery power of two electric cars is analysed and compared by data scientists after testing the cars under average city driving conditions. The average distance covered in 1 hour under such conditions is 40 km.

- The Zet car battery is fully charged with 80 kWh reducing at a rate of 20 kWh per hour.
- The Spark car battery is fully charged with 60 kWh reducing at a rate of 12 kWh per hour.

The data scientists wish to investigate the life of each battery charge and compare the performance of each car's battery within the limits of average city driving conditions.

a Find a rule for the battery power P (kWh) over t hours for:
  i the Zet car
  ii the Spark car.

b Find the battery power remaining (the value of P (kWh)) after 3 hours of average city driving for:
  i the Zet car
  ii the Spark car.

c After how long will there be 40 kWh of power remaining for:
  i the Zet car?
  ii the Spark car?

d Which car's battery has the longer travel time? Give reasons.

e If the cars are driven until their batteries are entirely depleted of power, how far could they travel under average city conditions? Recall that 40 km is covered in one hour.

f After how long will the cars have the same remaining battery power?

g A third car, the Watt, has a battery producing the following statistics:
  - 60 kWh after 1 hour of driving
  - 20 kWh after 4 hours of driving.

  Find a rule for the Watt's remaining battery power P (kWh) after t hours and calculate how far this car can travel on one full charge.

Temperature vs coffee

2 Lydia owns a coffee shop and notices a decline in coffee sales as the maximum daily temperature rises over summer. She collects the following statistics by selecting 5 days within a two-month period.

Data point	1	2	3	4	5
Max. temperature	15	18	27	31	39
Coffee sales	206	192	165	136	110

Lydia wishes to analyse her collected data and see if there is a clear relationship between the maximum temperature and coffee sales. She wants to be better informed, so she can predict income from coffee sales in the future.

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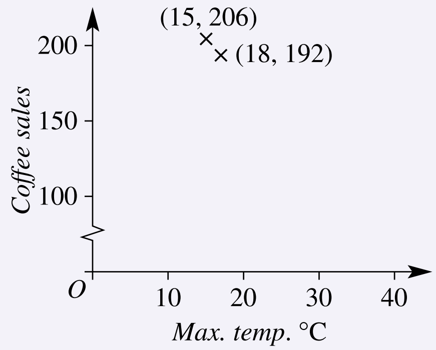


a Copy and complete this graph for the 5 data points. Two points have been done for you.

b Do you think there is a clear relationship between coffee sales and maximum temperature? Give a reason.

c Using the coordinates of data points 1 and 5, find a rule linking coffee sales, C, and maximum temperature, T.

d Using your rule from part **c**, predict the number of coffee sales for a day with the following maximum temperatures.

i 25°C **ii** 35°C

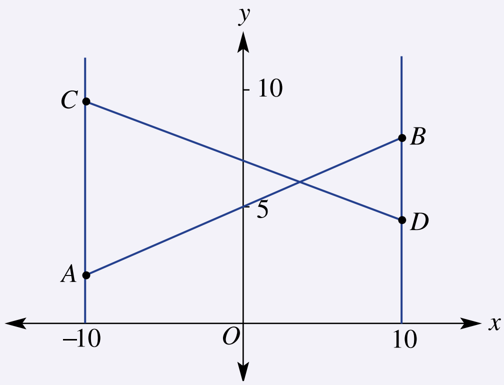
e Using your rule from part **c**, predict the maximum temperature for a day with the following number of coffee sales.

i 150 **ii** 202

f At what maximum temperature does your rule predict that there will be no coffee sales? Is this realistic?

Rope course

3 Douglas is designing a rope obstacle course between two vertical trees which are 20 m apart. To help analyse his design, he draws the two trees represented by $x = -10$ and $x = 10$ on a Cartesian plane as shown.
Two initial ropes AB and CD are used to start with. Douglas sets the heights of the points A, B, C and D as follows:
$A = 3$ m, $B = 7$ m, $C = 9$ m, $D = 4$ m



Douglas wants to be clear about his design so that he has a good idea about the amount of rope required and the positions of various key points on the obstacle course.

a Find the lengths of the following ropes correct to two decimal places.

i AB **ii** CD

b Find the equation of the lines which describes the following ropes.

i AB **ii** CD

c Use an accurate graph to determine the approximate position at which the ropes intersect.

d Douglas decides to place a third rope midway along and perpendicular to rope AB and run it from this position to the left tree.

i Find an equation for this third rope. **ii** How high up the left tree will it reach?

iii What will be the length of the rope correct to one decimal place?

e Douglas decides that he wants a fourth rope running from the point D on the right tree to the left tree, joining somewhere between point A and the midpoint of AC. What are the minimum and maximum lengths of rope that he might need? Round to one decimal place where necessary.

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4J Linear modelling

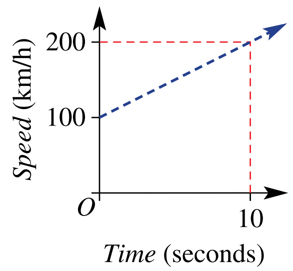
Learning intentions

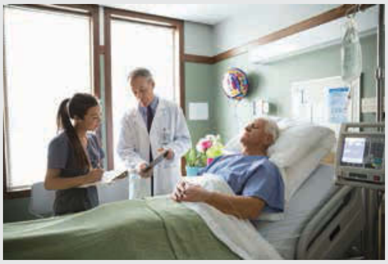
- To understand that many real-life situations can be modelled using a straight line
- To be able to find the equation linking two variables
- To know where the dependent and independent variable are positioned on a set of axes
- To be able to use an equation or graph to estimate the value of one variable given the other

Past, present and future learning

- This section addresses the Stage 5 Core Topic called Linear relationships A
- These concepts will be revised, briefly, in Chapter 1 of our Year 10 book and also in Stage 6 Advanced

If a relationship between two variables is linear, the graph will be a straight line and the equation linking the two variables can be written in gradient–intercept form. The process of describing and using such line graphs and rules for the relationship between two variables is called linear modelling. A test car, for example, increasing its speed from 100 km/h to 200 km/h in 10 seconds with constant acceleration could be modelled by the rule $s = 10t + 100$. This rule could then be used to calculate the speed at different times in the test run.





A doctor may prescribe 1400 mL of saline solution to be given intravenously over seven hours. The rate of infusion is 200 mL/h, and the remaining volume, V, after t hours has the linear equation: $V = -200t + 1400$.

Lesson starter: The test car

The graph shown above describes the speed of a racing car over a 10 second period.

- Explain why the rule is $s = 10t + 100$.
- Why might negative values of t not be considered for the graph?
- How could you accurately calculate the speed after 6.5 seconds?
- If the car continued to accelerate at the same rate, how could you accurately predict the car's speed after 13.2 seconds?

KEY IDEAS

- Many situations can often be **modelled** by using a linear rule or graph. The key elements of linear modelling include:
  - finding the rule linking the two variables
  - sketching a graph

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- using the graph or rule to predict or estimate the value of one variable given the other
- finding the rate of change of one variable with respect to the other variable – this is equivalent to finding the gradient.

BUILDING UNDERSTANDING

1 A person gets paid \$50 plus \$20 per hour. Decide which rule describes the relationship between the pay $\$P$ and number of hours worked, n.

A $P = 50 + n$
B $P = 50 + 20n$
C $P = 50n + 20$
D $P = 20 + 50$

2 The amount of money in a bank account is \$1000 and is increasing by \$100 per month.

a Find the amount of money in the account after:
 i 2 months **ii** 5 months **iii** 12 months

b State a rule for the amount of money A dollars after n months.

3 If $d = 5t - 4$, find:

a d when $t = 10$
b d when $t = 1.5$
c t when $d = 6$
d t when $d = 11$

Example 19 Applying linear relations

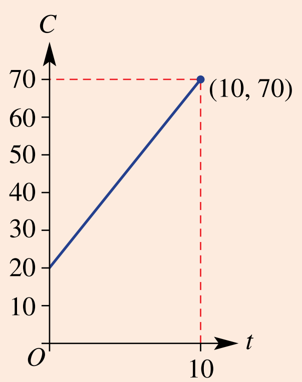
The deal offered by Bikeshare, a bicycle rental company, is a fixed charge of \$20 plus \$5 per hour of use.

a Write a rule for the total cost, $\$C$, of renting a bike for t hours.
b Sketch the graph of C versus t for $0 \leqslant t \leqslant 10$.
c What is the total cost when a bike is rented for 4 hours?
d If the total cost was \$50, for how many hours was a bike rented?

SOLUTION

a $C = 20 + 5t$

b C-intercept is at (0, 20)
At $t = 10$, $C = 20 + 5(10)$
$= 70$
$\therefore$ endpoint is (10, 70)



EXPLANATION

A fixed amount of \$20 plus \$5 for each hour.

Let $t = 0$ to find the C-intercept.
Letting $t = 10$ gives $C = 70$ and this gives the other endpoint.

Sketch the graph using the points (0, 20) and (10, 70).

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c $C = 20 + 5t$
$= 20 + 5(4)$
$= 40$
The cost is \$40.

Substitute $t = 4$ into the rule.

Answer the question using the correct units.

d $C = 20 + 5t$
$50 = 20 + 5t$
$30 = 5t$
$t = 6$
The bike was rented for 6 hours.

Write the rule and substitute $C = 50$. Solve the resulting equation for t by subtracting 20 from both sides then dividing both sides by 5.

Answer the question in words.

Now you try

A salesperson earns \$500 per week plus \$10 for each item sold.

a Write a rule for the total weekly wage, $\$W$, if the salesperson makes x sales.
b Draw a graph of W versus x for $0 \leqslant x \leqslant 100$.
c How much does the salesperson earn in a week if they make 25 sales?
d In a particular week the salesperson earns \$850. How many sales did they make?

Exercise 4J

FLUENCY	1–3	1, 2, 4	2, 4

Example 19

1 A sales representative earns \$400 a week plus \$20 for each sale she makes.
a Write a rule which gives the total weekly wage, $\$W$, if she makes x sales.
b Draw a graph of W versus x for $0 \leqslant x \leqslant 40$.
c How much does the sales representative earn in a particular week if she makes 12 sales?
d In a particular week, the sales representative earns \$1000. How many sales did she make?

2 A catering company charges \$500 for the hire of a marquee, plus \$25 per guest.
a Write a rule for the cost, $\$C$, of hiring a marquee and catering for n guests.
b Draw a graph of C versus n for $0 \leqslant n \leqslant 100$.
c How much would a party catering for 40 guests cost?
d If a party cost \$2250, how many guests were catered for?

3 A plumber charges a \$40 fee up-front and \$50 for each hour she works.
a Find a linear equation for the total charge, $\$C$, for n hours of work.
b What is the cost if she works 4 hours?
c If the plumber works on a job for two days and averages 6 hours per day, what will the total cost be?

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4 The cost, $\$C$, of recording a music CD is \$300, plus \$120 per hour of studio time.
 a Write a rule for the cost, $\$C$, of recording a CD requiring t hours of studio time.
 b Draw a graph of C versus t for $0 \leqslant t \leqslant 10$.
 c How much does a recording cost if it requires 6 hours of studio time?
 d If a recording cost \$660 to make, for how long was the studio used?

PROBLEM–SOLVING	5, 6	5, 6	6, 7

5 A petrol tank holds 66 litres of fuel. It contains 12 litres of petrol initially and the petrol pump fills it at 3 litres every 10 seconds.
 a Write a linear equation for the amount of fuel (F litres) in the tank after t minutes.
 b How long will it take to fill the tank?
 c How long will it take to add 45 litres into the petrol tank?

6 A tank is initially full with 4000 litres of water and water is being used at a rate of 20 litres per minute.
 a Write a rule for the volume, V litres, of water after t minutes.
 b Calculate the volume after 1.5 hours.
 c How long will it take for the tank to be emptied?
 d How long will it take for the tank to have only 500 litres?

7 A spa pool contains 1500 litres of water. It is draining at the rate of 50 litres per minute.
 a Draw a graph of the volume of water, V litres, remaining after t minutes.
 b Write a rule for the volume of water at time t minutes.
 c What does the gradient represent?
 d What is the volume of water remaining after 5 minutes?
 e After how many minutes is the pool half empty?

REASONING	8	8	8, 9

8 The rule for distance travelled d km over a given time t hours for a moving vehicle is given by $d = 50 + 80t$.
 a What is the speed of the vehicle?
 b If the speed was actually 70 km per hour, how would this change the rule? Give the new rule.

9 The altitude, h metres, of a helicopter t seconds after it begins its descent is given by $h = 350 - 20t$.
 a At what rate is the helicopter's altitude decreasing?
 b At what rate is the helicopter's altitude increasing?
 c What is the helicopter's initial altitude?
 d How long will it take for the helicopter to reach the ground?
 e If instead the rule was $h = 350 + 20t$, describe what the helicopter would be doing.

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ENRICHMENT: Sausages and cars	–	–	10, 11

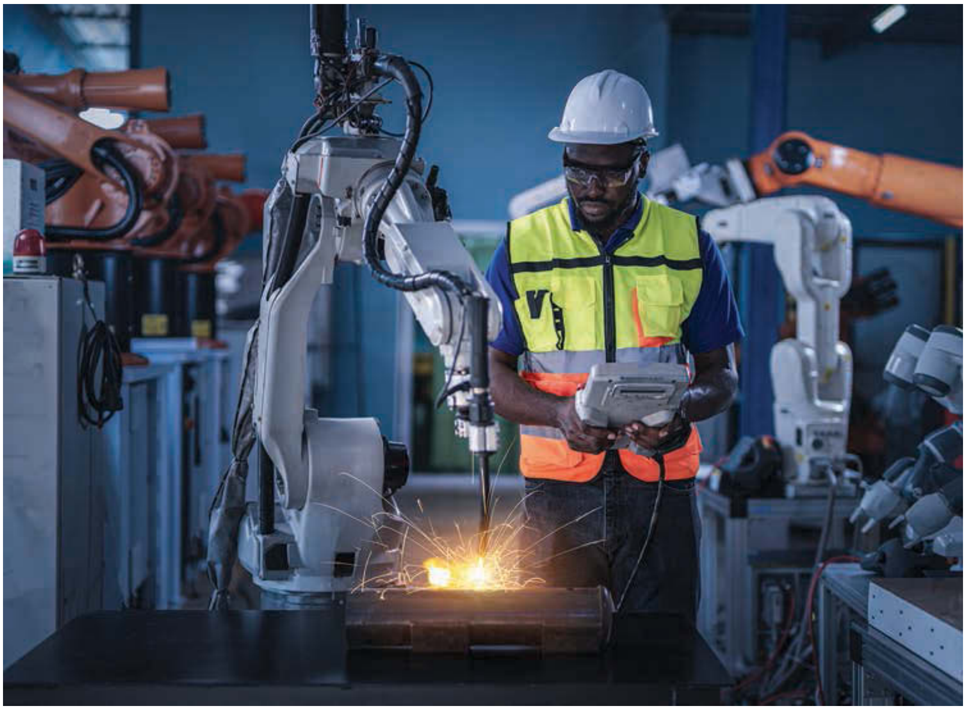
10 Joanne organised a sausage sizzle to raise money for her science club. The hire of the barbecue cost Joanne \$20, and the sausages cost 40c each.

a **i** Write a rule for the total cost, $\$C$, if Joanne buys and cooks n sausages.
ii If the total cost was \$84, how many sausages did Joanne buy?

b **i** If Joanne sells each sausage for \$1.20, write a rule to find her profit, $\$P$, after buying and selling n sausages.
ii How many sausages must she sell to 'break even'?
iii If Joanne's profit was \$76, how many sausages did she buy and sell?

11 The directors of a car manufacturing company calculate that the set-up cost for a new component is \$6700 and each component would cost \$10 to make.

a Write a rule for the total cost, $\$C$, of producing x components.
b Find the cost of producing 200 components.
c How many components could be produced for \$13 000?
d Find the cost of producing 500 components.
e If each component is able to be sold for \$20, how many must they sell to 'break even'?
f Write a rule for the profit, $\$P$, in terms of x.
g Write a rule for the profit, $\$T$, per component in terms of x.
h Find x, the number of components, if the profit per component is to be \$5.



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4K Graphical solutions to simultaneous equations

Learning intentions
- To understand that two straight line graphs intersect at exactly one point, unless the lines are parallel or identical
- To know that the point of intersection of two lines is the solution of the simultaneous equations
- To be able to use the graphs of two straight lines to read off the coordinates of the point of intersection
- To know how to use the equations to check if a point is at the intersection of the two lines

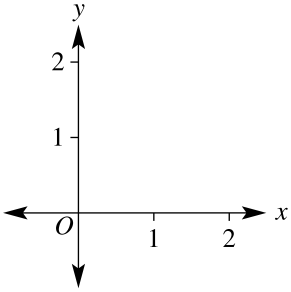
Past, present and future learning
- This section addresses the Stage 5 Path Topic called Equations C
- These concepts will be revised, briefly, in Chapter 1 of our Year 10 book and also in Stage 6 Advanced

To find a point that satisfies more than one equation involves finding the solution to simultaneous equations. An algebraic approach was considered in Chapter 2. A graphical approach involves locating an intersection point.

Revenue, R, and costs, C, often have linear equations, such as $R = 1000n$ and $C = 400n + 3000$, for producing n surfboards. The break-even point of a business is where these two linear graphs intersect; profit starts as revenue begins to exceed costs.

Lesson starter: Accuracy counts

Two graphs have the rules $y = x$ and $y = 2 - 4x$.
Accurately sketch the graphs of both rules on a large set of axes like the one shown.



- State the x-value of the point at the intersection of your two graphs.
- State the y-value of the point at the intersection of your two graphs.
- Discuss how you could use the rules to check if your point is correct.
- If your point does not satisfy both rules, check the accuracy of your graphs and try again.

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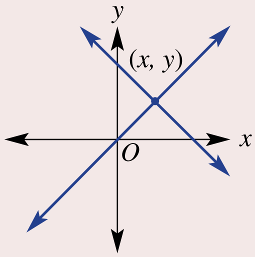
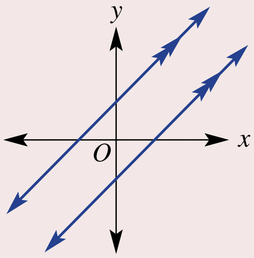
KEY IDEAS

- When we consider two or more equations at the same time, they are called **simultaneous equations**.
- To determine the **point of intersection** of two lines, we can use an accurate graph and determine its coordinates (x, y).
- Two situations can arise for the intersection of two different lines.
  - The two graphs intersect at one point only and there is one solution (x, y).
    - The point of intersection is simultaneously on both lines and is the **solution** to the simultaneous equations.



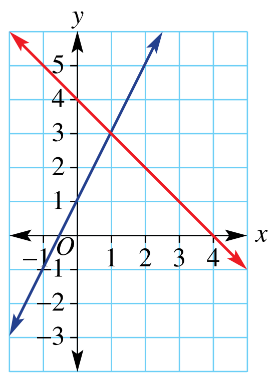
  - The two lines are parallel and there is no intersection.



BUILDING UNDERSTANDING

1 Give the coordinates of the point of intersection for these pairs of lines.

a



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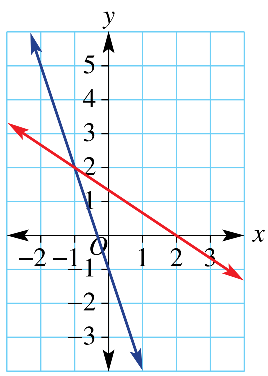
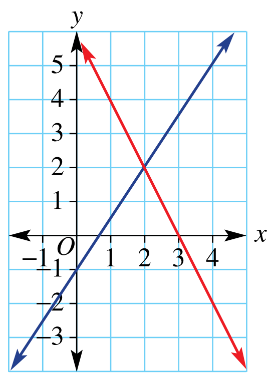


c



2 Decide if the point (1, 3) satisfies these equations. (*Hint*: Substitute (1, 3) into each equation to see if the equation is true.)

a $y = 2x + 1$ **b** $y = -x + 5$ **c** $y = -2x - 1$ **d** $y = 4x - 1$

3 Consider the two lines with the rules $y = 5x$ and $y = 3x + 2$ and the point (1, 5).

a Substitute (1, 5) into $y = 5x$. Does (1, 5) sit on the line with equation $y = 5x$?

b Substitute (1, 5) into $y = 3x + 2$. Does (1, 5) sit on the line with equation $y = 3x + 2$?

c Is (1, 5) the intersection point of the lines with the given equations?

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Example 20 Checking an intersection point

Decide if the given point is at the intersection of the two lines with the given equations.

a $y = 2x + 3$ and $y = -x$ with point $(-1, 1)$

b $y = -2x$ and $3x + 2y = 4$ with point $(2, -4)$

SOLUTION

a Substitute $x = -1$.

$$\begin{aligned} y &= 2x + 3 \\ &= 2 \times (-1) + 3 \\ &= -2 + 3 \\ &= 1 \end{aligned}$$

So $(-1, 1)$ satisfies $y = 2x + 3$.

$$\begin{aligned} y &= -x \\ &= -(-1) \\ &= 1 \end{aligned}$$

So $(-1, 1)$ satisfies $y = -x$.

$\therefore (-1, 1)$ is the intersection point.

b Substitute $x = 2$.

$$\begin{aligned} y &= -2x \\ &= -2 \times (2) \\ &= -4 \end{aligned}$$

So $(2, -4)$ satisfies $y = -2x$.

$$\begin{aligned} &3x + 2y \\ &= 3 \times (2) + 2 \times (-4) \\ &= 6 + (-8) \\ &= -2 \\ &\neq -4 \end{aligned}$$

So $(2, -4)$ is not on the line.

$\therefore (2, -4)$ is not the intersection point.

EXPLANATION

a Substitute $x = -1$ into $y = 2x + 3$ to see if $y = 1$. If so, then $(-1, 1)$ is on the line.

Repeat for $y = -x$.

If $(-1, 1)$ is on both lines then it must be the intersection point.

b Substitute $x = 2$ into $y = -2x$ to see if $y = -4$. If so, then $(2, -4)$ is on the line.

Substitute $x = 2$ and $y = -4$ to see if $3x + 2y = 4$ is true.
Clearly, the equation is not satisfied.

As the point is not on both lines, it cannot be the intersection point.

Now you try

Decide if the given point is at the intersection of the two lines with the given equations.

a $y = 3x - 5$ and $y = -2x$ with point $(1, -2)$

b $y = -4x$ and $3x - y = 2$ with point $(2, -8)$

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Example 21 Solving simultaneous equations graphically

Solve the simultaneous equations $y = 2x - 2$ and $x + y = 4$ graphically.

SOLUTION

$y = 2x - 2$

y-intercept (let $x = 0$):

$$y = 2 \times (0) - 2$$
$$y = -2 \therefore (0, -2)$$

x-intercept (let $y = 0$):

$$0 = 2x - 2$$
$$2 = 2x$$
$$x = 1 \therefore (1, 0)$$

EXPLANATION

Sketch each linear graph by first finding the y-intercept (substitute $x = 0$) and the x-intercept (substitute $y = 0$) and solve the resulting equation.

SOLUTION

$x + y = 4$

y-intercept (let $x = 0$):

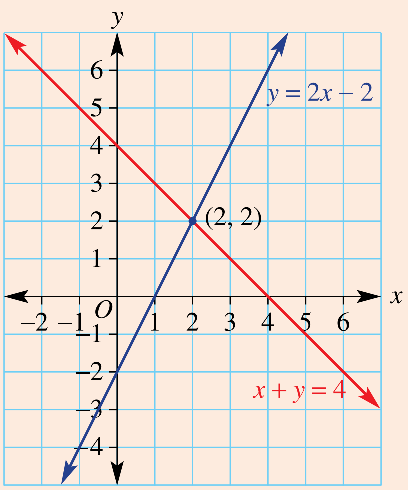
$$(0) + y = 4$$
$$y = 4 \therefore (0, 4)$$

x-intercept (let $y = 0$):

$$x + (0) = 4$$
$$x = 4 \therefore (4, 0)$$

EXPLANATION

Repeat the process for the second equation.



Sketch both graphs on the same set of axes by marking the intercepts and joining with a straight line.

Ensure that the axes are scaled accurately.

Locate the intersection point and read off the coordinates.

The intersection point is (2, 2).
The solution is $x = 2,\ y = 2$.

The point (2, 2) simultaneously belongs to both lines.

Now you try

Solve the simultaneous equations $y = -x + 1$ and $2x + y = 3$ graphically.

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Exercise 4K

FLUENCY	1, 2(½)	1–2(½)	1(½), 2(⅓)

Example 20

1 Decide if the given point is at the intersection of the two lines with the given equations.

a $y = 3x - 4$ and $y = 2x - 2$ with point (2, 2)

b $y = -x + 3$ and $y = -x$ with point (2, 1)

c $y = -4x + 1$ and $y = -x - 1$ with point (1, −3)

d $x - y = 10$ and $2x + y = 8$ with point (6, −4)

e $2x + y = 0$ and $y = 3x + 4$ with point (−1, 2)

f $x - 3y = 13$ and $y = -x - 1$ with point (4, −3)

Example 21

2 Solve these pairs of simultaneous equations graphically by finding the coordinates of the intersection point.

a $2x + y = 6$
$x + y = 4$

b $3x - y = 7$
$y = 2x - 4$

c $y = x - 6$
$y = -2x$

d $y = 2x - 4$
$x + y = 5$

e $2x - y = 3$
$3x + y = 7$

f $y = 2x + 1$
$y = 3x - 2$

g $x + y = 3$
$3x + 2y = 7$

h $y = x + 2$
$y = 3x - 2$

i $y = x - 3$
$y = 2x - 7$

j $y = 3$
$x + y = 2$

k $y = 2x - 3$
$x = -1$

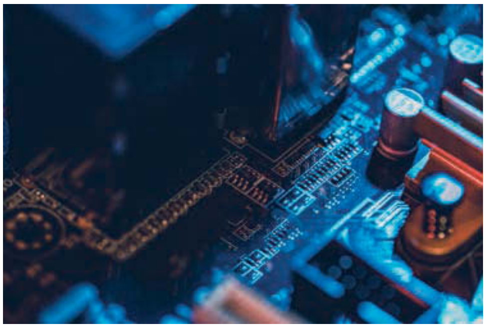
l $y = 4x - 1$
$y = 3$

PROBLEM–SOLVING	3, 4	3, 4	4, 5

3 A company manufactures electrical components. The cost, $\$C$ (including rent, materials and labour), is given by the rule $C = n + 3000$, and its revenue, $\$R$, is given by the rule $R = 5n$, where n is the number of components produced.

a Sketch the graphs of C and R on the same set of axes and determine the point of intersection.

b State the number of components, n, for which the costs $\$C$ are equal to the revenue $\$R$.



4 BasketCo. manufactures baskets. Its costs, $\$C$, are given by the rule $C = 4n + 2400$, and its revenue, $\$R$, is given by the rule $R = 6n$, where n is the number of baskets produced. Sketch the graphs of C and R on the same set of axes and determine the number of baskets produced if the costs equal the revenue.

5 Two asteroids are 1000 km apart and are heading straight for each other. One asteroid is travelling at 59 km per second and the other at 41 km per second. How long will it be before they collide?

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REASONING	6	6, 7	6–8

6 Explain why the graphs of the rules $y = 3x - 7$ and $y = 3x + 4$ have no intersection point.

7 For the following families of graphs, determine their points of intersection (if any).

a $y = x, y = 2x, y = 3x$

b $y = x, y = -2x, y = 3x$

c $y = x + 1, y = x + 2, y = x + 3$

d $y = -x + 1$ $y = -x + 2, y = -x + 3$

e $y = 2x + 1, y = 3x + 1, y = 4x + 1$

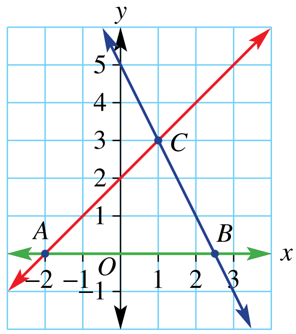
f $y = 2x + 3, y = 3x + 3, y = 4x + 3$

8 a If two lines have the equations $y = 3x + 1$ and $y = 2x + c$, find the value of c if the intersection point is at $x = 1$.

b If two lines have the equations $y = mx - 4$ and $y = -2x - 3$, find the value of m if the intersection point is at $x = -1$.

ENRICHMENT: Intersecting to find triangular areas	–	–	9, 10

9 The three lines with equations $y = 0$, $y = x + 2$ and $y = -2x + 5$ are illustrated here.



a State the coordinates of the intersection point of $y = x + 2$ and $y = -2x + 5$.

b Use $A = \frac{1}{2}bh$ to find the area of the enclosed triangle ABC.

10 Use the method outlined in Question 9 to find the area enclosed by these sets of three lines.

a $y = 0$, $y = x + 3$ and $y = -2x + 9$

b $y = 0$, $y = \frac{1}{2}x + 1$ and $y = -x + 10$

c $y = 2$, $x - y = 5$ and $x + y = 1$

d $y = -5$, $2x + y = 3$ and $y = x$

e $x = -3$, $y = -3x$ and $x - 2y = -7$

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Working mathematically

Swimming at number 4

At a swim meet, Jess is the fourth and fastest swimmer in her team of 4 which swims the medley relay. Her average speed is 1.5 metres per second. During the race, she often finds herself in second position just before she dives in and needs to catch up so that she wins the race for her team. Each swimmer needs to swim 50 metres.

In the relay Adrianna is the last swimmer of another team which is currently in the lead before she dives in. To win Jess needs to catch up and overtake Adrianna. Adrianna's average speed is 1.4 metres per second.

For this task, d metres is the distance travelled by a swimmer and t seconds is the time in seconds after Adrianna dives into the water.

Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate.

Routine problems

a Write a rule for Adrianna's distance (d metres) in terms of time (t seconds).

b Find how long it takes for Adrianna to complete the 50 metres to the nearest tenth of a second.

c If Jess dives into the water 2 seconds after Adrianna, then the rule describing Jess's distance after t seconds is given by: $d = 1.5t - 3$.

- **i** Find how long it takes for Jess to complete the 50 metres, after Adrianna dives into the water, to the nearest tenth of a second.
- **ii** Sketch a graph of d vs t for both Jess and Adrianna on the same set of axes. Show the coordinates of any endpoints and intercepts.
- **iii** Does Jess overtake Adrianna? If so, after how many seconds?

Non-routine problems

Explore and connect

a The problem is to determine the maximum head start that Adrianna can have so that Jess wins the relay for her team. Write down all the relevant information that will help solve this problem.

Choose and apply techniques

b Jess dives into the water 1 second after Adrianna. i.e. when $t = 1$ then $d = 0$ for Jess.

- **i** Find a rule for Jess's distance (d metres) after t seconds.
- **ii** Find the time taken for Jess to complete the 50 metres, after Adrianna dives into the water, to the nearest tenth of a second.
- **iii** Sketch a graph of d vs t for both Jess and Adrianna on the same set of axes. Show the coordinates of any endpoints and intercepts.
- **iv** Decide if Jess overtakes Adrianna. If so, after how many seconds?

c Repeat part **b** if Jess dives into the water 3 seconds after Adrianna.

Communicate thinking and reasoning

d If Jess catches up to Adrianna at exactly the time when Adrianna finishes the relay:

- **i** illustrate this situation with a graph
- **ii** determine the head start, in seconds to one decimal place, that Adrianna had in this situation.

e Decide under what circumstances will Jess win the relay for her team. Justify your answer.

f Summarise your results and describe any key findings.

Extension problems

Problem solve

a If Jess's swim speed was able to be increased to 1.6 m/s, investigate the maximum head start that Adrianna could have so that Jess wins the relay.

b If in one particular race Adrianna has a 2.1 second head start, determine Jess's minimum speed if she is to win the relay.

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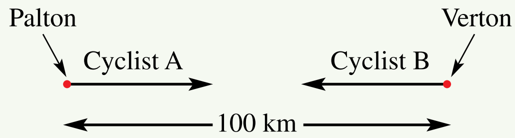
Investigation

Coming and going

The distance between two towns, Palton and Verton, is 100 km. Two cyclists travel in opposite directions between the towns, starting their journeys at the same time. Cyclist A travels from Palton to Verton at a speed of 20 km/h while cyclist B travels from Verton to Palton at a speed of 25 km/h.



Measuring the distance from Palton

a Using d_A km as the distance cyclist A is from Palton after t hours, explain why the rule connecting d_A and t is $d_A = 20t$.

b Using d_B km as the distance cyclist B is from Palton after t hours, explain why the rule connecting d_B and t is $d_B = 100 - 25t$.

Technology: Spreadsheet

c Instructions:

- Enter the time in hours into column A, starting at 0 hours.
- Enter the formulas for the distances d_A and d_B into columns B and C.
- Use the **Fill down** function to fill in the columns. Fill down until the distances show that both cyclists have completed their journey.

	A	B	C
1	0	= 20 * A1	= 100 – 25 * A1
2	= A1 + 1		
3			

d **i** Determine how long it takes for cyclist A to reach Verton.

ii Determine how long it takes for cyclist B to reach Palton.

iii After which hour are the cyclists the closest?

Investigating the intersection

a Change the time increment to a smaller unit. Try 0.5 hours using '= A1 + 0.5', or 0.1 hours using '= A1 + 0.1' in column A.

b Fill or scroll down to ensure that the distances show that both cyclists have completed their journey.

c Determine the time at which the cyclists are the closest.

d Continue altering the time increment until you are satisfied that you have found the time of intersection of the cyclists correct to one decimal place.

e *Extension*: Complete part **d** above but find an answer correct to three decimal places.

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Investigation

The graph

a Sketch a graph of d_A and d_B on the same set of axes. Scale your axes carefully to ensure that the full journey for both cyclists is represented.

b Determine the intersection point as accurately as possible on your graph and hence estimate the time when the cyclists meet.

c Use technology to confirm the point of intersection, and hence determine the time at which the cyclists meet, correct to three decimal places.

Algebra and proof

a At the point of intersection it could be said that $d_A = d_B$.
This means that $20t = 100 - 25t$.
Solve this equation for t.

b Find the exact distance from Palton at the point where the cyclists meet.

Reflection

Write a paragraph describing the journey of the two cyclists. Comment on:

- the speeds of the cyclists
- their meeting point
- the difference in computer and algebraic approaches in finding the time of the intersection point.

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1 A tank with 520 L of water begins to leak at a rate of 2 L per day. At the same time, a second tank is being filled at a rate of 1 L per hour starting at 0 L. How long does it take for the tanks to have the same volume?

2 Two cars travel towards each other on a 400 km stretch of road. One car travels at 90 km/h and the other at 70 km/h. How long does it take before they pass each other?

3 The points $(-1, 4)$, $(4, 6)$, $(2, 7)$ and $(-3, 5)$ are the vertices of a parallelogram. Find the midpoints of its diagonals. What do you notice?

4 A trapezium is enclosed by the straight lines $y = 0$, $y = 3$, $y = 7 - x$, and $x = k$, where k is a constant. Find the possible values of k given the trapezium has an area of 24 units^2.

5 Prove that the triangle with vertices at the points $A(-1, 3)$, $B(0, -1)$ and $C(3, 2)$ is isosceles.

6 Find the perimeter (to the nearest whole number) and area of the triangle enclosed by the lines with equations $x = -4$, $y = x$ and $y = -2x - 3$.

7 $ABCD$ is a parallelogram. A, B and C have coordinates $(5, 8)$, $(2, 5)$ and $(3, 4)$ respectively. Find the coordinates of D.

8 A kite is formed by joining the points $A(a, b)$, $B(-1, 3)$, $C(x, y)$ and $D(3, -5)$.

a Determine the equations of the diagonals BD and AC of this kite.

b Given $a = -5$ and $y = 4$, find the values of b and x.

c Find the area of the kite (without the use of a calculator).

9 A trapezium is enclosed by the straight lines $y = 0$, $y = 6$, $y = 8 - 2x$, and $y = x + k$, where k is a constant. Find the possible values of k given the trapezium has an area of 66 units^2.

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Chapter summary

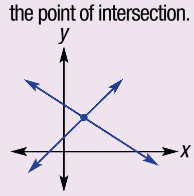
Linear relationships

Direct proportion

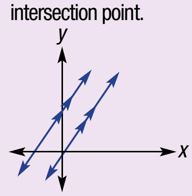
If two variables are directly proportional, their rule is of the form $y = mx$. The gradient of the graph represents the rate of change of y with respect to x; e.g. for a car travelling at 60 km/h for t hours, distance $= 60t$.

Graphical solutions of simultaneous equations

Graph each line and read off the point of intersection.



Parallel lines have no intersection point.

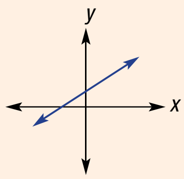


Linear modelling

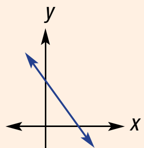
Define variables to represent the problem and write a rule relating the two variables. The rate of change of one variable with respect to the other is the gradient.

Gradient

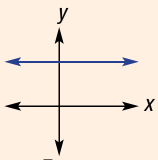
This measures the slope of a line.



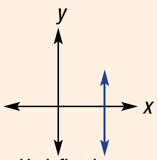
Positive



Negative



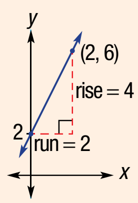
Zero



Undefined

Gradient $m = \frac{\text{rise}}{\text{run}}$

$m = \frac{4}{2} = 2$



Finding the equation of a line

Gradient–intercept form ($y = mx + c$)

Require gradient and y-intercept or any other point to substitute; e.g. line with gradient 3 passes through point (2, 1)

$\therefore\ y = 3x + c$

substitute $x = 2$, $y = 1$

$1 = 3(2) + c$

$1 = 6 + c$

$\therefore\ c = -5$

$y = 3x - 5$

Straight line

$y = mx + c$

gradient y-intercept

In the form $ax + by = d$, rearrange to $y = mx + c$ form to read off gradient and y-intercept.

Midpoint and length of a line segment

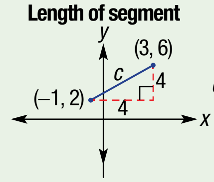
Midpoint is halfway between segment endpoints, e.g. midpoint of segment joining (–1, 2) and (3, 6).

$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$x = \frac{-1 + 3}{2} = 1$

$y = \frac{2 + 6}{2} = 4$, i.e. (1, 4)

Length of segment



$c^2 = 4^2 + 4^2$

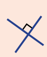
$c = \sqrt{32}$

$c = 5.7$ (to 1 d.p.)

Parallel and perpendicular lines

Parallel lines have the same gradient e.g. $y = 2x + 3$, $y = 2x - 1$ and $y = 2x$

Perpendicular lines are at right angles. Their gradients m_1 and m_2 are such that $m_1 m_2 = -1$, i.e. $m_2 = -\frac{1}{m_1}$.

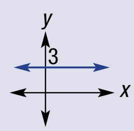


A line perpendicular to $y = 2x + 3$ has gradient $-\frac{1}{2}$.

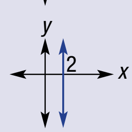
A linear relationship is made up of points (x, y) that form a straight line when plotted.

Special lines

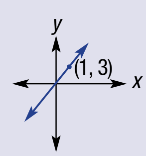
Horizontal line $y = c$
e.g. $y = 3$



Vertical line $x = k$
e.g. $x = 2$



$y = mx$ passes through the origin, substitute $x = 1$ to find another point, e.g. $y = 3x$



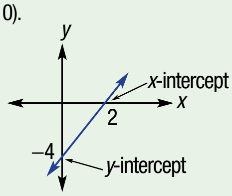
Sketching using intercepts

Sketch with two points. Often we find the x-intercept ($y = 0$) and y-intercept ($x = 0$).

e.g. $y = 2x - 4$

y-int: $x = 0$, $y = 2(0) - 4$
$= -4$

x-int: $y = 0$, $0 = 2x - 4$
$2x = 4$
$x = 2$



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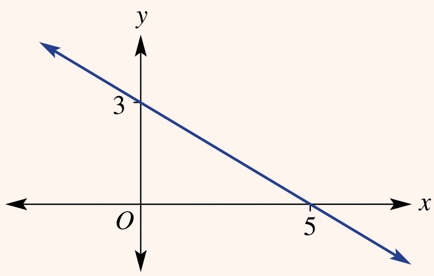
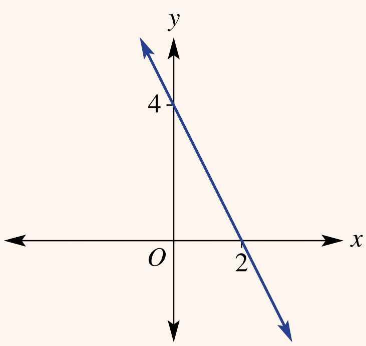
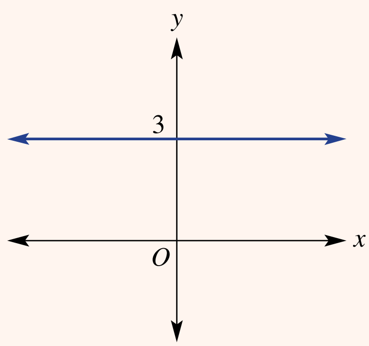
Chapter checklist with success criteria

A printable version of this checklist is available in the Interactive Textbook

		✔
4A	**1. I can plot points to graph a straight line.** e.g. Using $-3 \leqslant x \leqslant 3$, construct a table of values and plot a graph of $y = 2x - 4$.	☐
4A	**2. I can identify the *x*- and *y*-intercept.** e.g. Write down the coordinates of the x- and y-intercepts from this graph. 	☐
4A	**3. I can determine if a point is on a line.** e.g. Decide if the point $(-2, 9)$ is on the line with rule $y = -2x + 3$.	☐
4B	**4. I can sketch a graph of a straight line labelling intercepts.** e.g. Sketch the graphs of $y = -2x + 4$ and $3x - 4y = 12$ showing x- and y-intercepts.	☐
4C	**5. I can graph horizontal and vertical lines.** e.g. Sketch $y = -2$ and $x = 5$.	☐
4C	**6. I can sketch lines of the form $y = mx$.** e.g. Sketch the graph of $y = 2x$.	☐
4D	**7. I can find the gradient of a line.** e.g. For the graphs shown, state whether the gradient is positive, negative, zero or undefined and find the gradient.  	☐
4D	**8. I can find the gradient between two points.** e.g. Find the gradient of the line joining the points $(-1, 3)$ and $(4, 9)$.	☐
4E	**9. I can work with direct proportion.** e.g. A bathtub is being filled at a constant rate. It takes 10 minutes to fill the bath with 60 litres. Draw a graph of volume (V litres) vs time (t minutes) for $0 \leqslant t \leqslant 10$. Find the gradient of the graph and a rule for V. Use the rule to find the time for the bathtub to fill to 45 litres.	☐

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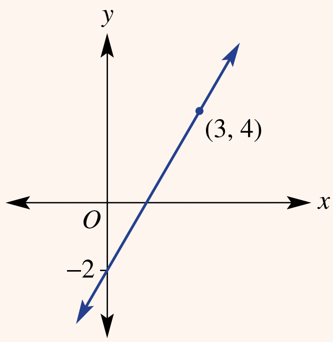
4F **10. I can rearrange linear equations.**
e.g. Rearrange $2x + 3y = 8$ to the form $y = mx + c$.

4F **11 I can state the gradient and y-intercept from a linear rule.**
e.g. State the gradient and y-intercept of $y = -2x + 3$ and $2y - 4x = 5$.

4F **12. I can use the gradient and y-intercept to sketch a graph.**
e.g. Use the gradient and y-intercept of $y = 3x - 2$ and $2x + 3y = 8$ to sketch their graphs.

4G **13. I can determine the equation of a straight line graph given the y-intercept.**
e.g. Determine the equation of the straight line shown here.



4G **14. I can determine the equation of a line given the gradient and a point.**
e.g. Find the equation of the line which has a gradient of 2 and passes through the point $(-2, 5)$.

4H **15. I can find the midpoint of a line segment.**
e.g. Find the midpoint $M(x, y)$ of the line segment joining the points $(2, -1)$ and $(6, 4)$.

4H **16. I can find the length of a line segment.**
e.g. Find the length of the segment joining $(-2, 1)$ and $(3, 5)$, correct to two decimal places.

4I **17. I can find the equation of a parallel line.**
e.g. Find the equation of a line that is parallel to the line $y = -2x + 3$ and passes through $(0, 1)$.

4I **18. I can find the equation of a perpendicular line.**
e.g. Find the equation of a line that is perpendicular to the line $y = 4x + 2$ and passes through $(0, -3)$.

4J **19. I can apply linear relations.**
e.g. The hire of a jumping castle involves a set-up cost of \$80 and a per hour rate of \$40. Write a rule for the total cost, \$$C$, of hiring a jumping castle for h hours and sketch its graph for $0 \leqslant h \leqslant 8$. Determine the total cost for hiring for 5 hours and how many hours it was hired for if the cost was \$200.

4K **20. I can check if a point is the intersection point of two lines.**
e.g. Determine if the point $(2, -1)$ is at the intersection of the lines with equations $y = -2x + 3$ and $2x - 3y = 7$.

4K **21. I can solve simultaneous equations graphically.**
e.g. Solve the simultaneous equations $y = 2x - 4$ and $x + y = 5$ graphically.

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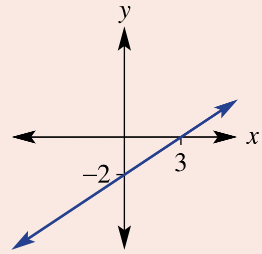
Short-answer questions

4A **1** Read off the coordinates of the x- and y-intercepts from the table and graph.

a

x	−2	−1	0	1	2
y	0	2	4	6	8

b



4B **2** Sketch the following linear graphs labelling x- and y-intercepts.

a $y = 2x - 4$ **b** $y = 3x + 9$ **c** $y = -2x + 5$ **d** $y = -x + 4$
e $2x + 4y = 8$ **f** $4x - 2y = 10$ **g** $2x - y = 7$ **h** $-3x + 6y = 12$

4B **3** Violet leaves her beach house by car and drives back to her home. Her distance d kilometres from her home after t hours is given by $d = 175 - 70t$.

a How far is her beach house from her home?
b How long does it take to reach her home?
c Sketch a graph of her journey between the beach house and her home.

4C **4** Sketch the following lines.

a $y = 3$ **b** $y = -2$ **c** $x = -4$
d $x = 5$ **e** $y = 3x$ **f** $y = -2x$

4D **5** By first plotting the given points, find the gradient of the line passing through the points.

a (3, 1) and (5, 5) **b** (2, 5) and (4, 3) **c** (1, 6) and (3, 1)
d (−1, 2) and (2, 6) **e** (−3, −2) and (1, 6) **f** (−2, 6) and (1, −4)

4E **6** An inflatable backyard swimming pool is being filled with water by a hose. It takes 4 hours to fill 8000 L.

a What is the rate at which water is poured into the pool?
b Draw a graph of volume (V litres) versus time (t hours) for $0 \leqslant t \leqslant 4$.
c Find the gradient of your graph and give the rule for V in terms of t.
d Use your rule to find the time to fill 5000 L.

4F **7** For each of the following linear relations, state the value of the gradient and the y-intercept, and then sketch using the gradient–intercept method.

a $y = 2x + 3$
b $y = -3x + 7$
c $2x + 3y = 9$
d $2y - 3x - 8 = 0$

4G **8** Give the equation of the straight line that:

a has gradient 3 and passes through the point (0, 2)
b has gradient −2 and passes through the point (3, 0)
c has gradient $\frac{4}{3}$ and passes through the point (6, 3).

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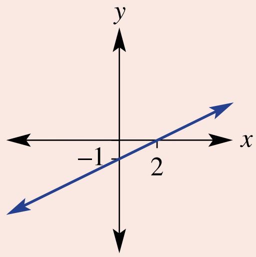
Chapter review

4G

9 Find the equations of the linear graphs below.

a



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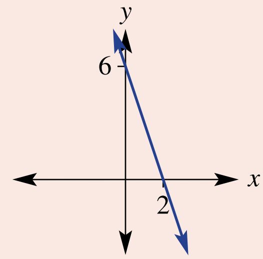

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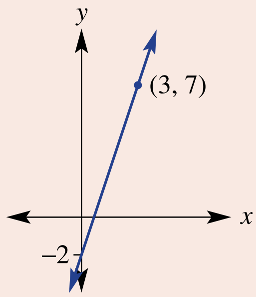
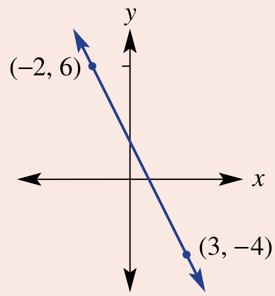


d



4H

10 For the line segment joining the following pairs of points, find:

i the midpoint

ii the length (to two decimal places where applicable).

a $(2, 4)$ and $(6, 8)$ **b** $(5, 2)$ and $(10, 7)$

c $(-2, 1)$ and $(2, 7)$ **d** $(-5, 7)$ and $(-1, -2)$

4H

11 Find the missing coordinate, n, if:

a the line joining $(-1, 3)$ and $(2, n)$ has gradient 2

b the line segment joining $(-2, 2)$ and $(4, n)$ has length 10, $n > 0$

c the midpoint of the line segment joining $(n, 1)$ and $(1, -3)$ is $(3.5, -1)$.

4I

12 Determine the linear equation that is:

a parallel to the line $y = 2x - 1$ and passes through the point $(0, 4)$

b parallel to the line $y = -x + 4$ and passes through the point $(0, -3)$

c perpendicular to the line $y = 2x + 3$ and passes through the point $(0, -1)$

d perpendicular to the line $y = -\frac{1}{3}x - 2$ and passes through the point $(0, 4)$

e parallel to the line $y = 3x - 2$ and passes through the point $(1, 4)$

f parallel to the line $3x + 2y = 10$ and passes through the point $(-2, 7)$.

4K

13 Determine if the point $(2, 5)$ is the intersection point of the graphs of the following pairs of equations.

a $y = 4x - 3$ and $y = -2x + 6$

b $3x - 2y = -4$ and $2x + y = 9$

4K

14 Find the point of intersection of the following straight lines using their graphs.

a $y = 2x - 4$ and $y = 6 - 3x$

b $2x + 3y = 8$ and $y = -x + 2$

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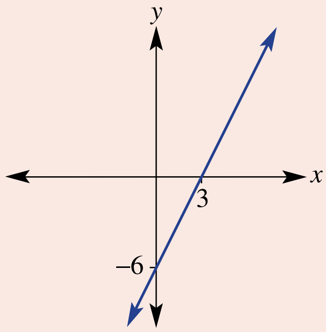
Multiple-choice questions

4A **1** The x- and y-intercepts of the graph of $2x + 4y = 12$ are respectively at:

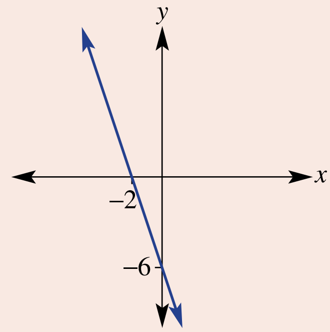
A (4, 0), (0, 2)
B (3, 0), (0, 2)
C (6, 0), (0, 3)
D (2, 0), (0, 12)
E (8, 0), (0, 6)

4B **2** The graph of $y = 3x - 6$ is represented by:

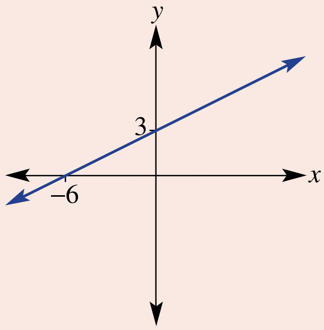
A



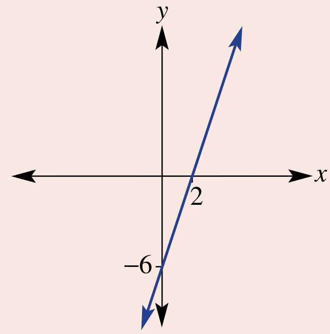
B



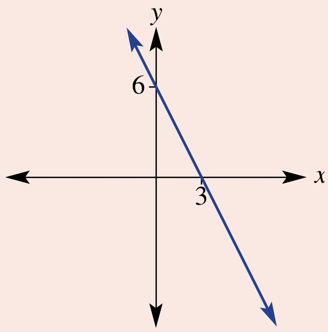
C



D



E



4D **3** The gradient of the line joining the points $(1, -2)$ and $(5, 6)$ is:

A 2
B 1
C 3
D -1
E $\frac{1}{3}$

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Chapter review

4A

4 The point that is not on the straight line $y = -2x + 3$ is:

A $(1, 1)$
B $(0, 3)$
C $(2, 0)$
D $(-2, 7)$
E $(3, -3)$

4F

5 The linear graph that *does not* have a gradient of 3 is:

A $y = 3x + 7$
B $\frac{y}{3} = x + 2$
C $2y - 6x = 1$
D $y - 3x = 4$
E $3x + y = -1$

4G

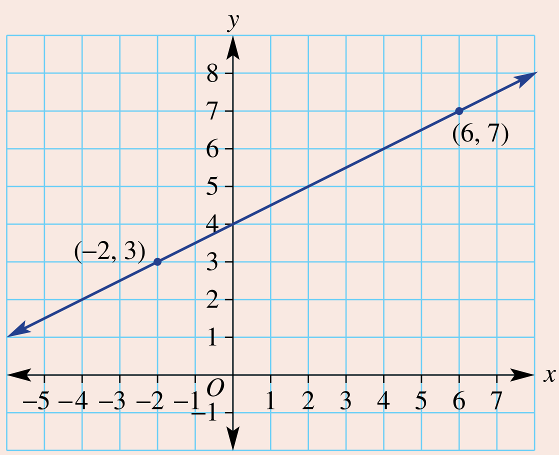
6 A straight line has a gradient of -2 and passes through the point $(0, 5)$. Its equation is:

A $2y = -2x + 10$
B $y = -2x + 5$
C $y = 5x - 2$
D $y - 2x = 5$
E $y = -2(x - 5)$

4G

7 The equation of the graph shown is:

A $y = x + 1$
B $y = \frac{1}{2}x + 4$
C $y = 2x + 1$
D $y = -2x - 1$
E $y = \frac{1}{2}x + 2$



4H

8 The length of the line segment joining the points $A(1, 2)$ and $B(4, 6)$ is:

A 5
B $\frac{4}{3}$
C 7
D $\sqrt{5}$
E $\sqrt{2}$

4I

9 The gradient of a line perpendicular to $y = 4x - 7$ would be:

A $\frac{1}{4}$
B 4
C -4
D $-\frac{1}{4}$
E -7

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4K

10 The point of intersection of $y = 2x$ and $y = 6 - x$ is:

A (6, 2)
B (−1, −2)
C (6, 12)
D (2, 4)
E (3, 6)

Extended-response questions

1 Joe requires an electrician to come to his house to do some work. He is trying to choose between two electricians he has been recommended.

a The first electrician's cost, $\$C$, is given by $C = 80 + 40n$ where n is the number of hours the job takes.

i State the hourly rate this electrician charges and his initial fee for coming to the house.
ii Sketch a graph of C versus n for $0 \leqslant n \leqslant 8$.
iii What is the cost of a job that takes 2.5 hours?
iv If the job costs \$280, how many hours did it take?

b The second electrician charges a callout fee of \$65 to visit the house and then \$45 per hour.

i Give the equation for the cost, $\$C$, of a job that takes n hours.
ii Sketch the graph of part **b i** for $0 \leqslant n \leqslant 8$ on the same axes as the graph in part **a**.

c Determine the point of intersection of the two graphs.

d After how many hours does the first electrician become the cheaper option?

2 Abby has set up a small business making clay vases. Her production costs include a fixed weekly cost for equipment hire and a cost per vase for clay.
Abby has determined that the total cost of producing 7 vases in a week was \$146 and the total cost of producing 12 vases in a week was \$186.

a Find a linear rule relating the production cost, $\$C$, to the number of vases produced, v. Start by plotting the given pair of points with the number of vases on the x-axis.

b Use your rule to state:

i the initial cost of materials each week
ii the ongoing cost of production per vase.

c At a selling price of \$12 per vase Abby determines her weekly profit to be given by $P = 4v - 90$.
How many vases must she sell in order to make a profit?

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5

Length, area, surface area and volume

Maths in context: Measurement skills are essential to our civilization

Our lifestyles depend on experts accurately using measurement skills to design and construct our:

- houses containing rectangular prism rooms with surface areas painted, rectangular prism concrete foundations, rectangular house block’s perimeter fenced.
- vehicles’ circular wheels, circumferences fitted with tyres, arc shaped curves driven using steering mechanisms, circle sectors cleaned by windscreen wipers.
- water supply and sewage systems, crucial to health, using cylindrical pipes.
- farms with rectangular paddock perimeters fenced and areas growing food crops, cylindrical silos for grain storage, circular and rectangular irrigation areas.

Sport is for our health and cultural enjoyment, and measurement skills are essential for the construction of:

- cricket ovals and rectangular football fields.
- basketball and netball rectangular courts with centre circle, goal semi-circles and circular goal rings.
- running, cycling and ice speed-skating tracks following the outer perimeter of a rectangle with semicircles at each end.
- shot put, discus and javelin throwing circles with sector landing areas.
- swimming and diving pools containing rectangular or trapezoidal prism shaped volumes of water.

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Chapter contents

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- solves problems involving the surface area of right prisms and practical problems involving the area of composite shapes and solids (MA5-ARE-C-01)
- solves problems involving the volume of composite solids consisting of right prisms and cylinders (MA5-VOL-C-01).

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

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5A Length and perimeter CONSOLIDATING

Learning intentions
- To review the metric units of length and be able to convert between them
- To be able to find the perimeter of closed shapes

Past, present and future learning
- This section consolidates Stage 4 concepts which are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced
- You may be expected to recall formulas from memory in the HSC Examination

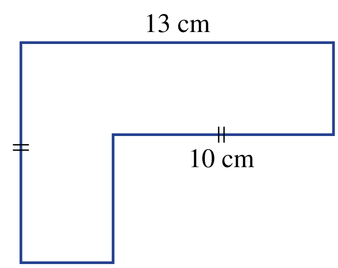
Length is at the foundation of measurement from which the concepts of perimeter, circumference, area and volume are developed. From the use of the royal cubit (distance from tip of middle finger to the elbow) used by the ancient Egyptians to the calculation of pi by computers, units of length have helped to create the world in which we live.

To determine the length of scaffolding required for a multistorey building, multiples of its perimeter are calculated.

Lesson starter: Not enough information?

All the angles at each vertex in this shape are 90° and the two given lengths are 10 cm and 13 cm.

- Do you think there is enough information to find the perimeter of the shape?
- If there is enough information, find the perimeter and discuss your method. If not, then list what information needs to be provided.



KEY IDEAS

- To convert between **metric units** of length, multiply or divide by the appropriate power of 10.

$$\text{km} \underset{\div 1000}{\overset{\times 1000}{\rightleftarrows}} \text{m} \underset{\div 100}{\overset{\times 100}{\rightleftarrows}} \text{cm} \underset{\div 10}{\overset{\times 10}{\rightleftarrows}} \text{mm}$$

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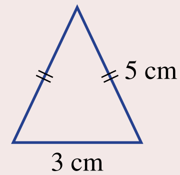
- **Perimeter** is the distance around the outside of a closed shape.
  - Sides with the same markings are of equal length.



$P = 2 \times 5 + 3$
$= 13$ cm

BUILDING UNDERSTANDING

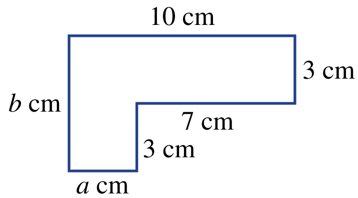
1 Convert the following length measurements to the units given in the brackets.

a 5 cm (mm) **b** 2.8 m (cm) **c** 521 mm (cm)
d 83.7 cm (m) **e** 4.6 km (m) **f** 2170 m (km)

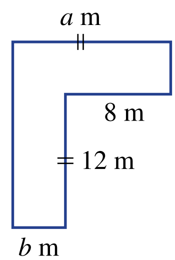
2 A steel beam is 8.25 m long and 22.5 mm wide. State the length and the breadth of the beam in centimetres.

3 Give the values of the pronumerals in these shapes. All angles at vertices are 90°.

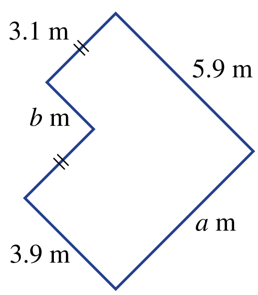
a



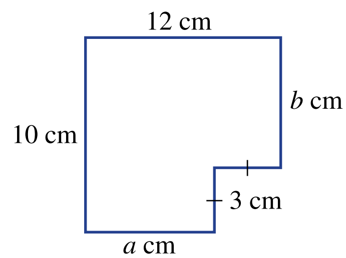
b



c



d



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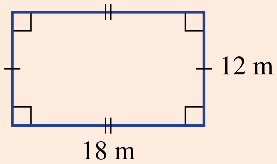
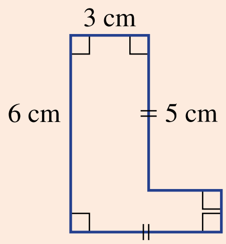
Example 1 Finding perimeters of simple shapes

Find the perimeter of each of the following shapes.

a



b



SOLUTION

a Perimeter $= 2 \times 12 + 2 \times 18$
$= 24 + 36$
$= 60$ m

b Perimeter $= (2 \times 5) + 6 + 3 + 2 + 1$
$= 22$ cm

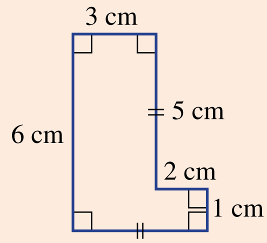
Alternative method:

Perimeter $= 2 \times 6 + 2 \times 5$
$= 22$ cm

EXPLANATION

There are two lengths of 12 m and two lengths of 18 m.

Missing sides are:
5 cm − 3 cm = 2 cm
6 cm − 5 cm = 1 cm

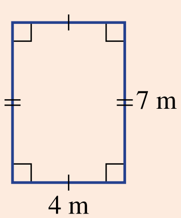


There are two opposite sides each of combined length 6 cm and two opposite sides each of combined length 5 cm.

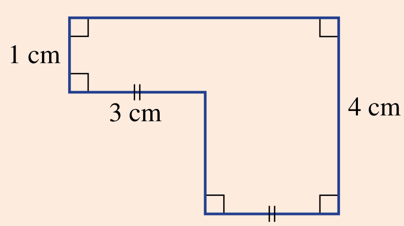
Now you try

Find the perimeter of each of the following shapes.

a



b



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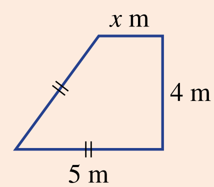
Example 2 Finding missing sides given a perimeter

Find the unknown side length in this shape with the given perimeter.



Perimeter = 16 m

SOLUTION

$$\begin{aligned} 2 \times 5 + 4 + x &= 16 \\ 14 + x &= 16 \\ x &= 2 \end{aligned}$$

$\therefore$ the missing length is 2 m.

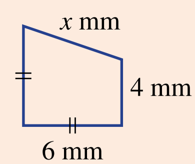
EXPLANATION

Add all the lengths and set equal to the given perimeter.

Solve for the unknown.

Now you try

Find the unknown side length in this shape with the given perimeter.



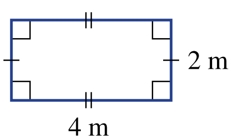
Perimeter = 23 mm

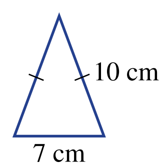
Exercise 5A

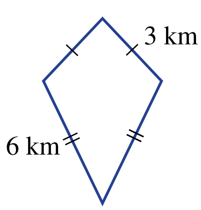
FLUENCY	1, 2(½), 3, 4	1–2(½), 3, 4, 5(½)	2(½), 3, 4, 5(½)

Example 1a

1 Find the perimeter of each of the following shapes.

a


b


c


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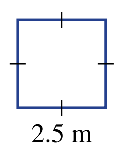

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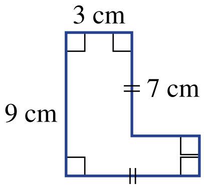
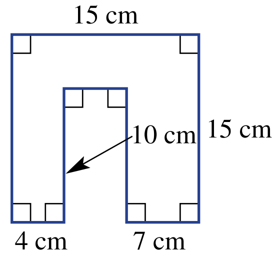
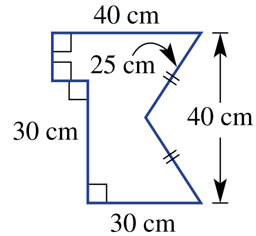
Example 1b

2 Find the perimeter of each of the following composite shapes.

a


b


c


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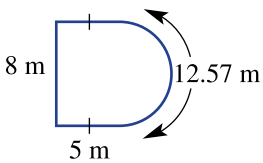
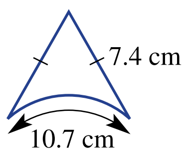
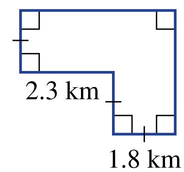
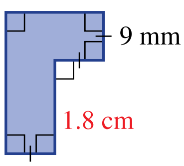
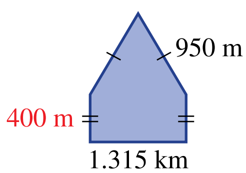


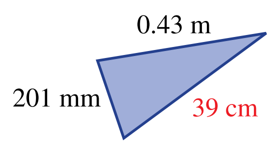
e


f


3 Find the perimeter of each of these shapes. You will need to convert the measurements to the same units. Give your answers in the units given in red.

a


b


c


4 Convert the following measurements to the units given in the brackets.

a 8 m (mm)

b 110 000 mm (m)

c 0.00001 km (cm)

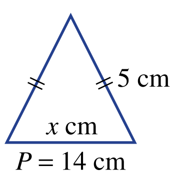
d 0.02 m (mm)

e 28 400 cm (km)

f 62 743 000 mm (km)

Example 2

5 Find the unknown side length in these shapes with the given perimeters.

a


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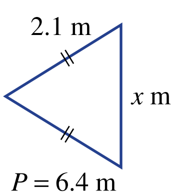

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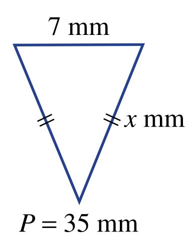
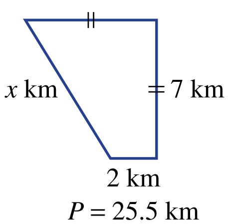
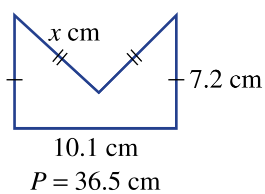


d


e


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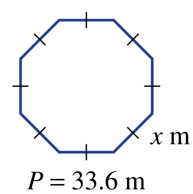

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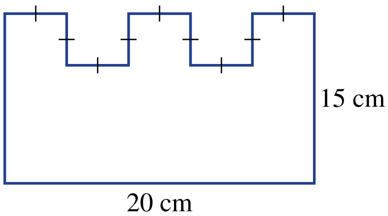
PROBLEM–SOLVING	6, 7	6–8	7–9

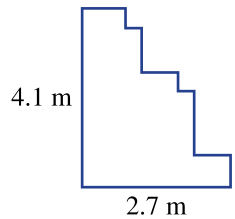
6 A lion enclosure is made up of five straight fence sections. Three sections are 20 m in length and the other two sections are 15.5 m and 32.5 m. Find the perimeter of the enclosure.

7 Find the perimeter of these shapes. Assume all angles are right angles.

a

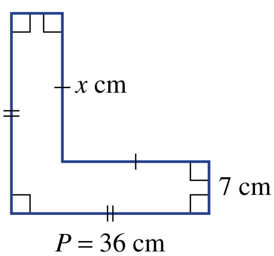


b

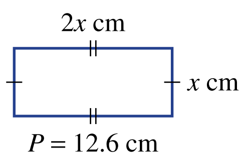


8 Find the value of x in these shapes with the given perimeter.

a



b



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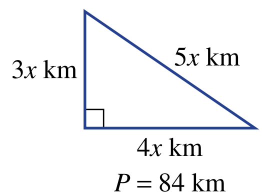
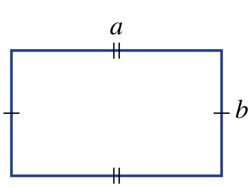


9 A photo 12 cm wide and 20 cm long is surrounded by a picture frame 3 cm wide. Find the outside perimeter of the framed photo.

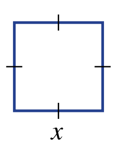
REASONING	10	10, 11	11–13

10 Give the rule for the perimeter of these shapes using the given pronumerals, e.g. $P = 3a + 2b$.

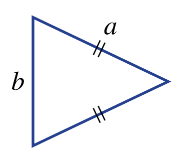
a



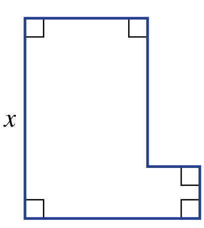
b



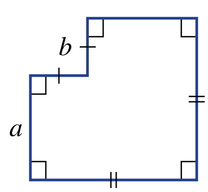
c



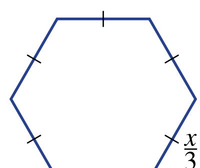
d



e



f



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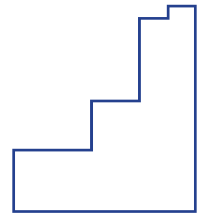
11 Explain why you do not need any more information to find the perimeter of this shape, assuming all angles are right angles.



12 A square vegetable patch has its side length incorrectly recorded as 2.6 m to one decimal place when it was really 2.4 m.

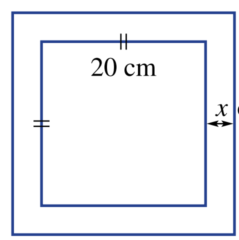
- **a** Calculate the percentage error for the recorded and actual measurement, correct to one decimal place.
- **b** Calculate the percentage error for the perimeter of the vegetable patch if the incorrect measurement is used. Round to one decimal place.
- **c** What do you notice about your answers to parts **a** and **b**?

13 A piece of string 1 m long is divided into parts according to the given ratios. Find the length of each part.

a 1 : 3　**b** 2 : 3　**c** 5 : 3　**d** 1 : 2 : 3 : 4

ENRICHMENT: Picture framing	–	–	14

14 A square picture of side length 20 cm is surrounded by a frame of breadth x cm.



- **a** Find the outside perimeter of the framed picture if:
  - **i** $x = 2$　**ii** $x = 3$　**iii** $x = 5$
- **b** Write a rule for the perimeter P of the framed picture in terms of x.
- **c** Use your rule to find the perimeter if:
  - **i** $x = 3.7$　**ii** $x = 7.05$
- **d** Use your rule to find the value of x if:
  - **i** the perimeter is 90 cm
  - **ii** the perimeter is 102 cm.
- **e** Is there a value of x for which the perimeter is 75 cm? Explain.

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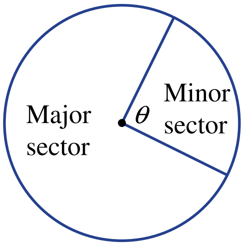
5B Circle circumference and perimeter of a sector CONSOLIDATING

Learning intentions
- To know the formula for the circumference of a circle
- To be able to calculate circumference of a circle and perimeter of semicircles and quadrants

Past, present and future learning
- This section consolidates Stage 4 concepts which are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced
- You may be expected to recall formulas from memory in the HSC Examination

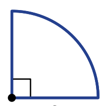
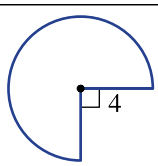
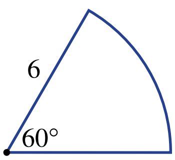
A portion of a circle enclosed by two radii and an arc is called a sector. The perimeter of a sector is made up of three components: two radii and the circular arc. Given an angle θ, it is possible to find the length of the arc using the rule for the circumference of a circle: $C = 2\pi r$ or $C = \pi d$.



Lesson starter: Perimeter of a sector

A sector is formed by dividing a circle with two radius cuts. The angle between the two radii determines the size of the sector. The perimeter will therefore depend on both the length of the radii and the angle between them.

- Complete this table to see if you can determine a rule for the perimeter of a sector. Remember that the circumference, C, of a circle is given by $C = 2\pi r$ where r is the radius length.

Shape	Fraction of full circle	Working and answer
	$\frac{90}{360} = \frac{1}{4}$	$P = 2 \times 3 + \frac{1}{4} \times 2\pi \times 3 \approx 10.71$
	$\frac{270}{360} =$	$P =$
		$P =$

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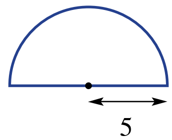
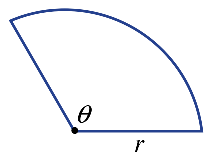
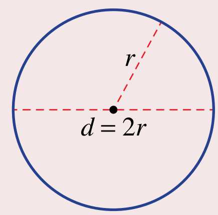
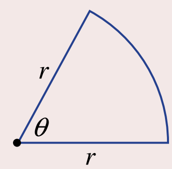
Shape	Fraction of full circle	Working and answer
		$P =$
		$P =$

KEY IDEAS

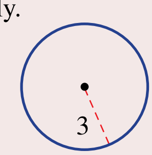


- **Circumference** of a circle $C = 2\pi r$ or $C = \pi d$.
  - $\frac{22}{7}$ or 3.14 can be used to approximate π or use technology for more precise calculations.
- A **sector** is a portion of a circle enclosed by two radii and an arc. Special sectors include:
  - a half circle, called a **semicircle**
  - a quarter circle, called a **quadrant**.
- The perimeter of a sector is given by $P = 2r + \frac{\theta}{360} \times 2\pi r$.



- The symbol for pi (π) can be used to write an answer exactly.

  For example: $C = 2\pi r$
  $= 2\pi \times 3$
  $= 6\pi$



BUILDING UNDERSTANDING

1 **a** What is the radius of a circle if its diameter is 5.6 cm?
b What is the diameter of a circle if its radius is 48 mm?

2 Simplify these numbers to give an exact answer. Do not evaluate with a calculator or round off. The first one is done for you.

a $2 \times 3 \times \pi = 6\pi$ **b** $6 \times 2\pi$ **c** $3 + \frac{1}{2} \times 4\pi$

d $2 \times 6 + \frac{1}{4} \times 12\pi$ **e** $2 \times 4 + \frac{90}{360} \times 2 \times \pi \times 4$ **f** $3 + \frac{270}{360} \times \pi$

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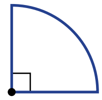
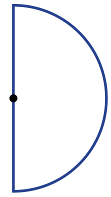
3 Determine the fraction of a circle shown in these sectors. Write the fraction in simplest form.

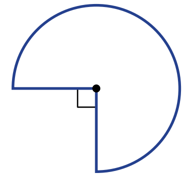
a



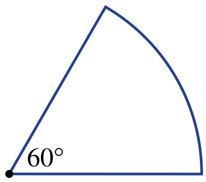
b



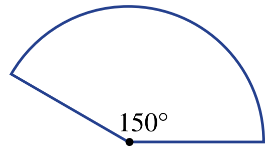
c



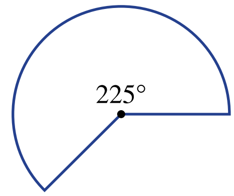
d



e



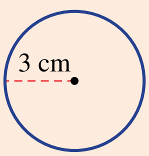
f



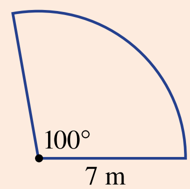
Example 3 Finding the circumference of a circle and the perimeter of a sector

Find the circumference of this circle and the perimeter of this sector correct to two decimal places.

a



b



SOLUTION

a
$$\begin{aligned} C &= 2\pi r \\ &= 2 \times \pi \times 3 \\ &= 6\pi \\ &= 18.85 \text{ cm (to 2 d.p.)} \end{aligned}$$

EXPLANATION

Use the formula $C = 2\pi r$ or $C = \pi d$ and substitute $r = 3$ (or $d = 6$).

6π would be the exact answer and 18.85 is the rounded answer. Give units.

SOLUTION

b
$$\begin{aligned} P &= 2r + \frac{\theta}{360} \times 2\pi r \\ &= 2 \times 7 + \frac{100}{360} \times 2 \times \pi \times 7 \\ &= 14 + \frac{35\pi}{9} \\ &= 26.22 \text{ m (to 2 d.p.)} \end{aligned}$$

EXPLANATION

Write the formula.

The fraction of the circle is $\frac{100}{360}\left(\text{or } \frac{5}{18}\right)$.

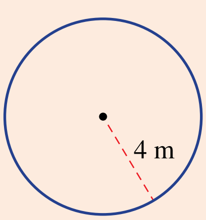
$14 + \frac{35\pi}{9}$ is the exact answer.

Use a calculator and round to two decimal places.

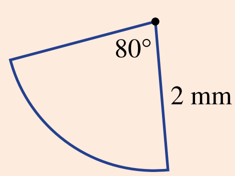
Now you try

Find the circumference of this circle and the perimeter of this sector correct to two decimal places.

a



b



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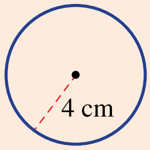
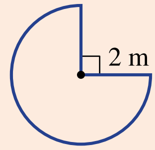
Example 4 Finding the exact perimeter of a shape

Give the exact circumference/perimeter of these shapes.

a



b



SOLUTION

a $C = 2\pi r$
$= 2 \times \pi \times 4$
$= 8\pi$ cm

b $P = 2 \times r + \frac{\theta}{360} \times 2\pi r$
$= 2 \times 2 + \frac{270}{360} \times 2\pi \times 2$
$= 4 + \frac{3}{4} \times 4\pi$
$= 4 + 3\pi$ m

EXPLANATION

a Write the formula with $r = 4$:
$2 \times \pi \times 4 = 2 \times 4 \times \pi$
Write the answer exactly in terms of π and include units.

b Use the perimeter of a sector formula.

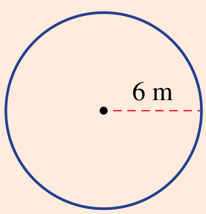
The angle inside the sector is 270° so the fraction is $\frac{270}{360} = \frac{3}{4}$.

$4 + 3\pi$ cannot be simplified further.

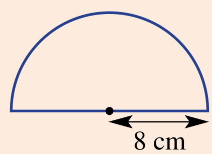
Now you try

Give the exact circumference/perimeter of these shapes.

a



b



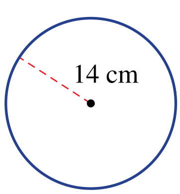
Exercise 5B

FLUENCY	1, 2–4(½)	1, 2–5(½)	2–5(½)

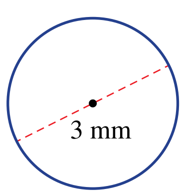
Example 3a

1 Find the circumference of these circles, correct to two decimal places. Use a calculator for the value of π.

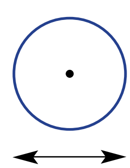
a



b



c



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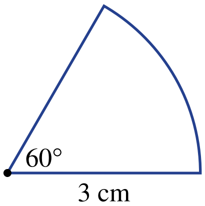
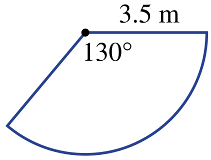
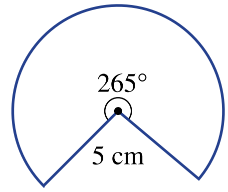
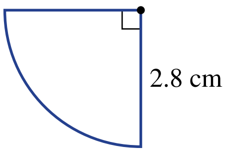
Example 3b

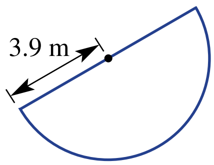
2 Find the perimeter of these sectors, correct to two decimal places.

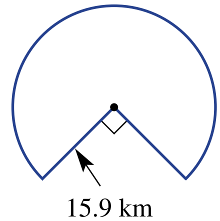
a


b


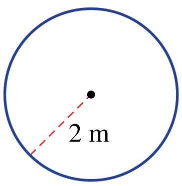
c


d (Quadrant)


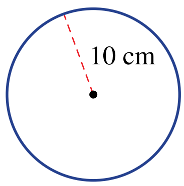
e (Semicircle)


f


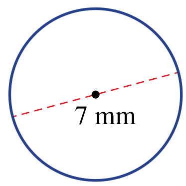
3 Find the circumference of these circles using the given approximation of π. Do not use a calculator.

a


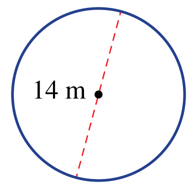
$\pi = 3.14$

b


$\pi = 3.14$

c


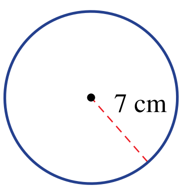
$\pi = \frac{22}{7}$

d


$\pi = \frac{22}{7}$

Example 4a

4 Give the exact circumference of these circles.

a


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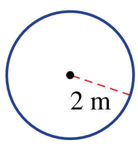

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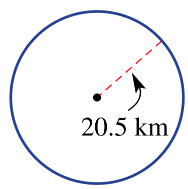

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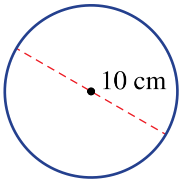

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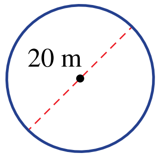

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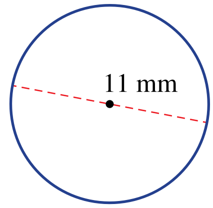
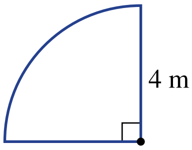


Example 4b

5 Give the exact perimeter of these sectors.

a


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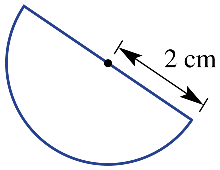

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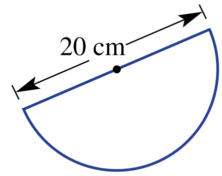

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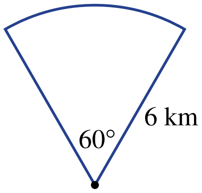

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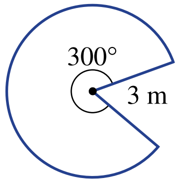

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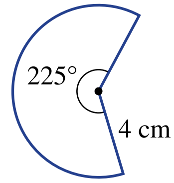

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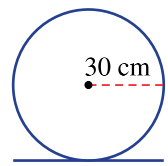
PROBLEM–SOLVING	6, 7	6–8	7–9

6 Find the distance around the outside of a circular pool of radius 4.5 m, correct to two decimal places.

7 Find the length of string required to surround the circular trunk of a tree that has a diameter of 1.3 m, correct to one decimal place.

8 The end of a cylinder has a radius of 5 cm. Find the circumference of the end of the cylinder, correct to two decimal places.

9 A wheel of radius 30 cm is rolled in a straight line.



a Find the circumference of the wheel correct to two decimal places.

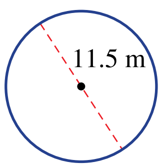
b How far, correct to two decimal places, has the wheel rolled after completing:

i 2 rotations? ii 10.5 rotations?

c Can you find how many rotations would be required to cover at least 1 km in length? Round to the nearest whole number.

REASONING	10(½)	10	10(½), 11

10 Give exact answers for the perimeter of these shapes.

a


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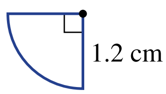

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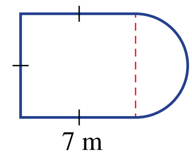

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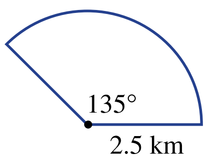
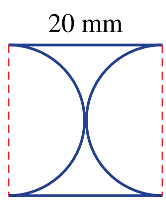


e


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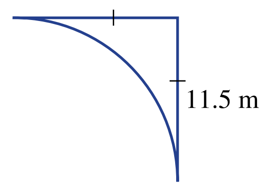

11 We know that the rule for the circumference of a circle is $C = 2\pi r$.

a Find a rule for r in terms of C.

b Find the radius of a circle to one decimal place if its circumference is:

i 10 cm ii 25 m

c Give the rule for the diameter of a circle in terms of its circumference C.

d After 1000 rotations a wheel has travelled 2.12 km. Find its diameter to the nearest centimetre.

ENRICHMENT: The ferris wheel	–	–	12

12 A large ferris wheel has a radius of 21 m. Round the answers to two decimal places for these questions.

a Find the distance a person will travel on one rotation of the wheel.

b A ride includes six rotations of the wheel. What distance is travelled in one ride?

c How many rotations would be required to ride a distance of:

i 500 m? ii 2 km?

d Another ferris wheel has a sign which reads, 'One ride of 10 rotations will cover 2 km'. What must be the diameter of the wheel?

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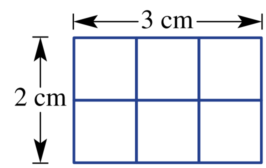
5C Area CONSOLIDATING

Learning intentions
- To be able to convert between metric units for area
- To know the formulas for the area of common shapes
- To be able to find the area of common shapes

Past, present and future learning
- This section consolidates Stage 4 concepts which are used in Stages 5 and 6
- Some of these questions are more challenging than those in our Stage 4 books
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced
- You may be expected to recall formulas from memory in the HSC Examination



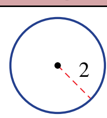
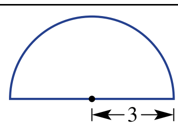
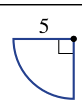
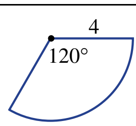
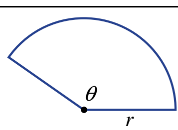
The number of square centimetres in this rectangle is 6; therefore the area is 6 cm^2.

A quicker way to find the number of squares is to note that there are two rows of three squares and hence the area is $2 \times 3 = 6$ cm^2. This leads to the formula $A = l \times b$ for the area of a rectangle.

For many common shapes, such as the parallelogram and trapezium, the rules for their area can be developed through consideration of simple rectangles and triangles. Shapes that involve circles or sectors rely on calculations involving pi (π).

Lesson starter: Formula for the area of a sector

We know that the area of a circle with radius r is given by the rule $A = \pi r^2$.
Complete this table of values to develop the rule for the area of a sector.

Shape	Fraction of full circle	Working and answer
	1	$A = \pi \times 2^2 \approx 12.57$ units^2
	$\frac{180}{360} =$	$A = \frac{1}{2} \times$
		$A =$
		$A =$
		$A =$

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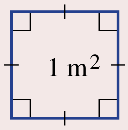
KEY IDEAS

- Conversion of area units

1 m = 100 cm



1 m = 100 cm

1 m^2 = 100^2 cm^2

$$\text{km}^2 \xrightarrow{\times 1000^2} \text{m}^2 \xrightarrow{\times 100^2} \text{cm}^2 \xrightarrow{\times 10^2} \text{mm}^2$$

$$\text{mm}^2 \xrightarrow{\div 10^2} \text{cm}^2 \xrightarrow{\div 100^2} \text{m}^2 \xrightarrow{\div 1000^2} \text{km}^2$$

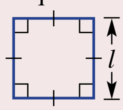
$10^2 = 10 \times 10 = 100$

$100^2 = 100 \times 100 = 10\,000$

$1000^2 = 1000 \times 1000 = 1\,000\,000$

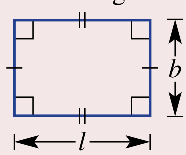
- The area of a two-dimensional shape is a measure of the space enclosed within its boundaries. Some common formulas are shown.

Square



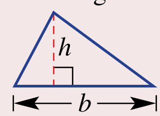
Area = l^2

Rectangle



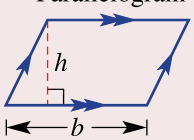
Area = lb

Triangle



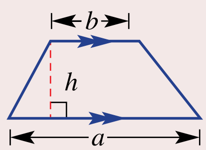
Area = $\frac{1}{2}bh$

Parallelogram



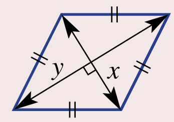
Area = bh

Trapezium



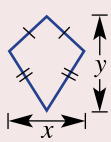
Area = $\frac{h}{2}(a + b)$

Rhombus



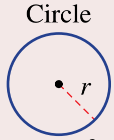
Area = $\frac{1}{2}xy$

Kite



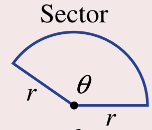
$A = \frac{1}{2}xy$

Circle



$A = \pi r^2$

Sector



$A = \frac{\theta}{360} \times \pi r^2$

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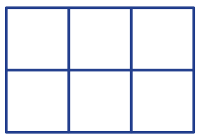
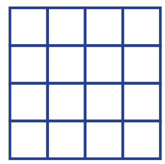
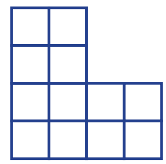
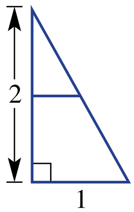
BUILDING UNDERSTANDING

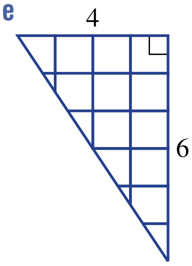
1 Count the number of squares to find the area of these shapes. Each square in each shape represents one square unit.

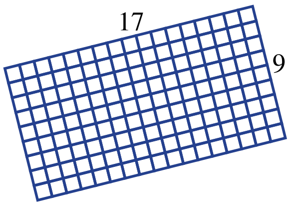
a 

b 

c 

d 

e 

f 

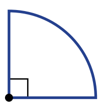
2 Name the shape that has the given area formula.

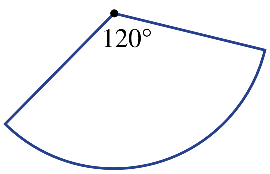
a $A = lb$ **b** $A = \pi r^2$ **c** $A = \frac{1}{2}xy$ (2 shapes)

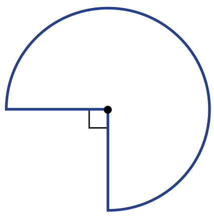
d $A = \frac{\theta}{360} \times \pi r^2$ **e** $A = \frac{1}{2}bh$ **f** $A = \frac{h}{2}(a + b)$

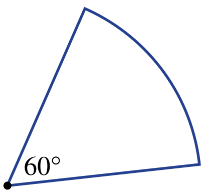
g $A = bh$ **h** $A = l^2$ **i** $A = \frac{1}{2}\pi r^2$

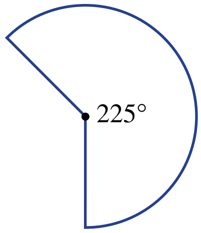
3 What fraction of a full circle is shown by these sectors? Simplify your fraction.

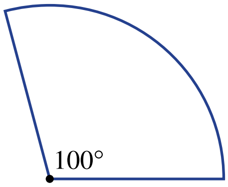
a 

b 

c 

d 

e 

f 

4 The shape below represents $3\,m^2$.

$1\,m^2$	$1\,m^2$	$1\,m^2$

a What is the length and breadth of the shape in cm?

b Hence, give the area of the shape in cm^2.

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Example 5 Converting units of area

Convert the following area measurements to the units given in the brackets.

a 859 mm^2 (cm^2) **b** 2.37 m^2 (cm^2)

SOLUTION

a $859\text{ mm}^2 = 859 \div 10^2\text{ cm}^2$
$= 8.59\text{ cm}^2$

b $2.37\text{ m}^2 = 2.37 \times 100^2\text{ cm}^2$
$= 23\,700\text{ cm}^2$

EXPLANATION

a $\text{cm}^2 \xleftarrow{\div 10^2 = 100} \text{mm}^2$ 859.

b $\text{m}^2 \xrightarrow{\times 100^2 = 10\,000} \text{cm}^2$ 2.3700

Now you try

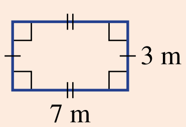
Convert the following area measurements to the units given in the brackets.

a 14 210 cm^2 (m^2) **b** 3.16 cm^2 (mm^2)

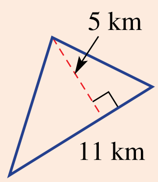
Example 6 Finding areas of rectangles, triangles and parallelograms

Find the area of each of the following plane figures.

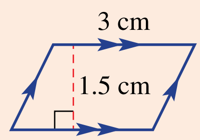
a



b



c



SOLUTION

a $\text{Area} = l \times b$
$= 7 \times 3$
$= 21\text{ m}^2$

b $\text{Area} = \frac{1}{2} \times b \times h$
$= \frac{1}{2} \times 11 \times 5$
$= 27.5\text{ km}^2$

c $\text{Area} = b \times h$
$= 3 \times 1.5$
$= 4.5\text{ cm}^2$

EXPLANATION

a Use the area formula for a rectangle.
Substitute $l = 7$ and $b = 3$.
Include the correct units.

b Use the area formula for a triangle.
Substitute $b = 11$ and $h = 5$.
Include the correct units.

c Use the area formula for a parallelogram.
Multiply the base length by the perpendicular height.

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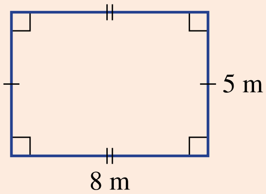
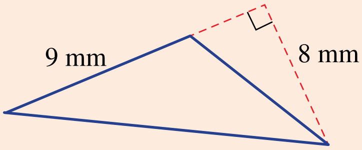
Now you try

Find the area of each of the following plane figures.

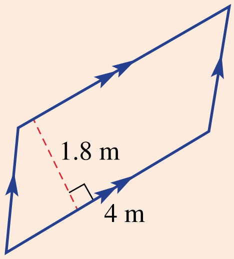
a



b



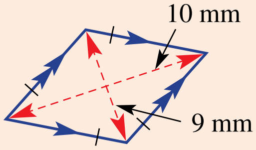
c



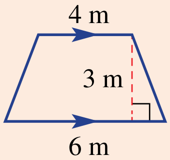
Example 7 Finding areas of rhombuses and trapeziums

Find the area of each of the following plane figures.

a



b



SOLUTION

a Area $= \frac{1}{2} \times x \times y$

$= \frac{1}{2} \times 10 \times 9$

$= 45\,\text{mm}^2$

b Area $= \frac{h}{2}(a + b)$

$= \frac{3}{2}(4 + 6)$

$= \frac{3}{2} \times 10$

$= 15\ \text{m}^2$

EXPLANATION

Use the area formula for a rhombus.

Substitute $x = 10$ and $y = 9$.

Include the correct units.

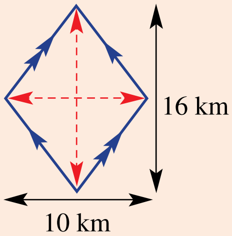
Use the area formula for a trapezium.

Substitute $a = 4$, $b = 6$ and $h = 3$ and evaluate.

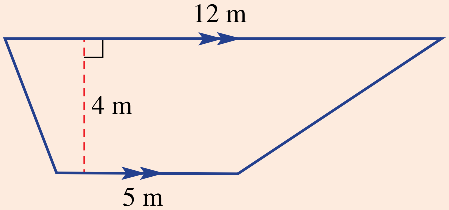
Now you try

Find the area of each of the following plane figures.

a



b



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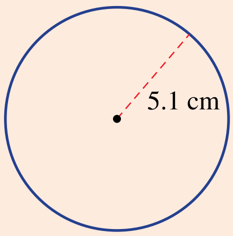
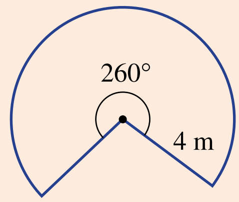
Example 8 Finding areas of circles and sectors

Find the area of this circle and sector correct to two decimal places.

a



b



SOLUTION

a $A = \pi r^2$

$= \pi \times (5.1)^2$

$= 81.71\ \text{cm}^2$ (to 2 d.p.)

b $A = \frac{\theta}{360} \times \pi r^2$

$= \frac{260}{360} \times \pi \times 4^2$

$= \frac{13}{18} \times \pi \times 16$

$= 36.30\ \text{m}^2$ (to 2 d.p.)

EXPLANATION

Write the rule and substitute $r = 5.1$.
81.7128… rounds to 81.71 as the third decimal place is 2.

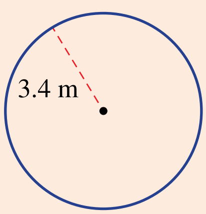
Use the sector formula.

The fraction of the full circle is $\frac{260}{360} = \frac{13}{18}$, so multiply this by πr^2 to get the sector area.
$\frac{104\pi}{9}\ \text{m}^2$ would be the exact answer.

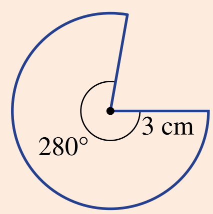
Now you try

Find the area of this circle and sector correct to two decimal places.

a



b



Exercise 5C

FLUENCY	1, 2–5(½), 6, 7(½)	2–5(½), 6, 7(½)	2–5(⅓), 6, 7(⅓)

1 Convert the following area measurements to the units given in the brackets.

Example 5a

a **i** $236\ \text{mm}^2$ (cm^2)

ii $48\,000\ \text{cm}^2$ (m^2)

Example 5b

b **i** $4.16\ \text{m}^2$ (cm^2)

ii $3.5\ \text{cm}^2$ (mm^2)

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Example 5

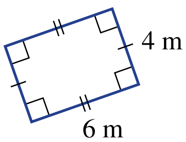
2 Convert the following area measurements to the units given in the brackets.

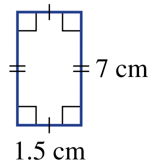
a 2 cm^2 (mm^2) **b** 500 mm^2 (cm^2) **c** 2.1 m^2 (cm^2)

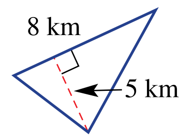
d 210 000 cm^2 (m^2) **e** 0.001 km^2 (m^2) **f** 3 200 000 m^2 (km^2)

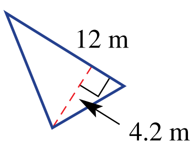
Example 6

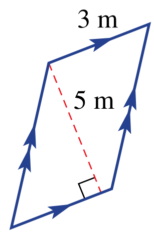
3 Find the area of each of the following plane figures.

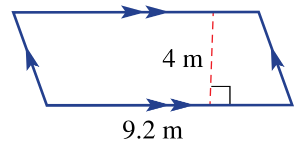
a


b


c


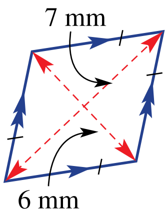
d


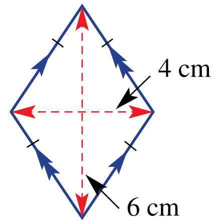
e


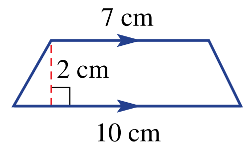
f


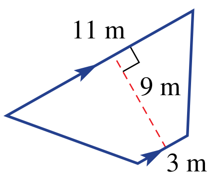
Example 7

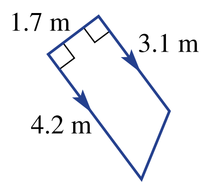
4 Find the area of each of the following plane figures.

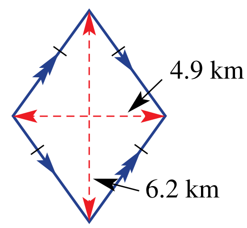
a


b


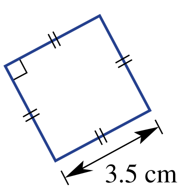
c


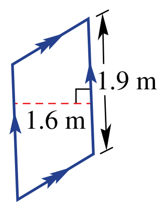
d


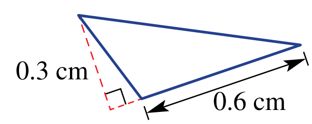
e


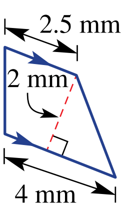
f


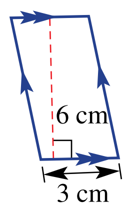
5 Find the area of each of the following mixed plane figures.

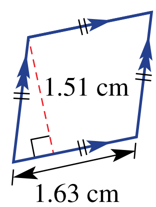
a


b


c


d


e


f


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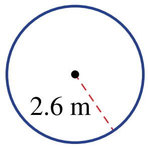
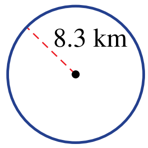
Example 8a

6 Find the area of these circles using the given value for pi (π). Round to two decimal places.

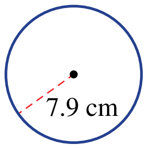
a $\pi = 3.14$



b $\pi = \frac{22}{7}$



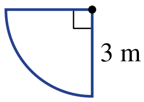
c π (from calculator)



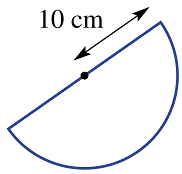
Example 8b

7 Find the area of these sectors rounding to two decimal places.

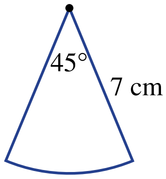
a



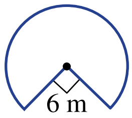
b



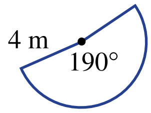
c



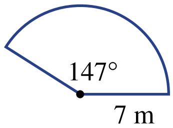
d



e



f



PROBLEM–SOLVING	8–10	8–11	8, 11, 12

8 Convert the following measurements to the units given in the brackets.

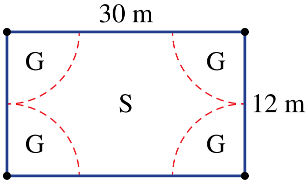
a $1.5\ \text{km}^2$ (cm^2) **b** $0.000005\ \text{m}^2$ (mm^2) **c** $75\,000\ \text{mm}^2$ (m^2)

9 A valuer tells you that your piece of land has an area of one-half a square kilometre ($0.5\ \text{km}^2$). How many square metres (m^2) do you own?

10 An old picture frame that was once square now leans to one side to form a rhombus. If the distances between pairs of opposite corners are 85 cm and 1.2 m, find the area enclosed within the frame in m^2.

11 A pizza shop is considering increasing the diameter of its family pizza tray from 32 cm to 34 cm. Find the percentage increase in area, correct to two decimal places.

12 A tennis court area is illuminated by four corner lights. The illumination of the sector area close to each light is considered to be good (G) while the remaining area is considered to be lit satisfactorily (S).
What percentage of the area is considered 'good'? Round to the nearest per cent.



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REASONING	13	13	14, 15

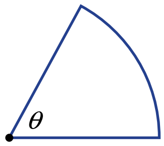
13 The rule for the area of a circle is given by $A = \pi r^2$.

a Rearrange this rule to find a rule for r in terms of A.

b Find the radius of a circle with the given areas. Round to one decimal place.

i 5 cm^2 **ii** 6.9 m^2 **iii** 20 km^2

14 A sector has a radius of 3 m.



a Find the angle θ, correct to the nearest degree, if its area is:

i 5 m^2 **ii** 25 m^2

b Explain why the area of the sector could not be 30 m^2.

15 The side length of a square is recorded as 6.4 cm.

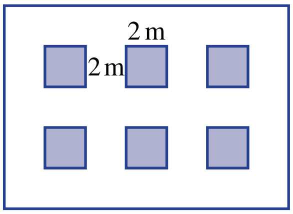
a Give the upper and lower limits of accuracy for this measurement.

b Find the upper and lower limits for this square's perimeter.

c Find the upper and lower limits for the area of this square, to one decimal place.

ENRICHMENT: Windows	–	–	16, 17

16 Six square windows of side length 2 m are to be placed into a 12 m wide by 8.5 m high wall as shown. The windows are to be positioned so that the vertical spacings between the windows and the wall edges are equal. Similarly, the horizontal spacings are also equal.



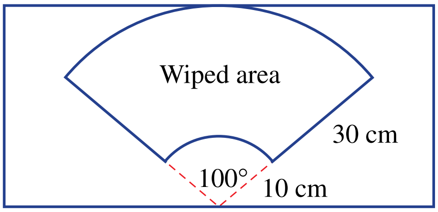
a **i** Find the horizontal distance between the windows.

ii Find the vertical distance between the windows.

b Find the area of the wall not including the window spaces.

c If the wall included 3 rows of 4 windows (instead of 2 rows of 3), investigate if it would be possible to space all the windows so that the horizontal and vertical spacings are uniform (although not necessarily equal to each other).

17 A rectangular window is wiped by a wiper blade forming the given sector shape.
What percentage area is cleaned by the wiper blade? Round to one decimal place.



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5D Perimeter and area of composite shapes

Learning intentions

- To be able to recognise the basic shapes that make up a composite shape
- To be able to find the perimeter and area of composite shapes

Past, present and future learning

- This section addresses the Stage 5 Core Topic called *Area and surface area A*
- These concepts will be revised, briefly, in Chapter 4 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced
- You may be expected to recall formulas from memory in the HSC Examination

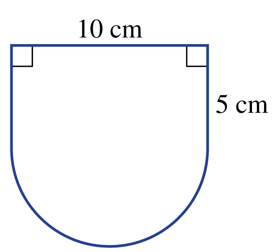
Composite shapes can be thought of as a combination of simpler shapes such as triangles and rectangles. Finding perimeters and areas of such shapes is a matter of identifying the more basic shapes they consist of and combining any calculations in an organised fashion.

For this high-rise office building in Oslo, Norway, the architect has designed windows with triangular, trapezoidal and composite shapes.

Lesson starter: Incorrect layout

Three students write their solution to finding the area of this shape on the board.

Chris	Matt	Moira
$A = l \times b$ $= 50 + \frac{1}{2}\pi r^2$ $= \frac{1}{2}\pi \times 5^2$ $= 39.27 + 50$ $= 89.27 \text{ cm}^2$	$A = \frac{1}{2}\pi \times 5^2$ $= 39.27 + 10 \times 5$ $= 89.27 \text{ cm}^2$	$A = l \times b + \frac{1}{2}\pi r^2$ $= 10 \times 5 + \frac{1}{2}\pi \times 5^2$ $= 89.27 \text{ cm}^2$



- All three students have the correct answer but only one student receives full marks. Who is it?
- Explain what is wrong with the layout of the other two solutions.

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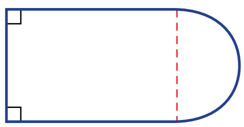
KEY IDEAS

- **Composite shapes** are made up of more than one basic shape.
- Addition and/or subtraction can be used to find areas and perimeters of composite shapes.
- The layout of the relevant mathematical working needs to make sense so that the reader of your work understands each step.

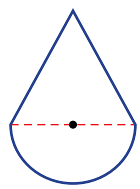
BUILDING UNDERSTANDING

1 Name the two different shapes that make up these composite shapes, e.g. square and semicircle.

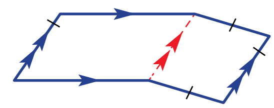
a



b

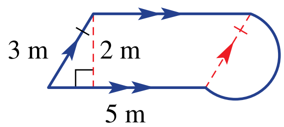


c



2 State the missing components to complete the working to find the perimeter and area of these composite shapes.

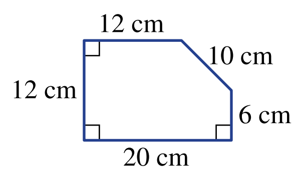
a



$$\begin{aligned} P &= 2 \times \underline{\qquad} + 3 + \tfrac{1}{2} \times 2\pi r \\ &= \underline{\qquad} + 3 + \underline{\qquad} \\ &= \underline{\qquad}\ \text{m} \end{aligned}$$

$$\begin{aligned} A &= bh + \tfrac{1}{2}\underline{\qquad} \\ &= 5 \times \underline{\qquad} + \tfrac{1}{2} \times \underline{\qquad} \\ &= \underline{\qquad} + \underline{\qquad} \\ &= \underline{\qquad}\ \text{m}^2 \end{aligned}$$

b



$$\begin{aligned} P &= 20 + 12 + \underline{\qquad} + \underline{\qquad} + \underline{\qquad} \\ &= \underline{\qquad}\ \text{cm} \end{aligned}$$

$$\begin{aligned} A &= lb - \tfrac{1}{2}bh \\ &= 12 \times \underline{\quad} - \tfrac{1}{2} \times 8 \times \underline{\quad} \\ &= \underline{\quad} - \underline{\quad} \\ &= \underline{\quad}\ \text{cm}^2 \end{aligned}$$

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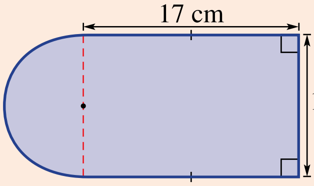
Example 9 Finding perimeters and areas of composite shapes

Find the perimeter and area of this composite shape, rounding answers to two decimal places.



SOLUTION

$$\begin{aligned} P &= 2 \times l + b + \frac{1}{2} \times 2\pi r \\ &= 2 \times 17 + 14 + \frac{1}{2} \times 2\pi \times 7 \\ &= 34 + 14 + \pi \times 7 \\ &= 69.99 \text{ cm (to 2 d.p.)} \end{aligned}$$

$$\begin{aligned} A &= l \times b + \frac{1}{2}\pi r^2 \\ &= 17 \times 14 + \frac{1}{2} \times \pi \times 7^2 \\ &= 238 + \frac{1}{2} \times \pi \times 49 \\ &= 314.97 \text{ cm}^2 \text{ (to 2 d.p.)} \end{aligned}$$

EXPLANATION

3 straight sides ⊐ + semicircle arc ⌒

Substitute $l = 17$, $b = 14$ and $r = 7$.

Simplify.

Calculate and round to two decimal places.

Area of rectangle ▭ + area of semicircle ⌓

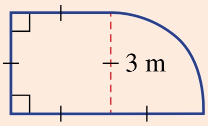
Substitute $l = 17$, $b = 14$ and $r = 7$.

Simplify.

Calculate and round to two decimal places.

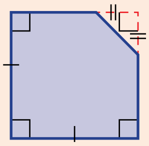
Now you try

Find the perimeter and area of this composite shape, rounding answers to two decimal places.



Example 10 Finding an area using subtraction

Find the shaded area of this composite shape.



SOLUTION

$$\begin{aligned} \text{Shaded area} &= l^2 - \frac{1}{2}bh \\ &= 6 \times 6 - \frac{1}{2} \times 2 \times 2 \\ &= 36 - 2 \\ &= 34 \text{ cm}^2 \end{aligned}$$

EXPLANATION

Use the full outer square and subtract the triangle area in the top right corner.

Include units.

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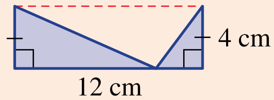
Now you try

Find the shaded area of this composite shape.

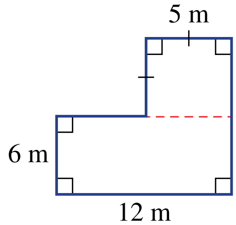


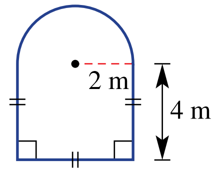
Exercise 5D

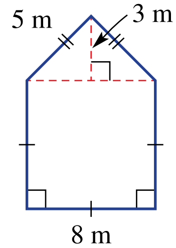
FLUENCY | 1–3(½) | 1–4(½) | 1–4(½)

Example 9

1 Find the perimeter and the area of each of these simple composite shapes, rounding answers to two decimal places where necessary.

a


b


c


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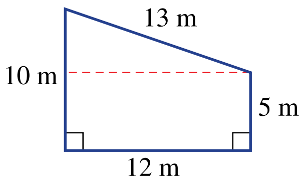

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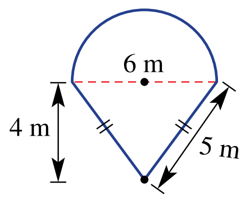

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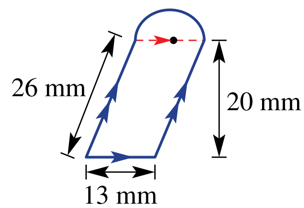
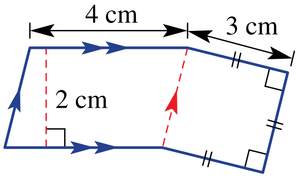


2 Find the area of the following composite shapes.

a


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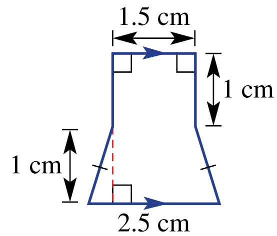

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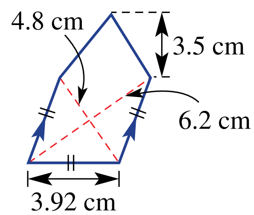

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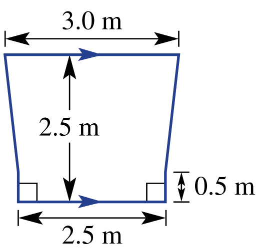

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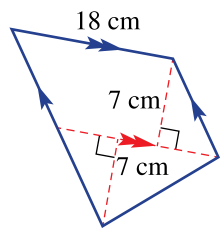

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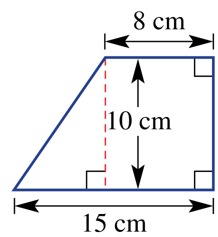

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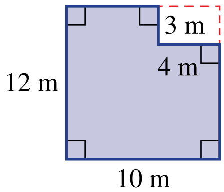
Example 10

3 Find the shaded area of each of these composite shapes.

a



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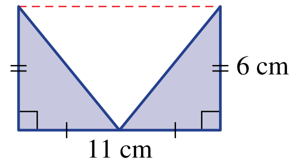

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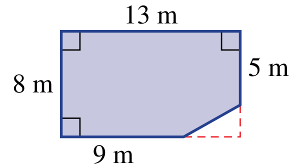
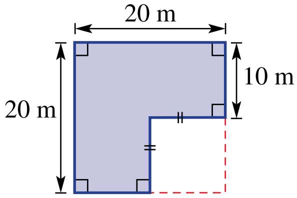
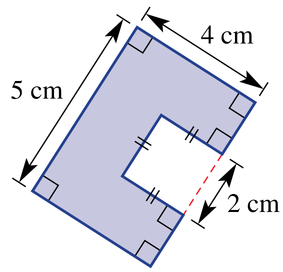


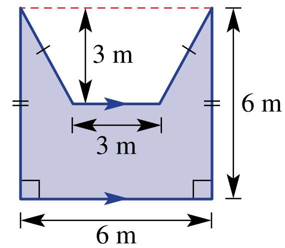
d



e

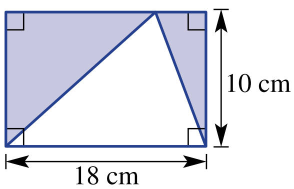


f



4 Find the area of the shaded region of the following shapes. Round to one decimal place where necessary.

a



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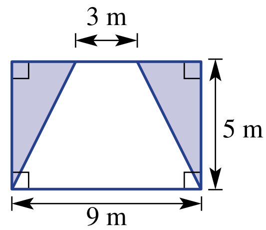

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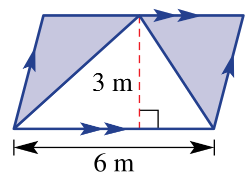

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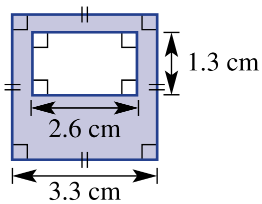

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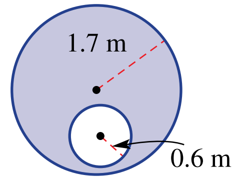

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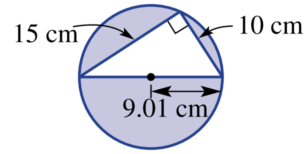
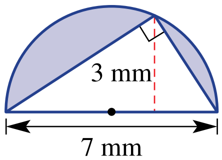
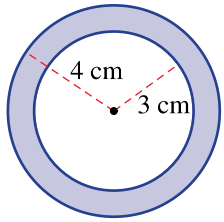


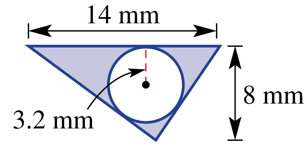
g



h



i



PROBLEM–SOLVING	5, 6	6, 7(½), 8	6–9

5 An area of lawn is made up of a rectangle measuring 10 m by 15 m and a semicircle of radius 5 m. Find the total area of lawn, correct to two decimal places.

6 Twenty circular pieces of pastry, each of diameter 4 cm, are cut from a rectangular sheet of pastry 20 cm long and 16 cm wide. What is the area, correct to two decimal places, of pastry remaining after the 20 pieces are removed?

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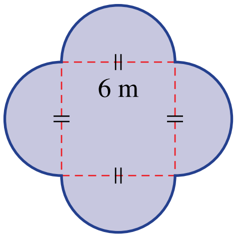
7 Find the perimeter and the area of the following composite shapes, correct to two decimal places where necessary.

a


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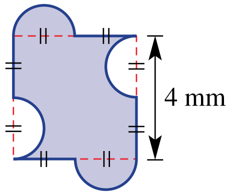

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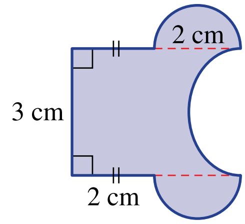

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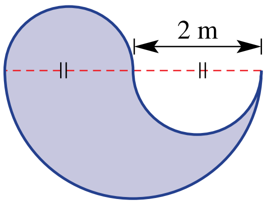

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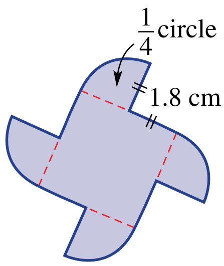

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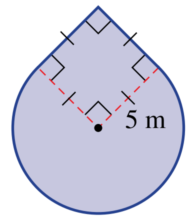
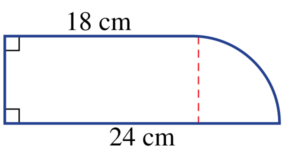


8 These shapes include sectors. Find their area, correct to one decimal place.

a


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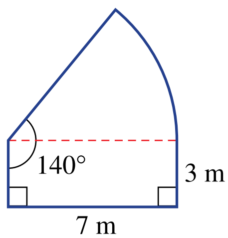

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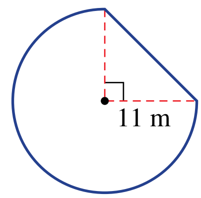
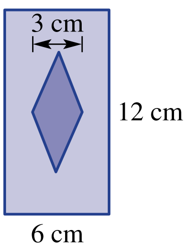


9 A new car manufacturer is designing a logo. It is in the shape of a diamond inside a rectangle. The diamond is to have a horizontal breadth of 3 cm and an area equal to one-sixth of the area of the rectangle. Find the required height of the diamond.



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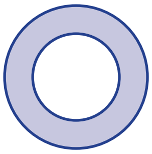
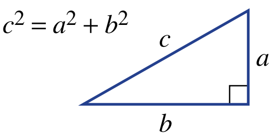
REASONING	10(½)	10, 11	10(½), 11, 12

10 Using exact values (e.g. $10 + 4\pi$), find the area of the shapes given in Question **7**.

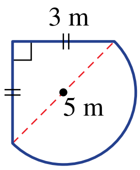
11 A circle of radius 10 cm has a hole cut out of its centre to form a ring. Find the radius of the hole if the remaining area is 50% of the original area. Round to one decimal place.



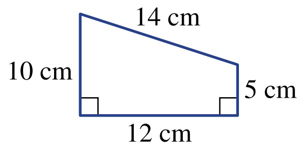
12 Use Pythagoras' theorem (illustrated in this diagram) to help explain why these composite shapes include incorrect information.



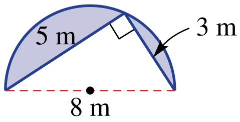
a



b

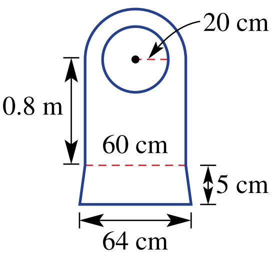


c

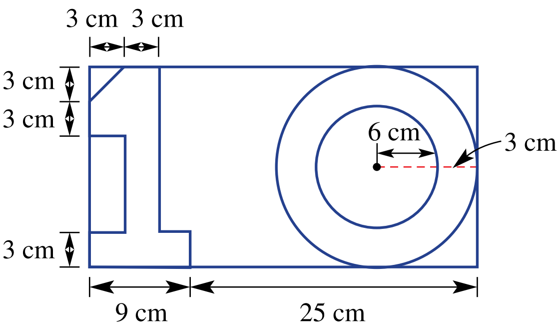


ENRICHMENT: Construction cut-outs	–	–	13, 14

13 The front of a grandfather clock consists of a timber board with dimensions as shown. A circle of radius 20 cm is cut from the board to form the clock face. Find the remaining area of the timber board correct to one decimal place.



14 The number 10 is cut from a rectangular piece of paper. The dimensions of the design are shown below.



a Find the length and breadth of the rectangular piece of paper.

b Find the sum of the areas of the two cut-out digits, 1 and 0, correct to one decimal place.

c Find the area of paper remaining after the digits have been removed (include the centre of the '0' in your answer) and round to one decimal place.

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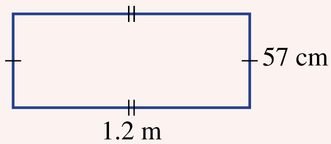
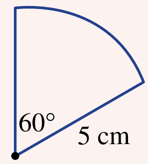
5A/B

1 Find the perimeter of each of the following shapes. Round to one decimal place in part **b**.

a



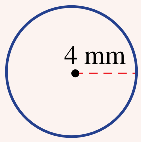
b



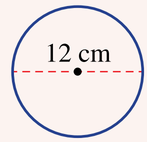
5B/C

2 Find, correct to one decimal place, the circumference and area of these circles.

a



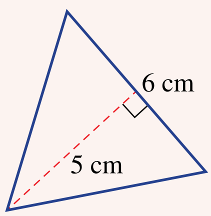
b



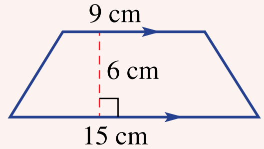
5C

3 Find the exact area of the following.

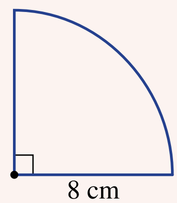
a



b



c



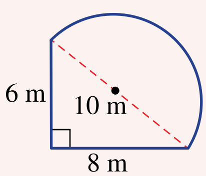
5C

4 Convert 85 000 mm^2 to cm^2.

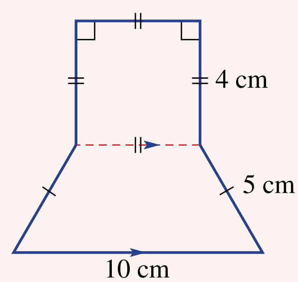
5D

5 Find the perimeter and area of these composite shapes, rounding answers to two decimal places in part **a**. (*Hint*: Use Pythagoras' theorem in part **b**.)

a

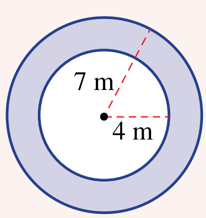


b



5D

6 Find, correct to two decimal places, the area of the shaded region.



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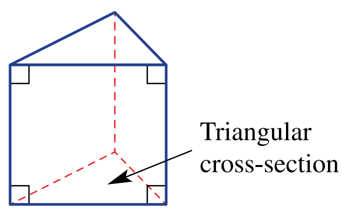
5E Surface area of prisms and pyramids

Learning intentions
- To understand how the surface area of a solid can be represented using a net
- To be able to calculate the surface area of prisms and pyramids

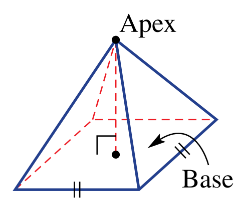
Past, present and future learning
- This section addresses the Stage 5 Path Topic called *Area and surface area B*
- These concepts will be revised, briefly, in Chapter 4 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced
- You may be expected to recall formulas from memory in the HSC Examination

Three-dimensional objects or solids have outside surfaces that together form the surface area. Nets are very helpful for determining the number and shape of the surfaces of a three-dimensional object.

For this section we will deal with right prisms and pyramids. A right prism has a uniform cross-section with two identical ends and the remaining sides are rectangles. A right pyramid has its apex sitting above the centre of its base.



Right triangular prism

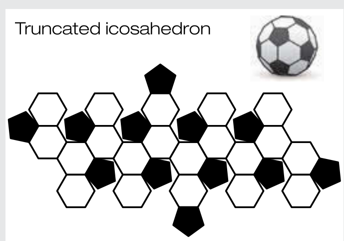


Right square-based pyramid

Lesson starter: Drawing prisms and pyramids

Prisms are named by the shape of their cross-section and pyramids by the shape of their base.

- Try to draw as many different right prisms and pyramids as you can.
- Describe the different kinds of shapes that make up the surface of your solids.
- Which solids are the most difficult to draw and why?



An icosahedron has 20 equilateral triangle faces. If its 12 vertices are truncated, i.e. sliced off, it then has 20 hexagon and 12 pentagon faces. These faces form the surface area net of a soccer ball.

KEY IDEAS

- The **surface area** (A) of a solid is the sum of the areas of all the surfaces.
- A **net** is a two-dimensional illustration of all the surfaces of a solid.
- A right **prism** is a solid with a uniform cross-section with two identical ends and the remaining sides are rectangles.
  - They are named by the shape of their cross-section.

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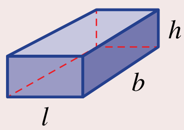
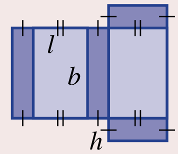
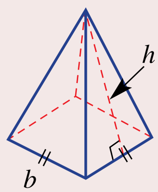
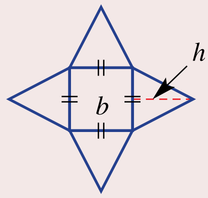
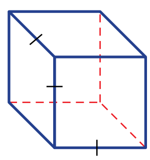
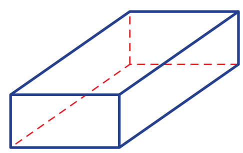
- The nets for a **rectangular prism** (**cuboid**) and square-based **pyramid** are shown here.

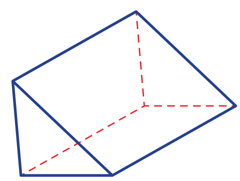
Solid	Net	A
Rectangular prism 		$A = 2 \times \text{top} + 2 \times \text{side} + 2 \times \text{end}$ $A = 2(lb + lh + hb)$
Square-based pyramid 		$A = \text{base} + 4 \times \text{side}$ $A = b^2 + 4\left(\frac{1}{2}bh\right)$ $= b^2 + 2bh$

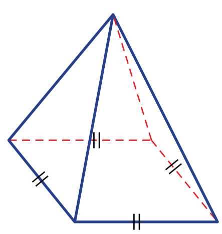
BUILDING UNDERSTANDING

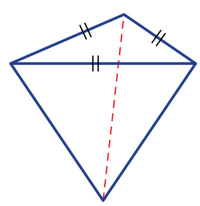
1 Sketch a suitable net for these prisms and pyramids and name each solid.

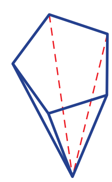
a 

b 

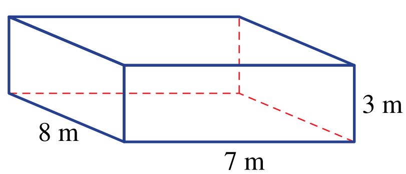
c 

d 

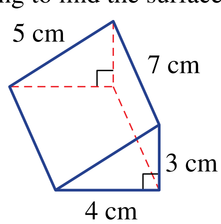
e 

f 

2 State the missing components to complete the working to find the surface area of these solids.

a 

$$\begin{aligned} A &= 2 \times 8 \times 7 + 2 \times 8 \times \_\_\_ \\ &\quad + 2 \times \_\_\_ \times \_\_\_ \\ &= \_\_\_ + \_\_\_ + \_\_\_ \\ &= \_\_\_ \text{ m}^2 \end{aligned}$$

b 

$$\begin{aligned} A &= 2 \times \frac{1}{2} \times 4 \times \_\_\_\_ + 5 \times 7 \\ &\quad + 4 \times \_\_\_\_ + \_\_\_\_ \times \_\_\_\_ \\ &= \_\_\_\_ + \_\_\_\_ + \_\_\_\_ + \_\_\_\_ \\ &= \_\_\_\_ \text{ cm}^2 \end{aligned}$$

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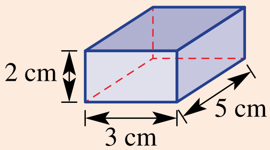
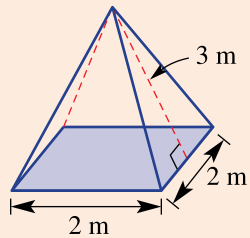
Example 11 Finding a surface area

Find the surface area of this right rectangular prism and this right square-based pyramid.

a



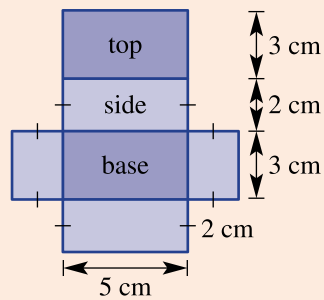
b



SOLUTION

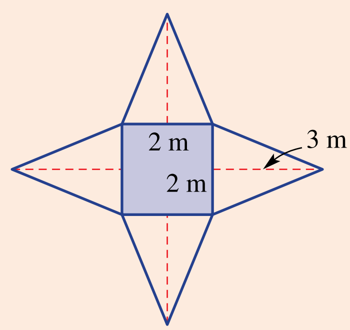
a $A = 2 \times \text{top} + 2 \times \text{side} + 2 \times \text{end}$
$= 2 \times (5 \times 3) + 2 \times (5 \times 2) + 2 \times (2 \times 3)$
$= 30 + 20 + 12$
$= 62\ \text{cm}^2$

EXPLANATION



b $A = \text{base} + 4 \times \text{side}$
$= 2 \times 2 + 4 \times \frac{1}{2} \times 2 \times 3$
$= 4 + 12$
$= 16\ \text{m}^2$

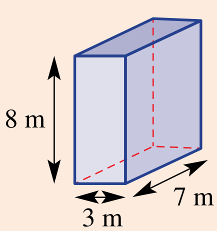
Square base + 4 identical triangles



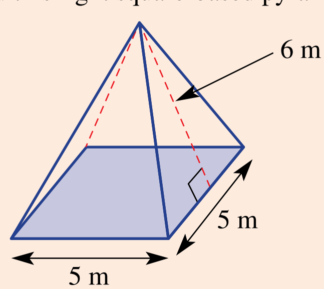
Now you try

Find the surface area of this right rectangular prism and this right square-based pyramid.

a



b



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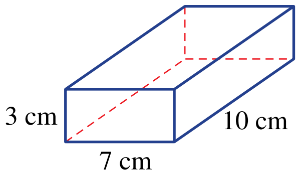
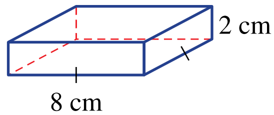
Exercise 5E

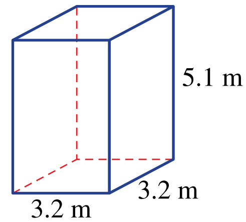
FLUENCY	1, 2, 3(½)	1, 2, 3(½), 4	1–2(⅓), 3(½), 4

Example 11a

1 Find the surface area of the following right rectangular prisms. Draw a net of the solid to help you.

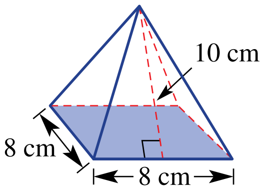
a


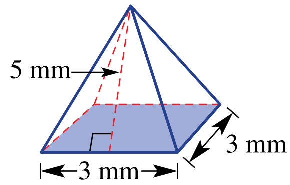
b


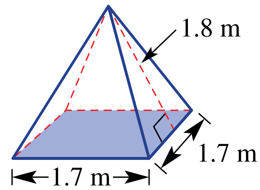
c


Example 11b

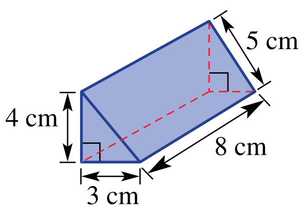
2 Find the surface area of each of these right square-based pyramids. Draw a net to help you.

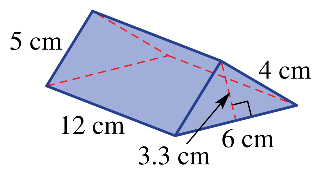
a


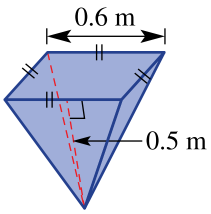
b


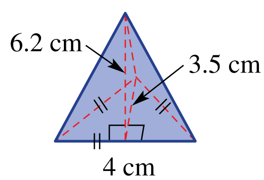
c


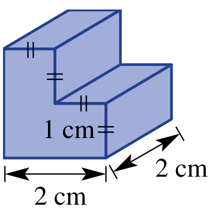
3 Find the surface area of each of the following solid objects.

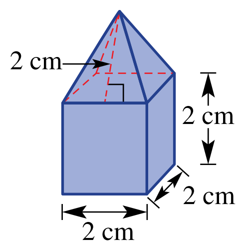
a


b


c


d


e


f


4 Find the surface area of a cube of side length 1 metre.

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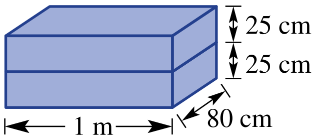
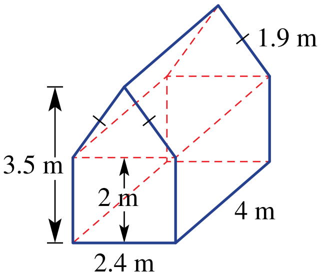
PROBLEM–SOLVING	5, 6	6, 7	7, 8

5 A rectangular box is to be covered in material. How much is required to cover the entire box if it has the dimensions 1.3 m, 1.5 m and 1.9 m?

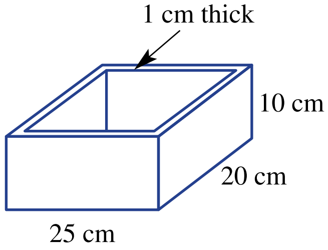
6 Two wooden boxes, both with dimensions 80 cm, 1 m and 25 cm, are placed on the ground, one on top of the other as shown. The entire outside surface is then painted. Find the area of the painted surface.



7 The outside four walls and roof of a barn (shown) are to be painted.
 a Find the surface area of the barn, not including the floor.
 b If 1 litre of paint covers 10 m^2, find how many litres are required to complete the job.



8 An open top rectangular box 20 cm wide, 25 cm long and 10 cm high is made from wood 1 cm thick. Find the surface area:
 a outside the box (do not include the top edge)
 b inside the box (do not include the top edge).



REASONING	9	9	9, 10

9 Draw the stack of 1 cm cube blocks that gives the minimum outside surface area and state this surface area if there are:
 a 2 blocks
 b 4 blocks
 c 8 blocks

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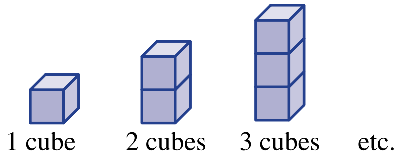
10 Cubes of side length one unit are stacked as shown.



a Complete this table.

Number of cubes (n)	1	2	3	4	5	6	7	8	9
Surface area (S)									

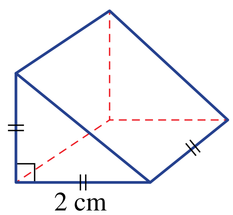
b Can you find the rule for the surface area (S) for n cubes stacked in this way? Write down the rule for S in terms of n.

c Investigate other ways of stacking cubes and look for rules for surface area in terms of n, the number of cubes.

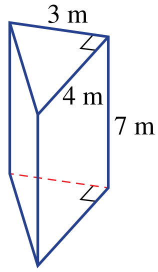
ENRICHMENT: Pythagoras required	–	–	11

11 For prisms and pyramids involving triangles, Pythagoras' theorem ($c^2 = a^2 + b^2$) can be used. Apply the theorem to help find the surface area of these solids. Round to one decimal place.

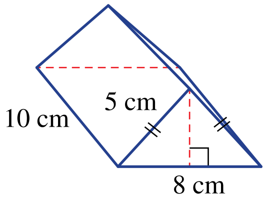
a



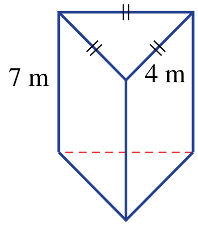
b



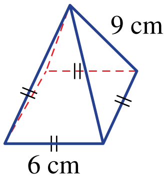
c



d



e



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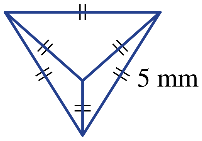

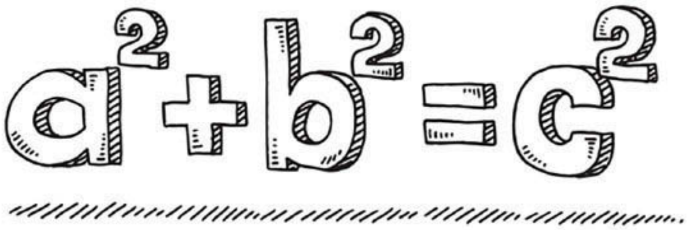

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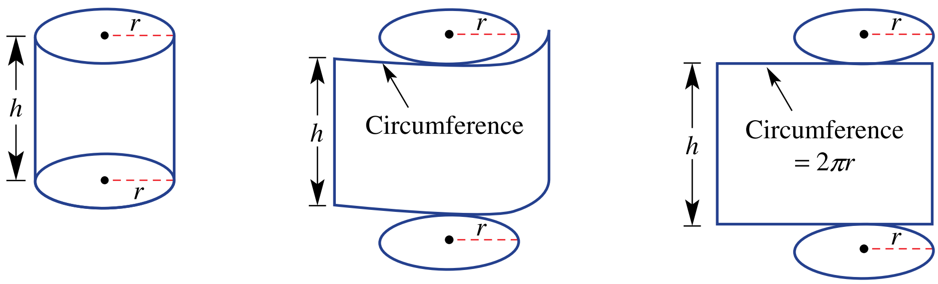
5F Surface area of cylinders

Learning intentions
- To understand how the net of a cylinder can be drawn to show the surface area
- To know the formula for the surface area of a cylinder
- To be able to calculate the surface area of a cylinder and sections of cylinders

Past, present and future learning
- This section addresses the Stage 5 Core Topic called *Area and surface area A*
- These concepts will be revised, briefly, in Chapter 4 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced
- You may be expected to recall formulas from memory in the HSC Examination

The net of a cylinder includes two circles and one rectangle with the length of the rectangle equal to the circumference of the circle. The sum of the area of all these shapes will give the surface area of a cylinder.



Lesson starter: Curved area

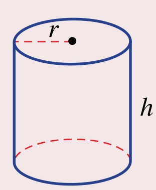
- Roll a piece of paper to form the curved surface of a cylinder.
- Do not stick the ends together because this will allow the paper to return to a flat surface.
- What shape is the paper when lying flat on a table?
- When curved to form the cylinder, what do the sides of the rectangle represent on the cylinder? How does this help to find the surface area of a cylinder?

KEY IDEAS

- Surface area of a **cylinder** = 2 circles + 1 rectangle

$$= 2 \times \pi r^2 + 2\pi r \times h$$

$$\therefore A = \underbrace{2\pi r^2}_{\text{2 circular ends}} + \underbrace{2\pi rh}_{\text{curved area}}$$



  - $A = 2\pi r^2 + 2\pi rh = 2\pi r(r + h)$

- In many problems, you will need to decide which parts of the surface of the cylinder should be included.

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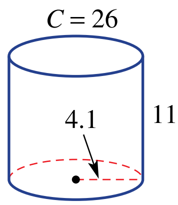
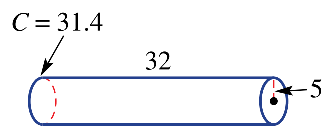
BUILDING UNDERSTANDING

1 Sketch a net suited to these cylinders. Label the sides using the given measurements. C represents the circumference.

a

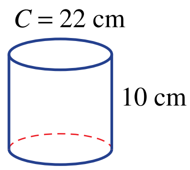


b

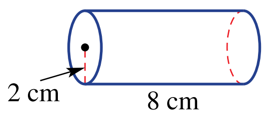


2 The curved surface of these cylinders is flattened out to form a rectangle. What would be the length and breadth of each rectangle? Round to two decimal places where necessary.

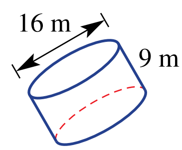
a



b

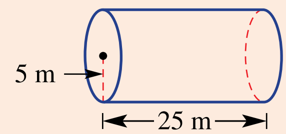


c



Example 12 Finding the surface area of a cylinder

Find the surface area of this cylinder, rounding to two decimal places.

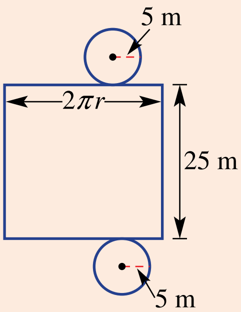


SOLUTION

$$
\begin{aligned}
A &= 2\pi r^2 + 2\pi rh \\
&= 2 \times \pi \times 5^2 + 2\pi \times 5 \times 25 \\
&= 50 \times \pi + 250 \times \pi \\
&= 50\pi + 250\pi \\
&= 300\pi \\
&= 942.48\ \text{m}^2\ (\text{to 2 d.p.})
\end{aligned}
$$

EXPLANATION

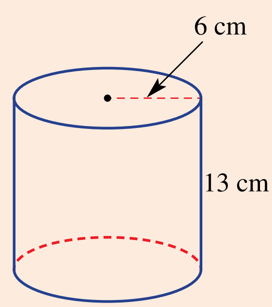
Surface area is made up of 2 circular ends and one rolled rectangular side.



Apply the formula with $r = 5$ and $h = 25$.
Round as required.

Now you try

Find the surface area of this cylinder, rounding to two decimal places.



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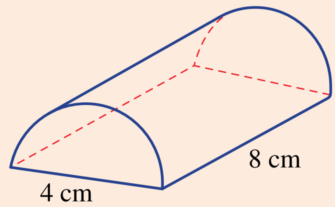
Example 13 Finding surface areas of cylindrical portions

Find the surface area of this half cylinder, rounding to two decimal places.

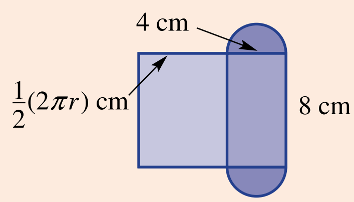


SOLUTION

$$\begin{aligned} A &= 2\left(\frac{1}{2}\pi r^2\right) + \frac{1}{2}(2\pi r) \times 8 + 4 \times 8 \\ &= 2 \times \frac{1}{2}\pi \times 2^2 + \frac{1}{2} \times 2 \times \pi \times 2 \times 8 + 32 \\ &= 20\pi + 32 \\ &= 94.83 \text{ cm}^2 \text{ (to 2 d.p.)} \end{aligned}$$

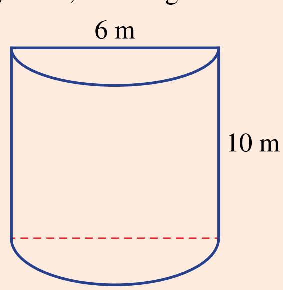
EXPLANATION

As well as half the curved surface of the cylinder, include the rectangular base (4×8) and the two semicircular ends $\left(\frac{1}{2}\pi r^2\right)$ each with $r = 2$.



Now you try

Find the surface area of this half cylinder, rounding to two decimal places.



If three tennis balls fit exactly into a cylindrical tube, which is greater, the tube's height or its top circumference?

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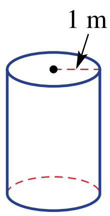
Exercise 5F

FLUENCY	1–3	1–4	2–4

Example 12

1 Find the surface area of these cylinders, rounding to two decimal places. Use a net to help.

a


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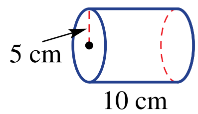
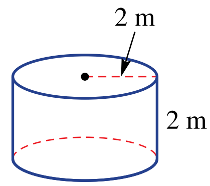
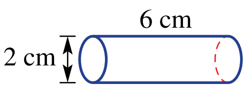


c


2 Find the surface area of these cylinders, rounding to one decimal place. Remember that the radius of a circle is half the diameter.

a


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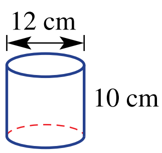

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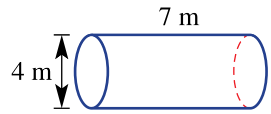
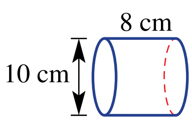


3 Find the surface area of a cylindrical plastic container of height 18 cm and with a circle of radius 3 cm at each end. Give the answer correct to two decimal places.

4 Find the area of the curved surface only for these cylinders, correct to two decimal places.

a


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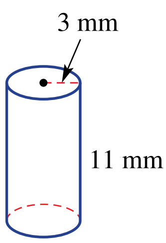

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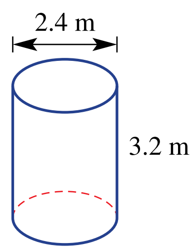
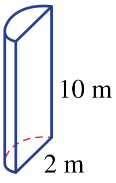


PROBLEM–SOLVING	5(½), 6	5(½), 6, 7	5(½), 7, 8

Example 13

5 Find the surface area of these solids, rounding to two decimal places.

a


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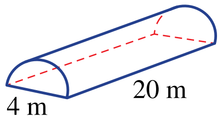

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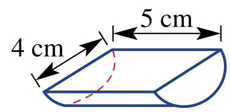

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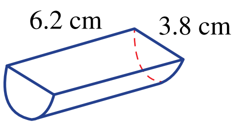

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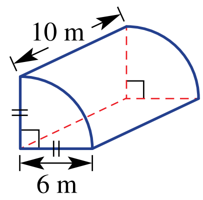

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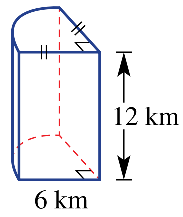

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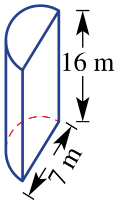
g



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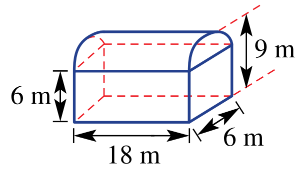
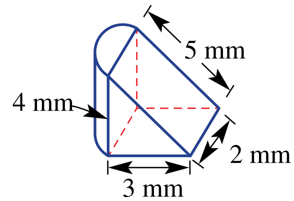
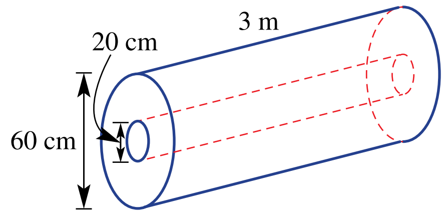


i

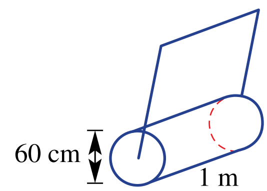


6 A water trough is in the shape of a half cylinder. Its semicircular ends have diameter 40 cm and the trough length is 1 m. Find the outside surface area in cm^2 of the curved surface plus the two semicircular ends, correct to two decimal places.

7 A log with diameter 60 cm is 3 m in length. Its hollow centre is 20 cm in diameter.
Find the surface area of the log in cm^2, including the ends and the inside, correct to one decimal place.



8 A cylindrical roller is used to press crushed rock in preparation for a tennis court. The rectangular tennis court area is 30 m long and 15 m wide. The roller has a breadth of 1 m and diameter 60 cm.



a Find the surface area of the curved part of the roller in cm^2 correct to three decimal places.

b Find the area, in m^2 to two decimal places, of crushed rock that can be pressed after:

i 1 revolution **ii** 20 revolutions.

c Find the minimum number of complete revolutions required to press the entire tennis court area.

REASONING	9	9, 10	9, 10

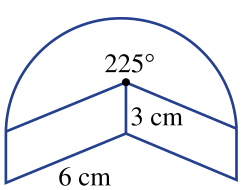
9 It is more precise to give exact values for calculations involving π, e.g. 24π.
Give the exact answers for the surface area of the cylinders in Question 1.

10 A cylinder cut in half gives half the volume but not half the surface area. Explain why.

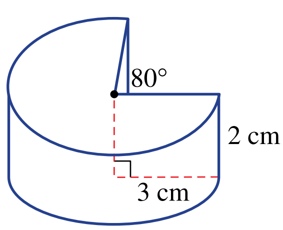
ENRICHMENT: Solid sectors	–	–	11

11 The sector area rule $A = \frac{\theta}{360} \times \pi r^2$ can be applied to find the surface areas of solids which have ends that are sectors. Find the exact surface area of these solids.

a



b



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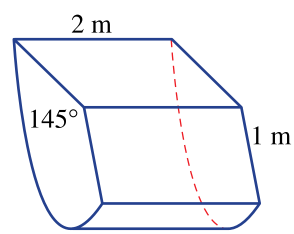

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The following problems will investigate practical situations drawing upon knowledge and skills developed throughout the chapter. In attempting to solve these problems, aim to identify the key information, use diagrams, formulate ideas, apply strategies, make calculations and check and communicate your solutions.

The penny-farthing

1 The penny-farthing was the first machine to be called a bicycle. It was a popular mode of transport in the late 19th century.
Edward's penny-farthing has a large wheel diameter of 135 cm and a small wheel diameter of 45 cm.

Edward is interested in the number of wheel rotations required over a certain distance and the relationship between the number of rotations of the small wheel and the large wheel.

- **a** Find the circumference of the following correct to two decimal places.
  - **i** the small wheel
  - **ii** the large wheel
- **b** To cover the same distance, how many times does the small wheel need to turn for each turn of the large wheel?
- **c** Edward rides the penny-farthing and the large wheel rotates 10 times.
  - **i** How far does he travel correct to the nearest cm?
  - **ii** How many times does the small wheel turn?
- **d** Edward wants to ride to school, which is a 1 km journey. How many rotations of the large wheel will occur? Round to the nearest integer.
- **e** On another journey, the small wheel makes 100 more rotations than the large wheel. How far did the penny-farthing travel? Round to the nearest metre.

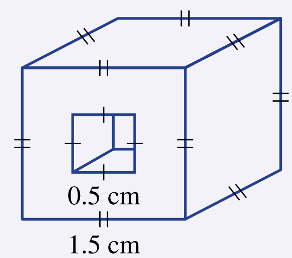
Fishing sinkers

2 Fishing lines often have a metal sinker attached to the line so that the hook and bait sink below the surface of the water. Each sinker must have a hole in it so the line can be properly attached.

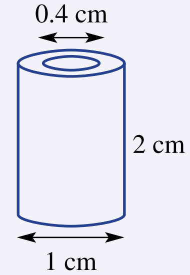
The 'Catch Fish' company produces sinkers of various shapes and sizes and coats them with a special anti-rust paint.

The Catch Fish company is interested in the surface area of each sinker so that it can calculate how much paint is required and the associated cost.

- **a** The most basic sinker is a cube of side length 1.5 cm with a single hole of breadth 0.5 cm as shown. Find the surface area of this sinker including the surface inside the hole.



- **b** A cylindrical sinker has dimensions as shown. Find the surface area of this sinker, including inside the hole, correct to two decimal places.



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c The company chooses to make 1000 of the cube sinkers (from part **a**) and 1000 of the cylindrical sinkers (from part **b** above). The paint costs \$120 per litre and each litre covers 10 m^2 or 100 000 cm^2.

- **i** What is the surface area for the 2000 sinkers? Round to the nearest cm^2.
- **ii** How much will it cost to paint all the sinkers? Assume that paint can be purchased in any quantity and round to the nearest dollar.

d The Catch Fish company decides to adjust the 2 cm height of the cylindrical sinker so that its surface area is the same as the cube sinker. Investigate to find the new height of the cylindrical sinker.

Circle bands

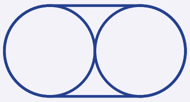
3 Bridget is playing with an elastic band and a number of one dollar coins, each of diameter 25 mm.

She lays a number of the coins flat on a table to form different shapes then places an elastic band around them.

Bridget is interested in the length of the elastic band around each shape formed by the coins and how much further the band needs to stretch as she adds more coins.

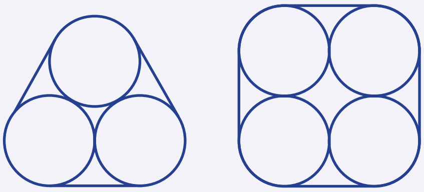
a What is the length of elastic band required to stretch around the following shapes? Round to the nearest mm.

- **i** A single dollar coin
- **ii** Two touching coins as shown



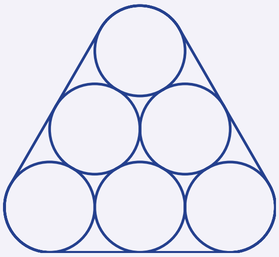
b Bridget tries placing a band around three and four coins.

How much further will the band need to stretch for the four-coin shape compared to the three-coin shape?



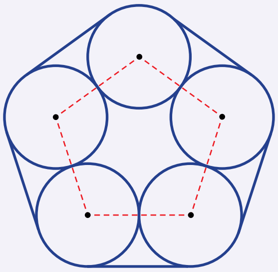
c Bridget arranges six coins in three rows to form a triangle as shown.

- **i** How much further does the band need to stretch compared to the three-coin shape above?
- **ii** If the triangle pattern had n rows, find the rule for the length, L mm, of the elastic band in terms of n.



d Bridget arranges five coins so that the line segments joining the centres form a regular pentagon (with internal angle sum 540°).

How far does the elastic band need to stretch? Round to the nearest mm.



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5G Volume of prisms

Learning intentions

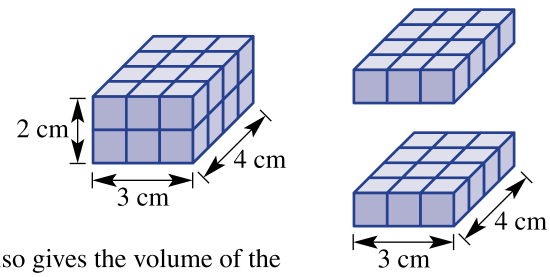
- To be able to convert between metric units of volume and capacity
- To understand how the volume of a solid relates to its constant cross-section and height
- To be able to calculate the volume of a solid with uniform cross-section

Past, present and future learning

- This section addresses the Stage 5 Core Topic called *Volume A*
- These concepts will be revised, briefly, in Chapter 4 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced
- You may be expected to recall formulas from memory in the HSC Examination

Volume is the number of cubic units contained within a three-dimensional object.

To find the volume for this solid we can count 24 individual cubic centimetres (24 cm^3) *or* multiply $3 \times 4 \times 2 = 24$ cm^3.



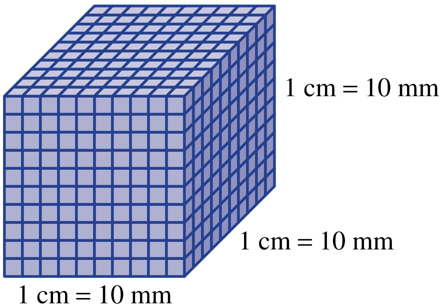
We can see that the area of the base ($3 \times 4 = 12$ cm^2) also gives the volume of the base layer 1 cm high. The number of layers equals the height, hence, multiplying the area of the base by the height will give the volume.

This idea can be applied to all right prisms provided a uniform cross-section can be identified. In such solids, the height or length used to calculate the volume is the length of the edge running perpendicular to the base or cross-section, as this measurement counts the number of layers.

Lesson starter: Cubic units

Consider this 1 cm cube (not to scale) divided into cubic millimetres.

- How many cubic mm sit on one edge of the 1 cm cube?
- How many cubic mm sit on one layer of the 1 cm cube?
- How many layers are there?
- How many cubic mm are there in total in the 1 cm cube?
- Complete this statement: 1 cm^3 = _____ mm^3
- Explain how you can find how many:
  - **a** cm^3 in 1 m^3
  - **b** m^3 in 1 km^3



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KEY IDEAS

- Common metric units for **volume** include **cubic kilometres** (km^3), **cubic metres** (m^3), **cubic centimetres** (cm^3) and **cubic millimetres** (mm^3).

$$\begin{tikzcd} \text{km}^3 \arrow[r, bend left, "\times 1000^3"] & \text{m}^3 \arrow[l, bend left, "\div 1000^3"] \arrow[r, bend left, "\times 100^3"] & \text{cm}^3 \arrow[l, bend left, "\div 100^3"] \arrow[r, bend left, "\times 10^3"] & \text{mm}^3 \arrow[l, bend left, "\div 10^3"] \end{tikzcd}$$

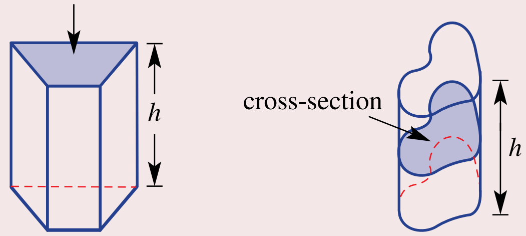
$$\begin{aligned} 1000^3 &= 1\,000\,000\,000 \\ 100^3 &= 1\,000\,000 \\ 10^3 &= 1000 \end{aligned}$$

- Common units of **capacity** include:
  - **megalitres** (ML) 1 ML = 1000 kL
  - **kilolitres** (kL) 1 kL = 1000 L
  - **litres** (L) 1 L = 1000 mL
  - **millilitres** (mL)

- Also 1 cm^3 = 1 mL so 1 L = 1000 cm^3 and 1 m^3 = 1000 L

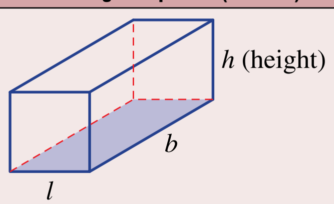
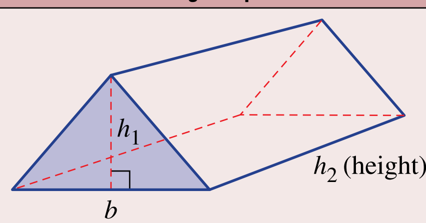
- For solids with a uniform **cross-section**:

$$\begin{aligned} \text{Volume} &= \text{area of cross-section} \times \text{perpendicular height} \\ V &= A \times \text{perp. height} \end{aligned}$$



- The perpendicular 'height' is the length of the edge that runs perpendicular to the cross-section. This edge can be the vertical height or the length of a prism.

- Some common formulas for volume include:

Rectangular prism (cuboid)	Triangular prism
 $V = A \times \text{perp. height}$ $= l \times b \times h$ $= lbh$	 $V = A \times \text{perp. height}$ $= \left(\frac{1}{2} \times b \times h_1\right) \times h_2$

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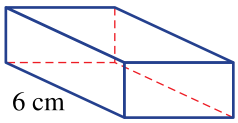
BUILDING UNDERSTANDING

1 Describe the cross-sectional shape for these prisms and state the perpendicular 'height' (i.e. the length of the edge perpendicular to the cross-section).

a



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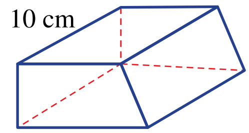

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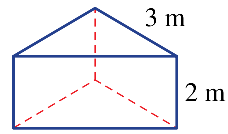

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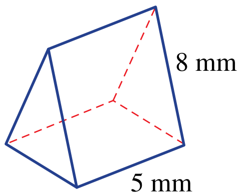

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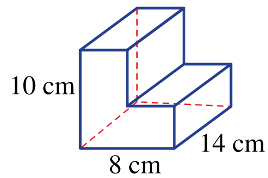

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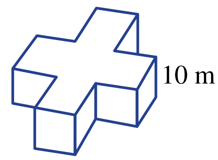
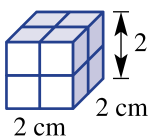


2 Find the volume of these three-dimensional rectangular prisms. Count the number of blocks if you wish.

a



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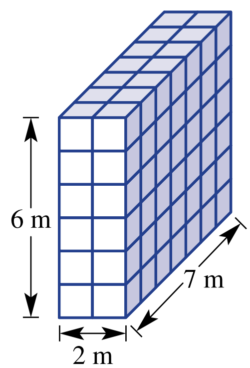

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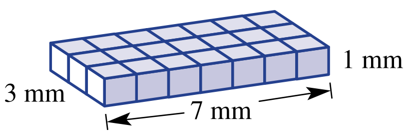

Example 14 Converting units of volume

Convert the following volume measurements to the units given in the brackets.

a 2.5 m^3 (cm^3) **b** 458 mm^3 (cm^3)

SOLUTION

a $2.5\text{ m}^3 = 2.5 \times 100^3\text{ cm}^3$
$= 2\,500\,000\text{ cm}^3$

b $458\text{ mm}^3 = 458 \div 10^3\text{ cm}^3$
$= 0.458\text{ cm}^3$

EXPLANATION

$\times 100^3 = 1\,000\,000$

$\text{m}^3 \rightarrow \text{cm}^3$

2.500000

$\text{cm}^3 \leftarrow \text{mm}^3$

$\div 10^3 = 1000$

Now you try

Convert the following volume measurements to the units given in the brackets.

a 4.8 cm^3 (mm^3) **b** 612 000 cm^3 (m^3)

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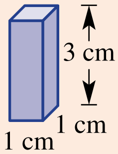
Example 15 Finding volumes of prisms and other solids

Find the volume of each of these three-dimensional objects.

a



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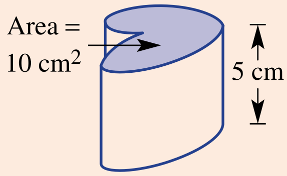

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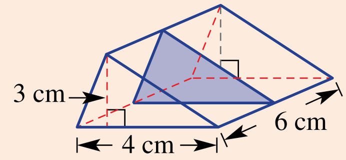


SOLUTION

a $V = l \times b \times h$

$= 1 \times 1 \times 3$

$= 3\ \text{cm}^3$

b $V = A \times \text{perp. height}$

$= 10 \times 5$

$= 50\ \text{cm}^3$

c $V = A \times \text{perp. height}$

$= \left(\frac{1}{2} \times b \times h_1\right) \times h_2$

$= \left(\frac{1}{2} \times 4 \times 3\right) \times 6$

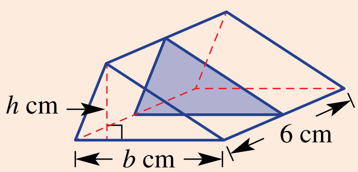
$= 36\ \text{cm}^3$

EXPLANATION

The solid is a rectangular prism.
Length = 1 cm, breadth = 1 cm and height = 3 cm

Volume = area of cross-section × perpendicular height
Substitute cross-sectional area = 10
and perpendicular height = 5.

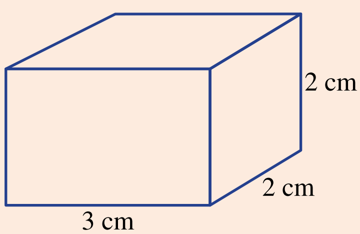
The cross-section is a triangle. The edge perpendicular to the cross-section is the length (6 cm).



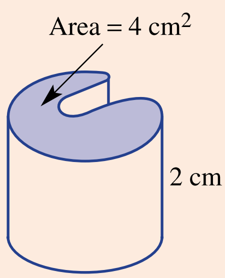
Now you try

Find the volume of each of these three-dimensional objects.

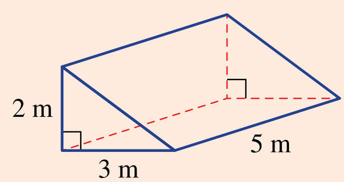
a



b



c



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Exercise 5G

FLUENCY	1–3(½), 4	1–3(½), 4, 5	1–3(⅓), 4, 5

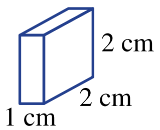
Example 14

1 Convert the following volume measurements to the units given in the brackets.

a 3 cm^3 (mm^3)	**b** 2000 mm^3 (cm^3)	**c** 8.7 m^3 (cm^3)
d 5900 cm^3 (m^3)	**e** 0.00001 km^3 (m^3)	**f** 21 700 m^3 (km^3)
g 3 L (mL)	**h** 0.2 kL (L)	**i** 3500 mL (L)
j 0.021 L (mL)	**k** 37 000 L (kL)	**l** 42 900 kL (ML)

Example 15a

2 Find the volume of these rectangular prisms.

a


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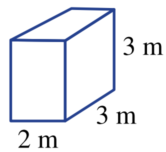

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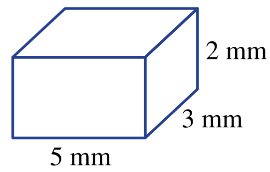

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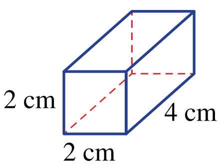

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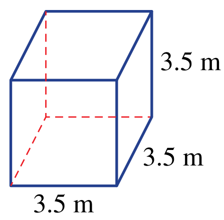

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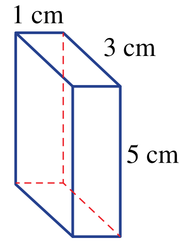
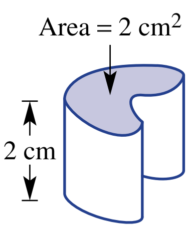


Example 15b

3 Find the volume of each of these three-dimensional objects. The cross-sectional area has been given.

a


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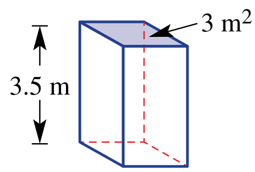

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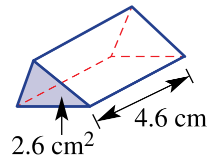

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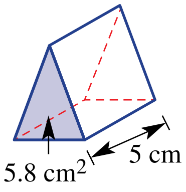

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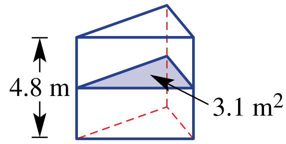

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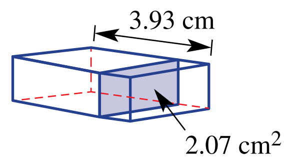

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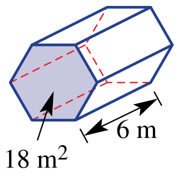
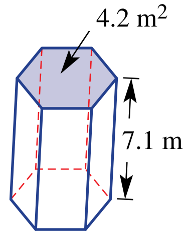
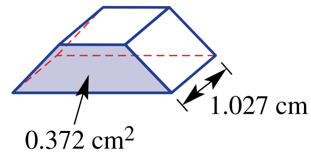


h


i


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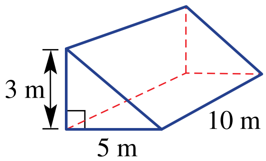
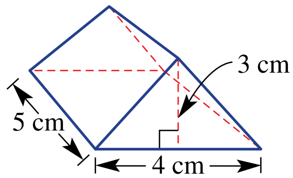
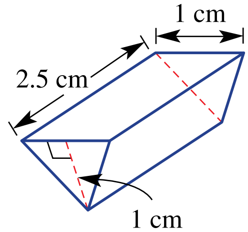
Example 17c

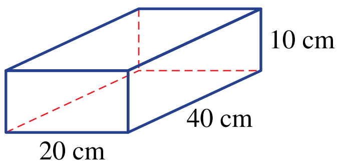
4 Find the volume of these triangular prisms.

a


b


c


5 Find the volume of these solids. Convert your answer to litres. Recall 1 cm^3 = 1 mL and 1 L = 1000 cm^3.

a


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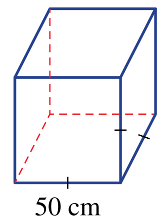
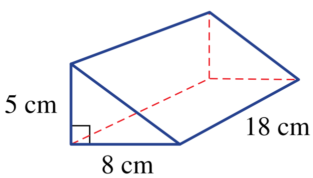


c


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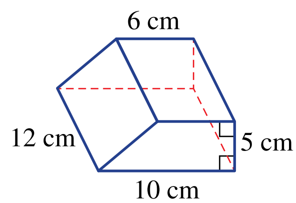


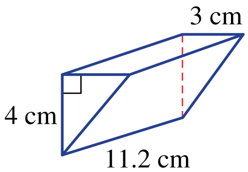
PROBLEM–SOLVING	6, 7	7–9	8–11

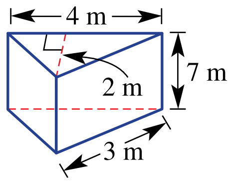
6 A brick is 10 cm wide, 20 cm long and 8 cm high. How much space would five of these bricks occupy?

7 How much air space is contained inside a rectangular cardboard box that has the dimensions 85 cm by 62 cm by 36 cm. Answer using cubic metres (m^3), correct to two decimal places.

8 25 L of water is poured into a rectangular fish tank which is 50 cm long, 20 cm wide and 20 cm high. Will it overflow?

9 Find the volume of each of the following solids, rounding to one decimal place where necessary.

a


b


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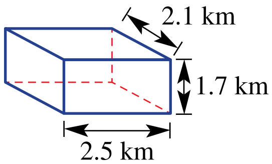

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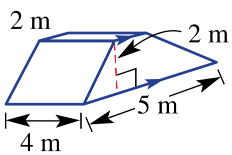

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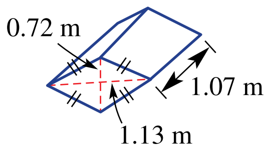

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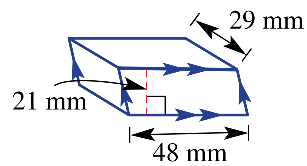

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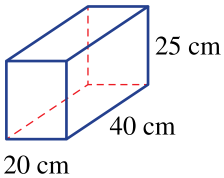
10 Use units for capacity to find the volume of these solids in litres.

a



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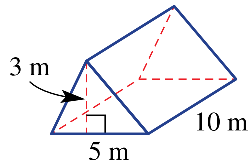

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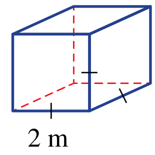
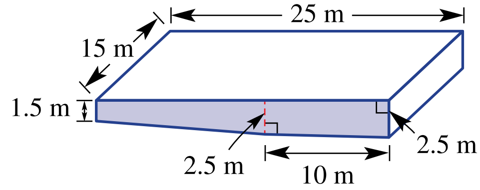


11 The given diagram is a sketch of a new 25 m swimming pool to be installed in a school sports complex.

a Find the area of the shaded side of the pool.

b Find the volume of the pool in litres.



REASONING	12	12	12, 13

12 **a** What single number do you multiply by to convert from:

i L to cm^3? **ii** L to m^3? **iii** mL to mm^3?

b What single number do you divide by to convert from:

i mm^3 to L? **ii** m^3 to ML? **iii** cm^3 to kL?

13 Write rules for the volume of these solids using the given pronumerals.

a A rectangular prism with length = breadth = x and height h

b A cube with side length s

c A rectangular prism with a square base (with side length t) and height *six* times the side length of the base

ENRICHMENT: Volume of a pyramid	–	–	14

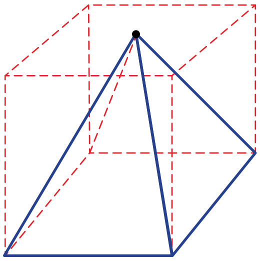
14 Earlier we looked at finding the surface area of a right pyramid like the one shown here.

Imagine the pyramid sitting inside a prism with the same base.

a Make an educated guess as to what fraction the volume of the pyramid is of the volume of the prism.

b Use the internet to find the actual answer to part **a**.

c Draw some pyramids and find their volume using the results from part **b**.



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5H Volume of cylinders

Learning intentions
- To know the formula for the volume of a cylinder
- To be able to find the volume and capacity of a cylinder

Past, present and future learning
- This section addresses the Stage 5 Core Topic called *Volume A*
- These concepts will be revised, briefly, in Chapter 4 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced
- You may be expected to recall formulas from memory in the HSC Examination

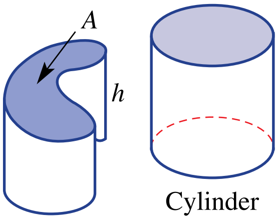
A right cylinder has its outside surface at right angles to its base and top. It has a uniform cross-section (a circle) and so the volume of a cylinder can be calculated in a similar way to that of a right prism. Cylindrical objects are commonly used to store gases and liquids, and so working out the volume of a cylinder is an important measurement calculation.

An oil refinery converts crude oil into many products including petrol, diesel, jet fuel, motor oils and asphalt base. Cylinder volumes are calculated for its storage tanks and pipe capacity per m^3.

Lesson starter: Writing the rule

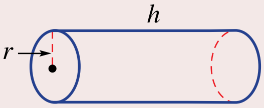
Previously we used the formula $V = A \times h$ to find the volume of solids with a uniform cross-section.

- Discuss any similarities between the two given solids.
- How can the rule $V = A \times h$ be developed further to find the rule for the volume of a cylinder?



KEY IDEAS

- The volume of a cylinder is given by
  $V = \pi r^2 \times h$ or $V = \pi r^2 h$
  - r is the radius of the circular ends.
  - h is the length or distance between the circular ends.



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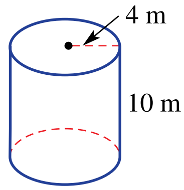
BUILDING UNDERSTANDING

1 Referring to the formula $V = \pi r^2 h$, state the values of r and h for these cylinders.

a


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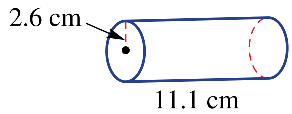

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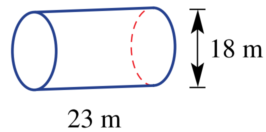
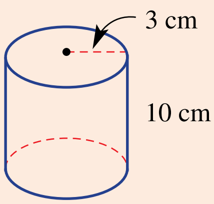


2 Convert the following to the units given in the brackets. Remember, $1 \text{ L} = 1000 \text{ cm}^3$ and $1 \text{ m}^3 = 1000 \text{ L}$.

a 2000 cm^3 (L)
b 4.3 cm^3 (mL)
c 3.7 L (cm^3)
d 1 m^3 (L)
e $38\,000$ L (m^3)
f 0.0002 m^3 (mL)

Example 16 Finding the volume of a cylinder

Find the volume of these cylinders correct to two decimal places.

a


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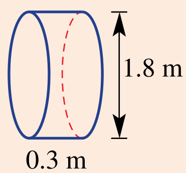


SOLUTION

a
$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \times (3)^2 \times 10 \\ &= 90\pi \\ &= 282.74 \text{ cm}^3 \text{ (to 2 d.p.)} \end{aligned}$$

b
$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \times (0.9)^2 \times 0.3 \\ &= 0.76 \text{ m}^3 \text{ (to 2 d.p.)} \end{aligned}$$

EXPLANATION

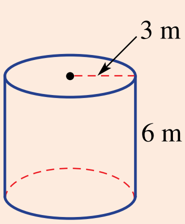
Substitute $r = 3$ and $h = 10$ into the rule.

$90\pi \text{ cm}^3$ would be the exact answer.
Include volume units.

The diameter is 1.8 m so $r = 0.9$.

Now you try

Find the volume of these cylinders correct to two decimal places.

a


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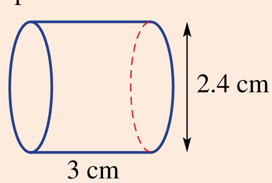

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Example 17 Finding the capacity of a cylinder

Find the capacity, in litres, of a cylinder with radius 30 cm and height 90 cm. Round to the nearest litre.

SOLUTION

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \times (30)^2 \times 90 \\ &= 254\,469 \text{ cm}^3 \\ &= 254 \text{ L (to the nearest litre)} \end{aligned}$$

EXPLANATION

Substitute $r = 30$ and $h = 90$.

There are 1000 cm^3 in 1 L so divide by 1000 to convert to litres. 254.469 to the nearest litre is 254 L.

Now you try

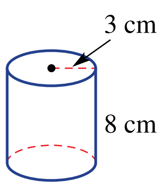
Find the capacity, in litres, of a cylinder with radius 40 cm and height 100 cm. Round to the nearest litre.

Exercise 5H

FLUENCY	1, 2(½)	1–2(½)	1–2(½)

Example 16

1 Find the volume of these cylinders correct to two decimal places.

a


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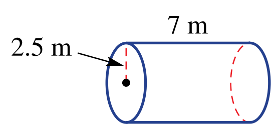

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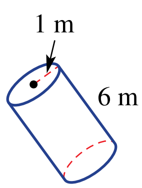

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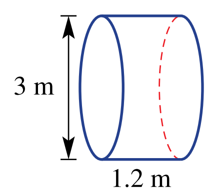

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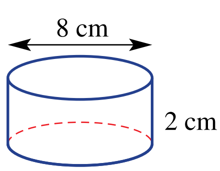

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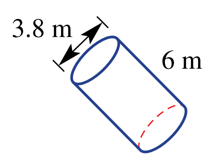

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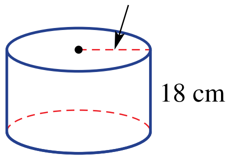
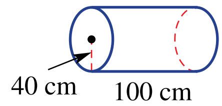
Example 17

2 Find the capacity, in litres, of these cylinders. Round to the nearest litre. Remember 1 L = 1000 cm^3 and 1 m^3 = 1000 L.

a


b


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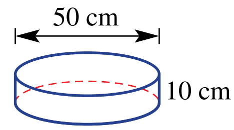

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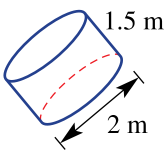

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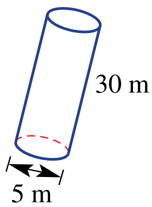

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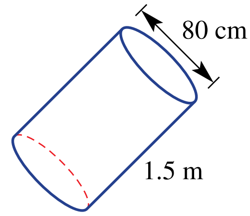


PROBLEM-SOLVING	3, 4	4, 5, 6(½)	5, 6

3 A cylindrical water tank has a radius of 2 m and a height of 2 m.

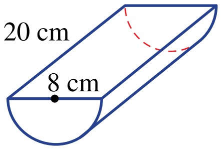
a Find its capacity, in m^3, rounded to three decimal places.

b Find its capacity, in litres, rounded to the nearest litre.

4 How many litres of gas can a tanker carry if it has a cylindrical tank that is 2 m in diameter and 12 m in length? Round to the nearest litre.

5 Of the following, determine which has the bigger volume and calculate the difference in volume to two decimal places: a cube with side length 1 m or a cylinder with radius 1 m and height 0.5 m.

6 Find the volume of these cylindrical portions correct to two decimal places.

a


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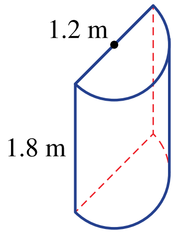

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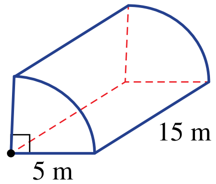

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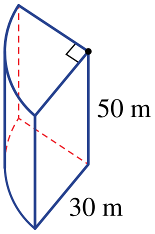

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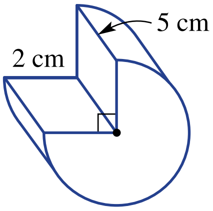

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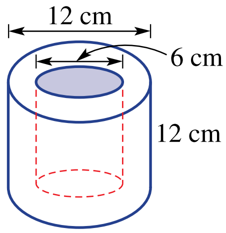

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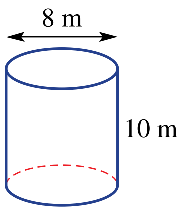
REASONING	7	7, 8(½)	7, 8(½), 9

7 The rule for the volume of a cylinder is $V = \pi r^2 h$. Show how you could use this rule to find, correct to three decimal places:

a h when $V = 20$ and $r = 3$ **b** r when $V = 100$ and $h = 5$.

8 Using exact values (e.g. 20π) find the volume of these solids.

a


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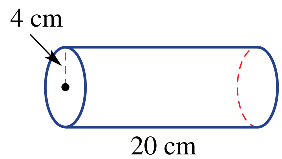

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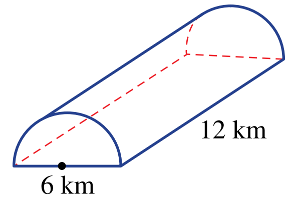

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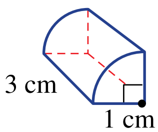

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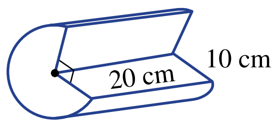

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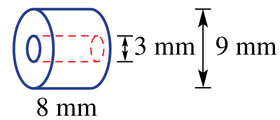
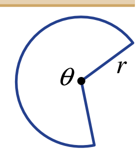


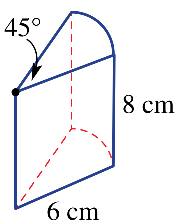
9 Draw a cylinder that has a circumference equal to its height. Try to draw it to scale.

ENRICHMENT: Solid sectors	–	–	10

10 You will recall that the area of a sector is given by $A = \frac{\theta}{360} \times \pi r^2$.



Use this fact to help find the volume of these solids, correct to two decimal places.

a


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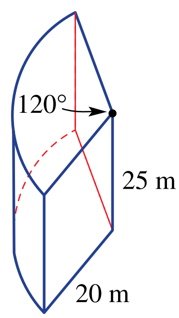

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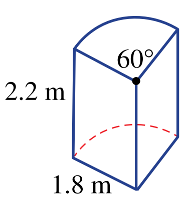

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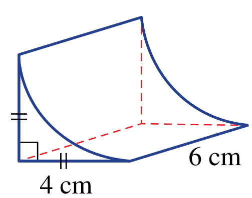

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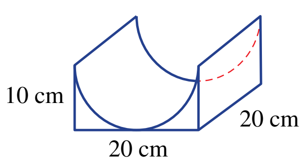

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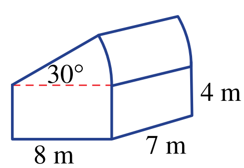

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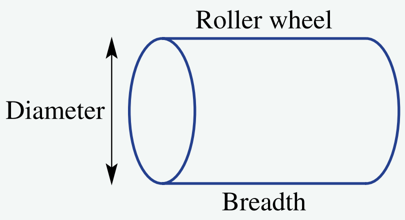
Pressing a new road

The construction of a new asphalt road involves pressing the surface with a roller. Two rollers are available, each with main roller wheels of varying diameter and breadth. The options are:

- Roller 1: Wheel diameter 1 metre and breadth 2 metres
- Roller 2: Wheel diameter 1.2 metres and breadth 2.5 metres.



The cost of hiring each roller depends on the number of revolutions in the following way:

- Roller 1: \$2.20 per revolution
- Roller 2: \$3 per revolution.

The rectangular section of road being built is 1 km long and 10 metres wide.

Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate.

Routine problems

a Find the following for Roller 1 correct to two decimal places:
  i the surface area of the roller wheel (curved surface area only)
  ii the area of ground pressed after 10 revolutions.

b Find the surface area in square metres of a section of road which is 1 km long and 10 metres wide.

c How many rotations of the wheel would need to occur if Roller 1 was to travel a single-length of the road 1 km long? Round to one decimal place.

Non-routine problems

Explore and connect

a The problem is to determine the cost of hiring each roller to press the entire 1 km stretch of road. Write down all the relevant information that will help solve this problem with the aid of diagrams.

Choose and apply techniques

b Using Roller 1 determine the total number of revolutions that the wheel needs to rotate if the entire 10 metre-wide road is to be pressed. Use the following assumptions.
  - It is not necessary to overlap one pressed strip with the next.
  - No revolutions are counted while the roller is manoeuvring into position at the start of each strip.

c Calculate the cost of hiring Roller 1 to press the 1 km stretch of road.

d Repeat parts **b** and **c** for Roller 2.

Communicate thinking and reasoning

e Assume that each strip rolled must now overlap the previous pressed strip by 10 cm. Determine the total number of revolutions required for each of the two rollers under these conditions.

f Compare and comment on the cost of hiring the rollers if:
  i no overlap is required between strips
  ii a 10 cm overlap is required between strips.

g Summarise your results and describe any key findings.

Extension problems

Problem solve

a If the overlap of each rolled strip is to be 50 cm instead, which roller would be cheaper to use? Give reasons.

b A third roller has wheel diameter 2 metres and breadth 4 metres and its cost per revolution is \$5. Will this roller be a cheaper option to press the 1 km stretch of road if a 10 cm overlap is required? Justify you answer.

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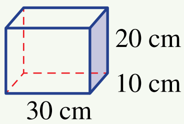
Investigation

Capacity and depth

Finding capacity

Find the capacity, in litres, of these containers (i.e. find the total volume of fluid they can hold).

a



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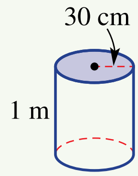
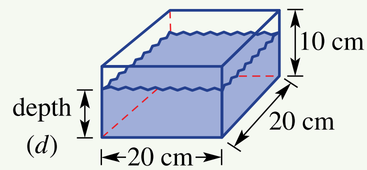


Remember: 1 millilitre (mL) of fluid occupies 1 cm^3 of space, therefore 1 litre (L) occupies 1000 cm^3 as there are 1000 mL in 1 litre.

Finding depth

The depth of water in a prism can be found if the base (cross-sectional) area and volume of water are given.



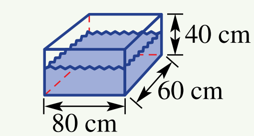
Consider a cuboid, as shown, with 2.4 L of water.

To find the depth of water:

- Convert the volume to cm^3: $2.4 \text{ L} = 2.4 \times 1000 \text{ cm}^3$
  $= 2400 \text{ cm}^3$
- Find the depth:
$$\begin{aligned} \text{Volume} &= \text{area of base} \times d \\ 2400 &= 20 \times 20 \times d \\ 2400 &= 400 \times d \\ \therefore d &= 6 \\ \therefore \ \text{depth is } 6\,\text{cm} \end{aligned}$$

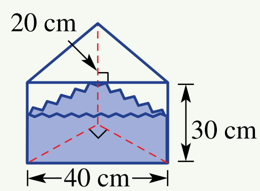
Use the above method to find the depth of water in these prisms.

a



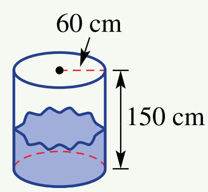
Volume = 96 L

b



Volume = 8 L

c



Volume = 500 L

Volumes of odd-shaped objects

Some solids may be peculiar in shape and their volumes may be difficult to measure.

a A rare piece of rock is placed into a cylindrical jug of water and the water depth rises from 10 cm to 11 cm. The radius of the jug is 5 cm.
  i Find the area of the circular base of the cylinder.
  ii Find the volume of water in the jug before the rock is placed in the jug.
  iii Find the volume of water in the jug including the rock.
  iv Hence find the volume of the rock.

b Use the procedure outlined in part **a** to find the volume of an object of your choice. Explain and show your working and compare your results with those of other students in your class if they are measuring the volume of the same object.

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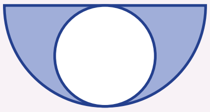
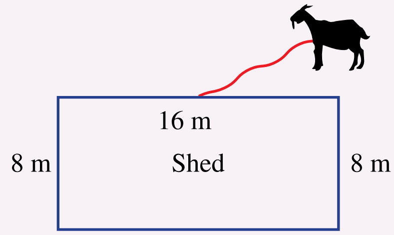
1 The 100 m^2 flat roof of a factory feeds all the water collected to a rainwater tank. If there is 1 mm of rainfall, how many litres of water go into the tank?

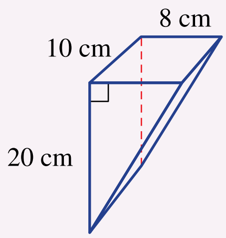
2 What is the relationship between the shaded and non-shaded regions in this circular diagram?



3 A goat is tethered to the centre of one side of a rectangular shed with a 10 m length of rope. What area of grass can the goat graze?



4 A rain gauge is in the shape of a triangular prism with dimensions as shown. What is the depth of water when it is half full?



5 A rectangular fish tank has base area 0.3 m^2 and height 30 cm and is filled with 80 L of water. Ten large fish, each with volume 50 cm^3, are placed into the tank. By how much does the water level rise?

6 Find the rule for the volume of a cylinder in terms of r only if the height is equal to its circumference.

7 The surface area of a cylinder is 2π square units. Find a rule for h in terms of r.

8 Give the dimensions of a cylinder that can hold 1 L of milk. What considerations are needed when designing containers for use by consumers?

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Chapter summary

Measurement

Length and perimeter

Circle circumference

$C = 2\pi r$ or πd

Units

$\times 1000$ $\times 100$ $\times 10$

km m cm mm

$\div 1000$ $\div 100$ $\div 10$

Sector perimeter

3 cm

$P = 2r + \frac{\theta}{360} \times 2\pi r$

$= 2 \times 3 + \frac{90}{360} \times 2\pi \times 3$

$= 6 + 1.5\pi$ (exact)

$= 10.71$ cm (to 2 d.p.)

Volume

Units

$\times 1000^3$ $\times 100^3$ $\times 10^3$

km^3 m^3 cm^3 mm^3

$\div 1000^3$ $\div 100^3$ $\div 10^3$

1 L = 1000 mL = 1000 cm^3

Prism or cylinder

$V = Ah$

A = area of cross-section

h = perp. height

$V = lbh$

$= 11 \times 6 \times 5$

$= 330$ m^3

5 m, 6 m, 11 m

$V = \pi r^2 h$

$= \pi(2)^2 7$

$= 28\pi$

$= 87.96$ cm^3

2 cm, 7 cm

Surface area

Prism or pyramid

Use a net to help find the sum of all surface areas.

5 m, 3 m

$A = b^2 + 4\left(\frac{1}{2}bh\right)$

$= 3^2 + 4 \times \frac{1}{2} \times 3 \times 5$

$= 39$ m^2

Cylinder

3 cm, 4 cm

$A = 2\pi r^2 + 2\pi rh$

$= 2\pi(3)^2 + 2\pi(3)(4)$

$= 18\pi + 24\pi$

$= 42\pi$

$= 131.95$ cm^2

Area

Triangle

$A = \frac{1}{2}bh$

Quadrilaterals

Square $A = l^2$

Rectangle $A = lb$

Parallelogram $A = bh$

Trapezium $A = \frac{h}{2}(a + b)$

Rhombus or kite $A = \frac{1}{2}xy$

Units

$\times 1000^2$ $\times 100^2$ $\times 10^2$

km^2 m^2 cm^2 mm^2

$\div 1000^2$ $\div 100^2$ $\div 10^2$

Circle

$A = \pi r^2$

sector $A = \frac{\theta}{360} \times \pi r^2$

$= \frac{120}{360} \times \pi \times 5^2$

$= \frac{1}{3} \times \pi \times 25$

$= 26.18$ m^2

120°, 5 m

Composite shapes

10 m, 8 m

$A = lb - \frac{1}{2}\pi r^2$

$= 10 \times 8 - \frac{1}{2}\pi \times 4^2$

$= 80 - 8\pi$

$= 54.87$ m^2

$P = 2 \times 10 + 8 + \frac{1}{2}2\pi(4)$

$= 28 + 4\pi$

$= 40.57$ m

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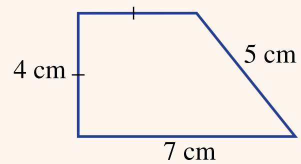
Chapter checklist with success criteria

A printable version of this checklist is available in the Interactive Textbook

✔

5A

1. I can find the perimeter of simple shapes.
e.g. Find the perimeter of the following shape.



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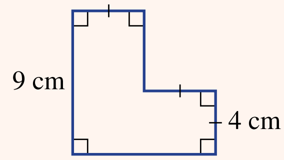
5A

2. I can find the perimeter of composite shapes.
e.g. Find the perimeter of the following shape.



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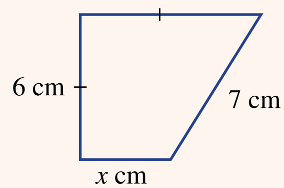
5A

3. I can find a missing side length given a perimeter.
e.g. Find the value of x if the perimeter of the given shape is 23 cm.



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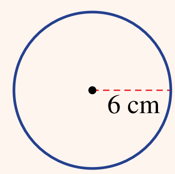
5B

4. I can find the circumference of a circle.
e.g. Find the circumference of this circle both in exact form and rounded to two decimal places.



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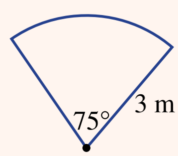
5B

5. I can find the perimeter of a sector.
e.g. Find the perimeter of this sector correct to two decimal places.



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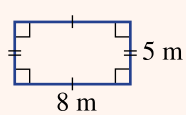
5C

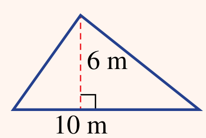
6. I can convert between units of area.
e.g. Convert 3200 cm^2 to m^2 and 2.4 cm^2 to mm^2.

☐

5C

7. I can find the area of simple shapes.
e.g. Find the area of this rectangle and triangle.





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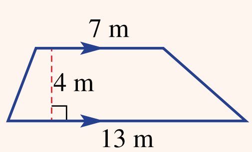
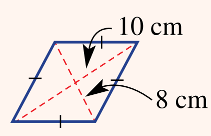
5C

8. I can find the area of a trapezium or rhombus.
e.g. Find the area of the following trapezium and rhombus.





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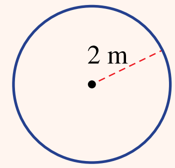
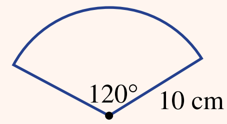
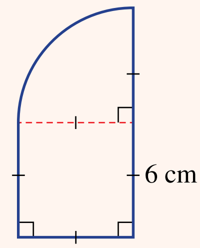
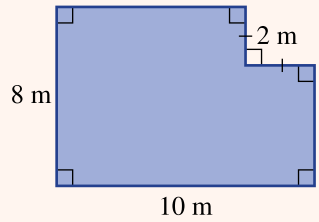
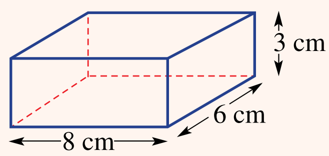
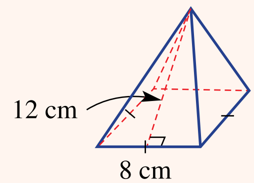
Chapter checklist

5C	**9. I can find the area of a circle.** e.g. Find the area of this circle correct to two decimal places. 	☐
5C	**10. I can find the area of a sector.** e.g. Find the area of this sector correct to two decimal places. 	☐
5D	**11. I can find the perimeter and area of composite shapes.** e.g. Find the perimeter and area of this composite shape, rounding answers to two decimal places. 	☐
5D	**12. I can find the area of a composite shape using subtraction.** e.g. Find the shaded area of this composite shape. 	☐
5E	**13. I can find the surface area of a prism.** e.g. Find the surface area of this right rectangular prism. 	☐
5E	**14. I can find the surface area of a pyramid.** e.g. Find the surface area of this right square-based pyramid. 	☐

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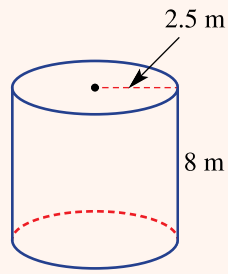
Chapter checklist

5F

15. I can find the surface area of a cylinder.
e.g. Find the surface area of this cylinder correct to two decimal places.



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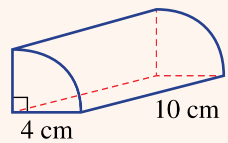
5F

16. I can find the surface area of cylindrical portions.
e.g. Find the surface area of this quarter-cylinder, rounding to two decimal places.



☐

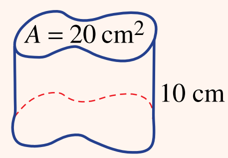
5G

17. I can convert between units of volume.
e.g. Convert 3 m^3 to cm^3 and 1276 mm^3 to cm^3.

☐

5G

18. I can find the volume of an object with a constant cross-section.
e.g. Find the volume of this solid object.



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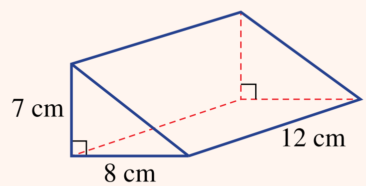
5G

19. I can find the volume of a prism.
e.g. Find the volume of this triangular prism.



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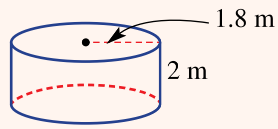
5H

20. I can find the volume of a cylinder.
e.g. Find the volume of this cylinder correct to two decimal places.



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5H

21. I can find volume in terms of capacity.
e.g. Find the capacity in litres of a cylinder with radius 20 cm and height 80 cm. Round to the nearest litre.

☐

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Chapter review

Short-answer questions

5A/C/G

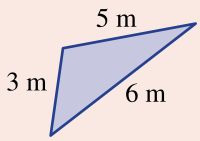
1 Convert the following measurements to the units given in brackets.

- **a** 3.8 m (cm)
- **b** 1.27 km (m)
- **c** 273 mm^2 (cm^2)
- **d** 5.2 m^2 (cm^2)
- **e** 0.01 m^3 (cm^3)
- **f** 53 100 mm^3 (cm^3)
- **g** 3100 mL (L)
- **h** 0.043 L (mL)
- **i** 2.83 kL (L)

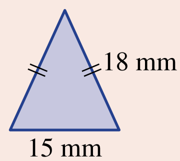
5A

2 Find the perimeter of each of the following shapes.

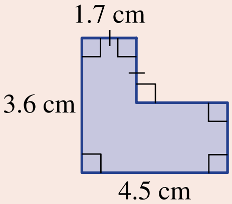
a



b



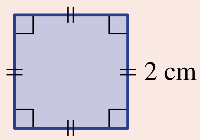
c



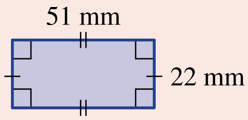
5C

3 Find the area of each of the following plane figures.

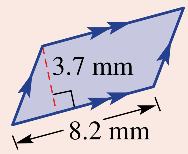
a



b



c



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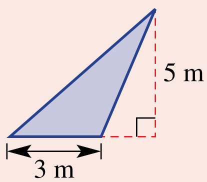
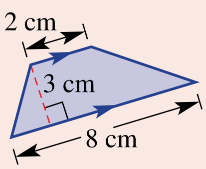
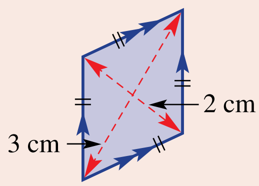


e



f



5D

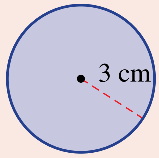
4 A new garden bed is to be constructed inside a rectangular lawn area of length 10.5 m and breadth 3.8 m. The garden bed is to be the shape of a triangle with base 2 m and height 2.5 m. With the aid of a diagram, find:

- **a** the area of the garden bed
- **b** the area of the lawn remaining around the garden bed.

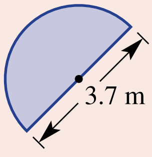
5B/D

5 Find the area and circumference or perimeter of each of the following shapes correct to two decimal places.

a



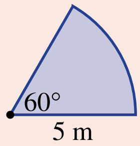
b



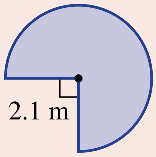
5B/C

6 Find the perimeter and area of these sectors. Round to two decimal places.

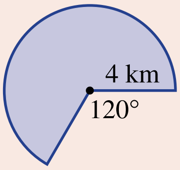
a



b



c



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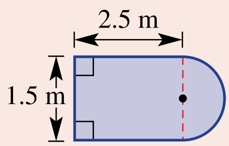
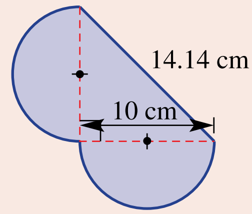
Chapter review

5D **7** Find the perimeter and area of each of the following composite shapes, correct to two decimal places.

a

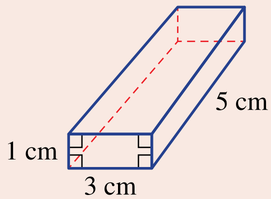


b

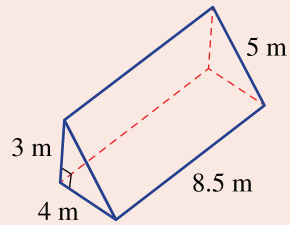


5E **8** Find the surface area of each of these solid objects.

a

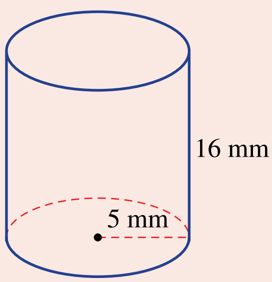


b

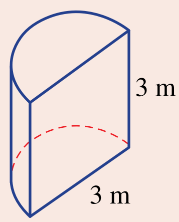


5F **9** Find the surface area of each of the following solid objects, correct to two decimal places.

a



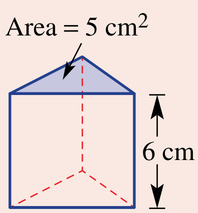
b



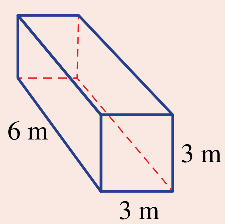
Half-cylinder

5G/H **10** Find the volume of each of these solid objects. Answer to the nearest litre in part **c**.

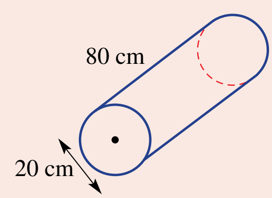
a



b



c



Multiple-choice questions

5A/C **1** If the area of a square field is 25 km^2, its perimeter is:

A 10 km **B** 20 km **C** 5 km **D** 50 km **E** 25 km

5C **2** 2.7 m^2 is the same as:

A 270 cm^2 **B** 0.0027 km^2 **C** 27 000 cm^2 **D** 2700 mm^2 **E** 27 cm^2

5C **3** A parallelogram has area 10 m^2 and base 5 m. Its perpendicular height is:

A 50 m **B** 2 m **C** 50 m^2 **D** 2 m^2 **E** 0.5 m

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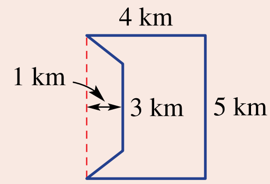
Chapter review

5D **4** This composite shape could be considered as a rectangle with an area in the shape of a trapezium removed. The shape's area is:

A 16 km^2 **B** 12 km^2 **C** 20 km^2
D 24 km^2 **E** 6 km^2

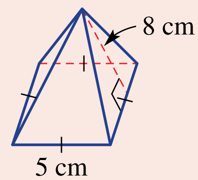


5B **5** A semicircular goal area has a diameter of 20 m. Its perimeter correct to the nearest metre is:

A 41 m **B** 36 m **C** 83 m **D** 51 m **E** 52 m

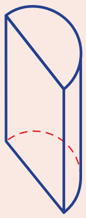
5E **6** The surface area of the pyramid shown is:

A 185 cm^2 **B** 105 cm^2 **C** 65 cm^2
D 100 cm^2 **E** 125 cm^2



5F **7** The area of the curved surface only of this half cylinder with radius 5 mm and height 12 mm is closest to:

A 942.5 mm^2 **B** 94.2 mm^2 **C** 377 mm^2
D 471.2 mm^2 **E** 188.5 mm^2



5G **8** The cross-sectional area of a prism is 100 m^2. If its volume is 6500 m^3, the prism's total perpendicular height would be:

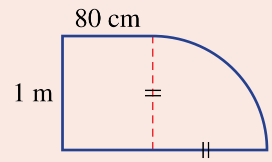
A 0.65 m **B** 650 000 m **C** 65 m **D** 6.5 m **E** 650 m

5H **9** The exact volume of a cylinder with radius 3 cm and height 10 cm is:

A 60π cm **B** 80π cm **C** 45π cm^3 **D** 30π cm^3 **E** 90π cm^3

Extended-response questions

1 An office receives five new desks with a bench shape made up of a rectangle and quarter-circle as shown.
The edge of the bench is lined with a rubber strip at a cost of \$2.50 per metre.



a Find the length, in centimetres, of the rubber edging strip for one desk correct to two decimal places.
b By converting your answer in part **a** to metres, find the total cost of the rubber strip for the five desks. Round to the nearest dollar.

The manufacturer claims that the desktop area space is more than 1.5 m^2.

c Find the area of the desktop in cm^2 correct to two decimal places.
d Convert your answer to m^2 and determine whether or not the manufacturer's claim is correct.

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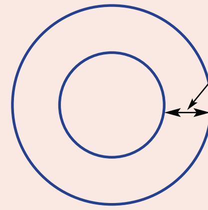
2 Circular steel railing of diameter 6 cm is to be used to fence the side of a bridge. The railing is hollow and the radius of the hollow circular space is 2 cm.



a By adding the given information to this diagram of the cross-section of the railing, determine the thickness of the steel.

b Determine, correct to two decimal places, the area of steel in the cross-section.

Eight lengths of railing each 10 m long are required for the bridge.

c Using your result from part **b**, find the volume of steel required for the bridge in cm^3.

d Convert your answer in part **c** to m^3.

The curved outside surface of the steel railings is to be painted to help protect the steel from the weather.

e Find the outer circumference of the cross-section of each railing correct to two decimal places.

f Find the surface area in m^2 of the eight lengths of railing that are to be painted. Round to the nearest m^2.

The cost of the railing paint is $80 per m^2.

g Using your answer from part **f**, find the cost of painting the bridge rails to the nearest dollar.

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Chapter 1: Computation and financial maths

Short-answer questions

1 Evaluate the following.

a $\frac{3}{7}+\frac{1}{4}$ b $2\frac{1}{3}-1\frac{5}{9}$ c $\frac{9}{10}\times\frac{5}{12}$ d $3\frac{3}{4}\div 2\frac{1}{12}$

2 Convert each of the following to a percentage.

a 0.6 b $\frac{5}{16}$ c 2 kg out of 20 kg d 75c out of \$3

3 Write these rates and ratios in simplest form.

a Prize money is shared between two people in the ratio 60 : 36.

b Jodie travels 165 km in 3 hours.

c 3 mL of rain falls in $1\frac{1}{4}$ hours.

4 Calculate the new value.

a Increase \$60 by 12% b Decrease 70 cm by 8%

5 Jeff earns a weekly retainer of \$400 plus 6% of the sales he makes. If he sells \$8200 worth of goods, how much will he earn for the week?

Multiple-choice questions

1 $-3+(4+(-10))\times(-2)$ is equal to:

A 21 B 9 C 18 D -15 E -6

2 The estimate of $221.7 \div 43.4 - 0.0492$ using one significant figure rounding is:

A 4.9 B 5.06 C 5.5 D 5 E 4.95

3 \$450 is divided in the ratio 4 : 5. The value of the smaller portion is:

A \$210 B \$250 C \$90 D \$200 E \$220

4 A book that costs \$27 is discounted by 15%. The new price is:

A \$20.25 B \$31.05 C \$4.05 D \$22.95 E \$25.20

5 Anna is paid a normal rate of \$12.10 per hour. If in a week she works 6 hours at the normal rate, 2 hours at time and a half and 3 hours at double time, how much does she earn?

A \$181.50 B \$145.20 C \$193.60 D \$175.45 E \$163.35

Extended-response question

Jim and Jill are trialling new banking arrangements.

a Jill plans to trial a simple interest plan. Before investing her money she increases the amount in her account by 20% to \$21 000.

i What was the original amount in her account?

ii Jill invests the \$21 000 for 4 years at an interest rate of 3% p.a. How much does she have in her account at the end of the 4 years?

iii Jill continues with this same plan and after a certain number of years has received \$5670 interest. For how many years has she had the money invested?

iv What percentage increase does this interest represent on her initial investment?

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b Jim is investing his \$21 000 in an account that compounds annually at 3% p.a. How much does he have after 4 years to the nearest cent?

c **i** Who had the more money in their investment after 4 years and by how much? Round to the nearest dollar.

ii Who will have the more money after 10 years and by how much? Round to the nearest dollar.

Chapter 2: Expressions, equations and inequalities

Short-answer questions

1 Solve the following equations and inequalities.

a $3x + 7 = 25$

b $\frac{2x - 1}{4} > 2$

c $4(2m + 3) = 15$

d $-3(2y + 4) - 2y = -4$

e $3(a + 1) \leqslant 4 - 8a$

f $3(2x - 1) = -2(4x + 3)$

2 Noah receives m dollars pocket money per week. His younger brother Jake gets \$6, which is half of \$3 less than Noah's amount.

a Write an equation to represent the problem.

b Solve the equation in part **a** to determine how much Noah receives each week.

3 The formula $S = \frac{n}{2}(a + l)$ gives the sum S of a sequence of n numbers with first term a and last term l.

a Find the sum of the sequence of ten terms 2, 5, 8, …, 29.

b Rearrange the formula to make l the subject.

c If a sequence of eight terms has a sum of 88 and a first term equal to 4, use your answer to part **b** to find the last term of this sequence.

4 Solve the following equations simultaneously.

a $x + 4y = 18$
$x = 2y$

b $7x - 2y = 3$
$y = 2x - 3$

c $2x + 3y = 4$
$x + y = 3$

d $3x + 4y = 7$
$5x + 2y = -7$

Multiple-choice questions

1 The simplified form of $5ab + 6a \div 2 + a \times 2b - a$ is:

A $10ab$ **B** $10ab + 3a$ **C** $13ab - a$
D $5ab + 3a + 2b$ **E** $7ab + 2a$

2 The expanded form of $-2(3m - 4)$ is:

A $-6m + 8$ **B** $-6m + 4$ **C** $-6m - 8$ **D** $-5m - 6$ **E** $5m + 8$

3 The solution to $\frac{d}{4} - 7 = 2$ is:

A $d = -20$ **B** $d = 15$ **C** $d = 36$ **D** $d = 1$ **E** $d = 30$

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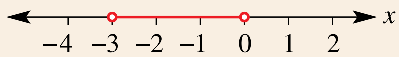
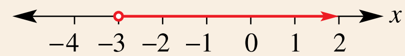
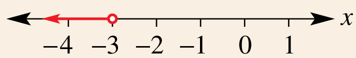
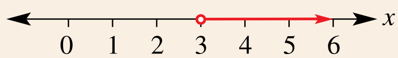
4 The solution to $1 - 3x < 10$ represented on a number line is:

A


B


C


D


E


5 The formula $m = \sqrt{\frac{b-1}{a}}$ with b as the subject is:

A $b = a\sqrt{m} + 1$ **B** $b = \frac{m^2}{a} + 1$ **C** $b = a^2m^2 + 1$

D $b = am^2 + 1$ **E** $b = m^2\sqrt{a} + 1$

Extended-response question

a Chris referees junior basketball games on a Sunday. He is paid \$20 plus \$12 per game he referees. He is trying to earn more than \$74 one Sunday. Let x be the number of games he referees.

- **i** Write an inequality to represent the problem.
- **ii** Solve the inequality to find the minimum number of games he must referee.

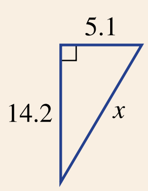
b Two parents support the game by buying raffle tickets and badges. One buys 5 raffle tickets and 2 badges for \$11.50 while the other buys 4 raffle tickets and 3 badges for \$12. Determine the cost of a raffle ticket and the cost of a badge by:

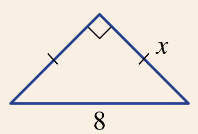
- **i** defining two variables
- **ii** setting up two equations to represent the problem
- **iii** solving your equations simultaneously.

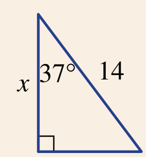
Chapter 3: Right-angled triangles: Pythagoras' theorem and trigonometry

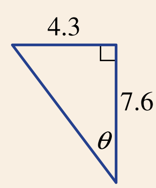
Short-answer questions

1 Find the value of the pronumerals, correct to one decimal place.

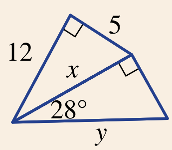
a


b


c


d


2 Find the value of the pronumerals. Round to one decimal place where necessary.

a


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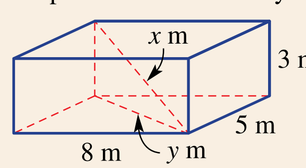

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3 A wire is to be connected from the edge of the top of a 28 m high building to the edge of the top of a 16 m high building. The buildings are 15 m apart.

a What length of wire, to the nearest centimetre, is required?

b What is the angle of depression from the top of the taller building to the top of the smaller building? Round to one decimal place.

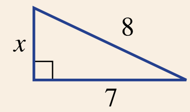
4 A yacht sails 18 km from its start location on a bearing of 295°T.

a How far east or west is it from its start location? Answer correct to one decimal place.

b On what true bearing would it need to sail to return directly to its start location?

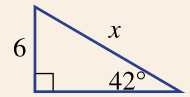
Multiple-choice questions

1 The exact value of x in the triangle shown is:



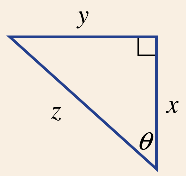
A 3.9 **B** $\sqrt{113}$ **C** $\sqrt{57}$
D $\sqrt{15}$ **E** 2.7

2 The correct expression for the triangle shown is:



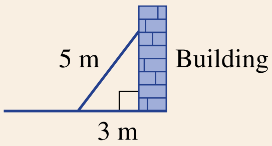
A $x = \dfrac{6}{\sin 42°}$ **B** $x = 6\tan 42°$ **C** $x = 6\sin 42°$
D $x = \dfrac{6}{\cos 42°}$ **E** $x = \dfrac{\sin 42°}{6}$

3 The correct expression for the angle θ is:



A $\theta = \tan^{-1}\left(\frac{x}{z}\right)$ **B** $\theta = \sin^{-1}\left(\frac{x}{z}\right)$ **C** $\theta = \tan^{-1}\left(\frac{y}{x}\right)$
D $\theta = \cos^{-1}\left(\frac{z}{x}\right)$ **E** $\theta = \cos^{-1}\left(\frac{x}{y}\right)$

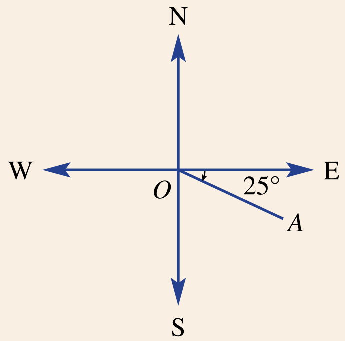
4 A 5 m plank of wood is leaning up against the side of a building as shown. If the plank touches the ground 3 m from the base of the building, the angle the wood makes with the building is closest to:



A 36.9° **B** 59° **C** 53.1° **D** 31° **E** 41.4°

5 The true bearing of A from O is:

A 025° **B** 125° **C** 155° **D** 065° **E** 115°



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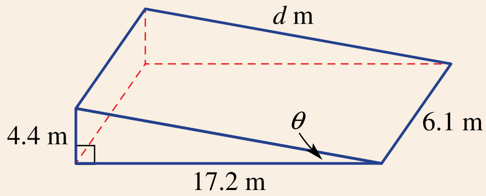
Extended-response question

A skateboard ramp is constructed as shown.

a Calculate the distance d metres up the ramp, correct to two decimal places.



b What is the angle of inclination (θ) between the ramp and the ground, correct to one decimal place?

c i A skateboarder rides from one corner of the ramp diagonally to the other corner. What distance would be travelled? Round to one decimal place.

ii If the skateboarder travels at an average speed of 10 km/h, how many seconds does it take to ride diagonally across the ramp? Answer correct to one decimal place.

Chapter 4: Linear relationships

Short-answer questions

1 Sketch the following linear graphs labelling x- and y-intercepts.

a $y = 2x - 6$

b $3x + 4y = 24$

c $y = 4x$

2 Find the gradient of each of the following.

a The line passing through the points $(-1, 2)$ and $(2, 4)$

b The line passing through the points $(-2, 5)$ and $(1, -4)$

c The line with equation $y = -2x + 5$

d The line with equation $-4x + 3y = 9$

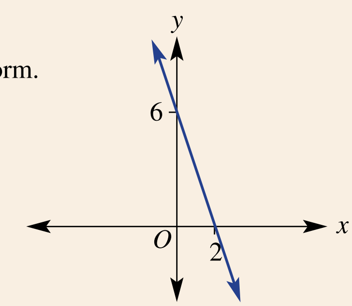
3 Give the equation of the following lines in gradient–intercept form.

a The line with the given graph

b The line with gradient 3 and passing through the point $(2, 5)$

c The line parallel to the line with equation $y = 2x - 1$ and passing through the origin

d The line perpendicular to the line with equation $y = 3x + 4$ and passing through the point $(0, 2)$



4 Solve the simultaneous equations $y = 2x - 4$ and $x + y = 5$ graphically by finding the coordinates of the point of intersection.

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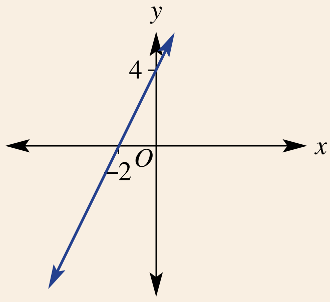
Multiple-choice questions

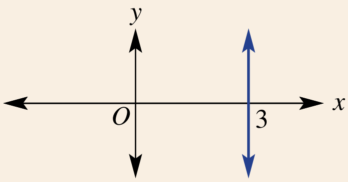
1 The coordinates of the x- and y-intercepts respectively for the graph shown are:

A $(-2, 4)$ and $(4, -2)$
B $(0, 4)$ and $(-2, 0)$
C $(-2, 0)$ and $(0, 4)$
D $(4, 0)$ and $(0, -2)$
E $(2, 0)$ and $(0, -4)$



2 The graph shown has equation:

A $y = 3x$
B $y = 3$
C $y = x + 3$
D $x = 3$
E $x + y = 3$



3 If the point $(-1, 3)$ is on the line $y = 2x + c$, the value of c is:

A 1 B 5 C -7 D -5 E -1

4 The line passing through the points $(-3, -1)$ and $(1, y)$ has gradient 2. The value of y is:

A 3 B 5 C 7 D 1 E 4

5 The midpoint and length (to one decimal place) of the line segment joining the points $(-2, 1)$ and $(4, 6)$ are:

A $(1, 3.5)$ and 7.8 B $(3, 5)$ and 5.4 C $(3, 3.5)$ and 6.1
D $(1, 3.5)$ and 3.3 E $(3, 3.5)$ and 3.6

Extended-response question

Doug works as a labourer. He is digging a trench and has 180 kg of soil to remove. He has taken 3 hours to remove 36 kg.

a What is the rate at which he is removing the soil?

b If he maintains this rate, write a rule for the amount of soil, S (kg), remaining after t hours.

c Draw a graph of your rule.

d How long will it take to remove all of the soil?

e Doug is paid \$40 for the job plus \$25 per hour worked.
 i Write a rule for his pay P dollars for working h hours.
 ii How much will he be paid to remove all the soil?

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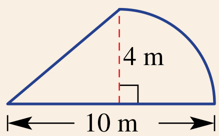
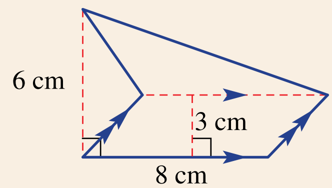
Chapter 5: Length, area, surface area and volume

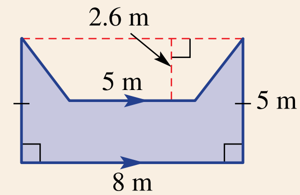
Short-answer questions

1 Find the area of each of the figures below. Round to two decimal places where necessary.

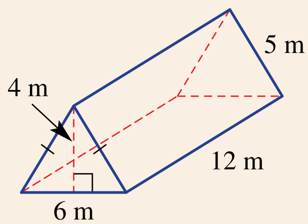
a


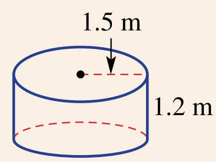
b


2 A tin of varnish for the timber (shaded) on the deck shown covers 6.2 square metres. How many tins will be required to completely varnish the deck?

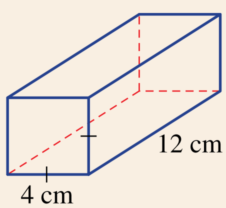


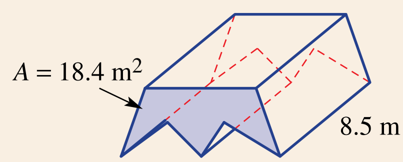
3 Find the surface area of these solid objects. Round to two decimal places where necessary.

a


b


4 Find the volume of the given prisms.

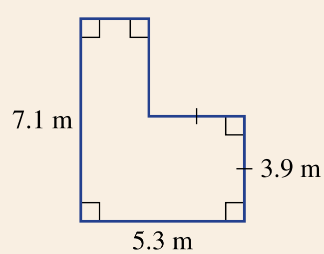
a


b


Multiple-choice questions

1 The perimeter and area of the figure shown are:

A 20.2 m, 22.42 m^2
B 24.8 m, 22.42 m^2
C 24.8 m, 25.15 m^2
D 20.2 m, 25.15 m^2
E 21.6 m, 24.63 m^2



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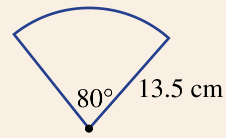
2 The exact perimeter in centimetres of this sector is:



A 127.2

B $\frac{81\pi}{2} + 27$

C $12\pi + 27$

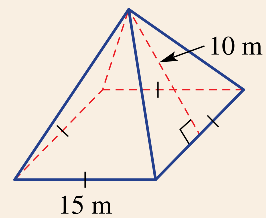
D 45.8

E $6\pi + 27$

3 420 cm^2 is equivalent to:

A 4.2 m^2 B 0.42 m^2 C 42 000 m^2 D 0.042 m^2 E 0.0042 m^2

4 This square pyramid has a surface area of:



A 525 m^2

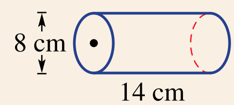
B 300 m^2

C 750 m^2

D 450 m^2

E 825 m^2

5 The volume of the cylinder shown is closest to:

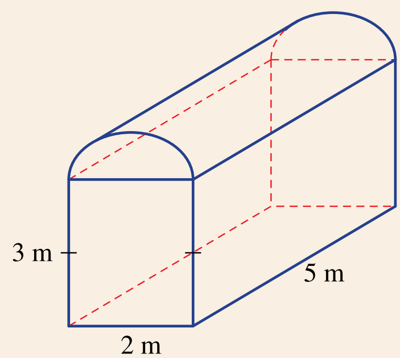


A 703.7 cm^3 B 351.9 cm^3 C 2814.9 cm^3

D 452.4 cm^3 E 1105.8 cm^3

Extended-response question

A barn in the shape of a rectangular prism with a semicylindrical roof and with the dimensions shown is used to store hay.



a The roof and the two long side walls of the barn are to be painted. Calculate the surface area to be painted correct to two decimal places.

b A cylindrical paint roller has a width of 20 cm and a radius of 3 cm.

i Find the area of the curved surface of the paint roller in m^2. Round to four decimal places.

ii Hence, state the area that the roller will cover in 100 revolutions.

c Find the minimum number of revolutions required to paint the area of the barn in part **a** with one coat.

d Find the volume of the barn correct to two decimal places.

e A rectangular bail of hay has dimensions 1 m by 40 cm by 40 cm. If there are 115 bails of hay in the barn, what volume of air space remains? Answer to two decimal places.

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6

Indices and surds

Maths in context: Indices and the frequencies of piano notes

An octave is group of consecutive notes (i.e., 12 semi-tones). If a C starts an octave, then this C's frequency is doubled for the next higher C. A piano's lowest C frequency multiplied by $2^1, 2^2, 2^3, 2^4$, *etc.*, calculates all its higher C frequencies. The highest C frequency multiplied by $2^{-1}, 2^{-2}, 2^{-3}, 2^{-4}$, *etc.*, gives all its lower C frequencies.

Within an octave, from one semitone to the next, the frequency increases by $\times\, 2^{\frac{1}{12}}$ because $\left(2^{\frac{1}{12}}\right)^{12} = 2^{\frac{1}{12}\times 12} = 2$. Hence in one octave, the 11 notes above C are found by multiplying the C frequency by $2^{\frac{1}{12}}, \left(2^{\frac{1}{12}}\right)^2, \ldots \left(2^{\frac{1}{12}}\right)^{10}, \left(2^{\frac{1}{12}}\right)^{11}$.

The Middle C frequency is 261.6 Hz (hertz = cycles/s).

The 2nd note, C sharp, has frequency $261.6 \times 2^{\frac{1}{12}} =$ 277.2 Hz.

The 3rd note, D, has frequency $261.6 \times \left(2^{\frac{1}{12}}\right)^2 =$ 293.6 Hz.

The 12th note, B, has frequency $261.6 \times \left(2^{\frac{1}{12}}\right)^{11} =$ 408.9 Hz.

The next higher C frequency is $261.6 \times \left(2^{\frac{1}{12}}\right)^{12} = 261.6 \times 2 = 523.2$ Hz.

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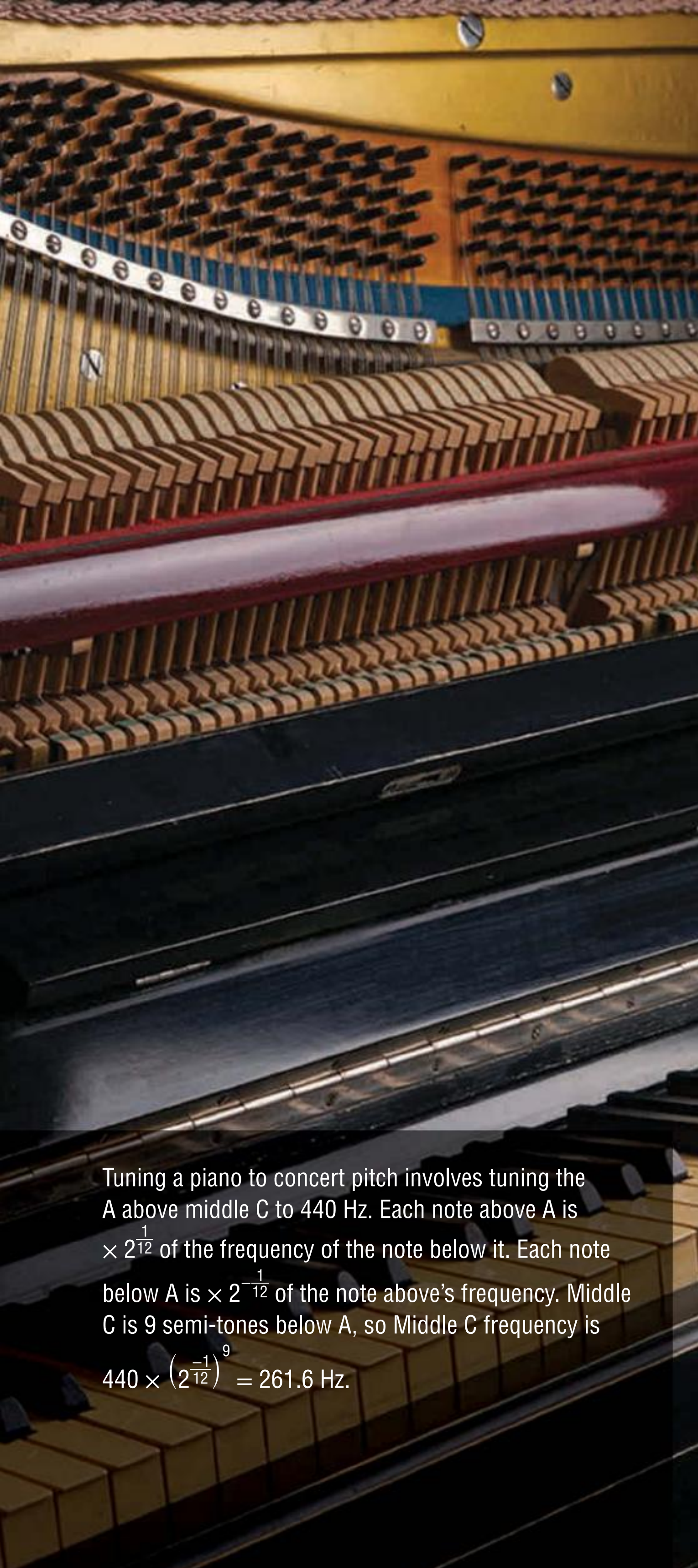

Tuning a piano to concert pitch involves tuning the A above middle C to 440 Hz. Each note above A is $\times 2^{\frac{1}{12}}$ of the frequency of the note below it. Each note below A is $\times 2^{-\frac{1}{12}}$ of the note above's frequency. Middle C is 9 semi-tones below A, so Middle C frequency is $440 \times \left(2^{\frac{-1}{12}}\right)^9 = 261.6$ Hz.

Chapter contents

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- simplifies algebraic fractions involving indices, and expands and factorises algebraic expressions (MA5-ALG-P-01)
- simplifies algebraic expressions involving positive-integer and zero indices, and establishes the meaning of negative indices for numerical bases (MA5-IND-C-01)
- applies the index laws to operate with algebraic expressions involving negative-integer indices (MA5-IND-P-01)
- describes and performs operations with surds and fractional indices (MA5-IND-P-02)
- solves measurement problems by using scientific notation to represent numbers and rounding to a given number of significant figures (MA5-MAG-C-01).

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

6A Index notation

Learning intentions

- To understand index notation and know that it is a way of writing repeated multiplication
- To be able to convert between expanded form and index form
- To be able to evaluate expressions written in index form
- To be able to express a number as a product of prime factors

Past, present and future learning

- This section addresses the Stage 5 Core Topic called *Indices A*
- These concepts will be revised, briefly, in Chapter 3 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

When a product includes the repeated multiplication of the same factor, indices can be used to produce a more concise expression. For example, $5 \times 5 \times 5$ can be written as 5^3 and $x \times x \times x \times x \times x$ can be written as x^5. The expression 5^3 is called a power, and we say '5 to the power of 3'. The 5 is called the base and the 3 is the index, exponent or power. Numbers written with indices are common in mathematics and can be applied to many types of problems. The mass of a 100 kg limestone block, for example, might decrease by 2 per cent per year for 20 years. The mass after 20 years could be calculated by multiplying 100 by 0.98, 20 times. This is written as $100 \times (0.98)^{20}$.

A wide range of people use the index laws in their profession, including scientists, chemists, radiographers, financial planners, economists, actuaries, astronomers, geologists, computer programmers, and medical, electrical, sound and aerospace engineers.

Lesson starter: Who has the most?

A person offers you one of two prizes.

Prize A
\$1000 now

Prize B
1 cent then doubled every day for 20 days

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- Which offer would you take?
- Try to calculate the final amount for prize B.
- How might you use indices to help calculate the value of prize B?
- How can a calculator help to find the amount for prize B using the power button $\boxed{\wedge}$?

KEY IDEAS

- **Indices** (plural of **index**) can be used to represent a product of the same factor.
- The **base** is the factor in the product.

Expanded form Index form

$$\overbrace{2 \times 2 \times 2 \times 2 \times 2} = 2^5 = 32$$

base index basic numeral

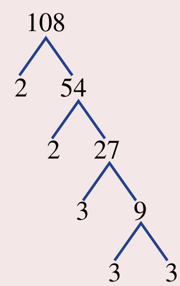
- The index (**exponent** or **power**) is the number of times the factor (base number) is written.

$$x \times x \times x \times x = x^4$$

base index

- Note that $a^1 = a$.
  For example: $5^1 = 5$
- **Prime factorisation** involves writing a number as a product of its prime factors.

$$108 = 2 \times 2 \times 3 \times 3 \times 3$$
$$= \underbrace{2^2 \times 3^3}_{\text{prime factor form}}$$



BUILDING UNDERSTANDING

1 State the missing index.

a $5^{\square} = 25$ b $2^{\square} = 8$

c $(-2)^{\square} = -8$ d $(-4)^{\square} = 16$

2 State the number or pronumeral that is the base in these expressions.

a 3^7 b $(-7)^3$

c $\left(\frac{2}{3}\right)^4$ d y^{10}

3 State the number that is the index in these expressions.

a 4^3 b $(-3)^7$

c $\left(\frac{1}{2}\right)^4$ d $(xy)^{13}$

4 State the prime factors of these numbers.

a 6 b 15

c 30 d 77

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Example 1 Writing in expanded form

Write in expanded form:

a a^3 **b** $(xy)^4$ **c** $2a^3b^2$

SOLUTION

a $a^3 = a \times a \times a$

b $(xy)^4 = xy \times xy \times xy \times xy$

c $2a^3b^2 = 2 \times a \times a \times a \times b \times b$

EXPLANATION

Factor a is written three times.

Factor xy is written four times.

Factor a is written three times and factor b is written twice. Factor 2 only appears once.

Now you try

Write in expanded form:

a b^4 **b** $(mn)^3$ **c** $5a^2b$

Example 2 Expanding and evaluating

Write each of the following in expanded form and then evaluate.

a 5^3 **b** $(-2)^5$ **c** $\left(\frac{2}{5}\right)^3$

SOLUTION

a $5^3 = 5 \times 5 \times 5$
$= 125$

b $(-2)^5 = (-2) \times (-2) \times (-2) \times (-2) \times (-2)$
$= -32$

c $\left(\frac{2}{5}\right)^3 = \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}$
$= \frac{8}{125}$

EXPLANATION

Write in expanded form with 5 written three times and evaluate.

Write in expanded form with -2 written five times and evaluate.

Write in expanded form. Evaluate by multiplying numerators and denominators.

Now you try

Write each of the following in expanded form and then evaluate.

a 2^4 **b** $(-3)^3$ **c** $\left(\frac{4}{3}\right)^2$

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Example 3 Writing in index form

Write each of the following in index form.

a $6 \times x \times x \times x \times x$

b $\frac{3}{7} \times \frac{3}{7} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}$

c $8 \times a \times a \times 8 \times b \times b \times a \times b$

SOLUTION

a $6 \times x \times x \times x \times x = 6x^4$

b $\frac{3}{7} \times \frac{3}{7} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = \left(\frac{3}{7}\right)^2 \times \left(\frac{4}{5}\right)^3$

c $8 \times a \times a \times 8 \times b \times b \times a \times b$
$= 8 \times 8 \times a \times a \times a \times b \times b \times b$
$= 8^2a^3b^3$

EXPLANATION

Factor x is written 4 times, 6 only once.

There are two groups of $\left(\frac{3}{7}\right)$ and three groups of $\left(\frac{4}{5}\right)$.

Group the numerals and like pronumerals and write in index form.

Now you try

Write each of the following in index form.

a $2 \times y \times y \times y$

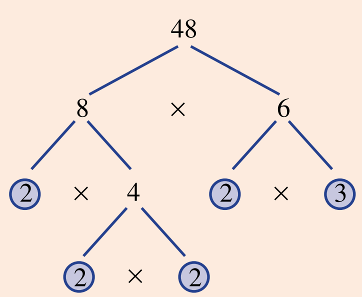
b $\frac{5}{9} \times \frac{5}{9} \times \frac{5}{9} \times \frac{3}{7} \times \frac{3}{7}$

c $3 \times a \times 3 \times 3 \times a$

Example 4 Finding the prime factor form

Express 48 as a product of its prime factors in index form.

SOLUTION



$\therefore 48 = 2 \times 2 \times 2 \times 2 \times 3$
$= 2^4 \times 3$

EXPLANATION

Choose a pair of factors of 48, for example 8 and 6.
Choose a pair of factors of 8, i.e. 2 and 4.
Choose a pair of factors of 6, i.e. 2 and 3.
Continue this process until the factors are all prime numbers (circled numbers).

Write the prime factors of 48.
Express in index notation.

Now you try

Express 60 as a product of its prime factors in index form.

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Exercise 6A

FLUENCY	1–6(½)	1–7(½)	1–7(⅓)

Example 1
1 Write each of the following in expanded form.
a b^3 b x^3 c $(5a)^4$ d $(3y)^3$
e $(pq)^2$ f $-3s^3t^2$ g $6x^3y^5$ h $5(yz)^6$

Example 2
2 Write each of the following in expanded form and then evaluate.
a 6^2 b 2^4 c 3^5 d 12^1
e $(-2)^3$ f $(-1)^7$ g $(-3)^4$ h $(-5)^2$
i $\left(\frac{2}{3}\right)^3$ j $\left(\frac{3}{4}\right)^2$ k $\left(\frac{1}{6}\right)^3$ l $\left(\frac{5}{2}\right)^2$
m $\left(\frac{2}{-3}\right)^3$ n $\left(\frac{-3}{4}\right)^4$ o $\left(\frac{-1}{4}\right)^2$ p $\left(\frac{5}{-2}\right)^5$

Example 3a
3 Write each of the following in index form.
a $3 \times 3 \times 3$ b $8 \times 8 \times 8 \times 8 \times 8 \times 8$
c $y \times y$ d $3 \times x \times x \times x$
e $4 \times c \times c \times c \times c \times c$ f $5 \times 5 \times 5 \times d \times d$
g $x \times x \times y \times y \times y$ h $7 \times b \times 7 \times b \times 7$

Example 3b
4 Write each of the following in index form.
a $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$ b $\frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}$
c $\frac{4}{7} \times \frac{4}{7} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}$ d $\frac{7x}{9} \times \frac{7x}{9} \times \frac{y}{4} \times \frac{y}{4} \times \frac{y}{4}$

Example 3c
5 Write each of the following in index form.
a $3 \times x \times y \times x \times 3 \times x \times 3 \times y$ b $3x \times 2y \times 3x \times 2y$
c $4d \times 2e \times 4d \times 2e$ d $6by(6by)(6y)$
e $3pq(3pq)(3pq)(3pq)$ f $7mn \times 7mn \times mn \times 7$

Example 4
6 Express each of the following as a product of prime factors in index form.
a 10 b 8 c 144
d 512 e 216 f 500

7 If $a = 3$, $b = 2$ and $c = -3$, evaluate these expressions.
a $(ab)^2$ b $(bc)^3$ c $\left(\frac{a}{c}\right)^4$ d $\left(\frac{b}{c}\right)^3$
e $(abc)^1$ f $c^2 + ab$ g ab^2c h c^2ab^3

PROBLEM–SOLVING	8, 9	8–10	9–11

8 Find the missing number.
a $3^{\square} = 81$ b $2^{\square} = 256$ c $\square^3 = 125$ d $\square^5 = 32$
e $\square^3 = -64$ f $\square^7 = -128$ g $\square^3 = \frac{1}{8}$ h $\left(\frac{2}{3}\right)^{\square} = \frac{16}{81}$

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9 A bacterium splits in two every 5 minutes. New cells also continue splitting in the same way. Use a table to help answer the following.
- a How long will it take for 1 cell to divide into:
  - i 4 cells?
  - ii 16 cells?
  - iii 64 cells?
- b A single cell is set aside to divide for 2 hours. How many cells will there be after this time?

10 A sharebroker says he can triple your money every year, so you invest $1000 with him.
- a How much should your investment be worth in 5 years' time?
- b For how many years should you invest if you were hoping for a total of at least $100 000? Give a whole number of years.

11 The weight of a 12 kg fat cat reduces by 10% each month. How long does it take for the cat to be at least 6 kg lighter than its original weight? Give your answer as a whole number of months.

REASONING	12, 13	12, 13	12, 13, 14(½)

12 a Evaluate the following.
- i 3^2
- ii $(-3)^2$
- iii $-(3)^2$
- iv $-(-3)^2$

b Explain why the answers to parts i and ii are positive.

c Explain why the answers to parts iii and iv are negative.

13 a Evaluate the following.
- i 2^3
- ii $(-2)^3$
- iii $-(2)^3$
- iv $-(-2)^3$

b Explain why the answers to parts i and iv are positive.

c Explain why the answers to parts ii and iii are negative.

14 It is often easier to evaluate a decimal raised to a power by first converting the decimal to a fraction, as shown on the right. Use this idea to evaluate these decimals as fractions.

$$\begin{aligned}(0.5)^4 &= \left(\frac{1}{2}\right)^4\\ &= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\\ &= \frac{1}{16}\end{aligned}$$

- a $(0.5)^3$
- b $(0.25)^2$
- c $(0.2)^3$
- d $(0.5)^6$
- e $(0.7)^2$
- f $(1.5)^4$
- g $(2.6)^2$
- h $(11.3)^2$
- i $(3.4)^2$

ENRICHMENT: LCM and HCF from prime factorisation	–	–	15

15 Last year you may have used prime factorisation to find the LCM (lowest common multiple) and the HCF (highest common factor) of two numbers. Here are the definitions.
- The LCM of two numbers in their prime factor form is the product of all the different primes raised to their highest power.
- The HCF of two numbers in their prime factor form is the product of all the common primes raised to their smallest power.

For example: $12 = 2^2 \times 3$ and $30 = 2 \times 3 \times 5$

The prime factors 2 and 3 are common.

$$\begin{aligned}\therefore \text{HCF} &= 2\times 3\\ &= 6\end{aligned} \qquad \begin{aligned}\therefore \text{LCM} &= 2^2\times 3\times 5\\ &= 60\end{aligned}$$

Find the LCM and HCF of these pairs of numbers by first writing them in prime factor form.
- a 4, 6
- b 42, 28
- c 24, 36
- d 10, 15
- e 40, 90
- f 100, 30
- g 196, 126
- h 2178, 1188

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6B Index laws for multiplying and dividing

Learning intentions

- To know the index laws for multiplication and division involving a common base
- To be able to apply the index laws for multiplication and division to simplify expressions

Past, present and future learning

- This section addresses the Stage 5 Core Topic called *Indices A*
- These concepts will be revised, briefly, in Chapter 3 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

An index law (or identity) is an equation that is true for all possible values of the variables in that equation. When multiplying or dividing numbers with the same base, index laws can be used to simplify the expression.

Consider $a^m \times a^n$.

Using expanded form:

$$a^m \times a^n = \underbrace{\overbrace{a \times a \times a \times \ldots \times a}^{m \text{ factors of } a} \times \overbrace{a \times a \times \ldots \times a}^{n \text{ factors of } a}}_{m+n \text{ factors of } a}$$

$$= a^{m+n}$$

So the total number of factors of a is $m + n$.
Also:

$$a^m \div a^n = \frac{\overbrace{a \times a \times \ldots \times \cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{a}}^{m \text{ factors of } a}}{\underbrace{\cancel{a} \times \ldots \times \cancel{a} \times \cancel{a}}_{n \text{ factors of } a}}$$

$$= a^{m-n}$$

So the total number of factors of a is $m - n$.

Biologists and statisticians use indices when modelling exponential population growth, such as, a bacteria population $P = 2^n$ after n generations, and Australia's projected population $P = 25 \times 1.04^n$ in millions, n years from 2018.

Lesson starter: Discovering laws for multiplication and division

Consider the two expressions $2^3 \times 2^5$ and $6^8 \div 6^6$.
Complete this working.

$$2^3 \times 2^5 = 2 \times \square \times \square \times 2 \times \square \times \square \times \square \times \square$$
$$= 2^{\square}$$

$$6^8 \div 6^6 = \frac{6 \times \square \times \square \times \square \times \square \times \square \times \square \times \square}{6 \times \square \times \square \times \square \times \square \times \square}$$
$$= \frac{6 \times 6}{1}$$
$$= 6^{\square}$$

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- What do you notice about the given expression and the answer in each case? Can you express this as a rule or law in words?
- Repeat the type of working given above and test your laws on these expressions.

a $3^2 \times 3^7$ **b** $4^{11} \div 4^8$

KEY IDEAS

- **Index law for multiplication:** $a^m \times a^n = a^{m+n}$
  - When multiplying terms with the same base, add the powers.
- **Index law for division:** $a^m \div a^n = \dfrac{a^m}{a^n} = a^{m-n}$
  - When dividing terms with the same base, subtract the powers.

BUILDING UNDERSTANDING

1 State the missing words.

a The index law for multiplication states that if you ____________ two terms with the same ____________ you ____________ the powers.

b The index law for division states that if you ____________ two terms with the same ____________ you ____________ the powers.

2 Give the missing numbers to complete the following.

a $3^2 \times 3^4 = 3 \times \square \times 3 \times \square \times \square \times \square$
$= 3^{\square}$

b $5^5 \div 5^3 = \dfrac{5 \times \square \times \square \times \square \times \square}{5 \times \square \times \square}$
$= 5^{\square}$

3 Decide if these statements are true or false.

a $5 \times 5 \times 5 \times 5 = 5^4$ **b** $2^6 \times 2^2 = 2^{6+2}$

c $7^2 \times 7^4 = 7^{4-2}$ **d** $8^4 \div 8^2 = 8^{4+2}$

e $a \times a^2 = a^3$ **f** $x^7 \div x = x^7$

Example 5 Using laws for multiplication and division with numerical bases

Simplify, giving your answer in index form.

a $3^6 \times 3^4$ **b** $7^9 \div 7^5$

SOLUTION	EXPLANATION
a $3^6 \times 3^4 = 3^{6+4}$	$a^m \times a^n = a^{m+n}$ (add the powers)
$= 3^{10}$	The base 3 is retained as the power acts as a counter.
b $7^9 \div 7^5 = 7^{9-5}$	$a^m \div a^n = a^{m-n}$ (subtract the powers)
$= 7^4$	

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Now you try

Simplify, giving your answer in index form.

a $2^4 \times 2^5$ **b** $9^7 \div 9^3$

Example 6 Using the index law for multiplication

Simplify each of the following using the index law for multiplication.

a $x^4 \times x^5$ **b** $x^3y^4 \times x^2y$ **c** $3m^4 \times 2m^5$

SOLUTION	EXPLANATION
a $x^4 \times x^5 = x^{4+5}$ $= x^9$	Use the index law for multiplication to add the indices.
b $x^3y^4 \times x^2y = x^{3+2}y^{4+1}$ $= x^5y^5$	Use the index law for multiplication to add the indices corresponding to each different base. Recall $y = y^1$.
c $3m^4 \times 2m^5 = 3 \times 2 \times m^{4+5}$ $= 6m^9$	Multiply the numbers then use the index law for multiplication to add the indices of the base m.

Now you try

Simplify each of the following using the index law for multiplication.

a $x^2 \times x^9$ **b** $a^2b^4 \times ab^3$ **c** $4m^2 \times 7m^3$

Example 7 Using the index law for division

Simplify each of the following using the index law for division.

a $x^{10} \div x^2$ **b** $\dfrac{8a^6b^3}{12a^2b^2}$

SOLUTION	EXPLANATION
a $x^{10} \div x^2 = x^{10-2}$ $= x^8$	Use the index law for division to subtract the indices.
b $\dfrac{8a^6b^3}{12a^2b^2} = \dfrac{{}^{2}\cancel{8}a^{6-2}b^{3-2}}{{}^{3}\cancel{12}}$ $= \dfrac{2a^4b}{3}$	Cancel the numbers using the highest common factor (4) and use the index law for division to subtract the indices for each different base.

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Now you try

Simplify each of the following using the index law for division.

a $a^7 \div a^3$ **b** $\dfrac{4x^6y^4}{10x^3y^3}$

Example 8 Combining the index laws for multiplication and division

Simplify each of the following using the index laws for multiplication and division.

a $x^2 \times x^3 \div x^4$ **b** $\dfrac{2a^3b \times 8a^2b^3}{4a^4b^2}$

SOLUTION

a $x^2 \times x^3 \div x^4 = x^5 \div x^4$
$= x$

b $\dfrac{2a^3b \times 8a^2b^3}{4a^4b^2} = \dfrac{{}^{4}\cancel{16}a^5b^4}{{}^{1}\cancel{4}a^4b^2}$
$= 4ab^2$

EXPLANATION

Add the indices for $x^2 \times x^3$.
Subtract the indices for $x^5 \div x^4$.

Multiply the numbers and use the index law for multiplication to add the indices for each different base in the numerator.
Use the index law for division to subtract the indices of each different base and cancel the 16 with the 4.

Now you try

Simplify each of the following using the index laws for multiplication and division.

a $a^4 \times a^3 \div a^2$ **b** $\dfrac{5a^2b^3 \times 4ab^2}{10ab^3}$

Exercise 6B

FLUENCY	1, 2–5(1/2)	2–6(1/3)	2–6(1/4)

1 Simplify, giving your answers in index form.

Example 5a
a **i** $2^4 \times 2^3$
ii $5^6 \times 5^3$

Example 5b
b **i** $3^7 \div 3^4$
ii $6^8 \div 6^3$

Example 5
2 Simplify, giving your answers in index form.

a $7^2 \times 7^4$ **b** $8^9 \times 8$ **c** $3^4 \times 3^4$ **d** $6^5 \times 6^9$
e $5^4 \div 5$ **f** $10^6 \div 10^5$ **g** $9^9 \div 9^6$ **h** $(-2)^5 \div (-2)^3$

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Example 6

3 Simplify each of the following using the index law for multiplication.

a $x^4 \times x^3$ **b** $a^6 \times a^3$ **c** $t^5 \times t^3$ **d** $y \times y^4$

e $d^2 \times d$ **f** $y^2 \times y \times y^4$ **g** $b \times b^5 \times b^2$ **h** $q^6 \times q^3 \times q^2$

i $x^3y^3 \times x^4y^2$ **j** $x^7y^3 \times x^2y$ **k** $5x^3y^5 \times xy^4$ **l** $xy^4z \times 4xy$

m $3m^3 \times 5m^2$ **n** $4e^4f^2 \times 2e^2f^2$ **o** $5c^4d \times 4c^3d$ **p** $9yz^2 \times 2yz^5$

Example 7

4 Simplify each of the following using the index law for division.

a $a^6 \div a^4$ **b** $x^5 \div x^2$ **c** $\frac{q^{12}}{q^2}$ **d** $\frac{d^7}{d^6}$

e $\frac{8b^{10}}{4b^5}$ **f** $\frac{12d^{10}}{36d^5}$ **g** $\frac{4a^{14}}{2a^7}$ **h** $\frac{18y^{15}}{9y^7}$

i $9m^3 \div m^2$ **j** $14x^4 \div x$ **k** $5y^4 \div y^2$ **l** $6a^6 \div a^5$

m $\frac{3m^7}{12m^2}$ **n** $\frac{5w^2}{25w}$ **o** $\frac{4a^4}{20a^3}$ **p** $\frac{7x^5}{63x}$

q $\frac{16x^8y^6}{12x^2y^3}$ **r** $\frac{6s^6t^3}{14s^5t}$ **s** $\frac{8m^5n^4}{6m^4n^3}$ **t** $-\frac{5x^2y}{xy}$

Example 8

5 Simplify each of the following using the index laws for multiplication and division.

a $b^5 \times b^2 \div b$ **b** $y^5 \times y^4 \div y^3$ **c** $c^4 \div c \times c^4$

d $x^4 \times x^2 \div x^5$ **e** $\frac{t^4 \times t^3}{t^6}$ **f** $\frac{p^2 \times p^7}{p^3}$

g $\frac{d^5 \times d^3}{d^2}$ **h** $\frac{x^9 \times x^2}{x}$ **i** $\frac{3x^3y^4 \times 8xy}{6x^2y^2}$

j $\frac{9b^4}{2g^3} \times \frac{4g^4}{3b^2}$ **k** $\frac{24m^7n^5}{5m^3n} \times \frac{5m^2n^4}{8mn^2}$ **l** $\frac{p^4q^3}{p^2q} \times \frac{p^6q^4}{p^3q^2}$

6 Simplify each of the following.

a $\frac{m^4}{n^2} \times \frac{m}{n^3}$ **b** $\frac{x}{y} \times \frac{x^3}{y}$ **c** $\frac{a^4}{b^3} \times \frac{b^6}{a}$

d $\frac{12a}{3c^3} \times \frac{6a^4}{4c^4}$ **e** $\frac{3f^2 \times 8f^7}{4f^3}$ **f** $\frac{4x^2b \times 9x^3b^2}{3xb}$

g $\frac{8k^4m^5}{5km^3} \times \frac{15km}{4k}$ **h** $\frac{12x^7y^3}{5x^4y} \times \frac{25x^2y^3}{8xy^4}$ **i** $\frac{9m^5n^2 \times 4mn^3}{12mn \times m^4n^2} \times \frac{m^3n^2}{2m^2n}$

PROBLEM–SOLVING	7–8(½)	7–8(½)	8(½), 9

7 Write the missing number.

a $2^7 \times 2^{\square} = 2^{19}$ **b** $6^{\square} \times 6^3 = 6^{11}$

c $11^6 \div 11^{\square} = 11^3$ **d** $19^{\square} \div 19^2 = 19$

e $x^6 \times x^{\square} = x^7$ **f** $a^{\square} \times a^2 = a^{20}$

g $b^{13} \div b^{\square} = b$ **h** $y^{\square} \div y^9 = y^2$

i $\square \times x^2 \times 3x^4 = 12x^6$ **j** $15y^4 \div (\square y^3) = y$

k $\square a^9 \div (4a) = \frac{a^8}{2}$ **l** $13b^6 \div (\square b^5) = \frac{b}{3}$

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8 Evaluate without using a calculator.

a $7^7 \div 7^5$
b $10^6 \div 10^5$
c $13^{11} \div 13^9$
d $2^{20} \div 2^{17}$
e $101^5 \div 101^4$
f $200^{30} \div 200^{28}$
g $7 \times 31^{16} \div 31^{15}$
h $3 \times 50^{200} \div 50^{198}$

9 If m and n are positive integers, how many combinations of m and n satisfy the following?

a $a^m \times a^n = a^8$
b $a^m \times a^n = a^{15}$

REASONING	10	10, 11(½)	11–12(½)

10 The given answers are incorrect. Give the correct answer and explain the error made.

a $a^4 \times a = a^4$
b $x^7 \div x = x^7$
c $3a^5 \div 6a^3 = 2a^2$
d $5x^7 \div 10x^3 = \dfrac{1}{2x^4}$
e $2x^7 \times 3x^4 = 5x^{11}$
f $a^5 \div a^2 \times a = a^5 \div a^3 = a^2$

11 Given that $a = 2x$, $b = 4x^2$ and $c = 5x^3$, find expressions for:

a $2a$
b $3b$
c $2c$
d $-2a$
e abc
f $\dfrac{c}{b}$
g $\dfrac{ab}{c}$
h $\dfrac{-2bc}{a}$

12 Simplify these expressions using the given pronumerals.

a $2^x \times 2^y$
b $5^a \times 5^b$
c $t^x \times t^y$
d $3^x \div 3^y$
e $10^p \div 10^y$
f $2^p \times 2^q \div 2^r$
g $10^p \div 10^q \div 10^r$
h $2^a \times 2^{a+b} \times 2^{3a-b}$
i $a^{x-2}b^x \times a^{2x}b^3$
j $a^xb^y \times a^yb^x$
k $a^xb^y \div (a^yb^x)$
l $w^{x+2}b^x \div w^{2x} \times b^3$
m $\dfrac{a^x \times 3a^y}{3a^2}$
n $\dfrac{4p^a \times 5q^b}{20q^5}$
o $\dfrac{10k^xm^y}{8km^3} \div \dfrac{5k^xm^{2x}}{16k}$

ENRICHMENT: Equal to *ab*	–	–	13, 14

13 Show working to prove that these expressions simplify to ab.

a $\dfrac{5a^2b^7}{9a^3b} \times \dfrac{9a^4b^2}{5a^2b^7}$
b $\dfrac{3a^5bc^3}{6a^4c} \times \dfrac{4b^3}{2abc} \times \dfrac{2a^3b^2c}{2a^2b^4c^2}$
c $\dfrac{3a^4b^5}{a^5b^2} \div \dfrac{6b^3}{2a^2b}$
d $\dfrac{2a}{3a^2b^3} \times \dfrac{9a^4b^7}{ab^5} \div \dfrac{6a}{b^2}$

14 Make up your own expressions which simplify to ab. Test them on a friend.

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6C The zero index and power of a power

Learning intentions

- To know the index law for raising a power to another power
- To know the rule for the zero index
- To be able to simplify expressions using the index laws and zero index

Past, present and future learning

- This section addresses the Stage 5 Core Topic called *Indices A*
- These concepts will be revised, briefly, in Chapter 3 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

Sometimes we find expressions already written in index form are raised to another power, such as $(2^3)^4$ or $(a^2)^5$.

Consider $(a^m)^n$.

$$\begin{aligned} \text{Using expanded form } (a^m)^n &= \overbrace{a^m \times a^m \times \ldots \times a^m}^{n \text{ factors of } a^m} \\ &= \underbrace{\overbrace{a \times a \times \ldots \times a}^{m \text{ factors of } a} \times \overbrace{a \times a \times \ldots \times a}^{m \text{ factors of } a} \times \ldots \times \overbrace{a \times a \times \ldots \times a}^{m \text{ factors of } a}}_{m \times n \text{ factors of } a} \\ &= a^{m \times n} \end{aligned}$$

So the total number of factors of a is $m \times n$.

$$\begin{aligned} \text{We also know that } a^m \div a^m &= \frac{\cancel{a} \times \cancel{a} \times \cancel{a} \times \cdots \times \cancel{a}}{\cancel{a} \times \cancel{a} \times \cancel{a} \times \cdots \times \cancel{a}} \\ &= \frac{1}{1} \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{But using the index law for division: } a^m \div a^m &= a^{m-m} \\ &= a^0 \end{aligned}$$

This implies that $a^0 = 1$.

Lesson starter: Discovering laws for power of a power and the zero index

Use the expanded form of 5^3 to simplify $(5^3)^2$ as shown.

$$\begin{aligned} (5^3)^2 &= (5 \times \square \times \square) \times (5 \times \square \times \square) \\ &= 5^{\square} \end{aligned}$$

- Repeat the above steps to simplify $(3^2)^4$ and $(x^4)^2$.
- What do you notice about the given expression and answer in each case? Can you express this as a law or rule in words?

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Now complete this table.

Index form	3^5	3^4	3^3	3^2	3^1	3^0
Basic numeral	243	81				

- What pattern do you notice in the basic numerals?
- What conclusion do you come to regarding 3^0?

KEY IDEAS

- **Index law power of a power:** $(a^m)^n = a^{mn}$
  - When raising a term in index form to another power, retain the base and multiply the indices. For example: $(x^2)^3 = x^{2\times 3} = x^6$.
- The **zero index:** $a^0 = 1$, where $a \neq 0$
  - Any expression except 0 raised to the power of zero is 1. For example: $(2a)^0 = 1$.

BUILDING UNDERSTANDING

1 State the missing words or numbers in these sentences.

a When raising a term or numbers in index form to another power, ____________ the indices.

b Any expression (except 0) raised to the power 0 is equal to ____________.

2 State the missing numbers in these tables.

a

Index form	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Basic numeral	64	32					

b

Index form	4^5	4^4	4^3	4^2	4^1	4^0
Basic numeral	1024	256				

3 State the missing numbers in this working.

a $(4^2)^3 = (4 \times \square) \times (4 \times \square) \times (4 \times \square)$
$= 4^{\square}$

b $(a^2)^5 = (a \times \square) \times (a \times \square) \times (a \times \square) \times (a \times \square) \times (a \times \square)$
$= a^{\square}$

Example 9 Using the index law for power of a power

Apply the index law for power of a power to simplify each of the following.

a $(x^5)^4$ **b** $3(y^5)^2$

SOLUTION

a $(x^5)^4 = x^{5\times 4}$
$= x^{20}$

EXPLANATION

Retain x as the base and multiply the indices.

Continued on next page

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b $3(y^5)^2 = 3y^{5\times 2}$
$= 3y^{10}$

Retain y and multiply the indices. The power only applies to the bracketed term.

Now you try

Apply the index law for power of a power to simplify each of the following.

a $(a^3)^2$

b $5(b^2)^7$

Example 10 Using the zero index

Apply the zero index rule to evaluate each of the following.

a $(-3)^0$

b $-(5x)^0$

c $2y^0 - (3y)^0$

SOLUTION

a $(-3)^0 = 1$

Any number raised to the power of 0 is 1.

b $-(5x)^0 = -1$

Everything in the brackets is to the power of 0 so $(5x)^0$ is 1.

c $2y^0 - (3y)^0 = 2 \times 1 - 1$
$= 2 - 1$
$= 1$

$2y^0$ has no brackets so the power applies to the y only, so $2y^0 = 2 \times y^0 = 2 \times 1$ while $(3y)^0 = 1$.

Now you try

Apply the zero index rule to evaluate each of the following.

a $(-2)^0$

b $-(4a)^0$

c $6x^0 - (2x)^0$

Example 11 Combining index laws

Simplify each of the following by applying the various index laws.

a $(x^2)^3 \times (x^3)^5$

b $\dfrac{(m^3)^4}{m^7}$

c $\dfrac{4x^2 \times 3x^3}{6x^5}$

SOLUTION

a $(x^2)^3 \times (x^3)^5 = x^6 \times x^{15}$
$= x^{21}$

Use the index law for power of a power to remove brackets first by multiplying indices. Then use the index law for multiplication to add indices.

b $\dfrac{(m^3)^4}{m^7} = \dfrac{m^{12}}{m^7}$
$= m^5$

Remove brackets by multiplying indices then simplify using the index law for division.

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c $\frac{4x^2 \times 3x^3}{6x^5} = \frac{12x^5}{6x^5}$
$= 2x^0$
$= 2 \times 1$
$= 2$

Simplify the numerator first by multiplying numbers and adding indices of base x.
Then cancel and subtract indices.
The zero index rule says $x^0 = 1$.

Now you try

Simplify each of the following by applying the various index laws.

a $(y^3)^4 \times (y^2)^3$ b $\frac{(t^2)^4}{t^3}$ c $\frac{5m^2 \times 2m^4}{10m^5}$

Exercise 6C

FLUENCY	1–5(½)	1–6(½)	1–6(⅓)

Example 9

1 Apply the index law for power of a power to simplify each of the following. Leave your answers in index form.

a $(x^2)^5$ b $(b^3)^4$ c $(a^4)^5$ d $(m^6)^2$
e $(3^2)^3$ f $(4^3)^5$ g $(3^5)^6$ h $(7^5)^2$
i $4(q^7)^4$ j $3(m^2)^5$ k $-3(c^2)^5$ l $-4(a^7)^3$

Example 10a, b

2 Evaluate each of the following.

a 5^0 b 9^0 c $(-6)^0$ d $(-3)^0$
e $-(4^0)$ f $\left(\frac{3}{4}\right)^0$ g $\left(-\frac{1}{7}\right)^0$ h $(4y)^0$

Example 10c

3 Evaluate each of the following.

a $5m^0$ b $-3p^0$ c $6x^0 - 2x^0$ d $-5n^0 - (8n)^0$
e $(3x^4)^0 + 3x^0$ f $1^0 + 2^0 + 3^0$ g $(1 + 2 + 3)^0$ h $100^0 - a^0$

Example 11a

4 Simplify each of the following by combining various index laws.

a $4 \times (4^3)^2$ b $(3^4)^2 \times 3$ c $x \times (x^0)^5$
d $y^5 \times (y^2)^4$ e $b^5 \times (b^3)^3$ f $(a^2)^3 \times a^4$
g $(d^3)^4 \times (d^2)^6$ h $(y^2)^6 \times (y)^4$ i $z^4 \times (z^3)^2 \times (z^5)^3$
j $a^3 f \times (a^4)^2 \times (f^4)^3$ k $x^2 y \times (x^3)^4 \times (y^2)^2$ l $(s^2)^3 \times 5(r^0)^3 \times rs^2$

Example 11b

5 Simplify each of the following.

a $7^8 \div (7^3)^2$ b $(4^2)^3 \div 4^5$ c $(3^6)^3 \div (3^5)^2$
d $(m^3)^6 \div (m^2)^9$ e $(y^5)^3 \div (y^6)^2$ f $(h^{11})^2 \div (h^5)^4$
g $\frac{(b^2)^5}{b^4}$ h $\frac{(x^4)^3}{x^7}$ i $\frac{(y^3)^3}{y^3}$

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Example 11c

6 Simplify each of the following using various index laws.

a $\frac{3x^4 \times 6x^3}{9x^{12}}$ **b** $\frac{5x^5 \times 4x^2}{2x^{10}}$ **c** $\frac{24(x^4)^4}{8(x^4)^2}$

d $\frac{4(d^4)^3 \times (e^4)^2}{8(d^2)^5 \times e^7}$ **e** $\frac{6(m^3)^2(n^5)^3}{15(m^5)^0(n^2)^7}$ **f** $\frac{2(a^3)^4(b^2)^6}{16(a)^0(b^6)^2}$

PROBLEM-SOLVING	7	7, 8	8, 9

7 There are 100 rabbits on Mt Burrow at the start of the year 2010. The rule for the number of rabbits N after t years (from the start of the year 2010) is $N = 100 \times 2^t$.

a Find the number of rabbits when:
i $t = 2$ **ii** $t = 6$ **iii** $t = 0$

b Find the number of rabbits at the beginning of:
i 2013 **ii** 2017 **iii** 2020

c How many years will it take for the population to first rise to more than 500 000? Give a whole number of years.

8 If m and n are positive integers, in how many ways can $(a^m)^n = a^{16}$?

9 Evaluate these without using a calculator.

a $(2^4)^8 \div 2^{30}$ **b** $(10^3)^7 \div 10^{18}$ **c** $(x^4)^9 \div x^{36}$

d $((-1)^{11})^2 \times ((-1)^2)^{11}$ **e** $-2((-2)^3)^3 \div (-2)^8$ **f** $\frac{(a^2)^3}{(b^4)^7} \times \frac{(b^7)^4}{(a^3)^2}$

REASONING	10	10, 11	11, 12

10 Explain the error made in the following problems then give the correct answer.

a $(a^4)^5 = a^9$ **b** $3(x^3)^2 = 9x^6$ **c** $(2x)^0 = 2$

11 a Simplify these by first working with the inner brackets. Leave your answer in index form.

i $((2^3)^4)^2$ **ii** $(((-2)^2)^5)^3$ **iii** $((x^6)^2)^7$ **iv** $(((a^2)^4)^3)^2$

b Simplify these expressions.

i $((2^a)^b)^c$ **ii** $((a^m)^n)^p$ **iii** $(x^{2y})^{3z}$

12 a Show that $\frac{5a^2b}{2ab^2} \div \frac{10a^4b^7}{4a^3b^8}$ is equal to 1.

b Make up your own expression like the one above for which the answer is equal to 1. Test it on a friend.

ENRICHMENT: Changing the base	–	–	13(½)

13 The base of a number in index form can be changed using the index law for power of a power.

For example: $8^2 = (2^3)^2$
$= 2^6$

Change the base numbers and simplify the following using the smallest possible base integer.

a 8^4 **b** 32^3 **c** 9^3

d 81^5 **e** 25^5 **f** 243^{10}

g 256^9 **h** 2401^{20} **i** $100\,000^{10}$

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6D Index laws extended

Learning intentions
- To know the index laws for removing brackets
- To be able to expand and simplify expressions using the index laws
- To be able to combine index laws to simplify complex expressions

Past, present and future learning
- This section addresses the Stage 5 Path Topic called *Indices B*
- These concepts will be revised, briefly, in Chapter 3 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

It is common to find expressions such as $(2x)^3$ and $\left(\frac{x}{3}\right)^4$ in mathematical problems. These differ from most of the expressions in previous sections in that they contain two or more numbers or pronumerals connected by multiplication or division and raised to a power. These expressions can also be simplified using two index laws, which effectively remove the brackets.

Consider $(a \times b)^m$.
Using expanded form:

$$\begin{aligned}(a \times b)^m &= \overbrace{ab \times ab \times ab \times \ \dots \ \times ab}^{m \text{ factors of } ab} \\ &= \underbrace{a \times a \times \ \dots \ \times a}_{m \text{ factors of } a} \times \underbrace{b \times b \times \ \dots \ \times b}_{m \text{ factors of } b} \\ &= a^m \times b^m\end{aligned}$$

So, this becomes a product of m factors of a and m factors of b.

Also:

$$\begin{aligned}\left(\frac{a}{b}\right)^m &= \overbrace{\frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \times \ \dots \ \times \frac{a}{b}}^{m \text{ factors of } \frac{a}{b}} \\ &= \frac{\overbrace{a \times a \times a \times \ \dots \ \times a}^{m \text{ factors of } a}}{\underbrace{b \times b \times b \times \ \dots \ \times b}_{m \text{ factors of } b}} \\ &= \frac{a^m}{b^m}\end{aligned}$$

So to remove the brackets, we can raise each of a and b to the power m.

Lesson starter: Discovering laws for brackets and fractions

Use the expanded form of $(2x)^3$ and $\left(\frac{x}{3}\right)^4$ to help simplify the expressions.

$$\begin{aligned}(2x)^3 &= 2x \times \square \times \square \\ &= 2 \times 2 \times 2 \times \square \times \square \times \square \\ &= 2^{\square} \times \square^{\square}\end{aligned}$$

$$\begin{aligned}\left(\frac{x}{3}\right)^4 &= \frac{x}{3} \times \square \times \square \times \square \\ &= \frac{x \times \square \times \square \times \square}{3 \times \square \times \square \times \square} \\ &= \frac{\square^{\square}}{\square^{\square}}\end{aligned}$$

- Repeat these steps to also simplify the expressions $(3y)^4$ and $\left(\frac{x}{2}\right)^5$.
- What do you notice about the given expressions and the answer in each case? Can you express this as a rule or law in words?

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KEY IDEAS

- **Index law for brackets:** $(a \times b)^m = (ab)^m = a^m b^m$
  - When multiplying two or more numbers raised to the power of m, raise each number in the brackets to the power of m. For example: $(2x)^2 = 2^2x^2 = 4x^2$.
- **Index law for fractions:** $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ and $b \neq 0$
  - When dividing two numbers raised to the power of m, raise each number in the brackets to the power of m. For example: $\left(\frac{y}{3}\right)^3 = \frac{y^3}{3^3} = \frac{y^3}{27}$.

BUILDING UNDERSTANDING

1 State the missing terms to complete the index laws for brackets and fractions.

a $(a \times b)^m = a^m \times \boxed{}$

b $\left(\frac{a}{b}\right)^m = \frac{a^m}{\boxed{}}$

2 State the missing terms to complete this working.

a
$$\begin{aligned}(5a)^3 &= 5a \times \boxed{} \times \boxed{} \\ &= 5 \times 5 \times 5 \times a \times \boxed{} \times \boxed{} \\ &= 5^3 \times \boxed{}\end{aligned}$$

b
$$\begin{aligned}\left(\frac{x}{6}\right)^3 &= \frac{x}{6} \times \boxed{} \times \boxed{} \\ &= \frac{x \times \boxed{} \times \boxed{}}{6 \times \boxed{} \times \boxed{}} \\ &= \frac{x^3}{\boxed{}}\end{aligned}$$

Example 12 Using the index law for brackets

Expand each of the following using the index law for brackets.

a $(5b)^3$ **b** $(-2x^3y)^4$ **c** $4(c^2d^3)^5$ **d** $3a^2(2a^3)^4$

SOLUTION	EXPLANATION
a $(5b)^3 = 5^3b^3$ $= 125b^3$	Raise each numeral and pronumeral in the brackets to the power of 3. Evaluate $5^3 = 5 \times 5 \times 5$.
b $(-2x^3y)^4 = (-2)^4(x^3)^4y^4$ $= 16x^{12}y^4$	Raise each value in the brackets to the power of 4. Evaluate $(-2)^4$ and simplify using the index law for power of a power.

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c $4(c^2d^3)^5 = 4(c^2)^5(d^3)^5$
$= 4c^{10}d^{15}$

Raise each value in the brackets to the power of 5.
Note that the coefficient (4) is not raised to the power of 5.
Simplify using index laws.

d $3a^2(2a^3)^4 = 3a^2(2)^4(a^3)^4$
$= 3a^2 \times 16 \times a^{12}$
$= 48a^{14}$

Raise each value in the brackets to the power of 4.
Use the index law for multiplication to combine terms with the same base: $a^2 \times a^{12} = a^{2+12}$.

Now you try

Expand each of the following using the index law for brackets.

a $(3a)^2$ **b** $(-3x^2y^3)^3$ **c** $5(a^2b)^4$ **d** $4b^3(2b^4)^2$

Example 13 Using the index law for fractions

Apply the index law for fractions to the following.

a $\left(\frac{6}{b}\right)^3$ **b** $\left(\frac{-2a^2}{3bc^3}\right)^4$ **c** $\left(\frac{x^2y^3}{c}\right)^3 \times \left(\frac{xc}{y}\right)^4$

SOLUTION

a $\left(\frac{6}{b}\right)^3 = \frac{6^3}{b^3}$
$= \frac{216}{b^3}$

b $\left(\frac{-2a^2}{3bc^3}\right)^4 = \frac{(-2)^4a^8}{3^4b^4c^{12}}$
$= \frac{16a^8}{81b^4c^{12}}$

c $\left(\frac{x^2y^3}{c}\right)^3 \times \left(\frac{xc}{y}\right)^4 = \frac{x^6y^9}{c^3} \times \frac{x^4c^4}{y^4}$
$= \frac{x^{10}y^9c^4}{c^3y^4}$
$= x^{10}y^5c$

EXPLANATION

a Raise each value in the brackets to the power of 3 and evaluate 6^3.

b Raise each value in the brackets to the power of 4.
Recall $(a^2)^4 = a^{2\times4}$ and $(c^3)^4 = c^{3\times4}$
Evaluate $(-2)^4$ and 3^4.

c Raise each value in the brackets to the power.
Multiply the numerators using the index law for multiplication then divide using the index law for division.

Now you try

Apply the index law for fractions to the following.

a $\left(\frac{2}{x}\right)^5$ **b** $\left(\frac{-3m^3}{2np^2}\right)^2$ **c** $\left(\frac{xy^2}{2}\right)^3 \times \left(\frac{x}{3y}\right)^2$

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Exercise 6D

FLUENCY	1–3(½)	1–4(½)	1–4(⅓)

Example 12a-c

1 Expand each of the following using the index law for brackets.

a $(5y)^2$ b $(4a)^3$ c $(-3r)^2$
d $-(3b)^4$ e $-(7r)^3$ f $(-2h^2)^4$
g $(5c^2d^3)^4$ h $(2x^3y^2)^5$ i $9(p^2q^4)^3$
j $(8t^2u^9v^4)^0$ k $-4(p^4qr)^2$ l $(-5s^7t)^2$

Example 12d

2 Simplify each of the following by applying the various index laws.

a $a(3b)^2$ b $a(3b^2)^3$ c $-3(2a^3b^4)^2a^2$
d $2(3x^2y^3)^3$ e $(-4b^2c^5d)^3$ f $a(2a)^3$
g $a(3a^2)^2$ h $5a^3(-2a^4b)^3$ i $-5(-2m^3pt^2)^5$
j $-(-7d^2f^4g)^2$ k $-2(-2^3x^4yz^3)^3$ l $-4a^2b^3(-2a^3b^2)^2$
m $(a^2b)^3 \times (ab^2)^4$ n $\dfrac{(2m^3n)^3}{m^4}$ o $\dfrac{3(2^2c^4d^5)^3}{(2cd^2)^4}$

Example 13a,b

3 Apply the index law for fractions to expand the following.

a $\left(\frac{p}{q}\right)^3$ b $\left(\frac{x}{y}\right)^4$ c $\left(\frac{4}{y}\right)^3$ d $\left(\frac{2}{r^3}\right)^2$
e $\left(\frac{s^3}{7}\right)^2$ f $\left(\frac{2m}{n}\right)^5$ g $\left(\frac{3n^3}{2m^4}\right)^3$ h $\left(\frac{-2r}{n}\right)^4$
i $\left(\frac{-3f}{2^3g^5}\right)^2$ j $\left(\frac{-3x}{2y^3g^5}\right)^2$ k $\left(\frac{3km^3}{4n^7}\right)^3$ l $-\left(\frac{-5w^4y}{2x^3}\right)^2$

Example 13c

4 Simplify each of the following.

a $\left(\frac{x^2}{y}\right)^3 \times \frac{2x}{y^4}$ b $\left(\frac{a^2b}{c}\right)^4 \times \left(\frac{b^2}{c}\right)^3$ c $\left(\frac{pq^3}{r^2}\right)^2 \times \left(\frac{p^0q^2}{r}\right)^4$
d $\left(\frac{a^3b}{c}\right)^3 \times \left(\frac{ac^4}{b}\right)^2$ e $\left(\frac{x^2z}{y}\right)^4 \times \left(\frac{xy^2}{z}\right)^3$ f $\left(\frac{r^3s}{t}\right)^2 \div \left(\frac{s}{rt^4}\right)^3$

PROBLEM-SOLVING	5	5, 6(½)	5, 6(½)

5 The rule for the number of seeds germinating in a glasshouse over a two-week period is given by $N = \left(\frac{t}{2}\right)^3$ where N is the number of germinating seeds and t is the number of days.

a Find the number of germinating seeds after:
  i 4 days ii 10 days
b Use the index law for fractions to rewrite the rule without brackets.
c Use your rule in part b to find the number of seeds germinating after:
  i 6 days ii 4 days
d Find the number of days required to germinate:
  i 64 seeds ii 1 seed

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6 Find the value of a that makes these equations true, given $a > 0$.

a $\left(\frac{a}{3}\right)^2 = \frac{4}{9}$
b $\left(\frac{a}{2}\right)^4 = 16$
c $(5a)^3 = 1000$
d $(2a)^4 = 256$
e $\left(\frac{2a}{3}\right)^2 = \frac{4}{9}$
f $\left(\frac{6a}{7}\right)^3 = 1728$

REASONING	7	7, 8	7, 8

7 Decide if the following are true or false. Give reasons.

a $(-2x)^2 = -(2x)^2$
b $(-3x)^3 = -(3x)^3$
c $\left(\frac{-5}{x}\right)^5 = -\left(\frac{5}{x}\right)^5$
d $\left(\frac{-4}{x}\right)^4 = -\left(\frac{4}{x}\right)^4$

8 Rather than evaluating $\frac{2^4}{4^4}$ as $\frac{16}{256} = \frac{1}{16}$, it is easier to evaluate $\frac{2^4}{4^4}$ in the way shown below.

$$\frac{2^4}{4^4} = \left(\frac{2}{4}\right)^4 = \left(\frac{1}{2}\right)^4 = \frac{1^4}{2^4} = \frac{1}{16}$$

a Explain why this method is helpful.
b Use this idea to evaluate these without the use of a calculator.

i $\frac{6^3}{3^3}$
ii $\frac{10^4}{5^4}$
iii $\frac{4^4}{12^4}$
iv $\frac{3^3}{30^3}$

ENRICHMENT: False laws	–	–	9

9 Consider the equation $(a + b)^2 = a^2 + b^2$.

a Using $a = 2$ and $b = 3$, evaluate $(a + b)^2$.
b Using $a = 2$ and $b = 3$, evaluate $a^2 + b^2$.
c Would you say that the equation is true for all values of a and b?
d Now decide whether $(a - b)^2 = a^2 - b^2$ for all values of a and b. Give an example to support your answer.
e Decide if these equations are true or false for all values of a and b.

i $(-ab)^2 = a^2b^2$
ii $-(ab)^2 = a^2b^2$
iii $\left(\frac{-a}{b}\right)^3 = \frac{-a^3}{b^3} (b \neq 0)$
iv $\left(\frac{-a}{b}\right)^4 = \frac{-a^4}{b^4} (b \neq 0)$

Index laws are routinely used in the finance industry, including by corporate and personal financial advisors and investment bankers. For example, \$$P$ invested at r% p.a. for n years increases to \$$A = P(1 + r)^n$.

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6E Negative indices

Learning intentions
- To know how negative indices can be equivalently expressed using positive indices
- To be able to express negative indices in terms of positive indices
- To be able to use the index laws with negative indices

Past, present and future learning
- This section addresses the Stage 5 Path Topic called *Indices B*
- These concepts will be revised, briefly, in Chapter 3 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

We know that $2^3 = 8$ and $2^0 = 1$, but what about 2^{-1} or 2^{-6}? Such numbers written in index form using negative indices also have meaning in mathematics.

Audio engineers who set up loud speaker systems for live performances know that, as sound travels from its source, its intensity, I, is inversely proportional ($\propto$) to the distance, d, squared, i.e. $I \propto \frac{1}{d^2}$ or $I \propto d^{-2}$

Consider $a^2 \div a^5$.

Method 1: Using the index law for division

$$\begin{aligned}\frac{a^2}{a^5} &= a^{2-5}\\ &= a^{-3}\end{aligned}$$

Method 2: By cancelling

$$\begin{aligned}\frac{a^2}{a^5} &= \frac{\cancel{a}^1 \times \cancel{a}^1}{a \times a \times a \times \cancel{a}_1 \times \cancel{a}_1}\\ &= \frac{1}{a^3}\end{aligned}$$

$$\therefore a^{-3} = \frac{1}{a^3}$$

Also, using the index law for multiplication we can write:

$$\begin{aligned}a^m \times a^{-m} &= a^{m+(-m)}\\ &= a^0\\ &= 1\end{aligned}$$

So dividing by a^m we have $a^{-m} = \frac{1}{a^m}$ or dividing by a^{-m} we have $a^m = \frac{1}{a^{-m}}$.

Lesson starter: Continuing the pattern

Explore the use of negative indices by completing this table.

Index form	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
Whole number or fraction	16	8					$\frac{1}{4} = \frac{1}{2^2}$	

÷2 ÷2 ÷2

- What do you notice about the numbers with negative indices in the top row in comparison to the fractions in the second row?
- Can you describe this connection formally in words?
- What might be another way of writing 2^{-7} or 5^{-4}?

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KEY IDEAS

- $a^{-m} = \frac{1}{a^m}$ For example: $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ (2^{-3} is the reciprocal of 2^3)
- $\frac{1}{a^{-m}} = a^m$ For example: $\frac{1}{2^{-3}} = 2^3 = 8$ (2^3 is the reciprocal of 2^{-3})
- a^{-1} is the reciprocal of a^1, which is $\frac{1}{a}$ For example: $5^{-1} = \frac{1}{5}$, which is the reciprocal of 5
- $\left(\frac{a}{b}\right)^{-1}$ is the reciprocal of $\frac{a}{b}$, which is $\frac{b}{a}$ For example: $\left(\frac{2}{3}\right)^{-1} = \frac{3}{2}$, which is the reciprocal of $\frac{2}{3}$
- $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n} = \frac{b^n}{a^n}$ For example: $\left(\frac{x^2}{3}\right)^{-4} = \left(\frac{3}{x^2}\right)^{4} = \frac{81}{x^8}$

BUILDING UNDERSTANDING

1 State the following using positive indices. For example: $\frac{1}{8} = \frac{1}{2^3}$.

a $\frac{1}{4}$ **b** $\frac{1}{9}$ **c** $\frac{1}{125}$ **d** $\frac{1}{27}$

2 State the missing numbers in these patterns.

a

Index form	3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}
Whole number or fraction	81	27					$\frac{1}{9} = \frac{1}{3^2}$	

÷3 ÷3 ÷3

b

Index form	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
Whole number or fraction	10 000							$\frac{1}{1000} = \frac{1}{10^3}$

÷10 ÷10

Example 14 Writing expressions using positive indices

Express each of the following with positive indices only.

a x^{-2} **b** $3a^{-2}b^4$

SOLUTION

a $x^{-2} = \frac{1}{x^2}$

b $3a^{-2}b^4 = \frac{3}{1} \times \frac{1}{a^2} \times \frac{b^4}{1}$

$= \frac{3b^4}{a^2}$

EXPLANATION

$a^{-m} = \frac{1}{a^m}$

Rewrite a^{-2} using a positive power and combine numerators and denominators.

Now you try

Express each of the following with positive indices only.

a a^{-3} **b** $2x^{-4}y^3$

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Example 15 Using $\frac{1}{a^{-m}} = a^m$

Express each of the following using positive indices only.

a $\frac{1}{c^{-2}}$ **b** $\frac{5}{x^3y^{-4}}$ **c** $\frac{x^{-3}}{y^{-5}}$

SOLUTION

a $\frac{1}{c^{-2}} = c^2$

b $\frac{5}{x^3y^{-4}} = \frac{5}{x^3} \times \frac{1}{y^{-4}}$
$= \frac{5y^4}{x^3}$

c $\frac{x^{-3}}{y^{-5}} = x^{-3} \times \frac{1}{y^{-5}}$
$= \frac{1}{x^3} \times \frac{y^5}{1}$
$= \frac{y^5}{x^3}$

EXPLANATION

$\frac{1}{a^{-m}} = a^m$

Express $\frac{1}{y^{-4}}$ as a positive power, y^4.

Express x^{-3} and $\frac{1}{y^{-5}}$ with positive indices using $a^{-m} = \frac{1}{a^m}$ and $\frac{1}{a^{-m}} = a^m$.

Now you try

Express each of the following using positive indices only.

a $\frac{1}{a^{-3}}$ **b** $\frac{6}{a^2b^{-5}}$ **c** $\frac{x^{-2}}{y^{-4}}$

Example 16 Evaluating without a calculator

Express using positive powers only, then evaluate without using a calculator.

a 3^{-4} **b** $\frac{-5}{3^{-2}}$ **c** $\left(\frac{2}{3}\right)^{-4}$

SOLUTION

a $3^{-4} = \frac{1}{3^4}$
$= \frac{1}{81}$

b $\frac{-5}{3^{-2}} = -5 \times \frac{1}{3^{-2}}$
$= -5 \times 3^2$
$= -5 \times 9$
$= -45$

EXPLANATION

Express 3^{-4} as a positive power and evaluate 3^4.

Express $\frac{1}{3^{-2}}$ as a positive power and simplify.

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c $\left(\frac{2}{3}\right)^{-4} = \frac{2^{-4}}{3^{-4}}$

$= 2^{-4} \times \frac{1}{3^{-4}}$

$= \frac{1}{2^4} \times 3^4$

$= \frac{3^4}{2^4} = \frac{81}{16}$

Apply the power to each numeral in the brackets using the index law for fractions.

Express 2^{-4} and $\frac{1}{3^{-4}}$ with positive indices and evaluate.

Now you try

Express using positive powers only, then evaluate without using a calculator.

a 2^{-5} b $\frac{-4}{2^{-3}}$ c $\left(\frac{5}{7}\right)^{-2}$

Exercise 6E

FLUENCY	1–3(½)	1–4(⅓)	1–4(¼)

Example 14

1 Express each of the following with positive indices only.

a x^{-5} b a^{-4} c b^{-6} d 5^{-2}
e 4^{-3} f 9^{-1} g $5x^{-2}$ h $4y^{-3}$
i $3m^{-5}$ j p^7q^{-2} k mn^{-4} l x^4y^{-4}
m $2a^{-3}b^{-1}$ n $7r^{-2}s^{-3}$ o $5^{-1}u^{-8}v^2$ p $9^{-1}m^{-3}n^{-5}$

Example 15a

2 Express each of the following using positive indices only.

a $\frac{1}{y^{-1}}$ b $\frac{1}{b^{-2}}$ c $\frac{1}{m^{-5}}$ d $\frac{1}{x^{-4}}$
e $\frac{7}{q^{-1}}$ f $\frac{3}{t^{-2}}$ g $\frac{5}{h^{-4}}$ h $\frac{4}{p^{-4}}$
i $\frac{a}{b^{-2}}$ j $\frac{e}{d^{-1}}$ k $\frac{2n^2}{m^{-3}}$ l $\frac{y^5}{3x^{-2}}$
m $\frac{-3}{7y^{-4}}$ n $\frac{-2}{b^{-8}}$ o $\frac{-3g}{4h^{-3}}$ p $\frac{(-3u)^2}{5t^{-2}}$

Example 15b

3 Express each of the following using positive indices only.

a $\frac{7}{x^{-4}y^3}$ b $\frac{1}{u^{-3}v^2}$ c $\frac{a^{-3}5^{-1}}{y^{-3}}$ d $\frac{2a^{-4}}{b^{-5}c^2}$
e $\frac{5a^2c^{-4}}{6b^{-2}d}$ f $\frac{5^{-1}h^3k^{-2}}{4^{-1}m^{-2}p}$ g $\frac{4t^{-1}u^{-2}}{3^{-1}v^2w^{-6}}$ h $\frac{4^{-1}x^2y^{-5}}{4m^{-1}n^{-4}}$

Example 15c

4 Express each of the following using positive indices only.

a $\frac{a^{-3}}{b^{-3}}$ b $\frac{x^{-2}}{y^{-5}}$ c $\frac{g^{-2}}{h^{-3}}$ d $\frac{m^{-1}}{n^{-1}}$
e $\frac{5^{-1}}{7^{-3}}$ f $\frac{3^{-2}}{4^{-3}}$ g $\frac{5^{-2}}{6^{-1}}$ h $\frac{4^{-3}}{8^{-2}}$

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PROBLEM–SOLVING	5(½)	5(½), 6	5(⅓), 6, 7

Example 16

5 Evaluate without the use of a calculator. (*Hint*: Write expressions using positive indices.)

a 5^{-1} **b** 3^{-2} **c** $(-4)^{-2}$

d -5^{-2} **e** 4×10^{-2} **f** -5×10^{-3}

g -3×2^{-2} **h** $8 \times (2^2)^{-2}$ **i** $6^4 \times 6^{-6}$

j $8^{-7} \times (8^2)^3$ **k** $(5^2)^{-1} \times (2^{-2})^{-1}$ **l** $(3^{-2})^2 \times (7^{-1})^{-1}$

m $\dfrac{1}{8^{-1}}$ **n** $\dfrac{1}{10^{-2}}$ **o** $\dfrac{-2}{5^{-3}}$

p $\dfrac{2}{2^{-3}}$ **q** $\dfrac{-5}{2^{-1}}$ **r** $\dfrac{2^3}{2^{-3}}$

s $\left(\dfrac{3}{8}\right)^{-2}$ **t** $\left(\dfrac{-4}{3}\right)^{-3}$ **u** $\dfrac{(-5)^2}{2^{-2}}$

v $\dfrac{(3^{-2})^3}{3^{-5}}$ **w** $\dfrac{(-2^{-3})^{-3}}{(2^{-2})^{-4}}$ **x** $\left(\dfrac{2^{-4}}{7^{-2}}\right)^{-1} \times \left(\dfrac{7^{-1}}{2^{-1}}\right)^{-4}$

6 The mass of a small insect is 2^{-9} kg. How many grams is this? Round to two decimal places.

7 Find the value of x in these equations.

a $2^x = \dfrac{1}{16}$ **b** $5^x = \dfrac{1}{625}$ **c** $(-3)^x = \dfrac{1}{81}$

d $(0.5)^x = 2$ **e** $(0.2)^x = 25$ **f** $3(2^{2x}) = 0.75$

REASONING	8	8	8, 9

8 Describe the error made in these problems then give the correct answer.

a $2x^{-2} = \dfrac{1}{2x^2}$ **b** $\dfrac{5}{a^4} = \dfrac{a^{-4}}{5}$ **c** $\dfrac{2}{(3b)^{-2}} = \dfrac{2b^2}{9}$

9 Consider the number $\left(\dfrac{2}{3}\right)^{-1}$.

a Complete this working: $\left(\dfrac{2}{3}\right)^{-1} = \dfrac{1}{\left(\frac{2}{3}\right)} = 1 \div \boxed{} = 1 \times \boxed{} = \dfrac{3}{2}$

b Show similar working as in part **a** to simplify these.

i $\left(\dfrac{5}{4}\right)^{-1}$ **ii** $\left(\dfrac{2}{7}\right)^{-1}$ **iii** $\left(\dfrac{x}{3}\right)^{-1}$ **iv** $\left(\dfrac{a}{b}\right)^{-1}$

c What conclusion can you come to regarding the simplification of fractions raised to the power -1?

d Simplify these fractions.

i $\left(\dfrac{2}{3}\right)^{-2}$ **ii** $\left(\dfrac{4}{5}\right)^{-2}$ **iii** $\left(\dfrac{1}{2}\right)^{-5}$ **iv** $\left(\dfrac{7}{3}\right)^{-3}$

ENRICHMENT: Exponential equations	–	–	10

10 To find x in $2^x = 32$ you could use trial and error; however, the following approach is more useful.

$$\begin{aligned} 2^x &= 32 \\ 2^x &= 2^5 \text{ (express 32 using a matching base)} \\ \therefore x &= 5 \end{aligned}$$

Use this idea to solve for x in these equations.

a $2^x = 16$ **b** $3^x = 81$ **c** $\left(\dfrac{1}{2}\right)^x = \dfrac{1}{8}$ **d** $\left(\dfrac{1}{7}\right)^x = \dfrac{1}{49}$

e $\left(\dfrac{2}{3}\right)^x = \dfrac{16}{81}$ **f** $4^{2x} = 64$ **g** $3^{x+1} = 243$ **h** $2^{3x-1} = 64$

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Progress quiz

6A **1** Write each of the following in expanded form and then evaluate where possible.
a a^4
b $5(hk)^3$
c 2^4
d $\left(\frac{-3}{4}\right)^3$

6A **2** Write each of the following in index form.
a $7 \times m \times m \times m \times m$
b $\frac{2}{3} \times \frac{2}{3} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}$
c $8h \times e \times 8h \times e \times 8h \times e \times e$

6B **3** Simplify each of the following using the index laws for multiplication and division.
a $2^5 \times 2^3$
b $a^3 \times a^5 \times a$
c $3k^3m \times 4mk^4$
d $5^{10} \div 5^2$
e $\frac{6a^8}{3a^2}$
f $\frac{8a^{10}m^6}{24a^5m^2}$

6B **4** Simplify each of the following using the index laws for multiplication and division.
a $x^4 \times x^5 \div x^3$
b $\frac{4x^3y^2 \times 6x^2y}{8x^4y^2}$

6C **5** Apply the index law for power of a power to simplify each of the following. Leave your answers in index form.
a $(x^3)^4$
b $-4\left(q^6\right)^7$

6C **6** Apply the zero index rule to evaluate each of the following.
a 8^0
b $(2x)^0$
c $7m^0$
d $-4n^0 - (5n)^0$

6C **7** Simplify each of the following by applying the various index laws.
a $(a^3)^2 \times (a^2)^4$
b $\frac{(m^4)^6}{m^2}$
c $\frac{3x^4 \times 12x^2}{9x^6}$
d $\frac{8(m^2)^4 \times 5(n^3)^2}{15(m^5)^0 \times 2n^2}$

6D **8** Expand each of the following using the index laws for brackets and fractions and simplify.
a $(2b)^3$
b $5\left(h^2j^3k\right)^3$
c $\left(-3x^4y^2\right)^3$
d $\left(\frac{4}{c}\right)^3$
e $\left(\frac{-2wx^3}{5y^2}\right)^3$
f $\left(\frac{a^2b}{c^2}\right)^4 \times \left(\frac{c^5}{b^2}\right)^2$

6E **9** Express each of the following using positive indices only.
a m^{-4}
b $7x^{-3}y^5$
c $\frac{1}{a^{-5}}$
d $\frac{a^{-2}}{c^{-3}}$
e $\frac{-13}{m^{-5}}$
f $\frac{-5t^{-2}u^3}{3^{-1}v^2w^{-3}}$

6E **10** Express using positive powers only, then evaluate without using a calculator.
a 4^{-2}
b $\frac{3}{3^{-2}}$
c $(-6)^{-2}$
d $\left(\frac{3}{2}\right)^{-3}$

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6F Scientific notation

Learning intentions
- To know that scientific notation is a way of representing very large and very small numbers
- To know the form of numbers written in scientific notation
- To be able to express numbers using scientific notation and as a basic numeral

Past, present and future learning
- This section addresses the Stage 5 Path Topic called *Numbers of any magnitude*
- These concepts will be revised, briefly, in Chapter 3 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

It is common in the practical world to be working with very large or very small numbers. For example, the number of cubic metres of concrete used to build the Hoover Dam in the United States was 3 400 000 m^3 and the mass of a molecule of water is 0.0000000000000000000000299 grams. Such numbers can be written more efficiently using powers of 10 with positive or negative indices. This is called scientific notation or standard form. The number is written using a number between 1 inclusive and 10 and this is multiplied by a power of 10. Such notation is also used to state very large and very small time intervals.

Nano-robotics is an area of experimental research for disease control, such as targeted cancer drug delivery. Nanobots are micrometres long, 3×10^{-6} m, with components nanometres long, e.g. (1×10^{-9}) m. One nanometre is around $\frac{1}{100\,000}$ of a hair's diameter.

Lesson starter: Building scientific notation

Use the information given to complete the table.

Decimal form	Working	Scientific notation
2 350 000	$2.35 \times 1\,000\,000$	2.35×10^6
502 170		
314 060 000		
0.000298	$2.98 \div 10\,000 = \frac{2.98}{10^4}$	2.98×10^{-4}
0.000004621		
0.003082		

- Discuss how each number using scientific notation is formed.
- When are positive indices used and when are negative indices used?
- Where does the decimal point appear to be placed when using scientific notation?

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KEY IDEAS

- Numbers written in **scientific notation** are expressed in the form $a \times 10^m$ where $1 \leqslant a < 10$ or $-10 < a \leqslant -1$ and m is an integer.
- Large numbers will use positive powers of 10.
  For example: 38 million years $= 38\,000\,000$ years
  $= 3.8 \times 10^7$ years
- Small numbers will use negative powers of 10.
  For example: 417 nanoseconds $= 0.000000417$ seconds
  $= 4.17 \times 10^{-7}$ seconds
- To write numbers using scientific notation, place the decimal point after the first non-zero digit then multiply by a power of 10.
- There are common prefixes for units that correspond to powers of 10.
  - tera: 10^{12} e.g. One terabyte $= 10^{12}$ bytes
  - giga: 10^9
  - mega: 10^6
  - kilo: 10^3
  - milli: 10^{-3} e.g. one milligram $= 10^{-3}$ grams
  - micro: 10^{-6}
  - nano: 10^{-9}
  - pico: 10^{-12}
- Examples of units for which very large or small numbers may be used:
  - 2178 km $= 2178 \times 10^3$ m $= 2\,178\,000$ m $= 2.178 \times 10^6$ metres
  - 2320 tonnes $= 2\,320\,000$ kg or 2.32×10^6 kg
  - 16 gigabytes $= 16\,000\,000\,000$ bytes or 1.6×10^{10} bytes
  - 109 milliseconds (thousandths of a second) $= 109 \times 10^{-3}$ seconds $= 0.109$ seconds or 1.09×10^{-1} seconds
  - 27 microns (millionth of a metre) $= 0.000027$ m or 2.7×10^{-5} metres
  - 54 nanoseconds (billionth of a second) $= 0.000000054$ or 5.4×10^{-8} seconds

BUILDING UNDERSTANDING

1 Which of the numbers 1000, 10 000 or 100 000 completes each equation?

a $6.2 \times$ _____ $= 62\,000$ **b** $9.41 \times$ _____ $= 9410$
c $1.03 \times$ _____ $= 103\,000$ **d** $3.2 \div$ _____ $= 0.0032$
e $5.16 \div$ _____ $= 0.0000516$ **f** $1.09 \div$ _____ $= 0.000109$

2 State the following as powers of 10.

a 100 000 **b** 100 **c** 1 000 000 000

3 If these numbers were written using scientific notation, would positive or negative indices be used?

a 2000 **b** 0.0004 **c** 19 300 **d** 0.00101431

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Example 17 Writing numbers using scientific notation

Write the following numbers using scientific notation.

a 4 500 000 b 0.0000004

SOLUTION

a $4\,500\,000 = 4.5 \times 10^6$

b $0.0000004 = 4 \times 10^{-7}$

EXPLANATION

Place the decimal point after the first non-zero digit (4) then multiply by 10^6 since the decimal place has been moved 6 places to the left.

The first non-zero digit is 4. Multiply by 10^{-7} since the decimal place has been moved 7 places to the right.

Now you try

Write the following numbers using scientific notation.

a 720 000 b 0.000003

Example 18 Writing numbers in decimal form

Express each of the following in decimal form.

a 9.34×10^5 b 4.71×10^{-5}

SOLUTION

a $9.34 \times 10^5 = 934\,000$

b $4.71 \times 10^{-5} = 0.0000471$

EXPLANATION

Move the decimal point 5 places to the right.

Move the decimal point 5 places to the left and insert zeros where necessary.

Now you try

Express each of the following in decimal form.

a 6.17×10^3 b 4.02×10^{-6}

Exercise 6F

FLUENCY	1–5(½)	1–5(½)	1–5(⅓)

Example 17a

1 Write the following using scientific notation.

a 43 000 b 2 300 000 000 000 c 16 000 000 000
d −7 200 000 e −3500 f −8 800 000
g 52 hundreds h 3 million i 21 thousands

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Example 17b

2 Write the following using scientific notation.

a 0.000003 **b** 0.0004 **c** −0.00876
d 0.00000000073 **e** −0.00003 **f** 0.000000000125
g −0.00000000809 **h** 0.000000024 **i** 0.0000345

3 Write each of the following numbers using scientific notation.

a 6000 **b** 720 000 **c** 324.5
d 7869.03 **e** 8459.12 **f** 0.2
g 0.000328 **h** 0.00987 **i** −0.00001
j −460 100 000 **k** 17 467 **l** −128

Example 18a

4 Express each of the following in decimal form.

a 5.7×10^4 **b** 3.6×10^6 **c** 4.3×10^8
d 3.21×10^7 **e** 4.23×10^5 **f** 9.04×10^{10}
g 1.97×10^8 **h** 7.09×10^2 **i** 6.357×10^5

Example 18b

5 Express each of the following in decimal form.

a 1.2×10^{-4} **b** 4.6×10^{-6} **c** 8×10^{-10}
d 3.52×10^{-5} **e** 3.678×10^{-1} **f** 1.23×10^{-7}
g 9×10^{-5} **h** 5×10^{-2} **i** 4×10^{-1}

PROBLEM-SOLVING	6–8(½)	6–8(½), 9, 10	6–8(½), 10, 11

6 Express each of the following approximate numbers using scientific notation.

a The mass of Earth is 6 000 000 000 000 000 000 000 000 kg.
b The diameter of Earth is 40 000 000 m.
c The diameter of a gold atom is 0.0000000001 m.
d The radius of Earth's orbit around the Sun is 150 000 000 km.
e The universal constant of gravitation is 0.0000000000667 Nm^2/kg^2.
f The half-life of polonium-214 is 0.00015 seconds.
g Uranium-238 has a half-life of 4 500 000 000 years.

7 Express each of the following in decimal form.

a Neptune is approximately 4.6×10^9 km from Earth.
b A population of bacteria contained 8×10^{12} organisms.
c The Moon is approximately 3.84×10^5 km from Earth.
d A fifty-cent coin is approximately 3.8×10^{-3} m thick.
e The diameter of the nucleus of an atom is approximately 1×10^{-14} m.
f The population of a city is 7.2×10^5.

8 Write the following using scientific notation in the units given in the brackets. Use the key ideas for help with the prefixes.

a 492 milliseconds (seconds) **b** 0.38 milliseconds (seconds)
c 2.1 microseconds (seconds) **d** 0.052 microseconds (seconds)
e 4 nanoseconds (seconds) **f** 139.2 nanoseconds (seconds)
g 4 megabytes (bytes) **h** 2 terabytes (bytes)
i 52 gigalitres (litres) **j** 0.182 megalitres (litres)
k 18 picograms (grams) **l** 0.4 micrograms (grams)

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9 When Sydney was planning for the 2000 Olympic Games, the Olympic Organising Committee made the following predictions.

- The cost of staging the games would be A\$1.7 billion ($\$1.7 \times 10^9$) (excluding infrastructure). In fact, \$140 million extra was spent on staging the games.
- The cost of constructing or upgrading infrastructure would be \$807 million.

Give each of the following answers in scientific notation.

a The actual total cost of staging the Olympic Games.
b The total cost of staging the games and constructing or upgrading the infrastructure.

10 Two planets are 2.8×10^8 km and 1.9×10^9 km from their closest sun. What is the difference between these two distances in scientific notation?

11 Two particles weigh 2.43×10^{-2} g and 3.04×10^{-3} g. Find the difference in their weight in scientific notation.

REASONING	12(½)	12(½)	12–13(⅓)

12 The number 47×10^4 is not written using scientific notation since 47 is not a number between 1 and 10. The following shows how to convert to scientific notation.

$$\begin{aligned} 47 \times 10^4 &= 4.7 \times 10 \times 10^4 \\ &= 4.7 \times 10^5 \end{aligned}$$

Write these numbers using scientific notation.

a 32×10^3 **b** 41×10^5 **c** 317×10^2 **d** 5714×10^2
e 0.13×10^5 **f** 0.092×10^3 **g** 0.003×10^8 **h** 0.00046×10^9
i 61×10^{-3} **j** 424×10^{-2} **k** 1013×10^{-6} **l** $490\,000 \times 10^{-1}$
m 0.02×10^{-3} **n** 0.0004×10^{-2} **o** 0.00372×10^{-1} **p** 0.04001×10^{-6}

13 Use the index law: $(a^m)^n = a^{m \times n}$ and the index law: $(a \times b)^m = a^m \times b^m$ to simplify these numbers. Give your answer using scientific notation.

a $(2 \times 10^2)^3$ **b** $(3 \times 10^4)^2$ **c** $(2.5 \times 10^{-2})^2$ **d** $(1.5 \times 10^{-3})^3$
e $(2 \times 10^{-3})^{-3}$ **f** $(5 \times 10^{-4})^{-2}$ **g** $\left(\frac{1}{3} \times 10^2\right)^{-2}$ **h** $\left(\frac{2}{5} \times 10^4\right)^{-1}$

ENRICHMENT: Scientific notation with index laws	–	–	14(½), 15, 16

14 Use index laws to simplify these and write using scientific notation.

a $(3 \times 10^2) \times (2 \times 10^4)$ **b** $(4 \times 10^4) \times (2 \times 10^7)$
c $(8 \times 10^6) \div (4 \times 10^2)$ **d** $(9 \times 10^{20}) \div (3 \times 10^{11})$
e $(7 \times 10^2) \times (8 \times 10^2)$ **f** $(1.5 \times 10^3) \times (8 \times 10^4)$
g $(6 \times 10^4) \div (0.5 \times 10^2)$ **h** $(1.8 \times 10^6) \div (0.2 \times 10^3)$
i $(3 \times 10^{-4}) \times (3 \times 10^{-5})$ **j** $(15 \times 10^{-2}) \div (2 \times 10^6)$
k $(4.5 \times 10^{-3}) \div (3 \times 10^2)$ **l** $(8.8 \times 10^{-1}) \div (8.8 \times 10^{-1})$

15 Determine, using index laws, how many seconds it takes for light to travel from the Sun to Earth given that Earth is 1.5×10^8 km from the Sun and the speed of light is 3×10^5 km/s.

16 Using index laws and the fact that the speed of light is equal to 3×10^5 km/s, determine:

a how far light travels in one nanosecond (1×10^{-9} seconds). Answer using scientific notation in kilometres then convert your answer to centimetres.
b how long light takes to travel 300 kilometres. Answer in seconds.

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6G Scientific notation using significant figures

Learning intentions
- To know how significant figures are counted
- To be able to round off a number to a given number of significant figures
- To be able to write a number in scientific notation to a given number of significant figures

Past, present and future learning
- This section addresses the Stage 5 Path Topic called *Numbers of any magnitude*
- These concepts will be revised, briefly, in Chapter 3 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

The number of digits used to record measurements depends on how accurately the measurements can be recorded. The volume of Earth, for example, has been calculated as 1 083 210 000 000 km^3. This appears to show six significant figures and could be written using scientific notation as 1.08321×10^{12}. A more accurate calculation may include more non-zero digits in the last seven places.

Palaeontologists, who study fossils, use 1 or 2 significant figures for the estimated time period when Tyrannosaurus Rex dinosaurs lived on Earth, i.e. from 7×10^7 years ago to 6.5×10^7 years ago.

The mass of a single oxygen molecule is known to be 0.000000000000000000000000053 g. This shows two significant figures and is written using scientific notation as 5.3×10^{-26}. On many calculators you will notice that very large or very small numbers are automatically converted to scientific notation using a certain number of significant figures. Numbers can also be entered into a calculator using scientific notation.

Lesson starter: Significant discussions

Begin a discussion regarding significant figures by referring to these questions.

- Why is the volume of Earth given as 1 083 210 000 000 km^3 written using seven zeros at the end of the number? Wouldn't the exact mass of Earth include some other digits in these places?
- Why is the mass of an oxygen molecule given as 5.3×10^{-26} written using only two significant digits? Wouldn't the exact mass of a water molecule include more decimal places?

KEY IDEAS

- **Significant figures** are counted from left to right starting at the first non-zero digit. Zeros with no non-zero digit on their right are not counted. For example:
  - 38 041 has five significant figures
  - 0.0016 has two significant figures
  - 3.21×10^4 has three significant figures.
- When using scientific notation the first significant figure sits to the left of the decimal point.

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- Calculators can be used to work with scientific notation.
  - $\boxed{\text{E}}$ or $\boxed{\text{EE}}$ or $\boxed{\text{EXP}}$ or $\boxed{\times 10^x}$ are common key names on calculators.
  - Pressing 2.37 $\boxed{\text{EE}}$ 5 gives 2.37×10^5 and 2.37E5 means 2.37×10^5.

BUILDING UNDERSTANDING

1 State the missing numbers in these tables, rounding each number to the given number of significant figures.

a 57 263

Significant figures	Rounded number
4	
3	57 300
2	
1	

b 0.0036612

Significant figures	Rounded number
4	
3	
2	
1	0.004

2 Decide if the following numbers are written using scientific notation with three significant figures.

a 4.21×10^4
b 32×10^{-3}
c 1803×10^6
d 0.04×10^2
e 1.89×10^{-10}
f 9.04×10^{-6}
g 5.56×10^{-14}
h 0.213×10^2
i 26.1×10^{-2}

Example 19 Stating the number of significant figures

State the number of significant figures given in these numbers.

a 451 **b** 0.005012 **c** 3.2×10^7

SOLUTION	EXPLANATION
a 3 significant figures	Starting at the '4' and moving right we count three digits.
b 4 significant figures	Start counting at the first non-zero digit.
c 2 significant figures	With scientific notation the first significant figure is to the left of the decimal point.

Now you try

State the number of significant figures given in these numbers.

a 4174 **b** 0.0103 **c** 5.81×10^4

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Example 20 Writing numbers in scientific notation using significant figures

Write these numbers using scientific notation and three significant figures.

a 2 183 000 **b** 0.0019482

SOLUTION

a $2\,183\,000 = 2.18 \times 10^6$ (to 3 sig. fig.)

EXPLANATION

Put the decimal point after the first non-zero digit. The decimal point has moved 6 places to the left, so multiply by 10^6. Round the third significant figure down since the following digit is less than 5.

b $0.0019482 = 1.95 \times 10^{-3}$ (to 3 sig. fig.)

Move the decimal point 3 places to the right and multiply by 10^{-3}. Round the third significant figure up to 5 since the following digit is greater than 4.

Now you try

Write these numbers using scientific notation and two significant figures.

a 487 130 **b** 0.06531

Example 21 Using a calculator with scientific notation

Use a calculator to evaluate each of the following, leaving your answers in scientific notation correct to four significant figures.

a $3.67 \times 10^5 \times 23.6 \times 10^4$ **b** $7.6 \times 10^{-3} + \sqrt{2.4 \times 10^{-2}}$

SOLUTION

a $3.67 \times 10^5 \times 23.6 \times 10^4$
$= 8.661 \times 10^{10}$ (to 4 sig. fig.)

EXPLANATION

Use a calculator and locate the button used to enter scientific notation.
Write using scientific notation with four significant figures.

b $7.6 \times 10^{-3} + \sqrt{2.4 \times 10^{-2}}$
$= 0.1625...$
$= 1.625 \times 10^{-1}$ (to 4 sig. fig.)

Use a calculator and locate the button used to enter scientific notation.
Write using scientific notation with a number between 1 and 10.

Now you try

Use a calculator to evaluate each of the following, leaving your answers in scientific notation correct to three significant figures.

a $2.1 \times 10^3 \times 36.7 \times 10^5$ **b** $1.04 \times 10^{-4} + \sqrt{4.8 \times 10^{-3}}$

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Exercise 6G

FLUENCY	1, 2–5(½)	2–6(⅓)	2–6(¼)

Example 19a

Example 19b

Example 19c

1 State the number of significant figures in these numbers.

a **i** 4361 **ii** 72

b **i** 0.016 **ii** 0.000749

c **i** 1.8×10^4 **ii** 1.402×10^{-3}

Example 19

2 State the number of significant figures in these numbers.

a 202 **b** 1007 **c** 30101 **d** 19

e 0.0183 **f** 0.2 **g** 0.706 **h** 0.00109

i 4.21×10^3 **j** 2.905×10^{-2} **k** 1.07×10^{-6} **l** 5.9×10^5

Example 20

3 Write these numbers using scientific notation and three significant figures.

a 242 300 **b** 171 325 **c** 2829 **d** 3 247 000

e 0.00034276 **f** 0.006859 **g** 0.01463 **h** 0.001031

i 23.41 **j** 326.042 **k** 19.618 **l** 0.172046

4 Write each number using scientific notation rounding to the number of significant figures given in the brackets.

a 47 760 (3) **b** 21 610 (2) **c** 4 833160 (4)

d 37.16 (2) **e** 99.502 (3) **f** 0.014427 (4)

g 0.00201 (1) **h** 0.08516 (1) **i** 0.0001010 (1)

Example 21a

5 Use a calculator to evaluate each of the following, leaving your answers in scientific notation correct to four significant figures.

a 4^{-6} **b** 78^{-3}

c $(-7.3 \times 10^{-4})^{-5}$ **d** $\dfrac{3.185}{7 \times 10^4}$

e $2.13 \times 10^4 \times 9 \times 10^7$ **f** $5.671 \times 10^2 \times 3.518 \times 10^5$

g $9.419 \times 10^5 \times 4.08 \times 10^{-4}$ **h** $2.85 \times 10^{-9} \times 6 \times 10^{-3}$

i $12\,345^2$ **j** 87.14^8

k $\dfrac{1.8 \times 10^{26}}{4.5 \times 10^{22}}$ **l** $\dfrac{-4.7 \times 10^2 \times 6.1 \times 10^7}{3.2 \times 10^6}$

Example 21b

6 Use a calculator to evaluate each of the following, leaving your answers in scientific notation correct to five significant figures.

a $\sqrt{8756}$ **b** $\sqrt{634 \times 7.56 \times 10^7}$

c $8.6 \times 10^5 + \sqrt{2.8 \times 10^{-2}}$ **d** $-8.9 \times 10^{-4} + \sqrt{7.6 \times 10^{-3}}$

e $\dfrac{5.12 \times 10^{21} - 5.23 \times 10^{20}}{2 \times 10^6}$ **f** $\dfrac{8.942 \times 10^{47} - 6.713 \times 10^{44}}{2.5 \times 10^{19}}$

g $\dfrac{2 \times 10^7 + 3 \times 10^8}{5}$ **h** $\dfrac{4 \times 10^8 + 7 \times 10^9}{6}$

i $\dfrac{6.8 \times 10^{-8} + 7.5 \times 10^{27}}{4.1 \times 10^{27}}$ **j** $\dfrac{2.84 \times 10^{-6} - 2.71 \times 10^{-9}}{5.14 \times 10^{-6} + 7 \times 10^{-8}}$

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PROBLEM–SOLVING	7, 8	8–10	9, 10

7 The mass of Earth is approximately 6 000 000 000 000 000 000 000 000 kg. Given that the mass of the Sun is 330 000 times the mass of Earth, find the mass of the Sun. Express your answer using scientific notation correct to three significant figures.

8 The diameter of Earth is approximately 12 756 000 m. If the Sun's diameter is 109 times that of Earth, calculate its diameter in kilometres. Express your answer using scientific notation correct to three significant figures.

9 Using the formula for the volume of a sphere, $V = \frac{4\pi r^3}{3}$, and, assuming Earth to be spherical, calculate the volume of Earth in km^3. Use the data given in Question **8** and express your answer using scientific notation correct to three significant figures.

10 Write these numbers from largest to smallest.
2.41×10^6, 24.2×10^5, 0.239×10^7, 2421×10^3, 0.02×10^8

REASONING	11	11, 12	12, 13

11 The following output is common on a number of different calculators and computers. Write down the number that you think they represent.

a 4.26E6
b 9.1E – 3
c 5.04EXP11
d 1.931EXP – 1
e 2.1^{06}
f 6.14^{-11}

12 Anton writes down $352\,000 \times 250\,000 = 8.8^{10}$. Explain his error.

13 **a** Round these numbers to three significant figures. Retain the use of scientific notation.
i 2.302×10^2
ii 4.9045×10^{-2}
iii 3.996×10^6

b What do you notice about the digit that is the third significant figure?

c Why do you think that it might be important to a scientist to show a significant figure that is a zero at the end of a number?

ENRICHMENT: Combining bacteria	–	–	14

14 A flask of type A bacteria contains 5.4×10^{12} cells and a flask of type B bacteria contains 4.6×10^8 cells. The two types of bacteria are combined in the same flask.

a How many bacterial cells are there in the flask?

b If type A bacterial cells double every 8 hours and type B bacterial cells triple every 8 hours how many cells are in the flask after:
i one day?
ii a week?
iii 30 days?

Express your answers using scientific notation correct to three significant figures.

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Applications and problem-solving

The following problems will investigate practical situations drawing upon knowledge and skills developed throughout the chapter. In attempting to solve these problems, aim to identify the key information, use diagrams, formulate ideas, apply strategies, make calculations and check and communicate your solutions.

Double the success

1 Through active investment, Angela's portfolio doubles in value every decade (10 years) and started with an initial investment of \$100 000.

Angela is interested in the value of her investment over time and the time that it takes for it to reach certain investment goals.

a Complete this table showing the value of Angela's investment over 4 decades.

Decade (n)	0	1	2	3	4
Value (\$$V$)	100 000				

b Which rule links the value of Angela's investment, \$$V$, over n decades?

A $V = 100\,000n$
B $V = 100\,000 \times 2^n$
C $V = 100\,000 \times n^2$
D $V = 2^n$

c Use the correct rule from part b to calculate the value of Angela's investment after:

i 6 decades
ii 1.5 decades (to the nearest cent).

d Use trial and error to find how long it takes for Angela's investment to grow in value to the following amounts. Round to the nearest year.

i \$150 000
ii \$1 000 000

e A friend of Angela is claiming that their investment of \$50 000 is tripling in value every decade.

i Find a rule for the value, \$$V$, of the friend's investment after n decades.
ii Assuming that the friend's investment started at the same time as Angela's, find how long it would take for the investments to be of the same value. Round to the nearest year. A trial and error approach is appropriate.

Rupert's Moon facts

2 Rupert is studying the Moon and finds these facts:

- Diameter 3475 km
- Mass: 7.35×10^{22} kg
- Average distance to Earth: 384 400 km
- Formation: 4.6 billion years ago
- NASA program cost: \$25.4 billion (about \$120 billion today)
- Dust particle diameter: 50 micrometres

Rupert is interested in the facts about the Moon and tries to make sense of all the numbers. He wants to use scientific notation to deal with the size of the numbers involved.

a Consider the average distance to Earth.

i Write this number using scientific notation.
ii If a rocket travels at 10 km per second, how long will it take to get to the Moon? Answer using hours and minutes.

b Consider the Moon's diameter.

i Write this number using scientific notation.
ii Find the difference between the Moon's diameter and Earth's diameter (12 742 km). Express your answer using scientific notation and two significant figures.

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c Consider the time of formation.
 i Write this number using scientific notation.
 ii Write in decimal form.

d Consider the program cost at the time. If an average car at the time cost \$2000, how many cars could have been purchased for the same amount of money?

e Consider the Moon's mass. How many times heavier is Earth if Earth's mass is 5.972×10^{24} kg? Round to the nearest integer.

f Consider the diameter of a dust particle.
 i Write this number in metres using scientific notation.
 ii How many dust particles sitting in a single row would you need to make a line 1 m long?

Computer depreciation

3 Reynah buys a new computer and at the end of each year, notices that the computer's value has dropped by 50% from the previous year's value.
The rule for linking the computer's value, \$$V$, n years after purchase is given by:

$$V = 3000 \times 0.5^n$$

Reynah is interested in the rate at which the computer's value is decreasing and the time that it takes for it to be valued at a certain price.

a Find the initial value of the computer (i.e. find the value of V when $n = 0$).

b Find the value of the computer after:
 i 2 years
 ii 5 years.

c Use trial and error to find how long it takes for the computer to decrease in value to the following amounts. Answer in years correct to one decimal place.
 i \$1000
 ii \$200

d The rule linking V with n can also be written as $V = 3000 \times 2^{-n}$.
 i Explain why the rule can also be written this way.
 ii Use this rule to verify your answers to part **b**.

e Another computer costs \$5000 and reduces in value so that each year its value is one third of the value from the previous year. Write a rule connecting the computer's value, \$$V$, after n years using:
 i a positive index
 ii a negative index.

f After how long will the two computers above have the same value, assuming they were purchased at the same time? Answer in years correct to one decimal place.

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6H Fractional indices and surds

Learning intentions

- To know the meaning of the terms rational number, irrational number and surd
- To know how to identify a number as rational or irrational

Past, present and future learning

- This section addresses the Stage 5 Path Topic called *Indices C*
- These concepts will be revised, briefly, in Chapter 3 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

So far we have considered indices including positive and negative integers and zero. Numbers can also be expressed using fractional indices. Two examples are $9^{\frac{1}{2}}$ and $5^{\frac{1}{3}}$.

Using the index law for multiplication: $9^{\frac{1}{2}} \times 9^{\frac{1}{2}} = 9^{\frac{1}{2}+\frac{1}{2}} = 9^1 = 9$

Since $\sqrt{9} \times \sqrt{9} = 9$ and $9^{\frac{1}{2}} \times 9^{\frac{1}{2}} = 9^1$ then $9^{\frac{1}{2}} = \sqrt{9}$.

Using the index law for multiplication: $5^{\frac{1}{3}} \times 5^{\frac{1}{3}} \times 5^{\frac{1}{3}} = 5^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = 5^1 = 5$

Since $\sqrt[3]{5} \times \sqrt[3]{5} \times \sqrt[3]{5} = 5$ and $5^{\frac{1}{3}} \times 5^{\frac{1}{3}} \times 5^{\frac{1}{3}} = 5$ then $5^{\frac{1}{3}} = \sqrt[3]{5}$.

This shows that numbers with fractional powers can be written using root signs. In the example above, $9^{\frac{1}{2}}$ is the square root of $9\left(\text{i.e. } \sqrt{9}\right)$ and $5^{\frac{1}{3}}$ is the cube root of $5\left(\text{i.e. } \sqrt[3]{5}\right)$.

You will have noticed that $9^{\frac{1}{2}} = \sqrt{9} = 3 = \frac{3}{1}$ and so $9^{\frac{1}{2}}$ is a rational number (a fraction), but $5^{\frac{1}{3}} = \sqrt[3]{5}$ does not appear to be able to be expressed as a fraction. In fact, $\sqrt[3]{5}$ is irrational and cannot be expressed as a fraction and is called a surd. As a decimal $\sqrt[3]{5} = 1.70997594668\ldots$, which is an infinite, non-recurring decimal with no repeated pattern. This is a characteristic of all surds.

Lesson starter: A surd or not?

Surds are numbers with a root sign that cannot be expressed as a fraction. As a decimal they are infinite and non-recurring (with no pattern).

Use a calculator to help complete the table on the right then decide if you think the numbers are surds.

Index form	With root sign	Decimal	Surd (Yes or No)
$2^{\frac{1}{2}}$	$\sqrt{2}$		
$4^{\frac{1}{2}}$	$\sqrt{4}$		
$11^{\frac{1}{2}}$			
$36^{\frac{1}{2}}$			
$\left(\frac{1}{9}\right)^{\frac{1}{2}}$			
$(0.1)^{\frac{1}{2}}$			
$3^{\frac{1}{3}}$	$\sqrt[3]{3}$		
$8^{\frac{1}{3}}$	$\sqrt[3]{8}$		

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Index form	With root sign	Decimal	Surd (Yes or No)
$15^{\frac{1}{3}}$			
$\left(\frac{1}{27}\right)^{\frac{1}{3}}$			
$5^{\frac{1}{4}}$			
$64^{\frac{1}{6}}$			

KEY IDEAS

- Numbers written with **fractional indices** can also be written using a root sign.
  - $a^{\frac{1}{m}} = \sqrt[m]{a}$
  - $\sqrt[2]{a}$ is written $\sqrt{a}$

  For example: $3^{\frac{1}{2}} = \sqrt{3}, 7^{\frac{1}{3}} = \sqrt[3]{7}, 2^{\frac{1}{5}} = \sqrt[5]{2}$
- Surds are irrational numbers written with a root sign.
  - Irrational numbers cannot be expressed as a fraction.
  - The decimal expansion is infinite and non-recurring with no pattern.

$\sqrt{2} = 1.41421356237\ldots$
$\sqrt[3]{10} = 2.15443469003\ldots$
$3^{\frac{1}{2}} = 1.73205080757\ldots$

BUILDING UNDERSTANDING

1 Evaluate these numbers. What do you notice?

a 2^2 and $\sqrt{4}$ **b** 2^3 and $\sqrt[3]{8}$ **c** 3^2 and $\sqrt{9}$
d 3^3 and $\sqrt[3]{27}$ **e** 4^2 and $\sqrt{16}$ **f** 4^3 and $\sqrt[3]{64}$

2 State whether the following are true or false.

a Rational numbers can be written as fractions.
b A surd is a rational number.
c A surd in decimal form will be infinite and non-recurring (with no pattern).
d $\sqrt{3} = 3^{\frac{1}{2}}$ **e** $\sqrt{8} = 8^{\frac{1}{3}}$ **f** $5^{\frac{1}{3}} = \sqrt{5}$ **g** $10^{\frac{1}{6}} = \sqrt{6}$

3 Use a calculator to evaluate these surds and round to four decimal places.

a $7^{\frac{1}{2}}$ (or $\sqrt{7}$) **b** $13^{\frac{1}{2}}$ (or $\sqrt{13}$) **c** $83^{\frac{1}{2}}$ (or $\sqrt{83}$)

A classical guitar, of scale length L mm, has each fret x mm from the bridge, where $x = L \times 2^{-\frac{n}{12}}$ for the nth fret from the nut. The 12th fret is half the scale length, doubling the pitch to one octave higher.

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Example 22 Writing numbers using a root sign

Write these numbers using a root sign.

a $6^{\frac{1}{2}}$

b $2^{\frac{1}{5}}$

SOLUTION

a $6^{\frac{1}{2}} = \sqrt{6}$

b $2^{\frac{1}{5}} = \sqrt[5]{2}$

EXPLANATION

$a^{\frac{1}{m}} = \sqrt[m]{a}$ so $6^{\frac{1}{2}} = \sqrt[2]{6}$ (or $\sqrt{6}$) the square root of 6.

$\sqrt[5]{2}$ is called the 5th root of 2.

Now you try

Write these numbers using a root sign.

a $7^{\frac{1}{2}}$

b $3^{\frac{1}{6}}$

Example 23 Evaluating numbers with fractional indices

Evaluate:

a $144^{\frac{1}{2}}$

b $27^{\frac{1}{3}}$

SOLUTION

a $144^{\frac{1}{2}} = \sqrt{144}$
$= 12$

b $27^{\frac{1}{3}} = \sqrt[3]{27}$
$= 3$

EXPLANATION

$a^{\frac{1}{m}} = \sqrt[m]{a}$ where $m = 2$ and the square root of 144 = 12 since $12^2 = 144$.

The cube root of 27 is 3 since $3^3 = 3 \times 3 \times 3 = 27$.

Now you try

Evaluate:

a $64^{\frac{1}{2}}$

b $216^{\frac{1}{3}}$

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Example 24 Using index laws with fractional indices

Use index laws to simplify these expressions.

a $a^{\frac{1}{2}} \times a^{\frac{3}{2}}$ **b** $\dfrac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}}$ **c** $\left(y^2\right)^{\frac{1}{4}}$

SOLUTION	EXPLANATION
a $a^{\frac{1}{2}} \times a^{\frac{3}{2}} = a^{\frac{1}{2}+\frac{3}{2}}$ $= a^{\frac{4}{2}}$ $= a^2$	When multiplying indices with the same base add the powers.
b $\dfrac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}} = x^{\frac{1}{2}-\frac{1}{3}}$ $= x^{\frac{1}{6}}$	When dividing indices with the same base, subtract the powers. $\frac{1}{2}-\frac{1}{3}=\frac{3}{6}-\frac{2}{6}=\frac{1}{6}$
c $\left(y^2\right)^{\frac{1}{4}} = y^{2\times\frac{1}{4}}$ $= y^{\frac{1}{2}}$	When raising a power to a power, multiply the indices: $2\times\frac{1}{4}=\frac{2}{4}=\frac{1}{2}$.

Now you try

Use index laws to simplify these expressions.

a $x^{\frac{1}{2}} \times x^{\frac{5}{2}}$ **b** $\dfrac{a^{\frac{2}{3}}}{a^{\frac{1}{2}}}$ **c** $\left(b^{\frac{1}{3}}\right)^6$

Exercise 6H

FLUENCY	1–4(½)	1–5(½)	1–5(⅓)

Example 22

1 Write these numbers using a root sign.

a $3^{\frac{1}{2}}$ **b** $7^{\frac{1}{2}}$ **c** $5^{\frac{1}{3}}$ **d** $12^{\frac{1}{3}}$

e $31^{\frac{1}{5}}$ **f** $18^{\frac{1}{7}}$ **g** $9^{\frac{1}{9}}$ **h** $3^{\frac{1}{8}}$

2 Write these numbers in index form given that $\sqrt[n]{a} = a^{\frac{1}{n}}$.

a $\sqrt{8}$ **b** $\sqrt{19}$ **c** $\sqrt[3]{10}$ **d** $\sqrt[3]{31}$

e $\sqrt[4]{5}$ **f** $\sqrt[5]{9}$ **g** $\sqrt[8]{11}$ **h** $\sqrt[11]{20}$

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Example 23

3 Without using a calculator, evaluate these numbers with fractional indices.

a $25^{\frac{1}{2}}$ **b** $49^{\frac{1}{2}}$ **c** $81^{\frac{1}{2}}$ **d** $169^{\frac{1}{2}}$

e $8^{\frac{1}{3}}$ **f** $64^{\frac{1}{3}}$ **g** $125^{\frac{1}{3}}$ **h** $1000^{\frac{1}{3}}$

i $16^{\frac{1}{4}}$ **j** $81^{\frac{1}{4}}$ **k** $625^{\frac{1}{4}}$ **l** $32^{\frac{1}{5}}$

Example 24

4 Use index laws to simplify these expressions. Leave your answer in index form.

a $a^{\frac{1}{2}} \times a^{\frac{1}{2}}$ **b** $a^{\frac{1}{3}} \times a^{\frac{1}{3}}$ **c** $a^{\frac{2}{3}} \times a^{\frac{4}{3}}$ **d** $a^2 \times a^{\frac{1}{2}}$

e $\dfrac{x^{\frac{2}{3}}}{x^{\frac{1}{3}}}$ **f** $\dfrac{x^{\frac{3}{2}}}{x^{\frac{1}{2}}}$ **g** $\dfrac{x^{\frac{7}{6}}}{x^{\frac{2}{6}}}$ **h** $\dfrac{x^{\frac{4}{3}}}{x^{\frac{1}{3}}}$

i $(y^2)^{\frac{1}{2}}$ **j** $(y^3)^{\frac{2}{3}}$ **k** $\left(y^{\frac{1}{2}}\right)^3$ **l** $\left(x^{\frac{1}{2}}\right)^{\frac{1}{2}}$

m $\left(x^{\frac{2}{3}}\right)^4$ **n** $\left(a^{\frac{2}{5}}\right)^{\frac{1}{3}}$ **o** $\left(a^{\frac{3}{4}}\right)^{\frac{1}{2}}$ **p** $\left(n^{\frac{2}{5}}\right)^{\frac{10}{3}}$

5 Use index laws to simplify these expressions.

a $a \times a^{\frac{1}{3}}$ **b** $a^{\frac{1}{2}} \times a^{\frac{1}{5}}$ **c** $a^{\frac{2}{3}} \times a^{\frac{3}{7}}$

d $a^5 \div a^{\frac{7}{3}}$ **e** $b^{\frac{2}{3}} \div b^{\frac{1}{2}}$ **f** $x^{\frac{4}{5}} \div x^{\frac{2}{3}}$

PROBLEM–SOLVING

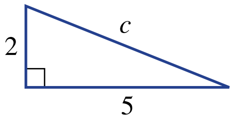
6–7(½) 6–8(½) 6–8(½)

6 Evaluate the following without a calculator. (*Hint*: First rewrite each question using positive indices using $a^{-m} = \frac{1}{a^m}$.)

a $4^{-\frac{1}{2}}$ **b** $8^{-\frac{1}{3}}$ **c** $32^{-\frac{1}{5}}$ **d** $81^{-\frac{1}{4}}$

e $25^{-\frac{1}{2}}$ **f** $27^{-\frac{1}{3}}$ **g** $1000^{-\frac{1}{3}}$ **h** $256^{-\frac{1}{4}}$

7 Find the length of the hypotenuse (c) in these right-angled triangles. Use Pythagoras' theorem $\left(c^2 = a^2 + b^2\right)$ and write your answers as surds.

a


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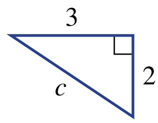

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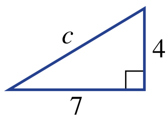
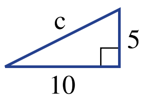
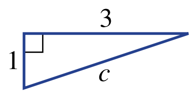
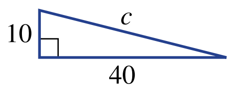


d


e


f


8 Note that
$$\begin{aligned} 8^{\frac{2}{3}} &= \left(8^{\frac{1}{3}}\right)^2 && \text{using the index law for power of a power} \\ &= \left(\sqrt[3]{8}\right)^2 && \text{since } 8^{\frac{1}{3}} = \sqrt[3]{8} \\ &= 2^2 && \sqrt[3]{8} = 2 \text{ since } 2^3 = 8 \\ &= 4 \end{aligned}$$

Use the approach shown in the example above to evaluate these numbers.

a $27^{\frac{2}{3}}$ **b** $64^{\frac{2}{3}}$ **c** $9^{\frac{3}{2}}$ **d** $25^{\frac{3}{2}}$

e $16^{\frac{5}{4}}$ **f** $4^{\frac{5}{2}}$ **g** $81^{\frac{3}{2}}$ **h** $125^{\frac{5}{3}}$

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REASONING	9	9, 10	10, 11

9 Show working to prove that the answers to all these questions simplify to 1. Remember $a^0 = 1$.

a $a^{\frac{1}{2}} \times a^{-\frac{1}{2}}$
b $a^{\frac{2}{3}} \times a^{-\frac{2}{3}}$
c $a^{\frac{4}{7}} \times a^{-\frac{4}{7}}$
d $a^{\frac{5}{6}} \div a^{\frac{5}{6}}$
e $\left(a^{\frac{1}{2}}\right)^{\frac{1}{2}} \div a^{\frac{1}{4}}$
f $a^2 \div (a^3)^{\frac{2}{3}}$

10 A student tries to evaluate $9^{\frac{1}{2}}$ on a calculator and types 9^1/2 and gets 4.5. You know that $9^{\frac{1}{2}} = \sqrt{9} = 3$. What has the student done wrong? (Note: ^ on some calculators is x^y.)

11 a Evaluate the following.
i $\sqrt{3^2}$
ii $\sqrt{5^2}$
iii $\sqrt{10^2}$

b Simplify $\sqrt{a^2}$ for $a \geqslant 0$.

c Use fractional indices to show that $\sqrt{a^2} = a$ if $a \geqslant 0$.

d Evaluate the following.
i $(\sqrt{4})^2$
ii $(\sqrt{9})^2$
iii $(\sqrt{36})^2$

e Simplify $(\sqrt{a})^2$.

f Use fractional indices to show that $(\sqrt{a})^2 = a$. Assume $a \geqslant 0$.

g Simplify:
i $\sqrt[3]{a^3}$
ii $\sqrt[5]{a^5}$
iii $(\sqrt[3]{a})^3$
iv $(\sqrt[6]{a})^6$

ENRICHMENT: Fractions raised to fractions	–	–	12(½), 13

12 Note, for example, that $\left(\frac{4}{9}\right)^{\frac{1}{2}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$ since $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$. Now evaluate the following.

a $\left(\frac{16}{25}\right)^{\frac{1}{2}}$
b $\left(\frac{9}{49}\right)^{\frac{1}{2}}$
c $\left(\frac{4}{81}\right)^{\frac{1}{2}}$
d $\left(\frac{8}{27}\right)^{\frac{1}{3}}$
e $\left(\frac{64}{125}\right)^{\frac{1}{3}}$
f $\left(\frac{16}{81}\right)^{\frac{1}{4}}$
g $\left(\frac{256}{625}\right)^{\frac{1}{4}}$
h $\left(\frac{1000}{343}\right)^{\frac{1}{3}}$

13 Note that $\left(\frac{4}{9}\right)^{-\frac{1}{2}} = \dfrac{1}{\left(\frac{4}{9}\right)^{\frac{1}{2}}} = \dfrac{1}{\sqrt{\frac{4}{9}}} = \dfrac{1}{\left(\frac{2}{3}\right)} = \frac{3}{2}$. Now evaluate the following.

a $\left(\frac{9}{4}\right)^{-\frac{1}{2}}$
b $\left(\frac{49}{144}\right)^{-\frac{1}{2}}$
c $\left(\frac{8}{125}\right)^{-\frac{1}{3}}$
d $\left(\frac{1296}{625}\right)^{-\frac{1}{4}}$

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6I Simple operations with surds

Learning intentions

- To understand that like surds can be combined under addition and subtraction
- To know how to add and subtract like surds
- To know how to multiply and divide surds
- To know simple rules related to surds

Past, present and future learning

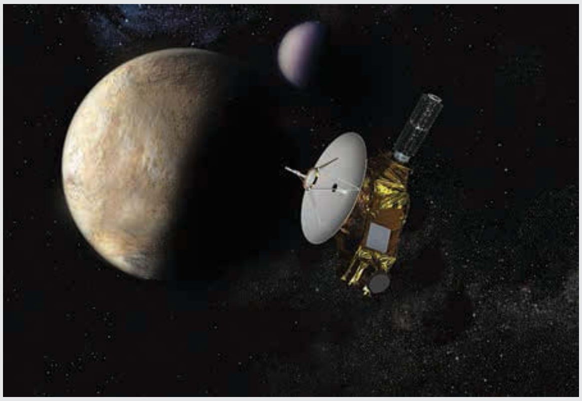
- This section addresses the Stage 5 Path Topic called *Indices C*
- These concepts will be revised and extended in Chapter 3 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

Since surds, such as $\sqrt{2}$ and $\sqrt{7}$, are numbers, they can be added, subtracted, multiplied or divided. Expressions with surds can also be simplified, but this depends on the surds themselves and the types of operations that sit between them.

$\sqrt{2}+\sqrt{3}$ cannot be simplified since $\sqrt{2}$ and $\sqrt{3}$ are not 'like' surds. This is like trying to simplify $x+y$. However, $\sqrt{2}+5\sqrt{2}$ simplifies to $6\sqrt{2}$; this is like simplifying $x+5x=6x$. Subtraction of surds is treated in the same manner.

Products and quotients involving surds can also be simplified as in these examples:

$$\sqrt{11}\times\sqrt{2}=\sqrt{22} \quad \text{and} \quad \sqrt{30}\div\sqrt{3}=\sqrt{10}$$



Engineers, architects, surveyors, navigators, physicists, geologists, astronomers and space scientists use surds, as they are exact values. For example, the formula for a spacecraft's escape velocity (from gravity) includes the number $\sqrt{2}$

$v=\dfrac{\sqrt{2}\sqrt{GM}}{\sqrt{R}}$.

Lesson starter: Rules for multiplication and division

Use a calculator to find a decimal approximation for each of the expressions in these pairs.

- $\sqrt{2}\times\sqrt{3}$ and $\sqrt{6}$
- $\sqrt{10}\times\sqrt{5}$ and $\sqrt{50}$

What does this suggest about the simplification of $\sqrt{a}\times\sqrt{b}$?

Repeat the above exploration for the following:

- $\sqrt{6}\div\sqrt{2}$ and $\sqrt{\dfrac{6}{2}}$
- $\sqrt{80}\div\sqrt{8}$ and $\sqrt{\dfrac{80}{8}}$

What does this suggest about the simplification of $\sqrt{a}\div\sqrt{b}$?

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KEY IDEAS

- **Surds** can be simplified using addition or subtraction if they are **'like' surds** (have the same numeral under the root sign).
  - $2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$
  - $11\sqrt{7} - 2\sqrt{7} = 9\sqrt{7}$
  - $\sqrt{3} + \sqrt{5}$ cannot be simplified.
- $(\sqrt{a})^2 = a$

  For example: $(\sqrt{5})^2 = 5$
- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

  For example: $\sqrt{5} \times \sqrt{3} = \sqrt{5 \times 3} = \sqrt{15}$
- $\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$

  For example: $\sqrt{10} \div \sqrt{5} = \sqrt{\frac{10}{5}} = \sqrt{2}$

BUILDING UNDERSTANDING

1 Decide if the following pairs of numbers contain 'like' surds.

a $3\sqrt{2}, 4\sqrt{2}$ **b** $5\sqrt{3}, 2\sqrt{3}$ **c** $4\sqrt{2}, 5\sqrt{7}$ **d** $\sqrt{3}, 2\sqrt{5}$

e $6\sqrt{6}, 3\sqrt{3}$ **f** $\sqrt{8}, 3\sqrt{8}$ **g** $19\sqrt{2}, -\sqrt{2}$ **h** $-3\sqrt{6}, 3\sqrt{5}$

2 **a** Use a calculator to find a decimal approximation for both $\sqrt{5} \times \sqrt{2}$ and $\sqrt{10}$. What do you notice?

b Use a calculator to find a decimal approximation for both $\sqrt{15} \div \sqrt{5}$ and $\sqrt{3}$. What do you notice?

Example 25 Adding and subtracting surds

Simplify:

a $2\sqrt{5} + 6\sqrt{5}$

b $\sqrt{3} - 5\sqrt{3}$

SOLUTION	EXPLANATION
a $2\sqrt{5} + 6\sqrt{5} = 8\sqrt{5}$	This is like simplifying $2x + 6x = 8x$. $2\sqrt{5}$ and $6\sqrt{5}$ contain like surds.
b $\sqrt{3} - 5\sqrt{3} = -4\sqrt{3}$	This is similar to $x - 5x = -4x$ in algebra.

Now you try

Simplify:

a $6\sqrt{3} + 2\sqrt{3}$

b $2\sqrt{7} - 5\sqrt{7}$

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Example 26 Multiplying and dividing surds

Simplify:

a $\sqrt{3} \times \sqrt{10}$

b $\sqrt{24} \div \sqrt{8}$

SOLUTION

a $\sqrt{3} \times \sqrt{10} = \sqrt{3 \times 10}$
$= \sqrt{30}$

b $\sqrt{24} \div \sqrt{8} = \sqrt{\frac{24}{8}}$
$= \sqrt{3}$

EXPLANATION

Use $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

Use $\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$

Now you try

Simplify:

a $\sqrt{5} \times \sqrt{3}$

b $\sqrt{60} \div \sqrt{10}$

Exercise 6I

FLUENCY	1–2(½)	1–2(½)	1–2(⅓)

Example 25

1 Simplify by collecting like surds.

a $3\sqrt{7} + 5\sqrt{7}$
b $2\sqrt{11} + 6\sqrt{11}$
c $\sqrt{5} + 8\sqrt{5}$
d $3\sqrt{6} + \sqrt{6}$
e $3\sqrt{3} + 2\sqrt{5} + 4\sqrt{3}$
f $5\sqrt{7} + 3\sqrt{5} + 4\sqrt{7}$
g $3\sqrt{5} - 8\sqrt{5}$
h $6\sqrt{7} - 10\sqrt{7}$
i $3\sqrt{7} - 2\sqrt{7} + 4\sqrt{7}$
j $5\sqrt{14} + \sqrt{14} - 7\sqrt{14}$
k $3\sqrt{2} - \sqrt{5} + 4\sqrt{2}$
l $6\sqrt{3} + 2\sqrt{7} - 3\sqrt{3}$

Example 26

2 Simplify:

a $\sqrt{5} \times \sqrt{6}$
b $\sqrt{3} \times \sqrt{7}$
c $\sqrt{10} \times \sqrt{7}$
d $\sqrt{8} \times \sqrt{2}$
e $\sqrt{12} \times \sqrt{3}$
f $\sqrt{2} \times \sqrt{11}$
g $\sqrt{3} \times \sqrt{3}$
h $\sqrt{12} \times \sqrt{12}$
i $\sqrt{36} \div \sqrt{12}$
j $\sqrt{20} \div \sqrt{2}$
k $\sqrt{42} \div \sqrt{6}$
l $\sqrt{60} \div \sqrt{20}$
m $\sqrt{45} \div \sqrt{5}$
n $\sqrt{32} \div \sqrt{2}$
o $\sqrt{49} \div \sqrt{7}$

PROBLEM-SOLVING	3–4(½)	3–4(½)	3–5(⅓)

3 Simplify:

a $2 - \sqrt{3} + 6 - 2\sqrt{3}$
b $\sqrt{2} - \sqrt{3} + 5\sqrt{2}$
c $7\sqrt{5} - \sqrt{2} + 1 + \sqrt{2}$
d $\frac{\sqrt{2}}{3} + \frac{\sqrt{2}}{2}$
e $\frac{\sqrt{7}}{2} + \frac{\sqrt{7}}{5}$
f $\frac{2\sqrt{6}}{7} - \frac{\sqrt{6}}{2}$
g $\sqrt{10} - \frac{\sqrt{10}}{3}$
h $5 - \frac{2\sqrt{3}}{3} + \sqrt{3}$
i $\frac{2\sqrt{8}}{7} - \frac{5\sqrt{8}}{8}$

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4 Note, for example, that $2\sqrt{3} \times 5\sqrt{2} = 2 \times 5 \times \sqrt{3} \times \sqrt{2}$
$= 10\sqrt{6}$

Now simplify the following.

a $5\sqrt{2} \times 3\sqrt{3}$
b $3\sqrt{7} \times 2\sqrt{3}$
c $4\sqrt{5} \times 2\sqrt{6}$
d $2\sqrt{6} \times 5\sqrt{3}$
e $10\sqrt{6} \div 5\sqrt{2}$
f $18\sqrt{12} \div 6\sqrt{2}$
g $20\sqrt{28} \div 5\sqrt{2}$
h $6\sqrt{14} \div 12\sqrt{7}$

5 Expand and simplify. Recall that $a(b + c) = a \times b + a \times c$.

a $2\sqrt{3}(3\sqrt{5} + 1)$
b $\sqrt{5}(\sqrt{2} + \sqrt{3})$
c $5\sqrt{6}(\sqrt{2} + 3\sqrt{5})$
d $7\sqrt{10}(2\sqrt{3} - \sqrt{10})$
e $\sqrt{13}(\sqrt{13} - 2\sqrt{3})$
f $\sqrt{5}(\sqrt{7} - 2\sqrt{5})$

REASONING	6(½)	6(½)	6–7(½)

6 Using $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$, the surd $\sqrt{18}$ can be simplified as shown.

$$\begin{aligned}\sqrt{18} &= \sqrt{9 \times 2}\\ &= \sqrt{9} \times \sqrt{2}\\ &= 3\sqrt{2}\end{aligned}$$

This simplification is possible because 18 has a factor that is a square number (9). Use this technique to simplify these surds.

a $\sqrt{8}$
b $\sqrt{12}$
c $\sqrt{27}$
d $\sqrt{45}$
e $\sqrt{75}$
f $\sqrt{200}$
g $\sqrt{60}$
h $\sqrt{72}$

7 Building on the idea discussed in Question 6, expressions such as $\sqrt{8} - \sqrt{2}$ can be simplified as shown:

$$\begin{aligned}\sqrt{8} - \sqrt{2} &= \sqrt{4 \times 2} - \sqrt{2}\\ &= 2\sqrt{2} - \sqrt{2}\\ &= \sqrt{2}\end{aligned}$$

Now simplify these expressions.

a $\sqrt{8} + 3\sqrt{2}$
b $3\sqrt{2} - \sqrt{8}$
c $\sqrt{18} + \sqrt{2}$
d $5\sqrt{3} - 2\sqrt{12}$
e $4\sqrt{8} - 2\sqrt{2}$
f $\sqrt{27} + 2\sqrt{3}$
g $3\sqrt{45} - 7\sqrt{5}$
h $6\sqrt{12} - 8\sqrt{3}$

ENRICHMENT: Binomial products	–	–	8(½)

8 Simplify the following by using the rule $(a + b)(c + d) = ac + ad + bc + bd$.
Note, $(a + b)^2 = (a + b)(a + b)$.

a $(\sqrt{2} + \sqrt{3})(\sqrt{2} + \sqrt{5})$
b $(\sqrt{3} - \sqrt{5})(\sqrt{3} + \sqrt{2})$
c $(2\sqrt{5} - 1)(3\sqrt{2} + 4)$
d $(1 - 3\sqrt{7})(2 + 3\sqrt{2})$
e $(2 - \sqrt{3})(2 + \sqrt{3})$
f $(\sqrt{5} - 1)(\sqrt{5} + 1)$
g $(3\sqrt{2} + \sqrt{3})(3\sqrt{2} - \sqrt{3})$
h $(8\sqrt{2} + \sqrt{5})(8\sqrt{2} - \sqrt{5})$
i $(1 + \sqrt{2})^2$
j $(\sqrt{6} - 3)^2$
k $(2\sqrt{3} - 1)^2$
l $(\sqrt{2} + 2\sqrt{5})^2$

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Working mathematically

Rabbits and hares

The population of rabbits and hares in a particular part of the countryside are booming because fox numbers are low. The population numbers in the table on the right were recorded at the beginning of January through to June.

Month	Jan	Feb	Mar	Apr	May	Jun
Rabbit population	500	1000	2000	4000	8000	16 000
Hare population	1	3	9	27	81	343

Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate.

Routine problems

a A population of a different colony of hares is modelled by the rule $P = 4^t$ where P is the population and t is in months. Find:
  i the population after 6 months
  ii the number of whole months for the population to reach at least 1000.

b A population of a different colony of rabbits is modelled by the rule $P = 200 \times 2^t$ where P is the population and t is in months. Find:
  i the population after 6 months
  ii the number of whole months for the population to reach at least 2000.

Non-routine problems

Explore and connect

a The problem is to determine the number of months required for the hare population to overtake the rabbit population. Write down all the relevant information that will help solve this problem.

b Copy and extend the given table showing the population of rabbits and hares in the first 12 months by continuing the pattern.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Rabbit population	500	1000	2000	4000	8000	16 000						
Hare population	1	3	9	27	81	343						

c The rule for the population of rabbits is of the form $P = 500 \times (?)^t$. State the missing number.

d The rule for the population of hares is of the form $P = (?)^t$. State the missing number.

Choose and apply techniques

e Use the rules outlined in parts **c** and **d** to confirm the population of:
  i rabbits initially (at $t = 0$) **ii** rabbits after 12 months **iii** hares after 12 months.

f Use the rules outlined in parts **c** and **d** to estimate the population of:
  i rabbits after 2 years **ii** hares after 3 years.

g Use your rules with a trial and error approach to determine the month in which the hare population overtakes the rabbit population.

Communicate thinking and reasoning

h Illustrate the increase in the population of rabbits and hares over two years by plotting a graph of the population of rabbits and hares. Use $0 \leqslant t \leqslant 12$ on the horizontal axis.

i If the rule for the population of hares is instead modelled by $P = 100 \times 2^t$, would there be a time when the hare population would overtake that of the rabbit population? Justify your response.

j Summarise your results and describe any key findings.

Extension problem

Problem solve

a Assume that the initial population of rabbits could vary where the population of rabbits is modelled by the rule $P = k \times 2^t$. Determine possible whole number values of k for which the population of hares overtakes that of rabbits after:
  i 12 months **ii** 6 months **iii** 5 years.

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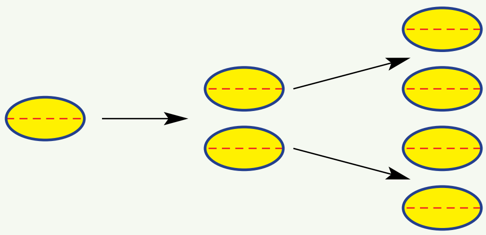
Cell growth

Many cellular organisms reproduce by a process of subdivision called mitosis. A single cell, for example, may divide into two every hour as shown below. After 2 hours the single starting cell has become four:



Dividing into two

A single cell divides into two every hour.

a How many cells will there be after the following number of hours? Explain how you obtained your answers.
 i 1 **ii** 2 **iii** 5

b Complete the table showing the number of cells after n hours.

n hours	0	1	2	3	4	5	6
Number of cells, N	1	2	4				
N in index form	2^0	2^1	2^2				

c Write a rule for the number of cells N after n hours.

d Use your rule from part **c** to find the number of cells after:
 i 8 hours **ii** 12 hours **iii** 2 days.

e Find how long it takes for a single cell to divide into a total of:
 i 128 cells **ii** 1024 cells **iii** 65 536 cells.

Dividing into three or more

a Complete a table similar to the table in the previous section for a cell that divides into 3 every hour.

b Write a rule for N in terms of n if a cell divides into 3 every hour, and then use the rule to find the number of cells after:
 i 2 hours **ii** 4 hours **iii** 8 hours.

c Write a rule for N in terms of n if a single cell divides into the following number of cells every hour.
 i 4 **ii** 5 **iii** 10

Cell cycle times

a If a single cell divides into two every 20 minutes, investigate how many cells there will be after 4 hours.

b If a single cell divides into three every 10 minutes, investigate how many cells there will be after 2 hours.

c Use the internet to research the cell cycle time and the types of division for at least two different types of cells. Describe the cells and explain the reproductive process.

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Problems and challenges

1 Determine the last digit of each of the following without using a calculator.
a 2^{222}
b 3^{300}
c 6^{87}

2 Determine the smallest value of n such that:
a $24n$ is a square number
b $750n$ is a square number.

3 Simplify $\left(\frac{32}{243}\right)^{-\frac{2}{5}}$ without using a calculator.

4 If $2^x = t$, express the following in terms of t.
a 2^{2x+1}
b 2^{1-x}

5 A single cell divides in two every 5 minutes and each new cell continues to divide every 5 minutes. How long does it take for the cell population to reach at least 1 million?

6 Find the value of x if $3^{3x-1} = \frac{1}{27}$.

7 a Write the following in index form.
i $\sqrt{2\sqrt{2}}$
ii $\sqrt{2\sqrt{2\sqrt{2}}}$
iii $\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}$
b What value do your answers to part **a** appear to be approaching?

8 Determine the highest power of 2 that divides exactly into 2 000 000.

9 Simplify these surds.
a $5\sqrt{8} - \sqrt{18}$
b $\frac{1}{\sqrt{2}} + \sqrt{2}$
c $(\sqrt{2} + 3\sqrt{5})^2 - (\sqrt{2} - 3\sqrt{5})^2$

10 Prove that:
a $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
b $\frac{3}{\sqrt{3}} = \sqrt{3}$
c $\frac{1}{\sqrt{2} - 1} = \sqrt{2} + 1$

11 Solve for x. There are two solutions for each. (*Hint*: Let $a = 2^x$ in part **a**.)
a $2^{2x} - 3 \times 2^x + 2 = 0$
b $3^{2x} - 12 \times 3^x + 27 = 0$

12 Given that $2^4 \times 3^2 \times 5 = 720$, find the smallest whole number x so that $720x$ is a perfect cube.

13 Given that $4^y \times 9^x \times 27 = 4 \times 2^x \times 3^{2y}$, find the values of x and y.

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Indices and surds

Index form

a^m ← index, base

e.g. $2^3 = 2 \times 2 \times 2$

$3^4 = 3 \times 3 \times 3 \times 3$

$a \times b \times a \times b \times b = a^2b^3$

Zero index

$a^0 = 1$

Any expression (except 0) to the power 0 equals 1.

e.g. $3x^0 = 3 \times 1$
$= 3$

while $(3x)^0 = 1$

Negative indices

$a^{-m} = \frac{1}{a^m}$

e.g. $2x^{-3} = \frac{2}{1} \times \frac{1}{x^3}$
$= \frac{2}{x^3}$

$\frac{1}{a^{-m}} = a^m$

e.g. $\frac{1}{(2x)^{-2}} = (2x)^2$
$= 4x^2$

Index laws for multiplication and division

$a^m \times a^n = a^{m+n}$

When multiplying terms with the same base, add indices.

e.g. $x^3 \times x^5 = x^{3+5}$
$= x^8$

$a^m \div a^n = a^{m-n}$

When dividing terms with the same base, subtract indices.

e.g. $\frac{x^7}{x^4} = x^{7-4}$
$= x^3$

Fractional indices

$a^{\frac{1}{n}} = \sqrt[n]{a}$

e.g. $a^{\frac{1}{2}} = \sqrt{a}$

$a^{\frac{1}{3}} = \sqrt[3]{a}$

e.g. $25^{\frac{1}{2}} = \sqrt{25} = 5$
since $5^2 = 25$

$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$
since $2^3 = 8$

Index law for power of a power

$(a^m)^n = a^{mn}$

To expand brackets, multiply indices.

e.g. $(x^2)^5 = x^{2 \times 5}$
$= x^{10}$

Scientific notation

In scientific notation, very large or very small numbers are written in the form $a \times 10^m$ where $1 \leqslant a < 10$ or $-10 < a \leqslant -1$. Large numbers will use positive powers of 10.

e.g. 2 350 000 kg $= 2.35 \times 10^6$ kg

Small numbers will use negative powers of 10.

e.g. 16 nano seconds
$= 16 \times 10^{-9}$ s
$= 0.000000016$ s
$= 1.6 \times 10^{-8}$ seconds

Index laws for brackets and fractions

$(ab)^m = a^mb^m$

e.g. $(3x^2)^3 = 3^3(x^2)^3$
$= 27x^6$

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

e.g. $\left(\frac{x}{2}\right)^4 = \frac{x^4}{2^4}$
$= \frac{x^4}{16}$

Operations with surds

'Like' surds can be added or subtracted.

$3\sqrt{7}$ and $5\sqrt{7}$ are 'like surds'

e.g. $3\sqrt{7} + 5\sqrt{7} = 8\sqrt{7}$

Multiply: $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

e.g. $\sqrt{3} \times \sqrt{10} = \sqrt{3 \times 10} = \sqrt{30}$

Divide: $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

e.g. $\frac{\sqrt{20}}{\sqrt{5}} = \sqrt{\frac{20}{5}}$
$= \sqrt{4} = 2$

Surds

A surd is a number written with a root sign which has a decimal representation that is infinite and non-recurring with no pattern. e.g. $\sqrt{2} = 1.41421356...$

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Chapter checklist

Chapter checklist with success criteria

A printable version of this checklist is available in the Interactive Textbook

		✔
6A	**1. I can write a power in expanded form.** e.g. Write $(ab)^3$ in expanded form.	☐
6A	**2. I can evaluate a power.** e.g. Write $(-4)^3$ in expanded form and then evaluate.	☐
6A	**3. I can write an expression in index form.** e.g. Write $6 \times x \times x \times y \times x \times y$ in index form.	☐
6A	**4. I can express a number in prime factor form.** e.g. Express 108 as a product of prime factors in index form.	☐
6B	**5. I can use the index laws for multiplication and division with numerical bases.** e.g. Simplify $5^4 \times 5^3$ and $3^7 \div 3^4$, answering in index form.	☐
6B	**6. I can use the index law for multiplication.** e.g. Simplify $3a^2b \times 4a^4b^3$.	☐
6B	**7. I can use the index law for division.** e.g. Simplify $\frac{18x^6y^4}{12xy^3}$.	☐
6B	**8. I can combine the index laws for multiplication and division.** e.g. Simplify $\frac{2a^5b^4 \times 6ab^2}{3a^4b^5}$.	☐
6C	**9. I can use the index law for power of a power.** e.g. Simplify $(m^6)^5$.	☐
6C	**10. I can use the zero index rule.** e.g. Evaluate 7^0 and $4x^0 - (5x)^0$.	☐
6D	**11. I can use the index law for brackets.** e.g. Expand $4(m^3n)^5$ using the index law for brackets.	☐
6D	**12. I can use the index law for fractions.** e.g. Apply the index law for fractions to $\left(\frac{-3}{b^5}\right)^4$.	☐
6D	**13. I can combine index laws.** e.g. Use index laws to simplify $\left(\frac{2}{a^0b^3}\right)^2 \times \frac{a^4(b^4)^3}{6ab^2}$.	☐
6E	**14. I can write expressions using only positive indices.** e.g. Express $2m^3n^{-2}$ and $\frac{5}{2x^{-3}}$ with positive indices only.	☐
6E	**15. I can evaluate negative powers without a calculator.** e.g. Express with positive powers and then evaluate 4^{-3} and $\frac{4}{3^{-2}}$.	☐
6F	**16. I can write numbers in scientific notation.** e.g. Write 123 000 and 0.00023 using scientific notation.	☐

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Chapter checklist

6F	**17. I can convert from scientific notation to decimal form.** e.g. Express 4.2×10^5 and 3.5×10^{-3} in decimal form.	☐
6G	**18. I can state the number of significant figures.** e.g. State the number of significant figures in 1027 and 0.024.	☐
6G	**19. I can write numbers in scientific notation using significant figures.** e.g. Write 1 738 212 in scientific notation using three significant figures.	☐
6G	**20. I can use a calculator with scientific notation.** e.g. Evaluate $7.2 \times 10^{-6} \times \sqrt{5.2 \times 10^3}$ using a calculator. Answer in scientific notation correct to three significant figures.	☐
6H	**21. I can express fractional indices using a root sign.** e.g. Write $10^{\frac{1}{3}}$ using a root sign.	☐
6H	**22. I can evaluate numbers with fractional indices.** e.g. Evaluate $81^{\frac{1}{2}}$.	☐
6H	**23. I can use index laws with fractional indices.** e.g. Simplify $\left(x^{\frac{1}{2}}\right)^3 \times x^{\frac{5}{2}}$.	☐
6I	**24. I can add and subtract like surds.** e.g. Simplify $4\sqrt{5} + 3\sqrt{5} - \sqrt{5}$.	☐
6I	**25. I can multiply and divide simple surds.** e.g. Simplify $\sqrt{5} \times \sqrt{6}$ and $\sqrt{14} \div \sqrt{7}$.	☐

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Chapter review

Short-answer questions

6A

1 Express each of the following in index form.

a $3 \times 3 \times 3 \times 3$

b $2 \times x \times x \times x \times y \times y$

c $3 \times a \times a \times a \times \frac{b}{a} \times b$

d $\frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{1}{7} \times \frac{1}{7}$

6A

2 Write the following as a product of prime factors in index form.

a 45

b 300

6B

3 Simplify using the index laws for multiplication and division.

a $x^3 \times x^7$

b $2a^3b \times 6a^2b^5c$

c $3m^2n \times 8m^5n^3 \times \frac{1}{2}m^3$

d $a^{12} \div a^3$

e $x^5y^3 \div (x^2y)$

f $\frac{5a^6b^3}{10a^8b}$

6C/D

4 Simplify:

a $(m^2)^3$

b $(3a^4)^2$

c $(-2a^2b)^5$

d $3a^0b$

e $2(3m)^0$

f $\left(\frac{a^2}{3}\right)^3$

6E

5 Express each of the following with positive indices.

a x^{-3}

b $4t^{-3}$

c $(3t)^{-2}$

d $\frac{2}{3}x^2y^{-3}$

e $5\left(\frac{x^2}{y^{-1}}\right)^{-3}$

f $\frac{5}{m^{-3}}$

6E

6 Fully simplify each of the following.

a $\frac{5x^8y^{-12}}{x^{10}} \times \frac{(x^2y^5)^2}{10}$

b $\left(\frac{(3x)^0}{3x^0y^2}\right)^4 \times \frac{9y^{10}}{x^{-3}}$

c $\frac{(4m^2n^3)^2}{2m^5n^4} \div \frac{mn^5}{(m^3n^2)^3}$

6F

7 Arrange the following numbers in ascending order.

$2.35, 0.007 \times 10^2, 0.0012, 3.22 \times 10^{-1}, 0.4, 35.4 \times 10^{-3}$

6F

8 Write the following numbers expressed using scientific notation in decimal form.

a 3.24×10^2

b 1.725×10^5

c 2.753×10^{-1}

d 1.49×10^{-3}

6G

9 Write each of the following values using scientific notation with three significant figures.

a The population of the world for 2020 was projected to reach 7 758 156 792.

b The area of the USA is 9 629 091 km^2.

c The time taken for light to travel 1 metre (in a vacuum) is 0.00000000333564 seconds.

d The wavelength of ultraviolet light from a fluorescent lamp is 0.000000294 m.

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6F

10 Write each of the following values using scientific notation in the units given in brackets.

a 25 years (hours). Assume 1 year = 365 days.

b 12 milliseconds (seconds)

c 432 nanograms (grams)

d 12.4 gigabytes (bytes)

6G

11 Use a calculator to evaluate the following, giving your answer using scientific notation correct to two significant figures.

a $m_s \times m_e$ where m_s(mass of Sun) $= 1.989 \times 10^{30}$ kg and m_e(mass of Earth) $= 5.98 \times 10^{24}$ kg

b The speed, v, in m/s of an object of mass $m = 2 \times 10^{-3}$ kg and kinetic energy $E = 1.88 \times 10^{-12}$ joules, where $v = \sqrt{\frac{2E}{m}}$

6H

12 Evaluate without using a calculator.

a $\sqrt[4]{16}$

b $\sqrt[3]{125}$

c $49^{\frac{1}{2}}$

d $81^{\frac{1}{4}}$

e $27^{-\frac{1}{3}}$

f $121^{-\frac{1}{2}}$

6H

13 Simplify the following expressing all answers in positive index form.

a $(s^6)^{\frac{1}{3}}$

b $3x^{\frac{1}{2}} \times 5x^2$

c $\left(3m^{\frac{1}{2}}n^2\right)^2 \times m^{-\frac{1}{4}}$

d $\dfrac{4}{a^{\frac{1}{3}}} \times \dfrac{\left(a^{\frac{1}{2}}\right)^4}{a}$

6I

14 Simplify the following operations with surds.

a $8\sqrt{7} - \sqrt{7} + 2$

b $2\sqrt{3} + 5\sqrt{2} - \sqrt{3} + 4\sqrt{2}$

c $\sqrt{8} \times \sqrt{8}$

d $\sqrt{5} \times \sqrt{3}$

e $\sqrt{7} \times \sqrt{2}$

f $\sqrt{2} \times \sqrt{11} \times \sqrt{2}$

g $\sqrt{42} \div \sqrt{7}$

h $\sqrt{75} \div \sqrt{3}$

i $2\sqrt{50} \div \sqrt{10}$

Multiple-choice questions

6B

1 $3x^7 \times 4x^4$ is equivalent to:

A $12x^7$ **B** $12x^{28}$ **C** $7x^{11}$ **D** $12x^{11}$ **E** $7x^3$

6C

2 $3\left(2y^2\right)^0$ simplifies to:

A 6 **B** 3 **C** $6y^2$ **D** $3y$ **E** 12

6D

3 $(2x^2)^3$ expands to:

A $2x^5$ **B** $2x^6$ **C** $6x^6$ **D** $8x^5$ **E** $8x^6$

6B

4 $2x^3y \times \dfrac{x^5y^2}{4x^2y}$ simplifies to:

A $\dfrac{x^6y^2}{2}$ **B** $2x^8y$ **C** $2x^6y^2$ **D** $\dfrac{x^4y^2}{2}$ **E** $8x^6y$

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Chapter review

6D **5** $\left(\dfrac{-x^2y}{3z^4}\right)^3$ is equal to:

A $\dfrac{x^6y^3}{3z^{12}}$ **B** $\dfrac{-x^5y^4}{9z^7}$ **C** $\dfrac{-x^6y^3}{27z^{12}}$ **D** $\dfrac{-x^2y^3}{3z^{12}}$ **E** $\dfrac{x^6y^3}{9z^{12}}$

6E **6** $2x^{-3}y^4$ expressed with positive indices is:

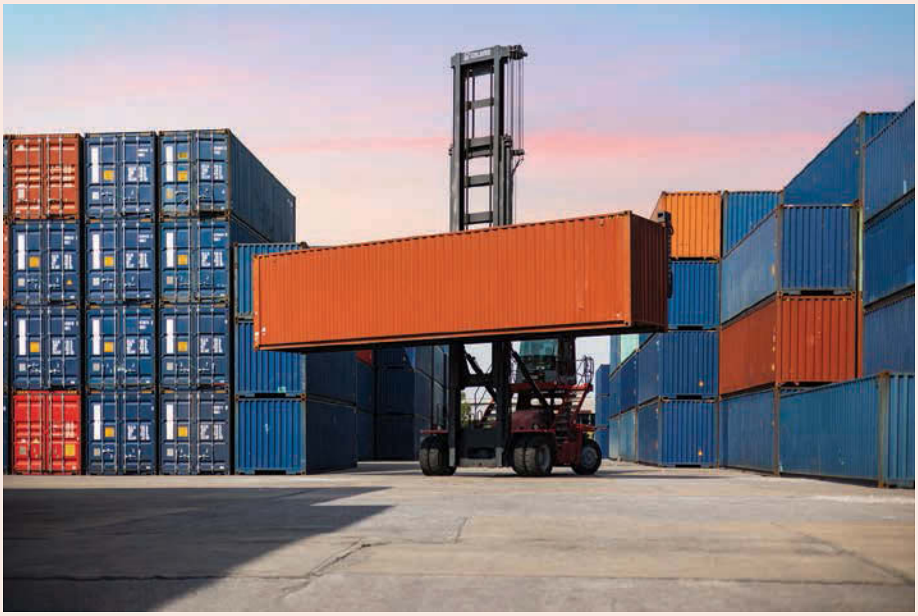
A $\dfrac{y^4}{2x^3}$ **B** $\dfrac{2y^4}{x^3}$ **C** $-2x^3y^4$ **D** $\dfrac{2}{x^3y^4}$ **E** $\dfrac{y^4}{8x^3}$

6E **7** $\dfrac{3}{(2x)^{-2}}$ is equivalent to:

A $\dfrac{-3}{(2x)^2}$ **B** $6x^2$ **C** $\dfrac{3x^2}{2}$ **D** $12x^2$ **E** $\dfrac{-3x^2}{4}$

6F **8** The weight of a cargo crate is 2.32×10^4 kg. In expanded form this weight in kilograms is:

A 2 320 000 **B** 232 **C** 23 200 **D** 0.000232 **E** 2320



6G **9** 0.00032761 using scientific notation rounded to three significant figures is:

A 328×10^{-5} **B** 3.27×10^{-4} **C** 3.28×10^4
D 3.30×10^4 **E** 3.28×10^{-4}

6H **10** $36^{\frac{1}{2}}$ is equal to:

A 18 **B** 6 **C** 1296 **D** 9 **E** 81

6I **11** The simplified form of $2\sqrt{7} - 3 + 4\sqrt{7}$ is:

A $-2\sqrt{7} - 3$ **B** $3\sqrt{7}$ **C** $6\sqrt{7} - 3$ **D** $\sqrt{7}$ **E** $8\sqrt{7} - 3$

6I **12** $\sqrt{3} \times \sqrt{7}$ is equivalent to:

A $\sqrt{21}$ **B** $\sqrt{10}$ **C** $2\sqrt{10}$ **D** $10\sqrt{21}$ **E** $21\sqrt{10}$

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Extended-response questions

1 Simplify each of the following, expressing answers with positive indices, using a combination of index laws.

a $\dfrac{(4x^2y)^3 \times x^2y}{12(xy^2)^2}$

b $\dfrac{2a^3b^4}{(5a^3)^2} \times \dfrac{20a}{3b^{-4}}$

c $\dfrac{(5m^4n^{-3})^2}{m^{-1}n^2} \div \dfrac{5(m^{-1}n)^{-2}}{mn^{-4}}$

d $\dfrac{(8x^4)^{\frac{1}{3}}}{2(y^3)^0} \times \dfrac{\left(3x^{\frac{1}{3}}\right)^2}{3(x^2)^{\frac{1}{2}}}$

2 The law of gravitational force is given by $F = \dfrac{Gm_1m_2}{d^2}$ where F is the magnitude of the gravitational force (in newtons, N) between two objects of mass m_1 and m_2 (in kilograms) a distance d (metres) apart. G is the universal gravitational constant, which is approximately 6.67×10^{-11} Nm^{2}kg^{-2}.

a If two objects of masses 2 kg and 4 kg are 3 m apart, calculate the gravitational force F between them. Answer using scientific notation correct to three significant figures.

b The average distance between Earth and the Sun is approximately 149 597 870 700 m.

i Write this distance using scientific notation with three significant figures.

ii Hence, if the mass of Earth is approximately 5.98×10^{24} kg and the mass of the Sun is approximately 1.99×10^{30} kg, calculate the gravitational force between them using scientific notation to two significant figures.

c The universal gravitational constant, G, is constant throughout the universe. However, acceleration due to gravity (a, units m/s^2) varies according to where you are in the solar system. Using the formula $a = \dfrac{Gm}{r^2}$ and the following table, work out and compare the acceleration due to gravity on Earth and on Mars. Answer to three significant figures.

Planet	Mass, m	Radius, r
Earth	5.98×10^{24} kg	6.375×10^6 m
Mars	6.42×10^{23} kg	3.37×10^6 m

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7

Properties of geometrical figures

Maths in context: Cantilever and truss bridges

Geometric knowledge, including congruent and similar triangles, Trigonometry, and Pythagoras' theorem, are essential skills used by civil engineers for designing and building structures, including bridges, cranes, roads, tunnels, airports, dams, communication towers, electricity pylons, water supply structures and sewage treatment systems.

The Tokyo Gate bridge (pictured) has a suspended central span, 440 m long, supported by two arms, or cantilevers, that reach out from each side of the bay to hold the central span in place. Each cantilever has vertical support only at one end, the concrete pillars.

Locals nickname it 'Dinosaur bridge', from the 'head' shape of each steel triangular support structure which are called trusses. Each truss weighs 6 000 tonnes, the bridge is 2.618 km long overall, 87.8 m high, and can support 32 000 vehicles/day.

The cantilevers, trusses and central span were built on land, moved onto a barge, then positioned with massive cranes. This style of bridge and its

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construction process allows shipping to continue without interruption, and with its height below 100 m, incoming aircraft are safe.

Other Cantilever bridges include the Forth bridge, Edinburgh, Scotland, 1890; Brisbane's Story Bridge, 1940; Brisbane's twin Gateway Motorway bridges, completed 2010; and Melbourne's Bolte Bridge, 1999.

Chapter contents

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- identifies and applies the properties of similar figures and scale drawings to solve problems (MA5-GEO-C-01)
- establishes conditions for congruent triangles and similar triangles and solves problems relating to properties of similar figures and plane shapes (MA5-GEO-P-01)
- constructs proofs involving congruent triangles and similar triangles and proves properties of plane shapes (MA5-GEO-P-02).

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

7A Angles and triangles CONSOLIDATING

Learning intentions
- To review the various types of angles and triangles
- To know the relationships between angles formed at a point
- To be able to find unknown angles in geometric diagrams
- To be able to use the exterior angle theorem for a triangle

Past, present and future learning
- This section consolidates Stage 4 concepts
- Some of these questions are more challenging than those in our Stage 4 books
- These concepts are assumed knowledge for Stage 6 Advanced and are occasionally useful

Geometry is all around us. The properties of the shape of a window, doorway or roofline depend on their geometry. When lines meet, angles are formed and it is these angles which help define the shape of an object. Fundamental to geometry are the angles formed at a point and the three angles in a triangle. Lines meeting at a point and triangles have special properties and will be revised in this section.


Geometry is an essential tool for architects, builders, navigators, astronomers, engineers and surveyors. The geometry of triangles has been applied by the architect of this 50 m high chimney at Greenwich, London

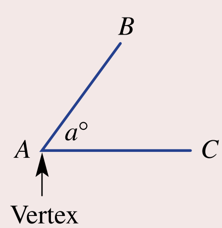
Lesson starter: Impossible triangles

Triangles are classified either by their side lengths or by their angles.

- First, write the list of three triangles which are classified by their side lengths and the three triangles that are classified by their angles.
- Now try to draw a triangle for the following descriptions. Decide which are possible and which are impossible.
  - Acute scalene triangle
  - Right equilateral triangle
  - Right isosceles triangle
  - Obtuse isosceles triangle
  - Obtuse scalene triangle
  - Acute equilateral triangle

KEY IDEAS

- When two **rays**, **lines** or **line segments** meet at a point, an **angle** is formed.
  - This angle is named $\angle A$ or $\angle BAC$ or $\angle CAB$.
  - The size of this angle in degrees is $a°$.



- Angle types
  - **Acute** between 0° and 90°
  - **Obtuse** between 90° and 180°
  - **Reflex** between 180° and 360°
  - **Right** 90°
  - **Straight** 180°
  - **Revolution** 360°

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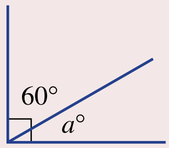
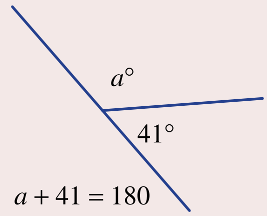
- Angles at a point
  - **Complementary** (sum to 90°)



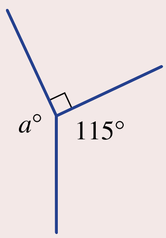
$a + 60 = 90$

  - **Supplementary** (sum to 180°)



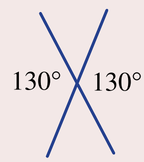
$a + 41 = 180$

  - **Revolution** (sum to 360°)

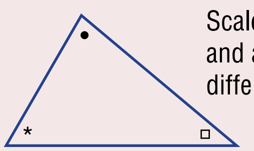
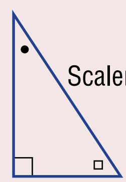
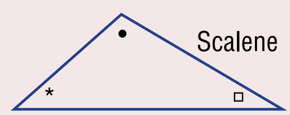
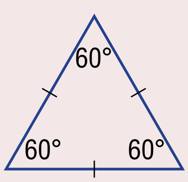
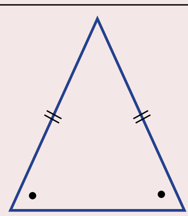
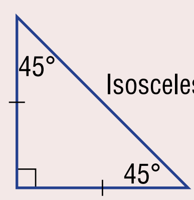
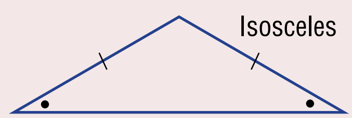


$a + 90 + 115 = 360$

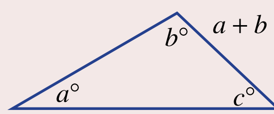
  - **Vertically opposite** (are equal)



- Types of triangles

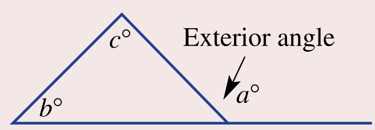
Acute angled (all angles < 90°)	Right-angled (includes a 90° angle)	Obtuse angled (one angle > 90°)
 Scalene (all sides and angles are different sizes)	 Scalene	 Scalene
 Equilateral (all angles 60° and all sides equal)  Isosceles (two angles equal and two sides equal)	 Isosceles	 Isosceles

  - The sum of the angles in a triangle is 180°.



$a + b + c = 180$

- An **exterior angle** is formed by extending one side of a shape.
  - **Exterior angle theorem of a triangle**: The exterior angle of a triangle is equal to the sum of the two opposite interior angles.



$a = b + c$

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BUILDING UNDERSTANDING

1 Choose a word or number to complete each sentence.

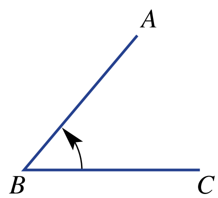
- **a** A 90° angle is called a _____ angle.
- **b** A _____ angle is called a straight angle.
- **c** A 360° angle is called a ___________.
- **d** _____ angles are between 90° and 180°.
- **e** _____ angles are between 0° and 90°.
- **f** Reflex angles are between _____ and 360°.
- **g** Complementary angles sum to _____.
- **h** ___________ angles sum to 180°.
- **i** The three angles in a triangle sum to _____.
- **j** Vertically opposite angles are _____.

2 Name the type of triangle that has:

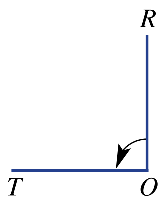
- **a** a pair of equal length sides
- **b** one obtuse angle
- **c** all angles 60°
- **d** one pair of equal angles
- **e** all angles acute
- **f** all sides of different length
- **g** one right angle.

3 Name the type of each angle, then estimate the size of the angle and use your protractor to determine an accurate measurement.

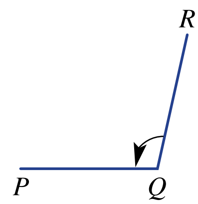
a



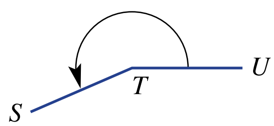
b



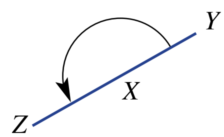
c



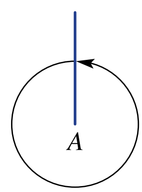
d



e



f



4 State whether each of the following pairs of angles is supplementary (S), complementary (C) or neither (N).

- **a** 30°, 60°
- **b** 45°, 135°
- **c** 100°, 90°
- **d** 50°, 40°

Example 1 Finding supplementary and complementary angles

For an angle of size 47° determine the size of its:

- **a** supplementary angle
- **b** complementary angle.

SOLUTION	EXPLANATION
a $180° - 47° = 133°$	Supplementary angles sum to 180°.
b $90° - 47° = 43°$	Complementary angles sum to 90°.

Now you try

For an angle of size 72° determine the size of its:

- **a** supplementary angle
- **b** complementary angle.

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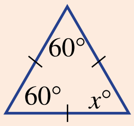
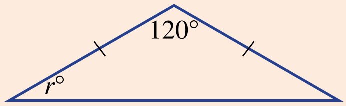
Example 2 Finding unknown angles in triangles

Name the types of triangles shown here and determine the values of the pronumerals.

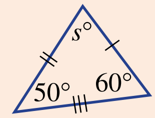
a



b



c



SOLUTION

a Equilateral triangle

$x = 60$

Explanation: All sides are equal, therefore all angles are equal.

b Obtuse isosceles triangle

$$\begin{aligned} 2r + 120 &= 180 \\ 2r &= 60 \\ r &= 30 \end{aligned}$$

Explanation: One angle is more than 90° and two sides are equal.
Angles in a triangle add to 180°. Since the triangle is isosceles, both base angles are $r°$.
Subtract 120° from both sides and then divide both sides by 2.

c Acute scalene triangle

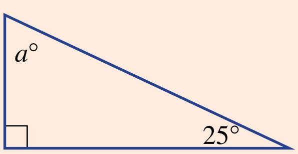
$$\begin{aligned} s + 50 + 60 &= 180 \\ s + 110 &= 180 \\ s &= 70 \end{aligned}$$

Explanation: All angles are less than 90° and all sides are of different length.
Angles in a triangle add to 180°. Simplify and solve for s.

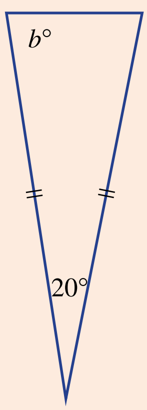
Now you try

Name the types of triangles shown here and determine the values of the pronumerals.

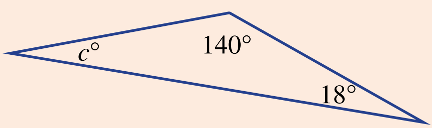
a



b



c



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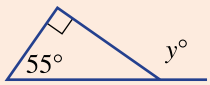
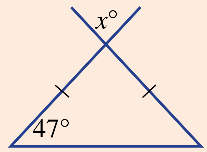
Example 3 Finding exterior and other angles

Find the value of each pronumeral. Give reasons for your answers.

a



b



SOLUTION

a $y = 90 + 55$
$= 145$

Alternative method:
Let a be the unknown angle.

$$\begin{aligned} a + 90 + 55 &= 180 \text{ (angle sum)} \\ a &= 35 \\ y + 35 &= 180 \\ y &= 145 \end{aligned}$$

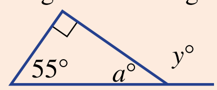
b

$$\begin{aligned} x + 47 + 47 &= 180 \text{ (angle sum)} \\ x + 94 &= 180 \\ x &= 86 \end{aligned}$$

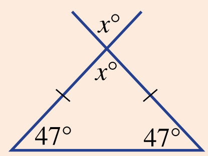
EXPLANATION

Use the exterior angle theorem, which states that the exterior angle is equal to the sum of the two opposite interior angles.

Alternatively:
Angles in a triangle sum to 180°.



Angles in a straight line are supplementary (sum to 180°).

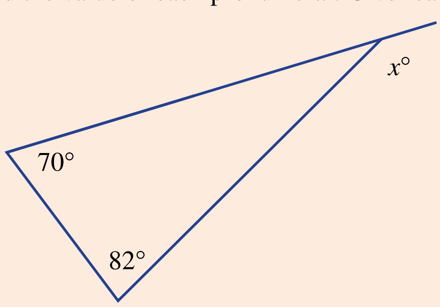


Note the isosceles triangle and vertically opposite angles.
Angles in a triangle add to 180° and vertically opposite angles are equal. Simplify and solve for x.

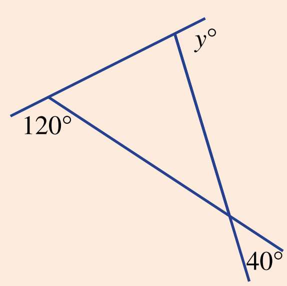
Now you try

Find the value of each pronumeral. Give reasons for your answers.

a



b



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Exercise 7A

FLUENCY	1–3(½)	1–4(½)	1–4(⅓)

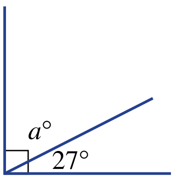
Example 1

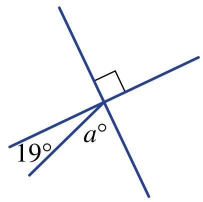
1 For each of the following angle sizes determine the size of:

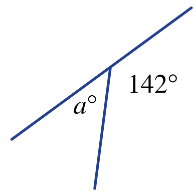
- **i** the supplementary angle
- **ii** the complementary angle.

a 55° **b** 74° **c** 89° **d** 22° **e** 38° **f** 47°

2 Find the value of a in these diagrams. Each diagram contains complementary or supplementary angles, or angles in a revolution. Refer to 'Angles at a point' in the key ideas if needed.

a


b


c


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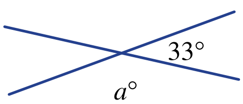

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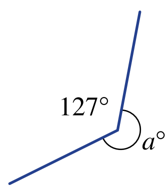

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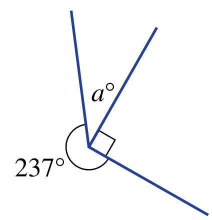
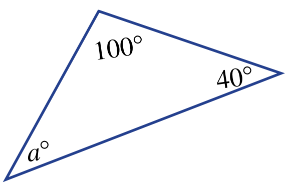


Example 2

3 Determine the values of the pronumerals and name the types of triangles shown here.

a


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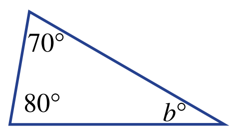

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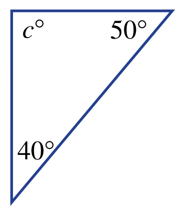

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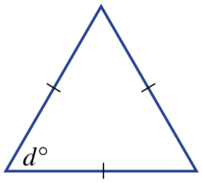

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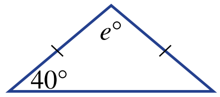

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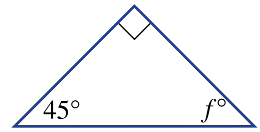
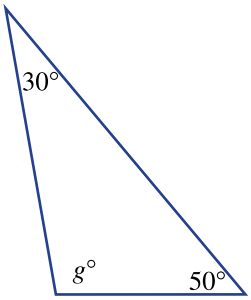


g


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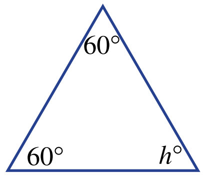

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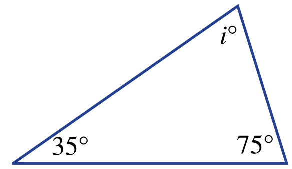

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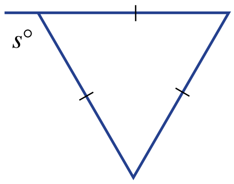
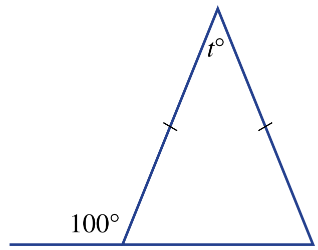
Example 3

4 Find the value of each pronumeral.

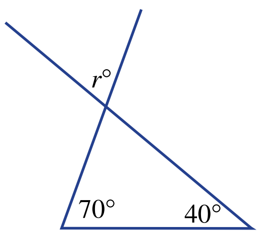
a



b



c



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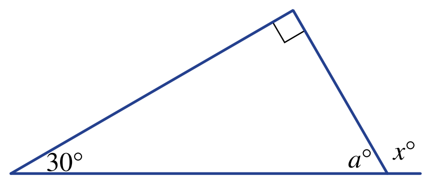
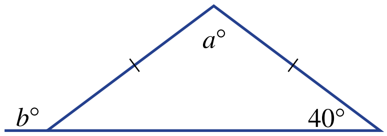
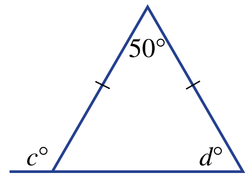


e



f



PROBLEM–SOLVING	5, 6	5(½), 6, 7(½)	6, 7(½)

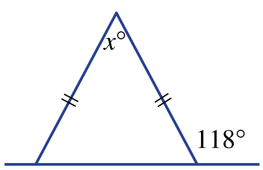
5 Calculate how many degrees the minute hand of a clock rotates in:

a 1 hour **b** $\frac{1}{4}$ of an hour **c** 10 minutes **d** 15 minutes

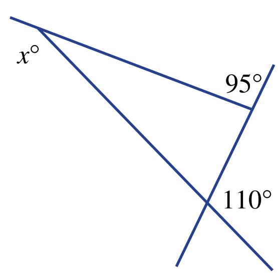
e 72 minutes **f** 1 minute **g** 2 hours **h** 1 day

6 Find the value of x in these diagrams.

a



b



7 Find the acute or obtuse angle between the hour and minute hands at these times. Remember to consider how the hour hand moves between each whole number.

a 3:00 p.m. **b** 5:00 a.m. **c** 6:30 p.m.

d 11:30 p.m. **e** 3:45 a.m. **f** 1:20 a.m.

g 4:55 a.m. **h** 2:42 a.m. **i** 9:27 a.m.

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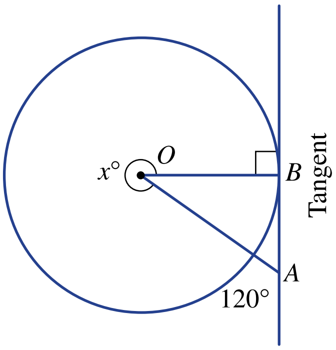
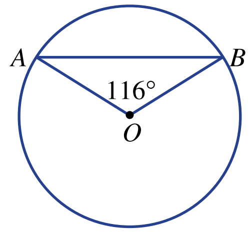
REASONING	8, 9	8, 9	9–11

8 A tangent to a circle is 90° to its radius. Explain why $x = 330$ in this diagram.

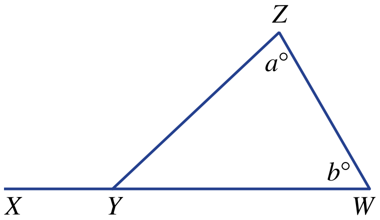


9 Explain why $\angle OAB$ is 32° in this circle. O is the centre of the circle.



10 In this diagram, $\angle XYZ$ is an exterior angle. *Do not* use the exterior angle theorem in the following.

- **a** If $a = 85$ and $b = 75$, find $\angle XYZ$.
- **b** If $a = 105$ and $b = 60$, find $\angle XYZ$.
- **c** Now using the pronumerals a and b, prove that $\angle XYZ = a° + b°$.

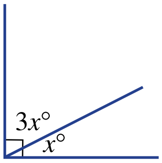


11 Prove that the three exterior angles of a triangle sum to 360°. Use the fact that the three interior angles sum to 180°.

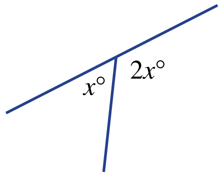
ENRICHMENT: Algebra in geometry	–	–	12

12 Write an equation for each diagram and solve it to find x.

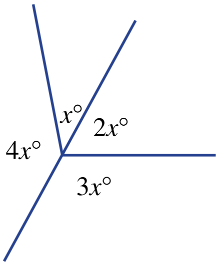
a



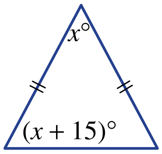
b



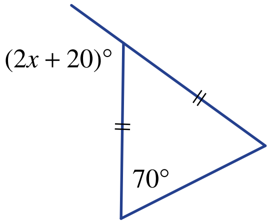
c



d



e



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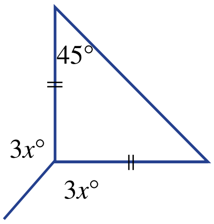

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7B Parallel lines CONSOLIDATING

Learning intentions
- To know the special pairs of angles formed when a transversal cuts two other lines
- To know the relationships between angles formed by parallel lines and a transversal
- To be able to calculate unknown angles associated with parallel lines

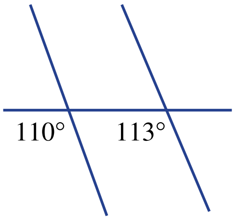
Past, present and future learning
- This section consolidates Stage 4 concepts
- Some of these questions are more challenging than those in our Stage 4 books
- These concepts are assumed knowledge for Stage 6 Advanced and are occasionally useful

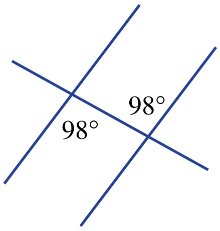
A line crossing two or more other lines (called a transversal) creates a number of special pairs of angles. If the transversal cuts two parallel lines, then these special pairs of angles will be either equal or supplementary.

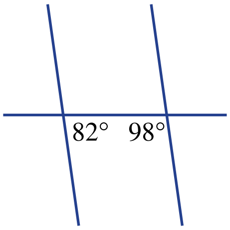
A piano has parallel keys, hammers and strings; however, a violin's strings are not parallel. Musicians write music using two sets of parallel lines: the treble clef (higher notes) and bass clef (lower notes).

Lesson starter: Are they parallel?

Here are three diagrams in which a transversal crosses two other lines.

1


2


3


- Decide if each diagram contains a pair of parallel lines. Give reasons for your answer.
- What words do you remember regarding the name given to each pair of angles shown in the diagrams?

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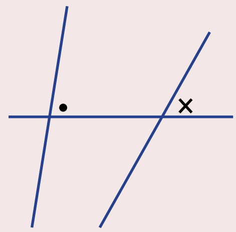
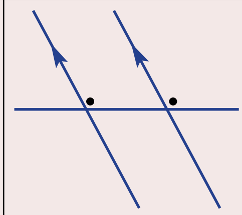
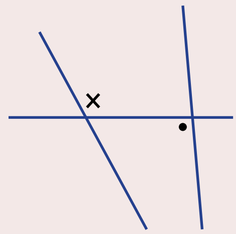
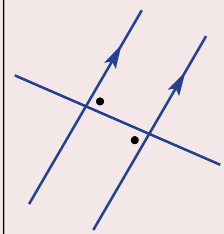
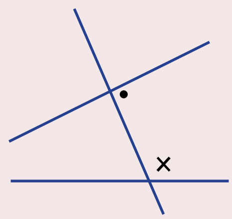
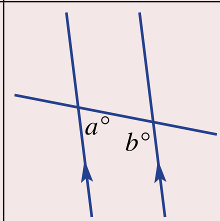
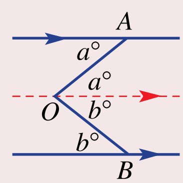
KEY IDEAS

- A **transversal** is a line crossing two or more other lines.

Pair of angles	Non-parallel lines	Parallel lines
Corresponding angles If lines are parallel corresponding angles are equal.		
Alternate angles If lines are parallel alternate angles are equal.		
Cointerior angles If lines are parallel cointerior angles are supplementary.		 $a + b = 180$

- If a line AB is parallel to a line CD we write $AB \parallel CD$.
- A parallel line can be added to a diagram to help find other angles.



$\angle AOB = a° + b°$

BUILDING UNDERSTANDING

1 Choose the word *equal* or *supplementary* to complete these sentences.

a If two lines are parallel corresponding angles are ________.

b If two lines are parallel alternate angles are ________.

c If two lines are parallel cointerior angles are ________.

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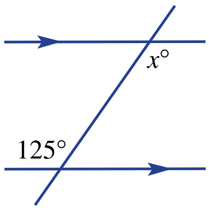
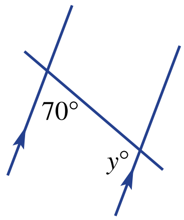
2 State the values of the pronumerals in these parallel lines. Give reasons for your answers.

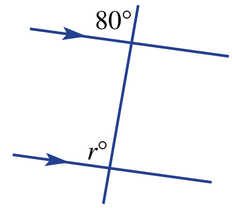
a



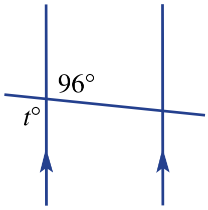
b



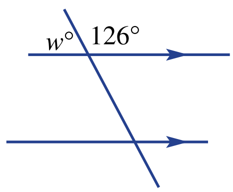
c



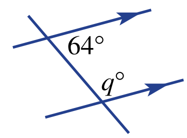
d



e



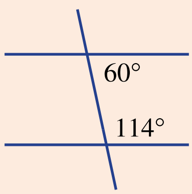
f



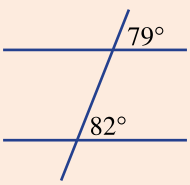
Example 4 Deciding if lines are parallel

Decide if each diagram contains a pair of parallel lines. Give a reason.

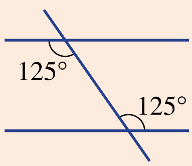
a



b



c



SOLUTION

a No. The two cointerior angles are not supplementary.

b No. The two corresponding angles are not equal.

c Yes. The two alternate angles are equal.

EXPLANATION

$60° + 114° = 174° \neq 180°$

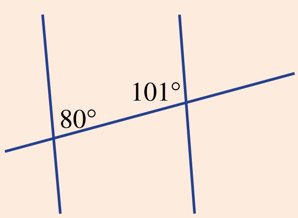
$79° \neq 82°$

If alternate angles are equal then the lines are parallel.

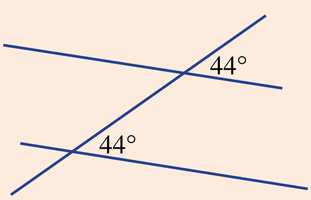
Now you try

Decide if each diagram contains a pair of parallel lines. Give a reason.

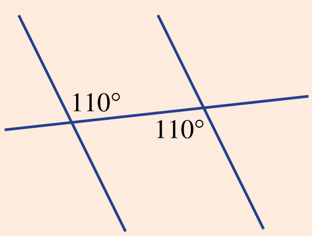
a



b



c



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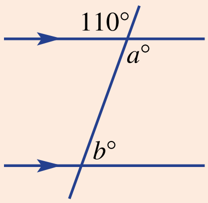
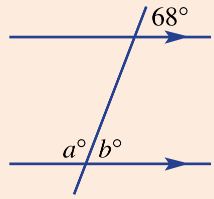
Example 5 Finding angles in parallel lines

Find the value of each of the pronumerals. Give reasons for your answers.

a



b



SOLUTION

a $a = 110$ (vertically opposite angles)

$b + 110 = 180$ (cointerior angles in parallel lines)

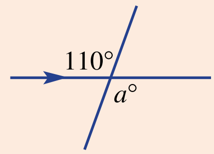
$b = 70$

b $b = 68$ (corresponding angles in parallel lines)

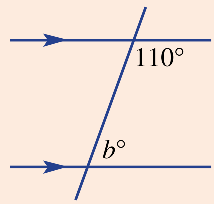
$a + 68 = 180$ (supplementary angles)

$a = 112$

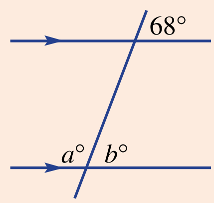
EXPLANATION



Vertically opposite angles are equal.



Cointerior angles add to 180° in parallel lines.



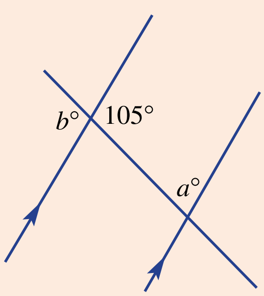
Corresponding angles are equal in parallel lines.

Supplementary angles add to 180° so $a + b = 180$.

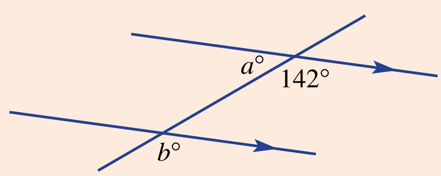
Now you try

Find the value of each of the pronumerals. Give reasons for your answers.

a



b



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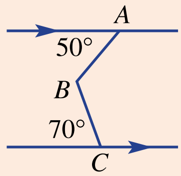
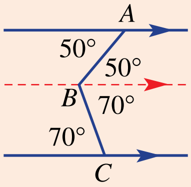
Example 6 Adding a third parallel line

Add a third parallel line to help find $\angle ABC$ in this diagram.



SOLUTION



$$\angle ABC = 50° + 70° \\ = 120°$$

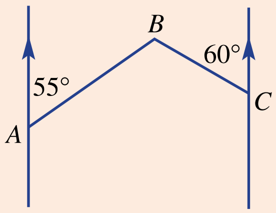
EXPLANATION

Add a third parallel line through B to create two pairs of equal alternate angles.

Add 50° and 70° to give the size of $\angle ABC$.

Now you try

Add a third parallel line to help find $\angle ABC$ in this diagram.



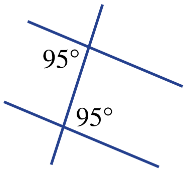
Exercise 7B

FLUENCY	1, 2(½)	1–2(½)	1–2(½)

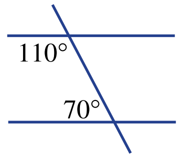
Example 4

1 Decide if each diagram contains a pair of parallel lines. Give a reason.

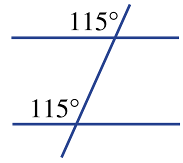
a



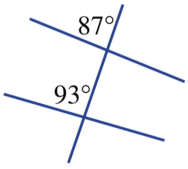
b



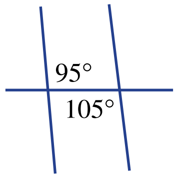
c



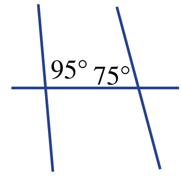
d



e



f



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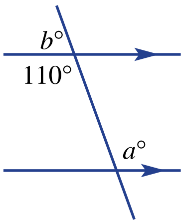
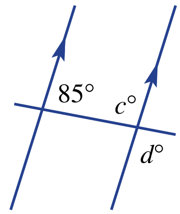
Example 5

2 Find the value of each pronumeral. Give reasons for your answers.

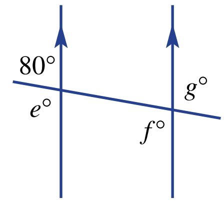
a



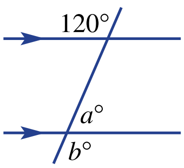
b



c



d



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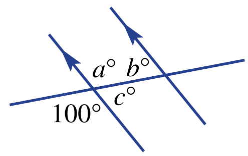

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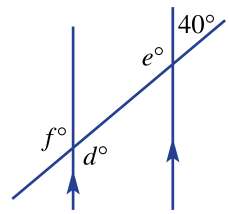
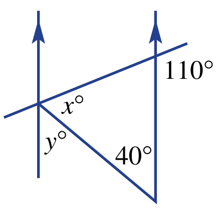


PROBLEM–SOLVING	3	3–4(½)	3–4(½), 5

3 Find the value of each pronumeral.

a



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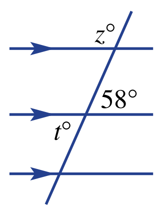

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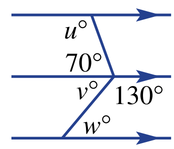

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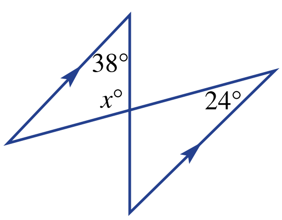

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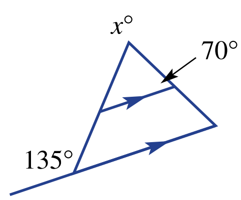

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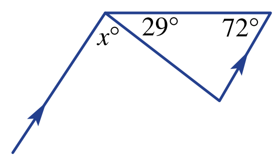
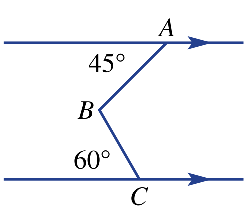


Example 6

4 Add a third parallel line to help find $\angle ABC$ in these diagrams.

a



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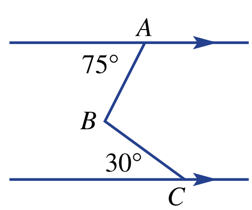

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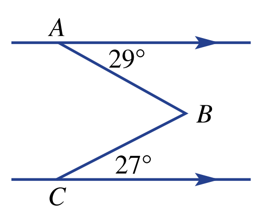
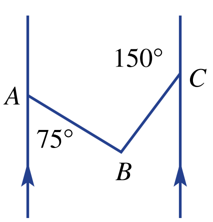
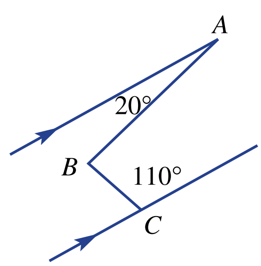


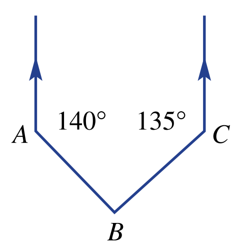
d



e



f



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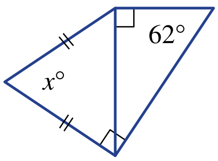
5 Find the value of x.

a



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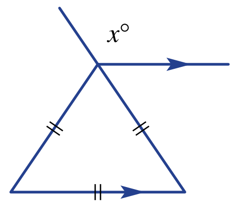

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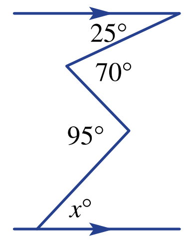
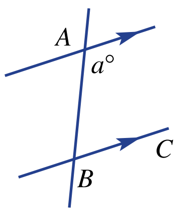


REASONING	6	6(½), 7	6(½), 7, 8

6 Write an expression (for example, $180^\circ - a^\circ$) for $\angle ABC$ in these diagrams.

a



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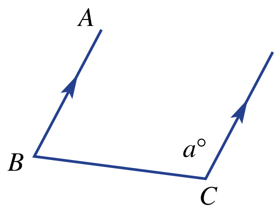

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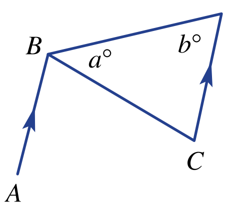

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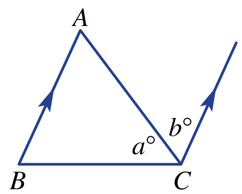
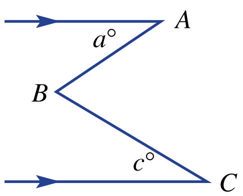


e



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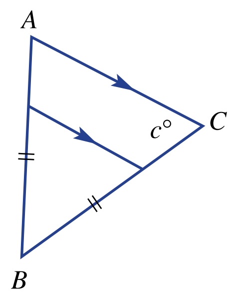
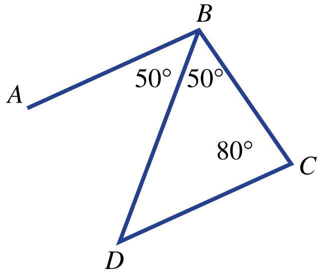


7 Give reasons why $AB \parallel DC$ (AB is parallel to DC) in this diagram.



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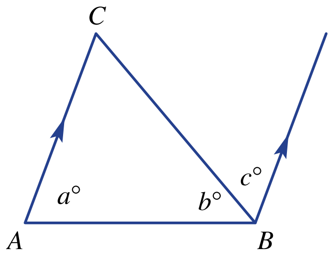
8 The diagram below includes a triangle and a pair of parallel lines.

a Using the parallel lines, explain why $a + b + c = 180$.

b Explain why $\angle ACB = c°$.

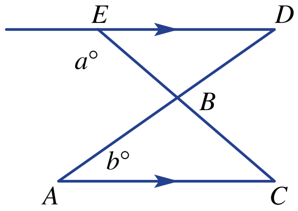
c Explain why this diagram helps to prove that the angle sum of a triangle is 180°.



ENRICHMENT: Proof in geometry	–	–	9

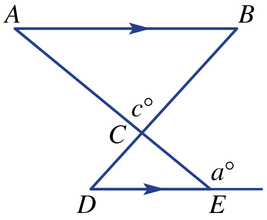
9 Here is a written proof showing that $\angle ABC = a° - b°$.

$$\begin{aligned}\angle BED &= 180° - a° && \text{(angle sum of a triangle)}\\ \angle BCA &= 180° - a° && \text{(alternate angles and } ED \parallel AC\text{)}\\ \angle ABC &= 180° - b° - (180 - a)° && \text{(angle sum of a triangle)}\\ &= 180° - b° - 180° + a°\\ &= -b° + a°\\ &= a° - b°\end{aligned}$$

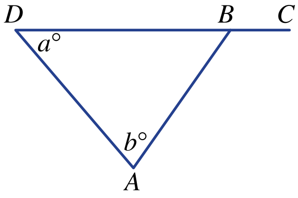


Write a similar proof for each of the following angles.

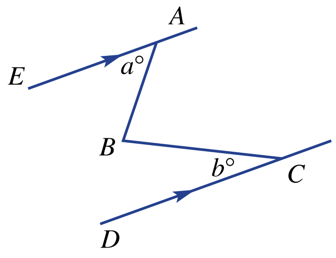
a $\angle ABC = a° - c°$



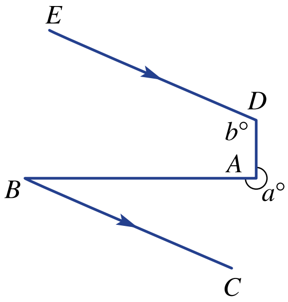
b $\angle ABC = a° + b°$



c $\angle ABC = a° + b°$



d $\angle ABC = 180° + b° - a°$



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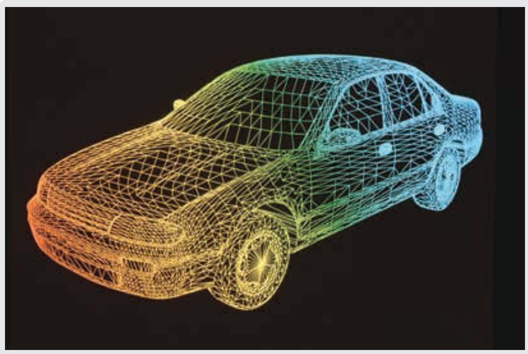
7C Quadrilaterals and other polygons

Learning intentions
- To know terms related to polygons such as regular, convex and non-convex
- To know how to find the interior angle sum of a polygon
- To know the properties of special quadrilaterals
- To know the angle sum of a quadrilateral
- To be able to calculate unknown angles inside polygons

Past, present and future learning
- This section addresses the Stage 5 Path Topics called *Properties of geometrical figures B and C*
- These concepts will be revised and extended in Chapter 2 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Extension

Closed two-dimensional shapes with straight sides are called polygons and are classified by their number of sides. Quadrilaterals have four sides and are classified further by their special properties.

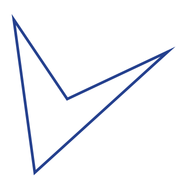


Computer programmers code 3D objects using a polygon mesh of triangles and quadrilaterals. Working with computer-aided design and machine learning, engineers can test and adjust the aerodynamic properties of a new car model before it is built.

Lesson starter: Draw that shape

Use your knowledge of polygons to draw each of the following shapes. Mark any features, including parallel sides and sides of equal length.

- convex quadrilateral
- non-convex pentagon
- regular hexagon
- square, rectangle, rhombus and parallelogram
- kite and trapezium

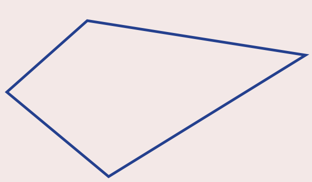


This is a non-convex quadrilateral.

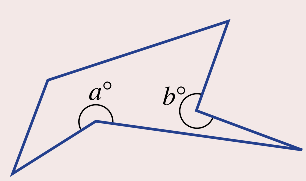
Compare the properties of each shape to ensure you have indicated each property on your drawings.

KEY IDEAS

- **Convex polygons** have all interior angles less than 180°. A **non-convex polygon** has at least one interior angle greater than 180°.



Convex quadrilateral



Non-convex hexagon

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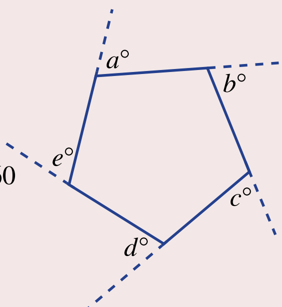
- **The sum of the interior angles**, S, in a polygon with n sides is given by $S = (n - 2) \times 180°$.

Polygon	Number of sides (n)	Angle sum (S)
Triangle	3	180°
Quadrilateral	4	360°
Pentagon	5	540°
Hexagon	6	720°
Heptagon	7	900°
Octagon	8	1080°
Nonagon	9	1260°
Decagon	10	1440°
Undecagon	11	1620°
Dodecagon	12	1800°
n-gon	n	$(n - 2) \times 180°$

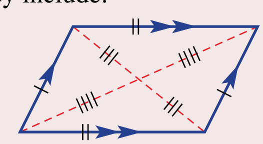
- **Regular polygons** have sides of equal length and equal interior angles.

- The sum of all the exterior angles of every polygon is 360°.
  - This example shows a pentagon but the 360° exterior sum is true for all polygons.

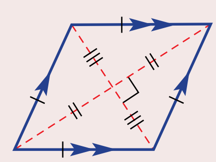
$a + b + c + d + e = 360$



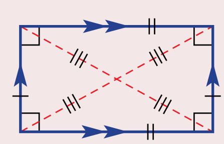
- **Parallelograms** are **quadrilaterals** with two pairs of parallel sides. They include:
  - Parallelogram: a quadrilateral with two pairs of parallel sides



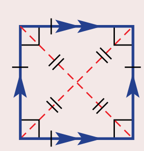
  - Rhombus: a parallelogram with all sides equal



  - Rectangle: a parallelogram with all angles 90°



  - Square: a rhombus with all angles 90°.



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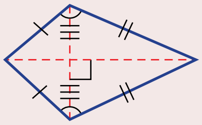
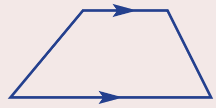
- The kite and trapezium are also special quadrilaterals.
  - Kite 
  - Trapezium 
  - Rhombuses and squares are kites
  - A trapezium, is a quadrilateral with at least one pain of parallel sides. So parallelograms including rhombuses, rectangles and squares, are trapezia.

BUILDING UNDERSTANDING

1 How many sides do these polygons have?

a Pentagon **b** Heptagon **c** Quadrilateral
d Undecagon **e** Nonagon **f** Dodecagon

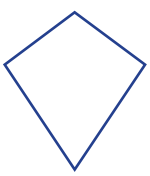
2 Use $S = (n - 2) \times 180°$ to find the angle sum, S, of these polygons, where n is the number of sides.

a Hexagon **b** Octagon **c** Undecagon

3 Choose the word to complete these sentences.

a A parallelogram has two pairs of ________ sides.
b The diagonals in a rhombus intersect at ________ angles.
c A ________ has at least one pair of parallel sides.
d The diagonals in a rectangle are ________ in length.

4 Name each of these shapes and describe them as convex or non convex.

a


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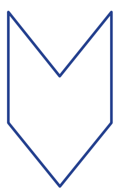
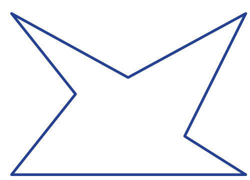
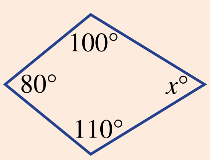


c


Example 7 Finding angles in quadrilaterals

Find the value of the pronumeral in each of these quadrilaterals.

a


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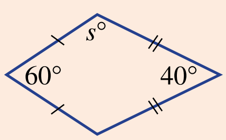
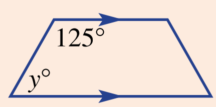


c


SOLUTION

a
$$\begin{aligned} x + 80 + 100 + 110 &= 360 \\ x + 290 &= 360 \\ x &= 70 \end{aligned}$$

EXPLANATION

The angles in a quadrilateral add to 360°.
Simplify and solve for x.

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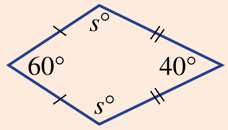
b $2s + 60 + 40 = 360$
$2s + 100 = 360$
$2s = 260$
$s = 130$



The angles in a quadrilateral add to 360° and the opposite angles (s°) are equal in a kite.

Simplify and solve for s.

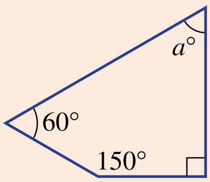
c $y + 125 = 180$
$y = 55$

Cointerior angles inside parallel lines are supplementary.

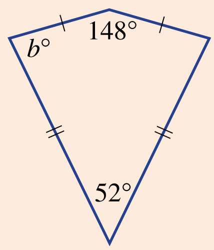
Now you try

Find the value of the pronumeral in each of these quadrilaterals.

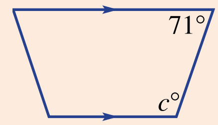
a



b



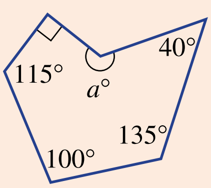
c



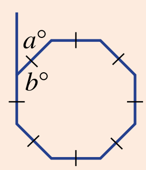
Example 8 Finding angles in polygons

For each polygon find the angle sum using $S = (n - 2) \times 180°$, then find the value of any pronumerals. The polygon in part **b** is regular.

a



b



SOLUTION

a $n = 6$ and $S = (n - 2) \times 180°$
$= (6 - 2) \times 180°$
$= 720$

$a + 90 + 115 + 100 + 135 + 40 = 720$
$a + 480 = 720$
$a = 240$

b $n = 8$ and $S = (n - 2) \times 180°$
$= (8 - 2) \times 180°$
$= 1080$

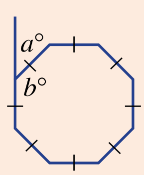
$8b = 1080$
$b = 135$
$a + 135 = 180$
$a = 45$

EXPLANATION

The shape is a hexagon with 6 sides so $n = 6$.

The sum of all angles is 720°. Simplify and solve for a.

The regular octagon has 8 sides, so use $n = 8$.



Each interior angle is equal to b°, so $8b$° is the angle sum.

a° is an exterior angle and a° and b° are supplementary.

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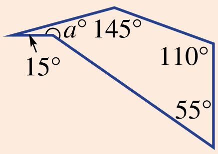
Now you try

For each polygon find the angle sum using $S = (n - 2) \times 180°$, then find the value of any pronumerals. The polygon in part **b** is regular.

a



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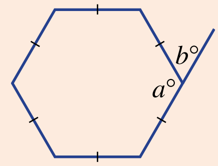


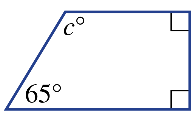
Exercise 7C

FLUENCY	1(½), 2, 3(½)	1–3(½)	2–3(½)

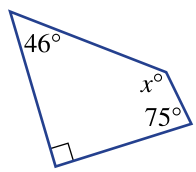
Example 7a

1 Find the values of the pronumerals in each of these quadrilaterals.

a



b



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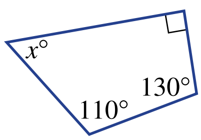
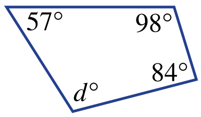
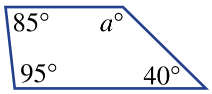


d



e



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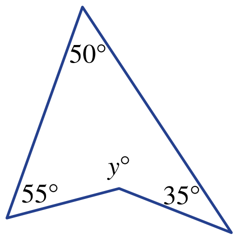
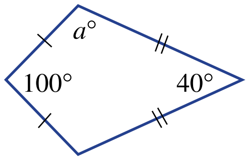


Example 7b,c

2 Find the value of the pronumerals.

a



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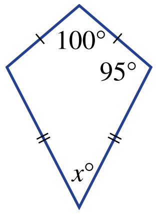

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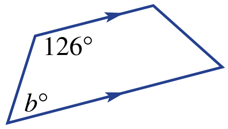

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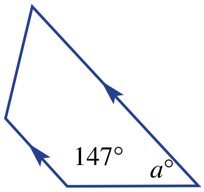

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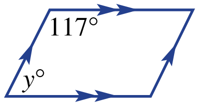

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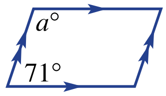

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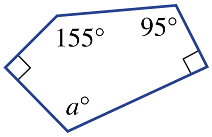
Example 8

3 For each polygon find the angle sum using $S = (n - 2) \times 180°$, then find the value of any pronumerals. The polygons in parts **e–i** are regular.

a


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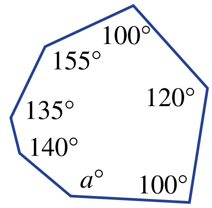
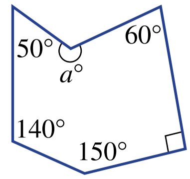


c


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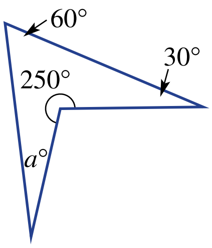
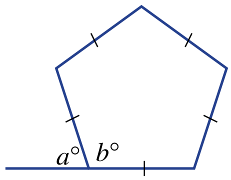


e


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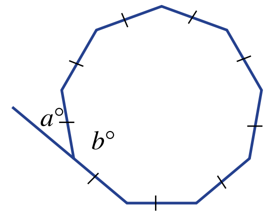
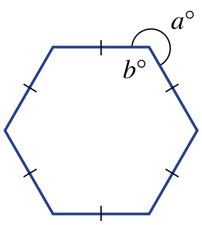
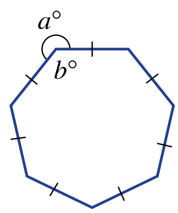
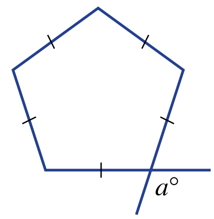


g


h


i


PROBLEM–SOLVING	4	4, 5, 6(½)	5, 6

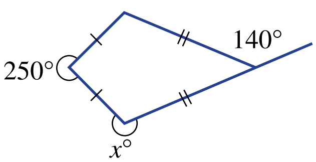
4 List all the quadrilaterals which have these properties.

a 2 pairs of equal length sides
b All interior angles 90°
c Diagonals of equal length
d Diagonals intersecting at right angles

5 Calculate the number of sides of a polygon that has the given angle sum. (*Hint:* Use the rule $S = (n - 2) \times 180°$.)

a 2520°
b 4140°
c 18 000°

6 Find the value of x in these diagrams.

a


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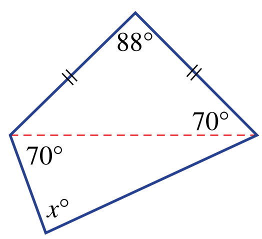

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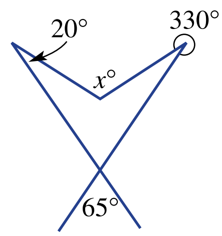

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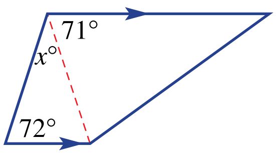

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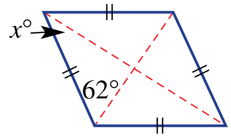

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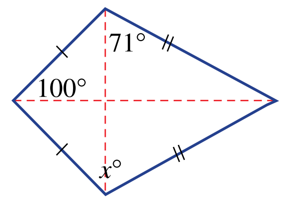

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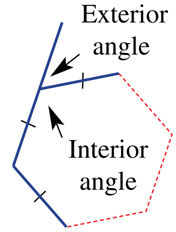
REASONING	7	7, 8	8, 9

7 Explain why a rectangle, a square and a rhombus are all parallelograms.



8 Consider a regular polygon with n sides.

a Write the rule for the sum of the interior angles ($S°$).

b Write the rule for the size of each interior angle ($I°$).

c Write the rule for the size of each exterior angle ($E°$).

d Use your rule from part **c** to find the size of the exterior angle of a regular decagon.

9 Recall that a non-convex polygon has at least one reflex interior angle.

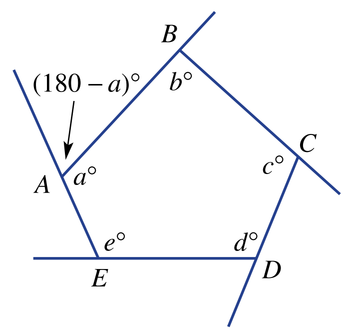
a What is the maximum number of interior reflex angles possible for these polygons?

i quadrilateral **ii** pentagon **iii** octagon

b Write an expression for the maximum number of interior reflex angles for a polygon with n sides.

ENRICHMENT: Angle sum proof	–	–	10

10 Note that if you follow the path around this pentagon starting and finishing at point A (provided you finish by pointing in the same direction as you started) you will have turned a total of 360°.



Complete this proof of the angle sum of a pentagon (540°).

$(180 - a) + (180 - b) + (\quad) + (\quad) + (\quad) = \_\_\_\_\_\_$

(since sum of exterior angles is ______)

$180 + 180 + \_\_\_\_\_\_ + \_\_\_\_\_\_ + \_\_\_\_\_\_ - (a + b + \_\_\_\_\_\_ + \_\_\_\_\_\_ + \_\_\_\_\_\_) = 360$

$\_\_\_\_\_\_ - (\qquad) = 360$

$(\qquad) = \_\_\_\_\_\_$

Now complete a similar proof for the angle sum of these polygons.

a Hexagon **b** Heptagon

(For an additional challenge, try to complete a similar proof for a polygon with n sides.)

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7D Congruent triangles

Learning intentions
- To know the meaning of the term congruent
- To be able to match corresponding sides and angles in congruent figures
- To know the four tests for congruence of triangles

Past, present and future learning
- This section addresses the Stage 5 Path Topics called *Properties of geometrical figures B and C*
- These concepts will be revised and extended in Chapter 2 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Extension

When two objects have the same shape and size we say they are congruent. Matching sides will be the same length and matching angles will be the same size. The areas of congruent shapes will also be equal. However, not every property of a pair of shapes needs to be known in order to determine their congruence. This is highlighted in the study of congruent triangles in which four tests can be used to establish congruence.

Engineers apply congruent triangle geometry in the design and construction of truss bridges, as shown on this railway truss bridge near Mt Fuji, Japan.

Lesson starter: Constructing congruent triangles

To complete this task you will need a ruler, pencil and protractor. (For accurate constructions you may wish to use compasses.) Divide these constructions up equally among the members of the class. Each group is to construct their triangle with the given properties.

1 Triangle ABC with $AB = 8$ cm, $AC = 5$ cm and $BC = 4$ cm

2 Triangle DEF with $DE = 7$ cm, $DF = 6$ cm and $\angle EDF = 40°$

3 Triangle GHI with $GH = 6$ cm, $\angle IGH = 50°$ and $\angle IHG = 50°$

4 Triangle JKL with $\angle JKL = 90°$, $JL = 5$ cm and $KL = 4$ cm

- Now compare all triangles with the vertices ABC. What do you notice? What does this say about two triangles that have three pairs of equal side lengths?
- Compare all triangles with the vertices DEF. What do you notice? What does this say about two triangles that have two pairs of equal side lengths and the included angles equal?
- Compare all triangles with the vertices GHI. What do you notice? What does this say about two triangles that have two equal corresponding angles and one corresponding equal length side?
- Compare all triangles with the vertices JKL. What do you notice? What does this say about two triangles that have one right angle, the hypotenuse and one other pair of corresponding sides equal?

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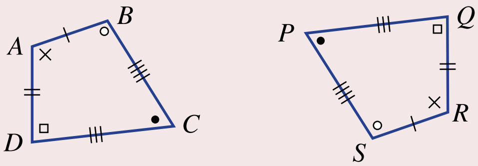
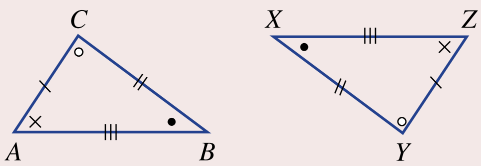
KEY IDEAS

- **Congruent figures** have the same shape and size.
  They are identical in every way.
  - If two figures are congruent, one of them can be transformed by using rotation, reflection and/or translation to match the other figure exactly.



- In the above diagram, AB and RS are called 'matching sides' or 'corresponding sides'.

- Consider these congruent triangles.



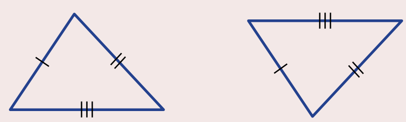
Matching (corresponding) sides	Matching (corresponding) angles
$AB = ZX$	$\angle A = \angle Z$
$BC = XY$	$\angle B = \angle X$
$AC = ZY$	$\angle C = \angle Y$

We write $\Delta ABC \equiv \Delta ZXY$ (not $\Delta ABC \equiv \Delta XYZ$)

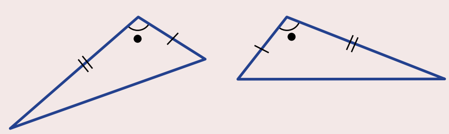
This is called a **congruence statement**. Note the order of the letters in the statement.

- **Corresponding** sides are opposite equal corresponding angles.

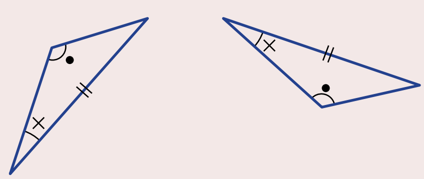
- Tests for triangle congruence.
  - **Side, Side, Side (SSS) test**
    If the three sides of a triangle are respectively equal to the three sides of another triangle, then the two triangles are congruent.



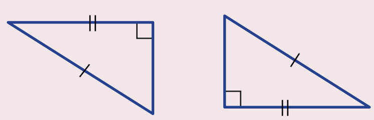
  - **Side, Angle, Side (SAS) test**
    If two sides and the included angle of a triangle are respectively equal to two sides and the included angle of another triangle, then the two triangles are congruent.



  - **Angle, Angle, Side (AAS) test**
    If two angles and one side of a triangle are respectively equal to two angles and the matching side of another triangle, then the two triangles are congruent.



  - **Right angle, Hypotenuse, Side (RHS) test**
    If the hypotenuse and second side of a right-angled triangle are respectively equal to the hypotenuse and a second side of another right-angled triangle, then the two triangles are congruent.



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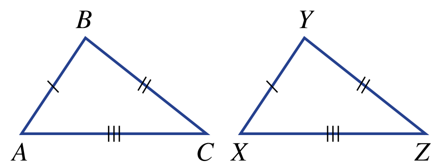
BUILDING UNDERSTANDING

1 State the missing terms to complete the sentences below.

- **a** Congruent figures are exactly the same shape and ________.
- **b** If triangle ABC is congruent to triangle STU then we write $\Delta ABC \equiv$ ________.
- **c** The abbreviated names of the four congruence tests for triangles are SSS, ___, ___ and ___.

2 These two triangles are congruent.



- **a** Name the side on ΔXYZ which corresponds to (matches):
  - **i** AB **ii** AC **iii** BC
- **b** Name the angle in ΔABC which corresponds to (matches):
  - **i** $\angle X$ **ii** $\angle Y$ **iii** $\angle Z$

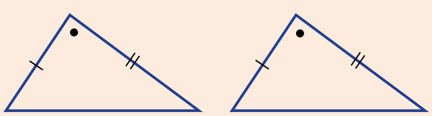
3 State a congruence statement (e.g. $\Delta ABC \equiv \Delta DEF$) given that:

- **a** triangle ABC is congruent to triangle FGH
- **b** triangle DEF is congruent to triangle STU.

Example 9 Choosing the appropriate congruence test

Which congruence test (SSS, SAS, AAS or RHS) would be used to show that these pairs of triangles are congruent?

a



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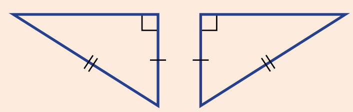
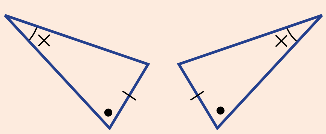


c



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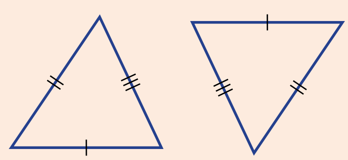

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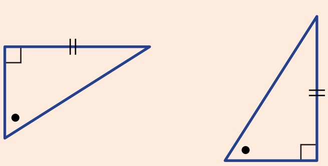

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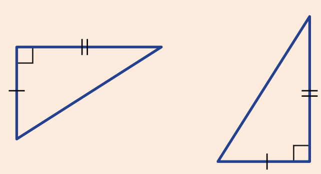

SOLUTION	EXPLANATION
a SAS	Two pairs of corresponding sides and the included angle are equal.
b RHS	A right angle, hypotenuse and one pair of corresponding sides are equal.
c AAS	Two pairs of angles and a pair of corresponding sides are equal.
d SSS	Three pairs of corresponding sides are equal.
e AAS	Two pairs of angles and a pair of matching sides are equal.
f SAS	Two pairs of matching sides and the included angles are equal.

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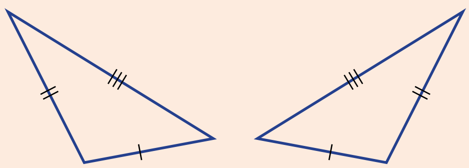
Now you try

Which congruence test (SSS, SAS, AAS or RHS) would be used to show that these pairs of triangles are congruent?

a


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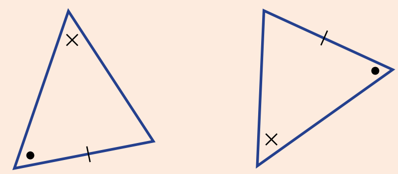

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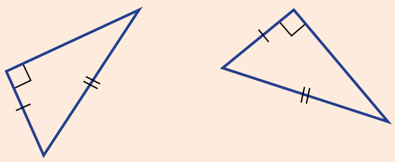

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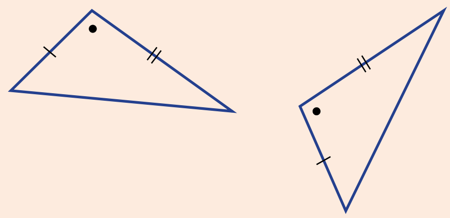

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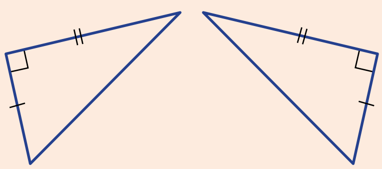

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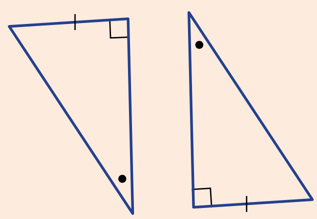

Example 10 Finding missing side lengths and angles using congruence

Find the values of the pronumerals in these pairs of congruent triangles.

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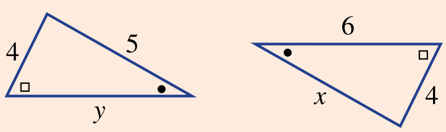

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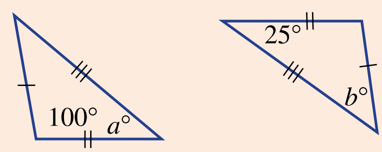

SOLUTION	EXPLANATION
a $x = 5$	The side of length x and the side of length 5 are in corresponding positions (opposite the □).
$y = 6$	The longest sides on the two triangles must be equal. The side of length y and the side of length 6 are corresponding sides.
b $a = 25$	The angle marked $a°$ corresponds to the 25° angle in the other triangle.
$b = 180 - 100 - 25$ $= 55$	The angle marked $b°$ corresponds to the missing angle in the first triangle. The sum of three angles in a triangle is 180°.

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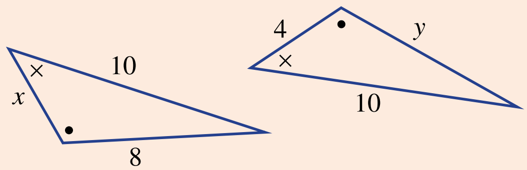
Now you try

Find the values of the pronumerals in these pairs of congruent triangles.

a



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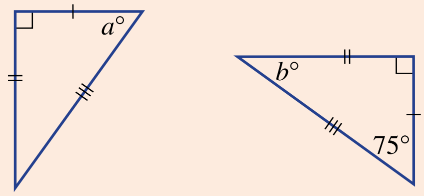


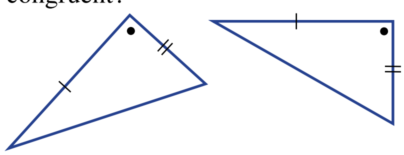
Exercise 7D

FLUENCY	1, 2(½)	1, 2(½)	1, 2(½)

Example 9

1 Which congruence test (SSS, SAS, AAS or RHS) would be used to show that these pairs of triangles are congruent?

a



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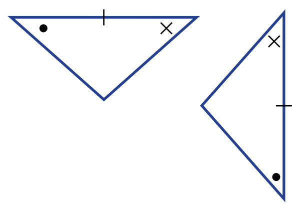

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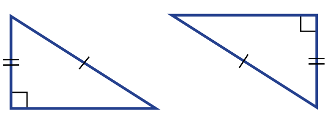

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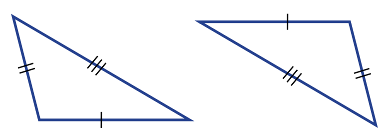

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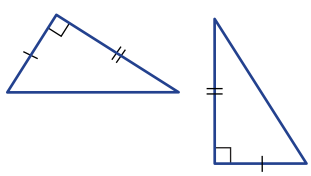

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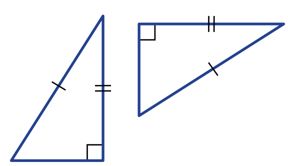
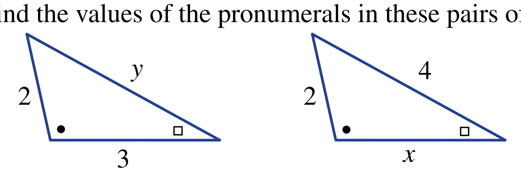


Example 10

2 Find the values of the pronumerals in these pairs of congruent triangles.

a



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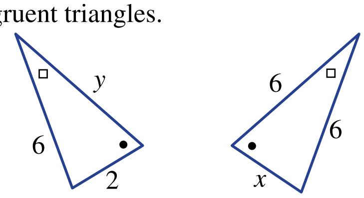

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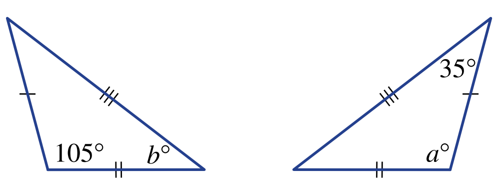

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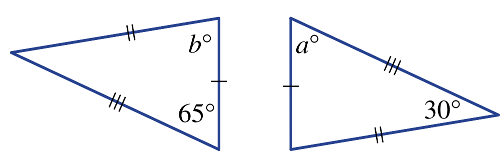

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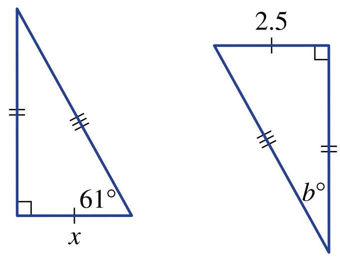

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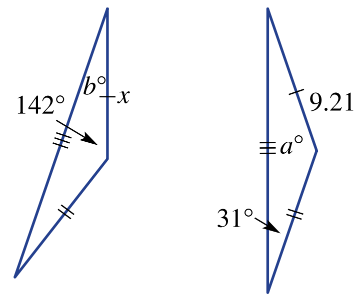

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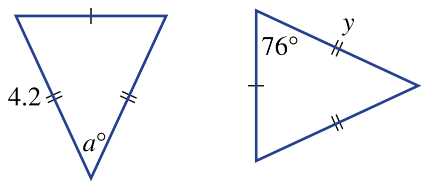

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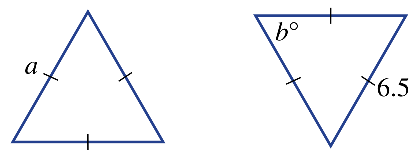
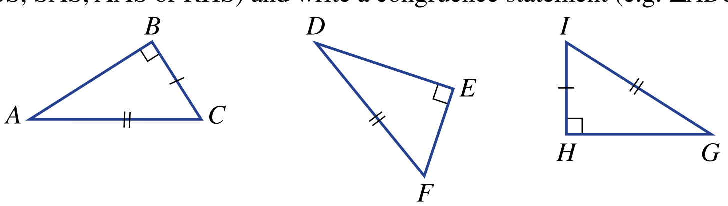


PROBLEM–SOLVING	3, 4	3, 4	4, 5

3 For each set of three triangles choose the two which are congruent. Give a reason (SSS, SAS, AAS or RHS) and write a congruence statement (e.g. $\Delta ABC \equiv \Delta FGH$).

a



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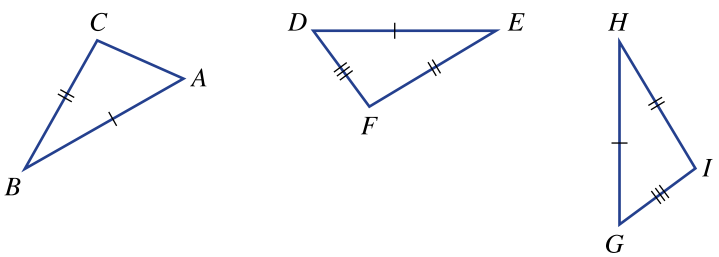

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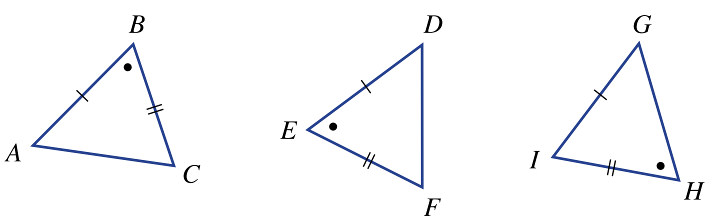

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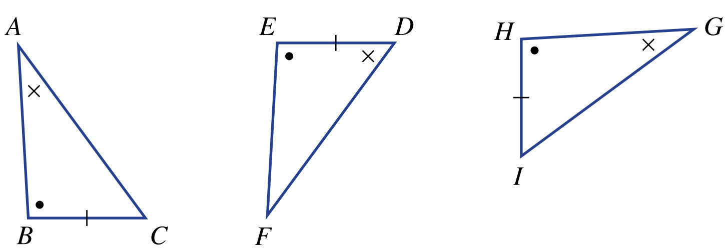

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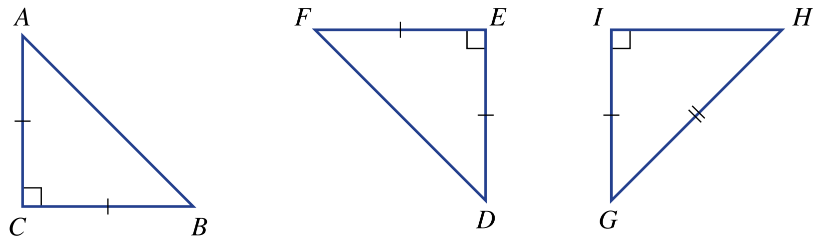
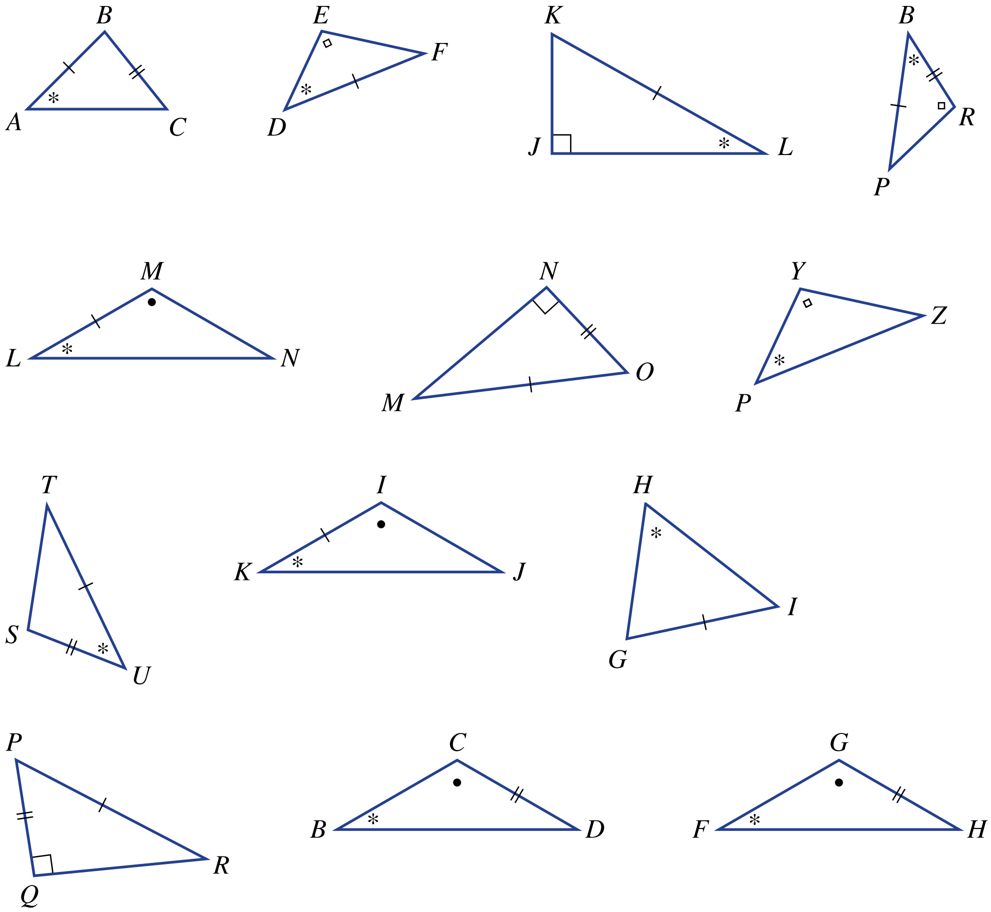
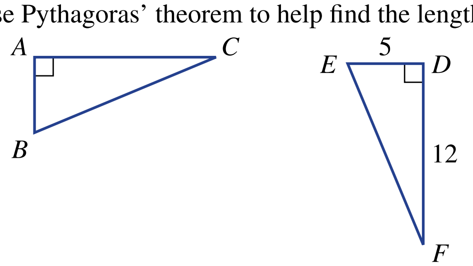


4 Identify all pairs of congruent triangles from those below. Angles with the same mark are equal.



5 Use Pythagoras' theorem to help find the length BC in these pairs of congruent triangles.

a



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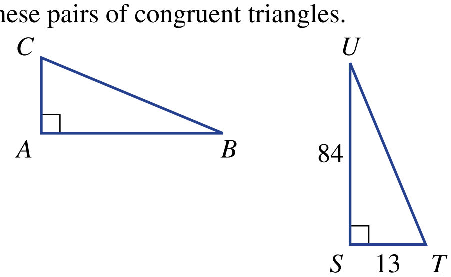

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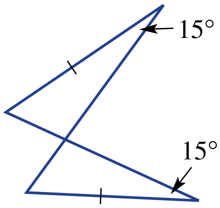
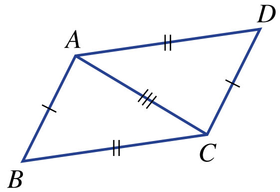
REASONING	6, 7	6–8	7–9

6 Are all triangles with three pairs of equal corresponding angles congruent? Explain why or why not.

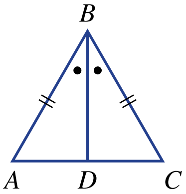
7 Consider this diagram of two triangles.
  a Explain why there are two pairs of equal matching angles.
  b Give the reason (SSS, SAS, AAS or RHS) why there are two congruent triangles.



8 *ABCD* is a parallelogram.
  a Give the reason why $\Delta ABC \equiv \Delta CDA$.
  b What does this say about $\angle B$ and $\angle D$?

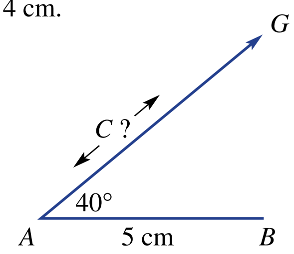


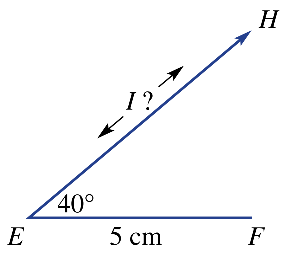
9 Consider the diagram shown at right.
  a Explain why there are two pairs of corresponding sides of equal length for the two triangles.
  b Give the reason (SSS, SAS, AAS or RHS) why there are two congruent triangles.
  c Write a congruence statement.
  d Explain why *AC* is perpendicular (90°) to *DB*.



ENRICHMENT: Why not angle, side, side?	–	–	10

10 Angle, Side, Side (ASS) is not a test for congruence of triangles. Complete these tasks to see why.
  a Draw two line segments *AB* and *EF* both 5 cm long.
  b Draw two rays *AG* and *EH* so that both $\angle A$ and $\angle E$ are 40°.
  c Now place a point *C* on ray *AG* so that $BC = 4$ cm.
  d Place a point *I* on ray *EH* so that *FI* is 4 cm but place it in a different position so that ΔABC is not congruent to ΔEFI.
  e Show how you could use compasses to find the two different places you could put the points *C* or *I* so that *BC* and *FI* are 4 cm.





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7E Using congruence in proof

Learning intentions

- To know the four tests for congruence of triangles
- To know how to prove that two triangles are congruent using one of the tests
- To be able to use congruence of triangles to prove other properties

Past, present and future learning

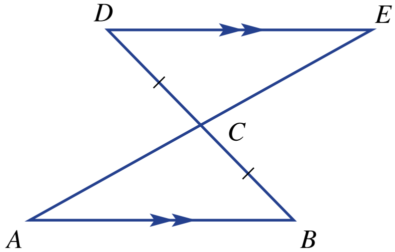
- This section addresses the Stage 5 Path Topics called *Properties of geometrical figures B and C*
- These concepts will be revised and extended in Chapter 2 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Extension

A mathematical proof is a sequence of correct statements that leads to a result. It should not contain any big 'leaps' and should provide reasons at each step. The proof that two triangles are congruent should list all the corresponding pairs of sides and angles. Showing that two triangles are congruent in more complex problems can then lead to the proof of other geometrical results.

Surveyors must take enough measurements to enable the calculation of every required length and angle. As polygons can be divided into triangles, surveyors use triangle congruence proofs to prove polygons are congruent.

Lesson starter: Complete the proof

Help complete the proof that $\Delta ABC \equiv \Delta EDC$ for this diagram. Give the missing reasons and congruent triangle in the final statement.



In ΔABC and ΔEDC:

$\angle DCE = \angle BCA$ (______________)

$\angle ABC = \angle EDC$ (______________)

$BC = DC$ (given equal sides)

$\therefore \Delta ABC \equiv$ _______ (AAS)

KEY IDEAS

- Prove that two triangles are congruent by listing all the known corresponding equal angles and sides.
  - Give reasons at each step.
  - Conclude by writing a congruence statement and the abbreviated reason (SSS, SAS, AAS or RHS).
  - Vertex labels are usually written in matching order.
- Other geometrical results can be proved by using the properties of congruent triangles.

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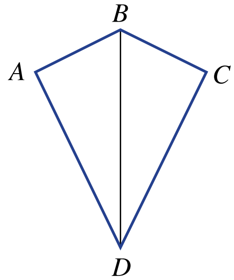
BUILDING UNDERSTANDING

1 Name the line segment which is common to both triangles in each diagram.

a



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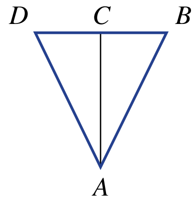

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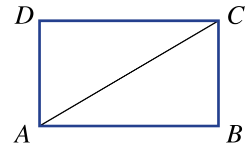
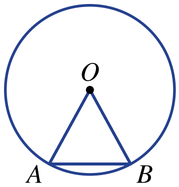


2 Why is ΔOAB an isosceles triangle in this circle? O is the centre of the circle.

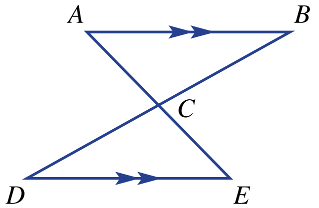


3 For this diagram name the angle which is:

a vertically opposite to $\angle ACB$

b alternate to $\angle CDE$

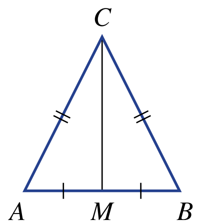
c alternate to $\angle BAC$.



4 For this isosceles triangle, CM is common to ΔAMC and ΔBMC.

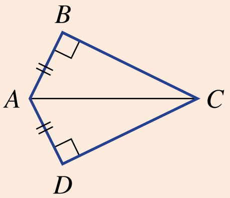
a Which congruence test would be used to prove that $\Delta AMC \equiv \Delta BMC$?

b Which angle in ΔBMC corresponds to $\angle AMC$ in ΔAMC?



Example 11 Proving that two triangles are congruent

Prove that $\Delta ABC \equiv \Delta ADC$.



SOLUTION

In ΔABC and ΔADC:

$\angle ABC = \angle ADC = 90°$ (given equal angles) (**R**)

AC is common (**H**)

$AB = AD$ (given equal sides) (**S**)

$\therefore \Delta ABC \equiv \Delta ADC$ (RHS)

EXPLANATION

Both triangles have a right angle.

AC is common to both triangles (hypotenuse).

AB and AD are marked as equal.

Write the congruence statement and the abbreviated reason. Write the vertex labels in matching order.

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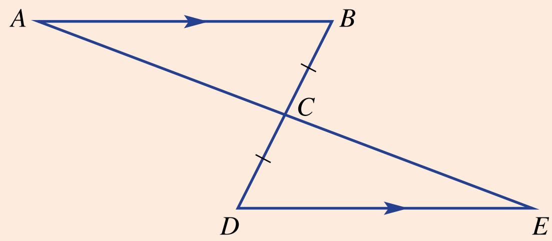
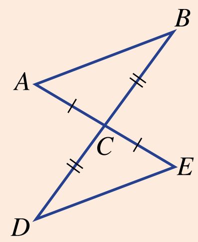
Now you try

Prove that $\Delta ABC \equiv \Delta EDC$.



Example 12 Proving geometrical results using congruence

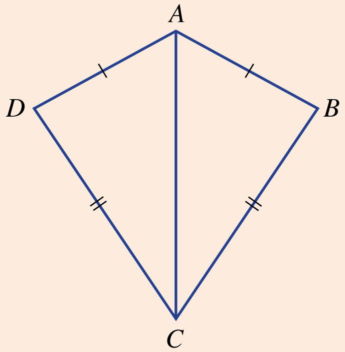
a Prove that $\Delta ABC \equiv \Delta EDC$.
b Hence prove that $AB \parallel DE$ (AB is parallel to DE).



SOLUTION	EXPLANATION
a In ΔABC and ΔEDC:	
$AC = EC$ (given equal sides) (**S**) $BC = DC$ (given equal sides) (**S**) $\angle ACB = \angle ECD$ (vertically opposite angles) (**A**) $\Delta ABC \equiv \Delta EDC$ (SAS)	List the given pairs of equal length sides and the vertically opposite angles. The included angle is between the given sides, hence SAS.
b $\angle BAC = \angle DEC$ (matching angles in congruent triangles)	All matching angles are equal.
$\therefore AB \parallel DE$ (alternate angles are equal)	If alternate angles are equal then AB and DE must be parallel.

Now you try

a Prove that $\Delta ABC \equiv \Delta ADC$.
b Hence prove that $\angle ABC = \angle ADE$
(i.e. that a kite has one pair of opposite equal angles).



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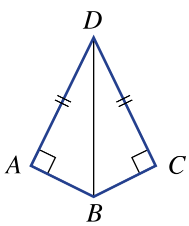
Exercise 7E

FLUENCY | 1(½) | 1(½) | 1(⅓)

Example 11

1 Prove that each pair of triangles is congruent. List your reasons and give the abbreviated congruence test.

a


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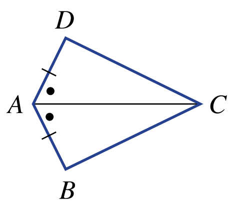

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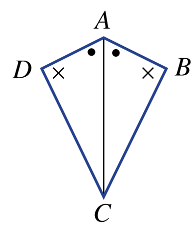

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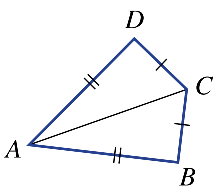

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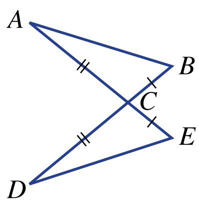

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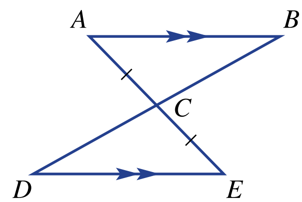

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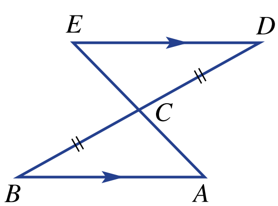

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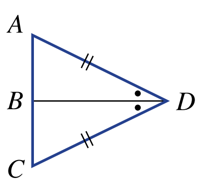

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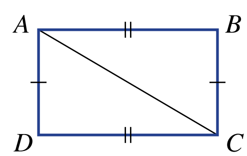

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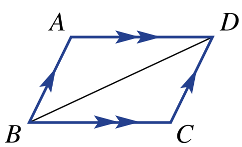

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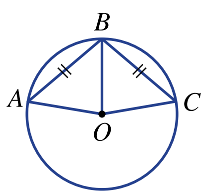

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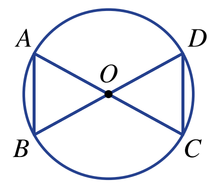
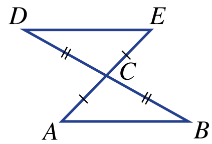


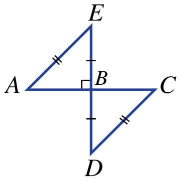
PROBLEM–SOLVING | 2, 3 | 2–5 | 4–7

Example 12

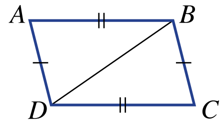
2 **a** Prove $\Delta ABC \equiv \Delta EDC$.
b Hence, prove $AB \parallel DE$.



3 **a** Prove $\Delta ABE \equiv \Delta CBD$.
b Hence, prove $AE \parallel CD$.



4 **a** Prove $\Delta ABD \equiv \Delta CDB$.
b Hence, prove $AD \parallel BC$.



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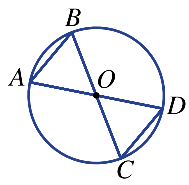
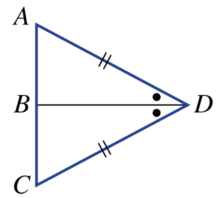
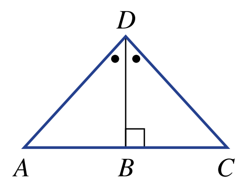
5 **a** Prove that $\Delta AOB \equiv \Delta DOC$. ($O$ is the centre of the circle.)
b Hence, prove that $AB \parallel CD$.



6 **a** Prove that $\Delta ABD \equiv \Delta CBD$.
b Hence, prove that AC is perpendicular to BD. ($AC \perp BD$)

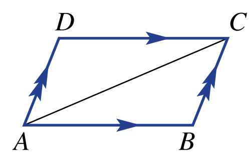


7 **a** Prove that $\Delta ABD \equiv \Delta CBD$.
b Hence, prove that ΔACD is isosceles.

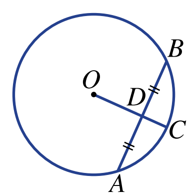


REASONING	8	8, 9	9–11

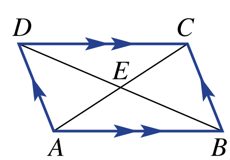
8 Use congruence to explain why $AD = BC$ and $AB = DC$ in this parallelogram.



9 Use congruence to explain why OC is perpendicular to AB in this diagram. (*Hint*: Form triangles.)



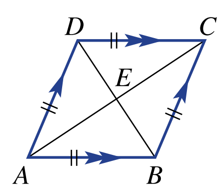
10 Use ΔABE and ΔCDE to explain why $AE = CE$ and $BE = DE$ in this parallelogram (i.e. the diagonals bisect each other).



11 Use congruence to show that the diagonals of any rectangle are equal in length.

ENRICHMENT: Extended proofs	–	–	12, 13

12 $ABCD$ is a rhombus. To prove that AC bisects BD at 90°, follow these steps.
a Prove that $\Delta ABE \equiv \Delta CDE$.
b Hence prove that AC bisects BD at 90°.



13 Use congruence to prove that the three angles in an equilateral triangle (given three equal side lengths) are all 60°.

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Progress quiz

7A

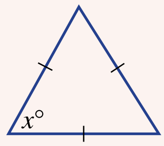
1 For an angle of size 53° determine the size of the:

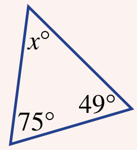
a complementary angle

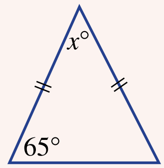
b supplementary angle.

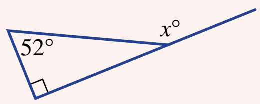
7A

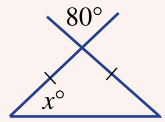
2 Find the value of x in each of these triangles.

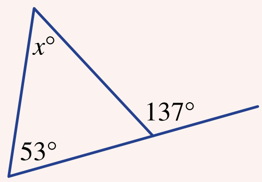
a


b


c


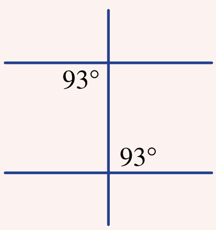
d


e


f


7B

3 Decide if each diagram contains a pair of parallel lines. Give a reason.

a


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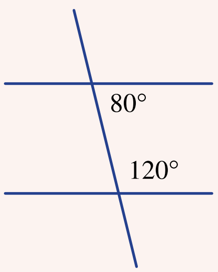
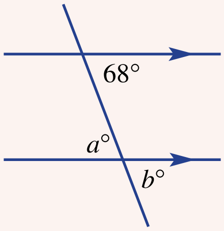
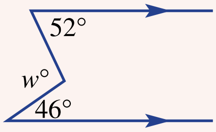


7B

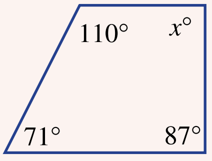
4 Find the value of each pronumeral. Give reasons for your answer.

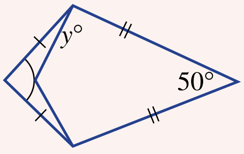
a


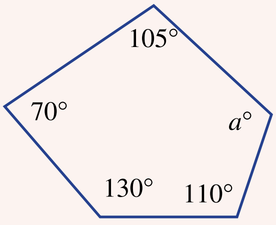
b


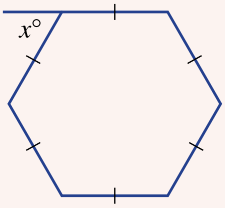
7C

5 Find the value of each pronumeral. Part **d** is regular.

a


b


c


d


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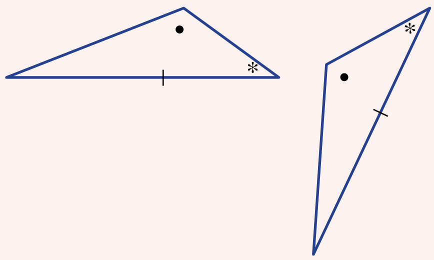
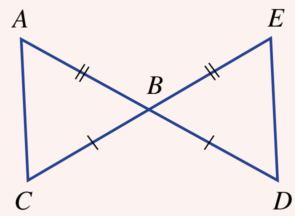
Progress quiz

7D **6** Which congruence test could be used to prove these triangles are congruent?

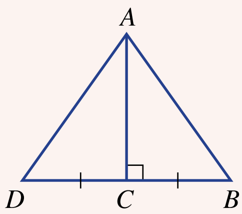
a



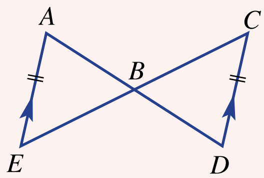
b



7E **7** Prove that triangle ABC is congruent to triangle ADC, hence prove that triangle ABD is isosceles.



7E **8** Consider the diagram below.



a Prove that $\Delta ABE \equiv \Delta DBC$.

b If $\angle EAB$ is $62°$, what is the size of angle BDC?

c If $AE = 7\,\text{cm}$, find the length of the side CD.

d If the area of triangle ABE is $21\,\text{cm}^2$, what is the area of triangle DBC?

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7F Enlargement and similar figures

Learning intentions
- To know that two similar figures are connected by a scale factor
- To be able to construct similar figures using the enlargement transformation
- To be able to calculate a scale factor from similar figures
- To be able to find unknown side lengths in similar figures

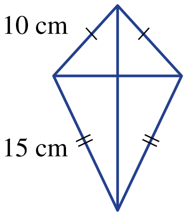
Past, present and future learning
- This section addresses the Stage 5 Core Topic called *Properties of geometrical figures A*
- These concepts will be revised and extended in Chapter 2 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

Similar figures have the same shape but not necessarily the same size. If two figures are similar then one of them can be enlarged or reduced so that it is identical (congruent) to the other. If a figure is enlarged by a scale factor greater than 1, the image will be larger than the original. If the scale factor is between 0 and 1, the image will be smaller.

A photo forms a similar image to the original, with all matching angles equal and matching lengths in the same ratio. Such as this photo of the Senanque Abbey, in fields of lavender, in southern France.

Lesson starter: Enlarging a kite

Mandy draws a kite design, then cuts out a larger shape to make the actual kite. The actual kite shape is to be similar to the design drawing. The 10 cm length on the drawing matches a 25 cm length on the kite.



- How should the interior angles compare between the drawing and the actual kite?
- By how much has the drawing been enlarged; that is, what is the scale factor? Explain your method to calculate the scale factor.
- What length on the kite matches the 15 cm length on the drawing?

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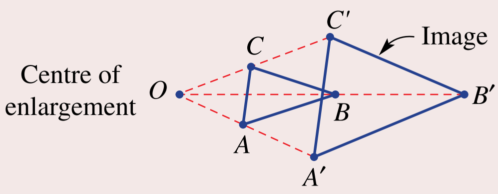
KEY IDEAS

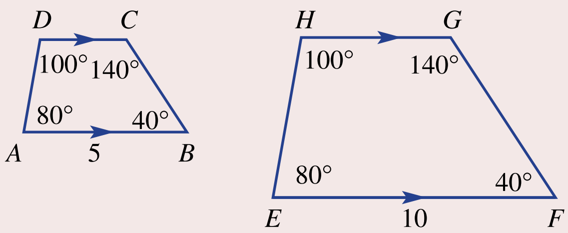
- **Enlargement** is a transformation which involves the increase or decrease in size of an object.
  - The ‘shape’ of the object is unchanged.
  - Enlargement uses a **centre of enlargement** and an **enlargement factor** or **scale factor**.



Scale factor

$$\frac{OA'}{OA} = \frac{OB'}{OB} = \frac{OC'}{OC}$$

- Two figures are **similar** if one can be enlarged to be congruent to the other.
  - Corresponding angles are equal.
  - Pairs of corresponding sides are in the same proportion or ratio.
- The **scale factor** $= \dfrac{\text{image length}}{\text{original length}}$.
  - If the scale factor is between 0 and 1, the image will be smaller than the original.
  - If the scale factor is greater than 1, the image will be larger than the original.
  - If the scale factor is equal to 1, the image will be congruent to the original.
- The symbols ||| and ~ are used to describe similarity.
  - We can write $ABCD\,|||\,EFGH$ or $ABCD \sim EFGH$.
  - The letters are usually written in matching order.



  - Scale factor $= \dfrac{EF}{AB} = \dfrac{10}{5} = 2$.

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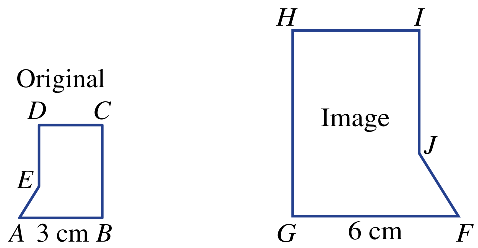
BUILDING UNDERSTANDING

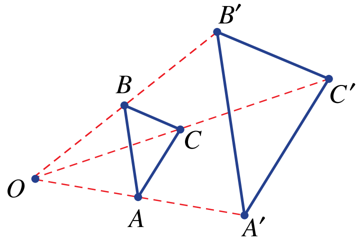
1 The two figures below are similar.



- **a** Name the angle in the larger figure which corresponds to $\angle A$.
- **b** Name the angle in the smaller figure which corresponds to $\angle I$.
- **c** Name the side in the larger figure which corresponds to BC.
- **d** Name the side in the smaller figure which corresponds to FJ.
- **e** Use FG and AB to find the scale factor.

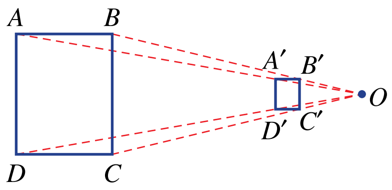
2 This diagram shows ΔABC enlarged to give the image $\Delta A'B'C'$.

- **a** Measure the lengths OA and OA'. What do you notice?
- **b** Measure the lengths OB and OB'. What do you notice?
- **c** Measure the lengths OC and OC'. What do you notice?
- **d** What is the scale factor?
- **e** Is $A'B'$ twice the length of AB? Measure to check.



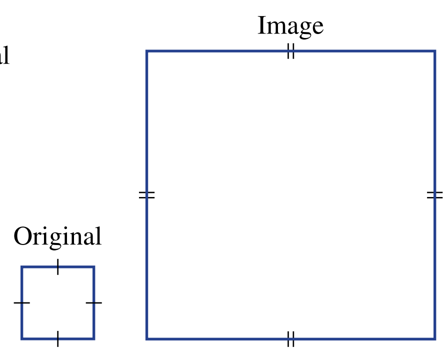
3 This diagram shows rectangle $ABCD$ enlarged (in this case reduced) to rectangle $A'B'C'D'$.

- **a** Measure the lengths OA and OA'. What do you notice?
- **b** Measure the lengths OD and OD'. What do you notice?
- **c** What is the scale factor?
- **d** Compare the lengths AD and $A'D'$. Is $A'D'$ one quarter of the length of AD?



4 A square is enlarged by a scale factor of 4.

- **a** Are the internal angles the same for both the original and the image?
- **b** If the side length of the original square was 2 cm, what would be the side length of the image square?
- **c** If the side length of the image square was 100 m, what would be the side length of the original square?



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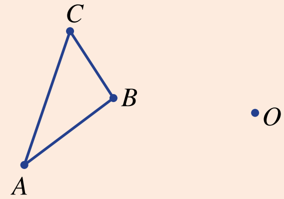
Example 13 Enlarging figures

Copy the given diagram using plenty of space. Use the given centre of enlargement (O) and these scale factors to enlarge ΔABC.

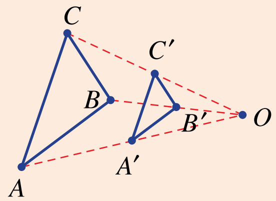
a Scale factor $\frac{1}{2}$

b Scale factor 3

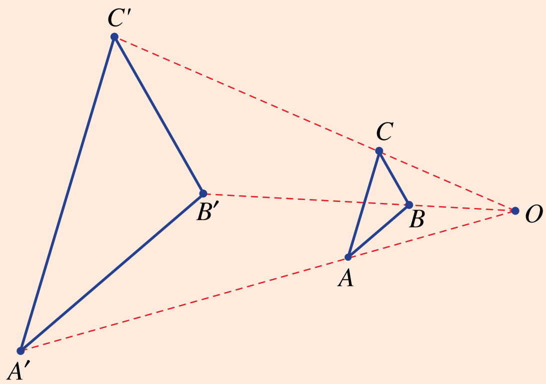


SOLUTION

a



b



EXPLANATION

Connect dashed lines between O and the vertices A, B and C.

Since the scale factor is $\frac{1}{2}$, place A' so that OA' is half of OA.

Repeat for B' and C'. Join vertices A', B' and C'.

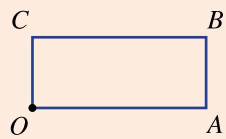
Draw dashed lines from O through A, B and C. Place A' so that $OA' = 3OA$. Repeat for B' and C' and form $\Delta A'B'C'$.

Now you try

Copy the given diagram using plenty of space. Use the given centre of enlargement (O) and these scale factors to enlarge the shape.

a Scale factor 2

b Scale factor $\frac{2}{3}$



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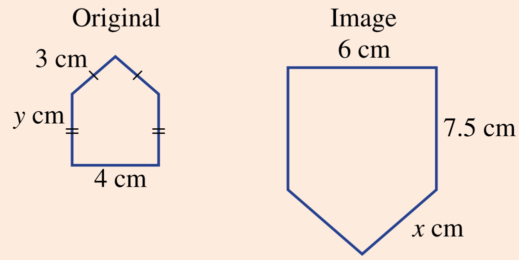
Example 14 Using the scale factor

These figures are similar.

a Find a scale factor.

b Find the value of x.

c Find the value of y.



SOLUTION

a Scale factor $= \frac{6}{4} = 1.5$

EXPLANATION

Choose two corresponding sides and use scale factor $= \frac{\text{image length}}{\text{original length}}$.

b $x = 3 \times 1.5$
$= 4.5$

Multiply the side lengths on the original by the scale factor to get the length of the corresponding side on the image.

c $y \times 1.5 = 7.5$
$y = 7.5 \div 1.5$
$= 5$

The corresponding side lengths are y cm and 7.5 cm. Divide the side lengths on the image by the scale factor to get the length of the corresponding side on the original.

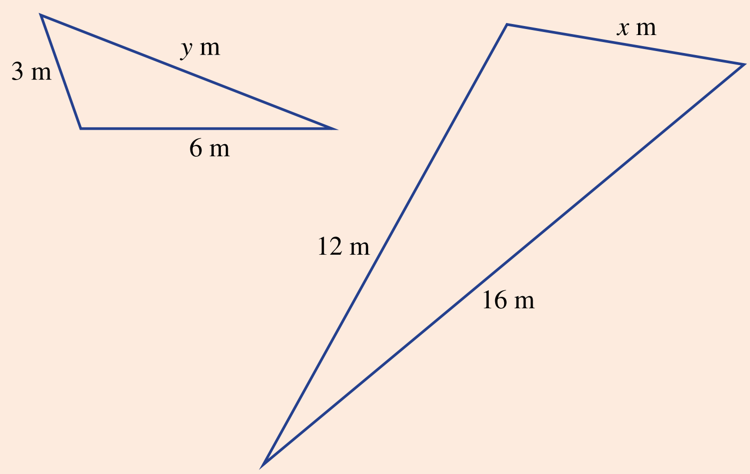
Now you try

These figures are similar.

a Find a scale factor.

b Find the value of x.

c Find the value of y.



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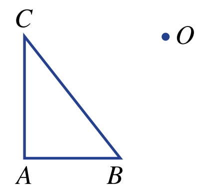
Exercise 7F

FLUENCY	1, 2, 3(½)	1, 3(½)	2, 3(½)

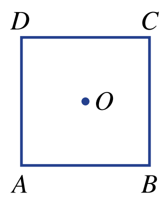
Example 13

1 Copy the given diagram leaving plenty of space around it, then use the centre of enlargement (O) and the given scale factors to enlarge ΔABC.

a Scale factor $\frac{1}{3}$ **b** Scale factor 2



2 This diagram includes a square with centre O and vertices $ABCD$.
Copy the diagram leaving plenty of space around it.



Enlarge square $ABCD$ by these scale factors and draw the image. Use O as the centre of enlargement.

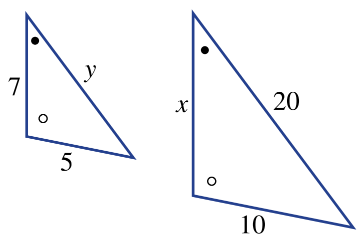
a $\frac{1}{2}$ **b** 1.5

Example 14

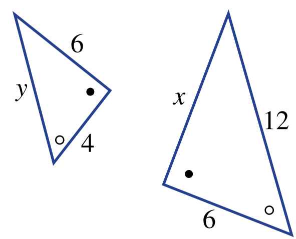
3 Each of the pairs of figures shown here are similar. For each pair, find:

i a scale factor **ii** the value of x **iii** the value of y.

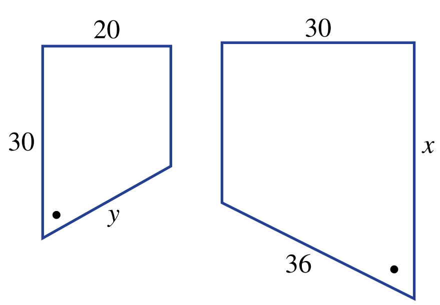
a



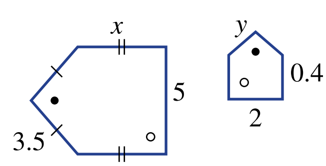
b



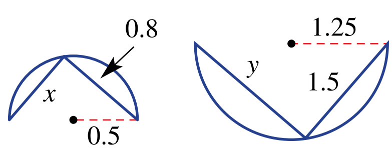
c



d



e



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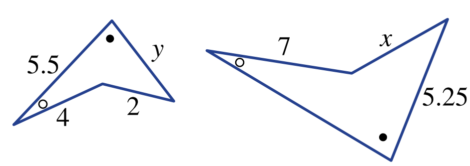

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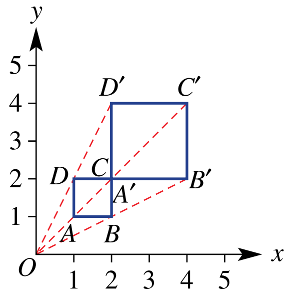
PROBLEM–SOLVING	4, 5	4(½), 5	5, 6

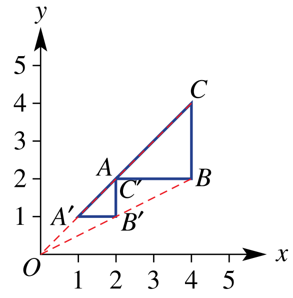
4 These diagrams show a shape and its image after enlargement. For each part, find:

- **i** the scale factor
- **ii** the coordinates (x, y) of the centre of enlargement.

a



b



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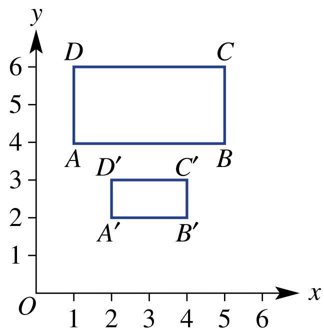

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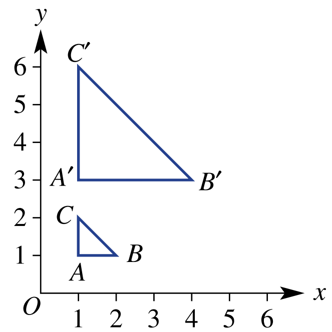
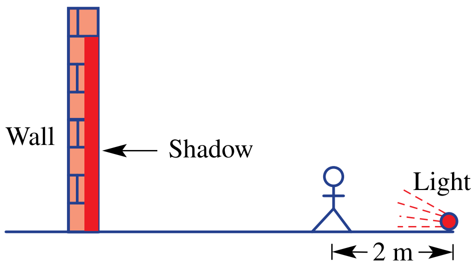


5 A person 1.8 m tall stands in front of a light that sits on the floor, and casts a shadow on the wall behind them. Form triangles to assist with the following questions.



- **a** How tall will the shadow be if the distance between the wall and the light is:
  - **i** 4 m? **ii** 10 m? **iii** 3 m?
- **b** How tall will the shadow be if the distance between the wall and the person is:
  - **i** 4 m? **ii** 5 m?
- **c** Find the distance from the wall to the person if the height of the shadow is:
  - **i** 7.2 m **ii** 4.5 m.

6 This truck is 12.7 m long.

- **a** Measure the length of the truck in the photo.
- **b** Measure the height of the truck in the photo.
- **c** Estimate the actual height of the truck.

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REASONING	7, 8	7–9	8–10

7 A figure is enlarged by a scale factor of a where $a > 0$.

a For what values of a will the image be larger than the original figure?
b For what values of a will the image be smaller than the original figure?
c For what value of a will the image be congruent to the original figure?

8 Explain why:

a any two squares are similar
b any two equilateral triangles are similar
c any two rectangles are not necessarily similar
d any two isosceles triangles are not necessarily similar.

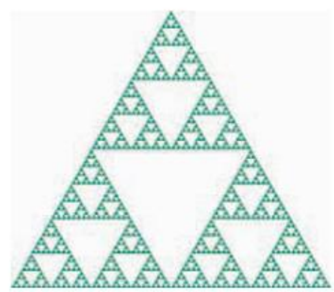
9 An object is enlarged by a factor of k. What scale factor should be used to reverse this enlargement?

10 A map has a scale ratio of 1:50 000.

a What length on the ground is represented by 2 cm on the map?
b What length on the map is represented by 12 km on the ground?

Enrichment: The Sierpinski triangle	–	–	11

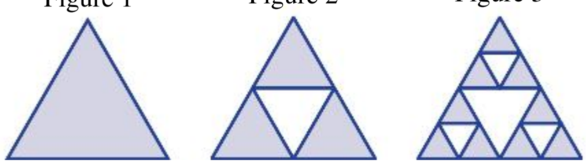
11 The Sierpinski triangle shown is a mathematically generated pattern. It is created by repeatedly enlarging triangles by a factor of $\frac{1}{2}$.



The steps are listed below.

1 Start with an equilateral triangle as in Figure 1.
2 Enlarge the triangle by a factor $\frac{1}{2}$.
3 Arrange three copies of the image as in Figure 2.
4 Continue repeating steps 2 and 3 with each triangle.

Figure 1 Figure 2 Figure 3



a Make a large copy of Figures 1 to 3 then draw the next two figures in the pattern.
b If the original triangle (Figure 1) had side length l, find the side length of the smallest triangle in:
i Figure 2
ii Figure 3
iii Figure 8 (assuming Figure 8 is the 8th diagram in the pattern).
c What fraction of the area is shaded in:
i Figure 2?
ii Figure 3?
iii Figure 6 (assuming Figure 6 is the 6th diagram in the pattern)?
d The Sierpinski triangle is one in which the process of enlargement and copying is continued forever. What is the area of a Sierpinski triangle?

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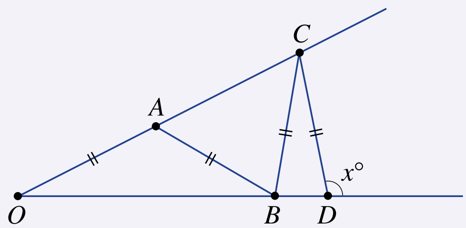
Applications and problem-solving

The following problems will investigate practical situations drawing upon knowledge and skills developed throughout the chapter. In attempting to solve these problems, aim to identify the key information, use diagrams, formulate ideas, apply strategies, make calculations and check and communicate your solutions.

Beam support

1 An angled beam is supported by three equal length struts (AB, BC and CD) as shown. The left strut is connected at point A such that $OA = AB$.

An engineer is interested in finding the possible position of angled beams, that can be supported by struts positioned in such a way.



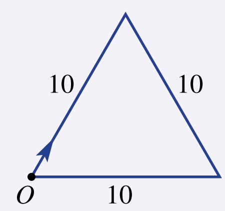
- **a** Giving reasons, find the value of x if $\angle AOB$ is equal to:
  - **i** $10°$
  - **ii** $20°$
- **b** Find an expression for x if $\angle AOB = a°$.
- **c** Try drawing a diagram using $\angle AOB = 40°$, labelling all angles. Describe the problem with the diagram if this angle is used.
- **d** Find the maximum angle of the beam ($\angle AOB$) so that all the struts AB, BC and CD can be positioned in the way illustrated in the original diagram.
- **e** What does your result from part **d** mean for the angle x?

Track navigation

2 A beginner computer programmer is using a simple platform to create a computer game involving different race tracks.

After writing step-by-step commands for polygon-shaped tracks, the programmer wants to be able to write as few commands as possible by using loops to account for commands that are repeated.

The tracks will involve regular polygons in which a car placed on the track has to be programmed to do laps of the track.

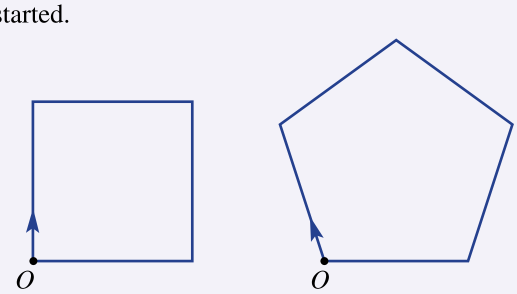


- **a** Consider the circuit shown, which is a regular triangle of side length 10 units. The car is placed at O facing in the direction of the track.

  The programmed command that can be used to travel along the track is: 'Move x, y', where x is the angle the car needs to first rotate in a clockwise direction so that it is facing in the direction of the track and y represents the distance to travel.

  The first command is: Move 0, 10

  Complete the final three instructions in the form 'Move __, __', so that the car completes one full circuit and finishes facing in the direction which it started.
- **b** For the two regular tracks shown, give the set of commands needed to get a car to complete one full lap. Use a side length of 10 units.



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The programmer notices that the set of instructions can be simplified by using a loop of the form 'Move x, y, z' for the repeating instructions, where z represents the number of times the command is repeated.

c Use the 'Move x, y, z' command to rewrite your commands to draw the square and pentagon in part **b**. Your first command needs to be 'Move 0, 10, 1'.

d Consider the sum of all the rotations used for each track. What do you notice?

e Using your result for part **d**, complete the following set of commands to complete one lap of an n-sided regular polygon of side length 10 units. Check that your commands are correct for $n = 8$.

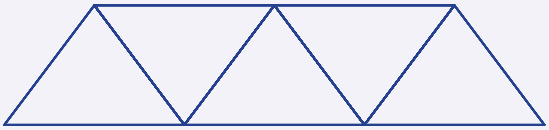
Move 0, 10, 1
Move ___, 10, ___
Move ___, 10, 1

Truss bridges

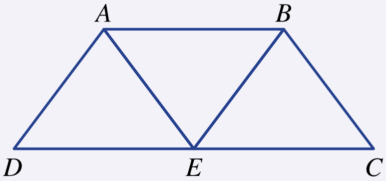
3 Truss bridges use triangles as supports above or below the deck to increase the amount of weight a bridge can hold. Triangles are used for their strength as, unlike a parallelogram, when compressed they maintain their shape and spread the force evenly.

An engineering student is looking at the geometrical structure of different truss designs before simulating experiments to see how the different arrangements affect the weight the bridge can handle.

The following design is a Warren Truss, with repeating triangular sections across the bridge.



Consider the portion of the design shown at right, in which the four diagonals are of equal length.

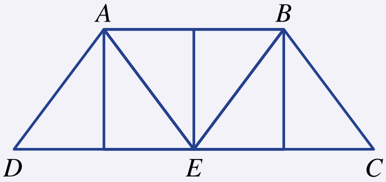


a Classify the type of triangles that have been formed.

b Which test proves that triangles DAE, BEA and EBC are congruent?

c Use the congruence of the triangles to prove that $AB \parallel DC$ and $AD \parallel BE$.

A Pratt Truss bridge involves further strengthening of the design by adding vertical supports from the vertex perpendicular to the base. The drawback of this design is the extra cost in materials.



d How many congruent triangles have now been formed in this diagram? Give reasons why the smaller triangles formed in this bridge frame are all congruent.

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7G Similar triangles

Learning intentions
- To know the four tests for similarity of triangles
- To know how to identify similar triangles
- To be able to use similarity of triangles to find unknown values

Past, present and future learning
- This section addresses the Stage 5 Path Topics called *Properties of geometrical figures B and C*
- These concepts will be revised and extended in Chapter 2 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Extension

Many geometric problems can be solved by using similar triangles. Shadows, for example, can be used to determine the height of a tall mast when the shadows form the base of two similar triangles. Solving such problems first involves the identification of two triangles and an explanation as to why they are similar. As with congruence of triangles, there is a set of minimum conditions to establish similarity in triangles.

Similar triangle geometry is the basis of trigonometry and both methods are used by astronomers, navigators, builders, architects, engineers and surveyors. Calculating the height of K2 in Pakistan, Earth's second highest mountain, would involve trigonometry.

Lesson starter: Are they similar?

Each point below describes two triangles. Accurately draw each pair and decide if they are similar (same shape but of different size).

- ΔABC with $AB = 2$ cm, $AC = 3$ cm and $BC = 4$ cm
  ΔDEF with $DE = 4$ cm, $DF = 6$ cm and $EF = 8$ cm
- ΔABC with $AB = 3$ cm, $AC = 4$ cm and $\angle A = 40^\circ$
  ΔDEF with $DE = 6$ cm, $DF = 8$ cm and $\angle D = 50^\circ$
- ΔABC with $\angle A = 30^\circ$ and $\angle B = 70^\circ$
  ΔDEF with $\angle D = 30^\circ$ and $\angle F = 80^\circ$
- ΔABC with $\angle A = 90^\circ$, $AB = 3$ cm and $BC = 5$ cm
  ΔDEF with $\angle D = 90^\circ$, $DE = 6$ cm and $EF = 9$ cm

Which pairs are similar and why? For the pairs that are not similar, what measurements could be changed so that they are similar?

KEY IDEAS

- Two triangles are **similar** if:
  - corresponding angles are equal
  - corresponding sides are in proportion (the same ratio).

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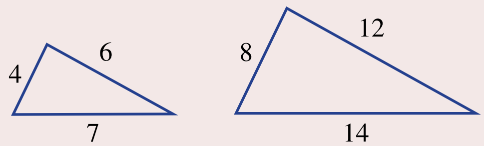
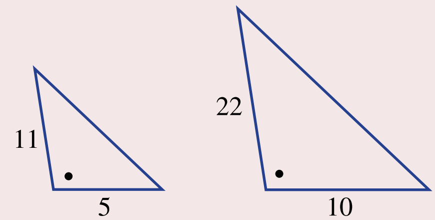
- The **similarity statement** for two similar triangles ΔABC and ΔDEF is:
  - $\Delta ABC \,|||\, \Delta DEF$ or
  - $\Delta ABC \sim \Delta DEF$

  Letters are usually written in matching order so AB corresponds to DE etc.

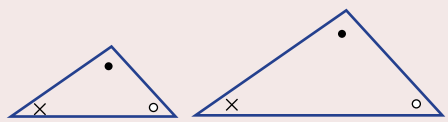
- **Tests for similar triangles**. (Not to be confused with the congruence tests for triangles).
  - Side, Side, Side (SSS)
    All three pairs of corresponding sides are in the same ratio.
    $$\frac{12}{6} = \frac{8}{4} = \frac{14}{7}$$



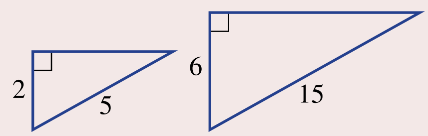
  - Side, Angle, Side (SAS)
    Two pairs of corresponding sides are in the same ratio and the included angle is equal.
    $$\frac{22}{11} = \frac{10}{5}$$



  - Angle, Angle, Angle (AAA or AA)
    All three corresponding angles are equal. (If there are two equal pairs then the third pair must be equal by the angle sum of a triangle.)



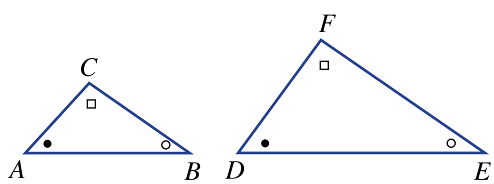
  - Right angle, Hypotenuse, Side (RHS)
    The hypotenuses of right-angled triangles and another corresponding pair of sides are in the same ratio.
    $$\frac{15}{5} = \frac{6}{2}$$



- The abbreviations such as RHS are not mentioned in the NSW Syllabus, but they are used in this section for convenience.

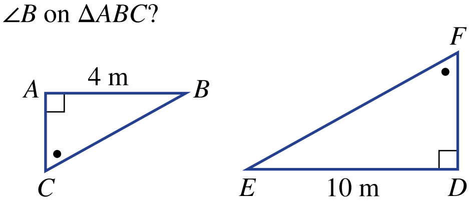
BUILDING UNDERSTANDING

1 These two triangles are similar.



a Which vertex on ΔDEF corresponds to (matches) vertex B on ΔABC?
b Which vertex on ΔABC corresponds to (matches) vertex F on ΔDEF?
c Which side on ΔDEF corresponds to (matches) side AC on ΔABC?
d Which side on ΔABC corresponds to (matches) side EF on ΔDEF?
e Which angle on ΔABC corresponds to (matches) $\angle D$ on ΔDEF?
f Which angle on ΔDEF corresponds to (matches) $\angle B$ on ΔABC?

2 What is the scale factor on this pair of similar triangles which enlarges ΔABC to ΔDEF?



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3 State the missing terms to complete the following sentences.

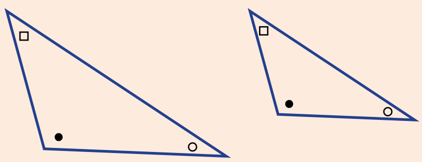
a The abbreviated tests for similar triangles are SSS, ______, ______ and ______.

b Similar figures have the same ______ but are not necessarily the same ______.

Example 15 Choosing a similarity test for triangles

Choose the similarity test which proves that these pairs of triangles are similar.

a



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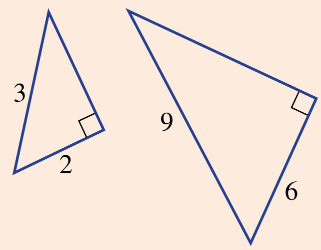

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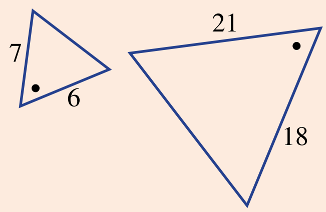

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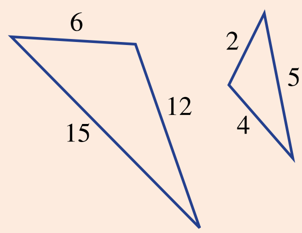
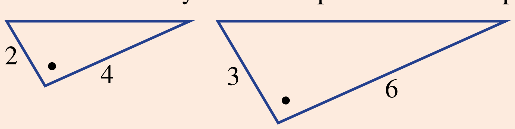


SOLUTION	EXPLANATION
a AAA	Three pairs of angles are equal.
b RHS	Both are right-angled triangles and the hypotenuses and another pair of sides are in the same ratio $\left(\frac{9}{3} = \frac{6}{2}\right)$.
c SAS	Two pairs of corresponding sides are in the same ratio $\left(\frac{21}{7} = \frac{18}{6}\right)$ and the included angles are equal.
d SSS	Three pairs of corresponding sides are in the same ratio $\left(\frac{15}{5} = \frac{12}{4} = \frac{6}{2}\right)$.

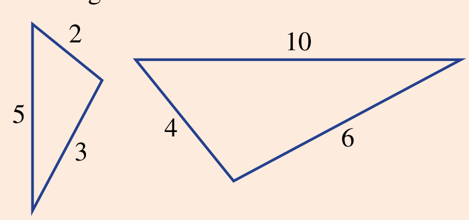
Now you try

Choose the similarity test which proves that these pairs of triangles are similar.

a



b



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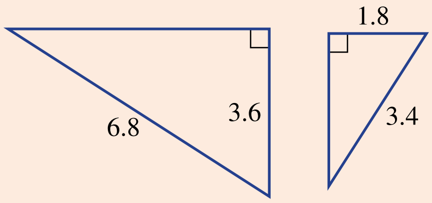
c



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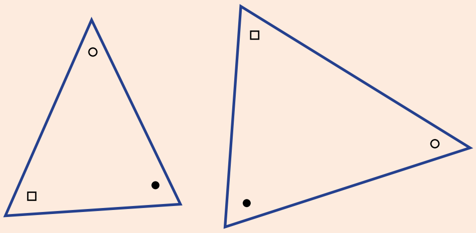
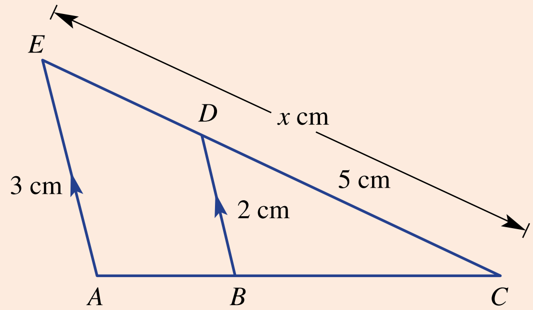


Example 16 Finding a missing length using similarity

For this pair of triangles:

a give a reason (SSS, SAS, AAA or RHS) why the two triangles are similar

b find the value of x.



SOLUTION

a AAA or just AA.

EXPLANATION

$\angle EAC = \angle DBC$ since AE is parallel to BD and $\angle C$ is common to both triangles. (Also $\angle AEC = \angle BDC$ since AE is parallel to BD).

b Scale factor $= \frac{3}{2} = 1.5$

$$\begin{aligned} \therefore x &= 5 \times 1.5 \\ &= 7.5 \end{aligned}$$

$\frac{AE}{BD} = \frac{3}{2}$.

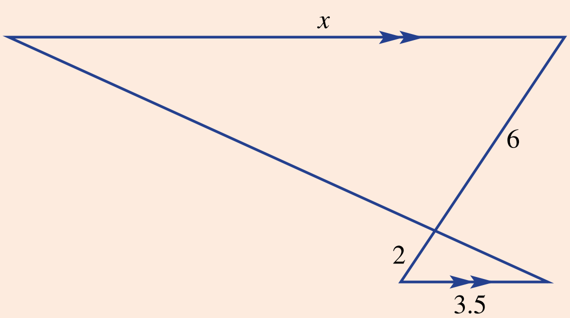
Multiply CD by the scale factor to find the length of the corresponding side CE.

Now you try

For this pair of triangles:

a give a reason (SSS, SAS, AAA or RHS) why the two triangles are similar

b find the value of x.



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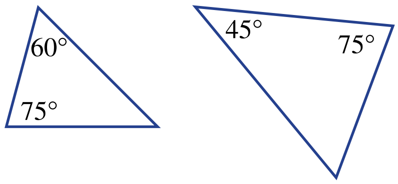
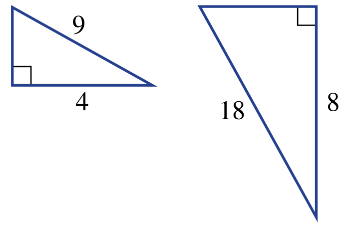
Exercise 7G

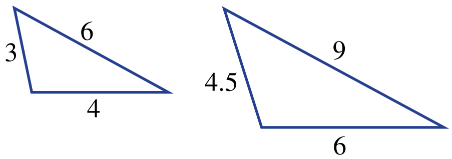
FLUENCY	1–4	1–2(½), 3, 5	1–2(½), 3, 5, 6

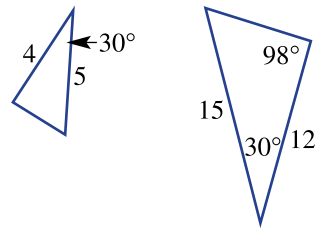
Example 15

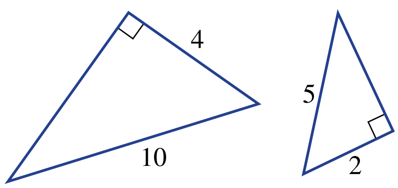
1 Choose the similarity test which proves that these pairs of triangles are similar.

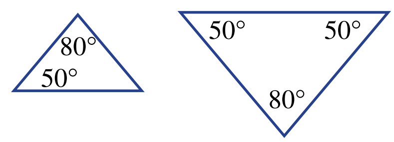
a 

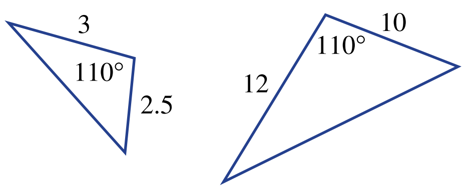
b 

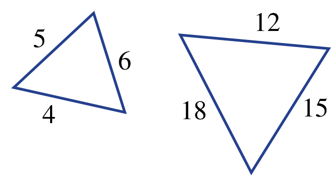
c 

d 

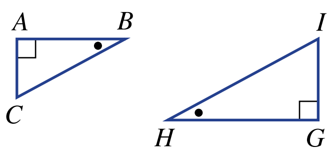
e 

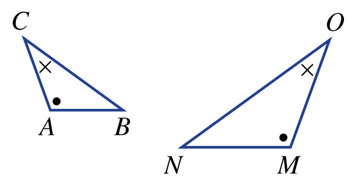
f 

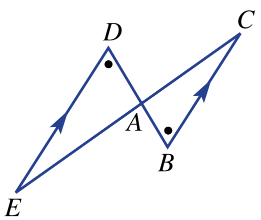
g 

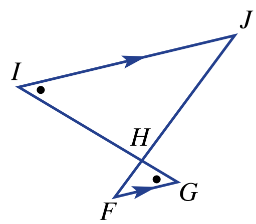
h 

2 Write similarity statements for these pairs of similar triangles. Write letters in matching order.

a 

b 

c 

d 

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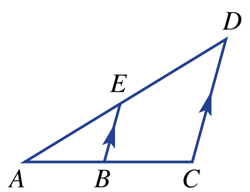

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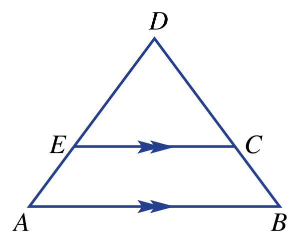
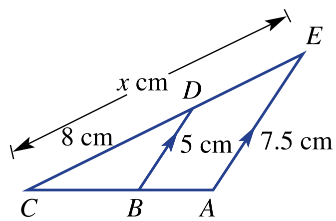


Example 16

3 For this pair of triangles:

a give a reason (SSS, SAS, AAA or RHS) why the two triangles are similar

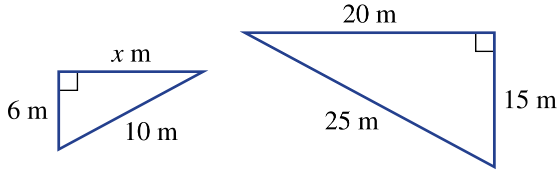
b find the value of x.



4 For this pair of triangles:

a give a reason (SSS, SAS, AAA or RHS) why the two triangles are similar

b find the value of x.

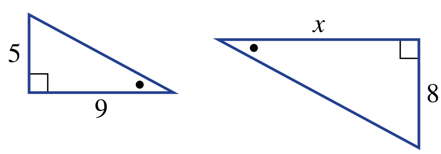


5 These pairs of triangles are similar. For each pair find:

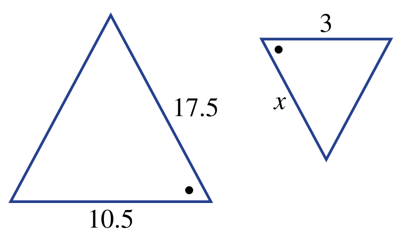
i the enlargement factor (scale factor) which enlarges the smaller triangle to the larger triangle

ii the value of x.

a



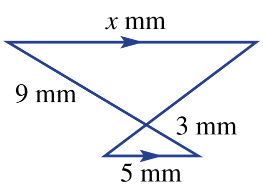
b



6 For this pair of triangles:

a give a reason (SSS, SAS, AAA or RHS) why the two triangles are similar

b find the value of x.



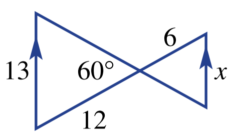
PROBLEM-SOLVING	7	7, 8	7–9

7 For each pair of similar triangles:

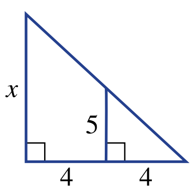
i give a reason (SSS, SAS, AAA or RHS) why the two triangles are similar

ii find the value of x.

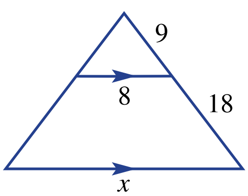
a



b



c



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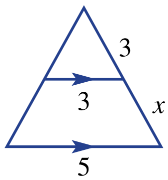
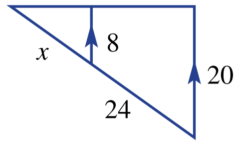
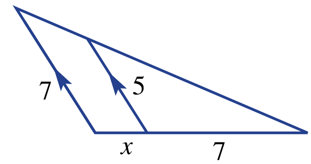
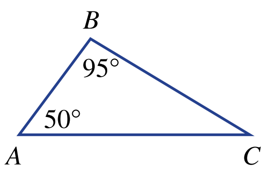
8 Find the value of x in these triangles.

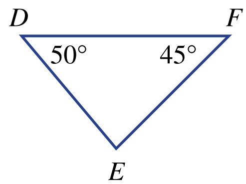
a


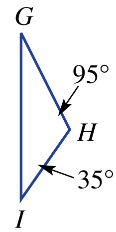
b


c


9 Name the triangle which is not similar to the other two in each group of three triangles.

a






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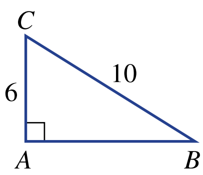
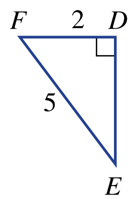
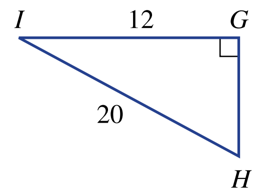
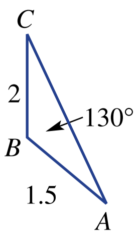
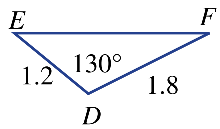
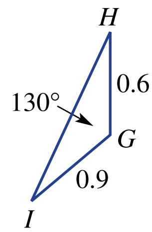


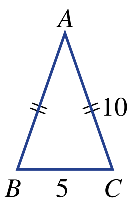


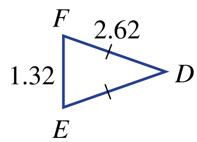


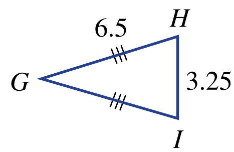
c






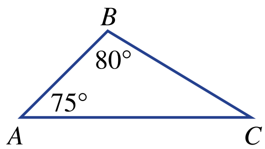
d


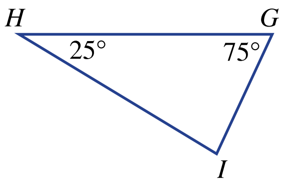




REASONING	10, 11	10–12	11–13

10 Give reasons why these two triangles are similar.





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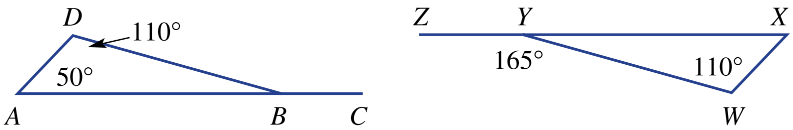
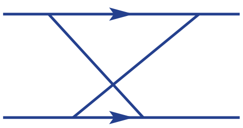
11 Give reasons why the two triangles in these diagrams are not similar.



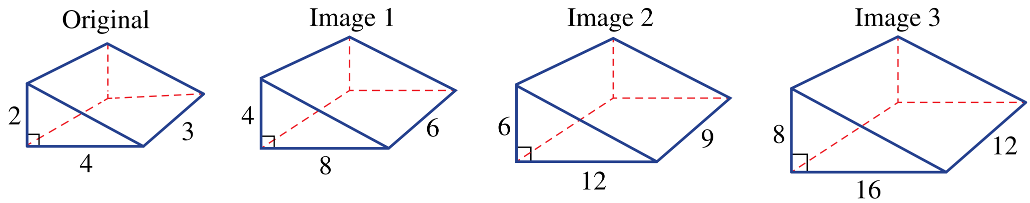
12 When two intersecting transversals join parallel lines, two triangles are formed. Explain why these two triangles are similar.



13 The four tests for similarity closely resemble the tests for congruence. Which similarity test closely matches the AAS congruence test? Explain the difference.

ENRICHMENT: Area and volume ratio	–	–	14

14 Consider these four similar triangular prisms (not drawn to scale).



a Complete this table.

Triangle	Original	Image 1	Image 2	Image 3
Length scale factor	1	2		
Area (cross-section)				
Area scale factor	1			
Volume				
Volume scale factor	1			

b What do you notice about the area scale factor compared to the length scale factor?

c What would be the area scale factor if the length scale factor is n?

d What would be the area scale factor if the length scale factor is:

i 10? **ii** 20? **iii** 100?

e What would be the area scale factor if the length scale factor is $\frac{1}{2}$?

f What do you notice about the volume scale factor compared to the length scale factor?

g What would be the volume scale factor if the length scale factor is n?

h What would be the volume scale factor if the length scale factor is:

i 5? **ii** 10? **iii** $\frac{1}{2}$?

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7H Proving and applying similar triangles

Learning intentions
- To know how to prove that two triangles are similar using the tests
- To be able to use similarity of triangles to find unknown values

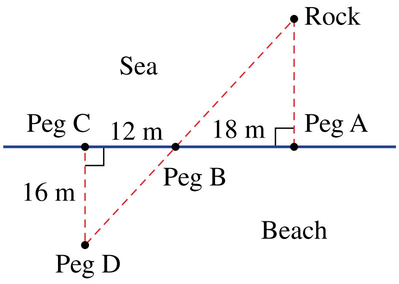
Past, present and future learning
- This section addresses the Stage 5 Path Topics called *Properties of geometrical figures B and C*
- These concepts will be revised and extended in Chapter 2 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Extension

Similar triangles can be used in many mathematical and practical problems. If two triangles are proved to be similar, then the properties of similar triangles can be used to find missing lengths or unknown angles. The approximate height of a tall object, or the width of a projected image, for example, can be found using similar triangles.


Similar triangles can be used to verify the height, 828 m, of the world's tallest building, the Burj Khalifa, Dubai. The Jeddah Tower, Saudi Arabia, is expected to be over 1 km tall when constructed.

Lesson starter: How far is the rock?

Ali is at the beach and decides to estimate how far an exposed rock is from seashore. He places four pegs in the sand as shown and measures the distance between them.



- Why do you think Ali has placed the four pegs in the way that is shown in the diagram?
- Why are the two triangles similar? Which test (SSS, SAS, AAA or RHS) could be used and why?
- How would Ali use the similar triangles to find the distance from the beach at peg A to the rock?

KEY IDEAS

- To prove triangles are similar, list any pairs of corresponding equal angles or pairs of sides in a given ratio.
  - Give reasons at each step.
  - Write a similarity statement, for example, $\Delta ABC ||| \Delta DEF$ or $\Delta ABC \sim \Delta DEF$.
  - Write the triangle similarity test in abbreviated form (SSS, SAS, AAA or RHS).
- To apply similarity in practical problems, follow these steps.
  - Prove two triangles are similar.
  - Find a scale factor.
  - Use the scale factor to find the value of any unknowns.

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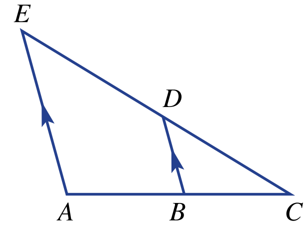
BUILDING UNDERSTANDING

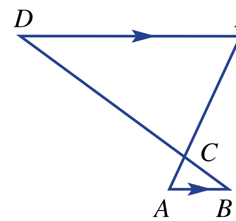
1 In this diagram, name the angle which is common to both ΔACE and ΔBCD.



2 In this diagram:

a name the pair of vertically opposite angles

b name the two pairs of equal alternate angles.

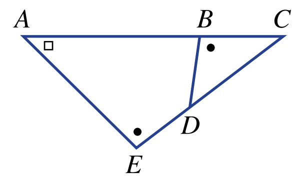


3 In this diagram:

a name the angle which is common for the two triangles

b name the side that corresponds to side

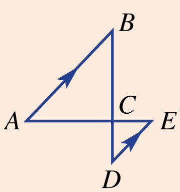
i DC **ii** AE



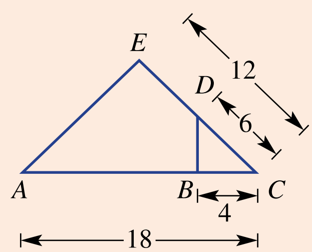
Example 17 Proving two triangles are similar

Prove that each pair of triangles is similar.

a



b

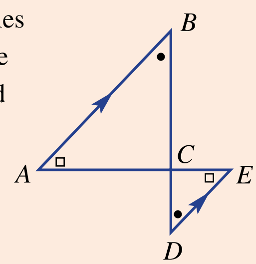


SOLUTION

a $\angle BAC = \angle DEC$
(alternate angles and $DE \parallel AB$)
$\angle ABC = \angle EDC$
(alternate angles and $DE \parallel AB$)
$\angle ACB = \angle ECD$
(vertically opposite angles)
$\therefore \Delta ABC \parallel\!\mid \Delta EDC$ (AAA)

EXPLANATION

Parallel lines cut by a transversal will create a pair of equal alternate angles.
Vertically opposite angles are also equal. Write the similarity statement and the abbreviated reason.



Continued on next page

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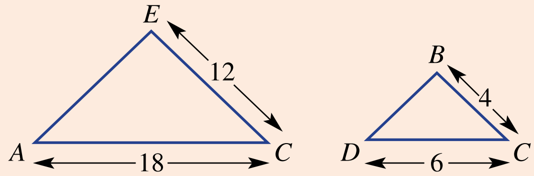
b $\angle ACE = \angle DCB$ (common)

$$\frac{AC}{DC} = \frac{18}{6} = 3$$

$$\frac{EC}{BC} = \frac{12}{4} = 3 = \frac{AC}{DC}$$

$\therefore \Delta ACE \mathrel{|||} \Delta DCB$ (SAS)

Redraw triangles to align corresponding sides.

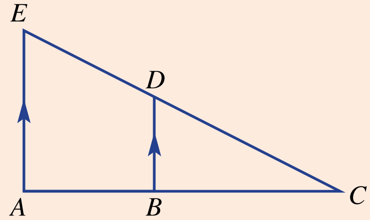


Note that there is a common angle and two pairs of corresponding sides. Find the scale factor for both pairs of sides to see if they are equal. Complete the proof with a similarity statement. The angle is the included angle.

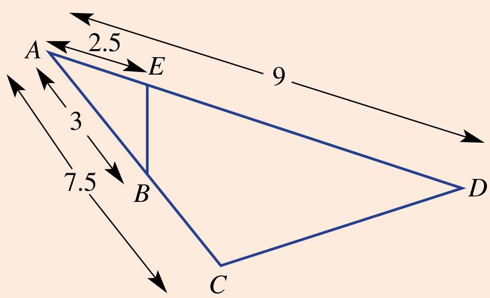
Now you try

Prove that each pair of triangles is similar.

a



b

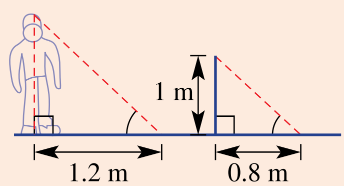


Example 18 Applying similarity

Chris' shadow is 1.2 m long when a 1 m vertical stick has a shadow 0.8 m long.

a Give a reason why the two triangles are similar.

b Determine Chris' height.



SOLUTION

a All angles are the same (AAA).

b

$$\begin{aligned} \text{Scale factor} &= \frac{1.2}{0.8} = 1.5 \\ \therefore \text{Chris' height} &= 1 \times 1.5 \\ &= 1.5\,\text{m} \end{aligned}$$

EXPLANATION

The sun's rays will pass over Chris and the stick and hit the ground at approximately the same angle.

First find the scale factor.

Multiply the height of the stick by the scale factor to find Chris' height.

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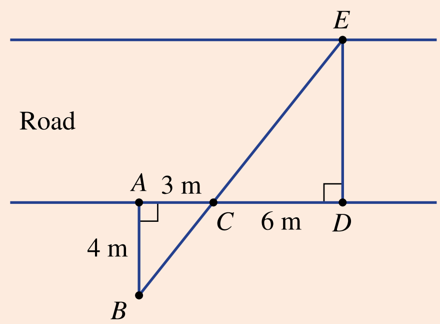
Now you try

Four pegs A, B, C and D are placed on one side of a road as shown.

a Give a reason why the two triangles are similar.

b Determine the distance across the road, DE.



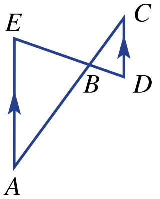
Exercise 7H

FLUENCY	1, 2	1(½), 2	1–2(½)

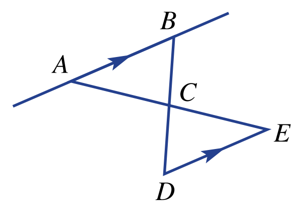
Example 17a

1 Prove that each pair of triangles is similar.

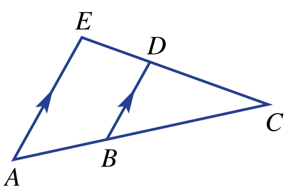
a



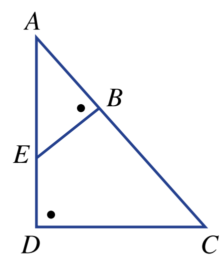
b



c



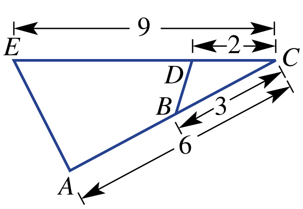
d



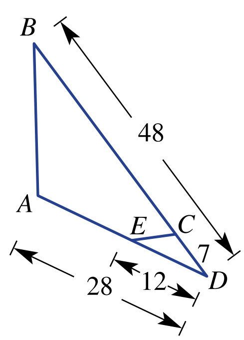
Example 17b

2 Prove that each pair of triangles is similar.

a



b



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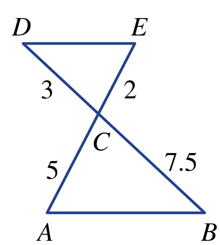

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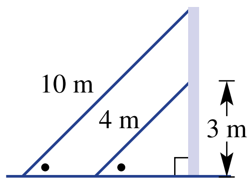
PROBLEM–SOLVING	3, 5, 6	3, 4, 6, 7	5–8

Example 18

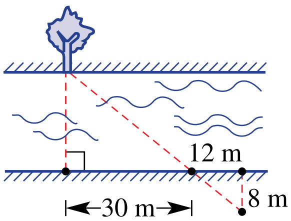
3 A tree's shadow is 20 m long, while a 2 m vertical stick has a shadow 1 m long.
  a Give a reason why the two triangles formed by the objects and their shadows are similar.
  b Find the height of the tree.

4 Two cables support a steel pole at the same angle as shown. The two cables are 4 m and 10 m in length, and the shorter cable reaches 3 m up the pole.
  a Give a reason why the two triangles are similar.
  b Find the height of the pole.

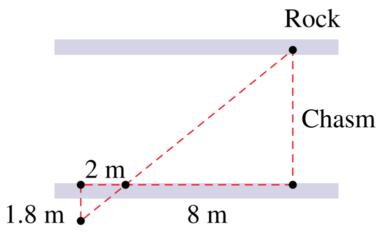


5 John stands 6 m from a vertical lamp post and casts a 2 m shadow. The shadow from the lamp post and from John end at the same place. Determine the height of the lamp post if John is 1.5 m tall.

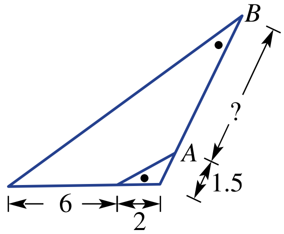
6 Joanne wishes to determine the breadth of a river without crossing it. She places four pegs as shown. Calculate the breadth of the river.



7 A deep chasm has a large rock sitting on one side as shown. Find the breadth of the chasm.

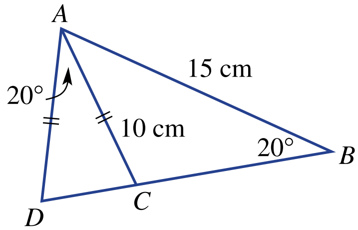


8 Find the length AB in this diagram if the two triangles are similar.



REASONING	9	9	9, 10

9 In this diagram, ΔADC is isosceles.
  a There are two triangles which are similar. Identify them and complete a proof. You may need to find another angle first.
  b Find the lengths DC and CB expressing your answers as fractions.



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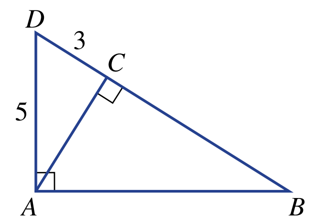
10 In this diagram, AC is perpendicular to BD and ΔABD is right-angled.



a Prove that ΔABD is similar to:

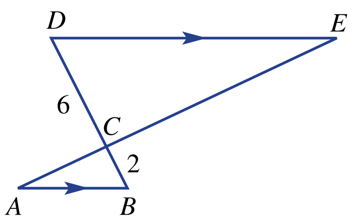
i ΔCBA **ii** ΔCAD

b Find these lengths:

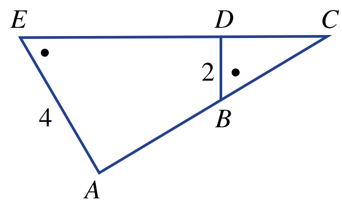
i BD **ii** AC **iii** AB

ENRICHMENT: Extended proofs	–	–	11

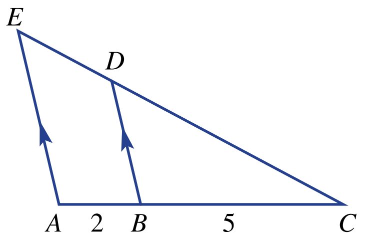
11 **a** Prove $AE = 4AC$.



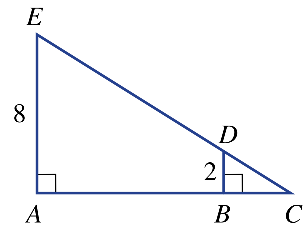
b Prove $BC = \frac{1}{2}CE$.

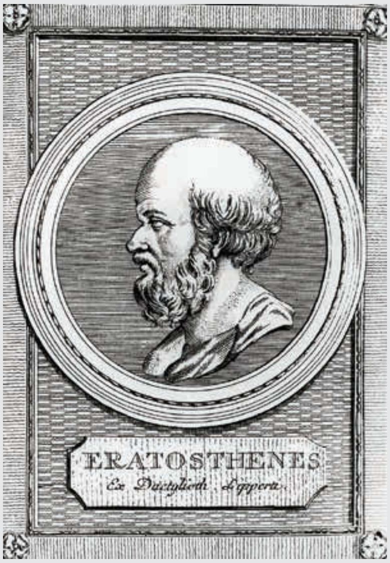


c Prove $CE = \frac{7}{5}CD$.



d Prove $AB = 3BC$.





Eratosthenes was a Greek mathematician, geographer and astronomer who, around 240 BCE, used similar triangles to quite accurately calculate Earth's circumference.

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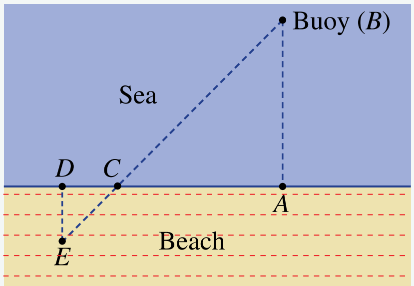
Working mathematically

Ocean ironman

The organisers of an ironman race want to estimate the distance from the beach to a floating buoy in the sea. Flags are placed on the beach by the organisers in an effort to create similar triangles as shown. The flag positions are at points A, C, D and E and the line segments AB and DE are assumed to be at right angles to the beach.



Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate.

Routine problems

One of the organisers measures the following:

- $AC = 200$ metres
- $CD = 5$ metres
- $DE = 4$ metres.

a Find the scale factor between the two similar triangles.

b Hence, estimate the distance from the beach to the buoy.

Non-routine problems

Explore and connect

a The problem is to determine the distance between the beach and the buoy using similar triangles. Write down all the relevant information that will help solve this problem with the aid of a diagram.

b Explain why the given positions of the flags form a pair of similar triangles.

Choose and apply techniques

c Two more attempts at positioning flags and measuring distances are undertaken by the organisers.

Jimmy's placement:

- $AC = 360$ metres
- $CD = 8$ metres
- $DE = 3$ metres.

Olivia's placement:

- $AC = 90$ metres
- $CD = 5$ metres
- $DE = 8$ metres.

i Use Jimmy's model to estimate the distance between the beach and the buoy.

ii Use Olivia's model to estimate the distance between the beach and the buoy.

Communicate thinking and reasoning

d Another organiser attempts to place the flags such that:

- $AC = 100$ metres
- $CD = 50$ centimetres
- $DE = 1.2$ metres.

Do you think that arranging the flags in this way is a good idea? Explain why.

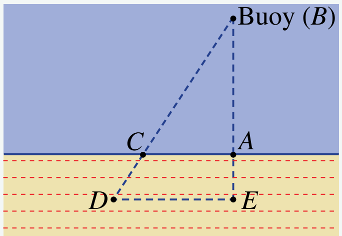
e If the actual distance from the beach to the buoy is 200 metres, determine an arrangement for the flags that would deliver the correct distance. Illustrate with a diagram including the measurements.

f Summarise your results and describe any key findings.

Extension problems

Problem solve

a Another way to arrange the flags is to use the model shown here. Using this model, identify the pair of similar triangles and explain why they are similar.



b Using the following measurements, estimate the distance from the beach to the buoy.

- $AC = 100$ metres
- $AE = 60$ metres
- $DE = 140$ metres

c Compare the two models studied in this task using the different diagrams. Do you have a preferred model? Explain why.

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Investigation

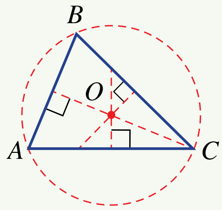
Triangle centres with technology

Use an interactive geometry package to construct the following shapes.

The circumcentre of a triangle

The point at which all perpendicular bisectors of the sides of a triangle meet is called the circumcentre.

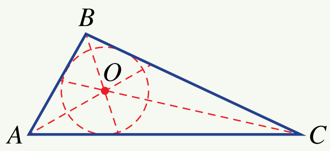
a Draw any triangle.
b Label the vertices A, B and C.
c Draw a perpendicular bisector for each side.
d Label the intersection point of the bisectors O.
e Using O as the centre, construct a circle that touches the vertices of the triangle.
f Drag any of the vertices and describe what happens to your construction.



The incentre of a triangle

The point at which all angle bisectors of a triangle meet is called the incentre.

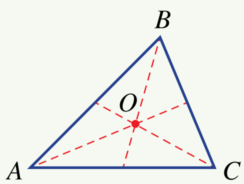
a Draw any triangle.
b Label the vertices A, B and C.
c Draw the three angle bisectors through the vertices.
d Label the intersection point of the bisectors O.
e Using O as your centre, construct a circle that touches the sides of the triangle.
f Drag any of the vertices and describe what happens to your construction.



The centroid of a triangle

The point of intersection of the three medians of a triangle is called the centroid. It can also be called the centre of gravity.

a Draw a triangle and label the vertices A, B and C.
b Find the midpoint of each line and draw a line segment from each midpoint to its opposite vertex.
c Label the intersection point of these lines O. This is the centroid of the triangle.
d Show your teacher the final construction and print it. Cut out the triangle and place a sharp pencil under the centroid. The triangle should balance perfectly.



The equilateral triangle: The special triangle

a Construct an equilateral triangle. Determine its incentre, circumcentre and centroid.
b What do you notice?

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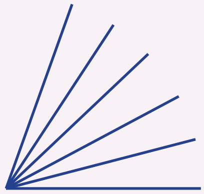
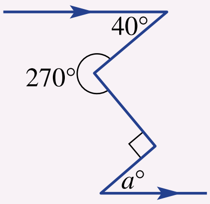
1 Use 12 matchsticks to make 6 equilateral triangles.

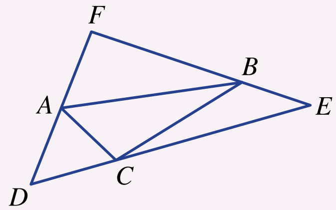
2 How many acute angles are there in the diagram shown below?



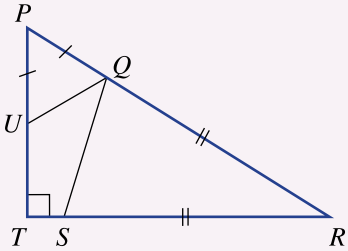
3 Find the value of a.



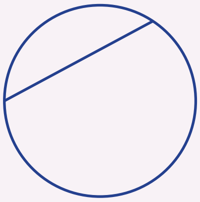
4 Explore (using interactive geometry software) where the points A, B and C should be on the sides of ΔDEF so that the perimeter of ΔABC is a minimum.



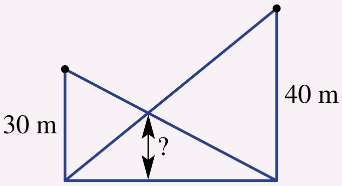
5 Find the size of angle UQS given the sides $PQ = PU$ and $QR = RS$. Angle PTR is 90°.



6 A circle is divided using chords (one chord is shown here). What is the maximum number of regions that can be formed if the circle is divided with 4 chords?



7 Two poles are 30 m and 40 m high. Cables connect the top of each vertical pole to the base of the other pole. How high is the intersection point of the cables above the ground?



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Geometry

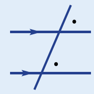
Angles

Acute $0° < \theta < 90°$
Right $90°$
Obtuse $90° < \theta < 180°$
Straight $180°$
Reflex $180° < \theta < 360°$
Revolution $360°$
Complementary angles sum to $90°$.
Supplementary angles sum to $180°$.
Vertically opposite angles are equal.

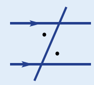
Parallel lines

If the lines are parallel:

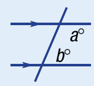
Corresponding angles are equal



Alternate angles are equal

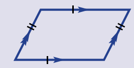


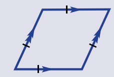
Cointerior angles are supplementary
$a + b = 180$

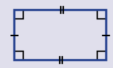


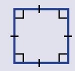
Quadrilaterals

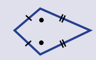
Four-sided figures – sum of the interior angles is $360°$

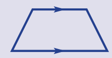
Parallelogram 

Rhombus 

Rectangle 

Square 

Kite 

Trapezium 

Congruent triangles

These are identical in shape and size. They may need to be rotated or reflected. We write $\Delta ABC \equiv \Delta DEF$.

$\equiv$: congruent to

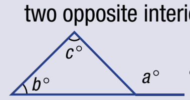
Tests for congruence:
SSS – three pairs of matching sides are equal
SAS – two pairs of matching sides and the included angles are equal
AAS – two pairs of angles and any pair of matching sides are equal
RHS – a right angle, hypotenuse and one other pair of matching sides are equal

Polygons

The sum of the interior angles in a polygon with n sides is
$S = (n - 2) \times 180°$
Regular polygons have all sides and angles equal.

Triangles

Sum of angles is $180°$.
Exterior angle equals sum of the two opposite interior angles.



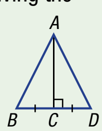
$a = b + c$
exterior angle

Types:
Acute-angled – all angles $< 90°$
Obtuse-angled – 1 angle $> 90°$
Right-angled – 1 angle $90°$
Equilateral – all angles $60°$ – all sides are equal
Isosceles – 2 angles and 2 sides are equal
Scalene – all sides and angles are different sizes

Proving congruence/similarity

List corresponding pairs of equal angles and pairs of sides that are equal/in same ratio. Give reasons for each pair. Write a congruence/similarity statement giving the abbreviated reason.

e.g.



AC (common)
$BC = DC$ (given)
$\angle ACD = \angle ACB = 90°$ (given)
$\therefore\ \Delta ACD \equiv \Delta ACB$ (SAS)

Similar figures

These are the same shape but different size.
Two figures are similar if one can be enlarged to be congruent to the other.
Enlargement uses a scale factor.

Scale factor $= \dfrac{\text{image length}}{\text{original length}}$

Similar triangles

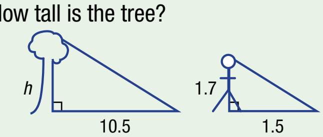
Similar triangles have corresponding angles equal and corresponding sides are in the same ratio.
We say $\Delta ABC\ |||\ \Delta DEF$ or $\Delta ABC \sim \Delta DEF$.

$|||$, $\sim$: similar to

Tests for similar triangles:
SSS – three pairs of matching sides in the same ratio
SAS – two pairs of matching sides in the same ratio and included angles equal
AAA or **AA** – all three pairs of matching angles are equal; two pairs is enough to prove this
RHS – the hypotenuses of right-angled triangles and another pair of matching sides are in the same ratio

Applying similar triangles

In practical problems, look to identify and prove pairs of similar triangles. Find a scale factor and use this to find the value of any unknowns.
e.g. shadow cast by a tree is 10.5 m while a person 1.7 m tall has a 1.5 m shadow. How tall is the tree?
Similar (AAA)
Scale factor $= \dfrac{10.5}{1.5} = 7$
$\therefore\ h = 1.7 \times 7$
$= 11.9$ m
Tree is 11.9 m tall.



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Chapter checklist

Chapter checklist with success criteria

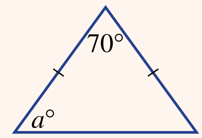
A printable version of this checklist is available in the Interactive Textbook

✔

7A **1. I can find supplementary and complementary angles.**
e.g. For an angle of size 62° determine the size of its supplementary angle and its complementary angle.

☐

7A **2. I can classify triangles and find an unknown angle.**
e.g. Name the type of triangle shown and determine the value of the pronumeral.



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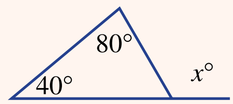
7A **3. I can find exterior angles.**
e.g. Find the value of the pronumeral in the diagram giving reasons.



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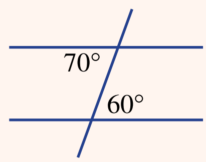
7B **4. I can determine if lines are parallel.**
e.g. Decide if the diagram contains a pair of parallel lines, giving a reason.



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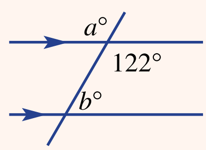
7B **5. I can find angles in parallel lines.**
e.g. Find the value of each of the pronumerals giving reasons.



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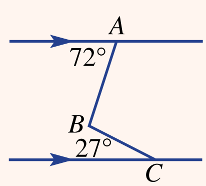
7B **6. I can find angles in parallel lines using a third parallel line.**
e.g. Add a third parallel line to the diagram to help find $\angle ABC$.



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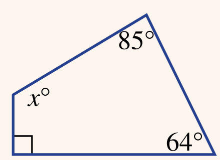
7C **7. I can find angles in quadrilaterals.**
e.g. Find the value of the pronumeral in the quadrilateral.



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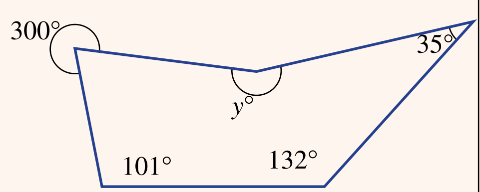
7C **8. I can find angles in polygons.**
e.g. For the polygon shown, find the angle sum and use this to find the value of the pronumeral.



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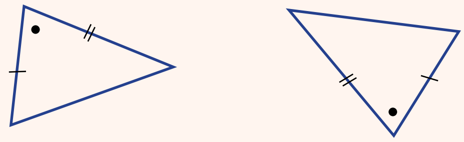
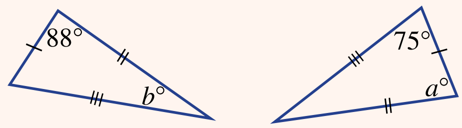
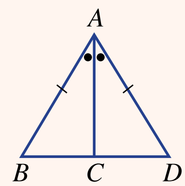
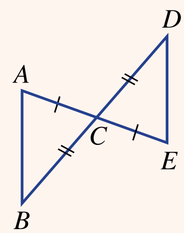
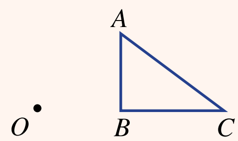
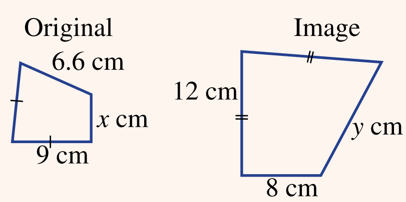
Chapter checklist

7D	**9. I can use the congruence tests to identify congruent triangles.** e.g. Which congruence test (SSS, SAS, AAS or RHS) would be used to show that these pairs of triangles are congruent? 	☐
7D	**10. I can use congruence to find missing side lengths or angles.** e.g. Find the value of the pronumerals in these congruent triangles. 	☐
7E	**11. I can prove that two triangles are congruent.** e.g. Prove that $\Delta ABC \equiv \Delta ADC$. 	☐
7E	**12. I can use congruence to prove geometrical results.** e.g. Prove that $\Delta ABC \equiv \Delta EDC$ and hence prove that $AB \parallel DE$. 	☐
7F	**13. I can enlarge a figure using a scale factor.** e.g. Copy the diagram and use the centre of enlargement (O) and each of the scale factors 2 and $\frac{1}{3}$ to enlarge ΔABC. 	☐
7F	**14. I can find and use the scale factor in similar figures.** e.g. For the similar figures shown, find the scale factor and use this to find the value of x and y. 	☐

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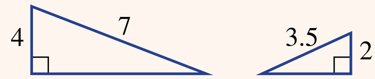
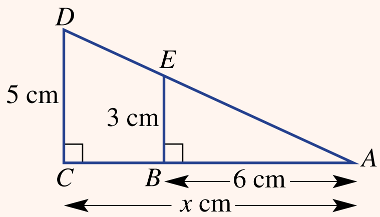
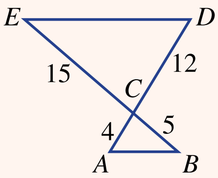
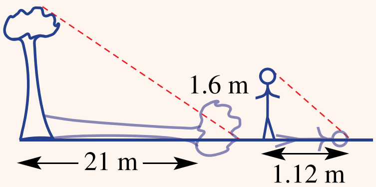
Chapter checklist

7G	**15. I can choose a similarity test for triangles.** e.g. Choose the similarity test which proves that these pairs of triangles are similar.		☐
7G	**16. I can use similarity to find a missing length.** e.g. For the pair of triangles shown, give a reason why the two triangles are similar and find the value of *x*.		☐
7H	**17. I can prove that two triangles are similar.** e.g. Prove that the two triangles are similar.		☐
7H	**18. I can apply similarity.** e.g. A 1.6 m tall girl has a shadow that is 1.12 m long. A tree's shadow is 21 m long. Give a reason why the two triangles in the diagram are similar and determine the tree's height.		☐

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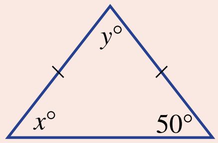
Short-answer questions

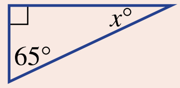
7A

1 Name the following triangles and find the value of the pronumerals.

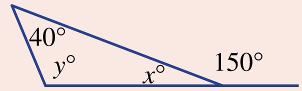
a



b



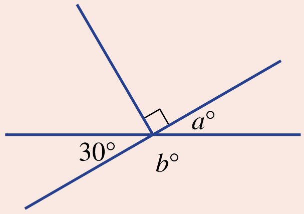
c



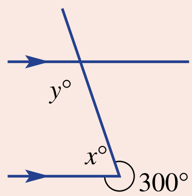
7A/B

2 Find the value of each pronumeral in the diagrams. Give reasons for your answers.

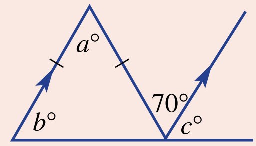
a



b

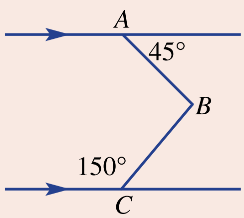


c



7B

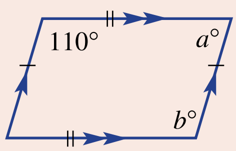
3 By adding a third parallel line to the diagram, find $\angle ABC$.



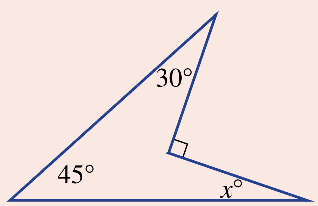
7C

4 Find the value of each pronumeral in the following polygons. The polygon in part **d** is regular.

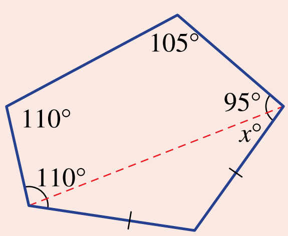
a



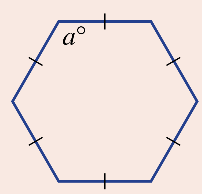
b



c



d



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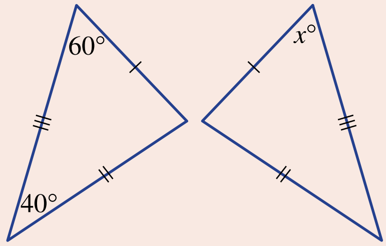
Chapter review

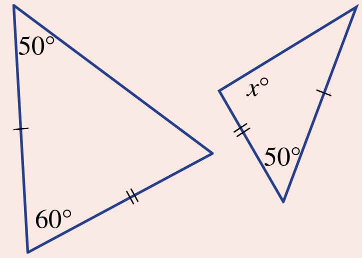
7D

5 Determine if the triangles in each pair are congruent. If congruent, give the abbreviated reason and state the value of any pronumerals.

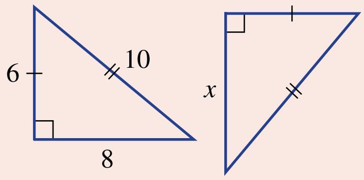
a



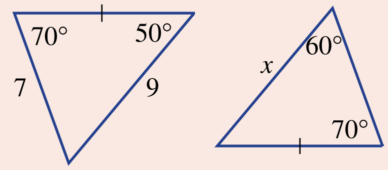
b



c

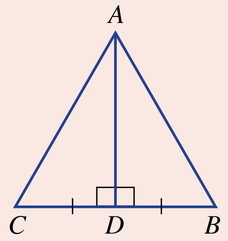


d



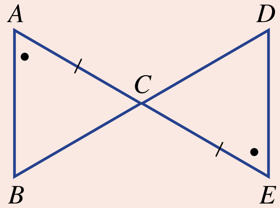
7E

6 **a** For the diagram below, prove that $\Delta ADB \equiv \Delta ADC$. List your reasons and give the abbreviated congruence test.



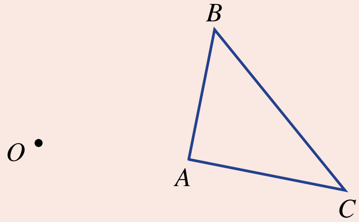
b **i** For the diagram at right, prove that $\Delta ACB \equiv \Delta ECD$. List your reasons and give the abbreviated congruence test.

ii Hence, prove that $AB \parallel DE$.



7F

7 Copy the given diagram using plenty of space. Using the centre of enlargement (O) and a scale factor of 3, enlarge ΔABC.



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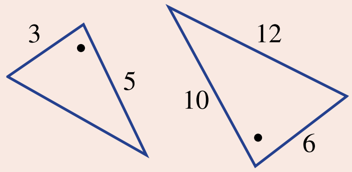
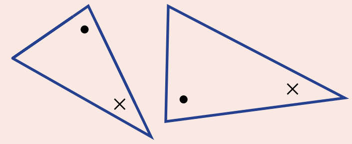
7G **8** Determine if the following pairs of triangles are similar, and state the similarity test which proves this.

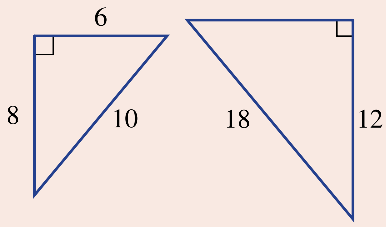
a



b

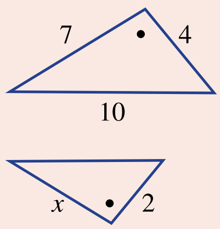


c

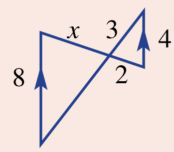


7G **9** For the following pairs of similar triangles, find the value of x.

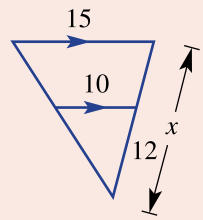
a



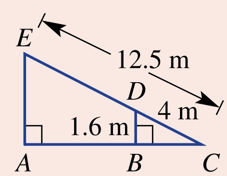
b



c



7H **10** A conveyor belt used to load luggage onto a plane is 12.5 m long. A vertical support 1.6 m high is placed under the conveyor belt such that it is 4 m along the conveyor belt as shown.



a Prove that $\Delta BCD \parallel\!\mid \Delta ACE$.

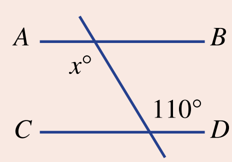
b Find the height (AE) of the luggage door above the ground.

Multiple-choice questions

7A **1** The angle that is supplementary to an angle of 55° is:

A 35° **B** 55° **C** 95°
D 125° **E** 305°

7B **2** What is the value of x if AB is parallel to CD?

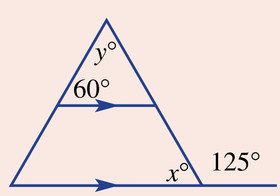


A 110 **B** 70 **C** 20
D 130 **E** 120

7A **3** If two angles in a triangle are complementary then the third angle is:

A acute **B** a right angle
C obtuse **D** reflex
E supplementary

7B **4** The values of x and y in the diagram are:



A $x = 35, y = 85$
B $x = 45, y = 45$
C $x = 55, y = 60$
D $x = 55, y = 65$
E $x = 65, y = 55$

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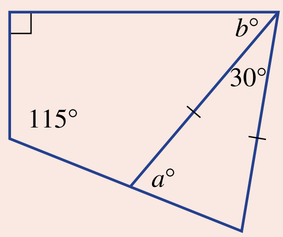
Chapter review

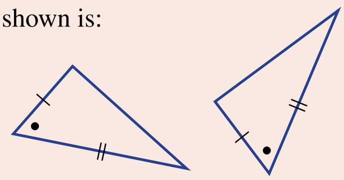
7C
5 The sum of the interior angles in a hexagon is:
A 1078°
B 360°
C 720°
D 900°
E 540°

7C
6 The quadrilateral with all sides equal, two pairs of opposite parallel sides and no right angles is:
A a kite
B a trapezium
C a parallelogram
D a rhombus
E a square

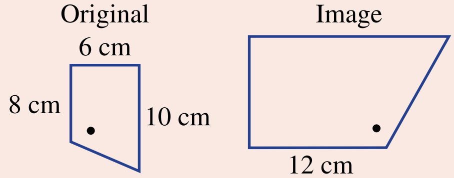
7C
7 The values of a and b in the diagram are:
A $a = 85, b = 60$
B $a = 75, b = 80$
C $a = 80, b = 55$
D $a = 70, b = 55$
E $a = 75, b = 50$



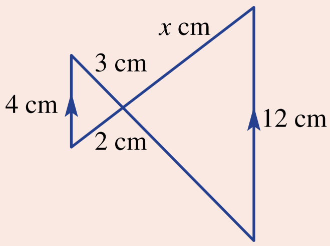
7D
8 The abbreviated reason for congruence in the two triangles shown is:
A AA
B SAS
C SSS
D AAS
E RHS



7G
9 The scale factor that enlarges the original figure to its image is:
A $\frac{2}{3}$
B 2
C 1.2
D 1.5
E 0.5



7G
10 The value of x in the diagram is:
A 6
B 9
C 10
D 8
E 7.5



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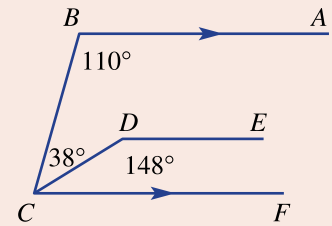
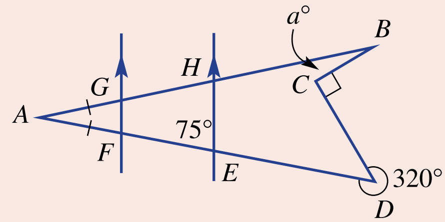
Extended-response questions

1 Complete the following.

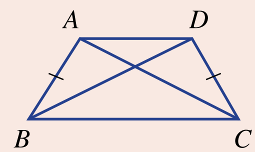
a Prove that $DE \parallel CF$.



b Show, with reasons, that $a = 20$.



c Use congruence to prove that $AC = BD$ in the diagram, given $AB = CD$ and $\angle ABC = \angle DCB$.

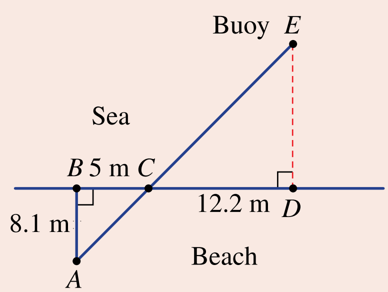


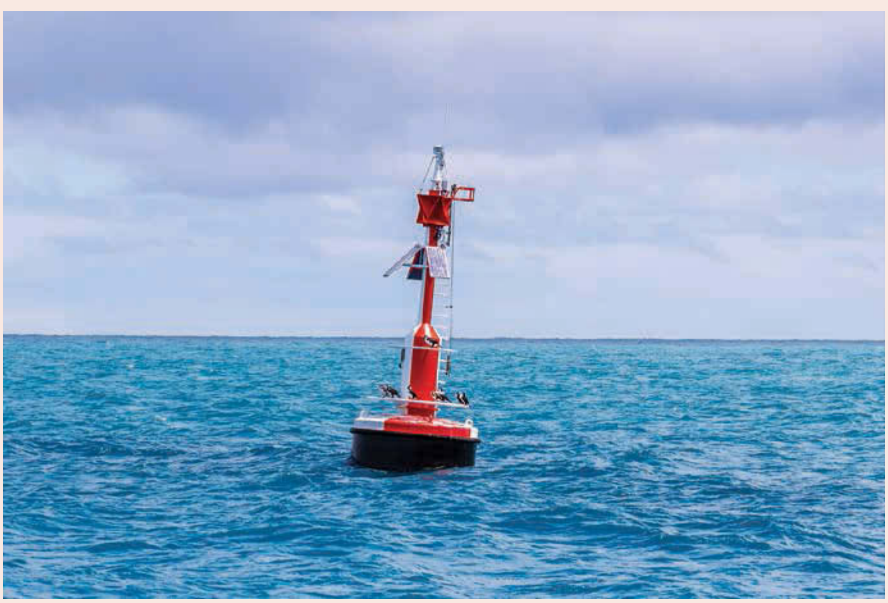
2 A buoy (E) is floating in the sea at some unknown distance from the beach as shown. The points A, B, C and D are measured and marked out on the beach as shown.

a Name the angle which is vertically opposite to $\angle ACB$.

b Explain, with reasons, why $\Delta ABC \ ||| \ \Delta EDC$.

c Find the distance from the buoy to the beach (ED), to one decimal place.





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8

Quadratic expressions and algebraic techniques

Maths in context: The story of algebra

Algebra follows rules of arithmetic, using pronumerals to represent unknown numbers. Algebraic notation is very efficient, but, for over 3500 years, solutions were written explanations of steps.

Babylonians, 2000 BCE, solved algebraic-like problems using complex arithmetic. Egyptians, 1650 BCE, solved questions with guess-and-check numerical solutions. The Greek Euclid, 350 BCE, solved algebra type questions using geometry explanations. Chinese, 250 BCE, made significant progress towards various equation solutions.

The Greek Diophantus, 275 CE, invented a geometric algebra using some symbols. The Indian Brahmagupta, 628 CE, solved linear and quadratic equations, including those with negative and irrational solutions. The Persian Al-Khwarizmi (shown in this image), 825 CE, wrote 'Al-jabra', introducing techniques of "reducing" and "balancing" to solve linear and quadratic equations. Al-Khwarizmi introduced algebra as a new branch of mathematics. He used the word 'roots' for unknowns, (like invisible tree roots), giving us 'equation roots' and 'square root' etc.

Around 1100 CE, European scholars first learned of algebra from the Latin translation of Al-Khwarizmi's Arabic book, now titled 'Algebrae' by 'Algoritmi', giving us 'algebra' and 'algorithm'. The Italian Fibonacci, 1225 CE, recorded many equation solving methods. In the 16th century, Italian mathematicians

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publicly competed to solve difficult algebraic equations, attracting enthusiastic crowds.

Our algebra notation started in the mid-1600s CE by the Europeans Descartes and Wallis. Today we enjoy efficient algebraic notation and techniques that are powerful problem-solving tools.

Nuclear fusion power offers a safe, reliable and unlimited source of clean energy. Development of technology to enable a fusion reaction to produce more energy than it uses is one of the world's greatest challenges for mathematicians, scientists and engineers to solve.

Chapter contents

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- simplifies algebraic fractions with numerical denominators and expands algebraic expressions (MA5-ALG-C-01)
- selects and applies appropriate algebraic techniques to operate with algebraic fractions, and expands, factorises and simplifies algebraic expressions (MA5-ALG-P-02).

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

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8A Expanding binomial products

Learning intentions
- To understand the distributive law for expanding binomial products
- To be able to expand and simplify binomial products

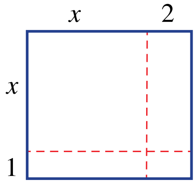
Past, present and future learning
- This section addresses the Stage 5 Core and Path Topics called *Algebraic techniques B and C*
- These concepts will be revised and extended in Chapter 5 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

A binomial is an expression with two terms such as $x + 5$ or $x^2 + 3$. You will recall from Chapter 2 that we looked at the product of a single term with a binomial expression, e.g. $2(x - 3)$ or $x(3x - 1)$. The product of two binomial expressions can also be expanded using the distributive law. This involves multiplying every term in one expression by every term in the other expression.

Business analysts expand algebraic products to model profit, as sales and profit per item can both depend on the selling price.
E.g. $P = 50(20 - c) \times (c - 4)$ for chocolates selling at \$$c$ per box.

Lesson starter: Rectangular expansions

If $(x + 1)$ and $(x + 2)$ are the side lengths of a rectangle as shown below, the total area can be found as an expression in two different ways.



- Write an expression for the total area of the rectangle using length = $(x + 2)$ and breadth = $(x + 1)$.
- Now find the area of each of the four parts of the rectangle and combine to give an expression for the total area.
- Compare your two expressions above and complete this equation:

$$(x + 2)(\_\_\_\_\_) = x^2 + \_\_\_\_\_ + \_\_\_\_\_.$$

- Can you explain a method for expanding the left-hand side to give the right-hand side?

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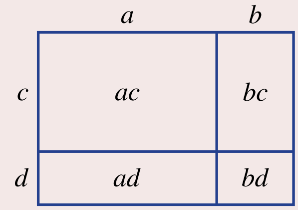
KEY IDEAS

- Expanding **binomial products** uses the **distributive law**.

$$(a+b)(c+d) = a(c+d) + b(c+d)$$
$$= ac + ad + bc + bd$$



- Diagrammatically $(a+b)(c+d) = ac + ad + bc + bd$

For example: $(x+1)(x+5) = x^2 + 5x + x + 5$
$= x^2 + 6x + 5$

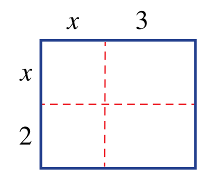
BUILDING UNDERSTANDING

1 The diagram on the right shows the area $(x+2)(x+3)$.

a State an expression for the area of each of the four regions inside the rectangle.

b State the missing terms.

$(x+2)(x+3) = \_\_\_ + 3x + \_\_\_ + 6$
$= \_\_\_ + 5x + \_\_\_$

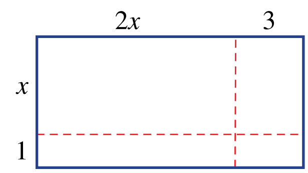


2 The diagram on the right shows the area $(2x+3)(x+1)$.

a State an expression for the area of each of the four regions inside the rectangle.

b State the missing terms.

$(2x+3)(\_\_\_\_\_\_) = 2x^2 + \_\_\_ + 3x + \_\_\_$
$= \_\_\_ + \_\_\_ + \_\_\_$



3 State the missing terms in these expansions.

a $(x+1)(x+5) = \_\_\_ + 5x + \_\_\_ + 5$
$= \_\_\_ + 6x + \_\_\_$

b $(x-3)(x+2) = \_\_\_ + \_\_\_ - 3x - \_\_\_$
$= \_\_\_ - x - \_\_\_$

c $(3x-2)(7x+2) = \_\_\_ + 6x - \_\_\_ - \_\_\_$
$= \_\_\_ - \_\_\_ - \_\_\_$

d $(4x-1)(3x-4) = \_\_\_ - \_\_\_ - 3x + \_\_\_$
$= \_\_\_ - 19x + \_\_\_$

Example 1 Expanding binomial products

Expand the following.

a $(x+3)(x+5)$ **b** $(x-4)(x+7)$ **c** $(2x-1)(x-6)$ **d** $(5x-2)(3x+7)$

SOLUTION

a $(x+3)(x+5) = x^2 + 5x + 3x + 15$
$= x^2 + 8x + 15$

EXPLANATION

Use the distributive law to expand the brackets and then collect the like terms $5x$ and $3x$.

Continued on next page

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b $(x-4)(x+7) = x^2 + 7x - 4x - 28$
$= x^2 + 3x - 28$

After expanding to get the four terms, collect the like terms $7x$ and $-4x$.

c $(2x-1)(x-6) = 2x^2 - 12x - x + 6$
$= 2x^2 - 13x + 6$

Remember $2x \times x = 2x^2$ and $-1 \times (-6) = 6$.

d $(5x-2)(3x+7) = 15x^2 + 35x - 6x - 14$
$= 15x^2 + 29x - 14$

Recall $5x \times 3x = 5 \times 3 \times x \times x = 15x^2$.

Now you try

Expand the following.

a $(x+2)(x+4)$ **b** $(x-3)(x+5)$ **c** $(2x-3)(x-4)$ **d** $(3x-5)(4x+3)$

Exercise 8A

FLUENCY	1–2(½)	1–2(½)	1–3(⅓)

Example 1a,b

1 Expand the following.

a $(x+2)(x+5)$ **b** $(b+3)(b+4)$ **c** $(t+8)(t+7)$
d $(a+1)(a+7)$ **e** $(y+10)(y+2)$ **f** $(m+4)(m+12)$
g $(x+3)(x-4)$ **h** $(x+4)(x-8)$ **i** $(x-6)(x+2)$
j $(x-1)(x+10)$ **k** $(x-1)(x-2)$ **l** $(x-4)(x-5)$

Example 1c,d

2 Expand the following.

a $(4x+3)(2x+5)$ **b** $(3x+2)(2x+1)$ **c** $(3x+1)(5x+4)$
d $(2x-3)(3x+5)$ **e** $(8x-3)(3x+4)$ **f** $(3x-2)(2x+1)$
g $(5x+2)(2x-7)$ **h** $(2x+3)(3x-2)$ **i** $(4x+1)(4x-5)$
j $(3x-2)(6x-5)$ **k** $(5x-2)(3x-1)$ **l** $(7x-3)(3x-4)$

3 Expand these binomial products.

a $(a+b)(a+c)$ **b** $(a-b)(a+c)$ **c** $(y-x)(z-y)$
d $(2x+y)(x-2y)$ **e** $(2a+b)(a-b)$ **f** $(3x-y)(2x+y)$
g $(2a-b)(3a+2)$ **h** $(4x-3y)(3x-4y)$ **i** $(xy-yz)(z+3x)$

PROBLEM–SOLVING	4	4, 5	5, 6

4 A rectangular room with dimensions 4 m by 5 m is to be extended. Both the length and the breadth are to be increased by x m.

a Find an expanded expression for the area of the new room.

b If $x = 3$:

i find the area of the new room

ii by how much has the area increased?

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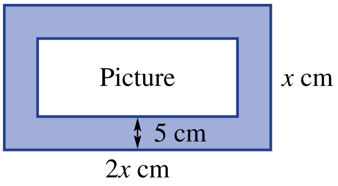
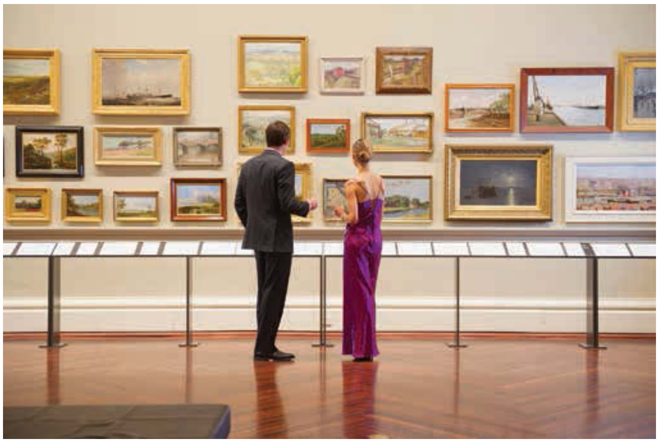
5 A rectangular picture frame 5 cm wide has a length which is twice the breadth x cm.

a Find an expression for the total area of the frame and picture.

b Find an expression in expanded form for the area of the picture only.

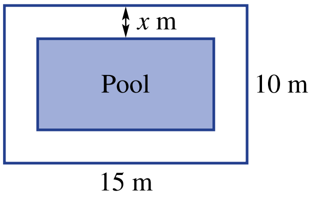




6 The outside edge of a path around a swimming pool is 15 m long and 10 m wide. The path is x metres wide.

a Find an expression for the area of the pool in expanded form.

b Find the area of the pool if $x = 2$.



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REASONING	7(½)	7–8(½)	7(⅓), 8(½)

7 Write the missing terms in these expansions.

a $(x+2)(x+\_\_\_)=x^2+5x+6$

b $(x+\_\_\_)(x+5)=x^2+7x+10$

c $(x+1)(x+\_\_\_)=x^2+7x+\_\_\_$

d $(x+\_\_\_)(x+9)=x^2+11x+\_\_\_$

e $(x+3)(x-\_\_\_)=x^2+x-\_\_\_$

f $(x-5)(x+\_\_\_)=x^2-2x-\_\_\_$

g $(x+1)(\_\_\_+3)=2x^2+\_\_\_+\_\_\_$

h $(\_\_\_-4)(3x-1)=9x^2-\_\_\_+\_\_\_$

i $(x+2)(\_\_\_+\_\_\_)=7x^2+\_\_\_+6$

j $(\_\_\_-\_\_\_)(2x-1)=6x^2-\_\_\_+4$

8 Consider the binomial product $(x+a)(x+b)$. Find the possible integer values of a and b if:

a $(x+a)(x+b)=x^2+5x+6$

b $(x+a)(x+b)=x^2-5x+6$

c $(x+a)(x+b)=x^2+x-6$

d $(x+a)(x+b)=x^2-x-6$

ENRICHMENT: Trinomial expansions	–	–	9(½), 10

9 Using the distributive law, $(a+b)(c+d+e)=ac+ad+ae+bc+bd+be$.
Use this knowledge to expand and simplify these products. (Note: $x \times x^2 = x^3$.)

a $(x+1)(x^2+x+1)$

b $(x-2)(x^2-x+3)$

c $(2x-1)(2x^2-x+4)$

d $(x^2-x+1)(x+3)$

e $(5x^2-x+2)(2x-3)$

f $(2x^2-x+7)(4x-7)$

g $(x+a)(x^2-ax+a)$

h $(x-a)(x^2-ax-a^2)$

i $(x+a)(x^2-ax+a^2)$

j $(x-a)(x^2+ax+a^2)$

10 Now try to expand $(x+1)(x+2)(x+3)$.

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8B Perfect squares and difference of two squares

Learning intentions

- To be able to recognise a perfect square
- To understand that expressions that are perfect squares can be expanded
- To be able to expand and simplify perfect squares
- To understand how a difference of two squares is formed
- To be able to expand and simplify to form a difference of two squares

Past, present and future learning

- This section addresses the Stage 5 Path Topic called *Algebraic techniques C*
- These concepts will be revised and extended in Chapter 5 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

$2^2 = 4$, $15^2 = 225$, x^2 and $(a+b)^2$ are all examples of perfect squares. To expand $(a+b)^2$ we multiply $(a+b)$ by $(a+b)$ and use the distributive law:

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= a^2 + ab + ba + b^2\\ &= a^2 + 2ab + b^2\end{aligned}$$

A similar result is obtained for the square of $(a-b)$:

$$\begin{aligned}(a-b)^2 &= (a-b)(a-b)\\ &= a^2 - ab - ba + b^2\\ &= a^2 - 2ab + b^2\end{aligned}$$

Another type of expansion involves the case that deals with the product of the sum and difference of the same two terms. The result is the difference of two squares:

$$\begin{aligned}(a+b)(a-b) &= a^2 - ab + ba - b^2\\ &= a^2 - b^2 \text{ (since } ab = ba\text{, the two middle terms cancel each other out)}\end{aligned}$$

Lesson starter: Seeing the pattern

Using $(a+b)(c+d) = ac + ad + bc + bd$, expand and simplify the binomial products in the two sets below.

Set A

$$\begin{aligned}(x+1)(x+1) &= x^2 + x + x + 1\\ &=\\ (x+3)(x+3) &=\\ &=\\ (x-5)(x-5) &=\\ &=\end{aligned}$$

Set B

$$\begin{aligned}(x+1)(x-1) &= x^2 - x + x - 1\\ &=\\ (x-3)(x+3) &=\\ &=\\ (x-5)(x+5) &=\\ &=\end{aligned}$$

- Describe the patterns you see in both sets of expansions above.
- Generalise your observations by completing the following expansions.

a $\begin{aligned}(a+b)(a+b) &= a^2 + \_\_ + \_\_ + \_\_\\ &= a^2 + \_\_ + \_\_\\ (a-b)(a-b) &=\\ &=\end{aligned}$

b $\begin{aligned}(a+b)(a-b) &= a^2 - \_\_ + \_\_ - \_\_\\ &=\end{aligned}$

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KEY IDEAS

- $3^2 = 9$, a^2, $(2y)^2$, $(x-1)^2$ and $(3-2y)^2$ are all examples of **perfect squares**.
- Expanding perfect squares
  - $$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= a^2+ab+ba+b^2\\ &= a^2+2ab+b^2\end{aligned}$$
  - $$\begin{aligned}(a-b)^2 &= (a-b)(a-b)\\ &= a^2-ab-ba+b^2\\ &= a^2-2ab+b^2\end{aligned}$$
- **Difference of two squares**
  - $$\begin{aligned}(a+b)(a-b) &= a^2-ab+ba-b^2\\ &= a^2-b^2\end{aligned}$$
  - $(a-b)(a+b)$ also expands to a^2-b^2
  - The result is a difference of two squares.

BUILDING UNDERSTANDING

1 State the missing terms in these expansions.

a $$\begin{aligned}(x+3)(x+3) &= x^2+3x+\_\_\_\_+\_\_\_\_\\ &= \_\_\_\_\_\_\_\_\_\_\_\_\end{aligned}$$

b $$\begin{aligned}(x+5)(x+5) &= x^2+5x+\_\_\_\_+\_\_\_\_\\ &= \_\_\_\_\_\_\_\_\_\_\_\_\end{aligned}$$

c $$\begin{aligned}(x-2)(x-2) &= x^2-2x-\_\_\_\_+\_\_\_\_\\ &= \_\_\_\_\_\_\_\_\_\_\_\_\end{aligned}$$

d $$\begin{aligned}(x-7)(x-7) &= x^2-7x-\_\_\_\_+\_\_\_\_\\ &= \_\_\_\_\_\_\_\_\_\_\_\_\end{aligned}$$

2 a Substitute the given value of b into $x^2+2bx+b^2$ and simplify.

i $b=3$ ii $b=11$ iii $b=15$

b Substitute the given value of b into $x^2-2bx+b^2$ and simplify.

i $b=2$ ii $b=9$ iii $b=30$

3 State the missing terms in these expansions.

a $$\begin{aligned}(x+4)(x-4) &= x^2-4x+\_\_\_\_-\_\_\_\_\\ &= \_\_\_\_\_\_\_\_\_\_\_\_\end{aligned}$$

b $$\begin{aligned}(x-10)(x+10) &= x^2+\_\_\_\_-\_\_\_\_-\_\_\_\_\\ &= \_\_\_\_\_\_\_\_\_\_\_\_\end{aligned}$$

c $$\begin{aligned}(2x-1)(2x+1) &= 4x^2+\_\_\_\_-\_\_\_\_-\_\_\_\_\\ &= \_\_\_\_\_\_\_\_\_\_\_\_\end{aligned}$$

d $$\begin{aligned}(3x+4)(3x-4) &= 9x^2-\_\_\_\_+\_\_\_\_-\_\_\_\_\\ &= \_\_\_\_\_\_\_\_\_\_\_\_\end{aligned}$$

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Example 2 Expanding perfect squares

Expand each of the following.

a $(x-4)^2$

b $(2x+3)^2$

SOLUTION

a $(x-4)^2 = (x-4)(x-4)$
$= x^2 - 4x - 4x + 16$
$= x^2 - 8x + 16$

Alternative method:

$(x-4)^2 = x^2 - 2 \times x \times 4 + 4^2$
$= x^2 - 8x + 16$

b $(2x+3)^2 = (2x+3)(2x+3)$
$= 4x^2 + 6x + 6x + 9$
$= 4x^2 + 12x + 9$

Alternative method:

$(2x+3)^2 = (2x)^2 + 2 \times 2x \times 3 + 3^2$
$= 4x^2 + 12x + 9$

EXPLANATION

Write in expanded form.
Use the distributive law.
Collect like terms.

Expand using $(a-b)^2 = a^2 - 2ab + b^2$ where $a = x$ and $b = 4$.

Write in expanded form.
Use the distributive law.
Collect like terms.

Expand using $(a+b)^2 = a^2 + 2ab + b^2$ where $a = 2x$ and $b = 3$. Recall $(2x)^2 = 2x \times 2x = 4x^2$.

Now you try

Expand each of the following.

a $(x-3)^2$

b $(3x+4)^2$

Example 3 Forming a difference of two squares

Expand and simplify the following.

a $(x+2)(x-2)$

b $(3x-2y)(3x+2y)$

SOLUTION

a $(x+2)(x-2) = x^2 - \cancel{2x} + \cancel{2x} - 4$
$= x^2 - 4$

Alternative method:

$(x+2)(x-2) = (x)^2 - (2)^2$
$= x^2 - 4$

b $(3x-2y)(3x+2y) = 9x^2 + \cancel{6xy} - \cancel{6xy} - 4y^2$
$= 9x^2 - 4y^2$

Alternative method:

$(3x-2y)(3x+2y) = (3x)^2 - (2y)^2$
$= 9x^2 - 4y^2$

EXPLANATION

Expand using the distributive law.
$-2x + 2x = 0$

$(a+b)(a-b) = a^2 - b^2$. Here $a = x$ and $b = 2$.

Expand using the distributive law.
$6xy - 6xy = 0$

$(a+b)(a-b) = a^2 - b^2$ with $a = 3x$ and $b = 2y$ here.

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Now you try

Expand and simplify the following.

a $(x+4)(x-4)$

b $(5x-3y)(5x+3y)$

Exercise 8B

FLUENCY	1–2(½), 4–5(½)	1–5(⅓)	1–5(⅓)

Example 2a

1 Expand each of the following perfect squares.

a $(x+1)^2$ b $(x+3)^2$ c $(x+2)^2$
d $(x+5)^2$ e $(x+4)^2$ f $(x+9)^2$
g $(x-2)^2$ h $(x-6)^2$ i $(x-1)^2$
j $(x-3)^2$ k $(x-9)^2$ l $(x-7)^2$

Example 2b

2 Expand each of the following perfect squares.

a $(2x+1)^2$ b $(2x+5)^2$ c $(3x+2)^2$
d $(3x+1)^2$ e $(5x+2)^2$ f $(4x+3)^2$
g $(7+2x)^2$ h $(5+3x)^2$ i $(2x-3)^2$
j $(3x-1)^2$ k $(4x-5)^2$ l $(2x-9)^2$
m $(3x+5y)^2$ n $(2x+4y)^2$ o $(7x+3y)^2$
p $(6x+5y)^2$ q $(4x-9y)^2$ r $(2x-7y)^2$
s $(3x-10y)^2$ t $(4x-6y)^2$ u $(9x-2y)^2$

3 Expand each of the following perfect squares.

a $(3-x)^2$ b $(5-x)^2$ c $(1-x)^2$
d $(6-x)^2$ e $(11-x)^2$ f $(8-2x)^2$
g $(2-3x)^2$ h $(9-2x)^2$ i $(10-4x)^2$

Example 3a

4 Expand and simplify the following to form a difference of two squares.

a $(x+1)(x-1)$ b $(x+3)(x-3)$ c $(x+8)(x-8)$
d $(x+4)(x-4)$ e $(x+12)(x-12)$ f $(x+11)(x-11)$
g $(x-9)(x+9)$ h $(x-5)(x+5)$ i $(x-6)(x+6)$
j $(5-x)(5+x)$ k $(2-x)(2+x)$ l $(7-x)(7+x)$

Example 3b

5 Expand and simplify the following.

a $(3x-2)(3x+2)$ b $(5x-4)(5x+4)$ c $(4x-3)(4x+3)$
d $(7x-3y)(7x+3y)$ e $(9x-5y)(9x+5y)$ f $(11x-y)(11x+y)$
g $(8x+2y)(8x-2y)$ h $(10x-9y)(10x+9y)$ i $(7x-5y)(7x+5y)$
j $(6x-11y)(6x+11y)$ k $(8x-3y)(8x+3y)$ l $(9x-4y)(9x+4y)$

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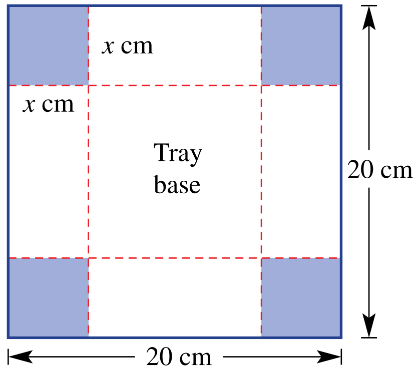
PROBLEM–SOLVING	6	6, 7	6, 7

6 Lara is x years old and her two best friends are $(x - 2)$ and $(x + 2)$ years old.
  a Write an expression for:
    i the square of Lara's age
    ii the product of the ages of Lara's best friends (in expanded form).
  b Are the answers from parts a i and ii equal? If not, by how much do they differ?

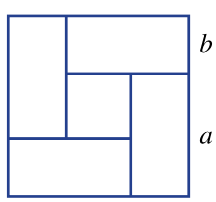
7 A square piece of tin of side length 20 cm has four squares of side length x cm removed from each corner. The sides are folded up to form a tray. The centre square forms the tray base.



  a Write an expression for the side length of the base of the tray.
  b Write an expression for the area of the base of the tray. Expand your answer.
  c Find the area of the tray base if $x = 3$.
  d Find the volume of the tray if $x = 3$.

REASONING	8	8, 9	9, 10

8 Four tennis courts are arranged as shown with a square storage area in the centre. Each court area has the dimensions $a \times b$.



  a Write an expression for the side length of the total area.
  b Write an expression for the total area.
  c Write an expression for the side length of the inside storage area.
  d Write an expression for the area of the inside storage area.
  e Subtract your answer to part d from your answer to part b to find the area of the four courts.
  f Find the area of one court. Does your answer confirm that your answer to part e is correct?

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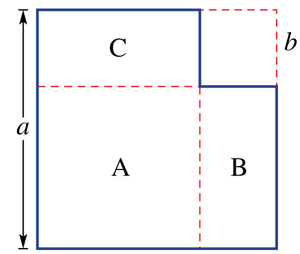
9 A square of side length x units has one side reduced by 1 unit and the other increased by 1 unit.

a Find an expanded expression for the area of the resulting rectangle.

b Is the area of the original square the same as the area of the resulting rectangle? Explain why/why not.

10 A square of side length b is removed from a square of side length a.



a Using subtraction write down an expression for the remaining area.

b Write expressions for the area of these regions:

i A

ii B

iii C

c Add all the expressions from part **b** to see if you get your answer from part **a**.

ENRICHMENT: Extended expansions	–	–	11(½)

11 Expand and simplify these expressions.

a $(x+2)^2-4$

b $(2x-1)^2-4x^2$

c $(x+3)(x-3)+6x$

d $1-(x+1)^2$

e $x^2-(x+1)(x-1)$

f $(x+1)^2-(x-1)^2$

g $(3x-2)(3x+2)-(3x+2)^2$

h $(5x-1)^2-(5x+1)(5x-1)$

i $(x+y)^2-(x-y)^2+(x+y)(x-y)$

j $(2x-3)^2+(2x+3)^2$

k $(2-x)^2-(2+x)^2$

l $(3-x)^2+(x-3)^2$

m $2(3x-4)^2-(3x-4)(3x+4)$

n $2(x+y)^2-(x-y)^2$

Engineers work with binomial products. A metal rectangle, with possible dimensions 16 cm by 24 cm, can have square corners removed and be folded to give a height of x cm. The box has base area $A = (16 - 2x)(24 - 2x)$.

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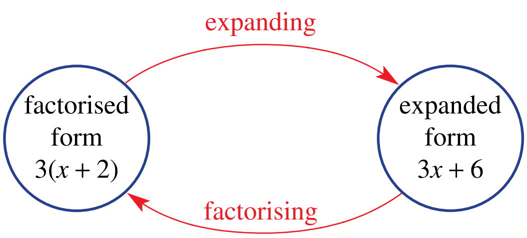
8C Factorising algebraic expressions

Learning intentions
- To understand that factorising and expanding are reverse processes
- To be able to identify the highest common factor
- To be able to factorise algebraic expressions involving a common factor

Past, present and future learning
- This section addresses the Stage 5 Core and Path Topics called *Algebraic techniques B and C*
- These concepts will be revised and extended in Chapter 5 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

The process of factorisation is a key step in the simplification of many algebraic expressions and in the solution of equations. It is the reverse process of expansion and involves writing an expression as a product of its factors.



Factorising is a key step when solving many of the equations used in business, engineering, technology and science. Fashion business analysts develop and solve equations to model and predict sales, revenue and profit.

Lesson starter: Which factorised form?

The product $x(4x + 8)$ when expanded gives $4x^2 + 8x$.

- Write down three other products that when expanded give $4x^2 + 8x$. (Do not use fractions.)
- Which of your products uses the highest common factor of $4x^2$ and $8x$? What is this highest common factor?

KEY IDEAS

- When **factorising** expressions with common factors, take out the highest common factor (HCF). The HCF could be:
  - a number
    For example: $2x + 10 = 2(x + 5)$
  - a pronumeral (or variable)
    For example: $x^2 + 5x = x(x + 5)$

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- the product of numbers and pronumerals
  For example: $2x^2 + 10x = 2x(x + 5)$

■ A factorised expression can be checked by using expansion.

For example: $2x(x + 5) = 2x^2 + 10x$.

BUILDING UNDERSTANDING

1 State the highest common factor (HCF) of these pairs of numbers.

a 8, 12 **b** 24, 30 **c** 100, 75 **d** 16, 24

2 Give the missing factor.

a $5 \times \_\_\_ = 5x$ **b** $5a \times \_\_\_ = 10a^2$

c $\_\_\_ \times 12x = -36x^2$ **d** $\_\_\_ \times -7xy = 14x^2y$

3 **a** State the missing factor in each part.

i $\_\_\_(x^2 + 2x) = 6x^2 + 12x$ **ii** $\_\_\_(2x + 4) = 6x^2 + 12x$

iii $\_\_\_(x + 2) = 6x^2 + 12x$

b Which equation above uses the HCF of $6x^2$ and $12x$?

c By looking at the terms left in the brackets, how do you know you have taken out the HCF?

Example 4 Finding the HCF

Determine the highest common factor (HCF) of the following.

a $6a$ and $8ab$ **b** $3x^2$ and $6xy$

SOLUTION	EXPLANATION
a $2a$	HCF of 6 and 8 is 2. HCF of a and ab is a.
b $3x$	HCF of 3 and 6 is 3. HCF of x^2 and xy is x.

Now you try

Determine the highest common factor (HCF) of the following.

a $4x$ and $10xy$ **b** $5x^2$ and $15xy$

Example 5 Factorising expressions

Factorise the following.

a $40 - 16b$ **b** $-8x^2 - 12x$

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SOLUTION

a $40 - 16b = 8(5 - 2b)$

EXPLANATION

The HCF of 40 and $16b$ is 8. Place 8 in front of the brackets and divide each term by 8.

b $-8x^2 - 12x = -4x(2x + 3)$

The HCF of the terms is $-4x$, including the common negative. Place the factor in front of the brackets and divide each term by $-4x$.

Now you try

Factorise the following.

a $28 - 21a$

b $-9x^2 - 15x$

Example 6 Taking out a binomial factor

Factorise the following.

a $3(x + y) + x(x + y)$

b $(7 - 2x) - x(7 - 2x)$

SOLUTION

a $3(x + y) + x(x + y)$
$= (x + y)(3 + x)$

EXPLANATION

HCF $= (x + y)$.
The second pair of brackets contains what remains when $3(x + y)$ and $x(x + y)$ are divided by $(x + y)$.

b $(7 - 2x) - x(7 - 2x)$
$= 1(7 - 2x) - x(7 - 2x)$
$= (7 - 2x)(1 - x)$

Insert 1 in front of the first bracket.
HCF $= (7 - 2x)$.
The second bracket must contain $1 - x$ after dividing $(7 - 2x)$ and $x(7 - 2x)$ by $(7 - 2x)$.

Now you try

Factorise the following.

a $4(a + b) + a(a + b)$

b $(4x + 3) - x(4x + 3)$

Exercise 8C

FLUENCY	1, 2–5(½)	2–6(⅓)	2–6(¼)

1 Determine the highest common factor (HCF) of the following.

Example 4a
a **i** $5a$ and $15ab$ **ii** $9b$ and $12ab$

Example 4b
b **i** $2x^2$ and $8xy$ **ii** $5x^2$ and $10x$

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Example 4

2 Determine the HCF of the following.

a $6x$ and $14xy$ **b** $12a$ and $18a$ **c** $10m$ and 4 **d** $12y$ and 8
e $15t$ and $6s$ **f** 15 and p **g** $9x$ and $24xy$ **h** $6n$ and $21mn$
i $10y$ and $2y$ **j** $8x^2$ and $14x$ **k** $4x^2y$ and $18xy$ **l** $5ab^2$ and $15a^2b$

Example 5a

3 Factorise the following.

a $7x + 7$ **b** $3x + 3$ **c** $4x - 4$ **d** $5x - 5$
e $4 + 8y$ **f** $10 + 5a$ **g** $3 - 9b$ **h** $6 - 2x$
i $12a + 3b$ **j** $6m + 6n$ **k** $10x - 8y$ **l** $4a - 20b$
m $x^2 + 2x$ **n** $a^2 - 4a$ **o** $y^2 - 7y$ **p** $x - x^2$
q $3p^2 + 3p$ **r** $8x - 8x^2$ **s** $4b^2 + 12b$ **t** $6y - 10y^2$
u $12a - 15a^2$ **v** $9m + 18m^2$ **w** $16xy - 48x^2$ **x** $7ab - 28ab^2$

Example 5b

4 Factorise the following by factoring out the negative sign as part of the HCF.

a $-8x - 4$ **b** $-4x - 2$ **c** $-10x - 5y$ **d** $-7a - 14b$
e $-9x - 12$ **f** $-6y - 8$ **g** $-10x - 15y$ **h** $-4m - 20n$
i $-3x^2 - 18x$ **j** $-8x^2 - 12x$ **k** $-16y^2 - 6y$ **l** $-5a^2 - 10a$
m $-6x - 20x^2$ **n** $-6p - 15p^2$ **o** $-16b - 8b^2$ **p** $-9x - 27x^2$

Example 6

5 Factorise the following which involve a binomial common factor.

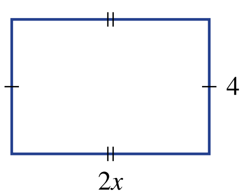
a $4(x + 3) + x(x + 3)$ **b** $3(x + 1) + x(x + 1)$
c $7(m - 3) + m(m - 3)$ **d** $x(x - 7) + 2(x - 7)$
e $8(a + 4) - a(a + 4)$ **f** $5(x + 1) - x(x + 1)$
g $y(y + 3) - 2(y + 3)$ **h** $a(x + 2) - x(x + 2)$
i $t(2t + 5) + 3(2t + 5)$ **j** $m(5m - 2) + 4(5m - 2)$
k $y(4y - 1) - (4y - 1)$ **l** $(7 - 3x) + x(7 - 3x)$

6 Factorise these mixed expressions.

a $6a + 30$ **b** $5x - 15$ **c** $8b + 18$
d $x^2 - 4x$ **e** $y^2 + 9y$ **f** $a^2 - 3a$
g $x^2y - 4xy + xy^2$ **h** $6ab - 10a^2b + 8ab^2$ **i** $m(m + 5) + 2(m + 5)$
j $x(x + 3) - 2(x + 3)$ **k** $b(b - 2) + (b - 2)$ **l** $x(2x + 1) - (2x + 1)$
m $y(3 - 2y) - 5(3 - 2y)$ **n** $(x + 4)^2 + 5(x + 4)$ **o** $(y + 1)^2 - 4(y + 1)$

PROBLEM–SOLVING	7	7(½), 8	7(⅓), 8, 9

7 Write down the perimeter of these shapes in factorised form.

a


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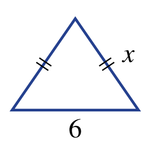

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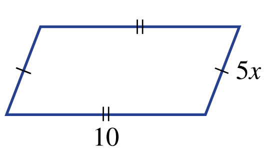
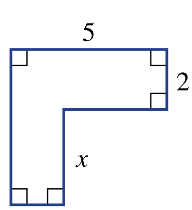


d


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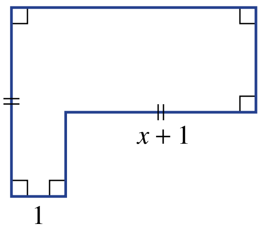

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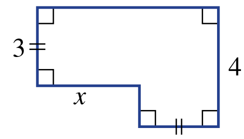

8 The expression for the area of a rectangle is $(4x^2 + 8x)$ square units. Find an expression for the breadth of the rectangle if its length is $(x + 2)$ units.

9 The height, above its starting point in metres, of a ball thrown in the air is given by $5t - t^2$, where t is the time in seconds.

- **a** Write an expression for the ball's height in factorised form.
- **b** Find the ball's height above its starting point at these times:
  - **i** $t = 0$
  - **ii** $t = 4$
  - **iii** $t = 2$
- **c** How long does it take for the ball's height to return to 0 metres (its starting height)? Use trial and error if required.

REASONING	10	10–11(½)	11–12(½)

10 $7 \times 9 + 7 \times 3$ can be evaluated by first factorising to $7(9 + 3)$. This gives $7 \times 12 = 84$. Use a similar technique to evaluate the following.

- **a** $9 \times 2 + 9 \times 5$
- **b** $6 \times 3 + 6 \times 9$
- **c** $-2 \times 4 - 2 \times 6$
- **d** $-5 \times 8 - 5 \times 6$
- **e** $23 \times 5 - 23 \times 2$
- **f** $63 \times 11 - 63 \times 8$

11 Common factors can also be removed from expressions with more than two terms.
For example: $2x^2 + 6x + 10xy = 2x(x + 3 + 5y)$
Factorise these expressions by taking out the HCF.

- **a** $3a^2 + 9a + 12$
- **b** $5z^2 - 10z + zy$
- **c** $x^2 - 2xy + x^2y$
- **d** $4by - 2b + 6b^2$
- **e** $-12xy - 8yz - 20xyz$
- **f** $3ab + 4ab^2 + 6a^2b$

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12 Sometimes we can choose to factor out a negative HCF or a positive HCF. Both factorisations are correct. For example:

$$\begin{aligned} -13x + 26 &= -13(x - 2) \quad \text{(HCF is } -13) \\ \text{OR } -13x + 26 &= 13(-x + 2) \quad \text{(HCF is 13)} \\ &= 13(2 - x) \end{aligned}$$

Factorise in two different ways: the first by factoring out a negative HCF and the second by factoring out a positive HCF.

a $-4x + 12$

b $-3x + 9$

c $-8n + 8$

d $-3b + 3$

e $-5m + 5m^2$

f $-7x + 7x^2$

g $-5x + 5x^2$

h $-4y + 22y^2$

ENRICHMENT: Factoring out a negative	–	–	13(½)

13 Using the fact that $a - b = -(b - a)$, you can factorise $x(x - 2) - 5(2 - x)$ by following these steps.

$$\begin{aligned} x(x - 2) - 5(2 - x) &= x(x - 2) + 5(x - 2) \\ &= (x - 2)(x + 5) \end{aligned}$$

Use this idea to factorise these expressions.

a $x(x - 4) + 3(4 - x)$

b $x(x - 5) - 2(5 - x)$

c $x(x - 3) - 3(3 - x)$

d $3x(x - 4) + 5(4 - x)$

e $3(2x - 5) + x(5 - 2x)$

f $2x(x - 2) + (2 - x)$

g $-4(3 - x) - x(x - 3)$

h $x(x - 5) + (10 - 2x)$

i $x(x - 3) + (6 - 2x)$

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8D Factorising the difference of two squares

Learning intentions
- To be able to recognise a difference of two squares
- To be able to express a difference of two squares in factorised form
- To know to check for a common factor first before further factorisation

Past, present and future learning
- This section addresses the Stage 5 Path Topic called *Algebraic techniques C*
- These concepts will be revised and extended in Chapter 5 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

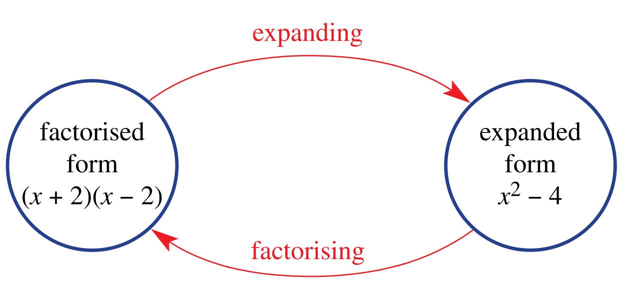
Recall that a difference of two squares is formed when expanding the product of the sum and difference of two terms. For example, $(x+2)(x-2)=x^2-4$. Reversing this process means that a difference of two squares can be factorised into two binomial expressions of the form $(a+b)$ and $(a-b)$.

Engineers use algebra when designing products used for bridges, buildings and machinery, such as this steel turbine for a power station. For example, a circle area difference: $A=\pi R^2-\pi r^2=\pi(R^2-r^2)$ $=\pi(R-r)(R+r)$.

Lesson starter: Expanding to understand factorising

Complete the steps in these expansions then write the conclusion.

- $$\begin{aligned}(x+3)(x-3) &= x^2-3x+\_\_\_-\_\_\_\\ &= x^2-\_\_\_\_\\ \therefore x^2-9 &= (\_\_\_+\_\_\_)(\_\_\_-\_\_\_)\end{aligned}$$
- $$\begin{aligned}(2x-5)(2x+5) &= 4x^2+10x-\_\_\_-\_\_\_\\ &= \_\_\_-\_\_\_\\ \therefore 4x^2-\_\_\_ &= (\_\_\_+\_\_\_)(\_\_\_-\_\_\_)\end{aligned}$$
- $$\begin{aligned}(a+b)(a-b) &= a^2-ab+\_\_\_-\_\_\_\\ &= \_\_\_-\_\_\_\\ \therefore a^2-\_\_\_ &= (\_\_\_+\_\_\_)(\_\_\_-\_\_\_)\end{aligned}$$



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KEY IDEAS

- Factorising the difference of two squares uses the rule $a^2 - b^2 = (a + b)(a - b)$.
  - $x^2 - 16 = x^2 - 4^2$
    $= (x + 4)(x - 4)$
  - $9x^2 - 100 = (3x)^2 - 10^2$
    $= (3x + 10)(3x - 10)$
  - $25 - 4y^2 = 5^2 - (2y)^2$
    $= (5 + 2y)(5 - 2y)$
- First take out common factors where possible.
  - $2x^2 - 18 = 2(x^2 - 9)$
    $= 2(x + 3)(x - 3)$

BUILDING UNDERSTANDING

1 Expand these binomial products to form a difference of two squares.

a $(x + 2)(x - 2)$
b $(x - 7)(x + 7)$
c $(3x - y)(3x + y)$

2 State the missing term. Assume it is a positive number.

a $(\_\_\_)^2 = 9$ **b** $(\_\_\_)^2 = 121$
c $(\_\_\_)^2 = 25b^2$ **d** $(\_\_\_)^2 = 49y^2$

3 State the missing terms to complete these factorisations.

a $x^2 - 16 = x^2 - 4^2$
$= (x + 4)(\_\_\_ - \_\_\_)$

b $x^2 - 144 = x^2 - (\_\_\_)^2$
$= (\_\_\_ + 12)(x - \_\_\_)$

c $16x^2 - 1 = (\_\_\_)^2 - (\_\_\_)^2$
$= (4x + \_\_\_)(\_\_\_ - 1)$

d $9a^2 - 4b^2 = (\_\_\_)^2 - (\_\_\_)^2$
$= (3a + \_\_\_)(\_\_\_ - 2b)$

Example 7 Factorising a difference of two squares

Factorise each of the following.

a $x^2 - 4$ **b** $9a^2 - 25$ **c** $81x^2 - y^2$

SOLUTION

a $x^2 - 4 = x^2 - 2^2$
$= (x + 2)(x - 2)$

EXPLANATION

Write as a difference of two squares (4 is the same as 2^2).
Write in factorised form:
$a^2 - b^2 = (a + b)(a - b)$
Here $a = x$ and $b = 2$.

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b $9a^2 - 25 = (3a)^2 - 5^2$
$= (3a + 5)(3a - 5)$

Write as a difference of two squares. $9a^2$ is the same as $(3a)^2$.
Write in factorised form.

c $81x^2 - y^2 = (9x)^2 - y^2$
$= (9x + y)(9x - y)$

$81x^2 = (9x)^2$
Use $a^2 - b^2 = (a + b)(a - b)$.

Now you try

Factorise each of the following.

a $x^2 - 16$ b $4a^2 - 9$ c $25a^2 - b^2$

Example 8 Factorising a more complex difference of two squares

Factorise each of the following.

a $2b^2 - 32$ b $(x + 1)^2 - 4$

SOLUTION

a $2b^2 - 32 = 2(b^2 - 16)$
$= 2(b^2 - 4^2)$
$= 2(b + 4)(b - 4)$

EXPLANATION

First, factor out the common factor of 2.
Write as a difference of two squares and then factorise.

b $(x + 1)^2 - 4 = (x + 1)^2 - 2^2$
$= ((x + 1) + 2)((x + 1) - 2)$
$= (x + 3)(x - 1)$

Write as a difference of two squares. In $a^2 - b^2$ here, a is the expression $x + 1$ and $b = 2$.
Write in factorised form and simplify.

Now you try

Factorise each of the following.

a $3a^2 - 27$ b $(x + 2)^2 - 9$

Exercise 8D

FLUENCY	1–3(½)	1–4(½)	1–2(¼), 3–4(½)

Example 7a

1 Factorise each of the following.

a $x^2 - 9$ b $y^2 - 25$ c $y^2 - 1$ d $x^2 - 64$
e $x^2 - 16$ f $b^2 - 49$ g $a^2 - 81$ h $x^2 - y^2$
i $a^2 - b^2$ j $16 - a^2$ k $25 - x^2$ l $1 - b^2$
m $36 - y^2$ n $121 - b^2$ o $x^2 - 400$ p $900 - y^2$

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Example 7b,c

2 Factorise each of the following.

a $4x^2 - 25$ **b** $9x^2 - 49$ **c** $25b^2 - 4$ **d** $4m^2 - 121$

e $100y^2 - 9$ **f** $81a^2 - 4$ **g** $1 - 4x^2$ **h** $25 - 64b^2$

i $16 - 9y^2$ **j** $36x^2 - y^2$ **k** $4x^2 - 25y^2$ **l** $64a^2 - 49b^2$

m $4p^2 - 25q^2$ **n** $81m^2 - 4n^2$ **o** $25a^2 - 49b^2$ **p** $100a^2 - 9b^2$

Example 8a

3 Factorise each of the following by first taking out the common factor.

a $3x^2 - 108$ **b** $10a^2 - 10$ **c** $6x^2 - 24$

d $4y^2 - 64$ **e** $98 - 2x^2$ **f** $32 - 8m^2$

g $5x^2y^2 - 5$ **h** $3 - 3x^2y^2$ **i** $63 - 7a^2b^2$

Example 8b

4 Factorise each of the following.

a $(x + 5)^2 - 9$ **b** $(x + 3)^2 - 4$ **c** $(x + 10)^2 - 16$

d $(x - 3)^2 - 25$ **e** $(x - 7)^2 - 1$ **f** $(x - 3)^2 - 36$

g $49 - (x + 3)^2$ **h** $4 - (x + 2)^2$ **i** $81 - (x + 8)^2$

PROBLEM–SOLVING	5	5, 6	5, 6

5 The height above ground (in metres) of an object thrown off the top of a building is given by $36 - 4t^2$ where t is in seconds.

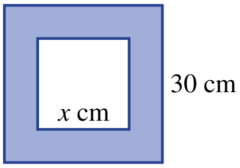
a Factorise the expression for the height of the object by first taking out the common factor.

b Find the height of the object:

i initially ($t = 0$) **ii** at 2 seconds ($t = 2$).

c How long does it take for the object to hit the ground? Use trial and error if you wish.

6 This 'multisize' square picture frame has side length 30 cm and can hold a square picture with any side length less than 26 cm.



a If the side length of the picture is x cm, write an expression for:

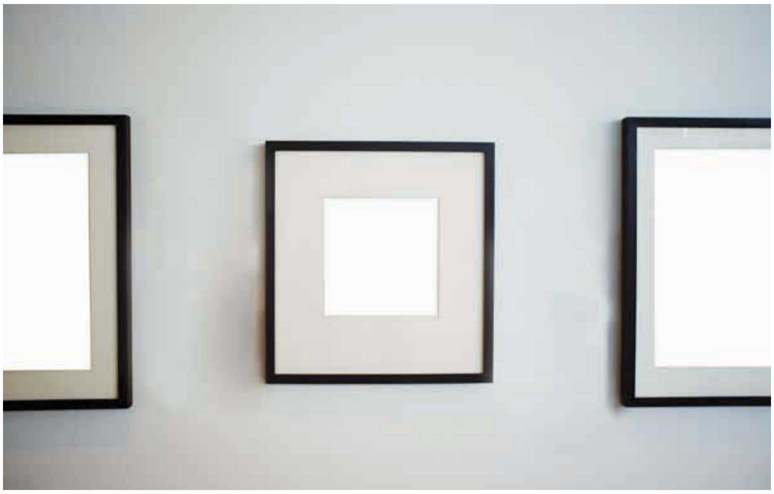
i the area of the picture

ii the area of the frame (in factorised form).

b Use your result from part **a ii** to find the area of the frame if:

i $x = 20$

ii the area of the picture is 225 cm^2.



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REASONING	7(½)	7(½), 8	7(½), 8, 9

7 Initially it may not appear that an expression such as $-4 + 9x^2$ is a difference of two squares. However, swapping the position of the two terms makes $-4 + 9x^2 = 9x^2 - 4$, which can be factorised to $(3x + 2)(3x - 2)$. Use this idea to factorise these difference of two squares.

a $-9 + x^2$
b $-121 + 16x^2$
c $-25a^2 + 4$
d $-y^2 + x^2$
e $-25a^2 + 4b^2$
f $-36a^2b^2 + c^2$
g $-16x^2 + y^2z^2$
h $-900a^2 + b^2$

8 Olivia factorises $16x^2 - 4$ to get $(4x + 2)(4x - 2)$, but the answer says $4(2x + 1)(2x - 1)$.

a What should Olivia do to get from her answer to the actual answer?
b What should Olivia have done initially to avoid this issue?

9 Find and explain the error in this working and correct it.

$$\begin{aligned} 9 - (x - 1)^2 &= (3 + x - 1)(3 - x - 1) \\ &= (2 + x)(2 - x) \end{aligned}$$

ENRICHMENT: Factorising with fractions and powers of 4	–	–	10(½)

10 Some expressions with fractions or powers of 4 can be factorised in a similar way. Factorise these.

a $x^2 - \frac{1}{4}$
b $x^2 - \frac{4}{25}$
c $25x^2 - \frac{9}{16}$
d $\frac{x^2}{9} - 1$
e $\frac{a^2}{4} - \frac{b^2}{9}$
f $\frac{5x^2}{9} - \frac{5}{4}$
g $\frac{7a^2}{25} - \frac{28b^2}{9}$
h $\frac{a^2}{8} - \frac{b^2}{18}$
i $x^4 - y^4$
j $2a^4 - 2b^4$
k $21a^4 - 21b^4$
l $\frac{x^4}{3} - \frac{y^4}{3}$

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8E Factorising by grouping in pairs

Learning intentions

- To know that a common factor can include a binomial term
- To be able to use the grouping method to factorise some four-term expressions
- To know that expressions can be rearranged to find common factors to use in the grouping process

Past, present and future learning

- This section addresses the Stage 5 Path Topic called *Algebraic techniques C*
- These concepts will be revised and extended in Chapter 5 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

When an expression can be written using four terms, such as $x^2 + 2x - x - 2$, it may be possible to factorise it into a product of two binomial terms such as $(x - 1)(x + 2)$. In such situations the method of grouping in pairs is often used.

Algebra is an essential tool for the engineers who work together making renewable energy possible, including: aerospace, civil, electrical, electronics, environmental, industrial, materials, mechanical and solar engineers.

Lesson starter: Two methods-Same result

The four-term expression $x^2 - 3x - 3 + x$ is written on the board.

Tommy chooses to rearrange the terms to give $x^2 - 3x + x - 3$ then factorises by grouping in pairs.

Sharon chooses to rearrange the terms to give $x^2 + x - 3x - 3$ then also factorises by grouping in pairs.

- Complete Tommy and Sharon's factorisation working.

Tommy

$$\begin{aligned} x^2 - 3x + x - 3 &= x(x - 3) + 1(\_\_\_\_) \\ &= (x - 3)(\_\_\_\_\_) \end{aligned}$$

Sharon

$$\begin{aligned} x^2 + x - 3x - 3 &= x(\_\_\_\_) - 3(\_\_\_\_) \\ &= (x + 1)(\_\_\_\_) \end{aligned}$$

- Discuss the differences in the methods. Is there any difference in their answers?
- Whose method do you prefer?

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KEY IDEAS

- **Factorisation by grouping in pairs** is a method that is often used to factorise a four-term expression.
  - Terms are grouped into pairs and factorised separately.
  - The common binomial factor is then taken out to complete the factorisation.
  - Terms can be rearranged to assist in the search of a common factor.

$$x^2 + 3x - 2x - 6$$
$$= x(x + 3) - 2(x + 3)$$
$$= (x + 3)(x - 2)$$

which could also be written as $(x - 2)(x + 3)$

BUILDING UNDERSTANDING

1 State each expanded expression.

a $2(x - 1)$
b $-5(1 - a)$
c $a(a + 5)$
d $y(4 - y)$
e $x(a + 1) + 2(a + 1)$
f $b(x - 2) - 3(x - 2)$

2 State the missing information.

a $2(x + 1) + x(x + 1) = (x + 1)(\_\_\_\_)$
b $5(x + 5) - x(x + 5) = (x + 5)(\_\_\_\_)$
c $a(x - 3) + (x - 3) = (x - 3)(\_\_\_\_)$
d $(x - 3) - a(x - 3) = (x - 3)(\_\_\_\_)$

3 Identify the common binomial term then factorise each expression.

a $x(x - 3) - 2(x - 3)$
b $x(x + 4) + 3(x + 4)$
c $3(2x + 1) - x(2x + 1)$
d $4(3x - 2) - x(3x - 2)$
e $3x(5 - x) + 2(5 - x)$
f $x(x - 2) + (x - 2)$

Example 9 Factorising by grouping in pairs

Use the method of grouping in pairs to factorise these expressions.

a $x^2 + 2x + 3x + 6$
b $x^2 + 3x - 5x - 15$

SOLUTION	EXPLANATION
a $x^2 + 2x + 3x + 6 = (x^2 + 2x) + (3x + 6)$	Group the first and second pair of terms.
$= x(x + 2) + 3(x + 2)$	Factorise each group.
$= (x + 2)(x + 3)$	Take the common factor $(x + 2)$ out of both groups.
b $x^2 + 3x - 5x - 15 = (x^2 + 3x) + (-5x - 15)$	Group the first and second pair of terms.
$= x(x + 3) - 5(x + 3)$	Factorise each group.
$= (x + 3)(x - 5)$	Take out the common factor $(x + 3)$.

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Now you try

Use the method of grouping in pairs to factorise these expressions.

a $x^2 + 4x + 2x + 8$ **b** $x^2 + 2x - 3x - 6$

Example 10 Rearranging an expression to factorise by grouping in pairs

Factorise $2x^2 - 9 - 18x + x$ using grouping in pairs.

SOLUTION

$$\begin{aligned} 2x^2 - 9 - 18x + x &= 2x^2 + x - 18x - 9 \\ &= x(2x + 1) - 9(2x + 1) \\ &= (2x + 1)(x - 9) \end{aligned}$$

Alternative method:

$$\begin{aligned} 2x^2 - 9 - 18x + x &= 2x^2 - 18x + x - 9 \\ &= 2x(x - 9) + 1(x - 9) \\ &= (x - 9)(2x + 1) \end{aligned}$$

EXPLANATION

Rearrange so that each group has a common factor. Factorise each group then take out $(2x + 1)$.

Alternatively, you can group in another order where each group has a common factor. Then factorise.

The answer will be the same.

Now you try

Factorise $2x^2 - 15 - 10x + 3x$ using grouping in pairs.

Exercise 8E

FLUENCY	1–2(½)	1–3(½)	1–3(⅓)

Example 9

1 Use the method of grouping in pairs to factorise these expressions.

a $x^2 + 3x + 2x + 6$ **b** $x^2 + 4x + 3x + 12$

c $x^2 + 7x + 2x + 14$ **d** $x^2 - 6x + 4x - 24$

e $x^2 - 4x + 6x - 24$ **f** $x^2 - 3x + 10x - 30$

g $x^2 + 2x - 18x - 36$ **h** $x^2 + 3x - 14x - 42$

i $x^2 + 4x - 18x - 72$ **j** $x^2 - 2x - xa + 2a$

k $x^2 - 3x - 3xc + 9c$ **l** $x^2 - 5x - 3xa + 15a$

2 Use the method of grouping in pairs to factorise these expressions. The HCF for each pair includes a pronumeral. For example, $2ab - 2ad + bc - cd = 2a(b - d) + c(b - d) = (b - d)(2a + c)$

a $3ab + 5bc + 3ad + 5cd$ **b** $4ab - 7ac + 4bd - 7cd$

c $2xy - 8xz + 3wy - 12wz$ **d** $5rs - 10r + st - 2t$

e $4x^2 + 12xy - 3x - 9y$ **f** $2ab - a^2 - 2bc + ac$

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3 Factorise these expressions. Remember to use a factor of 1 where necessary, for example, $x^2 - ax + x - a = x(x - a) + 1(x - a)$.

a $x^2 - bx + x - b$
b $x^2 - cx + x - c$
c $x^2 + bx + x + b$
d $x^2 + cx - x - c$
e $x^2 + ax - x - a$
f $x^2 - bx - x + b$

PROBLEM–SOLVING	4(½)	4–5(½)	4–6(½)

Example 10

4 Factorise these expressions by first rearranging the terms.

a $2x^2 - 7 - 14x + x$
b $5x + 2x + x^2 + 10$
c $2x^2 - 3 - x + 6x$
d $3x - 8x - 6x^2 + 4$
e $11x - 5a - 55 + ax$
f $12y + 2x - 8xy - 3$
g $6m - n + 3mn - 2$
h $15p - 8r - 5pr + 24$
i $16x - 3y - 8xy + 6$

5 What expanded expression factorises to the following?

a $(x - a)(x + 4)$
b $(x - c)(x - d)$
c $(x + y)(2 - z)$
d $(x - 1)(a + b)$
e $(3x - b)(c - b)$
f $(2x - y)(y + z)$
g $(3a + b)(2b + 5c)$
h $(m - 2x)(3y + z)$
i $(2p - 3q)(s + 2t)$

6 Note that $x^2 + 5x + 6 = x^2 + 2x + 3x + 6$, which can be factorised by grouping in pairs. Use a similar method to factorise the following.

a $x^2 + 7x + 10$
b $x^2 + 8x + 15$
c $x^2 + 10x + 24$
d $x^2 - x - 6$
e $x^2 + 4x - 12$
f $x^2 - 11x + 18$

REASONING	7	7	7, 8

7 $xa - 21 + 7a - 3x$ could be rearranged in two different ways before factorising.

Method 1

$$\begin{aligned} xa + 7a - 3x - 21 &= a(x + 7) - 3(\_\_\_\_\_\_) \\ &= \_\_\_\_\_\_\_\_\_\_\_\_ \end{aligned}$$

Method 2

$$\begin{aligned} xa - 3x + 7a - 21 &= x(a - 3) + 7(\_\_\_\_\_\_) \\ &= \_\_\_\_\_\_\_\_\_\_\_\_ \end{aligned}$$

a Copy and complete both methods for the above expression.

b Use different arrangements of the four terms to complete the factorisation of the following in two ways. Show working using both methods.

i $xb - 6 - 3b + 2x$
ii $xy - 8 + 2y - 4x$
iii $4m^2 - 15n + 6m - 10mn$
iv $2m + 3n - mn - 6$
v $4a - 6b^2 + 3b - 8ab$
vi $3ab - 4c - b + 12ac$

8 Make up at least three of your own four-term expressions that factorise to a binomial product. Describe the method that you used to make up each four-term expression.

ENRICHMENT: Grouping with more than four terms	–	–	9(½)

9 Factorise by grouping.

a $2(a - 3) - x(a - 3) - c(a - 3)$
b $b(2a + 1) + 5(2a + 1) - a(2a + 1)$
c $x(a + 1) - 4(a + 1) - ba - b$
d $3(a - b) - b(a - b) - 2a^2 + 2ab$
e $c(1 - a) - x + ax + 2 - 2a$
f $a(x - 2) + 2bx - 4b - x + 2$
g $a^2 - 3ac - 2ab + 6bc + 3abc - 9bc^2$
h $3x - 6xy - 5z + 10yz + y - 2y^2$
i $8z - 4y + 3x^2 + xy - 12x - 2xz$
j $-ab - 4cx + 3aby + 2abx + 2c - 6cy$

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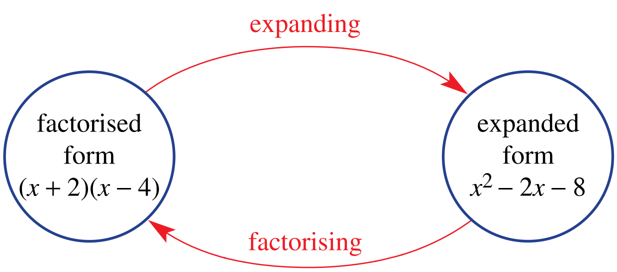
8F Factorising monic quadratic trinomials

Learning intentions
- To be able to identify a monic quadratic trinomial
- To understand the relationship between expanding brackets to form a trinomial and factorising a monic trinomial
- To know how to factorise a monic quadratic trinomial

Past, present and future learning
- This section addresses the Stage 5 Path Topic called *Algebraic techniques B*
- These concepts will be revised and extended in Chapter 5 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

An expression that takes the form $x^2 + bx + c$, where b and c are constants, is an example of a monic quadratic trinomial that has the coefficient of x^2 equal to 1. To factorise a quadratic expression, we need to use the distributive law in reverse. Consider the expansion shown below:



Quadratic equations are widely used in engineering, business and science, and factorisation is a key step when solving such equations. Self-driving cars are programmed to solve equations, including quadratic equations of motion.

If we examine the expansion above we can see how each term of the product is formed.

Product of x and x is x^2

$$(x + 2)(x - 4) = x^2 - 2x - 8$$

Product of 2 and -4 is -8
($2 \times (-4) = -8$, the constant term)

$x \times (-4) = -4x$

$$(x + 2)(x - 4) = x^2 - 2x - 8$$

$2 \times x = 2x$

Add $-4x$ and $2x$ to give the middle term, $-2x$
($-4 + 2 = -2$, the coefficient of x)

Lesson starter: So many choices

Mia says that as $-2 \times 3 = -6$ then $x^2 + 5x - 6$ must equal $(x - 2)(x + 3)$.

- Expand $(x - 2)(x + 3)$ to see if Mia is correct.
- What other pairs of numbers multiply to give -6?
- Which pair of numbers should Mia choose to correctly factorise $x^2 + 5x - 6$?
- What advice can you give Mia when trying to factorise these types of trinomials?

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KEY IDEAS

- To factorise a **quadratic trinomial** of the form $x^2 + bx + c$, find two numbers which:
  - multiply to give c and
  - add to give b.

For example, consider the expression $x^2 - 3x - 10$
Find two numbers that:
- multiply to give -10
- add to give -3.

- Use expansion to check your factorisation.

Choose -5 and $+2$ because
- $-5 \times 2 = -10$
- $-5 + 2 = -3$

- The answer can be written as $(x - 5)(x + 2)$ or $(x + 2)(x - 5)$.

$\therefore x^2 - 3x - 10 = (x - 5)(x + 2)$

- Use expansion to check your factorisation.

$$(x - 5)(x + 2) = x^2 - 3x - 10$$

- If c is a negative number, the signs in the factorised expression will be different from each other.

For example: $x^2 + 5x - 6 = (x + 6)(x - 1)$

c is negative

different signs

- If c is a positive number, the signs in the factorised expression will be the same as each other.

Example 1: $x^2 + 5x + 6 = (x + 2)(x + 3)$

c is positive

same signs

Example 2: $x^2 - 5x + 6 = (x - 2)(x - 3)$

BUILDING UNDERSTANDING

1 Give the result of expanding these binomial products.

a $(x + 2)(x + 7)$ **b** $(x + 3)(x + 11)$
c $(x + 12)(x - 5)$ **d** $(x - 2)(x - 6)$

2 Decide which two numbers multiply to give the first number and add to give the second number.

a 6, 5 **b** 12, 13 **c** 20, 9
d -15, 2 **e** -30, -1 **f** 18, -11

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Example 11 Factorising monic quadratic trinomials

Factorise each of the following quadratic expressions.

a $x^2 + 7x + 10$　　b $x^2 + 2x - 8$　　c $x^2 - 7x + 10$

SOLUTION	EXPLANATION
a $x^2 + 7x + 10 = (x + 5)(x + 2)$	Need two numbers that $\times$ to 10 and $+$ to 7. Factors of 10 include (10, 1) and (5, 2). The pair that adds to 7 is (5, 2).
b $x^2 + 2x - 8 = (x + 4)(x - 2)$	Need two numbers that $\times$ to -8 and $+$ to 2. Factors of -8 are $(-8, 1)$ or $(8, -1)$ or $(4, -2)$ or $(-4, 2)$ and $4 + (-2) = 2$, so choose $(4, -2)$.
c $x^2 - 7x + 10 = (x - 2)(x - 5)$	Need two numbers that $\times$ to 10 and $+$ to -7. Factors of 10 are (10, 1) or $(-10, -1)$ or (5, 2) or $(-5, -2)$. To add to a negative (-7), both factors must then be negative: $-5 + (-2) = -7$, so choose $(-5, -2)$.

Now you try

Factorise each of the following quadratic expressions.

a $x^2 + 9x + 14$　　b $x^2 + 4x - 12$　　c $x^2 - 7x + 12$

Example 12 Factorising with a common factor

Factorise the quadratic expression $2x^2 - 2x - 12$.

SOLUTION	EXPLANATION
$2x^2 - 2x - 12 = 2(x^2 - x - 6)$	First take out common factor of 2.
$= 2(x - 3)(x + 2)$	Factors of -6 are $(-6, 1)$ or $(6, -1)$ or $(-3, 2)$ or $(3, -2)$. $-3 + 2 = -1$ so choose $(-3, 2)$.

Now you try

Factorise the quadratic expression $3x^2 - 6x - 45$.

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Exercise 8F

FLUENCY	1–4(½)	1–5(⅓)	1–5(⅓)

Example 11a

1 Factorise each of the following quadratic expressions.

a $x^2 + 8x + 12$
b $x^2 + 10x + 9$
c $x^2 + 8x + 7$
d $x^2 + 15x + 14$
e $x^2 + 7x + 12$
f $x^2 + 10x + 16$
g $x^2 + 8x + 15$
h $x^2 + 9x + 20$
i $x^2 + 11x + 24$

Example 11b

2 Factorise each of the following quadratic expressions.

a $x^2 + 3x - 4$
b $x^2 + x - 2$
c $x^2 + 4x - 5$
d $x^2 + 5x - 14$
e $x^2 + 2x - 15$
f $x^2 + 8x - 20$
g $x^2 - 3x - 18$
h $x^2 - 7x - 18$
i $x^2 - x - 12$

Example 11c

3 Factorise each of the following quadratic expressions.

a $x^2 - 6x + 5$
b $x^2 - 2x + 1$
c $x^2 - 5x + 4$
d $x^2 - 9x + 8$
e $x^2 - 4x + 4$
f $x^2 - 8x + 12$
g $x^2 - 11x + 18$
h $x^2 - 10x + 21$
i $x^2 - 5x + 6$

4 Factorise each of the following quadratic expressions.

a $x^2 - 7x - 8$
b $x^2 - 3x - 4$
c $x^2 - 5x - 6$
d $x^2 - 6x - 16$
e $x^2 - 2x - 24$
f $x^2 - 2x - 15$
g $x^2 - x - 12$
h $x^2 - 11x - 12$
i $x^2 - 4x - 12$

Example 12

5 Factorise each of the following quadratic expressions by first taking out a common factor.

a $2x^2 + 10x + 8$
b $2x^2 + 22x + 20$
c $3x^2 + 18x + 24$
d $2x^2 + 14x - 60$
e $2x^2 - 14x - 36$
f $4x^2 - 8x + 4$
g $2x^2 + 2x - 12$
h $6x^2 - 30x - 36$
i $5x^2 - 30x + 40$
j $3x^2 - 33x + 90$
k $2x^2 - 6x - 20$
l $3x^2 - 3x - 36$

PROBLEM–SOLVING	6	6(½), 7	6(½), 7

6 Find the missing term in these trinomials if they are to factorise using positive integers. For example: the missing term in $x^2 + \square + 10$ could be $7x$ because $x^2 + 7x + 10$ factorises to $(x + 5)(x + 2)$ and 5 and 2 are integers. There may be more than one answer in each case.

a $x^2 + \square + 5$
b $x^2 - \square + 9$
c $x^2 - \square - 12$
d $x^2 + \square - 12$
e $x^2 + \square + 18$
f $x^2 - \square + 18$
g $x^2 - \square - 16$
h $x^2 + \square - 25$

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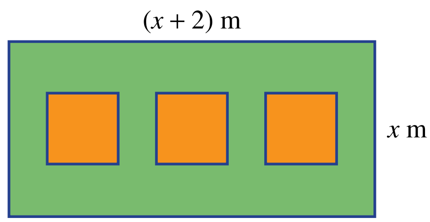
7 A rectangular backyard has a length 2 metres more than its breadth (x metres). Inside the rectangle are three paved squares each of area 5 m^2 as shown. The remaining area is lawn.



- **a** Find an expression for:
  - **i** the total backyard area
  - **ii** the area of lawn in expanded form
  - **iii** the area of lawn in factorised form.
- **b** Find the area of lawn if:
  - **i** $x = 10$
  - **ii** $x = 7$.

REASONING	8(½)	8(½)	8(½), 9

8 The expression $x^2 - 6x + 9$ factorises to $(x - 3)(x - 3) = (x - 3)^2$, which is a perfect square. Factorise these perfect squares. A common factor may also be involved.

- **a** $x^2 + 8x + 16$
- **b** $x^2 + 10x + 25$
- **c** $x^2 + 30x + 225$
- **d** $x^2 - 2x + 1$
- **e** $x^2 - 14x + 49$
- **f** $x^2 - 26x + 169$
- **g** $2x^2 + 4x + 2$
- **h** $5x^2 - 30x + 45$
- **i** $-3x^2 + 36x - 108$

9 Sometimes it is not possible to factorise quadratic trinomials using integers. Decide which of the following cannot be factorised using integers.

- **a** $x^2 - x - 56$
- **b** $x^2 + 5x - 4$
- **c** $x^2 + 7x - 6$
- **d** $x^2 + 3x - 108$
- **e** $x^2 + 3x - 1$
- **f** $x^2 + 12x - 53$

ENRICHMENT: Completing the square	–	–	10

10 It is useful to be able to write a simple quadratic trinomial in the form $(x + b)^2 + c$. This involves adding (and subtracting) a special number to form the first perfect square. This procedure is called completing the square. Here is an example.

$$\left(-\frac{6}{2}\right)^2 = 9$$

$$\begin{aligned} x^2 - 6x - 8 &= \underbrace{x^2 - 6x + 9}\ \underbrace{-\ 9 - 8} \\ &= (x - 3)(x - 3) - 17 \\ &= (x - 3)^2 - 17 \end{aligned}$$

Complete the square for these trinomials.

- **a** $x^2 - 2x - 8$
- **b** $x^2 + 4x - 1$
- **c** $x^2 + 10x + 3$
- **d** $x^2 - 16x - 3$
- **e** $x^2 + 18x + 7$
- **f** $x^2 - 32x - 11$

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8A **1** Expand the following.

- **a** $(x + 4)(x + 2)$
- **b** $(a - 5)(a + 8)$
- **c** $(2x - 3)(x + 6)$
- **d** $(3a - b)(a - 2b)$

8B **2** Expand each of the following perfect squares.

- **a** $(y + 4)^2$
- **b** $(x - 3)^2$
- **c** $(2a - 3)^2$
- **d** $(7k + 2m)^2$

8B **3** Expand and simplify the following.

- **a** $(x + 5)(x - 5)$
- **b** $(11x - 9y)(11x + 9y)$

8C **4** Factorise the following.

- **a** $25a - 15$
- **b** $x^2 - 7x$
- **c** $-12x^2 - 16x$
- **d** $2(a + 3) + a(a + 3)$
- **e** $7(8 + a) - a(8 + a)$
- **f** $k(k - 4) - (k - 4)$

8D **5** Factorise each of the following.

- **a** $x^2 - 81$
- **b** $16a^2 - 49$
- **c** $25x^2 - y^2$
- **d** $2a^2 - 50$
- **e** $12x^2y^2 - 12$
- **f** $(h + 3)^2 - 64$

8E **6** Use the method of grouping in pairs to factorise these expressions.

- **a** $x^2 + 7x + 2x + 14$
- **b** $a^2 + 5a - 4a - 20$
- **c** $x^2 - hx + x - h$

8E **7** Use grouping in pairs to factorise these expressions by first rearranging.

- **a** $2x^2 - 9 - 6x + 3x$
- **b** $3ap - 10 + 2p - 15a$

8F **8** Factorise these quadratic expressions.

- **a** $x^2 + 6x + 8$
- **b** $a^2 + 2a - 15$
- **c** $m^2 - 11m + 30$
- **d** $2k^2 + 2k - 24$

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8G Factorising non-monic quadratic trinomials

Learning intentions
- To understand the relationship between expansion and factorisation for binomial products
- To know and be able to apply the process for factorising non-monic quadratic trinomial

Past, present and future learning
- This section addresses the Stage 5 Path Topic called *Algebraic techniques C*
- These concepts will be revised and extended in Chapter 5 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

So far we have factorised quadratic trinomials in which the coefficient of x^2 is 1, such as $x^2 - 3x - 40$. These are called monic trinomials. We will now consider non-monic trinomials in which the coefficient of x is not equal to 1 and is also not a factor common to all three terms, such as in $6x^2 + x - 15$. The method used in this section uses grouping in pairs, which was discussed in **Section 8E**.

Flight simulators are programmed to replicate an aircraft's response to pilot input and weather conditions. Modelling plane flight includes quadratic equations, as the upward force on a plane's wing is proportional to the square of its airspeed.

Lesson starter: How the grouping in pairs method works

Consider the trinomial $2x^2 + 9x + 10$.

- First write $2x^2 + 9x + 10 = 2x^2 + 4x + 5x + 10$ then factorise by grouping in pairs.
- Note that $9x$ was split to give $4x + 5x$ and the product of $2x^2$ and 10 is $20x^2$. Describe the link between the pair of numbers $\{4, 5\}$ and the pair of numbers $\{2, 10\}$.
- Why was $9x$ split to give $4x + 5x$ and not, say, $3x + 6x$?
- Describe how the $13x$ should be split in $2x^2 + 13x + 15$ so it can be factorised by grouping in pairs.
- Now try your method for $2x^2 - 7x - 15$.

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KEY IDEAS

- To factorise a trinomial of the form $ax^2 + bx + c$, there are several different methods which can be used. Three methods are shown in this section.
- Method 1 involves grouping in pairs.
  For example:

$$5x^2 + 13x - 6$$
$$= 5x^2 - 2x + 15x - 6$$
$$= x(5x - 2) + 3(5x - 2)$$
$$\therefore 5x^2 + 13x - 6 = (5x - 2)(x + 3)$$
or
$$5x^2 + 13x - 6 = (x + 3)(5x - 2)$$

This expression has no common factors.
c is negative, so look for two numbers with different signs.
Multiply a and c to give -30. Factors of -30 are $(-1, 30)$, $(1, -30)$, $(-2, 15)$, $(2, -15)$ etc.
Look for two numbers that multiply to give -30 and add to give $+13$.
Those numbers are $(-2, 15)$.
Split the middle term into two terms.
Factorise the pairs and look for a common factor.
Take out the common factor. Check the answer by expanding.

- Method 2 involves the use of an algebraic fraction. For example:

$$5x^2 + 13x - 6$$
$$= \frac{(5x - 2)(5x + 15)}{5}$$
$$= \frac{(5x - 2)(\cancel{5}^{1}x + \cancel{15}^{3})}{\cancel{5}^{1}}$$
$$\therefore 5x^2 + 13x - 6 = (5x - 2)(x + 3)$$
$$\text{or } 5x^2 + 13x - 6 = (x + 3)(5x - 2)$$

This expression has no common factors.
c is negative, so the factors contain different signs.
Multiply a and c to give -30. Factors of -30 include $(-1, 30)$, $(1, -30)$, $(-2, 15)$, $(2, -15)$ etc.
Look for two numbers that multiply to give -30 and add to give $+13$.
Those numbers are $(-2, 15)$.
Set up a fraction and use the value of a in three places.
$\frac{(5x\quad)(5x\quad)}{5}$
Insert the two numbers, then try to simplify the fraction.
It should always be possible to simplify the denominator to 1.
It might require two steps.
Check the answer by expanding (same as method 1).

- Method 3 is often called the 'cross method'. It is explained in the Enrichment section at the end of Exercise 8G.

BUILDING UNDERSTANDING

1 State the two numbers that satisfy the given criteria.
- **a** Multiply to give 12 and add to give 8
- **b** Multiply to give -24 and add to give 5
- **c** Multiply to give 35 and add to give -12
- **d** Multiply to give -28 and add to give -3

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2 State the missing terms for these factorisations.

a $2x^2 + 7x + 5 = 2x^2 + 2x + \underline{\quad} + 5$
$= 2x(\underline{\qquad}) + 5(\underline{\qquad})$
$= (\underline{\qquad})(\underline{\qquad})$

b $2x^2 - 7x + 6 = 2x^2 - 3x - \underline{\quad} + 6$
$= x(\underline{\qquad}) - 2(\underline{\qquad})$
$= (\underline{\qquad})(\underline{\qquad})$

c $5x^2 + 9x - 2 = 5x^2 + 10x - \underline{\quad} - 2$
$= 5x(\underline{\qquad}) - 1(\underline{\qquad})$
$= (\underline{\qquad})(\underline{\qquad})$

d $4x^2 + 11x + 6 = 4x^2 + \underline{\quad} + 3x + 6$
$= (\underline{\quad})(x + 2) + 3(\underline{\qquad})$
$= (\underline{\qquad})(\underline{\qquad})$

Example 13 Factorising trinomials of the form $ax^2 + bx + c$

Factorise $2x^2 + 7x + 3$.

SOLUTION

$$\begin{aligned} 2x^2 + 7x + 3 &= 2x^2 + x + 6x + 3 \\ &= x(2x + 1) + 3(2x + 1) \\ &= (2x + 1)(x + 3) \end{aligned}$$

EXPLANATION

$a \times c = 2 \times 3 = 6$, then ask what factors of this number (6) add to 7. The answer is 1 and 6, so split $7x$ into $x + 6x$ and factorise by grouping in pairs.

Now you try

Factorise $3x^2 + 11x + 6$.

Example 14 Factorising trinomials involving negative numbers

Factorise these quadratic trinomials.

a $4x^2 + 5x - 6$

b $6x^2 - 17x + 12$

SOLUTION

a
$$\begin{aligned} 4x^2 + 5x - 6 &= 4x^2 + 8x - 3x - 6 \\ &= 4x(x + 2) - 3(x + 2) \\ &= (x + 2)(4x - 3) \end{aligned}$$

EXPLANATION

$4 \times (-6) = -24$, so ask what factors of -24 add to give 5. Choose 8 and -3. Then complete the factorisation by grouping in pairs.

SOLUTION

b
$$\begin{aligned} 6x^2 - 17x + 12 &= 6x^2 - 9x - 8x + 12 \\ &= 3x(2x - 3) - 4(2x - 3) \\ &= (2x - 3)(3x - 4) \end{aligned}$$

EXPLANATION

$6 \times 12 = 72$, so ask what factors of 72 add to give -17. Choose -9 and -8.
Complete a mental check.

$(2x - 3)(3x - 4)$
$-9x$
$-8x$ $\quad -9x - 8x = -17x$

Now you try

Factorise these quadratic trinomials.

a $4x^2 + 4x - 15$

b $8x^2 - 18x + 9$

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Exercise 8G

FLUENCY	1–2(½)	1–2(½)	1–2(⅓)

Example 13

1 Factorise these quadratic trinomials.

a $2x^2 + 9x + 4$ **b** $3x^2 + 7x + 2$ **c** $2x^2 + 7x + 6$
d $3x^2 + 8x + 4$ **e** $5x^2 + 12x + 4$ **f** $2x^2 + 11x + 12$
g $6x^2 + 13x + 5$ **h** $4x^2 + 5x + 1$ **i** $8x^2 + 14x + 5$

Example 14

2 Factorise these quadratic trinomials.

a $3x^2 + 2x - 5$ **b** $5x^2 + 6x - 8$ **c** $8x^2 + 10x - 3$
d $6x^2 - 13x - 8$ **e** $10x^2 - 3x - 4$ **f** $5x^2 - 11x - 12$
g $4x^2 - 16x + 15$ **h** $2x^2 - 15x + 18$ **i** $6x^2 - 19x + 10$
j $12x^2 - 13x - 4$ **k** $4x^2 - 12x + 9$ **l** $7x^2 + 18x - 9$
m $9x^2 + 44x - 5$ **n** $3x^2 - 14x + 16$ **o** $4x^2 - 4x - 15$

PROBLEM–SOLVING	3(½)	3–4(½)	3(½), 4(⅓), 5(½)

3 Factorise by first taking out a common factor.

a $30x^2 - 14x - 4$ **b** $12x^2 + 18x - 30$
c $27x^2 - 54x + 15$ **d** $21x^2 - 77x + 42$
e $36x^2 + 36x - 40$ **f** $50x^2 - 35x - 60$

4 Factorise these quadratic trinomials.

a $10x^2 + 27x + 11$ **b** $15x^2 + 14x - 8$
c $20x^2 - 36x + 9$ **d** $18x^2 - x - 5$
e $25x^2 + 5x - 12$ **f** $32x^2 - 12x - 5$
g $27x^2 + 6x - 8$ **h** $33x^2 + 41x + 10$
i $54x^2 - 39x - 5$ **j** $12x^2 - 32x + 21$
k $75x^2 - 43x + 6$ **l** $90x^2 + 33x - 8$

5 Factorise these trinomials.

a $-2x^2 + 7x - 6$ **b** $-5x^2 - 3x + 8$
c $-6x^2 + 13x + 8$ **d** $18 - 9x - 5x^2$
e $16x - 4x^2 - 15$ **f** $14x - 8x^2 - 5$

REASONING	6	6	6, 7

6 When splitting the $3x$ in $2x^2 + 3x - 20$, you could write:

A $2x^2 + 8x - 5x - 20$ or **B** $2x^2 - 5x + 8x - 20$.

a Complete the factorisation using **A**.
b Complete the factorisation using **B**.
c Does the order matter when you split the $3x$?
d Factorise these trinomials twice each. Factorise once by grouping in pairs then repeat but reverse the order of the two middle terms in the first line of working.

i $3x^2 + 5x - 12$ **ii** $5x^2 - 3x - 14$ **iii** $6x^2 + 5x - 4$

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7 Make up five non-monic trinomials in which the coefficient of x^2 is not equal to 1 and which factorise using the above method. Explain your method in finding these trinomials.

The stopping distance of a vehicle depends on its velocity and can be modelled by a quadratic equation.

ENRICHMENT: The cross method	–	–	8(⅓)

8 The cross method is another way to factorise trinomials of the form $ax^2 + bx + c$. It involves finding factors of ax^2 and factors of c then choosing pairs of these factors that add to bx.

For example: Factorise $6x^2 - x - 15$.

Factors of $6x^2$ include $(x, 6x)$ and $(2x, 3x)$.

Factors of -15 include $(15, -1)$, $(-15, 1)$, $(5, -3)$ and $(-5, 3)$.

We arrange a chosen pair of factors vertically then cross-multiply and add to get $-1x$.

$$\begin{array}{cc} x & 15 \\ 6x & -1 \end{array} \qquad \begin{array}{cc} x & 5 \\ 6x & -3 \end{array} \qquad \cdots\cdots \qquad \begin{array}{ccc} 2x & 3 & \longleftarrow \quad (2x+3) \\ 3x & -5 & \longleftarrow \quad (3x-5) \end{array}$$

$x \times (-1) + 6x \times 15$
$= 89x \neq -1x$

$x \times (-3) + 6x \times 5$
$= 27x \neq -1x$

$2x \times (-5) + 3x \times 3$
$= -1x$

You will need to continue until a particular combination works. The third cross-product gives a sum of $-1x$ so choose the factors $(2x + 3)$ and $(3x - 5)$, therefore:

$$6x^2 - x - 15 = (2x + 3)(3x - 5)$$

Try this method on the trinomials from Questions **2** and **4**.

Without air resistance, an object's path, subject to the Moon's gravity, can be modelled accurately using quadratic equations.

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8H Simplifying algebraic fractions: Multiplication and division

Learning intentions

- To know that expressions must be factorised before common factors can be cancelled
- To be able to simplify algebraic fractions by cancelling common factors
- To be able to multiply and divide algebraic fractions

Past, present and future learning

- This section addresses the Stage 5 Core and Path Topics called *Algebraic techniques B and C*
- These concepts will be revised and extended in Chapter 5 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

With a numerical fraction such as $\frac{6}{9}$, the highest common factor of 6 and 9 is 3, which can be cancelled: $\frac{6}{9} = \frac{{}^{1}\cancel{3} \times 2}{{}_{1}\cancel{3} \times 3} = \frac{2}{3}$. For algebraic fractions the process is the same. If expressions are in a factorised form, common factors can be easily identified and cancelled.

Lesson starter: Correct cancelling

Consider this cancelling attempt:

$$\frac{5x + \cancel{10}^{1}}{\cancel{20}_{2}} = \frac{5x + 1}{2}$$

- Substitute $x = 6$ into the left-hand side to evaluate $\frac{5x + 10}{20}$.
- Substitute $x = 6$ into the right-hand side to evaluate $\frac{5x + 1}{2}$.
- What do you notice about the two answers? How can you explain this?
- Decide how you might correctly cancel the expression on the left-hand side. Show your steps and check by substituting a value for x.

KEY IDEAS

- Simplify **algebraic fractions** by factorising and cancelling only common factors.

Incorrect	*Correct*
$\frac{2x + \cancel{4}^{2}}{\cancel{2}^{1}} = 2x + 2$	$\frac{2x + 4}{2} = \frac{{}^{1}\cancel{2}(x + 2)}{\cancel{2}^{1}} = x + 2$

- To multiply algebraic fractions:
  - factorise expressions where possible
  - cancel if possible
  - multiply the numerators and the denominators.

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- To divide algebraic fractions:
  - multiply by the reciprocal of the fraction following the division sign
  - follow the rules for multiplication after converting to the reciprocal.
    - The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.

BUILDING UNDERSTANDING

1 State the result after simplifying these fractions using cancelling.

a $\frac{5}{15}$ b $\frac{8x}{12}$ c $\frac{24}{8x}$ d $\frac{3(x+1)}{6}$

2 Factorise by taking out common factors.

a $3x + 6$ b $20 - 40x$ c $x^2 - 7x$ d $6x^2 + 24x$

3 State the missing terms.

a $\frac{2x-4}{8} = \frac{2(\boxed{})}{8} = \frac{\boxed{}}{4}$

b $\frac{12-18x}{2x-3x^2} = \frac{6(\boxed{})}{x(\boxed{})} = \frac{6}{x}$

4 Simplify:

a $\frac{6}{7} \times \frac{21}{2}$ b $\frac{5}{8} \div \frac{15}{2}$

Example 15 Simplifying algebraic fractions

Simplify the following by cancelling.

a $\frac{3(x+2)(x-4)}{6(x-4)}$ b $\frac{20-5x}{8-2x}$ c $\frac{x^2-4}{x+2}$

SOLUTION

a $\frac{{}^{1}\cancel{3}(x+2)\cancel{(x-4)}^{1}}{{}^{2}\cancel{6}\cancel{(x-4)}^{1}} = \frac{x+2}{2}$

b $\frac{20-5x}{8-2x} = \frac{5\cancel{(4-x)}^{1}}{2\cancel{(4-x)}^{1}}$
$= \frac{5}{2}$

c $\frac{x^2-4}{x+2} = \frac{{}^{1}\cancel{(x+2)}(x-2)}{\cancel{(x+2)}_{1}}$
$= x - 2$

EXPLANATION

a Cancel the common factors $(x - 4)$ and 3.

b Factorise the numerator and denominator then cancel common factor of $(4 - x)$.

c Factorise the difference of squares in the numerator then cancel the common factor.

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Now you try

Simplify the following by cancelling.

a $\dfrac{4(x+1)(x-3)}{12(x+1)}$ **b** $\dfrac{3-6x}{5-10x}$ **c** $\dfrac{x^2-9}{x+3}$

Example 16 Multiplying and dividing algebraic fractions

Simplify the following.

a $\dfrac{3(x-1)}{(x+2)} \times \dfrac{4(x+2)}{9(x-1)(x-7)}$

b $\dfrac{(x-3)(x+4)}{x(x+7)} \div \dfrac{3(x+4)}{x+7}$

c $\dfrac{x^2-4}{25} \times \dfrac{5x+5}{x^2-x-2}$

SOLUTION

a $\dfrac{{}^{1}\cancel{3}\cancel{(x-1)}^{1}}{\cancel{(x+2)}_{1}} \times \dfrac{4\cancel{(x+2)}^{1}}{{}_{3}\cancel{9}\cancel{(x-1)}_{1}(x-7)}$

$= \dfrac{1 \times 4}{1 \times 3(x-7)}$

$= \dfrac{4}{3(x-7)}$

b $\dfrac{(x-3)(x+4)}{x(x+7)} \div \dfrac{3(x+4)}{x+7}$

$= \dfrac{(x-3)\cancel{(x+4)}^{1}}{x\cancel{(x+7)}_{1}} \times \dfrac{\cancel{(x+7)}^{1}}{3\cancel{(x+4)}^{1}}$

$= \dfrac{x-3}{3x}$

c $\dfrac{x^2-4}{25} \times \dfrac{5x+5}{x^2-x-2}$

$= \dfrac{{}^{1}\cancel{(x-2)}(x+2)}{\cancel{25}^{5}} \times \dfrac{\cancel{5}^{1}\cancel{(x+1)}^{1}}{\cancel{(x-2)}\cancel{(x+1)}_{1}}$

$= \dfrac{x+2}{5}$

EXPLANATION

First, cancel any factors in the numerators with a common factor in the denominators. Then multiply the numerators and the denominators.

Multiply by the reciprocal of the fraction after the division sign.
Cancel common factors and multiply remaining numerators and denominators.

First factorise all the algebraic expressions.
Note that $x^2 - 4$ is a difference of two squares.
Then cancel as normal.

Now you try

Simplify the following.

a $\dfrac{2(x+1)}{(x-3)} \times \dfrac{9(x-3)}{10(x+1)(x-4)}$

b $\dfrac{4(x-2)(x+2)}{x(x-1)} \div \dfrac{2(x+2)}{x-1}$

c $\dfrac{x^2-5x+6}{18} \times \dfrac{6x+12}{x^2-4}$

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Exercise 8H

FLUENCY	1–3(½), 5–6(½)	1–6(½)	1–6(½)

Example 15a

1 Simplify the following by cancelling.

a $\frac{3(x+2)}{4(x+2)}$ b $\frac{x(x-3)}{3x(x-3)}$ c $\frac{20(x+7)}{5(x+7)}$

d $\frac{(x+5)(x-5)}{(x+5)}$ e $\frac{6(x-1)(x+3)}{9(x+3)}$ f $\frac{8(x-2)}{4(x-2)(x+4)}$

Example 15b

2 Simplify the following by factorising and then cancelling.

a $\frac{5x-5}{5}$ b $\frac{4x-12}{10}$ c $\frac{2x-4}{3x-6}$ d $\frac{12-4x}{6-2x}$

e $\frac{x^2-3x}{x}$ f $\frac{4x^2+10x}{5x}$ g $\frac{3x+3y}{2x+2y}$ h $\frac{4x-8y}{3x-6y}$

Example 15c

3 Simplify the following. These expressions involve difference of two squares.

a $\frac{x^2-100}{x+10}$ b $\frac{x^2-49}{x+7}$ c $\frac{x^2-25}{x+5}$

d $\frac{2(x-20)}{x^2-400}$ e $\frac{5(x-6)}{x^2-36}$ f $\frac{3x+27}{x^2-81}$

4 Simplify by first factorising the trinomials.

a $\frac{x^2-x-6}{x-3}$ b $\frac{x^2+8x+16}{x+4}$ c $\frac{x^2-7x+12}{x-4}$

d $\frac{x-2}{x^2+x-6}$ e $\frac{x+7}{x^2+5x-14}$ f $\frac{x-9}{x^2-19x+90}$

Example 16a

5 Simplify the following by cancelling.

a $\frac{2x(x-4)}{4(x+1)} \times \frac{(x+1)}{x}$ b $\frac{(x+2)(x-3)}{x-5} \times \frac{x-5}{x+2}$

c $\frac{x-3}{x+2} \times \frac{3(x+4)(x+2)}{x+4}$ d $\frac{2(x+3)(x+4)}{(x+1)(x-5)} \times \frac{(x+1)}{4(x+3)}$

Example 16b

6 Simplify the following by cancelling.

a $\frac{x(x+1)}{x+3} \div \frac{x+1}{x+3}$ b $\frac{x+3}{x+2} \div \frac{x+3}{2(x-2)}$

c $\frac{x-4}{(x+3)(x+1)} \div \frac{x-4}{4(x+3)}$ d $\frac{4x}{x+2} \div \frac{8x}{x-2}$

e $\frac{3(4x-9)(x+2)}{2(x+6)} \div \frac{9(x+4)(4x-9)}{4(x+2)(x+6)}$ f $\frac{5(2x-3)}{(x+7)} \div \frac{(x+2)(2x-3)}{x+7}$

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PROBLEM–SOLVING	7(½)	7(½)	7(½)

Example 16c

7 These expressions involve a combination of trinomials, difference of two squares and simple common factors. Simplify by first factorising where possible.

a $\dfrac{x^2+5x+6}{x+5} \div \dfrac{x+3}{x^2-25}$

b $\dfrac{x^2+6x+8}{x^2-9} \div \dfrac{x+4}{x-3}$

c $\dfrac{x^2+x-12}{x^2+8x+16} \times \dfrac{x^2-16}{x^2-8x+16}$

d $\dfrac{x^2+12x+35}{x^2-25} \times \dfrac{x^2-10x+25}{x^2+9x+14}$

e $\dfrac{9x^2-3x}{6x-45x^2}$

f $\dfrac{x^2-4x}{3x-x^2}$

g $\dfrac{3x^2-21x+36}{2x^2-32} \times \dfrac{2x+10}{6x-18}$

h $\dfrac{2x^2-18x+40}{x^2-x-12} \times \dfrac{3x+15}{4x^2-100}$

REASONING	8	8	8–9(½)

8 Just as $\dfrac{a^2}{2a}$ can be cancelled to $\dfrac{a}{2}$, $\dfrac{(a+5)^2}{2(a+5)}$ cancels to $\dfrac{a+5}{2}$. Use this idea to cancel these fractions.

a $\dfrac{(a+1)^2}{(a+1)}$

b $\dfrac{5(a-3)^2}{(a-3)}$

c $\dfrac{7(x+7)^2}{14(x+7)}$

d $\dfrac{3(x-1)(x+2)^2}{18(x-1)(x+2)}$

e $\dfrac{x^2+6x+9}{2x+6}$

f $\dfrac{11x-22}{x^2-4x+4}$

9 The expression $\dfrac{5-2x}{2x-5}$ can be written in the form $\dfrac{-1(-5+2x)}{2x-5} = \dfrac{-1(2x-5)}{2x-5}$, which can be cancelled to -1.

Use this idea to simplify these algebraic fractions.

a $\dfrac{7-3x}{3x-7}$

b $\dfrac{4x-1}{1-4x}$

c $\dfrac{8x+16}{-2-x}$

d $\dfrac{x+3}{-9-3x}$

e $\dfrac{5-3x}{18x-30}$

f $\dfrac{x^2-9}{3-x}$

ENRICHMENT: All in together	–	–	10(½)

10 Use your knowledge of factorisation and the ideas in Questions 8 and 9 above to simplify these algebraic fractions.

a $\dfrac{2x^2-2x-24}{16-4x}$

b $\dfrac{x^2-14x+49}{21-3x}$

c $\dfrac{x^2-16x+64}{64-x^2}$

d $\dfrac{4-x^2}{x^2+x-6} \times \dfrac{2x+6}{x^2+4x+4}$

e $\dfrac{2x^2-18}{x^2-6x+9} \times \dfrac{6-2x}{x^2+6x+9}$

f $\dfrac{x^2-2x+1}{4-4x} \div \dfrac{1-x^2}{3x^2+6x+3}$

g $\dfrac{4x^2-9}{x^2-5x} \div \dfrac{6-4x}{15-3x}$

h $\dfrac{x^2-4x+4}{8-4x} \times \dfrac{-2}{4-x^2}$

i $\dfrac{(x+2)^2-4}{(1-x)^2} \times \dfrac{x^2-2x+1}{3x+12}$

j $\dfrac{2(x-3)^2-50}{x^2-11x+24} \div \dfrac{x^2-4}{3-x}$

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8I Simplifying algebraic fractions: Addition and subtraction

Learning intentions
- To know the steps for adding and subtracting algebraic fractions
- To be able to find the lowest common denominator of fractions
- To be able to add and subtract algebraic fractions

Past, present and future learning
- This section addresses the Stage 5 Core and Path Topics called *Algebraic techniques B and C*
- These concepts will be revised and extended in Chapter 5 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

The process required for adding or subtracting algebraic fractions is similar to that used for fractions without pronumerals.

To simplify $\frac{2}{3} + \frac{4}{5}$, for example, you would find the lowest common multiple of the denominators (15) then express each fraction using this denominator. Adding the numerators completes the task. The same process is applied to $\frac{2x}{3} + \frac{4x}{5}$.

Financial analysts and economists use algebraic fractions when finding current amounts, such as a government's future income tax and future spending. Finding $\$P$ that compounds to $\$A$ at r% p.a. in n years, uses $A = P\left(1 + \frac{r}{100}\right)^n$.

Lesson starter: Compare the working

Here is the working for the simplification of the sum of a pair of numerical fractions and the sum of a pair of algebraic fractions.

$$\begin{aligned}\frac{2}{5} + \frac{3}{4} &= \frac{8}{20} + \frac{15}{20}\\ &= \frac{23}{20}\end{aligned} \qquad \begin{aligned}\frac{2x}{5} + \frac{3x}{4} &= \frac{8x}{20} + \frac{15x}{20}\\ &= \frac{23x}{20}\end{aligned}$$

- What type of steps were taken to simplify the algebraic fractions that are the same as for the numerical fractions?
- Write down the steps required to add (or subtract) algebraic fractions.

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KEY IDEAS

- To add or subtract algebraic fractions:
  - determine the lowest common denominator (LCD)
  - express each fraction using the LCD
  - add or subtract the numerators.

BUILDING UNDERSTANDING

1 Find the lowest common multiple of these pairs of numbers.

a 6, 8 **b** 3, 5 **c** 11, 13 **d** 12, 18

2 State the missing terms in the following expressions.

a $\dfrac{7x}{3} = \dfrac{\square}{9}$ **b** $\dfrac{3x+5}{11} = \dfrac{\square(3x+5)}{22}$

c $\dfrac{4}{x} = \dfrac{\square}{2x}$ **d** $\dfrac{30}{x+1} = \dfrac{\square}{3(x+1)}$

3 State the missing terms in these simplifications.

a $\dfrac{x}{4} + \dfrac{2x}{3} = \dfrac{\square}{12} + \dfrac{\square}{12} = \dfrac{\square}{12}$ **b** $\dfrac{5x}{7} - \dfrac{2x}{5} = \dfrac{\square}{35} - \dfrac{\square}{35} = \dfrac{\square}{35}$

c $\dfrac{x+1}{2} + \dfrac{2x+3}{4} = \dfrac{\square(x+1)}{4} + \dfrac{2x+3}{4} = \dfrac{\square + 2x + 3}{4} = \dfrac{\square}{4}$

4 State the LCD for these pairs of fractions.

a $\dfrac{x}{3}, \dfrac{2x}{5}$ **b** $\dfrac{3x}{7}, \dfrac{x}{2}$ **c** $\dfrac{2x}{3}, \dfrac{-5x}{6}$

Example 17 Adding and subtracting with numerals in the denominators

Simplify:

a $\dfrac{x}{4} - \dfrac{2x}{5}$ **b** $\dfrac{7x}{3} + \dfrac{x}{6}$ **c** $\dfrac{x+3}{2} + \dfrac{x-2}{5}$

SOLUTION

a $\dfrac{x}{4} - \dfrac{2x}{5} = \dfrac{5x}{20} - \dfrac{8x}{20}$

$= -\dfrac{3x}{20}$

EXPLANATION

Determine the LCD for 4 and 5, i.e. 20. Express each fraction as an equivalent fraction with a denominator of 20. Then subtract numerators.

Continued on next page

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b $\frac{7x}{3}+\frac{x}{6}=\frac{14x}{6}+\frac{x}{6}$
$=\frac{15x}{6}$
$=\frac{5x}{2}$

Note the LCD for 3 and 6 is 6 not $3 \times 6 = 18$.

Simplify $\frac{15}{6}$ to $\frac{5}{2}$ in the final step, dividing the numerator and denominator by 3.

c $\frac{x+3}{2}+\frac{x-2}{5}=\frac{5(x+3)}{10}+\frac{2(x-2)}{10}$
$=\frac{5x+15+2x-4}{10}$
$=\frac{7x+11}{10}$

The LCD for 2 and 5 is 10; write as equivalent fractions with denominator 10.
Expand the brackets and simplify the numerator by adding and collecting like terms.

Now you try

Simplify:

a $\frac{x}{3}-\frac{3x}{4}$

b $\frac{5x}{4}+\frac{x}{8}$

c $\frac{x+2}{3}+\frac{x-4}{2}$

Example 18 Adding and subtracting with algebraic terms in the denominators

Simplify:

a $\frac{2}{x}-\frac{5}{2x}$

b $\frac{2}{x}+\frac{3}{x^2}$

SOLUTION

a $\frac{2}{x}-\frac{5}{2x}=\frac{4}{2x}-\frac{5}{2x}$
$=-\frac{1}{2x}$

b $\frac{2}{x}+\frac{3}{x^2}=\frac{2x}{x^2}+\frac{3}{x^2}$
$=\frac{2x+3}{x^2}$

EXPLANATION

The LCD for x and $2x$ is $2x$, so rewrite the first fraction in an equivalent form with a denominator also of $2x$.

The LCD for x and x^2 is x^2, so rewrite the first fraction in an equivalent form so its denominator is also x^2, then add numerators.

Now you try

Simplify:

a $\frac{3}{x}-\frac{4}{3x}$

b $\frac{4}{x}+\frac{5}{x^2}$

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Exercise 8I

FLUENCY	1–4(½)	1–4(½)	1(¼), 2(⅓), 3–4(½)

Example 17a,b

1 Simplify:

a $\frac{x}{7}+\frac{x}{2}$ b $\frac{x}{3}+\frac{x}{15}$ c $\frac{x}{4}-\frac{x}{8}$ d $\frac{x}{9}+\frac{x}{5}$

e $\frac{y}{7}-\frac{y}{8}$ f $\frac{a}{2}+\frac{a}{11}$ g $\frac{b}{3}-\frac{b}{9}$ h $\frac{m}{3}-\frac{m}{6}$

i $\frac{m}{6}+\frac{3m}{4}$ j $\frac{a}{4}+\frac{2a}{7}$ k $\frac{2x}{5}+\frac{x}{10}$ l $\frac{p}{9}-\frac{3p}{7}$

m $\frac{b}{2}-\frac{7b}{9}$ n $\frac{9y}{8}+\frac{2y}{5}$ o $\frac{4x}{7}-\frac{x}{5}$ p $\frac{3x}{4}-\frac{x}{3}$

Example 17c

2 Simplify:

a $\frac{x+1}{2}+\frac{x+3}{5}$ b $\frac{x+3}{3}+\frac{x-4}{4}$ c $\frac{a-2}{7}+\frac{a-5}{8}$

d $\frac{y+4}{5}+\frac{y-3}{6}$ e $\frac{m-4}{8}+\frac{m+6}{5}$ f $\frac{x-2}{12}+\frac{x-3}{8}$

g $\frac{2b-3}{6}+\frac{b+2}{8}$ h $\frac{3x+8}{6}+\frac{2x-4}{3}$ i $\frac{2y-5}{7}+\frac{3y+2}{14}$

j $\frac{2t-1}{8}+\frac{t-2}{16}$ k $\frac{4-x}{3}+\frac{2-x}{7}$ l $\frac{2m-1}{4}+\frac{m-3}{6}$

Example 18a

3 Simplify:

a $\frac{3}{x}+\frac{5}{2x}$ b $\frac{7}{3x}-\frac{2}{x}$ c $\frac{7}{4x}-\frac{5}{2x}$ d $\frac{4}{3x}+\frac{2}{9x}$

e $\frac{3}{4x}-\frac{2}{5x}$ f $\frac{2}{3x}+\frac{1}{5x}$ g $\frac{-3}{4x}-\frac{7}{x}$ h $\frac{-5}{3x}-\frac{3}{4x}$

Example 18b

4 Simplify:

a $\frac{3}{x}+\frac{2}{x^2}$ b $\frac{5}{x^2}+\frac{4}{x}$ c $\frac{7}{x}+\frac{3}{x^2}$ d $\frac{4}{x}-\frac{5}{x^2}$

e $\frac{3}{x^2}-\frac{8}{x}$ f $-\frac{4}{x^2}+\frac{1}{x}$ g $\frac{3}{x}-\frac{7}{2x^2}$ h $-\frac{2}{3x}+\frac{3}{x^2}$

PROBLEM–SOLVING	5	5–6(½)	5(½), 6,7

5 Simplify these mixed algebraic fractions.

a $\frac{2}{x}+\frac{x}{4}$ b $\frac{-5}{x}+\frac{x}{2}$ c $\frac{-2}{x}-\frac{4x}{3}$ d $\frac{3}{2x}-\frac{5x}{4}$

e $\frac{3x}{4}-\frac{5}{6x}$ f $\frac{1}{3x}-\frac{x}{9}$ g $-\frac{2}{5x}+\frac{3x}{2}$ h $-\frac{5}{4x}-\frac{3x}{10}$

6 Find the missing algebraic fraction. The fraction should be in simplest form.

a $\frac{x}{2}+\frac{\square}{\square}=\frac{5x}{6}$ b $\frac{\square}{\square}+\frac{x}{4}=\frac{3x}{8}$

c $\frac{2x}{5}+\frac{\square}{\square}=\frac{9x}{10}$ d $\frac{2x}{3}-\frac{\square}{\square}=\frac{7x}{15}$

e $\frac{\square}{\square}-\frac{x}{3}=\frac{5x}{9}$ f $\frac{2x}{3}-\frac{\square}{\square}=\frac{5x}{12}$

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7 A hiker walks x metres up a hill at $\frac{6}{5}$ m/s and walks back down the same path to where they started at 2 m/s.

a Give a simplified expression for the total time in seconds of the walk in terms of x.

b Use your answer to part a to find the time taken if $x = 150$.

REASONING	8	8, 9	8, 9

8 Find and describe the error in each set of working. Then find the correct answer.

a $\frac{4x}{5} - \frac{x}{3} = \frac{3x}{2}$

b $\frac{x+1}{5} + \frac{x}{2} = \frac{2x+1}{10} + \frac{5x}{10}$
$= \frac{7x+1}{10}$

c $\frac{5x}{3} + \frac{x-1}{2} = \frac{10x}{6} + \frac{3x-1}{6}$
$= \frac{13x-1}{6}$

d $\frac{2}{x} - \frac{3}{x^2} = \frac{2}{x^2} - \frac{3}{x^2}$
$= \frac{-1}{x^2}$

9 A student thinks that the LCD to use when simplifying $\frac{x+1}{2} + \frac{2x-1}{4}$ is 8.

a Complete the simplification using a common denominator of 8.

b Now complete the simplification using the actual LCD of 4.

c How does your working for parts a and b compare? Which method is preferable and why?

ENRICHMENT: More than two fractions!	–	–	10(½)

10 Simplify by first finding the LCD.

a $\frac{2x}{5} - \frac{3x}{2} - \frac{x}{3}$

b $\frac{x}{4} - \frac{2x}{3} + \frac{5x}{6}$

c $\frac{5x}{8} - \frac{5x}{6} + \frac{3x}{4}$

d $\frac{x+1}{4} + \frac{2x-1}{3} - \frac{x}{5}$

e $\frac{2x-1}{3} - \frac{2x}{7} + \frac{x-3}{6}$

f $\frac{1-2x}{5} - \frac{3x}{8} + \frac{3x+1}{2}$

g $\frac{2}{3x} + \frac{5}{x} - \frac{1}{x}$

h $-\frac{1}{2x} + \frac{2}{x} - \frac{4}{3x}$

i $-\frac{4}{5x} - \frac{1}{2x} + \frac{3}{4x}$

j $\frac{4}{x^2} + \frac{3}{2x} - \frac{5}{3x}$

k $\frac{5}{x} - \frac{3}{2x^2} - \frac{5}{7x}$

l $\frac{2}{x^2} - \frac{4}{9x} - \frac{5}{3x^2}$

m $\frac{2}{x} + \frac{x}{5} - \frac{x}{3}$

n $\frac{3x}{2} - \frac{1}{2x} + \frac{x}{3}$

o $-\frac{4x}{9} + \frac{2}{5x} + \frac{2x}{5}$

Astronomers and astrophysicists use algebraic fractions when applying equations to calculate the speed of a star or galaxy. As a star moves towards or away from Earth, the wavelength of its light is changed, altering the colour. This is known as the Doppler effect.

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The following problems will investigate practical situations drawing upon knowledge and skills developed throughout the chapter. In attempting to solve these problems, aim to identify the key information, use diagrams, formulate ideas, apply strategies, make calculations and check and communicate your solutions.

Ice-cream profit

1 An ice-cream vendor has determined the profit equation for his business based on the selling price of his ice-cream cones. The price needs to be set high enough for the associated costs to be covered, but not too high such that sales start to decline.
The profit of the business, $\$P$, for an ice-cream selling price of $\$x$ is given by the equation $P = -120x^2 + 840x - 1200$.

With this knowledge, the vendor wants to analyse the rule to find which selling price leads to a profit and how he can set the selling price to maximise the profit of the business.

a Interpret the result from the rule for profit if the vendor gave the ice-cream cones away for free.

b Use the rule to determine the profit when:

i $x = 3$ **ii** $x = 4.5$

c Consider the results from part **b**. Describe what has happened to the company's profit as the ice-cream price increased from \$3 to \$4.50.

d By considering your results from part **b** and using increments of 0.5, find the selling price $\$x$ that gives the greatest profit. What is this profit?

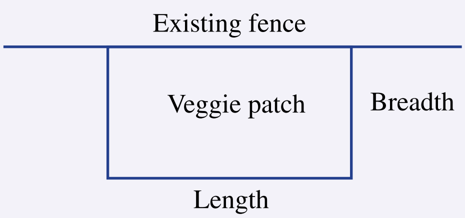
e The ice-cream prices are variable from day to day and the vendor will adjust to ensure that a profit is made each day.

i Factorise the rule for P by first removing the common factor.

ii Using your result from part **i**, consider which values of x would lead to a negative profit (i.e. a loss). Hence, between which values should the vendor maintain the ice-cream selling price to make a profit?

Veggie patch fencing

2 Farmer Jo is clearing an area of the farm for a new rectangular veggie patch. The area will need to be fenced off to keep some of the animals from trampling on it. Jo has 8 m of leftover chicken wire and some posts to use as fencing. She plans to use an existing fence on one side of the veggie patch as shown at right.



Jo is interested in looking at the different rectangular areas she can make with her 8 m of fencing material. In particular, she wishes to make the area for her veggie patch as large as possible.

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a For a veggie patch of breadth 1 m:
  i find the maximum possible length of the veggie patch
  ii determine the maximum area of the veggie patch.

b If the breadth of the veggie patch is 3.2 m, repeat steps **i** and **ii** from part **a**. What do you notice about the area of this veggie patch compared to the one in part **a** for the same 8 m perimeter?

c Let the breadth of the veggie patch be x m. Find expressions for the length and area of the veggie patch in terms of x.

d Use your area expression from part **c** to find the area of the veggie patch if $x = 1.5$.

e By considering the length and breadth dimensions of the veggie patch, what are the possible values that the breadth x m can take?

f Use a suitable method to predict the maximum possible area of the veggie patch and the dimensions that produce this area.

Tracking running progress

3 Oscar is starting a running program and is running for 48 minutes three times a week. In the first week he records that he walks a warm-up lap of a nearby oval at 2 km/h and jogs 6 laps at 4 km/h. Let d km be the distance around the oval.

Oscar is keen to analyse this data so that he can track his progress over time when he runs in different locations, and to set himself goals through the course of the program.

a Find an expression in terms of d for the time taken in hours to:
  i walk the warm-up lap **ii** jog the 6 laps.

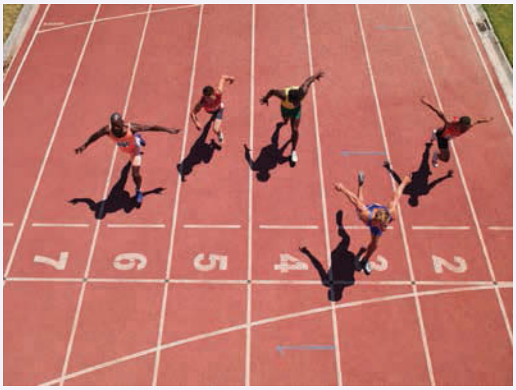
b Hence, find a simplified expression in terms of d for the total time taken.

c Given the warm-up lap and jog took 48 minutes, use your result from part **b** to help find the distance around the oval in metres.

As the training continues in week 2, Oscar increases his jogging speed to 5 km/h.

d He goes for a run on a different oval, completing the warm-up lap at 2 km/h and then jogging 8 laps at 5 km/h, all in 48 minutes. Repeat parts **a**–**c** to find the distance around this oval to the nearest metre.

e In week 3, Oscar returns to the first oval. The goal for the 48 minutes is to complete the regular warm-up lap and 8 laps jogging. Determine what his jogging speed in km/h will need to be to achieve this.



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8J Further addition and subtraction of algebraic fractions

Learning intentions
- To be able to simplify algebraic fractions with more complex numerators and denominators

Past, present and future learning
- This section addresses the Stage 5 Core and Path Topics called *Algebraic techniques B and C*
- These concepts will be revised and extended in Chapter 5 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

More complex addition and subtraction of algebraic fractions involves expressions such as the following:

$\frac{2x-1}{3} - \frac{x+4}{4}$ and $\frac{2}{x-3} - \frac{5}{(x-3)^2}$

In such examples, care needs to be taken at each step in the working to avoid common errors.

Electrical and biomedical engineers use algebraic fractions for many applications, including in the design of microcircuits and sensors for sophisticated medical equipment, such as MRI scanners.

Lesson starter: Three critical errors

The following simplification of algebraic fractions has three critical errors. Can you find them?

$$\begin{aligned}\frac{2x+1}{3} - \frac{x+2}{2} &= \frac{2x+1}{6} - \frac{3(x+2)}{6}\\ &= \frac{2x+1-3x+6}{6}\\ &= \frac{x+7}{6}\end{aligned}$$

The correct answer is $\frac{x-4}{6}$.

Fix the solution to obtain the correct answer.

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KEY IDEAS

- When combining algebraic fractions which involve subtraction signs, recall that:
  - the product of two numbers of opposite sign is a negative number
  - the product of two negative numbers is a positive number.

For example: $\frac{2(x-1)}{6} - \frac{3(x+2)}{6} = \frac{2(x-1) - 3(x+2)}{6}$

$= \frac{2x - 2 - 3x - 6}{6}$

and $\frac{5(1-x)}{8} - \frac{2(x-1)}{8} = \frac{5(1-x) - 2(x-1)}{8}$

$= \frac{5 - 5x - 2x + 2}{8}$

- A common denominator can be a product of two algebraic expressions.

For example: $\frac{2}{x+3} + \frac{3}{x-1} = \frac{2(x-1)}{(x+3)(x-1)} + \frac{3(x+3)}{(x+3)(x-1)}$

$= \frac{2x - 2 + 3x + 9}{(x+3)(x-1)}$

$= \frac{5x+7}{(x+3)(x-1)}$

BUILDING UNDERSTANDING

1. State the result of expanding the following.
   a $-2(x+3)$
   b $-7(2+3x)$
   c $-3(x-1)$
   d $-10(3-2x)$
2. State the LCD for these pairs of fractions.
   a $\frac{1}{3}, \frac{5}{9}$
   b $-\frac{5}{2x}, \frac{3}{2}$
   c $\frac{3}{x}, \frac{5}{x^2}$
   d $\frac{7}{x-2}, \frac{3}{x+3}$
3. Simplify:
   a $2(x+1) - 3(x+2)$
   b $5(x+2) - 2(x-3)$
   c $4 - 2(x-1)$

Problems that are solved using code often use formulas with algebraic fractions.

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Example 19 Simplifying with more complex numerators

Simplify:

a $\frac{x-1}{3} - \frac{x+4}{5}$

b $\frac{2x-3}{6} - \frac{3-x}{5}$

SOLUTION

a
$$\begin{aligned}\frac{x-1}{3} - \frac{x+4}{5} &= \frac{5(x-1)}{15} - \frac{3(x+4)}{15}\\ &= \frac{5x-5-3x-12}{15}\\ &= \frac{2x-17}{15}\end{aligned}$$

b
$$\begin{aligned}\frac{2x-3}{6} - \frac{3-x}{5} &= \frac{5(2x-3)}{30} - \frac{6(3-x)}{30}\\ &= \frac{10x-15-18+6x}{30}\\ &= \frac{16x-33}{30}\end{aligned}$$

EXPLANATION

a The LCD for 3 and 5 is 15. Insert brackets around each numerator when multiplying. (Note: $-3(x+4) = -3x - 12$ not $-3x + 12$.)

b Determine the LCD and express as equivalent fractions. Insert brackets.
Expand the brackets, recalling that $-6 \times (-x) = 6x$, and then simplify the numerator.

Now you try

Simplify:

a $\frac{x-4}{2} - \frac{x+2}{3}$

b $\frac{2x+1}{4} - \frac{2-x}{3}$

Example 20 Simplifying with more complex denominators

Simplify:

a $\frac{4}{x+1} + \frac{3}{x-2}$

b $\frac{3}{(x-1)^2} - \frac{2}{x-1}$

SOLUTION

a
$$\begin{aligned}&\frac{4}{x+1} + \frac{3}{x-2}\\ &= \frac{4(x-2)}{(x+1)(x-2)} + \frac{3(x+1)}{(x+1)(x-2)}\\ &= \frac{4x-8+3x+3}{(x+1)(x-2)}\\ &= \frac{7x-5}{(x+1)(x-2)}\end{aligned}$$

b
$$\begin{aligned}\frac{3}{(x-1)^2} - \frac{2}{x-1} &= \frac{3}{(x-1)^2} - \frac{2(x-1)}{(x-1)^2}\\ &= \frac{3-2x+2}{(x-1)^2}\\ &= \frac{5-2x}{(x-1)^2}\end{aligned}$$

EXPLANATION

a The LCD for $(x+1)$ and $(x-2)$ is $(x+1)(x-2)$. Rewrite each fraction as an equivalent fraction with this denominator, then add numerators.

b Just as the LCD for 3^2 and 3 is 3^2, the LCD for $(x-1)^2$ and $x-1$ is $(x-1)^2$.
Remember that $-2(x-1) = -2x + 2$.

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Now you try

Simplify:

a $\dfrac{3}{x+2}+\dfrac{2}{x-3}$

b $\dfrac{5}{(x-2)^2}-\dfrac{3}{x-2}$

Exercise 8J

FLUENCY	1–4(½)	1–4(½)	1–4(⅓)

Example 19a

1 Simplify:

a $\dfrac{x+3}{4}-\dfrac{x+2}{3}$
b $\dfrac{x-1}{3}-\dfrac{x+3}{5}$
c $\dfrac{x-4}{3}-\dfrac{x+1}{6}$
d $\dfrac{3-x}{5}-\dfrac{x+4}{2}$
e $\dfrac{5x-1}{4}-\dfrac{2+x}{8}$
f $\dfrac{3x+2}{14}-\dfrac{x+4}{4}$
g $\dfrac{1+3x}{4}-\dfrac{2x+3}{6}$
h $\dfrac{2-x}{5}-\dfrac{3x+1}{3}$
i $\dfrac{2x-3}{6}-\dfrac{4+x}{15}$

Example 19b

2 Simplify:

a $\dfrac{x+5}{3}-\dfrac{x-1}{2}$
b $\dfrac{x-4}{5}-\dfrac{x-6}{7}$
c $\dfrac{3x-7}{4}-\dfrac{x-1}{2}$
d $\dfrac{5x-9}{7}-\dfrac{2-x}{3}$
e $\dfrac{3x+2}{4}-\dfrac{5-x}{10}$
f $\dfrac{9-4x}{6}-\dfrac{2-x}{8}$
g $\dfrac{4x+3}{3}-\dfrac{5-2x}{9}$
h $\dfrac{2x-1}{4}-\dfrac{1-3x}{14}$
i $\dfrac{3x-2}{8}-\dfrac{4x-3}{7}$

Example 20a

3 Simplify:

a $\dfrac{3}{x-1}+\dfrac{4}{x+1}$
b $\dfrac{5}{x+4}+\dfrac{2}{x-3}$
c $\dfrac{3}{x-2}+\dfrac{4}{x+3}$
d $\dfrac{3}{x-4}+\dfrac{2}{x+7}$
e $\dfrac{7}{x+2}-\dfrac{3}{x+3}$
f $\dfrac{3}{x+4}-\dfrac{2}{x-6}$
g $\dfrac{-1}{x+5}+\dfrac{2}{x+1}$
h $\dfrac{-2}{x-3}-\dfrac{4}{x-2}$
i $\dfrac{3}{x-5}-\dfrac{5}{x-6}$

Example 20b

4 Simplify:

a $\dfrac{4}{(x+1)^2}-\dfrac{3}{x+1}$
b $\dfrac{2}{(x+3)^2}-\dfrac{4}{x+3}$
c $\dfrac{3}{x-2}+\dfrac{4}{(x-2)^2}$
d $\dfrac{-2}{x-5}+\dfrac{8}{(x-5)^2}$
e $\dfrac{-1}{x-6}+\dfrac{3}{(x-6)^2}$
f $\dfrac{2}{(x-4)^2}-\dfrac{3}{x-4}$
g $\dfrac{5}{(2x+1)^2}+\dfrac{2}{2x+1}$
h $\dfrac{9}{(3x+2)^2}-\dfrac{4}{3x+2}$
i $\dfrac{4}{(1-4x)^2}-\dfrac{5}{1-4x}$

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PROBLEM-SOLVING	5(½)	5–6(½)	5–6(½)

5 Simplify:

a $\frac{3x}{(x-1)^2}+\frac{2}{x-1}$ **b** $\frac{3x+2}{3x}+\frac{7}{12}$ **c** $\frac{2x-1}{4}+\frac{2-3x}{10x}$

d $\frac{2x}{x-5}-\frac{x}{x+1}$ **e** $\frac{3}{4-x}-\frac{2x}{x-1}$ **f** $\frac{5x+1}{(x-3)^2}+\frac{x}{x-3}$

g $\frac{3x-7}{(x-2)^2}-\frac{5}{x-2}$ **h** $\frac{-7x}{2x+1}+\frac{3x}{x+2}$ **i** $\frac{x}{x+1}-\frac{5x+1}{(x+1)^2}$

6 Simplify:

a $\frac{1}{(x+3)(x+4)}+\frac{2}{(x+4)(x+5)}$ **b** $\frac{3}{(x+1)(x+2)}-\frac{5}{(x+1)(x+4)}$

c $\frac{4}{(x-1)(x-3)}-\frac{6}{(x-1)(2-x)}$ **d** $\frac{5x}{(x+1)(x-5)}-\frac{2}{x-5}$

e $\frac{3}{x-4}+\frac{8x}{(x-4)(3-2x)}$ **f** $\frac{3x}{(x+4)(2x-1)}-\frac{x}{(x+4)(3x+2)}$

REASONING	7	7, 8(½)	8(½)

7 One of the most common errors made when subtracting algebraic fractions is hidden in this working shown below.

$$\begin{aligned}\frac{7x}{2}-\frac{x-2}{5}&=\frac{35x}{10}-\frac{2(x-2)}{10}\\&=\frac{35x-2x-4}{10}\\&=\frac{33x-4}{10}\end{aligned}$$

a What is the error and in which step is it made?

b By correcting the error how does the answer change?

8 Use the fact that $a-b=-1(b-a)$ to help simplify these.

a $\frac{3}{1-x}-\frac{2}{x-1}$ **b** $\frac{4x}{5-x}+\frac{3}{x-5}$ **c** $\frac{2}{7x-3}-\frac{7}{3-7x}$

d $\frac{1}{4-3x}+\frac{2x}{3x-4}$ **e** $\frac{-3x}{5-3x}-\frac{5}{3x-5}$ **f** $\frac{4}{x-6}+\frac{4}{6-x}$

ENRICHMENT: Factorise first	–	–	9(½)

9 Factorising a denominator before further simplification is a useful step. Simplify these by first factorising the denominators if possible.

a $\frac{3}{x+2}+\frac{5}{2x+4}$ **b** $\frac{7}{3x-3}-\frac{2}{x-1}$

c $\frac{3}{8x-4}-\frac{5}{1-2x}$ **d** $\frac{4}{x^2-9}-\frac{3}{x+3}$

e $\frac{5}{2x+4}+\frac{2}{x^2-4}$ **f** $\frac{10}{3x-4}-\frac{7}{9x^2-16}$

g $\frac{7}{x^2+7x+12}+\frac{2}{x^2-2x-15}$ **h** $\frac{3}{(x+1)^2-4}-\frac{2}{x^2+6x+9}$

i $\frac{3}{x^2-7x+10}-\frac{2}{10-5x}$ **j** $\frac{1}{x^2+x}-\frac{1}{x^2-x}$

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8K Equations involving algebraic fractions

Learning intentions
- To know how to solve equations with algebraic fractions by multiplying both sides by the lowest common denominator or by expressing the fractions with a common denominator

Past, present and future learning
- This section addresses the Stage 5 Path Topic called *Equations C*
- These concepts will be revised and extended in Chapter 5 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

For equations with more than one fraction, it is often best to try to simplify the equation by dealing with all the denominators at once. This involves finding and multiplying both sides by the lowest common denominator.


The study of optometry includes solving algebraic fractional equations that model the path of light through spectacle lenses.

Lesson starter: Why use the LCD?

For this equation follow each instruction.

$$\frac{x+1}{3} + \frac{x}{4} = 1$$

- Multiply every term in the equation by 3. What effect does this have on the fractions on the left-hand side?
- Starting with the original equation, multiply every term in the equation by 4. What effect does this have on the fractions on the left-hand side?
- Starting with the original equation, multiply every term in the equation by 12 and simplify.

Which instruction above does the best job in simplifying the algebraic fractions? Why?

KEY IDEAS

- For equations with more than one fraction, multiply both sides by the **lowest common denominator (LCD)**.
  - Multiply every term on both sides, not just the fractions.
  - Simplify the fractions and solve the equation using the methods learnt earlier.
- Alternatively, express each fraction using the same denominator then simplify by adding or subtracting.

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BUILDING UNDERSTANDING

1 State the lowest common denominator of all the fractions in each equation.

a $\frac{x}{3} - \frac{2x}{5} = 1$ **b** $\frac{x+1}{3} - \frac{x}{6} = 5$ **c** $\frac{2x-1}{7} + \frac{5x+2}{4} = 6$

2 Simplify the fractions by cancelling.

a $\frac{12x}{3}$ **b** $\frac{4(x+3)}{2}$ **c** $\frac{15x}{5x}$ **d** $\frac{36(x-7)(x-1)}{9(x-7)}$

3 Describe a method for solving the following equations.

a $\frac{2x+3}{5} = 3$ **b** $\frac{5}{x} = 2$

Example 21 Solving equations involving algebraic fractions

Solve each of the following equations.

a $\frac{2x}{3} + \frac{x}{2} = 7$ **b** $\frac{x-2}{5} - \frac{x-1}{3} = 1$ **c** $\frac{x-1}{3} = \frac{x}{4}$

SOLUTION / EXPLANATION

a

$$\begin{aligned} \frac{2x}{3} + \frac{x}{2} &= 7 \\ \frac{2x}{\cancel{3}_1} \times \cancel{6}^2 + \frac{x}{\cancel{2}_1} \times \cancel{6}^3 &= 7 \times 6 \\ 4x + 3x &= 42 \\ 7x &= 42 \\ x &= 6 \end{aligned}$$

Multiply each term by the LCD (LCD of 3 and 2 is 6) and cancel.
Simplify and solve for x.

$$\begin{aligned} \text{OR } \frac{2x}{3} + \frac{x}{2} &= 7 \\ \frac{4x}{6} + \frac{3x}{6} &= 7 \\ \frac{7x}{6} &= 7 \\ 7x &= 42 \\ x &= 6 \end{aligned}$$

Alternatively, write each fraction on the left-hand side using the LCD = 6.
Simplify by adding the numerators and solve the remaining equation.

b

$$\begin{aligned} \frac{x-2}{5} - \frac{x-1}{3} &= 1 \\ \frac{\cancel{15}^3(x-2)}{\cancel{5}_1} - \frac{\cancel{15}^5(x-1)}{\cancel{3}_1} &= 1 \times 15 \\ 3(x-2) - 5(x-1) &= 15 \\ 3x - 6 - 5x + 5 &= 15 \\ -2x - 1 &= 15 \\ -2x &= 16 \\ x &= -8 \end{aligned}$$

Multiply each term on both sides by 15 (LCD of 3 and 5 is 15) and cancel.
Expand the brackets and simplify by combining like terms. (Note: $-5(x-1) = -5x + 5$ not $-5x - 5$.)

Alternatively, write each fraction using the LCD = 15 then combine the numerators and solve $\frac{3(x-2)}{15} - \frac{5(x-1)}{15} = 1$.

Continued on next page

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c

$$\frac{x-1}{3} = \frac{x}{4}$$

Multiply both sides by 12 (LCD of 3 and 4 is 12) and cancel.

$$\frac{\cancel{12}^4(x-1)}{\cancel{3}_1} = \frac{\cancel{12}^3 x}{\cancel{4}_1}$$

Expand brackets and solve for x.

$$\begin{aligned} 4x - 4 &= 3x \\ x - 4 &= 0 \\ x &= 4 \end{aligned}$$

OR $\frac{x-1}{3} = \frac{x}{4}$

$$\begin{aligned} 4(x-1) &= 3x \\ 4x - 4 &= 3x \\ x &= 4 \end{aligned}$$

Alternatively, you can cross multiply since each side is a single fraction: $\frac{x-1}{3} \times \frac{x}{4}$.

Now you try

Solve each of the following equations.

a $\frac{x}{4} + \frac{2x}{3} = 11$ **b** $\frac{x+3}{5} - \frac{x-2}{2} = -2$ **c** $\frac{x+2}{5} = \frac{x}{3}$

Example 22 Solving equations that include algebraic denominators

Solve each of the following equations.

a $\frac{5}{2x} - \frac{4}{3x} = 2$ **b** $\frac{3}{x+1} = \frac{2}{x+4}$

SOLUTION

a

$$\frac{5}{2x} - \frac{4}{3x} = 2$$

LCD of $2x$ and $3x$ is $6x$.

$$\frac{5}{\cancel{2x}_1} \times \cancel{6x}^3 - \frac{4}{\cancel{3x}_1} \times \cancel{6x}^2 = 2 \times 6x$$

$$\begin{aligned} 15 - 8 &= 12x \\ 7 &= 12x \\ x &= \frac{7}{12} \end{aligned}$$

Multiply each term by $6x$. Cancel and simplify. Solve for x, leaving the answer in fraction form.

(Alternative solution: $\frac{15}{6x} - \frac{8}{6x} = 2$)

b

$$\frac{3}{x+1} = \frac{2}{x+4}$$

$$\frac{3\cancel{(x+1)}(x+4)}{\cancel{(x+1)}} = \frac{2(x+1)\cancel{(x+4)}}{\cancel{(x+4)}}$$

Multiply each term by the common denominator $(x + 1)(x + 4)$.

$$\begin{aligned} 3(x+4) &= 2(x+1) \\ 3x + 12 &= 2x + 2 \\ x + 12 &= 2 \\ x &= -10 \end{aligned}$$

Expand the brackets.
Subtract $2x$ from both sides to gather x terms on one side, then subtract 12 from both sides.

OR $\frac{3}{x+1} = \frac{2}{x+4}$

$$\begin{aligned} 3(x+4) &= 2(x+1) \\ 3x + 12 &= 2x + 2 \\ x + 12 &= 2 \\ x &= -10 \end{aligned}$$

Since each side is a single fraction you can 'cross-multiply': $\frac{3}{x+1} \times \frac{2}{x+4}$

This gives the same result as above.

EXPLANATION

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Now you try

Solve each of the following equations.

a $\dfrac{3}{4x} - \dfrac{2}{5x} = 2$ **b** $\dfrac{3}{x+2} = \dfrac{2}{x-1}$

Exercise 8K

FLUENCY	1–3(½)	1–4(⅓), 5(½)	1–5(⅓)

Example 21a

1 Solve each of the following equations.

a $\dfrac{x}{2} + \dfrac{x}{5} = 7$ **b** $\dfrac{x}{2} + \dfrac{x}{3} = 10$ **c** $\dfrac{y}{3} + \dfrac{y}{4} = 14$

d $\dfrac{x}{2} - \dfrac{3x}{5} = -1$ **e** $\dfrac{5m}{3} - \dfrac{m}{2} = 1$ **f** $\dfrac{3a}{5} - \dfrac{a}{3} = 2$

g $\dfrac{3x}{4} - \dfrac{5x}{2} = 14$ **h** $\dfrac{8a}{3} - \dfrac{2a}{5} = 34$ **i** $\dfrac{7b}{2} + \dfrac{b}{4} = 15$

Example 21b

2 Solve each of the following equations.

a $\dfrac{x-1}{2} + \dfrac{x+2}{3} = 11$ **b** $\dfrac{b+3}{2} + \dfrac{b-4}{3} = 1$ **c** $\dfrac{n+2}{3} + \dfrac{n-2}{2} = 1$

d $\dfrac{a+1}{5} - \dfrac{a+1}{6} = 2$ **e** $\dfrac{x+5}{2} - \dfrac{x-1}{4} = 3$ **f** $\dfrac{x+3}{2} - \dfrac{x+1}{3} = 2$

g $\dfrac{m+4}{3} - \dfrac{m-4}{4} = 3$ **h** $\dfrac{2a-8}{2} + \dfrac{a+7}{6} = 1$ **i** $\dfrac{2y-1}{4} - \dfrac{y-2}{6} = -1$

Example 21c

3 Solve each of the following equations.

a $\dfrac{x+1}{2} = \dfrac{x}{3}$ **b** $\dfrac{x-2}{3} = \dfrac{x}{2}$ **c** $\dfrac{n+3}{4} = \dfrac{n-1}{2}$

d $\dfrac{a+2}{3} = \dfrac{a+1}{2}$ **e** $\dfrac{3+y}{2} = \dfrac{2-y}{3}$ **f** $\dfrac{2m+4}{4} = \dfrac{m+6}{3}$

Example 22a

4 Solve each of the following equations.

a $\dfrac{3}{4x} - \dfrac{1}{2x} = 4$ **b** $\dfrac{2}{3x} - \dfrac{1}{2x} = 2$ **c** $\dfrac{4}{2m} - \dfrac{2}{5m} = 3$

d $\dfrac{1}{2x} - \dfrac{1}{4x} = 9$ **e** $\dfrac{1}{2b} + \dfrac{1}{b} = 2$ **f** $\dfrac{1}{2y} + \dfrac{1}{3y} = 4$

g $\dfrac{1}{3x} + \dfrac{1}{2x} = 2$ **h** $\dfrac{2}{3x} - \dfrac{1}{x} = 2$ **i** $\dfrac{7}{2a} - \dfrac{2}{3a} = 1$

Example 22b

5 Solve each of the following equations.

a $\dfrac{3}{x+1} = \dfrac{1}{x+2}$ **b** $\dfrac{2}{x+3} = \dfrac{3}{x+2}$

c $\dfrac{2}{x+5} = \dfrac{3}{x-2}$ **d** $\dfrac{1}{x-3} = \dfrac{1}{2x+1}$

e $\dfrac{2}{x-1} = \dfrac{1}{2x+1}$ **f** $\dfrac{1}{x-2} = \dfrac{2}{3x+2}$

PROBLEM–SOLVING	6, 7	6, 7, 8(½)	7, 8(½), 9

6 Half of a number (x) plus one-third of twice the same number is equal to 4.

a Write an equation describing the situation.

b Solve the equation to find the number.

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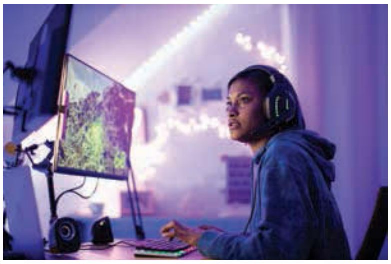
7 Molly and Billy each have the same number of computer games (x computer games each). Hazel, who doesn't have any games, takes one-third of Molly's computer games and a quarter of Billy's computer games to give her a total of 77 computer games.



a Write an equation describing the total number of computer games for Hazel.
b Solve the equation to find how many computer games Molly and Billy each had.

8 Use your combined knowledge of all the methods learnt earlier to solve these equations with algebraic fractions.

a $\dfrac{2x+3}{1-x}=4$
b $\dfrac{5x+2}{x+2}=3$
c $\dfrac{3x-2}{x-1}=2$
d $\dfrac{2x}{3}+\dfrac{x-1}{4}=2x-1$
e $\dfrac{3}{x^2}-\dfrac{2}{x}=\dfrac{5}{x}$
f $\dfrac{1-3x}{x^2}+\dfrac{3}{2x}=\dfrac{4}{x}$
g $\dfrac{x-1}{2}+\dfrac{3x-2}{4}=\dfrac{2x}{3}$
h $\dfrac{4x+1}{3}-\dfrac{x-3}{6}=\dfrac{x+5}{6}$
i $\dfrac{1}{x+2}-\dfrac{2}{x-3}=\dfrac{5}{(x+2)(x-3)}$
j $\dfrac{2}{x+4}-\dfrac{1}{x-1}=\dfrac{3}{(x+4)(x-1)}$

9 One number is twice another number and the sum of their reciprocals is $\frac{3}{10}$.
a Write an equation describing the situation.
b Solve the equation to find the two numbers.

REASONING	10, 11	10, 11	11, 12(½)

10 A common error when solving equations with algebraic fractions is made in this working. Find the error and explain how to avoid it.

$$\begin{aligned}\frac{3x-1}{4}+2x &= \frac{x}{3} \quad \text{(LCD = 12)}\\ \frac{12(3x-1)}{4}+2x &= \frac{12x}{3}\\ 3(3x-1)+2x &= 4x\\ 9x-3+2x &= 4x\\ 7x &= 3\\ x &= \frac{3}{7}\end{aligned}$$

11 Another common error is made in this working. Find and explain how to avoid this error.

$$\begin{aligned}\frac{x}{2}-\frac{2x-1}{3} &= 1 \quad \text{(LCD = 6)}\\ \frac{6x}{2}-\frac{6(2x-1)}{3} &= 6\\ 3x-2(2x-1) &= 6\\ 3x-4x-2 &= 6\\ -x &= 8\\ x &= -8\end{aligned}$$

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12 Some equations with decimals can be solved by first multiplying by a power of 10. Here is an example.

$$\begin{aligned} 0.8x - 1.2 &= 2.5 \quad \text{(Multiply both sides by 10 to remove all decimals.)} \\ 8x - 12 &= 25 \\ 8x &= 37 \\ x &= \frac{37}{8} \end{aligned}$$

Solve these decimal equations using the same idea. For parts **d–f** you will need to multiply by 100.

a $0.4x + 1.4 = 3.2$

b $0.3x - 1.3 = 0.4$

c $0.5 - 0.2x = 0.2$

d $1.31x - 1.8 = 2.13$

e $0.24x + 0.1 = 3.7$

f $2 - 3.25x = 8.5$

ENRICHMENT: Literal equations	–	–	13(½)

13 Solve each of the following equations for x in terms of the other pronumerals.
(*Hint:* You may need to use factorisation to make x the subject in your working.)

a $\frac{x}{a} - \frac{x}{2a} = b$

b $\frac{ax}{b} - \frac{cx}{2} = d$

c $\frac{x-a}{b} = \frac{x}{c}$

d $\frac{x+a}{b} = \frac{d+e}{c}$

e $\frac{ax+b}{4} = \frac{x+c}{3}$

f $\frac{x+a}{3b} + \frac{x-a}{2b} = 1$

g $\frac{2a-b}{a} + \frac{a}{x} = a$

h $\frac{1}{a} - \frac{1}{x} = \frac{1}{c}$

i $\frac{a}{x} = \frac{b}{c}$

j $\frac{a}{x} + b = \frac{c}{x}$

k $\frac{ax-b}{x-b} = c$

l $\frac{cx+b}{x+a} = d$

m $\frac{2a+x}{a} = b$

n $\frac{1}{x-a} = \frac{1}{ax+b}$

o $\frac{a}{b} - \frac{a}{a+x} = 1$

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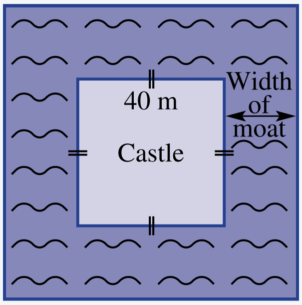
Working mathematically

Investing in art

Moats were constructed around castles to help protect its occupants from invaders. A square castle of side length 40 metres is to be surrounded by a square moat of a certain breadth as shown.



Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate.

Routine problems

a Find the area of land occupied by the 40-metre square castle.
b If the breadth of the moat is 10 metres, find the surface area of the water in the moat.
c Find the outside perimeter of the moat if the breadth of the moat is 8.5 metres.

Non-routine problems

Explore and connect

a The problem is to determine a breadth for the moat so that the surface area of water is at least 10 times the area of the castle. Write down all the relevant information that will help solve this problem with the aid of a diagram.
b If the breadth of the moat is x metres, find expressions in terms of x for the following:
  i the perimeter of the outside of the moat
  ii the total area occupied by the moat and the castle.

Choose and apply techniques

c Use your expressions from part **b ii** to show that the area occupied by the moat, not including the castle, can be written in the form $4x(40 + x)$.
d Using the expression from part **c**, find the area occupied by the moat for the following values of x.
  i 5 **ii** 8 **iii** 12
e Determine the smallest integer value of x such that the area of the moat is at least four times the area of the castle.

Communicate thinking and reasoning

f Assuming that the lord of the castle wants the area of the moat to be as close as possible to four times the area of the castle, investigate if this is possible using non-integer values of x.
g Instead, the lord wants to build a square castle of side length x metres inside a square lake of side length 50 metres.
  i Show that the remaining lake area forming the moat is given by the expression $(50 + x)(50 - x)$.
  ii Use a trial and error approach to determine the breadth of the castle that delivers a moat area which is four times the area of the castle. Give your answer correct to one decimal place.
h Summarise your results and describe any key findings.

Extension problems

A square castle of breadth a metres is surrounded by a moat of breadth x metres as per the castle in the introduction of this task.

Problem solve

a Find an expression for the area of the moat in terms of a and x.
b Find the value of a such that a moat of breadth 10 metres occupies four times the area of the castle.
Give your answer correct to one decimal place. Illustrate the solution with a drawing.

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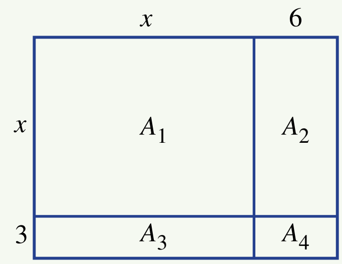
Investigation

Expanding quadratics using areas

Consider the expansion of the quadratic $(x + 3)(x + 6)$. This can be represented by finding the area of the rectangle shown.



$$\begin{aligned}\text{Total area} &= A_1 + A_2 + A_3 + A_4\\ &= x^2 + 6x + 3x + 18\end{aligned}$$

Therefore:

$(x + 3)(x + 6) = x^2 + 9x + 18$

Expanding with positive signs

a Draw a diagram and calculate the area to determine the expansion of the following quadratics.

i $(x + 4)(x + 5)$ **ii** $(x + 7)(x + 8)$

iii $(x + 3)^2$ **iv** $(x + 5)^2$

b Using the same technique establish the rule for expanding $(a + b)^2$.

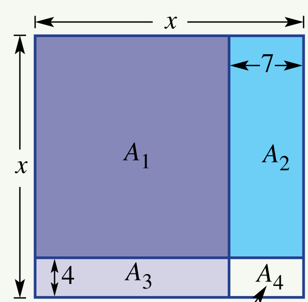
Expanding with negative signs

Consider the expansion of $(x - 4)(x - 7)$.

$$\begin{aligned}\text{Area required} &= \text{total area} - (A_2 + A_3 + A_4)\\ &= x^2 - [(A_2 + A_4) + (A_3 + A_4) - A_4]\\ &= x^2 - (7x + 4x - 28)\\ &= x^2 - 11x + 28\end{aligned}$$

Therefore:

$(x - 4)(x - 7) = x^2 - 11x + 28$



This area is counted twice when we add $7x + 4x$.

a Draw a diagram and calculate the area to determine the expansion of the following quadratics.

i $(x - 3)(x - 5)$

ii $(x - 6)(x - 4)$

iii $(x - 4)^2$

iv $(x - 2)^2$

b Using the same technique, establish the rule for expanding $(a - b)^2$.

Difference of two squares

Using a diagram to represent $(a - b)(a + b)$, determine the appropriate area and establish a rule for the expansion of $(a - b)(a + b)$.

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Problems and challenges

1 a The difference between the squares of two consecutive numbers is 97. What are the two numbers?

b The difference between the squares of two consecutive odd numbers is 136. What are the two numbers?

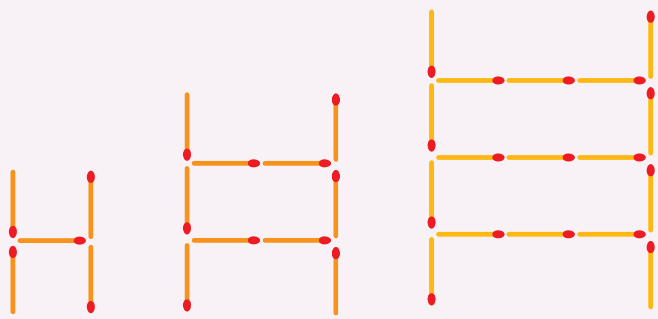
c The difference between the squares of two consecutive multiples of 3 is 81. What are the two numbers?

2 a If $x^2 + y^2 = 6$ and $(x + y)^2 = 36$, find the value of xy.

b If $x + y = 10$ and $xy = 2$, find the value of $\frac{1}{x} + \frac{1}{y}$.

3 Find the values of the different digits a, b, c and d if the four-digit number $abcd$ is such that $abcd \times 4 = dcba$.

4 a Find the quadratic rule that relates the width, n, to the number of matches in the pattern below.



b Draw a possible pattern for these rules.

i $n^2 + 3$

ii $n(n - 1)$

5 Factorise $n^2 - 1$ and use the factorised form to explain why when n is prime and greater than 3, $n^2 - 1$ is:

a divisible by 4

b divisible by 3

c thus divisible by 12.

6 Prove that this expression is equal to 1.

$$\frac{2x^2 - 8}{5x^2 - 5} \div \frac{x - 2}{5x - 5} \div \frac{2x^2 - 10x - 28}{x^2 - 6x - 7}$$

7 Prove that $4x^2 - 4x + 1 \geqslant 0$ for all x.

8 Ryan and Susan competed in a race over 4 km. Ryan ran at a constant speed. Sophie, however, ran the first 2 km at a speed 1 km/h more than Ryan and ran the second 2 km at a speed 1 km/h less than Ryan. Who won the race?

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Chapter summary

Algebraic techniques

Monic quadratic trinomials

These are of the form:

$x^2 + bx + c$

Require two numbers that multiply to c and add to b

e.g. $x^2 - 7x - 18 = (x - 9)(x + 2)$

since $-9 \times 2 = -18$

and $-9 + 2 = -7$

Difference of two squares

$a^2 - b^2 = (a - b)(a + b)$

e.g. $x^2 - 16 = x^2 - 4^2$

$= (x - 4)(x + 4)$

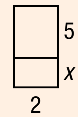
e.g. $4x^2 - 9 = (2x)^2 - 3^2$

$= (2x - 3)(2x + 3)$

Expansion

The process of removing brackets.

e.g. $2(x + 5) = 2x + 10$



$(x + 3)(2x - 4) = x(2x - 4) + 3(2x - 4)$

$= 2x^2 - 4x + 6x - 12$

$= 2x^2 + 2x - 12$

Special cases

Difference of two squares

$(a - b)(a + b) = a^2 - b^2$

e.g. $(x - 5)(x + 5) = x^2 + \cancel{5x} - \cancel{5x} - 25$

$= x^2 - 25$

Perfect squares

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

e.g. $(2x + 3)^2 = (2x + 3)(2x + 3)$

$= 4x^2 + 6x + 6x + 9$

$= 4x^2 + 12x + 9$

Factorisation

The reverse process of expansion. Always remove the highest common factor first.

e.g. $2ab + 8b$ HCF $= 2b$

$= 2b(a + 4)$

e.g. $3(x + 2) + y(x + 2)$ HCF $= x + 2$

$= (x + 2)(3 + y)$

By grouping in pairs

If there are four terms, it may be possible to group into two binomial terms.

e.g. $5x + 10 - 3x^2 - 6x$

$= 5(x + 2) - 3x(x + 2)$

$= (x + 2)(5 - 3x)$

Trinomials of the form $ax^2 + bx + c$

Can be factorised using grouping also:

e.g. $3x^2 + 7x - 6$ $3 \times (-6) = -18$

$= 3x^2 + 9x - 2x - 6$ Use 9 and -2

$= 3x(x + 3) - 2(x + 3)$ since

$= (x + 3)(3x - 2)$ $9 \times (-2) = -18$

and $9 + (-2) = 7$

Add/subtract

Find the lowest common denominator (LCD) before applying operation.

e.g. $\frac{3x}{4} + \frac{x - 1}{3}$

$= \frac{9x}{12} + \frac{4(x - 1)}{12}$

$= \frac{9x + 4x - 4}{12}$

$= \frac{13x - 4}{12}$

Algebraic fractions

These involve algebraic expressions in the numerator and/or denominator.

e.g. $\frac{x}{3}, \frac{2x}{x - 7}, \frac{3}{5x}, \frac{7}{(x - 1)^2}$

Multiply/divide

To divide, multiply by the reciprocal. Factorise all expressions, cancel and then multiply.

e.g. $\frac{3}{x^2 - 9} \div \frac{15}{2x + 6}$

$= \frac{\cancel{3}^{1}}{(x - 3)\cancel{(x + 3)}_{1}} \times \frac{\cancel{2(x + 3)}^{1}}{\cancel{15}_{5}}$

$= \frac{2}{5(x - 3)}$

Solving equations with algebraic fractions

Find the lowest common denominator (LCD) and multiply every term by the LCD or express each fraction using the LCD.

e.g. LCD of $2x$ and $3x$ is $6x$

LCD of $(x - 1)$ and $(x + 3)$ is $(x - 1)(x + 3)$

LCD of $2x$ and x^2 is $2x^2$

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Chapter checklist

Chapter checklist with success criteria

A printable version of this checklist is available in the Interactive Textbook

		✔
8A	**1. I can expand binomial products.** e.g. Expand $(2x - 3)(x + 4)$.	☐
8B	**2. I can expand a perfect square and form a difference of two squares.** e.g. Expand $(x - 6)^2$ and $(4x - 5)(4x + 5)$.	☐
8C	**3. I can factorise using the HCF.** e.g. Factorise fully $12x^2 - 8x$ and $x(2x + 3) + 5(2x + 3)$.	☐
8D	**4. I can factorise a difference of two squares.** e.g. Factorise $x^2 - 100$ and $4a^2 - 9$.	☐
8E	**5. I can factorise by grouping in pairs.** e.g. Use the method of grouping in pairs to factorise $x^2 + 6x - 3x - 18$.	☐
8E	**6. I can factorise using grouping in pairs by first rearranging.** e.g. Factorise $2x^2 - 9 - 6x + 3x$.	☐
8F	**7. I can factorise a quadratic trinomial.** e.g. Factorise $x^2 + 4x - 21$.	☐
8D/F	**8. I can factorise with trinomials or difference of two squares by first taking out a common factor.** e.g. Fully factorise $2x^2 - 18$ and $3x^2 - 21x + 36$.	☐
8G	**9. I can factorise trinomials of the form $ax^2 + bx + c$.** e.g. Factorise $3x^2 + 10x + 9$ and $4x^2 - 4x - 3$.	☐
8H	**10. I can simplify algebraic fractions by factorising and cancelling.** e.g. Simplify $\frac{3x + 6}{8x + 16}$.	☐
8H	**11. I can multiply and divide algebraic fractions by first cancelling.** e.g. Simplify $\frac{2(x + 5)}{x + 4} \times \frac{5(x + 4)(x - 1)}{6(x + 5)}$.	☐
8H	**12. I can multiply and divide algebraic fractions by factorising.** e.g. Simplify $\frac{x^2 - 16}{x + 4} \div \frac{6x - 24}{3}$.	☐
8I	**13. I can add and subtract algebraic fractions with numeric denominators.** e.g. Simplify $\frac{x + 4}{2} + \frac{x - 2}{3}$.	☐
8I	**14. I can add and subtract algebraic fractions with algebraic terms in the denominator.** e.g. Simplify $\frac{4}{x} - \frac{2}{3x}$.	☐
8J	**15. I can subtract algebraic fractions.** e.g. Simplify $\frac{2x + 2}{3} - \frac{x - 5}{4}$.	☐
8J	**16. I can add or subtract algebraic fractions with binomial denominators.** e.g. Simplify $\frac{3}{x + 2} + \frac{2}{x - 4}$.	☐
8K	**17. I can solve equations involving algebraic fractions.** e.g. Solve $\frac{3x}{2} + \frac{x}{5} = 3$ and $\frac{2}{x + 1} = \frac{6}{x - 5}$.	☐

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Chapter review

Short-answer questions

8A

1 Expand the following binomial products.

a $(x-3)(x+4)$
b $(x-7)(x-2)$
c $(2x-3)(3x+2)$
d $3(x-1)(3x+4)$

8B

2 Expand the following.

a $(x+3)^2$
b $(x-4)^2$
c $(3x-2)^2$
d $(x-5)(x+5)$
e $(7-x)(7+x)$
f $(11x-4)(11x+4)$

8C

3 Write the following in fully factorised form by removing the highest common factor.

a $4a+12b$
b $6x-9x^2$
c $-5x^2y-10xy$
d $3(x-7)+x(x-7)$
e $x(2x+1)-(2x+1)$
f $(x-2)^2-4(x-2)$

8D

4 Factorise the following difference of two squares.

a x^2-100
b $3x^2-48$
c $25x^2-y^2$
d $49-9x^2$
e $(x-3)^2-81$
f $1-x^2$

8E

5 Factorise the following by grouping in pairs.

a $x^2-3x+6x-18$
b $4x^2+10x-2x-5$
c $3x-8b+2bx-12$

8F/G

6 Factorise the following trinomials.

a $x^2+8x+15$
b $x^2-3x-18$
c x^2-7x+6
d $3x^2+15x-42$
e $2x^2+16x+32$
f $5x^2+17x+6$
g $4x^2-4x-3$
h $6x^2-17x+12$

8H

7 Simplify the following.

a $\dfrac{3x+12}{3}$
b $\dfrac{2x-16}{3x-24}$
c $\dfrac{x^2-9}{5(x+3)}$

8H

8 Simplify the following algebraic fractions by first factorising and cancelling where possible.

a $\dfrac{3}{2x}\times\dfrac{x}{6}$
b $\dfrac{x(x-4)}{8(x+1)}\times\dfrac{4(x+1)}{x}$
c $\dfrac{x^2+3x}{3x+6}\times\dfrac{x+2}{x+3}$
d $\dfrac{2x}{5x+20}\div\dfrac{x}{x+4}$
e $\dfrac{4x^2-9}{10x^2}\div\dfrac{10x-15}{x}$
f $\dfrac{x^2+5x+6}{x+3}\div\dfrac{x^2-4}{4x-8}$

8I/J

9 Simplify the following by first finding the lowest common denominator.

a $\dfrac{x}{4}+\dfrac{2x}{3}$
b $\dfrac{3}{4x}+\dfrac{1}{2x}$
c $\dfrac{7}{x}-\dfrac{2}{x^2}$
d $\dfrac{x-1}{6}-\dfrac{x+3}{8}$
e $\dfrac{3}{x+1}+\dfrac{5}{x+2}$
f $\dfrac{7}{(x-4)^2}-\dfrac{2}{x-4}$

8K

10 Solve the following equations involving fractions.

a $\dfrac{x}{4}+\dfrac{2x}{5}=13$
b $\dfrac{4}{x}-\dfrac{2}{3x}=20$
c $\dfrac{x+3}{2}+\dfrac{x-4}{3}=6$
d $\dfrac{4}{1-x}=\dfrac{5}{x+4}$

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Chapter review

Multiple-choice questions

8A **1** $(2x-1)(x+5)$ in expanded and simplified form is:

A $2x^2+9x-5$ **B** $x^2+11x-5$ **C** $4x^2-5$

D $3x^2-2x+5$ **E** $2x^2+4x-5$

8B **2** $(3a+2b)^2$ is equivalent to:

A $9a^2+6ab+4b^2$ **B** $9a^2+4b^2$ **C** $3a^2+6ab+2b^2$

D $3a^2+12ab+2b^2$ **E** $9a^2+12ab+4b^2$

8D **3** $16x^2-49$ in factorised form is:

A $(4x-7)^2$ **B** $(16x-49)(16x+49)$ **C** $(2x-7)(8x+7)$

D $(4x-7)(4x+7)$ **E** $4(4x^2-49)$

8E **4** The factorised form of $x^2+3x-2x-6$ is:

A $(x-3)(x+2)$ **B** $x-2(x+3)^2$ **C** $(x+3)(x-2)$

D $x-2(x+3)$ **E** $x(x+3)-2$

8F **5** If $(x-2)$ is a factor of $x^2+5x-14$ the other factor is:

A x **B** $x+7$ **C** $x-7$ **D** $x-16$ **E** $x+5$

8G **6** The factorised form of $3x^2+10x-8$ is:

A $(3x+1)(x-8)$

B $(x-4)(3x+2)$

C $(3x+2)(x+5)$

D $(3x-2)(x+4)$

E $(x+1)(3x-8)$

8H **7** The simplified form of $\dfrac{3x+6}{(x+5)(x+1)}\times\dfrac{x+5}{x+2}$ is:

A $\dfrac{3}{x+1}$ **B** $\dfrac{15}{2x^2+5}$ **C** $x+3$ **D** $\dfrac{5}{x+2}$ **E** $3(x+5)$

8I **8** $\dfrac{x+2}{5}+\dfrac{2x-1}{3}$ written as a single fraction is:

A $\dfrac{11x+1}{15}$ **B** $\dfrac{11x+9}{8}$ **C** $\dfrac{3x+1}{8}$ **D** $\dfrac{11x+7}{15}$ **E** $\dfrac{13x+1}{15}$

8J **9** The LCD of $\dfrac{3x+1}{2x}$ and $\dfrac{4}{x+1}$ is:

A $8x$ **B** $2x(x+1)$ **C** $(x+1)(3x+1)$

D $8x(3x+1)$ **E** $x(x+1)$

8K **10** The solution to $\dfrac{3}{1-x}=\dfrac{4}{2x+3}$ is:

A $x=-\frac{1}{2}$ **B** $x=-\frac{9}{11}$ **C** $x=\frac{1}{7}$ **D** $x=2$ **E** $x=-\frac{4}{5}$

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Extended-response questions

1 A pig pen for a small farm is being redesigned. It is originally a square of side length x m.

a In the planning, the length is initially kept as x m and the breadth altered such that the area of the pen is $(x^2 + 3x)$ square metres. What is the new breadth?

b Instead, it is determined that the original length will be increased by 1 metre and the original breadth will be decreased by 1 metre.

i What effect does this have on the perimeter of the pig pen compared with the original size?

ii Determine an expression in expanded form for the new area of the pig pen. How does this compare with the original area?

c The final set of dimensions requires an extra 8 m of fencing to go around the pen compared with the original pen. If the length of the pen has been increased by 7 m, then the breadth of the pen must decrease. Find:

i the change that has been made to the breadth of the pen

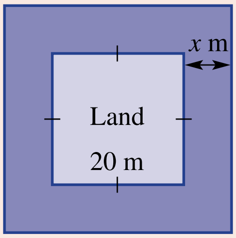
ii the new area enclosed by the pen

iii what happens when $x = 3$.

2 The security tower for a palace is on a small square piece of land 20 m by 20 m with a moat of breadth x metres the whole way around it as shown.

a State the area of the piece of land.

b **i** Give expressions for the length and the breadth of the combined moat and land.



ii Find an expression, in expanded form, for the entire area occupied by the moat and the land.

c If the tower occupies an area of $(x + 10)^2$ m^2, what fraction of the total area in part **b ii** is this?

d Use your answers to parts **a** and **b** to give an expression for the area occupied by the moat alone, in factorised form.

e Use trial and error to find the value of x such that the area of the moat alone is 500 m^2.

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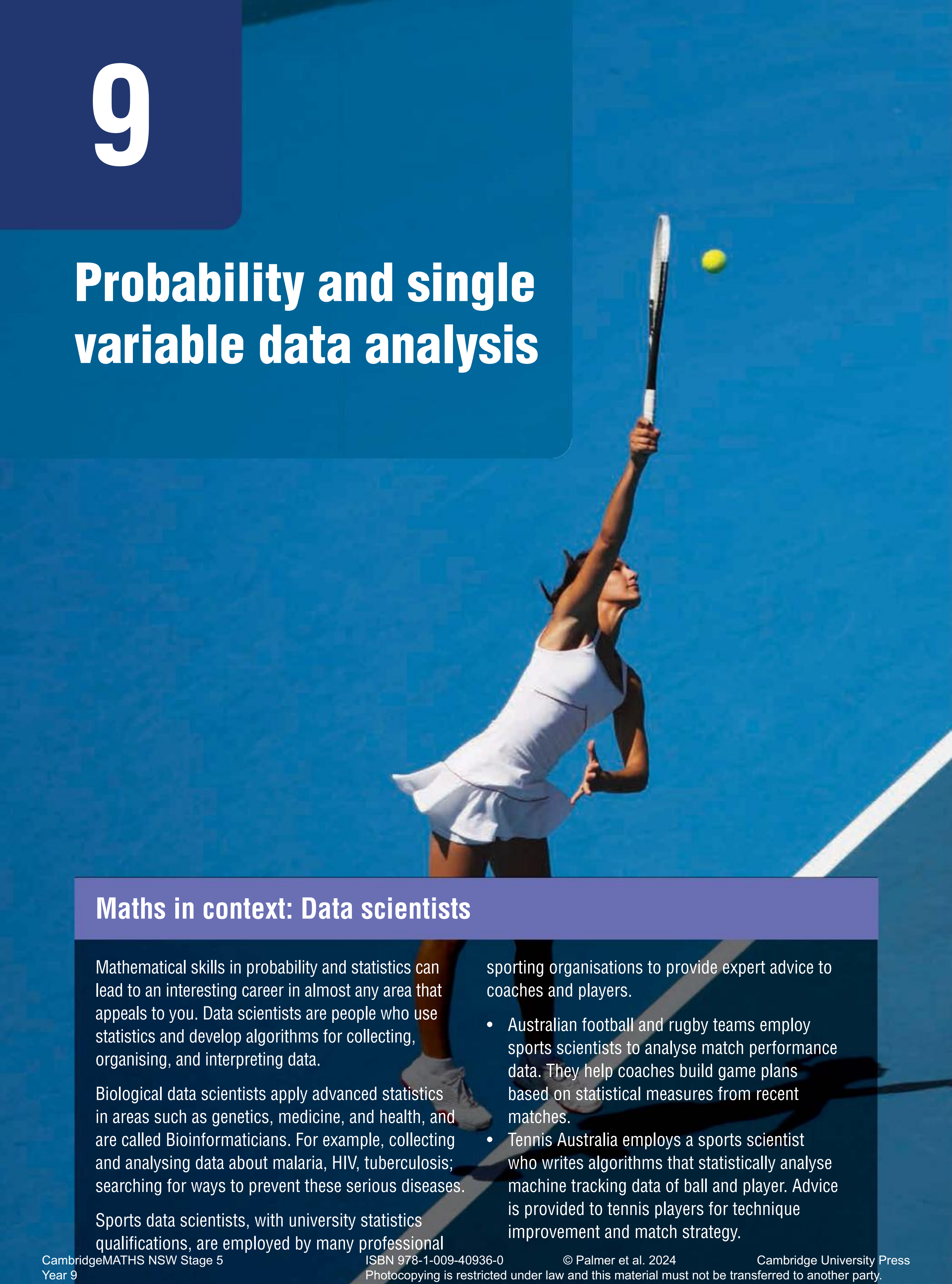

9

Probability and single variable data analysis

Maths in context: Data scientists

Mathematical skills in probability and statistics can lead to an interesting career in almost any area that appeals to you. Data scientists are people who use statistics and develop algorithms for collecting, organising, and interpreting data.

Biological data scientists apply advanced statistics in areas such as genetics, medicine, and health, and are called Bioinformaticians. For example, collecting and analysing data about malaria, HIV, tuberculosis; searching for ways to prevent these serious diseases.

Sports data scientists, with university statistics qualifications, are employed by many professional sporting organisations to provide expert advice to coaches and players.

- Australian football and rugby teams employ sports scientists to analyse match performance data. They help coaches build game plans based on statistical measures from recent matches.
- Tennis Australia employs a sports scientist who writes algorithms that statistically analyse machine tracking data of ball and player. Advice is provided to tennis players for technique improvement and match strategy.

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Chapter contents

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- solves problems involving probabilities in multistage chance experiments and simulations (MA5-PRO-C-01)
- solves problems involving Venn diagrams, 2-way tables and conditional probability (MA5-PRO-P-01)
- compares and analyses datasets using summary statistics and graphical representations (MA5-DAT-C-01).

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

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9A Review of probability CONSOLIDATING

Learning intentions
- To understand the concept of probability and how it is described numerically
- To know the terminology of probability
- To be able to find the probability of an event in an experiment with equally likely outcomes
- To be able to find the probability of the complement of an event

Past, present and future learning
- This section consolidates Stage 4 concepts which are used in Stage 5 and Stage 6 Advanced
- Some of these questions are more challenging than those in our Stage 4 books
- These concepts will be revised and extended in Chapter 8 of our Year 10 book and also in Stage 6

The mathematics associated with describing chance is called probability. We can precisely calculate the chance of some events occurring, such as rolling a sum of 12 from two dice or flipping three heads if a coin is tossed five times. To do this we need to know how many outcomes there are in total and how many of the outcomes are favourable (i.e. which match the result we are interested in). The number of favourable outcomes in comparison to the total number of outcomes will determine how likely it is that the favourable event will occur.

Tennis matches are won or lost on the average probability, p, of a player winning each point. Probability calculations show that the chance of winning a match increases rapidly if p increases slightly. E.g. 50% chance if $p = 0.5$; 75% if $p = 0.52$; and 95% if $p = 0.55$.

Lesson starter: Choose an event

As a class group, write down and discuss at least three events which have the following chance of occurring.

- impossible chance
- very low chance
- medium to low chance
- even (50–50) chance
- medium to high chance
- very high chance
- certain chance

KEY IDEAS

- A **chance** or **random experiment** is an activity that may produce a variety of different results (**outcomes**) which occur randomly. Rolling a die is a random experiment.
- The **sample space** is the list of all possible outcomes from an experiment.
- Set brackets $\{\ldots\}$ are used to list sets of numbers or other objects.
- An **event** is a collection of outcomes resulting from an experiment.
  For example, rolling a die is a random experiment with six possible outcomes: $\{1, 2, 3, 4, 5, 6\}$. The event 'rolling a number greater than 4' includes the outcomes 5 and 6. This is an example of a **compound event** because it contains more than one element from the sample space.

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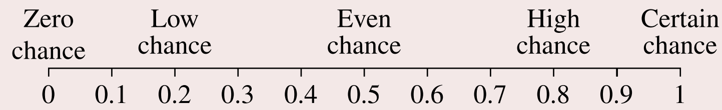
- The probability of an event in which all outcomes are **equally likely** is given by:

  $P(\text{Event}) = \dfrac{\text{Number of outcomes in which event occurs}}{\text{Total number of outcomes}}$

- Probabilities are numbers between 0 and 1 inclusive, and can be written as a decimal, fraction or percentage. For example: 0.55 or $\frac{11}{20}$ or 55%

- For all events, $0 \leqslant P(\text{Event}) \leqslant 1$.



- The **complement** of an event A is the event in which A does not occur.

  $P(\text{not A}) = 1 - P(\text{A})$

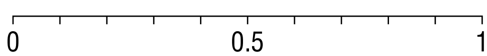
BUILDING UNDERSTANDING

1 Jim believes that there is a 1 in 4 chance that the flower on his prized rose will bloom tomorrow.

a Express the chance '1 in 4' as:

i a fraction **ii** a decimal **iii** a percentage.

b Indicate the level of chance described by Jim on this number line.



2 State the components missing from this table.

	Percentage	Decimal	Fraction	Number line
	50%	0.5	$\frac{1}{2}$	0 0.5 1 (× at 0.5)
a	25%			0 0.5 1
b				0 0.2 0.5 1 (× at 0.2)
c		0.6		0 0.2 0.5 1
d			$\frac{17}{20}$	0 0.2 0.5 1

3 Ten people make the following guesses of the chance that they will get a salary bonus this year.

0.7, $\frac{2}{5}$, 0.9, $\frac{1}{3}$, 2 in 3, $\frac{3}{7}$, 1 in 4, 0.28, $\frac{2}{9}$, 0.15

Can you order their chances from lowest to highest? (*Hint*: Change each into a decimal.)

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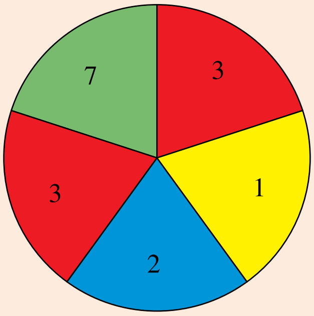
Example 1 Finding probabilities of events

This spinner has five equally divided sections.

a List the sample space using the given numbers.
b Find $P(3)$.
c Find $P(\text{not a } 3)$.
d Find $P(\text{a } 3 \text{ or a } 7)$.
e Find $P(\text{a number which is at least a } 3)$.

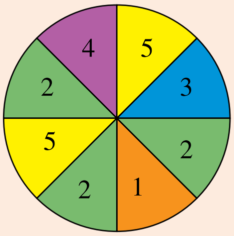


SOLUTION	EXPLANATION
a $\{1, 2, 3, 7\}$	Use set brackets and list all the possible outcomes in any order.
b $P(3) = \frac{2}{5}$ or 0.4	$P(3) = \frac{\text{number of sections labelled 3}}{\text{number of equal sections}}$
c $P(\text{not a } 3) = 1 - P(3)$ $= 1 - \frac{2}{5}$ or $1 - 0.4$ $= \frac{3}{5}$ or 0.6	'Not a 3' is the complementary event of obtaining a 3. Alternatively, count the number of sectors which are not 3.
d $P(\text{a } 3 \text{ or a } 7) = \frac{3}{5}$	There are two 3s and one 7 in the five sections.
e $P(\text{at least a } 3) = \frac{3}{5}$	Three of the sections have the numbers 3 or 7, which are 3 or more.

Now you try

This spinner has eight equally divided sections.

a List the sample space using the given numbers.
b Find $P(2)$.
c Find $P(\text{not a } 2)$.
d Find $P(\text{a } 2 \text{ or a } 5)$.
e Find $P(\text{a number which is at least a } 2)$.



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Example 2 Choosing letters from a word

A letter is randomly chosen from the word PROBABILITY. Find the following probabilities.

a $P(\text{L})$
b $P(\text{not L})$
c $P(\text{vowel})$
d $P(\text{consonant})$
e $P(\text{vowel or a B})$
f $P(\text{vowel or consonant})$

SOLUTION	EXPLANATION
a $P(\text{L}) = \frac{1}{11}$	One of the 11 letters in PROBABILITY is an L.
b $\begin{aligned} P(\text{not L}) &= 1 - \frac{1}{11} \\ &= \frac{10}{11} \end{aligned}$	The event 'not L' is the complement of the event selecting an L. Complementary events sum to 1.
c $P(\text{vowel}) = \frac{4}{11}$	There are 4 vowels: O, A and two letter Is.
d $\begin{aligned} P(\text{consonant}) &= 1 - \frac{4}{11} \\ &= \frac{7}{11} \end{aligned}$	The events 'vowel' and 'consonant' are complementary.
e $P(\text{vowel or a B}) = \frac{6}{11}$	There are 4 vowels and 2 letter Bs.
f $P(\text{vowel or consonant}) = 1$	This event includes all possible outcomes.

Now you try

A letter is randomly chosen from the word AEROPLANE. Find the following probabilities.

a $P(\text{R})$
b $P(\text{not R})$
c $P(\text{vowel})$
d $P(\text{consonant})$
e $P(\text{consonant or an A})$
f $P(\text{vowel or consonant})$

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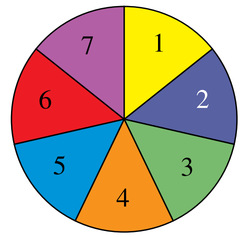
Exercise 9A

FLUENCY	1, 2–3 (½), 5, 6	1, 2–4 (½), 5, 6	2–4 (½), 5, 6 (½)

Example 1

1 This spinner has seven equally divided sections.

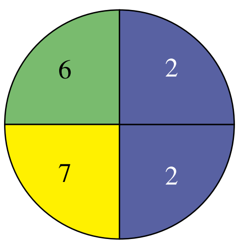


- **a** List the sample space using the given numbers.
- **b** Find $P(2)$.
- **c** Find $P(\text{not a } 2)$.
- **d** Find $P(\text{a } 2 \text{ or a } 3)$.
- **e** Find $P(\text{a number which is at least a } 2)$.

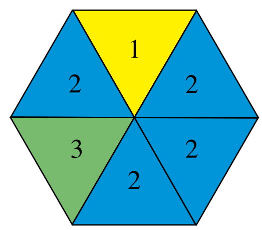
2 The spinners below have equally divided sections. Complete the following for each spinner.

- **i** List the sample space using the given numbers.
- **ii** Find $P(2)$.
- **iii** Find $P(\text{not a } 2)$.
- **iv** Find $P(\text{a } 2 \text{ or a } 3)$.
- **v** Find $P(\text{a number which is at least a } 2)$.

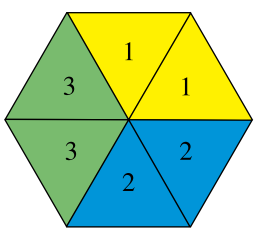
a



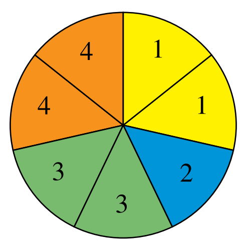
b



c



d



3 Find the probability of obtaining a blue ball when a ball is selected at random from a box which contains:

- **a** 4 blue balls and 4 red balls
- **b** 3 blue balls and 5 red balls
- **c** 1 blue ball, 3 red balls and 2 white balls
- **d** 8 blue balls, 15 black balls and 9 green balls
- **e** 15 blue balls only
- **f** 5 yellow balls and 2 green balls.

4 Find the probability of *not* selecting a blue ball when a ball is selected at random from a box containing the balls described in Question **3** parts **a** to **f** above.

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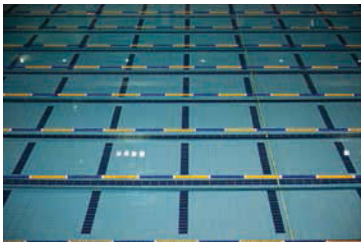
5 A swimming pool has eight lanes and each of eight swimmers has an equal chance of being placed in lane 1. Find the probability that a particular swimmer:

a will swim in lane 1

b will not swim in lane 1.



Example 2

6 A letter is chosen at random from the word ALPHABET. Find the following probabilities.

a $P(\text{L})$
b $P(\text{A})$
c $P(\text{A or L})$
d $P(\text{vowel})$
e $P(\text{consonant})$
f $P(\text{vowel or consonant})$
g $P(\text{Z})$
h $P(\text{A or Z})$
i $P(\text{not an A})$
j $P(\text{letter from the first half of the alphabet})$

PROBLEM–SOLVING	7, 8	8, 9	8–9(½), 10

7 The school captain is to be chosen at random from four candidates. Two are already 18 years old (Hayley and Rocco) and two are 17 years old (Alisa and Stuart).

a List the sample space.

b Find the probability that the school captain will be:

i Hayley

ii currently 17 years old

iii neither Stuart nor Hayley.

8 A six-sided die is rolled and the uppermost face is observed and recorded. Find the following probabilities.

a $P(6)$
b $P(3)$
c $P(\text{not a } 3)$
d $P(1 \text{ or } 2)$
e $P(\text{a number less than } 5)$
f $P(\text{an even number or odd number})$
g $P(\text{a square number})$
h $P(\text{not a prime number})$
i $P(\text{a number greater than } 1)$

9 A card is drawn at random from a deck of 52 playing cards. The deck includes 13 black spades, 13 black clubs, 13 red hearts and 13 red diamonds. This includes four of each of ace, king, queen, jack, 2, 3, 4, 5, 6, 7, 8, 9 and 10. Find the probability that the card will be:

a the queen of diamonds

b an ace

c a red king

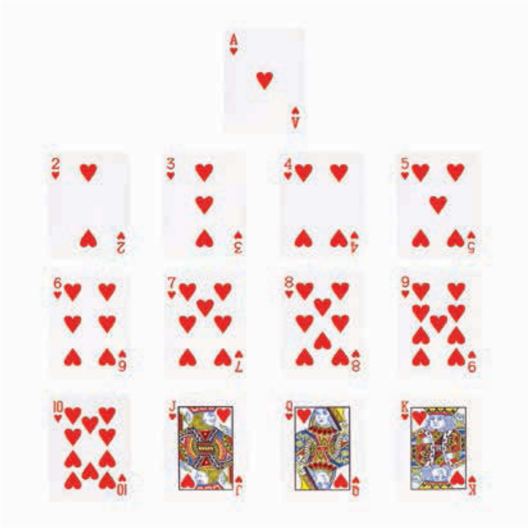
d a red card

e a jack or a queen

f any card except a 2

g any card except a jack or a black queen

h not a black ace.



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10 A letter is chosen at random from the word PROBABILITY. Find the probability that the letter will be:

a a B
b not a B
c a letter belonging to one of the first five letters in the alphabet
d a letter from the word RABBIT
e a letter that is not in the word RABBIT.

REASONING	11, 12	11, 12	12, 13

11 Amanda selects a letter at random from the word SOLO and writes $P(S) = \frac{1}{3}$. Explain her error.

12 A six-sided die is rolled. Which of the following events have a probability equal to $\frac{1}{3}$?

A more than 4
B at least 4
C less than or equal to 3
D no more than 2
E at most 4
F less than 3

13 A number is selected at random from the set $\{1, 2, 3, \ldots, 25\}$. Find the probability that the number chosen is:

a a multiple of 2
b a factor of 24
c a square number
d a prime number
e divisible by 3
f divisible by 3 or 2
g divisible by 3 and 2
h divisible by 2 or 3 or 7
i divisible by 13 and 7.

ENRICHMENT: Faulty iPhone	–	–	14

14 Xavier selects his favourite album on his iPhone and taps the shuffle function so songs are randomly played. The time length for each track is as shown in the table on the right.
The shuffle function is faulty and begins playing randomly at an unknown place somewhere on the album, not necessarily at the beginning of a track.

Track	Time (minutes)
1	3
2	4
3	4
4	5
5	4
6	3
7	4
8	4

a Find the total number of minutes of music available on this album.
b Find the probability that the iPhone will begin playing on track 1.
c Find the probability that the iPhone will begin on:
 i track 2
 ii track 3
 iii a track that is 4 minutes long
 iv track 4
 v track 7 or 8
 vi a track that is not 4 minutes long.

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9B Venn diagrams and two-way tables

Learning intentions
- To know how Venn diagrams and two-way tables show all combinations of two attributes
- To be able to use a Venn diagram to display the distribution of two sets
- To be able to fill out a two-way table either from a problem or from a Venn diagram
- To be able to use a Venn diagram or two-way table to calculate probabilities of events

Past, present and future learning
- This section addresses the Stage 5 Path Topic called *Probability B*
- These concepts will be revised and extended in Chapter 8 of our Year 10 book and also in Stage 6 Advanced

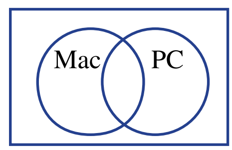
When the results of an experiment involve overlapping categories, it can be very helpful to organise the information into a Venn diagram or two-way table. Probabilities of particular events can easily be calculated from these types of diagrams.

Lesson starter: Mac or PC

Twenty people were surveyed to find out whether they owned a Mac or PC computer at home. The survey revealed that 8 people owned a Mac and 15 people owned a PC. Everyone surveyed owned at least one type of computer.

- Do you think some people owned both a Mac and PC? Discuss.
- Use these diagrams to help organise the number of people who own Macs and PCs.
- Use your diagrams to describe the proportion (fraction) of people owning Macs and/or PCs for all the different areas in the diagrams.

Venn diagram



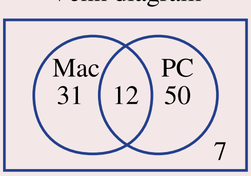
Two-way table

	Mac	No Mac	Total
PC			
No PC			
Total			

KEY IDEAS

- A **Venn diagram** and a **two-way table** help to organise outcomes into different categories. This example shows the type of computers owned by 100 people.

Venn diagram



Two-way table

	Mac	No Mac	Total
PC	12	50	62
No PC	31	7	38
Total	43	57	100

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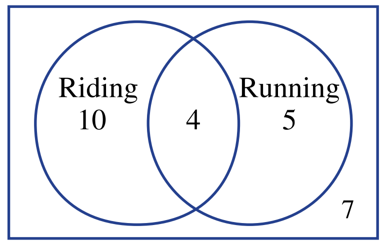
These diagrams show, for example, that:

- 12 people own both a Mac and a PC
- 62 people own a PC
- 57 people do not own a Mac
- $P(\text{Mac}) = \frac{43}{100}$
- $P(\text{only Mac}) = \frac{31}{100}$
- $P(\text{Mac or PC}) = \frac{93}{100}$
- $P(\text{Mac and PC}) = \frac{12}{100} = \frac{3}{25}$

BUILDING UNDERSTANDING

1 This Venn diagram shows the number of people who enjoy riding and running.



a How many people in total are represented by this Venn diagram?

b How many people enjoy:

- **i** only riding?
- **ii** running (in total)?
- **iii** both riding and running?
- **iv** neither riding nor running?
- **v** riding or running?

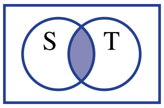
c How many people do not enjoy:

- **i** riding?
- **ii** running?

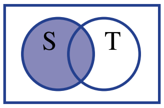
2 Match the diagrams **A**, **B**, **C** or **D** with the given description.

a S **b** S only **c** S and T **d** S or T

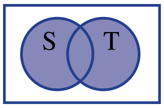
A



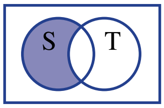
B



C



D



3 State the missing numbers in these two-way tables.

a

	A	Not A	Total
B	7	8	
Not B		1	
Total	10		

b

	A	Not A	Total
B	2		7
Not B		4	
Total			20

Example 3 Using a Venn diagram

A survey of 30 people found that 21 like AFL and 12 like soccer. Also, 7 people like both AFL and soccer and 4 like neither AFL nor soccer.

a Construct a Venn diagram for the survey results.

b How many people:

- **i** like AFL or soccer?
- **ii** do not like soccer?
- **iii** like only AFL?

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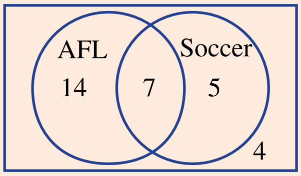
c If one of the 30 people was randomly selected, find:
  i P(like AFL and soccer)
  ii P(like neither AFL nor soccer)
  iii P(like only soccer).

SOLUTION

EXPLANATION

a 

Place the appropriate number in each category ensuring that:
- the total who like AFL is 21
- the total who like soccer is 12
- there are 30 in total.

b **i** 26

The total number of people who like AFL, soccer or both is $14 + 7 + 5 = 26$.

ii $30 - 12 = 18$

12 like soccer so 18 do not.

iii 14

21 like AFL, but 7 of these also like soccer.

c **i** $P(\text{like AFL and soccer}) = \frac{7}{30}$

7 of the 30 people like AFL and soccer.

ii $P(\text{like neither AFL nor soccer}) = \frac{4}{30} = \frac{2}{15}$

The 4 people who like neither AFL nor soccer sit outside both categories.

iii $P(\text{like only soccer}) = \frac{5}{30} = \frac{1}{6}$

5 people like soccer but not AFL.

Now you try

A survey of 25 people found that 18 people like comedies and 12 like action movies. Also, 8 people like both comedies and action movies and 3 like neither comedies nor action movies.

a Construct a Venn diagram for the survey results.
b How many people:
  i like comedies or action movies?
  ii do not like action movies?
  iii like only comedies?
c If one of the 25 people was randomly selected, find:
  i P(likes comedies and action movies)
  ii P(likes neither comedies nor action movies)
  iii P(likes only action movies).

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Example 4 Using a two-way table

At a car yard, 24 cars are tested for fuel used: 18 of the cars run on petrol, 8 cars run on gas and 3 cars can run on both petrol and gas.

a Illustrate the situation using a two-way table.

b How many of the cars:

- **i** do not run on gas?
- **ii** run on neither petrol nor gas?

c Find the probability that a randomly selected car:

- **i** runs on gas
- **ii** runs on only gas
- **iii** runs on gas or petrol.

SOLUTION

a

	Gas	Not gas	Total
Petrol	3	15	18
Not petrol	5	1	6
Total	8	16	24

EXPLANATION

Set up a table as shown and enter the numbers (in black) from the given information.
Fill in the remaining numbers (in red) ensuring that each column and row adds to the correct total.

b **i** 16

The total at the base of the 'Not gas' column is 16.

ii 1

The number at the intersection of the 'Not gas' column and the 'Not petrol' row is 1.

c **i** $P(\text{gas}) = \frac{8}{24} = \frac{1}{3}$

8 cars in total run on gas out of the 24 cars.

ii $P(\text{only gas}) = \frac{5}{24}$

Of the 8 cars that run on gas, 5 of them do not also run on petrol.

iii $P(\text{gas or petrol}) = \frac{15 + 5 + 3}{24} = \frac{23}{24}$

Of the 24 cars, some run on petrol only (15), some run on gas only (5) and some run on gas and petrol (3).

Now you try

Thirty people at a gym are asked about their gym usage. Of these, 18 people attend classes, 19 people use the gym equipment and 8 people attend classes and use the equipment.

a Illustrate the situation using a two-way table.

b How many of the people:

- **i** do not attend classes?
- **ii** neither attend classes nor use the equipment?

c Find the probability that a randomly selected person:

- **i** uses the gym equipment
- **ii** only attends classes
- **iii** attends classes or uses the gym equipment.

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Exercise 9B

FLUENCY	1–5	1, 3–5	2, 4, 5

Example 3

1 In a class of 30 students, 22 carried a phone and 9 carried an iPad, 3 carried both a phone and an iPad and 2 students carried neither.

- **a** Represent the information using a Venn diagram.
- **b** How many people:
  - **i** carried a phone or an iPad or both?
  - **ii** did not carry an iPad?
  - **iii** carried only an iPad?
- **c** If one of the 30 people was selected at random, find the following probabilities.
  - **i** P(carries a phone and an iPad)
  - **ii** P(carries neither a phone nor an iPad)
  - **iii** P(carries only a phone)

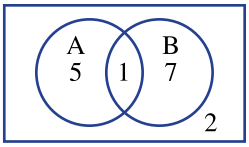
2 In a survey of 20 people, it is found that 13 people like toast for breakfast and 10 like cereal. Also 6 people like both toast and cereal and 3 like neither toast nor cereal.

- **a** Represent the information using a Venn diagram.
- **b** How many people:
  - **i** like toast or cereal for breakfast?
  - **ii** do not like toast?
  - **iii** like only toast?
- **c** If one of the 20 people was randomly selected, find:
  - **i** P(likes toast and cereal)
  - **ii** P(likes only cereal).

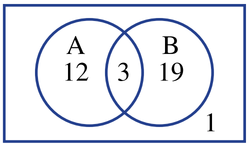
3 For each Venn diagram, find the following probabilities. You will need to calculate the total number in the sample first.

- **i** P(A)
- **ii** P(A only)
- **iii** P(not B)
- **iv** P(A and B)
- **v** P(A or B)
- **vi** P(neither A nor B)

a



b



Example 4

4 Of the 50 desserts served at a restaurant one evening, 25 were served with ice-cream, 21 were served with cream and 5 were served with both cream and ice-cream.

- **a** Illustrate the situation using a two-way table.
- **b** How many of the desserts:
  - **i** did not have cream?
  - **ii** had neither cream nor ice-cream?
- **c** Find the probability that a randomly chosen dessert:
  - **i** had cream
  - **ii** had only cream
  - **iii** had cream or ice-cream.

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5 Find the following probabilities using each of the given tables. First fill in the missing numbers.

- **i** $P(\text{A})$
- **ii** $P(\text{not A})$
- **iii** $P(\text{A and B})$
- **iv** $P(\text{A or B})$
- **v** $P(\text{B only})$
- **vi** $P(\text{neither A nor B})$

a

	A	Not A	Total
B	3	1	
Not B	2		4
Total			

b

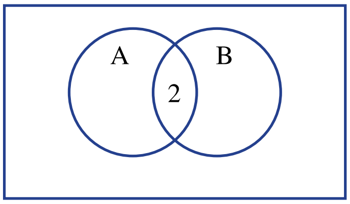
	A	Not A	Total
B		4	15
Not B	6		
Total			26

PROBLEM–SOLVING 6–8 6–9 6, 8, 10

6 For each two-way table, fill in the missing numbers then transfer the information to a Venn diagram.

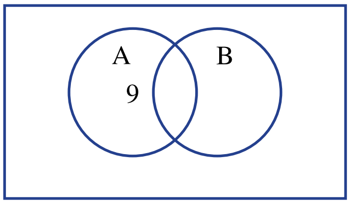
a

	A	Not A	Total
B	2		8
Not B			
Total		7	12



b

	A	Not A	Total
B		4	
Not B	9		13
Total	12		



7 Of a group of 10 people, 6 rented their house, 4 rented a car and 3 did not rent either a car or their house.

- **a** Draw a Venn diagram.
- **b** How many people rented both a car and their house?
- **c** Find the probability that one person rented only a car.

8 One hundred people were surveyed regarding their use of water for their garden. Of that group, 23 said that they used rainwater, 48 said that they used tap water and 41 said that they did not water at all.

- **a** Represent this information in a two-way table.
- **b** How many people used both rain and tap water?
- **c** What is the probability that one person selected at random uses only tap water?
- **d** What is the probability that one person selected at random uses tap water or rainwater?

9 All members of a ski club enjoy skiing and/or snowboarding. Of those members, 7 enjoy only snowboarding, 16 enjoy skiing and 4 enjoy both snowboarding and skiing. How many people are in the ski club?

10 Of 30 cats, 24 eat tinned or dry food, 10 like dry food and 5 like both tinned and dry food. Find the probability that a randomly selected cat likes only tinned food.

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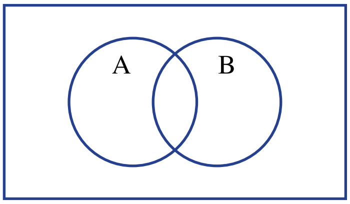
REASONING	11	11, 12	12–14

11 Complete the two-way table and transfer to a Venn diagram using the pronumerals w, x, y and z.

	A	Not A	Total
B	x	y	
Not B	z	w	
Total			



12 The total number of people in a survey is T. The number of Victorians in the survey is x and the number of doctors is y. The number of doctors who are Victorian is z. Write algebraic expressions for the following using any of the variables x, y, z and T.

a The number who are neither Victorian nor a doctor
b The number who are not Victorian
c The number who are not doctors
d The number who are Victorian but not a doctor
e The number who are a doctor but not Victorian
f The number who are not Victorian and a doctor
g The number who are not Victorian nor a doctor

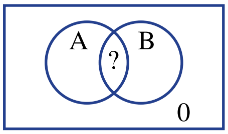
13 Explain what is wrong with this two-way table. Try to complete it to find out.

	A	Not A	Total
B		12	
Not B			7
Total	11		19

14 What is the minimum number of numbers that need to be given in a two-way table so that all numbers in the table can be calculated?

ENRICHMENT: Finding a rule for A and B	–	–	15

15 Two overlapping events, A and B, include 20 elements with 0 elements in the 'neither A nor B' region.



a Draw a Venn diagram for the following situations.
i The number in A is 12 and the number in B is 10.
ii The number in A is 15 and the number in B is 11.
iii The number in A is 18 and the number in B is 6.

b If the total number in A or B is now 100 (not 20), complete a Venn diagram for the following situations.
i The number in A is 50 and the number in B is 60.
ii The number in A is 38 and the number in B is 81.
iii The number in A is 83 and the number in B is 94.

c Now describe a method that finds the number in the common area for A and B. Your method should work for all the above examples.

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9C Using set notation

Learning intentions

- To know the symbols and notation used to describe different sets of data
- To be able to list sets from a description or diagram
- To be able to interpret symbols used to represent sets of data and find associated probabilities

Past, present and future learning

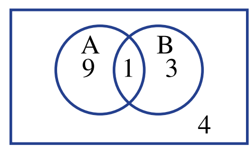
- This section goes beyond the Stage 5 Path Topic called *Probability B*
- These concepts will be revised and extended in Chapter 8 of our Year 10 book and also in Stage 6 Advanced

Using symbols to describe different sets of objects can make the writing of mathematics more efficient and easier to read. For example, *the probability that a randomly chosen person likes both apples and bananas* could be written $P(A \cap B)$, provided the events A and B are clearly defined.

Set notation is useful when analysing statistical data, such as: police data of car, bike and pedestrian accidents related to alcohol or speed or both; and energy company data of household numbers that use mains power or solar power or both.

Lesson starter: English language meaning to mathematical meaning

The number of elements in two events called A and B are illustrated in this Venn diagram. Use your understanding of the English language meaning of the given words to match with one of the mathematical terms and a number from the Venn diagram. They are in jumbled order.



English	Mathematical	Number in Venn diagram
a not A	**A** A union B	**i** 1
b A or B	**B** sample space	**ii** 7
c A and B	**C** complement of A	**iii** 13
d anyone	**D** A intersection B	**iv** 17

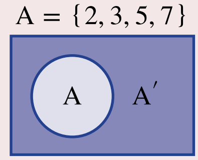
KEY IDEAS

- The **sample space** (list of all possible outcomes) is sometimes called the universal set and is given the symbol S, Ω, U or ξ.

  For example: $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

- Set A is a particular **subset** (⊂) of the sample space if all the elements in A are contained in the sample space. For example, $\{2, 3, 5, 7\} \subset \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

  $A = \{2, 3, 5, 7\}$

  $A' = \{1, 4, 6, 8, 9, 10\}$

- A′ is the complement of A and contains the elements not in A.

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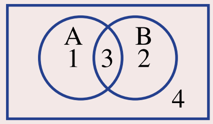
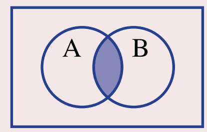
- $5 \in$ A means that 5 is **an element of** A.
- $\varnothing$ is the **null** or **empty** set and contains no elements; $\therefore \varnothing = \{\ \}$.
- $n(\text{A})$ is the **cardinal number** of A and means the number of elements in A, e.g. $n(\text{A}) = 4$.
- A Venn diagram can be used to illustrate how different subsets in the sample space are grouped.

For example: $\text{A} = \{2, 3, 5, 7\}$
$\text{B} = \{1, 3, 5, 7, 9\}$

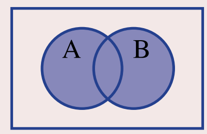


	A	A′	Total
B	3	2	5
B′	1	4	5
Total	4	6	10

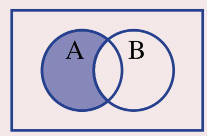
- We usually write the cardinal numbers inside a Venn diagram.
- $\text{A} \cap \text{B}$ means A **and** B, which means the **intersection** of A and B and includes the elements in common with both sets: $\therefore \text{A} \cap \text{B} = \{3, 5, 7\}$.



- $\text{A} \cup \text{B}$ means A **or** B, which means the **union** of A and B and includes the elements in either A or B or both: $\therefore \text{A} \cup \text{B} = \{1, 2, 3, 5, 7, 9\}$.



- A **only** is the elements in A but not in B: $\therefore$ A only $= \{2\}$ and $n(\text{A only}) = 1$.



BUILDING UNDERSTANDING

1 Match each of the terms **a–f** with the symbols **A–F**.

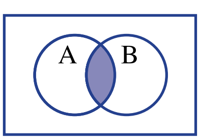
a complement **b** union **c** element of
d intersection **e** empty or null set **f** number of elements

A $\cap$ **B** $n(\text{A})$ **C** A′ **D** $\cup$ **E** $\in$ **F** $\varnothing$

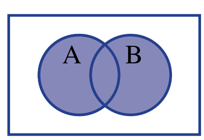
2 Choose a diagram **A–D** that matches each of these sets **a–d**.

a $\text{A} \cup \text{B}$ **b** A′ **c** $\text{A} \cap \text{B}$ **d** B only

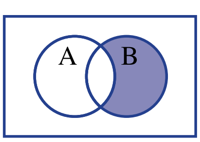
A



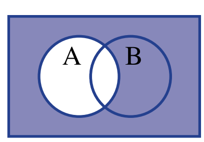
B



C



D



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3 Using the given two-way table, state the following.

a $n(A \cap B)$
b $n(B)$
c $n(A)$
d $n(A')$

	A	A′	Total
B	2	8	10
B′	5	1	6
Total	7	9	16

Example 5 Using set notation

A number is chosen from the set of positive integers between 1 and 8 inclusive. A is the set of odd numbers between 1 and 8 inclusive and B is the set of prime numbers between 1 and 8 inclusive.

a List these sets.
 i the sample space **ii** A **iii** B

b List these sets.
 i $A \cap B$ **ii** $A \cup B$ **iii** A' **iv** B only

c Draw a Venn diagram using the number of elements in each set.

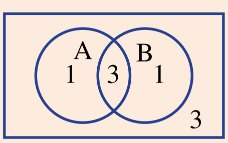
d Find:
 i $n(A)$ **ii** $P(A)$ **iii** $n(A \cap B)$ **iv** $P(A \cap B)$

SOLUTION / EXPLANATION

a **i** $\{1, 2, 3, 4, 5, 6, 7, 8\}$ — List all the numbers, using set brackets.
ii $A = \{1, 3, 5, 7\}$ — A includes all the odd numbers.
iii $B = \{2, 3, 5, 7\}$ — B includes all the prime numbers. 1 is not prime.

b **i** $A \cap B = \{3, 5, 7\}$ — $\{3, 5, 7\}$ are common to both A and B.
ii $A \cup B = \{1, 2, 3, 5, 7\}$ — $\{1, 2, 3, 5, 7\}$ are in either A or B or both.
iii $A' = \{2, 4, 6, 8\}$ — A' means the elements not in A.
iv B only $= \{2\}$ — B only means the elements in B but not in A.

c



Place each cardinal number into the appropriate region, i.e. there are 3 numbers common to sets A and B so 3 is placed in the overlapping region.

d **i** $n(A) = 4$ — $n(A)$ is the cardinal number of A. There are four elements in A.
ii $P(A) = \frac{4}{8} = \frac{1}{2}$ — $P(A)$ means the chance that the element will belong to A. There are 4 numbers in A compared with 8 in the sample space.
iii $n(A \cap B) = 3$ — There are three elements in $A \cap B$.
iv $P(A \cap B) = \frac{3}{8}$ — Three of eight elements are in $A \cap B$.

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Now you try

A number is chosen from the set of positive integers between 1 and 15 inclusive. A is the set of odd numbers between 1 and 15 inclusive and B is the set of prime numbers between 1 and 15 inclusive.

a List these sets.
  i the sample space **ii** A **iii** B
b List these sets.
  i $A \cap B$ **ii** $A \cup B$ **iii** A' **iv** B only
c Draw a Venn diagram using the number of elements in each set.
d Find:
  i $n(A)$ **ii** $P(A)$ **iii** $n(A \cap B)$ **iv** $P(A \cap B)$

Exercise 9C

FLUENCY	1–3	1–4	2–4

Example 5

1 A number is chosen from the set of positive integers between 1 and 10 inclusive. A is the set of odd numbers between 1 and 10 inclusive and B is the set of prime numbers between 1 and 10 inclusive.
  a List these sets.
    i the sample space **ii** A **iii** B
  b List these sets.
    i $A \cap B$ **ii** $A \cup B$ **iii** A' **iv** B only
  c Draw a Venn diagram using the number of elements in each set
  d Find:
    i $n(A)$ **ii** $P(A)$ **iii** $n(A \cap B)$ **iv** $P(A \cap B)$

2 A number is chosen from the set of positive integers between 1 and 20 inclusive. A is the set of multiples of 3 that are less than 20 and B is the set of factors of 15.
  a List the sets A and B and then list these sets.
    i $A \cap B$ **ii** $A \cup B$ **iii** A' **iv** B only
  b Draw a Venn diagram using the number of elements in each set.
  c Find:
    i $n(B)$ **ii** $P(B)$ **iii** $n(A \cap B)$
    iv $P(A \cap B)$ **v** $n(A \cup B)$ **vi** $P(A \cup B)$

3 Consider the sample space $\Omega = \{1, 2, 3, 4, 5, 6\}$ and the sets $A = \{1, 4, 6\}$ and $B = \{2, 3, 5\}$.
  a State whether the following are true (T) or false (F).
    i $A \subset \Omega$ **ii** $B \subset \Omega$
    iii $3 \in A$ **iv** $5 \in A$
    v $n(B) = 3$ **vi** $n(B') = 2$
    vii $A \cap B = \varnothing$ **viii** $A \cup B = \varnothing$
  b Find these probabilities.
    i $P(B)$ **ii** $P(B')$ **iii** $P(A \cap B)$

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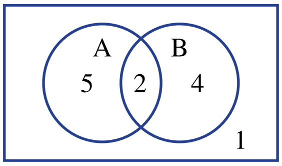
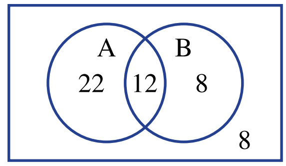
4 For each diagram or table, find the following probabilities.

i $P(A \cap B)$ **ii** $P(A \cup B)$ **iii** $P(A')$

a



b



c

	A	A′	Total
B	5	2	
B′			
Total	9		15

d

	A	A′	Total
B		8	13
B′	4		
Total		12	

PROBLEM–SOLVING	5–7	5–7	6, 7

5 Four students have the names FRED, RON, RACHEL and HELEN while the sets A, B and C are defined by:

A = {students with a name including the letter R}
B = {students with a name including the letter E}
C = {students with a name including the letter Z}

a List these sets.

i A **ii** B **iii** C **iv** $A \cap B$

b If a student is chosen at random from the group, find these probabilities.

i $P(A)$ **ii** $P(A')$ **iii** $P(C)$
iv $P(C')$ **v** $P(A \cap B)$ **vi** $P(A \cup B)$

6 Consider all the letters of the alphabet. Let A = {the set of vowels} and B = {different letters of the word MATHEMATICS}. Find:

a n(sample space) **b** $n(A)$ **c** $n(A \cap B)$ **d** $n(B')$
e $P(A)$ **f** $P(A')$ **g** $P(A \cap B)$ **h** $P(A \cup B)$

7 For 50 people who all have at least a cat or a dog, let A be the set of all pet owners who have a dog and B be the set of all pet owners who have a cat. If $n(A) = 32$ and $n(B) = 29$, find the following.

a $n(A \cap B)$ **b** $n(A \text{ only})$ **c** $P(A \cup B)$ **d** $P(A')$

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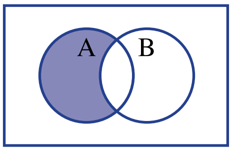
REASONING	8, 9	8–10	10–12

8 From Question 7, A is the set of dog owners and B is the set of cat owners. Write a brief description of the following groups of people.

a $A \cap B$ b $A \cup B$ c B' d B only

9 The region 'A only' could be thought of as the intersection of A and the complement of B so, A only $= A \cap B'$. Use set notation to describe the region 'B only'.



10 a Is $n(A) + n(B)$ ever equal to $n(A \cup B)$? If so, show an example using a Venn diagram.

b Is $n(A) + n(B)$ ever less than $n(A \cup B)$? Explain.

11 The four inner regions of a two-way table can be described using intersections. The A only region is described by $n(A \cap B')$. Describe the other three regions using intersections.

	A	A′	Total
B			$n(B)$
B′	$n(A \cap B')$		$n(B')$
Total	$n(A)$	$n(A')$	$n(\Omega)$

12 Research what it means if we say that two events A and B are **mutually exclusive**. Give a brief description.

ENRICHMENT: How are $A' \cap B'$ and $(A \cup B)'$ related?	–	–	13

13 Consider the set of integers $\{1, 2, 3, \ldots, 20\}$. Let $A = \{\text{prime numbers less than } 20\}$ and $B = \{\text{factors of } 12\}$.

a List these sets.

i A ii B iii $A \cap B$
iv $A \cup B$ v $A' \cap B$ vi $A \cap B'$
vii $A' \cup B$ viii $A \cup B'$ ix $A' \cup B'$

b Find the following probabilities.

i $P(A \cup B)$ ii $P(A \cap B)$ iii $P(A' \cap B)$ iv $P(A' \cap B')$
v $P(A \cup B')$ vi $P((A \cap B)')$ vii $P(A' \cup B')$ viii $P((A \cup B)')$

c Draw Venn diagrams to shade the regions $A' \cap B'$ and $(A \cup B)'$. What do you notice?

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9D Using arrays for two-step experiments

Learning intentions

- To know how to list the sample space of a two-step experiment in an array
- To understand the difference between 'with replacement' and 'without replacement'
- To be able to construct arrays and then find associated probabilities

Past, present and future learning

- This section addresses the Stage 5 Core Topic called *Probability A*
- These concepts will be revised and extended in Chapter 8 of our Year 10 book and also in Stage 6 Advanced

When an experiment consists of two steps such as rolling two dice or selecting two people from a group, we can use an array (or table) to systematically list the sample space.

Genetic arrays find probabilities of inherited traits, such as for the melanin pigments that control hair, eye and skin colour. Fairer skin has a higher risk of skin cancer: 30% of melanomas arise from moles, 70% from normal skin.

Lesson starter: Is it a 1 in 3 chance?

Billy tosses two coins on the kitchen table at home and asks what the chance is of getting two tails.

- Dad says that there are 3 outcomes: two heads, two tails or one of each, so there is a 1 in 3 chance.
- Mum says that with coins, all outcomes have a 1 in 2 chance of occurring.
- Billy's sister Betty says that there are 4 outcomes so it's a 1 in 4 chance.

Can you explain who is correct and why?

KEY IDEAS

- Example of two-step experiments include:
  - tossing two coins (or tossing one coin twice)
  - choosing two socks from a drawer full of socks
  - rolling two dice
  - choosing two cards from a pack
  - rolling a die and then tossing a coin.
- An **array** or table is often used to list the sample space for experiments with two steps.
- When listing outcomes it is important to be consistent with the order for each outcome. For example: the outcome (heads, tails) should be distinguished from the outcome (tails, heads).
- Some experiments are conducted **without replacement**, which means some outcomes that may be possible **with replacement** are not possible.
  For example: Two letters are chosen from the word CAT.

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With replacement

		1st		
		C	A	T
2nd	C	(C, C)	(A, C)	(T, C)
	A	(C, A)	(A, A)	(T, A)
	T	(C, T)	(A, T)	(T, T)

9 outcomes

Without replacement

		1st		
		C	A	T
2nd	C	×	(A, C)	(T, C)
	A	(C, A)	×	(T, A)
	T	(C, T)	(A, T)	×

6 outcomes

BUILDING UNDERSTANDING

1 State the missing outcomes, then count the total number of outcomes. Part **a** is *with replacement* and part **b** is *without replacement*.

a

		1st		
		1	2	3
2nd	1	(1, 1)	(2, 1)	
	2			
	3			

b

		1st		
		A	B	C
2nd	A	×	(B, A)	(C, A)
	B		×	
	C			×

2 These two tables list the outcomes for the selection of two letters at random from the word MAT.

	Table A		
	M	A	T
M	(M, M)	(A, M)	(T, M)
A	(M, A)	(A, A)	(T, A)
T	(M, T)	(A, T)	(T, T)

	Table B		
	M	A	T
M	×	(A, M)	(T, M)
A	(M, A)	×	(T, A)
T	(M, T)	(A, T)	×

a Which table shows selection where replacement is allowed (with replacement)?

b Which table shows selection where replacement is not allowed (without replacement)?

c What is the probability of choosing the outcome (T, M) from:

i Table A? **ii** Table B?

d How many outcomes include the letter A using:

i Table A? **ii** Table B?

Example 6 Finding the sample space for events with replacement

Two coins are tossed.

a Draw a table to list the sample space.

b Find the probability of obtaining (H, T).

c Find $P(1\text{ head})$.

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SOLUTION

EXPLANATION

a

		Toss 1	
		H	T
Toss 2	H	(H, H)	(T, H)
	T	(H, T)	(T, T)

Represent the results of each coin toss.

Sample space = {(H, H), (H, T), (T, H), (T, T)}

The table shows four possible outcomes.

b $P(\text{H, T}) = \frac{1}{4}$

One of the four outcomes is (H, T).

c $P(1 \text{ head}) = \frac{2}{4} = \frac{1}{2}$

Two outcomes have one head: (H, T), (T, H).

Now you try

Two identical counters have a red side (R) and a blue side (B). They are each tossed.

a Draw a table to list the sample space.

b Find the probability of obtaining (B, B).

c Find $P(1\,\text{red side})$.

Example 7 Finding the sample space for events without replacement

Two letters are chosen at random from the word TREE without replacement.

a List the outcomes in a table.

b Find the probability that the two letters chosen are both E.

c Find the probability that at least one of the letters is an E.

SOLUTION

EXPLANATION

a

		1st			
		T	R	E	E
2nd	T	×	(R, T)	(E, T)	(E, T)
	R	(T, R)	×	(E, R)	(E, R)
	E	(T, E)	(R, E)	×	(E, E)
	E	(T, E)	(R, E)	(E, E)	×

List all the outcomes maintaining a consistent order. Note that the same letter cannot be chosen twice.
Both Es need to be listed so that each outcome in the sample space is equally likely.

b $P(\text{E, E}) = \frac{2}{12} = \frac{1}{6}$

As there are 2 Es in the word TREE, it is possible to obtain the outcome (E, E) in two ways.

c $P(\text{at least one E}) = \frac{10}{12} = \frac{5}{6}$

10 of the 12 outcomes contain at least one E.

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Now you try

Two letters are chosen at random from the word DATA without replacement.

a List the outcomes in a table.

b Find the probability that the two letters chosen are both A.

c Find the probability that at least one of the letters is an A.

Exercise 9D

FLUENCY	1–3	1–4	1, 3, 4

Example 6

1 A six-sided die is rolled twice.

a Complete a table like the one shown.

b What is the total number of outcomes?

c Find the probability that the outcome is:

i (1, 1)

ii a double (two of the same number)

iii (3, 1), (2, 2) or (1, 3)

iv any outcome containing a 1 or a 6.

		1st					
		1	2	3	4	5	6
2nd	1	(1, 1)	(2, 1)				
	2						
	3						
	4						
	5						
	6						

2 Two dots are selected, one from each of the sets A and B, where A = {•, ∘} and B = {•, ∘, ∘}.

a Complete a table like this one, showing all the possible outcomes.

b State the total number of outcomes.

c Find the probability that the outcome will:

i be (•, ∘)

ii contain one black dot

iii contain two of the same dots.

		A	
		•	∘
B	•	(•, •)	(∘, •)
	∘		
	∘		

Example 7

3 Two letters are chosen at random from the word DOG *without replacement*.

a Complete a table like the one shown.

b Find the probability of obtaining the (G, D) outcome.

c Find the probability of obtaining an outcome with an O in it.

		1st		
		D	O	G
2nd	D	×	(O, D)	(G, D)
	O		×	
	G			×

4 Two digits are selected at random *without replacement* from the set {1, 2, 3, 4}.

a Draw a table to show the sample space. Remember doubles such as (1, 1), (2, 2), etc. are not allowed.

b Find:

i $P(1, 2)$

ii $P(4, 3)$.

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c Find the probability that:

i both numbers will be at least 3
ii the outcome will contain a 1 or a 4
iii the outcome will contain a 1 and a 4
iv the outcome will not contain a 3.

PROBLEM–SOLVING	5, 6	5, 6	6, 7

5 The total sum is recorded from rolling two four-sided dice.

a Copy and complete this table, showing all possible totals that can be obtained.

		Roll 1			
		1	2	3	4
Roll 2	1	2	3		
	2				
	3				
	4				

b Find the probability that the total sum is:

i 2
ii 2 or 3
iii less than or equal to 4
iv more than 6
v at most 6.

6 Jill guesses the answers to two multiple-choice questions with options A, B, C, D or E.

a Copy and complete this table, showing all possible guesses that can be obtained.

		Guess 1				
		A	B	C	D	E
Guess 2	A	(A, A)	(B, A)			
	B					
	C					
	D					
	E					

b Find the probability that she will guess:

i (D, A)
ii the same letter
iii different letters.

c Find the probability that Jill will get:

i exactly one of her answers correct
ii both of her answers correct.

7 Many board games involve the rolling of two six-sided dice.

a Use a table to help find the probability that the sum of the two dice is:

i 12
ii 2 or 3
iii 11 or 12
iv less than or equal to 7
v less than 7
vi at least 10
vii at most 4
viii 1.

b Which total sum has the highest probability and what is the probability of rolling that sum?

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REASONING	8, 9	9–11	10–12

8 Two letters are chosen at random from the word MATHEMATICIAN.

a How many outcomes sit in the sample space if selections are made:

i with replacement? **ii** without replacement?

b How many of the outcomes contain the same letter if selection is made:

i with replacement? **ii** without replacement?

9 Two letters are chosen from the word WOOD without replacement. Is it possible to obtain the outcome (O, O)? Explain why.

10 In a bag are five counters each of a different colour: green (G), yellow (Y), red (R), blue (B) and purple (P).

a A counter is drawn from the bag, replaced and then a second is selected. Find the probability that a green counter then a blue counter are selected; that is, find P(G, B).

b The counter selected is not replaced before the second is selected. Find the probability that a green counter then a blue counter are selected.

11 A six-sided die and a ten-sided die have been rolled simultaneously. What total sum(s) has/have the highest probability?

12 A spinner numbered 1 to 50 with equal size sectors is spun twice. Find the probability that the total for the two spins is:

a 100 **b** 51 **c** 99 **d** 52 **e** 55

ENRICHMENT: Two cards from the deck	–	–	13

13 Two cards are dealt to you from a pack of playing cards that includes four suits (Hearts, Diamonds, Clubs and Spades), with each suit containing {2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A}. You keep both cards.

a Does this situation involve with replacement or without replacement?

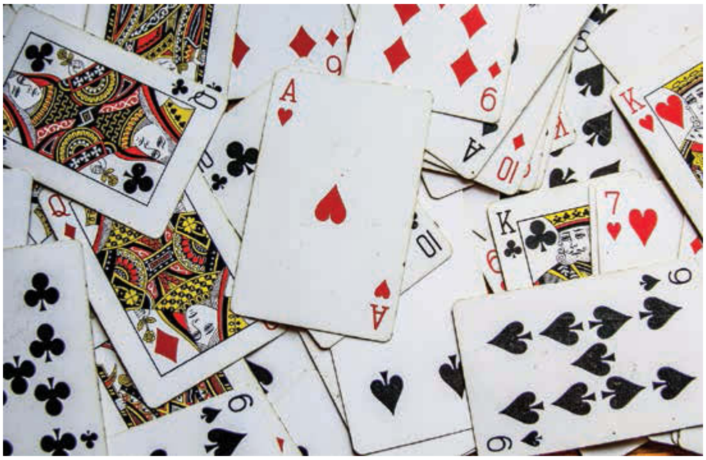
b How many outcomes would there be in your sample space (all possible selections of two cards)?

c What is the probability of receiving an ace of diamonds and an ace of hearts?

d Find the probability of obtaining two cards that:

i are both twos

ii are both hearts.



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9E Using tree diagrams

Learning intentions
- To know how to use a tree diagram to list the sample space from multistage experiments
- To be able to construct and use a tree diagram to find the probability of certain events

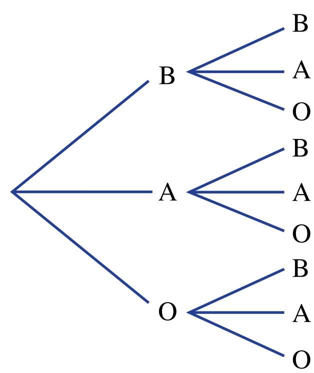
Past, present and future learning
- This section addresses the Stage 5 Core Topic called *Probability A*
- These concepts will be revised and extended in Chapter 8 of our Year 10 book and also in Stage 6 Advanced

When experiments consist of two or more steps, a tree diagram can be used to list the sample space. Tables are often used for two-step experiments, but a tree diagram can be extended for experiments with any number of steps.

The spread of an infectious disease can be simply modelled with a tree diagram. If each ill person infects 4 other people, after 10 infection 'waves', over 1 million people are infected. Fortunately, vaccination strengthens the immune system, reducing infection rates.

Lesson starter: What's the difference?

You are offered a choice of two pieces of fruit from a banana, an apple and an orange. You choose two at random. This tree diagram shows selection with replacement.



- How many outcomes will there be?
- How many of the outcomes contain two round fruits?
- How would the tree diagram change if the selection was completed without replacement? Would there be any difference in the answers to the above two questions? Discuss.

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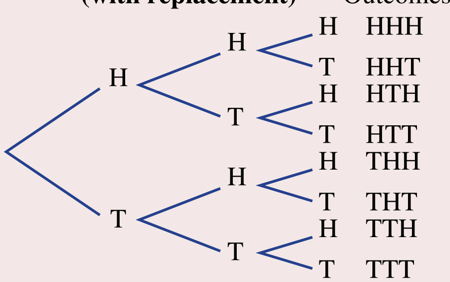
KEY IDEAS

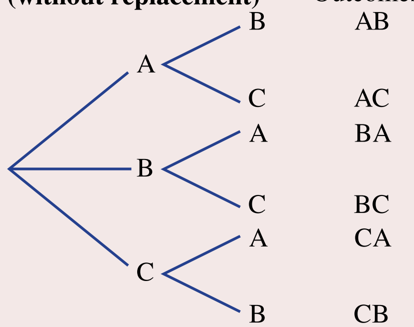
- **Tree diagrams** are used to list the sample space for experiments with two or more steps.
  - The outcomes for each stage of the experiment are listed vertically and each stage is connected with branches.

    For example:

Tossing a coin 3 times
(with replacement)



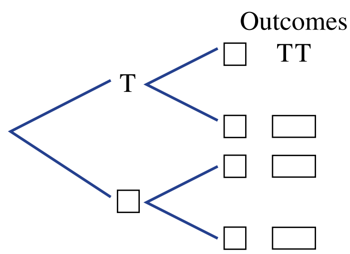
Selecting 2 letters from {A, B, C}
(without replacement)



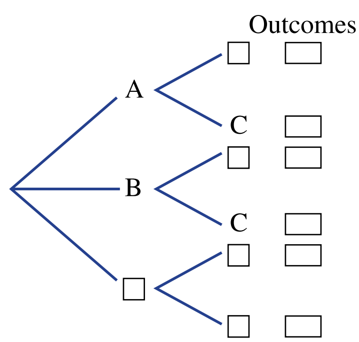
In these examples, each set of branches produces outcomes that are all equally likely.

BUILDING UNDERSTANDING

1. State the missing components for this tree diagram for selecting two letters *with replacement* from the word TO and list the outcomes.



2. State the missing components for this tree diagram to find the outcomes of selecting two people from a group of three (A, B and C) *without replacement*.



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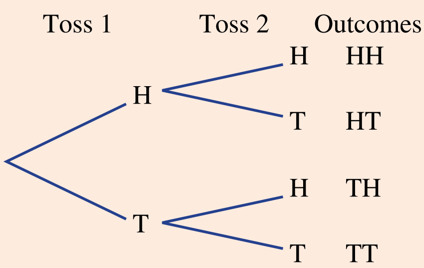
Example 8 Constructing a tree diagram

An experiment involves tossing two coins.

a Complete a tree diagram to show all possible outcomes.
b What is the total number of outcomes?
c Find the probability of tossing:
- **i** two tails
- **ii** one tail
- **iii** at least one head.

SOLUTION

a



b The total number of outcomes is 4.

c **i** $P(\text{TT}) = \frac{1}{4}$

ii $P(1\text{ tail}) = \frac{2}{4} = \frac{1}{2}$

iii $P(\geqslant 1\text{ head}) = \frac{3}{4}$

EXPLANATION

The tree diagram shows two coin tosses one after the other resulting in $2 \times 2 = 4$ outcomes.

There are four possibilities in the outcomes column.

One of the four outcomes is TT.

Two outcomes have one tail: {HT, TH}.

Three outcomes have at least one head: {HH, HT, TH}.

Now you try

Two regular six-sided die are rolled. It is noted if they show odd (O) or even (E) numbers.

a Complete a tree diagram to show all possible odd (O) and even (E) outcomes.
b What is the total number of outcomes?
c Find the probability of rolling:
- **i** two even numbers
- **ii** one even number
- **iii** at least one odd number.

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Example 9 Constructing a tree diagram without replacement

Two people are selected at random without replacement from a group of three: Annabel (A), Brodie (B) and Chris (C).

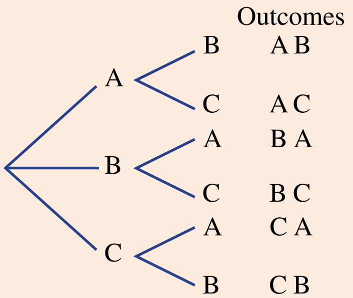
a List all the possible outcomes for the selection using a tree diagram.

b Find the probability that the selection will contain:

i Annabel and Brodie **ii** Chris **iii** Chris or Brodie.

SOLUTION

a



b i $P(\text{Annabel and Brodie}) = \frac{2}{6}$

$= \frac{1}{3}$

ii $P(\text{Chris}) = \frac{4}{6}$

$= \frac{2}{3}$

iii $P(\text{Chris or Brodie}) = \frac{6}{6}$

$= 1$

EXPLANATION

On the first choice there are three options (A, B or C), but on the second choice there are only two remaining.

2 of the 6 outcomes contain Annabel and Brodie (A B) and (B A).

4 out of the 6 outcomes contain Chris.

All of the outcomes contain at least one of Chris or Brodie.

Now you try

Two lollipops are selected at random without replacement from a group of three: strawberry (S), grape (G) and cola (C).

a List all the possible outcomes for the selection using a tree diagram.

b Find the probability that the selection will contain:

i strawberry and cola **ii** grape **iii** grape or cola.

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Exercise 9E

FLUENCY	1–4	1–4	2, 4, 5

Example 8

1 A coin is tossed twice.

a Complete this tree diagram to show all the possible outcomes.

b What is the total number of outcomes?

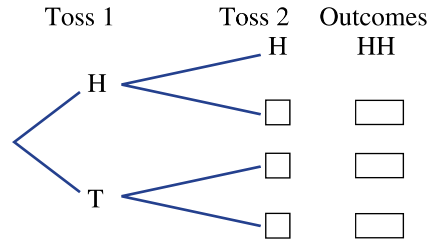
c Find the probability of obtaining:

i two heads

ii one head

iii at least one head

iv at least one tail.



2 A spinner with three numbers, 1, 2 and 3, is spun twice. Each number is equally likely to be spun on a single spin.

a List the set of possible outcomes, using a tree diagram.

b What is the total number of possible outcomes?

c Find the probability of spinning:

i two 3s

ii at least one 3

iii no more than one 2

iv two odd numbers.

Example 9

3 Two people are selected at random without replacement from a group of three: Donna (D), Elle (E) and Fernando (F).

a List all the possible outcomes for the selection using a tree diagram.

b Find the probability that the selection will contain:

i Donna and Elle

ii Fernando

iii Fernando or Elle.

4 A drawer contains two red socks (R), one blue sock (B) and one yellow sock (Y). Two socks are selected at random without replacement.

a Copy and complete this tree diagram.

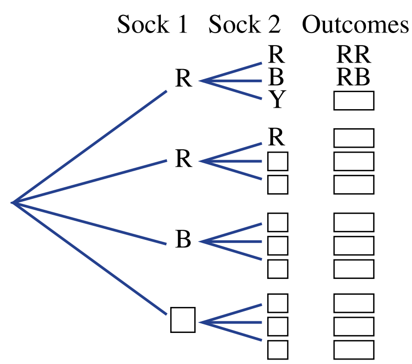
b Find the probability of obtaining:

i a red sock and a blue sock

ii two red socks

iii any pair of socks of the same colour

iv any pair of socks of different colour.



5 A student who has not studied for a multiple-choice test decides to guess the answers for every question. There are three questions, and three choices of answer (A, B and C) for each question. Given that only one of the possible choices (A, B or C) is correct for each question, state the probability that the student guesses:

a 1 correct answer

b 2 correct answers

c 3 correct answers

d 0 correct answers.

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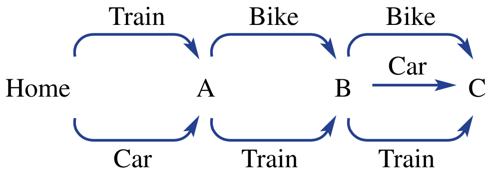
PROBLEM–SOLVING	6	6, 7	6, 7

6 Michael needs to deliver parcels to three places (A, B and C in order) in the city. This diagram shows the different ways in which he can travel.



a Draw a tree diagram showing all the possible outcomes of transportation options.
b What is the total number of possible outcomes?
c If Michael randomly chooses one of these outcomes, find the probability that he will use:
  i the train all three times
  ii the train exactly twice
  iii his bike exactly once
  iv different transport each time
  v a car at least once.

7 A shelf in a discount supermarket contains an equal number of tins of tomatoes and tins of peaches – all mixed together and without labels. You select four tins in a hurry. Use a tree diagram to help find the probability of selecting the correct number of tins of tomatoes and/or peaches for each of these recipe requirements. Assume that the shelves are continually refilled so that probabilities remain constant.
a You need four tins of tomatoes for a stew.
b You need four tins of peaches for a peach crumble.
c You need at least three tins of tomatoes for a bolognese.
d You need at least two tins of peaches for a fruit salad.
e You need at least one tin of tomatoes for a vegetable soup.

REASONING	8	8, 9	9, 10

8 You toss a coin four times. Use a tree diagram to find the probability that you toss:
a 0 tails
b 1 tail
c 2 tails
d 3 tails
e 4 tails.

9 a A coin is tossed 5 times. How many outcomes will there be?
b A coin is tossed n times. Write a rule for the number of outcomes in the sample space.

10 Use a tree diagram to investigate the probability of selecting two counters from a bag of three black and two white counters if the selection is drawn:
a with replacement
b without replacement.

Is there any difference?

ENRICHMENT: Selecting matching clothes	–	–	11

11 A man randomly selects a tie from his collection of one green and two red ties, a shirt from a collection of one red and two white shirts, and either a red or a black hat. Use a tree diagram to help find the probability that he selects a tie, shirt and hat according to the following descriptions:
a red tie, red shirt and black hat
b all three items red
c one item red
d two items red
e at least two items red
f a green hat
g a green tie and a black hat
h a green tie or a black hat
i not a red item
j a red tie or white shirt or black hat.

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9F Using relative frequencies to estimate probabilities

Learning intentions
- To understand how relative frequency can be used to estimate probability
- To be able to calculate observed probability
- To be able to calculate the expected number of occurrences

Past, present and future learning
- This section addresses the Stage 5 Core Topic called *Probability A*
- These concepts will be revised and extended in Chapter 8 of our Year 10 book and also in Stage 6 Advanced

In some situations it may not be possible to list all the outcomes in the sample space and find theoretical probabilities. If this is the case then an experiment, survey or simulation can be conducted to find experimental probabilities. Provided the experiment includes a sufficient number of trials, these probabilities can be used to estimate the chance of particular events. Experimental probability is frequently used in science experiments and for research in medicine and economics.

An actuary uses past data to calculate the probability of insurance claims for property loss, accidents, sickness or death. Based on these risk probabilities, actuaries determine the insurance premiums for different age groups and various activities.

Lesson starter: Newspaper theories

A tabloid newspaper reports that of 10 people interviewed in the street, five had a dose of the flu. At about the same time a medical student tested 100 people and found that 21 had the flu.

- What is the experimental probability of having the flu, according to the newspaper's survey?
- What is the experimental probability of having the flu, according to the medical student's results?
- Which of the two sets of results would be more reliable and why? Discuss the reasons.
- Using the medical student's results, how many people would you expect to have the flu in a group of 1000 and why?

KEY IDEAS

- **Experimental probability** or **relative frequency** is calculated using the results of an experiment or survey.

  $$\text{Experimental probability} = \frac{\text{number of times the outcome occurs}}{\text{total number of trials in the experiment}} = \text{relative frequency}$$

- The **long-run proportion** is the experimental probability for a sufficiently large number of trials.
- The **expected number of occurrences** = probability × number of trials.

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BUILDING UNDERSTANDING

1 This table shows the results of three different surveys of how many people in Perth use public transport (PT).

Survey	Number who use PT	Survey size	Experimental probability
A	2	10	$\frac{2}{10} = 0.2$
B	5	20	____________
C	30	100	____________

a What are the two missing numbers in the experimental probability list?
b Which survey should be used to estimate the probability that a person uses public transport and why?

2 The experimental probability of Jess hitting a bullseye on a dartboard is 0.05 (or $\frac{5}{100}$). How many bullseyes would you expect Jess to get if she threw the following number of darts?
a 100 darts **b** 400 darts **c** 80 darts

Example 10 Finding the experimental probability

A box contains an unknown number of coloured balls and a ball is drawn from the box and then replaced. The procedure is repeated 100 times and the colour of the ball drawn is recorded each time. Twenty-five red balls were recorded.
a Find the experimental probability for selecting a red ball.
b Find the expected number of red balls drawn if the procedure was repeated 500 times.

SOLUTION

a $P(\text{red ball}) = \frac{25}{100}$
$= 0.25$

b Expected number of red balls in 500
$= 0.25 \times 500$
$= 125$

EXPLANATION

$P(\text{red ball}) = \frac{\text{number of red balls drawn}}{\text{total number of balls drawn}}$

There are 25 red balls drawn and 100 balls in total.

Expected number of occurrences
= probability × number of trials

Now you try

A biscuit jar contains an unknown number of biscuits and a biscuit is drawn from the jar and then replaced. The procedure is repeated 50 times and the type of biscuit drawn is recorded each time. Thirty choc-chip biscuits were recorded.
a Find the experimental probability for selecting a choc-chip biscuit.
b Find the expected number of choc-chip biscuits drawn if the procedure was repeated 200 times.

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Exercise 9F

FLUENCY	1, 2	1–3	2, 3

Example 10

1 A bag contains an unknown number of counters, and a counter is selected from the bag and then replaced. The procedure is repeated 100 times and the colour of the counter is recorded each time. Sixty of the counters drawn were blue.

- **a** Find the experimental probability for selecting a blue counter.
- **b** Find the expected number of blue counters drawn if the procedure was repeated:
  - **i** 100 times **ii** 200 times **iii** 600 times.

2 In an experiment involving 200 people chosen at random, 175 people said that they owned a home computer.

- **a** Calculate the experimental probability of choosing a person who owns a home computer.
- **b** Find the expected number of people who own a home computer from the following group sizes.
  - **i** 400 people **ii** 5000 people **iii** 40 people

3 By calculating the experimental probability, estimate the chance that each of the following events will occur.

- **a** Nat will walk to work today, given that she walked to work five times in the last 75 working days.
- **b** Mike will win the next game of cards, given that of the last 80 games he has won 32.
- **c** Brett will hit the bullseye on the dartboard with his next attempt, given that of the last 120 attempts he hit the bullseye 22 times.

PROBLEM–SOLVING	4, 5	4–6	5–7

4 The colour of cars along a highway was noted over a short period of time and summarised in this frequency table.

Colour	White	Silver	Blue	Green
Frequency	7	4	5	4

- **a** How many cars had their colour recorded?
- **b** Find the experimental probability that a car's colour is:
  - **i** blue **ii** white.
- **c** If the colour of 100 cars was recorded, what would be the expected number of:
  - **i** blue cars?
  - **ii** green cars?
  - **iii** blue or green cars?

5 A six-sided die is rolled 120 times. How many times would you expect the following events to occur?

- **a** a 6
- **b** a 1 or a 2
- **c** a number less than 4
- **d** a number that is at least 5

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6 The letters from a three-letter word are written separately onto cards. A card is chosen at random and replaced and this is repeated 100 times. The results are shown in the table.

Letter	E	S
Frequency	64	36

a Find the experimental probability of obtaining an E.
b Based on these results what is the three-letter word likely to be?

7 A spinner is divided into three regions not necessarily of equal size. The regions are numbered 1, 2 and 3 and the spinner is spun 50 times. The table shows the results.

Number	1	2	3
Frequency	26	11	13

a Find the experimental probability of obtaining:
i a 1 **ii** at least a 2 **iii** a 1 or a 3.
b Based on these results, how many 3s would you expect if the spinner is spun 300 times?
c In fact, the spinner is divided up using simple and common fractions. Give a likely example describing how the regions might be divided.

REASONING 8–10 8–11 10–12

8 The results of throwing a drawing pin and observing how many times the pin lands with the spike pointing up are shown in the table. Results are recorded at different stages of the experiment.

Number of throws	Frequency (spike up)	Experimental probability
1	1	1.00
5	2	0.40
10	5	0.50
20	9	0.45
50	18	0.36
100	41	0.41

Which experimental probability would you choose to best represent the probability that the pin will land spike up? Why?

9 Phil rolled a fair six-sided die 10 times and, incredibly, receives 9 sixes.
a Find the experimental probability of rolling a six.
b Is it likely that Phil will receive the same number of sixes if he rolled the die 10 more times? Explain.

10 Do you think that a coin is fair or biased given the following experimental results? Give reasons.
a 47 tails out of 100 tosses
b 23 heads out of 24 tosses
c 1 tail out of 1 toss

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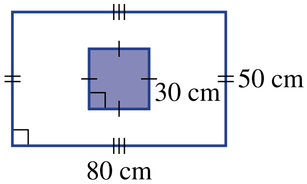
11 Decide if the following statements are true.

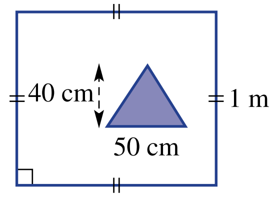
- **a** The experimental probability is always equal to the theoretical probability.
- **b** The experimental probability can be greater than the theoretical probability.
- **c** If the experimental probability is zero then the theoretical probability is zero.
- **d** If the theoretical probability is zero then the experimental probability is zero.

12 One hundred darts are randomly thrown at the given dartboards. No darts miss the dartboard entirely. How many darts do you expect to hit the blue shaded region? Give reasons.

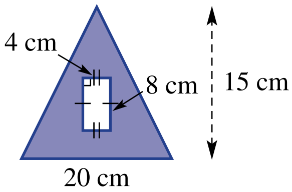
a



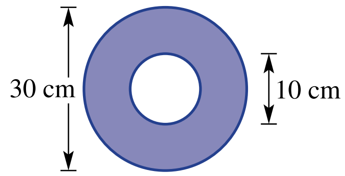
b



c



d



ENRICHMENT: More than a guessing game	–	–	13, 14

13 A bag of 10 counters includes counters of four different colours.
The results from drawing and replacing one counter at a time for 80 trials are shown in this table.
Use the given information to find out how many counters of each colour were likely to be in the bag.

Colour	Total
Blue	26
Red	17
Green	29
Yellow	8

14 A box of 12 chocolates of the same size and shape includes five different centres. The results from selecting and replacing one chocolate at a time for 60 trials are shown in this table.
Use the given information to find out how many chocolates of each type were likely to be in the box.

Centre	Total
Strawberry	11
Caramel	14
Coconut	9
Nut	19
Mint	7

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9A **1** List the sample space for:

a rolling a standard die

b the win/loss result of two matches for a team.

9A **2** Find the probability of choosing a vowel when choosing one letter at random from the word MATHEMATICS.

9B **3** A survey of 40 sporty people found that 28 liked tennis, 25 liked squash and 13 liked both.

a Construct a Venn diagram for these results.

b How many people liked both tennis and squash?

c If one of the 40 people is chosen at random, what is the probability that they like only one sport?

9B **4** Use this two-way table to find the probability that a randomly selected car has both a sunroof and air-conditioning.

	Aircon	No aircon	Total
Sunroof		9	
No sunroof	55	6	61
Total		15	100

9C **5** A number is chosen from the set of positive integers between 10 and 20, inclusive. A is the set of even numbers between 10 and 20, inclusive, and B is the set of numbers divisible by 3 between 10 and 20, inclusive.

a List these sets.

i the sample space

ii A

iii B

iv $A \cap B$

b Find:

i $n(A \cap B)$

ii $P(A \cap B)$

iii $P(A')$.

9D **6** Find the probability of:

a obtaining a head and a tail if two coins are tossed

b choosing two vowels from the word MATE if two letters are chosen at random, without replacement

c rolling a double 6 or a double 3 if two standard dice are rolled.

9E **7** Two letters are chosen, without replacement, from the letters of the word WAY.

a Use a tree diagram to list all the possible combinations for the selection.

b Find the probability that the selection will contain:

i the letter A

ii a W or a Y

iii not a Y

iv a W as the first letter.

9F **8** A drawer contains an unknown number of blue, black and white single socks. One sock is chosen at random from the drawer and its colour noted before the sock is returned to the drawer. The outcomes of this experiment are recorded in the table below.

Colour	Blue	Black	White
Frequency	16	44	40

Find the experimental probability of choosing:

a a white sock

b a blue or a white sock.

9F **9** If a regular six-sided die is rolled 420 times, how many times would you expect a 1 or a 2 to appear?

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9G Data and sampling

Learning intentions
- To know the meaning of key words associated with data and sampling
- To be able to classify the different types of data
- To know the common types of sampling methods
- To be able to recognise when bias may exist when collecting data

Past, present and future learning
- This section addresses the Stage 5 Path Topic called *Data analysis C*
- These concepts will be revised and extended in Chapter 9 of our Year 10 book and also in Stage 6 Advanced

Statistics is an essential part of how science collects, analyses and interprets data. We use data to support ideas and uncover new theories. We also use data using tables and graphs to help display information in an organised and understandable way. Data can also be used to influence decision making and help sell products, which is why it is very important to think about how data is collected, how it is analysed and the conclusions that might be communicated.

We use technology to help collect and analyse large data samples.

Lesson starter: Is there bias?

A data collection company has designed a survey to collect information about people's preferences regarding how they communicate with friends and family. The company is interested in feedback from the entire population of a town. Three employees, Jemma, Karri and Moshi, are asked to survey a range of individuals and report back with their data samples.

- Jemma's method involves messaging all her friends on her contact list.
- Karri surveys people in-person in a busy street at around 3:30 p.m. once school is finished for the day.
- Moshi uses a list of mobile phone numbers which belong to people who have agreed to complete surveys for a small fee.
- Discuss each of the methods employed by Jemma, Karri and Moshi. Do you think any of the survey methods would lead to bias? Give reasons.
- Discuss alternative methods to collect the data that might be less biased.

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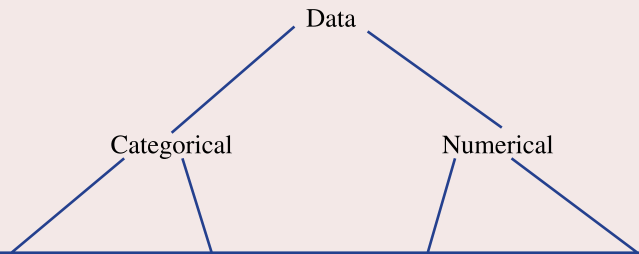
KEY IDEAS

- Key words associated with statistics
  - **Population** is a complete group with at least one characteristic in common. This group could be people, animals, objects or any event.
  - **Sample** is a selection or subset of the population.
  - **Survey** is an investigation or tool to collect information about a given sample or population.
    - A survey of the entire population is called a **census**.
  - **Bias** is a tendency which causes a difference in the results of a survey and the true facts.

- **Types of data**
  - **Categorical** (or qualitative) data can be sorted into groups or categories. For example: people's favourite types of animals.
  - **Numerical** (or quantitative) data are expressed as numbers. For example: the heights of twenty 15-year-old students.



Nominal	Ordinal	Discrete	Continuous
Categories have no order; e.g. colours: red, blue, yellow	Categories can be ordered; e.g. low, medium, high	Data can take on only a limited number of values; e.g. number of children in a family	Data can take on any value in a given range; e.g. time taken to complete a race

- **Types of sampling methods**
  - **Simple random sampling**: Each individual in the population has an equal chance of being chosen. For example, each individual is assigned a number and a random number generator selects a number.
  - **Systematic sampling**: Individuals are selected at regular intervals. For example, every 20th car is stopped on a highway and the driver is surveyed regarding alcohol consumption.
  - **Stratified sampling**: The population is divided into subgroups and individuals are surveyed within that group. For example, a population is surveyed regarding their heart health but are first divided into age groups.

- **Errors** may occur in sampling. Examples include:
  - Sample size is too small.
  - The sample does not represent the entire population.
  - Poor or inaccurate measurements are recorded.
  - An inappropriate sampling method is chosen.

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BUILDING UNDERSTANDING

1. Give the key statistical word which matches each definition,
   - a An investigation or tool to collect information about a given sample or population.
   - b A tendency which causes a difference in the results of a survey and the true facts.
   - c A complete group with at least one characteristic in common.
   - d A selection or subset of the population.
2. Give the name of the type of data matching the description.
   - a Data that can be sorted into groups or categories.
   - b Data which can be expressed as numbers.
3. Give the name of the type of sampling method matching the description.
   - a The population is divided into subgroups and individuals are surveyed within that group.
   - b Each individual in the population has an equal chance of being chosen.
   - c Individuals are selected at regular intervals.

Example 11 Classifying data

Classify the data obtained from the following survey as either numerical discrete, numerical continuous, categorical ordinal or categorical nominal.

- a The mass of each shell in a collection of sea shells
- b The results of a survey asking people to rate a TV program as good, satisfactory or poor
- c The colours of the next 10 cars that pass on a busy road
- d The number of computers in a household

SOLUTION	EXPLANATION
a Numerical continuous	The data includes numbers which could take on any number within a given range. The recordings may depend on the accuracy of the measuring instrument.
b Categorical ordinal	The data includes words which are categories and have a clear order.
c Categorical nominal	The data includes colours which are categories with no particular order.
d Numerical discrete	The data includes numbers from a limited range of values, in this case, the natural numbers 1, 2, 3, …

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Now you try

Classify the data obtained from the following survey as either numerical discrete, numerical continuous, categorical ordinal or categorical nominal.

a The number of students in all the academic classes in a school
b The type of caterpillar found in a number of vegetable garden beds
c The time taken to complete a task
d The type of safety vest used with one of three sizes available including large, medium or small

Example 12 Choosing a sampling method

a Identify the sampling method used in each of these surveys.
  i A car factory assembly line chooses every fifth car to check the quality of paint.
  ii One hundred email addresses are randomly selected from a list of all email addresses in a workplace.
  iii Fifty dogs are first grouped into breed type then tested for fleas.
b Eighty people on the street at 7 p.m. are surveyed about how often they go out for dinner. This data is then used to make conclusions about the population of that suburb and how often they go out to dinner. Do you think the sample will contain bias? If so, explain why.

SOLUTION	EXPLANATION
a i Systematic	Individual cars are selected at regular intervals.
ii Simple random	Each individual in the workplace has an equal chance of being chosen.
iii Stratified	The dogs are divided into subgroups (breeds) and then dogs are tested for fleas within that breed.
b Yes, it is perhaps more likely that the people surveyed tend to go out for dinner more often.	People who are already out in the street at 7 p.m. are perhaps more likely to go out to dinner on any night of the week.

Now you try

a Identify the sampling method used in each of these surveys.
  i Athletes are divided into sport type then tested for heart fitness.
  ii Every fifth competition paper ordered by surnames is selected for analysis regarding the working shown.
  iii Every family in an apartment block is assigned a number and then five random numbers are chosen so that families can be surveyed regarding their thoughts on the standard of the cleaning in the block.
b Wasram is to seek opinions about what people in the community think of the most recent Soccer World Cup. He surveys his team-mates at the soccer club using a questionnaire. Do you think the sample will contain bias? If so, explain why.

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Exercise 9G

FLUENCY	1–5	2–5	3, 5

Example 11

1 Classify the following as numerical or categorical data.

- **a** The number of pets in a household
- **b** The colour of hair on the heads of 13 models
- **c** The type of mark on a test as either answer, method or consequential
- **d** The number of litres of drink in 10 bottles

Example 11

2 Classify the data obtained from the following survey as either numerical discrete, numerical continuous, categorical ordinal or categorical nominal.

- **a** The style of jacket on a wall in a shop
- **b** The level of firmness of a bed described as hard, medium or soft
- **c** The weight of 25 packets of chips delivered to a shop
- **d** The shirt size of a range of clothing using the integers 36, 37, 38, ...

3 Classify the data obtained from the following survey as either numerical discrete, numerical continuous, categorical ordinal or categorical nominal.

- **a** The type of food purchased at a supermarket
- **b** The time taken to complete a 100 m sprint
- **c** The rating of a guide's use of the English language with the options: excellent, good, fair and poor
- **d** The scores out of 20 using whole numbers only for an assignment

Example 12

4 **a** Identify the sampling method used in each of these surveys.

- **i** A random selection of passport numbers are selected from a government list then checked for people who travelled to the United States.
- **ii** People at a hospital are sorted into age groups then tested for their reaction to a particular type of medicine.
- **iii** Every tenth person in the queue at a football match is surveyed regarding their thoughts about who will win the match.

b A lecturer wants to learn about the opinions of his students and asks them in a lecture to put their hand up and respond verbally to a couple of questions. Do you think the sample will contain bias? If so, explain why.

5 **a** Identify the sampling method used in each of these surveys.

- **i** Every eighth Christmas tree sold at a shop is tested for bugs.
- **ii** A school surveys students about the new uniform and selects a random sample of 20 students from each year level.
- **iii** A random selection of people chosen from the electoral role are surveyed regarding their opinion on how the election was conducted.

b Matty records information about the bird population on her farm and sits by the lake to wait for the birds to come along before recording the bird type and colour. Do you think the sample will contain bias? If so, explain why.

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PROBLEM–SOLVING	6, 7	6–9	7–9

6 In a survey involving stratified sampling, 10 people are to be selected from a company including 30 people working in the sales department and 70 people working in the administration department. How many people should be selected from each department?

7 The following statistics were collected about Wally's pet cat.
- Length: 81.45 cm
- Weight: 14.72 kg
- Favourite food: Dine
- Number of toys: 8
- Type of scratching post: short, standard and tall

Describe the type of data for each of the five characteristics.

8 What advice would you give to the following people who are trying to generate unbiased data?
a Hudson wants to form a view as to the difficulty of a test and talks to his five closest friends.
b Molly, a vet, wants to review the time that it takes to serve customers and looks at the data for her most experienced vet in the practice.
c Prabhdeep is researching the cost of bread in a town and records the cost of all the types of bread in a supermarket.
d Zach wants to assess the amount of study students put into their mathematics. He surveys students exiting the Extension maths exam for the Accelerated students.

9 A school has 50 junior school staff and 70 senior school staff. Of the staff, 24 are to be selected to gauge staff opinions regarding the sport program. How many staff from the junior school should they select?

REASONING	10	10, 11	11, 12

10 Shoe sizes are usually recorded with only whole numbers and sometimes halves. Describe the type of data produced after surveying people's shoe size.

11 Explain why stratified sampling might be used to explore people's resting heart rates.

12 A special type of sampling is called voluntary response sampling. This involves members of the population who have volunteered to participate in a research study. Explain why such a method could lead to bias.

ENRICHMENT: The Challenger disaster	–	–	13

13 In 1985 the Challenger space shuttle disintegrated soon after launch and killed all seven crew members. The cause of the disaster was an O-ring which failed and allowed pressurised gasses to escape and cause a chain reaction.
Statistics had been collected and analysed before the launch which collected data regarding the variables *temperature* and *the number of times the O-ring had failed*. The conclusion was that there was no correlation between the two variables. Unfortunately, they did not look at the data collected from when the O-rings had not failed and after including this data there proved to be a more significant correlation between the variables and if discovered earlier, may have stopped the launch.
Use the internet to research a different statistical error which led to a disaster or embarrassing mistake made by a company or researcher. Comment on the following:
a The statistical error made
b The consequences of the error
c Suggestions regarding what could have been done to avoid the error.

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9H Mean, median and mode CONSOLIDATING

Learning intentions

- To know that mean, median and mode are used to summarise a dataset
- To be able to find the mean, median and mode of a dataset
- To know how the median is found for datasets with an odd or even number of values
- To understand that an outlier is a value much smaller or larger than the rest of the data

Past, present and future learning

- This section consolidates Stage 4 concepts which are used in Stage 5 and Stage 6 Advanced
- Some of these questions are more challenging than those in our Stage 4 books
- These concepts will be revised and extended in Chapter 9 of our Year 10 book and also in Stage 6

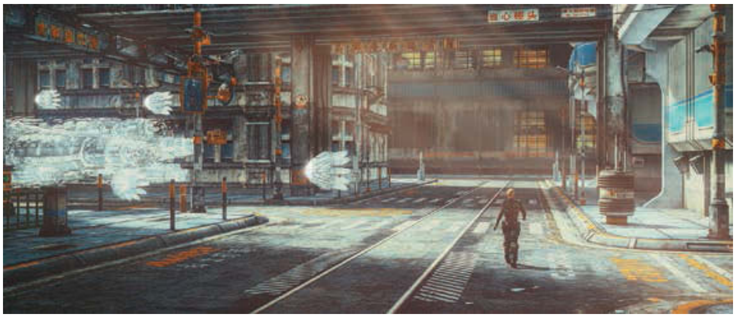
The discipline of statistics involves collecting and summarising data. It also involves drawing conclusions and making predictions, which is why many of the decisions we make today are based on statistical analysis. The type and amount of product stocked on supermarket shelves, for example, is determined by the sales statistics and other measures such as average cost and price range.

The Dengue fever virus (pictured) infects an average of 390 million people per year. Control of viral diseases improves when the molecular structures are known. Bioinformatics combines statistics, biology and information technology and analyses molecular biological data.

Lesson starter: Game purchase

Arathi purchases 7 computer games at a sale: 3 games cost \$20 each, 2 games cost \$30, 1 game costs \$50 and the last game costs \$200.

- Recall and discuss the meaning of the words mean, median and mode.
- Can you work out the mean, median or mode for the cost of Arathi's games?
- Which of the mean, median or mode gives the best 'average' for the cost of Arathi's games?
- Why is the mean greater than the median in this case?



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KEY IDEAS

- Statisticians use **summary statistics** to highlight important aspects of a data set. These are summarised below.
- Some summary statistics are called **measures of location**.
  - The two most commonly used measures of location are the **mean** and the **median**. These are also called '**measures of centre**' or '**measures of central tendency**'. Mean and median can only be applied to numerical data.
  - The **mean** is sometimes called the '**arithmetic mean**' or the '**average**'. The formula used for calculating the mean, $\bar{x}$, is:

    $$\bar{x} = \frac{\text{sum of data values}}{\text{number of data values}}$$

    For example, in the following data set

5	7	2	5	1

    the mean is $\frac{5+7+2+5+1}{5} = 20 \div 5 = 4$
  - The **median** divides an ordered data set into two sets, each of which contain the same number of data values. It is often called the 'middle value'. The median is found by firstly ensuring the data values are in ascending order, then selecting the 'middle' value.

    If the number of values is odd, simply choose the one in the middle.

    2 2 3 4 6 9 9

    median = 4

    If the number of values is even, find the average of the two in the middle.

    2 2 3 4 7 9 9 9

    Median = $(4 + 7) \div 2 = 5.5$
  - The **mode** of a data set is the most frequently occurring data value. There can be more than one mode. When there are two modes, the data is said to be **bimodal**. The mode can be very useful for categorical data. It can also be used for numerical data, but it may not be an accurate measure of centre. For example, in the data set below the mode is 10, but it is the largest data value.

1	2	3	10	10
  - An **outlier** is a data value which is significantly greater than or less than the other data values in the data set. The inclusion of an outlier in a data set can inflate or deflate the mean and give a false impression of the 'average'.
    For example:
    In the data set 1, 2, 3, 4, 5, the mean is 3 and median is 3.
    If the data value 5 is changed to 20, it is now an outlier.
    In the data set 1, 2, 3, 4, 20, the median is still 3, but the mean is now 6, which is greater than four of the five data values. In this case, the median is a better measure of centre than the mean.

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BUILDING UNDERSTANDING

1. State the missing word.
   a The mode is the most ________ value.
   b The median is the ________ value.
   c To calculate the ________, you add up all the values and divide by the number of values.

2. State the mean, median and mode for these simple ordered data sets.
   a 1 2 2 2 4 4 6
   b 7 11 14 18 20 20

Example 13 Finding measures of centre

For the given data sets, find the following:

i the mean
ii the median
iii the mode.

a 5 2 4 10 6 1 2 9 6
b 17 13 26 15 9 10

SOLUTION

a i $\text{Mean} = \frac{5+2+4+10+6+1+2+9+6}{9}$
$= 5$

Find the sum of all the numbers and divide by the number of values.

ii 1 2 2 4 5 6 6 9 10
Median = 5

First, order the data.
The median is the middle value.

iii Mode = 2 and 6

The data set is bi-modal as there are two numbers with the highest frequency.

b i $\text{Mean} = \frac{17+13+26+15+9+10}{6}$
$= 15$

The sum is 90 and there are 6 values.

ii 9 10 13 15 17 26
14

First, order the data.

$\text{Median} = \frac{13+15}{2}$
$= 14$

Since there are two values in the middle find the mean of them.

iii No mode

None of the values are repeated so there is no mode.

Now you try

For the given data sets, find the following:

i the mean
ii the median
iii the mode.

a 4 8 10 2 3 6 2
b 12 15 21 24 18 16 17 13

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Example 14 Finding a data value for a required mean

The hours a shop assistant spends cleaning the store in eight successive weeks are:
8, 9, 12, 10, 10, 8, 5, 10.

a Calculate the mean for this set of data.

b How many hours would the shop assistant need to clean in the ninth week for the mean to equal 10?

SOLUTION

a $\text{Mean} = \dfrac{8+9+12+10+10+8+5+10}{8}$

$= 9$

Sum of the 8 data values is 72.

b Let a be the number of hours in the ninth week.

$$\text{Require } \frac{72+a}{8+1} = 10$$
$$\frac{72+a}{9} = 10$$
$$72 + a = 90$$
$$a = 18$$

$72 + a$ is the total of the new data and $8 + 1$ is the new total number of data values. Set this equal to the required mean of 10. Solve for a.

The number of hours would need to be 18.

Write the answer.

Now you try

The amount of money a teenager earned in her last six babysitting appointments is:
\$15, \$25, \$35, \$18, \$20, \$25.

a Calculate the mean for this set of data.

b How much would the teenager need to earn on the next babysitting appointment for the mean to equal \$25?

Exercise 9H

FLUENCY	1, 2, 3(½), 4	1–3(½), 4	1–3(½), 4

Example 13

1 For the given data sets, find:

i the mean **ii** the median **iii** the mode.

a 6 13 5 4 16 10 3 5 10

b 10 17 5 16 4 14

c 3.5 2.1 4.0 8.3 2.1

d 0.7 3 2.9 10.4 6 7.2 1.3 8.5

e 6 0 −3 8 2 −3 9 5

f 3 −7 2 3 −2 −3 4

2 Decide if the following data sets are bi-modal.

a 2 7 9 5 6 2 8 7 4

b 1 6 2 3 3 1 5 4 1 9

c 10 15 12 11 18 13 9 16 17

d 23 25 26 23 19 24 28 26 27

3 These data sets include an outlier (an extreme value compared to the rest of the data). State the outlier, then calculate the mean and the median. Include the outlier in your calculations.

a 5 7 7 8 12 33

b 1.3 1.1 1.0 1.7 1.5 1.6 −1.1 1.5

c −58 −60 −59 −4 −64

d −7.5 −2.4 −5.6 −1.2 10

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4 In three races Paula recorded the times 25.1 seconds, 24.8 seconds and 24.1 seconds.
  a What is the mean time of the races? Round to two decimal places.
  b Find the median time.

PROBLEM–SOLVING	5, 6	5, 6, 8(½)	5, 7, 8

Example 14

5 A netball player scored the following number of goals in her 10 most recent games:
  15 14 16 14 15 12 16 17 16 15
  a What is her mean score?
  b What number of goals does she need to score in the next game for the mean of her scores to be 16?

6 Stevie obtained the following scores on her first five Mathematics tests:
  92 89 94 82 93
  a What is her mean test score?
  b If there is one more test left to complete, and she wants to achieve an average of at least 85, what is the lowest score Stevie can obtain for her final test?

7 Seven numbers have a mean of 8. Six of the numbers are 9, 7, 6, 4, 11 and 10. Find the seventh number.

8 Write down a set of 5 numbers which has the following values:
  a a mean of 5, median of 6 and mode of 7
  b a mean of 5, median of 4 and mode of 8
  c a mean of 4, median of 4 and mode of 4
  d a mean of 4.5, median of 3 and mode of 2.5
  e a mean of 1, median of 0 and mode of 5
  f a mean of 1, median of $1\frac{1}{4}$ and mode of $1\frac{1}{4}$.

REASONING	9, 10	9–11	10–12

9 The prices of six houses in Darwin are listed.
  \$324 000 \$289 000 \$431 000 \$295 000 \$385 000 \$1 700 000
  a What is the median house price?
  b The data set includes an outlier. Which price would be considered the outlier?
  c If the outlier was removed from the data set, by how much would the median change?
  d Find the mean house price to the nearest dollar.
  e If the outlier was removed from the data set, by how much would the mean change, to the nearest dollar?

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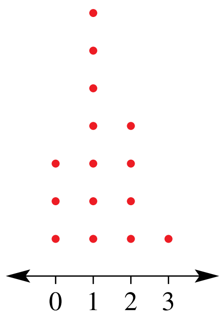
10 Explain why outliers significantly affect the mean but not the median.

11 This dot plot shows the frequency of households with 0, 1, 2 or 3 pets.



- **a** How many households were surveyed?
- **b** Find the mean number of pets.
- **c** Find the median number of pets.
- **d** Find the mode.
- **e** Another household with 7 pets is added to the list. Does this change the median? Explain.

12 This simple data set contains nine numbers: 1 2 2 2 2 2 3 4 5

- **a** Find the median.
- **b** How many numbers greater than 5 need to be added to the list to change the median? (Give the least number.)
- **c** How many numbers less than 1 need to be added to the list to change the median? (Give the least number.)

ENRICHMENT: Formula to get an A	–	–	13

13 A school awards grades in Mathematics each semester according to this table.

Average score	Grade
90–100	A^+
80–99	A
70–89	B^+
60–79	B
50–69	C^+
0–49	C

Ryan has scored the following results for four topics this semester and has one topic to go: 75 68 85 79

- **a** What is Ryan's mean score so far?
- **b** What grade will Ryan get for the semester if his fifth score is:
  - **i** 50? **ii** 68? **iii** 94?
- **c** Find the maximum average score Ryan can receive for the semester. Is it possible for him to get an A^+?
- **d** Find the least score that Ryan needs in his fifth topic for him to receive an average of:
  - **i** B^+ **ii** A.
- **e** Write a rule for the mean score M for the semester if Ryan receives a mark of m in the fifth topic.
- **f** Write a rule for the mark m in the fifth topic if Ryan scores an average of M for the semester.

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9I Stem-and-leaf plots CONSOLIDATING

Learning intentions
- To be able to construct and interpret a stem-and-leaf plot, both single and back-to-back
- To be able to interpret the shape of a stem-and-leaf plot in terms of symmetry and skewness

Past, present and future learning
- This section consolidates Stage 4 concepts which are used in Stage 5 and Stage 6 Advanced
- Some of these questions are more challenging than those in our Stage 4 books
- These concepts will be revised and extended in Chapter 9 of our Year 10 book and also in Stage 6

Stem-and-leaf plots (or stem plots) are commonly used to display a single data set or two related data sets. They help to show how the data is distributed, as in a histogram, but retain all the individual data elements so no detail is lost. The median and mode can be easily read from a stem-and-leaf plot because all the data values sit in order.

Japanese train timetables use a stem-and-leaf plot to display departure times. The 'stems' are hours, and each 'leaf' represents a train's departure time in minutes past the hour.

Lesson starter: Ships vs Chops

At a school, Ms Ships' class and Mr Chops' class sit the same exam. The scores are displayed using this back-to-back stem-and-leaf plot. Discuss the questions that follow.

Ms Ships' class		Mr Chops' class
3 1	5	0 1 1 3 5 7
8 8 7 5	6	2 3 5 5 7 9 9
6 4 4 2 1	7	8 9 9
7 4 3	8	0 3
6	9	1

7 | 8 means 78

- Which class had the greater number of students?
- What were the lowest and highest scores for each class?
- What were the median scores for each class?
- The results of which class could be described as symmetrical and which as skewed?
- Which class had the better results?

KEY IDEAS

- Stem-and-leaf plots are graphs used to display data involving a varying number of digits such as ages or test results.
- A **stem-and-leaf plot** uses a stem number and a leaf number to represent data.
  - The data is shown in two parts: a stem and a leaf.
  - The 'key' tells you how the plot is to be read.

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- The graph is similar to a histogram on its side or a bar graph with class intervals, but there is no loss of detail of the original data.

Ordered stem-and-leaf plot

Stem	Leaf
1	2 6
2	2 3 4 7
3	1 2 4 7 8 9
4	2 3 4 5 8
5	7 9

2 | 4 represents 24 people

A key is added to show the place value of the stems and leaves.

■ **Back-to-back stem-and-leaf plots** can be used to compare two sets of data. The stem is drawn in the middle with the leaves on either side.

Scores for the last 30 AFL games

Winning scores		Losing scores
	7	4 5 8 8 9
1	8	0 0 3 3 6 7
9 7 5	9	1 2 3 6
8 4 4 1	10	3 9
9 5 0	11	1
1	12	

10 | 9 represents 109

81 lowest winning score

Symmetrical

Positively Skewed

111 highest losing score

■ A **cluster** is a set of data values that are similar to each other. They form the peak of the distribution. In the data sets above, the losing scores are clustered around the high 70s and low 80s.

■ **Symmetrical data** will produce a graph that is evenly distributed above and below the central cluster.

■ **Skewed data** will produce a graph which includes data bunched to one side of the centre.
- Positively skewed data is bunched left of the centre
- Negatively skewed data is bunched right of the centre.

BUILDING UNDERSTANDING

1 This stem-and-leaf plot shows the length of time, in minutes, Alexis spoke on her phone for a number of calls.

a How many calls are represented by the stem-and-leaf plot?

b What is the length of the:

  i shortest phone call?

  ii longest phone call?

c What is the mode (the most common call time)?

Stem	Leaf
0	8
1	5 9
2	1 1 3 7
3	4 5

2 | 1 means 21 minutes

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2 This back-to-back stem-and-leaf plot shows the thickness of tyre tread on a selection of cars from the city and the country.

a How many car tyres were tested altogether?

b Is the distribution of tread thickness for city cars more symmetrical or skewed?

c Is the distribution of tread thickness for country cars more symmetrical or skewed?

City		Country
8 7 3 1 0 0 0	0	6 8 8 9
8 6 3 1 1 0	1	4 5 5 6 9
1	2	3 4 4

1 | 3 means 13 mm

Example 15 Constructing and using a stem-and-leaf plot

Consider this set of data.

0.3	2.5	4.1	3.7	2.0	3.3	4.8	3.3	4.6	0.1	4.1	7.5	1.4	2.4
5.7	2.3	3.4	3.0	2.3	4.1	6.3	1.0	5.8	4.4	0.1	6.8	5.2	1.0

a Organise the data into an ordered stem-and-leaf plot.

b Find the median.

c Find the mode.

d Describe the data as symmetrical or skewed.

SOLUTION

a

Stem	Leaf
0	1 1 3
1	0 0 4
2	0 3 3 4 5
3	0 3 3 4 7
4	1 1 1 4 6 8
5	2 7 8
6	3 8
7	5

3 | 4 means 3.4

EXPLANATION

The minimum is 0.1 and the maximum is 7.5 so stems range from 0 to 7.

Place leaves in order from smallest to largest. As some numbers appear more than once, e.g. 0.1, their leaf (1) appears the same number of times.

b Median $= \dfrac{3.3 + 3.4}{2}$

$= 3.35$

There are 28 data values. The median is the average of the two middle values (the 14th and 15th values).

c Mode is 4.1.

The most common value is 4.1.

d Data is approximately symmetrical.

The distribution of numbers is approximately symmetrical about the stem containing the median.

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Now you try

Consider this set of data.

1.4	2.0	6.4	0.2	3.5	2.3	4.7	3.1	5.6	4.8
5.1	0.3	4.3	3.4	1.4	2.5	3.8	4.4	3.9	5.7

a Organise the data into an ordered stem-and-leaf plot.
b Find the median.
c Find the mode.
d Describe the data as symmetrical or skewed.

Example 16 Constructing back-to-back stem-and-leaf plots

A shop owner has two jeans shops. The daily sales in each shop over a 16-day period are monitored and recorded as follows.

Shop A: 3 12 12 13 14 14 15 15 21 22 24 24 24 26 27 28
Shop B: 4 6 6 7 7 8 9 9 10 12 13 14 14 16 17 27

a Draw a back-to-back stem-and-leaf plot with an interval of 10.
b Compare and comment on differences between the sales made by the two shops.

SOLUTION

a

Shop A		Shop B
3	0	4 6 6 7 7 8 9 9
5 5 4 4 3 2 2	1	0 2 3 4 4 6 7
8 7 6 4 4 4 2 1	2	7

1 | 3 means 13 sales

b Shop A has the highest number of daily sales. Its sales are generally between 12 and 28, with one day of very low sales of 3.
Shop B sales are generally between 4 and 17 with only one high sale day of 27.

EXPLANATION

The data for each shop is already ordered. Stems are in intervals of 10. Record leaf digits for Shop A on the left and Shop B on the right.

Look at both sides of the plot for the similarities and differences.

Now you try

Two basketball teams scored the following points in their 10-game season:

Taipans: 71 62 75 88 73 67 64 72 68 78
JackJumpers: 87 76 82 94 88 75 81 76 97 89

a Draw a back to back stem-and-leaf plot with an interval of 10.
b Compare and comment on differences between the scores of the two teams.

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Exercise 9I

FLUENCY	1(½), 2–4	1(½), 2–4	1(½), 3–5

Example 15

1 For each of the following data sets:

- **i** organise the data into an ordered stem-and-leaf plot
- **ii** find the median
- **iii** find the mode
- **iv** describe the data as symmetrical or positively or negatively skewed.

a 2.3 4.5 6.2 2.4 4.1 3.8 3.2 2.3 5.5 5.9 1.8 3.6 4.8 3.7

b 41 33 28 24 19 32 54 35 26 28 19 23 32 26 28

c 31 33 23 35 15 23 48 50 35 42 15 21
51 31 34 23 42 50 26 30 45 37 39 45 45

d 34.5 34.9 33.7 34.5 35.8 33.8 34.3 35.2 37.0 34.7
35.2 34.4 35.5 36.5 36.1 33.3 35.4 32.0 36.3 34.8

e 167 159 159 193 161 164 167 157 158 175 177 185
177 202 185 187 159 189 167 159 173 198 200

2 The number of vacant rooms in a motel each week over a 20-week period is shown below.

12 8 11 10 21 12 6 11 12 16 14 22 5 15 20 6 17 8 14 9

a Draw a stem-and-leaf plot of this data.

b In how many weeks were there fewer than 12 vacant rooms?

c Find the median number of vacant rooms.

Example 16

3 For each of the following sets of data:

- **i** draw a back-to-back stem-and-leaf plot.
- **ii** compare and comment on the difference between the two data sets.

a **Set A:** 46 32 40 43 45 47 53 54 40 54 33 48 39 43
Set B: 48 49 31 40 43 47 48 41 49 51 44 46 53 44

b **Set A:** 0.7 0.8 1.4 8.8 9.1 2.6 3.2 0.3 1.7 1.9 2.5 4.1 4.3 3.3 3.4
3.6 3.9 3.9 4.7 1.6 0.4 5.3 5.7 2.1 2.3 1.9 5.2 6.1 6.2 8.3
Set B: 0.1 0.9 0.6 1.3 0.9 0.1 0.3 2.5 0.6 3.4 4.8 5.2 8.8 4.7 5.3
2.6 1.5 1.8 3.9 1.9 0.1 0.2 1.2 3.3 2.1 4.3 5.7 6.1 6.2 8.3

4 **a** Draw back-to-back stem-and-leaf plots for the final scores of St Kilda and Collingwood in the 24 games given here.

St Kilda:
126 68 78 90 87 118 88 125 111 117 82 82
80 66 84 138 109 113 122 80 94 83 106 68

Collingwood:
104 80 127 88 103 95 78 118 89 82 103 115
98 77 119 91 71 70 63 89 103 97 72 68

b In what percentage of games did each team score more than 100 points?

c Comment on the distribution of the scores for each team.

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5 The data below gives the maximum temperature each day for a three-week period in spring.

18	18	15	17	19	17	21
20	15	17	15	18	19	19
20	22	19	17	19	15	17

Use a stem-and-leaf plot to determine:

a the number of days on which the temperature was higher than 18°C

b the median temperature

c the difference between the minimum and maximum temperatures.

PROBLEM–SOLVING 6 | 6 | 6, 7

6 This stem-and-leaf plot shows the time taken, in seconds, by Helena to run 100 m in her last 25 races.

a Find Helena's median time.

b What is the difference between the slowest and fastest times?

c In her 26th race her time was 14.8 seconds. If this was added to the stem-and-leaf plot, would her median time change? If so, by how much?

Stem	Leaf
14	9
15	4 5 6 6 7 7 7 8 9
16	0 0 1 1 2 2 3 4 4 5 5 5 7 7
17	2

14 | 9 represents 14.9 seconds

7 Two brands of batteries were tested to determine their lifetimes in hours. The data below shows the lifetimes of 20 batteries of each brand.

Brand A:	7.3	8.2	8.4	8.5	8.7	8.8	8.9	9.0	9.1	9.2
	9.3	9.4	9.4	9.5	9.5	9.6	9.7	9.8	9.9	9.9
Brand B:	7.2	7.3	7.4	7.5	7.6	7.8	7.9	7.9	8.0	8.1
	8.3	9.0	9.1	9.2	9.3	9.4	9.5	9.6	9.8	9.8

a Draw a back-to-back stem-and-leaf plot for this data.

b How many batteries of each brand lasted more than 9 hours?

c Compare the two sets of data and comment on any similarities or differences.

REASONING 8, 9 | 8–10 | 8, 10, 11

8 This ordered stem-and-leaf plot has some unknown digits.

Stem	Leaf
0	2 3 8
1	1 5 a 8
c	0 b 6 6 2
3	2 5 9

3 | 5 means 0.35

a What is the value of c?

b What is the smallest number in the data set?

c What values could the following pronumerals take?

i a ii b

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9 The back-to-back stem-and-leaf plot below shows the birth weight in kilograms of babies born to mothers who do or don't smoke.

Birth weight of babies

Smoking mothers		Non-smoking mothers
4 3 2 2	2	4
9 9 8 7 6 6 5 5	2*	8 9
4 3 2 1 1 1 0 0 0	3	0 0 1 2 2 3
6 5 5	3*	5 5 5 6 6 7 7 8
1	4	
	4*	5 5 6

2 | 4 means 2.4 kg

2* | 5 means 2.5 kg

a What percentage of babies born to smoking mothers have a birth weight of less than 3 kg?
b What percentage of babies born to non-smoking mothers have a birth weight of less than 3 kg?
c Compare and comment on the differences between the birth weights of babies born to mothers who smoke and those born to mothers who don't smoke.

10 Explain why in a symmetrical distribution the mean is close to the median.

11 Find the median if all the data in each back-to-back stem-and-leaf plot was combined.

a

5	3	8 9
9 7 7 1	4	0 2 2 3 6 8
8 6 5 2 2	5	3 3 7 9
7 4 0	6	1 4

4 | 2 means 42

b

3	16	0 3 3 6 7 9
9 6 6 1	17	0 1 1 4 8 8
8 7 5 5 4 0	18	2 2 6 7
2	19	0 1

16 | 3 means 16.3

ENRICHMENT: How skewed? – – 12

12 As seen skewness can be positive or negative. If the tail of the distribution is pointing up in a stem-and-leaf plot (towards the smaller numbers) then we say the data is negatively skewed.

Stem	Leaf
1	3
2	1 4
3	0 2 7
4	1 1 3 8 9 9
5	0 4 4 5 7

Negatively skewed data

3 | 2 means 32

If the tail is pointing in the reverse direction then the data is positively skewed.
a Find the mean (correct to two decimal places) and the median for the above data.
b Which of the mean or median is higher for the given data? Can you explain why?
c Which of the mean or median would be higher for a set of positively skewed data? Why?
d What type of distribution would lead to the median and mean being quite close?

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The following problems will investigate practical situations drawing upon knowledge and skills developed throughout the chapter. In attempting to solve these problems, aim to identify the key information, use diagrams, formulate ideas, apply strategies, make calculations and check and communicate your solutions.

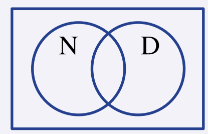
Medical trials

1 A team of medical researchers is involved in two trials. One trial will record the effect of a new drug on a group of patients with a certain condition and the second trial involves testing for a particular condition in its early stages.

The researchers need to be able to approach pharmaceutical companies to fund their drug trials and testing. To do so they need to be able to show some evidence of the impact of each of trial to show that it is worth pursuing.

a For the first phase of the trial, 60 patients with the condition were involved in a drug trial. Half were given the new drug and half were given a placebo. The result recorded indicated whether or not a patient was still showing symptoms after one week.
After one week it was found that 32 patients showed no symptoms, with 10 of those on the placebo.

i Use this Venn diagram to summarise the results. D represents the set of patients taking the drug and N represents the set of patients with no symptoms after a week.



ii Find the probability that a randomly selected patient took the drug and had no symptoms after one week.

iii Find the probability that a randomly selected patient did not take the drug and had no symptoms after one week.

iv Based on your results to parts **ii** and **iii**, can you hypothesise whether you think the drug had an impact?

v If this drug was used to treat all patients who have the condition, based on the data what percentage could be expected to show no symptoms within a week? Round to the nearest per cent.

vi If you had to promote this drug to a pharmaceutical company or in an advertising campaign, which two statistics would you use based on the data?

The drug was also trialled on a group of patients who did not have the condition and some side effects were noted. The researchers determine that to be prescribed the drug, a test must be used in the early stages to try to determine if a patient's symptoms indicate they have the condition.
The researchers must first consider the accuracy of the test.

b The test was carried out on a group of 40 patients – some who went on to develop the condition and some who did not. The results of the test were that 29 people tested positive, and of those 27 people had the condition. In total, 30 people had the condition.

i Summarise these findings in a two-way table where C is the set of patients who did have the condition and T is the set of patients who tested positive to the condition.

	T	Not T	Total
C			
Not C			
Total			

ii Based on the results, determine the probability of 'C and T' and the probability of 'C or T' and interpret these sets of patients.

iii A 'false positive' is a result in which a person tests positive but does not have the condition. From this data, what percentage of tests resulted in a false positive?

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iv Based on this data, in what percentage of patients did the test not detect the condition when it was actually present?

v The researchers will receive extra funding to refine the test if fewer than 10% of patients get an incorrect result. Will the researchers get the extra funding?

Raising the average

2 Statisticians are now commonly connected to both sporting and commentary teams. Statistics are used to compare performances and to analyse opposition players and teams.

For batsmen in cricket, averages are particularly important. In cricket, if you finish an innings 'not out', the runs are added to your total runs but the average is calculated by dividing by the number of 'outs', not innings. For example, for the scores 30, 26 and 10 the average is 22, but for scores of 30, 26 and 10 not out the average is 33.

In a season of cricket in Australia, sports journalists spend time analysing statistics to highlight under-performing players and discuss possible selection issues by looking at scores achieved and averages, and by comparing players.

a A batsman is under pressure heading into the final Test match. He only gets to bat once in the match and he is keen to raise his average above 30. From the six previous completed innings he has played, he has averaged 26.5. What is the lowest score he can get to achieve this average if:

i he gets out in the innings?

ii he is not out at the end of the innings?

b Give a possible set of scores the batsman in part **a** had for his 7 innings if his mode score was 22, his median was 25, he achieved his average of at least 30 and he was out in each innings.

The opening batsman combination is considered not to have worked throughout the series. The team batted 10 times, but one of the batsman was injured in one game and only batted in 9 innings. Both openers were out each time they batted. Their scores are shown below.

Opener 1: 18 6 21 18 31 42 23 25 31

Opener 2: 4 4 11 24 107 52 13 27 8 6

c i Display the data using a back-to-back stem-and-leaf plot and compare/comment on their performances.

ii The commentators are calling for opener 1 to be left out of the team. Do you think this is reasonable? What statistic might they be basing this on?

iii The data for opener 1 was incorrectly entered. The score of 21 is incorrect. Once the new score is entered, the median changes to 25 but the range is unchanged. What are the possible scores that opener 1 obtained in this innings?

d After the next three Test match series, a batsman is dropped from the team. In the two series this batsman had averaged 23 from 8 completed innings in the first series and 18.2 from 5 completed innings in the second series. What was the combined average for the two series, correct to one decimal place?

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Winning odds

3 A betting agency sets odds on a range of sports around the world. They do so by analysing past performances and then setting the odds – trying to keep them in their favour!

A particular up-and-coming betting agency is analysing the previous head-to-head performances of tennis players, in order to set the odds for the winner of the match, as well as updating the odds throughout a match. They use various simulations to predict the chances of different outcomes.

Two particular players have split the sets they have played against each other in the past 50–50. The match can be simulated with a coin, as each player is equally likely to win a set.

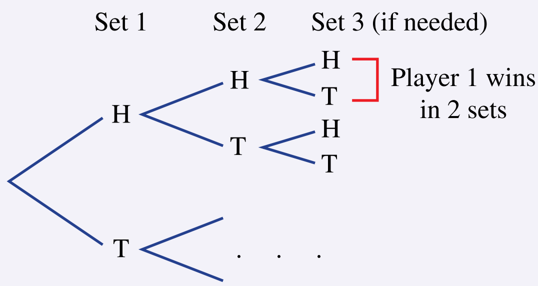
In women's matches, the winner is the first to win two sets.

a Complete the tree diagram of a coin toss simulating the three possible sets. Player 1 wins the set if heads and player 2 wins if tails.

Use the tree diagram to find:

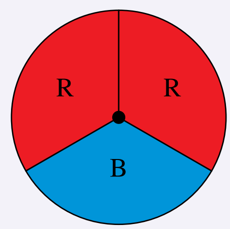
i the probability that the match is completed in two sets

ii the probability the winner of the match lost the first set.



In the next round, the head-to-head battle shows that one player has won two-thirds of the sets she has played against her opponent. This can be simulated with a spinner which has three equal regions, two coloured red and one coloured blue.

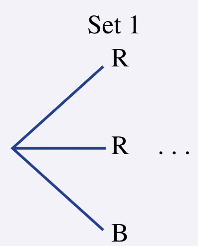
The spinner is spun to determine the winner of each set. The third set will not be played if a winner has been determined.



b Complete a tree diagram to find the probability that:

i the match lasted three sets

ii the non-favourite wins at least one set.



c At tournaments around the world, the scoring system in the final set and the tiebreaker is being altered. At a particular tournament the women's game is trialling a first to 7 tiebreaker that doesn't require a two-point advantage. Two close rivals are considered equally likely to win a point in a tiebreaker whether they are serving or not. Calculate the percentage chance of each player winning from the following positions:

i 6 points to 5 (*Hint*: At most two points will need to be played.)

ii 6 points to 4 (*Hint*: Consider how many points might be needed and how to simulate this.)

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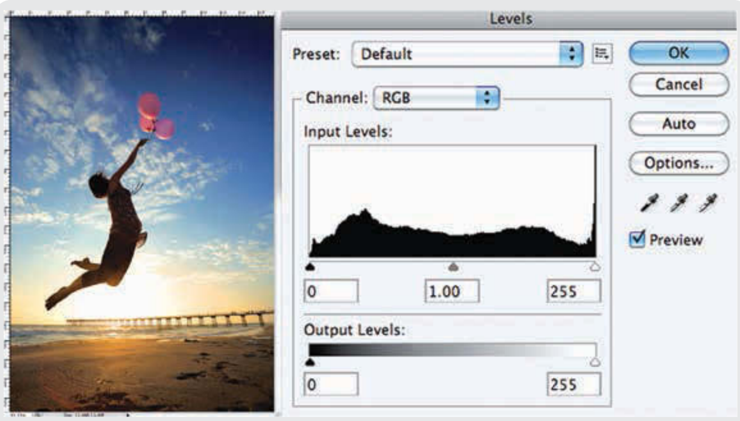
9J Grouping data into classes EXTENDING

Learning intentions
- To be able to construct a frequency table from a set of data
- To be able to construct and analyse a histogram or percentage frequency histogram

Past, present and future learning
- This section goes beyond the Stage 5 Path Topic called *Data Analysis C*
- These concepts will be revised and extended in Chapter 9 of our Year 10 book and also in Stage 6 Advanced

For some data, especially large sets, it makes sense to group the data and then record the frequency for each group to produce a frequency table. For numerical data, a graph generated from a frequency table gives a histogram. Like a stem-and-leaf plot, a histogram shows how the data is distributed across the full range of values. A histogram, for example, is used to display the level of exposure of the pixels in an image in digital photography. It uses many narrow columns to show how the luminance values are distributed across the scale from black to white.

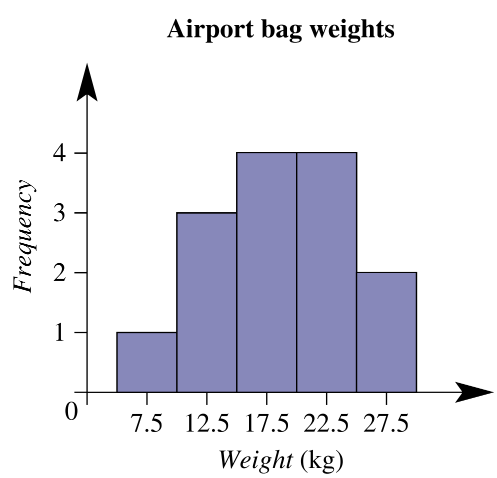


A luminance value histogram is shown as used in digital photography software. The horizontal axis shows brightness levels in a range starting at zero (pure black) on the left and finishing at 255 (pure white) on the right. The vertical axis shows the number of pixels of each brightness value that the photo contains.

Lesson starter: Baggage check

The histogram shows the distribution of the weight of a number of bags checked at the airport.

- How many bags had a weight in the range 10 to 15 kg?
- How many bags were checked in total?
- Is it possible to determine the exact mean, median or mode of the weight of the bags by looking at the histogram? Discuss.
- Describe the distribution of checked bag weights for the given graph.



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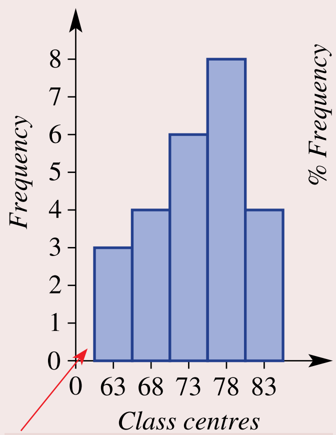
KEY IDEAS

- A **frequency table** shows the number of values within a set of categories or **class intervals**.
- Grouped numerical data can be illustrated using a **histogram**.
  - The height of a column corresponds to the frequency of values in that class interval.
  - There are usually no gaps between columns.
  - The scales are evenly spread with each bar spreading across the boundaries of the class interval.
  - A **percentage frequency histogram** shows the frequencies as percentages of the total.
- Like a stem-and-leaf plot, a histogram can show if the data is **skewed** or **symmetrical**.

Frequency table

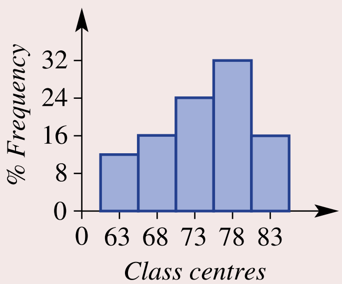
Class	Class centre	Frequency	Percentage frequency
61–65	63	3	12
66–70	68	4	16
71–75	73	6	24
76–80	78	8	32
81–85	83	4	16
	Total	25	100

Frequency histogram



This gap should be half as wide as the columns

Percentage frequency histogram



- In this frequency table, 70– includes numbers from 70 to less than 75.

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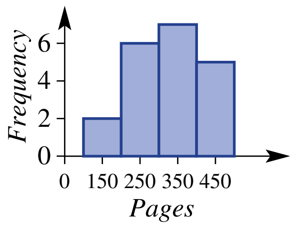
BUILDING UNDERSTANDING

1 State the missing numbers in this frequency table, i.e. find the values of the pronumerals.

Class interval	Frequency	Percentage frequency
0–4	1	10
5–9	3	c
10–14	4	d
a–19	b	e
Total	10	f

2 This frequency histogram shows the number of books in which the number of pages is within a given interval for some textbooks selected from a school library.



a How many textbooks had between 100 and 200 pages?
b How many textbooks were selected from the library?
c What percentage of textbooks had between:
  i 200 and 300 pages?
  ii 200 and 400 pages?

Example 17 Constructing frequency tables and histograms

The data below shows the number of hamburgers sold each hour by a 24-hour fast-food store during a 50-hour period.

1	10	18	14	20	11	19	10	17	21
5	16	7	15	21	15	10	22	11	18
12	12	3	12	8	12	6	5	14	14
14	4	9	15	17	19	6	24	16	17
14	11	17	18	19	19	19	18	18	20

a Set up and complete a grouped frequency table, using class intervals 0–4, 5–9, etc. Include a percentage frequency column.
b Construct a frequency histogram.
c In how many hours did the fast-food store sell:
  i fewer than 10 hamburgers?
  ii at least 15 hamburgers?

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SOLUTION

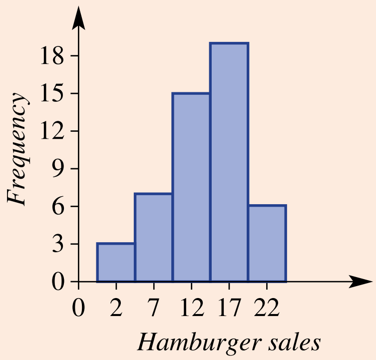
EXPLANATION

a

Class interval	Frequency	Percentage frequency
0–4	3	6%
5–9	7	14%
10–14	15	30%
15–19	19	38%
20–24	6	12%
Total	50	100%

Create class intervals of 5 from 0 up to 25, since 24 is the maximum number. Record the number of data values in each interval in the frequency column. Convert to a percentage by dividing by the total (50) and multiplying by 100.

b



Create a frequency histogram with frequency on the vertical axis and the class intervals on the horizontal axis. The height of the column shows the frequency of that interval.

c **i** $3 + 7 = 10$ hours

Fewer than 10 hamburgers covers the 0–4 and 5–9 intervals.

ii $19 + 6 = 25$ hours

At least 15 hamburgers covers the 15–19 and 20–24 intervals.

Now you try

The heights of 40 students were measured to the nearest centimetre and recorded.

160	149	153	143	179	159	152	161	169	157
167	163	148	175	173	150	175	160	170	162
145	158	163	164	172	178	169	171	153	165
152	161	170	174	166	165	158	157	152	148

a Set up and complete a grouped frequency table, using class intervals 140–144, 145–149, etc. Include a percentage frequency column.

b Construct a frequency histogram.

c How many students are:

i shorter than 150 cm?

ii at least 170 cm tall?

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Exercise 9J

FLUENCY	1, 2, 4	1, 3, 4	1, 3, 4

Example 17

1 The data below shows the number of ice-creams sold from an ice-cream van over a 50-day period.

0	5	0	35	14	15	18	21	21	36
45	2	8	2	2	3	17	3	7	28
35	7	21	3	46	47	1	1	3	9
35	22	7	18	36	3	9	2	2	11
37	37	45	11	12	14	17	22	1	2

a Set up and complete a grouped frequency table using class intervals 0–9, 10–19, etc.
Include a percentage frequency column.

b Construct a frequency histogram.

c On how many days did the ice-cream van sell:

i fewer than 20 ice-creams?

ii at least 30 ice-creams?

2 The data below shows the mark out of 100 on the Science exam for 60 Year 9 students.

50	67	68	89	82	81	50	50	89	52	60	82	52	60	87	89	71	73	75	83
86	50	52	71	80	95	87	87	87	74	60	60	61	63	63	65	82	86	96	88
50	94	87	64	64	72	71	72	88	86	89	69	71	80	89	92	89	89	60	83

a Set up and complete a grouped frequency table, using class intervals 50–59, 60–69, etc.
Include a percentage frequency column, rounding to two decimal places where necessary.

b Construct a frequency histogram.

c **i** How many marks were less than 70 out of 100?

ii What percentage of marks were at least 80 out of 100? Answer correct to one decimal place.

3 The number of goals kicked by a country footballer in each of his last 30 football matches is given below.

8	9	3	6	12	14	8	3	4	5	2	5	6	4	13
8	9	12	11	7	12	14	10	9	8	12	10	11	4	5

a Organise the data into a grouped frequency table using a class interval width of 3 starting at 0.

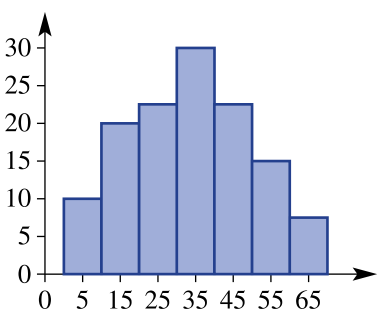
b Draw a frequency histogram for the data.

c In how many games did the player kick fewer than six goals?

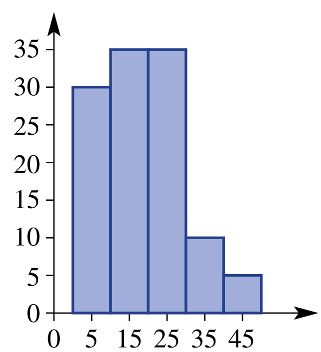
d In how many games did he kick more than 11 goals?

4 Which one of these histograms illustrates a symmetrical data set and which one shows a skewed data set?

a



b



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PROBLEM–SOLVING	5, 6	5, 6	5–7

5 Find the unknown numbers (pronumerals) in these frequency tables.

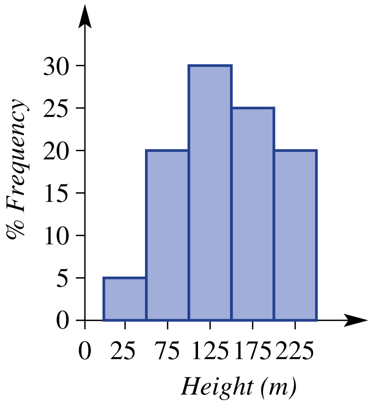
a

Class interval	Frequency	Percentage frequency
10–14	a	15
15–19	11	b
20–24	7	c
25–29	d	10
30–34	e	f
Total	40	g

b

Class interval	Frequency	Percentage frequency
0–2	a	2
3–5	9	b
6–8	c	16
9–11	12	d
12–14	e	f
Total	50	g

6 This percentage frequency histogram shows the heights of office towers in a city.



a What percentage of office towers have heights of:
- **i** between 50 m and 100 m?
- **ii** less than 150 m?
- **iii** no more than 200 m?
- **iv** at least 100 m?
- **v** between 100 m and 150 m or greater than 200 m?

b If there are 100 office towers in the city, how many have a height of:
- **i** between 100 m and 150 m?
- **ii** at least 150 m?

c If the city had 40 office towers, how many have a height of:
- **i** between 0 m and 50 m?
- **ii** no more than 150 m?

7 The data below shows the length of overseas phone calls (in minutes) made by a particular household over a six-week period.

1.5	1	1.5	1.4	8	4	4	10.1	9.5	1	3
8	5.9	6	6.4	7	3.5	3.1	3.6	3	4.2	4.3
4	12.5	10.2	10.3	4.5	4.5	3.4	3.5	3.5	5	3.5
3.6	4.5	4.5	12	11	12	14	14	12	13	10.8
12.1	2.4	3.8	4.2	5.6	10.8	11.2	9.3	9.2	8.7	8.5

What percentage of phone calls were more than 3 minutes in length? Answer to one decimal place.

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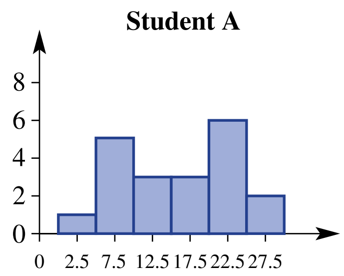
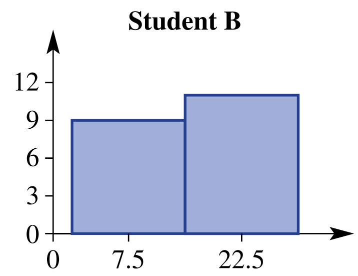
REASONING	8	8, 9	8, 10

8 Explain why you cannot work out the exact value for the mean, median and mode for a set of data presented in a histogram.

9 What can you work out from a frequency histogram that you cannot work out from a percentage frequency histogram? Completing Question 6 will provide a clue.

10 Two students show different histograms for the same set of data.





a Which histogram is more useful in helping to analyse the data?
b What would you advise student B to do differently when constructing the histogram?

ENRICHMENT: The distribution of weekly wages	–	–	11

11 The data below shows the weekly wages of 50 people in dollars.

400	500	552	455	420	424	325	204	860	894	464	379	563
940	384	370	356	345	380	720	540	654	678	628	656	670
740	750	730	766	760	700	700	768	608	576	890	920	874
450	674	725	612	605	600	548	670	230	725	860		

a What is the minimum weekly wage and the maximum weekly wage?
b i Organise the data into up to 10 class intervals.
ii Draw a frequency histogram for the data.
c i Organise the data into up to 5 class intervals.
ii Draw a frequency histogram for the data.
d Discuss the shapes of the two graphs. Which graph better represents the data and why?

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9K Measures of spread: range and interquartile range

Learning intentions
- To understand the concept of quartiles for a set of data
- To be able to find the 5-number summary for a set of data
- To understand how the range and interquartile range describe the spread of a data set

Past, present and future learning
- This section addresses the Stage 5 Core Topic called *Data Analysis A*
- These concepts will be revised and extended in Chapter 9 of our Year 10 book and also in Stage 6 Advanced

The mean, median and mode are three numbers that help define the centre of a data set; however, it is also important to describe the spread. Two teams of swimmers from different countries, for example, might have similar mean race times but the spread of race times for each team could be very different.

Planning future food supply requires statistics. Weather statisticians analyse rainfall and temperatures, providing measures of centre and spread, and climate trends. Agricultural statisticians analyse land use data, such as these irrigated vineyards near Mildura, Victoria.

Lesson starter: Swim times

Two Olympic swimming teams are competing in the 4×100 m relay. The 100 m personal best times, in seconds, for the four members of each team are given.

Team A: 48.3 48.5 48.9 49.2

Team B: 47.4 48.2 49.0 51.2

- Find the mean time for each team.
- Which team's times are more spread out?
- What does the difference in the range of values for each team tell you about the spread?

KEY IDEAS

- Two measures that help to describe how data is spread are the **range** and **interquartile range**. These are called measures of spread.
- The **range** is the difference between the maximum and minimum values.
  - Range = maximum value − minimum value
- If a set of numerical data is placed in order from smallest to largest, then:
  - the middle number of the lower half is called the **lower quartile** (Q_1)
  - the middle number of the data is called the median (Q_2)
  - the middle number of the upper half is called the **upper quartile** (Q_3)

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- the difference between the upper quartile and lower quartile is called the **interquartile range** (IQR).

$$IQR = Q_3 - Q_1$$

- If there is an odd number of values, remove the middle value (the median) before calculating Q_1 and Q_3.
- An **outlier** is a value that is not in the vicinity of the rest of the data.

BUILDING UNDERSTANDING

1 State the missing words.

- **a** To find the range, you subtract the __________ from the __________.
- **b** To find the interquartile range, you subtract the __________ from the __________.

2 This ordered data set shows the number of hours of sleep 11 people had the night before.

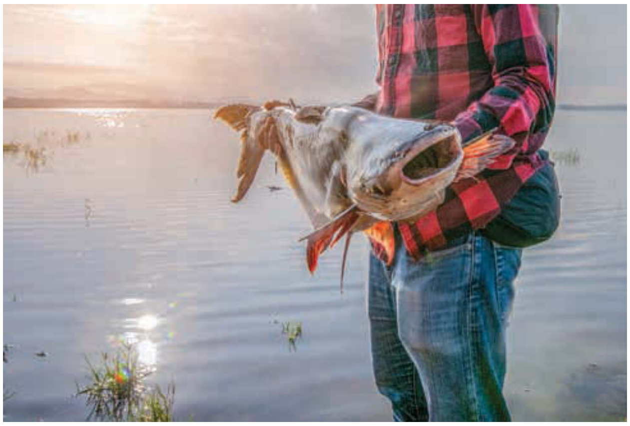
3 5 5 6 7 7 7 8 8 8 9

- **a** State the range (the difference between the maximum and minimum values).
- **b** State the median (Q_2, the middle number).
- **c** Remove the middle number then find:
  - **i** the lower quartile (Q_1, the middle of the lower half)
  - **ii** the upper quartile (Q_3, the middle of the upper half).
- **d** Find the interquartile range (IQR, the difference between the upper quartile and lower quartile).

3 This ordered data set shows the number of fish Daniel caught in the 12 weekends that he went fishing during the year.

1 2 3 3 4 4 6 7 7 9 11 13

- **a** Find the range (the difference between the maximum and minimum values).
- **b** Find the median (Q_2, the middle number).
- **c** Split the data in half then find:
  - **i** the lower quartile (Q_1, the middle of the lower half)
  - **ii** the upper quartile (Q_3, the middle of the upper half).
- **d** Find the interquartile range (IQR, the difference between the upper quartile and lower quartile).



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Example 18 Finding the range and quartiles for an odd number of data values

The following data values are the results for a school Mathematics test.

67 96 62 85 73 56 79 19 76 23 68 89 81

a List the data in order from smallest to largest.

b Find the range.

c Find:

i the median (Q_2)

ii the lower quartile (Q_1)

iii the upper quartile (Q_3)

iv the interquartile range (IQR).

SOLUTION

a $19\ 23\ \underbrace{56\ 62}_{Q_1}\ 67\ 68\ \underbrace{73}_{Q_2}\ 76\ 79\ \underbrace{81\ 85}_{Q_3}\ 89\ 96$

b $\text{Range} = 96 - 19$
$= 77$

c **i** $Q_2 = 73$

ii $Q_1 = \dfrac{56 + 62}{2}$
$= 59$

iii $Q_3 = \dfrac{81 + 85}{2}$
$= 83$

iv $\text{IQR} = 83 - 59$
$= 24$

EXPLANATION

Order the data.

Range = maximum value − minimum value.

The middle number is 73.

Q_1 (the middle value of the lower half) is halfway between 56 and 62. As there is an odd number of values, exclude the number 73 before finding Q_1 and Q_3.

Q_3 (the middle value of the upper half) is halfway between 81 and 85.

$\text{IQR} = Q_3 - Q_1$

Now you try

The following data values are the percentage results for a student's spelling tests over a 9-week term.

83 90 76 81 72 84 92 78 86

a List the data in order from smallest to largest.

b Find the range.

c Find:

i the median (Q_2)

ii the lower quartile (Q_1)

iii the upper quartile (Q_3)

iv the interquartile range (IQR).

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Example 19 Finding quartiles for an even number of data values

Here is a set of measurements collected by measuring the lengths, in metres, of 10 long-jump attempts.

6.7 9.2 8.3 5.1 7.9 8.4 9.0 8.2 8.8 7.1

a List the data in order from smallest to largest.

b Find the range.

c Find:

  i the median (Q_2)

  ii the lower quartile (Q_1)

  iii the upper quartile (Q_3)

  iv the interquartile range (IQR).

d Interpret the IQR.

SOLUTION	EXPLANATION
a 5.1 6.7 $\underset{Q_1}{7.1}$ 7.9 $\underbrace{8.2 \mid 8.3}_{Q_2}$ 8.4 $\underset{Q_3}{8.8}$ 9.0 9.2	Order the data to locate Q_1, Q_2 and Q_3.
b Range $= 9.2 - 5.1$ $= 4.1$ m	Range = maximum value − minimum value.
c i $Q_2 = \dfrac{8.2 + 8.3}{2}$ $= 8.25$ m	Q_2 is halfway between 8.2 and 8.3.
ii $Q_1 = 7.1$ m	The middle value of the lower half is 7.1.
iii $Q_3 = 8.8$ m	The middle value of the upper half is 8.8.
iv IQR $= 8.8 - 7.1$ $= 1.7$ m	IQR is the difference between Q_1 and Q_3.
d The middle 50% of jumps differed by less than 1.7 m.	The IQR is the range of the middle 50% of the data.

Now you try

The weights, in kg, of 14 parcels lodged at the post office an hour before closing are listed:

1.6 0.8 2.3 2.6 3.1 1.2 1.8 2.0 2.3 3.5 1.7 1.1 0.4 2.8

a List the data in order from smallest to largest.

b Find the range.

c Find:

  i the median (Q_2)

  ii the lower quartile (Q_1)

  iii the upper quartile (Q_3)

  iv the interquartile range (IQR).

d Interpret the IQR.

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Exercise 9K

FLUENCY	1(½), 2, 3	1(½), 2, 3	1(⅓), 3

Example 18

1 Consider the sets of data given below.
   i List the set of data in order from smallest to largest.
   ii Find the range.
   iii Find the median (Q_2).
   iv Find the lower quartile (Q_1).
   v Find the upper quartile (Q_3).
   vi Find the interquartile range (IQR).

a 5 7 3 6 2 1 9 7 11 9 0 8 5
b 38 36 21 18 27 41 29 35 37 30 30 21 26
c 180 316 197 176 346 219 183 253 228
d 256 163 28 520 854 23 367 64 43 787 12 343 76 3 28
e 1.8 1.9 1.3 1.2 2.1 1.2 0.9 1.7 0.8
f 10 35 0.1 2.3 23 12 0.02

Example 19

2 The top 10 highest box office takings for films, in millions of dollars, are given:
   1405 1519 1672 2049 2187 2788 2068 1342 1516 1347
a Find the range.
b Find:
   i the median (Q_2)
   ii the lower quartile (Q_1)
   iii the upper quartile (Q_3)
   iv the interquartile range (IQR).
c Interpret the IQR.

3 The running time, in minutes, of 16 movies at the cinema were as follows:
   123 110 98 120 102 132 112 140 120 139 42 96 152 115 119 128
a Find the range.
b Find:
   i the median (Q_2)
   ii the lower quartile (Q_1)
   iii the upper quartile (Q_3)
   iv the interquartile range (IQR).
c Interpret the IQR.

PROBLEM-SOLVING	4, 5	4–6	5, 6

4 The following set of data represents the sale price, in thousands of dollars, of 14 vintage cars.
   89 46 76 41 12 52 76 97 547 59 67 76 78 30
a For the 14 vintage cars, find:
   i the lowest price paid
   ii the highest price paid
   iii the median price
   iv the lower quartile of the data
   v the upper quartile of the data
   vi the IQR.
b Interpret the IQR for the price of the vintage cars.
c If the price of the most expensive vintage car increased, what effect would this have on Q_1, Q_2 and Q_3? What effect would it have on the mean price?

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5 Find the interquartile range for the data in these stem-and-leaf plots.

a

Stem	Leaf
3	4 8 9
4	1 4 8 8
5	0 3 6 9
6	2 6

5 | 2 means 52

b

Stem	Leaf
17	5 8
18	0 4 6 7
19	1 1 2 9 9
20	4 4 7 8
21	2 6 8

21 | 3 means 21.3

6 Over a period of 30 days, Lara records how many fairies she sees in the garden each day. The data is organised into this frequency table.

Number of fairies each day	0	1	2	3	4	5
Frequency	7	4	8	4	6	1

a Find the median number of fairies seen in the 30 days.
b Find the interquartile range.
c If Lara changes her mind about the day she saw 5 fairies and now says that she saw 10 fairies, would this change the IQR? Explain.

REASONING	7	7, 8	8, 9

7 Two data sets have the same range. Does this mean they have the same lowest and highest values? Explain.

8 A car yard has more than 10 cars listed for sale. One expensive car is priced at \$600 000 while the remaining cars are all priced near \$40 000. The salesperson realises that there is an error in the price for the expensive car – there is one too many zeros printed on the price.
a Does the change in price for the expensive car change the value of the range? Give reasons.
b Does the change in price for the expensive car change the value of the median? Give reasons.
c Does the change in price for the expensive car change the value of the IQR? Give reasons.

9 a Is it possible for the range to equal the IQR? If so, give an example.
b Is it possible for the IQR to equal zero? If so, give an example.

ENRICHMENT: How many lollies in the jar?	–	–	10

10 The following two sets of data represent the number of jelly beans found in 10 jars purchased from two different confectionery stores, A and B.

Shop A: 25 26 24 24 28 26 27 25 26 28
Shop B: 22 26 21 24 29 19 25 27 31 22

a Find Q_1, Q_2 and Q_3 for:
i shop A
ii shop B.
b The top 25% of the data is above which value for shop A?
c The lowest 25% of the data is below which value for shop B?
d Find the interquartile range (IQR) for the number of jelly beans in the 10 jars from:
i shop A
ii shop B.
e By looking at the given sets of data, why should you expect there to be a significant difference between the IQR of shop A and the IQR of shop B?
f Which shop offers greater consistency in the number of jelly beans in each jar it sells?

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9L Box plots

Learning intentions

- To understand the features of a box plot in describing the spread of a set of data
- To know how to construct a box plot with outliers
- To be able to compare data sets using parallel box plots

Past, present and future learning

- This section addresses the Stage 5 Core Topic called *Data Analysis A*
- These concepts will be revised and extended in Chapter 9 of our Year 10 book and also in Stage 6 Advanced

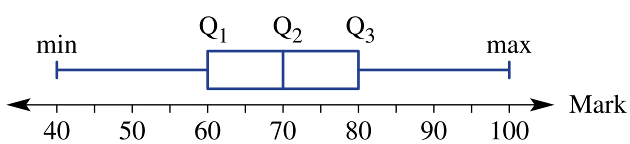
A box plot is a commonly used graph for a data set showing the maximum and minimum values, the median and the upper and lower quartiles. Box plots are often used to show how a data set is distributed and how two sets compare. Box plots are used, for example, to compare a school's examination performance against the performance of all schools in a state.

Every business seeks a competitive advantage from the statistical analysis of its data, including marketing, production, sales and profit. Data scientists who have computer algorithm skills in statistics and analytics are in high demand.

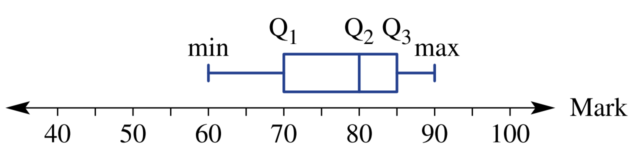
Lesson starter: School performance

These two box plots show the performance of two schools on their English exams.

Box High School



Plot Secondary College



- Which school produced the highest mark?
- Which school produced the highest median (Q_2)?
- Which school produced the highest IQR?
- Which school produced the highest range?
- Describe the performance of Box High School against Plot Secondary College. Which school has better overall results and why?

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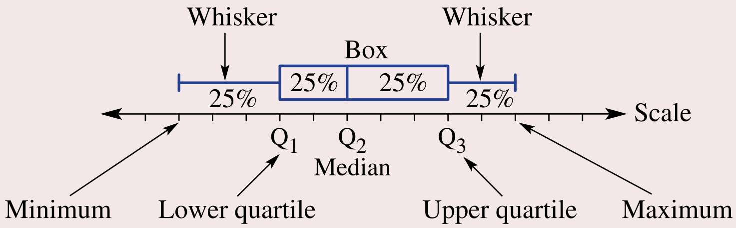
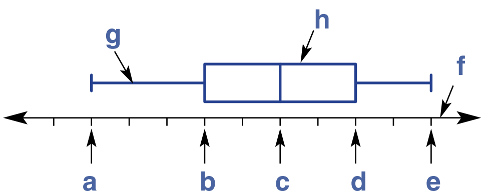
KEY IDEAS

- A **box plot** (also called a box-and-whisker plot) is a graph which shows:
  - maximum and minimum values
  - median (Q_2)
  - lower and upper quartiles (Q_1 and Q_3).
- A quarter (25%) of the data is spread across each of the four sections of the graph.



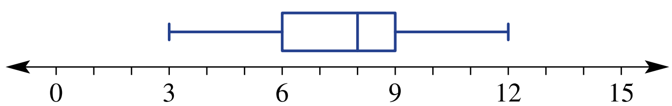
BUILDING UNDERSTANDING

1 State the names of the features labelled **a** to **h** on this box plot.

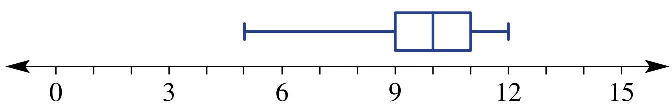


2 Choose the correct box plot for a data set which has all these measures:
- minimum = 3
- maximum = 12
- median = 9
- lower quartile = 5
- upper quartile = 11

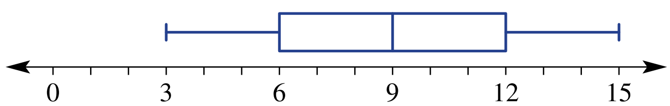
A



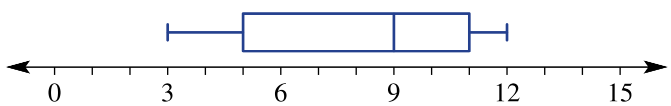
B



C



D



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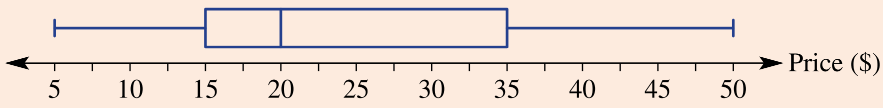
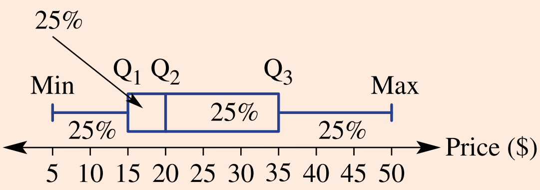
Example 20 Interpreting a box plot

This box plot summarises the price of all the books in a book shop.



a State the minimum and maximum book prices.
b Find the range of the book prices.
c State the median book price.
d Find the interquartile range.
e Fifty per cent of the books are priced below what amount?
f Twenty-five per cent of the books are priced above what amount?
g If there were 1000 books in the store, how many would be priced below $15?



SOLUTION	EXPLANATION
a Minimum book price = $5 Maximum book price = $50	
b Range = $50 − $5 = $45	Range = maximum price − minimum price
c Median book price = $20	The median is Q_2.
d Interquartile range = $35 − $15 = $20	Interquartile range = $Q_3 - Q_1$
e $20	50% of books are below Q_2.
f $35	The top 25% of books are above Q_3.
g $0.25 \times 1000 = 250$ 250 books would be below $15.	25% of books are priced below $15.

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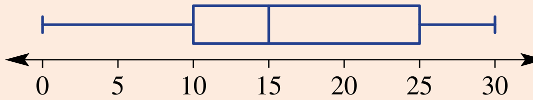
Now you try

This box plot summarises the number of air-conditioning units sold at an electrical store each day in summer.



a State the minimum and maximum number of units sold in a day.
b Find the range of the number of sales.
c State the median number of sales.
d Find the interquartile range.
e Fifty per cent of days had sales below what number?
f Twenty-five per cent of days had sales above what number?
g If there were 80 days of sales, on how many days were there fewer than 10 sales?

Example 21 Constructing a box plot

The following set of data represents the 11 scores resulting from throwing two dice and adding their scores.

7 10 7 12 8 9 6 6 5 4 8

a Find:
- **i** the minimum value
- **ii** the maximum value
- **iii** the median
- **iv** the lower quartile
- **v** the upper quartile.

b Draw a box plot to represent the data.

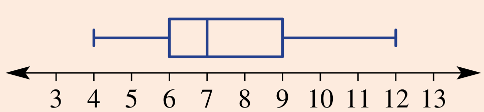
SOLUTION

a

		Q_1			Q_2			Q_3		
4	5	6	6	7	7	8	8	9	10	12

- **i** Min. value = 4
- **ii** Max. value = 12
- **iii** $Q_2 = 7$
- **iv** $Q_1 = 6$
- **v** $Q_3 = 9$

b Box plot: Throwing two dice



EXPLANATION

Order the data.

Determine the minimum and maximum value.

The median is the middle value.

Remove the middle value then locate the lower quartile and upper quartile.

Draw a scaled horizontal axis.

Place the box plot above the axis marking in the five key statistics from part **a**.

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Now you try

The following set of data represents the hours worked by a shift worker in their last 15 shifts.

6 8 4 2 6 6 7 3 5 5 6 8 7 3 5

a Find:

i the minimum value **ii** the maximum value **iii** the median

iv the lower quartile **v** the upper quartile.

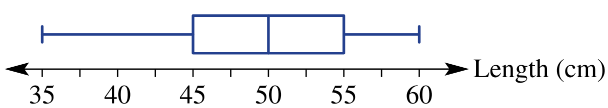
b Draw a box plot to represent the data.

Exercise 9L

FLUENCY	1, 2, 3(½)	1, 3(½)	2, 3(½)

Example 20

1 This box plot summarises the lengths of babies born in a particular week in a hospital.



a State the minimum and maximum lengths of babies.

b Find the range of the length of the babies.

c State the median baby length.

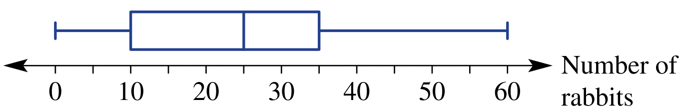
d Find the interquartile range.

e Fifty per cent of the babies were born shorter than what length?

f Twenty-five per cent of the babies were born longer than what length?

g If there were 80 babies born in the week, how many would be expected to be less than 45 cm in length?

2 This box plot summarises the number of rabbits spotted per day in a paddock over a 100-day period.



a State the minimum and maximum number of rabbits spotted.

b Find the range of the number of rabbits spotted.

c State the median number of rabbits spotted.

d Find the interquartile range.

e On 75% of the days, the number of rabbits spotted is below what number?

f On 50% of the days, the number of rabbits spotted is more than what number?

g On how many days was the number of rabbits spotted less than 10?

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Example 21

3 Consider the sets of data below.

- **i** State the minimum value.
- **ii** State the maximum value.
- **iii** Find the median (Q_2).
- **iv** Find the lower quartile (Q_1).
- **v** Find the upper quartile (Q_3).
- **vi** Draw a box plot to represent the data.

a 2 2 3 3 4 6 7 7 7 8 8 8 8 9 11 11 13 13 13

b 43 21 65 45 34 42 40 28 56 50 10 43 70 37 61 54 88 19

c 435 353 643 244 674 364 249 933 523 255 734

d 0.5 0.7 0.1 0.2 0.9 0.5 1.0 0.6 0.3 0.4 0.8 1.1 1.2 0.8 1.3 0.4 0.5

PROBLEM–SOLVING	4, 5	4, 5	5, 6

4 The following set of data describes the number of cars parked in a street on 18 given days.

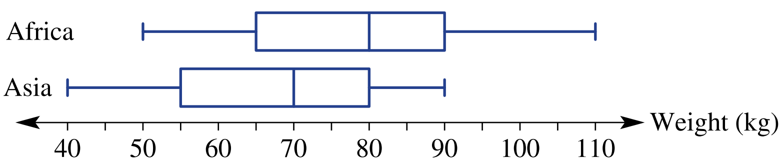
14 26 39 46 13 30 5 46 37 26 39 8 8 9 17 48 29 27

a Represent the data as a box plot.

b State the percentage of days on which the number of cars parked on the street was between:

i 4 and 48 **ii** 13 and 39 **iii** 5 and 39 **iv** 39 and 48.

5 The weights of a sample of adult leopards from Africa and Asia are summarised in these box plots.



a Which leopard population sample has the highest minimum weight?

b What is the difference between the ranges for the two population samples?

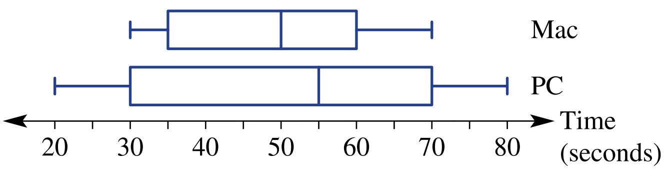
c Is the IQR the same for both leopard samples? If so, what is it?

d **i** What percentage of African leopards have a weight less than 80 kg?

ii What percentage of Asian leopards have a weight less than 80 kg?

e A leopard has a weight of 90 kg. Is it likely to be an Asian or African leopard?

6 The time that it takes for a sample of computers to start up is summarised in these box plots.



a What type of computer has the lowest median?

b What percentage of Mac computers started up in less than 1 minute?

c What percentage of PC computers took longer than 55 seconds to start up?

d What do you notice about the range for Mac computers and the IQR for PC computers? What does this mean?

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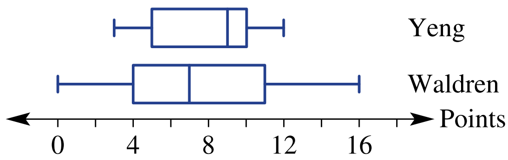
REASONING	7	7–9	8–10

7 The number of points per game for two basketball players over a season is summarised in these box plots.



a Which player has the highest maximum?
b Which player has the highest median?
c Which player has the smallest IQR?
d Which player is a more consistent basketball scorer? Give reasons.
e Which player is more likely to have scored the greatest number of points? Give reasons.

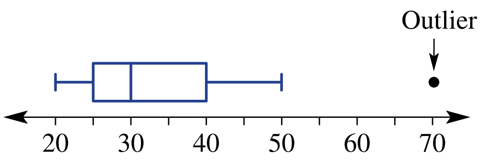
8 Give an example of a small data set which has the following.
a Maximum = upper quartile
b Median = lower quartile

9 Does the median always sit exactly in the middle of a box on a box plot? Explain.

10 Could the mean of a data set be greater than Q_3? Explain.

ENRICHMENT: Outliers and battery life	–	–	11

11 Outliers on box plots are shown as separate points.



The life, in months, of a particular battery used in a high-powered calculator is shown in this data set.

3 3 3 4 4 5 6 6 6 7 8 8 9 17

a Use all the values to calculate Q_1, Q_2 and Q_3 for the data set.
b Do any of the values appear to be outliers?
c Not including the outlier, what is the next highest value?
d Draw a box plot for the data using a dot (•) to show the outlier.
e Can you give a logical reason for the outlier?

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Working mathematically

The missing data

Gregory is a marine biologist and is collecting and analysing data regarding shark and dolphin numbers in a particular area of the Great Barrier Reef.

He collects data each day for 14 days, but unfortunately two pieces of data are lost due to a computer glitch as shown.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Sharks	15	12	4	$?_1$	15	11	19	5	1	10	12	6	24	11
Dolphins	4	7	2	3	2	6	4	2	$?_2$	4	7	2	4	6

Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate.

Routine problems

a If Gregory assumes that $?_1 = 10$, find the following correct to one decimal place.
   i The mean and the median number of sharks recorded.
   ii The range and interquartile range for the number of sharks recorded.

b If Gregory assumes that $?_2 = 5$, find the following correct to one decimal place.
   i The mean and the median number of dolphins recorded.
   ii The range and interquartile range for the number of dolphins recorded.

c Which set of data (dolphin or shark) appears to be more inconsistent and spread out?

Non-routine problems

Explore and connect

a The problem is to determine the possible values of $?_1$ and $?_2$ so that Gregory can complete his report based on some statistics that he can remember. Write down all the relevant information that will help solve this problem.

Choose and apply techniques

b Gregory seems to remember that the range for the number of dolphins was 7. Determine the value of $?_2$ if Gregory's memory is correct.

c From memory, Gregory thought that the mean number of sharks over the 14 days was approximately 12. Determine the likely value for $?_1$.

d From memory, Gregory thought that the mean number of dolphins over the 14 days was approximately 5. Determine the likely value for $?_2$.

e If the median for the number of sharks was equal to 11, find the possible values of $?_1$.

f If the interquartile range of the number of dolphins is 4, determine the possible values of $?_2$.

Communicate thinking and reasoning

g If in fact the mean number of dolphins over the 14-day period was 4 after rounding to the nearest integer, find all the possible values of $?_2$.

h If in fact the interquartile range for the number of sharks was 9, find all the possible values of $?_1$.

i Summarise your results and describe any key findings.

Extension problems

Problem solve

a Decide if it is possible for the median of either set of data to be anything other than an integer. If so, determine what values of $?_1$ and $?_2$ allow this to happen.

b Is it possible to choose values for $?_1$ and $?_2$ so that the interquartile range for sharks is exactly double that for dolphins? If so, explain how this is possible.

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How many in the bag?

For this activity you will need:

- a bag or large pocket
- different-coloured counters
- paper and pen.

Five counters

a Form pairs and then, without watching, have a third person (e.g. a teacher) place five counters of two different colours into the bag or pocket. An example of five counters could be two red and three blue, but at this point in the activity you will not know this.

b Without looking, one person selects a counter from the bag while the other person records its colour. Replace the counter in the bag.

c Repeat part **b** for a total of 100 trials. Record the results in a table similar to this one.

Colour	Tally	Frequency
Red	卌 III …	…
Blue	卌 卌 II …	…
Total	100	100

d Find the experimental probability for each colour. For example, if 42 red counters were recorded, then the experimental probability $= \frac{42}{100} = 0.42$.

e Use these experimental probabilities to help estimate how many of each colour of counter are in the bag. For example, 0.42 is close to $0.4 = \frac{2}{5}$, therefore guess two red and three blue counters. Use this table to help.

Colour	Frequency	Experimental probability	Closest multiple of 0.2, e.g. 0.2, 0.4, …	Guess of how many counters of this colour
Total	100	1	1	5

f Now take the counters out of the bag to see if your estimate is correct.

More colours and counters

a Repeat the steps above but this time use three colours and 8 counters.

b Repeat the steps above but this time use four colours and 12 counters.

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Problems and challenges

1 A fair coin is tossed 5 times.
   a How many outcomes are there?
   b Find the probability of obtaining at least 4 tails.
   c Find the probability of obtaining at least 1 tail.

2 Three cards, A, B and C, are randomly arranged in a row.
   a Find the probability that the cards will be arranged in the order A B C from left to right.
   b Find the probability that the B card will take the right-hand position.

3 Four students, Rick, Belinda, Katie and Chris, are the final candidates for the selection of school captain and vice-captain. Two of the four students will be chosen at random to fill the positions.
   a Find the probability that Rick will be chosen for:
      i captain
      ii captain or vice-captain.
   b Find the probability that Rick and Belinda will be chosen for the two positions.
   c Find the probability that Rick will be chosen for captain and Belinda will be chosen for vice-captain.
   d Find the probability that the two positions will be filled by Rick and Chris or Belinda and Katie.

4 State what would happen to the mean, median and range of a data set in these situations.
   a Five is added to each value in the data set.
   b Each value in the data set is doubled.
   c Each value in the data set is doubled and then decreased by 1.

5 Three pieces of fruit have an average weight of m grams. After another piece of fruit is added, the average weight doubles. Find the weight of the extra piece of fruit in terms of m.

6 a Five different data values have a range and median equal to 7. If two of the values are 3 and 5, what are the possible combinations of values?
   b Four data values have a range of 10, a mode of 2 and a median of 5. What are the four values?

7 Five integer scores out of 10 are all greater than 0. If the median is x, the mode is one more than the median and the mean is one less than the median, find all the possible sets of values if $x < 7$.

8 Thomas works in the school office for work experience. He is given four letters and four addressed envelopes. What is the probability that Thomas, who is not very good at his job, places none of the four letters into its correct envelope?

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Chapter summary

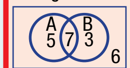
Probability and statistics

Probability review

The sample space is the list of all possible outcomes of an experiment.
For equally likely outcomes:
$P(\text{event}) = \dfrac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$
e.g. roll a normal six-sided die
$P(>4) = \frac{2}{6} = \frac{1}{3}$
$0 \le P(\text{event}) \le 1$
$P(\text{not } A) = 1 - P(A)$

Venn diagrams and two-way tables

These organise data from two or more categories.



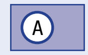
Venn diagram

	A	A′	
B	7	3	10
B′	5	6	11
	12	9	21

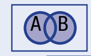
i.e. 6 is in neither category, 7 is in both categories
$P(A) = \frac{12}{21} = \frac{4}{7}$
$P(B \text{ only}) = \frac{3}{21} = \frac{1}{7}$

Set notation

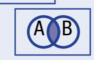
Within a sample space are a number of subsets.
A' = not A



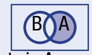
$A \cup B$ means A or B; the union of A and B.



$A \cap B$ means A and B; the intersection of A and B.



A only is the elements in A but not in B.



$n(A)$ is the number of elements in A
ϕ is the empty or null set
e.g. $A = \{1, 2, 3, 4, 5\}$ $B = \{2, 4, 6, 8\}$
$A \cup B = \{1, 2, 3, 4, 5, 6, 8\}$
$A \cap B = \{2, 4\}$ $n(B) = 4$
B only $= \{6, 8\}$
$3 \in A$ means 3 is an element of A

Measures of spread

Range = maximum value − minimum value
Interquartile range (IQR) = upper quartile (Q_3) − lower quartile (Q_1)
IQR is the range of the middle 50% of data.
e.g. To find Q_1 and Q_3, first locate median (Q_2) then find middle values of upper and lower half.

e.g. 1 2 5 6 7 | 9 10 12 12 15
Q_1 (5), $Q_2 = 8$, Q_3 (12)

IQR = 12 − 5
= 7
For odd number of values exclude median from each half.

Two-step experiments

These can be represented by tables or tree diagrams. They involve more than one component and can occur with or without replacement.
e.g. a bag contains 2 blue counters and 1 green; 2 are selected at random.

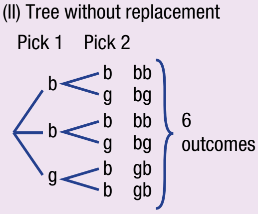
(I) Table with replacement

		Pick 1		
		b	b	g
Pick 2	b	(b, b)	(b, b)	(g, b)
	b	(b, b)	(b, b)	(g, b)
	g	(b, g)	(b, g)	(g, g)

Sample space of 9 outcomes

$P(\text{2 of same colour}) = \frac{5}{9}$

(II) Tree without replacement



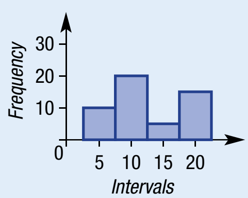
$P(\text{2 of same colour}) = \frac{2}{6} = \frac{1}{3}$

Data and sampling

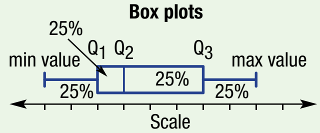
A survey can be used to collect information using a sample or population.
A sample needs to be chosen carefully to be representative and to avoid bias.
The type of data can be:
- numerical: discrete, continuous
- categorical: ordinal, nominal

Grouped data

Data values can be grouped into class intervals, e.g. 0–4, 5–9, etc. and recorded in a frequency table.
The frequency or percentage frequency of each interval can be recorded in a histogram.
e.g.



Box plots



25% of data is in each of the four sections.

Stem-and-leaf plots

These display all the data values using a stem and a leaf.
An ordered back-to-back stem-and-leaf plot compares two data sets:

Leaf (skewed)	Stem	Leaf (symmetrical)
9 8 7 2	1	0 3
7 4 3 3	2	2 2 4
5 2 1	3	3 6 7 8
7	4	4 5 9
0	5	0

3 | 5 means 35 (key)

Mean, median and mode

Mode is the most common value (there can be more than one). Two modes means the data is bi-modal.
Mean = average
$= \dfrac{\text{sum of all values}}{\text{number of values}}$
Median is the middle value of data that is ordered.

odd data set: 2 4 7 10 12 — median 7
even data set: 2 4 6 10 15 18 — median 8

An outlier is a value that is not in the vicinity of the rest of the data.

Experimental probability

This is calculated from results of an experiment or survey as the relative frequency.
Experimental probability
$= \dfrac{\text{number of times event occurs}}{\text{total number of trials}}$
Expected number of occurrences
= probability × number of trials

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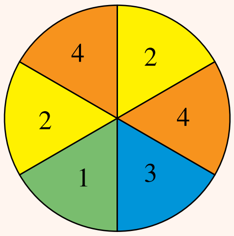
Chapter checklist

Chapter checklist with success criteria

A printable version of this checklist is available in the Interactive Textbook

		✔
9A	**1. I can find probabilities of events.** e.g. This spinner has six equally divided sections. List the sample space from a spin and find both P(not a 2) and P(a 1 or a 2). 	☐
9B	**2. I can use a Venn diagram.** e.g. A survey of 30 people in 2019 found that 12 thought Nick Kyrgios would win a grand slam tennis event in his career and 16 thought Alex de Minaur would win a grand slam tennis event in his career. Of those, 6 thought both would win a grand slam event while 8 thought neither would win one. Construct a Venn diagram. Use this to state how many people do not think Nick Kyrgios will win one as well as the probability that a randomly chosen person thinks only Alex de Minaur will win one.	☐
9B	**3. I can use a two-way table.** e.g. On a particular day at a car service station, 20 cars are serviced: 12 cars need a wheel alignment, 10 cars need their brake fluid replaced and 4 require both. Illustrate the situation using a two-way table. Use the table to find how many cars require neither, and the probability that a randomly selected car needs only a wheel alignment.	☐
9C	**4. I can use set notation.** e.g. A number is chosen from the set of integers between 1 and 10 inclusive. Let A be the set of even numbers between 1 and 10 inclusive and B the set of factors of 24 between 1 and 10 inclusive. Draw a Venn diagram showing the number of elements in sets A and B. i. List the sets $A \cup B$ and B only. ii. Find $n(A')$ and $P(A \cap B)$.	☐
9D	**5. I can use an array to find the sample space for events with replacement.** e.g. Two four-sided dice numbered 1, 2, 3, 4 are rolled. Draw a table to list the sample space and find the probability of obtaining two even numbers.	☐
9D	**6. I can use an array to find the sample space for events without replacement.** e.g. Two letters are chosen at random from the word GREEN without replacement. List the outcomes in a table and find the probability that at least one of the letters is an E.	☐
9E	**7. I can construct a tree diagram.** e.g. A game involves tossing three coins. Complete a tree diagram to show all possible outcomes and find the probability of tossing exactly 2 heads.	☐
9E	**8. I can construct a tree diagram involving without replacement.** e.g. Two donuts are selected from a box containing three donuts with strawberry (S), chocolate (C) and banana (B) icing. List all the possible outcomes for the selection in a tree diagram and find the probability that the selection will contain the chocolate iced donut.	☐
9F	**9. I can find the experimental probability.** e.g. A container contains an unknown number of coloured counters. A counter is drawn from the container and then replaced. This is repeated 50 times and the colour of the counter is recorded each time. Given that 20 green counters were recorded, find the experimental probability for selecting a green counter, and find the expected number of green counters if there were 300 counters in total in the container.	☐

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Chapter checklist

9G	**10. I can classify data.** e.g. Classify the data type from the following surveys. i. The number of Christmas presents received ii. The breakfast buffet at a hotel: poor, average, good, excellent	☐
9G	**11. I can choose a sampling method.** e.g. Identify the sampling method used in each of these surveys. i. An airline chooses every sixth passenger on a flight ii. One hundred students are grouped into year level and then tested on spelling words	☐
9G	**12. I can identify bias in a data collection method.** e.g. One hundred people at the airport are surveyed about how often they travel. This data is used to make conclusions about that city and how often people travel. Do you think this survey will contain bias? If so, explain why.	☐
9H	**13. I can find the mean, median and mode.** e.g. Find the mean, median and mode for this data set: 8 8 3 6 4 4.	☐
9H	**14. I can find a data value for a required mean.** e.g. A student's quiz marks for the first 7 weeks of term were: 4, 8, 6, 5, 6, 7, 6. Calculate the mean of these marks and find the score needed in the 8th week for the mean to equal 6.5.	☐
9I	**15. I can construct and use a stem-and-leaf plot.** e.g. Organise the data below into an ordered stem-and-leaf plot and find the median and mode. State whether the data is symmetrical or skewed. 0.4 3.2 2.4 1.7 4.4 2.1 4.1 2.7 3.2	☐
9I	**16. I can construct a back-to-back stem-and-leaf plot.** e.g. The maximum daily temperatures for two world cities over a two-week period are recorded as: **City A:** 21 18 24 26 30 26 22 19 18 24 19 28 27 32 **City B:** 12 12 9 15 10 13 20 21 21 17 22 18 17 9 Draw a back-to-back stem-and-leaf plot with an interval of 10 and compare and comment on the differences between the maximum temperatures for the two cities.	☐
9J	**17. I can construct frequency tables and histograms.** e.g. For the data below showing the ice-cream sales each hour of operation over a week, set up and complete a grouped frequency table using class intervals 0–4, 5–9, etc. 15 22 1 0 20 3 10 2 23 3 11 14 19 3 16 14 24 7 1 15 5 7 2 29 28 3 2 13 26 20 22 25 6 7 12 6 13 7 7 20 11 11 From the table, construct a frequency histogram and state for how many hours there were sales of 15 or more ice-creams.	☐
9K	**18. I can find the range.** e.g. Find the range for this data set: 7 17 21 34 22 18 8 3 16	☐
9K	**19. I can find the quartiles for an odd number of data values.** e.g. List the data below in order from smallest to largest and find the median (Q_2), the lower quartile (Q_1), the upper quartile (Q_3) and the interquartile range (IQR). 7 17 21 34 22 18 8 3 16	☐

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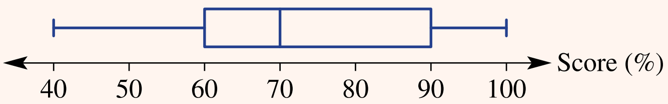
Chapter checklist

9K	**20. I can find the quartiles for an even number of data values.** e.g. List the data below in order from smallest to largest and find the median (Q_2), the lower quartile (Q_1), the upper quartile (Q_3) and the interquartile range (IQR). Interpret the IQR. 12 21 18 8 4 16 24 11 27 10	☐
9L	**21. I can interpret a box plot.** e.g. This box plot summarises the test results (%) of a class of students.  i. Find the range and interquartile range of the results. ii. Twenty-five per cent of results were above what number? iii. If there were 24 students in the class, how many scored above 70%?	☐
9L	**22. I can construct a box plot.** e.g. For the data shown below, find the minimum and maximum values, the median and lower and upper quartiles and draw a box plot to represent the data. 18 15 13 14 21 25 17 11 15 16 18 20 24 19	☐

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Short-answer questions

9A

1 Determine the probability of each of the following.

a Rolling more than 2 on a normal six-sided die

b Selecting a vowel from the word EDUCATION

c Selecting a pink or white jelly bean from a packet containing 4 pink, 2 white and 4 black jelly beans

9B

2 From a survey of 50 people, 30 have the newspaper delivered, 25 read it online, 10 do both and 5 do neither.

a Construct a Venn diagram for the survey results.

b How many people only read the newspaper online?

c If one of the 50 people was randomly selected, find:

i P(have paper delivered and read it online)

ii P(don't have it delivered)

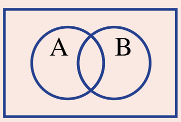
iii P(only read it online).

9B/C

3 **a** Copy and complete this two-way table.

	A	A′	Total
B		16	
B′	8		20
Total	17		

b Convert the information into a Venn diagram like that shown.



c Find the following probabilities.

i $P(\text{B}')$

ii $P(\text{A} \cap \text{B})$

iii $n(\text{A only})$

iv $n(\text{A} \cup \text{B})$

9D

4 A spinner with equal areas of red, green and blue is spun and a four-sided die numbered 1 to 4 is rolled.

a Complete a table like the one shown and state the number of outcomes in the sample space.

		Die			
		1	**2**	**3**	**4**
Spinner	**red**	(red, 1)	(red, 2)		
	green				
	blue				

b Find the probability that the outcome:

i is red and an even number

ii is (blue, 4) or (green, 4)

iii does not involve blue.

9E

5 Libby randomly selects two coins from her pocket *without replacement*. Her pocket contains a \$1 coin, and two 10-cent coins.

a List all the possible combinations using a tree diagram.

b If a chocolate bar costs \$1.10, find the probability that she can hand over the two coins to pay for it.

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Chapter review

9F

6 A quality controller records the frequency of the types of chocolates from a sample of 120 from its production line.

Centre	Soft	Hard	Nut
Frequency	50	22	48

a What is the experimental probability of randomly selecting a nut centre?
b In a box of 24 chocolates, how many would be expected to have a non-soft centre?

9G

7 Jody is planning to run a survey asking people in her year about their current favourite Netflix show.
a Classify the type of data to be collected.
b Identify the sampling method in each of the following survey methods she is considering.
  i Assigning each student a number and generating some random numbers
  ii Surveying every fifth person in the school canteen queue.
c After planning, Jody surveys everyone in her extension science class, and then will use the data to make conclusions about her year level. Do you think this survey will contain bias? If so, explain why.

9H

8 Claudia records the number of emails she receives each weekday for two weeks as follows.
30 31 33 23 29 31 21 15 24 23
Find:
a the mean
b the median
c the mode.

9H

9 Jacob achieved scores of 12, 9, 7 and 12 on his last four language vocabulary tests. What score must he get on the fifth test to have a mean of 11?

9I

10 Two mobile phone salespeople are both aiming for a promotion to be the new assistant store manager. The best salesperson over a 15-week period will get the promotion. The number of mobile phones they sold each week is recorded below.

Employee 1: 21 34 40 38 46 36 23 51 35 25 39 19 35 53 45
Employee 2: 37 32 29 41 24 17 28 20 37 48 42 38 17 40 45

a Draw an ordered back-to-back stem-and-leaf plot for the data.
b For each employee, find:
  i the median number of sales
  ii the mean number of sales.
c By comparing the two sets of data, state, with reasons, who you think would get the promotion.
d Describe each employee’s data as approximately symmetrical or skewed.

9J

11 The data below represents the finish times, in minutes, of 25 competitors in a local car rally.
134 147 162 164 145 159 151 143 136 155 163 157 168
128 144 161 158 136 178 152 167 154 161 152 171
a Record the data in a frequency table in class intervals of 10 minutes. Include a percentage frequency column.
b Construct a frequency histogram.
c Determine:
  i the number of competitors who finished in less than 140 minutes
  ii the percentage of competitors who finished with times of between 130 and 160 minutes.

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9K/L

12 Scott scores the following runs in each of his innings across the course of a cricket season:

20 5 34 42 10 3 29 55 25 37 51 12 34 22

a Find the range.

b Construct a box plot to represent the data by first finding the quartiles.

c From the box plot, 25% of his innings were above what number of runs?

Multiple-choice questions

9A

1 A letter is randomly chosen from the word XYLOPHONE. The probability that it is an O is:

A $\frac{1}{8}$ **B** $\frac{2}{9}$ **C** $\frac{1}{4}$ **D** $\frac{1}{9}$ **E** $\frac{1}{3}$

9B

2 The values of x and y in the two-way table are:

A $x = 12, y = 8$

B $x = 12, y = 11$

C $x = 16, y = 4$

D $x = 10, y = 1$

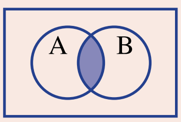
E $x = 14, y = 6$

	A	not A	Total
B		5	9
not B	8	y	
Total	x		25

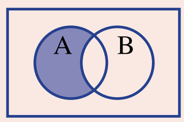
9C

3 Which shaded region represents $A \cup B$?

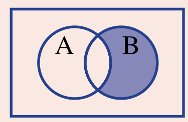
A



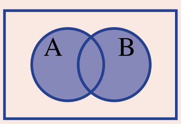
B



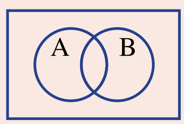
C



D



E



9D/E

4 A bag contains 2 green balls and 1 red ball. Two balls are randomly selected *without replacement*. The probability of selecting one of each colour is:

A $\frac{1}{2}$ **B** $\frac{2}{3}$ **C** $\frac{5}{6}$ **D** $\frac{1}{3}$ **E** $\frac{3}{4}$

9F

5 Students roll a biased die and find that the experimental probability of rolling a 5 is 0.3. From 500 rolls of the die, the expected number of 5s would be:

A 300 **B** 167 **C** 180 **D** 150 **E** 210

9I

6 The median of the data in this stem-and-leaf plot is:

A 74 **B** 71 **C** 86

D 65 **E** 70

Stem	Leaf
5	3 5 8
6	1 4 7
7	0 2 4 7 9
8	2 6 6

7 | 4 means 74

9G

7 The type of data generated by the survey question 'How long is your arm length?' is:

A a random number **B** categorical ordinal

C categorical nominal **D** numerical discrete

E numerical continuous

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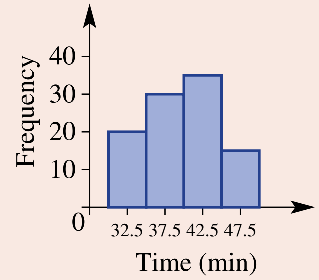
Chapter review

9J

8 This frequency histogram shows the times of competitors in a fun run. The percentage of competitors who finished in better than 40 minutes was:



A 55% **B** 85% **C** 50%
D 62.5% **E** 60%

9K

9 Consider the set of ordered data below.

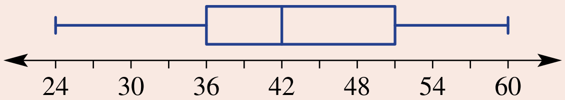
1.1 2.3 2.4 2.8 3.1 3.4 3.6 3.8 3.8 4.1 4.5 4.7 4.9

The interquartile range is:

A 2–5 **B** 1.7 **C** 3.8 **D** 1.3 **E** 2.1

9L

10 Choose the incorrect statement about the box plot below.



A The range is 36.
B Fifty per cent of values are between 36 and 51.
C The median is 42.
D Twenty-five per cent of values are below 36.
E The interquartile range is 20.

Extended-response questions

1 The local Sunday market has a number of fundraising activities.

a For $1 you can spin a spinner numbered 1–5 twice. If you spin two even numbers you receive $2 (your dollar back plus an extra dollar), if you spin two odd numbers you get your dollar back and otherwise you lose your dollar.

		First spin				
		1	**2**	**3**	**4**	**5**
Second spin	**1**	(1, 1)	(2, 1)			
	2					
	3					
	4					
	5					

i Complete the table shown to list the sample space.
ii What is the probability of losing your dollar?
iii What is the probability of making a dollar profit?
iv In 50 attempts, how many times would you expect to lose your dollar?
v If you start with $100 and have 100 attempts, how much money would you expect to end up with?

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b Forty-five people were surveyed as they walked through the market as to whether they bought a sausage and/or a drink from the sausage sizzle. Twenty-five people bought a sausage, 30 people bought a drink, with 15 buying both. Let S be the set of people who bought a sausage and D the set of people who bought a drink.

- **i** Construct a Venn diagram to represent this information.
- **ii** How many people did not buy either a drink or a sausage?
- **iii** How many people bought a sausage only?
- **iv** If a person was randomly selected from the 45 people surveyed, what is the probability they bought a drink but not a sausage?
- **v** Find $P(S')$.
- **vi** Find $P(S' \cup D)$ and state what this probability represents.

2 The data below represents the data collected over a month of 30 consecutive days of the delay time (in minutes) of the flight departures of the same evening flight for two rival airlines.

Airline A

2	11	6	14	18	1	7	4	12	14	9	2	13	4	19
13	17	3	52	24	19	12	14	0	7	13	18	1	23	8

Airline B

6	12	9	22	2	15	10	5	10	19	5	12	7	11	18
21	15	10	4	10	7	18	1	18	8	25	4	22	19	26

a Does the data for airline A appear to have any outliers (numbers not near the majority of data elements)?

b Remove any outliers listed in part **a**, and find the following values for airline A:

- **i** median (Q_2)
- **ii** lower quartile (Q_1)
- **iii** upper quartile (Q_3).

c Hence, complete a box plot of the delay times for airline A.

d Airline A reports that half its flights for that month had a delay time of less than 10 minutes. Is this claim correct? Explain.

e On the same axis as in part **c**, construct a box plot for the delay times for airline B.

f Find the range and interquartile range of the data for the two airlines and comment on the spread of the delay times for each company.

g Use your answers to the previous question parts to explain which airline you would choose on the basis of their delay times and why.

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10

Quadratic equations and graphs of parabolas

Maths in context: Projectiles, parabolas, and price point for maximum revenue

Galileo (17th century) discovered that under the influence of gravity the trajectory (flight path) of a projectile (a launched object) follows a parabola shaped curve modelled by a quadratic equation.

In many sports, projectiles follow parabolic trajectories, including in basketball, volleyball, soccer, cricket, golf, shotput, discus, javelin, ski jumping, long and high jumping, and horse jumping. The launching angle influences the trajectory's maximum height and range. Generally, a launch angle of 45° will give maximum range, with steeper angles resulting in a greater maximum height but shorter range. Sports Scientists can use quadratic equations to analyse an athlete's performance trajectories, studying details for possible technique improvement.

Quadratic equations are formed when two linear expressions are multiplied. Financial analysts use quadratic equations to model sales and price. If selling n items, each priced at p dollars, a linear formula for n using p is developed.

Revenue = unit price (p) × n (number sold)

Revenue = p × (*formula for n using p*)

E.g., $R = p \times (60 - 0.1p) = 60p - 0.1p^2$

The revenue equation is a quadratic, and the graph of 'Revenue' (y-axis) vs 'Price' (x-axis) is an inverted parabola. As the price p increases, revenue initially increases to a maximum, then it decreases with continuing decline in sales. The parabola's turning point locates the item's price point for maximum revenue.

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Chapter contents

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- identifies connections between algebraic and graphical representations of quadratic and exponential relationships in various contexts (MA5-NLI-C-01)
- identifies and compares features of parabolas and exponential curves in various contexts (MA5-NLI-C-02)
- solves monic quadratic equations, linear inequalities and cubic equations (MA5-EQU-P-01).

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

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10A Quadratic equations

Learning intentions
- To be able to identify a quadratic equation
- To know how to write a quadratic equation in standard form
- To understand the Null Factor Law
- To be able to apply the Null Factor Law to solve an equation in factorised form

Past, present and future learning
- This section addresses concepts from the Stage 5 Path Topics *Equations B and C*
- These concepts will be revised and extended in Chapter 5 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

Quadratic equations are commonplace in theoretical and practical applications of mathematics. They are used to solve problems in geometry and measurement as well as in number theory and physics. The path that a projectile takes while flying through the air, for example, can be analysed using quadratic equations.

Quadratic equations are widely used in many areas of work including: sports scientists analysing the trajectory of a javelin or discus, electronic engineers finding currents and voltages, and financial analysts modelling future revenue and profit.

A quadratic equation can be written in the form $ax^2 + bx + c = 0$ where a, b and c are constants and $a \neq 0$. Examples include $x^2 - 2x + 1 = 0$, $5x^2 - 3 = 0$ and $-0.2x^2 + 4x = 0$. Unlike linear equations which have a single solution, quadratic equations can have zero, one or two solutions. For example, $x = 2$ and $x = -1$ are solutions to the quadratic equation $x^2 - x - 2 = 0$ since $2^2 - 2 - 2 = 0$ and $(-1)^2 - (-1) - 2 = 0$. One method for finding the solutions to quadratic equations involves the use of the Null Factor Law where each factor of a factorised quadratic expression is equated to zero.

Lesson starter: Exploring the Null Factor Law

$x = 1$ is not a solution to the quadratic equation $x^2 - x - 12 = 0$ since $1^2 - 1 - 12 \neq 0$.

- Use trial and error to find at least one of the two numbers that are solutions to $x^2 - x - 12 = 0$.
- Rewrite the equation in factorised form.

$$x^2 - x - 12 = 0 \text{ becomes } (\qquad)(\qquad) = 0$$

- Now repeat the first task above to find solutions to the equation using the factorised form.
- Was the factorised form easier to work with? Discuss.

KEY IDEAS

- A **quadratic equation** can be written in the form $ax^2 + bx + c = 0$.
  - This is called **standard form**.
  - a, b and c are constants and $a \neq 0$.

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- The **Null Factor Law** states that if the product of two numbers is zero, then either or both of the two numbers is zero.
  - If $p \times q = 0$ then $p = 0$ or $q = 0$.
  - If $(x + 1)(x - 3) = 0$ then $x + 1 = 0$ (so $x = -1$) or $x - 3 = 0$ (so $x = 3$).
  - $x + 1$ and $x - 3$ are the linear factors of $x^2 - 2x - 3$, which factorises to $(x + 1)(x - 3)$.

BUILDING UNDERSTANDING

1 Evaluate these quadratic expressions by substituting the x-value given in the brackets.

a $x^2 - 5$ $(x = 1)$ **b** $x^2 + 2x$ $(x = -1)$ **c** $3x^2 - x + 2$ $(x = 5)$

2 Decide if the following equations are quadratics (yes or no).

a $1 - 2x = 0$ **b** $x^2 + 3x + 1 = 0$ **c** $x^2 - 1 = 0$

d $x^3 + x^2 - 2 = 0$ **e** $x^5 + 1 - x = 0$ **f** $x + 2 = 4x^2$

3 Solve these linear equations.

a $x - 1 = 0$ **b** $x + 3 = 0$ **c** $2x + 4 = 0$

4 State the missing components to complete the following working, which uses the Null Factor Law.

a $x(x - 5) = 0$

$x = 0$ or ____ $= 0$

$x = 0$ or $x =$ ____

b $(2x + 1)(x - 3) = 0$

____ $= 0$ or $x - 3 = 0$

$2x =$ ____ or $x =$ ____

$x =$ ____ or $x = 3$

Example 1 Writing in standard form

Write these quadratic equations in standard form: $ax^2 + bx + c = 0$, $a > 0$.

a $x^2 = 2x + 7$ **b** $2(x^2 - 3x) = 5$ **c** $2x - 7 = -3x^2$

SOLUTION	EXPLANATION
a $x^2 = 2x + 7$ $x^2 - 2x - 7 = 0$	We require the form $ax^2 + bx + c = 0$. Subtract $2x$ and 7 from both sides to move all terms to the left-hand side.
b $2(x^2 - 3x) = 5$ $2x^2 - 6x = 5$ $2x^2 - 6x - 5 = 0$	First expand brackets then subtract 5 from both sides.
c $2x - 7 = -3x^2$ $3x^2 + 2x - 7 = 0$	Add $3x^2$ to both sides, to make the x^2 coefficient positive.

Now you try

Write these quadratic equations in standard form: $ax^2 + bx + c = 0$, $a > 0$.

a $x^2 = 3x + 5$ **b** $2(x^2 + 4x) = 7$ **c** $3x - 4 = -2x^2$

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Example 2 Testing for a solution

Substitute the given x-value into the equation and say whether or not it is a solution.

a $x^2 + x - 6 = 0 \quad (x = 2)$

b $2x^2 + 5x - 3 = 0 \quad (x = -4)$

SOLUTION

a
$$\begin{aligned} x^2 + x - 6 &= 2^2 + 2 - 6 \\ &= 6 - 6 \\ &= 0 \end{aligned}$$
$\therefore\ x = 2$ is a solution

EXPLANATION

Substitute $x = 2$ into the equation to see if the left-hand side equals zero.
$x = 2$ satisfies the equation, so $x = 2$ is a solution.

b
$$\begin{aligned} 2x^2 + 5x - 3 &= 2(-4)^2 + 5(-4) - 3 \\ &= 2 \times 16 + (-20) - 3 \\ &= 32 - 20 - 3 \\ &= 9 \end{aligned}$$
$\therefore\ x = -4$ is not a solution

Substitute $x = -4$. Recall that $(-4)^2 = 16$ and $5 \times (-4) = -20$.
The equation is not satisfied since it is $\neq 0$ so $x = -4$ is not a solution.

Now you try

Substitute the given x-value into the equation and say whether or not it is a solution.

a $x^2 + x - 3 = 0 \quad (x = 1)$

b $3x^2 + 4x - 15 = 0 \quad (x = -3)$

Example 3 Using the Null Factor Law

Use the Null Factor Law to solve these equations.

a $x(x + 2) = 0$

b $(x - 1)(x + 5) = 0$

c $(2x - 1)(5x + 3) = 0$

SOLUTION

a $x(x + 2) = 0$
$x = 0$ or $x + 2 = 0$
$x = 0$ or $x = -2$

EXPLANATION

The factors are x and $(x + 2)$. Solve each linear factor equal to zero.
Check each solution by substituting into the original equation.

b $(x - 1)(x + 5) = 0$
$x - 1 = 0$ or $x + 5 = 0$
$x = 1$ or $x = -5$

Solve each factor $(x - 1)$ and $(x + 5)$ equal to zero.
Check your solutions using substitution.

c $(2x - 1)(5x + 3) = 0$
$2x - 1 = 0$ or $5x + 3 = 0$
$2x = 1$ or $5x = -3$
$x = \frac{1}{2}$ or $x = -\frac{3}{5}$

The two factors are $(2x - 1)$ and $(5x + 3)$.
Each one results in a two-step linear equation.
Check your solutions using substitution.

Now you try

Use the Null Factor Law to solve these equations.

a $x(x + 5) = 0$

b $(x + 2)(x - 3) = 0$

c $(3x - 2)(2x + 5) = 0$

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Exercise 10A

FLUENCY	1–2(½), 3, 5(½)	1–2(½), 4, 5–6(½)	1(½), 2(⅓), 4, 5–6(⅓)

Example 1

1 Write these quadratic equations in standard form: $ax^2 + bx + c = 0$, $a > 0$.

a $x^2 = 7x + 2$ **b** $x^2 - 5x = -2$ **c** $x^2 = 4x - 1$
d $3(x^2 - 2x) = 4$ **e** $2(x^2 + x) + 1 = 0$ **f** $3(x^2 - x) = -4$
g $4 = -3x^2$ **h** $3x = x^2 - 1$ **i** $5x = 2(-x^2 + 5)$

Example 2

2 Substitute the given x-value into the quadratic equation and say whether or not it is a solution.

a $x^2 - 1 = 0\ (x = 1)$ **b** $x^2 - 25 = 0\ (x = 5)$ **c** $x^2 - 4 = 0\ (x = 1)$
d $2x^2 + 1 = 0\ (x = 0)$ **e** $x^2 - 9 = 0\ (x = -3)$ **f** $x^2 - x - 12 = 0\ (x = 5)$
g $x^2 + 2x + 1 = 0\ (x = -1)$ **h** $2x^2 - x + 3 = 0\ (x = -1)$ **i** $5 - 2x + x^2 = 0\ (x = -2)$

3 Substitute $x = -2$ and $x = 5$ into the equation $x^2 - 3x - 10 = 0$. What do you notice?

4 Substitute $x = -3$ and $x = 4$ into the equation $x^2 - x - 12 = 0$. What do you notice?

Example 3a,b

5 Use the Null Factor Law to solve these equations.

a $x(x + 1) = 0$ **b** $x(x + 5) = 0$ **c** $x(x - 2) = 0$
d $x(x - 7) = 0$ **e** $(x + 1)(x - 3) = 0$ **f** $(x - 4)(x + 2) = 0$
g $(x + 7)(x - 3) = 0$ **h** $\left(x + \frac{1}{2}\right)\left(x - \frac{1}{2}\right) = 0$ **i** $2x(x + 5) = 0$
j $5x\left(x - \frac{2}{3}\right) = 0$ **k** $\frac{x}{3}\left(x + \frac{2}{3}\right) = 0$ **l** $\frac{2x}{5}(x + 2) = 0$

Example 3c

6 Use the Null Factor Law to solve these equations.

a $(2x - 1)(x + 2) = 0$ **b** $(x + 2)(3x - 1) = 0$ **c** $(5x + 2)(x + 4) = 0$
d $(x - 1)(3x - 1) = 0$ **e** $(x + 5)(7x + 2) = 0$ **f** $(3x - 2)(5x + 1) = 0$
g $(11x - 7)(2x - 13) = 0$ **h** $(4x + 9)(2x - 7) = 0$ **i** $(3x - 4)(7x + 1) = 0$

PROBLEM-SOLVING	7	7–8(½)	8–9(½)

7 Find the numbers which satisfy the given condition.

a The product of x and a number 3 more than x is zero.
b The product of x and a number 7 less than x is zero.
c The product of a number 1 less than x and a number 4 more than x is zero.
d The product of a number 1 less than twice x and 6 more than x is zero.
e The product of a number 3 more than twice x and 1 less than twice x is zero.

8 Write these equations as quadratics in standard form. Remove any brackets and fractions.

a $5x^2 + x = x^2 - 1$ **b** $2x = 3x^2 - x$
c $3(x^2 - 1) = 1 + x$ **d** $2(1 - 3x^2) = x(1 - x)$
e $\frac{x}{2} = x^2 - \frac{3}{2}$ **f** $\frac{4x}{3} - x^2 = 2(1 - x)$
g $\frac{5}{x} + 1 = x$ **h** $\frac{3}{x} + \frac{5}{2} = 2x$

9 These quadratic equations have two integer solutions between -5 and 5. Use trial and error to find them.

a $x^2 - x - 2 = 0$ **b** $x^2 - 4x + 3 = 0$ **c** $x^2 - 4x = 0$
d $x^2 + 3x = 0$ **e** $x^2 + 3x - 4 = 0$ **f** $x^2 - 16 = 0$

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REASONING	10	10, 11	11, 12

10 Consider the quadratic equation $(x + 2)^2 = 0$.

a Write the equation in the form $(\_\_\_\_)(\_\_\_\_) = 0$.

b Use the Null Factor Law to find the solutions to the equation. What do you notice?

c Now solve these quadratic equations.

i $(x + 3)^2 = 0$
ii $(x - 5)^2 = 0$
iii $(2x - 1)^2 = 0$
iv $(5x - 7)^2 = 0$

11 Consider the equation $3(x - 1)(x + 2) = 0$.

a First divide both sides of the equation by 3. Write down the new equation.

b Solve the equation using the Null Factor Law.

c Compare the given original equation with the equation found in part **a**. Explain why the solutions are the same.

d Solve these equations.

i $7(x + 2)(x - 3) = 0$
ii $11x(x + 2) = 0$
iii $\dfrac{(x + 1)(x - 3)}{4} = 0$
iv $-2(x + 2)(x - 5) = 0$

12 Consider the equation $(x - 3)^2 + 1 = 0$.

a Substitute these x-values to decide if they are solutions to the equation.

i $x = 3$
ii $x = 4$
iii $x = 0$
iv $x = -2$

b Do you think the equation will have a solution? Explain why.

ENRICHMENT: Polynomials	–	–	13, 14

13 Polynomials are sums of integer powers of x. They are given names according to the highest power of x in the polynomial expression.

Example	Polynomial name
2	Constant
$2x + 1$	Linear
$x^2 - 2x + 5$	Quadratic
$x^3 - x^2 + 6x - 1$	Cubic
$7x^4 + x^3 + 2x - x + 4$	Quartic
$4x^5 - x + 1$	Quintic

Name these polynomial equations.

a $3x - 1 = 0$
b $x^2 + 2 = 0$
c $x^5 - x^4 + 3 = 0$
d $5 - 2x + x^3 = 0$
e $3x - 2x^4 + x^2 = 0$
f $5 - x^5 = x^4 + x$

14 Solve these polynomial equations using the Null Factor Law.

a $(x + 1)(x - 3)(x + 2) = 0$

b $(x - 2)(x - 5)(x + 11) = 0$

c $(2x - 1)(3x + 2)(5x - 1) = 0$

d $(3x + 2)(5x + 4)(7x + 10)(2x - 13) = 0$

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10B Solving $ax^2 + bx = 0$ and $x^2 - d = 0$ by factorising

Learning intentions
- To know the steps required to solve a quadratic equation
- To be able to factorise a quadratic equation in order to apply the Null Factor Law
- To be able to find the solutions of quadratic equations involving common factors or perfect squares

Past, present and future learning
- This section addresses concepts from the Stage 5 Path Topics *Equations B and C*
- These concepts will be revised and extended in Chapter 5 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

When using the Null Factor Law, we notice that equations must first be expressed as a product of two factors. Hence, any equation not in this form must first be factorised. Two types of quadratic equations are studied here. The first is of the form $ax^2 + bx = 0$, where x is a common factor, and the second is of the form $x^2 = d$.

Aerospace engineers use quadratic equations, such as for the trajectory of a rocket launching a communications satellite. An object launched vertically with velocity, u, moving under gravity, with acceleration, g, reaches a maximum height, h, where $u^2 = 2gh$.

Lesson starter: Which factorisation technique?

These two equations may look similar but they are not the same: $x^2 - 9x = 0$ and $x^2 - 9 = 0$.

- Discuss how you could factorise each expression on the left-hand side of the equations.
- How does the factorised form help to solve the equations? What are the solutions? Are the solutions the same for both equations?
- By rearranging $x^2 - 9 = 0$ into $x^2 = 9$, can you explain how to find the two solutions you found above?

KEY IDEAS

- When solving an equation of the form $ax^2 + bx = 0$, factorise by taking out common factors including x.

$$\begin{aligned} 2x^2 - 8x &= 0 \\ 2x(x - 4) &= 0 \\ 2x = 0 \text{ or } x - 4 &= 0 \\ x = 0 \text{ or } x &= 4 \end{aligned}$$

- When solving an equation of the form $ax^2 = d$, divide both sides by a and then take the square root of each side.

$$\begin{aligned} 5x^2 &= 20 \\ x^2 &= 4 \\ x = 2 &\text{ or } x = -2 \end{aligned}$$

- $x^2 = d^2$ could also be rearranged to $x^2 - d^2 = 0$ and then factorised as a difference of two squares.

$$\begin{aligned} x^2 - 4 &= 0 \\ (x + 2)(x - 2) &= 0 \\ x = -2 &\text{ or } x = 2 \end{aligned}$$

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BUILDING UNDERSTANDING

1 State the highest common factor of these pairs of terms.

a $16x$ and 24 **b** x^2 and $2x$ **c** $3x^2$ and $6x$

2 Factorise these expressions fully by first taking out the highest common factor.

a $x^2 - 3x$ **b** $6x^2 + 4x$
c $4x - 16x^2$ **d** $2x^2 - 8$

3 Use the Null Factor Law to state the solutions to these equations.

a $x(x - 3) = 0$ **b** $4x(x + 1) = 0$

4 Explain why $x = 3$ and $x = -3$ are solutions to $x^2 = 9$.

Example 4 Solving quadratic equations of the form $ax^2 + bx = 0$

Solve each of the following equations.

a $x^2 + 4x = 0$ **b** $2x^2 = 8x$

SOLUTION	EXPLANATION
a $x^2 + 4x = 0$ $x(x + 4) = 0$ $x = 0$ or $x + 4 = 0$ $x = 0$ or $x = -4$	Factorise by taking out the common factor x. Using the Null Factor Law, set each factor, x and $(x + 4)$, equal to 0. Solve for x. Check your solutions using substitution.
b $2x^2 = 8x$ $2x^2 - 8x = 0$ $2x(x - 4) = 0$ $2x = 0$ or $x - 4 = 0$ $x = 0$ or $x = 4$	Make the right-hand side equal to zero by subtracting $8x$ from both sides. Factorise by taking out the common factor of $2x$ and apply the Null Factor Law to solve. For $2 \times x = 0$, x must equal 0.

Now you try

Solve each of the following equations.

a $x^2 + 6x = 0$ **b** $3x^2 = 15x$

Example 5 Solving equations of the form $ax^2 = d$

Solve these equations.

a $x^2 = 16$ **b** $3x^2 = 18$

SOLUTION	EXPLANATION
a $x^2 = 16$ $x = 4$ or $x = -4$	As this is a perfect square, take the square root of both sides. $x = 4$ or -4 since $4^2 = 16$ and $(-4)^2 = 16$.

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Alternative method:

$$\begin{aligned} x^2 &= 16 \\ x^2 - 16 &= 0 \\ (x+4)(x-4) &= 0 \end{aligned}$$
$x + 4 = 0$ or $x - 4 = 0$
$x = -4$ or $x = 4$

Rearrange into standard form.
Factorise using $a^2 - b^2 = (a + b)(a - b)$ then use the Null Factor Law to find the solutions.

b $3x^2 = 18$
$x^2 = 6$
$x = \sqrt{6}$ or $x = -\sqrt{6}$

Divide both sides by 3 and then take the square root of each side.
As 6 is not a square number leave answers in exact form as $\sqrt{6}$ and $-\sqrt{6}$.

Alternative method:

$$\begin{aligned} 3x^2 &= 18 \\ 3x^2 - 18 &= 0 \\ 3(x^2 - 6) &= 0 \end{aligned}$$
$3(x + \sqrt{6})(x - \sqrt{6}) = 0$
$x + \sqrt{6} = 0$ or $x - \sqrt{6} = 0$
$x = -\sqrt{6}$ or $x = \sqrt{6}$

Alternatively, express in standard form and then note the common factor of 3.
Treat $x^2 - 6 = x^2 - (\sqrt{6})^2$ as a difference of squares.

Apply the Null Factor Law and solve for x.

Now you try

Solve these equations.

a $x^2 = 49$ **b** $2x^2 = 14$

Exercise 10B

FLUENCY	1–4(½)	1–5(½)	1–5(½)

Example 4a

1 Solve these equations.
a $x^2 + 3x = 0$ **b** $x^2 + 7x = 0$ **c** $x^2 + 4x = 0$ **d** $x^2 - 5x = 0$
e $x^2 - 8x = 0$ **f** $x^2 - 2x = 0$ **g** $x^2 + \frac{1}{3}x = 0$ **h** $x^2 - \frac{1}{2}x = 0$

2 Solve these equations by first taking out the highest common factor.
a $2x^2 - 6x = 0$ **b** $3x^2 - 12x = 0$ **c** $4x^2 + 20x = 0$
d $6x^2 - 18x = 0$ **e** $-5x^2 + 15x = 0$ **f** $-2x^2 - 8x = 0$

Example 4b

3 Solve the following equations by first rearranging into standard form.
a $x^2 = 3x$ **b** $5x^2 = 10x$ **c** $4x^2 = 16x$
d $3x^2 = -9x$ **e** $2x^2 = -8x$ **f** $7x^2 = -21x$

Example 5a

4 Solve the following equations.
a $x^2 = 9$ **b** $x^2 = 36$ **c** $x^2 = 25$
d $x^2 - 144 = 0$ **e** $x^2 - 81 = 0$ **f** $x^2 - 400 = 0$

Example 5b

5 Solve each of the following equations by first noting the common factor.
a $7x^2 = 28$ **b** $5x^2 = 45$ **c** $2x^2 = 50$ **d** $6x^2 = 24$
e $2x^2 = 12$ **f** $3x^2 = 15$ **g** $5x^2 - 35 = 0$ **h** $8x^2 - 24 = 0$

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PROBLEM–SOLVING	6(½)	6(½), 7	7, 8

6 Solve these equations by first rearranging where necessary.

a $4 = x^2$
b $-x^2 + 25 = 0$
c $-x^2 = -100$
d $-3x^2 = 21x$
e $-5x^2 + 35x = 0$
f $1 - x^2 = 0$
g $12x = 18x^2$
h $2x^2 + x = 7x^2 - 2x$
i $9 - 2x^2 = x^2$

7 Write an equation and solve it to find the number.

a The square of the number is 7 times the same number.
b The difference between the square of a number and 64 is zero.
c 3 times the square of a number is equal to -12 times the number.

8 Remove brackets or fractions to help solve these equations.

a $x - \frac{4}{x} = 0$
b $\frac{36}{x} - x = 0$
c $\frac{3}{x^2} = 3$
d $5(x^2 + 1) = 3x^2 + 7$
e $x(x - 3) = 2x^2 + 4x$
f $3x(5 - x) = x(7 - x)$

REASONING	9	9	9, 10

9 Consider the equation $x^2 + 4 = 0$.

a Explain why it cannot be written in the form $(x + 2)(x - 2) = 0$.
b Are there any solutions to the equation $x^2 + 4 = 0$? Why/Why not?

10 An equation of the form $ax^2 + bx = 0$ will always have two solutions if a and b are not zero.

a Explain why one of the solutions will always be $x = 0$.
b Write the rule for the second solution in terms of a and b.

ENRICHMENT: More quadratic equation forms	–	–	11–12(½)

11 Note, for example, that $4x^2 = 9$ becomes $x^2 = \frac{9}{4}$ and then $x = \frac{3}{2}$ or $x = -\frac{3}{2}$, since $\sqrt{\frac{9}{4}} = \frac{3}{2}$.

Now solve these equations.

a $9x^2 = 16$
b $25x^2 = 36$
c $4 = 100x^2$
d $81 - 25x^2 = 0$
e $64 - 121x^2 = 0$
f $-49x^2 + 144 = 0$

12 Note, for example, that $(x - 1)^2 - 4 = 0$ becomes $(x - 1)^2 = 4$ with $x - 1 = 2$ or $x - 1 = -2$, giving $x = 3$ and $x = -1$. Now solve these equations.

a $(x - 2)^2 = 9$
b $(x + 5)^2 = 16$
c $(2x + 1)^2 = 1$
d $(5x - 3)^2 - 25 = 0$
e $(4 - x)^2 - 9 = 0$
f $(3 - 7x)^2 - 100 = 0$

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10C Solving $x^2 + bx + c = 0$ by factorising

Learning intentions

- To be able to solve quadratic equations involving monic trinomials
- To understand that trinomials that factorise to perfect squares will have only one solution

Past, present and future learning

- This section addresses concepts from the Stage 5 Path Topics *Equations B and C*
- These concepts will be revised and extended in Chapter 5 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

Earlier in Chapter 8 we learnt to factorise quadratic trinomials with three terms. For example, $x^2 + 5x + 6$ factorises to $(x + 2)(x + 3)$. This means that the Null Factor Law can be used to solve equations of the form $x^2 + bx + c = 0$.

Mechanical engineers design industrial ventilation systems and can solve quadratic equations to calculate dimensions. For a duct with a rectangular cross-section area 1500 cm^2, a possible equation is: $x(x - 20) = 1500$, i.e. $x^2 - 20x - 1500 = 0$.

Lesson starter: Remembering how to factorise quadratic trinomials

First expand these quadratics using the distributive law.

Distributive law: $(a + b)(c + d) = ac + ad + bc + bd$

- $(x + 1)(x + 2)$
- $(x - 3)(x + 4)$
- $(x - 5)(x - 2)$

Now factorise these expressions.

- $x^2 + 5x + 6$
- $x^2 - x - 12$
- $x^2 - 8x + 7$

Discuss your method for finding the factors of each quadratic above.

KEY IDEAS

- Solve quadratics of the form $x^2 + bx + c = 0$ by factorising the quadratic trinomial.
  - Ask 'What factors of c add to give b?'
  - Then use the Null Factor Law.

$$\begin{aligned} x^2 - 3x - 28 &= 0 \\ (x - 7)(x + 4) &= 0 \\ x - 7 = 0 \text{ or } x + 4 &= 0 \\ x = 7 \text{ or } x &= -4 \end{aligned} \quad \text{and} \quad \begin{aligned} -7 \times 4 &= -28 \\ -7 + 4 &= -3 \end{aligned}$$

- Equations involving perfect squares will give only one solution.

$$\begin{aligned} x^2 - 6x + 9 &= 0 \\ (x - 3)(x - 3) &= 0 \\ x - 3 &= 0 \\ x &= 3 \end{aligned}$$

- $x^2 + bx + c$ is called a **monic** quadratic since the coefficient of x^2 is 1.

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BUILDING UNDERSTANDING

1. Decide which two factors of the first number add to give the second number.
   - a $6, 5$
   - b $10, -7$
   - c $-5, 4$
   - d $-12, -1$
2. Factorise each of the following.
   - a $x^2 + 12x + 35$
   - b $x^2 + 4x - 45$
   - c $x^2 - 10x + 16$
3. State the missing components to complete the working to solve each equation.
   - a $x^2 + 9x + 20 = 0$
     $(x + 5)(\_\_\_) = 0$
     $x + 5 = 0$ or $\_\_\_\_ = 0$
     $x = \_\_\_$ or $x = \_\_\_$
   - b $x^2 - 2x - 24 = 0$
     $(x - 6)(\_\_\_) = 0$
     $x - 6 = 0$ or $\_\_\_\_ = 0$
     $x = \_\_\_$ or $x = \_\_\_$

Example 6 Solving equations involving quadratic trinomials

Solve these quadratic equations.

a $x^2 + 7x + 12 = 0$
b $x^2 - 2x - 8 = 0$
c $x^2 - 8x + 15 = 0$

SOLUTION	EXPLANATION
a $x^2 + 7x + 12 = 0$ $(x + 3)(x + 4) = 0$ $x + 3 = 0$ or $x + 4 = 0$ $x = -3$ or $x = -4$	Factors of 12 which add to 7 are 3 and 4. $3 \times 4 = 12, 3 + 4 = 7$ Use the Null Factor Law to solve the equation.
b $x^2 - 2x - 8 = 0$ $(x - 4)(x + 2) = 0$ $x - 4 = 0$ or $x + 2 = 0$ $x = 4$ or $x = -2$	Factors of -8 which add to -2 are -4 and 2. $-4 \times 2 = -8, -4 + 2 = -2$ Solve using the Null Factor Law.
c $x^2 - 8x + 15 = 0$ $(x - 5)(x - 3) = 0$ $x - 5 = 0$ or $x - 3 = 0$ $x = 5$ or $x = 3$	The factors of 15 must add to give -8. $-5 \times (-3) = 15$ and $-5 + (-3) = -8$, so -5 and -3 are the two numbers.

Now you try

Solve these quadratic equations.

a $x^2 + 6x + 8 = 0$
b $x^2 - 3x - 10 = 0$
c $x^2 - 8x + 12 = 0$

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Example 7 Solving with perfect squares and trinomials not in standard form

Solve these quadratic equations.

a $x^2 - 8x + 16 = 0$

b $x^2 = x + 6$

SOLUTION

a
$$\begin{aligned} x^2 - 8x + 16 &= 0 \\ (x-4)(x-4) &= 0 \\ x - 4 &= 0 \\ x &= 4 \end{aligned}$$

b
$$\begin{aligned} x^2 &= x + 6 \\ x^2 - x - 6 &= 0 \\ (x-3)(x+2) &= 0 \\ x - 3 &= 0 \text{ or } x + 2 = 0 \\ x &= 3 \text{ or } x = -2 \end{aligned}$$

EXPLANATION

Factors of 16 which add to -8 are -4 and -4.
$(x-4)(x-4) = (x-4)^2$ is a perfect square so there is only one solution.

First make the right-hand side equal zero by subtracting x and 6 from both sides. This is now in standard form. (When rearranging to standard form, keep the x^2 term positive.)
Factors of -6 which add to -1 are -3 and 2.

Now you try

Solve these quadratic equations.

a $x^2 - 10x + 25 = 0$

b $x^2 = 3x + 40$

Exercise 10C

FLUENCY	1, 2–3(½)	2–4(⅓)	2–4(⅓)

Example 6a

1 Solve these quadratic equations.

a $x^2 + 8x + 12 = 0$ b $x^2 + 11x + 24 = 0$ c $x^2 + 7x + 10 = 0$

Example 6

2 Solve these quadratic equations.

a $x^2 + 10x + 24 = 0$ b $x^2 + 9x + 18 = 0$ c $x^2 + 4x + 3 = 0$

d $x^2 - 12x + 32 = 0$ e $x^2 - 9x + 18 = 0$ f $x^2 - 10x + 21 = 0$

g $x^2 - 2x - 15 = 0$ h $x^2 - 6x - 16 = 0$ i $x^2 - 4x - 45 = 0$

j $x^2 - 10x + 24 = 0$ k $x^2 - x - 42 = 0$ l $x^2 + 5x - 84 = 0$

Example 7a

3 Solve these quadratic equations which include perfect squares.

a $x^2 + 6x + 9 = 0$ b $x^2 + 4x + 4 = 0$ c $x^2 + 14x + 49 = 0$

d $x^2 + 24x + 144 = 0$ e $x^2 - 10x + 25 = 0$ f $x^2 - 16x + 64 = 0$

g $x^2 - 12x + 36 = 0$ h $x^2 - 18x + 81 = 0$ i $x^2 - 20x + 100 = 0$

Example 7b

4 Solve these quadratic equations by first rearranging to standard form.

a $x^2 = 3x + 10$ b $x^2 = 7x - 10$ c $x^2 = 6x - 9$

d $x^2 = 4 - 3x$ e $14 - 5x = x^2$ f $x^2 + 16 = 8x$

g $x^2 - 12 = -4x$ h $6 - x^2 = 5x$ i $15 = 8x - x^2$

j $16 - 6x = x^2$ k $-6x = x^2 + 8$ l $-x^2 - 7x = -18$

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PROBLEM–SOLVING	5(½), 6	5(½), 6, 7(½)	5(⅓), 6, 7–8(½)

5 Solve these equations by taking out a common factor as the first factorising step.

a $2x^2 - 2x - 12 = 0$ **b** $3x^2 + 24x + 45 = 0$ **c** $4x^2 - 24x - 64 = 0$
d $4x^2 - 20x + 24 = 0$ **e** $2x^2 - 8x + 8 = 0$ **f** $3x^2 + 6x + 3 = 0$
g $7x^2 - 70x + 175 = 0$ **h** $2x^2 + 12x = -18$ **i** $5x^2 = 35 - 30x$

6 The temperature in °C inside a room at a scientific base in Antarctica after 10 a.m. is given by the expression $t^2 - 9t + 8$ where t is in hours. Find the times in the day when the temperature is 0°C.

7 Write down a quadratic equation in standard form which has the following solutions.

a $x = 1$ and $x = 2$ **b** $x = 3$ and $x = -2$ **c** $x = -4$ and $x = 1$
d $x = -3$ and $x = 10$ **e** $x = 5$ only **f** $x = -11$ only

8 Remove brackets, decimals or fractions and write in standard form to help solve these equations.

a $x^2 = 5(x - 1.2)$ **b** $2x(x - 3) = x^2 - 9$
c $3(x^2 + x - 10) = 2x^2 - 5(x + 2)$ **d** $x - 2 = \frac{35}{x}$
e $2 + \frac{1}{x} = -x$ **f** $\frac{x}{4} = 1 - \frac{1}{x}$

REASONING	9	9, 10	10, 11

9 Consider this equation and solution.

a Explain what is wrong with the solution.
b Find the correct solutions to the equation.

$$\begin{aligned} x^2 - x - 12 &= -6 \\ (x - 4)(x + 3) &= -6 \\ x - 4 = -6 \text{ or } x + 3 &= -6 \\ x = -2 \text{ or } x &= -9 \end{aligned}$$

10 Explain why $x^2 - 2x + 1 = 0$ only has one solution.

11 Write down a quadratic equation that has these solutions. Write in factorised form.

a $x = a$ only **b** $x = a$ and $x = b$

ENRICHMENT: Solving non-monics	–	–	12(½)

12 Many quadratic equations have a coefficient of x^2 not equal to 1. These are called non-monic quadratics. Factorisation can be trickier but is still possible. You may have covered this in Chapter 8. Here is an example.

$$\begin{aligned} 2x^2 + 5x - 3 &= 0 && \text{(Factors of } 2 \times (-3) = -6 \text{ that add to give 5 are } -1 \text{ and } 6.) \\ 2x^2 - x + 6x - 3 &= 0 && (6x - 1x = 5x) \\ x(2x - 1) + 3(2x - 1) &= 0 && \text{Group in pairs.} \\ (2x - 1)(x + 3) &= 0 \\ 2x - 1 = 0 \text{ or } x + 3 &= 0 \\ x = \frac{1}{2} \text{ or } x &= -3 \end{aligned}$$

Solve these quadratic equations.

a $5x^2 + 16x + 3 = 0$ **b** $3x^2 + 4x - 4 = 0$ **c** $6x^2 + x - 1 = 0$
d $10x^2 + 5x - 5 = 0$ **e** $2x^2 = 15 - 7x$ **f** $4x^2 = 12x - 9$
g $x = 3x^2 - 14$ **h** $17x = 12 - 5x^2$ **i** $25 = -9x^2 - 30x$

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10D Using quadratic equations to solve problems

Learning intentions
- To be able to form an equation from a word problem using a variable for the unknown
- To be able to recognise a quadratic equation and apply the appropriate steps to solve
- To understand that solutions will need to be checked for validity in the context of the problem

Past, present and future learning
- This section addresses concepts from the Stage 5 Path Topics *Equations B and C*
- These concepts will be revised and extended in Chapter 5 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

When using mathematics to solve problems, we often arrive at a quadratic equation. In solving the quadratic equation we obtain the solutions to the problem. Setting up an equation and then interpreting the solution are important parts of the problem-solving process.

When dividing up land for new suburbs, surveyors apply area formulas which form quadratic equations when dimensions are expressed using the same variable. Surveyors re-arrange the quadratic to equal zero and solve it for the unknown variable.

Lesson starter: Solving for the unknown number

The product of a positive number and 6 more than the same number is 16.

- Using x as the unknown number, write an equation describing the given condition.
- Solve your equation for x.
- Are both solutions feasible (allowed)?
- Discuss how this method compares with the method of trial and error.

KEY IDEAS

- When using quadratic equations to solve problems, follow these steps.
  - Define your variable.
    - Write 'Let x be …' e.g. Let x be the breadth in cm.
  - Use your variable to define any other quantity, e.g. length $= x + 7$.
  - Write an equation relating the facts given in the question.
  - Solve your equation using the Null Factor Law.
  - Check that your solutions are feasible.
    - Some problems may not allow solutions that are negative numbers or fractions.
  - Answer the original question in words and check that your answer seems reasonable.

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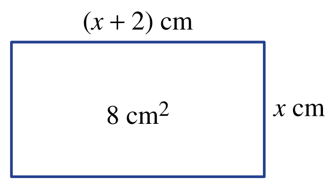
BUILDING UNDERSTANDING

1 Solve these equations for x by first expanding and producing a zero on the right-hand side.

- **a** $x(x+3)=18$
- **b** $x(x-1)=20$
- **c** $(x-1)(x+4)=6$

2 This rectangle has an area of 8 cm^2 and a length that is 2 cm more than its breadth.



- **a** Using length × breadth = area, give an equation.
- **b** Solve your equation by expanding and subtracting 8 from both sides. Then use the Null Factor Law.
- **c** Which of your two solutions is feasible for the breadth of the rectangle?
- **d** State the dimensions (breadth and length) of the rectangle.

Example 8 Solving area problems with quadratic equations

The length of a book is 4 cm more than its breadth and the area of the face of the book is 320 cm^2. Find the dimensions of the face of the book.

SOLUTION	EXPLANATION
Let x cm be the breadth of the book face. Length $= (x+4)$ cm	Define a variable for breadth then write the length in terms of the breadth.
Area: $x(x+4)=320$ $x^2+4x-320=0$	Write an equation to suit the given situation. Expand, then subtract 320 from both sides.
$(x+20)(x-16)=0$	Factorise the quadratic trinomial
$x+20=0$ or $x-16=0$ $x=-20$ or $x=16$ $\therefore x=16$ since $x>0$	Solve using the Null Factor Law, but note that a breadth of -20 cm is not feasible.
$\therefore$ breadth $=16$ cm and length $=20$ cm	Finish by writing the dimensions – breadth and length – as required. Length $= x+4=16+4=20$ cm

Now you try

The length of a rectangular jigsaw puzzle is 12 cm more than its breadth and the area of the jigsaw is 640 cm^2. Find the dimensions of the jigsaw puzzle.

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Exercise 10D

FLUENCY	1–4, 6	1, 3, 5, 6	2, 4–6

Example 8

1 The length of a rectangular magazine is 8 cm more than its breadth and the area of the magazine is 240 cm^2. Find the dimensions of the magazine.

2 The length of a rectangular brochure is 5 cm more than its breadth and the area of the face of the brochure is 36 cm^2. Find the dimensions of the face of the brochure.

3 The product of a number and 2 more than the same number is 48. Write an equation and solve to find the two possible solutions.

4 The product of a number and 7 less than the same number is 60. Write an equation and solve to find the two possible solutions.

5 The product of a number and 13 less than the same number is 30. Write an equation and solve to find the two possible solutions.

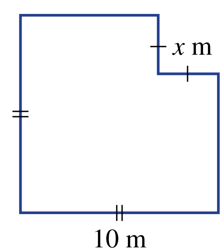
6 The length of a small kindergarten play area is 20 metres less than its breadth and the area is 69 m^2. Find the dimensions of the play area.

PROBLEM–SOLVING	7, 8	7–9	8–10

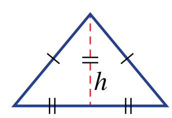
7 A square of side length 10 metres has a square of side length x metres removed from one corner.

a Write an expression for the area remaining after the square of side length x metres is removed. (*Hint*: Use subtraction.)

b Find the value of x if the area remaining is 64 m^2.



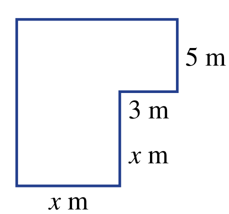
8 An isosceles triangle has height (h) equal to half its base length. Find the value of h if the area is 25 square units.



9 A rectangular farm shed (3 m by 5 m) is to be extended to form an ‘L’ shape as shown.

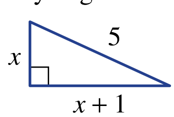
a Write an expression for the total area of the extended farm shed.

b Find the value of x if the total area is to be 99 m^2.



10 Use Pythagoras’ theorem to find the value of x in these right-angled triangles.

a



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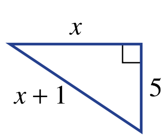

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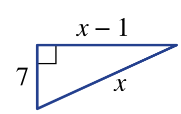

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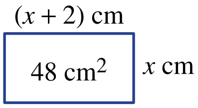
REASONING	11, 12	12, 13	12–14

11 The equation for the area of this rectangle is $x^2 + 2x - 48 = 0$, which has solutions $x = -8$ and $x = 6$. Which solution do you ignore and why?



12 The product of an integer and one less than the same integer is 6. The equation for this is $x^2 - x - 6 = 0$. How many different solutions are possible and why?

13 This table shows the sum of the first n positive integers. If $n = 3$ then the sum is $1 + 2 + 3 = 6$.

n	1	2	3	4	5	6
Sum	1	3	6			

a Write the sum for $n = 4$, $n = 5$ and $n = 6$.

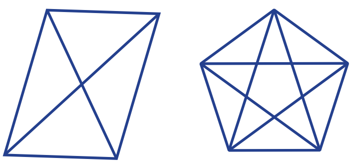
b The expression for the sum is given by $\frac{n(n+1)}{2}$. Use this expression to find the sum if:

i $n = 7$ ii $n = 20$

c Use the expression to find n for these sums. Write an equation and solve it.

i sum = 45 ii sum = 120

14 The number of diagonals in a polygon with n sides is given by $\frac{n}{2}(n - 3)$. Shown here are the diagonals for a quadrilateral and a pentagon.



n	4	5	6	7
Diagonals	2	5		

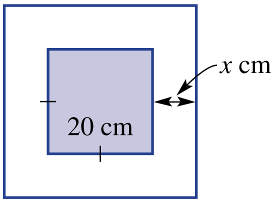
a Use the given expression to find the two missing numbers in the table.

b Find the value of n if the number of diagonals is:

i 20 ii 54

ENRICHMENT: Picture frames	–	–	15, 16

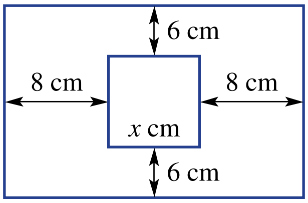
15 A square picture is to be edged with a border of breadth x cm. The inside picture has side length of 20 cm.



a Write an expression for the total area.

b Find the breadth of the frame if the total area of picture and frame is to be 1600 cm^2.

16 A square picture is surrounded by a rectangular frame as shown. The total area is to be 320 cm^2. Find the side length of the picture.



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Progress quiz

10A 1 Write these quadratic equations in standard form.
a $x^2 - 3x = 8$
b $3(x^2 - 2) = 4x$
c $2(x^2 - 5) = 2x - 7$

10A 2 Substitute the given x-value into the equation and say whether or not it is a solution.
a $x^2 - x - 6 = 0 \quad (x = 2)$
b $x^2 - x - 6 = 0 \quad (x = -2)$

10A 3 Use the Null Factor Law to solve these equations.
a $x(x - 7) = 0$
b $(x - 5)(x + 2) = 0$
c $(3x - 1)(4x - 3) = 0$

10B 4 Solve the following equations.
a $x^2 + 11x = 0$
b $3x^2 - 18x = 0$
c $5x^2 = 20x$

10B 5 Solve these equations.
a $x^2 = 49$
b $x^2 - 1 = 0$
c $3x^2 = 12$
d $7x^2 - 63 = 0$

10C 6 Solve these quadratic equations.
a $x^2 + 11x + 24 = 0$
b $x^2 - 15x + 36 = 0$
c $x^2 - 2x - 35 = 0$

10C 7 Solve these quadratic equations.
a $x^2 + 8x + 16 = 0$
b $x^2 - 14x + 49 = 0$
c $x^2 - 24 = 5x$

10D 8 A rectangular lawn is 3 m longer than it is wide and has an area of 54 m^2. Write an equation and solve it to find the dimensions of this lawn.

10D 9 The product of a number and 5 more than the same number is 84. Write an equation and solve to find the two possible values of the number.

10C 10 Write down a quadratic equation in standard form which has the following solutions.
a $x = 3$ and $x = 2$
b $x = 6$ and $x = -1$
c $x = -3$ only

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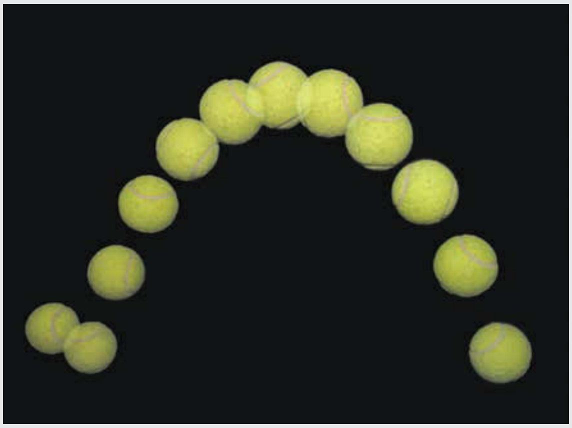
10E The parabola

Learning intentions
- To know that a quadratic relation produces a graph called a parabola
- To know the key features of a parabola including symmetry and the vertex
- To understand that a parabola can have a minimum or maximum vertex
- To be able to identify the key features of a parabola from a graph
- To be able to plot a parabola from a rule by creating a table of values and plotting points

Past, present and future learning
- This section addresses concepts from the Stage 5 Path Topic *Non-linear relationships C*
- These concepts will be revised and extended in Chapter 7 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

Relations that have rules in which the highest power of x is 2, such as $y = x^2$, $y = 2x^2 - 3$ and $y = 3x^2 + 2x - 4$, are called quadratics and their graphs are called parabolas. Parabolic shapes can be seen in many modern-day objects or situations such as the arches of bridges, the paths of projectiles and the surfaces of reflectors.



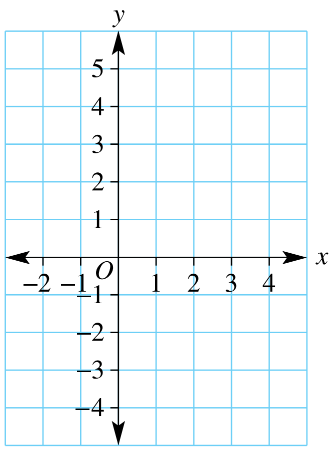
When hitting a ball, tennis players, cricketers and golfers can vary the force and launching angle, hence modifying the direction, maximum height and range of the ball's parabolic flight path.

Lesson starter: Finding features

A quadratic is given by the equation $y = x^2 - 2x - 3$. Complete these tasks to discover its graphical features.

- Use the rule to complete this table of values.

x	−2	−1	0	1	2	3	4
y							



- Plot your points on a copy of the axes shown at right and join them to form a smooth curve.
- Describe these features:
  - minimum turning point
  - axis of symmetry
  - coordinates of the y-intercept
  - coordinates of the x-intercepts

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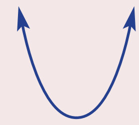
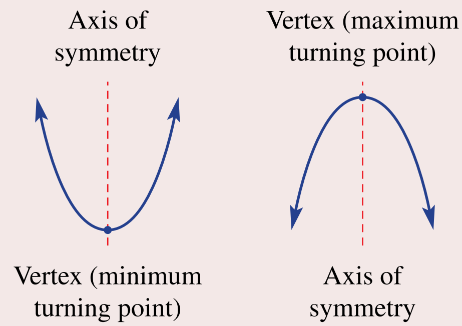
KEY IDEAS

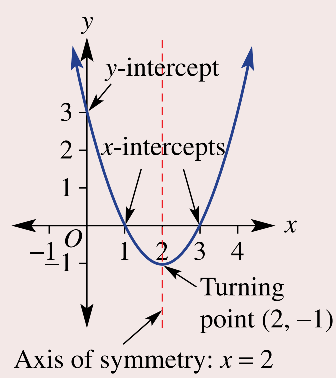
- The graph of a quadratic relation is called a **parabola**. Its basic shape is shown here.
  - The basic quadratic rule is $y = x^2$.



- The general equation of a quadratic is $y = ax^2 + bx + c$.
- A parabola is symmetrical about a line called the **axis of symmetry** and it has one **turning point** (a vertex), which may be a **maximum** or a **minimum**.
  - If a is positive the parabola is concave up.
  - If a is negative the parabola is concave down.



Here is an example of a parabola with equation $y = x^2 - 4x + 3$, showing all the key features.



BUILDING UNDERSTANDING

1 Choose a word from this list to complete each sentence.

lowest, parabola, vertex, highest, intercepts, zero

- **a** A maximum turning point is the ___________ point on the graph.
- **b** The graph of a quadratic is called a ___________.
- **c** The x-___________ are the points where the graph cuts the x-axis.
- **d** The axis of symmetry is a vertical line passing through the ___________.
- **e** A minimum turning point is the ___________ point on the graph.
- **f** The y-intercept is at x equals ___________.

2 State the equation of a vertical line (e.g. $x = 2$) that passes through these points.

- **a** $(3, 0)$
- **b** $(1, 5)$
- **c** $(-2, 4)$

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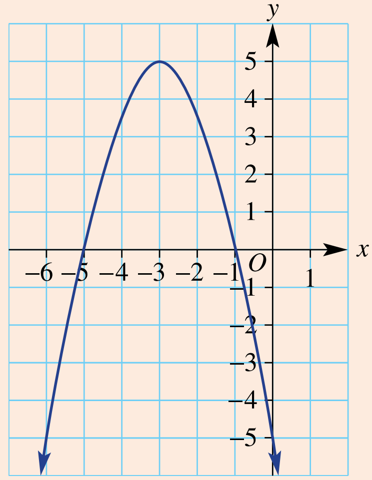
Example 9 Identifying the features of a parabola

For this graph state the:

a equation of the axis of symmetry
b type of turning point
c coordinates of the turning point
d coordinates of the x-intercepts
e coordinates of the y-intercept.



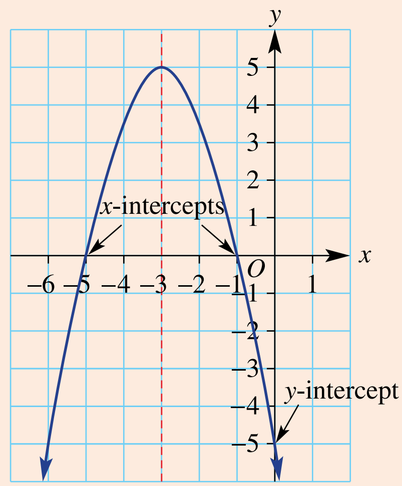
SOLUTION

a $x = -3$
b Maximum turning point
c Turning point is $(-3, 5)$
d x-intercepts: $(-5, 0)$ and $(-1, 0)$
e y-intercept: $(0, -5)$

EXPLANATION

Graph is symmetrical about the vertical line $x = -3$. Graph has its highest y-coordinate at the turning point, so it is a maximum point.

Highest point: turning point $= (-3, 5)$



Axis of symmetry: $x = -3$

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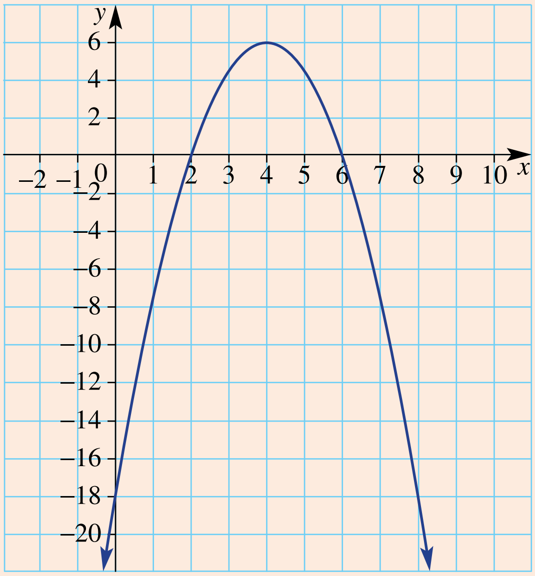
Now you try

For this graph state the:

a equation of the axis of symmetry
b type of turning point
c coordinates of the turning point
d coordinates of the x-intercepts
e coordinates of the y-intercept.



The cross-section of a satellite dish is parabolic.

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Example 10 Plotting a parabola

Use the quadratic rule $y = x^2 - 4$ to complete these tasks.

a Complete this table of values.

x	−3	−2	−1	0	1	2	3
y							

b Draw a set of axes using a scale that suits the numbers in your table. Then plot the points to form a parabola.

c State these features.

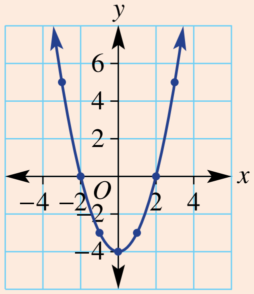
- **i** Type of turning point
- **ii** Axis of symmetry
- **iii** Coordinates of the turning point
- **iv** The coordinates of the y-intercept
- **v** The coordinates of the x-intercepts.

SOLUTION

a

x	−3	−2	−1	0	1	2	3
y	5	0	−3	−4	−3	0	5

b



c **i** Minimum turning point

ii $x = 0$ is the axis of symmetry

iii Turning point $(0, -4)$

iv y-intercept: $(0, -4)$

v x-intercepts: $(-2, 0)$ and $(2, 0)$

EXPLANATION

Substitute each x-value into the rule to find each y-value: e.g. $x = -3$,
$$y = (-3)^2 - 4 = 9 - 4 = 5$$

Plot each coordinate pair and join to form a smooth curve.

The turning point at $(0, -4)$ has the lowest y-coordinate for the entire graph.
The vertical line $x = 0$ divides the graph like a mirror line. The y-intercept is at $x = 0$.
The x-intercepts are at $y = 0$ on the x-axis.

Now you try

Use the quadratic rule $y = x^2 - 9$ to complete these tasks.

a Complete this table of values.

x	−3	−2	−1	0	1	2	3
y							

b Draw a set of axes using a scale that suits the numbers in your table. Then plot the points to form a parabola.

c State these features.

- **i** Type of turning point
- **ii** Axis of symmetry
- **iii** Coordinates of the turning point
- **iv** The coordinates of the y-intercept
- **v** The coordinates of the x-intercepts.

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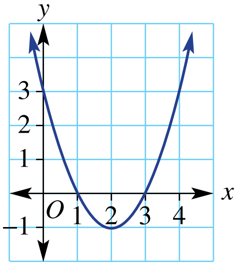
Exercise 10E

FLUENCY	1(½), 2, 3	1(½), 2–4	1(⅓), 2, 3, 5

Example 9

1 For each of the following graphs, state:
   - i the equation of the axis of symmetry
   - ii the type of turning point (maximum or minimum)
   - iii the coordinates of the turning point
   - iv the coordinates of the x-intercepts
   - v the coordinates of the y-intercept.

a


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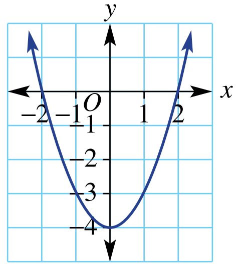

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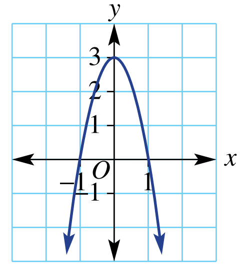

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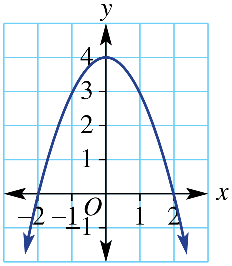

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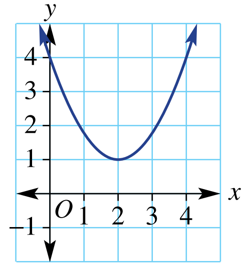

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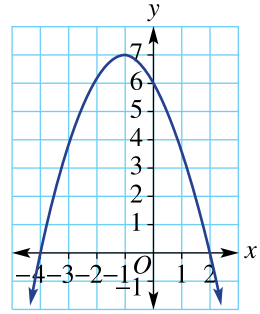

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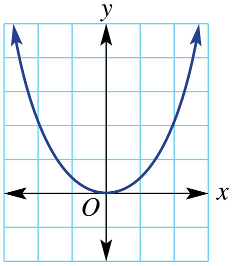

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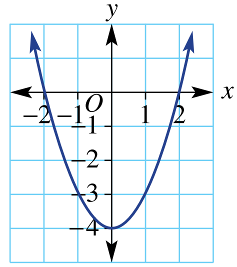

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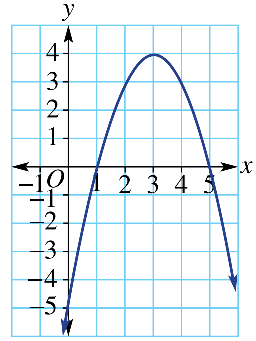

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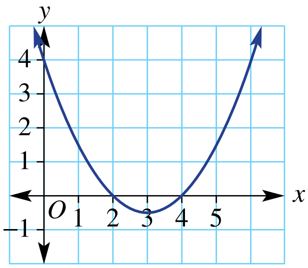

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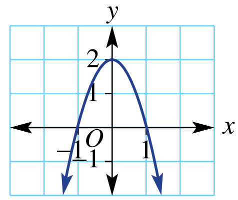

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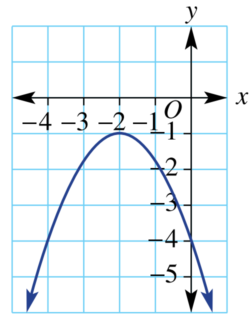

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Example 10

2 Use the quadratic rule $y = x^2 - 1$ to complete these tasks.

a Complete the table of values.

x	−2	−1	0	1	2
y					

b Draw a set of axes using a scale that suits the numbers in your table. Then plot the points to form a parabola.

c State these features.

i Type of turning point

ii Axis of symmetry

iii Coordinates of the turning point

iv The coordinates of the y-intercept

v The coordinates of the x-intercepts

3 Use the quadratic rule $y = x^2 + 2x - 3$ to complete these tasks.

a Complete the table of values.

x	−4	−3	−2	−1	0	1	2
y							

b Draw a set of axes using a scale that suits the numbers in your table. Then plot the points to form a parabola.

c State these features.

i Type of turning point

ii Axis of symmetry

iii Coordinates of the turning point

iv The coordinates of the y-intercept

v The coordinates of the x-intercepts

4 Use the quadratic rule $y = 9 - x^2$ to complete these tasks.

a Complete the table of values.

x	−3	−2	−1	0	1	2	3
y							

b Draw a set of axes using a scale that suits the numbers in your table. Then plot the points to form a parabola.

c State these features.

i Type of turning point

ii Axis of symmetry

iii Coordinates of the turning point

iv The coordinates of the y-intercept

v The coordinates of the x-intercepts

5 Use the quadratic rule $y = -x^2 + x + 2$ to complete these tasks. Recall that $-x^2 = -1 \times x^2$.

a Complete the table of values.

x	−2	−1	0	1	2	3
y						

b Draw a set of axes using a scale that suits the numbers in your table. Then plot the points to form a parabola.

c State these features.

i Type of turning point

ii Axis of symmetry

iii Coordinates of the turning point

iv The coordinates of the y-intercept

v The coordinates of the x-intercepts

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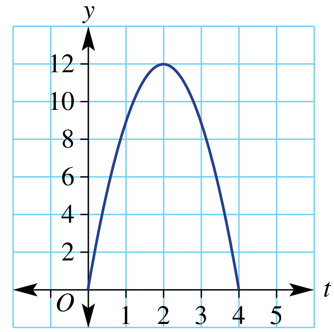
PROBLEM–SOLVING	6, 7	6, 8, 9	7, 9, 10

6 This graph shows the height of a cricket ball, y metres, as a function of time t seconds.



a **i** At what times is the ball at a height of 9 m?

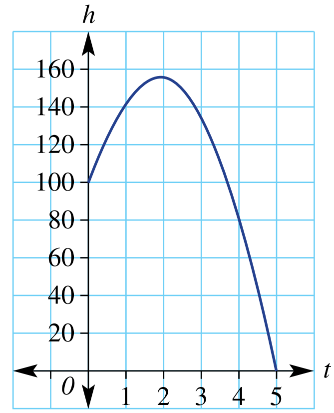
ii Why are there two different times?

b **i** At what time is the ball at its greatest height?

ii What is the greatest height the ball reaches?

iii After how many seconds does it hit the ground?

7 The graph gives the height, h m, at time t seconds, of a rocket which is fired up in the air.



a From what height is the rocket launched?

b What is the approximate maximum height that the rocket reaches?

c For how long is the rocket in the air?

d What is the difference in time for when the rocket is going up and when it is going down?

8 A parabola has x-intercepts at $(-2, 0)$ and $(4, 0)$. The y-coordinate of the turning point is -3.

a What is the equation of its axis of symmetry?

b What are the coordinates of the turning point?

9 A parabola has a turning point at $(1, 3)$ and one of its x-intercepts at $(0, 0)$.

a What is the equation of its axis of symmetry?

b What are the coordinates of the other x-intercept?

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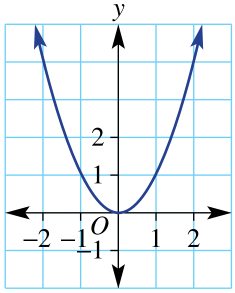
10 Write the rule for these parabolas. Use trial and error to help and check your rule by substituting a known point.

a



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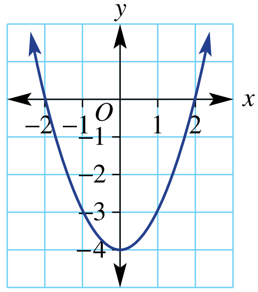

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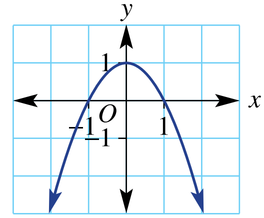

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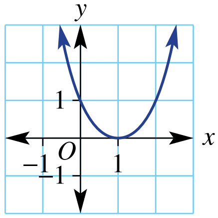

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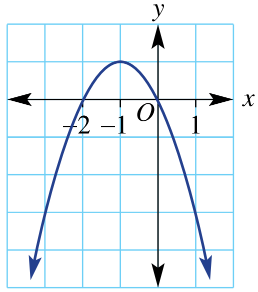

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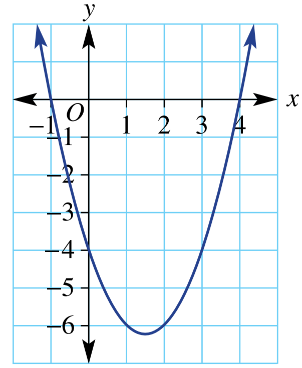


REASONING | 11, 12 | 11–13 | 11, 13, 14

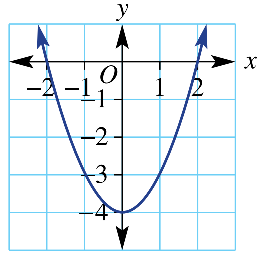
11 Is it possible to draw a parabola with the following properties? If yes, draw an example.

a Two x-intercepts
b One x-intercept
c No x-intercepts
d No y-intercept

12 **a** Mal calculates the y-value for $x = 2$ using $y = -x^2 + 2x$ and gets $y = 8$. Explain his error.

b Mai calculates the y-value for $x = -3$ using $y = x - x^2$ and gets $y = 6$. Explain her error.

13 This graph shows the parabola for $y = x^2 - 4$.



a For what values of x is $y = 0$?
b For what value of x is $y = -4$?
c How many values of x give a y-value which is:
  i greater than -4?
  ii equal to -4?
  iii less than -4?

14 This table corresponds to the rule $y = x^2 - 2x$.

x	-1	0	1	2	3	4
y	3	0	-1	0	3	8

a Use this table to solve these equations
  i $0 = x^2 - 2x$
  ii $3 = x^2 - 2x$

b How many solutions would there be to the equation $8 = x^2 - 2x$? Why?
c How many solutions would there be to the equation $-1 = x^2 - 2x$? Why?
d How many solutions would there be to the equation $-2 = x^2 - 2x$? Why?

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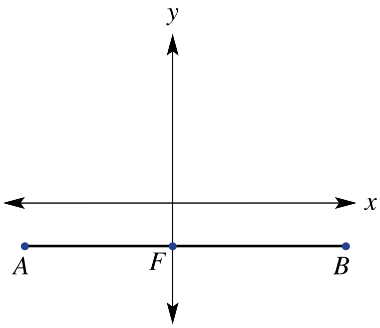
ENRICHMENT: Using software to construct a parabola	–	–	15

15 Follow the steps below to construct a parabola using an **interactive geometry package**.

Step 1. Show the coordinate axes system by selecting **Show Axes** from the **Draw** toolbox.

Step 2. Construct a line which is parallel to the x-axis and passes through a point F on the y-axis near the point $(0, -1)$.

Step 3. Construct a line segment AB on this line as shown in the diagram.

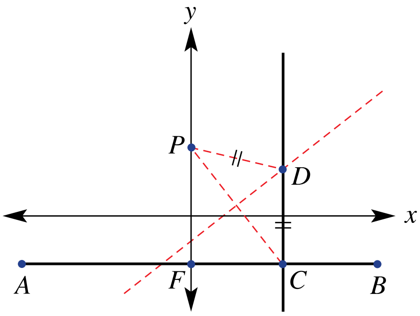


Step 4. Hide the line AB and then construct:

- a point C on the line segment AB
- a point P on the y-axis near the point $(0, 1)$.

Step 5. Construct a line which passes through the point C and is perpendicular to AB.

Step 6. Construct the point D which is equidistant from point P and segment AB. (*Hint*: Use the perpendicular bisector of PC.)

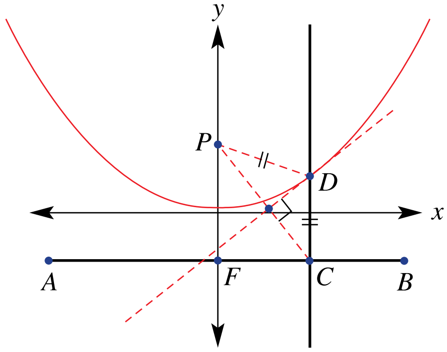


Step 7. Select **Trace** from the **Display** toolbox and click on the point D.

Step 8. Animate point C and observe what happens.

Step 9. Select **Locus** from the **Construct** toolbox and click at D and then at C.

Step 10. Drag point P and/or segment AB (by dragging F). (Clear the trace points by selecting **Refresh** drawing from the **Edit** menu.) What do you notice?



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10F Sketching $y = ax^2$ with dilations and reflections

Learning intentions
- To understand the effect of the value and sign of the coefficient a on the shape and features of the parabola

Past, present and future learning
- This section addresses concepts from the Stage 5 Path Topic *Non-linear relationships C*
- These concepts will be revised and extended in Chapter 7 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

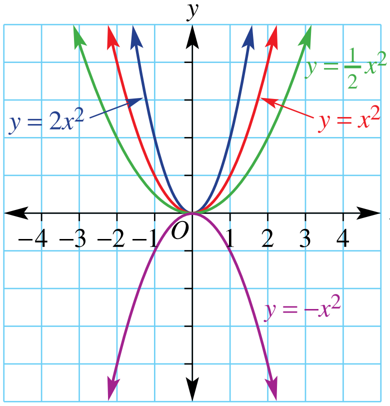
In geometry we know that shapes can be transformed by applying reflections, rotations, translations and dilations (enlargement). The same types of transformations can also be applied to graphs, including parabolas. Altering the value of a in $y = ax^2$ causes both dilations and reflections.

Suspension bridges have supporting cables that form parabolic curves, such as those on this New York bridge. Engineers model such parabolas with quadratic equations that are then used to calculate the vertical support heights.

Lesson starter: What is the effect of *a*?

This table and graph show a number of examples of $y = ax^2$ with varying values of a. They could also be produced using technology.

x	−2	−1	0	1	2
$y = x^2$	4	1	0	1	4
$y = 2x^2$	8	2	0	2	8
$y = \frac{1}{2}x^2$	2	$\frac{1}{2}$	0	$\frac{1}{2}$	2
$y = -x^2$	−4	−1	0	−1	−4



- Discuss how the different values of a affect the y-values in the table.
- Discuss how the different values of a affect the shape of the graph.
- How would the graphs of the following rules compare to the graphs shown above?

a $y = 3x^2$ **b** $y = \frac{1}{4}x^2$ **c** $y = -\frac{1}{2}x^2$

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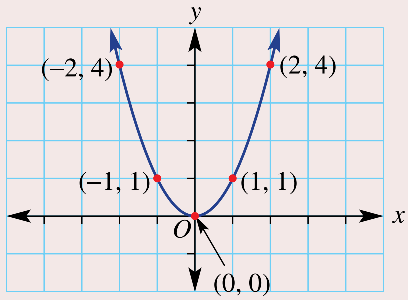
KEY IDEAS

- The most basic parabola is the graph of $y = x^2$. It can be graphed quickly using the five key points below.

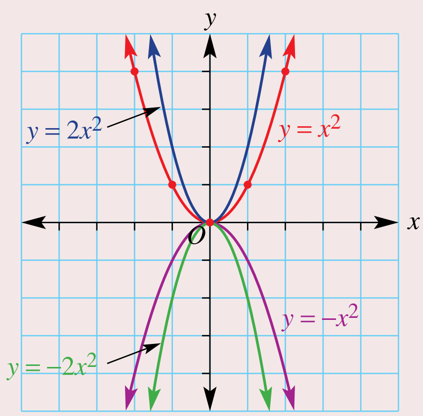


If you memorise these five points, you can use them to draw many other parabolas.

- The equation $y = ax^2$ is used to describe the family of parabolas that includes $y = x^2$, $y = 2x^2$, $y = \frac{1}{2}x^2$, $y = -x^2$ etc.
- All the parabolas in this family possess the following features.
  - The vertex (or turning point) is (0, 0).
  - The axis of symmetry is $x = 0$.
  - If $a > 0$, the graph is **upright** (or **concave up**) and has a minimum turning point.
  - If $a < 0$, the graph is **inverted** (or **concave down**) and has a maximum turning point.

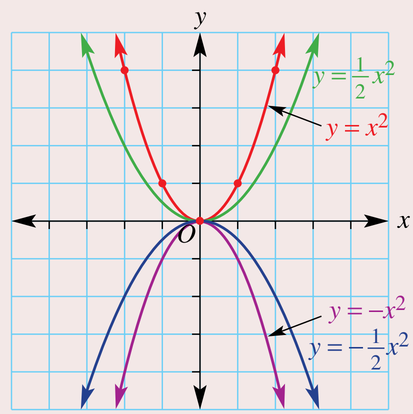
- If $a > 1$ or $a < -1$:
  the graph appears narrower than either $y = x^2$ or $y = -x^2$.

  For example: $y = 2x^2$ or $y = -2x^2$



- If $-1 < a < 1$:
  the graph appears wider than either $y = x^2$ or $y = -x^2$.

  For example: $y = \frac{1}{2}x^2$ or $y = -\frac{1}{2}x^2$



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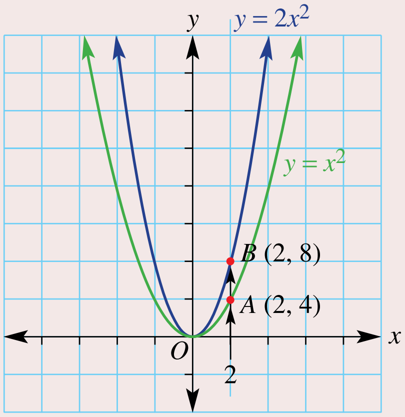
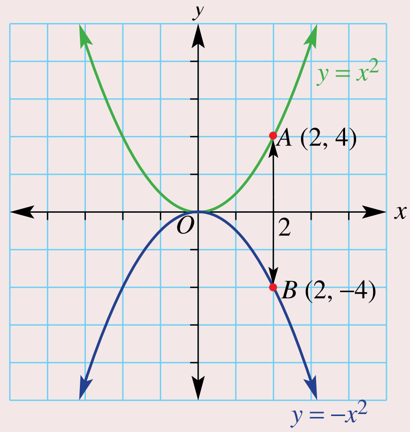
- For $y = 2x^2$ we say that the graph of $y = x^2$ is **dilated** from the x-axis by a factor of 2. For example: the point $A(2, 4)$ on $y = x^2$ is dilated to the point $B(2, 8)$ on $y = 2x^2$.



This dilation makes $y = 2x^2$ appear narrower than $y = x^2$.

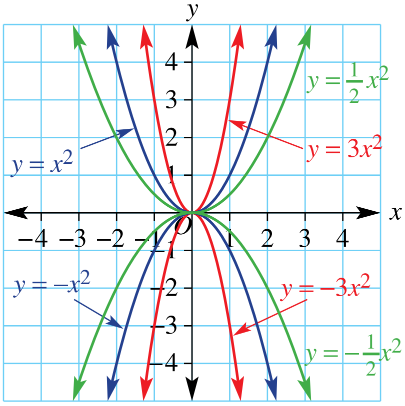
- For $y = -x^2$ we say that the graph of $y = x^2$ is **reflected** in the x-axis. For example: the point $A(2, 4)$ on $y = x^2$ is reflected across the x-axis to $B(2, -4)$ on $y = -x^2$.



$y = x^2$ is concave up and $y = -x^2$ is concave down.

BUILDING UNDERSTANDING

1 Shown here are the graphs of $y = x^2$, $y = 3x^2$, $y = \frac{1}{2}x^2$, $y = -x^2$, $y = -3x^2$ and $y = -\frac{1}{2}x^2$.



a State the rules of the three graphs which have a minimum turning point.
b State the rules of the three graphs which have a maximum turning point.
c What are the coordinates of the turning point for all the graphs?
d What is the equation of the axis of symmetry for all the graphs?

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e State the rule of the graph which is:
  i upright (concave up) and narrower than $y = x^2$
  ii upright (concave up) and wider than $y = x^2$
  iii inverted (concave down) and narrower than $y = -x^2$
  iv inverted (concave down) and wider than $y = -x^2$.

f State the rule of the graph which is:
  i a reflection of $y = x^2$ in the x-axis
  ii a reflection of $y = 3x^2$ in the x-axis
  iii a reflection of $y = -\frac{1}{2}x^2$ in the x-axis.

2 Select the word *positive* or *negative* to suit each sentence.

a The graph of $y = ax^2$ will be upright (concave up) with a minimum turning point if a is __________.

b The graph of $y = ax^2$ will be inverted (concave down) with a maximum turning point if a is __________.

Example 11 Comparing graphs of $y = ax^2$, $a > 0$

Complete the following for $y = x^2$, $y = 2x^2$ and $y = \frac{1}{2}x^2$.

a Draw up and complete a table of values for $-2 \leqslant x \leqslant 2$.

b Plot their graphs on the same set of axes.

c Write down the equation of the axis of symmetry and the coordinates of the turning point.

d **i** Does the graph of $y = 2x^2$ appear wider or narrower than the graph of $y = x^2$?

  ii Does the graph of $y = \frac{1}{2}x^2$ appear wider or narrower than the graph of $y = x^2$?

SOLUTION

a $y = x^2$

x	-2	-1	0	1	2
y	4	1	0	1	4

$y = 2x^2$

x	-2	-1	0	1	2
y	8	2	0	2	8

$y = \frac{1}{2}x^2$

x	-2	-1	0	1	2
y	2	$\frac{1}{2}$	0	$\frac{1}{2}$	2

EXPLANATION

Substitute each x-value into $y = x^2$, $y = 2x^2$ and $y = \frac{1}{2}x^2$.

e.g. for $y = 2x^2$, if $x = 2$,

$$\begin{aligned} y &= 2(2)^2 \\ &= 2(4) \\ &= 8 \end{aligned}$$

If $x = -1$, $\quad \begin{aligned} y &= 2(-1)^2 \\ &= 2(1) \\ &= 2 \end{aligned}$

Continued on next page

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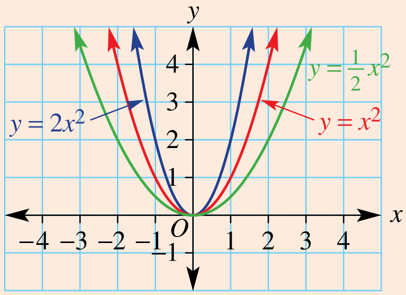
b



Plot the points for each graph using the coordinates from the tables and join them with a smooth curve.

c Axis of symmetry: y-axis ($x = 0$)
Turning point: minimum at (0, 0)

Look at graphs to see symmetry about the y-axis and a minimum turning point at the origin.

d **i** The graph of $y = 2x^2$ appears narrower than the graph of $y = x^2$.

For each value of x, $2x^2$ is twice that of x^2; hence, the graph (y-values) of $y = 2x^2$ rises more quickly.

ii The graph of $y = \frac{1}{2}x^2$ appears wider than the graph of $y = x^2$.

For each value of x, $\frac{1}{2}x^2$ is half that of x^2; hence, the graph of $y = \frac{1}{2}x^2$ rises more slowly.

Now you try

Complete the following for $y = x^2$, $y = 4x^2$ and $y = \frac{1}{4}x^2$.

a Draw up and complete a table of values for $-2 \leqslant x \leqslant 2$.
b Plot their graphs on the same set of axes.
c Write down the equation of the axis of symmetry and the coordinates of the turning point.
d **i** Does the graph of $y = 4x^2$ appear wider or narrower than the graph of $y = x^2$?
ii Does the graph of $y = \frac{1}{4}x^2$ appear wider or narrower than the graph of $y = x^2$?

Example 12 Comparing graphs of $y = ax^2$, $a < 0$

Complete the following for $y = -x^2$, $y = -3x^2$ and $y = -\frac{1}{2}x^2$.

a Draw up and complete a table of values for $-2 \leqslant x \leqslant 2$.
b Plot their graphs on the same set of axes.
c Write down the the equation of the axis of symmetry and the coordinates of the turning point.
d **i** Does the graph of $y = -3x^2$ appear wider or narrower than the graph of $y = -x^2$?
ii Does the graph of $y = -\frac{1}{2}x^2$ appear wider or narrower than the graph of $y = -x^2$?

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SOLUTION

a $y = -x^2$

x	−2	−1	0	1	2
y	−4	−1	0	−1	−4

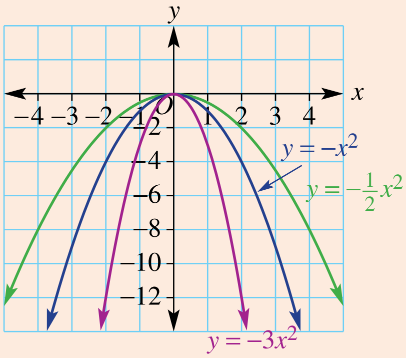
$y = -3x^2$

x	−2	−1	0	1	2
y	−12	−3	0	−3	−12

$y = -\frac{1}{2}x^2$

x	−2	−1	0	1	2
y	−2	$-\frac{1}{2}$	0	$-\frac{1}{2}$	−2

b



c Axis of symmetry: y-axis ($x = 0$)
Turning point: maximum at (0, 0)

d **i** The graph of $y = -3x^2$ appears narrower than the graph of $y = -x^2$.

ii The graph of $y = -\frac{1}{2}x^2$ appears wider than the graph of $y = -x^2$.

EXPLANATION

Substitute each x-value into $y = -x^2$, $y = -3x^2$ and $y = -\frac{1}{2}x^2$.

e.g. for $y = -3x^2$, if $x = 2$,

$$\begin{aligned} y &= -3(2)^2 \\ &= -3(4) \\ &= -12 \end{aligned}$$

$$\begin{aligned} \text{If } x = -1,\ y &= -3(-1)^2 \\ &= -3(1) \\ &= -3 \end{aligned}$$

Plot the coordinates for each graph from the tables and join them with a smooth curve.

Graphs are symmetrical about the y-axis with a maximum turning point at the origin.

For each value of x, $-3x^2$ is three times that of $-x^2$; hence, the graph of $y = -3x^2$ gets larger in the negative direction more quickly.

For each value of x, $-\frac{1}{2}x^2$ is half that of $-x^2$.

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Now you try

Complete the following for $y = -x^2$, $y = -4x^2$ and $y = -\frac{1}{4}x^2$.

a Draw up and complete a table of values for $-2 \leqslant x \leqslant 2$.
b Plot their graphs on the same set of axes.
c Write down the equation of the axis of symmetry and the coordinates of the turning point.
d i Does the graph of $y = -4x^2$ appear wider or narrower than the graph of $y = -x^2$?
ii Does the graph of $y = -\frac{1}{4}x^2$ appear wider or narrower than the graph of $y = -x^2$?

Exercise 10F

FLUENCY	1, 2–3(½)	1, 2–3(½)	2–3(½)

Example 11

1 Complete the following for $y = x^2$, $y = 3x^2$ and $y = \frac{1}{3}x^2$.
a Draw up and complete a table of values for $-2 \leqslant x \leqslant 2$.
b Plot their graphs on the same set of axes.
c Write down the equation of the axis of symmetry and the coordinates of the turning point.
d i Does the graph of $y = 3x^2$ appear wider or narrower than the graph of $y = x^2$?
ii Does the graph of $y = \frac{1}{3}x^2$ appear wider or narrower than the graph of $y = x^2$?

2 For the equations given below, complete these tasks.
i Draw up and complete a table of values for $-2 \leqslant x \leqslant 2$.
ii Plot the graphs of the equations on the same set of axes.
iii Write down the coordinates of the turning point and the equation of the axis of symmetry.
iv Determine whether the graphs of the equations each appear wider or narrower than the graph of $y = x^2$.

a $y = 6x^2$ b $y = 5x^2$ c $y = \frac{1}{6}x^2$ d $y = \frac{1}{5}x^2$

Example 12

3 For the equations given below, complete these tasks.
i Draw up and complete a table of values for $-2 \leqslant x \leqslant 2$.
ii Plot the graphs of the equations on the same set of axes.
iii List the key features for each graph, such as the axis of symmetry, turning point, x-intercept and y-intercept.
iv Determine whether the graphs of the equations each appear wider or narrower than the graph of $y = -x^2$.

a $y = -2x^2$ b $y = -3x^2$ c $y = -\frac{1}{2}x^2$ d $y = -\frac{1}{3}x^2$

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PROBLEM-SOLVING	4	4, 5	5

4 Here are eight quadratics of the form $y = ax^2$.

A $y = 6x^2$ **B** $y = -7x^2$ **C** $y = 4x^2$ **D** $y = \frac{1}{9}x^2$

E $y = \frac{x^2}{7}$ **F** $y = 0.3x^2$ **G** $y = -4.8x^2$ **H** $y = -0.5x^2$

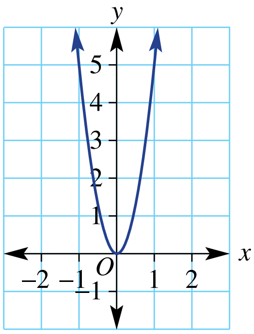
a Which rule would give a graph which is upright (concave up) and the narrowest?

b Which rule would give a graph which is inverted (concave down) and the widest?

5 Match each of the following parabolas with the appropriate equation from the list below. Do a mental check by substituting the coordinates of a known point.

a $y = 3x^2$ **b** $y = -x^2$ **c** $y = 5x^2$ **d** $y = \frac{1}{2}x^2$ **e** $y = -5x^2$ **f** $y = 2x^2$

A



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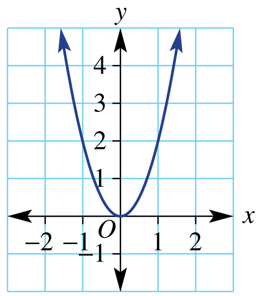

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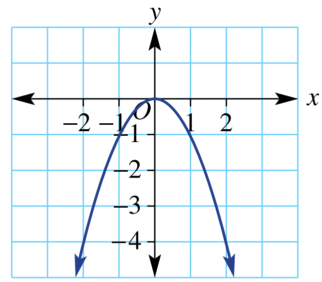

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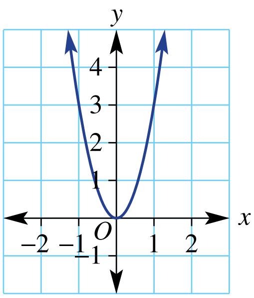

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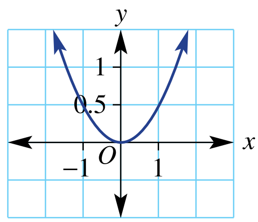
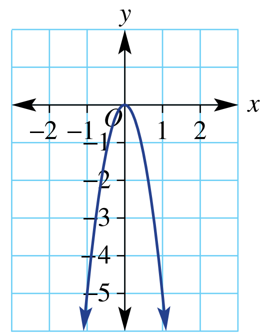


F

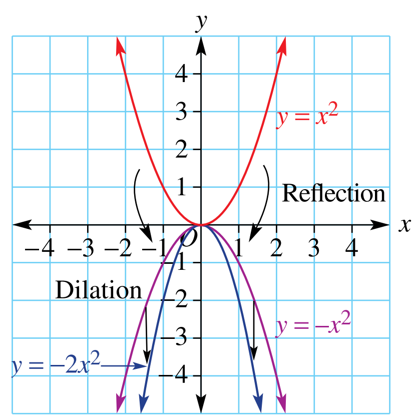


REASONING	6	6, 7	6(½), 7–9

6 The graph of $y = -2x^2$ can be obtained from $y = x^2$ by conducting these transformations:

- reflection in the x-axis
- dilation by a factor of 2 from the x-axis.

In the same way as above, describe the two transformations which take:



a $y = x^2$ to $y = -3x^2$ **b** $y = x^2$ to $y = -6x^2$

c $y = x^2$ to $y = -\frac{1}{2}x^2$ **d** $y = -x^2$ to $y = 2x^2$

e $y = -x^2$ to $y = 3x^2$ **f** $y = -x^2$ to $y = \frac{1}{3}x^2$

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7 Write the rule for the graph after each set of transformations.

a The graph of $y = x^2$ is reflected in the x-axis then dilated by a factor of 4 from the x-axis.

b The graph of $y = -x^2$ is reflected in the x-axis then dilated by a factor of $\frac{1}{3}$ from the x-axis.

c The graph of $y = 2x^2$ is reflected in the x-axis then dilated by a factor of 2 from the x-axis.

d The graph of $y = \frac{1}{3}x^2$ is reflected in the x-axis then dilated by a factor of 4 from the x-axis.

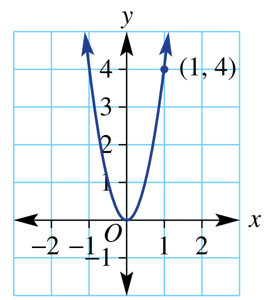
8 The graph of $y = ax^2$ is reflected in the x-axis and dilated from the x-axis by a given factor. Does it matter which transformation is completed first? Explain.

9 The graph of the rule $y = ax^2$ is reflected in the y-axis. What is the new rule of the graph?

ENRICHMENT: Substitute to find the rule	–	–	10(½), 11

10 If a rule is of the form $y = ax^2$ and it passes through a point, say (1, 4), we can substitute this point to find the value of a.

$$y = ax^2$$
$$4 = a \times (1)^2$$
$$\therefore a = 4 \text{ and } y = 4x^2$$



Use this method to determine the equation of a quadratic relation that has an equation of the form $y = ax^2$ and passes through:

a (1, 5)
b (1, 7)
c (−1, −1)
d (−2, 7)
e (−5, 4)
f (3, 26)
g (4, 80)
h (−1, −52)

11 The rule of the form $y = ax^2$ models the shape of the parabolic cables of the Golden Gate Bridge. If the cable is centred at (0, 0) and the top of the right pylon has the coordinates (492, 67), find a possible equation that describes this shape. The numbers given are in metres.

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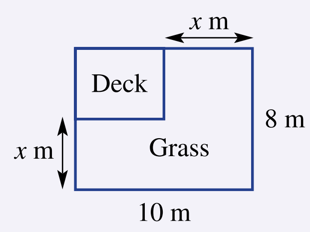
The following problems will investigate practical situations drawing upon knowledge and skills developed throughout the chapter. In attempting to solve these problems, aim to identify the key information, use diagrams, formulate ideas, apply strategies, make calculations and check and communicate your solutions.

Backyard blitz

1 Jamie is planning to renovate his rectangular backyard. He wishes to have a rectangular deck that is positioned against the house and a fence and has synthetic grass of breadth x m, on the other sides as shown. The backyard has dimensions 10 m by 8 m.

The synthetic grass he has chosen costs \$35 per square metre, and the timber for the deck is \$200 per square metre. Jamie's budget is \$4000.



Jamie is investigating possible designs for his backyard by varying the breadth of the grassed area to suit his budget of \$4000 for the grass and timber materials.

a Give expressions in metres in terms of x for:
- **i** the length (longer side) of the deck
- **ii** the breadth of the deck.

b Give an expression for the area in expanded and simplified form of:
- **i** the deck
- **ii** the grassed area.

c One design Jamie is considering is such that the area of the deck occupies one-tenth of the backyard area.
- **i** By solving an appropriate equation, determine the value of x that satisfies this design.
- **ii** What is the cost of the synthetic grass for this value of x?
- **iii** Will Jamie come in under budget using this model?

d Give a rule in terms of x for the cost, \$$C$, of renovating the backyard. Expand and simplify your rule and use it to verify your answer to part **c iii**.

e If one quote for the cost of materials is \$5275, use your rule in part **d** to solve an equation to find the value of x and hence the deck area that gives this quote. (*Hint*: Remove the coefficient of x^2 as the common factor when factorising.)

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Computer golf

2 A computer game designer is using quadratic rules to model the path of a golf ball for a variety of shot types in a new golf game. Based on the user's selection of club and other parameters, including distance from the hole, a quadratic rule is assigned to simulate their shot. The rule models the ball's path from when it is struck until when it first bounces.

The game designer is in the testing phase and is checking some of the rules for accuracy and to see how they can be manipulated to create other scenarios in the game.

On the easy level of the game, the rules used give the ball a direct path in line with the hole.

One such rule used to model certain pitch shots from the fairway onto the putting green is given by $y = -\frac{1}{75}x(x - 60)$, where x is the horizontal distance in metres from where the ball was hit and y is the vertical height of the ball above ground in metres.

A pitch shot using this model is taken 80 m from the hole.

a Use the rule to find:
- **i** the height above ground of the ball after it has travelled 15 m horizontally
- **ii** the height above ground from which the ball is struck.

b Use an equation to determine how far from the hole the ball first bounces.

c Using some of the key points found in parts **a** and **b**:
- **i** plot a graph showing the path of the ball until it first bounces
- **ii** use the graph and symmetry of the flight path to determine how far the ball has travelled horizontally when it reaches its maximum height
- **iii** hence use the rule to find the maximum height reached by the ball.

The game designer adds some trees to the golf course for added difficulty. From where the above shot is taken, one particular tree is placed in line with the hole 18 m from where the shot is taken. The tree is 10.8 m high.

d Using the given rule for the shot simulation, determine if the ball will clear the tree.

e Different flight paths are obtained by adjusting the rule for the shot.
- **i** Adjust the given rule so that the ball is still hit from ground level but first bounces 12 m from the hole.
- **ii** Check algebraically whether or not this shot will clear the tree.

f A water hazard is also placed on the course in line with the ball and the hole. The hazard covers a distance of between 48 m and 55 m from where the shot is being taken. Write a rule that the programmer could use such that the shot lands in the water before it bounces and reaches a maximum height of at least 13 m.

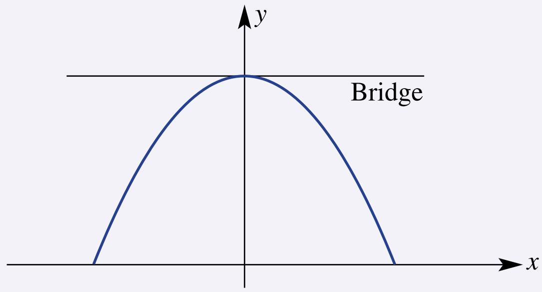
ISBN 978-1-009-40936-0

Bridges and barges

3 A pre-existing bridge across a river is to be replaced by a new bridge further downstream. The new bridge will be wider to ease traffic congestion.

Barges are moved by tugboats under this bridge to deliver cargo to the port. The design of new barges which are being built cannot fit under the current bridge. The new barge is rectangular and has a breadth of 6 m and a height, including cargo, of 5 m.

The current bridge is supported by a parabolic arch and the design is modelled on a Cartesian plane with rule $y = -\frac{1}{2}x^2 + 8$, as shown below. The y-axis indicates the middle of the bridge and the x-axis represents water level. The parabolic arch is at water level at the edges of the river.



The designer's aim is to keep the current features of the bridge and parabolic arch design, but to ensure that the new barges fit under the bridge.

a Use the model of the current bridge to determine the height of the bridge above water in the middle of the bridge.

b By solving an appropriate equation, determine the breadth of the river.

c Verify that the new barge design will not be able to pass under the arch of the bridge.

The specifications for the parabolic arch of the new bridge are a maximum height of 8 m and a breadth of 6 m at a height of 5 m above water level.

d **i** Draw a model of the new bridge on a set of axes with the y-axis indicating the centre of the bridge and the x-axis at water level.

ii Find a quadratic rule in the form $y = ax^2 + c$ that fits the arch specifications.

This bridge is placed further down the river where the river is wider.

iii Using your rule, find the breadth of this bridge. Answer correct to one decimal place.

e The bridge designers are looking to the future and decide they should leave more room for extra cargo to fit under the bridge. Find a rule for a parabolic arch under the bridge that will be 10 m high and that will allow the total height above water of the barge to be at least 8 m on a 6 m wide barge.

ISBN 978-1-009-40936-0

Applications and problem-solving

10G Sketching translations of $y = x^2$

Learning intentions

- To understand what it means for a graph to be translated
- To know what causes a graph to be translated vertically and horizontally
- To understand the vertex form of a quadratic equation
- To be able to sketch a graph of a quadratic equation expressed in vertex form, labelling the vertex and y-intercept

Past, present and future learning

- This section addresses concepts from the Stage 5 Path Topic *Non-linear relationships C*
- These concepts will be revised and extended in Chapter 7 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

As well as reflection and dilation, there is a third type of transformation called translation. Translation involves a shift of every point on the graph horizontally and/or vertically. Unlike reflections and dilations, a translation alters the coordinates of the turning point. The shape of the curve is unchanged, but its position in relation to the origin is altered.

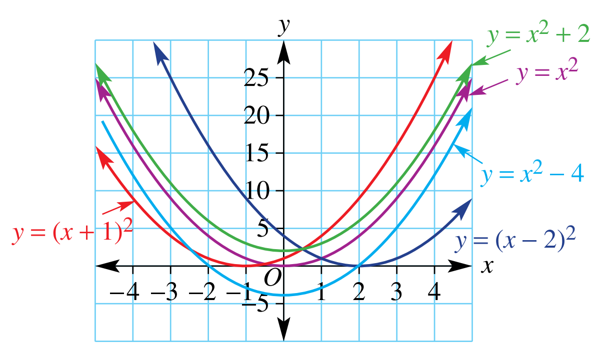
Manufacturers employ mathematicians to develop profit equations, which are quadratics when sales and profit per item are linear relations of the selling price. As the selling price increases, profit increases to the parabola's maximum point and then decreases.

Lesson starter: Which way: Left, right, up or down?

This table and graph shows the quadratics $y = x^2$, $y = (x - 2)^2$, $y = (x + 1)^2$, $y = x^2 - 4$ and $y = x^2 + 2$. The table could also be produced using technology.

- Discuss what effect the different numbers in the rules had on the y-values in the table.
- Also discuss what effect the numbers in the rules have on each graph. How are the coordinates of the turning point changed?
- What conclusions could you draw on the effect of h in the rule $y = (x - h)^2$?
- What conclusions could you draw on the effect of k in the rule $y = x^2 + k$?
- What if the rule was $y = -x^2 + 2$ or $y = -(x + 1)^2$? Describe how the graphs would look.

x	−3	−2	−1	0	1	2	3
$y = x^2$	9	4	1	0	1	4	9
$y = (x - 2)^2$	25	16	9	4	1	0	1
$y = (x + 1)^2$	4	1	0	1	4	9	16
$y = x^2 - 4$	5	0	−3	−4	−3	0	5
$y = x^2 + 2$	11	6	3	2	3	6	11



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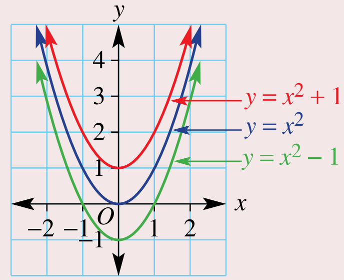
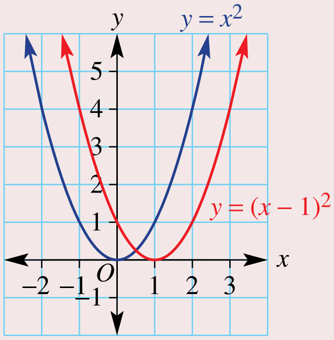
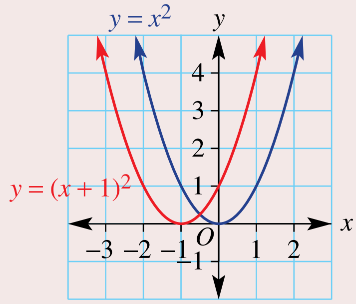
KEY IDEAS

- A **translation** of a graph involves a shift of every point horizontally and/or vertically.
- Vertical translations: $y = x^2 + k$
  - If $k > 0$, the graph is translated k units up (red curve).
  - If $k < 0$, the graph is translated k units down (green curve).
  - The turning point is $(0, k)$ for all curves.
  - The axis of symmetry is the line $x = 0$ for all curves.
  - The y-intercept is $(0, k)$ for all curves.



- Horizontal translations: $y = (x - h)^2$
  - If $h > 0$, the graph is translated h units to the right.
  - If $h < 0$, the graph is translated h units to the left.





  - The turning point is $(h, 0)$ in both cases.
  - The axis of symmetry is the line $x = h$ in both cases.
  - The y-intercept is $(0, h^2)$ in both cases.
- The **turning point form** of a quadratic is given by:

$$y = a(x - h)^2 + k$$

$a > 0$ upright (concave up) graph

$a < 0$ graph inverted (concave down)

Translates the graph up or down

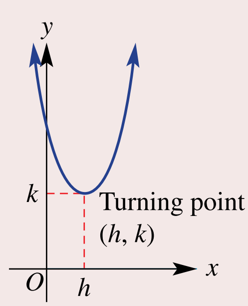
$k > 0$ ↑ $k < 0$ ↓

Translates the graph left or right:

$h > 0$ ⟶

$h < 0$ ⟵

Axis of symmetry: $x = h$



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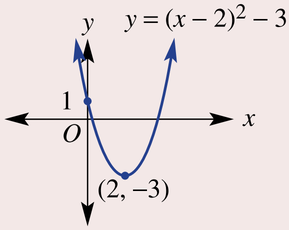
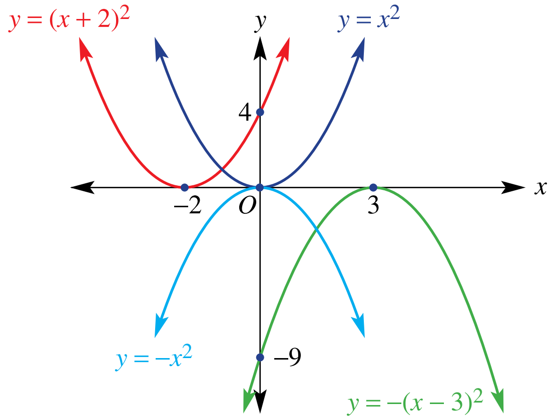
- To sketch a graph of a quadratic equation in turning point form, follow these steps.
  - Draw and label a set of axes.
  - Identify important points including the turning point and y-intercept.
  - Sketch the curve connecting the key points and making the curve symmetrical.



BUILDING UNDERSTANDING

1 This diagram shows the graphs of $y = x^2$, $y = -x^2$, $y = (x + 2)^2$ and $y = -(x - 3)^2$.

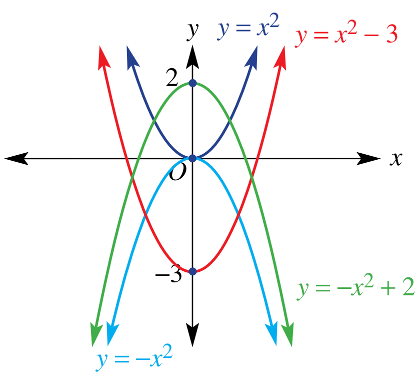


a State the coordinates of the turning point of the graph of:
  i $y = (x + 2)^2$
  ii $y = -(x - 3)^2$

b Compared to the graph of $y = x^2$, which way has the graph of $y = (x + 2)^2$ been translated (left or right)?

c Compared to the graph of $y = -x^2$ which way has the graph of $y = -(x - 3)^2$ been translated (left or right)?

2 This diagram shows the graphs of $y = x^2$, $y = -x^2$, $y = x^2 - 3$ and $y = -x^2 + 2$.



a State the coordinates of the turning point of the graph of:
  i $y = x^2 - 3$
  ii $y = -x^2 + 2$

b Compared to the graph of $y = x^2$, which way has the graph of $y = x^2 - 3$ been translated (up or down)?

c Compared to the graph of $y = -x^2$, which way has the graph of $y = -x^2 + 2$ been translated (up or down)?

3 Substitute $x = 0$ to find the coordinates of the y-intercept for these rules.

a $y = x^2 + 3$ **b** $y = -x^2 - 4$

c $y = -(x - 2)^2$ **d** $y = (x + 5)^2$

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Example 13 Sketching with horizontal and vertical translations

Sketch the graphs of these rules showing the y-intercept and the coordinates of the turning point.

a $y = x^2 + 2$

b $y = -x^2 - 1$

c $y = (x - 3)^2$

d $y = -(x + 2)^2$

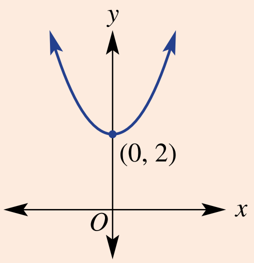
SOLUTION

a $y = x^2 + 2$

Turning point is (0, 2).

y-intercept ($x = 0$): $y = (0)^2 + 2 = 2$

$\therefore (0, 2)$



EXPLANATION

For $y = x^2 + k$, $k = 2$, so the graph of $y = x^2$ is translated 2 units up.

The point (0, 0) shifts to (0, 2).

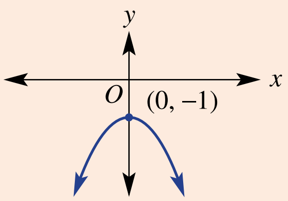
SOLUTION

b $y = -x^2 - 1$

Turning point is (0, −1).

y-intercept ($x = 0$): $y = -(0)^2 - 1 = -1$

$\therefore (0, -1)$



EXPLANATION

The graph of $y = -x^2$ is a reflection of the graph of $y = x^2$ in the x-axis.

For $y = -x^2 + k$, $k = -1$ so the graph of $y = -x^2$ is translated down 1 unit.

SOLUTION

c $y = (x - 3)^2$

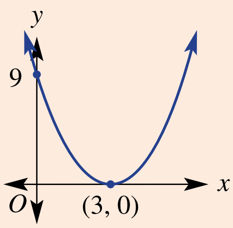
Turning point is (3, 0).

y-intercept ($x = 0$): $y = (0 - 3)^2$

$= (-3)^2$

$= 9$

$\therefore (0, 9)$



EXPLANATION

For $y = (x - h)^2$, $h = 3$, so the graph of $y = x^2$ is translated 3 units to the right.

The y-intercept is found by substituting $x = 0$ into the rule.

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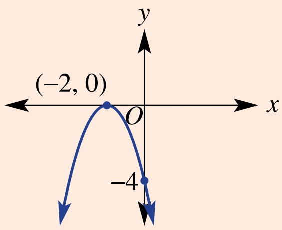
d $y = -(x + 2)^2$

Turning point is $(-2, 0)$.

y-intercept $(x = 0)$: $y = -(0 + 2)^2$
$= -(2)^2$
$= -4$
$\therefore (0, -4)$



For $y = -(x - h)^2$, $h = -2$ since $-(x - (-2))^2 = -(x + 2)^2$. So the graph of $y = -x^2$ is translated 2 units to the left.
The y-intercept is found by substituting $x = 0$.

The negative sign in front means the graph is inverted.

Now you try

Sketch the graphs of these rules showing the y-intercept and the coordinates of the turning point.

a $y = x^2 + 4$ **b** $y = -x^2 - 2$
c $y = (x - 1)^2$ **d** $y = -(x + 3)^2$

Example 14 Sketching with combined translations

Sketch these graphs showing the y-intercept and the coordinates of the turning point.

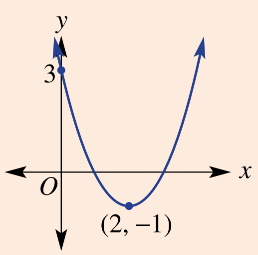
a $y = (x - 2)^2 - 1$ **b** $y = -(x + 3)^2 + 2$

SOLUTION

a $y = (x - 2)^2 - 1$

Turning point is $(2, -1)$.

y-intercept $(x = 0)$: $y = (0 - 2)^2 - 1$
$= 4 - 1$
$= 3$
$\therefore (0, 3)$



EXPLANATION

For $y = (x - h)^2 + k$, $h = 2$ and $k = -1$, so the graph of $y = x^2$ is shifted 2 to the right and 1 down.
Substitute $x = 0$ for the y-intercept.

First, position the coordinates of the turning point and y-intercept, then join to form the curve.

Continued on next page

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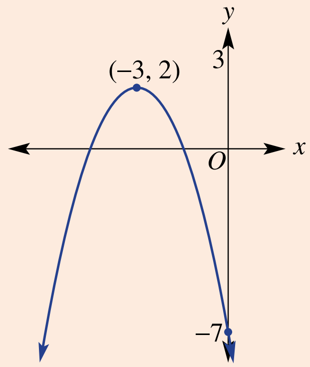
b $y = -(x + 3)^2 + 2$
Turning point is $(-3, 2)$.
y-intercept $(x = 0)$: $y = -(0 + 3)^2 + 2$
$= -9 + 2$
$= -7$
$\therefore (0, -7)$

For $y = -(x - h)^2 + k$, $h = -3$ and $k = 2$, so the graph of $y = x^2$ is shifted 3 to the left and 2 up.



First, position the coordinates of the turning point and y-intercept then join to form the curve.

Now you try

Sketch these graphs showing the y-intercept and the coordinates of the turning point.

a $y = (x - 3)^2 - 2$ **b** $y = -(x + 2)^2 + 1$

Exercise 10G

FLUENCY	1–3, 4(½)	1–2(½), 3, 4(½)	1–2(½), 4(½)

1 Sketch the graphs of these rules showing the y-intercept and the coordinates of the turning point.

Example 13a
Example 13b

a $y = x^2 + 1$ **b** $y = x^2 + 3$ **c** $y = x^2 - 2$
d $y = -x^2 + 4$ **e** $y = -x^2 + 1$ **f** $y = -x^2 - 5$

2 Sketch the graphs of these rules showing the y-intercept and the coordinates of the turning point.

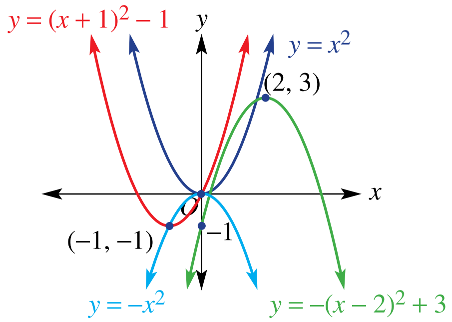
Example 13c
Example 13d

a $y = (x - 2)^2$ **b** $y = (x - 4)^2$ **c** $y = (x + 3)^2$
d $y = -(x - 3)^2$ **e** $y = -(x + 6)^2$ **f** $y = -(x + 2)^2$

3 The below diagram shows the graphs of $y = x^2$, $y = -x^2$, $y = -(x - 2)^2 + 3$ and $y = (x + 1)^2 - 1$.

a State the coordinates of the turning point of the graph of:
  i $y = -(x - 2)^2 + 3$
  ii $y = (x + 1)^2 - 1$

b State the missing words and numbers.
  i Compared to the graph of $y = x^2$, the graph of $y = (x + 1)^2 - 1$ has to be translated ______ unit to the ______ and ______ unit ______.
  ii Compared to the graph of $y = -x^2$, the graph of $y = -(x - 2)^2 + 3$ has to be translated ______ units to the ______ and ______ units ______.



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Example 14

4 Sketch each graph showing the y-intercept and the coordinates of the turning point.

a $y = (x - 3)^2 + 2$ **b** $y = (x - 1)^2 - 1$ **c** $y = (x + 2)^2 - 3$

d $y = (x + 1)^2 + 7$ **e** $y = -(x - 2)^2 + 1$ **f** $y = -(x - 5)^2 + 3$

g $y = -(x + 3)^2 - 4$ **h** $y = -(x + 1)^2 - 5$ **i** $y = -(x - 3)^2 - 6$

PROBLEM–SOLVING	5	5, 6	5, 6(½), 7

5 Match each parabola with the appropriate equation from the list below.

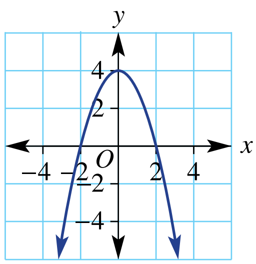
a $y = x^2$ **b** $y = (x + 2)^2$ **c** $y = x^2 - 4$

d $y = -x^2 - 3$ **e** $y = (x - 2)^2$ **f** $y = x^2 + 4$

g $y = 4 - x^2$ **h** $y = (x + 3)^2$ **i** $y = (x - 5)^2$

j $y = x^2 + 3$ **k** $y = -(x + 3)^2$ **l** $y = -x^2 + 3$

A



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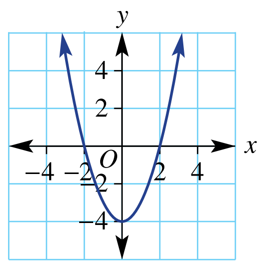

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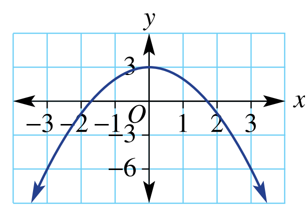

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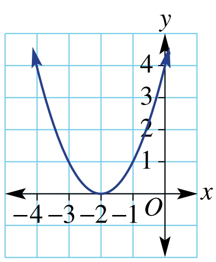
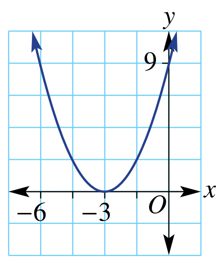


E



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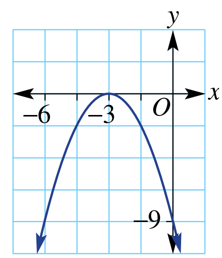

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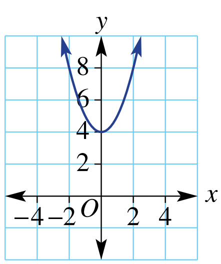

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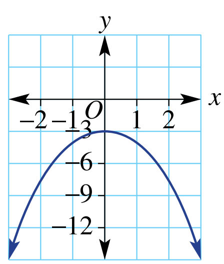

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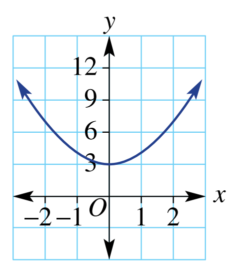
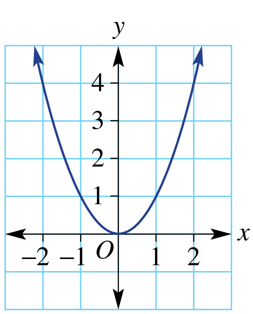


J



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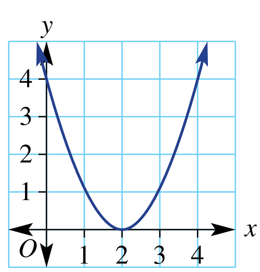

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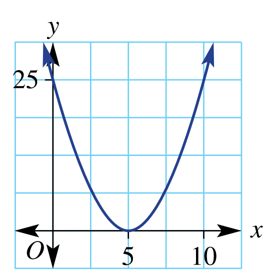

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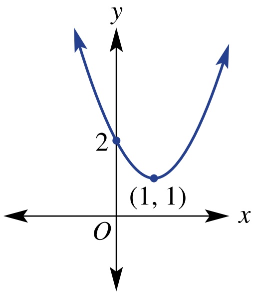
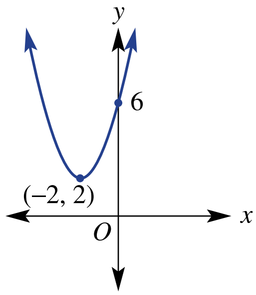
6 Write the rule for each graph in the turning point form: $y = (x - h)^2 + k$.

a



b



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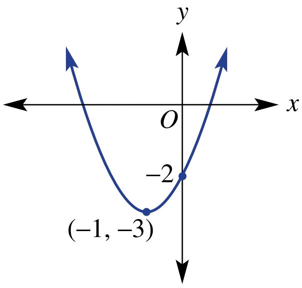

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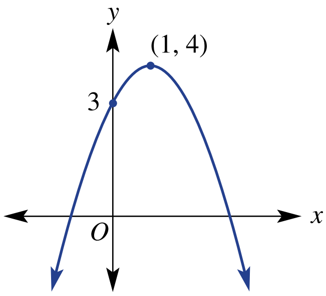

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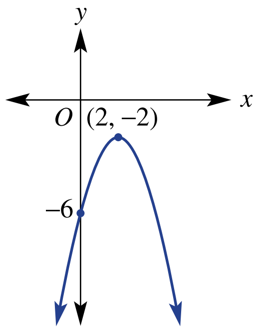

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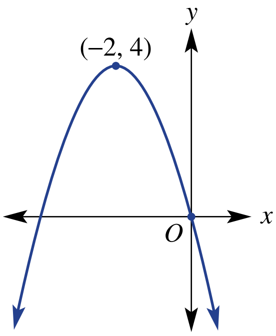

7 A bike track can be modelled approximately by combining two different quadratic equations.
The first part of the bike path can be modelled by the equation $y = -(x - 2)^2 + 9$ for $-2 \leqslant x \leqslant 5$.
The second part of the bike track can be modelled by the equation $y = (x - 7)^2 - 4$ for $5 \leqslant x \leqslant 10$.

a Find the turning point of the graph of each quadratic equation.

b Sketch each graph on the same set of axes. On your sketch of the bike path you need to show the coordinates of the start and finish of the track and where it crosses the x-axis.

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REASONING	8	8, 9	9, 10

8 Written in the form $y = a(x - h)^2 + k$, the rule $y = 4 - (x + 2)^2$ could be rearranged to give $y = -(x + 2)^2 + 4$.

a Rearrange these rules and write in the form $y = a(x - h)^2 + k$.

i $y = 3 - (x + 1)^2$

ii $y = 4 + (x + 3)^2$

iii $y = -3 + (x - 1)^2$

iv $y = -7 - (x - 5)^2$

v $y = -2 - x^2$

vi $y = -6 + x^2$

b Write down the coordinates of the turning point for each of the quadratics in part a.

9 A quadratic has the rule $y = a(x - h)^2 + k$.

a What are the coordinates of the turning point?

b Write an expression for the y-coordinate of the y-intercept.

10 Investigate and explain how the graph of:

a $y = (2 - x)^2$ compares to the graph of $y = (x - 2)^2$

b $y = (1 - x)^2$ compares to the graph of $y = (x - 1)^2$.

ENRICHMENT: Finding rules	–	–	11–14(½)

11 Find the equation of the quadratic relation which is of the form $y = x^2 + c$ and passes through:

a $(1, 4)$
b $(3, 5)$
c $(2, 1)$
d $(2, -1)$

12 Find the equation of the quadratic relation which is of the form $y = -x^2 + c$ and passes through:

a $(1, 3)$
b $(-1, 3)$
c $(3, 15)$
d $(-2, 6)$

13 Find the possible equations of each of the following quadratics if their equation is of the form $y = (x - h)^2$ and their graph passes through the point:

a $(1, 16)$
b $(3, 1)$
c $(-1, 9)$
d $(3, 9)$

14 Find the rule for each of these graphs with the given turning point (TP) and y-intercept.

a TP = $(1, 1)$, y-intercept = $(0, 0)$

b TP = $(-2, 0)$, y-intercept = $(0, 4)$

c TP = $(3, 0)$, y-intercept = $(0, -9)$

d TP = $(-3, 2)$, y-intercept = $(0, -7)$

e TP = $(-1, 4)$, y-intercept = $(0, 5)$

f TP = $(3, -9)$, y-intercept = $(0, 0)$

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10H Sketching parabolas using intercept form

Learning intentions

- To understand how a parabola can have 0, 1 or 2 x-intercepts
- To be able to find the x-intercepts of a parabola by solving a quadratic equation
- To know how to use symmetry and the x-intercepts to help locate the vertex
- To be able to sketch a quadratic graph in expanded form using factorisation

Past, present and future learning

- This section addresses concepts from the Stage 5 Path Topics *Non-linear relationships B and C*
- These concepts will be revised and extended in Chapter 7 of our Year 10 book
- Expertise with these concepts is assumed knowledge for Stage 6 Advanced

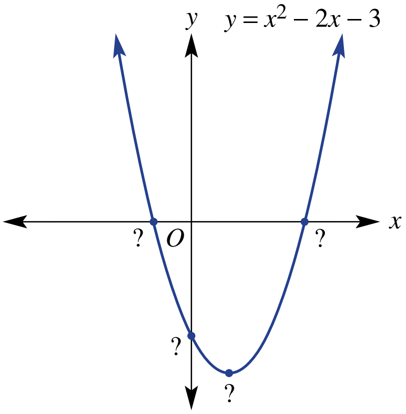
So far we have sketched parabolas using rules of the form $y = a(x - h)^2 + k$ where the coordinates of the turning point can be determined directly from the rule. An alternative method for sketching parabolas uses the factorised form of the quadratic rule and the Null Factor Law to find the x-intercepts. The turning point can be found by considering the axis of symmetry, which sits halfway between the two x-intercepts.

Parallel lines reflected from a parabola, and parallel light or radio waves reflected from a parabolic dish, meet at a focal point above the vertex. Scientists use CSIRO's six radio telescopes, with parabolic dishes, to research objects in outer space.

Lesson starter: From x-intercepts to turning point

This graph has the rule $y = x^2 - 2x - 3$ but all its important features are not shown.
Find the features by discussing these questions.

- Can the quadratic rule be factorised?
- What is the value of y at the x-intercepts?
- How can the factorised form of the rule help to find the x-intercepts?
- How does the x-coordinate of the turning point relate to the x-intercepts?
- Discuss how the y-coordinate of the turning point can be found.
- Finish by finding the y-intercept.



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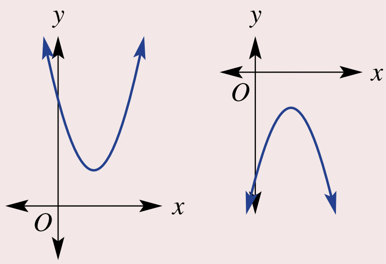
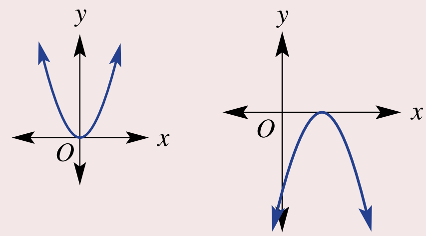
KEY IDEAS

- All parabolas have one y-intercept and can have zero, one or two x-intercepts.

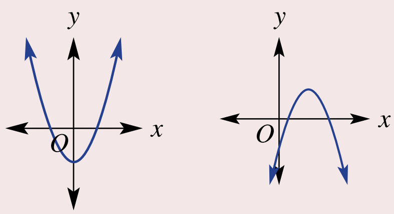
Zero x-intercepts

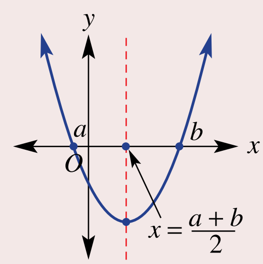


One x-intercept



Two x-intercepts





- x-intercepts can be found by substituting $y = 0$ and using the Null Factor Law.
- If the graph has two x-intercepts (at $x = a$ and $x = b$), the turning point can be found by:
  - calculating the x-coordinate of the turning point, which is the midpoint of a and b; that is, $x = \frac{a+b}{2}$
  - calculating the y-coordinate of the turning point by substituting the x-coordinate into the rule for the quadratic.

$$\begin{aligned} y &= (x-a)(x-b) \\ x\text{-intercepts: } 0 &= (x-a)(x-b) \\ x-a &= 0 \text{ or } x-b=0 \\ x &= a \text{ or } x = b \end{aligned}$$

BUILDING UNDERSTANDING

1 Factorise these quadratics.

a $x^2 + 2x$ **b** $x^2 - 3x$ **c** $x^2 - 9$

d $x^2 - 49$ **e** $x^2 - x - 12$ **f** $x^2 - 4x + 4$

2 For these factorised quadratics, use the Null Factor Law to solve for x, then find the x-value halfway between the two values.

a $0 = (x-2)(x+2)$ **b** $0 = (x+1)(x+5)$

c $0 = (x-1)(x+3)$ **d** $0 = (x-2)(x+3)$

3 Use substitution to find the y-coordinate of the turning point of these quadratics. The x-coordinate of the turning point is given.

a $y = x^2 - 4, x = 0$ **b** $y = x^2 - 6x, x = 3$ **c** $y = x^2 + 4x + 3, x = -2$

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Example 15 Finding intercepts of quadratic graphs

For each of the following quadratic relations, find:

- **i** the coordinates of the x-intercepts
- **ii** the coordinates of the y-intercept.

a $y = x(x + 1)$

b $y = 2(x + 2)(x - 3)$

SOLUTION

a **i** x-intercepts (let $y = 0$):

$x(x + 1) = 0$

$x = 0$ or $x + 1 = 0$

$x = 0$ or $x = -1$

$\therefore (0, 0)$ and $(-1, 0)$

ii y-intercept (let $x = 0$):

$y = 0(0 + 1)$

$y = 0$

$\therefore (0, 0)$

b **i** $y = 2(x + 2)(x - 3)$ has two x-intercepts.

x-intercepts (let $y = 0$):

$2(x + 2)(x - 3) = 0$

$x + 2 = 0$ or $x - 3 = 0$

$x = -2$ or $x = 3$

$\therefore (-2, 0)$ and $(3, 0)$

ii y-intercept (let $x = 0$):

$y = 2(0 + 2)(0 - 3)$

$y = 2(2)(-3)$

$y = -12$

$\therefore (0, -12)$

EXPLANATION

Let $y = 0$ to find the x-intercepts.

Apply the Null Factor Law to set each factor equal to 0 and solve.

Let $x = 0$ to find the y-intercept.

There are two different factors.

Let $y = 0$ to find the x-intercepts. Set each factor equal to 0 and solve.

Let $x = 0$ to find the y-intercept.

Now you try

For each of the following quadratic relations, find:

- **i** the coordinates of the x-intercepts
- **ii** the coordinates of the y-intercept.

a $y = x(x + 5)$

b $y = 3(x + 1)(x - 4)$

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Example 16 Sketching using intercept form

Consider the quadratic relation $y = x^2 - 2x$.

a Find the coordinates of the y-intercept.
b Factorise the relation.
c Find the coordinates of the x-intercepts.
d Find the axis of symmetry.
e Find the turning point.
f Sketch the graph clearly showing all the key features.

SOLUTION / EXPLANATION

a y-intercept (let $x = 0$): $y = 0$
$\therefore (0, 0)$

Let $x = 0$ to find the y-intercept. $y = 0$ since $0^2 - 2(0) = 0$.

b
$$\begin{aligned} y &= x^2 - 2x \\ &= x(x - 2) \end{aligned}$$

Take out the common factor of x.

c x-intercepts (let $y = 0$):
$0 = x(x - 2)$
$x = 0$ or $x - 2 = 0$
$x = 0$ or $x = 2$
$\therefore (0, 0)$ and $(2, 0)$

Let $y = 0$ to find the x-intercepts. Set each factor equal to 0 and solve.

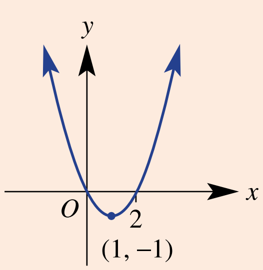
d Axis of symmetry: $x = \dfrac{0 + 2}{2}$
$\therefore x = 1$

The axis of symmetry is halfway between the x-intercepts.

e Turning point occurs when $x = 1$.
When $x = 1$, $y = (1)^2 - 2(1)$
$y = -1$
$\therefore$ there is a minimum turning point at $(1, -1)$.

Substitute $x = 1$ into $y = x^2 - 2x$ to find the y-coordinate.
$x = 1$ and $y = -1$
The coefficient of x^2 is positive, therefore the basic shape is .

f



Sketch the graph, labelling the key features found above.

Now you try

Consider the quadratic relation $y = x^2 - 4x$.

a Find the coordinates of the y-intercept.
b Factorise the relation.
c Find the coordinates of the x-intercepts.
d Find the axis of symmetry.
e Find the turning point.
f Sketch the graph clearly showing all the key features.

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Example 17 Sketching a quadratic trinomial

Consider the quadratic relation $y = x^2 + 2x - 8$.

a Find the coordinates of the y-intercept.
b Factorise the relation.
c Find the coordinates of the x-intercepts.
d Find the axis of symmetry.
e Find the turning point.
f Sketch the graph clearly showing all the key features.

SOLUTION / EXPLANATION

a y-intercept (let $x = 0$): $y = -8$
$\therefore (0, -8)$

Let $x = 0$ to find the y-intercept.

b $y = x^2 + 2x - 8$
$\quad = (x + 4)(x - 2)$

Factorise the quadratic trinomial:
$4 \times (-2) = -8$ and $4 + (-2) = 2$.

c x-intercepts (let $y = 0$):
$0 = (x + 4)(x - 2)$
$x + 4 = 0$ or $x - 2 = 0$
$x = -4$ or $x = 2$
$\therefore (-4, 0)$ and $(2, 0)$

Let $y = 0$ to find the x-intercepts.

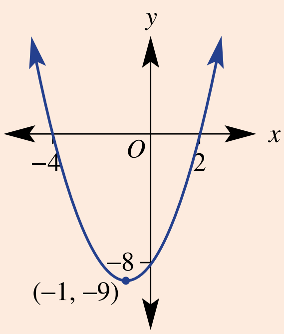
d Axis of symmetry: $x = \dfrac{-4 + 2}{2}$
$\therefore x = -1$

The axis of symmetry is halfway between the x-intercepts.

e Turning point occurs when $x = -1$.
When $x = -1$, $y = (-1)^2 + 2(-1) - 8$
$y = -9$
$\therefore$ there is a minimum turning point at $(-1, -9)$.

Substitute $x = -1$ into $y = x^2 + 2x - 8$ to find the y-coordinate. $y = 1 - 2 - 8$
$x = -1$ and $y = -9$
The coefficient of x^2 is positive, therefore the basic shape is $\cup$.

f



Sketch the graph showing the key features.

Now you try

Consider the quadratic relation $y = x^2 - 4x - 12$.

a Find the coordinates of the y-intercept.
b Factorise the relation.
c Find the coordinates of the x-intercepts.
d Find the axis of symmetry.
e Find the turning point.
f Sketch the graph clearly showing all the key features.

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Exercise 10H

FLUENCY	1–3(½)	1–4(½)	1–4(⅓)

Example 15

1 For each of the following quadratic relations, find:

i the coordinates of the x-intercepts

ii the coordinates of the y-intercept.

a $y = x(x + 7)$
b $y = x(x + 3)$
c $y = x(x + 4)$
d $y = (x - 4)(x + 2)$
e $y = (x + 2)(x - 5)$
f $y = (x - 7)(x + 3)$
g $y = 2(x + 3)(x - 1)$
h $y = 3(x + 4)(x + 1)$
i $y = (2 - x)(3 - x)$

Example 16

2 Consider each of the following quadratic relations.

i Find the coordinates of the y-intercept.

ii Factorise the relation.

iii Find the coordinates of the x-intercepts.

iv Find the axis of symmetry.

v Find the turning point.

vi Sketch the graph clearly showing all the key features.

a $y = x^2 - 5x$
b $y = x^2 + x$
c $y = x^2 - 3x$
d $y = 2x + x^2$
e $y = 5x + x^2$
f $y = 3x - x^2$
g $y = -x^2 - 8x$
h $y = -2x - x^2$
i $y = -x^2 + x$

Example 17

3 Consider each of the following quadratic relations.

i Find the coordinates of the y-intercept.

ii Factorise the relation.

iii Find the coordinates of the x-intercepts.

iv Find the axis of symmetry.

v Find the turning point.

vi Sketch each graph clearly showing all the key features.

a $y = x^2 - 3x + 2$
b $y = x^2 - 4x + 3$
c $y = x^2 + 2x - 3$
d $y = x^2 + 4x + 4$
e $y = x^2 + 2x + 1$
f $y = x^2 + 2x - 8$

4 For each of the following relations, sketch the graph, clearly showing the x- and y-intercepts and the turning point.

a $y = x^2 - 1$
b $y = 9 - x^2$
c $y = x^2 + 5x$
d $y = 2x^2 - 6x$
e $y = 4x^2 - 8x$
f $y = x^2 - 3x - 4$
g $y = x^2 + x - 12$
h $y = x^2 + 2x + 1$
i $y = x^2 - 6x + 9$

Design structures, such as this building at LAX airport, can be modelled on a set of axes.

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PROBLEM–SOLVING	5	5, 6	5(½), 6, 7

5 State the missing number (?) in these quadratic graphs.

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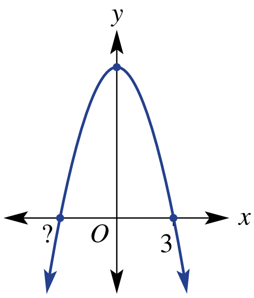

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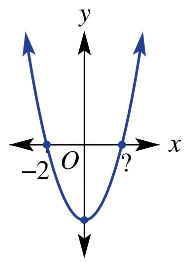

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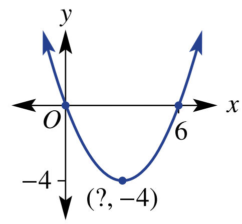

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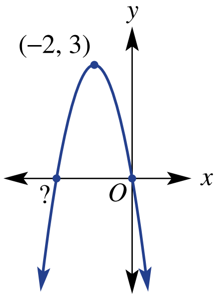

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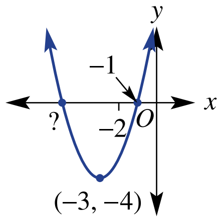

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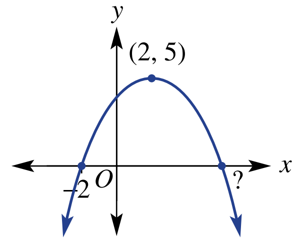

6 The path of a golf ball is given by the rule $y = 30x - x^2$, where y is the height in metres above the ground and x is the horizontal distance in metres. Find:

a how far the ball travels horizontally

b how high the ball reaches mid-flight.

7 A test rocket is fired and follows a path described by $y = 0.1x(200 - x)$. The height is y metres above ground and x is the horizontal distance in metres.

a How far does the rocket travel horizontally?

b How high does the rocket reach mid-flight?

REASONING	8	8, 9	9, 10

8 Explain why the coordinates of the x-intercept and the turning point for $y = (x - 2)^2$ are the same.

9 Write down an expression for the y-coordinate of the y-intercept for these quadratics.

a $y = ax^2 + bx + c$

b $y = (x - a)(x - b)$

10 $y = x^2 - 2x - 15$ can also be written in the form $y = (x - 1)^2 - 16$.

a Use the second rule to state the coordinates of the turning point.

b Use the first rule to find the x-intercepts then the turning point. Check you get the same result.

c $y = x^2 - 4x - 45$ can be written in the form $y = (x - h)^2 + k$. Find the value of h and k.

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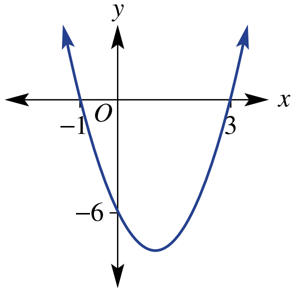
ENRICHMENT: Rule finding using x-intercepts — – — – — 11(½)

11 Find the rule for these graphs using intercept form. The first one is done for you.

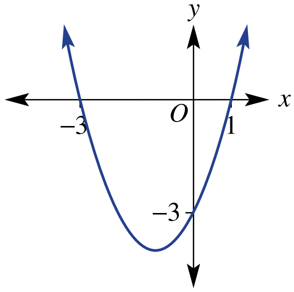
a



$$\begin{aligned} y &= a(x+1)(x-3) \\ \text{Substitute } (0, -6) \quad -6 &= a(0+1)(0-3) \\ -6 &= a(1) \times (-3) \\ -6 &= -3a \\ a &= 2 \end{aligned}$$

$\therefore y = 2(x+1)(x-3)$

b



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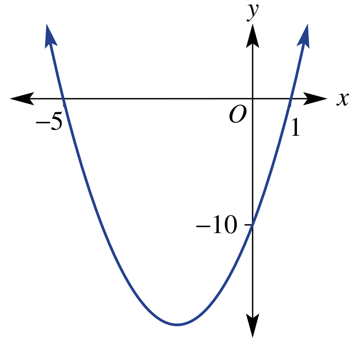

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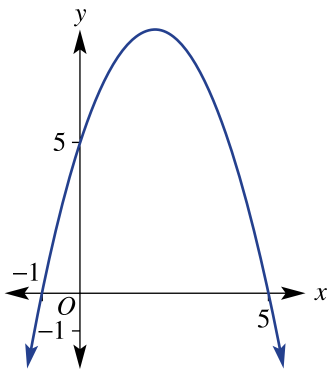

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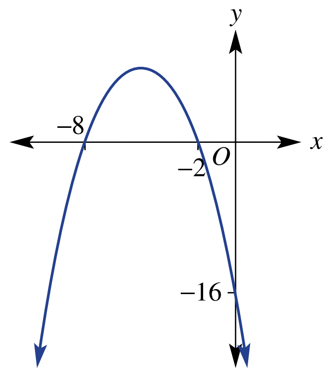

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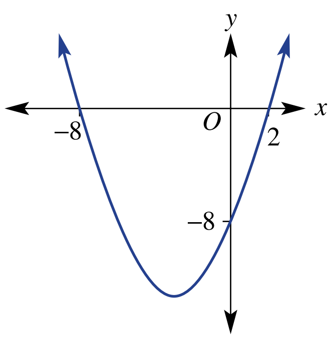

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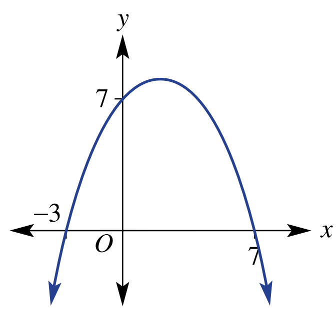

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Bridge support arch

Parabolic shapes are often used as bridge arches due to their self-supporting properties. Using the coordinates (0, 0) for one side of a bridge arch, a model for a bridge parabolic curve is given by $y = kx(a - x)$ where:

- y metres is the vertical distance above water level
- x metres is the horizontal distance from the left-hand side
- a and k are positive numbers.

Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate.

Routine problems

a If $k = 0.5$ and $a = 30$:

i construct the rule connecting y and x

ii sketch a graph representing the bridge arch by firstly finding the x-intercepts

iii determine the maximum height of the arch.

b Repeat part **a** for $k = 0.2$ and $a = 50$.

Non-routine problems

Explore and connect

a The problem is to determine a rule of a new bridge design which clears a river of a certain breadth and has a maximum arch height of less than or equal to 40 metres. Write down all the relevant information that will help solve this problem with the aid of a rough sketch.

b Construct the rule connecting y and x using the following values for a and k.

i $a = 50$ and $k = 0.1$ **ii** $a = 70$ and $k = 0.02$

Choose and apply techniques

c If $a = 50$ and $k = 0.1$, sketch a graph of the bridge arch by locating and labelling the following:

i x-intercepts **ii** turning point.

d Decide if an arch that is designed using $a = 50$ and $k = 0.1$ meets the requirement that the maximum height is to be less than or equal to 40 metres.

e If $a = 70$ and $k = 0.02$, sketch a graph of the bridge arch by locating and labelling the following:

i x-intercepts **ii** turning point.

f Decide if an arch that is designed using $a = 70$ and $k = 0.02$ meets the requirement that the maximum height is to be less than or equal to 40 metres.

Communicate thinking and reasoning

g Choose your own values of a and k so that a bridge that spans a 60 metre-wide river has a maximum arch height of less than 40 metres. Justify your choice by showing your calculations and a graph.

h Choose your own values of a and k so that a bridge that spans a 60 metre-wide river has a maximum arch height of exactly 40 metres. Justify your choice by showing your calculations and a graph.

i Summarise your results and describe any key findings.

Extension problems

Problem solve

a Show that the rule for k in terms of a is $k = \dfrac{160}{a^2}$ if the maximum height of the arch is to be 40 metres.

b Use the rule for k in part **a** to determine the value of k if the breadth of the river is 100 m.

c Use the rule for k in part **a** to determine the breadth of the river, correct to one decimal place, if $k = 0.3$.

Working mathematically

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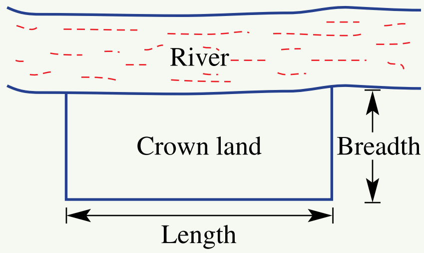
Investigation

Grazing Crown land

Land along the side of rivers is usually owned by the government and is sometimes called Crown land. Farmers can often lease this land to graze their sheep or cattle.



A farmer has a permit to fence off a rectangular area of land alongside the river. She has 400 m of fencing available and does not need to fence along the river.

A given breadth

a Find the length of the rectangular area when the breadth is:
 i 50 m **ii** 120 m **iii** 180 m

b Find the area of the land when the breadth is:
 i 50 m **ii** 120 m **iii** 180 m

c Which breadth from part **b** above gave the largest area? Explain why the area decreases for small and large values for the breadth.

The variable breadth

a Using x metres to represent the breadth, write an expression for the length showing working.

b Write an expression for the area of the land in terms of x.

c Use your area expression from part **b** to find the area of the land when x is:
 i 20 **ii** 80 **iii** 160

The graph

a Using your expression from part **b** above, sketch a graph of Area (A) versus x. You should find the following to help complete the graph.
 i y-intercept **ii** x-intercepts
 iii axis of symmetry **iv** turning point

b What value of x gives the maximum area of land for grazing? Explain your choice and give the dimensions of the rectangular area of land.

General observations

a What do you notice about the breadth and the length when the area is a maximum?

b See if the same is true if the farmer had 600 m of fencing instead. Show your expressions and graph.

c Prove your observation to parts **a** and **b** above by finding the x-value that gives a maximum area using k metres of fencing. (*Hint*: Use $A = x(k - 2x)$.)

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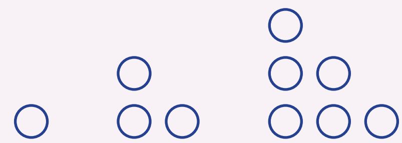
1 a What do you notice about the sum of these numbers?
 i $1 + 3$
 ii $1 + 3 + 5$
 iii $1 + 3 + 5 + 7$
 iv $1 + 3 + 5 + 7 + 9$
 b Find the sum of the first 100 odd integers.

2 Solve these equations.
 a $6x^2 = 35 - 11x$
 b $\frac{2}{x^2} = 1 - \frac{1}{x}$
 c $\sqrt{x - 1} = \frac{2}{x - 1}$

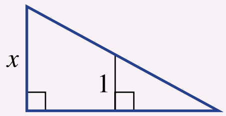
3 The height of a projectile (in metres) above the ground is given by the expression $t(14 - t)$ where time t is in seconds. For how long is the projectile at a height of above 40 m?

4 Find the quadratic rule that relates the number of balls to the term number (n) in the pattern below.

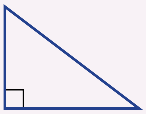


If there are 66 balls in the pattern, what term number is it?

5 Find the value of x in this diagram.



6 a A right-angled triangle's shortest side is of length x, the hypotenuse is 9 units longer than the shortest side and the other side is 1 unit longer than the shortest side. Find the side lengths of the triangle.
 b The area of a right-angled triangle is 60 square units and the lengths of the two shorter sides differ by 7 units. Find the length of the hypotenuse.



7 Given $a > 0$, for what values of k does $y = a(x - h)^2 + k$ have:
 a two x-intercepts? b one x-intercept? c no x-intercepts?

8 For the following equations, list the possible values of a that will give integer solutions for x.
 a $x^2 + ax + 24 = 0$ b $x^2 + ax - 24 = 0$

9 Solve the equation $(x^2 - 3x)^2 - 16(x^2 - 3x) - 36 = 0$ for all values of x. (*Hint*: Let $a = x^2 - 3x$.)

10 Solve the equation $x^4 - 10x^2 + 9 = 0$ for all values of x. (*Hint*: Let $a = x^2$.)

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Chapter summary

Introduction to quadratic equations and graphs

Turning point form

$y = a(x - h)^2 + k$
turning point (h, k)
e.g. $y = 2(x + 1)^2 + 3$
TP: $(-1, 3)$
y-int: $x = 0$, $y = 2(0 + 1)^2 + 3$
$= 2(1)^2 + 3$
$= 5$
$\therefore (0, 5)$

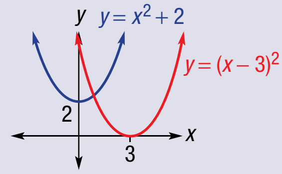
Standard form

A quadratic is of the form
$y = ax^2 + bx + c$, $a \neq 0$

Translation

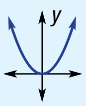
$y = x^2 + k$ $\quad k > 0$, graph shifts up k units
$k < 0$, graph shifts down k units
$y = (x - h)^2$ $\quad h > 0$, graph shifts h units right
$h < 0$, graph shift h units left

e.g.

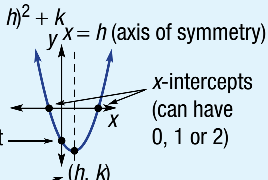


Parabola

Basic form $y = x^2$

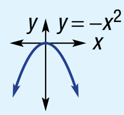


Turning point form
$y = a(x - h)^2 + k$



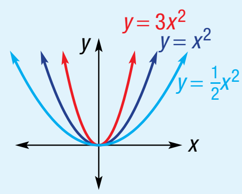
Dilation and reflection

$y = ax^2$
$a < 0$, graph is reflected in x-axis



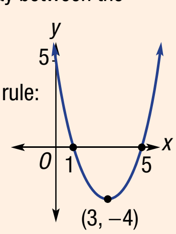
$-1 < a < 1$:
graph appears wider than $y = x^2$ or $y = -x^2$

$a > 1$ or $a < -1$:
graph appears narrower than $y = x^2$ or $y = -x^2$
e.g.



Sketching $y = ax^2 + bx + c$

Find x- and y-intercepts and turning point then sketch.
e.g. $y = x^2 - 6x + 5$
y-int: $x = 0$, $y = 0^2 - 6(0) + 5$
$= 5$
x-int: $y = 0$, $0 = x^2 - 6x + 5$
$0 = (x - 1)(x - 5)$
$x - 1 = 0$ or $x - 5 = 0$
$x = 1$ or $x = 5$
Turning point: halfway between the x-intercepts:
$x = \frac{1 + 5}{2} = 3$
Substitute $x = 3$ into rule:
$y = 3^2 - 6(3) + 5$
$= -4$
$(3, -4)$ minimum turning point



Applications

Define variable.
Set up equation.
Solve using Null Factor Law.
Check answer makes sense.
Answer the question in words.

Solving quadratic equations

- Rewrite equation in standard form: i.e. $ax^2 + bx + c = 0$
- Factorise
- Apply Null Factor Law: If $p \times q = 0$, $p = 0$ or $q = 0$.
- Solve remaining equation:
  e.g. $x^2 + 5x = 0$
  $x(x + 5) = 0$
  $x = 0$ or $x + 5 = 0$
  $x = 0$ or $x = -5$
  e.g. $x^2 - 12 = 4x$
  $x^2 - 4x - 12 = 0$
  $(x - 6)(x + 2) = 0$
  $x - 6 = 0$ or $x + 2 = 0$
  $x = 6$ or $x = -2$
- If $x^2 = 16$, square root both sides so $x = 4$ or $x = -4$.

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Chapter checklist with success criteria

A printable version of this checklist is available in the Interactive Textbook ✔

10A **1. I can write a quadratic equation in standard form.**
e.g. Write $x(x - 3) = 6$ in standard form. ☐

10A **2. I can determine if a value is a solution of an equation.**
e.g. Use substitution to determine if $x = -2$ is a solution of $2x^2 + 3x - 2 = 0$. ☐

10A **3. I can use the Null Factor Law to solve an equation.**
e.g. Use the Null Factor Law to solve $(x + 2)(x - 5) = 0$. ☐

10B **4. I can solve a quadratic equation with a common factor.**
e.g. Solve $2x^2 - 10x = 0$. ☐

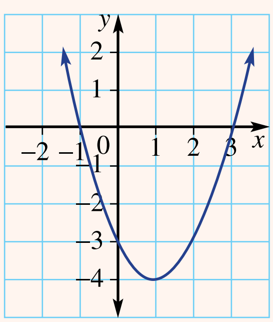
10B **5. I can solve a quadratic equation of the form $ax^2 = d$.**
e.g. Solve $6x^2 = 24$. ☐

10C **6. I can solve equations involving quadratic trinomials.**
e.g. Solve $x^2 - 3x - 70 = 0$. ☐

10C **7. I can solve quadratic equations not in standard form.**
e.g. Solve $x^2 = 9x - 20$. ☐

10D **8. I can set up and solve a quadratic equation from a word problem.**
e.g. The length of a rectangular postcard is 5 cm more than its breadth and the area of the postcard is 84 cm^2. Find the dimensions of the postcard. ☐

10E **9. I can identify the key features of a parabola.**
e.g. Consider this graph.



State:
i. the equation of the axis of symmetry
ii. the type and coordinates of the turning point
iii. the x- and y-intercept coordinates. ☐

10E **10. I can plot a parabola from a table of values.**
e.g. For $y = x^2 - 4x + 4$, fill in the table of values and plot the points on a set of axes to form a parabola. ☐

x							
y							

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Chapter checklist

10E	**11. I can compare graphs of $y = ax^2$, $a > 0$.** e.g. Sketch $y = x^2$, $y = 3x^2$ and $y = \frac{1}{3}x^2$ by first completing the table of values below.	☐

x					
$y = x^2$					
$y = 3x^2$					
$y = \frac{1}{3}x^2$					

State the turning point of each graph and whether the graphs of $y = 3x^2$ and $y = \frac{1}{3}x^2$ are wider or narrower than the graph of $y = x^2$.

10F	**12. I can compare graphs of $y = ax^2$, $a < 0$.** e.g. Sketch $y = -x^2$, $y = -4x^2$ and $y = -\frac{1}{4}x^2$ by first completing the table of values below.	☐

x					
$y = -x^2$					
$y = -4x^2$					
$y = -\frac{1}{4}x^2$					

State the turning point of each graph and whether the graphs of $y = -4x^2$ and $y = -\frac{1}{4}x^2$ are wider or narrower than the graph of $y = -x^2$.

10G	**13. I can sketch parabolas with horizontal and vertical translations.** e.g. Sketch $y = x^2 + 3$ and $y = -(x + 1)^2$ showing the y-intercept and turning point.	☐
10G	**14. I can sketch parabolas with combined translations.** e.g. Sketch $y = (x - 1)^2 - 3$ showing the y-intercept and turning point.	☐
10H	**15. I can sketch parabolas using intercept form.** e.g. For the quadratic relation $y = x^2 - 6x$, find the x- and y-intercepts and the turning point and sketch its graph, labelling key features.	☐
10H	**16. I can sketch a quadratic trinomial.** e.g. For the quadratic relation $y = x^2 - 4x - 12$, find the x- and y-intercepts and the turning point and sketch its graph, labelling key features.	☐

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Short-answer questions

10E

1 Consider the quadratic $y = x^2 - 2x - 3$.

a Complete this table of values for the equation.

x	-3	-2	-1	0	1	2	3
y							

b Plot the points in part **a** on a Cartesian plane and join in a smooth curve.

10A

2 Use the Null Factor Law to solve the following equations.

a $x(x + 2) = 0$

b $3x(x - 4) = 0$

c $(x + 3)(x - 7) = 0$

d $(x - 2)(2x + 4) = 0$

e $(x + 1)(5x - 2) = 0$

f $(2x - 1)(3x - 4) = 0$

10B/C

3 Solve the following quadratic equations by first factorising.

a $x^2 + 3x = 0$ **b** $2x^2 - 8x = 0$ **c** $x^2 = 25$

d $x^2 = 81$ **e** $5x^2 = 20$ **f** $3x^2 - 30 = 0$

g $x^2 + 10x + 21 = 0$ **h** $x^2 - 3x - 40 = 0$ **i** $x^2 - 8x + 16 = 0$

10C

4 Write the following quadratic equations in standard form and solve for x.

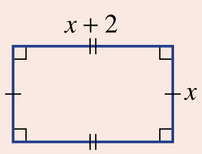
a $x^2 = 5x$ **b** $3x^2 = 18x$ **c** $x^2 + 12 = -8x$

d $2x + 15 = x^2$ **e** $x^2 + 15 = 8x$ **f** $4 - x^2 = 3x$

10D

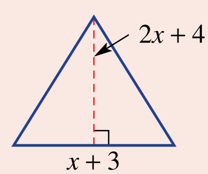
5 Set up and solve a quadratic equation to determine the value of x that gives the specified area of the shapes below.

a



$A = 80$ units^2

b



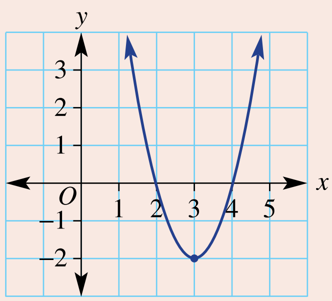
$A = 30$ units^2

10E

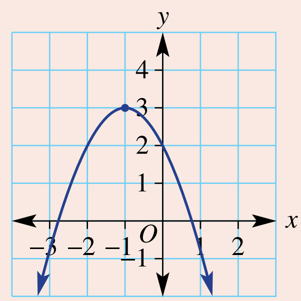
6 For the following graphs state:

i the axis of symmetry **ii** the turning point and its type.

a



b



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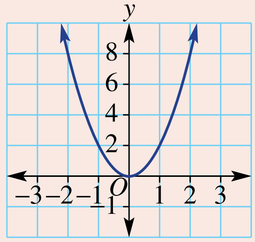
10F/G

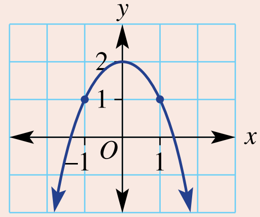
7 Match the following quadratic rules with their graphs.

$y = x^2 - 4$, $y = 2x^2$, $y = (x + 1)^2$, $y = (x - 3)^2$, $y = -x^2 + 2$

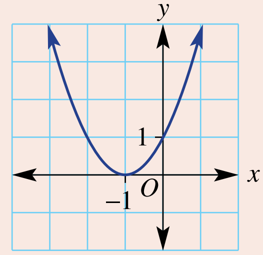
a



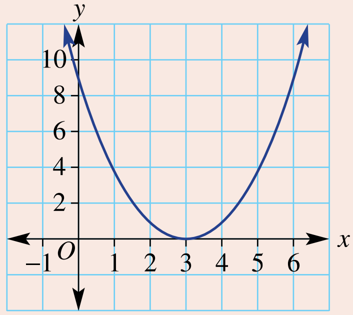
b



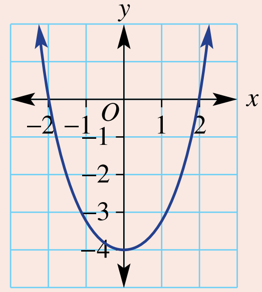
c



d



e



10G

8 Sketch the following graphs labelling the turning point and y-intercept.

a $y = x^2 + 2$ **b** $y = -x^2 - 5$ **c** $y = (x + 2)^2$

d $y = -(x - 3)^2$ **e** $y = (x - 1)^2 + 3$ **f** $y = 2(x + 2)^2 - 4$

10G

9 State the transformations that take $y = x^2$ to each of the graphs in Question **8**.

10H

10 Sketch the following graphs labelling the y-intercept, turning point and x-intercepts.

a $y = x^2 - 8x + 12$ **b** $y = x^2 + 10x + 16$

c $y = x^2 + 2x - 15$ **d** $y = x^2 + 4x - 5$

Multiple-choice questions

10A

1 (1, 3) is a point on a curve with which equation?

A $y = x^2$ **B** $y = (x - 3)^2$ **C** $y = x^2 + 2x - 3$

D $y = x^2 + 2$ **E** $y = 6 - x^2$

10A

2 The solution(s) to $2x(x - 3) = 0$ is/are:

A $x = -3$ **B** $x = 0$ or $x = 3$ **C** $x = 2$ or $x = 3$

D $x = 0$ or $x = -3$ **E** $x = 0$

10A

3 The quadratic equation $x^2 = 7x - 12$ in standard form is:

A $x^2 - 7x - 12 = 0$ **B** $-x^2 + 7x + 12 = 0$ **C** $x^2 - 7x + 12 = 0$

D $x^2 + 7x - 12 = 0$ **E** $x^2 + 7x + 12 = 0$

10B

4 The solution(s) to $x(x + 2) = 2x + 9$ is/are:

A $x = -3$ or $x = 3$ **B** $x = 0$ or $x = -2$ **C** $x = 3$

D $x = 9$ or $x = -1$ **E** $x = 9$ or $x = 0$

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The following applies to Questions **5** and **6**.

The height, h metres, of a toy rocket above the ground t seconds after launch is given by $h = 6t - t^2$.

10D **5** The rocket returns to ground level after:

A 5 seconds **B** 3 seconds **C** 12 seconds
D 6 seconds **E** 8 seconds

10H **6** The rocket reaches its maximum height after:

A 6 seconds **B** 3 seconds **C** 10 seconds
D 4 seconds **E** 9 seconds

10G **7** The turning point of $y = (x - 2)^2 - 4$ is:

A a maximum at $(2, 4)$
B a minimum at $(-2, 4)$
C a maximum at $(-2, 4)$
D a minimum at $(-2, -4)$
E a minimum at $(2, -4)$

10G **8** The transformation of the graph of $y = x^2$ to $y = x^2 - 2$ is described by:

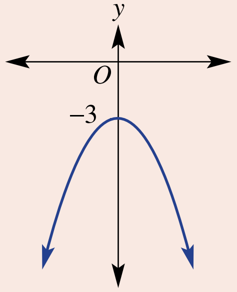
A a translation of 2 units to the left
B a translation of 2 units to the right
C a translation of 2 units down
D a translation of 2 units up
E a translation of 2 units right and 2 units down

10F **9** Compared to $y = x^2$, the narrowest graph is:

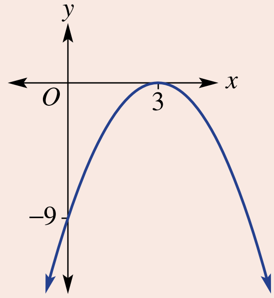
A $y = 5x^2$ **B** $y = 0.2x^2$ **C** $y = 2x^2$
D $y = \frac{1}{2}x^2$ **E** $y = 3.5x^2$

10G **10** The graph of $y = -(x - 3)^2$ is:

A



B



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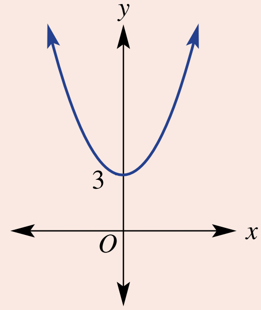
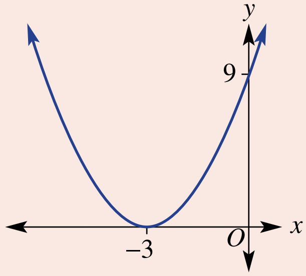
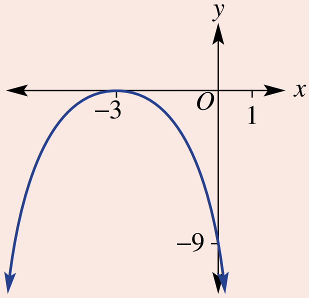


D



E



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Extended-response questions

1 A sail of a yacht is in the shape of a right-angled triangle. It has a base length of $2x$ metres and its height is 5 metres more than half its base.

- a Write an expression for the height of the sail.
- b Give an expression for the area of the sail in expanded form.
- c If the area of the sail is 14 m^2, find the value of x.
- d Hence, state the dimensions of the sail.

2 Connor and Sam are playing in the park with toy rockets they have made. They launch their rockets at the same time to see whose is better.

- a The path of Sam's rocket is modelled by the equation $h = 12t - 2t^2$, where h is the height of the rocket in metres after t seconds.
  - i Find the coordinates of the axis intercepts.
  - ii Find the turning point.
  - iii Sketch a graph of the height of Sam's rocket over time.
- b The path of Connor's rocket is modelled by the equation $h = -(t - 4)^2 + 16$, where h is the height of the rocket in metres after t seconds.
  - i Find the h-intercept coordinates (i.e. $t = 0$).
  - ii State the turning point.
  - iii Use the answers from parts b i and ii to state the coordinates of the two t-intercepts.
  - iv Sketch a graph of the height of Connor's rocket over time.
- c
  - i Whose rocket was in the air for longer?
  - ii Whose rocket reached the greater height and by how much?
  - iii How high was Sam's rocket when Connor's rocket was at its maximum height?

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Chapter 6: Indices and surds

Short-answer questions

1 Use index laws to simplify the following.

a $\dfrac{9a^6b^3}{18a^4b^2}$ b $\dfrac{\left(-3x^4y^2\right)^2 \times 6xy^2}{27x^6y}$ c $\left(2x^2\right)^3 - 3x^0 + (5x)^0$

2 Write each of the following using positive indices and simplify.

a $\dfrac{5}{m^{-2}}$ b $\dfrac{4a^6b^{-4}}{6a^{-2}b}$ c $3\left(x^{\frac{1}{2}}\right)^3 y^{-\frac{1}{3}} \times x^{\frac{1}{2}} y^{-\frac{2}{3}}$

3 Convert these numbers to the units given in brackets. Write your answer using scientific notation with three significant figures.

a 30.71 g (kg)

b 4236 tonnes (kg)

c 3.4 hours (seconds)

d 235 nanoseconds (seconds)

4 Simplify the following.

a $144^{\frac{1}{2}}$ b $27^{\frac{1}{3}}$

c $2\sqrt{3} + 3\sqrt{5} + 4\sqrt{5}$ d $\sqrt{35} \div \sqrt{5}$

Multiple-choice questions

1 $3a^2b^3 \times 4ab^2$ is equivalent to:

A $12a^2b^6$ B $7a^3b^5$ C $12a^3b^5$ D $12a^4b^5$ E $7a^2b^6$

2 $\left(\dfrac{2x}{5}\right)^3$ is equivalent to:

A $\dfrac{6x^3}{5}$ B $\dfrac{8x^3}{125}$ C $\dfrac{2x^3}{5}$ D $\dfrac{2x^4}{15}$ E $\dfrac{2x^3}{125}$

3 4^{-2} can be expressed as:

A $\dfrac{1}{4^{-2}}$ B $\dfrac{1}{8}$ C -16 D $\dfrac{1}{16}$ E -8

4 $3x^{-4}$ written with positive indices is:

A $-3x^4$ B $\dfrac{1}{3x^4}$ C $-\dfrac{3}{x^4}$ D $\dfrac{1}{3x^{-4}}$ E $\dfrac{3}{x^4}$

5 0.00371 in scientific notation is:

A 0.371×10^{-3} B 3.7×10^{-2} C 3.71×10^{-3}

D 3.71×10^3 E 371×10^3

Extended-response question

The average human body contains about 74 billion cells.

a Write this number of cells:

i in decimal form ii using scientific notation.

b If the population of a particular city is 2.521×10^6, how many human cells are there in the city? Give your answer using scientific notation correct to three significant figures.

c If the average human weighs 64.5 kilograms, what is the average mass of one cell in grams? Give your answer using scientific notation correct to three significant figures.

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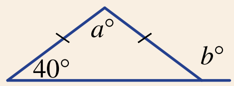
Chapter 7: Properties of geometrical figures

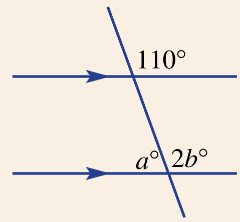
Short-answer questions

1 Find the value of each pronumeral in the following.

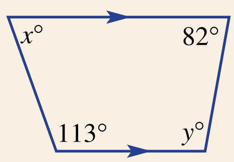
a



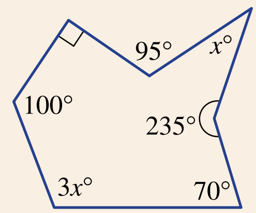
b



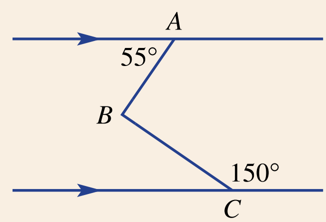
c



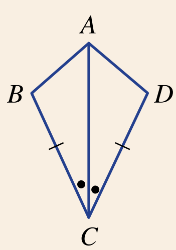
d



2 Find the value of $\angle ABC$ by adding a third parallel line.



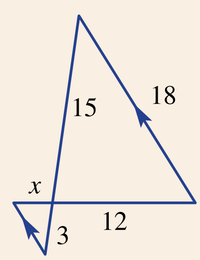
3 Prove $\Delta ABC \equiv \Delta ADC$.



4 Consider the pair of triangles shown at right.

a Give a reason (SSS, SAS, AAA or RHS) why the two triangles are similar.

b Find the value of x.



Multiple-choice questions

1 The supplementary angle to 55° is:

A 55° **B** 35° **C** 125° **D** 135° **E** 70°

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2 A quadrilateral with all four sides equal and opposite sides parallel is best described by a:

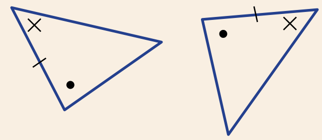
A parallelogram B rhombus C rectangle
D trapezium E kite

3 The size of the interior angle in a regular pentagon is:

A 108° B 120° C 96° D 28° E 115°

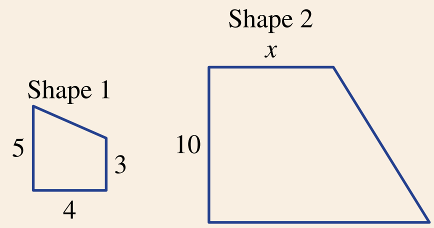
4 The test that proves congruence in these two triangles is:

A SAS B RHS
C AAA D SSS
E AAS



5 What is the scale factor that enlarges shape 1 to shape 2 in these similar figures, and what is the value of x?

A 2 and $x = 8$
B 2.5 and $x = 7.5$
C 3.33 and $x = 13.33$
D 2.5 and $x = 12.5$
E 2 and $x = 6$



Extended-response question

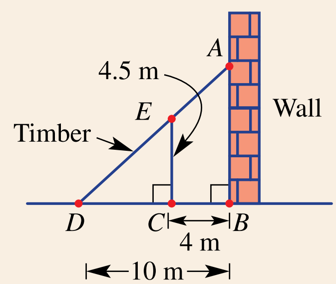
A vertical wall is being supported by a piece of timber that touches the ground 10 m from the base of the wall. A vertical metal support 4.5 m high is placed under the timber support 4 m from the wall.

a Prove $\Delta ABD ||| \Delta ECD$.

b Find how far up the wall the timber reaches.

c How far above the ground is the point halfway along the timber support?

d The vertical metal support is moved so that the timber support is able to reach one metre higher up the wall. If the piece of timber now touches the ground 9.2 m from the wall, find how far the metal support is from the wall. Give your answer correct to one decimal place.

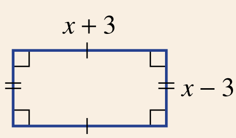


Chapter 8: Quadratic expressions and algebraic fractions

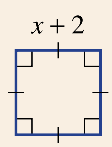
Short-answer questions

1 Find the area of the following shapes in expanded form.

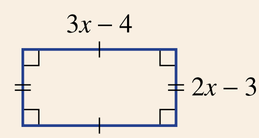
a



b



c



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2 Factorise each of the following fully.

a $8ab + 2a^2b$ **b** $9m^2 - 25$ **c** $3b^2 - 48$

d $(a + 7)^2 - 9$ **e** $x^2 + 6x + 9$ **f** $x^2 + 8x - 20$

g $2x^2 - 16x + 30$ **h** $2x^2 - 11x + 12$ **i** $6x^2 + 5x - 4$

3 Factorise the following by grouping in pairs.

a $x^2 - 3x - x + 3$ **b** $2x^2 - 10 - 5x + 4x$

4 **a** Simplify these algebraic fractions.

i $\dfrac{3x + 24}{2x + 16}$ **ii** $\dfrac{12}{x^2 - 9} \div \dfrac{3}{x - 3}$ **iii** $\dfrac{x + 3}{2} + \dfrac{3x}{7}$

iv $\dfrac{2}{x} - \dfrac{5}{3x}$ **v** $\dfrac{7}{x + 1} + \dfrac{4}{x - 2}$ **vi** $\dfrac{5}{x - 5} - \dfrac{3}{x + 2}$

b Solve these equations involving algebraic fractions.

i $\dfrac{5}{2x} + \dfrac{1}{3x} = 2$ **ii** $\dfrac{4}{x - 5} = \dfrac{2}{x + 3}$

Multiple-choice questions

1 The expanded form of $(x + 4)(3x - 2)$ is:

A $3x^2 + 12x - 8$ **B** $4x^2 + 14x - 8$ **C** $3x^2 + 12x - 10$

D $3x^2 - 8$ **E** $3x^2 + 10x - 8$

2 $2(x + 2y) - x(x + 2y)$ factorises to:

A $2x(x + 2y)$ **B** $(x + 2y)^2(2 - x)$ **C** $(2 + x)(x + 2y)$

D $(x + 2y)(2 - x)$ **E** $(x + 2y)\left(2 - x^2 - 2xy\right)$

3 $x^2 - 2x - 24$ in factorised form is:

A $(x - 2)(x + 12)$ **B** $(x - 6)(x + 4)$ **C** $(x - 8)(x + 3)$

D $(x + 6)(x - 4)$ **E** $(x - 6)(x - 4)$

4 $\dfrac{3x + 6}{(x - 2)(x - 4)} \times \dfrac{x^2 - 4}{(x + 2)^2}$ is equivalent to:

A $\dfrac{3}{x - 4}$ **B** $\dfrac{3(x - 2)}{(x - 4)(x + 2)}$ **C** $\dfrac{3x - 2}{(x + 1)^2}$

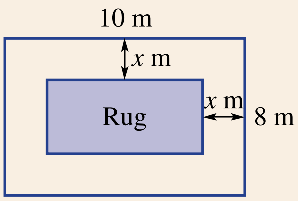
D $\dfrac{3}{(x + 2)(x - 2)}$ **E** $\dfrac{3x^2}{(x - 2)(x + 2)}$

5 $\dfrac{7}{(x + 1)^2} - \dfrac{4}{x + 1}$ simplifies to:

A $\dfrac{3x + 1}{(x + 1)^2}$ **B** $\dfrac{6 - 4x}{(x + 1)^2}$ **C** $\dfrac{-9}{(x + 1)^2}$ **D** $\dfrac{3 - 4x}{(x + 1)^2}$ **E** $\dfrac{11 - 4x}{(x + 1)^2}$

Extended-response question

A rectangular room 10 metres long and 8 metres wide has a rectangular rug in the middle of it that leaves a border x metres wide all the way around it as shown.



a Write expressions for the length and the breadth of the rug.

b Write an expression for the area of the rug in expanded form.

c What is the area of the rug when $x = 1$?

d Fully factorise your expression in part **b** by first removing the common factor.

e What happens when $x = 4$?

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Chapter 9: Probability and single variable data analysis

Short-answer questions

1 In a survey of 30 people, 18 people drink coffee, 14 people drink tea and 8 people drink both. Let C be the set of people who drink coffee and T the set of people who drink tea.
  a Construct a Venn diagram for the survey results.
  b Find:
    i $n(C \cup T)$
    ii $n(T')$.
  c If one of the 30 people was randomly selected, find:
    i P(drinks neither coffee nor tea)
    ii P(drinks coffee only).

2 Two ice-creams are randomly selected without replacement from a box containing one vanilla (V), two strawberry (S) and one chocolate (C) flavoured ice-creams.
  a Draw a tree diagram to show each of the possible outcomes.
  b What is the probability of selecting:
    i a vanilla and a strawberry flavoured ice-cream?
    ii two strawberry flavoured ice-creams?
    iii no vanilla flavoured ice-cream?

3 Identify the sampling method used in each of these surveys.
  a Employees are selected from each department in a workplace and surveyed on their job satisfaction.
  b Every tenth customer in a store is surveyed regarding their thoughts on customer service in the store.

4 The data below shows the number of aces served by a player in each of their grand slam tennis matches for the year.

15 22 11 17 25 25 12 31 26 18 32 11 25 32 13 10

  a Classify the type of data collected.
  b Construct a stem-and-leaf plot for the data.
  c From the stem-and-leaf plot, find the mode and median number of aces.
  d Is the data symmetrical or skewed?

5 The frequency table shows the number of visitors, in intervals of fifty, to a theme park each day in April.
  a Complete the frequency table shown. Round to one decimal place where necessary.
  b Construct a frequency histogram.
  c i On how many days were there fewer than 100 visitors?
    ii On what percentage of days were there between 50 and 200 visitors?

Class interval	Class centre	Frequency	Percentage frequency
0–49		2	
50–99		4	
100–149		5	
150–199		9	
200–249			
250–299		3	
Total		30	

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Multiple-choice questions

1 The probability of not rolling a number less than 3 on a normal six-sided die is:

A $\frac{1}{3}$ **B** 4 **C** $\frac{1}{2}$ **D** $\frac{2}{3}$ **E** 3

2 From the two-way table, $P(A \cap B)$ is:

A $\frac{1}{5}$ **B** 4 **C** $\frac{9}{20}$
D $\frac{1}{4}$ **E** 16

	A	A′	Total
B		7	
B′	5		
Total		11	20

3 In the selection of 40 marbles, 28 were blue. The experimental probability of the next one selected being blue is:

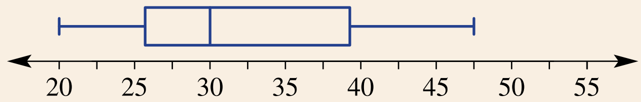
A 0.28 **B** 0.4 **C** 0.7 **D** 0.54 **E** 0.75

4 The median, mean and range of the data set 12 3 1 6 10 1 5 18 11 15 are, respectively:

A 5.5, 8.2, 1 – 18 **B** 8, 8.2, 17 **C** 5.5, 8, 17
D 8, 8.2, 1 – 18 **E** 8, 74.5, 18

5 The interquartile range of the data in the box plot shown is:

A 10 **B** 27
C 13 **D** 30
E 17



Extended-response question

A game at the school fair involves randomly selecting a green ball and a red ball each numbered 1, 2 or 3.

a List the outcomes in a table.

b What is the probability of getting an odd and an even number?

c Participants win $1 when they draw two balls showing the same number.
  i What is the probability of winning $1?
  ii If someone wins six times, how many games are they likely to have played?

d The ages of those playing the game in the first hour are recorded and are shown below.

12 16 7 24 28 9 11 17 18 18 37 9 40 16 32 42 14

  i Draw a box plot to represent the data.
  ii Twenty-five per cent of the participants are below what age?
  iii If this data is used as a model for the 120 participants throughout the day, how many would be expected to be aged less than 30?

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Chapter 10: Quadratic equations and graphs of parabolas

Short-answer questions

1 Solve the following quadratic equations.
 a $(x + 5)(x - 3) = 0$
 b $(2x - 1)(3x + 5) = 0$
 c $4x^2 + 8x = 0$
 d $5x^2 = 45$
 e $x^2 - 9x + 14 = 0$
 f $8x = -x^2 - 16$

2 The length of a rectangular swimming pool is x m and its breadth is 7 m less than its length. If the area occupied by the pool is 120 m^2, solve an appropriate equation to find the dimensions of the pool.

3 Sketch the following graphs showing the y-intercept and turning point, and state the transformations that have taken place from the graph of $y = x^2$.
 a $y = x^2 + 3$
 b $y = -(x + 4)^2$
 c $y = (x - 2)^2 + 5$

4 Consider the quadratic relation $y = x^2 - 4x - 12$.
 a Find the coordinates of the y-intercept.
 b Factorise the relation and find the coordinates of the x-intercepts.
 c Find the coordinates of the turning point.
 d Sketch the graph.

Multiple-choice questions

1 The solution(s) to $3x(x + 5) = 0$ is/are:
 A $x = -5$
 B $x = 0$ or $x = -5$
 C $x = 3$ or $x = 5$
 D $x = 5$
 E $x = 0$ or $x = 5$

2 $x^2 = 3x - 2$ is the same as the equation:
 A $x^2 + 3x - 2 = 0$
 B $x^2 - 3x + 2 = 0$
 C $x^2 - 3x - 2 = 0$
 D $-x^2 + 3x + 2 = 0$
 E $x^2 + 3x + 2 = 0$

3 The *incorrect* statement about the graph of $y = 2x^2$ is:
 A the graph is the shape of a parabola
 B the point $(-2, 8)$ is on the graph
 C its turning point is at $(0, 0)$
 D it has a minimum turning point
 E the graph is wider than the graph of $y = x^2$

4 The type and coordinates of the turning point of the graph of $y = -(x + 3)^2 + 2$ are:
 A a minimum at $(3, 2)$
 B a maximum at $(3, 2)$
 C a maximum at $(-3, 2)$
 D a minimum at $(-3, 2)$
 E a minimum at $(3, -2)$

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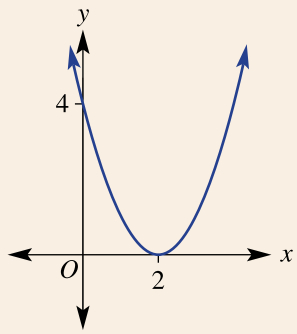
5 The graph shown has the equation:

A $y = 4x^2$
B $y = x^2 + 4$
C $y = (x + 2)^2$
D $y = x^2 + 2$
E $y = (x - 2)^2$



Extended-response question

The flight path of a soccer ball kicked upwards from the ground is given by the equation $y = 120x - 20x^2$, where y is the height of the ball above the ground in centimetres at any time x seconds.

a Find the x-intercepts to determine when the ball lands on the ground.

b Find the coordinates of the turning point and state:
  i the maximum height reached by the ball
  ii after how many seconds the ball reaches this maximum height.

c At what times was the ball at a height of 160 cm?

d Sketch a graph of the path of the ball until it returns to the ground.

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Chapter 1

1A

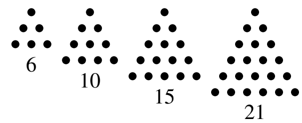
Building understanding

1 a 1, 2, 4, 7, 8, 14, 28, 56 b HCF = 8
c 3, 6, 9, 12, 15, 18, 21 d LCM = 15
e 2, 3, 5, 7, 11, 13, 17, 19, 23, 29
2 a 121 b 12 c 27 d 2
3 a –1 b –1 c 15
d –6 e 22 f –9

Now you try

Example 1
a 5 b 4 c –65

Exercise 1A

1 a 2 b 2 c 10 d 16
e –9 f –3 g –4 h 10
i –11 j 2 k –3 l 4
m –23 n 10 o 0 p 3
q –9 r 7 s –1 t 4
2 a 28 b 24 c 187 d 30
3 a 4 b 5 c 1 d 23
4 a 4 b 23 c –3
d 2 e –2 f –18
g –6 h –1 i 1
5 a –2 b –38 c –8
d 27 e 1 f 24
g 21 h 0 i 24
6 a $-2 \times [11 + (-2)] = -18$ b $[-6 + (-4)] \div 2 = -5$
c $[2 - 5] \times (-2) = 6$ d $-10 \div [3 + (-5)] = 5$
e $3 - [(-2) + 4] \times 3 = -3$
f $[(-2)^2 + 4] \div (-2) = -2^2$
7 252 days
8 4
9 a 7 and –2 b –5 and 2
10 8
11 a i 16 ii 16
b $a = \pm 4$
c $a = 3$
d The square of a negative number has negative signs occurring in pairs and will create a positive answer.
e –3
f The squaring of any number produces a positive answer.
g i –4 ii –125
iii –9 iv –16
h no
i yes
j A prime number has only two factors – itself and 1, therefore the only common factor for any pair of prime numbers is 1.
k Again as there are only two factors of any prime, the LCM must be the multiple of primes.
12 a false b false c true
d true e false f true
13 a i $1 + 2 + 3 = 6$
ii $28 (1 + 2 + 4 + 7 + 14 = 28)$
iii $1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248 = 496$
b i


ii 28, 36
c i 0, 1, 1, 2, 3, 5, 8, 13, 21, 34
ii –21, 13, –8, 5, –3, 2, –1, 1, 0, 1…
$\therefore$ –21, –8, –3, –1

1B

Building understanding

1 a 40 b 270 c 7.9 d 0.04
2 a 32 100 b 432 c 5.89 d 0.443
3 a 60 b 57.375 c 2.625

Now you try

Example 2
a 138.47 b 0.03 c 2.80
Example 3
a 3600 b 49 000 c 0.029
Example 4
26, –25.85792

Exercise 1B

1 a i 124.27 ii 830.44
b i 0.02 ii 0.07
c i 7.69 ii 13.50
2 a 17.96 b 11.08 c 72.99
d 47.86 e 63.93 f 23.81
g 804.53 h 500.57 i 821.27
j 5810.25 k 1005.00 l 2650.00
3 a 7 b 73 c 130 d 36 200
4 a 0.333 b 0.286 c 1.182 d 13.793
5 a 2400 b 35 000 c 0.060 d 34
e 110 000 f 0.0025 g 2.1 h 0.71
6 a 30 000 b 200 c 0.05 d 0.0006
7 a 3600, 3693 b 760, 759.4
c 4000, 4127.16 d 3000, 3523.78
e 0, 0.72216 f 4, 0.716245
g 0.12, 0.1186 h 0.02, 0.02254
i 10, 8.4375 j 1600, 1683.789156
k 0.08, 0.074957... l 11, 10.25538...
8 a A: 54.3, B: 53.8, 0.5 b A: 54.28, B: 53.79, 0.49
c A: 54, B: 54, 0 d A: 50, B: 50, 0
9 a each to 1 significant figure
b each to 2 significant figures
c each to the nearest integer

Answers

1A

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10 8.33 m
11 0.143 tonnes
12 2.14999 is closer to 2.1 correct to 1 decimal place
$\therefore$ round down
13 as magnesium in this case would be zero if rounded to two decimal places rather than 2 significant figures
14 a i 50 ii 624
b 50
c 600
d The addition is the same as the original but the multiplication is lower ($20 \times 30 < 24 \times 26$). When rounding the two numbers to one sig fig, you are changing the numbers by the same amount in opposite directions, so the sum will stay the same, but product will differ
15 a 0.18181818
b i 8 ii 1 iii 8
c 0.1428571428571
d i 4 ii 2 iii 8
e not possible

1C

Building understanding

1 a $1\frac{2}{5}$ b $4\frac{1}{3}$ c $4\frac{4}{11}$
2 a $\frac{11}{7}$ b $\frac{16}{3}$ c $\frac{19}{2}$
3 a $\frac{2}{5}$ b $\frac{4}{29}$ c $4\frac{1}{6}$
4 a 9 b 24 c 3 d 10

Now you try

Example 5
a 4.625 b $0.\overline{18}$

Example 6
a $\frac{9}{20}$ b $1\frac{17}{40}$

Example 7
$\frac{7}{11}$

Exercise 1C

1 a 2.75 b 0.35 c 2.3125
d 1.875 e 2.625 f 3.8
g 3.4 h 1.21875
2 a $0.\overline{27}$ b $0.\dot{7}$ c $1.\overline{285714}$
d $0.41\dot{6}$ e $1.\dot{1}$ f $3.8\dot{3}$
g $7.2\dot{6}$ h $2.\overline{63}$
3 a $\frac{7}{20}$ b $\frac{3}{50}$ c $3\frac{7}{10}$
d $\frac{14}{25}$ e $1\frac{7}{100}$ f $\frac{3}{40}$
g $3\frac{8}{25}$ h $7\frac{3}{8}$ i $2\frac{1}{200}$
j $10\frac{11}{250}$ k $6\frac{9}{20}$ l $2\frac{101}{1000}$
4 a $\frac{5}{6}$ b $\frac{13}{20}$ c $\frac{7}{10}$
d $\frac{5}{12}$ e $\frac{7}{16}$ f $\frac{11}{14}$
g $\frac{19}{30}$ h $\frac{11}{27}$
5 a $\frac{5}{12}, \frac{7}{18}, \frac{3}{8}$ b $\frac{5}{24}, \frac{3}{16}, \frac{1}{6}$ c $\frac{7}{12}, \frac{23}{40}, \frac{8}{15}$
6 a $\frac{9}{20}$ b $\frac{3}{20}$ c $\frac{32}{45}$ d $\frac{23}{75}$
7 a $\frac{11}{6}, \frac{7}{3}$ b $\frac{2}{5}, \frac{2}{15}$ c $\frac{11}{12}, \frac{12}{12}$ d $\frac{5}{7}, \frac{11}{14}$
8 Weather forecast
9 a $\frac{3}{5}$ b $\frac{5}{9}$ c $\frac{8}{13}$ d $\frac{23}{31}$
10 a 31, 32 b 36, 37, 38, …, 55
c 4, 5 d 2, 3
e 43, 44, 45, …, 55 f 4, 5, 6
11 $\frac{ac+b}{c}$
12 a yes, e.g. $\frac{7}{14} = \frac{1}{2}$ and 7 is prime
b no as a and b will have no common factors other than one
c no as then a factor of 2 can be used to cancel
d yes, e.g. $\frac{5}{7}$
13 a $\frac{8}{9}$ b $\frac{2}{9}$ c $\frac{9}{11}$ d $3\frac{43}{99}$
e $9\frac{25}{33}$ f $\frac{44}{333}$ g $2\frac{917}{999}$ h $13\frac{8125}{9999}$

1D

Building understanding

1 a 6 b 63 c 4 d 60
2 a $\frac{7}{3}$ b $\frac{39}{5}$ c $\frac{41}{4}$
3 a $\frac{9}{6} + \frac{8}{6} = \frac{17}{6}$ b $\frac{4}{3} - \frac{2}{5} = \frac{20}{15} - \frac{6}{15} = \frac{14}{15}$
c $\frac{5}{3} \times \frac{7}{2} = \frac{35}{6}$
4 a $\frac{1}{5}$ b 4 c $\frac{3}{2}$ d $\frac{4}{5}$

Now you try

Example 8
a $\frac{11}{15}$ b $\frac{41}{8}$ or $5\frac{1}{8}$ c $\frac{37}{20}$ or $1\frac{17}{20}$

Example 9
a $\frac{12}{35}$ b $\frac{23}{3}$ or $7\frac{2}{3}$

Example 10
a $\frac{5}{6}$ b $\frac{46}{33}$ or $1\frac{13}{33}$

Exercise 1D

1 a $\frac{3}{5}$ b $\frac{4}{9}$ c $1\frac{2}{7}$ d $\frac{19}{20}$
e $\frac{19}{21}$ f $1\frac{7}{40}$ g $\frac{7}{10}$ h $\frac{17}{27}$
2 a $\frac{2}{5}$ b $\frac{1}{45}$ c $\frac{11}{20}$ d $\frac{1}{10}$
e $\frac{1}{18}$ f $\frac{1}{8}$ g $\frac{13}{72}$ h $\frac{5}{48}$

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3 a 5 b $3\frac{2}{5}$ c $5\frac{1}{7}$
d $6\frac{11}{15}$ e $7\frac{17}{63}$ f $17\frac{13}{16}$

4 a $1\frac{1}{2}$ b $\frac{3}{4}$ c $\frac{13}{20}$
d $\frac{29}{40}$ e $\frac{5}{6}$ f $1\frac{16}{77}$

5 a $\frac{6}{35}$ b $\frac{1}{2}$ c $\frac{1}{12}$ d $\frac{4}{9}$
e $4\frac{1}{2}$ f $5\frac{1}{3}$ g $7\frac{1}{2}$ h 5
i 15 j 26 k $\frac{2}{3}$ l $\frac{5}{6}$
m $2\frac{1}{4}$ n $3\frac{1}{2}$ o 6 p $1\frac{1}{3}$

6 a $\frac{20}{21}$ b $1\frac{1}{8}$ c $\frac{45}{56}$ d $\frac{27}{28}$
e $1\frac{1}{3}$ f $1\frac{1}{2}$ g 6 h $\frac{7}{8}$
i 18 j 9 k 16 l 64
m $\frac{1}{10}$ n $\frac{1}{12}$ o $\frac{4}{27}$ p $3\frac{1}{3}$
q 4 r $\frac{1}{6}$ s $1\frac{1}{2}$ t $\frac{7}{8}$

7 $\frac{7}{8}$ tonnes

8 $5\frac{29}{56}$ tonnes

9 $\frac{5}{12}$ hours (25 min)

10 a $\frac{2}{7}$ b $\frac{8}{15}$ c 16
d $\frac{66}{85}$ e $2\frac{10}{21}$ f $\frac{3}{13}$

11 7 truckloads

12 3 hours

13 $\frac{5}{6}$, problem is the use of negatives in the method since $\frac{1}{3} < \frac{1}{2}$

14 a $\frac{b}{a}$ b $\frac{c}{ac + b}$

15 a 1 b $\frac{a^2}{b^2}$ c 1
d $\frac{c}{a}$ e a f $\frac{c}{b}$

16 a $-\frac{2}{3}$ b $\frac{5}{4}$ c $\frac{83}{10}$ d 1
e $\frac{50}{31}$ f $\frac{81}{400}$ g $\frac{329}{144}$ h $\frac{969}{100}$
i $\frac{583}{144}$ j $\frac{5}{11}$

1E

Building understanding

1 a 4 b 12 c 11 d 3

2 a 9 b $\frac{4}{9}$ c $\frac{5}{9}$ d 8 e 10

3 a i 240 km ii 40 km iii 520 km
b i 5 h ii $4\frac{1}{2}$ h iii 15 min

4 a \$4 b \$3 c \$2.50

Now you try

Example 11
a 3 : 2 b 16 : 9 c 5 : 3

Example 12
\$200

Example 13
a 35 km/h b 400 revs/min

Example 14
a 7 kg bag b juice B

Exercise 1E

1 a 1 : 5 b 2 : 5 c 3 : 2 d 4 : 3
e 9 : 20 f 45 : 28 g 3 : 14 h 22 : 39
i 1 : 3 j 1 : 5 k 20 : 7 l 10 : 3

2 a 1 : 10 b 1 : 5 c 2 : 3
d 7 : 8 e 25 : 4 f 1 : 4
g 1 : 4 h 24 : 5 i 4 : 20 : 5
j 4 : 3 : 10 k 5 : 72 l 3 : 10 : 40

3 a \$200, \$300 b \$150, \$350
c \$250, \$250 d \$175, \$325

4 a \$10, \$20, \$40 b \$14, \$49, \$7 c \$40, \$25, \$5

5 a 15 km/h b 2000 rev/min
c 45 strokes/min d 14 m/s
e 8 mL/h f 92 beats/min

6 a 55 km b $16\frac{1}{2}$ km c $5\frac{1}{2}$ km

7 a 3 kg deal b Red Delicious
c 2.4 L d 0.7 GB

8 a coffee A: \$3.60, coffee B: \$3.90
Therefore, coffee A is the best buy.
b pasta A: \$1.25, pasta B: \$0.94
Therefore, pasta B is the best buy.
c cereal A: \$0.37, cereal B: \$0.40
Therefore, cereal A is the best buy.

9 120

10 \$3000, \$1200, \$1800 respectively

11 \$15.90

12 108 L

13 \$1.62

14 1 : 4

15 36°, 72°, 108°, 144°

16 Find cost per kilogram or number of grams per dollar.
Cereal A is the best buy.

17 a false b false c true d true

18 a $a + b$ b $\frac{a}{a + b}$ c $\frac{b}{a + b}$

19 a i 100 mL ii 200 mL
b i 250 mL ii 270 mL
c i 300 mL ii 1 : 4
d i 1 : 3 ii 7 : 19
iii 26 : 97 iv 21 : 52
e jugs 3 and 4

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1F

Building understanding

1 a $\frac{3}{100}$ b $\frac{11}{100}$ c $\frac{7}{20}$ d $\frac{2}{25}$
2 a 0.04 b 0.23 c 0.86 d 0.463
3 a 50% b 25% c 75% d 20%

Now you try

Example 15
a 75% b 0.6 c $\frac{11}{400}$

Example 16
12.5%

Example 17
$8.40

Example 18
$300

Exercise 1F

1 a 34% b 40% c 6% d 70%
e 100% f 132% g 109% h 310%
2 a 0.67 b 0.3 c 2.5 d 0.08
e 0.0475 f 0.10625 g 0.304 h 0.4425
3 a $\frac{67}{100}$ b $\frac{3}{10}$ c $2\frac{1}{2}$ d $\frac{2}{25}$
e $\frac{19}{400}$ f $\frac{17}{160}$ g $\frac{38}{125}$ h $\frac{177}{400}$
4

Percentage	Fraction	Decimal
10%	$\frac{1}{10}$	0.1
50%	$\frac{1}{2}$	0.5
5%	$\frac{1}{20}$	0.05
25%	$\frac{1}{4}$	0.25
20%	$\frac{1}{5}$	0.2
12.5%	$\frac{1}{8}$	0.125
1%	$\frac{1}{100}$	0.01
$11.\dot{1}$%	$\frac{1}{9}$	$0.\dot{1}$
$22.\dot{2}$%	$\frac{2}{9}$	$0.\dot{2}$
75%	$\frac{3}{4}$	0.75
15%	$\frac{3}{20}$	0.15
90%	$\frac{9}{10}$	0.9
37.5%	$\frac{3}{8}$	0.375
$33\frac{1}{3}$%	$\frac{1}{3}$	$0.\dot{3}$
$66\frac{2}{3}$%	$\frac{2}{3}$	$0.\dot{6}$
62.5%	$\frac{5}{8}$	0.625
$16.\dot{6}$%	$\frac{1}{6}$	$0.1\dot{6}$

5 a 25% b $33\frac{1}{3}$% c 16%
d 200% e 2800% f 25%
6 a $36 b $210 c 48 kg
d 30 km e 15 apples f 350 m
g 250 people h 200 cars i $49
7 a $120 b $700 c $300
d $7 e $0.20 f $400
8 a $540 b $600 c $508
d $1250 e $120 f $40
9 $16\frac{2}{3}$%
10 $6\frac{1}{4}$%
11 48 kg
12 15 students
13 9 students
14 $1150
15 a $P = 100$ b $P < 100$ c $P > 100$
16 a $x = 2y$ b $x = 5y$
c $x = \frac{3}{5}y$ (or $5x = 3y$) d $14x = 5y$
17 a 72 b $\frac{10}{11}$ c 280%
d $3\frac{1}{4}$ e 150%

1G

Building understanding

1 a 1.4 b 21% c 0.27 d 6%
2 a $30 b 25%
3 a 12 kg b 11.1%

Now you try

Example 19
$50

Example 20
$4.76

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Example 21

a 25% b 12%

Example 22

227 m^2

Exercise 1G

1 a $52.50 b 37.8 min (37 min 48 s)
c 375 mL d 1.84 m
e 27.44 kg f 36 watts
g $13 585 h $1322.40

2 a 19.2 cm b 24.5 cm
c 39.06 kg d 48.4 min (49 m in 24 s)
e $78.48 f 202.4 mL
g 18°C h $402.36

3 50% 4 44% 5 28% 6 4%

7 a 22.7% b 26.7% c 30.9% d 38.4%

8 $21.50 9 30 068 10 $14 895

11 193 474 ha

12 $10.91

13 $545.45

14 a $900
b $990
c As 10% of 1000 = 100 but 10% of 900 = 90

15 25% 16 100% 17 42.86%

18 a $635.58 b $3365.08
c $151.20 d $213.54

19 a 79.86 g b $97 240.50
c $336 199.68 d 7.10 cm

Progress quiz

1 a −54 b 8

2 a 3.46 b 45.9
c 0.0079 d 46 800 000

3 a 0.75 b 0.8 c 0.35 d $0.\dot{3}$

4 a $\frac{9}{10}$ b $\frac{17}{20}$ c $\frac{1}{8}$

5 a $1\frac{1}{3}$ b $\frac{4}{21}$ c $1\frac{1}{5}$ d $2\frac{1}{12}$

6 a 5 : 9 b 30 : 1 c 20 : 3 d 3 : 32

7 a $250, $150
b 1.8 kg, 4.2 kg
c $266\frac{2}{3}$ cm, $333\frac{1}{3}$ cm, 400 cm

8 a $70/h b 80 km/h

9 B

10 a 80% b 96% c 375% d 8%

11 2040 cm

12 $400

13 a $504 b 475 kg

14 40%

15 850 L

1H

Building understanding

1 $3 profit, $2.50 loss, $7.30, $2070

2 a 90% b 85%

Now you try

Example 23

a $144 b 45%

Example 24

$16.80

Example 25

$171.36

Example 26

$40 000

Exercise 1H

1 a $75 b 15%

2 a i $2 ii 20%
b i $5 ii 25%
c i $16.80 ii 14%
d i $2450 ii 175%

3 166.67% 4 40% 5 $37.50

6 $1001.25 7 42.3% 8 $148.75

9 $760.50 10 $613.33 11 $220

12 increased by 4%

13 25%

14 a $54.75 b 128%

15 No, both methods give the same price. Whichever order you use you'll still multiply the original amount by the same two values, and multiplication is commutative.

16 $2100

17 a i $54 187.50 ii $33 277.90
b 10 years

18 a $34 440 b $44 000 c $27 693.75
d $32 951.10 e $62 040 f $71 627.10

1I

Building understanding

1 a $3952 b $912 c $24

2 a $79.80 b $62.70 c $91.20 d $102.60

3 a $200 b $900

Now you try

Example 27

a $2394.23 b Gary c $67 080

Example 28

a $418 b $16.50

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Answers

1I

Example 29

a $1299.50   b $20

Exercise 1I

1 a $1826.92   b Georgia   c $37 606.40

2 a i $19.50   ii $27   iii $42.50
b i $30 201.60   ii $44 044   iii $20 134.40

3 a $165.60   b $239.20   c $368
d $552   e $515.20   f $809.60

4 a 7   b 18   c 33
d 25   e 37   f 40

5 $14.50 per hour
6 $12.20 per hour
7 $490
8 $4010
9 $0.06
10 $25
11 Cate, Adam, Ed, Diana, Bill
12 $839.05
13 $1239.75
14 4.58%
15 $437.50
16 a 12 hours   b 9 and 2, 6 and 4, 3 and 6
17 a i $920   ii $1500
b i $A = 0.02x$
ii $A = 1200 + 0.025(x - 60\,000)$ or $0.025x - 300$
18 a $5500
b Choose plan A if you expect that you will sell less than $5500 worth of jewellery in a week and plan B if you expect to sell more than $5500.
19 a i Superannuation is a system in which part of your salary is set aside for your retirement.
ii It is compulsory in the hope that when workers retire they will have enough funds for a comfortable retirement and will not require the age pension, which is funded by taxpayers.
iii 9.5%
iv It is invested by the people who manage superannuation funds, in the hope that the funds will grow rapidly over time.
b No
c $8312
d 27.02% (to 2 decimal places)
e 27

1J

Building understanding

1 Taxable income = **gross** income minus **tax deductions**.
2 False
3 Anything from $0 to $18 200
4 30 cents

Now you try

Example 30

a $128 200   b $30 052
c $2564   d $32 616
e 25.4% (to 1 d.p.)
f. Bella receives a tax refund of $2384.

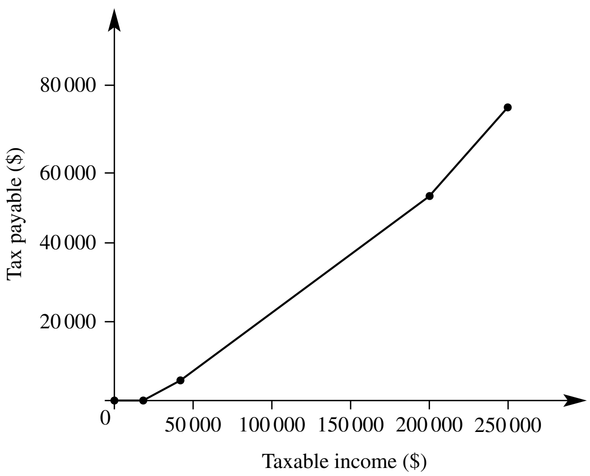
Exercise 1J

1 a. $91 600   b. $19 072   c. $1832
d. $20 904   e. 22.8%
f. He needs to pay another $904.

2 a $2242   b $9592
c $36 592   d $51 592

3

Taxable income	$0	$18 200	$45 000	$200 000	$250 000
Tax payable	$0	$0	$5092	$51 592	$74 092



4 $5092
5 a $65 625
b $11 279.50
c $13 12.50
d $12 592
e 19.2% (to 1 decimal place)
f Refund due of $478
6 $87 500
7 $6172.84
8 $103 766.67
9 a i $2000   ii $11 500   iii $35 000
b

Income	Rate	Tax payable
$40 001 – $90 000	25%	$3750 + 25% of (income – $40 000)
$90 001 –	33%	$16 250 + 33% of (income – $90 000)

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c i $2000 ii $6300
iii $24 000 iv $40 000.50
d An extra dollar of income can push you into a higher tax bracket where you don't just pay the higher tax rate on the dollar but on your entire income. No incentive to earn more.

10 Gross income is the total income earned before tax is deducted. Taxable income is found by subtracting tax deductions from gross income.

11 If a person pays too much tax during the year, they will receive a tax refund. If they do not pay enough tax during the year, they will have a tax liability to pay.

12 They only pay 45 cents for every dollar over $200 000.

13 a Answers will vary.
b i $15 592
ii $30, so this means the $100 donation really only cost you $70.

1K

Building understanding

1 a i 3 ii 1.5 iii 5.5
b i 48 ii 30 iii 87
2 a $50 b $60 c $240
3 a $12 000 b 6% p.a. c 3.5 years
d $720 e $1440 f $2520

Now you try

Example 31
$420

Example 32
$4550, $24 550

Example 33
9 months

Exercise 1K

1 a $3600 b $416
2 $600
3 $2700, $17 700
4 $1980, $23 980
5 $2560
6 9 months
7 16 months
8 $2083.33
9 Choice 2
10 a $14 400 b $240
11 10%
12 a $$P$ b 12.5%
c i 20 years ii 40 years iii double
13 a $51 000 b 4 years
14 a $P = \frac{I}{rn}$ b $n = \frac{I}{Pr}$ c $R = \frac{I}{Pn}$
15 a $1750 a month b $18 000
c $6000 d 2%

1L

Building understanding

1 a $200 b $2200 c $220
d $2420 e $242 f $2662
2 a 2731.82 b 930.44
c 2731.82 d 930.44
3 a $\$4000 \times (1.2)^3$
b $\$15000 \times (1.07)^6$
c $\$825 \times (1.11)^4$

Now you try

Example 34
$6749.18

Example 35
$25 207

Example 36
$1654.10

Exercise 1L

1 $4282.32
2 a $6515.58 b $10 314.68
c $34 190.78 d $5610.21
3 $293 865.62
4 a 21.7% b 19.1% c 136.7% d 33.5%
5 $33 776
6 a $23 558 b $33 268
c $28 879 d $25 725
7 a $480 b $22 185
c $9844
8 $543 651
9 6142 people
10 6.54 kg
11 trial and error gives 12 years
12 trial and error gives 5 years
13 a 35%
b 40.26%
c It calculates each years' interest on the original $400 not the accumulated total that compound interest uses.
14 a 15.76% b 25.44%
c 24.02% d 86.96%
e $((1 + r)^t - 1) \times 100\%$
15 a $7509.25 b 9.39% p.a.
16 a 70.81% b 7.08%
17 a 5.39% p.a.
b 19.28% p.a.

Answers

1J

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Answers

1M

1M

Building understanding

1 C

2

Year	Value at start of year	10% depreciation	Value at end of year
1	\$20 000	\$2000	\$18 000
2	\$18 000	\$1800	\$16 200
3	\$16 200	\$1620	\$14 580
4	\$14 580	\$1458	\$13 122

3 C

Now you try

Example 37
a \$6215.95 b \$2215.95

Example 38
\$28 892.30

Example 39
\$29 524.50

Exercise 1M

1 a \$8508.54 b \$1508.54
2 a \$5909.82 b \$2341.81
c \$31 737.49 d \$10770.33
e \$1270.29 f \$399.07
g \$87 045.75 h \$2 704 813.83
3 a \$6448.90 b \$5635.80
c \$19 629.68 d \$1119.92
e \$100 231.95 f \$35 905.58
4 a \$116 622 b \$18 946
c \$9884 d \$27 022
e \$18 117 f \$18 301
5 a \$48 b \$49.44 c \$50.86
6 a \$555.13
b 5.7 years
c 7.4% p.a.
7 a \$3.83 b \$4.10 c \$2.00
d \$133.05 e \$8.86
8 \$20 887.38
9 a i \$1236
ii \$1255.09
iii \$1264.93
iv \$1271.60
v \$1274.86
b The more compounding periods, the more interest is earned over time.
10 9.01% p.a.
11 Answers can vary – some examples of possible scenarios are shown here.
a \$300 at 7% p.a. for 12 years compounded annually or \$300 at 84% p.a. compounded monthly for 1 year
b \$300 at 2% p.a. compounded annually for 36 years or \$300 at 24% p.a. compounded monthly for 3 years or \$300 at 8% p.a. compounded quarterly for 9 years
c \$1000 at 0.5% p.a. compounded annually for 48 years or \$1000 at 6% p.a. compounded monthly for 4 years
d \$1000 depreciating at 5% p.a. for 5 years
e \$9000 depreciating at 20% p.a. for 3 years
12 a \$2155.06
b \$4310.13, yes
c \$2100. no
d Doubling the principal doubles the interest.
e Doubling the interest rate and halving the time period does not produce the same amount of interest on the investment.
13 a i \$60 ii \$60.90 iii \$61.36
iv \$61.68 v \$61.76 vi \$61.83
b As the interest is being added back onto the principal at a faster rate, then the more compounding periods used, the more interest is earned over time.
14 a

n	1	2	3	4	5
$(1.09)^n$	1.09	1.188	1.295	1.412	1.539
	6	7	8	9	
	1.677	1.828	1.993	2.17	

b i Approx. 9 years
ii Approx. 6 years
iii Approx. 4 years
c 26% p.a. compounded annually
d 72 (it is known as the rule of 72)
e i 36% p.a ii 16% p.a. iii 7.2% p.a.
f years $\times$ rate $=$ 144
g The rule of 72 still applies.

Problems and challenges

1 Discuss the answer with classmates as more than one answer for each may be possible. Some suggestions are given below (be creative).
$(4-4)\times(4+4)=0$ $(4-4)+(4\div 4)=1$
$4\div 4+4\div 4=2$ $\sqrt{4\times 4}-(4\div 4)=3$
$4+(4-4)\times 4=4$ $\sqrt{4\times 4}+4\div 4=5$
$(4+4+4)\div\sqrt{4}=6$ $4+4-4\div 4=7$
$\sqrt{4}+\sqrt{4}+\sqrt{4}+\sqrt{4}=8$ $4+4+4\div 4=9$
$\sqrt{4}\times\sqrt{4}\times\sqrt{4}+\sqrt{4}=10$
2 12
3 a $\frac{4}{7}$ b $\frac{14}{17}$
4 125 mL
5 40.95% reduction

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6 7.91% p.a.
7 a 44% b 10%
8 200 000 cm^2 (20 m^2)
9 9, 7, 2, 14, 11, 5, 4, 12, 13, 3, 6, 10, 15, 1, 8

Success criteria example questions

1 −23
2 1
3 3.46
4 25 000; 0.042
5 4000
6 $0.58\dot{3}$
7 $\frac{47}{20} = 2\frac{7}{20}$
8 $\frac{11}{15} > \frac{18}{25}$
9 $\frac{19}{24}$
10 $\frac{13}{20}$
11 $\frac{16}{3} = 5\frac{1}{3}$
12 $\frac{21}{20} = 1\frac{1}{20}$
13 8:5; 5:2
14 250 cm
15 $35/hour
16 2 kg of rice for $3.60
17 0.245; $\frac{2}{25}$
18 85%
19 $21.60
20 $600
21 $32.40; $34
22 75% increase
23 $1.38
24 75%
25 $1092
26 $80
27 $40.84
28 $899.10
29 $850
30 $24 592
31 $240
32 $2\frac{1}{3}$ years = 2 years 4 months
33 $14 599.83
34 $91 776
35 $31 663.92
36 $9549.11
37 $8939.07
38 $2509.27

Short-answer questions

1 a −16 b 2 c 0
d 10 e −23 f 1
2 a 21.5 b 29 100 c 0.153 d 0.00241
3 a 200 b 60 c 2
4 a 2.125 b $0.8\dot{3}$ c $1.\overline{857142}$
5 a $\frac{3}{4}$ b $1\frac{3}{5}$ c $2\frac{11}{20}$
6 a $\frac{1}{2}$ b $2\frac{1}{6}$ c $\frac{7}{24}$
d 2 e $3\frac{3}{4}$ f $2\frac{19}{28}$
7 a 5 : 2 b 16 : 9 c 75 : 14
8 a 50, 30 b 25, 55 c 10, 20, 50
9 a store A: $2.25/kg; store B: $2.58/kg ∴ A is best buy
b store A: 444 g/$; store B:388 g/$
10

Decimal	Fraction	Percentage
0.6	$\frac{3}{5}$	60%
$0.\dot{3}$	$\frac{1}{3}$	$33\frac{1}{3}\%$
0.0325	$\frac{13}{400}$	$3\frac{1}{4}\%$
0.75	$\frac{3}{4}$	75%
1.2	$1\frac{1}{5}$	120%
2	2	200%

11 a $77.50 b 1.65
12 a 150 b 25
13 a 72 b 1.17 c 20%
14 12.5 kg
15 $1800
16 a $25 b $16\frac{2}{3}\%$
17 a $18.25 b $14.30
18 $27 592
19 $525
20 $4\frac{1}{2}$ years
21 $63 265.95
22 $39 160
23 $1857.62
24 $31 407.49

Multiple-choice questions

1 D 2 B 3 C 4 A 5 E 6 E
7 D 8 E 9 C 10 A 11 C 12 B

Extended-response questions

1 a $231 b $651
c i $63 ii $34.65
2 a i $26 625 ii $46 928.44
b 87.71% c $82 420
d 26.26%
e 7.4% p.a.

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Chapter 2

2A

Building understanding

1 a 2 b 2 c 3 d 1
2 a C b D c B
d A e F f E
3 a 5 b −2 c $\frac{1}{3}$ d $-\frac{2}{5}$

Now you try

Example 1
a $a+7$ b $1.2n$ c $\frac{60}{p}$

Example 2
a $x+6$ b $3x-1$ c $\frac{a-2}{3}$ d $\sqrt{a+b}$

Example 3
a 8 b 13

Exercise 2A

1 a i $4+r$ ii $t+2$
iii $b+g$ iv $x+y+z$
b i $6P$ ii $10n$
iii $2D$ iv $5P+2D$
c $\frac{500}{C}$
2 a $2+x$ b $ab+y$
c $x-5$ d $3x$
e $3x-2y$ or $2y-3x$ f $3p$
g $2x+4$ h $\frac{x+y}{5}$
i $4x-10$ j $(m+n)^2$
k m^2+n^2 l $\sqrt{x+y}$
m $a+\frac{1}{a}$ n $(\sqrt{x})^3$
3 a −31 b −25 c −33 d −19
e $\frac{1}{2}$ f 4 g 1 h 85
4 a $1\frac{1}{6}$ b $4\frac{1}{4}$ c $\frac{1}{6}$ d $-1\frac{1}{3}$
5 a 60 m^2
b length $= 12+x$, width $= 5-y$
c $A=(12+x)(5-y)$
6 a 18 square units b 1, 2, 3, 4, 5
7 a $\frac{P}{10}$ b $\frac{nP}{10}$
8 a i $P=2x+2y$ ii $A=xy$
b i $P=4p$ ii $A=p^2$
c i $P=x+y+5$ ii $A=\frac{5x}{2}$
9 a A:$2(x+y)$; B:$2x+y$, different
b same
10 a B b A:$c^2=(a+b)^2$
11 a $\frac{n(n+1)}{2}$
b i 10 ii 55
c $\frac{n^2}{2}+\frac{n}{2}$ d 10, 55
e Half the sum of n and the square of n.

2B

Building understanding

1 a variable (pronumeral) b $5x$
c unlike
2 a $\frac{5}{2}$ b 3 c $\frac{1}{5}$ d $-\frac{1}{20}$
3 a like b unlike c unlike
d unlike e like f unlike

Now you try

Example 4
a $35n$ b $-18x^2y$

Example 5
a $\frac{y}{2}$ b $-2b$

Example 6
a $5a+2$ b $6m+7n$ c $3a^2b-2ab$

Exercise 2B

1 a $18r$ b $16b$ c $-14x$ d $-15p$
e $-12cd$ f $-15mn$ g $-24rs$ h $-40jk$
2 a $21a^2b$ b $-15mn^2$ c $18gh^2$
d $12x^2y^2$ e $8a^2b^2$ f $-6m^2n^2$
3 a $4b$ b $-\frac{a}{3}$ c $\frac{2ab}{3}$
d $\frac{m}{2}$ e $-\frac{x}{4}$ f $\frac{5s}{3}$
g uv h $\frac{5rs}{8}$ i $\frac{5ab}{9}$
4 a $\frac{2x}{5}$ b $\frac{4}{3a}$ c $\frac{11mn}{3}$ d $6ab$
e $-\frac{5}{gh}$ f 8 g −3 h $\frac{7n}{3}$
i $-\frac{9q}{2}$ j $3b$ k $-5x$ l $\frac{m}{2}$
5 a $\frac{4x}{y}$ b $\frac{5p}{2}$ c $-6ab$ d $-\frac{3a}{2b}$
e $-\frac{7n}{5m}$ f $\frac{10s}{t}$ g $8n$ h $3y$
i $4b$ j $6xy$ k $5m$ l $3pq^2$
6 a $10a$ b $7n$ c $8y$
d $11x$ e $3ab$ f $7mn$
g $y+8$ h $3x+5$ i $7xy+4y$
j $12ab+3$ k $2-6m$ l $4-x$
7 a $5a+9b$ b $6x+5y$
c $4t+6$ d $11x+4$
e $5xy+4x$ f $7mn-9$
g $5ab-a$ h 0
8 a xy^2 b $7a^2b$
c $3m^2n$ d p^2q^2
e $7x^2y-4xy^2$ f $13rs^2-6r^2s$
g $-7x-2x^2$ h $4a^2b-3ab^2$
i $7pq^2-8pq$ j $8m^2n^2-mn^2$
9 a $x+y$ b $4x+2y$
10 a $8x$ b $3x^2$

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11 a $30x$ cm b $30x^2$ cm^2

12 $\frac{20x + 75}{21}$

13 a true b false c true
d false e false f true

14 a $(4x + 4y)$ km b $(8x + 4)$ km c $(2x - 2)$ km

15 a a^3 b $\frac{b^2}{3}$ c $\frac{b^2}{3}$ d $\frac{3ab}{8}$
e $-\frac{2a^3}{3}$ f $-\frac{2}{5a^2}$ g $\frac{2}{5a^5}$ h $\frac{a^2}{4b^3}$
i $3a^3b$ j $\frac{4b^3}{a}$ k $\frac{a^3}{3b}$ l $-\frac{a}{2b^3}$

2C

Building understanding

1 a i $5x$ ii 10
b $5x + 10$ c $x + 2$ d $5(x + 2)$ e $5x + 10$

2 a $3 \times x + 3 \times 5$
$= 3x + 15$
b $4 \times y + 4 \times (-3)$
$= 4y - 12$
c $-2 \times x + (-2) \times 3$
$= -2x - 6$
d $-3 \times x + (-3) \times (-1)$
$= -3x + 1$

Now you try

Example 7
a $5x + 10$ b $3x - 21$ c $-3x + 6$

Example 8
a $3x + 21y$ b $-6a^2 + 12a$

Example 9
a $2y + 2$ b $x - 9y$

Exercise 2C

1 a $2x + 6$ b $5x + 60$ c $2x + 14$ d $7x - 63$
e $3x - 6$ f $2x - 12$ g $28 - 4x$ h $21 - 7x$

2 a $-3x - 6$ b $-2x - 22$ c $-5x + 15$
d $-6x + 36$ e $-8 + 4x$ f $-65 - 13x$
g $-180 - 20x$ h $-300 + 300x$

3 a $2a + 4b$ b $15a - 10$ c $12m - 15$
d $-16x - 40$ e $-12x - 15$ f $-4x^2 + 8xy$
g $-18ty + 27t$ h $3a^2 + 4a$ i $2d^2 - 5d$
j $-6b^2 + 10b$ k $8x^2 + 2x$ l $5y - 15y^2$

4 a $2x + 11$ b $6x - 14$
c $15x - 3$ d $3x + 1$
e $4x - 5$ f $2x + 1$
g $-3x - 4$ h $-5x - 19$
i $11 - x$ j $12 - x$
k $2 - 2x$ l $6 - 3x$

5 a $5x + 12$ b $4x - 8$
c $11x - 2$ d $17x - 7$
e $-4x - 6$ f $x - 7$
g $2x - 4$ h $-27x - 4$
i $-4x - 18$ j $-13x - 7$
k $11x - 7$ l $2x - 15$

6 $A = x^2 + 4x$

7 a $2x + 2$ b $2x^2 - 3x$ c $6x^2 - 2x$
d $2x^2 + 4x$ e $x^2 + 2x + 6$ f $20 + 8x - x^2$

8 $20n - 200$

9 $0.2x - 2000$

10 a $2x + 12$ b $x^2 - 4x$ c $-3x - 12$
d $-7x + 49$ e $19 - 2x$ f $x - 14$

11 a $6(50 + 2) = 312$ b $9(100 + 2) = 918$
c $5(90 + 1) = 455$ d $4(300 + 26) = 1304$
e $3(100 - 1) = 297$ f $7(400 - 5) = 2765$
g $9(1000 - 10) = 8910$ h $6(900 - 21) = 5274$

12 a $a = \$6000$, $b = \$21\,000$
b i \$3000 ii \$12 600 iii \$51 000
c i 0
ii $0.2x - 4000$
iii $0.3x - 9000$
iv $0.5x - 29\,000$
d Answers may vary.
e Answers may vary.

2D

Building understanding

1 a 3 b 40 c 5 d 6

2 a 3 b 2 c 12 d 6

3 B, C

Now you try

Example 10
a $x = 3$ b $x = -1$

Example 11
a $x = 22$ b $x = \frac{6}{5}$ c $x = 17$

Exercise 2D

1 a 2 b 1 c 4 d -4
e $-3\frac{2}{3}$ f $1\frac{2}{3}$ g $4\frac{1}{2}$ h $\frac{1}{2}$
i $1\frac{1}{3}$ j $-\frac{5}{7}$ k $-\frac{1}{8}$ l $-\frac{3}{20}$

2 a -3 b -1 c 2 d 8
e $-1\frac{2}{5}$ f $-2\frac{5}{7}$ g $\frac{3}{8}$ h $1\frac{3}{4}$

3 a -6 b -6 c 30
d -15 e -8 f 12
g 20 h -5

4 a 9 b 6 c $-6\frac{3}{4}$
d $-7\frac{1}{2}$ e $\frac{2}{3}$ f $-\frac{4}{25}$
g 12 h 12 i -9
j -8 k 6 l $-1\frac{1}{2}$

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5 a 11 b 6 c −10 d −12
e −5 f −1 g 7 h 5
i $4\frac{2}{7}$ j $5\frac{1}{3}$ k −7 l 3

6 a 26 b 28 c 3
d 28 e 8 f $3\frac{3}{7}$

7 7

8 $900

9 a should have +1 before ÷2
b should have × 3 before −2
c need to ÷ −1 as $-x = 7$
d should have +4 before ×3

10 a i 5 ii −3 iii $\frac{1}{5}$
iv 3 v $-\frac{5}{6}$ vi $\frac{3}{10}$
b when the common factor divides evenly into the RHS

11 a $a = b + c$ b $a = \frac{c - b}{2}$
c $a = \frac{c - 2d}{b}$ d $a = c(b + d)$
e $a = -\frac{cd}{b}$ f $a = \frac{b}{2c}$
g $a = \frac{c(3 - d)}{2b}$ h $a = \frac{2d(b - 3)}{c}$
i $a = cd - b$ j $a = b + cd$
k $a = \frac{2be + 6c}{d}$ l $a = \frac{d - 3ef}{4c}$

2E

Building understanding

1 a $4x - 12$ b 2 c $5x - 9$

2 a $2x + 6 = 5$ b $5 + 2x - 2 = 7$
c $2x + 1 = -6$ d $2x - 3 = 1$

Now you try

Example 12
a $x = \frac{7}{6}$ b $x = 3$

Example 13
a $x = -3$ b $x = 14$

Exercise 2E

1 a $6\frac{1}{3}$ b $9\frac{2}{5}$ c $3\frac{3}{4}$ d $9\frac{1}{2}$
e $\frac{1}{2}$ f $-5\frac{1}{2}$ g $-\frac{4}{5}$ h $\frac{1}{14}$
i $3\frac{1}{6}$ j $\frac{2}{3}$ k $\frac{9}{10}$ l $1\frac{1}{6}$

2 a 2 b −10 c 1 d 5
e 3 f 3 g 1 h −2

3 a 1 b 4 c 2
d 8 e 8 f 3
g 4 h −1 i $\frac{5}{11}$

4 a −1 b $3\frac{1}{2}$ c −9 d −14
e −16 f 3 g 19 h −3
i −13 j −26 k $-2\frac{2}{3}$ l $-1\frac{1}{10}$

5 a $\frac{1}{2}$ b $2\frac{2}{3}$ c $1\frac{1}{2}$ d $-\frac{5}{6}$
e $\frac{2}{5}$ f $1\frac{1}{2}$ g −6

6 Let $$w$ be the wage, $2(3w - 6) = 18$, $5

7 11 marbles

8 a $x = 5$ b $x = 5$
c Dividing both sides by 3 is faster because 9 ÷ 3 is an integer.

9 a $x = 4\frac{1}{3}$ b $x = 4\frac{1}{3}$
c Expanding the brackets is faster because 7 ÷ 3 gives a fraction answer.

10 a $x = 4$ b $x = 4$
c Method a is preferable as you don't have to deal with negatives.
Final step in method a: divide both sides by a positive number.
Final step in method b: divide both sides by a negative number.

11 a $x = \frac{d}{a - b}$
b $x = \frac{2}{a - b}$
c $x = \frac{c}{5a - b}$
d $x = -\frac{6}{3a - 4b}$ or $\frac{6}{4b - 3a}$
e $x = -c$
f $x = b$
g $x = \frac{d + bd + c}{a - b}$
h $x = -\frac{ab + bc}{a - b + 1}$ or $\frac{ab + bc}{b - a - 1}$

2F

Building understanding

1 a $2x - 1 = 11$ b $2x + 100 = 2200$
c $m + (m + 12) = 60$ d $80n + 100 = 480$

Now you try

Example 14
5

Example 15
$187 and $239

Exercise 2F

1 a $2x = x + 5$ b $x - 8 = 3x + 2$
c $x - 3 = 4x - 9$ d $3(x + 7) = 9$

2 a Let e be the number of goals for Emma.
b $e + 8$
c $e + e + 8 = 28$
d $e = 10$
e Emma scored 10 goals, Leonie scored 18 goals.

3 a Let w be the width in centimetres.
b length = $4w$
c $2w + 2(4w) = 560$
d $w = 56$
e length = 224 cm, width = 56 cm

4 $40x + 50 = 290$, 6 days.

5 $x + 2x + 2 = 32$, 10 km, 20 km

6 $x + x + 280 = 1000$, $360, $640.

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7 15, 45
8 Andrew \$102.50, Brenda \$175, Cammi \$122.50
9 19 km
10 I am 10 years old.
11 first leg = 54 km, second leg = 27 km, third leg = 18 km, fourth leg = 54 km
12 Eric is 18 years old now.
13 4 fiction, 8 non-fiction books
14 2 hours
15 8 p.m.
16 rectangle $L = 55$ m, $W = 50$ m; triangle side = 70 m
17 a 27, 28, 29
b i $x, x+2, x+4$ ii 4, 6, 8
c i $x, x+2, x+4$ ii 15, 17, 19
d i $x, x+3, x+6$ ii 24, 27, 30
18 a $T = 8x + 7200$ b 300
c $R = 24x$ d $x = 350$
e 3825
19 \$15 200
20 438 km
21 Anna 6; Henry 4; Chloe 12; twins 11

Progress quiz

1 a $5 + x + y$ b $4y - 15$ c $k^2 + p^2$
d $(m-n)^2$ or $(n-m)^2$ e $\sqrt{x+16}$
2 a 12 b -28 c $\frac{1}{2}$ d -12
3 a $14ab$ b $-40p^2q$
c $\frac{4}{m}$ d $-6y$
4 a $2x + 7$ b $11 + km$
c $5xy - x$ d $a^2b + ab^2 + b$
5 a $4a + 28$ b $-6x^2 + 15x$
6 a $3a + 4$ b $5x - 2y$
7 a $x = 4$ b $a = 21$ c $p = -3\frac{1}{2}$
d $t = 20$ e $x = 11$ f $k = 7$
8 a $x = 6\frac{1}{3}$ b $a = 3$ c $m = -2$ d $y = 9\frac{1}{2}$
9 a $2(x+5) = 14$ b $x - 9 = 4x + 6$
10 Let g be Jed's team's goals: $g + g + 14 = 30$; 22 goals
11 Let a be my age, $4(a+5) = 88$, 17 years old

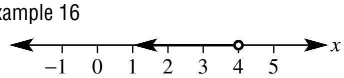
2G

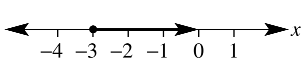
Building understanding

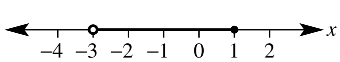
1 a $3 > 2$ b $-1 < 4$
c $-7 < -3$ d $5 > -50$
2 a C b A c B

Now you try

Example 16

a


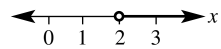
b


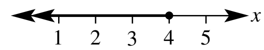
c


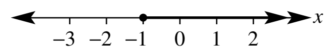
Example 17

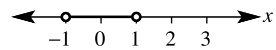
a $x < 11$ b $x < -1$ c $a \geqslant -9$ d $a \geqslant -\frac{7}{2}$

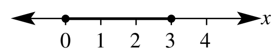
Exercise 2G

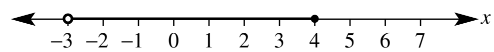
1 a


b


c


d


e


f


2 a $x < 3$ b $b > 5$ c $y > 6$
d $x \geqslant 3$ e $t > -5$ f $y \geqslant -10$
g $m < 6$ h $a \geqslant 1\frac{1}{2}$ i $x < 1$
3 a $x < 4$ b $n \leqslant -1$ c $x \geqslant \frac{3}{5}$
d $a \geqslant 4$ e $x \geqslant -6$ f $x \geqslant 10$
g $x < -6$ h $t \leqslant -2$ i $m > -4\frac{5}{6}$
4 a $x \leqslant 16$ b $x \leqslant -9$ c $x \leqslant 20$
d $x < 11$ e $x > -2\frac{2}{3}$ f $x \geqslant -2$
5 a $x < 1$ b $a < -8$ c $x \leqslant -2$
d $x < 2\frac{1}{2}$ e $y < -3\frac{1}{5}$ f $x < -\frac{4}{7}$
6 a $x \geqslant 2\frac{1}{2}$ b $t > -\frac{3}{5}$ c $y \leqslant \frac{3}{8}$
d $a < 1\frac{1}{5}$ e $m \geqslant 6$ f $b < 5\frac{1}{2}$
7 less than 18
8 less than 13 cm
9 a 3 b 2 c 4 d 0
10 $x = 3$ or $x = 4$ or $x = 5$
11 399 km
12 a i $-0.9, 0, 0.5, 1, 1.8$ etc
ii Numbers must be less than 2.
b i $-4, -2, 0, 1, 5$ etc
ii Numbers must be greater than -5.
c i $x < a$ ii $x > -a$
13 a $x < 3$ b $x < 3$
c Reverse the inequality sign when dividing by a negative number.
14 a $x < -13$ b $x \geqslant -3$ c $x > \frac{4}{7}$
d $x \leq \frac{13}{5}$ e $x > \frac{10}{17}$ f $x \geqslant \frac{3}{4}$
15 a $x > \frac{b-c}{a}$ b $x \geqslant b - a$ c $x \leqslant a(b+c)$
d $x \leqslant \frac{ac}{b}$ e $x < \frac{cd-b}{a}$ f $x \geqslant \frac{b-cd}{2}$
g $x < \frac{c}{a} - b$ h $x > \frac{b-cd}{a}$ i $x < b - \frac{c}{a}$
j $x \leqslant \frac{b+c}{1-a}$ k $x < \frac{b+1}{b-a}$ l $x \leqslant \frac{c-b}{b-a}$

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2H

Building understanding

1 a A b D c M d A
2 a subtract 3 b multiply by 4
 c square both sides d subtract c
3 a i subtract 5, divide by 3
 ii subtract b, divide by a
 b i divide by 3, take square root of both sides
 ii divide by a, take square root of both sides

Now you try

Example 18
a 10 b 180

Example 19
21

Example 20
a $a=\frac{b}{2}+c$ or $a=\frac{b+2c}{2}$ b $a=\sqrt{c+b}$

Exercise 2H

1 a 21 b 24 c 2 d 6
 e 452.39 f 249.86 g 25.06 h 14.95
2 a 36 b 5 c 20 d 4.14
 e 3.39 f 18.67 g 0.06
3 a $t=\frac{v-u}{a}$ b $x=\frac{2A}{y}$
 c $n=\frac{p}{m}-x$ d $x=\frac{cd-a}{b}$
 e $r=\frac{A}{2\pi h}$ f $r=\frac{100I}{Pt}$
 g $r=\sqrt{\frac{V}{\pi h}}$ h $v=\sqrt{PR}$
 i $h=\frac{S-2\pi r^2}{2\pi r}$ j $p=-q\pm\sqrt{A}$
 k $A=(4C-B)^2$ l $g=\frac{4\pi^2 l}{T^2}$
4 a 88.89 km/h
 b i $d=st$ ii 285 km
5 a i 212°F ii 100.4°F
 b $C=\frac{5}{9}(F-32)$
 c i −10°C ii 36.7°C
6 a 35 m/s b 2 s
7 a decrease b 3988 L
 c 6 hours 57 minutes d 11 hours 7 minutes
8 a $D=\frac{c}{100}$ b $d=100e$ c $D=0.7m$
 d $V=1.15P$ e $C=50+18t$ f $d=42-14t$
 g $C=\frac{c}{b}$
9 a $a=\frac{P}{4}$ b $a=180-b$ c $a=90-b$
 d $a=\frac{180-b}{2}$ e $a=\sqrt{c^2-b^2}$ f $a=\sqrt{\frac{4A}{\pi}}$
10 a 73 b 7 c 476.3

2I

Building understanding

1 a $y=5$ b $x=-3$ c $x=10$
2 a A b C
3 a no b yes

Now you try

Example 21
a $x=2, y=6$ b $x=-1, y=4$ c $x=2, y=-3$

Exercise 2I

1 a $x=1, y=2$ b $x=5, y=1$ c $x=\frac{1}{2}, y=\frac{3}{2}$
2 a $x=3, y=9$ b $x=-1, y=3$
 c $x=1, y=0$ d $x=2, y=1$
 e $x=1, y=3$ f $x=0, y=4$
 g $x=3, y=-2$ h $x=4, y=1$
 i $x=-1, y=4$
3 a $x=6, y=11$ b $x=4, y=3$
 c $x=12, y=-3$ d $x=-18, y=-4$
 e $x=-2, y=10$ f $x=2, y=-4$
4 17, 31
5 10 tonnes, 19 tonnes
6 width = $1\frac{2}{3}$ cm, length = $3\frac{5}{6}$ cm
7 $x-(3x-1)=x-3x+1$; to avoid sign error use brackets when substituting
8 a $x=\frac{1}{3}, y=2\frac{1}{3}$ b $x=-\frac{5}{6}, y=6\frac{1}{3}$
 c $x=-22, y=-7$
9 a $x=\frac{b}{a+b}, y=\frac{b^2}{a+b}$
 b $x=\frac{b}{a+1}, y=\frac{1}{a+1}$
 c $x=\frac{a-b}{2}, y=\frac{a+b}{2}$
 d $x=\frac{a-ab}{a-b}, y=\frac{a-ab}{a-b}-a$
 e $x=\frac{2a}{a-b}, y=\frac{2ab}{a-b}+a$
 f $x=\frac{2a+ab}{a-b}-b, y=\frac{2a+ab}{a-b}$

2J

Building understanding

1 a − b + c + d −
2 a i subtraction ii addition
 iii subtraction
 b i addition ii subtraction
 iii addition

Now you try

Example 22
a $x=1, y=2$ b $x=-1, y=4$

Example 23
a $x=2, y=-1$
b $x=3, y=-2$

Exercise 2J

1 a $x=1, y=1$ b $x=2, y=2$
 c $x=2, y=-1$

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2 a $x = 2, y = 4$ b $x = 8, y = -1$
c $x = -2, y = 6$ d $x = -3, y = -13$
e $x = 2, y = 3$ f $x = 1, y = 3$

3 a $x = -3, y = 4$ b $x = 2, y = 1$
c $x = 2, y = 1$ d $x = -2, y = 3$
e $x = 7, y = -5$ f $x = 5, y = 4$
g $x = 5, y = -5$ h $x = -3, y = -2$
i $x = -1, y = -3$

4 a $x = 3, y = -5$ b $x = 2, y = -3$
c $x = 2, y = 4$ d $x = 3, y = 1$
e $x = -1, y = 4$ f $x = 3, y = -2$
g $x = 5, y = -3$ h $x = -2, y = -2$
i $x = 2, y = 1$ j $x = -5, y = -3$
k $x = -3, y = 9$ l $x = -1, y = -2$

5 21 and 9

6 102°, 78°

7 $L = 261.5$ m, $W = 138.5$ m

8 11 mobile phones, 6 iPads

9 a $x = 2, y = 1$ b $x = 2, y = 1$
c Method b is preferable as it avoids the use of a negative coefficient.

10 Rearrange one equation to make x or y the subject.
a $x = 4, y = 1$ b $x = -1, y = -1$

11 a no solution b no solution

12 a $x = \frac{a+b}{2}, y = \frac{a-b}{2}$ b $x = \frac{b}{2a}, y = -\frac{b}{2}$
c $x = -\frac{a}{2}, y = -\frac{3a}{2b}$ d $x = 0, y = b$
e $x = \frac{1}{3}, y = \frac{b}{3a}$ f $x = -\frac{4}{a}, y = 6$
g $x = \frac{3b}{7a}, y = \frac{b}{7}$ h $x = \frac{3b}{7a}, y = -\frac{b}{7}$
i $x = 0, y = \frac{b}{a}$ j $x = -\frac{c}{a}, y = c$
k $x = \frac{b-4}{ab-4}, y = \frac{1-a}{ab-4}$ l $x = 1, y = 0$
m $x = \frac{c-bd}{a(b+1)}, y = \frac{c+d}{b+1}$ n $x = \frac{a+2b}{a-b}, y = \frac{3a}{a-b}$
o $x = \frac{3b}{a+3b}, y = \frac{3b-2a}{a+3b}$ p $x = -\frac{b}{a-bc}, y = \frac{bc-2a}{a-bc}$
q $x = \frac{c+f}{a+d}, y = \frac{cd-af}{b(a+d)}$ r $x = \frac{c-f}{a-d}, y = \frac{af-cd}{b(a-d)}$

2K

Building understanding

1 a $x + y = 42, x - y = 6$
b elimination

2 a $l = 3w, 2l + 2w = 120$
b substitution

Now you try

Example 24
The dimensions of the rectangle are 16 cm by 10 cm.

Example 25
hat \$8.40, scarf \$11.20

Exercise 2K

1 a Let l m be the length
Let w m be the width
b $l = w + 10, 2l + 2w = 80$
c $w = 15, l = 25$
d The length of the pool is 25 m and the width is 15 m.

2 potatoes 480 ha; corn 340 ha

3 a Let \$$m$ be the cost of milk.
Let \$$c$ be the cost of chips.
b $3m + 4c = 17, m + 5c = 13$
c $m = 3, c = 2$
d A bottle of milk costs \$3 and a bag of chips \$2.

4 a Let \$$g$ be the cost of lip gloss.
Let \$$e$ be the cost of eye shadow.
b $7g + 2e = 69, 4g + 3e = 45$
c $g = 9, e = 3$
d A lip gloss costs \$9 and eye shadow \$3

5 cricket ball \$12; tennis ball \$5

6 4 buckets of chips; 16 hot dogs

7 300 adults, 120 children

8 11 five-cent and 16 twenty-cent coins

9 Michael is 35 years old now.

10 Jenny \$100, Kristy \$50

11 160 adult tickets and 80 child tickets

12 5 hours

13 jogging 3 km/h; cycling 9 km/h

14 a Malcolm is 14 years old.
b The second digit of Malcolm's age is 3 more than the first digit.
c 6:14, 25, 36, 47, 58 or 69

15 original number is 37

16 any two-digit number that has the first digit 2 more than the second (e.g. 42 or 64 etc.)

2L

Building understanding

1 a 4 b 0 c None

2 a
$$\begin{aligned} 2x^2 - 16 &= 0 \\ 2x^2 &= 16 \\ x^2 &= 8 \\ x &= \pm\sqrt{8} \end{aligned}$$
b
$$\begin{aligned} 2x^2 + 16 &= 16 \\ 2x^2 &= 0 \\ x^2 &= 0 \\ x &= 0 \end{aligned}$$
c
$$\begin{aligned} 2x^2 + 16 &= 0 \\ 2x^2 &= -16 \\ x^2 &= -8 \end{aligned}$$
No solutions

3 a Two b One c None
d Two e None f One

Now you try

Example 26
a ± 7 b ± 3 c ± 12

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Example 27

a $\pm\sqrt{29}$ b ± 3.3

Example 28

a ± 11 b No solution c 5 d $-\sqrt{13}$

Exercise 2L

1 a $x = \pm 3$ b $x = \pm 11$ c $x = \pm 1$ d $x = \pm 20$
e $x = \pm 2$ f $x = \pm 10$ g $x = \pm 5$ h $x = \pm 7$
i $x = \pm 6$ j $x = \pm 4$ k $x = \pm 8$ l $x = \pm 9$
m $x = \pm 11$ n $x = \pm 4$ o $x = \pm\frac{1}{12}$ p $x = \pm\frac{1}{10}$

2 a i $x = \pm\sqrt{15}$ ii $x = \pm\sqrt{10}$
iii $x = \pm\sqrt{42}$ iv $x = \pm\sqrt{3}$
v $x = \pm\sqrt{7}$ vi $x = \pm\sqrt{11}$
vii $x = \pm\sqrt{34}$ viii $x = \pm\sqrt{39}$
b i $x = \pm 2.2$ ii $x = \pm 4.1$
iii $x = \pm 6.3$ iv $x = \pm 11.8$

3 a $x = \pm 7$ b $x = \pm 6$
c $x = \pm\sqrt{13}$ d No solutions
e No solutions f $x = \pm\sqrt{22}$
g $x = 5$ h $x = \sqrt{2}$
i $x = \pm\sqrt{15}$ j $x = 22$
k $x = 2\sqrt{22}$ l No solution

4 a $x = \pm 3$
b $x = \pm 4$
c $x = \pm 2$
d $x = \pm 8$
e $x = \sqrt{6}$
f No solution
g $x = -1$
h $x = -\sqrt{2}$

5 9 or 99

6 a $x^2 = 14$, $x = \pm 3.7$
b Side length must be positive so the value of x will be 3.7 m.

7 a 2.8 m since $r > 0$
b 3.8 cm since $r > 0$
c 2 m since $r > 0$

8 a i $c = 10$
ii $a = \sqrt{7}$
iii $b = \sqrt{56} (= 2\sqrt{14})$
b $x = \sqrt{32} (= 4\sqrt{2})$

9 8.5 m

10 a 15 m
b 3.46 seconds
c 2 seconds

11 x^2 is positive for all values of x, since a positive × a positive = a positive and a negative × a negative = a positive.

12 a $c > 0$ b $c = 0$ c $c < 0$

13 $x^2 \geq 0$ for all values of x so $x^2 + 10 \leq 10$ for all values of x so it cannot equal 6.

14 a i $x = \pm 4$ ii $x = 4$
b Different since in part ii, just $+\sqrt{16}$ is required not $-\sqrt{16}$ also.

15 10 seconds

16 6.4 seconds

Problems and challenges

1 a $x = \frac{k+4}{w-a}$ b $K = \frac{a-b}{y}$ c $a = \frac{w}{1-km^2}$

2 39

3 \$140

4 8

5 a i > ii < iii > iv <
b c, b, a, d

6 $x = 1, y = -2, z = 5$

7 a $x = \frac{ab}{a-b}$ b $x = \frac{10}{3}$ or $3\frac{1}{3}$
c $x = -\frac{7}{11}$ d $x = \frac{29}{4}$ or $7\frac{1}{4}$

Success criteria example questions

1 $20t$

2 10

3 $28x^2y$

4 $\frac{2y}{3}$

5 $3xy - 2x^2y$

6 $12x^2 - 20x$

7 $-9x + 13$

8 $x = -3$

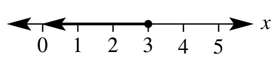
9 $x = 5$

10 $x = \frac{1}{8}$

11 $x = -6$

12 $x + 4 = 2x - 3$

13 Ava has 16 and Elise has 11

14 

15 $x > -4$

16 $c = 15$

17 $a = 5$

18 $r = \sqrt{\frac{V}{\pi h}}$

19 $x = 2, y = 1$

20 $x = -1, y = 3$

21 A meat pie costs \$6 and a bottle of water costs \$3

22 $x^2 - 2 = 0$

Short-answer questions

1 a $7m$ b $2(x + y)$ c $3m$ d $\frac{n}{4} - 3$

2 a -7 b 7 c 24 d 8

3 a $8mn$ b $\frac{xy}{3}$
c $6b^2$ d $4 - 3b$
e $2mn + 2m - 1$ f $2p + 4q$

4 a $2x + 14$ b $-6x - 15$ c $6x^2 - 8x$
d $-10a + 8a^2$ e $13 - 4x$ f $27x - 10$

5 a $x = 9$ b $x = 26$ c $x = 15$
d $x = 1$ e $x = 2$ f $x = -9$
g $x = -10$ h $x = \frac{5}{7}$

6 a $2n + 3 = 21, n = 9$ b $\frac{l-5}{3} = 7, l = 26$
c $\frac{x}{4} - 5 = 0, x = 20$

7 a $x = 5$ b $x = 1\frac{5}{6}$ c $x = 4$
d $x = -5$ e $x = 1$ f $x = 4$

8 \$260

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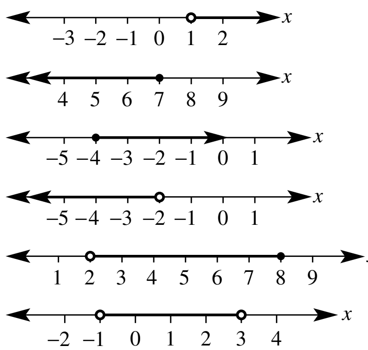
9 a b c d e f



10 a $x < 12$ b $m > -1$ c $y \geqslant -6$
d $x < 17$ e $a > 4$ f $x \geqslant 2$

11 n (sales) $\geqslant$ \$24 000

12 a $E = 60$ b $a = 5$ c $h = 10$

13 a $x = \dfrac{v^2 - u^2}{2a}$ b $\theta = \dfrac{2A}{r^2}$
c $I = \sqrt{\dfrac{P}{R}}$ d $a = \dfrac{2S}{n} - l = \dfrac{2S - nl}{n}$

14 a $x = 8, y = 2$ b $x = -\dfrac{3}{5}, y = -1\dfrac{3}{5}$
c $x = 4, y = 11$ d $x = 11, y = 4$
e $x = -3, y = -5$ f $x = 4, y = 1$

15 Billy is 20 years old and his sister is 12 years old

16 a $x = \pm 2$ b $x = \pm\sqrt{7}$
c $x = \pm\sqrt{10}$ d $x = -1$

Multiple-choice questions

1 C	2 D	3 D	4 C	5 B	6 A
7 A	8 E	9 B	10 A	11 B	12 D

Extended-response questions

1 a $0 < b \leqslant 10$
b $b = \dfrac{2A}{h} - a$ or $b = \dfrac{2A - ah}{h}$
c 8 m d $h = \dfrac{2A}{a + b}$ e 8 m

2 a \$5 per ride
b i Let \$$c$ be the cost of a bucket of chips.
Let \$$d$ be the cost of a drink.
ii $2d + c = 11$, $3d + 2c = 19$
iii $d = 3$, $c = 5$
iv Chips cost \$5 per bucket and drinks cost \$3 each.

Chapter 3

3A

Building understanding

1 a 17 b 50 c $\sqrt{8}$
2 a $c^2 = a^2 + b^2$ b $x^2 = y^2 + z^2$ c $j^2 = k^2 + l^2$
3 a 10.24 b 13 c 10 d 4.90

Now you try

Example 1
a 5 b 5.59

Example 2
$\sqrt{58}$

Exercise 3A

1 a $c = 26$ b $c = 17$ c $c = 15$
d $c = 25$ e $c = 41$ f $c = 50$

2 a 4.47 b 3.16 c 15.62
d 11.35 e 7.07 f 0.15

3 a $\sqrt{5}$ b $\sqrt{58}$ c $\sqrt{34}$
d $\sqrt{37}$ e $\sqrt{109}$ f $\sqrt{353}$

4 a 21.63 mm b 150 mm c 50.99 mm
d 155.32 cm e 1105.71 m f 0.02 m

5 a hypotenuse = 3.61 m; perimeter = 8.61 m
b hypotenuse = 2.24 m; perimeter = 5.24 m
c hypotenuse = 5.67 cm; perimeter = 13.19 cm
d hypotenuse = 0.08 m; perimeter = 0.19 m
e hypotenuse = 7.07 mm; perimeter = 17.07 mm
f hypotenuse = 4.53 cm; perimeter = 10.93 cm

6 4.4 m
7 42 m
8 250 m
9 2.4 m
10 495 m

11 a 5 cm b $\dfrac{25}{3}$ or $8\dfrac{1}{3}$ cm

12 5.83 m

13 a no b no c yes
d no e yes f yes

14 a 9.4 m
b i acute-angled ii obtuse-angled
c i obtuse-angled ii acute-angled

15 a 5.66 cm b 5.66 cm c yes

16 a 77.78 cm b 1.39 m
c reduce is by 7.47 cm d 43.73 cm

17 42.43 cm, 66.49 cm

3B

Building understanding

1 a 14 b 12 c 4 d 8
2 a true b false c false
d true e true f false

Now you try

Example 3
a $a = 7$ b $b = 10.27$
c $x = \sqrt{\dfrac{49}{2}}\left(= \dfrac{7}{\sqrt{2}} = \dfrac{7\sqrt{2}}{2}\right)$

Exercise 3B

1 a 16 b 24 c 6
d 21 e 60 f 27

2 a 8.66 b 11.31 c 5.11
d 17.55 e 7.19 f 0.74

3 a 30 cm b 149.67 cm c 1.65 cm
d 2.24 km e 12 cm f 52.92 mm

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4 a $\sqrt{\frac{25}{2}}\left(=\frac{5\sqrt{2}}{2}\right)$ b $\sqrt{8}\,(=2\sqrt{2})$
c $\sqrt{\frac{1521}{200}}\left(=\frac{39\sqrt{2}}{20}\right)$
5 5.3 m
6 14.2 m
7 a $\sqrt{187}$ b $\sqrt{567}$ or $9\sqrt{7}$ c 40
8 49 cm
9 1.86 m
10 a $\sqrt{\frac{81}{5}}\left(=\frac{9\sqrt{5}}{5}\right)$ b $\sqrt{5}$
c $\sqrt{\frac{5}{2}}$ or $\sqrt{\frac{10}{2}}$
11 a $\sqrt{\frac{5}{2}}, 3\sqrt{\frac{5}{2}}$ b $\frac{10}{\sqrt{13}}, \frac{15}{\sqrt{13}}$ c $\frac{25}{\sqrt{74}}, \frac{35}{\sqrt{74}}$
12 a the side c
b $\sqrt{6}$
c no, c^2 is enough
d error due to rounding, x would not be exact
13 a i $\sqrt{8}$ or $2\sqrt{2}$ ii $\sqrt{7}$ iii $\sqrt{6}$
b OB: 2.60, OC: 2.79, OD: 2.96
c differ by 0.04; the small difference is the result of rounding errors
d Answers may vary.

3C

Building understanding

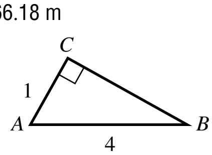
1 a C, II b A, III c B, I

Now you try

Example 4
40.45 m

Exercise 3C

1 3.0 m
2 142.9 m
3 3823 mm
4 1060 m
5 a 6.4 m b 5.7 cm c 5.4 m d 6.0 m
6 466.18 m
7 a



b 3.9 cm
8 a 47 m b 89.8 m
9 171 cm
10 a $\frac{5}{\sqrt{2}}$ or $\frac{5\sqrt{2}}{2}$ b $6+\frac{5}{\sqrt{2}}$ or $6+\frac{5\sqrt{2}}{2}$
c 10.2
11 a 1.1 km b 1.1 km c 3.8 km
12 a 28.28 cm
b i 80 cm ii 68.3 cm iii 48.3 cm
c Answers may vary.
d Answers may vary.

3D

Building understanding

1 a yes b yes c no
d yes e yes f no
g yes h no i yes

Now you try

Example 5
7.60 cm

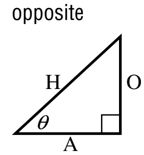
Exercise 3D

1 11.18 mm
2 a 0.34 m b 19.75 cm
3 a 3.0 cm b 2.5 m c 3.1 cm
4 a 18.0 cm b 7 mm c 0.037 m
5 a $\sqrt{2}$ cm b 1.7 cm
6 a $\sqrt{208}$ cm or $4\sqrt{13}$ cm b 15.0 cm
7 84 m
8 2.86 cm
9 $\sqrt{3} : 2$
10 a $\sqrt{65}$ b 8.06
c $\sqrt{69}$, 8.31 d 8.30
e Rounding errors have accumulated.
11 a i $\sqrt{8}$ cm or $2\sqrt{2}$ cm ii 3.74 cm
b i $\sqrt{50}$ cm or $5\sqrt{2}$ cm ii 5.45 cm
12 a i 11.82 m ii 12.15 m
iii 11.56 m iv 11.56 m
b Shortest distance is 10.44 m.

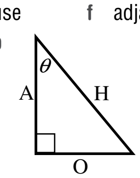
3E

Building understanding

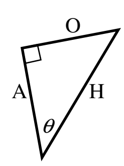
1 a hypotenuse b opposite c adjacent
d opposite e hypotenuse f adjacent
2 a



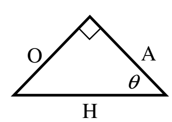
b



c



d



3 a 5 b 4 c 3 d 3 e 4

Now you try

Example 6
a $\tan\theta=\frac{6}{5}$ c $\sin\theta=\frac{2}{3}$
b $\cos\theta=\frac{3}{8}$

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Exercise 3E

1 a $\sin\theta = \frac{4}{7}$ b $\tan\theta = \frac{5}{4}$ c $\cos\theta = \frac{3}{5}$
d $\sin\theta = \frac{2}{3}$ e $\tan\theta = 1$ f $\sin\theta = \frac{x}{y}$
g $\tan\theta = \frac{4}{5}$ h $\cos\theta = \frac{a}{2b}$ i $\sin\theta = \frac{5y}{3x}$

2 a i $\frac{5}{13}$ ii $\frac{5}{13}$ iii the same
b i $\frac{12}{13}$ ii $\frac{12}{13}$ iii the same
c i $\frac{5}{12}$ ii $\frac{5}{12}$ iii the same

3 a $\sin\theta = \frac{3}{5}, \cos\theta = \frac{4}{5}, \tan\theta = \frac{3}{4}$
b $\sin\theta = \frac{5}{13}, \cos\theta = \frac{12}{13}, \tan\theta = \frac{5}{12}$
c $\sin\theta = \frac{12}{13}, \cos\theta = \frac{5}{13}, \tan\theta = \frac{12}{5}$

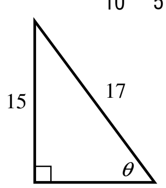
4 a $\frac{4}{5}$ b $\frac{3}{5}$ c $\frac{3}{5}$
d $\frac{3}{4}$ e $\frac{4}{5}$ f $\frac{4}{3}$

5 $\frac{3}{4}$

6 a $\frac{7}{25}$ b $\frac{24}{25}$ c $\frac{7}{24}$

7 a i 5
ii $\sin\theta = \frac{3}{5}, \cos\theta = \frac{4}{5}, \tan\theta = \frac{3}{4}$
b i 25
ii $\sin\theta = \frac{7}{25}, \cos\theta = \frac{24}{25}, \tan\theta = \frac{7}{24}$
c i 15
ii $\sin\theta = \frac{9}{15} = \frac{3}{5}, \cos\theta = \frac{12}{15} = \frac{4}{5}, \tan\theta = \frac{9}{12} = \frac{3}{4}$
d i 10
ii $\sin\theta = \frac{8}{10} = \frac{4}{5}, \cos\theta = \frac{6}{10} = \frac{3}{5}, \tan\theta = \frac{8}{6} = \frac{4}{3}$

8 a



b 8
c $\sin\theta = \frac{15}{17}, \cos\theta = \frac{8}{17}, \tan\theta = \frac{15}{8}$

9 a i $\frac{1}{2}$ ii $\frac{\sqrt{3}}{2}$ iii $\frac{1}{\sqrt{3}}$
iv $\frac{\sqrt{3}}{2}$ v $\frac{1}{2}$ vi $\sqrt{3}$
b i They are equal. ii They are equal.

10 a Answers may vary.
b i 0.766 ii 0.643 iii 0.839
iv 0.766 v 1.192 vi 0.643
c $\sin 40^\circ = \cos 50^\circ, \sin 50^\circ = \cos 40^\circ,$
$\tan 50^\circ = \frac{1}{\tan 40^\circ}, \tan 40^\circ = \frac{1}{\tan 50^\circ}$

11 a yes, any isosceles right-angled triangle
b no, as it would require the hypotenuse (the longest side) to equal the opposite
c no, adjacent side can't be zero
d no; the numerator < denominator for sin and cos as the hypotenuse is the longest side

12 a $\sin\theta = \frac{3}{5}, \tan\theta = \frac{3}{4}$
b i $\cos\theta = \frac{\sqrt{3}}{2}, \tan\theta = \frac{1}{\sqrt{3}}$
ii $\tan\theta = \sqrt{3}, \sin\theta = \frac{\sqrt{3}}{2}$
iii $\sin\theta = \frac{1}{\sqrt{2}}, \cos\theta = \frac{1}{\sqrt{2}}$
c equals one
d $(\sin\theta)^2 + (\cos\theta)^2 = 1$ (the Pythagorean identity)

3F

Building understanding

1 a A b O c H
2 a sin b tan c cos

Now you try

Example 7
a 0.94 b 0.87 c 1.39 d 0.08

Example 8
3.21

Example 9
a 2.12 b 4.49 c 2.02

Exercise 3F

1 a 0.73 b 0.10 c 0.25 d 0.46
2 a 3.06 b 18.94 c 5.03
d 0.91 e 1.71 f 9.00
g 2.36 h 4.79 i 7.60
3 a 5.95 b 0.39 c 12.59 d 3.83
e 8.40 f 1.36 g 29.00 h 1.62
i 40.10 j 4.23 k 14.72 l 13.42
m 17.62 n 5.48 o 10.02 p 1.02
4 1.12 m
5 44.99 m
6 10.11 m
7 a 20.95 m b 10 cm
8 a 65° b 1.69 c 1.69
d They are the same as both are suitable methods for finding x.
9 a i 80° ii 62° iii 36° iv 9°
b i both 0.173... ii both 0.469...
iii both 0.587... iv both 0.156...
c $\sin\theta^\circ = \cos(90^\circ - \theta^\circ)$
d i 70° ii 31° iii 54° iv 17°
10 a $\sqrt{2}$
b i $\frac{1}{\sqrt{2}}$ ii $\frac{1}{\sqrt{2}}$ iii 1
c $\sqrt{3}$
d i $\frac{1}{2}$ ii $\frac{\sqrt{3}}{2}$ iii $\frac{1}{\sqrt{3}}$
iv $\frac{\sqrt{3}}{2}$ v $\frac{1}{2}$ vi $\sqrt{3}$

Progress quiz

1 a 6.66 b 7.88
2 a $\sqrt{41}$ b $\sqrt{50}\ (= 5\sqrt{2})$
3 224.5 cm
4 a 10.4 cm b 10.8 cm
5 a AC b BC c $\frac{3}{5}$ d $\frac{4}{3}$

Answers

3E

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6 a 7.46 b 6.93
7 a 13.80 b 25.27 c 2.50
8 a 3.464 cm b 6.9 cm^2

3G

Building understanding

1 a $x = 2$ b $x = 5$ c $x = 7$ d $x = 4$
2 a N b D c D

Now you try

Example 10
3.40

Example 11
a 8.00 b $x = 7.58$, $y = 23.27$

Exercise 3G

1 a 0.98 b 12.80 c 14.43
d 9.52 e 114.83 f 22.05
2 a 13.45 b 16.50 c 57.90 d 26.33
e 15.53 f 38.12 g 9.15 h 32.56
i 21.56 j 47.79 k 45.57 l 23.33
3 a $x = 7.5$, $y = 6.4$ b $a = 7.5$, $b = 10.3$
c $a = 6.7$, $b = 7.8$ d $x = 9.5$, $y = 12.4$
e $x = 12.4$, $y = 9.2$ f $x = 21.1$, $y = 18.8$
g $m = 56.9$, $n = 58.2$ h $x = 15.4$, $y = 6.0$
4 40 m
5 3848 m
6 a 23.7 m b 124.9 m
7 a B, as student B did not use an approximation in their working out
b Use your calculator and do not round sin 31° during working.
c i difference of 0.42 ii difference of 0.03
8 a i 10.990 ii 11.695
b i 0.34 ii 0.94 iii 0.36
c equal to tan 20°
d i $\frac{b}{c}$ ii $\frac{a}{c}$ iii $\frac{b}{a}$ iv $\frac{b}{a}$
e same as tan θ

3H

Building understanding

1 a 11.54 b 41.41 c 26.57
2 a $\frac{1}{2}$
b 50°
c $45° = \tan^{-1}(1)$
3 a sine b cosine c tan

Now you try

Example 12
a 65° b 59.0°

Example 13
34°

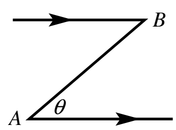
Exercise 3H

1 a 30° b 45° c 30°
d 90° e 50° f 90°
2 a 34.85° b 19.47° c 64.16°
d 75.52° e 36.87° f 38.94°
g 30.96° h 57.99° i 85.24°
3 a 43°3' b 31°9' c 40°36' d 15°36'
e 55°18' f 50°29' g 48°35' h 41°25'
4 17°
5 25.4°
6 26.6°
7 23.13°
8 a 128.7° b 72.5° c 27.3°
9 a 90°, 37°, 53° b 90°, 23°, 67° c 90°, 16°, 74°
10 45°
11 $\angle ACM = 18.4°$ $\angle ACB = 33.7°$, no it is not half
12 a 18° b 27° c 45°
d 5.67 m e up to 90°

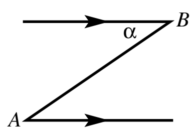
3I

Building understanding

1 $a = 65$, $b = 25$
2 a 22° b 22°
3 a i



ii



b yes, $\theta = \alpha$, alternate angles are equal on parallel lines

Now you try

Example 14
31 m

Example 15
11 m

Example 16
22°

Example 17
yes, 7 m above

Exercise 3I

1 29 m
2 16 m
3 157 m
4 38 m
5 90 m
6 37°
7 6°
8 10°
9 yes by 244.8 m
10 a 6° b 209 m
11 4634 mm
12 15°, 4319 mm
13 1.25 m
14 a 6.86 m b 26.6°

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c i $m = h + y$ ii $y = x\tan\theta$
iii $m = h + x\tan\theta$

15 $A = \frac{1}{2}a^2\tan\theta$

16 a i 47.5 km ii 16.25 km
b no, $\sin(2 \times \theta) < 2 \times \sin\theta$
c yes, $\sin\left(\frac{1}{2}\theta\right) > \frac{1}{2}\sin\theta$

17 a yes b 12.42 km
c no, after 10 min the plane will be above 4 km
d 93 km/h or more

18 a 1312 m b 236 km/h

3J

Building understanding

1 a 0° b 045° c 090° d 135°
e 180° f 225° g 270° h 315°

2 a 070° b 130° c 255° d 332°

Now you try

Example 18
a 230°T b 050°T

Example 19
5.7 km

Example 20
a 1.7 km b 300°T

Exercise 3J

1 a i 040°T ii 220°T
b i 142°T ii 322°T
c i 125°T ii 305°T
d i 067°T ii 247°T
e i 330°T ii 150°T
f i 228°T ii 048°T

2 3.28 km

3 59.45 km

4 3.4 km

5 11 km

6 a 39 km b 320° T

7 a 11.3 km b 070° T

8 310 km

9 3.6 km

10 6.743 km

11 a $180° + a°$ b $a° - 180°$

12 a 320° T b 245° T c 065° T
d 238° T e 278° T

13 a 620 km b 606 km c 129 km

14 a i 115 km ii 96 km
b 158 km
c i 68 km ii 39.5 min

Problems and challenges

1 $P = 4 + 2\sqrt{8}$

2 $10\,\text{m}^2$

3 15 cm

4 010° T

5 round peg square hole

6 122° T

7 a i 3, 4, 5 ii 5, 12, 13
b $m^2 + n^2$

Success criteria example questions

1 $\sqrt{137}$; 11.7

2 8.7

3 10.1 m

4 13 m

5 $\cos\theta = \frac{6}{7}$

6 0.82

7 12.7

8 6.7

9 $x = 25.7$, $y = 28.4$

10 47.6°

11 61°

12 11 m

13 53°

14 120° and 300°

15 5 km; 020° T

Short-answer questions

1 a 37 b $\sqrt{12}$ or $2\sqrt{3}$ c $\sqrt{50}$ or $5\sqrt{2}$

2 4.49 m

3 a 13 cm b 13.93 cm

4 19 m

5 a 4.91 m b $x = 2.83$, $h = 2.65$

6 a 0.64 b 2.25 c 0.72 d 0.55

7 a 7.83 b 48.02 c 50.71

8 28.01 m

9 177.91 m

10 a 59.45 km b 53.53 km

11 053.13°

12 63.2 m

13 5.3 m

14 a 52.5 km b 13.59 km

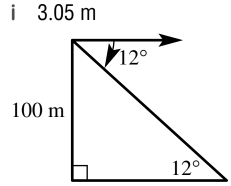
Multiple-choice questions

1 A 2 B 3 A 4 C 5 B
6 C 7 D 8 C 9 A 10 B

Extended-response questions

1 a 2.15 m
b 0.95 m
c i 3.05 m ii yes

2 a



b 470 m c 3° d 1530 m

3 a 35 km b 399 km

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Chapter 4

4A

Building understanding

1 a A (4, 1) B (2, 3) C (0, 3)
D (−2, 2) E (−3, 1) F (−1, 0)
G (−3, −2) H (−2, −4) I (0, −3)
J (2, −2) K (3, −4) L (2, 0)
b i F, L ii C, I
c i D, E ii J, K

2 a 2 b −7

3 a 1 b 21

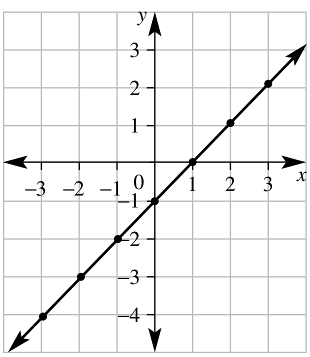
4 Answers may vary; e.g. (1, 4), (–1, 2) etc.

Now you try

Example 1

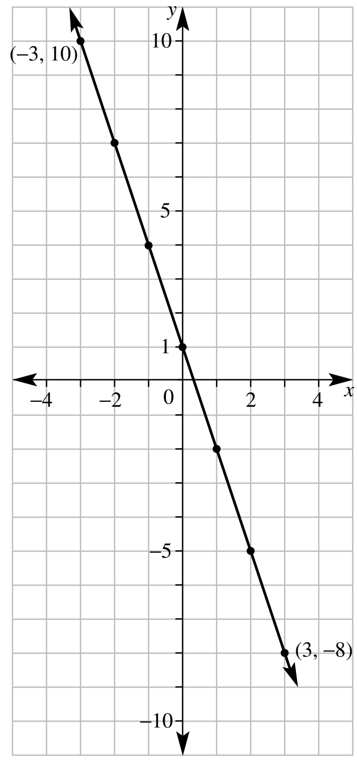
a

x	−3	−2	−1	0	1	2	3
y	−4	−3	−2	−1	0	1	2



b

x	−3	−2	−1	0	1	2	3
y	10	7	4	1	−2	−5	−8



Example 2

a x-intercept is at (1, 0), y-intercept is at (0, 1)

b x-intercept is at (5, 0), y-intercept is at (0, −3)

Example 3

a yes

b no

Exercise 4A

1 a $y = x - 1$

x	−3	−2	−1	0	1	2	3
y	−4	−3	−2	−1	0	1	2

b $y = 2x - 3$

x	−3	−2	−1	0	1	2	3
y	−9	−7	−5	−3	−1	1	3

c $y = -x + 4$

x	−3	−2	−1	0	1	2	3
y	7	6	5	4	3	2	1

d $y = -3x$

x	−3	−2	−1	0	1	2	3
y	9	6	3	0	−3	−6	−9

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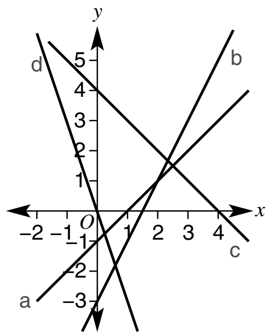


2 a (1, 0), (0, 1) b (−2, 0), (0, 2)
c (4, 0), (0, 8) d (−5, 0), (0, 10)
e (2, 0), (0, 3) f (7, 0), (0, −3)
g (−11, 0), (0, 5) h (−2, 0), (0, −5)

3 a yes b no c yes
d yes e no f no

4 a no b yes c no
d no e yes f no

5 a $y = x + 2$ b $y = 2x$
c $y = 2x + 1$ d $y = -x + 2$

6 a C b D c B d A

7 a yes b no c no d yes

8 a For graph to move 1 down from (1, 1) it moves $\frac{1}{2}$ across.
b For graph to move 1 up from (0, −1) it moves $1\frac{1}{2}$ across.

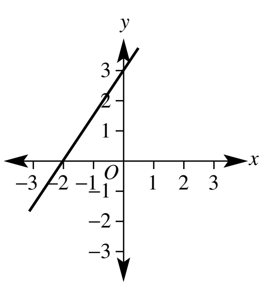
9 a false b true c true d false

10 a $y = 2x + 7$ b $y = -x + 20$
c $y = -3x - 10$ d $y = -5x + 4$
e $y = \frac{1}{2}x + \frac{1}{2}$ f $y = -\frac{1}{2}x - \frac{3}{2}$

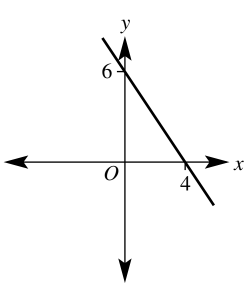
4B

Building understanding

1 a



b



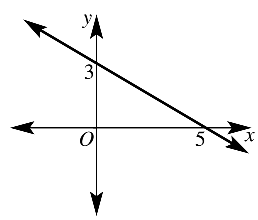
2 a i 3 ii 4 iii −6
b i 10 ii 1 iii 6

3 a (0, 4) b (0, −5) c (0, 3)
4 a (−2, 0) b (3, 0) c (2, 0)

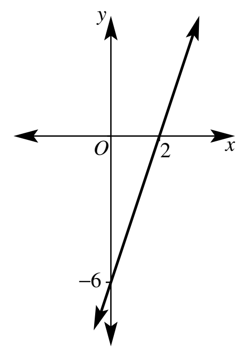
Now you try

Example 4

a

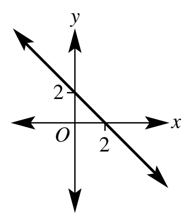


b

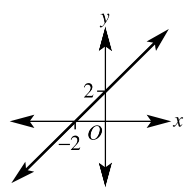


Exercise 4B

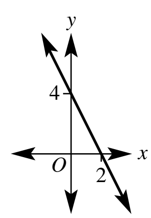
1 a



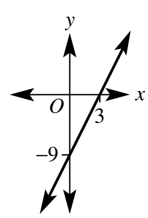
b



c



d



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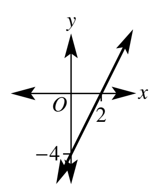

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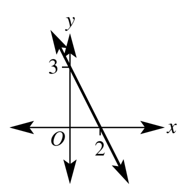
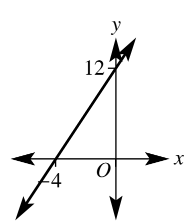
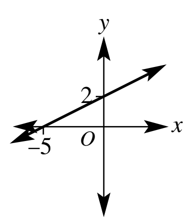


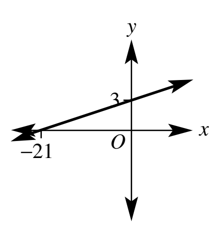
g



h



i



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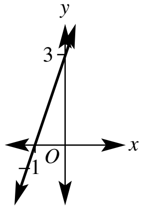
2 a



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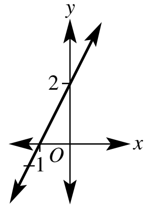

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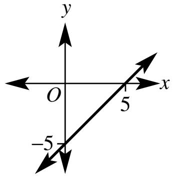

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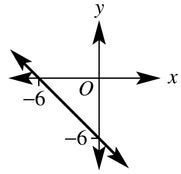

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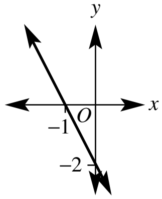

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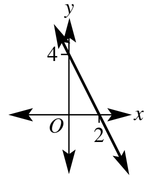
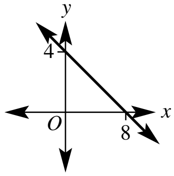


3 a



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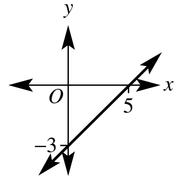

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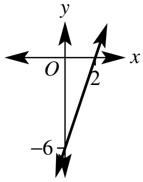

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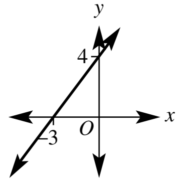

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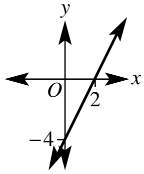

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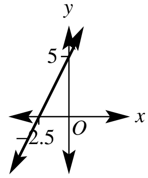
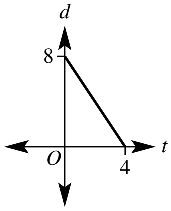


4 a 8 m **b** 4 seconds

c



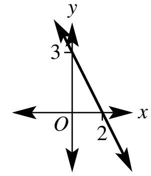
5 a 100 m **b** 12.5 s

6 a $\left(1\frac{2}{3}, 0\right), \left(0, -2\frac{1}{2}\right)$ **b** $(-7, 0), \left(0, -1\frac{2}{5}\right)$

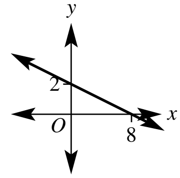
c $\left(6\frac{1}{2}, 0\right), (0, -13)$ **d** $\left(-\frac{1}{2}, 0\right), (0, -1)$

e $(3, 0), \left(0, -1\frac{1}{2}\right)$ **f** $\left(\frac{1}{3}, 0\right), \left(0, -\frac{1}{7}\right)$

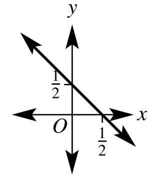
7 a



b



c



8 (0, 0) is the x- and y-intercept for all values of a and b

9 a $x + y = 4$ **b** $x + y = 2$

c $x - y = 3$ **d** $x - y = -1$

e $x + y = k$ **f** $x + y = -k$

10 a $\left(\frac{c}{a}, 0\right), \left(0, \frac{c}{b}\right)$ **b** $\left(-\frac{cb}{a}, 0\right), (0, c)$

c $\left(\frac{c}{a}, 0\right), \left(0, -\frac{c}{b}\right)$ **d** $\left(-\frac{c}{b}, 0\right), \left(0, \frac{c}{a}\right)$

e $\left(-\frac{c}{b}, 0\right), \left(0, \frac{c}{a}\right)$ **f** $\left(\frac{bc}{a}, 0\right), \left(0, \frac{bc}{a}\right)$

4C

Building understanding

1 a (2, 0) **b** (−3, 0) **c** (0, 0)

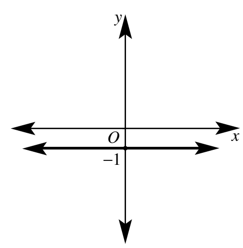
2 a (0, 3) **b** (0, −2) **c** (0, 0)

3 a 5 **b** $\frac{1}{3}$ **c** −4 **d** −0.1

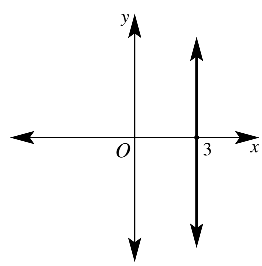
Now you try

Example 5

a



b



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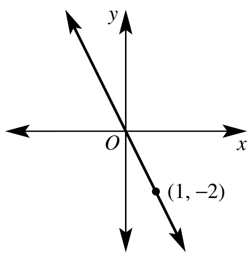
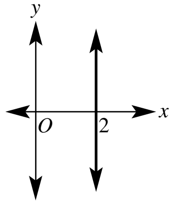
Example 6

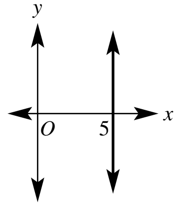


Exercise 4C

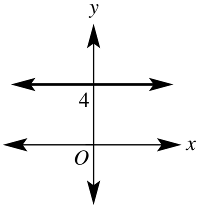
1 a



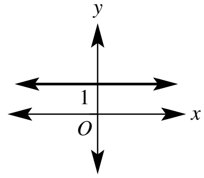
b



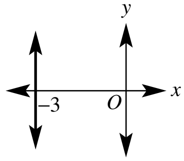
c



d



e



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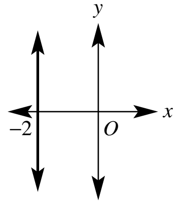
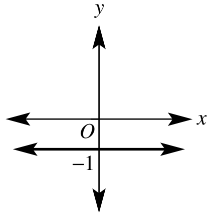
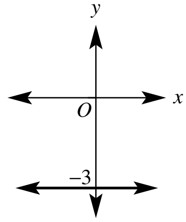


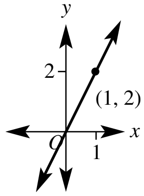
g



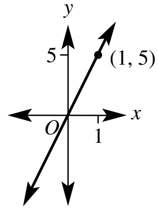
h



2 a



b



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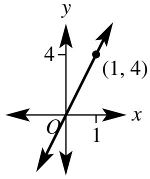

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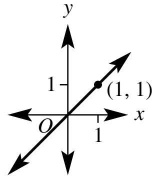

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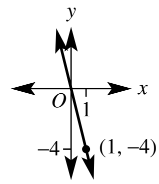
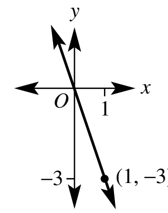


f



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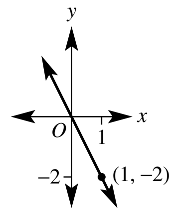

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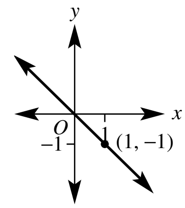

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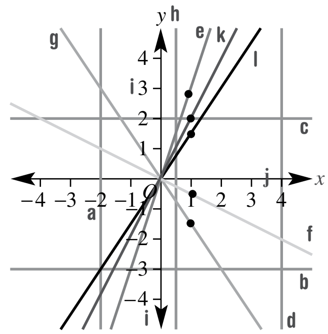

4 a $y = -2$ b $y = 4$ c $x = -2$
d $x = 5$ e $y = 1.5$ f $x = -6.7$

5 a $y = 3$ b $x = 5$ c $x = -2$ d $y = 0$

6 a $y = 250$ b $y = -45$

7 a $(1, 2)$ b $(-3, 5)$ c $(0, -4)$
d $(4, 0)$ e $(0, 0)$ f $(1, 3)$
g $(3, -27)$ h $(5, 40)$

8 a $A = 15$ square units b $A = 68$ square units

9 a i $y = 1$ or $y = -5$
ii $y = 0$ or $y = -4$
iii $y = 3\frac{1}{2}$ or $y = -7\frac{1}{2}$
b i $y = 1$ or $y = -5$
ii $y = 7$ or $y = -11$
iii $y = 9\frac{1}{2}$ or $y = -13\frac{1}{2}$

10 a $y = 2x$ b $y = -x$
c $y = 3x$ d $y = -3x$

11 a $y = 3x$ b $y = 4x$
c $y = -5x$ d $y = -2x$

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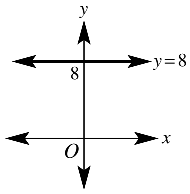
12 a


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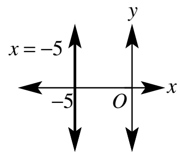

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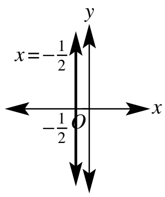

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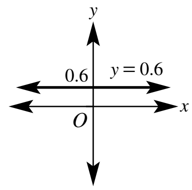

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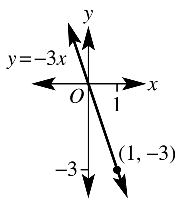

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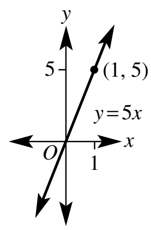

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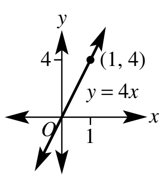

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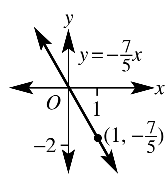


13 $x = -1, y = -5, y = 5x$

14 a (k, c) b $m = \frac{c}{k}$

15 $m = \frac{1}{2}$

4D

Building understanding

1 a 2 b $\frac{1}{2}$ c 3
d −1 e −3 f $-\frac{1}{4}$

2 a zero b negative
c positive d undefined

3 a 2 b 0.5 c 6 d −6

Now you try

Example 7

a negative, gradient $= -\frac{2}{3}$ b zero, gradient = 0
c undefined d positive, gradient $= \frac{3}{5}$

Example 8

a $m = 2$ b $m = -\frac{4}{7}$

Exercise 4D

1 a positive, 2 b zero
c negative, $-\frac{3}{4}$ d undefined
e positive, $\frac{1}{2}$ f positive, 3
g positive, 2 h negative, −4

2 a 2 b $\frac{4}{3}$ c 2 d 1
e $-\frac{1}{4}$ f −2 g −1 h $\frac{1}{2}$
i $\frac{3}{2}$ j $-\frac{5}{2}$ k $\frac{1}{3}$ l $\frac{5}{2}$

3 *A* 1 *B* −2 *C* 1 *D* $\frac{1}{2}$ *E* −2 *F* 3

4 a $-\frac{1}{20}$ b $\frac{35}{2}$ c $-\frac{1}{5}$ d $\frac{15}{2}$

5 a 6 b −1 c 4 d $-\frac{3}{2}$

6 300 m

7 A 9 B −4 C −1
D 4 E −0.4 F 2.4

8 $\frac{7}{11} = \frac{35}{55}, \frac{3}{5} = \frac{33}{55}$ hence $\frac{7}{11} > \frac{3}{5}$ so $\frac{7}{11}$ is steeper

9 a run $= x_2 - x_1$ b rise $= y_2 - y_1$ c $m = \frac{y_2 - y_1}{x_2 - x_1}$
d i $m = \frac{3}{2}$ ii $m = \frac{5}{4}$
iii $m = -\frac{5}{3}$ iv $m = \frac{4}{3}$
e yes, the rise and the run work out regardless

10 a i −1 ii −3
b (4.5, 0)
c i (6, 0) ii (2.4, 0)
iii $\left(\frac{9}{7}, 0\right)$ iv (7.5, 0)

4E

Building understanding

1 a i 10 km ii 20 km iii 30 km
b 10 km/h c 10
d They are the same.

2 a 15 mm
b i 6 days ii 20 days
c

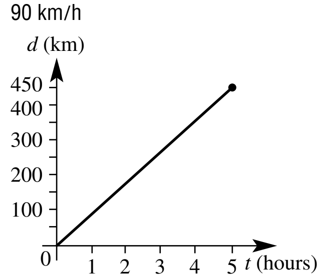
t	0	1	2	3	4
h	0	5	10	15	20

d B e 5

Now you try

Example 9

a 90 km/h

b


c i 90 ii $d = 90t$

d i 315 km ii 1.5 hours

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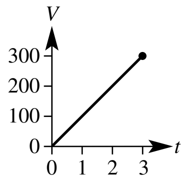
Exercise 4E

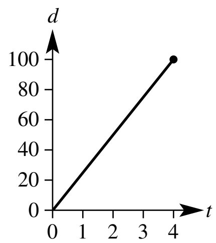
1 a 100 L/hour

b 

c i 100 ii $V = 100t$

d i $V = 150$ L ii $t = 2$ hours

2 a 25 km/h

b 

c i 25 ii $d = 25t$

d i 62.5 km

ii 1.6 hours or 1 h 36 min

3 a $d = 50t$ b $g = 2t$

c $C = 1.25n$ d $P = 20t$

4 a 100 km/h b 7 cm/s

c 2.5 cm/min d 49 mm/min

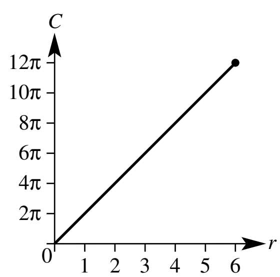
5 a 10.2 L b 72.25 L c 800 km

6 Sally

7 Leopard

8 25 months

9 a i 0 ii 4π iii 12π

b 

c gradient is 2π, the coefficient of r

10 No, $A = \pi r^2$ so area (A) is proportional to square of radius (r^2).

11 a $A = 2h$

b 2 cm^2 increase for each 1 cm increase in height

12 yes, for any fixed time, e.g. $t = 5$, $s = \frac{d}{5}$

13 1.2 minutes or 72 seconds

14 $\frac{5}{12}$ km

4F

Building understanding

1 a $y = 2x + 5$ b $y = -2x + 3$

c $y = -x - 2$ d $y = -\frac{1}{2}x - 10$

2 a 1 b -5 c 9

3 a $y = x + 7$ b $y = -x + 3$ c $y = 2x + 5$

Now you try

Example 10

a gradient $= 5$
y-intercept $= (0, -2)$

b gradient $= 6$
y-intercept $= (0, 0)$

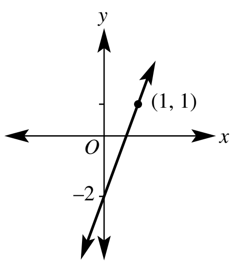
Example 11

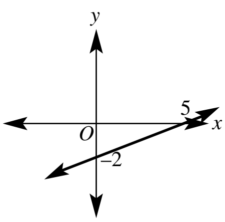
a $y = -5x + 10$ b $2x + y = 3$

Example 12

a gradient $= 3$
y-intercept $= (0, -2)$

b gradient $= \frac{2}{5}$
y-intercept $= (0, -2)$





Exercise 4F

1 a gradient $= 3$, y-intercept $= (0, 4)$

b gradient $= -5$, y-intercept $= (0, -2)$

c gradient $= -2$, y-intercept $= (0, 3)$

d gradient $= \frac{1}{3}$, y-intercept $= (0, -4)$

e gradient $= -4$, y-intercept $= (0, 0)$

f gradient $= 2$, y-intercept $= (0, 0)$

g gradient $= 1$, y-intercept $= (0, 0)$

h gradient $= -0.7$, y-intercept $= (0, 0)$

2 a $y = -2x + 3$

b $y = 3x - 1$

c $y = -3x + 2$

d $y = x + 2$

e $2x - y = 1$ or $-2x + y = -1$

f $3x + y = 4$ or $-3x - y = -4$

g $x - 3y = 1$ or $-x + 3y = -1$

h $2x - 7y = -2$ or $-2x + 7y = 2$

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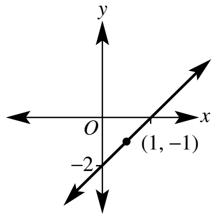
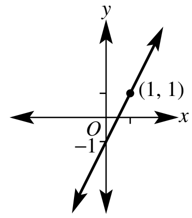
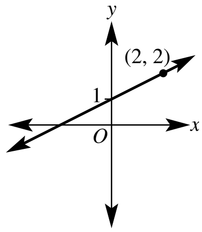
3 a 1, (0, −2)



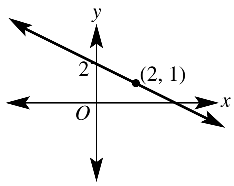
b 2, (0, −1)



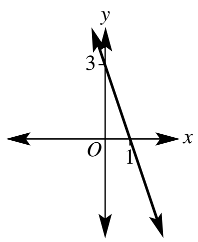
c $\frac{1}{2}$, (0, 1)



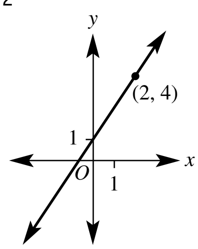
d $-\frac{1}{2}$, (0, 2)



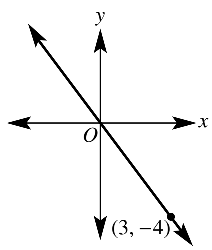
e −3, (0, 3)



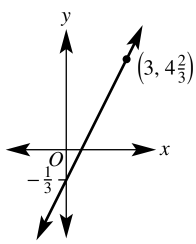
f $\frac{3}{2}$, (0, 1)



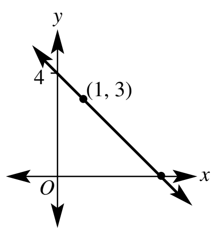
g $-\frac{4}{3}$, (0, 0)



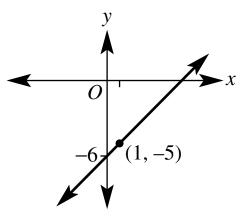
h $\frac{5}{3}$, $\left(0, -\frac{1}{3}\right)$



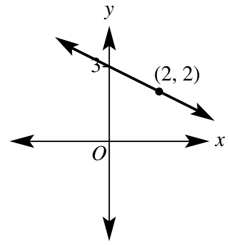
4 a −1, (0, 4)



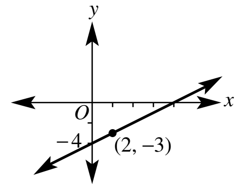
b 1, (0, −6)



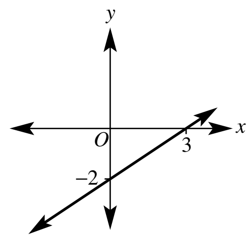
c $-\frac{1}{2}$, (0, 3)



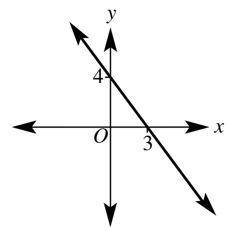
d $\frac{1}{2}$, (0, −4)



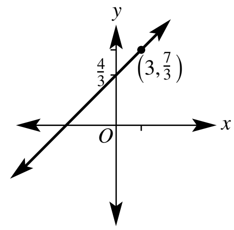
e $\frac{2}{3}$, (0, −2)



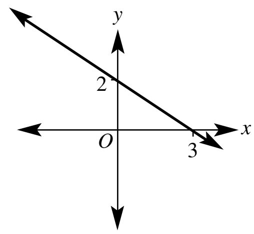
f $-\frac{4}{3}$, (0, 4)



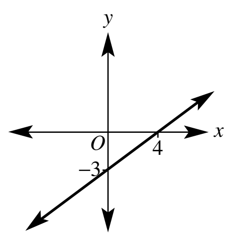
g $\frac{1}{3}$, $\left(0, \frac{4}{3}\right)$



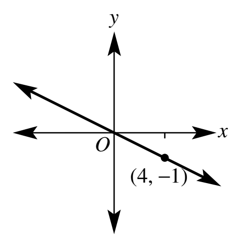
h $-\frac{2}{3}$, (0, 2)



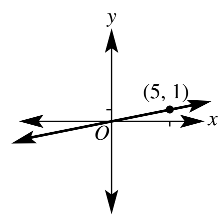
i $\frac{3}{4}$, (0, −3)



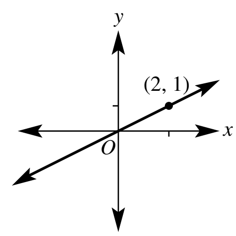
j $-\frac{1}{4}$, (0, 0)



k $\frac{1}{5}$, (0, 0)



l $\frac{1}{2}$, (0, 0)



5 a D **b** F **c** C
d B **e** E **f** A

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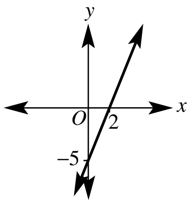
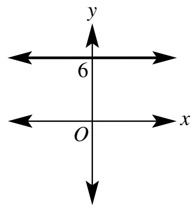
6 a


b


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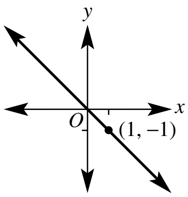

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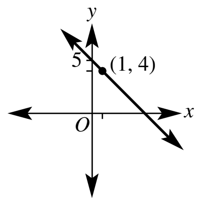

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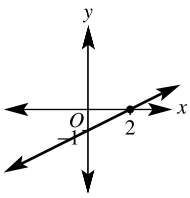

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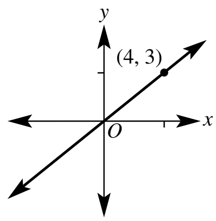

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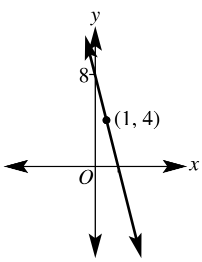

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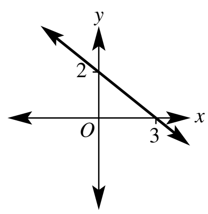


7 C, D, F, H

8 a $y = 2x + 2$, y-intercept is $(0, 2)$

b expand brackets and simplify

9 $y = k$

10 $y = -\frac{a}{b}x + \frac{d}{b}$, $m = -\frac{a}{b}$, y − int$\left(0, \frac{d}{b}\right)$

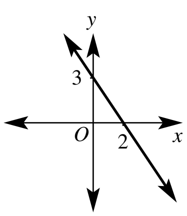
11 a $y = 2x + 3$ b $y = -x + 2$

c $y = \frac{5}{3}x + \frac{2}{3}$ d $y = -\frac{2}{5}x - \frac{1}{5}$

Progress quiz

1 a $(2, 0), (0, -4)$ b $(3, 0), (0, 6)$

2 a not on line b on line

3 a


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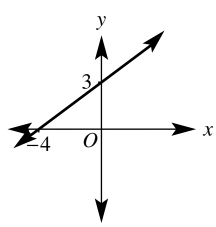
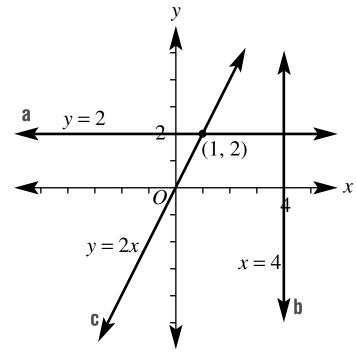


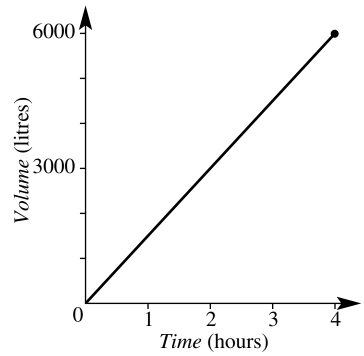
4


5 a positive, 2 b undefined

c zero d negative, $-\frac{2}{3}$

6 a $m = 2$ b $m = -3$

7 a 1500 L/h

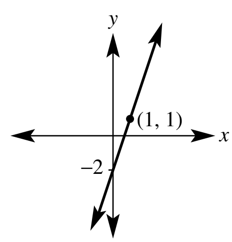
b


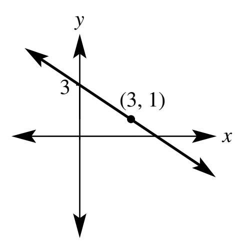
c i 1500 ii $V = 1500t$

d i 4500 L ii 3.5 hours

8 a $y = -5x + 8$ b $-3x + y = -5$

9 a 3, $(0, -2)$ b $-\frac{2}{3}$, $(0, 3)$





4G

Building understanding

1 a $y = 2x + 5$ b $y = 4x - 1$ c $y = -2x + 5$

2 a 1 b 9 c 5

Now you try

Example 13

$y = -2x + 4$

Example 14

$y = \frac{2}{3}x + 6$

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Exercise 4G

1 a $y = x + 3$ b $y = 2x - 2$
c $y = 8x + 8$ d $y = -x + 4$

2 a $y = 3x + 5$ b $y = -2x - 1$ c $y = -3x + 8$
d $y = x - 3$ e $y = -3x + 3$ f $y = 5x - 1$
g $y = -x + 8$ h $y = -3x + 6$ i $y = -2x + 2$
j $y = -4x - 9$

3 a $y = \frac{3}{4}x + 3$ b $y = -\frac{3}{4}x + 3$ c $y = -\frac{5}{4}x + 3$
d $y = \frac{3}{2}x + 4$ e $y = \frac{3}{5}x$ f $y = -\frac{1}{3}x - 1$

4 a i 2 ii $y = 2x + 2$
b i -1 ii $y = -x + 3$
c i -4 ii $y = -4x + 11$
d i 1 ii $y = x - 4$

5 $\left(2\frac{1}{2}, 0\right)$

6 $\left(\frac{3}{5}, 0\right), \left(0, -\frac{3}{4}\right)$

7 $V = -20t + 120$, $V = 120$ L initially

8 a $y = 5x$ b $y = 6.5x + 2$

9 a 1 b 8 c -6 d $\frac{5}{2}$

10 a $c = 3$ b $c = 3$
c no, y-intercept is fixed for any given line

11 $y = \frac{b}{a}x$

12 a $y = 2x$ b $y = -2x + 4$
c $y = 2x + 5$ d $y = -\frac{3}{2}x + 2$
e $y = \frac{5}{7}x + \frac{1}{7}$ f $y = -\frac{13}{3}x - \frac{11}{3}$

4H

Building understanding

1 a 4 b 8 c 1 d -3

2 a 5.5 b 2.5 c -1.5 d -2.5

3 a 2.24 b 8.60 c 10.20

Now you try

Example 15
a (2, 3.5) b (−1, −1.5)

Example 16
8.06

Exercise 4H

1 a (3, 3) b (2, 2)
c (4, 1) d (−1, 3)
e (−1, −1) f (1.5, 1.5)
g (0.5, 3) h (−1.5, −2.5)
i (−3, −8.5) j (0.5, −2.5)

2 a 5.10 b 2.83 c 5.39
d 4.47 e 3.61 f 2.83
g 8.94 h 7.21 i 6.71

3 $B(8, 0)$; $A(-6, 5)$, $A(-6, 9)$

4 a (−3, 1) b (1, −4) c (8, 2.5)

5 a 12.8 b 24.2

6 (0, 0), (0, 4), (2, 0), (2, 4)

7 a $x = \frac{x_1 + x_2}{2}$ b $y = \frac{y_1 + y_2}{2}$
c $M(1, -0.5)$

8 a i $x_2 - x_1$ ii $y_2 - y_1$
iii $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
b Answers may vary.

9 a $\frac{1}{3}$ b $\frac{1}{3}$
c i (1, 1) ii (−2, −0.5) iii (2, 1.5)
d i (−1, 1) ii (−2, 5)
iii (0.4, −1.8) iv (−2.4, 2.6)

4I

Building understanding

1 a yes b no c yes d no

2 a $-\frac{1}{5}$ b $\frac{1}{3}$

3 a yes b no c no d yes

Now you try

Example 17
$y = 4x - 2$

Example 18
$y = \frac{1}{3}x + 4$

Exercise 4I

1 a $y = 4x + 8$ b $y = -2x - 7$
c $y = \frac{2}{3}x - 5$ d $y = -\frac{4}{5}x + \frac{1}{2}$

2 a $y = -\frac{1}{3}x + 3$ b $y = -\frac{1}{5}x + 7$
c $y = \frac{1}{2}x - 4$ d $y = x + 4$
e $y = \frac{1}{7}x - \frac{1}{2}$ f $y = -x + \frac{5}{4}$

3 a i $y = 1$ ii $y = -2$
b i $x = 3$ ii $x = -3$
c i $x = 2$ ii $x = 0$
d i $y = 7$ ii $y = -\frac{1}{2}$

4 a $y = -3x + 9$ b $y = \frac{1}{2}x + \frac{5}{2}$
c $y = -\frac{1}{5}x + 6\frac{1}{5}$ d $y = x + 5$

5 $y = 0$, $y = x + 3$, $y = -x + 3$

6 $y = 2x - 10$ or $y = 2x + 10$

7 a i $-\frac{3}{2}$ ii -5
iii 7 iv $\frac{11}{3}$
b $-\frac{b}{a}$

8 a i $-\frac{1}{2}$ ii 3
b i 1 ii $-\frac{1}{7}$

9 a $y = -\frac{1}{2}x + 4$ b $y = \frac{2}{3}x + 1\frac{2}{3}$
c $y = 2x - 3$ d $y = -2x - 5\frac{1}{2}$
e $y = \frac{3}{7}x - \frac{5}{7}$ f $y = -\frac{5}{6}x + \frac{1}{6}$

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4J

Building understanding

1 B

2 a i \$1200 ii \$1500 iii \$2200
  b $A = 1000 + 100n$

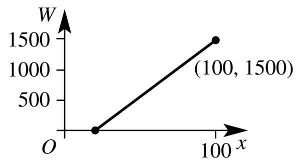
3 a 46 b 3.5 c 2 d 3

Now you try

Example 19

a $W = 500 + 10x$

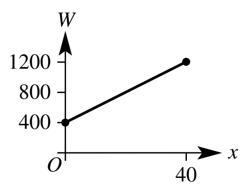
b



c \$750 d 35

Exercise 4J

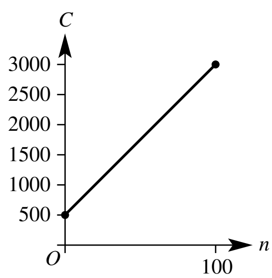
1 a $W = 20x + 400$
  b



(Note: This graph should really be a series of points but a full line segment is used as a guide.)

  c \$640 d 30

2 a $C = 25n + 500$
  b

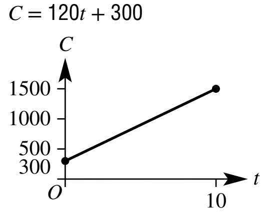


(Note: This graph should really be a series of points but a full line segment is used as a guide.)

  c \$1500 d 70

3 a $C = 50n + 40$ b \$240 c \$640

4 a $C = 120t + 300$
  b

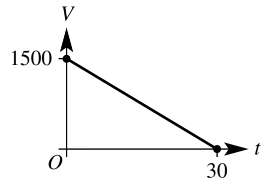


  c \$1020 d 3 hours

5 a $F = 18t + 12$ b 3 minutes c 2.5 minutes

6 a $V = 4000 - 20t$ b 2200 L
  c 200 minutes or 3 hours 20 minutes
  d 2 hours 55 minutes or 175 minutes

7 a



  b $V = 1500 - 50t$
  c the number of litres drained per minute
  d 1250 L e 15 minutes

8 a 80 km/h
  b rate changes, i.e. new gradient = 70, $d = 50 + 70t$

9 a 20 m/s b -20 m/s
  c 350 m d 17.5 s
  e increasing altitude at rate of 20 m/s

10 a i $C = 0.4n + 20$ ii 160
   b i $P = 0.8n - 20$ ii 25
   iii 120

11 a $C = 10x + 6700$ b \$8700
   c 630 d \$11 700
   e 670 f $P = 10x - 6700$
   g $T = \dfrac{10x - 6700}{x}$ h 1340

4K

Building understanding

1 a (1, 3) b (−1, 2) c (2, 2)

2 a true b false c false d true

3 a yes b yes c yes

Now you try

Example 20

a yes b no

Example 21

(2, −1)

Exercise 4K

1 a yes b no c no
  d yes e no f no

2 a (2, 2) b (3, 2) c (2, −4)
  d (3, 2) e (2, 1) f (3, 7)
  g (1, 2) h (2, 4) i (4, 1)
  j (−1, 3) k (−1, −5) l (1, 3)

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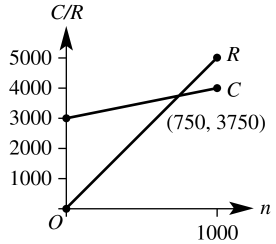
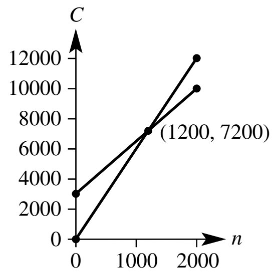
3 a 

(Note: This graph should really be a series of points but a full line segment is used as a guide.)

b 750

4 1200 baskets



(Note: This graph should really be a series of points but a full line segment is used as a guide.)

5 10 seconds

6 parallel lines, i.e. same gradient

7 a (0, 0) b (0, 0)
c no intersection d no intersection
e (0, 1) f (0, 3)

8 a 2 b −3

9 a (1, 3) b $A = 6.75$ units^2

10 a 18.75 units^2 b 24 units^2 c 16 units^2
d 27 units^2 e 7 units^2

Problems and challenges

1 20 days

2 2.5 hours

3 (0.5, 5.5), diagonals intersect at their midpoint

4 $k = -2.5$ or 13.5

5 length AB = length AC

6 $A = 13.5$ square units, $P = 20$ units

7 (6, 7)

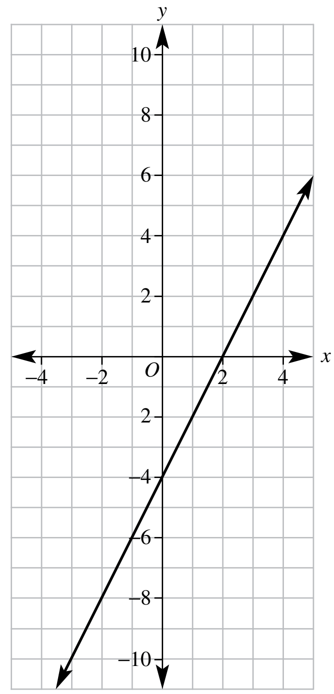
8 a BD: $y = -2x + 1$, AC: $y = \frac{1}{2}x - \frac{3}{2}$
b $b = -4$, $x = 11$ c 80 units^2

9 $k = 11.5$ or -10.5

Success criteria example questions

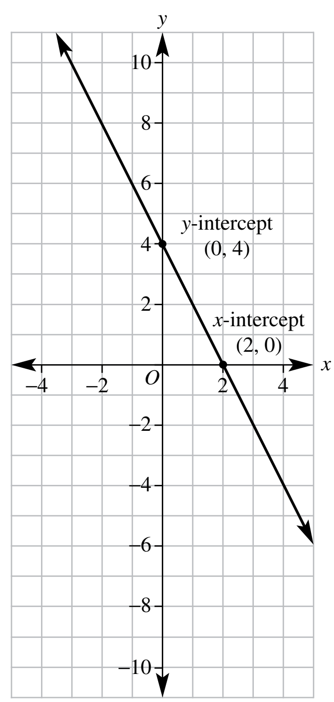
1

x	−3	−2	−1	0	1	2	3
y	−10	−8	−6	−4	−2	0	2



2 The x-intercept is at (5, 0); the y-intercept is at (0, 3)

3 The point is not on the line

4 

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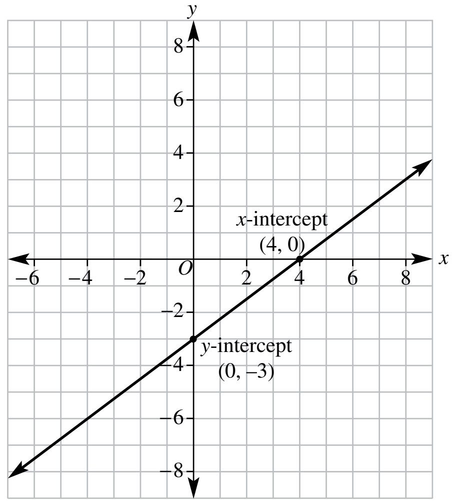
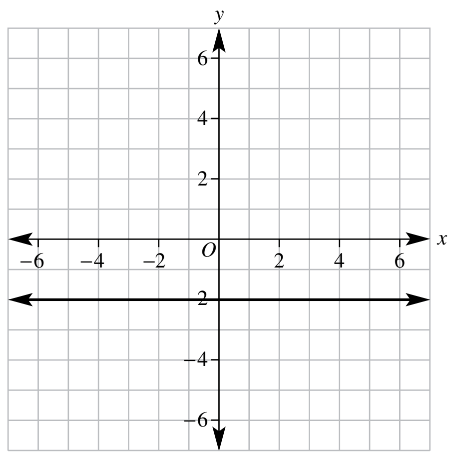
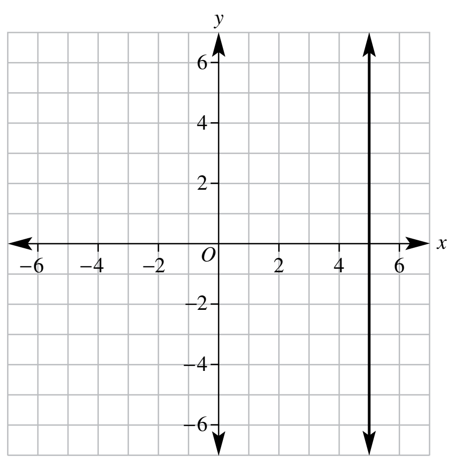
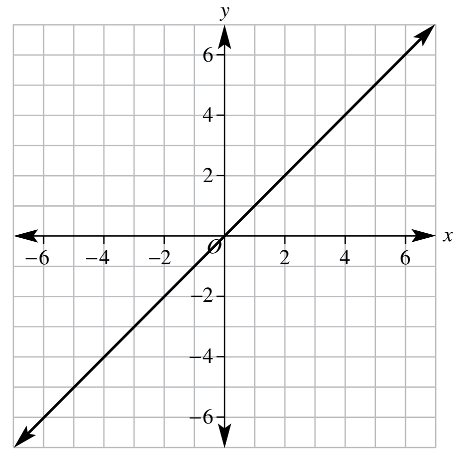


5





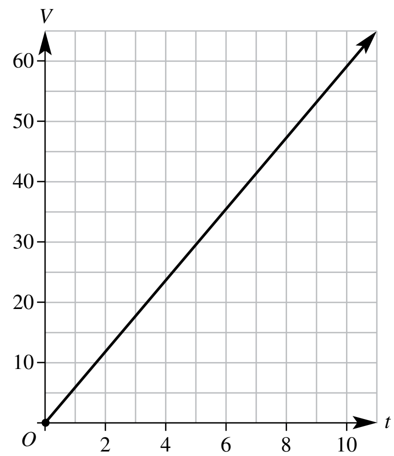
6



7 Negative, -2; zero, 0

8 $\frac{6}{5}$

9



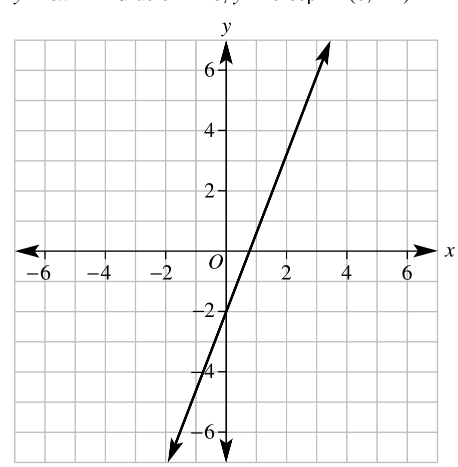
$V = 6t$; it takes $7\frac{1}{2}$ minutes to fill to 45 L

10 $y = -\frac{2}{3}x + \frac{8}{3}$

11 $y = -2x + 3$: Gradient $= -2$, y-intercept $= (0, 3)$

$2y - 4x = 5$: Gradient $= 2$, y-intercept $= \left(0, \frac{5}{2}\right)$

12 $y = 3x - 2$: Gradient $= 3$, y-intercept $= (0, -2)$



$2x + 3y = 8$: Gradient $= -\frac{2}{3}$, y-intercept $= \left(0, \frac{8}{3}\right)$

Answers Ch4 review

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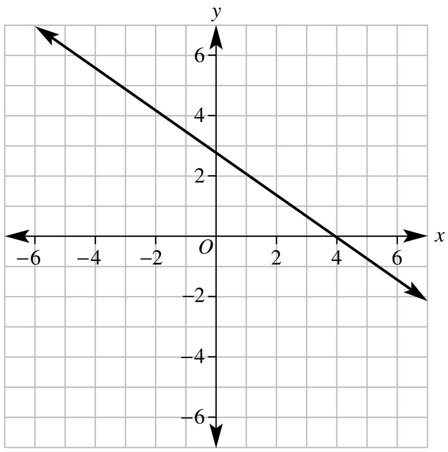


13 $y = 2x - 2$

14 $y = 2x + 9$

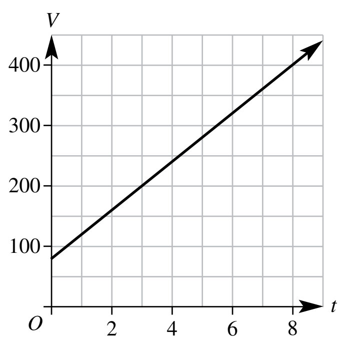
15 $\left(4, \frac{3}{2}\right)$

16 6.40

17 $y = -2x + 1$

18 $y = -\frac{1}{4}x - 3$

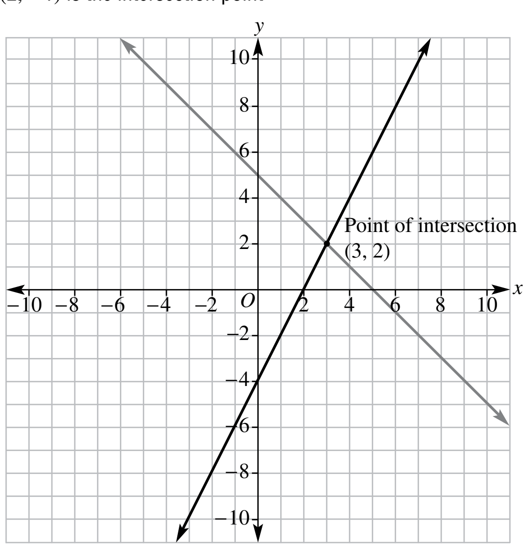
19 $C = 80 + 40h$



It would cost $280 for 5 hours; for 3 hours it would cost $200

20 $(2, -1)$ is the intersection point

21

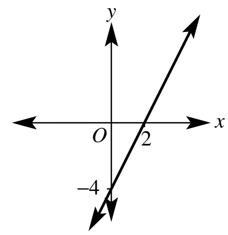


The intersection point is $(3, 2)$

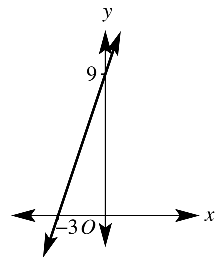
Short-answer questions

1 a $(-2, 0)$, $(0, 4)$ b $(3, 0)$, $(0, -2)$

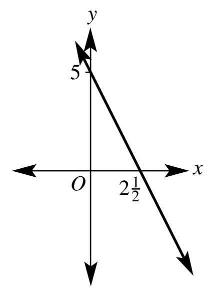
2 a



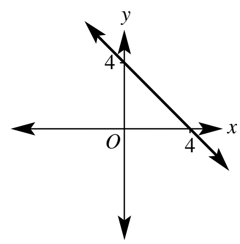
b



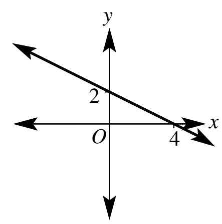
c



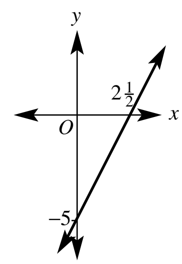
d



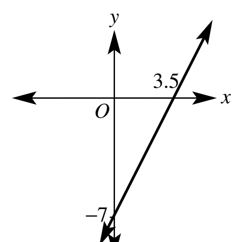
e



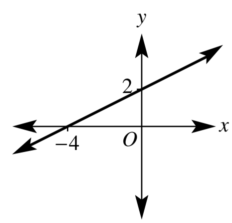
f



g



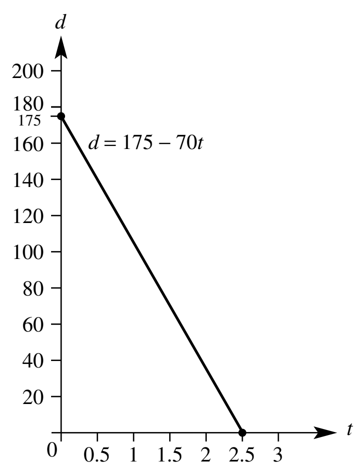
h



3 a 175 km

b 2.5 hours

c



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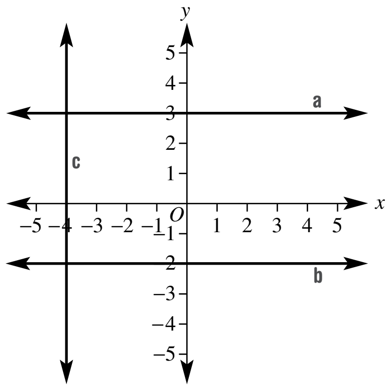
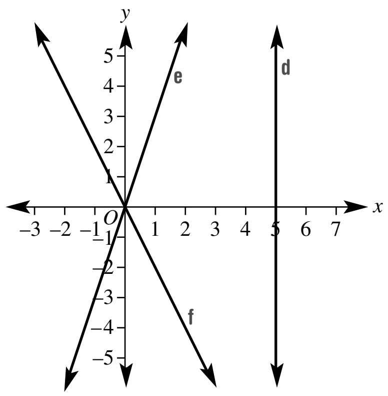
4 **a** $y = 3$ **b** $y = -2$ **c** $x = -4$



d $x = 5$ **e** $y = 3x$ **f** $y = -2x$

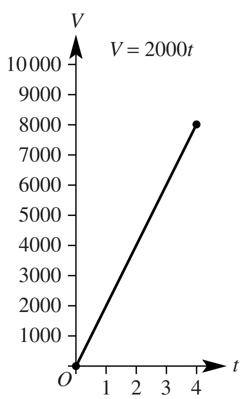


5 **a** 2 **b** −1 **c** $-\frac{5}{2}$

d $\frac{4}{3}$ **e** 2 **f** $-\frac{10}{3}$

6 **a** 2000 L/h

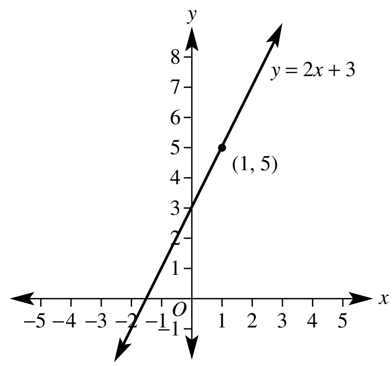
b



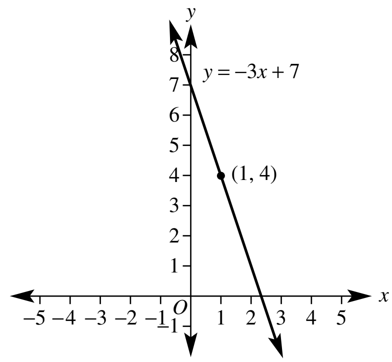
c $V = 2000t$

d 2.5 hours

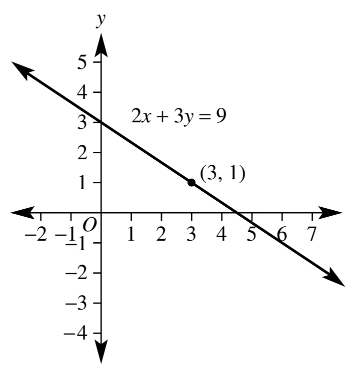
7 **a** gradient = 2 y-intercept = 3



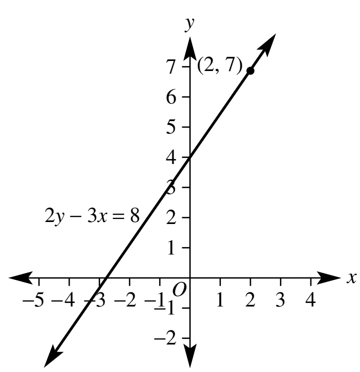
b gradient = −3 y-intercept = 7



c gradient = $-\frac{2}{3}$ y-intercept = 3



d gradient = $\frac{3}{2}$ y-intercept = 4



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8 a $y = 3x + 2$ b $y = -2x + 6$ c $y = \frac{4}{3}x - 5$

9 a $y = \frac{x}{2} - 1$ b $y = -3x + 6$
c $y = 3x - 2$ d $y = -2x + 2$

10 a i $M(4, 6)$ ii 5.66
b i $M(7.5, 4.5)$ ii 7.07
c i $M(0, 4)$ ii 7.21
d i $M(-3, 2.5)$ ii 9.85

11 a $n = 9$ b $n = 10$ c $n = 6$

12 a $y = 2x + 4$ b $y = -x - 3$
c $y = -\frac{1}{2}x - 1$ d $y = 3x + 4$
e $y = 3x + 1$ f $3x + 2y = 8$ or $y = -\frac{3x}{2} + 4$

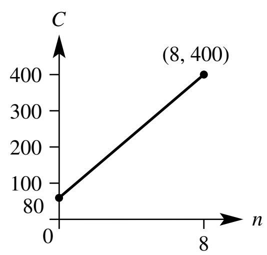
13 a no b yes

14 a $(2, 0)$ b $(-2, 4)$

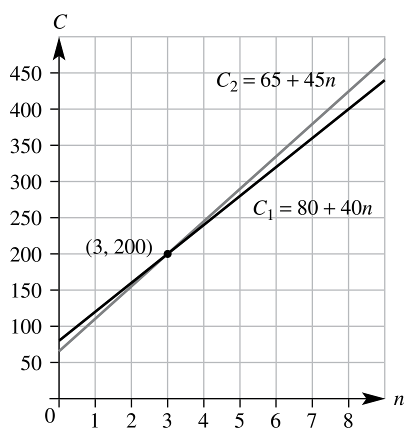
Multiple-choice questions

1 C	2 D	3 A	4 C	5 E
6 B	7 B	8 A	9 D	10 D

Extended-response questions

1 a i \$40/h, \$80
ii


iii \$180 iv 5 hours

b i $C = 65 + 45n$
ii


c $(3, 200)$ d after 3 hours

2 a $C = 8v + 90$
b i \$90 ii \$8 per vase
c 23 vases

Chapter 5

5A

Building understanding

1 a 50 mm b 280 cm c 52.1 cm
d 0.837 m e 4600 m f 2.17 km

2 825 cm, 2.25 cm

3 a $a = 3, b = 6$ b $a = 12, b = 4$
c $a = 6.2, b = 2$ d $a = 9, b = 7$

Now you try

Example 3
a 22 m b 20 cm

Example 4
7 mm

Exercise 5A

1 a 12 m b 27 cm c 18 km d 10 m

2 a 32 cm b 80 cm c 170 cm
d 30.57 m e 25.5 cm f 15.4 km

3 a 9 cm b 4015 m c 102.1 cm

4 a 8000 mm b 110 m c 1 cm
d 20 mm e 0.284 km f 62.743 km

5 a $x = 4$ b $x = 2.2$ c $x = 14$
d $x = 9.5$ e $x = 6$ f $x = 4.2$

6 108 m

7 a 86 cm b 13.6 m

8 a $x = 2$ b $x = 2.1$ c $x = 7$

9 88 cm

10 a $P = 2a + 2b$ b $P = 4x$ c $P = 2a + b$
d $P = 2x + 2y$ e $P = 4(a + b)$ f $P = 2x$

11 All vertical sides add to 13 cm and all horizontal sides add to 10 cm.

12 a 8.3% b 8.3%
c The percentage error is the same.

13 a 25 cm, 75 cm
b 40 cm, 60 cm
c 62.5 cm, 37.5 cm
d 10 cm, 20 cm, 30 cm, 40 cm

14 a i 96 cm ii 104 cm iii 120 cm
b $P = 4(20 + 2x)$, $\therefore P = 8x + 80$
c i 109.6 cm ii 136.4 cm
d i $x = 1.25$ ii $x = 2.75$
e no, as with no frame the picture has a perimeter of 80 cm

5B

Building understanding

1 a 2.8 cm b 96 mm

2 a 6π b 12π c $3 + 2\pi$
d $12 + 3\pi$ e $8 + 2\pi$ f $3 + \frac{3\pi}{4}$

3 a $\frac{1}{4}$ b $\frac{1}{2}$ c $\frac{3}{4}$
d $\frac{1}{6}$ e $\frac{5}{12}$ f $\frac{5}{8}$

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Now you try

Example 5
a 25.13 m b 6.79 mm

Example 6
a 12π m b $16 + 8\pi$ cm

Exercise 5B

1 a 87.96 cm b 9.42 mm c 12.57 km
2 a 9.14 cm b 14.94 m c 33.13 cm
d 10.00 cm e 20.05 m f 106.73 km
3 a 12.56 m b 62.8 cm c 22 mm d 44 m
4 a 14π cm b 4π m c 41π km
d 10π cm e 20π m f 11π mm
5 a $8 + 2\pi$ m b $4 + 2\pi$ cm c $10\pi + 20$ cm
d $12 + 2\pi$ km e $5\pi + 6$ m f $5\pi + 8$ cm
6 28.27 m
7 4.1 m
8 31.42 cm
9 a 188.50 cm
b i 376.99 cm ii 1979.20 cm
c 531
10 a 11.5π m b $2.4 + 0.6\pi$ cm c $21 + 3.5\pi$ m
d $5 + 1.875\pi$ km e $40 + 20\pi$ mm f $23 + 5.75\pi$ m
11 a $r = \dfrac{C}{2\pi}$
b i 1.6 cm ii 4.0 m
c $d = \dfrac{C}{\pi}$ d 67 cm
12 a 131.95 m b 791.68 m
c i 3.79 ii 15.16
d 63.66 m

5C

Building understanding

1 a 6 b 16 c 12
d 1 e 12 f 153
2 a rectangle b circle c rhombus/kite
d sector of circle e triangle f trapezium
g parallelogram h square i semicircle
3 a $\frac{1}{4}$ b $\frac{1}{3}$ c $\frac{3}{4}$
d $\frac{1}{6}$ e $\frac{5}{8}$ f $\frac{5}{18}$
4 a 300 cm and 100 cm
b 30 000 cm^2

Now you try

Example 7
a 1.421 m^2 b 316 mm^2

Example 8
a 40 m^2 b 36 mm^2 c 7.2 m^2

Example 9
a 80 km^2 b 34 m^2

Example 10
a 36.32 m^2 b 21.99 cm^2

Exercise 5C

1 a i 2.36 cm^2 ii 4.8 m^2
b i 41 600 cm^2 ii 350 mm^2
2 a 200 mm^2 b 5 cm^2 c 21 000 cm^2
d 21 m^2 e 1000 m^2 f 3.2 km^2
3 a 24 m^2 b 10.5 cm^2 c 20 km^2
d 25.2 m^2 e 15 m^2 f 36.8 m^2
4 a 21 mm^2 b 12 cm^2 c 17 cm^2
d 63 m^2 e 6.205 m^2 f 15.19 km^2
5 a 12.25 cm^2 b 3.04 m^2 c 0.09 cm^2
d 6.5 mm^2 e 18 cm^2 f 2.4613 cm^2
6 a 21.23 m^2 b 216.51 km^2 c 196.07 cm^2
7 a 7.07 m^2 b 157.08 cm^2 c 19.24 cm^2
d 84.82 m^2 e 26.53 m^2 f 62.86 m^2
8 a 1.5×10^{10}(15 000 000 000)cm^2
b 5 mm^2 c 0.075 m^2
9 500 000 m^2
10 0.51 m^2
11 12.89%
12 31%
13 a $r = \sqrt{\dfrac{A}{\pi}}$
b i 1.3 cm ii 1.5 m iii 2.5 km
14 a i 64° ii 318°
b as angle would be greater than 360°, which is not possible (28.3 m^2 is the largest area possible, i.e. full circle)
15 a 6.35 cm and 6.45 cm
b 25.4 cm and 25.8 cm
c 40.3 cm^2 and 41.6 cm^2
16 a i 1.5 m ii 1.5 m
b 78 m^2
c yes
17 46.7%

5D

Building understanding

1 a semicircle and rectangle b triangle and semicircle
c rhombus and parallelogram

2 a
$$\begin{aligned} P &= 2 \times 5 + 3 + \frac{1}{2} \times 2\pi r \\ &= 10 + 3 + 1.5\pi \\ &= 13 + 1.5\pi \\ &= 17.7 \text{ m} \end{aligned} \qquad \begin{aligned} A &= bh + \frac{1}{2}\pi r^2 \\ &= 5 \times 2 + \frac{1}{2} \times \pi \times 1.5^2 \\ &= 10 + 1.125\pi \\ &= 13.5 \text{ m}^2 \end{aligned}$$

b
$$\begin{aligned} P &= 20 + 12 + 12 + 10 + 6 \\ &= 60 \text{ cm} \end{aligned} \qquad \begin{aligned} A &= lb - \frac{1}{2}bh \\ &= 12 \times 20 - \frac{1}{2} \times 8 \times 6 \\ &= 240 - 24 \\ &= 216 \text{ cm}^2 \end{aligned}$$

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Now you try

Example 11
$P = 16.71$ m, $A = 16.07$ m^2

Example 12
24 cm^2

Exercise 5D

1 a 46 m, 97 m^2
 b 18.28 m, 22.28 m^2
 c 34 m, 76 m^2
 d 40 m, 90 m^2
 e 19.42 m, 26.14 m^2
 f 85.42 mm, 326.37 mm^2

2 a 17 cm^2 b 3.5 cm^2 c 21.74 cm^2
 d 6.75 m^2 e 189 cm^2 f 115 cm^2

3 a 108 m^2 b 33 cm^2 c 98 m^2
 d 300 m^2 e 16 cm^2 f 22.5 m^2

4 a 90 cm^2 b 15 m^2 c 9 m^2
 d 7.51 cm^2 e 7.95 m^2 f 180.03 cm^2
 g 8.74 mm^2 h 21.99 cm^2 i 23.83 mm^2

5 189.27 m^2

6 68.67 cm^2

7 a 37.70 m, 92.55 m^2
 b 20.57 mm, 16 mm^2
 c 18.00 cm, 11.61 cm^2
 d 12.57 m, 6.28 m^2
 e 25.71 cm, 23.14 cm^2
 f 33.56 m, 83.90 m^2

8 a 136.3 cm^2 b 42.4 m^2 c 345.6 m^2

9 8 cm

10 a $36 + 18\pi$ b 16 c $12 - \frac{\pi}{8}$
 d 2π e $12.96 + 3.24\pi$ f $25 + \frac{75\pi}{4}$

11 7.1 cm

12 a hypotenuse (diameter) would equal 4.24 not 5
 b hypotenuse (sloped edge) should be 13 cm not 14 cm
 c hypotenuse (diameter) should be 5.83 not 8 m

13 5267.1 cm^2

14 a 34 cm, 18 cm b 226.9 cm^2 c 385.1 cm^2

Progress quiz

1 20%

2 25.35 cm and 25.45 cm

3 a 3.54 m b 15.2 cm

4 a $C = 25.1$ mm, $A = 50.3$ mm^2
 b $C = 37.7$ cm, $A = 113.1$ cm^2

5 a 15 cm^2 b 72 cm^2 c 16π cm^2

6 850 cm^2

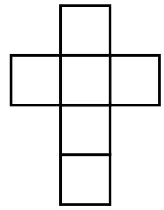
7 a $P = 29.71$ m, $A = 63.27$ m^2
 b $P = 32$ cm, $A = 44$ cm^2

8 103.67 m^2

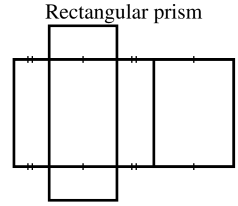
5E

Building understanding

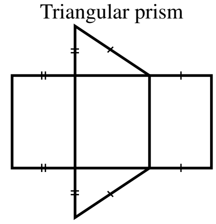
1 a Cube



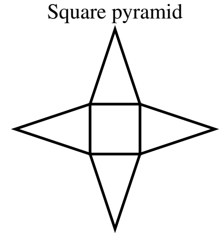
b Rectangular prism



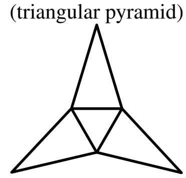
c Triangular prism



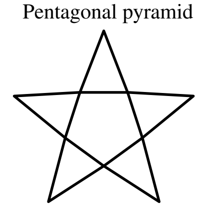
d Square pyramid



e Tetrahedron (triangular pyramid)



f Pentagonal pyramid



2 a
$$\begin{aligned} \text{TSA} &= 2 \times 8 \times 7 + 2 \times 8 \times 3 + 2 \times 7 \times 3 \\ &= 112 + 48 + 42 \\ &= 202\ \text{m}^2 \end{aligned}$$

b
$$\begin{aligned} \text{TSA} &= 2 \times \frac{1}{2} \times 4 \times 3 + 5 \times 7 + 4 \times 7 + 3 \times 7 \\ &= 12 + 35 + 28 + 21 \\ &= 96\ \text{cm}^2 \end{aligned}$$

Now you try

Example 13
a 202 m^2 b 85 m^2

Exercise 5E

1 a 242 cm^2 b 192 cm^2 c 85.76 m^2

2 a 224 cm^2 b 39 mm^2 c 9.01 m^2

3 a 108 cm^2 b 199.8 cm^2 c 0.96 m^2
 d 44.2 m^2 e 22 cm^2 f 28 cm^2

4 6 m^2

5 14.54 m^2

6 34 000 cm^2

7 a 44.4 m^2 b 4.44 L

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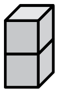
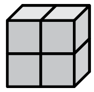
8 a $1400\,\text{cm}^2$ b $1152\,\text{cm}^2$

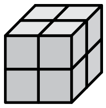
9 a $10\,\text{cm}^2$



b $16\,\text{cm}^2$



c $24\,\text{cm}^2$



10 a [6, 10, 14, 18, 22, 26, 30, 34, 38]

b $S = 4n + 2$

c Answers may vary.

11 a $17.7\,\text{cm}^2$

b $96\,\text{m}^2$

c $204\,\text{cm}^2$

d $97.9\,\text{m}^2$

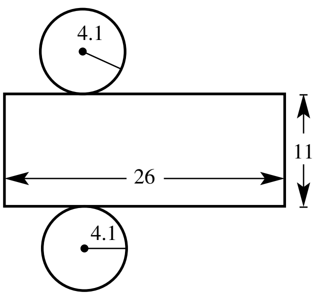
e $137.8\,\text{cm}^2$

f $43.3\,\text{mm}^2$

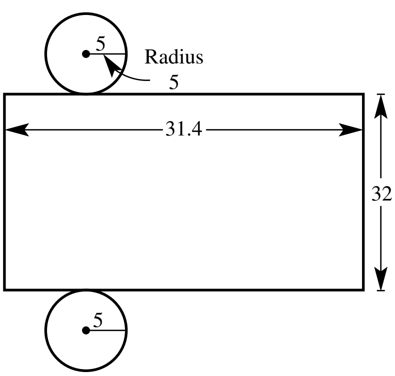
5F

Building understanding

1 a



b



2 a 22 cm by 10 cm

b 12.57 cm by 8 cm

c 50.27 m by 9 m

Now you try

Example 14

$716.28\,\text{cm}^2$

Example 15

$182.52\,\text{m}^2$

Exercise 5F

1 a $25.13\,\text{m}^2$ b $471.24\,\text{cm}^2$ c $50.27\,\text{m}^2$

2 a $44.0\,\text{cm}^2$ b $603.2\,\text{cm}^2$ c $113.1\,\text{m}^2$

3 $395.84\,\text{cm}^2$

4 a $251.33\,\text{cm}^2$ b $207.35\,\text{mm}^2$ c $24.13\,\text{m}^2$

5 a $54.56\,\text{m}^2$ b $218.23\,\text{m}^2$ c $63.98\,\text{cm}^2$

d $71.91\,\text{cm}^2$ e $270.80\,\text{m}^2$ f $313.65\,\text{km}^2$

g $326.41\,\text{m}^2$ h $593.92\,\text{m}^2$ i $43.71\,\text{mm}^2$

6 $7539.82\,\text{cm}^2$

7 $80\,424.8\,\text{cm}^2$

8 a $18\,849.556\,\text{cm}^2$

b i $1.88\,\text{m}^2$ ii $37.70\,\text{m}^2$

c 239

9 a $8\pi\,\text{m}^2$ b $150\pi\,\text{cm}^2$ c $16\pi\,m^2$

10 Surface area of half cylinder is more than half surface area of a cylinder as it includes new rectangular surface.

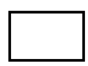
11 a $\left(\frac{135\pi}{2} + 36\right)\text{cm}^2$ b $\left(\frac{70\pi}{3} + 12\right)\text{cm}^2$

c $\left(\frac{29\pi}{12} + 4\right)\text{m}^2$

5G

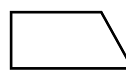
Building understanding

1 a



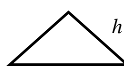
$h = 6$ cm

b



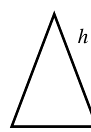
$h = 10$ cm

c



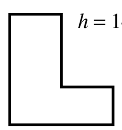
$h = 2$ m

d



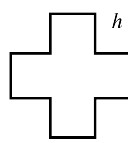
$h = 5$ mm

e



$h = 14$ cm

f



$h = 10$ m

2 a $8\,\text{cm}^3$ b $84\,\text{m}^3$ c $21\,\text{mm}^3$

Now you try

Example 16

a $4800\,\text{mm}^3$

b $0.612\,\text{m}^3$

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Example 17

a 12 cm^3 b 8 cm^3 c 15 m^3

Exercise 5G

1 a 3000 mm^3 b 2 cm^3
c $87\,00\,000\text{ cm}^3$ d 0.0059 m^3
e $10\,000\text{ m}^3$ f 0.0000217 km^3
g 3000 mL h 200 L
i 3.5 L j 21 mL
k 37 kL l 42.9 mL

2 a 4 cm^3 b 18 m^3
c 30 mm^3 d 16 cm^3
e 42.875 m^3 f 15 cm^3

3 a 4 cm^3 b 10.5 m^3
c 11.96 cm^3 d 29 cm^3
e 14.88 m^3 f 8.1351 cm^3
g 108 m^3 h 29.82 m^3
i 0.382044 cm^3

4 a 75 m^3 b 30 cm^3 c 1.25 cm^3

5 a 8 L b 125 L
c 0.36 L d 0.48 L

6 8000 cm^3

7 0.19 m^3

8 Yes, the tank only holds 20 L

9 a 67.2 cm^3 b 28 m^3
c 8.9 km^3 d 28 m^3
e 0.4 m^3 f $29\,232\text{ mm}^3$

10 a 20 L b 75 000 L c 8000 L

11 a 55 m^2 b 825 000 L

12 a i 1000 ii $\frac{1}{1000}$ iii 1000
b i 1 000 000 ii 1000
iii 1 000 000

13 a $V = x^2h$ b $V = s^3$ c $V = 6t^3$

14 a Answers may vary. b $\frac{1}{3}$
c Answers may vary.

5H

Building understanding

1 a $r = 4, h = 10$ b $r = 2.6, h = 11.1$
c $r = 9, h = 23$

2 a 2 L b 4.3 mL c 3700 cm^3
d 1000 L e 38 m^3 f 200 mL

Now you try

Example 18

a 169.65 m^3 b 13.57 cm^3

Example 19

503 L

Exercise 5H

1 a 226.19 cm^3
b 137.44 m^3
c 18.85 m^3
d 8.48 m^3
e 100.53 cm^3
f 68.05 m^3

2 a 13 L b 503 L c 20 L
d 4712 L e 589 049 L f 754 L

3 a 25.133 m^3 b 25 133 L

4 37 699 L

5 cylinder by 0.57 m^3

6 a 502.65 cm^3
b 1.02 m^3
c 294.52 m^3
d 35342.92 m^3
e 47.12 cm^3
f 1017.88 cm^3

7 a 0.707 b 2.523

8 a $160\pi\text{ m}^3$
b $320\pi\text{ cm}^3$
c $54\pi\text{ km}^3$
d $\frac{3\pi}{4}\text{ cm}^3$
e $1500\pi\text{ cm}^3$
f $144\pi\text{ mm}^3$

9 Answers may vary, but require $h = 2\pi r$.

10 a 113.10 cm^3
b $10\,471.98\text{ m}^3$
c 3.73 m^3
d 20.60 cm^3
e 858.41 cm^3
f 341.29 m^3

Problems and challenges

1 100 L

2 non-shaded is half the shaded area

3 157.1 m^2

4 $\sqrt{200}$ cm = 14.14 cm

5 $\frac{1}{6}$ cm

6 $V = 2\pi^2r^3$

7 $h = \frac{1 - r^2}{r}$

8 Answers may vary, but $1000 = \pi r^2h$ needs to hold true for r and h in centimetres. Designers need to consider production costs, material costs and keeping the surface area to a miniumum so that they can maximise profits, as well as the ability to use, stack and market their products. If the container is for cold storage, then fitting into a standard fridge door is also a consideration.

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Success criteria example questions

1 23%
2 6.65 kg and 6.75 kg
3 20 cm
4 34 cm
5 $x = 4$
6 37.70 cm
7 9.93 m
8 0.32 m^2; 240 mm^2
9 40 m^2; 30 m^2
10 40 m^2; 40 cm^2
11 12.57 m^2
12 104.72 cm^2
13 Perimeter = 33.42 cm; area = 64.27 cm^2
14 76 m^2
15 180 cm^2
16 256 cm^2
17 164.93 m^2
18 167.96 cm^2
19 3 000 000 cm^3; 1.276 cm^3
20 200 cm^3
21 336 cm^3
22 20.36 m^2
23 101 L

Short-answer questions

1 a 380 cm b 1270 m c 2.73 cm^2
d 52 000 cm^2 e 10 000 cm^3 f 53.1 cm^3
g 3.1 L h 43 mL i 2830 L
2 a 4% b 40%
3 a 14 m b 51 mm c 16.2 cm
4 a 4 cm^2 b 1122 mm^2 c 30.34 mm^2
d 7.5 m^2 e 15 cm^2 f 3 cm^2
5 a 2.5 m^2 b 37.4 m^2
6 a $A = 28.27$ cm^2, $P = 18.85$ cm
b $A = 5.38$ m^2, $P = 9.51$ m
7 a $P = 15.24$ m, $A = 13.09$ m^2
b $P = 14.10$ m, $A = 10.39$ m^2
c $P = 24.76$ km, $A = 33.51$ km^2
8 a 8.86 m, 4.63 m^2 b 45.56 cm, 128.54 cm^2
9 a 46 cm^2 b 114 m^2
10 a 659.73 mm^2 b 30.21 m^2
11 a 30 cm^3 b 54 m^3 c 25 L

Multiple-choice questions

1 E 2 B 3 C 4 B 5 A
6 D 7 B 8 E 9 C 10 E

Extended-response questions

1 a 517.08 cm b $65 c 15 853.98 cm^2
d 1.58 m^2; claim is correct
2 a 1 cm b 15.71 cm^2 c 125 680 cm^3
d 0.125 68 m^3 e 18.85 cm f 15 m^2
g $1200

Semester review 1

Computation and financial maths

Short-answer questions

1 a $\frac{19}{28}$ b $\frac{7}{9}$ c $\frac{3}{8}$ d $1\frac{4}{5}$
2 a 60% b 31.25% c 10% d 25%
3 a 5 : 3 b 55 km/h c 2.4 mL/h
4 a $67.20 b 64.4 cm
5 $892

Multiple-choice questions

1 B 2 E 3 D 4 D 5 A

Extended-response question

a i $17 500
ii $23 520
iii 9 years
iv 27%
b $23 635.69
c i Jim by $116 ii Jim by $922

Expressions, equations and inequalities

Short-answer questions

1 a $x = 6$ b $x > \frac{9}{2}$
c $m = \frac{3}{8}$ d $y = -1$
e $a \leqslant \frac{1}{11}$ f $x = \frac{-1}{3}$
2 a $\frac{m-3}{2} = 6$
b Noah gets $15 pocket money
3 a 155
b $I = \frac{2S}{n} - a$
c 18
4 a $x = 6, y = 3$
b $x = -1, y = -5$
c $x = 5, y = -2$
d $x = -3, y = 4$

Multiple-choice questions

1 E 2 A 3 C 4 B 5 D

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Extended-response question

a i $12x + 20 > 74$ ii 5 games

b i Let $\$x$ be the cost of a raffle ticket and $\$y$ the cost of a badge.

ii $5x + 2y = 11.5$ and $4x + 3y = 12$

iii A raffle ticket costs $1.50 and a badge costs $2.

Right-angled triangles: Pythagoras' theorem and trigonometry

Short-answer questions

1 a $x = 15.1$ b $x = 5.7$

c $x = 11.2$ d $\theta = 29.5°$

2 a $x = 13, y = 14.7$ b $y = 9.4, x = 9.9$

3 a 19.21 m b 38.7°

4 a 16.3 km west b 115°

Multiple-choice questions

1 D 2 A 3 C 4 A 5 E

Extended-response question

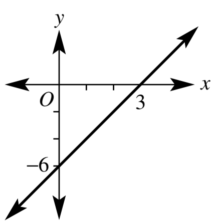
a 17.75 m b 14.3°

c i 18.8 m ii 6.8 s

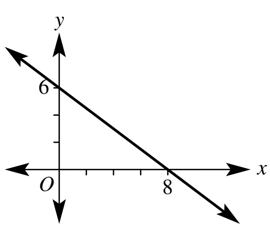
Linear relationships

Short-answer questions

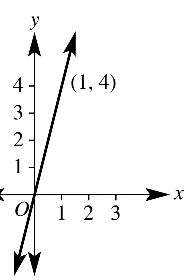
1 a



b



c



2 a $\frac{2}{3}$ b -3 c -2 d $\frac{4}{3}$

3 a $y = -3x + 6$ b $y = 3x - 1$

c $y = 2x$ d $y = -\frac{1}{3}x + 2$

4 (3, 2)

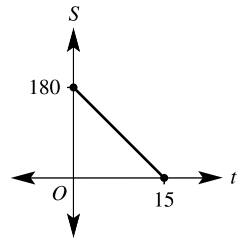
Multiple-choice questions

1 C 2 D 3 B 4 C 5 A

Extended-response question

a 12 kg/h b $S = 180 - 12t$

c



d 15 hours

e i $P = 40 + 25h$ ii $415

Length, area, surface area and volume

Short-answer questions

1 a 6.25% b 6.35 kg, 6.45 kg

2 a 24.57 m^2 b 36 cm^2

3 4 tins

4 a 216 m^2 b 25.45 m^2

5 a 192 cm^3 b 156.4 cm^3

Multiple-choice questions

1 C 2 E 3 D 4 A 5 A

Extended-response question

a 45.71 m^2

b i 0.0377 m^2 ii 3.77 m^2

c 1213 d 37.85 m^3 e 19.45 m^3

Chapter 6

6A

Building understanding

1 a 2 b 3 c 3 d 2

2 a 3 b -7 c $\frac{2}{3}$ d y

3 a 3 b 7 c 4 d 13

4 a 2, 3 b 3, 5 c 2, 3, 5 d 7, 11

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Now you try

Example 1

a $b \times b \times b \times b$ b $mn \times mn \times mn$ c $5 \times a \times a \times b$

Example 2

a 16 b -27 c $\frac{16}{9}$

Example 3

a $2y^3$ b $\left(\frac{5}{9}\right)^3 \times \left(\frac{3}{7}\right)^2$ c 3^3a^2

Example 4

$2^2 \times 3 \times 5$

Exercise 6A

1 a $b \times b \times b$ b $x \times x \times x$
 c $5 \times a \times 5 \times a \times 5 \times a \times 5 \times a$
 d $3 \times y \times 3 \times y \times 3 \times y$ e $p \times q \times p \times q$
 f $-3 \times s \times s \times s \times t \times t$
 g $6 \times x \times x \times x \times y \times y \times y \times y \times y$
 h $5 \times y \times z \times y \times z \times y \times z \times y \times z \times y \times z \times y \times z$

2 a 36 b 16 c 243 d 12
 e -8 f -1 g 81 h 25
 i $\frac{8}{27}$ j $\frac{9}{16}$ k $\frac{1}{216}$ l $\frac{25}{4}$
 m $-\frac{8}{27}$ n $\frac{81}{256}$ o $\frac{1}{16}$ p $-\frac{3125}{32}$

3 a 3^3 b 8^6 c y^2 d $3x^3$
 e $4c^5$ f 5^3d^2 g x^2y^3 h 7^3b^2

4 a $\left(\frac{2}{3}\right)^4$ b $\left(\frac{3}{5}\right)^5$
 c $\left(\frac{4}{7}\right)^2\left(\frac{1}{5}\right)^4$ d $\left(\frac{7x}{9}\right)^2\left(\frac{y}{4}\right)^3$

5 a $3^3x^3y^2$ b $(3x)^2(2y)^2$ or $3^22^2x^2y^2$
 c $(4d)^2(2e)^2$ or $4^22^2d^2e^2$ d $6^3b^2y^3$
 e $(3pq)^4$ or $3^4p^4q^4$ f $(7mn)^3$ or $7^3m^3n^3$

6 a 2×5 b 2^3 c $2^4 \times 3^2$
 d 2^9 e $2^3 \times 3^3$ f $2^2 \times 5^3$

7 a 36 b -216 c 1 d $-\frac{8}{27}$
 e -18 f 15 g -36 h 216

8 a 4 b 8 c 5 d 2
 e -4 f -2 g $\frac{1}{2}$ h 4

9 a i 10 min ii 20 min iii 30 min
 b $2^{24} = 16\,777\,216$ cells

10 a $1000 \times 3^5 = \$243\,000$ b 5 years

11 7 months

12 a i 9 ii 9 iii -9 iv -9
 b Same signs give positive when multiplying
 c A positive answer is multiplied by the -1 outside the bracket.

13 a i 8 ii -8 iii -8 iv 8
 b A positive cubed is positive.
 A negative answer is multiplied by negative.
 c A negative number cubed will be negative.
 A positive answer is multiplied by negative one.

14 a $\frac{1}{8}$ b $\frac{1}{16}$ c $\frac{1}{125}$
 d $\frac{1}{64}$ e $\frac{49}{100}$ f $\frac{81}{16}$
 g $\frac{169}{25}$ h $\frac{12769}{100}$ i $\frac{289}{25}$

15 a LCM = 12, HCF = 2 b LCM = 84, HCF = 14
 c LCM = 72, HCF = 12 d LCM = 30, HCF = 5
 e LCM = 360, HCF = 10 f LCM = 300, HCF = 10
 g LCM = 1764, HCF = 14 h LCM = 13 068, HCF = 198

6B

Building understanding

1 a multiply, base, add b divide, base, subtract

2 a $3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$
 b $\frac{5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5} = 5^2$

3 a true b true c false
 d false e true f false

Now you try

Example 5

a 2^9 b 9^4

Example 6

a x^{11} b a^3b^7 c $28m^5$

Example 7

a a^4 b $\frac{2x^3y}{5}$

Example 8

a a^5 b $2a^2b^2$

Exercise 6B

1 a i 2^7 ii 5^9
 b i 3^3 ii 6^5

2 a 7^6 b 8^{10} c 3^8 d 6^{14}
 e 5^3 f 10 g 9^3 h $(-2)^2$

3 a x^7 b a^9 c t^8 d y^5
 e d^3 f y^7 g b^8 h q^{11}
 i x^7y^5 j x^9y^4 k $5x^4y^9$ l $4x^2y^5z$
 m $15m^5$ n $8e^6f^4$ o $20c^7d^2$ p $18y^2z^7$

4 a a^2 b x^3 c q^{10} d d
 e $2b^5$ f $\frac{d^5}{3}$ g $2a^7$ h $2y^8$
 i $9m$ j $14x^3$ k $5y^2$ l $6a$
 m $\frac{m^5}{4}$ n $\frac{w}{5}$ o $\frac{a}{5}$ p $\frac{x^4}{9}$
 q $\frac{4x^6y^3}{3}$ r $\frac{3st^2}{7}$ s $\frac{4mn}{3}$ t $-5x$

5 a b^6 b y^6 c c^7 d x
 e t f p^6 g d^6 h x^{10}
 i $4x^2y^3$ j $6b^2g$ k $3m^5n^6$ l p^5q^4

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6 a $\frac{m^5}{n^5}$ b $\frac{x^4}{y^2}$ c a^3b^3
d $\frac{6a^5}{c^7}$ e $6f^6$ f $12x^4b^2$
g $6k^3m^3$ h $\frac{15x^4y}{2}$ i $\frac{3m^2n^3}{2}$

7 a 12 b 8 c 3 d 3
e 1 f 18 g 12 h 11
i 4 j 15 k 2 l 39

8 a $7^2 = 49$ b 10
c $13^2 = 169$ d $2^3 = 8$
e 101 f $200^2 = 40\,000$
g $7 \times 31 = 217$ h $43 \times 50^2 = 7500$

9 a 7 combinations b 14 combinations

10 a a^5, power of one not added
b x^6, power of one not subtracted
c $\frac{a^2}{2}$, $3 \div 6$ is $\frac{1}{2}$ not 2
d $\frac{x^4}{2}$, numerator power is larger hence x^4 in numerator
e $6x^{11}$, mutiply coefficients not add
f $a^3 \times a = a^4$, order of operations done incorrectly

11 a $4x$ b $12x^2$ c $10x^3$ d $-4x$
e $40x^6$ f $\frac{5x}{4}$ g $\frac{8}{5}$ h $-20x^4$

12 a 2^{x+y} b 5^{a+b} c t^{x+y}
d 3^{x-y} e 10^{p-y} f 2^{p+q-r}
g 10^{p-q-r} h 2^{5a} i $a^{3x-2}b^{x+3}$
j $a^{x+y}b^{x+y}$ k $a^{x-y}b^{y-x}$ l $w^{2-x}b^{x+3}$
m a^{x+y-2} n p^aq^{b-5} o $4m^{y-3-2x}$

13 Answers may vary.
14 Answers may vary.

6C

Building understanding

1 a multiply b 1
2 a 16, 8, 4, 2, 1 b 64, 16, 4, 1
3 a $(4 \times 4) \times (4 \times 4) \times (4 \times 4) = 4^6$
b $(a \times a) \times (a \times a) \times (a \times a) \times (a \times a) \times (a \times a) = a^{10}$

Now you try

Example 9
a a^6 b $5b^{14}$

Example 10
a 1 b −1 c 5

Example 11
a y^{18} b t^5 c m

Exercise 6C

1 a x^{10} b b^{12} c a^{20} d m^{12}
e 3^6 f 4^{15} g 3^{30} h 7^{10}
i $4q^{28}$ j $3m^{10}$ k $-3c^{10}$ l $-4a^{21}$

2 a 1 b 1 c 1 d 1
e −1 f 1 g 1 h 1

3 a 5 b −3 c 4 d −6
e 4 f 3 g 1 h 0

4 a 4^7 b 3^9 c x d y^{13}
e b^{14} f a^{10} g d^{24} h y^{16}
i z^{25} j $a^{11}f^{13}$ k $x^{14}y^5$ l $5rs^8$

5 a 7^2 b 4 c 3^8 d 1 e y^3
f h^2 g b^6 h x^5 i y^6

6 a $\frac{2}{x^5}$ b $\frac{10}{x^3}$ c $3x^8$
d $\frac{d^2e}{2}$ e $\frac{2m^6n}{5}$ f $\frac{a^{12}}{8}$

7 a i 400 ii 6400 iii 100
b i 800 ii 12 800 iii 102 400
c 13 years

8 5 ways

9 a 4 b 1000 c 1
d 1 e 4 f 1

10 a 4×5 not $4 + 5$, a^{20}
b power of 2 only applies to x^3, $3x^6$
c power zero applies to whole bracket, 1

11 a i 2^{24} ii $(-2)^{30} = 2^{30}$
iii x^{84} iv a^{48}
b i 2^{abc} ii a^{mnp} iii x^{6yz}

12 a Answers may vary. b Answers may vary.

13 a 2^{12} b 2^{15} c 3^6 d 3^{20} e 5^{10}
f 3^{50} g 2^{72} h 7^{80} i 10^{50}

6D

Building understanding

1 a $a^m \times b^m$ b $\frac{a^m}{b^m}$

2 a $5a \times 5a \times 5a$
$= 5 \times 5 \times 5 \times a \times a \times a$
$= 5^3 \times a^3$
b $\frac{x}{6} \times \frac{x}{6} \times \frac{x}{6}$
$= \frac{x \times x \times x}{6 \times 6 \times 6}$
$= \frac{x^3}{6^3}$

Now you try

Example 12
a $9a^2$ b $-27x^6y^9$ c $5a^8b^4$ d $16b^{11}$

Example 13
a $\frac{32}{x^5}$ b $\frac{9m^6}{4n^2p^4}$ c $\frac{x^5y^4}{72}$

Exercise 6D

1 a $25y^2$ b $64a^6$
c $9r^2$ d $-81b^4$
e $-343r^3$ f $(-2)^4h^8 = 16h^8$

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g $625c^8d^{12}$
h $32x^{15}y^{10}$
i $9p^6q^{12}$
j 1
k $-4p^8q^2r^2$
l $25s^{14}t^2$

2 a $9ab^2$
b $27ab^6$
c $-12a^8b^8$
d $54x^6y^9$
e $-64b^6c^{15}d^3$
f $8a^4$
g $9a^5$
h $-40a^{15}b^3$
i $160m^{15}p^5t^{10}$
j $-49d^4f^8g^2$
k $1024x^{12}y^3z^9$
l $-16a^8b^7$
m $a^{10}b^{11}$
n $8m^5n^3$
o $12c^8d^7$

3 a $\frac{p^3}{q^3}$
b $\frac{x^4}{y^4}$
c $\frac{64}{y^3}$
d $\frac{4}{r^6}$
e $\frac{s^6}{49}$
f $\frac{32m^5}{n^5}$
g $\frac{27n^9}{8m^{12}}$
h $\frac{16r^4}{n^4}$
i $\frac{9f^2}{64g^{10}}$
j $\frac{9x^2}{4y^6g^{10}}$
k $\frac{27k^3m^9}{64n^{21}}$
l $-\frac{25w^8y^2}{4x^6}$

4 a $\frac{2x^7}{y^7}$
b $\frac{a^8b^{10}}{c^7}$
c $\frac{p^2q^{14}}{r^8}$
d $a^{11}bc^5$
e $x^{11}y^2z$
f $\frac{r^9t^{10}}{s}$

5 a i 8
ii 125
b $N = \frac{t^3}{8}$
c i 27
ii 8
d i 8
ii 2

6 a 2
b 4
c 2
d 2
e 1
f 14

7 a false, $(-2)^2 = +(2)^2$
b true, $(-3)^3 = -(3)^3$
c true, $(-5)^5 = -(5)^5$
d false, $(-4)^4 = +(4)^4$

8 a By simplifying first, there are smaller numbers to raise to powers.
b i 8
ii 16
iii $\frac{1}{81}$
iv $\frac{1}{1000}$

9 a 25
b 13
c no
d no, $(3-2)^2 \neq 3^2 - 2^2$
e i true
ii false
iii true
iv false

6E

Building understanding

1 a $\frac{1}{2^2}$
b $\frac{1}{3^2}$
c $\frac{1}{5^3}$
d $\frac{1}{3^3}$

2 a

Index form	3^4	3^3	3^2	3^1
Whole number or fraction	81	27	9	3

Index form	3^0	3^{-1}	3^{-2}	3^{-3}
Whole number or fraction	1	$\frac{1}{3}$	$\frac{1}{9} = \frac{1}{3^2}$	$\frac{1}{27} = \frac{1}{3^3}$

b

Index form	10^4	10^3	10^2	10^1
Whole number or fraction	10 000	1000	100	10

Index form	10^0	10^{-1}	10^{-2}	10^{-3}
Whole number or fraction	1	$\frac{1}{10}$	$\frac{1}{100} = \frac{1}{10^2}$	$\frac{1}{1000} = \frac{1}{10^3}$

Now you try

Example 14

a $\frac{1}{a^3}$
b $\frac{2y^3}{x^4}$

Example 15

a a^3
b $\frac{6b^5}{a^2}$
c $\frac{y^4}{x^2}$

Example 16

a $\frac{1}{32}$
b -32
c $\frac{49}{25}$

Exercise 6E

1 a $\frac{1}{x^5}$
b $\frac{1}{a^4}$
c $\frac{1}{b^6}$
d $\frac{1}{25}$
e $\frac{1}{64}$
f $\frac{1}{9}$
g $\frac{5}{x^2}$
h $\frac{4}{y^3}$
i $\frac{3}{m^5}$
j $\frac{p^7}{q^2}$
k $\frac{m}{n^4}$
l $\frac{x^4}{y^4}$
m $\frac{2}{a^3b}$
n $\frac{7}{r^2s^3}$
o $\frac{v^2}{5u^8}$
p $\frac{1}{9m^3n^5}$

2 a y
b b^2
c m^5
d x^4
e $7q$
f $3t^2$
g $5h^4$
h $4p^4$
i ab^2
j de
k $2m^3n^2$
l $\frac{x^2y^5}{3}$
m $-\frac{3y^4}{7}$
n $-2b^8$
o $-\frac{3gh^3}{4}$
p $\frac{9u^2t^2}{5}$

3 a $\frac{7x^4}{y^3}$
b $\frac{u^3}{v^2}$
c $\frac{y^3}{5a^3}$
d $\frac{2b^5}{a^4c^2}$
e $\frac{5a^2b^2}{6c^4d}$
f $\frac{4h^3m^2}{5k^2p}$
g $\frac{12w^6}{tu^2v^2}$
h $\frac{mn^4x^2}{16y^5}$

4 a $\frac{b^3}{a^3}$
b $\frac{y^5}{x^2}$
c $\frac{h^3}{g^2}$
d $\frac{n}{m}$
e $\frac{343}{5}$
f $\frac{64}{9}$
g $\frac{6}{25}$
h 1

5 a $\frac{1}{5}$
b $\frac{1}{9}$
c $\frac{1}{16}$
d $-\frac{1}{25}$
e $\frac{1}{25}$
f $-\frac{1}{200}$
g $-\frac{3}{4}$
h $\frac{1}{2}$

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i $\frac{1}{36}$ j $\frac{1}{8}$ k $\frac{4}{25}$ l $\frac{7}{81}$
m 8 n 100 o -250 p 16
q -10 r 64 s $\frac{64}{9}$ t $-\frac{27}{64}$
u 100 v $\frac{1}{3}$ w -2 x 49

6 1.95 g

7 a -4 b -4 c -4
d -1 e -2 f -1

8 a negative power only applies to x, $\frac{2}{x^2}$
b $5 = 5^1$ has a positive power, $5a^{-4}$
c $\frac{2}{3^{-2}b^{-2}} = 2 \times 3^2 \times b^2 = 18b^2$

9 a $1 \div \frac{2}{3} = 1 \times \frac{3}{2}$
b i $\frac{4}{5}$ ii $\frac{7}{2}$ iii $\frac{3}{x}$ iv $\frac{b}{a}$
c (fraction)$^{-1}$ = reciprocal of fraction
d i $\frac{9}{4}$ ii $\frac{25}{16}$ iii 32 iv $\frac{27}{343}$

10 a 4 b 4 c 3 d 2
e 4 f $\frac{3}{2}$ g 4 h $\frac{7}{3}$

Progress quiz

1 a $a \times a \times a \times a$ b $5 \times hk \times hk \times hk$
c $2 \times 2 \times 2 \times 2 = 16$ d $\frac{-3}{4} \times \frac{-3}{4} \times \frac{-3}{4} = -\frac{27}{64}$

2 a $7m^4$ b $\left(\frac{2}{3}\right)^2 \times \left(\frac{1}{5}\right)^3$
c $8^3e^4h^3 = (8h)^3e^4$

3 a 2^8 b a^9 c $12k^7m^2$
d 5^8 e $2a^6$ f $\frac{a^5m^4}{3}$

4 a x^6 b $3xy$

5 a x^{12} b $-4q^{42}$

6 a 1 b 1 c 7 d -5

7 a a^{14} b m^{22} c 4 d $\frac{4m^8n^4}{3}$

8 a $8b^3$ b $5h^6j^9k^3$ c $-27x^{12}y^6$
d $\frac{64}{c^3}$ e $\frac{-8w^3x^9}{125y^6}$ f a^8c^2

9 a $\frac{1}{m^4}$ b $\frac{7y^5}{x^3}$ c a^5
d $\frac{c^3}{a^2}$ e $-13m^5$ f $\frac{-15u^3w^3}{t^2v^2}$

10 a $\frac{1}{16}$ b 27 c $\frac{1}{36}$ d $\frac{8}{27}$

6F

Building understanding

1 a 10 000 b 1000 c 100 000
d 1000 e 100 000 f 10 000

2 a 10^5 b 10^2 c 10^9

3 a positive b negative c positive d negative

Now you try

Example 17
a 7.2×10^5 b 3×10^{-6}

Example 18
a 6170 b 0.00000402

Exercise 6F

1 a 4.3×10^4 b 2.3×10^{12} c 1.6×10^{10}
d -7.2×10^6 e -3.5×10^3 f -8.8×10^6
g 5.2×10^3 h 3×10^6 i 2.1×10^4

2 a 3×10^{-6} b 4×10^{-4} c -8.76×10^{-3}
d 7.3×10^{-10} e -3×10^{-5} f 1.25×10^{-10}
g -8.09×10^{-9} h 2.4×10^{-8} i 3.45×10^{-5}

3 a 6×10^3 b 7.2×10^5 c 3.245×10^2
d 7.86903×10^3 e 8.45912×10^3 f 2×10^{-1}
g 3.28×10^{-4} h 9.87×10^{-3} i -1×10^{-5}
j -4.601×10^8 k 1.7467×10^4 l -1.28×10^2

4 a 57 000 b 3 600 000
c 430 000 000 d 32 100 000
e 423 000 f 90 400 000 000
g 197 000 000 h 709
i 635 700

5 a 0.00012 b 0.0000046
c 0.0000000008 d 0.0000352
e 0.3678 f 0.000000123
g 0.00009 h 0.05
i 0.4

6 a 6×10^{24} b 4×10^7 c 1×10^{-10}
d 1.5×10^8 e 6.67×10^{-11} f 1.5×10^{-4}
g 4.5×10^9

7 a 4 600 000 000 b 8 000 000 000 000
c 384 000 d 0.0038
e 0.00000000000001 f 720 000

8 a 4.92×10^{-1} b 3.8×10^{-4} c 2.1×10^{-6}
d 5.2×10^{-8} e 4×10^{-9} f 1.392×10^{-7}
g 4×10^6 h 2×10^{12} i 5.2×10^7
j 1.82×10^5 k 1.8×10^{-11} l 4×10^{-7}

9 a $\$1.84 \times 10^9$ b $\$2.647 \times 10^9$

10 1.62×10^9 km

11 2.126×10^{-2} g

12 a 3.2×10^4 b 4.1×10^6 c 3.17×10^4
d 5.714×10^5 e 1.3×10^4 f 9.2×10^1
g 3×10^5 h 4.6×10^5 i 6.1×10^{-2}
j 4.24 k 1.013×10^{-3} l 4.9×10^{-4}
m 2×10^{-5} n 4×10^{-6} o 3.72×10^{-4}
p 4.001×10^{-8}

13 a 8×10^6 b 9×10^8 c 6.25×10^{-4}
d 3.375×10^{-9} e 1.25×10^8 f 4×10^6
g 9×10^{-4} h 2.5×10^{-4}

14 a 6×10^6 b 8×10^{11} c 2×10^4
d 3×10^9 e 5.6×10^5 f 1.2×10^8
g 1.2×10^3 h 9×10^3 i 9×10^{-9}
j 7.5×10^{-8} k 1.5×10^{-5} l 1

15 $5 \times 10^2 = 500$ seconds

16 a 3×10^{-4} km = 30 cm
b 1×10^{-3} seconds (one thousandth of a second)
= 0.001 seconds

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6G

Building understanding

1 a 57 260, 57 300, 57 000, 60 000
b 0.003661, 0.00366, 0.0037, 0.004

2 a yes b no c no d no e yes
f yes g yes h no i no

Now you try

Example 19
a 4 b 3 c 3

Example 20
a 4.9×10^5 b 6.5×10^{-2}

Example 21
a 7.71×10^9 b 6.94×10^{-2}

Exercise 6G

1 a i 4 ii 2
b i 2 ii 3
c i 2 ii 4

2 a 3 b 4 c 5 d 2
e 3 f 1 g 3 h 3
i 3 j 4 k 3 l 2

3 a 2.42×10^5 b 1.71×10^5 c 2.83×10^3
d 3.25×10^6 e 3.43×10^{-4} f 6.86×10^{-3}
g 1.46×10^{-2} h 1.03×10^{-3} i 2.34×10^1
j 3.26×10^2 k 1.96×10^1 l 1.72×10^{-1}

4 a 4.78×10^4 b 2.2×10^4 c 4.833×10^6
d 3.7×10^1 e 9.95×10^1 f 1.443×10^{-2}
g 2×10^{-3} h 9×10^{-2} i 1×10^{-4}

5 a 2.441×10^{-4} b 2.107×10^{-6} c -4.824×10^{15}
d 4.550×10^{-5} e 1.917×10^{12} f 1.995×10^8
g 3.843×10^2 h 1.710×10^{-11} i 1.524×10^8
j 3.325×10^{15} k 4.000×10^3 l -8.959×10^3

6 a 9.3574×10^1 b 2.1893×10^5 c 8.6000×10^5
d 8.6288×10^{-2} e 2.2985×10^{15} f 3.5741×10^{28}
g 6.4000×10^7 h 1.2333×10^9 i 1.8293
j 5.4459×10^{-1}

7 1.98×10^{30} kg

8 1.39×10^6 km

9 1.09×10^{12} km^3

10 0.02×10^8, 0.239×10^7, 2.41×10^6, 24.2×10^5, 2421×10^3

11 a 4.26×10^6 b 9.1×10^{-3}
c 5.04×10^{11} d 1.931×10^{-1}
e 2.1×10^6 f 6.14×10^{-11}

12 should be 8.8×10^{10}

13 a i 2.30×10^2 ii 4.90×10^{-2} iii 4.00×10^6
b It is zero
c It clarifies the precision of the number

14 a 5.40046×10^{12}
b i 4.32×10^{13} ii 1.61×10^{19} iii 4.01×10^{51}

6H

Building understanding

1 a 4, 2 b 8, 2 c 9, 3
d 27, 3 e 16, 4 f 64, 4

2 a true b false c true d true
e false f false g false

3 a 2.6458 b 3.6056 c 9.1104

Now you try

Example 22
a $\sqrt{7}$ b $\sqrt[6]{3}$

Example 23
a 8 b 6

Example 24
a x^3 b $a^{\frac{1}{6}}$ c b^2

Exercise 6H

1 a $\sqrt{3}$ b $\sqrt{7}$ c $\sqrt[3]{5}$ d $\sqrt[3]{12}$
e $\sqrt[5]{31}$ f $\sqrt[7]{18}$ g $\sqrt[9]{9}$ h $\sqrt[8]{3}$

2 a $8^{\frac{1}{2}}$ b $19^{\frac{1}{2}}$ c $10^{\frac{1}{3}}$ d $31^{\frac{1}{3}}$
e $5^{\frac{1}{4}}$ f $9^{\frac{1}{5}}$ g $11^{\frac{1}{8}}$ h $20^{\frac{1}{11}}$

3 a 5 b 7 c 9 d 13
e 2 f 4 g 5 h 10
i 2 j 3 k 5 l 2

4 a a b $a^{\frac{2}{3}}$ c a^2 d $a^{\frac{5}{2}}$
e $x^{\frac{1}{3}}$ f x g $x^{\frac{5}{6}}$ h x
i y j y^2 k $y^{\frac{3}{2}}$ l $x^{\frac{1}{4}}$
m $x^{\frac{8}{3}}$ n $a^{\frac{2}{15}}$ o $a^{\frac{3}{8}}$ p $n^{\frac{4}{3}}$

5 a $a^{\frac{4}{3}}$ b $a^{\frac{7}{10}}$ c $a^{\frac{23}{21}}$
d $a^{\frac{8}{3}}$ e $b^{\frac{1}{6}}$ f $x^{\frac{2}{15}}$

6 a $\frac{1}{2}$ b $\frac{1}{2}$ c $\frac{1}{2}$ d $\frac{1}{3}$
e $\frac{1}{5}$ f $\frac{1}{3}$ g $\frac{1}{10}$ h $\frac{1}{4}$

7 a $\sqrt{29}$ b $\sqrt{13}$ c $\sqrt{65}$
d $\sqrt{125}$ e $\sqrt{10}$ f $\sqrt{1700}$

8 a 9 b 16 c 27 d 125
e 32 f 32 g 729 h 3125

9 a $a^{\frac{1}{2}+\left(-\frac{1}{2}\right)} = a^0 = 1$
b $a^{\frac{2}{3}+\left(-\frac{2}{3}\right)} = a^0 = 1$

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c $a^{\frac{4}{7}-\frac{4}{7}} = a^0 = 1$
d $a^{\frac{5}{6}-\frac{5}{6}} = a^0 = 1$
e $a^{\frac{1}{4}} \times a^{-\frac{1}{4}} = a^{\frac{1}{4}+\left(-\frac{1}{4}\right)} = a^0 = 1$
f $a^2 \div a^2 = a^{2-2} = a^0 = 1$

10 Brackets needed for fractional power, $9^{\frac{1}{2}} = 3$

11 a i 3 ii 5 iii 10
b a
c $(a^2)^{\frac{1}{2}} = a^{2\times\frac{1}{2}} = a$
d i 4 ii 9 iii 36
e a
f $\left(a^{\frac{1}{2}}\right)^2 = a^{\frac{1}{2}\times 2} = a$
g i a ii a iii a iv a

12 a $\frac{4}{5}$ b $\frac{3}{7}$ c $\frac{2}{9}$ d $\frac{2}{3}$
e $\frac{4}{5}$ f $\frac{2}{3}$ g $\frac{4}{5}$ h $\frac{10}{7}$

13 a $\frac{2}{3}$ b $\frac{12}{7}$ c $\frac{5}{2}$ d $\frac{5}{6}$

6I

Building understanding

1 a like b like c unlike d unlike
e unlike f like g like h unlike
2 a both = 3.162 b both = 1.732

Now you try

Example 25
a $8\sqrt{3}$ b $-3\sqrt{7}$

Example 26
a $\sqrt{15}$ b $\sqrt{6}$

Exercise 6I

1 a $8\sqrt{7}$ b $8\sqrt{11}$ c $9\sqrt{5}$
d $4\sqrt{6}$ e $7\sqrt{3} + 2\sqrt{5}$ f $9\sqrt{7} + 3\sqrt{5}$
g $-5\sqrt{5}$ h $-4\sqrt{7}$ i $5\sqrt{7}$
j $-\sqrt{14}$ k $7\sqrt{2} - \sqrt{5}$ l $3\sqrt{3} + 2\sqrt{7}$
2 a $\sqrt{30}$ b $\sqrt{21}$ c $\sqrt{70}$
d 4 e 6 f $\sqrt{22}$
g 3 h 12 i $\sqrt{3}$
j $\sqrt{10}$ k $\sqrt{7}$ l $\sqrt{3}$
m 3 n 4 o $\sqrt{7}$
3 a $8 - 3\sqrt{3}$ b $6\sqrt{2} - \sqrt{3}$ c $7\sqrt{5} + 1$
d $\frac{5\sqrt{2}}{6}$ e $\frac{7\sqrt{7}}{10}$ f $-\frac{3\sqrt{6}}{14}$
g $\frac{2\sqrt{10}}{3}$ h $5 + \frac{\sqrt{3}}{3}$ i $-\frac{19\sqrt{8}}{56}$
4 a $15\sqrt{6}$ b $6\sqrt{21}$ c $8\sqrt{30}$
d $10\sqrt{18}$ e $2\sqrt{3}$ f $3\sqrt{6}$
g $4\sqrt{14}$ h $\frac{\sqrt{2}}{2}$
5 a $6\sqrt{15} + 2\sqrt{3}$ b $\sqrt{10} + \sqrt{15}$
c $5\sqrt{12} + 15\sqrt{30}$ d $14\sqrt{30} - 70$
e $13 - 2\sqrt{39}$ f $\sqrt{35} - 10$
6 a $2\sqrt{2}$ b $2\sqrt{3}$ c $3\sqrt{3}$ d $3\sqrt{5}$
e $5\sqrt{3}$ f $10\sqrt{2}$ g $2\sqrt{15}$ h $6\sqrt{2}$
7 a $5\sqrt{2}$ b $\sqrt{2}$ c $4\sqrt{2}$ d $\sqrt{3}$
e $6\sqrt{2}$ f $5\sqrt{3}$ g $2\sqrt{5}$ h $4\sqrt{3}$
8 a $2 + \sqrt{10} + \sqrt{6} + \sqrt{15}$
b $3 + \sqrt{6} - \sqrt{15} - \sqrt{10}$
c $6\sqrt{10} + 8\sqrt{5} - 3\sqrt{2} - 4$
d $2 + 3\sqrt{2} - 6\sqrt{7} - 9\sqrt{14}$
e 1
f 4
g 15
h 123
i $3 + 2\sqrt{2}$
j $15 - 6\sqrt{6}$
k $13 - 4\sqrt{3}$
l $22 + 4\sqrt{10}$

Problems and challenges

1 a 4 b 1 c 6
2 a 6
b 30
3 $\frac{9}{4}$
4 a $2t^2$
b $\frac{2}{t}$
5 100 minutes
6 $-\frac{2}{3}$
7 a i $2^{\frac{3}{4}}$ ii $2^{\frac{7}{8}}$ iii $2^{\frac{15}{16}}$
b 2
8 2^7
9 a $7\sqrt{2}$ b $\frac{3}{\sqrt{2}}$ or $\frac{3\sqrt{2}}{2}$ c $12\sqrt{10}$
10 **a–c** Answers may vary.
11 a $x = 0$ or 1 b $x = 1$ or 2
12 $x = 2^2 \times 3 \times 5^2 = 300$
13 $x = -1, y = \frac{1}{2}$

Success criteria example questions

1 $ab \times ab \times ab$
2 $(-4) \times (-4) \times (-4) = -64$
3 $6x^3y^2$
4 $2^2 \times 3^3$
5 $5^7; 3^3$
6 $12a^6b^4$
7 $\frac{3x^5y}{2}$
8 $4a^2b$
9 m^{30}

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10 1; 3
11 $4m^{15}n^5$
12 $\frac{81}{b^{20}}$
13 $\frac{2a^3b^4}{3}$
14 $\frac{2m^3}{n^2}; \frac{5x^3}{2}$
15 $\frac{1}{4^3} = \frac{1}{64}$; $4 \times 3^2 = 36$
16 1.23×10^5; 2.3×10^{-4}
17 420 000; 0.0035
18 4; 2
19 1.74×10^6
20 5.19×10^{-4}
21 $\sqrt[3]{10}$
22 9
23 x^4
24 $6\sqrt{5}$
25 $\sqrt{30}$; $\sqrt{2}$

Short-answer questions

1 a 3^4 b $2x^3y^2$
c $3a^2b^2$ d $\left(\frac{3}{5}\right)^3 \times \left(\frac{1}{7}\right)^2$
2 a $3^2 \times 5$ b $2^2 \times 3 \times 5^2$
3 a x^{10} b $12a^5b^6c$ c $12m^{10}n^4$
d a^9 e x^3y^2 f $\frac{b^2}{2a^2}$
4 a m^6 b $9a^8$ c $-32a^{10}b^5$
d $3b$ e 2 f $\frac{a^6}{27}$
5 a $\frac{1}{x^3}$ b $\frac{4}{t^3}$ c $\frac{1}{9t^2}$
d $\frac{2x^2}{3y^3}$ e $\frac{5}{x^6y^3}$ f $5m^3$
6 a $\frac{x^2}{2y^2}$ b $\frac{x^3y^2}{9}$ c $8m^7n^3$
7 0.0012, 35.4×10^{23}, 3.22×10^{21}, 0.4, 0.007×10^2, 2.35
8 a 324 b 172 500
c 0.2753 d 0.00149
9 a 7.76×10^9 people b 9.63×10^6 km^2
c 3.34×10^{-9} seconds d 2.94×10^{-7} m
10 a 2.19×10^5 b 1.2×10^{-2}
c 4.32×10^{-7} d 1.24×10^{10}
11 a 1.2×10^{55} b 4.3×10^{-5}
12 a 2 b 5 c 7
d 3 e $\frac{1}{3}$ f $\frac{1}{11}$
13 a s^2 b $15x^{\frac{5}{2}}$ c $9m^{\frac{3}{4}}n^4$ d $4a^{\frac{2}{3}}$
14 a $7\sqrt{7} + 2$ b $\sqrt{3} + 9\sqrt{2}$ c 8
d $\sqrt{15}$ e $\sqrt{14}$ f $2\sqrt{11}$
g $\sqrt{6}$ h 5 i $2\sqrt{5}$

Multiple-choice questions

1 D 2 B 3 E 4 A
5 C 6 B 7 D 8 C
9 E 10 B 11 C 12 A

Extended-response questions

1 a $\frac{16x^6}{3}$ b $\frac{8b^8}{15a^2}$ c $\frac{5m^8}{n^{10}}$ d $3x$
2 a 5.93×10^{-11} N
b i 1.50×10^{11} m ii 3.5×10^{22} N
c Earth $a = 9.81$ m/s^2, Mars $a = 3.77$ m/s^2
Acceleration due to gravity on Earth is more than $2\frac{1}{2}$ times that on Mars.

Chapter 7

7A

Building understanding

1 a right b 180°
c revolution d obtuse
e acute f 180°
g 90° h supplementary
i 180° j equal
2 a isosceles triangle b obtuse angled triangle
c equilateral triangle d isosceles triangle
e acute angled triangle f scalene triangle
g right-angled triangle
3 a acute, 50° b right, 90°
c obtuse, 101° d reflex, 202°
e straight, 180° f revolution, 360°
4 a C b S c N d C

Now you try

Example 1
a 108° b 18°
Example 2
a right-angled scalene triangle, 65
b acute isosceles triangle, 80
c obtuse scalene triangle, 22
Example 3
a 152 b 100

Exercise 7A

1 a i 125° ii 35°
b i 106° ii 16°
c i 91° ii 1°
d i 158° ii 68°
e i 142° ii 52°
f i 133° ii 43°

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2 a $a = 63$ b $a = 71$ c $a = 38$
d $a = 147$ e $a = 233$ f $a = 33$

3 a obtuse isosceles, $a = 40$
b acute scalene, $b = 30$
c right-angled scalene, $c = 90$
d equilateral, $d = 60$
e obtuse isosceles, $e = 100$
f right-angled isosceles, $f = 45$
g obtuse scalene, $g = 100$
h equilateral, $h = 60$
i acute scalene, $i = 70$

4 a $s = 120$ b $t = 20$
c $r = 70$ d $a = 60, x = 120$
e $a = 100, b = 140$ f $c = 115, d = 65$

5 a 360° b 90° c 60° d 90°
e 432° f 6° g 720° h 8640°

6 a $x = 56$ b $x = 155$

7 a 90° b 150° c 15°
d 165° e 157.5° f 80°
g 177.5° h 171° i 121.5°

8
$$\begin{aligned} \angle AOB + \angle ABO &= 120° \text{ (exterior angle of triangle)} \\ \angle AOB &= 30° \\ \text{(reflex) } x &= 330 \end{aligned}$$

9 $AO = BO$ (radii)
ΔAOB is isoceles, 2 sides equal, $\angle AOB = 116°$
$\therefore \angle OAB = 32°$, base angles of isosceles triangle

10 a 160°
b 165°
c
$$\begin{aligned} \angle WYZ + a° + b° &= 180° \text{ angle sum of a triangle} \\ \angle XYZ + \angle WYZ &= 180° \text{ straight line} \\ \therefore \angle XYZ &= a° + b° \end{aligned}$$

11 Let the interior angles of any triangle be $a°$, $b°$ and $c°$.
Now $a + b + c = 180$
The exterior angles become $180° - a°, 180° - b°, 180° - c°$ (straight line)
$$\begin{aligned} \text{Exterior sum} &= (180 - a)° + (180 - b)° + (180 - c)° \\ &= 540° - a° - b° - c° \\ &= 540° - (a + b + c)° \\ &= 540° - 180° \\ &= 360° \end{aligned}$$

12 a $4x = 90, x = 22.5$
b $3x = 180, x = 60$
c $10x = 360, x = 36$
d $2(x + 15) + x = 180, x = 50$
e $2x + 20 = 140, x = 60$
f $6x + 90 = 360, x = 45$

7B

Building understanding

1 a equal
b equal
c supplementary

2 a 125, alternate angles in || lines
b 110, cointerior angles in || lines
c 80, corresponding angles in || lines
d 96, vertically opposite
e 54, supplementary angles
f 116, cointerior angles in || lines

Now you try

Example 4
a No, cointerior angles are not supplementary.
b Yes, corresponding angles are equal.
c Yes, alternate angles are equal.

Example 5
a $a = 75$ (cointerior angles in parallel lines),
$b = 105$ (vertically opposite angles)
b $a = 38$ (supplementary angles),
$b = 142$ (corresponding angles in parallel lines)

Example 6
115°

Exercise 7B

1 a Yes, alternate angles are equal.
b Yes, cointerior angles add to 180°.
c Yes, corresponding angles are equal.
d No, corresponding angles are not equal.
e No, alternate angles are not equal.
f No, cointerior angles do not add to 180°.

2 a $a = 110, b = 70$
b $c = 95, d = 95$
c $e = 100, f = 100, g = 100$
d $a = 60, b = 120$
e $a = 100, b = 80, c = 80$
f $e = 140, f = 140, d = 140$

3 a $x = 70, y = 40$ b $t = 58, z = 122$
c $u = 110, v = 50, w = 50$ d $x = 118$
e $x = 295$ f $x = 79$

4 a 105° b 105° c 56°
d 105° e 90° f 85°

5 a 56 b 120 c 50

6 a $180° - a°$ b $180° - a°$
c $180° - (a° + b°)$ d $180° - (a° + b°)$
e $a° + c°$ f $180° - 2c°$

7 $\angle ABC = 100°, \angle BCD = 80°, \angle ABC + \angle BCD = 180°$
$\therefore AB \parallel DC$ as cointerior angles are supplementary.

8 a cointerior angles on parallel lines add to 180°
b alternate angles are equal, on parallel lines
c $\Delta ABC => a + b + c = 180$ and these are the three angles of the triangle

9 a
$$\begin{aligned} \angle BAE &= 180° - a° \text{ (alternate angles and } AB \parallel DE) \\ \angle ABC &= 180° - c° - (180 - a)° \text{ (angle sum of a triangle)} \\ &= 180° - c° - 180° + a° \\ &= -c° + a° \\ &= a° - c° \end{aligned}$$
b $\angle ABD = 180° - (a° + b°)$ (angle sum of triangle ΔABD)
$$\begin{aligned} \angle ABC + \angle ABD &= 180° \text{ (straight line)} \\ \therefore \angle ABC &= a° + b° \end{aligned}$$

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c construct XY through B parallel to AE
$\therefore \angle ABY = a°$ (alternate angles, $AE \parallel XY$)
$\therefore \angle CBY = b°$ (alternate angles, $DC \parallel XY$)
$\therefore \angle ABC = a° + b°$

d construct XY through A parallel to ED
$\angle XAD = 180° - b°$ (cointerior angles, $ED \parallel XY$)
$\angle DAB = 360° - a°$ (revolution)
$\therefore \angle XAB = 360° - a° - (180° - b°)$
$= 180° + b° - a°$
$\angle ABC = \angle XAB$ (alternate angles and $XY \parallel BC$)
$\therefore \angle ABC = 180° + b° - a°$

7C

Building understanding

1 a 5 b 7 c 4 d 11 e 9 f 12
2 a 720° b 1080° c 1620°
3 a parallel b right
c trapezium d equal
4 a convex quadrilateral b non-convex hexagon
c non-convex heptagon

Now you try

Example 7
a 60 b 80 c 109

Example 8
a $a = 215$ b $a = 120, b = 60$

Exercise 7C

1 a 115 b 149 c 30
d 121 e 140 f 220
2 a 110 b 70 c 54
d 33 e 63 f 109
3 a 110 b 150 c 230 d 20
e $b = 108, a = 72$
f $b = 140, a = 40$
g $b = 120, a = 240$
h $b = 128\frac{4}{7}, a = 231\frac{3}{7}$
i 108
4 a parallelogram, rectangle, kite
b rectangle, square
c square, rectangle
d square, rhombus, kite
5 a 16 b 25 c 102
6 a 255 b 86 c 115
d 37 e 28 f 111
7 A parallelogram has opposite sides parallel and equal. Rectangles, squares and rhombi have these properties (and more) and are therefore all parallelograms.
8 a $S = (n - 2) \times 180°$ b $I = \dfrac{(n-2) \times 180°}{n}$
c $E = \dfrac{360}{n}$ d 36°
9 a i one ii two iii five
b $(n - 3)$
10 $(180 - a) + (180 - b) + (180 - c) + (180 - d) + (180 - e) = 360$
(sum of exterior angles is 360°)

$$180 + 180 + 180 + 180 + 180 - (a + b + c + d + e) = 360$$
$$900 - (a + b + c + d + e) = 360$$
$$a + b + c + d + e = 540$$

a $a + b + c + d + e + f = 720$
b $a + b + c + d + e + f + g = 900$

7D

Building understanding

1 a size
b $\Delta ABC \equiv \Delta STU$
c SAS, RHS, AAS
2 a i XY ii XZ iii YZ
b i $\angle A$ ii $\angle B$ iii $\angle C$
3 a $\Delta ABC \equiv \Delta FGH$ b $\Delta DEF \equiv \Delta STU$

Now you try

Example 9
a SSS b AAS c RHS
d SAS e SAS f AAS

Example 10
a $x = 4, y = 8$ b $a = 75, b = 15$

Exercise 7D

1 a SAS b AAS c RHS d SSS
e SAS f RHS g SSS
2 a $x = 3, y = 4$ b $x = 2, y = 6$
c $a = 105, b = 40$ d $a = 65, b = 85$
e $x = 2.5, b = 29$ f $a = 142, x = 9.21, b = 7$
g $y = 4.2, a = 28$ h $a = 6.5, b = 60$
3 a $\Delta ABC \equiv \Delta GHI$ (RHS) b $\Delta DEF \equiv \Delta GHI$ (SSS)
c $\Delta ABC \equiv \Delta DEF$ (SAS) d $\Delta ABC \equiv \Delta GHI$ (AAS)
e $\Delta ACB \equiv \Delta DEF$ (SAS)
4 $\Delta DEF \equiv \Delta BRP$
$\Delta LMN = \Delta KIJ$
$\Delta BCD \equiv \Delta FGH$
$\Delta MNO \equiv \Delta RQP$
5 a $BC = 13$ b $BC = 85$
6 No, they can all be different sizes, one might have all sides 2 cm and another all sides 5 cm.
7 a one given, the other pair are vertically opposite
b AAS
8 a SSS b equal
9 a one given ($BA = BC$) and side BD is common
b SAS
c $\Delta ABD \equiv \Delta CBD$
d $\angle ADB = \angle CDB$ (corresponding angles in congruent triangles)
but $\angle ADB + \angle CDB = 180°$ (straight angle)
$\therefore \angle ADB = \angle CDB = 90°$
and AC is perpendicular to DB
10 a–e Answers may vary.

Answers

7B

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7E

Building understanding

1 a BD b AC c AC
2 $OA = OB$ radii of circle centre O
3 a $\angle ECD$ b $\angle CBA$ c $\angle DEC$
4 a SSS b $\angle BMC$

Now you try

Example 11
In ΔABC and ΔEDC:
$\angle ACB = \angle ECD$ (vertically opposite) (A)
$\angle ABC = \angle EDC$ (alternate angles in parallel lines) (A)
$BC = DC$ (given) (S)
$\therefore \Delta ABC \equiv \Delta EDC$ (AAS)

Example 12
a In ΔABC and ΔADC:
$AB = AD$ (given) (S)
$BC = DC$ (given) (S)
AC is common (S)
$\therefore \Delta ABC \equiv \Delta ADC$ (SSS)
b Since $\Delta ABC \equiv \Delta ADC$ then $\angle ABC = \angle ADC$.

Exercise 7E

1 a $AD = CD$ (given)
$\angle DAB = \angle DCB = 90°$ (given)
DB is common
$\therefore \Delta\ ABD \equiv \Delta CBD$ (RHS)
b AC is common
$AD = AB$ (given)
$\angle DAC = \angle BAC$ (given)
$\therefore \Delta\ ADC \equiv \Delta ABC$ (SAS)
c AC is common
$\angle ADC = \angle ABC$ (given)
$\angle DAC = \angle BAC$ (given)
$\therefore \Delta ADC \equiv \Delta ABC$ (AAS)
d AC is common
$AD = AB$ (given)
$DC = BC$ (given)
$\therefore \Delta ADC \equiv \Delta ABC$ (SSS)
e $AC = DC$ (given)
$BC = EC$ (given)
$\angle ACB \equiv \angle DCE$ (vertically opposite)
$\therefore \Delta\ ABC \equiv \Delta DEC$ (SAS)
f $AC = EC$ (given)
$\angle CAB = \angle CED$ (alternate angles, $AB \parallel DE$)
$\angle ACB = \angle ECD$ (vertically opposite)
(or $\angle CBA = \angle CDE$ (alternate angles, $AB \parallel DE$))
$\therefore \Delta ABC \equiv \Delta EDC$ (AAS)
g $DC = BC$ (given)
$\angle EDC = \angle ABC$ (alternate angles, $DE \parallel AB$)
$\angle DCE = \angle BCA$ (vertically opposite)
(or $\angle DEC = \angle BAC$ (alternate angles, $AB \parallel DE$))
$\therefore \Delta CDE \equiv \Delta CBA$ (AAS)
h BD is common
$AD = CD$ (given)
$\angle ADB = \angle CDB$ (given)
$\therefore \Delta ABD \equiv \Delta CBD$ (SAS)
i AC is common
$AB = CD$ (given)
$BC = DA$ (given)
$\therefore \Delta ABC \equiv \Delta CDA$ (SSS)
j BD is common
$\angle ABD = \angle CDB$ (alternate angles, $AB \parallel CD$)
$\angle ADB = \angle CBD$ (alternate angles, $AD \parallel CB$)
$\therefore \Delta ABD \equiv \Delta CDB$ (AAS)
k $OA = OC$ (radii)
OB is common
$AB = CB$ (given)
$\therefore \Delta AOB \equiv \Delta COB$ (SSS)
l $OA = OD$ and $OB = OC$ (radii)
$\angle AOB = \angle COD$ (vertically opposite)
$\Delta AOB \equiv \Delta COD$ (SAS)
2 a $DC = BC$ (given)
$EC = AC$ (given)
$\angle DCE = \angle BCA$ (vertically opposite)
$\therefore \Delta ABC \equiv \Delta EDC$ (SAS)
b $\angle EDC = \angle ABC$ (corresponding angles in congruent triangles)
$\therefore AB \parallel DE$ (alternate angles are equal)
3 a $AE = CD$ (given)
$BE = BD$ (given)
$\angle ABE = \angle CBD$ (vertically opposite with $\angle ABE$ given 90°)
$\therefore \Delta ABE \equiv \Delta CBD$ (RHS)
b $\angle EAB = \angle DCB$ (corresponding angles in congruent triangles)
$\therefore AE \parallel CD$ (alternate angles equal)
4 a DB is common
$AB = CD$ (given)
$AD = CB$ (given)
$\therefore \Delta ABD \equiv \Delta CDB$ (SSS)
b $\angle ADB = \angle CBD$ (corresponding angles in congruent triangles)
$\therefore AD \parallel BC$ (alternate angles equal)
5 a $OB = OC$ (radii)
$OA = OD$ (radii)
$\angle AOB = \angle DOC$ (vertically opposite)
$\therefore \Delta AOB \equiv \Delta DOC$ (SAS)
b $\angle ABO = \angle DCO$ (corresponding angles in congruent triangles)
$\therefore AB \parallel CD$ (alternate angles equal)
6 a BD is common
$AD = CD$ (given)
$\angle ADB = \angle CDB$ (given)
$\therefore \Delta ABD \equiv \Delta CBD$ (SAS)
b $\angle ABD = \angle CBD$ (corresponding angles in congruent triangles)
and $\angle ABD + \angle CBD = 180°$ (straight line)
$\therefore \angle ABD = \angle CBD = 90°$ and AC is perpendicular to BD

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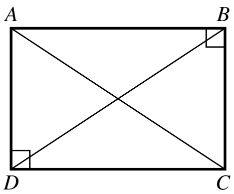
7 **a** DB is common
$\angle ABD = \angle CBD$ (given 90°)
$\angle ADB = \angle CDB$ (given)
$\therefore \Delta ABD \equiv \Delta CBD$ (AAS)
b $AD = CD$ (corresponding side in congruent triangles)
$\therefore \Delta ACD$ is isosceles (2 equal sides)

8 Consider ΔADC and ΔCBA
AC is common
$\angle DAC = \angle BCA$ (alternate angles, $AD \parallel BC$)
$\angle DCA = \angle BAC$ (alternate angles, $DC \parallel AB$)
$\therefore \Delta\ ADC \equiv \Delta CBA$ (AAS)
So $AD = BC$, $AB = DC$ are equal corresponding sides in congruent triangles.

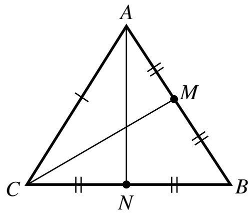
9 Consider ΔOAD and ΔOBD
OD is common
$OA = OB$ (radii)
$AD = BD$ (given)
$\therefore \Delta OAD \equiv \Delta OBD$ (SSS)
$\angle ODA = \angle ODB = 90°$ (corresponding angles in congruent triangles are equal and supplementary to a straight line)
$\therefore OC \perp AB$

10 $AB = DC$ (opposite sides of parallelogram)
$\angle AEB = \angle CED$ (vertically opposite)
$\angle BAE = \angle DCE$ (alternate angles $DC \parallel AB$)
$\therefore \Delta\ ABE \equiv \Delta CDE$ (AAS)
So $AE = CE$ and $BE = DE$, corresponding sides in congruent triangles

11


Consider ΔADC and ΔBCD
$AD = CB$ and DC is common (opposite sides of a rectangle are equal)
$\angle ADC = \angle BCD = 90°$ (angles of a rectangle)
$\therefore \Delta ADC \equiv \Delta BCD$ (SAS)
So $AC = BD$ (corresponding sides in congruent triangles)
$\therefore$ The diagonals of a rectangle are equal

12 **a** Consider ΔABE and ΔCDE
$AB = CD$ (sides of a rhombus)
$\angle ABE = \angle CDE$ (alternate angles, $AB \parallel CD$)
$\angle BAE = \angle DCE$ (alternate angles, $AB \parallel CD$)
$\therefore \Delta ABE \equiv \Delta CDE$ (AAS)
b Consider ΔDCE and ΔBCE
CE is common
$DC = BC$ (sides of a rhombus)
$DE = BE$ (corresponding sides in congruent triangles)
$\therefore \Delta DCE \equiv \Delta BCE$ (SSS)
$\angle DEC = \angle BEC$ (corresponding angles in congruent triangles)
$\angle DEC + \angle BEC = 180°$ (straight line)
$\therefore \angle DEC = \angle BEC = 90°$
and $AE = CE$ (corresponding sides in congruent triangles)
$\therefore AC$ bisects BD at 90°

13


Let ΔABC be any equilateral triangle $AB = CB = AC$.
Step one
Join C to M, the midpoint of AB.
Prove $\Delta CAM \equiv \Delta CBM$ (SSS).
$\therefore \angle CAM = \angle CBM$ (corresponding angles in congruent triangles)
Step two
Join A to N, the midpoint of CB.
Prove $\Delta ANC \equiv \Delta ANB$.
$\therefore \angle ACN = \angle ABN$ (corresponding angles in congruent triangles)
Now $\angle CAB = \angle ABC = \angle ACB$
and as $\angle CAB + \angle ABC + \angle ACB = 180°$
(angle sum of ΔABC)
$\angle CAB = \angle ABC = \angle ACB = 60°$

Progress quiz

1 **a** 37° **b** 127°
2 **a** 60 **b** 56 **c** 50
d 142 **e** 50 **f** 84
3 **a** yes, as there is a pair of equal alternate angles
b no, as the pair of cointerior angles are not supplementary
4 **a** $a = 68$ (alternate angles equal in parallel lines)
$b = 68$ (corresponding angles equal in parallel lines)
b $w = 98$ (alternate angles equal in parallel lines)
5 **a** $x = 92$ (angle sum of a quadrilateral)
b $y = 110$ (kite, angle sum of quadrilateral)
c $a = 125$ (angle sum of a pentagon)
d $x = 60$ (exterior angle of a regular hexagon)
6 **a** AAS
b SAS
7 AC is common
$DC = BC$ (given)
$\angle ACD = \angle ACB$ (given 90°)
$\therefore \Delta ABC \equiv \Delta ADC$ (SAS)
$AB = AD$ (corresponding sides in congruent triangles)
$\therefore \Delta ABD$ is isosceles
8 **a** $AE = DC$ (given)
$\angle ABE = \angle DBC$ (vertically opposite angles)
$\angle EAB = \angle CDB$ (alternate angles, $AE \parallel CD$)
$\therefore \Delta ABE \equiv \Delta DBC$ (AAS)
b 62° **c** 7 cm **d** 21 cm^2

7F

Building understanding

1 **a** $\angle F$ **b** $\angle D$ **c** GH **d** AE **e** 2

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2 a double b double c double
d 2 e yes

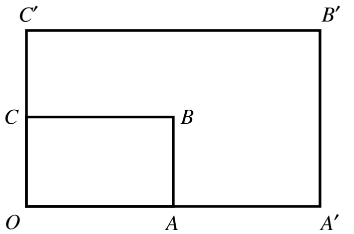
3 a OA' is a quarter of OA b OD' is a quarter of OD
c $\frac{1}{4}$ d yes

4 a yes b 8 cm c 25 m

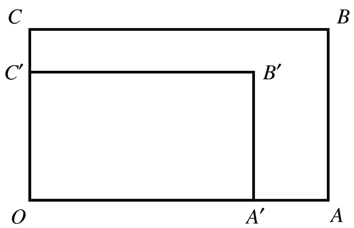
Now you try

Example 13

a



b



Example 14

a 2 b 6 c 8

Exercise 7F

1 a $\Delta A'B'C'$ should have sides $\frac{1}{3}$ that of ΔABC
b $\Delta A'B'C'$ should have sides double that of ΔABC

2 a $A'B'C'D'$ should have sides lengths $\frac{1}{2}$ that of $ABCD$
b $A'B'C'D'$ should have side lengths 1.5 times that of $ABCD$

3 a i 2 ii 14 iii 10
b i 1.5 ii 9 iii 8
c i 1.5 ii 45 iii 24
d i 0.4 ii 1 iii 1.4
e i 2.5 ii 0.6 iii 2
f i 1.75 ii 3.5 iii 3

4 a i 2 ii (0, 0)
b i 0.5 ii (0, 0)
c i 0.5 ii (3, 0)
d i 3 ii (1, 0)

5 a i 3.6 m ii 9 m iii 2.7 m
b i 5.4 m ii 6.3 m
c i 6 m ii 3 m

6 a 12.7 cm b 3 cm c 3 m

7 a $a > 1$ b $a < 1$ c $a = 1$

8 a all angles of any square equal 90°, with only 1 side length
b all angles in any equilateral triangle equal 60° with only 1 side length
c length and width might be multiplied by different numbers
d Two isosceles triangles do not have to have the same base angles

9 $\frac{1}{k}$

10 a 100 000 cm = 1 km b 24 cm

11 a N/A
b i $\frac{l}{2}$ ii $\frac{l}{4}$ iii $\frac{l}{128}$
c i $\frac{3}{4}$ ii $\frac{9}{16}$ iii $\frac{243}{1024}$
d zero

7G

Building understanding

1 a E b C c DF
d BC e $\angle A$ f $\angle E$

2 2.5

3 a SAS, AAA, and RHS b shape, size

Now you try

Example 15

a SAS b SSS c RHS d AAA

Example 16

a AAA b 10.5

Exercise 7G

1 a AAA b RHS c SSS d SAS
e RHS f AAA g SAS h SSS

2 a $\Delta ABC \,|||\, \Delta GHI$ b $\Delta ABC \,|||\, \Delta MNO$
c $\Delta ABC \,|||\, \Delta ADE$ d $\Delta HFG \,|||\, \Delta HJI$
e $\Delta ADC \,|||\, \Delta AEB$ f $\Delta ABD \,|||\, \Delta ECD$

3 a AAA b 12

4 a RHS b 8

5 a i 1.6 ii 14.4
b i 3.5 ii 5

6 a AAA b 15

7 a i AAA ii 6.5
b i AAA ii 10
c i AAA ii 24

8 a 2 b 16 c 2.8

9 a ΔDEF b ΔDEF c ΔABC d ΔDEF

10 $\angle ACB = 25°$, AAA

11 $\angle WXY = 55°$, not similar as angles not equal

12 2 pairs of equal alternate angles are always formed

13 AAA, in congruency a side length is needed for the triangles to be the same size, in similarity it is not needed.

14 a

Triangle	Original	Image 1	2	3
Length scale factor	1	2	3	4
Area	4	16	36	64
Area scale factor	1	4	9	16
Volume	12	96	324	768
Volume scale factor	1	8	27	64

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b Area scale factor = (length scale factor)^2

c n^2

d **i** 100 **ii** 400 **iii** 10 000

e $\frac{1}{4}$

f volume scale factor = (length scale factor)^3

g n^3

h **i** 125 **ii** 1000 **iii** $\frac{1}{8}$

7H

Building understanding

1 $\angle C$

2 **a** $\angle ACB$ and $\angle ECD$

b $\angle BAC = \angle DEC$ and $\angle CBA = \angle CDE$

3 **a** $\angle C$

b **i** AC **ii** DB

Now you try

Example 17

a $\angle ACE = \angle BCD$ (common)
$\angle CAE = \angle CBD$ (corresponding angles in parallel lines)
$\angle AEC = \angle BDC$ (corresponding angles in parallel lines)
$\therefore \triangle ACE \;|||\; \triangle BCD$ (AAA)

b $\angle CAD = \angle EAB$ (common)
$\frac{AD}{BA} = \frac{9}{3} = 3$
$\frac{AC}{AE} = \frac{7.5}{2.5} = 3$
$\therefore \triangle ACD \;|||\; \triangle AEB$ (SAS)

Example 18

a All angles are the same (AAA).

b 8 m

Exercise 7H

1 **a** $\angle AEB = \angle CDB$ (alternate angles, $EA \parallel DC$)
$\angle EAB = \angle DCB$ (alternate angles, $EA \parallel DC$)
$\angle EBA = \angle DBC$ (vertically opposite)
$\therefore \triangle AEB \;|||\; \triangle CDB$ (AAA)

b $\angle BAC = \angle DEC$ (alternate angles, $AB \parallel DE$)
$\angle ABC = \angle EDC$ (alternate angles, $AB \parallel DE$)
$\angle ACB = \angle ECD$ (vertically opposite)
$\therefore \triangle ACB \;|||\; \triangle ECD$ (AAA)

c $\angle C$ is common
$\angle CDB = \angle CEA$ (corresponding angles, $AE \parallel BD$)
$\angle CBD = \angle CAE$ (corresponding angles, $AE \parallel BD$)
$\therefore \triangle CBD \;|||\; \triangle CAE$ (AAA)

d $\angle A$ is common
$\angle ABE = \angle ADC$ (given)
$\therefore \triangle ABE \;|||\; \triangle ADC$ (AA)
(note 2 angles is enough – $\angle AEB = \angle ACD$ (angle sum of a triangle))

2 **a** $\angle C$ is common
$\frac{CA}{CD} = \frac{6}{2} = 3$
$\frac{CE}{CB} = \frac{9}{3} = 3$
$\therefore \triangle CDB \;|||\; \triangle CAE$ (SAS)

b $\angle D$ is common
$\frac{AD}{CD} = \frac{28}{7} = 4$
$\frac{DB}{DE} = \frac{48}{12} = 4$
$\therefore \triangle ABD \;|||\; \triangle CED$ (SAS)

c $\angle DCE = \angle BCA$ (vertically opposite)
$\frac{EC}{AC} = \frac{2}{5}$
$\frac{DC}{BC} = \frac{3}{7.5} = \frac{2}{5}$
$\therefore \triangle DCE \;|||\; \triangle BCA$ (SAS)

3 **a** AAA **b** 40 m

4 **a** AAA **b** 7.5 m

5 6 m **6** 20 m **7** 7.2 m **8** $\frac{55}{6}$

9 **a** $\angle ADC = \angle ACD = 80°$ (base angles of isosceles $\triangle ADC$)
$\angle ACB = 100°$ (straight angle)
$\angle CAB = 60°$ (angle sum of $\triangle ACB$)
Now $\angle DAB = 80°$
$\angle DAC = \angle DBA$ (given 20°)
$\angle D$ is common
$\angle ACD = \angle BAD$ (both 80°)
$\therefore \triangle ACD \;|||\; \triangle BAD$ (AAA)

b $DC = \frac{20}{3}$ $CB = \frac{25}{3}$

10 **a** **i** $\angle B$ is common
$\angle DAB = \angle ACB$ (given 90°)
$\therefore \triangle ABD \;|||\; \triangle CBA$ (AAA)

ii $\angle D$ is common
$\angle DCA = \angle DAB$ (given 90°)
$\therefore \triangle ABD \;|||\; \triangle CAD$ (AAA)

b **i** $BD = \frac{25}{3}$ **ii** $AC = 4$ **iii** $AB = \frac{20}{3}$

11 **a** $\angle ACB = \angle ECD$ (vertically opposite)
$\angle CAB = \angle CED$ (alternate angles, $DE \parallel BA$)
$\therefore \triangle ABC \;|||\; \triangle EDC$ (AAA)
$\therefore \frac{DC}{BC} = \frac{EC}{AC}$ (ratio of corresponding sides in similar triangles)
$\frac{6}{2} = \frac{EC}{AC}$
$\therefore 3AC = CE$
as $AC + CE = AE$
$AE = 4AC$

b $\angle C$ is common
$\angle DBC = \angle AEC$ (given)
$\therefore \triangle CBD \;|||\; \triangle CEA$ (AAA)
$\therefore \frac{DB}{AE} = \frac{2}{4} = \frac{BC}{CE}$ (ratio of corresponding sides in similar triangles)
$\therefore 4BC = 2CE$
$BC = \frac{1}{2}CE$

c $\angle C$ is common
$\angle CBD = \angle CAE$ (corresponding angles, $BD \parallel AE$)
$\therefore \triangle CBD \;|||\; \triangle CAE$ (AAA)
$\therefore \frac{CB}{CA} = \frac{CD}{CE}$ (ratio of corresponding sides in similar triangles)
$\therefore \frac{5}{7} = \frac{CD}{CE}$
$\therefore 5CE = 7CD$
$\therefore CE = \frac{7}{5}CD$

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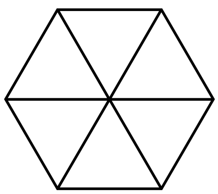
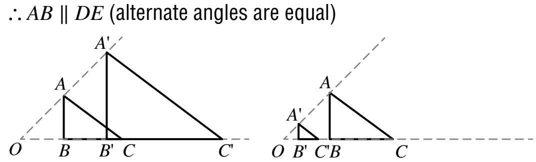
d $\angle C$ is common
$\angle CBD = \angle CAE$ (given 90°)
$\therefore \Delta CBD \mathbin{|||} \Delta CAE$ (AAA)
$\therefore \dfrac{BD}{AE} = \dfrac{CB}{CA}$ (ratio of corresponding sides in similar triangles)
$\dfrac{2}{8} = \dfrac{CB}{CA}$
$\dfrac{1}{4} = \dfrac{CB}{CB + AB}$
$CB + AB = 4CB$
$\therefore AB = 3CB$

Problems and challenges

1 

2 15
3 40°
4 A, B and C should be placed where the three altitudes of the triangle intersect the three sides.
5 45°
6 11
7 $\dfrac{120}{7}$

Answers to success criteria example questions

1 118°; 28°
2 Isosceles; $a = 55$
3 $x = 120$
4 No, alternate angles are not equal
5 $a = 122$ (vertically opposite angles are equal); $b = 58$ (cointerior angles are supplementary)
6 99°
7 $x = 121$
8 540°; $y = 212$
9 SAS
10 $a = 88$; $b = 17$
11 $AB = AD$ (given equal sides) (S)
$\angle BAC = \angle DAC$ (given equal angles) (A)
AC is common (S)
$\therefore \Delta ABC \equiv \Delta ADC$ (SAS)
12 $AC = EC$ (given equal sides) (S)
$BC = DC$ (given equal sides) (S)
$\angle ACB = \angle ECD$ (vertically opposite angles) (A)
$\Delta ABC \equiv \Delta EDC$ (SAS)
$\angle BAC = \angle DEC$ (matching angles in congruent triangles)
$\therefore AB \parallel DE$ (alternate angles are equal)
13 

14 $x = 6$; $y = 8.8$
15 RHS
16 AA; $x = 10$
17 $\dfrac{DC}{AC} = \dfrac{12}{4} = 3$
$\angle ECD = \angle BCA$ (vertically opposite angles are equal)
$\dfrac{EC}{BC} = \dfrac{15}{5} = 3 = \dfrac{DC}{AC}$
$\therefore \Delta ACB \mathbin{|||} \Delta DCE$ (SAS)
18 All angles are the same (AAA); 30 m

Short-answer questions

1 a isosceles, $x = 50$, $y = 80$
b right angled, $x = 25$
c obtuse angled, $x = 30$, $y = 110$
2 a $a = 30$ (vertically opposite) $b = 150$ (straight angle)
b $x = 60$ (revolution) $y = 120$ (cointerior angles in parallel lines)
c $a = 70$ (alternate angles and parallel lines)
$b = 55$ (angle sum of isosceles triangle)
$c = 55$ (corresponding angles and parallel lines)
3 $\angle ABC = 75°$
4 a $a = 70$, $b = 110$ b $x = 15$
c $x = 30$ d $a = 120$
5 a SSS, $x = 60$ b not congruent
c RHS, $x = 8$ d AAS, $x = 9$
6 a AD is common
$\angle ADC = \angle ADB$ (given 90°)
$CD = BD$ (given)
$\therefore \Delta ADC \equiv \Delta ADB$ (SAS)
b i $AC = EC$ (given)
$\angle BAC = \angle DEC$ (given)
$\angle ACB = \angle ECD$ (vertically opposite)
$\therefore \Delta ACB \equiv \Delta ECD$ (AAS)
ii as $\angle BAC = \angle DEC$ (alternate angles are equal)
$\therefore AB \parallel DE$
7 For image $\Delta A'B'C'$, $OA' = 3OA$, $OB' = 3OB$, $OC' = 3OC$
8 a yes, SAS b yes, AAA c not similar
9 a 3.5 b 4 c 18
10 a $\angle C$ is common
$\angle DBC = \angle EAC = 90°$ (given)
$\therefore \Delta BCD \mathbin{|||} \Delta ACE$ (AAA)
b 5 m

Multiple-choice questions

1 D	2 A	3 B	4 D	5 C
6 D	7 E	8 B	9 D	10 A

Extended-response questions

1 a $\angle ABC + \angle BCF = 180°$ (cointerior angles, $AB \parallel CF$)
$\therefore \angle BCF = 70°$
$\therefore \angle DCF = 32°$
Now $\angle EDC + \angle DCF = 32° + 148°$
$= 180°$
$\therefore DE \parallel CF$ as cointerior angles add to 180°

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b $\angle CDA = 40°$ (revolution)
reflex $\angle BCD = 270°$ (revolution)
$\angle DAB = 30°$ (angle sum in isosceles triangle)
$BADC$ quadrilateral angle sum 360°
$\therefore a = 360 - (30 + 270 + 40)$
$= 20$
or other proof

c $AB = CD$ (given)
$\angle ABC = \angle DCB$ (given)
BC is common
$\therefore \Delta ABC \equiv \Delta DCB$(SAS)
$\therefore AC = BD$ (corresponding sides in congruent triangles)

2 a $\angle ECD$
b $\angle ABC = \angle EDC$ (given 90°)
$\angle ACB = \angle ECD$ (vertically opposite)
$\therefore \Delta ABC \,|||\, \Delta EDC$ (AAA)
c 19.8 m

Chapter 8

8A

Building understanding

1 a $x^2, 2x, 3x, 6$
b $x^2 + 3x + 2x + 6 = x^2 + 5x + 6$

2 a $2x^2, 2x, 3x, 3$
b $(2x + 3)(x + 1) = 2x^2 + 2x + 3x + 3$
$= 2x^2 + 5x + 3$

3 a $x^2 + 5x + x + 5 = x^2 + 6x + 5$
b $x^2 + 2x - 3x - 6 = x^2 - x - 6$
c $21x^2 + 6x - 14x - 4 = 21x^2 - 8x - 4$
d $12x^2 - 16x - 3x + 4 = 12x^2 - 19x + 4$

Now you try

Example 1
a $x^2 + 6x + 8$
b $x^2 + 2x - 15$
c $2x^2 - 11x + 12$
d $12x^2 - 11x - 15$

Exercise 8A

1 a $x^2 + 7x + 10$ b $b^2 + 7b + 12$
c $t^2 + 15t + 56$ d $a^2 + 8a + 7$
e $y^2 + 12y + 20$ f $m^2 + 16m + 48$
g $x^2 - x - 12$ h $x^2 - 4x - 32$
i $x^2 - 4x - 12$ j $x^2 + 9x - 10$
k $x^2 - 3x + 2$ l $x^2 - 9x + 20$

2 a $8x^2 + 26x + 15$ b $6x^2 + 7x + 2$
c $15x^2 + 17x + 4$ d $6x^2 + x - 15$
e $24x^2 + 23x - 12$ f $6x^2 - x - 2$
g $10x^2 - 31x - 14$ h $6x^2 + 5x - 6$
i $16x^2 - 16x - 5$ j $18x^2 - 27x + 10$
k $15x^2 - 11x + 2$ l $21x^2 - 37x + 12$

3 a $a^2 + ac + ab + bc$ b $a^2 + ac - ab - bc$
c $yz - y^2 - xz + xy$ d $2x^2 - 3xy - 2y^2$
e $2a^2 - ab - b^2$ f $6x^2 + xy - y^2$
g $6a^2 + 4a - 3ab - 2b$ h $12x^2 - 25xy + 12y^2$
i $3x^2y - yz^2 - 2xyz$

4 a $x^2 + 9x + 20$
b i 56 m^2 ii 36 m^2

5 a $2x^2$ b $2x^2 - 30x + 100$

6 a $150 - 50x + 4x^2$ b 66 m^2

7 a 3 b 2
c 6, 6 d 2, 18
e 2, 6 f 3, 15
g $2x, 5x, 3$ h $3x, 15x, 4$
i $7x, 3, 17x$ j $3x, 4, 11x$

8 a $a = 3, b = 2$ or $a = 2, b = 3$
b $a = -3, b = -2$ or $a = -2, b = -3$
c $a = 3, b = -2$ or $a = -2, b = 3$
d $a = 2, b = -3$ or $a = -3, b = 2$

9 a $x^3 + 2x^2 + 2x + 1$ b $x^3 - 3x^2 + 5x - 6$
c $4x^3 - 4x^2 + 9x - 4$ d $x^3 + 2x^2 - 2x + 3$
e $10x^3 - 17x^2 + 7x - 6$ f $8x^3 - 18x^2 + 35x - 49$
g $x^3 + ax - a^2x + a^2$ h $x^3 - 2ax^2 + a^3$
i $x^3 + a^3$ j $x^3 - a^3$

10 $x^3 + 6x^2 + 11x + 6$

8B

Building understanding

1 a $+3x + 9 = x^2 + 6x + 9$
b $+5x + 25 = x^2 + 10x + 25$
c $-2x + 4 = x^2 - 4x + 4$
d $-7x + 49 = x^2 - 14x + 49$

2 a i $x^2 + 6x + 9$
ii $x^2 + 22x + 121$
iii $x^2 + 30x + 225$
b i $x^2 - 4x + 4$
ii $x^2 - 18x + 81$
iii $x^2 - 60x + 900$

3 a $+4x - 16 = x^2 - 16$
b $-10x - 100 = x^2 - 100$
c $+2x - 2x - 1 = 4x^2 - 1$
d $-12x + 12x - 16 = 9x^2 - 16$

Now you try

Example 2
a $x^2 - 6x + 9$ b $9x^2 + 24x + 16$

Example 3
a $x^2 - 16$ b $25x^2 - 9y^2$

Exercise 8B

1 a $x^2 + 2x + 1$ b $x^2 + 6x + 9$
c $x^2 + 4x + 4$ d $x^2 + 10x + 25$
e $x^2 + 8x + 16$ f $x^2 + 18x + 81$
g $x^2 - 4x + 4$ h $x^2 - 12x + 36$

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i $x^2 - 2x + 1$
j $x^2 - 6x + 9$
k $x^2 - 18x + 81$
l $x^2 - 14x + 49$

2 a $4x^2 + 4x + 1$
b $4x^2 + 20x + 25$
c $9x^2 + 12x + 4$
d $9x^2 + 6x + 1$
e $25x^2 + 20x + 4$
f $16x^2 + 24x + 9$
g $49 + 28x + 4x^2$
h $25 + 30x + 9x^2$
i $4x^2 - 12x + 9$
j $9x^2 - 6x + 1$
k $16x^2 - 40x + 25$
l $4x^2 - 36x + 81$
m $9x^2 + 30xy + 25y^2$
n $4x^2 + 16xy + 16y^2$
o $49x^2 + 42xy + 9y^2$
p $36x^2 + 60xy + 25y^2$
q $16x^2 - 72xy + 81y^2$
r $4x^2 - 28xy + 49y^2$
s $9x^2 - 60xy + 100y^2$
t $16x^2 - 48xy + 36y^2$
u $81x^2 - 36xy + 4y^2$

3 a $9 - 6x + x^2$
b $25 - 10x + x^2$
c $1 - 2x + x^2$
d $36 - 12x + x^2$
e $121 - 22x + x^2$
f $64 - 32x + 4x^2$
g $4 - 12x + 9x^2$
h $81 - 36x + 4x^2$
i $100 - 80x + 16x^2$

4 a $x^2 - 1$
b $x^2 - 9$
c $x^2 - 64$
d $x^2 - 16$
e $x^2 - 144$
f $x^2 - 121$
g $x^2 - 81$
h $x^2 - 25$
i $x^2 - 36$
j $25 - x^2$
k $4 - x^2$
l $49 - x^2$

5 a $9x^2 - 4$
b $25x^2 - 16$
c $16x^2 - 9$
d $49x^2 - 9y^2$
e $81x^2 - 25y^2$
f $121x^2 - y^2$
g $64x^2 - 4y^2$
h $100x^2 - 81y^2$
i $49x^2 - 25y^2$
j $36x^2 - 121y^2$
k $64x^2 - 9y^2$
l $81x^2 - 16y^2$

6 a i x^2
ii $x^2 - 4$
b No, they differ by 4.

7 a $20 - 2x$
b $(20 - 2x)(20 - 2x) = 400 - 80x + 4x^2$
c 196 cm^2
d 588 cm^3

8 a $a + b$
b $(a + b)(a + b) = a^2 + 2ab + b^2$
c $a - b$
d $(a - b)(a - b) = a^2 - 2ab + b^2$
e $4ab$
f ab, so yes area of four courts is $4ab$

9 a $x^2 - 1$
b No, area of rectangle is 1 square unit less.

10 a $a^2 - b^2$
b i $(a - b)^2 = a^2 - 2ab + b^2$
ii $b(a - b) = ab - b^2$
iii $b(a - b) = ab - b^2$
c yes, $a^2 - 2ab + b^2 + ab - b^2 + ab - b^2 = a^2 - b^2$

11 a $x^2 + 4x$
b $-4x + 1$
c $x^2 + 6x - 9$
d $-x^2 - 2x$
e 1
f $4x$
g $-12x - 8$
h $-10x + 2$
i $x^2 + 4xy - y^2$
j $8x^2 + 18$
k $-8x$
l $2x^2 - 12x + 18$
m $9x^2 - 48x + 48$
n $x^2 + 6xy + y^2$

8C

Building understanding

1 a 4
b 6
c 25
d 8

2 a x
b $2a$
c $-3x$
d $-2x$

3 a i 6
ii $3x$
iii $6x$
b iii
c terms have no common factor

Now you try

Example 4
a $2x$
b $5x$

Example 5
a $7(4 - 3a)$
b $-3x(3x + 5)$

Example 6
a $(a + b)(4 + a)$
b $(4x + 3)(1 - x)$

Exercise 8C

1 a i $5a$
ii $3b$
b i $2x$
ii $5x$

2 a $2x$
b $6a$
c 2
d 4
e 3
f 1
g $3x$
h $3n$
i $2y$
j $2x$
k $2xy$
l $5ab$

3 a $7(x + 1)$
b $3(x + 1)$
c $4(x - 1)$
d $5(x - 1)$
e $4(1 + 2y)$
f $5(2 + a)$
g $3(1 - 3b)$
h $2(3 - x)$
i $3(4a + b)$
j $6(m + n)$
k $2(5x - 4y)$
l $4(a - 5b)$
m $x(x + 2)$
n $a(a - 4)$
o $y(y - 7)$
p $x(1 - x)$
q $3p(p + 1)$
r $8x(1 - x)$
s $4b(b + 3)$
t $2y(3 - 5y)$
u $3a(4 - 5a)$
v $9m(1 + 2m)$
w $16x(y - 3x)$
x $7ab(1 - 4b)$

4 a $-4(2x + 1)$
b $-2(2x + 1)$
c $-5(2x + y)$
d $-7(a + 2b)$
e $-3(3x + 4)$
f $-2(3y + 4)$
g $-5(2x + 3y)$
h $-4(m + 5n)$
i $-3x(x + 6)$
j $-4x(2x + 3)$
k $-2y(8y + 3)$
l $-5a(a + 2)$
m $-2x(3 + 10x)$
n $-3p(2 + 5p)$
o $-8b(2 + b)$
p $-9x(1 + 3x)$

5 a $(x + 3)(4 + x)$
b $(x + 1)(3 + x)$
c $(m - 3)(7 + m)$
d $(x - 7)(x + 2)$
e $(a + 4)(8 - a)$
f $(x + 1)(5 - x)$
g $(y + 3)(y - 2)$
h $(x + 2)(a - x)$
i $(2t + 5)(t + 3)$
j $(5m - 2)(m + 4)$
k $(4y - 1)(y - 1)$
l $(7 - 3x)(1 + x)$

6 a $6(a + 5)$
b $5(x - 3)$
c $2(4b + 9)$
d $x(x - 4)$
e $y(y + 9)$
f $a(a - 3)$
g $xy(x - 4 + y)$
h $2ab(3 - 5a + 4b)$
i $(m + 5)(m + 2)$
j $(x + 3)(x - 2)$

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k $(b-2)(b+1)$ l $(2x+1)(x-1)$
m $(3-2y)(y-5)$ n $(x+4)(x+9)$
o $(y+1)(y-3)$

7 a $4(x+2)$ b $2(x+3)$ c $10(x+2)$
d $2(x+7)$ e $2(2x+3)$ f $2(x+7)$

8 $4x$

9 a $t(5-t)$
b i 0 m ii 4 m iii 6 m
c 5 seconds

10 a 63 b 72 c −20
d −70 e 69 f 189

11 a $3(a^2+3a+4)$ b $z(5z-10+y)$
c $x(x-2y+xy)$ d $2b(2y-1+3b)$
e $-4y(3x+2z+5xz)$ f $ab(3+4b+6a)$

12 a $-4(x-3)=4(3-x)$
b $-3(x-3)=3(3-x)$
c $-8(n-1)=8(1-n)$
d $-3(b-1)=3(1-b)$
e $-5m(1-m)=5m(m-1)$
f $-7x(1-x)=7x(x-1)$
g $-5x(1-x)=5x(x-1)$
h $-2y(2-11y)=2y(11y-2)$

13 a $(x-4)(x-3)$ b $(x-5)(x+2)$
c $(x-3)(x+3)$ d $(x-4)(3x-5)$
e $(2x-5)(3-x)$ f $(x-2)(2x-1)$
g $(x-3)(4-x)$ h $(x-5)(x-2)$
i $(x-3)(x-2)$

8D

Building understanding

1 a x^2-4 b x^2-49 c $9x^2-y^2$

2 a 3 b 11 c $5b$ d $7y$

3 a $(x+4)(x-4)$
b $x^2-(12)^2=(x+12)(x-12)$
c $(4x)^2-(1)^2=(4x+1)(4x-1)$
d $(3a)^2-(2b)^2=(3a+2b)(3a-2b)$

Now you try

Example 7
a $(x+4)(x-4)$ b $(2a+3)(2a-3)$
c $(5a+b)(5a-b)$

Example 8
a $3(a+3)(a-3)$ b $(x-1)(x+5)$

Exercise 8D

1 a $(x+3)(x-3)$ b $(y+5)(y-5)$
c $(y+1)(y-1)$ d $(x+8)(x-8)$
e $(x+4)(x-4)$ f $(b+7)(b-7)$
g $(a+9)(a-9)$ h $(x+y)(x-y)$
i $(a+b)(a-b)$ j $(4+a)(4-a)$
k $(5+x)(5-x)$ l $(1+b)(1-b)$
m $(6+y)(6-y)$ n $(11+b)(11-b)$
o $(x+20)(x-20)$ p $(30+y)(30-y)$

2 a $(2x+5)(2x-5)$ b $(3x+7)(3x-7)$
c $(5b+2)(5b-2)$ d $(2m+11)(2m-11)$
e $(10y+3)(10y-3)$ f $(9a+2)(9a-2)$
g $(1+2x)(1-2x)$ h $(5+8b)(5-8b)$
i $(4+3y)(4-3y)$ j $(6x+y)(6x-y)$
k $(2x+5y)(2x-5y)$ l $(8a+7b)(8a-7b)$
m $(2p+5q)(2p-5q)$ n $(9m+2n)(9m-2n)$
o $(5a+7b)(5a-7b)$ p $(10a+3b)(10a-3b)$

3 a $3(x+6)(x-6)$ b $10(a+1)(a-1)$
c $6(x+2)(x-2)$ d $4(y+4)(y-4)$
e $2(7+x)(7-x)$ f $8(2+m)(2-m)$
g $5(xy+1)(xy-1)$ h $3(1+xy)(1-xy)$
i $7(3+ab)(3-ab)$

4 a $(x+8)(x+2)$ b $(x+5)(x+1)$
c $(x+14)(x+6)$ d $(x+2)(x-8)$
e $(x-6)(x-8)$ f $(x+3)(x-9)$
g $(10+x)(4-x)$ h $-x(x+4)$
i $(17+x)(1-x)$

5 a $4(3+t)(3-t)$
b i 36 m ii 20 m
c 3 seconds

6 a i x^2 ii $(30+x)(30-x)$
b i 500 cm^2 ii 675 cm^2

7 a $(x+3)(x-3)$ b $(4x+11)(4x-11)$
c $(2+5a)(2-5a)$ d $(x+y)(x-y)$
e $(2b+5a)(2b-5a)$ f $(c+6ab)(c-6ab)$
g $(yz+4x)(yz-4x)$ h $(b+30a)(b-30a)$

8 a Factorise each binomial:
$$(4x+2)(4x-2) = 2(2x+1)2(2x-1) = 4(2x+1)(2x-1)$$
b Take out common factor of 4.

9 $9-(x-1)^2$
$=(3+x-1)(3-(x-1))$ insert brackets when subtracting a binomial
$=(2+x)(3-x+1)$ remember $-1\times -1=+1$
$=(2+x)(4-x)$

10 a $\left(x+\frac{1}{2}\right)\left(x-\frac{1}{2}\right)$ b $\left(x+\frac{2}{5}\right)\left(x-\frac{2}{5}\right)$
c $\left(5x+\frac{3}{4}\right)\left(5x-\frac{3}{4}\right)$ d $\left(\frac{x}{3}+1\right)\left(\frac{x}{3}-1\right)$
e $\left(\frac{a}{2}+\frac{b}{3}\right)\left(\frac{a}{2}-\frac{b}{3}\right)$ f $5\left(\frac{x}{3}+\frac{1}{2}\right)\left(\frac{x}{3}-\frac{1}{2}\right)$
g $7\left(\frac{a}{5}+\frac{2b}{3}\right)\left(\frac{a}{5}-\frac{2b}{3}\right)$ h $\frac{1}{2}\left(\frac{a}{2}+\frac{b}{3}\right)\left(\frac{a}{2}-\frac{b}{3}\right)$
i $(x+y)(x-y)(x^2+y^2)$ j $2(a+b)(a-b)(a^2+b^2)$
k $21(a+b)(a-b)(a^2+b^2)$ l $\frac{1}{3}(x+y)(x-y)(x^2+y^2)$

8E

Building understanding

1 a $2x-2$ b $-5+5a$
c a^2+5a d $4y-y^2$
e $ax+x+2a+2$ f $bx-2b-3x+6$

2 a $2+x$ b $5-x$
c $a+1$ d $1-a$

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3 a $(x-3)(x-2)$ b $(x+4)(x+3)$
c $(2x+1)(3-x)$ d $(3x-2)(4-x)$
e $(5-x)(3x+2)$ f $(x-2)(x+1)$

Now you try

Example 9
a $(x+4)(x+2)$ b $(x+2)(x-3)$

Example 10
$(x-5)(2x+3)$

Exercise 8E

1 a $(x+3)(x+2)$ b $(x+4)(x+3)$
c $(x+7)(x+2)$ d $(x-6)(x+4)$
e $(x-4)(x+6)$ f $(x-3)(x+10)$
g $(x+2)(x-18)$ h $(x+3)(x-14)$
i $(x+4)(x-18)$ j $(x-2)(x-a)$
k $(x-3)(x-3c)$ l $(x-5)(x-3a)$

2 a $(3a+5c)(b+d)$ b $(4b-7c)(a+d)$
c $(y-4z)(2x+3w)$ d $(s-2)(5r+t)$
e $(x+3y)(4x-3)$ f $(2b-a)(a-c)$

3 a $(x-b)(x+1)$ b $(x-c)(x+1)$
c $(x+b)(x+1)$ d $(x+c)(x-1)$
e $(x+a)(x-1)$ f $(x-b)(x-1)$

4 a $(x-7)(2x+1)$ b $(x+5)(x+2)$
c $(x+3)(2x-1)$ d $(1-2x)(3x+4)$
e $(x-5)(11+a)$ f $(3-2x)(4y-1)$
g $(n+2)(3m-1)$ h $(3-r)(5p+8)$
i $(2-y)(8x+3)$

5 a $x^2+4x-ax-4a$ b $x^2-dx-cx+cd$
c $2x-xz+2y-yz$ d $ax+bx-a-b$
e $3cx-3bx-bc+b^2$ f $2xy+2xz-y^2-yz$
g $6ab+15ac+2b^2+5bc$ h $3my+mz-6xy-2xz$
i $2ps+4pt-3qs-6qt$

6 a $(x+2)(x+5)$ b $(x+3)(x+5)$
c $(x+4)(x+6)$ d $(x-3)(x+2)$
e $(x+6)(x-2)$ f $(x-9)(x-2)$

7 a Method 1: $a(x+7)-3(x+7)=(x+7)(a-3)$
Method 2: $x(a-3)+7(a-3)=(a-3)(x+7)$
b i $(x-3)(b+2)$ ii $(x+2)(y-4)$
iii $(2m+3)(2m-5n)$ iv $(2-n)(m-3)$
v $(1-2b)(4a+3b)$ vi $(3a-1)(b+4c)$

8 Answers may vary.

9 a $(a-3)(2-x-c)$ b $(2a+1)(b+5-a)$
c $(a+1)(x-4-b)$ d $(a-b)(3-b-2a)$
e $(1-a)(c-x+2)$ f $(x-2)(a+2b-1)$
g $(a-3c)(a-2b+3bc)$ h $(1-2y)(3x-5z+y)$
i $(x-4)(3x+y-2z)$ j $(ab-2c)(2x+3y-1)$

8F

Building understanding

1 a $x^2+9x+14$ b $x^2+14x+33$
c $x^2+7x-60$ d $x^2-8x+12$

2 a 3, 2 b 12, 1 c 4, 5
d 5, −3 e −6, 5 f −9, −2

Now you try

Example 11
a $(x+2)(x+7)$ b $(x+6)(x-2)$
c $(x-3)(x-4)$

Example 12
$3(x+3)(x-5)$

Exercise 8F

1 a $(x+6)(x+2)$ b $(x+9)(x+1)$
c $(x+7)(x+1)$ d $(x+14)(x+1)$
e $(x+3)(x+4)$ f $(x+8)(x+2)$
g $(x+5)(x+3)$ h $(x+4)(x+5)$
i $(x+8)(x+3)$

2 a $(x+4)(x-1)$ b $(x+2)(x-1)$
c $(x+5)(x-1)$ d $(x+7)(x-2)$
e $(x+5)(x-3)$ f $(x+10)(x-2)$
g $(x+6)(x-3)$ h $(x+9)(x-2)$
i $(x+4)(x-3)$

3 a $(x-5)(x-1)$ b $(x-1)(x-1)$
c $(x-1)(x-4)$ d $(x-8)(x-1)$
e $(x-2)(x-2)$ f $(x-6)(x-2)$
g $(x-9)(x-2)$ h $(x-7)(x-3)$
i $(x-3)(x-2)$

4 a $(x-8)(x+1)$ b $(x-4)(x+1)$
c $(x-6)(x+1)$ d $(x-8)(x+2)$
e $(x-6)(x+4)$ f $(x-5)(x+3)$
g $(x-4)(x+3)$ h $(x-12)(x+1)$
i $(x-6)(x+2)$

5 a $2(x+4)(x+1)$ b $2(x+10)(x+1)$
c $3(x+2)(x+4)$ d $2(x+10)(x-3)$
e $2(x-9)(x+2)$ f $4(x-1)(x-1)$
g $2(x-2)(x+3)$ h $6(x-6)(x+1)$
i $5(x-4)(x-2)$ j $3(x-5)(x-6)$
k $2(x-5)(x+2)$ l $3(x-4)(x+3)$

6 a $6x$ b $6x$ or $10x$
c $x, 4x, 11x$ d $x, 4x, 11x$
e $9x, 11x, 19x$ f $9x, 11x, 19x$
g $0x, 6x, 15x$ h $0x, 24x$

7 a i $x(x+2)$ ii $x^2+2x-15$
iii $(x+5)(x-3)$
b i 105 m^2 ii 48 m^2

8 a $(x+4)^2$ b $(x+5)^2$ c $(x+15)^2$
d $(x-1)^2$ e $(x-7)^2$ f $(x-13)^2$
g $2(x+1)^2$ h $5(x-3)^2$ i $-3(x-6)^2$

9 B, C, E, F

10 a $(x-1)^2-9$ b $(x+2)^2-5$
c $(x+5)^2-22$ d $(x-8)^2-67$
e $(x+9)^2-74$ f $(x-16)^2-267$

Progress quiz

1 a x^2+6x+8 b $a^2+3a-40$
c $2x^2+9x-18$ d $3a^2-7ab+2b^2$

2 a $y^2+8y+16$ b x^2-6x+9
c $4a^2-12a+9$ d $49k^2+28km+4m^2$

3 a x^2-25 b $121x^2-81y^2$

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4 a $5(5a-3)$ b $x(x-7)$
c $-4x(3x+4)$ d $(a+3)(2+a)$
e $(8+a)(7-a)$ f $(k-4)(k-1)$
5 a $(x+9)(x-9)$ b $(4a+7)(4a-7)$
c $(5x+y)(5x-y)$ d $2(a+5)(a-5)$
e $12(xy+1)(xy-1)$ f $(h+11)(h-5)$
6 a $(x+7)(x+2)$ b $(a+5)(a-4)$
c $(x-h)(x+1)$
7 a $(x-3)(2x+3)$ b $(3a+2)(p-5)$
8 a $(x+4)(x+2)$ b $(a+5)(a-3)$
c $(m-6)(m-5)$ d $2(k-3)(k+4)$

8G

Building understanding

1 a 2, 6 b 8, −3
c −5, −7 d −7, 4

2 a $= 2x^2+2x+5x+5$
$= 2x(x+1)+5(x+1)$
$= (x+1)(2x+5)$

b $= 2x^2-3x-4x+6$
$= x(2x-3)-2(2x-3)$
$= (2x-3)(x-2)$

c $= 5x^2+10x-x-2$
$= 5x(x+2)-1(x+2)$
$= (x+2)(5x-1)$

d $= 4x^2+8x+3x+6$
$= 4x(x+2)+3(x+2)$
$= (x+2)(4x+3)$

Now you try

Example 13
$(3x+2)(x+3)$

Example 14
a $(2x+5)(2x-3)$ b $(4x-3)(2x-3)$

Exercise 8G

1 a $(2x+1)(x+4)$ b $(3x+1)(x+2)$
c $(2x+3)(x+2)$ d $(3x+2)(x+2)$
e $(5x+2)(x+2)$ f $(2x+3)(x+4)$
g $(3x+5)(2x+1)$ h $(4x+1)(x+1)$
i $(4x+5)(2x+1)$
2 a $(3x+5)(x-1)$ b $(5x-4)(x+2)$
c $(2x+3)(4x-1)$ d $(2x+1)(3x-8)$
e $(2x+1)(5x-4)$ f $(x-3)(5x+4)$
g $(2x-5)(2x-3)$ h $(x-6)(2x-3)$
i $(2x-5)(3x-2)$ j $(3x-4)(4x+1)$
k $(2x-3)(2x-3)$ l $(x+3)(7x-3)$
m $(x+5)(9x-1)$ n $(x-2)(3x-8)$
o $(2x-5)(2x+3)$
3 a $2(3x-2)(5x+1)$ b $6(x-1)(2x+5)$
c $3(3x-5)(3x-1)$ d $7(x-3)(3x-2)$
e $4(3x-2)(3x+5)$ f $5(2x-3)(5x+4)$
4 a $(2x+1)(5x+11)$ b $(3x+4)(5x-2)$
c $(2x-3)(10x-3)$ d $(2x+1)(9x-5)$
e $(5x-3)(5x+4)$ f $(4x+1)(8x-5)$
g $(3x+2)(9x-4)$ h $(3x+1)(11x+10)$
i $(6x-5)(9x+1)$ j $(2x-3)(6x-7)$
k $(3x-1)(25x-6)$ l $(6x-1)(15x+8)$
5 a $-(x-2)(2x-3)$ b $-(x-1)(5x+8)$
c $-(2x+1)(3x-8)$ d $-(x+3)(5x-6)$
e $-(2x-5)(2x-3)$ f $-(2x-1)(4x-5)$

6 a $(x+4)(2x-5)$
b $(2x-5)(x+4)$
c No, you get the same result.
d i $(x+3)(3x-4)$
ii $(x-2)(5x+7)$
iii $(2x-1)(3x+4)$
7 Answers may vary.
8 See answers to Questions 2 and 4.

8H

Building understanding

1 a $\frac{1}{3}$ b $\frac{2x}{3}$ c $\frac{3}{x}$ d $\frac{x+1}{2}$
2 a $3(x+2)$ b $20(1-2x)$
c $x(x-7)$ d $6x(x+4)$
3 a $\frac{2(x-2)}{8}=\frac{x-2}{4}$ b $\frac{6(2-3x)}{x(2-3x)}$
4 a 9 b $\frac{1}{12}$

Now you try

Example 15
a $\frac{x-3}{3}$ b $\frac{3}{5}$ c $x-3$
Example 16
a $\frac{9}{5(x-4)}$ b $\frac{2(x-2)}{x}$ c $\frac{x-3}{3}$

Exercise 8H

1 a $\frac{3}{4}$ b $\frac{1}{3}$ c 4
d $x-5$ e $\frac{2(x-1)}{3}$ f $\frac{2}{x+4}$
2 a $x-1$ b $\frac{2(x-3)}{5}$ c $\frac{2}{3}$
d 2 e $x-3$ f $\frac{2(2x+5)}{5}$
g $\frac{3}{2}$ h $\frac{4}{3}$
3 a $x-10$ b $x-7$ c $x-5$
d $\frac{2}{x+20}$ e $\frac{5}{x+6}$ f $\frac{3}{x-9}$
4 a $x+2$ b $x+4$ c $x-3$
d $\frac{1}{x+3}$ e $\frac{1}{x-2}$ f $\frac{1}{x-10}$
5 a $\frac{x-4}{2}$ b $x-3$
c $3(x-3)$ d $\frac{x+4}{2(x-5)}$
6 a x b $\frac{2(x-2)}{x+2}$
c $\frac{4}{x+1}$ d $\frac{x-2}{2(x+2)}$
e $\frac{2(x+2)^2}{3(x+4)}$ f $\frac{5}{x+2}$
7 a $(x+2)(x-5)$ b $\frac{x+2}{x+3}$
c $\frac{x-3}{x-4}$ d $\frac{x-5}{x+2}$
e $\frac{3x-1}{2-15x}$ f $\frac{x-4}{3-x}$
g $\frac{x+5}{2(x+4)}$ h $\frac{3}{2(x+3)}$

Answers

Progress quiz

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8 a $a+1$ b $5(a-3)$ c $\frac{x+7}{2}$
d $\frac{x+2}{6}$ e $\frac{x+3}{2}$ f $\frac{11}{x-2}$

9 a -1 b -1 c -8
d $-\frac{1}{3}$ e $-\frac{1}{6}$ f $-(x+3)$

10 a $-\frac{x+3}{2}$ b $-\frac{x-7}{3}$
c $-\frac{x-8}{x+8}$ d $-\frac{2}{x+2}$
e $-\frac{4}{x+3}$ f $\frac{3(x+1)}{4}$
g $\frac{3(2x+3)}{2x}$ h $-\frac{1}{2(x+2)}$
i $\frac{x}{3}$ j $-\frac{2}{x-2}$

8I

Building understanding

1 a 24 b 15 c 143 d 36
2 a $21x$ b 2 c 8 d 90
3 a $\frac{3x}{12}+\frac{8x}{12}=\frac{11x}{12}$
b $\frac{25x}{35}-\frac{14x}{35}=\frac{11x}{35}$
c $\frac{2(x+1)}{4}+\frac{(2x+3)}{4}=\frac{2x+2+2x+3}{4}=\frac{4x+5}{4}$
4 a 15 b 14 c 6

Now you try

Example 17
a $-\frac{5x}{12}$ b $\frac{11x}{8}$ c $\frac{5x-8}{6}$

Example 18
a $\frac{5}{3x}$ b $\frac{4x+5}{x^2}$

Exercise 8I

1 a $\frac{9x}{14}$ b $\frac{2x}{5}$ c $\frac{x}{8}$ d $\frac{14x}{45}$
e $\frac{y}{56}$ f $\frac{13a}{22}$ g $\frac{2b}{9}$ h $\frac{m}{6}$
i $\frac{11m}{12}$ j $\frac{15a}{28}$ k $\frac{x}{2}$ l $-\frac{20p}{63}$
m $-\frac{5b}{18}$ n $\frac{61y}{40}$ o $\frac{13x}{35}$ p $\frac{5x}{12}$

2 a $\frac{7x+11}{10}$ b $\frac{7x}{12}$ c $\frac{15a-51}{56}$
d $\frac{11y+9}{30}$ e $\frac{13m+28}{40}$ f $\frac{5x-13}{24}$
g $\frac{11b-6}{24}$ h $\frac{7x}{6}$ i $\frac{7y-8}{14}$
j $\frac{5t-4}{16}$ k $\frac{34-10x}{21}$ l $\frac{8m-9}{12}$

3 a $\frac{11}{2x}$ b $\frac{1}{3x}$ c $-\frac{3}{4x}$
d $\frac{14}{9x}$ e $\frac{7}{20x}$ f $\frac{13}{15x}$
g $-\frac{31}{4x}$ h $-\frac{29}{12x}$

4 a $\frac{3x+2}{x^2}$ b $\frac{5+4x}{x^2}$ c $\frac{7x+3}{x^2}$
d $\frac{4x-5}{x^2}$ e $\frac{3-8x}{x^2}$ f $\frac{x-4}{x^2}$
g $\frac{6x-7}{2x^2}$ h $\frac{9-2x}{3x^2}$

5 a $\frac{8+x^2}{4x}$ b $\frac{x^2-10}{2x}$
c $\frac{-6-4x^2}{3x}$ d $\frac{6-5x^2}{4x}$
e $\frac{9x^2-10}{12x}$ f $\frac{3-x^2}{9x}$
g $\frac{15x^2-4}{10x}$ h $\frac{-25-6x^2}{20x}$

6 a $\frac{x}{3}$ b $\frac{x}{8}$ c $\frac{x}{2}$
d $\frac{x}{5}$ e $\frac{8x}{9}$ f $\frac{x}{4}$

7 a $\frac{4x}{3}$ b 200 seconds

8 a $\frac{8x+2}{8}=\frac{2(4x+1)}{8}=\frac{4x+1}{4}$
b $\frac{4x+1}{4}$
c Using denominator 8 does not give answer in simplified form and requires extra steps. Preferable to use actual LCD.

9 a didn't make a common denominator, $\frac{7x}{15}$
b didn't use brackets: $2(x+1)=2x+2$, $\frac{7x+2}{10}$
c didn't use brackets: $3(x-1)=3x-3$, $\frac{13x-3}{6}$
d didn't multiply numerator in $\frac{2}{x}$ by x as well as denominator, $\frac{2x-3}{x^2}$

10 a $-\frac{43x}{30}$ b $\frac{5x}{12}$ c $\frac{13x}{24}$
d $\frac{43x-5}{60}$ e $\frac{23x-35}{42}$ f $\frac{29x+28}{40}$
g $\frac{14}{3x}$ h $\frac{1}{6x}$ i $-\frac{11}{20x}$
j $\frac{24-x}{6x^2}$ k $\frac{60x-21}{14x^2}$ l $\frac{3-4x}{9x^2}$
m $\frac{30-2x^2}{15x}$ n $\frac{11x^2-3}{6x}$ o $\frac{18-2x^2}{45x}$

8J

Building understanding

1 a $-2x-6$ b $-14-21x$
c $-3x+3$ d $-30+20x$
2 a 9 b $2x$
c x^2 d $(x-2)(x+3)$
3 a $-x-4$ b $3x+16$ c $6-2x$

Now you try

Example 19
a $\frac{x-16}{6}$ b $\frac{10x-5}{12}$

Example 20
a $\frac{5x-5}{(x+2)(x-3)}$ b $\frac{11-3x}{(x-2)^2}$

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Exercise 8J

1 a $\frac{1-x}{12}$ b $\frac{2x-14}{15}$ c $\frac{x-9}{6}$
d $\frac{-7x-14}{10}$ e $\frac{9x-4}{8}$ f $\frac{-x-24}{28}$
g $\frac{5x-3}{12}$ h $\frac{1-18x}{15}$ i $\frac{8x-23}{30}$

2 a $\frac{13-x}{6}$ b $\frac{2x+2}{35}$ c $\frac{x-5}{4}$
d $\frac{22x-41}{21}$ e $\frac{17x}{20}$ f $\frac{30-13x}{24}$
g $\frac{14x+4}{9}$ h $\frac{20x-9}{28}$ i $\frac{10-11x}{56}$

3 a $\frac{7x-1}{(x-1)(x+1)}$ b $\frac{7x-7}{(x+4)(x-3)}$
c $\frac{7x+1}{(x-2)(x+3)}$ d $\frac{5x+13}{(x-4)(x+7)}$
e $\frac{4x+15}{(x+2)(x+3)}$ f $\frac{x-26}{(x+4)(x-6)}$
g $\frac{x+9}{(x+5)(x+1)}$ h $\frac{16-6x}{(x-3)(x-2)}$
i $\frac{7-2x}{(x-5)(x-6)}$

4 a $\frac{1-3x}{(x+1)^2}$ b $\frac{-4x-10}{(x+3)^2}$ c $\frac{3x-2}{(x-2)^2}$
d $\frac{18-2x}{(x-5)^2}$ e $\frac{9-x}{(x-6)^2}$ f $\frac{14-3x}{(x-4)^2}$
g $\frac{4x+7}{(2x+1)^2}$ h $\frac{1-12x}{(3x+2)^2}$ i $\frac{20x-1}{(1-4x)^2}$

5 a $\frac{5x-2}{(x-1)^2}$ b $\frac{19x+8}{12x}$
c $\frac{10x^2-11x+4}{20x}$ d $\frac{x^2+7x}{(x-5)(x+1)}$
e $\frac{2x^2-5x-3}{(4-x)(x-1)}$ f $\frac{x^2+2x+1}{(x-3)^2}$
g $\frac{3-2x}{(x-2)^2}$ h $\frac{-x^2-11x}{(2x+1)(x+2)}$
i $\frac{x^2-4x-1}{(x+1)^2}$

6 a $\frac{3x+11}{(x+3)(x+4)(x+5)}$ b $\frac{2-2x}{(x+1)(x+2)(x+4)}$
c $\frac{26-10x}{(x-1)(x-3)(2-x)}$ d $\frac{3x-2}{(x+1)(x-5)}$
e $\frac{2x+9}{(x-4)(3-2x)}$ f $\frac{7x^2+7x}{(x+4)(2x-1)(3x+2)}$

7 a second line $-2 \times (-2) = +4$ not -4
b $\frac{33x+4}{10}$

8 a $\frac{5}{1-x}$ b $\frac{4x-3}{5-x}$ c $\frac{9}{7x-3}$
d $\frac{1-2x}{4-3x}$ e 1 f 0

9 a $\frac{11}{2(x+2)}$ b $\frac{1}{3(x-1)}$
c $\frac{23}{4(2x-1)}$ d $\frac{13-3x}{(x+3)(x-3)}$
e $\frac{5x-6}{2(x+2)(x-2)}$ f $\frac{30x+33}{(3x-4)(3x+4)}$
g $\frac{9x-27}{(x+3)(x+4)(x-5)}$ h $\frac{x+11}{(x-1)(x+3)^2}$
i $\frac{2x+5}{5(x-2)(x-5)}$ j $\frac{-2}{x(x-1)(x+1)}$

8K

Building understanding

1 a 15 b 6 c 28
2 a $4x$ b $2(x+3)$
c 3 d $4(x-1)$
3 a multiply by 5, subtract 3 then divide by 2
b multiply by x then divide by 2

Now you try

Example 21
a $x = 12$ b $x = 12$ c $x = 3$

Example 22
a $x = \frac{7}{40}$ b $x = 7$

Exercise 8K

1 a 10 b 12 c 24
d 10 e $\frac{6}{7}$ f $7\frac{1}{2}$
g -8 h 15 i 4

2 a 13 b 1 c $1\frac{3}{5}$
d 59 e 1 f 5
g 8 h $3\frac{2}{7}$ i $-3\frac{1}{4}$

3 a -3 b -4 c 5
d 1 e -1 f 6

4 a $\frac{1}{16}$ b $\frac{1}{12}$ c $\frac{8}{15}$
d $\frac{1}{36}$ e $\frac{3}{4}$ f $\frac{5}{24}$
g $\frac{5}{12}$ h $-\frac{1}{6}$ i $2\frac{5}{6}$

5 a $-\frac{5}{2}$ b -5 c -19
d -4 e -1 f -6

6 a $\frac{x}{2} + \frac{2x}{3} = 4$ b $x = 3\frac{3}{7}$

7 a $\frac{x}{3} + \frac{x}{4} = 77$ b 132 games

8 a $\frac{1}{6}$ b 2 c 0
d $\frac{9}{13}$ e $\frac{3}{7}$ f $\frac{2}{11}$
g $1\frac{5}{7}$ h 0 i -12
j 9

9 a $\frac{1}{x} + \frac{1}{2x} = \frac{3}{10}$ b 5 and 10

10 On the second line, not every term has been multiplied by 12. The $2x$ should be $24x$ to give an answer $x = \frac{3}{29}$.

11 On third line of working, $-2 \times (-1) = +2$ not -2, giving answer $x = -4$.

Answers

8J

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12 a $4\frac{1}{2}$ b $5\frac{2}{3}$ c $1\frac{1}{2}$
d 3 e 15 f -2

13 a $x = 2ab$ b $x = \frac{2bd}{2a - bc}$
c $x = \frac{ac}{c - b}$ d $x = \frac{bd + be - ac}{c}$
e $x = \frac{4c - 3b}{3a - 4}$ f $x = \frac{6b + a}{5}$
g $x = -\frac{a^2}{2a - b - a^2}$ or $\frac{a^2}{a^2 - 2a + b}$
h $x = \frac{ac}{c - a}$ i $x = \frac{ac}{b}$
j $x = \frac{c - a}{b}$ k $x = \frac{b - bc}{a - c}$
l $x = \frac{ad - b}{c - d}$ m $x = ab - 2a$
n $x = \frac{a + b}{1 - a}$ o $x = \frac{2ab - a^2}{a - b}$

Problems and challenges

1 a 48, 49 b 33, 35 c 12, 15
2 a 15 b 5
3 $a = 2$, $b = 1$, $c = 7$ and $d = 8$
4 a $(n + 1)^2 + 1$ or $n^2 + 2n + 2$
b Answers may vary.
5 $(n - 1)(n + 1)$
a $n - 1$ and $n + 1$ are both even, since they are consecutive even numbers one of them is divisible by 4 hence their product is also divisible by 4
b $n - 1$, n, $n + 1$ are 3 consecutive numbers, one of them must be divisible by 3. Since n is prime it must be $n - 1$ or $n + 1$ so their product is divisible by 3
c $n^2 - 1$ is divisible by 3 and 4 and since they have no common factor it must also be divisible by $3 \times 4 = 12$
6 Factorise each expression and cancel.
7 $4x^2 - 4x + 1 = (2x - 1)^2$, which is always greater than or equal to zero
8 Ryan

Success criteria example questions

1 $2x^2 + 5x - 12$
2 $x^2 - 12x + 36$; $16x^2 - 25$
3 $4x(3x - 2)$; $(2x + 3)(x + 5)$
4 $(x - 10)(x + 10)$; $(2a - 3)(2a + 3)$
5 $(x + 6)(x - 3)$
6 $(x - 3)(2x + 3)$
7 $(x + 7)(x - 3)$
8 $2(x - 3)(x + 3)$; $3(x - 3)(x - 4)$
9 $3x^2 + 11x + 6$; $(2x + 1)(2x - 3)$
10 $\frac{3}{8}$
11 $\frac{5(x - 1)}{3}$
12 $\frac{1}{2}$
13 $\frac{5x + 8}{6}$
14 $\frac{10}{3x}$
15 $\frac{5x + 23}{12}$
16 $\frac{5x - 8}{(x + 2)(x - 4)}$
17 $x = \frac{30}{17}$; $x = -4$

Short-answer questions

1 a $x^2 + x - 12$ b $x^2 - 9x + 14$
c $6x^2 - 5x - 6$ d $9x^2 + 3x - 12$
2 a $x^2 + 6x + 9$ b $x^2 - 8x + 16$
c $9x^2 - 12x + 4$ d $x^2 - 25$
e $49 - x^2$ f $121x^2 - 16$
3 a $4(a + 3b)$ b $3x(2 - 3x)$
c $-5xy(x + 2)$ d $(x - 7)(x + 3)$
e $(2x + 1)(x - 1)$ f $(x - 2)(x - 6)$
4 a $(x + 10)(x - 10)$ b $3(x + 4)(x - 4)$
c $(5x + y)(5x - y)$ d $(7 + 3x)(7 - 3x)$
e $(x - 12)(x + 6)$ f $(1 - x)(1 + x)$
5 a $(x - 3)(x + 6)$ b $(2x + 5)(2x - 1)$
c $(x - 4)(3 + 2b)$
6 a $(x + 3)(x + 5)$ b $(x - 6)(x + 3)$
c $(x - 6)(x - 1)$ d $3(x + 7)(x - 2)$
e $2(x + 4)^2$ f $(5x + 2)(x + 3)$
g $(2x - 3)(2x + 1)$ h $(3x - 4)(2x - 3)$
7 a $x + 4$ b $\frac{2}{3}$ c $\frac{x - 3}{5}$
8 a $\frac{1}{4}$ b $\frac{x - 4}{2}$ c $\frac{x}{3}$
d $\frac{2}{5}$ e $\frac{2x + 3}{50x}$ f 4
9 a $\frac{11x}{12}$ b $\frac{5}{4x}$
c $\frac{7x - 2}{x^2}$ d $\frac{x - 13}{24}$
e $\frac{8x + 11}{(x + 1)(x + 2)}$ f $\frac{15 - 2x}{(x - 4)^2}$
10 a $x = 20$ b $x = \frac{1}{6}$
c $x = 7$ d $x = -1\frac{2}{9}$

Multiple-choice questions

1 A	2 E	3 D	4 C	5 B
6 D	7 A	8 E	9 B	10 A

Extended-response questions

1 a $(x + 3)$ m
b i no change
ii $(x^2 - 1)$ m^2, 1 square metre less in area
c i width $= (x - 3)$ m, decreased by 3 metres
ii $A = (x + 7)(x - 3) = (x^2 + 4x - 21)$ m^2
iii $A = 0$ m^2
2 a 400 m^2
b i $l = w = (20 + 2x)$ m
ii $(4x^2 + 80x + 400)$ m^2
c $\frac{1}{4}$
d $4x(x + 20)$ m^2
e $x = 5$

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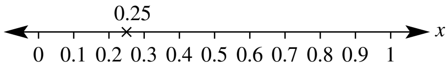
Chapter 9

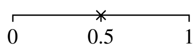
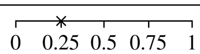
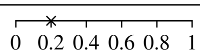
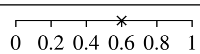
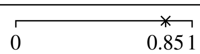
9A

Building understanding

1 a i $\frac{1}{4}$ ii 0.25 iii 25%

b 

2

	Percentage	Decimal	Fraction	Number line
	50%	0.5	$\frac{1}{2}$	
a	25%	0.25	$\frac{1}{4}$	
b	20%	0.2	$\frac{1}{5}$	
c	60%	0.6	$\frac{3}{5}$	
d	85%	0.85	$\frac{17}{20}$	

3 0.15, $\frac{2}{9}$, 1 in 4, 0.28, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{7}$, 2 in 3, 0.7, 0.9

Now you try

Example 1

a {1, 2, 3, 4, 5} b $\frac{3}{8}$ c $\frac{5}{8}$

d $\frac{5}{8}$ e $\frac{7}{8}$

Example 2

a $\frac{1}{9}$ b $\frac{8}{9}$ c $\frac{5}{9}$ d $\frac{4}{9}$ e $\frac{2}{3}$ f 1

Exercise 9A

1 a {1, 2, 3, 4, 5, 6, 7} b $\frac{1}{7}$

c $\frac{6}{7}$ d $\frac{2}{7}$

e $\frac{6}{7}$

2 a i {2, 6, 7} ii $\frac{1}{2}$ iii $\frac{1}{2}$

iv $\frac{1}{2}$ v 1

b i {1, 2, 3} ii $\frac{2}{3}$ iii $\frac{1}{3}$

iv $\frac{5}{6}$ v $\frac{5}{6}$

c i {1, 2, 3} ii $\frac{1}{3}$ iii $\frac{2}{3}$

iv $\frac{2}{3}$ v $\frac{2}{3}$

d i {1, 2, 3, 4} ii $\frac{1}{7}$ iii $\frac{6}{7}$

iv $\frac{3}{7}$ v $\frac{5}{7}$

3 a $\frac{1}{2}$ b $\frac{3}{8}$ c $\frac{1}{6}$

d $\frac{1}{4}$ e 1 f 0

4 a $\frac{1}{2}$ b $\frac{5}{8}$ c $\frac{5}{6}$

d $\frac{3}{4}$ e 0 f 1

5 a $\frac{1}{8}$ b $\frac{7}{8}$

6 a $\frac{1}{8}$ b $\frac{1}{4}$ c $\frac{3}{8}$ d $\frac{3}{8}$ e $\frac{5}{8}$

f 1 g 0 h $\frac{1}{4}$ i $\frac{3}{4}$ j $\frac{3}{4}$

7 a {Hayley, Alisa, Rocco, Stuart}

b i $\frac{1}{4}$ ii $\frac{1}{2}$ iii $\frac{1}{2}$

8 a $\frac{1}{6}$ b $\frac{1}{6}$ c $\frac{5}{6}$ d $\frac{1}{3}$ e $\frac{2}{3}$

f 1 g $\frac{1}{3}$ h $\frac{1}{2}$ i $\frac{5}{6}$

9 a $\frac{1}{52}$ b $\frac{1}{13}$ c $\frac{1}{26}$ d $\frac{1}{2}$

e $\frac{2}{13}$ f $\frac{12}{13}$ g $\frac{23}{26}$ h $\frac{25}{26}$

10 a $\frac{2}{11}$ b $\frac{9}{11}$ c $\frac{3}{11}$

d $\frac{7}{11}$ e $\frac{4}{11}$

11 Amanda has not taken into account the fact there are two O's.

12 A, D, F

13 a $\frac{12}{25}$ b $\frac{8}{25}$ c $\frac{1}{5}$ d $\frac{9}{25}$ e $\frac{8}{25}$

f $\frac{16}{25}$ g $\frac{4}{25}$ h $\frac{17}{25}$ i 0

14 a 31 min b $\frac{3}{31}$

c i $\frac{4}{31}$ ii $\frac{4}{31}$ iii $\frac{20}{31}$

iv $\frac{5}{31}$ v $\frac{8}{31}$ vi $\frac{11}{31}$

9B

Building understanding

1 a 26

b i 10 ii 9 iii 4

iv 7 v 19

c i 12 ii 17

2 a B b D c A d C

3 a

	A	Not A	Total
B	7	8	15
Not B	3	1	4
Total	10	9	19

b

	A	Not A	Total
B	2	5	7
Not B	9	4	13
Total	11	9	20

Answers

9A

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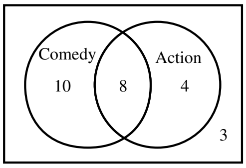
Now you try

Example 3

a



b i 22 ii 13 iii 10

c i $\frac{8}{25}$ ii $\frac{3}{25}$ iii $\frac{4}{25}$

Example 4

a

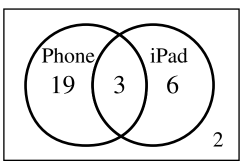
	Classes	Not classes	Total
Equipment	8	11	19
Not equipment	10	1	11
Total	18	12	30

b i 12 ii 1

c i $\frac{19}{30}$ ii $\frac{1}{3}$ iii $\frac{29}{30}$

Exercise 9B

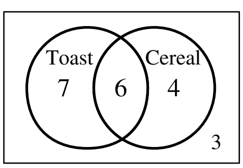
1 a



b i 28 ii 21 iii 6

c i $\frac{1}{10}$ ii $\frac{1}{15}$ iii $\frac{19}{30}$

2 a



b i 17 ii 7 iii 7

c i $\frac{3}{10}$ ii $\frac{1}{5}$

3 a i $\frac{2}{5}$ ii $\frac{1}{3}$ iii $\frac{7}{15}$

iv $\frac{1}{15}$ v $\frac{13}{15}$ vi $\frac{2}{15}$

b i $\frac{3}{7}$ ii $\frac{12}{35}$ iii $\frac{13}{35}$

iv $\frac{3}{35}$ v $\frac{34}{35}$ vi $\frac{1}{35}$

4 a

	Cream	Not cream	Total
Ice-cream	5	20	25
Not ice-cream	16	9	25
Total	21	29	50

b i 29 ii 9

c i $\frac{21}{50}$ ii $\frac{8}{25}$ iii $\frac{41}{50}$

5 a i $\frac{5}{8}$ ii $\frac{3}{8}$ iii $\frac{3}{8}$

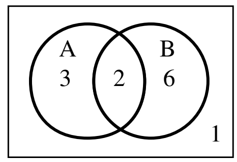
iv $\frac{3}{4}$ v $\frac{1}{8}$ vi $\frac{1}{4}$

b i $\frac{17}{26}$ ii $\frac{9}{26}$ iii $\frac{11}{26}$

iv $\frac{21}{26}$ v $\frac{2}{13}$ vi $\frac{5}{26}$

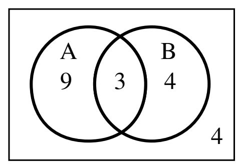
6 a

	A	Not A	Total
B	2	6	8
Not B	3	1	4
Total	5	7	12

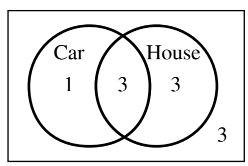


b

	A	Not A	Total
B	3	4	7
Not B	9	4	13
Total	12	8	20



7 a



b 3 c $\frac{1}{10}$

8 a

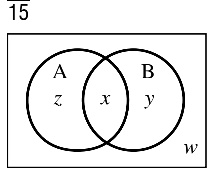
	Rainwater	Not rainwater	Total
Tap water	12	36	48
Not tap water	11	41	52
Total	23	77	100

b 12 c $\frac{9}{25}$ d $\frac{59}{100}$

9 23

10 $\frac{7}{15}$

11



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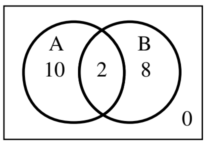
12 a $T - x - y + z$ b $T - x$ c $T - y$
d $x - z$ e $y - z$ f $y - z$
g $T - x + z$

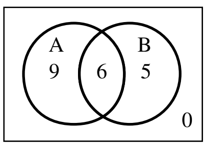
13

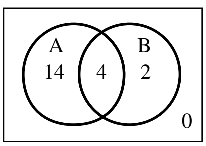
	A	Not A	Total
B	0	12	12
Not B	11	−4	7
Total	11	8	19

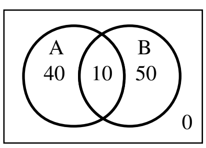
Filling in table so totals add up requires a negative number, which is impossible!

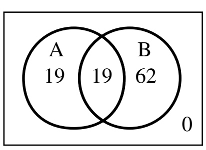
14 4

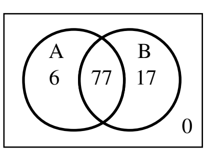
15 a i


ii


iii


b i


ii


iii


c overlap = (A + B) − total

9C

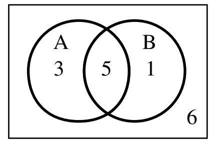
Building understanding

1 a C b D c E d A
e F f B
2 a B b D c A d C
3 a 2 b 10 c 7 d 9

Now you try

Example 5

a i {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}
ii {1, 3, 5, 7, 9, 11, 13, 15}
iii {2, 3, 5, 7, 11, 13}

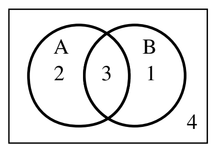
b


c i {3, 5, 7, 11, 13} ii {1, 2, 3, 5, 7, 9, 11, 13, 15}
iii {2, 4, 6, 8, 10, 12, 14} iv {2}

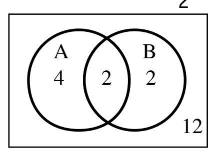
d i 8 ii $\frac{8}{15}$ iii 5 iv $\frac{1}{3}$

Exercise 9C

1 a i {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
ii {1, 3, 5, 7, 9} iii {2, 3, 5, 7}
b i {3, 5, 7} ii {1, 2, 3, 5, 7, 9}
iii {2, 4, 6, 8, 10} iv {2}

c


d i 5 ii $\frac{1}{2}$ iii 3 iv $\frac{3}{10}$

2 a


A = {3, 6, 9, 12, 15, 18}
B = {1, 3, 5, 15}

b i {3, 15} ii {1, 3, 5, 6, 9, 12, 15, 18}
iii {1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20}
iv {1, 5}

c i 4 ii $\frac{1}{5}$ iii 2
iv $\frac{1}{10}$ v 8 vi $\frac{2}{5}$

3 a i true ii true iii false iv false
v true vi false vii true viii false

b i $\frac{1}{2}$ ii $\frac{1}{2}$ iii 0

4 a i $\frac{1}{6}$ ii $\frac{11}{12}$ iii $\frac{5}{12}$
b i $\frac{6}{25}$ ii $\frac{21}{25}$ iii $\frac{8}{25}$
c i $\frac{1}{3}$ ii $\frac{11}{15}$ iii $\frac{2}{5}$
d i $\frac{5}{21}$ ii $\frac{17}{21}$ iii $\frac{4}{7}$

5 a i {Fred, Ron, Rachel} ii {Fred, Rachel, Helen}
iii { } iv {Fred, Rachel}
b i $\frac{3}{4}$ ii $\frac{1}{4}$ iii 0
iv 1 v $\frac{1}{2}$ vi 1

6 a 26 b 5 c 3 d 18
e $\frac{5}{26}$ f $\frac{21}{26}$ g $\frac{3}{26}$ h $\frac{5}{13}$

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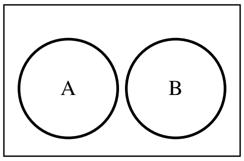
7 a 11 b 21 c 1 d $\frac{9}{25}$

8 a own both a dog and a cat
b own dogs or cats or both
c does not own a cat
d owns a cat but no dogs

9 B only = B ∩ A′

10 a yes

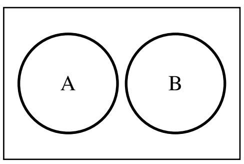


b No, as A ∪ B includes only elements from sets A and B.

11

	A	A′	Total
B	n(A ∩ B)	n(B ∩ A)′	n(B)
B′	n(A ∩ B′)	n(A′∩ B′)	n(B′)
Total	n(A)	n(A′)	n(sample space)

12 Mutually exclusive events have no common elements, i.e. A ∩ B = ∅.



13 a i {2, 3, 5, 7, 11, 13, 17, 19}
ii {1, 2, 3, 4, 6, 12} iii {2, 3}
iv {1, 2, 3, 4, 5, 6, 7, 11, 12, 13, 17, 19}
v {1, 4, 6, 12} vi {5, 7, 11, 13, 17, 19}
vii {1, 2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20}
viii {2, 3, 5, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20}
ix {1, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}

b i $\frac{3}{5}$ ii $\frac{1}{10}$ iii $\frac{1}{5}$ iv $\frac{2}{5}$
v $\frac{4}{5}$ vi $\frac{9}{10}$ vii $\frac{9}{10}$ viii $\frac{2}{5}$

c They are equal.

9D

Building understanding

1 a 9 outcomes

	1	2	3
1	(1, 1)	(2, 1)	(3, 1)
2	(1, 2)	(2, 2)	(3, 2)
3	(1, 3)	(2, 3)	(3, 3)

b 6 outcomes

	A	B	C
A	X	(B, A)	(C, A)
B	(A, B)	X	(C, B)
C	(A, C)	(B, C)	X

2 a table A b table B
c i $\frac{1}{9}$ ii $\frac{1}{6}$
d i 5 ii 4

Now you try

Example 6

a

		Counter 1	
		R	**B**
Counter 2	**R**	(R, R)	(B, R)
	B	(R, B)	(B, B)

b $\frac{1}{4}$ c $\frac{1}{2}$

Example 7

a

		1st			
		D	**A**	**T**	**A**
2nd	**D**	×	(A, D)	(T, D)	(A, D)
	A	(D, A)	×	(T, A)	(A, A)
	T	(D, T)	(A, T)	×	(A, T)
	A	(D, A)	(A, A)	(T, A)	×

b $\frac{1}{6}$ c $\frac{5}{6}$

Exercise 9D

1 a

	1	2	3	4	5	6
1	(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)
2	(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
3	(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)
4	(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)	(6, 4)
5	(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)	(6, 5)
6	(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	(6, 6)

b 36

c i $\frac{1}{36}$ ii $\frac{1}{6}$ iii $\frac{1}{12}$ iv $\frac{5}{9}$

2 a

		A	
		•	**∘**
B	**•**	(•, •)	(∘, •)
	∘	(•, ∘)	(∘, ∘)
	∘	(•, ∘)	(∘, ∘)

b 6

c i $\frac{1}{3}$ ii $\frac{1}{2}$ iii $\frac{1}{2}$

3 a

	D	O	G
D	X	(O, D)	(G, D)
O	(D, O)	X	(G, O)
G	(D, G)	(O, G)	X

b $\frac{1}{6}$ c $\frac{2}{3}$

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4 a

		1st			
		1	2	3	4
2nd	1	X	(2, 1)	(3, 1)	(4, 1)
	2	(1, 2)	X	(3, 2)	(4, 2)
	3	(1, 3)	(2, 3)	X	(4, 3)
	4	(1, 4)	(2, 4)	(3, 4)	X

b i $\frac{1}{12}$ ii $\frac{1}{12}$

c i $\frac{1}{6}$ ii $\frac{5}{6}$ iii $\frac{1}{6}$ iv $\frac{1}{2}$

5 a

	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

b i $\frac{1}{16}$ ii $\frac{3}{16}$ iii $\frac{3}{8}$

iv $\frac{3}{16}$ v $\frac{13}{16}$

6 a

	A	B	C	D	E
A	(A, A)	(B, A)	(C, A)	(D, A)	(E, A)
B	(A, B)	(B, B)	(C, B)	(D, B)	(E, B)
C	(A, C)	(B, C)	(C, C)	(D, C)	(E, C)
D	(A, D)	(B, D)	(C, D)	(D, D)	(E, D)
E	(A, E)	(B, E)	(C, E)	(D, E)	(E, E)

b i $\frac{1}{25}$ ii $\frac{1}{5}$ iii $\frac{4}{5}$

c i $\frac{8}{25}$ ii $\frac{1}{25}$

7 a i $\frac{1}{36}$ ii $\frac{1}{12}$ iii $\frac{1}{12}$ iv $\frac{7}{12}$

v $\frac{5}{12}$ vi $\frac{1}{6}$ vii $\frac{1}{6}$ viii 0

b 7, $\frac{1}{6}$

8 a i 169 ii 156

b i 25 ii 12

9 Yes; if one O is removed another remains to be used.

10 a $\frac{1}{25}$ b $\frac{1}{20}$

11 7, 8, 9, 10, 11

12 a $\frac{1}{2500}$ b $\frac{1}{50}$ c $\frac{1}{1250}$ d $\frac{49}{2500}$

e $\frac{23}{1250}$

13 a without replacement b 2652

c $\frac{2}{52} \times \frac{1}{51} + \frac{2}{52} \times \frac{1}{51}$

d i $\frac{1}{221}$ ii $\frac{1}{17}$

9E

Building understanding

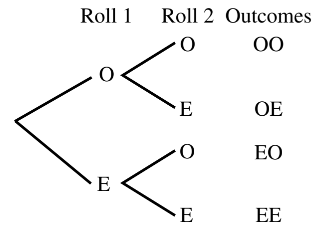
1 TT, TO, OT, OO

2 AB, AC, BA, BC, CA, CB

Now you try

Example 8

a

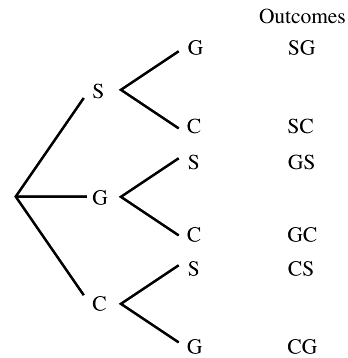


b 4

c i $\frac{1}{4}$ ii $\frac{1}{2}$ iii $\frac{3}{4}$

Example 9

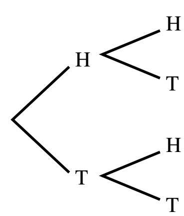
a



b i $\frac{1}{3}$ ii $\frac{2}{3}$ iii 1

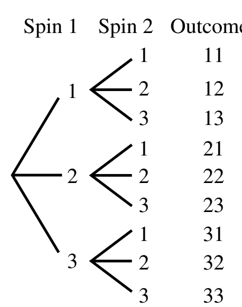
Exercise 9E

1 a HH, HT, TH, TT b 4



c i $\frac{1}{4}$ ii $\frac{1}{2}$ iii $\frac{3}{4}$ iv $\frac{3}{4}$

2 a



b 9

c i $\frac{1}{9}$ ii $\frac{5}{9}$ iii $\frac{8}{9}$ iv $\frac{4}{9}$

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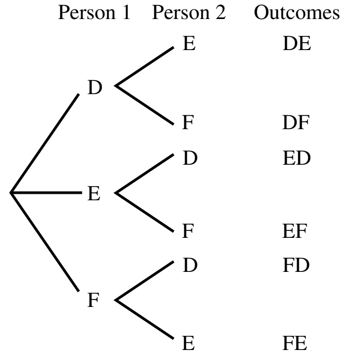
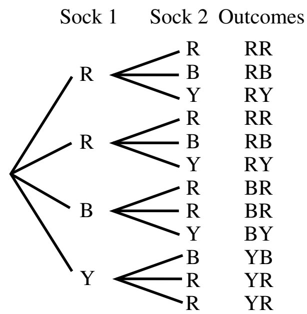
3 a



b i $\frac{1}{3}$ ii $\frac{2}{3}$ iii 1

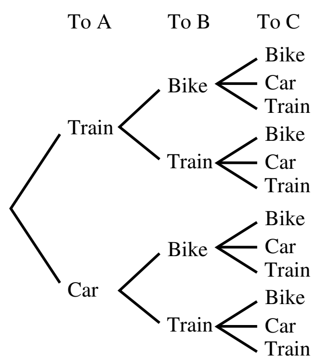
4 a



b i $\frac{1}{3}$ ii $\frac{1}{6}$ iii $\frac{1}{6}$ iv $\frac{5}{6}$

5 a $\frac{4}{9}$ b $\frac{2}{9}$ c $\frac{1}{27}$ d $\frac{8}{27}$

6 a



b 12

c i $\frac{1}{12}$ ii $\frac{1}{3}$ iii $\frac{1}{2}$ iv $\frac{1}{4}$

v $\frac{2}{3}$

7 a $\frac{1}{16}$ b $\frac{1}{16}$ c $\frac{5}{16}$ d $\frac{11}{16}$

e $\frac{15}{16}$

8 a $\frac{1}{16}$ b $\frac{1}{4}$ c $\frac{3}{8}$ d $\frac{1}{4}$

e $\frac{1}{16}$

9 a 32 b 2^n

10 Yes, there is a difference; the probability of obtaining two of the same colour is lower without replacement.

11 a $\frac{1}{9}$ b $\frac{1}{9}$ c $\frac{7}{18}$ d $\frac{7}{18}$ e $\frac{1}{2}$

f 0 g $\frac{1}{6}$ h $\frac{2}{3}$ i $\frac{1}{9}$ j $\frac{17}{18}$

9F

Building understanding

1 a B: $\frac{5}{20} = 0.25$, C: $\frac{30}{100} = 0.3$

b C, as it is a larger sample size

2 a 5 b 20 c 4

Now you try

Example 10

a 0.6 b 120

Exercise 9F

1 a 0.6

b i 60 ii 120 iii 360

2 a $\frac{7}{8}$

b i 350 ii 4375 iii 35

3 a $\frac{1}{15}$ b $\frac{2}{5}$ c $\frac{11}{60}$

4 a 20

b i $\frac{1}{4}$ ii $\frac{7}{20}$

c i 25 ii 20 iii 45

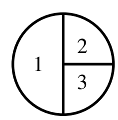
5 a 20 b 40 c 60 d 40

6 a 0.64 b SEE

7 a i 0.52 ii 0.48 iii 0.78

b 78

c $\frac{1}{2}, \frac{1}{4}, \frac{1}{4}$



8 0.41, from the 100 throws as the more times an experiment is carried out the closer the experimental probability becomes to the actual/theoretical probability

9 a $\frac{9}{10}$

b No, as the number of throws increases, the experiment should produce results closer to the theoretical probability $\left(\frac{1}{6}\right)$.

10 a fair, close to 0.5 chance of tails

b biased, nearly all results are heads

c can't determine on such a small sample

11 a no b true c no d true

12 a $\frac{\text{shaded area}}{\text{total area}} = 0.225 \therefore 100 \text{ shots} \approx 23$

b $\frac{1}{10} \times 100 = 10$

c $\frac{150 - 32}{150} \times 100 \approx 79$

d $\frac{225\pi - 25\pi}{225\pi} \times 100 \approx 89$

13 3 blue, 2 red, 4 green, 1 yellow

14 2 strawberry, 3 caramel, 2 coconut, 4 nut, 1 mint

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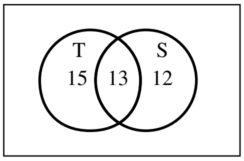
Progress quiz

1 a {1, 2, 3, 4, 5, 6} b {WL, LW, WW, LL}

2 $\frac{4}{11}$

3 a



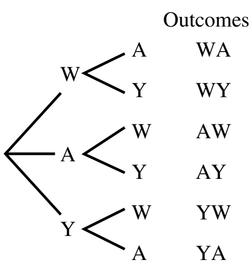
b 13 c $\frac{27}{40}$

4 $\frac{3}{10}$ or 0.3

5 a i {10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}
ii {10, 12, 14, 16, 18, 20}
iii {12, 15, 18}
iv {12, 18}
b i 2 ii $\frac{2}{11}$ iii $\frac{5}{11}$

6 a $\frac{1}{2}$ b $\frac{1}{6}$ c $\frac{1}{18}$

7 a



b i $\frac{2}{3}$ ii 1 iii $\frac{1}{3}$ iv $\frac{1}{3}$

8 a 0.4 b 0.56

9 140

9G

Building understanding

1 a survey b bias
c population d sample
2 a categorical b numerical
3 a stratified
b simple random
c systematic

Now you try

Example 11
a numerical discrete b categorical nominal
c numerical continuous d categorical ordinal

Example 12
a i stratified
ii systematic
iii simple random
b Yes. Wasram's team-mates at the soccer club are unlikely to represent the views of the wider community.

Exercise 9G

1 a numerical b categorical
c categorical d numerical
2 a categorical nominal b categorical ordinal
c numerical continuous d numerical discrete
3 a categorical nominal b numerical continuous
c categorical ordinal d numerical discrete
4 a i simple random
ii stratified
iii systematic
b Yes. Only the people who are confident in answering the questions will raise their hand.
5 a i systematic
ii stratified
iii simple random
b Yes. There is a good chance that more water birds are in that area.
6 Sales (3), Administration (7)
7 Length: Numerical continuous
Weight: Numerical continuous
Favourite food: Categorical nominal
Number of toys: Numerical discrete
Type of scratching post: Categorical ordinal
8 a expand his sample to include other member of the class
b survey other vets in the practice too
c consider the price of bread in other types of shops
d survey other students not in the extension program
9 10 Junior
10 numerical discrete
11 The data might be only relevant if sorted into age groups
12 There is a good chance that the people who volunteer for surveys do not represent the wider community.
13 Answers may vary

9H

Building understanding

1 a common/frequent
b middle
c mean

2

	Mean	Median	Mode
a	3	2	2
b	15	16	20

Now you try

Example 13
a i 5 ii 4 iii 2
b i 17 ii 16.5 iii no mode

Example 14
a \$23 b \$37

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Exercise 9H

1

	Mean	Median	Mode
a	8	6	5, 10
b	11	12	none
c	4	3.5	2.1
d	5	4.5	none
e	3	3.5	−3
f	0	2	3

2 a yes b no c no d yes

3 a outlier = 33, mean = 12, median = 7.5
 b outlier = −1.1, mean = 1.075, median = 1.4
 c outlier = −4, mean = −49, median = −59
 d outlier = 10, mean = −1.34, median = −2.4

4 a 24.67 s b 24.8 s

5 a 15 b 26

6 a 90 b 60

7 9

8 Answers may vary; examples below.
 a 1, 4, 6, 7, 7 is a set b 2, 3, 4, 8, 8 is a set
 c 4, 4, 4, 4, 4 is a set d 2.5, 2.5, 3, 7, 7.5 is a set
 e −3, −2, 0, 5, 5 is a set f $0, \frac{1}{2}, 1\frac{1}{4}, 1\frac{1}{4}, 2$ is a set

9 a $354 500 b $1 700 000
 c ($354 500 and $324 000) drops $30 500
 d $570 667
 e ($570 667 and $344 800) drops $225 867

10 An outlier has a large impact on the addition of all the scores and therefore significantly affects the mean. An outlier does not move the middle of the group of scores significantly.

11 a 15 b 1.2 c 1 d 1
 e No, as it will still lie in the 1 column.

12 a 2 b 3 c 7

13 a 76.75
 b i 71.4, B^+ ii 75, B^+ iii 80.2, A
 c 81.4; he cannot get an A^+
 d i 43 ii 93
 e $M = \frac{307 + m}{5}$
 f $m = 5M - 307$

9I

Building understanding

1 a 9
 b i 8 min ii 35 min
 c 21 min

2 a 26
 b skewed
 c symmetrical

Now you try

Example 15

a

Stem	Leaf
0	2 3
1	4 4
2	0 3 5
3	1 4 5 8 9
4	3 4 7 8
5	1 6 7
6	4

2 | 3 means 2.3

b 3.65 c 1.4

d Data is approximately symmetrical.

Example 16

a

Taipans		JackJumpers
8 7 4 2	6	
8 5 3 2 1	7	5 6 6
8	8	1 2 7 8 9
	9	4 7

7 | 2 means 72

b The JackJumpers have the highest scores, with scores between 75 and 97. The scores for the Taipans are generally between 62 and 78, with one high score of 88.

Exercise 9I

1 a i

Stem	Leaf
1	8
2	3 3 4
3	2 6 7 8
4	1 5 8
5	5 9
6	2

2 | 3 means 2.3

 ii 3.75 iii 2.3
 iv almost symmetrical

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b **i**

Stem	Leaf
1	9 9
2	3 4 6 6 8 8 8
3	2 2 3 5
4	1
5	4

2 | 3 means 23

ii 28 **iii** 28

iv positively skewed

c **i**

Stem	Leaf
1	5 5
2	1 3 3 3 6
3	0 1 1 3 4 5 5 7 9
4	2 2 5 5 5 8
5	0 0 1

3 | 1 means 31

ii 35 **iii** 23, 45

iv almost symmetrical

d **i**

Stem	Leaf
32	0
33	3 7 8
34	3 4 5 5 7 8 9
35	2 2 4 5 8
36	1 3 5
37	0

33 | 7 means 33.7

ii 34.85 **iii** 35.2, 34.5

iv almost symmetrical

e **i**

Stem	Leaf
15	7 8 9 9 9 9
16	1 4 7 7 7
17	3 5 7 7
18	5 5 7 9
19	3 8
20	0 2

16 | 4 means 164

ii 173 **iii** 159

iv positively skewed

2 **a**

Stem	Leaf
0	5 6 6 8 8 9
1	0 1 1 2 2 2 4 4 5 6 7
2	0 1 2

1 | 2 means 12

b 9 **c** 12

3 **a** **i**

Set A		Set B
3 2	3	1
9	3	
3 3 0 0	4	0 1 3 4 4
8 7 6 5	4	6 7 8 8 9 9
4 4 3	5	1 3

4 | 6 means 46

ii Set A has values spread between 32 and 54 while set B has most of its values between 40 and 53 with an outlier at 31.

b **i**

Set A		Set B
8 7 4 3	0	1 1 1 2 3 6 6 9 9
9 9 7 6 4	1	2 3 5 8 9
6 5 3 1	2	1 5 6
9 9 6 4 3 2	3	3 4 9
7 3 1	4	3 7 8
7 3 2	5	2 3 7
2 1	6	1 2
8 3	8	3 8
1	9	

1 | 4 means 1.4

ii Sets A and B are similar. The frequency decreases as the numbers increase.

4 **a**

Collingwood		St Kilda
8 3	6	6 8 8
8 7 2 1 0	7	8
9 9 8 2 0	8	0 0 2 2 3 4 7 8
8 7 5 1	9	0 4
4 3 3 3	10	6 9
9 8 5	11	1 3 7 8
7	12	2 5 6
	13	8

8 | 2 means 82

b Collingwood $33\frac{1}{3}\%$, St Kilda $41\frac{2}{3}\%$

c Data for Collingwood is almost symmetrical and based on these results seems to be consistent. St Kilda has groups of similar scores and although less consistent, it has higher scores.

5 **a** 9 days **b** 18°C **c** 7°C

6 **a** 16.1 s **b** 2.3 s

c yes 0.05 s lower

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7 a **Battery lifetime**

Brand A		Brand B
3	7	2 3 4
	7	5 6 8 9 9
4 2	8	0 1 3
9 8 7 5	8	
4 4 3 2 1 0	9	0 1 2 3 4
9 9 8 7 6 5 5	9	5 6 8 8

8 | 3 represents 8.3 hours

b Brand A, 12; Brand B, 8

c Brand A consistently performs better than Brand B.

8 a $c = 2$ **b** 0.02

c i 5, 6, 7 or 8 **ii** 0, 1, 2, 3, 4, 5 or 6

9 a 48%

b 15%

c In general, birth weights of babies are lower for mothers who smoke.

10 In symmetrical data the mean is close to the median, as the data is spread evenly from the centre with an even number of data values with a similar difference from the mean above and below it.

11 a 50.5 **b** 17.8

12 a mean = 41.06, median = 43

b The median is higher, as more of the scores exist in the higher stems but a few low scores lower the mean.

c The mean is higher for positively skewed data, as the majority of the scores are in the lower stems; however, a few high scores increase the mean.

d symmetrical data

9J

Building understanding

1 $a = 15$, $b = 2$, $c = 30$, $d = 40$, $e = 20$, $f = 100$

2 a 2 **b** 20

c i 30% **ii** 65%

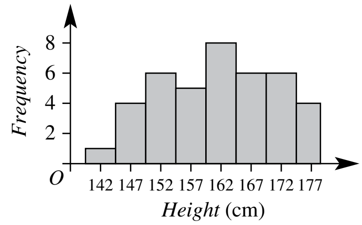
Now you try

Example 17

a

Class interval	Frequency	Percentage frequency
140 – 144	1	2.5
145 – 149	4	10
150 – 154	6	15
155 – 159	5	12.5
160 – 164	8	20
165 – 169	6	15
170 – 174	6	15
175 – 179	4	10
Total	40	100

b



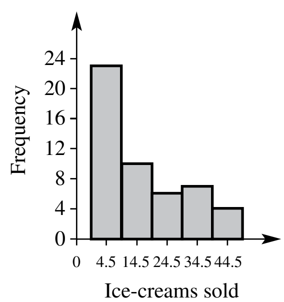
c i 5 **ii** 10

Exercise 9J

1 a

Class	Frequency	Percentage frequency
0–9	23	46%
10–19	10	20%
20–29	6	12%
30–39	7	14%
40–49	4	8%
	50	100%

b

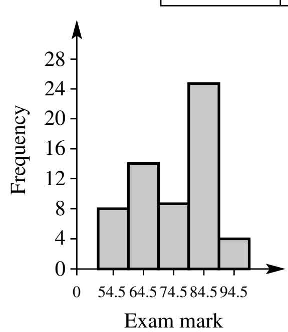


c i 33 **ii** 11

2 a

Class	Frequency	Percentage frequency
50–59	8	13.33%
60–69	14	23.33%
70–79	9	15%
80–89	25	41.67%
90–99	4	6.67%
	60	100%

b



c i 22 **ii** 48.3%

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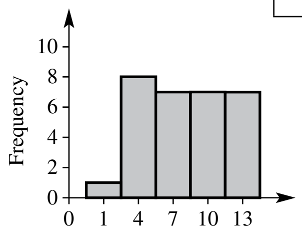
3 a

Number of goals	Frequency
0–2	1
3–5	8
6–8	7
9–11	7
12–14	7
	30

b



c 9 d 7

4 a symmetrical data b skewed data

5 a $a = 6, b = 27.5, c = 17.5, d = 4, e = 12, f = 30, g = 100$

b $a = 1, b = 18, c = 8, d = 24, e = 20, f = 40, g = 100$

6 a i 20% ii 55% iii 80% iv 75%

v 50%

b i 30 ii 45

c i 2 ii 22

7 85.5%

8 because you only have the number of scores in the class interval not the individual scores

9 the number of data items within each class

10 a student A

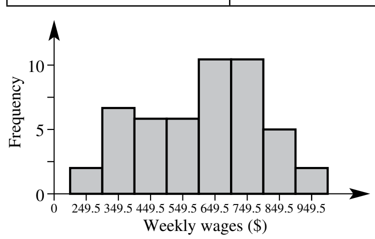
b Make the intervals for their groups of data smaller so that the graph conveys more information.

11 a minimum wage: \$204; maximum wage: \$940

b i

Weekly wages (\$)	Frequency
200 – 299	2
300 – 399	7
400 – 499	6
500 – 599	6
600 – 699	11
700 – 799	11
800 – 899	5
900 – 999	2

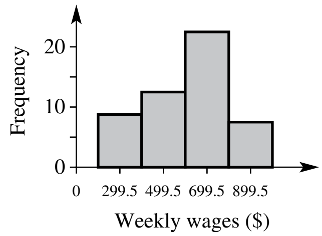
ii



c i

Weekly wages	Frequency
200 – 399	9
400 – 599	12
600 – 799	22
800 – 999	7

ii



d The graph with more intervals shows greater detail. As the first graph has each pair of intervals quite similar, these two graphs are quite similar.

9K

Building understanding

1 a minimum, maximum

b lower quartile, Q_1, upper quartile, Q_3

2 a 6 b 7

c i 5 ii 8

d 3

3 a 12 b 5

c i 3 ii 8

d 5

Now you try

Example 18

a 72 76 78 81 83 84 86 90 92

b 20

c i 83 ii 77 iii 88 iv 11

Example 19

a 0.4 0.8 1.1 1.2 1.6 1.7 1.8 2.0 2.3 2.3 2.6 2.8 3.1 3.5

b 3.1

c i 1.9 ii 1.2 iii 2.6 iv 1.4

d The middle 50% of packages differed by less than 1.4 kg.

Exercise 9K

1

	Range	Q_2	Q_1	Q_3	IQR
a	11	6	2.5	8.5	6
b	23	30	23.5	36.5	13
c	170	219	181.5	284.5	103
d	851	76	28	367	339
e	1.3	1.3	1.05	1.85	0.8
f	34.98	10	0.1	23	22.9

2 a 1446 million

b i 1595.5 million ii 1405 million

iii 2068 million iv 663 million

c The middle 50% of films varied in takings by 663 million.

3 a 110 min

b i 119.5 min ii 106 min

iii 130 min iv 24 min

c The middle 50% of videos rented varied in length by 24 min.

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4 a i \$12 000 **ii** \$547 000 **iii** \$71 500
iv \$46 000 **v** \$78 000 **vi** \$32 000
b The middle 50% of prices differs by no more than \$32 000.
c No effect on Q_1, Q_2 or Q_3 but the mean would increase.
5 a 17.5 **b** 2.1
6 a 2 **b** 2
c No, only the maximum value has changed, so no impact on IQR.
7 No, as the range is the difference between the highest and lowest scores and different sets of two numbers can have the same difference ($10 - 8 = 2$, $22 - 20 = 2$).
8 a Yes, as lowering the highest price reduces the range.
b No, the middle price will not change.
c No, as only one value; the highest has changed, yet it still remains the highest, Q_1 and Q_3 remain unchanged.
9 a Yes (3, 3, 3, 4, 4, 4: $IQR = 4 - 3 = 1$: range = 1)
b Yes (4, 4, 4, 4, 4 has $IQR = 0$)
10 a i $Q_1 = 25$; $Q_2 = 26$; $Q_3 = 27$
ii $Q_1 = 22$; $Q_2 = 24.5$; $Q_3 = 27$
b 27 jelly beans **c** 22 jelly beans
d i $IQR = 2$ **ii** $IQR = 5$
e Shop B is less consistent than shop A and its data is more spread out.
f shop A

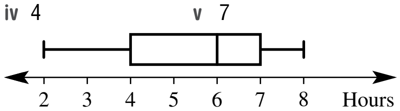
9L

Building understanding

1 a minimum value **b** lower quartile, Q_1
c median, Q_2 **d** upper quartile, Q_3
e maximum value **f** scale
g whisker **h** box
2 D

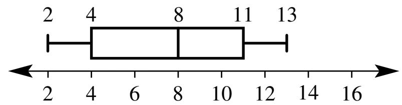
Now you try

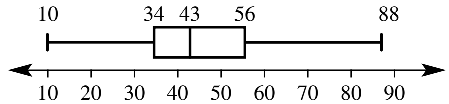
Example 20
a min = 0, max = 30 **b** 30
c 15 **d** 15
e 15 **f** 25
g 20

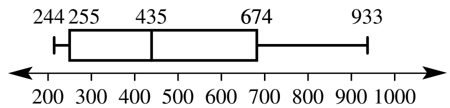
Example 21
a i 2 **ii** 8 **iii** 6
iv 4 **v** 7
b


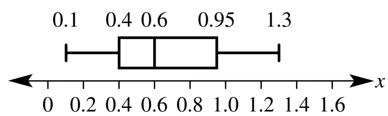
Exercise 9L

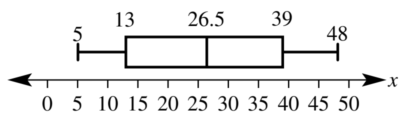
1 a min = 35 cm, max = 60 cm
b 25 cm **c** 50 cm **d** 10 cm
e 50 cm **f** 55 cm **g** 20 babies
2 a min = 0 rabbits, max = 60 rabbits
b 60 **c** 25 **d** 25
e 35 **f** 25 **g** 25

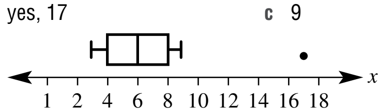
3 a i 2 **ii** 13 **iii** 8
iv 4 **v** 11
vi


b i 10 **ii** 88 **iii** 43
iv 34 **v** 56
vi


c i 244 **ii** 933 **iii** 435 **iv** 255
v 674
vi


d i 0.1 **ii** 1.3 **iii** 0.6 **iv** 0.4
v 0.95
vi


4 a


b i 100% **ii** 50%
iii 75% **iv** 25%
5 a Africa **b** 10 kg **c** yes, 25 kg
d i 50% **ii** 75%
e African
6 a Mac **b** 75% **c** 50%
d They are the same. Mac has 100% of times within same range as the middle 50% of PC start up times. Mac is more consistent.
7 a Waldren **b** Yeng **c** Yeng
d Yeng, smaller range and IQR
e Yeng, higher median
8 a 1, 3, 5, 6, 6 **b** 1, 4, 4, 4, 5, 6
9 No, the median is anywhere within the box, including at times at its edges.
10 Yes, one reason is if an outlier is the highest score, then the mean > Q_3; e.g. [1, 1, 3, 5, 6, 7, 256].
11 a $Q_1 = 4$, $Q_2 = 6$, $Q_3 = 8$
b yes, 17 **c** 9
d


e This calculator may have been used less often than the others, so the battery lasted longer.

Problems and challenges

1 a $2^5 = 32$ **b** $\frac{3}{16}$ **c** $\frac{31}{32}$
2 a $\frac{1}{6}$ **b** $\frac{1}{3}$

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3 a i $\frac{1}{4}$ ii $\frac{1}{2}$
b $\frac{1}{6}$ c $\frac{1}{12}$ d $\frac{1}{3}$

4 a mean and median increase by 5, range unchanged
b mean, median and range all double
c mean and median double and decrease 1, range doubles

5 $5m$ grams

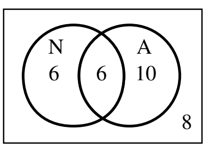
6 a 3, 5, 7, 8, 10 or 3, 5, 7, 9, 10
b 2, 2, 8, 12

7 1, 4, 6, 7, 7; 2, 3, 6, 7, 7 and 1, 2, 5, 6, 6

8 $\frac{3}{8}$

Success criteria example questions

1 Sample space: {1, 2, 4, 2, 4, 3}; Pr(not a 2) $= \frac{2}{3}$;
Pr(a 1 or a 2) $= \frac{1}{2}$

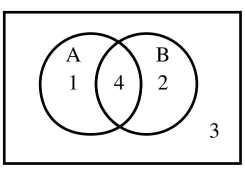
2 

; 18; $\frac{1}{3}$

3

	W	W'	
B	4	6	10
B'	8	2	10
	12	8	20

; 2; $\frac{2}{5}$

4 

;
i {1, 2, 3, 4, 6, 8, 10}; {1, 3}
ii 5; $\frac{2}{5}$

5

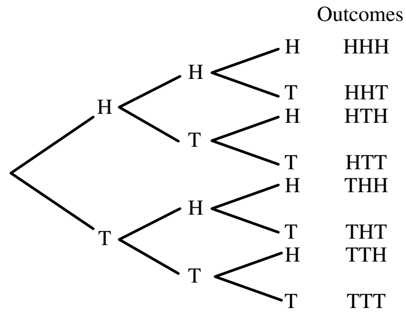
	1	2	3	4
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)

P(2 even numbers) $= \frac{1}{4}$

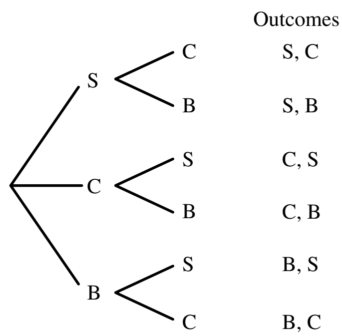
6

	G	R	E	E	N
G	X	(G, R)	(G, E)	(G, E)	(G, N)
R	(R, G)	X	(R, E)	(R, E)	(R, N)
E	(E, G)	(E, R)	X	(E, E)	(E, N)
E	(E, G)	(E, R)	(E, E)	X	(E, N)
N	(N, G)	(N, R)	(N, E)	(N, E)	X

P(at least one of the letters is an E) $= \frac{7}{10}$

7 

; Pr(tossing exactly 2 heads) $= \frac{3}{8}$

8 

; Pr(the selection will contain the chocolate iced donut) $= \frac{2}{3}$

9 $\frac{2}{5}$; 120

10 i numerical discrete;
ii Categorical ordinal

11 i systematic;
ii stratified

12 Yes, it is more likely that people at the airport travel more often.

13 Mean = 5.5, median = 5, mode = 4 & 8

14 6; 10

15

0	4
1	7
2	1 4 7
3	2 2
4	1 4

Key
0 | 4 = 0.4

; median = 2.7, mode = 3.2; skewed

16

City A		City B
	0	9 9
9 9 8 8	1	0 2 2 3 5 7 7 8
8 7 6 6 4 4 2 1	2	0 1 1 2
2 0	3	

Key
0 | 9 = 9
City A has the highest recorded maximum daily temperature of 32°C. Its maximum daily temperatures are generally between 18°C and 28°C. City B's maximum daily temperatures are lower than City A, being generally between 10°C and 22°C

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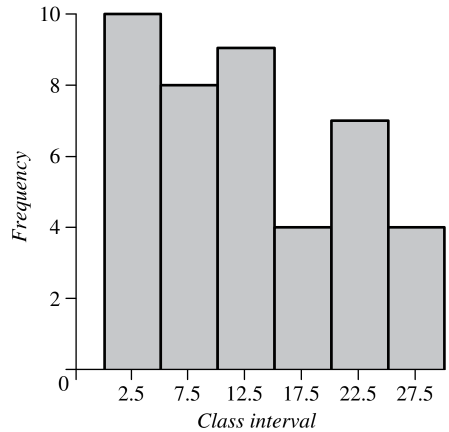
17 

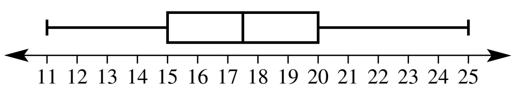
There are 15 hours of sales of 15 or more ice creams.

18 31

19 $Q_1 = 7.5$, $Q_2 = 17$, $Q_3 = 21.5$, $IQR = 14$

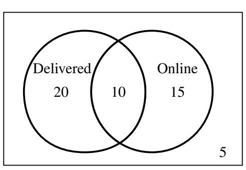
20 $Q_1 = 10$, $Q_2 = 14$, $Q_3 = 21$, $IQR = 11$; the middle 50% of the data differed by less than 11

21 **i** range = 60, IQR = 30;
ii 90;
iii 12

22 

Short-answer questions

1 **a** $\frac{2}{3}$ **b** $\frac{5}{9}$ **c** $\frac{3}{5}$

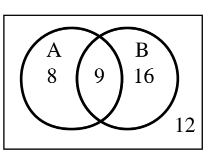
2 **a** 

b 15

c **i** $\frac{1}{5}$ **ii** $\frac{2}{5}$ **iii** $\frac{3}{10}$

3 **a**

	A	A′	Total
B	9	16	25
B′	8	12	20
Total	17	28	45

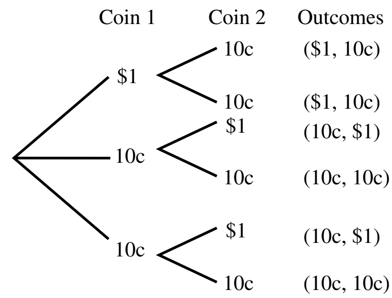
b 

c **i** $\frac{4}{9}$ **ii** $\frac{1}{5}$ **iii** 8 **iv** 33

4 **a** 12 outcomes

	1	2	3	4
Red	(red, 1)	(red, 2)	(red, 3)	(red, 4)
Green	(green, 1)	(green, 2)	(green, 3)	(green, 4)
Blue	(blue, 1)	(blue, 2)	(blue, 3)	(blue, 4)

b **i** $\frac{1}{6}$ **ii** $\frac{1}{6}$ **iii** $\frac{2}{3}$

5 **a** 

b $\frac{2}{3}$

6 **a** $\frac{48}{120} = \frac{2}{5}$ **b** 14

7 **a** Categorical nominal
b **i** simple random **ii** systematic
c Yes, her extension science class may have different interests to others in the year level.

8 **a** 26 **b** 26.5 **c** 23, 31

9 15

10 **a**

Employee 1		Employee 2
9	1	7 7
5 3 1	2	0 4 8 9
9 8 6 5 5 4	3	2 7 7 8
6 5 0	4	0 1 2 5 8
3 1	5	

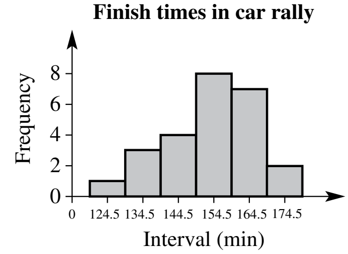
2 | 4 means 24

b **i** Employee 1: 36, Employee 2: 37
ii Employee 1: 36, Employee 2: 33
c Employee 1, they have a higher mean and more sales at the high end.
d Employee 1 symmetrical, Employee 2 skewed

11 **a**

Class interval	Class centre	Frequency	Percentage frequency
120 – 129	124.5	1	4
130 – 139	134.5	3	12
140 – 149	144.5	4	16
150 – 159	154.5	8	32
160 – 169	164.5	7	28
170–179	174.5	2	8
Total		25	100%

b **Finish times in car rally**



c **i** 4 **ii** 60%

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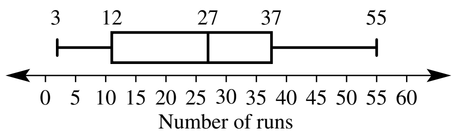
12 a 52 runs

b 

c 37 runs

Multiple-choice questions

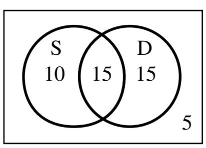
1 B	2 A	3 D	4 B	5 D
6 B	7 E	8 C	9 B	10 E

Extended-response questions

1 a i

		1st spin				
		1	2	3	4	5
2nd spin	1	(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)
	2	(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)
	3	(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)
	4	(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)
	5	(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)

ii $\frac{12}{25}$ iii $\frac{4}{25}$ iv 24 v \$68

b i 

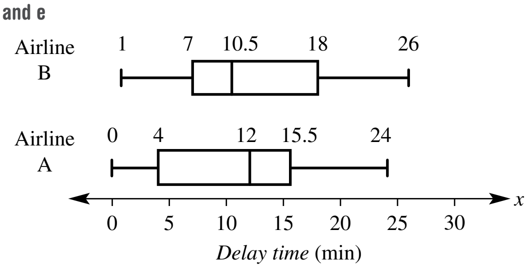
ii 5 iii 10 iv $\frac{1}{3}$ v $\frac{4}{9}$

vi $\frac{7}{9}$, the probability a person did not buy only a sausage

2 a Yes, 52 min

b i 12 ii 4 iii 15.5

c and e



d No, the median time is 12 min, so half the flights have less than a 12 minute delay.

f Airline A: range = 24, IQR = 11.5
Airline B: range = 25, IQR = 11
Both airlines have a very similar spread of their data when the outlier is removed.

g There is not much difference when airline A's outlier is removed. It has a marginally better performance with 75% of flights delayed less than 15.5 min compared with 18 min for airline B.

Chapter 10

10A

Building understanding

1 a -4 b -1 c 72
2 a no b yes c yes
d no e no f yes
3 a 1 b -3 c -2
4 a $x - 5 = 0$, $x = 5$
b $2x + 1 = 0$, $2x = -1$ or $x = 3$, $x = -\frac{1}{2}$ or $x = 3$

Now you try

Example 1
a $x^2 - 3x - 5 = 0$ b $2x^2 + 8x - 7 = 0$
c $2x^2 + 3x - 4 = 0$

Example 2
a not a solution b is a solution

Example 3
a $x = 0, -5$ b $x = -2, 3$ c $x = \frac{2}{3}, -\frac{5}{2}$

Exercise 10A

1 a $x^2 - 7x - 2 = 0$ b $x^2 - 5x + 2 = 0$
c $x^2 - 4x + 1 = 0$ d $3x^2 - 6x - 4 = 0$
e $2x^2 + 2x + 1 = 0$ f $3x^2 - 3x + 4 = 0$
g $3x^2 + 4 = 0$ h $x^2 - 3x - 1 = 0$
i $2x^2 + 5x - 10 = 0$
2 a yes b yes c no d no e yes
f no g yes h no i no
3 both are solutions
4 both are solutions
5 a 0, -1 b 0, -5 c 0, 2 d 0, 7
e -1, 3 f 4, -2 g -7, 3 h $-\frac{1}{2}, \frac{1}{2}$
i 0, -5 j 0, $\frac{2}{3}$ k 0, $-\frac{2}{3}$ l 0, -2
6 a $\frac{1}{2}, -2$ b $-2, \frac{1}{3}$ c $-\frac{2}{5}, -4$
d $1, \frac{1}{3}$ e $-5, -\frac{2}{7}$ f $\frac{2}{3}, -\frac{1}{5}$
g $\frac{7}{11}, \frac{13}{2}$ h $-\frac{9}{4}, \frac{7}{2}$ i $\frac{4}{3}, -\frac{1}{7}$
7 a 0, -3 b 0, 7 c 1, -4
d $\frac{1}{2}, -6$ e $-\frac{3}{2}, \frac{1}{2}$
8 a $4x^2 + x + 1 = 0$ b $3x^2 - 3x = 0$
c $3x^2 - x - 4 = 0$ d $5x^2 + x - 2 = 0$
e $2x^2 - x - 3 = 0$ f $3x^2 - 10x + 6 = 0$
g $x^2 - x - 5 = 0$ h $4x^2 - 5x - 6 = 0$
9 a -1, 2 b 1, 3 c 0, 4
d 0, -3 e -4, 1 f -4, 4
10 a $(x + 2)(x + 2) = 0$
b both solutions are the same, $x = -2$
c i -3 ii 5 iii $\frac{1}{2}$ iv $\frac{7}{5}$
11 a $(x - 1)(x + 2) = 0$ b $x = 1$, $x = -2$
c Multiplying by a constant doesn't change a zero value.
d i -2, 3 ii 0, -2 iii -1, 3 iv -2, 5

Answers
Ch9 review

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12 a i no ii no iii no iv no
b It has no solutions as $(x-3)^2$ is always $\geqslant 0$, so $(x-3)^2+1 \geqslant 1$

13 a linear b quadratic c quintic
d cubic e quartic f quintic

14 a $-2, -1, 3$ b $-11, 2, 5$
c $-\frac{2}{3}, \frac{1}{5}, \frac{1}{2}$ d $-\frac{10}{7}, -\frac{4}{5}, -\frac{2}{3}, \frac{13}{2}$

10B

Building understanding

1 a 8 b x c $3x$

2 a $x(x-3)$ b $2x(3x+2)$
c $4x(1-4x)$ d $2(x-2)(x+2)$

3 a $x=0, x=3$ b $x=0, x=-1$

4 $3^2 = 3 \times 3 = 9$ and $(-3)^2 = -3 \times (-3) = 9$

Now you try

Example 4
a $x=0, -6$ b $x=0, 5$

Example 5
a $x=-7, 7$
b $x=-\sqrt{7}, \sqrt{7}$

Exercise 10B

1 a $x=0, x=-3$ b $x=0, x=-7$ c $x=0, x=-4$
d $x=0, x=5$ e $x=0, x=8$ f $x=0, x=2$
g $x=0, x=-\frac{1}{3}$ h $x=0, x=\frac{1}{2}$

2 a $x=0, x=3$ b $x=0, x=4$ c $x=0, x=-5$
d $x=0, x=3$ e $x=0, x=3$ f $x=0, x=-4$

3 a $x=0, x=3$ b $x=0, x=2$ c $x=0, x=4$
d $x=0, x=-3$ e $x=0, x=-4$ f $x=0, x=-3$

4 a $x=3, x=-3$ b $x=6, x=-6$
c $x=5, x=-5$ d $x=12, x=-12$
e $x=9, x=-9$ f $x=20, x=-20$

5 a $x=2, x=-2$ b $x=3, x=-3$
c $x=5, x=-5$ d $x=2, x=-2$
e $x=\sqrt{6}, x=-\sqrt{6}$ f $x=\sqrt{5}, x=-\sqrt{5}$
g $x=\sqrt{7}, x=-\sqrt{7}$ h $x=\sqrt{3}, x=-\sqrt{3}$

6 a $x=2, x=-2$ b $x=5, x=-5$
c $x=10, x=-10$ d $x=0, x=-7$
e $x=0, x=7$ f $x=-1, x=1$
g $x=0, x=\frac{2}{3}$ h $x=0, x=\frac{3}{5}$
i $x=-\sqrt{3}, x=\sqrt{3}$

7 a 0 or 7 b 8 or -8 c 0 or -4

8 a $x=-2, x=2$ b $x=-6, x=6$
c $x=-1, x=1$ d $x=-1, x=1$
e $x=0, x=-7$ f $x=0, x=4$

9 a $(x+2)(x-2) = x^2-4$ not x^2+4
b no, $x^2 \geqslant 0$, so $x^2+4 \geqslant 4$

10 a $ax^2+bx = x(ax+b) = 0$, $x=0$ is always one solution
b $x=-\frac{b}{a}$

11 a $x=-\frac{4}{3}, x=\frac{4}{3}$ b $x=-\frac{6}{5}, x=\frac{6}{5}$
c $x=-\frac{1}{5}, x=\frac{1}{5}$ d $x=-\frac{9}{5}, x=\frac{9}{5}$
e $x=-\frac{8}{11}, x=\frac{8}{11}$ f $x=-\frac{12}{7}, x=\frac{12}{7}$

12 a $x=-1, x=5$ b $x=-1, x=-9$
c $x=-1, x=0$ d $x=-\frac{2}{5}, x=\frac{8}{5}$
e $x=1, x=7$ f $x=-1, x=\frac{13}{7}$

10C

Building understanding

1 a 3, 2 b $-5, -2$ c $5, -1$ d $-4, 3$

2 a $(x+5)(x+7)$ b $(x+9)(x-5)$
c $(x-2)(x-8)$

3 a $(x+5)(x+4)=0$, $x+5=0$ or $x+4=0$, $x=-5$ or $x=-4$
b $(x-6)(x+4)=0$, $x-6=0$ or $x+4=0$, $x=6$ or $x=-4$

Now you try

Example 6
a $x=-2, -4$ b $x=5, -2$ c $x=6, 2$

Example 7
a $x=5$ b $x=8, -5$

Exercise 10C

1 a $-6, -2$ b $-8, -3$ c $-5, -2$

2 a $-4, -6$ b $-6, -3$ c $-3, -1$ d 4, 8
e 3, 6 f 3, 7 g $-3, 5$ h $-2, 8$
i $-5, 9$ j 4, 6 k $-6, 7$ l $-12, 7$

3 a -3 b -2 c -7 d -12 e 5
f 8 g 6 h 9 i 10

4 a $-2, 5$ b 2, 5 c 3 d $-4, 1$
e $-7, 2$ f 4 g $-6, 2$ h $-6, 1$
i 3, 5 j $-8, 2$ k $-4, -2$ l $-9, 2$

5 a $-2, 3$ b $-5, -3$ c $-2, 8$
d 2, 3 e 2 f -1
g 5 h -3 i $-7, 1$

6 11 am, 6 pm

7 a $x^2-3x+2=0$ b $x^2-x-6=0$
c $x^2+3x-4=0$ d $x^2-7x-30=0$
e $x^2-10x+25=0$ f $x^2+22x+121=0$

8 a 2, 3 b 3 c $-10, 2$
d $-5, 7$ e -1 f 2

9 a Equation is not written in standard form $x^2+bx+c=0$ so cannot apply Null Factor Law in this form.
b $x=-2, x=3$

10 It is a perfect square: $(x-1)(x-1)$, $(x-1)^2=0$, $x=1$

11 a $(x-a)(x-a)=0$ or $(x-a)^2=0$
b $(x-a)(x-b)=0$

12 a $x=-3$ or $x=-\frac{1}{5}$ b $x=\frac{2}{3}$ or $x=-2$
c $x=\frac{1}{3}$ or $x=-\frac{1}{2}$ d $x=\frac{1}{2}$ or $x=-1$

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e $x = \frac{3}{2}$ or $x = -5$ f $x = \frac{3}{2}$
g $x = \frac{7}{3}$ or $x = -2$ h $x = \frac{3}{5}$ or $x = -4$
i $x = -\frac{5}{3}$

10D

Building understanding

1 a −6, 3 b 5, −4 c 2, −5
2 a $x(x + 2) = 8$ b 2, −4
c 2, breadth > 0 d $l = 4$ cm, $b = 2$ cm

Now you try

Example 8
breadth = 20 cm and length = 32 cm

Exercise 10D

1 $l = 20$ cm, $b = 12$ cm
2 $l = 9$ cm, $b = 4$ cm
3 −8, 6
4 −5, 12
5 −2, 15
6 $L = 3$ m, $B = 23$ m
7 a $A = 100 - x^2$ b $x = 6$
8 $h = 5$
9 a $A = x^2 + 5x + 15$ b $x = 7$
10 a $x = 3$ (−4 not valid)
b $x = 12$ c $x = 25$
11 −8 not valid because dimensions must be > 0
12 $x = -2$ or $x = 3$; both valid as both are integers
13 a 10, 15, 21
b i 28 ii 210
c i 9 ii 15
14 a 9, 14
b i 8 ii 12
15 a $A = (20 + 2x)^2 = 4x^2 + 80x + 400$
b 10 cm
16 4 cm

Progress quiz

1 a $x^2 - 3x - 8 = 0$ b $3x^2 - 4x - 6 = 0$
c $2x^2 - 2x - 3 = 0$
2 a $x = 2$ is not a solution b $x = -2$ is a solution
3 a $x = 0$ or $x = 7$ b $x = 5$ or $x = -2$
c $x = \frac{1}{3}$ or $x = \frac{3}{4}$
4 a $x = 0$ or $x = -11$ b $x = 0$ or $x = 6$
c $x = 0$ or $x = 4$
5 a $x = 7$ or $x = -7$ b $x = 1$ or $x = -1$
c $x = 2$ or $x = -2$ d $x = 3$ or $x = -3$
6 a $x = -3$ or $x = -8$ b $x = 3$ or $x = 12$
c $x = -5$ or $x = 7$
7 a $x = -4$ b $x = 7$
c $x = -3$ or $x = 8$
8 length 9 m; breadth 6 m
9 number is −12 or 7
10 a $x^2 - 5x + 6 = 0$ b $x^2 - 5x - 6 = 0$
c $x^2 + 6x + 9 = 0$

10E

Building understanding

1 a highest b parabola c intercepts
d vertex e lowest f zero
2 a $x = 3$ b $x = 1$ c $x = -2$

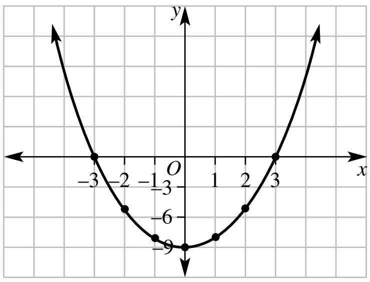
Now you try

Example 9
a $x = 4$ b maximum c (4, 6)
d (2, 0), (6, 0) e (0, −18)

Example 10
a

x	−3	−2	−1	0	1	2	3
y	0	−5	−8	−9	−8	−5	0

b



c i minimum ii $x = 0$ iii (0, −9)
iv (0, −9)
v (−3, 0) and (3, 0)

Exercise 10E

1

	i	ii	iii	iv	v
a	$x = 2$	minimum	(2, −1)	(1, 0), (3, 0)	(0, 3)
b	$x = 0$	minimum	(0, −4)	(−2, 0), (2, 0)	(0, −4)
c	$x = 0$	maximum	(0, 3)	(−1, 0), (1, 0)	(0, 3)
d	$x = 0$	maximum	(0, 4)	(−2, 0), (2, 0)	(0, 4)
e	$x = 2$	minimum	(2, 1)	none	(0, 4)
f	$x = -1$	maximum	(−1, 7)	(−4, 0), (2, 0)	(0, 6)
g	$x = 0$	minimum	(0, 0)	(0, 0)	(0, 0)
h	$x = 0$	minimum	(0, −4)	(−2, 0), (2, 0)	(0, −4)
i	$x = 3$	maximum	(3, 4)	(1, 0), (5, 0)	(0, −5)
j	$x = 3$	minimum	$\left(3, -\frac{1}{2}\right)$	(2, 0), (4, 0)	(0, 4)
k	$x = 0$	maximum	(0, 2)	(−1, 0), (1, 0)	(0, 2)
l	$x = -2$	maximum	(−2, −1)	none	(0, −4)

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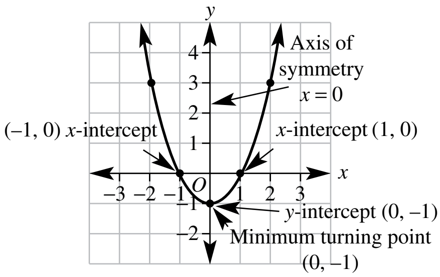
2 **a** $y = x^2 - 1$

x	−2	−1	0	1	2
y	3	0	−1	0	3

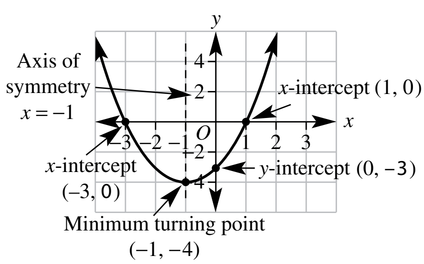
b, c



3 **a** $y = x^2 + 2x - 3$

x	−4	−3	−2	−1	0	1	2
y	5	0	−3	−4	−3	0	5

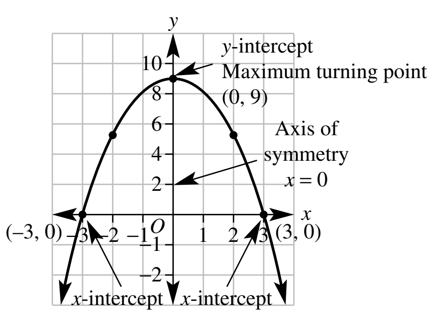
b, c



4 **a** $y = 9 - x^2$

x	−3	−2	−1	0	1	2	3
y	0	5	8	9	8	5	0

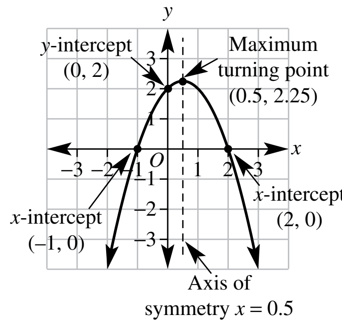
b, c



5 **a** $y = -x^2 + x + 2$

x	−2	−1	0	1	2	3
y	−4	0	2	2	0	−4

b, c



6 **a** **i** 1 s, 3 s
ii one time is on the way up, the other is on the way down
b **i** 2 s **ii** 12 m **iii** 4 s

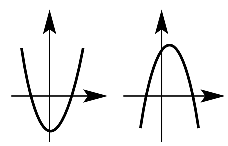
7 **a** 100 m **b** 155 m **c** 5 s
d journey takes 1 s longer to go down to the ground

8 **a** $x = 1$ **b** $(1, -3)$

9 **a** $x = 1$ **b** $(2, 0)$

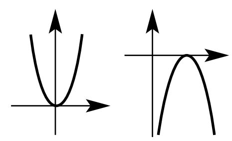
10 **a** $y = x^2$ **b** $y = x^2 - 4$
c $y = 1 - x^2$ **d** $y = x^2 - 2x + 1$
e $y = -x^2 - 2x$ **f** $y = x^2 - 3x - 4$

11 **a** Yes



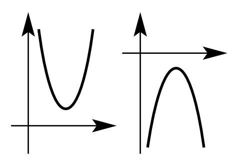
Two x-intercepts

b Yes



One x-intercept

c Yes



Zero x-intercepts

d No

12 **a** $-2^2 = -1 \times 2^2 = -4$. It is not $(-2)^2 = 4$, so correct solution is $y = -2^2 + 2 \times 2 = -4 + 4 = 0$
b $-(-3)^2 = -1 \times 9 = -9$. It is not $(-3)^2 = +9$, so correct solution is $y = -3 - (-3)^2 = -3 - 9 = -12$

13 **a** $x = -2, x = 2$ **b** $x = 0$
c **i** infinite **ii** one **iii** none

14 **a** **i** $x = 0, x = 2$ **ii** $x = -1, x = 3$
b two; a parabola is symmetrical
c one; this is the minimum turning point
d none; −1 is the minimum y-value so there are no values of y less than −1

15 Answers may vary.

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10F

Building understanding

1 a $y = 3x^2, y = x^2, y = \frac{1}{2}x^2$

b $y = -3x^2, y = -x^2, y = -\frac{1}{2}x^2$

c $(0, 0)$ d $x = 0$

e i $y = 3x^2$ ii $y = \frac{1}{2}x^2$

iii $y = -3x^2$ iv $y = -\frac{1}{2}x^2$

f i $y = -x^2$ ii $y = -3x^2$ iii $y = \frac{1}{2}x^2$

2 a positive

b negative

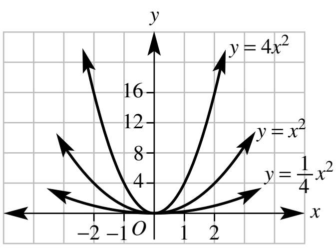
Now you try

Example 11

a

x	−2	−1	0	1	2
$y = x^2$	4	1	0	1	4
$y = 4x^2$	16	4	0	4	16
$y = \frac{1}{4}x^2$	1	$\frac{1}{4}$	0	$\frac{1}{4}$	1

b



c axis of symmetry: $x = 0$, turning point $(0, 0)$

d i narrower

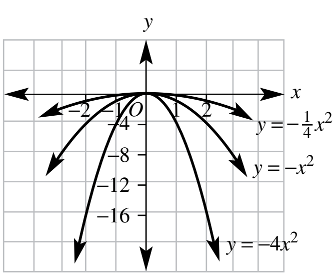
ii wider

Example 12

a

x	−2	−1	0	1	2
$y = -x^2$	−4	−1	0	−1	−4
$y = -4x^2$	−16	−4	0	−4	−16
$y = -\frac{1}{4}x^2$	−1	$-\frac{1}{4}$	0	$-\frac{1}{4}$	−1

b



c axis of symmetry: $x = 0$, turning point $(0, 0)$

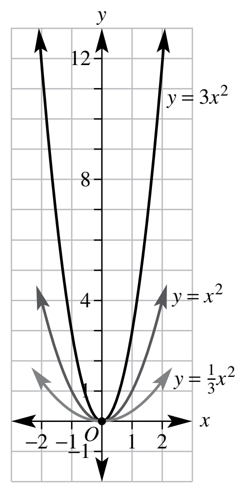
d i narrower ii wider

Exercise 10F

1 a

x	−2	−1	0	1	2
$y = x^2$	4	1	0	1	4
$y = 3x^2$	12	3	0	3	12
$y = \frac{1}{3}x^2$	$\frac{4}{3}$	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{4}{3}$

b



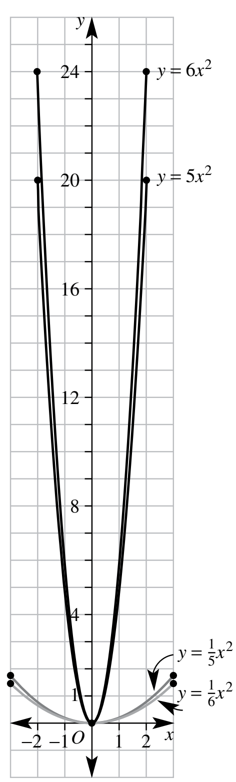
c For all three graphs, the turning point is a minimum at $(0, 0)$ and the axis of symmetry is $x = 0$.

d i narrower ii wider

2 a i

x	−2	−1	0	1	2
y	24	6	0	6	24

ii



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iii axis of symmetry: $x = 0$; turning point: $(0, 0)$
iv narrower

b i

x	−2	−1	0	1	2
y	20	5	0	5	20

ii See part **b**.
iii axis of symmetry: $x = 0$; turning point: $(0, 0)$; x- and y-intercept: $(0, 0)$
iv narrower

c i

x	−2	−1	0	1	2
y	$\frac{2}{3}$	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{2}{3}$

ii See part **a**.
iii axis of symmetry: $x = 0$; turning point: $(0, 0)$
iv wider

d i

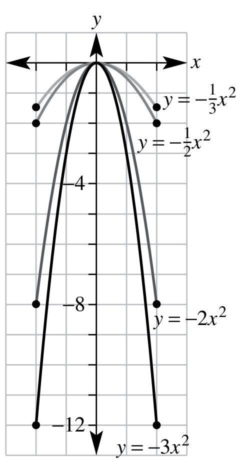
x	−2	−1	0	1	2
y	$\frac{4}{5}$	$\frac{1}{5}$	0	$\frac{1}{5}$	$\frac{4}{5}$

ii See part **a**.
iii axis of symmetry: $x = 0$; turning point: $(0, 0)$
iv wider

3 a i

x	−2	−1	0	1	2
y	−8	−2	0	−2	−8

ii



iii axis of symmetry: $x = 0$; turning point: $(0, 0)$; x- and y-intercept: $(0, 0)$
iv narrower

b i

x	−2	−1	0	1	2
y	−12	−3	0	−3	−12

ii See part **a**.
iii axis of symmetry: $x = 0$; turning point: $(0, 0)$; x- and y-intercept: $(0, 0)$
iv narrower

c i

x	−2	−1	0	1	2
y	−2	$-\frac{1}{2}$	0	$-\frac{1}{2}$	−2

ii See part **a**.
iii axis of symmetry: $x = 0$; turning point: $(0, 0)$; x- and y-intercept: $(0, 0)$
iv wider

d i

x	−2	−1	0	1	2
y	$-\frac{4}{3}$	$-\frac{1}{3}$	0	$-\frac{1}{3}$	$-\frac{4}{3}$

ii See part **a**.
iii axis of symmetry: $x = 0$; turning point: $(0, 0)$; x- and y-intercept: $(0, 0)$
iv wider

4 a A b H

5 a D b C c A
d E e F f B

6 a reflection in the x-axis, dilaton by a factor of 3 from the x-axis
b reflection in the x-axis, dilaton by a factor of 6 from the x-axis
c reflection in the x-axis, dilaton by a factor of $\frac{1}{2}$ from the x-axis
d reflection in the x-axis, dilaton by a factor of 2 from the x-axis
e reflection in the x-axis, dilaton by a factor of 3 from the x-axis
f reflection in the x-axis, dilaton by a factor of $\frac{1}{3}$ from the x-axis

7 a $y = -4x^2$ b $y = \frac{1}{3}x^2$
c $y = -4x^2$ d $y = -\frac{4}{3}x^2$

8 No, because both transformations are multiplying to 'a' and multiplication is commutative: $bc = cb$

9 $y = ax^2$, has y-axis as axis of symmetry so it is symmetrical about the y-axis

10 a $y = 5x^2$ b $y = 7x^2$
c $y = x^2$ d $y = \frac{7}{4}x^2$
e $y = \frac{4}{25}x^2$ f $y = \frac{26}{9}x^2$
g $y = 5x^2$ h $y = -52x^2$

11 $y = \frac{67}{242\,064}x^2$

10G

Building understanding

1 a i $(-2, 0)$ ii $(3, 0)$
b left c right

2 a i $(0, -3)$ ii $(0, 2)$
b down c up

3 a $(0, 3)$ b $(0, -4)$ c $(0, -4)$ d $(0, 25)$

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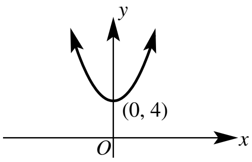
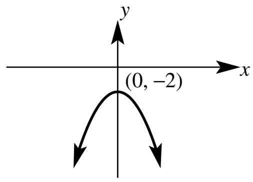
Now you try

Example 13

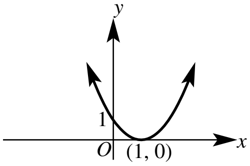
a



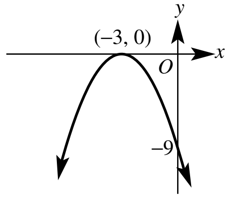
b



c

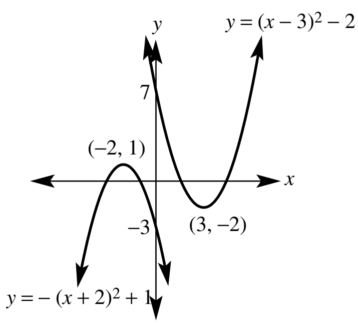


d



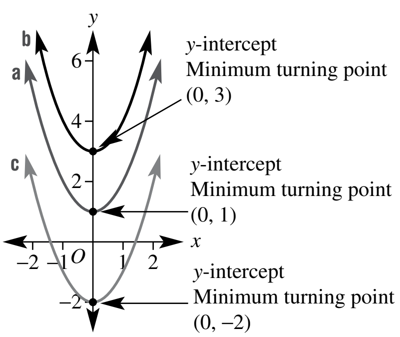
Example 14

a and b

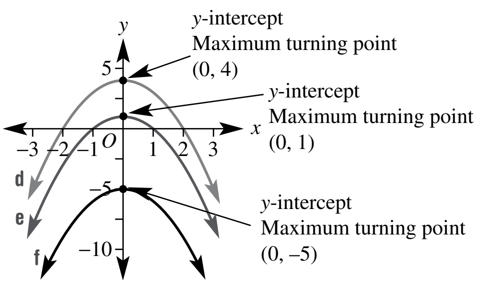


Exercise 10G

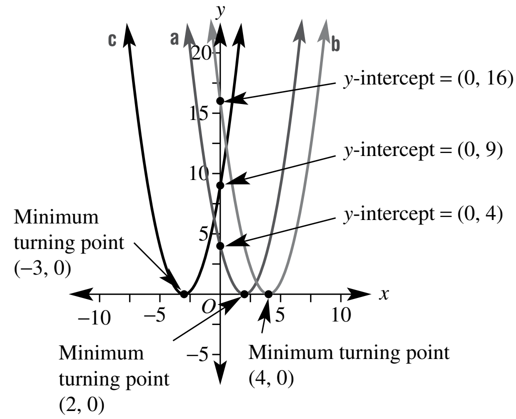
1 **a** $y = x^2 + 1$ **b** $y = x^2 + 3$ **c** $y = x^2 - 2$



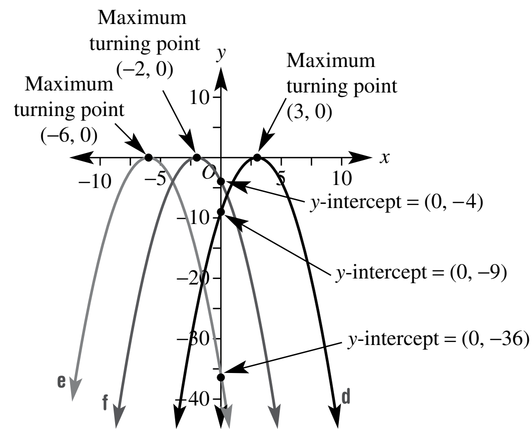
d $y = -x^2 + 4$ **e** $y = -x^2 + 1$ **f** $y = -x^2 - 5$



2 **a** $y = (x - 2)^2$ **b** $y = (x - 4)^2$ **c** $y = (x + 3)^2$



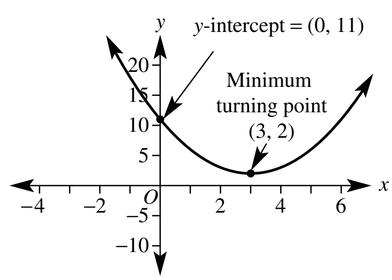
d $y = -(x - 3)^2$ **e** $y = -(x + 6)^2$ **f** $y = -(x + 2)^2$



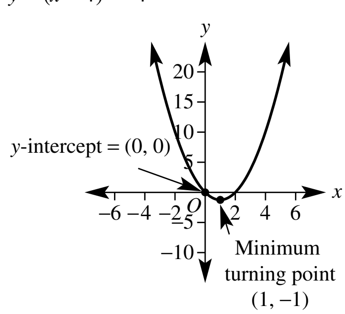
3 **a** **i** (2, 3) **ii** (–1, –1)

b **i** one, left, one, down **ii** two, right, three, up

4 **a** $y = (x - 3)^2 + 2$



b $y = (x - 1)^2 - 1$



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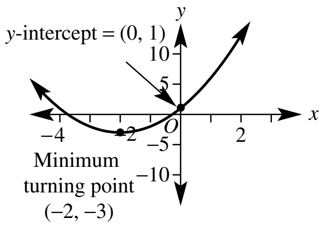
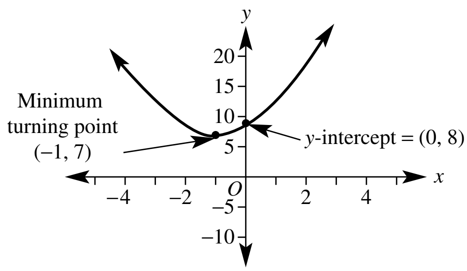
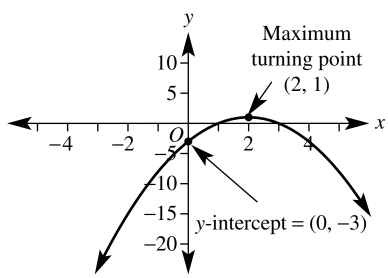
c $y = (x + 2)^2 - 3$



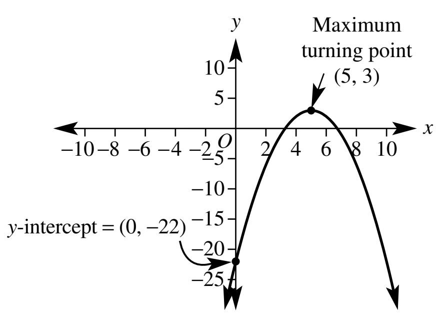
d $y = (x + 1)^2 + 7$



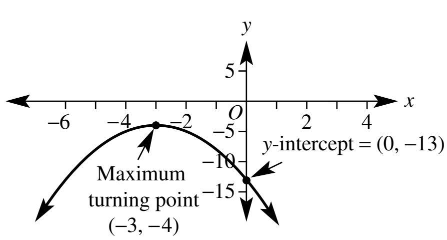
e $y = -(x - 2)^2 + 1$



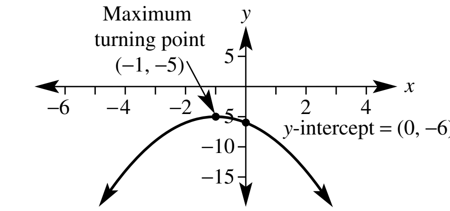
f $y = -(x - 5)^2 + 3$



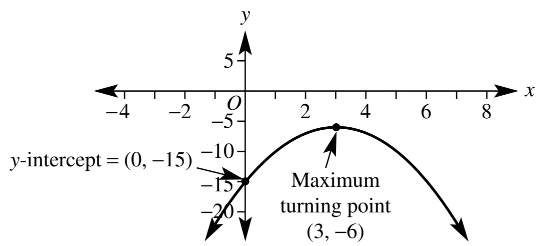
g $y = -(x + 3)^2 - 4$



h $y = -(x + 1)^2 - 5$



i $y = -(x - 3)^2 - 6$

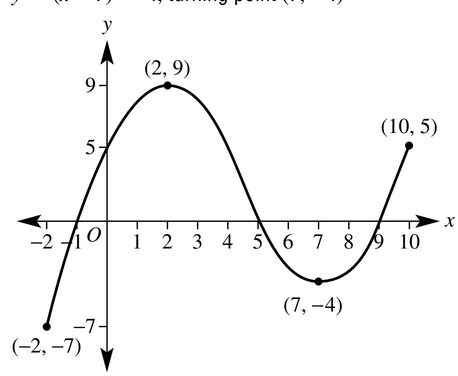


5 **a** J **b** D **c** B **d** H **e** K **f** G
g A **h** E **i** L **j** I **k** F **l** C

6 **a** $y = (x - 1)^2 + 1$ **b** $y = (x + 2)^2 + 2$
c $y = (x + 1)^2 - 3$ **d** $y = -(x - 1)^2 + 4$
e $y = -(x - 2)^2 - 2$ **f** $y = -(x + 2)^2 + 4$

7 **a** $y = -(x - 2)^2 + 9$, turning point (2, 9)
$y = (x - 7)^2 - 4$, turning point (7, −4)

b



8 **a** **i** $y = -(x + 1)^2 + 3$ **ii** $y = (x + 3)^2 + 4$
iii $y = (x - 1)^2 - 3$ **iv** $y = -(x - 5)^2 - 7$
v $y = -x^2 - 2$ **vi** $y = x^2 - 6$
b **i** (−1, 3) **ii** (−3, 4) **iii** (1, −3)
iv (5, −7) **v** (0, −2) **vi** (0, −6)

9 **a** (h, k) **b** $y = ah^2 + k$

10 **a** $(2 - x)^2 = (-1(x - 2))^2 = (-1)^2(x - 2)^2 = (x - 2)^2$, so graphs are the same
b graphs are the same

11 **a** $y = x^2 + 3$ **b** $y = x^2 - 4$
c $y = x^2 - 3$ **d** $y = x^2 - 5$

12 **a** $y = -x^2 + 4$ **b** $y = -x^2 + 4$
c $y = -x^2 + 24$ **d** $y = -x^2 + 10$

13 **a** $y = (x + 3)^2$ or $y = (x - 5)^2$
b $y = (x - 2)^2$ or $y = (x - 4)^2$

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c $y = (x + 4)^2$ or $y = (x - 2)^2$
d $y = x^2$ or $y = (x - 6)^2$

14 a $y = -(x - 1)^2 + 1$ b $y = (x + 2)^2$
c $y = -(x - 3)^2$ d $y = -(x + 3)^2 + 2$
e $y = (x + 1)^2 + 4$ f $y = (x - 3)^2 - 9$

10H

Building understanding

1 a $x(x + 2)$ b $x(x - 3)$
c $(x + 3)(x - 3)$ d $(x + 7)(x - 7)$
e $(x - 4)(x + 3)$ f $(x - 2)(x - 2) = (x - 2)^2$

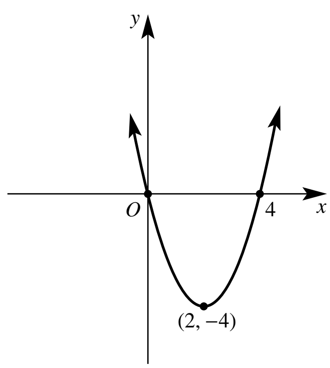
2 a 0 b -3
c -1 d -0.5

3 a -4 b -9 c -1

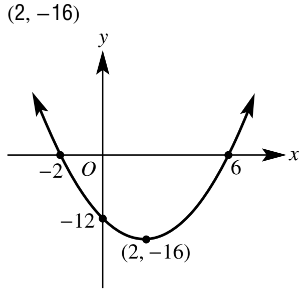
Now you try

Example 15
a i (0, 0), (−5, 0) ii (0, 0)
b i (−1, 0), (4, 0) ii (0, −12)

Example 16
a (0, 0) b $y = x(x - 4)$ c (0, 0), (4, 0)
d $x = 2$ e (2, −4)
f



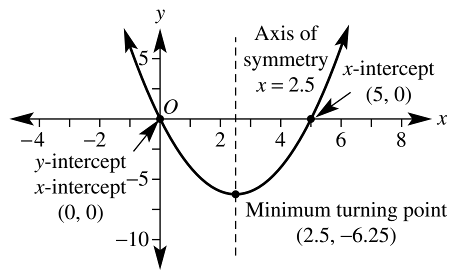
Example 17
a (0, −12) b $y = (x - 6)(x + 2)$
c (6, 0), (−2, 0) d $x = 2$
e (2, −16)
f



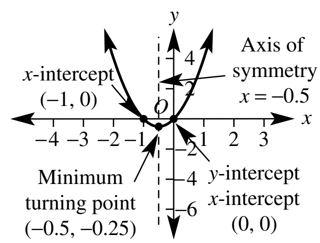
Exercise 10H

1 a i (0, 0), (−7, 0) ii (0, 0)
b i (0, 0), (−3, 0) ii (0, 0)
c i (0, 0), (−4, 0) ii (0, 0)
d i (4, 0), (−2, 0) ii (0, −8)
e i (−2, 0), (5, 0) ii (0, −10)
f i (−3, 0), (7, 0) ii (0, −21)
g i (−3, 0), (1, 0) ii (0, −6)
h i (−4, 0), (−1, 0) ii (0, 12)
i i (2, 0), (3, 0) ii (0, 6)

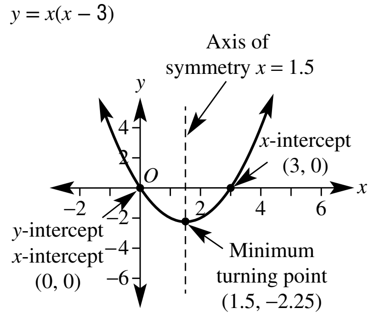
2 a $y = x(x - 5)$



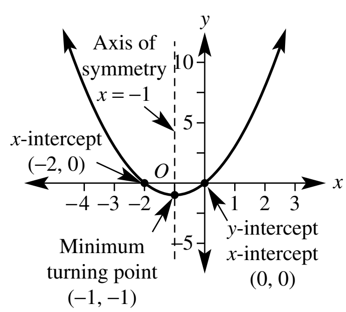
b $y = x(x + 1)$



c $y = x(x - 3)$



d $y = x(2 + x)$



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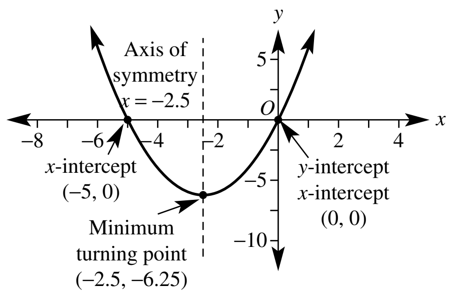
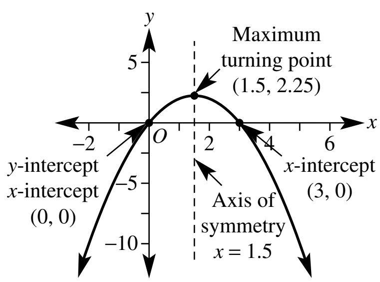
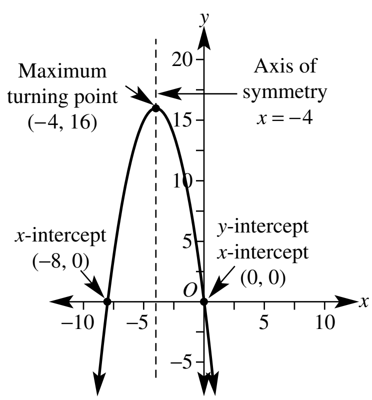
e $y = x(5 + x)$



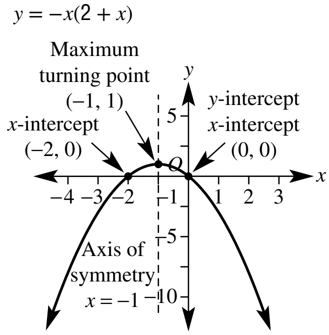
f $y = x(3 - x)$



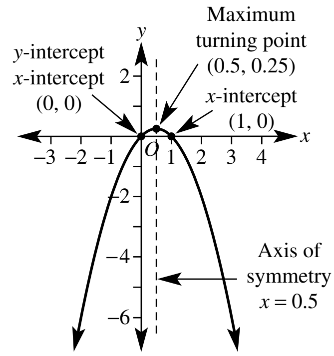
g $y = -x(x + 8)$



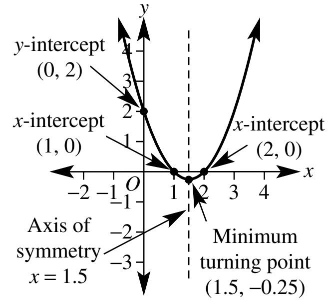
h $y = -x(2 + x)$



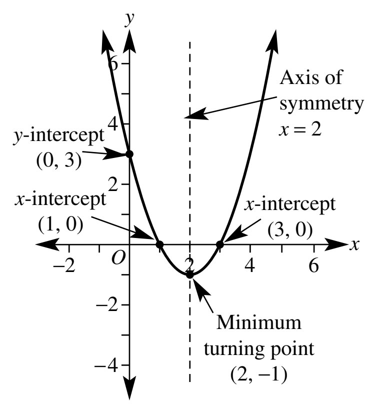
i $y = -x(x - 1)$



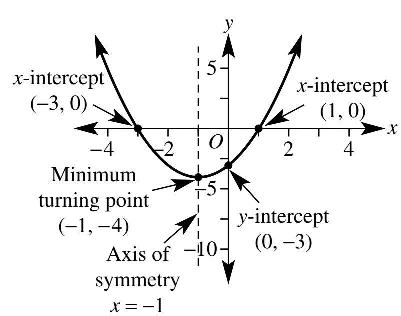
3 a $y = (x - 2)(x - 1)$



b $y = (x - 3)(x - 1)$



c $y = (x + 3)(x - 1)$



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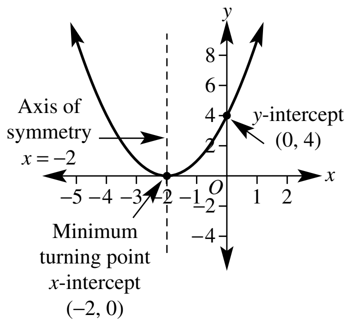
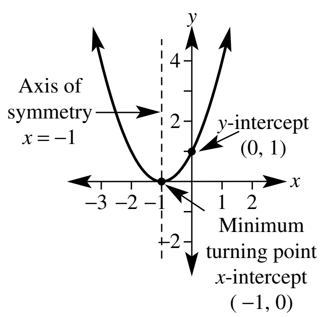
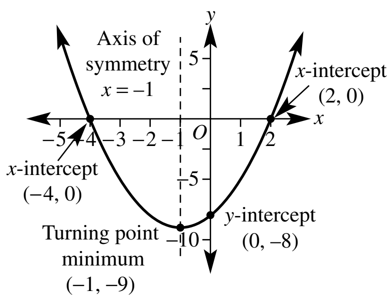
d $y = (x + 2)^2$



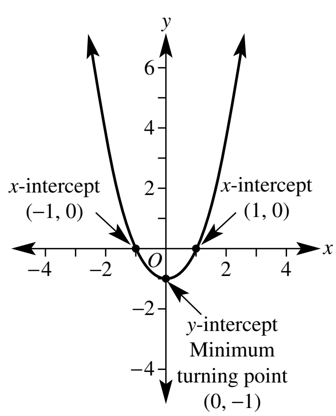
e $y = (x + 1)^2$



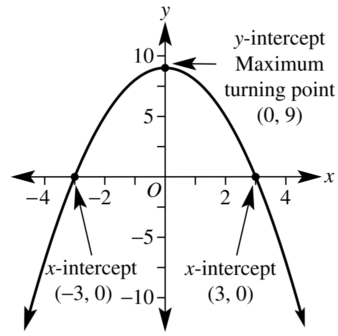
f $y = (x + 4)(x - 2)$



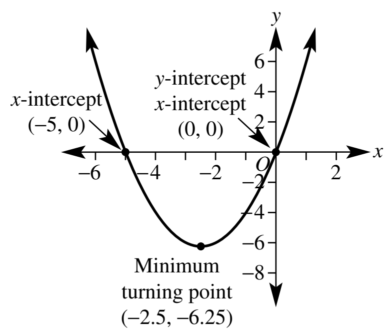
4 a $y = x^2 - 1$



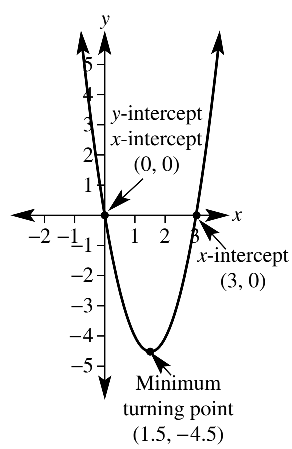
b $y = 9 - x^2$



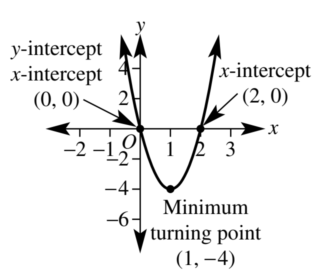
c $y = x^2 + 5x$



d $y = 2x^2 - 6x$



e $y = 4x^2 - 8x$



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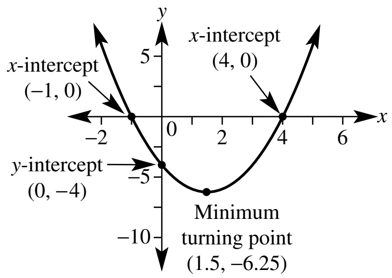
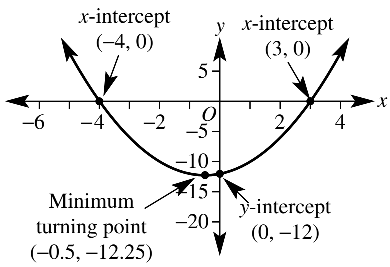
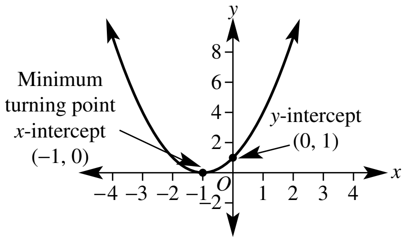
f $y = x^2 - 3x - 4$



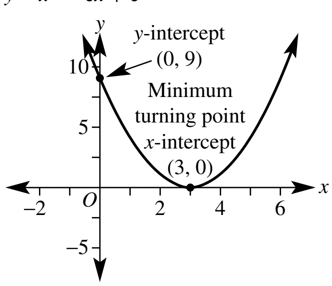
g $y = x^2 + x - 12$



h $y = x^2 + 2x + 1$



i $y = x^2 - 6x + 9$



5 a -3 b 2 c 3
d -4 e -5 f 6

6 a 30 m b 225 m

7 a 200 m b 1000 m

8 There is only one x-intercept so this must be the turning point.

9 a $y = c$
b $y = ab$

10 a $(1, -16)$
b $(5, 0), (-3, 0)$, turning point x-coordinate: $\frac{5 + (-3)}{2} = 1$, turning point$(1, -16)$
c $y = (x - 2)^2 - 49$, $h = 2$, $k = -49$

11 b $y = (x + 3)(x - 1)$ c $y = 2(x + 5)(x - 1)$
d $y = -(x + 1)(x - 5)$ e $y = -(x + 8)(x + 2)$
f $y = \frac{1}{2}(x + 8)(x - 2)$ g $y = -\frac{1}{3}(x + 3)(x - 7)$

Problems and challenges

1 a They are square numbers.
b $100^2 = 10\,000$

2 a $\frac{5}{3}, -\frac{7}{2}$ b $2, -1$ c $2^{\frac{2}{3}} + 1$

3 6 seconds

4 $\frac{n(n + 1)}{2}$; $n = 11$

5 $x = 2$

6 a 20 units, 21 units, 29 units
b 17 units

7 a $k < 0$ b $k = 0$ c $k > 0$

8 a 10, −10, 11, −11, 14, −14, 25, −25
b 2, −2, 5, −5, 10, −10, 23, −23

9 $x = -3, 1, 2, 6$

10 $x = -3, -1, 1, 3$

Success criteria example questions

1 $x^2 - 3x - 6 = 0$

2 Yes, $x = -2$ is a solution

3 $x = -2$ or $x = 5$

4 $x = 0$ or $x = 5$

5 $x = \pm 2$

6 $x = 10$ or $x = -7$

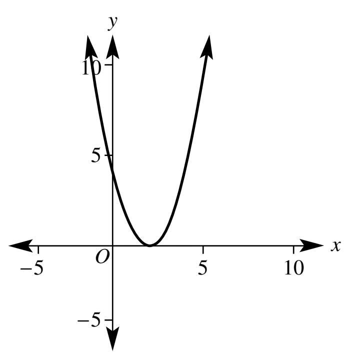
7 $x = 4$ or $x = 5$

8 Width = 7 cm, length = 12 cm

9 i $x = 1$
ii Minimum turning point $(1, -4)$
iii x-intercepts: $(-1, 0)$ and $(3, 0)$; y-intercept: $(0, -3)$

10

x	−3	−2	−1	0	1	2	3
y	25	16	9	4	1	0	1



11

x	−2	−1	0	1	2
$y = x^2$	4	1	0	1	4
$y = 3x^2$	12	3	0	3	12
$y = \frac{1}{3}x^2$	$1\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{1}{3}$	$1\frac{1}{3}$

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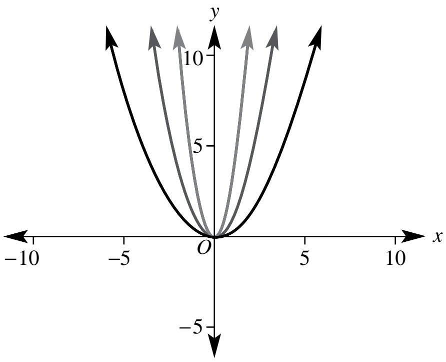
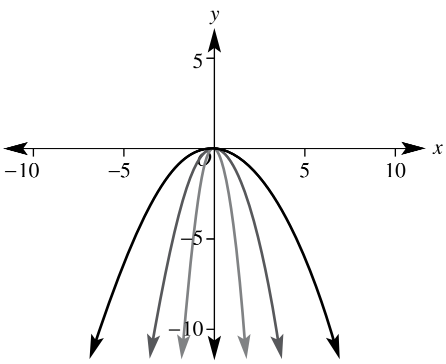


Turning point of each graph: minimum at (0, 0)
The graph of $y = 3x^2$ appears narrower than the graph of $y = x^2$
The graph of $y = \frac{1}{3}x^2$ appears wider than the graph of $y = x^2$

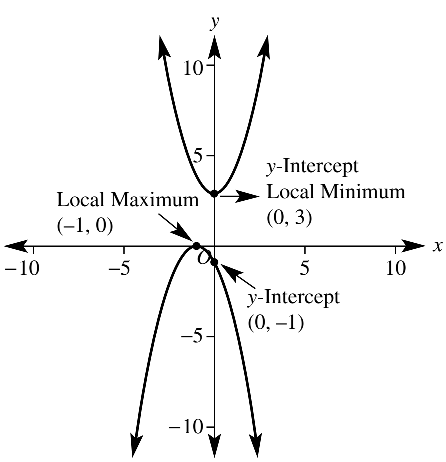
12

x	-2	-1	0	1	2
$y = -x^2$	-4	-1	0	-1	-4
$y = -4x^2$	-16	-4	0	-4	-16
$y = -\frac{1}{4}x^2$	-1	$-\frac{1}{4}$	0	$-\frac{1}{4}$	-1

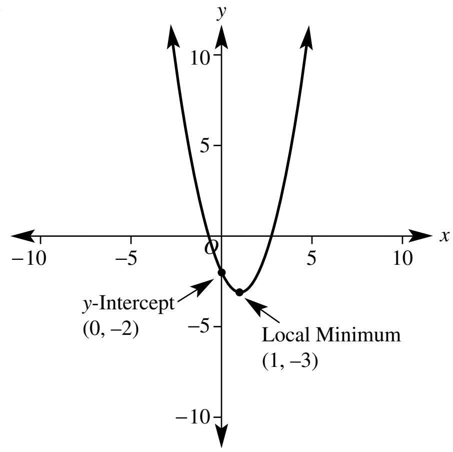


Turning point of each graph: maximum at (0, 0)
The graph of $y = -4x^2$ appears narrower than the graph of $y = -x^2$
The graph of $y = -\frac{1}{4}x^2$ appears wider than the graph of $y = -x^2$

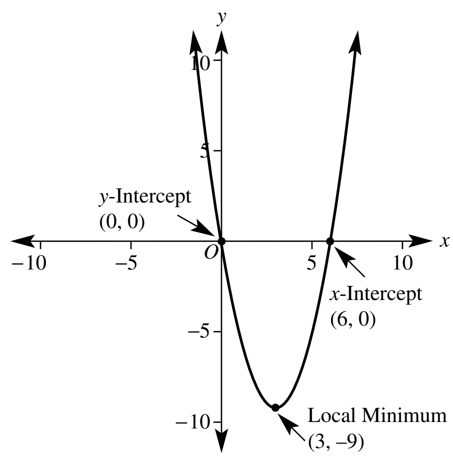
13



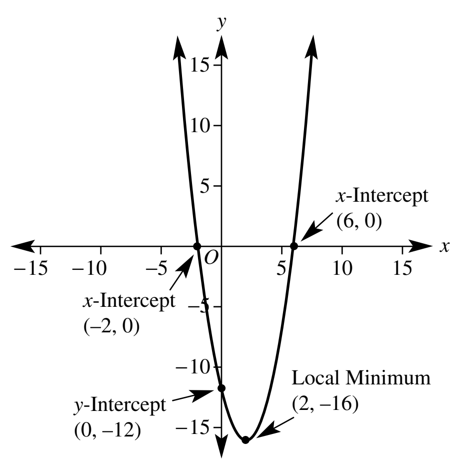
14



15 x-intercepts: (0, 0) and (6, 0)
y-intercept: (0, 0)
turning point: (3, −9)



16 x-intercepts: (−2, 0) and (6, 0)
y-intercept: (0, −12)
turning point: (2, −16)



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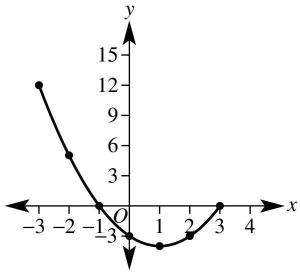
Short-answer questions

1 **a** $y = x^2 - 2x - 3$

x	−3	−2	−1	0	1	2	3
y	12	5	0	−3	−4	−3	0

b



2 **a** 0, −2 **b** 0, 4 **c** −3, 7
d 2, −2 **e** $-1, \frac{2}{5}$ **f** $\frac{1}{2}, \frac{4}{3}$

3 **a** 0, −3 **b** 0, 4 **c** −5, 5
d −9, 9 **e** −2, 2 **f** $-\sqrt{10}, \sqrt{10}$
g −3, −7 **h** −5, 8 **i** 4

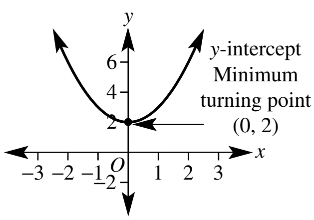
4 **a** $x^2 - 5x = 0$; $x = 0, 5$
b $3x^2 - 18x = 0$; $x = 0, 6$
c $x^2 + 8x + 12 = 0$; $x = -6, -2$
d $x^2 - 2x - 15 = 0$; $x = -3, 5$
e $x^2 - 8x + 15 = 0$; $x = 3, 5$
f $x^2 + 3x - 4 = 0$; $x = -4, 1$

5 **a** $x^2 + 2x - 80 = 0$; $x = 8$ units
b $x^2 + 5x - 24 = 0$; $x = 3$ units

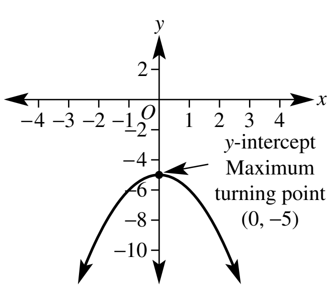
6 **a** **i** $x = 3$ **ii** minimum (3, −2)
b **i** $x = -1$ **ii** maximum (−1, 3)

7 **a** $y = 2x^2$ **b** $y = -x^2 + 2$ **c** $y = (x + 1)^2$
d $y = (x - 3)^2$ **e** $y = x^2 - 4$

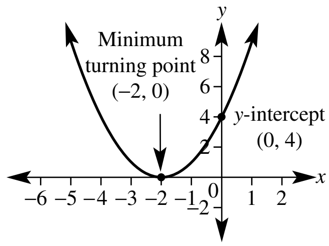
8 **a** $y = x^2 + 2$



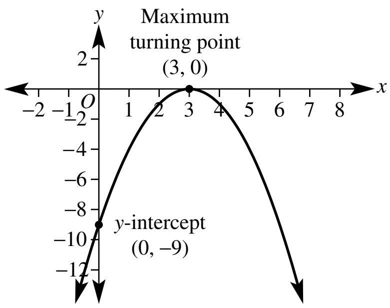
b $y = -x^2 - 5$



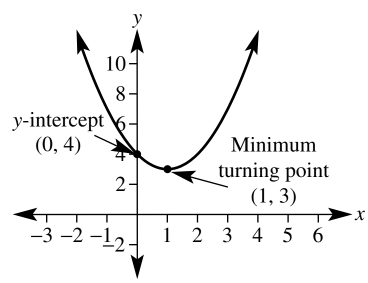
c $y = (x + 2)^2$



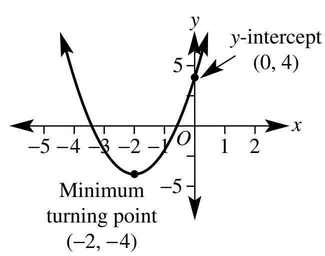
d $y = -(x - 3)^2$



e $y = (x - 1)^2 + 3$



f $y = 2(x + 2)^2 - 4$



9 **a** $y = x^2$ translated up 2 units
b $y = x^2$ reflected in the x-axis and translated down 5 units
c $y = x^2$ translated 2 units left
d $y = x^2$ reflected in the x-axis then translated 3 units right
e $y = x^2$ translated 1 unit right then 3 units up
f $y = x^2$ dilated by a factor of 2 from the x-axis, translated 2 units left and then down 4 units

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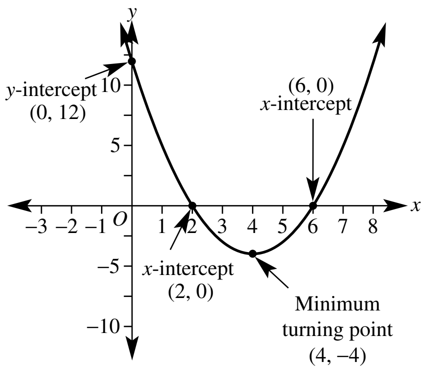
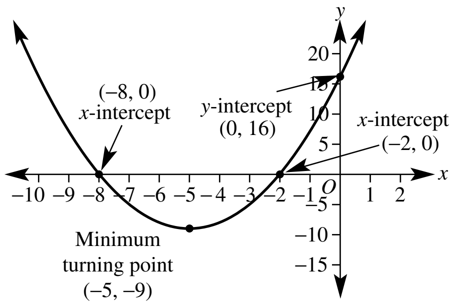
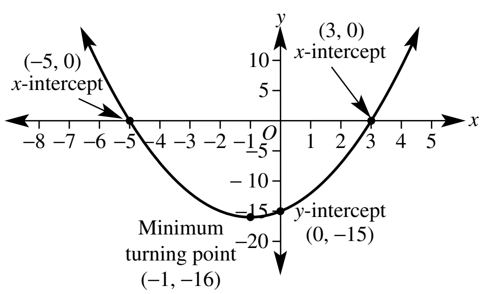
10 a $y = x^2 - 8x + 12$



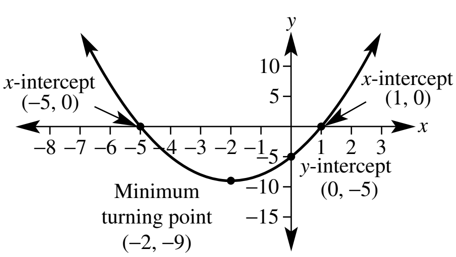
b $y = x^2 + 10x + 16$



c $y = x^2 + 2x - 15$



d $y = x^2 + 4x - 5$



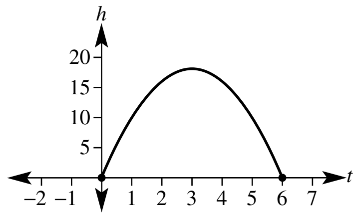
Multiple-choice questions

1 D 2 B 3 C 4 A 5 D
6 B 7 E 8 C 9 A 10 B

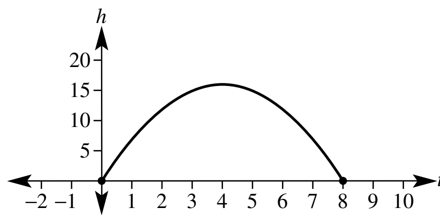
Extended-response questions

1 a $x + 5$ b $A = x^2 + 5x$
c $x = 2$ ($x = -7$ not valid as $x > 0$)
d base = 4 m, height = 7 m

2 a i (0, 0), (6, 0) ii maximum at (3, 18)
iii



b i (0, 0) ii (4, 16) iii (0, 0), (8, 0)
iv



c i Connor's rocket, 8 s
ii Sam's rocket, 2 m higher
iii 16 m, the same height

Semester review 2

Indices and surds

Short-answer questions

1 a $\frac{a^2b}{2}$ c $8x^6 - 2$
b $2x^3y^5$

2 a $5m^2$ b $\frac{2a^8}{3b^5}$ c $\frac{3x^2}{y}$

3 a 3.07×10^{-2} kg b 4.24×10^6 kg
c 1.22×10^4 seconds d 2.35×10^{-7} seconds

4 a 12 b 3
c $2\sqrt{3} + 7\sqrt{5}$ d $\sqrt{7}$

Multiple-choice questions

1 C 2 B 3 D 4 E 5 C

Extended-response questions

a i 74 000 000 000 ii 7.4×10^{10}
b 1.87×10^{17} c 8.72×10^{-7}

Properties of geometrical figures

Short-answer questions

1 a $a = 100$, $b = 140$ b $a = 70$, $b = 55$
c $x = 67$, $y = 98$ d $x = 35$

2 85°

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3 $CB = CD$ (given equal sides)
$\angle ACB = \angle ACD$ (given equal angles)
AC is common
$\therefore \Delta ABC \mathrel{|||} \Delta ADC$ (SAS)

4 a AAA b 2.4

Multiple-choice questions

1 C 2 B 3 A 4 E 5 B

Extended-response question

a $\angle ABD = \angle ECD$ (given right angles)
$\angle ADB = \angle EDC$ (common angle)
$\angle DAB = \angle DEC$ (corresponding angles in parallel lines are equal)
$\therefore \Delta ABD \equiv \Delta ECD$ (AAA)

b 7.5 m c 3.75 m d 4.3 m

Quadratic expressions and algebraic techniques

Short-answer questions

1 a $x^2 - 9$ b $x^2 + 4x + 4$
c $6x^2 - 17x + 12$

2 a $2ab(4 + a)$ b $(3m - 5)(3m + 5)$
c $3(b - 4)(b + 4)$ d $(a + 4)(a + 10)$
e $(x + 3)^2$ f $(x - 2)(x + 10)$
g $2(x - 3)(x - 5)$ h $(2x - 3)(x - 4)$
i $(2x - 1)(3x + 4)$

3 a $(x - 3)(x - 1)$ b $(x + 2)(2x - 5)$

4 a i $\frac{3}{2}$ ii $\frac{4}{x+3}$
iii $\frac{13x+21}{14}$ iv $\frac{1}{3x}$
v $\frac{11x-10}{(x+1)(x-2)}$ vi $\frac{2x+25}{(x-5)(x+2)}$
b i $x = \frac{17}{12}$ ii $x = -11$

Multiple-choice questions

1 E 2 D 3 B 4 A 5 D

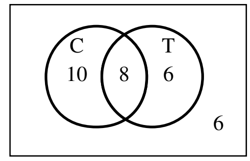
Extended-response question

a $10 - 2x$ and $8 - 2x$
b $(10 - 2x)(8 - 2x) = 80 - 36x + 4x^2$
c 48 m^2
d $4(x - 4)(x - 5)$
e Area of rug is 0 as it has no width.

Probability and single variable data analysis

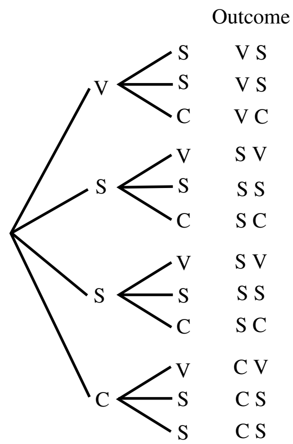
Short-answer questions

1 a



b i 24 ii 16
c i $\frac{1}{5}$ ii $\frac{1}{3}$

2 a



b i $\frac{1}{3}$ ii $\frac{1}{6}$ iii $\frac{1}{2}$

3 a stratified b systematic

4 a numerical discrete
b

Stem	Leaf
1	0 1 1 2 3 5 7 8
2	2 5 5 5 6
3	1 2 2

1 | 3 means 13 aces

c mode = 25, median = 20
d positively skewed

5 a

Class interval	Frequency	Percentage frequency
0–49	2	6.7
50–99	4	13.3
100–149	5	16.7
150–199	9	30
200–249	7	23.3
250–299	3	10
Total	30	100

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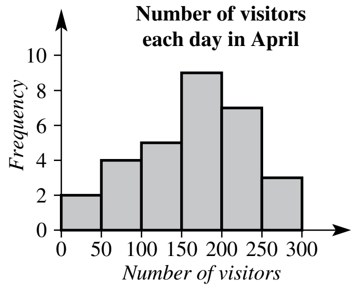
b



c **i** 6 **ii** 60%

Multiple-choice questions

1 D **2** A **3** C **4** B **5** C

Extended-response question

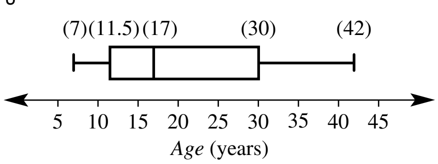
a

		Red		
		1	**2**	**3**
Green	**1**	(1, 1)	(1, 2)	(1, 3)
	2	(2, 1)	(2, 2)	(2, 3)
	3	(3, 1)	(3, 2)	(3, 3)

b $\frac{4}{9}$

c **i** $\frac{1}{3}$ **ii** 18

d **i**



ii 11.5 years **iii** 90

Quadratic equations and graphs of parabolas

Short-answer questions

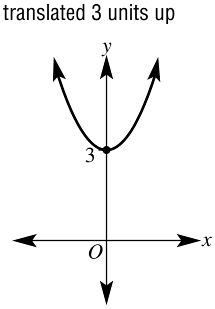
1 **a** $x = -5$ or $x = 3$ **b** $x = \frac{1}{2}$ or $x - \frac{5}{3}$

c $x = 0$ or $x = -2$ **d** $x = -3$ or $x = 3$

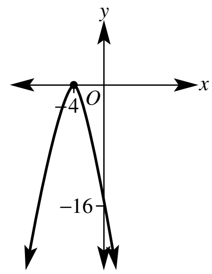
e $x = 2$ or $x = 7$ **f** $x = -4$

2 15 m by 8 m

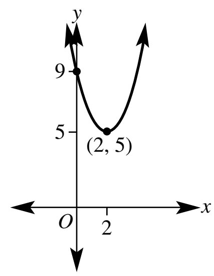
3 **a** translated 3 units up



b reflected in the x-axis and translated 4 units left

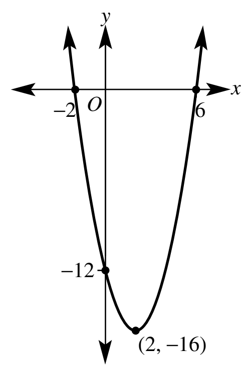


c translated 2 units right and 5 units up



4 **a** (0, −12) **b** (−2, 0), (6, 0) **c** (2, −16)

d



Multiple-choice questions

1 B **2** B **3** E **4** C **5** E

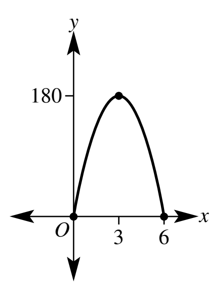
Extended-response question

a Lands after 6 seconds.

b **i** 180 cm **ii** 3 seconds

c after 2 seconds and 4 seconds

d



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Answers

Semester review 2

11

Algorithmic thinking

Maths in context: Algorithms and networks

For a modern society to function effectively we know that road and rail networks, electrical circuits and various communication systems need to operate efficiently. The study of how such systems work is called Networks and is a key application of mathematics, computer science and engineering. The optimisation of networks involves the use of algorithms which aim to produce the best performance using the minimum amount of resources.

The capacity of a rail network, for example, depends largely on the effectiveness of the timetable. However, these can be difficult to construct by hand given the vast number of possible combinations of factors which influence the performance of the network. Algorithms aided by computer programs are used in conjunction with network mathematics to search for optimal network solutions.

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Optimal solutions to network problems help find a shortest path, maximum flow, minimum cost or maximum profit, all helping to improve the society in which we live.

Chapter contents

Australian Curriculum 9.0

design, test and refine algorithms involving a sequence of steps and decisions based on geometric constructions and theorems; discuss and evaluate refinements (AC9M9SP04)

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

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Introduction

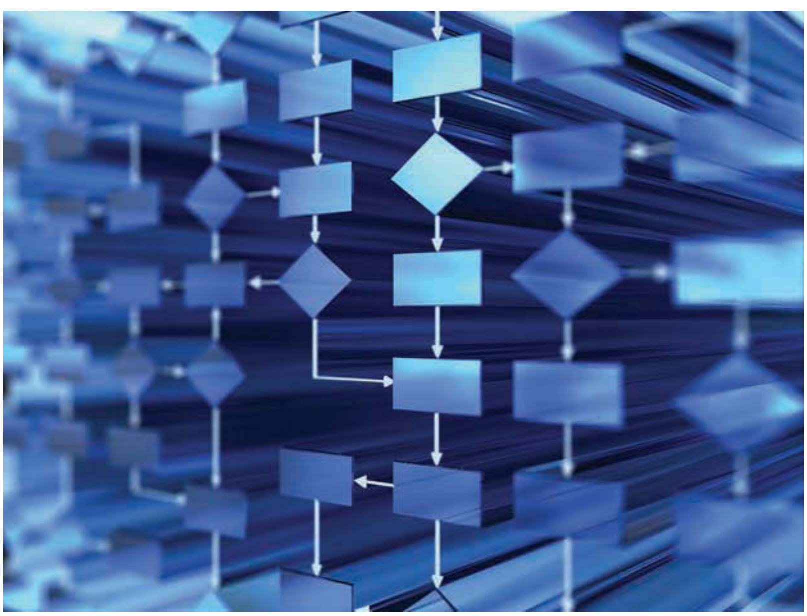
An **algorithm** is a sequence of steps that when followed, lead to the solution of a problem. It has a defined set of inputs and delivers an output. Each step in the algorithm leads to another step or completes the algorithm.

Algorithms occur in mathematics and computing, as well as in simple areas of daily life such as following a recipe. Algorithmic thinking is a type of thinking that involves designing algorithms to solve problems. The algorithms we design can then be written in a way that a computer program will understand, so that the computer does the hard computational work.

In the following activities you will carry out some algorithms as well as think about the design, analysis and implementation of your own algorithms.

The algorithms in these activities will be described through the use of spreadsheets, flow charts (a way of writing an algorithm in the form of a diagram), programming language and simulations. The following symbols will be used in the flow charts with arrows to connect each stage:

for input/output stages | for process stages | for decision stages



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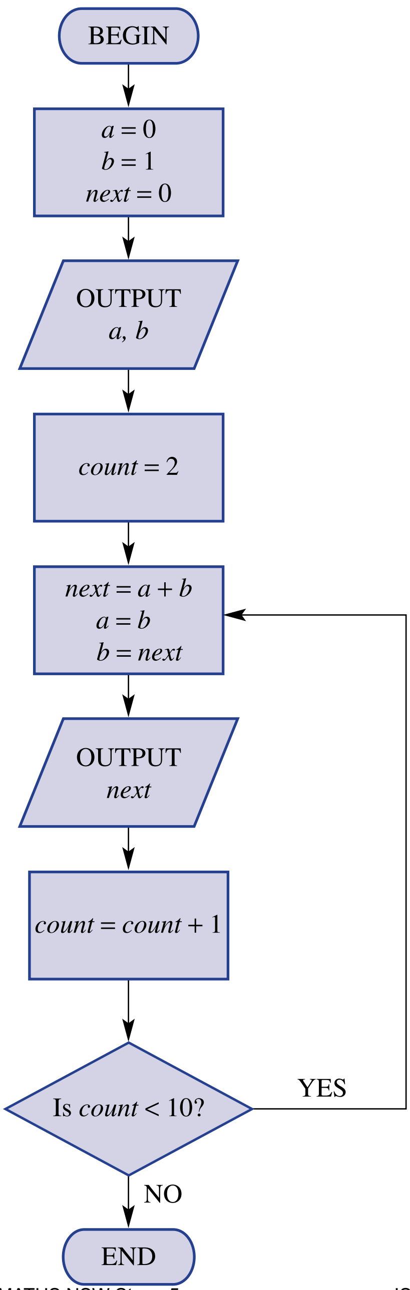
Activity 1: Algorithms for number patterns and financial maths

NUMBER AND ALGEBRA

Algorithms can be used to generate number patterns as well as to carry out tasks in the financial world. Some examples are seen in the following parts.

1.1 Number patterns

Consider the flowchart algorithm shown below.



a Trace through the algorithm by completing a table like the one shown below, updating the values of the variables as you go. Add any output to the line at the bottom of the table. The first set of values is done for you.

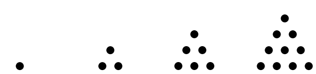
a	*b*	*next*	*count*
0	1	0	2
.	.	.	.
.	.	.	.
.	.	.	.
Output: 0, 1,			

b What is the special sequence of numbers generated in the output of part **a**?

c Which part of the algorithm controls how many numbers are displayed?

The triangular numbers are another number sequence as shown below.

Term: 1st 2nd 3rd 4th . . .



1 3 6 10 . . .

Consider the pattern in the triangular numbers.

d Set up a spreadsheet to generate the first 20 triangular numbers based on the term number. In the first column enter the numbers 1 to 20 for the term number and in the second column set up a way of generating the triangular numbers.

e Complete a flowchart that generates the first 10 triangular numbers. Use the flowchart from part **a** as a guide. Test it on a friend using a table to see if they get the correct list of numbers as they trace through the algorithm.

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1.2 Algorithms in finance

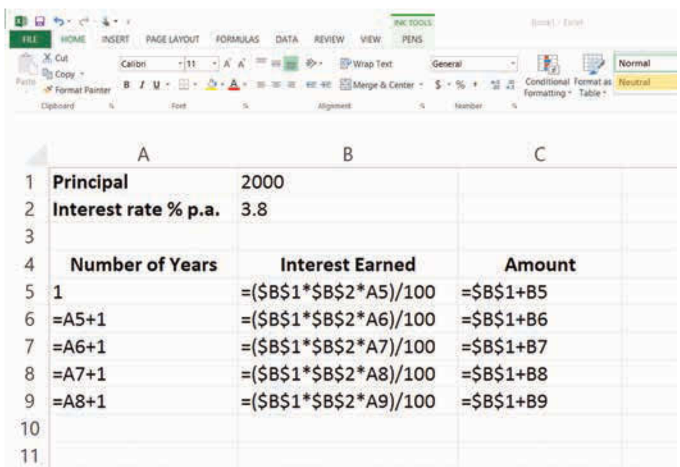
a Simple interest

In **Section 1K** you would have studied simple interest. Recall, that this is the interest (I) calculated at a set rate ($r\%$), over a certain period of time (t) on a principal amount (P). When calculating simple interest, the interest is the same for each time period.

The formula is $I = \frac{Prt}{100}$.

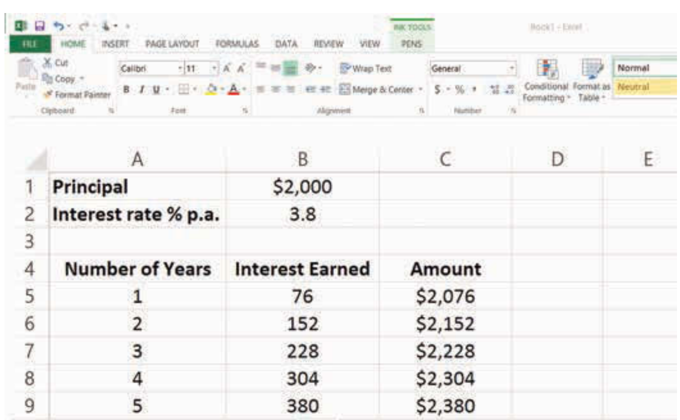
Consider the case in which a sum of $2000 is invested in a simple interest bank account with an interest rate of 3.8% p.a. for 5 years.

i Set-up a spreadsheet like the one shown in Figure 1 below to display the total amount of interest earned after each year and the account balance. Once the initial formulas are entered, fill down the columns until $t = 5$.



	A	B	C
1	Principal	2000	
2	Interest rate % p.a.	3.8	
3			
4	Number of Years	Interest Earned	Amount
5	1	=($B$1*$B$2*A5)/100	=$B$1+B5
6	=A5+1	=($B$1*$B$2*A6)/100	=$B$1+B6
7	=A6+1	=($B$1*$B$2*A7)/100	=$B$1+B7
8	=A7+1	=($B$1*$B$2*A8)/100	=$B$1+B8
9	=A8+1	=($B$1*$B$2*A9)/100	=$B$1+B9
10			
11			

Figure 1: Showing formulas



	A	B	C	D	E
1	Principal	$2,000			
2	Interest rate % p.a.	3.8			
3					
4	Number of Years	Interest Earned	Amount		
5	1	76	$2,076		
6	2	152	$2,152		
7	3	228	$2,228		
8	4	304	$2,304		
9	5	380	$2,380		

Figure 2: Calculated values

The spreadsheet in Figure 2 could also be generated in programming languages. The first stage is to design the flowchart algorithm.

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ii The steps of the algorithm are listed below. From these steps construct a flowchart for the algorithm.

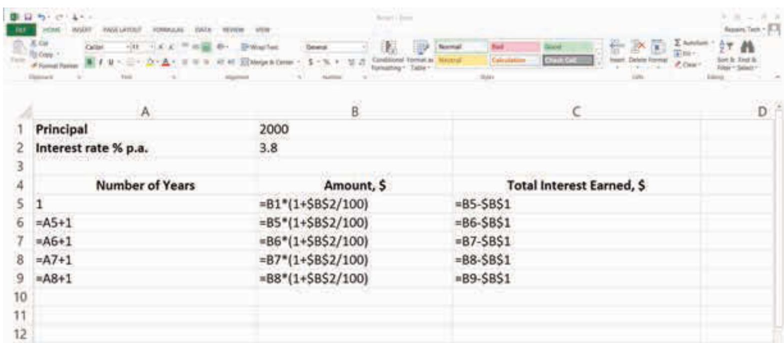
Step 1. Receive inputs P, r and t.
Step 2. Set the counter n to $n = 1$ to represent the end of the first year.
Step 3. Calculate the simple interest and the current balance for year n.
Step 4. Display the current year n, the interest earned and the balance.
Step 5. Increase n by 1.
Step 6. If $n < t$ repeat steps 3 to 6 else End.

iii Use your algorithm in part **ii** above to find the total amount after 5 years if $P = 5000$ and $r = 4$.

b **Compound interest**

A second type of interest, which you may have studied in **Section 1L**, is compound interest. Compound interest is such that interest is calculated on both the principal amount and the previously accrued interest. This is achieved by increasing the current year's amount by the percentage rate.

i Set-up the spreadsheet below to display the balance each year and the interest earned in total for an investment of $2000 compounding annually at 3.8%.



	A	B	C	D
1	**Principal**	2000		
2	**Interest rate % p.a.**	3.8		
3				
4	**Number of Years**	**Amount, $**	**Total Interest Earned, $**	
5	1	=B1*(1+$B$2/100)	=B5-$B$1	
6	=A5+1	=B5*(1+$B$2/100)	=B6-$B$1	
7	=A6+1	=B6*(1+$B$2/100)	=B7-$B$1	
8	=A7+1	=B7*(1+$B$2/100)	=B8-$B$1	
9	=A8+1	=B8*(1+$B$2/100)	=B9-$B$1	
10				
11				
12				

ii Experiment with your spreadsheet, using different principal amounts, interest rates and time periods.

iii Write out the steps of the compound interest algorithm, as for simple interest in part **a ii**, then create a flowchart that could be used to program this algorithm.

Ext **iv** Redesign your algorithm from part **iii** so that it determines and outputs the number of full years it takes for the amount in the account to be double the initial investment.

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1.3 Income tax calculation algorithm

You may have seen tax tables used to calculate how much tax a person must pay based on their taxable income.

The taxable income is taken as the gross income minus deductions. An example of a tax table with different tax brackets is shown below. Here you are taxed at a higher rate for dollars earned over certain amounts.

Taxable income	Tax on this income
0–$18 200	Nil
$18 201–$37 000	19c for each $1 over $18 200
$37 001–$80 000	$3572 plus 32.5c for each $1 over $37 000
$80 001–$1 80 000	$17 547 plus 37c for each $1 over $80 000
$180 001 and over	$54 547 plus 45c for each $1 over $180 000

a Using the table above, find the tax payable for these taxable incomes.
 i $1000
 ii $25 000
 iii $50 000
 iv $100 000
 v $200 000

b Write an algorithm in flowchart form that determines the tax payable based on input of a person's gross income and deductions total.

c Have a friend trace through your algorithm using a gross income of $62 487 and deductions of $350. Do they complete the algorithm with the correct answer?

d Carefully choose four other values to test that your algorithm works correctly. How did you make your choice?

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Activity 2: Minimising and maximising

MEASUREMENT AND GEOMETRY

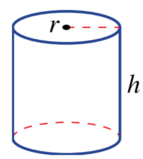
In the following we will consider two common scenarios. The first scenario is for a fixed volume of an object and determining the dimensions which minimise the surface area (i.e. material required to form it). The second is for a fixed surface area (amount of material), and determining the dimensions which maximise the volume.

2.1 Cylindrical cans

A company packages tinned food in cylindrical cans. They require a fixed volume for these cans and wish to use dimensions that give the minimum surface area to minimise the cost of production.

Recall the following formulas for a cylinder:

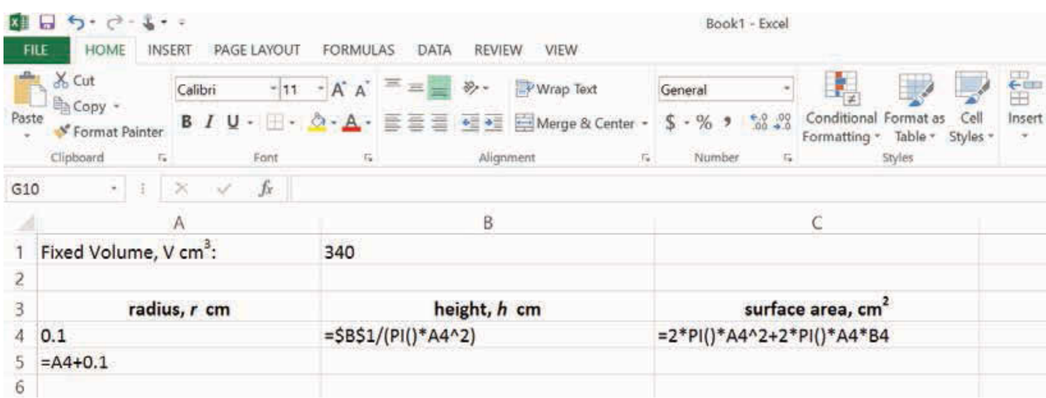
$$\text{Volume: } V = \pi r^2 h$$
$$\text{Surface area: } A = 2\pi r^2 + 2\pi r h$$



Let the required volume of the cans be $340\,\text{cm}^3$.

a In $V = \pi r^2 h$, let $V = 340$. Rearrange the formula to make h the subject to show that $h = \dfrac{340}{\pi r^2}$.

b In a spreadsheet make a column for r and create a column to calculate h (using the formula from part **a**). Also create a column for the surface area. Start at $r = 0.1$ and increment (increase) your r values by 0.1. Using a smaller increment will give greater accuracy.
In Microsoft Excel, the PI() function accesses π.
Fill down each column.



c Use the graphing features of your spreadsheet to sketch a graph of surface area against radius. You should notice a minimum (lowest point) on your graph. Refer to your spreadsheet and write down the closest r and h values that give the minimum surface area.

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d Repeat parts **b** and **c** using the following different volumes. (In your spreadsheet you will only need to change cell B1).

i $V = 400\ \text{cm}^3$ **ii** $V = 500\ \text{cm}^3$

e What do you notice about the relationship between the r and h values of a cylinder that minimise its surface area? Do you see many tin cans like this?

Ext **f** Alter your increment size in your spreadsheet to see if you can find the r and h values that deliver the minimum surface area to a great level of accuracy.

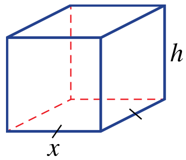
2.2 Cardboard boxes

A packaging company has $3200\ \text{cm}^2$ of cardboard to be used to form each box. The boxes are required to have square bases. The box will use all the cardboard, thus the $3200\ \text{cm}^2$ will be the surface area.

The dimensions of the box are to be set so that the box has the largest possible volume.

a For the square-based rectangular prism shown, write down the formula for:

i its volume **ii** its surface area.



b Using the surface area of $3200\ \text{cm}^2$, rearrange the surface area formula in part **a ii** to make h the subject.

c Create a spreadsheet like the one in **2.1** part **b** with columns for x, h and volume, and investigate the box dimensions (x and h) that maximise the volume of the box.

d Use your spreadsheet from part **c** for different fixed surface areas and find the dimensions that deliver a maximum volume.

e In conclusion, what shaped box maximises the volume for a given surface area?

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Activity 3: Sorting, simulations and sampling

STATISTICS AND PROBABILITY

3.1 Ways of sorting data to find the median

Many statistical calculations have a set process or algorithm for calculating them. For example, the mean is found by adding the data values and dividing by the number of values. The range is found by finding the maximum and minimum values and calculating the difference between them.

Recall that the process for finding the median of a set of data is:

Step 1. Sort the data in ascending order.
Step 2. Locate the middle value: If there is an odd number of data values this will be the median value; if there is an even number of data values, average the two middle values.

From an algorithm point of view, the most involved part of this process is sorting the list of data.

There are a number of different sorting algorithms already in existence. Here we will compare two different types: selection sort (which you may have seen in Year 8) and bubble sort.

Selection sort: This method involves finding the smallest (or largest) element in a list and swapping it with the first (or last) item in the list and then moving along the list until it is sorted.

For example, the data set 7 8 6 12 2 3 9 after one pass through becomes 2 8 6 12 7 3 9 where the minimum value 2 moves to the start of the list and swaps with the 7. The next pass through becomes 2 3 6 12 7 8 9 and so on.

a Use the data sets listed below to trace through the selection sort algorithm. Complete a table like the one shown to show the algorithm in action. Record the number of comparisons (comparing data elements to find the minimum) and the number of swaps made along the way. The first row for the first data set is completed for you.

i 7 3 9 5 **ii** 6 2 8 5 4

Pass number	List order	Number of comparisons made	Number of swaps made
1	3 7 9 5	3	1
⋮			
		Total:	**Total:**

Bubble sort: this method works through the list by comparing each adjacent pair of numbers along the way and swapping them if necessary so that the smaller number is on the left. One pass through the list ensures the biggest number is at the end of the list. The process continues on the list until the data sorted.

For example, for the data set 3 8 6 4 7 the first pass involves the following stages:

3 8 6 4 7
3 8 6 4 7
3 6 8 4 7
3 6 4 8 7
3 6 4 7 8

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Each adjacent pair was compared (3 and 8 stayed where they were as they were already in order). This involved four comparisons and three swaps. Pass 2 will then work on the list 3 6 4 7, since 8 is now in the correct position.

b Use the bubble sort algorithm for the two data sets in part **a**, by setting up a similar table.

c Compare your tables from parts **a** and **b**.

- **i** What do you notice about the number of comparisons required in the two algorithms? How does this compare to the number of data elements?
- **ii** What do you notice about the number of swaps between the two algorithms?
- **iii** Hence, in general, which algorithm do you think is considered more efficient (runs faster or requires less processing)?

d A flowchart can be used to represent these algorithms as well as pseudo code (an informal programming language that can eventually be implemented in code).
The example below shows pseudo code for the selection sort algorithm.

$$
\begin{array}{l}
\textit{for } i = 1 \rightarrow n-1 \\
\quad \textit{min} \leftarrow i \\
\quad\quad \textit{for } j = i+1 \rightarrow n \\
\quad\quad\quad \textit{if } a_j < a_{min} \textit{ then} \\
\quad\quad\quad\quad \textit{min} \leftarrow j \\
\textit{swap } a_i \textit{ and } a_{min}
\end{array}
$$

The data is input as a list/array in which the position in the list is referenced as a_1 for the first element, a_2 for the second element up to the nth (last) element a_n.
The 'for loop' is used to work through the repetitive nature of the process. Inside the 'for loop' each value of the counter i increases by 1 each time after each process. The $\leftarrow$ updates the value of the variable with a new value.

Read through the above code and explain how it works. Try working through the code with the data list in the table at right.

5	4	6	3
a_1	a_2	a_3	a_4

Set-up a table similar to the one below to keep track of the pronumeral values and the list order. In this list $n = 4$. The first run through with $i = 1$ is done for you.

i	min	j	Test condition	min
1	1	2 (i.e. $i + 1$)	$a_2 < a_1$–yes	2 (updated)
1	2	3	$a_3 < a_2$–no	No change
1	2	4 (i.e. n)	$a_4 < a_2$–yes	4 (updated)
$j = n$, break out of for loop, update list order swapping a_i and a_{min}: 3 4 6 5				
2	2	3	…	…
⋮				

e Using the format below, complete the pseudo code for the bubble sort algorithm by filling in the boxes.
j iterates from 1 to $n - i$ as the biggest numbers will be being sorted at the end of the list first this time.

$$
\begin{array}{l}
\textit{for } i = 1 \rightarrow n-1 \\
\quad\quad \textit{for } j = 1 \rightarrow n-i \\
\quad\quad\quad\quad \textit{if } \boxed{} \textit{ then} \\
\quad\quad\quad\quad\quad \boxed{}
\end{array}
$$

Ext **f** The efficiency of the bubble sort algorithm can be improved by stopping when the list is sorted (i.e. if no more swaps are taking place as you pass through the list). Consider ways you could implement this in your code in part **e** so that when the list is sorted it breaks out of the 'for loop'.

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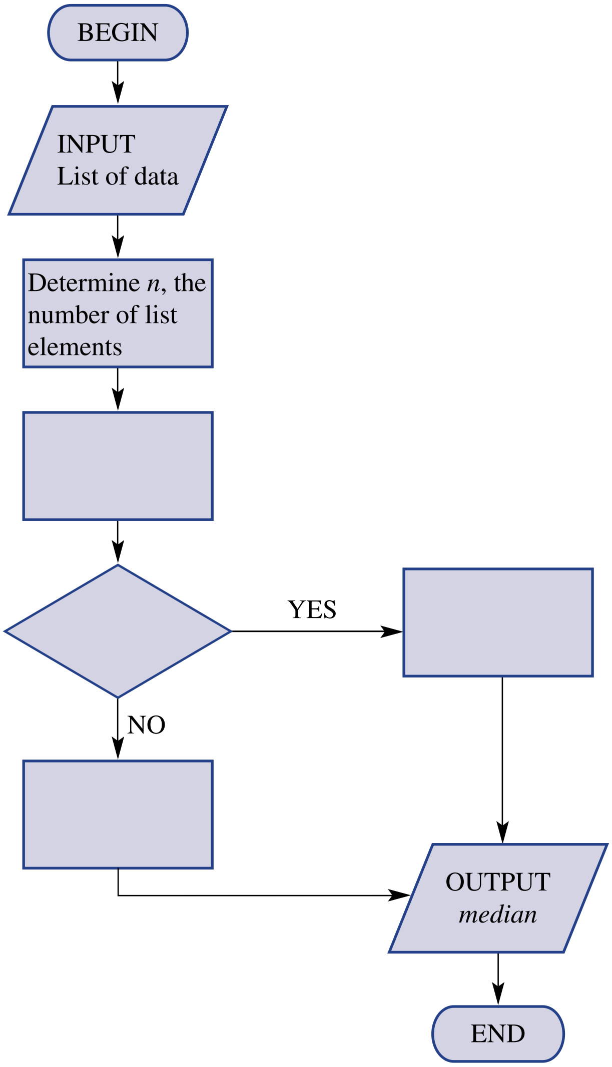
Now that we have algorithms for sorting a list of data, we can use these to help us find the median of a set of data.

g **i** Copy and complete the empty boxes in the median flowchart algorithm below. Use the box shapes as a guide for the task.



ii Turn this flowchart into pseudo code, detailing exactly how you will locate the median for an odd and even number of data values.

Ext **iii** Extend your code in part **ii** to also find the interquartile range (recall from **Section 9K** this is $Q_3 - Q_1$).

3.2 Simulations in probability calculations

Simulations to determine good approximations for probabilities can be carried out using different techniques. For experiments with two equally likely outcomes, a coin is a good device which can be used to carry out the simulation. Other devices include dice, spinners, random number generators such as calculators and computers and random number tables.

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The following tasks will explore the use of some of these.

a **Simulation devices**
Consider the following scenarios and describe how you could use the given device to carry out the simulation.
- i Simulate the gender of the next baby born, using a coin.
- ii Simulate the result of the next penalty shot from a soccer player who has a poor record (1 out of every 3), using a six-sided die.
- iii Simulate getting the correct answer from a random guess on a multiple-choice question with four options, using a deck of cards.
- iv Simulate the situation '2 in 5 chocolate bars wins a prize', using a die.
- v Based on the number of paths through a maze, a mouse has a 1 in 8 chance of exiting through the path with a cheese reward at the end. Simulate, using three coins.

b **Technology and random numbers**
You may have noticed in part **a** that some probabilities were easier to simulate than others with certain devices. The use of random numbers on calculators and computers makes this more achievable. For example, to represent a 2 in 5 chance you can generate a random integer between 1 and 5 and assign the numbers 1 and 2 to represent a win and 3, 4 and 5 a loss.
Many scientific calculators have a randint() function that can generate a random integer in a specified range. In Microsoft Excel RANDBETWEEN(1, 5) will generate a random integer between 1 and 5.

Consider a basketball player who gets on average 75% of her free throws in.
- i Consider how you could use a random number generator to simulate her next free throw.

Technology would be helpful in the following but coins or a die could also be used effectively.
- ii Using your chosen device, estimate the probability that the player makes 5 or 6 of the 6 free throw shots she takes in a game by following these steps.
  - Generate 6 random numbers and count if 5 or 6 of them are in. Use a table like the one below. Repeat for 50 trials.

Trial number	Number of free throw shots in out of 6	5 or 6 shots in? (Yes/No)
1		
2		
⋮		
50		

  - Calculate the probability estimate: $\dfrac{\text{number of times 5 or 6 in (yes)}}{\text{number of trials}}$
- iii The theoretical probability is 0.534. How does your answer compare? Compare with other members of the class.

c **Random number tables**
The simulation in part **b** could also be carried out using a random number table.
The random number table shown on the next page groups digits in groups of five but is generated with a random digit between 0 and 9 at each place. These numbers can then be grouped together to represent 1, 2, 3 digit numbers and more.

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58315	10578	77473	16526	53775
22646	82056	42313	50814	60650
07419	77083	11543	26629	08313
76940	62101	86568	08456	53641
15963	82704	06272	58036	61078
80644	86510	78615	06079	60154
30571	92400	07305	64811	03054
09182	14662	10472	97400	65696
34361	67837	16869	00904	79928
56845	84009	10030	28001	64238
64794	74684	75213	38693	27053
78586	70912	18697	55949	39557
92444	18934	51892	24300	20247
10287	04605	29245	52500	90501
75233	47520	50251	75778	69504
89599	84143	14821	26191	18076
35287	40045	24635	45782	12217
07320	03379	80797	26408	75542
43560	20875	53010	23045	23634
90623	75405	18139	31992	64709
06655	97645	90074	57757	48091
59959	72048	57584	39509	43253
88358	78732	45246	00345	02690
11524	56474	20503	02944	86411
45850	83072	27083	65098	92990
05013	21373	93138	21196	91294
80820	27119	29971	73672	21843
30859	16362	07037	82057	67908
19037	36352	26371	72254	33515
94869	80499	48086	43199	55744
02516	67531	73014	46866	52298
46238	37370	63515	37083	33247
26103	81339	93391	43856	95475
07664	85034	46581	88772	93372
18305	71127	91648	96303	65869
04887	10435	77400	30370	20995
08597	14871	08080	99425	73733
01876	18260	04657	87735	07273
16680	12966	75383	87195	86948
43454	22639	45772	62461	67602
89229	90868	03485	85955	73123
70283	78014	64377	40020	73714
19609	05831	80438	76003	50046
87442	56988	25210	21541	81928
43409	93065	52495	77536	81227

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Consider the following scenario:
Over his career, an AFL player has a set shot for goal record of 68% goals, 24% behinds and 8% out on the full.
As we have percentages, we can use random numbers between 0 (i.e. 00) and 99.

i Decide which sets of two-digit numbers you would use to represent a goal, a behind and out on the full.

Two-digit numbers can be drawn from the random number table by selecting a starting point in the table and reading off two digits at a time to form a number.
For example, for the block of numbers 58315 10578 you would get the numbers 58, 31, 51, 05 (i.e. 5) and 78.

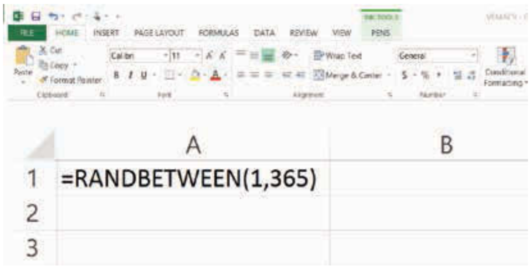
ii Use the random number table (starting from any location in the table) to carry out 20 trials to estimate the probability that in 5 set shots on goal the player scores at least 3 goals.
- Record your results in a table like the one shown on the right.
- Estimate the probability: $\dfrac{\text{number of 'yes'}}{\text{number of trials}}$
- Compare your result with those of other members of the class.

Trial number	Number of goals	At least 3 goals? (Yes/No)
1		
2		
⋮		
20		

d **The birthday problem**

A famous problem in probability is the question of how many people you need in a room for there to be more than a 50–50 chance of two people sharing the same birthday.
In Senior Mathematics courses you can prove that you need 23 people in a room for this to be the case. We will investigate this using a simulation.

i Use a spreadsheet to get a sample of 23 random integers from 1 to 365 (representing the different birthdays across the year – February 29 is left out!). Fill down the formula shown to get 23 numbers.



ii Repeat part **i** 50 times by filling across to column AX. Cut and paste your data values onto a new sheet so that they do not keep changing.

iii Sort each column from smallest to largest in order to quickly compare if there are any duplicates (repeat numbers representing the same birthday).

iv Count how many columns have duplicates and calculate your probability. Is it over 50%?

v As a class, record all of your probabilities and find the mean of these. Does this support the result outlined?

vi Repeat the process in parts **i–v** for 22 people in the room to see if this probability falls below 50%.

Ext **vii** Use these methods to investigate how many people you would need in the room for there to be a probability of more than 90% that two people will share a birthday.

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